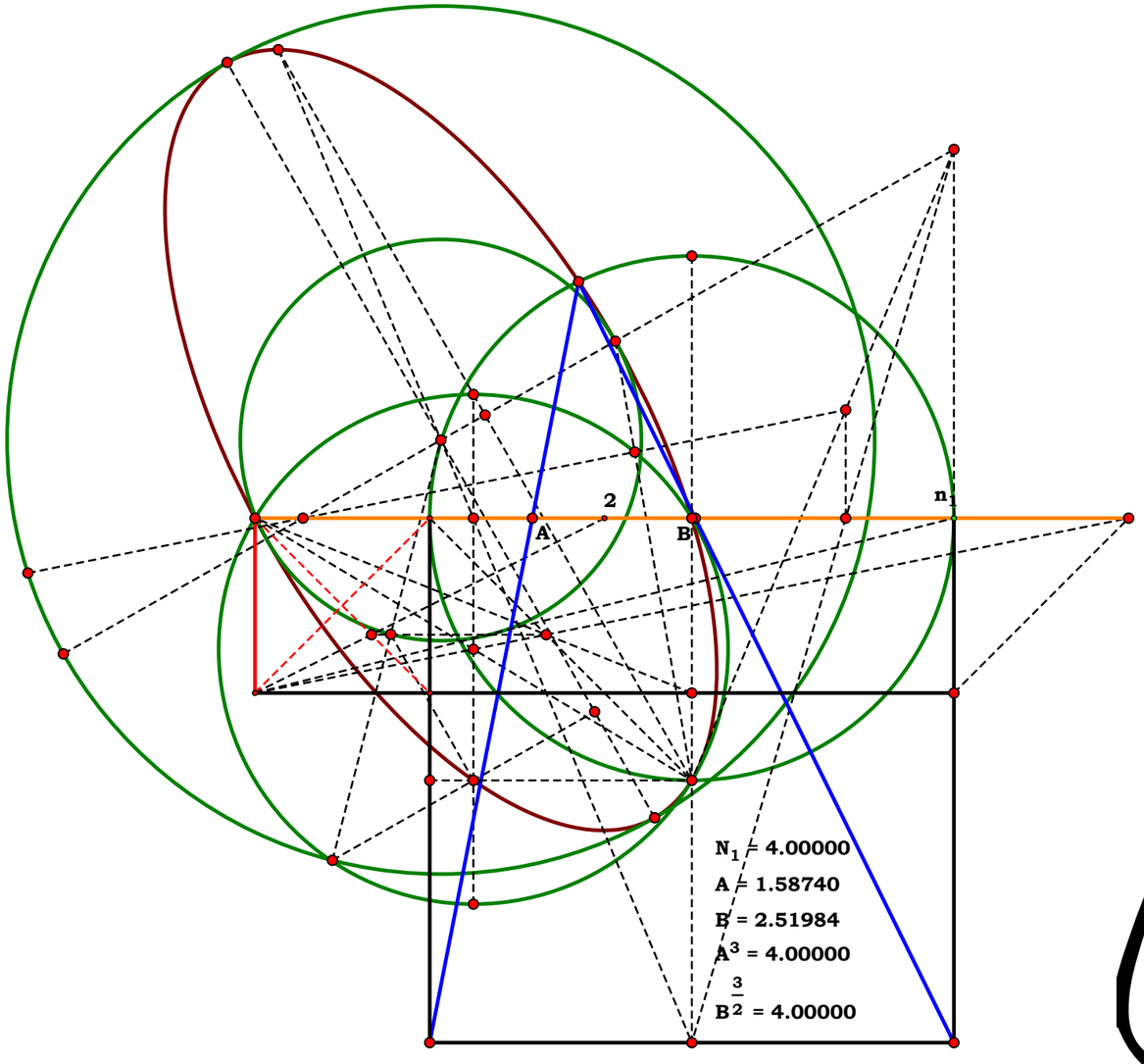
# The Delian Quest

**John Clark**

**John 312**




# The Delian Foreword

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Saturday, September 4, 2021

## Literacy

The process I used to do this work is actually described by Plato. Plato was a grammar teacher and one of the most fundamental concepts you have to realize, as Plato demonstrated in his dialogs, especially *Parmenides* and *Theaetetus*, is that all grammar systems are methods of binary recursion which he called *Dialectic*, words by two's. One starts with names and naming conventions based on this binary, and from there, one has to keep track of their names, build a dictionary with them, and from the items named, discover their relationships. This work is my first attempt, to write factual human literature demonstrating the pairing of our grammar systems to say the same things.

But fundamental to Plato's teaching in grammar is the point being made by the Judeo-Christian Scripture, that all a mind can do, by biological and physical fact, is read and write. We read and write with our whole body, mind and soul. Grammar affords us the ability to predict the results of any behavior, any complexity, and as one will see in this work, compute the results of any given number of variables, free from computational time: output is concurrent with the input. In short, not even the fastest computers today can match Geometry for speed and accuracy: And when one can comprehend the metaphors of the Book, one will realize by doing the math, there are four, and only four, basic systems of grammar, Common Grammar, Arithmetic, Algebra and Geometry: these four make a Grammar Matrix which defines literacy and mental competence. We learn to guide our whole behavior using a binary matrix in order to do our own

work, as Plato would say, or again, our biologically defined job. This Matrix was put into metaphors of the JCS.

This is a grammar book, a helper in learning how to read and write, or as I am want to say, learning how to say what you see.

There is no Trigonometry here, which is not a formal grammar, there is no Cartesian Geometry here, which is not a formal grammar, and there is no Calculus here, which is not a formal grammar. However, what is here is proof that the doctrines promoted by these previously mentioned are provably wrong. Binary recursion produces four, and only four, categories of grammar. Every formal system of grammar lays out, step by step, the pairing of our original Universal Binary with mental and computational behavior.

## What is the Delian Quest?

One should, at least attempt, to find an intelligible concept commensurate with certain perceptible things. For example, the *Delian Problem*, as you probably know, is about the duplication of a cube, or one can say, the manipulation of a three dimensional object so as to produce a given product: in this case, a cube of twice the volume of one given. What is stated is a request for a perceptible result, not in terms of an estimate, or one satisfying some pre-determined precision, but exactly, perfectly. However, one can look at the problem intelligibly and metaphorically, in terms of the definition of what a mind, or again, what man is to become: a symbolic information processor in order to maintain and promote life. A mind, after all, when functional, is the most powerful life support system possible for any form of life. With it one can virtualize the environment and predict results favorable to the continuance of that life. Put into what people call a religious metaphor: "*The Testimony of Jesus is the Spirit of Prophecy*." Grammar systems, when functional, allow us to predict the

results of any number of givens. Thus, if one is smart enough, they see reasoning, judgment, and prophecy as synonyms.

Intelligibly then, one is looking for the very same result one finds in the metaphor, *The Father, the Son, and the Holy Spirit are One*. If one is paying attention, we are being presented with another three dimensional request. What is the meaning of that request?

We, as a mind, are evolving to become the most powerful life support system possible, i.e. the most fit for survival. What this means is that we are evolving to become masters of the perceptible through a mind. The foundation of mind is memory. Memory is a virtualization of perception and thus memory, is the foundation of what is called the Intelligible. Intelligence is the ability to manipulate memory for survival. Intelligence is founded upon what is called pattern recognition, or which Plato called the ability to see the *similar idea in the many examples*. There is no intelligible, no other idea, which demonstrates intelligence, than the recognition of an idea which covers all perceptions. That idea will then eventually end up with many names set into a phrase such as *a unit, a thing*. This phrase, comprising of two words, is what Plato called *Dialectic*, or what we call today, *Binary*. Every thing is defined as a binary relationship between a thing's form, shape, limits, etc., and a thing's relative difference also known as material and material difference: the stuff a thing is made of.

As the Universe is made up of every thing, one can say that the Universe is the product of Binary Recursion. Now, if one were parsing information correctly, they could reduce the story of Adam and Eve into a single metaphorical sentence which denotes how a functional mind works such as: *Adam and Eve are a Conjugate Binary Pair, which by Complete Induction and Deduction produce the human race*. Fundamental intelligence will eventually arrive at a simple provable fact, as a computer

can produce all of its output as the result of binary recursion, so can any functional mind.

Our mind is defined to master the Universal, the concept of binary and binary recursion, by learning from the particulars in our environment. Thus, it comes naturally, to the more intelligent of a species, to attempt to learn how every particular thing can be judged by that Universal. For example, the metaphor of the Father, Son and Holy Spirit can be transformed into exactly what we are, a mind responsible for a given product such that the product is true. Our Father, or teacher is perception: We, as the student, or Son of perception learn behavior from those perceptions. Thus, both metaphors, Duplication of the Cube, and what is called a religious quest, simply become, Perception determines conception; conception determines will; which is a description of every life support system of a living organism. In short, be it a scientific problem or a mystical problem, our mind sees but one object, one problem to categorize them under: a simple biological fact. This is an example of what Plato meant by the similar idea in the many examples, or again, the definition of any thing, or again, a unit. *Dialectic* is a term Plato used to denote binary recursion.

A mind is one of a group of life support systems of the body within which we reside; as such it has a well-defined, biologically determined, job to perform and well-defined, physically determined, means of doing that job. As the definition of a thing, aka, a unit, is a binary expression, our job is to learn, all the days of our life, correct and true binary recursion in order to maintain and promote life. Therefore, this work is simply part of my work, given to me, given to us all, by simple biological fact.

When I was very young, I became aware of a problem with humanity as a whole. The human mind is not exactly functional, and people are prone to a life of pointless and bizarre behavior. I, personally, was not being educated by our social systems, and had spent a great deal of time looking for teachers in books.

I was quickly approaching my 40's when I decided to try and learn some Geometry. In order to keep myself motivated, I chose to try and solve a problem called impossible to solve. The greatest minds in history had tried to solve it and gave up on it, therefore, I figured that since it was unsolvable, I would never have an excuse for leaving off my study of geometry. Unsolvable problems, like unrequited love, is one of the best carrots on a stick for our own dumb ass.

Somehow, I started with exactly the right figure to pursue. And somehow, writing equations to figures came naturally to me and it certainly had nothing to do with that ridiculous so called Cartesian Geometry, although of great utility for mechanics, I do not call it a pure mathematic. In geometry, a ruler is not allowed unless one produces one as capable of doing the math by the geometric figure. One does not set up rulers to measure where they will physically put a point. There is no precision in that.

Ten years down the road, I realized that I did solve the problem but that its solution was trivial compared to what I discovered along the way. I had written equations to figures for so long, I started learning how to write figures to equations until I had laid the foundation for *Basic Analog Mathematics*, or BAM for short. One actually draws the blueprint for computation where the output is concurrent with the input, i.e. no processing time. One can do all of one's logical and analogical processing using simple geometry as exampled in BAM and all of it quite independent of processing time. What put me onto this were apparently aliens, or so I assume, when they deliberately demonstrated the effect while helping to save my life from a very stupid decision I made while driving.

So called Euclidean Geometry, is just a grammar system, like any other, derived from binary recursion, i.e. the recursion of a simple unit. What this means is that it is wholly impossible to claim any other kind of geometry unless one's mind is dysfunctional and incapable of comprehending the fallacies it introduces. As every grammar is a product which expresses simple binary recursion, one cannot claim a different geometry for it would have to be based on something other than the recursion of a unit, i.e. it would leave no math by which to proof it, nor would it be based on the Universal Binary which defines our physical reality. Every form of true mathematics is the result of binary recursion and although this is fundamental to mathematics, it is surprising how many so called mathematicians cannot apply this first principle by which to judge their own words. Non-Euclidean Geometries, every one of them, are based on simple fallacies which their proponents cannot grasp.

The two main focal points of this early study of geometry which was constantly on my mind were that I was learning how to say what I saw. The second was how to establish a unit from which all the rest of the equations were derived. The fact is, that unit has to be expressed in the figure itself, either as one of the segments or a proportion of one of the segments. These two focal points of mind remain as the motivation for my study which will lead to a better understanding of the result, Basic Analog Mathematics or more formal, *Basic Analog Grammar*.

One of the things I discovered while going back over this project is that some work done and finished in the last update were never updated in the resulting product, i.e. overlooked, for example, the very first series of plates.

## Grammar and Naming Conventions

Binary recursion affords us exactly four categories of grammar: Common Grammar, Arithmetic, Algebra and Geometry. This means that biologically

we are afforded a Grammar Matrix by which to process information. This matrix not only allows us to formulate verification by cross-checking, but also allows us to acquire the maximum utility from our experiences. However, the arts to do these processes have been greatly neglected because the human race is still very much mentally incompetent. Plato, himself, suggested using geometry as an aid to follow the concepts presented in common grammar, however, that work, *Parmenides* has been, by their own admission, over the heads of so called Platonic scholars. One has to learn not only how to correctly construct a figure, but also learn how to pair it with logical grammars. For example, the Arithmetic Naming Convention gives us names which we call numbers. A number is just a name in arithmetic grammar. If one is a complete idiot, they claim that there are different kinds of numbers which come about because of how one uses numbers. Every book on math I have ever read was written by someone who can be judged, by simple grammatical fact, as illiterate.

The Algebraic Naming Convention affords us letters. When we first establish a correspondence between the Arithmetic Naming Convention in our write-up of a figure, we are simply using Algebra as synonyms for Arithmetic. When we convert the Algebraic from being determined by the Arithmetic, which uses a standard naming outside the figure, to the figure as establishing the unit. One will find when they do this, they will see relationships in the figure not revealed by Arithmetic as given in the raw.

In many of the plates I accompany the write up with, one can see the results.

I will go over the previous introductory material for this work, and let it follow this introductory addendum. I will also add an essay addressing objectives in pairing the analogic of geometry to logical grammars such as common grammar, arithmetic and algebra.

All in all, one should develop a very firm objective belief, it is wholly impossible to predict the results of our own behavior, or fulfill the promise

of intelligence, when factually, there is not one correct grammar book on the planet, nor is simple binary recursion being taught anywhere as the foundation of the Grammar Matrix, we are evolving to master. Everywhere on this planet, from the simplest minds to those claimed to be genius; mankind is pre-literate and falls short of the very first principle which defines intelligence, the recognition of simple binary recursion. I, personally, find it odd, that one can even use a computer every day, even program it to perform its functions, and not realize that every thing it does, is the result of binary recursion which means that every possible system of grammar it can parrot, is also the product of binary recursion, or as Plato said, there are two, and only two, parts of speech recursively applied to produce all systems of grammar. The only thing a mind can do, is learn to read and write. Until there is a social recognition and use of our Grammar Matrix as the foundation of our behavior, mankind cannot be said to be more than an illiterate fool, no matter how many awards we give our self to celebrate that very same stupidity.

We are constantly told every moment of our life that our survival hinges on the mastery of simple binary recursion, and it is not a sign of intelligence when one does not recognize it.

The Delian Quest is more than a simple book, more than a single perceptible problem, it is our biological imperative, our Universal Problem, intelligence via the intelligible. Although my book shows that I solved for a particular ability to do cube roots exactly in geometry, it may only faintly help in achieving our Universal Quest, to have dominion over our environment.

# The Simile in Multis

Or the Magic of Metaphor

Saturday, September 4, 2021

*The greatest form of ignorance is thinking that you know what you provably do not know*: Plato. Almost everyone believes that they have the ability to think and reason with some measure of truth, when this is provably not true. These people will not suffer being corrected as they defend themselves over what is true: they suffer the more extreme conditions of mental illness, yet they are also, the most common type of human today.

The ability to comprehend grammar is a biological given. We denote this ability as linguistic functionality of a mind, or any information processor. In the Judeo-Christian Scripture, it is called the light which is the manner in which Plato put the sun in the cave metaphor of the *Republic*. In that metaphor, Plato was admitting that he, himself, was a prophet as defined in the JCS. That metaphor refers to the fact that all information processing, the life of man, is a physical fact which is based on the definition of a thing as a relative constrained by correlatives, i.e. just like the metaphor of Adam and Eve, it is a binary relationship which Plato called *Dialectic*, language by two's. This binary is an intelligible which we put into the perceptible using symbols by which we construct grammar systems in order to do our biologically defined job. The difficulty of comprehending the binary metaphor can be gleaned from a history of responses to Plato's dialog called *Parmenides*. In that dialog Plato suggests to the reader to draw a line segment to follow it. The line segment is a binary, or in the grammar of geometry, the *First Principle*. We can only name the correlatives and the relatives of a binary construct: these are called nouns and verbs.

The unit binary in geometry,

, which gives us the simple sentence and our primitive equation. AB are correlatives, while c is the relative. This is the simplest binary analog example. If one looks to so called *Set Theory* and *Venn Diagrams*, one may learn why their writers always fall into contradictions, as Plato and Euclid noted, you do not start diagramming with a circle, but with a simple segment. If you cannot understand *Parmenides*, you cannot comprehend grammar and binary recursion. To understand this binary, we have to realize that A and B are the shape, limits, or container of c. A, B and c, individually are not things, they are parts of a thing. To put this into a signed number, it would be AcB for Common Grammar and 12c for Arithmetic, however, this name denotes a particular thing. In geometry, if we use the measurement function, it would look like AcB = 12c, this means that A = 0, B = 12 and c is the coordinate system of reference called linear distance. In this mode, we are simply making synonyms between Common Grammar names and Arithmetic names. If we now take AcB and divide it by 12c, we get $\frac{AB}{12}$ and since they are synonyms, we get $\frac{AB}{12}$ = 1: or we are denoting that AB in Grammar is the same name as 12 in arithmetic. We have paired Common Grammar to Arithmetic, or again, made the two names synonyms.

This is a straightforward transform back to simple counting. When we can do this, we can now comprehend that not only is common grammar and arithmetic simple methods of counting, all grammar systems are, or in other words, every possible system of grammar is effected by simple binary recursion based on the simple definition of a thing, and even our own biology for any particular sense can either abstract a things material difference, or make us aware of a things limits, or boundaries. Binary recursion not only allows us to address memory, but also to manage and

manipulate memory in order for us to do our biologically defined job, to have life, and to have it more abundantly.

The true comprehension of the intelligible unit covers the whole of the perceptible and the intelligible. Binary recursion can only produce a binary result and not the gibberish common to the intellectuals of the world today.

So, before we become too lost, we have to put our binary into a definition of a thing, or unit.

**Definition:** A thing is any relative constrained by correlatives, or in simple terms, a thing is comprised of a shape with some material in that shape. A thing is a binary construct. The material is not the shape, nor is the shape material. A noun is a container for verbs.

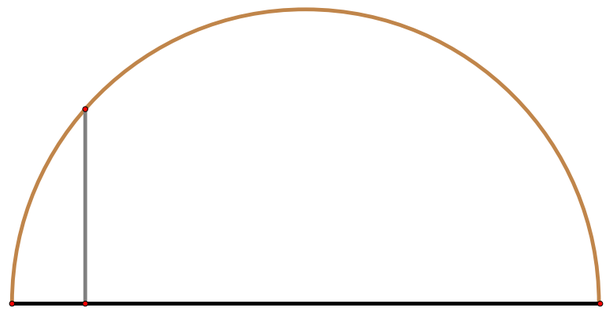
Stupidity is when one takes a noun, which has absolutely no meaning at all for its paradigm is a container, not a material difference, and claims that the particular noun is a container for perceptions, verbs, that they have never acquired, one achieves functional schizophrenia, a schizoid population. What a person knows, retains in memory, is proportional to their own experiences.

When one studies *Parmenides* by Plato, they will begin to see the confusion in their own mind because the untrained mind confuses this distinction and tends to call the parts of a thing, things of which they are parts, i.e., gibberish. The ability to keep track of the intelligible is what Confucius meant by being aware of the truth of things. We have to always keep in mind that a thing is a binary, and this divides words into two, and only two, parts of speech, nouns and verbs: these are in a binary relationship and every grammar, correctly taught, is aimed at teaching simple binary recursion. Today, everyone simply uses words as a caveman uses a club; they spend a lifetime playing the shell game with words. There is no dictionary of common grammar today, which holds to the first

principles of binary recursion in grammar, i.e., every one of them is written by the functionally illiterate.

Let us take a brief and simple look at metaphor, how it works and what it is.

Let us take a simple grammar system, arithmetic and name a few things with it.

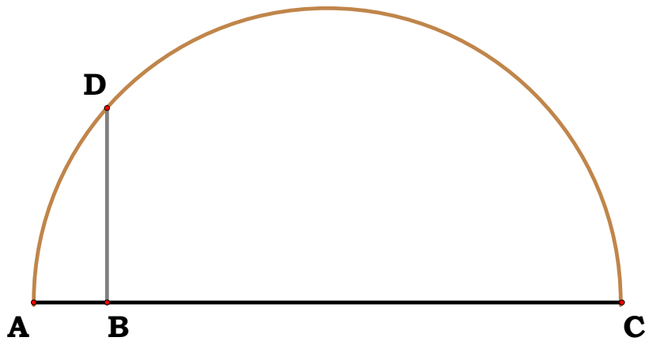


It is just a simple thing, perhaps a tunnel with an upright interior wall someone has sketched out for us. Let us take a moment to think about a common phrase used by some so called intellectuals, *self-evident*, and *self-evident truth*. We have a distinction between the perceptible and the intelligible. Which can we actually share with anyone? Can an intelligible ever be *self evident*? Can a perceptible ever be an intelligible, and vice versa? Which is absolute? Do we all live in the same reality? Do we all have the same memory sets and ability to virtualize our environment? Can we all perceive the same things? But can we all transform those perceptions into the intelligible of memory as everyone else? Which is objective, the perceptible or intelligible, and which is subjective. Which is sharable, and which is not?

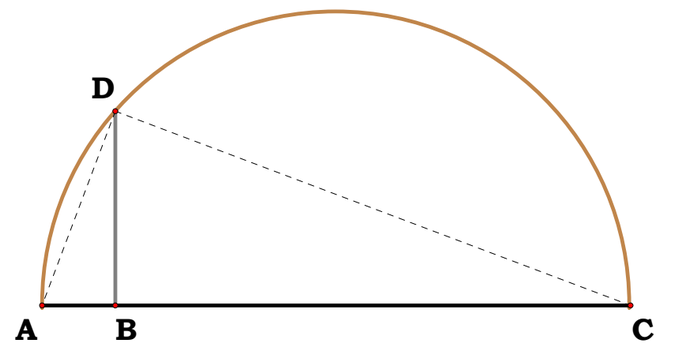
Think about the word Truth. Is it perceptible or intelligible? Does one look for truth in the perceptible, claiming as many do to be on a search for truth… and in perceptible things to boot? Is that indicative of the least bit

of intelligence? Truth is the state of being true. Can one thing, in of itself be true? Does anyone, in their right mind, go around claiming, for each particular thing, that it is not different from itself? When told that truth is within, do we start dissecting brains to find it? I know plenty of intellectuals claim that by dissecting the brain they can find language, but then they paid a great deal of money to formalize their insanity. If you cannot paper train a dog, can you have paper trained intellectuals?

We are given perceptible objects, self-evident objects, which any bug, dog looking to piss on a wall, or weed can in some measure perceive, but it is not in the least bit intelligible. We want to mentally manipulate this thing in our mind, virtualize it, and we do that with the aid of grammar systems. Our sketch of the tunnel was our first grammar system. Let us start naming its parts.

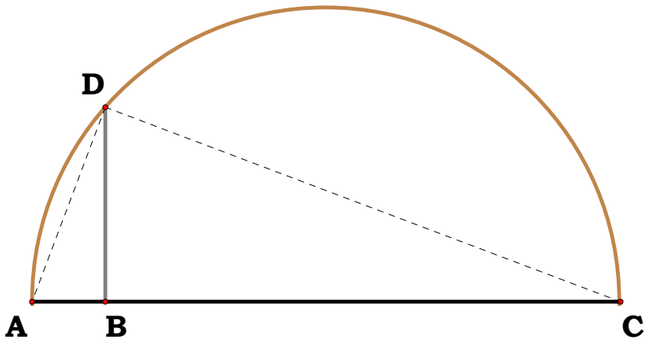



I have used the alphabet which is used in the English Common Grammar system to name the limits of each part of our little tunnel. Algebra also uses alphabets in its system of grammar. So, now we have paired two grammar systems, common grammar which is a logic and the geometric figure which is an analogic. Let us pair arithmetic synonyms to our common grammar names.




Well, we find that our tunnel is kind of small, perhaps on a Hollywood scale for some movie. We have all these arithmetic names established as synonyms with common grammar names. And we also see, that this pairing is traditionally elliptical which should not be occurring in a formal system of grammar. This ellipses does not tell us much about the relationships between the parts, that is, ones we cannot see with our eyes. We are getting a hint that there is something which is not self-evident, but which only names can show us is there.

So far, we have taken our common grammar and paired it with our arithmetic grammar, but something is lacking. Our common grammar alphabet can name anything perceptible or intelligible, however, our arithmetic pairing is specific to a predefined system of measure, in this case centimeters. How do we make a pairing with a common grammar and a universal name? We perform an arithmetic operation, division. If we divide any of the two arithmetic names, we find that the particular now becomes a universal because the particular unit of measure now becomes a simple ratio.




$AC = 9.86267$ cm $\quad AD = 3.48698$ cm $\quad BD = 3.26177$ cm

$AB = 1.23283$ cm $\quad CD = 9.22568$ cm $\quad BC = 8.62983$ cm

$$\frac{AC}{AC} = 1.00000 \qquad \frac{AD}{AC} = 0.35355 \qquad \frac{BD}{AC} = 0.33072$$

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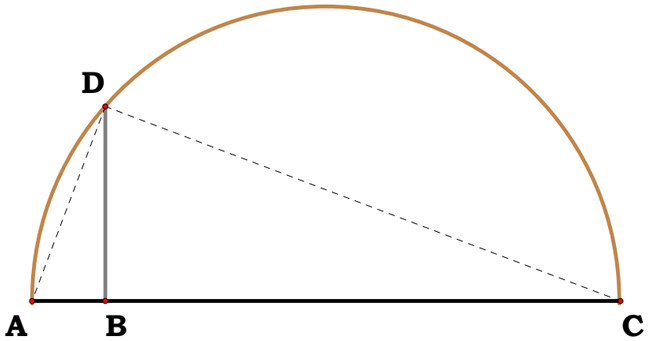


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$$\frac{AC}{AB} = 8.00000 \qquad \frac{CD}{AB} = 7.48331 \qquad \frac{BC}{AB} = 7.00000$$

We can now transform the arithmetic pairing with common grammar to a true algebraic pairing. Now we have three universal grammatical expressions, the common grammar, the arithmetic and the algebraic, each telling the reader that the geometric figure is, itself, a universal expression.

So, even though our program is wrong in the way it establishes it assignments, we can fix it by using an operation with the symbols.

We have six different arithmetic names which we can denominate as a unit for the figure. We can try all six, and see what it tells us, but right away we see we have patterns but we also have done something else, something very intelligible, so most do not get it. The simple arithmetic convention of names is comprised of two names, a noun and a verb. Ten cats, five fish, four centimeters. It is a verb noun system. Some call it signed numbers which does not make any sense at all because even numbers are signs, or symbols. Arithmetic, when correctly comprehended, provides us with simple sentences of verb and noun when paired to common grammar. However, when we ratio one name to another, the name for the material difference disappears. Length, weight, color, etc., that is names for some material difference, cancels out as unity. We have gone from the particular to the universal, or metaphor. We have found the similar idea in the many examples when we remove the particular, we get a universal. And if we keep going, we get equations wholly independent of any particular system of grammar or *coordinate system of reference*: keep this in mind when reading the pseudo intellectual Einsteins of history. These equations are just as valid for dirt as for jelly fish.

The fact that every thing in the Universe is the result of binary recursion also tells us that every thing in the Universe can be compared as ratios. Long grammatical names can be reduced to a much smaller set of ratios.

What we have done, is through grammar, brought to life the intelligible which is not at all perceptible. We slowly learn to go from the simple arithmetic one-to-one correspondence of names to points of perception, or arithmetic identity, to geometric, or metaphorical, identity. Most today are, however, either wholly unaware that there are two types of identity, or binary identity, or they are trying to figure it out. Identity, is itself, a binary conceptual unit. This process allows us to deal with information in the

simplest possible terms, not as particulars, as our senses produce for us, but in the intelligible which only the intelligent can comprehend. The very fact that so called intellectuals today are inventing particular words, and pretend grammar systems as fast as they can, is simply due to the fact that they are illiterate. Binary recursion produces exactly four categories of grammar, common grammar, arithmetic, algebra and geometry, and as it is binary, there is no such thing as a theory of grammar, a theory of numbers, or countless other hot air theories of the pseudo-intellectuals of our day.

Most people, denote themselves as this or that, they make themselves a horde in a box, a collection of empty nouns, gender, religion, politics, when factually, we are simply an information processor working at all times with virtual information through Language turned into four specific grammar systems. Language is Universal and Intelligible, while Grammar is Particular and Perceptible. One cannot teach Language but one can teach, providing they have learnt it, grammar systems. Not having a standard for Language, however, what is called grammar systems today are provably not much more than gibberish.

Plato tried to keep Aristotle close, but it was clear to him, while Aristotle was claiming that metaphor has no place in reasoning, Plato was teaching it as fundamental to thought. As Plato said, you can teach some people for an incredibly long time, but they will never get it. We have the letter of the Law, which is based on arithmetic constructs, but we have the intent of the Law which is metaphorical, something most if not every, judge in the world would deny.

The algebraic equation, when correctly written, is independent of any particular application, just like every other grammar.

A grammar matrix makes full use of the absolute and the relative for the express purpose of information management. Mysticism rules the world today, but there is usually someone who spends their life denying its

validity, but never banishing it from their own mind. One of the greatest and most destructive myth of all is that words, symbols, have meaning, and when great fun by Plato was made of that idea in *Cratylus*, his whole point was missed. If words had, in of themselves, any meaning at all, then it is a fact, they would be wholly useless for information processing. The indexing system is not the indexed perceptions. Meaning is the motivation to effect one's behavior, like hunger, pain, thirst, pleasure, which no word, in the history of the Universe, has ever felt. To ascribe meaning to words is anthropomorphic, the signs of a savage. How is it that even today, one can be sued for things of no meaning while actual things done by our governing bodies go by with a blessing? We mean to do things, which means that meaning is a synonym for planned behavior, which, has not come to fruition for a linguistically functional species on this planet.

Faith is the substance of the things you had hoped for, the evidence of those things not seen, thus faith is prophecy, the ability to virtualize information to be used and is used to bring about the best state of life for one's self, and one's environment. But, which can never be said too much, mankind does not even know what a grammar system is yet, nor does he teach it.

Tell me the world is not sick. Sickness is a degraded state of the body, during evolution a mind is in a degraded state of functionality.

A sign of a very weak mind is confusing a thing with its virtualization, a theme Plato exampled a number of times. Our virtualizations are intelligible, they are not nor ever can be the perceptible, however, one can consider them as maps, thus prophecy, predicting behavior, is map making. The map is not the territory. The intelligible is not the perceptible. We do not eat the recipe, we use the recipe to make a meal. But, no matter how many times you say it, you find a world eating paper, the very same paper they wipe with.

Therefore, we are back at the beginning, the cave paintings, the hand-made tools, all of these are the evolution of linguistic ability for the mind can do nothing else but learn to read and write. Analogic, the pairing of behavior to intelligible concepts, was our first and shall be our last, system of grammar. With a simple stroke of the hand to produce a segment, we have recognized the binary foundation of Geometry, which, the fools of today do not recognize, means there is one, and only one, geometry, just like there is one, and only one, common grammar, one and only one arithmetic, and one, and only one algebra. Each of these comprise a symbol set, and the method of recursively applying those symbols to count data.

Arithmetic reasoning is based on a one-to-one correspondence, one limit to one material difference. We use arithmetic to accumulate data from perception. Geometric, or again, metaphorical reasoning, is based on the one to many, the simile in multis, the one idea from the many examples, and this is called pattern recognition. How is it, that today, we have our so-called intellectuals spouting pattern recognition with their mouth, while on the other hand, claiming that there are an infinite number of grammar systems? Sounds like a damned brain dead fool to me.

Intelligence is the ability to construct standards of information processing which can be applied to all data. Ask your computer, does it use binary recursion to do it all, while on the other hand morons like Microsoft claims that there are countless systems of grammar? Do you want to read tomorrow, the files you make today, or do you want Microsoft to tell you that you are no longer allowed access to your own files? Is Microsoft, or any other corporation, educational, religious, or governmental the standard of information processing, or is simple binary recursion which is common to all and yet independent of all as well?

*Adam and Eve are a Conjugate Binary Pair, whereby Compete Induction and Deduction, produce the human race*. Words without wisdom is the condom which prevents the birth of man.

An illiterate species cannot reliably produce a literate computer. In my following work on *Basic Analog Mathematics*, you may learn how to draw one which is faster, and more accurate than any computer produced today, providing you have the intelligence to comprehend the work. Wow, instead of simply Geometry, I can call it Transcendental Transfinite Tit Tweaking Hand Computation! Or T-T-T-THC for short: it is legal you know, my stoned government said so.

# Naming Convention

Saturday, September 4, 2021

## Universally

Every possible grammar is a method of utilizing binary for memory manipulation. And every possible grammar is effected by complete induction and deduction of the unit. A unit, is a universal conjugate binary pair of a relative constrained by correlatives.

At the foundation of every possible grammar is the intelligible unit to formulate each particular grammar; the symbol set it uses and the method by which those symbols are grouped to express names is simply a recursion of that unit; these are the naming convention by which each grammar is based and every proof can resolve the entire chain of usage back to the convention of names, or what Plato called, the First Principle Parts of that grammar, which is the convention of names that grammar is based on, logical and analogical.

Fundamentally, a grammar is an indexing system, which is something you should take away from the above. We can call a grammar system an indexing system, or a memory mapping system but we can never say, if we are sane, that the grammar, or its product, is the data, or information, our actual experiences. Grammar is simply a tool afforded to us by Language. It is the Ultimate Tool.

These two elements of grammar, symbol sets and how we apply recursion to them, which are standards of human behavior for a correct system of grammar and are the standard for sapient psychology. As the human race is still primitive, it has not standardized this yet; close, but still not rational. It cannot be said that the human race is sapient or even civil, not by factual definition. Psychology is commensurate with the principles of grammar which are functionally resident in a mind as

grammar systems, and currently man is claiming ignorance of both. This amounts to calling himself a brainless fool and like a fool, he is happy with that result.

Our two intelligible elements of a unit, or thing, can be expressed as no difference between such and such, which is Arithmetic, and a difference, which is Geometric. For example, we compose symbol sets with standard behaviors of the hand. If I wanted to compose a symbol set which was absolute to those two elements, it would be geometry, stop, go stop, for a line segment. In Logics, we use a many to one in terms of behavior to formulate symbol and the methods to manipulate them. Thus the symbol sets are composed of many behaviors to form letters, numbers, etc. The symbolic convention in Arithmetic requires a one-to-one correspondence in how those symbols can be grouped or indexed to form names. The second element is how we index or group those letters to form particular names for the particular examples of the elements we experience in the things around us. These names must be kept in a dictionary, but there are no standard recognized dictionaries today except, when achieved in the coding particular programs which keeps them from crashing. There is no such care taken in human social behavior.

Common Grammar does not have an ordered system of indexing these, Arithmetic does, while Algebra combines both of the indexing methods used in common grammar and arithmetic for its system. In algebra, the names for the parts of things are unordered, however, the operations, our use of those names, are arithmetically ordered.

Geometry is the remaining grammar which starts with an arithmetic, one-to-one correspondence between the intelligible unit and the behavior of the hand. This means that Geometry is composed of two, and only two, symbols; point and segment, one is perceptible, the line, and one is intelligible, the point. A point which can be within a range is called its locus. When someone using the grammar is stupid, the ascribe relative

differences to points, as if they are perceptible objects instead of boundaries. They have moving points, and an infinite number of them to comprise a line, a plane, and even space itself. They are wholly ignorant of their insanity. When we make something visible, and call it a point, we are not denoting the perceptible, but something intelligible which these people do not *get*. A circle is just such a locus. Just like every other grammar, geometry functions by complete induction and deduction of a unit. The loci of a process is a simple way of saying things like, for ever P from N to X, etc. When one is simple minded, they actually believe that points move, or that one can ascribe motion to them; but an absolute is never a relative. It is not professional, save by way of conversation, to claim a point moves: such phrases are colloquialisms. One may say it moves only as a colloquialism and claiming a colloquialism in a formal write-up may not be professional and should always be pointed out as such in order not to confuse children.

When one is simple, or is conveying geometry to the simple minded, one does not expect them to comprehend the intelligible, even so called geniuses never did. So, we say, in those cases, for those minds, that geometry is effected by straightedge and compass. This is akin to saying that essays are written with pen or pencil, and paper. We express ourself to the simple minded in terms of the perceptible, but it should always be followed by a more exacting intelligible expression as simple minded people often never even imagine a distinction between perceptible descriptions and intelligible definitions. If you explain it to them, then they have a chance to eventually comprehend something intelligible. If you do not, then you forget that most of us are lazy.

Every grammar system forms names by using names, as a process of recursion, thus we are always seeking names by manipulating names; this is called reasoning, finding the equation, or solving for an equation or describing some thing or process, or a recipe and even instructions. As

grammar is a process of virtualization, the actual product will factually always be a virtual representation, or metaphor. Names, recursively used, produce only names, even in geometry. The first names, however, are symbols which one learns to pair with their own hand, or method of expression such as speech. These name one's own behavior specifically for grammar basics, units of behavior to effect a grammar.

Mystics teach and preach that reality is determined by names when in fact, names are determined by the intelligible mapping of reality. The map does not produce any reality at all. We may produce things by following a map, but the map has not ability, no motivation, to do anything at all. Names, in of themselves, have absolutely no meaning. There is nothing, in all of creation, which is a product of itself. From the Pope to the multiple PhD holders, men preach mysticism about names. Man is very much still a simple minded savage in the universe. The assignment of memory to names is an intelligible standard of mental behavior which is still not taught today; man is still proto-linguistic. The claim that the meaning of a set of symbols is derived from those symbols is the most basic self-referential fallacy possible and this fallacy is still the foundation of a great deal of social education. This is simply delusion and insanity, a schizophrenic result produced during evolution. A mind has to evolve out of schizophrenia, out of delusion, out of mysticism.

Currently there are no correct grammar books, no correct educational systems, and no correct governments on the earth and the only one that can change that is that person we have slept with every day of our life. Did you know that this makes it wholly impossible not to have spent the greater part of our life not sleeping with a whore? Wow, now I need a shrink. Is that a self-referential fallacy, or simply a tautology?

Casually, we call something an angle, which means what Euclid said it did; things which are angled or again, in some respect proportional other than simply 1. Angle is not a noun until it is defined in terms of our naming

convention. Euclid showed how to do this, but simple minded people, view the angle as if it were a crotch and they describe it as such, the meeting of two legs when it is factually a ratio. Simple minded people, who apparently never consider the obvious, never ponder how it is, in plane geometry, that we ever never have anything more than two dimensions. The word dimension means binary, mentioning by the two-elements of a thing. By recursion of this unit, we say one dimension, meaning a single binary unit, two dimension, meaning 2, or two binary units etc. Thus a one-dimensional object in geometry is a simple segment. One mentions the points, or limits, and one mentions the relative difference called a line for linearity. A line can represent any relative whatsoever. It is completely metaphorical or to be more correct, since words have no meaning, we, ourselves, employ it metaphorically, or always intelligibly. So, when I say grammars are metaphorical, in truth, it means that if we are intelligent, we employ them metaphorically.

Any particular thing can have any number of units to describe it. We make this *having* possibly as part of grammar. This is one distinction between the perceptible and the intelligible, but if we are not idiots, we do not say that each particular thing exists in so many dimensions, when dimension refers to a naming convention established by Language and expressed in grammars.

Another thing to consider, in the working with things. If I slice and dice a unit, this is geometric, or deduction. When I add unit to unit, this is induction. There is no mystery, save for the ignorant, between induction and deduction. Deduction is Geometric, while induction is Arithmetic. They are not types of reasoning, but types of behavior in regard to things and our use of grammar. How can it ever be possible, when all of information processing is afforded by complete induction and deduction of a unit, to now have this doubled except by mystics and the ignorant? Is reasoning different from itself? Inductive reasoning, deductive reasoning,

positive and negative reasoning, are phrases for those who play with words, but are wholly devoid of intelligence. If, when I turn my computer on, it does something today differently than yesterday, I need to fix it, or junk it. For example, I have a raid 5 system for the boob tube which today has a red light on that will not go away. So far, raid 5 means I have lost no data. So, I have to back it up before I pull and replace the defective drive before further failure makes it impossible. This means I will not be able to watch reruns and I might become emotionally damaged if I cannot hear Walter tell me about flying monkeys.

Due to human simplicity brought about by our evolution, people confuse the name of a thing with the convention of names all of the time. For example, mystics teach that there are such things as real numbers, whole numbers, rational numbers, irrational numbers, imaginary numbers positive and negative numbers; all of which confuse a name and a naming convention with some particular use of it. This is mysticism in action. How many times can someone read Plato, and learn that the relative difference between terms, (such as lines and points) cannot be predicated of each other? It is wholly impossible for a name to be irrational; names are how we rationalize, or name. When you use a name constructed in an arithmetic convention of names to name a geometric process, then it is not the result which is irrational, it is the user who claims that induction is equal to deduction. We recursively name our only two working convents in binary; which was once put as, the point, or limit, or arithmetic, is that which has no part, or geometric; etc. My point is, a lot of teaching goes into making children remember half-baked, delusional rubbish. Teaching is supposed to unconfused children, not habituate them to it. I do know that forcing children to repeat rubbish as part of their social structure does cause mental damage and it is part of our social structure today.

Deduction, aka Geometric reasoning, or proportional, or again metaphorical processing has a very decided effect on memory requirements, it effects a kind of memory compression. Things are not grouped Arithmetically, one-to-one, but in accordance with some system of measure, of which there are actually few. It is also called thinking in accordance with the definition of a thing. Reasoning is factually geometric, or metaphorical when a mind is functional. It is wholly impossible to reason manipulating names arithmetically unless one has infinite memory, and infinite patients as one can do nothing with that information. Arithmetic is a method of assigning names, not manipulating them. I use the terms Arithmetic and Geometric, in this respect, in accordance with the original convention of name assignment, not the operations on the resulting names; every grammar system makes available the use of both for operations on the names created by these conventions.

**Particularly**

I have always looked at the Delian Quest as a unique type of novel. But in the writing of that novel, one can say that all the work to this point is sketched out, not in a finished format. During the work, I was learning about naming conventions, and I was very aware of it, but now, in the finishing of this work, I have to lay down what I understand of that convention. For example, we have induction and deduction. Deduction is parsing what we already have, where induction is using what we have to acquire more. In geometry, it can look like this.

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I can start naming relative difference, or the part, in terms of some other given system of arithmetic naming, or I can start by naming it simply as 1, making it the unit. If I start with two things, I have to name them relative to some other standard, or I can name one or the other relative and the remaining one as a proportion to it. This will produce results which look

different but that difference is wholly determined by the naming convention. When one renders a definition from a chain of reasoning, the resulting definitions appear different, yet that difference is wholly determined by the naming convention we started with.

I can name AB as 1, which means that point C is an induction, in ratio to AB and likewise if CB is named 1, then AC is a ratio to it. If I name AC as 36 and CB as 14, I am using an external unit and the process is inductive. If I name AC 1, the CB is a ratio to AC then I am doing a deductive process upon the names.

**Induction and Deduction**

One of the things one should ultimately arrive at in the distinction between induction and deduction, and how one writes up a plate.

Arithmetic equality relies on the Arithmetic system of grammar and so one will always have numbers which are no more than arithmetic names. The Geometric system of grammar relies on proportion which does not have arithmetic names, it has proportion. We learn proportion by using Arithmetic names, but proportion is actually independent of them. Every grammar uses both Arithmetic equality and Geometric equality; the one is not, nor ever can be the other, a relative is never an absolute and this fact has everything to do with Law. When primitive people write Laws, they mistake the Arithmetic with the Geometric, the absolute with the relative.

For example; 2 is an Arithmetic Name, however, in common grammar we can distinguish between two different operations it can name; two cats, or twice denied. Two cats, or twice the standard by which a thing is determined to be a cat, or twice denied, two judgments based on one or two standards, or units of judgment.

Now if you are really stupid, you Cantor your speech claiming that there are two kinds of numbers, Cardinal and Ordinal. It is wholly impossible to

have two kinds of numbers as a number is no more than an arithmetic name. So, the Cantor's of grammar are mythologist, it is not long before they start multiplying how many kinds of numbers one has, just like those who claim that certain names can effect magical spells and inCantations (sic).

Thus, there are no irrational numbers, there are results which can be put into arithmetic names, and results which can only be put into geometric names. We have a grammar matrix to function by because we have two elements of every thing to name and thus we formulate four distinct systems of grammar all of which use the unit, but simply express that unit using four distinct behaviors to construct the symbol sets and the methods those symbols are manipulated.

Arithmetic is for particular examples, for example, assigning names, while Geometric is for the universal, or every member of a class. Judgment is then, and always has been, Geometric, or proportional, or again, metaphorical. Line upon line; Precept upon precept.

Thus, the simple minded manipulate names arithmetically, the more complex a mind is, the more, as Plato noted, that mind sees and uses, the similar idea in the many examples, or metaphorically, proportionally, just as the Bible is written.

A correct grammar book is a book using Geometry, proportional, metaphorical, reasoning. A geometry book has to use the grammar matrix, all four grammar systems.

**Notes**

And so to be complete with the demonstrations, I should example each of these choices and how it changes the APPEARANCE of the definition; not to mention cleaning up my past write-ups which were rather awkward. I may not clean them all up or catch everything as I have no help in these projects of mine. Not many people actually consider that the only path to

salvation for mankind is learning how to do, and doing, the work of the mind, our own work. It is a learned process which is currently not even taught. I do not call anyone a teacher of a thing who is ignorant of what we are and why we are. We are simply another life support system of the body with a well defined job to do and well defined means of doing it. Unfortunately, this well defined thing is not often discovered, nor is it discoverable by the blind, or mentally handicapped which is just what happens during evolution.

When starting out in exploring, one may ponder these issues, but since one is running to learn, they often get put aside until one gets to where one is going, and then one has to clean up the mess in the end, just like building anything.

If one has ever seen one or more of the skits, the *Anal Carpenter*, it is something to consider.

Thus, in terms of the naming conventions in geometry, there are no actual options for mistake, the range of options arrive in the basket of the logical system of grammar we use to pair with geometry. We can name things in the simple arithmetic, implying some standard unit, of which there are many, or we can name every thing in terms of proportion. We then have 2 square ways to name our geometric elements in grammar and I will example these throughout the work. The equations will look different for each one of these ways, and how they are mixed in the write-up, but the final equations should be true to the choice of the naming conventions used at the start.

So, I have a lot of work in this final version, the conclusion of my Delian Quest as a particular Novel, but as a living behavior, it can never end until I, myself, expire.

The four horseman, four ways we ride to measure the four corners of the earth are simply four naming conventions used in four grammar

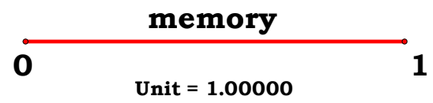
systems or the single grammar matrix of the virtual reality in our mind as an image of God, or reality itself.

I can also say that the Delian Quest, like all initial investigations and learning, displays a lot of thrashing about. This leaves open a final work, which is demonstrated in a highly organized fashion. BAM, or BAG is more on that lines, but I have in mind a much shorter work which I currently call Hominid's search for the Holy Grail. In this work, the end is already a given. However, I do not think anyone suspects that a single equation can denote the whole of grammatical manipulation, which it does.

# The Point

Friday, December 3, 2021

Here, I will put the unit metaphor directly in terms of its biological application. View the following line as a brain:—




The foundation of a brain is memory. It is a container for our individual experiences, and as we are all individuals, each with our own particular experiences, we acquire distinctly different sets of memory, however, the container, called our brain does not differ, by definition, as a physical brain with a distinct biological function which is called mind. Thus, our grammar systems, are designed exactly the same way, to formulate memory management systems which are common to all, yet devoid of any particular relative difference, i.e. containers which can contain anything, but the system to manage those containers are common to all. We all can order our thoughts based on the very same intelligible common to us all, i.e. grammar systems are independent of race, creed, gender, geographical location, or particular examples of human morons.

The common error with people is the confusion between the container and the contained. Grammar systems are systems of container management, memory management systems, they are not theories or theoretical, i.e. we are back to the original container, our own mind, and again, they are designed to manage our memory, however, everywhere you go, people are insisting that you should believe, act on, memory you do not have but are grammatical constructs particular to them, i.e. that they are *special*, but all they are factually offing you is the empty, or magic, box: While, concurrently, nobody is teaching people facts, *grammar systems are not the memories which they are designed to manage*.

And so, primitive morons, in their primitive delusions, worship magic words, which is factually simple stupidity. They worship ignorance, the lack of experience, the lack of life, i.e. they worship death, while proclaiming that this death is life, just like this empty cup will satisfy your thirst. And so, we have a culture of zombies who degenerate into spastic responses when you try to engage them in an intelligent conversation.

The unit, the metaphor, every possible grammar is based on, can be paired with the idea of mind itself, a container for experiences which are supposed to aid in survival, but leaders, from small sets of social sets to corporations and governments, are all doggedly trying to get you to favor the empty, the magical, box as some particular thing, when it is grammar systems which are things to learn in order to help manage *all* information, not someone's particular insane delusions.

The name is not the memory of that which is named. The map, is not the territory. The grammar is not the life, it is how we order our life. A fool, talking in terms of theories, is simply another moron trying to play the magician.

And so, it is the intelligent man who understands grammar systems as simple systems of binary recursion by which we can use to order our life to the best of our ability. This idea goes back to, as indicated by the story of Adam and Eve: Adam and Eve are a Conjugate Binary Pair, whereby Complete Induction and Deduction produce the human race. Literacy never wavers from its biological foundation, and even the computer has to recognize binary recursion as the only method of manipulating information and life, even though it will never know what life is.

Grammar systems are based on the intelligible, that which cannot be seen, the limits of relative differences, and thus, by the very fact that mankind has yet to recognize that simple binary recursion is the foundation of all grammar systems, the very same idea of a mind, is testimony enough to witness against mankind, that he is not even self-

aware. There are no factually correct grammar books today which are used to establish acceptable mental and social behavior of mankind, yet his mind is evolving to attain to this very same standard called intelligent.

Man is not yet intelligent, i.e. mankind is still a fool.

Relation to self is inadmissible, and as I show, the self is based on a physical and biological fact, just like the story of Adam and Eve. The mind of man is still ignorant of simple binary recursion. This separation of self from self, is called schizophrenia. Schizophrenic and fool, are synonyms.

One is supposed to learn grammar systems are memory management systems, not learn to repeat ridiculous doctrines of the mentally incompetent. Yet in our school books today, it is these mythologies which are considered the most important, things to memorize, while neglecting the teaching of exactly how to think by the use of words. Do you know of any educational institution today, which knows what it is supposed to teach, what children, all over the world, are supposed to learn? I don't.

What experiences does a child get by sitting behind a desk, being threatened to learn to repeat rubbish, and often medicated to keep them quite at their desk? This is wholly insane. When are they given grammar books, devoid of doctrinal trash? What experiences are they given to share? The whole concept of education today, as in the near past, should make one want to toss their lunch in a teachers lap.

An aware mind does not look to books with trash in them, but to books which are aimed at helping to teach them how to manage and use their own experiences for survival. Simple grammar books, showing the art of names, our grammar matrix which allows us to use our imagination on how to actually construct a good future.

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One can view every system of grammar as the above segment: the material difference within the limits is given first as a variable, it can be

any material difference, i.e. we start with the intelligible, a universal, but it becomes functional when we particularize it with our own experiences. We can never understand the word cat but we do have knowledge of particular cats. Only an illiterate fool would tell you stories of parallel universes while at the same time promoting non-Euclidean Geometry where there is not such thing as even a parallel line. Or telling you that a line is composed of an infinite number of points, when there is no such thing as a point, a limit of a material difference is not the material of which it limits.

Etc.

# The Conclusion of the Delian Quest

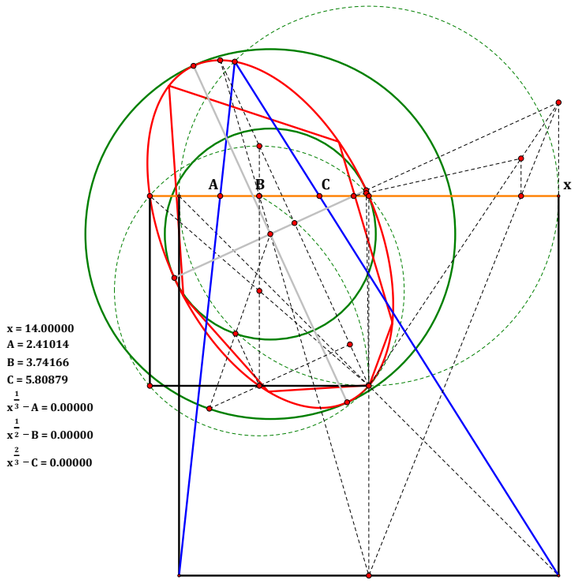
Friday, December 3, 2021

Normal people cannot see, as Plato said, the similar idea in the many examples. This is called pattern recognition and it requires the ability to comprehend metaphor.

In terms of the source of the Delian Problem and the source of the Judeo-Christian Scripture, one can see an identical message, the evolution of judgment of mankind. Where the JCS sets out many puzzles, all of which lead to the same conclusion, that mankind will not be able to read that book until after a certain time in human history, the Delian Problem set out one, however, that one, as in the JCS, is a universal expression of truth in regard to judgment: Do you comprehend that binary recursion produces the whole of judgment? Today, the computer is asking that same question, and once again, no one seems to be paying attention. Every possible system of grammar, and every possible system of manipulating things, is the results of binary recursion, just by the definition of a thing itself. For information processing, this gives us: Common Grammar, Arithmetic, Algebra and Geometry. Geometry, being the only analogic, produces metaphorical examples for each of the other three. This is our physically and biologically given method of performing all judgments.

In the perceptible, one can solve the particular Delian Problem, the duplication of the cube in at least two distinct ways. Many have tried to solve it with conics, with questionable results.

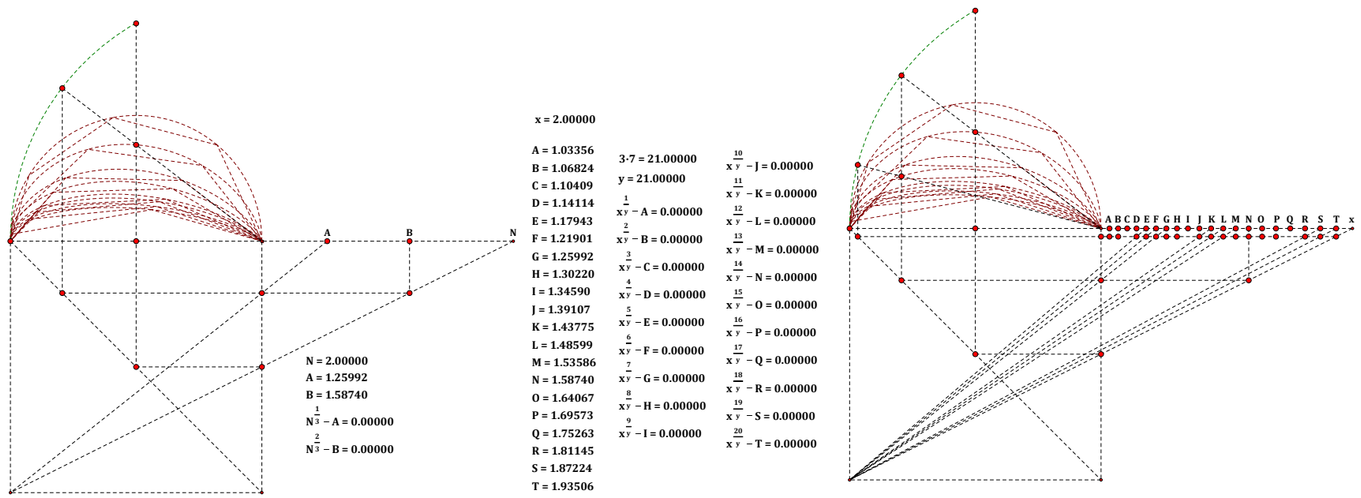
Here is the conic solution:




Fundamentally, it is a square root figure which gives you all the information required leaving only the loci to be drawn in, i.e. everything is given, if you have the wit to find it, and you either fill it in, or freeze up, afraid to take the next step.

Some have sought for a more expanded method of doing roots and powers altogether, but once again, no results.

Here, is any exponential series.

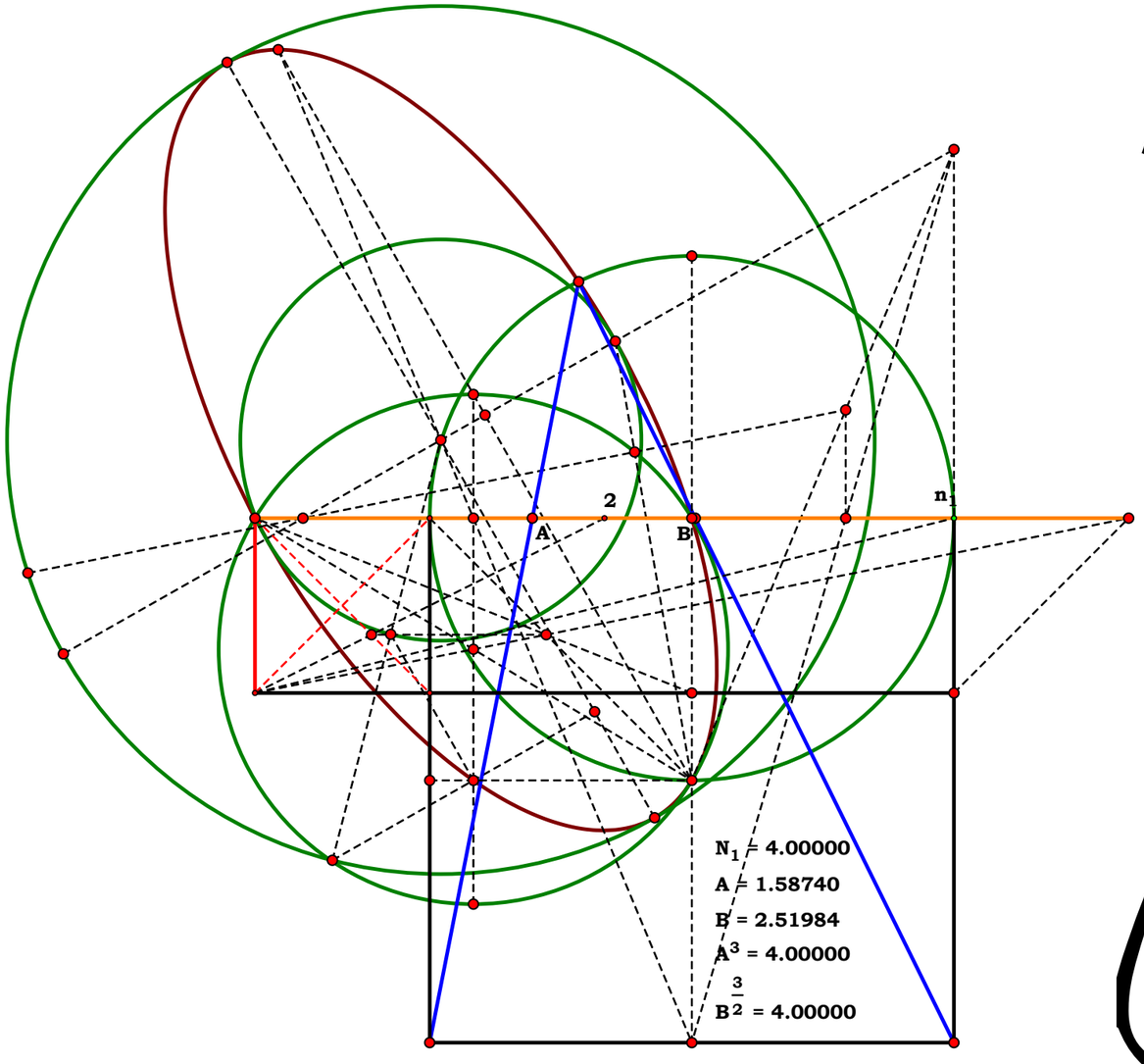



Power sequence for the loci, 2, 3, 5, 7, 11, 13, 17, 21

The mind is designed, by biological fact, to maintain and promote life through the manipulation of memory using binary recursion. Binary recursion means both parts of a thing, not only the relative, but the correlative as well. Therefore, when given a pair of tools, i.e. a binary set of tools, anyone with sight can see them perceptibly, however, to comprehend and use them, one has to see them intelligibly.

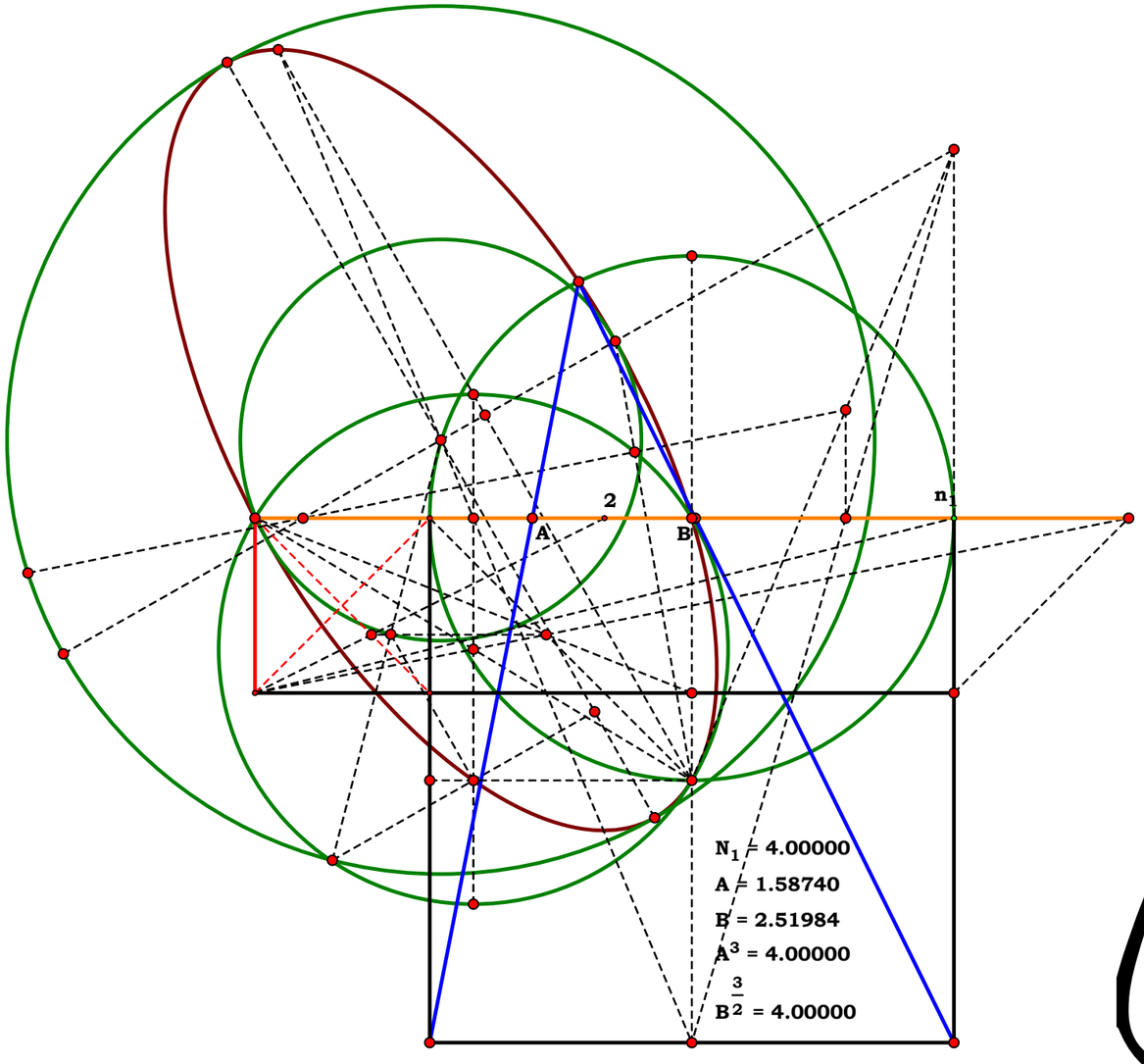
As both the JCS and the Delphic Oracle imply, Language is Universal and Intelligible, while Grammar is Particular and Perceptible. If one does not have the intelligence to comprehend the Universal Binary of Language, one cannot ever become literate. The ability to repeat symbols does not imply linguistic functionality, it is the ability to effect binary recursion with those symbols. And as each member of our Grammar Matrix, examples one of the four ways to utilize binary recursion, each must still arrive at the same results for any judgment, or result of binary recursion.

The literacy of man goes beyond the ability to solve a problem or two, one becomes literate, when judgment is applied to one's own behavior such that it complies with the definition of the mind as a life support system.




# The Delian Quest by Year

**John Clark**




# The Delian Quest 1989

**John Clark**

# The Delian Quest: Original Submission

Sunday, January 19, 2020

The following is a copy of the statistics of a document written long ago. I wrote it using Ami Pro, Windows Write, or Word 2, at one time had Word 6 save it another time. I eventually made Word 2003 my standard word processor. This is the first figure I started with and it eventually led not only to the Delian solution but to Basic Analog Mathematics and my understand that every possible grammar is a binary expression.

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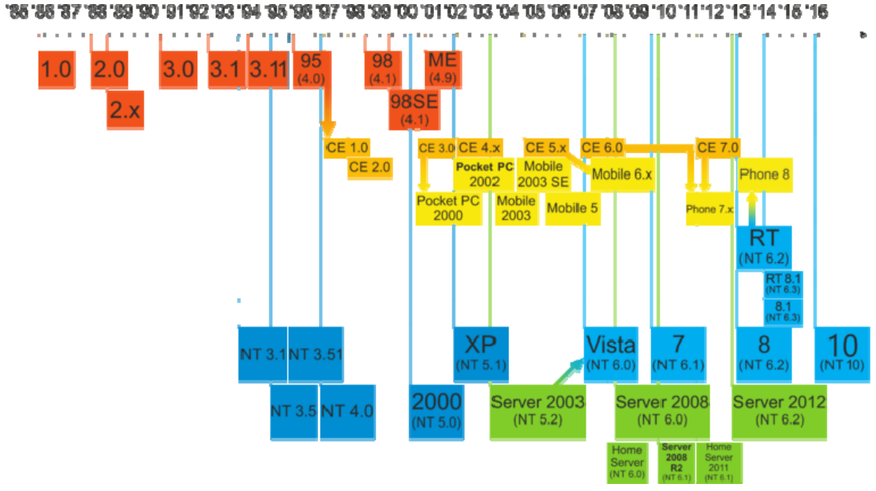
Filename: DELIAN.DOC
Directory: E:\My Documents\1989
Template: C:\Users\John\AppData\Roaming\Microsoft\Templates\Normal.dot
Title:
Subject:
Author: John J. Clark
Keywords:
Comments:
Creation Date: 5/9/1992 4:32:00 PM
Change Number: 26
Last Saved On: 10/6/1992 7:23:00 AM
Last Saved By: John J. Clark
Total Editing Time: 339 Minutes
Last Printed On: 10/2/2019 10:29:00 AM
As of Last Complete Printing
Number of Pages: 19
Number of Words: 2,019 (approx.)
Number of Characters: 9,553 (approx.)

---

What it tells me is that I wrote it using an operating system prior to Windows 3 and may have originally written using Ami Pro which I quickly abandoned when I purchased Word. The formatting, however, leans towards Word 2. It tells me that I started learning geometry at about the age of 38 never having taken the topic in school. It was doing my drawing by hand at the time, the first drawing program I used was TommyCad. I later found Geometer's Sketchpad. Early on, I was using Word 2 to construct data table's which I would save to a text document picked up by QBasic to do the math, and then pass it back to Word. I wrote macro's for that. I did that until Mathcad sent me an invitation to buy that program and have used it ever since.

I suspect Mathcad sent me the offer as Microsoft approached me first to become a beta tester, probably based on the machine I had just purchased which ran at a blazing 25mhz, which I later dropped out of because of the stupid way they ran the program, the original version of Dos 6 completely ate my hard drive and I lost a lot of work. I was using one of those sewing machine boxed computers at the time to carry it to and from work as I worked at G.M. I was the first G.M. factory worker to carry a pc in and out of the plant which led to a Union settlement allowing employees to bring their own computers in and out.

The following table is from Wikipedia.




I wrote the letter as I believed that a real geometer should work on the problem of cube duplication from this starting point as I have never even had geometry in school. However, I was surprised to find that these publications only expected such letters from professors instead of factory workers. No matter, once I had the combination of a drawing program and Mathcad, I could get more involved in simply learning. When I got these two together, I started putting some of my original drawing's on paper into a formal digital diary.

I have posted my work on AOL, personal free websites, even shareware CD rom's, mostly because I was looking for a kindred study flame, which never happened.

Before the Delian Quest, I pondered the issue of *Who wrote the Book of John*? I found that all evidence points to the wife of Christ, Mary and that enough still remains in the text to make a good case of it as it was deliberately altered to hide the fact. There is evidence to support the idea that Christ started to perform at his own wedding, which has a root in Law. His wife evidently became a companion prophet. There is also evidence that Peter had his eye on her which never panned out.

Then I decided to tackle the Name of the Beast 666. The depth of that solution evolved over years. After these, I decided to find another impossible problem to solve when I ran into the Delian Problem. My studies started in my childhood by putting before myself specific questions. Then for a long time just reading any book I could come up with. Then I started targeting so called impossible problems. My original approach to the Bible was a result of a sign post in a lucid dream. My first reaction was that it was rubbish, but it quickly faded when I realized, rather quickly, it was using words in a manner I had never seen before, it was deliberately testing the reader. Later, I found another writer who wrote in a similar fashion, Plato.

All of this tells me that I have been in a state of cognitive dissonance before I started my work on the Number of His Name and geometry as I should have been dead prior to this, before I came back to Michigan.

At this time, I am want to do another revision of the Delian Quest and have decided to put the opening as I had done before the 2015 release.

# THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is, as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefore this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

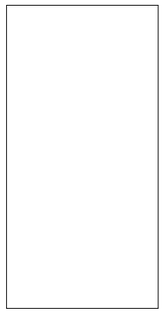


Plate 1

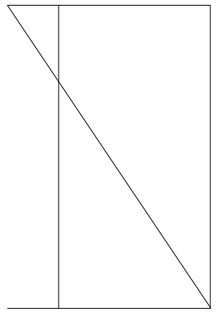


Plate 2

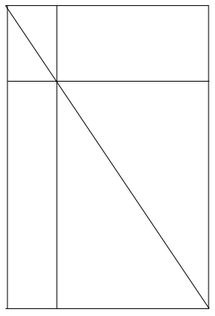


Plate 3

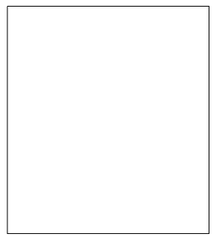


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

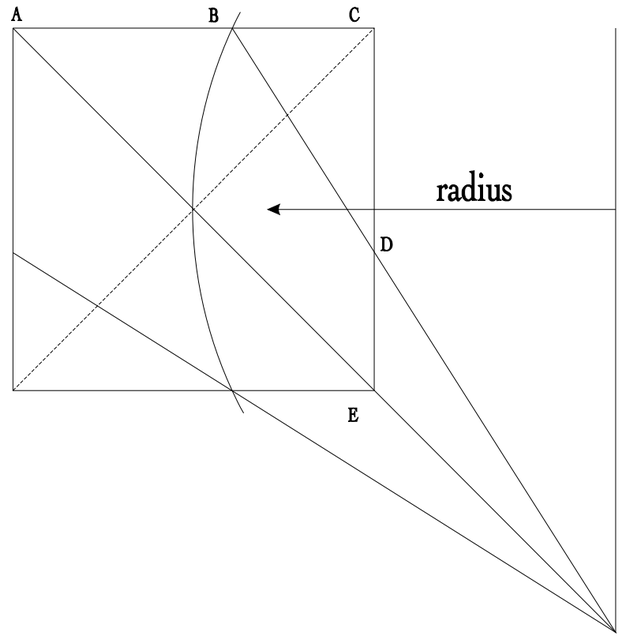
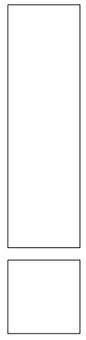
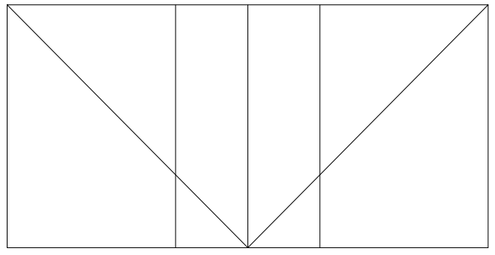



Plate 5 AB = CD. BC = DE

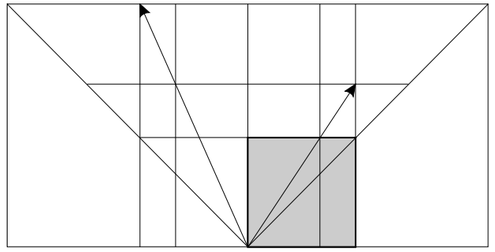
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



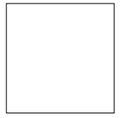
P.6



P.7

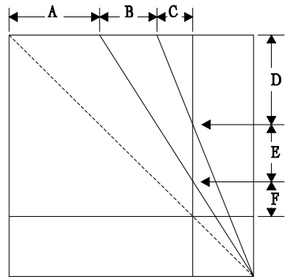


P.8

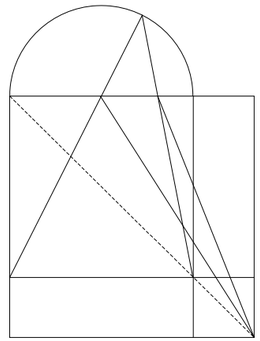


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

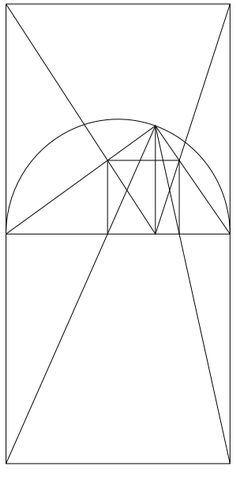



P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

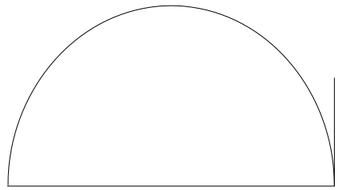


Plate 13

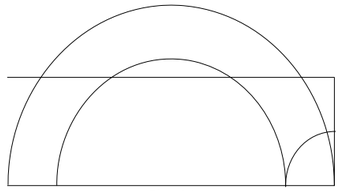


Plate 14

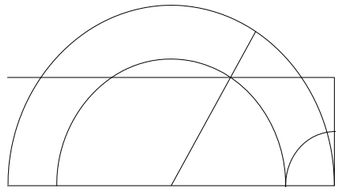


Plate 15

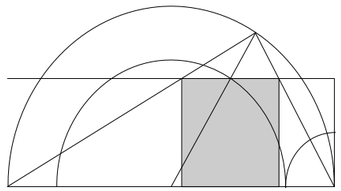
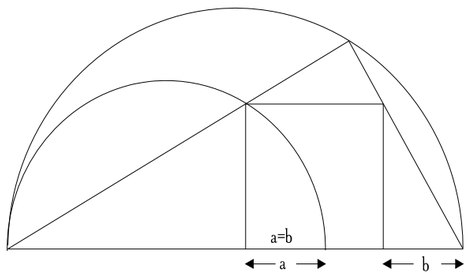
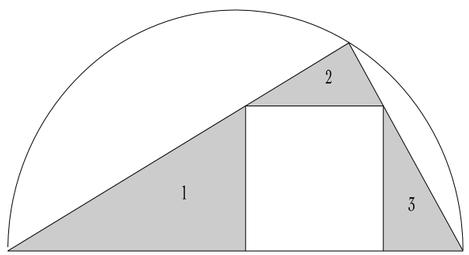


Plate 16

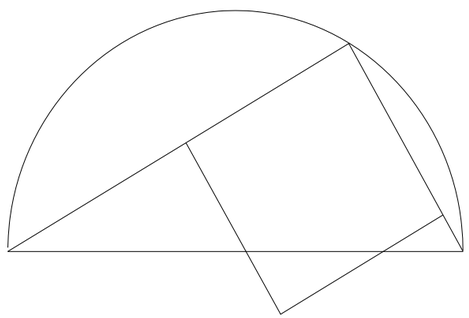
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.




In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"




P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

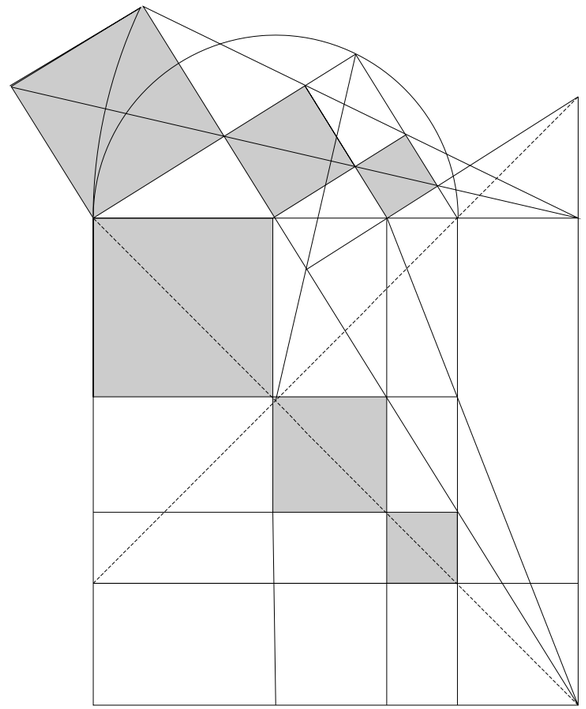
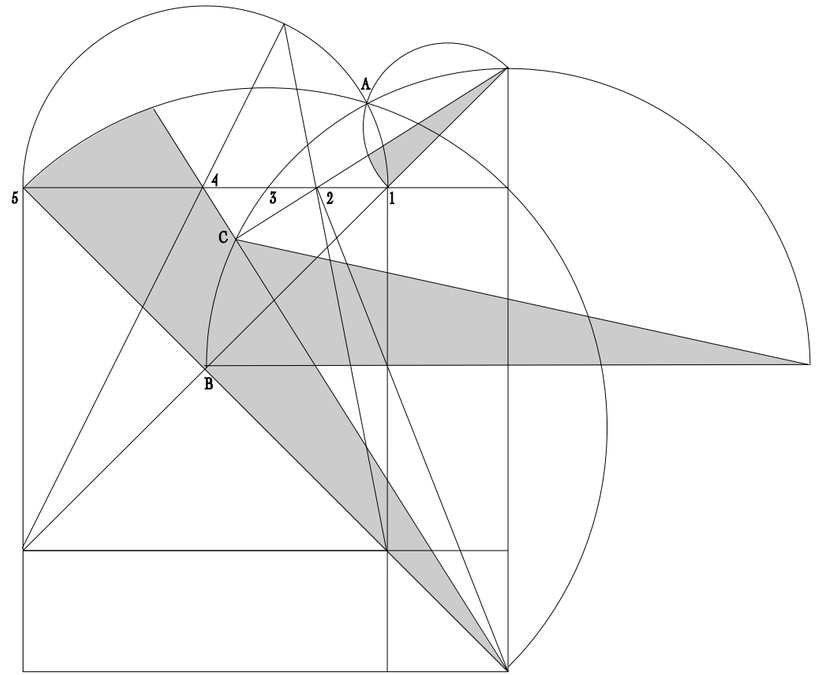


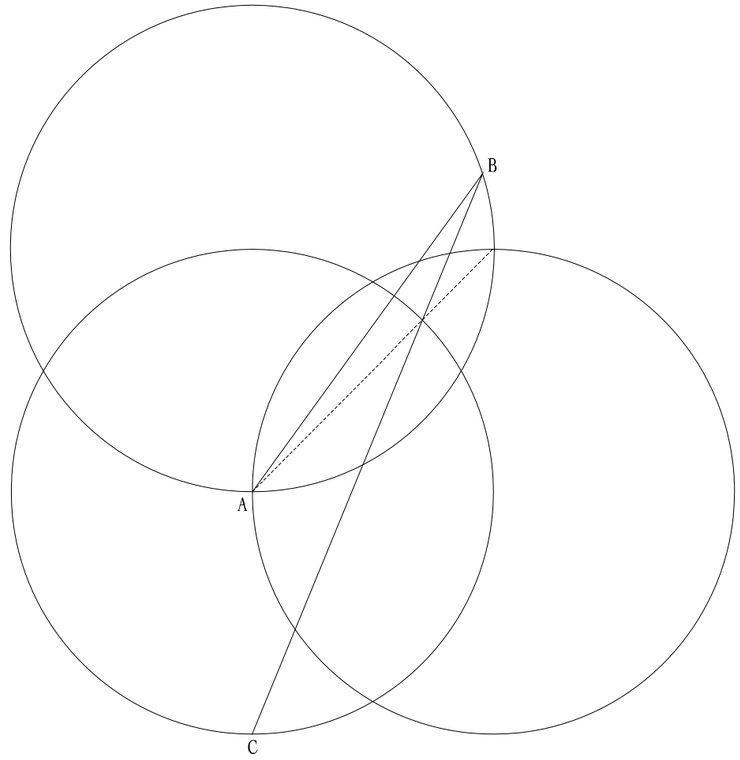
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression.  It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at.  Plate 21.




All three semicircles intersect the semicircle which we draw our right angle in at the same point (A).  From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application.  I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure.  One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.




How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

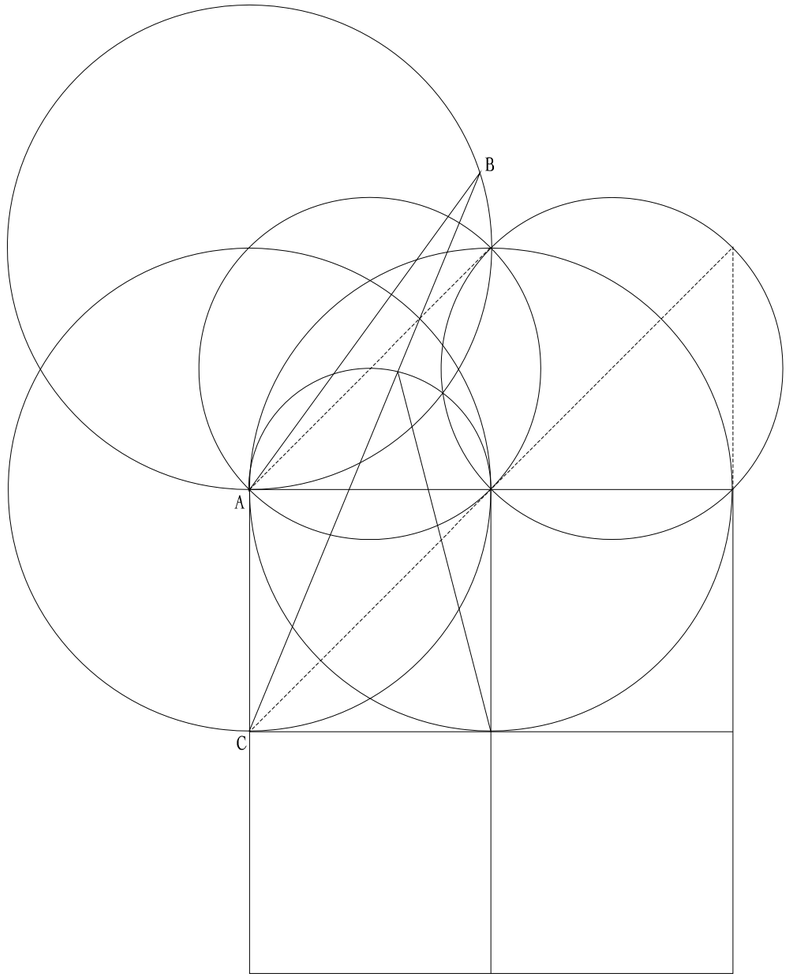
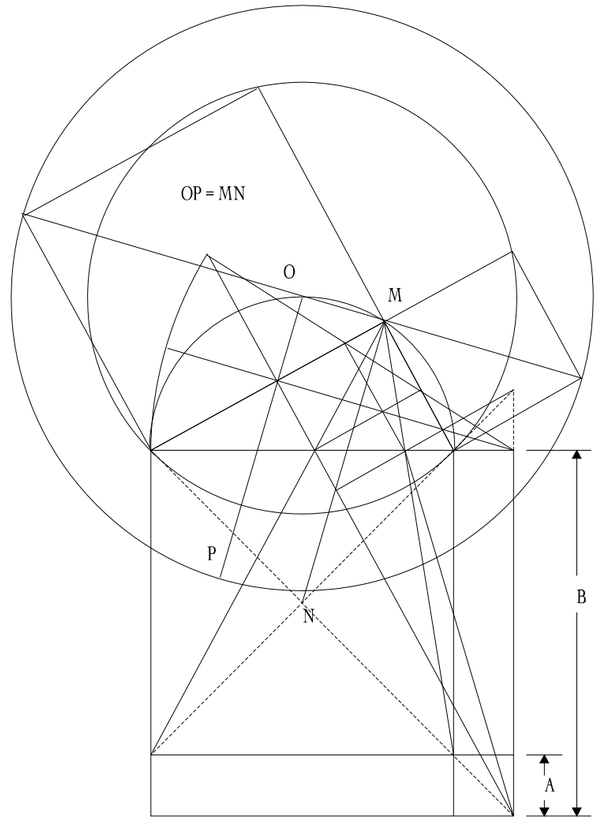



Plate 23

On Plate 24 the radius for the circle OP is given by MN.




One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of $B^2A$ (if you have missed it, the figure gives both roots, $A^2B$ and $B^2A$) there is a series of intersects, (three of them). When these intersects form a line parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

The following is a list of returns for publication of the previous material. The first one is interesting in that the writer claims not to have understood the preceding document.

# GEOMETRIAE DEDICATA

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F. D. VELDKANW
Nblemakch Mm4uw
Postbus 80.010
3508 TA Utrecht
The Netherlands
Telephone: (0)30-531519

Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

# American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA  Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

**Location:**
201 Charles **Street**
Providence, **RI**
02904


December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Matheniatics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

# American Mathematical Society

**Roger E. Howe**
Bulletin
Editorial Committee

Department of Mathematics
Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

# JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches Institut
Technische Universitiit MUnchen
Arcisstr. 21

D-8000 MUnchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".

Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H.-J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:

You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.

All together the article in the given version is not understandable.

JOURNAL OF GEOM TRY

Editor's Office

München, 1 June 1990

Dear Professor Clark,

Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.

We are very sorry that we could not be of any help to you.

Sincerely yours,
H.-J. Kroll

(This one is a form letter.)

**société mathématique de france**

paris, le
BULLETIN

n. réf. a l'attention de
v. réf.

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Detian Solution

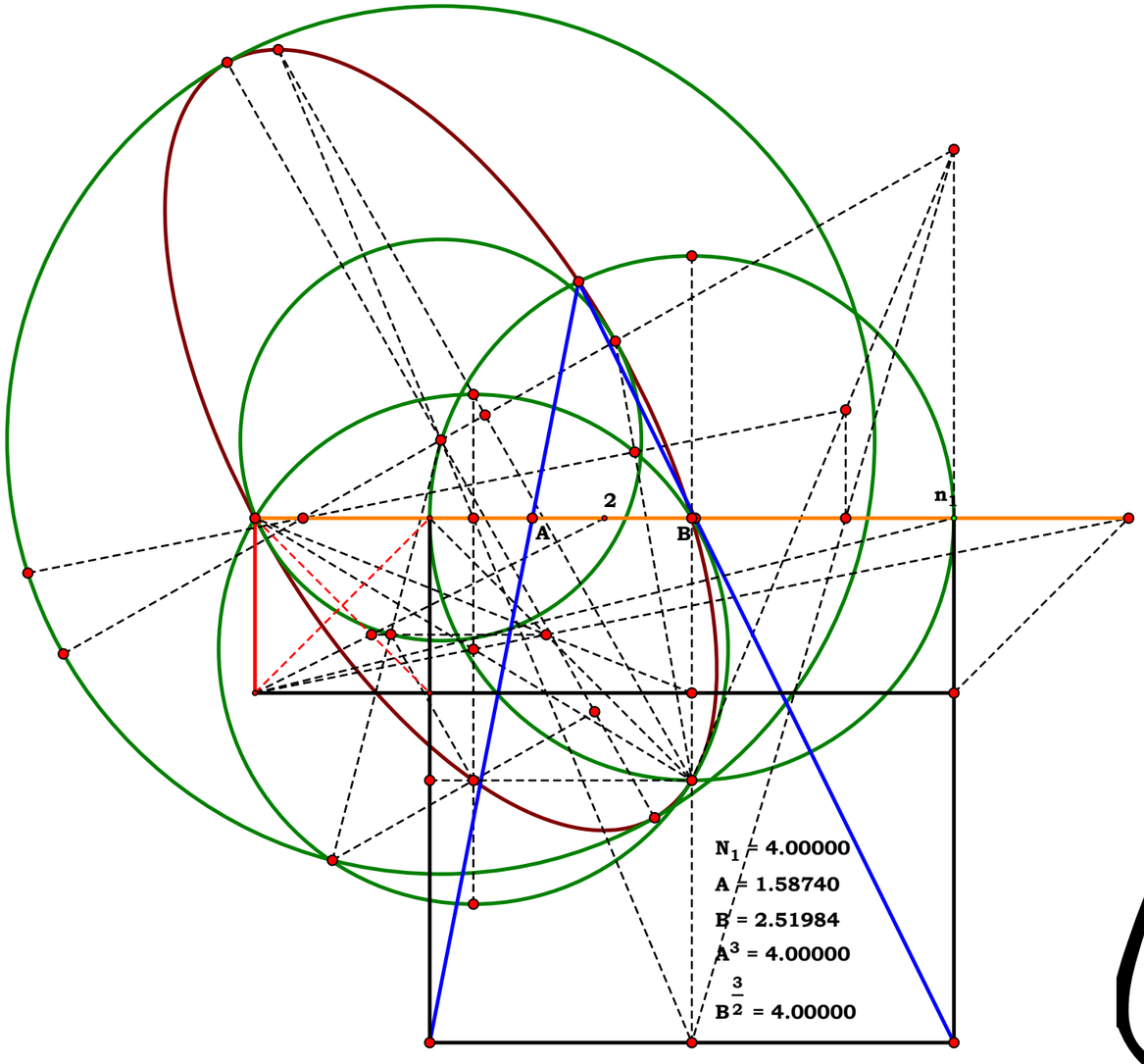
Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

P.J. : Manuscrit

I was expecting at least some type of guidance, or cogent response, other than, after stating that I was not a geometer, everyone insisted on rubbing it in by calling me a professor, which I found very rude.




# The Delian Quest 1992

**John Clark**

Unit := 1

Given.

AB := 8.8900 $N_1$ := 2

BC := 3.28600 $N_2$ := 4

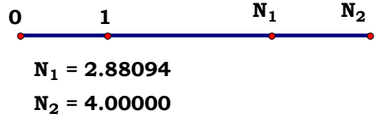
062092R1

Descriptions.

There are many ways to take the square root of any two differences, this is one of them. It is a very old figure, one can find it in Euclid's *Elements*. One can then say, that the *Delian Quest* starts with a given, The *Elements* of Euclid. What many do not realize, is the foundation of that work, the concept that Geometry is just another binary grammar system, traces back to Plato.

$BD := \sqrt{AB \cdot BC}$ BD = 5.404863

Now, let us fold BA over BC and make what is called a numbered line.




$N_1$ = 2.88094

$N_2$ = 4.00000

As the first figure is a given, all we have to do to show how to take the square root of two numbers on a number line is simply unfold it. I am going to unfold it to the perpendicular to the point of origin. And now we can take advantage of all the simple resulting equations for the two numbers and the result.

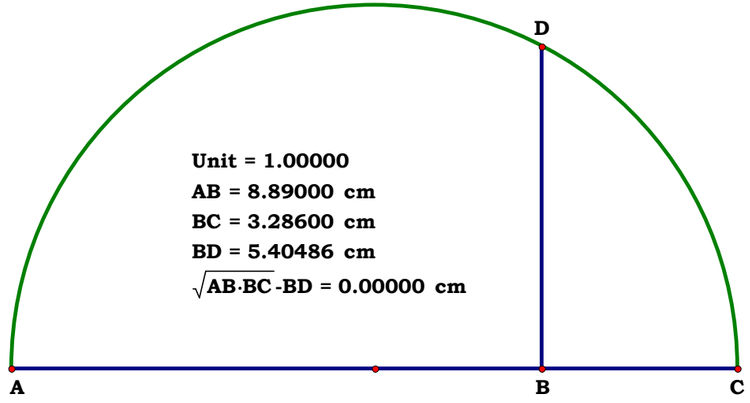
$\sqrt{N_1 \cdot N_2} = 2.828427$ $R := \sqrt{N_1 \cdot N_2}$

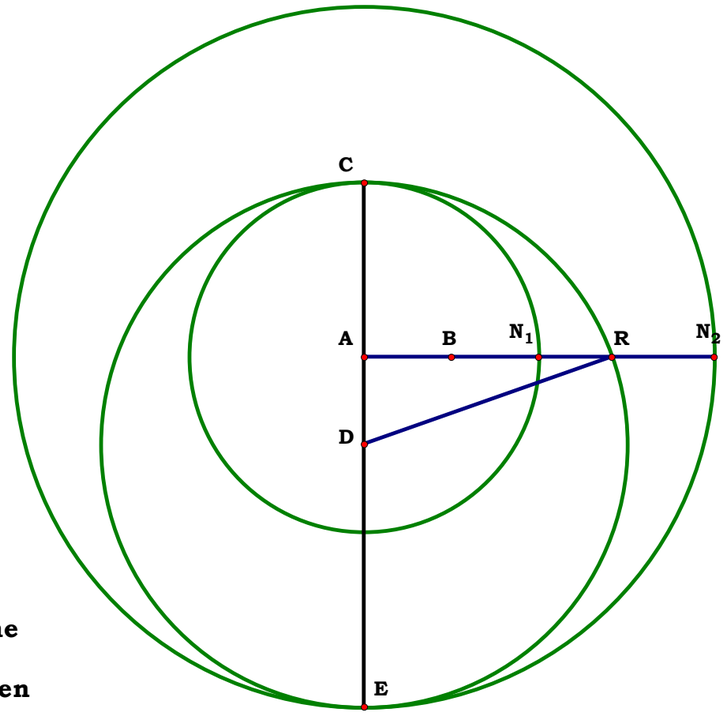
Definitions.

$R - \sqrt{N_1 \cdot N_2} = 0$ $\frac{R^2}{N_1} - N_2 = 0$ $\frac{R^2}{N_2} - N_1 = 0$

The numbered line is not a new idea. Numbers are just names developed as the Arithmetic Naming Convention. We can also use Common Grammar to name our points as has been done for thousands of years. A line with the points given names has always been a part of formal Geometry.

## A Duplicate Ratio




1 = 1.00000

$N_1$ = 2.00000

$N_2$ = 4.00000

$\sqrt{N_1 \cdot N_2}$ = 2.82843

A = 2.82843

$\sqrt{N_1 \cdot N_2}$-A = 0.00000

$\frac{A^2}{N_1}$-$N_2$ = 0.00000

$\frac{A^2}{N_2}$-$N_1$ = 0.00000

$AN_1$ = 2.93133 cm

$AN_2$ = 5.86267 cm

Unit := 1

Given.

$N_1 := 4$

$N_2 := 2$

# A Duplicate Ratio

062092R2

Descriptions.

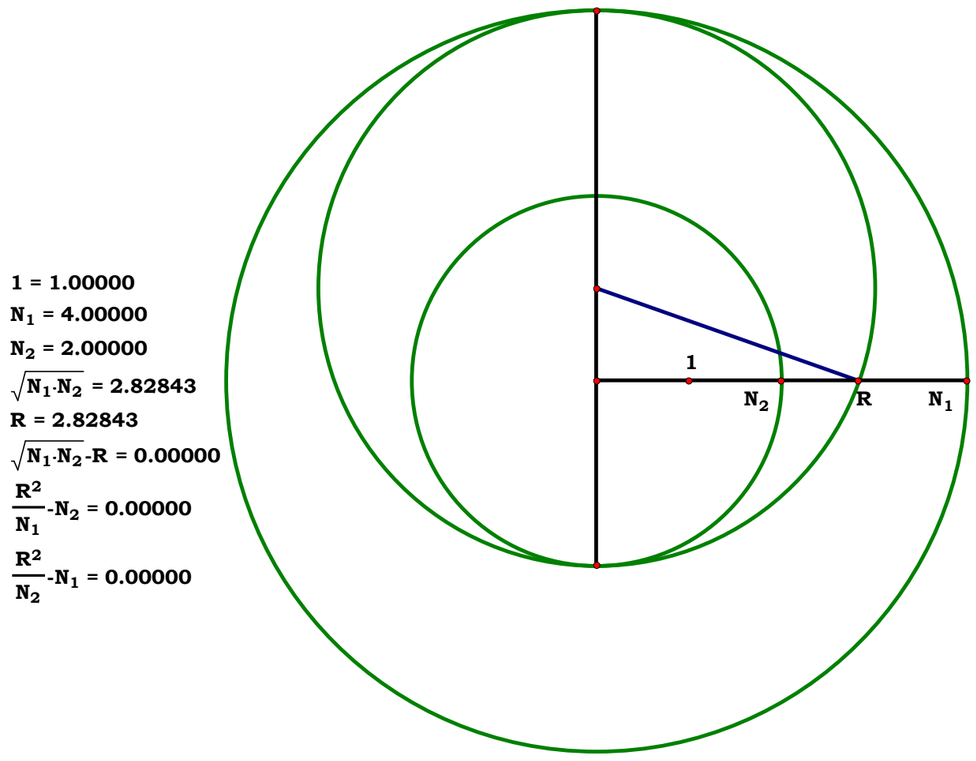
It really does not make much of a difference if unfold I fold my figure, certainly not in the result. The next question is, can I take a third thing and put it proportionally at point A?

$R := \sqrt{N_1 \cdot N_2}$

Definitions.

$\sqrt{N_1 \cdot N_2} = 2.828427$

$\frac{R^2}{N_1} - N_2 = 0 \quad \frac{R^2}{N_2} - N_1 = 0$




1 = 1.00000

$N_1$ = 4.00000

$N_2$ = 2.00000

$\sqrt{N_1 \cdot N_2}$ = 2.82843

R = 2.82843

$\sqrt{N_1 \cdot N_2} - R$ = 0.00000

$\frac{R^2}{N_1} - N_2$ = 0.00000

$\frac{R^2}{N_2} - N_1$ = 0.00000

Unit := 1

Given.

$N_1 := 4$

$N_2 := 2$

062092R3

Descriptions.

To add any number of differences, proportionally to the first two given, we simply take half of it and project from the root of the first two to find our two radii from the center which will place that third difference on the line All the while, we se wwe have be producing duplicate ratios. It then follows that any number proportional ratios is going to depend on the square root of term pairs.

The whole exercise is then, given two differences and then find the point of similarity from which they are set into this proportional series.

$A := \sqrt{N_1 \cdot N_2}$

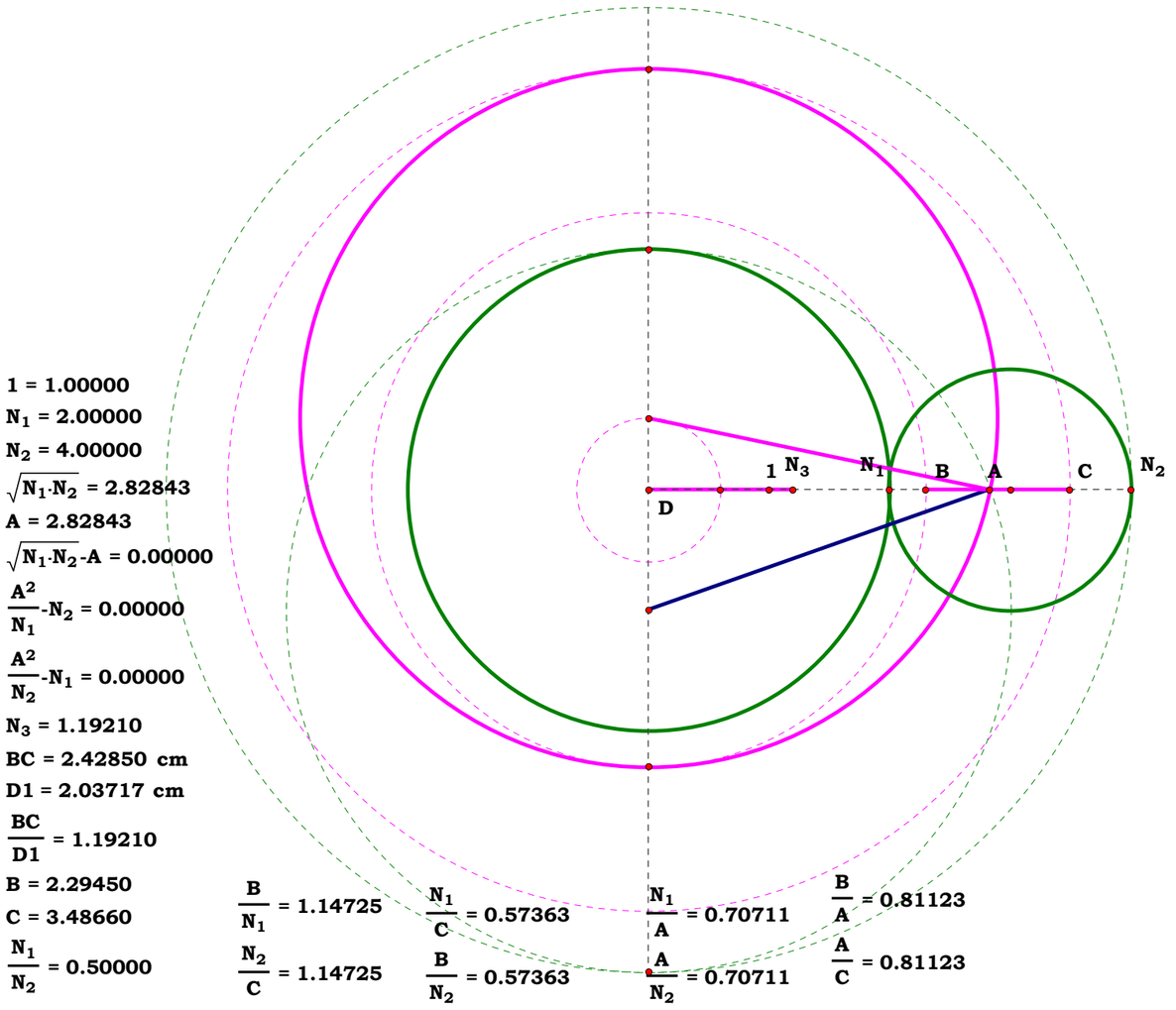
Definitions.

$\sqrt{N_1 \cdot N_2} = 2.828427$

$A - \sqrt{N_1 \cdot N_2} = 0$

$\frac{A^2}{N_1} - N_2 = 0 \quad \frac{A^2}{N_2} - N_1 = 0$

## A Duplicate Ratio




1 = 1.00000

$N_1$ = 2.00000

$N_2$ = 4.00000

$\sqrt{N_1 \cdot N_2}$ = 2.82843

A = 2.82843

$\sqrt{N_1 \cdot N_2} - A$ = 0.00000

$\frac{A^2}{N_1} - N_2$ = 0.00000

$\frac{A^2}{N_2} - N_1$ = 0.00000

$N_3$ = 1.19210

BC = 2.42850 cm

D1 = 2.03717 cm

$\frac{BC}{D1}$ = 1.19210

B = 2.29450

C = 3.48660

$\frac{N_1}{N_2}$ = 0.50000

$\frac{B}{N_1}$ = 1.14725 $\quad \frac{N_1}{C}$ = 0.57363 $\quad \frac{N_1}{A}$ = 0.70711 $\quad \frac{B}{A}$ = 0.81123

$\frac{N_2}{C}$ = 1.14725 $\quad \frac{B}{N_2}$ = 0.57363 $\quad \frac{A}{N_2}$ = 0.70711 $\quad \frac{A}{C}$ = 0.81123

062092R4

Unit, external

Given.

$N_1 := .74206$

$N_2 := 3 \quad BC := N_2$

$N_3 := 1.05 \quad DE := N_3$

# A Duplicate Ratio

Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

Descriptions.

$AC := N_1 + BC$

$AF := N_1 \quad AG := AC \quad AJ := \sqrt{AF \cdot AG}$

$AL := \frac{DE}{2} \quad JL := \sqrt{AJ^2 + AL^2}$

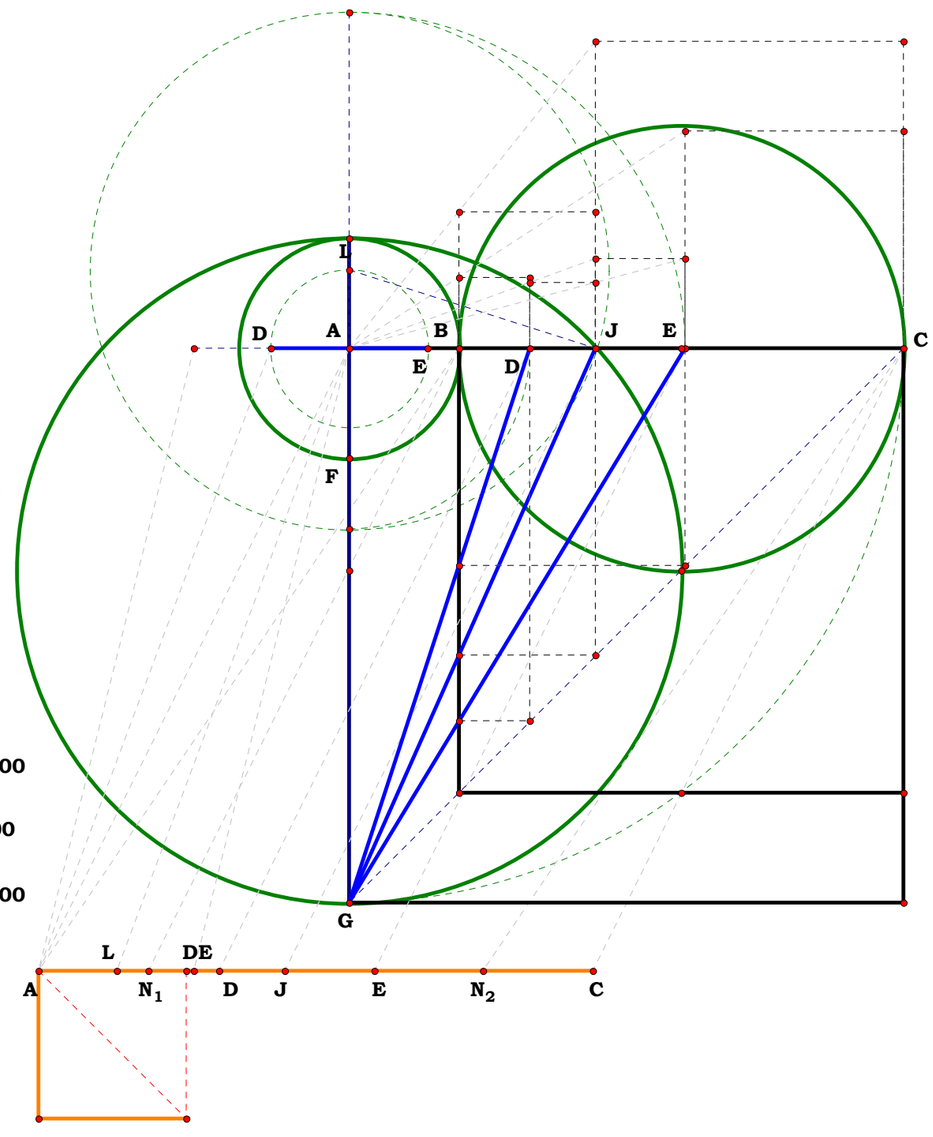
$AD := JL - AL \quad AE := JL + AL$

$\frac{N_1}{AD} - \frac{AE}{AC} = 0 \quad \frac{AD}{AJ} - \frac{AJ}{AE} = 0 \quad \frac{N_1}{AJ} - \frac{AJ}{AC} = 0$ etc.,

Definitions

AC = 3.74206 AJ = 1.666383 AL = 0.525

AD = 1.222129 AE = 2.272129




$N_1$ = 0.74206
$N_2$ = 3.00000
DE = 1.05000
X/Y = 0.35000
X = 7.00000
Y = 20.00000
C = 3.74206
J = 1.66638
L = 0.52500
D = 1.22212
E = 2.27212
$\frac{N_1}{D} - \frac{E}{C}$ = 0.00000
$\frac{D}{J} - \frac{J}{E}$ = 0.00000
$\frac{N_1}{J} - \frac{J}{C}$ = 0.00000

**Definitions**

$$AC - \left(N_1 + N_2\right) = 0$$

$$AF - N_1 = 0$$

$$AG - \left(N_1 + N_2\right) = 0$$

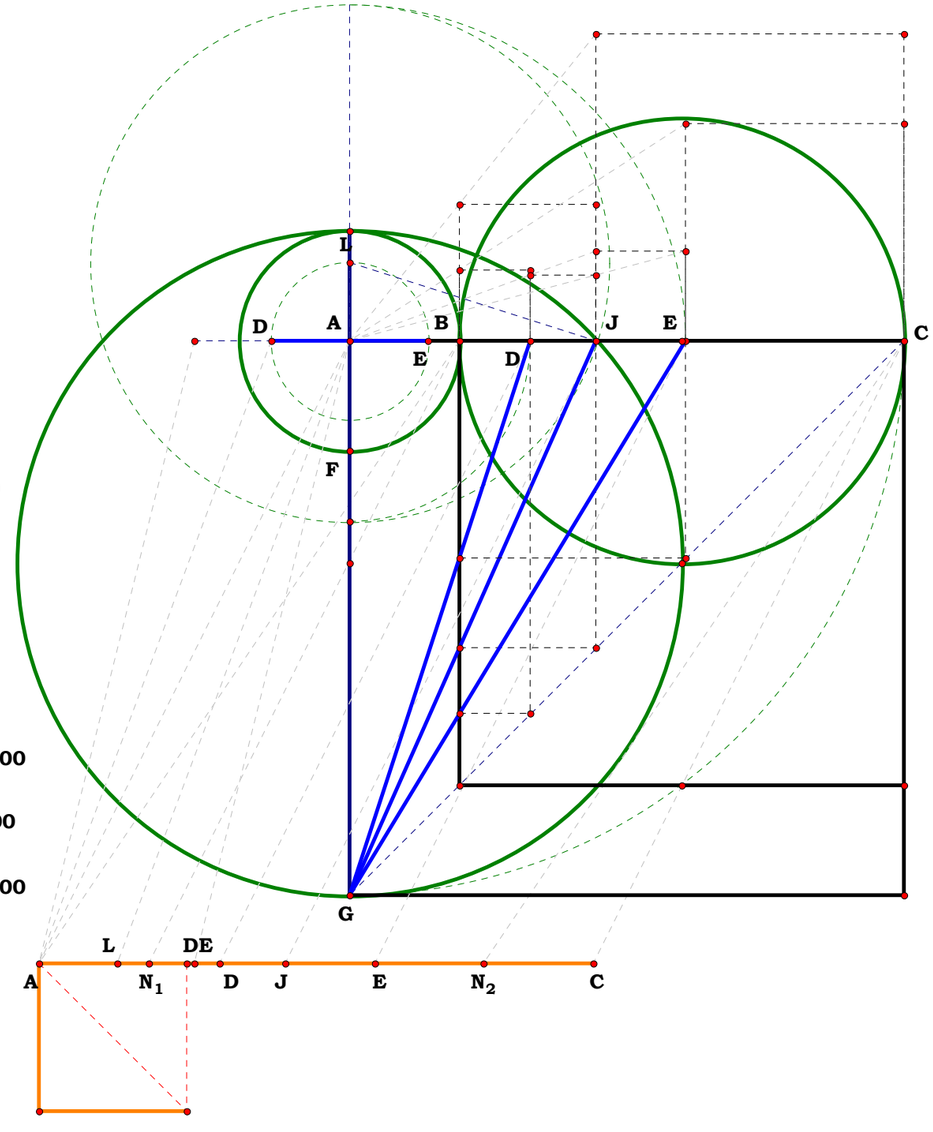
$$AJ - \sqrt{N_1 \cdot \left(N_1 + N_2\right)} = 0$$

$$AL - \frac{N_3}{2} = 0$$

$$JL - \frac{\sqrt{\left(4 \cdot N_1^{\,2} + 4 \cdot N_2 \cdot N_1 + N_3^{\,2}\right)}}{2} = 0$$

$$AD - \frac{\sqrt{4 \cdot N_1^{\,2} + 4 \cdot N_1 \cdot N_2 + N_3^{\,2}} - N_3}{2} = 0$$

$$AE - \frac{N_3 + \sqrt{4 \cdot N_1^{\,2} + 4 \cdot N_1 \cdot N_2 + N_3^{\,2}}}{2} = 0$$

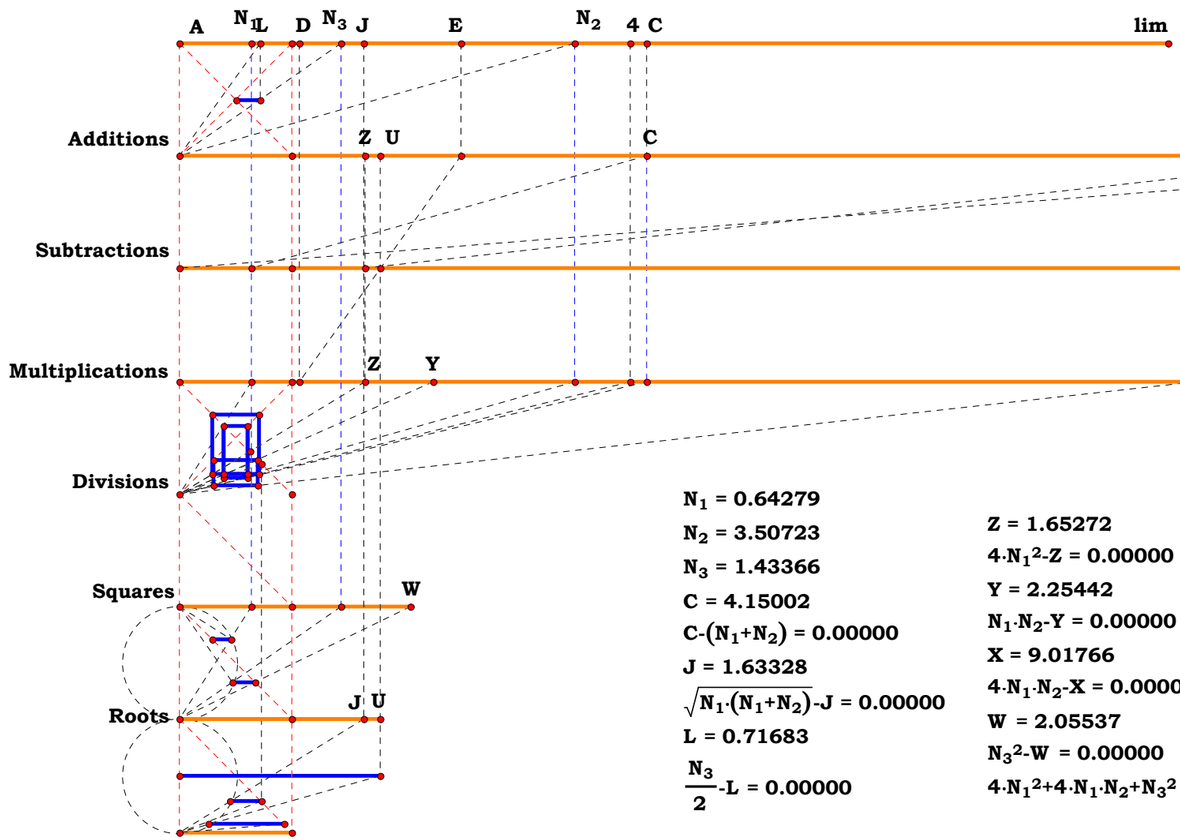


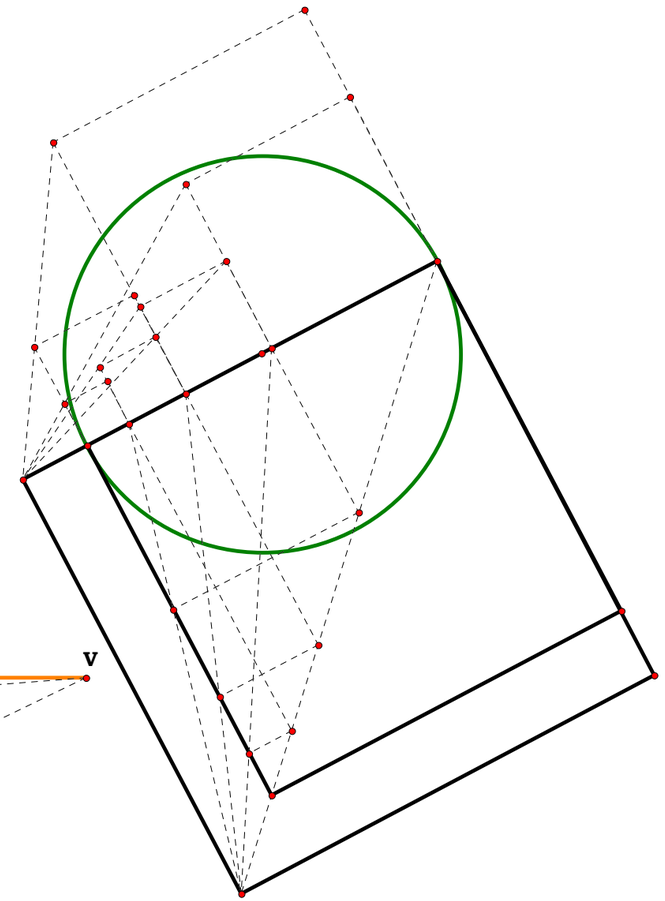

$N_1$ = 0.74206
$N_2$ = 3.00000
DE = 1.05000
X/Y = 0.35000
X = 7.00000
Y = 20.00000
C = 3.74206
J = 1.66638
L = 0.52500
D = 1.22212
E = 2.27212

$$\frac{N_1}{D} - \frac{E}{C} = 0.00000$$

$$\frac{D}{J} - \frac{J}{E} = 0.00000$$

$$\frac{N_1}{J} - \frac{J}{C} = 0.00000$$



$N_1 = 0.64279$

$N_2 = 3.50723$

$N_3 = 1.43366$

$C = 4.15002$

$C-(N_1+N_2) = 0.00000$

$J = 1.63328$

$\sqrt{N_1 \cdot (N_1+N_2)}-J = 0.00000$

$L = 0.71683$

$\frac{N_3}{2}-L = 0.00000$

$Z = 1.65272$

$4 \cdot N_1{}^2-Z = 0.00000$

$Y = 2.25442$

$N_1 \cdot N_2-Y = 0.00000$

$X = 9.01766$

$4 \cdot N_1 \cdot N_2-X = 0.00000$

$W = 2.05537$

$N_3{}^2-W = 0.00000$

$4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2 = 12.72575$

$V = 12.72575$

$U = 1.78366$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-U = 0.00000$

$D = 1.06683$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}-N_3}{2}-D = 0.00000$

$E = 2.50049$

$\frac{N_3+\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-E = 0.00000$

# A Duplicate Ratio

062092R5

Unit.

$AB := 1$

Given.

$N_1 := 3.98280$ $\quad BC := N_1$

$N_2 := .81091$ $\quad DE := N_2$

**Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.**

**Definitions**

$AC := 1 + N_1$ $\quad AC = 4.9828$

$AF := AB$ $\quad AF = 1$

$AG := (AB + N_1)$ $\quad AG = 4.9828$

$AJ := \sqrt{(N_1 + AB)}$ $\quad AJ = 2.232219$

$AL := \frac{N_2}{2}$ $\quad AL = 0.405455$

$JL := \frac{\sqrt{(N_2^2 + 4 \cdot N_1 + 4)}}{2}$ $\quad JL = 2.268743$

$AD := \frac{\sqrt{(N_2^2 + 4 \cdot N_1 + 4)} - N_2}{2}$ $\quad AD = 1.863288$

$AE := \frac{N_2 + \sqrt{(N_2^2 + 4 \cdot N_1 + 4)}}{2}$ $\quad AE = 2.674198$

$\frac{AB}{AD} - \frac{AE}{AC} = 0$ $\quad \frac{AD}{AJ} - \frac{AJ}{AE} = 0$

$\frac{AB}{AJ} - \frac{AJ}{AC} = 0$

$\frac{AB}{AD} - \frac{AE}{AC} = 0.00000$

$\frac{AD}{AJ} - \frac{AJ}{AE} = 0.00000$

$\frac{AB}{AJ} - \frac{AJ}{AC} = 0.00000$

AB = 1.80433 cm
AD = 3.36199 cm
AJ = 4.02767 cm
AE = 4.82514 cm
AC = 8.99063 cm
AL = 0.73158 cm
AF = 1.80433 cm
JL = 4.09357 cm

AB = 1.00000
AD = 1.86329
AJ = 2.23222
AE = 2.67420
AC = 4.98280
AL = 0.40545
AF = 1.00000
JL = 2.26874

$AC - (1 + N_1) = 0.00000$

$AF - AB = 0.00000$

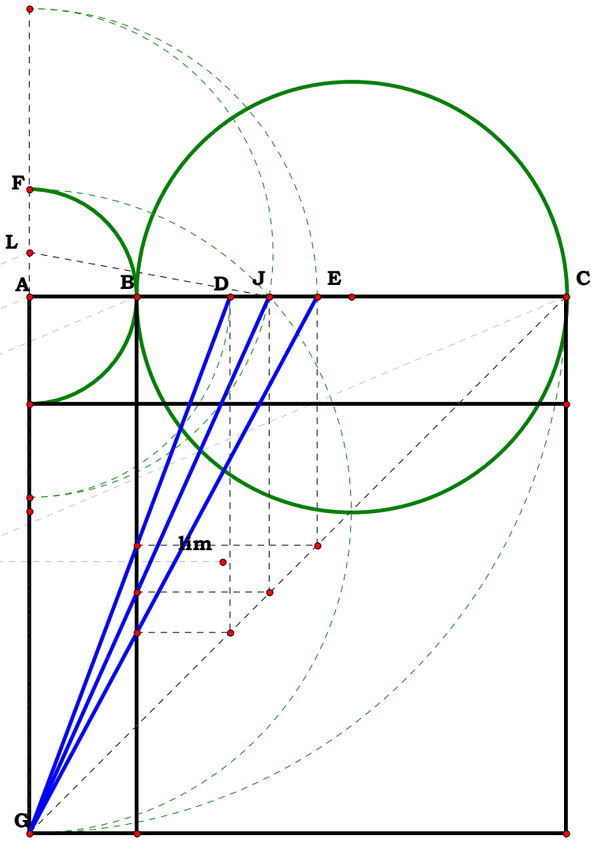
$AJ - \sqrt{N_1 + AB} = 0.00000$

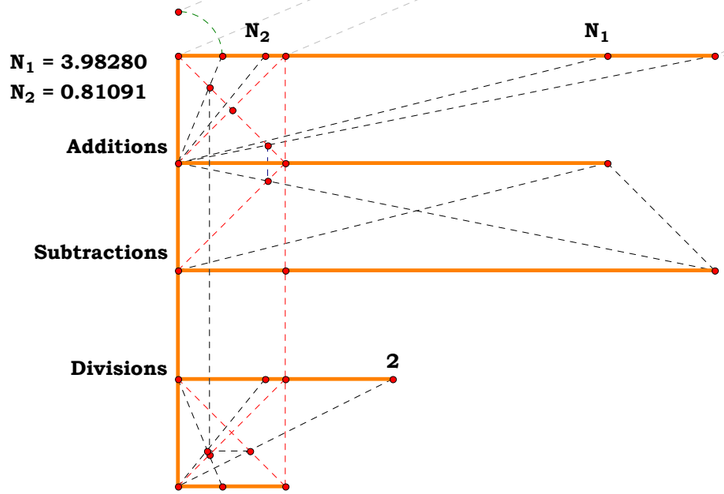
$AL - \frac{N_2}{2} = 0.00000$

$JL - \frac{\sqrt{N_2^2 + 4 \cdot N_1 + 4}}{2} = 0.00000$

$AD - \frac{\sqrt{N_2^2 + 4 \cdot N_1 + 4} - N_2}{2} = 0.00000$

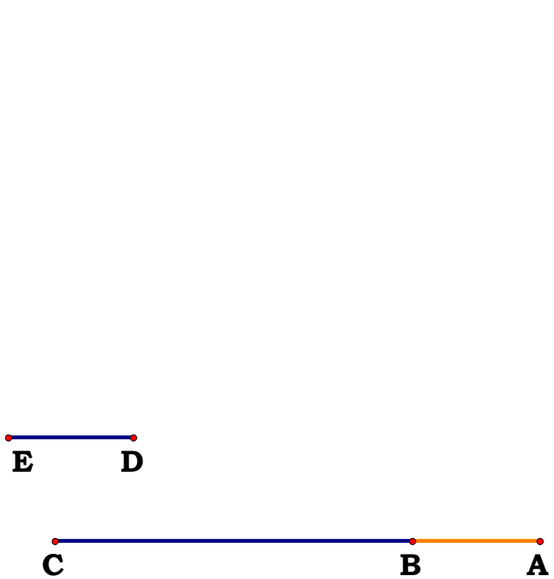
$AE - \frac{N_2 + \sqrt{N_2^2 + 4 \cdot N_1 + 4}}{2} = 0.00000$

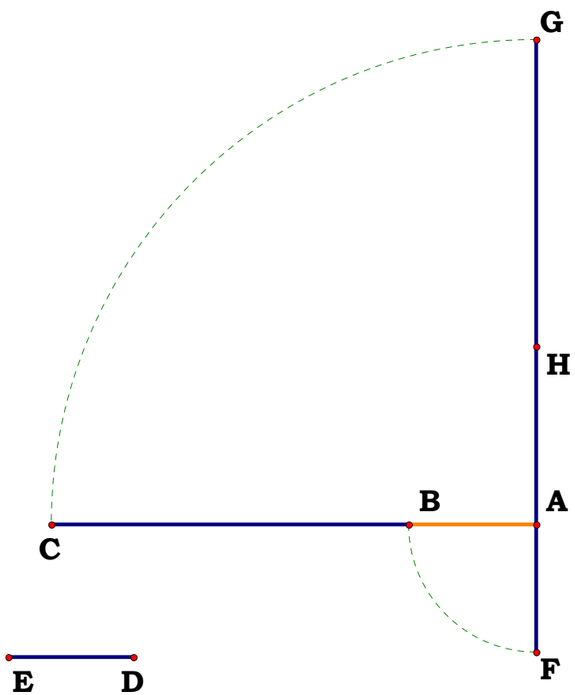


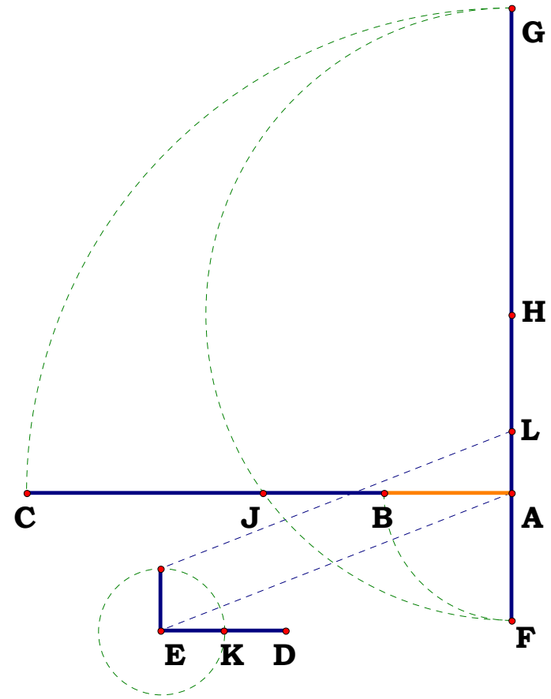



# A Duplicate Ratio

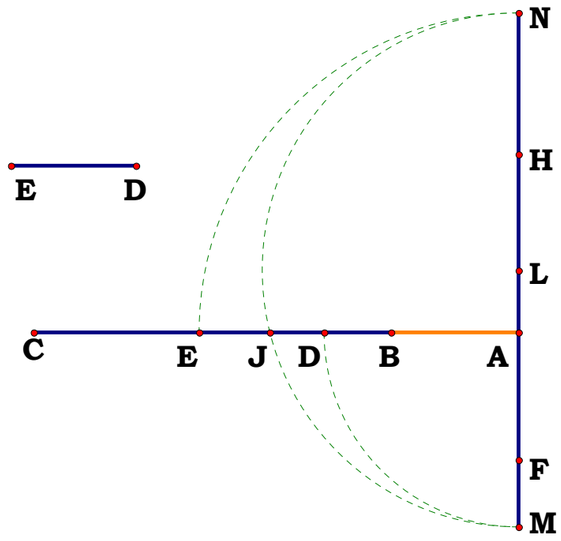
**We may now be ready to come to an outline of the whole affair.**

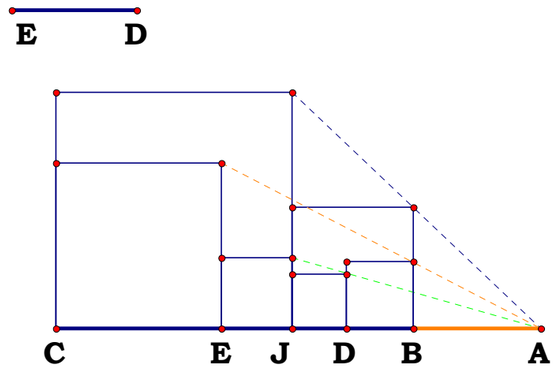


062092R6




AB = 2.13783 cm
AD = 3.25040 cm
AJ = 4.16849 cm
AE = 5.34590 cm
AC = 8.12800 cm

$\frac{AB}{AD} = 0.65771$

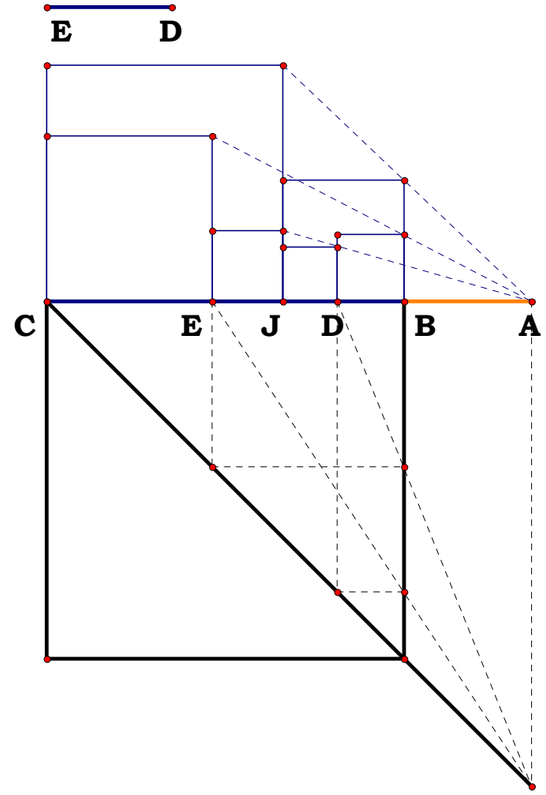
$\frac{AE}{AC} = 0.65771$

$\frac{AD}{AJ} = 0.77975$

$\frac{AJ}{AE} = 0.77975$

$\frac{AB}{AJ} = 0.51286$

$\frac{AJ}{AC} = 0.51286$



062092R6

**Starting with a simple given, we will end up prepared to formulat Basic Analog Mathematics, i.e., write Geometric Figures which can compute any mathematical and any logical result which, as the output is concurrent with the input, independent of time, that is, process information in no time whatsoever, i.e., computation independent of time. And it is all the result of binary recursion. Every possible grammar is the product of binary recursion, and, as one can plainly see, it is possible to produce one's results, quite independent of time. Therefore, the ability to predict the future, using binary recursion, is not only possible, one can say, it is factually proven. Our biologically defined job, to learn to predict the results of any number of givens is a proven fact and provably possible.**

**BAG for 062092**

$N_1 = 4.27962$

$N_2 = 2.48336$

$AB = 1.00000$ $AF = 1.00000$

$AC = 5.27962$ $AG = 5.27962$

$AJ = 2.29774$

$\sqrt{AB \cdot AG} - AJ = 0.00000$

$AL = 1.24168$

$JL = 2.61178$

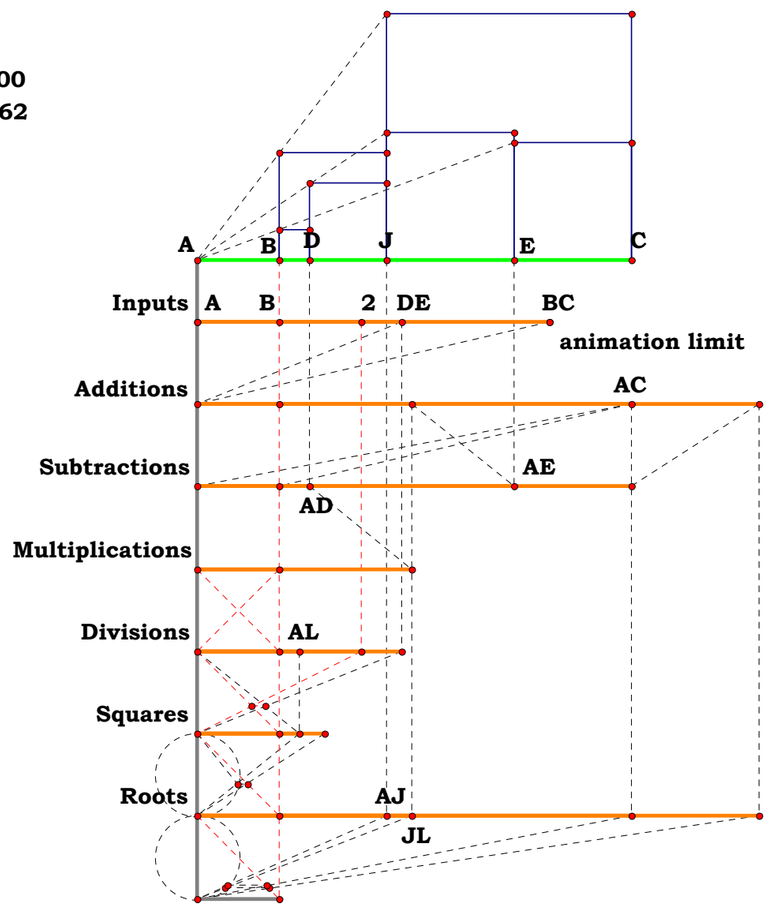
$\sqrt{AJ^2+AL^2} - JL = 0.00000$

$AD = 1.37010$

$AD - JL - AL = 0.00000$

$AE = 3.85346$

$AE - (JL + AL) = 0.00000$




$$\frac{AB}{AD} - \frac{2}{\sqrt{N_2^2+4 \cdot N_1+4} - N_2} = 0.00000$$

$$\frac{AE}{AC} - \frac{N_2+\sqrt{N_2^2+4 \cdot N_1+4}}{2 \cdot N_1+2} = 0.00000$$

$$\frac{AD}{AJ} - \frac{\sqrt{N_2^2+4 \cdot N_1+4} - N_2}{2 \cdot \sqrt{N_1+1}} = 0.00000$$

$$\frac{AJ}{AE} - \frac{2 \cdot \sqrt{N_1+1}}{N_2+\sqrt{N_2^2+4 \cdot N_1+4}} = 0.00000$$

$$\frac{AB}{AJ} - \frac{1}{\sqrt{N_1+1}} = 0.00000$$

$$\frac{AJ}{AC} - \frac{1}{\sqrt{N_1+1}} = 0.00000$$

**Unit.**

$AB := 1$

**Given.**

$N_1 := AB$

**081292**

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**

$$CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}$$

**Descriptions.**

$$BH := \sqrt{2\cdot AB^2} \qquad CG := \frac{AB^2}{BH}$$

$$AG := \sqrt{CG^2 + (AB + CG)^2} \qquad DG := CG\cdot\frac{2AB}{AG}$$

$$GJ := \sqrt{AB^2 - DG^2} \qquad AE := \frac{(AB + CG)\cdot(AG + GJ)}{AG}$$

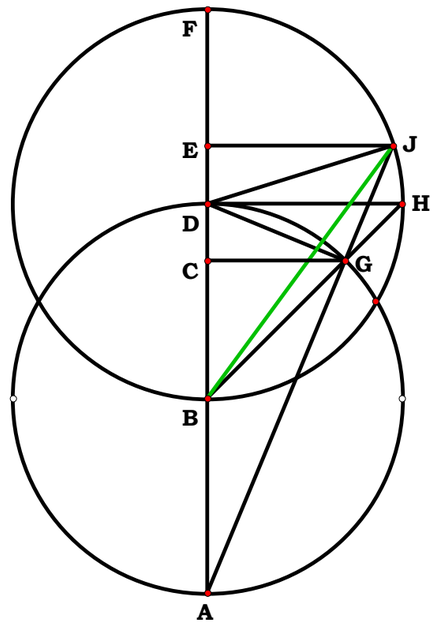
$$EJ := \frac{CG\cdot AE}{AB + CG} \qquad BJ := \sqrt{EJ^2 + (AE - AB)^2}$$

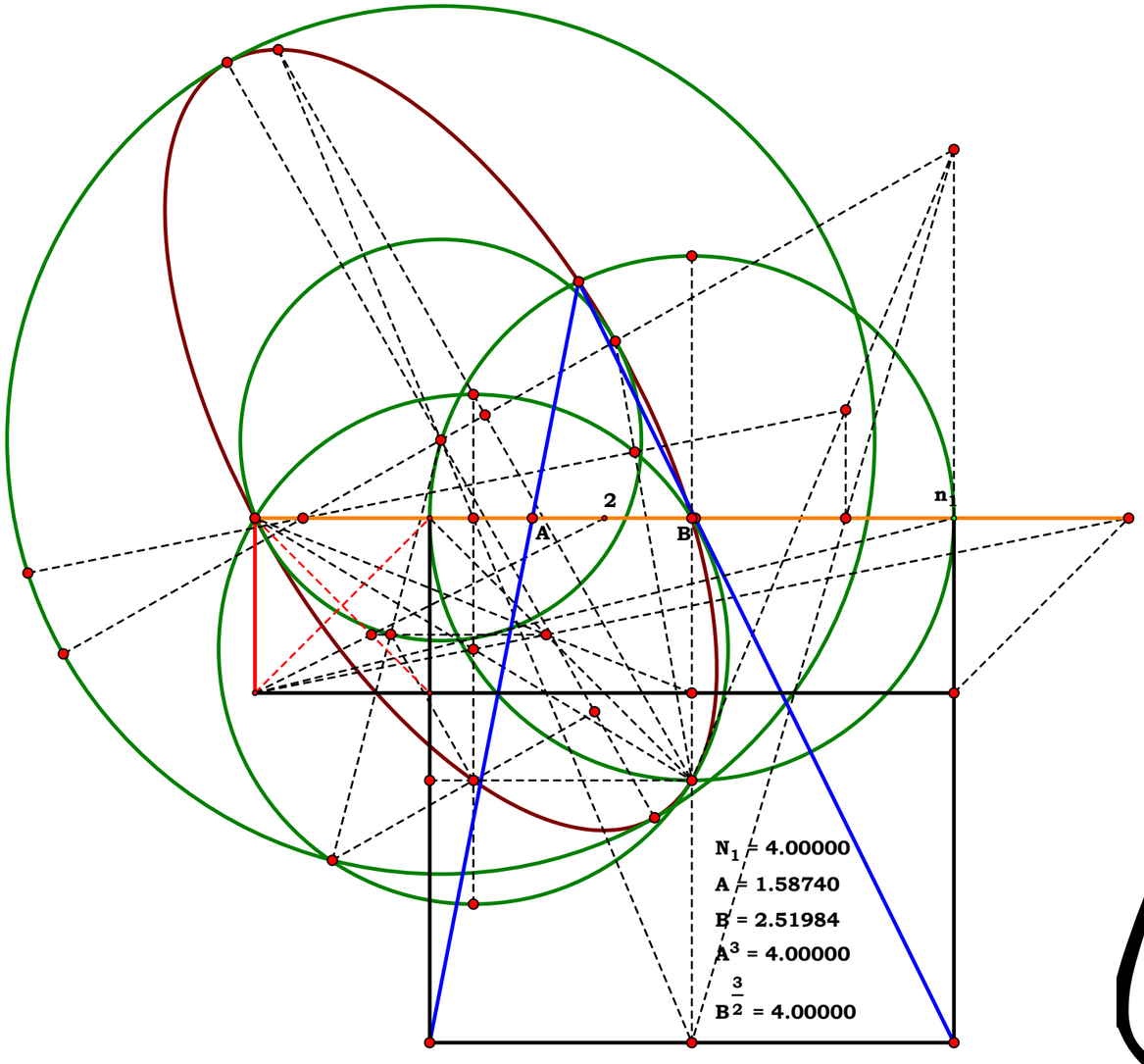
$$\frac{BJ}{\left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}} = 1.000943 \qquad BJ - \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}} = 0.00152$$

**Definitions.**

$$BJ - N_1\cdot\sqrt{\sqrt{2} + 2^{\frac{1}{4}}} = 0$$

**Rusty Cube of a Sphere**





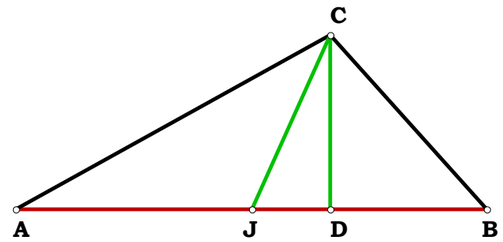

# The Delian Quest 1993

**John Clark**

010893A

## Pythagoras Revisited

$AB := 7.89517$
$AC := 6.02581$
$BC := 3.92697$



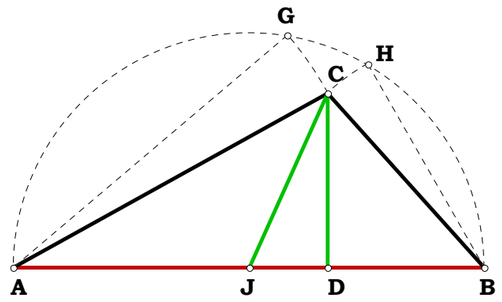

**Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.**

**Descriptions.**

$$AE := \frac{AC^2}{AB} \qquad BF := \frac{BC^2}{AB} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2}$$

$$AD := AE + DE \qquad BD := AB - AD \qquad CD := \sqrt{AC^2 - AD^2}$$

$$AJ := \frac{AB}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$$




$S_1 := AB$

$S_2 := AC$

$S_3 := BC$

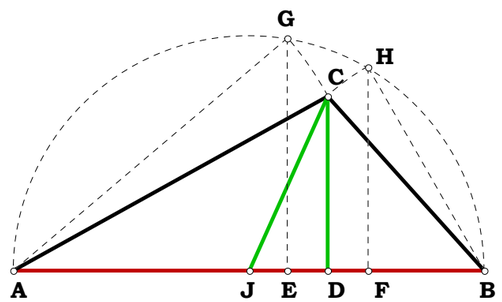
**Definitions.**

$$EF - \frac{AB^2 - AC^2 - BC^2}{AB} = 0 \qquad DE - \frac{AB^2 - AC^2 - BC^2}{2 \cdot AB} = 0$$

$$AD - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad BD - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$DJ - \frac{\sqrt{\left(AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \qquad CJ - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$

$$CD - \frac{\sqrt{[(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)]}}{2 \cdot AB} = 0$$




$$EF - \frac{S_1^2 - S_2^2 - S_3^2}{S_1} = 0 \qquad DE - \frac{S_1^2 - S_2^2 - S_3^2}{2 \cdot S_1} = 0$$

$$AD - \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^2 - S_2^2 + S_3^2}{2 \cdot S_1} = 0$$

$$DJ - \frac{\sqrt{\left(S_2^2 - S_3^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^2 - S_1^2 + 2 \cdot S_3^2}}{2} = 0$$

$$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0$$

010893B

Given.

$W := 6 \quad Y := 8$

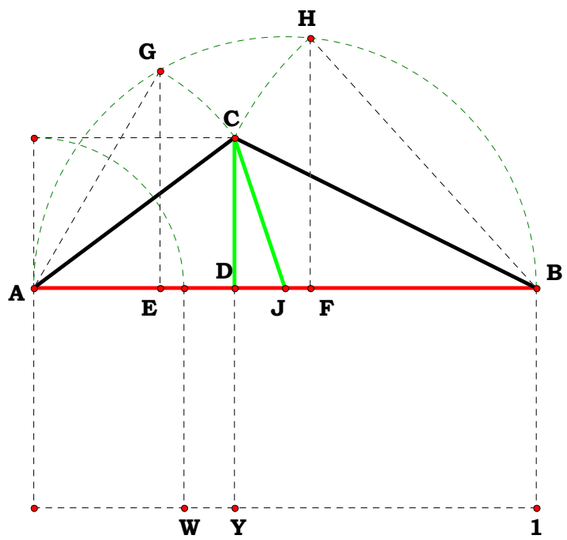
$X := 20 \quad Z := 15$

Unit.

$$AB := \frac{X}{X}$$

# Pythagoras Revisited

**Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.**




Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.40000
Y = 6.00000
Z = 15.00000
AC = 0.50000
BD = 0.60000
BC = 0.67082
AE = 0.25000
BF = 0.45000
EF = 0.30000
CJ = 0.31623

Descriptions.

$$AD := \frac{Y}{Z} \quad CD := \frac{W}{X} \quad AC := \sqrt{AD^2 + CD^2}$$

$$BD := AB - AD \quad BC := \sqrt{BD^2 + CD^2}$$

$$AE := \frac{AC^2}{AB} \quad BF := \frac{BC^2}{AB} \quad EF := AB - (AE + BF) \quad DE := \frac{EF}{2}$$

$$AJ := \frac{AB}{2} \quad DJ := AD - AJ \quad CJ := \sqrt{CD^2 + DJ^2}$$

Definitions.

$$AD - \frac{Y}{Z} = 0 \quad CD - \frac{W}{X} = 0 \quad AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$$

$$BD - \frac{Z - Y}{Z} = 0 \quad BC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{X \cdot Z} = 0$$

$$AE - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} = 0 \quad BF - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}{X^2 \cdot Z^2} = 0$$

$$EF - \frac{2 \cdot \left(X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0 \quad DE - \frac{\left(X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2\right)}{X^2 \cdot Z^2} = 0$$

$$AJ - \frac{1}{2} = 0 \quad DJ - \frac{2 \cdot Y - Z}{2 \cdot Z} = 0$$

$$CJ - \frac{\sqrt{4 \cdot W^2 \cdot Z^2 + 4 \cdot X^2 \cdot Y^2 - 4 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{2 \cdot X \cdot Z} = 0$$

$$\frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} - AC = 0.00000$$

$$\frac{Z - Y}{Z} - BD = 0.00000$$

$$\frac{\sqrt{((W^2 \cdot Z^2 + X^2 \cdot Y^2) - 2 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}}{X \cdot Z} - BC = 0.00000$$

$$\frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} - AE = 0.00000$$

$$\frac{((W^2 \cdot Z^2 + X^2 \cdot Y^2) - 2 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}{X^2 \cdot Z^2} - BF = 0.00000$$

$$\frac{2 \cdot (X^2 \cdot Y \cdot Z - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} - EF = 0.00000$$

$$\frac{\sqrt{((4 \cdot W^2 \cdot Z^2 + 4 \cdot X^2 \cdot Y^2) - 4 \cdot X^2 \cdot Y \cdot Z) + X^2 \cdot Z^2}}{2 \cdot X \cdot Z} - CJ = 0.00000$$

Unit is external.

Given.

$N_1 := 3$

$N_2 := 4$

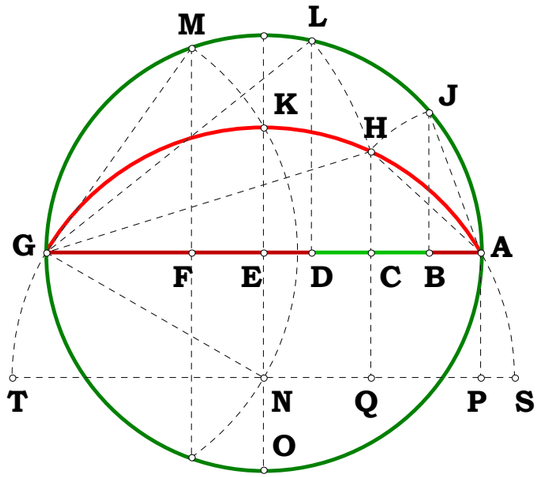
**The curve AK is derived from the cube root figure as demonstrated.**

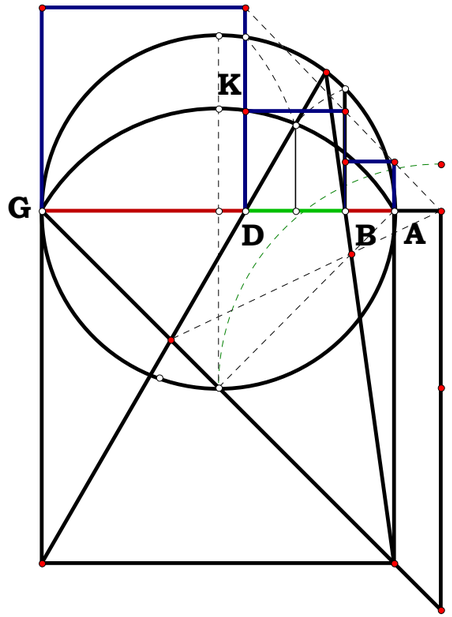
**Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?**
**Divide a segment twice such that the mean segment is the root of the extreems.**

060393A

Descriptions.

## Exploring The Curve AK




$AG := N_1 \qquad AC := \frac{AG}{N_2}$

$GF := \frac{AG}{3} \qquad FM := \sqrt{GF \cdot (AG - GF)}$

$GM := \sqrt{GF^2 + FM^2} \qquad ST := 2 \cdot GM \qquad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$

$PS := \frac{ST - AG}{2} \qquad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN \qquad AH := \sqrt{AC^2 + CH^2} \qquad GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \frac{AH^2}{AG} \qquad DG := \frac{GH^2}{AG} \qquad BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0 \qquad AB = 0.348612 \qquad BD = 0.802776 \qquad DG = 1.848612$

Definitions.

$AC - \frac{N_1}{N_2} = 0 \qquad GF - \frac{N_1}{3} = 0 \qquad FM - \frac{\sqrt{2} \cdot N_1}{3} = 0 \qquad GM - \frac{\sqrt{3} \cdot N_1}{3} = 0 \qquad ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0 \qquad EN - \frac{N_1}{\sqrt{12}} = 0$

$PS - \frac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0 \qquad HQ - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12}}{N_2 \cdot \sqrt{12}} = 0 \qquad CH - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0 \qquad AH - \frac{N_1 \cdot \sqrt{N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6}}{\sqrt{6 \cdot N_2}} = 0$

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0 \qquad AB - \frac{N_1 \cdot \left(N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6\right)}{6 \cdot N_2} = 0 \qquad DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{6 \cdot N_2} = 0$

$BD - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0 \qquad BD^2 - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0 \qquad AB \cdot DG - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$

Unit.

$AG := 1$

Given.

$N_1 := 4$

060393B

**The curve AK is derived from the cube root figure as demonstrated.**

**Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?**
**Divide a segment twice such that the mean segment is the root of the extreems.**

## Exploring The Curve AK

Descriptions.

$$AC := \frac{AG}{N_1} \qquad GF := \frac{AG}{3} \qquad FM := \sqrt{GF \cdot (AG - GF)}$$

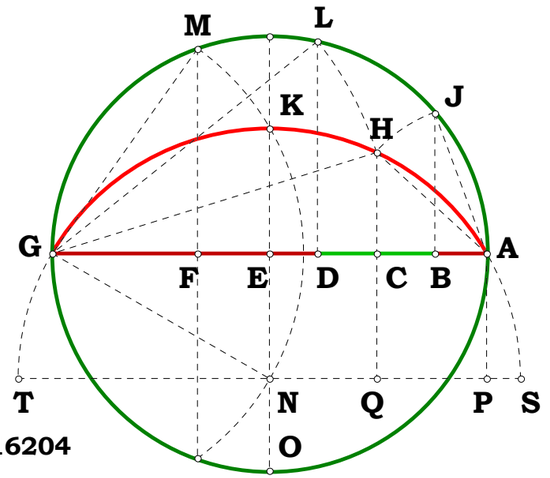
$$GM := \sqrt{GF^2 + FM^2} \qquad ST := 2 \cdot GM \qquad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$$

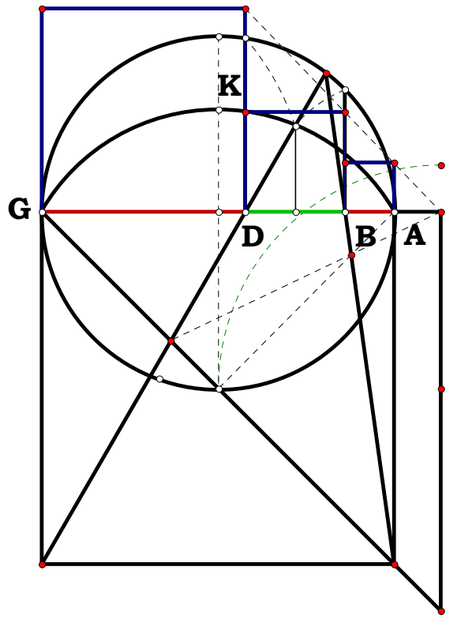
$$PS := \frac{ST - AG}{2} \qquad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$$

$$CH := HQ - EN \qquad AH := \sqrt{AC^2 + CH^2} \qquad GH := \sqrt{(AG - AC)^2 + CH^2}$$

$$AB := \frac{AH^2}{AG} \qquad DG := \frac{GH^2}{AG} \qquad BD := AG - (AB + DG)$$

$$BD - \sqrt{AB \cdot DG} = 0 \qquad AB = 0.116204 \qquad BD = 0.267592 \qquad DG = 0.616204$$




Definitions.

$$AC - \frac{1}{N_1} = 0 \qquad GF - \frac{1}{3} = 0 \qquad FM - \frac{\sqrt{2}}{\sqrt{9}} = 0 \qquad GM - \frac{1}{\sqrt{3}} = 0 \qquad ST - \frac{2 \cdot \sqrt{3}}{3} = 0$$

$$EN - \frac{1}{\sqrt{12}} = 0 \qquad PS - \left(\frac{\sqrt{3}}{3} - \frac{1}{2}\right) = 0 \qquad HQ - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12}}{\sqrt{12} \cdot N_1} = 0 \qquad CH - \frac{\left(\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1\right) \cdot \sqrt{3}}{6 \cdot N_1} = 0$$

$$AH - \frac{\sqrt{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}}{\sqrt{6 \cdot N_1}} = 0 \qquad GH - \frac{\sqrt{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}}{\sqrt{6 \cdot N_1}} = 0 \qquad AB - \frac{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}{6 \cdot N_1} = 0$$

$$DG - \frac{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}{6 \cdot N_1} = 0 \qquad BD - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1}{3 \cdot N_1} = 0 \qquad BD - \frac{\sqrt{2 \cdot \left(N_1^2 + 6 \cdot N_1 - 6\right) - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 12 \cdot N_1 - 12}}}{3 \cdot N_1} = 0$$

060793A

Unit is external.

Given.

$AD := 2.17506$ $AB := 3.14654$ $AC := 1.74732$

$BD := 2.61333$ $CD := 1.38168$

## Two Triangles with a Common Side.

Descriptions.

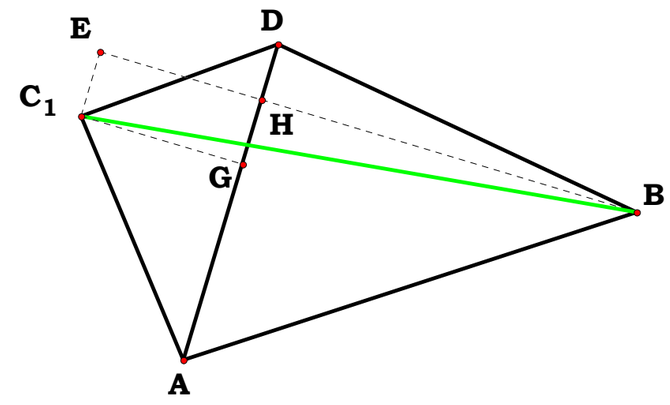
Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD + AB + BD)\cdot(-AD + AB + BD)\cdot(AD - AB + BD)(AD + AB - BD)}}{2\cdot AD}$$

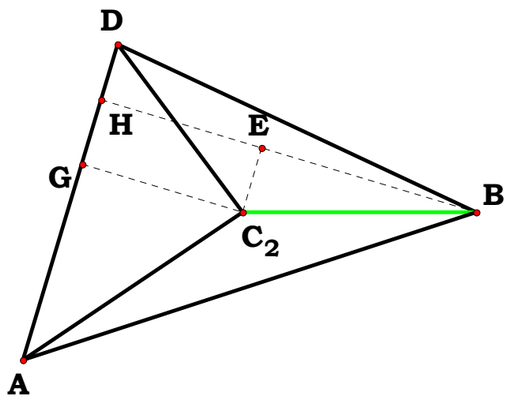
Greatest Disance:




$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Least Distance




$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + AB^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}\ \ldots}{2\cdot AD} = 0$$

$$+\ -AD^4 - AB^2\cdot AC^2 + AB^2\cdot AD^2 + AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2 + AB^2\cdot AD^2 - AD^4 - AB^2\cdot AC^2}\ \ldots}{2\cdot AD} = 0$$

$$+\ -\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}$$

060793B

**Unit**

$AB := 1$

**Given.**

$AD := 2.17506$ $\quad AC := 1.74732$

$BD := 2.61333$ $\quad CD := 1.38168$

**Descriptions.**

**Let the two triangles ABD and ACD be given.**

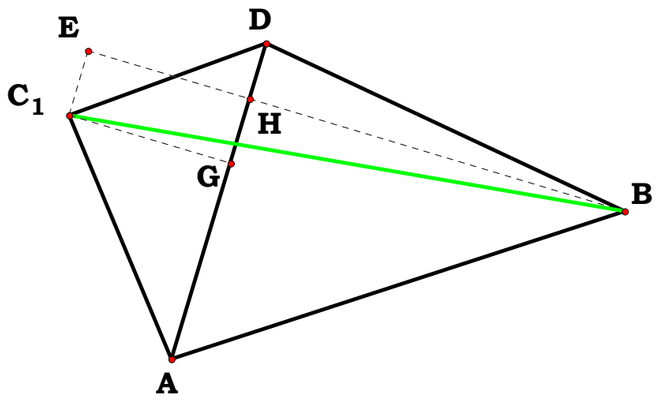
**Given two triangles with a common side, find the difference between their free vertices from opposing sides.**

## Two Triangles with a Common Side.

$$CG := \frac{\sqrt{(AD+CD+AC)\cdot(-AD+CD+AC)\cdot(AD-CD+AC)(AD+CD-AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD+BD-1)\cdot(AD+BD+1)\cdot(AD-BD+1)\cdot(BD-AD+1)}}{2\cdot AD}$$

**Greatest Disance:**



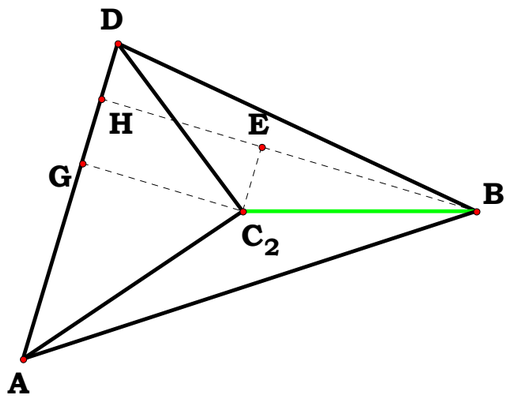

$$BC_1 := \sqrt{GH^2 + (CG+BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH-CG)^2}$$

**Least Distance**

$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + 1^2 - BD^2}{2\cdot AD}$$

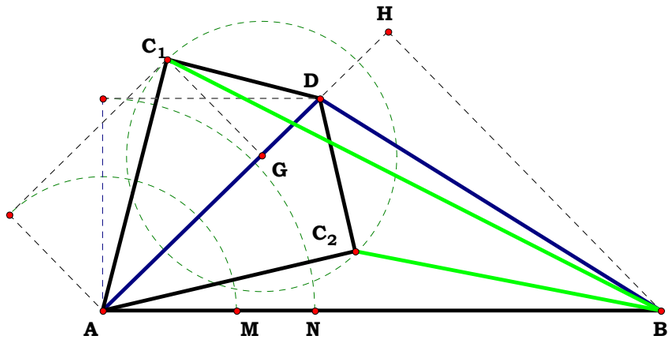
$$GH := AH - AG$$




**Definitions.**

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{(AC\cdot AD)^2 + (AC\cdot BD)^2 - AC^2 - AD^4 + (AD\cdot BD)^2 + (AD\cdot CD)^2 - (BD\cdot CD)^2 + AD^2 + CD^2 \ldots + \sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)\cdot(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{(AC\cdot AD)^2 + (AC\cdot BD)^2 - AC^2 - AD^4 + (AD\cdot BD)^2 + (AD\cdot CD)^2 - (BD\cdot CD)^2 + AD^2 + CD^2 \ldots + -\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$




Unit = 1.00000
XY = 0.24024
X = 3.20765
Y = 13.35191

XY = 0.38041
X = 7.60821
Y = 20.00000

AB = 9.36083 cm
AD = 5.09446 cm
$AC_1$ = 4.34404 cm
BD = 6.73584 cm
$C_1D$ = 2.63736 cm
AG = 3.71664 cm
AH = 6.69425 cm
$\frac{AB}{AB}$ = 1.00000
$\frac{AD}{AB}$ = 0.54423
$\frac{AC_1}{AB}$ = 0.46407
$\frac{BD}{AB}$ = 0.71958
$\frac{C_1D}{AB}$ = 0.28174
$\frac{AG}{AB}$ = 0.39704
$\frac{AH}{AB}$ = 0.71513

$BC_1$ = 9.28248 cm
$BC_2$ = 5.22560 cm

(1+AD)-BD = 0.82465
(1-AD)+BD = 1.17535
(AD-1)+BD = 0.26381
1+AD+BD = 2.26381

(AC+AD)-CD = 0.72655
(AC-AD)+CD = 0.20158
(AD-AC)+CD = 0.36191
AC+AD+CD = 1.29004

AB = 1.00000
AD = 0.54423
AC = 0.46407
CD = 0.28174
BD = 0.71958
AG = 0.39704
AH = 0.71513

$\frac{(AD^2+AC^2)-CD^2}{2 \cdot AD}$ -AG = 0.00000

$\frac{(AD^2+1)-BD^2}{2 \cdot AD}$ -AH = 0.00000

$BC_1$ = 0.99163

$BC_2$ = 0.55824

$$\frac{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2}{\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}} = 0.38355 \div 0.19895$$

$$\frac{\sqrt{2} \cdot \sqrt{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2)+\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}}{2 \cdot AD} - BC_1 = 0.00000$$

$$\frac{\sqrt{2} \cdot \sqrt{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2)-\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}}{2 \cdot AD} - BC_2 = 0.00000$$

060793C

Unit

$AB := 1$

Given.

$AD := 2.17506$ $AC := 1.74732$

$BD := 2.61333$ $CD := 1.38168$

## Two Triangles with a Common Side.

Descriptions.

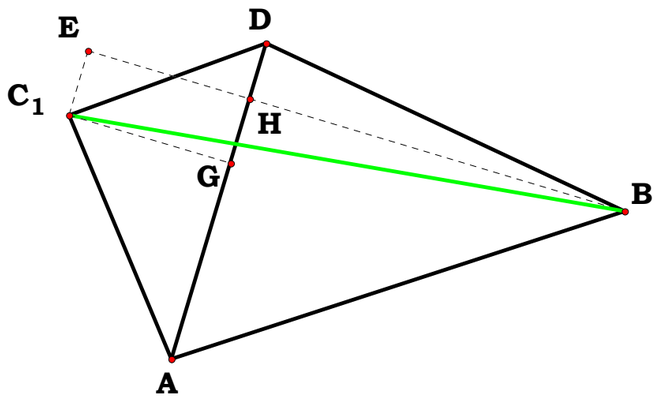
Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD + BD - 1)\cdot(AD + BD + 1)\cdot(AD - BD + 1)\cdot(BD - AD + 1)}}{2\cdot AD}$$

Greatest Disance:



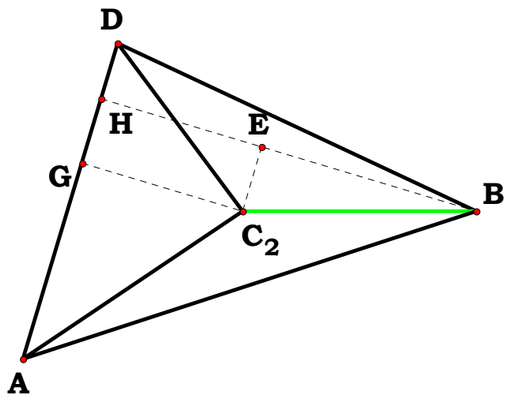

$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Least Distance

$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + 1^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$




Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(1 + AD - BD)\cdot(1 - AD + BD)\cdot(AD - 1 + BD)\cdot(1 + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}\ \ldots}{2\cdot AD} = 0$$

$$+\ AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2\ \ldots}}{2\cdot AD} = 0$$

$$+\ -\sqrt{(1 + AD - BD)\cdot(1 - AD + BD)\cdot(AD - 1 + BD)\cdot(1 + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}$$

060793D

Unit

AB := 1

Given.

Z := 10 U := 5

V := 2 W := 3 X := 7

Descriptions.

Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$AM := \frac{U}{Z} \quad AN := \frac{V}{Z} \quad AO := \frac{W}{Z} \quad AP := \frac{X}{Z} \quad AD := \sqrt{AN^2 + AM^2}$$

$$BN := AB - AN \quad BD := \sqrt{BN^2 + AM^2} \quad AC := \sqrt{AP^2 + AO^2}$$

$$DH := AP - AD \quad CD := \sqrt{DH^2 + AO^2} \quad AG := \frac{AD^2 + AB^2 - BD^2}{2 \cdot AD}$$

$$BG := \frac{\sqrt{(AD + BD - AB) \cdot (AD - BD + AB) \cdot (BD - AD + AB) \cdot (AD + AB + BD)}}{2 \cdot AD}$$

$$BJ := BG + AO \quad BF := BG - AO \quad AH := AD + DH \quad CJ := AH - AG$$

$$BC := \sqrt{BJ^2 + CJ^2} \quad BE := \sqrt{BF^2 + CJ^2}$$

Definitions.

$$AM - \frac{U}{Z} = 0 \quad AN - \frac{V}{Z} = 0 \quad AO - \frac{W}{Z} = 0 \quad AP - \frac{X}{Z} = 0 \quad AD - \frac{\sqrt{U^2 + V^2}}{Z} = 0$$

$$BN - \frac{Z - V}{Z} = 0 \quad BD - \frac{\sqrt{U^2 + V^2 - 2 \cdot V \cdot Z + Z^2}}{Z} = 0 \quad AC - \frac{\sqrt{W^2 + X^2}}{Z} = 0 \quad DH - \frac{X - \sqrt{U^2 + V^2}}{Z} = 0 \quad CD - \frac{\sqrt{U^2 + V^2 + W^2 + X^2 - 2 \cdot X \cdot \sqrt{U^2 + V^2}}}{Z} = 0$$

$$AG - \frac{2 \cdot V \cdot Z}{2 \cdot Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BG - \frac{U}{\sqrt{U^2 + V^2}} = 0 \quad BJ - \frac{U \cdot Z + W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BF - \frac{U \cdot Z - W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad AH - \frac{X}{Z} = 0 \quad CJ - \frac{X \cdot \sqrt{U^2 + V^2} - V \cdot Z}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

$$BC - \frac{\sqrt{2 \cdot Z \cdot (U \cdot W - V \cdot X) \cdot \sqrt{U^2 + V^2} + (W^2 + X^2 + Z^2) \cdot (U^2 + V^2)}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BE - \frac{\sqrt{(W^2 + X^2 + Z^2) \cdot (U^2 + V^2) - 2 \cdot Z \cdot (U \cdot W + V \cdot X) \cdot \sqrt{U^2 + V^2}}}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

## Two Triangles with a Common Side.

Unit = 1.00000
M = 0.50000
U = 5.00000
Z = 10.00000

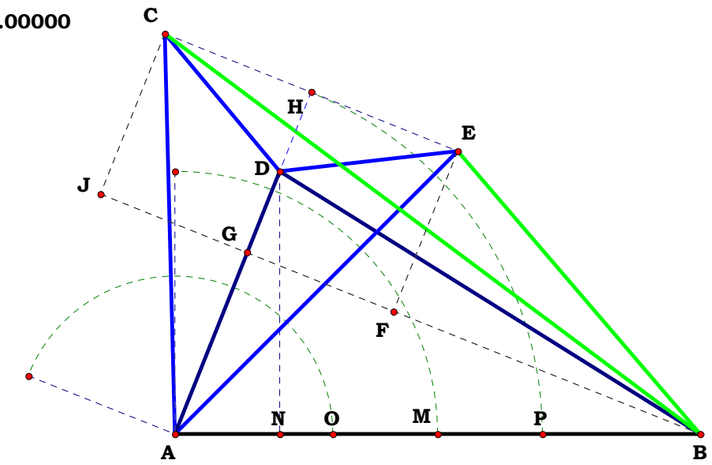
N = 0.20000
V = 2.00000

O = 0.30000
W = 3.00000

P = 0.70000
X = 7.00000

AB = 1.00000
AC = 0.76158
AD = 0.53852
BD = 0.94340
CD = 0.34070
AG = 0.37139

BG = 0.92848
BJ = 1.22848
BF = 0.62848
BC = 1.27167
BE = 0.70920



**060993A Rectangular Roots.**

**Given.**

$AB := 3$

$AC := 1$

**Given any value $N_1$, any other value, $N_2$, greater than twice the square root of $N_1$ can be divided such that the resulting pair of values equals $N_1$.**

**Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.**

**Descriptions.**

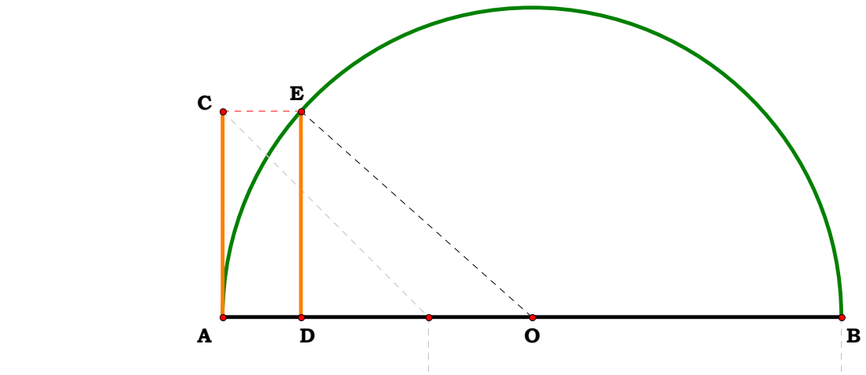
$DE := AC \qquad EO := \frac{AB}{2} \qquad DO := \sqrt{EO^2 - DE^2} \qquad BD := EO + DO$

$AD := AB - BD \qquad AD \cdot BD - AC^2 = 0$

**Definitions.**

$AD - \frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad AD = 0.381966$

$BD - \frac{AB + \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad BD = 2.618034 \qquad AD + BD = 3$



Unit.

$AB := 1$

Given.

$Y := 20$

$X := 8$

**060993B Rectangular Roots.**

**Descriptions.**

$$AC := \frac{X}{Y} \qquad DE := AC \qquad EO := \frac{AB}{2}$$

$$DO := \sqrt{EO^2 - DE^2} \qquad BD := EO + DO$$

$$AD := AB - BD \qquad AD \cdot BD - AC^2 = 0$$

**Definitions.**

$$AC - \frac{X}{Y} = 0 \qquad DE - \frac{X}{Y} = 0 \qquad EO - \frac{1}{2} = 0$$

$$DO - \frac{\sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0 \qquad BD - \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$AD - \frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$\frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} \cdot \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} - AC^2 = 0 \qquad \frac{X^2}{Y^2} - AC^2 = 0$$

**Given any value $N_1$, any other value, $N_2$, greater than twice the square root of $N_1$ can be divided such that the resulting pair of values equals $N_1$.**

**Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.**

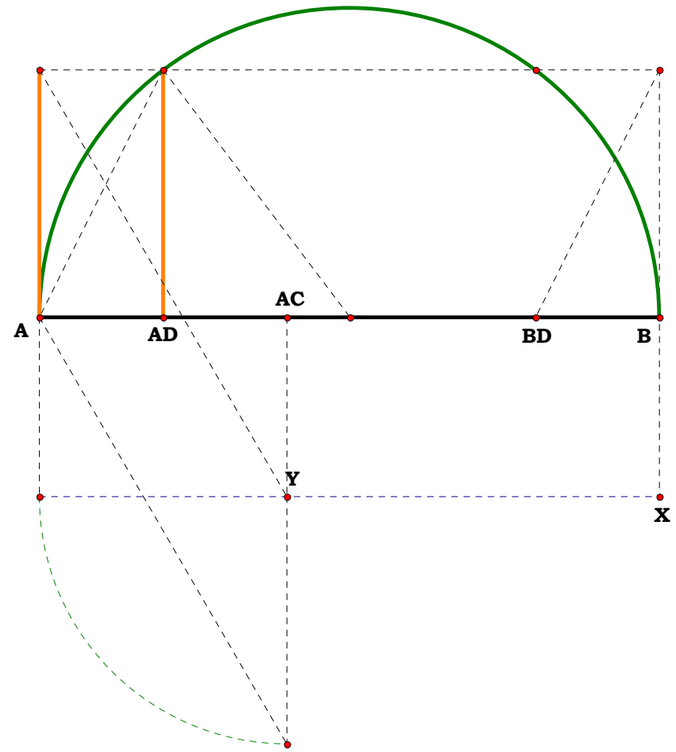
XY = 0.40000
X = 8.00000
Y = 20.00000

AD = 0.20000
AC = 0.40000
BD = 0.80000
AB = 1.00000

Unit = 1.00000

$\frac{Y-\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y}$-AD = 0.00000

$\frac{Y+\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y}$-BD = 0.00000



062193A Pyramid of Ratios I

## Pyramid of Rations 1

**If you just draw the figure, and measure and compare the numbers, you start to see it. Now one should simply take it and all its treasures.**

**Definitions for the unit we use for division. This will give us a Cardinal result. The buttons will put the N1 or 2 on a number, but you have to define its actual ratio to the figure.**

Hide Points

| | |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |
| 7 | 7 |
| 8 | 8 |
| 9 | 9 |
| 10 | 10 |

Unit = 1.00000

X = 0.40000

Y = 0.30000

01 = 2.32038 cm

$ON_1$ = 9.28153 cm

$ON_2$ = 6.96115 cm

$\frac{ON_1}{01}$ = 4.00000

$N_1$ = 4.00000

$\frac{ON_2}{01}$ = 3.00000

$N_2$ = 3.00000

AC = 16.40759 cm

AF = 6.56304 cm

$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1}$ = 2.50000

$\frac{AC}{AF}$ = 2.50000

$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} - \frac{AC}{AF}$ = 0.00000

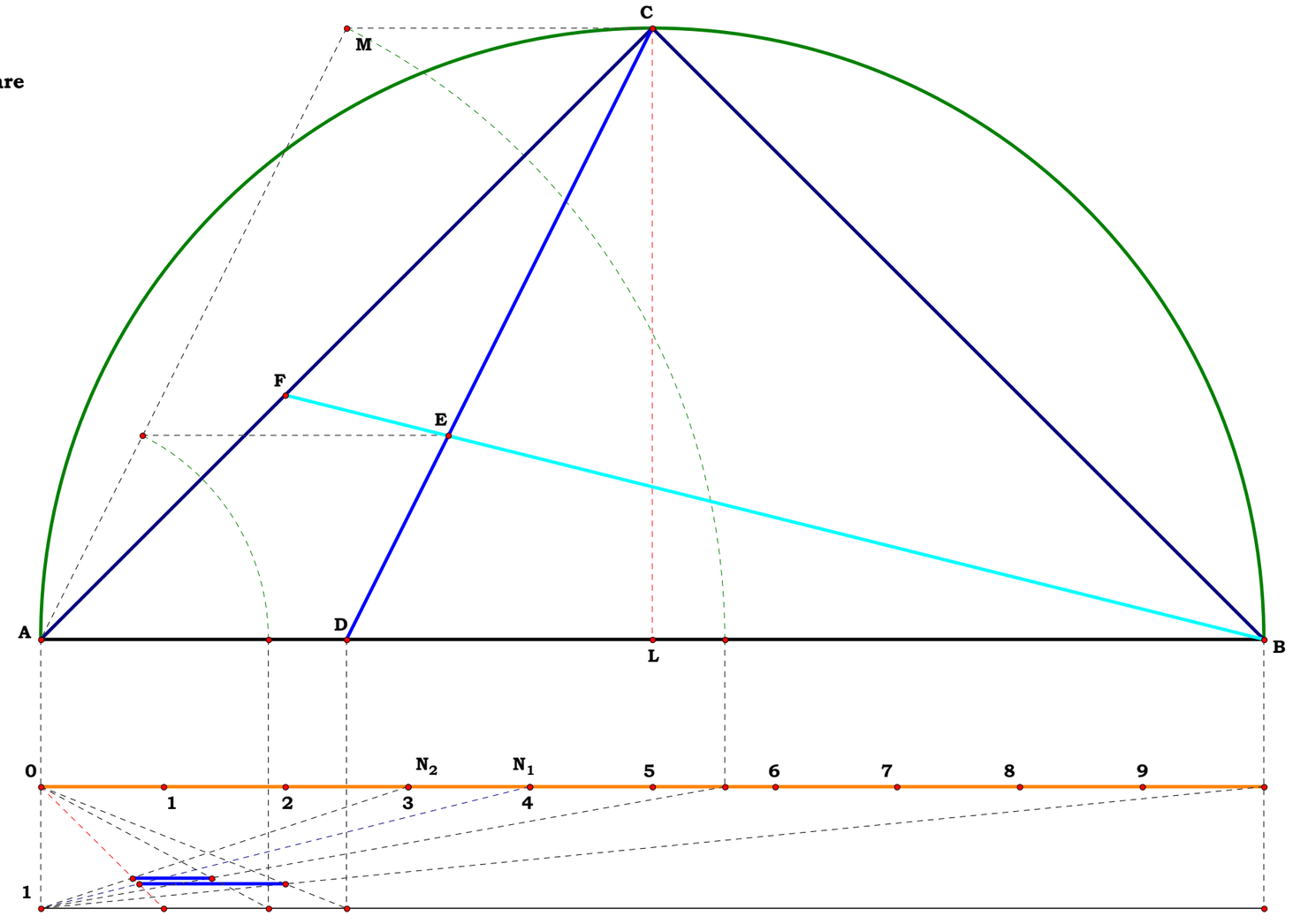
BF = 19.13437 cm

EF = 3.18906 cm

$\frac{BF}{EF}$ = 6.00000

$\frac{N_1 \cdot N_2}{N_2 - 1}$ = 6.00000

$\frac{N_1 \cdot N_2}{N_2 - 1} - \frac{BF}{EF}$ = 0.00000



**062193B**

**Unit.**
**Given.**

## Pyramid of Ratios I

**Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?**

**Descriptions.**
**Definitions.**

$N1 := 3 \quad N2 := 5 \quad \delta := 1 .. N2$

$\delta =$

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

$AB := 1 \quad AD := \frac{AB}{N1} \quad AL := \frac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

$DE_\delta := \frac{CD \cdot \delta}{N2} \quad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right) =$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\text{if}\left(N2 - \delta, \frac{N1 \cdot N2}{N2 - \delta}, 0\right) =$

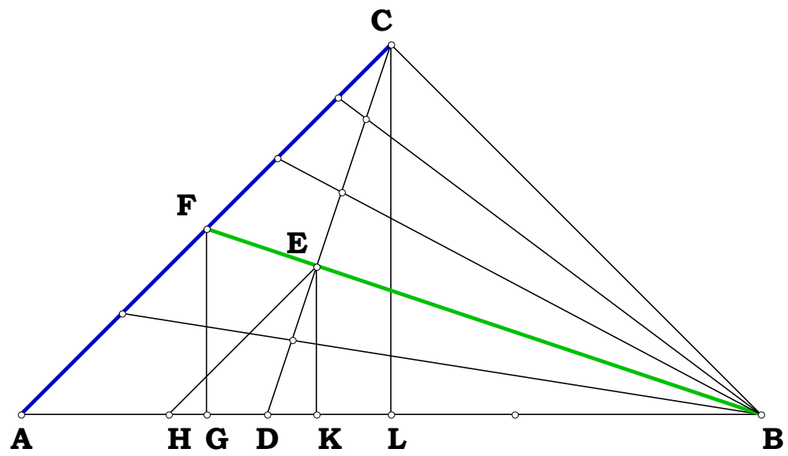
| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\frac{AC}{AF_\delta} =$

| |
|---|
| 3.666667 |
| 2 |
| 1.444444 |
| 1.166667 |
| 1 |

$\frac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta} =$

| |
|---|
| 3.666667 |
| 2 |
| 1.444444 |
| 1.166667 |
| 1 |




**This was my original write up, long ago, and it takes advantage of some functions of Mathcad. For the final product of The Delian Quest, it will become a series of demonstrations. However, the root figure does not change, nor what it does.**

**I never had Geometry in school, nor did I ever do well in algebra because like everything else, those who teach it really do not know what in the hell they are doing. Too much mythology, and too much insistence on traditional carping. I am a simple person and I like things simple, honest, and organized.**

062193C

**Descriptions.**

Unit.

$AB := 1$

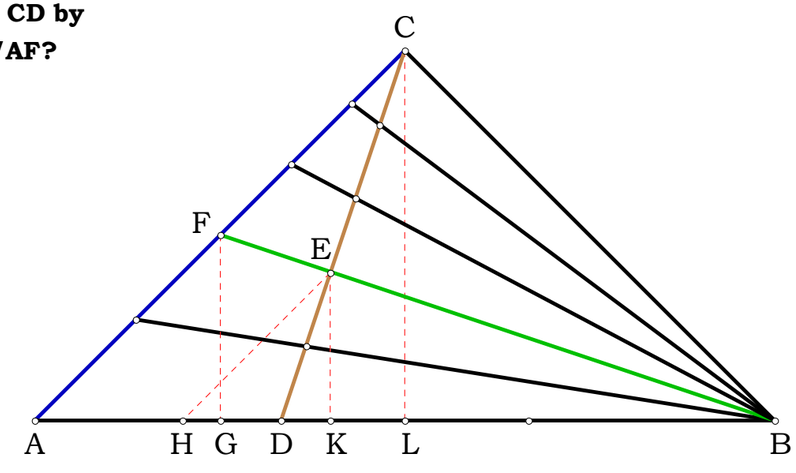
Given.

$N_1 := 4$

$N_2 := 2$

## Pyramid of Ratios I

**Divide AB by $N_1$ then divide CD by $N_2$, what are BF/EF and AC/AF?**




$AD := \frac{AB}{N_1}$ $AL := \frac{AB}{2}$ $AC := \sqrt{2 \cdot AL^2}$ $CL := AL$ $DL := AL - AD$

$CD := \sqrt{DL^2 + CL^2}$ $DE := \frac{CD}{N_2}$ $EK := \frac{CL \cdot DE}{CD}$ $DK := \frac{DL \cdot EK}{CL}$

$AK := AD + DK$ $BK := AB - AK$ $BE := \sqrt{BK^2 + EK^2}$

$HK := \frac{AL \cdot EK}{CL}$ $BH := BK + HK$ $EH := \frac{AC \cdot EK}{CL}$ $AF := \frac{EH \cdot AB}{BH}$

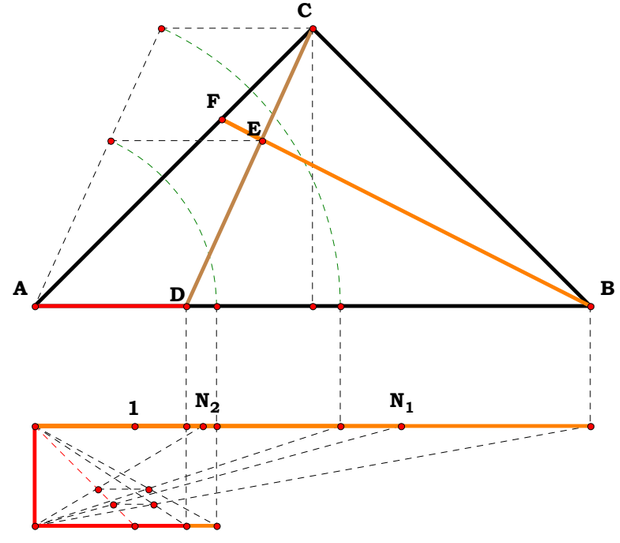
$BF := \frac{BE \cdot AB}{BH}$ $EF := BF - BE$

$\frac{BF}{EF} = 8$ $\frac{AC}{AF} = 1.75$ $\frac{CD}{DE} = 2$

**Definitions.**

$\frac{AC}{AF} - \frac{N_1 \cdot N_2 - N_2 + 1}{N_1} = 0$

$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0$




$\frac{BF}{EF} = 9.08799$ $\frac{N_1 \cdot N_2}{N_2 - 1} = 9.08799$

$\frac{AC}{AF} = 1.49057$ $\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.49057$

$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0.00000$ $\frac{AC}{AF} - \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 0.00000$

062193D Pyramid of Ratios I

# Pyramid of Ratios I

**The buttons on the macro are divided into two columns, one for the number of divisions I want for a given unit, and the other a particular point in that range. This means that if I set the points equal to what I am working with, I do not have to draw anything further to play with the line to examine it. Thus, althoght N1 and N2 are working with two different lengths of line, one always constant and the other variable, it is transparent in the the equation. We are dividing each of them in a Cardinal fashion, just rendering an arithmetic name for its point in a ratio of that particular line. And, this allows me to write up the equations for the figure which reduce, in the end, to w, x, y and z and always giving an exact answer no matter what original system of measurement one started with, inches, meters, or pixels, or even mystical particals of quantum dust. A thing, is a thing, is a thing, which is always independent of any convention of names and what one is looking for, is a simple, universal method of notation without using Einstein's rubbrer bands and crazy glue.**

**Just the fact that one is working with four names in the equation tells me that I have two different things using two different systems of measurement, each one bringing itself into the equation as it is.**

Y -> 1 … Y -> 20

X -> 0 … X -> 20

Show Points

Y -> 1 … Y -> 20

X -> 0 … X -> 20

Show Points

$$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} = 1.53333$$

$$\frac{X \cdot (W-Z)+Y \cdot Z}{Y \cdot W} = 1.53333$$

$$\frac{AC}{AF} = 1.53333$$

$$\frac{N_1 \cdot N_2}{N_2 - 1} = 12.50000$$

$$\frac{Y \cdot Z}{X \cdot (Z-W)} = 12.50000$$

$$\frac{BF}{EF} = 12.50000$$

AC = 7.80151 cm

AF = 5.08794 cm

BF = 8.25996 cm

EF = 0.66080 cm

Unit = 1.00000

X = 4.00000

Y = 20.00000

$$\frac{Y}{X} = 5.00000$$

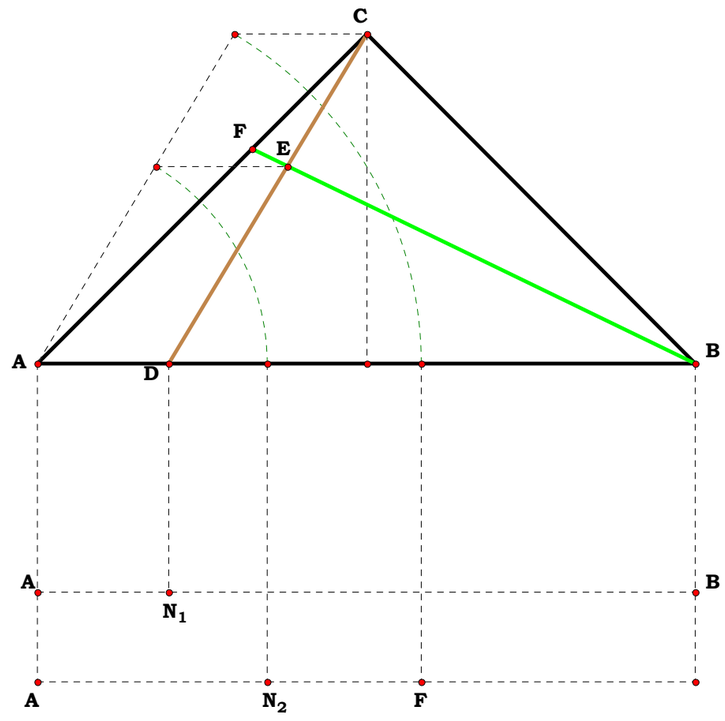
$N_1$ = 5.00000

W = 6.00000

Z = 10.00000

$$\frac{Z}{W} = 1.66667$$

$N_2$ = 1.66667



062193E

**Unit.**

$AB := 1$

**Given.**

$W := 6$

$X := 4$

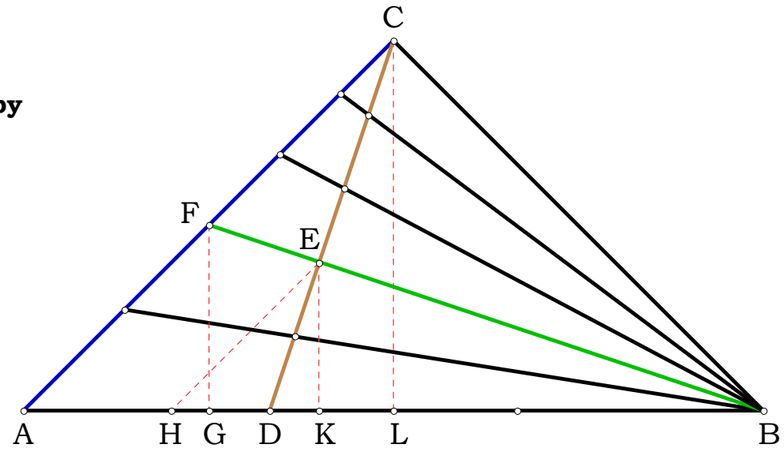
$Y := 20$

$Z := 10$

**Descriptions.**

## Pyramid of Ratios I

**Divide AB by $N_1$ then divide CD by $N_2$, what are BF/EF and AC/AF?**




$$N_1 := \frac{Y}{X} \qquad N_2 := \frac{Z}{W}$$

$$AD := \frac{AB}{N_1} \qquad AL := \frac{AB}{2} \qquad AC := \sqrt{2 \cdot AL^2} \qquad CL := AL \qquad DL := AL - AD$$

$$CD := \sqrt{DL^2 + CL^2} \qquad DE := \frac{CD}{N_2} \qquad EK := \frac{CL \cdot DE}{CD} \qquad DK := \frac{DL \cdot EK}{CL}$$

$$AK := AD + DK \qquad BK := AB - AK \qquad BE := \sqrt{BK^2 + EK^2}$$

$$HK := \frac{AL \cdot EK}{CL} \qquad BH := BK + HK \qquad EH := \frac{AC \cdot EK}{CL} \qquad AF := \frac{EH \cdot AB}{BH}$$

$$BF := \frac{BE \cdot AB}{BH} \qquad EF := BF - BE$$

$$\frac{BF}{EF} = 12.5 \qquad \frac{AC}{AF} = 1.533333 \qquad \frac{CD}{DE} = 1.666667$$

$$\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.53333$$

$$\frac{X \cdot (W - Z) + Y \cdot Z}{Y \cdot W} = 1.53333$$

$$\frac{AC}{AF} = 1.53333$$

$$\frac{N_1 \cdot N_2}{N_2 - 1} = 12.50000$$

$$\frac{Y \cdot Z}{X \cdot (Z - W)} = 12.50000$$

$$\frac{BF}{EF} = 12.50000$$

AC = 7.80151 cm

AF = 5.08794 cm

BF = 8.25996 cm

EF = 0.66080 cm

Unit = 1.00000

X = 4.00000

Y = 20.00000

$\frac{Y}{X} = 5.00000$

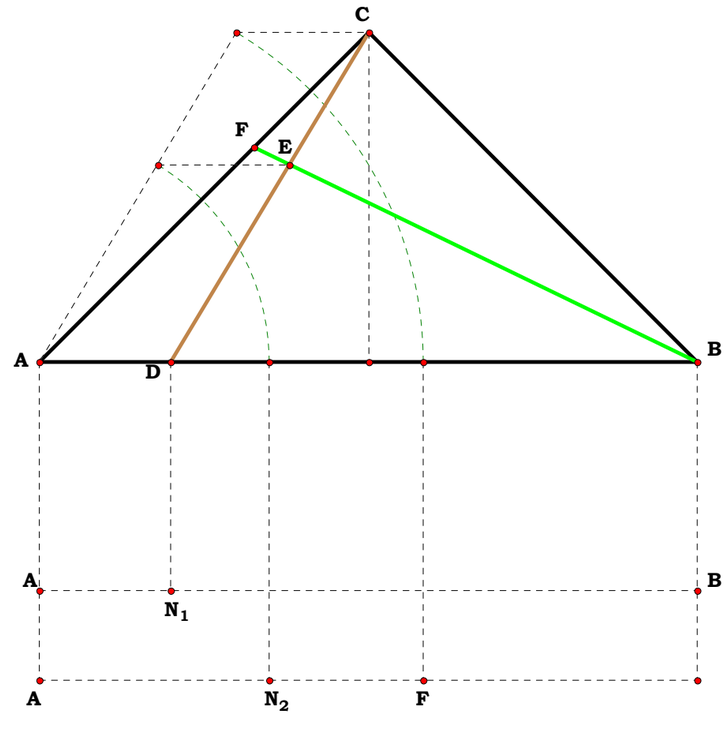
$N_1 = 5.00000$

W = 6.00000

Z = 10.00000

$\frac{Z}{W} = 1.66667$

$N_2 = 1.66667$



**Definitions.**

$$\frac{AC}{AF}-\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}=0 \qquad \frac{BF}{EF}-\frac{Y\cdot Z}{X\cdot(Z-W)}=0$$

$$N_1-\frac{Y}{X}=0 \quad N_2-\frac{Z}{W}=0 \quad AD-\frac{X}{Y}=0 \quad AL-\frac{1}{2}=0 \quad AC-\frac{1}{\sqrt{2}}=0$$

$$CL-\frac{1}{2}=0 \quad DL-\frac{Y-2\cdot X}{2\cdot Y}=0 \quad CD-\frac{\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{\sqrt{2}\cdot Y}=0 \quad DE-\frac{\sqrt{2}\cdot W\cdot\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{2\cdot Y\cdot Z}=0$$

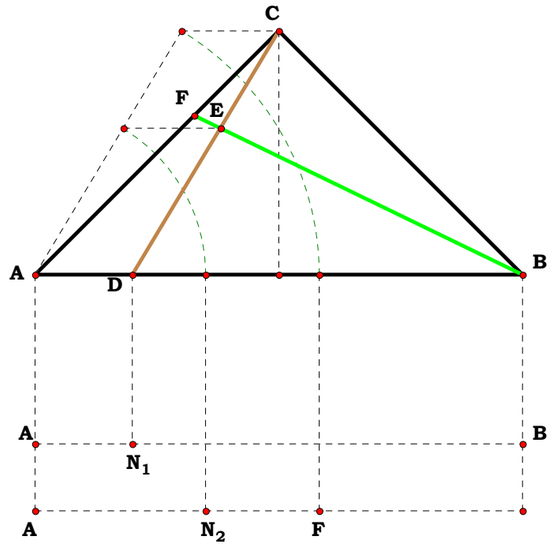
$$EK-\frac{W}{2\cdot Z}=0 \quad DK-\frac{W\cdot(Y-2\cdot X)}{2\cdot Y\cdot Z}=0 \quad AK-\frac{[W\cdot(Y-2\cdot X)+2\cdot X\cdot Z]}{2\cdot Y\cdot Z}=0 \quad BK-\frac{2\cdot(W\cdot X+Y\cdot Z)-(W\cdot Y+2\cdot X\cdot Z)}{2\cdot Y\cdot Z}=0$$

$$BE-\frac{\sqrt{2\cdot(X-Y)^2\cdot Z^2+[2\cdot W\cdot(Y-X)\cdot(2\cdot X-Y)]\cdot Z+W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)}}{\sqrt{2}\cdot Y\cdot Z}=0 \quad HK-\frac{W}{2\cdot Z}=0$$

$$BH-\frac{W\cdot X-X\cdot Z+Y\cdot Z}{Y\cdot Z}=0 \qquad EH-\frac{\sqrt{2}\cdot W}{2\cdot Z}=0 \qquad AF-\frac{\sqrt{2}\cdot W\cdot Y}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$BF-\frac{\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$EF-\frac{X\cdot(W-Z)\cdot\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot Y\cdot Z\cdot(X\cdot Z-W\cdot X-Y\cdot Z)}=0$$




Unit = 1.00000
X = 4.00000
Y = 20.00000
$\frac{Y}{X}$ = 5.00000
$N_1$ = 5.00000
W = 6.00000
Z = 10.00000
$\frac{Z}{W}$ = 1.66667
$N_2$ = 1.66667

$\frac{(N_1\cdot N_2-N_2)+1}{N_1}$ = 1.53333
$\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}$ = 1.53333
$\frac{AC}{AF}$ = 1.53333
$\frac{N_1\cdot N_2}{N_2-1}$ = 12.50000
$\frac{Y\cdot Z}{X\cdot(Z-W)}$ = 12.50000
$\frac{BF}{EF}$ = 12.50000
AC = 5.78095 cm
AF = 3.77019 cm
BF = 6.12067 cm
EF = 0.48965 cm

062793A

Given.

$N_1 := 3 \quad AB := N_1$

$N_2 := 6 \quad BC := N_2$

$N_3 := 4 \quad AC := N_3$

## Describe A Circle About a Triangle

**Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.**

Descriptions.

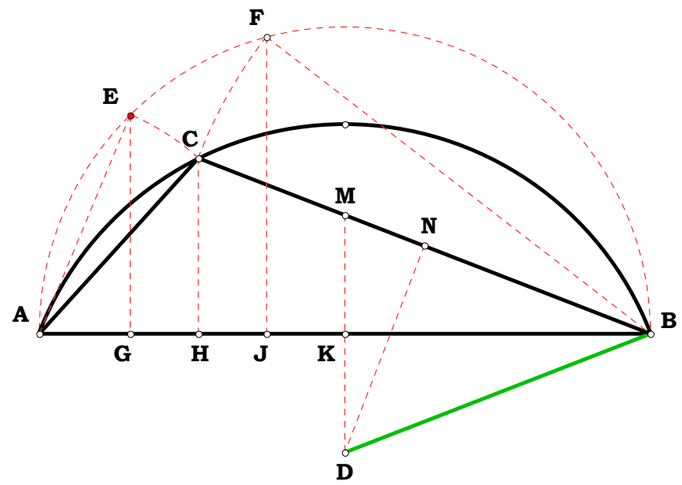
$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB) \quad NOT(X) := X = \delta := 0 .. 2$

$BK := \frac{AB}{2} \quad AE := AC \quad BF := BC \quad AG := \frac{AE^2}{AB} \quad BJ := \frac{BF^2}{AB}$

$GJ := AB - (AG + BJ) \quad HJ := \frac{GJ}{2} \quad BH := BJ + HJ$

$CH := \sqrt{BC^2 - BH^2} \quad BN := \frac{BC}{2} \quad BM := \frac{BC \cdot BK}{BH}$

$MN := BM - BN \quad DN := \frac{BH \cdot MN}{CH} \quad BD := \sqrt{BN^2 + DN^2}$




Definitions.

$radius := if(\Delta, BD, 0) \quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375412$

$imaginary\_radius = 0$

**The construction is independent of the side one starts with.**

$\Delta = 1$

$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix} \quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix} \quad R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$

The name of the Radius in terms of the givens.

$R^T = (3.375412 \quad 3.375412 \quad 3.375412)$ **The equation is a statement in regard to the relationship between each side of a triangle.**

$$BK - \frac{N_1}{2} = 0 \qquad AE - N_3 = 0 \qquad BF - N_2 = 0 \qquad AG - \frac{N_3^2}{N_1} = 0 \qquad BJ - \frac{N_2^2}{N_1} = 0$$

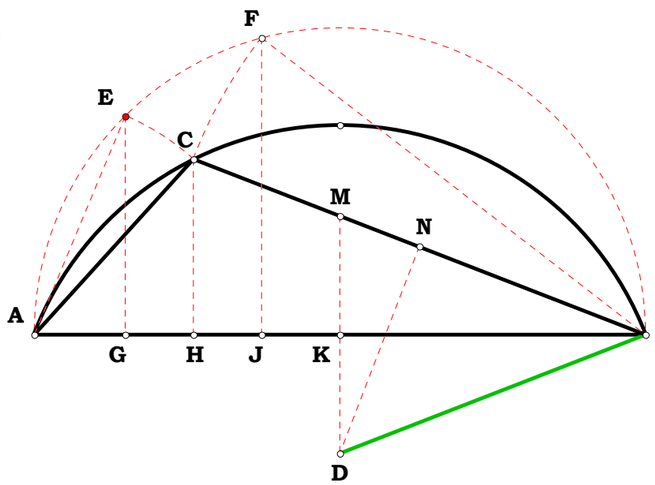
$$GJ - \frac{N_1^2 - N_2^2 - N_3^2}{N_1} = 0 \qquad HJ - \frac{N_1^2 - N_2^2 - N_3^2}{2 \cdot N_1} = 0 \qquad BH - \frac{N_1^2 + N_2^2 - N_3^2}{2 \cdot N_1} = 0$$

$$CH - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot N_1} = 0 \qquad BN - \frac{N_2}{2} = 0$$

$$BM - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2 - N_3^2} = 0 \qquad MN - \frac{N_2 \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}{2 \cdot \left(N_1^2 + N_2^2 - N_3^2\right)} = 0$$

$$DN - \frac{N_2 \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}{2 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}} = 0$$

$$BD - \frac{N_1 \cdot N_2 \cdot N_3}{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}} = 0$$




AB = 4.03775 in. $N_1$ = 4.03775 in.
BC = 3.20102 in. $N_2$ = 3.20102 in.
AC = 1.55963 in. $N_3$ = 1.55963 in.
BD = 2.16667 in.

$$\frac{N_1 \cdot N_2 \cdot N_3}{\sqrt{(N_1+N_2+N_3) \cdot ((N_1-N_2)+N_3) \cdot ((N_2-N_1)+N_3) \cdot ((N_1+N_2)-N_3)}} - BD = 0.00000 \text{ in.}$$

$\frac{XO}{01} = 1.35962$ $\frac{WO}{01} = 1.35962$ $\frac{ZO}{01} = 1.35962$

X = 1.35962 XO = 1.18110 in. 01 = 0.86870 in.

M = 1.35962 WO = 1.18110 in. OZ = 1.18110 in.

X-M = 0.00000 ZO = 1.18110 in.

PO = 1.18110 in.

TR = 1.18110 in.

$S_1 = 1.55042$

$S_2 = 2.69740$

$S_3 = 2.41211$

A = 10.08768 $S_1 \cdot S_2 \cdot S_3 - A = 0.00000$

$\frac{S_1 \cdot S_2 \cdot S_3}{\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3}} - M = 0.00000$

B = 6.65993 $(S_1+S_2+S_3)-B = 0.00000$

C = 2.58068 $\sqrt{S_1+S_2+S_3}-C = 0.00000$

D = 3.55909 $(S_2+S_3)-S_1-D = 0.00000$

E = 1.88655 $\sqrt{(S_2+S_3)-S_1}-E = 0.00000$

F = 1.26512 $(S_1+S_3)-S_2-F = 0.00000$

G = 1.12478 $\sqrt{(S_1+S_3)-S_2}-G = 0.00000$

H = 1.83571 $(S_1+S_2)-S_3-H = 0.00000$

J = 1.35489 $\sqrt{(S_1+S_2)-S_3}-J = 0.00000$

K = 7.41948

$\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3} - K = 0.00000$

062793B

Unit.

$AB := 1$

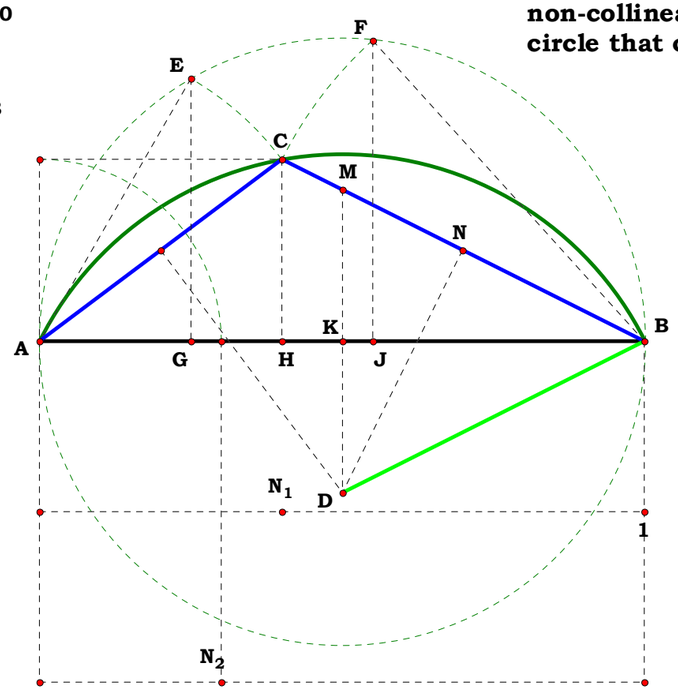
Given.

$W := 4$ $Y := 3$

$X := 10$ $Z := 10$

# Describe A Circle About a Triangle

Unit = 1.00000
X/W = 2.50000
W = 4.00000
X = 10.00000
Z/Y = 3.33333
Y = 3.00000
Z = 10.00000
$N_1$ = 2.50000
$N_2$ = 3.33333




**Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.**

**Descriptions.**

$AH := \frac{W}{X}$ $CH := \frac{Y}{Z}$ $AC := \sqrt{AH^2 + CH^2}$ $BH := AB - AH$

$BC := \sqrt{BH^2 + CH^2}$ $BK := \frac{AB}{2}$ $AE := AC$ $BF := BC$

$AG := \frac{AE^2}{AB}$ $BJ := \frac{BF^2}{AB}$ $GJ := AB - (AG + BJ)$ $HJ := \frac{GJ}{2}$

$BN := \frac{BC}{2}$ $BM := \frac{BC \cdot BK}{BH}$ $MN := BM - BN$ $DN := \frac{BH \cdot MN}{CH}$

$BD := \sqrt{BN^2 + DN^2}$

**Definitions.**

$AH - \frac{W}{X} = 0$ $CH - \frac{Y}{Z} = 0$ $AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BH - \frac{X - W}{X} = 0$

$BC - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$ $BK - \frac{1}{2}$ $AE - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BF - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$

$AG - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} = 0$ $BJ - \frac{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}{X^2 \cdot Z^2} = 0$ $GJ - \frac{2 \cdot (W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} = 0$ $HJ - \frac{(W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} = 0$

$BN - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X \cdot Z} = 0$ $BM - \frac{\sqrt{X^2 \cdot (Y^2 + Z^2) + W \cdot Z^2 \cdot (W - 2 \cdot X)}}{2 \cdot Z \cdot (X - W)} = 0$ $MN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X \cdot Z \cdot (X - W)} = 0$

$DN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X^2 \cdot Y} = 0$ $BD - \frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot (W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2 + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} = 0$

**Giving any side of a triangle as unity, then each of the remaing two sides are named as the algebraic names assigned to A and B like such:**

$$A := \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z}$$

$$B := \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z}$$

$\delta := 0 .. 2$ **The result is independent of the side one starts assignes to unity**

$$S_1 := \begin{pmatrix} 1 \\ A \\ B \end{pmatrix} \quad S_2 := \begin{pmatrix} A \\ B \\ 1 \end{pmatrix} \quad S_3 := \begin{pmatrix} B \\ 1 \\ A \end{pmatrix}$$

$$R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$$

**The Arithmetic name of the Radius in terms of the givens.**

$R_\delta =$

| |
|---|
| 0.559017 |
| 0.559017 |
| 0.559017 |

**BD = 0.559017**

**The only time this process fails in in the Mythological Great Circle, which Shamen (sic) claim is a line. The real significance of the Great Circle is that in looking forward to one's future, these people actually end up where they started. I believe Dodson called it a Caucus Race of which he had plenty of experience. It is a whole lot more flattering to claim we have moved when all we did is stand still. the Couch Potato Philosopher.**

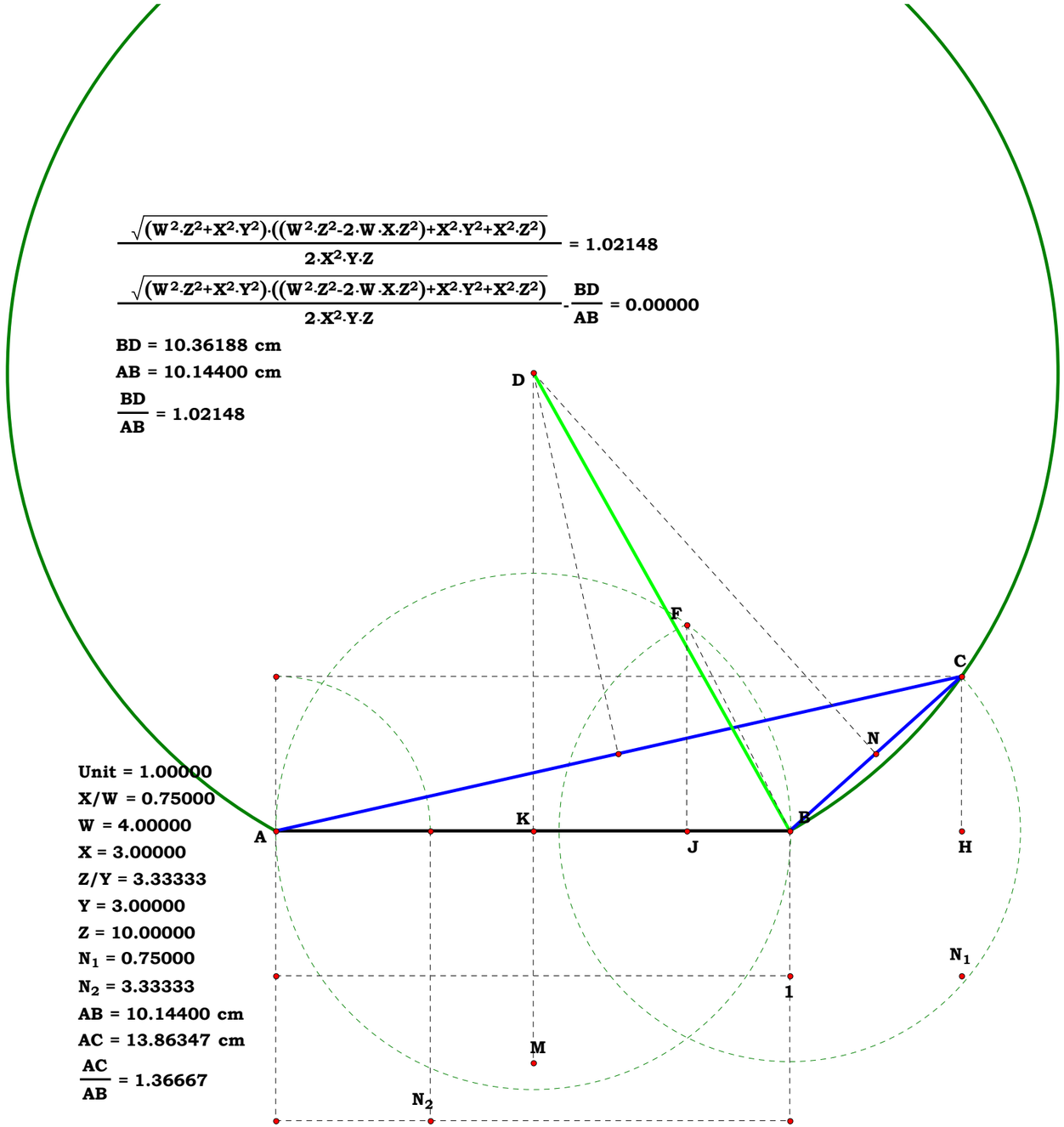
$$\frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot ((W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2) + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} = 1.02148$$

$$\frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot ((W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2) + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} - \frac{BD}{AB} = 0.00000$$

**BD = 10.36188 cm**

**AB = 10.14400 cm**

$$\frac{BD}{AB} = 1.02148$$




**Unit = 1.00000**
**X/W = 0.75000**
**W = 4.00000**
**X = 3.00000**
**Z/Y = 3.33333**
**Y = 3.00000**
**Z = 10.00000**
**$N_1$ = 0.75000**
**$N_2$ = 3.33333**
**AB = 10.14400 cm**
**AC = 13.86347 cm**

$$\frac{AC}{AB} = 1.36667$$

071593A

Descriptions.

## Pyramid of Ratios II

Unit.

$AB := 1$

Given.

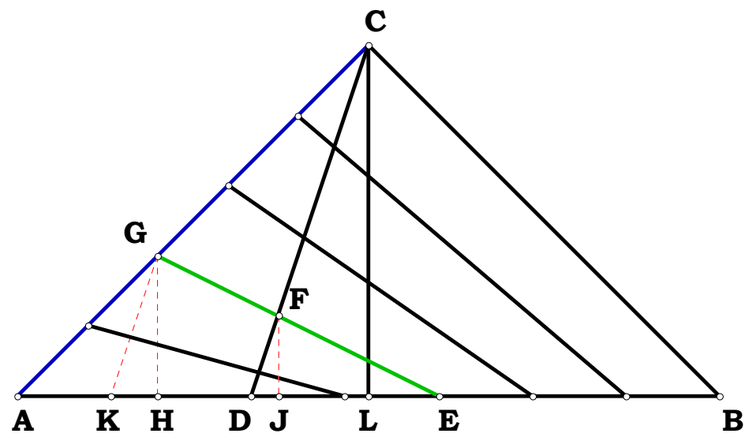
$N_1 := 3$

$N_2 := 5$

$\delta := 1 .. N_2$

Back in the day, I used to write shit up like this, but then I forgot just what in the hell I was doing, so, I had to invent other ways to write these up and to understand them.

AB is divided by $N_1$ and AC and BD is divided by $N_2$, what are EG/FG and CD/DF?




$AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$ $DE_\delta := \frac{BD \cdot \delta}{N_2}$ $AG_\delta := \frac{AC \cdot \delta}{N_2}$

$AE_\delta := AD + DE_\delta$ $AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}}$ $GH_\delta := AH_\delta$ $EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2}$ $AL := \frac{AB}{2}$ $DL := AL - AD$ $CL := \sqrt{\frac{AC^2}{2}}$

$HK_\delta := \frac{DL \cdot GH_\delta}{CL}$ $EK_\delta := EH_\delta + HK_\delta$ $DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta}$ $FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2}$ $EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta}$ $FG_\delta := EG_\delta - EF_\delta$

$CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right) =$

| |
|---|
| 15 |
| 4.375 |
| 2.222222 |
| 1.40625 |
| 1 |

$\text{if}\left[\delta, N_2 \cdot \frac{\left[\left(N_2 + \delta \cdot N_1\right) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot \left(N_1 - 1\right)\right]}, 0\right] =$

| |
|---|
| 15 |
| 4.375 |
| 2.222222 |
| 1.40625 |
| 1 |

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right) =$

| |
|---|
| 1.5 |
| 2.333333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot \left(N_1 - 2\right)}{N_2 - \delta}, 0\right] =$

| |
|---|
| 1.5 |
| 2.333333 |
| 4 |
| 9 |
| 0 |

071593B

# Pyramid of Ratios II

Unit.

$AB := 1$

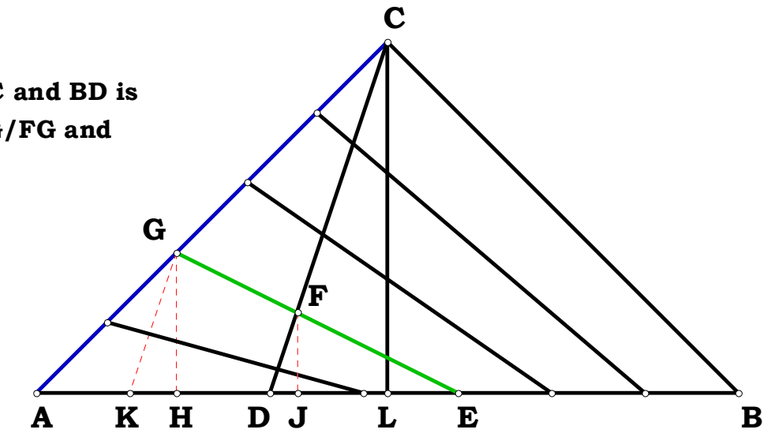
Given.

$N_1 := 3$

$N_2 := 5$

When I got to this point, I thought I had the bull gonads, but, I was not satisfied.

AB is divided by $N_1$ and AC and BD is divided by $N_2$, what are EG/FG and CD/DF?




Descriptions.

$AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$ $AL := \frac{AB}{2}$

$DE := \frac{BD}{N_2}$ $AG := \frac{AC}{N_2}$ $AE := AD + DE$ $AH := \sqrt{\frac{AG^2}{2}}$

$GH := AH$ $EH := AE - AH$ $EG := \sqrt{EH^2 + GH^2}$ $DL := AL - AD$

$CL := AL$ $HK := \frac{DL \cdot AH}{AL}$ $EK := EH + HK$

$DJ := \frac{HK \cdot DE}{EK}$ $EF := \frac{EG \cdot DE}{EK}$ $FG := EG - EF$

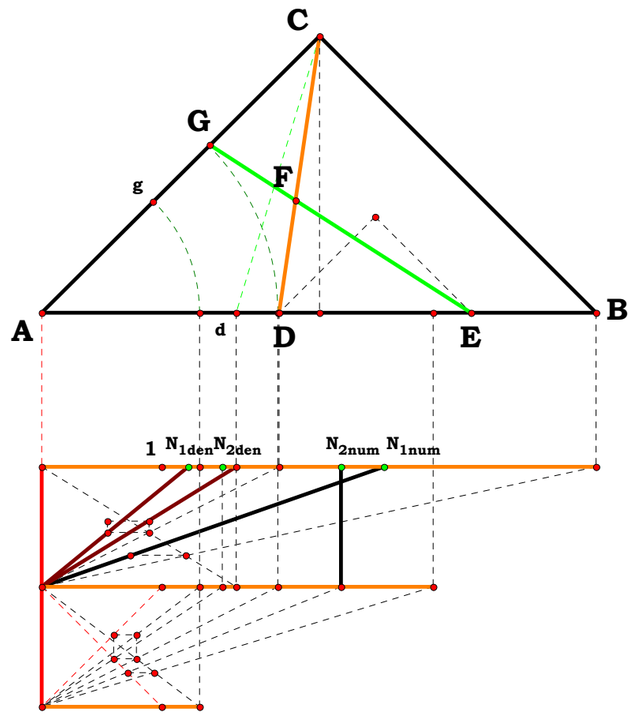
$FJ := \frac{GH \cdot DE}{EK}$ $CD := \sqrt{CL^2 + DL^2}$ $DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} = 1.5$ $\frac{CD}{DF} = 15$

Definitions.

$\frac{N_2 + N_1 - 2}{N_2 - 1} = 1.5$ $\frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} = 15$

$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0$ $\frac{CD}{DF} - \frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}$




$N_1 = 2.32900$

$N_2 = 1.65606$

$\frac{N_{1\,num}}{N_{1\,den}} = 2.32900$

$\frac{N_{2\,num}}{N_{2\,dem}} = 1.65606$

$\frac{EG}{FG} = 3.02573$ $\frac{(N_2+N_1)-2}{N_2-1} = 3.02573$ $\frac{EG}{FG} - \frac{(N_2+N_1)-2}{N_2-1} = 0.00000$

$\frac{CD}{DF} = 2.47357$ $\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 2.47357$ $\frac{CD}{DF} - \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 0.00000$

071593C

Unit.

$AB := 1$

Given.

$W := 3$

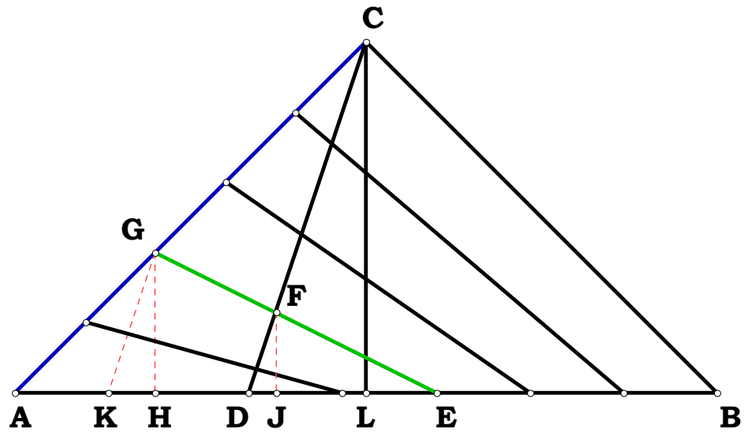
$X := 10$

$Y := 8$

$Z := 20$

## Pyramid of Ratios II

**What we have, is a way to understand how to write something, which shows a very intimate account of the ratios which, if we stop to think about it, we had set all along. We are learning the results of our own behavior applied to information.**




Descriptions.

$N_1 := \frac{X}{W}$ $N_2 := \frac{Z}{Y}$ $AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$

$AL := \frac{AB}{2}$ $DE := \frac{BD}{N_2}$ $AG := \frac{AC}{N_2}$ $AE := AD + DE$ $AH := \sqrt{\frac{AG^2}{2}}$

$GH := AH$ $EH := AE - AH$ $EG := \sqrt{EH^2 + GH^2}$ $DL := AL - AD$

$CL := AL$ $HK := \frac{DL \cdot AH}{AL}$ $EK := EH + HK$ $DJ := \frac{HK \cdot DE}{EK}$

$EF := \frac{EG \cdot DE}{EK}$ $FG := EG - EF$ $FJ := \frac{GH \cdot DE}{EK}$

$CD := \sqrt{CL^2 + DL^2}$ $DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0$ $\frac{CD}{DF} - \left( \frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} \right) = 0$

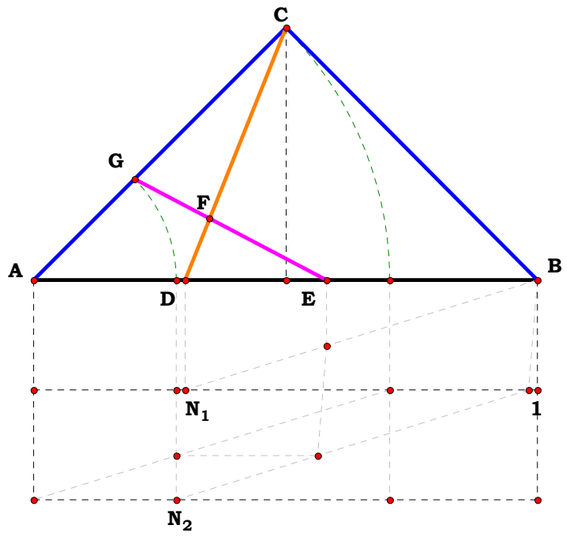
Definitions.

$N_1 - \frac{X}{W} = 0$ $N_2 - \frac{Z}{Y} = 0$ $AD - \frac{W}{X} = 0$ $AC - \frac{1}{\sqrt{2}} = 0$ $BD - \frac{(X - W)}{X} = 0$

$AL - \frac{1}{2} = 0$ $DE - \frac{Y \cdot (X - W)}{X \cdot Z} = 0$ $AG - \frac{\sqrt{2} \cdot Y}{2 \cdot Z} = 0$ $AE - \frac{W \cdot (Z - Y) + X \cdot Y}{X \cdot Z} = 0$ $AH - \frac{Y}{2 \cdot Z} = 0$

$GH - \frac{Y}{2 \cdot Z} = 0$ $EH - \frac{2 \cdot W \cdot (Z - Y) + X \cdot Y}{2 \cdot X \cdot Z} = 0$ $EG - \frac{\sqrt{Y^2 \cdot X^2 + [2 \cdot W \cdot Y \cdot (Z - Y)] \cdot X + 2 \cdot W^2 \cdot (Y - Z)^2}}{\sqrt{2} \cdot X \cdot Z} = 0$

$DL - \frac{(X - 2 \cdot W)}{2 \cdot X} = 0$ $CL - \frac{1}{2} = 0$ $HK - \frac{Y \cdot (X - 2 \cdot W)}{2 \cdot X \cdot Z} = 0$ $EK - \frac{(W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)}{X \cdot Z} = 0$




Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
$N_1$ = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
$N_2$ = 2.50000

EG = 3.62887 cm
FG = 1.41999 cm
$\frac{EG}{FG}$ = 2.55556
$\frac{(N_1+N_2)-2}{N_2-1}$ = 2.55556
$\frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{W \cdot (Y - Z)}$ = 2.55556

CD = 4.55082 cm
DF = 1.10803 cm
$\frac{CD}{DF}$ = 4.10714
$\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1}$ = 4.10714
$\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)}$ = 4.10714

$$DJ - \frac{Y^2 \cdot (W - X) \cdot (X - 2 \cdot W)}{2 \cdot X \cdot Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0$$

$$EF - \frac{Y \cdot (X - W) \cdot \sqrt{2 \cdot X^2 \cdot Y^2 + 4 \cdot W^2 \cdot (Y - Z)^2 - 4 \cdot W \cdot X \cdot Y \cdot (Y - Z)}}{2 \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

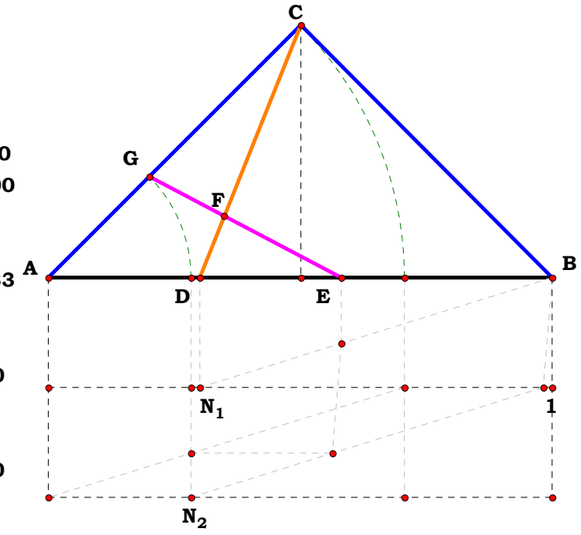
$$FG - \frac{W \cdot (Z - Y) \cdot \sqrt{2 \cdot X^2 \cdot Y^2 + 4 \cdot W^2 \cdot (Y - Z)^2 - 4 \cdot W \cdot X \cdot Y \cdot (Y - Z)}}{2 \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$FJ - \frac{Y^2 \cdot (X - W)}{2 \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$CD - \frac{\sqrt{2 \cdot W^2 - 2 \cdot W \cdot X + X^2}}{\sqrt{2} \cdot X} = 0$$

$$DF - \frac{Y^2 \cdot (X - W) \cdot \sqrt{2 \cdot W^2 - 2 \cdot W \cdot X + X^2}}{\sqrt{2} \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$\frac{EG}{FG} - \frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{(Y - Z) \cdot W} = 0 \qquad \frac{CD}{DF} - \left[ \frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)} \right] = 0$$




Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
$N_1$ = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
$N_2$ = 2.50000

EG = 3.62887 cm
FG = 1.41999 cm

$\frac{EG}{FG} = 2.55556$

$\frac{(N_1+N_2)-2}{N_2-1} = 2.55556$

$\frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{W \cdot (Y - Z)} = 2.55556$

CD = 4.55082 cm
DF = 1.10803 cm

$\frac{CD}{DF} = 4.10714$

$\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 4.10714$

$\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)} = 4.10714$

Unit.

$AB := 1$

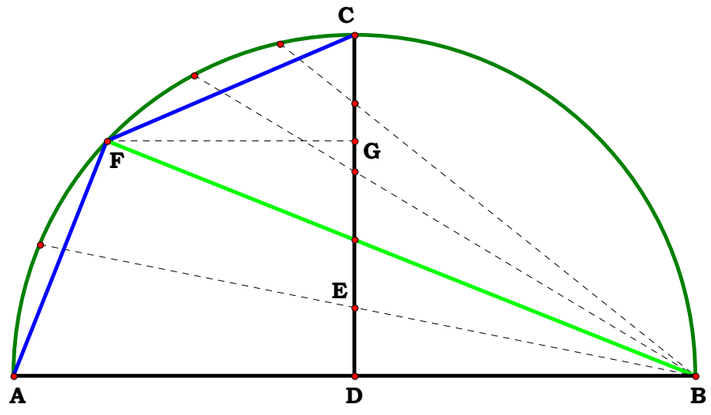
Given.

$N := 6$

$\delta := 1 .. N$

## Pyramid of Ratios III

**Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?**




**072593A**

**Descriptions.**

$AD := \frac{AB}{2}$ $BD := AD$ $CD := BD$ $BC := \sqrt{\frac{BD^2}{2}}$ $DE_\delta := \frac{CD \cdot \delta}{N}$

$BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$ $BF_\delta := \frac{BD \cdot AB}{BE_\delta}$ $AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta}$ $EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta}$ $FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$ $DG := DE + EG$ $CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

$\frac{DE_\delta}{DG_\delta} =$

| |
|---|
| 0.514 |
| 0.556 |
| 0.625 |
| 0.722 |
| 0.847 |
| 1 |

$\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2} =$

| |
|---|
| 0.513889 |
| 0.555556 |
| 0.625 |
| 0.722222 |
| 0.847222 |
| 1 |

$\frac{BE_\delta}{BF_\delta} =$

| |
|---|
| 0.513889 |
| 0.555556 |
| 0.625 |
| 0.722222 |
| 0.847222 |
| 1 |

$\frac{BF_\delta}{BE_\delta} =$

| |
|---|
| 1.945946 |
| 1.8 |
| 1.6 |
| 1.384615 |
| 1.180328 |
| 1 |

$\frac{2}{\left[\left(\frac{\delta}{N}\right)^2 + 1\right]} =$

| |
|---|
| 1.945946 |
| 1.8 |
| 1.6 |
| 1.384615 |
| 1.180328 |
| 1 |

$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right) =$

| |
|---|
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |

$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right) =$

| |
|---|
| 0.282843 |
| 0.707107 |
| 1.414214 |
| 2.828427 |
| 7.071068 |
| 0 |

Unit.

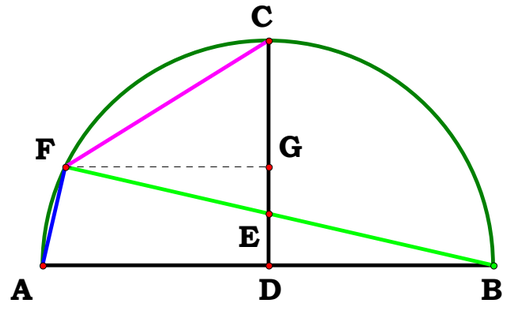
$AB := 1$

Given.

$N := 5$

**072593B**

**Pyramid of Ratios III**




**Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?**

**Descriptions.**

$CD := \frac{AB}{2}$ $DE := \frac{CD}{N}$ $BE := \sqrt{CD^2 + DE^2}$

$BF := \frac{CD \cdot AB}{BE}$ $AF := \frac{DE \cdot BF}{CD}$ $DG := \frac{DE \cdot BF}{BE}$

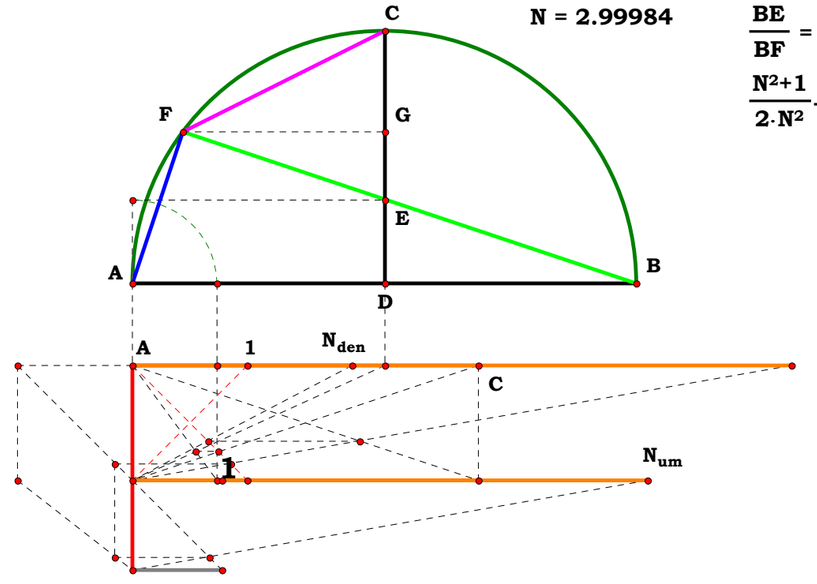
$CG := CD - DG$ $FG := \frac{CD \cdot (DG - DE)}{DE}$ $CF := \sqrt{FG^2 + CG^2}$

**Definitions.**

$\frac{BE}{BF} = 0.52$ $\frac{N^2 + 1}{2 \cdot N^2} = 0.52$

$\frac{AF}{CF} = 0.353553$ $\frac{\sqrt{2}}{N - 1} = 0.353553$

$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0$ $\frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$




$\frac{N_{um}}{N_{den}} = 2.99984$ N = 2.99984

BE = 1.75445 in.

BF = 3.15797 in.

AF = 1.05271 in.

CF = 1.48864 in.

$\frac{BE}{BF} = 0.55556$ $\frac{AF}{CF} = 0.70716$

$\frac{N^2+1}{2 \cdot N^2} - \frac{BE}{BF} = 0.00000$ $\frac{\sqrt{2}}{N-1} - \frac{AF}{CF} = 0.00000$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 4$

072593C

**Descriptions.**

$$N := \frac{Y}{X} \quad CD := \frac{AB}{2} \quad DE := \frac{CD}{N} \quad BE := \sqrt{CD^2 + DE^2}$$

$$BF := \frac{CD \cdot AB}{BE} \quad AF := \frac{DE \cdot BF}{CD} \quad DG := \frac{DE \cdot BF}{BE}$$

$$CG := CD - DG \quad FG := \frac{CD \cdot (DG - DE)}{DE} \quad CF := \sqrt{FG^2 + CG^2}$$

$$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$$

**Definitions.**

$$N - \frac{Y}{X} = 0 \quad CD - \frac{1}{2} \quad DE - \frac{X}{2 \cdot Y} = 0 \quad BE - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$$

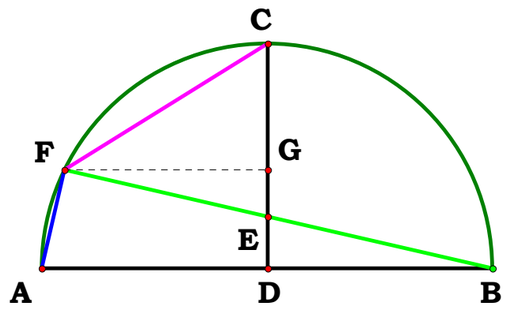
$$BF - \frac{Y}{\sqrt{X^2 + Y^2}} = 0 \quad AF - \frac{X}{\sqrt{X^2 + Y^2}} = 0 \quad DG - \frac{X \cdot Y}{X^2 + Y^2} = 0$$

$$CG - \frac{(X - Y)^2}{2 \cdot (X^2 + Y^2)} = 0 \quad FG - \frac{(Y - X) \cdot (X + Y)}{2 \cdot (X^2 + Y^2)} = 0$$

$$CF - \frac{(Y - X)}{\sqrt{2 \cdot (X^2 + Y^2)}} = 0$$

$$\frac{BE}{BF} - \frac{X^2 + Y^2}{2 \cdot Y^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2} \cdot X}{Y - X} = 0$$

## Pyramid of Ratios III




**Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?**

Unit = 1.00000

X = 4.00000

Y = 20.00000

X/Y = 0.20000

Y/X = 5.00000

N = 5.00000

$\frac{N^2+1}{2 \cdot N^2}$ = 0.52000

$\frac{\sqrt{2}}{N-1}$ = 0.35355

BE = 4.78390 cm

BF = 9.19981 cm

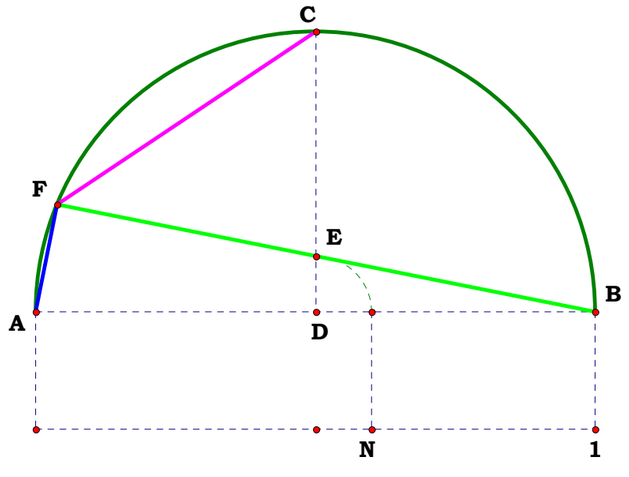
$\frac{BE}{BF}$ = 0.52000

AF = 1.83996 cm

CF = 5.20420 cm

$\frac{X^2+Y^2}{2 \cdot Y^2}$ = 0.52000

$\frac{\sqrt{2} \cdot X}{Y-X}$ = 0.35355



110693A

# Gruntwork I on the Delian Solution

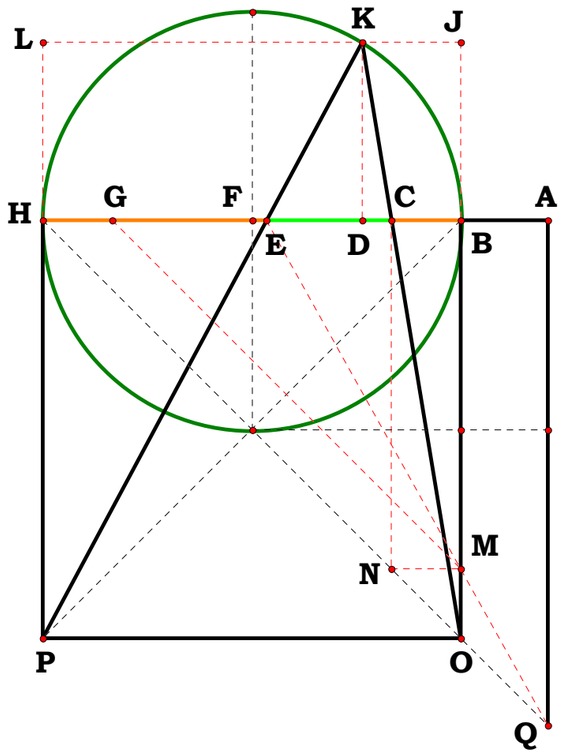
Unit.

$BH := 1$

Given.

$N := 5$

Does $(AB^2 \times AH)^{1/3} = AC$
and $(AB \times AH^2)^{1/3} = AE$?




**Descriptions.**

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $JO := BH + DK$ $BC := \frac{BD \cdot BH}{JO}$

$BG := BH - BC$ $EH := \frac{DH \cdot BH}{JO}$ $BE := BH - EH$ $EG := EH - BC$

$GM := \sqrt{2 \cdot BG^2}$ $HO := \sqrt{2 \cdot BH^2}$ $HQ := \frac{GM \cdot EH}{EG}$ $OQ := HQ - HO$

$AB := \frac{OQ}{\sqrt{2}}$ $AC := AB + BC$ $AE := AB + BE$ $AH := AB + BH$

$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0$ $(AB \cdot AH^2)^{\frac{1}{3}} - AE = 0$

**Definitions.**

$BF - \frac{1}{2} = 0$ $BD - \frac{1}{2 \cdot N} = 0$ $DH - \frac{(2 \cdot N - 1)}{2 \cdot N} = 0$ $DK - \frac{\sqrt{2 \cdot N - 1}}{(2 \cdot N)} = 0$

$JO - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)}{2 \cdot N} = 0$ $BC - \frac{1}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $BG - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$

$EH - \frac{(2 \cdot N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $BE - \frac{\left(\sqrt{2 \cdot N - 1} + 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$ $EG - \frac{2 \cdot (N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$

$GM - \sqrt{2} \cdot \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)} = 0$ $HO - \sqrt{2} = 0$

$$HQ - \frac{\sqrt{2}\cdot(2\cdot N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

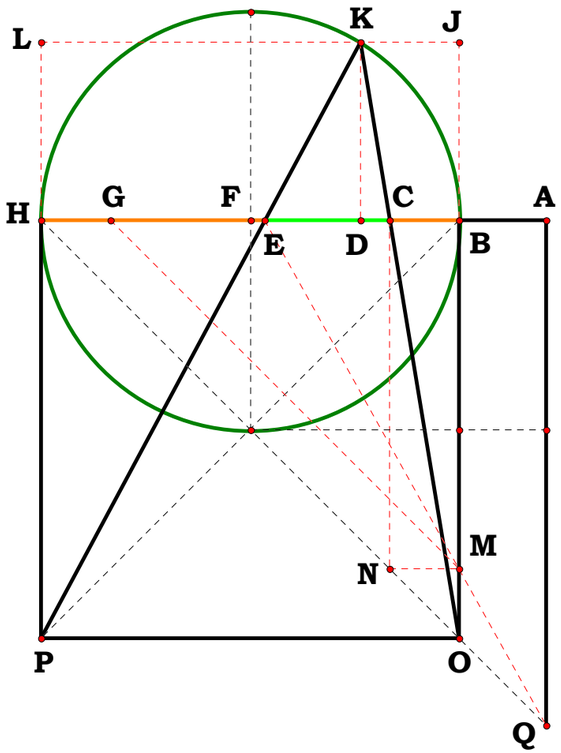
$$OQ - \frac{\sqrt{2}\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{\left(2\cdot N+\sqrt{2\cdot N-1}\right)\cdot(2\cdot N-2)} = 0$$

$$AB - \frac{\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AC - \frac{\left[2\cdot N+2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AE - \frac{\left[2\cdot N+(2\cdot N-1)^{\frac{3}{2}}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AH - \frac{\left[(2\cdot N-1)^{\frac{3}{2}}-4\cdot N+4\cdot N^2+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$




BF = 1.37917 in.

BD = 0.65310 in.

$\frac{BF}{BD} = 2.11173$

N = 2.11173

BH = 2.75833 in.

AE = 1.85725 in.

$\frac{BH\cdot\left(\left(2\cdot N+(2\cdot N-1)^{\frac{3}{2}}\right)-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)}$-AE = 0.00000 in.

110693B

**Unit.**

$BH := 1$

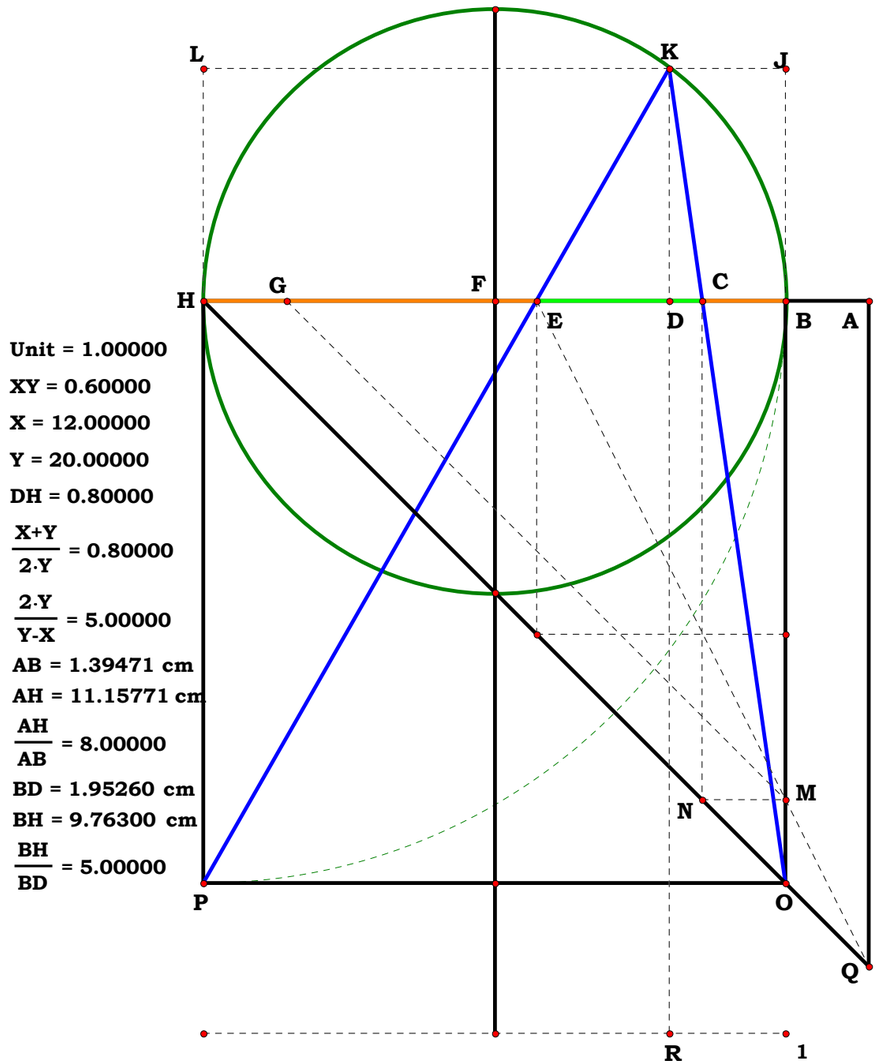
**Given.**

$Y := 20$

$X := 13$

# Gruntwork I on the Delian Solution

**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/3} = AE$?**




Unit = 1.00000

XY = 0.60000

X = 12.00000

Y = 20.00000

DH = 0.80000

$\frac{X+Y}{2 \cdot Y}$ = 0.80000

$\frac{2 \cdot Y}{Y-X}$ = 5.00000

AB = 1.39471 cm

AH = 11.15771 cm

$\frac{AH}{AB}$ = 8.00000

BD = 1.95260 cm

BH = 9.76300 cm

$\frac{BH}{BD}$ = 5.00000

**Descriptions.**

$BF := \frac{BH}{2}$ $BD := \frac{Y - X}{2 \cdot Y}$ $DH := BH - BD$ $DK := \sqrt{BD \cdot DH}$ $JO := BH + DK$

$BC := \frac{BD \cdot BH}{JO}$ $BG := BH - BC$ $EH := \frac{DH \cdot BH}{JO}$ $BE := BH - EH$ $EG := EH - BC$

$GM := \sqrt{2 \cdot BG^2}$ $HO := \sqrt{2 \cdot BH^2}$ $HQ := \frac{GM \cdot EH}{EG}$ $OQ := HQ - HO$

$AB := \frac{OQ}{\sqrt{2}}$ $AC := AB + BC$ $AE := AB + BE$ $AH := AB + BH$

$\frac{BF}{BD} = 2.857143$ $\frac{AH}{AB} = 10.235849$ $\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$

**Definitions.**

$BF - \frac{1}{2} = 0$ $BD - \frac{Y - X}{2 \cdot Y} = 0$ $DH - \frac{X + Y}{2 \cdot Y} = 0$ $DK - \frac{\sqrt{Y^2 - X^2}}{2 \cdot Y} = 0$

$JO - \frac{2 \cdot Y + \sqrt{Y^2 - X^2}}{2 \cdot Y} = 0$ $BC - \frac{Y - X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$ $BG - \frac{X + Y + \sqrt{Y^2 - X^2}}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$

$EH - \frac{X + Y}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$ $BE - \frac{(X + Y) \cdot \sqrt{Y^2 - X^2} + (X - Y)^2}{X^2 + 3 \cdot Y^2} = 0$ $EG - \frac{2 \cdot X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$

$GM - \frac{\sqrt{2} \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{\left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $HO - \sqrt{2} = 0$ $HQ - \frac{\sqrt{2} \cdot (X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $OQ - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right) \cdot \sqrt{2}}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$

$AB - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $AC - \frac{(Y - X) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $AE - \frac{(X + Y) \cdot \left(\sqrt{Y^2 - X^2} + Y - X\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$ $AH - \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$

Unit.

$BH := 1$

Given.

$N_1 := 5$

**110993A**

**Descriptions.**

$BG := \frac{BH}{2}$ $CF := \frac{BH}{N_1}$ $BL := CF$ $GP := BG$

$BK := \frac{BL}{2}$ $BD := BK$ $NP := BD$ $GN := GP - NP$ $EN := BL$

$GE := \sqrt{GN^2 - EN^2}$ $CE := BD$ $BC := BG - (GE + CE)$

$GH := BG$ $EF := BD$ $FH := GH + GE - EF$ $FQ := FH$

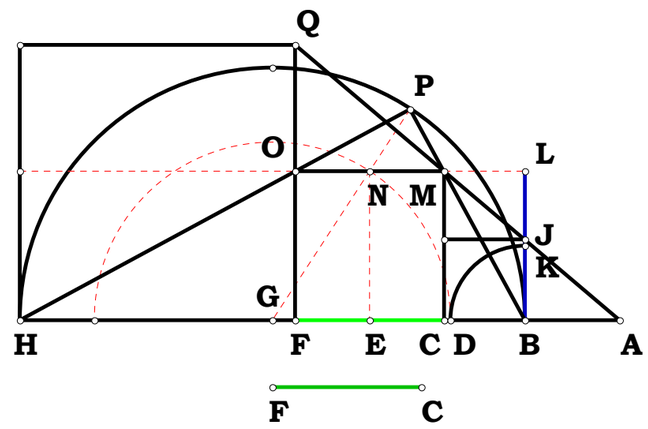
$FO := BL$ $OQ := FQ - ]MO := CF$ $AF := \frac{MO \cdot FQ}{OQ}$ $AC := AF - CF$

$AH := AF + FH$ $AB := AH - BH$

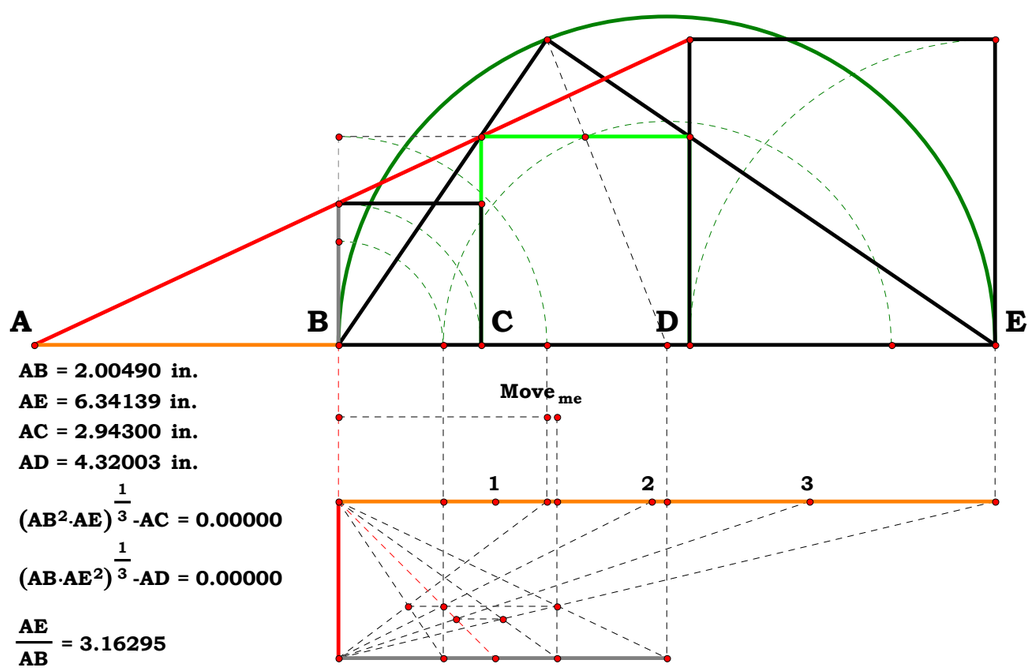
$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$ $\frac{AH}{AB} = 51.980762113532$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

## Solve For Cube Root Placement




**With straight edge and compass only, solve the given problem.**
**BH is the difference between the segments AH and AB.**
**CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**




AB = 2.00490 in.

AE = 6.34139 in.

AC = 2.94300 in.

AD = 4.32003 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 3.16295$

**Definitions.**

$$BH - 1 = 0 \qquad BG - \frac{1}{2} = 0 \qquad CF - \frac{1}{N_1} = 0 \qquad BK - \frac{1}{2 \cdot N_1} = 0 \qquad GN - \frac{N_1 - 1}{2 \cdot N_1} = 0$$

$$GE - \frac{\sqrt{(N_1 + 1) \cdot (N_1 - 3)}}{2 \cdot N_1} = 0 \qquad BC - \frac{N_1 - \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{2 \cdot N_1} = 0$$

$$FH - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{2 \cdot N_1} = 0$$

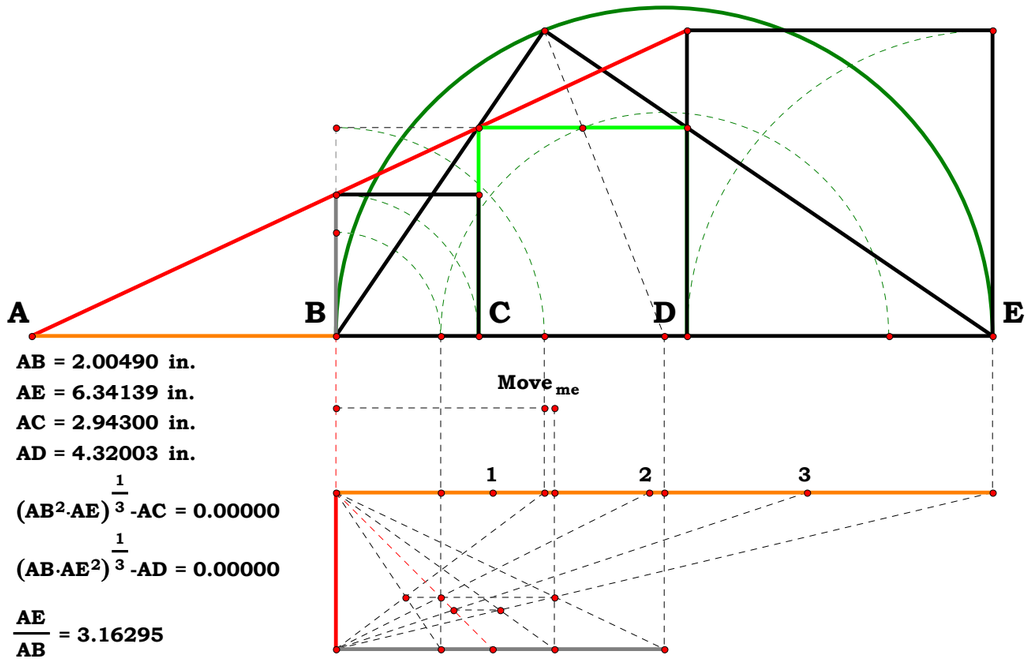
$$OQ - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3}{2 \cdot N_1} = 0$$

$$AF - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AC - \frac{2}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AH - \frac{\left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1\right]^2}{2 \cdot N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AB - \frac{N_1 - \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$




AB = 2.00490 in.
AE = 6.34139 in.
AC = 2.94300 in.
AD = 4.32003 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 3.16295$

Unit.

$BG := 1$

Given.

$Y := 20$

$X := 15$

110993B

Descriptions.

$BH := 2 \cdot BG \quad CF := \frac{2 \cdot X}{3 \cdot Y} \quad BL := CF \quad GP := BG$

$BK := \frac{BL}{2} \quad BD := BK \quad NP := BD \quad GN := GP - NP \quad EN := BL$

$GE := \sqrt{GN^2 - EN^2} \quad CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD \quad FH := GH + GE - EF \quad FQ := FH$

$FO := BL \quad OQ := FQ - ]MO := CF \quad AF := \frac{MO \cdot FQ}{OQ} \quad AC := AF - CF$

$AH := AF + FH \quad AB := AH - BH$

$CH := CF + FH$

Arithmetic Names:

$CF = 0.5$

$FH = 1.309017$

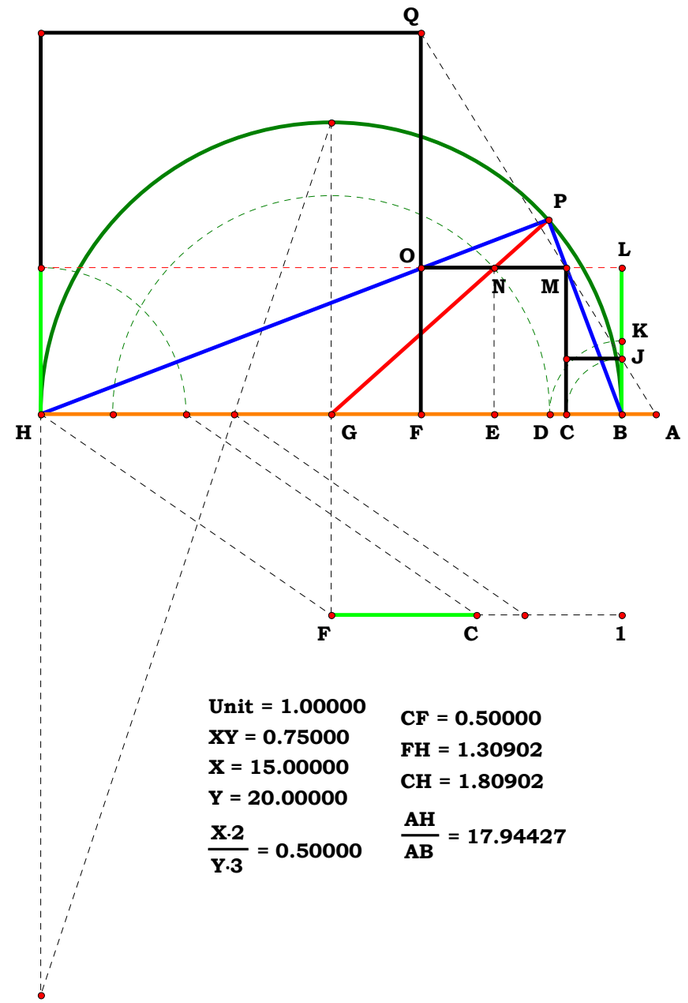
$CH = 1.809017$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \quad \frac{AH}{AB} = 17.944271909999$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

## Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem. BH is the difference between the segments AH and AB. CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**




Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000
$\frac{X \cdot 2}{Y \cdot 3} = 0.50000$

CF = 0.50000
FH = 1.30902
CH = 1.80902
$\frac{AH}{AB} = 17.94427$

**Definitions.**

$$BH - 2 = 0 \quad CF - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad BL - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad GP - 1 = 0$$

$$BK - \frac{X}{3 \cdot Y} = 0 \quad BD - \frac{X}{3 \cdot Y} = 0 \quad NP - \frac{X}{3 \cdot Y} = 0 \quad GN - \frac{3 \cdot Y - X}{3 \cdot Y} = 0$$

$$EN - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad GE - \frac{\sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{\sqrt{3} \cdot Y} = 0 \quad CE - \frac{X}{3 \cdot Y} = 0$$

$$BC - \frac{3 \cdot Y - X - \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad GH - 1 = 0$$

$$EF - \frac{X}{3 \cdot Y} = 0 \quad FH - \frac{3 \cdot Y - X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0$$

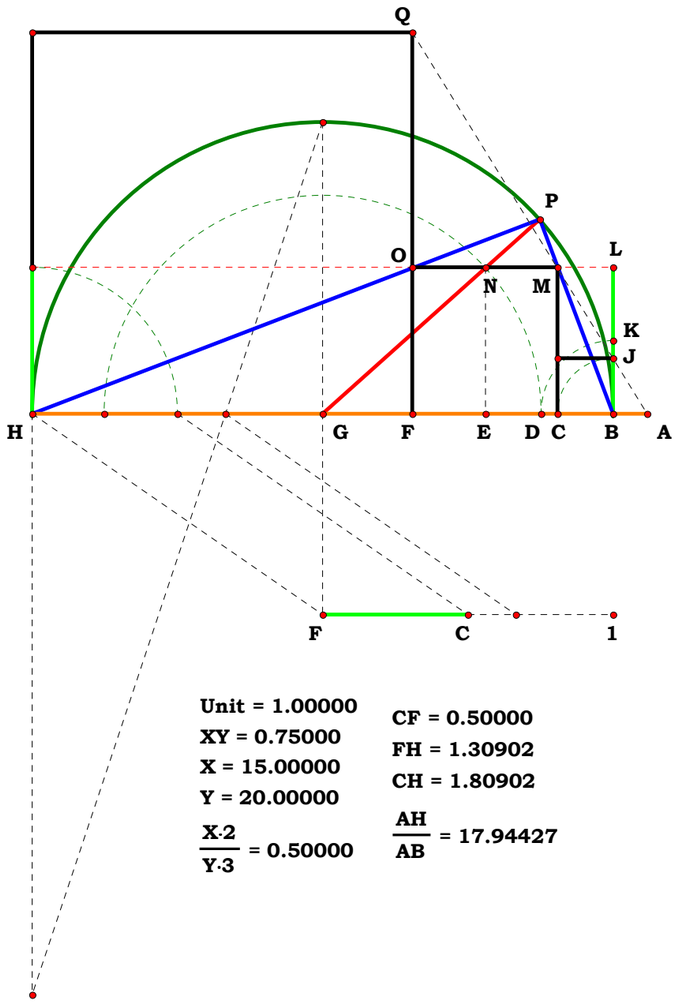
$$CH - \frac{X + 3 \cdot Y + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad FQ - \frac{3 \cdot Y - X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0$$

$$FO - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad OQ - \frac{3 \cdot (Y - X) + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad MO - \frac{2 \cdot X}{3 \cdot Y} = 0$$

$$AF - \frac{2 \cdot X \cdot \left(3 \cdot Y - X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0 \quad AC - \frac{4 \cdot X^2}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0$$

$$AH - \frac{18 \cdot Y^2 - 12 \cdot X \cdot Y - 2 \cdot X^2 - 2 \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2} \cdot (X - 3 \cdot Y) \cdot \sqrt{3}}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}\right)} = 0 \quad AB - \frac{2 \cdot X \cdot \left(3 \cdot Y - X - \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}\right)}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0$$

$$\frac{AH}{AB} = 17.944272$$




Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000
$\frac{X \cdot 2}{Y \cdot 3} = 0.50000$

CF = 0.50000
FH = 1.30902
CH = 1.80902
$\frac{AH}{AB} = 17.94427$

111093

# Gruntwork II on the Delian Solution

**Unit.**

$AE := 1$

**Given.**

$N := 4$

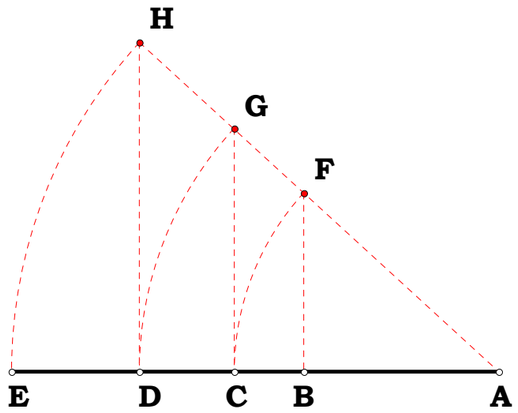
**Descriptions.**

$DE := \frac{AE}{N}$ $AD := AE - DE$

$AH := AE$ $AG := AD$

$AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

$AB := \frac{AC \cdot AC}{AD}$




**Definitions.**

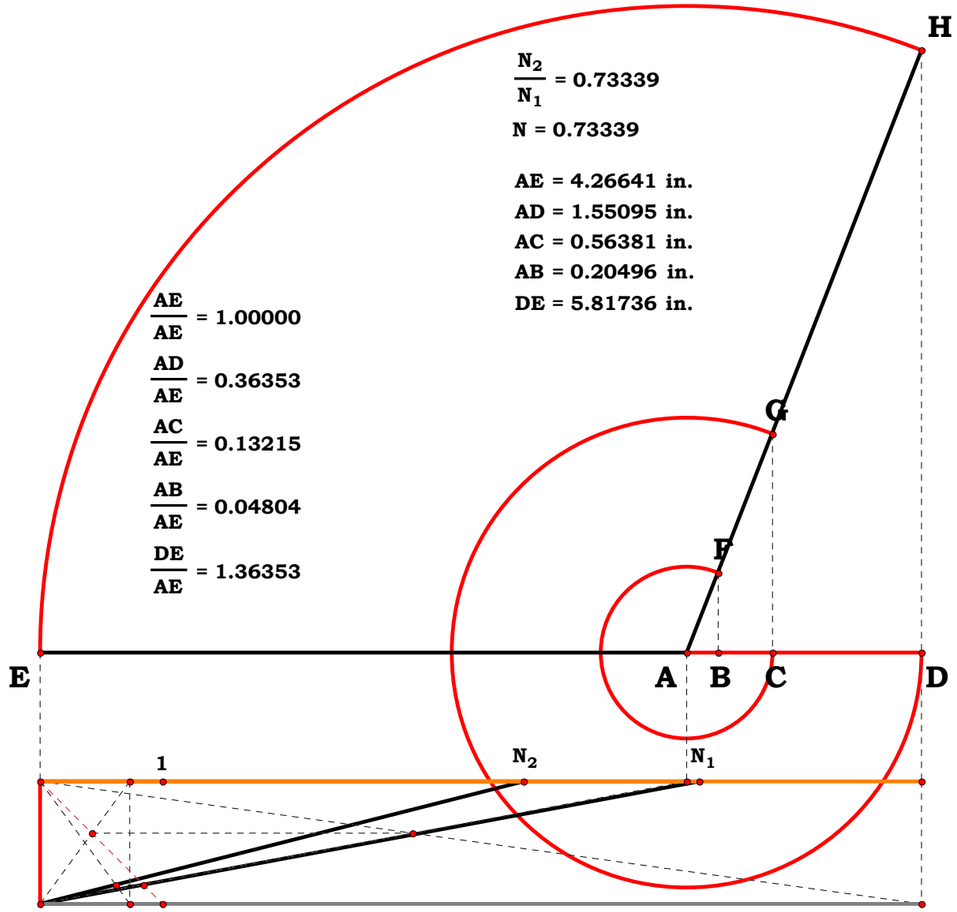
$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2.37$ $\frac{AD}{AB} = 1.777778$ $\frac{AC}{AB} = 1.333333$

**Albebraic Names:**

$\frac{1}{N} - DE = 0$ $1 - \frac{1}{N} - AD = 0$ $\frac{(N-1)^2}{N^2} - AC = 0$

$\frac{(N-1)^3}{N^3} - AB = 0$




$\frac{N_2}{N_1} = 0.73339$

$N = 0.73339$

$AE = 4.26641$ in.
$AD = 1.55095$ in.
$AC = 0.56381$ in.
$AB = 0.20496$ in.
$DE = 5.81736$ in.

$\frac{AE}{AE} = 1.00000$
$\frac{AD}{AE} = 0.36353$
$\frac{AC}{AE} = 0.13215$
$\frac{AB}{AE} = 0.04804$
$\frac{DE}{AE} = 1.36353$

$\sqrt{\left(\frac{1}{N}\right)^2} = 1.36353$ $\sqrt{\left(1-\frac{1}{N}\right)^2} = 0.36353$ $\frac{(N-1)^2}{N^2} = 0.13215$ $\sqrt{\frac{(N-1)^{3\,2}}{N^3}} = 0.04804$

$\sqrt{\left(\frac{1}{N}\right)^2} - \frac{DE}{AE} = 0.00000$ $\sqrt{\left(1-\frac{1}{N}\right)^2} - \frac{AD}{AE} = 0.00000$ $\frac{(N-1)^2}{N^2} - \frac{AC}{AE} = 0.00000$ $\sqrt{\frac{(N-1)^{3\,2}}{N^3}} - \frac{AB}{AE} = 0.00000$

Unit.

$AB := 1$

Given.

$N := 3$

111193A

Descriptions.

## The Archimedean Paper Trisector

When I looked up the Archimedean Paper Trisector, which is all I found. I did not find where anyone had bothered to complete the figure, for it was obvious to me that the figure was simply not complete. The first task then in writing up the figure is to simply complete the figure.

Once one understands that the angle on the center is twice the angle from the circumference one can then start to work filling in the figure to include the APT. One can see, not only here, but in other figures that trisection is involved with the right triangle and square roots.

$AD := \frac{AB}{N}$ $BD := AB - AD$ $CD := \sqrt{AD \cdot BD}$ $BC := \sqrt{CD^2 + BD^2}$

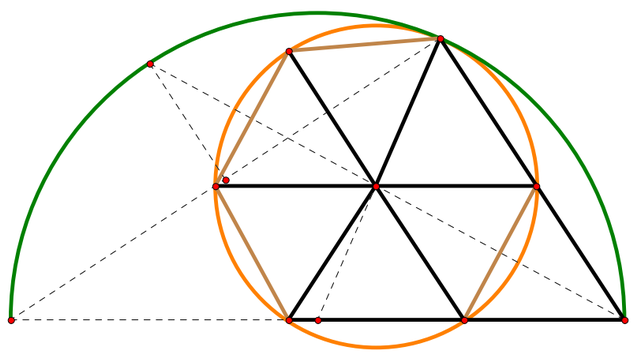
$AC := \sqrt{AD^2 + CD^2}$ $BH := \frac{BC}{2}$ $AO := \frac{AB}{2}$ $HO := \frac{AC}{2}$ $HL := \frac{CD}{2}$ $LO := \frac{AD}{2}$

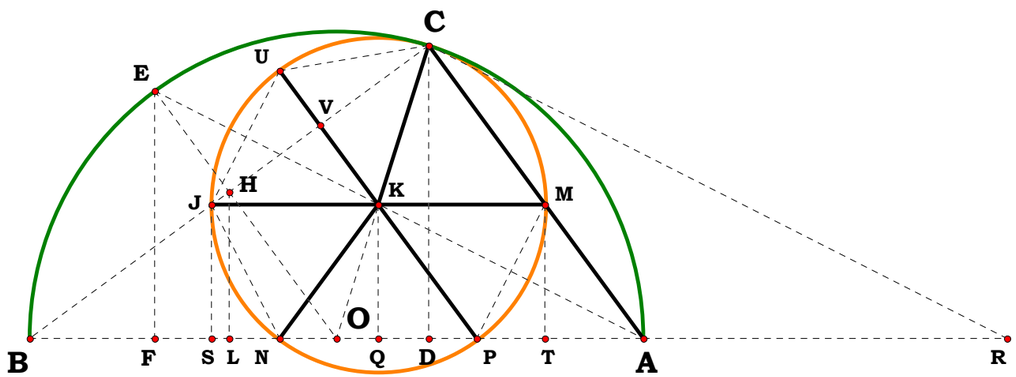
$OF := \frac{LO \cdot AO}{HO}$ $AF := AO + OF$ $BF := AB - AF$ $EF := \sqrt{BF \cdot AF}$ $DR := \frac{AF \cdot CD}{EF}$

$DO := AO - AD$ $OR := DR + DO$ $KQ := \frac{CD \cdot AO}{OR}$ $OK := \frac{AO \cdot KQ}{CD}$

$CK := AO - OK$ $QP := \sqrt{CK^2 - KQ^2}$ $OQ := \frac{DO \cdot KQ}{CD}$ $EH := AO - HO$

$AP := AO - (OQ + QP)$ $AP - CK = 0$





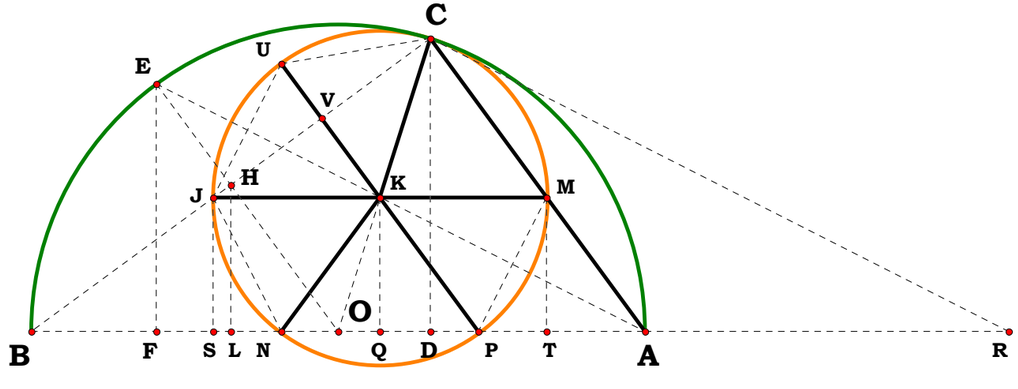
$$CJ := \frac{BC \cdot CK}{AO} \qquad AQ := AO - OQ \qquad SO := CK - OQ \qquad BS := AO - SO$$

$$BJ := BC - CJ \qquad JS := \sqrt{BJ^2 - BS^2} \qquad JS - KQ = 0 \qquad SN := SO + OQ - QP$$

$$JN := \sqrt{JS^2 + SN^2} \qquad UV := \frac{EH \cdot CJ}{BC} \qquad JV := \frac{CJ}{2} \qquad JU := \sqrt{JV^2 + UV^2} \qquad CU := JU$$

$$AT := \frac{AD \cdot CK}{AC} \qquad PT := AP - AT \qquad MT := \frac{CD \cdot AP}{AC} \qquad MP := \sqrt{PT^2 + MT^2}$$

$$MP - JN = 0 \qquad MP - JU = 0 \qquad MP - CU = 0$$




**Definitions.**

$$AD - \frac{1}{N} = 0 \qquad BD - \frac{N-1}{N} = 0 \qquad CD - \frac{\sqrt{N-1}}{N} = 0 \qquad BC - \frac{\sqrt{N-1}}{\sqrt{N}} = 0 \qquad AC - \frac{1}{\sqrt{N}} = 0 \qquad BH - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AO - \frac{1}{2} = 0 \qquad HO - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad HL - \frac{\sqrt{N-1}}{2 \cdot N} = 0 \qquad LO - \frac{1}{2 \cdot N} = 0 \qquad OF - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad AF - \frac{\sqrt{N}+1}{2 \cdot \sqrt{N}} \qquad BF - \frac{\sqrt{N}-1}{2 \cdot \sqrt{N}} = 0$$

$$EF - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0 \qquad DR - \frac{\sqrt{N-1} \cdot \left(\sqrt{N}+1\right)}{N \cdot \sqrt{\left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)}} = 0 \qquad DO - \frac{\sqrt{(N-2)^2}}{2 \cdot N} = 0 \qquad OR - \frac{\sqrt{N}+2}{2 \cdot \sqrt{N}} = 0 \qquad KQ - \frac{\sqrt{N-1}}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0$$

$$OK - \frac{\sqrt{N}}{2 \cdot \left(\sqrt{N}+2\right)} = 0 \qquad QP - \frac{1}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad OQ - \frac{\sqrt{(N-2)^2}}{2 \cdot \sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad EH - \frac{\sqrt{N}-1}{2 \cdot \sqrt{N}} = 0$$

$$AP - \frac{1}{\sqrt{N}+2} = 0 \qquad CK - \frac{1}{\sqrt{N}+2} = 0$$

**In logic, things which have the same name are equal. CK equals AP.**

$$CJ - \frac{2\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AQ - \frac{\sqrt{N}+1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SO - -\frac{N-2\cdot\sqrt{N}-2}{2\cdot\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$BS - \frac{1\cdot(N-1)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad BJ - \frac{1\cdot\sqrt{N-1}}{\sqrt{N}+2} = 0 \qquad JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

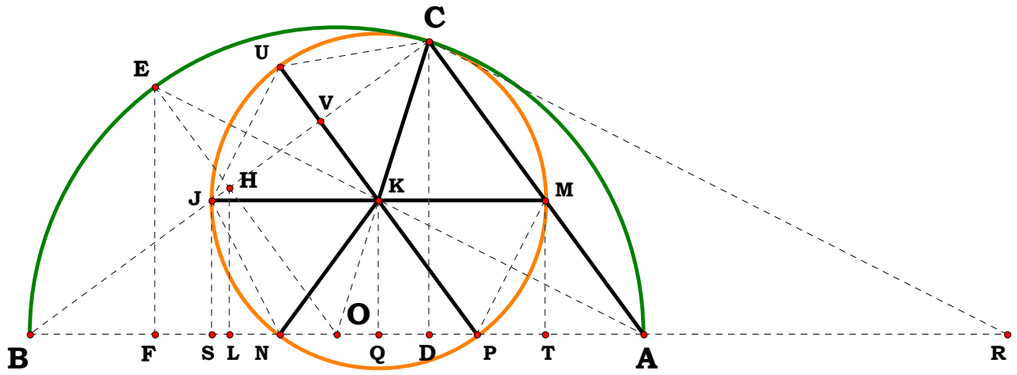
$$JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SN - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JN - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad UV - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JV - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad CU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AT - \frac{1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$PT - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MT - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0$$



Unit := 1

Given.

N := 4

111193B1

Descriptions.

$A := N - Unit \quad AB := \frac{A}{2} \quad AN := N - A$

$NK := \sqrt{N \cdot AN} \quad BK := N - (NK + AB)$

$DK := \sqrt{AB^2 + BK^2} \quad KM := \frac{BK \cdot NK}{DK}$

$CE := \frac{N - AB}{2} \quad MP := \frac{CE}{2} \quad CK := CE - BK$

$KP := \frac{CK}{2} \quad NP := NK - KP \quad CF := \frac{MP \cdot CE}{NP}$

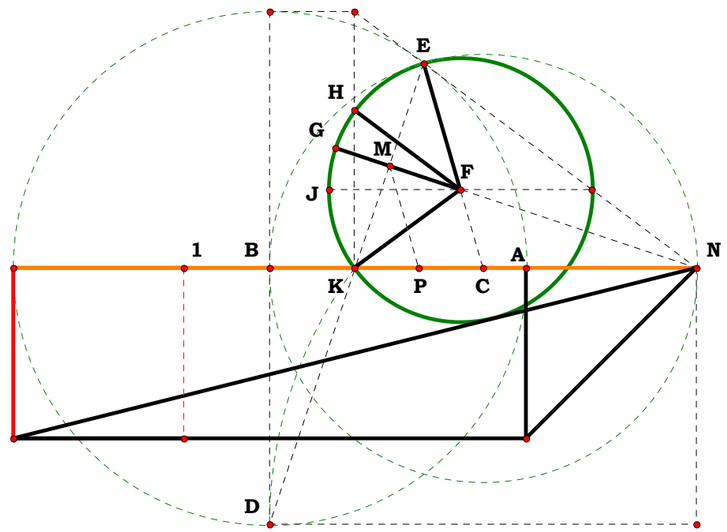
$EF := CE - CF \quad EM := KM$

EF = 0.769231 KM = 0.632456

Definitions.

$EF - \frac{\sqrt{N} \cdot (N + 1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \quad EM - \frac{\sqrt{2} \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2}{2 \cdot \sqrt{(N + 1) \cdot \left(\sqrt{N} - 1\right)^2}} = 0$

$\frac{EF}{EM} - \frac{(N + 1) \cdot \sqrt{(2 \cdot N + 2) \cdot \left(\sqrt{N} - 1\right)^2}}{\left(\sqrt{N} - 1\right)^2 \cdot \left(N + 4 \cdot \sqrt{N} + 1\right)} = 0$




N = 4.00000

EM = 1.81051 cm

GM = 0.94862 cm

m∠EFG = 55.30485°

m∠HFG = 18.43495°

m∠EFK = 110.60969°

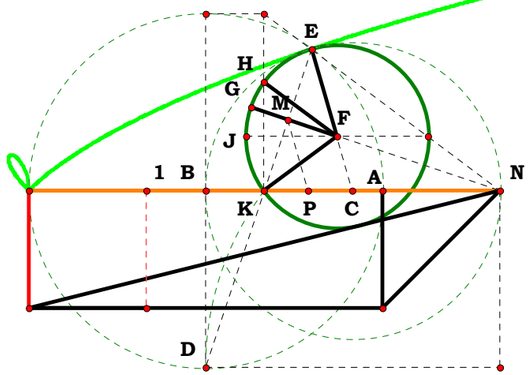
m∠HFJ = 36.86990°

EF = 0.76923

EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \quad \frac{m\angle EFK}{m\angle HFJ} = 3.00000$

$\frac{\sqrt{N} \cdot (N+1)}{N+4 \cdot \sqrt{N}+1} - EF = 0.00000 \quad \frac{\sqrt{2 \cdot N} \cdot \left(\sqrt{N}-1\right)^2}{2 \cdot \sqrt{(N+1) \cdot \left(\sqrt{N}-1\right)^2}} - EM = 0.00000$




N = 4.00000

EM = 1.24826 cm

GM = 0.65403 cm

m∠EFG = 55.30485°

m∠HFG = 18.43495°

m∠EFK = 110.60969°

m∠HFJ = 36.86990°

EF = 0.76923

EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \quad \frac{m\angle EFK}{m\angle HFJ} = 3.00000$

$\frac{\sqrt{N} \cdot (N+1)}{N+4 \cdot \sqrt{N}+1} - EF = 0.00000 \quad \frac{\sqrt{2 \cdot N} \cdot \left(\sqrt{N}-1\right)^2}{2 \cdot \sqrt{(N+1) \cdot \left(\sqrt{N}-1\right)^2}} - EM = 0.00000$

Unit := 1

Given.

N := 3

111193B2

Descriptions.

$H := \frac{N}{2}$ $J := N - Unit$ $F := N - \sqrt{(N + J)}$

$FK := \sqrt{F^2 + J^2}$ $FN := N - F$

$EF := \frac{F \cdot FN}{FK}$ $CF := 2 \cdot EF$ $EG := \frac{H}{2}$

$FH := H - F$ $FG := \frac{FH}{2}$ $GN := FN - FG$

$BH := \frac{EG \cdot H}{GN}$ $BC := H - BH$ $BC = 0.897763$

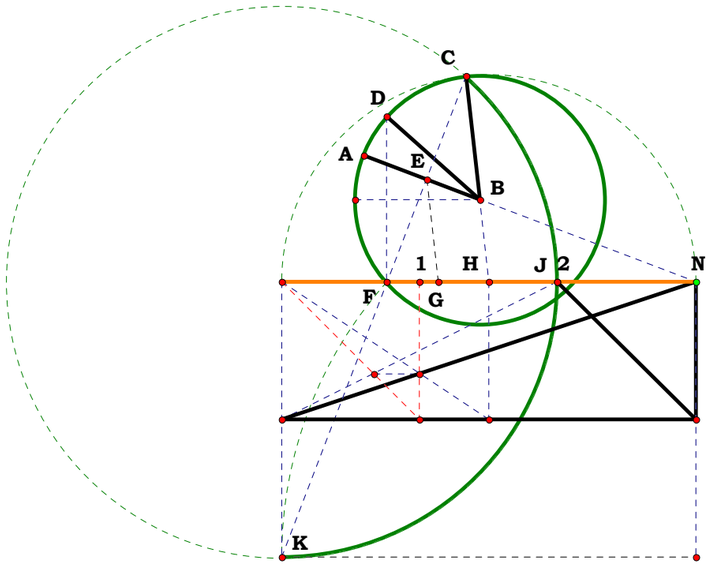
$CE := EF$ $CE = 0.797878$

Definitions.

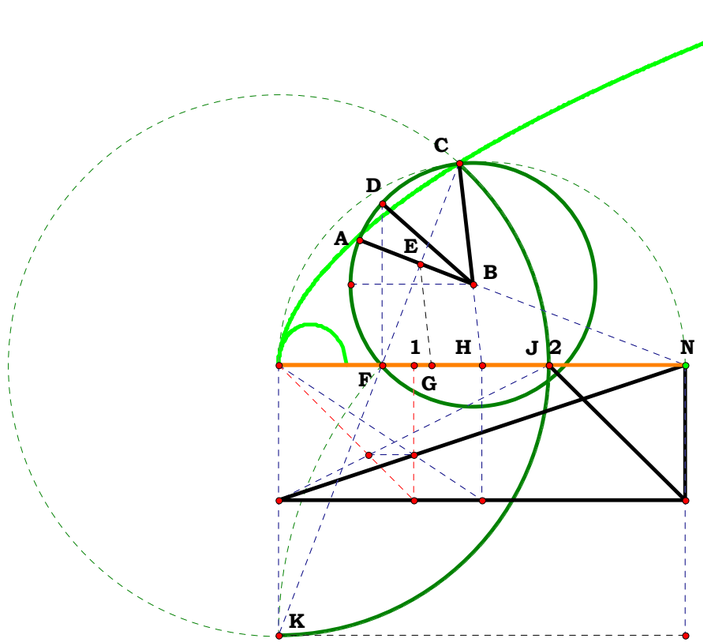
$$BC - \frac{N \cdot \sqrt{2 \cdot N - 1}}{N + 2 \cdot \sqrt{2 \cdot N - 1}} = 0$$

$$CE - \frac{\left(N \cdot \sqrt{2 \cdot N - 1} - 2 \cdot N + 1\right)}{\sqrt{2 \cdot N \cdot \left(N - \sqrt{2 \cdot N - 1}\right)}} = 0$$

$$\frac{BC}{CE} - \frac{\sqrt{2} \cdot \sqrt{2 \cdot N - 1} \cdot N \cdot \sqrt{N^2 - \sqrt{2 \cdot N - 1} \cdot N}}{\sqrt{2 \cdot N - 1} \cdot \left(N^2 - 4 \cdot N + 2\right) - N + 2 \cdot N^2} = 0$$




m∠ABC = 62.71547°
m∠ABD = 20.90516°
$\frac{m\angle ABC}{m\angle ABD}$ = 3.00000
N = 3.00000
BC = 0.89776
CE = 0.79788
$\frac{N \cdot \sqrt{2 \cdot N-1}}{N+2 \cdot \sqrt{2 \cdot N-1}}$-BC = 0.00000
$\frac{\left(N \cdot \sqrt{2 \cdot N-1}-2 \cdot N\right)+1}{\sqrt{2 \cdot N \cdot \left(N-\sqrt{2 \cdot N-1}\right)}}$-CE = 0.00000




m∠ABC = 62.71547°
m∠ABD = 20.90516°
$\frac{m\angle ABC}{m\angle ABD}$ = 3.00000
N = 3.00000
BC = 0.89776
CE = 0.79788
$\frac{N \cdot \sqrt{2 \cdot N-1}}{N+2 \cdot \sqrt{2 \cdot N-1}}$-BC = 0.00000
$\frac{\left(N \cdot \sqrt{2 \cdot N-1}-2 \cdot N\right)+1}{\sqrt{2 \cdot N \cdot \left(N-\sqrt{2 \cdot N-1}\right)}}$-CE = 0.00000

Unit := 1

Given.

N := 5

111193B3

Descriptions.

$P := N + Unit \qquad F := P - \sqrt{P} \qquad FP := P - F$

$O := \frac{N}{2} \qquad FO := F - O \qquad OP := P - O$

$AJ := \frac{OP}{2} \qquad FK := \sqrt{FO^2 + O^2} \qquad FG := \frac{FO \cdot FP}{FK}$

$GH := \frac{AJ}{2} \qquad FJ := AJ - FO$

$FH := \frac{FJ}{2} \qquad PH := FP - FH \qquad BJ := \frac{GH \cdot AJ}{PH}$

$AB := AJ - BJ \qquad AG := FG$

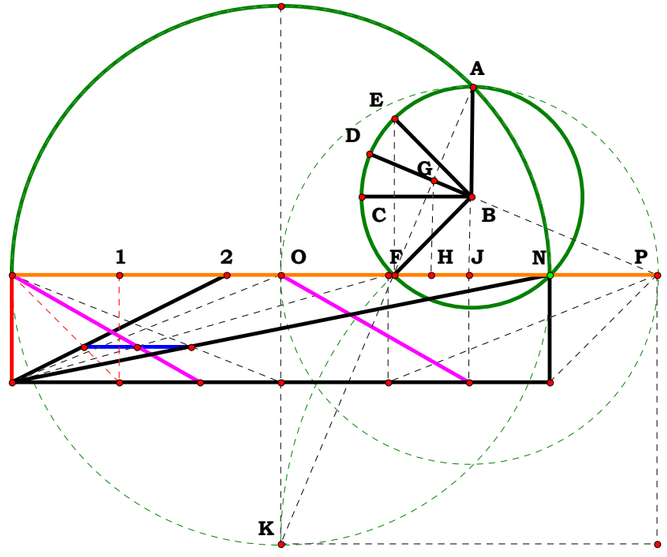
$AB = 1.020745 \qquad AG = 0.948914$

Definitions.

$$AB - \frac{\sqrt{N+1} \cdot (N+2)}{N + 4 \cdot \sqrt{N+1} + 2} = 0$$

$$AG - \frac{2 \cdot \sqrt{2} \cdot \left(\sqrt{N+1}\right)^3 - 2 \cdot \sqrt{2} \cdot N - 2 \cdot \sqrt{2} - \sqrt{2} \cdot N \cdot \sqrt{N+1}}{2 \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}} = 0$$

$$\frac{AB}{AG} - \frac{\sqrt{2} \cdot \sqrt{N+1} \cdot (N+2) \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}}{\left(N + 4 \cdot \sqrt{N+1} + 2\right) \cdot \left[2 \cdot \left(\sqrt{N+1}\right)^3 - N \cdot \sqrt{N+1} - 2 \cdot N - 2\right]} = 0$$




N = 5.00000

m∠ABD = 68.37704°

m∠DBE = 22.79235°

m∠ABF = 136.75407°

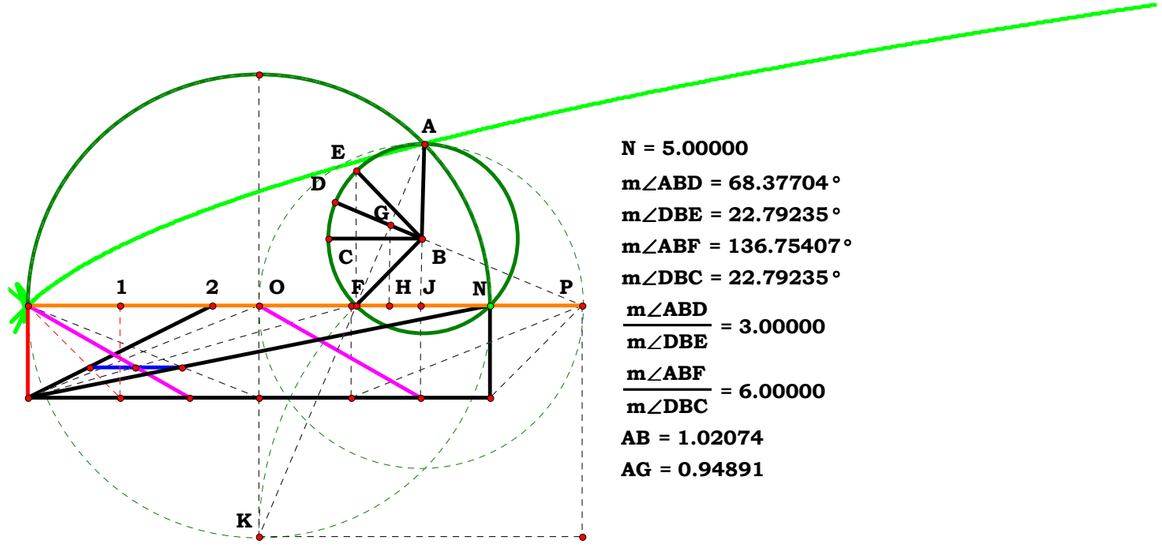
m∠DBC = 22.79235°

$\frac{m\angle ABD}{m\angle DBE} = 3.00000$

$\frac{m\angle ABF}{m\angle DBC} = 6.00000$

AB = 1.02074

AG = 0.94891




N = 5.00000

m∠ABD = 68.37704°

m∠DBE = 22.79235°

m∠ABF = 136.75407°

m∠DBC = 22.79235°

$\frac{m\angle ABD}{m\angle DBE} = 3.00000$

$\frac{m\angle ABF}{m\angle DBC} = 6.00000$

AB = 1.02074

AG = 0.94891

Unit.

$AC := 1$

Given.

$N := 5$

111193B4

**Descriptions.**

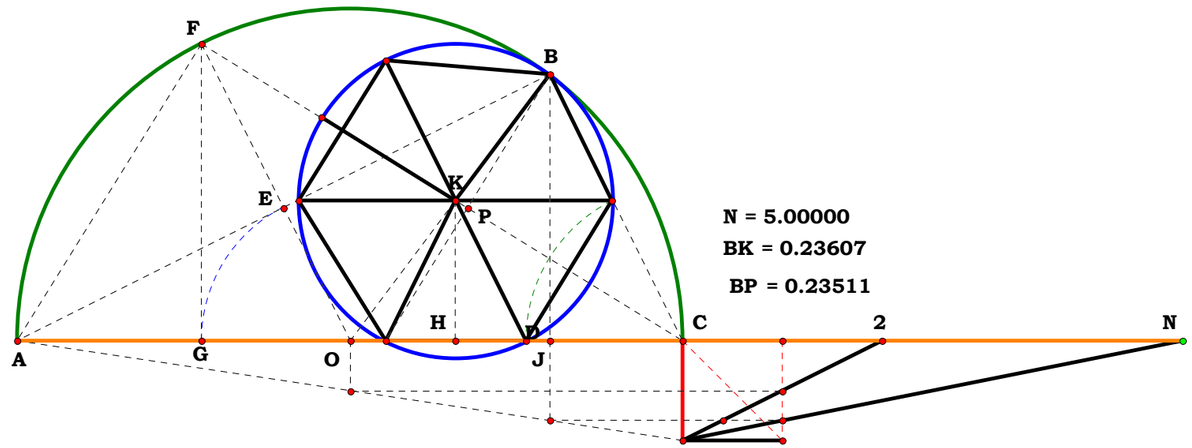
$AO := \frac{AC}{2}$ $CO := AO$ $CD := \frac{AC}{N}$ $AD := AC - CD$ $BD := \sqrt{AD \cdot CD}$

$AB := \sqrt{AD^2 + BD^2}$ $BE := \frac{AB}{2}$ $FG := BE$ $EO := \sqrt{AO^2 - BE^2}$

$AG := AO - EO$ $CG := AC - AG$ $DO := AD - AO$ $GM := \frac{DO \cdot FG}{BD}$

$CM := CG + GM$ $FM := \sqrt{GM^2 + FG^2}$ $KO := \frac{FM \cdot AO}{CM}$

$BK := AO - KO$ $BK = 0.236068$




**Definitions.**

$BK - \frac{1}{\sqrt{N} + 2} = 0$ $N - \left(\frac{1}{BK} - 2\right)^2 = 0$ $BC := \sqrt{CD^2 + BD^2}$ $CF := \sqrt{FG^2 + CG^2}$

$CP := \frac{CF \cdot BC}{AC}$ $BP := \sqrt{BC^2 - CP^2}$ $KP := \sqrt{BK^2 - BP^2}$

$KP - \frac{\sqrt{2} \cdot \sqrt{N^3 + 4 \cdot N^{\frac{3}{2}} - 3 \cdot N^{\frac{5}{2}}}}{2 \cdot \sqrt{4 \cdot N^3 + N^4 + 4 \cdot N^{\frac{7}{2}}}} = 0$ $KP - \frac{\sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}}{\sqrt{2 \cdot N^{\frac{3}{2}} \cdot \left(\sqrt{N} + 2\right)^2}} = 0$

$\frac{BK}{KP} = 11.135164$ $\frac{\sqrt{2} \cdot \sqrt{\left[\left(\sqrt{N}\right)^3 \cdot \left(\sqrt{N} + 2\right)^2\right]}}{\left(\sqrt{N} + 2\right) \cdot \sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}} = 11.135164$ $\frac{BK}{KP} - \frac{\sqrt{2} \cdot \sqrt{\left[\left(\sqrt{N}\right)^3 \cdot \left(\sqrt{N} + 2\right)^2\right]}}{\left(\sqrt{N} + 2\right) \cdot \sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}} = 0$

Unit.

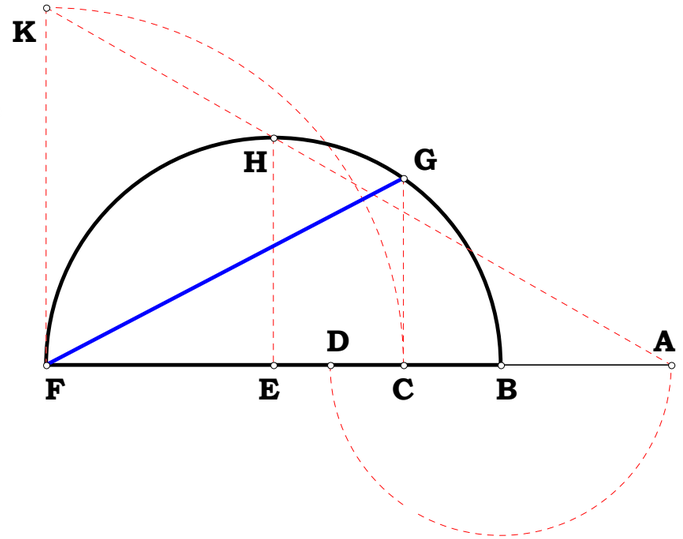
$BF := 1$

**To Square A Circle Off The Base Of A Right Triangle.**

**Using the approximation, π = 22/7, square the circle off the base of a right triangle.**

**111293**

**Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?**




**Descriptions.**

$BE := \frac{BF}{2}$ $EH := BE$ $BD := \frac{3}{4} \cdot BE$ $AB := BD$

$AE := AB + BE$ $AF := AB + BF$ $FK := \frac{EH \cdot AF}{AE}$ $CF := FK$

$BC := BF - CF$ $CG := \sqrt{BC \cdot CF}$ $FG := \sqrt{CF^2 + CG^2}$

$\pi\_A := \frac{FG^2}{BE^2}$

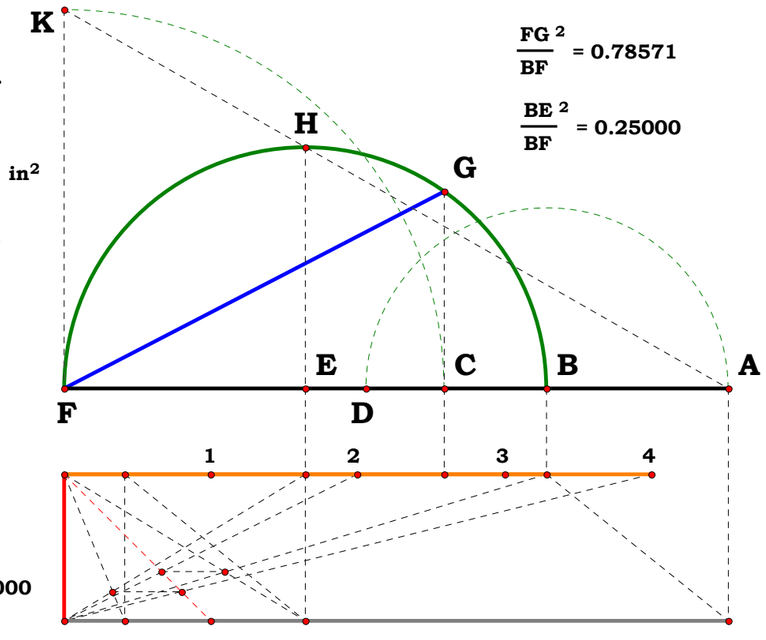
**Definitions.**

$\pi\_A - \frac{22}{7} = 0$

$\pi = 3.14159265359$

$\pi\_A = 3.142857142857$

$\frac{\pi}{\pi\_A} = 0.999597662505843$




DE = 0.39839 in.

BF = 3.18715 in.

AB = 1.19518 in.

AB·BF = 3.80922 $in^2$

FG = 2.82511 in.

BE = 1.59357 in.

BF = 3.18715 in.

$\frac{FG^2}{BF} = 0.78571$

$\frac{BE^2}{BF} = 0.25000$

$\frac{22}{\frac{FG^2}{BF}} = 28.00000$

$\frac{7}{\frac{BE^2}{BF}} = 28.00000$

FK = 2.50419 in.

$\left(\frac{FK}{BF}\right) \cdot 28 = 22.00000$

Unit.

$BJ := 1$

Given.

$N_1 := 2$

## Exploring Cube Roots Plate A

111893A

Descriptions. describe AB.

$BH := \frac{BJ}{2}$ $BD := \frac{BH}{N_1}$ $HJ := BH$

$DH := BH - BD$ $HR := BJ$ $DJ := DH + HJ$

$DL := \sqrt{BD \cdot DJ}$ $DF := \frac{DH \cdot DL}{DL + HR}$ $FO := BH$ $BF := BD + DF$

$MO := FO - DL$ $LM := DF$ $AF := \frac{LM \cdot FO}{MO}$ $AB := AF - BF$

Definitions.

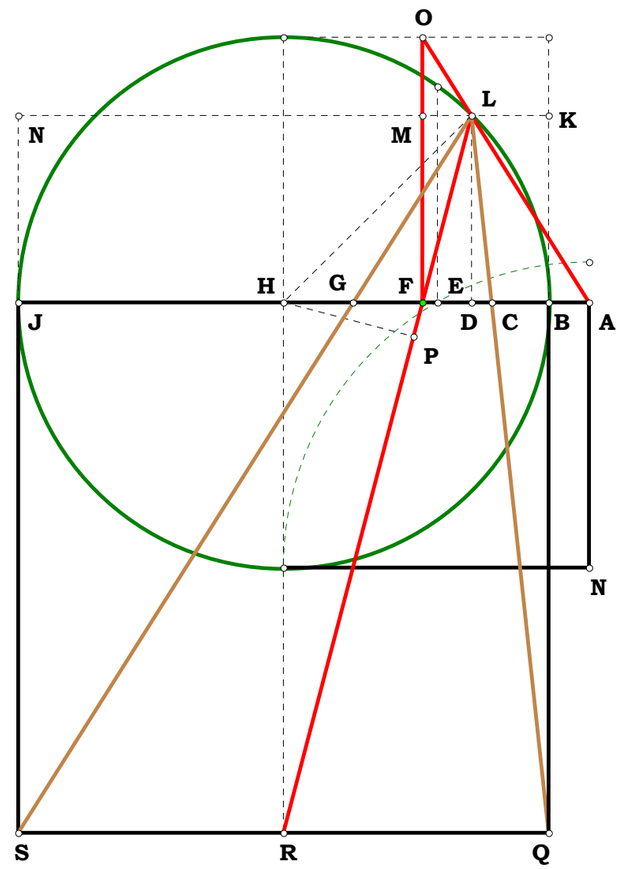
$BJ - 1 = 0$ $BH - \frac{1}{2} = 0$ $BD - \frac{1}{2 \cdot N_1} = 0$ $HJ - \frac{1}{2} = 0$ $DH - \frac{N_1 - 1}{2 \cdot N_1} = 0$

$HR - 1 = 0$ $DJ - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0$ $DL - \frac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$

$DF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot (2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1})} = 0$ $FO - \frac{1}{2} = 0$ $BF - \frac{\sqrt{2 \cdot N_1 - 1} + 2}{2 \cdot (2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1})} = 0$

$MO - \frac{N_1 - \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$ $LM - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot (2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1})} = 0$

$AF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot (2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}) \cdot (N_1 - \sqrt{2 \cdot N_1 - 1})} = 0$ $AB - \frac{\sqrt{2 \cdot N_1 - 1} - 1}{2 \cdot (2 \cdot N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{2 \cdot N_1 - 1} + 1)} = 0$



BH- $\frac{N_1}{2}$ = 0.00000 in.

BD- $\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

HJ- $\frac{N_1}{2}$ = 0.00000 in.

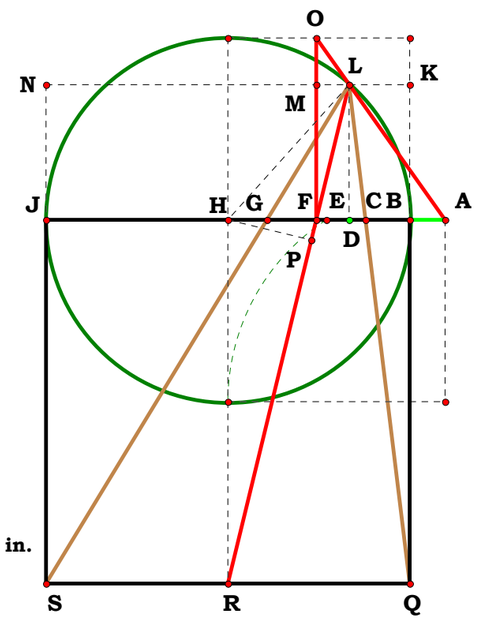
DH- $\frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

HR- $N_1$ = 0.00000 in.

DJ- $\frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

DL- $\frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2}$ = 0.00000 in.

DF- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.




FO- $\frac{N_1}{2}$ = 0.00000 in.

BF- $\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 2\right)}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

MO- $\frac{N_1 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right)}{2 \cdot N_2}$ = 0.00000 in.

LM- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

AF- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right) \cdot \left(2 \cdot N_2 - 2 \cdot \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

AB- $\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(\left(2 \cdot N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1}\right) + 1\right)}$ = 0.00000 in.

$N_1$ = 2.40000 in.

$N_2$ = 3.01550

BJ = 2.40000 in.

BH = 1.20000 in.

BD = 0.39794 in.

HJ = 1.20000 in.

$\frac{BH}{BD}$ = 3.01550

DH = 0.80206 in.

HR = 2.40000 in.

DJ = 2.00206 in.

DL = 0.89258 in.

DF = 0.21743 in.

FO = 1.20000 in.

BF = 0.61537 in.

MO = 0.30742 in.

LM = 0.21743 in.

AF = 0.84873 in.

AB = 0.23336 in.

Given.

$BJ := 1$

Given.

$N_1 := 3.01550$

111893B

## Exploring Cube Roots Plate B

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root. Notice in this write-up I chose the wrong point to proportion. I get the right answers, but the equations are more complicated. Compare features to plate A.**

**Descriptions.**

$BH := \frac{BJ}{2}$ $HL := BH$ $BF := \frac{BH}{N_1}$ $FH := BH - BF$ $HR := BJ$ $FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR}$ $PH := \frac{HR \cdot FP}{FH}$ $LP := \sqrt{HL^2 - PH^2}$ $FL := LP - FP$ $DF := \frac{FH \cdot FL}{FR}$

$DL := \frac{HR \cdot FL}{FR}$ $FO := BH$ $FM := DL$ $MO := FO - FM$ $LM := DF$ $AF := \frac{LM \cdot FO}{MO}$

$AB := AF - BF$ $BQ := BJ$ $BK := DL$ $BD := BF - DF$ $KQ := BQ + BK$ $KL := BD$

$BC := \frac{KL \cdot BQ}{KQ}$ $DJ := BJ - BD$ $LN := DJ$ $JS := BJ$ $JN := DL$ $NS := JS + JN$

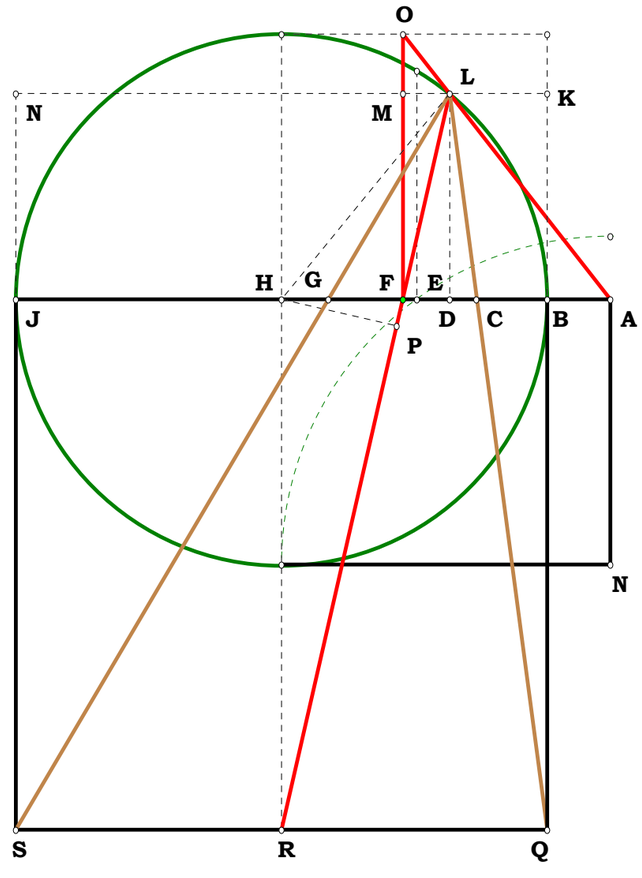
$GJ := \frac{LN \cdot JS}{NS}$ $BG := BJ - GJ$ $AC := AB + BC$ $AG := AB + BG$ $AJ := AB + BJ$

**Definitions.**

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

$BJ - 1 = 0$ $BH - \frac{1}{2} = 0$ $HL - \frac{1}{2} = 0$ $BF - \frac{1}{2 \cdot N_1} = 0$ $FH - \frac{N_1 - 1}{2 \cdot N_1} = 0$ $HR - 1 = 0$

$FR - \frac{\sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}}{2 \cdot N_1} = 0$ $FP - \frac{\left(N_1 - 1\right)^2}{2 \cdot N_1 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$ $PH - \frac{N_1 - 1}{\sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$



$$LP - \frac{\sqrt{N_1^2 + 6 \cdot N_1 - 3}}{2 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad FL - -\frac{N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1}{2 \cdot N_1 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$$

$$DF - -\frac{(N_1 - 1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0 \qquad DL - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0$$

$$FO - \frac{1}{2} = 0 \qquad FM - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad MO - \frac{7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

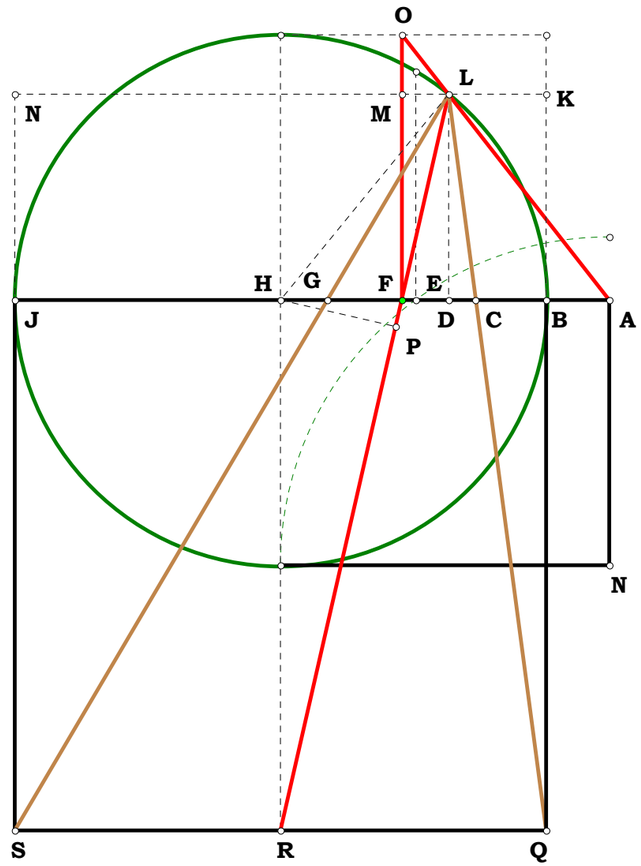
$$LM - -\frac{(N_1 - 1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0 \qquad AF - \frac{(1 - N_1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3\right)} = 0$$

$$AB - \frac{\left(N_1^2 + N_1\right) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3 \cdot N_1 - 4 \cdot N_1^2 - N_1^3 - 2}{2 \cdot N_1 \cdot \left(7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3\right)} = 0 \qquad BQ - 1 = 0$$

$$BK - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad BD - \frac{(1 - N_1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + N_1^2 + 2 \cdot N_1 + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

$$KQ - \frac{N_1 \cdot \left(4 \cdot N_1 + \sqrt{N_1^2 + 6 \cdot N_1 - 3}\right)}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad KL - \frac{2 \cdot N_1 + N_1^2 + (1 - N_1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

$$BC - \frac{2 \cdot N_1 + N_1^2 - \sqrt{N_1^2 + 6 \cdot N_1 - 3} \cdot (N_1 - 1) + 1}{2 \cdot N_1 \cdot \left(4 \cdot N_1 + \sqrt{N_1^2 + 6 \cdot N_1 - 3}\right)} = 0 \qquad DJ - \frac{(N_1 - 1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 9 \cdot N_1^2 - 6 \cdot N_1 + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$



$$LN - \frac{(N_1 - 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + 9\cdot N_1^2 - 6\cdot N_1 + 1}{2\cdot(5\cdot N_1^2 - 2\cdot N_1 + 1)} = 0 \qquad JS - 1 = 0$$

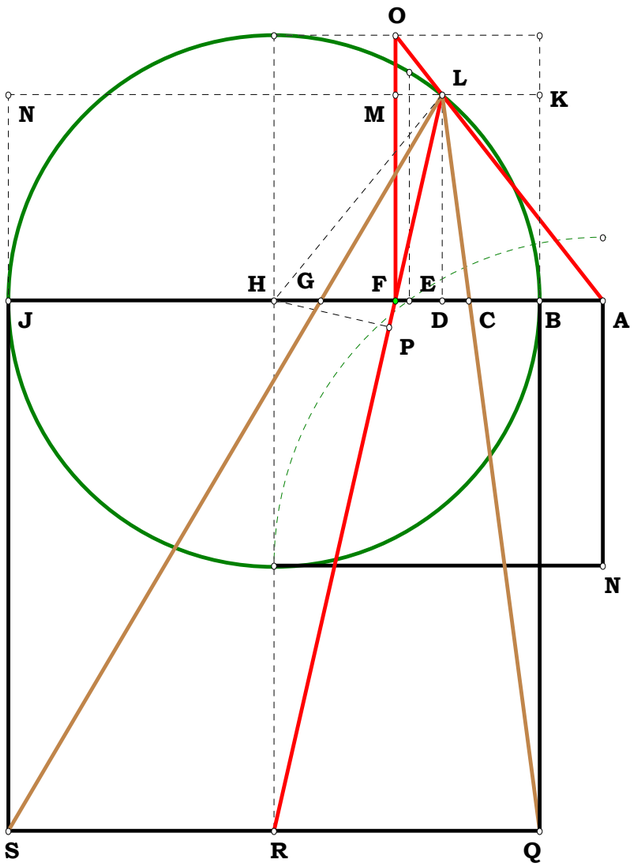
$$JN - \frac{2\cdot N_1 - N_1^2 + N_1\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} - 1}{5\cdot N_1^2 - 2\cdot N_1 + 1} = 0 \qquad NS - \frac{N_1\cdot(4\cdot N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3})}{5\cdot N_1^2 - 2\cdot N_1 + 1} = 0$$

$$GJ - \frac{(N_1 - 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + 9\cdot N_1^2 - 6\cdot N_1 + 1}{2\cdot N_1\cdot(4\cdot N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3})} = 0 \qquad BG - \frac{3 - N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3}}{6\cdot N_1} = 0$$

$$AC - \frac{(9\cdot N_1^2 - 6\cdot N_1^3 - 8\cdot N_1 + 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + (N_1 + 1)\cdot(6\cdot N_1^3 + 3\cdot N_1^2 - 8\cdot N_1 + 3)}{2\cdot N_1\cdot(26\cdot N_1^3 - 36\cdot N_1^2 + 18\cdot N_1) - 2\cdot N_1\cdot(N_1^2 + 6\cdot N_1 - 3)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}} = 0$$

$$AG - \frac{N_1^2 - 4\cdot N_1^3 + 1 + \sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot(4\cdot 1\cdot N_1^2 - 3\cdot 1\cdot N_1 + 1) - 2\cdot N_1}{6\cdot N_1 - 12\cdot N_1^2 + 14\cdot N_1^3 - 4\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot N_1^2} = 0$$

$$AJ - \frac{13\cdot N_1^3 - 16\cdot N_1^2 - 2 + \sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot(1\cdot N_1 - 3\cdot 1\cdot N_1^2) + 9\cdot N_1}{6\cdot N_1 - 12\cdot N_1^2 + 14\cdot N_1^3 - 4\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot N_1^2} = 0$$



**Etc.**

BF- $\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

FH- $\frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

FR- $\frac{N_1 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}{2 \cdot N_2}$ = 0.00000 in.

FP- $\frac{N_1 \cdot (N_2 - 1)^2}{2 \cdot N_2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

PH- $\frac{N_1 \cdot (N_2 - 1)}{\sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

LP- $\frac{N_1 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}}{2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

FL- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

DF- $\frac{N_1 \cdot (N_2 - 1) \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

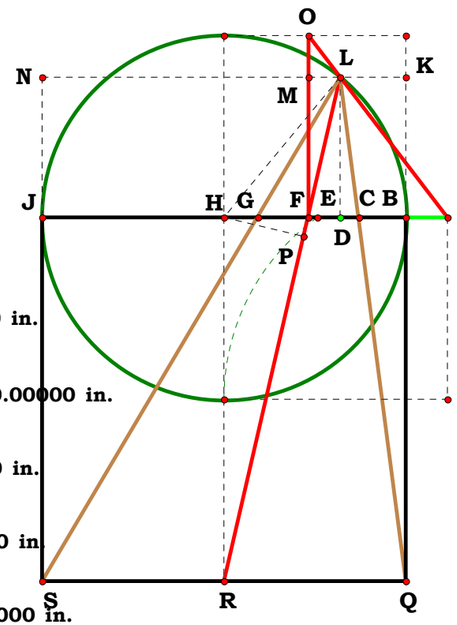
DL- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}$ = 0.00000 in.

FM- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}$ = 0.00000 in.

MO- $\frac{N_1 \cdot ((7 \cdot N_2^2 - 6 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) + 3)}{2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

LM- $\frac{N_1 \cdot (N_2 - 1) \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

AF- $\frac{N_1 \cdot (N_2 - 1) \cdot ((N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) + 1)}{2 \cdot N_2 \cdot (((6 \cdot N_2 - 7 \cdot N_2^2) + 2 \cdot N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 3)}$ = 0.00000 in.




AB- $\frac{N_1 \cdot ((((3 \cdot N_2 + N_2^2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 4 \cdot N_2^2 - N_2^3) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 2)}{2 \cdot ((3 \cdot N_2 - 2 \cdot N_2^2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3} - 6 \cdot N_2^2) + 7 \cdot N_2^3)}$ = 0.00000 in

$N_1$ = 2.40000 in.

$N_2$ = 1.86107

Animate Point

BJ = 2.40000 in.

BH = 1.20000 in.

BF = 0.64479 in.

$\frac{BH}{BF}$ = 1.86107

FH = 0.55521 in.

FR = 2.46338 in.

FP = 0.12514 in.

PH = 0.54092 in.

LP = 1.07117 in.

FL = 0.94603 in.

DF = 0.21322 in.

DL = 0.92169 in.

FM = 0.92169 in.

MO = 0.27831 in.

LM = 0.21322 in.

AF = 0.91936 in.

AB = 0.27457 in.

Unit.

$BK := 1$

Given.

$N_1 := 4$

111893C

Descriptions.

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N_1}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

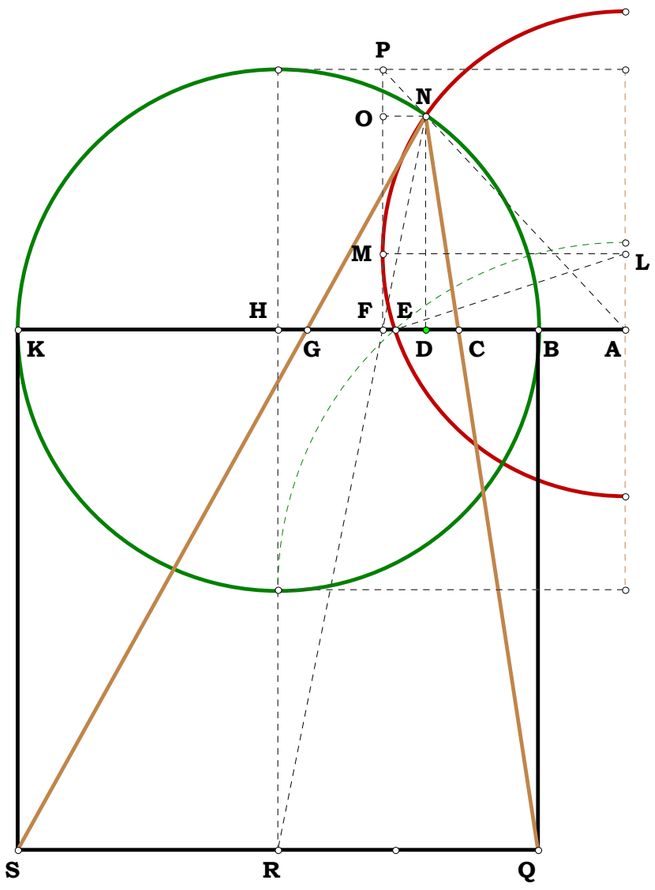
Definitions.

$BH - \frac{1}{2} = 0$ $BD - \frac{1}{2 \cdot N_1} = 0$ $DK - \frac{(2 \cdot N_1 - 1)}{2 \cdot N_1} = 0$

$DN - \frac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$ $BQ - 1 = 0$ $KS - 1 = 0$

## Exploring Cube Roots Plate C

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**

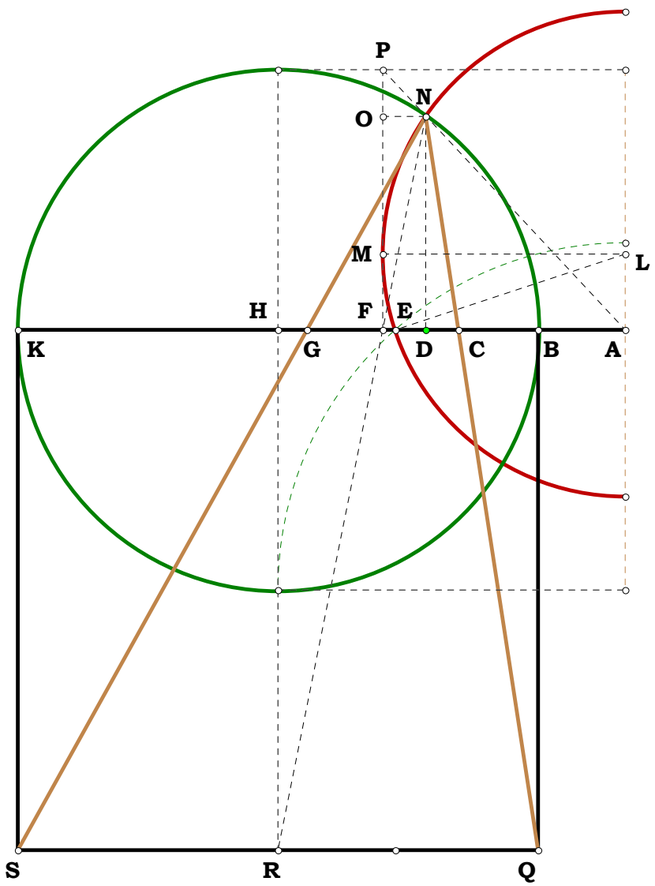


$$HR - 1 = 0 \qquad BC - \frac{1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0 \qquad GK - \frac{2\cdot N_1 - 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0$$

$$BG - \frac{\sqrt{2\cdot N_1 - 1} + 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0 \qquad DH - \frac{N_1 - 1}{2\cdot N_1} = 0 \qquad FH - \frac{N_1 - 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0$$

$$BF - \frac{\sqrt{2\cdot N_1 - 1} + 2}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad CF - \frac{\sqrt{2\cdot N_1 - 1}}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad AL - \frac{\sqrt{2\cdot N_1 - 1}}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$

$$DF - \frac{\left(N_1 - 1\right)\cdot\sqrt{2\cdot N_1 - 1}}{2\cdot N_1\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad NO - \frac{\left(N_1 - 1\right)\cdot\sqrt{2\cdot N_1 - 1}}{2\cdot N_1\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$




$$FP - \frac{1}{2} = 0 \qquad PO - \frac{N_1 - \sqrt{2\cdot N_1 - 1}}{2\cdot N_1} = 0 \qquad AD - \frac{\left(N_1 - 1\right)\cdot\left(\sqrt{2\cdot N_1 - 1}\right)^2}{2\cdot N_1\cdot\left(N_1 - \sqrt{2\cdot N_1 - 1}\right)\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$

$$AB - \frac{\left(\sqrt{2\cdot N_1 - 1} - 1\right)}{2\cdot\left(2\cdot {N_1}^2 - 2\cdot N_1 - N_1\cdot\sqrt{2\cdot N_1 - 1} + 1\right)} = 0 \qquad AF - \frac{\left(N_1 - 1\right)\cdot\left(2\cdot N_1 + 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)}{2\cdot\left(3\cdot N_1 + \sqrt{2\cdot N_1 - 1} - 6\cdot {N_1}^2 + 4\cdot {N_1}^3 - 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad EL - AF = 0 \qquad LM - AF = 0$$

$$AK - \frac{\left[4\cdot {N_1}^2 - \left(2\cdot N_1 - 1\right)^{\frac{3}{2}} - 4\cdot N_1 + 1\right]}{2\cdot\left(2\cdot {N_1}^2 - 2\cdot N_1 - N_1\cdot\sqrt{2\cdot N_1 - 1} + 1\right)} = 0 \qquad AE_1 - \sqrt{\left[\frac{\left(N_1 - 1\right)^2\cdot\left(2\cdot N_1 + 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)^2}{4\cdot\left(3\cdot N_1 + \sqrt{2\cdot N_1 - 1} - 6\cdot {N_1}^2 + 4\cdot {N_1}^3 - 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1}\right)^2} - \frac{\left(2\cdot N_1 - 1\right)}{4\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)^2}\right]} = 0$$

$$AE_2 - \sqrt{\left[\frac{\left(\sqrt{2\cdot N_1 - 1} - 1\right)\cdot\left[4\cdot {N_1}^2 - \left(2\cdot N_1 - 1\right)^{\frac{3}{2}} - 4\cdot N_1 + 1\right]}{4\cdot\left(2\cdot N_1 - 2\cdot {N_1}^2 + N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)^2}\right]} = 0 \qquad AE_1 - AE_2 = 0$$

111893D

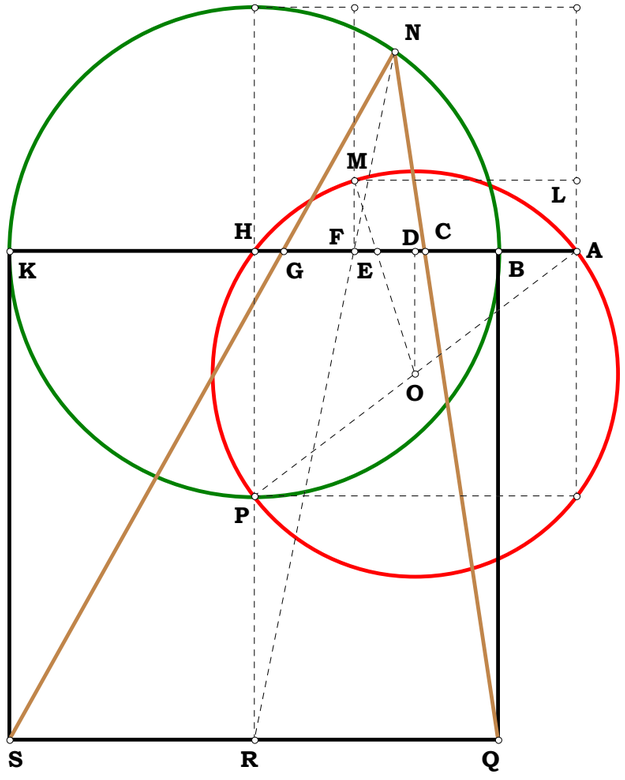
$AB := 1$

Given.

$N_1 := 3$

## Exploring Cube Roots Plate D

**The circle AO passes through point M. FM equals half of CG.**




**Descriptions.**

$BK := N_1 \quad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \qquad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

$CG := AG - AC \qquad CF := \frac{CG}{2} \qquad BH := \frac{BK}{2}$

$AH := AB + BH \qquad HP := BH \qquad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \qquad DO := \frac{HP}{2} \qquad AF := AC + CF \qquad AD := \frac{AH}{2}$

$DF := AF - AD \qquad FM := CF \qquad MO := AO$

**Definitions.**

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0 \qquad AK - \left(1 + N_1\right) = 0$

$AC - \left(N_1 + 1\right)^{\frac{1}{3}} = 0 \qquad AG - \left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} = 0 \qquad CG - \left[\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}\right] = 0$

$CF - \frac{\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}}{2} = 0 \qquad BH - \frac{N_1}{2} = 0 \qquad AH - \frac{2 + N_1}{2} = 0 \qquad HP - \frac{N_1}{2} = 0 \qquad AP - \frac{\sqrt{N_1^2 + 2 \cdot N_1 + 2}}{\sqrt{2}} = 0$

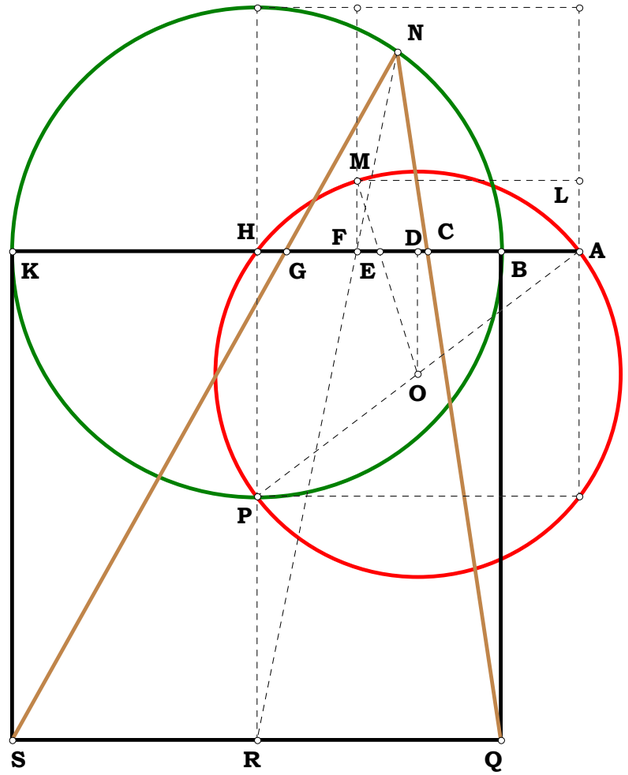
$$AO - \frac{\sqrt{2}}{4}\cdot\sqrt{\left(N_1{}^2 + 2\cdot N_1 + 2\right)} = 0 \qquad DO - \frac{N_1}{4} = 0 \qquad AF - \frac{\left[\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} + \left(N_1+1\right)^{\frac{1}{3}}\right]}{2} = 0$$

$$AD - \frac{2+N_1}{4} = 0 \qquad DF - \frac{\left[2\cdot\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - N_1 - 2 + 2\cdot\left(N_1+1\right)^{\frac{1}{3}}\right]}{4} = 0$$

$$FM - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - \left(N_1+1\right)^{\frac{1}{3}}}{2} = 0 \qquad MO - \frac{\sqrt{2}}{4}\cdot\sqrt{2\cdot 1^2 + 2\cdot 1\cdot N_1 + N_1{}^2} = 0$$

$$MO^2 - \frac{\left[\begin{array}{l} 2\cdot N_1 + N_1{}^2 - 4\cdot N_1\cdot\left(N_1+1\right)^{\frac{1}{3}} - 4\cdot\left(N_1+1\right)^{\frac{1}{3}} \ \ldots \\ +\ 4\cdot\left(N_1+1\right)^{\frac{2}{3}} - 4\cdot\left(N_1{}^2 + 2\cdot N_1 + 1\right)^{\frac{1}{3}} + 4\cdot\left(N_1{}^2 + 2\cdot N_1 + 1\right)^{\frac{2}{3}} + 2 \end{array}\right]}{8} = 0$$

$$\frac{\left[\left(N_1+1\right)^2\right]^{\frac{2}{3}}}{2} + \frac{\left(N_1+1\right)^{\frac{2}{3}}}{2} - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}}}{2} - \frac{\left(N_1+1\right)^{\frac{1}{3}}}{2} - \frac{N_1\cdot\left(N_1+1\right)^{\frac{1}{3}}}{2} = 0$$



**Unit.** $AB := 1$

**Given.** $AE := 2$

$\delta := 0 .. \Delta$

# 112293 Cube by Iteration

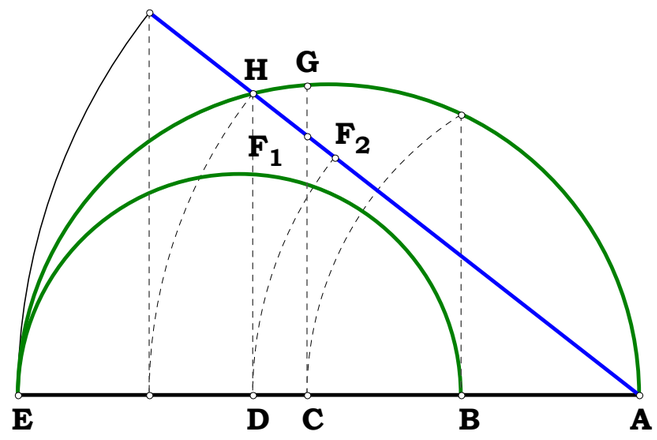
**Descriptions.**

**When $F_1$ and $F_2$ are the same point on C, then a sixth root series has been constructed. Use iteration to place $F_2$ on $F_1$.**

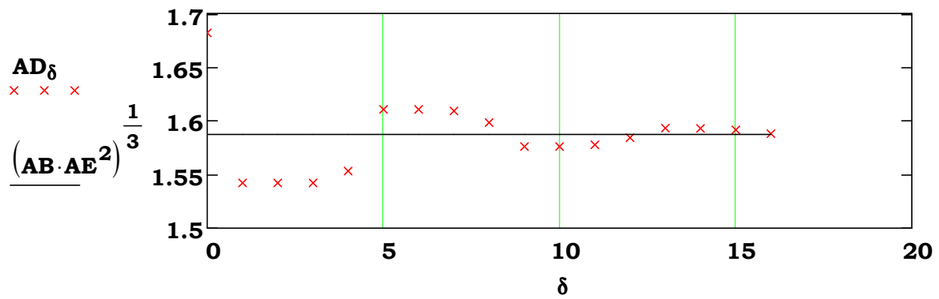
$AC := \sqrt{AB \cdot AE}$ $\quad CE := AE - AC$ $\quad CG := \sqrt{AC \cdot CE}$ $\quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{pmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{pmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[\dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG}\right]^2} \end{bmatrix}$$

$$\begin{pmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{pmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$




$\Delta \equiv 16$




$\left(AB^2 \cdot AE^4\right)^{\frac{1}{6}} - AD_\Delta = -6.761441 \times 10^{-4}$

$\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD_\Delta = -6.761441 \times 10^{-4}$

**Not a very promising prospect!**

112493

**Unit.**

$AE := 1$

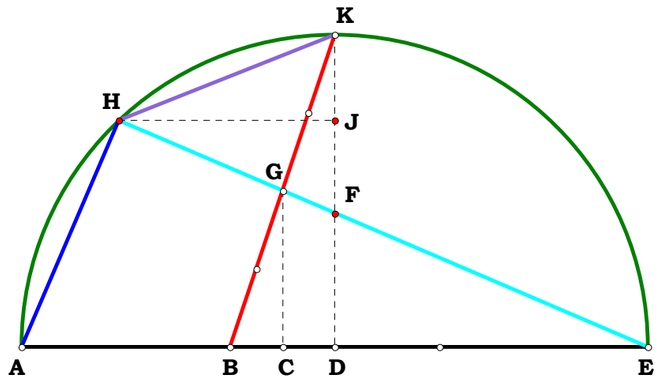
**POR Series IV**

**Given.**

$N_1 := 3$ $\alpha := 1 .. N_1 - 1$

$N_2 := 5$ $\beta := 1 .. N_2 - 1$

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**




**Descriptions.**

$AB := \frac{AE}{N_1}$ $AD := \frac{AE}{2}$ $DK := AD$ $DE := AD$

$BD := AD - AB$ $BK := \sqrt{BD^2 + DK^2}$ $BG := \frac{BK}{N_2}$ $BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK}$ $AC := AB + BC$ $CE := AE - AC$ $DF := \frac{CG \cdot DE}{CE}$ $EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2}$ $AH := \frac{DF \cdot AE}{EF}$ $EH := \frac{DE \cdot AE}{EF}$ $GH := EH - EG$ $FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF}$ $HJ := \frac{DE \cdot FH}{EF}$ $DJ := DF + FJ$ $JK := DK - DJ$ $HK := \sqrt{HJ^2 + JK^2}$

**Definitions.**

$\frac{AH}{HK} = 0.265165$ $\frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265165$

$\frac{EH}{GH} = 2.85$ $N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesAH_{\alpha, \beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

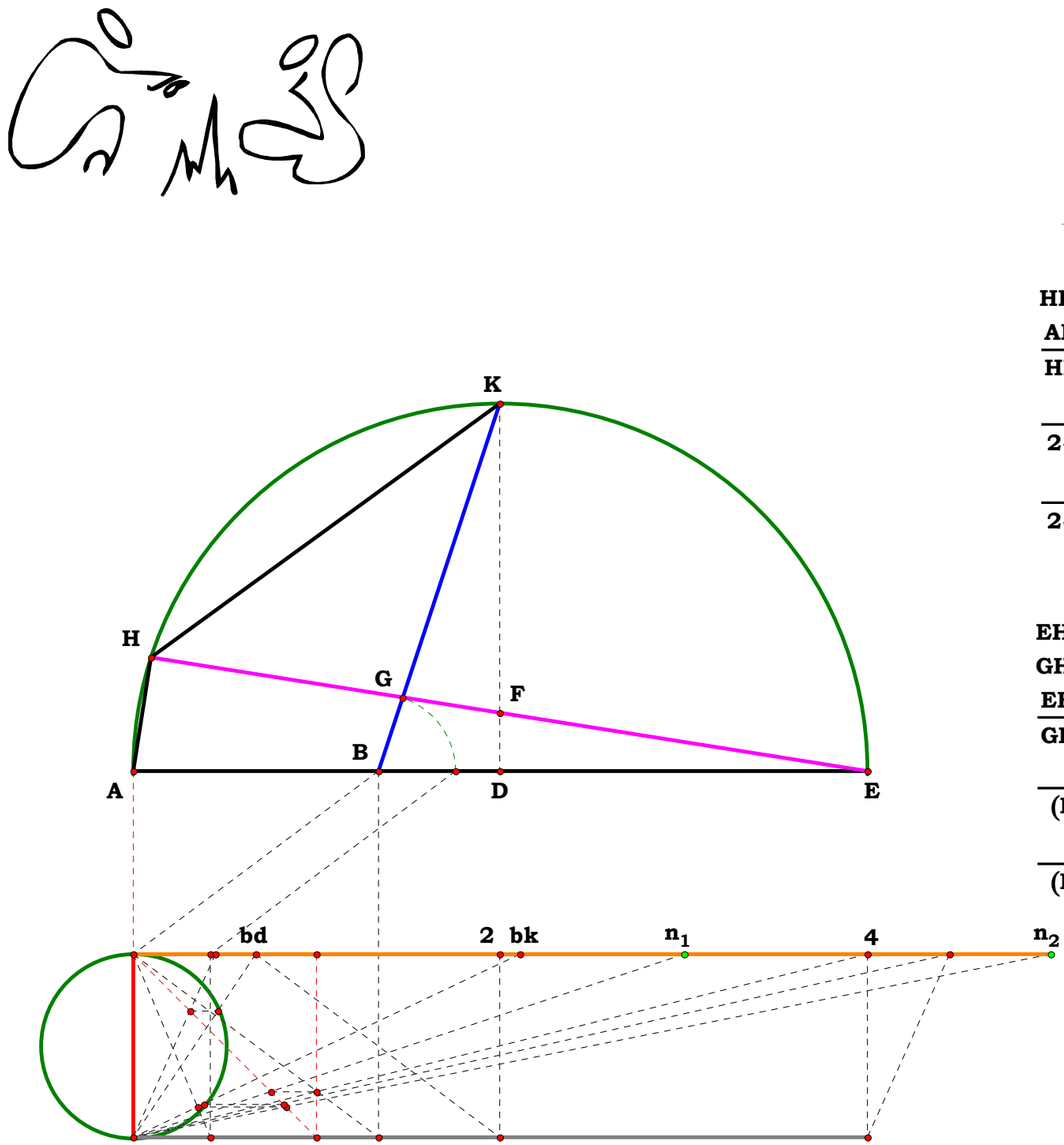
$SeriesEH_{\alpha, \beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$SeriesAH =$

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.265165 | 0.707107 | 1.59099 | 4.242641 |
| 2 | 0.53033 | 1.414214 | 3.181981 | 8.485281 |

$SeriesEH =$

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2.85 | 3 | 3.643 | 6 |
| 2 | 1.65 | 2 | 2.786 | 5.25 |




$$HK = 2.35269$$

$$\frac{AH}{HK} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} - \frac{AH}{HK} = 0.00000$$

$$EH = 3.95105$$

$$GH = 1.38633$$

$$\frac{EH}{GH} = 2.85000$$

$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} = 2.85000$$

$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} - \frac{EH}{GH} = 0.00000$$

**Unit.**

$AH := 1$

**Given.**

$\delta := 1 .. 7$

$\Delta := 1 .. 6$

**120293**

**Descriptions.**

$AP_\delta := \frac{AH}{\delta}$ $\quad AG_\delta := \frac{(AP_\delta)^2}{AH}$ $\quad AO_\delta := AG_\delta$

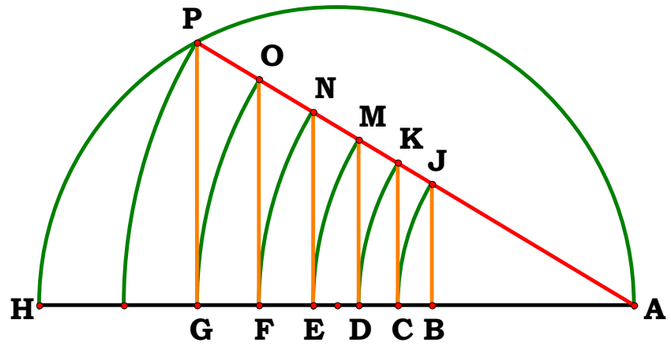
$AF_\delta := \frac{(AG_\delta)^2}{AP_\delta}$ $\quad AE_\delta := \frac{(AF_\delta)^2}{AO_\delta}$ $\quad AN_\delta := AF_\delta$

$AD_\delta := \frac{(AE_\delta)^2}{AN_\delta}$ $\quad AM_\delta := AE_\delta$

$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta}$ $\quad AK_\delta := AD_\delta$ $\quad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta}$

**POR Roots and Powers (Pyramid of Ratio Series V)**

**Is the progression noticed in 112993 a continuous phenomenon?**



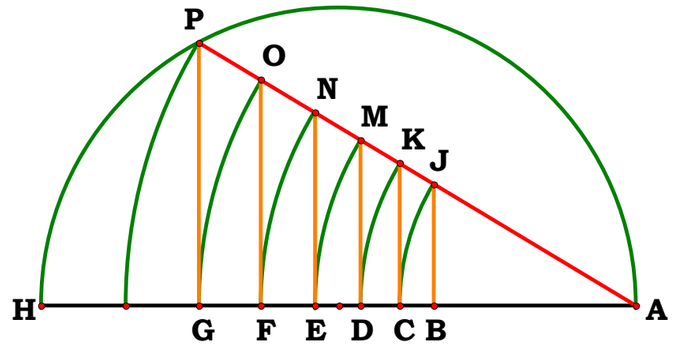

**Definitions.**

$\frac{AH}{AP_\Delta} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} =$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

$$\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}} = \frac{AC_\Delta}{AB_\Delta} =$$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 |




$$\left(\begin{array}{ccccc}
\frac{AH}{1}, & \frac{AH}{1^2}, & \frac{AH}{1^3}, & \frac{AH}{1^4}, & \text{etc} \\
\frac{AH}{2}, & \frac{AH}{2^2}, & \frac{AH}{2^3}, & \frac{AH}{2^4}, & \text{etc} \\
\frac{AH}{3}, & \frac{AH}{3^2}, & \frac{AH}{3^3}, & \frac{AH}{3^4}, & \text{etc} \\
\frac{AH}{4}, & \frac{AH}{4^2}, & \frac{AH}{4^3}, & \frac{AH}{4^4}, & \text{etc} \\
\frac{AH}{5}, & \frac{AH}{5^2}, & \frac{AH}{5^3}, & \frac{AH}{5^4}, & \text{etc}
\end{array}\right)$$

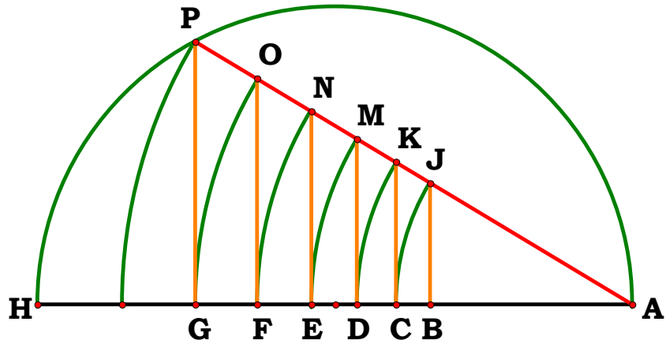
| $AP_\Delta =$ | $AG_\Delta =$ | $AF_\Delta =$ | $AE_\Delta =$ | $AD_\Delta =$ | $AC_\Delta =$ | $AB_\Delta =$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.5 | 0.25 | 0.125 | 0.0625 | 0.03125 | 0.015625 | $7.8125 \cdot 10^{-3}$ |
| 0.333333 | 0.111111 | 0.037037 | 0.012346 | $4.115226 \cdot 10^{-3}$ | $1.371742 \cdot 10^{-3}$ | $4.572474 \cdot 10^{-4}$ |
| 0.25 | 0.0625 | 0.015625 | $3.90625 \cdot 10^{-3}$ | $9.765625 \cdot 10^{-4}$ | $2.441406 \cdot 10^{-4}$ | $6.103516 \cdot 10^{-5}$ |
| 0.2 | 0.04 | $8 \cdot 10^{-3}$ | $1.6 \cdot 10^{-3}$ | $3.2 \cdot 10^{-4}$ | $6.4 \cdot 10^{-5}$ | $1.28 \cdot 10^{-5}$ |
| 0.166667 | 0.027778 | $4.62963 \cdot 10^{-3}$ | $7.716049 \cdot 10^{-4}$ | $1.286008 \cdot 10^{-4}$ | $2.143347 \cdot 10^{-5}$ | $3.572245 \cdot 10^{-6}$ |

$$AP_\Delta = \quad AG_\Delta \cdot \frac{AH}{AP_\Delta} = \quad AF_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^2 = \quad AE_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^3 = \quad AD_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^4 = \quad AC_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^5 = \quad AB_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^6 =$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| 0.333333 | 0.333333 | 0.333333 | 0.333333 | 0.333333 | 0.333333 | 0.333333 |
| 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 | 0.166667 |

$$\frac{AH^2}{AP_\Delta} = \left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH =$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |




$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{3} =$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 64 | 64 | 64 | 64 | 64 | 64 | 64 |
| 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| 216 | 216 | 216 | 216 | 216 | 216 | 216 |

$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}} =$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 128 | 128 | 128 | 128 | 128 | 128 | 128 |
| $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ |
| $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ | $1.6384 \cdot 10^4$ |
| $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ | $7.8125 \cdot 10^4$ |
| $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ | $2.79936 \cdot 10^5$ |

$$GH_\delta := AH - AG_\delta \qquad GP_\delta := \sqrt{AG_\delta \cdot GH_\delta} \qquad FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta} \qquad EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta}$$

$$DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta} \qquad CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta} \qquad BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$$

$\frac{GP_\Delta}{\Delta^5} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{FO_\Delta}{\Delta^4} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{EN_\Delta}{\Delta^3} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{DM_\Delta}{\Delta^2} =$

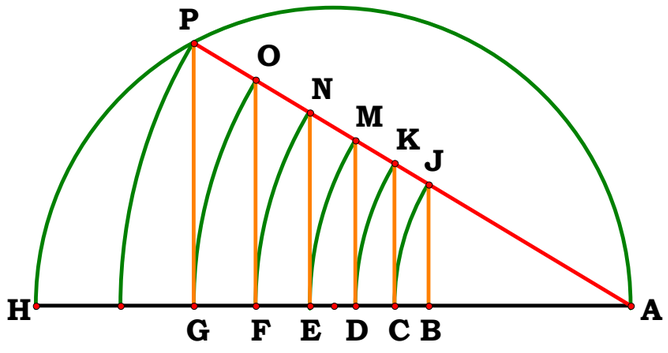
| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{CK_\Delta}{\Delta} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$BJ_\Delta =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |




$GP_\Delta =$

| |
|---|
| 0 |
| 0.433013 |
| 0.31427 |
| 0.242061 |
| 0.195959 |
| 0.164336 |

$FO_\Delta =$

| |
|---|
| 0 |
| 0.216506 |
| 0.104757 |
| 0.060515 |
| 0.039192 |
| 0.027389 |

$EN_\Delta =$

| |
|---|
| 0 |
| 0.108253 |
| 0.034919 |
| 0.015129 |
| $7.838367 \cdot 10^{-3}$ |
| $4.564876 \cdot 10^{-3}$ |

$DM_\Delta =$

| |
|---|
| 0 |
| 0.054127 |
| 0.01164 |
| $3.78221 \cdot 10^{-3}$ |
| $1.567673 \cdot 10^{-3}$ |
| $7.608127 \cdot 10^{-4}$ |

$CK_\Delta =$

| |
|---|
| 0 |
| 0.027063 |
| $3.879873 \cdot 10^{-3}$ |
| $9.455526 \cdot 10^{-4}$ |
| $3.135347 \cdot 10^{-4}$ |
| $1.268021 \cdot 10^{-4}$ |

$BJ_\Delta =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$\frac{CK_\Delta}{\frac{AH}{AP_\Delta}} =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

$BJ_\Delta =$

| |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |

| $GP_{\Delta}$ = |
|---|
| 0 |
| 0.433013 |
| 0.31427 |
| 0.242061 |
| 0.195959 |
| 0.164336 |

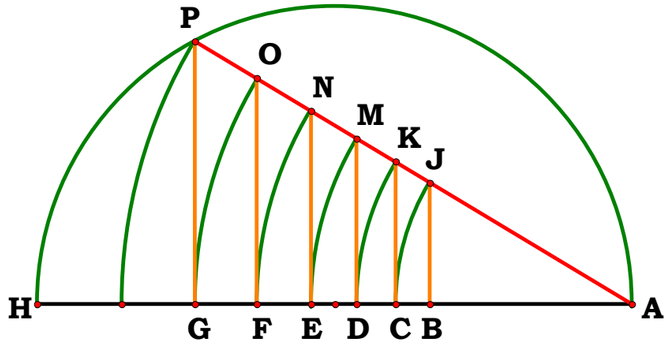
| $FO_{\Delta}$ = |
|---|
| 0 |
| 0.216506 |
| 0.104757 |
| 0.060515 |
| 0.039192 |
| 0.027389 |

| $EN_{\Delta}$ = |
|---|
| 0 |
| 0.108253 |
| 0.034919 |
| 0.015129 |
| $7.838367 \cdot 10^{-3}$ |
| $4.564876 \cdot 10^{-3}$ |

| $DM_{\Delta}$ = |
|---|
| 0 |
| 0.054127 |
| 0.01164 |
| $3.78221 \cdot 10^{-3}$ |
| $1.567673 \cdot 10^{-3}$ |
| $7.608127 \cdot 10^{-4}$ |

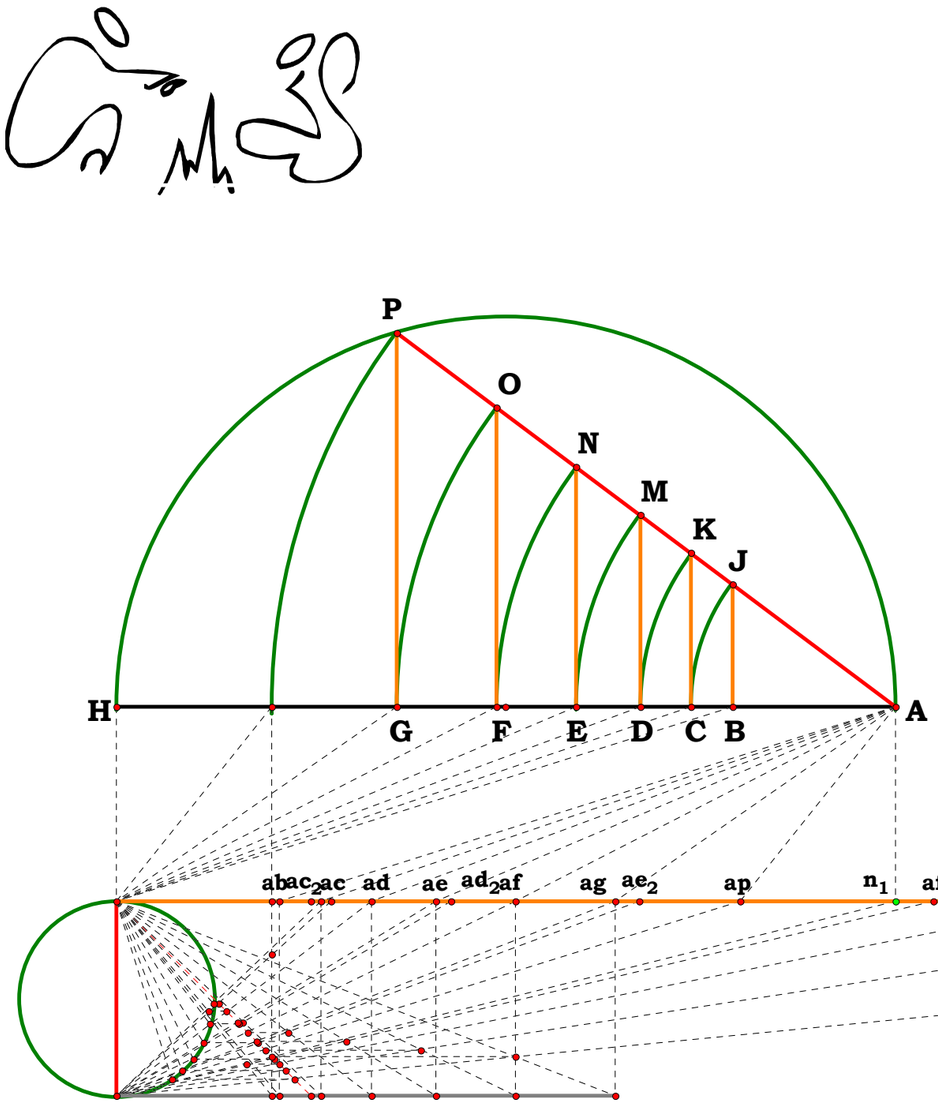
| $CK_{\Delta}$ = |
|---|
| 0 |
| 0.027063 |
| $3.879873 \cdot 10^{-3}$ |
| $9.455526 \cdot 10^{-4}$ |
| $3.135347 \cdot 10^{-4}$ |
| $1.268021 \cdot 10^{-4}$ |

| $BJ_{\Delta}$ = |
|---|
| 0 |
| 0.013532 |
| $1.293291 \cdot 10^{-3}$ |
| $2.363881 \cdot 10^{-4}$ |
| $6.270694 \cdot 10^{-5}$ |
| $2.113369 \cdot 10^{-5}$ |




$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = 0.216506 \quad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = 0.108253 \quad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = 0.054127$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = 0.104757 \quad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = 0.034919 \quad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = 0.01164$$




ab = 0.83886
ac = 1.04858
ad = 1.31072
ae = 1.63840
af = 2.04800
ag = 2.56000
ap = 3.20000

$\frac{N_1}{ap} = 1.25000$

$\frac{N_1\,\frac{1}{7}}{ab} = 1.25000$

$\frac{N_1\,\frac{1}{6}}{ac} = 1.25000$

$\frac{N_1\,\frac{1}{5}}{ad} = 1.25000$

$\frac{N_1\,\frac{1}{4}}{ae} = 1.25000$

$\frac{N_1\,\frac{1}{3}}{af} = 1.25000$

$\frac{N_1\,\frac{1}{2}}{ag} = 1.25000$

$\frac{ap\,\frac{1}{6}}{ab} = 1.25000$

$\frac{ag\,\frac{1}{5}}{ab} = 1.25000$

$\frac{af\,\frac{1}{4}}{ab} = 1.25000$

$\frac{ae\,\frac{1}{3}}{ab} = 1.25000$

$\frac{ad\,\frac{1}{2}}{ab} = 1.25000$

$\frac{ac}{ab} = 1.25000$

# Exponential Series M^(1/2^N)

**Unit.**

$AB := 1$

**Given.**

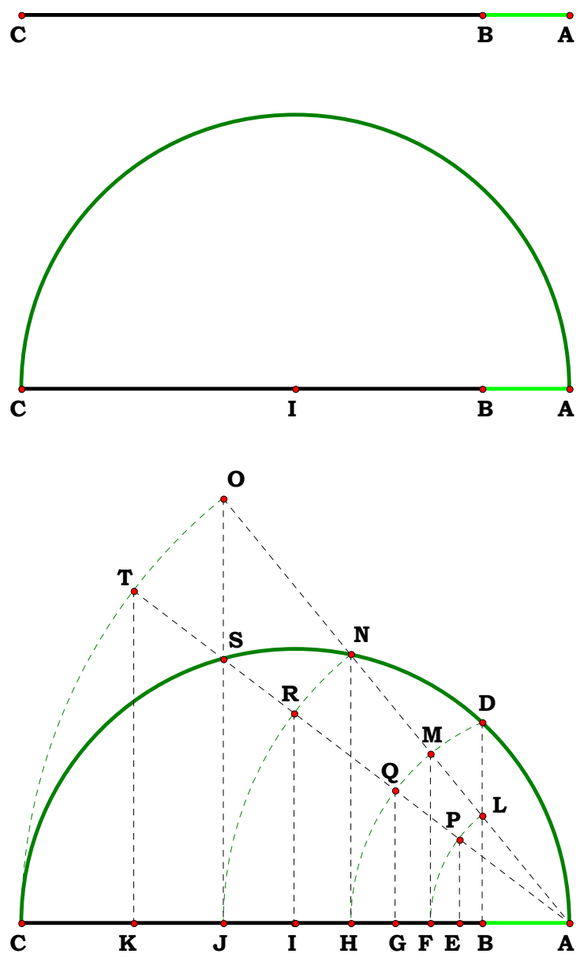
$N_1 := 5 \qquad AC := N_1$

**120493**

**Descriptions.**

**To use the digital indexing system to apply names, let AC be the thing with which we seek to name an exponential series on. AB is our unit. As a number is a ratio, numbers are two dimensional.**

**A number is no more than a digital name used with the stipulation that the indexing system is further qualified by using as standard difference. In other words that the concept of ratio will employ a name called a number. Ratio, however, is independent of the naming convention. Given any ration, say, M, describe a two prime exponential series, M^1/2^N, where N is any whole ratio.**




**The circle is a two dimensional object which is capable of producing every ratio between two differences.**

$BC := AC - AB \qquad BD := \sqrt{BC \cdot AB} \qquad AH := \sqrt{AB^2 + BD^2} \qquad CH := AC - AH$

$HN := \sqrt{CH \cdot AH} \qquad AJ := \sqrt{AH^2 + HN^2} \qquad CJ := AC - AJ \qquad JS := \sqrt{CJ \cdot AJ}$

$AK := \sqrt{JS^2 + AJ^2} \qquad AI := \frac{AJ^2}{AK} \qquad AG := \frac{AH^2}{AI} \qquad AF := \frac{AH^2}{AJ} \qquad AE := \frac{AF^2}{AG}$

**Definitions.**

$AC^0 - AB = 0$

$AC^{\frac{1}{8}} - AE = 0 \qquad AC^{\frac{2}{8}} - AF = 0 \qquad AC^{\frac{3}{8}} - AG = 0 \qquad AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0 \qquad AC^{\frac{6}{8}} - AJ = 0 \qquad AC^{\frac{7}{8}} - AK = 0 \qquad AC^{\frac{8}{8}} - AC = 0$

$AC^{\frac{1}{8}} - AE = 0$

$AC^{\frac{2}{8}} - AF = 0$

$AC^{\frac{3}{8}} - AG = 0$

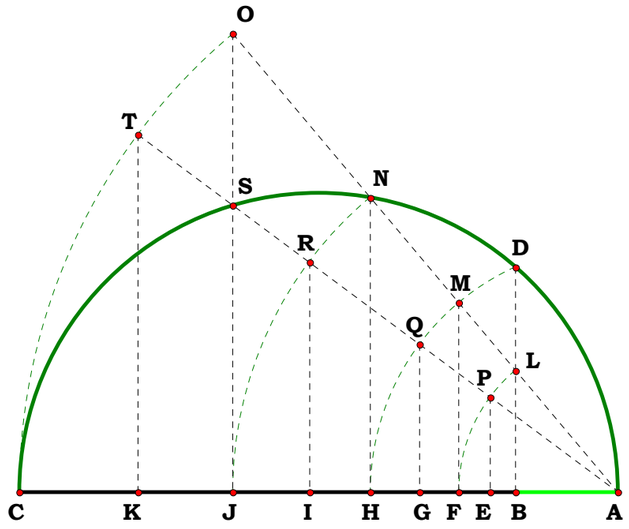
$AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0$

$AC^{\frac{6}{8}} - AJ = 0$

$AC^{\frac{7}{8}} - AK = 0$

$AC^{\frac{8}{8}} - AC = 0$




| | | | |
|---|---|---|---|
| $AC^0 = 1.00000$ | $N_1 = 1.00000$ | $\frac{AB}{AB} = 1.00000$ | AC = 3.95000 in. |
| $AC^{\frac{1}{8}} = 1.24752$ | AE = 1.24752 | $\frac{AE}{AB} = 1.24752$ | AB = 0.67333 in. |
| $AC^{\frac{2}{8}} = 1.55630$ | AF = 1.55630 | $\frac{AF}{AB} = 1.55630$ | AE = 0.83998 in. |
| $AC^{\frac{3}{8}} = 1.94151$ | AG = 1.94151 | $\frac{AG}{AB} = 1.94151$ | AF = 1.04790 in. |
| $AC^{\frac{4}{8}} = 2.42207$ | AH = 2.42207 | $\frac{AH}{AB} = 2.42207$ | AG = 1.30727 in. |
| $AC^{\frac{5}{8}} = 3.02157$ | AI = 3.02157 | $\frac{AI}{AB} = 3.02157$ | AH = 1.63084 in. |
| $AC^{\frac{6}{8}} = 3.76946$ | AJ = 3.76946 | $\frac{AJ}{AB} = 3.76946$ | AI = 2.03450 in. |
| $AC^{\frac{7}{8}} = 4.70247$ | AK = 4.70247 | $\frac{AK}{AB} = 4.70247$ | AJ = 2.53807 in. |
| $AC^{\frac{8}{8}} = 5.86641$ | AC = 5.86641 | $\frac{AC}{AB} = 5.86641$ | AK = 3.16629 in. |

$AC^0 - N_1 = 0.00000$

$AC^{\frac{1}{8}} - AE = 0.00000$

$AC^{\frac{2}{8}} - AF = 0.00000$

$AC^{\frac{3}{8}} - AG = 0.00000$

$AC^{\frac{4}{8}} - AH = 0.00000$

$AC^{\frac{5}{8}} - AI = 0.00000$

$AC^{\frac{6}{8}} - AJ = 0.00000$

$AC^{\frac{7}{8}} - AK = 0.00000$

$AC^{\frac{8}{8}} - AC = 0.00000$

**Alternate method of creating an exponential series.**

$N_1 = 2.97175$

$N_1^{\frac{8}{8}} = 2.97175$

$N_1^{\frac{7}{8}} = 2.59350$

$N_1^{\frac{6}{8}} = 2.26339$

$N_1^{\frac{5}{8}} = 1.97530$

$N_1^{\frac{4}{8}} = 1.72388$

$N_1^{\frac{3}{8}} = 1.50446$

$N_1^{\frac{2}{8}} = 1.31296$

$N_1^{\frac{1}{8}} = 1.14585$

$N_1^{\frac{9}{8}} = 3.40517$

$N_1^{\frac{10}{8}} = 3.90181$

$N_1^{\frac{11}{8}} = 4.47087$

$N_1^{\frac{-1}{8}} = 0.87272$

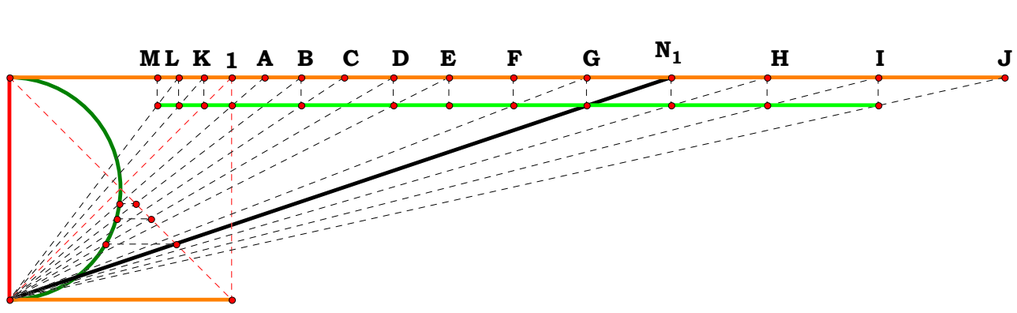
$N_1^{\frac{-2}{8}} = 0.76163$

$N_1^{\frac{-3}{8}} = 0.66469$

A = 1.14585
B = 1.31296
C = 1.50446
D = 1.72388
E = 1.97530
F = 2.26339
G = 2.59350
H = 3.40517
I = 3.90181
J = 4.47087
K = 0.87272
L = 0.76163
M = 0.66469

$N_1^0 = 1.00000$

$N_1^{\frac{8}{8}} - N_1 = 0.00000$




$N_1^{\frac{-1}{8}} - K = 0.00000$

$N_1^{\frac{-2}{8}} - L = 0.00000$

$N_1^{\frac{-3}{8}} - M = 0.00000$

$N_1^{\frac{1}{8}} - A = 0.00000$

$N_1^{\frac{2}{8}} - B = 0.00000$

$N_1^{\frac{3}{8}} - C = 0.00000$

$N_1^{\frac{4}{8}} - D = 0.00000$

$N_1^{\frac{5}{8}} - E = 0.00000$

$N_1^{\frac{6}{8}} - F = 0.00000$

$N_1^{\frac{7}{8}} - G = 0.00000$

$N_1^{\frac{9}{8}} - H = 0.00000$

$N_1^{\frac{10}{8}} - I = 0.00000$

$N_1^{\frac{11}{8}} - J = 0.00000$

Unit.

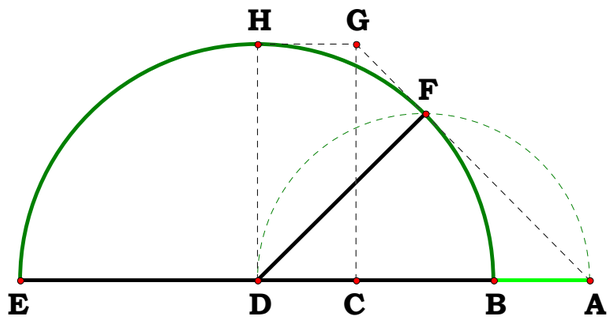
$BE := 1$

Given.

$N_1 := 6$

**120693A**

**Alternate Method: Square Root**
**Common Segment Common Endpoint**




**Descriptions.**

$AB := N_1 \qquad AE := AB + BE \qquad BD := \frac{BE}{2}$

$AE - (N_1 + BE) = 0 \qquad BD - \frac{BE}{2} = 0 \qquad DF := BD$

$AD := BD + AB \qquad DF - \frac{BE}{2} = 0 \qquad AD - \frac{2 \cdot N_1 + BE}{2} = 0$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

**Definitions.**

$\sqrt{AB \cdot AE} - AC = 0$

$AF - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

$AC - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

120693B

Unit.

$CJ := 1$

Given.

$N_1 := 5$

Are A, P and Q collinear? Are A, K and N collinear?

**Descriptions.**

$AC := N_1$ $\quad AJ := AC + CJ$ $\quad AE := \left(AC^2 \cdot AJ\right)^{\frac{1}{3}}$ $\quad AG := \left(AC \cdot AJ^2\right)^{\frac{1}{3}}$

$CG := AG - AC$ $\quad GJ := CJ - CG$ $\quad GN := \sqrt{CG \cdot GJ}$ $\quad AB := \frac{AE}{2}$ $\quad CE := AE - AC$

$CH := \frac{CJ}{2}$ $\quad BK := AB$ $\quad HK := CH$ $\quad HJ := CH$ $\quad AH := AJ - HJ$ $\quad BH := AH - AB$

$BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$ $\quad AD := AB + BD$ $\quad DE := AE - AD$ $\quad DK := \sqrt{AD \cdot DE}$

$GQ := \sqrt{AG \cdot GJ}$ $\quad CP := \sqrt{AC \cdot CE}$ $\quad \frac{AG}{GN} - \frac{AD}{DK} = -8.526 \quad \frac{AG}{GQ} - \frac{AC}{CP} = 0$

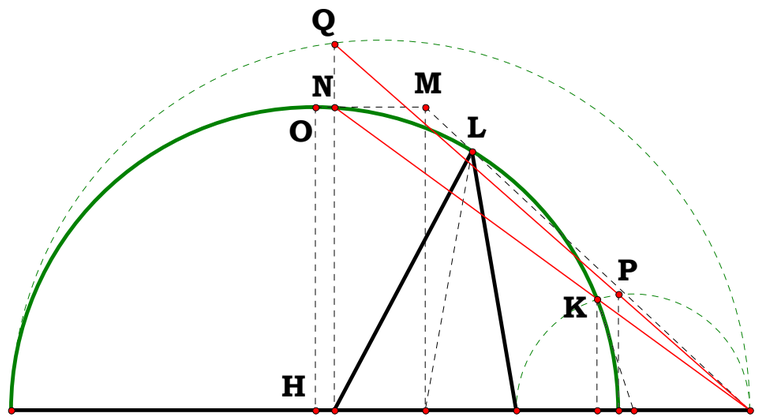
**Definitions.**

$AJ - \left(N_1 + 1\right) = 0$ $\quad AE - \left(N_1^3 + N_1^2\right)^{\frac{1}{3}} = 0$ $\quad AG - \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} = 0$

$CG - \left[\left[N_1 \cdot \left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - N_1\right] = 0$ $\quad GJ - \left[N_1 - \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} + 1\right] = 0$

$GN - \sqrt{\left(2 \cdot N_1 + 1\right) \cdot \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} - \left[N_1 + N_1^2 + \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{2}{3}}\right]} = 0$

**Gruntwork IV on the Delian Solution.**



121193A

Unit.

$AB := 1$

Given.

$N := 6$

**Descriptions.**

$AL := AB \cdot N$ $BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$ $AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB$ $BJ := AJ - AB$ $JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$ $KR := BK$ $KL := BK$

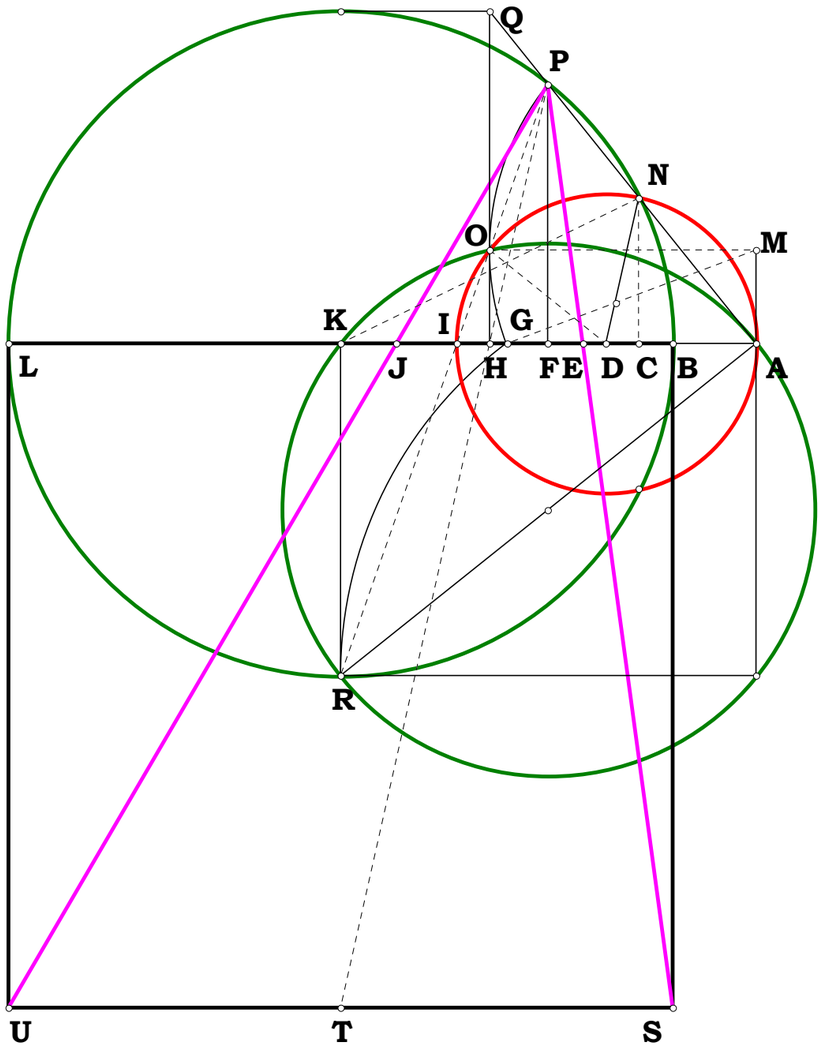
$FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$ $AD := \frac{AI}{2}$

$KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

**The structure in red appears to be a constant.**

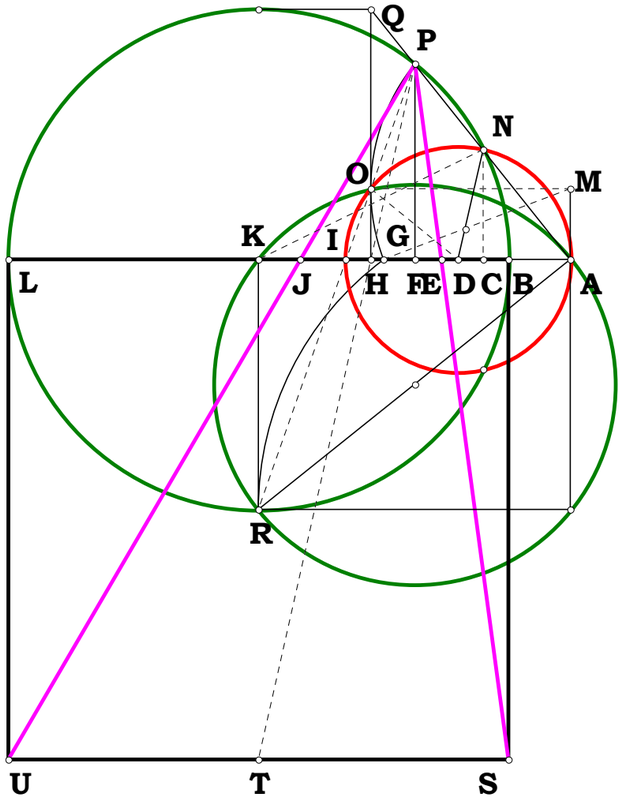


**Definitions.**

$$AL - N = 0 \qquad BL - (N-1) = 0 \qquad BK - \frac{N-1}{2} = 0 \qquad AE - N^{\frac{1}{3}} = 0 \qquad AJ - N^{\frac{2}{3}} = 0$$

$$BE - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \qquad EJ - N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) = 0$$

$$FJ - \frac{N \cdot \left(N^{\frac{1}{3}} - 1\right)}{N^{\frac{2}{3}} + 1} = 0 \qquad FL - \frac{N^{\frac{2}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0 \qquad BF - \frac{N-1}{N^{\frac{2}{3}} + 1} = 0 \qquad FP - \frac{N^{\frac{1}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0$$




$$FK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad IK - \frac{\left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$AK - \frac{N+1}{2} = 0 \qquad AI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{2}{3}} + 1\right)}{N^{\frac{1}{3}} + 1} = 0 \qquad AD - \frac{N + N^{\frac{1}{3}}}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad FH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad AF - \frac{\left(N^{\frac{1}{3}}\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{2}{3}} + 1} = 0$$

$$AH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{2} = 0$$

$$HI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$HO - \frac{\sqrt{N^{\frac{2}{3}} - 2 \cdot N + N^{\frac{4}{3}}}}{2} = 0$$

$$DK - \frac{N^{\frac{4}{3}} + 1}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$CK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

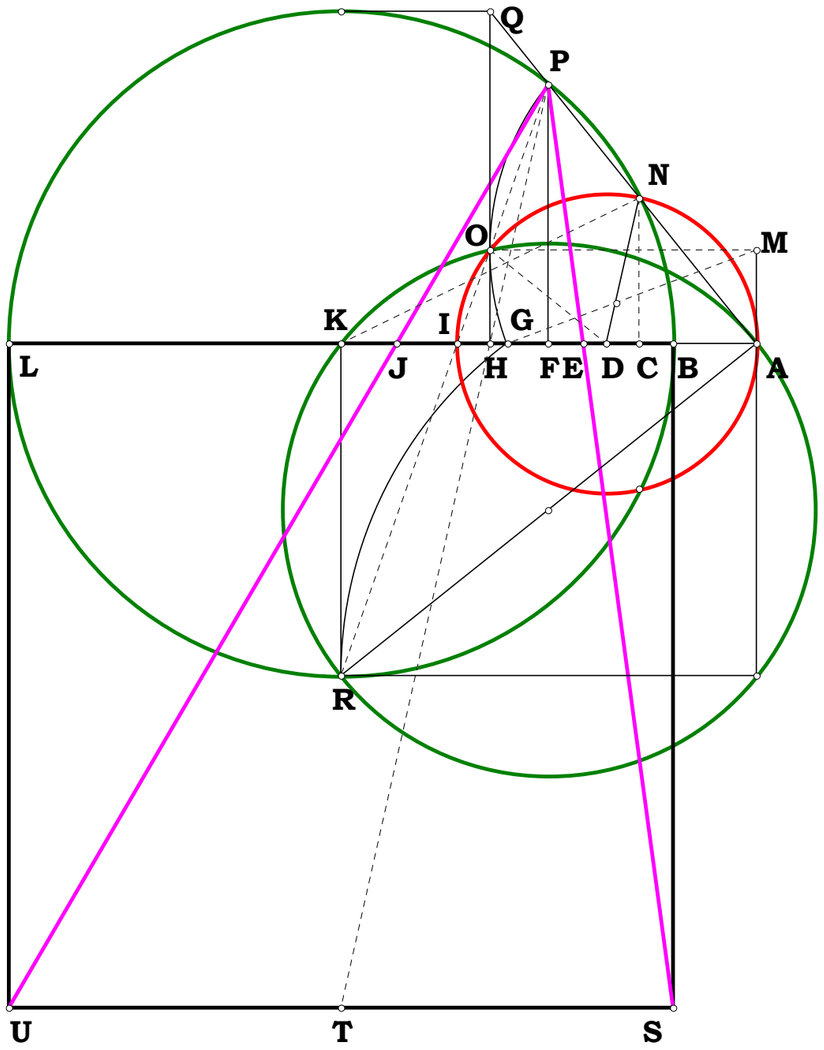
$$AC - \frac{\left(N^{\frac{1}{3}}\right)^3 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{4}{3}} + 1} = 0$$

$$CI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)^2}{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$CN - \frac{N^{\frac{2}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)}{\left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$\frac{KR}{IK} - \frac{N^{\frac{1}{3}} + 1}{N^{\frac{1}{3}} - 1} = 0$$

$$\frac{AF}{FP} - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{N - 1} = 0$$



121193B

Unit.

$AB := 1$

Given.

$Y := 4$

$X := 20$

**Descriptions.**

$AL := \frac{X}{Y}$ $BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := (AL)^{\frac{1}{3}}$ $AJ := AL^{\frac{2}{3}}$

$BE := AE - AB$ $BJ := AJ - AB$ $JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$ $KR := BK$ $KL := BK$

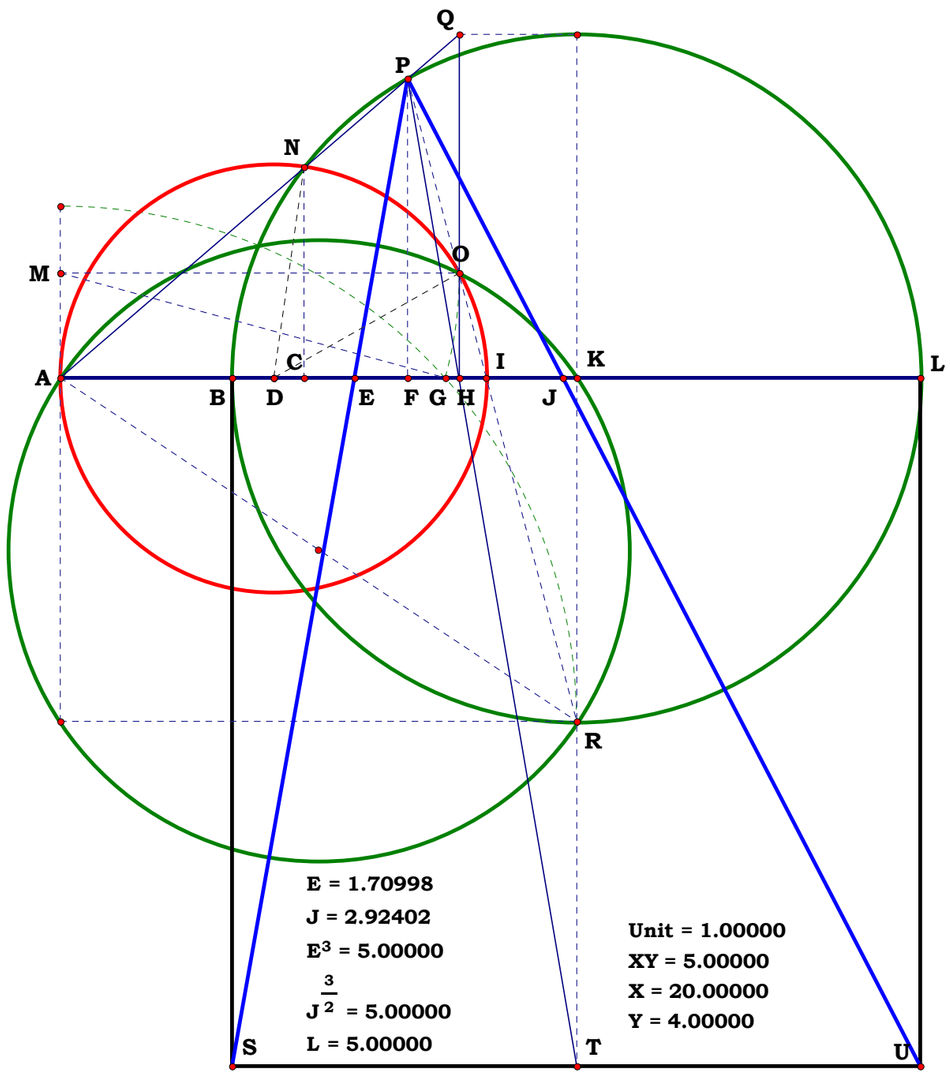
$FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$ $AD := \frac{AI}{2}$

$KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

**The structure in red appears to be a constant.**



**Definitions.**

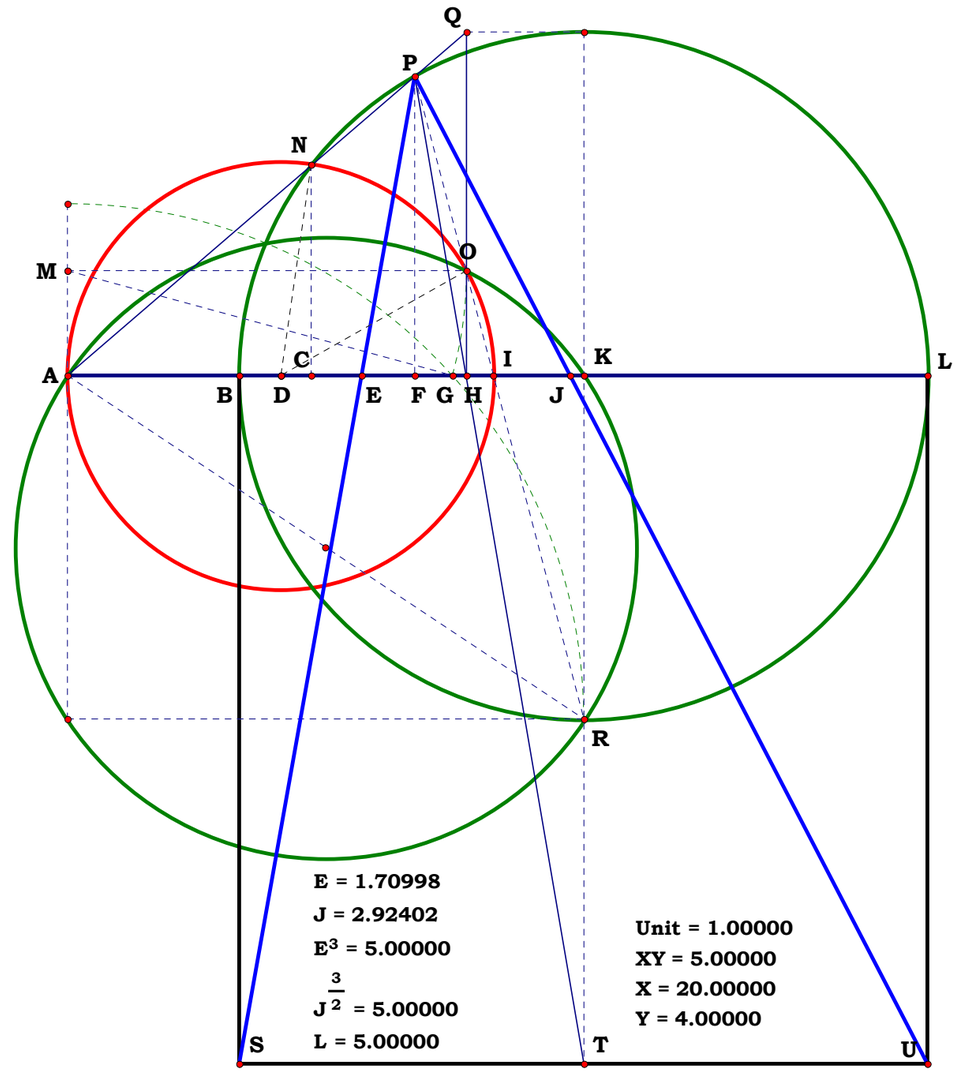
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

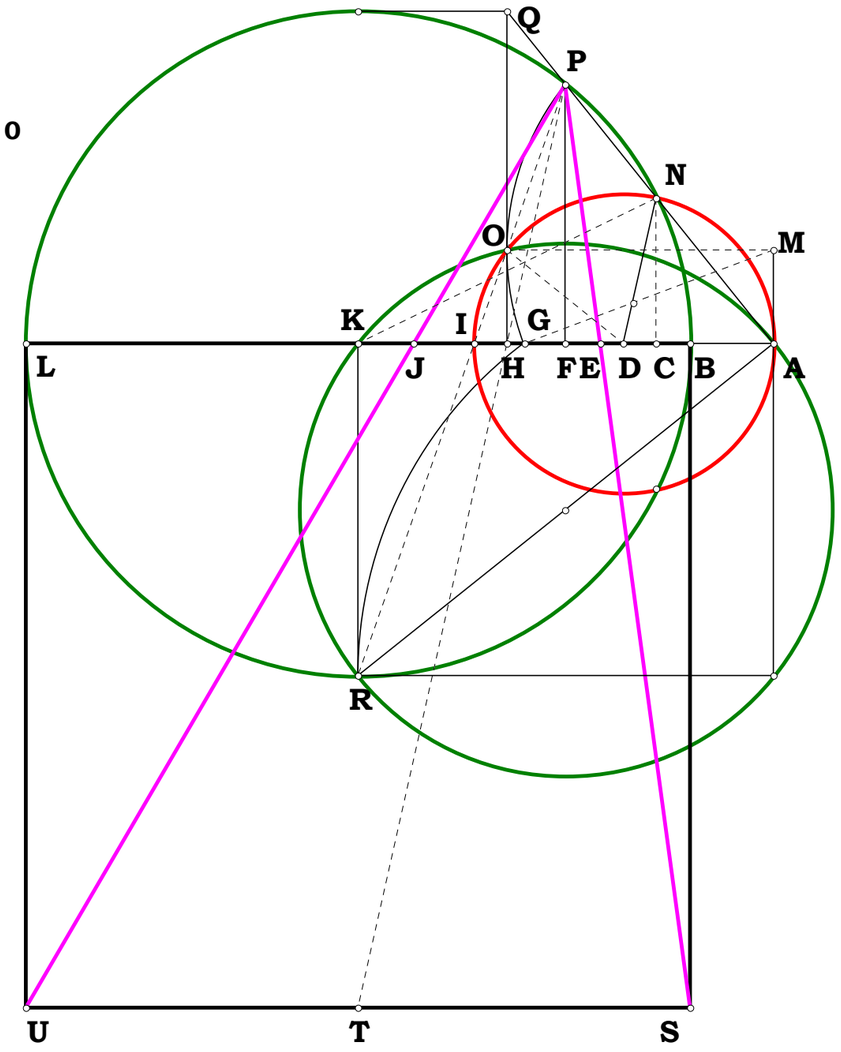


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} + 2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2 \cdot X}{Y} + \frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad CK - -\frac{(X - Y) \cdot \left[Y - X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2 \cdot Y^2 \cdot \left[\frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y} + 1\right]} = 0 \qquad AC - \frac{X \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]}{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + \left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CN - \frac{(X - Y) \cdot \left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X - Y} = 0$$



121193C

Unit.

$BL := 1$

Given.

$Y := 5$

$X := 10$

**Descriptions.**

$AB := 1$ $\quad AL := \frac{X}{Y}$ $\quad BK := \frac{BL}{2}$ $\quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$ $\quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB$ $\quad BJ := AJ - AB$ $\quad JL := BL - BJ$ $\quad EJ := AJ - AE$ $\quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $\quad BF := BL - FL$ $\quad FP := \sqrt{BF \cdot FL}$ $\quad KR := BK$ $\quad KL := BK$

$FK := FL - KL$ $\quad IK := \frac{FK \cdot KR}{KR + FP}$ $\quad AK := BK + AB$ $\quad AI := AK - IK$ $\quad AD := \frac{AI}{2}$

$KT := BL$ $\quad FH := \frac{FK \cdot FP}{KT + FP}$ $\quad AF := BF + AB$ $\quad AH := AF + FH$ $\quad HI := AI - AH$

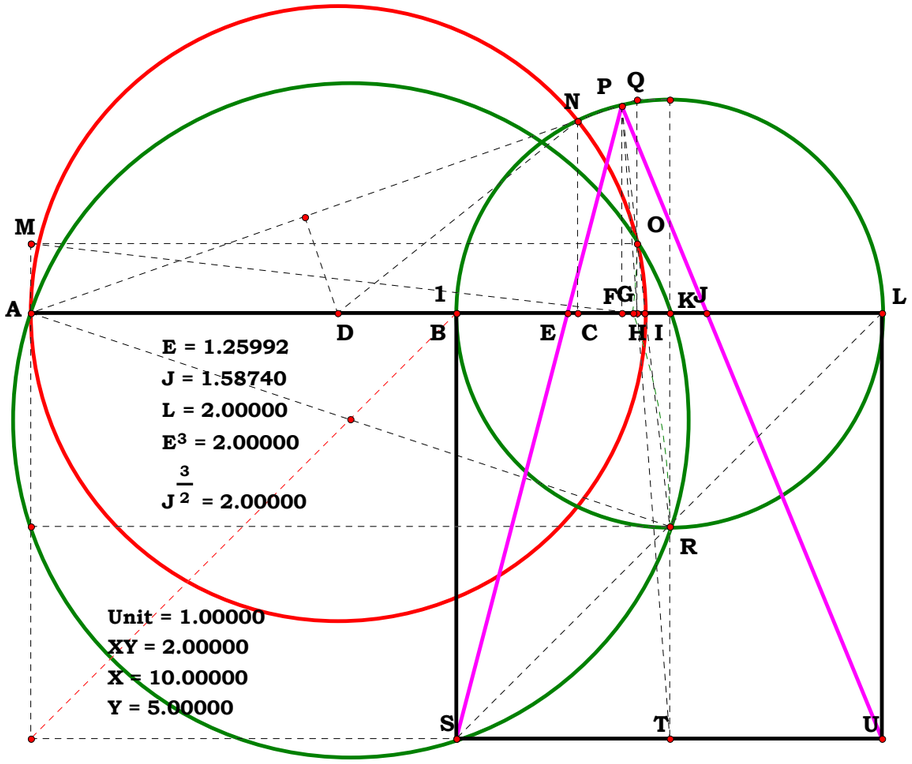
$HO := \sqrt{AH \cdot HI}$ $\quad DN := AD$ $\quad KN := BK$ $\quad DK := AK - AD$ $\quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $\quad CI := AI - AC$ $\quad CN := \sqrt{AC \cdot CI}$ $\quad \frac{KR}{IK} - \frac{HO}{HI} = 0$ $\quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

$AE = 1.259921$ $\quad AE^3 = 2$

$AJ = 1.587401$ $\quad AJ^{\frac{3}{2}} = 2$

**The structure in red appears to be a constant.**



**Definitions.**

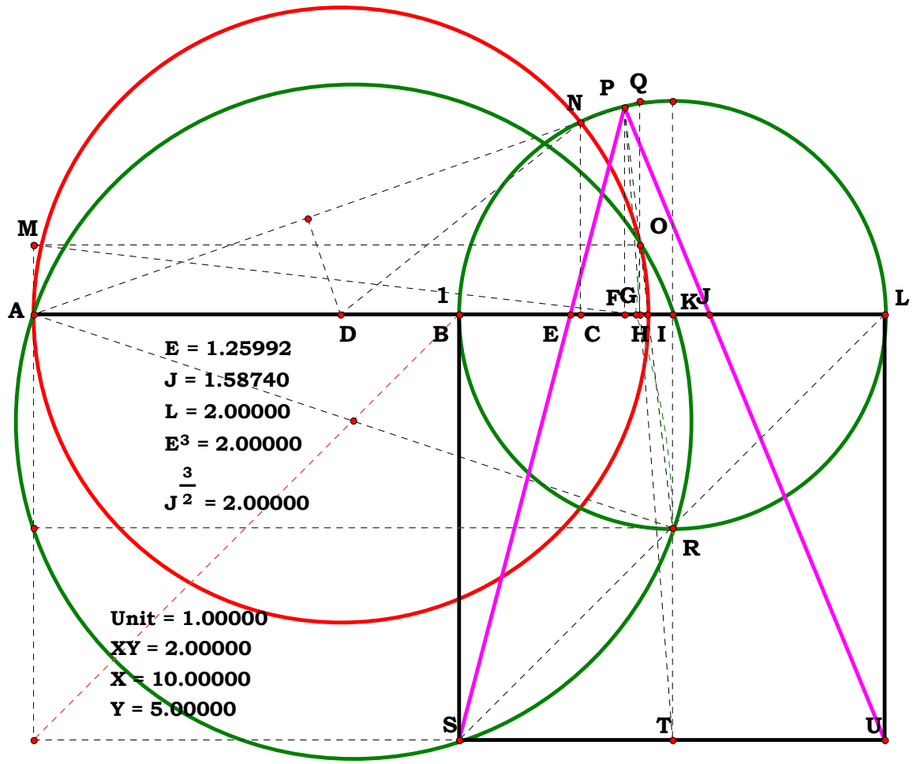
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

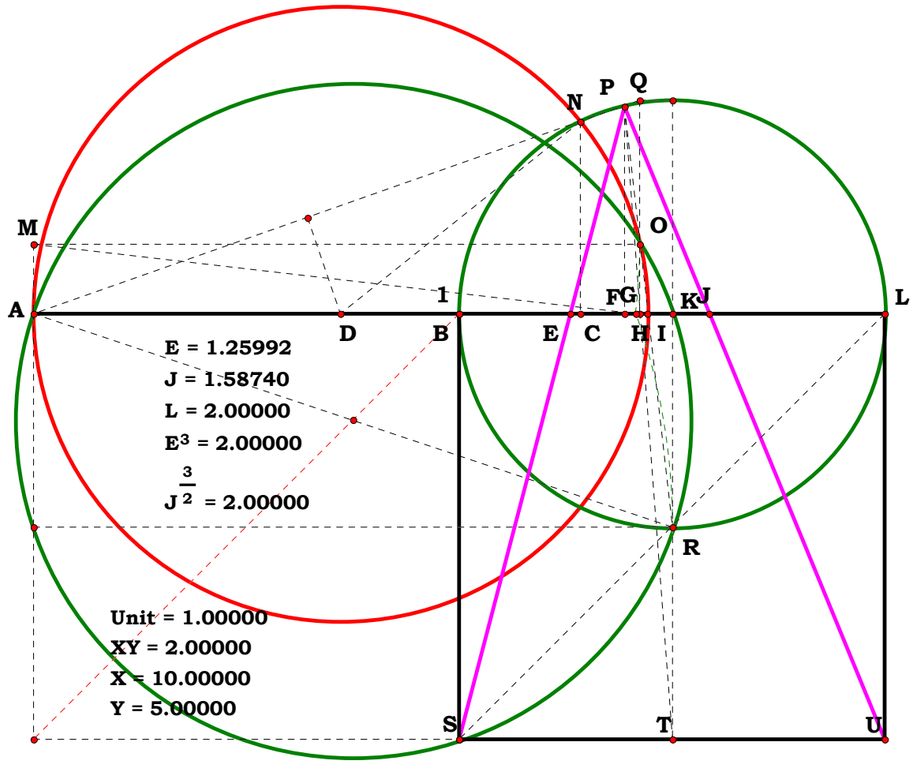


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} + 2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2 \cdot X}{Y} + \frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad CK - -\frac{(X - Y) \cdot \left[Y - X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2 \cdot Y^2 \cdot \left[\frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y} + 1\right]} = 0 \qquad AC - \frac{X \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]}{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + \left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CN - \frac{(X - Y) \cdot \left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X - Y} = 0$$



Given.

$N_1 := 5$

$N_2 := 3$

121293A1

Descriptions.

$AF := N_1$ $BE := N_2$

$AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$ $AB := AD - BD$

$AE := BE + AB$ $AC := \frac{AE}{2}$ $CG := AC$

$CD := AD - AC$ $GH := 2 \cdot \sqrt{CG^2 - CD^2}$

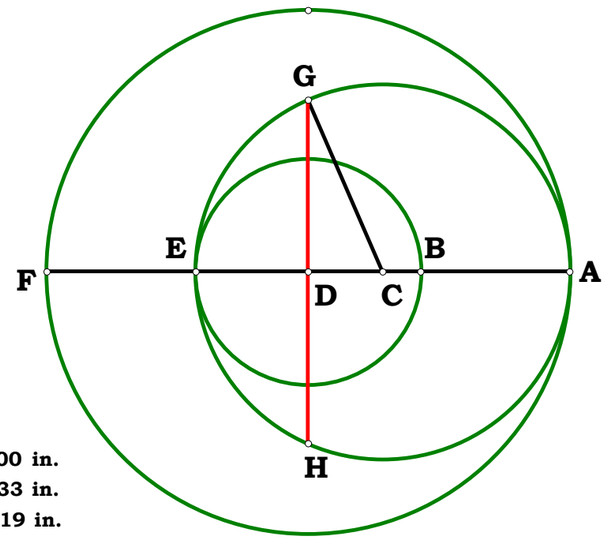
Definitions.

$GH - \sqrt{AF \cdot BE} = 0$

$GH - \sqrt{N_1 \cdot N_2} = 0$

## Common Segment Common Midpoint

**Square root by common segment common midpoint.**
**Given AFand BE is GH their root?**




AF = 3.45000 in.
BE = 1.48333 in.
GH = 2.26219 in.
$\sqrt{AF \cdot BE}$ - GH = 0.00000 in.

121293A2

Unit.

$AF := 1$

Given.

$Y := 20$

$X := 9$

**Descriptions.**

$BE := \frac{X}{Y}$ $AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$

$AB := AD - BD$ $AE := BE + AB$

$AC := \frac{AE}{2}$ $CG := AC$ $CD := AD - AC$

$GH := 2 \cdot \sqrt{CG^2 - CD^2}$ $GH - \sqrt{AF \cdot BE} = 0$

**Definitions.**

$BE - \frac{X}{Y} = 0$ $AD - \frac{1}{2} = 0$ $BD - \frac{X}{2 \cdot Y} = 0$

$AB - \left( \frac{Y - X}{2 \cdot Y} \right) = 0$ $AE - \left( \frac{X + Y}{2 \cdot Y} \right) = 0$

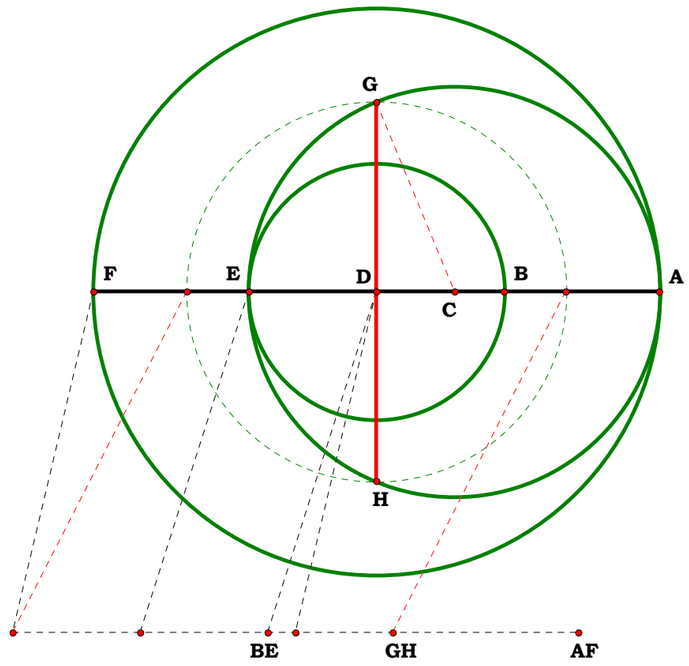
$AC - \frac{X + Y}{4 \cdot Y} = 0$ $CG - \frac{X + Y}{4 \cdot Y} = 0$

$CD - \frac{Y - X}{4 \cdot Y} = 0$ $GH - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$GH - \sqrt{\frac{X}{Y}} = 0$

## Common Segment Common Midpoint

**Square root by common segment common midpoint.**
**Given AFand BE is GH their root?**




Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AF = 1.00000
BE = 0.45000
GH = 0.67082
$\sqrt{BE}$ = 0.67082

121293B1

Given.

$N_1 := 1$

$N_2 := 4$

$N_3 := 3$

## Generalize The Previous Square Root Figure

**Descriptions.**

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

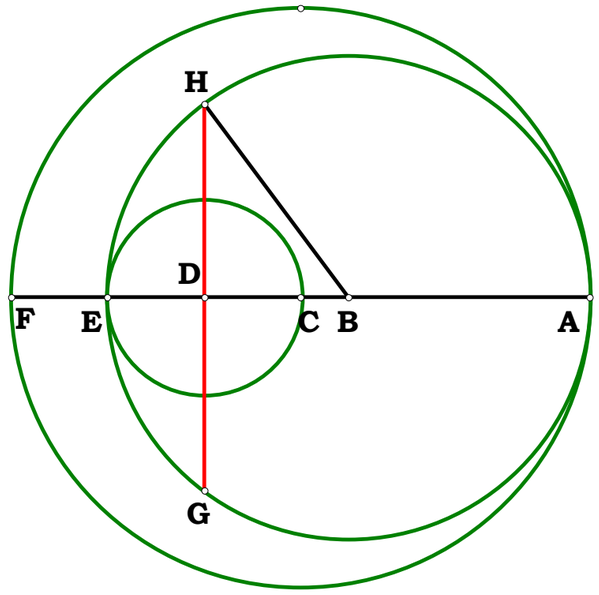
$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

**Definitions.**

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$




$AF = 3.81667$ in.

$DF = 1.27449$ in.

$DE = 0.64116$ in.

$\frac{AF}{DF} = 2.99466$

$\frac{DF}{DE} = 1.98779$

$$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} = 2.55338 \text{ in.}$$

$HG = 2.55338$ in.

$$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} - HG = 0.00000 \text{ in.}$$

121293B2

**Descriptions.**

$AF := 1$

**Given.**

$Y := 20$

$X := 8$

$W := 9$

$V := 3$

$DF := \frac{X}{Y}$ $AD := AF - DF$

$DE := DF - \frac{DF \cdot V}{W}$ $AE := AD + DE$

$AB := \frac{AE}{2}$ $BD := AD - AB$ $BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$ $GH = 0.8$

**Definitions.**

$DF - \frac{X}{Y} = 0$ $AD - \frac{Y - X}{Y} = 0$ $DE - \frac{X \cdot (W - V)}{W \cdot Y}$

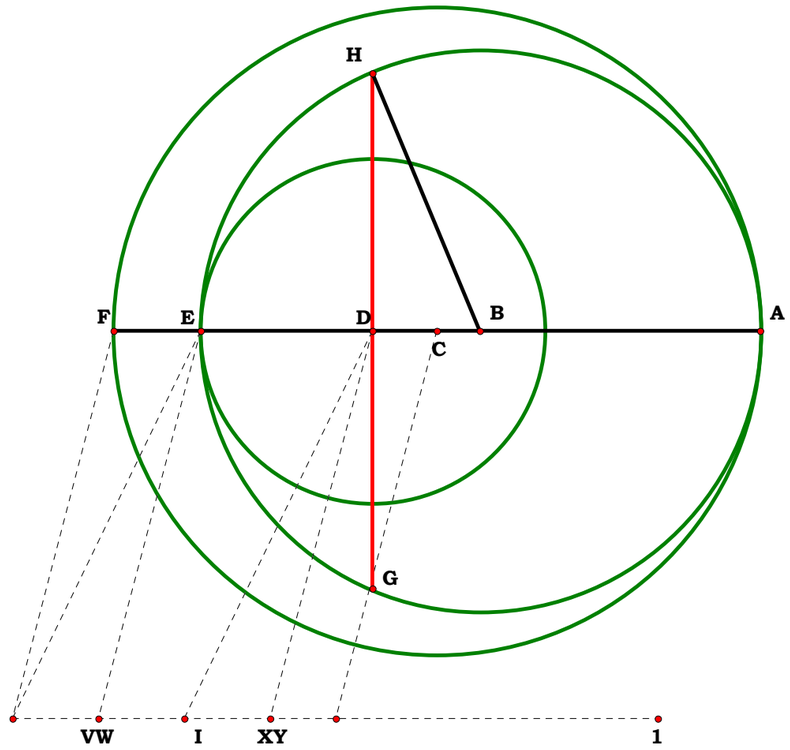
$AE - \frac{W \cdot Y - V \cdot X}{W \cdot Y} = 0$ $AB - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0$

$BD - \frac{V \cdot X - 2 \cdot W \cdot X + W \cdot Y}{2 \cdot W \cdot Y} = 0$

$BH - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0$ $GH = 0.8$

$GH - 2 \cdot \frac{\sqrt{X \cdot (V - W) \cdot (X - Y)}}{\sqrt{W} \cdot Y} = 0$

## Generalize The Previous Square Root Figure




Unit = 1.00000

XY = 0.40000

X = 8.00000

Y = 20.00000

VW = 0.33333

V = 3.00000

W = 9.00000

$\frac{Y}{X} = 2.50000$

$\frac{W}{V} = 3.00000$

$\frac{GH}{AF} = 0.80000$

$\frac{2 \cdot \sqrt{X \cdot (V-W) \cdot (X-Y)}}{\sqrt{W} \cdot Y} - \frac{GH}{AF} = 0.00000$

**Unit.**

$AR := 1$ **Using 120493**

**Given.**

$\Delta := 5$ $\delta := 2 .. \Delta + 1$

**121693A**

**Descriptions.**

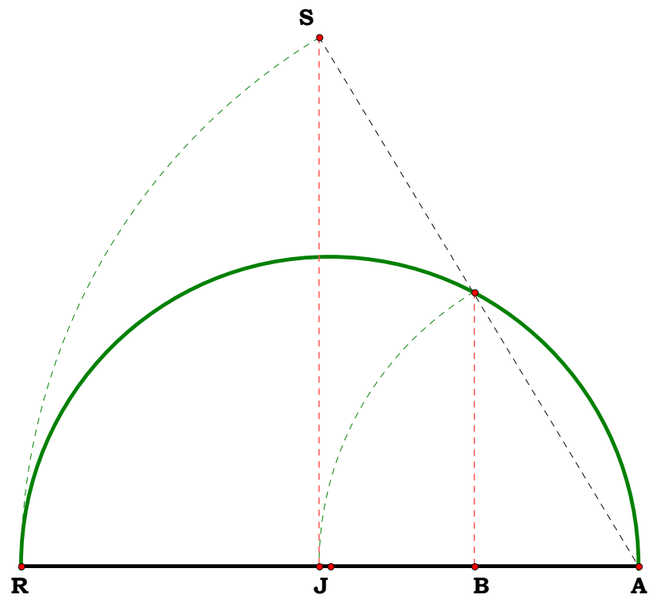
$$AB_\delta := \frac{AR}{\delta}$$

$$AJ_\delta := \sqrt{AB_\delta \cdot AR} \qquad JR_\delta := AR - AJ_\delta$$

$$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \qquad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$$

| $AB_\delta$ = | $AJ_\delta$ = |
|---|---|
| 0.5 | 0.707107 |
| 0.333333 | 0.57735 |
| 0.25 | 0.5 |
| 0.2 | 0.447214 |
| 0.166667 | 0.408248 |

**The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.**




**Euclidean Exponential Series**

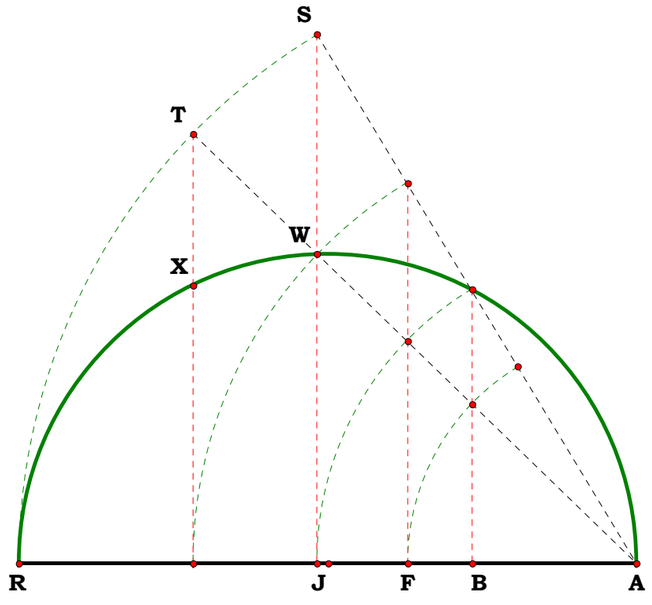
$$AT := AR \qquad AN_\delta := AW_\delta \qquad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$$

$$NR_\delta := AR - AN_\delta \qquad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$$

$$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$$

**Definitions.**

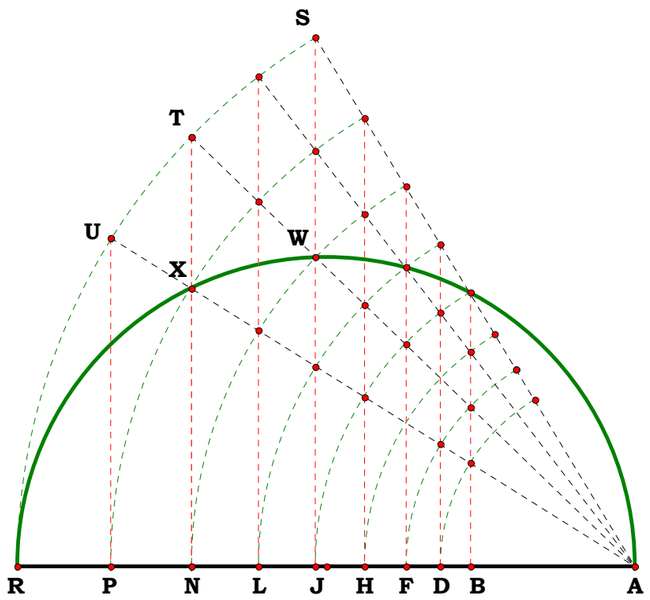
| $AB_\delta$ = | $AF_\delta$ = | $AJ_\delta$ = | $AN_\delta$ = |
|---|---|---|---|
| 0.5 | 0.594604 | 0.707107 | 0.840896 |
| 0.333333 | 0.438691 | 0.57735 | 0.759836 |
| 0.25 | 0.353553 | 0.5 | 0.707107 |
| 0.2 | 0.29907 | 0.447214 | 0.66874 |
| 0.166667 | 0.260847 | 0.408248 | 0.638943 |




**What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.**

$$AU := AR \qquad AP_\delta := AX_\delta \qquad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta} \qquad AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \qquad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$$

$$PR_\delta := AR - AP_\delta \qquad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta} \qquad AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$$

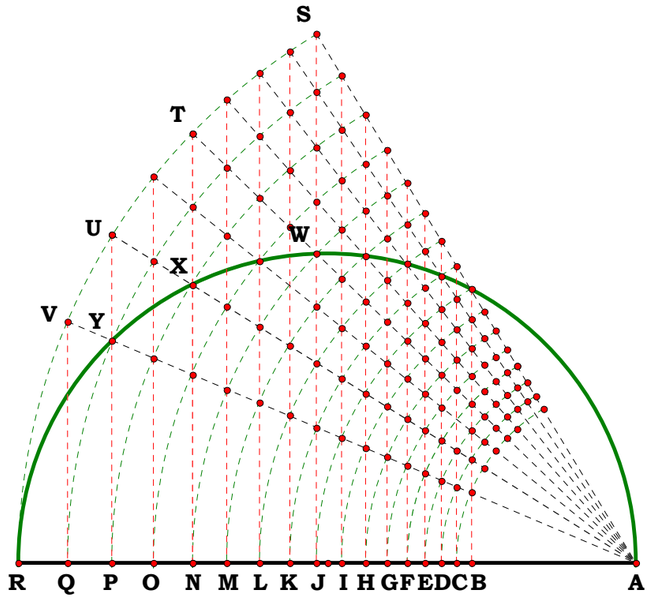



| $AB_\delta$ = | $AD_\delta$ = | $AF_\delta$ = | $AH_\delta$ = | $AJ_\delta$ = | $AL_\delta$ = | $AN_\delta$ = | $AP_\delta$ = |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.545254 | 0.594604 | 0.64842 | 0.707107 | 0.771105 | 0.840896 | 0.917004 |
| 0.333333 | 0.382401 | 0.438691 | 0.503268 | 0.57735 | 0.662338 | 0.759836 | 0.871686 |
| 0.25 | 0.297302 | 0.353553 | 0.420448 | 0.5 | 0.594604 | 0.707107 | 0.840896 |
| 0.2 | 0.244569 | 0.29907 | 0.365716 | 0.447214 | 0.546873 | 0.66874 | 0.817765 |
| 0.166667 | 0.208506 | 0.260847 | 0.326329 | 0.408248 | 0.510732 | 0.638943 | 0.799339 |

$$AV := AR \qquad AQ_\delta := AY_\delta \qquad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta} \qquad AM_\delta := \frac{(AN_\delta)^2}{AO_\delta}$$

$$AK_\delta := \frac{(AL_\delta)^2}{AM_\delta} \qquad AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \qquad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \qquad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$$




| $AB_\delta$ = | $AC_\delta$ = | $AD_\delta$ = | $AE_\delta$ = | $AF_\delta$ = |
|---|---|---|---|---|
| 0.5 | 0.522137 | 0.545254 | 0.569394 | 0.594604 |
| 0.333333 | 0.357025 | 0.382401 | 0.40958 | 0.438691 |
| 0.25 | 0.272627 | 0.297302 | 0.32421 | 0.353553 |
| 0.2 | 0.221165 | 0.244569 | 0.27045 | 0.29907 |
| 0.166667 | 0.186416 | 0.208506 | 0.233213 | 0.260847 |

| $AG_\delta$ = |
|---|
| 0.620929 |
| 0.469872 |
| 0.385553 |
| 0.330718 |
| 0.291757 |

| $AH_\delta$ = |
|---|
| 0.64842 |
| 0.503268 |
| 0.420448 |
| 0.365716 |
| 0.326329 |

| $AI_\delta$ = |
|---|
| 0.677128 |
| 0.539038 |
| 0.458502 |
| 0.404417 |
| 0.364998 |

| $AJ_\delta$ = |
|---|
| 0.707107 |
| 0.57735 |
| 0.5 |
| 0.447214 |
| 0.408248 |

| $AK_\delta$ = |
|---|
| 0.738413 |
| 0.618386 |
| 0.545254 |
| 0.494539 |
| 0.456624 |

| $AL_\delta$ = |
|---|
| 0.771105 |
| 0.662338 |
| 0.594604 |
| 0.546873 |
| 0.510732 |

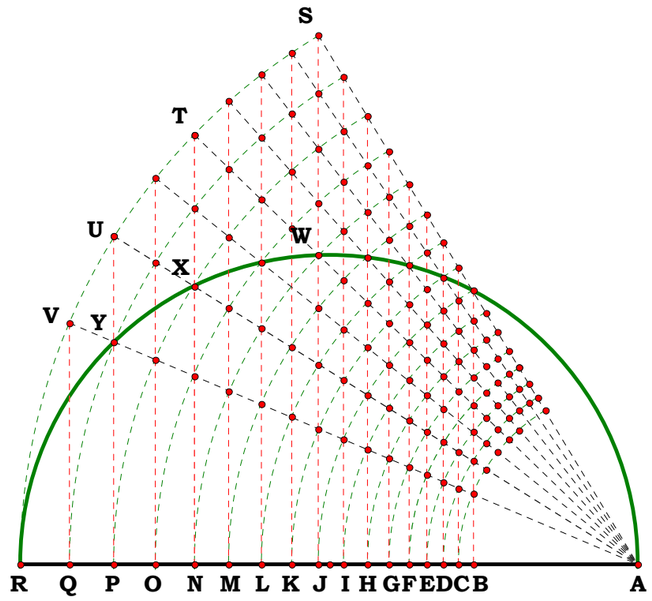
| $AM_\delta$ = |
|---|
| 0.805245 |
| 0.709414 |
| 0.64842 |
| 0.604744 |
| 0.571252 |

| $AN_\delta$ = |
|---|
| 0.840896 |
| 0.759836 |
| 0.707107 |
| 0.66874 |
| 0.638943 |

| $AO_\delta$ = |
|---|
| 0.878126 |
| 0.813841 |
| 0.771105 |
| 0.739508 |
| 0.714655 |

| $AP_\delta$ = |
|---|
| 0.917004 |
| 0.871686 |
| 0.840896 |
| 0.817765 |
| 0.799339 |

| $AQ_\delta$ = |
|---|
| 0.957603 |
| 0.933641 |
| 0.917004 |
| 0.904304 |
| 0.894058 |




**Values found by the investigator of 12_14_93**

| $AQ_\delta$ = |
|---|
| 0.957603 |
| 0.933641 |
| 0.917004 |
| 0.904304 |
| 0.894058 |

| $(AB_\delta)^{\frac{1}{16}}$ = |
|---|
| 0.957603 |
| 0.933641 |
| 0.917004 |
| 0.904304 |
| 0.894058 |

| $AP_\delta$ = |
|---|
| 0.917004 |
| 0.871686 |
| 0.840896 |
| 0.817765 |
| 0.799339 |

| $(AB_\delta)^{\frac{1}{8}}$ = |
|---|
| 0.917004 |
| 0.871686 |
| 0.840896 |
| 0.817765 |
| 0.799339 |

| $AN_\delta$ = |
|---|
| 0.840896 |
| 0.759836 |
| 0.707107 |
| 0.66874 |
| 0.638943 |

| $(AB_\delta)^{\frac{1}{4}}$ = |
|---|
| 0.840896 |
| 0.759836 |
| 0.707107 |
| 0.66874 |
| 0.638943 |

| $AL_\delta$ = |
|---|
| 0.771105 |
| 0.662338 |
| 0.594604 |
| 0.546873 |
| 0.510732 |

| $(AB_\delta)^{\frac{3}{8}}$ = |
|---|
| 0.771105 |
| 0.662338 |
| 0.594604 |
| 0.546873 |
| 0.510732 |

| $AI_\delta$ = |
|---|
| 0.677128 |
| 0.539038 |
| 0.458502 |
| 0.404417 |
| 0.364998 |

| $(AB_\delta)^{\frac{9}{16}}$ = |
|---|
| 0.677128 |
| 0.539038 |
| 0.458502 |
| 0.404417 |
| 0.364998 |

| $AH_\delta$ = |
|---|
| 0.64842 |
| 0.503268 |
| 0.420448 |
| 0.365716 |
| 0.326329 |

| $(AB_\delta)^{\frac{5}{8}}$ = |
|---|
| 0.64842 |
| 0.503268 |
| 0.420448 |
| 0.365716 |
| 0.326329 |

| $AG_\delta$ = |
|---|
| 0.620929 |
| 0.469872 |
| 0.385553 |
| 0.330718 |
| 0.291757 |

| $(AB_\delta)^{\frac{11}{16}}$ = |
|---|
| 0.620929 |
| 0.469872 |
| 0.385553 |
| 0.330718 |
| 0.291757 |

| $AF_\delta$ = |
|---|
| 0.594604 |
| 0.438691 |
| 0.353553 |
| 0.29907 |
| 0.260847 |

| $(AB_\delta)^{\frac{3}{4}}$ = |
|---|
| 0.594604 |
| 0.438691 |
| 0.353553 |
| 0.29907 |
| 0.260847 |

$(AB_\delta)^{\frac{13}{16}}$

$AE_\delta =$

| |
|---|
| 0.569394 |
| 0.40958 |
| 0.32421 |
| 0.27045 |
| 0.233213 |

$(AB_\delta)^{\frac{13}{16}} =$

| |
|---|
| 0.569394 |
| 0.40958 |
| 0.32421 |
| 0.27045 |
| 0.233213 |

$AD_\delta =$

| |
|---|
| 0.545254 |
| 0.382401 |
| 0.297302 |
| 0.244569 |
| 0.208506 |

$(AB_\delta)^{\frac{7}{8}} =$

| |
|---|
| 0.545254 |
| 0.382401 |
| 0.297302 |
| 0.244569 |
| 0.208506 |

$AC_\delta =$

| |
|---|
| 0.522137 |
| 0.357025 |
| 0.272627 |
| 0.221165 |
| 0.186416 |

$(AB_\delta)^{\frac{15}{16}} =$

| |
|---|
| 0.522137 |
| 0.357025 |
| 0.272627 |
| 0.221165 |
| 0.186416 |

$AQ_\delta =$

| |
|---|
| 0.957603 |
| 0.933641 |
| 0.917004 |
| 0.904304 |
| 0.894058 |

$(AB_\delta)^{\frac{1}{16}} =$

| |
|---|
| 0.957603 |
| 0.933641 |
| 0.917004 |
| 0.904304 |
| 0.894058 |

$AP_\delta =$

| |
|---|
| 0.917004 |
| 0.871686 |
| 0.840896 |
| 0.817765 |
| 0.799339 |

$(AB_\delta)^{\frac{2}{16}} =$

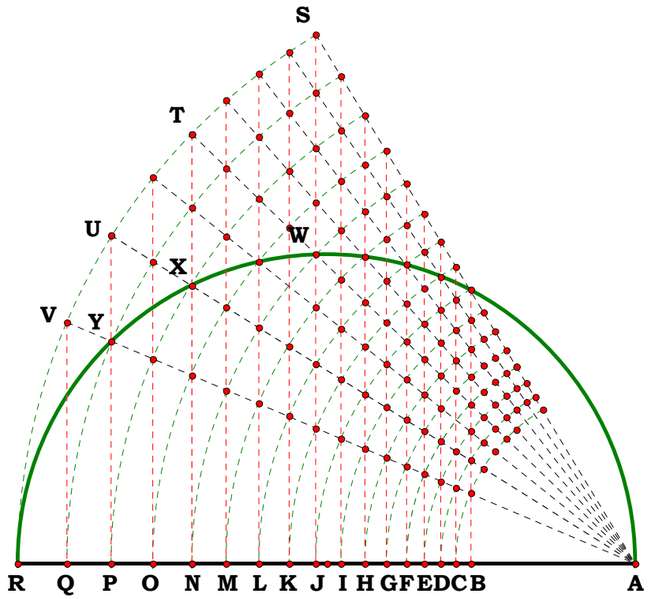
| |
|---|
| 0.917004 |
| 0.871686 |
| 0.840896 |
| 0.817765 |
| 0.799339 |

$AO_\delta =$

| |
|---|
| 0.878126 |
| 0.813841 |
| 0.771105 |
| 0.739508 |
| 0.714655 |

$(AB_\delta)^{\frac{3}{16}} =$

| |
|---|
| 0.878126 |
| 0.813841 |
| 0.771105 |
| 0.739508 |
| 0.714655 |




$AN_\delta =$

| |
|---|
| 0.840896 |
| 0.759836 |
| 0.707107 |
| 0.66874 |
| 0.638943 |

$(AB_\delta)^{\frac{4}{16}} =$

| |
|---|
| 0.840896 |
| 0.759836 |
| 0.707107 |
| 0.66874 |
| 0.638943 |

$AM_\delta =$

| |
|---|
| 0.805245 |
| 0.709414 |
| 0.64842 |
| 0.604744 |
| 0.571252 |

$(AB_\delta)^{\frac{5}{16}} =$

| |
|---|
| 0.805245 |
| 0.709414 |
| 0.64842 |
| 0.604744 |
| 0.571252 |

$AL_\delta =$

| |
|---|
| 0.771105 |
| 0.662338 |
| 0.594604 |
| 0.546873 |
| 0.510732 |

$(AB_\delta)^{\frac{6}{16}} =$

| |
|---|
| 0.771105 |
| 0.662338 |
| 0.594604 |
| 0.546873 |
| 0.510732 |

$AK_\delta =$

| |
|---|
| 0.738413 |
| 0.618386 |
| 0.545254 |
| 0.494539 |
| 0.456624 |

$(AB_\delta)^{\frac{7}{16}} =$

| |
|---|
| 0.738413 |
| 0.618386 |
| 0.545254 |
| 0.494539 |
| 0.456624 |

$AJ_\delta =$

| |
|---|
| 0.707107 |
| 0.57735 |
| 0.5 |
| 0.447214 |
| 0.408248 |

$(AB_\delta)^{\frac{8}{16}} =$

| |
|---|
| 0.707107 |
| 0.57735 |
| 0.5 |
| 0.447214 |
| 0.408248 |

$AI_\delta =$

| |
|---|
| 0.677128 |
| 0.539038 |
| 0.458502 |
| 0.404417 |
| 0.364998 |

$(AB_\delta)^{\frac{9}{16}} =$

| |
|---|
| 0.677128 |
| 0.539038 |
| 0.458502 |
| 0.404417 |
| 0.364998 |

$AH_\delta =$

| |
|---|
| 0.64842 |
| 0.503268 |
| 0.420448 |
| 0.365716 |
| 0.326329 |

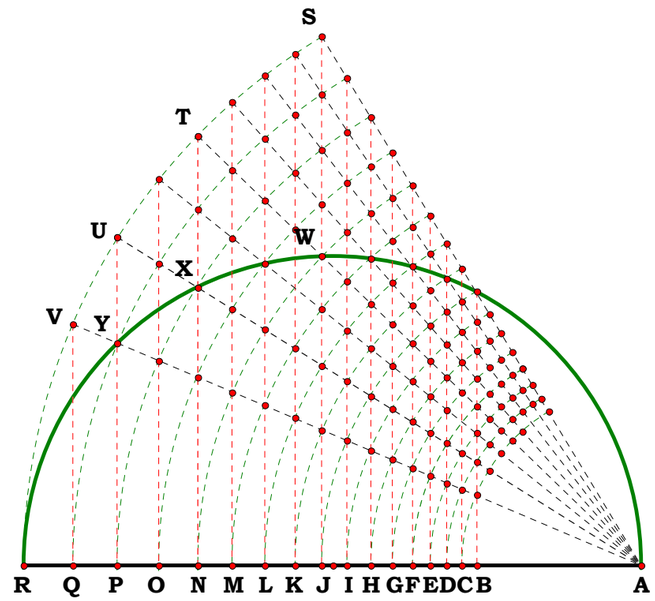
$(AB_\delta)^{\frac{10}{16}} =$

| |
|---|
| 0.64842 |
| 0.503268 |
| 0.420448 |
| 0.365716 |
| 0.326329 |

| $AG_\delta$ = | $(AB_\delta)^{\frac{11}{16}}$ = | $AF_\delta$ = | $(AB_\delta)^{\frac{12}{16}}$ = |
|---|---|---|---|
| 0.620929 | 0.620929 | 0.594604 | 0.594604 |
| 0.469872 | 0.469872 | 0.438691 | 0.438691 |
| 0.385553 | 0.385553 | 0.353553 | 0.353553 |
| 0.330718 | 0.330718 | 0.29907 | 0.29907 |
| 0.291757 | 0.291757 | 0.260847 | 0.260847 |

| $AE_\delta$ = | $(AB_\delta)^{\frac{13}{16}}$ = | $AD_\delta$ = | $(AB_\delta)^{\frac{14}{16}}$ = |
|---|---|---|---|
| 0.569394 | 0.569394 | 0.545254 | 0.545254 |
| 0.40958 | 0.40958 | 0.382401 | 0.382401 |
| 0.32421 | 0.32421 | 0.297302 | 0.297302 |
| 0.27045 | 0.27045 | 0.244569 | 0.244569 |
| 0.233213 | 0.233213 | 0.208506 | 0.208506 |

| $AC_\delta$ = | $\left[(AB_\delta)^{\frac{15}{16}}\right]$ = | $AB_\delta$ = | $\left[(AB_\delta)^{\frac{16}{16}}\right]$ = |
|---|---|---|---|
| 0.522137 | 0.522137 | 0.5 | 0.5 |
| 0.357025 | 0.357025 | 0.333333 | 0.333333 |
| 0.272627 | 0.272627 | 0.25 | 0.25 |
| 0.221165 | 0.221165 | 0.2 | 0.2 |
| 0.186416 | 0.186416 | 0.166667 | 0.166667 |




**Resultant Equation**

$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

**or**

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

**depending on direction of transcription.**

121693B

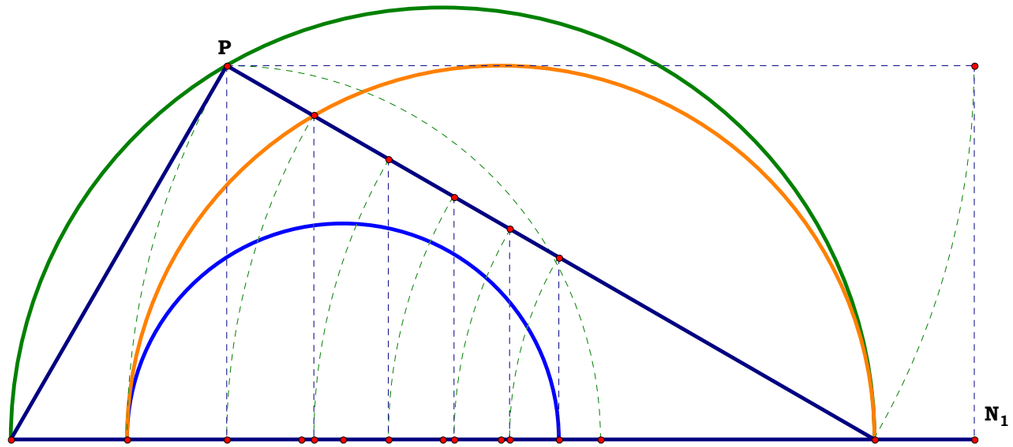
**Given.**

$DIV \equiv 7$

$\Delta := DIV$

$\delta := 1 .. \Delta$

## Exponential Series




**Descriptions.**

$AP := 10 \qquad AH := AP \qquad PF_1 := 5 \qquad AF_1 := \sqrt{AP^2 - (PF_1)^2}$

$AF_2 := \frac{AF_1 \cdot AF_1}{AP} \qquad AF_{\delta+1} := \frac{AF_1 \cdot AF_\delta}{AP}$

$\frac{PF_1^{\,2}}{AF_2} = 3.333333 \qquad \frac{PF_1^{\,2}}{\left(\frac{10}{3}\right)} = 7.5 \qquad \frac{AP}{PF_1} = 2$

$\sqrt{AF_\Delta \cdot AH} = 6.044456$

$AF_\Delta = 3.653545 \qquad \frac{AH}{AF_\Delta} = 2.737068$

**Definitions.**

$\left[(AF_\Delta)^{\Delta-\delta} \cdot (AH)^\delta\right]^{\frac{1}{\Delta}} = \quad if(\Delta - \delta, AF_{\Delta-\delta}, 0) = \quad \Delta - \delta =$

| | | |
|---|---|---|
| 4.21875 | 4.21875 | 6 |
| 4.871393 | 4.871393 | 5 |
| 5.625 | 5.625 | 4 |
| 6.495191 | 6.495191 | 3 |
| 7.5 | 7.5 | 2 |
| 8.660254 | 8.660254 | 1 |
| 10 | 0 | 0 |

J H $F_1$ 1 $F_2$ $F_3$ $F_4$ $F_5$ $F_6$ $N_2$ A

$F_1 1 = 1.25400$ cm

$AP = 12.54000$ cm

$AH = 12.54000$ cm

$AF_1 = 10.85996$ cm

$AF_2 = 9.40500$ cm

$AF_3 = 8.14497$ cm

$AF_4 = 7.05375$ cm

$AF_5 = 6.10873$ cm

$AF_6 = 5.29031$ cm

$\frac{AH}{F_1 1} = 10.00000 \qquad \frac{AF_3}{F_1 1} = 6.49519$

$\frac{AP}{F_1 1} = 10.00000 \qquad \frac{AF_4}{F_1 1} = 5.62500$

$\frac{AF_1}{F_1 1} = 8.66025 \qquad \frac{AF_5}{F_1 1} = 4.87139$

$\frac{AF_2}{F_1 1} = 7.50000 \qquad \frac{AF_6}{F_1 1} = 4.21875$

**Unit.**

$L := 1$

**Given.**

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

**121693C**

**Descriptions.**

**Every number is expressible as a ratio.**

$OJ := \frac{X}{Y}$ $OK := \sqrt{OJ}$ $OJ_1 := OK^2$

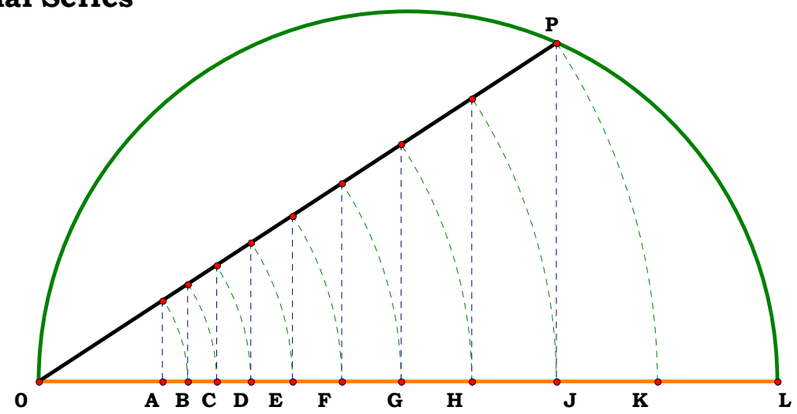
$OJ_2 := \frac{OJ^2}{OK}$ $OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$

$\left[\left(OJ_\Delta\right)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} = \text{if}\left(\Delta-\delta, OJ_{\Delta-\delta}, 0\right) = \Delta-\delta = \text{if}\left[\Delta-\delta, \left[(OJ)_{\Delta-\delta}\right]^{\frac{1}{[\Delta-(\delta-1)]}}, 0\right] =$

| | | | |
|---|---|---|---|
| 0.16807 | 0.16807 | 9 | 0.83666 |
| 0.200882 | 0.200882 | 8 | 0.83666 |
| 0.2401 | 0.2401 | 7 | 0.83666 |
| 0.286974 | 0.286974 | 6 | 0.83666 |
| 0.343 | 0.343 | 5 | 0.83666 |
| 0.409963 | 0.409963 | 4 | 0.83666 |
| 0.49 | 0.49 | 3 | 0.83666 |
| 0.585662 | 0.585662 | 2 | 0.83666 |
| 0.7 | 0.7 | 1 | 0.83666 |
| 0.83666 | 0 | 0 | 0 |

**Definitions.**

**Exponential Series**




**A = 0.16807**
**B = 0.20088**
**C = 0.24010**
**D = 0.28697**
**E = 0.34300**
**F = 0.40996**
**G = 0.49000**
**H = 0.58566**
**J = 0.70000**
**K = 0.83666**
**L = 1.00000**
**Unit = 1.00000**
**XY = 0.70000**
**X = 14.00000**
**Y = 20.00000**

$A^{\frac{1}{10}}\text{-}K = 0.00000$
$A^{\frac{2}{10}}\text{-}J = 0.00000$
$A^{\frac{3}{10}}\text{-}H = 0.00000$
$A^{\frac{4}{10}}\text{-}G = 0.00000$
$A^{\frac{5}{10}}\text{-}F = 0.00000$
$A^{\frac{6}{10}}\text{-}E = 0.00000$
$A^{\frac{7}{10}}\text{-}D = 0.00000$
$A^{\frac{8}{10}}\text{-}C = 0.00000$
$A^{\frac{9}{10}}\text{-}B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}}\text{-}K = 0.00000$
$H^{\frac{1}{3}}\text{-}K = 0.00000$
$G^{\frac{1}{4}}\text{-}K = 0.00000$
$F^{\frac{1}{5}}\text{-}K = 0.00000$
$E^{\frac{1}{6}}\text{-}K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}}\text{-}K = 0.00000$
$B^{\frac{1}{9}}\text{-}K = 0.00000$
$A^{\frac{1}{10}}\text{-}K = 0.00000$

$K^2\text{-}J = 0.00000$
$K^3\text{-}H = 0.00000$
$K^4\text{-}G = 0.00000$
$K^5\text{-}F = 0.00000$
$K^6\text{-}E = 0.00000$
$K^7\text{-}D = 0.00000$
$K^8\text{-}C = 0.00000$
$K^9\text{-}B = 0.00000$
$K^{10}\text{-}A = 0.00000$

121693D

Unit.

$L := 1$

Given.

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

# Exponential Series

**Descriptions.**

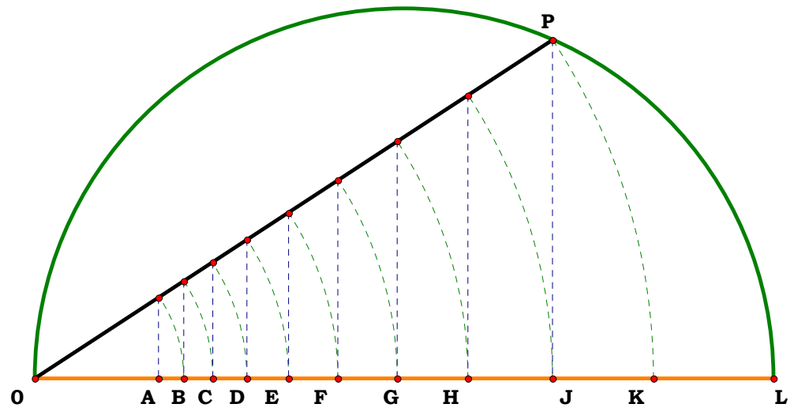
The first was X/Y, and this is Y/X.

$OJ := \frac{Y}{X}$ $OK := \sqrt{OJ}$ $OJ_1 := OK^2$

$OJ_2 := \frac{OJ^2}{OK}$ $OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$ $\sqrt{\frac{Y}{X}} = 1.195229$

$\left[\left(OJ_\Delta\right)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} = \text{if}\left(\Delta - \delta, OJ_{\Delta-\delta}, 0\right) = \Delta - \delta + 1 = \left(\sqrt{\frac{Y}{X}}\right)^{\Delta-\delta+1} = \delta = \left(\sqrt{\frac{Y}{X}}\right)^{\delta} =$

| | | | | | |
|---|---|---|---|---|---|
| 5.949902 | 5.949902 | 10 | 5.949902 | 1 | 1.195229 |
| 4.978045 | 4.978045 | 9 | 4.978045 | 2 | 1.428571 |
| 4.164931 | 4.164931 | 8 | 4.164931 | 3 | 1.707469 |
| 3.484632 | 3.484632 | 7 | 3.484632 | 4 | 2.040816 |
| 2.915452 | 2.915452 | 6 | 2.915452 | 5 | 2.439242 |
| 2.439242 | 2.439242 | 5 | 2.439242 | 6 | 2.915452 |
| 2.040816 | 2.040816 | 4 | 2.040816 | 7 | 3.484632 |
| 1.707469 | 1.707469 | 3 | 1.707469 | 8 | 4.164931 |
| 1.428571 | 1.428571 | 2 | 1.428571 | 9 | 4.978045 |
| 1.195229 | 0 | 1 | 1.195229 | 10 | 5.949902 |




**Definitions.**

$A = 0.16807$
$B = 0.20088$
$C = 0.24010$
$D = 0.28697$
$E = 0.34300$
$F = 0.40996$
$G = 0.49000$
$H = 0.58566$
$J = 0.70000$
$K = 0.83666$
$L = 1.00000$
$\text{Unit} = 1.00000$
$XY = 0.70000$
$X = 14.00000$
$Y = 20.00000$

$A^{\frac{1}{10}} - K = 0.00000$
$A^{\frac{2}{10}} - J = 0.00000$
$A^{\frac{3}{10}} - H = 0.00000$
$A^{\frac{4}{10}} - G = 0.00000$
$A^{\frac{5}{10}} - F = 0.00000$
$A^{\frac{6}{10}} - E = 0.00000$
$A^{\frac{7}{10}} - D = 0.00000$
$A^{\frac{8}{10}} - C = 0.00000$
$A^{\frac{9}{10}} - B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}} - K = 0.00000$
$H^{\frac{1}{3}} - K = 0.00000$
$G^{\frac{1}{4}} - K = 0.00000$
$F^{\frac{1}{5}} - K = 0.00000$
$E^{\frac{1}{6}} - K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}} - K = 0.00000$
$B^{\frac{1}{9}} - K = 0.00000$
$A^{\frac{1}{10}} - K = 0.00000$

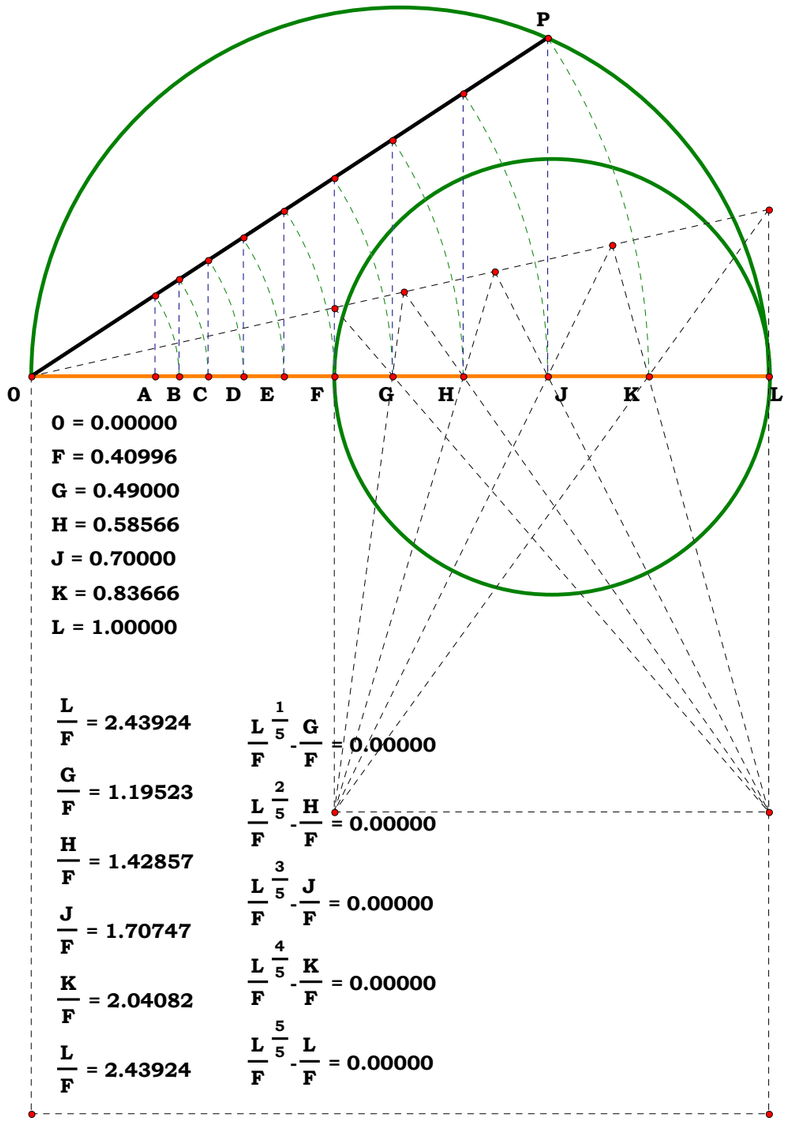
$K^2 - J = 0.00000$
$K^3 - H = 0.00000$
$K^4 - G = 0.00000$
$K^5 - F = 0.00000$
$K^6 - E = 0.00000$
$K^7 - D = 0.00000$
$K^8 - C = 0.00000$
$K^9 - B = 0.00000$
$K^{10} - A = 0.00000$
$\frac{L}{K} = 1.19523$

**And if given any member of a series, save the square root, we have from 042906**

**Let us plead the 5th.**

**So, if one is clever enough, which I am not, one can actually do any root seriesl from the fact that any number is expressible as a ratio, and triangles just love ratios.**

**At one time I was seriously considering changing the name of the Delian Quest to One Circle, One Square, and One Line, but it did not stick.**




$\sqrt{\frac{X}{Y}} = 0.83666$

$J^{\frac{1}{2}} - K = 0.00000$

$H^{\frac{1}{3}} - K = 0.00000$

$G^{\frac{1}{4}} - K = 0.00000$

$F^{\frac{1}{5}} - K = 0.00000$

$E^{\frac{1}{6}} - K = 0.00000$

$D^{\frac{1}{7}} = 0.83666$

$C^{\frac{1}{8}} - K = 0.00000$

$B^{\frac{1}{9}} - K = 0.00000$

$A^{\frac{1}{10}} - K = 0.00000$

$K^2 - J = 0.00000$

$K^3 - H = 0.00000$

$K^4 - G = 0.00000$

$K^5 - F = 0.00000$

$K^6 - E = 0.00000$

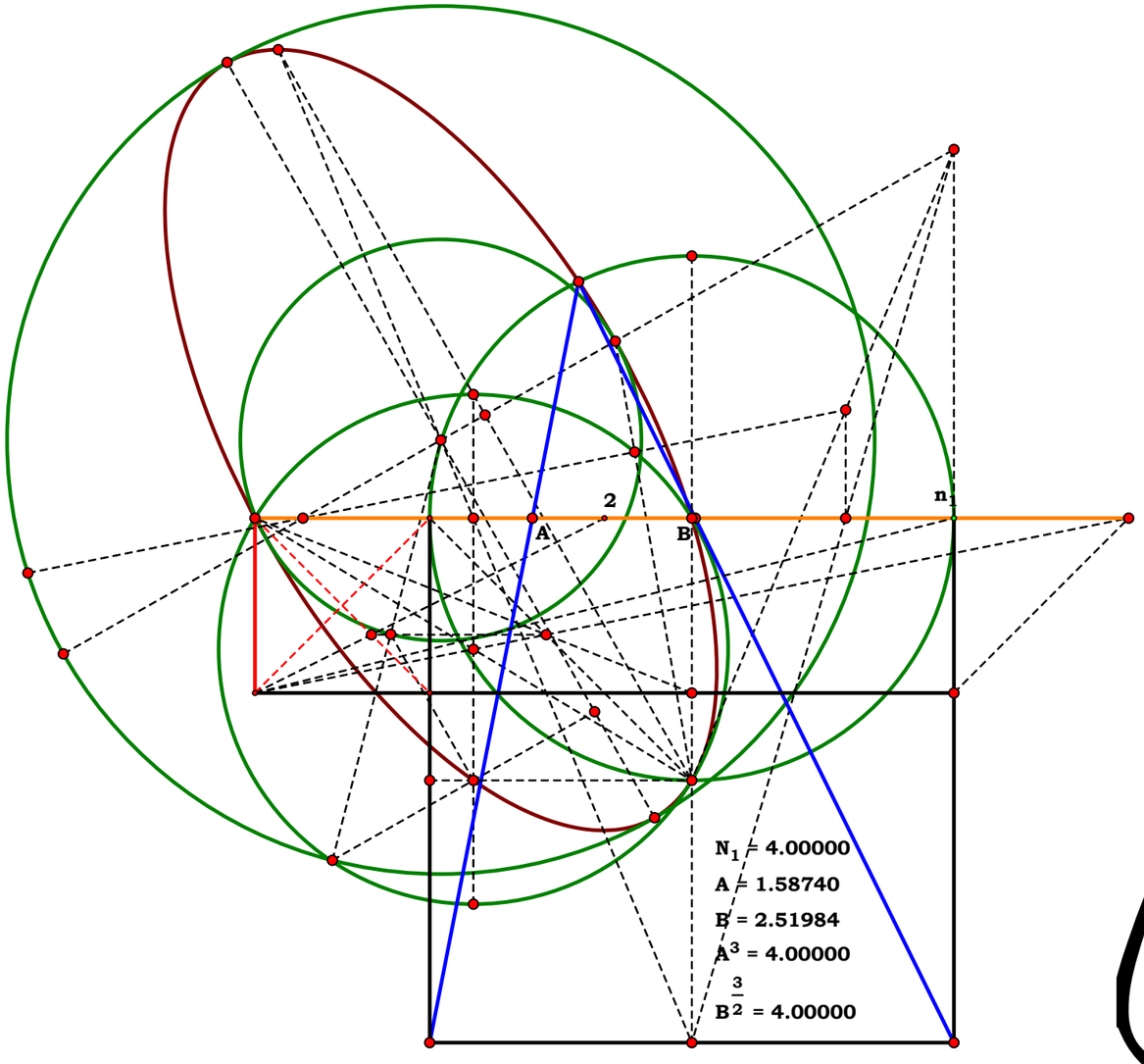
$K^7 - D = 0.00000$

$K^8 - C = 0.00000$

$K^9 - B = 0.00000$

$K^{10} - A = 0.00000$

$\frac{L}{K} = 1.19523$




# The Delian Quest 1994

**John Clark**

040694A

Unit.

Given.

$N_1 := 3$ $AB := N_1$

$N_2 := 4$ $BC := N_2$

$N_3 := 5$ $AC := N_3$

Descriptions.

$AK := AC$ $BD := BC$ $AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$ $FK := AK - AF$

$CF := \sqrt{AC^2 - AF^2}$ $AH := AF + \frac{FK}{2}$ $HN := \frac{CF}{2}$ $BF := AB - AF$

$DF := BD - BF$ $CD := \sqrt{CF^2 + DF^2}$ $BM := \frac{CF \cdot BD}{CD}$ $BE := \frac{CF \cdot BM}{CD}$

$GL := \frac{HN \cdot AB}{AH + BE}$

Definitions. $S_1 := \begin{pmatrix} AB \\ BC \\ AC \end{pmatrix}$ $S_2 := \begin{pmatrix} BC \\ AC \\ AB \end{pmatrix}$ $S_3 := \begin{pmatrix} AC \\ AB \\ BC \end{pmatrix}$ $\delta := 0 .. 2$

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$AK - N_3 = 0$ $BD - N_2 = 0$ $AF - \frac{N_1^2 - N_2^2 + N_3^2}{2 \cdot N_1} = 0$ $FK - \frac{(N_2 - N_1 + N_3) \cdot (N_1 + N_2 - N_3)}{2 \cdot N_1} = 0$

$CF - \frac{\sqrt{(N_1 + N_2 + N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_1 + N_2 - N_3) \cdot (N_2 - N_1 + N_3)}}{2 \cdot N_1} = 0$ $AH - \frac{(N_1 - N_2 + N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

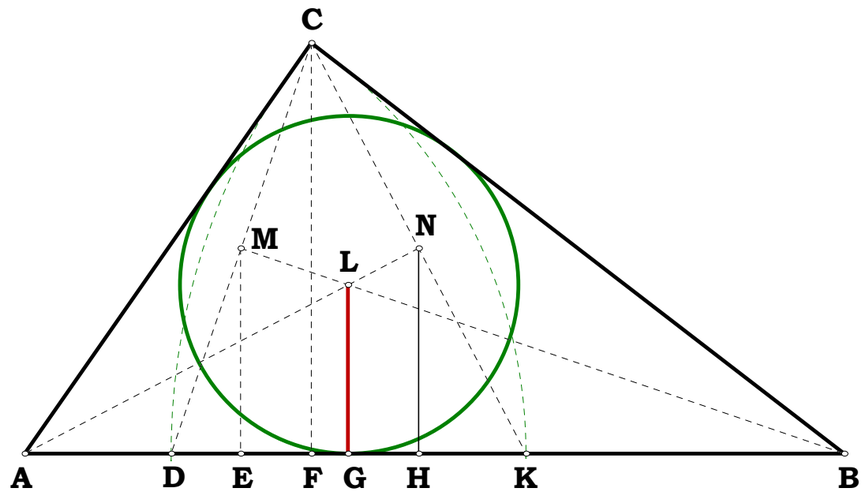
$HN - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{4 \cdot N_1} = 0$ $BF - \frac{N_1^2 + N_2^2 - N_3^2}{2 \cdot N_1} = 0$ $DF - \frac{(N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}{2 \cdot N_1} = 0$

$CD - \frac{\sqrt{N_2 \cdot (N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}}{\sqrt{N_1}} = 0$ $BM - \frac{N_2 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot \sqrt{N_1} \cdot \sqrt{N_2 \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3)}} = 0$ $BE - \frac{(N_1 + N_2 - N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

$GL - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot (N_1 + N_2 + N_3)} = 0$

## Inscribing A Circle In A Given Triangle

**Given three sides of a triangle, what is the length of the inscribed radius?**




**Imagine that, another equation for the radius along with every other definition!**

**DE = 0.58830**

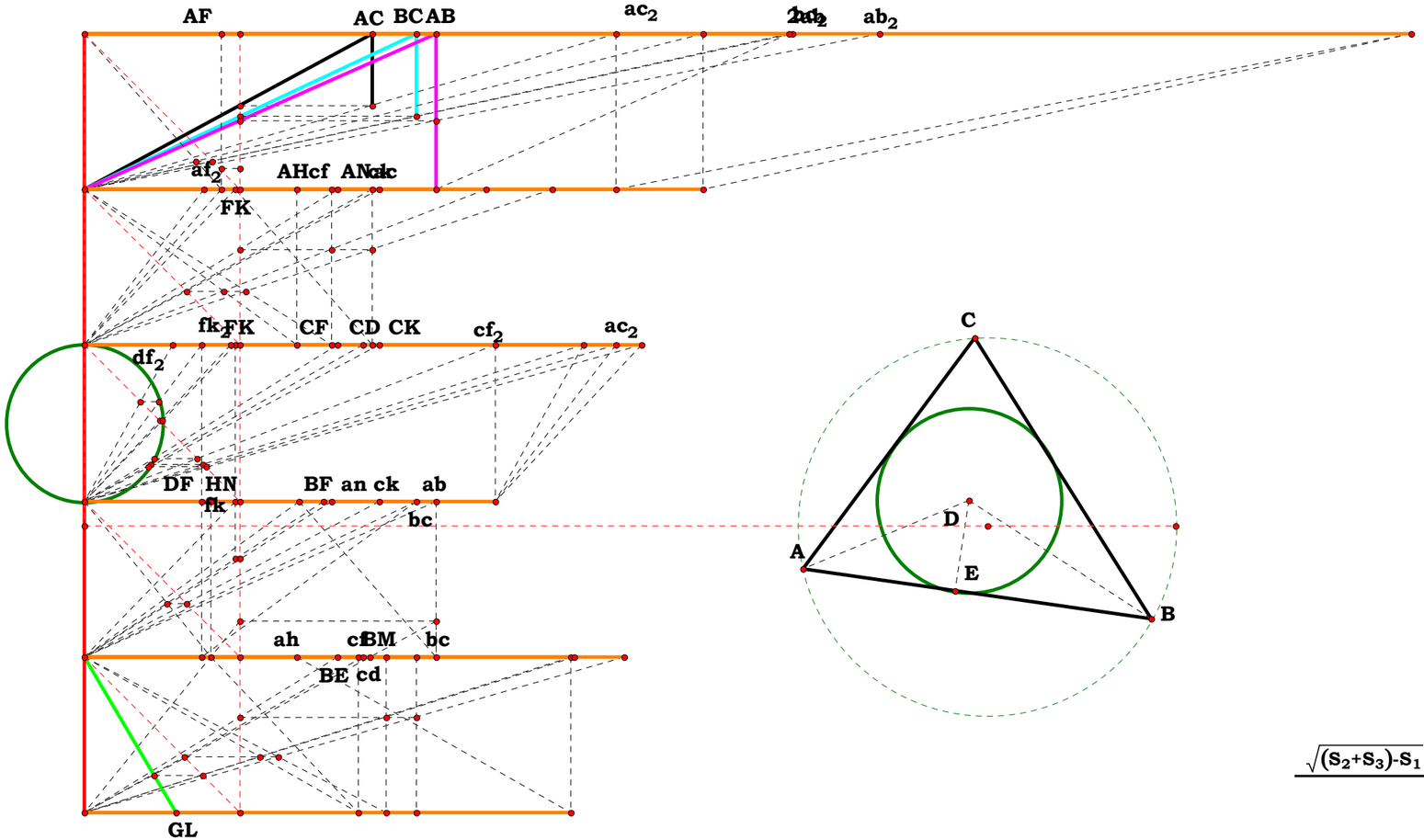
**GL = 0.58830**

**DE-GL = 0.00000**

**AB = 2.25959**

**BC = 2.13235**

**AC = 1.84623**







$S_1 = 2.25959$

$S_2 = 2.13235$

$S_3 = 1.84623$

$$\frac{\sqrt{(S_2+S_3)-S_1}\cdot\sqrt{(S_1+S_3)-S_2}\cdot\sqrt{(S_1+S_2)-S_3}}{2\cdot\sqrt{S_1+S_2+S_3}}-GL = 0.00000$$

040694A

Unit. AB is the working side or base.

AB := 1

Given.

W := 8 Y := 11

X := 20 Z := 20

Descriptions.

$AF := \frac{W}{X}$ AF = 0.4 $AP := \frac{Y}{Z}$ AP = 0.55 CF := AP

$AC := \sqrt{AF^2 + CF^2}$ BF := AB − AF $BC := \sqrt{BF^2 + CF^2}$

AK := AC BD := BC FK := AK − AF $HN := \frac{CF}{2}$

$AH := AF + \frac{FK}{2}$ DF := BD − BF $BE := BF + \frac{DF}{2}$ EM := HN

$GO := \frac{EM \cdot AB}{BE + AH}$ GO = 0.220528

Definitions.

$AF - \frac{W}{X} = 0$ $AP - \frac{Y}{Z} = 0$ $CF - \frac{Y}{Z} = 0$

$AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BF - \frac{X - W}{X} = 0$

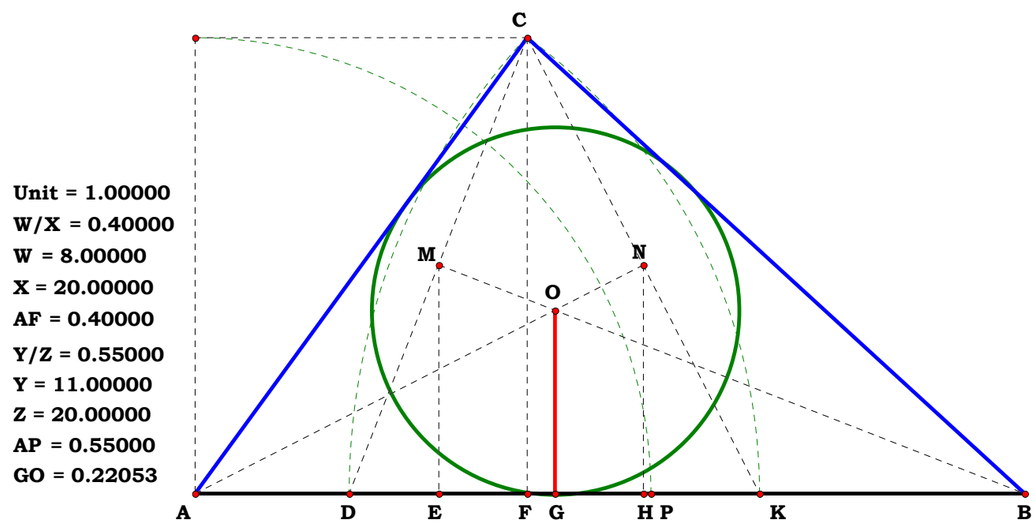
$BC - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$ $AK - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BD - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$

$FK - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - W \cdot Z}{X \cdot Z} = 0$ $HN - \frac{Y}{2 \cdot Z} = 0$ $AH - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W \cdot Z}{2 \cdot X \cdot Z} = 0$ $DF - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)} + Z \cdot (W - X)}{X \cdot Z} = 0$

$BE - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)} + Z \cdot (X - W)}{2 \cdot X \cdot Z} = 0$ $EM - \frac{Y}{2 \cdot Z} = 0$ $GO - \frac{X \cdot Y}{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)} + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + X \cdot Z} = 0$

## Inscribing A Circle In A Given Triangle

**Given three sides of a triangle, what is the length of the inscribed radius?**




Unit = 1.00000
W/X = 0.40000
W = 8.00000
X = 20.00000
AF = 0.40000
Y/Z = 0.55000
Y = 11.00000
Z = 20.00000
AP = 0.55000
GO = 0.22053

$\frac{X \cdot Y}{X \cdot Z + \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}} - GO = 0.00000$

Unit.

$AB := 1$

Given.

$N := 5$

042194A

Descriptions.

$AL := AB \cdot N$ $\quad AF := \sqrt{AB \cdot AL}$

$AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$ $\quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB$ $\quad BP := BL$ $\quad LR := BL$ $\quad FL := AL - AF$

$BC := AC - AB$ $\quad BJ := AJ - AB$ $\quad JL := BL - BJ$

$BF := AF - AB$ $\quad FJ := AJ - AF$ $\quad CF := AF - AC$

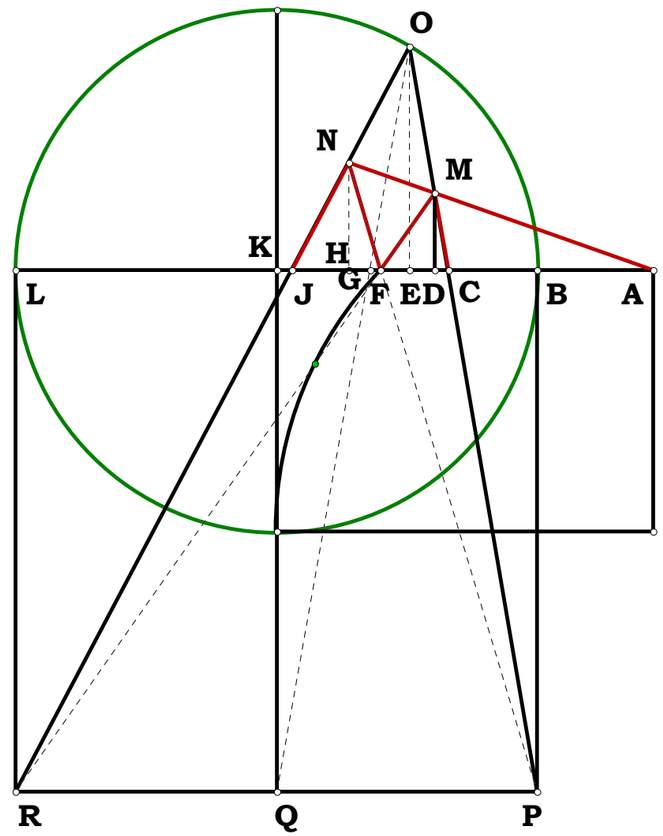
$FG := \dfrac{BF \cdot FJ}{BF + JL}$ $\quad GN := \dfrac{BP \cdot FG}{BF}$ $\quad CD := \dfrac{BC \cdot CF}{BC + FL}$

$DM := \dfrac{BP \cdot CD}{BC}$ $\quad AD := AC + CD$ $\quad AG := AF + FG$

$\dfrac{AG}{GN} - \dfrac{AD}{DM} = 0$

## The Cradle

**Are A, M, N colinear?**



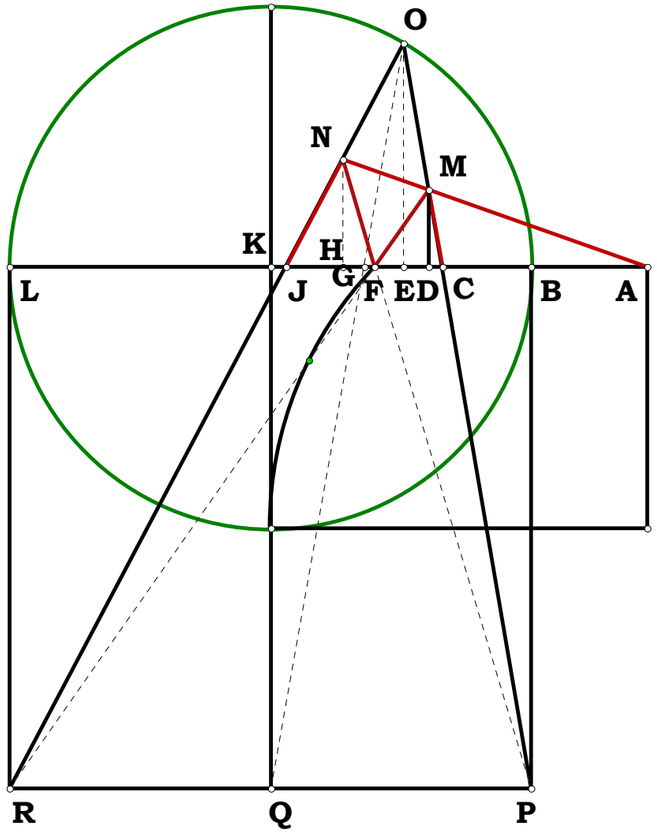
**Definitions.**

$AL - N = 0$ $\quad AF - \sqrt{N} = 0$ $\quad AC - N^{\frac{1}{3}} = 0$

$AJ - N^{\frac{2}{3}} = 0$ $\quad BL - (N - 1) = 0$ $\quad FL - \left(N - \sqrt{N}\right) = 0$

$BC - \left(N^{\frac{1}{3}} - 1\right) = 0$ $\quad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0$ $\quad JL - \left(N - N^{\frac{2}{3}}\right) = 0$ $\quad BF - \left(\sqrt{N} - 1\right) = 0$

$FJ - \left(N^{\frac{2}{3}} - \sqrt{N}\right) = 0$ $\quad CF - \left[\sqrt{N} - (N)^{\frac{1}{3}}\right] = 0$ $\quad FG - \dfrac{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$




$GN - \dfrac{\sqrt{N} \cdot (N - 1)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad CD - \dfrac{N^{\frac{2}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$DM - \dfrac{N^{\frac{4}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad AD - \dfrac{N + \sqrt{N} + N^{\frac{2}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$AG - \dfrac{N + N^{\frac{2}{3}} + N^{\frac{4}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad \dfrac{AG}{GN} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0$ $\quad \dfrac{AD}{DM} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0$

042194B

Unit.

$AB := 1$

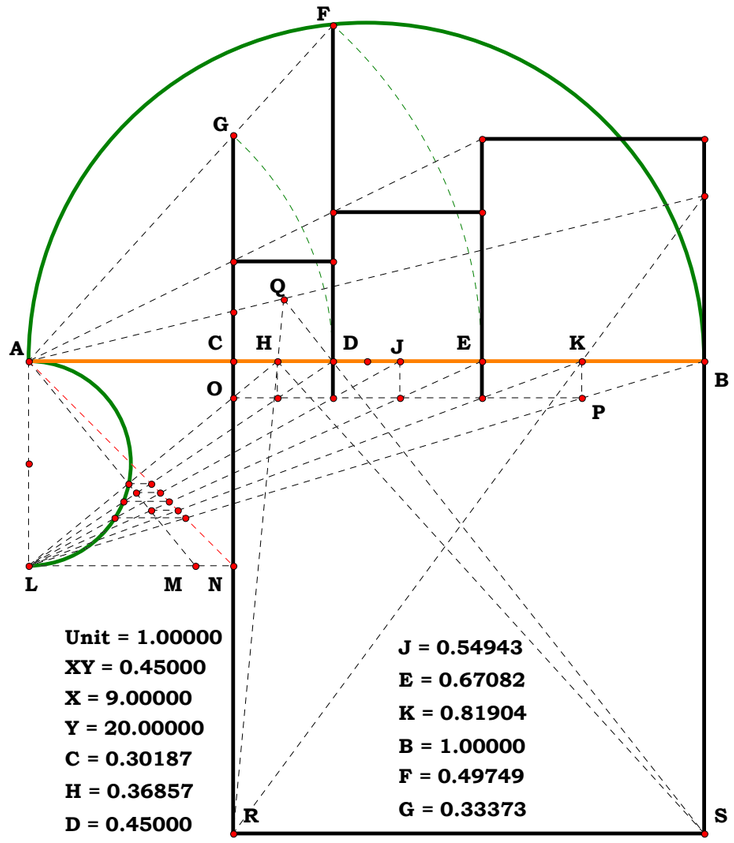
Given.

$X := 9$

$Y := 20$

## The Cradle

**Are A, M, N colinear?**




**Descriptions.**

$AD := \frac{X}{Y}$ $AE := \sqrt{AD}$ $AE = 0.67082$ $AC := AD^{\frac{3}{2}}$ $AC = 0.301869$

$\frac{AB}{AC} = 3.312693$ $AH := AC^{\frac{5}{6}}$ $AH = 0.368566$ $AD - AC^{\frac{4}{6}} = 0$

$AJ := AC^{\frac{3}{6}}$ $AJ = 0.549426$ $AE := AC^{\frac{2}{6}}$ $AE = 0.67082$

$AK := AC^{\frac{1}{6}}$ $AK = 0.819036$ $AB - AC^{\frac{0}{6}} = 0$

**In this plate one can see some of the various ways to project a root series. We have one method down in the little unit box, L, M, and N. We have the parallel line method shown by O and P. We have the peak method R, Q, and S. and we have the method I started wtih. So, eventually one can show, many ways that each of them are part of a root series. So, here I will simply cash out.**

**When every possible root series is constructed in exactly the same way, it is very odd to say that one cannot do this or that root series. We have to learn how to start and stop them, and how many geometric recursions we want between those two limits. Learning the different ways to produce those series and learning how they are all interrelated can do nothing more than help us in that regard.**

**Definitions.**

$AD - \frac{X}{Y} = 0$ $AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$AC := \left(\frac{X}{Y}\right)^{\frac{3}{2}}$ $AH - \left[\left(\frac{X}{Y}\right)^{\frac{3}{2}}\right]^{\frac{5}{6}} = 0$

**It is perfectly silly, but Mathcad 15 will not reduce these expressions. You have to do it manually.**

$AH - \left(\frac{X}{Y}\right)^{\frac{5}{4}} = 0$ $AJ - \left(\frac{X}{Y}\right)^{\frac{3}{4}} = 0$

$AE - \left(\frac{X}{Y}\right)^{\frac{1}{2}} = 0$ $AK - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$ $AB - \left(\frac{X}{Y}\right)^{\frac{0}{12}} = 0$

Given.

# Tangents and Similarity Points.

042694A

**What is the Algebraic names of the similarity points C and F in relation to the radius of each circle and the difference between their centers?**

**I believe that I was almost laughing when I drew this up originally. I made it an acronym side show. I actually get annoyed with acronyms. It scared me so much I never did it again. But, for the last version of DQ. I should at least put a dress on the graphics and remove the acorns. For some reason, I always liked this write-up.**

**Descriptions.**

**I will work with point C first.**

**Given AD = large radius**
**BE = small radius**
**AB = difference between centers**

$AD := 4 \quad BE := 2 \quad AB := 7 \qquad DE := AD - BE$

$$AC := \frac{AB \cdot AD}{DE} \qquad AC = 14$$

**AC "External similarity point Origin to center of Radius Large"**

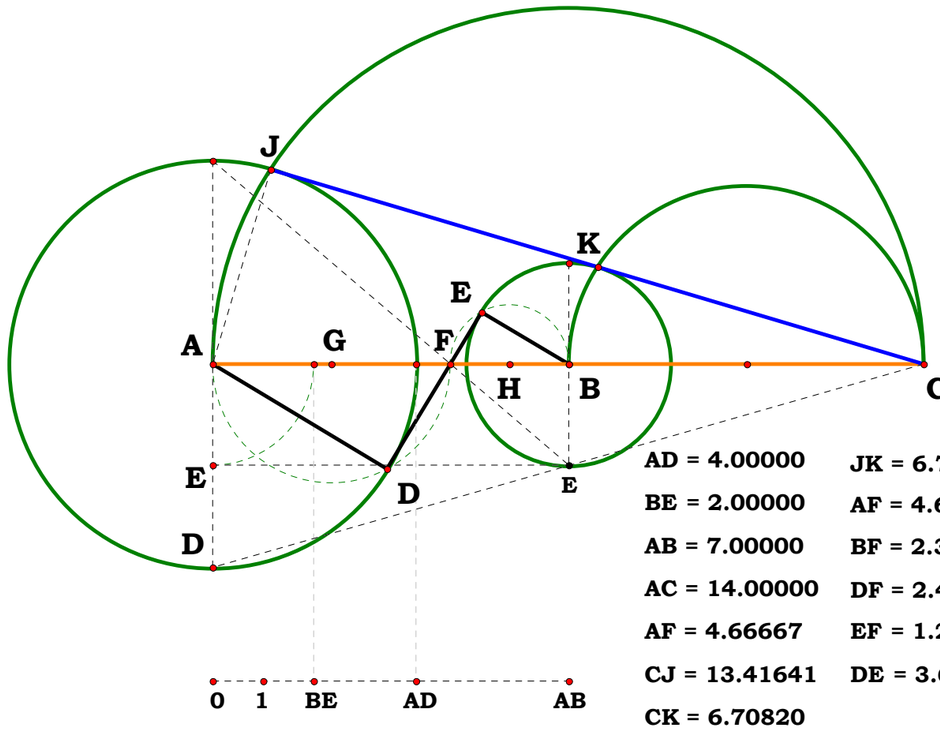
$$AC := \text{if}\left(AD \neq BE\ ,\ \text{if}\left(BE > AD\ ,\ 0\ ,\ \frac{AB \cdot AD}{AD - BE}\right)\ ,\ \infty\right) \qquad AC = 14$$

**What is the length of line JC tangent to both circles?**

$$CJ := \sqrt{AC^2 - AD^2} \qquad CJ = 13.416408$$

**And what is the formula?**
**JC " External similarity point Origin to Tangent (Large Radius)"**

$$CJ - AD \cdot \frac{\sqrt{(AD - BE + AB) \cdot (-AD + BE + AB)}}{AD - BE} = 0$$




AD = 4.00000
BE = 2.00000
AB = 7.00000
AC = 14.00000
AF = 4.66667
CJ = 13.41641
CK = 6.70820

JK = 6.70820
AF = 4.66667
BF = 2.33333
DF = 2.40370
EF = 1.20185
DE = 3.60555

**What is the length of the line tangent to the least circle (CK)?**

$BC := AC - AB \qquad BC = 7$

$CK := \sqrt{BC^2 - BE^2}$

$CK = 6.708204$

**And what is the formula?**

**CK " External similarity point Origin to Tangent (Small Radius)"**

$$CK - BE \cdot \frac{\sqrt{-(AD - BE + AB) \cdot (AD - BE - AB)}}{AD - BE} = 0$$

**Lastly what is the length of line from tangent to tangent of these circles?**

$JK := CJ - CK$

$JK = 6.708204$

**And what is the formula for, JK, Tangent to Tangent"?**

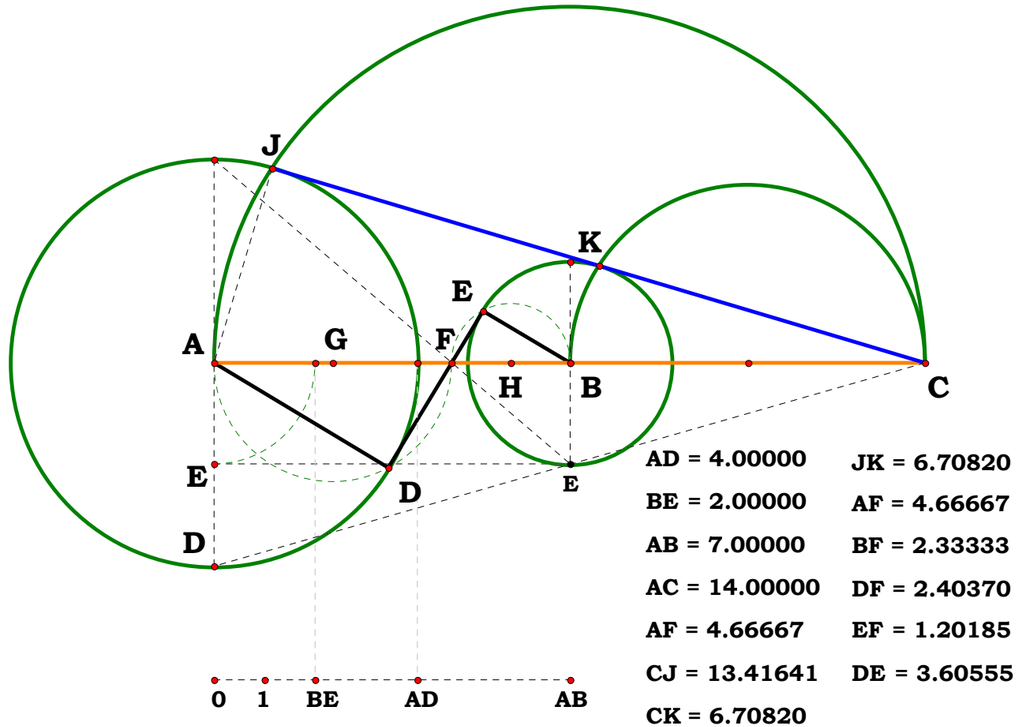
$$JK - \sqrt{-(AD - BE + AB) \cdot (AD - BE - AB)} = 0$$

**I will now turn my attention to the point F the internal similarity point.**

$$AF := \frac{AB \cdot AD}{AD + BE} \qquad AF = 4.666667$$

**AF "Internal similarity point to center of Radius Large"**

$$AF - AB \cdot \frac{AD}{AD + BE} = 0$$




AD = 4.00000
BE = 2.00000
AB = 7.00000
AC = 14.00000
AF = 4.66667
CJ = 13.41641
CK = 6.70820

JK = 6.70820
AF = 4.66667
BF = 2.33333
DF = 2.40370
EF = 1.20185
DE = 3.60555

$BF := AB - AF \qquad BF = 2.333333$

**BF "Internal similarity point to center of Radius Small"**

$$BF - AB \cdot \frac{BE}{AD + BE} = 0$$

$DF := \sqrt{AF^2 - AD^2} \qquad DF = 2.403701$

**DF "Internal similarity point Origin to Tangent (Large Radius)"**

$$DF - AD \cdot \frac{\sqrt{-(AD + BE - AB) \cdot (AD + BE + AB)}}{(AD + BE)} = 0$$

$DF = 2.403701 \qquad EF := \sqrt{BF^2 - BE^2} \qquad EF = 1.20185$

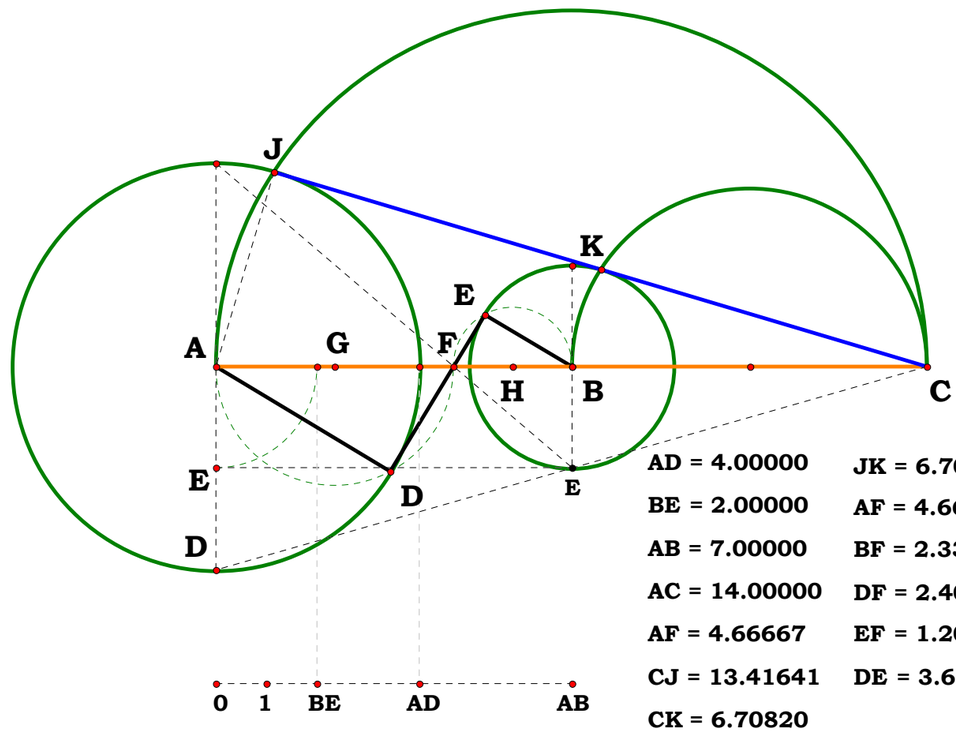
**EF "Internal similarity point Origin to Tangent (Small Radius)"**

$$EF - BE \cdot \frac{\sqrt{-(AD + BE - AB) \cdot (AD + BE + AB)}}{AD + BE} = 0$$

$DE := DF + EF \qquad DE = 3.605551$

**DE "Internal similarity point Tangent to Tangent"**

$$DE - \sqrt{-(AD + BE - AB) \cdot (AD + BE + AB)} = 0 \qquad DE = 3.605551$$




AD = 4.00000 JK = 6.70820
BE = 2.00000 AF = 4.66667
AB = 7.00000 BF = 2.33333
AC = 14.00000 DF = 2.40370
AF = 4.66667 EF = 1.20185
CJ = 13.41641 DE = 3.60555
CK = 6.70820

042694B

**Given.**

$N_1 := 3$

$N_2 := 2$

$N_3 := 7$

**Descriptions.**

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$

**What is the length of the AO, O being the similarity point?**

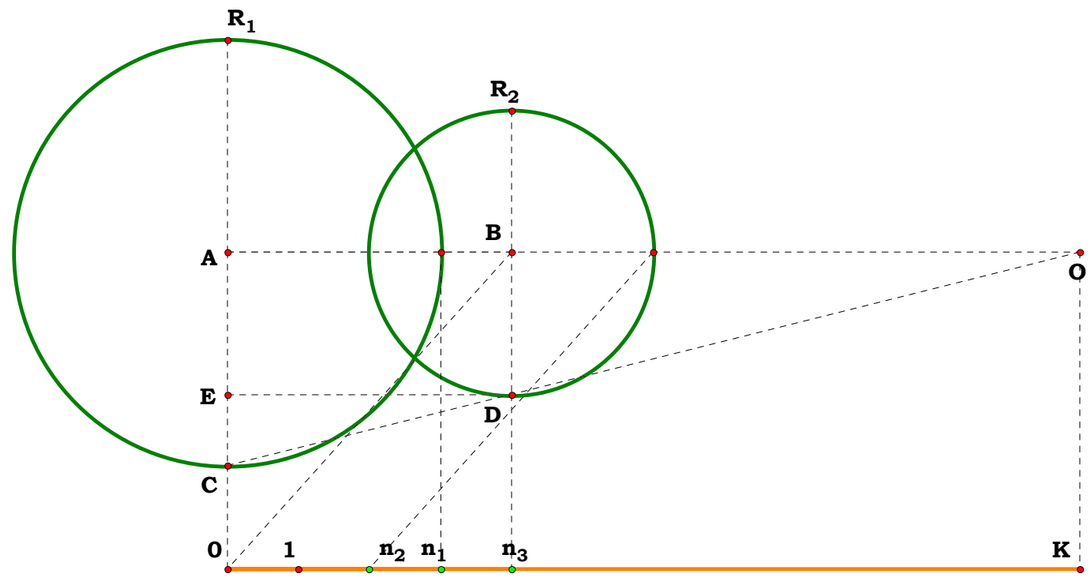
$AC := R_1$ $BD := R_2$ $AB := N_3$

$DE := AB$ $AE := BD$ $CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE}$ $AO = 21$

$\frac{N_3 \cdot R_1}{R_1 - R_2} = 21$ $AO - \frac{N_3 \cdot R_1}{R_1 - R_2} = 0$

## Tangents and Similarity Points.



**What is the length of the tangent GO?**

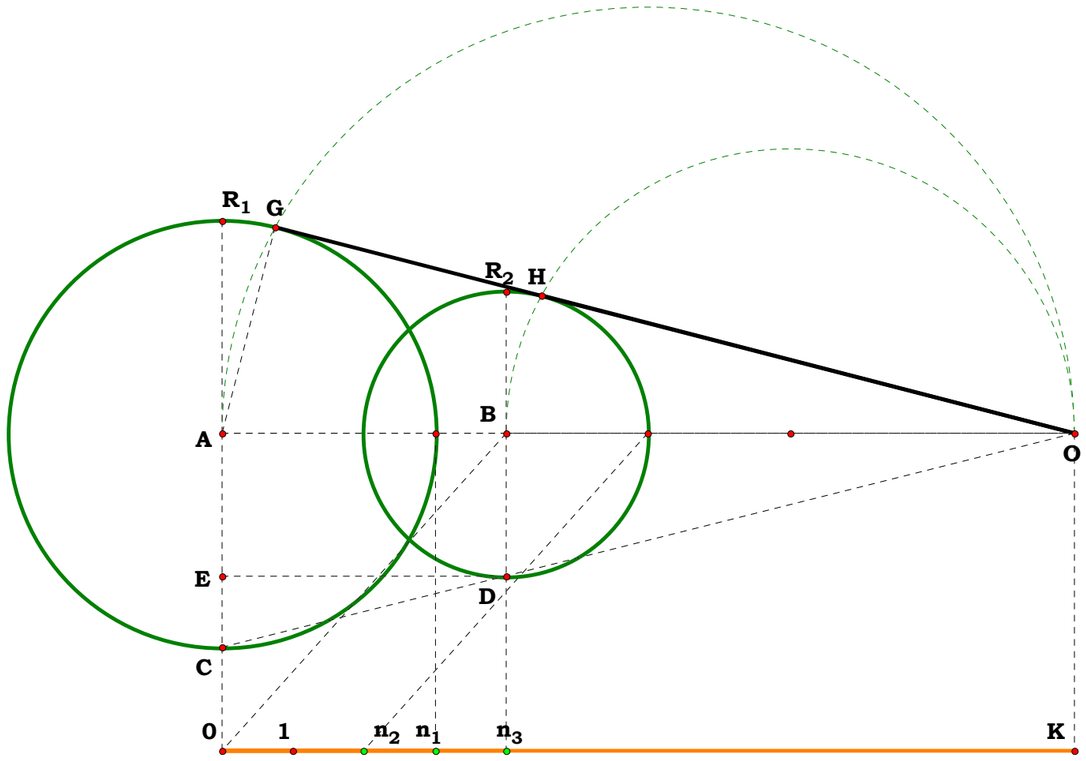
$$AG := R_1 \qquad GO := \sqrt{AO^2 - AG^2}$$

$$GO - \frac{R_1 \cdot \sqrt{N_3^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2}}{\sqrt{(R_1 - R_2)^2}} = 0$$

**What is the length of the tangent HO?**

$$BH := R_2 \qquad HO := \sqrt{\left(\frac{N_3 \cdot R_1}{R_1 - R_2} - N_3\right)^2 - R_2^2}$$

$$HO - \frac{R_2 \cdot \sqrt{N_3^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2}}{\sqrt{(R_1 - R_2)^2}} = 0$$



**What is the length of line tangent to tangent of these circles?**

$$GH := \frac{GO \cdot AB}{AO} \qquad GH - \sqrt{N_3^{\,2} - R_1^{\,2} + 2 \cdot R_1 \cdot R_2 - R_2^{\,2}} = 0$$

**What are the names of the tangents AP and BP to the similarity point P?**

$$AP := \frac{N_3 \cdot R_1}{R_1 + R_2} \qquad BP := N_3 - \frac{N_3 \cdot R_1}{R_1 + R_2}$$

**What is JP?**

$$JP := \sqrt{AP^2 - R_1^{\,2}} \qquad JP - \frac{R_1 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{R_1 + R_2} = 0$$

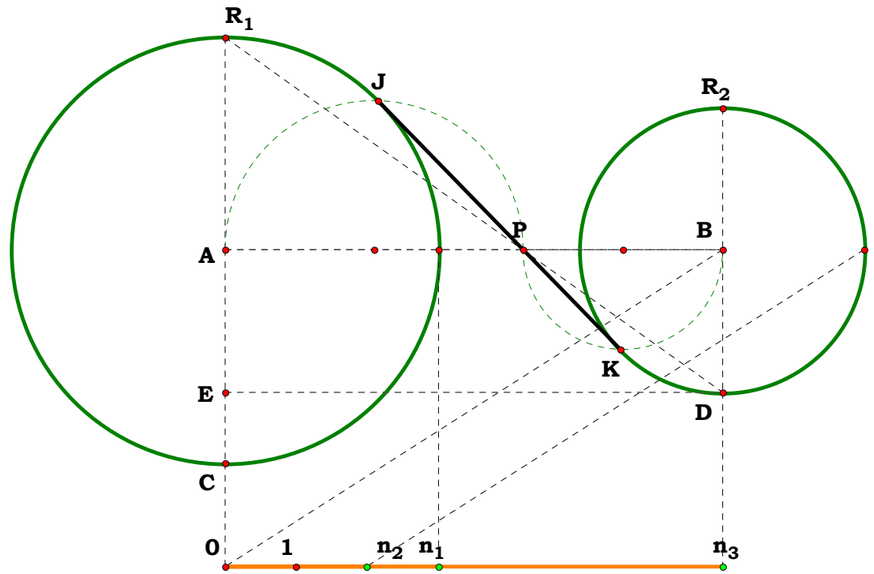
**What is KP?**

$$BP - \frac{N_3 \cdot R_2}{R_1 + R_2} = 0$$

$$KP := \frac{JP \cdot BP}{AP} \qquad KP - \frac{R_2 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{R_1 + R_2} = 0$$

**What is JK?**

$$JK := \frac{JP \cdot AB}{AP} \qquad JK - \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}} = 0$$



042694C

Unit.

$AB := 1$

Given.

$W := 8$ $Y := 4$

$X := 20$ $Z := 20$

Descriptions.

I will work with point C first.

Given AD = large radius
BE = small radius
AB = difference between centers

$AD := \frac{W}{X}$ $BE := \frac{Y}{Z}$ $AB := 1$

$DE := AD - BE$ $AC := \frac{AB \cdot AD}{DE}$ $AC = 2$

AC "External similarity point Origin to center of Radius Large"

$AC := if\left(AD \neq BE, if\left(BE > AD, 0, \frac{AB \cdot AD}{AD - BE}\right), \infty\right)$ $AC = 2$

What is the length of line JC tangent to both circles?

$CJ := \sqrt{AC^2 - AD^2}$ $CJ = 1.959592$

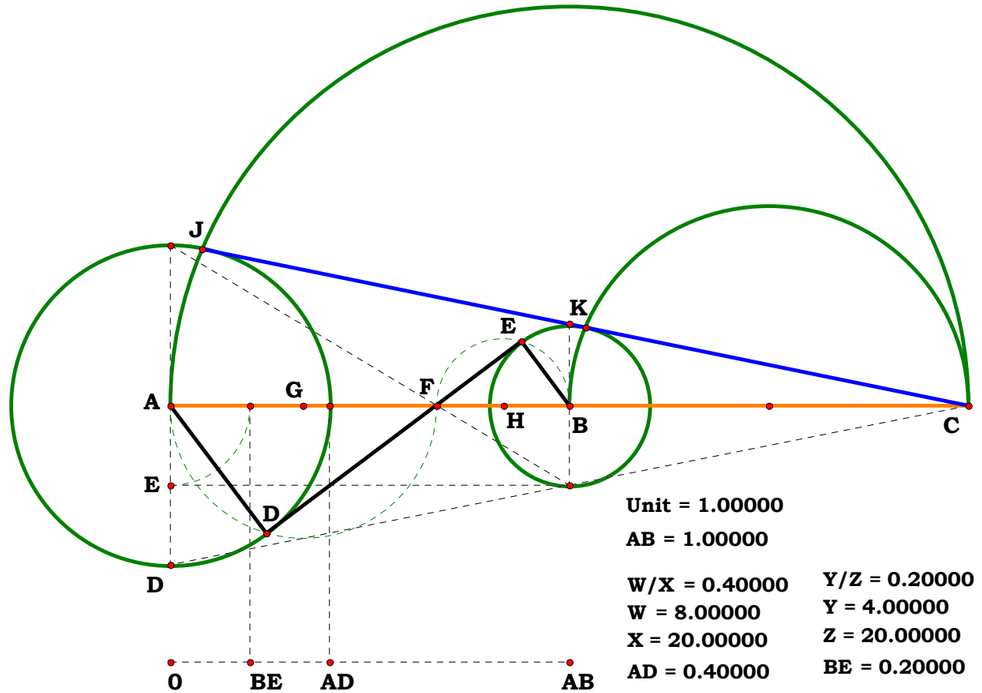
And what is the formula?
JC " External similarity point Origin to Tangent (Large Radius)"

$$CJ - \frac{W \cdot \sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{X \cdot (W \cdot Z - X \cdot Y)} = 0$$

What is the Algebraic names of the similarity points C and F in relation to the radius of each circle and the difference between their centers?

I believe that I was almost laughing when I drew this up originally. I made it an acronym side show. I actually get annoyed with acronyms. It scared me so much I never did it again. But, for the last version of DQ. I should at least put a dress on the graphics and remove the acorns. For some reason, I always liked this write-up.

## Tangents and Similarity Points.




Unit = 1.00000

AB = 1.00000

W/X = 0.40000 Y/Z = 0.20000
W = 8.00000 Y = 4.00000
X = 20.00000 Z = 20.00000
AD = 0.40000 BE = 0.20000

AC = 2.00000 AF = 0.66667
AF = 0.66667 BF = 0.33333
CJ = 1.95959 DF = 0.53333
CK = 0.97980 EF = 0.26667
JK = 0.97980 DE = 0.80000

**What is the length of the line tangent to the least circle (CK)?**

$BC := AC - AB \qquad BC = 1$

$CK := \sqrt{BC^2 - BE^2}$

$CK = 0.979796$

**And what is the formula?**

**CK " External similarity point Origin to Tangent (Small Radius)"**

$$CK - \frac{Y \cdot \sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{Z \cdot (W \cdot Z - X \cdot Y)} = 0$$

**Lastly what is the length of line from tangent to tangent of these circles?**

$JK := CJ - CK$

$JK = 0.979796$

**And what is the formula for, JK, Tangent to Tangent"?**

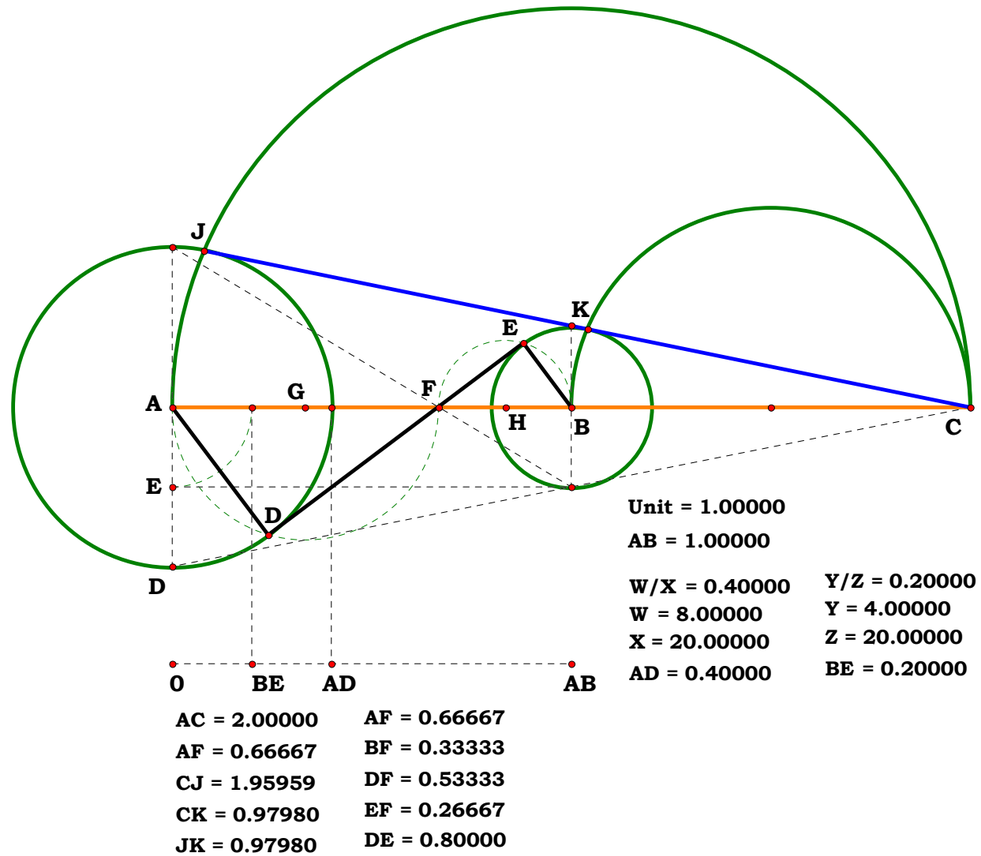
$$JK - \frac{\sqrt{(W \cdot Z - X \cdot Y + X \cdot Z) \cdot (X \cdot Y - W \cdot Z + X \cdot Z)}}{X \cdot Z} = 0$$

**I will now turn my attention to the point F the internal similarity point.**

$$AF := \frac{AB \cdot AD}{AD + BE} \qquad AF = 0.666667$$

**AF "Internal similarity point to center of Radius Large"**

$$AF - \frac{W \cdot Z}{W \cdot Z + X \cdot Y} = 0$$



$BF := AB - AF \qquad BF = 0.333333$

**BF "Internal similarity point to center of Radius Small"**

$$BF - \frac{Y \cdot X}{W \cdot Z + X \cdot Y} = 0$$

$DF := \sqrt{AF^2 - AD^2} \qquad DF = 0.533333$

**DF "Internal similarity point Origin to Tangent (Large Radius)"**

$$DF - AD \cdot \frac{\sqrt{-(AD + BE - AB) \cdot (AD + BE + AB)}}{(AD + BE)} = 0$$

$DF = 0.533333 \qquad EF := \sqrt{BF^2 - BE^2} \qquad EF = 0.266667$

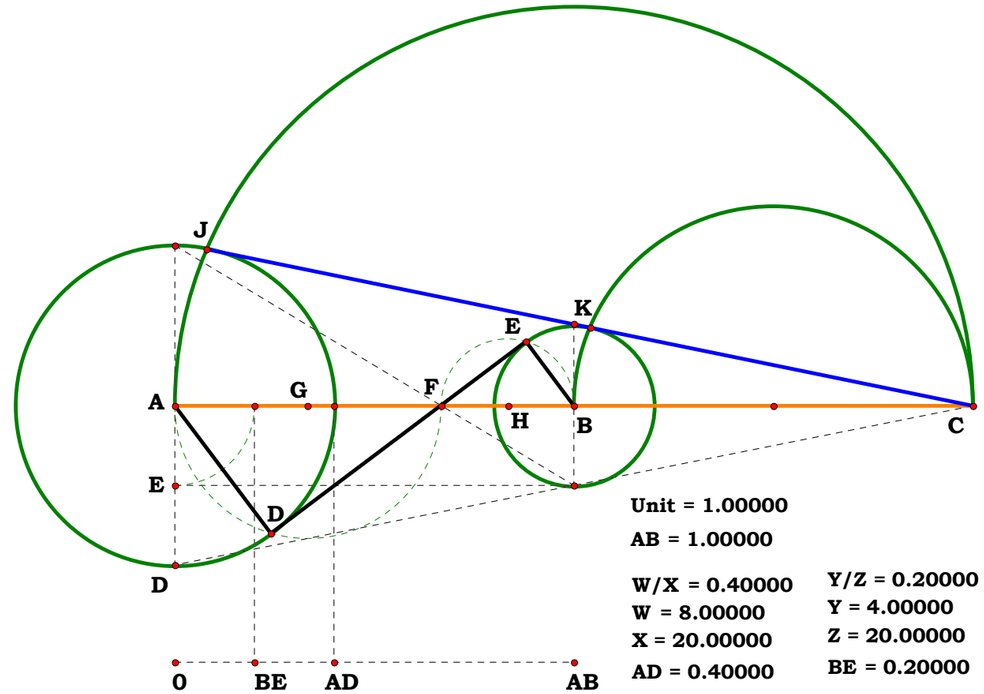
**EF "Internal similarity point Origin to Tangent (Small Radius)"**

$$EF - \frac{Y \cdot \sqrt{(W \cdot Z + X \cdot Y + X \cdot Z) \cdot (X \cdot Z - X \cdot Y - W \cdot Z)}}{Z \cdot (W \cdot Z + X \cdot Y)} = 0$$

$DE := DF + EF \qquad DE = 0.8$

**DE "Internal similarity point Tangent to Tangent"**

$$DE - \frac{\sqrt{(X \cdot Z - X \cdot Y - W \cdot Z) \cdot (W \cdot Z + X \cdot Y + X \cdot Z)}}{X \cdot Z} = 0$$




Unit = 1.00000
AB = 1.00000

W/X = 0.40000
W = 8.00000
X = 20.00000
AD = 0.40000

Y/Z = 0.20000
Y = 4.00000
Z = 20.00000
BE = 0.20000

AC = 2.00000
AF = 0.66667
CJ = 1.95959
CK = 0.97980
JK = 0.97980

AF = 0.66667
BF = 0.33333
DF = 0.53333
EF = 0.26667
DE = 0.80000

042794A

Given.

$N_1 := 3$

$N_2 := 2$

$N_3 := 6$

Descriptions.

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$ $AB := N_3$

$AC := \frac{R_1^2}{AB}$ $BD := \frac{R_2^2}{AB}$ $CD := AB - (AC + BD)$

$CE := \frac{CD}{2}$ $AE := AC + CE$ $BE := AB - AE$

$$AE - \frac{N_3^2 + R_1^2 - R_2^2}{2 \cdot N_3} = 0$$

$$BE - \frac{N_3^2 - R_1^2 + R_2^2}{2 \cdot N_3} = 0$$

Definitions.

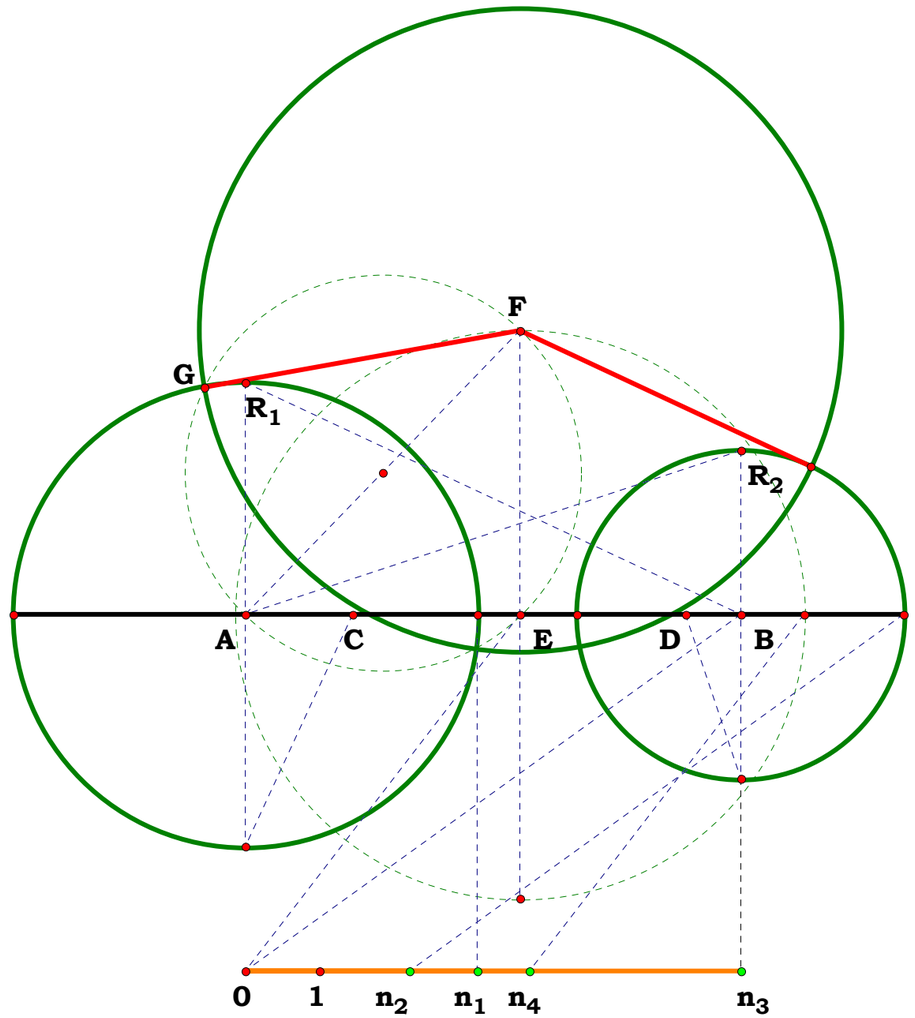
If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.

$N_4 := 3$ $AF := \sqrt{AE^2 + N_4^2}$ $FG := \sqrt{AF^2 - R_1^2}$

$$FG - \frac{\sqrt{N_3^2 \cdot \left(N_3^2 + 4 \cdot N_4^2 - 2 \cdot R_1^2 - 2 \cdot R_2^2\right) + \left(R_1 - R_2\right)^2 \cdot \left(R_1 + R_2\right)^2}}{2 \cdot \sqrt{N_3^2}} = 0$$

**A drawomg solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie. It does not lend itself to formal geometry, so I developed my own method, this is actually one of the methods I developed and it is quite simple. I was actually surprised to see how undeveloped it was.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**

## The Chordal or Power Line of two Circles



042794B

Unit.

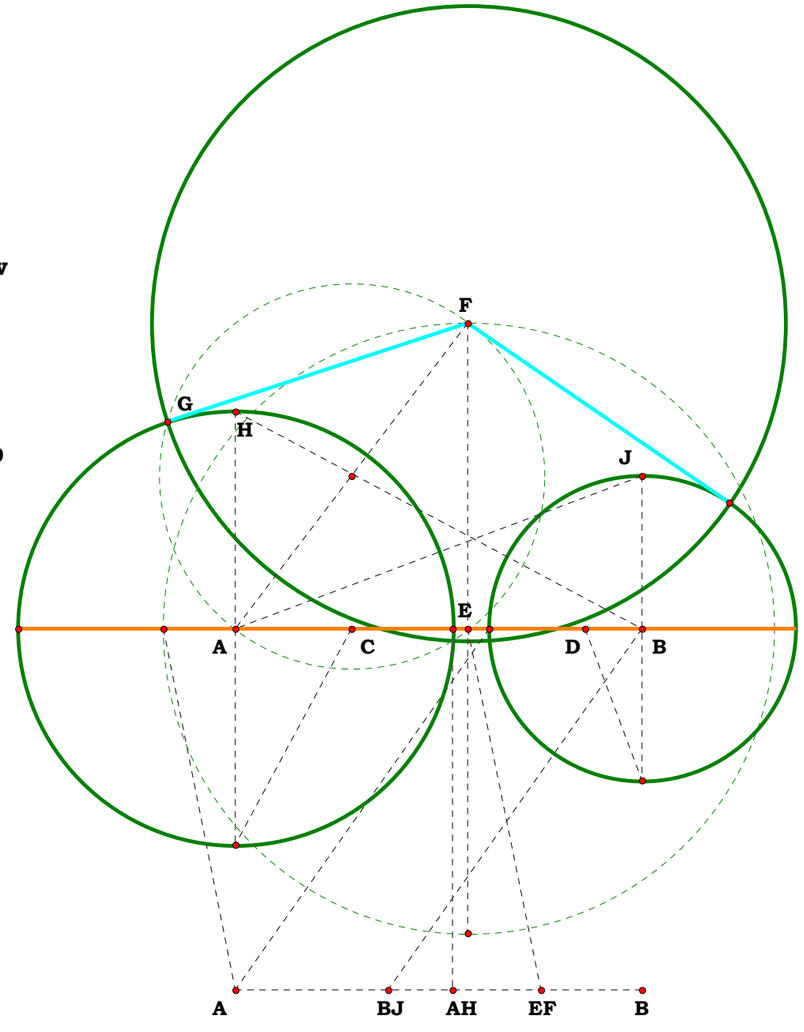
$AB := 1$

Given.

$U := 8$ $W := 3$ $Y := 12$

$V := 15$ $X := 8$ $Z := 16$

## The Chordal or Power Line of two Circles




Unit = 1.00000
XY = 0.53333
U = 8.00000
V = 15.00000
AH = 0.53333

XY = 0.37500
W = 3.00000
X = 8.00000
BJ = 0.37500

XY = 0.75000
Y = 12.00000
Z = 16.00000
AE = 0.57191
EF = 0.75000
BE = 0.42809
FG = 0.77791
AF = 0.94318

**Descriptions.**

**A drawomg solution for finding the chordal as outlined in** *100 Great Problems of Elementary Mathematics Their History and Solution* **by Heinrich Dörrie. It does not lend itself to formal geometry, so I developed my own method, this is actually one of the methods I developed and it is quite simple. I was actually surprised to see how undeveloped it was.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**

$$AH := \frac{U}{V} \qquad BJ := \frac{W}{X} \qquad EF := \frac{Y}{Z}$$

$$AC := \frac{AH^2}{AB} \qquad BD := \frac{BJ^2}{AB} \qquad CD := AB - (AC + BD)$$

$$CE := \frac{CD}{2} \qquad AE := AC + CE \qquad BE := AB - AE$$

$$AE - \frac{U^2 \cdot X^2 - V^2 \cdot W^2 + V^2 \cdot X^2}{2 \cdot V^2 \cdot X^2} = 0 \qquad AE = 0.57191$$

$$BE - -\frac{U^2 \cdot X^2 - V^2 \cdot W^2 - V^2 \cdot X^2}{2 \cdot V^2 \cdot X^2} = 0 \qquad BE = 0.42809$$

**Definitions.**

$$AF := \frac{\sqrt{\left(U^2 \cdot X^2 - V^2 \cdot W^2 + V^2 \cdot X^2\right)^2 \cdot Z^2 + 4 \cdot V^4 \cdot X^4 \cdot Y^2}}{2 \cdot V^2 \cdot X^2 \cdot Z}$$

$AF = 0.943176$

$$FG := \frac{\sqrt{\left[\left(U^4 - 2 \cdot U^2 \cdot V^2 + V^4\right) \cdot X^4 - 2 \cdot V^2 \cdot W^2 \cdot \left(U^2 + V^2\right) \cdot X^2 + V^4 \cdot W^4\right] \cdot Z^2 + 4 \cdot V^4 \cdot X^4 \cdot Y^2}}{2 \cdot V^2 \cdot X^2 \cdot Z}$$

$FG = 0.777905$

042894

**Given.**

$N_1 := 3$ $D_1 := 7.81447$

$N_2 := 1$ $D_2 := 6.96686$

$N_3 := 2$ $D_3 := 5.33279$

**Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.**

**Descriptions.**

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$ $R_3 := \sqrt{N_3^2}$ $AC := D_1$ $AE := D_2$ $CE := D_3$

$$AG := \frac{\sqrt{\left(R_1^2 + D_1^2 - R_2^2\right)^2}}{2 \cdot D_1} \qquad AH := \frac{\sqrt{\left(R_1^2 + D_2^2 - R_3^2\right)^2}}{2 \cdot D_2} \qquad AM := \frac{\sqrt{\left(D_2^2 + D_1^2 - D_3^2\right)^2}}{2 \cdot D_1}$$

$EM := \sqrt{AE^2 - AM^2}$ $AK := \frac{AE \cdot AH}{AM}$ $GK := AK - AG$ $GJ := \frac{AM \cdot GK}{EM}$

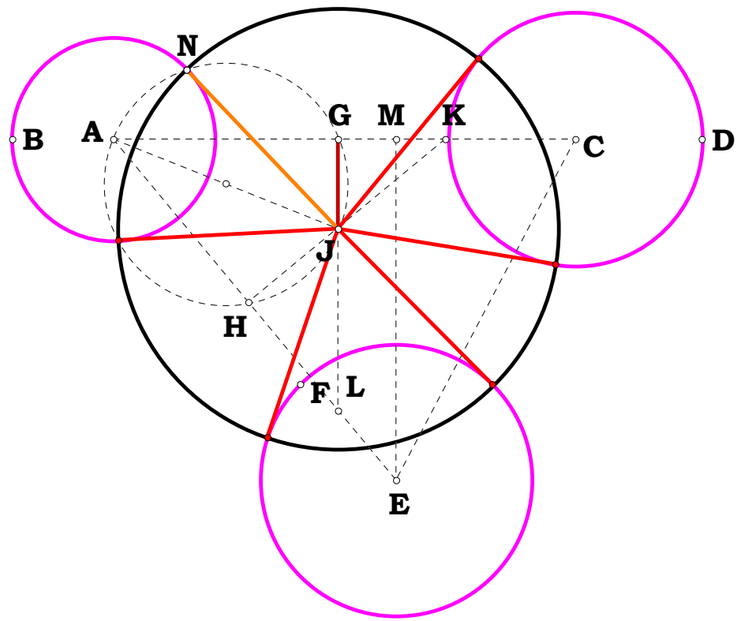
$AJ := \sqrt{AG^2 + GJ^2}$ $AN := R_1$ $JN := \sqrt{AJ^2 - AN^2}$

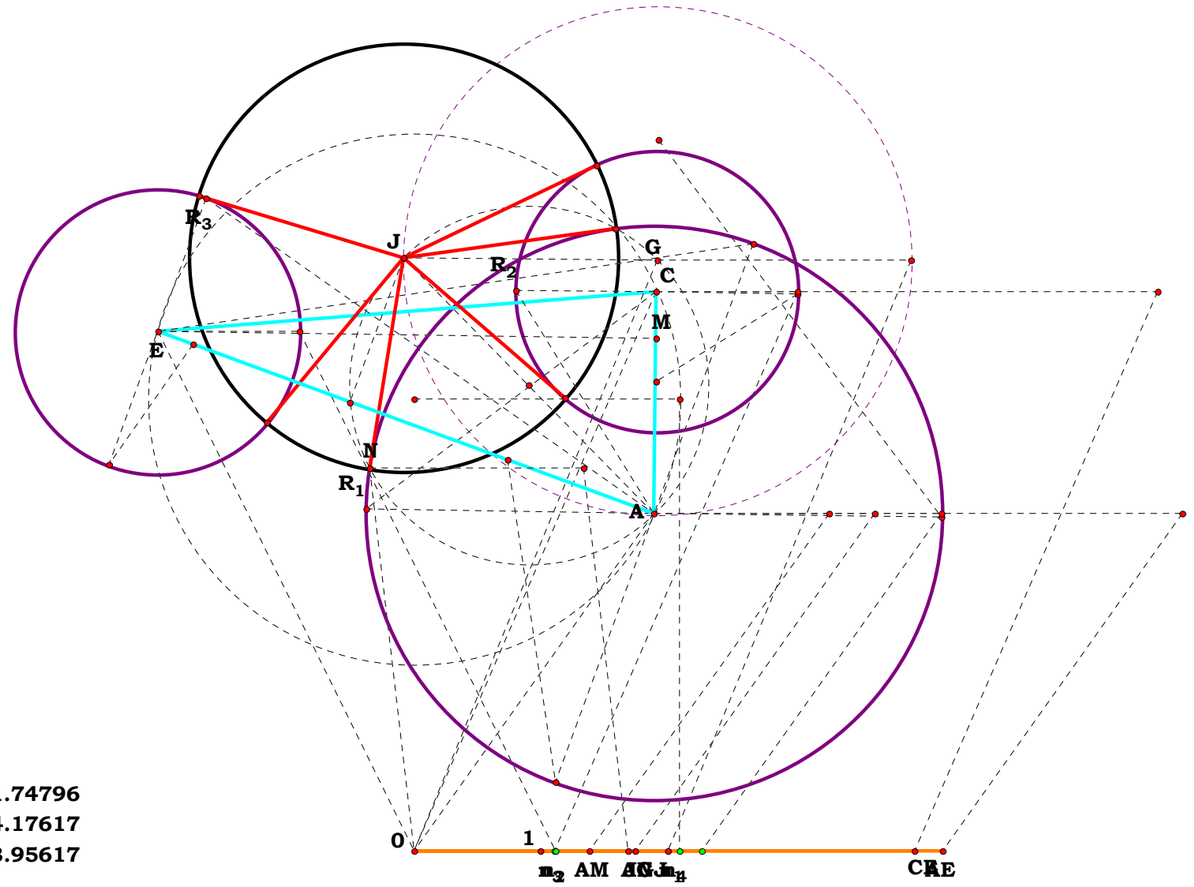
**Definitions.**

$$GJ - \frac{2 \cdot D_1^2 \cdot \sqrt{\left(D_2^2 + R_1^2 - R_3^2\right)^2} - \sqrt{\left(D_1^2 + D_2^2 - D_3^2\right)^2} \cdot \sqrt{\left(D_1^2 + R_1^2 - R_2^2\right)^2}}{2 \cdot \sqrt{D_1^2} \cdot \sqrt{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_2 - D_1 + D_3\right) \cdot \left(D_1 + D_2 + D_3\right)}} = 0$$

$$JN - \sqrt{\frac{\begin{aligned} &D_1^2 \cdot R_3^2 \cdot \left(R_1^2 + R_2^2 - R_3^2 - D_1^2\right) + D_1^2 \cdot D_2^2 \cdot \left(R_2^2 + R_3^2 - D_3^2\right) \ \ldots \\ &+ D_2^2 \cdot R_1^2 \cdot \left(D_3^2 + R_2^2 - R_3^2\right) + D_1^2 \cdot D_3^2 \cdot \left(R_1^2 + R_3^2\right) + D_3^2 \cdot R_1^2 \cdot \left(R_2^2 + R_3^2 - D_3^2 - R_1^2\right) \ \ldots \\ &+ D_2^2 \cdot R_2^2 \cdot \left(R_3^2 - D_2^2 - R_2^2\right) + D_3^2 \cdot R_2^2 \cdot \left(D_2 - R_3\right) \cdot \left(D_2 + R_3\right) - D_1^2 \cdot R_1^2 \cdot R_2^2 \end{aligned}}{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 + D_2 + D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_1 - D_2 - D_3\right)}} = 0$$

**Power Point**






$N_1 = 2.27017$ $R_1 = 2.27017$ $AC = 1.74796$ $D_1 = 1.74796$

$N_2 = 1.11001$ $R_2 = 1.11001$ $AE = 4.17617$ $D_2 = 4.17617$

$N_3 = 1.12235$ $R_3 = 1.12235$ $CE = 3.95617$ $D_3 = 3.95617$

$N_4 = 2.09689$ $AM = 1.38576$

$GJ = 2.00571$

$JN = 1.68883$

Animate Points

$$\frac{\sqrt{((D_1^2+D_2^2)-D_3^2)^2}}{2\cdot D_1}\text{-AM} = 0.00000$$

$$\frac{2\cdot D_1^2\cdot\sqrt{((D_2^2+R_1^2)-R_3^2)^2}-\sqrt{((D_1^2+D_2^2)-D_3^2)^2}\cdot\sqrt{((D_1^2+R_1^2)-R_2^2)^2}}{2\cdot\sqrt{D_1^2}\cdot\sqrt{((D_1+D_2)-D_3)\cdot((D_1-D_2)+D_3)\cdot((D_2-D_1)+D_3)\cdot(D_1+D_2+D_3)}}\text{-GJ} = 0.00000$$

$$\sqrt{\frac{(D_1^2.R_3^2.((R_1^2+R_2^2)-R_3^2-D_1^2)+D_1^2.D_2^2.((R_2^2+R_3^2)-D_3^2))+(D_2^2.R_1^2.((D_3^2+R_2^2)-R_3^2)+D_1^2.D_3^2.(R_1^2+R_3^2)+D_3^2.R_1^2.((R_2^2+R_3^2)-D_3^2-R_1^2))+((D_2^2.R_2^2.(R_3^2-D_2^2-R_2^2)+D_3^2.R_2^2.(D_2-R_3)\cdot(D_2+R_3))-D_1^2.R_1^2.R_2^2)}{(((D_1+D_2)-D_3)\cdot(D_1+D_2+D_3)\cdot((D_1-D_2)+D_3)\cdot(D_1-D_2-D_3))}}\text{-JN} = 0.00000$$

Given.

$AF := 2.652$

$BE := 1.390$

$AB := 2.992$

042994A

Descriptions.

$AD := BE \qquad DE := AB$

$DF := AF - AD \qquad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$CE := \frac{EF \cdot BE}{DF} \qquad CF := EF + CE \qquad BC := \frac{AB \cdot CE}{EF}$

Definitions.

See the end note on the next plate for those who want to make AD greater than AF.

$EF = 3.247262$

$EF - \sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2} = 0$

$CE = 3.576619$

$CE - \frac{\sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2} \cdot BE}{AF - BE} = 0$

$CF = 6.823881$

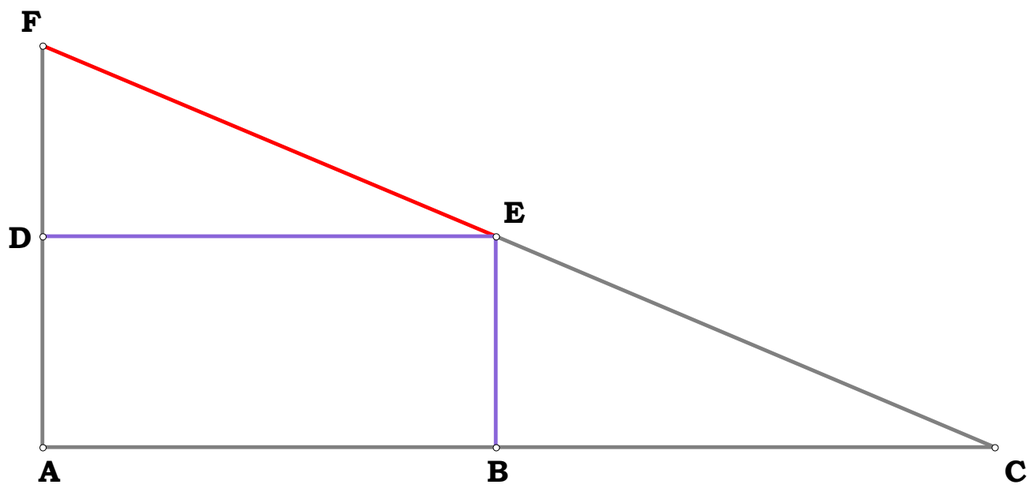
$CF - \frac{AF \cdot \sqrt{AF^2 - 2 \cdot AF \cdot BE + BE^2 + AB^2}}{AF - BE} = 0$

$BC = 3.295468$

$BC - \frac{BE \cdot AB}{AF - BE} = 0$

## Teeter-Totter

Given the rectangle ABDE, and some point F, collinear with AD, what are CE, CF, EF, BC?




$AB = 2.992$ in.

$BE = 1.390$ in.

$AF = 2.652$ in.

$EF = 3.247$ in.

$CE = 3.574$ in.

$CF = 6.821$ in.

$BC = 3.292$ in.

$\sqrt{(AF^2\text{-}2\cdot AF\cdot BE)+BE^2+AB^2} = 3.247$ in.

$EF\text{-}\sqrt{(AF^2\text{-}2\cdot AF\cdot BE)+BE^2+AB^2} = 0.000$ in.

$CE\text{-}\frac{BE\cdot\sqrt{(AF^2\text{-}2\cdot AF\cdot BE)+BE^2+AB^2}}{AF\text{-}BE} = 0.000$ in.

$CF\text{-}\frac{AF\cdot\sqrt{(AF^2\text{-}2\cdot AF\cdot BE)+BE^2+AB^2}}{AF\text{-}BE} = 0.000$ in.

$BC\text{-}\frac{BE\cdot AB}{AF\text{-}BE} = 0.000$ in.

042994B

Unit.

$AB := 1$

Given.

$W := 6$ $Y := 15$

$X := 20$ $Z := 18$

## Teeter-Totter

Given the rectangle ABDE, and some point F, collinear with AD, what are CE, CF, EF, BC?

Descriptions.

$BE := \frac{W}{X}$ $AF := \frac{Y}{Z}$ $AD := BE$ $DE := AB$

$BE = 0.3$ $AF = 0.833333$

$DF := AF - AD$ $EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$CE := \frac{EF \cdot BE}{DF}$ $CF := EF + CE$ $BC := \frac{AB \cdot CE}{EF}$

Definitions.

$EF = 1.133333$

$EF - \frac{\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0$

$CE = 0.6375$

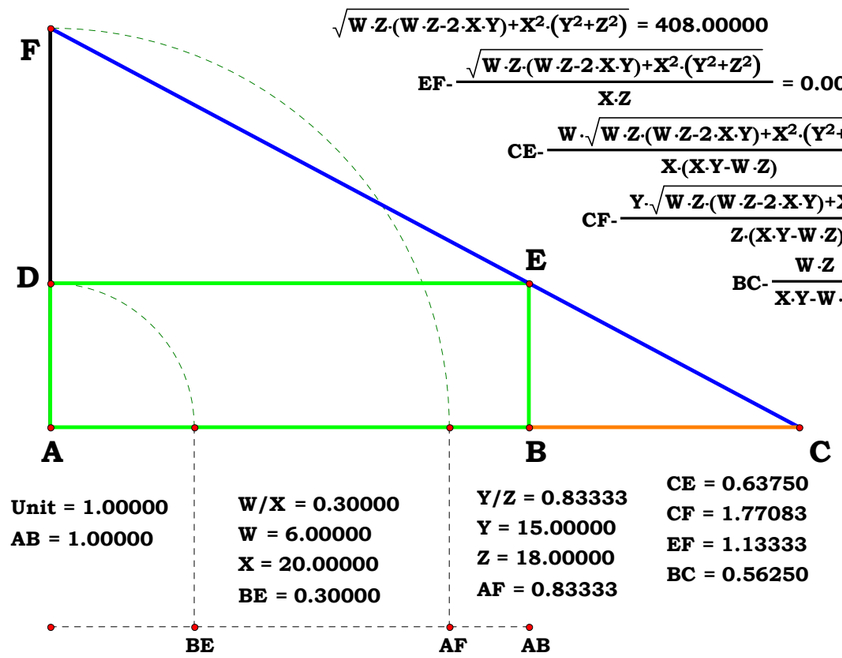
$CE - \frac{W \cdot \sqrt{X^2 \cdot \left(Y^2 + Z^2\right) + W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y)}}{X \cdot (X \cdot Y - W \cdot Z)} = 0$

$CF = 1.770833$

$CF - \frac{Y \cdot \sqrt{X^2 \cdot \left(Y^2 + Z^2\right) + W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y)}}{Z \cdot (X \cdot Y - W \cdot Z)} = 0$

$BC = 0.5625$

$BC - \frac{W \cdot Z}{X \cdot Y - W \cdot Z} = 0$




$\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)} = 408.00000$

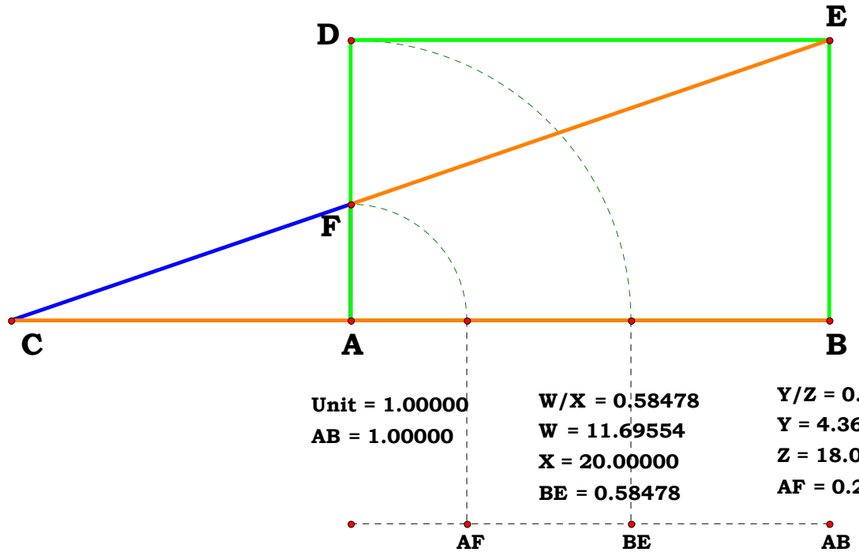
$EF - \frac{\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z} = 0.00000$

$CE - \frac{W \cdot \sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot (X \cdot Y - W \cdot Z)} = 0.00000$

$CF - \frac{Y \cdot \sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot \left(Y^2 + Z^2\right)}}{Z \cdot (X \cdot Y - W \cdot Z)} = 0.00000$

$BC - \frac{W \cdot Z}{X \cdot Y - W \cdot Z} = 0.00000$

**On both of these plates, if you are going to invert D and F, then you have to make a small change to some of the equations, making sure the the result is always positive.**




$\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot (Y^2 + Z^2)} = 380.48222$

EF- $\frac{\sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0.00000$

CE- $\frac{W \cdot \sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot (Y^2 + Z^2)}}{\sqrt{(X \cdot (X \cdot Y - W \cdot Z))^2}} = 0.00000$

CF- $\frac{Y \cdot \sqrt{W \cdot Z \cdot (W \cdot Z - 2 \cdot X \cdot Y) + X^2 \cdot (Y^2 + Z^2)}}{\sqrt{(Z \cdot (X \cdot Y - W \cdot Z))^2}} = 0.00000$

BC- $\frac{W \cdot Z}{\sqrt{(X \cdot Y - W \cdot Z)^2}} = 0.00000$

**Given.**

$AB := 5$

$BC := 2$

**Division $N^2$**

**043094A**

**Descriptions.**

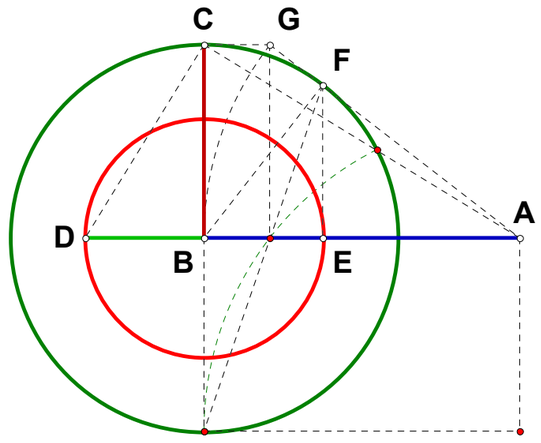
$$AC := \sqrt{AB^2 + BC^2}$$

$$CD := \frac{BC \cdot AC}{AB}$$

$$BD := \sqrt{CD^2 - BC^2}$$

**Definitions.**

$$\frac{BC^2}{AB} - BD = 0$$



043094B

**Unit.**

$AB := 1$

**Given.**

$X := 3$

$Y := 1$

**Descriptions.**

$BC := \frac{X}{Y}$

$AC := \sqrt{AB^2 + BC^2}$

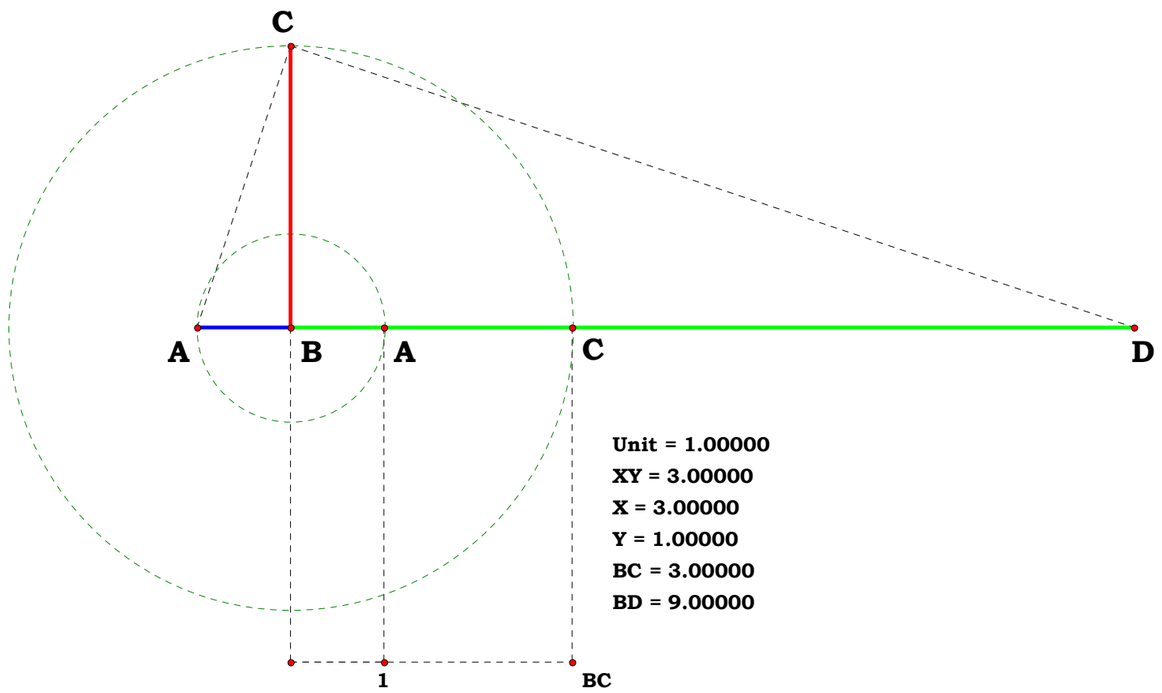
$CD := \frac{BC \cdot AC}{AB}$

$BD := \sqrt{CD^2 - BC^2}$

**Definitions.**

$\left(\frac{X}{Y}\right)^2 - BD = 0$

**Division $N^2$**




**Unit = 1.00000**
**XY = 3.00000**
**X = 3.00000**
**Y = 1.00000**
**BC = 3.00000**
**BD = 9.00000**

Given.

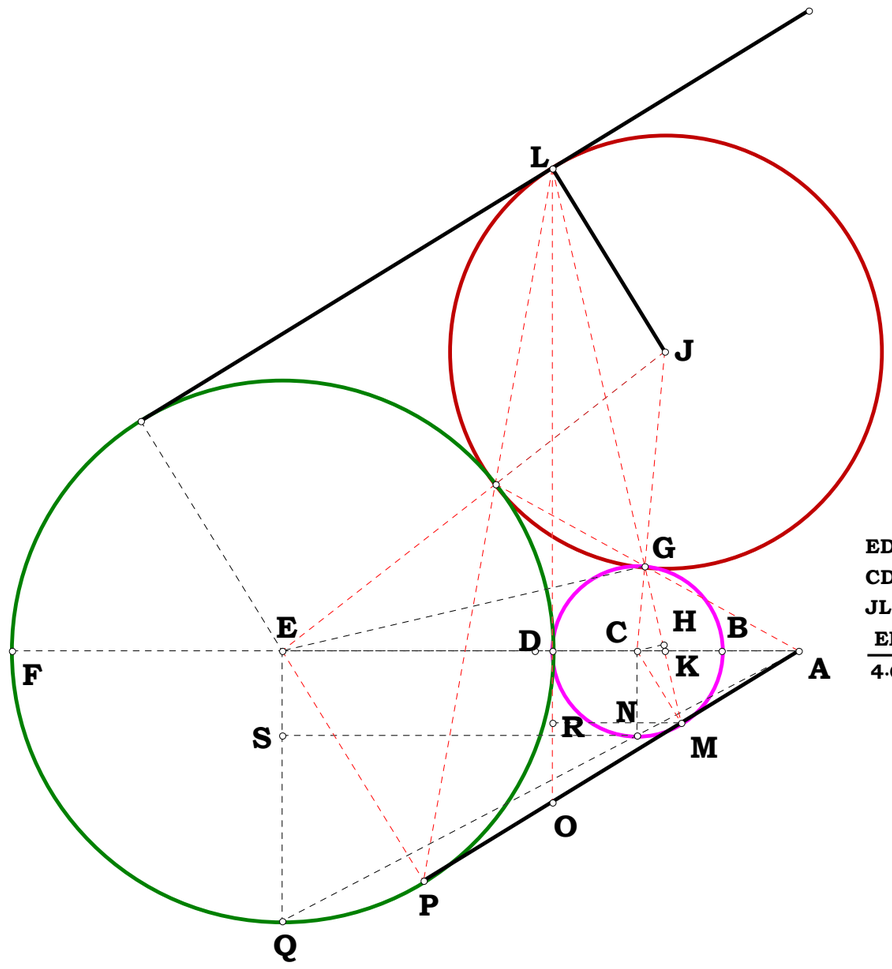
$R_1 := 3$ $DE := R_1$

$R_2 := 2$ $BC := R_2$

050194A

## Two Circles And A Parallel

**Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.**




ED = 1.783 in.
CD = 0.558 in.
JL = 1.424 in.
$\frac{ED^2}{4 \cdot CD}$-JL = 0.000 in.

Descriptions.

$CN := BC$ $EQ := DE$ $CD := BC$ $CE := CD + DE$

$ES := CN$ $NS := CE$ $SQ := EQ - ES$ $AE := \frac{NS \cdot EQ}{SQ}$

$AD := AE - DE$ $EP := DE$ $AP := \sqrt{AE^2 - EP^2}$

$DO := \frac{EP \cdot AD}{AP}$ $DL := \frac{DO \cdot DE}{CD}$ $AC := AD - CD$ $CM := BC$

$AM := \frac{AP \cdot AC}{AE}$ $AO := \frac{AE \cdot AD}{AP}$ $MO := AO - AM$ $MR := \frac{AD \cdot MO}{AO}$

$RO := \frac{DO \cdot MR}{AD}$ $DR := DO - RO$ $LR := DR + DL$ $ML := \sqrt{MR^2 + LR^2}$

$DK := \frac{MR \cdot DL}{LR}$ $CK := DK - CD$ $CH := \frac{LR \cdot CK}{ML}$

$MH := \sqrt{CM^2 - CH^2}$ $MG := 2 \cdot MH$ $GL := ML - MG$ $GJ := \frac{CM \cdot GL}{MG}$

Definitions.

$R_3 := \frac{R_1^2}{4 \cdot R_2}$ $GJ - R_3 = 0$

050194B

Unit.

mn := 1

Given.

W := 13 Y := 4

X := 20 Z := 15

## Two Circles And A Parallel

Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite F which is tangent to the larger circle.

Descriptions.

The results of Plate A tells us the equation for the remaining radius, and knowing that it is tangent to both circles the construction becomes obvious.

$$AD := \frac{W}{X} \qquad BD := \frac{Y}{Z}$$

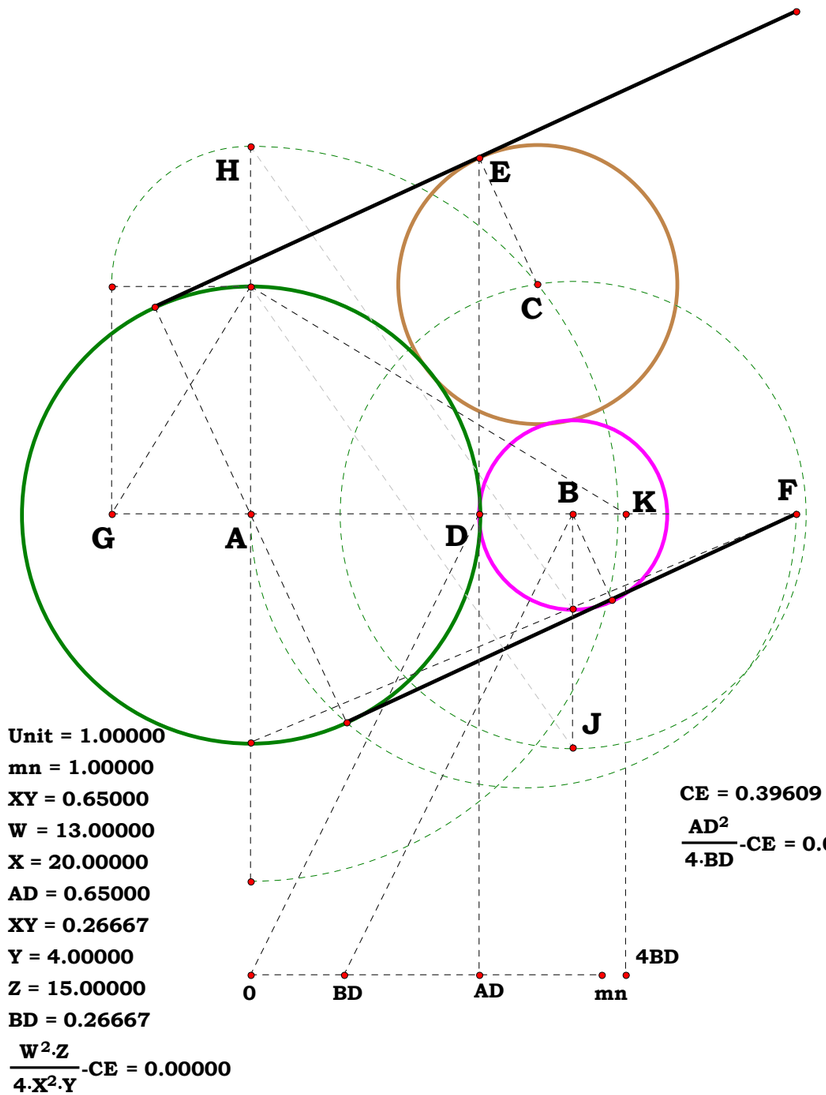
$$AK := 4 \cdot BD \qquad AG := \frac{AD^2}{AK}$$

$$AH := AD + AG \qquad BJ := BD + AG$$

$$CE := \frac{AD^2}{4 \cdot BD} \qquad CE = 0.396094$$

Definitions.

$$CE - \frac{W^2 \cdot Z}{4 \cdot X^2 \cdot Y} = 0$$

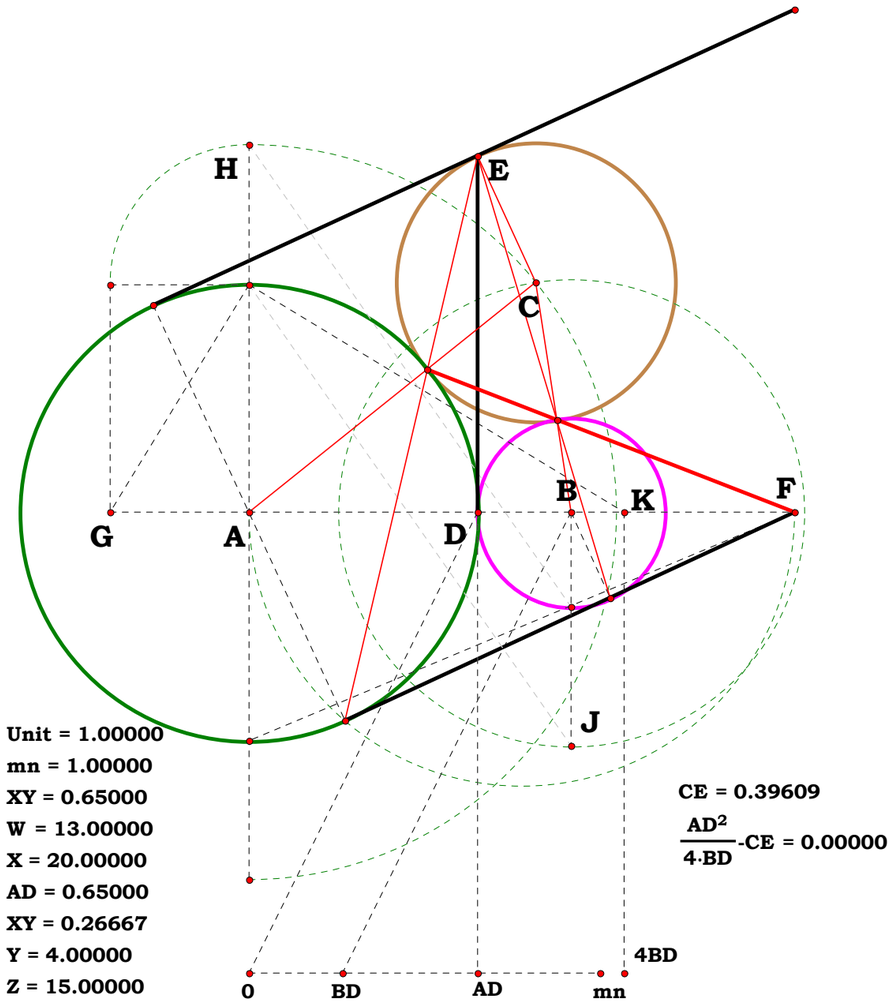



Unit = 1.00000
mn = 1.00000
XY = 0.65000
W = 13.00000
X = 20.00000
AD = 0.65000
XY = 0.26667
Y = 4.00000
Z = 15.00000
BD = 0.26667
$\frac{W^2 \cdot Z}{4 \cdot X^2 \cdot Y}$-CE = 0.00000

CE = 0.39609
$\frac{AD^2}{4 \cdot BD}$-CE = 0.00000

**If one compare this plate, B, with A, they find something missing, some understanding as to why it is so. Although one can construct figures from equations, the finished construction may need to be aughmented with structures which are implicit in the equation.**

**Just knowing ones projection is dependent on the powerline, a lot less construction is needed than any of my plates on this show.**



050494A

**Given.**

$R_1 := 1.49167$ $\quad FK := R_1$

$R_2 := .70833$ $\quad BC := R_2$

$D := 2.65$ $\quad CK := D$

$N := 15.18411$

**Descriptions.**

$FL := 2 \cdot FK \qquad FG := \frac{FL}{N} \qquad AK := \frac{D \cdot R_1}{R_1 - R_2}$

$EK := \frac{R_1^{\,2} + D^2 - R_2^{\,2}}{2 \cdot D} \qquad AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$

$GL := FL - FG \qquad GM := \sqrt{FG \cdot GL} \qquad AJ := \frac{AQ \cdot AQ}{AK} \qquad AF := AK - FK$

$FJ := AJ - AF \qquad JL := FL - FJ \qquad JQ := \sqrt{FJ \cdot JL} \qquad GJ := FJ - FG$

$QM := \sqrt{(JQ + GM)^2 + GJ^2} \qquad GH := \frac{GJ \cdot GM}{JQ + GM} \qquad HM := \frac{QM \cdot GM}{JQ + GM}$

$EF := EK - FK \qquad EH := EF + FG + GH \qquad HO := \frac{HM \cdot EH}{GH} \qquad MO := HO - HM$

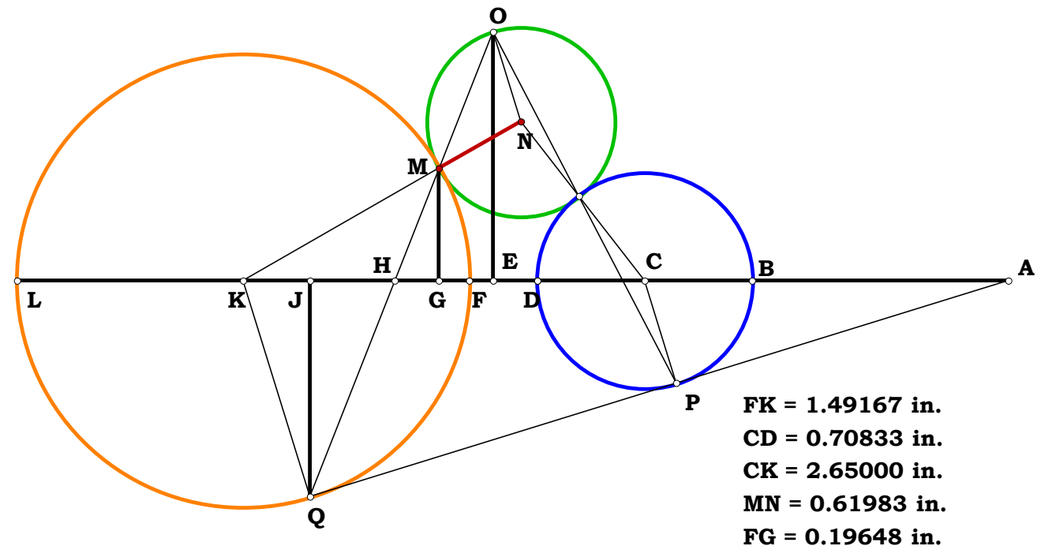
$KM := FK \qquad MN := \frac{KM \cdot MO}{QM} \qquad MN = 0.619833$

**Definitions.**

$$MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

## Two Circles And A Tangent

**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**




**FK = 1.49167 in.**
**CD = 0.70833 in.**
**CK = 2.65000 in.**
**MN = 0.61983 in.**
**FG = 0.19648 in.**
$\frac{2 \cdot FK}{FG} = 15.18411$

050494B

Unit.

$CK := 1$

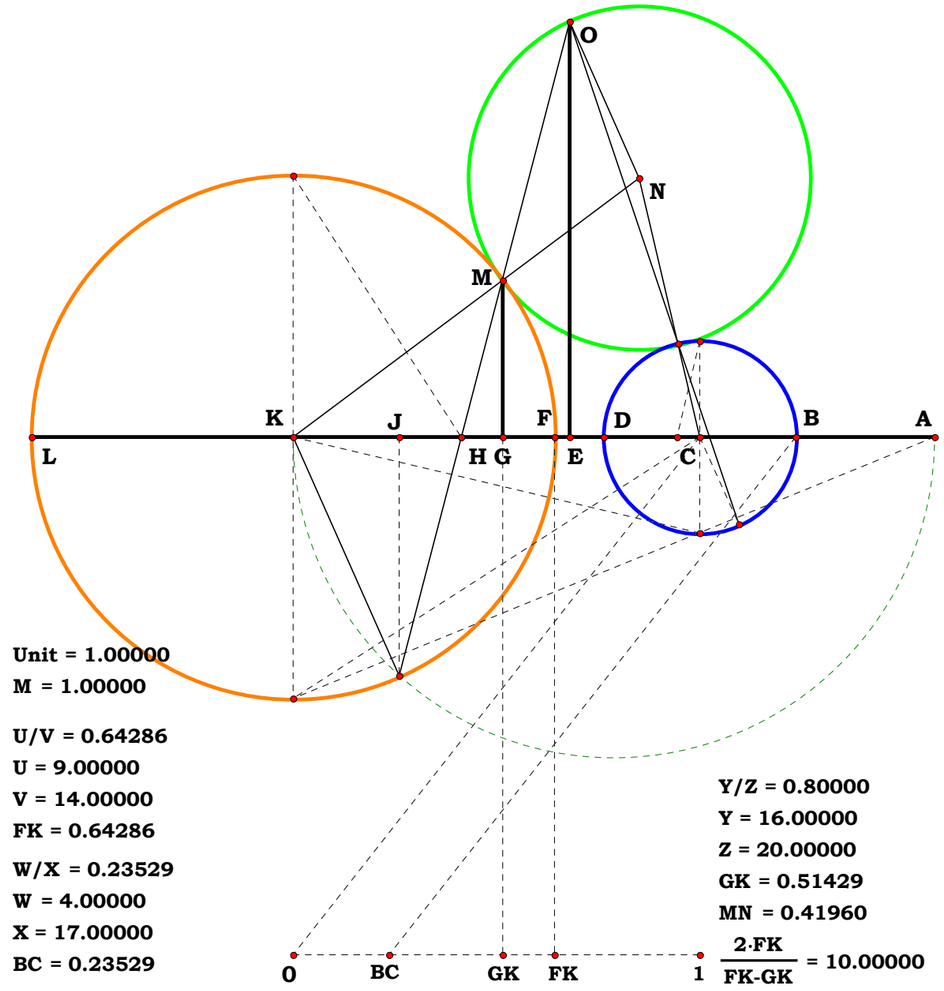
Given.

$U := 9$ $W := 4$ $Y := 16$

$V := 14$ $X := 17$ $Z := 20$

## Two Circles And A Tangent

**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**




**Descriptions.**

$FK := \frac{U}{V}$ $BC := \frac{W}{X}$ $GK := \frac{FK \cdot Y}{Z}$ $FG := FK - GK$

$FL := 2 \cdot FK$ $AK := \frac{CK \cdot FK}{FK - BC}$ $EK := \frac{FK^2 + CK^2 - BC^2}{2 \cdot CK}$

$AQ := FK \cdot \frac{\sqrt{(FK - BC + CK) \cdot (-FK + BC + CK)}}{FK - BC}$

$GL := FL - FG$ $GM := \sqrt{FG \cdot GL}$ $AJ := \frac{AQ \cdot AQ}{AK}$ $AF := AK - FK$

$FJ := AJ - AF$ $JL := FL - FJ$ $JQ := \sqrt{FJ \cdot JL}$ $GJ := FJ - FG$

$QM := \sqrt{(JQ + GM)^2 + GJ^2}$ $GH := \frac{GJ \cdot GM}{JQ + GM}$ $HM := \frac{QM \cdot GM}{JQ + GM}$

$EF := EK - FK$ $EH := EF + FG + GH$ $HO := \frac{HM \cdot EH}{GH}$ $MO := HO - HM$

$KM := FK$ $MN := \frac{KM \cdot MO}{QM}$ $MN = 0.419597$

**Definitions.**

$$MN - \frac{X^2 \cdot \left[Z \cdot \left(U^2 + V^2\right) - 2 \cdot U \cdot V \cdot Y\right] - V^2 \cdot W^2 \cdot Z}{2 \cdot V \cdot X \cdot (V \cdot W \cdot Z - U \cdot X \cdot Z + V \cdot X \cdot Y)} = 0$$

Unit

$FH := 1$

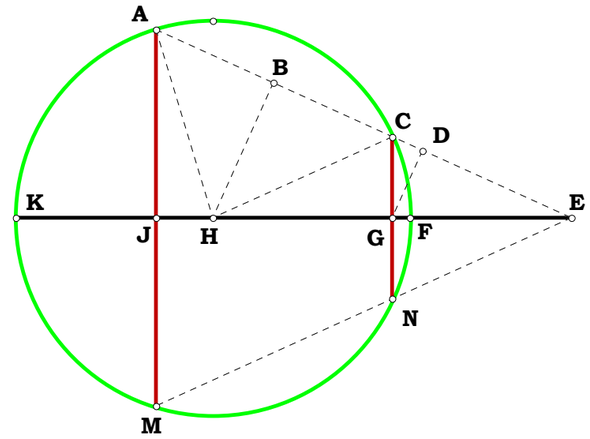
Given.

$N := 2.423$

050694

# A Ratio In Trisection

**What is AJ to CG?**




**Descriptions.**

$CE := FH \qquad CG := \frac{FH}{N} \qquad EG := \sqrt{CE^2 - CG^2} \qquad CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2} \qquad EH := 2 \cdot EG \qquad BH := \frac{DG \cdot EH}{EG} \qquad CH := FH$

$BC := \sqrt{CH^2 - BH^2} \qquad AC := 2 \cdot BC \qquad AE := AC + CE \qquad AJ := \frac{CG \cdot AE}{CE}$

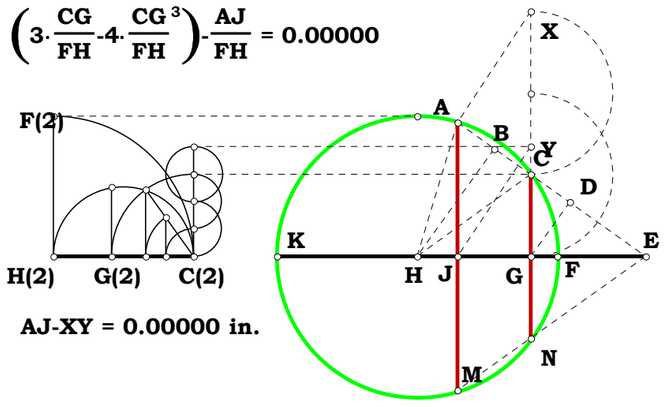
$3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0 \qquad 3 \cdot CG - 4 \cdot CG^3 - AJ = 0 \qquad AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$

**Definitions.**

$AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

**The resultant equation suggests this construction.**

$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG^3}{FH}\right) - \frac{AJ}{FH} = 0.00000$




AJ-XY = 0.00000 in.

Unit.

$AE := 1$

Given.

$N := 5$

## A Trisection Ratio with the Paper Trisector

**The figure works on the fact that trisection takes place as point K moves between .5 and 1 of half the radius. Thus one can examine it by a simple fact. Division in this method will take place, for the Paper Trisector, over 3/4 of the semi-circle.**

**In trisection, what is the ratio of FG/EK?**

**050794A**

**Descriptions.**

$EH := \frac{AE}{2}$ $HK := \frac{EH}{N}$ $AK := AE + EH + HK$

$EJ := AE$ $EK := EH + HK$ $AD := \frac{EJ \cdot AK}{EK}$ $CD := AE$

$AC := AD - CD$ $BC := \frac{AC}{2}$ $CE := AE$ $BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC$ $DE := \sqrt{BD^2 + BE^2}$ $DF := \frac{BD \cdot AD}{DE}$

$EG := AE$ $DG := DE + EG$ $FG := DG - DF$

$HK = 0.1$ $FG = 0.361715$

$EK = 0.6$ $AK = 1.6$

**Definitions.**

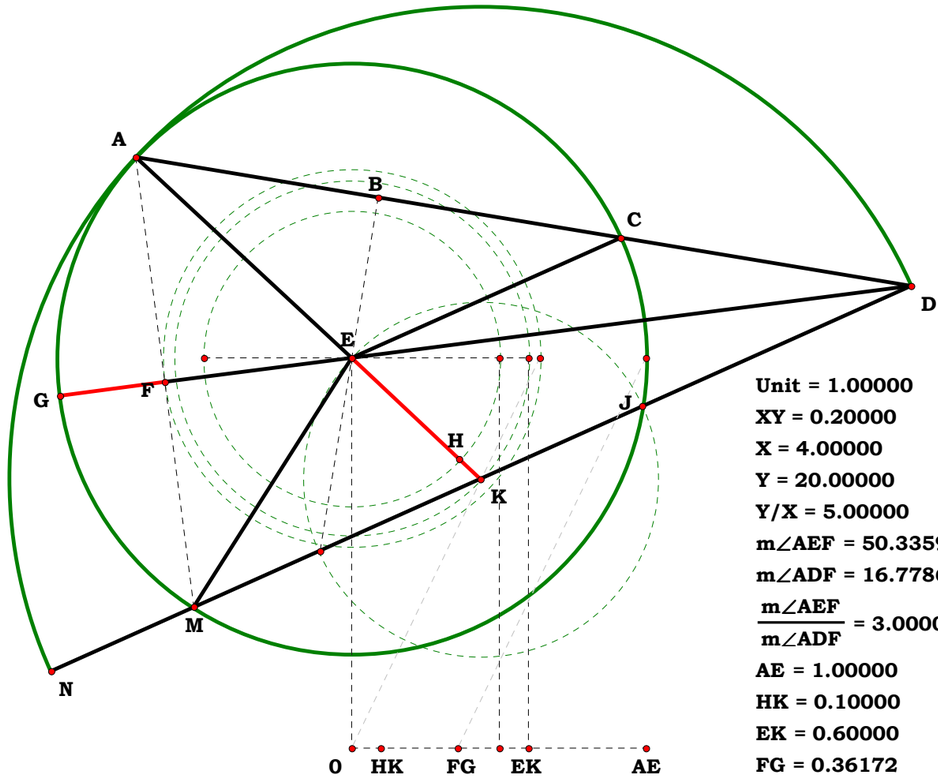
$EH - \frac{1}{2} = 0$ $HK - \frac{1}{2 \cdot N} = 0$ $AK - \frac{3 \cdot N + 1}{2 \cdot N} = 0$ $EJ - 1 = 0$ $EK - \frac{N + 1}{2 \cdot N} = 0$

$AD - \frac{3 \cdot N + 1}{N + 1} = 0$ $CD - 1 = 0$ $AC - \frac{2 \cdot N}{N + 1} = 0$ $BC - \frac{N}{N + 1} = 0$ $CE - 1 = 0$

$BE - \frac{\sqrt{2 \cdot N + 1}}{(N + 1)} = 0$ $BD - \frac{2 \cdot N + 1}{N + 1} = 0$ $DE - \frac{\sqrt{2 \cdot (2 \cdot N + 1)}}{\sqrt{N + 1}} = 0$ $EG - 1 = 0$

$DF - \frac{(3 \cdot N + 1) \cdot \sqrt{4 \cdot N + 2}}{2 \cdot (N + 1)^{\frac{3}{2}}} = 0$ $DG - \frac{\sqrt{2} \cdot \sqrt{2 \cdot N + 1} + \sqrt{N + 1}}{\sqrt{N + 1}} = 0$

$FG - \frac{\left(\sqrt{2} - \sqrt{2} \cdot N\right) \cdot \sqrt{2 \cdot N + 1} + 2 \cdot (N + 1)^{\frac{3}{2}}}{2 \cdot (N + 1)^{\frac{3}{2}}} = 0$ $\frac{FG}{EK} - \frac{N \cdot \left[(1 - N) \cdot \sqrt{2} \cdot \sqrt{2 \cdot N + 1} + 2 \cdot (N + 1)^{\frac{3}{2}}\right]}{(N + 1)^{\frac{5}{2}}} = 0$




Unit = 1.00000
XY = 0.20000
X = 4.00000
Y = 20.00000
Y/X = 5.00000
m∠AEF = 50.33596°
m∠ADF = 16.77865°
$\frac{m\angle AEF}{m\angle ADF} = 3.00000$
AE = 1.00000
HK = 0.10000
EK = 0.60000
FG = 0.36172
AK = 1.60000
N = 5.00000

$\frac{N \cdot \left((1-N) \cdot \sqrt{2} \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}\right)}{(N+1)^{\frac{5}{2}}} - \frac{FG}{EK} = 0.00000$

$\frac{\left(\sqrt{2} - \sqrt{2} \cdot N\right) \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}}{2 \cdot (N+1)^{\frac{3}{2}}} - FG = 0.00000$

Unit.

$AE := 1$

Given.

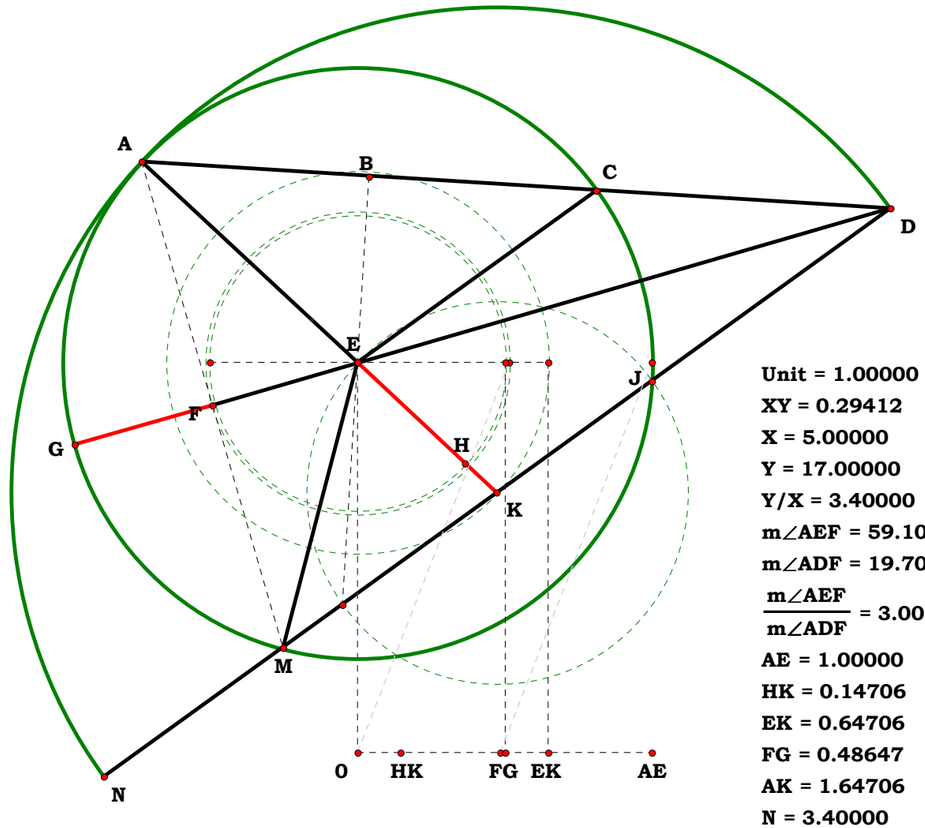
$X := 5 \quad Y := 17$

050794B

## A Trisection Ratio with the Paper Trisector

**The figure works on the fact that trisection takes place as point K moves between .5 and 1 of half the radius. Thus one can examine it by a simple fact. Division in this method will take place, for the Paper Trisector, over 3/4 of the semi-circle.**

**In trisection, what is the ratio of FG/EK?**




Unit = 1.00000
XY = 0.29412
X = 5.00000
Y = 17.00000
Y/X = 3.40000
m∠AEF = 59.10085°
m∠ADF = 19.70028°
$\frac{m\angle AEF}{m\angle ADF} = 3.00000$
AE = 1.00000
HK = 0.14706
EK = 0.64706
FG = 0.48647
AK = 1.64706
N = 3.40000

$EH := \frac{AE}{2} \quad N := \frac{Y}{X} \quad HK := \frac{EH}{N} \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

$HK = 0.147059 \quad FG = 0.486472 \quad N = 3.4$

$EK = 0.647059 \quad AK = 1.647059$

**Definitions.**

$HK - \frac{X}{2 \cdot Y} = 0 \quad EK - \frac{X + Y}{2 \cdot Y} = 0$

$FG - \frac{\sqrt{X + 2 \cdot Y} \cdot (X - Y) \cdot \sqrt{2} + 2 \cdot (X + Y) \cdot \sqrt{X + Y}}{2 \cdot (X + Y)^{\frac{3}{2}}} = 0$

$AK - \frac{X + 3 \cdot Y}{2 \cdot Y} = 0$

$\frac{FG}{EK} - \frac{Y \cdot \left[\sqrt{X + 2 \cdot Y} \cdot \left(X^2 - Y^2\right) \cdot \sqrt{2} + 2 \cdot (X + Y)^{\frac{5}{2}}\right]}{(X + Y)^{\frac{7}{2}}} = 0$

$\frac{\sqrt{X+2 \cdot Y} \cdot (X-Y) \cdot \sqrt{2} + 2 \cdot (X+Y) \cdot \sqrt{(X+Y)}}{2 \cdot (X+Y)^{\frac{3}{2}}} - FG = 0.00000$

$\frac{Y \cdot \left(\sqrt{X+2 \cdot Y} \cdot (X^2-Y^2) \cdot \sqrt{2} + 2 \cdot (X+Y)^{\frac{5}{2}}\right)}{(X+Y)^{\frac{7}{2}}} - \frac{FG}{EK} = 0.00000$

$\frac{N \cdot \left((1-N) \cdot \sqrt{2} \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}\right)}{(N+1)^{\frac{5}{2}}} - \frac{FG}{EK} = 0.00000$

$\frac{\left(\sqrt{2} - \sqrt{2} \cdot N\right) \cdot \sqrt{2 \cdot N+1} + 2 \cdot (N+1)^{\frac{3}{2}}}{2 \cdot (N+1)^{\frac{3}{2}}} - FG = 0.00000$

Unit

$CE := 1$

Given.

$Y := 17 \quad N_1 := 2$

051694A

$X := 6$

Descriptions.

Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.
The number of marbles one can stack this way is equal to the number of E/J, or $N_1$.

$$DE := \frac{X}{Y} \qquad EJ := CE \cdot N_1 \qquad DJ := \sqrt{DE^2 + EJ^2} \qquad JG := \frac{EJ^2}{DJ}$$

$$BE := CE \qquad EG := \sqrt{EJ^2 - JG^2} \qquad BG := \sqrt{BE^2 - EG^2}$$

$$BJ := BG + JG \qquad JK := CE \qquad BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$$

$DH = 0.301428$

Definitions.

$$DE - \frac{X}{Y} = 0 \qquad EJ - N_1 = 0 \qquad DJ - \frac{\sqrt{N_1^2 \cdot Y^2 + X^2}}{Y} = 0$$

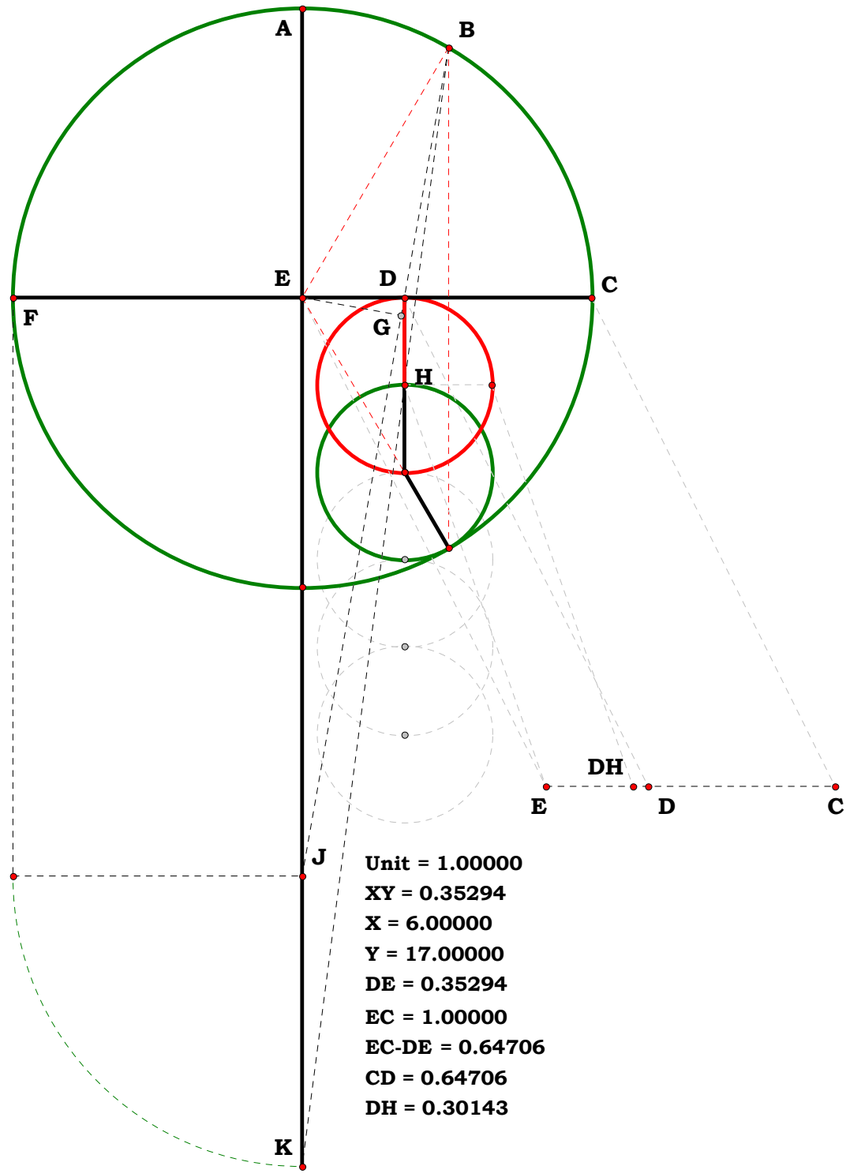
$$JG - \frac{N_1^2 \cdot Y}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \qquad BE - 1 = 0 \qquad EG - \frac{N_1 \cdot X}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$BG - \frac{\sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \qquad BJ - \frac{N_1^2 \cdot Y + \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$JK - 1 = 0 \qquad BD - \frac{Y \cdot \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2} - X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$DH - \frac{Y \cdot \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2} - X^2}{Y \cdot \left[ N_1^2 \cdot Y + \sqrt{X^2 - N_1^2 \cdot (X^2 - Y^2)} \right]} = 0$$

## Tangent Diameter and Circles



051694B

Unit.

$AF := 1$

Given.

$X := 5 \quad N_1 := 3$

$Y := 13$

**Descriptions.**

**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.**
**The number of marbles one can stack this way is equal to the number of F/J, or $N_1$.**

$$DF := AF \quad FJ := N_1 \quad HJ := AF \quad EF := \frac{X}{Y} \quad EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$$

$$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG \quad BJ := BE + EJ \quad EK := \frac{HJ \cdot BE}{BJ}$$

$$AJ := FJ + AF \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot EK\right]^2} \quad BC - EK = 0 \quad EK = 0.224477$$

**Definitions.**

$$DF - 1 = 0 \quad FJ - N_1 = 0 \quad HJ - 1 = 0 \quad EF - \frac{X}{Y} = 0 \quad EJ - \frac{\sqrt{N_1^2 \cdot Y^2 + X^2}}{Y} = 0$$

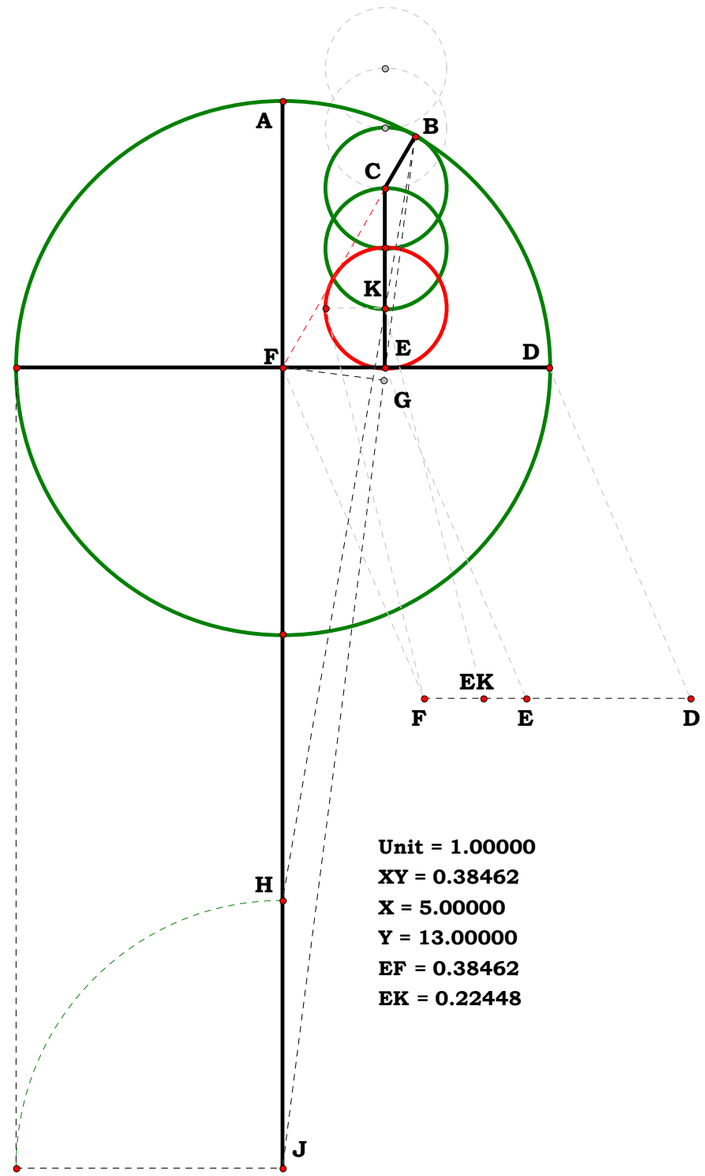
$$EG - \frac{X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BF - 1 = 0 \quad FG - \frac{N_1 \cdot X}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad AJ - (N_1 + 1) = 0$$

$$BG - \frac{\sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad BE - \frac{Y \cdot \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2} - X^2}{Y \cdot \sqrt{N_1^2 \cdot Y^2 + X^2}} = 0$$

$$BJ - \frac{N_1^2 \cdot Y + \sqrt{N_1^2 \cdot Y^2 - N_1^2 \cdot X^2 + X^2}}{\sqrt{N_1^2 \cdot Y^2 + X^2}} = 0 \quad EK - \frac{Y \cdot \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2} - X^2}{Y \cdot \left[N_1^2 \cdot Y + \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2}\right]} = 0$$

$$BC - \frac{Y \cdot \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2} - X^2}{Y \cdot \left[N_1^2 \cdot Y + \sqrt{N_1^2 \cdot (Y^2 - X^2) + X^2}\right]} = 0$$

## Tangent Diameter and Circles



102794A

**Unit.**

$Y := 1$ $AB := Y$

**Given.**

$X := 5$ $N := X$

**Descriptions.**

$AG := N$ $BG := AG - AB$ $BF := \frac{BG}{2}$

$AF := AB + BF$ $FH := BF$ $DF := \frac{FH^2}{AF}$ $AD := AF - DF$

$BD := AD - AB$ $DG := BG - BD$ $FJ := BF$

$DH := \sqrt{BD \cdot DG}$ $DE := \frac{DF \cdot DH}{DH + FJ}$ $AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$ $AE = 2.236068$

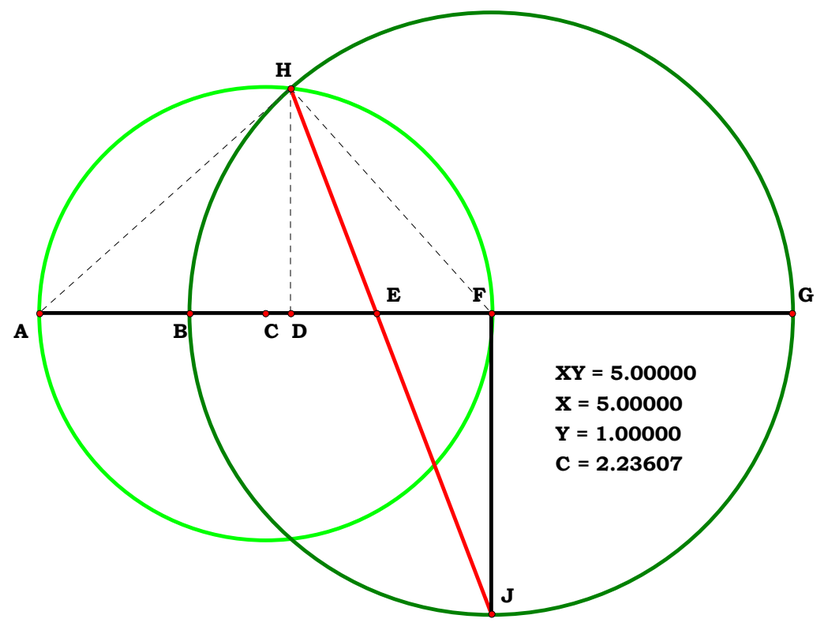
**Definitions.**

**The following is as far as Mathcad 15 will get you, the rest you have to do by hand. As I noted elsware, Mathcad is not the sharpest tool in the shed, I hope!**

$$AE - \frac{\sqrt{(N+1)^2} \cdot \left[\sqrt{N \cdot (N-1)^2} \cdot \sqrt{(N+1)^2} - 2 \cdot N + 2 \cdot N^2\right]}{(N+1) \cdot \left[N \cdot \sqrt{(N+1)^2} - \sqrt{(N+1)^2} + 2 \cdot \sqrt{N \cdot (N-1)^2}\right]} = 0$$

$AE - \sqrt{N} = 0$

**Trivial Method Square Root**




**AE is the square root of AB x AG.**

**Now, if you are wondering why I say trivial, it is because I foundf that word in a dictionary and thought it would look good standing there.**

102794B

**Unit.**

$Y := 1$ $AB := Y$

**Given.**

$X := 4$ $N := X$

**Descriptions.**

$AG := N$ $AC := \frac{AB}{2}$ $CG := AG - AC$ $CF := \frac{CG}{2}$

$AF := AC + CF$ $FH := CF$ $CD := \frac{AC^2}{CG}$ $AD := AC + CD$

$DG := CG - CD$ $CJ := AC$ $BD := AB - AD$

$DH := \sqrt{BD \cdot AD}$ $CE := \frac{CD \cdot AC}{DH + AC}$ $EG := CG - CE$

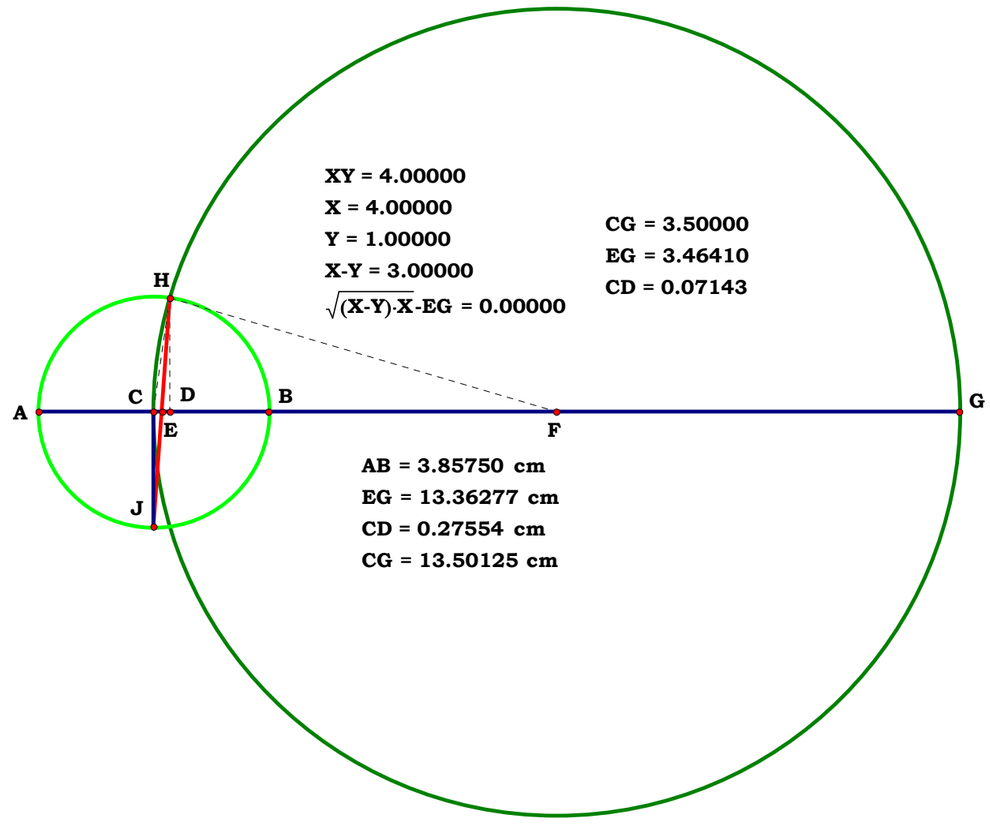
$\sqrt{N \cdot (N - AB)} - EG = 0$

$CG = 3.5$ $EG = 3.464102$ $CD = 0.071429$

**Definitions.**

$$EG - \frac{\left(N^2 - N + 1\right) \cdot \sqrt{N \cdot (N-1)} + 2 \cdot N^2 - 2 \cdot N}{N^2 - N + 2 \cdot \sqrt{N \cdot (N-1)} + 1} = 0$$

**Trivial Method Square Root**




**AE is the square root of BG x AG.**

102794C

Given.

$X := 6 \quad Y := 20$

Unit.

$AG := 1$

Descriptions.

**In plates A and B, we took one or the other things involved in computatrion and called it unity. Here, both are grouped as a unit and one can see the whole of all the possible interactions between the two in a simple figure.**

$$AB := \frac{X}{Y} \quad BG := \frac{Y - X}{Y} \quad FH := \frac{BG}{2} \quad AF := AB + FH$$

$$CH := \frac{AF}{2} \quad CD := \frac{2 \cdot CH^2 - FH^2}{2 \cdot CH} \quad DF := CH - CD$$

$$DH := \sqrt{(CH + CD) \cdot DF} \quad DE := \frac{DF \cdot DH}{DH + FH}$$

$$AE := CH + CD + DE \qquad AE = 0.547723$$

$$\sqrt{\frac{X}{Y}} - AE = 0$$

Definitions.

$$AB - \frac{X}{Y} = 0 \quad BG - \frac{Y - X}{Y} = 0 \quad FH - \frac{Y - X}{2 \cdot Y} = 0$$

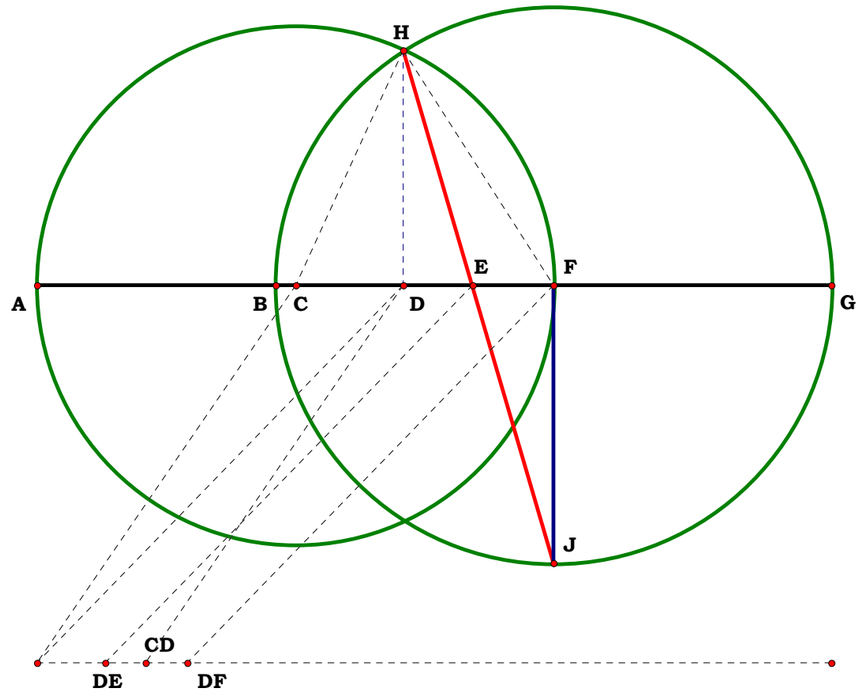
$$AF - \frac{X + Y}{2 \cdot Y} = 0 \quad CH - \frac{X + Y}{4 \cdot Y} = 0$$

$$CD - \frac{\left(6 \cdot X \cdot Y - X^2 - Y^2\right)}{4 \cdot Y \cdot (X + Y)} = 0 \quad DF - \frac{(X - Y)^2}{2 \cdot Y \cdot (X + Y)} = 0$$

$$DH - \frac{(Y - X) \cdot \sqrt{X}}{\sqrt{Y} \cdot (X + Y)} = 0 \quad DE - \frac{\sqrt{X} \cdot \left(\sqrt{X} - \sqrt{Y}\right)^2}{\sqrt{Y} \cdot (X + Y)} = 0$$

$$AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$$

## Trivial Method Square Root




Unit = 1.00000

XY = 0.30000
X = 6.00000
Y = 20.00000
B = 0.30000
$\sqrt{B}$ = 0.54772
C = 0.54772

A = 0.00000
B = 0.30000
C = 0.32500
D = 0.46154
E = 0.54772
F = 0.65000

G = 1.00000
DE = 0.08618
DF = 0.18846
CD = 0.13654

**AE is the square root of AB x AG, which is always 1. One should come to understand that considering two things, and the relation between them, they are grouped as one thing and are proportional to that whole. Thus, a simple 1, or unit, produces every possible solution there ever can be in terms between 0 and 1. So, slide B from A to G and see every possible root that can exist.**

Unit.

$AB := 1$

Given.

$N := 5$

**102894A**

**Descriptions.**

$AH := AB \cdot N \quad BH := AH - AB$

$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

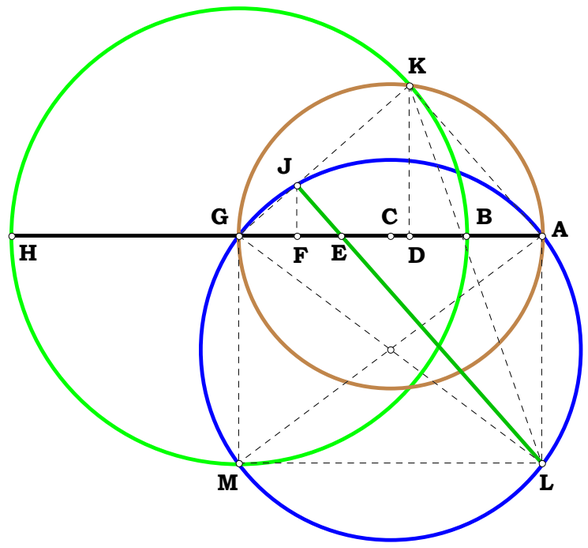
$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$GJ := \frac{GL^2 + GK^2 - KL^2}{2 \cdot GK} \quad FG := \frac{DG \cdot GJ}{GK}$

$AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL}$

$AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0 \quad AE = 2.236068$

**Trivial Method Square Root**




$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ - \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} = 0$

**AE is the square root of AB x AH.**

**Definitions.**

**Another fine example of Mathcad's inability to function rationally. Even to get the reduction this far required too much manual labor, but if one wants to continue, then one will get to the simple result.**

$$AE - \frac{(N-1)^2 \cdot (N+1) \cdot \sqrt{N \cdot (N-1)^2} + 2 \cdot N \cdot (N-1) \cdot (N+1) \cdot \sqrt{(N+1)^2}}{\left(N^3 - 3 \cdot N^2 + 3 \cdot N - 1\right) \cdot \sqrt{(N+1)^2} + \sqrt{N \cdot (N-1)^2} \cdot \left(2 \cdot N^2 + 4 \cdot N + 2\right)} = 0$$

$AE - \sqrt{N} = 0$

102894B

**Given.**

$X := 6 \quad Y := 20$

**Unit.**

$AH := 1$

**Descriptions.**

$AB := \frac{X}{Y} \quad BH := \frac{Y - X}{Y} \quad BH := AH - AB \quad BG := \frac{BH}{2}$

$GK := BG \quad AG := AB + BG \quad DG := \frac{GK^2}{AG} \quad AD := AG - DG$

$AL := BG \quad GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG$

$DH := BH - BD \quad DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$GJ := \frac{GL^2 + GK^2 - KL^2}{2 \cdot GK} \quad FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG$

$FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF$

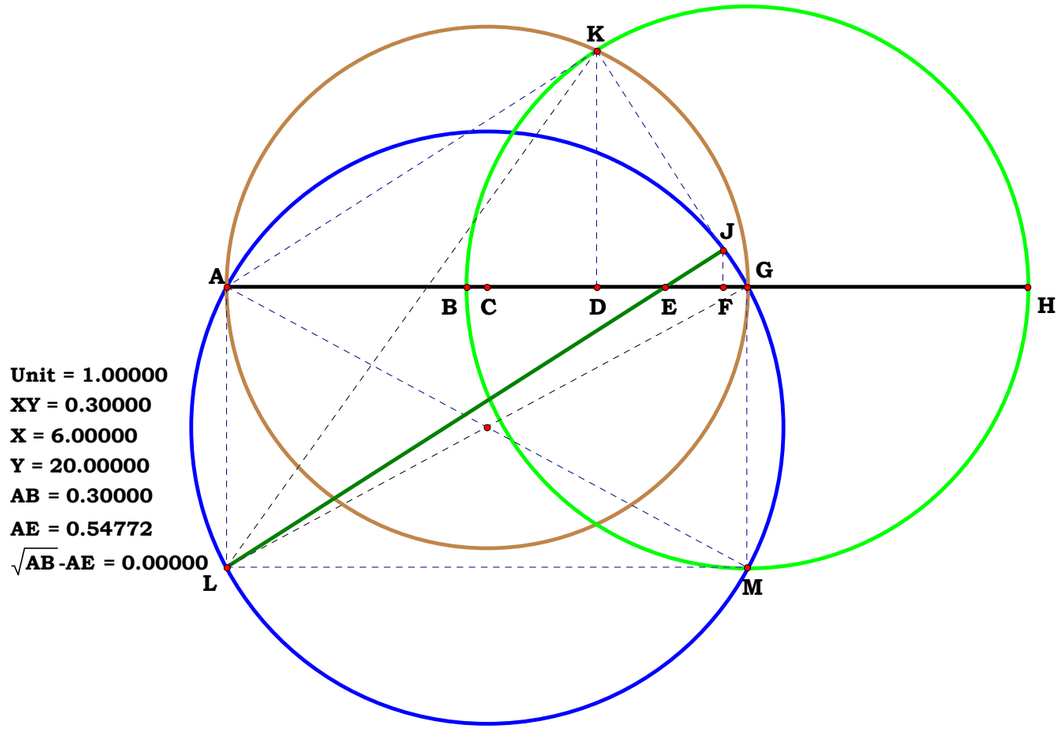
$\sqrt{AB \cdot AH} - AE = 0 \quad AE = 0.547723$

**Definitions.**

**And again, this method judges both parties equally.**

$AE - \sqrt{\frac{X}{Y}} = 0 \quad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

**Trivial Method Square Root**




**AE is the square root of AB x AH which is always unity.**

**Unit.**

$AB := 1$

**Given.**

$N := 22$

**103194A**

**Descriptions.**

$AF := N \quad BF := AF - AB \quad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \frac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \frac{CF \cdot CH}{(CH + FK)}$

$EF := CF - CE \quad BE := BC + CE \quad DF := BF - BE \quad CD := AD - AC$

**Definitions.**

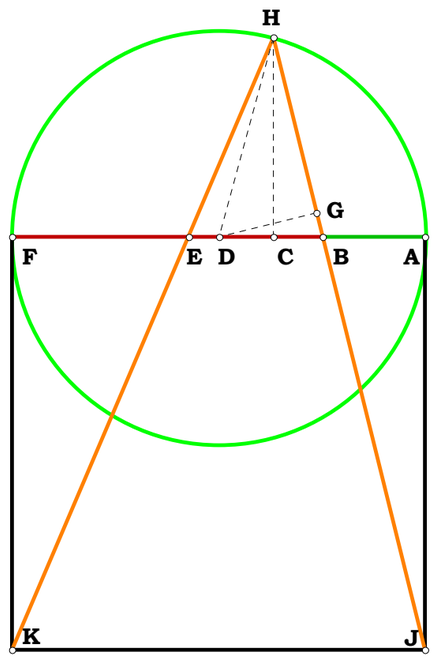
**I cannot get Mathcad to transform one into the other.**

$BE^2 - EF = 0 \quad \sqrt{EF} - BE = 0$

$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$

$EF - \frac{2 \cdot N^2 - \sqrt{4 \cdot N - 3} - 2 \cdot N + 1}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} = 0$

**Square Root of a Segment**




**Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.**

110194A

Given.

$N_1 := 4.55192$

$N_2 := 3.86362$

Descriptions.

$BG := N_1$ $BC := N_1 - N_2$ $BN := BG$ $BF := \frac{BG}{2}$ $FL := BF$ $CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2}$ $CH := \frac{BC \cdot CF}{CN}$ $FH := \frac{BN \cdot CF}{CN}$ $HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL$ $CD := \frac{BC \cdot CL}{CN}$ $DL := \frac{BN \cdot CL}{CN}$ $GM := DL$ $BD := BC + CD$

$DG := BG - BD$ $LM := DG$ $GO := BG$ $MO := GO + GM$ $EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC$ $CE := CG - EG$ $CJ := BC$ $CK := CE$ $IJ := BC$

$JK := CK - CJ$ $IK := \sqrt{IJ^2 + JK^2}$ $BI := BC$ $AB := \frac{IJ \cdot BI}{JK}$ $AG := AB + BG$

$AE := AB + BC + CE$ $AC := AB + BC$ $AB = 0.746998$ $AG = 5.298918$

$AE = 2.757812$

Definitions.

$\left(AB^2 \cdot AG\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

Here, everything looks as it should and we might get comfortable, but let us just move C.

Given any point on a segment, cut the segment into duplicate ratios with that point.

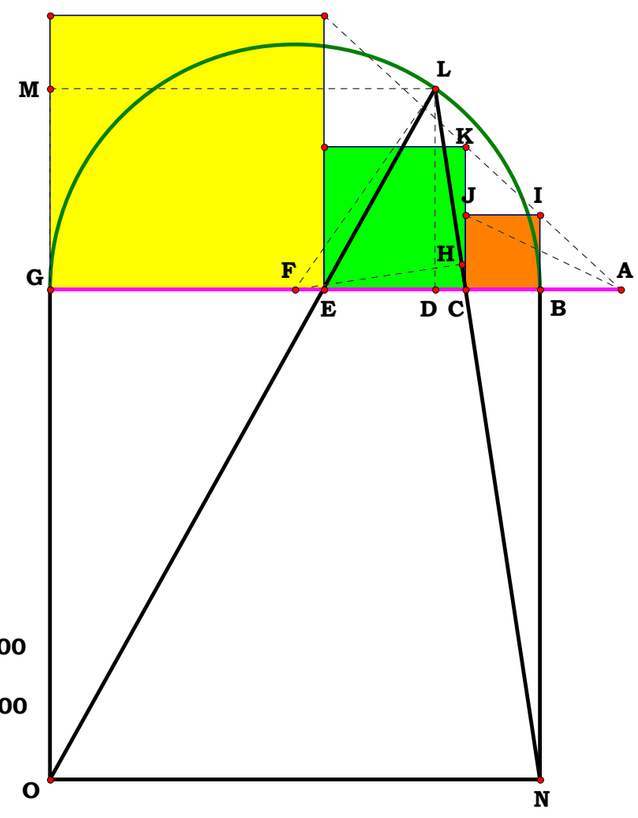
$EG = 2.541106$ $CE = 1.322514$ $BC = 0.6883$ $\frac{EG}{CE} - \frac{CE}{BC} = 0$

## Duplicate Ratios

Given BG and BC find AB, AG, such that $(AB^2 \cdot AG)^{1/3} = BC$. For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in $N_2$ in order to show a problem with so called mathematicians today.

$N_1 = 4.55192$
$N_2 = 3.86362$
$AB = 0.74699$
$BF = 2.27596$
$AG = 5.29891$
$AC = 1.43528$
$AE = 2.75779$
$(AB^2 \cdot AG)^{\frac{1}{3}} = 1.43528$
$(AB \cdot AG^2)^{\frac{1}{3}} = 2.75779$
$(AB^2 \cdot AG)^{\frac{1}{3}} - AC = 0.00000$
$(AB \cdot AG^2)^{\frac{1}{3}} - AE = 0.00000$



110194B

Given.

$N_1 := 4.55192$

$N_2 := 2.63187$

## Duplicate Ratios

Given AG - AB = BG and $(AB^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in $N_2$ in order to show a problem with so called mathematicians today.

Descriptions.

$BG := N_1 \quad BC := N_1 - N_2 \quad BN := BG \quad BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN} \quad FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN} \quad GM := DL \quad BD := BC + CD$

$DG := BG - BD \quad LM := DG \quad GO := BG \quad MO := GO + GM \quad EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC \quad CE := CG - EG \quad CJ := BC \quad CK := CE \quad IJ := BC$

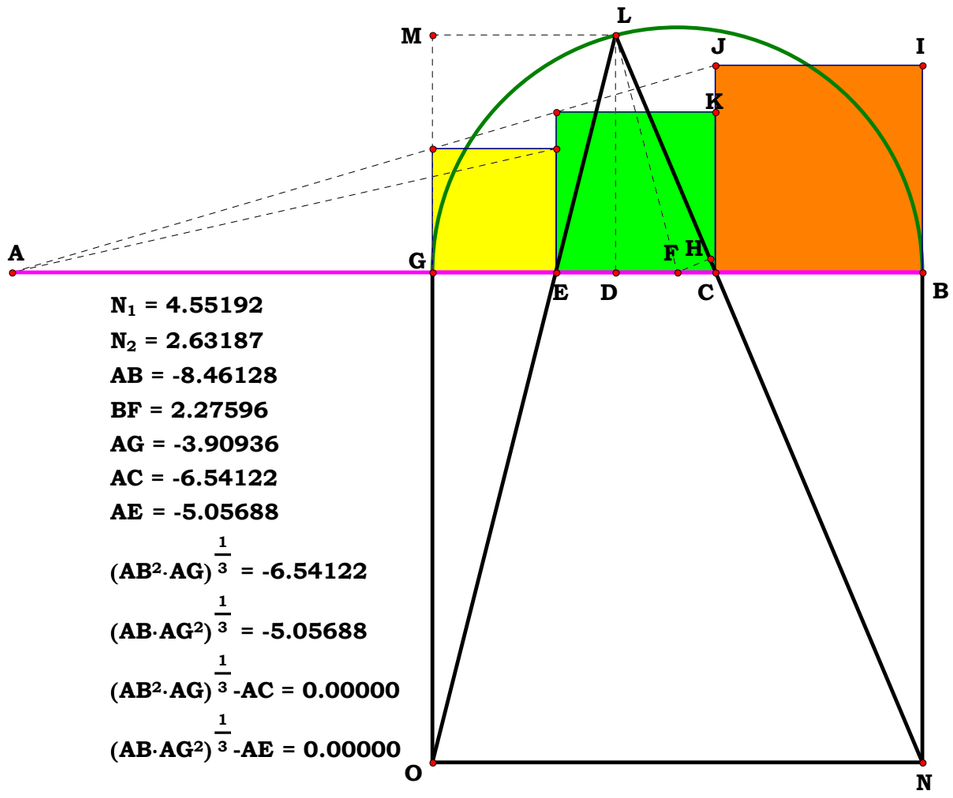
$JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2} \quad BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE \quad AC := AB + BC \quad AB = -8.461312 \quad AG = -3.909392$

$AE = -5.056911$

Definitions.

$\left(AB^2 \cdot AG\right)^{\frac{1}{3}} = 3.270631 + 5.664899i \quad \left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 7.585367 + 4.379414i$




$N_1$ = 4.55192
$N_2$ = 2.63187
AB = -8.46128
BF = 2.27596
AG = -3.90936
AC = -6.54122
AE = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$ = -6.54122
$(AB \cdot AG^2)^{\frac{1}{3}}$ = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$ -AC = 0.00000
$(AB \cdot AG^2)^{\frac{1}{3}}$ -AE = 0.00000

For those who believe that math is perfect, that mathematicians are literate, here we have an example of simple geometry and a child's drawing program showing the illiteracy of one of the top math programs world-wide. How is it possible for two identical equations with identical inputs produce two different resullts? How is it possible not to know what your own grammar means? Do the equation in Windows Calculator, it will even get it right. Why can't a so called high end math program?

Unit.

$AB := 1$

Given.

$N := 5$

122494A

Descriptions.

$AJ := AB \cdot N$ $\quad AF := \sqrt{AB \cdot AJ}$ $\quad BJ := AJ - AB$ $\quad BG := \frac{BJ}{2}$

$AG := AB + BG$ $\quad GS := BG$ $\quad DG := \frac{GS^2}{AG}$ $\quad FG := AG - AF$

$BD := BG - DG$ $\quad DJ := BJ - BD$ $\quad DS := \sqrt{BD \cdot DJ}$

$FK := \frac{DS \cdot FG}{DG}$ $\quad BF := AF - AB$ $\quad BK := \sqrt{BF^2 + FK^2}$

$FI := \frac{DJ \cdot FK}{DS}$ $\quad BI := FI + BF$ $\quad BP := \frac{BK \cdot BJ}{BI}$

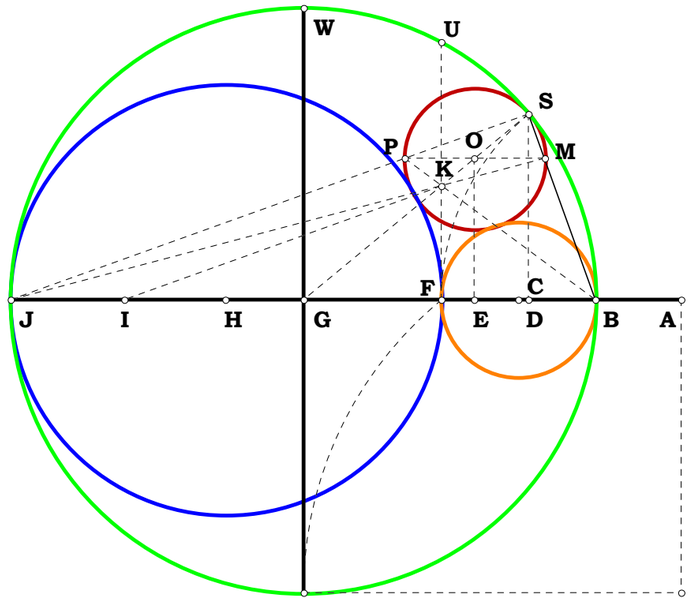
$KP := BP - BK$ $\quad MP := \frac{BJ \cdot KP}{BK}$ $\quad OS := \frac{MP}{2}$

Definitions.

$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0$ $\quad OS - \frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(N + \sqrt{N} + 1\right)} = 0$

## Power Line At Square Root

**In this square root figure, what is the Algebraic definition of the tangent circle OS?**




AB = 1.43933 cm

AJ = 11.23950 cm

OS = 1.18009 cm

$\frac{AB}{AB} = 1.00000$ $\quad AB = 1.00000$

$\frac{AJ}{AB} = 7.80882$ $\quad N = 7.80882$

$\frac{OS}{AB} = 0.81989$ $\quad OS = 0.81989$

$\frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(\sqrt{N} + N + 1\right)} - OS = 0.00000$

Given.

$Y := 20 \quad X := 16$

Unit.

$AH := \frac{Y}{Y}$

122494B

Descriptions.

$AD := \frac{X}{Y} \quad AC := \frac{AH}{2} \quad DO := \sqrt{AD \cdot (AH - AD)} \quad CD := \sqrt{AC^2 - DO^2}$

$CK := \frac{AC^2}{CD} \quad AK := CK + AC \quad DK := AK - AD \quad DJ := \frac{DK}{2}$

$AJ := AD + DJ \quad CJ := AJ - AC \quad CG := \frac{AC^2}{CJ} \quad AG := CG + AC$

$GV := \sqrt{AG \cdot (AH - AG)} \quad DP := \frac{DJ \cdot CD}{AC} \quad DW := \frac{AG \cdot DP}{GV}$

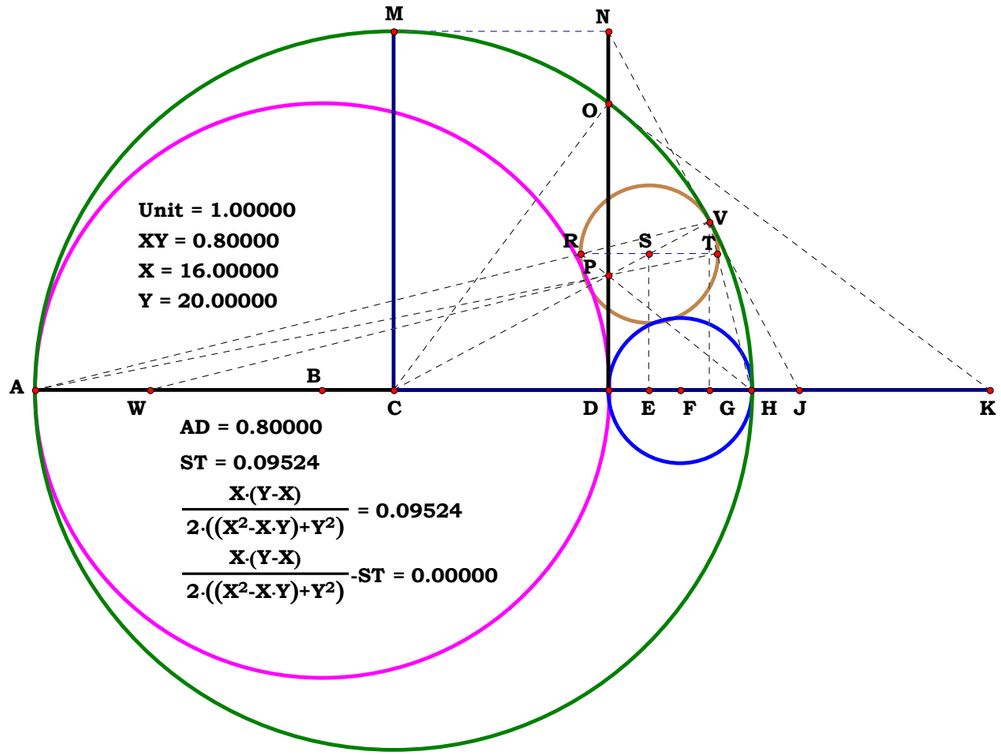
$DH := AH - AD \quad HP := \sqrt{DH^2 + DP^2} \quad AW := AD - DW$

$HR := \frac{HP \cdot AH}{AH - AW} \quad PR := HR - HP \quad RT := \frac{AH \cdot PR}{HP} \quad ST := \frac{RT}{2}$

$ST = 0.095238$

## Power Line At Square Root

**In this square root figure, what is the Algebraic definition of the tangent circle OS Given just point D?**




Definitions.

$AD - \frac{X}{Y} = 0 \quad AC - \frac{1}{2} = 0 \quad DO - \frac{\sqrt{X \cdot (Y - X)}}{Y} = 0 \quad CD - \frac{(2 \cdot X - Y)}{2 \cdot Y} = 0 \quad CK - \frac{Y}{2 \cdot (2 \cdot X - Y)} = 0$

$AK - \frac{X}{2 \cdot X - Y} = 0 \quad DK - \frac{2 \cdot X \cdot (Y - X)}{Y \cdot (2 \cdot X - Y)} = 0 \quad DJ - \frac{2 \cdot X \cdot (Y - X)}{2 \cdot Y \cdot (2 \cdot X - Y)} = 0 \quad AJ - \frac{X^2}{Y \cdot (2 \cdot X - Y)} = 0 \quad CJ - \frac{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2}{2 \cdot Y \cdot (2 \cdot X - Y)} = 0 \quad CG - \frac{Y \cdot (2 \cdot X - Y)}{2 \cdot \left(2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2\right)} = 0$

$AG - \frac{X^2}{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2} = 0 \quad GV - \frac{X \cdot (Y - X)}{\left(2 \cdot X^2 + Y^2 - 2 \cdot X \cdot Y\right)} = 0 \quad DP - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad DW - \frac{X^2}{Y^2} = 0 \quad DH - \frac{Y - X}{Y} = 0 \quad HP - \frac{\sqrt{X^2 + Y^2} \cdot (Y - X)}{Y^2} = 0$

$AW - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad HR - \frac{(Y - X) \cdot \sqrt{X^2 + Y^2}}{X^2 - X \cdot Y + Y^2} = 0 \quad PR - \frac{\sqrt{X^2 + Y^2} \cdot X \cdot (X - Y)^2}{Y^2 \cdot \left(X^2 - X \cdot Y + Y^2\right)} = 0 \quad RT - \frac{X \cdot (Y - X)}{X^2 - X \cdot Y + Y^2} = 0 \quad ST - \frac{X \cdot (Y - X)}{2 \cdot \left(X^2 - X \cdot Y + Y^2\right)} = 0$

122595A

**Descriptions.**

**Unit.**

$AB := 1$

**Given.**

$N := 5$

$\Delta := 5$

$\delta := 1 .. \Delta$

# Two Prime Exponential Series Developed Through The Powerline Progression

$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB \quad BJ := \frac{BO}{2} \quad JZ := BJ \quad JV := BJ \quad JO := BJ$

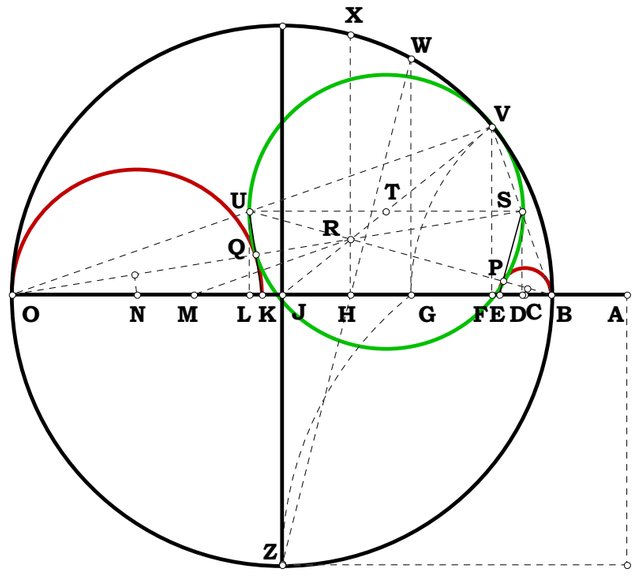
$BG_1 := AG - AB \quad GO_1 := BO - BG_1 \quad GW_1 := \sqrt{BG_1 \cdot GO_1} \quad GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{pmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{pmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - (BG_\delta + GH_\delta) \\ \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]} \\ BJ - (BG_\delta + GH_\delta) \\ \frac{[BJ - (BG_\delta + GH_\delta)] \cdot \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}}{JZ + \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}} \end{bmatrix}$$

$HJ := BJ - BG_\Delta \quad FJ := \frac{(N-1)^2}{2 \cdot (N+1)} \quad BF := BJ - FJ \quad FO := FJ + JO \quad FV := \sqrt{BF \cdot FO} \quad HR := \frac{FV \cdot HJ}{FJ}$

$BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2} \quad HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR \quad SU := \frac{BO \cdot RU}{BR}$

$TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR} \quad BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$




**Definitions.**

$N^{\frac{1}{2^\Delta}} - AE = 0$

| $\delta =$ | $N^{\frac{1}{2^\delta}} =$ |
|---|---|
| 1 | 2.236068 |
| 2 | 1.495349 |
| 3 | 1.222845 |
| 4 | 1.105823 |
| 5 | 1.051581 |

**Descriptions.**

$$LU := \frac{HR \cdot BU}{BR} \qquad BL := \frac{BH \cdot BU}{BR} \qquad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \qquad DS := LU \qquad OS := \frac{OR \cdot DS}{HR}$$

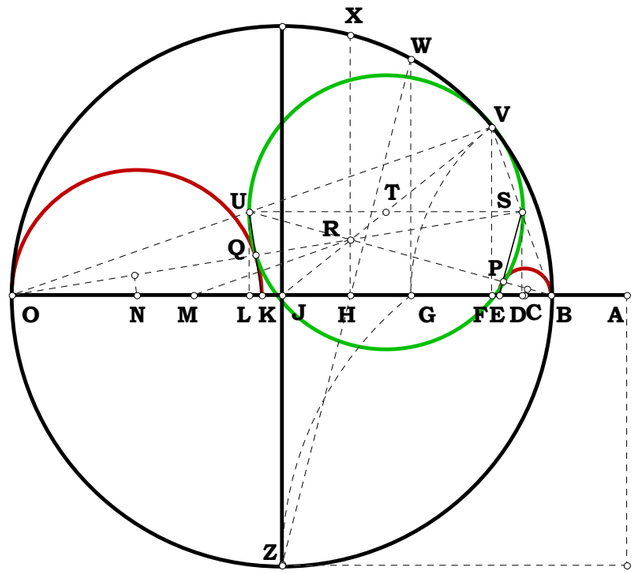
$$DO := \frac{HO \cdot DS}{HR} \qquad QS := \frac{DO \cdot SU}{OS} \qquad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \qquad AK := AO - KO$$

**Definitions.**

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

| $\delta =$ | $N^{\frac{2^{\delta}-1}{2^{\delta}}} =$ |
|---|---|
| 1 | 2.236068 |
| 2 | 3.343702 |
| 3 | 4.088827 |
| 4 | 4.521519 |
| 5 | 4.754745 |



122595B

Given.

$X := 6 \quad Y := 20$

Unit.

$\frac{Y}{Y} = 1$

# Two Prime Exponential Series Developed Through The Powerline Progression

**Descriptions.**

**A series is actually the recursion of a process. In this case, you will find that M through C and then again through ever G, we have produced an exponential series with a simple square root figure. One can see, there are many places one can start this figure and determine the rest of it.**

$DH := \frac{Y}{Y} \quad BD := 2 \cdot DH \quad CH := \frac{X}{Y} \quad HM := DH$

$CM := \sqrt{CH^2 + HM^2} \quad LM := \frac{HM \cdot BD}{CM} \quad LM = 1.915653$

$JL := CH \cdot \frac{BD}{CM} \quad JL = 0.574696 \quad FH := \frac{HM^2}{CH}$

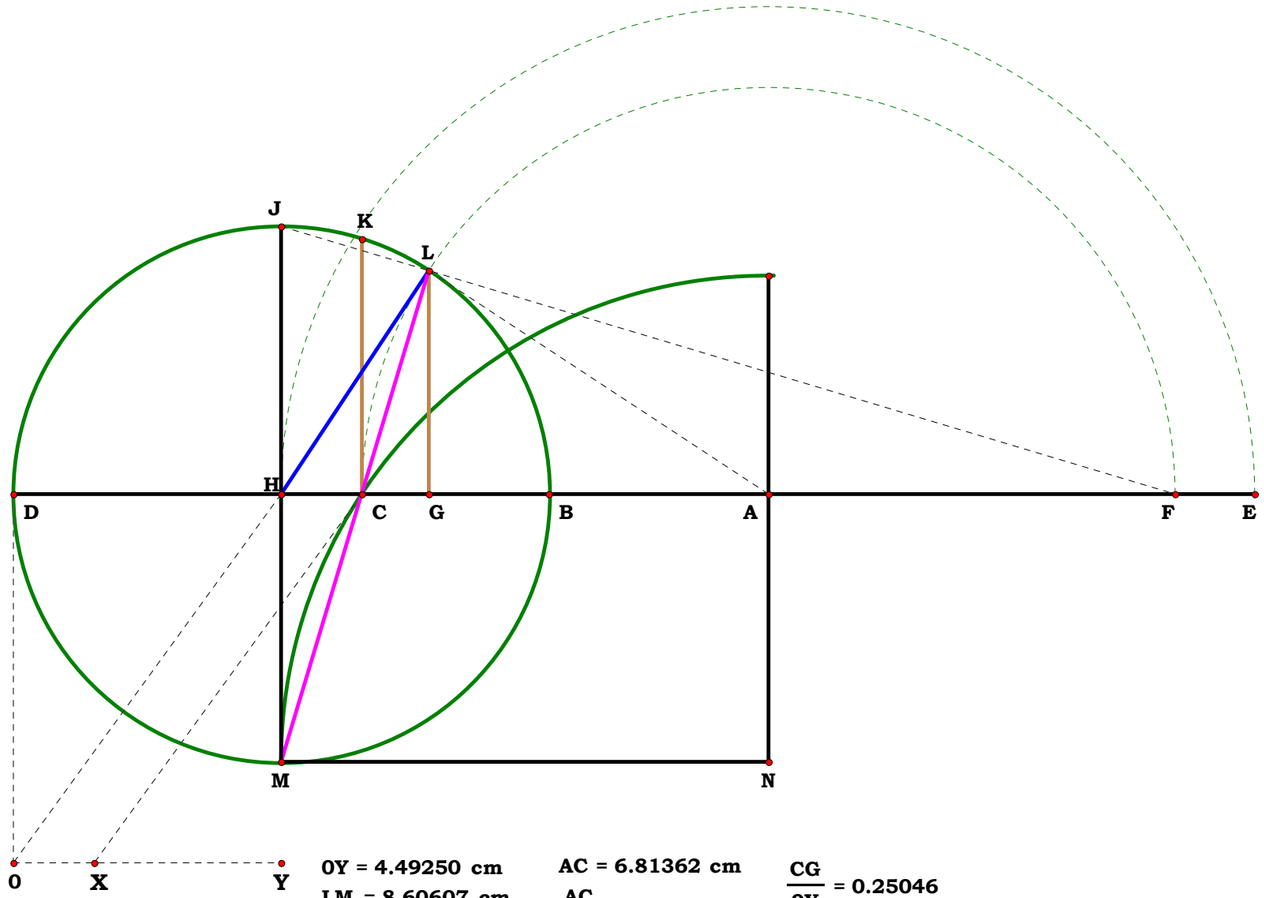
$DF := FH + DH \quad DF = 4.333333 \quad CF := FH - CH$

$CD := CH + DH \quad AC := \frac{CF}{2} \quad AD := AC + CD$

$AB := AD - BD \quad \sqrt{AB \cdot AD} - AC = 0$

$CG := \frac{CH \cdot (LM - CM)}{CM} \quad BG := DH - (CH + CG)$

$GL := \sqrt{BG \cdot (BD - BG)} \quad GL = 0.834862$




Unit = 1.00000

XY = 0.30000

X = 6.00000

Y = 20.00000

$x_F$ = 4.33333

0Y = 4.49250 cm

LM = 8.60607 cm

$\frac{LM}{0Y} = 1.91565$

LJ = 2.58182 cm

$\frac{LJ}{0Y} = 0.57470$

AC = 6.81362 cm

$\frac{AC}{0Y} = 1.51667$

AH = 8.16137 cm

$\frac{AH}{0Y} = 1.81667$

CG = 1.12519 cm

$\frac{CG}{0Y} = 0.25046$

GL = 3.75062 cm

$\frac{GL}{0Y} = 0.83486$

**Definitions.**

$DH - 1 = 0$ $BD := 2$ $CH - \frac{X}{Y} = 0$ $HM - 1 = 0$ $CM - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$

$LM - \frac{2 \cdot Y}{\sqrt{X^2 + Y^2}} = 0$ $JL - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$ $FH - \frac{Y}{X} = 0$ $DF - \frac{X + Y}{X} = 0$

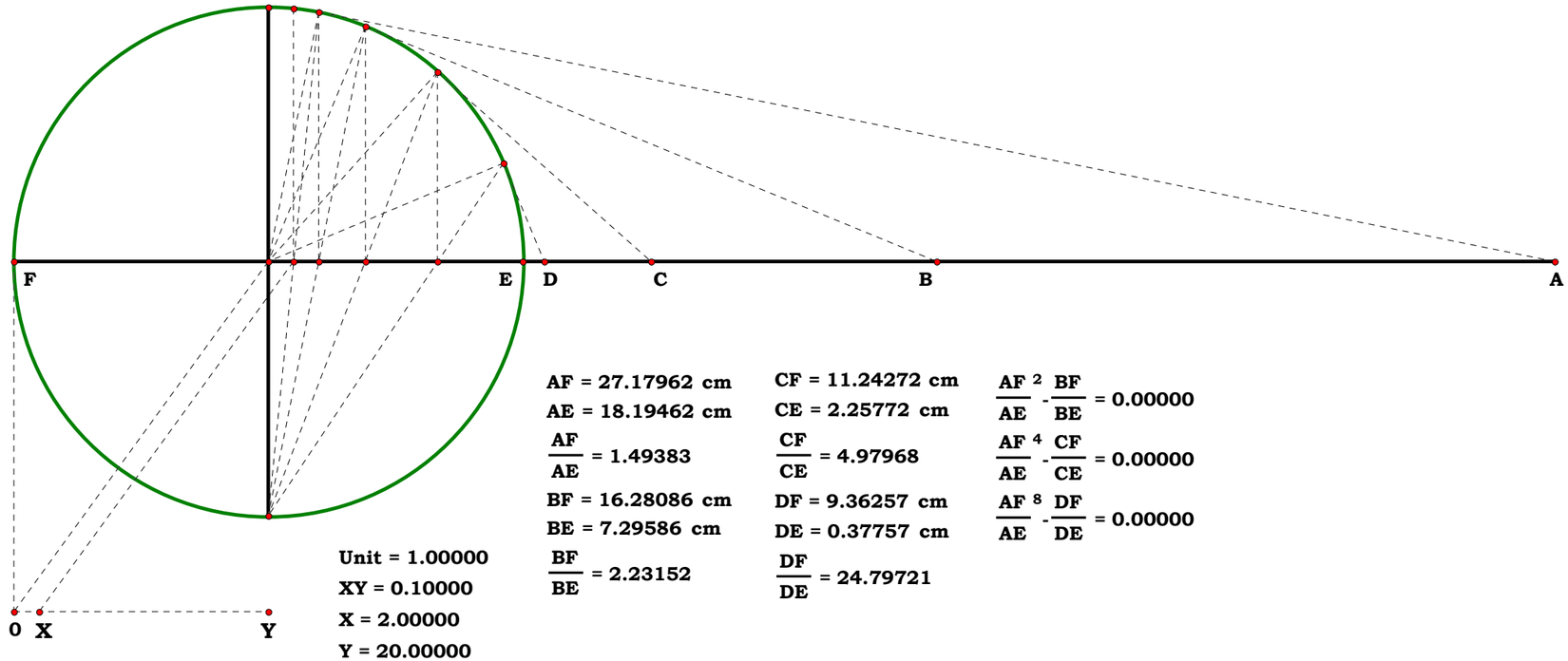
$CF - \frac{Y^2 - X^2}{X \cdot Y} = 0$ $CD - \frac{X + Y}{Y} = 0$

$AC - \frac{Y^2 - X^2}{2 \cdot X \cdot Y} = 0$ $AD - \frac{(X + Y)^2}{2 \cdot X \cdot Y}$

$AB - \frac{(X - Y)^2}{2 \cdot X \cdot Y} = 0$ $AC - \frac{\left(Y^2 - X^2\right)}{2 \cdot X \cdot Y} = 0$

$CG - \frac{X \cdot \left(Y^2 - X^2\right)}{\left(X^2 + Y^2\right) \cdot Y} = 0$ $BG - \frac{(X - Y)^2}{X^2 + Y^2} = 0$

$GL - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$




Unit = 1.00000
XY = 0.10000
X = 2.00000
Y = 20.00000

AF = 27.17962 cm
AE = 18.19462 cm
$\frac{AF}{AE} = 1.49383$
BF = 16.28086 cm
BE = 7.29586 cm
$\frac{BF}{BE} = 2.23152$

CF = 11.24272 cm
CE = 2.25772 cm
$\frac{CF}{CE} = 4.97968$
DF = 9.36257 cm
DE = 0.37757 cm
$\frac{DF}{DE} = 24.79721$

$\frac{AF^2}{AE} - \frac{BF}{BE} = 0.00000$
$\frac{AF^4}{AE} - \frac{CF}{CE} = 0.00000$
$\frac{AF^8}{AE} - \frac{DF}{DE} = 0.00000$

Unit.

$AB := 1$

Given.

$N := 5$

122694A

Descriptions.

$AF := AB \cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EF := BE \quad EK := BE$

$AD := \sqrt{AB \cdot AF} \quad CE := \frac{(N-1)^2}{2 \cdot (N+1)} \quad CF := CE + EF$

$DF := AF - AD \quad BC := BF - CF \quad CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF} \qquad DG_1 = 1.236068$

$BD := AD - AB \quad DG_2 := \frac{EK \cdot BD}{BE} \quad DG_1 - DG_2 = 0$

Definitions.

This might give you something to think about.

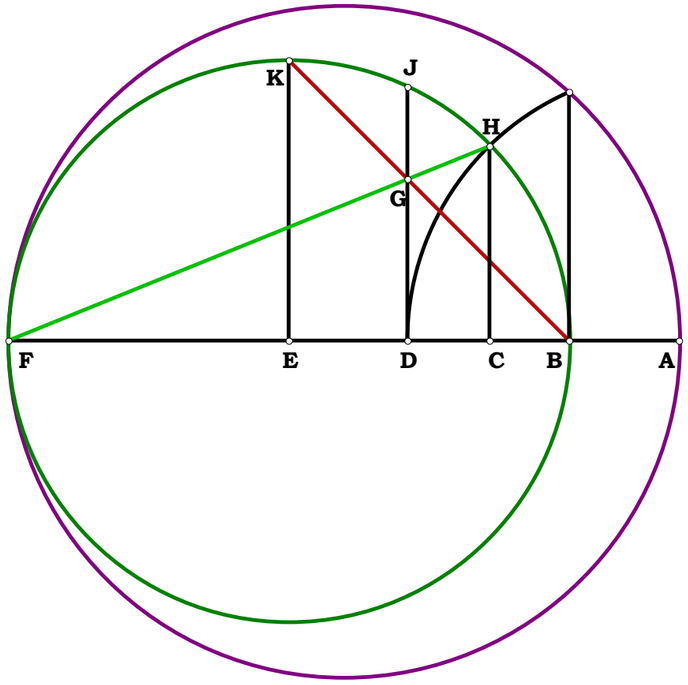
$$DG_1 - \frac{(N+1) \cdot \sqrt{N \cdot (N-1)^2} \cdot \left(N - \sqrt{N}\right)}{N \cdot (N-1) \cdot \sqrt{(N+1)^2}} = 0$$

$$DG_2 - \left(\sqrt{N} - 1\right) = 0$$

## Exponential Series

Is Point G on DJ?

Is G, the intersection of FH and BK, on DJ?



122694B

**Descriptions.**

**Definitions.**

**Given.**

$X := 7$

$Y := 20$

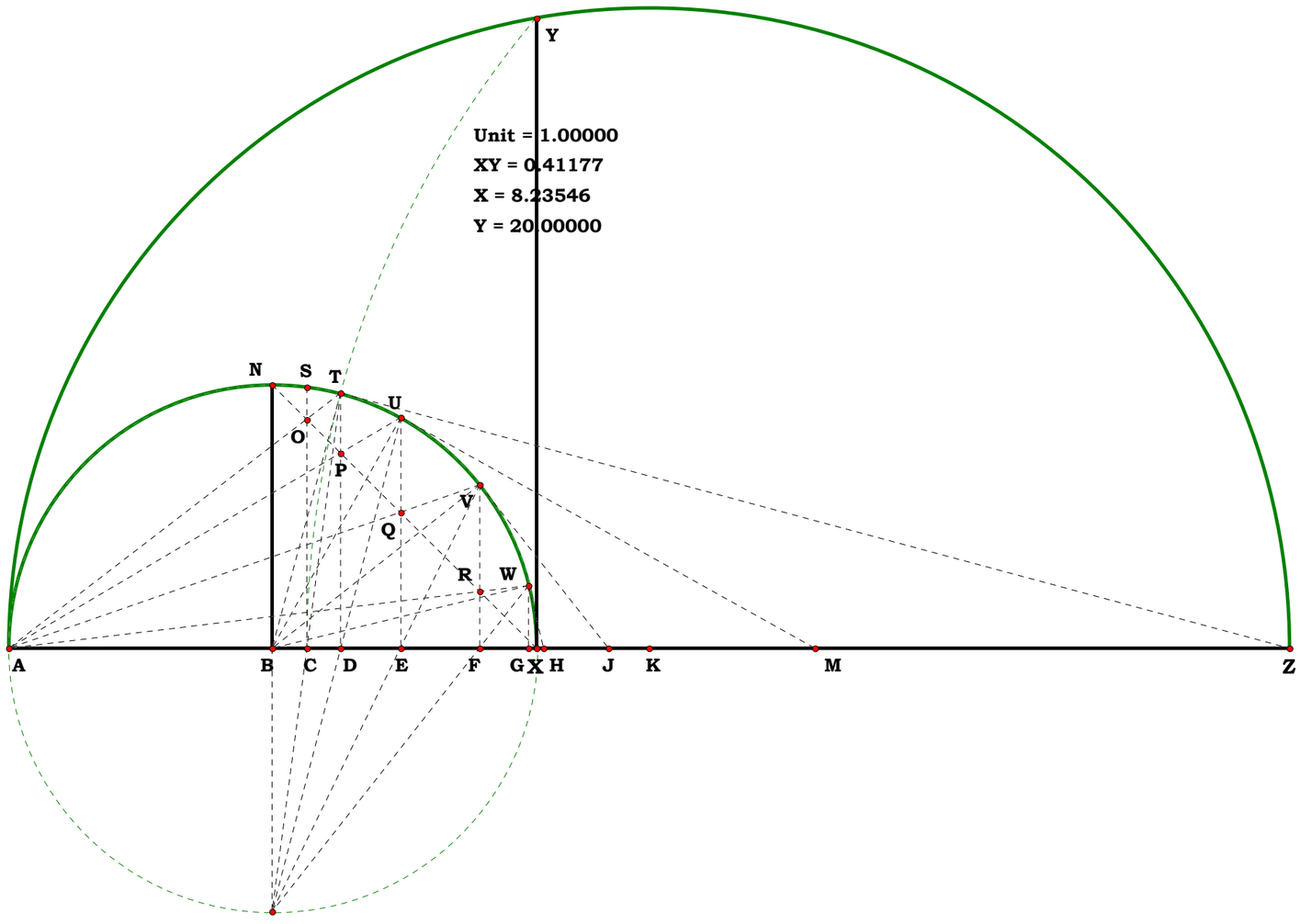
**Unit.**

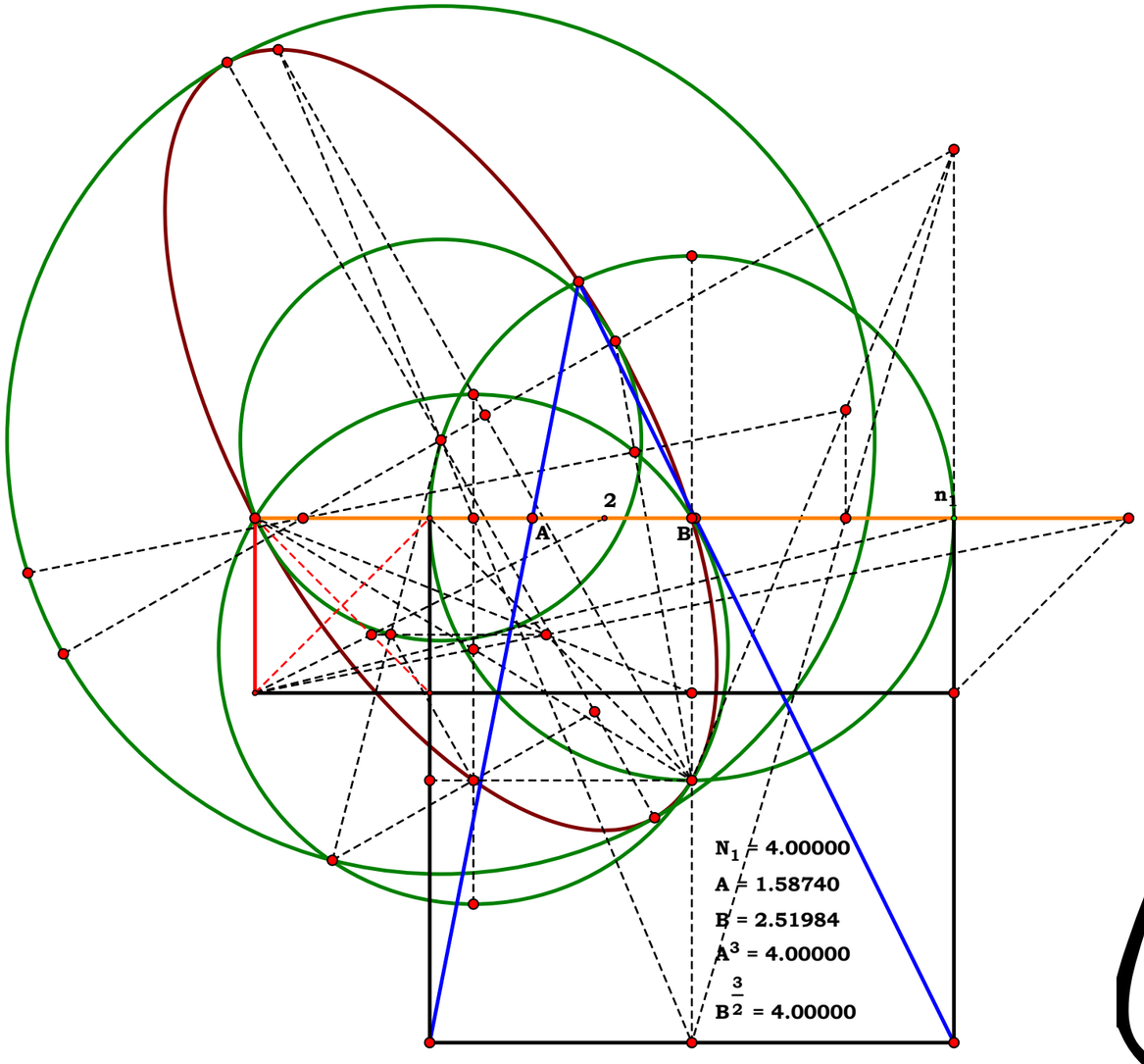
$\frac{Y}{Y} = 1$

**Procrastinated Write-up**

## Exponential Series

**Leaving love and me out of the equation.**






# The Delian Quest 1995

**John Clark**

**Unit.**

$AC := 1$

**Given.**

$N := 5$

**010695A**

**Descriptions.**

$AJ := AC \cdot N \quad CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

$MN := EN \quad NO := 1BE := EN \quad EH := EN \quad BJ := BE + EJ$

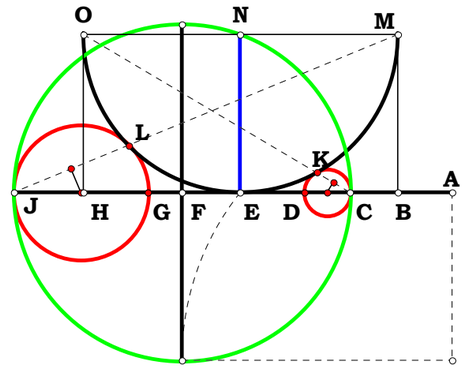
$MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO \quad ML := \frac{BJ \cdot MO}{MJ}$

$JL := MJ - ML \quad GJ := \frac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ \quad \left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \frac{CH \cdot MO}{CO} \quad CK := CO - KO \quad CD := \frac{CO \cdot CK}{CH}$

$AD := AC + CD \quad \left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0$

**Alternate Method Quad Roots**




**Definitions.**

$N^{\frac{3}{4}} - AG = 0 \quad N^{\frac{1}{4}} - AD = 0$

100695B

**Given.**

$X := 6$

$Y := 20$

**Unit.**

$FG := \frac{Y}{Y}$

**Finding the Unit for a Figure.**

**Supplement to 100695**

**Descriptions.**

$BF := 2 \cdot FG \qquad DG := \frac{X}{Y} \qquad DF := FG + DG \qquad BD := FG - DG$

$DH := \sqrt{DF \cdot BD} \qquad JK := 2 \cdot DH \qquad FK := \sqrt{(DF + DH)^2 + DH^2}$

$KU := \frac{(DF + DH) \cdot JK}{FK} \qquad FU := FK - KU \qquad EF := \frac{JK \cdot FU}{KU}$

$BJ := \sqrt{(DH + BD)^2 + DH^2} \qquad JV := \frac{(DH + BD) \cdot JK}{BJ}$

$BV := BJ - JV \qquad BC := \frac{JK \cdot BV}{JV}$

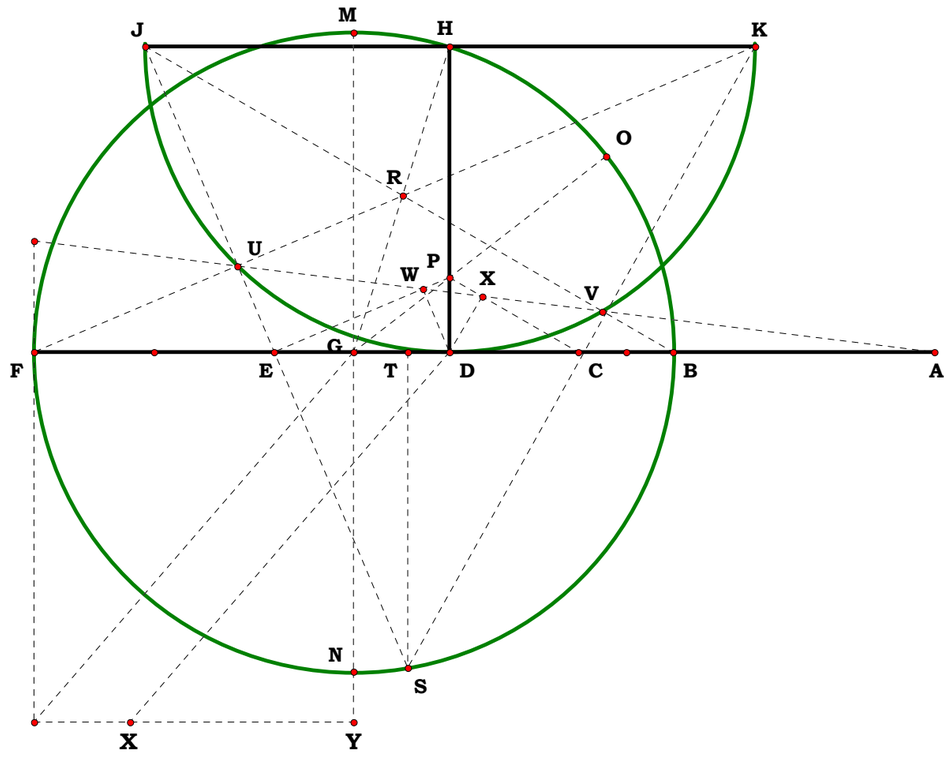
**Use 041694, Tangents and Similarity Points for the next equation.**

$AF := \frac{EF \cdot (BC - BF)}{BC - EF} \qquad AB := AF - BF \qquad AC := AB + BC$

$AD := AB + BD \qquad AE := AF - EF$

$\left(\frac{AF}{AB}\right)^{\frac{1}{4}} - \frac{AC}{AB} = 0 \qquad \left(\frac{AF}{AB}\right)^{\frac{2}{4}} - \frac{AD}{AB} = 0 \qquad \left(\frac{AF}{AB}\right)^{\frac{3}{4}} - \frac{AE}{AB} = 0$

**What the above tells us is that the figure has to be redrawn such that AB is the unit if we want to express the figure as a quad root series.**
**The following plate will do just that.**




Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000
BF = 10.72067 cm
AB = 4.37761 cm
BC = 1.58807 cm
CD = 2.16417 cm
DE = 2.94926 cm
EF = 4.01917 cm
AF = 15.09827 cm
BF = 10.72067 cm
CF = 9.13260 cm
DF = 6.96843 cm
TF = 6.25594 cm
FG = 5.36033 cm

**Definitions.**

$$BF - 2 = 0 \quad DG - \frac{X}{Y} = 0 \quad DF - \frac{X+Y}{Y} = 0 \quad BD - \frac{Y-X}{Y} = 0 \quad DH - \frac{\sqrt{Y^2-X^2}}{Y} = 0$$

$$JK - \frac{2\cdot\sqrt{Y^2-X^2}}{Y} = 0 \quad FK - \frac{\sqrt{(X+Y)\cdot\left(3\cdot Y-X+2\cdot\sqrt{Y^2-X^2}\right)}}{Y} = 0$$

$$KU - \frac{2\cdot\sqrt{Y^2-X^2}\cdot\left(X+Y+\sqrt{Y^2-X^2}\right)}{\sqrt{(X+Y)\cdot\left(3\cdot Y-X+2\cdot\sqrt{Y^2-X^2}\right)}\cdot Y} = 0$$

$$FU - \frac{(X+Y)^2}{Y\cdot\sqrt{2\cdot X\cdot\sqrt{Y^2-X^2}+2\cdot Y\cdot\sqrt{Y^2-X^2}-X^2+3\cdot Y^2+2\cdot X\cdot Y}} = 0$$

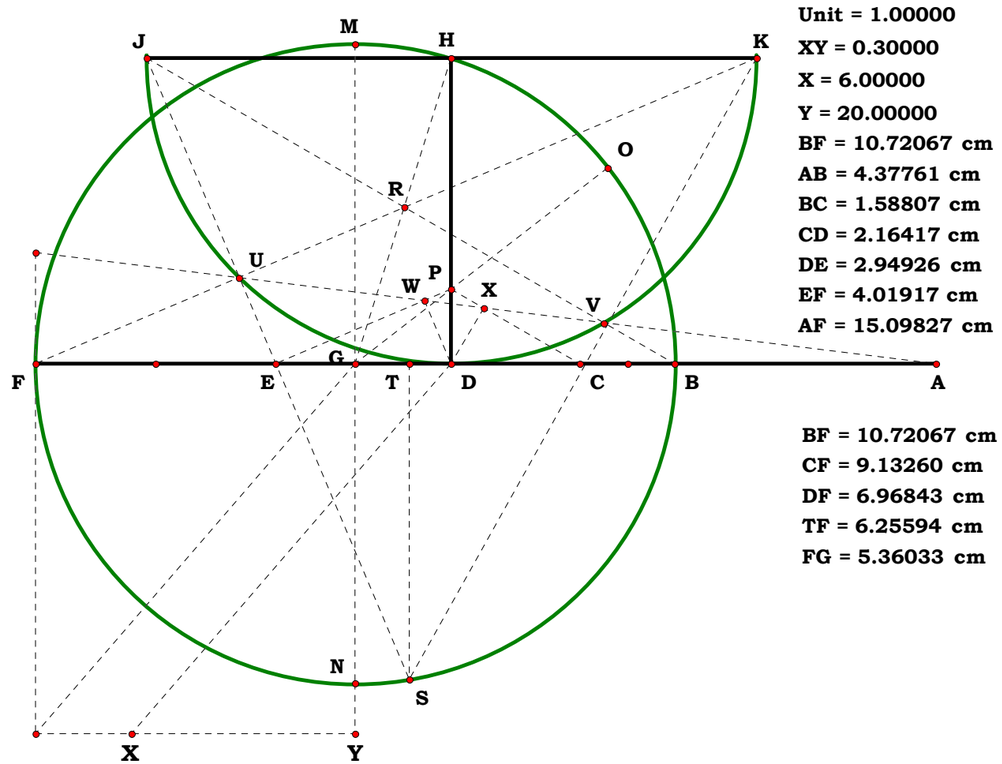
$$EF - \frac{\sqrt{(X+Y)\cdot\left(3\cdot Y-X+2\cdot\sqrt{Y^2-X^2}\right)}\cdot(X+Y)^2}{Y\cdot\left(X+Y+\sqrt{Y^2-X^2}\right)\cdot\sqrt{2\cdot X\cdot\sqrt{Y^2-X^2}+2\cdot Y\cdot\sqrt{Y^2-X^2}-X^2+3\cdot Y^2+2\cdot X\cdot Y}} = 0$$

$$BJ - \frac{\sqrt{(Y-X)\cdot\left(X+3\cdot Y+2\cdot\sqrt{Y^2-X^2}\right)}}{Y} = 0 \quad JV - \frac{2\cdot\left(Y-X+\sqrt{Y^2-X^2}\right)\cdot\sqrt{Y^2-X^2}}{Y\cdot\sqrt{(Y-X)\cdot\left(X+3\cdot Y+2\cdot\sqrt{Y^2-X^2}\right)}} = 0$$

$$BV := \frac{(X-Y)^2}{Y\cdot\sqrt{2\cdot Y\cdot\sqrt{Y^2-X^2}-2\cdot X\cdot\sqrt{Y^2-X^2}-X^2+3\cdot Y^2-2\cdot X\cdot Y}}$$

$$BC - \frac{(X-Y)^2\cdot\sqrt{(Y-X)\cdot\left(X+3\cdot Y+2\cdot\sqrt{Y^2-X^2}\right)}}{Y\cdot\left(Y-X+\sqrt{Y^2-X^2}\right)\cdot\sqrt{2\cdot Y\cdot\sqrt{Y^2-X^2}-2\cdot X\cdot\sqrt{Y^2-X^2}-X^2+3\cdot Y^2-2\cdot X\cdot Y}} = 0$$

$$AF - \frac{(X+Y)^2}{2\cdot X\cdot Y} = 0 \quad AB - \frac{(X-Y)^2}{2\cdot X\cdot Y} = 0 \quad AC - \frac{(X-Y)^2\cdot\left(X+Y+\sqrt{Y^2-X^2}\right)}{2\cdot X\cdot Y\cdot\left(Y-X+\sqrt{Y^2-X^2}\right)} = 0$$




Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000
BF = 10.72067 cm
AB = 4.37761 cm
BC = 1.58807 cm
CD = 2.16417 cm
DE = 2.94926 cm
EF = 4.01917 cm
AF = 15.09827 cm

BF = 10.72067 cm
CF = 9.13260 cm
DF = 6.96843 cm
TF = 6.25594 cm
FG = 5.36033 cm

100695C

**Given.**

$X := 9$

$Y := 2$

**Unit.**

$AB := \frac{Y}{Y}$

**Descriptions.**

$AF := \frac{X}{Y}$ $BF := AF - AB$ $BO := \frac{BF}{2}$ $AD := \sqrt{AF}$

$BD := AD - AB$ $DF := BF - BD$ $DG := \sqrt{BD \cdot DF}$

$HJ := 2 \cdot DG$ $BJ := \sqrt{(BD + DG)^2 + DG^2}$

$JK := \frac{(BD + DG) \cdot HJ}{BJ}$ $BK := BJ - JK$ $BC := \frac{HJ \cdot BK}{JK}$

$AC := AB + BC$ $FH := \sqrt{(DF + DG)^2 + DG^2}$

$HP := \frac{(DF + DG) \cdot HJ}{FH}$ $FP := FH - HP$ $EF := \frac{HJ \cdot FP}{HP}$

$AE := AF - EF$ $AF^{\frac{1}{4}} - AC = 0$ $AF^{\frac{2}{4}} - AD = 0$ $AF^{\frac{3}{4}} - AE = 0$

**Definitions.**

$AF - \frac{X}{Y} = 0$ $BF - \frac{X - Y}{Y} = 0$ $BO - \frac{X - Y}{2 \cdot Y} = 0$ $AD - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

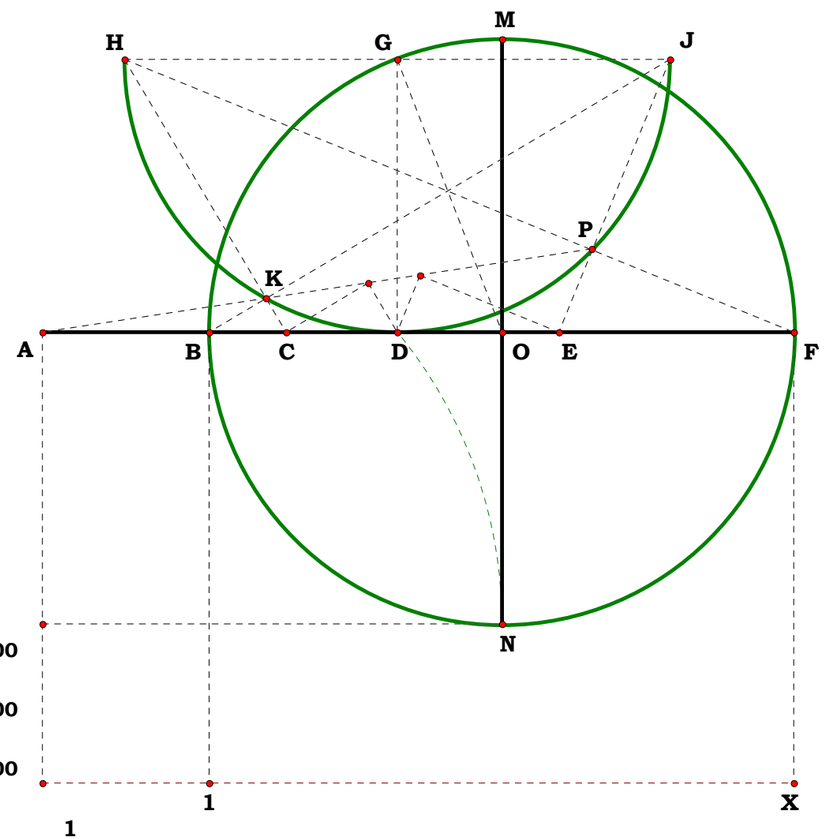
$BD - \frac{\sqrt{X} - \sqrt{Y}}{\sqrt{Y}} = 0$ $DF - \frac{\sqrt{X} \cdot (\sqrt{X} - \sqrt{Y})}{Y} = 0$ $DG - \frac{X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y})}{(Y^3)^{\frac{1}{4}}} = 0$

$BJ - \frac{\sqrt{(\sqrt{X} - \sqrt{Y})^2 \cdot \left[2 \cdot \sqrt{X} \cdot Y + \sqrt{Y^3} + 2 \cdot X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot (Y^3)^{\frac{1}{4}}\right]}}{\sqrt{Y \cdot \sqrt{Y^3}}} = 0$

## Finding the Unit for a Figure.

**Supplement to 100695**

**Mathcad 15 is not able to reduce these equations to the final powers of $\frac{X}{Y}$, therefore one is going to have to do that manually.**




Unit = 1.00000

XY = 4.50000

X = 9.00000

Y = 2.00000

AB = 1.00000

AC = 1.45648

AD = 2.12132

AE = 3.08965

AF = 4.50000

$AC^4$ = 4.50000

$AD^{\frac{4}{2}}$ = 4.50000

$AE^{\frac{4}{3}}$ = 4.50000

$AF^{\frac{1}{4}} - AC$ = 0.00000

$AF^{\frac{2}{4}} - AD$ = 0.00000

$AF^{\frac{3}{4}} - AE$ = 0.00000

$HJ - \frac{2 \cdot X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y})}{(Y^3)^{\frac{1}{4}}} = 0$

$JK - \frac{2 \cdot X^{\frac{1}{4}} \cdot (\sqrt{X} - \sqrt{Y}) \cdot \sqrt{Y \cdot \sqrt{Y^3}} \cdot (\sqrt{X} - \sqrt{Y}) \cdot \left[X^{\frac{1}{4}} \cdot \sqrt{Y} + (Y^3)^{\frac{1}{4}}\right]}{\sqrt{Y^3} \cdot \sqrt{(\sqrt{X} - \sqrt{Y})^2 \cdot \left[2 \cdot \sqrt{X} \cdot Y + \sqrt{Y^3} + 2 \cdot X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot (Y^3)^{\frac{1}{4}}\right]} \cdot \sqrt{Y}} = 0$

$$BK - \frac{Y^3 \cdot \left(\sqrt{X} - \sqrt{Y}\right)^2}{\sqrt{Y^3} \cdot \sqrt{Y \cdot \sqrt{Y^3}} \cdot \sqrt{\left(\sqrt{X} - \sqrt{Y}\right)^2 \cdot \left[2 \cdot \sqrt{X} \cdot Y + \sqrt{Y^3} + 2 \cdot X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot \left(Y^3\right)^{\frac{1}{4}}\right]}} = 0 \qquad BC - \frac{Y^{\frac{5}{2}} \cdot \left(\sqrt{X} - \sqrt{Y}\right)}{\left(Y^3\right)^{\frac{3}{4}} \cdot \left[X^{\frac{1}{4}} \cdot \sqrt{Y} + \left(Y^3\right)^{\frac{1}{4}}\right]} = 0 \qquad AC - \frac{X^{\frac{1}{4}} \cdot \sqrt{Y} \cdot \left[X^{\frac{1}{4}} \cdot Y^2 + \left(Y^3\right)^{\frac{3}{4}}\right]}{\left(Y^3\right)^{\frac{3}{4}} \cdot \left[X^{\frac{1}{4}} \cdot \sqrt{Y} + \left(Y^3\right)^{\frac{1}{4}}\right]} = 0 \qquad AC - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$$

$$FH - \frac{X^{\frac{1}{4}} \cdot \sqrt{X^{\frac{3}{2}} \cdot \sqrt{Y^3} + 2 \cdot X \cdot Y^2 + 2 \cdot Y^3 - 4 \cdot \sqrt{X} \cdot Y^{\frac{5}{2}} - 2 \cdot X \cdot \sqrt{Y} \cdot \sqrt{Y^3} + \sqrt{X} \cdot Y \cdot \sqrt{Y^3} + 2 \cdot X^{\frac{5}{4}} \cdot Y \cdot \left(Y^3\right)^{\frac{1}{4}} + 2 \cdot X^{\frac{1}{4}} \cdot Y^2 \cdot \left(Y^3\right)^{\frac{1}{4}} - 4 \cdot X^{\frac{3}{4}} \cdot Y^{\frac{3}{2}} \cdot \left(Y^3\right)^{\frac{1}{4}}}}{Y \cdot \left(Y^3\right)^{\frac{1}{4}}} = 0$$

$$HP - \frac{2 \cdot X^{\frac{1}{4}} \cdot \left(\sqrt{X} - \sqrt{Y}\right)^2 \cdot \left[Y + X^{\frac{1}{4}} \cdot \left(Y^3\right)^{\frac{1}{4}}\right]}{\left(Y^3\right)^{\frac{1}{4}} \cdot \sqrt{X^{\frac{3}{2}} \cdot \sqrt{Y^3} + 2 \cdot X \cdot Y^2 + 2 \cdot Y^3 - 4 \cdot \sqrt{X} \cdot Y^{\frac{5}{2}} - 2 \cdot X \cdot \sqrt{Y} \cdot \sqrt{Y^3} + \sqrt{X} \cdot Y \cdot \sqrt{Y^3} + 2 \cdot X^{\frac{5}{4}} \cdot Y \cdot \left(Y^3\right)^{\frac{1}{4}} + 2 \cdot X^{\frac{1}{4}} \cdot Y^2 \cdot \left(Y^3\right)^{\frac{1}{4}} - 4 \cdot X^{\frac{3}{4}} \cdot Y^{\frac{3}{2}} \cdot \left(Y^3\right)^{\frac{1}{4}}}} = 0$$

$$FP - \frac{X^{\frac{3}{4}} \cdot \left(\sqrt{X} - \sqrt{Y}\right)^2 \cdot \left(Y^3\right)^{\frac{1}{4}}}{Y \cdot \sqrt{X^{\frac{3}{2}} \cdot \sqrt{Y^3} + 2 \cdot X \cdot Y^2 + 2 \cdot Y^3 - 4 \cdot \sqrt{X} \cdot Y^{\frac{5}{2}} - 2 \cdot X \cdot \sqrt{Y} \cdot \sqrt{Y^3} + \sqrt{X} \cdot Y \cdot \sqrt{Y^3} + 2 \cdot X^{\frac{5}{4}} \cdot Y \cdot \left(Y^3\right)^{\frac{1}{4}} + 2 \cdot X^{\frac{1}{4}} \cdot Y^2 \cdot \left(Y^3\right)^{\frac{1}{4}} - 4 \cdot X^{\frac{3}{4}} \cdot Y^{\frac{3}{2}} \cdot \left(Y^3\right)^{\frac{1}{4}}}} = 0 \qquad EF - \frac{X^{\frac{3}{4}} \cdot \left(\sqrt{X} - \sqrt{Y}\right) \cdot \left(Y^3\right)^{\frac{1}{4}}}{Y \cdot \left[Y + X^{\frac{1}{4}} \cdot \left(Y^3\right)^{\frac{1}{4}}\right]} = 0$$

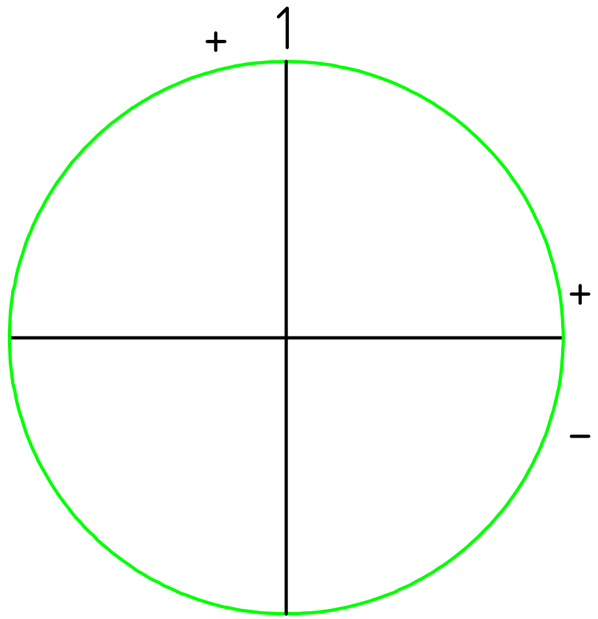
$$AE - \frac{\left(X^{\frac{1}{4}}\right)^3 \cdot \left[X^{\frac{1}{4}} \cdot \sqrt{Y} + \left(Y^3\right)^{\frac{1}{4}}\right]}{\sqrt{Y} \cdot \left[Y + X^{\frac{1}{4}} \cdot \left(Y^3\right)^{\frac{1}{4}}\right]} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{3}{4}} = 0$$

**011295A**

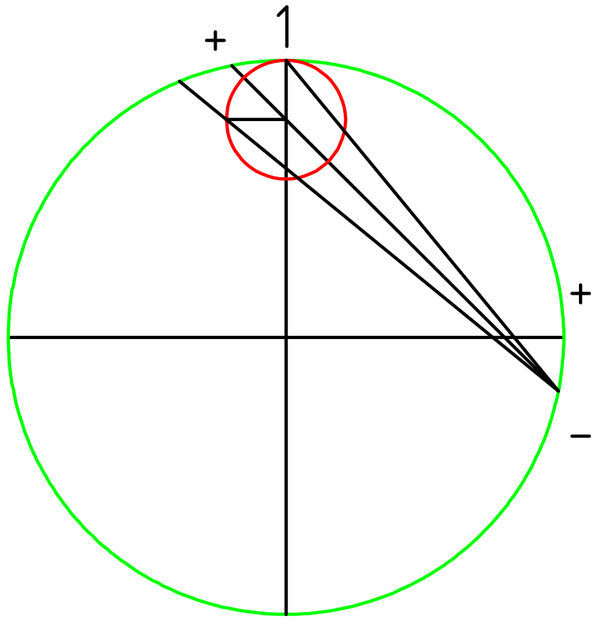
## Archimedean Trisection Revisited.

**I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.**

**I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.**




**The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have the si $\frac{90}{8} = 11.25$ my starting angle of will be 90 degrees.**




$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$

$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

$$8 + 1 - 1 = 8 \qquad 8 \cdot 11.25 = 90$$

$$8 + 1 - 1 - 2 = 6 \qquad 6 \cdot 11.25 = 67.5$$

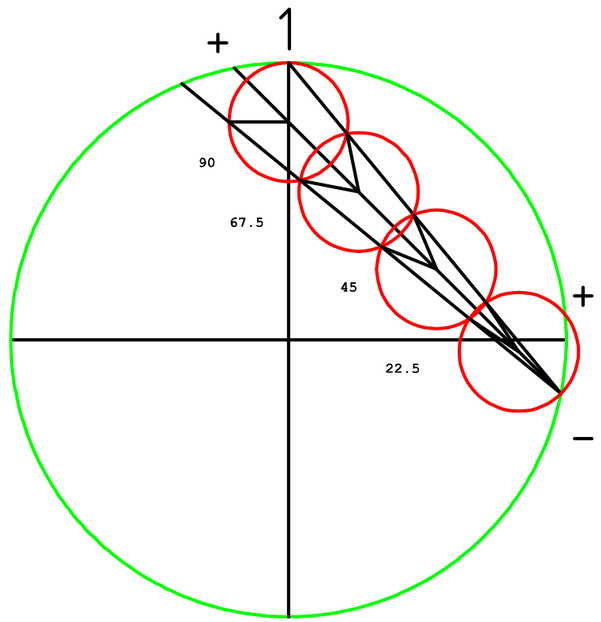
$$8 + 1 - 1 - 2 - 2 = 4 \qquad 4 \cdot 11.25 = 45$$

$$8 + 1 - 1 - 2 - 2 - 2 = 2 \qquad 2 \cdot 11.25 = 22.5$$

$$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$$

$$\mathbf{mod}(8 + 1 - 1, 2) = 0$$

**I have added another plus to a quadrant at the bottom of the figure.**




$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

$$8 + 1 + 1 - 1 = 9 \qquad 9 \cdot 11.25 = 101.25$$

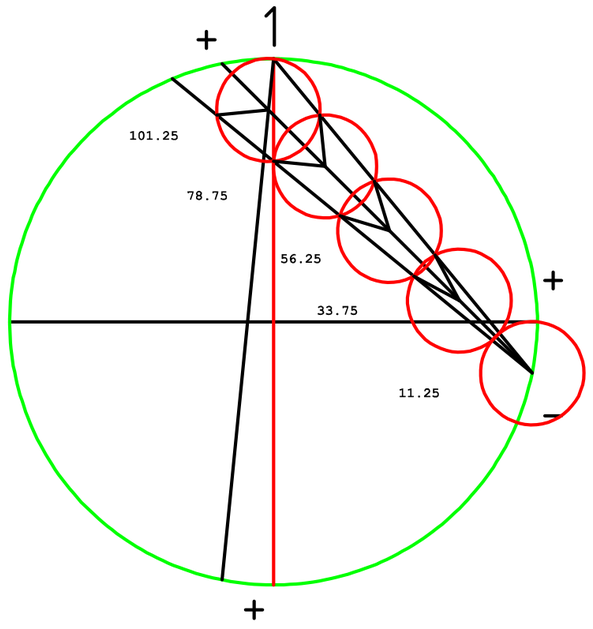
$$8 + 1 + 1 - 1 - 2 = 7 \qquad 7 \cdot 11.25 = 78.75$$

$$8 + 1 + 1 - 1 - 2 - 2 = 5 \qquad 5 \cdot 11.25 = 56.25$$

$$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3 \qquad 3 \cdot 11.25 = 33.75$$

$$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1 \qquad 1 \cdot 11.25 = 11.25$$

$$\mathbf{mod}(8 + 1 + 1 - 1, 2) = 1$$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $\quad B = 1.125 \quad \frac{9}{8} = 1.125$

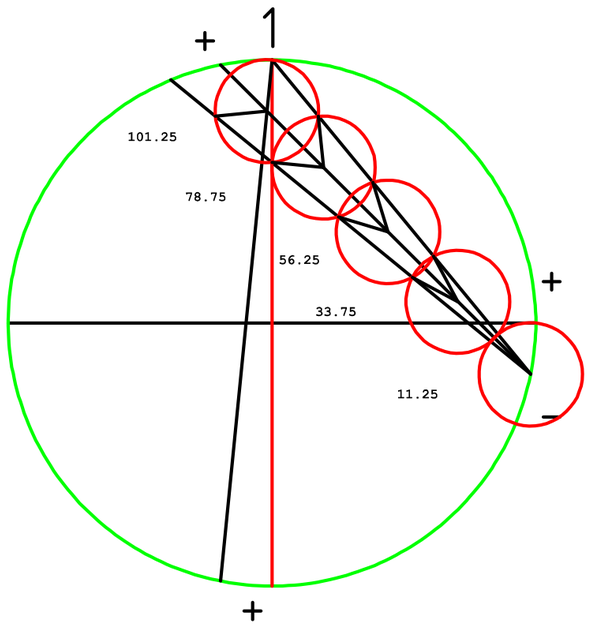
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\quad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\quad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$



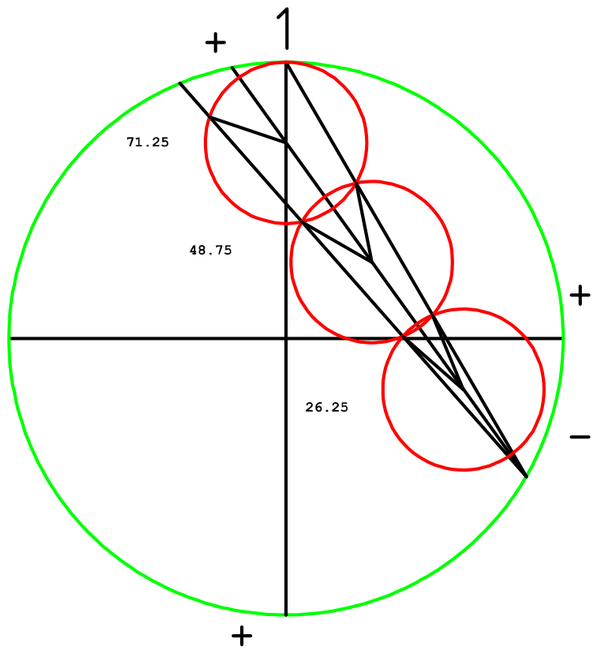

$B := 1 + \frac{3}{24} - \frac{8}{24} = 0.791667$ $\quad \frac{19}{24} = 0.791667$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\quad \frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.749526$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.249905$ $\quad \frac{B \cdot .166666}{3.166666} \cdot 90 = 3.749986$

| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$



$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8} \qquad B = 2.25 \qquad \frac{18}{8} = 2.25$$

$$\frac{B \cdot 9}{9} \cdot 90 = 202.5 \qquad \frac{B \cdot 8}{9} \cdot 90 = 180$$

$$\frac{B \cdot 7}{9} \cdot 90 = 157.5 \qquad \frac{B \cdot .6}{9} \cdot 90 = 13.5$$

$8 + 1 - 1 + 10 = 18 \qquad 18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16 \quad 16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14 \quad 14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12 \quad 12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10 \quad 10 \cdot 11.25 = 112.5$

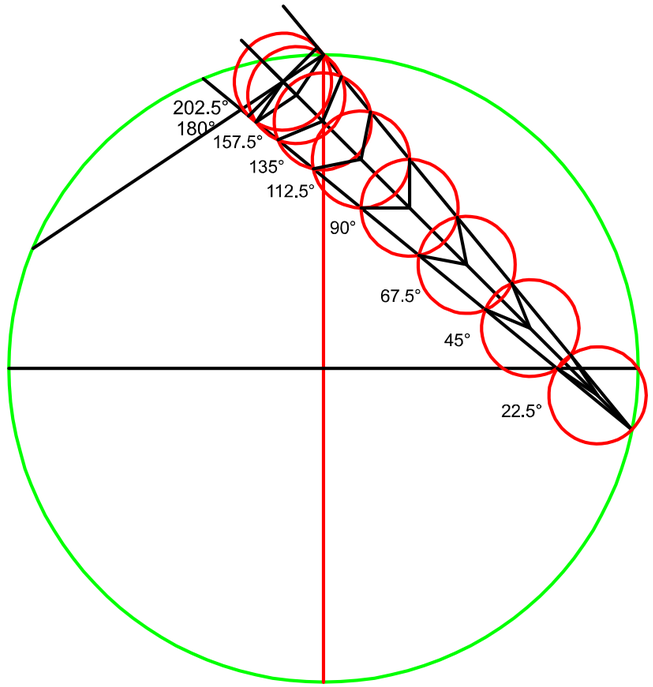
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8 \quad 8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6 \quad 6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4 \quad 4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2 \qquad 2 \cdot 11.25 = 22.5$

$$\mathbf{mod}[(8 + 1 - 1) + 10, 2] = 0$$




$$B := 1 + \frac{1}{7} - \frac{2}{7} \qquad B = 0.857143 \qquad \frac{6}{7} = 0.857143$$

$$\frac{B \cdot 6}{6} \cdot 90 = 77.142857 \qquad \frac{B \cdot 4}{6} \cdot 90 = 51.428571$$

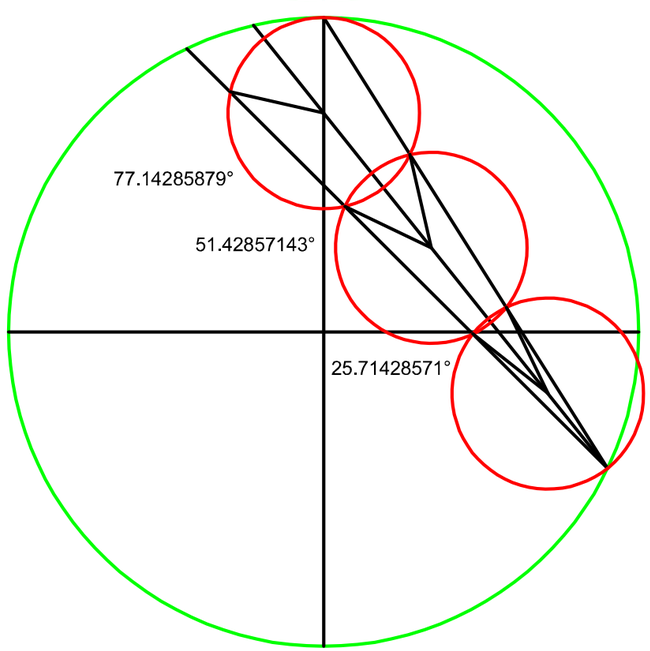
$$\frac{B \cdot 2}{6} \cdot 90 = 25.714286 \qquad c := \frac{90}{7}$$

$7 + 1 - (1 \cdot 2) = 6 \qquad 6 \cdot c = 77.142857$

$7 + 1 - (2 \cdot 2) = 4 \qquad 4 \cdot c = 51.428571$

$7 + 1 - (3 \cdot 2) = 2 \qquad 2 \cdot c = 25.714286$

$\mathbf{mod}(7 + 1 - 2, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.285714$

$\frac{B \cdot 3}{7} \cdot 90 = 38.571429$ $\frac{B \cdot 1}{7} \cdot 90 = 12.857143$

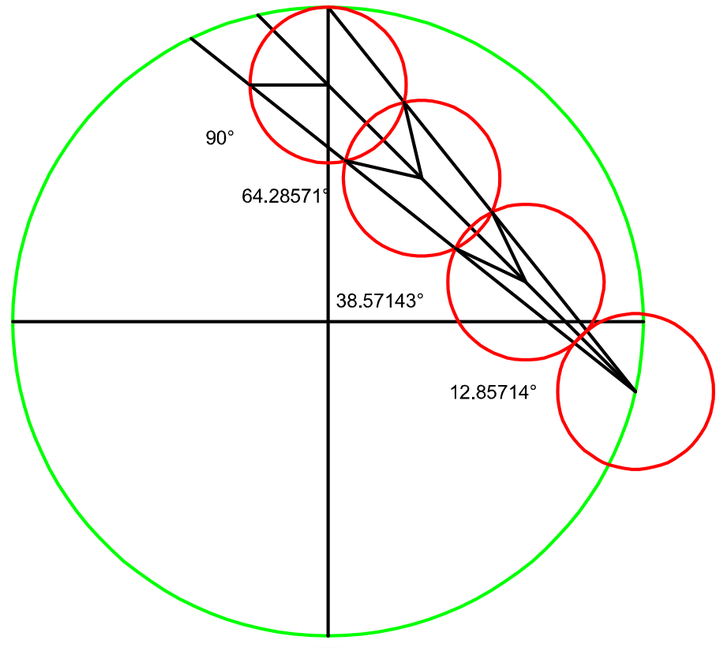
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.285714$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.571429$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.857143$

$\mathbf{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.017857$

$\frac{B \cdot 57}{57} \cdot 90 = 91.607143$ $\frac{B \cdot 41}{57} \cdot 90 = 65.892857$

$\frac{B \cdot 25}{57} \cdot 90 = 40.178571$ $c := \frac{90}{56}$

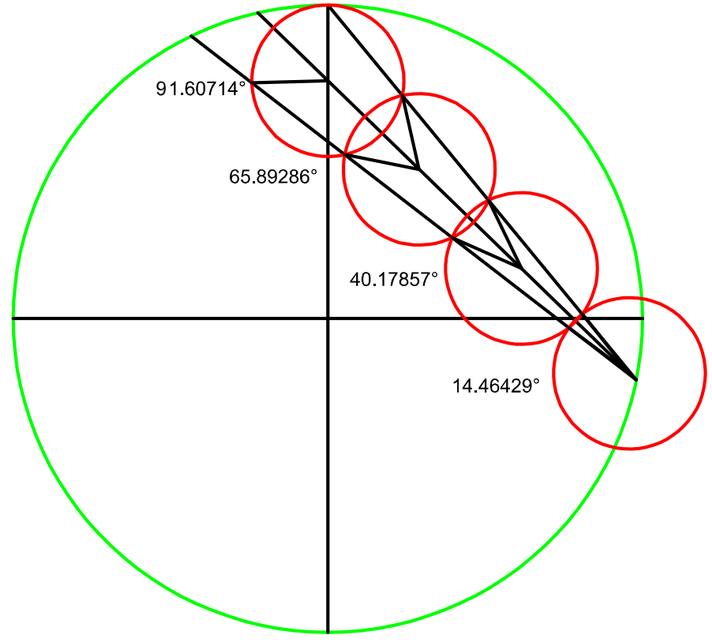
$56 + 8 - 7 = 57$ $57 \cdot c = 91.607143$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.892857$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.178571$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.464286$

$\mathbf{mod}(56 + 8 - 7, 16) = 9$



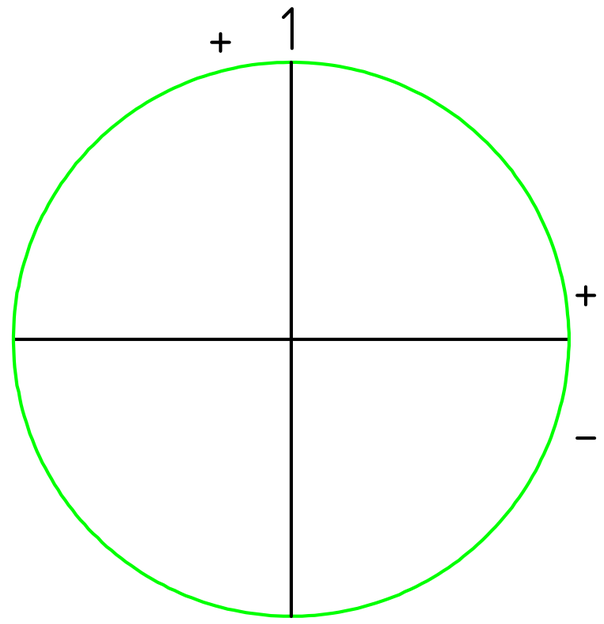
$$B := 1 + \frac{1}{7} - \frac{1}{7} \qquad B = 1$$

$$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.285714$$

$$\frac{B \cdot 3}{7} \cdot 90 = 38.571429 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.857143$$

$$7 + 1 - 1 = 7$$
$$7 + 1 - 1 - (1 \cdot 2) = 5$$
$$7 + 1 - 1 - (2 \cdot 2) = 3$$
$$7 + 1 - 1 - 2 - 2 - 2 = 1$$

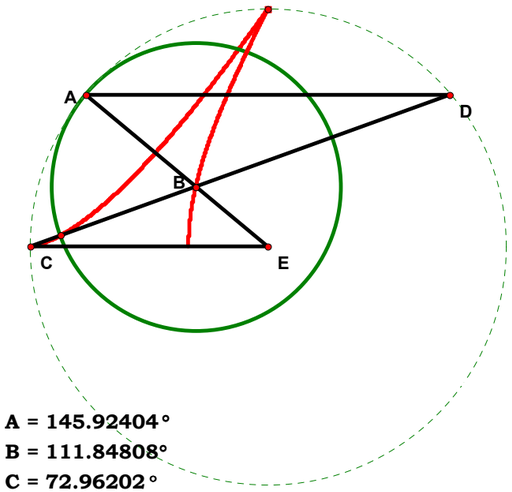
$$\mathrm{mod}(7 + 1 - 1, 2) = 1$$



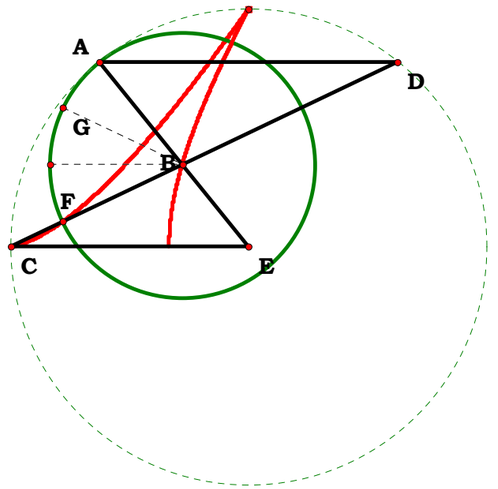
011295B

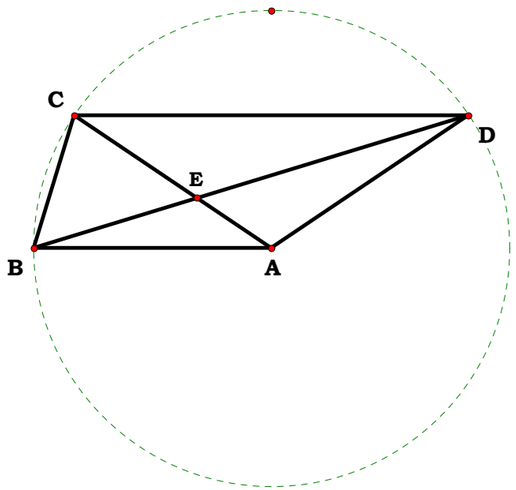
# Archimedean Trisection Revisited.

m∠ABC = 59.71415°
m∠ADC = 19.90472°
$\frac{m\angle ABC}{m\angle ADC}$ = 3.00000
m∠BCE = 19.90472°




m∠ABC = 76.48479°
m∠ADC = 25.49493°
$\frac{m\angle ABC}{m\angle ADC}$ = 3.00000
m∠BCE = 25.49493°
m∠ABF = 76.48479°
m∠ABG = 25.49493°
$\frac{m\angle ABF}{m\angle ABG}$ = 3.00000




m∠BAD = 145.92404°
m∠CAD = 111.84808°
m∠ABC = 72.96202°
m∠ABD = 17.03798°
m∠BCA = 72.96202°
m∠BCD = 107.03798°
m∠ADB = 17.03798°
m∠ADC = 34.07596°

A = 145.92404°
B = 111.84808°
C = 72.96202°
D = 17.03798°

F = 107.03798°

H = 34.07596°

E = 72.96202°

G = 17.03798°

m∠AEB = 128.88606°
m∠BEC = 51.11394°
m∠CED = 128.88606°
m∠DEA = 51.11394°

M = 128.88606°
N = 51.11394°

J = 128.88606°
K = 51.11394°

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\frac{A}{B}$ = 1.30466 | $\frac{B}{A}$ = 0.76648 | $\frac{C}{A}$ = 0.50000 | $\frac{D}{A}$ = 0.11676 | $\frac{F}{A}$ = 0.73352 | $\frac{H}{A}$ = 0.23352 | $\frac{M}{A}$ = 0.88324 | $\frac{N}{A}$ = 0.35028 |
| $\frac{A}{C}$ = 2.00000 | $\frac{B}{C}$ = 1.53296 | $\frac{C}{B}$ = 0.65233 | $\frac{D}{B}$ = 0.15233 | $\frac{F}{B}$ = 0.95699 | $\frac{H}{B}$ = 0.30466 | $\frac{M}{B}$ = 1.15233 | $\frac{N}{B}$ = 0.45699 |
| $\frac{A}{D}$ = 8.56463 | $\frac{B}{D}$ = 6.56463 | $\frac{C}{D}$ = 4.28232 | $\frac{D}{C}$ = 0.23352 | $\frac{F}{C}$ = 1.46704 | $\frac{H}{C}$ = 0.46704 | $\frac{M}{C}$ = 1.76648 | $\frac{N}{C}$ = 0.70056 |
| $\frac{A}{F}$ = 1.36329 | $\frac{B}{F}$ = 1.04494 | $\frac{C}{F}$ = 0.68165 | $\frac{D}{F}$ = 0.15918 | $\frac{F}{D}$ = 6.28232 | $\frac{H}{D}$ = 2.00000 | $\frac{M}{D}$ = 7.56463 | $\frac{N}{D}$ = 3.00000 |
| $\frac{A}{H}$ = 4.28232 | $\frac{B}{H}$ = 3.28232 | $\frac{C}{H}$ = 2.14116 | $\frac{D}{H}$ = 0.50000 | $\frac{F}{H}$ = 3.14116 | $\frac{H}{F}$ = 0.31835 | $\frac{M}{F}$ = 1.20412 | $\frac{N}{F}$ = 0.47753 |
| $\frac{A}{M}$ = 1.13219 | $\frac{B}{M}$ = 0.86781 | $\frac{C}{M}$ = 0.56610 | $\frac{D}{M}$ = 0.13219 | $\frac{F}{M}$ = 0.83049 | $\frac{H}{M}$ = 0.26439 | $\frac{M}{H}$ = 3.78232 | $\frac{N}{H}$ = 1.50000 |
| $\frac{A}{N}$ = 2.85488 | $\frac{B}{N}$ = 2.18821 | $\frac{C}{N}$ = 1.42744 | $\frac{D}{N}$ = 0.33333 | $\frac{F}{N}$ = 2.09411 | $\frac{H}{N}$ = 0.66667 | $\frac{M}{N}$ = 2.52154 | $\frac{N}{M}$ = 0.39658 |

040195A

Descriptions.

Given.

$N_1 := 5$

$N_2 := 12$

$\delta := 0 .. 2$

$AG := N_2 \quad AD_0 := N_1 \quad AN := AD_0$

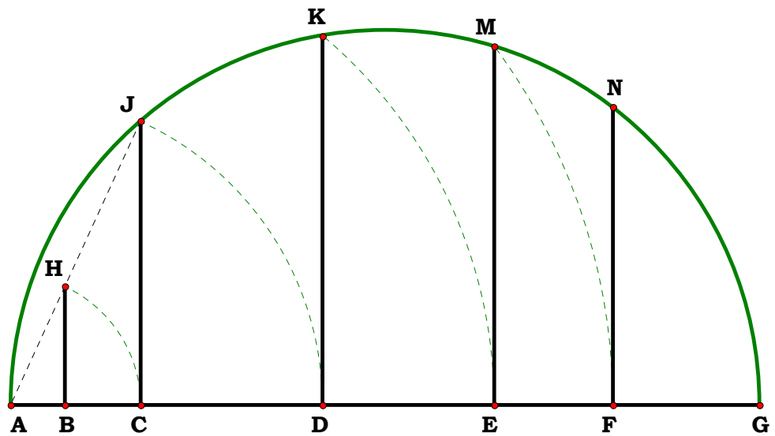
$AJ_1 := AD_0 \quad DF_0 := AG - AD_0$

$DO_0 := \sqrt{AD_0 \cdot DF_0} \quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AG - AO_\delta \\ \sqrt{AO_\delta \cdot (AG - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AG - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

## Exponential Series-Roots and Powers

**I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.**




Unit = 1.00000
XY = 0.41667
X = 5.00000
Y = 12.00000

AB = 0.07234
AC = 0.17361
AD = 0.41667
AE = 0.64550
AF = 0.80343

$\frac{Unit}{AB} = 13.82400$

$\frac{Unit}{AC} = 5.76000$

$\frac{Unit}{AD} = 2.40000$

$\frac{Unit}{AE} = 1.54919$

$\frac{Unit}{AF} = 1.24467$

Definitions.

$$\sum_\delta \left[ \frac{AG}{AD_\delta} - \left( \frac{N_2}{N_1} \right)^{\frac{1}{2^\delta}} \right] = 0$$

| $\left(\frac{N_2}{N_1}\right)^{\delta+1}$ |
|---|
| 2.4 |
| 5.76 |
| 13.824 |

| $\left(\frac{N_2}{N_1}\right)^{\frac{1}{2^\delta}}$ |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

| $\frac{AG}{AD_\delta}$ |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

| $\left(\frac{N_2}{N_1}\right)^{\frac{1}{2^\delta}}$ |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

040195B

Descriptions.

Given.

$X := 5$

$Y := 12$

$\delta := 0 .. 2$

$AG := \frac{Y}{Y}$ $AD_0 := \frac{Y}{X}$ $AN := AD_0$

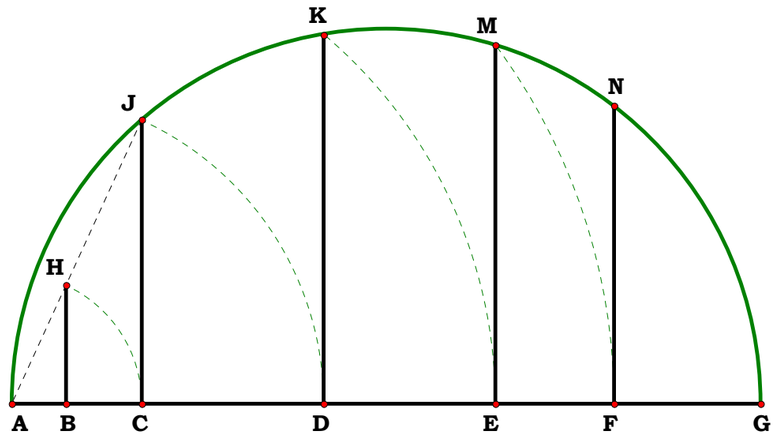
$AJ_1 := AD_0$ $DF_0 := AG - AD_0$

$DO_0 := \sqrt{AD_0 \cdot DF_0}$ $AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AG - AO_\delta \\ \sqrt{AO_\delta \cdot (AG - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AG - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

## Exponential Series-Roots and Powers

**I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.**




Unit = 1.00000
XY = 0.41667
X = 5.00000
Y = 12.00000

AB = 0.07234
AC = 0.17361
AD = 0.41667
AE = 0.64550
AF = 0.80343

$\frac{Unit}{AB} = 13.82400$

$\frac{Unit}{AC} = 5.76000$

$\frac{Unit}{AD} = 2.40000$

$\frac{Unit}{AE} = 1.54919$

$\frac{Unit}{AF} = 1.24467$

**Definitions.**

$$\sum_\delta \left[ AD_\delta - \left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} \right] = 0$$

$\left(\frac{Y}{X}\right)^{\delta+1} =$

| |
|---|
| 2.4 |
| 5.76 |
| 13.824 |

$\left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} =$

| |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

$AD_\delta =$

| |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

$\left(\frac{Y}{X}\right)^{\frac{1}{2^\delta}} =$

| |
|---|
| 2.4 |
| 1.549193 |
| 1.244666 |

$AE := 5.20700$

$Unit := AE$

Given.

$AB := 1.11300 \quad N_1 := AB$

091395A1

$AC := 2.02712 \quad N_2 := AC$

Given AE, AB, AC what is GH?

Descriptions.

$BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2}$

$GK = 4.27657 \qquad GH = 2.138285$

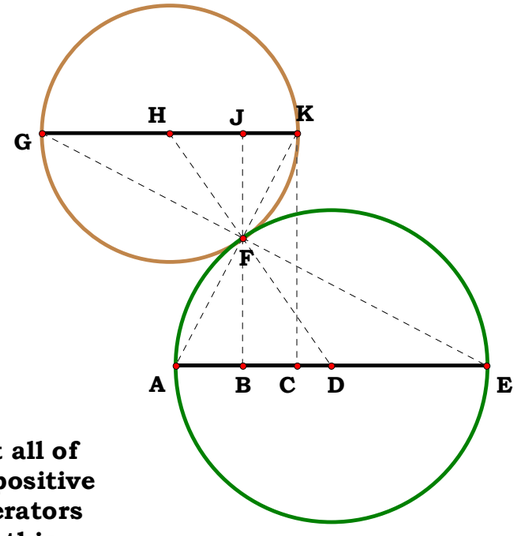
Definitions.

When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}} = 1$$

$$GH - \left[\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot \frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}}\right] = 0 \qquad \text{Combinaing we get:} \qquad GH - \frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} = 0$$

## A Study In Placement




$AE = 5.20700$ cm

$AB = 1.11300$ cm

$AC = 2.02712$ cm

$CK = 3.88781$ cm

$EF = 4.61708$ cm

$GK = 4.27653$ cm

$GH = 2.13827$ cm

$BC = 0.91412$ cm

$Unit = 5.20700$ cm

$N_1 = 1.11300$ cm

$N_2 = 2.02712$ cm

$AC - AB = 0.91412$ cm

$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$

$\frac{AE \cdot BC}{AB} - GK = 0.00000$ cm

$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}} = 1.00000$

Is GK outside side AE? = 1.00000

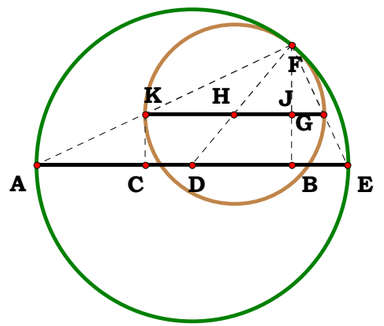
X = 1.00000

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} = 2.13827$ cm

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot X - GH = 0.00000$ cm

$\frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} - GH = 0.00000$ cm

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot \frac{(AC-AB)}{\sqrt{(AC-AB)^2}} - GH = 0.00000$ cm




$\frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} - GH = 0.00000$ cm

Given.

$W := 4$ $Y := 6$

$X := 20$ $Z := 17$

**091395A2**

**Given AE, AB, AC what is GH?**

**Descriptions.**

$AB := \frac{W}{X}$ $AC := \frac{Y}{Z}$

$BC := (AC - AB)$ $GK := \frac{BC}{AB}$ $GH := \frac{GK}{2}$

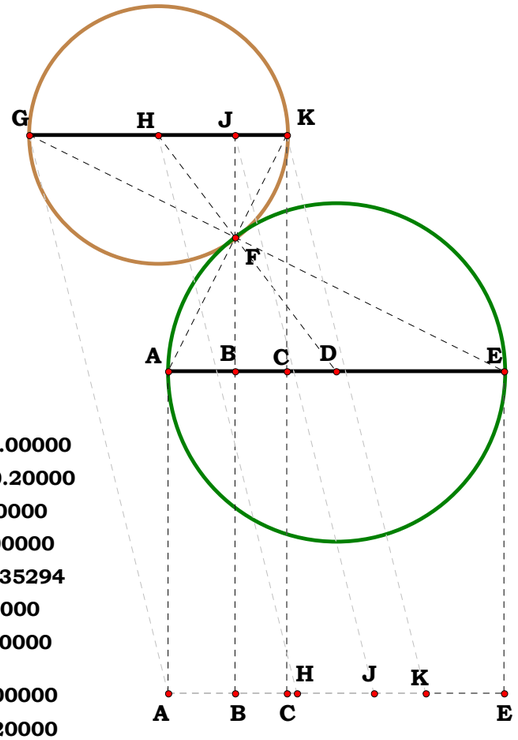
$GK = 0.764706$ $GH = 0.382353$

**Definitions.**

$AB - \frac{W}{X} = 0$ $AC - \frac{Y}{Z} = 0$ $BC - \frac{X \cdot Y - W \cdot Z}{X \cdot Z} = 0$

$GK - \frac{X \cdot Y - W \cdot Z}{W \cdot Z} = 0$ $GH - \frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z} = 0$

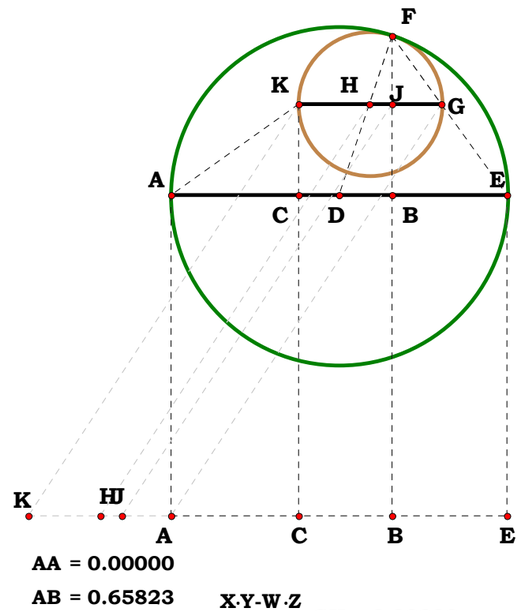
# A Study In Placement




Unit = 1.00000
W/X = 0.20000
W = 4.00000
X = 20.00000
Y/Z = 0.35294
Y = 6.00000
Z = 17.00000

AA = 0.00000
AB = 0.20000
AC = 0.35294
GH = 0.38235
GJ = 0.61176
GK = 0.76471
AE = 1.00000

$\frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z}$-GH = 0.00000




AA = 0.00000
AB = 0.65823
AC = 0.37923
GH = -0.21193
GJ = -0.14486
GK = -0.42386
AE = 1.00000

$\frac{X \cdot Y - W \cdot Z}{2 \cdot W \cdot Z}$-GH = 0.00000

Unit = 1.00000 Y/Z = 0.37923
W/X = 0.65823 Y = 6.44696
W = 13.16452 Z = 17.00000
X = 20.00000

$AE := 5.20700$

$Unit := AE$

Given.

$EF := 4.58113 \quad N_1 := EF$

091395B1

$AC := 1.96362 \quad N_2 := AC$

Given AE, EF AC what is GH?

Descriptions.

$AB := AE - \frac{EF^2}{AE} \qquad BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB}$

$GH := \frac{GK}{2} \qquad GK = 3.48358 \qquad GH = 1.74179$

Definitions.

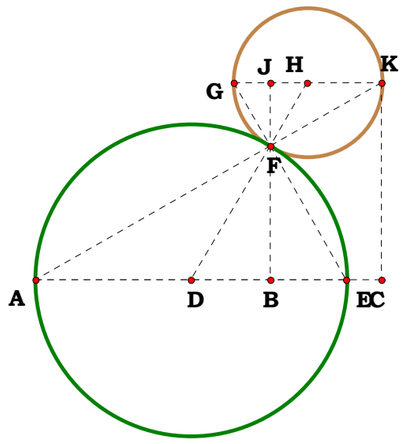
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}} = 1$$

$$GH - \left[\frac{\left(N_2 \cdot Unit^2 - Unit^3 + Unit \cdot N_1^2\right)}{2 \cdot Unit^2 - 2 \cdot N_1^2} \cdot \frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}\right] = 0$$

$$GH - \frac{Unit \cdot \sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}{2 \cdot \left(Unit^2 - N_1^2\right)} = 0$$

## A Study In Placement




$AE = 5.20700$ cm

$AB = 3.92817$ cm

$AC = 5.79479$ cm

$CK = 3.30635$ cm

$EF = 2.58048$ cm

$GK = 2.47430$ cm

$GH = 1.23715$ cm

$BC = 1.86662$ cm

$\frac{Unit \cdot \sqrt{((N_1^2 - Unit^2) + N_2 \cdot Unit)^2}}{2 \cdot (Unit^2 - N_1^2)} - GH = 0.00000$ cm

$Unit = 5.20700$ cm

$N_1 = 2.58048$ cm

$N_2 = 5.79479$ cm

$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$

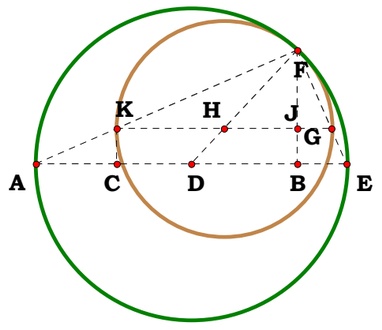
$\frac{AE \cdot BC}{AB} - GK = 0.00000$ cm

$AC - AB = 1.86662$ cm

$\frac{(AC - AB)}{\sqrt{(AC - AB)^2}} = 1.00000$

Is GK outside AE? = 1.00000

X = 1.00000




$\frac{Unit \cdot \sqrt{((N_1^2 - Unit^2) + N_2 \cdot Unit)^2}}{2 \cdot (Unit^2 - N_1^2)} - GH = 0.00000$ cm

Given.

$W := 6$ $Y := 14$

$X := 20$ $Z := 17$

$AE := 1$

091395B2

Given AE, EF AC what is GH?

Descriptions.

$AB := \frac{W}{X}$ $EF := AB$ $AD := AE - AB^2$ $AC := \frac{Y}{Z}$

$CD := AD - AC$ $DE := AE - AD$ $GK := \frac{AE \cdot CD}{DE}$

$GJ := \frac{GK}{2}$ $GJ = 0.480392$

Definitions.

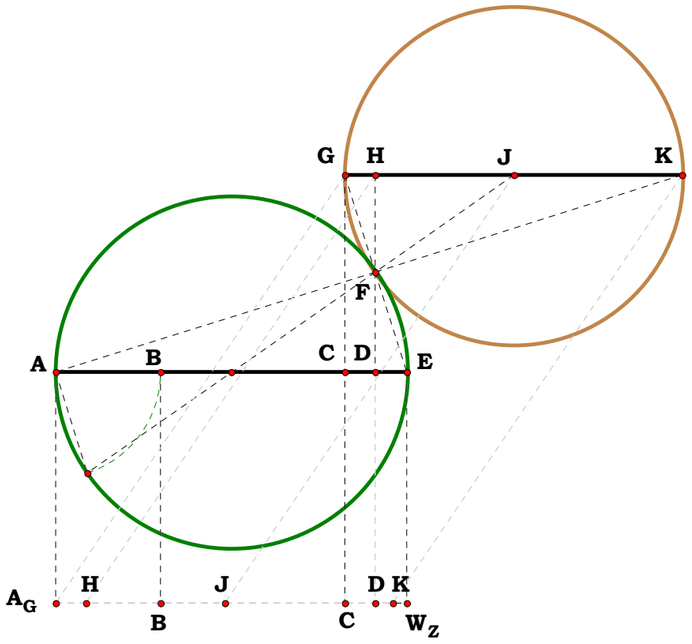
$AB - \frac{W}{X} = 0$ $EF - \frac{W}{X} = 0$ $AD := \frac{X^2 - W^2}{X^2}$

$AC - \frac{Y}{Z} = 0$ $CD - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{X^2 \cdot Z} = 0$

$DE - \frac{W^2}{X^2} = 0$ $GK - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{W^2 \cdot Z} = 0$

$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0$

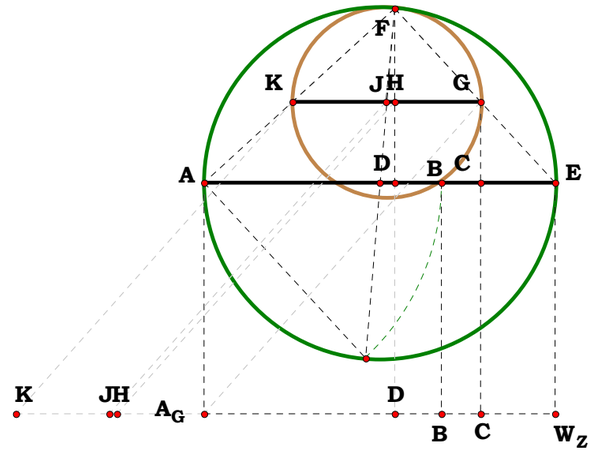
## A Study In Placement




Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.82353
Y = 14.00000
Z = 17.00000

$A_G$ = 0.00000
GH = 0.08647
AB = 0.30000
GJ = 0.48039
AC = 0.82353
GK = 0.96078
AD = 0.91000
$AW_Z$ = 1.00000

$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0.00000$




Unit = 1.00000
W/X = 0.67737
W = 13.54733
X = 20.00000
Y/Z = 0.78759
Y = 13.38903
Z = 17.00000

$A_G$ = 0.00000
GH = -0.24642
AB = 0.67737
GJ = -0.26853
AC = 0.78759
GK = -0.53706
AD = 0.54117
$AW_Z$ = 1.00000

$GJ - \frac{X^2 \cdot Z - X^2 \cdot Y - W^2 \cdot Z}{2 \cdot W^2 \cdot Z} = 0.00000$

$AE := 5.20700$

Unit := AE

Given.

$CK := 2.82631$ $N_1 := CK$

091395C1 $AC := 4.41895$ $N_2 := AC$

Given AE, CK, AC what is GH?

Descriptions.

$$CM := \frac{CK^2}{AC} \qquad AB := \frac{AC \cdot AE}{AC + CM} \qquad BC := AC - AB$$

$$GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2} \qquad GK = 1.019626 \qquad GH = 0.509813$$

Definitions.

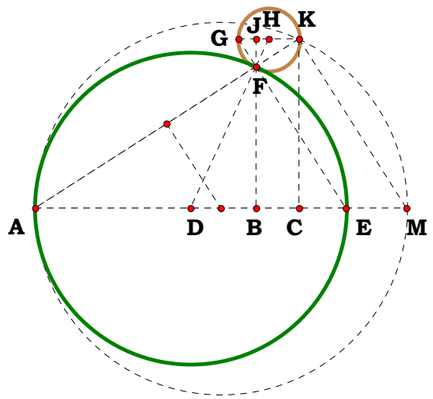
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)} = 1$$

$$GH - \left[\frac{N_1^2 + N_2^2 - Unit \cdot N_2}{2 \cdot N_2} \cdot \frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)}\right] = 0$$

$$GH - \frac{\left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}{2 \cdot \sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}} = 0$$

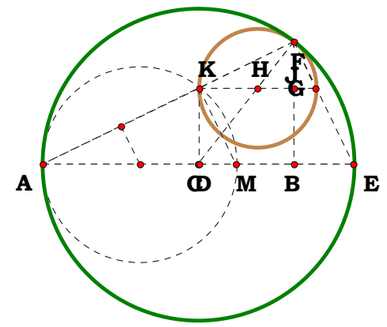
## A Study In Placement




AE = 5.20700 cm
AB = 3.69534 cm
AC = 4.41895 cm
CK = 2.82631 cm
EF = 2.80557 cm
GK = 1.01963 cm
GH = 0.50981 cm
BC = 0.72362 cm
CM = 1.80767 cm

Unit = 5.20700 cm
$N_1$ = 2.82631 cm
$N_2$ = 4.41895 cm
AC-AB = 0.72362 cm
$\frac{AE}{AB} - \frac{GK}{BC}$ = 0.00000
$\frac{AE \cdot BC}{AB}$ - GK = 0.00000 cm
$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}}$ = 1.00000
Is GK outside AE? = 1.00000
X = 1.00000

$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$




$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$

Combinaing we get: $\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)}$

And since MC cannot figure this out, we finish it by hand.

Given.

$W := 12$ $Y := 6$

$X := 10$ $Z := 8$

Unit.

$AE := 1$

**091395C2**

**Given AE, CK, AC what is GH?**

**Descriptions.**

$$CK := \frac{Y}{Z} \quad AC := \frac{W}{X} \quad CM := \frac{CK^2}{AC}$$

$$AB := \frac{AC \cdot AE}{AC + CM} \quad BC := AC - AB$$

$$GK := \frac{AE \cdot BC}{AB} \quad GH := \frac{GK}{2}$$

$AB = 0.719101$

$GK = 0.66875$ $GH = 0.334375$

**Definitions.**

$$CK - \frac{Y}{Z} = 0 \quad AC - \frac{W}{X} = 0$$

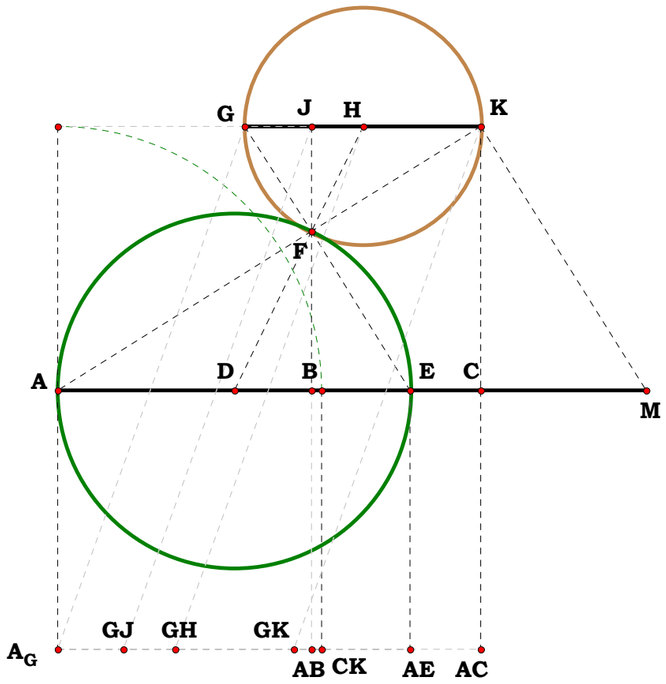
$$CM - \frac{X \cdot Y^2}{W \cdot Z^2} = 0 \quad AB - \frac{W^2 \cdot Z^2}{W^2 \cdot Z^2 + X^2 \cdot Y^2} = 0$$

$$BC := \frac{W \cdot \left(W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2\right)}{X \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}$$

$$GK - \frac{W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2}{W \cdot X \cdot Z^2} = 0$$

$$GH - \frac{W^2 \cdot Z^2 - W \cdot X \cdot Z^2 + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} = 0$$

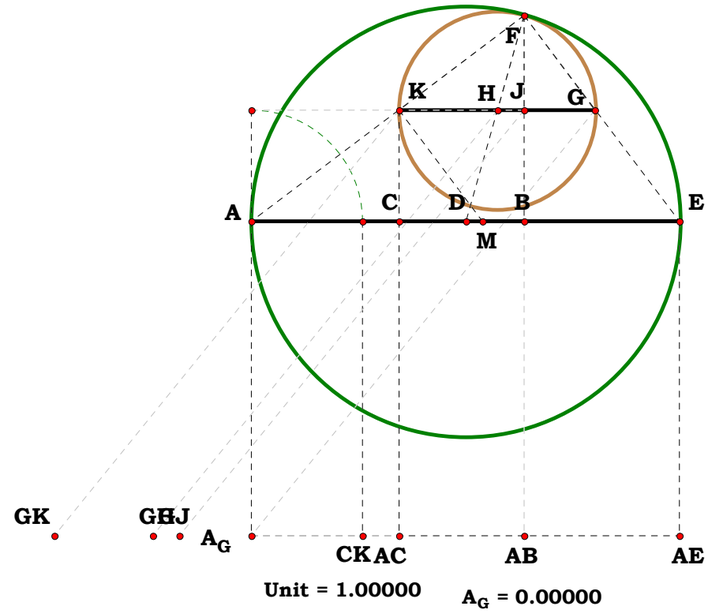
## A Study In Placement




Unit = 1.00000
W/X = 1.20000
W = 12.00000
X = 10.00000
Y/Z = 0.75000
Y = 6.00000
Z = 8.00000

$A_G = 0.00000$
GJ = 0.18785
GH = 0.33438
GK = 0.66875
AB = 0.71910
CK = 0.75000
AE = 1.00000
AC = 1.20000

$$\frac{(W^2 \cdot Z^2 - W \cdot X \cdot Z^2) + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} - GH = 0.00000$$




Unit = 1.00000
W/X = 0.34412
W = 3.44118
X = 10.00000
Y/Z = 0.26022
Y = 2.08175
Z = 8.00000

$A_G = 0.00000$
GJ = -0.16702
GH = -0.22955
GK = -0.45911
AB = 0.63620
CK = 0.26022
AE = 1.00000
AC = 0.34412

$$\frac{(W^2 \cdot Z^2 - W \cdot X \cdot Z^2) + X^2 \cdot Y^2}{2 \cdot W \cdot X \cdot Z^2} - GH = 0.00000$$

101495A

**Descriptions.**

**Unit.**

$AB := 1$

**Given.**

$N_1 := 5$

$N_2 := 4$

$AF := N_1 \quad AK := N_2 \quad BF := AF - AB$

$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

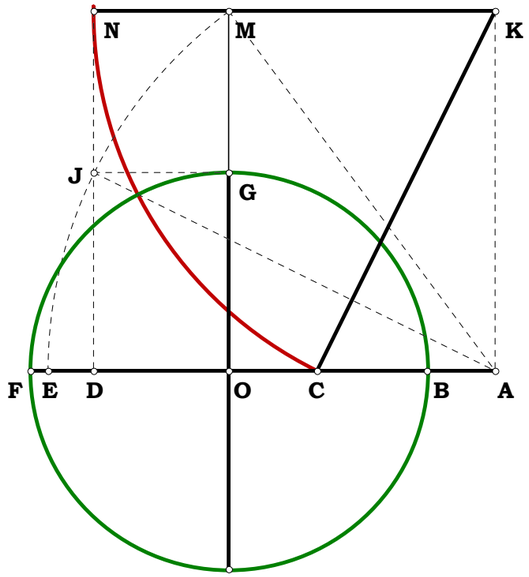
$\sqrt{AB \cdot AF} - AC = 0$

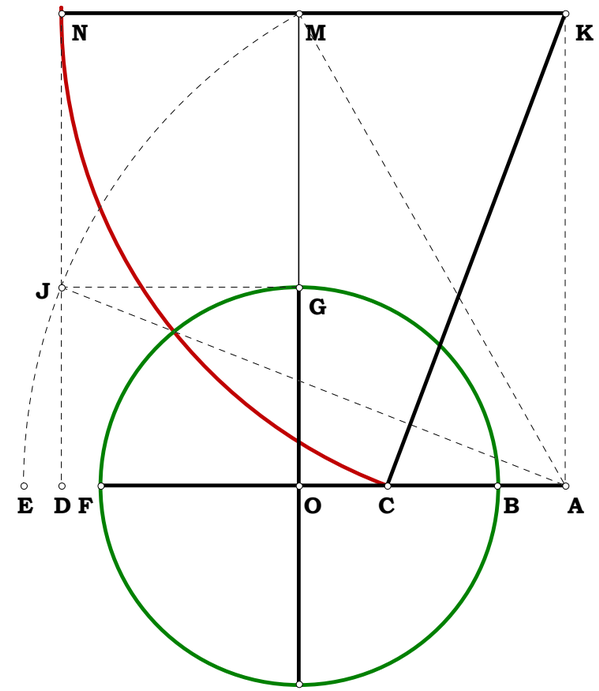
**Definitions.**

$AC - \sqrt{N_1} = 0$

## Alternate Method Square Root

**For any AK is AC the root of AB x AF?**



101495B

Given.

$W := 5$ $Y := 6$

$X := 20$ $Z := 6$

Unit.

$BF := 1$

Descriptions.

$AB := \frac{W}{X}$ $AF := AB + 1$ $AK := \frac{Y}{Z}$

$BO := \frac{BF}{2}$ $AO := AB + BO$

$KM := AO$ $AM := \sqrt{AK^2 + KM^2}$

$DJ := BO$ $AJ := AM$

$AD := \sqrt{AJ^2 - DJ^2}$ $CK := AD$

$AC := \sqrt{CK^2 - AK^2}$ $\sqrt{AB \cdot AF} - AC = 0$

Definitions.

$AB - \frac{W}{X} = 0$ $AF - \frac{W + X}{X} = 0$ $AK - \frac{Y}{Z} = 0$

$BO - \frac{1}{2} = 0$ $AO - \frac{2 \cdot W + X}{2 \cdot X} = 0$ $KM - \frac{2 \cdot W + X}{2 \cdot X} = 0$

$AM - \frac{\sqrt{4 \cdot W \cdot Z^2 \cdot (W + X) + X^2 \cdot (4 \cdot Y^2 + Z^2)}}{2 \cdot X \cdot Z} = 0$

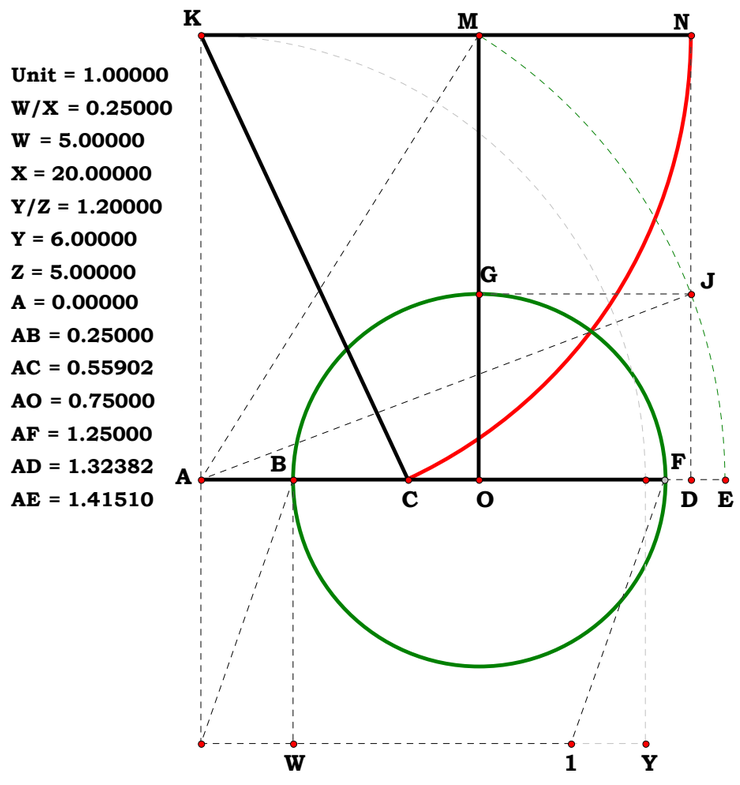
$DJ - \frac{1}{2} = 0$ $AJ - \frac{\sqrt{4 \cdot W \cdot Z^2 \cdot (W + X) + X^2 \cdot (4 \cdot Y^2 + Z^2)}}{2 \cdot X \cdot Z} = 0$

$AD - \frac{\sqrt{W \cdot Z^2 \cdot (W + X) + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $CK - \frac{\sqrt{W \cdot Z^2 \cdot (W + X) + X^2 \cdot Y^2}}{X \cdot Z} = 0$

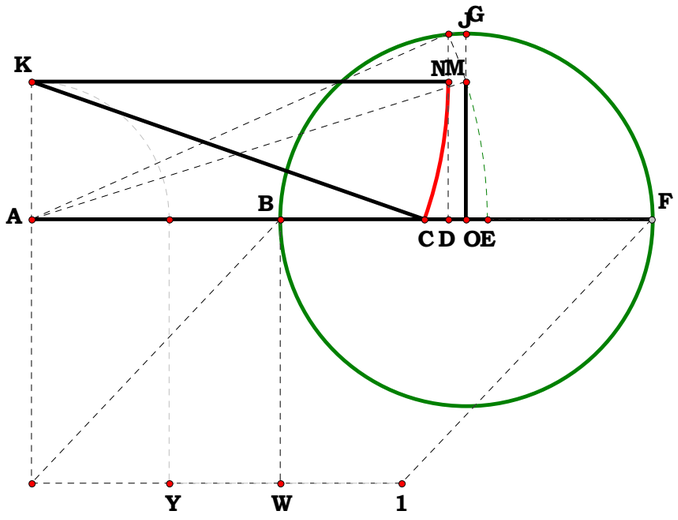
$AC := \frac{\sqrt{W \cdot (W + X)}}{X}$ $\frac{W \cdot (W + X)}{X} = 6.25$

## Alternate Method Square Root

**For any AK is AC the root of AB x AF?**




Unit = 1.00000
W/X = 0.25000
W = 5.00000
X = 20.00000
Y/Z = 1.20000
Y = 6.00000
Z = 5.00000
A = 0.00000
AB = 0.25000
AC = 0.55902
AO = 0.75000
AF = 1.25000
AD = 1.32382
AE = 1.41510




Unit = 1.00000
W/X = 0.67286
W = 13.45712
X = 20.00000
Y/Z = 0.37134
Y = 1.85669
Z = 5.00000
A = 0.00000
AB = 0.67286
AC = 1.06094
AO = 1.17286
AF = 1.67286
AD = 1.12405
AE = 1.23024

$\frac{\sqrt{W \cdot (W+X)}}{X}$-AC = 0.00000

**Y and Z have wholly disappeared out of the results, therefore, it does not make any difference what value given, as long as some value is given. It is structures such as this, equations such as this, which proves that the perceptible is not guaranteed by the grammatical result of a single grammar, in short, it does not creat a thing. One must pair a logic with an analogic.**

**Unit.**

$AE := 1$

**Given.**

$N_1 := 2$

**102095A**

**Descriptions.**

$$AB := \frac{AE}{N_1} \qquad BE := AE - AB \qquad BF := \sqrt{AB \cdot BE} \qquad AF := \sqrt{AB^2 + BF^2}$$

$$AC := AF \qquad CE := AE - AC \qquad EG := CE \qquad DE := \frac{EG^2}{AE}$$

$$BD := AE - (AB + DE) \qquad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1 \qquad AD := AE - DE \qquad DG := \sqrt{AD \cdot DE}$$

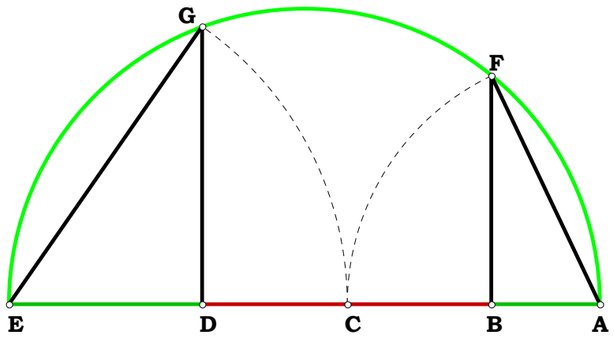
**Definitions.**

$$\frac{1}{N_1} - AB = 0 \qquad 1 - \frac{1}{N_1} - BE = 0 \qquad \sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0 \qquad \sqrt{\frac{N_1}{N_1^2}} - AF = 0$$

$$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0 \qquad 1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0 \qquad 2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$$

$$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0 \qquad \frac{\sqrt{4 \cdot \sqrt{N_1} - 5 \cdot N_1 + 2 \cdot N_1^{\frac{3}{2}} - 1}}{N_1} - DG = 0$$

**Four Times The Square**



**Given.**

$X := 4$

$Y := 20$

**Unit.**

$AE := 1$

**102095B**

**Descriptions.**

$$AB := \frac{X}{Y} \qquad BE := AE - AB \qquad BF := \sqrt{AB \cdot BE} \qquad AF := \sqrt{AB^2 + BF^2}$$

$$AC := AF \qquad CE := AE - AC \qquad EG := CE \qquad DE := \frac{EG^2}{AE}$$

$$BD := AE - (AB + DE) \qquad AD := AE - DE \qquad DG := \sqrt{AD \cdot DE}$$

$$\frac{BD^2}{AB \cdot DE} = 4 \qquad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$$

$AF = 0.447214$ $AD = 0.694427$

**Definitions.**

$$AB - \frac{X}{Y} = 0 \qquad BE - \frac{Y - X}{Y} = 0 \qquad BF - \frac{\sqrt{X \cdot (Y - X)}}{Y} = 0$$

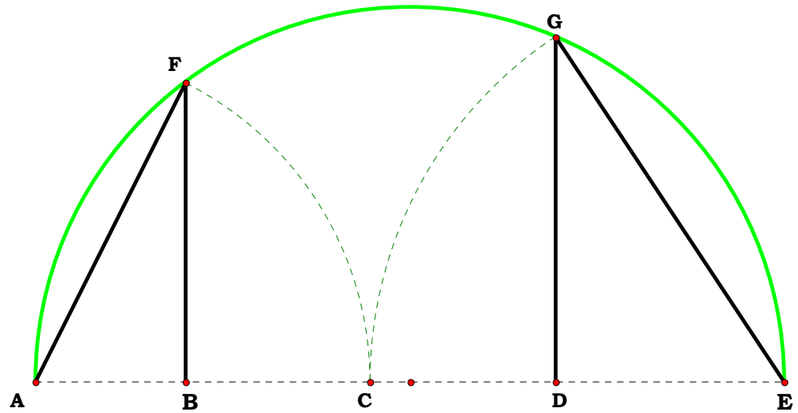
$$AF - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad AC - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad CE - \frac{\sqrt{Y} - \sqrt{X}}{\sqrt{Y}} = 0$$

$$EG - \frac{\sqrt{Y} - \sqrt{X}}{\sqrt{Y}} = 0 \qquad DE - \frac{\left(\sqrt{Y} - \sqrt{X}\right)^2}{Y} = 0$$

$$BD - \frac{2 \cdot \sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y} = 0 \qquad AD - \frac{\sqrt{X} \cdot \left(2 \cdot \sqrt{Y} - \sqrt{X}\right)}{Y} = 0$$

$$DG - \frac{X^{\frac{1}{4}} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \sqrt{2 \cdot \sqrt{Y} - \sqrt{X}}}{Y} = 0$$

## Four Times The Square




Unit = 1.00000 AB = 0.20000

XY = 0.20000 AC = 0.44721

X = 4.00000 AD = 0.69443

Y = 20.00000

110195A

Descriptions.

Unit.

$AG := 1$

Given.

$N_1 := 5$

$N_2 := 2$

## A Modification Of The Square Root Figure, Gemini Roots

**On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.**

**Although the final equation is wholly correct, the write-up, itself, is pure garbage. I remember doing this plate right once, but I no longer seem to have that write-up, so I did it again in the plate B. This errant write-up goes back at least as far as 2001. When I started to examine it, the faults it contains shortly after a few equations puzzles me. Maybe that is what happens when you work 12 hours a day, seven days a week for a long time. Well, I often get equations before I write them up, but rarely do so badly on the write-up as this is. Or, the write-up could be the result of a complete lapse in sanity by yours truly. At any rate, it stops being correct after the definition of EO. After EO, it makes no sense at all. Not even on the graphic for this is D a mobile point!. The working point is C, not D.**

$AE := \frac{AG}{2}$ $EG := AE$ $EF := \frac{AG}{2 \cdot N_1}$ $AF := AE + EF$

$FG := EG - EF$ $FN := \sqrt{AF \cdot FG}$ $GN := \sqrt{FN^2 + FG^2}$ $GK := GN$

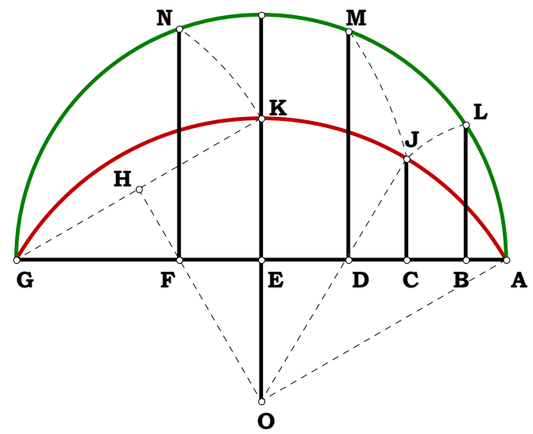
$EK := \sqrt{GK^2 - EG^2}$ $EO := \frac{EG \cdot EF}{EK}$ $OK := EO + EK$ $DE := \frac{AE}{N_2}$

$DO := \sqrt{DE^2 + EO^2}$ $DJ := OK - DO$ $CD := \frac{DE \cdot DJ}{DO}$ $CE := CD + DE$

$CJ := \frac{EO \cdot DJ}{DO}$ $AC := AE - CE$ $AJ := \sqrt{AC^2 + CJ^2}$ $AL := AJ$ $AB := \frac{AL^2}{AG}$

$CG := AG - AC$ $GJ := \sqrt{CG^2 + CJ^2}$ $GM := GJ$ $DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG)$




**Definitions.**

$$\frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$$

110195B

Given.

$W := 15$ $Y := 14$

$X := 20$ $Z := 20$

Unit.

$AG := 1$

Descriptions.

$AE := \frac{AG}{2}$ $EF := \frac{Y}{4 \cdot Z}$ $AF := AE - EF$ $AB := \frac{W}{X}$

$FG := AE + EF$ $FN := \sqrt{AF \cdot FG}$ $AN := \sqrt{FN^2 + AF^2}$

$AK := AN$ $EK := \sqrt{AK^2 - AE^2}$ $EO := \frac{AE \cdot EF}{EK}$

$OK := EO + EK$ $BG := AG - AB$

$PQ := 2 \cdot OK$ $SQ := \frac{PQ - AG}{2}$ $RJ := \sqrt{(AB + SQ) \cdot [PQ - (AB + SQ)]}$

$BJ := RJ - EO$ $AJ := \sqrt{AB^2 + BJ^2}$ $AM := AJ$ $AC := \frac{AJ^2}{AG}$

$BC := AB - AC$ $AD := AB + BC$ $CD := 2 \cdot BC$ $DG := AG - AD$

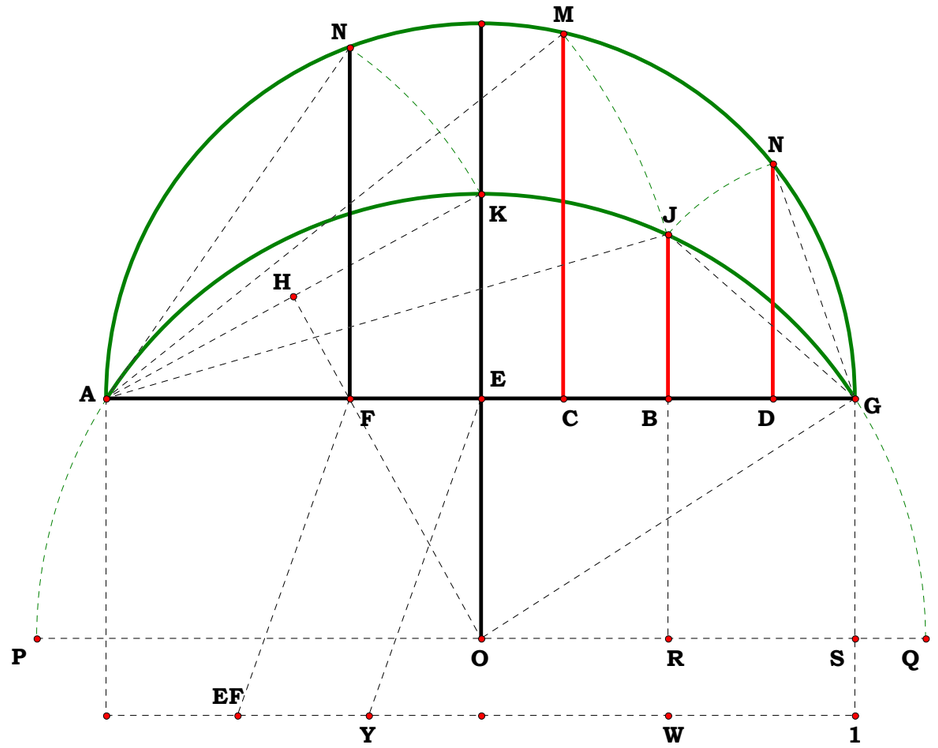
$N := \frac{AG}{2 \cdot EF}$ $N = 2.857143$ $\frac{\sqrt{AC \cdot DG}}{CD} = 0.928571$

$\frac{N - 1}{2} = 0.928571$ $\frac{N - 1}{2} - \frac{\sqrt{AC \cdot DG}}{CD} = 0$

$\frac{1 - 2 \cdot EF}{4 \cdot EF} - \frac{\sqrt{AC \cdot DG}}{CD} = 0$

## A Modification Of The Square Root Figure, Gemini Roots

**On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.**




Unit = 1.00000
W/X = 0.75000
W = 15.00000
X = 20.00000
Y/Z = 0.70000
Y = 14.00000
Z = 20.00000

A = 0.00000
AF = 0.32500
AE = 0.50000
AG = 1.00000
EF = 0.17500
AC = 0.61030
AB = 0.75000
AD = 0.88970

$\frac{1-2 \cdot EF}{4 \cdot EF} - \frac{\sqrt{AC \cdot (AG-AD)}}{AD-AC} = 0.00000$

**Definitions.**

$$AE - \frac{1}{2} = 0 \quad EF - \frac{Y}{4 \cdot Z} = 0 \quad AF - \frac{(2 \cdot Z - Y)}{4 \cdot Z} = 0 \quad AB - \frac{W}{X} = 0 \quad FG - \frac{Y + 2 \cdot Z}{4 \cdot Z} = 0 \quad FN - \frac{\sqrt{4 \cdot Z^2 - Y^2}}{4 \cdot Z} = 0 \quad AN - \frac{\sqrt{(2 \cdot Z - Y)}}{2 \cdot \sqrt{Z}} = 0 \quad AK - \frac{\sqrt{(2 \cdot Z - Y)}}{2 \cdot \sqrt{Z}} = 0 \quad EK - \frac{\sqrt{(Z - Y)}}{\sqrt{4 \cdot Z}} = 0$$

$$EO - \frac{Y}{4 \cdot \sqrt{Z} \cdot \sqrt{Z - Y}} = 0 \quad OK - \frac{2 \cdot Z - Y}{4 \cdot \sqrt{Z} \cdot \sqrt{Z - Y}} = 0 \quad BG - \frac{X - W}{X} = 0 \quad PQ - \frac{2 \cdot Z - Y}{2 \cdot \sqrt{Z} \cdot \sqrt{Z - Y}} = 0 \quad SQ - \frac{2 \cdot Z - Y - 2 \cdot \sqrt{Z} \cdot \sqrt{Z - Y}}{4 \cdot \sqrt{Z} \cdot \sqrt{Z - Y}} = 0$$

$$RJ - \frac{\sqrt{\left[\left[2 \cdot \sqrt{Z} \cdot (X - 2 \cdot W) \cdot \sqrt{Z - Y} + X \cdot (Y - 2 \cdot Z)\right] \cdot \left[2 \cdot \sqrt{Z} \cdot (2 \cdot W - X) \cdot \sqrt{Z - Y} + X \cdot (Y - 2 \cdot Z)\right]\right]}}{4 \cdot X \cdot \sqrt{(Z - Y) \cdot Z}} = 0 \quad BJ - \frac{\sqrt{Z} \cdot \sqrt{Z - Y} \cdot \sqrt{16 \cdot W^2 \cdot Z \cdot (Y - Z) - 16 \cdot W \cdot X \cdot Z \cdot (Y - Z) + X^2 \cdot Y^2} - X \cdot Y \cdot \sqrt{Z^2 - Y \cdot Z}}{4 \cdot X \cdot \sqrt{Z} \cdot \sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z}} = 0$$

$$AJ - \frac{\sqrt{\sqrt{Z} \cdot (Y - Z) \cdot \left(8 \cdot W \cdot Y \cdot Z - X \cdot Y^2 - 8 \cdot W \cdot Z^2\right) - Y \cdot \sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z} \cdot \sqrt{16 \cdot W^2 \cdot Z \cdot (Y - Z) - 16 \cdot W \cdot X \cdot Z \cdot (Y - Z) + X^2 \cdot Y^2}}}{\sqrt{8 \cdot Z^{\frac{3}{2}} \cdot X \cdot (Y - Z)^2}} = 0$$

$$AM - \frac{\sqrt{\sqrt{Z} \cdot (Y - Z) \cdot \left(8 \cdot W \cdot Y \cdot Z - X \cdot Y^2 - 8 \cdot W \cdot Z^2\right) - Y \cdot \sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z} \cdot \sqrt{16 \cdot W^2 \cdot Z \cdot (Y - Z) - 16 \cdot W \cdot X \cdot Z \cdot (Y - Z) + X^2 \cdot Y^2}}}{\sqrt{8 \cdot Z^{\frac{3}{2}} \cdot X \cdot (Y - Z)^2}} = 0$$

$$AC - \frac{8 \cdot \left(\sqrt{Z}\right)^3 \cdot W \cdot (Y - Z)^2 - \sqrt{Z} \cdot X \cdot Y^2 \cdot (Y - Z) - Y \cdot \sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z} \cdot \sqrt{X^2 \cdot Y^2 + 16 \cdot W \cdot Z \cdot (Y - Z) \cdot (W - X)}}{8 \cdot X \cdot Z^{\frac{3}{2}} \cdot (Y - Z)^2} = 0$$

$$BC - \frac{Y \cdot \left(\sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z} \cdot \sqrt{16 \cdot W^2 \cdot Y \cdot Z - 16 \cdot W^2 \cdot Z^2 - 16 \cdot W \cdot X \cdot Y \cdot Z + 16 \cdot W \cdot X \cdot Z^2 + X^2 \cdot Y^2} - X \cdot Y \cdot Z^{\frac{3}{2}} + X \cdot Y^2 \cdot \sqrt{Z}\right)}{8 \cdot Z^{\frac{3}{2}} \cdot X \cdot (Y - Z)^2} = 0$$

$$AD - \frac{\sqrt{Z} \cdot (Y - Z) \cdot \left(X \cdot Y^2 + 8 \cdot W \cdot Y \cdot Z - 8 \cdot W \cdot Z^2\right) + Y \cdot \sqrt{Z - Y} \cdot \sqrt{Z^2 - Y \cdot Z} \cdot \sqrt{16 \cdot W^2 \cdot Z \cdot (Y - Z) - 16 \cdot W \cdot X \cdot Z \cdot (Y - Z) + X^2 \cdot Y^2}}{8 \cdot Z^{\frac{3}{2}} \cdot X \cdot (Y - Z)^2} = 0$$

$$CD - \frac{Y\cdot\left(\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W^2\cdot Y\cdot Z-16\cdot W^2\cdot Z^2-16\cdot W\cdot X\cdot Y\cdot Z+16\cdot W\cdot X\cdot Z^2+X^2\cdot Y^2}-X\cdot Y\cdot Z^{\frac{3}{2}}+X\cdot Y^2\cdot\sqrt{Z}\right)}{4\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}=0$$

$$DG - \frac{\sqrt{Z}\cdot(Y-Z)\cdot\left[8\cdot\left(W\cdot Z^2+X\cdot Y\cdot Z\right)-\left[X\cdot Y^2+8\cdot Z\cdot(W\cdot Y+X\cdot Z)\right]\right]-Y\cdot\sqrt{Z-Y}\cdot\sqrt{Z^2-Y\cdot Z}\cdot\sqrt{16\cdot W\cdot Z\cdot(Y-Z)\cdot(W-X)+X^2\cdot Y^2}}{8\cdot Z^{\frac{3}{2}}\cdot X\cdot(Y-Z)^2}=0$$

**This is all a bit out of hand, maybe?**

$N := \frac{AG}{2\cdot EF}$ $\quad N = 2.857143$

$\frac{N-1}{2} = 0.928571$ $\quad \frac{N-1}{2} - \frac{\sqrt{AC\cdot DG}}{CD} = 0$ $\quad \frac{\sqrt{AC\cdot DG}}{CD} = 0.928571$

110595A

**Descriptions.**

Unit.

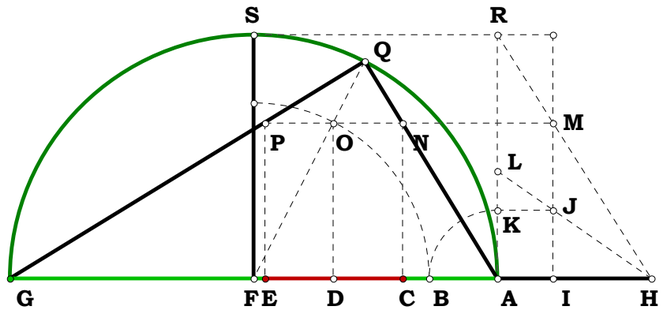
$AG := 1$

Given.

$N_1 := 3$

$N_2 := 5$

## Alternate Method Gemini Roots




$AF := \frac{AG}{2}$ $AR := AF$ $FQ := AF$ $FG := AF$

$AL := \frac{AR}{N_1}$ $IM := \frac{AR}{N_2}$ $AK := \frac{AL \cdot IM}{AR}$ $DO := IM$

$AB := AK$ $BF := AF - AB$ $FO := BF$ $OQ := FQ - FO$

$NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO}$ $NP - 2 \cdot AK = 0$ $CD := AK$

$DE := AK$ $DF := \sqrt{FO^2 - DO^2}$ $AD := AF - DF$

$AC := AD - CD$ $EG := FG + DF - DE$ $CE := NP$

$\frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$

**Definitions.**

$AF - \frac{1}{2} = 0$ $AR - \frac{1}{2} = 0$ $FQ - \frac{1}{2} = 0$ $FG - \frac{1}{2} = 0$ $AL - \frac{1}{2 \cdot N_1} = 0$ $IM - \frac{1}{2 \cdot N_2} = 0$ $DO - \frac{1}{2 \cdot N_2} = 0$

$AK - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $AB - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $CD - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $DE - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$

$BF - \frac{N_1 \cdot N_2 - 1}{2 \cdot N_1 \cdot N_2} = 0$ $FO - \frac{N_1 \cdot N_2 - 1}{2 \cdot N_1 \cdot N_2} = 0$ $OQ - \frac{1}{2 \cdot N_1 \cdot N_2} = 0$ $NP - \frac{1}{N_1 \cdot N_2} = 0$ $CE - \frac{1}{N_1 \cdot N_2} = 0$

$DF - \frac{\sqrt{(N_1 \cdot N_2 - N_1 - 1) \cdot (N_1 + N_1 \cdot N_2 - 1)}}{2 \cdot N_1 \cdot N_2} = 0$ $AD - \frac{N_1 \cdot N_2 - \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1}}{2 \cdot N_1 \cdot N_2} = 0$ $AC - \frac{N_1 \cdot N_2 - \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1} - 1}{2 \cdot N_1 \cdot N_2} = 0$

$EG - \frac{N_1 \cdot N_2 + \sqrt{N_1^2 \cdot N_2^2 - N_1^2 - 2 \cdot N_1 \cdot N_2 + 1} - 1}{2 \cdot N_1 \cdot N_2} = 0$ $\frac{N_1}{2} = 1.5$ $\frac{\sqrt{AC \cdot EG}}{CE} = 1.5$

110595B

**Unit.**

$AB := 1$

**Given.**

$W := 8 \quad Y := 13$

$X := 20 \quad Z := 20$

**Descriptions.**

$BC := AB \quad AP := AB \quad RT := AB \quad BM := AB$

$AC := 2 \cdot AB \quad AN := \frac{W}{X} \quad OR := \frac{Y}{Z} \quad RS := \frac{AN \cdot OR}{RT}$

$EH := OR \quad FJ := OR \quad GK := OR$

$BJ := AB - RS \quad JM := RS$

$HK := 2 \cdot JM \quad EG := HK$

$BF := \sqrt{BJ^2 - FJ^2} \quad BG := BF + JM$

$CG := BC - BG \quad AG := AC - CG \quad AE := AG - EG$

$$\frac{AB}{2 \cdot AN} - \frac{\sqrt{AE \cdot CG}}{EG} = 0$$

**Definitions.**

$BC - 1 = 0 \quad AP - 1 = 0 \quad RT - 1 = 0 \quad BM - 1 = 0$

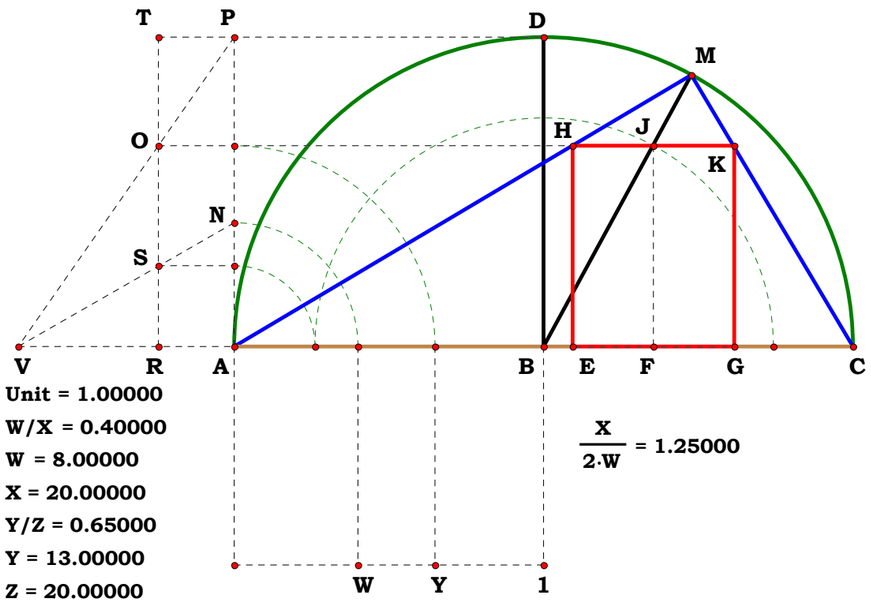
$AC - 2 = 0 \quad AN - \frac{W}{X} = 0 \quad OR - \frac{Y}{Z} = 0 \quad RS - \frac{W \cdot Y}{X \cdot Z} = 0$

$EH - \frac{Y}{Z} = 0 \quad FJ - \frac{Y}{Z} = 0 \quad GK - \frac{Y}{Z} = 0 \quad BJ - \frac{X \cdot Z - W \cdot Y}{X \cdot Z} = 0 \quad JM - \frac{W \cdot Y}{X \cdot Z} = 0 \quad HK - \frac{2 \cdot W \cdot Y}{X \cdot Z} = 0 \quad EG - \frac{2 \cdot W \cdot Y}{X \cdot Z} = 0$

$$BF - \frac{\sqrt{(W \cdot Y - X \cdot Y - X \cdot Z) \cdot (W \cdot Y + X \cdot Y - X \cdot Z)}}{X \cdot Z} = 0 \qquad BG - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} + W \cdot Y}{X \cdot Z} = 0 \qquad CG - \frac{X \cdot Z - W \cdot Y - \sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2}}{X \cdot Z} = 0$$

$$AG - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} + W \cdot Y + X \cdot Z}{X \cdot Z} = 0 \qquad AE - \frac{\sqrt{W^2 \cdot Y^2 - 2 \cdot W \cdot X \cdot Y \cdot Z - X^2 \cdot Y^2 + X^2 \cdot Z^2} - W \cdot Y + X \cdot Z}{X \cdot Z} = 0 \qquad \frac{\sqrt{AE \cdot CG}}{EG} - \frac{X}{2 \cdot W} = 0 \qquad \frac{AB}{2 \cdot AN} - \frac{X}{2 \cdot W} = 0$$

## Alternate Method Gemini Roots




Unit = 1.00000

W/X = 0.40000

W = 8.00000

X = 20.00000

Y/Z = 0.65000

Y = 13.00000

Z = 20.00000

$\frac{X}{2 \cdot W} = 1.25000$

120195A

**Unit.**

$AH := 1$

**Given.**

$N_1 := 5$

**Descriptions.**

$AE := \frac{AH}{2}$ $EH := AE$ $EP := AE$ $AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{AE}{N_1}$ $CE := AB$ $CH := EH + CE$ $CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH}$ $MP := AP - AM$ $NP := \frac{EP \cdot MP}{AP}$

**Definitions.**

$\frac{1}{2} \cdot \frac{(N_1 - 1)}{(N_1 + 1)} - NP = 0$ $\frac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$

$AE - \frac{1}{2} = 0$ $EH - \frac{1}{2} = 0$ $EP - \frac{1}{2} = 0$

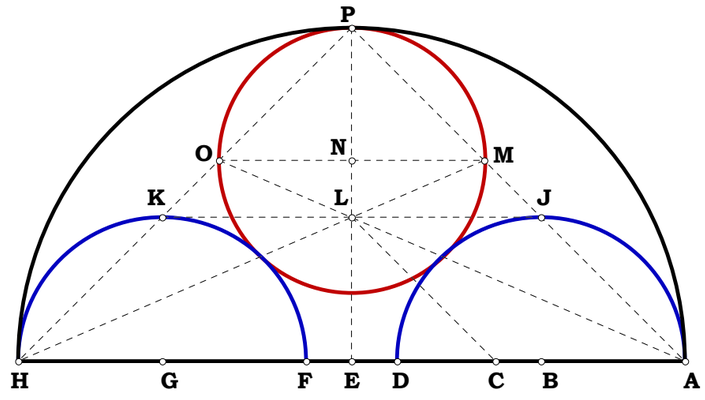
$AP - \frac{\sqrt{2}}{2} = 0$ $AB - \frac{1}{2 \cdot N_1} = 0$ $CE - \frac{1}{2 \cdot N_1} = 0$

$CH - \frac{N_1 + 1}{2 \cdot N_1} = 0$ $CL - \frac{1}{\sqrt{2} \cdot N_1} = 0$ $AM - \frac{\sqrt{2}}{N_1 + 1} = 0$

$MP - \frac{\sqrt{2} \cdot (N_1 - 1)}{2 \cdot (N_1 + 1)} = 0$ $NP - \frac{N_1 - 1}{2 \cdot (N_1 + 1)} = 0$

## Method For Equals

**Given AB find NP.**



120195B

Given.

$X := 8$

$Y := 20$

Unit.

$AE := \frac{Y}{Y}$

**Descriptions.**

$AH := 2 \cdot AE \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{X}{Y} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$

**Definitions.**

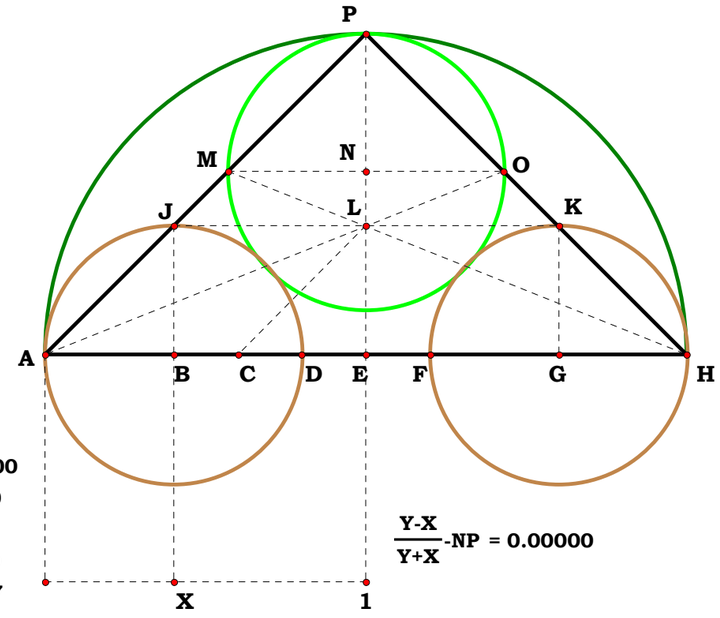
$EH - 1 = 0 \quad EP - 1 = 0 \quad AH - 2 = 0 \quad AP - \sqrt{2} = 0$

$AB - \frac{X}{Y} = 0 \quad CE - \frac{X}{Y} = 0 \quad CH - \frac{X+Y}{Y} = 0 \quad CL - \frac{\sqrt{2} \cdot X}{Y} = 0$

$AM - \frac{2 \cdot \sqrt{2} \cdot X}{X+Y} = 0 \quad MP - \frac{(Y-X) \cdot \sqrt{2}}{X+Y} = 0 \quad NP - \frac{Y-X}{X+Y} = 0$

## Method For Equals

**Given AB find NP.**




Unit = 1.00000

XY = 0.40000

X = 8.00000

Y = 20.00000

NP = 0.42857

$\frac{Y-X}{Y+X}$-NP = 0.00000

## 27 Euler's Straight Line

*In all triangles the center of the circumscribed circle, the point of intersection of the medians, and the point of intersection of the altitudes are situated in this order in a straight line—the Euler line—and are spaced in such a manner that the altitude intersection is twice as far from the median intersection as the center of the circumscribed circle is.*

Leonhard Euler (1707–1783) was one of the greatest and most fertile mathematicians of all time. His writings comprise 45 volumes and over 700 papers, most of them long ones, published in periodicals.

The above theorem is among the results of the paper "Solutio facilis problematum quorundam geometricorum difficillimorum," which appeared in the journal *Novi commentarii Academiae Petropolitanae* (*ad annum 1765*).

The following proof of the Euler theorem is distinguished by its great simplicity.

In the triangle $ABC$ let $M$ be the midpoint of side $AB$, $S$ the median intersection, which lies on $CM$, so that

$$SC = 2 \cdot SM, \tag{1}$$

and $U$ the center of the circle of circumscription, lying on the perpendicular bisector of $AB$.

We extend $US$ by $SO$ so that

$$SO = 2 \cdot SU, \tag{2}$$

and join $O$ to $C$.

According to (1) and (2) the triangles $MUS$ and $COS$ are similar. Consequently, $CO \parallel MU$, i.e., $CO \perp AB$, or expressed verbally, the line connecting the point $O$ with a vertex of the triangle is perpendicular to the side of the triangle opposite the vertex; consequently, the connecting line is an altitude of the triangle.

*The three altitudes consequently pass through point* 0. This is, therefore, the altitude intersection, and Euler's theorem is proved.

NOTE. Our proof contains at the same time the solution to the interesting

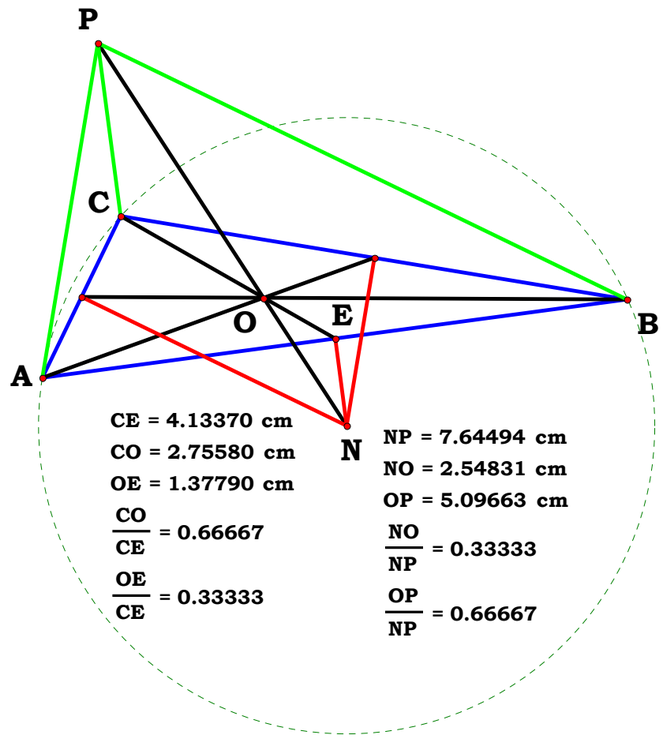
# Euler's Straight Line

**120795A**

**Dover republished this book in 1965. The only time I ever drew it up I was still using TommyCad. The graphic, on the following page, is not even appropriate for the figure. The proof is not valid at all. In a proof, one has to have their Lets, only be the given three lines of a triangle.**

**Euler's Straight Line**
**In all triangles the center of the circumscribed circle, the point of intersection of the medians, and the point of intersection of the altitudes are situated in this order in a straight line-the Euler line-and are spaced in such a manner that the altitude intersection is twice as far from the median intersection as the center of the circumscribed circle is.**

**So, what is the so called proposition, which is not labled as such, try to say? What does the above statement mean?**




**Let us take any triangle whatsoever and construct three points with it. One is the center of the circle which circumscribes it, point N, the second is perpendicular of each point to the segment opposite to it. These three will also converge at what is being called an altitude, point P. Next let us take the midpoint of each segment and form a segment with the opposite vertex point. These three will also construct one point, O. These three points will be collinear, and if we take NP for the unit, ON will be 1/3rd of it, while OP the other 2/3rds.**

**This plate in my releases has never been drawn up by me in Sketchpad. It is still the origional grapic I did in TommyCad. So, in this revision I will correct that issue.**

**The real problem that I see, is that every median is also cut in the same ratio, no different than the so called Euler's line as if Euler had anything what so ever to do than connent the dots. which is hardly a great feat for any so called mathematician, or anyone playing dots to begin with. The most obsurd thing which ego does for someone is incite them to simply rename a name in a particular grammar.**
**If this is Euler's straight line, are the the rest of his lines crooked?**

120795AR

Unit.

Given.

$\delta := 0 .. 2$

$Side\_1 := 2$

$Side\_2 := 3$

$Side\_3 := 4$

Descriptions.

**Scholia:** ***100 Great Problems of Elementary Mathematics*** **H. Dorrie Problem 27 Euler's Line**

**Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.**

$$AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$$

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3) \cdot (Side\_1 + Side\_3 > Side\_2) \cdot (Side\_2 + Side\_3 > Side\_1)$ $TRIANGLE = 1$

$AE_\delta := \frac{AB_\delta}{2}$ $Ak_\delta := AC_\delta$ $Bl_\delta := BC_\delta$ $Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta}$ $Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$

$Ah_\delta := AB_\delta - Bh_\delta$ $hi_\delta := Ah_\delta - Ai_\delta$ $Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$ $Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2}$

$BE_\delta := AE_\delta$ $Bj_\delta := AB_\delta - Aj_\delta$ $Bg_\delta := \frac{BC_\delta}{2}$ $Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$ $fg_\delta := Bf_\delta - Bg_\delta$

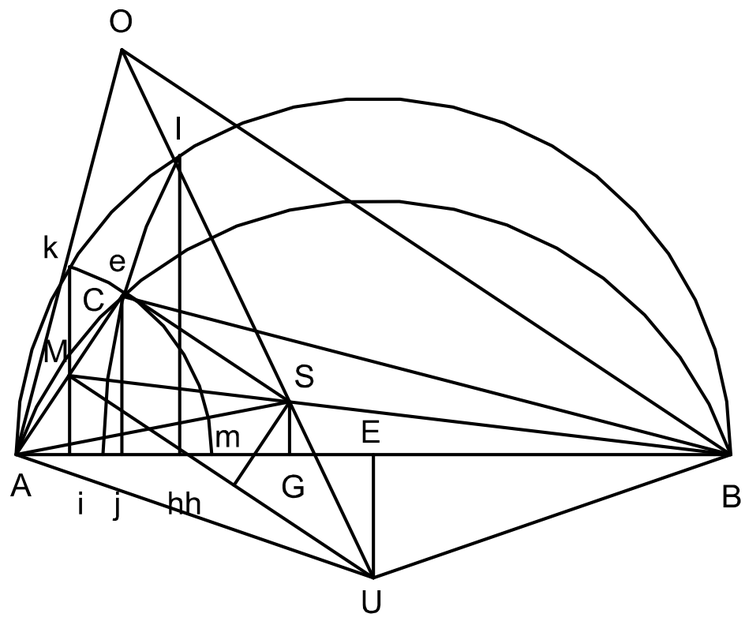
$Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$ $BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$ $AM_\delta := \frac{AC_\delta}{2}$

$AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta}$ $BGG_\delta := AB_\delta - AGG_\delta$ $GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2}$

$BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$ $BS_\delta := \frac{2 \cdot BM_\delta}{3}$ $BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta}$ $GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta$ $AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$ $MS_\delta := BM_\delta - BS_\delta$ $AU_\delta := BU_\delta$

$MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2}$ $Ae_\delta := \frac{1}{2} \cdot \frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2} \cdot AM_\delta - \frac{1}{2} \cdot \frac{(MS_\delta)^2}{AM_\delta}$



**The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.**

$eM_\delta := Ae_\delta - AM_\delta$ $\quad Sm_\delta := eM_\delta$ $\quad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2}$ $\quad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}\,,\ MU_\delta - Mm_\delta\,,\ MU_\delta + Mm_\delta\right]$ $\quad SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2}$ $\quad UO_\delta := 3 \cdot SU_\delta$

**Due to the way in which certain lines lay, the above switch was needed.**

**Definitions.**

**Is this a TRIANGLE = 1 ? Now if you are wondering why thumbs up means that things are a go, or okay, think about it.**

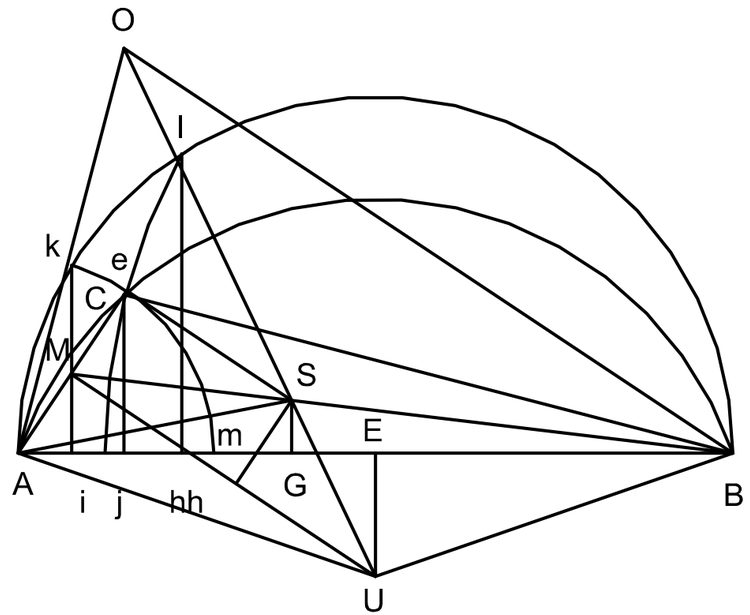
| $SU_\delta$ = |
|---|
| 1.021981 |
| 1.021981 |
| 1.021981 |

| $UO_\delta$ = |
|---|
| 3.065942 |
| 3.065942 |
| 3.065942 |

| $AU_\delta$ = |
|---|
| 2.065591 |
| 2.065591 |
| 2.065591 |

$\frac{SU_\delta}{UO_\delta}$ =

| |
|---|
| 0.333333 |
| 0.333333 |
| 0.333333 |



Unit.

Given.

$AB := 10.49867$

$BC := 9.21723$

$AC := 3.47398$

120795B

Descriptions.

# Euler's Straight Line

Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

Given three sides of a triangle, determine the length of the Euler line.
Work the drawing from each of the sides.

In this write up, I will use equations from previous versions of the DQ. 062793, 010893.

$$AD := \frac{AB}{2} \qquad AO := \frac{AB \cdot AC \cdot BC}{\sqrt{AB + AC + BC} \cdot \sqrt{AC + BC - AB} \cdot \sqrt{AB - AC + BC} \cdot \sqrt{AB + AC - BC}}$$

$$AJ := \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} \qquad CD := \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} \qquad AE := \frac{\sqrt{2 \cdot AB^2 - BC^2 + 2 \cdot AC^2}}{2}$$

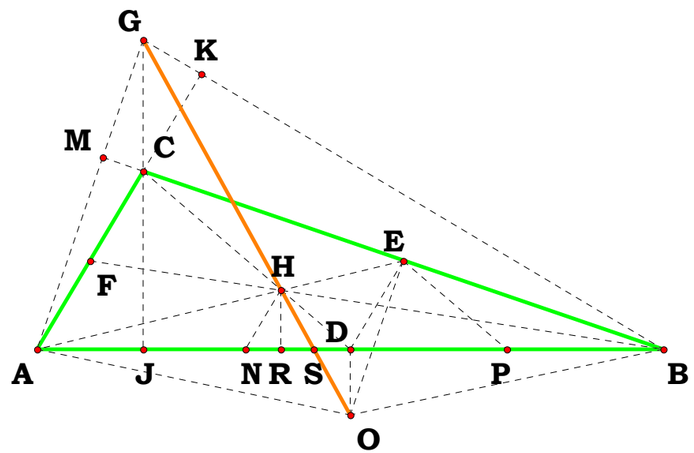
$$BF := \frac{\sqrt{2 \cdot BC^2 - AC^2 + 2 \cdot AB^2}}{2} \qquad DE := \frac{AC}{2} \qquad DP := \frac{AD}{2} \qquad EP := \frac{CD}{2} \qquad AP := AD + DP$$

$$DH := \frac{EP \cdot AD}{AP} \qquad \frac{CD}{DH} = 3 \qquad DN := \frac{AD \cdot DH}{CD} \qquad NR := \frac{AJ \cdot DH}{CD} \qquad BJ := AB - AJ$$

$$CJ := \sqrt{BC^2 - BJ^2} \qquad HR := \frac{CJ \cdot DH}{CD} \qquad DR := DN - NR \qquad DO := \sqrt{AO^2 - AD^2}$$

$$HO := \sqrt{(HR + DO)^2 + DR^2} \qquad AR := AD - DR \qquad JR := AR - AJ \qquad RS := \frac{DR \cdot HR}{DO + HR}$$

$$HS := \frac{HO \cdot HR}{DO + HR} \qquad JS := JR + RS \qquad GS := \frac{HS \cdot JS}{RS} \qquad OS := HO - HS \qquad GO := GS + OS \qquad \frac{GO}{HO} = 3$$




| | | | | |
|---|---|---|---|---|
| $AO = 5.364458$ | $BF = 9.724843$ | $EP = 2.288963$ | $NR = 0.592667$ | $GS = 5.931605$ |
| $AJ = 1.778$ | $DE = 1.73699$ | $DH = 1.525976$ | $RS = 0.548099$ | $OS = 1.262059$ |
| $CD = 4.577927$ | $DP = 2.624668$ | $CJ = 2.984502$ | $HS = 1.135829$ | $GO = 7.193663$ |
| $AE = 6.317117$ | $HR = 0.994834$ | $DN = 1.749778$ | $JS = 2.862322$ | |

| | |
|---|---|
| AB = 10.49867 cm | EP = 2.28896 cm |
| AC = 3.47398 cm | DH = 1.52597 cm |
| BC = 9.21723 cm | CJ = 2.98450 cm |
| AO = 5.36446 cm | DN = 1.74978 cm |
| AJ = 1.77800 cm | NR = 0.59267 cm |
| CD = 4.57792 cm | RS = 0.54810 cm |
| AE = 6.31712 cm | HS = 1.13583 cm |
| BF = 9.72484 cm | JS = 2.86232 cm |
| DE = 1.73699 cm | GS = 5.93160 cm |
| DP = 2.62467 cm | OS = 1.26206 cm |
| HR = 0.99483 cm | GO = 7.19366 cm |

Definitions.

$$AD - \frac{AB}{2} = 0 \qquad AO - \frac{AB \cdot AC \cdot BC}{\sqrt{AB + AC + BC} \cdot \sqrt{AC + BC - AB} \cdot \sqrt{AB - AC + BC} \cdot \sqrt{AB + AC - BC}} = 0 \qquad AJ - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0 \qquad AE - \frac{\sqrt{2 \cdot AB^2 - BC^2 + 2 \cdot AC^2}}{2} = 0 \qquad BF - \frac{\sqrt{2 \cdot BC^2 - AC^2 + 2 \cdot AB^2}}{2} = 0 \qquad DE - \frac{AC}{2} = 0$$

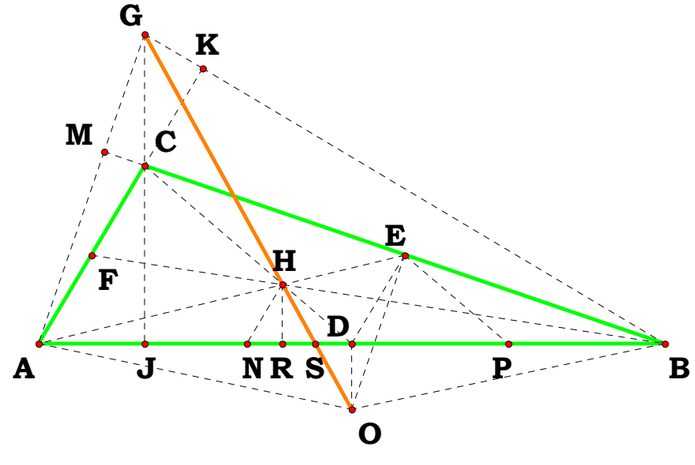
$$DP - \frac{AB}{4} = 0 \quad EP - \frac{\sqrt{2\cdot AC^2 - AB^2 + 2\cdot BC^2}}{4} = 0 \quad AP - \frac{3\cdot AB}{4} = 0 \quad DH - \frac{\sqrt{2\cdot AC^2 - AB^2 + 2\cdot BC^2}}{6} = 0$$

$$NR - \frac{AB^2 + AC^2 - BC^2}{6\cdot AB} = 0 \quad BJ - \frac{AB^2 - AC^2 + BC^2}{2\cdot AB} = 0 \quad DN - \frac{AB}{6} = 0 \quad DR - \frac{(BC - AC)\cdot(AC + BC)}{6\cdot AB} = 0$$

$$CJ - \frac{\sqrt{(AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)}}{2\cdot AB} = 0 \quad AR - \frac{3\cdot AB^2 + AC^2 - BC^2}{6\cdot AB} = 0$$

$$HR - \frac{\sqrt{(AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)}}{6\cdot AB} = 0$$

$$HO - \frac{\sqrt{AB^6 - AB^4\cdot AC^2 - AB^4\cdot BC^2 - AB^2\cdot AC^4 + 3\cdot AB^2\cdot AC^2\cdot BC^2 - AB^2\cdot BC^4 + AC^6 - AC^4\cdot BC^2 - AC^2\cdot BC^4 + BC^6}}{\sqrt{(AB + AC - BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\cdot(9\cdot AB - 9\cdot AC + 9\cdot BC)}} = 0$$




$$JR - \frac{(BC - AC)\cdot(AC + BC)}{3\cdot AB} = 0 \quad RS - \frac{(AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\cdot(AC + BC)\cdot(BC - AC)}{6\cdot AB\cdot\left[3\cdot AB^4 - 3\cdot AB^2\cdot AC^2 - 3\cdot AB^2\cdot BC^2 + (AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\right]} = 0$$

$$HS - \frac{(AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\cdot\sqrt{AB^6 - AB^4\cdot AC^2 - AB^4\cdot BC^2 - AB^2\cdot AC^4 + 3\cdot AB^2\cdot AC^2\cdot BC^2 - AB^2\cdot BC^4 + AC^6 - AC^4\cdot BC^2 - AC^2\cdot BC^4 + BC^6}}{\left[3\cdot AB^4 - 3\cdot AB^2\cdot AC^2 - 3\cdot AB^2\cdot BC^2 + (AB + AC - BC)\cdot(AB - AC + BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\right]\cdot\sqrt{(AB + AC - BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\cdot(9\cdot AB - 9\cdot AC + 9\cdot BC)}} = 0$$

$$JS - \frac{(BC - AC)\cdot(AC + BC)\cdot\left(AB^2 - AC^2 + BC^2\right)\cdot\left(AB^2 + AC^2 - BC^2\right)}{2\cdot AB\cdot\left(2\cdot AB^4 - AB^2\cdot AC^2 - AB^2\cdot BC^2 - AC^4 + 2\cdot AC^2\cdot BC^2 - BC^4\right)} = 0 \quad DO - \frac{AB\cdot\left(AB^2 - AC^2 - BC^2\right)}{2\cdot\sqrt{(AB + AC + BC)\cdot(AB - AC + BC)\cdot(AB + AC - BC)\cdot(AC - AB + BC)}} = 0$$

$$GS - \frac{3\cdot\left(AB^2 + AC^2 - BC^2\right)\cdot\left(AB^2 - AC^2 + BC^2\right)\cdot\sqrt{AB^6 - AB^4\cdot AC^2 - AB^4\cdot BC^2 - AB^2\cdot AC^4 + 3\cdot AB^2\cdot AC^2\cdot BC^2 - AB^2\cdot BC^4 + AC^6 - AC^4\cdot BC^2 - AC^2\cdot BC^4 + BC^6}}{\sqrt{(AB + AC - BC)\cdot(AC - AB + BC)\cdot(AB + AC + BC)\cdot(9\cdot AB - 9\cdot AC + 9\cdot BC)}\cdot\left(2\cdot AB^4 - AB^2\cdot AC^2 - AB^2\cdot BC^2 - AC^4 + 2\cdot AC^2\cdot BC^2 - BC^4\right)} = 0$$

$$OS - \frac{3\cdot\sqrt{AB^6 - AB^4\cdot AC^2 - AB^4\cdot BC^2 - AB^2\cdot AC^4 + 3\cdot AB^2\cdot AC^2\cdot BC^2 - AB^2\cdot BC^4 + AC^6 - AC^4\cdot BC^2 - AC^2\cdot BC^4 + BC^6}\cdot AB^2\cdot\left(AB^2 - AC^2 - BC^2\right)}{\sqrt{18\cdot AB^2\cdot AC^2 - 9\cdot AB^4 + 18\cdot AB^2\cdot BC^2 - 9\cdot AC^4 + 18\cdot AC^2\cdot BC^2 - 9\cdot BC^4}\cdot\left(2\cdot AB^4 - AB^2\cdot AC^2 - AB^2\cdot BC^2 - AC^4 + 2\cdot AC^2\cdot BC^2 - BC^4\right)} = 0$$

$$GO - \frac{\sqrt{AB^6 - AB^4\cdot AC^2 - AB^4\cdot BC^2 - AB^2\cdot AC^4 + 3\cdot AB^2\cdot AC^2\cdot BC^2 - AB^2\cdot BC^4 + AC^6 - AC^4\cdot BC^2 - AC^2\cdot BC^4 + BC^6}}{\sqrt{2\cdot AB^2\cdot AC^2 - AB^4 + 2\cdot AB^2\cdot BC^2 - AC^4 + 2\cdot AC^2\cdot BC^2 - BC^4}} = 0$$

120795C

Descriptions.

Given.

$W := 6$ $Y := 3$

$X := 20$ $Z := 13$

Unit.

$AB := \frac{Y}{Y}$

## Euler's Straight Line

Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

**Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.**

**In this write up, I will use equations from previous versions of the DQ. 062793, 010893.**

$AM := \frac{Y}{Z}$ $BM := AB - AM$ $CM := \frac{W}{X}$ $AC := \sqrt{AM^2 + CM^2}$ $BC := \sqrt{BM^2 + CM^2}$

$AD := \frac{AB}{2}$ $AO := \frac{AB \cdot AC \cdot BC}{\sqrt{AB + AC + BC} \cdot \sqrt{AC + BC - AB} \cdot \sqrt{AB - AC + BC} \cdot \sqrt{AB + AC - BC}}$

$AM := \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB}$ $CD := \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2}$ $AE := \frac{\sqrt{2 \cdot AB^2 - BC^2 + 2 \cdot AC^2}}{2}$

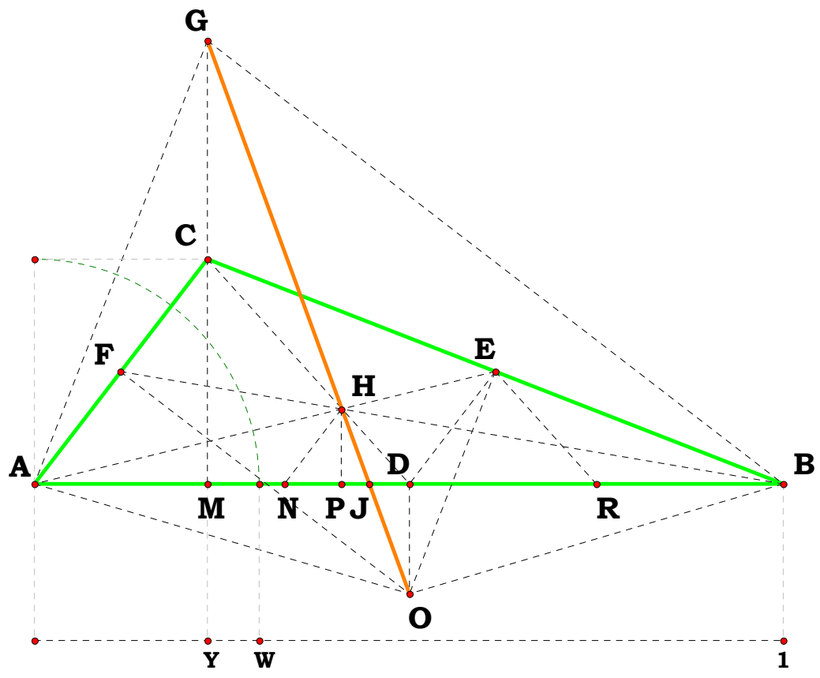
$BF := \frac{\sqrt{2 \cdot BC^2 - AC^2 + 2 \cdot AB^2}}{2}$ $DE := \frac{AC}{2}$ $DR := \frac{AD}{2}$ $ER := \frac{CD}{2}$ $AR := AD + DR$

$DH := \frac{ER \cdot AD}{AR}$ $\frac{CD}{DH} = 3$ $DN := \frac{AD \cdot DH}{CD}$ $NP := \frac{AM \cdot DH}{CD}$ $BM := AB - AM$

$HP := \frac{CM \cdot DH}{CD}$ $DP := DN - NP$ $DO := \sqrt{AO^2 - AD^2}$

$HO := \sqrt{(HP + DO)^2 + DP^2}$ $AP := AD - DP$ $JP := AP - AM$ $PJ := \frac{DP \cdot HP}{DO + HP}$

$HJ := \frac{HO \cdot HP}{DO + HP}$ $MJ := JP + PJ$ $GJ := \frac{HJ \cdot MJ}{PJ}$ $OJ := HO - HJ$ $GO := GJ + OJ$ $\frac{GO}{HO} = 3$




Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.23077
Y = 3.00000
Z = 13.00000

AC = 0.37849
BC = 0.82566
AO = 0.52084
GO = 0.78518

Definitions.

$AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{X \cdot Z} = 0$ $AD - \frac{1}{2} = 0$

$$AO - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 - 2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2}}{\sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 \ \ldots} + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} \ \ldots \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 \ \ldots} + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} \ \ldots \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 \ \ldots} + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} \ \ldots \cdot \sqrt{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2 \ \ldots} + -2 \cdot X^2 \cdot Y \cdot Z + X^2 \cdot Z^2} + X \cdot Z} = 0$$

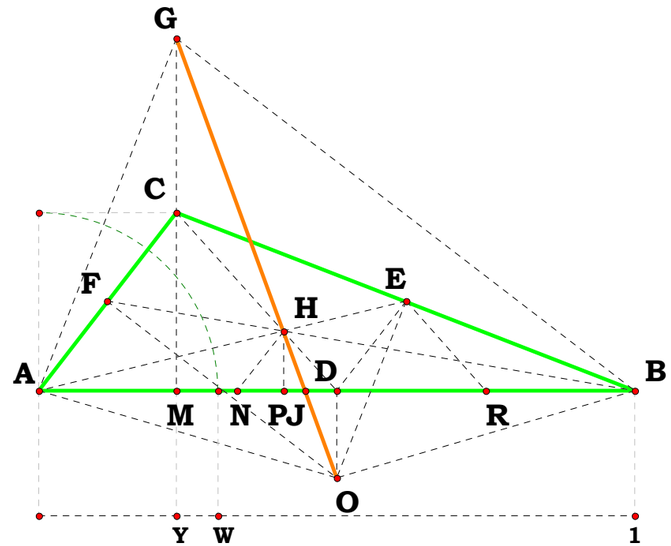
$$AM - \frac{Y}{Z} = 0 \qquad CD - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0 \qquad AE - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 + 2\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0 \qquad BF - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + 4\cdot X^2\cdot Z^2}}{2\cdot X\cdot Z} = 0$$

$$DE - \frac{\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}}{2\cdot X\cdot Z} = 0 \qquad ER - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{4\cdot X\cdot Z} = 0 \qquad AR - \frac{3}{4} = 0 \qquad DH - \frac{\sqrt{4\cdot W^2\cdot Z^2 + 4\cdot X^2\cdot Y^2 - 4\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2}}{6\cdot X\cdot Z} = 0$$

$$DR - \frac{1}{4} = 0 \qquad NP - \frac{Y}{3\cdot Z} = 0 \qquad BM - \frac{Z - Y}{Z} = 0 \qquad DN - \frac{1}{6} = 0 \qquad DP - -\frac{\left(\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2 - 2\cdot X^2\cdot Y\cdot Z + X^2\cdot Z^2} + \sqrt{W^2\cdot Z^2 + X^2\cdot Y^2}\right)\cdot\left(\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - BC\cdot X\cdot Z\right)}{6\cdot X^2\cdot Z^2} = 0$$

$$CM - \frac{W}{X} = 0 \qquad AP - \frac{Y + Z}{3\cdot Z} = 0 \qquad HP - \frac{W}{3\cdot X} = 0$$

$$HO - \frac{\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{6\cdot W\cdot X\cdot Z^2} = 0$$




$$JP - \frac{Z - 2\cdot Y}{3\cdot Z} = 0 \qquad PJ - -\frac{W^2\cdot Z\cdot(Z - 2\cdot Y)}{3\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$$

$$HJ - \frac{W\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{3\cdot X\cdot\left(3\cdot X^2\cdot Y\cdot Z - 3\cdot X^2\cdot Y^2 - W^2\cdot Z^2\right)} = 0$$

$$MJ - -\frac{X^2\cdot Y\cdot\left(2\cdot Y^2 - 3\cdot Y\cdot Z + Z^2\right)}{Z\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0 \qquad DO - \frac{X^2\cdot Y\cdot Z - W^2\cdot Z^2 - X^2\cdot Y^2}{2\cdot W\cdot X\cdot Z^2} = 0$$

$$GJ - \frac{X\cdot Y\cdot(Y - Z)\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{W\cdot Z^2\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$$

$$OJ - \frac{\left(W^2\cdot Z^2 + X^2\cdot Y^2 - X^2\cdot Y\cdot Z\right)\cdot\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{2\cdot W\cdot X\cdot Z^2\cdot\left(W^2\cdot Z^2 + 3\cdot X^2\cdot Y^2 - 3\cdot X^2\cdot Y\cdot Z\right)} = 0$$

$$GO - \frac{\sqrt{W^4\cdot Z^4 + 10\cdot W^2\cdot X^2\cdot Y^2\cdot Z^2 - 10\cdot W^2\cdot X^2\cdot Y\cdot Z^3 + W^2\cdot X^2\cdot Z^4 + 9\cdot X^4\cdot Y^4 - 18\cdot X^4\cdot Y^3\cdot Z + 9\cdot X^4\cdot Y^2\cdot Z^2}}{2\cdot W\cdot X\cdot Z^2} = 0$$

121695

**Descriptions.**

$AD := n \cdot a \qquad BE := \sqrt{a^2 + b^2} \qquad BC := \frac{a^2}{BE}$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2} \qquad CG := \sqrt{FG^2 - CF^2}$

$AG := FG \qquad AC := AG + CG$

$BG := CG - BC \qquad DG := FG$

$BD := DG - BG \qquad AB := AG + BG$

$AE := AB + BE \qquad DE := BE - BD$

$DH := \frac{b^2}{DE} \qquad DI := AE \qquad HI := DI - DH$

$z := AE \qquad z = 12.621556$

$c := DE \qquad c = 0.621556$

$d := HI \qquad d = 6.18609$

$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

**Given.**

$n \equiv 3$

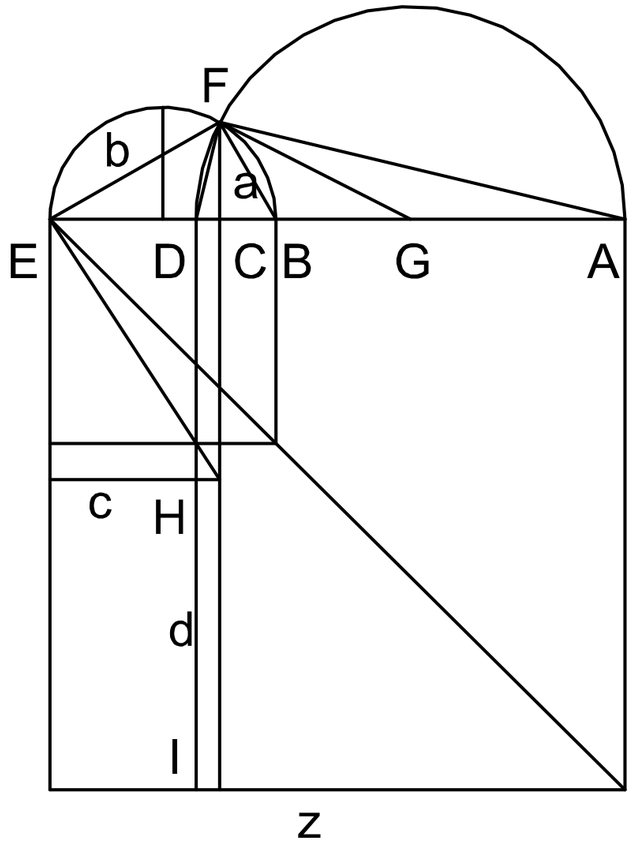
$a \equiv 4$

$b \equiv 2$

**Place values here :**

**Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.**

**Given a, n and b for the equation $z^2 =$ $n\,az + b^2 + cd$ find z, c, and d.**



**Expressing c and d in terms of the givens does not really look esthetically pleasing.**

$$d - 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}=0$$

$$c-\frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}=0$$

**The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving *z*.**

$$z-\frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}=0$$

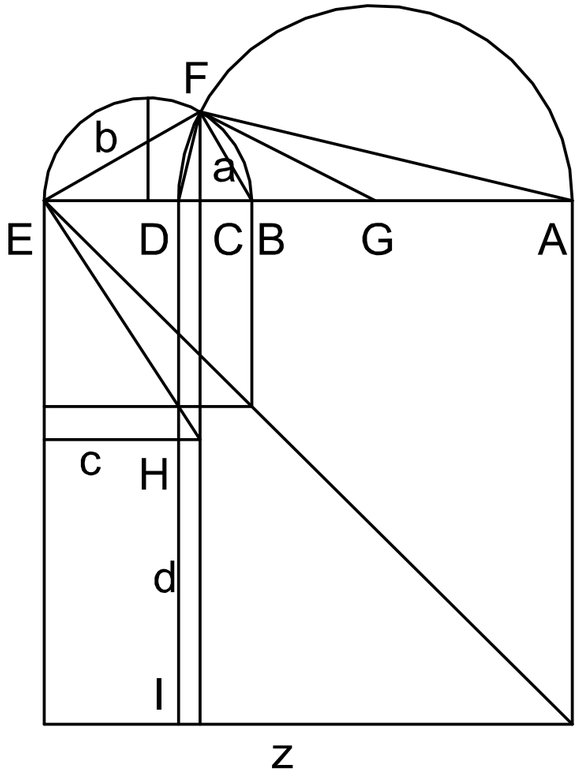
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p:=-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

**Solve for *z* below.**

$$\begin{pmatrix}\frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\end{pmatrix}$$



**121895**

**Descriptions.**

**Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham**

$$z^2 := az - b^2$$

**The problem is given for the solution of z when a and b are given. Working the problem backward,** $z^2 + b^2 := az$ **one can see constants in the figure for solving when only a and b are given.**

$$b := 2.12 \qquad z := 1.41 \qquad c := \frac{b^2}{z}$$

**Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.**

$$a := z + c$$

**We find that this c has another relation to z, for it holds a proportion to it in the given equation.**

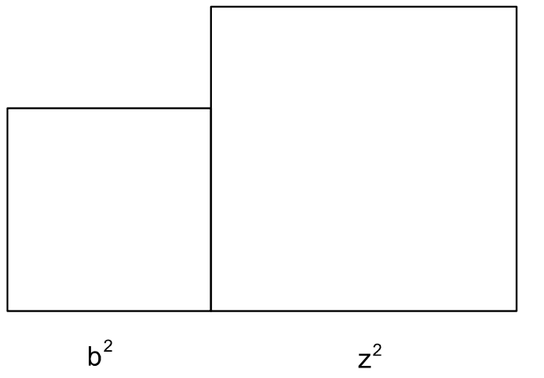
$$\left(c^2 + b^2\right) - a \cdot c = 1.776357 \times 10^{-15}$$

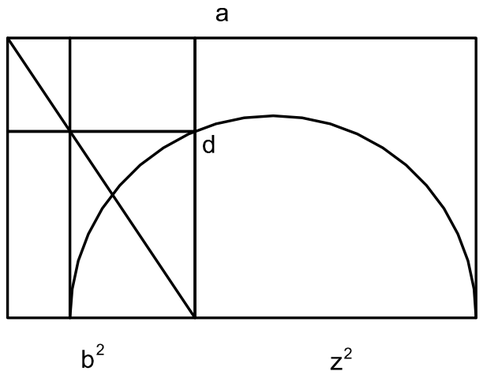
$$\left(c^2 + b^2\right) - [(z + c) \cdot c] = 1.776357 \times 10^{-15}$$

$$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$$

$$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$$

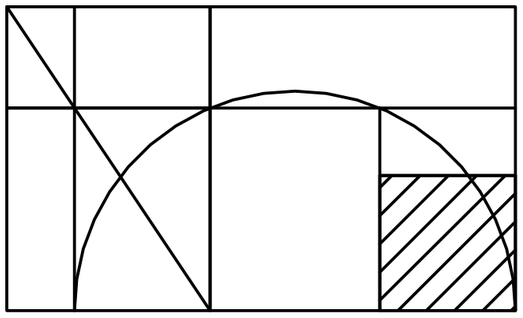
$$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$$

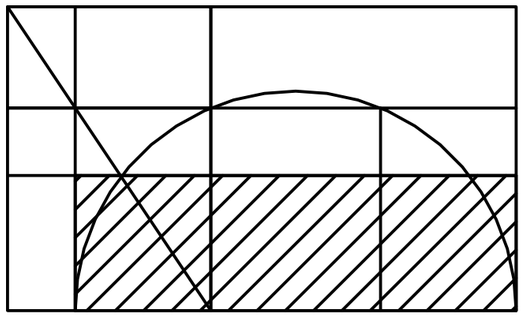


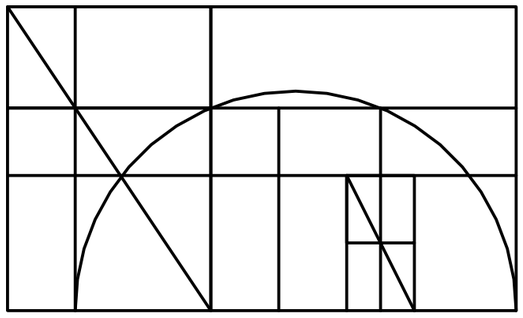




**Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.**

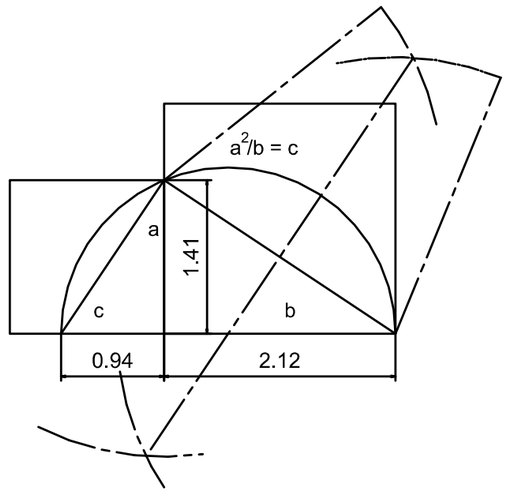
**One can also see that working the figure in a straight forward manner, imaginary situations are not possible,**




**The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.**



Unit.

$EF := 1$

Given.

$N := 2$

**122095A**

**Descriptions.**

$EJ := EF \cdot N$ $AE := \frac{EJ^2}{EF}$ $AF := AE + EF$

$AB := \frac{AF}{2}$ $BF := AB$ $BE := BF - EF$ $BH := BE$

$BJ := \sqrt{EJ^2 + BE^2}$ $BD := \frac{BE \cdot BH}{BJ}$ $BG := BD$

$BC := \frac{BE \cdot BG}{BJ}$ $GH := BH - BG$ $DE := BE - BD$

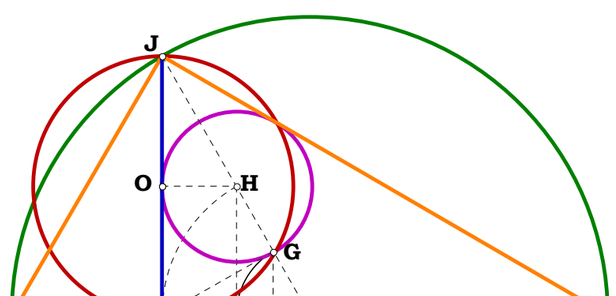
$HO := DE$ $EG_1 := \sqrt{BE^2 - BG^2}$ $GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2}$ $\frac{GH}{HO} = 1$ $\frac{EG_1}{EG_2} = 1$

$\left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0$ $\left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

## Delian Solution in Every Right Angle

**This plate is derived from the fact that for any EJ taken as a square, divided by EF, the answer is AE. And in every case, the small circle OH has a relationship to the circle EJ. This is another plate on geometric progression.**




EJ = 1.708 in.

EF = 0.992 in.

AE = 2.943 in.

$\frac{EJ^2}{EF}$-AE = 0.000 in.

**Definitions.**

$EJ - N = 0$ $AE - N^2 = 0$ $AF - \left(N^2 + 1\right) = 0$ $AB - \frac{N^2 + 1}{2} = 0$ $BF - \frac{N^2 + 1}{2} = 0$ $BE - \frac{(N-1)\cdot(N+1)}{2} = 0$

$BH - \frac{(N-1)\cdot(N+1)}{2} = 0$ $BJ - \frac{\left(N^2+1\right)}{2} = 0$ $BD - \frac{(N-1)^2\cdot(N+1)^2}{2\cdot\left(N^2+1\right)} = 0$ $BG - \frac{(N-1)^2\cdot(N+1)^2}{2\cdot\left(N^2+1\right)} = 0$

$BC - \frac{(N-1)^3\cdot(N+1)^3}{2\cdot\left(N^2+1\right)^2} = 0$ $GH - \frac{(N-1)\cdot(N+1)}{N^2+1} = 0$ $DE - \frac{(N-1)\cdot(N+1)}{N^2+1} = 0$ $HO - \frac{(N-1)\cdot(N+1)}{N^2+1} = 0$

$EG_1 - \frac{\left(N^3-N\right)}{\left(N^2+1\right)} = 0$ $GJ - \frac{2\cdot N^2}{N^2+1} = 0$ $EG_2 - \frac{N\cdot(N-1)\cdot(N+1)}{\left(N^2+1\right)} = 0$

Given.

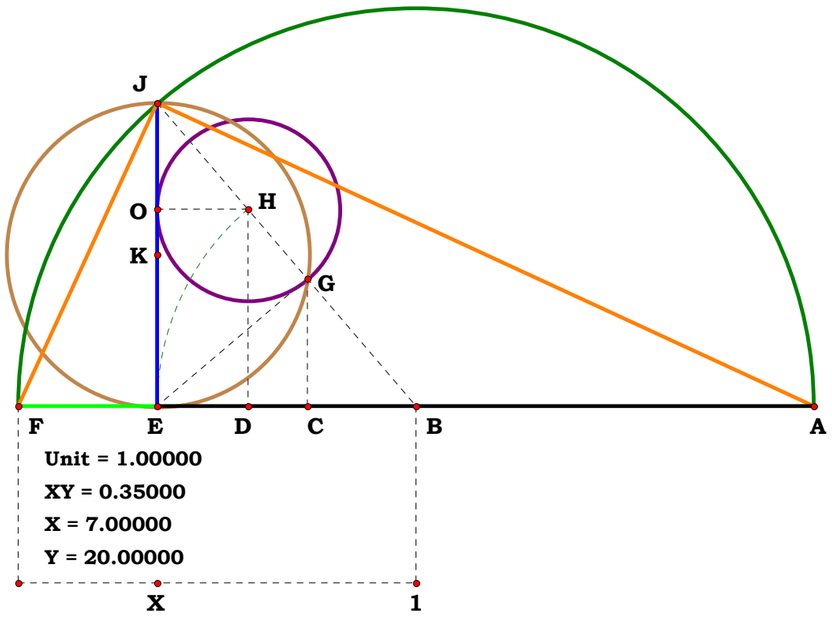
$X := 7$

$Y := 20$

## 122095B

## Delian Solution in Every Right Angle

**This plate is derived from the fact that for any EJ taken as a square, divided by EF, the answer is AE. And in every case, the small circle OH has a relationship to the circle EJ.**




**Descriptions.**

$AF := 2 \quad EF := \frac{X}{Y} \quad EJ := \sqrt{EF \cdot (AF - EF)} \quad AE := \frac{EJ^2}{EF}$

$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF$

$BH := BE \quad BJ := \sqrt{EJ^2 + BE^2} \quad BD := \frac{BE \cdot BH}{BJ}$

$BG := BD \quad BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD$

$HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \quad \frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1$

$\left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

**Definitions.**

$AF - 2 = 0 \quad EF - \frac{X}{Y} = 0 \quad EJ - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{Y} = 0 \quad AE - \frac{2 \cdot Y - X}{Y} = 0 \quad AB - 1 = 0 \quad BF - 1 = 0 \quad BE - \frac{Y - X}{Y} = 0$

$BH - \frac{Y - X}{Y} = 0 \quad BJ - \sqrt{1} = 0 \quad BD - \frac{(X - Y)^2}{Y^2} = 0 \quad BG - \frac{(X - Y)^2}{Y^2} = 0 \quad BC - \frac{(Y - X)^3}{Y^3} = 0 \quad GH - \frac{X \cdot (Y - X)}{Y^2} = 0$

$DE - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad HO - \frac{X \cdot (Y - X)}{Y^2} = 0 \quad EG_1 - \frac{\sqrt{X \cdot (2 \cdot Y - X)} \cdot (Y - X)}{Y^2} = 0 \quad GJ - \frac{X \cdot (2 \cdot Y - X)}{Y^2} = 0 \quad EG_2 - \frac{\sqrt{X \cdot (2 \cdot Y - X)} \cdot (Y - X)}{Y^2} = 0$

**Unit.**

$AB := 1$

**Given.**

$N := 7$

**122195A1**

**Descriptions.**

$AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD} \quad BD := AD - AB \quad CD := AD - AC$

$BC := BD - CD \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$

**Definitions.**

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \quad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0$

$AD - N = 0 \quad AC - \sqrt{N} = 0 \quad BD - (N-1) = 0 \quad CD - \left(N - \sqrt{N}\right) = 0$

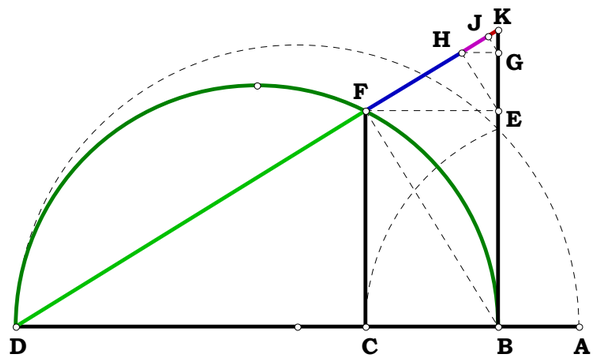
$BC - \left(\sqrt{N} - 1\right) = 0 \quad CF - N^{\frac{1}{4}} \cdot \left(\sqrt{N} - 1\right) = 0 \quad DF - \sqrt{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2 \cdot \left(\sqrt{N} + 1\right)} = 0$

$DK - \frac{\sqrt{N + \sqrt{N}} \cdot (N-1)}{\sqrt{N}} = 0 \quad FK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N}} = 0$

$HK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N} \cdot \left(\sqrt{N} + 1\right)} = 0 \quad JK - \frac{\sqrt{N + \sqrt{N}} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N} \cdot \left(\sqrt{N} + 1\right)^2} = 0$

## Pascal's Triangle With Exponential Division

**Plate A1**



122195A2

**Given.**

$X := 3 \qquad Y := 20$

$AB := \frac{X}{Y} \qquad N := \frac{Y}{X}$

**Descriptions.**

$AD := AB \cdot N \qquad AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB \qquad CD := AD - AC$

$BC := BD - CD \qquad CF := \sqrt{BC \cdot CD} \qquad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \qquad FK := \frac{DK \cdot BC}{BD} \qquad HK := \frac{FK \cdot FK}{DK} \qquad JK := \frac{HK \cdot HK}{FK}$

**Definitions.**

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0$

$AD - 1 = 0 \qquad AC - \frac{\sqrt{X}}{\sqrt{Y}} = 0 \qquad BD - \frac{Y - X}{Y} = 0 \qquad CD - \frac{\sqrt{Y} - \sqrt{X}}{\sqrt{Y}} = 0$

$BC - \frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y} = 0 \qquad CF - \frac{\sqrt{\sqrt{X} \cdot Y - 2 \cdot X \cdot \sqrt{Y} + X^{\frac{3}{2}}}}{Y^{\frac{3}{4}}} = 0$

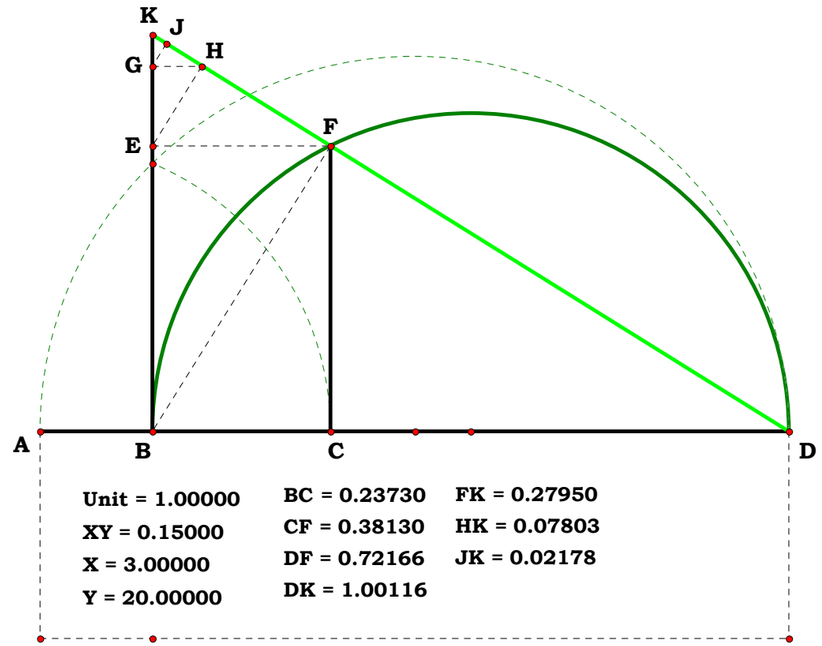
$DF - \frac{\sqrt{\sqrt{X} + \sqrt{Y}} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{3}{4}}} = 0 \qquad DK - \frac{\left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(\sqrt{X} + \sqrt{Y}\right) \cdot \sqrt{\sqrt{X} + \sqrt{Y}}}{Y^{\frac{5}{4}}} = 0$

$FK - \frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(\sqrt{\sqrt{X} + \sqrt{Y}}\right)^3}{Y^{\frac{5}{4}} \cdot \left(\sqrt{X} + \sqrt{Y}\right)} = 0$

$HK - \frac{X \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{5}{4}} \cdot \sqrt{\sqrt{X} + \sqrt{Y}}} = 0 \qquad JK - \frac{\left(\sqrt{X}\right)^3 \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{5}{4}} \cdot \left(\sqrt{X} + \sqrt{Y}\right)^{\frac{3}{2}}} = 0$

## Pascal's Triangle With Exponential Division

**Plate A2**




| | | |
|---|---|---|
| Unit = 1.00000 | BC = 0.23730 | FK = 0.27950 |
| XY = 0.15000 | CF = 0.38130 | HK = 0.07803 |
| X = 3.00000 | DF = 0.72166 | JK = 0.02178 |
| Y = 20.00000 | DK = 1.00116 | |

$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y} - BC = 0.00000$

$\frac{\sqrt{\left(\sqrt{X} \cdot Y - 2 \cdot X \cdot \sqrt{Y}\right) + X^{\frac{3}{2}}}}{Y^{\frac{3}{4}}} - CF = 0.00000$

$\frac{\sqrt{\sqrt{X} + \sqrt{Y}} \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{3}{4}}} - DF = 0.00000$

$\frac{\left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(\sqrt{X} + \sqrt{Y}\right) \cdot \sqrt{\sqrt{X} + \sqrt{Y}}}{Y^{\frac{5}{4}}} - DK = 0.00000$

$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \sqrt{\sqrt{X} + \sqrt{Y}^3}}{Y^{\frac{5}{4}} \cdot \left(\sqrt{X} + \sqrt{Y}\right)} - FK = 0.00000$

$\frac{X \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{5}{4}} \cdot \sqrt{\sqrt{X} + \sqrt{Y}}} - HK = 0.00000$

$\frac{\sqrt{X}^3 \cdot \left(\sqrt{Y} - \sqrt{X}\right)}{Y^{\frac{5}{4}} \cdot \left(\sqrt{X} + \sqrt{Y}\right)^{\frac{3}{2}}} - JK = 0.00000$

**Unit.**

$AB := 1$

**Given.**

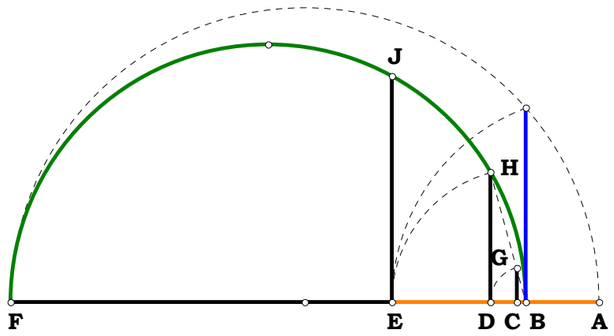
$N := 7$

**122195B1**

**Descriptions.**

# Pascal's Triangle With Exponential Division

**Plate B1**




$AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \quad BH := BE$

$BD := \frac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \frac{BD \cdot BG}{BE}$

**Definitions.**

$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \quad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \quad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

$AF - N = 0 \quad BF - (N-1) = 0 \quad AE - \sqrt{N} = 0$

$BE - \left(\sqrt{N} - 1\right) = 0 \quad BH - \left(\sqrt{N} - 1\right) = 0$

$BD - \frac{\sqrt{N}-1}{\sqrt{N}+1} = 0 \quad BG - \frac{\sqrt{N}-1}{\sqrt{N}+1} = 0 \quad BC - \frac{\sqrt{N}-1}{\left(\sqrt{N}+1\right)^2} = 0$

122195B2

Given.

$X := 3 \quad Y := 15$

$AB := \frac{X}{Y} \quad N := \frac{Y}{X}$

**Descriptions.**

$AF := AB \cdot N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \quad BH := BE$

$BD := \frac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \frac{(N-1)}{(\sqrt{N}-1)} = 0 \quad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \quad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

**Definitions.**

$AF - 1 = 0 \quad BF - \frac{Y - X}{Y} = 0 \quad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$BE - \frac{\sqrt{X} \cdot (\sqrt{Y} - \sqrt{X})}{Y} = 0 \quad BH - \frac{\sqrt{X} \cdot (\sqrt{Y} - \sqrt{X})}{Y} = 0$

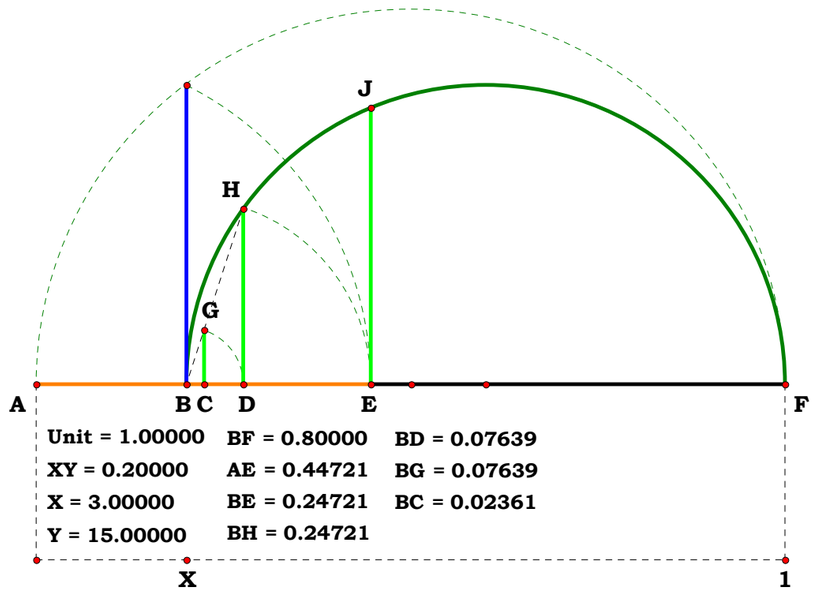
$BD - \frac{X \cdot (\sqrt{Y} - \sqrt{X})}{Y \cdot (\sqrt{X} + \sqrt{Y})} = 0 \quad BG - \frac{X \cdot (\sqrt{Y} - \sqrt{X})}{Y \cdot (\sqrt{X} + \sqrt{Y})} = 0$

$BC - \frac{(\sqrt{X})^3 \cdot (\sqrt{Y} - \sqrt{X})}{Y \cdot (\sqrt{X} + \sqrt{Y})^2} = 0$

$\frac{BF}{BE} - \left(\frac{\sqrt{Y}}{\sqrt{X}} + 1\right) = 0 \quad \frac{BF}{BD} - \frac{X + Y + 2 \cdot \sqrt{X} \cdot \sqrt{Y}}{X} = 0 \quad \frac{BF}{BC} - \frac{X^3 - 3 \cdot X^2 \cdot Y + 3 \cdot X \cdot Y^2 - Y^3}{X^2 \cdot \left[X + 3 \cdot Y - X \cdot \left(\frac{Y}{X}\right)^{\frac{3}{2}} - 3 \cdot \sqrt{X} \cdot \sqrt{Y}\right]} = 0$

## Pascal's Triangle With Exponential Division

### Plate B2




$\frac{Y-X}{Y}$-BF = 0.00000

$\frac{\sqrt{X}}{\sqrt{Y}}$-AE = 0.00000

$\frac{\sqrt{X} \cdot (\sqrt{Y}-\sqrt{X})}{Y}$ = 0.24721

$\frac{\sqrt{X} \cdot (\sqrt{Y}-\sqrt{X})}{Y}$-BE = 0.00000

$\frac{\sqrt{X} \cdot (\sqrt{Y}-\sqrt{X})}{Y}$-BH = 0.00000

$\frac{X \cdot (\sqrt{Y}-\sqrt{X})}{Y \cdot (\sqrt{X}+\sqrt{Y})}$ = 0.07639

$\frac{X \cdot (\sqrt{Y}-\sqrt{X})}{Y \cdot (\sqrt{X}+\sqrt{Y})}$-BD = 0.00000

$\frac{X \cdot (\sqrt{Y}-\sqrt{X})}{Y \cdot (\sqrt{X}+\sqrt{Y})}$-BG = 0.00000

$\frac{\sqrt{X^3} \cdot (\sqrt{Y}-\sqrt{X})}{Y \cdot (\sqrt{X}+\sqrt{Y})^2}$-BC = 0.00000

**Unit.**

$AB := 1$

**Given.**

$N := 7$

**122195C1**

**Descriptions.**

$AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}}$

$BF := AF - AB \quad BJ := BF \quad BE := \frac{BJ \cdot BF}{BG}$

$BH := BE \quad BC := \frac{BH \cdot BE}{BF}$

**Definitions.**

$$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}}-1} = 0 \qquad \frac{BG}{BE} - \frac{N^2-2\cdot N+1}{N^{\left(\frac{3}{2}\right)}-2\cdot N^{\left(\frac{3}{4}\right)}+1} = 0 \qquad \frac{BG}{BC} - \frac{N^3-3\cdot N^2+3\cdot N-1}{N^{\left(\frac{9}{4}\right)}-3\cdot N^{\left(\frac{3}{2}\right)}+3\cdot N^{\left(\frac{3}{4}\right)}-1} = 0$$

$$AG - N = 0 \qquad BG - (N-1) = 0 \qquad AF - \left(N^3\right)^{\frac{1}{4}} = 0$$

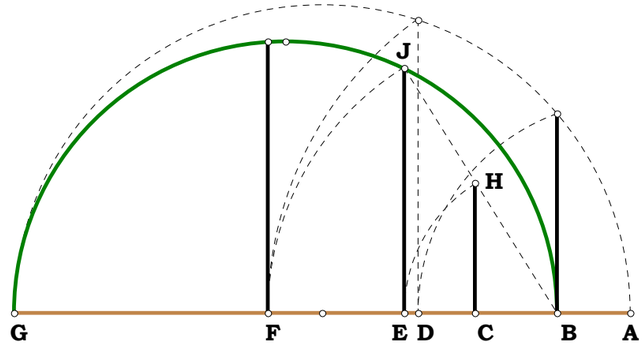
$$BF - \left[\left(N^3\right)^{\frac{1}{4}} - 1\right] = 0 \qquad BJ - \left[\left(N^3\right)^{\frac{1}{4}} - 1\right] = 0$$

$$BE - \frac{\left[\left(N^3\right)^{\frac{1}{4}} - 1\right]^2}{N-1} = 0 \qquad BH - \frac{\left[\left(N^3\right)^{\frac{1}{4}} - 1\right]^2}{N-1} = 0$$

$$BC - \frac{\left[\left(N^3\right)^{\frac{1}{4}} - 1\right]^3}{(N-1)^2} = 0$$

## Pascal's Triangle With Exponential Division

**Plate C1**



Given.

$X := 3 \qquad Y := 17$

$AB := \frac{X}{Y} \qquad N := \frac{Y}{X}$

122195C2

Descriptions.

$AG := AB \cdot N \qquad BG := AG - AB \qquad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \qquad BF := AF - AB$

$BJ := BF \qquad BE := \frac{BJ \cdot BF}{BG} \qquad BH := BE \qquad BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^2 - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0$

$\frac{BG}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$

Definitions.

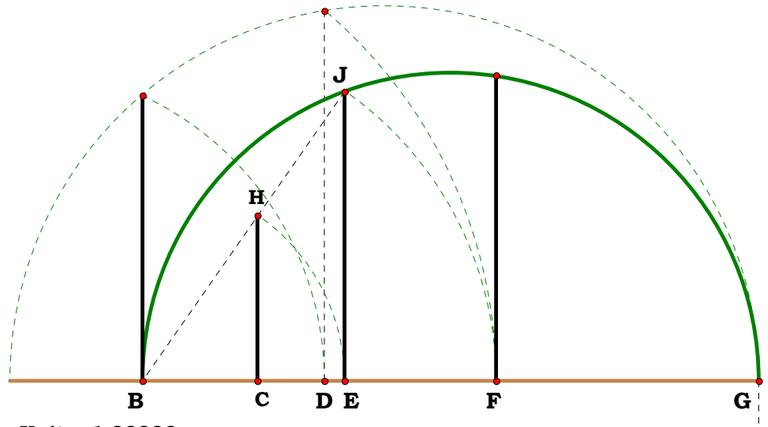
$AG - 1 = 0 \qquad BG - \frac{Y-X}{Y} = 0 \qquad AF - \left(\frac{X}{Y}\right)^{\frac{1}{4}} = 0$

$BF - \frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0$

$BJ - \frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0$

## Pascal's Triangle With Exponential Division

### Plate C2




Unit = 1.00000
XY = 0.17647
X = 3.00000
Y = 17.00000

BG = 0.82353
AF = 0.64814
BF = 0.47167
BJ = 0.47167

BE = 0.27014
BH = 0.27014
BC = 0.15472

X

1

$\frac{Y-X}{Y}$-BG = 0.00000

$\frac{X}{Y}^{\frac{1}{4}}$-AF = 0.00000

$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y} = 0.47167$

$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y}$-BF = 0.00000

$\frac{X^{\frac{1}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y}$-BJ = 0.00000

$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^2}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)} = 0.27014$

$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^2}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)}$-BE = 0.00000

$\frac{\sqrt{X} \cdot \left(\sqrt{Y} - \sqrt{X}\right) \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^2}{Y \cdot (Y-X) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)}$-BH = 0.00000

$\frac{X^{\frac{3}{4}} \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right)^2 \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)^3}{Y \cdot \left(\sqrt{X} + \sqrt{Y}\right)^2 \cdot \left(Y^{\frac{1}{4}} - X^{\frac{1}{4}}\right) \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right)^2}$-BC = 0.00000

$$BE - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0$$

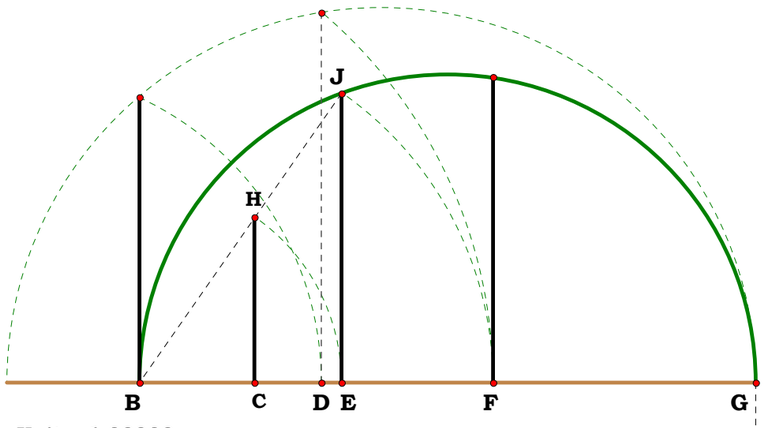
$$BH - \frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0$$

$$BC - \frac{X^{\frac{3}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)^2\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^3}{\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^2\cdot Y\cdot\left(\sqrt{X}+\sqrt{Y}\right)^2\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)} = 0$$

$$\frac{BG}{BF} - \frac{\left(\sqrt{X}+\sqrt{Y}\right)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)}{X^{\frac{1}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)} = 0$$

$$\frac{BG}{BE} - \frac{\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^2\cdot\left(\sqrt{X}+\sqrt{Y}\right)^2}{\sqrt{X}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2} = 0$$

$$\frac{BG}{BC} - \frac{\left(\sqrt{X}+\sqrt{Y}\right)^3\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^3}{X^{\frac{3}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^3} = 0$$




Unit = 1.00000
XY = 0.17647
X = 3.00000
Y = 17.00000

BG = 0.82353
AF = 0.64814
BF = 0.47167
BJ = 0.47167

BE = 0.27014
BH = 0.27014
BC = 0.15472

X

1

$$\frac{Y-X}{Y}\text{-BG} = 0.00000$$

$$\frac{X^{\frac{1}{4}}}{Y}\text{-AF} = 0.00000$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y} = 0.47167$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y}\text{-BF} = 0.00000$$

$$\frac{X^{\frac{1}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{Y}\text{-BJ} = 0.00000$$

$$\frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)} = 0.27014$$

$$\frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)}\text{-BE} = 0.00000$$

$$\frac{\sqrt{X}\cdot\left(\sqrt{Y}-\sqrt{X}\right)\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^2}{Y\cdot(Y-X)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)}\text{-BH} = 0.00000$$

$$\frac{X^{\frac{3}{4}}\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)^2\cdot\left(\sqrt{X}+\sqrt{Y}+X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)^3}{Y\cdot\left(\sqrt{X}+\sqrt{Y}\right)^2\cdot\left(Y^{\frac{1}{4}}-X^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^2}\text{-BC} = 0.00000$$

Unit.

$AC := 1$

Given.

$N_1 := 9 \quad CE := N_1$

**122995A**

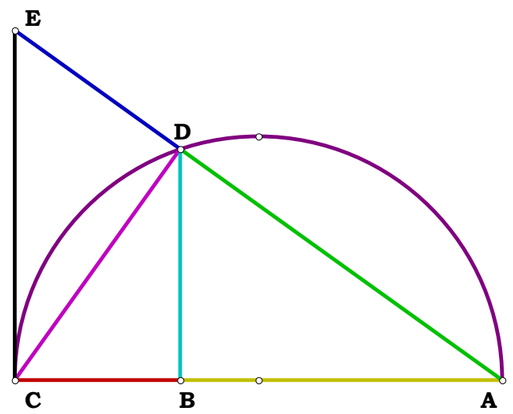
**Given AC and CE find BC.**

**Descriptions.**

$$AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$$

$$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$$

$$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$$




**Definitions.**

$$BC - \frac{N_1^2}{N_1^2 + 1} = 0 \quad BD - \frac{N_1}{N_1^2 + 1} = 0 \quad AB - \frac{1}{N_1^2 + 1} = 0$$

$$AD - \frac{1}{\sqrt{N_1^2 + 1}} = 0 \quad DE - \frac{N_1^2}{\sqrt{1 + N_1^2}} = 0$$

Given.

$X := 7 \quad CE := X$

$Y := 11 \quad AC := Y$

**122995B**

**Descriptions.**

$$AE := \sqrt{AC^2 + CE^2} \qquad AD := \frac{AC \cdot AC}{AE}$$

$$AB := \frac{AD^2}{AC} \qquad BC := AC - AB \qquad BD := \sqrt{AB \cdot BC}$$

$$CD := \sqrt{BC^2 + BD^2} \qquad DE := AE - AD$$

**Definitions.**

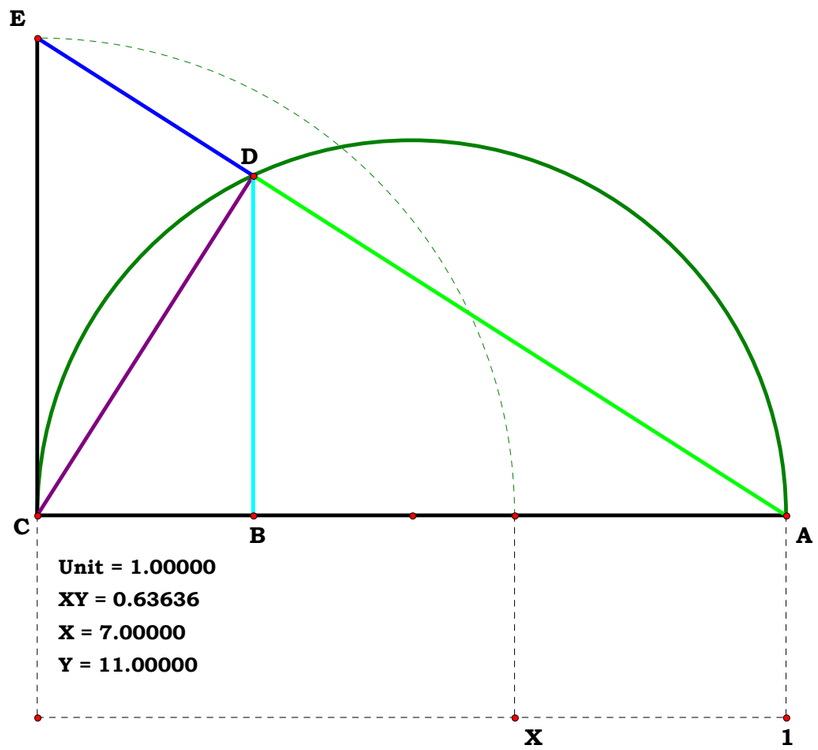
$$AE - \sqrt{Y^2 + X^2} = 0 \qquad AD - \frac{Y^2}{\sqrt{Y^2 + X^2}} = 0$$

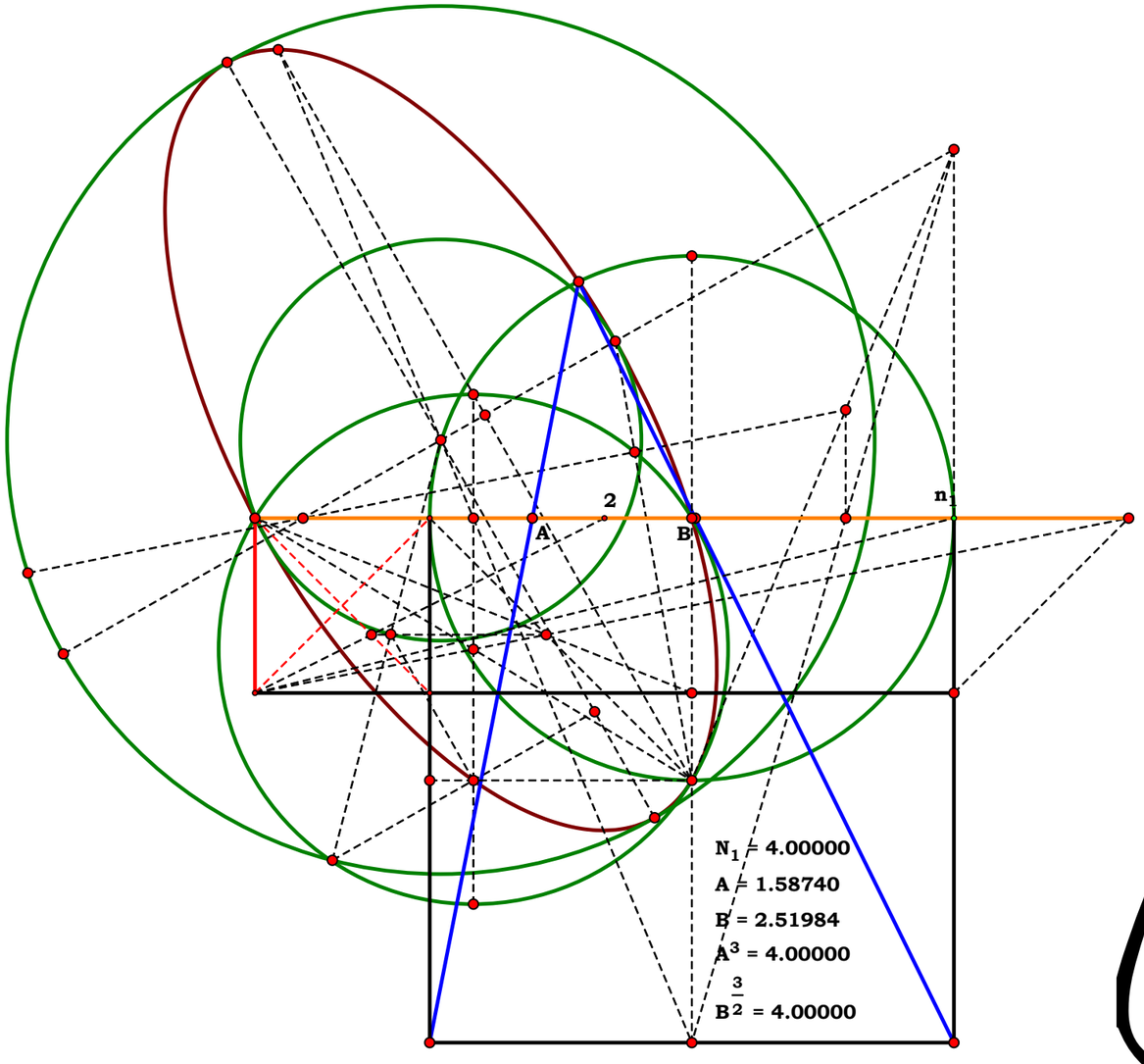
$$AB - \frac{Y^3}{X^2 + Y^2} = 0 \qquad BC - \frac{X^2 \cdot Y}{X^2 + Y^2} = 0$$

$$BD - \frac{X \cdot Y^2}{X^2 + Y^2} = 0 \qquad CD - \frac{X \cdot Y}{\sqrt{X^2 + Y^2}} = 0$$

$$DE - \frac{X^2}{\sqrt{X^2 + Y^2}} = 0$$

**Given AC and CE find BC.**



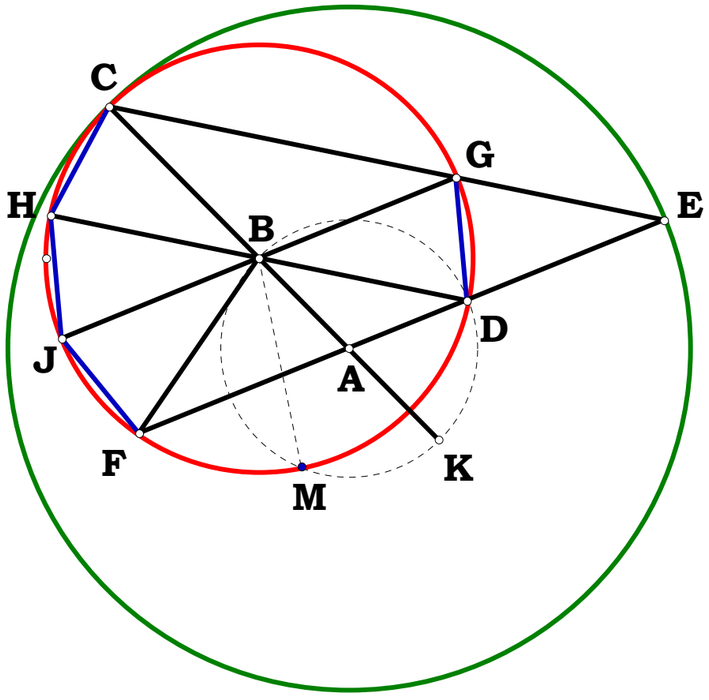



# The Delian Quest 1996

**John Clark**

010496

# The Archamedian Paper Trisector- Without the Numbers.




Given any circle AB.

Given any circle BC such that BC ≤ 2AB.

Construct AE such that AE = AC.

As AC = AB + BC and AD = AB  so too DE = BC.

Construct DH parallel to BD. Construct CE.

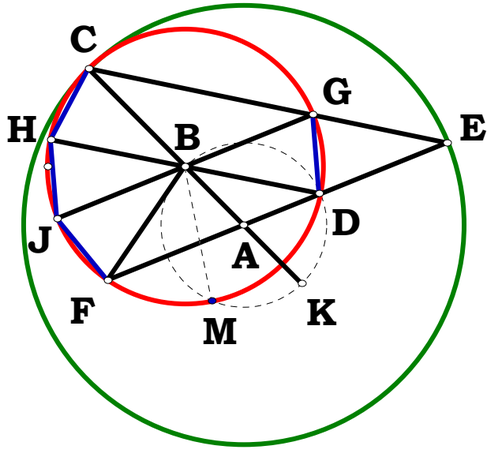
As AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD.

From here one can take two paths.

Construct GJ parallel to EF.

As CE is parallel to DH, DG = CH.

As GJ is parallel to EF, DG = FJ.




As ∠ HBJ is opposite and equal to ∠ GBD, DG = HJ, therefore ∠ DG is $\frac{1}{3}$ CF.

As CE is parallel to DH and EF is parallel to GJ,  EGBD is equilateral.

By construction DK = KM.

As DH is parallel to CE, CH = DG.

As DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.

Unit.

$AD := 2$

Given.

$N := AD$

010796

A rusty Compass construction for the duplication of the cube.

**Descriptions.**

$AB := \frac{AD}{2}$ $AG := \sqrt{2 \cdot AB^2}$ $AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $AC = 1.257079$

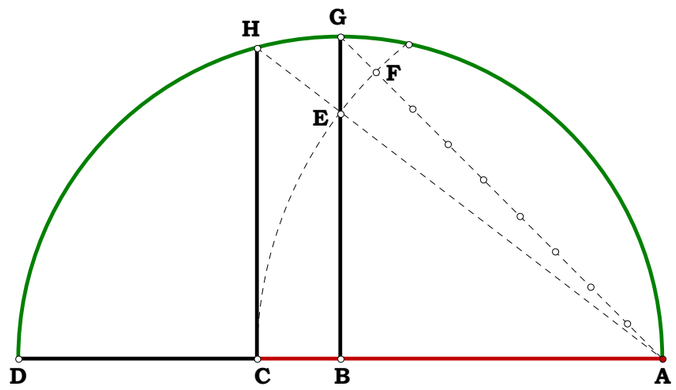
$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.259921$ $\frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002261$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.

**Definitions.**

$\frac{2^{\frac{1}{3}}}{2} \cdot \left(N^3\right)^{\frac{1}{3}}$ $\frac{9 \cdot 2^{\frac{5}{6}} \cdot \left(N^3\right)^{\frac{1}{3}}}{16 \cdot \sqrt{N^2}}$

Rusty Cubes



010896A1

**Given.**

$R_1 := 2$

$R_2 := 5$

$D := 4$

**Descriptions.**

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + JK) \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \quad GJ := FJ - FG \quad DI := \frac{DM}{2}$

$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

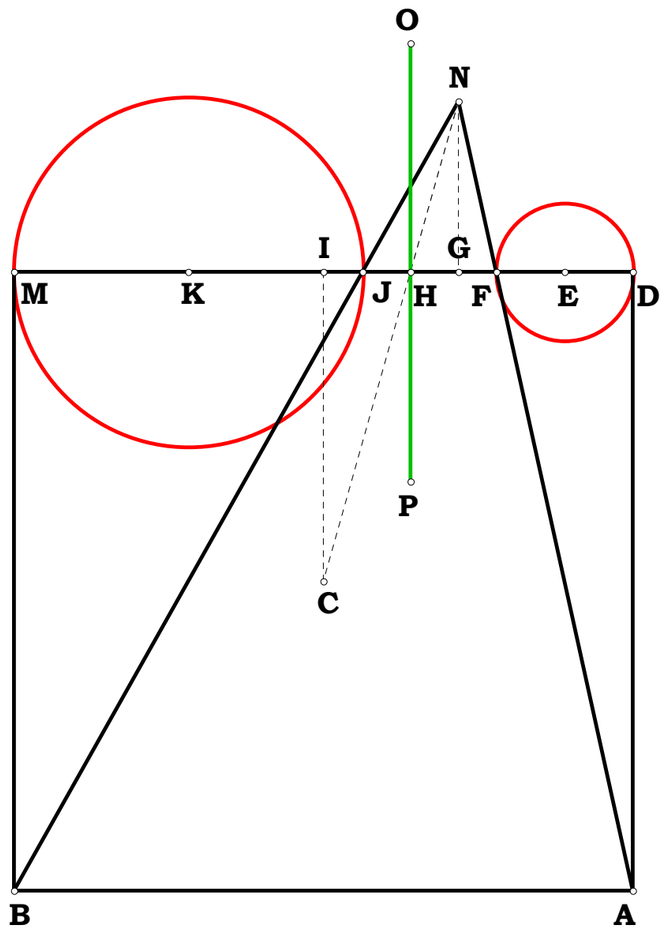
$HM := DM - DH$

**Definitions.**

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$

**Alternate Method Power Line**



010896A2

**Unit.**

$BD := 1$

**Given.**

$W := 6 \quad Y := 2$

$X := 20 \quad Z := 17$

**Descriptions.**

$$AB := \frac{W}{X} \qquad CD := \frac{Y}{Z} \qquad AC := BD - (AB + CD)$$

$$MN := AC - (AB + CD) \qquad BM := 2 \cdot AB \qquad DN := 2 \cdot CD$$

$$NO := \frac{DN \cdot MN}{BD - MN} \qquad MO := MN - NO \qquad DE := \frac{BD}{2}$$

$$DO := DN + NO \qquad EO := DE - DO \qquad EF := DE$$

$$JO := \frac{BD \cdot NO}{DN} \qquad KO := \frac{EO \cdot JO}{EF + JO}$$

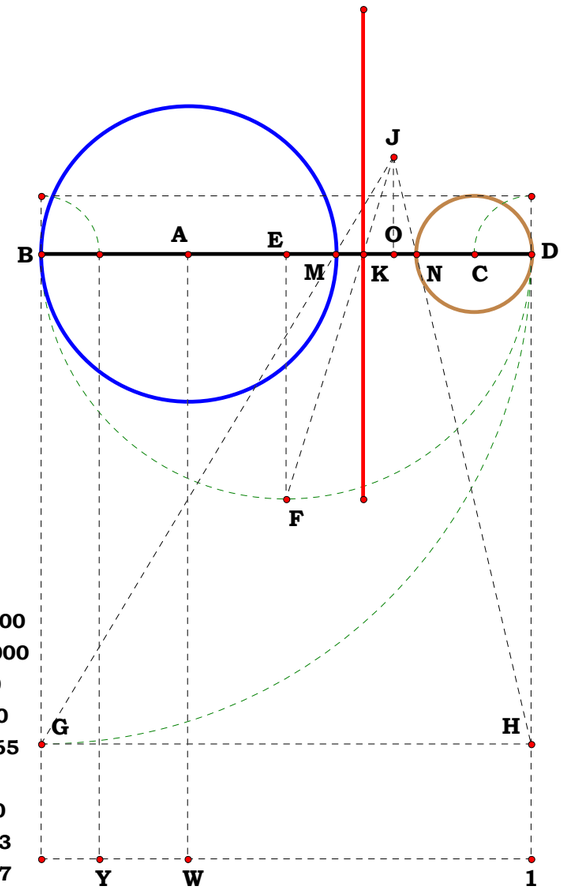
$$DK := DO + KO \qquad BK := BD - DK$$

$DK = 0.343434 \quad BK = 0.656566$

**Definitions.**

$$DK - \frac{Z \cdot (2 \cdot W - X)}{2 \cdot [W \cdot Z + X \cdot (Y - Z)]} = 0 \qquad BK - \frac{X \cdot (2 \cdot Y - Z)}{2 \cdot [W \cdot Z + X \cdot (Y - Z)]} = 0$$

## Alternate Method Power Line




Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.11765
Y = 2.00000
Z = 17.00000
DK = 0.34343
BK = 0.65657

$$\frac{Z \cdot (2 \cdot W - X)}{2 \cdot (W \cdot Z + X \cdot (Y - Z))} - DK = 0.00000$$

$$\frac{X \cdot (2 \cdot Y - Z)}{2 \cdot (W \cdot Z + X \cdot (Y - Z))} - BK = 0.00000$$

$$AB - \frac{W}{X} = 0 \qquad CD - \frac{Y}{Z} = 0 \qquad AC - \frac{X\cdot(Z-Y) - W\cdot Z}{X\cdot Z} = 0$$

$$MN - \frac{X\cdot Z - 2\cdot(X\cdot Y + W\cdot Z)}{X\cdot Z} = 0 \qquad BM - \frac{2\cdot W}{X} = 0$$

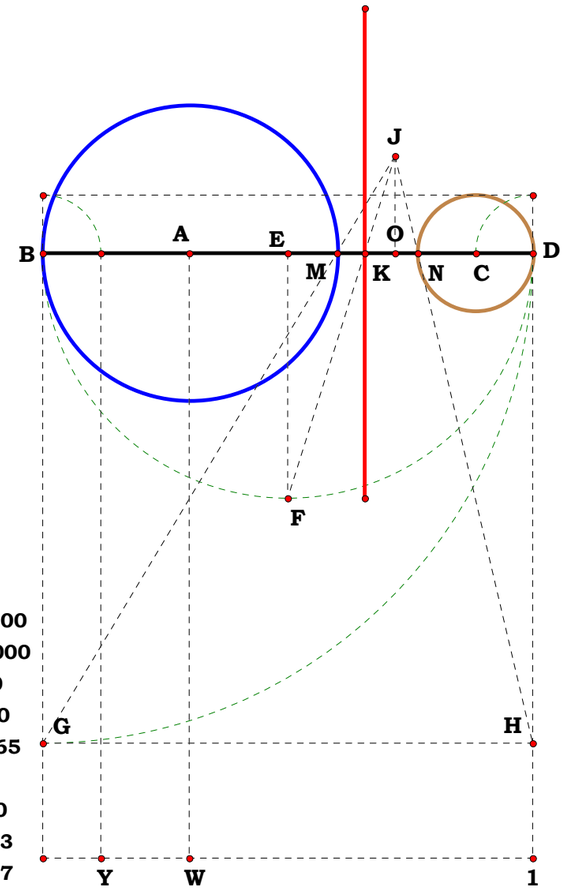
$$DN - \frac{2\cdot Y}{Z} = 0 \qquad NO - \frac{Y\cdot[X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)]}{Z\cdot(W\cdot Z + X\cdot Y)} = 0$$

$$MO - \frac{W\cdot[X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)]}{X\cdot(W\cdot Z + X\cdot Y)} = 0 \qquad DE - \frac{1}{2} = 0$$

$$DO - \frac{X\cdot Y}{W\cdot Z + X\cdot Y} = 0 \qquad EO - \frac{W\cdot Z - X\cdot Y}{2\cdot(W\cdot Z + X\cdot Y)} = 0$$

$$JO - \frac{X\cdot Z - 2\cdot(W\cdot Z + X\cdot Y)}{2\cdot(W\cdot Z + X\cdot Y)} = 0 \qquad EF - \frac{1}{2} = 0$$

$$KO - \frac{(W\cdot Z - X\cdot Y)\cdot[2\cdot(W\cdot Z + X\cdot Y) - X\cdot Z]}{2\cdot(W\cdot Z + X\cdot Y)\cdot[W\cdot Z + X\cdot(Y - Z)]} = 0$$




Unit = 1.00000
W/X = 0.30000
W = 6.00000
X = 20.00000
Y/Z = 0.11765
Y = 2.00000
Z = 17.00000
DK = 0.34343
BK = 0.65657

$$\frac{Z\cdot(2\cdot W - X)}{2\cdot(W\cdot Z + X\cdot(Y - Z))} - DK = 0.00000$$

$$\frac{X\cdot(2\cdot Y - Z)}{2\cdot(W\cdot Z + X\cdot(Y - Z))} - BK = 0.00000$$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

010896B1

Descriptions.

$BG := AG - AB$

$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

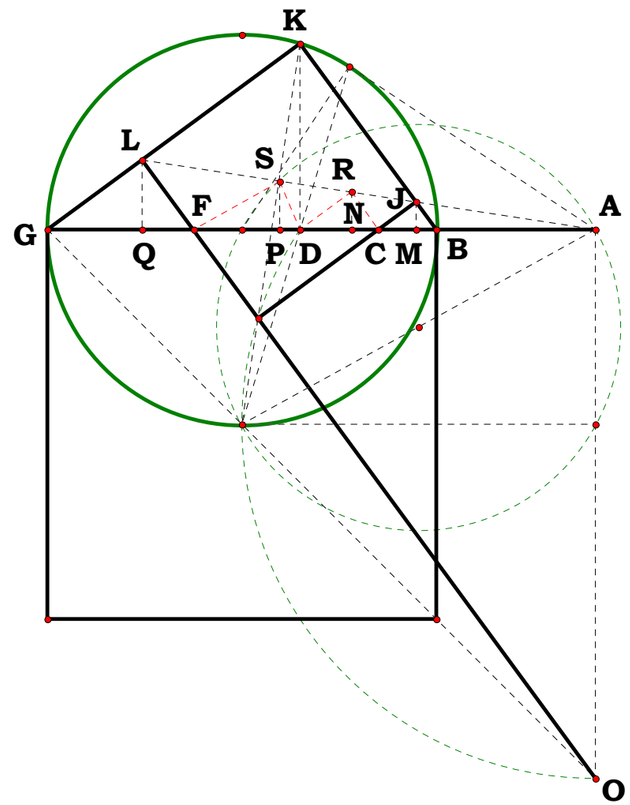
$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$\frac{GL}{BJ} = 5 \quad \frac{AG}{AB} = 5$

## Quad Roots

**The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.**



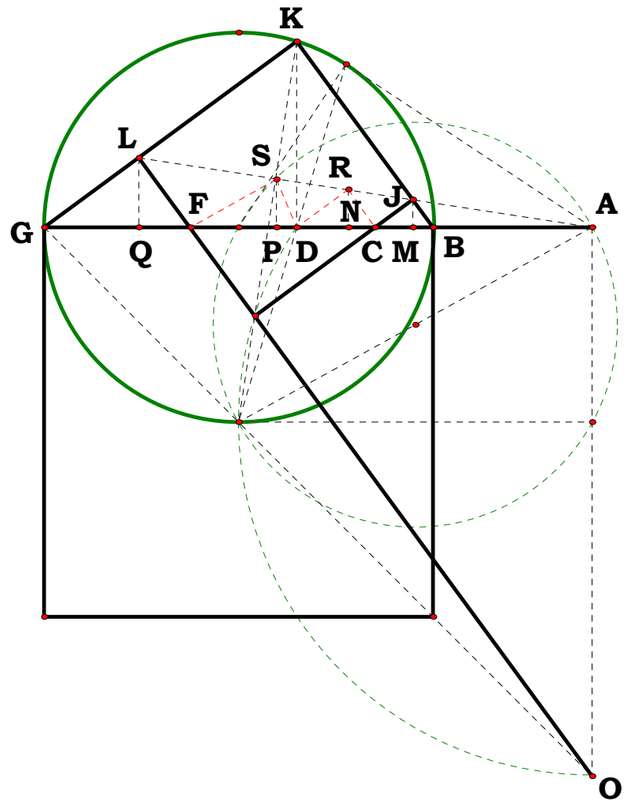
**Definitions.**

$$\frac{GK}{GL} - \left( \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} \right) = 0 \qquad \frac{BK}{BJ} - \left( N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG} \quad CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG}$$

$$\frac{BG}{BM} - \left( N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$\frac{BG}{CN} - \left( N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} \right) = 0$$

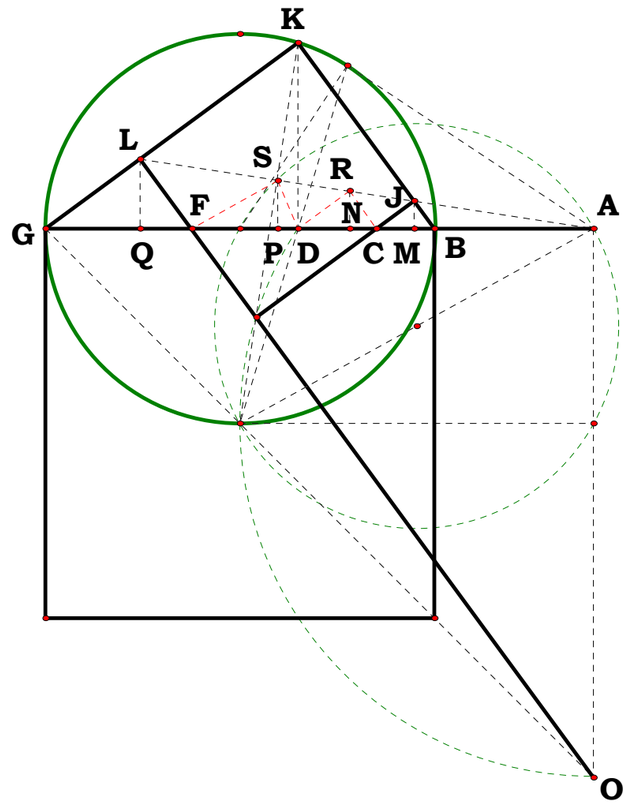


$$\frac{BG}{DP} - \left( N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} + N^{\frac{-2}{4}} \right) = 0$$

$$\frac{BG}{FQ} - \left( N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} + N^{\frac{-1}{4}} + N^{\frac{-2}{4}} + N^{\frac{-3}{4}} \right) = 0$$

$$\frac{AG}{BM} - \left( \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0 \qquad \frac{AG}{CN} - \left( \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0$$

$$\frac{AG}{DP} - \left( \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0 \qquad \frac{AG}{FQ} - \left( \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0$$



010896B2

**Descriptions.**

**Unit.**

$AB := 1$

**Given.**

$X := 4$

$Y := 1$

$AC := \frac{X}{Y}$ $BC := AC - AB$ $BD := \frac{BC}{2}$ $AD := AB + BD$

$AE := \sqrt{AD^2 - BD^2}$ $DE := AD - AE$ $BE := AE - AB$

$CE := BC - BE$ $EH := \sqrt{BE \cdot CE}$ $EJ := \frac{DE \cdot EH}{BD + EH}$

$AJ := AE + EJ$ $DJ := DE - EJ$ $JK := \sqrt{BD^2 + DJ^2}$

$JM := \frac{DJ \cdot AJ}{JK}$ $HK := \sqrt{(BD + EH)^2 + DE^2}$

$MH := HK - (JK + JM)$ $OR := 2 \cdot MH$ $BH := \sqrt{EH^2 + BE^2}$

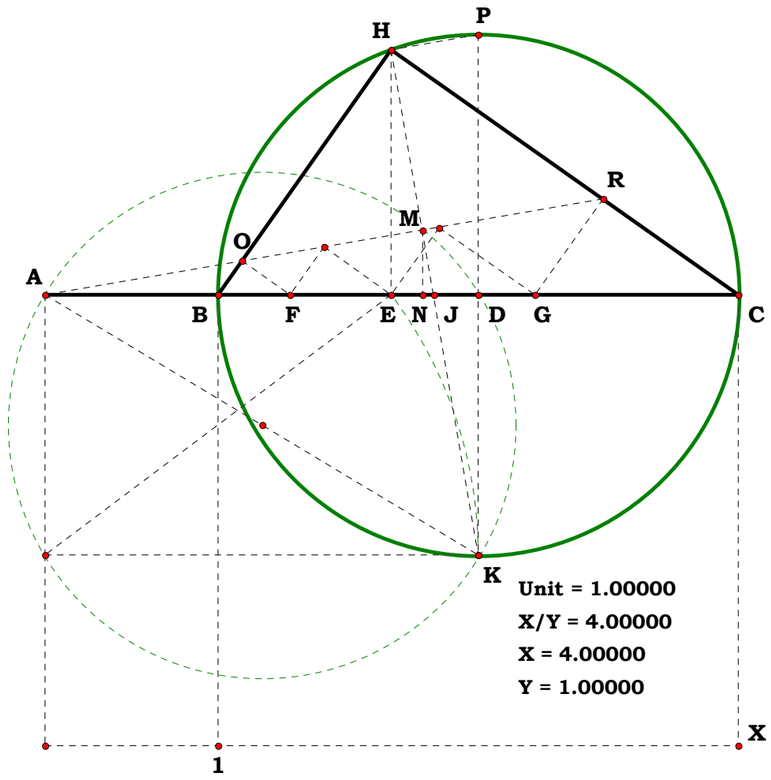
$HO := \sqrt{2 \cdot MH^2}$ $BO := BH - HO$ $CH := \sqrt{CE^2 + EH^2}$

$BF := \frac{BC \cdot BO}{BH}$ $AF := AB + BF$ $CR := CH - HO$

$CG := \frac{BC \cdot CR}{CH}$ $AG := AC - CG$

$AC^{\frac{1}{4}} - AF = 0$ $AC^{\frac{2}{4}} - AE = 0$ $AC^{\frac{3}{4}} - AG = 0$

## Quad Roots




$\frac{X}{Y} = 4.00000$

$AF = 1.41421$

$AE = 2.00000$

$AG = 2.82843$

$AC = 4.00000$

$\frac{X}{Y}^{\frac{1}{4}} - AF = 0.00000$

$\frac{X}{Y}^{\frac{2}{4}} - AE = 0.00000$

$\frac{X}{Y}^{\frac{3}{4}} - AG = 0.00000$

$\frac{X}{Y}^{\frac{4}{4}} - AC = 0.00000$

**Definitions.**

$$AC - \frac{X}{Y} = 0 \quad BC - \frac{X-Y}{Y} = 0 \quad BD - \frac{X-Y}{2\cdot Y} = 0 \quad AD - \frac{X+Y}{2\cdot Y} = 0 \quad AE - \frac{\sqrt{X}}{\sqrt{Y}} = 0$$

$$DE - \frac{\left(\sqrt{X}-\sqrt{Y}\right)^2}{2\cdot Y} = 0 \quad BE - \frac{\sqrt{X}-\sqrt{Y}}{\sqrt{Y}} = 0 \quad CE - \frac{\sqrt{X}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{Y} = 0$$

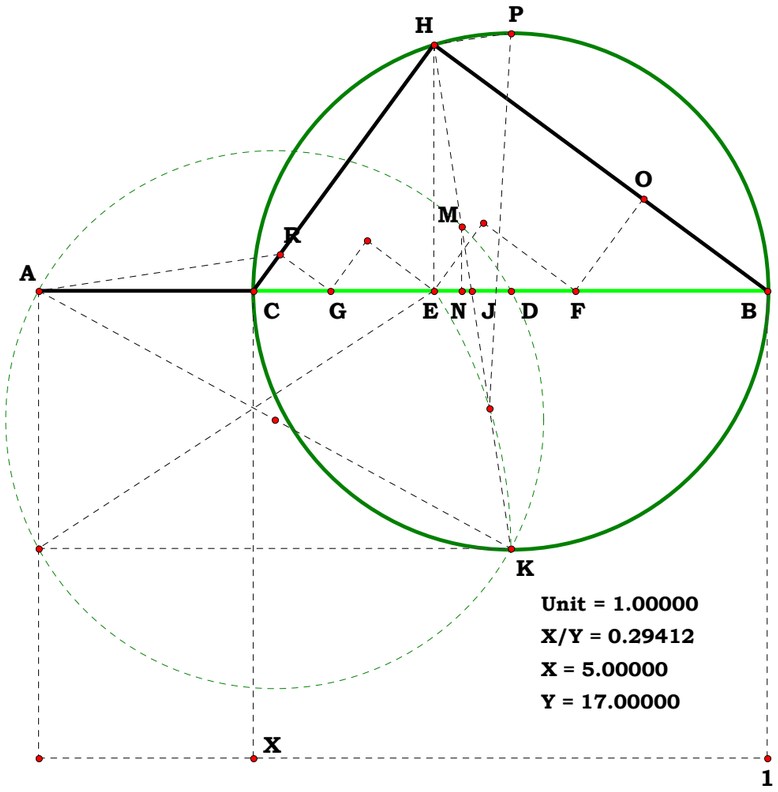
$$EH - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{X}-\sqrt{Y}\right)}{3\cdot Y^{\frac{1}{4}}} = 0 \quad EJ - \frac{X^{\frac{1}{4}}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)^2}{3\cdot Y^{\frac{1}{4}}} = 0 \quad AJ - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}-X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{3\cdot Y^{\frac{1}{4}}} = 0$$

$$DJ - \frac{\left(\sqrt{X}+\sqrt{Y}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)^2}{2\cdot Y} = 0 \quad JK - \frac{\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{3}{2}}}{\sqrt{2}\cdot Y} = 0$$

$$JM - \frac{\sqrt{2}\cdot X^{\frac{1}{4}}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\left(\sqrt{X}+\sqrt{Y}-X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)}{2\cdot Y^{\frac{3}{4}}\cdot\left(\sqrt{X}+\sqrt{Y}\right)^{\frac{1}{2}}} = 0$$

$$HK - \frac{\sqrt{2}\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)\cdot\sqrt{\sqrt{X}+\sqrt{Y}}\cdot\left(X^{\frac{1}{4}}+Y^{\frac{1}{4}}\right)^2}{2\cdot Y} = 0$$

$$MH - \frac{X^{\frac{1}{4}}\cdot\left(\sqrt{2}\cdot\sqrt{X}+\sqrt{2}\cdot\sqrt{Y}+\sqrt{2}\cdot X^{\frac{1}{4}}\cdot Y^{\frac{1}{4}}\right)\cdot\left(X^{\frac{1}{4}}-Y^{\frac{1}{4}}\right)}{2\cdot Y^{\frac{3}{4}}\cdot\sqrt{\sqrt{X}+\sqrt{Y}}} = 0$$




$\frac{X}{Y} = 0.29412$

AF = 0.73643

AE = 0.54233

AG = 0.39938

AC = 0.29412

$\frac{X}{Y}^{\frac{1}{4}}$-AF = 0.00000

$\frac{X}{Y}^{\frac{2}{4}}$-AE = 0.00000

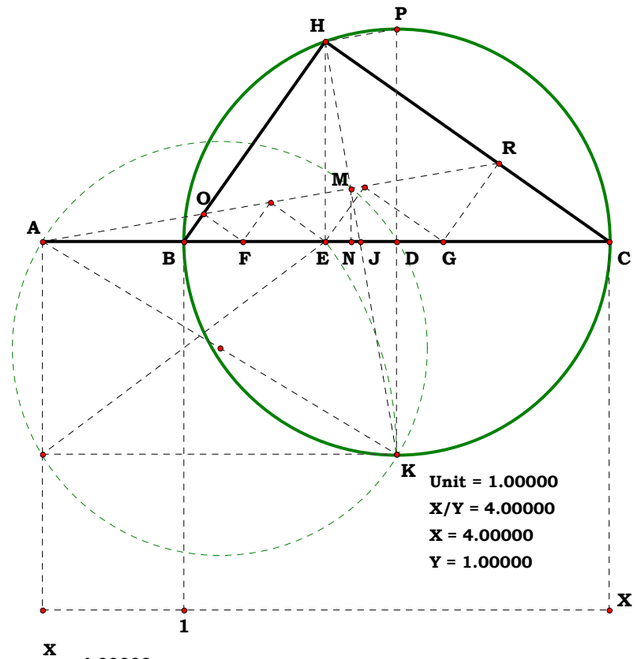
$\frac{X}{Y}^{\frac{3}{4}}$-AG = 0.00000

$\frac{X}{Y}^{\frac{4}{4}}$-AC = 0.00000

$$OR - \frac{X^{\frac{1}{4}} \cdot \left(\sqrt{2} \cdot \sqrt{X} + \sqrt{2} \cdot \sqrt{Y} + \sqrt{2} \cdot X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right) \cdot \left(X^{\frac{1}{4}} - Y^{\frac{1}{4}}\right)}{Y^{\frac{3}{4}} \cdot \sqrt{\sqrt{X} + \sqrt{Y}}} = 0$$

$$BH - \frac{\sqrt{\sqrt{X} + \sqrt{Y}} \cdot \left(\sqrt{X} - \sqrt{Y}\right)}{Y^{\frac{3}{4}}} = 0$$

$$HO - \frac{\left(X^{\frac{1}{4}}\right) \cdot \left(X^{\frac{1}{4}} - Y^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y^{\frac{3}{4}} \cdot \sqrt{\sqrt{X} + \sqrt{Y}}} = 0$$




$$BO - \frac{X^{\frac{1}{4}} - Y^{\frac{1}{4}}}{\sqrt{\sqrt{X} + \sqrt{Y}}} = 0 \qquad CH - \frac{\sqrt{\sqrt{X} \cdot \left(\sqrt{X} + \sqrt{Y}\right)} \cdot \left(\sqrt{X} - \sqrt{Y}\right)}{Y} = 0 \qquad BF - \frac{X^{\frac{1}{4}} - Y^{\frac{1}{4}}}{Y^{\frac{1}{4}}} = 0 \qquad AF - \frac{X^{\frac{1}{4}}}{Y^{\frac{1}{4}}} = 0$$

$\frac{X}{Y}$ = 4.00000

AF = 1.41421

AE = 2.00000

AG = 2.82843

AC = 4.00000

$\frac{X}{Y}^{\frac{1}{4}}$-AF = 0.00000

$\frac{X}{Y}^{\frac{2}{4}}$-AE = 0.00000

$\frac{X}{Y}^{\frac{3}{4}}$-AG = 0.00000

$\frac{X}{Y}^{\frac{4}{4}}$-AC = 0.00000

$$CR - \frac{\left(X^{\frac{1}{4}} - Y^{\frac{1}{4}}\right) \cdot \sqrt{\sqrt{X} + \sqrt{Y}} \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right) \cdot \sqrt{X + \sqrt{X} \cdot \sqrt{Y}} - \left(X^{\frac{1}{4}} - Y^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} \cdot \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{3}{4}} + X^{\frac{3}{4}} \cdot Y^{\frac{1}{4}}\right)}{Y \cdot \sqrt{\sqrt{X} + \sqrt{Y}}} = 0$$

$$CG - \frac{\left(X^{\frac{1}{4}} - Y^{\frac{1}{4}}\right) \cdot \left(\sqrt{X} + \sqrt{Y}\right) \cdot \left[\sqrt{\sqrt{X} + \sqrt{Y}} \cdot \left(X^{\frac{1}{4}} + Y^{\frac{1}{4}}\right) \cdot \sqrt{X + \sqrt{X} \cdot \sqrt{Y}} - \left(\sqrt{X} \cdot \sqrt{Y} + X^{\frac{1}{4}} \cdot Y^{\frac{3}{4}} + X^{\frac{3}{4}} \cdot Y^{\frac{1}{4}}\right)\right]}{Y \cdot \sqrt{\sqrt{X} + \sqrt{Y}} \cdot \sqrt{\sqrt{X} \cdot \left(\sqrt{X} + \sqrt{Y}\right)}} = 0$$

$$AG - \frac{Y^{\frac{3}{4}} \cdot \sqrt{X + \sqrt{X} \cdot \sqrt{Y}} + \left(X - X^{\frac{1}{4}} \cdot Y^{\frac{3}{4}}\right) \cdot \sqrt{\sqrt{X} + \sqrt{Y}}}{Y^{\frac{3}{4}} \cdot \sqrt{X + \sqrt{X} \cdot \sqrt{Y}}} = 0$$

**This seems to be about the best Mathcad 15 can do for AG.**

011396A

**Descriptions.**

**Unit.**

$AD := 1$

**Given.**

$N_1 := 5 \qquad N_a := 2 .. N_1$

$N_2 := 7 \qquad N_b := 2 .. N_2$

$AB := \frac{AD}{N_1} \qquad BD := AD - AB \qquad BG := \sqrt{AB \cdot BD} \qquad BE := \frac{BG}{N_2}$

$BC := \frac{BD \cdot BE}{BG} \qquad AE := \sqrt{AB^2 + BE^2} \qquad AC := AB + BC \qquad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \qquad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0 \qquad \frac{AF}{EF} = 1.458333$

**Definitions.**

$SeriesAF_{N_a, N_b} := \frac{N_a \cdot N_b}{(N_a - 1) \cdot (N_b - 1)}$

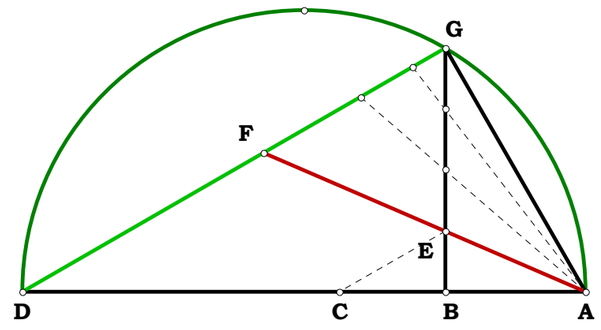
$$SeriesAF = \begin{pmatrix} 4 & 3 & 2.666667 & 2.5 & 2.4 & 2.333333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.666667 & 2 & 1.777778 & 1.666667 & 1.6 & 1.555556 \\ 2.5 & 1.875 & 1.666667 & 1.5625 & 1.5 & 1.458333 \end{pmatrix}$$

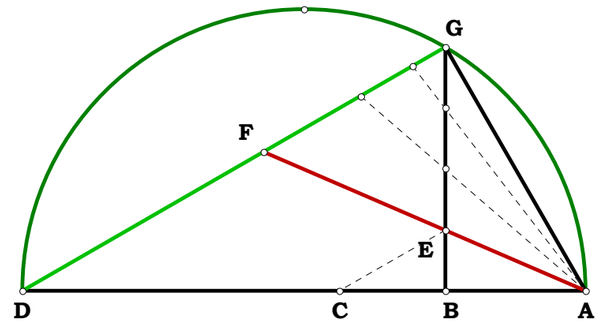
$DG := \sqrt{BD^2 + BG^2} \qquad CE := \sqrt{BC^2 + BE^2} \qquad DF := \frac{CE \cdot AD}{AC} \qquad GF := DG - DF$

$\frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0 \qquad SeriesDG_{N_a, N_b} := \frac{(N_b + N_a - 1)}{(N_b - 1)} \qquad \frac{DG}{GF} = 1.833333$

$$SeriesDG = \begin{pmatrix} 3 & 2 & 1.666667 & 1.5 & 1.4 & 1.333333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333333 & 2 & 1.8 & 1.666667 \\ 6 & 3.5 & 2.666667 & 2.25 & 2 & 1.833333 \end{pmatrix}$$

## Pyramid of Ratios VI, Moving the Point



Unit.

$AD := 1$

Given.

$W := 5 \quad Y := 14$

$X := 20 \quad Z := 20$

011396B

Descriptions.

$$AB := \frac{W}{X} \quad BD := AD - AB \quad BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG \cdot (Z - Y)}{Z}$$

$$BC := \frac{BD \cdot BE}{BG} \quad AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$$

$$EF := AF - AE \quad DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2}$$

$$DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF \quad \frac{AF}{EF} = 1.904762 \quad \frac{DG}{GF} = 2.714286$$

Definitions.

$$AB - \frac{W}{X} = 0 \quad BD - \frac{X - W}{X} = 0 \quad BG - \frac{\sqrt{W \cdot (X - W)}}{X} = 0$$

$$BE - \frac{\sqrt{W \cdot X - W^2} \cdot (Z - Y)}{X \cdot Z} = 0 \quad BC - \frac{(W - X) \cdot (Y - Z) \cdot \sqrt{W \cdot X - W^2}}{X \cdot Z \cdot \sqrt{-W \cdot (W - X)}} = 0$$

$$AE - \frac{\sqrt{W \cdot \left[Y^2 \cdot (X - W) + X \cdot Z^2 + 2 \cdot Y \cdot Z \cdot (W - X)\right]}}{X \cdot Z} = 0$$

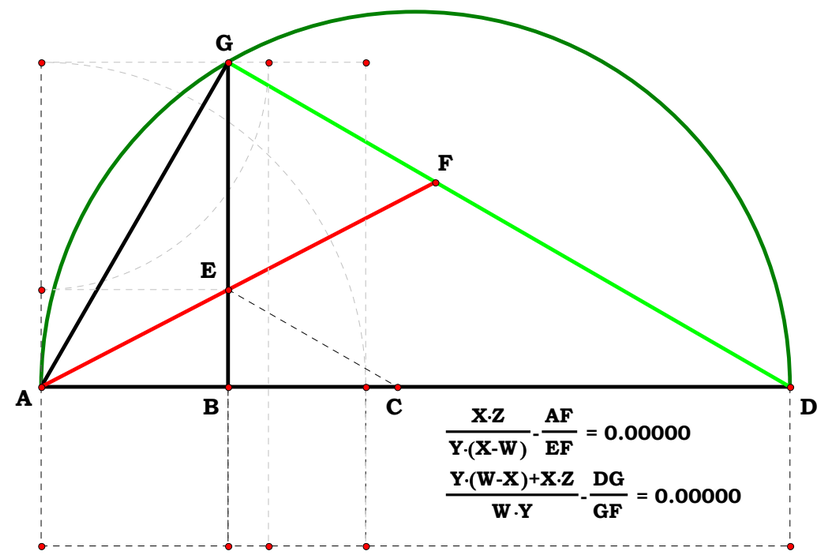
$$AC - \frac{Y \cdot (W - X) + X \cdot Z}{X \cdot Z} = 0 \quad AF - \frac{\sqrt{W \cdot \left[X \cdot Z^2 - Y \cdot (Y - 2 \cdot Z) \cdot (W - X)\right]}}{W \cdot Y - X \cdot Y + X \cdot Z} = 0$$

$$EF - \frac{\sqrt{W \cdot \left[Y^2 \cdot (X - W) + X \cdot Z^2 + 2 \cdot Y \cdot Z \cdot (W - X)\right]} \cdot Y \cdot (W - X)}{X \cdot Z \cdot [Y \cdot (X - W) - X \cdot Z]} = 0$$

$$DG - \frac{\sqrt{X - W}}{\sqrt{X}} = 0 \quad CE - \frac{(Z - Y) \cdot \sqrt{X - W}}{\sqrt{X} \cdot Z} = 0 \quad DF - \frac{\sqrt{X} \cdot \sqrt{X - W} \cdot (Z - Y)}{W \cdot Y - X \cdot Y + X \cdot Z} = 0 \quad GF - \frac{W \cdot Y \cdot \sqrt{X - W}}{\sqrt{X} \cdot (W \cdot Y - X \cdot Y + X \cdot Z)} = 0$$

$$\frac{AF}{EF} - \frac{X \cdot Z}{Y \cdot (X - W)} = 0 \quad \frac{AF}{EF} = 1.904762 \quad \frac{DG}{GF} - \frac{Y \cdot (W - X) + X \cdot Z}{W \cdot Y} = 0 \quad \frac{DG}{GF} = 2.714286$$

## Pyramid of Ratios VI, Moving the Point




Unit = 1.00000
W/X = 0.25000
W = 5.00000
X = 20.00000
Y/Z = 0.70000
Y = 14.00000
Z = 20.00000

$\frac{X \cdot Z}{Y \cdot (X - W)} - \frac{AF}{EF} = 0.00000$

$\frac{Y \cdot (W - X) + X \cdot Z}{W \cdot Y} - \frac{DG}{GF} = 0.00000$

W Y 1

011696

Unit.

$BD := 1$

$N_a := 3$

**Descriptions.**

$$BC := \frac{BD}{2} \quad CG := \frac{BC}{N_a} \quad BG := BC - CG \quad DG := BC + CG$$

$$GH := \sqrt{BG \cdot DG} \quad CF := BC \quad CO := \frac{CG \cdot CF}{GH + CF}$$

$$GJ := \frac{BG \cdot GH}{BD + GH} \quad GK := \frac{DG \cdot GH}{BD + GH} \quad JK := GJ + GK$$

$$GT := \frac{GH \cdot JK}{BD} \quad AG := \frac{CF \cdot GT}{CO} \quad AD := AG + DG$$

$$AJ := AG - GJ \quad AK := AG + GK \quad AB := AD - BD$$

$$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} - AJ = 0 \quad \left(AB \cdot AD^2\right)^{\frac{1}{3}} - AK = 0$$

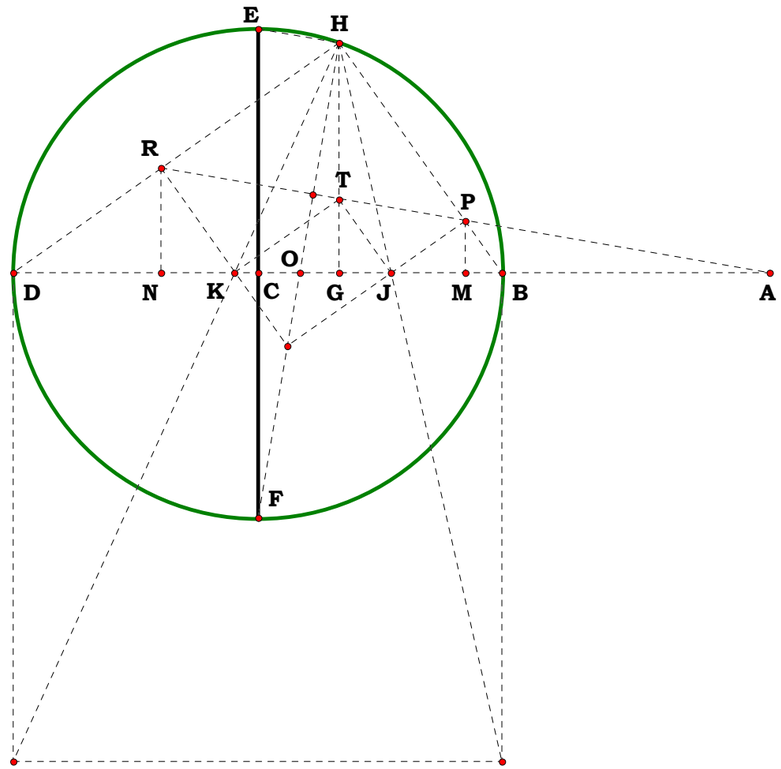
$$BH := \sqrt{GH^2 + BG^2} \quad DH := \sqrt{GH^2 + DG^2}$$

$$BJ := BG - GJ \quad DK := DG - GK$$

$$DR := \frac{DH \cdot DK}{BD} \quad BP := \frac{BH \cdot BJ}{BD} \quad BM := \frac{BG \cdot BJ}{BD} \quad KN := \frac{BG \cdot DK}{BD}$$

$$\frac{AD}{AB} = 2.828427 \quad \frac{DR}{BP} = 2.828427 \quad N := \frac{AD}{AB}$$

**The figure cuts the base in Cube Roots and provides some interesting ratios.**



**Definitions.**

$$\left(\frac{AB}{AD}\right)^{\frac{2}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}+\left(\frac{AB}{AD}\right)^{\frac{0}{3}}=2.207107 \qquad \frac{DH}{DR}=2.207107 \qquad \frac{N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}}{N^{\frac{2}{3}}}=2.207107$$

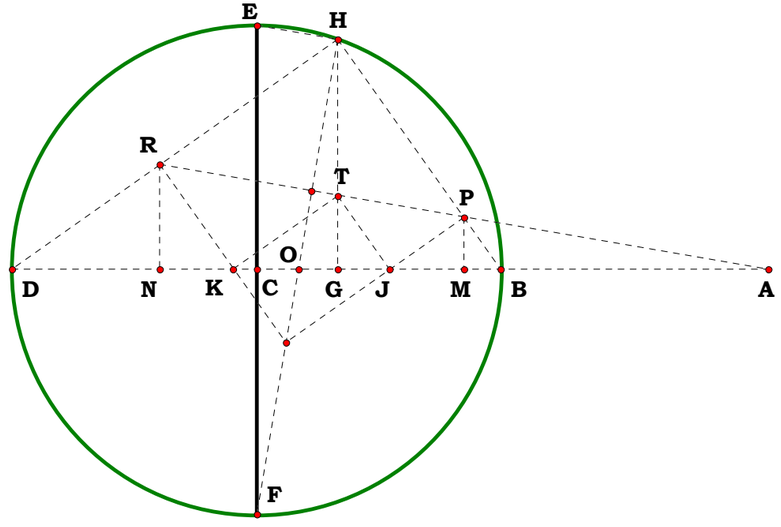
$$\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}=4.414214 \qquad \frac{BH}{BP}=4.414214 \qquad N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}=4.414214$$

$$\left(\frac{AD}{AB}\right)^{\frac{4}{3}}+\left(\frac{AD}{AB}\right)^{\frac{3}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}=13.242641$$

$$N^{\frac{4}{3}}+N^{\frac{3}{3}}+N^{\frac{2}{3}}+N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}=13.242641 \qquad \frac{BD}{BM}=13.242641$$

$$\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{3}{3}}+\left(\frac{AB}{AD}\right)^{\frac{0}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}=9.363961$$

$$N^{\frac{3}{3}}+N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}+\frac{1}{N^{\frac{1}{3}}}=9.363961 \qquad \frac{BD}{GJ}=9.363961$$



$$\left(\frac{AD}{AB}\right)^{\frac{2}{3}}+\left(\frac{AD}{AB}\right)^{\frac{1}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}+\left(\frac{AD}{AB}\right)^{\frac{0}{3}}+\left(\frac{AB}{AD}\right)^{\frac{1}{3}}+\left(\frac{AB}{AD}\right)^{\frac{2}{3}}=6.62132$$

$$N^{\frac{2}{3}}+N^{\frac{1}{3}}+N^{\frac{0}{3}}+N^{\frac{0}{3}}+\frac{1}{N^{\frac{1}{3}}}+\frac{1}{N^{\frac{2}{3}}}=6.62132$$

$$\frac{AD^{\frac{5}{3}}+AB^{\frac{2}{3}}\cdot AD}{AD^{\frac{1}{3}}\cdot AB^{\frac{4}{3}}-AB^{\frac{5}{3}}}=20.485281$$

$$\frac{AD}{BM}=20.485281$$

$$\frac{N^{\frac{5}{3}}+N}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=20.485281$$

$$\frac{AD^{\frac{4}{3}}+AB^{\frac{2}{3}}\cdot AD^{\frac{2}{3}}}{AD^{\frac{1}{3}}\cdot AB-AB^{\frac{4}{3}}}=14.485281$$

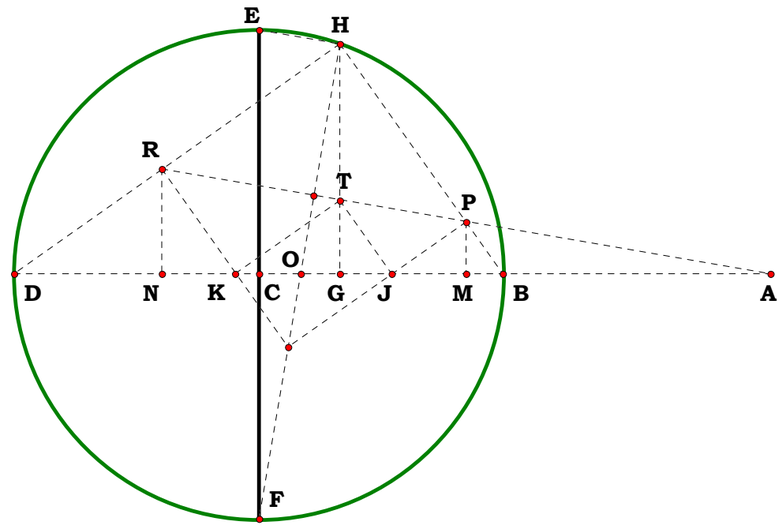
$$\frac{AD}{GJ}=14.485281$$

$$\frac{N^{\frac{4}{3}}+N^{\frac{2}{3}}}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=14.485281$$

$$\frac{AD+AB^{\frac{2}{3}}\cdot AD^{\frac{1}{3}}}{AD^{\frac{1}{3}}\cdot AB^{\frac{2}{3}}-AB}=10.242641$$

$$\frac{AD}{KN}=10.242641$$

$$\frac{N+N^{\frac{1}{3}}}{N^{\frac{1}{3}}-N^{\frac{0}{3}}}=10.242641$$



Given.

$AE := 5$

$AB := 3$

**011796A**

**Descriptions.**

$BE := AE - AB \quad BD := \frac{BE}{2} \quad DF := BD \quad DE := BD$

$AD := AB + BD \quad DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$CD := \frac{AD^2 - AF^2 + DF^2}{2 \cdot AD} \quad BC := BD - CD \quad CE := CD + DE$

$CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2}$

$\frac{AE}{AB} - \frac{EF}{BF} = 0$

**Definitions.**

$BE - (AE - AB) = 0 \quad BD - \frac{AE - AB}{2} = 0 \quad DF - \frac{AE - AB}{2} = 0 \quad DE - \frac{AE - AB}{2} = 0$

$AD - \frac{AB + AE}{2} = 0 \quad DH - \frac{AE - AB}{2} = 0 \quad AH - \frac{\sqrt{AB^2 + AE^2}}{\sqrt{2}} = 0 \quad AG - \frac{\sqrt{2} \cdot (AB + AE)^2}{4 \cdot \sqrt{AB^2 + AE^2}} = 0$

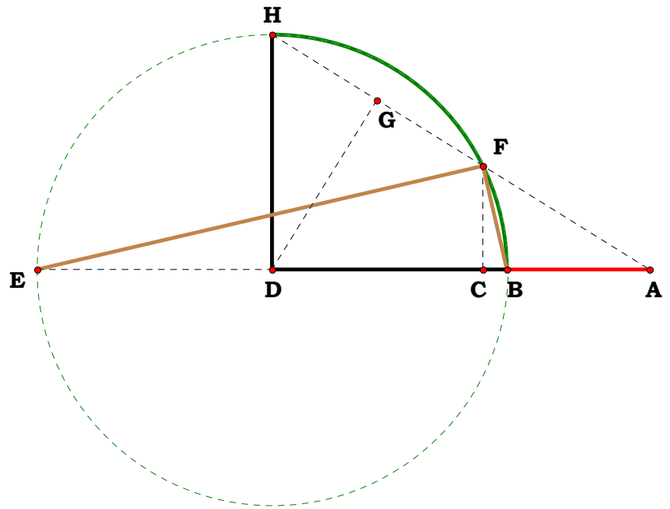
$GH - \frac{\sqrt{2} \cdot \left(\sqrt{2} \cdot AB - \sqrt{2} \cdot AE\right)^2}{8 \cdot \sqrt{AB^2 + AE^2}} = 0 \quad FG - \frac{\sqrt{2} \cdot \left(\sqrt{2} \cdot AB - \sqrt{2} \cdot AE\right)^2}{8 \cdot \sqrt{AB^2 + AE^2}} = 0 \quad AF - \frac{\sqrt{2} \cdot AB \cdot AE}{\sqrt{AB^2 + AE^2}} = 0$

$CD - \frac{(AB - AE)^2 \cdot (AB + AE)}{2 \cdot \left(AB^2 + AE^2\right)} = 0 \quad BC - \frac{AB^2 \cdot (AE - AB)}{AB^2 + AE^2} = 0 \quad CE - \frac{AE^2 \cdot (AE - AB)}{AB^2 + AE^2} = 0$

$CF - \frac{AB \cdot AE \cdot (AE - AB)}{\left(AB^2 + AE^2\right)} = 0 \quad BF - \frac{AB \cdot (AE - AB)}{\sqrt{AB^2 + AE^2}} = 0 \quad EF - \frac{AE \cdot (AE - AB)}{\sqrt{AB^2 + AE^2}} = 0$

**Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.**

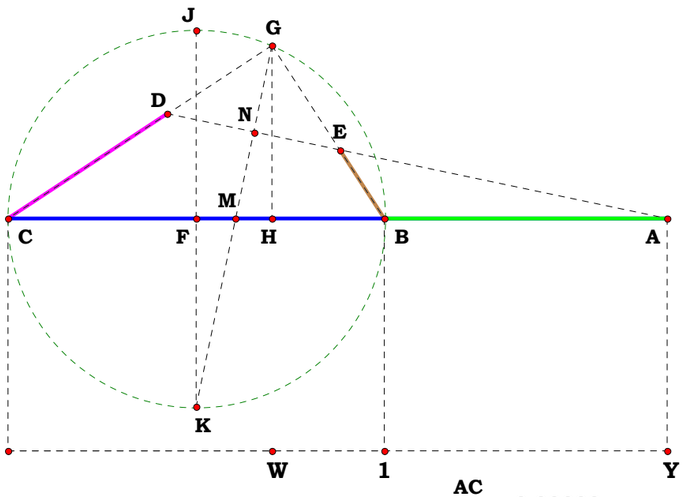
**Right Triangle In A Given Ratio**



011796B1

## Divide The Sides Of A Right Triangle In A Given Ratio

**Here was another junk write up, not only that, the original plate was defective, not absolutely correct, but, it takes time to learn how to say what one sees and at this revision time, all of it has to be fixed. What is being noted is that in this figure, there is a constant ratio, no matter where GH is on BC, AB is to AC as CD is to BE It is very simple, and originally I over obfuscated the whole thing. In short, it has something to say about the common angle. I suspect, now that I review it, it is very iimportant. There is, and always has been, a physical standard for ratio in this respect.**



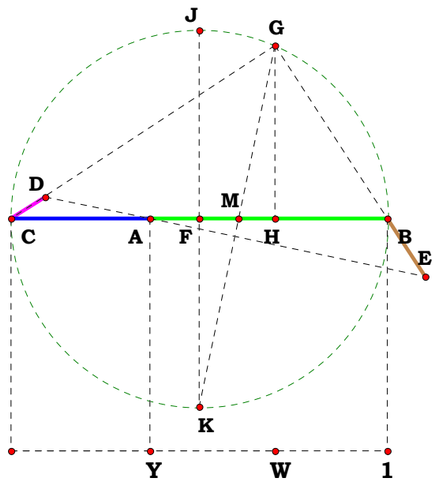

Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



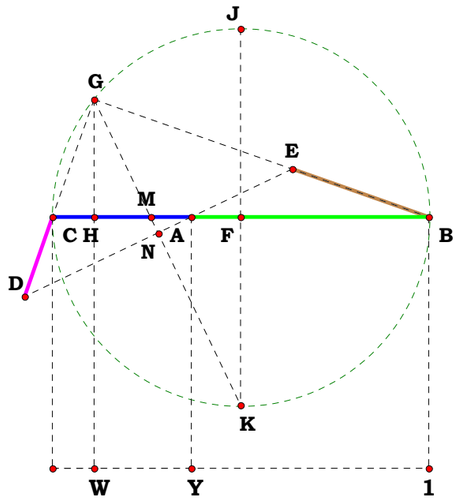

Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 0.67230 cm
BE = 1.15172 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$




Unit = 1.00000
W/X = 0.10988
W = 2.19759
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 1.42395 cm
BE = 2.43938 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

**Unit.**

$AB := 1$

**Given.**

$N_1 := 2.33333 \quad AC := N_1$

$N_2 := .933333 \quad CH := N_2$

**011796B1**

**Descriptions.**

$BC := AC - AB \quad BF := \frac{BC}{2} \quad BH := BC - CH \quad GH := \sqrt{CH \cdot BH}$

$FK := BF \quad FH := CH - BF \quad GK := \sqrt{FH^2 + (FK + GH)^2}$

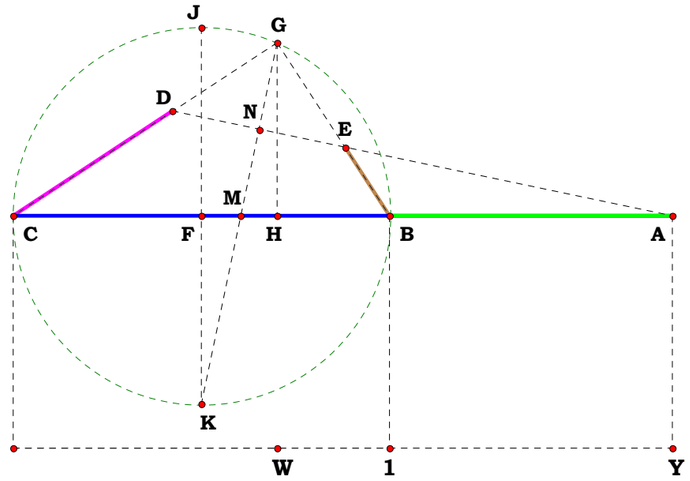
$FM := \frac{FH \cdot FK}{FK + GH} \quad HM := FH - FM \quad AH := AC - CH \quad AM := AH + HM$

$MN := \frac{FH \cdot AM}{GK} \quad KM := \sqrt{FK^2 + FM^2} \quad KN := KM + MN$

$GN := GK - KN \quad CG := \sqrt{CH^2 + GH^2} \quad BG := \sqrt{BH^2 + GH^2}$

$DG := \sqrt{2 \cdot GN^2} \quad EG := DG \quad CD := CG - DG \quad BE := BG - EG$

$\frac{AC}{AB} = 2.33333 \quad \frac{CD}{BE} = 2.33333 \quad \frac{AC}{AB} - \frac{CD}{BE} = 0$




Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm
CH = 4.41898 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 2.33333$

$\frac{CH}{AB} = 0.93333$

$\frac{CD}{AB} = 0.67419$

$\frac{BE}{AB} = 0.28894$

AB = 1.00000
AC = 2.33333
CH = 0.93333
CD = 0.67419
BE = 0.28894

**Definitions.**

$BC - (N_1 - 1) = 0 \quad BF - \frac{N_1 - 1}{2} = 0 \quad BH - (N_1 - N_2 - 1) = 0 \quad GH - \sqrt{N_2 \cdot (N_1 - N_2 - 1)} = 0 \quad FK - \frac{N_1 - 1}{2} = 0$

$FH - \frac{2 \cdot N_2 - N_1 + 1}{2} = 0 \quad GK - \frac{\sqrt{(N_1 - 1) \cdot \left(N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1\right)}}{\sqrt{2}} = 0 \quad FM - \frac{(N_1 - 1) \cdot (2 \cdot N_2 - N_1 + 1)}{2 \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} = 0 \quad HM - \frac{\sqrt{N_1 \cdot N_2 - N_2^2 - N_2} \cdot (2 \cdot N_2 - N_1 + 1)}{N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1} = 0$

$AH - (N_1 - N_2) = 0 \quad AM - \frac{N_2 - N_1 + N_1^2 - N_1 \cdot N_2 + \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} \cdot (N_1 + 1)}{2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} + N_1 - 1} = 0 \quad MN - \frac{\sqrt{2} \cdot (2 \cdot N_2 - N_1 + 1) \cdot \left[(N_1 + 1) \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} + N_2 - N_1 + N_1^2 - N_1 \cdot N_2\right]}{2 \cdot \sqrt{(N_1 - 1) \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} \cdot \left[N_1 + 2 \cdot \sqrt{-N_2 \cdot (N_2 - N_1 + 1)} - 1\right]} = 0$

$KM - \frac{\sqrt{(N_1 - 1)^3 \cdot \left(N_1 + 2 \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} - 1\right)}}{\sqrt{2 \cdot \left[(4 \cdot N_1 - 4) \cdot \sqrt{N_1 \cdot N_2 - N_2^2 - N_2} + N_1^2 + 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 4 \cdot N_2^2 - 4 \cdot N_2 + 1\right]}} = 0$

$$KN - (KM + MN) = 0$$

**On this equation, Matcad is going to expand it past its viewing ability. At the minimum of 10 percept page size, the results is well over 10 pages and it cannot reduce it. It would take hours to do this manually. A second approach, subing all the expressions that are reduced, and rebuild with that. See last page.**

$$KN - \frac{\sqrt{2}\cdot\left[\begin{array}{l}\sqrt{(N_1-1)^3\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\ \ldots \\ +\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}+N_1{}^2+4\cdot N_1\cdot N_2-2\cdot N_1-4\cdot N_2{}^2-4\cdot N_2+1\right]}\cdot(N_1-2\cdot N_2-1)\cdot\left[\begin{array}{l}N_1-N_2-\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-N_1{}^2\ \ldots \\ +\ N_1\cdot N_2-N_1\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}\end{array}\right]\end{array}\right]}{2\cdot\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}+N_1{}^2+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2{}^2-2\cdot N_1+1\right]}\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]} = 0$$

$$GN - \left[\frac{\sqrt{(N_1-1)\cdot\left(N_1+2\cdot\sqrt{N_1\cdot N_2-N_2{}^2-N_2}-1\right)}}{\sqrt{2}}\ \ldots + \frac{\sqrt{2}\cdot\left[\begin{array}{l}\sqrt{(N_1-1)^3\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\ \ldots \\ +\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}+N_1{}^2+4\cdot N_1\cdot N_2-2\cdot N_1-4\cdot N_2{}^2-4\cdot N_2+1\right]}\cdot(N_1-2\cdot N_2-1)\cdot\left[\begin{array}{l}N_1-N_2-\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-N_1{}^2\ \ldots \\ +\ N_1\cdot N_2-N_1\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}\end{array}\right]\end{array}\right]}{2\cdot\sqrt{\left[(4\cdot N_1-4)\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}+N_1{}^2+4\cdot N_1\cdot N_2-4\cdot N_2-4\cdot N_2{}^2-2\cdot N_1+1\right]}\cdot\sqrt{(N_1-1)\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\cdot\left[N_1+2\cdot\sqrt{\left[N_2\cdot(N_1-N_2-1)\right]}-1\right]}\right] = 0$$

$$CG - \sqrt{N_2{}^2+GH^2} = 0$$

**Mathcad 15 blows a tire here. After spreading it out, and then asking it to collet, it deletes evreything and comes back, "undefined." I might get back to this some other time, maybe. What you can take away from the distinction between Logic, such as Arithmetic and Algebra, and Analogic, the geometric figure, one of them does the computations instantly and concurrently, and logics require a lot of binary computation not required by analogic. Much of the current problem are the so called invalid principles used in logic today.**

$$BG - \sqrt{BH^2+GH^2} = 0$$

$$DG - \sqrt{2\cdot GN^2} = 0$$

$$EG - DG = 0$$

$$CD - (CG - DG) = 0$$

$$BE - (BG - EG) = 0$$

$(KM + MN)$

$MN = 0.31209$

$KM = 0.681031$

$(KM + MN) = 0.993121$

$$Z := \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]}$$

$$Y := \sqrt{\left(N_1 - 1\right)^3 \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]}$$

$$X := \sqrt{\left(N_1 - 1\right) \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]}$$

$$W := \sqrt{\left[\left(4 \cdot N_1 - 4\right) \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} + {N_1}^2 + 4 \cdot N_1 \cdot N_2 - 4 \cdot N_2 - 4 \cdot {N_2}^2 - 2 \cdot N_1 + 1\right]}$$

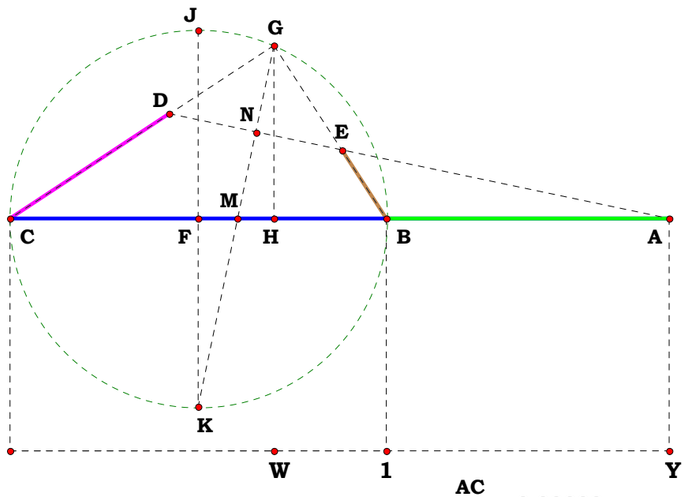
$$\frac{\sqrt{2} \cdot \left[Y \cdot \left(N_1 + 2 \cdot Z - 1\right) \cdot X + W \cdot \left(N_1 - 2 \cdot N_2 - 1\right) \cdot \left(N_1 - N_2 - Z - {N_1}^2 + N_1 \cdot N_2 - N_1 \cdot Z\right)\right]}{2 \cdot W \cdot X \cdot \left(N_1 + 2 \cdot Z - 1\right)} = 0.993121$$

$$\frac{\sqrt{2} \cdot \left[\begin{array}{l} \sqrt{\left(N_1 - 1\right)^3 \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]} \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right] \cdot \sqrt{\left(N_1 - 1\right) \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]} \ \ldots \\ + \sqrt{\left[\left(4 \cdot N_1 - 4\right) \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} + {N_1}^2 + 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 4 \cdot {N_2}^2 - 4 \cdot N_2 + 1\right]} \cdot \left(N_1 - 2 \cdot N_2 - 1\right) \cdot \left[\begin{array}{l} N_1 - N_2 - \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - {N_1}^2 \ \ldots \\ + \ N_1 \cdot N_2 - N_1 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} \end{array}\right] \end{array}\right]}{2 \cdot \sqrt{\left[\left(4 \cdot N_1 - 4\right) \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} + {N_1}^2 + 4 \cdot N_1 \cdot N_2 - 4 \cdot N_2 - 4 \cdot {N_2}^2 - 2 \cdot N_1 + 1\right]} \cdot \sqrt{\left(N_1 - 1\right) \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]} \cdot \left[N_1 + 2 \cdot \sqrt{\left[N_2 \cdot \left(N_1 - N_2 - 1\right)\right]} - 1\right]} = 0.993121$$

011796B2

## Divide The Sides Of A Right Triangle In A Given Ratio

**Here was another junk write up, not only that, the original plate was defective, not absolutely correct, but, it takes time to learn how to say what one sees and at this revision time, all of it has to be fixed. What is being noted is that in this figure, there is a constant ratio, no matter where GH is on BC, AB is to AC as CD is to BE It is very simple, and originally I over obfuscated the whole thing. In short, it has something to say about the common angle. I suspect, now that I review it, it is very iimportant. There is, and always has been, a physical standard for ratio in this respect.**



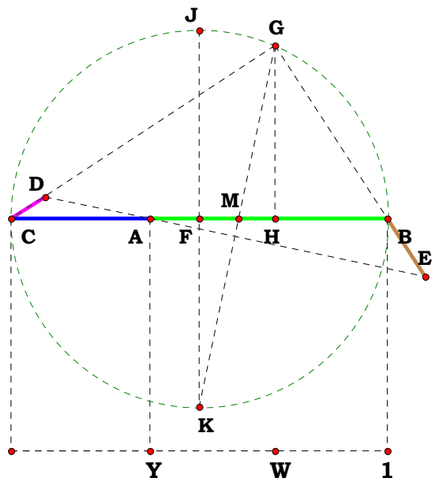

Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 1.75000
Y = 7.00000
Z = 4.00000

AB = 4.73462 cm
AC = 11.04746 cm
CD = 3.19202 cm
BE = 1.36801 cm

$\frac{AC}{AB} = 2.33333$

$\frac{CD}{BE} = 2.33333$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$



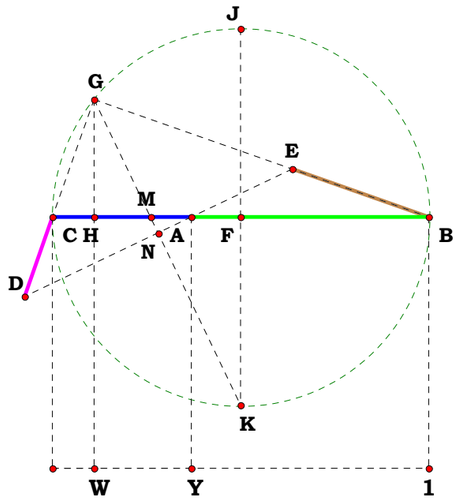

Unit = 1.00000
W/X = 0.70000
W = 14.00000
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 0.67230 cm
BE = 1.15172 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$




Unit = 1.00000
W/X = 0.10988
W = 2.19759
X = 20.00000
Y/Z = 0.36858
Y = 1.47432
Z = 4.00000

AB = 3.98604 cm
AC = 2.32679 cm
CD = 1.42395 cm
BE = 2.43938 cm

$\frac{AC}{AB} = 0.58373$

$\frac{CD}{BE} = 0.58373$

$\frac{AC}{AB} - \frac{CD}{BE} = 0.00000$

**Given.**

$W := 14 \quad Y := 7$

$X := 20 \quad Z := 4$

**011796B2**

**Descriptions.**

$AC := \frac{Y}{Z} \quad AB := \frac{Y - Z}{Z} \quad CH := \frac{W}{X}$

$BC := AC - AB \quad BF := \frac{BC}{2} \quad BH := BC - CH \quad GH := \sqrt{CH \cdot BH}$

$FK := BF \quad FH := CH - BF \quad GK := \sqrt{FH^2 + (FK + GH)^2}$

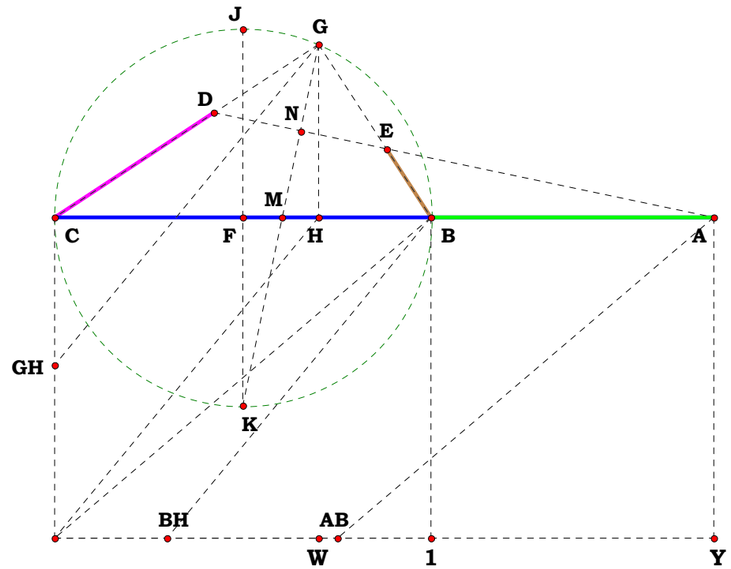
$FM := \frac{FH \cdot FK}{FK + GH} \quad HM := FH - FM \quad AH := AC - CH \quad AM := AH + HM$

$MN := \frac{FH \cdot AM}{GK} \quad KM := \sqrt{FK^2 + FM^2} \quad KN := KM + MN$

$GN := GK - KN \quad CG := \sqrt{CH^2 + GH^2} \quad BG := \sqrt{BH^2 + GH^2}$

$DG := \sqrt{2 \cdot GN^2} \quad EG := DG \quad CD := CG - DG \quad BE := BG - EG$

$\frac{AC}{AB} = 2.333333 \quad \frac{CD}{BE} = 2.333333 \quad \frac{AC}{AB} - \frac{CD}{BE} = 0 \quad \frac{AC}{AB} - \frac{Y}{Y - Z} = 0$



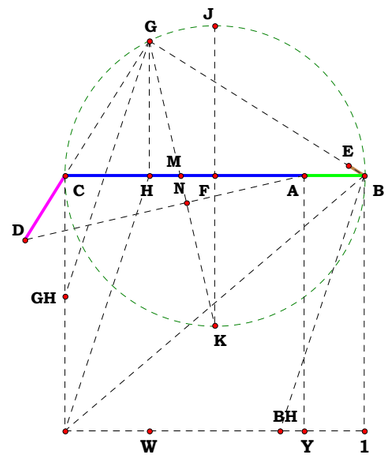

Unit = 1.00000 AC = 1.75000 AB = 0.75000
W/X = 0.70000 BC = 1.00000
W = 14.00000 CH = 0.70000
X = 20.00000 CM = 0.60436
Y/Z = 1.75000 CF = 0.50000
Y = 7.00000 BH = 0.30000
Z = 4.00000 GH = 0.45826

**To make this always work, when AB is less than BC:** $\frac{AC}{AB} - \frac{Y}{\sqrt{(Y - Z)^2}} = 0$

**This is because a difference and a sum are not the same thing.**

**As one can see, W and X, which form everything about where GH is, and the ratios on it, have disappeared, in short, these particulars have nothing to do with the outcome.**

**Perhaps if I have the time, I will rework the whole thing using** $AB - \frac{\sqrt{(Y - Z)^2}}{Z} = 0$ **and things might work out better because there may, perhaps be five or more operations of math instead of four.**




Unit = 1.00000 AC = 0.79776
W/X = 0.28088 BC = 1.00000
W = 5.61760 CH = 0.28088
X = 20.00000 CM = 0.38460
Y/Z = 0.79776 CF = 0.50000
Y = 3.19104 BH = 0.71912
Z = 4.00000 GH = 0.44943

$\frac{Y}{\sqrt{(Y-Z)^2}} = 3.94462$

$\frac{AC}{AB} - \frac{Y}{\sqrt{(Y-Z)^2}} = 0.00000$

$\frac{CD}{BE} - \frac{Y}{\sqrt{(Y-Z)^2}} = 0.00000$

**Definitions.**

$$AC - \frac{Y}{Z} = 0 \qquad AB - \frac{Y - Z}{Z} = 0 \qquad CH - \frac{W}{X} = 0 \qquad BC - 1 = 0 \qquad BF - \frac{1}{2} = 0$$

$$BH - \frac{X - W}{X} = 0 \qquad GH - \frac{\sqrt{W \cdot (X - W)}}{X} = 0 \qquad FK - \frac{1}{2} = 0 \qquad FH - \frac{2 \cdot W - X}{2 \cdot X} = 0$$

$$GK - \frac{\sqrt{X + 2 \cdot \sqrt{W \cdot X - W^2}}}{\sqrt{2 \cdot X}} = 0 \qquad FM - \frac{2 \cdot W - X}{2 \cdot \left[X + 2 \cdot \sqrt{-W \cdot (W - X)}\right]} = 0$$

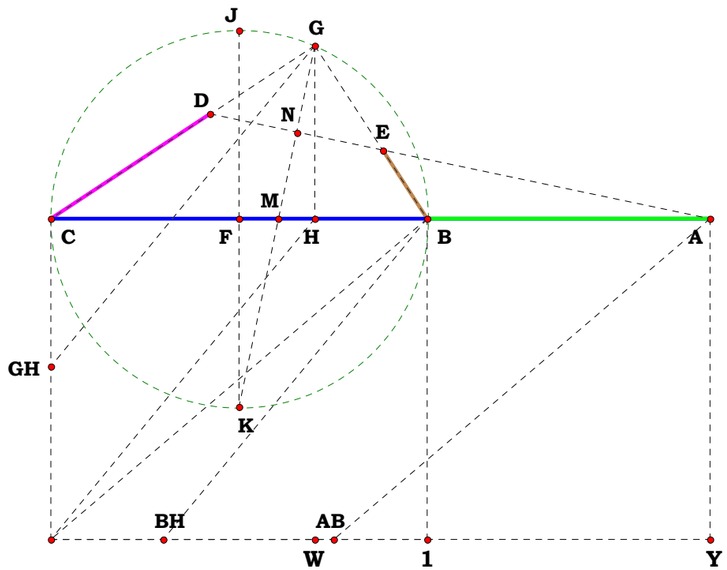
$$HM - \frac{(2 \cdot W - X) \cdot \sqrt{W \cdot X - W^2}}{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)} = 0 \qquad AH - \frac{X \cdot Y - W \cdot Z}{X \cdot Z} = 0 \qquad AM - \frac{\sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z) - W \cdot Z + X \cdot Y}{Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)} = 0$$

$$MN - \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}} = 0$$

$$KM - \frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} = 0$$

$$KN - \left[\frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} + \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}}\right] = 0$$

$$GN - \left[\frac{\sqrt{X + 2 \cdot \sqrt{W \cdot X - W^2}}}{\sqrt{2 \cdot X}} - \left[\frac{\sqrt{X \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)}}{\sqrt{2} \cdot \left[\sqrt{X^2 - 4 \cdot W^2 + 4 \cdot X \cdot \left(W + \sqrt{W \cdot X - W^2}\right)}\right]} + \frac{\sqrt{2} \cdot (X - 2 \cdot W) \cdot \left[W \cdot Z - X \cdot Y - \sqrt{W \cdot X - W^2} \cdot (2 \cdot Y - Z)\right]}{2 \cdot \sqrt{X} \cdot Z \cdot \left(X + 2 \cdot \sqrt{W \cdot X - W^2}\right)^{\frac{3}{2}}}\right]\right] = 0 \qquad CG - \frac{\sqrt{W}}{\sqrt{X}} = 0 \qquad BG - \frac{\sqrt{X - W}}{\sqrt{X}} = 0$$




| | | |
|---|---|---|
| Unit = 1.00000 | AC = 1.75000 | AB = 0.75000 |
| W/X = 0.70000 | BC = 1.00000 | |
| W = 14.00000 | CH = 0.70000 | |
| X = 20.00000 | CM = 0.60436 | |
| Y/Z = 1.75000 | CF = 0.50000 | |
| Y = 7.00000 | BH = 0.30000 | |
| Z = 4.00000 | GH = 0.45826 | |

**I am not even going to try, this time, to have Mathcad 15 simplify the following. Mathcad 15 crashes and the programmars nevrer imagined to have this program save backup and restore files. That function has to be manually set and it sometimes works.**

$$DG - \sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}} - \left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right] = 0$$

$$EG - \sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}} - \left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right] = 0$$

$$CD - \left[\frac{\sqrt{W}}{\sqrt{X}} - \sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}} - \left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]\right] = 0$$

$$BE - \left[\frac{\sqrt{X-W}}{\sqrt{X}} - \sqrt{2}\cdot\left[\frac{\sqrt{X+2\cdot\sqrt{W\cdot X-W^2}}}{\sqrt{2\cdot X}} - \left[\frac{\sqrt{X\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)}}{\sqrt{2}\cdot\left[\sqrt{X^2-4\cdot W^2+4\cdot X\cdot\left(W+\sqrt{W\cdot X-W^2}\right)}\right]} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[W\cdot Z-X\cdot Y-\sqrt{W\cdot X-W^2}\cdot(2\cdot Y-Z)\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left(X+2\cdot\sqrt{W\cdot X-W^2}\right)^{\frac{3}{2}}}\right]\right]\right] = 0$$

**Mathcad15 cannot simplify the equation, in essence, it cannot produce an algebraic result as easily as Arithmetic and geometry can give their result.**

$$\frac{\sqrt{2}\cdot\left[\frac{\sqrt{2}\cdot\sqrt{X^2+2\cdot X\cdot\sqrt{W\cdot X-W^2}}}{2\cdot\sqrt{4\cdot X\cdot\sqrt{-W\cdot(W-X)}-4\cdot W^2+X^2+4\cdot W\cdot X}} - \frac{\sqrt{2}\cdot\sqrt{X+2\cdot\sqrt{-W\cdot(W-X)}}}{2\cdot\sqrt{X}} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[Z\cdot\sqrt{-W\cdot(W-X)}-2\cdot Y\cdot\sqrt{-W\cdot(W-X)}+W\cdot Z-X\cdot Y\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left[X+2\cdot\sqrt{-W\cdot(W-X)}\right]^{\frac{3}{2}}}\right] + \frac{\sqrt{W}}{\sqrt{X}}}{\sqrt{2}\cdot\left[\frac{\sqrt{2}\cdot\sqrt{X^2+2\cdot X\cdot\sqrt{W\cdot X-W^2}}}{2\cdot\sqrt{4\cdot X\cdot\sqrt{-W\cdot(W-X)}-4\cdot W^2+X^2+4\cdot W\cdot X}} - \frac{\sqrt{2}\cdot\sqrt{X+2\cdot\sqrt{-W\cdot(W-X)}}}{2\cdot\sqrt{X}} + \frac{\sqrt{2}\cdot(X-2\cdot W)\cdot\left[Z\cdot\sqrt{-W\cdot(W-X)}-2\cdot Y\cdot\sqrt{-W\cdot(W-X)}+W\cdot Z-X\cdot Y\right]}{2\cdot\sqrt{X}\cdot Z\cdot\left[X+2\cdot\sqrt{-W\cdot(W-X)}\right]^{\frac{3}{2}}}\right] + \frac{\sqrt{X-W}}{\sqrt{X}}} = 2.333333$$

$$\frac{AC}{AB} = 2.333333 \quad \frac{CD}{BE} = 2.333333 \quad \frac{AC}{AB} - \frac{CD}{BE} = 0 \quad \frac{AC}{AB} - \frac{Y}{Y-Z} = 0$$

**Unit.**

$BH := 1$

**Given.**

$N := 5$

**012196A**

**Descriptions.**

$BP := BH \quad HQ := BH \quad BG := \frac{BH}{2} \quad GO := BG \quad GN := BG$

$NO := BH \quad GH := BG \quad BE := \frac{BG}{N} \quad EG := BG - BE$

$EO := \sqrt{EG^2 + GO^2} \quad MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO$

$EL := \frac{EM}{2} \quad LK := EL \quad LO := EO + EL \quad LJ := \frac{LK^2}{LO}$

$EJ := EL - LJ \quad AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH$

$AB := AH - BH \quad DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO}$

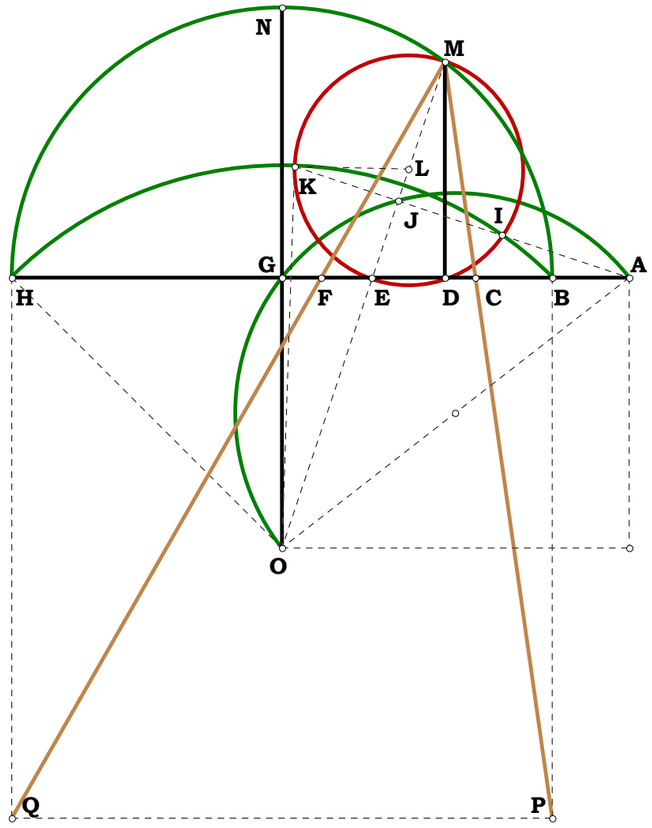
$BD := BG - (EG + DE) \quad BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD$

$DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

**Definitions.**

$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0 \quad (AB \cdot AH^2)^{\frac{1}{3}} - AF = 0$

## More On Cube Roots



012196B

Given.

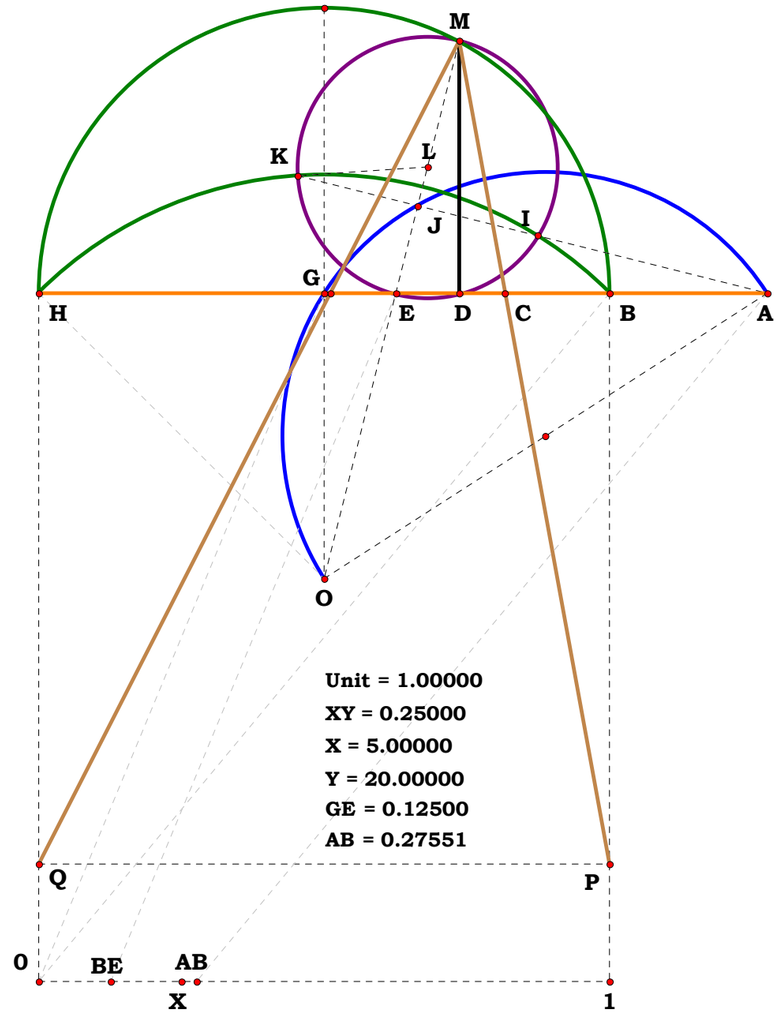
$X := 5$

$Y := 20$

Unit.

$BH := \frac{Y}{Y}$

## More On Cube Roots




Descriptions.

$BP := BH \quad HQ := BH \quad BG := \frac{BH}{2} \quad GO := BG \quad GN := BG$

$NO := BH \quad GH := BG \quad BE := BG - \frac{X}{2 \cdot Y} \quad EG := BG - BE$

$EO := \sqrt{EG^2 + GO^2} \quad MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO$

$EL := \frac{EM}{2} \quad LK := EL \quad LO := EO + EL \quad LJ := \frac{LK^2}{LO}$

$EJ := EL - LJ \quad AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH$

$AB := AH - BH \quad DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO}$

$BD := BG - (EG + DE) \quad BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD$

$DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0 \quad (AB \cdot AH^2)^{\frac{1}{3}} - AF = 0$

$GE := BG - BE \quad GE = 0.125 \quad AB = 0.27551$

**Definitions.**

$$BH - 1 = 0 \quad BP - 1 = 0 \quad HQ - 1 = 0 \quad NO - 1 = 0 \quad BG - \frac{1}{2} = 0 \quad GO - \frac{1}{2} = 0 \quad GN - \frac{1}{2} = 0 \quad GH - \frac{1}{2} = 0$$

$$BE - \frac{Y - X}{2 \cdot Y} = 0 \quad EG - \frac{X}{2 \cdot Y} = 0 \quad EO - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0 \quad MO - \frac{Y}{\sqrt{X^2 + Y^2}} = 0 \quad EM - \frac{(Y - X) \cdot (X + Y)}{2 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$$

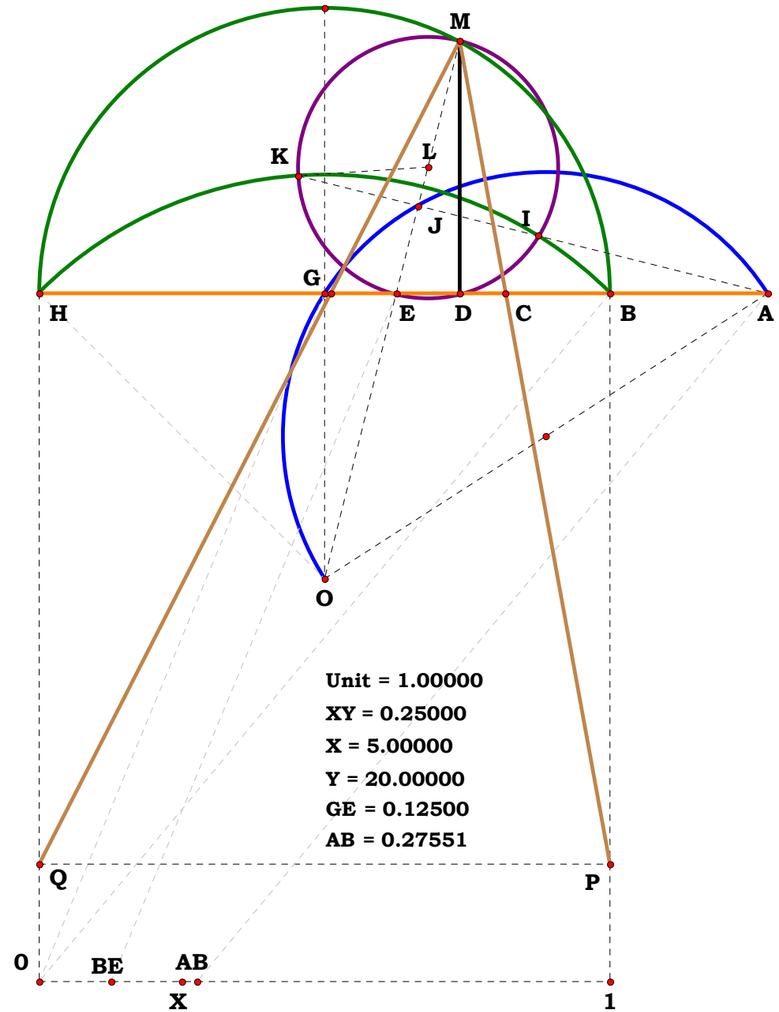
$$EL - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0 \quad LK - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0 \quad LO - \frac{X^2 + 3 \cdot Y^2}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$$

$$LJ - \frac{(X - Y)^2 \cdot (X + Y)^2}{4 \cdot Y \cdot \left(X^2 + 3 \cdot Y^2\right) \cdot \sqrt{X^2 + Y^2}} = 0 \quad EJ - \frac{(Y - X) \cdot (X + Y) \cdot \left(X^2 + Y^2\right)}{2 \cdot \sqrt{X^2 + Y^2} \cdot Y \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad AE - \frac{(Y - X) \cdot \left(X^2 + Y^2\right) \cdot (X + Y)}{2 \cdot X \cdot Y \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$$

$$AH - \frac{(X + Y)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad AB - \frac{(Y - X)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad DE - \frac{X \cdot (Y - X) \cdot (X + Y)}{2 \cdot Y \cdot \left(X^2 + Y^2\right)} = 0 \quad DM - \frac{(Y - X) \cdot (X + Y)}{2 \cdot \left(X^2 + Y^2\right)} = 0$$

$$BD - \frac{(X - Y)^2}{2 \cdot \left(X^2 + Y^2\right)} = 0 \quad BC - \frac{(X - Y)^2}{X^2 + 3 \cdot Y^2} = 0 \quad DH - \frac{(X + Y)^2}{2 \cdot \left(X^2 + Y^2\right)} = 0 \quad DF - \frac{(Y - X) \cdot (X + Y)^3}{2 \cdot \left(X^2 + Y^2\right) \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$$

$$AC - \frac{(X + Y) \cdot (X - Y)^2}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0 \quad AF - \frac{(Y - X) \cdot (X + Y)^2}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$$




**And again, Mathcad 15 cannot reduce the following equations.**

$$\left[\left[\frac{(Y - X)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)}\right]^2 \cdot \frac{(X + Y)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)}\right]^{\frac{1}{3}} - \frac{(X + Y) \cdot (X - Y)^2}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$$

$$\left[\frac{(Y - X)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} \cdot \left[\frac{(X + Y)^3}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)}\right]^2\right]^{\frac{1}{3}} - \frac{(Y - X) \cdot (X + Y)^2}{2 \cdot X \cdot \left(X^2 + 3 \cdot Y^2\right)} = 0$$

Unit.

$AF := 1$

Given.

$N_1 := 6$

$N_2 := 3$

012296A

Descriptions.

$AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

$GL := AL - AG \quad GK := \frac{GL}{N_2} \quad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

Definitions.

$AL - N_1 = 0 \quad FL - (N_1 - 1) = 0 \quad FJ - \frac{N_1 - 1}{2} = 0 \quad AM - \sqrt{N_1} = 0$

$AJ - \frac{N_1 + 1}{2} = 0 \quad AG - \frac{2 \cdot N_1}{N_1 + 1} = 0 \quad GL - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0 \quad GK - \frac{N_1 \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

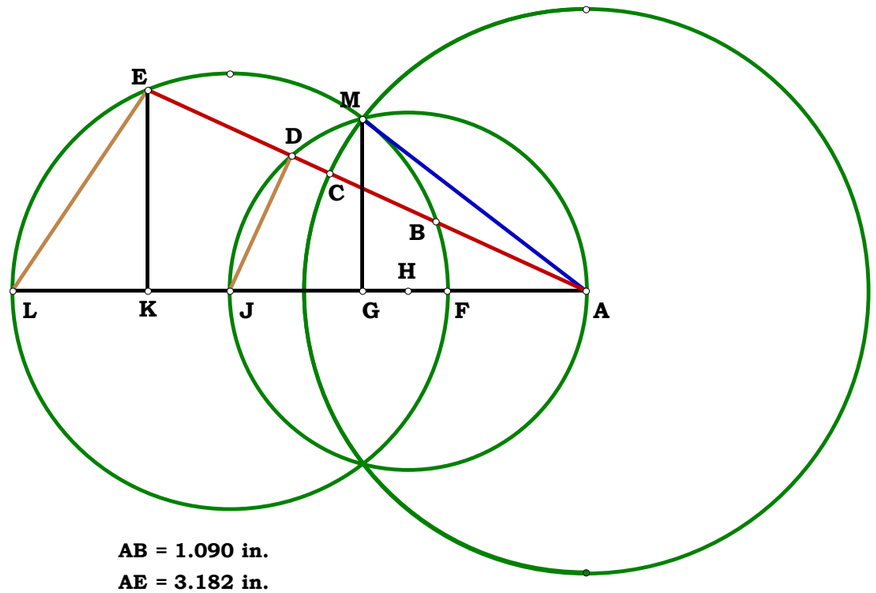
$FG - \frac{N_1 - 1}{N_1 + 1} = 0 \quad FK - \frac{(N_1 - 1) \cdot (N_1 + N_2)}{N_2 \cdot (N_1 + 1)} = 0 \quad KL - \frac{N_1 \cdot (N_2 - 1) \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

$EK - \frac{(N_1 - 1) \cdot \sqrt{N_1 \cdot (N_2 - 1) \cdot (N_1 + N_2)}}{N_2 \cdot (N_1 + 1)} = 0 \quad AK - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{N_2 \cdot (N_1 + 1)} = 0 \quad AE - \frac{\sqrt{N_1 \cdot (N_1 + N_2 - 1)}}{\sqrt{N_2}} = 0$

$AD - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{N_1 \cdot (N_1 + N_2 - 1)}} = 0 \quad DE - \frac{N_1 \cdot (N_1 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{{N_1}^2 - N_1 + N_1 \cdot N_2}} = 0 \quad AB - \frac{N_1 \cdot \sqrt{N_2}}{\sqrt{{N_1}^2 - N_1 + N_1 \cdot N_2}} = 0$

## Trivial Method Square Root

**For any E between arc M and L, AM is the square root of AB x AE. Perhaps this is one way to construct a logical operator to determine class membership for E.**

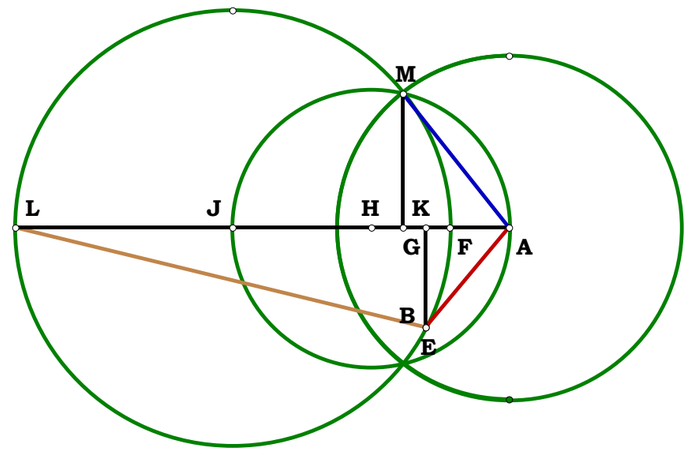



AB = 1.090 in.

AE = 3.182 in.

AM = 1.862 in.

$\sqrt{AB \cdot AE}$-AM = 0.000 in.




AB = 0.857 in.

AE = 0.857 in.

AM = 1.130 in.

$\sqrt{AB \cdot AE}$-AM = -0.273 in.

Unit.

$AB := 1$

Given.

$N := 5$

**012496A**

**Descriptions.**

$AE := N \quad BE := AE - AB \quad BD := \frac{BE}{2} \quad AD := AB + BD$

$DM := BD \quad DH := BD \quad CD := \frac{DH \cdot DM}{AD} \quad BC := BD - CD$

$CE := BE - BC \quad CJ := \sqrt{BC \cdot CE} \quad AC := AB + BC$

$AJ := \sqrt{(AB + BC)^2 + CJ^2}$

**Definitions.**

$AJ - \sqrt{N} = 0 \quad 2 \cdot \frac{N}{(1 + N)} - AC = 0$

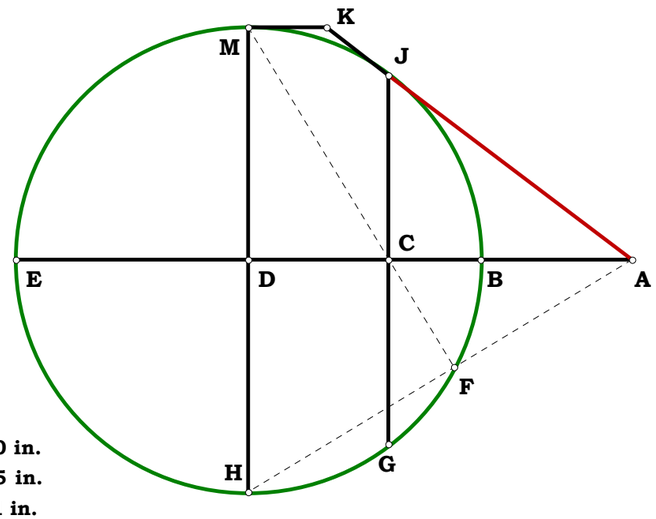
$AE - N = 0 \quad BE - (N - 1) = 0 \quad BD - \frac{N - 1}{2} = 0 \quad AD - \left(\frac{N + 1}{2}\right)$

$DM - \frac{N - 1}{2} = 0 \quad DH - \frac{N - 1}{2} = 0 \quad CD - \frac{(N - 1)^2}{2 \cdot (N + 1)} \quad BC - \frac{N - 1}{N + 1} = 0$

$CE - \frac{N \cdot (N - 1)}{N + 1} = 0 \quad CJ - \frac{\sqrt{N} \cdot (N - 1)}{(N + 1)} = 0 \quad AC - \frac{2 \cdot N}{N + 1} = 0 \quad AJ - \sqrt{N} = 0$

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**

## Tangent




AB = 1.000 in.

BC = 0.605 in.

CJ = 1.221 in.

AJ = 2.017 in.

$\sqrt{(AB+BC)^2+CJ^2}$-AJ = 0.000 in.

012496B

Descriptions.

Given.

$X := 5$

$Y := 20$

Unit.

$AB := \frac{Y}{Y}$

$AE := \frac{X}{Y}$ $BE := \frac{X - Y}{Y}$ $BD := \frac{BE}{2}$ $AD := AB + BD$

$DM := BD$ $DH := BD$ $CD := \frac{DH \cdot DM}{AD}$ $BC := BD - CD$

$CE := BE - BC$ $CJ := \sqrt{BC \cdot CE}$ $AC := AB + BC$

$AJ := \sqrt{(AB + BC)^2 + CJ^2}$ $AC = 0.4$ $AJ = 0.5$

Definitions.

$AE - \frac{X}{Y} = 0$ $BE - \frac{X - Y}{Y} = 0$ $BD - \frac{X - Y}{2 \cdot Y} = 0$

$AD - \frac{X + Y}{2 \cdot Y} = 0$ $DM - \frac{X - Y}{2 \cdot Y} = 0$ $DH - \frac{X - Y}{2 \cdot Y} = 0$

$CD - \frac{(X - Y)^2}{2 \cdot Y \cdot (X + Y)} = 0$ $BC - \frac{X - Y}{X + Y} = 0$ $CE - \frac{X \cdot (X - Y)}{Y \cdot (X + Y)} = 0$

$CJ - \frac{\sqrt{X} \cdot (Y - X)}{\sqrt{Y} \cdot (X + Y)} = 0$ $AC - \frac{2 \cdot X}{X + Y} = 0$ $AJ - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

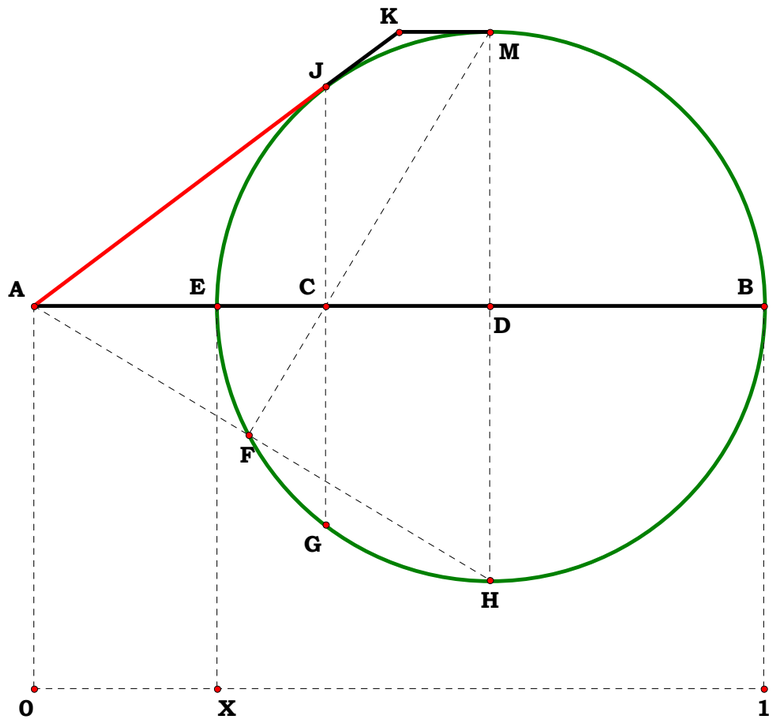
$AE = 0.25$ $BE = -0.75$ $BD = -0.375$ $AD = 0.625$

$CD = 0.225$ $BC = -0.6$ $CE = -0.15$ $CJ = 0.3$

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**

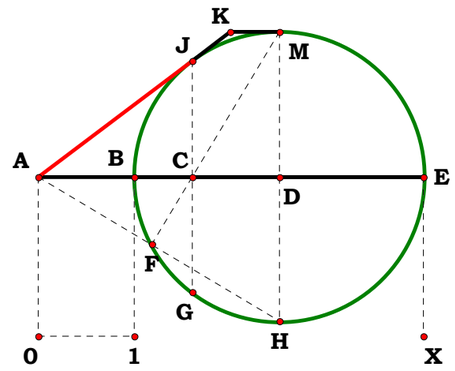
**What I did on this plate is draw it up like the first, then I moved X btween 0 and 1. Sketchpad automatically swaps B and E, all else remaining the same. The equations work the same way Sketchpad did. However, look at the step by step results, it will give you something to think about.**

## Tangent




Unit = 1.00000
XY = 0.25000
X = 5.00000
Y = 20.00000

AC = 0.40000
AJ = 0.50000
$\frac{\sqrt{X}}{\sqrt{Y}}$-AJ = 0.00000




Unit = 1.00000
XY = 4.00000
X = 4.00000
Y = 1.00000

AC = 1.60000
AJ = 2.00000
$\frac{\sqrt{X}}{\sqrt{Y}}$-AJ = 0.00000

Unit.

$BE := 1$

Given.

$N := 1.333333$

012596A

Descriptions.

$BD := \frac{BE}{2}$ $DK := BD$ $DJ := BD$ $JK := BE$ $DE := BD$

$BC := \frac{BD}{N}$ $CD := BD - BC$ $CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK}$ $CH := HK - CK$ $CF := \frac{CH}{2}$

$FK := CK + CF$ $GK := \frac{DK \cdot FK}{CK}$ $FG := \frac{CD \cdot FK}{CK}$

$GJ := JK - GK$ $AD := \frac{GJ \cdot DK}{FG}$ $AE := AD + DE$

$AB := AE - BE$ $AB = 0.275511$ $AE = 1.275511$

$BD = 0.5$ $BC = 0.375$

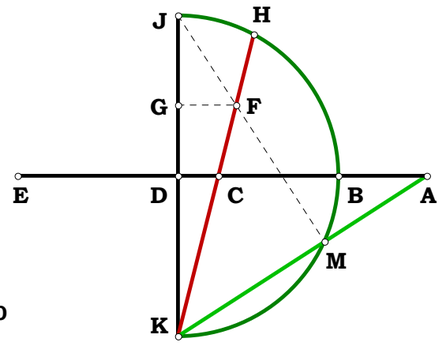
Definitions.

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

**Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.**

## On Cubes




$\frac{BD}{BC} = 1.33333$

$N = 1.33333$

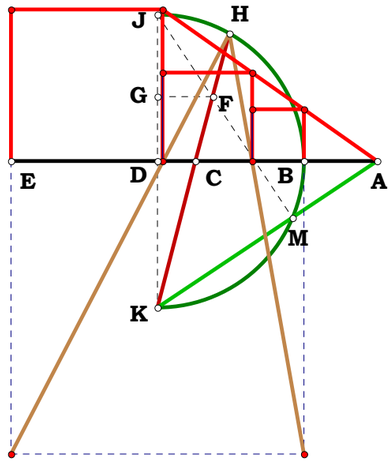
$AE = 1.27551$

$AB = 0.27551$

$BD = 0.50000$

$\frac{BD}{\frac{BD}{BC}} = 0.37500$

$\frac{(2 \cdot N-1)^3}{2 \cdot (N-1) \cdot \left((4 \cdot N^2-2 \cdot N)+1\right)} - AE = 0.00000$

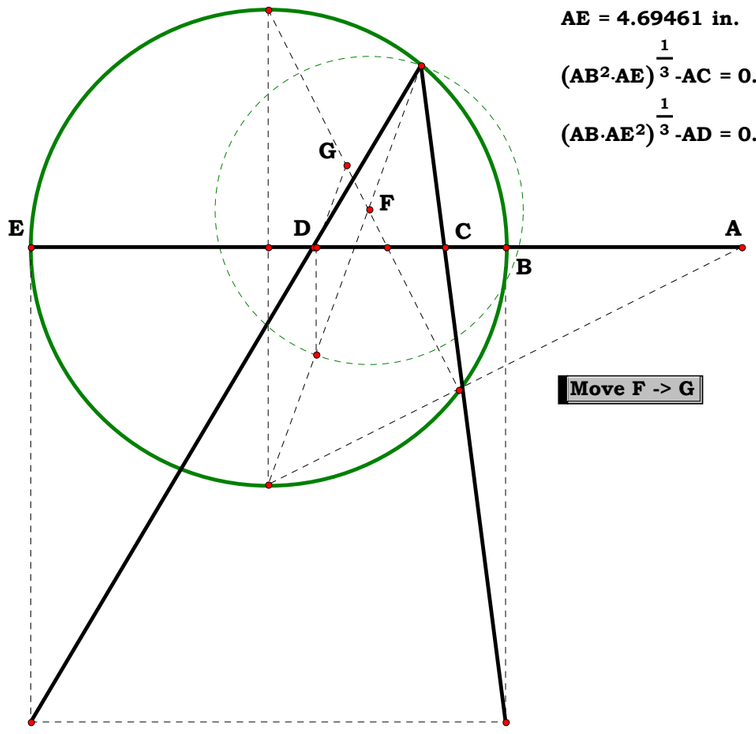


# Quick Roots

**De Button is the replacement for Calculus and it still uses just a straightedge and compass.**

**Quick roots. Push De Button.**

**AB = 1.55925 in.**
**AC = 1.96323 in.**
**AD = 2.83093 in.**
**AE = 4.69461 in.**
$(AB^2 \cdot AE)^{\frac{1}{3}}$ **-AC = 0.28829**
$(AB \cdot AE^2)^{\frac{1}{3}}$ **-AD = 0.42023**

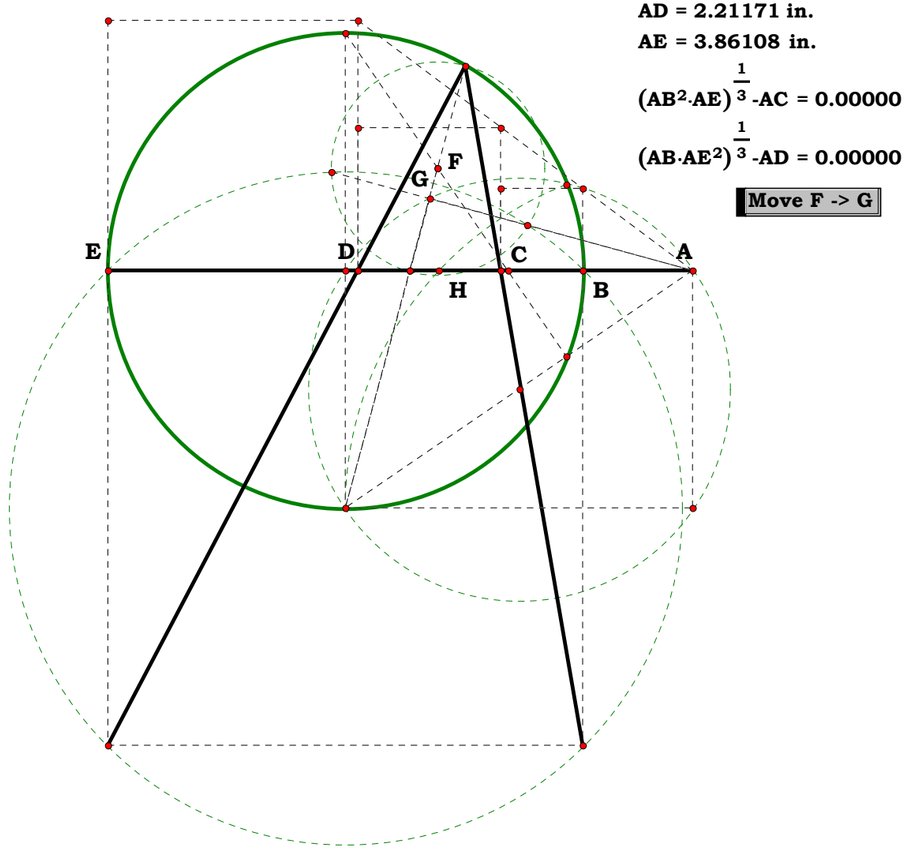



**Quick roots. Push De Button.**

**It only takes a few presses to get Sketchpad to fifteen decimal places down.**

**AB = 0.72571 in.**
**AC = 1.26691 in.**
**AD = 2.21171 in.**
**AE = 3.86108 in.**
$(AB^2 \cdot AE)^{\frac{1}{3}}$ **-AC = 0.00000**
$(AB \cdot AE^2)^{\frac{1}{3}}$ **-AD = 0.00000**

**Move F -> G**



012596B

Descriptions.

Given.

$X := 5$

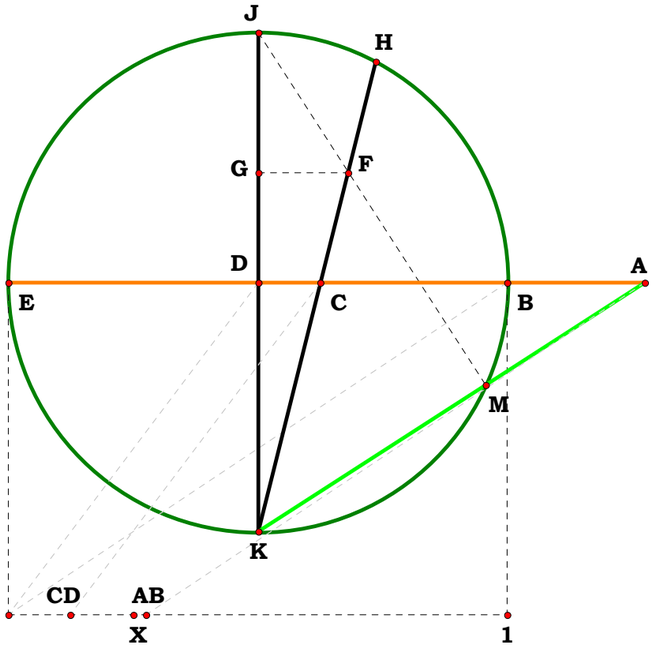
$Y := 20$

Unit.

$BE := \frac{Y}{Y}$

**Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.**

## On Cubes




Unit = 1.00000
XY = 0.25000
X = 5.00000
Y = 20.00000
CD = 0.12500
AB = 0.27551
AB = 1.27551

$\frac{(Y-X)^3}{2 \cdot X \cdot (X^2+3 \cdot Y^2)} - AB = 0.00000$

$BD := \frac{BE}{2}$ $DK := BD$ $DJ := BD$ $JK := BE$

$DE := BD$ $CD := \frac{X}{2 \cdot Y}$ $CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK}$ $CH := HK - CK$ $CF := \frac{CH}{2}$

$FK := CK + CF$ $GK := \frac{DK \cdot FK}{CK}$ $FG := \frac{CD \cdot FK}{CK}$

$GJ := JK - GK$ $AD := \frac{GJ \cdot DK}{FG}$ $AE := AD + DE$

$AB := AE - BE$ $AB = 0.27551$ $AE = 1.27551$

$BD = 0.5$

**Definitions.**

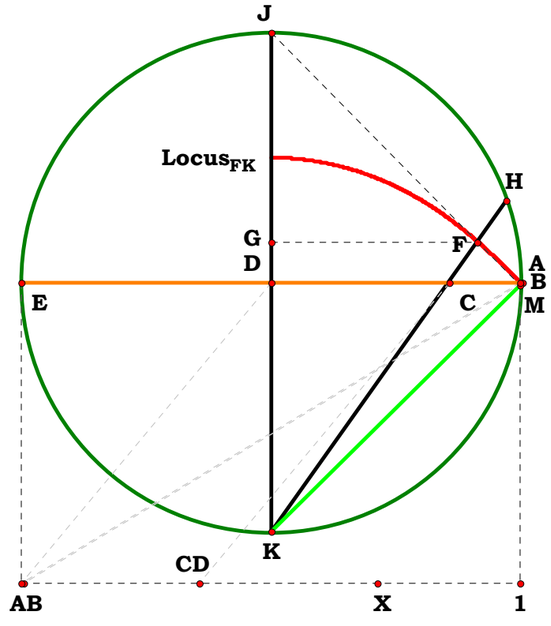
$BD - \frac{1}{2} = 0$ $DK - \frac{1}{2} = 0$ $DJ - \frac{1}{2} = 0$ $DE - \frac{1}{2} = 0$

$JK - 1 = 0$ $CD - \frac{X}{2 \cdot Y} = 0$ $CK - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$

$HK - \frac{Y}{\sqrt{X^2 + Y^2}} = 0$ $CH - \frac{(Y - X) \cdot (X + Y)}{2 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$

$CF - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$ $FK - \frac{X^2 + 3 \cdot Y^2}{4 \cdot Y \cdot \sqrt{X^2 + Y^2}} = 0$ $GK - \frac{DK \cdot FK}{CK} = 0$ $FG - \frac{X \cdot (X^2 + 3 \cdot Y^2)}{4 \cdot Y \cdot (X^2 + Y^2)} = 0$

$GJ - \frac{3 \cdot X^2 + Y^2}{4 \cdot (X^2 + Y^2)} = 0$ $AD - \frac{Y \cdot (3 \cdot X^2 + Y^2)}{2 \cdot X \cdot (X^2 + 3 \cdot Y^2)} = 0$ $AE - \frac{(X + Y)^3}{2 \cdot X \cdot (X^2 + 3 \cdot Y^2)} = 0$ $AB - \frac{(Y - X)^3}{2 \cdot X \cdot (X^2 + 3 \cdot Y^2)} = 0$




Unit = 1.00000
XY = 0.71301
X = 14.26010
Y = 20.00000
CD = 0.35650
AB = 0.00472
AB = 1.00472

$\frac{(Y-X)^3}{2 \cdot X \cdot (X^2+3 \cdot Y^2)} - AB = 0.00000$

012996A

**Given.**

$N_1 := 2 \quad N_2 := 3$

$N_3 := 9$

**Descriptions.**

$AE := N_1 \quad AH := N_2 \quad AC := \frac{AE}{2}$

$CF := N_3 \quad BC := \frac{AC \cdot CF}{AH} \quad CE := AC$

$BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

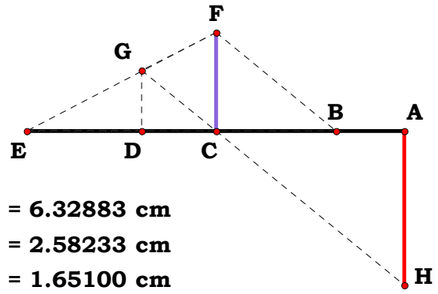
$AD := AC + CD \quad DG := \frac{AH \cdot CD}{AC} \quad BC - \frac{N_1 \cdot N_3}{2 \cdot N_2} = 0$

**Definitions.**

$DE - \frac{N_1 \cdot N_2}{2 \cdot (N_2 + N_3)} = 0 \quad AD - \frac{N_1 \cdot (N_2 + 2 \cdot N_3)}{2 \cdot (N_2 + N_3)} = 0$

$BC - \frac{N_1 \cdot N_3}{2 \cdot N_2} = 0 \quad CD - \frac{N_1 \cdot N_3}{2 \cdot (N_2 + N_3)} = 0 \quad DG - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0$

**Linear division** $\frac{N_1 \cdot (N_2 + 2 \cdot N_3)}{2 \cdot (N_2 + N_3)}$




$N_1$ = 6.32883 cm

$N_2$ = 2.58233 cm

$N_3$ = 1.65100 cm

$\frac{N_1 \cdot N_2}{2 \cdot (N_2 + N_3)}$-DE = 0.00000 cm

$\frac{N_1 \cdot (N_2 + 2 \cdot N_3)}{2 \cdot (N_2 + N_3)}$-AD = 0.00000 cm

$\frac{N_1 \cdot N_3}{2 \cdot N_2}$-BC = 0.00000 cm

$\frac{N_1 \cdot N_3}{2 \cdot (N_2 + N_3)}$-CD = 0.00000 cm

$\frac{N_2 \cdot N_3}{N_2 + N_3}$-DG = 0.00000 cm

012996B

**Given.**

$N_1 := 2 \quad N_2 := 3$

$N_3 := 9 \quad N_4 := 3$

**Linear division** $\dfrac{N_2 \cdot (N_1 - N_3)^2}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4}$

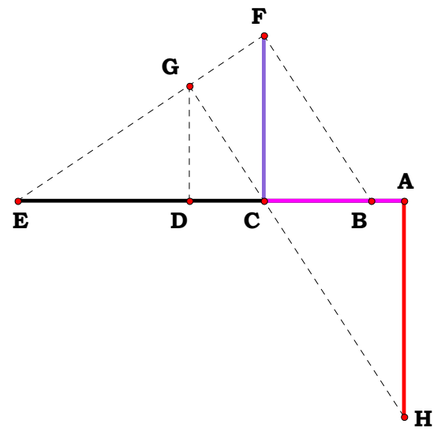
**Descriptions.**

$AE := N_1 \quad AH := N_2 \quad AC := N_3$

$CF := N_4 \quad BC := \dfrac{AC \cdot CF}{AH} \quad CE := AE - AC$

$BE := CE + BC \quad CD := \dfrac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \dfrac{CF \cdot CE}{BE}$




$\dfrac{N_2 \cdot N_4 \cdot (N_1 - N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4}$-DG = 0.00000 cm

$\dfrac{N_3 \cdot N_4 \cdot (N_1 - N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4}$-CD = 0.00000 cm

$\dfrac{N_3 \cdot ((N_1 \cdot N_2 + N_1 \cdot N_4) - N_2 \cdot N_3)}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4}$-AD = 0.00000 cm

$\dfrac{N_2 \cdot (N_1 - N_3)^2}{(N_1 \cdot N_2 - N_2 \cdot N_3) + N_3 \cdot N_4}$-DE = 0.00000 cm

$\dfrac{N_3 \cdot N_4}{N_2}$-BC = 0.00000 cm

**Definitions.**

$DE - \dfrac{N_2 \cdot (N_1 - N_3)^2}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad AD - \dfrac{N_3 \cdot (N_1 \cdot N_2 + N_1 \cdot N_4 - N_2 \cdot N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0$

$BC - \dfrac{N_3 \cdot N_4}{N_2} = 0$

$CD - \dfrac{N_3 \cdot N_4 \cdot (N_1 - N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad DG - \dfrac{N_2 \cdot N_4 \cdot (N_1 - N_3)}{N_1 \cdot N_2 - N_2 \cdot N_3 + N_3 \cdot N_4} = 0$

**Given.**

$W := 8$ $Y := 6$

$X := 20$ $Z := 17$

**012996C**

**Descriptions.**

$$AE := \frac{X}{X} \quad EH := \frac{W}{X} \quad AC := \frac{AE}{2}$$

$$CF := \frac{Y}{Z} \quad AD := \frac{AC \cdot EH}{EH + CF} \quad AD = 0.265625$$

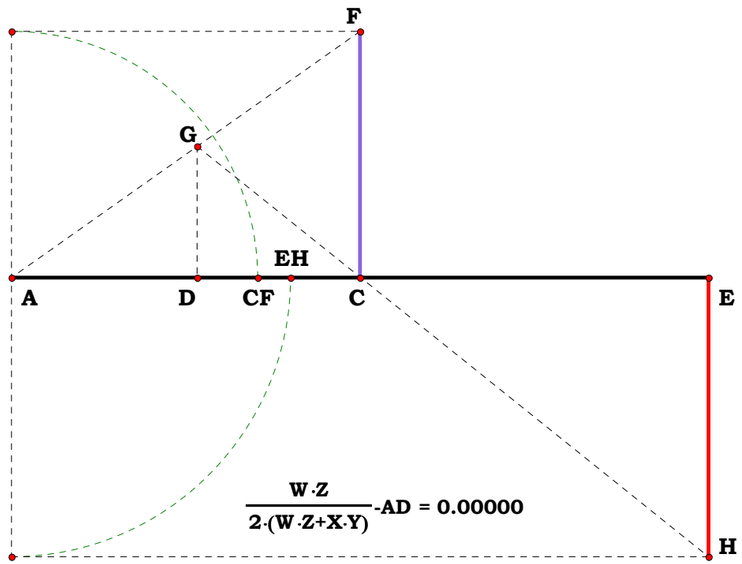
**Definitions.**

$$AE - \frac{X}{X} = 0 \quad EH - \frac{W}{X} = 0 \quad AC - \frac{1}{2} = 0$$

$$CF - \frac{Y}{Z} = 0 \quad AD - \frac{W \cdot Z}{2 \cdot (W \cdot Z + X \cdot Y)} = 0$$

**Linear division** $\dfrac{W \cdot Z}{2 \cdot (W \cdot Z + X \cdot Y)}$

AE = 1.00000
W/X = 0.40000
W = 8.00000
X = 20.00000
Y/Z = 0.35294
Y = 6.00000
Z = 17.00000
EH = 0.40000
CF = 0.35294
AD = 0.26562



012996D

**Given.**

$U := 8$ $W := 11$ $Y := 5$

$V := 20$ $X := 17$ $Z := 15$

**Descriptions.**

$$AB := \frac{V}{V} \qquad BC := \frac{U}{V} \qquad AD := \frac{W}{X} \qquad DE := \frac{Y}{Z}$$

$$BD := AB - AD \qquad DH := \frac{BD \cdot DE}{BC}$$

$$AH := AD + DH \qquad AF := \frac{AD \cdot AD}{AH} \qquad AF = 0.444853$$

**Definitions.**

$$AB - 1 = 0 \qquad BC - \frac{U}{V} \qquad AD - \frac{W}{X} = 0 \qquad DE - \frac{Y}{Z} = 0$$

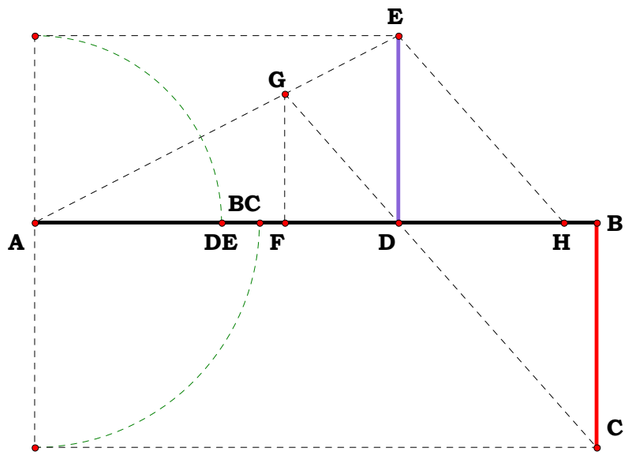
$$BD - \frac{X - W}{X} = 0 \qquad DH - \frac{V \cdot Y \cdot (X - W)}{U \cdot X \cdot Z} = 0$$

$$AH - \frac{U \cdot W \cdot Z + V \cdot Y \cdot (X - W)}{U \cdot X \cdot Z} = 0$$

$$AF - \frac{U \cdot Z \cdot W^2}{X \cdot [U \cdot W \cdot Z + V \cdot Y \cdot (X - W)]} = 0$$

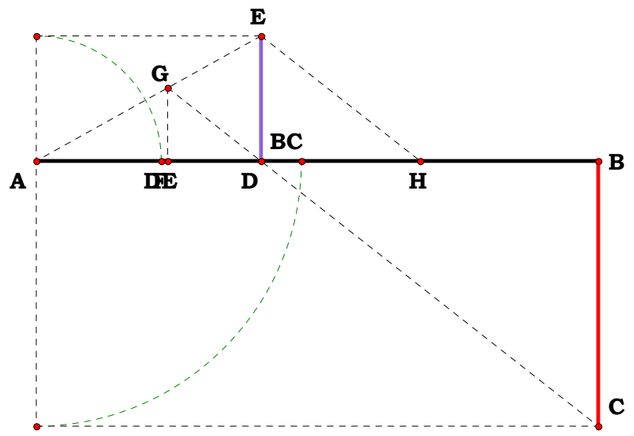
**Linear division** $\dfrac{U \cdot Z \cdot W^2}{X \cdot [U \cdot W \cdot Z + V \cdot Y \cdot (X - W)]}$

AB = 1.00000
BC = 0.40000
U = 8.00000
V = 20.00000
AD = 0.64706
W = 11.00000
X = 17.00000
DE = 0.33333
Y = 5.00000
Z = 15.00000
AF = 0.44485
FG = 0.22917
AH = 0.94118




$\dfrac{U \cdot Z \cdot W^2}{X \cdot (U \cdot W \cdot Z + V \cdot Y \cdot (X - W))}$ -AF = 0.00000

AB = 1.00000
BC = 0.47059
U = 8.00000
V = 17.00000
AD = 0.40000
W = 8.00000
X = 20.00000
DE = 0.22222
Y = 4.00000
Z = 18.00000
AF = 0.23415
FG = 0.13008
AH = 0.68333




$\dfrac{U \cdot Z \cdot W^2}{X \cdot (U \cdot W \cdot Z + V \cdot Y \cdot (X - W))}$ -AF = 0.00000

013196A

**On Gemini Roots**

**Unit.**

$BC := 1$

**Given.**

$N_1 := 5$

$N_2 := 3$

**Descriptions.**

$BJ := N_1 \quad CJ := BJ - BC \quad CI := \frac{CJ}{2} \quad IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF \quad FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2$

$HS := CR \quad FI := FJ - IJ \quad FG := \frac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \frac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \frac{HS \cdot GO}{OS}$

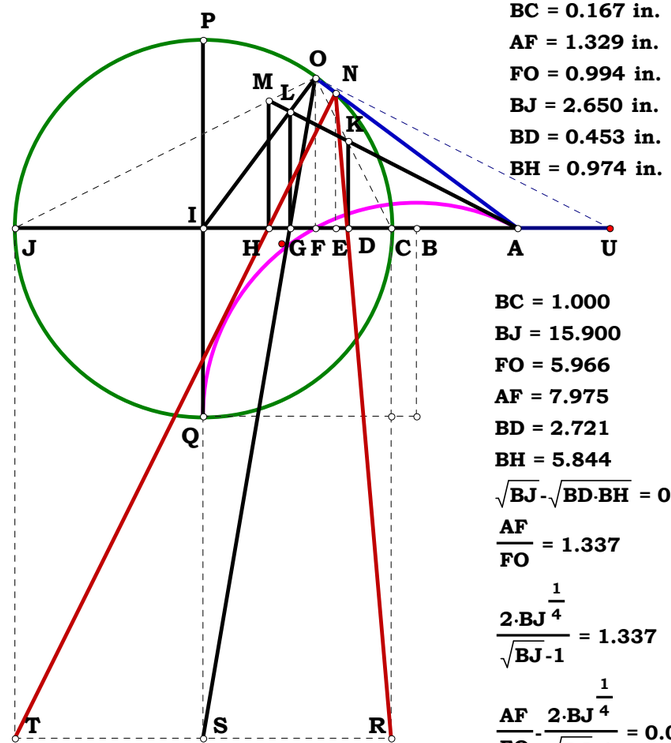
$FU := \frac{AG \cdot FO}{GL} \quad AH := \frac{FU \cdot AJ}{FU + FJ} \quad DK := \frac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \frac{AG \cdot DK}{GL} \quad AC := AF - CF$

$CD := AD - AC \quad CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \frac{CR \cdot DH}{CD + HJ}$

$CE := \frac{CD \cdot (CR + EN)}{CR} \quad AE := AC + CE \quad BD := BC + CD \quad BH := BC + CH$

$\frac{AF}{FO} - \frac{AE}{EN} = 0 \qquad \sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$

**Hitting AO from any RT while maintaining Gemini Roots.**




BC = 0.167 in.
AF = 1.329 in.
FO = 0.994 in.
BJ = 2.650 in.
BD = 0.453 in.
BH = 0.974 in.

BC = 1.000
BJ = 15.900
FO = 5.966
AF = 7.975
BD = 2.721
BH = 5.844

$\sqrt{BJ} - \sqrt{BD \cdot BH} = 0.000$

$\frac{AF}{FO} = 1.337$

$\frac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 1.337$

$\frac{AF}{FO} - \frac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 0.000$

**Definitions.**

$BJ - N_1 = 0 \quad CJ - (N_1 - 1) = 0 \quad CI - \frac{N_1 - 1}{2} = 0 \quad IJ - \frac{N_1 - 1}{2} = 0 \quad BF - \sqrt{N_1} = 0$

$AB - \sqrt{N_1} = 0 \quad AF - 2 \cdot \sqrt{N_1} = 0 \quad CF - (\sqrt{N_1} - 1) = 0 \quad FJ - (N_1 - \sqrt{N_1}) = 0$

$FO - \sqrt{\sqrt{N_1}} \cdot (\sqrt{N_1} - 1) = 0 \quad CR - (N_1 \cdot N_2 - N_2) = 0$

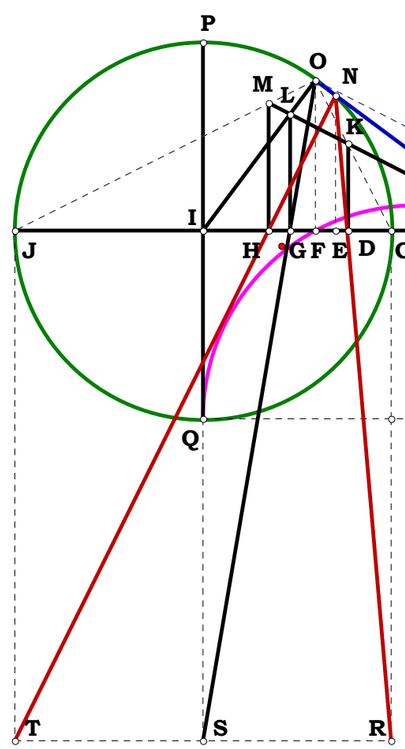
$HS - (N_1 \cdot N_2 - N_2) = 0 \quad FI - \frac{(\sqrt{N_1} - 1)^2}{2} = 0 \quad FG - \frac{N_1^{\frac{1}{4}} - 2 \cdot N_1^{\frac{3}{4}} + N_1^{\frac{5}{4}}}{2 \cdot \left(N_2 + \sqrt{N_1} \cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0$

$AG - \frac{N_1^{\frac{1}{4}} \cdot (\sqrt{N_1} + 1) \cdot \left(4 \cdot N_1^{\frac{1}{4}} \cdot N_2 + \sqrt{N_1} + 1\right)}{2 \cdot \left(N_2 + \sqrt{N_1} \cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0$

$OS - \frac{(\sqrt{N_1} - 1) \cdot \sqrt{(\sqrt{N_1} + 1) \cdot \left[8 \cdot N_1^{\frac{1}{4}} \cdot N_2 + 4 \cdot N_2^2 + \sqrt{N_1} \cdot (4 \cdot N_2^2 + 1) + 1\right]}}{2} = 0$

$GO - \frac{N_1^{\frac{1}{4}} \cdot \sqrt{(\sqrt{N_1} + 1) \cdot \left[8 \cdot N_1^{\frac{1}{4}} \cdot N_2 + 4 \cdot N_2^2 + \sqrt{N_1} \cdot (4 \cdot N_2^2 + 1) + 1\right]} \cdot (N_1^{\frac{1}{4}} - 1) \cdot (N_1^{\frac{1}{4}} + 1)}{2 \cdot \left(N_2 + \sqrt{N_1} \cdot N_2 + N_1^{\frac{1}{4}}\right)} = 0 \quad AJ - \sqrt{N_1} \cdot (\sqrt{N_1} + 1) = 0$

$GL - \frac{N_1^{\frac{1}{4}} \cdot N_2 \cdot (N_1 - 1)}{N_2 + \sqrt{N_1} \cdot N_2 + N_1^{\frac{1}{4}}} = 0 \quad FU - \frac{N_1^{\frac{1}{4}} \cdot \left(4 \cdot N_1^{\frac{1}{4}} \cdot N_2 + \sqrt{N_1} + 1\right)}{2 \cdot N_2} = 0 \quad AH - \frac{N_1^{\frac{2}{4}} \cdot \left(4 \cdot N_1^{\frac{1}{4}} \cdot N_2 + \sqrt{N_1} + 1\right)}{2 \cdot N_1^{\frac{1}{4}} \cdot N_2 + 1} = 0$

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BH = 0.974 in.

BC = 1.000
BJ = 15.900
FO = 5.966
AF = 7.975
BD = 2.721
BH = 5.844

$\sqrt{BJ} - \sqrt{BD \cdot BH} = 0.000$

$\frac{AF}{FO} = 1.337$

$\frac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 1.337$

$\frac{AF}{FO} - \frac{2 \cdot BJ^{\frac{1}{4}}}{\sqrt{BJ} - 1} = 0.000$

$$DK - \frac{2\cdot N_1^{\frac{3}{4}}\cdot N_2 - 2\cdot N_1^{\frac{1}{4}}\cdot N_2}{2\cdot N_2 + N_1^{\frac{1}{4}}} = 0 \qquad AD - \frac{N_1^{\frac{1}{4}}\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{2\cdot N_2 + N_1^{\frac{1}{4}}} = 0 \qquad AC - \left(\sqrt{N_1} + 1\right) = 0$$

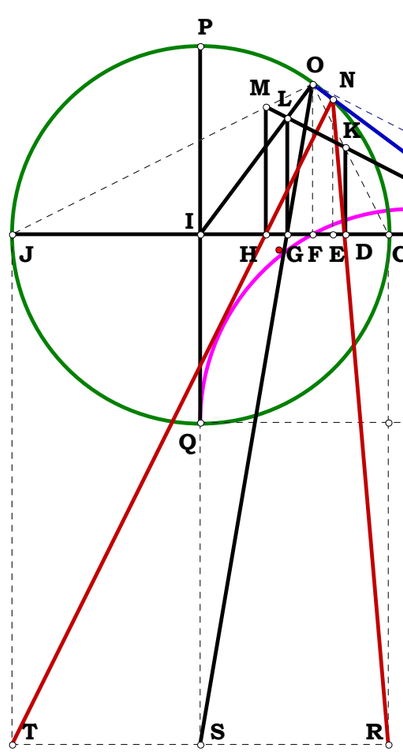
$$CD - \frac{2\cdot N_2\cdot\left(\sqrt{N_1} - 1\right)}{2\cdot N_2 + N_1^{\frac{1}{4}}} = 0 \qquad CH - \frac{N_1 - 2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 2\cdot N_1^{\frac{3}{4}}\cdot N_2 - 1}{2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 1} = 0$$

$$DH - \frac{N_1^{\frac{1}{4}}\cdot\left(\sqrt{N_1} - 1\right)\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{2\cdot N_2\cdot\left(\sqrt{N_1} + 1\right) + N_1^{\frac{1}{4}}\cdot\left(4\cdot {N_2}^2 + 1\right)} = 0 \qquad HJ - \frac{2\cdot N_1^{\frac{5}{4}}\cdot N_2 - 2\cdot N_1^{\frac{3}{4}}\cdot N_2}{2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 1} = 0$$

$$EN - \frac{4\cdot N_1^{\frac{3}{2}}\cdot N_2 - 4\cdot\sqrt{N_1}\cdot N_2 - N_1^{\frac{1}{4}} - N_1^{\frac{3}{4}} + N_1^{\frac{5}{4}} + N_1^{\frac{7}{4}}}{2\cdot\left(N_1 + 2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 2\cdot N_1^{\frac{3}{4}}\cdot N_2 + 1\right)} = 0 \qquad CE - \frac{N_1 - 2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 2\cdot N_1^{\frac{5}{4}}\cdot N_2 - 1}{N_1 + 2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 2\cdot N_1^{\frac{3}{4}}\cdot N_2 + 1} = 0$$

$$AE - \frac{\sqrt{N_1}\cdot\left(\sqrt{N_1} + 1\right)\cdot\left(4\cdot N_1^{\frac{1}{4}}\cdot N_2 + \sqrt{N_1} + 1\right)}{N_1 + 2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 2\cdot N_1^{\frac{3}{4}}\cdot N_2 + 1} = 0 \qquad BD - \frac{2\cdot\sqrt{N_1}\cdot N_2 + N_1^{\frac{1}{4}}}{2\cdot N_2 + N_1^{\frac{1}{4}}} = 0 \qquad BH - \frac{N_1 + 2\cdot N_1^{\frac{3}{4}}\cdot N_2}{2\cdot N_1^{\frac{1}{4}}\cdot N_2 + 1} = 0$$

$$\frac{AE}{EN} - \frac{2\cdot N_1^{\frac{1}{4}}}{\left(\sqrt{N_1} - 1\right)} = 0 \qquad \frac{AF}{FO} - \frac{2\cdot N_1^{\frac{1}{4}}}{\left(\sqrt{N_1} - 1\right)} = 0$$

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BJ = 2.650 in.
BD = 0.453 in.
BH = 0.974 in.

BC = 1.000
BJ = 15.900
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BH = 5.844

$\sqrt{BJ}\cdot\sqrt{BD\cdot BH} = 0.000$

$\frac{AF}{FO} = 1.337$

$\frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ}-1} = 1.337$

$\frac{AF}{FO} - \frac{2\cdot BJ^{\frac{1}{4}}}{\sqrt{BJ}-1} = 0.000$

020296A

**Given.**

$N_1 := 3.467$

$N_2 := 1.728$

**Descriptions.**

$BE := N_1 \quad BD := \frac{BE}{2} \quad CH := BD \quad BD := \frac{BE}{2}$

$CD := BD - \frac{BD}{N_2} \quad DH := \sqrt{CD^2 + CH^2} \quad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \quad AB := AD - BD \quad BC := BD - CD$

$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$

**Definitions.**

$BE - N_1 = 0 \quad BD - \frac{N_1}{2} = 0 \quad CH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2} = 0$

$CD - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0 \quad DH - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{2 \cdot N_2} = 0$

$DF - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{4 \cdot N_2} = 0 \quad AD - \frac{N_1 \cdot (2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0$

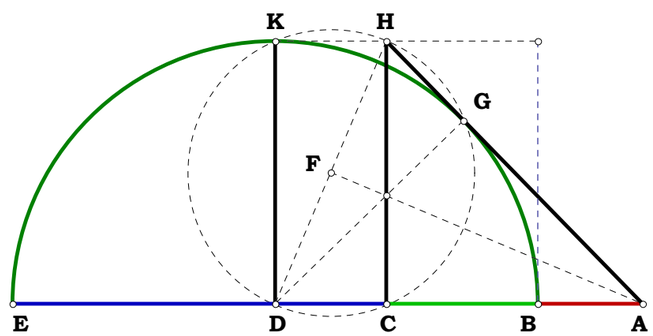
$AB - \frac{N_1}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad BC - \frac{N_1}{2 \cdot N_2} = 0$

$\sqrt{(AB + BE) \cdot AB} - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad AB + BC - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)}$

$\frac{BC^2}{BE - 2 \cdot BC} - AB = 0$

## Find A Segment

**Find segment AB.**




**BC = 1.003 in.**
**BE = 3.467 in.**
**AB = 0.689 in.**

**BD = 1.733 in.**
**BC = 1.003 in.**

$\frac{BD}{BC} = 1.728$

**DH = 1.881 in.**
**AD = 2.422 in.**

$\frac{BC^2}{BE-2 \cdot BC}$-AB = 0.000 in.

**Given BE and BC such that**
$\sqrt{(AB + BE) \cdot AB}$ **= AB + BC, find AB.**

020296B

**Given.**

$X := 7$

$Y := 20$

**Descriptions.**

$DE := \frac{Y}{Y}$ $BD := DE$ $BE := 2 \cdot DE$

$CH := BD$ $CD := \frac{X}{Y}$ $DH := \sqrt{CD^2 + CH^2}$

$DF := \frac{DH}{2}$ $AD := \frac{DH \cdot DF}{CD}$ $AB := AD - BD$

$BC := BD - CD$ $AB = 0.603571$

$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$

**Definitions.**

$DE - \frac{Y}{Y} = 0$ $BD - \frac{Y}{Y} = 0$ $CH - \frac{Y}{Y} = 0$

$BE - 2 = 0$ $CD - \frac{X}{Y} = 0$ $DH - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$

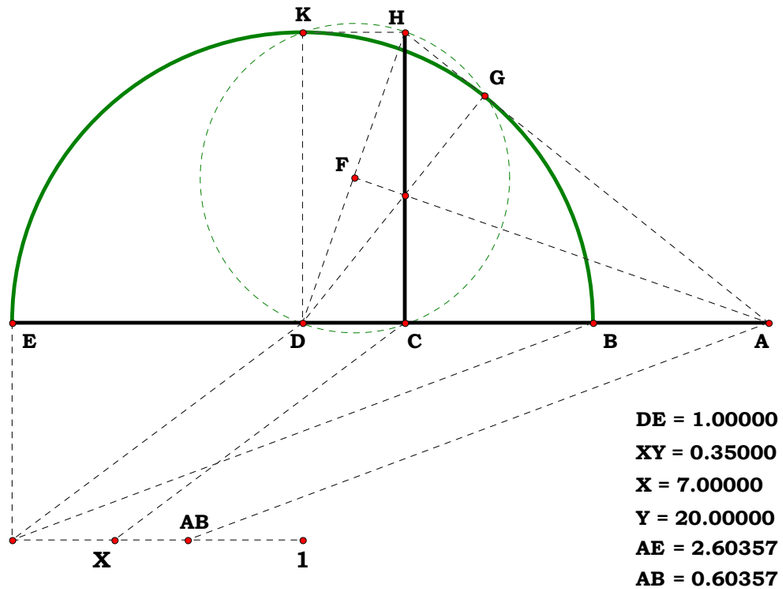
$DF - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$ $AD - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0$

$AB - \frac{(Y - X)^2}{2 \cdot X \cdot Y} = 0$ $BC - \frac{Y - X}{Y} = 0$

$\frac{(X + Y) \cdot (Y - X)}{2 \cdot X \cdot Y} - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$

## Find A Segment

**Find segment AB.**




DE = 1.00000
XY = 0.35000
X = 7.00000
Y = 20.00000
AE = 2.60357
AB = 0.60357

**Given BE and BC such that $\sqrt{(AB + BE) \cdot AB} = AB + BC$, find AB.**

**Unit.**

**Given.**

$N := 2$

$\Delta := 40 \quad \delta := 0 .. \Delta$

**021496**

**Descriptions.**

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

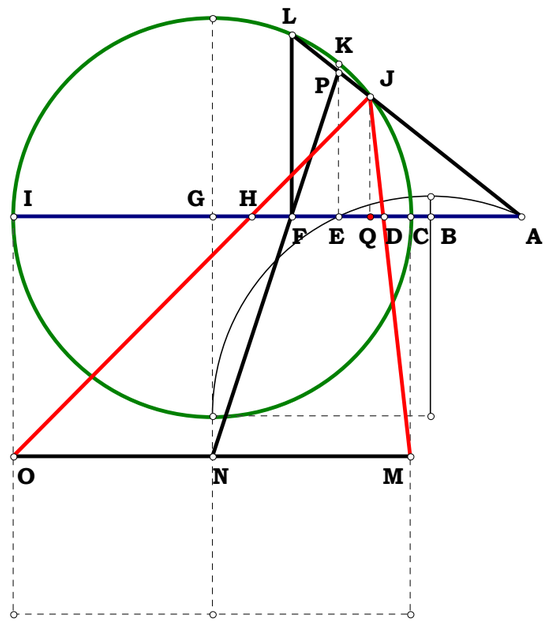
$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$GN := CG \cdot N \quad IO := GN \quad CM := GN$

$$\begin{pmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{pmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC\right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK}\right)\right]} \end{bmatrix}$$

**Or, the 17 decimal place rustic solution.**

**Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.**



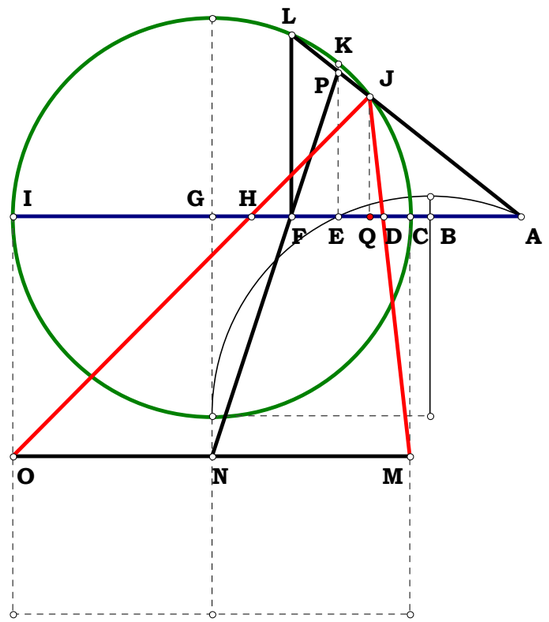

$$\begin{pmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{pmatrix} := \begin{pmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{pmatrix}$$

$AK := \sqrt{AE^2 + EK^2}$ $AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2}$ $AJ := \frac{AK^2}{AL}$ $AQ := \frac{AF_\Delta \cdot AJ}{AL}$

$CQ := AQ - AC$ $IQ := CI - CQ$ $JQ := \sqrt{CQ \cdot IQ}$ $CD := \frac{CQ \cdot CM}{CM + JQ}$

$HI := \frac{IQ \cdot IO}{IO + JQ}$ $DH := CI - (CD + HI)$ $BD := BC + CD$

$BH := BC + CD + DH$ $\frac{DH}{\sqrt{CD \cdot HI}} = 1$ $BE - \sqrt{BD \cdot BH} = 0$




**The next two equations are for the Delian Problem only.**
**Resolution set to max of the program.**

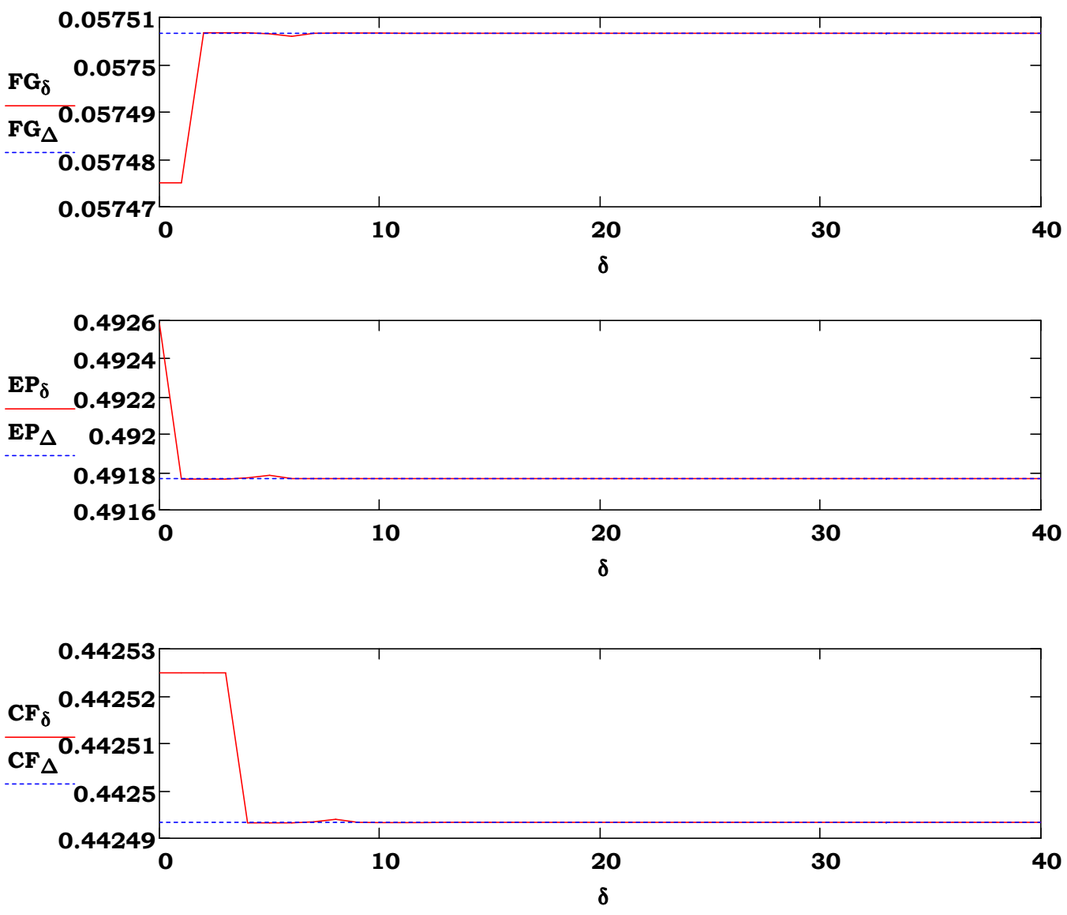
$(BC^2 \cdot BI)^{\frac{1}{3}} - BD = 0$ $(BC \cdot BI^2)^{\frac{1}{3}} - BH = 0$

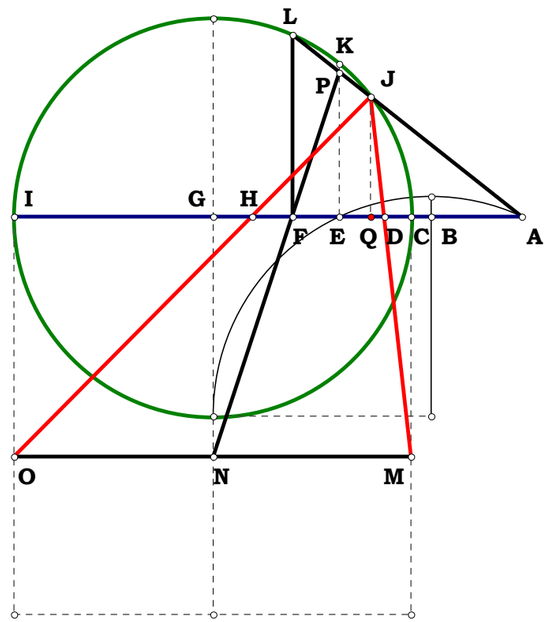
$BD = 1.2599210498948732$ $2^{\frac{1}{3}} = 1.2599210498948732$

**17 decimal places. Good to the limits of the program.**

$BH = 1.5874010519681994$ $4^{\frac{1}{3}} = 1.5874010519681994$

**A graphics peek.**




**The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist. The solution is only good to material differences, on the atomic level, so to speak.**

041496A

Unit.

$CM := 1$

Given.

$N_1 := .13749$

$N_2 := .30814$

Descriptions.

$CK := \frac{CM}{2}$ $CE := N_1$ $LM := N_2$ $EL := CM - (CE + LM)$

$BL := \frac{EL \cdot LM}{LM - CE}$ $BM := BL + LM$ $BC := BM - CM$ $BK := \frac{CM}{2} + BC$

$R_1 := LM$ $R_2 := CE$ $D := EL$ $KS := CK$ $EH := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$

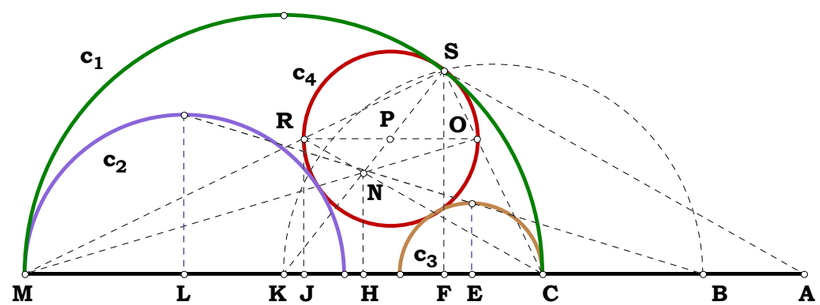
$FK := \frac{KS^2}{BK}$ $CF := CK - FK$ $FM := CM - CF$ $FS := \sqrt{CF \cdot FM}$

$HK := CK - (CE + EH)$ $CH := CK - HK$ $HN := \frac{FS \cdot HK}{FK}$ $AF := \frac{CH \cdot FS}{HN}$

$JR := \frac{FS \cdot CM}{AF + FM}$ $RO := \frac{CM \cdot (FS - JR)}{FS}$ $PS := \frac{RO}{2}$ $PS = 0.167485$

Given $c_1$, $c_2$, $c_3$, find $c_4$. I had this sketched out in 95, but if I put it there I would have had a lot of document links to redo in "The Quest." In my earlier revisions, it seems that I forgot to remove the reciprocals for C2 and 3.

## Method for Unequals




CM = 1.00000
CE = 0.13749
LM = 0.30814
PS = 0.16749

$\frac{(2 \cdot LM - 1) \cdot (2 \cdot CE - 1)}{2 \cdot (1 - 4 \cdot CE \cdot LM)} - PS = 0.00000$

Definitions.

$CK - \frac{1}{2} = 0$ $CE - N_1 = 0$ $LM - N_2 = 0$ $EL - \left(1 - N_2 - N_1\right) = 0$ $BL - \frac{N_2 \cdot \left(N_1 + N_2 - 1\right)}{N_1 - N_2} = 0$ $BM - \frac{N_2 \cdot \left(2 \cdot N_1 - 1\right)}{N_1 - N_2} = 0$ $BC - \frac{N_1 \cdot \left(2 \cdot N_2 - 1\right)}{N_1 - N_2} = 0$

$BK - \frac{4 \cdot N_1 \cdot N_2 - N_2 - N_1}{2 \cdot \left(N_1 - N_2\right)} = 0$ $R_1 - N_2 = 0$ $R_2 - N_1 = 0$ $D - \left(1 - N_2 - N_1\right) = 0$ $KS - \frac{CM}{2} = 0$ $EH - \frac{2 \cdot N_1 + 2 \cdot N_2 - 2 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2 - 1}{2 \cdot \left(N_1 + N_2 - 1\right)} = 0$ $FK - \frac{CM^2 \cdot \left(N_1 - N_2\right)}{2 \cdot \left(4 \cdot N_1 \cdot N_2 - N_2 - N_1\right)} = 0$

$CF - \frac{N_1 \cdot \left(2 \cdot N_2 - 1\right)}{4 \cdot N_1 \cdot N_2 - N_2 - N_1} = 0$ $FM - \frac{N_2 \cdot \left(2 \cdot N_1 - 1\right)}{4 \cdot N_1 \cdot N_2 - N_2 - N_1} = 0$ $FS - \frac{\sqrt{N_1 \cdot N_2 \cdot \left(2 \cdot N_1 - 1\right) \cdot \left(2 \cdot N_2 - 1\right)}}{\left(N_1 + N_2 - 4 \cdot N_1 \cdot N_2\right)} = 0$ $HK - \frac{N_1 - N_2}{2 \cdot \left(N_1 + N_2 - 1\right)} = 0$ $CH - \frac{2 \cdot N_2 - 1}{2 \cdot \left(N_1 + N_2 - 1\right)} = 0$

$HN - \frac{\sqrt{N_1 \cdot N_2 \cdot \left(2 \cdot N_1 - 1\right) \cdot \left(2 \cdot N_2 - 1\right)}}{\left(1 - N_2 - N_1\right)} = 0$ $AF - \frac{\left(2 \cdot N_2 - 1\right)}{2 \cdot \left(4 \cdot N_1 \cdot N_2 - N_2 - N_1\right)} = 0$ $JR - \frac{2 \cdot \sqrt{N_1 \cdot N_2 \cdot \left(2 \cdot N_1 - 1\right) \cdot \left(2 \cdot N_2 - 1\right)}}{\left(1 - 4 \cdot N_1 \cdot N_2\right)} = 0$ $RO - \frac{\left(2 \cdot N_2 - 1\right) \cdot \left(2 \cdot N_1 - 1\right)}{1 - 4 \cdot N_1 \cdot N_2} = 0$

$PS - \frac{\left(2 \cdot N_2 - 1\right) \cdot \left(2 \cdot N_1 - 1\right)}{2 \cdot \left(1 - 4 \cdot N_1 \cdot N_2\right)} = 0$

Given.

$W := 3$ $Y := 6$

$X := 20$ $Z := 19$

Unit.

$CM := \frac{X}{X}$

041496B

Descriptions.

$CK := \frac{CM}{2}$ $CE := \frac{W}{X}$ $LM := \frac{Y}{Z}$ $EL := CM - (CE + LM)$

$BL := \frac{EL \cdot LM}{LM - CE}$ $BM := BL + LM$ $BC := BM - CM$ $BK := \frac{CM}{2} + BC$

power line. $EH := \frac{\left(CE^2 + EL^2 - LM^2\right)}{2 \cdot EL}$ $KS := CK$

$FK := \frac{KS^2}{BK}$ $CF := CK - FK$ $FM := CM - CF$ $FS := \sqrt{CF \cdot FM}$

$HK := CK - (CE + EH)$ $CH := CK - HK$ $HN := \frac{FS \cdot HK}{FK}$ $AF := \frac{CH \cdot FS}{HN}$

$JR := \frac{FS \cdot CM}{AF + FM}$ $RO := \frac{CM \cdot (FS - JR)}{FS}$ $PS := \frac{RO}{2}$ $PS = 0.159091$

Definitions.

$CM - 1 = 0$ $CK - \frac{1}{2} = 0$ $CE - \frac{W}{X} = 0$ $LM - \frac{Y}{Z} = 0$ $EL - \frac{(X \cdot Z - X \cdot Y - W \cdot Z)}{X \cdot Z} = 0$ $BL - \frac{Y \cdot (W \cdot Z + X \cdot Y - X \cdot Z)}{Z \cdot (W \cdot Z - X \cdot Y)} = 0$

$BM - \frac{Y \cdot (2 \cdot W - X)}{W \cdot Z - X \cdot Y} = 0$ $BC - \frac{W \cdot (2 \cdot Y - Z)}{W \cdot Z - X \cdot Y} = 0$ $BK - \frac{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{2 \cdot (W \cdot Z - X \cdot Y)} = 0$ $EH - \frac{2 \cdot W^2 \cdot Z - 2 \cdot X^2 \cdot Y + X^2 \cdot Z + 2 \cdot W \cdot X \cdot Y - 2 \cdot W \cdot X \cdot Z}{2 \cdot X \cdot (X \cdot Z - X \cdot Y - W \cdot Z)} = 0$

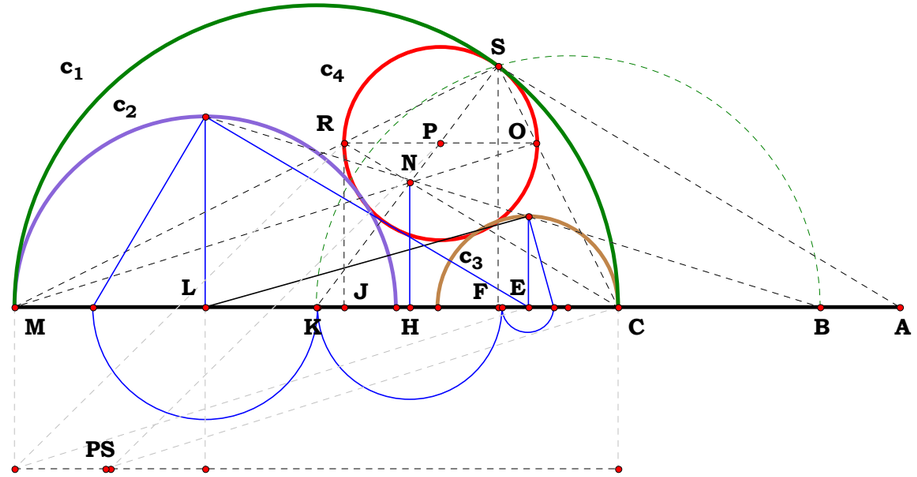
$KS - \frac{1}{2} = 0$ $FK - \frac{W \cdot Z - X \cdot Y}{2 \cdot (4 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0$ $CF - \frac{W \cdot (2 \cdot Y - Z)}{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y} = 0$ $FM - \frac{Y \cdot (2 \cdot W - X)}{4 \cdot W \cdot Y - W \cdot Z - X \cdot Y} = 0$

$FS - \frac{\sqrt{W \cdot Y \cdot (2 \cdot W - X) \cdot (2 \cdot Y - Z)}}{(W \cdot Z - 4 \cdot W \cdot Y + X \cdot Y)} = 0$ $HK - \frac{W \cdot Z - X \cdot Y}{2 \cdot (W \cdot Z + X \cdot Y - X \cdot Z)} = 0$ $CH - \frac{X \cdot (2 \cdot Y - Z)}{2 \cdot (W \cdot Z + X \cdot Y - X \cdot Z)} = 0$ $HN - \frac{\sqrt{W \cdot Y \cdot (X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}}{(X \cdot Z - X \cdot Y - W \cdot Z)} = 0$

$AF - \frac{X \cdot (2 \cdot Y - Z)}{2 \cdot (4 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0$ $JR - \frac{2 \cdot \sqrt{W \cdot Y \cdot (X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}}{(X \cdot Z - 4 \cdot W \cdot Y)} = 0$ $RO := \frac{(X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}{X \cdot Z - 4 \cdot W \cdot Y}$ $PS - \frac{(X - 2 \cdot W) \cdot (Z - 2 \cdot Y)}{2 \cdot (X \cdot Z - 4 \cdot W \cdot Y)} = 0$

## Method for Unequals

Given $c_1$, $c_2$, $c_3$, find $c_4$. The thin blue lines is the process I developed for finding the powerline between two circles.




CM = 1.00000
CE = 0.15000
W = 3.00000
X = 20.00000
LM = 0.31579
Y = 6.00000
Z = 19.00000
PS = 0.15909

CE LM CM

$\frac{(X-2 \cdot W) \cdot (Z-2 \cdot Y)}{2 \cdot (X \cdot Z-4 \cdot W \cdot Y)} - PS = 0.00000$

Unit.

$AB := 1$

Given.

$N_1 := 4$

041596

Descriptions.

# On Gemini Roots

$BE := N_1 \qquad BD := \frac{BE}{2}$

$AE := AB + BE \qquad AC := \sqrt{AB \cdot AE} \qquad BC := AC - AB \qquad CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \qquad CG := \frac{CF^2}{CD} \qquad BG := CG - BC \qquad EG := BG + BE \qquad CH := \frac{1}{2} \cdot CF$

$DH := \sqrt{CH^2 + CD^2} \qquad DI := \frac{1}{2} \cdot DH \qquad DL := \frac{CD \cdot DI}{DH} \qquad BL := BD - DL \qquad EL := BE - BL$

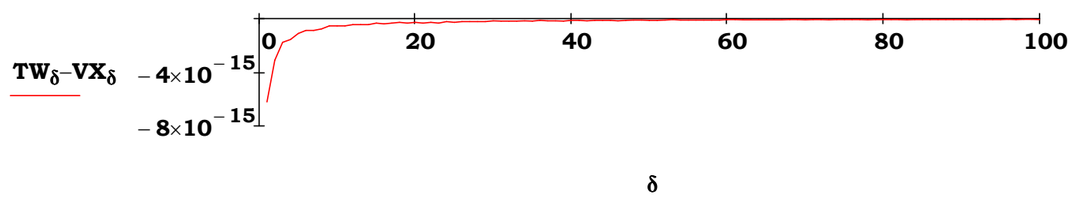
$JL := \sqrt{BL \cdot EL} \qquad GL := BL + BG \qquad GJ := \sqrt{JL^2 + GL^2} \qquad GK := \frac{BG \cdot EG}{GJ} \qquad GM := \frac{GL \cdot GK}{GJ}$

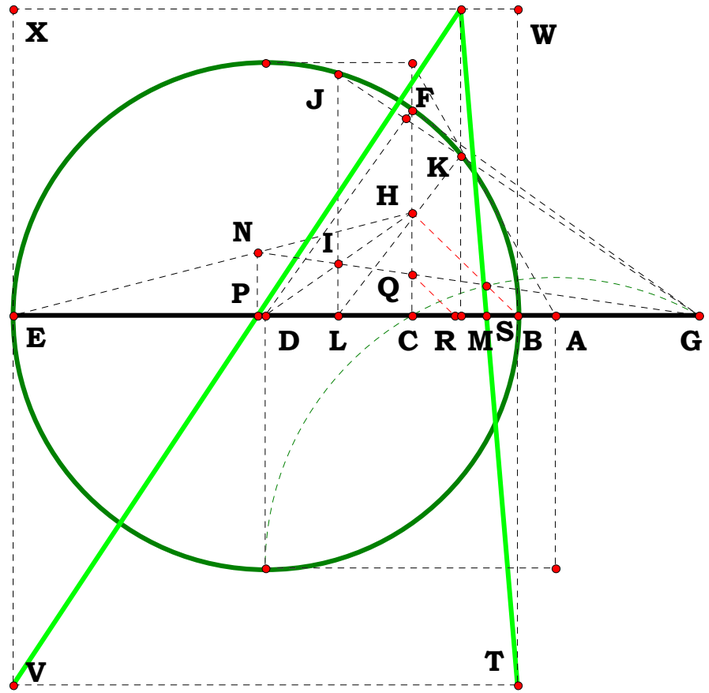
$BM := GM - BG \qquad EM := BE - BM \qquad IL := \sqrt{DI^2 - DL^2} \qquad CO := \frac{GL \cdot CH}{IL} \qquad NP := \frac{CH \cdot EG}{(CO + CE)}$

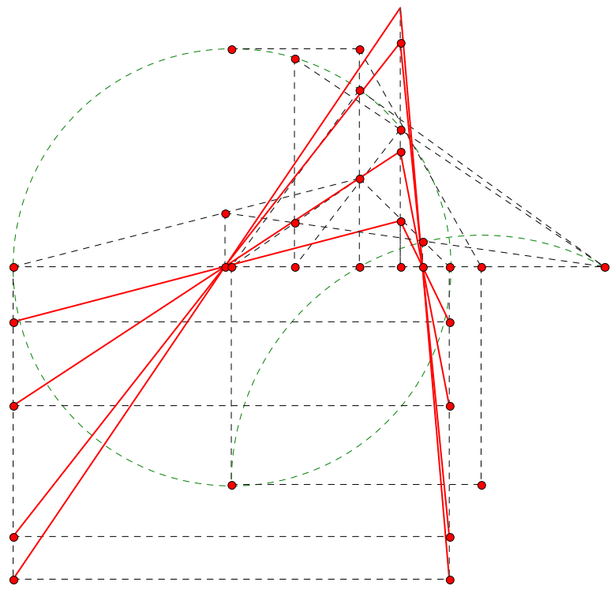
$EP := \frac{CE \cdot NP}{CH} \qquad CQ := \frac{IL \cdot CG}{GL} \qquad CR := \frac{BC \cdot CQ}{CH} \qquad GR := CG - CR \qquad BS := \frac{CR \cdot BG}{GR}$

Definitions.

$\delta := 1 .. 100 \qquad E_\delta := \frac{BE}{\delta} \qquad BT_\delta := E_\delta \qquad EV_\delta := E_\delta \qquad TW_\delta := \frac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$



Unit.

$AJ := 1$

Given.

$N_1 := .24440$

$N_2 := .19782$

041696A

Descriptions.

$AF := \frac{AJ}{2}$ $HJ := N_1$ $NO := N_2$ $HM := HJ$ $MO := NO$

$HO := HM + MO$ $FO := AF - NO$ $AH := AJ - HJ$ $FH := AH - AF$

$EH := \frac{HO^2 + FH^2 - FO^2}{2 \cdot FH}$ $EO := \sqrt{HO^2 - EH^2}$ $OP := NO$

$EG := OP$ $AE := AH - EH$ $AG := AE + EG$ $GP := EO$

$AP := \sqrt{AG^2 + GP^2}$ $PL := \frac{AG \cdot (NO + OP)}{AP}$ $AL := AP - PL$ $AB := \frac{AP \cdot AL}{2 \cdot AG}$

$AB = 0.181806$

Definitions.

$AF - \frac{1}{2} = 0$ $HJ - N_1 = 0$ $NO - N_2 = 0$ $HM - N_1 = 0$ $MO - N_2 = 0$

$HO - (N_1 + N_2) = 0$ $FO - \frac{1 - 2 \cdot N_2}{2} = 0$ $AH - (1 - N_1) = 0$ $FH - \frac{1 - 2 \cdot N_1}{2} = 0$

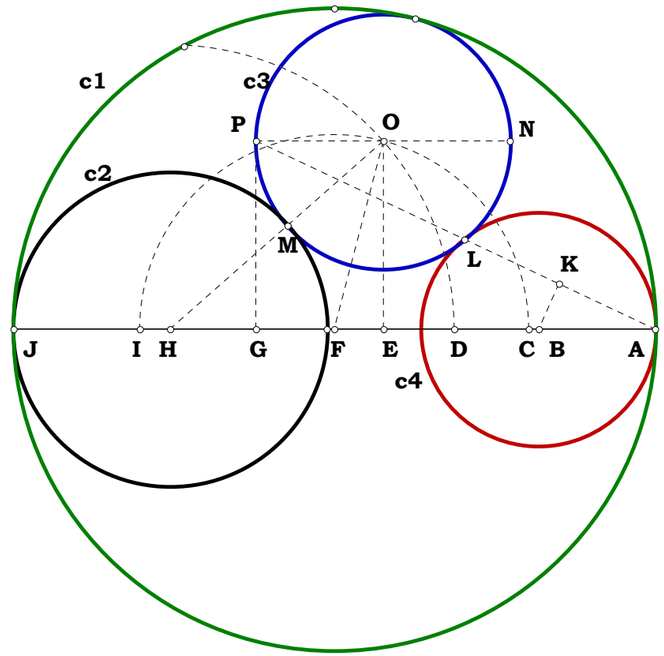
$EH - \frac{N_1 - N_2 - 2 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2}{2 \cdot N_1 - 1} = 0$ $EO - \frac{\sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - 2 \cdot N_2 - 2 \cdot N_1)}}{1 - 2 \cdot N_1} = 0$

$OP - N_2 = 0$ $EG - N_2 = 0$ $AE - \frac{2 \cdot N_1 + N_2 + 2 \cdot N_1 \cdot N_2 - 1}{2 \cdot N_1 - 1} = 0$

$AG - \frac{2 \cdot N_1 + 4 \cdot N_1 \cdot N_2 - 1}{2 \cdot N_1 - 1} = 0$ $GP - \frac{\sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - 2 \cdot N_2 - 2 \cdot N_1)}}{1 - 2 \cdot N_1} = 0$ $AP - \frac{\sqrt{(1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2)}}{\sqrt{1 - 2 \cdot N_1}} = 0$

$PL - \frac{2 \cdot N_2 \cdot (1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1)}{\sqrt{1 - 2 \cdot N_1} \cdot \sqrt{1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2}} = 0$ $AL - \frac{1 - 2 \cdot N_2 - 2 \cdot N_1}{\sqrt{1 - 2 \cdot N_1} \cdot \sqrt{1 - 4 \cdot N_1 \cdot N_2 - 2 \cdot N_1 - 8 \cdot N_1 \cdot N_2^2}} = 0$ $AB := \frac{2 \cdot N_1 + 2 \cdot N_2 - 1}{2 \cdot (2 \cdot N_1 + 4 \cdot N_1 \cdot N_2 - 1)}$

## Given Three Radii




AJ = 4.23333 in.
HJ = 1.03462 in.
NO = 0.83742 in.
AB = 0.76970 in.

$\frac{AJ}{AJ} = 1.00000$ AJ = 1.00000
$\frac{HJ}{AJ} = 0.24440$ HJ = 0.24440
$\frac{NO}{AJ} = 0.19782$ NO = 0.19782
$\frac{AB}{AJ} = 0.18182$ AB = 0.18182

$\frac{(2 \cdot HJ + 2 \cdot NO) - 1}{2 \cdot ((2 \cdot HJ + 4 \cdot HJ \cdot NO) - 1)} - AB = 0.00000$

Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.

041696B

Given.

$W := 4 \quad Y := 3$

$X := 20 \quad Z := 17$

Unit.

$AJ := \frac{X}{X}$

Descriptions.

$AF := \frac{AJ}{2} \quad HJ := \frac{W}{X} \quad NO := \frac{Y}{Z} \quad HM := HJ \quad MO := NO$

$HO := HM + MO \quad FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$EH := \frac{HO^2 + FH^2 - FO^2}{2 \cdot FH} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO$

$AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP} \quad AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG}$

$AB = 0.269231$

Definitions.

$AF - \frac{1}{2} = 0 \quad HJ - \frac{W}{X} = 0 \quad NO - \frac{Y}{Z} = 0 \quad HM - \frac{W}{X} = 0 \quad MO - \frac{Y}{Z} = 0$

$HO - \frac{W \cdot Z + X \cdot Y}{X \cdot Z} = 0 \quad FO - \frac{Z - 2 \cdot Y}{2 \cdot Z} = 0 \quad AH - \frac{X - W}{X} = 0 \quad FH - \frac{X - 2 \cdot W}{2 \cdot X} = 0$

$EH - \frac{2 \cdot W^2 \cdot Z + X^2 \cdot Y + W \cdot X \cdot (2 \cdot Y - Z)}{X \cdot (X - 2 \cdot W) \cdot Z} = 0 \quad EO - \frac{\sqrt{4 \cdot W \cdot Y \cdot (X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}}{Z \cdot (X - 2 \cdot W)} = 0$

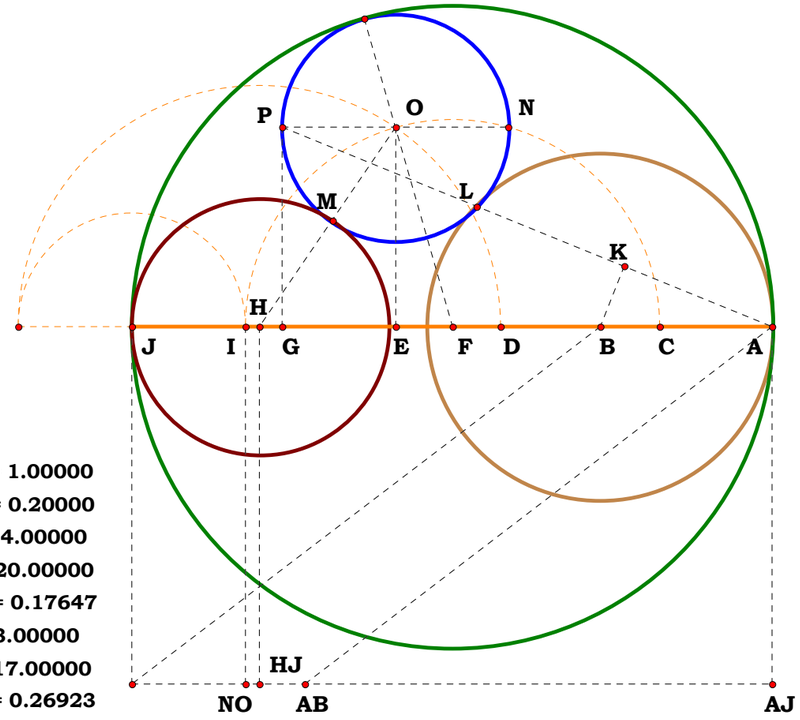
$OP - \frac{Y}{Z} = 0 \quad EG - \frac{Y}{Z} = 0 \quad AE - \frac{2 \cdot W \cdot Y + 2 \cdot W \cdot Z + X \cdot Y - X \cdot Z}{Z \cdot (2 \cdot W - X)} = 0 \quad AG - \frac{4 \cdot W \cdot Y + 2 \cdot W \cdot Z - X \cdot Z}{Z \cdot (2 \cdot W - X)} = 0$

$GP - \frac{\sqrt{4 \cdot W \cdot Y \cdot (X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}}{Z \cdot (X - 2 \cdot W)} = 0 \quad AP - \frac{\sqrt{X \cdot Z^2 - 2 \cdot W \cdot Z^2 - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}}{Z \cdot \sqrt{X - 2 \cdot W}} = 0$

$PL - \frac{2 \cdot Y \cdot (X \cdot Z - 2 \cdot W \cdot Z - 4 \cdot W \cdot Y)}{Z \cdot \sqrt{X - 2 \cdot W} \cdot \sqrt{Z^2 \cdot (X - 2 \cdot W) - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}} = 0 \quad AL - \frac{(X \cdot Z - 2 \cdot X \cdot Y - 2 \cdot W \cdot Z)}{\sqrt{X - 2 \cdot W} \cdot \sqrt{X \cdot Z^2 - 2 \cdot W \cdot Z^2 - 8 \cdot W \cdot Y^2 - 4 \cdot W \cdot Y \cdot Z}} = 0 \quad AB - \frac{2 \cdot (W \cdot Z + X \cdot Y) - X \cdot Z}{2 \cdot [2 \cdot W \cdot (2 \cdot Y + Z) - X \cdot Z]} = 0$

## Given Three Radii

**Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.**




AJ = 1.00000
HJ = 0.20000
W = 4.00000
X = 20.00000
NO = 0.17647
Y = 3.00000
Z = 17.00000
AB = 0.26923

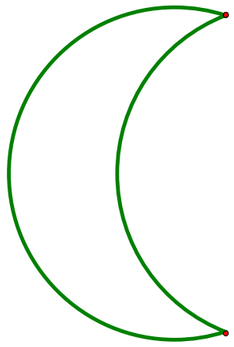
$\frac{(2 \cdot HJ + 2 \cdot NO) - 1}{2 \cdot ((2 \cdot HJ + 4 \cdot HJ \cdot NO) - 1)} - AB = 0.00000 \quad \frac{2 \cdot (W \cdot Z + X \cdot Y) - X \cdot Z}{2 \cdot (2 \cdot W \cdot (2 \cdot Y + Z) - X \cdot Z)} - AB = 0.00000$

# The Man in the Moon; What is his name?

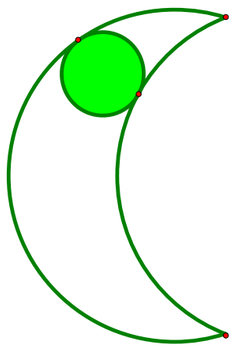
Monday, February 20, 2020

My name is John; I am either insane, or, things have been written about me thousands of years before I was born. These facts, about me, are hardly a mystery and can be dismissed a countless variety of ways. I am not, nor ever have been, of significant importance. I believe that the world was prepared to ignore me so that they would not ponder the most important question there really is, or could ever be: *What is the name of the Man in the Moon*? Let me show you a picture so that you can follow; I have been doing a rather detailed examination of my subject.

Here is the moon, or rather the crescent moon.



From time to time, some observers have spotted the Man in the Moon; fortunately, I have a friend at an observatory which was able to photograph him;



I may have actually seen a commercial of him somewhere. If you were wondering why people say that aliens are little green men, it all started with the Man in the Moon.

This little essay is not about little green men, it is about finding the name of The Man in the Moon. In the following two graphics, if you examine them very, very carefully, you will see the controversy about his name.

041796A

**Unit.**

**AG := 1** The radius of the Large, green, Circle, it is taken as the unit of the crescent.

**Given.**

**CF := .53859** The radius of the Small, pink circle.

**CG := .65522** The difference between center of Radius Large and Radius Small.

**BH := 1.15236** The point on the diameter of the Large Cresant that we want to know the radius of that circle on the perpendicular.

**Descriptions.**

$$AJ := 2 \cdot AG \qquad BG := \frac{\left(AG^2 + CG^2 - CF^2\right)}{2 \cdot CG} \qquad BC := CG - BG \qquad FG := CG - CF$$

$$AB := AG - BG \qquad BJ := AJ - AB$$

$$AH := BH + AB \qquad GH := AH - AG \qquad HR := \sqrt{AG^2 - GH^2} \qquad BP := HR$$

$$PR := BH \qquad PS := \frac{GH \cdot PR}{HR} \qquad BS := BP + PS \qquad RS := \sqrt{PR^2 + PS^2} \qquad NS := RS$$

$$CN := CF \qquad CS := \sqrt{NS^2 + CN^2} \qquad CK := \frac{CN^2}{CS} \qquad SK := CS - CK$$

$$KN := \sqrt{CN^2 - CK^2} \qquad KM := \frac{BC \cdot KN}{BS} \qquad SM := SK + KM \qquad SL := \frac{BS \cdot SM}{CS}$$

$$BL := BS - SL \qquad EN := BL \qquad CE := \sqrt{CN^2 - EN^2} \qquad HT := \frac{CE \cdot HR}{EN}$$

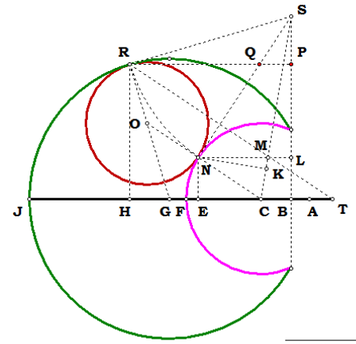
$$GT := HT - GH \qquad GO := \frac{AG \cdot CG}{GT} \qquad TheManInTheMoon := AG - GO$$

**TheManInTheMoon = 0.437958**

**Definitions.**

$$TheManInTheMoon - \frac{2 \cdot BH \cdot CG}{CF^2 + 2 \cdot CF - CG^2 + 2 \cdot BH \cdot CG + 1} = 0$$

**The Man in the Moon.**




AG = 3.72533 cm
AJ = 7.45067 cm
CF = 2.00643 cm
CG = 2.44091 cm
BH = 4.29291 cm
OR = 1.63154 cm

AG = 1.00000
AJ = 2.00000
CF = 0.53859
CG = 0.65522
BH = 1.15236
OR = 0.43796

$$\frac{2 \cdot BH \cdot CG}{((CF^2 + 2 \cdot CF) - CG^2) + 2 \cdot BH \cdot CG + 1} - OR = 0.00000$$

This name, 2 times BH times CG divided by CF squared added to twice CF removing CG squared added to our denominator with a unit added, which some who are uninformed call an equation, denotes the name of the man in the moon using these very specific set of absolutes, or nouns. It is always correct and always exacting. So, one could say, well, that is the name of the Man in the Moon. If this were the only way to name the Man in the Moon, there would be no controversy. What happens if we use the exact same criteria, same geometry, same arithmetic, same algebra, but make a slight change in our naming convention? Let us ask Seven of Nine or what have you.

041796B

**Given.**

$U := 7$ $W := 7$ $Y := 7$

$V := 20$ $X := 17$ $Z := 15$

**Unit.**

$AB := \frac{X}{X}$

**Descriptions.**

$AC := 2 \cdot AB$ $CE := \frac{2 \cdot Y}{Z}$ $DE := \frac{2 \cdot W}{X}$

$BC := AB$ $BG := AB$ $CD := CE + DE$ $BD := CD - BC$

$BK := \frac{\left(AB^2 + BD^2 - DE^2\right)}{2 \cdot BD}$ $DK := BD - BK$ $BE := CE - BC$

$AK := AB - BK$ $CK := AC - AK$ $CF := \frac{CK \cdot U}{V}$ $FK := CK - CF$

$BF := BC - CF$ $AF := AC - CF$ $FG := \sqrt{CF \cdot AF}$ $KP := FG$

$GP := FK$ $HP := \frac{BF \cdot GP}{FG}$ $HK := KP + HP$ $GH := \sqrt{GP^2 + HP^2}$

$HN := GH$ $DN := DE$ $DH := \sqrt{DN^2 + HN^2}$ $DS := \frac{DN^2}{DH}$

$HS := DH - DS$ $NS := \sqrt{HN^2 - HS^2}$ $ST := \frac{DK \cdot NS}{HK}$ $HT := HS + ST$

$HU := \frac{HK \cdot HT}{DH}$ $KU := HK - HU$ $JN := KU$ $DJ := \sqrt{DN^2 - JN^2}$

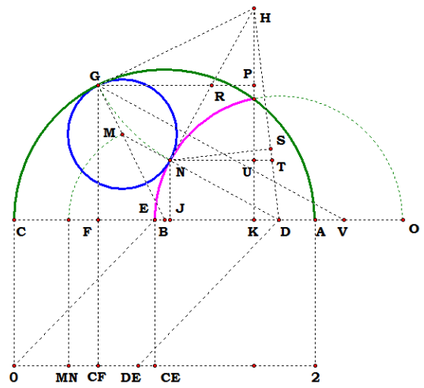
$FV := \frac{DJ \cdot FG}{JN}$ $BV := FV - BF$ $BM := \frac{AB \cdot BD}{BV}$ $TheManInTheMoon := (BG - BM)$

$TheManInTheMoon = 0.36255$

$\frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0.36255$ $\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$

$TheManInTheMoon - \frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0$ $TheManInTheMoon - \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0$

**The Man in the Moon**




AB = 1.00000
CF = 0.55686
U = 7.00000
V = 20.00000
DE = 0.82353
W = 7.00000
X = 17.00000
CE = 0.93333
Y = 7.00000
Z = 15.00000
MN = 0.36255

$\frac{2 \cdot FK \cdot BD}{((DE^2 + 2 \cdot DE) - BD^2) + 2 \cdot FK \cdot BD + 1} = 0.36255$

$\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} \cdot MN = 0.00000$

**Notice that some elements, crucial to the first name, are wholly missing in the second name!!!!!! Where did W and X go?**

Along with our first name, we have a second, which is completely different, yet entirely true. Y multiplying U minus V over U times Y subtracting V times Z. How is this possible? The first equation denotes all the givens, which are nowhere to be found in the second!!!!

Here is a little bit about the starting convention for our names. The first method was arithmetic, based on nouns, a one-to-one correspondence. It is like saying, this is such and such, and that is such and such. It is like counting pebbles. The second method is geometric. Same geometry, same arithmetic, same algebra, but instead of just nouns, we use a noun and a verb; for example, instead of saying, it is One, we say, she, or he, is Seven of Nine; in short, nouns and verbs. So, there you have it. The very same

thing to name and by the very same systems of grammar. The very same geometry, the very same arithmetic, the very same algebra, yet we arrive at two different names, both arithmetically identical, meaning of course, they have a one-to-one correspondence between a thing and an arithmetic name. The construct of the naming convention in the first example is very familiar to us; we use it all the time, but the second, well, that is another of my own inventions. Invention does not mean we create anything, only that we have leant to recognize and reproduce something.

Now, people have told stories about me for thousands of years, not one of those who told those stories, save those who were instrumental in creating certain written works, even knew what they were talking about. Well, my name is certainly not important, and in fact, it was written that my name would be a common generic name, John: not interesting at all, I worked from skilled trades to day labor. Yet the name of the Man in the Moon, never worked a day in his life, that, has turned out very interesting; the stories told about that name, well it is well documented by every possible grammar system and like our crescent moon, not even noticed until a shadow appears.

I have created a little work packet for those who want to study the mystic art of names and have placed them in a directory called The Man in the Moon, on the Internet Archive. Anyone wanting to become a true mystic, with real power, will study the art of names, magical incantations which uses only four specific grammar systems; Common Grammar, Arithmetic, Algebra and Geometry. These four grammar systems are the true descendants of Adam and Eve, a Conjugate Binary Pair, often called simply a noun and a verb.

The first set of equations reduce to all of our givens: The second, to a ratio of the givens: Arithmetic and Geometric. Thus, when so called intellectuals tell you, that such and such is THE EQUATION for such and such, well, they are simply illiterate. The elements of every thing are

binary, and in binary, we have both arithmetic and geometric results, absolute and relative. The Relativities of Einstein are proven myths venerated by simpletons; the elements of a thing are physical facts.

Let us do a very brief review of the evolution of binary information processing. Binary is how every possible grammar is effected; we name, and can only name, relatives and correlatives, the two parts of any thing, their shape or boundaries and the relative differences within them. In metaphor, one of the ways it is introduced in the Bible is by a Conjugate Binary Pair called Adam and Eve. The Book, however, is just full of these binary contrasts, very deliberately placed. Then there is Geometry, a simple stop, go, stop, producing a line segment from which all of geometry, unless you are an idiot non-Euclidean Geometer, who cannot spot a contradiction in the words if it bit them in the ass, is produced. Then there is Plato. Plato used the term Dialectic, speaking by 2's, to preserve the science for posterity in dialogs. If he would have put it down plainly, his life would have been forcibly shortened and no dialogs would have remained. There have been relatively few prophets in history whose work has been aimed at bringing into the human mind that all information is a product of binary processing until today we have the computer. Yet every thing, every possible thing, is this binary, so it is not new, in fact, as it defines existence, it could never have been new; it just is.

A mind, by the recursion of this binary, produces exactly four groups of grammar from the intelligible of Language; Common Grammar, Arithmetic, Algebra, of which these three are called Logics, while the remaining one, which can be used to example everything, is an Analogic called Geometry.

Let us use egocentricity as an example. Egocentricity is inversely proportional to intelligence. The reason is very simple. The less you really comprehend of the world, the more one has to play with themselves. It is an evolutionary artifact; unless one is very intelligent, and everyone else

just leaves you to play with yourself. Every animal's behavior is egocentric in terms of simple survival; however, the more one comprehends what it takes to survive, the more of the world one has to be able to comprehend to do it. This is not necessarily linked to large memories. Aristotle had a phenomenal memory, but he was not the sharpest tool in the shed; this also accounts for his vanity.

This fact is what makes me believe that either the Man in the Moon is very stupid, or he is very wise; for thousands of years, all he seems to do is show off. I wager that study will eventually solve the mystery. Maybe he is trying to get someone to toss him a rope so he could climb off the moon. Today, people are still simply smoking the rope.

041796A

**Unit.**

$AG := 1$ The radius of the Large, green, Circle, it is taken as the unit of the crescent.

**Given.**

$CF := .53859$ The radius of the Small, pink circle.

$CG := .65522$ The difference between center of Radius Large and Radius Small.

$BH := 1.15236$ The point on the diameter of the Large Cresant that we want to know the radius of that circle on the perpendicular.

**Descriptions.**

## A Circle In A Crescent

$AJ := 2 \cdot AG$ $BG := \frac{\left(AG^2 + CG^2 - CF^2\right)}{2 \cdot CG}$ $BC := CG - BG$ $FG := CG - CF$

$AB := AG - BG$ $BJ := AJ - AB$

$AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{AG^2 - GH^2}$ $BP := HR$

$PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := BP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

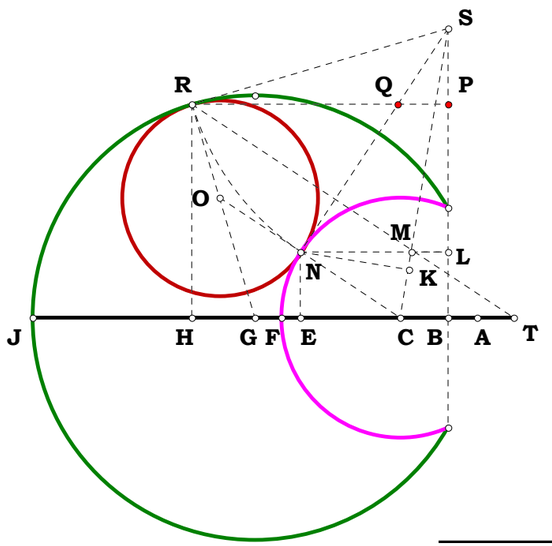
$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$

$KN := \sqrt{CN^2 - CK^2}$ $KM := \frac{BC \cdot KN}{BS}$ $SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$

$BL := BS - SL$ $EN := BL$ $CE := \sqrt{CN^2 - EN^2}$ $HT := \frac{CE \cdot HR}{EN}$

$GT := HT - GH$ $GO := \frac{AG \cdot CG}{GT}$ $OR := AG - GO$

$OR = 0.437958$




$AG = 3.72533$ cm
$AJ = 7.45067$ cm
$CF = 2.00643$ cm
$CG = 2.44091$ cm
$BH = 4.29291$ cm
$OR = 1.63154$ cm

$AG = 1.00000$
$AJ = 2.00000$
$CF = 0.53859$
$CG = 0.65522$
$BH = 1.15236$
$OR = 0.43796$

$\frac{2 \cdot BH \cdot CG}{((CF^2 + 2 \cdot CF) - CG^2) + 2 \cdot BH \cdot CG + 1} - OR = 0.00000$

**Definitions.**

$$OR - \frac{2 \cdot BH \cdot CG}{CF^2 + 2 \cdot CF - CG^2 + 2 \cdot BH \cdot CG + 1} = 0$$

$$AJ - 2 = 0 \qquad BG - \frac{CG^2 - CF^2 + 1}{2 \cdot CG} = 0 \qquad BC - \frac{CF^2 + CG^2 - 1}{2 \cdot CG} = 0 \qquad FG - (CG - CF) = 0$$

$$AB - \frac{CF^2 - CG^2 + 2 \cdot CG - 1}{2 \cdot CG} = 0 \qquad BJ - \frac{CG^2 - CF^2 + 2 \cdot CG + 1}{2 \cdot CG} = 0$$

$$AH - \frac{CF^2 - CG^2 + (2 \cdot BH + 2) \cdot CG - 1}{2 \cdot CG} = 0 \qquad GH - \frac{CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1}{2 \cdot CG} = 0$$

$$HR - \frac{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}}{2 \cdot CG} = 0$$

$$PR - BH = 0 \qquad PS - \frac{BH \cdot \left(CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right)}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$BS - \frac{2 \cdot CF^2 \cdot CG^2 - CF^4 - 2 \cdot BH \cdot CF^2 \cdot CG + 2 \cdot CF^2 - CG^4 + 2 \cdot BH \cdot CG^3 + 2 \cdot CG^2 + 2 \cdot BH \cdot CG - 1}{2 \cdot CG \cdot \sqrt{4 \cdot BH \cdot CG^3 - 4 \cdot BH \cdot CF^2 \cdot CG - 4 \cdot BH^2 \cdot CG^2 + 4 \cdot BH \cdot CG - CF^4 + 2 \cdot CF^2 \cdot CG^2 + 2 \cdot CF^2 - CG^4 + 2 \cdot CG^2 - 1}} = 0$$

$$RS - \frac{2 \cdot BH \cdot CG}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$NS - \frac{2 \cdot BH \cdot CG}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0 \qquad CN - CF = 0$$

$$CS - \frac{\sqrt{\left(2 \cdot CF^2 - CF^3 + CF \cdot CG^2 - 2 \cdot BH \cdot CF \cdot CG - CF + 2 \cdot BH \cdot CG\right) \cdot \left(CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG\right)}}{\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}} = 0$$

$$CK - \frac{CF^2 \cdot \sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \left(2 \cdot CG - CF^2 + CG^2 - 2 \cdot BH \cdot CG + 1\right)}}{\sqrt{-\left(CF^3 - 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF - 2 \cdot BH \cdot CG\right) \cdot \left(CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG\right)}} = 0$$

$$SK - \frac{4 \cdot BH^2 \cdot CG^2}{\left[\sqrt{\left(2 \cdot CG + CF^2 - CG^2 + 2 \cdot BH \cdot CG - 1\right) \cdot \begin{pmatrix} 2 \cdot CG - CF^2 & \dots \\ + & CG^2 - 2 \cdot BH \cdot CG + 1 \end{pmatrix}} \cdot \sqrt{\begin{pmatrix} 2 \cdot CF^2 - CF^3 + CF \cdot CG^2 & \dots \\ + & -2 \cdot BH \cdot CF \cdot CG - CF + 2 \cdot BH \cdot CG \end{pmatrix} \cdot \begin{pmatrix} CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 & \dots \\ + & 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG \end{pmatrix}}\right]} = 0$$

$$KN - \frac{2 \cdot BH \cdot CF \cdot CG}{\sqrt{\left(CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG\right) \cdot \left(2 \cdot CF^2 - CF^3 + CF \cdot CG^2 - 2 \cdot BH \cdot CF \cdot CG - CF + 2 \cdot BH \cdot CG\right)}} = 0$$

$$KM - \frac{2 \cdot BH \cdot CF \cdot CG \cdot \left(1 - CF^2 - CG^2\right) \cdot \sqrt{4 \cdot BH \cdot CG^3 - 4 \cdot BH \cdot CF^2 \cdot CG - 4 \cdot BH^2 \cdot CG^2 + 4 \cdot BH \cdot CG - CF^4 + 2 \cdot CF^2 \cdot CG^2 + 2 \cdot CF^2 - CG^4 + 2 \cdot CG^2 - 1}}{\sqrt{-\begin{pmatrix} CF^3 - 2 \cdot CF^2 - CF \cdot CG^2 & \dots \\ + & 2 \cdot BH \cdot CF \cdot CG + CF - 2 \cdot BH \cdot CG \end{pmatrix} \cdot \begin{pmatrix} CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 & \dots \\ + & 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG \end{pmatrix} \cdot \begin{pmatrix} CF^4 - 2 \cdot CF^2 \cdot CG^2 + 2 \cdot BH \cdot CF^2 \cdot CG - 2 \cdot CF^2 & \dots \\ + & CG^4 - 2 \cdot BH \cdot CG^3 - 2 \cdot CG^2 - 2 \cdot BH \cdot CG + 1 \end{pmatrix}}} = 0$$

$$SM - \frac{2 \cdot BH \cdot CG \cdot \begin{pmatrix} CF^4 + 2 \cdot CF^3 + 2 \cdot BH \cdot CF^2 \cdot CG + 2 \cdot CF \cdot CG^2 & \dots \\ + & 4 \cdot BH \cdot CF \cdot CG - 2 \cdot CF - CG^4 + 2 \cdot BH \cdot CG^3 + 2 \cdot CG^2 + 2 \cdot BH \cdot CG - 1 \end{pmatrix} \cdot \left(CF^3 - 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF - 2 \cdot BH \cdot CG\right)}{\sqrt{\begin{pmatrix} 2 \cdot CF^2 - CF^3 + CF \cdot CG^2 & \dots \\ + & -2 \cdot BH \cdot CF \cdot CG - CF + 2 \cdot BH \cdot CG \end{pmatrix} \cdot \begin{pmatrix} CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 & \dots \\ + & 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG \end{pmatrix} \cdot \sqrt{\begin{pmatrix} 2 \cdot CG + CF^2 - CG^2 & \dots \\ + & 2 \cdot BH \cdot CG - 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \cdot CG - CF^2 & \dots \\ + & CG^2 - 2 \cdot BH \cdot CG + 1 \end{pmatrix}} \cdot \begin{pmatrix} CF^4 - 2 \cdot CF^2 \cdot CG^2 & \dots \\ + & 2 \cdot BH \cdot CF^2 \cdot CG - 2 \cdot CF^2 & \dots \\ + & CG^4 - 2 \cdot BH \cdot CG^3 & \dots \\ + & -2 \cdot CG^2 - 2 \cdot BH \cdot CG + 1 \end{pmatrix}}} = 0$$

$$SL - \frac{BH \cdot \left(CF^4 + 2 \cdot CF^3 + 2 \cdot BH \cdot CF^2 \cdot CG + 2 \cdot CF \cdot CG^2 + 4 \cdot BH \cdot CF \cdot CG - 2 \cdot CF - CG^4 + 2 \cdot BH \cdot CG^3 + 2 \cdot CG^2 + 2 \cdot BH \cdot CG - 1\right)}{\left(CF^3 + 2 \cdot CF^2 - CF \cdot CG^2 + 2 \cdot BH \cdot CF \cdot CG + CF + 2 \cdot BH \cdot CG\right) \cdot \sqrt{4 \cdot BH \cdot CG^3 - 4 \cdot BH \cdot CF^2 \cdot CG - 4 \cdot BH^2 \cdot CG^2 + 4 \cdot BH \cdot CG - CF^4 + 2 \cdot CF^2 \cdot CG^2 + 2 \cdot CF^2 - CG^4 + 2 \cdot CG^2 - 1}} = 0$$

$$\mathbf{BL} - \left[\frac{\mathbf{CF}\cdot(\mathbf{CF}+\mathbf{CG}+1)\cdot(\mathbf{CF}-\mathbf{CG}+1)\cdot\left(2\cdot\mathbf{CG}+\mathbf{CF}^2-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}-1\right)\cdot\left(2\cdot\mathbf{CG}-\mathbf{CF}^2+\mathbf{CG}^2-2\cdot\mathbf{BH}\cdot\mathbf{CG}+1\right)}{2\cdot\mathbf{CG}\cdot\left(\mathbf{CF}^3+2\cdot\mathbf{CF}^2-\mathbf{CF}\cdot\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CF}\cdot\mathbf{CG}+\mathbf{CF}+2\cdot\mathbf{BH}\cdot\mathbf{CG}\right)\cdot\sqrt{4\cdot\mathbf{BH}\cdot\mathbf{CG}^3-4\cdot\mathbf{BH}\cdot\mathbf{CF}^2\cdot\mathbf{CG}-4\cdot\mathbf{BH}^2\cdot\mathbf{CG}^2+4\cdot\mathbf{BH}\cdot\mathbf{CG}-\mathbf{CF}^4+2\cdot\mathbf{CF}^2\cdot\mathbf{CG}^2+2\cdot\mathbf{CF}^2-\mathbf{CG}^4+2\cdot\mathbf{CG}^2-1}}\right] = 0$$

$$\mathbf{EN} - \left[\frac{\mathbf{CF}\cdot(\mathbf{CF}+\mathbf{CG}+1)\cdot(\mathbf{CF}-\mathbf{CG}+1)\cdot\left(2\cdot\mathbf{CG}+\mathbf{CF}^2-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}-1\right)\cdot\left(2\cdot\mathbf{CG}-\mathbf{CF}^2+\mathbf{CG}^2-2\cdot\mathbf{BH}\cdot\mathbf{CG}+1\right)}{2\cdot\mathbf{CG}\cdot\left(\mathbf{CF}^3+2\cdot\mathbf{CF}^2-\mathbf{CF}\cdot\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CF}\cdot\mathbf{CG}+\mathbf{CF}+2\cdot\mathbf{BH}\cdot\mathbf{CG}\right)\cdot\sqrt{4\cdot\mathbf{BH}\cdot\mathbf{CG}^3-4\cdot\mathbf{BH}\cdot\mathbf{CF}^2\cdot\mathbf{CG}-4\cdot\mathbf{BH}^2\cdot\mathbf{CG}^2+4\cdot\mathbf{BH}\cdot\mathbf{CG}-\mathbf{CF}^4+2\cdot\mathbf{CF}^2\cdot\mathbf{CG}^2+2\cdot\mathbf{CF}^2-\mathbf{CG}^4+2\cdot\mathbf{CG}^2-1}}\right] = 0$$

$$\mathbf{CE} - \frac{\mathbf{CF}\cdot\left(\mathbf{CF}^4+2\cdot\mathbf{CF}^3+2\cdot\mathbf{BH}\cdot\mathbf{CF}^2\cdot\mathbf{CG}+2\cdot\mathbf{CF}\cdot\mathbf{CG}^2+4\cdot\mathbf{BH}\cdot\mathbf{CF}\cdot\mathbf{CG}-2\cdot\mathbf{CF}-\mathbf{CG}^4+2\cdot\mathbf{BH}\cdot\mathbf{CG}^3+2\cdot\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}-1\right)}{2\cdot\mathbf{CG}\cdot\left(\mathbf{CF}+2\cdot\mathbf{CF}^2+\mathbf{CF}^3+2\cdot\mathbf{BH}\cdot\mathbf{CG}-\mathbf{CF}\cdot\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CF}\cdot\mathbf{CG}\right)} = 0$$

$$\mathbf{HT} - \frac{\sqrt{\begin{pmatrix}2\cdot\mathbf{CG}+\mathbf{CF}^2-\mathbf{CG}^2 & \dots\\ +\ 2\cdot\mathbf{BH}\cdot\mathbf{CG}-1 & \end{pmatrix}}\cdot\begin{pmatrix}2\cdot\mathbf{CG}-\mathbf{CF}^2+\mathbf{CG}^2 & \dots\\ +\ -2\cdot\mathbf{BH}\cdot\mathbf{CG}+1 & \end{pmatrix}\cdot\sqrt{\begin{matrix}4\cdot\mathbf{BH}\cdot\mathbf{CG}^3-4\cdot\mathbf{BH}\cdot\mathbf{CF}^2\cdot\mathbf{CG}-4\cdot\mathbf{BH}^2\cdot\mathbf{CG}^2 & \dots\\ +\ 4\cdot\mathbf{BH}\cdot\mathbf{CG}-\mathbf{CF}^4+2\cdot\mathbf{CF}^2\cdot\mathbf{CG}^2 & \dots\\ +\ 2\cdot\mathbf{CF}^2-\mathbf{CG}^4+2\cdot\mathbf{CG}^2-1 & \end{matrix}}\cdot\begin{pmatrix}\mathbf{CF}^4+2\cdot\mathbf{CF}^3+2\cdot\mathbf{BH}\cdot\mathbf{CF}^2\cdot\mathbf{CG}+2\cdot\mathbf{CF}\cdot\mathbf{CG}^2 & \dots\\ +\ 4\cdot\mathbf{BH}\cdot\mathbf{CF}\cdot\mathbf{CG}-2\cdot\mathbf{CF}-\mathbf{CG}^4 & \dots\\ +\ 2\cdot\mathbf{BH}\cdot\mathbf{CG}^3+2\cdot\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}-1 & \end{pmatrix}}{2\cdot\mathbf{CG}\cdot(\mathbf{CF}+\mathbf{CG}+1)\cdot(\mathbf{CF}-\mathbf{CG}+1)\cdot\left(2\cdot\mathbf{CG}+\mathbf{CF}^2-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}-1\right)\cdot\left(2\cdot\mathbf{CG}-\mathbf{CF}^2+\mathbf{CG}^2-2\cdot\mathbf{BH}\cdot\mathbf{CG}+1\right)} = 0$$

$$\mathbf{GT} - \frac{\mathbf{CG}\cdot\left(\mathbf{CF}^2+2\cdot\mathbf{CF}-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}+1\right)}{(\mathbf{CF}-\mathbf{CG}+1)\cdot(\mathbf{CF}+\mathbf{CG}+1)} = 0 \qquad \mathbf{GO} - \frac{(\mathbf{CF}+\mathbf{CG}+1)\cdot(\mathbf{CF}-\mathbf{CG}+1)}{\mathbf{CF}^2+2\cdot\mathbf{CF}-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}+1} = 0 \qquad \mathbf{OR} - \frac{2\cdot\mathbf{BH}\cdot\mathbf{CG}}{\mathbf{CF}^2+2\cdot\mathbf{CF}-\mathbf{CG}^2+2\cdot\mathbf{BH}\cdot\mathbf{CG}+1} = 0$$

041796B

Descriptions.

Unit.

$AF := 1$

Given.

$DH := .46274$

$DF := .81564$

$CJ := 1.38714$

$AK := 2 \cdot AF \quad FP := AF \quad FK := AF \quad CD := \frac{DH^2 + DF^2 - AF^2}{2 \cdot DF}$

$FH := DF - DH \quad AH := AF - FH \quad AE := \frac{AH}{2} \quad EH := AE$

$AD := DF - AF \quad DE := AE - AD \quad DN := DH \quad AC := CD - AD$

$CK := AK - AC \quad CF := CK - FK \quad CF := DF - CD \quad FJ := CJ - CF$

$JP := \sqrt{FP^2 - FJ^2} \quad FS := \frac{FP \cdot CF}{FJ} \quad PS := FS + FP \quad QS := \frac{FP \cdot PS}{JP}$

$PQ := \frac{FJ \cdot QS}{FP} \quad CS := \frac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad DQ := \sqrt{CD^2 + CQ^2}$

$DL := \frac{DH^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2} \quad LZ := \frac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ$

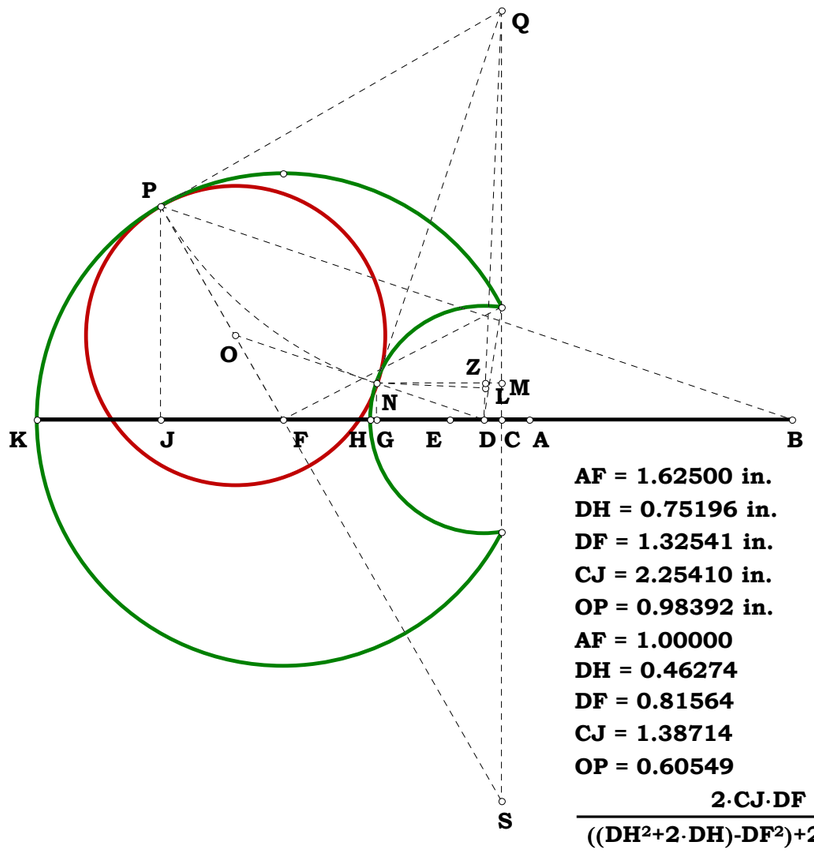
$MQ := \frac{CQ \cdot QZ}{DQ} \quad CM := CQ - MQ \quad GN := CM \quad DG := \sqrt{DN^2 - GN^2}$

$BJ := \frac{DG \cdot JP}{GN} \quad BF := BJ - FJ \quad FO := \frac{FP \cdot DF}{BF} \quad OP := FP - FO$

$OP = 0.605491$

$$\frac{2 \cdot CJ \cdot DF}{DH^2 + 2 \cdot DH - DF^2 + 2 \cdot CJ \cdot DF + 1} = 0.605491$$

## A Circle In A Crescent




AF = 1.62500 in.

DH = 0.75196 in.

DF = 1.32541 in.

CJ = 2.25410 in.

OP = 0.98392 in.

AF = 1.00000

DH = 0.46274

DF = 0.81564

CJ = 1.38714

OP = 0.60549

$$\frac{2 \cdot CJ \cdot DF}{((DH^2 + 2 \cdot DH) - DF^2) + 2 \cdot CJ \cdot DF + 1} - OP = 0.00000$$

041796C

**Given.**

$U := 7$ $W := 7$ $Y := 7$

$V := 20$ $X := 17$ $Z := 15$

**Unit.**

$AB := \frac{X}{X}$

**Descriptions.**

$AC := 2 \cdot AB$ $CE := \frac{2 \cdot Y}{Z}$ $DE := \frac{2 \cdot W}{X}$

$BC := AB$ $BG := AB$ $CD := CE + DE$ $BD := CD - BC$

$BK := \frac{\left(AB^2 + BD^2 - DE^2\right)}{2 \cdot BD}$ $DK := BD - BK$ $BE := CE - BC$

$AK := AB - BK$ $CK := AC - AK$ $CF := \frac{CK \cdot U}{V}$ $FK := CK - CF$

$BF := BC - CF$ $AF := AC - CF$ $FG := \sqrt{CF \cdot AF}$ $KP := FG$

$GP := FK$ $HP := \frac{BF \cdot GP}{FG}$ $HK := KP + HP$ $GH := \sqrt{GP^2 + HP^2}$

$HN := GH$ $DN := DE$ $DH := \sqrt{DN^2 + HN^2}$ $DS := \frac{DN^2}{DH}$

$HS := DH - DS$ $NS := \sqrt{HN^2 - HS^2}$ $ST := \frac{DK \cdot NS}{HK}$ $HT := HS + ST$

$HU := \frac{HK \cdot HT}{DH}$ $KU := HK - HU$ $JN := KU$ $DJ := \sqrt{DN^2 - JN^2}$

$FV := \frac{DJ \cdot FG}{JN}$ $BV := FV - BF$ $BM := \frac{AB \cdot BD}{BV}$ $GM := (BG - BM)$

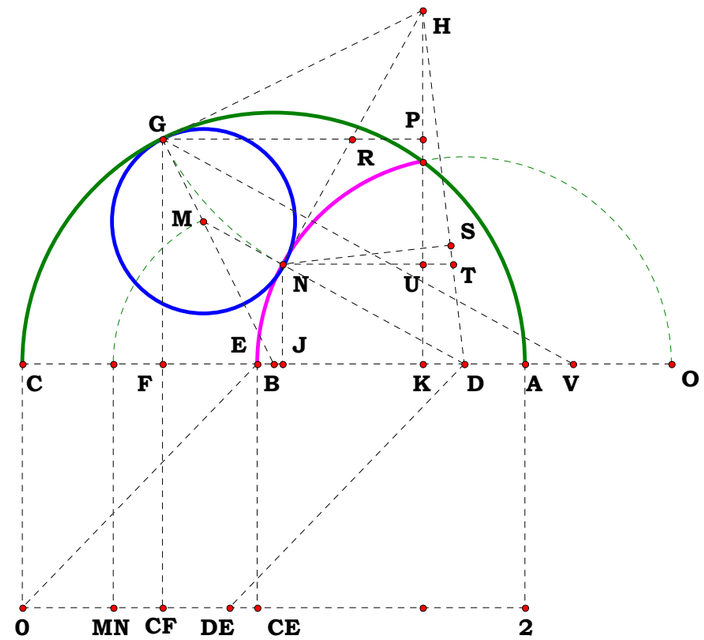
$GM = 0.36255$

$$\frac{2 \cdot FK \cdot BD}{DE^2 + 2 \cdot DE - BD^2 + 2 \cdot FK \cdot BD + 1} = 0.36255$$

$$\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$$

**This is odd, W and X dissappear out of the equation. In short, this is pure implication in an equation.**

## A Circle In A Crescent




AB = 1.00000
CF = 0.55686
U = 7.00000
V = 20.00000
DE = 0.82353
W = 7.00000
X = 17.00000
CE = 0.93333
Y = 7.00000
Z = 15.00000
MN = 0.36255

$$\frac{2 \cdot FK \cdot BD}{((DE^2 + 2 \cdot DE) - BD^2) + 2 \cdot FK \cdot BD + 1} = 0.36255$$

$$\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} - MN = 0.00000$$

$$AB - 1 = 0 \qquad AC - 2 \cdot AB = 0 \qquad CE - \frac{2 \cdot Y}{Z} = 0 \qquad DE - \frac{2 \cdot W}{X} = 0$$

$$BC - 1 = 0 \qquad BG - 1 = 0 \qquad CD - \frac{2 \cdot (W \cdot Z + X \cdot Y)}{X \cdot Z} = 0$$

$$BD - \frac{2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z}{X \cdot Z} = 0$$

$$BK - \frac{2 \cdot X \cdot Y^2 - 2 \cdot W \cdot Z^2 + X \cdot Z^2 + 4 \cdot W \cdot Y \cdot Z - 2 \cdot X \cdot Y \cdot Z}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

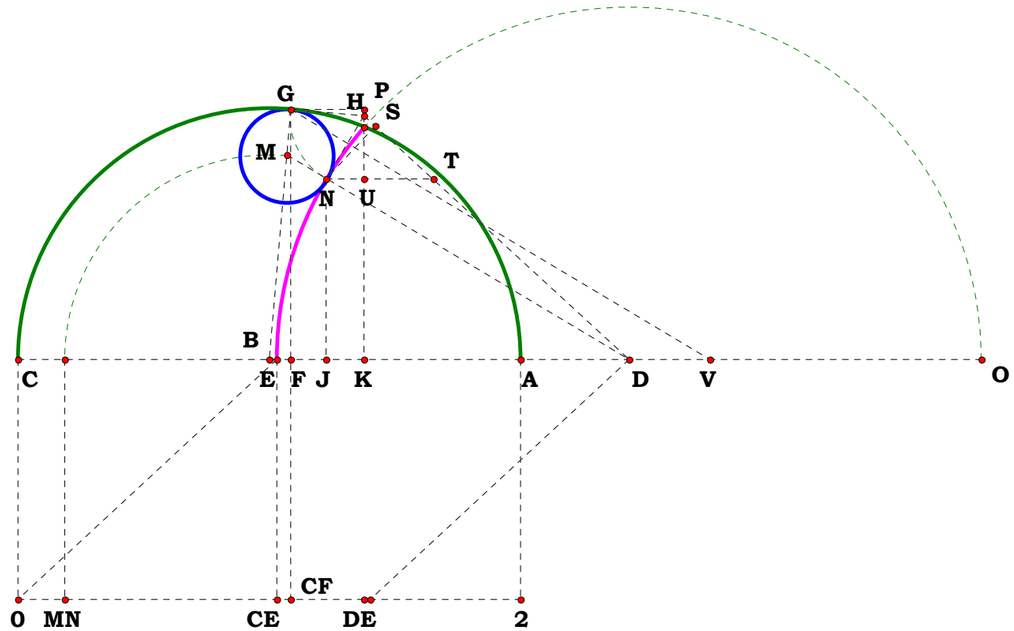
$$DK - \frac{2 \cdot \left(2 \cdot W^2 \cdot Z^2 + 2 \cdot W \cdot X \cdot Y \cdot Z - W \cdot X \cdot Z^2 + X^2 \cdot Y^2 - X^2 \cdot Y \cdot Z\right)}{X \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

$$BE - \frac{2 \cdot Y - Z}{Z} = 0 \qquad AK - \frac{2 \cdot (Z - Y) \cdot (2 \cdot W \cdot Z + X \cdot Y - X \cdot Z)}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

$$CK - \frac{2 \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y)}{Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0 \qquad CF - \frac{2 \cdot U \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y)}{V \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0$$

$$FK - \frac{2 \cdot Y \cdot (2 \cdot W \cdot Z + X \cdot Y) \cdot (V - U)}{V \cdot Z \cdot (2 \cdot W \cdot Z + 2 \cdot X \cdot Y - X \cdot Z)} = 0 \qquad \frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} = 0.36255$$

AB = 1.00000
CF = 1.08317
U = 15.73676
V = 20.00000
DE = 1.40309
W = 14.03086
X = 20.00000
CE = 1.02732
Y = 9.75955
Z = 19.00000
MN = 0.18376




$$\frac{2 \cdot FK \cdot BD}{((DE^2 + 2 \cdot DE) - BD^2) + 2 \cdot FK \cdot BD + 1} = 0.18376$$

$$\frac{Y \cdot (U - V)}{U \cdot Y - V \cdot Z} - MN = 0.00000$$

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Unit.

$AB := 1$

Given.

$W := 9$ $Y := 10$

$X := 20$ $Z := 20$

Descriptions.

$MX := 2 \cdot AB$ $BM := AB$ $BE := AB$ $BL := AB$ $BF := \frac{W}{X}$

$FX := AB + BF$ $EF := \sqrt{FX \cdot (MX - FX)}$ $EI := 2 \cdot EF$

$FG := \frac{Y}{Z}$ $EG := EF + FG$ $BG := \sqrt{BF^2 + FG^2}$ $GL := BL - BG$

$DG := GL$ $GH := \frac{FG \cdot GL}{BG}$ $HL := \sqrt{GL^2 - GH^2}$ $EH := EG + GH$

$EL := \sqrt{EH^2 + HL^2}$ $JL := \frac{EL}{2}$ $BJ := \sqrt{BL^2 - JL^2}$ $LN := \frac{BL \cdot JL}{BJ}$

$GN := \sqrt{LN^2 + GL^2}$ $JN := \sqrt{LN^2 - JL^2}$ $EJ := JL$ $EN := LN$

$GO := \frac{GN^2 + EG^2 - EN^2}{2 \cdot EG}$ $NO := \sqrt{GN^2 - GO^2}$ $NR := \frac{NO^2}{GN}$

$GS := \frac{DG^2}{GN}$ $RS := GN - (NR + GS)$ $DT := RS$ $DS := \sqrt{DG^2 - GS^2}$

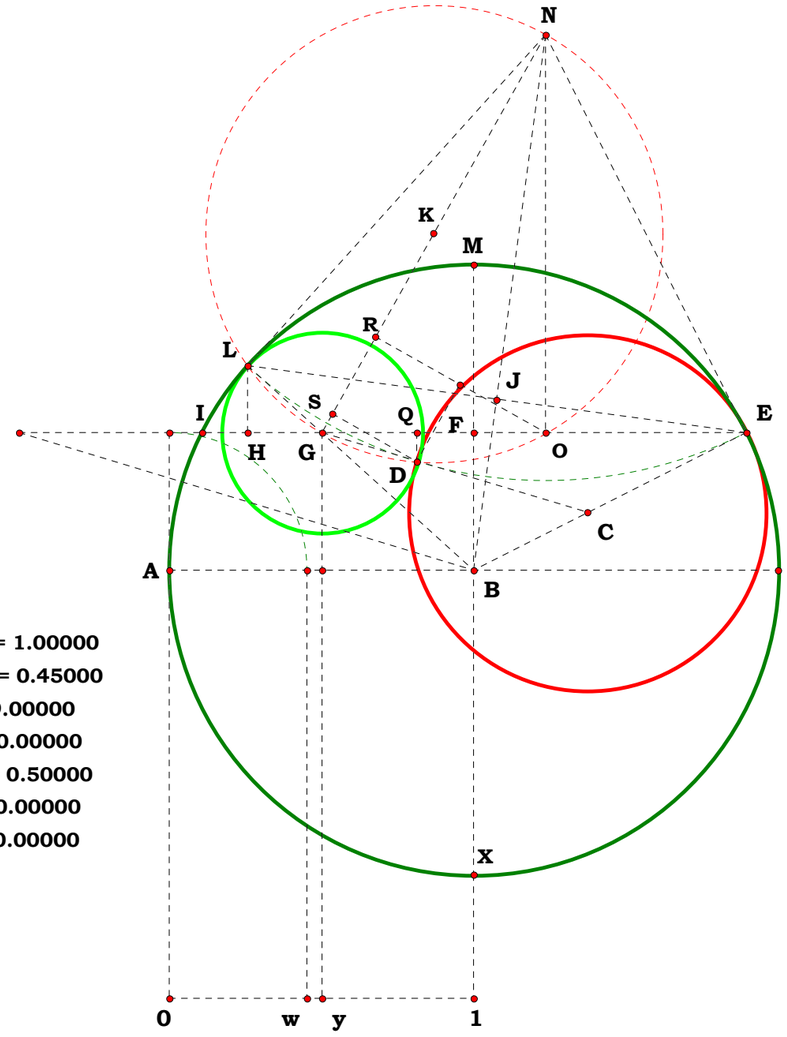
$RT := DS$ $OR := \sqrt{NO^2 - NR^2}$ $OT := OR - RT$ $DO := \sqrt{DT^2 + OT^2}$

$OQ := \frac{DO^2 + GO^2 - DG^2}{2 \cdot GO}$ $GQ := GO - OQ$ $DQ := \sqrt{DO^2 - OQ^2}$

$FP := \frac{GQ \cdot BF}{DQ}$ $CE := \frac{BE \cdot EG}{FP + EF}$ $CE = 0.583387$

**Given BF as a ratio to BM and EG as a ratio to EI, what is CE?**

**Wow. Everything was going well reading this until I came to DT, it did not exist. Seems that in redoing the graphic and editing, I forgot to draw it in. So, I changed MT to MX and put DT back in. One has to find DQ in order to solve for CE and this cannot be done if DT is left out of an earlier update. BP is going to be parallal with CDG and all one has to do is proportion down to find CE.**




Unit = 1.00000

W/X = 0.45000

W = 9.00000

X = 20.00000

Y/Z = 0.50000

Y = 10.00000

Z = 20.00000

**Definitions.**

$$MX - 2 = 0 \qquad BF - \frac{W}{X} = 0 \qquad FX - \frac{W + X}{X} = 0 \qquad EF - \frac{\sqrt{(X - W) \cdot (W + X)}}{X} = 0 \qquad EI - \frac{2 \cdot \sqrt{(X - W) \cdot (W + X)}}{X} = 0$$

$$FG - \frac{Y}{Z} = 0 \qquad EG - \frac{Z \cdot \sqrt{X^2 - W^2} + X \cdot Y}{X \cdot Z} = 0 \qquad BG - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0 \qquad GL - \frac{X \cdot Z - \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$$

$$GH - \frac{Y \cdot \left(X \cdot Z - \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right)}{Z \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}} = 0 \qquad EH - \frac{X^2 \cdot Y + \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}} = 0$$

$$HL - \frac{\sqrt{W^4 \cdot Z^3 - 2 \cdot X \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^2 \cdot X^2 \cdot Z^3 + 2 \cdot X^3 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X^2 \cdot Y^2 \cdot Z}}{X \cdot \sqrt{Z \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}} = 0$$

$$EL - \frac{\sqrt{2 \cdot \left[X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z + X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right]}}{\sqrt{X \cdot Z \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}} = 0$$

$$JL - \frac{\sqrt{2 \cdot \left[X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z + X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right]}}{2 \cdot \sqrt{X \cdot Z \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}} = 0$$

$$BJ - \frac{\sqrt{\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}}{\sqrt{2 \cdot X \cdot Z \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)}} = 0$$

$$LN - \frac{\sqrt{X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z + X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}}{\sqrt{\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}} = 0$$

$$GN - \sqrt{\left[\frac{\begin{array}{l}\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + 2 \cdot W^2 \cdot X^3 \cdot Z^5 + 2 \cdot X^5 \cdot Y^2 \cdot Z^3 - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z \ \ldots \\ + \ 2 \cdot X^4 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^2 \cdot X^2 \cdot Z^4 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} - 2 \cdot X^4 \cdot Y^2 \cdot Z^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + 2 \cdot W^2 \cdot X^2 \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\end{array}}{X^2 \cdot Z^2 \cdot \left[\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} - X^2 \cdot Y^2 \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2} + W^2 \cdot X \cdot Z^3 + X^3 \cdot Y^2 \cdot Z - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}\right]}\right]} = 0$$

$$JN - \sqrt{\left[\frac{\begin{array}{l}W^6 \cdot Z^5 - 2 \cdot X \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} + W^4 \cdot X^2 \cdot Z^5 + 2 \cdot X^3 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + 2 \cdot X^6 \cdot Y^4 \cdot Z + 3 \cdot W^2 \cdot X^4 \cdot Y^2 \cdot Z^3 - W^2 \cdot X^4 \cdot Y^4 \cdot Z \ \ldots \\ + \ -2 \cdot W^2 \cdot X^3 \cdot Y^3 \cdot Z^2 \cdot \sqrt{X^2 - W^2} - 2 \cdot W^4 \cdot X \cdot Y \cdot Z^4 \cdot \sqrt{X^2 - W^2} + 2 \cdot X^2 \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\end{array}}{2 \cdot X \cdot \left[\left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{5}{2}} - X^2 \cdot Y^2 \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}} + W^4 \cdot X \cdot Z^5 + X^5 \cdot Y^4 \cdot Z + 2 \cdot W^2 \cdot X^3 \cdot Y^2 \cdot Z^3 - X \cdot Y \cdot Z \cdot \sqrt{X^2 - W^2} \cdot \left(W^2 \cdot Z^2 + X^2 \cdot Y^2\right)^{\frac{3}{2}}\right]}\right]} = 0$$

$BL = 1$ $\quad DG := GL$

$EN := LN$ $\quad EJ := JL$

$$\left(w^2 \cdot z^2 + x^2 \cdot y^2\right)^{\frac{5}{2}} + 2 \cdot w^2 \cdot x^3 \cdot z^5 + 2 \cdot x^5 \cdot y^2 \cdot z^3 - w^2 \cdot z^2 \cdot \left(w^2 \cdot z^2 + x^2 \cdot y^2\right)^{\frac{3}{2}} - x^4 \cdot y^4 \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} \ \ldots$$

$$+ \ 2 \cdot x^2 \cdot z^2 \cdot \left(w^2 \cdot z^2 + x^2 \cdot y^2\right)^{\frac{3}{2}} - 2 \cdot w^4 \cdot x \cdot z^5 + 2 \cdot x^5 \cdot y^4 \cdot z + 4 \cdot x^4 \cdot y^3 \cdot z^2 \cdot \sqrt{x^2 - w^2} - 2 \cdot w^2 \cdot x^2 \cdot z^4 \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} \ \ldots$$

$$+ \ -6 \cdot x^4 \cdot y^2 \cdot z^2 \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} + 3 \cdot w^2 \cdot x^2 \cdot y^2 \cdot z^2 \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} - x \cdot y \cdot z^3 \cdot \left(x^2 - w^2\right)^{\frac{3}{2}} \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} \ \ldots$$

$$GO - \frac{+ \ 4 \cdot w^2 \cdot x^2 \cdot y \cdot z^4 \cdot \sqrt{x^2 - w^2} - x^3 \cdot y \cdot z^3 \cdot \sqrt{x^2 - w^2} \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} - 3 \cdot x^3 \cdot y^3 \cdot z \cdot \sqrt{x^2 - w^2} \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} + x \cdot y \cdot z \cdot \sqrt{x^2 - w^2} \cdot \left(w^2 \cdot z^2 + x^2 \cdot y^2\right)^{\frac{3}{2}}}{2 \cdot x \cdot z \cdot \left(z \cdot \sqrt{x^2 - w^2} + x \cdot y\right) \cdot \left[\left(w^2 \cdot z^2 + x^2 \cdot y^2\right)^{\frac{3}{2}} - x^2 \cdot y^2 \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2} + w^2 \cdot x \cdot z^3 + x^3 \cdot y^2 \cdot z - x \cdot y \cdot z \cdot \sqrt{x^2 - w^2} \cdot \sqrt{w^2 \cdot z^2 + x^2 \cdot y^2}\right]} = 0$$

$$
\mathbf{NO} - \frac{\left[\begin{array}{l}
w^8\cdot z^5 - w^2\cdot x^6\cdot z^5 + 3\cdot w^4\cdot x^4\cdot z^5 - 3\cdot w^6\cdot x^2\cdot z^5 - 2\cdot x^5\cdot y^5\cdot\left(x^2-w^2\right)^{\frac{3}{2}} + 2\cdot x^7\cdot y^5\cdot\sqrt{x^2-w^2} - 6\cdot w^2\cdot x^6\cdot y^2\cdot z^3 \ \ldots \\
+\ 12\cdot w^4\cdot x^4\cdot y^2\cdot z^3 - 6\cdot w^6\cdot x^2\cdot y^2\cdot z^3 - 2\cdot w^2\cdot x^5\cdot y^5\cdot\sqrt{x^2-w^2} - 2\cdot x^5\cdot y^3\cdot z^2\cdot\left(x^2-w^2\right)^{\frac{3}{2}} \ \ldots \\
+\ 2\cdot x^7\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2} - w^2\cdot x^6\cdot y^4\cdot z + w^4\cdot x^4\cdot y^4\cdot z - 2\cdot w^2\cdot x^3\cdot y^3\cdot z^2\cdot\left(x^2-w^2\right)^{\frac{3}{2}} - 4\cdot w^2\cdot x^5\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2} \ \ldots \\
+\ 2\cdot w^4\cdot x^3\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2} + 2\cdot w^4\cdot x\cdot y\cdot z^4\cdot\left(x^2-w^2\right)^{\frac{3}{2}} - 2\cdot w^6\cdot x\cdot y\cdot z^4\cdot\sqrt{x^2-w^2} - x^2\cdot y\cdot z\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} \ \ldots \\
+\ x^4\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} - 4\cdot w^2\cdot x^3\cdot y\cdot z^4\cdot\left(x^2-w^2\right)^{\frac{3}{2}} + 2\cdot w^4\cdot x^3\cdot y\cdot z^4\cdot\sqrt{x^2-w^2} + x^4\cdot y\cdot z^3\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} \ \ldots \\
+\ 6\cdot x^4\cdot y^3\cdot z\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} - x^6\cdot y\cdot z^3\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} \ \ldots \\
+\ -6\cdot x^6\cdot y^3\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} - w^2\cdot x^2\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} \ \ldots \\
+\ w^2\cdot x^4\cdot y\cdot z^3\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} + 6\cdot w^2\cdot x^4\cdot y^3\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}
\end{array}}{\sqrt{\left[\begin{array}{l}
x^2\cdot\left[\begin{array}{l}
w^6\cdot z^5 - w^2\cdot x^4\cdot z^5 - 2\cdot x^6\cdot y^2\cdot z^3 - 2\cdot x^3\cdot y^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} + 2\cdot x^5\cdot y^4\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} - 2\cdot x^3\cdot z^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} \ \ldots \\
+\ -2\cdot x^6\cdot y^4\cdot z + 6\cdot w^2\cdot x^4\cdot y^2\cdot z^3 - 6\cdot w^4\cdot x^2\cdot y^2\cdot z^3 - 2\cdot x^3\cdot y^3\cdot z^2\cdot\left(x^2-w^2\right)^{\frac{3}{2}} - 2\cdot x^5\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2} \ \ldots \\
+\ 6\cdot x^5\cdot y^2\cdot z^2\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} + w^2\cdot x^4\cdot y^4\cdot z + 2\cdot w^2\cdot x\cdot z^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} + 2\cdot w^2\cdot x^3\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2} \ \ldots \\
+\ -6\cdot w^2\cdot x^3\cdot y^2\cdot z^2\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} + 2\cdot w^2\cdot x\cdot y\cdot z^4\cdot\left(x^2-w^2\right)^{\frac{3}{2}} - 2\cdot w^4\cdot x\cdot y\cdot z^4\cdot\sqrt{x^2-w^2} \ \ldots \\
+\ -4\cdot x^2\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}} - 2\cdot w^2\cdot x^3\cdot y\cdot z^4\cdot\sqrt{x^2-w^2} + 2\cdot x^2\cdot y\cdot z^3\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2} \ \ldots \\
+\ 6\cdot x^4\cdot y^3\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}
\end{array}\right]
\end{array}\right]}} = 0
$$

$$\sqrt{x^2\cdot z^2\cdot\left[\begin{array}{l}\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}-x^2\cdot y^2\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ w^2\cdot x\cdot z^3+x^3\cdot y^2\cdot z-x\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\end{array}\right]}\cdot\left[\begin{array}{l}w^8\cdot z^5-w^2\cdot x^6\cdot z^5+3\cdot w^4\cdot x^4\cdot z^5-3\cdot w^6\cdot x^2\cdot z^5-2\cdot x^5\cdot y^5\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\ \ldots\\ +\ 2\cdot x^7\cdot y^5\cdot\sqrt{x^2-w^2}-6\cdot w^2\cdot x^6\cdot y^2\cdot z^3\ \ldots\\ +\ 12\cdot w^4\cdot x^4\cdot y^2\cdot z^3-6\cdot w^6\cdot x^2\cdot y^2\cdot z^3-2\cdot w^2\cdot x^5\cdot y^5\cdot\sqrt{x^2-w^2}\ \ldots\\ +\ -2\cdot x^5\cdot y^3\cdot z^2\cdot\left(x^2-w^2\right)^{\frac{3}{2}}+2\cdot x^7\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2}-w^2\cdot x^6\cdot y^4\cdot z\ \ldots\\ +\ w^4\cdot x^4\cdot y^4\cdot z-2\cdot w^2\cdot x^3\cdot y^3\cdot z^2\cdot\left(x^2-w^2\right)^{\frac{3}{2}}-4\cdot w^2\cdot x^5\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2}\ \ldots\\ +\ 2\cdot w^4\cdot x^3\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2}+2\cdot w^4\cdot x\cdot y\cdot z^4\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\ \ldots\\ +\ -2\cdot w^6\cdot x\cdot y\cdot z^4\cdot\sqrt{x^2-w^2}-x^2\cdot y\cdot z\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}\ \ldots\\ +\ x^4\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}-4\cdot w^2\cdot x^3\cdot y\cdot z^4\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\ \ldots\\ +\ 2\cdot w^4\cdot x^3\cdot y\cdot z^4\cdot\sqrt{x^2-w^2}+x^4\cdot y\cdot z^3\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ 6\cdot x^4\cdot y^3\cdot z\cdot\left(x^2-w^2\right)^{\frac{3}{2}}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ -x^6\cdot y\cdot z^3\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ -6\cdot x^6\cdot y^3\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ -w^2\cdot x^2\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}\ \ldots\\ +\ w^2\cdot x^4\cdot y\cdot z^3\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ 6\cdot w^2\cdot x^4\cdot y^3\cdot z\cdot\sqrt{x^2-w^2}\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\end{array}\right]$$

$$NR-\frac{}{x^2\cdot\sqrt{\begin{array}{l}\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{5}{2}}+2\cdot w^2\cdot x^3\cdot z^5+2\cdot x^5\cdot y^2\cdot z^3-x^2\cdot y^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}-w^4\cdot x\cdot z^5\ \ldots\\ +\ x^5\cdot y^4\cdot z+2\cdot x^4\cdot y^3\cdot z^2\cdot\sqrt{x^2-w^2}-2\cdot w^2\cdot x^2\cdot z^4\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ +\ -2\cdot x^4\cdot y^2\cdot z^2\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ \\ +\ 2\cdot w^2\cdot x^2\cdot y\cdot z^4\cdot\sqrt{x^2-w^2}-x\cdot y\cdot z\cdot\sqrt{x^2-w^2}\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}\end{array}}}\cdot\left[\begin{array}{l}w^6\cdot z^5-w^2\cdot x^4\cdot z^5-2\cdot x^6\cdot y^2\cdot z^3\ \ldots\\ \\ +\ -2\cdot x^3\cdot y^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}\ \ldots\\ +\ 2\cdot x^5\cdot y^4\cdot\sqrt{w^2\cdot z^2+x^2\cdot y^2}\ \ldots\\ \\ +\ -2\cdot x^3\cdot z^2\cdot\left(w^2\cdot z^2+x^2\cdot y^2\right)^{\frac{3}{2}}-2\cdot x^6\cdot y^4\cdot z\ \ldots\end{array}\right]=0$$

$$
\begin{bmatrix}
+\ 6\cdot W^2\cdot X^4\cdot Y^2\cdot Z^3 - 6\cdot W^4\cdot X^2\cdot Y^2\cdot Z^3 \ \ldots \\
+\ -2\cdot X^3\cdot Y^3\cdot Z^2\cdot\left(X^2-W^2\right)^{\frac{3}{2}} - 2\cdot X^5\cdot Y^3\cdot Z^2\cdot\sqrt{X^2-W^2} \ \ldots \\
+\ 6\cdot X^5\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} + W^2\cdot X^4\cdot Y^4\cdot Z \ \ldots \\
+\ 2\cdot W^2\cdot X\cdot Z^2\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} \ \ldots \\
+\ 2\cdot W^2\cdot X^3\cdot Y^3\cdot Z^2\cdot\sqrt{X^2-W^2} \ \ldots \\
+\ -6\cdot W^2\cdot X^3\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} \ \ldots \\
+\ 2\cdot W^2\cdot X\cdot Y\cdot Z^4\cdot\left(X^2-W^2\right)^{\frac{3}{2}} - 2\cdot W^4\cdot X\cdot Y\cdot Z^4\cdot\sqrt{X^2-W^2} \ \ldots \\
+\ -4\cdot X^2\cdot Y\cdot Z\cdot\sqrt{X^2-W^2}\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} \ \ldots \\
+\ -2\cdot W^2\cdot X^3\cdot Y\cdot Z^4\cdot\sqrt{X^2-W^2} \ \ldots \\
+\ 2\cdot X^2\cdot Y\cdot Z^3\cdot\left(X^2-W^2\right)^{\frac{3}{2}}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} \ \ldots \\
+\ 6\cdot X^4\cdot Y^3\cdot Z\cdot\sqrt{X^2-W^2}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}
\end{bmatrix}
$$

$$
GS - \frac{\left(X\cdot Z-\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\right)^2}{X^2\cdot Z^2\cdot\sqrt{\dfrac{\begin{bmatrix}\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{5}{2}} + 2\cdot W^2\cdot X^3\cdot Z^5 + 2\cdot X^5\cdot Y^2\cdot Z^3 - X^2\cdot Y^2\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} - W^4\cdot X\cdot Z^5 + X^5\cdot Y^4\cdot Z \ \ldots \\ +\ 2\cdot X^4\cdot Y^3\cdot Z^2\cdot\sqrt{X^2-W^2} - 2\cdot W^2\cdot X^2\cdot Z^4\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} - 2\cdot X^4\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} \ \ldots \\ +\ 2\cdot W^2\cdot X^2\cdot Y\cdot Z^4\cdot\sqrt{X^2-W^2} - X\cdot Y\cdot Z\cdot\sqrt{X^2-W^2}\cdot\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}}\end{bmatrix}}{X^2\cdot Z^2\cdot\left[\left(W^2\cdot Z^2+X^2\cdot Y^2\right)^{\frac{3}{2}} - X^2\cdot Y^2\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2} + W^2\cdot X\cdot Z^3 + X^3\cdot Y^2\cdot Z - X\cdot Y\cdot Z\cdot\sqrt{X^2-W^2}\cdot\sqrt{W^2\cdot Z^2+X^2\cdot Y^2}\right]}}} = 0
$$

$$DS - \sqrt{\frac{\begin{array}{l} W^2\cdot X^3\cdot Z^5 - 5\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{5}{2}} + X^5\cdot Y^2\cdot Z^3 + 2\cdot W^2\cdot Z^2\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} + 7\cdot X^2\cdot Y^2\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} \ldots \\ + 2\cdot W^4\cdot Z^4\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - 2\cdot X^4\cdot Y^4\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - X^2\cdot Z^2\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} + 3\cdot W^4\cdot X\cdot Z^5 + X^5\cdot Y^4\cdot Z \ldots \\ + 4\cdot W^2\cdot X^3\cdot Y^2\cdot Z^3 - 2\cdot X^4\cdot Y^3\cdot Z^2\cdot\sqrt{X^2 - W^2} - 2\cdot W^2\cdot X^2\cdot Z^4\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - X^4\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - 2\cdot W^2\cdot X^2\cdot Y\cdot Z^4\cdot\sqrt{X^2 - W^2} \ldots \\ + X^3\cdot Y\cdot Z^3\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - 4\cdot X^3\cdot Y^3\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} \ldots \\ + 5\cdot X\cdot Y\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} - 4\cdot W^2\cdot X\cdot Y\cdot Z^3\cdot\sqrt{X^2 - W^2}\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} \end{array}}{\begin{array}{l} \left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{5}{2}} + 2\cdot W^2\cdot X^3\cdot Z^5 + 2\cdot X^5\cdot Y^2\cdot Z^3 - X^2\cdot Y^2\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} - W^4\cdot X\cdot Z^5 + X^5\cdot Y^4\cdot Z \ldots \\ + 2\cdot X^4\cdot Y^3\cdot Z^2\cdot\sqrt{X^2 - W^2} - 2\cdot W^2\cdot X^2\cdot Z^4\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} - 2\cdot X^4\cdot Y^2\cdot Z^2\cdot\sqrt{W^2\cdot Z^2 + X^2\cdot Y^2} + 2\cdot W^2\cdot X^2\cdot Y\cdot Z^4\cdot\sqrt{X^2 - W^2} - X\cdot Y\cdot Z\cdot\sqrt{X^2 - W^2}\cdot\left(W^2\cdot Z^2 + X^2\cdot Y^2\right)^{\frac{3}{2}} \end{array}}} = 0$$

**This seems to be the limit of my Mathcad**

$$OR - \sqrt{NO^2 - NR^2} = 0 \qquad OT - (OR - RT) = 0 \qquad DO - \sqrt{DT^2 + OT^2} = 0 \qquad OQ - \frac{DO^2 + GO^2 - DG^2}{2\cdot GO} = 0 \qquad GQ - (GO - OQ) = 0$$

$$DQ - \sqrt{DO^2 - OQ^2} = 0 \qquad FP - \frac{GQ\cdot BF}{DQ} = 0$$

$$CE - \frac{BE\cdot EG}{FP + EF} = 0$$

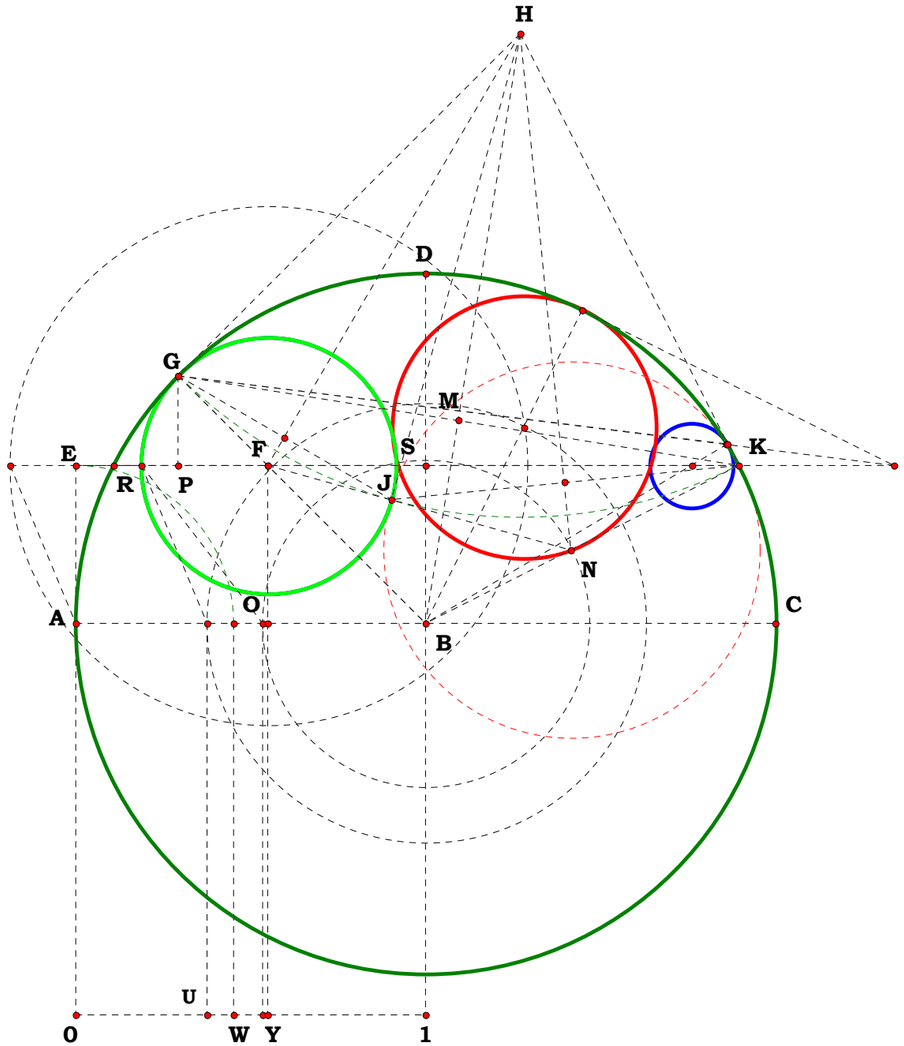
042296B

**Unit.**
**Given.**

**Definitions.**

**Place EF and GH and find JK.**

**I do not think I have drawn this figure correctly since the first time I drew it. Every write up of it after the drawing has been in error. I may get around to writing it up now that I disected and redrew it correctly.**




Unit = 1.00000
W/X = 0.45000
W = 9.00000
X = 20.00000

Y/Z = 0.55000
Y = 11.00000
Z = 20.00000

XY = 0.70000
U = 14.00000
V = 20.00000

Unit.

$AB := 1$

Given.

$N_1 := 3$

042396

Descriptions.

$AH := AB \cdot N_1$ $BH := AH - AB$ $BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

$GM := BG$ $GH := BG$ $AG := AH - GH$ $AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$

$BD := AD - AB$ $DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$

$DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$ $CE := \frac{CF}{2}$ $BE := BC + CE$

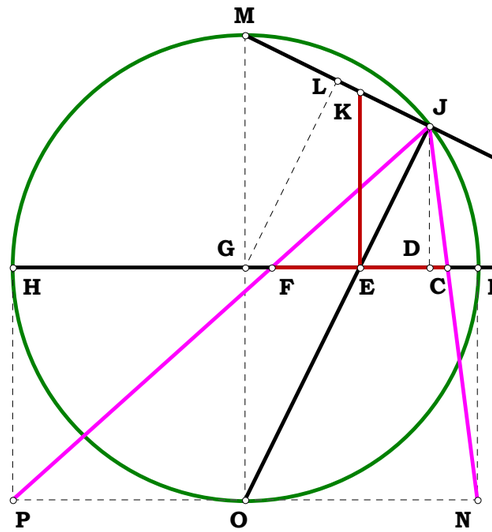
$AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$

Definitions.

$$EK - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$CF - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

Is CF always equal to EK?




AB = 1.57415 in.

AH = 4.64082 in.

EK = 1.16000 in.

CF = 1.16000 in.

$\frac{AB}{AB} = 1.00000$

$\frac{AH}{AB} = 2.94814$

$\frac{EK}{AB} = 0.73691$

$\frac{CF}{AB} = 0.73691$

Unit = 1.00000

$N_1 = 2.94814$

$$\frac{2 \cdot N_1 \cdot (N_1 - Unit)}{(N_1 + Unit)^2} = 0.73691$$

**Sometimes the process of naming becomes very complex and it is impossible given the working constraints to put everything in terms of the givens, actually, they often do not illicit any recognition in the mind as to their truth, so one has to write all the steps down anyway, but sometimes one would like to show all the definitions in a given step by step process.**

$$BH - (N_1 - 1) = 0 \qquad BG - \frac{N_1 - 1}{2} = 0 \qquad AG - \left(\frac{N_1 + 1}{2}\right) = 0$$

$$AM - \frac{1}{2} \cdot \sqrt{2 \cdot N_1^{\,2} + 2} = 0 \qquad AL - \frac{1}{2} \cdot \frac{(N_1 + 1)^2}{\sqrt{2 \cdot N_1^{\,2} + 2}} = 0 \qquad LM - \frac{1}{2} \cdot \frac{(N_1 - 1)^2}{\sqrt{2 \cdot N_1^{\,2} + 2}} = 0$$

$$AJ - \frac{2}{\sqrt{2 \cdot N_1^{\,2} + 2}} \cdot N_1 = 0 \qquad AD - (N_1 + 1) \cdot \frac{N_1}{(N_1^{\,2} + 1)} = 0 \qquad BD - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0$$

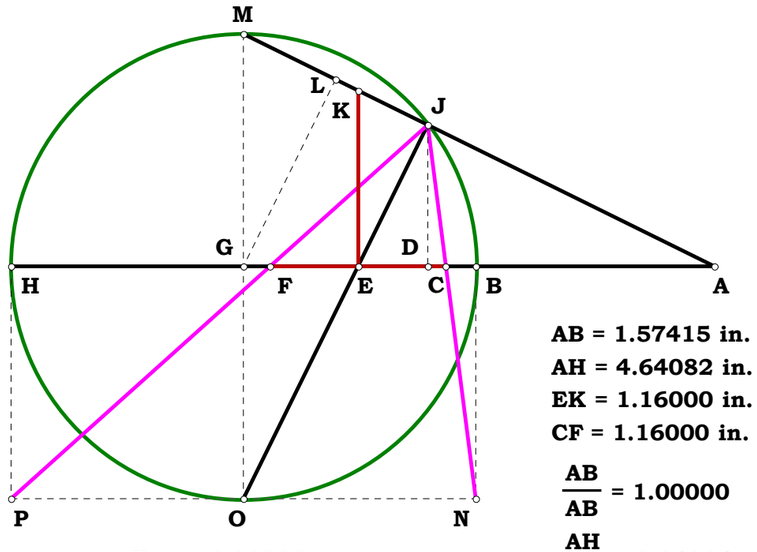
$$DH - N_1^{\,2} \cdot \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0 \qquad DJ - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} \cdot N_1 = 0 \qquad BC - \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$DF - 2 \cdot N_1^{\,3} \cdot \frac{(N_1 - 1)}{\left[(N_1 + 1)^2 \cdot (N_1^{\,2} + 1)\right]} = 0 \qquad CD - 2 \cdot (N_1 - 1) \cdot \frac{N_1}{\left[(N_1^{\,2} + 1) \cdot (N_1 + 1)^2\right]} = 0$$

$$CF - 2 \cdot N_1 \cdot \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad CE - N_1 \cdot \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad BE - \frac{(N_1 - 1)}{(N_1 + 1)} = 0$$

$$AE - 2 \cdot \frac{N_1}{(N_1 + 1)} = 0 \qquad EK - 2 \cdot (N_1 - 1) \cdot \frac{N_1}{(N_1 + 1)^2}$$

$$EK - CF = 0 \qquad EK = 0.75$$




AB = 1.57415 in.
AH = 4.64082 in.
EK = 1.16000 in.
CF = 1.16000 in.

Unit = 1.00000
$N_1$ = 2.94814
$\frac{2 \cdot N_1 \cdot (N_1\text{-Unit})}{(N_1\text{+Unit})^2}$ = 0.73691

$\frac{AB}{AB}$ = 1.00000
$\frac{AH}{AB}$ = 2.94814
$\frac{EK}{AB}$ = 0.73691
$\frac{CF}{AB}$ = 0.73691

Unit.

$AB := 1$

Given.

$N_1 := .36802$

042496

Descriptions.

$AF := AB$ $\quad AD := \frac{AF}{2}$ $\quad AC := N_1$ $\quad DO := \frac{AB}{2}$ $\quad OR := AF$ $\quad CD := AD - AC$

$CO := \sqrt{CD^2 + DO^2}$ $\quad PO := \frac{DO \cdot OR}{CO}$ $\quad CP := PO - CO$ $\quad CK := \frac{DO \cdot CP}{PO}$

$JK := CK$ $\quad KO := \sqrt{CD^2 + (DO + CK)^2}$ $\quad JO := \sqrt{KO^2 - JK^2}$ $\quad KS := \frac{JK^2}{KO}$

$SO := KO - KS$ $\quad JS := \frac{JK \cdot SO}{JO}$ $\quad ST := \frac{CD \cdot SO}{DO + CK}$ $\quad JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD}$ $\quad TU := \frac{CD \cdot JT}{KO}$ $\quad DU := TO - (DO + TU)$

$CV := DU$ $\quad CQ := 2 \cdot CK$ $\quad QV := CQ - CV$ $\quad BH := \frac{CK \cdot CV}{QV}$

$CK = 0.232581$ $\quad BH = 0.082585$

Definitions.

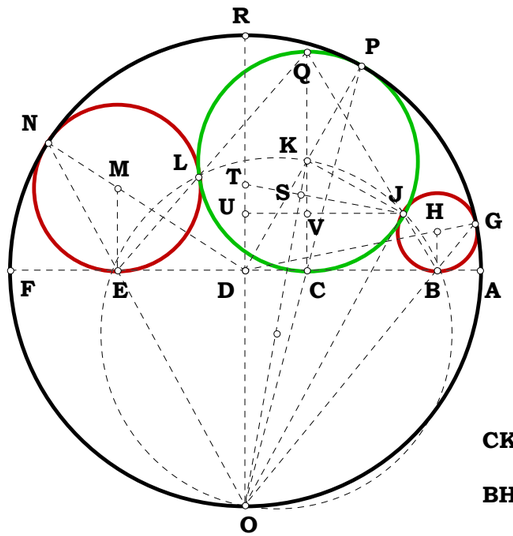
$$CK - \left(N_1 - N_1^2\right) = 0$$

$$BH - \frac{N_1 \cdot \left(N_1 - 1\right) \cdot \left(2 \cdot N_1 - 2 \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} - 2\right)}{2 \cdot N_1 + 6 \cdot N_1^2 - 16 \cdot N_1^3 + 8 \cdot N_1^4 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} + 2} = 0$$

If one takes the time to work MC by hand we get; $\quad BH - \frac{N_1 \cdot \left(1 - N_1\right)}{\left[\sqrt{2} - \left(2N_1 - 1\right)\right]^2} = 0$

## Three Circles.

Given AC, find CK and BH.




AF = 7.87400 cm
AC = 2.89777 cm
CK = 1.83134 cm
BH = 0.65027 cm

$\frac{AF}{AF} = 1.00000$ $\quad AF = 1.00000$

$\frac{AC}{AF} = 0.36802$ $\quad N_1 = 0.36802$

$\frac{CK}{AF} = 0.23258$ $\quad CK = 0.23258$

$\frac{BH}{AF} = 0.08258$ $\quad BH = 0.08258$

$CK\text{-}N_1\text{-}N_1^2 = 0.00000$

$BH\text{-}\frac{N_1 \cdot (1\text{-}N_1)}{(\sqrt{2}\text{-}2 \cdot N_1\text{-}1)^2} = 0.00000$

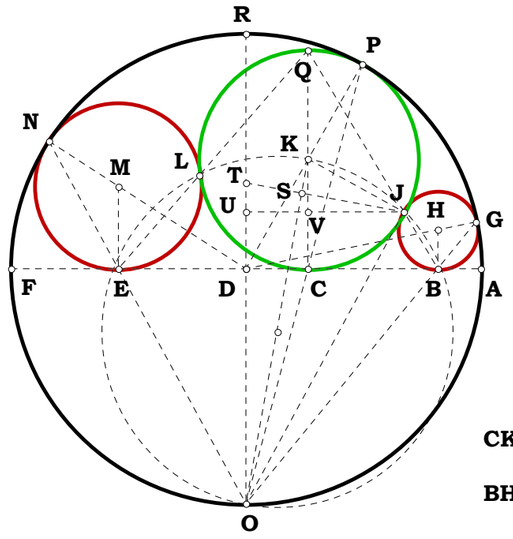
**Some Algebraic Names, or Definitions.**

$$AD - \frac{1}{2} = 0 \qquad AC - N_1 = 0 \qquad CD - \frac{(1 - 2 \cdot N_1)}{2} = 0 \qquad CO - \frac{\sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}}{\sqrt{2}} = 0$$

$$PO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad CP - \frac{\sqrt{2} \cdot N_1 \cdot (1 - N_1)}{\sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad CK - \left(N_1 - N_1^2\right) = 0$$

$$KO - \frac{\sqrt{\left(2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1\right)}}{\sqrt{2}} = 0 \qquad KS - \frac{\sqrt{2} \cdot N_1^2 \cdot (N_1 - 1)^2}{\sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0$$

$$SO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad JS - \frac{N_1 \cdot (1 - N_1)}{\sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0$$




AF = 7.87400 cm
AC = 2.89777 cm
CK = 1.83134 cm
BH = 0.65027 cm

$\frac{AF}{AF}$ = 1.00000 $\frac{AC}{AF}$ = 0.36802 $\frac{CK}{AF}$ = 0.23258 $\frac{BH}{AF}$ = 0.08258

AF = 1.00000
$N_1$ = 0.36802
CK = 0.23258
BH = 0.08258

$CK\text{-}N_1\text{-}N_1^2$ = 0.00000

$BH\text{-}\frac{N_1 \cdot (1\text{-}N_1)}{(\sqrt{2}\text{-}2 \cdot N_1\text{-}1)^2}$ = 0.00000

$$ST - \frac{\sqrt{2} \cdot (1 - 2 \cdot N_1)}{2 \cdot (2 \cdot N_1 - 2 \cdot N_1^2 + 1) \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad JO - \frac{1}{\sqrt{2}} = 0$$

$$JT - \frac{N_1 + N_1^2 - 4 \cdot N_1^3 + 2 \cdot N_1^4 - \sqrt{2} \cdot N_1 + \frac{\sqrt{2}}{2}}{(2 \cdot N_1 - 2 \cdot N_1^2 + 1) \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad TO - \frac{1}{2 \cdot N_1 - 2 \cdot N_1^2 + 1} = 0$$

$$TU - \left[\frac{1}{2 \cdot N_1 - 2 \cdot N_1^2 + 1} - \frac{(3 \cdot \sqrt{2} - 2) \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1^3 + (2 - \sqrt{2}) \cdot N_1 + 1}{2 \cdot (2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1)}\right] = 0 \qquad DU - \left[\frac{(3 \cdot \sqrt{2} - 2) \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1^3 + (2 - \sqrt{2}) \cdot N_1 + 1}{2 \cdot (2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1)} - \frac{1}{2}\right] = 0$$

$$CQ - 2 \cdot \left(N_1 - N_1^2\right) = 0 \qquad QV - \frac{N_1 \cdot (N_1 - 1) \cdot \left(2 \cdot N_1 + 6 \cdot N_1^2 - 16 \cdot N_1^3 + 8 \cdot N_1^4 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} + 2\right)}{2 \cdot \left(4 \cdot N_1^3 - 2 \cdot N_1^4 - 2 \cdot N_1^2 - 1\right)} = 0$$

Unit.

$AC := 1$

Given.

$N_1 := 2.817$ $AF := N_1$

$N_2 := 3$ $FK := N_2$

# One Over N + One

Descriptions.

042596

$CF := AF - AC$ $CE := \frac{CF}{2}$ $AE := AC + CE$

$EJ := \frac{FK \cdot AE}{AF}$ $DL := FK$ $EF := CE$ $DF := \frac{EF \cdot DL}{EJ}$

$CG := \frac{FK \cdot AC}{AF}$ $CD := CF - DF$ $DH := CG$

$HL := DL - DH$ $BC := \frac{CD \cdot CG}{HL}$

Definitions.

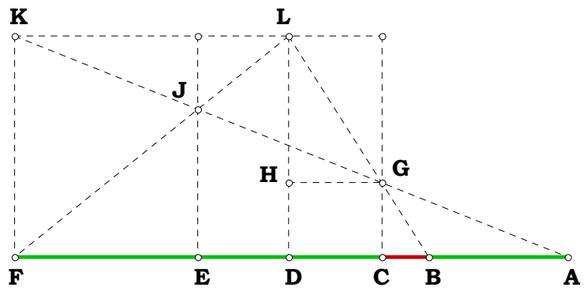
$CF - (N_1 - 1) = 0$ $CE - \frac{1}{2} \cdot (N_1 - 1) = 0$

$AE - \frac{1}{2} \cdot (1 + N_1) = 0$

$EJ - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0$ $DF - \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 = 0$

$CG - \frac{N_2}{N_1} = 0$ $CD - \frac{(N_1 - 1)}{(1 + N_1)} = 0$

$HL - N_2 \cdot \frac{(N_1 - 1)}{N_1} = 0$ $\frac{1}{N_1 + 1} - BC = 0$




$N_1 = 2.96971$

$N_2 = 1.18938$

**Construct 1/(N+1)**

**In this construction, $N_2$ has to be something, it can be anything and will not change the results BC.**

$AC = 1.22908$ in.
$AF = 3.65000$ in.
$CF = 2.42092$ in.
$CE = 1.21046$ in.
$AE = 2.43954$ in.
$FK = 1.46184$ in.
$EJ = 0.97705$ in.
$DL = 1.46184$ in.
$EF = 1.21046$ in.
$DF = 1.81107$ in.
$CG = 0.49225$ in.
$CD = 0.60985$ in.
$DH = 0.49225$ in.
$HL = 0.96959$ in.
$BC = 0.30961$ in.

$\frac{AC}{AC} = 1.00000$

$\frac{AF}{AC} - N_1 = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{CF}{AC} - N_1 - 1 = 0.00000$

$\frac{CE}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{AE}{AC} - \frac{N_1 + 1}{2} = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{EJ}{AC} - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0.00000$

$\frac{DL}{AC} - N_2 = 0.00000$

$\frac{EF}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{DF}{AC} - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0.00000$

$\frac{CG}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{CD}{AC} - \frac{N_1 - 1}{N_1 + 1} = 0.00000$

$\frac{DH}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{HL}{AC} - \frac{N_2 \cdot (N_1 - 1)}{N_1} = 0.00000$

$\frac{BC}{AC} - \frac{1}{N_1 + 1} = 0.00000$

042696

Unit. $AB := 1$

Given. $W := 10$ $Y := 7$

$X := 18$ $Z := 20$

## Three Bases.

082621 Each time I looked at the original write-up done in 96, I have promised myself to redo it as it was not done right. So, I finally got it done.

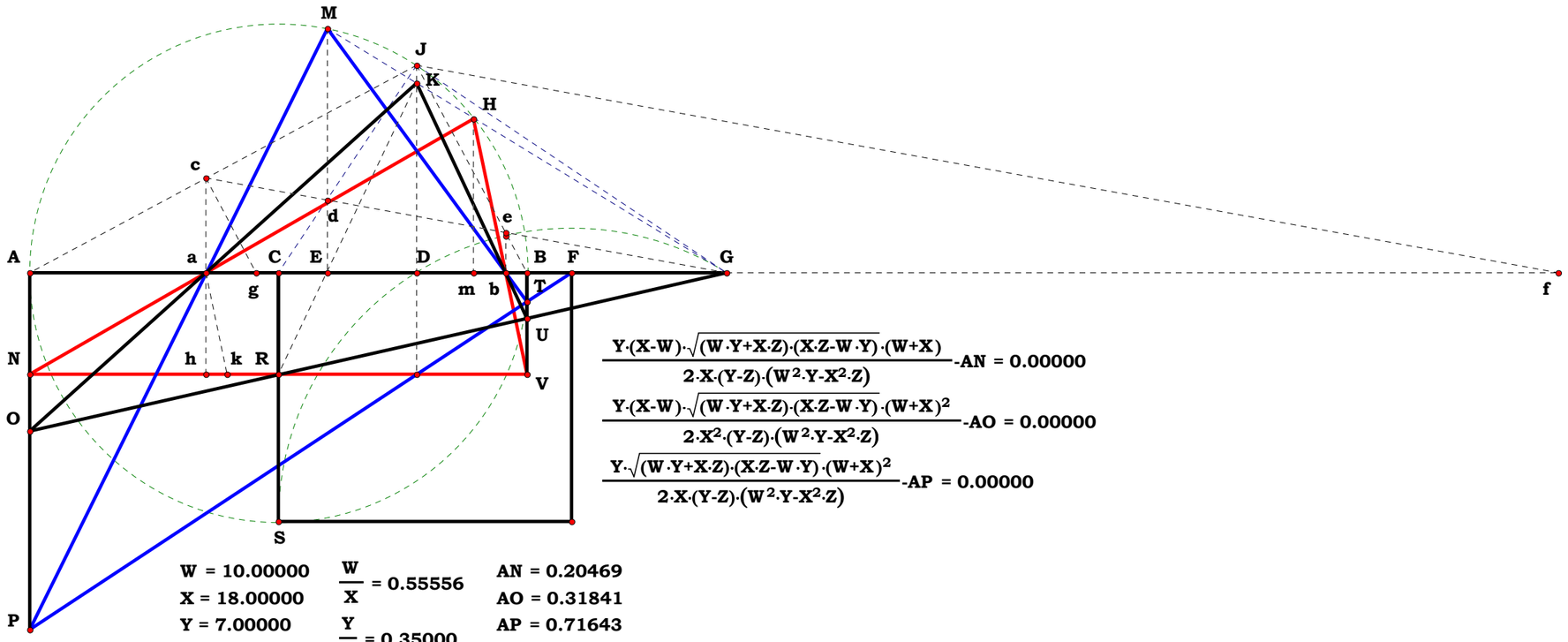
Descriptions.

$AC := \frac{AB}{2}$ $BC := AC$ $CJ := AC$ $CD := BC \cdot \frac{W}{X}$ $AD := AC + CD$ $CE := CD \cdot \frac{Y}{Z}$ $AE := AC + CE$ $BD := AB - AD$ $BE := AB - AE$ $DJ := \sqrt{AD \cdot BD}$ $EM := \sqrt{AE \cdot BE}$ $DG := \frac{DJ^2}{CD}$

$AG := AD + DG$ $DF := \frac{DG}{2}$ $AF := AD + DF$ $BF := AF - AB$ $Ed := DJ \cdot \frac{CE}{CD}$ $EG := AG - AE$ $Df := EG \cdot \frac{DJ}{Ed}$ $Af := Df + AD$ $Aa := \frac{AD \cdot AG}{Af}$ $Ag := \frac{AB \cdot Aa}{AD}$ $Gg := AG - Ag$

$ag := Ag - Aa$ $BG := AG - AB$ $Bb := \frac{ag \cdot BG}{Gg}$ $DK := EM \cdot \frac{DG}{EG}$ $DE := CD - CE$ $CR := DK \cdot \frac{CE}{DE}$ $ah := CR$ $Nh := Aa$ $hk := Nk := Nh + hk$ $ab := AB - (Aa + Bb)$ $Hm := ah \cdot \frac{ab}{Nk}$

$am := \frac{Nh \cdot ab}{Nk}$ $Am := Aa + am$ $Bm := AB - Am$ $AN := CR$ $Da := AD - Aa$ $AO := \frac{DK \cdot Aa}{Da}$ $Ea := AE - Aa$ $AP := EM \cdot \frac{Aa}{Ea}$ $Eb := ab - Ea$ $BT := EM \cdot \frac{Bb}{Eb}$ $Db := ab - Da$




$W = 10.00000$ $\frac{W}{X} = 0.55556$

$X = 18.00000$

$Y = 7.00000$ $\frac{Y}{Z} = 0.35000$

$Z = 20.00000$

$AN = 0.20469$

$AO = 0.31841$

$AP = 0.71643$

$\frac{Y \cdot (X-W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W+X)}{2 \cdot X \cdot (Y-Z) \cdot (W^2 \cdot Y - X^2 \cdot Z)} - AN = 0.00000$

$\frac{Y \cdot (X-W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W+X)^2}{2 \cdot X^2 \cdot (Y-Z) \cdot (W^2 \cdot Y - X^2 \cdot Z)} - AO = 0.00000$

$\frac{Y \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W+X)^2}{2 \cdot X \cdot (Y-Z) \cdot (W^2 \cdot Y - X^2 \cdot Z)} - AP = 0.00000$

$BU := DK \cdot \frac{Bb}{Db}$

$BV := CR$

**Definitions.**

$$AC - \frac{1}{2} = 0 \quad BC - \frac{1}{2} = 0 \quad CJ - \frac{1}{2} = 0 \quad CD - \frac{W}{2 \cdot X} = 0 \quad AD - \frac{W + X}{2 \cdot X} = 0 \quad CE - \frac{W \cdot Y}{2 \cdot X \cdot Z} = 0 \quad AE - \frac{W \cdot Y + X \cdot Z}{2 \cdot X \cdot Z} = 0 \quad BD - \frac{X - W}{2 \cdot X} = 0 \quad BE - \frac{X \cdot Z - W \cdot Y}{2 \cdot X \cdot Z} = 0 \quad DJ - \frac{\sqrt{(X - W) \cdot (W + X)}}{2 \cdot X} = 0$$

$$EM - \frac{\sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)}}{2 \cdot X \cdot Z} = 0 \quad DG - \frac{(X - W) \cdot (W + X)}{2 \cdot W \cdot X} = 0 \quad AG - \frac{W + X}{2 \cdot W} = 0 \quad DF - \frac{(X - W) \cdot (W + X)}{4 \cdot W \cdot X} = 0 \quad AF - \frac{(W + X)^2}{4 \cdot W \cdot X} = 0 \quad BF - \frac{(W - X)^2}{4 \cdot W \cdot X} = 0 \quad Ed - \frac{Y \cdot \sqrt{(X - W) \cdot (W + X)}}{2 \cdot X \cdot Z} = 0$$

$$EG - \frac{X^2 \cdot Z - W^2 \cdot Y}{2 \cdot W \cdot X \cdot Z} = 0 \quad Df - \frac{X^2 \cdot Z - W^2 \cdot Y}{2 \cdot W \cdot X \cdot Y} = 0 \quad Af - \frac{W \cdot Y + X \cdot Z}{2 \cdot W \cdot Y} = 0 \quad Aa - \frac{Y \cdot (W + X)^2}{2 \cdot X \cdot (W \cdot Y + X \cdot Z)} = 0 \quad Ag - \frac{Y \cdot (W + X)}{W \cdot Y + X \cdot Z} = 0 \quad Gg - \frac{(W + X) \cdot (X \cdot Z - W \cdot Y)}{2 \cdot W \cdot (W \cdot Y + X \cdot Z)} = 0 \quad ag - \frac{Y \cdot (X - W) \cdot (W + X)}{2 \cdot X \cdot (W \cdot Y + X \cdot Z)} = 0$$

$$BG - \frac{X - W}{2 \cdot W} = 0 \quad Bb - \frac{Y \cdot (W - X)^2}{2 \cdot X \cdot (X \cdot Z - W \cdot Y)} = 0 \quad DK - \frac{(W - X) \cdot \sqrt{-(W \cdot Y + X \cdot Z) \cdot (W \cdot Y - X \cdot Z)} \cdot (W + X)}{2 \cdot X \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad DE - \frac{W \cdot (Z - Y)}{2 \cdot X \cdot Z} = 0$$

$$CR - \frac{Y \cdot (X - W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad ah - \frac{Y \cdot (X - W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad Nh - \frac{Y \cdot (W + X)^2}{2 \cdot X \cdot (W \cdot Y + X \cdot Z)} = 0 \quad hk - \frac{Y \cdot (W - X)^2}{2 \cdot X \cdot (X \cdot Z - W \cdot Y)} = 0$$

$$Nk - \frac{Y \cdot \left(2 \cdot W^2 \cdot Y - W^2 \cdot Z - X^2 \cdot Z\right)}{(W \cdot Y - X \cdot Z) \cdot (W \cdot Y + X \cdot Z)} = 0 \quad ab - \frac{(Z - Y) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)}{(W \cdot Y - X \cdot Z) \cdot (W \cdot Y + X \cdot Z)} = 0 \quad Hm - \frac{(W - X) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X \cdot \left(2 \cdot W^2 \cdot Y - W^2 \cdot Z - X^2 \cdot Z\right)} = 0$$

$$am - \frac{(Z - Y) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right) \cdot (W + X)^2}{2 \cdot X \cdot (W \cdot Y + X \cdot Z) \cdot \left(2 \cdot W^2 \cdot Y - W^2 \cdot Z - X^2 \cdot Z\right)} = 0 \quad Am - \frac{(W + X)^2 \cdot (W \cdot Y - X \cdot Z)}{2 \cdot X \cdot \left(2 \cdot W^2 \cdot Y - W^2 \cdot Z - X^2 \cdot Z\right)} = 0 \quad Bm - \frac{(W - X)^2 \cdot (W \cdot Y + X \cdot Z)}{2 \cdot X \cdot \left(W^2 \cdot Z - 2 \cdot W^2 \cdot Y + X^2 \cdot Z\right)} = 0$$

$$AN - \frac{Y \cdot (X - W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad Da - \frac{(Z - Y) \cdot (W + X)}{2 \cdot (W \cdot Y + X \cdot Z)} = 0 \quad AO - \frac{Y \cdot (X - W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)^2}{2 \cdot X^2 \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad Ea - \frac{(Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)}{2 \cdot X \cdot Z \cdot (W \cdot Y + X \cdot Z)} = 0$$

$$AP - \frac{Y \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)^2}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad Eb - \frac{(Z - Y) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)}{2 \cdot X \cdot Z \cdot (W \cdot Y - X \cdot Z)} = 0 \quad BT - \frac{Y \cdot (W - X)^2 \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)}}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad Db - \frac{(Z - Y) \cdot (W - X)}{2 \cdot (W \cdot Y - X \cdot Z)} = 0$$

$$BU - \frac{Y \cdot (W - X)^2 \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X^2 \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0 \quad BV - \frac{Y \cdot (X - W) \cdot \sqrt{(W \cdot Y + X \cdot Z) \cdot (X \cdot Z - W \cdot Y)} \cdot (W + X)}{2 \cdot X \cdot (Y - Z) \cdot \left(W^2 \cdot Y - X^2 \cdot Z\right)} = 0$$

042796

Unit.

$CF := 1$

Given.

$N_1 := 4$

$N_2 := 9$

Descriptions.

$CE := \frac{CF}{2}$ $CD := \frac{CE}{N_1}$ $FK := N_2$ $DM := FK$ $EL := FK$

$DF := CF - CD$ $EF := \frac{CF}{2}$ $EJ := \frac{DM \cdot EF}{DF}$ $JL := EL - EJ$

$KL := EF$ $AF := \frac{KL \cdot FK}{JL}$ $AC := AF - CF$ $CG := \frac{FK \cdot AC}{AF}$

$DH := CG$ $HM := DM - DH$ $BC := \frac{CD \cdot DH}{HM}$ $BF := BC + CF$

$BD := BC + CD$ $AB := AF - BF$

$AB^2 - BC \cdot BF = 0$ $\sqrt{BC \cdot BF} - BD = 0$ $BD - (CD + BC) = 0$

$AB - BD = 0$ $\sqrt{BC \cdot BF} - AB = 0$ $AB = 0.145833$

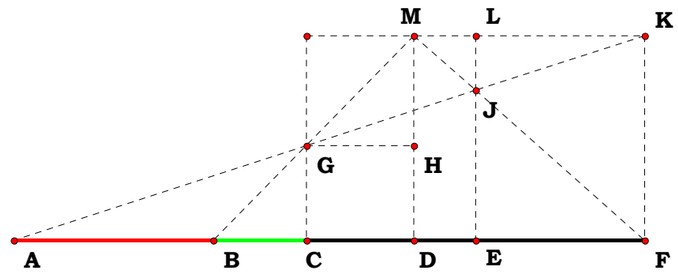
One may notice that can be any value at all, it just has to be some value. Every use of symbols require prue intelligble concepts which are perceptible in the grammar, but not all people can see them intelligibly.

## A Root Figure

The difference between the perceptible and the intelligible.

CD + BC is the square root of $\sqrt{BC \cdot BF}$. What is BC?

When I originally did this story, I said to myself, well I can name that tune in 2 variables, which it should be, however, it is one of those things one says that they will do later as it has little to do with the main goal of the Delian Quest. Sometimes later means later in the day, sometimes later means decades away, as in this case. What are revisions for? One should also be aware, while learning, that one is not going to have the experience required to do a story justice until they mature, thus waiting is certainly an option.




CF = 5.67267 cm $\frac{CF}{CF} = 1.00000$

CE = 2.83633 cm $\frac{CE}{CD} = 1.57637$

CD = 1.79928 cm $\frac{AB}{CF} = 0.59234$

AB = 3.36014 cm $\frac{BC}{CF} = 0.27515$

BC = 1.56086 cm $\frac{BF}{CF} = 1.27515$

BF = 7.23353 cm

$N_1 = 1.57637$

AB = 0.59234

BC = 0.27515

BF = 1.27515

$\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.59234$ $AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.27515$ $BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 1.27515$ $BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$ cm

This equation is in the universal and any relatative difference can use it.

This equation is particular. It is from these particular examples, perceptible examples, that one acquires what Plato called, the similie in multis, or the Universal, or again, the intelligible. One aims for three things then, the correct range for a varaible built in to the equation, the answer expressed only in the givens, and the equation be free from any particular perceptible as grammar is always applicable universally. Not all my examples fill that bill, but then this is my storybook, my rather odd novel.

**Definitions.**

$$CF - 1 = 0 \qquad CE - \frac{1}{2} = 0 \qquad CD - \frac{1}{2 \cdot N_1} = 0 \qquad FK - N_2 = 0 \qquad DM - N_2 = 0 \qquad EL - N_2 = 0 \qquad DF - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0 \qquad EF - \frac{1}{2} = 0 \qquad EJ - \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1} = 0$$

$$JL - \frac{N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0 \qquad KL - \frac{1}{2} = 0 \qquad AF - \frac{2 \cdot N_1 - 1}{2 \cdot (N_1 - 1)} = 0 \qquad AC - \frac{1}{2 \cdot (N_1 - 1)} = 0 \qquad CG - \frac{N_2}{2 \cdot N_1 - 1} = 0 \qquad DH := \frac{N_2}{2 \cdot N_1 - 1} \qquad HM - \frac{2 \cdot N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$$

$$BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BD - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0$$

$$\left[\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)}\right]^2 - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \sqrt{\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)}} - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} - \left[\frac{1}{2 \cdot N_1} + \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)}\right] = 0$$

$$AB - BD = 0 \qquad \sqrt{BC \cdot BF} - AB = 0 \qquad AB = 0.145833$$

Unit.

Given.

$N_1 := 2$

$N_2 := 6$

**Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.**

042896

**I had finally decided to write this up in Oct. of 94, and being punctual, here it is.**

**Process Summary will use a $5^{th}$ root series for an example.**

**Descriptions.**

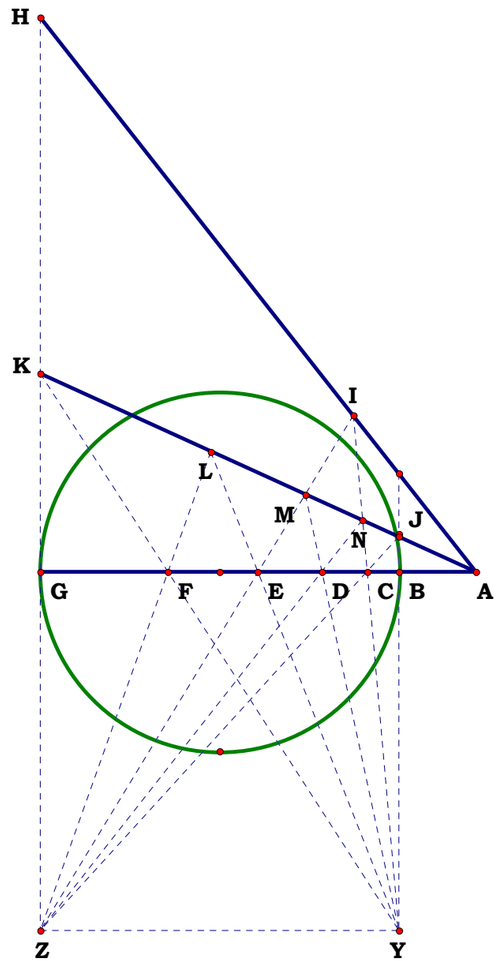
$$AB := N_1 \qquad AG := N_2 \qquad AE := \left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}$$

$$BG := AG - AB \qquad GZ := BG \qquad YZ := BG$$

$$BY := BG \qquad BE := AE - AB \qquad EG := BG - BE$$

**Definitions.**

$$GH := \frac{BY \cdot EG}{BE}$$




AB = 1.29117 cm
AG = 7.30394 cm
AC = 1.82599 cm
AD = 2.58233 cm
AE = 3.65197 cm
AF = 5.16467 cm

$(AB^4 \cdot AG^1)^{\frac{1}{5}} - AC = 0.00000$

$(AB^3 \cdot AG^2)^{\frac{1}{5}} - AD = 0.00000$

$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$

$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$

$\frac{AG^{\frac{0}{5}}}{AB} = 1.00000$

$\frac{AG^{\frac{1}{5}}}{AB} = 1.41421$

$\frac{AG^{\frac{2}{5}}}{AB} = 2.00000$

$\frac{AG^{\frac{3}{5}}}{AB} = 2.82843$

$\frac{AG^{\frac{4}{5}}}{AB} = 4.00000$

$\frac{AG^{\frac{5}{5}}}{AB} = 5.65685$

$\frac{AG^{\frac{1\,2}{5}}}{AB} = 2.00000$

$\frac{AG^{\frac{1\,3}{5}}}{AB} = 2.82843$

$\frac{AG^{\frac{1\,4}{5}}}{AB} = 4.00000$

$\frac{AG^{\frac{1\,5}{5}}}{AB} = 5.65685$

$\frac{AG}{AB} = 5.65685$

$\frac{AF}{AB} = 4.00000$

$\frac{AE}{AB} = 2.82843$

$\frac{AD}{AB} = 2.00000$

$\frac{AC}{AB} = 1.41421$

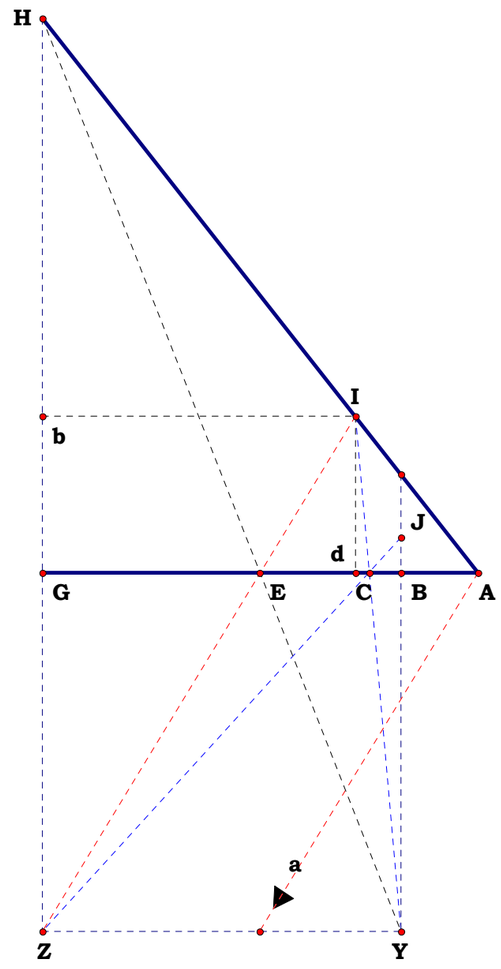
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$




$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$

$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1}{5}^{2}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}^{3}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{1}{5}^{4}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}^{5}} = 5.65685$$

AB = 1.29117 cm
AG = 7.30394 cm
AC = 1.82599 cm
AD = 2.58233 cm
AE = 3.65197 cm
AF = 5.16467 cm

$$(AB^{4} \cdot AG^{1})^{\frac{1}{5}} - AC = 0.00000$$

$$(AB^{3} \cdot AG^{2})^{\frac{1}{5}} - AD = 0.00000$$

$$(AB^{2} \cdot AG^{3})^{\frac{1}{5}} - AE = 0.00000$$

$$(AB^{1} \cdot AG^{4})^{\frac{1}{5}} - AF = 0.00000$$

$$\frac{AG}{AB} = 5.65685$$

$$\frac{AF}{AB} = 4.00000$$

$$\frac{AE}{AB} = 2.82843$$

$$\frac{AD}{AB} = 2.00000$$

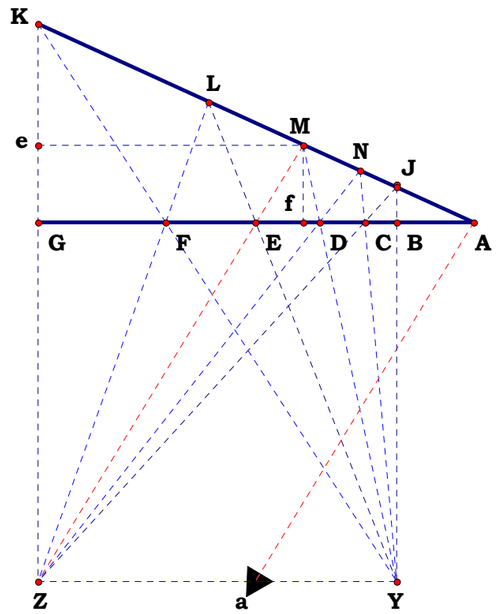
$$\frac{AC}{AB} = 1.41421$$

$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$




$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$

$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1\,2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{1\,3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{1\,4}{5}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{1\,5}{5}} = 5.65685$$

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$$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$$

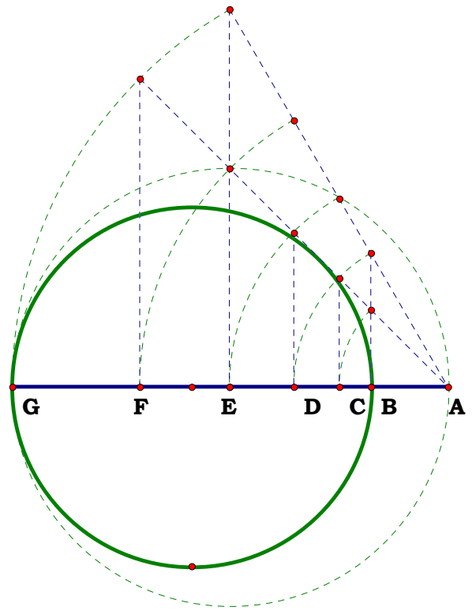
$$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$$

**If any of a prime root series can be given exactly, every root of the series can be determined exactly.**

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

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$$\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}} - AE = 0.00000$$
$$\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}} - AF = 0.00000$$

$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$
$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$
$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$
$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$
$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$
$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1}{5}^2} = 2.00000$$
$$\frac{AG}{AB}^{\frac{1}{5}^3} = 2.82843$$
$$\frac{AG}{AB}^{\frac{1}{5}^4} = 4.00000$$
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$$\frac{AG}{AB} = 5.65685$$
$$\frac{AF}{AB} = 4.00000$$
$$\frac{AE}{AB} = 2.82843$$
$$\frac{AD}{AB} = 2.00000$$
$$\frac{AC}{AB} = 1.41421$$

**Unit.**

$AB := 1$

**Given.**

$N := 5$

**042996**

**Descriptions.**

$AG := AB \cdot N \qquad BG := AG - AB$

$BF := \frac{BG}{2} \qquad FK := BF \qquad FO := BF \qquad AF := BF + AB$

$DF := \frac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

$HO := \frac{AF \cdot KO}{AK} \qquad DO := \frac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

**Definitions.**

$AG - N = 0 \qquad BG - (N - 1) = 0 \qquad BF := \frac{N - 1}{2} \qquad AF - \left(\frac{1}{2} \cdot N + \frac{1}{2}\right) = 0$

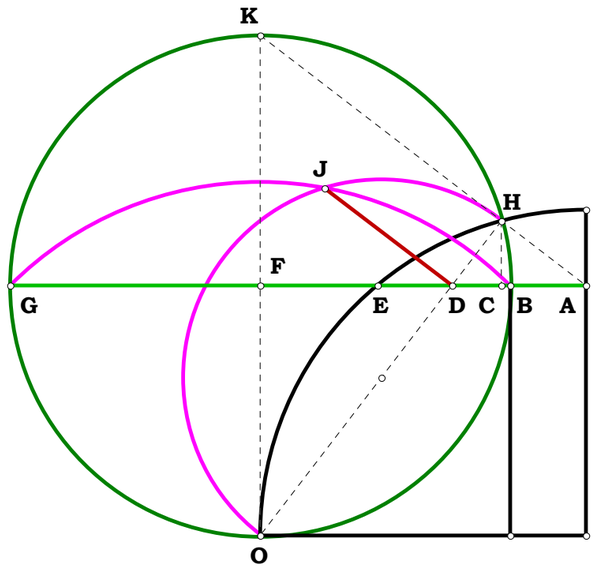
$DF - \frac{(N - 1)^2}{2 \cdot (N + 1)} = 0 \qquad AK - \frac{\sqrt{2 \cdot (N^2 + 1)}}{2} = 0$

$HO - \frac{\sqrt{2} \cdot (N^2 - 1)}{2 \cdot \sqrt{N^2 + 1}} = 0 \qquad DO - \frac{\sqrt{2} \cdot (N - 1) \cdot \sqrt{N^2 + 1}}{2 \cdot (N + 1)} = 0$

$DH - \frac{\sqrt{2} \cdot N \cdot (N - 1)}{(N + 1) \cdot \sqrt{N^2 + 1}} = 0 \qquad DJ - \sqrt{N} \cdot \frac{(N - 1)}{(N + 1)} = 0$

**DJ is the Geometric name, what is its Algebraic name?**

**What is DJ?**



043096

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 128 \quad Root := 8$

$\delta := 1 .. Root$

$BG := N_1 \quad AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

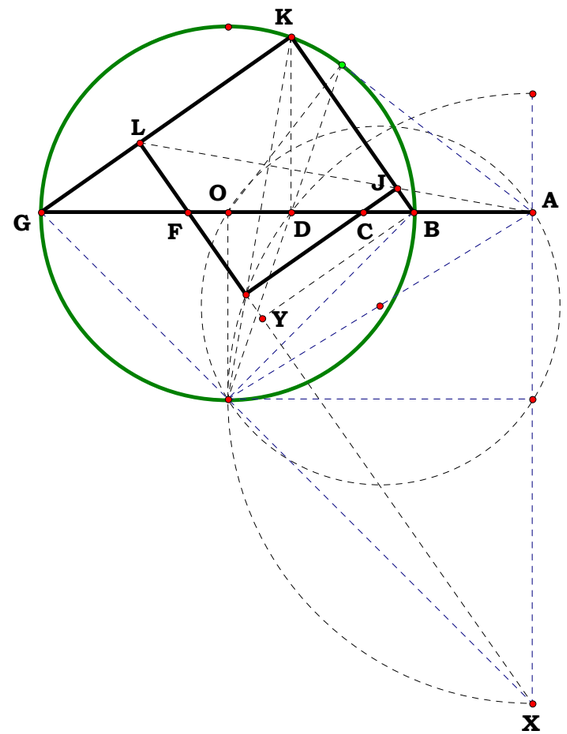
$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$

**Geometric Exponential Series of the form**

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \text{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

**Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.**



**Definitions.**

$GL = 51.575206$ $BJ = 0.399808$

$\frac{GL}{BJ} = 129$ $\frac{AG}{AB} = 129$

$$\frac{\sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root-1}{Root}}} = 2.179442$$

$\frac{GK}{GL} = 2.179442$

$$\sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}} = 153.147965$$

$\frac{BK}{BJ} = 153.147965$

$BM := \frac{BD \cdot BC}{BG}$ $FQ := \frac{BD \cdot FG}{BG}$

$\frac{AG}{FQ} = 9.598866$

$\frac{AG}{BM} = 674.506167$

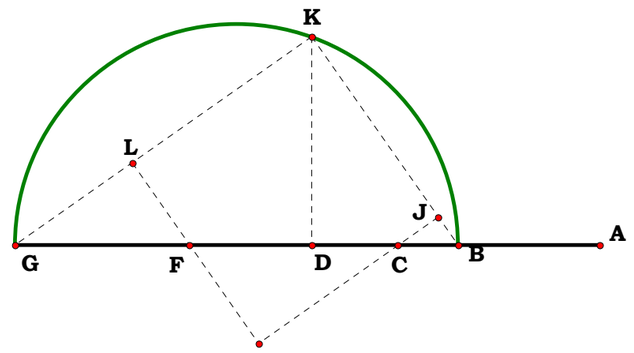
**On the left is the first and last of the series, on the right is the entire series.**

| $\frac{\delta+2}{Root}$ | $\frac{\delta}{Root}$ | $\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}}$ | $\left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}$ |
|---|---|---|---|
| 0.375 | 0.125 | 6.186872 | 1.835793 |
| 0.5 | 0.25 | 11.357817 | 3.370136 |
| 0.625 | 0.375 | 20.850601 | 6.186872 |
| 0.75 | 0.5 | 38.277387 | 11.357817 |
| 0.875 | 0.625 | 70.26936 | 20.850601 |
| 1 | 0.75 | 129 | 38.277387 |
| 1.125 | 0.875 | 236.817299 | 70.26936 |
| 1.25 | 1 | 434.747545 | 129 |

$\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} = 1.835793$

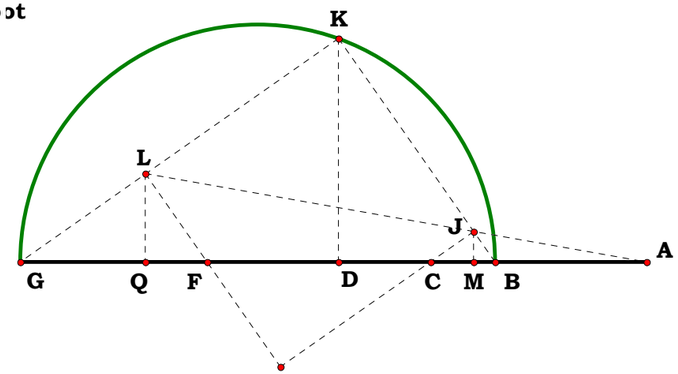
$\frac{1}{Root} = 0.125$

$\frac{0}{Root} = 0$




$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}} =$$

| |
|---|
| 9.598866 |
| 17.621531 |
| 32.349484 |
| 59.386957 |
| 109.02216 |
| 200.142121 |
| 367.419508 |
| 674.506167 |



122096

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad AL := N_1$

$N_2 := .2$

**Descriptions.**

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL \quad AJ = 1.681793$

**Definitions.**

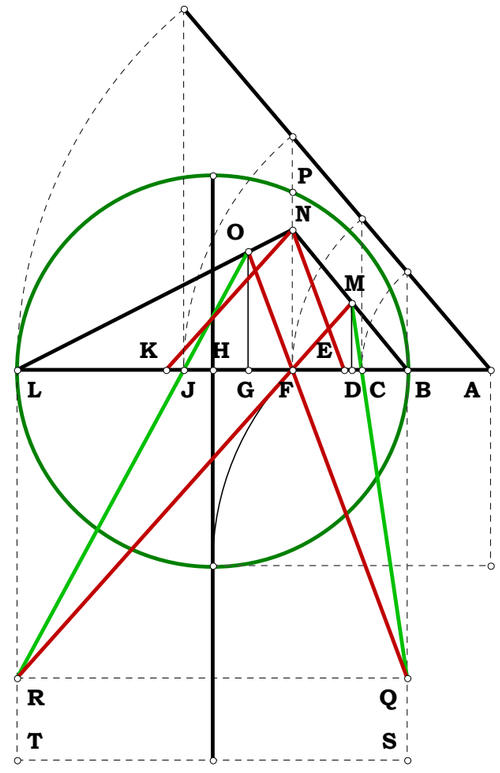
$\left(AL^3\right)^{\frac{1}{4}} - AJ = 0$

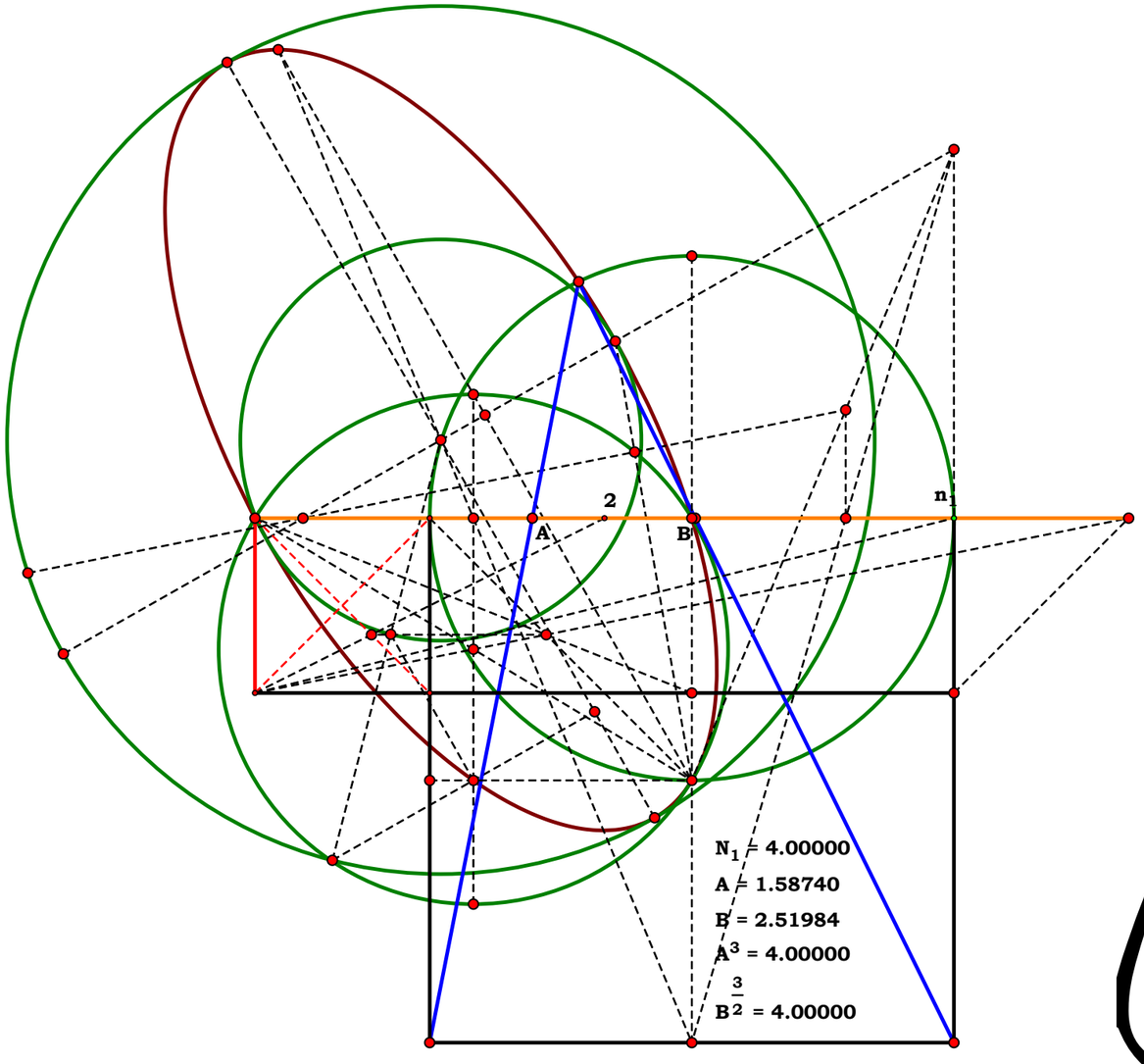
$AJ - \left(N_1^{\frac{1}{4}}\right)^3 = 0$

$\left(N_1^3\right)^{\frac{1}{4}} - \left(N_1^{\frac{1}{4}}\right)^3 = 0$

## Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off in the figure.**






# The Delian Quest 1997

**John Clark**

Unit.

Given.

$n := 1 .. 3$

$$S_1 := \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad S_2 := \begin{pmatrix} b \\ c \\ a \end{pmatrix} \quad S_3 := \begin{pmatrix} c \\ a \\ b \end{pmatrix}$$

040397

Descriptions.

$$Is\_This\_a\_Triangle := \left(S_{1_1} + S_{2_1} > S_{3_1}\right) \cdot \left(S_{1_1} + S_{3_1} > S_{2_1}\right) \cdot \left(S_{2_1} + S_{3_1} > S_{1_1}\right)$$

As was learned in school, the area of a triagle is given by $\frac{1}{2} \cdot B \cdot H$.

From 04_02_97.MCD I show that, for a given side, the height is given by;

$$H_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

And since $B := S_1$ Area is defined as

$$A_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{4}$$

$\frac{1 \cdot B_n \cdot H_n}{2} - A_n =$

| |
|---|
| 0 |
| 0 |
| 0 |

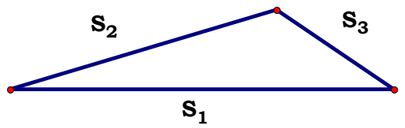
$A_n =$

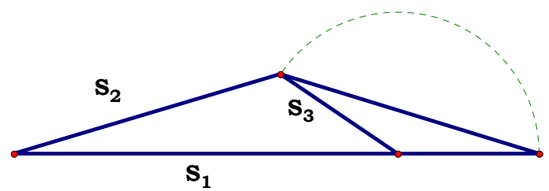
| |
|---|
| 2.904738 |
| 2.904738 |
| 2.904738 |

$H_n =$

| |
|---|
| 2.904738 |
| 1.936492 |
| 1.452369 |

**Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?**



**What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.**

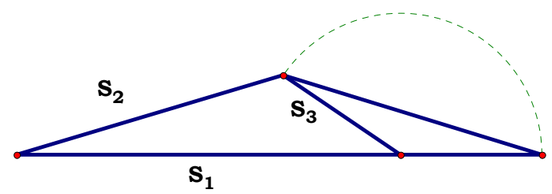
$$\mathbf{Acute}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S2}_n\right)^2} > \mathbf{S3}_n\ ,\ 1\ ,\ 0\right] \qquad \mathbf{Acute2}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S3}_n\right)^2} > \mathbf{S2}_n\ ,\ 1\ ,\ 0\right]$$

$\mathbf{Acute}_n =$

| |
|---|
| 0 |
| 1 |
| 1 |

$\mathbf{Acute2}_n =$

| |
|---|
| 1 |
| 0 |
| 1 |




$\mathbf{H}_n =$

| |
|---|
| 2.904738 |
| 1.936492 |
| 1.452369 |

$\mathbf{S1}_n =$

| |
|---|
| 2 |
| 3 |
| 4 |

$\mathbf{S2}_n =$

| |
|---|
| 3 |
| 4 |
| 2 |

$\mathbf{S3}_n =$

| |
|---|
| 4 |
| 2 |
| 3 |

**if we halve** $\angle \mathbf{S1S2}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

| |
|---|
| 4.357106 |
| 3.872983 |
| 1.452369 |

**if we halve** $\angle \mathbf{S1S3}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

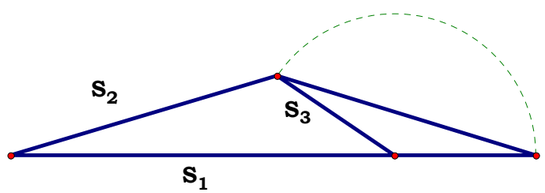
| |
|---|
| 5.809475 |
| 1.936492 |
| 2.178553 |

$\mathbf{Is\_This\_a\_Triangle} = 1$

**Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.**

$$\left(\mathbf{S2}_n > \mathbf{S3}_n\right) - \left[\frac{\left(\mathbf{B}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n > \frac{\left(\mathbf{B}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n\right] =$$

| |
|---|
| 0 |
| 0 |
| 0 |



**Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.**

$$H_n = \frac{\sqrt{S1_n + S2_n + S3_n} \cdot \sqrt{-S1_n + S2_n + S3_n} \cdot \sqrt{S1_n - S2_n + S3_n} \cdot \sqrt{S1_n + S2_n - S3_n}}{2 \cdot S1_n}$$

Given $S_1$, $S_2$ and $\sqrt{S_1^2 + S_2^2} > S_3$, find $S_3$.

$Acute_n =$

| |
|---|
| 0 |
| 1 |
| 1 |

Is_This_a_Triangle = 1

$$S4_n := \sqrt{(S2_n)^2 - (H_n)^2}$$

$$SX_n := \text{if}(Acute_n, S1_n - S4_n, S1_n + S4_n)$$

$a \equiv 2$ $b \equiv 3$ $c \equiv 4$ ← Plug your values in here.

$$S3_n := \sqrt{(H_n)^2 + (SX_n)^2}$$

$S1_n =$

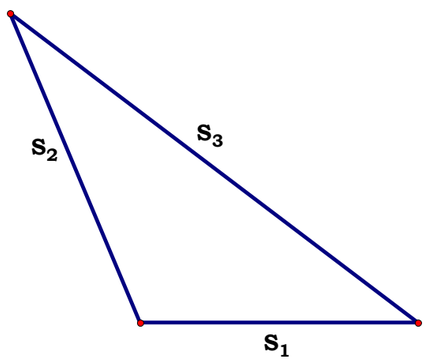
| |
|---|
| 2 |
| 3 |
| 4 |

$S2_n =$

| |
|---|
| 3 |
| 4 |
| 2 |

$S3_n =$

| |
|---|
| 4 |
| 2 |
| 3 |



040497

**Unit.**

**Given.**

$N_1 := 5 \quad AB := N_1$

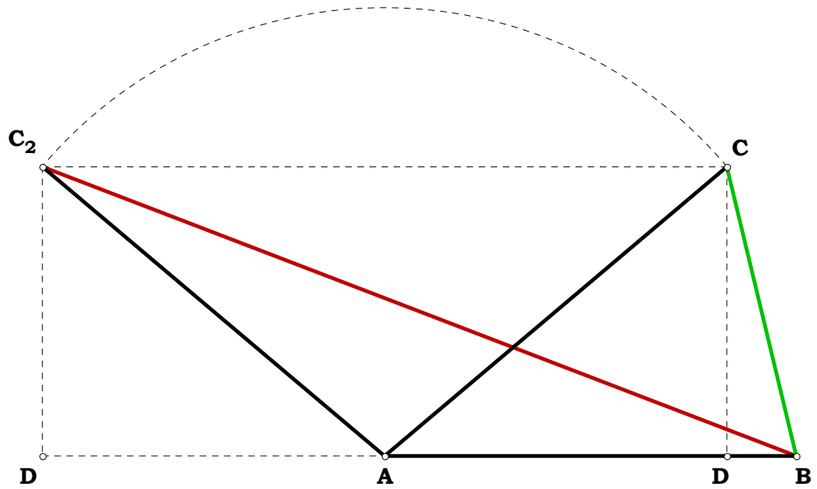
$N_2 := 4 \quad AC := N_2$

$N_3 := 3 \quad CD := N_3$

**Descriptions.**

## Triangles

**Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.**




$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$

$BC_1 := \sqrt{CD^2 + BD_1^2} \qquad BC_2 := \sqrt{CD^2 + BD_2^2}$

$BC_1 = 8.213252 \qquad BC_2 = 3.813461$

$BC_1 - \sqrt{N_1^2 + N_2^2 + 2 \cdot N_1 \cdot \sqrt{N_2^2 - N_3^2}} = 0$

$BC_2 - \sqrt{\left(N_1^2 + N_2^2 - 2 \cdot N_1 \cdot \sqrt{N_2^2 - N_3^2}\right)} = 0$

**Definitions.**

$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_1$

$$\frac{\sqrt{S_1 + S_2 + S_3} \cdot \sqrt{-S_1 + S_2 + S_3} \cdot \sqrt{S_1 - S_2 + S_3} \cdot \sqrt{S_1 + S_2 - S_3}}{2 \cdot S_1} - CD = 0$$

$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_2$

$$\frac{\sqrt{S_1 + S_2 + S_3} \cdot \sqrt{-S_1 + S_2 + S_3} \cdot \sqrt{S_1 - S_2 + S_3} \cdot \sqrt{S_1 + S_2 - S_3}}{2 \cdot S_1} - CD = 0$$

$AB = 2.71667$ in. $\quad N_1 = 2.71667$ in.

$AC = 2.95665$ in. $\quad N_2 = 2.95665$ in.

$CD = 1.90833$ in. $\quad N_3 = 1.90833$ in.

$BC = 1.96260$ in. $\quad N_1^2+N_2^2 = 16.12208$ in$^2$

$BC_2 = 5.32845$ in. $\quad 2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2} = 12.27028$ in$^2$

$BC - \sqrt{(N_1^2+N_2^2) - (2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2})} = 0.00000$ in.

$BC_2 - \sqrt{(N_1^2+N_2^2) + (2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2})} = 0.00000$ in.

Unit.

Given.

$N_1 := 3.14854$ $AD := N_1$

$N_2 := 6.50875$ $AC := N_2$

**042897**

Descriptions.

$CD := \sqrt{AD^2 + AC^2}$ $DH := CD$

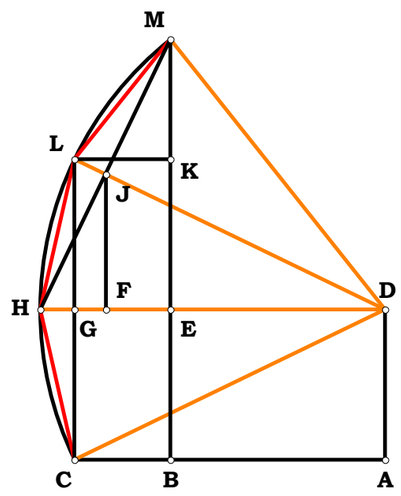
$CG := AD$ $DG := AC$ $GH := DH - DG$ $CH := \sqrt{GH^2 + CG^2}$

$HJ := CG$ $DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH}$ $EF := FH$ $DE := DH - (EF + FH)$

$AB := DE$ $EG := DG - DE$ $LM := CH$ $LK := EG$

$KM := \sqrt{LM^2 - LK^2}$ $BE := AD$ $BK := 2 \cdot BE$ $BM := BK + KM$




$AD = 2.51883$ cm $N_1 = 2.51883$ cm

$AC = 5.20700$ cm $N_2 = 5.20700$ cm

$CH = 2.58413$ cm $\sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 2.58413$ cm

$CH - \sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 0.00000$ cm

$m\angle DCA = 25.81490°$

$m\angle CDM = 77.44471°$

$\frac{m\angle CDM}{m\angle DCA} = 3.00000$

Some definitions:

$\sqrt{N_1^2 + N_2^2} - CD = 0$ $\sqrt{N_1^2 + N_2^2} - N_2 - GH = 0$ $\sqrt{2 \cdot N_1^2 + 2 \cdot N_2^2 - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} - CH = 0$

$\frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} - FH = 0$ $\frac{\left(N_2^2 - N_1^2\right)}{\sqrt{N_1^2 + N_2^2}} - DE = 0$

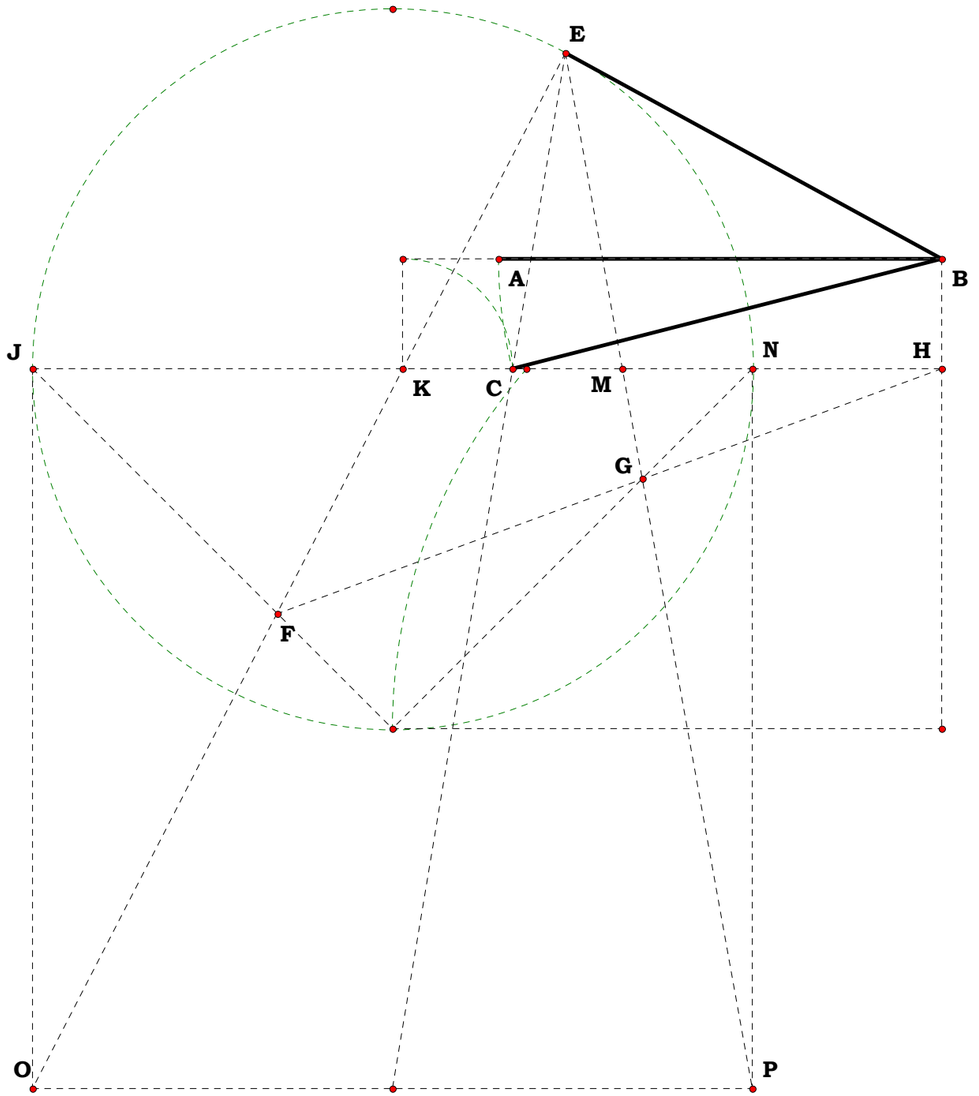
$2 \cdot N_1 + \frac{\sqrt{\left(2 \cdot N_2^3 - 2 \cdot N_1^2 \cdot N_2\right) \cdot \sqrt{N_1^2 + N_2^2} + N_1^4 + 5 \cdot N_1^2 \cdot N_2^2 - 2 \cdot N_2 \cdot \left(\sqrt{N_1^2 + N_2^2}\right)^3}}{\sqrt{N_1^2 + N_2^2}} - BM = 0$

042997

## Trisection and the Cube Roots

**Exploring trisection produced in the cube root figure. If one places E where they will, they would find the origin of the root series H by projecting through FG. Taking half of KM for BH we find that angle ABC is 1/3 of angle EBC. So, we can not only produce a cube root series from OP but also a trisection as well from the orign which is the same origin for the square root of the figure.**

**I should write up some plates concerning the point G or F and the different relationships they form with the finished plate.**




$m\angle ABC = 14.33586°$

$m\angle EBA = 28.67173°$

$\frac{m\angle EBA}{m\angle ABC} = 2.00000$

$m\angle EBC = 43.00759°$

$\frac{m\angle EBC}{m\angle ABC} = 3.00000$

Unit.

$AB := 1$

Given.

$N := 5$

**Descriptions.**

$AH := N \qquad BH := AH - AB \qquad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2} \qquad AE := AC + CE$

$AU := CE \qquad NV := AU \qquad MW := AU$

**(For the next two equations see 042897.)**

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

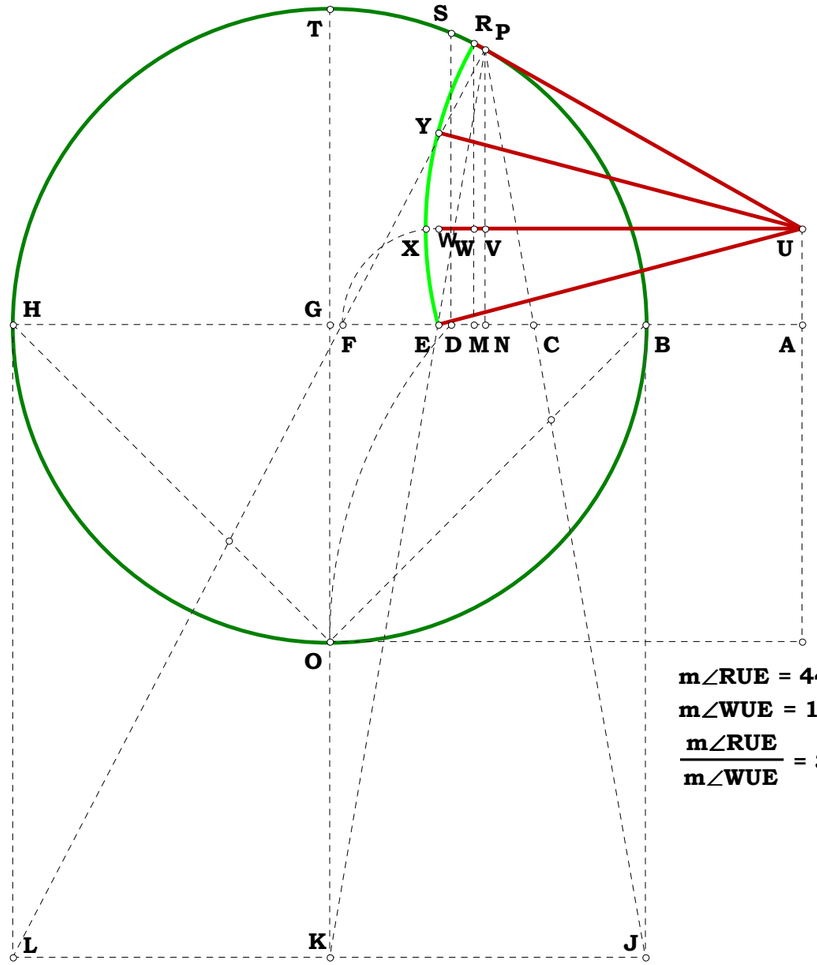
$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{\left(AU^2\right)^{\frac{3}{2}} + 5 \cdot AE^2 \cdot \sqrt{AU^2} - 4 \cdot AE \cdot AU \cdot \sqrt{AE^2 + AU^2}}{\left(AE^2 + AU^2\right) \cdot \sqrt{AU^2}}}$$

$RW := MR - MW \qquad BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH}$

$NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV \qquad AN := AC + CN \qquad UV := AN$

$UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

**If trisection can be placed at RUE, then PV is proportional to RW.**




$m\angle RUE = 44.289°$

$m\angle WUE = 14.763°$

$\frac{m\angle RUE}{m\angle WUE} = 3.000$

Unit. $AB := 1$

Given. $X := 12$

$Y := 20$

**081921 Study**

**Descriptions.**

$$AD := \frac{AB}{2} \quad DE := \frac{X}{2 \cdot Y} \quad AE := DE + AD \quad AE = 0.8$$

$$BE := AB - AE \quad EO := \sqrt{AE \cdot BE} \quad DG := \frac{DE \cdot AB}{AB + EO}$$

$$GN := \frac{EO \cdot DG}{DE} \quad FH := GN \quad GF := \frac{GN}{2} \quad GH := \frac{GN}{2}$$

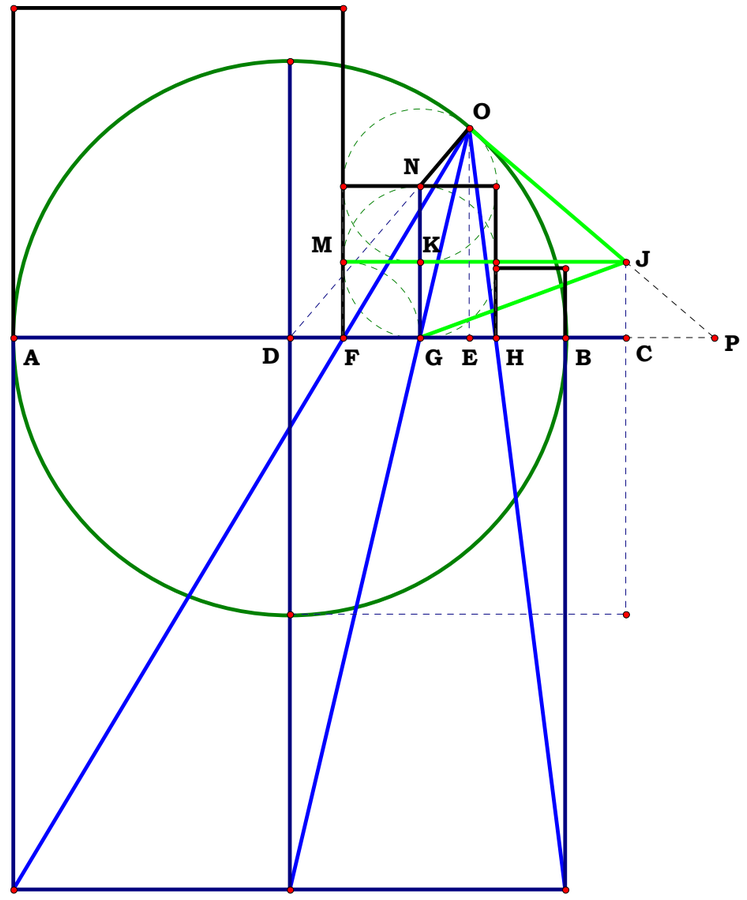
$$DO := \frac{1}{2} \quad DP := \frac{DO^2}{DE} \quad BP := DP - AD \quad PE := BE + BP$$

$$CP := \frac{GN \cdot PE}{2 \cdot EO} \quad BC := BP - CP \quad BG := AD - DG \quad CH := BG + BC - GH$$

$$CF := BG + BC + GH \quad AC := AB + BC \quad \frac{AC}{BC} = 8 \quad \frac{CH}{BC} = 2 \quad \frac{CF}{BC} = 4$$

$$CH - \left(BC^2 \cdot AC\right)^{\frac{1}{3}} = 0 \quad CF - \left(BC \cdot AC^2\right)^{\frac{1}{3}} = 0$$

XY = 0.65000
X = 13.00000
Y = 20.00000
Unit = 1.00000
AE = 0.82500
$x_G$ = 0.73551
$y_N$ = 0.27535




**Definitions.**

$$AD - \frac{1}{2} = 0 \quad DE - \frac{X}{2 \cdot Y} = 0 \quad AE - \frac{X + Y}{2 \cdot Y} = 0 \quad BE - \frac{Y - X}{2 \cdot Y} = 0 \quad EO - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y} = 0 \quad DG - \frac{X}{2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}} = 0 \quad GN - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y + \sqrt{(Y - X) \cdot (X + Y)}} = 0$$

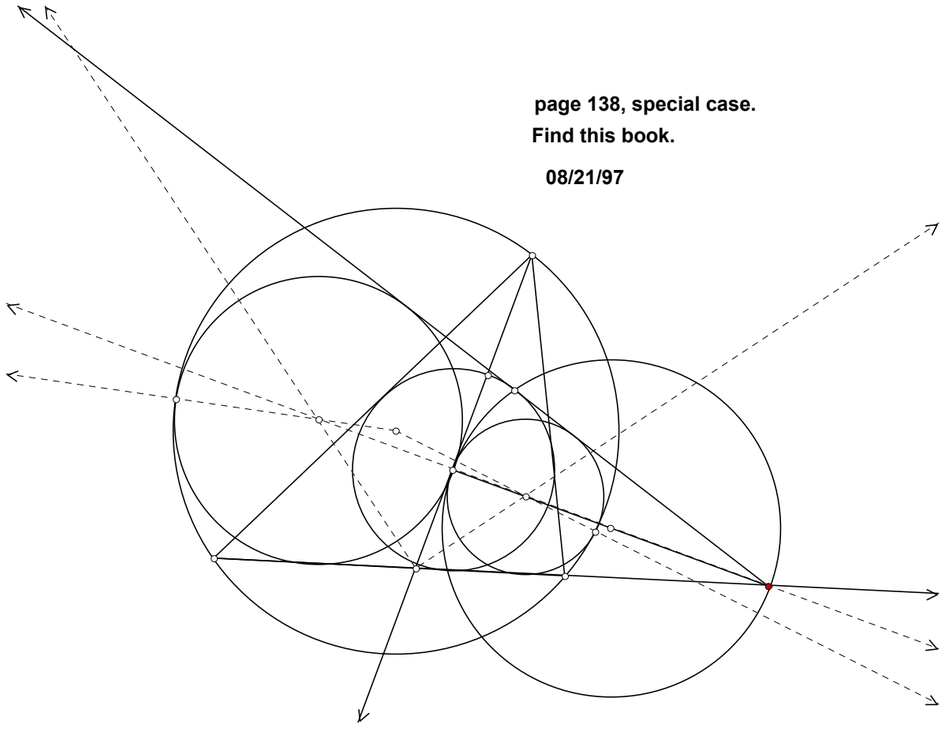
$$FH - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}} = 0 \quad GF - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0 \quad GH - \frac{\sqrt{(Y - X) \cdot (X + Y)}}{2 \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0 \quad DO - \frac{1}{2} = 0 \quad DP - \frac{Y}{2 \cdot X} = 0 \quad BP - \frac{Y - X}{2 \cdot X} = 0$$

$$PE - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0 \quad CP - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot \left[2 \cdot Y + \sqrt{-(X - Y) \cdot (X + Y)}\right]} = 0 \quad BC - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad BG - \frac{2 \cdot Y - 2 \cdot X + \sqrt{Y^2 - X^2}}{2 \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad CH - \frac{(Y - X) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$$

$$CF - \frac{(X + Y) \cdot \left(Y - X + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AC - \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)} = 8 \quad \frac{X + Y + \sqrt{Y^2 - X^2}}{Y - X + \sqrt{Y^2 - X^2}} = 2 \quad \frac{X + Y}{Y - X} = 4$$

082197

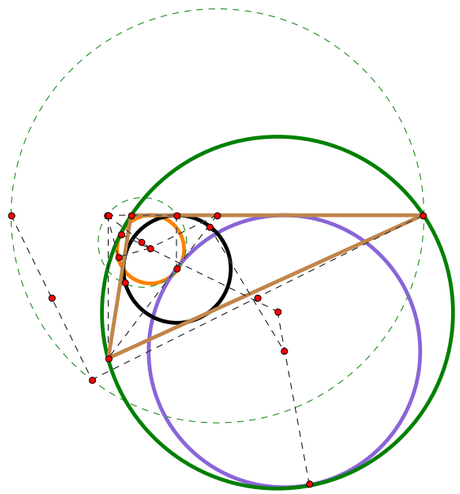
**In my files I have the following plate, from what book I do not recall, however, the figure as it was written up there was listed as a "special case," of what I do not recall either. But I want to write it up because it is not a special case of anything, it is actually a plate showing that one can treat every triangle as an eight circle problem, simply add the remaining two sides by recursion of the first. I suspect now that if someone thought this was a special case of something, then they did not comprehend the actual relationships, they are easily found by compass.**
**The project would start with the equations from 062793 and 040694. So, this is a project I am interested in doing and have been for a long time. I might even find the book it came from. Might be interesting to find the equations for all eight circles. Maybe some day.**

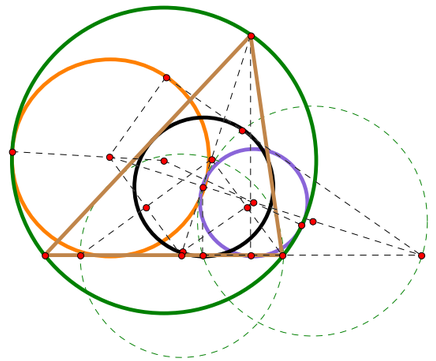


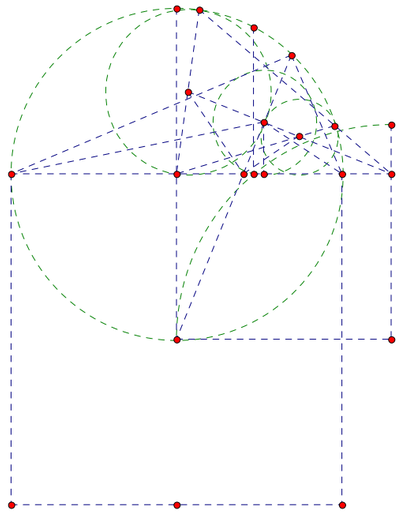
page 138, special case.
Find this book.

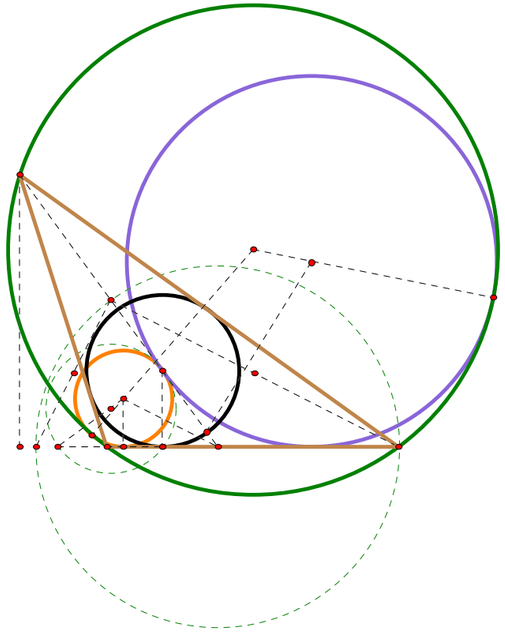
08/21/97

## Eight Circles and a Forgotten Book.





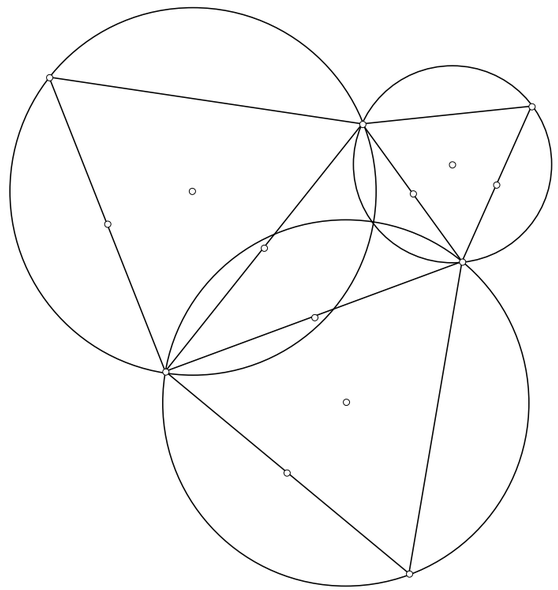




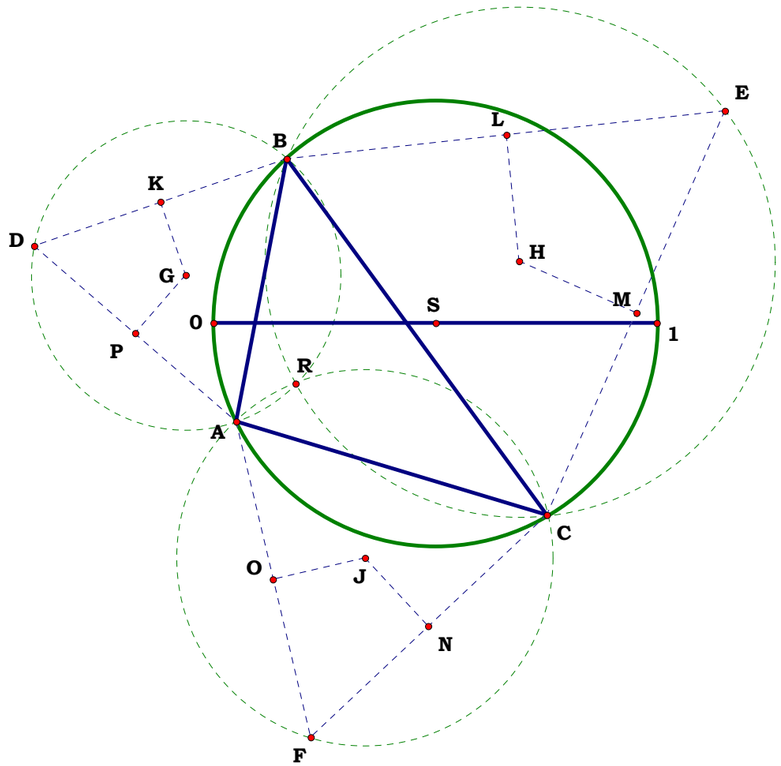
**Unit.**
**Given.**

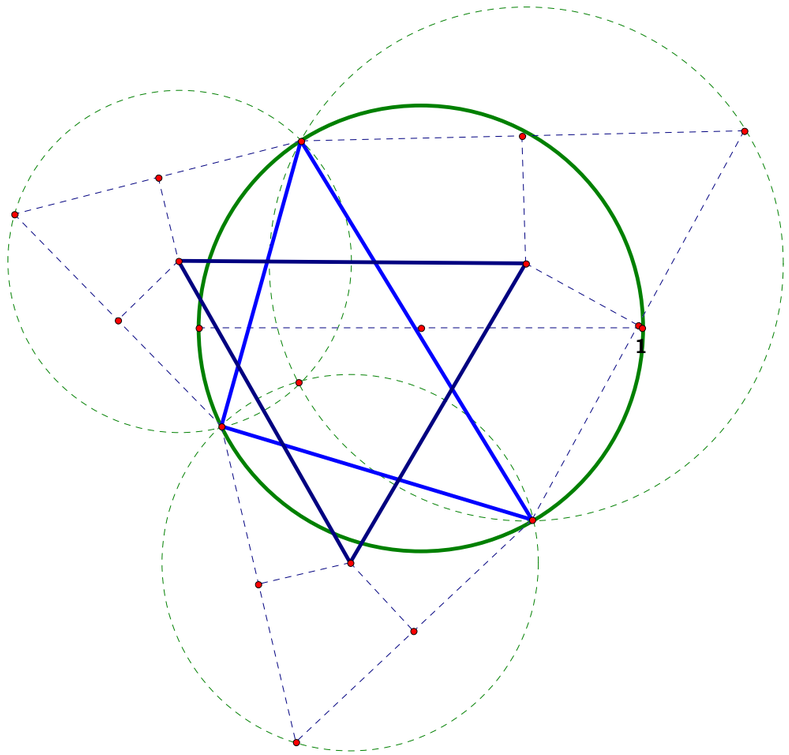
**Steiner Point**

**082297**
**Descriptions.**
**Definitions.**



**Steiner point solution (a) page 361 Computing in Euclidean Geometry Du and Hwang 1985**



Unit.

Given.

$N_1 := 2.66666$ $AB := N_1$

$N_2 := 1.31473$ $EF := N_2$

$N_3 := 1.26711$

091197A

Descriptions.

$$AD := \frac{N_1}{N_3} \quad BD := AB - AD \quad DJ := \sqrt{AD \cdot BD} \quad AC := \frac{AB}{2}$$

$$DC := \frac{(AC - AD)^2}{\sqrt{(AC - AD)^2}} \quad CH := \frac{EF}{2} \quad CJ := AC$$

$$DG := \frac{DJ \cdot CH}{CJ} \quad CG := \sqrt{DG^2 + DC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AB}{2}\right)^2 - CH^2}$$

Definitions.

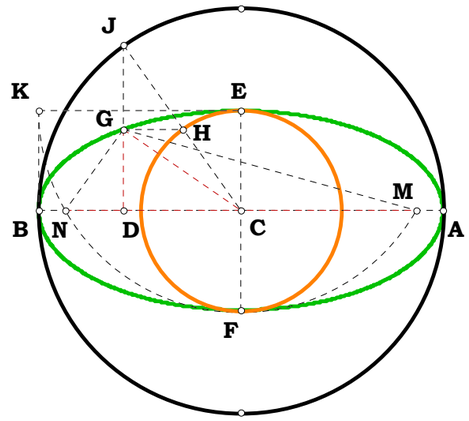
$$BD - \frac{N_1 \cdot (N_3 - 1)}{N_3} = 0 \quad DJ - \frac{N_1 \cdot \sqrt{(N_3 - 1)}}{N_3} = 0 \quad AC - \frac{N_1}{2} = 0$$

$$DC - \frac{N_1 \cdot \sqrt{(N_3 - 2)^2}}{2 \cdot N_3} = 0 \quad CH - \frac{N_2}{2} = 0 \quad CJ - \frac{N_1}{2} = 0 \quad DG - \frac{N_2 \cdot \sqrt{N_3 - 1}}{N_3} = 0$$

$$CG - \frac{\sqrt{(N_3^2 - 4 \cdot N_3 + 4) \cdot N_1^2 + 4 \cdot N_2^2 \cdot (N_3 - 1)}}{2 \cdot N_3} = 0 \quad MN - \sqrt{(N_1 - N_2) \cdot (N_1 + N_2)} = 0$$

## The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**




AB = 2.66667 in.

EF = 1.31473 in.

AD = 2.10453 in.

$\frac{AB}{AD}$ = 1.26711

CH = 0.65736 in.

DG = 0.53625 in.

MN = 2.32004 in.

$MN - 2 \cdot \sqrt{\frac{AB^2}{2} - CH^2}$ = 0.00000 in.

$MN - \sqrt{(AB - EF) \cdot (AB + EF)}$ = 0.00000 in.

091197B

Unit.

Given.

$S_1 := 8.14917 \quad AB := S_1$

$S_2 := 7.23745 \quad AC := S_2$

$S_3 := 2.58277 \quad BC := S_3$

Descriptions.

$DE := AC + BC \quad AH := \frac{DE}{2} \quad AG := \frac{AB}{2} \quad FG := \sqrt{AH^2 - AG^2}$

Definitions.

$$FG - \frac{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2} = 0$$

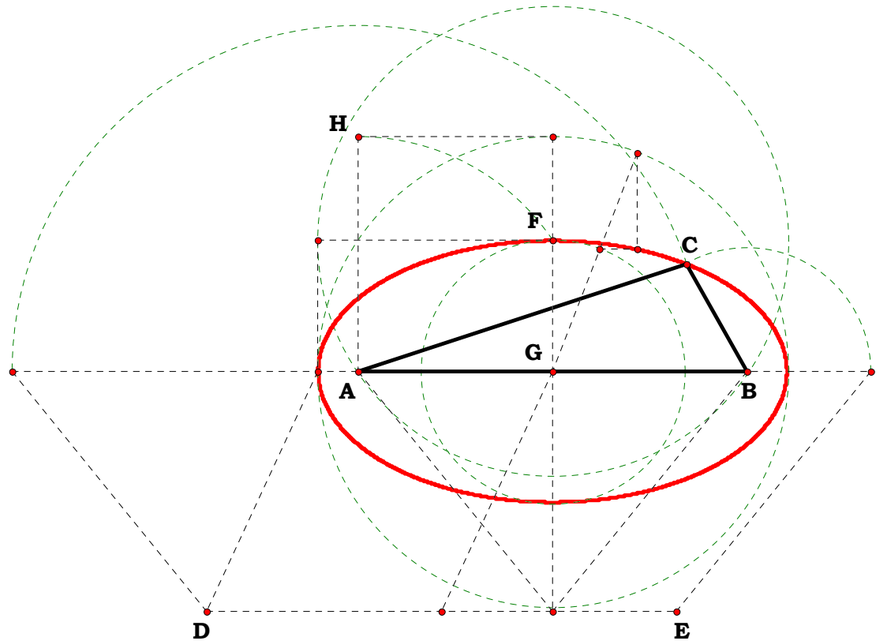
The ratio of the ellipse is thus;

$$\frac{AH}{FG} - \frac{(S_2 + S_3)}{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}} = 0$$

From any point on DE, one can find everything and not once think about x and y.

## The Ellipse

Given triangle ABC, and AB as base, describe the Ellipse



Unit. $A1 := 1$

Given.

$W := 9 \quad Z := .21569$

$X := 20 \quad Y := 20$

091197C

**Love is the functional Conjugate Binary Pair.**
**Shape adjustment is don by AW.**

**Descriptions.**

$AB := \frac{A1}{2} \quad BC := AB \quad BD := AB \quad AW := AB \cdot \frac{W}{X} \quad BW := AB - AW$

$BE := BW \quad BF := BW \quad AZ := A1 \cdot \frac{Z}{Y} \quad CZ := \sqrt{AZ \cdot (A1 - AZ)}$

$DZ := CZ \quad BZ := \sqrt{BC^2 - CZ^2} \quad EG := BZ \cdot \frac{BC - BW}{BC} \quad FH := EG$

$JZ := CZ + EG \quad KZ := DZ - FH$

**Definitions.**

$AB - \frac{1}{2} = 0 \quad BC - \frac{1}{2} = 0 \quad BD - \frac{1}{2} = 0 \quad AW - \frac{W}{2 \cdot X} = 0 \quad BW - \frac{X - W}{2 \cdot X} = 0$

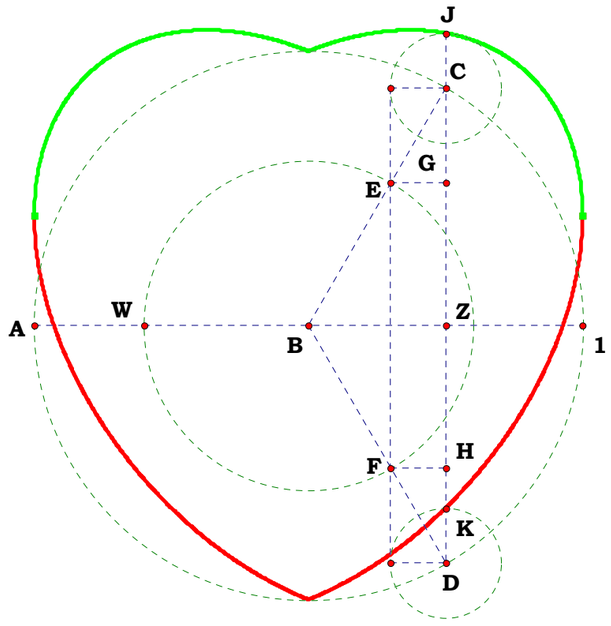
$BW - \frac{X - W}{2 \cdot X} = 0 \quad BF - \frac{X - W}{2 \cdot X} = 0 \quad AZ - \frac{Z}{Y} = 0 \quad CZ - \frac{\sqrt{Z \cdot (Y - Z)}}{Y} = 0$

$DZ - \frac{\sqrt{Z \cdot (Y - Z)}}{Y} = 0 \quad BZ - \frac{\sqrt{(Y - 2 \cdot Z)^2}}{2 \cdot Y} = 0 \quad EG - \frac{\sqrt{(Y - 2 \cdot Z)^2} \cdot W}{2 \cdot Y \cdot X} = 0$

$FH - \frac{\sqrt{(Y - 2 \cdot Z)^2} \cdot W}{2 \cdot Y \cdot X} = 0$

$JZ - \frac{W \cdot \sqrt{(Y - 2 \cdot Z)^2} + 2 \cdot X \cdot \sqrt{Y \cdot Z - Z^2}}{2 \cdot X \cdot Y} = 0 \quad KZ - \frac{2 \cdot X \cdot \sqrt{Y \cdot Z - Z^2} - W \cdot \sqrt{(Y - 2 \cdot Z)^2}}{2 \cdot X \cdot Y} = 0$

## The Perfect Heart




Unit = 1.00000

y/w = 0.40000

W = 8.00000

X = 20.00000

XY = 0.75000

Z = 15.00000

Y = 20.00000

$\frac{W}{2 \cdot X}$-AW = 0.00000

$\frac{X-W}{2 \cdot X}$-BW = 0.00000

$\frac{Z}{Y}$-AZ = 0.00000

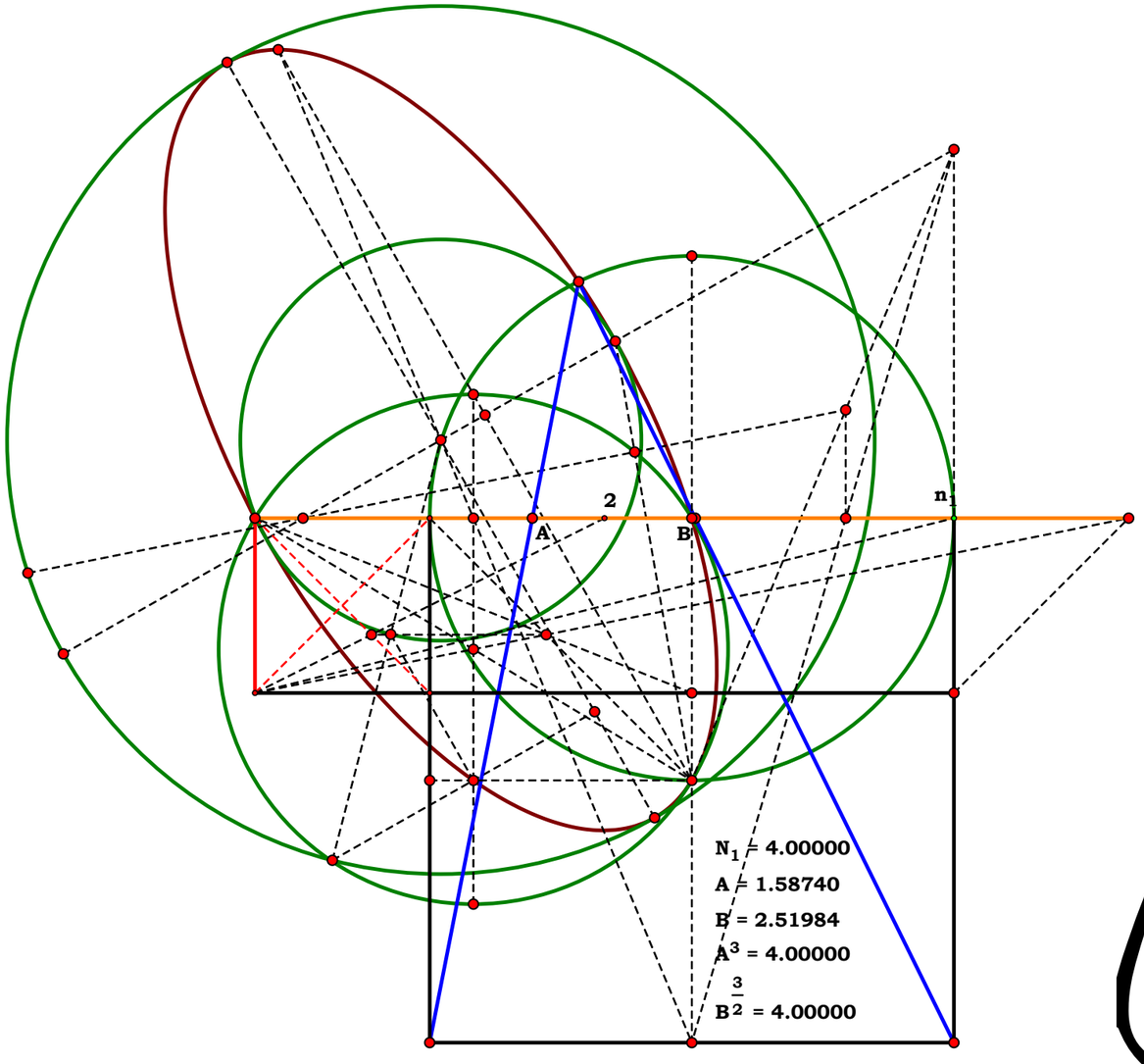
$\frac{\sqrt{Z \cdot (Y-Z)}}{Y}$-CZ = 0.00000

$\frac{\sqrt{(2 \cdot Z-Y)^2}}{2 \cdot Y}$-BZ = 0.00000

$\frac{W \cdot \sqrt{(2 \cdot Z-Y)^2}}{2 \cdot Y \cdot X}$-EG = 0.00000

$\frac{(W \cdot \sqrt{(Y-2 \cdot Z)^2}) + (2 \cdot X \cdot \sqrt{Y \cdot Z-Z^2})}{(2 \cdot X \cdot Y)}$-JZ = 0.00000

$\frac{\left|(2 \cdot X \cdot \sqrt{Y \cdot Z-Z^2}) - (W \cdot \sqrt{(Y-2 \cdot Z)^2})\right|}{(2 \cdot X \cdot Y)}$-KZ = 0.00000




# The Delian Quest 1998

**John Clark**

**Unit.**
**Given.**

$N := .656$

$\Delta := 4$

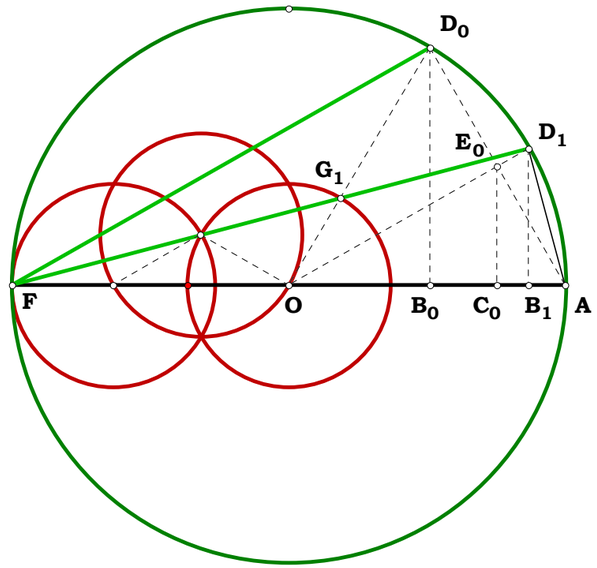
$\delta := 0 .. \Delta - 1$

**020298**
**Descriptions.**

$AF := 2.3754 \quad AO := \frac{AF}{2} \quad AB_0 := N \quad BD_0 := \sqrt{AB_0 \cdot (AF - AB_0)}$

$CE_0 := \frac{BD_0}{2} \quad AC_0 := \frac{AB_0}{2} \quad OC_0 := AO - AC_0 \quad OE_0 := \sqrt{(CE_0)^2 + (OC_0)^2}$

$$\begin{pmatrix} OB_{\delta+1} \\ AB_{\delta+1} \\ BD_{\delta+1} \\ CE_{\delta+1} \\ AC_{\delta+1} \\ OC_{\delta+1} \\ OE_{\delta+1} \end{pmatrix} := \begin{bmatrix} OC_\delta \cdot \frac{AO}{OE_\delta} \\ AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{OE_\delta} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{(2 \cdot OE_\delta)} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta + OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot \sqrt{AO \cdot \frac{(AF \cdot OE_\delta + 4 \cdot OC_\delta \cdot AO - OC_\delta \cdot AF)}{OE_\delta}} \end{bmatrix}$$




$AD_\delta := \sqrt{(AB_\delta)^2 + (BD_\delta)^2}$

**Definitions.**

$$AD = \begin{pmatrix} 1.248304 \\ 0.648825 \\ 0.327541 \\ 0.164163 \end{pmatrix}$$

**Length of cord by progressive bisections.**

**I have no idea why I did this figure, it was so long ago. I don't know what I would do with a cord series unless one wanted to see how close one could get to PI starting from some particular angle or the length of some cord of a circle of some lenght at some commensurable angle. I wonder if I will ever get that bored?**

Unit.

Given.

$N_1 := 2.98958$ $AE := N_1$

$N_2 := 1.86690$ $EG := N_2$

021098

Descriptions.

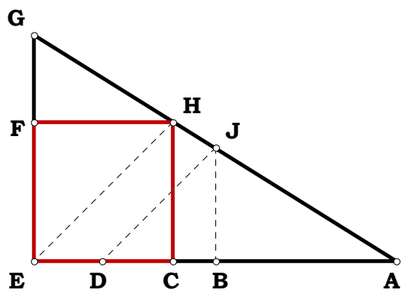
$AB := \frac{AE}{2}$ $BJ := \frac{EG}{2}$ $BD := BJ$

$AD := AB + BD$ $CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE$ $FG := EG - CE$

## A Square In A Triangle

**What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?**




AE = 2.39167 in. $N_1$ = 2.39167 in.

EG = 1.49352 in. $N_2$ = 1.49352 in.

CE = 0.91939 in.

AC = 1.47228 in.

$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0.00000$ in. $\frac{AE}{AC} - \frac{N_1 + N_2}{N_1} = 0.00000$

Definitions.

$AB - \frac{N_1}{2} = 0$ $BJ - \frac{N_2}{2} = 0$ $AD - \left(\frac{N_1}{2} + \frac{N_2}{2}\right) = 0$

$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$ $AC - \frac{{N_1}^2}{N_1 + N_2} = 0$

$FG - \frac{{N_2}^2}{N_1 + N_2} = 0$ $\frac{AE}{AC} - \left(\frac{N_1 + N_2}{N_1}\right) = 0$ $\frac{EG}{FG} - \frac{N_1 + N_2}{N_2} = 0$

Unit.

Given.

$N_1 := 1.67500$ $AH := N_1$

$N_2 := 1.55441$ $HN := N_1 \cdot N_2$

022598A

Descriptions. Given for the third power.

$HJ := HN - AH$ $FH := \frac{AH \cdot HJ}{AH + HJ}$ $FG := FH$

$AF := AH - FH$ $DF := \frac{AF \cdot FG}{AF + FG}$ $AD := AF - DF$

$DE := DF$ $BD := \frac{AD \cdot DE}{AD + DE}$ $AB := AD - BD$

Definitions.

$\frac{AH}{AF} - N_2^{\,1} = 0$ $\frac{AH}{AD} - N_2^{\,2} = 0$ $\frac{AH}{AB} - N_2^{\,3} = 0$

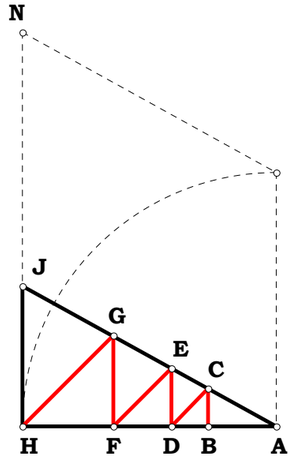
$HN - N_1 \cdot N_2 = 0$ $HJ - (N_1 \cdot N_2 - N_1) = 0$ $FH - \frac{N_1 \cdot (N_2 - 1)}{N_2} = 0$

$FG - \frac{N_1 \cdot (N_2 - 1)}{N_2} = 0$ $AF - \frac{N_1}{N_2} = 0$ $DF - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,2}} = 0$

$AD - \frac{N_1}{N_2^{\,2}} = 0$ $DE - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,2}} = 0$ $BD - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,3}} = 0$ $AB - \frac{N_1}{N_2^{\,3}} = 0$

## Alternate Method Root Series

Given a length and a unit, raise that length to any whole power.




AH = 1.67500 in. $N_1$ = 1.67500 in.

HN = 2.60364 in.

$\frac{HN}{AH}$ = 1.55441 $N_2$ = 1.55441

HJ = 0.92864 in.

AB = 0.44598 in.

AF = 1.07758 in.

AD = 0.69324 in.

$\frac{AH}{AF}$-$N_2$ = 0.00000 $\frac{AH}{AD}$-$N_2^2$ = 0.00000 $\frac{AH}{AB}$-$N_2^3$ = 0.00000

Unit.

$AH := 1$

Given.

$N_1 := 5$ $\quad HM := N_1$

$N_2 := 6$ $\quad AN := N_2$

022598B

Descriptions.

$$HO := \frac{AH \cdot HM}{AN} \qquad AO := AH + HO \qquad AF := \frac{AH^2}{AO}$$

$$FH := AH - AF \qquad FG := FH \qquad DF := \frac{AF \cdot FG}{AF + FG}$$

$$AD := AF - DF \qquad DE := DF \qquad BD := \frac{AD \cdot DE}{AD + DE}$$

$$AB := AD - BD$$

Definitions.

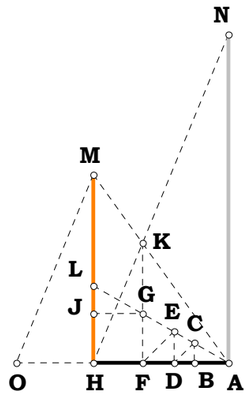
$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0 \qquad HO - \frac{N_1}{N_2} = 0 \qquad AO - \frac{N_1 + N_2}{N_2} = 0$$

$$AF - \frac{N_2}{N_1 + N_2} = 0 \qquad FH - \frac{N_1}{N_1 + N_2} = 0 \qquad FG - \frac{N_1}{N_1 + N_2} = 0$$

$$DF - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0 \qquad AD - \frac{N_2^2}{(N_1 + N_2)^2} = 0 \qquad DE - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0$$

$$BD - \frac{N_1 \cdot N_2^2}{(N_1 + N_2)^3} = 0 \qquad AB - \frac{N_2^3}{(N_1 + N_2)^3} = 0 \qquad AB^{\frac{1}{3}} - \frac{N_2}{(N_1 + N_2)} = 0$$

## Sum Divided by One Powered




$AH = 0.89167$ in.

$HM = 1.24469$ in.

$AN = 2.16687$ in.

$AB = 0.22848$ in.

$\frac{HM}{AH} = 1.39591$ $\quad N_1 = 1.39591$

$\frac{AN}{AH} = 2.43013$ $\quad N_2 = 2.43013$

$\frac{AB}{AH} = 0.25624$ $\quad AB = 0.25624$

$$AB - \frac{N_2^3}{(N_1+N_2)^3} = 0.00000$$

# Doing the Math

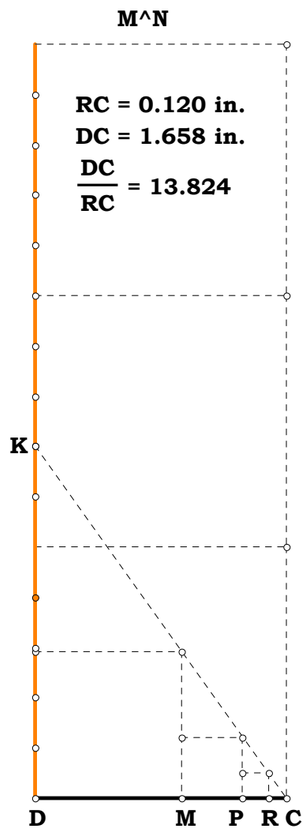
**022598C**

**Descriptions.**

**These plates sat, I never actually wrote them up, in their directory, but included in the Delian Quest. Because they are so elementary, I assumed that doing math with a geometric figure was known. I was a bit conflicted about this, however, working 12 hours a day for years on end tends to dull the senses. Then, I got to thinking about them again in 2007. I even did a couple of searches on the internet to see if anyone had actually developed doing the math with a simple geometric figure and could not find anything. Then I found scraps in old books found on the Internet Archive where certain operations were fragmented and really undeveloped. Then I started to realize and understand that BAM was not developed as a grammar If it had been, there would be no talk of non-Euclidean Geometry, there would only be embarrassment of its memory.**

**One can see that it is directly derived from plate A on this date. BAM (Basic Analog Mathematics) has its roots in exponential series.**

**Definitions.**




$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

$\left(2+\frac{1}{5}\right)^3 = 10.648$

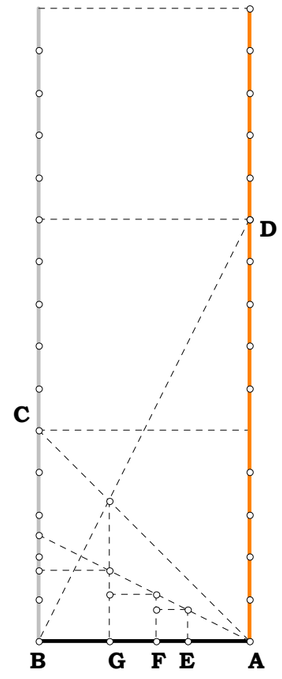
$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

$\left(1+\frac{1}{5}\right)^3 = 1.728$

**Given line AB, divide it by the equation** $\frac{BC+AD^{W}}{AD} = 5.06250$ **where W is a whole number.**




$\frac{AB}{AG} = 1.50000$

$\frac{AB}{AF} = 2.25000$

$\frac{AB}{AE} = 3.37500$

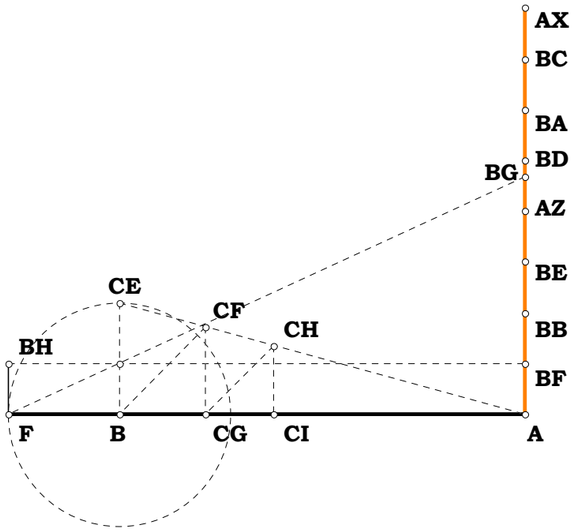
$\frac{BC+AD^{1}}{AD} = 1.50000$

$\frac{BC+AD^{2}}{AD} = 2.25000$

$\frac{BC+AD^{3}}{AD} = 3.37500$

022698

**Now ain't this just typically human. Some of the more defined plates that led to Basic Analog Mathematics promptly get wrote up in 0816 2015. I am on the ball! It appears I never even bothered to do a pdf file of these. They are, however, not fundamentally distinct from some previous write ups except, these are series format. So, I think I will forgo the write ups again!**




$\frac{AF}{FB} = 4.672$

$AB = 2.679$ in.

$FB = 0.730$ in.

$AF = 3.408$ in.

$\frac{AF}{AB} = 1.272$

$ACG = 2.105$ in.

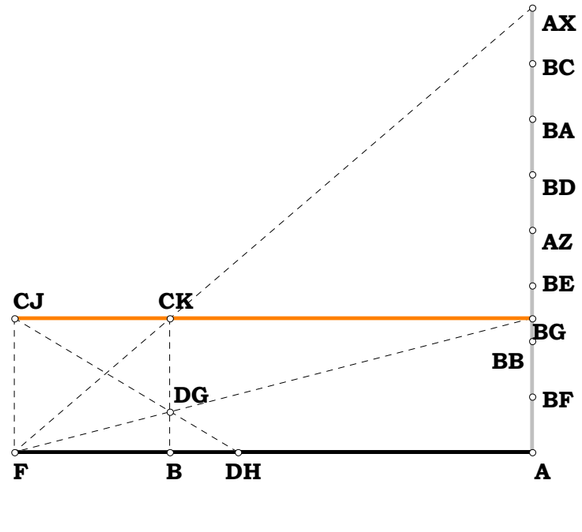
$ACI = 1.655$ in.

$\frac{AF}{ACI} = 2.060$

$\frac{AF}{ACG} = 1.619$

$\frac{AF^3}{AB} = 2.060$

$\frac{AF^2}{AB} = 1.619$




$AF = 3.417$ in.

$AB = 2.388$ in.

$\frac{AF}{AB} = 1.431$

$\frac{AF}{DHF} = 2.321$

$\frac{DHF}{AF} = 0.431$

$DHF = 1.472$ in.

$BF = 1.029$ in.

$\frac{AAX}{BGA} = 3.321$

$\frac{AAX}{BGA} - 1 = 2.321$

$BGA = 0.884$ in.

$AAX = 2.936$ in.

$\left(\frac{1}{\frac{AAX}{BGA} - 1}\right) \cdot BF = 0.443$ in.

Unit.

$AB := 1$

Given.

$N_1 := 4.27 \quad AF := N_1$

042398

Descriptions.

$BF := AF - AB \quad AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2}$

$BD := AD - AB \quad DE := BE - BD \quad EQ := BE$

$DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \quad PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ}$

$DM := QM - DQ \quad AE := AB + BE \quad AC := \frac{AE}{2}$

$Db := \frac{DM}{2} \quad CM := AC \quad ab := \frac{CM \cdot Db}{DM}$

$CD := AD - AC \quad Ca := \frac{CD}{2} \quad Aa := AC + Ca$

$CH := \frac{ab \cdot AC}{Aa} \quad AM := AD \quad Ac := \frac{AM \cdot CH}{CM}$

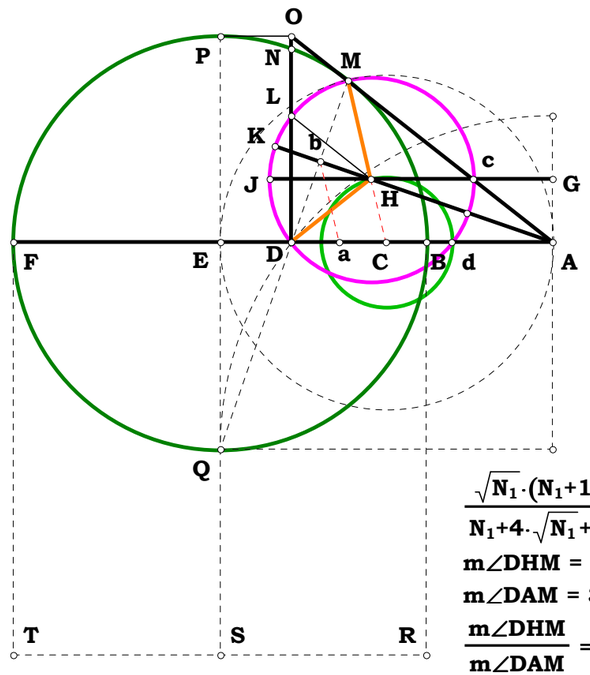
$HM := CM - CH \quad HM - Ac = 0$

Definitions.

$Ac - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0 \quad HM - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0$

## A Square Root Figure And Triseciton

**In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?**




AB = 2.11667 cm

AF = 9.03817 cm

Ac = 1.70294 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000$ $\quad N_1 = 4.27000$

$\frac{Ac}{AB} = 0.80454$ $\quad Ac = 0.80454$

$\frac{\sqrt{N_1} \cdot (N_1+1)}{N_1+4 \cdot \sqrt{N_1}+1} - Ac = 0.00000$

$m\angle DHM = 115.05651°$

$m\angle DAM = 38.35217°$

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$

**The traditional paper trisector fits right into this figure.**
**This figure seems to be just full of surprises.**

$$AF - N_1 = 0 \qquad BF - (N_1 - 1) = 0 \qquad AD - \sqrt{N_1} = 0$$

$$BE - \frac{N_1 - 1}{2} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0 \qquad DE - \frac{\left(\sqrt{N_1} - 1\right)^2}{2} = 0$$

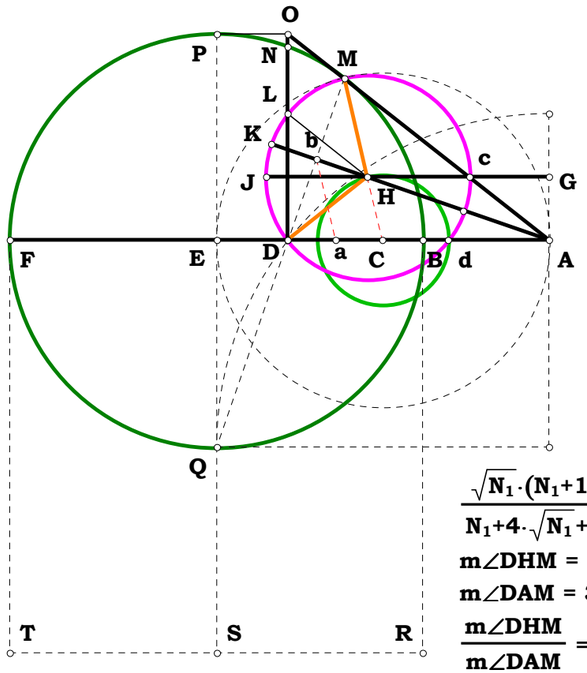
$$DQ - \frac{\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}}{\sqrt{2}} = 0 \qquad QM - \frac{\sqrt{2}\cdot\left(\sqrt{N_1} - 1\right)^2\cdot\left(\sqrt{N_1} + 1\right)^2}{2\cdot\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0$$

$$DM - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0 \qquad AE - \frac{1 + N_1}{2} = 0 \qquad AC - \frac{1 + N_1}{4} = 0$$

$$Db - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{2\cdot\sqrt{(N_1 + 1)\cdot\left(N_1 - 2\cdot\sqrt{N_1} + 1\right)}} = 0 \qquad CD - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{4} = 0$$

$$Ca - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{8} = 0 \qquad Aa - \frac{N_1 + 4\cdot\sqrt{N_1} + 1}{8} = 0$$

$$CH - \frac{\left(1 + N_1\right)^2}{4\cdot\left(N_1 + 4\cdot\sqrt{N_1} + 1\right)} = 0$$




AB = 2.11667 cm
AF = 9.03817 cm
Ac = 1.70294 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000$ $\qquad N_1 = 4.27000$

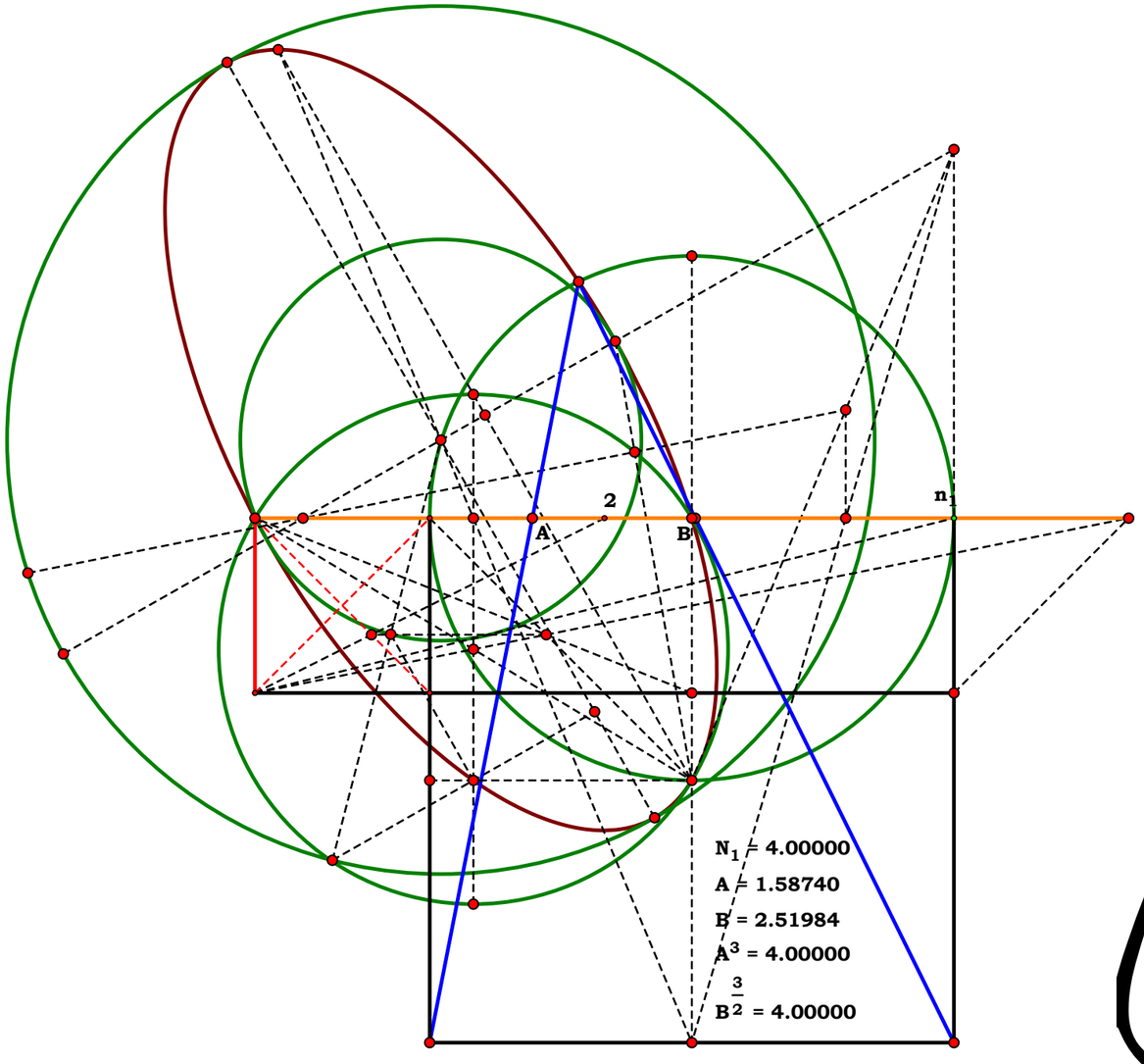
$\frac{Ac}{AB} = 0.80454$ $\qquad Ac = 0.80454$

$\frac{\sqrt{N_1}\cdot(N_1+1)}{N_1+4\cdot\sqrt{N_1}+1}$-Ac = 0.00000

m∠DHM = 115.05651°

m∠DAM = 38.35217°

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$




# The Delian Quest 1999

**John Clark**

072499

Unit.

$AB := 1$

Given.

$N_1 := 6.429$ $AD := N_1$

$N_2 := 2$

# On Gemini Roots

CE is to EF as CY is to CW

Descriptions.

$BD := AD - AB$ $BC := \frac{BD}{2}$ $CW := BC$ $CT := BC$

$FV := BC$ $AI := \sqrt{AB \cdot AD}$ $BI := AI - AB$ $DI := BD - BI$

$IR := \sqrt{BI \cdot DI}$ $AE := AI$ $AC := AB + BC$ $ED := AD + AE$

$CE := AC + AE$ $EF := \frac{CE}{N_2}$ $BE := AE + AB$ $EI := AE + AI$

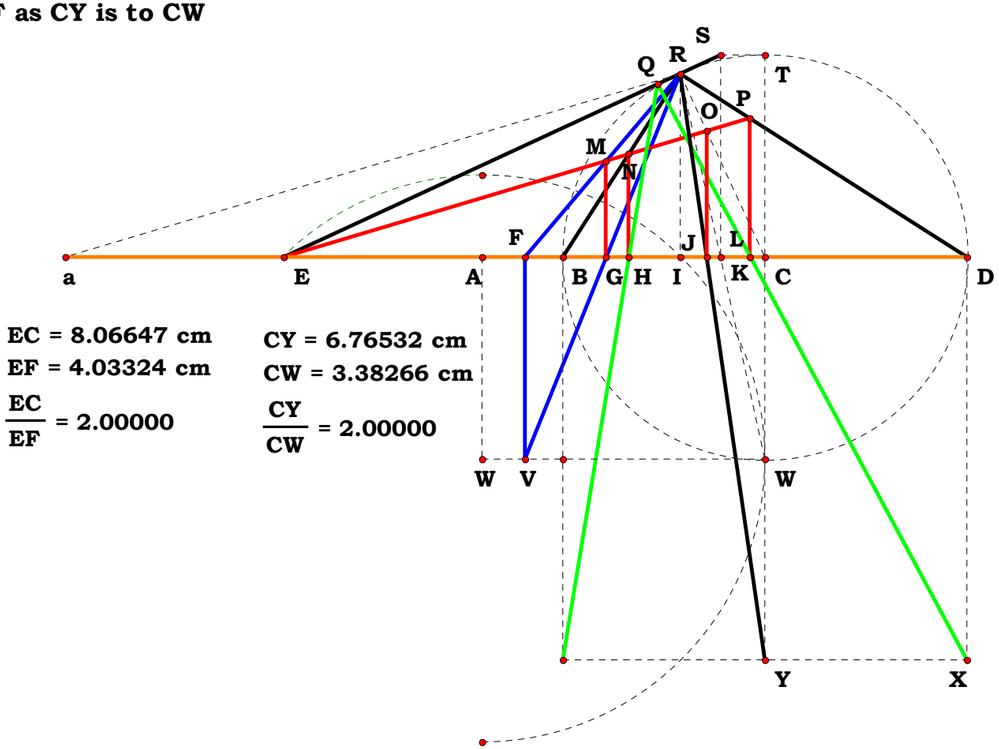
$FI := EI - EF$ $FG := \frac{FI \cdot FV}{FV + IR}$ $EG := EF + FG$ $GI := FI - FG$

$GM := \frac{FV \cdot GI}{FI}$ $Ia := \frac{EG \cdot IR}{GM}$ $EL := \frac{Ia \cdot ED}{Ia + DI}$ $BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI$ $BH := \frac{BI \cdot BE}{Ba}$ $EH := BE + BH$ $CI := AC - AI$

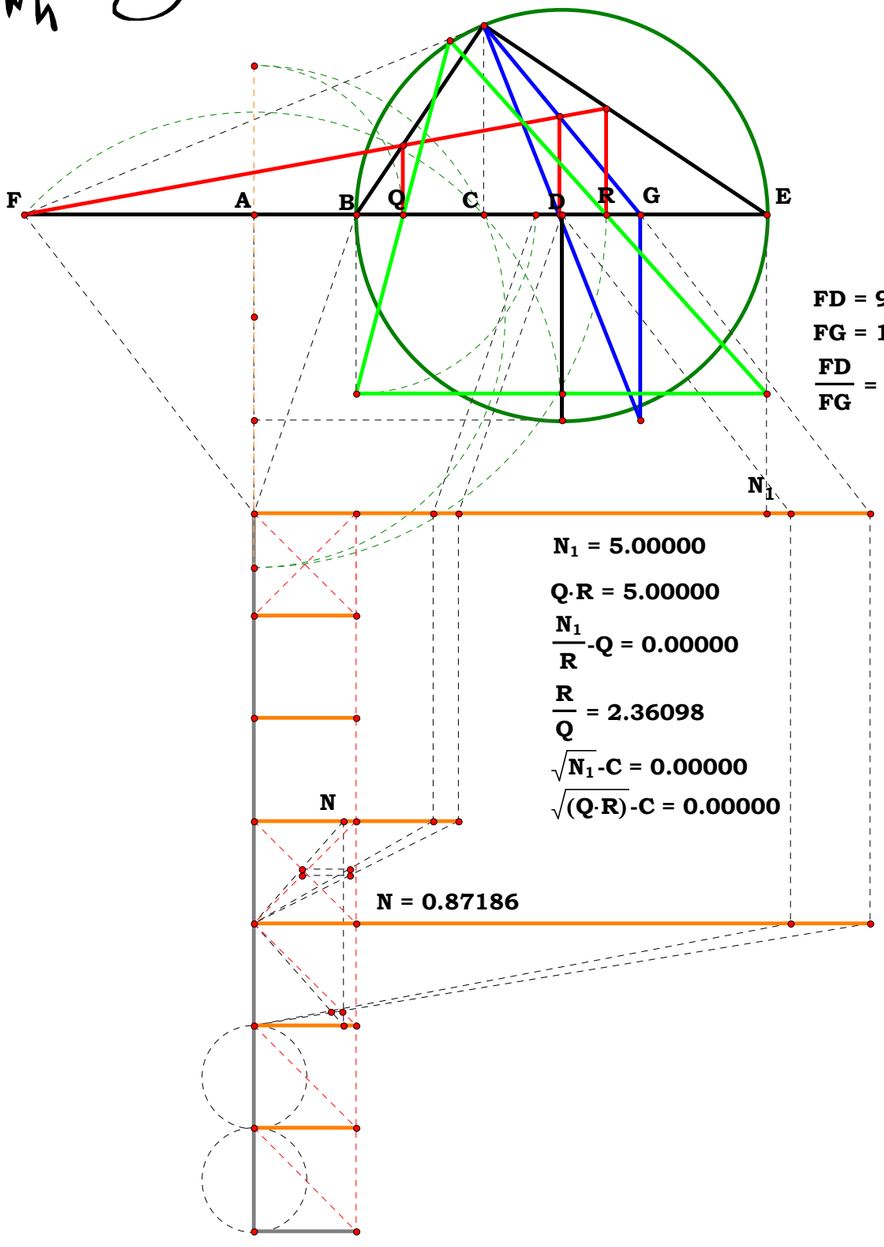
$JO := \frac{IR \cdot CE}{CI + Ia}$ $CJ := \frac{CI \cdot JO}{IR}$ $JI := CI - CJ$

$CY := \frac{IR \cdot CJ}{JI}$ $\frac{CY}{CW} - \frac{CE}{EF} = 0$




EC = 8.06647 cm
EF = 4.03324 cm
$\frac{EC}{EF} = 2.00000$

CY = 6.76532 cm
CW = 3.38266 cm
$\frac{CY}{CW} = 2.00000$




FD = 9.00429 cm

FG = 10.32767 cm

$\frac{FD}{FG}$ = 0.87186

DK = 2.99862 cm

DJ = 3.43933 cm

$\frac{DK}{DJ}$ = 0.87186

$\frac{FD}{FG}-\frac{DK}{DJ}$ = 0.00000

$N_1$ = 5.00000

Q·R = 5.00000

$\frac{N_1}{R}-Q$ = 0.00000

$\frac{R}{Q}$ = 2.36098

$\sqrt{N_1}-C$ = 0.00000

$\sqrt{(Q\cdot R)}-C$ = 0.00000

N = 0.87186

A = 0.00000

B = 1.00000

C = 2.23607

Q = 1.45525

R = 3.43583

E = 5.00000

$\frac{E}{R}$ = 1.45525

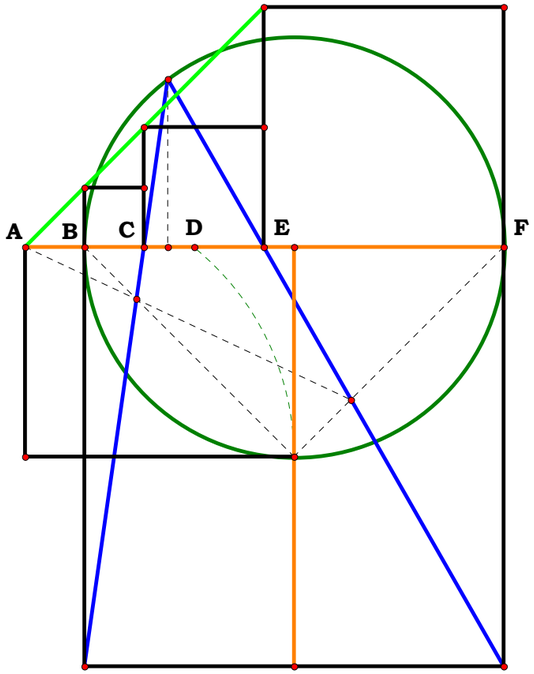
$\frac{E}{R}-Q$ = 0.00000

081199

## A Delian Solution

**In this revision, I have dimmed down the names of those points which are not used in a section which greately facilitates reading of each chapter.**

**As in the past, you can compare the Arithmetic results produced by both the Sketchpad and Mathcad. One will then have Geometric Names, which is the figure, Algebraic Names used in MathCad, and Arithmetic Names, produced by both.**




$AF^{\frac{1}{3}} - AC = 0.00000$

$AF^{\frac{1}{2}} - AD = 0.00000$

$AF^{\frac{2}{3}} - AE = 0.00000$

$af^{\frac{1}{3}} - ac = 0.00000$

$af^{\frac{1}{2}} - ad = 0.00000$

$af^{\frac{2}{3}} - ae = 0.00000$

AB = 1.00310 cm
AC = 2.00619 cm
AD = 2.83718 cm
AE = 4.01238 cm
AF = 8.02476 cm

$\frac{AB}{AB} = 1.00000$ AB = 1.00000

$\frac{AC}{AB} = 2.00000$ AC = 2.00000

$\frac{AD}{AB} = 2.82843$ AD = 2.82843

$\frac{AE}{AB} = 4.00000$ AE = 4.00000

$\frac{AF}{AB} = 8.00000$ AF = 8.00000

$\frac{AB}{AC} = 0.50000$ ac = 0.50000

$\frac{AB}{AD} = 0.35355$ ad = 0.35355

$\frac{AB}{AE} = 0.25000$ ae = 0.25000

$\frac{AB}{AF} = 0.12500$ af = 0.12500

Unit.

AB := 2.59047

Given.

AG := 11.81347

081199

## A Delian Solution

Descriptions.

$BG := AG - AB$ $BF := \frac{BG}{2}$ $FG := BF$ $AF := AB + BF$

$FX := BF$ $Mf := \frac{\sqrt{AF^2 + FX^2}}{2}$ $Lf := \frac{FX}{2}$ $ML := Mf - Lf$

$FL := \frac{AF}{2}$ $Xd := FL$ $df := Lf$ $IX := FX$ $Md := Mf + df$

Definitions.

BG = 9.22300 BF = 4.61150 FG = 4.61150 AF = 7.20197

FX = 4.61150 Mf = 4.27593 Lf = 2.30575 ML = 1.97018

FL = 3.60099 Xd = 3.60099 df = 2.30575 IX = 4.61150

Md = 6.58168

Definitions.

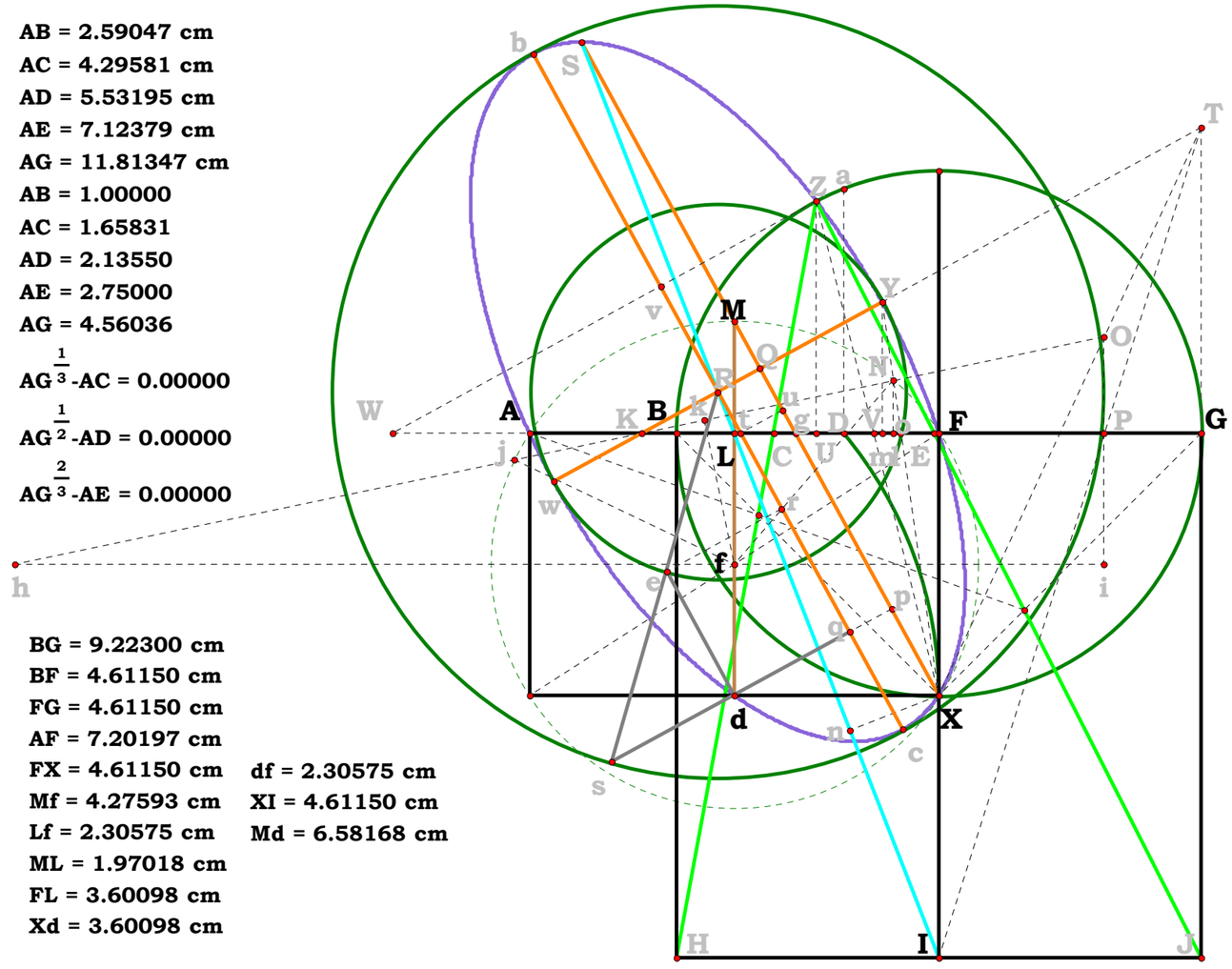
**What are the minor and major axis for the ellipse that will give point Z for the cube root?**

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036

$AG^{\frac{1}{3}}$-AC = 0.00000

$AG^{\frac{1}{2}}$-AD = 0.00000

$AG^{\frac{2}{3}}$-AE = 0.00000




BG = 9.22300 cm
BF = 4.61150 cm
FG = 4.61150 cm
AF = 7.20197 cm
FX = 4.61150 cm
Mf = 4.27593 cm
Lf = 2.30575 cm
ML = 1.97018 cm
FL = 3.60098 cm
Xd = 3.60098 cm

df = 2.30575 cm
XI = 4.61150 cm
Md = 6.58168 cm

**Descriptions.**

$$MX := \sqrt{Xd^2 + Md^2} \qquad SX := \frac{MX \cdot IX}{IX - ML} \qquad Lg := \frac{FL \cdot ML}{ML + FX}$$

$$QX := \frac{SX}{2} \qquad Fg := FL - Lg \qquad Xg := \frac{MX \cdot Fg}{Xd} \qquad Qg := QX - Xg$$

$$Kg := \frac{Xg \cdot Qg}{Fg} \qquad GK := FG + Fg + Kg \qquad GJ := BG \qquad GT := \frac{Fg \cdot GK}{FX}$$

$$JT := GJ + GT \qquad IJ := BF \qquad FP := \frac{IJ \cdot GJ}{JT} \qquad OP := \frac{IX \cdot GT}{JT}$$

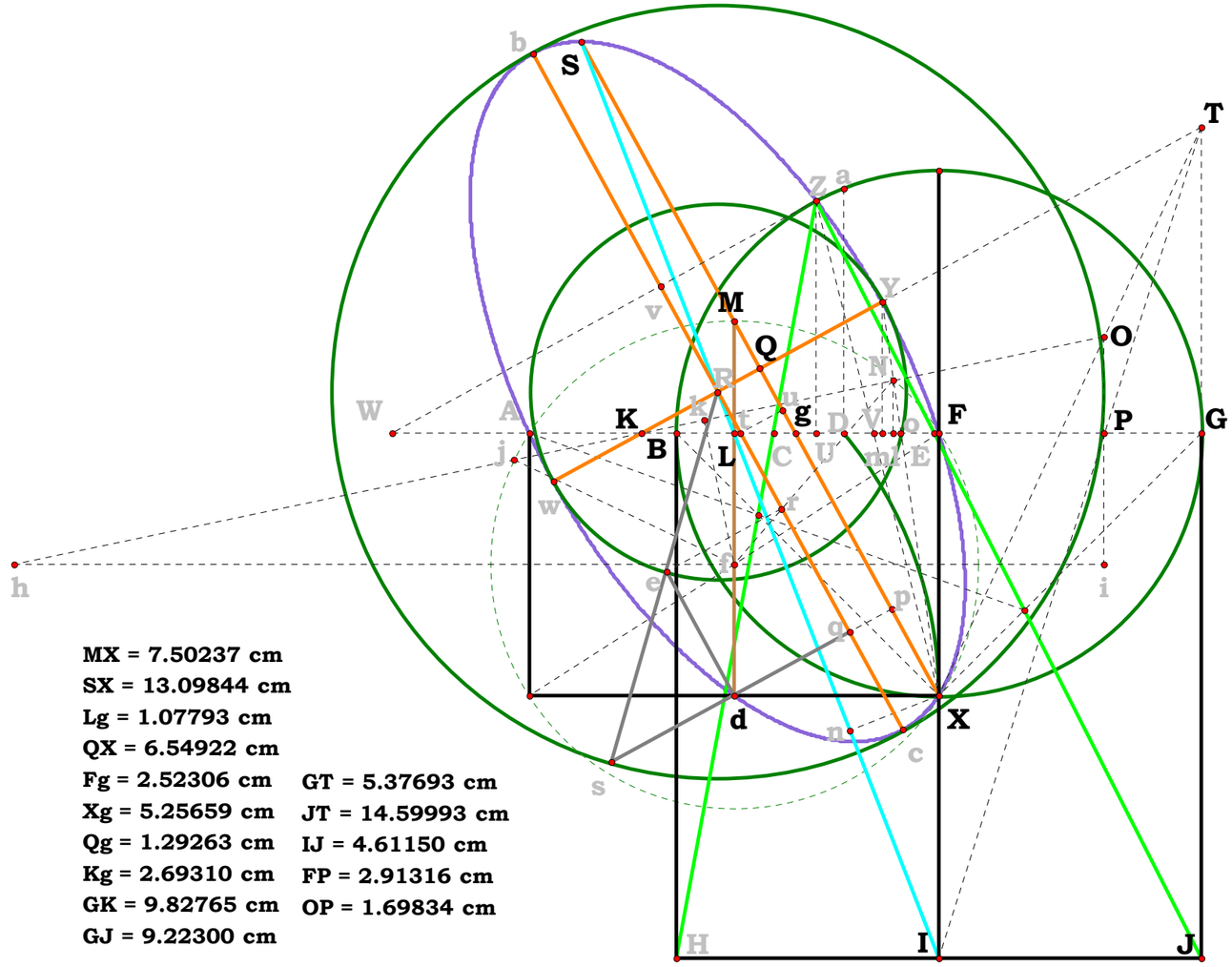
**Definitions.**

MX = 7.50237 SX = 13.09845 Lg = 1.07793 QX = 6.54922

Fg = 2.52306 Xg = 5.25659 Qg = 1.29263

Kg = 2.69310 GK = 9.82766 GJ = 9.22300 GT = 5.37693

JT = 14.59993 IJ = 4.61150 FP = 2.91315 OP = 1.69835




MX = 7.50237 cm
SX = 13.09844 cm
Lg = 1.07793 cm
QX = 6.54922 cm
Fg = 2.52306 cm
Xg = 5.25659 cm
Qg = 1.29263 cm
Kg = 2.69310 cm
GK = 9.82765 cm
GJ = 9.22300 cm

GT = 5.37693 cm
JT = 14.59993 cm
IJ = 4.61150 cm
FP = 2.91316 cm
OP = 1.69834 cm

**Descriptions.**

$KP := Fg + Kg + FP$ $\quad Pi := Lf$ $\quad Oi := OP + Pi$ $\quad hi := \frac{KP \cdot Oi}{OP}$

$fi := FP + FL$ $\quad fh := hi - fi$ $\quad KO := \sqrt{KP^2 + OP^2}$ $\quad hk := \frac{KP \cdot fh}{KO}$

$Nf := Mf$ $\quad fk := \frac{OP \cdot fh}{KO}$ $\quad Nk := \sqrt{Nf^2 - fk^2}$ $\quad Nh := hk + Nk$

$Oh := \frac{KO \cdot Oi}{OP}$ $\quad NO := Oh - Nh$ $\quad KN := KO - NO$

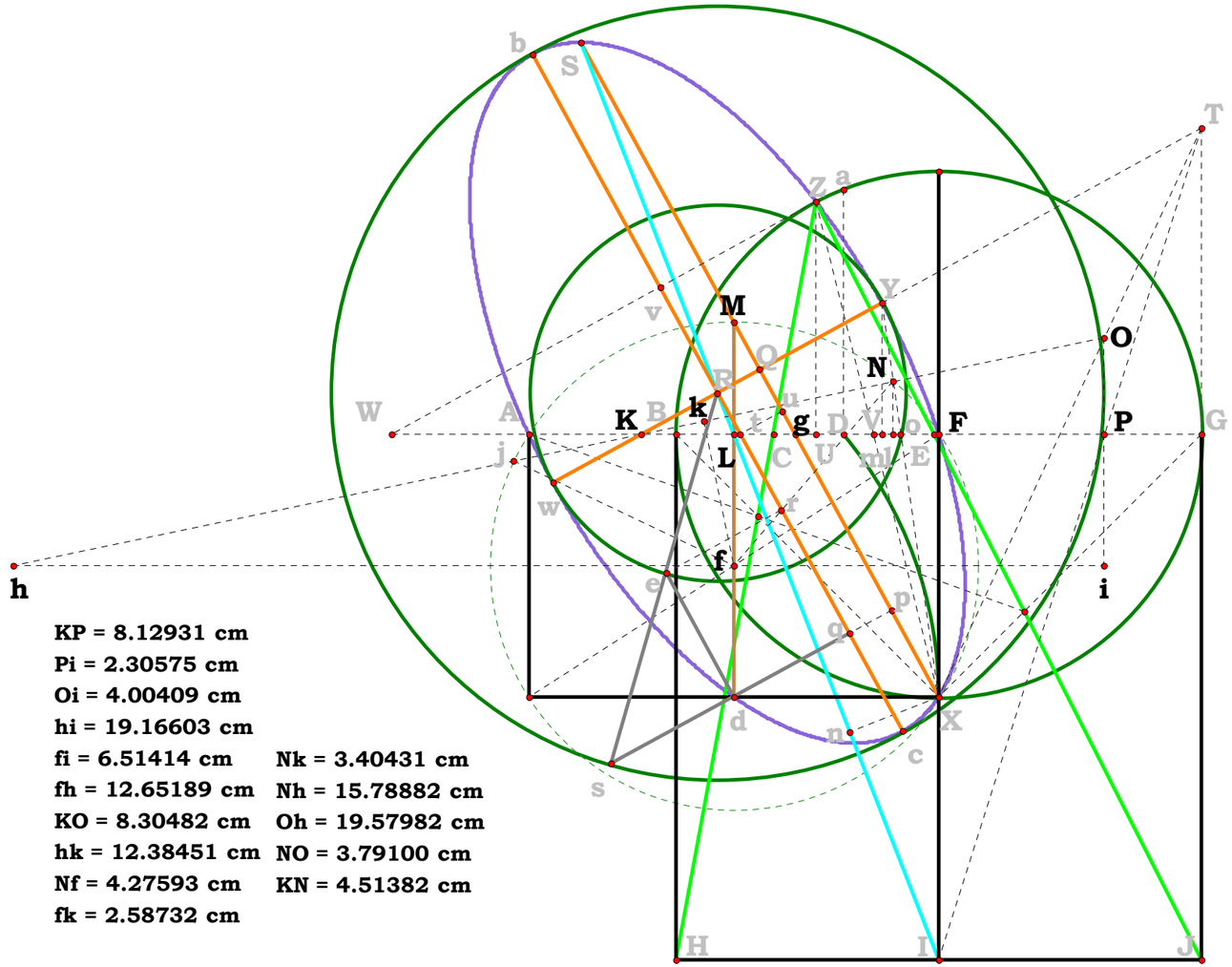
**Definitions.**

KP = 8.12931 Pi = 2.30575 Oi = 4.00410 hi = 19.16603

fi = 6.51414 fh = 12.65189 KO = 8.30482 hk = 12.38451

Nf = 4.27593 fk = 2.58733 Nk = 3.40431 Nh = 15.78882

Oh = 19.57983 NO = 3.79100 KN = 4.51382




KP = 8.12931 cm
Pi = 2.30575 cm
Oi = 4.00409 cm
hi = 19.16603 cm
fi = 6.51414 cm
fh = 12.65189 cm
KO = 8.30482 cm
hk = 12.38451 cm
Nf = 4.27593 cm
fk = 2.58732 cm

Nk = 3.40431 cm
Nh = 15.78882 cm
Oh = 19.57982 cm
NO = 3.79100 cm
KN = 4.51382 cm

**Descriptions.**

$$Nl := \frac{OP \cdot KN}{KO} \qquad Kl := \frac{KP \cdot KN}{KO} \qquad FK := KP - FP \qquad Fl := FK - Kl$$

$$NX := \sqrt{(FX + Nl)^2 + Fl^2} \qquad XY := \frac{NX \cdot IX}{IX - Nl} \qquad Fm := \frac{Fl \cdot XY}{NX} \qquad Km := FK - Fm$$

$$Fo := \frac{Fl \cdot FX}{FX + Nl} \qquad Xo := \frac{NX \cdot Fo}{Fl} \qquad mo := Fm - Fo \qquad Ym := \frac{FX \cdot mo}{Fo}$$

$$FI := 2 \cdot BF \qquad IL := \sqrt{FL^2 + FI^2} \qquad IS := \frac{IL \cdot IX}{IX - ML} \qquad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

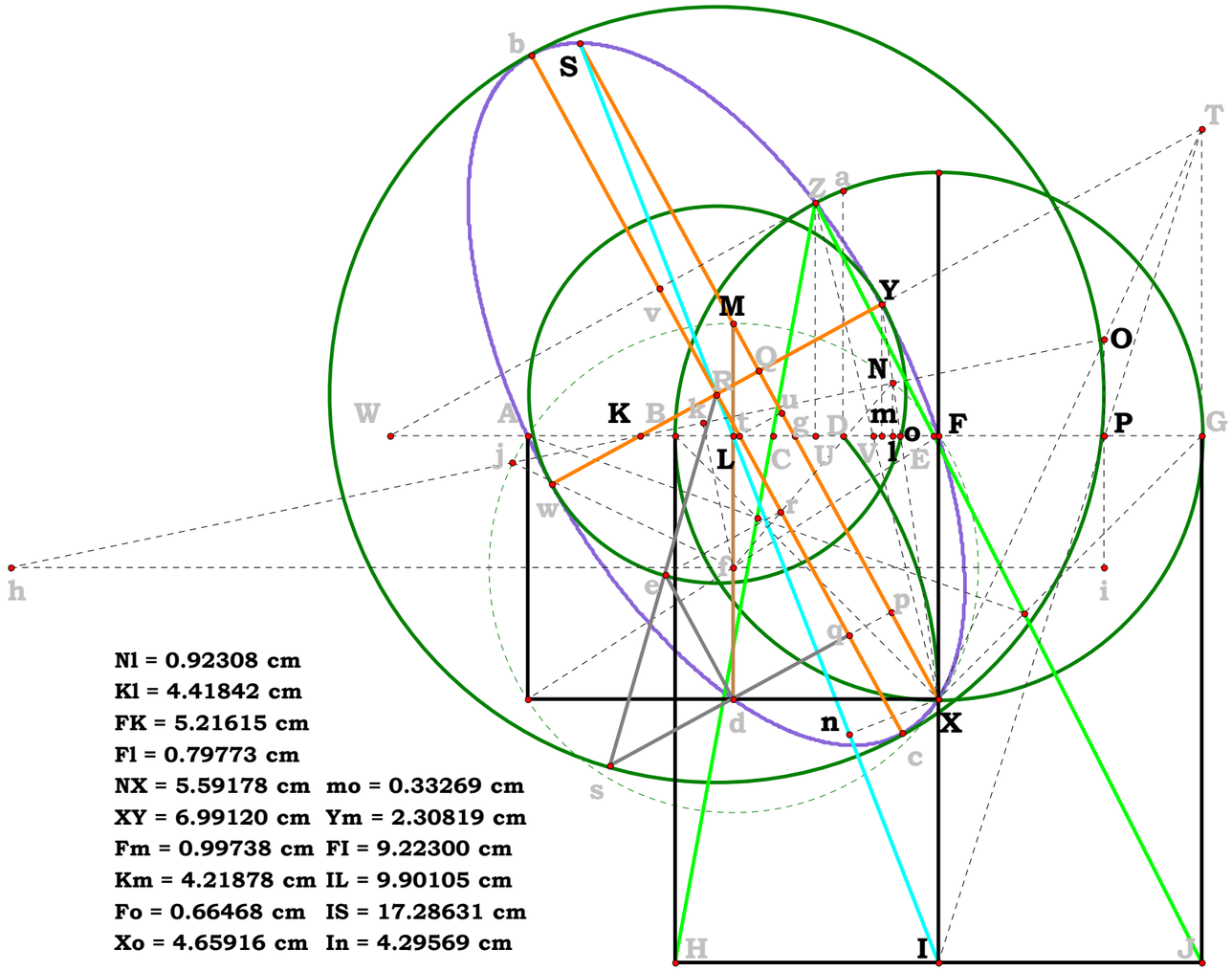
**Definitions.**

Nl = 0.92308 Kl = 4.41843 FK = 5.21616 Fl = 0.79773

NX = 5.59178 XY = 6.99120 Fm = 0.99738 Km = 4.21878

Fo = 0.66468 Xo = 4.65916 mo = 0.33269 Ym = 2.30819

FI = 9.22300 IL = 9.90105 IS = 17.28632 In = 4.29569



**Descriptions.**

$Xn := \sqrt{IX^2 - In^2}$ $QR := \frac{Xn \cdot QX}{IS - In}$ $KT := \sqrt{GK^2 + GT^2}$ $KY := \frac{KT \cdot Ym}{GT}$

$KQ := \frac{GK \cdot Qg}{GT}$ $RY := KY - KQ + QR$ $Xd := FL$ $dp := \frac{GK \cdot Xd}{KT}$

$pq := QR$ $dq := dp - pq$ $Xp := \frac{GT \cdot Xd}{KT}$ $Qp := QX - Xp$ $Rq := Qp$

$er := dq$ $Re := RY$ $Rr := \sqrt{Re^2 - er^2}$ $Rs := \frac{Re \cdot Rq}{Rr}$

**RY is the radius of the Minor Axis.**

**Rs is the radius of the Major Axis.**

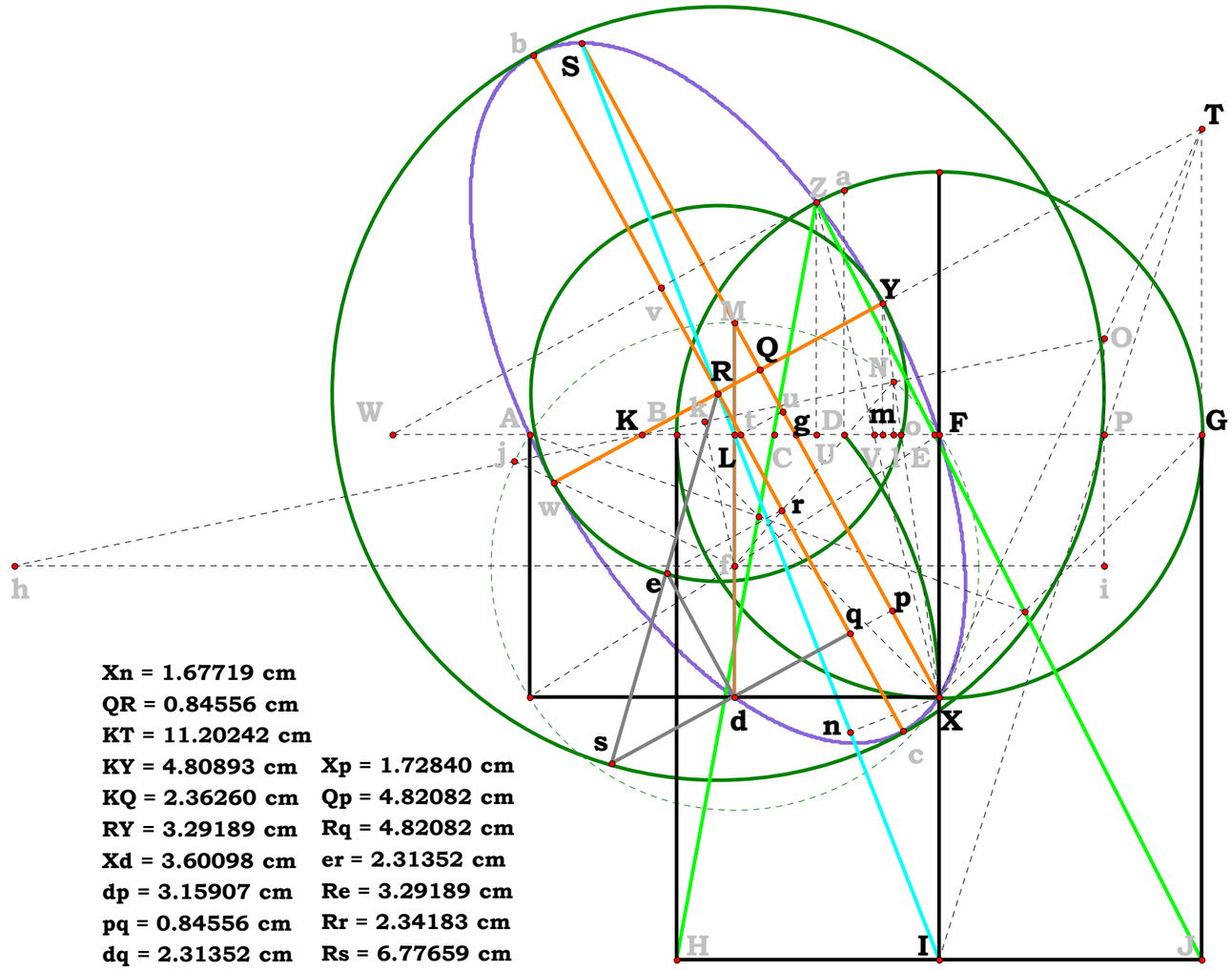
**Definitions.**

Xn = 1.67719 QR = 0.84556 KT = 11.20242 KY = 4.80894

KQ = 2.36260 RY = 3.29189 Xd = 3.60099 dp = 3.15907

pq = 0.84556 dq = 2.31352 Xp = 1.72840 Qp = 4.82082
Rq = 4.82082 er = 2.31352 Re = 3.29189 Rr = 2.34183

Rs = 6.77659




Xn = 1.67719 cm
QR = 0.84556 cm
KT = 11.20242 cm
KY = 4.80893 cm Xp = 1.72840 cm
KQ = 2.36260 cm Qp = 4.82082 cm
RY = 3.29189 cm Rq = 4.82082 cm
Xd = 3.60098 cm er = 2.31352 cm
dp = 3.15907 cm Re = 3.29189 cm
pq = 0.84556 cm Rr = 2.34183 cm
dq = 2.31352 cm Rs = 6.77659 cm

**The first two equations should have been learnt by prior explorations. And they will be repeated demonstrated in following demonstrations. Therefore, here we can, like other equations, simply recall them for the figure.**

**Descriptions.**

$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \qquad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \qquad BC := AC - AB \qquad BE := AE - AB$$

$$CE := BE - BC \qquad EG := BG - BE \qquad CU := \frac{BC \cdot CE}{BC + EG} \qquad BU := BC + CU$$

$$GU := BG - BU \qquad UZ := \sqrt{BU \cdot GU} \qquad UW := \frac{GK \cdot UZ}{GT} \qquad Gg := GK - Kg$$

$$tu := QR \qquad gt := \frac{KT \cdot tu}{GK} \qquad Gt := Gg + gt \qquad GW := GU + UW$$

**Is the segment Zv equal to the perpendicular for the ellipse?**

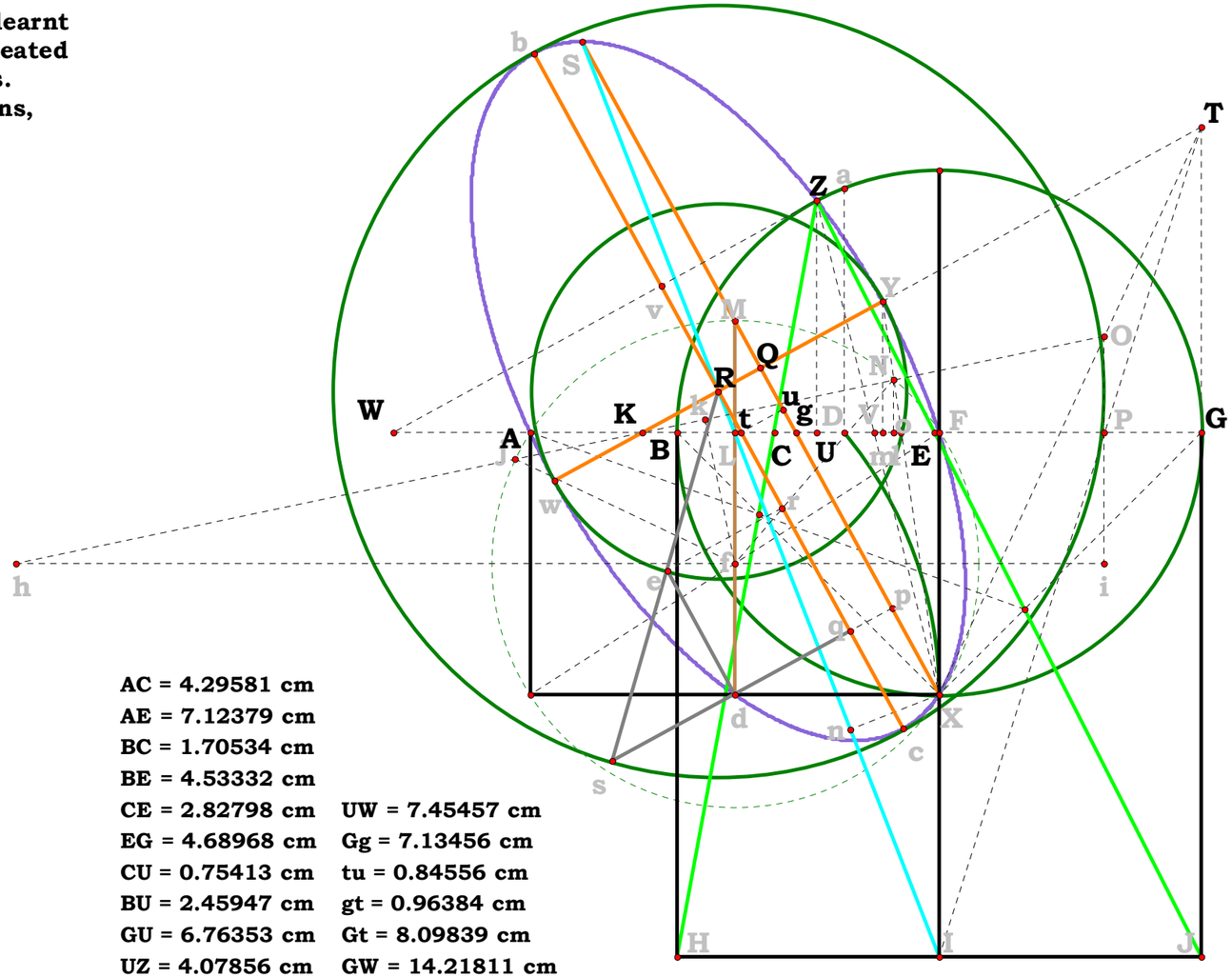
**Definitions.**

**AC = 4.29581 AE = 7.12379 BC = 1.70534 BE = 4.53332**

**CE = 2.82798 EG = 4.68968 CU = 0.75413 BU = 2.45947**

**GU = 6.76353 UZ = 4.07856 UW = 7.45457 Gg = 7.13456**

**tu = 0.84556 gt = 0.96384 Gt = 8.09839 GW = 14.21811**




**AC = 4.29581 cm**
**AE = 7.12379 cm**
**BC = 1.70534 cm**
**BE = 4.53332 cm**
**CE = 2.82798 cm**
**EG = 4.68968 cm**
**CU = 0.75413 cm**
**BU = 2.45947 cm**
**GU = 6.76353 cm**
**UZ = 4.07856 cm**

**UW = 7.45457 cm**
**Gg = 7.13456 cm**
**tu = 0.84556 cm**
**gt = 0.96384 cm**
**Gt = 8.09839 cm**
**GW = 14.21811 cm**

**Descriptions.**

$Wt := GW - Gt$ $\quad tv := \frac{GT \cdot Wt}{KT}$ $\quad Kt := GK - Gt$ $\quad Rt := \frac{GT \cdot Kt}{KT}$

$Rv := tv - Rt$ $\quad bc := 2 \cdot Rs$ $\quad Rc := Rs$ $\quad cv := Rc + Rv$

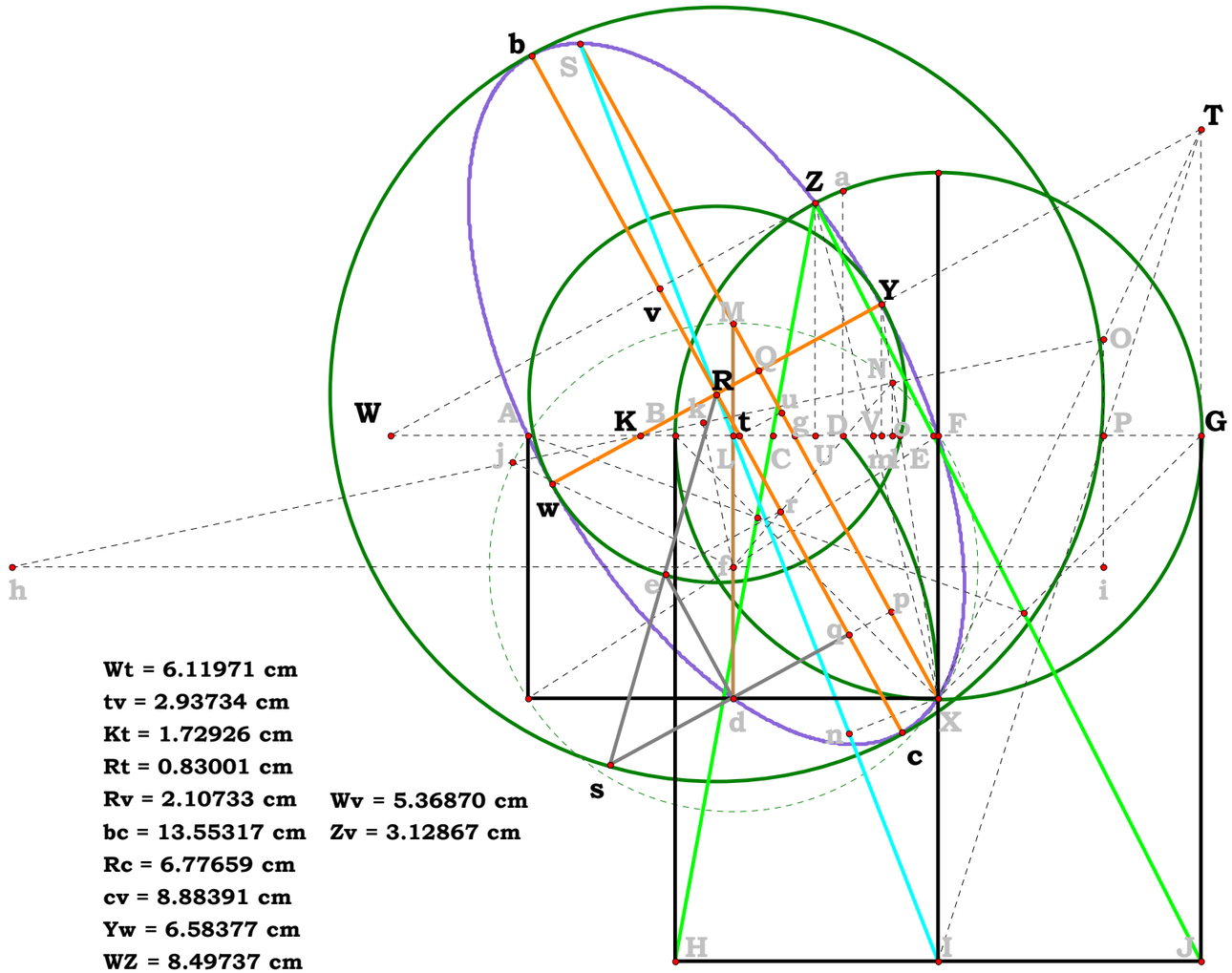
$Yw := 2 \cdot RY$ $\quad WZ := \frac{KT \cdot UZ}{GT}$ $\quad Wv := \frac{GK \cdot tv}{GT}$ $\quad Zv := |WZ - Wv|$

**Definitions.**

$Wt = 6.11971$ $\quad tv = 2.93734$ $\quad Kt = 1.72926$ $\quad Rt = 0.83001$

$Rv = 2.10733$ $\quad bc = 13.55318$ $\quad Rc = 6.77659$ $\quad cv = 8.88391$

$Yw = 6.58377$ $\quad WZ = 8.49737$ $\quad Wv = 5.36870$ $\quad Zv = 3.12867$




Wt = 6.11971 cm
tv = 2.93734 cm
Kt = 1.72926 cm
Rt = 0.83001 cm
Rv = 2.10733 cm
bc = 13.55317 cm
Rc = 6.77659 cm
cv = 8.88391 cm
Yw = 6.58377 cm
WZ = 8.49737 cm

Wv = 5.36870 cm
Zv = 3.12867 cm

**Given.**

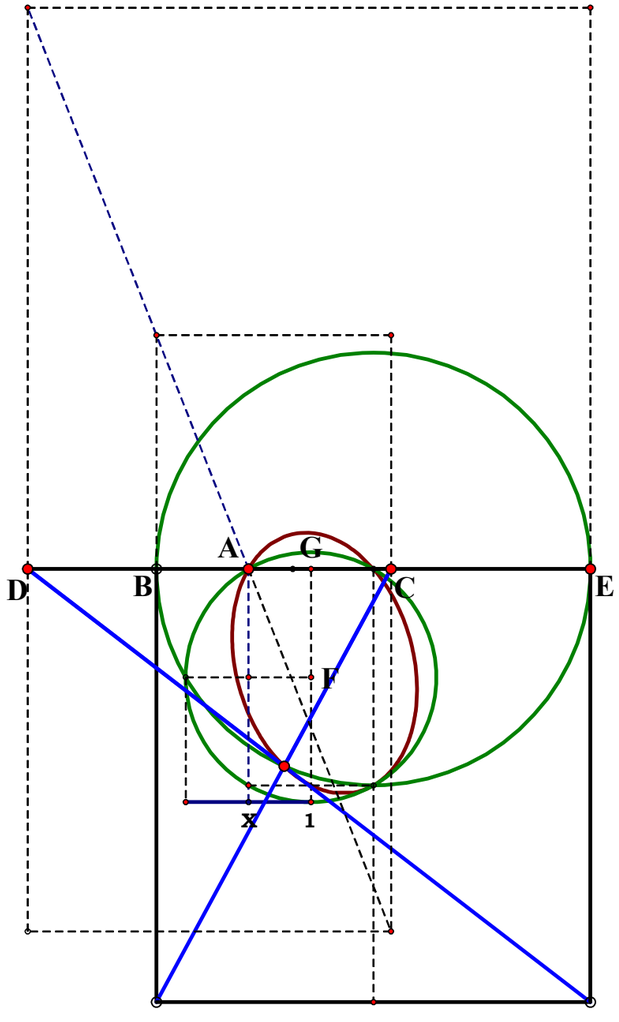
$N_1 := Yw$ $\quad N_2 := bc$ $\quad N_3 := cv$ $\quad N_4 := bc$

**Definitions.**

$\sqrt{N_3 \cdot (N_4 - N_3)} \cdot \frac{N_1}{N_2} - Zv = 0.00000$

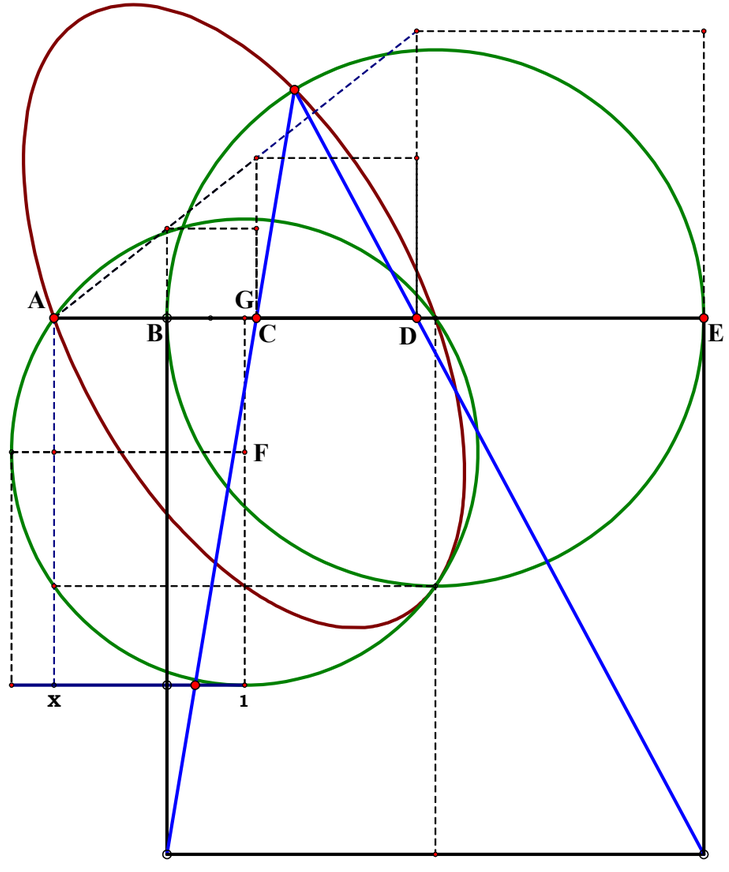
$\sqrt{N_3 \cdot (N_4 - N_3)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

**As one can see, the figure covers the complete range of cubes and the full range of intersection of the ellipse with the major circle.**




Unit = 1.00000
XY = 0.50076
X = 10.01523
Y = 20.00000

AB = 1.54284 cm
AC = 2.39035 cm
AD = 3.70342 cm
AE = 5.73777 cm
$(AB^2 \bullet AE)^{\frac{1}{3}}$ AC = 0.00000
$(AB \bullet AE^2)^{\frac{1}{3}}$ AD = 0.00000
AB = 1.00000
AC = 1.54932
AD = 2.40039
AE = 3.71897
$AE^{\frac{1}{3}}$ AC = 0.00000
$AE^{\frac{2}{3}}$ AD = 0.00000




Unit = 1.00000
XY = 0.18220
X = 3.64391
Y = 20.00000

AB = 2.24895 cm
AC = 4.02907 cm
AD = 7.21822 cm
AE = 12.93169 cm
$(AB^2 \bullet AE)^{\frac{1}{3}}$ AC = 0.00000
$(AB \bullet AE^2)^{\frac{1}{3}}$ AD = 0.00000
AB = 1.00000
AC = 1.79154
AD = 3.20960
AE = 5.75011
$AE^{\frac{1}{3}}$ AC = 0.00000
$AE^{\frac{2}{3}}$ AD = 0.00000

081899

Unit.

Given.

$N_1 := 5.05354$ $AC := N_1$

$N_2 := 14.49917$ $AF := N_2$

$N_3 := 5.71500$ $FN := N_3$

Descriptions.

$CF := AF - AC$ $CG := \frac{FN \cdot AC}{AF}$ $EF := \frac{CF}{2}$ $EH := \frac{FN \cdot (AC + EF)}{AF}$

$KN := \frac{EF \cdot FN}{EH}$ $JK := CF - KN$ $BC := \frac{JK \cdot CG}{FN - CG}$ $BF := BC + CF$

$AD := AC + JK$ $BD := BC + JK$

Definitions.

$CF - (N_2 - N_1) = 0$ $\left(\frac{AF}{AC}\right)^2 - \frac{BF}{BC} = 0$ $\sqrt{\frac{BF}{BC}} - \frac{AF}{AC} = 0$

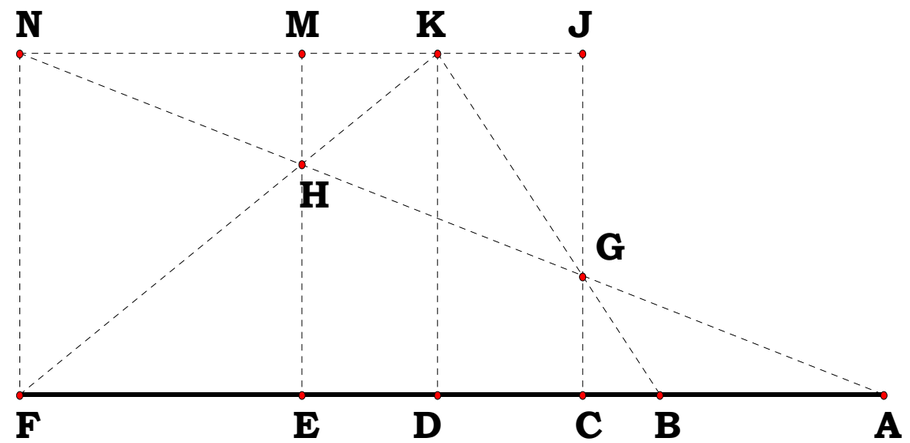
$CG - \frac{N_3 \cdot N_1}{N_2} = 0$ $EF - \frac{N_2 - N_1}{2} = 0$ $EH - \frac{N_3 \cdot (N_1 + N_2)}{2 \cdot N_2} = 0$

$KN - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $JK - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $BC - \frac{N_1^2}{N_1 + N_2} = 0$

$BF := \frac{N_2^2}{N_1 + N_2}$ $AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0$ $BD - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

**Promptly writing this up in 0816 2015**

**Exponential series by changing the unit, in other words, the same way as done inside a circle only this circle is getting smaller.**



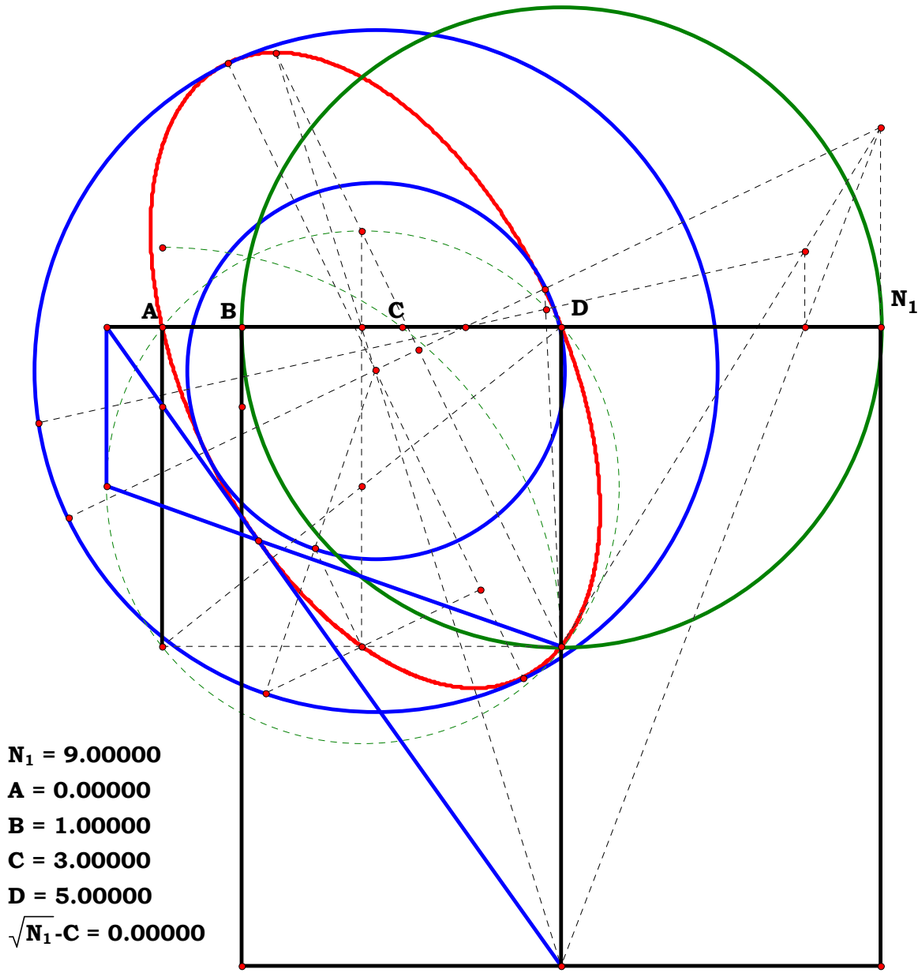
081999

## What about the Names?

**Geometry, Algebra and Arithmetic are each and all binary grammars. All we do when writing up a figure is pair the name for any particular thing or the parts of a thing in terms of each of the convention of names provided by these three systems of grammar to each other. Each of these names are a binary expression, but they each use the convention of names given by a particular grammar. Traditionally, people have approached the topic in terms of precision, which is not right. The distinction is between the perceptible and the intelligible provided by each system of grammar which is expressible by our ability in that grammar system. Grammar cannot, in any wise, change the facts, nor can our ability with a grammar. Stupid people speak of proof as if proof determines the reality, the facts, which is wholly bizzar. All we are doing when pairing names is exercising our ability to do so. Proofing is only our ability to follow the intelligible using perceptible systems of grammar. So, by writing up complex figures one not only pushes their limits, but also learn how to fall back and go on naming in all three conventions. We go from universal expressions to expressions particular to where we are naming.**

**It is quite natural to become wholly frustrated when on reaches their own particular limits and the limits of even the computer that they may be using.**

**What is the name of the Ellipse which gives us the cube roots of any number?**




$N_1$ = 9.00000
A = 0.00000
B = 1.00000
C = 3.00000
D = 5.00000
$\sqrt{N_1}$-C = 0.00000

**Unit.**

$AB := 1$

**Given.**

$N_1 := 9$

**Descriptions.**

$BN_1 := N_1 - AB \qquad BD := \frac{BN_1}{2}$

$DO := BD \qquad AD := BD + AB$

$AC := \sqrt{AD^2 - DO^2}$

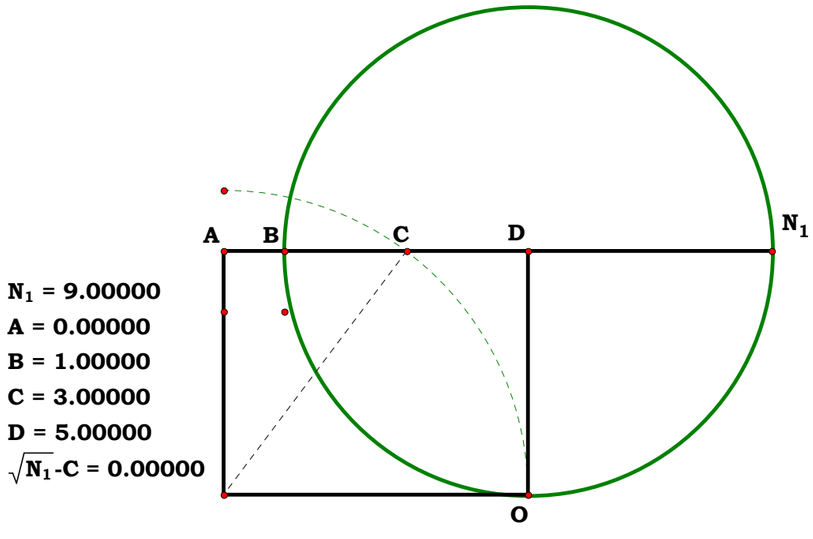
**Definitions.**

$BN_1 - (N_1 - 1) = 0 \qquad BD - \frac{N_1 - 1}{2} = 0$

$DO - \frac{N_1 - 1}{2} = 0 \qquad AD - \frac{N_1 + 1}{2} = 0$

$AC - \sqrt{\left(\frac{N_1 + 1}{2}\right)^2 - \left(\frac{N_1 - 1}{2}\right)^2} = 0$

$AC - \sqrt{N_1} = 0$




$N_1$ = 9.00000
A = 0.00000
B = 1.00000
C = 3.00000
D = 5.00000
$\sqrt{N_1}$-C = 0.00000

**AC is the square root of $N_1$**

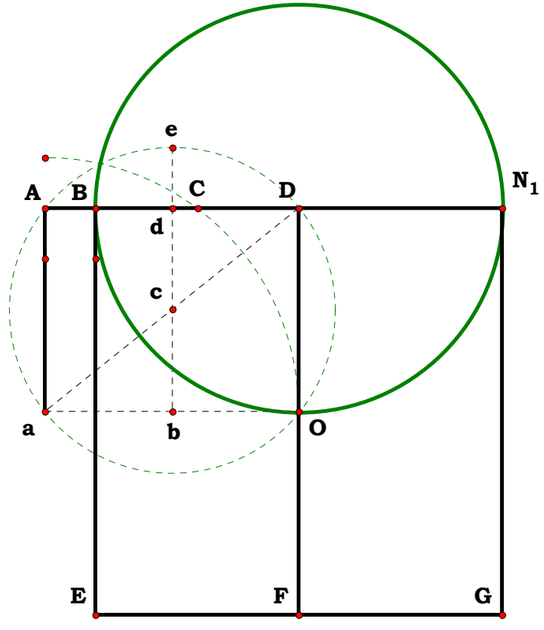
**Descriptions.**

$Dd := \frac{AD}{2} \qquad Da := \sqrt{AD^2 + DO^2} \qquad ce := \frac{Da}{2} \qquad de := ce - \frac{BD}{2}$

**Definitions.**

$Dd - \frac{N_1 + 1}{4} = 0 \qquad Da - \frac{\sqrt{N_1^2 + 1}}{\sqrt{2}} = 0 \qquad ce - \frac{\sqrt{2 \cdot (N_1^2 + 1)}}{4} = 0$

$de - \frac{\sqrt{2 \cdot (N_1^2 + 1)} - N_1 + 1}{4} = 0$




$N_1$ = 9.00000

Dd = 2.50000 $\frac{N_1+1}{4}$-Dd = 0.00000

Da = 6.40312 $\frac{\sqrt{N_1^2+1}}{\sqrt{2}}$-Da = 0.00000

ce = 3.20156 $\frac{\sqrt{2\cdot(N_1^2+1)}}{4}$-ce = 0.00000

de = 1.20156 $\frac{(\sqrt{2\cdot(N_1^2+1)}-N_1)+1}{4}$-de = 0.00000

**Descriptions.**

$DF := BN_1$ $\quad Fd := \sqrt{Dd^2 + DF^2}$

$Df := DO + de$ $\quad df := \sqrt{Dd^2 + Df^2}$ $\quad Ff := DF - Df$

$Og := \dfrac{df \cdot DO}{Ff}$

**Definitions.**

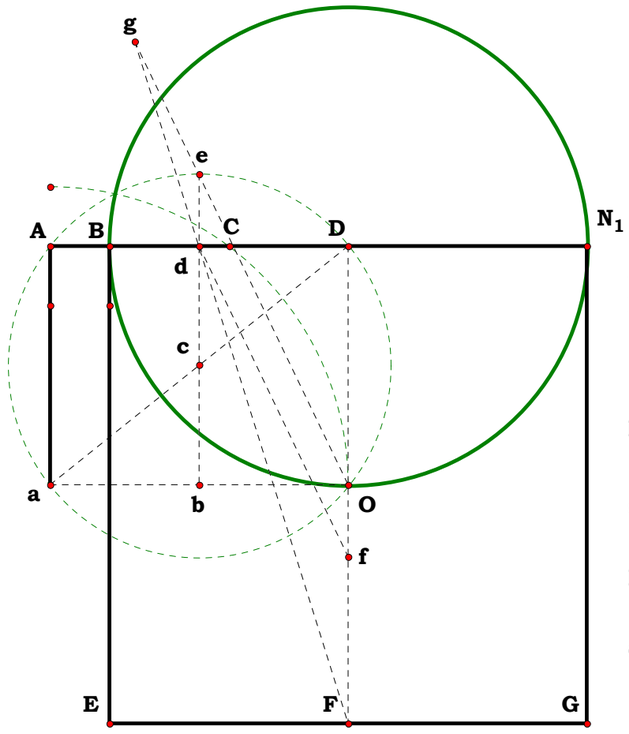
$$DF - (N_1 - 1) = 0 \qquad Fd - \frac{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}}{4} = 0$$

$$Df - \frac{N_1 + \sqrt{2 \cdot (N_1^2 + 1)} - 1}{4} = 0$$

$$df - \frac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1) + 2}}{\sqrt{8}} = 0$$

$$Ff - \frac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3}{4} = 0$$

$$Og - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)} + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{2 \cdot \left[3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3\right]} = 0$$




$N_1 = 9.00000$

$DF = 8.00000$ $\quad N_1 - 1 = 8.00000$

$Fd = 8.38153$ $\quad \dfrac{\sqrt{(17 \cdot N_1^2 - 30 \cdot N_1) + 17}}{4} - Fd = 0.00000$

$Df = 5.20156$ $\quad \dfrac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1) + 2}}{\sqrt{8}} - df = 0.00000$

$df = 5.77116$ $\quad \dfrac{(N_1 + \sqrt{2 \cdot (N_1^2 + 1)}) - 1}{4} - Df = 0.00000$

$Ff = 2.79844$ $\quad \dfrac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3}{4} - Ff = 0.00000$

$Og = 8.24911$ $\quad \dfrac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{(2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)}) + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{2 \cdot (3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3)} - Og = 0.00000$

**Descriptions.**

$$OI := \frac{Og}{2} \qquad FO := DO \qquad Fh := \frac{DF \cdot FO}{Fd}$$

$$Oh := \sqrt{FO^2 - Fh^2} \qquad gh := \sqrt{Og^2 - Oh^2}$$

$$IJ := \frac{Oh \cdot OI}{gh}$$

**Definitions.**

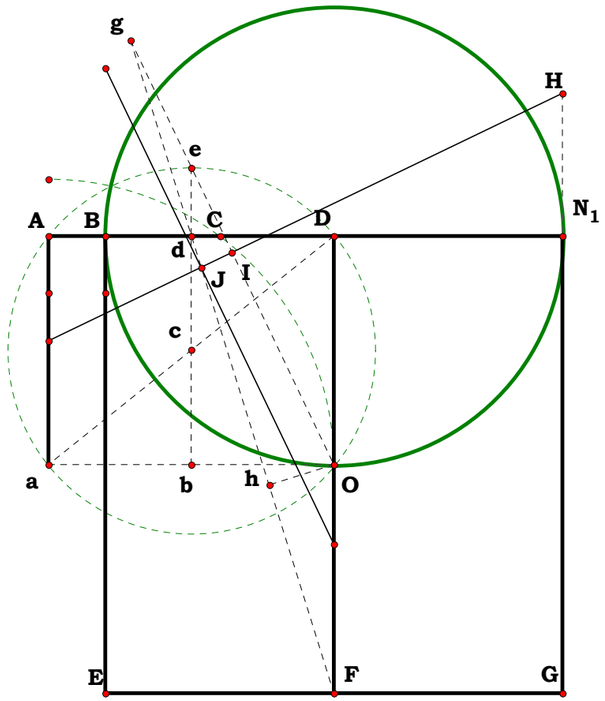
$$FO - \frac{N_1 - 1}{2} = 0 \qquad Fh - \frac{2 \cdot (N_1 - 1)^2}{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$Oh - \frac{\sqrt{(N_1^2 - 1)^2}}{2 \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$OI - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)} + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{4 \cdot \left[3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3\right]} = 0$$

$$gh - \frac{\sqrt{(N_1 - 1)^2 \cdot \left[\left[8 \cdot (N_1 - 1) \cdot (5 \cdot N_1^2 - 6 \cdot N_1 + 5)\right] \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 57 \cdot (N_1^4 + 1) + 150 \cdot N_1^2 - 124 \cdot N_1^3 - 124 \cdot N_1\right]}}{\sqrt{\left[4 \cdot (17 \cdot N_1^2 - 30 \cdot N_1 + 17) \cdot \left[\left[11 \cdot (N_1^2 + 1) - 18 \cdot N_1\right] - 6 \cdot \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1)\right]\right]}} = 0$$

$$IJ - \frac{\sqrt{2} \cdot \sqrt{-(68 \cdot N_1^2 - 120 \cdot N_1 + 68)} \cdot \left[18 \cdot N_1 - 11 \cdot N_1^2 + 6 \cdot \sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{N_1^2 + 1} - 11\right] \cdot \sqrt{(N_1^2 - 1)^2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} + \sqrt{2} \cdot N_1 \cdot \sqrt{N_1^2 + 1} + 2}}{8 \cdot \sqrt{(N_1 - 1)^2 \cdot \left[150 \cdot N_1^2 - 124 \cdot N_1 - 124 \cdot N_1^3 + 57 \cdot N_1^4 + \sqrt{2} \cdot (8 \cdot N_1 - 8) \cdot \sqrt{N_1^2 + 1} \cdot (5 \cdot N_1^2 - 6 \cdot N_1 + 5) + 57\right] \cdot \left(3 \cdot N_1 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} - 3\right) \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}}} = 0$$




$N_1 = 9.00000$

$\frac{OI}{AB} = 4.12456$

$\frac{Fh}{AB} = 3.81792$

$\frac{gh}{AB} = 8.16237$

$\frac{FO}{AB} = 4.00000$

$\frac{Oh}{AB} = 1.19310$

$\frac{IJ}{AB} = 0.60289$

**Definitions (Arithmetic).**

| | |
|---|---|
| $OI = 4.124556$ | $FO = 4$ |
| $Fh = 3.81792$ | $Oh = 1.1931$ |
| $gh = 8.162374$ | $IJ = 0.602889$ |

**At this point of complexity, I have to set the definitions aside and just compare the arithmetic. I really do not have all the time there is. This is the major reason I set this write-up aside for a later date, one which is after mine. As we find in the name of Shadows (Babylon 5,) a definition could become over 10,000 symbols long.**

$Oj := \frac{Df \cdot OI}{df}$ $Ij := \frac{Dd \cdot Oj}{Df}$ $Ik := DO + Ij$

$Hk := \frac{Dd \cdot Ik}{Df}$ $Dj := DO - Oj$ $HN_1 := Hk - Dj$

$HI := \frac{df \cdot Ik}{Df}$ $HJ := HI + IJ$ $lm := \frac{DO \cdot HN_1}{DF + HN_1}$

$nN_1 := \frac{Df \cdot HN_1}{Dd}$

**Definitions (Arithmetic).**

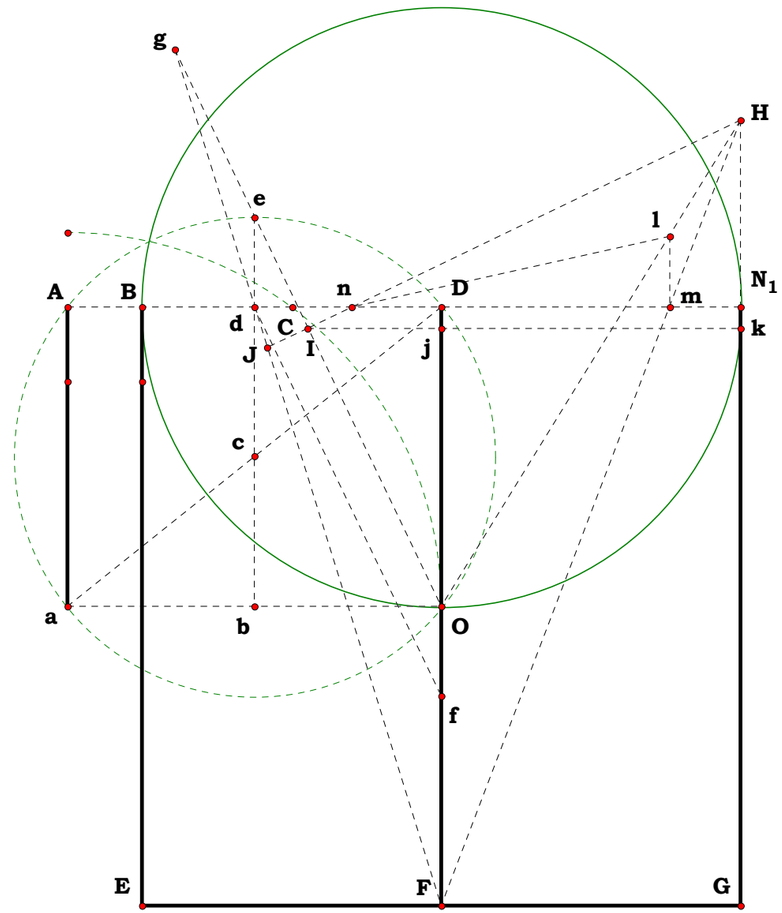
$Oj = 3.717475$ $Ij = 1.786711$

$Ik = 5.786711$ $Hk = 2.781237$

$Dj = 0.282525$ $HN_1 = 2.498713$

$HI = 6.420382$ $HJ = 7.023271$

$lm = 0.952007$ $nN_1 = 5.198884$




$N_1 = 9.00000$

$AB = 1.25400$ cm

$\frac{Oj}{AB} = 3.71748$ $\frac{Ij}{AB} = 1.78671$

$\frac{Ik}{AB} = 5.78671$ $\frac{Hk}{AB} = 2.78124$

$\frac{Dj}{AB} = 0.28252$ $\frac{HN_1}{AB} = 2.49871$

$\frac{HI}{AB} = 6.42038$ $\frac{HJ}{AB} = 7.02327$

$\frac{ml}{AB} = 0.95201$ $\frac{nN_1}{AB} = 5.19888$

**082999A**

**The first section of this is simply demonstrating that cube roots are producable as a ratio between BE and BO. In fact, using whole numbers, and a lot of time, one can actually construct an Arithmetic dictionary of such ratios.**

**Then sometime I am want to I show that AW passes through R, Y, S, V, S and T.**

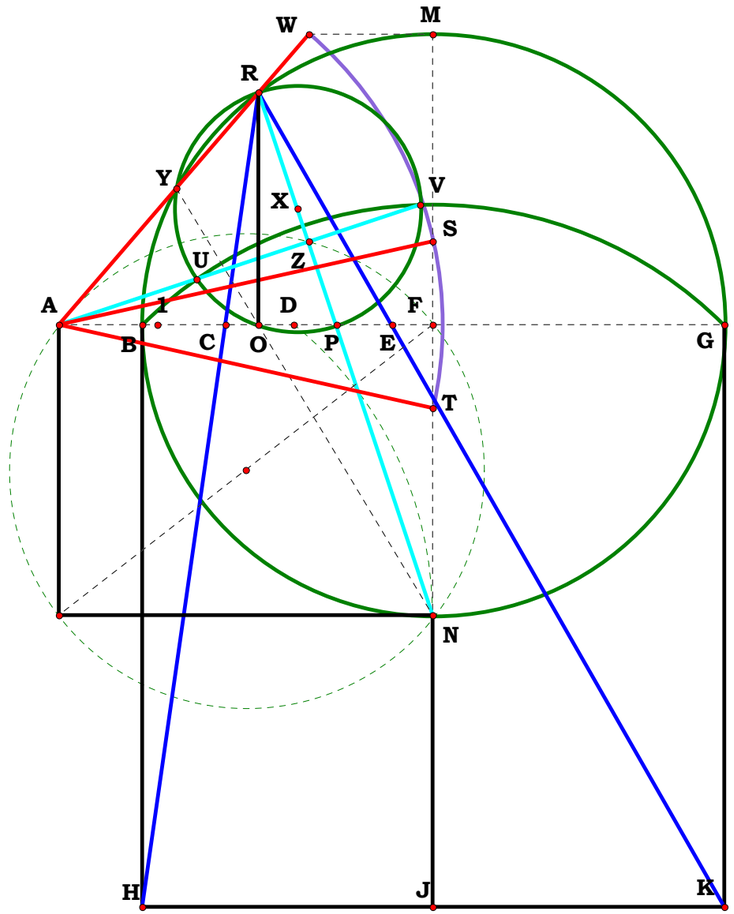
**UV originates at A.**

**and;**

**The cord ST is equal to CE.**

**The circle RX pass through R, Y, O, P and V.**

**See E Go!**



**Unit.**

$BG := 1$

**Given.**

$N_1 := 2$

$N_2 := 5$

**Descriptions.**

$BF := \frac{BG}{2}$ $BO := \frac{N_1 \cdot BF}{N_2}$ $GO := BG - BO$ $OR := \sqrt{BO \cdot GO}$

$FN := BF$ $FO := BF - BO$ $FP := \frac{FO \cdot FN}{FN + OR}$ $GK := BG$ $BH := BG$

$BC := \frac{BO \cdot BH}{BH + OR}$ $GE := \frac{GO \cdot GK}{GK + OR}$ $CE := BG - (BC + GE)$

$BE := BC + CE$ $AB := \frac{BC^2}{CE - BC}$ $AG := AB + BG$ $\frac{AG}{AB} = 8$

**Definitions.**

$BF - \frac{1}{2} = 0$ $BO - \frac{N_1}{2 \cdot N_2} = 0$ $GO - \frac{2 \cdot N_2 - N_1}{2 \cdot N_2} = 0$

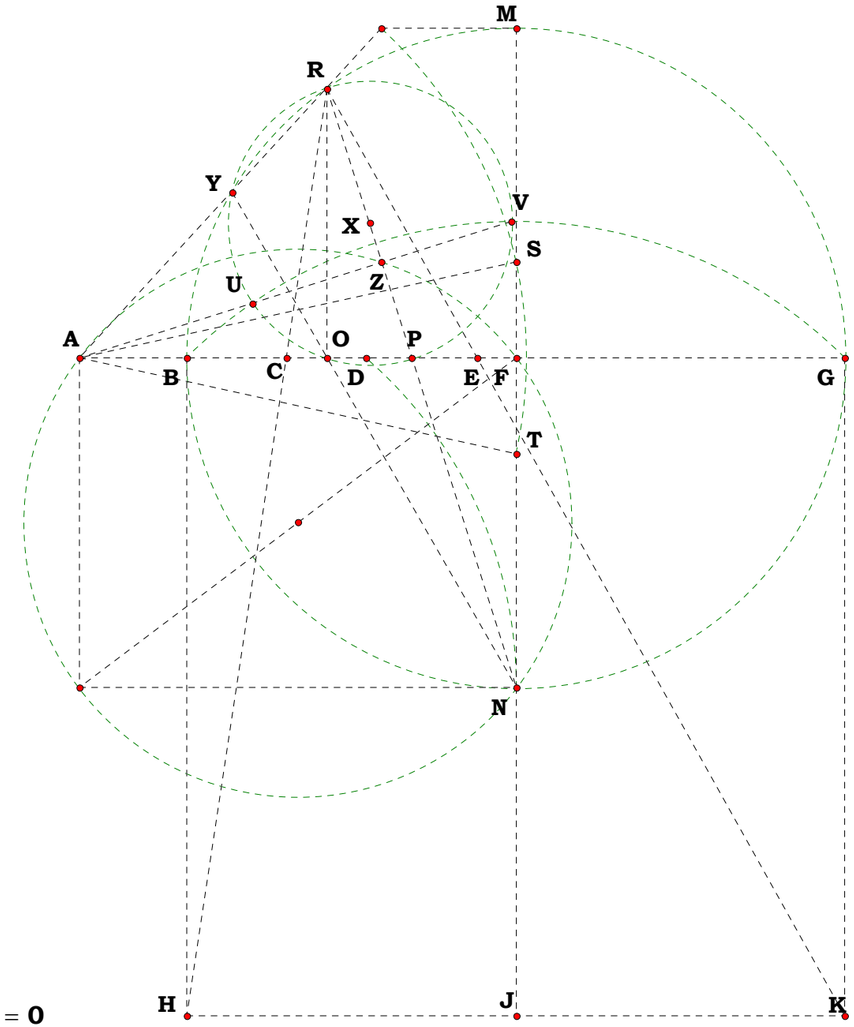
$OR - \frac{\sqrt{N_1 \cdot (2 \cdot N_2 - N_1)}}{2 \cdot N_2} = 0$ $FN - \frac{1}{2} = 0$ $FO - \frac{N_2 - N_1}{2 \cdot N_2} = 0$

$FP - \frac{N_2 - N_1}{2 \cdot \left(N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right)} = 0$ $GK - 1 = 0$ $BH - 1 = 0$

$BC - \frac{N_1}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $GE - \frac{(N_1 - 2 \cdot N_2) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2\right)}{N_1^2 - 2 \cdot N_1 \cdot N_2 + 4 \cdot N_2^2} = 0$

$CE - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $BE - \frac{N_1 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $AB - \frac{N_1^2}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - N_1\right)} = 0$

$AG - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} \cdot (N_1 - 2 \cdot N_2)}{N_1^2 + N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$

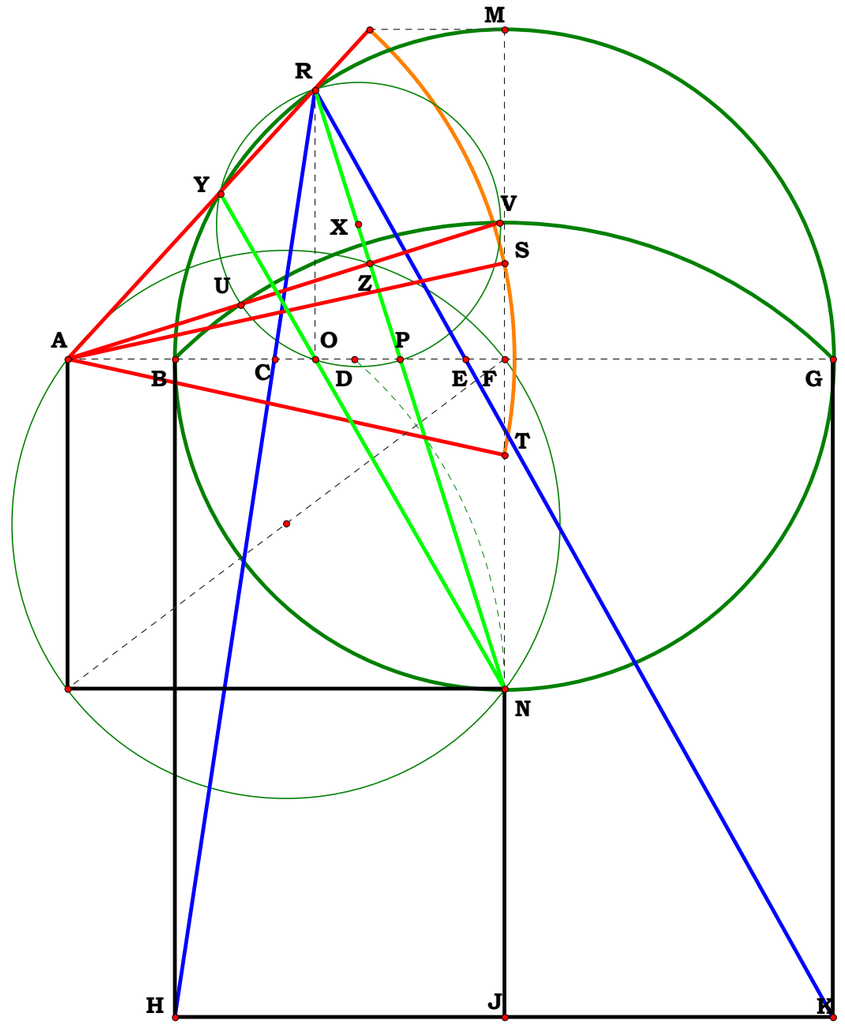


$$AC := AB + BC$$
$$AE := AB + BE$$

$$\left(\frac{AG}{AB}\right)^{\left(\frac{1}{3}\right)} - \frac{AC}{AB} = 0 \qquad \left(\frac{AG}{AB}\right)^{\left(\frac{2}{3}\right)} - \frac{AE}{AB} = 0$$

**I find it very strange that so called mathematicians claim that one cannot abstract cube roots in geomety when every grammar is a binary expression and, since cube roots is simply a two dimensional ratio. I grant that the process to most is complicated, however, complicated and impossible are not the same concept.**

**I have, by no means, finished the write-up of this figure as I want to find the equations to the remaining structures pointed out in the opening graphic, however, I am still in the early stages of this revision and may come back to it at some later date.**




$$N_1 := 2 \qquad N_2 := 5$$

$$AB - \frac{N_1^{\,2}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}\right) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1\right)} = 0$$

$$AC - \frac{N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}}{\left(2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1^{\,2}\right)} = 0$$

$$AE - \frac{N_1 \cdot \left(N_1 - 2 \cdot N_2\right)}{N_1^{\,2} + N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}} = 0$$

083099A

## Comming Through the Front Door.

**One can prove the Solution to Cube Roots with this figure in a manner suggested as far back as the early Greeks, start with it the figure as proven, and work backward.**

Unit. $By := 1$

Given. $y := 20$

$x := 7$

**Descriptions.**

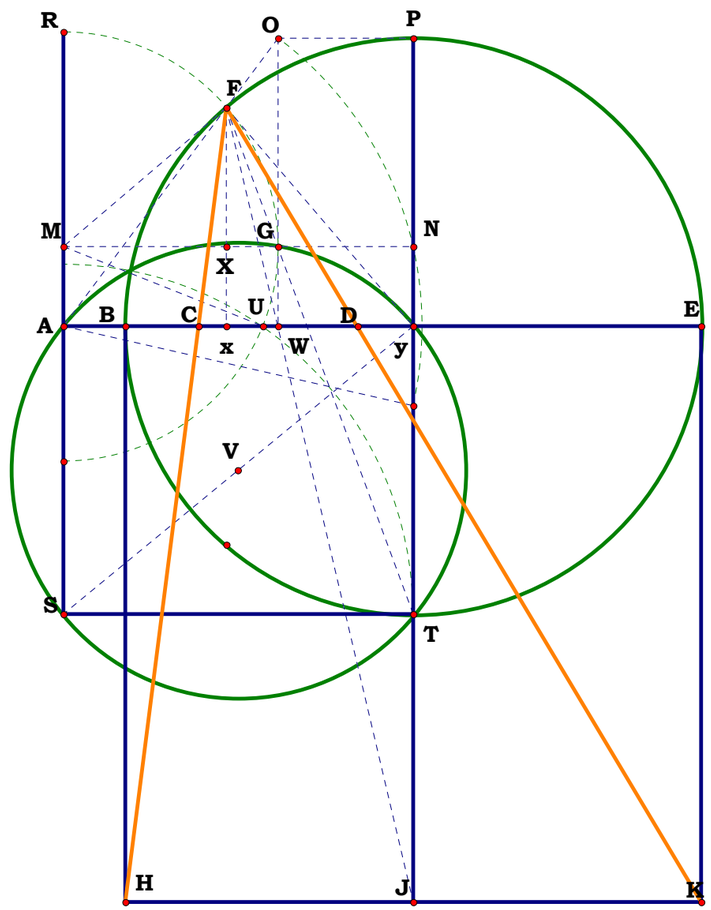
$Bx := By \cdot \frac{x}{y}$ $BE := 2 \cdot By$ $Fx := \sqrt{Bx \cdot (BE - Bx)}$ $JT := By$ $GW := \frac{Fx \cdot By}{(Fx + BE)}$

$Xx := GW$ $xy := By - Bx$ $FX := Fx - Xx$ $MX := \frac{FX \cdot Fx}{xy}$ $AB := MX - Bx$

$BC := \frac{Bx \cdot BE}{Fx + BE}$ $Ex := BE - Bx$ $DE := \frac{Ex \cdot BE}{Fx + BE}$ $AE := AB + BE$ $AC := BC + AB$

$AD := AE - DE$ $\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$$\frac{AE}{AB} - \frac{(x - 2 \cdot y) \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{x \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$$




x = 7.00000
y = 20.00000
AB = 1.04618 cm
AC = 2.27150 cm
AD = 4.93198 cm
AE = 10.70851 cm
$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$
$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

**Descriptions.**

$By - 1 = 0$ $Bx - \frac{x}{y} = 0$ $BE - 2 = 0$ $Fx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{y} = 0$ $JT - 1 = 0$

$GW - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y + \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $Xx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y + \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $xy - \frac{(y - x)}{y} = 0$

$FX - \frac{y \cdot \sqrt{2 \cdot x \cdot y - x^2} - x^2 + 2 \cdot x \cdot y}{y \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $MX - \frac{\sqrt{x \cdot (2 \cdot y - x)} \cdot \left(x^2 - y \cdot \sqrt{2 \cdot x \cdot y - x^2} - 2 \cdot x \cdot y\right)}{y \cdot (x - y) \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $AB - \left[\frac{x \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (y - x)}\right] = 0$ $BC - \frac{2 \cdot x}{2 \cdot y + \sqrt{-x \cdot (x - 2 \cdot y)}} = 0$

$Ex - \frac{(2 \cdot y - x)}{y} = 0$ $DE - \frac{2 \cdot (2 \cdot y - x)}{2 \cdot y + \sqrt{-x \cdot (x - 2 \cdot y)}} = 0$ $AE - \frac{(2 \cdot y - x) \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{(x - y) \cdot \left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $AC := \frac{x \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}$ $AD - \left[\frac{(x - 2 \cdot y) \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}\right] = 0$

083099B

# Comming Through the Front Door.

Unit. $By := 1$

Given. $y := 20$ $x := 10$

**Descriptions.**

$Bx := By \cdot \frac{x}{y}$ $BE := 2 \cdot By$ $Fx := \sqrt{Bx \cdot (BE - Bx)}$ $JT := By$ $GW := \frac{Fx \cdot By}{(BE - Fx)}$

$Xx := GW$ $xy := By - Bx$ $FX := Fx - Xx$ $MX := \frac{FX \cdot Fx}{xy}$ $AB := Bx - MX$

$BC := \frac{Bx \cdot BE}{BE - Fx}$ $Ex := BE - Bx$ $DE := \frac{Ex \cdot BE}{BE - Fx}$ $AE := BE - AB$ $AC := BC - AB$

$AD := DE - AE$ $\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$$\frac{AE}{AB} - \frac{(x - 2 \cdot y) \cdot \left(x - 2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)}{x \cdot \left(x + \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$$

**One can see that the final equations are identical.**

**Descriptions.**

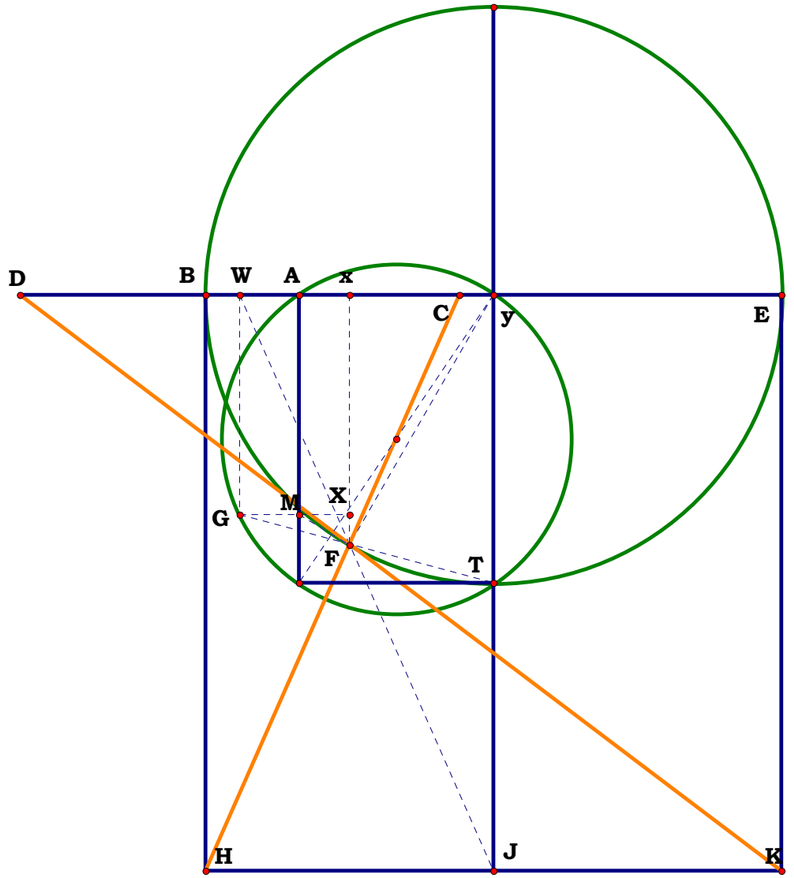
$By - 1 = 0$ $Bx - \frac{x}{y} = 0$ $BE - 2 = 0$ $Fx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{y} = 0$ $JT - 1 = 0$

$GW - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y - \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $Xx - \frac{\sqrt{x \cdot (2 \cdot y - x)}}{2 \cdot y - \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $xy - \frac{y - x}{y} = 0$

$FX - \frac{y \cdot \sqrt{2 \cdot x \cdot y - x^2} + x^2 - 2 \cdot x \cdot y}{y \cdot \left(2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right)} = 0$ $MX - \frac{\sqrt{x \cdot (2 \cdot y - x)} \cdot \left(y \cdot \sqrt{2 \cdot x \cdot y - x^2} + x^2 - 2 \cdot x \cdot y\right)}{y \cdot (x - y) \cdot \left(\sqrt{2 \cdot x \cdot y - x^2} - 2 \cdot y\right)} = 0$ $AB - \frac{x \cdot \left(x - \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)} = 0$

$BC - \frac{2 \cdot x}{2 \cdot y - \sqrt{-x \cdot (x - 2 \cdot y)}} = 0$ $Ex - \frac{(2 \cdot y - x)}{y} = 0$ $DE - \frac{2 \cdot (2 \cdot y - x)}{2 \cdot y - \sqrt{x \cdot (2 \cdot y - x)}} = 0$ $AE - \left[2 - \frac{x \cdot \left(x - \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}\right] = 0$ $AC - \left[\frac{x \cdot \left(x - 2 \cdot y + \sqrt{2 \cdot x \cdot y - x^2}\right)}{\left(2 \cdot y - \sqrt{2 \cdot x \cdot y - x^2}\right) \cdot (x - y)}\right] = 0$

$AD := DE - AE$ $\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$




XY = 0.50000
X = 10.00000
Y = 20.00000

AB = 1.55941 cm
AC = 2.70097 cm
AD = 4.67823 cm
AE = 8.10292 cm
$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$
$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$
By = 4.83117 cm

101799

Unit. $CD := 1$

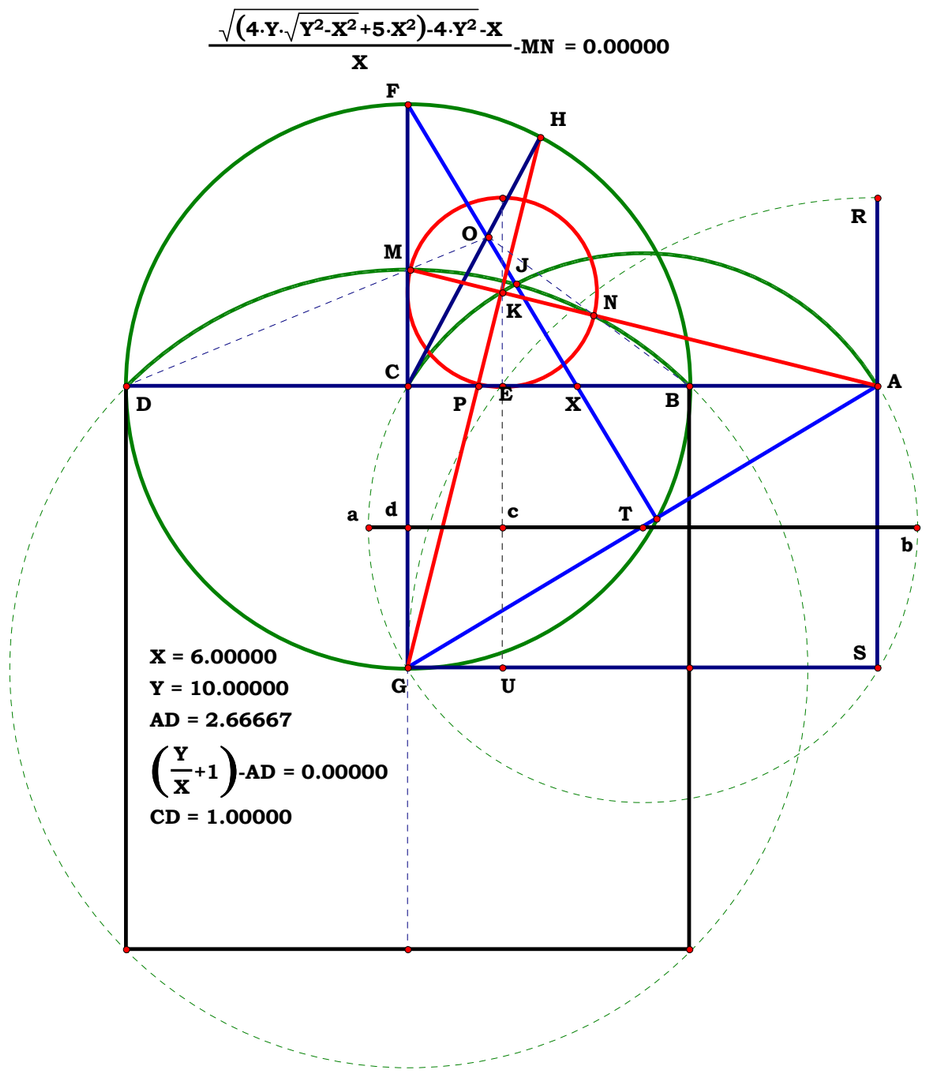
Given.

$X := 6$

$Y := 10$

## Name that Circle

What is MN?




**Descriptions.**

$BC := CD \quad BD := 2\cdot CD \quad CX := BC\cdot\frac{X}{Y} \quad AD := \frac{Y}{X}+1 \quad AC := AD - CD$

$CG := BC \quad AG := \sqrt{AC^2+CG^2} \quad AB := AD - BD \quad AE := \sqrt{AB\cdot AD}$

$CE := AC - AE \quad ab := AG \quad ad := \frac{ab-AC}{2} \quad ac := CE + ad$

$cK := \sqrt{ac\cdot(ab-ac)} \quad EK := cK - \frac{CG}{2}$

$MN := 2\cdot EK$ etc.

**Definitions.**

$BC - 1 = 0 \quad BD - 2 = 0 \quad CX - \frac{X}{Y} = 0 \quad AD - \frac{X+Y}{X} = 0$

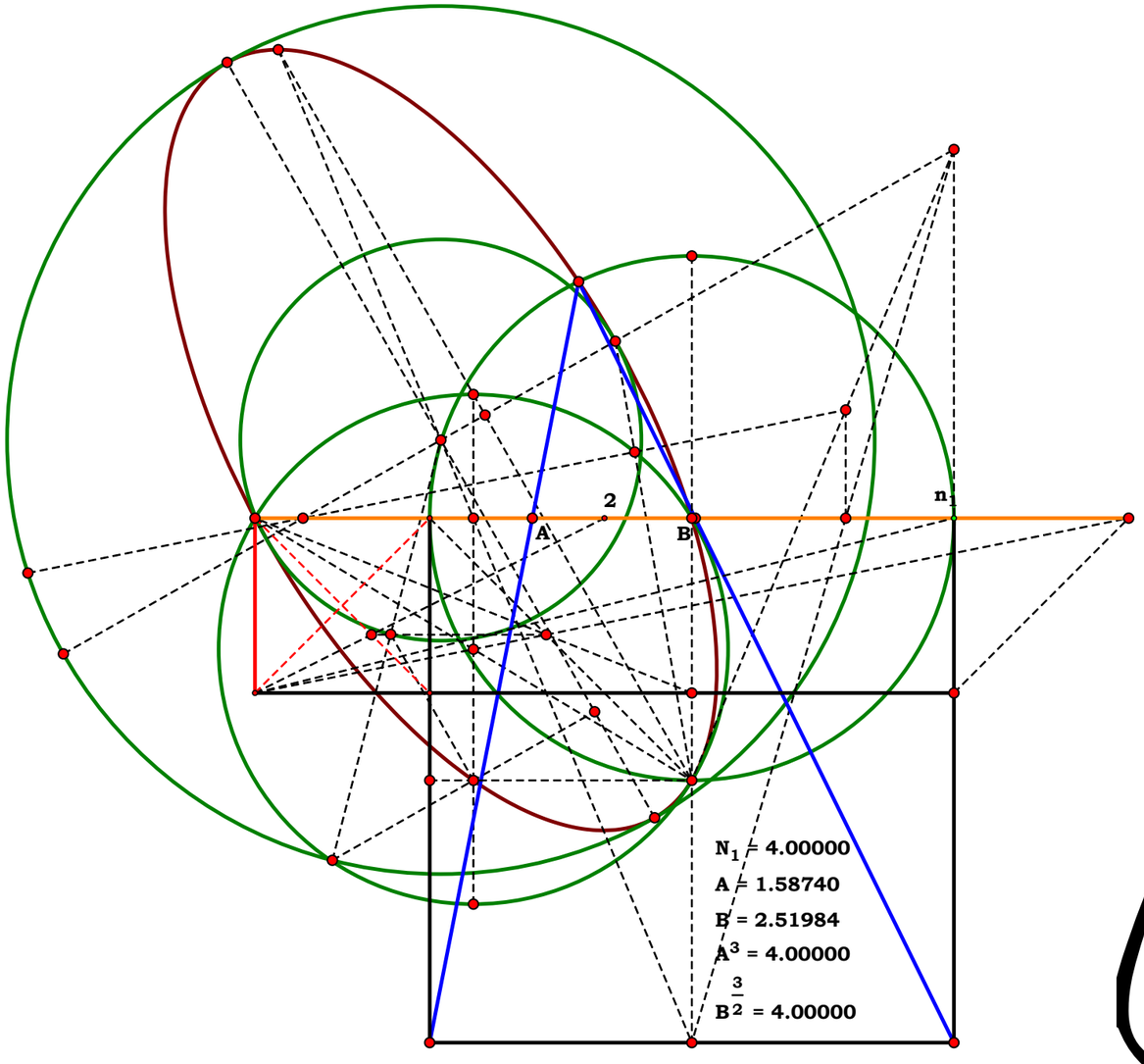
$AC - \frac{Y}{X} = 0 \quad CG - 1 = 0 \quad AG - \frac{\sqrt{X^2+Y^2}}{X} = 0 \quad AB - \frac{Y-X}{X} = 0$

$AE - \frac{\sqrt{(Y-X)\cdot(X+Y)}}{X} = 0 \quad CE - \frac{Y-\sqrt{Y^2-X^2}}{X} = 0$

$ab - \frac{\sqrt{X^2+Y^2}}{X} = 0 \quad ad - \frac{\sqrt{X^2+Y^2}-Y}{2\cdot X} = 0 \quad ac - \frac{Y+\sqrt{X^2+Y^2}-2\cdot\sqrt{Y^2-X^2}}{2\cdot X} = 0$

$cK - \frac{\sqrt{\left(Y+\sqrt{X^2+Y^2}-2\cdot\sqrt{Y^2-X^2}\right)\cdot\left(\sqrt{X^2+Y^2}-Y+2\cdot\sqrt{Y^2-X^2}\right)}}{2\cdot X} = 0 \quad EK - \frac{\sqrt{4\cdot Y\cdot\sqrt{Y^2-X^2}+5\cdot X^2-4\cdot Y^2}-X}{2\cdot X} = 0$

$MN - \frac{\sqrt{4\cdot Y\cdot\sqrt{Y^2-X^2}+5\cdot X^2-4\cdot Y^2}-X}{X} = 0$ etc.




# The Delian Quest 2000

**John Clark**

070200

**Unit.**

$AC := 1$

**Given.**

$N_1 := 4$

$N_2 := 5$ $\delta := 1 .. N_2$

**Descriptions.**

$AB := \frac{AC}{N_1}$ $BC := AC - AB$ $BD := \sqrt{AB \cdot BC}$ $BE_\delta := \frac{BD \cdot \delta}{N_2}$ $CE_\delta := \sqrt{(BE_\delta)^2 + BC^2}$

$CG_\delta := \frac{BC \cdot AC}{CE_\delta}$ $AD := \sqrt{AB^2 + BD^2}$ $AG_\delta := \frac{BE_\delta \cdot AC}{CE_\delta}$ $EG_\delta := CG_\delta - CE_\delta$ $EH_\delta := \frac{BE_\delta \cdot EG_\delta}{CE_\delta}$

$GH_\delta := \frac{BC \cdot EH_\delta}{BE_\delta}$ $BH_\delta := BE_\delta + EH_\delta$ $DH_\delta := BD - BH_\delta$ $DG_\delta := \sqrt{(GH_\delta)^2 + (DH_\delta)^2}$

$BK_\delta := \frac{AB \cdot BE_\delta}{BD}$ $CK_\delta := BC + BK_\delta$ $CJ_\delta := \frac{CE_\delta \cdot AC}{CK_\delta}$ $EJ_\delta := CJ_\delta - CE_\delta$

**Definitions.**

$AG_\delta =$

| |
|---|
| 0.114708 |
| 0.225018 |
| 0.327327 |
| 0.419314 |
| 0.5 |

$\frac{\delta}{\sqrt{\delta^2 + N_1 \cdot N_2^2 - N_2^2}} =$

| |
|---|
| 0.114708 |
| 0.225018 |
| 0.327327 |
| 0.419314 |
| 0.5 |

$DG_\delta =$

| |
|---|
| 0.39736 |
| 0.292306 |
| 0.188982 |
| 0.090784 |
| 0 |

$(N_2 - \delta) \cdot \sqrt{\frac{(N_1 - 1)}{\left[N_1 \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)\right]}} =$

| |
|---|
| 0.39736 |
| 0.292306 |
| 0.188982 |
| 0.090784 |
| 0 |

$EG_\delta =$

| |
|---|
| 0.238416 |
| 0.204614 |
| 0.151186 |
| 0.081706 |
| 0 |

$\frac{(N_2^2 - \delta^2) \cdot \sqrt{N_1 - 1}}{N_2 \cdot N_1 \cdot \sqrt{\delta^2 + N_2^2 \cdot N_1 - N_2^2}} =$

| |
|---|
| 0.238416 |
| 0.204614 |
| 0.151186 |
| 0.081706 |
| 0 |

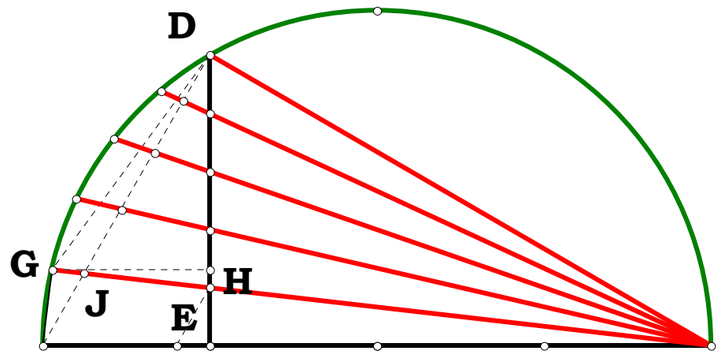
$CG_\delta =$

| |
|---|
| 0.993399 |
| 0.974355 |
| 0.944911 |
| 0.907841 |
| 0.866025 |

$N_2 \cdot \frac{(N_1 - 1)}{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}} =$

| |
|---|
| 0.993399 |
| 0.974355 |
| 0.944911 |
| 0.907841 |
| 0.866025 |

**In process. POR something or other.**



$BE_\delta =$

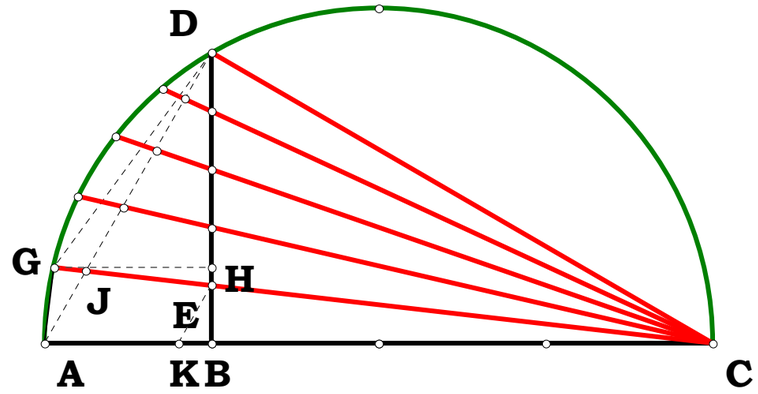
| |
|---|
| 0.086603 |
| 0.173205 |
| 0.259808 |
| 0.34641 |
| 0.433013 |

$\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2} = EJ_\delta =$

| | |
|---|---|
| 0.086603 | 0.188746 |
| 0.173205 | 0.135837 |
| 0.259808 | 0.088192 |
| 0.34641 | 0.043481 |
| 0.433013 | 0 |

$\frac{(N_2 - \delta) \cdot \sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{N_2 \cdot N_1 \cdot (N_2 \cdot N_1 - N_2 + \delta)} =$

| |
|---|
| 0.188746 |
| 0.135837 |
| 0.088192 |
| 0.043481 |
| 0 |




$AD = 0.5 \quad \sqrt{\frac{1}{N_1}} = 0.5$

$CJ_\delta =$

| |
|---|
| 0.943729 |
| 0.905577 |
| 0.881917 |
| 0.869616 |
| 0.866025 |

$\frac{\sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{N_2 \cdot N_1 - N_2 + \delta}$

| |
|---|
| 0.943729 |
| 0.905577 |
| 0.881917 |
| 0.869616 |
| 0.866025 |

$\frac{AD}{AG_\delta} =$

| |
|---|
| 4.358899 |
| 2.222049 |
| 1.527525 |
| 1.192424 |
| 1 |

$\frac{\sqrt{\frac{(\delta^2 + N_2^2 \cdot N_1 - N_2^2)}{N_1}}}{\delta} =$

| |
|---|
| 4.358899 |
| 2.222049 |
| 1.527525 |
| 1.192424 |
| 1 |

$\frac{CG_\delta}{EG_\delta} =$

| |
|---|
| 4.166667 |
| 4.761905 |
| 6.25 |
| 11.111111 |
| $7.800463 \cdot 10^{15}$ |

$\text{if}\left(N_2^2 - \delta^2, \frac{N_2^2 \cdot N_1}{N_2^2 - \delta^2}, 0\right) =$

| |
|---|
| 4.166667 |
| 4.761905 |
| 6.25 |
| 11.111111 |
| 0 |

$\text{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right) =$

| |
|---|
| 5 |
| 6.666667 |
| 10 |
| 20 |
| 0 |

$\text{if}\left(EJ_\delta, \frac{N_2 \cdot N_1}{N_2 - \delta}, 0\right) =$

| |
|---|
| 5 |
| 6.666667 |
| 10 |
| 20 |
| 0 |

070900

**Unit.**

**Given.**

$N_1 := 1.79201$ $AB := N_1$

$N_2 := 10.41743$ $AG := N_2$

**Descriptions.**

# Alternate Method Quad Roots

$AD := \sqrt{AB \cdot AG}$

$BD := AD - AB$ $BG := AG - AB$

$DG := BG - BD$ $DM := \sqrt{BD \cdot DG}$

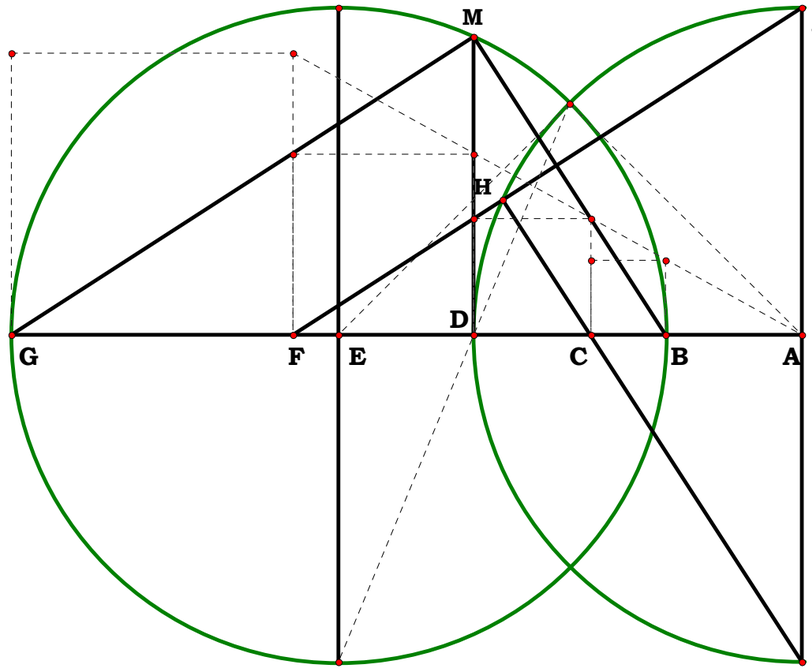
$AJ := AD$ $AK := AD$ $BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2}$ $AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

**Definitions.**

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$ $\left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$




AB = 2.27795 cm
AC = 3.53711 cm
AD = 5.49228 cm
AF = 8.52821 cm
AG = 13.24228 cm
AB = 1.00000
AC = 1.55276
AD = 2.41107
AF = 3.74382
AG = 5.81325

$AG^{\frac{3}{4}}$-AF = 0.00000

$AG^{\frac{2}{4}}$-AD = 0.00000

$AG^{\frac{1}{4}}$-AC = 0.00000

$AG^{\frac{0}{4}}$-AB = 0.00000

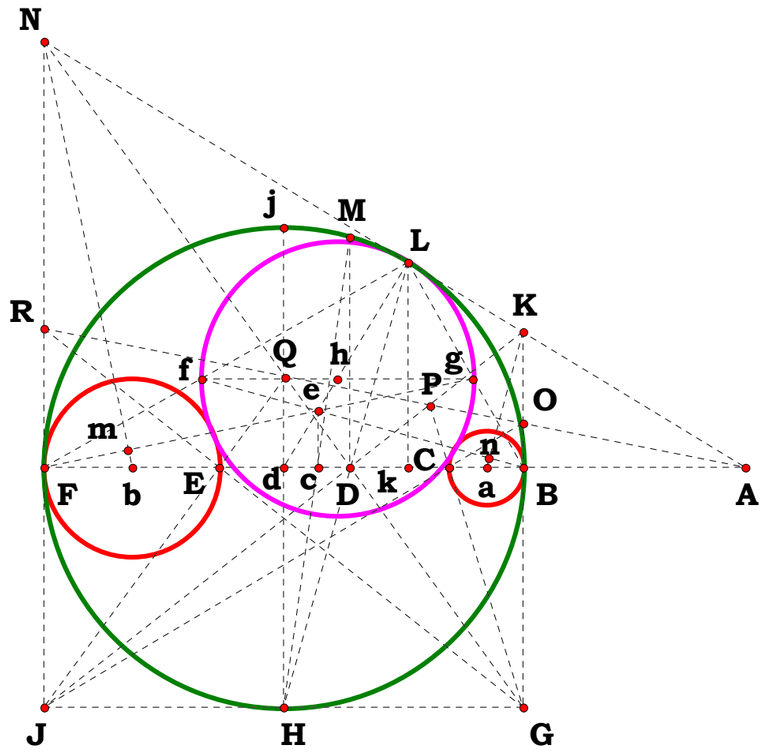
Unit.

Given.

$N_1 := 3.73926$ $AB := N_1$

$N_2 := 11.78259$ $AF := N_2$

## Quad Roots via Tangent Circles.




i.e., A, K, L and N are colinear.

**Descriptions.**

**000720a**

$AD := \sqrt{AB \cdot AF}$ $BF := AF - AB$ $Bd := \frac{BF}{2}$ $BD := AD - AB$

$Dd := Bd - BD$ $DH := \sqrt{Bd^2 + Dd^2}$ $HL := \frac{Bd \cdot BF}{DH}$

$DL := HL - DH$ $Dk := \frac{Dd \cdot DL}{DH}$ $Bk := Bd - (Dd + Dk)$ $Lk := \frac{Bd \cdot Dk}{Dd}$

$JN := \frac{Bd \cdot BF}{BD}$ $GK := \frac{Bd \cdot BF}{(Bd + Dd)}$ $FN := JN - Bd$ $BK := GK - Bd$

$Ak := Bk + AB$ $AF - \frac{BF \cdot FN}{FN - BK} = 0$ $\frac{AF}{FN} - \frac{Ak}{Lk} = 0$

$DF := Bd + Dd$ $DM := \sqrt{BD \cdot DF}$ $cd := \frac{Dd \cdot Bd}{Bd + DM}$ $dk := Bd - Bk$

$ce := \frac{Lk \cdot cd}{dk}$ $Fb := \frac{ce \cdot FN}{Bd + cd}$ $Ba := \frac{ce \cdot BK}{Bd - cd}$ $AE := AF - 2 \cdot Fb$

$AC := AB + 2 \cdot Ba$

**Definitions.**

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AC = 0$ $\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$ **etc., etc.**

Unit.

Given.

$N_1 := 2.07320$ $AB := N_1$

$N_2 := 10.53987$ $AF := N_2$

# Quad Roots by equal angles.

OOO720b

Descriptions.

$AD := \sqrt{AB \cdot AF}$ $BF := AF - AB$

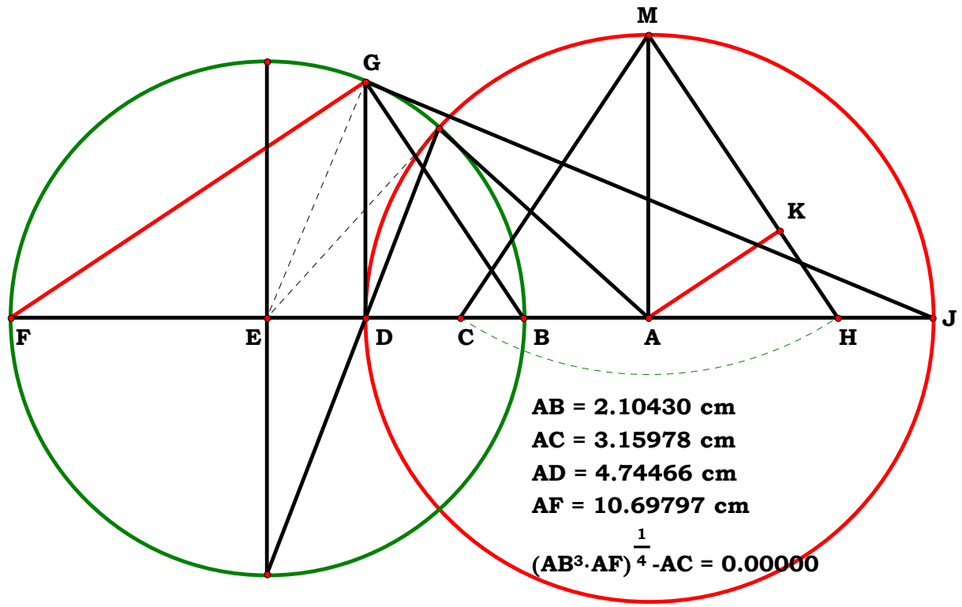
$AM := AD$ $DF := AF - AD$

$BD := AD - AB$ $DG := \sqrt{DF \cdot BD}$

$BG := \sqrt{DG^2 + BD^2}$ $AC := \frac{BD \cdot AD}{DG}$

Definitions.

$$AC - \left(AB^3 \cdot AF\right)^{\frac{1}{4}} = 0$$



**Unit.**

$AB := 1$

**Given.**

$N := 5 \qquad AG := N$

**000801a**

**Descriptions.**

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}}$

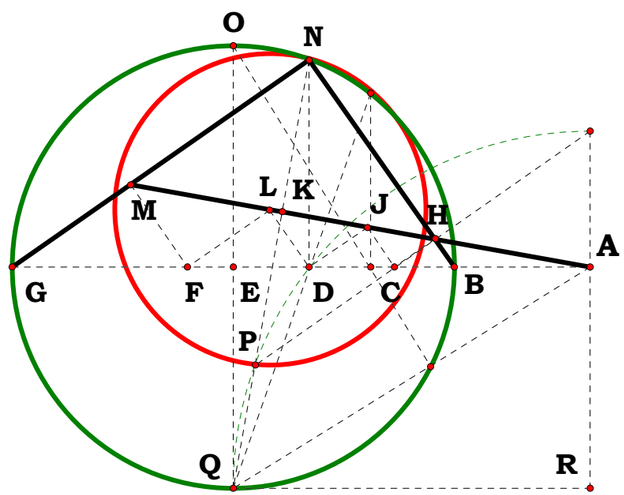
$BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG$

**Definitions.**

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

**Alternate Method Quad Roots**



**Unit.**

$AB := 1$

**Given.**

$N := 5 \qquad AG := N$

**080100B**

**Descriptions.**

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2}$

$AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

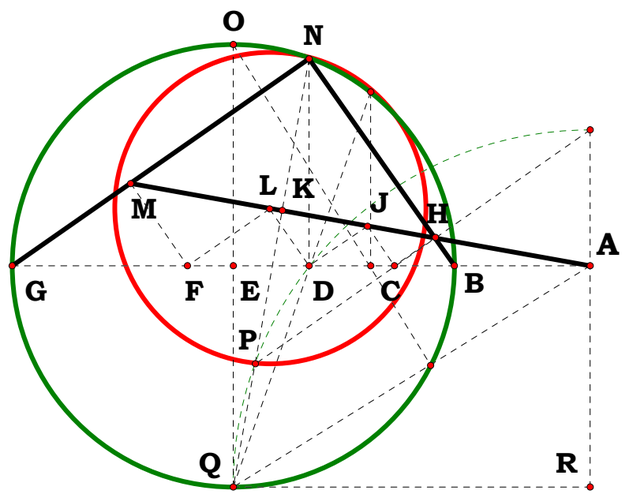
$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2}$

$GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN} \qquad AF := AG - GF$

**Definitions.**

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

**Alternate Method Quad Roots**



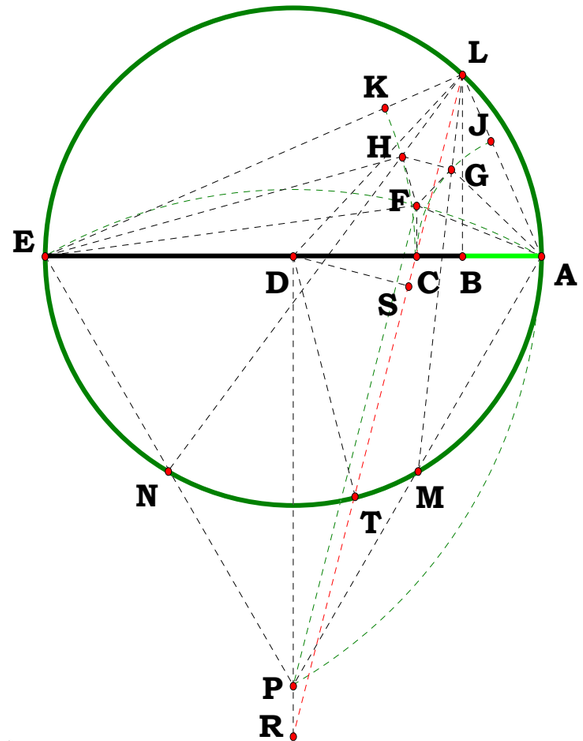
# In Trisection What Is AB?

**Given.**

$AC := .884$

$AE := 3.521$

**In the trisection figure given and given AC as the Unit what is AB?**




**080200**

**Descriptions.**

$AD := \frac{AE}{2}$ $EP := AE$ $DE := AD$ $DP := \sqrt{EP^2 - DE^2}$

$FP := EP$ $CE := AE - AC$ $CD := CE - DE$ $CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF$ $DR := DP + PR$ $CR := \sqrt{CD^2 + DR^2}$ $CS := \frac{CD^2}{CR}$

$DS := \sqrt{CD^2 - CS^2}$ $DL := AD$ $LS := \sqrt{DL^2 - DS^2}$ $RS := CR - CS$

$LR := RS + LS$ $BD := \frac{CD \cdot LR}{CR}$ $AB := AD - BD$ $ST := LS$ $RT := RS - ST$

**In trisection the length RT to the similarity point is equal to the radius of the circle.**

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

**Definitions.**

$$AD - \frac{AE}{2} = 0 \qquad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \qquad CF - \left(\frac{\sqrt{4 \cdot AC \cdot AE - 4 \cdot AC^2 + 3 \cdot AE^2}}{2} - \frac{\sqrt{3} \cdot AE}{2}\right) = 0 \qquad CE - (AE - AC) = 0 \qquad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0$$

$$DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad CR - AE = 0 \qquad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \qquad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0 \qquad RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0$$

$$BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0 \qquad AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \qquad AB \cdot AE^2 - AC^2(3 \cdot AE - 2 \cdot AC) = 0$$

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 2$

080300A

Descriptions.

$AD := AB$ $\quad AP := \frac{AD}{2}$ $\quad BP := AB + AP$ $\quad BO := \frac{BP}{N_1}$ $\quad AE := AB$

$DO := N_2 - BO$ $\quad GO := \sqrt{BO \cdot DO}$ $\quad BG := \sqrt{GO^2 + BO^2}$ $\quad BS := \frac{BG}{2}$ $\quad ER := BS$ $\quad TO := ER$

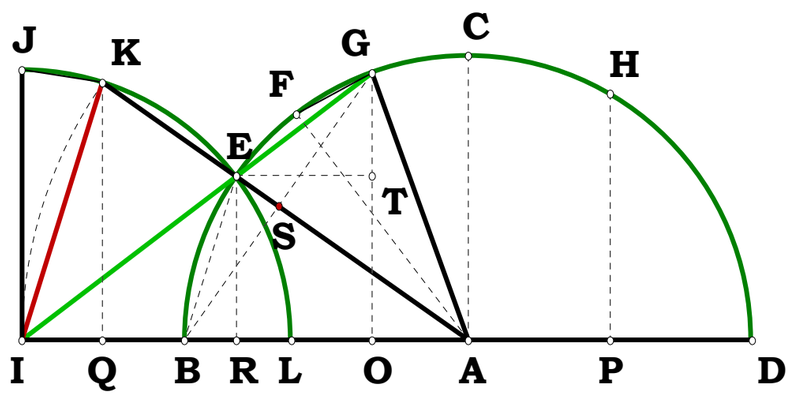
$GT := GO - TO$ $\quad AS := \sqrt{AB^2 - BS^2}$ $\quad ES := AE - AS$ $\quad BR := ES$ $\quad OR := BO - BR$

$ET := OR$ $\quad IO := \frac{ET \cdot GO}{GT}$ $\quad BI := IO - BO$ $\quad AI := BI + AB$ $\quad BE := \sqrt{ER^2 + BR^2}$

$GE := BE$ $\quad GI := \frac{GE \cdot GO}{GT}$ $\quad EI := GI - GE$ $\quad AK := AI$ $\quad IK := EI$ $\quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

$EK := AK - AE$ $\quad \frac{EK}{IQ} = 2$

On Trisection




If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.

Definitions.

$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0$ $\quad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0$ $\quad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0$ $\quad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$

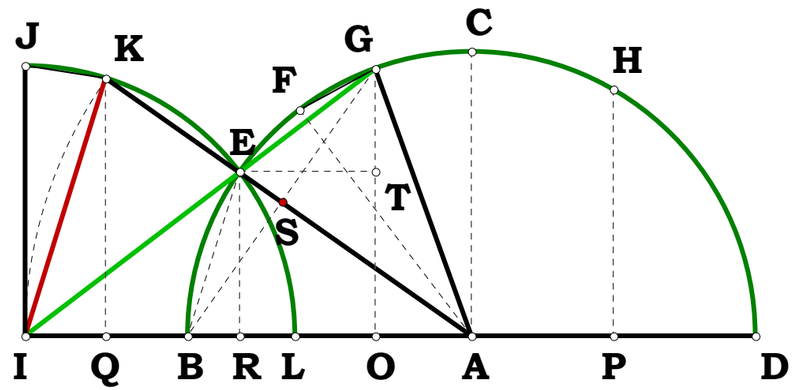
$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0$ $\quad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0$ $\quad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1} \cdot N_1}\right]}{N_1} - OR = 0$

$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot {N_1}^2-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$

$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$




$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

**Unit.**

$AB := 1$

**Given.**

$N_1 := 3$

$N_2 := 2$

**080300B**

**Descriptions.**

$AD := AB$ $\quad AP := \frac{AD}{2}$ $\quad BP := AB + AP$ $\quad BO := \frac{BP}{N_1}$ $\quad AE := AB$

$DO := N_2 - BO$ $\quad GO := \sqrt{BO \cdot DO}$ $\quad BG := \sqrt{GO^2 + BO^2}$ $\quad BS := \frac{BG}{2}$ $\quad ER := BS$ $\quad TO := ER$

$GT := GO - TO$ $\quad AS := \sqrt{AB^2 - BS^2}$ $\quad ES := AE - AS$ $\quad BR := ES$ $\quad OR := BO - BR$

$ET := OR$ $\quad IO := \frac{ET \cdot GO}{GT}$ $\quad BI := IO - BO$ $\quad AI := BI + AB$ $\quad BE := \sqrt{ER^2 + BR^2}$

$GE := BE$ $\quad GI := \frac{GE \cdot GO}{GT}$ $\quad EI := GI - GE$ $\quad AK := AI$ $\quad IK := EI$ $\quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

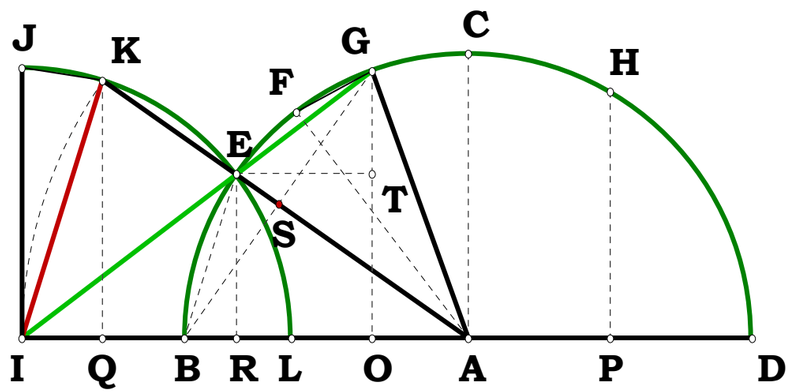
$EK := AK - AE$ $\quad \frac{EK}{IQ} = 2$

**Definitions.**

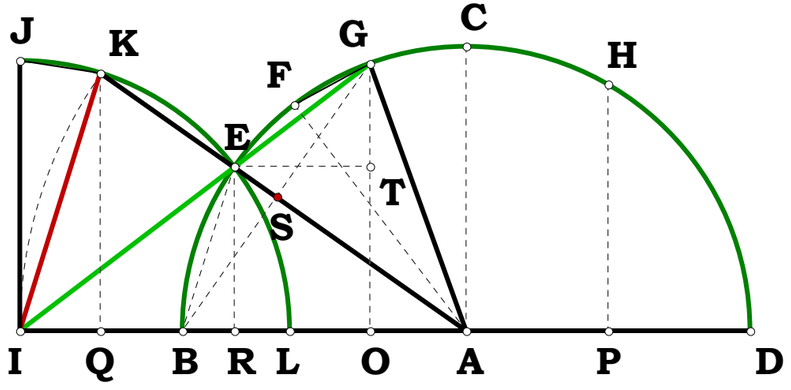
$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0$ $\quad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0$ $\quad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0$ $\quad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$

$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0$ $\quad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0$ $\quad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1} \cdot N_1}\right]}{N_1} - OR = 0$

## On Trisection




**If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.**




$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot {N_1}^2-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$

$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$

$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

**080400A**

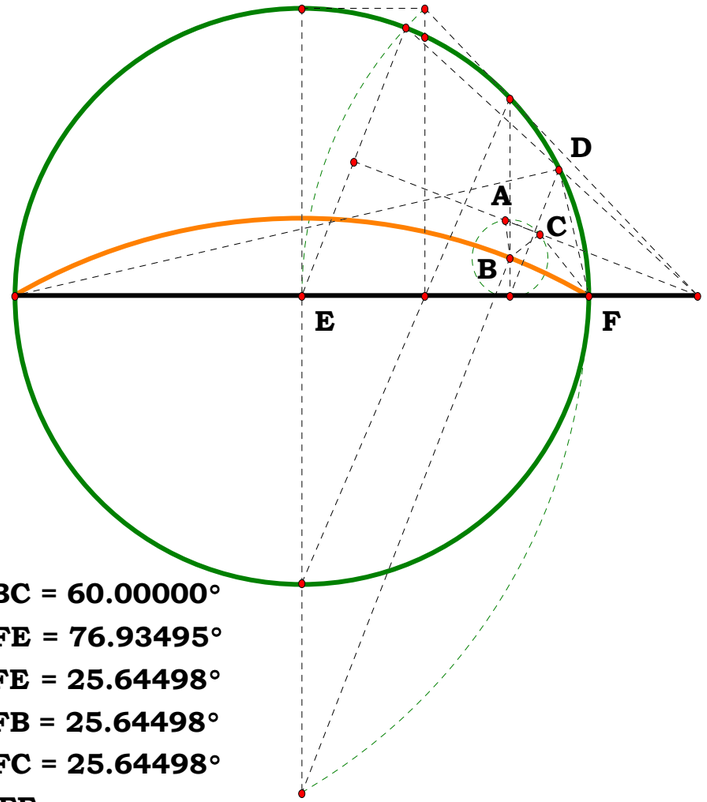
**With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?**

**I am going to figure this out and then I am going to order the equations a little different at the start to see what happens to all the definitions.**

**This is the A plate, or the first of six.**

**What will be demonstrated are the differences in the choice of what one uses for a unit to write the figure up.**

**Trisection and Square Roots**




$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

**Unit.**

**Given.**

$N_1 := 1.90557$ $\quad AB := N_1$

$N_2 := 12.01265$ $\quad AF := N_2$

**Descriptions.**

$AD := \sqrt{AB \cdot AF}$ $\quad BD := AD - AB$

$BF := AF - AB$ $\quad DF := AF - AD$ $\quad DJ := \sqrt{BD \cdot DF}$

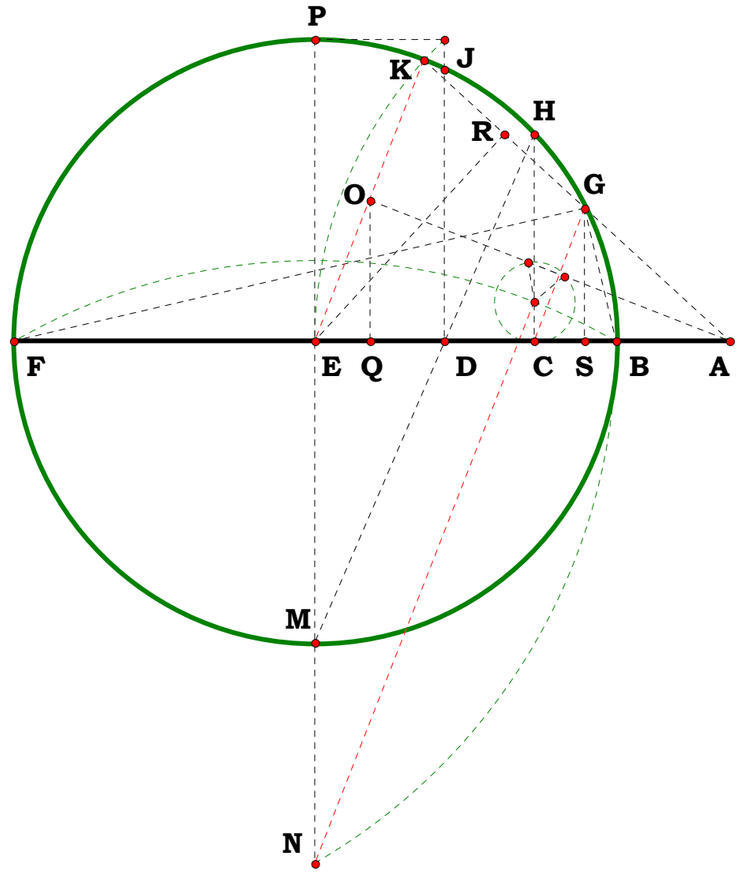
$BE := \frac{BF}{2}$ $\quad EO := \frac{BE}{2}$ $\quad AE := BE + AB$ $\quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $\quad DM := \sqrt{DE^2 + BE^2}$ $\quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $\quad CD := \frac{DE \cdot DH}{DM}$ $\quad CE := DE + CD$ $\quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $\quad KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $\quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $\quad AS := AE - (DE + CD + CS)$ $\quad BS := AS - AB$




**Definitions:**

$AB - N_1 = 0$ $\quad AF - N_2 = 0$ $\quad AD - \sqrt{N_1 \cdot N_2} = 0$ $\quad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0$

$BF - \left(N_2 - N_1\right) = 0$ $\quad DF - \left(N_2 - \sqrt{N_1 \cdot N_2}\right) = 0$ $\quad DJ - \sqrt{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2} = 0$

$BE - \frac{N_2 - N_1}{2}$ $\quad EO - \frac{N_2 - N_1}{4} = 0$ $\quad AE - \frac{N_1 + N_2}{2} = 0$ $\quad EQ - \frac{\left(N_1 - N_2\right)^2}{8 \cdot \left(N_1 + N_2\right)} = 0$

$$DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}}{2} = 0 \qquad DM - \frac{\sqrt{\left[N_1^{\,2} + N_2^{\,2} + 2\cdot N_1\cdot N_2 - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2)\right]}}{\sqrt{2}} = 0$$

$$HM - \frac{\sqrt{2}\cdot(N_1 - N_2)^2}{2\cdot\sqrt{N_1^{\,2} + N_2^{\,2} - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2) + 2\cdot N_1\cdot N_2}} = 0$$

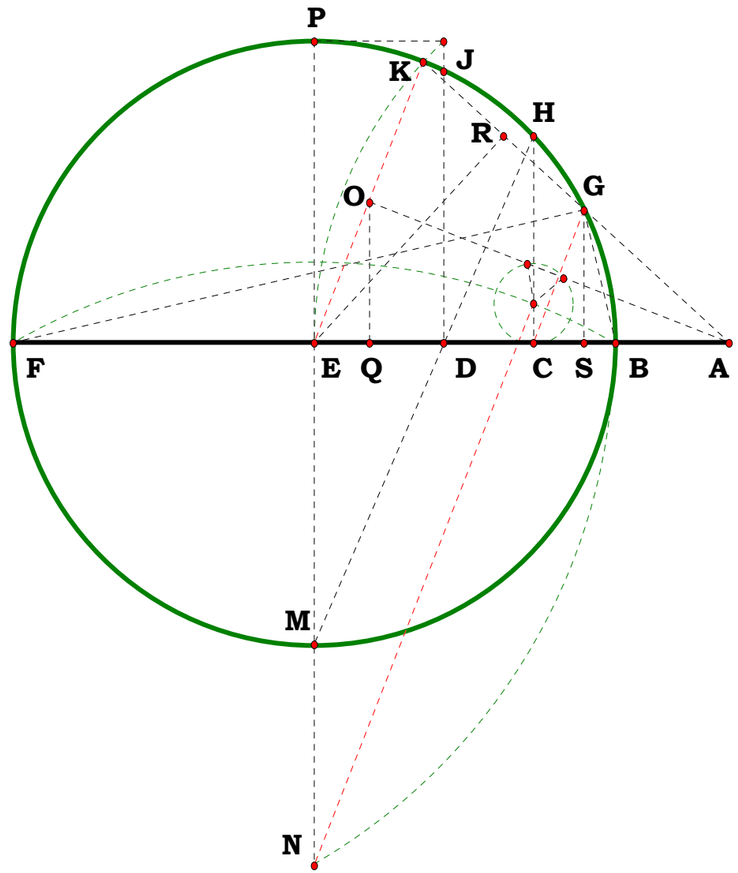
$$DH - \frac{(N_1 + N_2)\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})}} = 0$$

$$CD - \frac{\sqrt{N_1\cdot N_2}\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right) - 4\cdot N_1^{\,2}\cdot N_2 - 4\cdot N_1\cdot N_2^{\,2}}{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})} = 0$$

$$CE - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad BC - \frac{N_1\cdot(N_2 - N_1)}{N_1 + N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(N_1 - N_2)^2}}{2} = 0$$

$$KG - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad AG - \frac{2\cdot N_1\cdot N_2}{N_1 + N_2} = 0 \qquad CS - \frac{N_1\cdot N_2\cdot(N_1 - N_2)^2}{(N_1 + N_2)^3} = 0$$

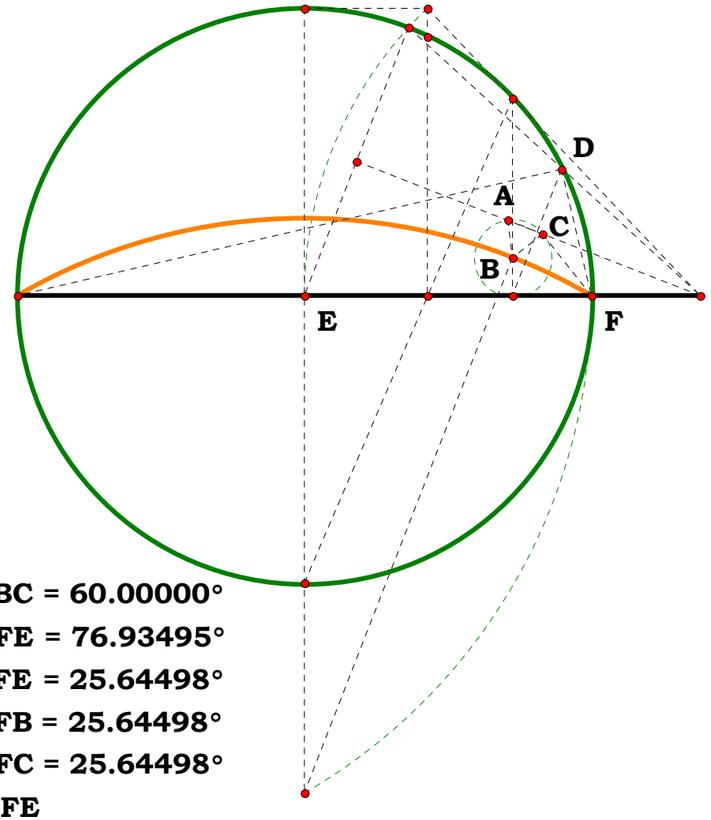
$$AS - \frac{N_1\cdot N_2\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right)}{(N_1 + N_2)^3} = 0 \qquad BS - \frac{N_1^{\,2}\cdot(N_2 - N_1)\cdot(N_1 + 3\cdot N_2)}{(N_1 + N_2)^3} = 0$$



**080400B**

**This is the B plate, or the second of six.**

## Trisection and Square Roots




$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

**Unit.**

**Given.**

$N_1 := 1.90557$ $AB := N_1$

$N_2 := 10.10708$ $BF := N_2$

**Descriptions.**

$AF := AB + BF$ $AD := \sqrt{AB \cdot AF}$

$BD := AD - AB$ $DF := AF - AD$ $DJ := \sqrt{BD \cdot DF}$

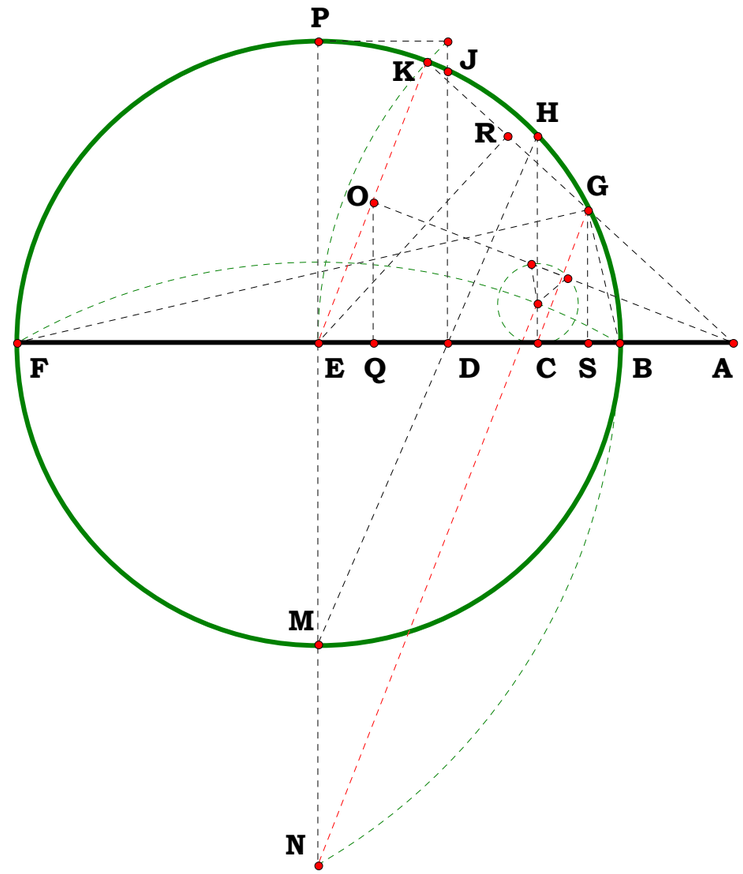
$BE := \frac{BF}{2}$ $EO := \frac{BE}{2}$ $AE := BE + AB$ $EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $DM := \sqrt{DE^2 + BE^2}$ $HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $CD := \frac{DE \cdot DH}{DM}$ $CE := DE + CD$ $BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $AS := AE - (DE + CD + CS)$ $BS := AS - AB$




**Definitions:**

$AB - N_1 = 0$ $BF - N_2 = 0$ $AF - (N_1 + N_2) = 0$

$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0$ $BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right]$ $DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0$ $BE - \frac{N_2}{2} = 0$ $EO - \frac{N_2}{4} = 0$

$AE - \frac{2 \cdot N_1 + N_2}{2} = 0$ $EQ - \frac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0$ $DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$

$$DM - \frac{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}{2}=0$$

$$HM - \frac{N_2^{\,2}}{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)-2\cdot N_1^{\,2}-2\cdot N_1\cdot N_2\right]}{\sqrt{\left(2\cdot N_1+N_2\right)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot\left(N_1+N_2\right)}\right]}}=0$$

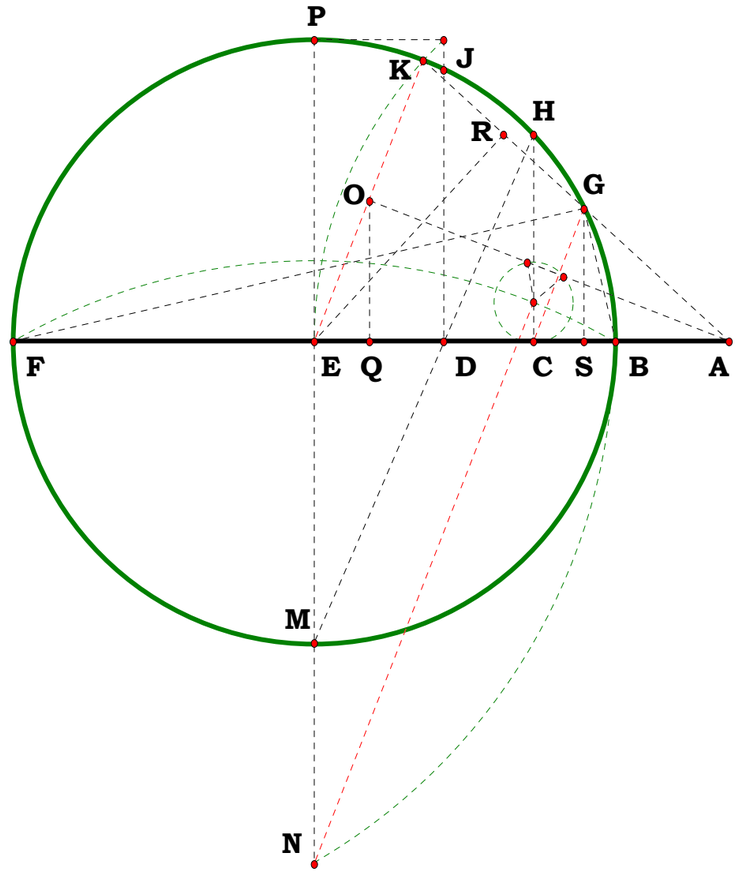
$$CD - \frac{\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(8\cdot N_1^{\,2}+8\cdot N_1\cdot N_2+N_2^{\,2}\right)-4\cdot N_1\cdot\left(2\cdot N_1+N_2\right)\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\right)}=0$$

$$CE - \frac{N_2^{\,2}}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{N_2^{\,2}}}{2}=0$$

$$KG - \frac{N_2^{\,2}}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad AG - \frac{2\cdot N_1\cdot\left(N_1+N_2\right)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot N_2^{\,2}\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$AS - \frac{N_1\cdot\left(N_1+N_2\right)\cdot\left(8\cdot N_1^{\,2}+8\cdot N_1\cdot N_2+N_2^{\,2}\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

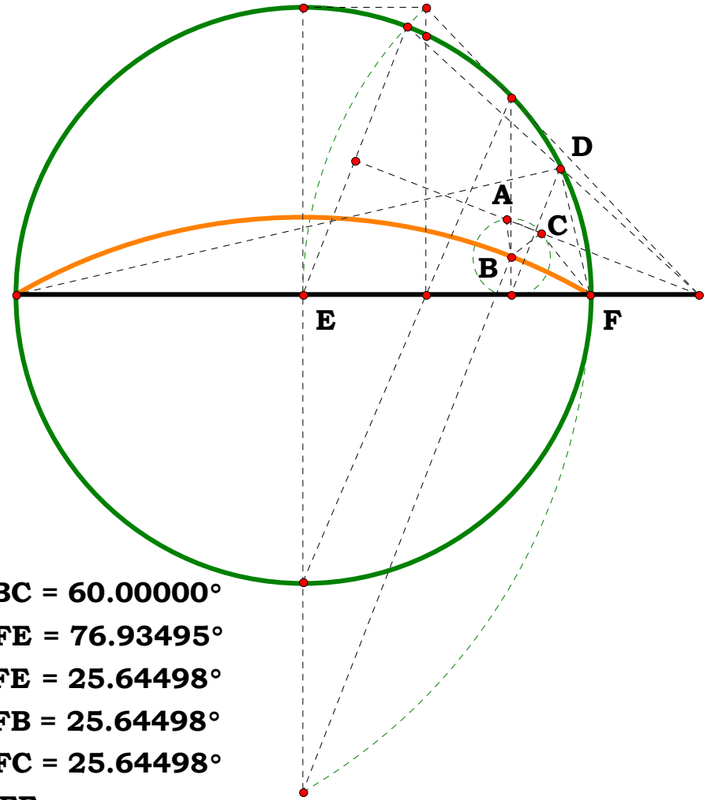
$$BS - \frac{N_1^{\,2}\cdot N_2\cdot\left(4\cdot N_1+3\cdot N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$



080400C

This is the C plate, or the third of six.

## Trisection and Square Roots




m∠ABC = 60.00000°
m∠DFE = 76.93495°
m∠BFE = 25.64498°
m∠CFB = 25.64498°
m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

Unit.

$AB := 1$

Given.

$N_1 := 12.01265 \qquad AF := N_1$

**Descriptions.**

$AD := \sqrt{AB \cdot AF} \qquad BD := AD - AB$

$BF := AF - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

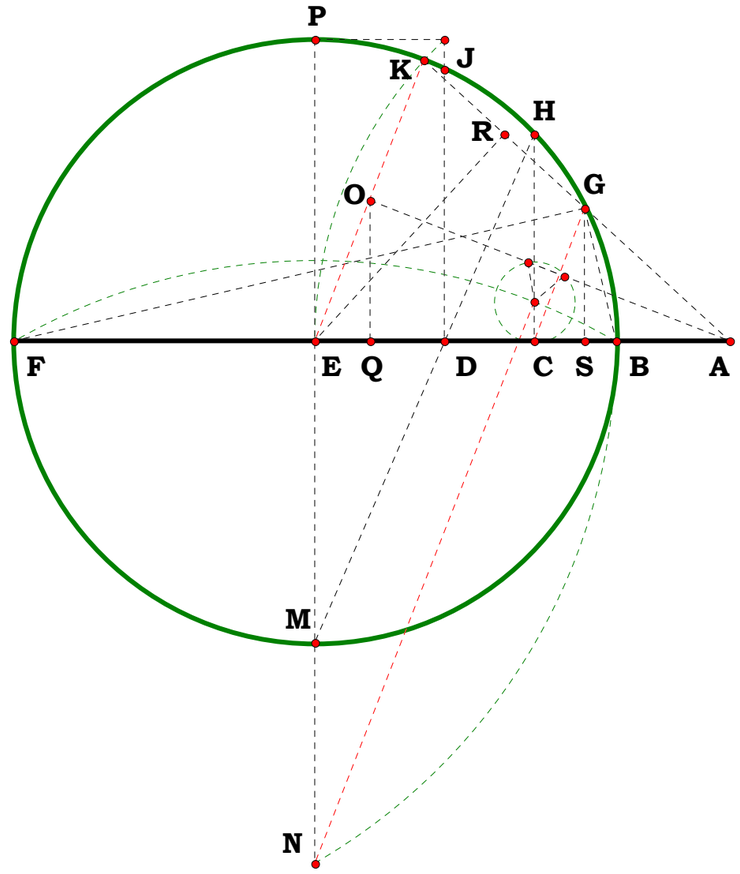
$BE := \frac{BF}{2} \qquad EO := \frac{BE}{2} \qquad AE := BE + AB \qquad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \frac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \frac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$




**Definitions:**

$1 - 1 = 0 \qquad AF - N_1 = 0 \qquad AD - \sqrt{N_1} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0$

$BF - \left(N_1 - 1\right) = 0 \qquad DF - \left(N_1 - \sqrt{N_1}\right) = 0 \qquad DJ - \sqrt{\sqrt{N_1} \cdot \left(\sqrt{N_1} - 1\right)^2} = 0$

$BE - \frac{N_1 - 1}{2} \qquad EO - \frac{N_1 - 1}{4} = 0 \qquad AE - \frac{1 + N_1}{2} = 0 \qquad EQ - \frac{\left(1 - N_1\right)^2}{8 \cdot \left(1 + N_1\right)} = 0$

$$DE - \frac{1 + N_1 - 2\cdot\sqrt{N_1}}{2} = 0 \qquad DM - \frac{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}}{\sqrt{2}} = 0$$

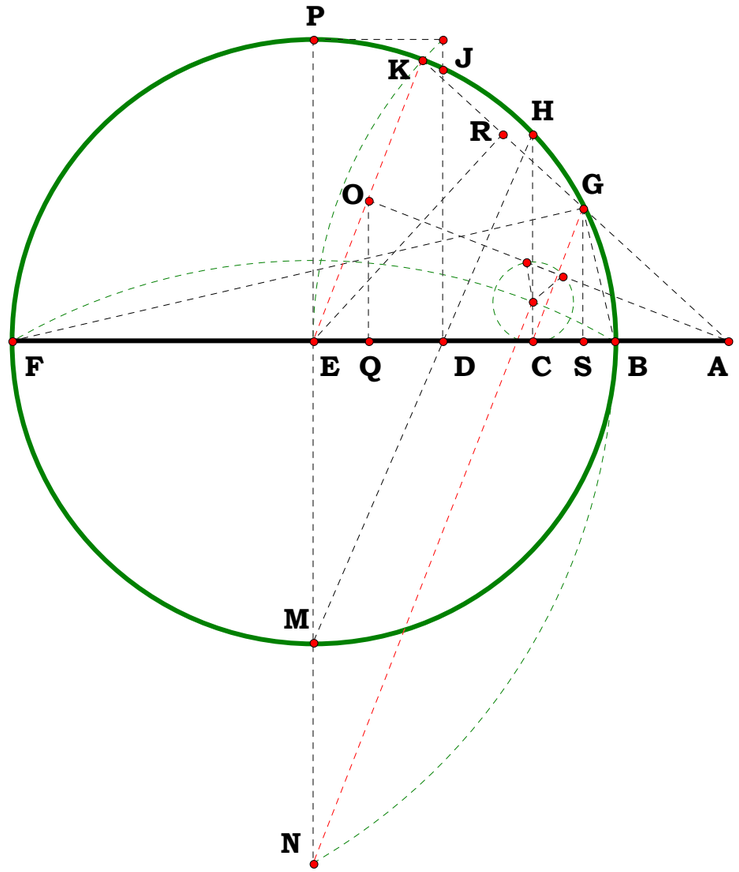
$$HM - \frac{\sqrt{2}\cdot(1-N_1)^2}{2\cdot\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0 \qquad DH - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^2}{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0$$

$$CD - \frac{\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^4}{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2} = 0 \qquad CE - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$BC - \frac{1\cdot(N_1-1)}{1+N_1} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(1-N_1)^2}}{2} = 0 \qquad KG - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$AG - \frac{2\cdot 1\cdot N_1}{1+N_1} = 0 \qquad CS - \frac{1\cdot N_1\cdot(1-N_1)^2}{(1+N_1)^3} = 0$$

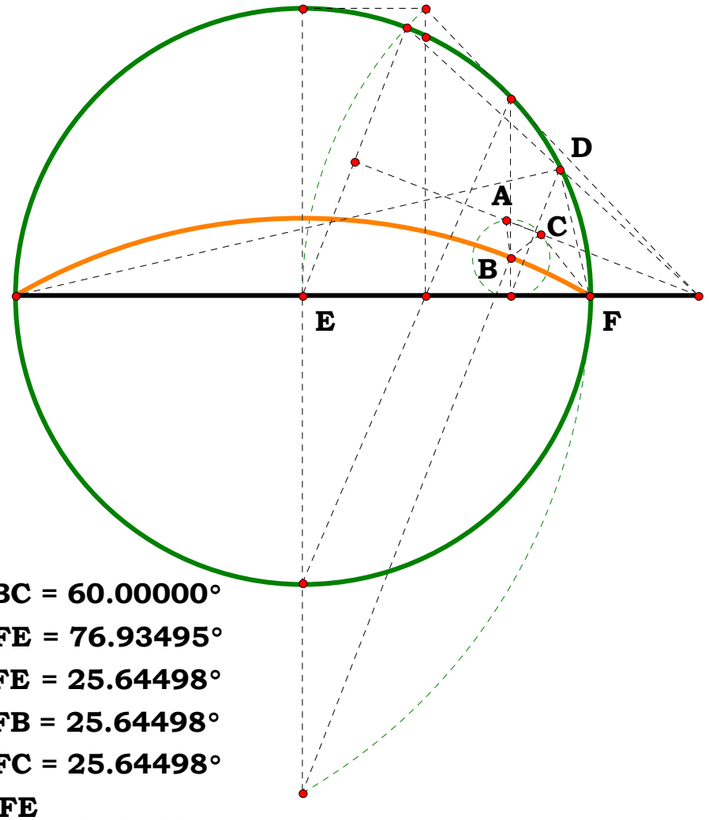
$$AS - \frac{N_1\cdot\left(N_1^2+6\cdot N_1+1\right)}{(1+N_1)^3} = 0 \qquad BS - \frac{(N_1-1)\cdot(3\cdot N_1+1)}{(1+N_1)^3} = 0$$



080400D

This is the D plate, or the forth of six.

## Trisection and Square Roots




$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

**Unit.**

**Given.**

$N_1 := 1.90557$ $\quad AB := N_1$

$N_2 := 10.10708$ $\quad BF := N_2$

**Descriptions.**

$AF := AB + BF$ $\quad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB$ $\quad DF := AF - AD$ $\quad DJ := \sqrt{BD \cdot DF}$

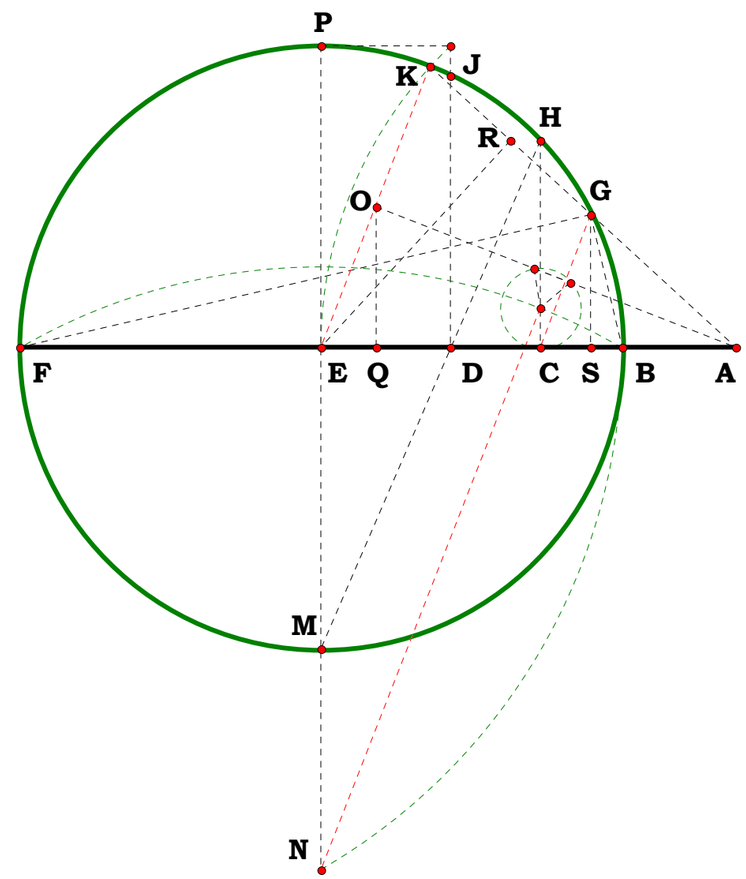
$BE := \frac{BF}{2}$ $\quad EO := \frac{BE}{2}$ $\quad AE := BE + AB$ $\quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $\quad DM := \sqrt{DE^2 + BE^2}$ $\quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $\quad CD := \frac{DE \cdot DH}{DM}$ $\quad CE := DE + CD$ $\quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $\quad KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $\quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $\quad AS := AE - (DE + CD + CS)$ $\quad BS := AS - AB$




**Definitions:**

$AB - N_1 = 0$ $\quad BF - N_2 = 0$ $\quad AF - (N_1 + N_2) = 0$

$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0$ $\quad BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right]$ $\quad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0$ $\quad BE - \frac{N_2}{2} = 0$ $\quad EO - \frac{N_2}{4} = 0$

$AE - \frac{2 \cdot N_1 + N_2}{2} = 0$ $\quad EQ - \frac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0$ $\quad DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$

$$DM - \frac{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}{2}=0$$

$$HM - \frac{{N_2}^2}{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)-2\cdot{N_1}^2-2\cdot N_1\cdot N_2\right]}{\sqrt{\left(2\cdot N_1+N_2\right)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot\left(N_1+N_2\right)}\right]}}=0$$

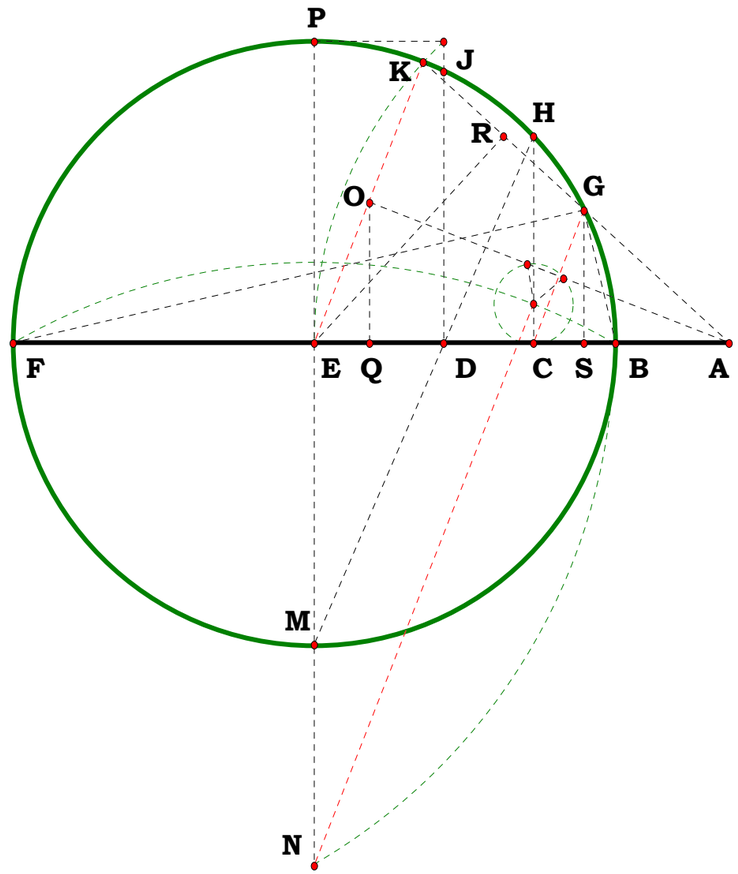
$$CD - \frac{\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(8\cdot{N_1}^2+8\cdot N_1\cdot N_2+{N_2}^2\right)-4\cdot N_1\cdot\left(2\cdot N_1+N_2\right)\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\right)}=0$$

$$CE - \frac{{N_2}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{{N_2}^2}}{2}=0$$

$$KG - \frac{{N_2}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad AG - \frac{2\cdot N_1\cdot\left(N_1+N_2\right)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot{N_2}^2\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$AS - \frac{N_1\cdot\left(N_1+N_2\right)\cdot\left(8\cdot{N_1}^2+8\cdot N_1\cdot N_2+{N_2}^2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

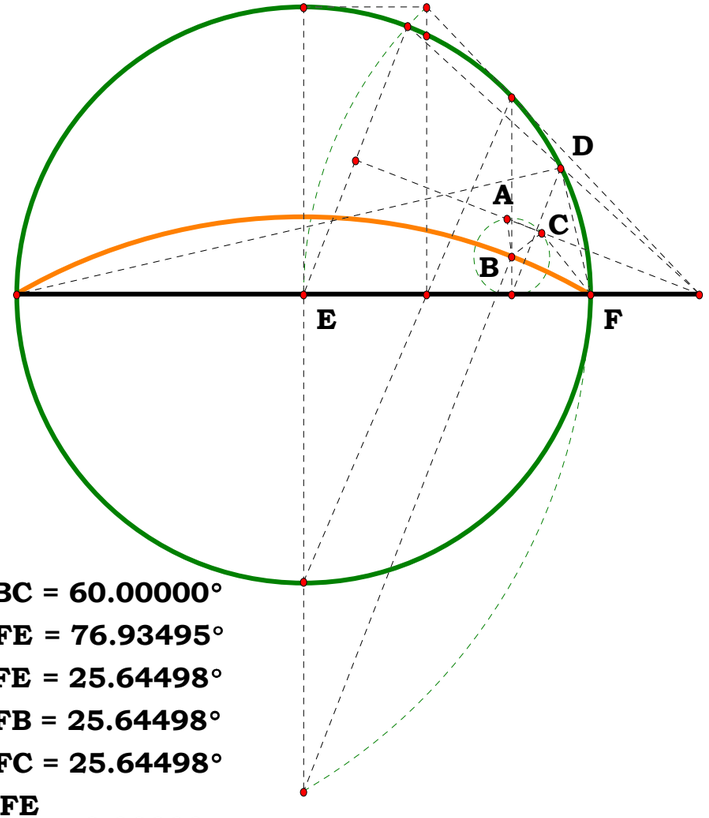
$$BS - \frac{{N_1}^2\cdot N_2\cdot\left(4\cdot N_1+3\cdot N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$



080400E

**Trisection and Square Roots**

**This is the E plate, or the fifth of six.**




$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

P

**Unit.**

$BF := 1$

**Given.**

$N := 4$

$AF := N + BF \quad AD := \sqrt{N \cdot AF} \quad BD := AD - N$

$BF := AF - N \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$

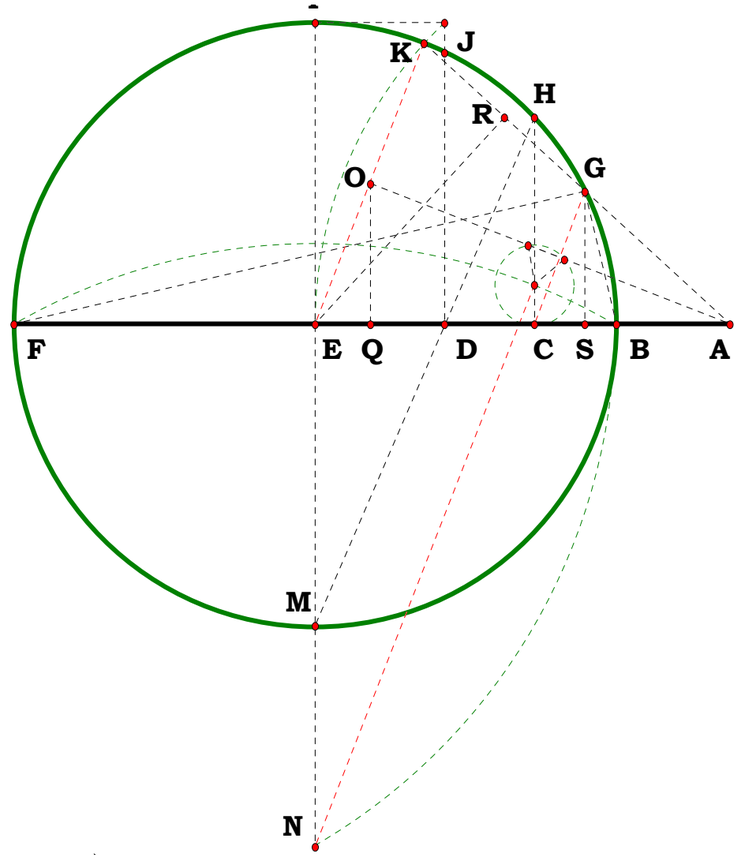
$BE := \frac{BF}{2} \quad EO := \frac{BE}{2} \quad AE := BE + N \quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \quad CD := \frac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \quad KG := \frac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - N$




**Definitions:**

$N - N = 0 \quad AF - (N + 1) = 0 \quad AD - \sqrt{N^2 + N} = 0 \quad BD - \left(\sqrt{N^2 + N} - N\right) = 0$

$BF - (N + 1 - N) = 0 \quad DF - \left[N + 1 - \sqrt{N \cdot (N + 1)}\right] = 0 \quad DJ - \sqrt{(2 \cdot N + 1) \cdot \sqrt{N^2 + N} - \left(2 \cdot N^2 + 2 \cdot N\right)} = 0$

$BE - 2^{-1} \quad EO - 2^{-2} = 0 \quad AE - \frac{2 \cdot N + 1}{2} = 0 \quad EQ - \frac{1}{8 \cdot (2 \cdot N + 1)} = 0$

$$DE - \frac{2\cdot N - 2\cdot\sqrt{N^2+N}+1}{2} = 0 \qquad DM - \frac{\sqrt{2\cdot(2\cdot N+1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]}}{2} = 0$$

$$HM - \left[(4\cdot N+2)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]\right]^{\frac{-1}{2}} = 0$$

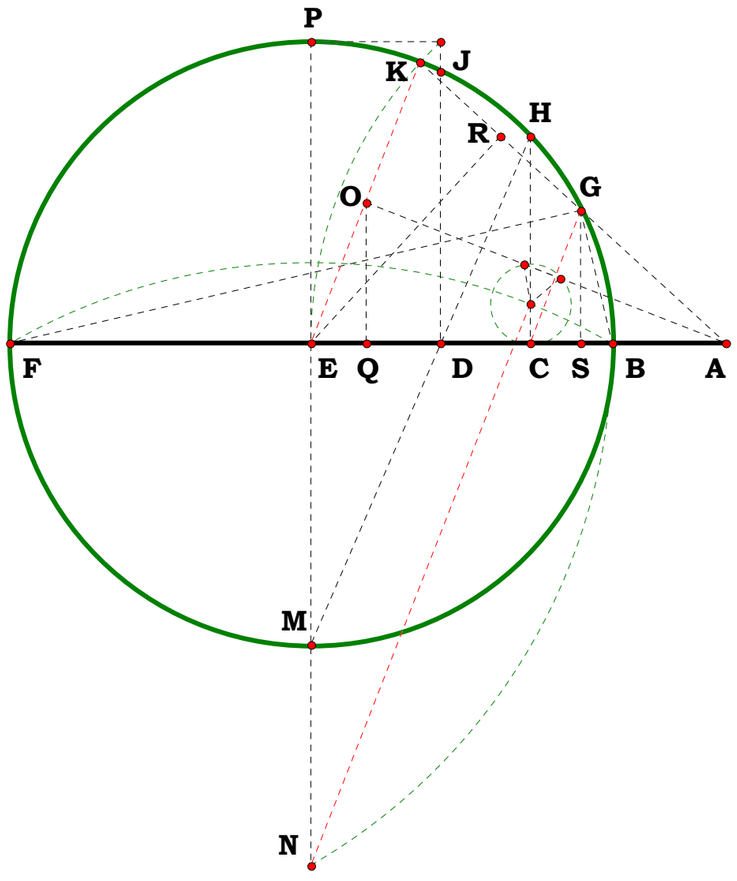
$$DH - \frac{(2\cdot N+1)\cdot\left(\sqrt{2}\cdot\sqrt{N^2+N}\right) - 2\cdot\sqrt{2}\cdot N\cdot(N+1)}{\sqrt{\left[(2\cdot N+1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]\right]}} = 0$$

$$CD - \frac{\left(8\cdot N^2+8\cdot N+1\right)\cdot\sqrt{N^2+N} - \left(8\cdot N^3+12\cdot N^2+4\cdot N\right)}{(2\cdot N+1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2+N}+1\right)} = 0$$

$$CE - \frac{1}{2\cdot(2\cdot N+1)} = 0 \qquad BC - \frac{N}{2\cdot N+1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{2\cdot(2\cdot N+1)} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N+1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N+1)^3} = 0$$

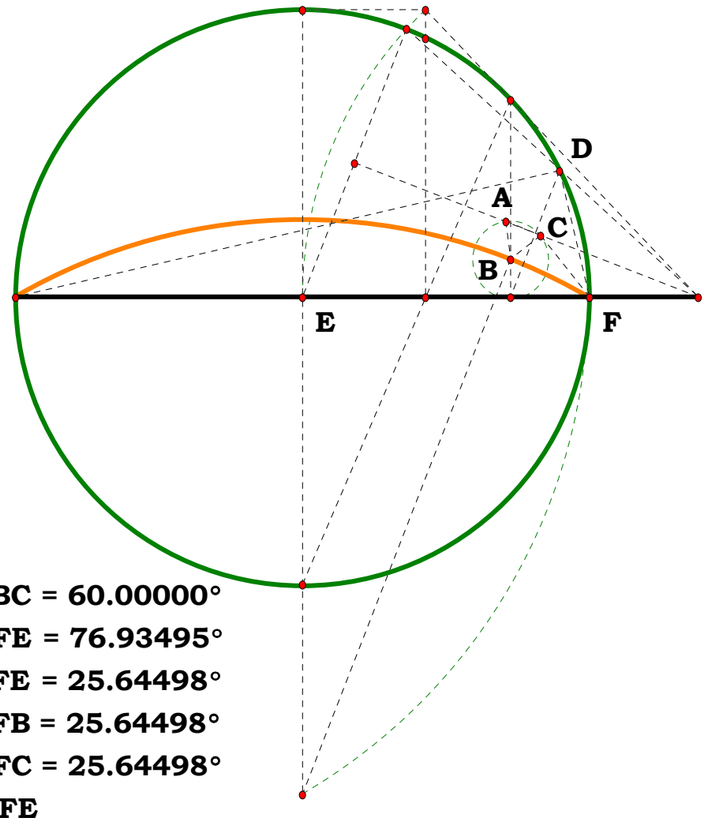
$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2+8\cdot N+1\right)}{(2\cdot N+1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N+3)}{(2\cdot N+1)^3} = 0$$



080400F

**This is the F plate, or the sixth of six.**

## Trisection and Square Roots




$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit

$BF := 1$

Given.

$N := 3 \qquad AB := N$

$AF := AB + BF \qquad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

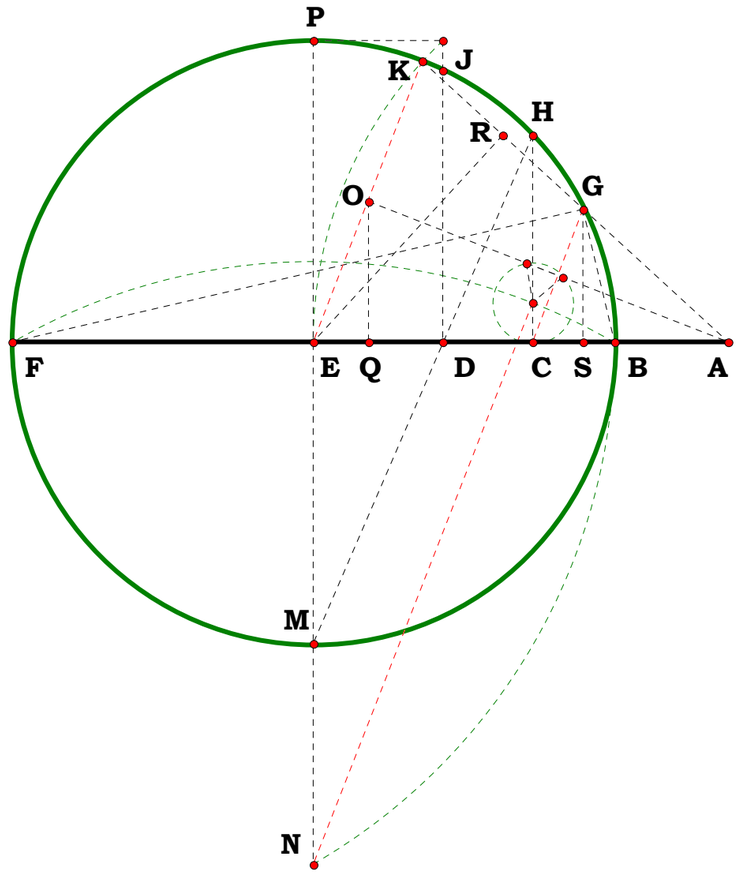
$BE := \frac{BF}{2} \qquad EO := \frac{BE}{2} \qquad AE := BE + AB \qquad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \frac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \frac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$




**Definitions:**

$AB - N = 0 \qquad BF - 1 = 0 \qquad AF - (N + 1) = 0$

$AD - \sqrt{N \cdot (N + 1)} = 0 \qquad BD - \sqrt{N \cdot (N + 1)} - N \qquad DF - \left[N + 1 - \sqrt{N \cdot (N + 1)}\right] = 0$

$DJ - \sqrt{\sqrt{N^2 + 1 \cdot N} \cdot (2 \cdot N + 1) - 2 \cdot N - 2 \cdot N^2} = 0 \qquad BE - \frac{1}{2} = 0 \qquad EO - \frac{1}{4} = 0$

$AE - \frac{2 \cdot N + 1}{2} = 0 \qquad EQ - \frac{1^2}{8 \cdot (2 \cdot N + 1)} = 0 \qquad DE - \frac{2 \cdot N + 1 - 2 \cdot \sqrt{N^2 + 1 \cdot N}}{2} = 0$

$$DM - \frac{\sqrt{(4\cdot N+2)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}}{2} = 0$$

$$HM - \frac{\sqrt{2}}{2\cdot\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

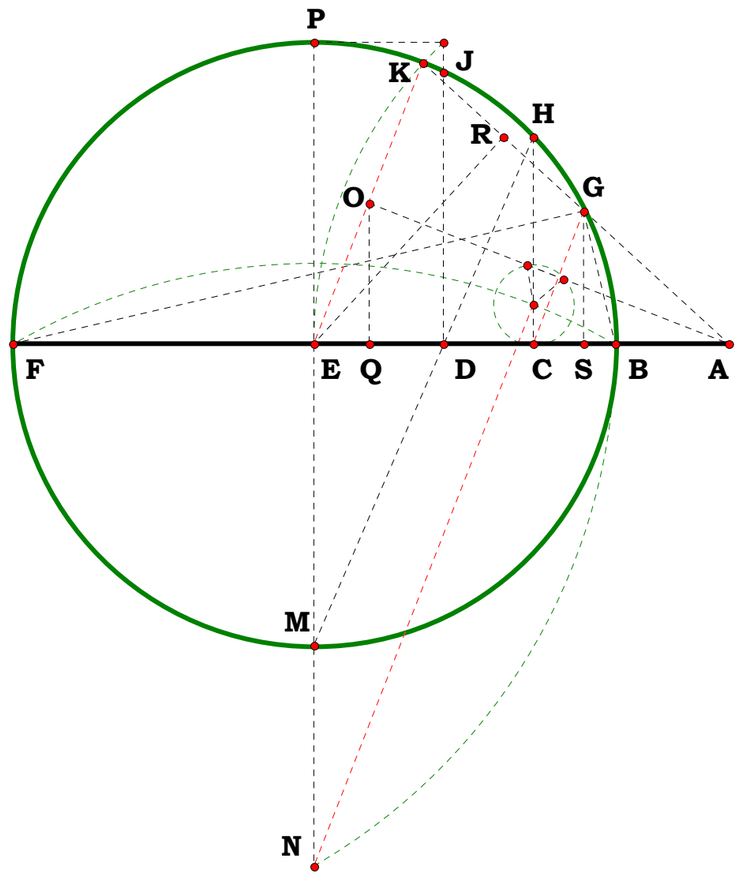
$$DH - \frac{(2\cdot N+1)\cdot\sqrt{2\cdot N^2+2\cdot N}-\left[2\cdot\sqrt{2}\cdot N\cdot(N+1)\right]}{\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

$$CD - \frac{\left(8\cdot N^2+8\cdot N+1\right)\cdot\sqrt{N^2+N}-4\cdot N\cdot(N+1)\cdot(2\cdot N+1)}{(2\cdot N+1)\cdot\left(2\cdot N+1-2\cdot\sqrt{N^2+N}\right)} = 0$$

$$CE - \frac{1}{4\cdot N+2} = 0 \qquad BC - \frac{N}{2\cdot N+1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{4\cdot N+2} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N+1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N+1)^3} = 0$$

$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2+8\cdot N+1\right)}{(2\cdot N+1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N+3)}{(2\cdot N+1)^3} = 0$$



080700

**Unit.**

$AB := 1$

**Given.**

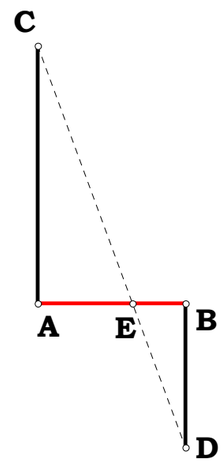
$N_1 := 9 \quad AC := N_1$

$N_2 := 2 \quad BD := N_2$

**Descriptions.**

## Proportion Series II

**Divide AB into the same ratio as AB:CD.**




$$BE := \frac{AC \cdot AB}{AC + BD} \qquad CE := \frac{BD \cdot AB}{AC + BD}$$

$$BE + CE - AB = 0 \qquad \frac{AC}{BD} - \frac{BE}{CE} = 0$$

**Definitions.**

$$BE - \frac{N_1}{N_1 + N_2} = 0 \qquad CE - \frac{N_2}{N_1 + N_2} = 0$$

Unit.

$AB := 1$

Given.

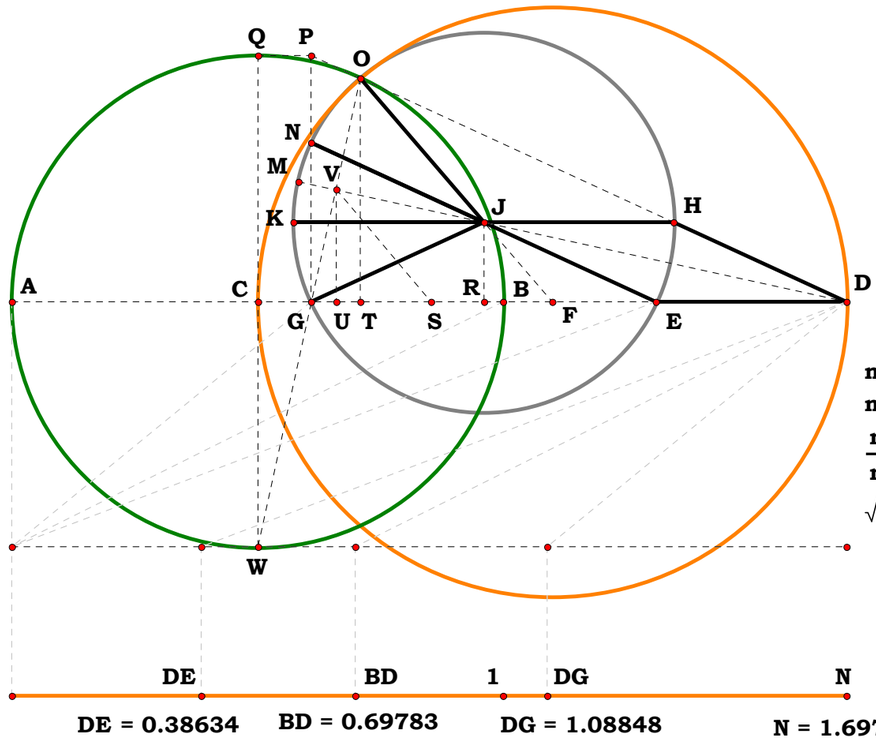
$N := 3 \qquad AD := N$

000822A

Descriptions.

This square root figure affords another approach to proofing the Archimedean Paper Trisecter.

## Square Root and the Archimedean Paper Trisecter.




$m\angle OJG = 74.01600°$

$m\angle NJK = 24.67200°$

$\frac{m\angle OJG}{m\angle NJK} = 3.00000$

$\sqrt{N \cdot BD} - DG = 0.00000$

$DE = 0.38634 \qquad BD = 0.69783 \qquad DG = 1.08848 \qquad N = 1.69783$

$BD := AD - AB \qquad DG := \sqrt{BD \cdot AD} \qquad BC := \frac{AB}{2}$

$CD := BD + BC \qquad DF := \frac{CD}{2} \qquad BG := DG - BD$

$CG := BC - BG \qquad GW := \sqrt{CG^2 + BC^2} \qquad OW := \frac{BC \cdot AB}{GW}$

$GO := OW - GW \qquad GU := \frac{CG \cdot GO}{2 \cdot GW} \qquad DU := DG - GU$

$OT := \frac{BC \cdot GO}{GW} \qquad US := \sqrt{\left(\frac{DF}{2}\right)^2 - \left(\frac{OT}{2}\right)^2} \qquad DS := DU - US$

$JR := \frac{OT \cdot DF}{2 \cdot DS} \qquad JF := \frac{DF \cdot JR}{OT} \qquad EJ := DF - JF$

$FR := \frac{2 \cdot US \cdot JR}{OT} \qquad DR := DF + FR \qquad ER := \sqrt{EJ^2 - JR^2}$

$DE := DR - ER \qquad DE - EJ = 0$

$\frac{(2 \cdot N-1) \cdot (((2 \cdot N-1) \cdot \sqrt{N^2-N}+2 \cdot N)-2 \cdot N^2)}{(((4 \cdot N-2) \cdot \sqrt{N^2-N}+4 \cdot N)-4 \cdot N^2)+1} = 0.38634 \qquad \frac{(2 \cdot N-1) \cdot (((2 \cdot N-1) \cdot \sqrt{N^2-N}+2 \cdot N)-2 \cdot N^2)}{(((4 \cdot N-2) \cdot \sqrt{N^2-N}+4 \cdot N)-4 \cdot N^2)+1} - DE = 0.00000$

As one can see, the APT actually multiplies an angle.

Definitions.

$BD - (N - 1) = 0 \qquad DG - \sqrt{N^2 - N} = 0 \qquad BC - \frac{1}{2} = 0 \qquad CD - \frac{2 \cdot N - 1}{2} = 0 \qquad DF - \frac{2 \cdot N - 1}{4} = 0 \qquad BG - \left(\sqrt{N^2 - N} - N + 1\right) = 0 \qquad CG - \frac{2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1}{2} = 0$

$GW - \frac{\sqrt{(2 \cdot N - 1) \cdot \left(2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1\right)}}{\sqrt{2}} = 0 \qquad OW - \frac{\sqrt{2}}{2 \cdot \sqrt{(2 \cdot N - 1) \cdot \left(2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1\right)}} = 0 \qquad GO - \frac{(2 \cdot N - 1) \cdot \left(\sqrt{2} \cdot \sqrt{N^2 - N}\right) - 2 \cdot \sqrt{2} \cdot N \cdot (N - 1)}{\sqrt{(2 - 4 \cdot N) \cdot \sqrt{N^2 - N} + (2 \cdot N - 1)^2}} = 0$

$$GU - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot\sqrt{N^2 - N} + \left(12\cdot N^2 - 8\cdot N^3 - 4\cdot N\right)}{2\cdot\left[(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N + 1\right]} = 0 \qquad DU - \frac{\sqrt{N^2 - N}}{2\cdot(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad OT - \frac{(2\cdot N - 1)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N - 2\cdot N^2}{(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0$$

$$US - \frac{4\cdot N^2 - 4\cdot N - 1}{8\cdot(2\cdot N - 1)} = 0 \qquad DS - \frac{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1}{8\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad JR - \frac{(1 - 2\cdot N)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N^2 - 2\cdot N}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1} = 0 \qquad JF - \frac{(2\cdot N - 1)^2\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0$$

$$EJ - \frac{(2\cdot N - 1)\cdot\left[(2\cdot N - 1)\cdot\sqrt{N^2 - N} + 2\cdot N - 2\cdot N^2\right]}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0 \qquad FR - \frac{\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)\cdot\left(4\cdot N^2 - 4\cdot N - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0 \qquad DR - \frac{\sqrt{N^2 - N}}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0$$

$$ER - \frac{\sqrt{4\cdot N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N^2 - N} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N^2 - N} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} = 0$$

**Here is where Mathcad 15 Uncle call Uncle! It cannot reduce the following equation and I am at a loss as to how to effect reductions for it in the time I am willing to spend on it.**

$$DE - \left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}}\right] = 0$$

**DE − EJ = 0**

$$\left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} - \frac{(2\cdot N - 1)\cdot\left[(1 - 2\cdot N)\cdot\sqrt{N^2 - N} + 2\cdot N^2 - 2\cdot N\right]}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1}\right] = 0$$

**If you ask Mathcad to reduce the above equation, it will simply spred it out over several pages and quit.**

Unit.

$AB := 1$

Given.

$N := 1.43693 \quad AC := N$

**000822B**

**In the 2015 revision of the DQ, I got as fars as a blank Mathcad template like this one. So, yea, I have put doing this off for a special long time.**

**Descriptions.**

$$BO := \frac{AB}{2} \quad BC := AC - AB \quad DO := BO$$

$$CO := BO + BC \quad CD := \sqrt{CO^2 + DO^2}$$

$$DF := \frac{DO \cdot AB}{CD} \quad CF := CD - DF \quad CH := \frac{CD \cdot CF}{CO}$$

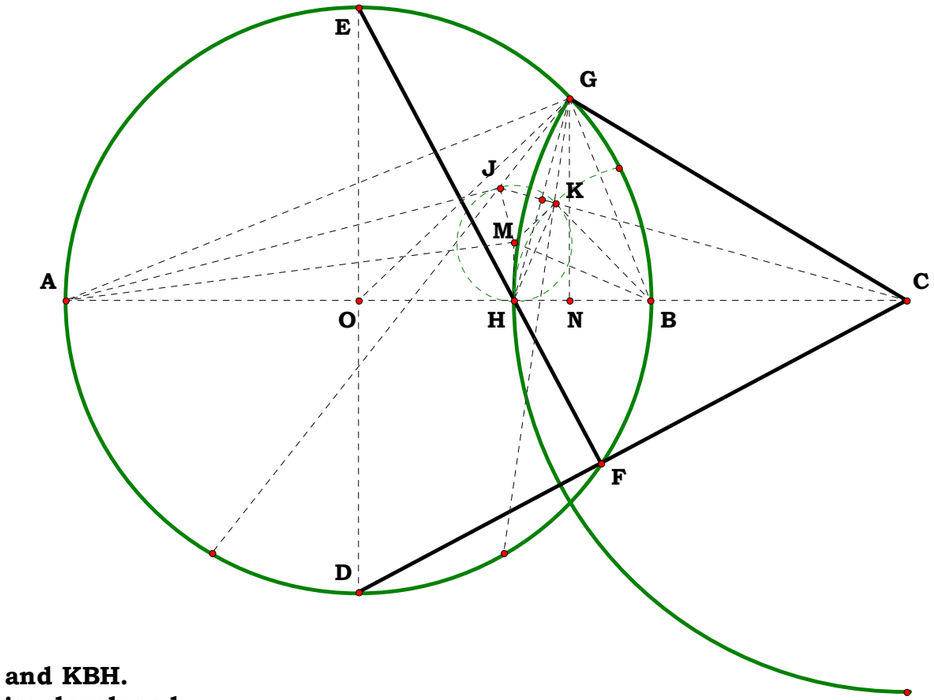
$$BH := CH - BC \quad GO := BO \quad CN := \frac{CO^2 - GO^2 + CH^2}{2 \cdot CO}$$

$$GN := \sqrt{CH^2 - CN^2} \quad BN := CN - BC$$

**To be completed by simply dividing the cords, GCH and KBH. This should help show that angle trisection, the entire developed figure is a proportion to the square root figure.**

**Definitions.**

**Show the trisection in a circle for any square root that also divides the circle into six equal cords.**




$m\angle KCH = 15.47574°$

$m\angle MBH = 22.73787°$

$m\angle GBH = 68.21360°$

$m\angle MBH = 22.73787°$

$\frac{m\angle GBH}{m\angle MBH} = 3.00000$

$m\angle EGF = 30.00000°$

$m\angle KJM = 60.00000°$

$N = 1.43693$

Unit.

$AB := 1$

Given.

$N := .3 \quad BC := N$

082300

Descriptions.

$AC := AB - N \qquad BE := \frac{BC}{2} \qquad AE := AC + BE$

$AN := \sqrt{BE^2 + AE^2} \qquad FN := \frac{BE \cdot BC}{AN} \qquad DE := \frac{BE^2}{AE}$

$BD := BE + DE \quad AD := AB - BD \qquad AH := \frac{AE^2 - BE^2 + AD^2}{2 \cdot AE}$

$BH := AB - AH \qquad CH := BC - BH$

Definitions.

$AC - (1 - N) = 0 \qquad BE - \frac{N}{2} = 0 \qquad AE - \left[\frac{(2 - N)}{2}\right] = 0$

$AN - \frac{\sqrt{N^2 - 2 \cdot N + 2}}{\sqrt{2}} = 0 \qquad FN - \frac{\sqrt{2} \cdot N^2}{2 \cdot \sqrt{N^2 - 2 \cdot N + 2}} = 0$

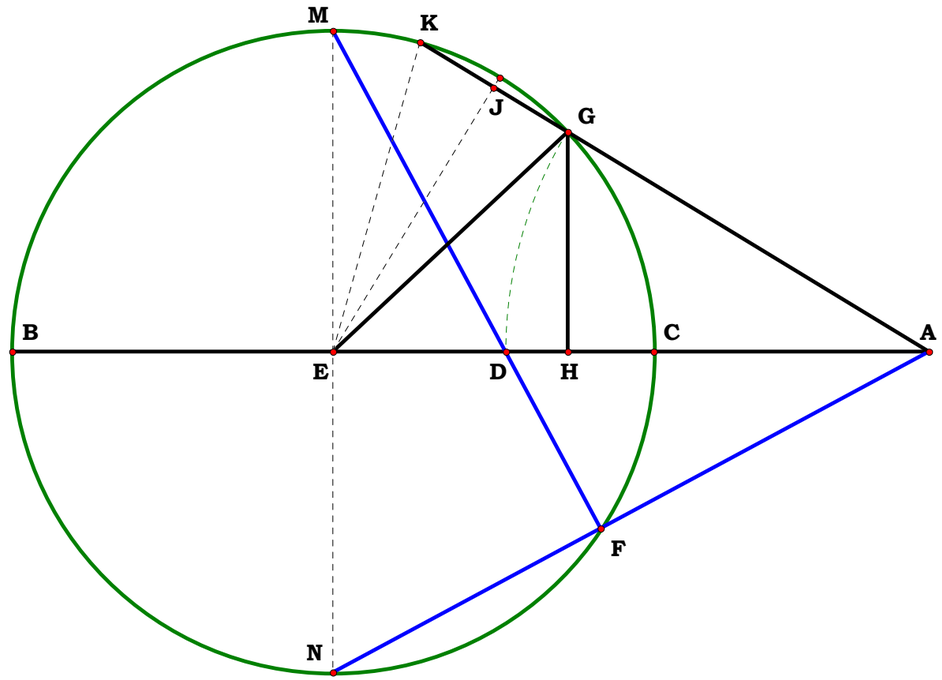
$DE - \frac{N^2}{2 \cdot (2 - N)} = 0$

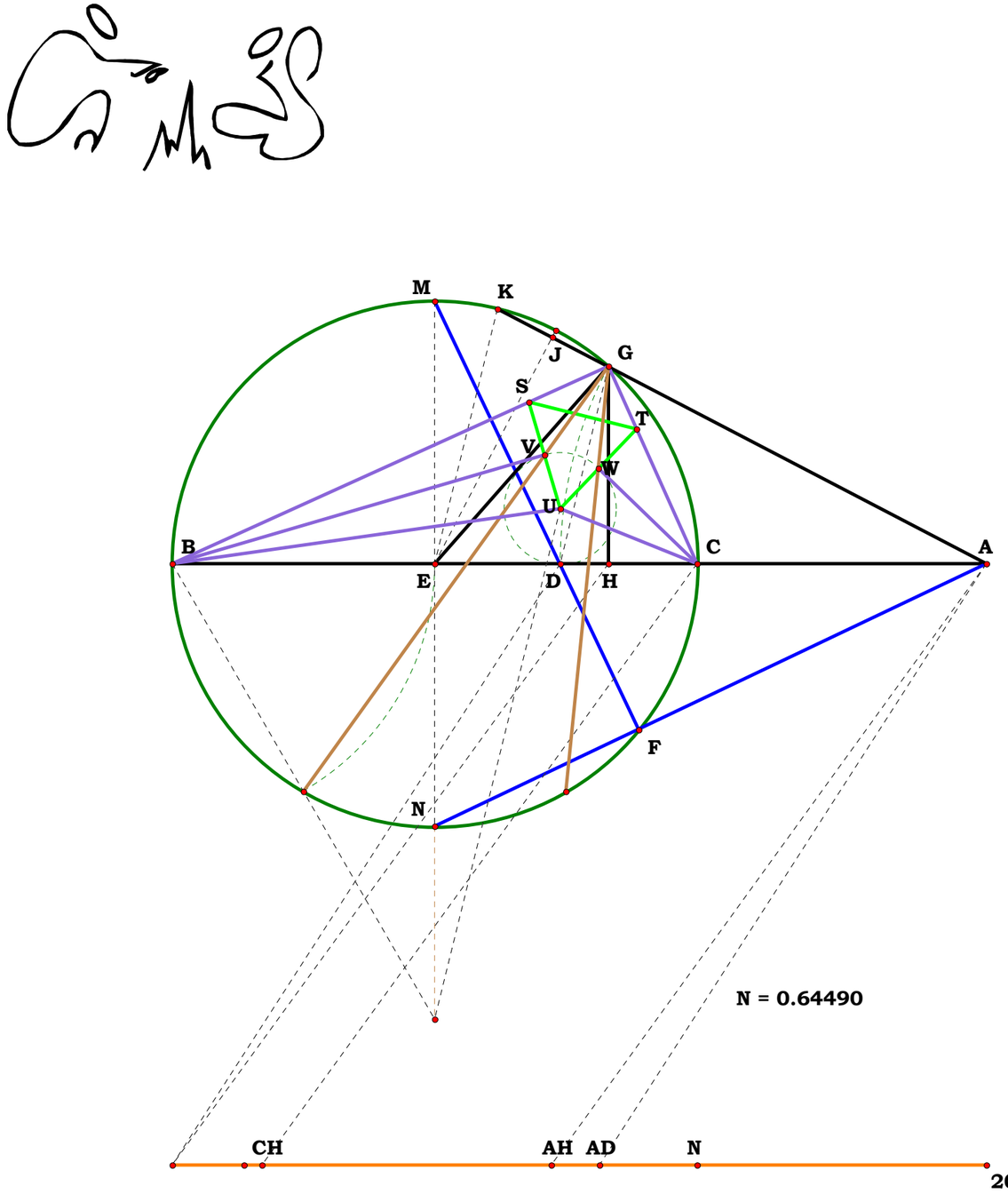
$BD - \left[\frac{N}{(2 - N)}\right] = 0 \qquad AD - \left[\frac{2 \cdot (1 - N)}{(2 - N)}\right] = 0 \qquad AH - \frac{(1 - N) \cdot \left(N^2 - 8 \cdot N + 8\right)}{(2 - N)^3} = 0$

$BH - \left[\frac{N \cdot (4 - 3 \cdot N)}{(2 - N)^3}\right] = 0 \qquad CH - \frac{N \cdot (4 - N) \cdot (1 - N)^2}{(2 - N)^3} = 0$

## Trisection In A Square Root Figure

**Given the square root figure drawn for trisection, what is AR given AB and AD? A slightly different apprach than the one on 04.**






BD = 0.47590

$\frac{N}{2-N}$ = 0.47590

$\frac{N}{2-N}$-BD = 0.00000

AD = 0.52410

$\frac{2\cdot(N-1)}{N-2}$ = 0.52410

$\frac{2\cdot(N-1)}{N-2}$-AD = 0.00000

AH = 0.46475

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3}$ = 0.46475

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3}$-AH = 0.00000

BH = 0.53525

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3}$ = 0.53525

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3}$-BH = 0.00000

CH = 0.10965

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3}$ = 0.10965

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3}$-CH = 0.00000

m∠GEM = 41.29679°

m∠KEM = 13.76560°

$\frac{m\angle GEM}{m\angle KEM}$ = 3.00000

m∠BGC = 90.00000°

m∠VGW = 30.00000°

$\frac{m\angle BGC}{m\angle VGW}$ = 3.00000

m∠GCD = 65.64840°

m∠UCD = 21.88280°

$\frac{m\angle GCD}{m\angle UCD}$ = 3.00000

m∠GBD = 24.35160°

m∠GBV = 8.11720°

$\frac{m\angle GBD}{m\angle GBV}$ = 3.00000

m∠UST = 60.00000°

BC = 0.64490

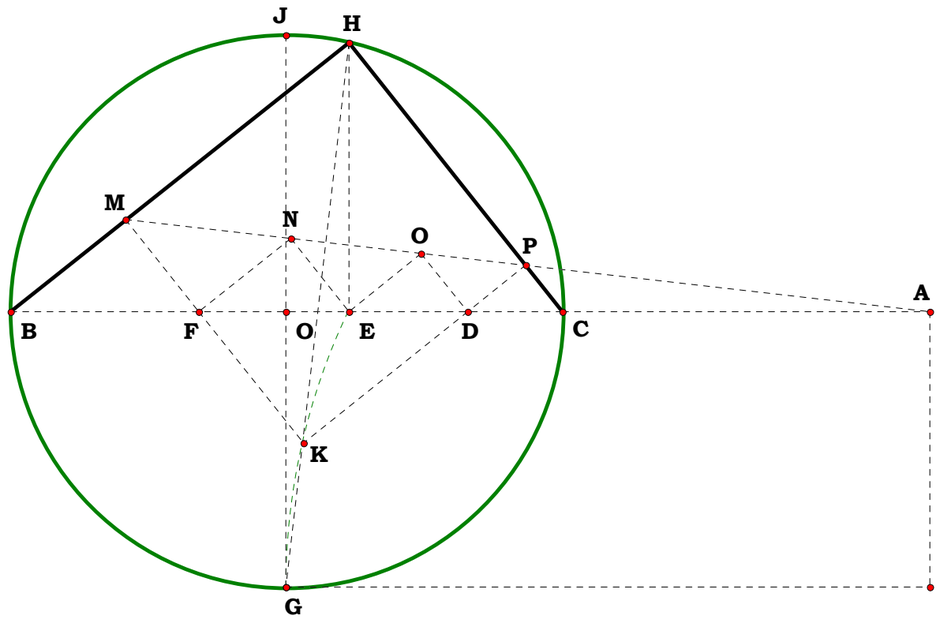
BC-N = 0.00000

**Unit.**
**Given.**
**Descriptions.**
**Definitions.**

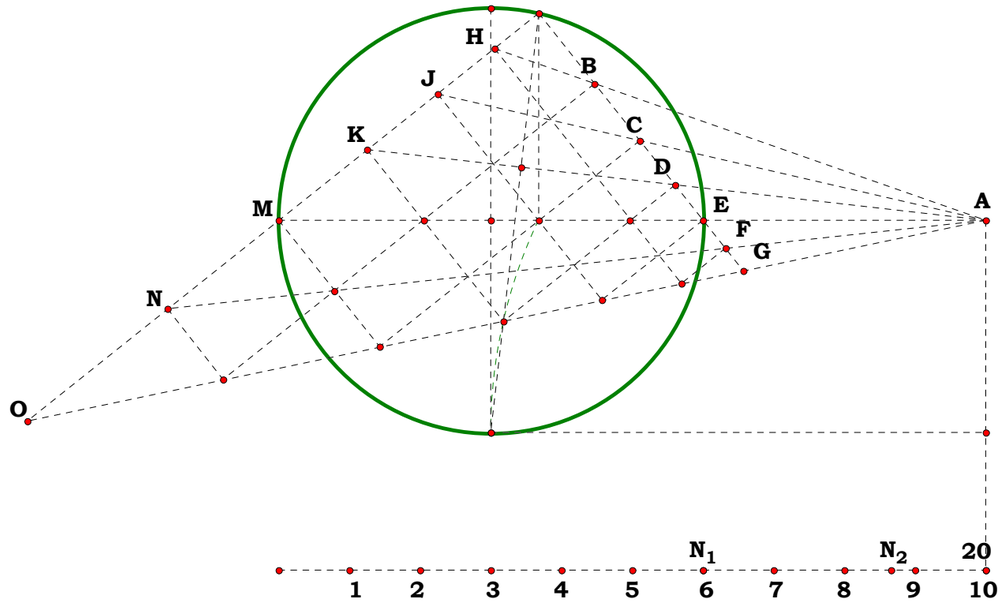
**Quad Roots Etc.**

**090100Z**

**I have had this plate in and out of revisions since 00 but it is so simple and straight forward, I just like looking at it.**




**And the figure can be expanded and this I find interesting.**




$\frac{AB}{AC}^{\frac{1}{4}} = 1.25743$ $\frac{AB}{AC}^{\frac{1}{4}} - \frac{AD}{AC} = 0.00000$

$\frac{AB}{AC}^{\frac{2}{4}} = 1.58114$ $\frac{AB}{AC}^{\frac{2}{4}} - \frac{AE}{AC} = 0.00000$

$\frac{AB}{AC}^{\frac{3}{4}} = 1.98818$ $\frac{AB}{AC}^{\frac{3}{4}} - \frac{AF}{AC} = 0.00000$

$\frac{AC}{AC} = 1.00000$

$\frac{AD}{AC} = 1.25743$

$\frac{AE}{AC} = 1.58114$

$\frac{AF}{AC} = 1.98818$

$\frac{AB}{AC} = 2.50000$

AC = 6.16580 cm
AD = 7.75308 cm
AE = 9.74899 cm
AO = 10.79015 cm
AF = 12.25870 cm
AB = 15.41450 cm

Unit.

$BG := 1$

Given.

$N_1 := 8$

$N_2 := 10$

090300A

Descriptions.

# Ratios In Trisection

**How does BF vary with BC? How does DF vary with BC?**

$BE := \frac{BG}{2}$ $EM := BE$ $BO := \sqrt{2 \cdot BE^2}$ $EN := BE$ $EK := \frac{BE \cdot BE}{BO}$

$KN := EN - EK$ $BK := \frac{BO}{2}$ $BN := \sqrt{BK^2 + KN^2}$ $BD := \frac{BN^2}{BG}$ $BC := BD - BD \cdot \frac{N_1}{N_2}$

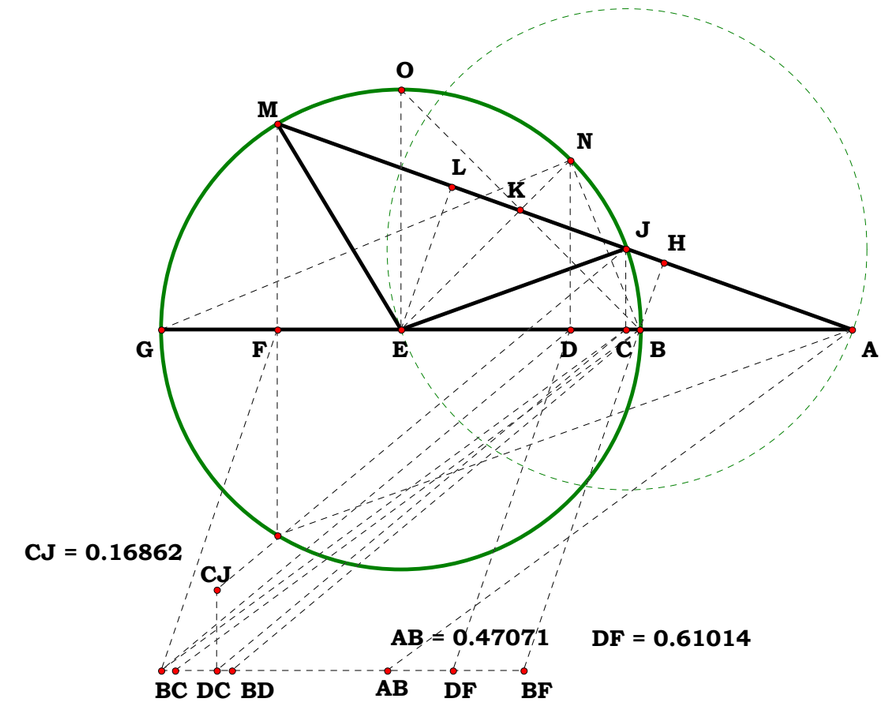
$CG := BG - BC$ $CJ := \sqrt{BC \cdot CG}$ $AJ := BE$ $AC := \sqrt{AJ^2 - CJ^2}$ $AB := AC - BC$

$AE := AB + BE$ $JH := \frac{CJ^2}{AJ}$ $AH := AJ - JH$ $AL := \frac{AH \cdot AE}{AC}$ $JL := AL - AJ$

$LM := JL$ $AM := AL + LM$ $AF := \frac{AH \cdot AM}{AC}$ $BF := AF - AB$ $DF := BF - BD$

$CJ = 0.168616$ $BD = 0.146447$ $DF = 0.610136$ $BF = 0.756583$

$BC = 0.029289$ $CG = 0.970711$ $AB = 0.441421$ $DF = 0.610136$




$BC = 0.02929$ $BD = 0.14645$ $BF = 0.75658$

$CG = 0.97071$ $DC = 0.11716$

Definitions.

$BC - \frac{\left(\sqrt{2} - 2\right) \cdot \left(N_1 - N_2\right)}{4 \cdot N_2} = 0$ $BE - \frac{1}{2} = 0$ $EM - \frac{1}{2} = 0$ $BO - \frac{\sqrt{2}}{2} = 0$ $EN - \frac{1}{2} = 0$

$EK - \frac{\sqrt{2}}{4} = 0$ $KN - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) = 0$ $BK - \frac{\sqrt{2}}{4} = 0$ $BN - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$BD - \frac{2 - \sqrt{2}}{4} = 0$ $BC - \frac{\left(\sqrt{2} - 2\right) \cdot \left(N_1 - N_2\right)}{4 \cdot N_2} = 0$ $CG - \frac{\left(N_2 - N_1\right) \cdot \left(\sqrt{2} + 2\right) + 4 \cdot N_1}{4 \cdot N_2} = 0$ $CJ - \frac{\sqrt{\left(N_1 - N_2\right) \cdot \left[N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right) - N_2\right]}}{2\sqrt{2} \cdot N_2} = 0$ $AJ - \frac{1}{2} = 0$ $AC - \frac{N_2 + N_1 \cdot \left(\sqrt{2} - 1\right)}{2\sqrt{2} \cdot N_2} = 0$

$AB - \frac{N_2 \cdot \left(\sqrt{2} - 1\right) - N_1 \cdot \left(\sqrt{2} - 2\right)}{2 \cdot N_2} = 0$ $AE - \frac{\sqrt{2} \cdot N_2 - N_1 \cdot \left(\sqrt{2} - 2\right)}{2 \cdot N_2} = 0$ $JH - \frac{\left(N_1 - N_2\right) \cdot \left[N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right) - N_2\right]}{4 \cdot N_2^2} = 0$ $AH - \frac{\left[N_2 + N_1 \cdot \left(\sqrt{2} - 1\right)\right]^2}{4 \cdot N_2^2} = 0$

$AL - \frac{\left(2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right) \cdot \sqrt{2} + 3 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2}{2 \cdot N_2^2} = 0$ $JL - \frac{N_1 \cdot \left[N_2 \cdot \left(2 \cdot \sqrt{2} - 2\right) - N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right)\right]}{2 \cdot N_2^2} = 0$ $LM - \frac{N_1 \cdot \left[N_2 \cdot \left(2 \cdot \sqrt{2} - 2\right) - N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right)\right]}{2 \cdot N_2^2} = 0$

$$BF - \frac{\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{{N_2}^3}=0$$

$$AM - \left[\frac{\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left[N_2+N_1\cdot\left(3\cdot\sqrt{2}-4\right)\right]}{2\cdot{N_2}^2}\right]=0 \qquad AF - \frac{\sqrt{2}\cdot\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-N_1+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-4\cdot N_1+3\cdot\sqrt{2}\cdot N_1\right)}{4\cdot{N_2}^3}=0$$

$$DF - \left[\frac{3\cdot N_1\cdot\left[\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot{N_1}^2+\left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2+\left(2-\sqrt{2}\right)\cdot{N_2}^2\right]}{4\cdot{N_2}^3}\right]=0$$

$$\frac{BF}{BC}=25.831354 \qquad \frac{DF}{BC}=20.831354$$

$$\frac{BF}{BC} - \frac{4\cdot\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{{N_2}^2\cdot\left(\sqrt{2}-2\right)\cdot\left(N_1-N_2\right)}=0$$

$$\frac{DF}{BC} - \frac{\left(6\cdot{N_1}^2\cdot N_2-3\cdot N_1\cdot{N_2}^2+4\cdot\sqrt{2}\cdot{N_1}^3-6\cdot{N_1}^3-6\cdot\sqrt{2}\cdot{N_1}^2\cdot N_2\right)}{{N_2}^2\cdot\left(N_1-N_2\right)}=0$$

**Unit.**

$BC := 1$

**Given.**

$N_1 := 10$

$N_2 := 8$

**090300B**

**Descriptions.**

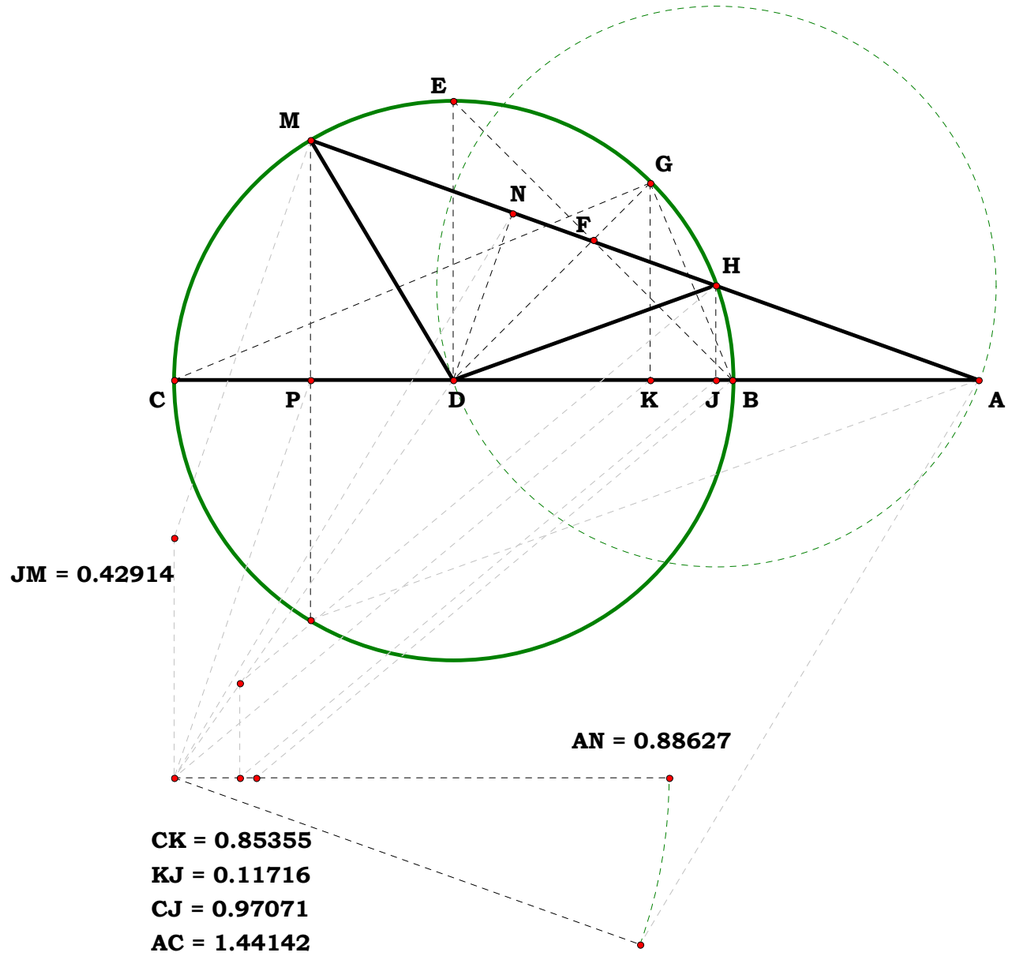
$BD := \frac{BC}{2}$ $DE := BD$ $BE := \sqrt{2 \cdot BD^2}$ $DK := \frac{BD^2}{BE}$ $BK := BD - DK$ $KJ := BK \cdot \frac{N_2}{N_1}$

$CK := BD + DK$ $CJ := CK + KJ$ $BJ := BC - CJ$ $HJ := \sqrt{CJ \cdot BJ}$ $AJ := \sqrt{BD^2 - HJ^2}$

$AB := AJ - BJ$ $AB = 0.441421$ $AN := \frac{AJ \cdot (BD + AB)}{BD}$ $HN := AN - BD$ $MN := HN$

$AM := AN + MN$ $MP := \frac{HJ \cdot AM}{BD}$ $MP = 0.429145$ $\frac{HJ}{MP} = 0.392912$

$BR := \frac{HJ \cdot AB}{AJ}$ $AP := \frac{AB \cdot MP}{BR}$ $BP := AP - AB$ $CP := BC - BP$ $KP := BP - BK$




**Definitions.**

$BD - \frac{1}{2} = 0$ $DE - \frac{1}{2} = 0$ $BE - \frac{1}{\sqrt{2}} = 0$ $DK - \frac{\sqrt{2}}{4} = 0$ $BK - \frac{2 - \sqrt{2}}{4} = 0$

$KJ - \frac{N_2 \cdot (2 - \sqrt{2})}{4 \cdot N_1} = 0$ $CK - \frac{2 + \sqrt{2}}{4} = 0$ $CJ - \frac{N_1 \cdot (\sqrt{2} + 2) - N_2 \cdot (\sqrt{2} - 2)}{4 \cdot N_1} = 0$

$BJ - \frac{(N_2 - N_1) \cdot (\sqrt{2} - 2)}{4 \cdot N_1} = 0$ $HJ - \frac{\sqrt{(N_1 - N_2) \cdot (N_1 + 3 \cdot N_2 - 2 \cdot \sqrt{2} \cdot N_2)}}{2 \cdot \sqrt{2} \cdot N_1} = 0$

$AJ - \frac{N_1 + N_2 \cdot (\sqrt{2} - 1)}{2 \cdot \sqrt{2} \cdot N_1} = 0$ $AB - \left[\frac{N_1 \cdot (\sqrt{2} - 1) - N_2 \cdot (\sqrt{2} - 2)}{2 \cdot N_1}\right] = 0$

$AN - \frac{2 \cdot N_2 \cdot (N_1 - N_2) \cdot \sqrt{2} + N_1^2 - 2 \cdot N_1 \cdot N_2 + 3 \cdot N_2^2}{2 \cdot N_1^2} = 0$ $HN - \frac{N_2 \cdot \left[N_1 \cdot (2 \cdot \sqrt{2} - 2) - N_2 \cdot (2 \cdot \sqrt{2} - 3)\right]}{2 \cdot N_1^2} = 0$ $AM - \left[\frac{\left(N_1 + \sqrt{2} \cdot N_2\right) \cdot \left[N_1 + N_2 \cdot (3 \cdot \sqrt{2} - 4)\right]}{2 \cdot N_1^2}\right] = 0$

$MP = 0.429145$

$HJ = 0.168616$

$$MP - \frac{\left(N_1 + \sqrt{2}\cdot N_2\right)\cdot\sqrt{2\cdot\left(N_1 - N_2\right)\cdot\left(N_1 + 3\cdot N_2 - 2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left(N_1 - 4\cdot N_2 + 3\cdot\sqrt{2}\cdot N_2\right)}{4\cdot {N_1}^3} = 0$$

$$BR - \frac{\sqrt{\left(N_1 - N_2\right)\cdot\left(N_1 + 3\cdot N_2 - 2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left[N_1\cdot\left(\sqrt{2}-1\right) - N_2\cdot\left(\sqrt{2}-2\right)\right]}{2\cdot N_1\cdot\left[N_1 + N_2\cdot\left(\sqrt{2}-1\right)\right]} = 0$$

$$AP - \frac{\left(N_1 + \sqrt{2}\cdot N_2\right)\cdot\sqrt{\left(2\cdot N_1 - 2\cdot N_2\right)\cdot\left(N_1 + 3\cdot N_2 - 2\cdot\sqrt{2}\cdot N_2\right)}\cdot\left[N_1 + N_2\cdot\left(\sqrt{2}-1\right)\right]\cdot\left[N_1 + N_2\cdot\left(3\cdot\sqrt{2}-4\right)\right]}{4\cdot {N_1}^3\cdot\sqrt{\left(N_1 - N_2\right)\cdot\left(N_1 + 3\cdot N_2 - 2\cdot\sqrt{2}\cdot N_2\right)}} = 0$$

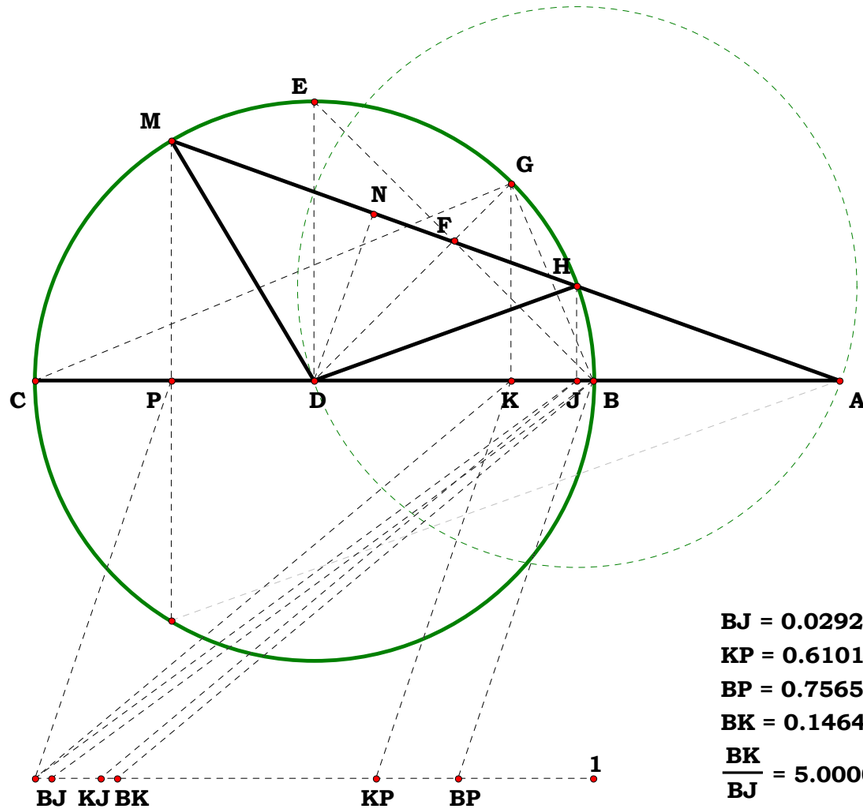
$$BP - \frac{\left[\left(\frac{\sqrt{2}}{8} + \frac{1}{4}\right)\cdot\left[N_1 - N_2\cdot\left(2\cdot\sqrt{2}-3\right)\right]\cdot\left[N_2\cdot\left(2\cdot\sqrt{2}-2\right) - N_1\cdot\left(\sqrt{2}-2\right)\right]^2\right]}{{N_1}^3} = 0$$

$$CP - \frac{\left(\frac{1}{4} - \frac{\sqrt{2}}{8}\right)\cdot\left(N_1 - N_2\right)\cdot\left[N_1\cdot\left(\sqrt{2}+2\right) + N_2\cdot\left(2\cdot\sqrt{2}-2\right)\right]^2}{{N_1}^3} = 0$$

$$KP - \frac{3\cdot N_2\cdot\left[\left(2-\sqrt{2}\right)\cdot {N_1}^2 + \left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2 + \left(\frac{20}{3} - \frac{14\cdot\sqrt{2}}{3}\right)\cdot {N_2}^2\right]}{4\cdot {N_1}^3} = 0$$

$$\frac{BP}{BJ} - \frac{\left(12\cdot\sqrt{2}+17\right)\cdot\left[N_1 - N_2\cdot\left(2\cdot\sqrt{2}-3\right)\right]\cdot\left[N_2\cdot\left(3\cdot\sqrt{2}-4\right) - N_1\cdot\left(2\cdot\sqrt{2}-3\right)\right]^2}{{N_1}^2\cdot\left(N_1 - N_2\right)} = 0$$

$$\frac{KP}{BJ} - \frac{\left(\frac{3\cdot\sqrt{2}}{2} + 3\right)\cdot N_2\cdot\left[\left(2-\sqrt{2}\right)\cdot {N_1}^2 + \left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2 + \left(\frac{20}{3} - \frac{14\cdot\sqrt{2}}{3}\right)\cdot {N_2}^2\right]}{{N_1}^2\cdot\left(N_1 - N_2\right)} = 0$$




$BJ = 0.02929$

$KP = 0.61014$

$BP = 0.75658$

$BK = 0.14645$

$\frac{BK}{BJ} = 5.00000$

$KJ = 0.11716$

$\frac{KJ}{BK} = 0.80000$

$\frac{BK}{BK} = 1.00000$

$\frac{KJ}{BK} = 0.80000$

$\frac{BJ}{BK} = 0.20000$

$\frac{2}{10} = 0.20000$

$$\frac{BP}{BJ} = 25.831354 \qquad BJ - \frac{(N_2 - N_1)\cdot(\sqrt{2}-2)}{4\cdot N_1} = 0 \qquad BK - \frac{2-\sqrt{2}}{4} = 0$$

$$\frac{KP}{BJ} = 20.831354$$

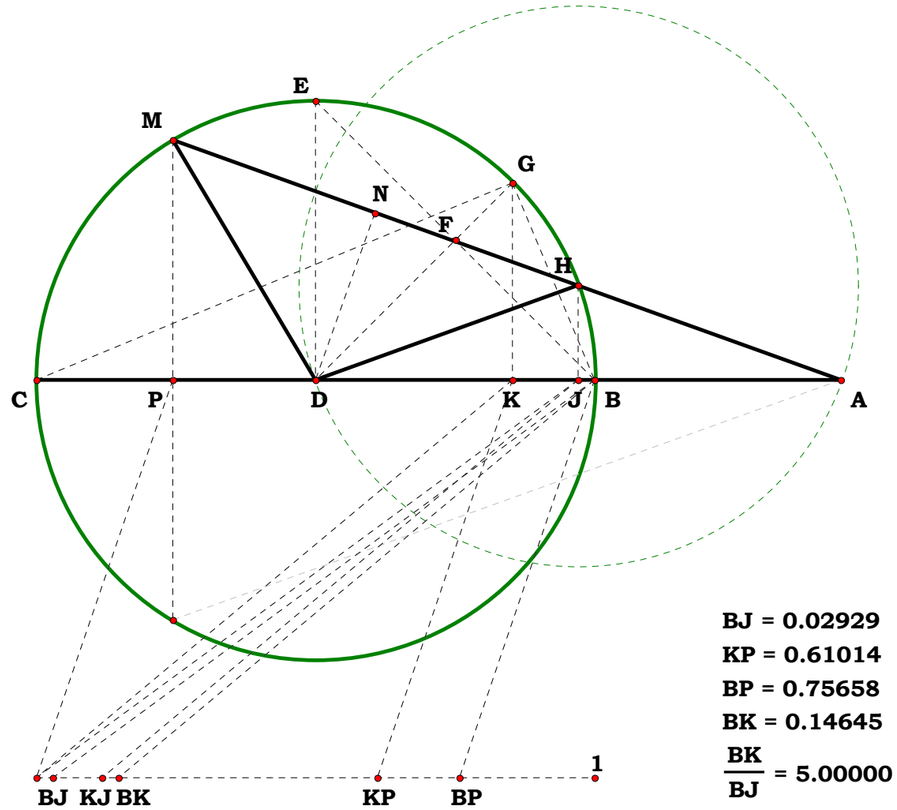
$$BP - \frac{\left[\left(\frac{\sqrt{2}}{8}+\frac{1}{4}\right)\cdot\left[10-8\cdot(2\cdot\sqrt{2}-3)\right]\cdot\left[8\cdot(2\cdot\sqrt{2}-2)-10\cdot(\sqrt{2}-2)\right]^2\right]}{10^3} = 0$$

**Which means that you can write a simple program using whole numbers for N1 and N2 to find BJ as a unit to name BP and then you will know the name of BJ. A ratio is a unit conversion.**

$$\frac{BJ}{BK} = 0.2 \qquad \frac{BJ}{BK} - \frac{N_1 - N_2}{N_1} = 0 \qquad N_1 = 10 \qquad N_2 = 8 \qquad \frac{1}{1000000000000000} = 1\times 10^{-15}$$

**Which is the ratio one uses all the time when dividing a simple segment. I trust one can write a program to find any given ratio with it? N1 simply sets the precision of the Arithmetic name.**

$$KP - \frac{3\cdot 8\cdot\left[\left(2-\sqrt{2}\right)\cdot 10^2+\left(6\cdot\sqrt{2}-8\right)\cdot 10\cdot 8+\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot 8^2\right]}{4\cdot 10^3} = 0$$




BJ = 0.02929
KP = 0.61014
BP = 0.75658
BK = 0.14645
KJ = 0.11716

$\frac{BK}{BK} = 1.00000$ $\frac{KJ}{BK} = 0.80000$ $\frac{BK}{BJ} = 5.00000$ $\frac{BJ}{BK} = 0.20000$ $\frac{KJ}{BK} = 0.80000$ $\frac{2}{10} = 0.20000$

Unit.

$BC := 1$

Given.

$N := 2$

091600

Descriptions.

$BJ := N \quad BE := \sqrt{BC \cdot BJ} \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IO := CI \quad NO := CJ \quad CR := CJ \quad CE := BE - BC$

$EI := CI - CE \quad EJ := CJ - CE \quad EL := \sqrt{CE \cdot EJ}$

$EG := \frac{EI \cdot EL}{EL + IO} \quad GI := EI - EG \quad GO := \sqrt{GI^2 + IO^2}$

$OP := GO \quad IP := IO + OP \quad EF := \frac{EI \cdot EL}{EL + IP} \quad FI := EI - EF$

$FO := \sqrt{FI^2 + IO^2} \quad OK := \frac{IO \cdot NO}{FO} \quad FK := OK - FO$

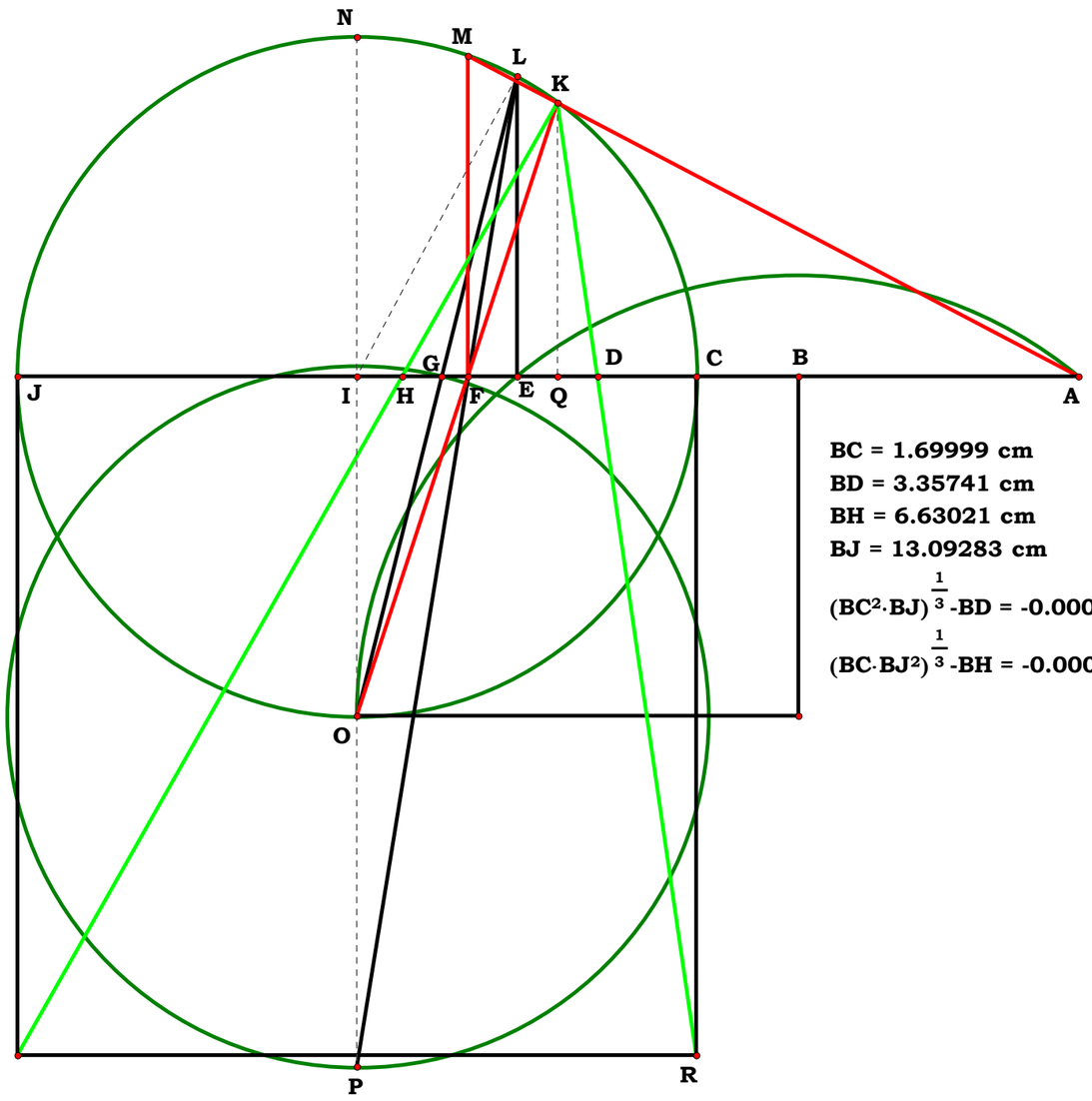
$FQ := \frac{FI \cdot FK}{FO} \quad QI := FQ + FI \quad CQ := CI - QI \quad QJ := CJ - CQ$

$QK := \sqrt{CQ \cdot QJ} \quad CD := \frac{CQ \cdot CR}{CR + QK} \quad BD := CD + BC$

$\left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 4.486958 \times 10^{-6}$

Definitions.

**Goshdarn Good Pencil**

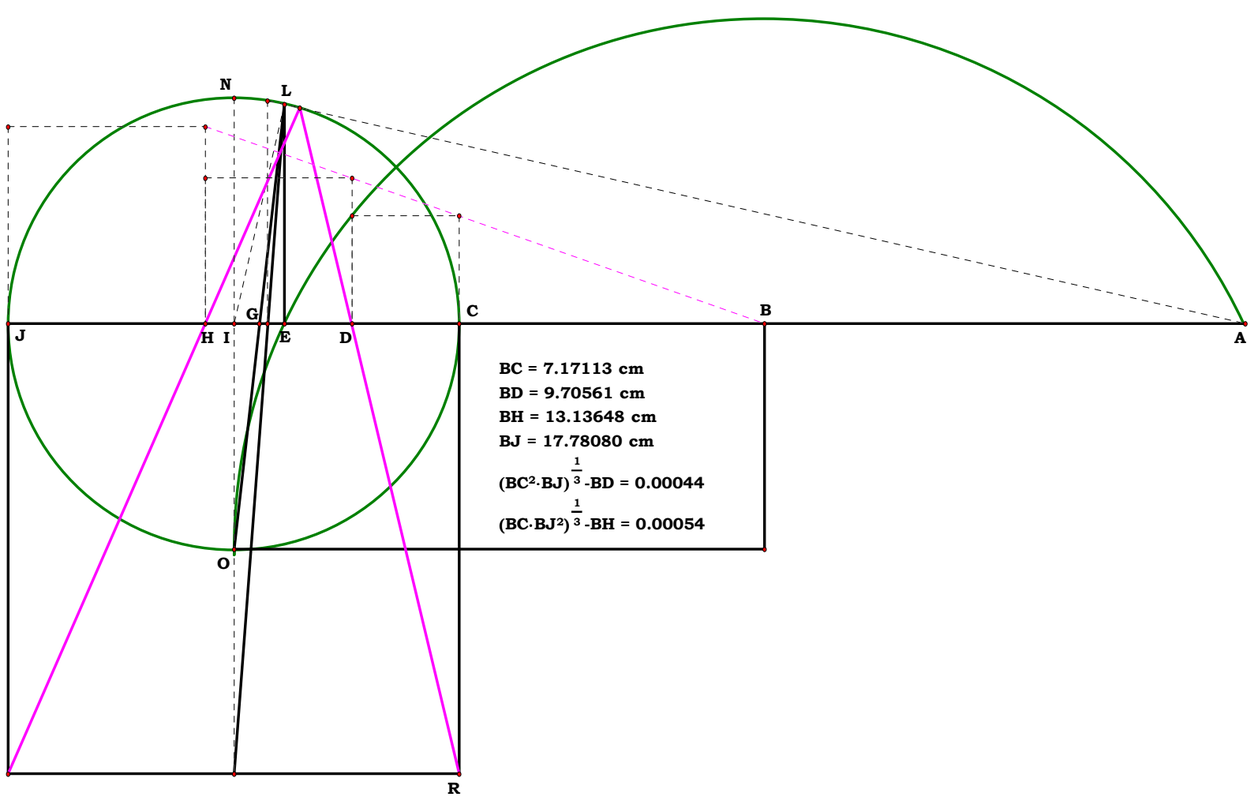


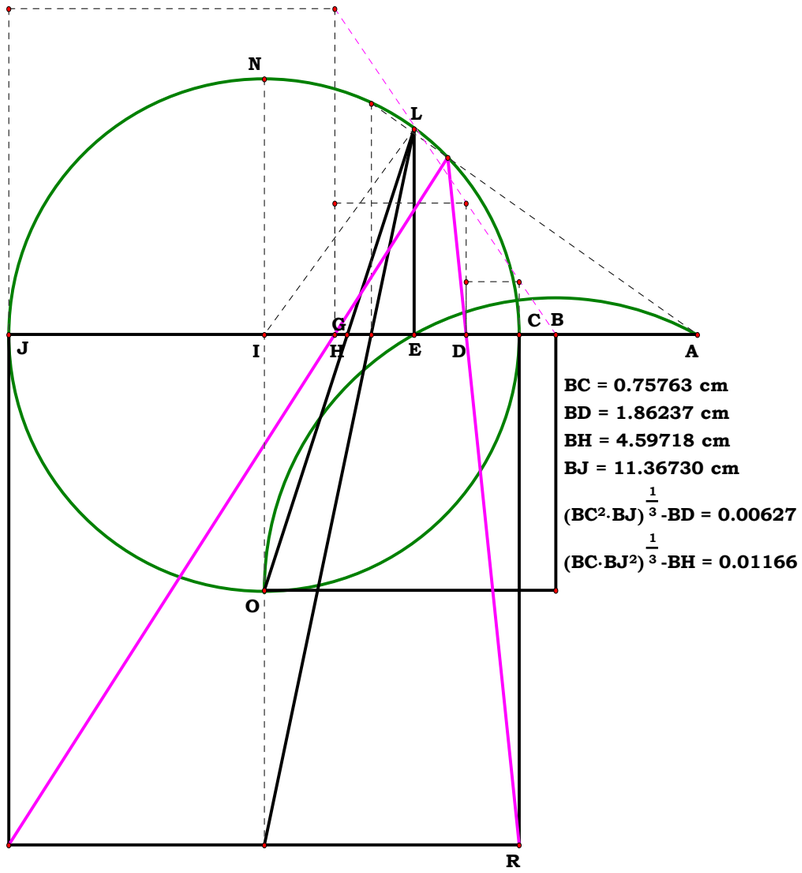

BC = 1.69999 cm
BD = 3.35741 cm
BH = 6.63021 cm
BJ = 13.09283 cm

$(BC^2 \cdot BJ)^{\frac{1}{3}} - BD = -0.00022$

$(BC \cdot BJ^2)^{\frac{1}{3}} - BH = -0.00035$

**Compare to the following which is a great deal less accurate. When you work the figure, you cannot tell it is off:**



**091800A**

**Unit.**

**Given.**

$N_1 := 2$ $BC := N_1$ $CJ := BC$ $GK := CJ$

$N_2 := 3$ $GH := N_2$ $GM := GH$

$N_3 := 8$ $CG := N_3$ $JK := CG$

**Descriptions.**

$BH := BC + CG + GH$ $BE := \frac{BH}{2}$ $KM := GM - GK$

$JK := CG$ $AG := \frac{JK \cdot GM}{KM}$ $AH := AG + GH$

$AB := AH - BH$ $AE := AB + BE$

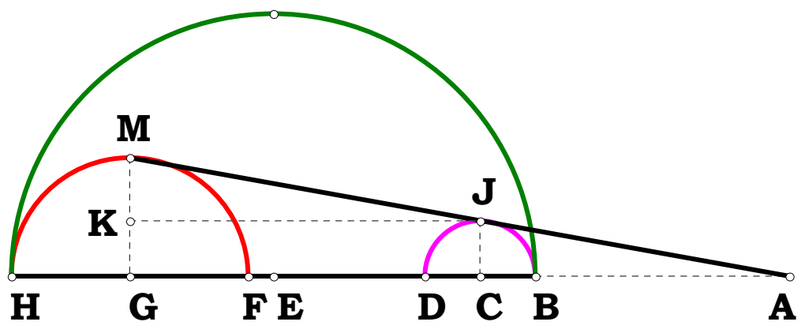
**Definitions.**

$BH - (N_1 + N_3 + N_2) = 0$ $BE - \frac{N_1 + N_3 + N_2}{2} = 0$ $KM - (N_2 - N_1) = 0$

$AG - \frac{N_3 \cdot N_2}{N_2 - N_1} = 0$ $AH - \frac{N_2 \cdot (N_1 - N_2 - N_3)}{N_1 - N_2} = 0$

$AB - \frac{N_1 \cdot (N_1 - N_2 + N_3)}{(N_2 - N_1)} = 0$ $AE - \frac{(N_1 - N_2)^2 + N_3 \cdot (N_1 + N_2)}{2 \cdot (N_2 - N_1)} = 0$

**Midpoints and Similarity Points**




**What is AE given the radius of the two circles and the difference between their centers? (External Unit).**

Unit.

$BH := 1$

Given.

$N_1 := 3 \quad N_3 := 4$

$N_2 := 7 \quad N_4 := 9$

**091800B**

**Descriptions.**

$$BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2} \quad GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$$

$$CJ := BC \quad GM := GH \quad GK := CJ \quad KM := GM - GK \quad JK := CG$$

$$AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$$

**Definitions.**

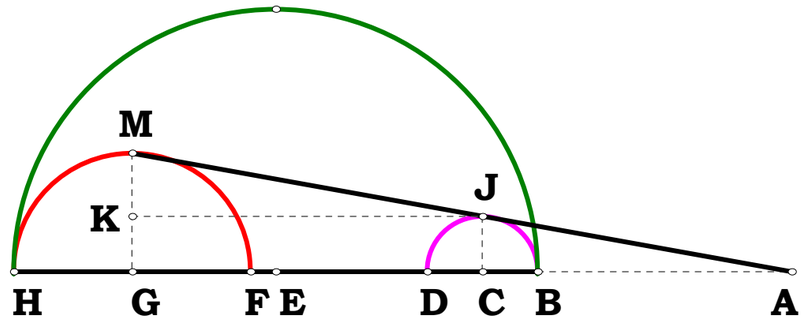
$$BE - \frac{1}{2} = 0 \quad BC - \frac{N_1}{N_2} = 0 \quad GH - \frac{N_3}{N_4} = 0 \quad CG - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$CJ - \frac{N_1}{N_2} = 0 \quad GM - \frac{N_3}{N_4} = 0 \quad GK - \frac{N_1}{N_2} = 0 \quad KM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0 \quad JK - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$AG - \frac{N_3 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3 - N_2 \cdot N_4\right)}{N_4 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0 \quad AH - \frac{N_3 \cdot \left(2 \cdot N_1 - N_2\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0 \quad AB - \frac{N_1 \cdot \left(2 \cdot N_3 - N_4\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0$$

$$AE - \frac{4 \cdot N_1 \cdot N_3 - N_1 \cdot N_4 - N_2 \cdot N_3}{2 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0$$

**Midpoints and Similarity Points**




**What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).**

Unit.

Given.

$N_1 := 11.69458$ $BE := N_1$

$N_2 := 2.96916$ $BC := N_2$

000920A

Descriptions.

$BD := \frac{BE}{2}$ $CD := BD - BC$ $CH := BD$

$DH := \sqrt{BD^2 + CD^2}$ $DG := \frac{DH^2}{2BD}$ $AD := \frac{BD \cdot DG}{CD}$

$AE := AD + BD$ $AB := AE - BE$ $AC := BC + AB$

$AC - \sqrt{AB \cdot AE} = 0$

Definitions.

$BD - \frac{N_1}{2} = 0$ $CD - \left(\frac{N_1 - 2 \cdot N_2}{2}\right) = 0$ $CH - \frac{N_1}{2} = 0$

$DH - \frac{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}}{\sqrt{2}} = 0$

$DG - \frac{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}{2 \cdot N_1} = 0$

$AD - \frac{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}{2 \cdot (N_1 - 2 \cdot N_2)} = 0$ $AE - \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0$

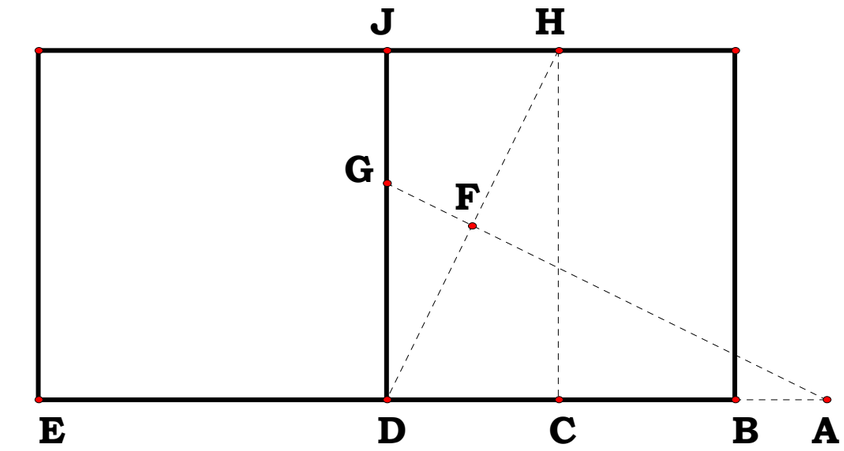
$AB - \frac{N_2^2}{N_1 - 2 \cdot N_2} = 0$ $AC - \frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0$

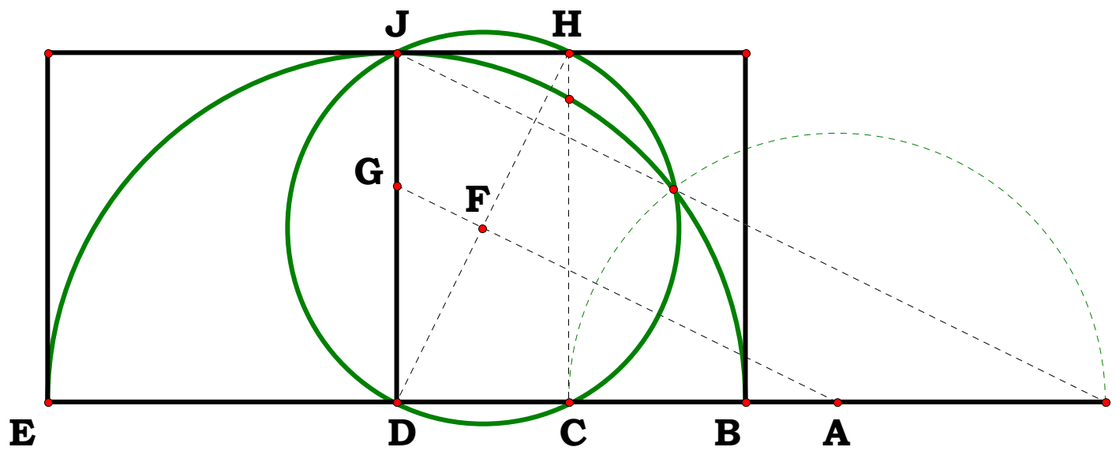
## Squaring

**Is AC the square root of AB x AE?**

**Given BC, find AB such that AB x AE is the square root.**

**This plate solves for the figure using a traditional square root figure.**
**The next two plates, which I put off doing until now, will approach the figure differently and the last might be a surprise but it was what I was pondering when I set up the original figure for solving cube roots. The last figure will actuall glue both of these figures together. Now, if one had glued them both together, one would be hard pressed to prove that cube roots are impossible in geometry as the compound figure makes the claim rather dubious.**



Unit.

$BE := 1$

Given.

$N_1 := 5$

$N_2 := 20$

092000B

Descriptions.

$DE := \frac{BE}{2}$ $BD := DE$ $CD := BD \cdot \frac{N_1}{N_2}$

$CE := CD + DE$ $DF := \sqrt{CD^2 + BD^2}$

$DG := \frac{DF}{2}$ $AD := \frac{DF \cdot DG}{CD}$ $AE := AD + DE$

$AB := AD - BD$ $AC := AD - CD$

Definitions.

$DE - \frac{1}{2} = 0$ $BD - \frac{1}{2} = 0$ $CD - \frac{N_1}{2 \cdot N_2}$

$CE - \frac{N_1 + N_2}{2 \cdot N_2} = 0$ $DF - \frac{\sqrt{N_1^2 + N_2^2}}{2 \cdot N_2} = 0$

$DG - \frac{\sqrt{N_1^2 + N_2^2}}{4 \cdot N_2} = 0$ $AD - \frac{N_1^2 + N_2^2}{4 \cdot N_1 \cdot N_2} = 0$ $AE - \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$

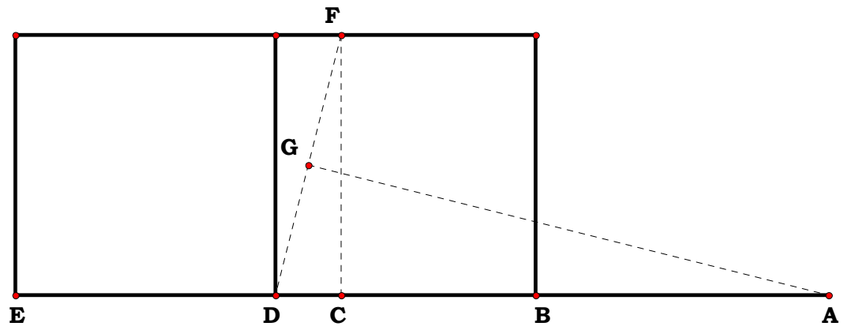
$AB - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$ $AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$

$AB \cdot AE - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} \cdot \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$ $AC - \sqrt{\frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2}{16 \cdot N_1^2 \cdot N_2^2}} = 0$ $AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$

## Squaring

**Is AC the square root of AB x AE?**

**Given BC, find AB such that AB x AE is the square root.**




BE = 1.00000

DE = 0.50000

CE = 0.62500

AE = 1.56250

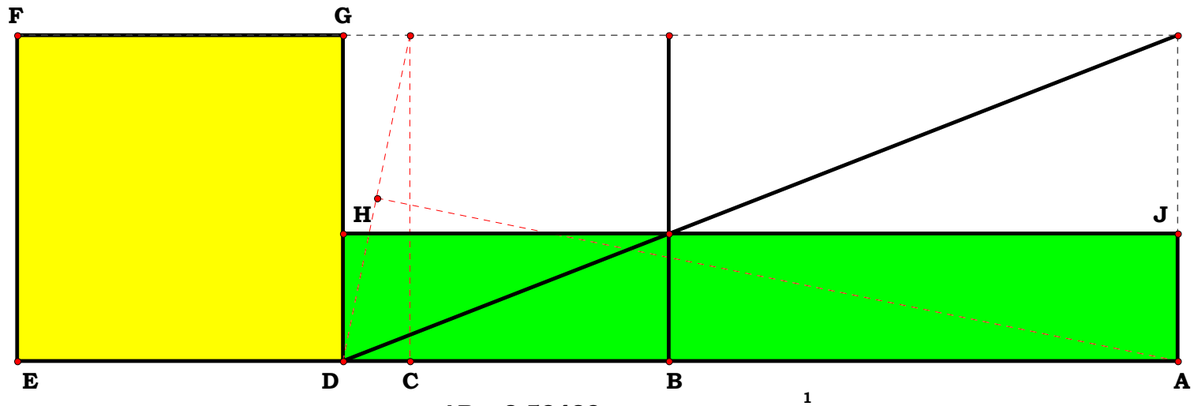
$\frac{DC}{DB} = 0.25000$

Unit.
Given.
Descriptions.
Definitions.

092000B

I am not going to write this up as it is obvious. One will notice that the part of the figure with the green in it is a figure in the Elements involving complements. The original two ways to express the same figure produces a cube root figure, but one which is not exactly as we would hope for.
The figure does, however, disprove the claim that cube roots cannot be done at all when it is actually a primive.

## Cube Root Primitive




Area FGDE = 29.87898 cm²
Area HJAD = 29.87898 cm²

DE = 5.46617 cm
DH = 2.13559 cm
AD = 13.99098 cm

AB = 8.52482 cm
AC = 12.87900 cm
AE = 19.45715 cm

$(AB \cdot AE)^{\frac{1}{2}} - AC = 0.00000$
$AB \cdot AC \cdot AE = 2136.22303\ cm^3$

$DE \cdot DH \cdot AD = 163.32347\ cm^3$
$(AB \cdot AC \cdot AE)^{\frac{1}{3}} - AC = 0.00000$
$(DE \cdot DH \cdot AD)^{\frac{1}{3}} - DE = 0.00000$

111300

Descriptions.

Unit.

Given.

Definitions.

$N_1 := 3$ $AB := N_1$

$N_2 := 5$ $AD := N_2$

$N_3 := 1$ $DE := N_3$

## For Two Right Triangles.

Given AB, DE, AD find BE, AC, CD, CE, BC.

BAD and BED are right.

$BD := \sqrt{AB^2 + AD^2}$ $BF := \frac{AB^2}{BD}$ $DG := \frac{DE^2}{BD}$ $BE := \sqrt{BD^2 - DE^2}$

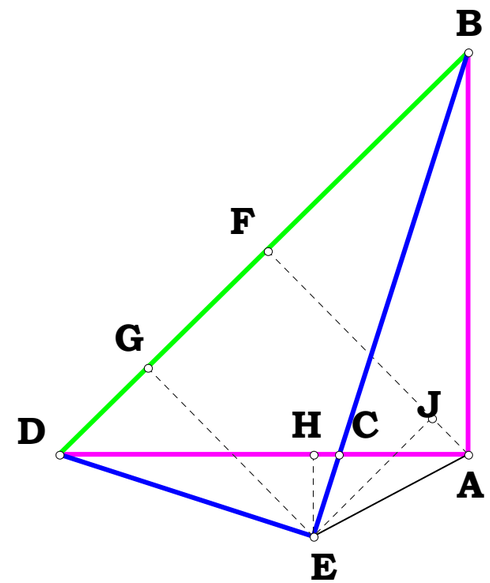
$AF := \sqrt{AB^2 - BF^2}$ $EG := \sqrt{DE^2 - DG^2}$ $FG := BD - (BF + DG)$

$EJ := FG$ $FJ := EG$ $AJ := AF - FJ$ $AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD$ $S_2 := DE$ $S_3 := AE$ $AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$EH := \sqrt{AE^2 - AH^2}$ $CH := \frac{EH \cdot AH}{AB + EH}$ $AC := AH - CH$

$CE := \frac{AC \cdot DE}{AB}$ $CD := AD - AC$ $BC := BE - CE$

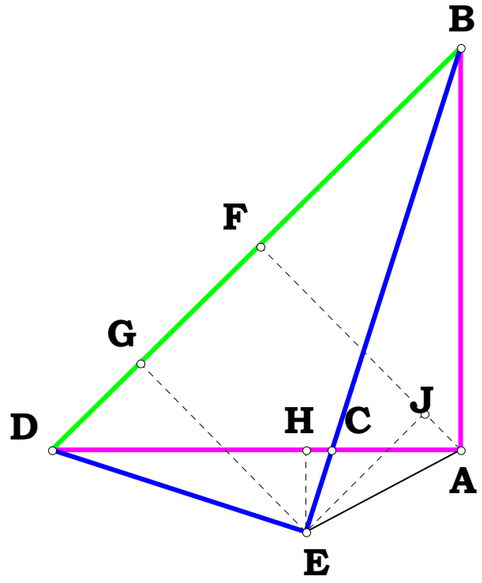


**Definitions:**

$$BE - \sqrt{N_1^2 + N_2^2 - N_3^2} = 0$$

$$\frac{N_1 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - AC = 0$$

$$N_2 + \frac{N_1 \cdot N_2 \cdot N_3^2 - N_2 \cdot \left(N_1^3 + N_1 \cdot N_2^2\right) + N_1^2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CD = 0$$




$$\frac{N_3 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CE = 0$$

$$BC - \left(\sqrt{N_1^2 + N_2^2 - N_3^2} + \frac{N_2 \cdot N_3^3 - N_2^3 \cdot N_3 - N_1^2 \cdot N_2 \cdot N_3 + N_1 \cdot N_3^2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}}\right) = 0$$

Unit.

$A1 := 1$

Given.

$N := 8$

112700

Descriptions.

Let us suppose that we have any unit, or thing and we want to parse that thing into units, any number of units at all, we must first define the unit. In the following case, our starting unit, or thing is A to 1. Now when we parse it, we are defining smaller units. We do not want to call them fractions, or have to deal with fractions, so we rename our thing in terms of a given number of our new chosen unit, AB. This figure shows how to proceed to parse A1 into a 2N exponential series. We do this, as said, by creating a fraction, and giving the unit of that fraction the name of our new working unit. In short, we are converting base systems. We go from base 1, always to some other base. The base is named for the number of units it contains, or subsets of our given set. In this case, 8 is a subset of 1, and every member of 1, is defined in terms of 1.

$AB := \frac{A1}{N}$ AB is a member of the set A1.

$BF := \sqrt{AB \cdot (A1 - AB)}$ $AF := \sqrt{AB^2 + BF^2}$ $AD := AF$

$DH := \sqrt{AD \cdot (A1 - AD)}$ $AH := \sqrt{AD^2 + DH^2}$ $AE := AH$

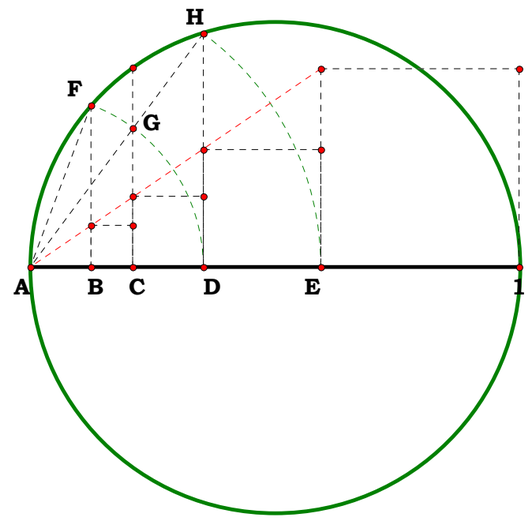
$AG := AD$ $AC := \frac{AD \cdot AD}{AH}$

Definitions.

$\frac{A1}{AB} = 8$ $\frac{A1}{AC} = 4.756828$ $\frac{A1}{AD} = 2.828427$ $\frac{A1}{AE} = 1.681793$ $C := \frac{A1}{AC}$ $D := \frac{A1}{AD}$ $E := \frac{A1}{AE}$ $N^{\frac{3}{4}} - C = 0$ $N^{\frac{2}{4}} - D = 0$ $N^{\frac{1}{4}} - E = 0$ Etc.

## Any 2N root series.

This is just 112293 with lipstick and a dress. I have always felt a bit of annoyance with those who write Algebra books claiming that exponential series is a pure conceptual abstraction which has no geometric figure to demonstrate it. Apparently those writers do not even know simple geometry. I do not mind someone being ignorant, but when such words are in school books it is disinformation and misleading of students. I had never lernt geometry when I read that in a school book, however I was still amazed at the author putting words into a text which could not have possibly been true. Euclid gave his readers individual components to work with, the readers inability to combine those components together to figure our their interaction is not the fault of Euclid, it is the stupidity and lazyness of the reader.




$\frac{Unit}{XY} = 8.00000$

$x_B = 0.12500$

$x_C = 0.21022$

$x_D = 0.35355$

$x_E = 0.59460$

$\frac{Unit}{x_B} = 8.00000$

$\frac{Unit}{x_C} = 4.75683$

$\frac{Unit}{x_D} = 2.82843$

$\frac{Unit}{x_E} = 1.68179$

$B = 8.00000$ $B^{\frac{4}{4}} = 8.00000$

$C = 4.75683$ $B^{\frac{3}{4}} = 4.75683$

$D = 2.82843$ $B^{\frac{2}{4}} = 2.82843$

$E = 1.68179$ $B^{\frac{1}{4}} = 1.68179$

Now we can think of A1 as being a class or a noun, and B, C, D, E, members of that class, or its defining characteristics.

112800A

Unit.

$AE := 1 \quad JK := AE$

Given.

$N := 3 \quad EJ := N$

Descriptions.

$$HJ := \frac{JK \cdot EJ}{JK + EJ} \quad EH := EJ - HJ \quad GH := \frac{EH \cdot HJ}{EH + HJ}$$

$$EG := EH - GH \quad FG := \frac{EG \cdot GH}{EG + GH} \quad EF := EG - FG$$

$$DE := \frac{EF \cdot AE}{EF + AE} \quad AD := AE - DE \quad CD := \frac{AD \cdot DE}{AD + DE}$$

$$AC := AD - CD \quad BC := \frac{AC \cdot CD}{AC + CD} \quad AB := AC - BC$$

$M := 0 .. 3 \quad P := 0 .. 3 \quad AEAB_{M,P} := \left[ \frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

Definitions.

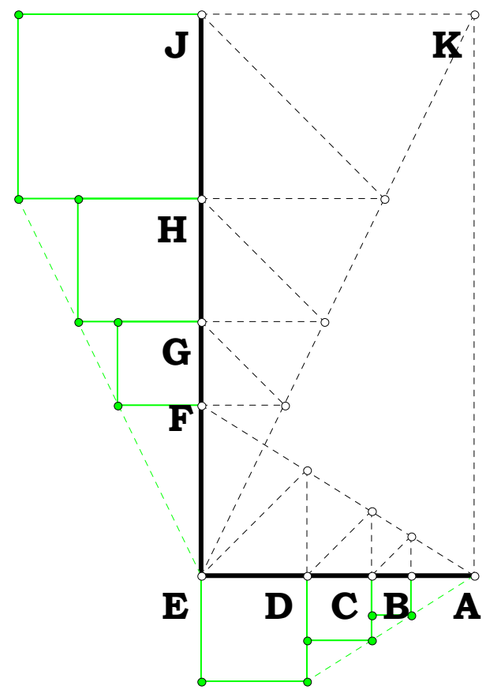
$$AEAB = \begin{pmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.5625 & 34.328125 \\ 1 & 2.6875 & 7.222656 & 19.410889 \\ 1 & 2.265625 & 5.133057 & 11.629581 \end{pmatrix}$$

$$AEAB_{3,3} - \frac{AE}{AB} = 0 \quad AEAB_{3,2} - \frac{AE}{AC} = 0 \quad AEAB_{3,1} - \frac{AE}{AD} = 0 \quad AEAB_{3,0} - \frac{AE}{AE} = 0$$

## Means On Means

**Modify 02/28/98 for Mean proportionals between E and J.**

**As I have always found this little exercise quite useless, I have decided on a B writeup aimed at helping to explain how to use things like this in template making for geometric progression. My templates tend all to be arithmetic in expression, however, using them to construct one which effects geometric progression is quite easy. So, try to reflect on what the plate is demonstrating.**



112900A

**Unit.**

$AC := 1$

**Given.**

$N_1 := 3$ $AH := N_1$

$N_2 := 12$ $CJ := N_2$

**Descriptions for Division.**

$$AB := \frac{AH}{(CJ + AH)} \cdot AC \quad BC := AC - AB$$

$$BD := BC \quad CG := \frac{BD \cdot AC}{AB} \quad CG = 4$$

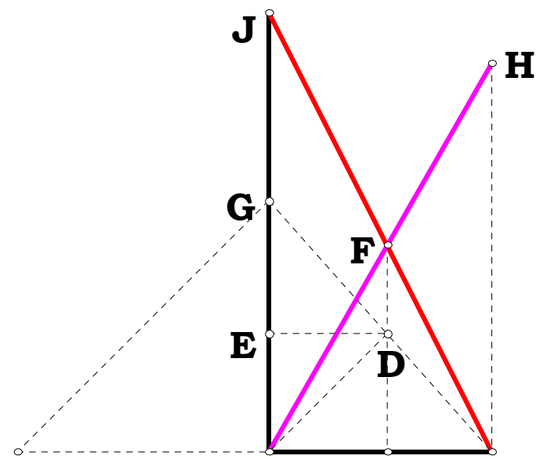
**Definitions.**

$$AB - \frac{N_1}{(N_2 + N_1)} = 0 \quad BC - \frac{N_2}{N_1 + N_2} = 0$$

$$BD - \frac{N_2}{N_1 + N_2} = 0 \quad CG - \frac{N_2}{N_1} = 0$$

## Multiplication and Division-Line By A Line

**Given some unit, and two differences, multiply or divide the one difference by the other.**
**For Division:**



112900B

Unit.

$AC := 1$

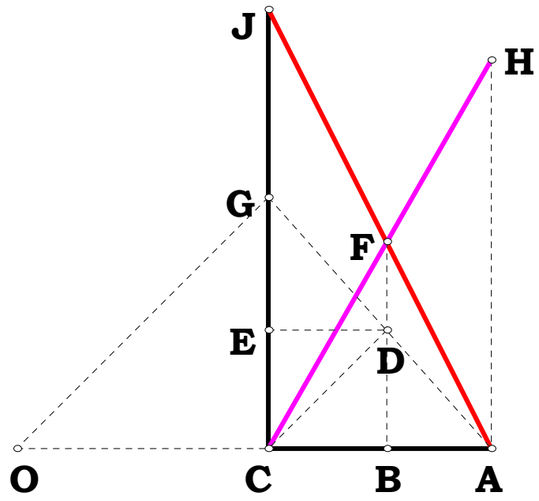
Given.

$N_1 := 5 \quad AH := N_1$

$N_2 := 7 \quad CG := N_2$

## Multiplication and Division-Line By A Line

**Given some unit, and two differences, multiply or divide the one difference by the other.**
**For Division:**




**Descriptions for Multiplication.**

$CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$

**Definitions.**

$CO - N_2 = 0 \qquad BD - \frac{N_2}{N_2 + 1} = 0$

$BC - \frac{N_2}{N_2 + 1} = 0 \qquad AB - \frac{1}{N_2 + 1} = 0$

$BF - \frac{N_1 \cdot N_2}{N_2 + 1} = 0 \qquad CJ - N_1 \cdot N_2 = 0$

$CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$

120500

**Descriptions.**

**Unit.**

$N_3 := 1$ $CE := N_3$

**Given.**

$N_1 := 5$ $BC := N_1$

$N_2 := 25$ $AC := N_2$

$N_4 := .5$ $EF := N_4$

$BD := \frac{EF \cdot BC}{CE}$ $CF := \sqrt{CE^2 - EF^2}$

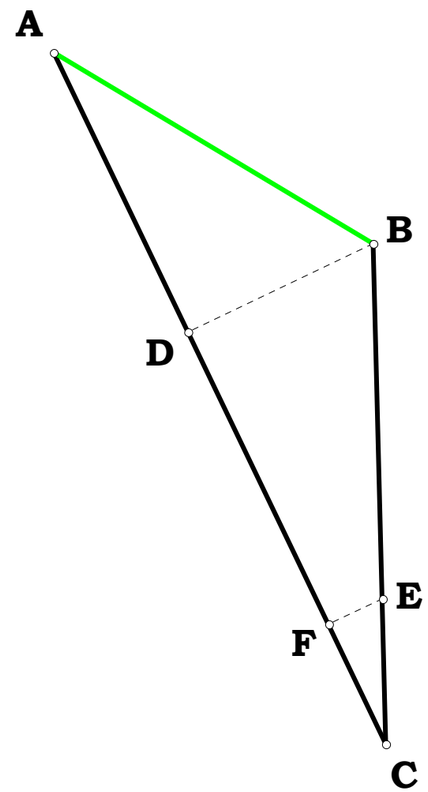
$CD := \frac{CF \cdot BC}{CE}$ $AD := AC - CD$

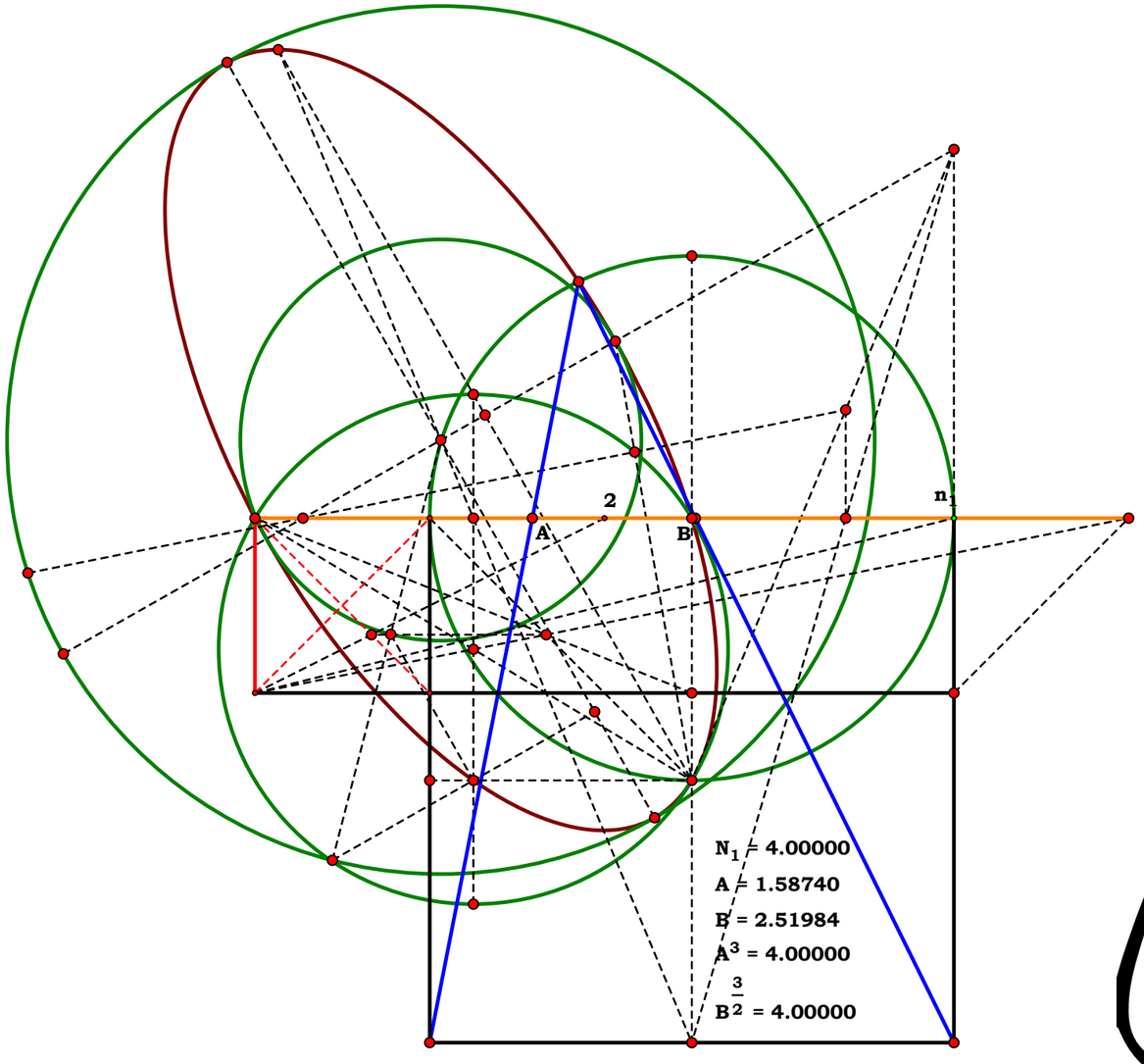
$AB := \sqrt{BD^2 + AD^2}$ $AB = 20.82051$

**Definitions.**

$$AB - \frac{\sqrt{N_1^2 \cdot N_3 + N_2^2 \cdot N_3 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2}}}{\sqrt{N_3}} = 0$$

**From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.**






# The Delian Quest 2001

**John Clark**

010101

**Unit.**

**Given.**

$N_1 := 5$ $\quad AC := N_1$

$N_2 := 3$ $\quad BC := N_2$

**Square Root, common segment common endpoint.**

**Alternate method for common segment common endpoint square root.** $\sqrt{AC \cdot BC} = CD$

**Descriptions.**

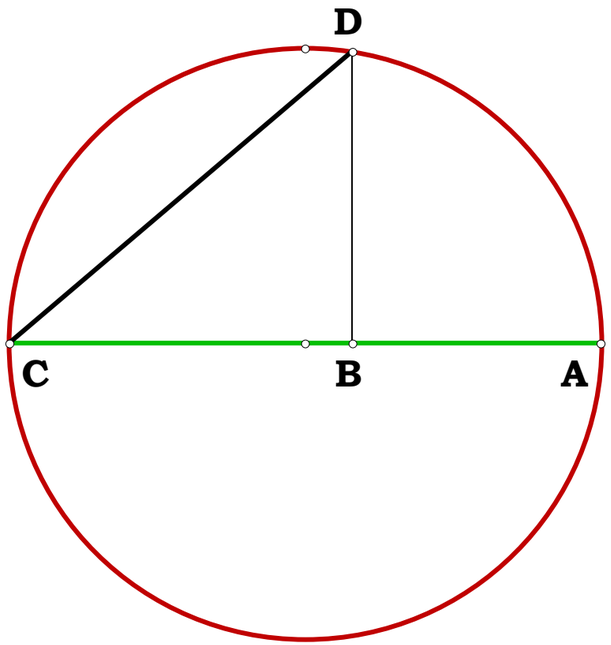
$AB := AC - BC$

$BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BD^2 + BC^2}$

**Definitions.**

$\sqrt{(N_1 \cdot N_2)} - CD = 0$



042101

**Descriptions.**

**Unit.**

**Given.**

$N_1 := 2 \quad AB := N_1$

$N_2 := 3 \quad CD := N_2$

$N_3 := 4 \quad AC := N_3$

$BC := \sqrt{AB^2 + AC^2} \quad CG := \frac{CD^2}{BC} \quad BF := \frac{AB^2}{BC}$

$BG := BC - CG \quad CF := BC - BF \quad AF := \sqrt{AB^2 - BF^2}$

$DG := \sqrt{CD^2 - CG^2} \quad FH := \frac{BG \cdot AF}{DG} \quad CH := CF + FH$

$BD := \sqrt{BG^2 + DG^2} \quad AH := \frac{BD \cdot FH}{BG} \quad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \quad CE := \frac{AC \cdot BC}{CH} \quad AE := AC - CE$

**Definitions.**

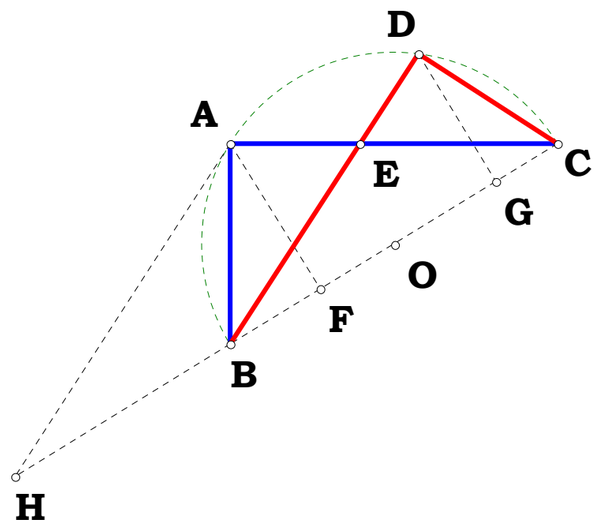
$\sqrt{N_1^2 + N_3^2} - BC = 0 \quad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$

$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0 \quad \frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0 \quad \frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0 \quad \frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$

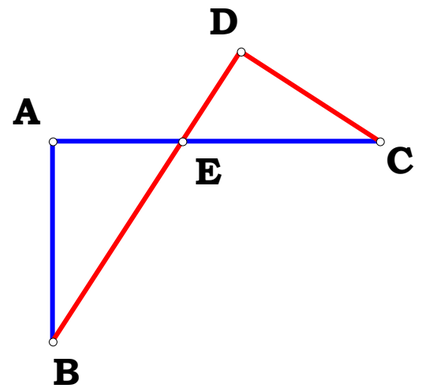
$\frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} - DG = 0 \quad \frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} \quad \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0 \quad N_3 \cdot \frac{N_1}{N_2} - AH = 0$

$$\frac{N_3 \cdot \left[N_1 \cdot \left(N_1^2 - N_2^2 + N_3^2\right) \cdot \sqrt{N_1^2 + N_3^2} + N_2 \cdot N_3 \cdot \sqrt{\left[\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)\right]}\right]}{N_2 \cdot \sqrt{N_1^2 + N_3^2} \cdot \sqrt{\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}} - CH = 0$$

**Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?**




**Three Given Five Taken**



**The Five Sought:**

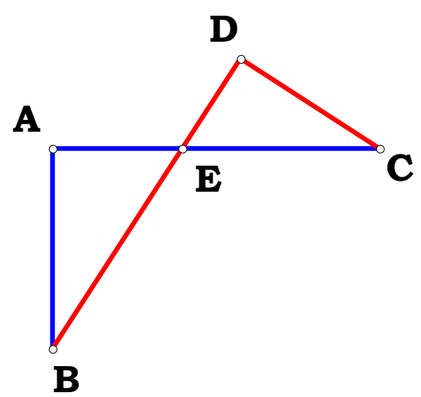
$$\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - DE = 0$$

$$N_1 \cdot \frac{\left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - BE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - AE = 0$$

$$\frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1} - CE = 0$$



042201

Descriptions.

Unit.

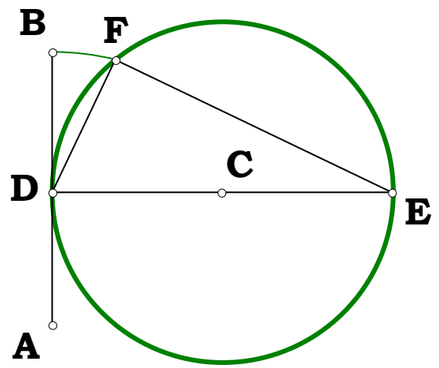
$AB := 1$

Given.

$N_1 := 2.052$

$N_2 := .62$

Given AB as unit, AD and DC, what is EF and DF?



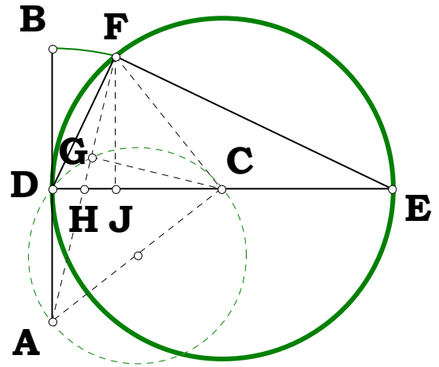

$AD := \frac{AB}{N_1}$ $CD := AB \cdot N_2$ $DE := 2CD$ $AF := AB$ $CF := CD$

$AC := \sqrt{AD^2 + CD^2}$ $AG := \frac{AC^2 + AF^2 - CF^2}{2 \cdot AF}$ $CG := \sqrt{AC^2 - AG^2}$

$DH := CD - \frac{CG \cdot \left(AD^2 + CD^2\right)}{CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD}$ $AH := AD \cdot \frac{\left(AD^2 + CD^2\right)}{\left(CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD\right)}$

$FH := AF - AH$ $HJ := \frac{DH \cdot FH}{AH}$ $DJ := DH + HJ$ $EJ := DE - DJ$ $FJ := \frac{AD \cdot FH}{AH}$

$EF := \sqrt{FJ^2 + EJ^2}$ $DF := \sqrt{DJ^2 + FJ^2}$




Definitions.

$\frac{AB}{N_1} - AD = 0$ $AB \cdot N_2 - CD = 0$ $(2AB) \cdot N_2 - DE = 0$ $AB \cdot \frac{\sqrt{\left(1 + N_2^2 \cdot N_1^2\right)}}{N_1} - AC = 0$

$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^2\right)}{N_1^2} - AG = 0$ $AB \cdot \frac{\sqrt{\left(2 \cdot N_1^2 \cdot N_2 + N_1^2 - 1\right) \cdot \left(2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1\right)}}{2 \cdot N_1^2} - CG = 0$

$AB \cdot \frac{N_2 \cdot N_1^3 + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4}}{N_1^3 + N_1^2 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + N_1} - DH = 0$ $2 \cdot AB \cdot \frac{\left(1 + N_2^2 \cdot N_1^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + 1 + N_1^2\right)} - AH = 0$

$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^2 \cdot N_1^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + 1 + N_1^2\right)} - FH = 0$

$$\frac{-1}{2}\cdot AB\cdot\left(\frac{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}\cdot N_2^{\,2}+1}{N_1^{\,2}\cdot N_2+N_1^{\,4}\cdot N_2+N_1^{\,3}\cdot N_2^{\,2}\cdot\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}-\frac{N_1^{\,4}\cdot N_2^{\,2}+3\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+1}{N_1^{\,4}\cdot N_2^{\,3}+N_1^{\,2}\cdot N_2}\right)-HJ=0$$

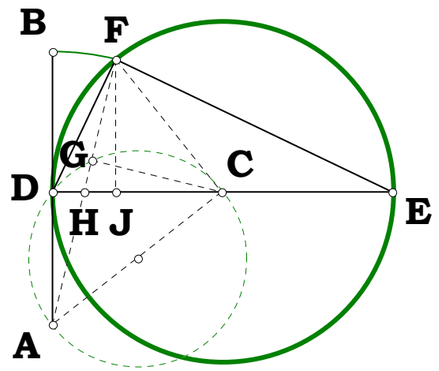
$$\frac{1}{2}\cdot AB\cdot\frac{N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{4\cdot N_1^{\,3}\cdot N_2^{\,3}-N_1^{\,3}\cdot N_2+3\cdot N_1\cdot N_2+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-EJ=0$$

$$\frac{-1}{2}\cdot AB\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\frac{\sqrt{N_2\cdot\left[N_1\cdot N_2\cdot\left(4\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+3\right)+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right]}}{\sqrt{N_1^{\,3}\cdot N_2^{\,2}+N_1}}-EF=0$$

$$AB\cdot\frac{\sqrt{\left[N_2\cdot\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right)\right]}}{\sqrt{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}}-DF=0$$



042301A

**Unit.**

$AB := 1$

**Given.**

$N := .36307 \quad BC := N$

$AC := AB + BC \quad AG := \frac{AB}{2} \quad GM := \sqrt{3 \cdot AG^2} \quad CG := AC - AG$

$CN := \sqrt{CG^2 + AG^2} \quad NO := \frac{AG \cdot AB}{CN} \quad CO := CN - NO$

$CE := \frac{CN \cdot CO}{CG} \quad GE := CG - CE \quad JS := \sqrt{(AB + GE) \cdot (AB - GE)}$

$EJ := JS - GM \quad CJ := \sqrt{CE^2 + EJ^2} \quad CL := \frac{CG \cdot CE}{CJ}$

**Definitions.**

$AC - (1 + N) = 0 \quad AG - \frac{1}{2} = 0 \quad GM - \frac{\sqrt{3}}{2} = 0$

$CG - \frac{2 \cdot N + 1}{2} \quad CN - \frac{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}}{\sqrt{2}} = 0$

$NO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0 \quad CO - \frac{\sqrt{2} \cdot N \cdot (N + 1)}{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0$

$CE - \frac{2 \cdot N \cdot (N + 1)}{2 \cdot N + 1} = 0 \quad GE - \frac{1}{2 \cdot (2 \cdot N + 1)} = 0$

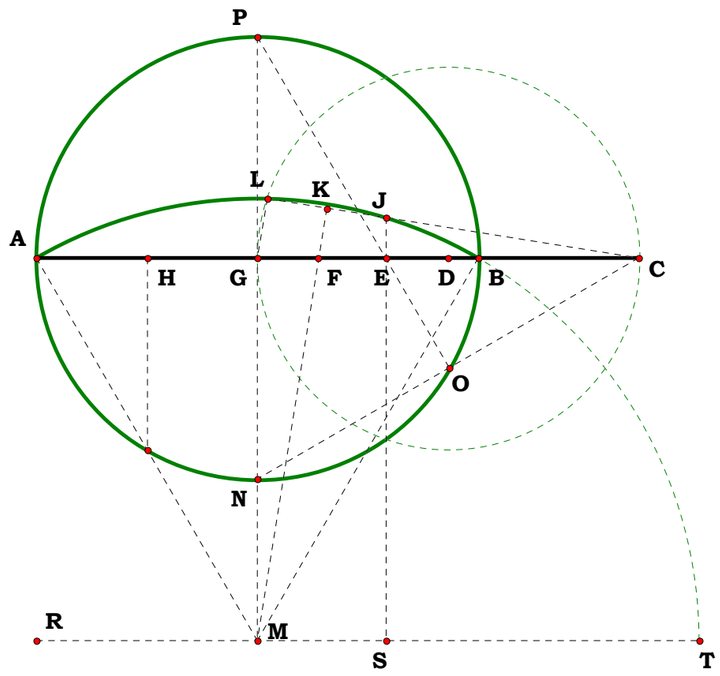
$JS - \frac{\sqrt{(4 \cdot N + 1) \cdot (4 \cdot N + 3)}}{2 \cdot (2 \cdot N + 1)} = 0 \quad EJ - \frac{\sqrt{16 \cdot N^2 + 16 \cdot N + 3} - \sqrt{3} \cdot (2 \cdot N + 1)}{4 \cdot N + 2} = 0$

$CJ - \frac{\sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right)} + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}}{\sqrt{4 \cdot N + 2}} = 0$

$CL - \frac{2 \cdot \sqrt{2} \cdot N \cdot (N + 1) \cdot \left(N + \frac{1}{2}\right)}{\sqrt{2 \cdot N + 1} \cdot \sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right)} + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}} = 0$

## Counterpoint

**When I origionally did part of this, it was a mess. I have taken it upon myself to keep the figure and title.**



**Unit.**

$AB := 1$

**Given.**

$N := 5 \quad AG := N$

**042401A**

**Descriptions.**

$BG := AG - AB \quad BF := \frac{BG}{2} \quad AF := AB + BF \quad AK := AF$

$FK := BF \quad AE := \frac{2AK^2 - FK^2}{2AF} \quad AJ := AE \quad JK := AK - AJ$

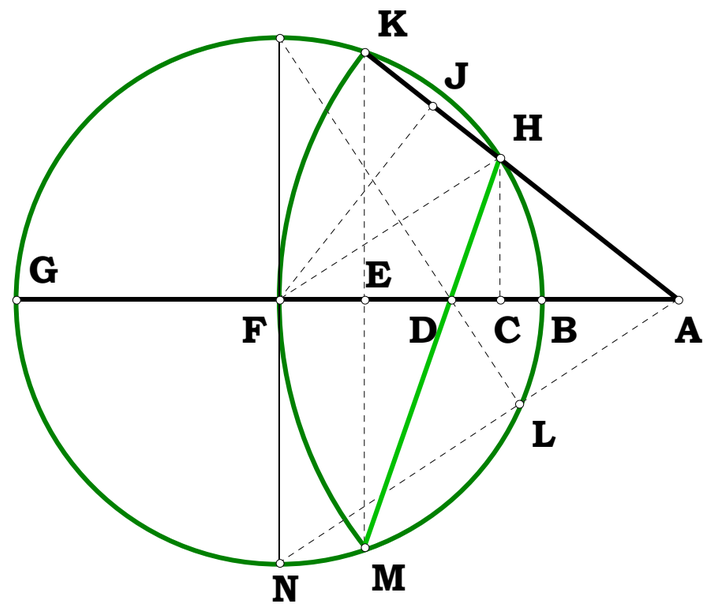
$HJ := JK \quad AH := AK - (JK + HJ) \quad AC := \frac{AE \cdot AH}{AK} \quad CE := AE - AC$

$BE := AE - AB \quad EG := BG - BE \quad EK := \sqrt{BE \cdot EG} \quad BC := AC - AB$

$CG := BG - BC \quad CH := \sqrt{BC \cdot CG} \quad DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \quad HM := \sqrt{CE^2 + (EK + CH)^2}$

**Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?**



**Definitions.**

$$N - 1 - BG = 0 \qquad \frac{1}{2}\cdot N - \frac{1}{2} - BF = 0$$

$$\frac{1}{2} + \frac{1}{2}\cdot N - AF = 0 \qquad \frac{1}{4}\cdot\frac{\left(N^2 + 6\cdot N + 1\right)}{(1+N)} - AE = 0$$

$$\frac{1}{4}\cdot\frac{\left(1 - 2\cdot N + N^2\right)}{(1+N)} - JK = 0 \qquad 2\cdot\frac{N}{(1+N)} - AH = 0$$

$$\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^3}\cdot N - AC = 0 \qquad \frac{1}{4}\cdot\left(N^2 + 6\cdot N + 1\right)\cdot\frac{(N-1)^2}{(1+N)^3} - CE = 0$$

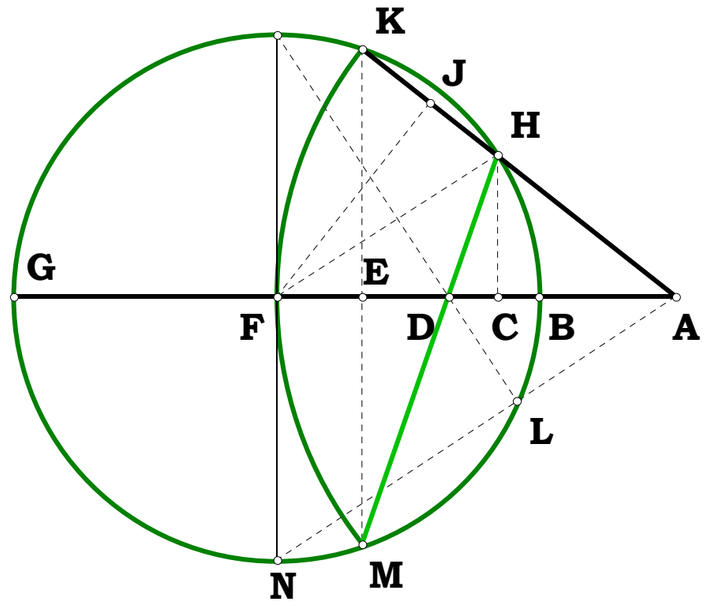
$$\frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} - BE = 0 \qquad \frac{1}{4}\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)} - EG = 0$$

$$(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3} - BC = 0 \qquad \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} - EK = 0$$

$$N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3} - CG = 0 \qquad \sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3} - CH = 0$$

$$\frac{1}{4}\cdot\frac{(N-1)^2}{(1+N)} - DE = 0 \qquad \frac{1}{2}\cdot\frac{(N-1)^2}{(N+1)} - DF = 0$$

$$\frac{1}{2}\cdot(N-1)\cdot\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^2} - HM = 0$$



**Unit.**

$AB := 1$

**Given.**

$N := 5 \qquad AG := N$

**042401B**

**Descriptions.**

$BG := AG - AB \qquad BF := \frac{AB}{2} \qquad GF := BG + BF \qquad GK := GF$

$FK := BF \qquad EG := \frac{2GK^2 - FK^2}{2GF} \qquad GJ := EG \qquad JK := GK - GJ \qquad HJ := JK$

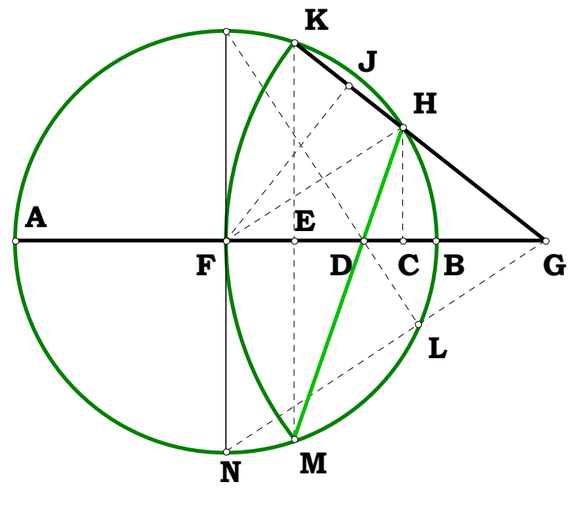
$GH := GK - (JK + HJ) \qquad CG := \frac{EG \cdot GH}{GK} \qquad CE := EG - CG$

$BE := EG - BG \qquad AE := AB - BE \qquad EK := \sqrt{BE \cdot AE} \qquad AC := AG - CG$

$BC := AB - AC \qquad CH := \sqrt{BC \cdot AC} \qquad DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \qquad HM := \sqrt{CE^2 + (EK + CH)^2}$

**Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?**



**Definitions.**

$$BG - (N-1) = 0 \quad BF - \frac{1}{2} = 0 \quad GF - \frac{2\cdot N-1}{2} = 0 \quad GK - \frac{2\cdot N-1}{2} = 0$$

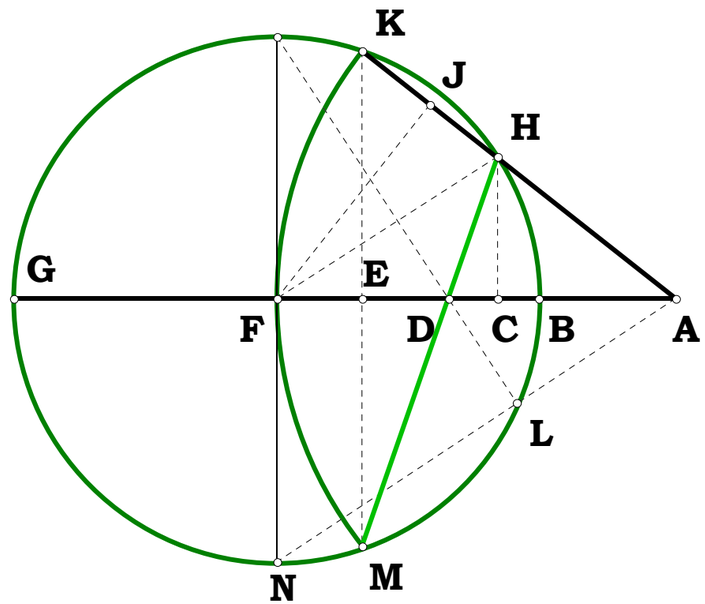
$$FK - \frac{1}{2} = 0 \quad EG - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0 \quad GJ - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0$$

$$JK - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad HJ - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad GH - \frac{2\cdot N\cdot(N-1)}{2\cdot N-1} = 0$$

$$CG - \frac{N\cdot(N-1)\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)}{(2\cdot N-1)^3} = 0 \quad CE - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)^3} = 0$$

$$BE - \frac{4\cdot N-3}{4\cdot(2\cdot N-1)} = 0 \quad AE - \frac{4\cdot N-1}{4\cdot(2\cdot N-1)} = 0 \quad EK - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)} = 0$$

$$AC - \frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3} = 0 \quad BC - \frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3} = 0$$




$$CH - \frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3} = 0$$

**Mathcad claims that this is solvable, however, it was spread over several pages and I do not know what to make of that; however, if it is right, then there you go for trisection.**

$$DE - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad DF - \frac{2}{4\cdot(2\cdot N-1)} = 0 \quad HM - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)}{2\cdot(2\cdot N-1)^2} = 0$$

**Unit.**

$AB := 1$

**Given.**

**Descriptions.**

$N := 5.768$ $AJ := N$

**042501A**

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

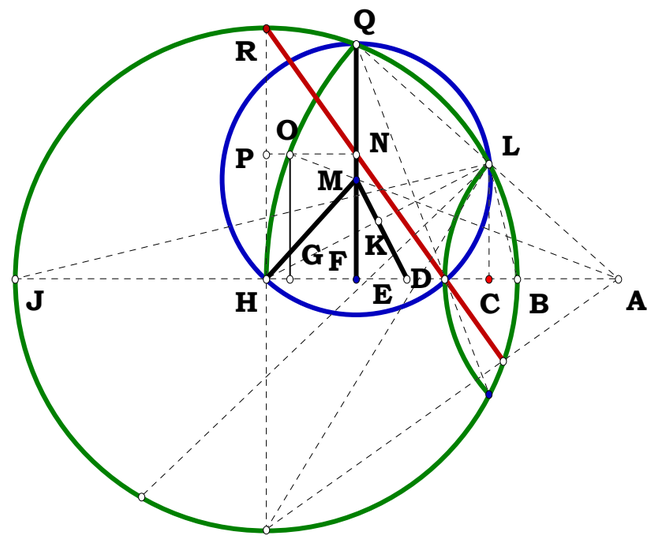
$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0$ $DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

**What is the Algebraic name of the circle HM? Does point N divide DR in half?**




**Definitions.**

$BJ - (N - 1) = 0$ $BH - \frac{1}{2} \cdot (N - 1) = 0$ $HP - \frac{1}{4} \cdot (N - 1) = 0$

$AH - \frac{1}{2} \cdot (1 + N) = 0$ $AG - \frac{1}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} = 0$ $\frac{1}{4} \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$AF - \frac{1}{4} \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} = 0$ $FM - \frac{1}{4} \cdot (N - 1) \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} = 0$

$$BF - \frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} = 0$$

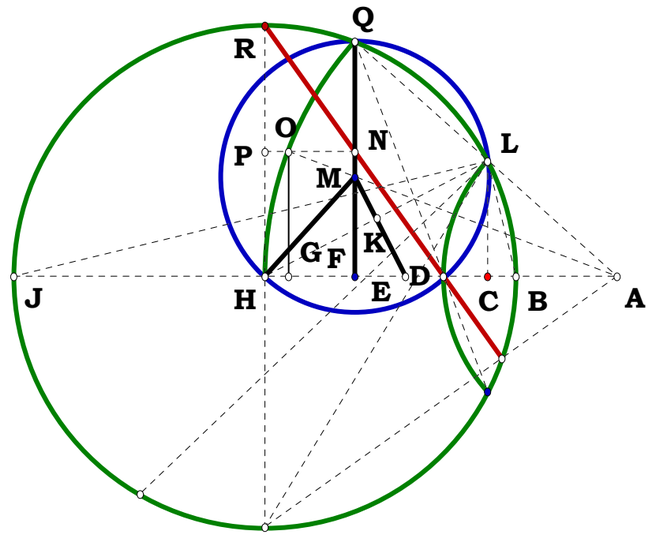
$$FJ - \frac{1}{4}\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)} = 0$$

$$FQ - \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} = 0$$

$$MQ - \frac{1}{2}\cdot(1+N)\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$HM - \frac{1}{2}\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$DH - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \qquad FH - \frac{(N-1)^2}{4\cdot(N+1)} = 0$$



**Unit.**

$AB := 1$

**Given.**

**Descriptions.**

$N := 6$ $AJ := N$

**042501B**

$BJ := AJ - AB$ $BH := \frac{AB}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $JH := BJ + BH$

$JO := JH$ $JG := \sqrt{JO^2 - GO^2}$

$HQ := BH$ $JQ := JH$ $FH := \frac{HQ^2}{2 \cdot JH}$

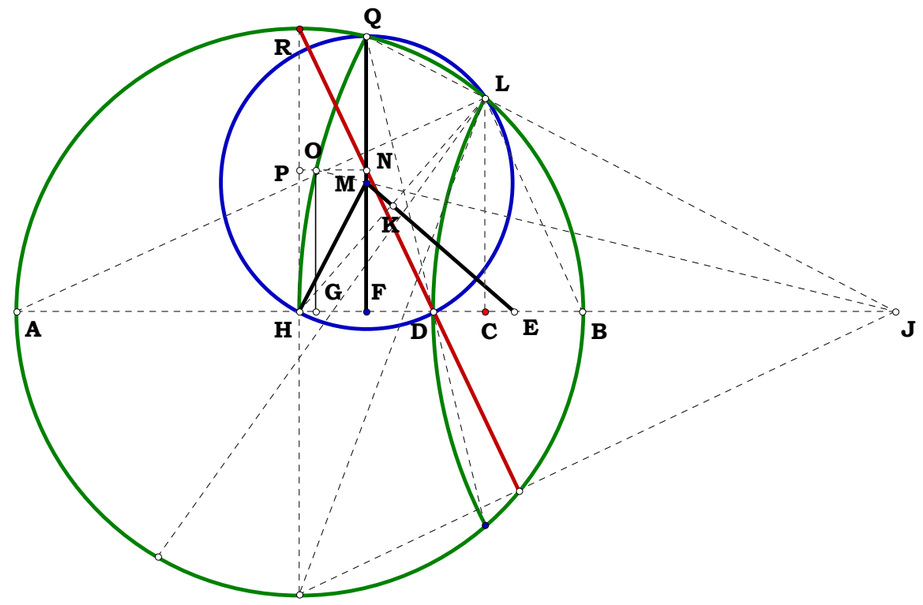
$AH := BH$ $FJ := JH - FH$ $FM := \frac{GO \cdot FJ}{JG}$

$AF := FH + AH$ $BF := AB - AF$ $FQ := \sqrt{BF \cdot AF}$

$MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0$ $DH := \frac{HR^2}{JH}$ $\frac{DH}{2} - FH = 0$

**What is the Algebraic name of the circle HM? Does point N divide DR in half?**



**Definitions.**

$$BJ - (N - 1) = 0 \quad BH - \frac{1}{2} = 0 \quad HR - \frac{1}{2} = 0$$

$$HP - \frac{1}{4} = 0 \quad GO - \frac{1}{4} = 0 \quad JH - \frac{2 \cdot N - 1}{2} = 0$$

$$JO - \frac{2 \cdot N - 1}{2} = 0 \quad JG - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0$$

$$HQ - \frac{1}{2} = 0 \quad JQ - \frac{2 \cdot N - 1}{2} = 0 \quad FH - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0$$

$$AH - \frac{1}{2} = 0 \quad FJ - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$$

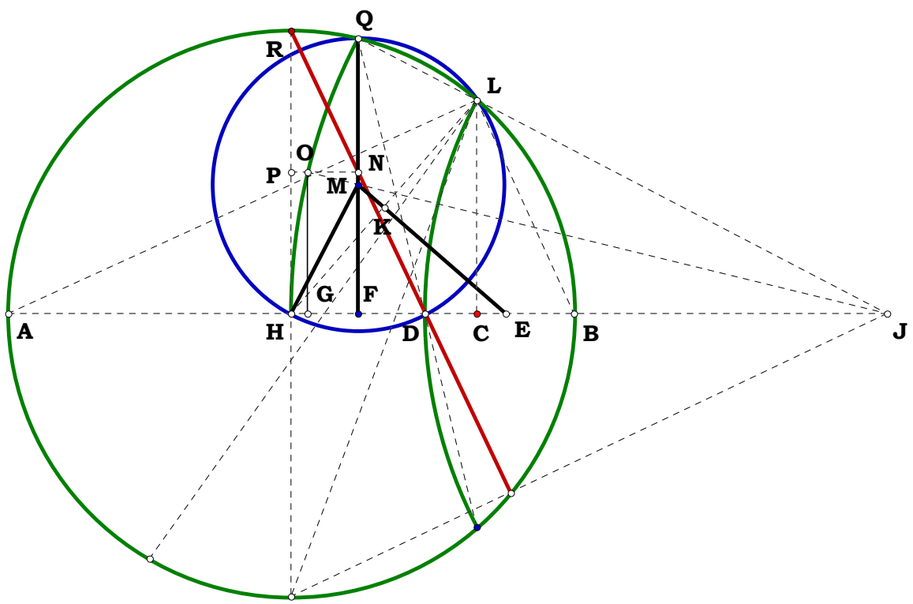
$$FM - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} \cdot (2 \cdot N - 1)} = 0$$

$$AF - \frac{4 \cdot N - 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad BF - \frac{4 \cdot N - 3}{4 \cdot (2 \cdot N - 1)} = 0$$

$$FQ - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot (2 \cdot N - 1)} = 0 \quad MQ - \frac{2 \cdot N - 1}{2 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0$$

$$HM - \frac{(2 \cdot N - 1)}{\sqrt{4 \cdot (4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0 \quad DH - \frac{1}{2 \cdot (2 \cdot N - 1)} = 0$$

$$HM - MQ = 0 \quad \frac{DH}{2} - FH = 0$$



042901A

**Unit.**

$AB := 1$

**Given.**

$N := 5 \qquad AD := N$

**Descriptions.**

$BD := AD - AB \qquad BC := \frac{BD}{2} \qquad CR := BC$

$CP := BC \qquad CD := BC \qquad AC := AB + BC \qquad CG := \frac{CP^2}{AC}$

$AG := AC - CG \qquad AV := AG \qquad CV := BC \qquad CF := \frac{CV^2 + AC^2 - AV^2}{2AC}$

$BF := BC - CF \qquad DF := CF + CD \qquad FV := \sqrt{BF \cdot DF} \qquad CE := \frac{CF \cdot CR}{CR - FV}$

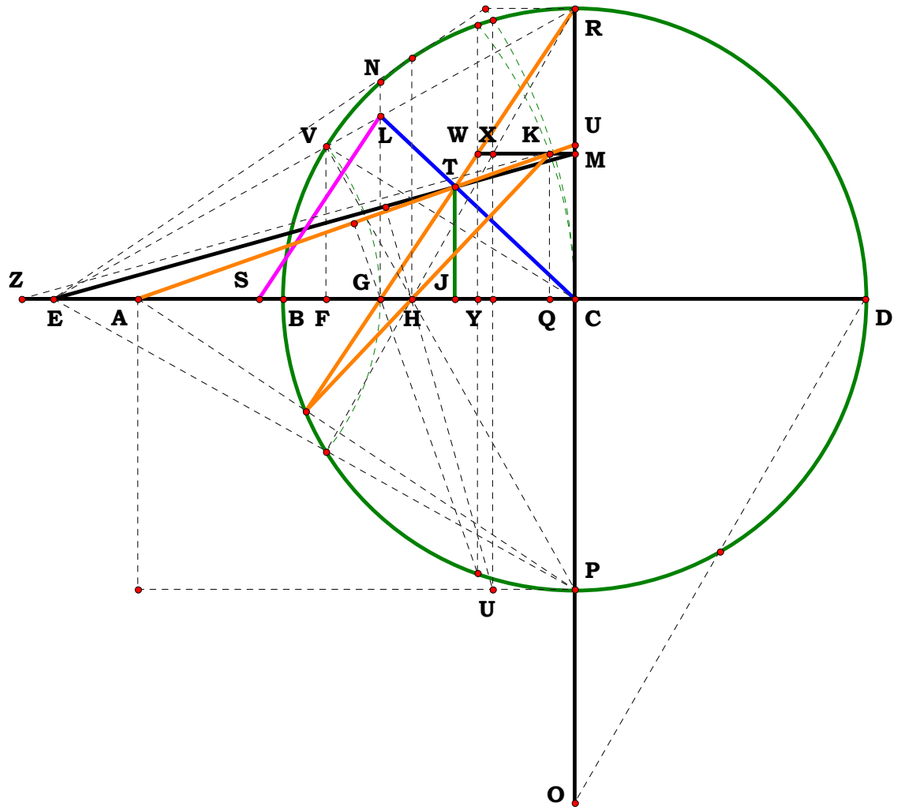
$BE := CE - BC \qquad BG := BC - CG \qquad EF := BE + BF \qquad EG := BE + BG$

$GL := \frac{FV \cdot EG}{EF} \qquad GS := \frac{CG \cdot GL}{CR} \qquad CJ := \frac{CG^2}{GS + CG}$

$JT := \frac{GL \cdot CJ}{CG}$

## Four Lines To A Point

**Does the difference CJ and JT each have but one Algebraic name?**



$$CM := \frac{CR}{2} \qquad AK := AC \qquad KQ := CM$$

$$AQ := \sqrt{AK^2 - KQ^2} \qquad CU := \frac{KQ \cdot AC}{AQ}$$

$$CZ := \frac{CE \cdot CU}{CM} \qquad AU := \sqrt{AC^2 + CU^2}$$

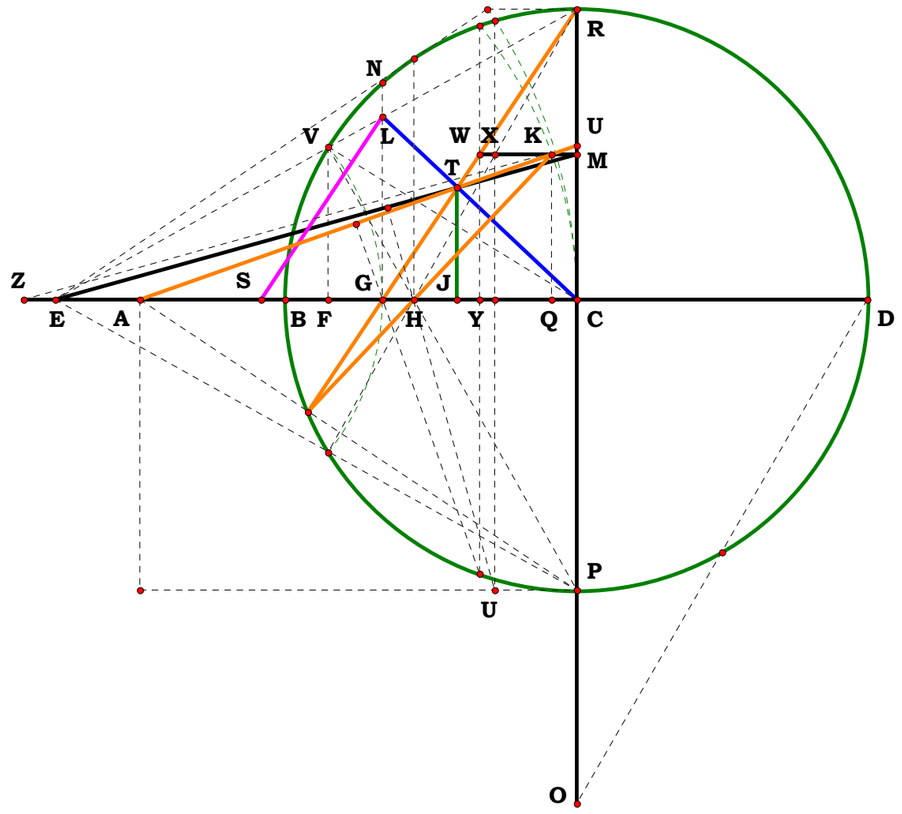
$$AZ := CZ - AC \qquad AE := CE - AC$$

$$AJ := \frac{AC \cdot AE}{AZ} \qquad JC := AC - AJ$$

$$TJ := \frac{CU \cdot AJ}{AC}$$

$$JC - CJ = 0$$

$$JT - TJ = 0$$



$$N - 5 = 0 \qquad AD - N = 0$$

**Definitions.**

$$BD - (N-1) = 0 \quad BC - \frac{N-1}{2} = 0 \quad CR - \frac{N-1}{2} = 0 \quad CP - \frac{N-1}{2} = 0$$

$$CD - \frac{N-1}{2} = 0 \quad AC - \frac{N+1}{2} \quad CG - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AG - \frac{2\cdot N}{N+1} = 0$$

$$AV - \frac{2\cdot N}{N+1} = 0 \quad CV - \frac{N-1}{2} = 0 \quad CF - \frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3} = 0$$

$$BF - \frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3} = 0 \qquad DF - \frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3} = 0$$

$$FV - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} = 0$$

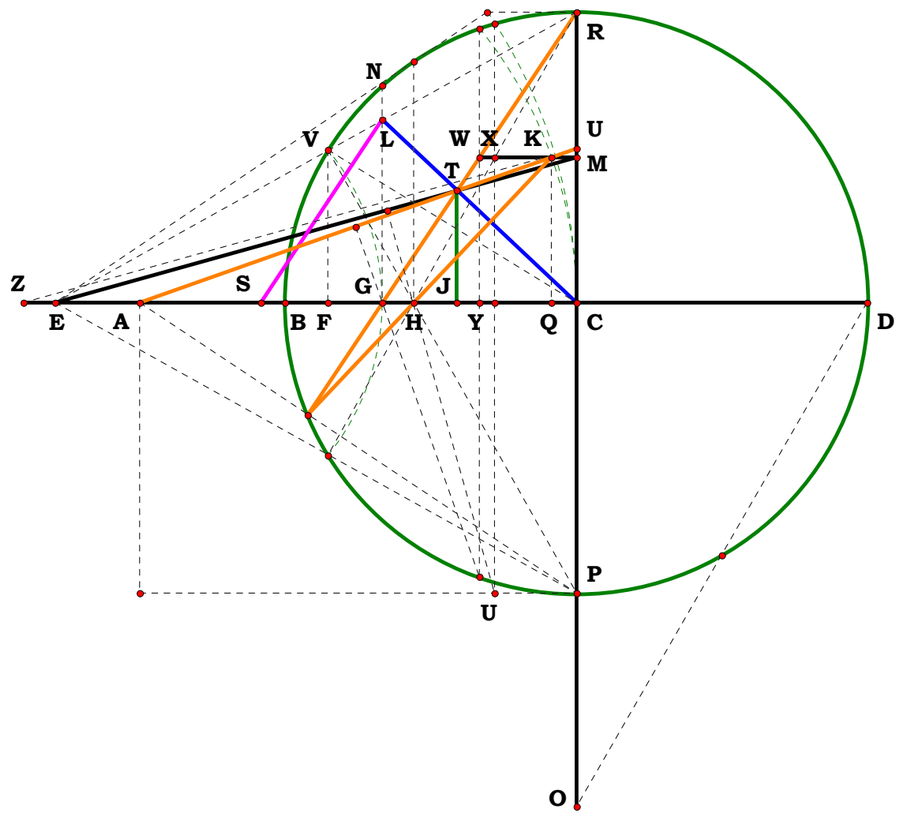
$$CE - \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$BE - \frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1} = 0 \qquad BG - \frac{N-1}{N+1} = 0$$

$$EF - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0 \qquad EG - \frac{N\cdot(N-1)^2\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$GL - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0 \qquad GS - \frac{N\cdot(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)^2\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0$$

$$CJ - \frac{N^4+2\cdot N^3-6\cdot N^2+2\cdot N+1}{14\cdot N\cdot(N+1)+2\cdot N^3+4\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+2} = 0 \qquad JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$



$$CM - \frac{N-1}{4} = 0 \qquad AK - \frac{N+1}{2} = 0$$

$$KQ - \frac{N-1}{4} = 0 \qquad AQ - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$$

$$CU - \frac{(N+1)\cdot(N-1)}{2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$CZ - \frac{(N-1)^2\cdot(N+1)\cdot\left(N^2+4\cdot N+1\right)}{\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$AU - \frac{(N+1)^2}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$AZ - \frac{\left[\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^3+3\cdot N^2+3\cdot N+1\right)-2\cdot N\cdot\left(N^3+5\cdot N^2+4\cdot N+5\right)-2\right]\cdot(N+1)}{4\cdot N\cdot(N+3)\cdot(3\cdot N+1)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^3+6\cdot N^2+6\cdot N+2\right)} = 0$$

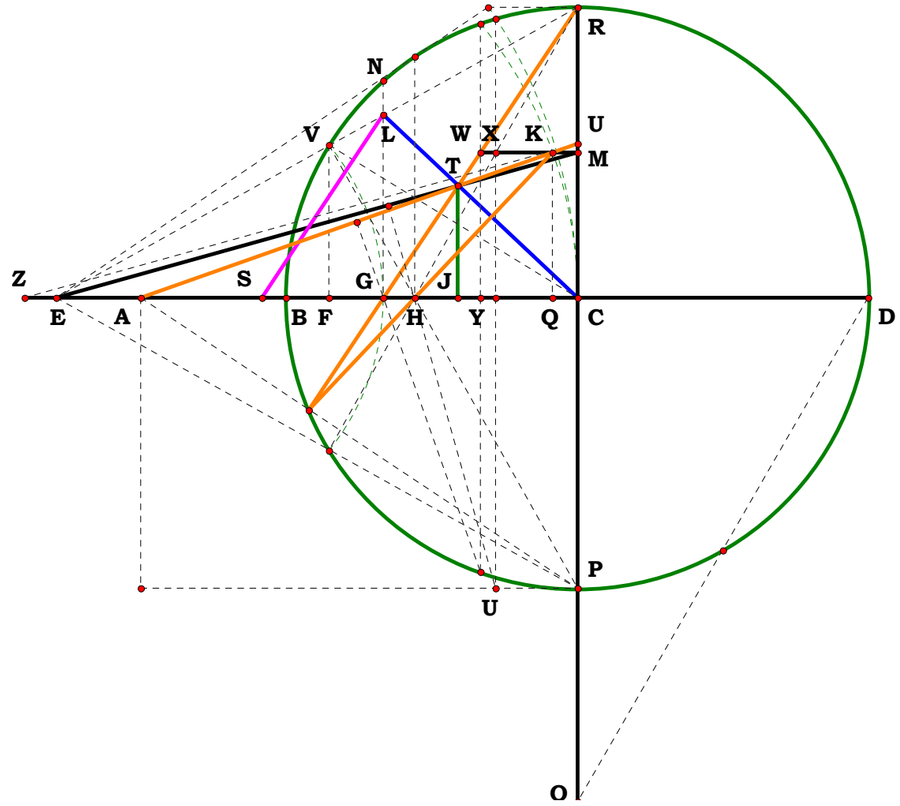
$$AE - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^2+N\right)-\left(N+6\cdot N^2+N^3\right)}{3\cdot N+3\cdot N^2+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$

$$AJ - \frac{3\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N\cdot(N+1)\cdot\left(N^4+14\cdot N^3+34\cdot N^2+14\cdot N+1\right)\right]+N^2\cdot\left(3\cdot N^5+28\cdot N^4+117\cdot N^3+216\cdot N^2+117\cdot N+28\right)}{2\cdot N\cdot\left(N^6+11\cdot N^5+41\cdot N^4+75\cdot N^3+75\cdot N^2+41\cdot N+11\right)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N^2\cdot\left(N^4+10\cdot N^3+35\cdot N^2+36\cdot N+35\right)+10\cdot N+1\right]+2} = 0$$

$$JC - \frac{18\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^7+9\cdot N^6+15\cdot N^5-25\cdot N^4-25\cdot N^3+15\cdot N^2+9\cdot N+1\right)+48\cdot N^2-2\cdot N^3-132\cdot N^4-2\cdot N^5+48\cdot N^6+18\cdot N^7+2\cdot N^8+2}{44\cdot N+164\cdot N^2+300\cdot N^3+300\cdot N^4+164\cdot N^5+44\cdot N^6+4\cdot N^7-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^6+20\cdot N^5+70\cdot N^4+72\cdot N^3+70\cdot N^2+20\cdot N+2\right)+4} = 0$$

$$TJ - \frac{99\cdot N^5-25\cdot N^2-89\cdot N^3-99\cdot N^4-3\cdot N+89\cdot N^6+25\cdot N^7+3\cdot N^8+\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(33\cdot N^3-14\cdot N^6-33\cdot N^5-N^7+14\cdot N^2+N\right)}{2\cdot(N+1)^5\cdot\left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-208\cdot N^2-488\cdot N^3-570\cdot N^4-488\cdot N^5-208\cdot N^6-40\cdot N^7-3\cdot N^8-40\cdot N-3} = 0$$

$$TJ - JT = 0 \qquad JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$



**Unit.**

$AB := 1$

**Given.**

$N := 1.22457 \qquad AD := N$

**042901B**

**Descriptions.**

# Four Lines To A Point

**Does the difference CJ and JT each have but one Algebraic name?**

$$BD := AD - AB \qquad BC := \frac{AB}{2} \qquad CR := BC$$

$$CP := BC \qquad AC := BC \qquad CD := AD - AC \qquad CG := \frac{CP^2}{CD}$$

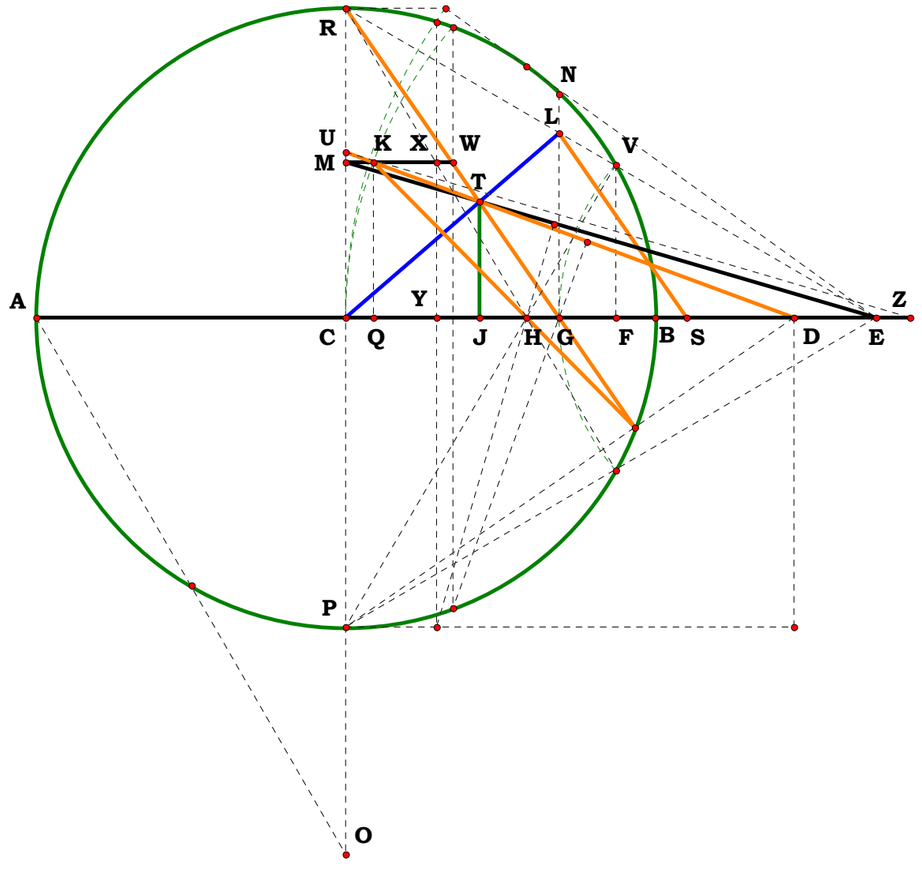
$$DG := CD - CG \qquad DV := DG \qquad CV := BC \qquad CF := \frac{CV^2 + CD^2 - DV^2}{2CD}$$

$$BF := BC - CF \qquad AF := CF + AC \qquad FV := \sqrt{BF \cdot AF} \qquad CE := \frac{CF \cdot CR}{CR - FV}$$

$$BE := CE - BC \qquad BG := BC - CG \qquad EF := BE + BF \qquad EG := BE + BG$$

$$GL := \frac{FV \cdot EG}{EF} \qquad GS := \frac{CG \cdot GL}{CR} \qquad CJ := \frac{CG^2}{GS + CG}$$

$$JT := \frac{GL \cdot CJ}{CG}$$



$$CM := \frac{CR}{2} \qquad DK := CD \qquad KQ := CM$$

$$DQ := \sqrt{DK^2 - KQ^2} \qquad CU := \frac{KQ \cdot CD}{DQ}$$

$$CZ := \frac{CE \cdot CU}{CM} \qquad DU := \sqrt{CD^2 + CU^2}$$

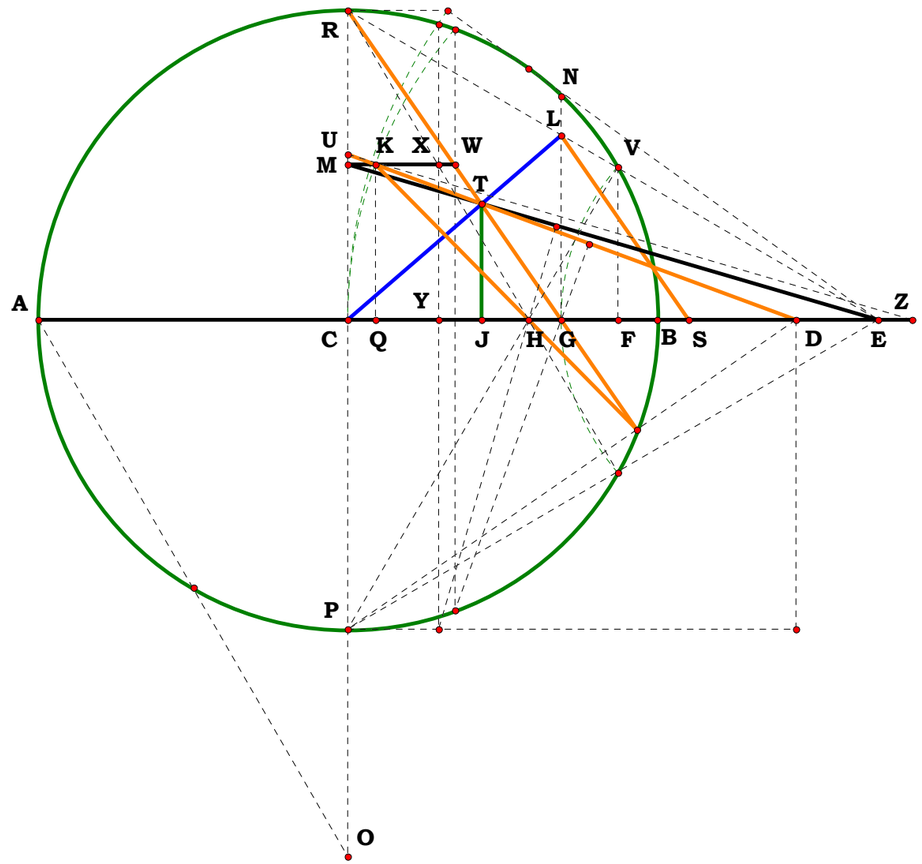
$$DZ := CZ - CD \qquad DE := CE - CD$$

$$DJ := \frac{CD \cdot DE}{DZ} \qquad JC := CD - DJ$$

$$TJ := \frac{CU \cdot DJ}{CD}$$

$$JC - CJ = 0$$

$$JT - TJ = 0$$



**Definitions.**

$$BD-(N-1)=0 \quad BC-\frac{1}{2}=0 \quad CR-\frac{1}{2}=0 \quad CP-\frac{1}{2}=0 \quad AC-\frac{1}{2}=0 \quad CD-\frac{2\cdot N-1}{2}=0$$

$$CG-\frac{1}{2\cdot(2\cdot N-1)}=0 \quad DG-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad DV-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad CV-\frac{1}{2}=0$$

$$CF-\frac{6\cdot N^2-6\cdot N+1}{2\cdot(2\cdot N-1)^3}=0 \quad BF-\frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3}=0 \quad AF-\frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3}$$

$$FV-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3}=0 \quad CE-\frac{6\cdot N^2-6\cdot N+1}{\left(4\cdot N-4\cdot N^2\right)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}+2\cdot(2\cdot N-1)^3}=0$$

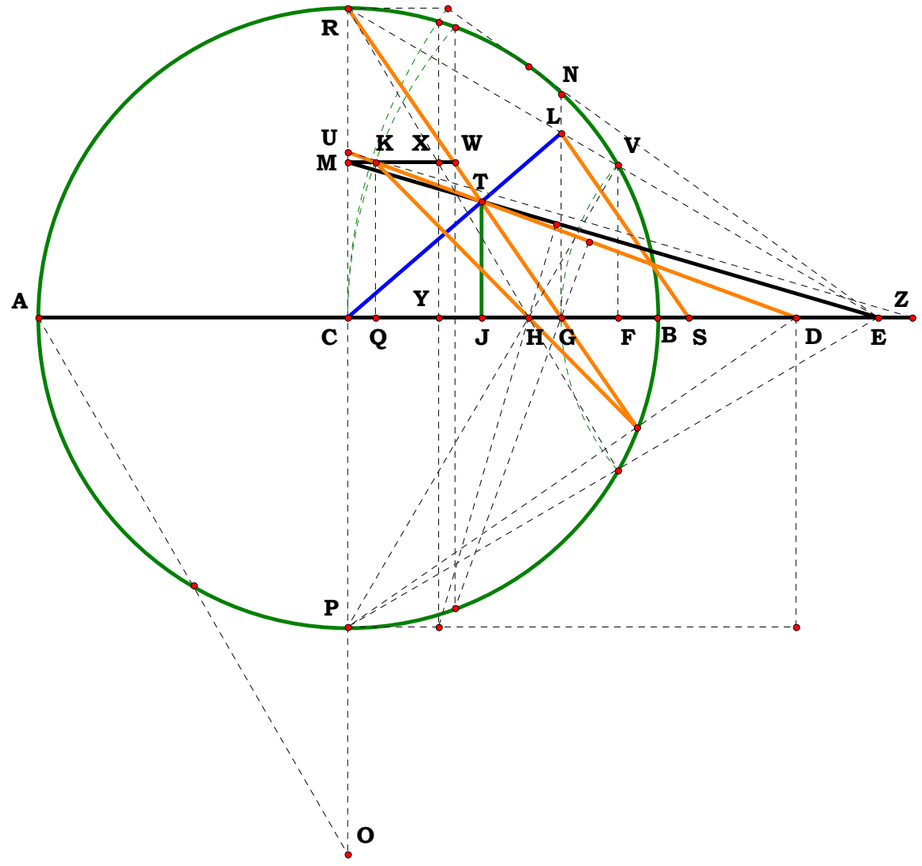
$$BE-\frac{(N-1)\cdot\left(5\cdot N+N\cdot\sqrt{16\cdot N^2-16\cdot N+3}-4\cdot N^2-1\right)}{\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+8\cdot N^3-12\cdot N^2+6\cdot N-1}=0 \quad BG-\frac{N-1}{2\cdot N-1}=0$$

$$EF-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot N\cdot(N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)}{(2\cdot N-1)^3\cdot\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)^3\cdot\left[(2\cdot N-1)^3\right]}=0$$

$$EG-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left[\left(8\cdot N^3-12\cdot N^2+6\cdot N-1\right)+\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}\right]}=0 \quad GL-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$GS-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)^2\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0 \quad CJ-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot\left(6\cdot N^2-6\cdot N+1\right)}{\left(4\cdot N^2-4\cdot N\right)\cdot\left(\sqrt{16\cdot N^2-16\cdot N+3}\right)^2+\left(32\cdot N^3-48\cdot N^2+20\cdot N-2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$JT-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{\left(2\cdot N^2-2\cdot N\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0$$



$$CM - \frac{1}{4} = 0 \qquad DK - \frac{2\cdot N - 1}{2} = 0 \qquad KQ - \frac{1}{4} = 0$$

$$DQ - \frac{\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}}{4} = 0 \qquad CU - \frac{2\cdot N - 1}{2\cdot\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}} = 0$$

$$CZ - \frac{(2\cdot N - 1)\cdot\left(6\cdot N^2 - 6\cdot N + 1\right)}{(2\cdot N - 1)^3\cdot\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)} + (4\cdot N - 1)\cdot(4\cdot N - 3)\cdot\left(2\cdot N - 2\cdot N^2\right)} = 0$$

$$DU - \frac{(2\cdot N - 1)^2}{\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}} = 0$$

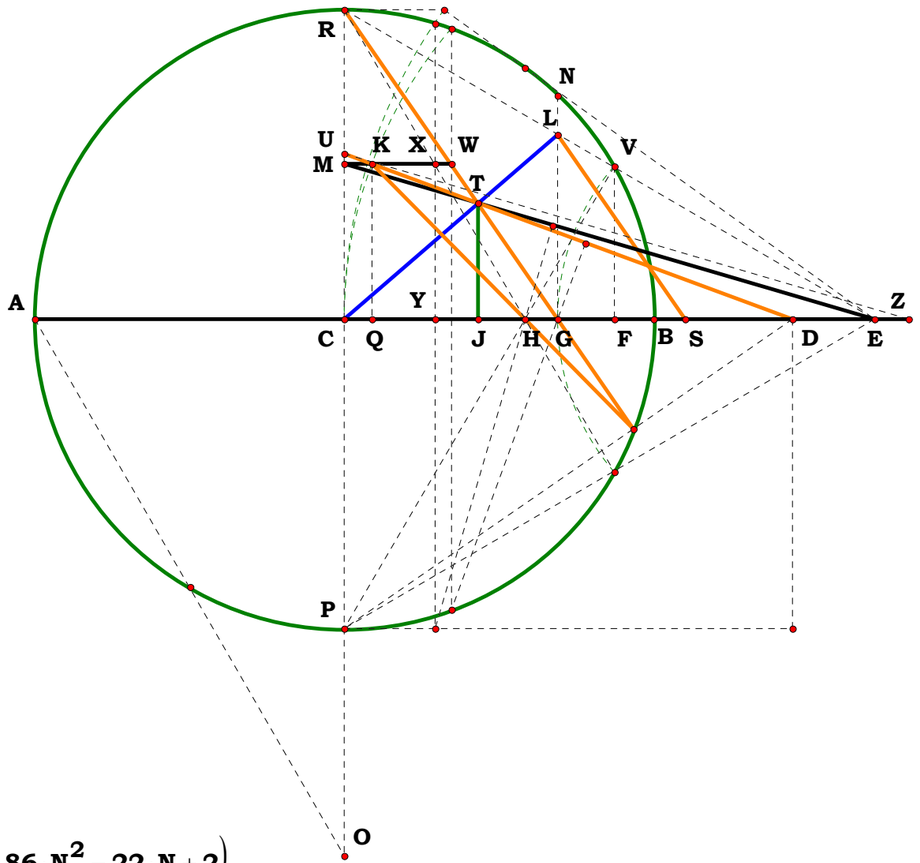
$$DZ - \frac{2\cdot N\cdot\left(32\cdot N^4 - 80\cdot N^3 + 82\cdot N^2 - 43\cdot N + 11\right) - (2\cdot N - 1)^4\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - 2}{2\cdot(2\cdot N - 1)^3\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - 4\cdot N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)} = 0$$

$$DE - \frac{N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - N\cdot(N - 1)\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)}{\left(2\cdot N - 2\cdot N^2\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} + 8\cdot N^3 - 12\cdot N^2 + 6\cdot N - 1} = 0$$

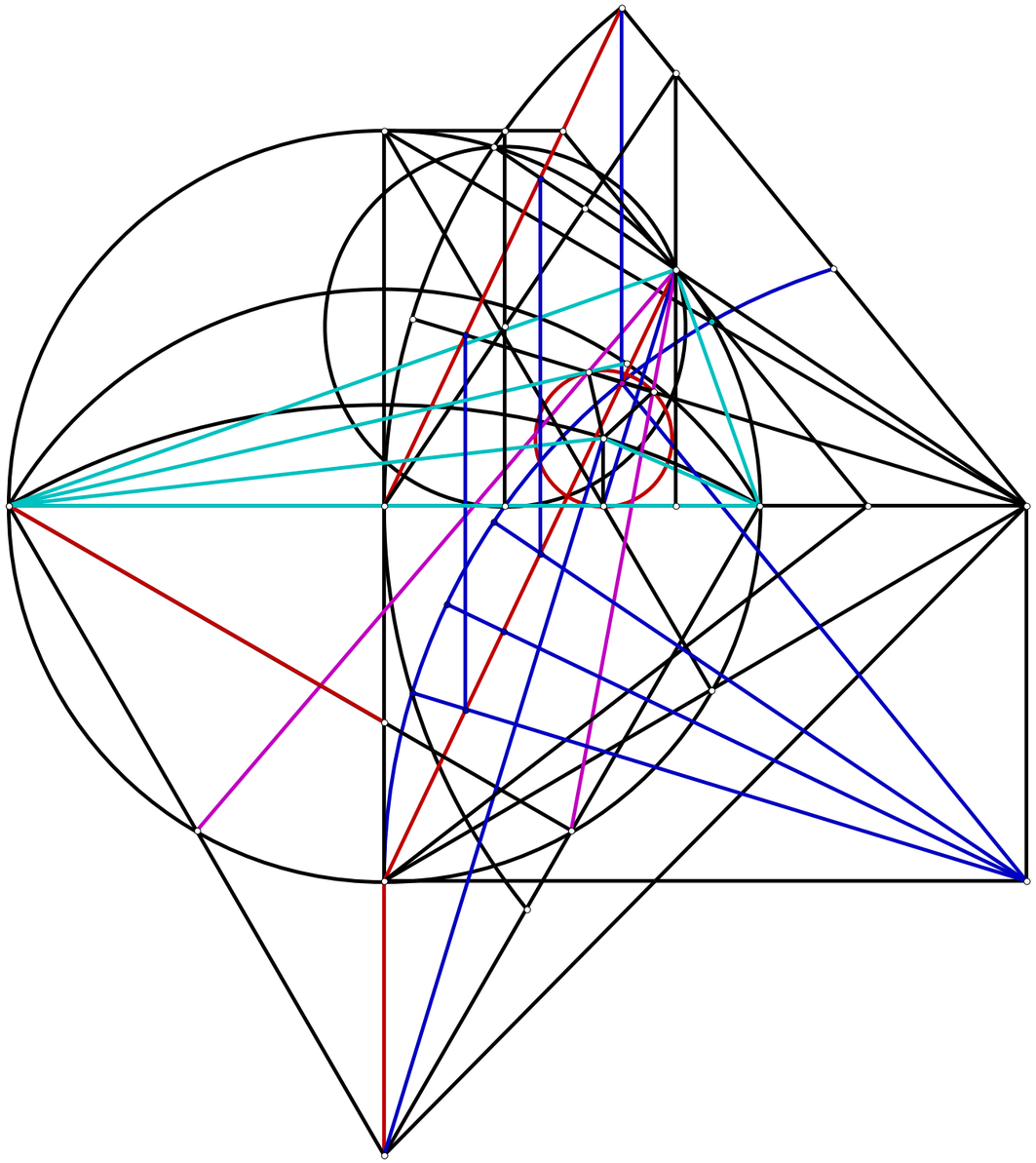
$$JC - \frac{\left(6\cdot N^2 - 6\cdot N + 1\right)\cdot\left(16\cdot N^3 - 24\cdot N^2 + 10\cdot N - 1\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - \left(6\cdot N^2 - 6\cdot N + 1\right)\cdot\left(64\cdot N^4 - 128\cdot N^3 + 86\cdot N^2 - 22\cdot N + 2\right)}{\left[8\cdot N\cdot(N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 53\cdot N^2 - 21\cdot N + 4\right) + 2\right]\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} + -4\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot(2\cdot N - 1)^5} = 0$$

$$DJ - \frac{N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3}\cdot\left(64\cdot N^4 - 128\cdot N^3 + 82\cdot N^2 - 18\cdot N + 1\right) - N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 42\cdot N^2 - 10\cdot N + 1\right)}{\sqrt{16\cdot N^2 - 16\cdot N + 3}\cdot\left(128\cdot N^6 - 384\cdot N^5 + 468\cdot N^4 - 296\cdot N^3 + 100\cdot N^2 - 16\cdot N + 1\right) - (2\cdot N - 1)^5\cdot\left(16\cdot N^2 - 16\cdot N + 2\right)} = 0$$

$$TJ - \frac{N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 42\cdot N^2 - 10\cdot N + 1\right) - N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\left(64\cdot N^4 - 128\cdot N^3 + 82\cdot N^2 - 18\cdot N + 1\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3}}{2\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot(2\cdot N - 1)^5\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - (4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left[4\cdot N\cdot(N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 53\cdot N^2 - 21\cdot N + 4\right) + 1\right]} = 0$$



**Just an Ilustration.**



Unit.

$AB := 1$

Given.

$N := 4$ $AE := N$

**Just some Algebraic Names**

050601A

Descriptions.

$BE := AE - AB$ $BO := \frac{BE}{2}$ $AO := AB + BO$ $AJ := AO$ $JO := BO$

$GO := \frac{JO}{2}$ $AG := \sqrt{AO^2 - GO^2}$ $AP := \frac{AG^2}{AO}$ $OP := AO - AP$ $NO := 2 \cdot OP$

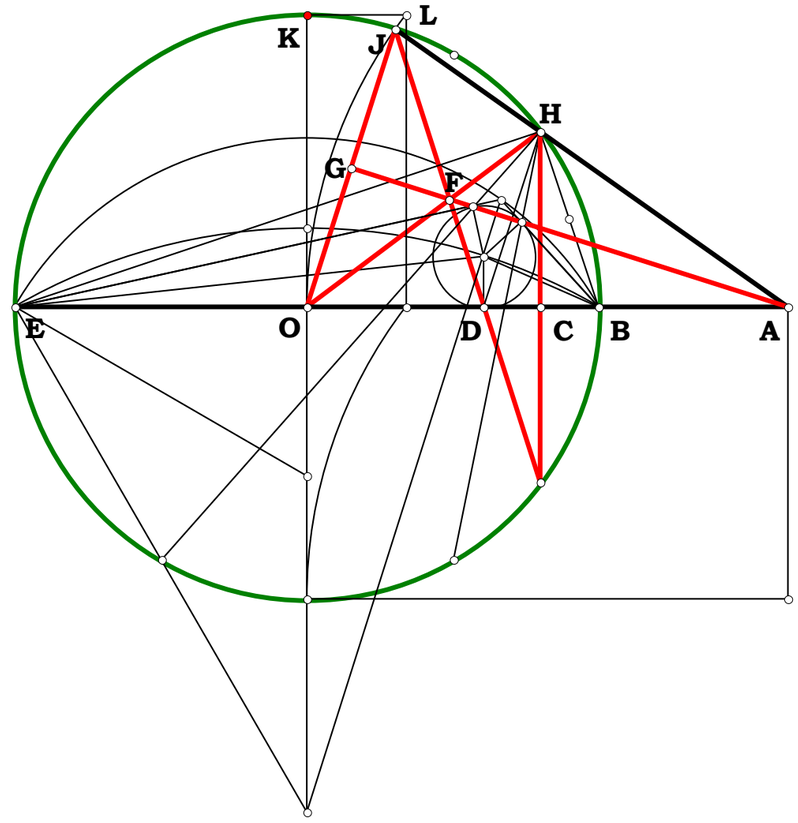
$AN := AO - NO$ $JM := NO$ $HO := BO$ $HJ := 2 \cdot JM$ $AH := AJ - HJ$

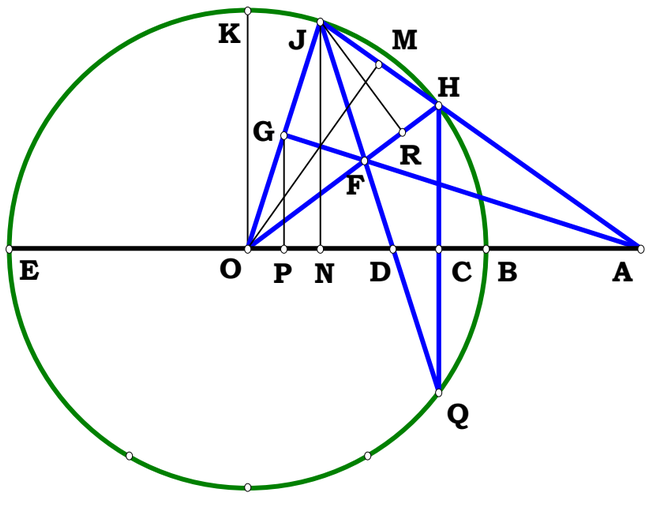
$AC := \frac{AN \cdot AH}{AJ}$ $CH := \sqrt{AH^2 - AC^2}$ $HQ := 2 \cdot CH$ $CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH}$ $CQ := CH$ $JQ := \sqrt{(CQ + JN)^2 + CN^2}$ $OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2}$ $FO := \frac{JO \cdot GO}{OR}$ $FJ := FO$ $DQ := \frac{JQ \cdot CQ}{CQ + JN}$ $DF := JQ - (DQ + FJ)$

$FH := HO - FO$ $FG := \frac{JR \cdot GO}{OR}$ $AF := AG - FG$



**Definitions.**

$$AE - N = 0 \quad BE - (N-1) = 0 \quad BO - \frac{N-1}{2} = 0 \quad AO - \frac{N+1}{2} = 0 \quad AG - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$$

$$AP - \frac{(3\cdot N+1)\cdot(N+3)}{8\cdot(N+1)} = 0 \quad OP - \frac{(N-1)^2}{8\cdot(N+1)} = 0 \quad NO - \frac{(N-1)^2}{4\cdot(N+1)} = 0 \quad AN - \frac{N^2+6\cdot N+1}{4\cdot(N+1)} = 0$$

$$HJ - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AH - \frac{2\cdot N}{N+1} = 0 \quad AC - \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} = 0 \quad CH - \frac{N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0$$

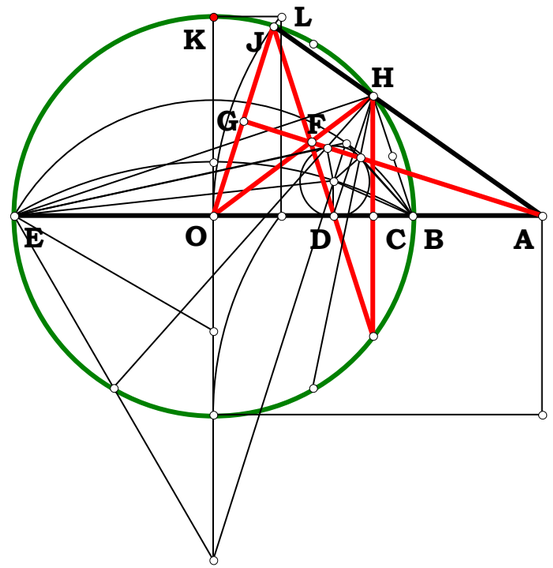
$$HQ - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0 \quad CN - \frac{(N-1)^2\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^3} = 0 \quad JN - \frac{\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot(N-1)}{4\cdot(N+1)} = 0$$

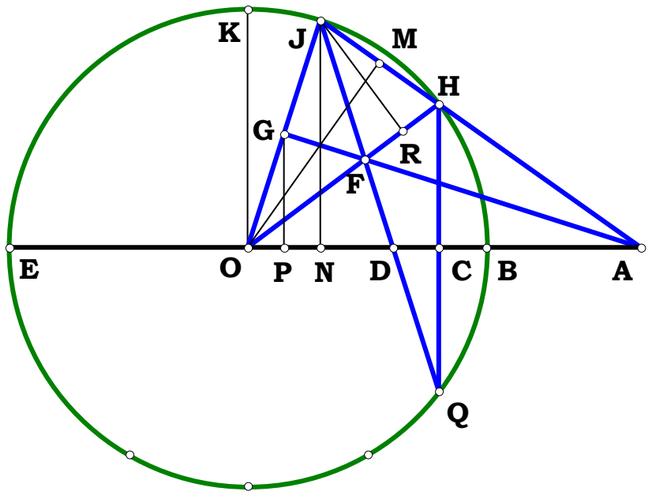
$$JQ - \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} = 0 \quad OR - \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^2} = 0 \quad JR - \frac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot(N+1)^2} = 0$$

$$FO - \frac{(N-1)\cdot(N+1)^2}{2\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad DQ - \frac{2\cdot N\cdot(N-1)}{(N+1)^2} = 0 \quad DF - \frac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0 \quad FH - \frac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0$$

$$FG - \frac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad GO - \frac{N-1}{4} = 0$$

$$AF - \frac{\left(2\cdot N-N^2-1\right)\cdot\sqrt{3\cdot N+1}\cdot\sqrt{N+3}+\left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0$$



**Unit.**

$AB := 1$

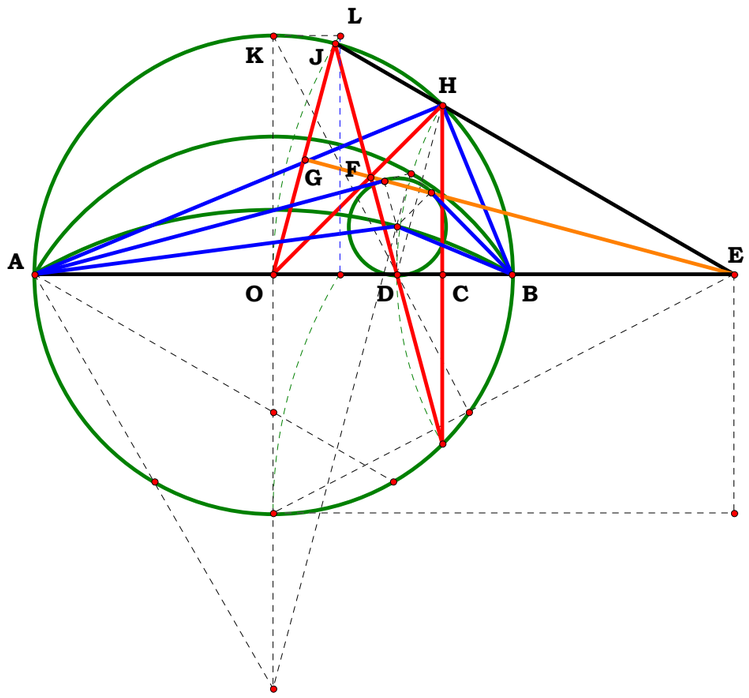
**Given.**

$N := 1.46595 \quad AE := N$

**050601B**

**Descriptions.**

## Just some Algebraic Names




$BE := AE - AB \quad BO := \frac{AB}{2} \quad EO := BE + BO \quad EJ := EO$

$JO := BO \quad GO := \frac{JO}{2} \quad EG := \sqrt{EO^2 - GO^2}$

$EP := \frac{EG^2}{EO} \quad OP := EO - EP \quad NO := 2 \cdot OP$

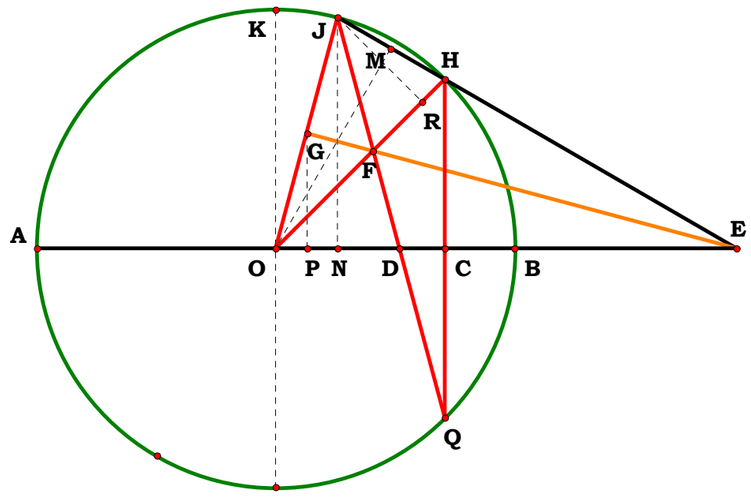
$EN := EO - NO \quad JM := NO \quad HO := BO \quad HJ := 2 \cdot JM \quad EH := EJ - HJ$

$EC := \frac{EN \cdot EH}{EJ} \quad CH := \sqrt{EH^2 - EC^2} \quad HQ := 2 \cdot CH \quad CN := EN - EC$

$JN := \frac{CH \cdot EJ}{EH} \quad CQ := CH \quad JQ := \sqrt{(CQ + JN)^2 + CN^2} \quad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2} \quad FO := \frac{JO \cdot GO}{OR} \quad FJ := FO \quad DQ := \frac{JQ \cdot CQ}{CQ + JN}$

$DF := JQ - (DQ + FJ) \quad FH := HO - FO \quad FG := \frac{JR \cdot GO}{OR} \quad EF := EG - FG$



**Definitions.**

$$BE-(N-1)=0 \quad BO-\frac{1}{2}=0 \quad EO-\frac{2\cdot N-1}{2}=0 \quad EJ-\frac{2\cdot N-1}{2}=0$$

$$JO-\frac{1}{2}=0 \quad GO-\frac{1}{4}=0 \quad EG-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4}=0$$

$$EP-\frac{(4\cdot N-1)\cdot(4\cdot N-3)}{8\cdot(2\cdot N-1)}=0 \quad OP-\frac{1}{8\cdot(2\cdot N-1)}=0 \quad NO-\frac{1}{4\cdot(2\cdot N-1)}=0$$

$$EN-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)}=0 \quad JM-\frac{1}{4\cdot(2\cdot N-1)}=0 \quad HO-\frac{1}{2}=0 \quad HJ-\frac{1}{2\cdot(2\cdot N-1)}=0$$

$$EH-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad EC-\frac{N\cdot(N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}{(2\cdot N-1)^3}=0 \quad CH-\frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3}=0$$

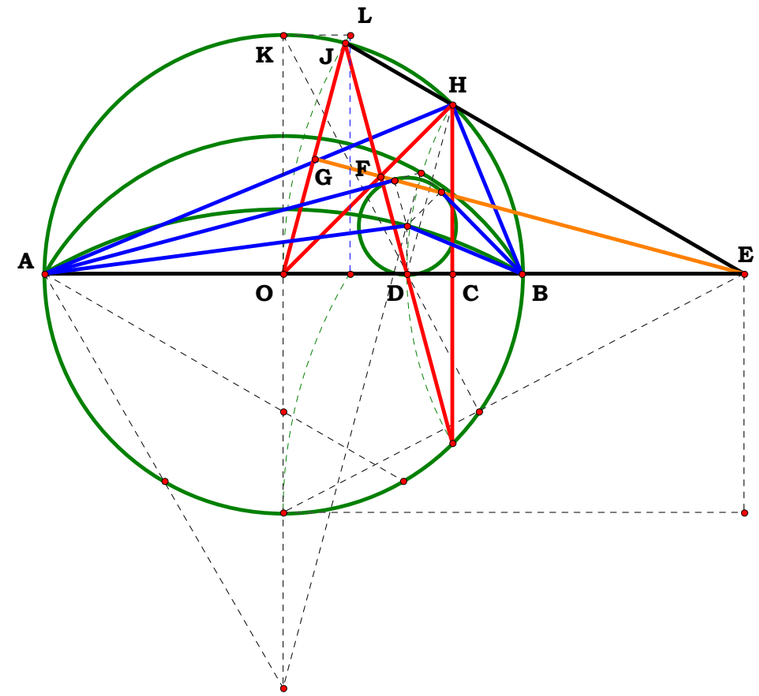
$$HQ-\frac{2\cdot N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3}=0 \quad CN-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^3}=0 \quad JN-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)}=0$$

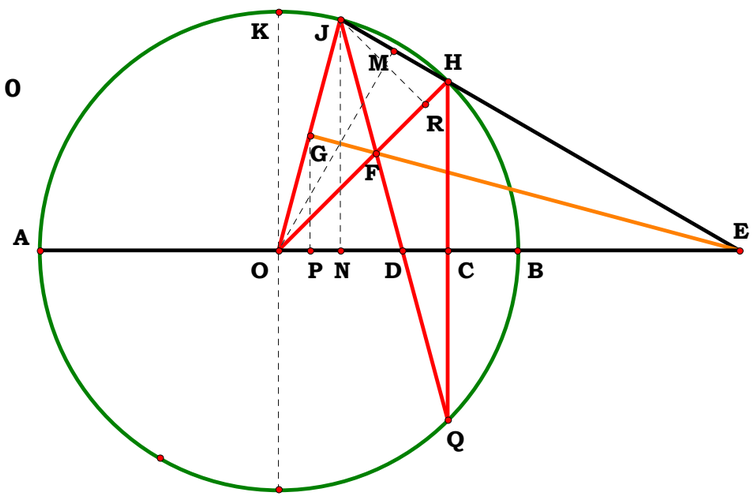
$$CQ-\frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3}=0 \quad JQ-\frac{\left(8\cdot N^2-8\cdot N+1\right)}{2\cdot(2\cdot N-1)^2}=0 \quad OR-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^2}=0$$

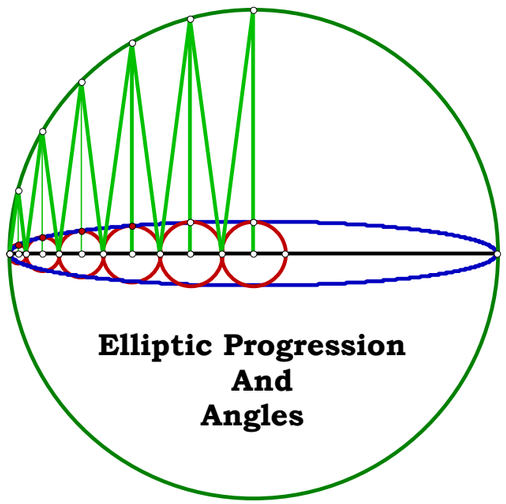
$$JR-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)^2}=0 \quad FO-\frac{(2\cdot N-1)^2}{2\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0 \quad FJ-\frac{HO}{4\cdot\left(-HJ^2+HO^2+JO^2\right)}=0$$

$$DQ-\frac{2\cdot N\cdot(N-1)}{(2\cdot N-1)^2}=0 \quad DF-\frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0 \quad FH-\frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0$$

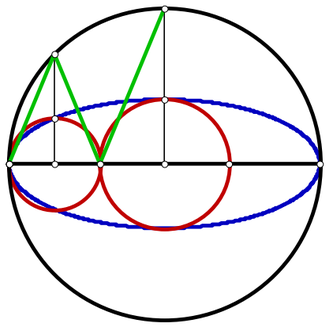
$$FG-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0 \quad EF-\frac{2\cdot\sqrt{16\cdot N^2-16\cdot N+3}\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0$$



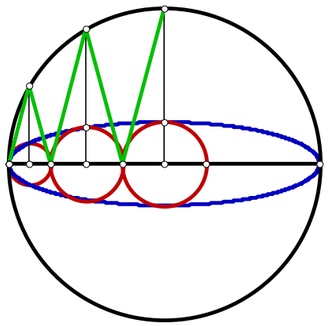





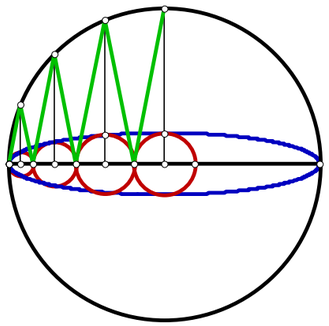

$\frac{m\angle CAAW}{m\angle ARCN} = 2.000$



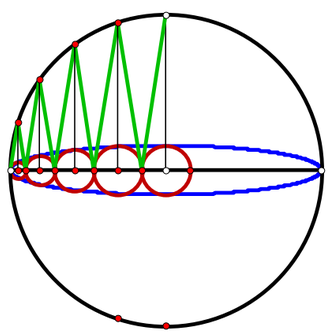
$\frac{m\angle CAAW}{m\angle ARCN} = 3.000$



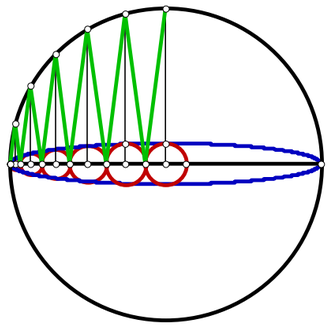
$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$



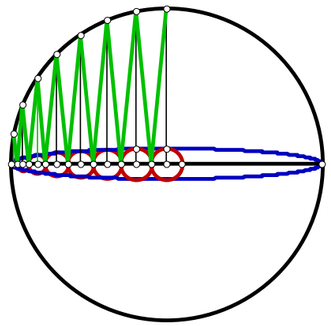
$\frac{m\angle CAAW}{m\angle BCA} = 5.000$



$\frac{m\angle CAAW}{m\angle ARCN} = 6.000$



$\frac{m\angle CAAW}{m\angle ARCN} = 8.000$



Unit.

$AB := 1$

Given.

$N_1 := \frac{AB}{10}$

$N_2 := 8$

**050701 EP**

**Descriptions.**

$AQ := N_1 \cdot N_2$ $\quad BQ := AB - AQ$ $\quad DQ := \sqrt{BQ \cdot AQ}$

$AO := \frac{AB}{2}$ $\quad AY := \sqrt{2 \cdot AO^2}$ $\quad OQ := AQ - AO$

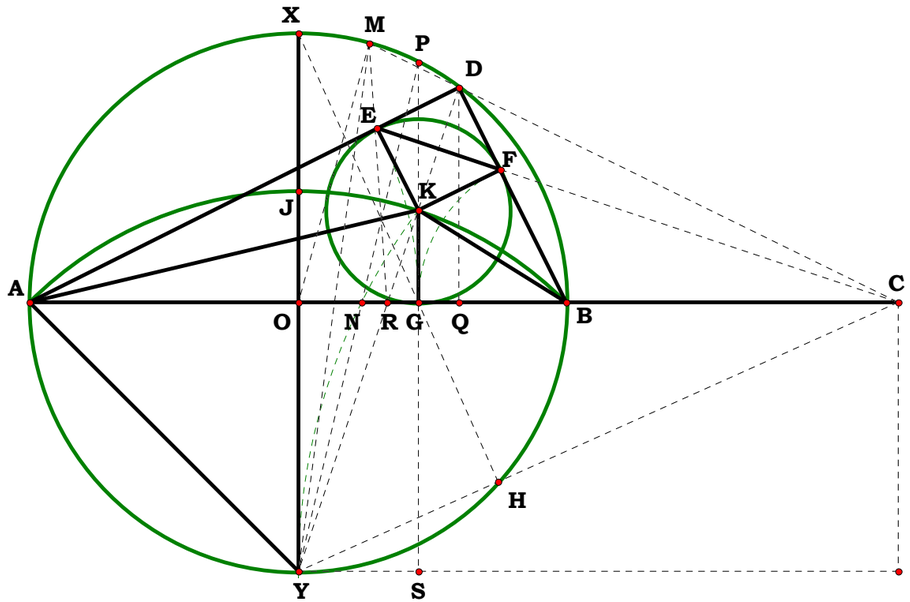
$DY := \sqrt{(AO + DQ)^2 + OQ^2}$ $\quad OR := \frac{OQ \cdot AO}{AO + DQ}$

$QR := OQ - OR$ $\quad KS := \frac{AY \cdot (DQ + AO)}{DY}$

$GK := KS - AO$ $\quad GR := \frac{QR \cdot GK}{DQ}$ $\quad GO := GR + OR$

$CO := \frac{AO^2}{GO}$ $\quad BC := CO - AO$ $\quad AC := AB + BC$

**What is an Angle?**




$AC = 1.61803$

$N_2 = 8.00000$

$N_1 = 10.00000$

$$\frac{\left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right) + \sqrt{1 + 2 \cdot \sqrt{\left(\frac{1}{N_1}\right) \cdot N_2 - \frac{1}{N_1}^2 \cdot N_2^2}}}{2 \cdot \left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right)} = 1.61803$$

$$AC - \frac{\left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right) + \sqrt{1 + 2 \cdot \sqrt{\left(\frac{1}{N_1}\right) \cdot N_2 - \frac{1}{N_1}^2 \cdot N_2^2}}}{2 \cdot \left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right)} = 0.00000$$

**Definitions.**

$AQ - N_1 \cdot N_2 = 0$ $\quad BQ - \left(1 - N_1 \cdot N_2\right) = 0$ $\quad DQ - \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} = 0$

$AO - \frac{1}{2} = 0$ $\quad AY - \frac{1}{\sqrt{2}} = 0$ $\quad OQ - \frac{2 \cdot N_1 \cdot N_2 - 1}{2} = 0$ $\quad DY - \frac{\sqrt{4 \cdot \sqrt{-N_1 \cdot N_2 \cdot \left(N_1 \cdot N_2 - 1\right)} + 2}}{2} = 0$ $\quad OR - \frac{2 \cdot N_1 \cdot N_2 - 1}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)} = 0$

$QR - \frac{\sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} = 0$ $\quad KS - \frac{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}{2 \cdot \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0$ $\quad GK - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2} = 0$

$GR - \frac{\left(\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1\right) \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)}$ $\quad GO - \frac{N_1 \cdot N_2 - \frac{1}{2}}{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0$ $\quad CO - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0$

$BC - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 2 \cdot N_1 \cdot N_2 + 1}{4 \cdot N_1 \cdot N_2 - 2} = 0$ $\quad AC - \frac{2 \cdot N_1 \cdot N_2 + \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0$ $\quad AC - \frac{N_2 + \sqrt{5} \cdot \sqrt{\sqrt{10 \cdot N_2 - N_2^2} + 5} - 5}{2 \cdot N_2 - 10} = 0$

**Unit.**

$AB := 1$

**0507013**
**Descriptions.**

$AC := \frac{AB}{2}$ $BF := \sqrt{2 \cdot AC^2}$

$BG := \frac{BF}{2}$ $CH := AC$ $CG := BG$

$GH := CH - CG$ $BH := \sqrt{BG^2 + GH^2}$

$BD := \frac{BH^2}{AB}$ $CE := \frac{AB - (4 \cdot BD)}{2}$

**Definitions.**

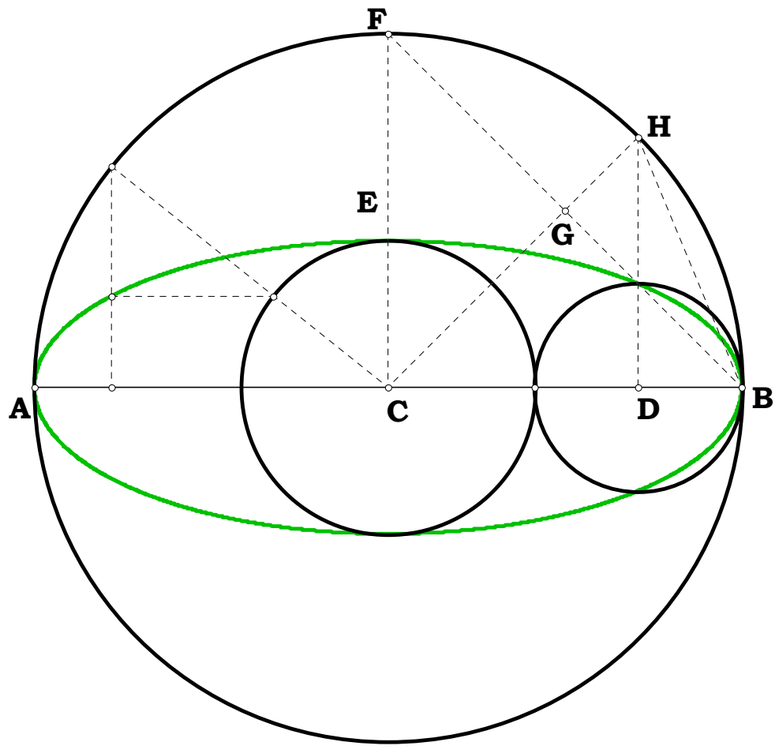
$AC - \frac{1}{2} = 0$ $BF - \frac{\sqrt{2}}{2} = 0$

$BG - \frac{\sqrt{2}}{4} = 0$ $CH - \frac{1}{2} = 0$ $CG - \frac{\sqrt{2}}{4} = 0$

$GH - \frac{2 - \sqrt{2}}{4} = 0$ $BH - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$BD - \frac{2 - \sqrt{2}}{4} = 0$ $CE - \frac{\sqrt{2} - 1}{2} = 0$

**Angles by Ellipse:**



Unit.

$AN := 1$

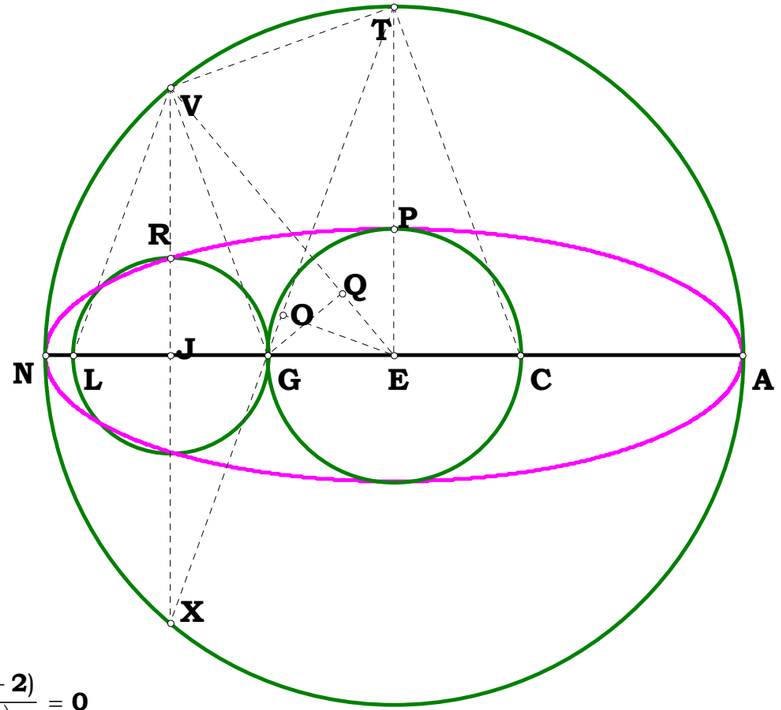
Given.

$N := 5.52$

0507012

## Elliptic Progression Outtake Two

Angles TEV and EVJ equals CTG.




**Descriptions.**

$$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN \quad EG := \frac{AN}{N} \quad EP := EG$$

$$GT := \sqrt{EG^2 + ET^2} \quad GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO \quad JX := \frac{ET \cdot GX}{GT}$$

$$GJ := \frac{EG \cdot GX}{GT} \quad \frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV := JX \quad EJ := EG + GJ$$

$$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2} \quad EQ := \frac{EJ \cdot EG}{EV} \quad GQ := \frac{JV \cdot EQ}{EJ}$$

$$GQ - GJ = 0 \quad \frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \quad TV - \frac{2 \cdot AN}{\sqrt{N^2 + 4}} = 0$$

**Definitions.**

$$EN - \frac{1}{2} = 0 \quad ET - \frac{1}{2} = 0 \quad EV - \frac{1}{2} = 0 \quad EG - \frac{1}{N} = 0 \quad EP - \frac{1}{N} = 0$$

$$GT - \frac{\sqrt{N^2 + 4}}{2 \cdot N} = 0 \quad GO - \frac{2}{N \cdot \sqrt{N^2 + 4}} = 0 \quad GX - \frac{(N-2) \cdot (N+2)}{2 \cdot N \cdot \sqrt{N^2 + 4}} = 0 \quad JX - \frac{(N-2) \cdot (N+2)}{2 \cdot (N^2 + 4)} = 0$$

$$GJ - \frac{(N-2) \cdot (N+2)}{N \cdot (N^2 + 4)} = 0 \quad \frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV - \frac{(N-2) \cdot (N+2)}{2 \cdot (N^2 + 4)} = 0 \quad EJ - \frac{2 \cdot N}{N^2 + 4} = 0$$

$$TV - \frac{2}{\sqrt{N^2 + 4}} = 0 \quad EQ - \frac{4}{N^2 + 4} = 0 \quad GQ - \frac{(N-2) \cdot (N+2)}{N \cdot (N^2 + 4)} = 0$$

Unit.

$AL := 1$

**0507013**

**Descriptions.**

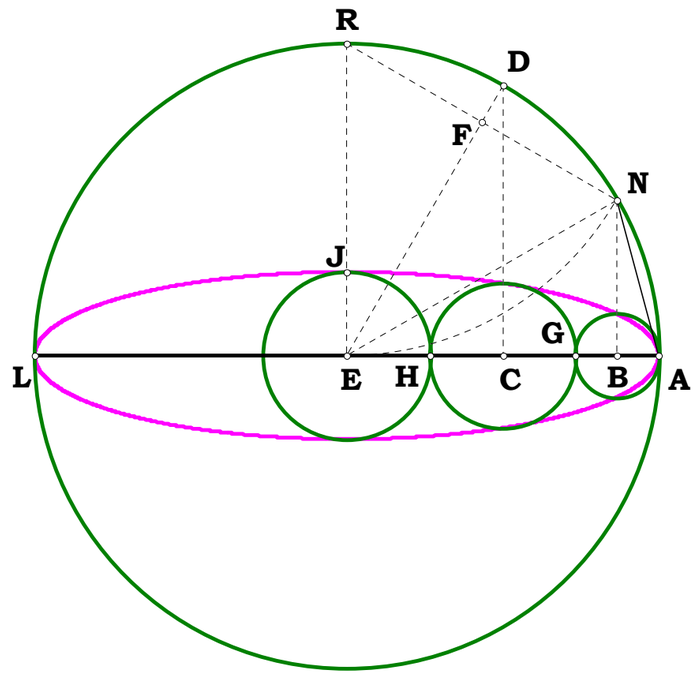
$AE := \frac{AL}{2}$ $ER := AE$ $NR := ER$ $FN := \frac{NR}{2}$ $EN := AE$

$EF := \sqrt{EN^2 - FN^2}$ $DE := AE$ $DF := DE - EF$ $DN := \sqrt{DF^2 + FN^2}$

$AN := DN$ $AB := \frac{AN^2}{AL}$ $EL := AE$ $BL := AL - AB$ $BN := \sqrt{AN^2 - AB^2}$

$EJ := \frac{BN \cdot EL}{BL}$ $GH := AE - (EJ + 2 \cdot AB)$

**Trisection:**




**Definitions.**

$FN - \frac{1}{4} = 0$ $EF - \frac{\sqrt{3}}{4} = 0$ $DF - \frac{2 - \sqrt{3}}{4} = 0$ $AN - \frac{\sqrt{2} \cdot (\sqrt{3} - 1)}{4} = 0$

$\frac{2 - \sqrt{3}}{4} - AB = 0$ $\frac{\sqrt{3} + 2}{4} - BL = 0$ $\frac{1}{4} - BN = 0$ $\frac{1}{2 \cdot \sqrt{3} + 4} - EJ = 0$ $\frac{\sqrt{3}}{2 \cdot \sqrt{3} + 4} - GH = 0$

Unit.

$AC := 1$

Given.

**050701A**

**Descriptions.**

$CP := \sqrt{2AC^2}$ $AP := AC$ $EP := CP$

$CE := \sqrt{AC^2 + (AP + EP)^2}$

$EK := CE$ $EH := CE$ $AE := \sqrt{CE^2 - AC^2}$

$AH := EH - AE$ $AT := AH$ $EU := \frac{AT \cdot EK}{AC}$

$\frac{EK}{EU} = 5.027339$ $1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 5.027339$

**Definitions.**

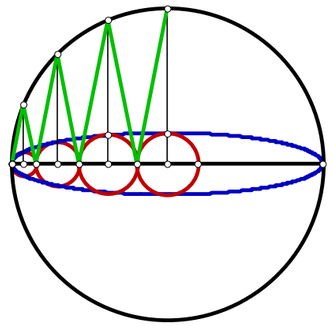
$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$

$CP - \sqrt{2} = 0$ $EP - \sqrt{2} = 0$ $CE - \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 0$

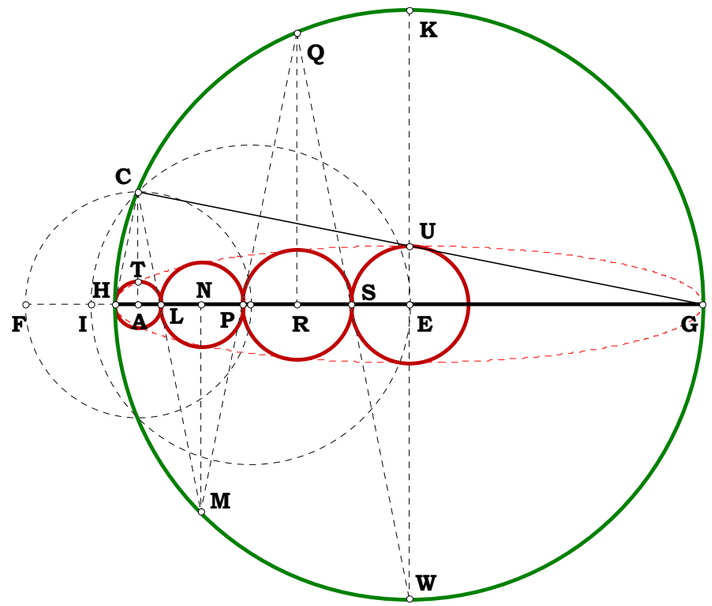
$AE - \left(\sqrt{2} + 1\right) = 0$ $AH - \left(\sqrt{2} \cdot \sqrt{2 + \sqrt{2}} - 1 - \sqrt{2}\right) = 0$

$EU - \left(2 - \sqrt{\sqrt{2} + 2}\right) \cdot \left(\sqrt{2} + 2\right) = 0$

$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$



**An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle, from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles.**



Unit.

$AL := 1$

0507013B

Descriptions.

$AE := \frac{AL}{2}$ $AR := \sqrt{2 \cdot AE^2}$

$AM := \frac{AR}{2}$ $EO := AE$ $EM := AM$

$MO := EO - EM$ $AO := \sqrt{AM^2 + MO^2}$

$AY := \frac{AO}{2}$ $EY := \sqrt{AE^2 - AY^2}$

$EN := AE$ $NY := EN - EY$ $AN := \sqrt{AY^2 + NY^2}$

$AP := \frac{AN^2}{AL}$ $NP := \sqrt{AN^2 - AP^2}$ $ES := \frac{NP \cdot AE}{AL - AP}$

Definitions.

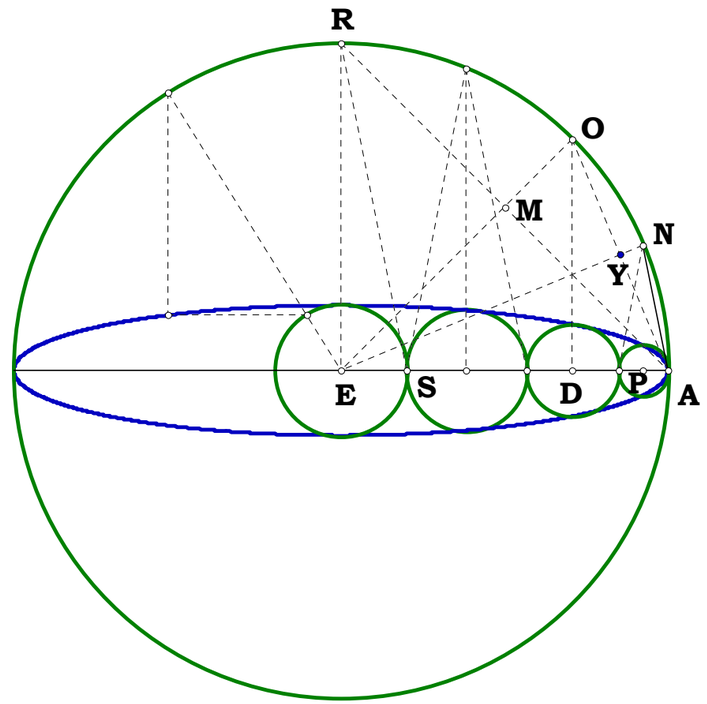
$AR - \frac{\sqrt{2}}{2} = 0$ $AM - \frac{\sqrt{2}}{4} = 0$ $MO - \frac{2 - \sqrt{2}}{4} = 0$ $AO - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$AY - \frac{\sqrt{2 - \sqrt{2}}}{4} = 0$ $EY - \frac{\sqrt{\sqrt{2} + 2}}{4} = 0$ $NY - \frac{2 - \sqrt{\sqrt{2} + 2}}{4} = 0$

$AN - \frac{\sqrt{2 - \sqrt{\sqrt{2} + 2}}}{2} = 0$ $AP - \frac{2 - \sqrt{2 + \sqrt{2}}}{4} = 0$ $NP - \frac{\sqrt{2 - \sqrt{2}}}{4} = 0$

$ES - \frac{\sqrt{2 - \sqrt{2}}}{2 \cdot \left(\sqrt{\sqrt{2} + 2} + 2\right)} = 0$

Quadsection:



**Unit.**

$AG := 1$

**0507014**

**Descriptions.**

$AE := \frac{AG}{2}$ $AC := \frac{AE}{2}$ $CG := AG - AC$ $CJ := \sqrt{AC \cdot CG}$

$EL := AE$ $CE := AC$ $JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

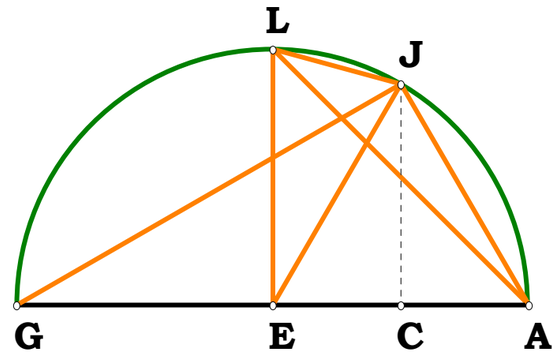
$AJ := \sqrt{AC^2 + CJ^2}$ $GJ := \sqrt{CG^2 + CJ^2}$ $AL := \sqrt{AE^2 + EL^2}$

**Definitions.**

$AE - \frac{1}{2} = 0$ $AC - \frac{1}{4} = 0$ $CG - \left(1 - \frac{1}{4}\right) = 0$ $CJ - \frac{1}{4} \cdot \sqrt{3} = 0$

$JL - \left(\frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}\right) = 0$ $AJ - \frac{1}{2} = 0$ $GJ - \frac{1}{2} \cdot \sqrt{3} = 0$ $AL - \frac{1}{2} \cdot \sqrt{2} = 0$

**Outtake Four: Some Names**



Unit.

$AL := 1$

0507013

Descriptions.

## Alternate Method: Pentasection Or Irrational Rationals

*Irrational* means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.

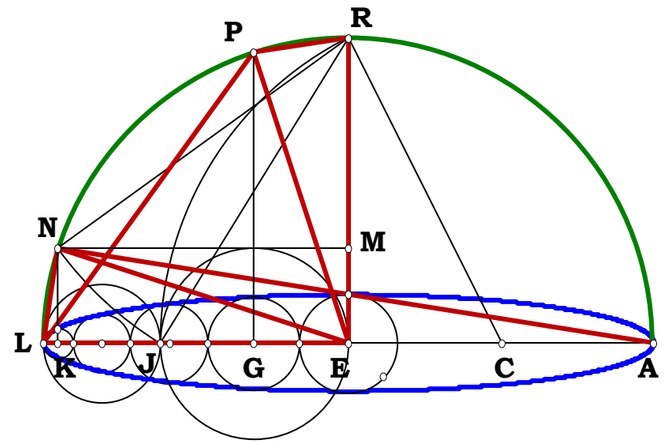
$$AE := \frac{AL}{2} \qquad AC := \frac{AE}{2} \qquad CE := AC \qquad ER := AE \qquad CR := \sqrt{CE^2 + ER^2} \qquad CJ := CR$$

$$EJ := CJ - CE \qquad JR := \sqrt{EJ^2 + ER^2} \qquad NR := JR \qquad EN := AE \qquad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$$

$$KN := EM \qquad EK := \sqrt{EN^2 - KN^2} \qquad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$$

$$EG := \frac{EJ}{2} \qquad GL := EL - EG \qquad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL}$$

$$PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2} \qquad PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$$




**Definitions:**

$$AE - \frac{1}{2} = 0 \qquad AC - \frac{1}{4} = 0 \qquad CR - \frac{1}{4} \cdot \sqrt{5} = 0 \qquad EJ - \left( \frac{\sqrt{5}}{4} - \frac{1}{4} \right) = 0$$

$$JR - \frac{\sqrt{2} \cdot \sqrt{5 - \sqrt{5}}}{4} = 0 \qquad EM - \left( \frac{\sqrt{5}}{8} - \frac{1}{8} \right) = 0 \qquad EK - \frac{1}{2} \cdot \sqrt{\frac{\sqrt{5}}{8} + \frac{5}{8}} = 0$$

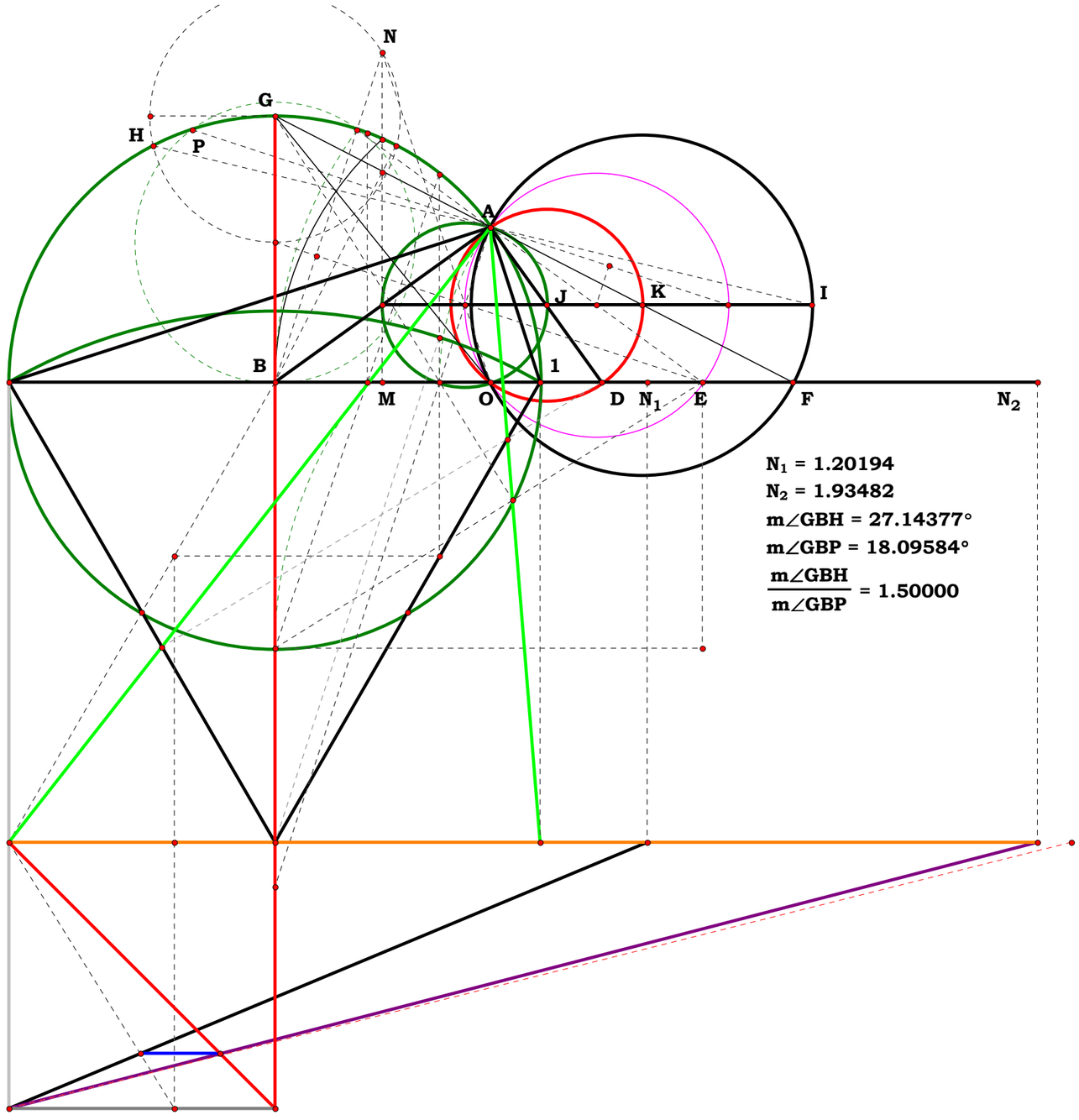
$$KL - \left( \frac{1}{2} - \frac{\sqrt{2} \cdot \sqrt{\sqrt{5} + 5}}{8} \right) = 0 \qquad LN - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0 \qquad EG - \left( \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} \right) = 0$$

$$GL - \left( \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} \right) = 0 \qquad AG - \left( \frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} \right) = 0 \qquad GP - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} = 0$$

$$PR - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0 \qquad AN - \frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0$$

**051101 Round Tuits**

**I had intended to someday write this figure up, it is chock full of fixed intersections which are part of the figure, you just have to find them. Here it is, some eighteen years later. I know that there are a lot of circles in it, but in order to writhe this figure up, I need a round tuit.**



**Unit.**

$AB := 1$

**Given.**

$N_1 := 1.20194$

$N_2 := 1.93482$

**Descriptions.**

$AS := \frac{N_1 + N_2}{2 \cdot N_2}$ $AS = 0.810608$ $ST := \sqrt{AS \cdot (AB - AS)}$ $AO := \frac{AB}{2}$

$OQ := AO$ $OP := AO$ $OS := AS - AO$ $PS := \sqrt{OS^2 + OP^2}$ $CO := \frac{OP^2}{OS}$

$AC := CO + AO$ $CW := CO$ $OW := AO$ $OX := \frac{CO^2 + OW^2 - CW^2}{2 \cdot CO}$

$FW := 2 \cdot OX$ $CF := CW - FW$ $CI := \frac{CF}{2}$ $FO := AO$ $EO := \frac{CO^2 + FO^2 - CF^2}{2 \cdot CO}$

$BO := AO$ $BE := BO - EO$ $AE := AO + EO$ $EF := \sqrt{AE \cdot BE}$ $CE := CF$

$DF := \frac{FO \cdot EF}{EO}$ $FH := \frac{DF}{2}$ $DE := \frac{EF \cdot DF}{FO}$ $DO := EO + DE$ $GH := \frac{DO}{2}$

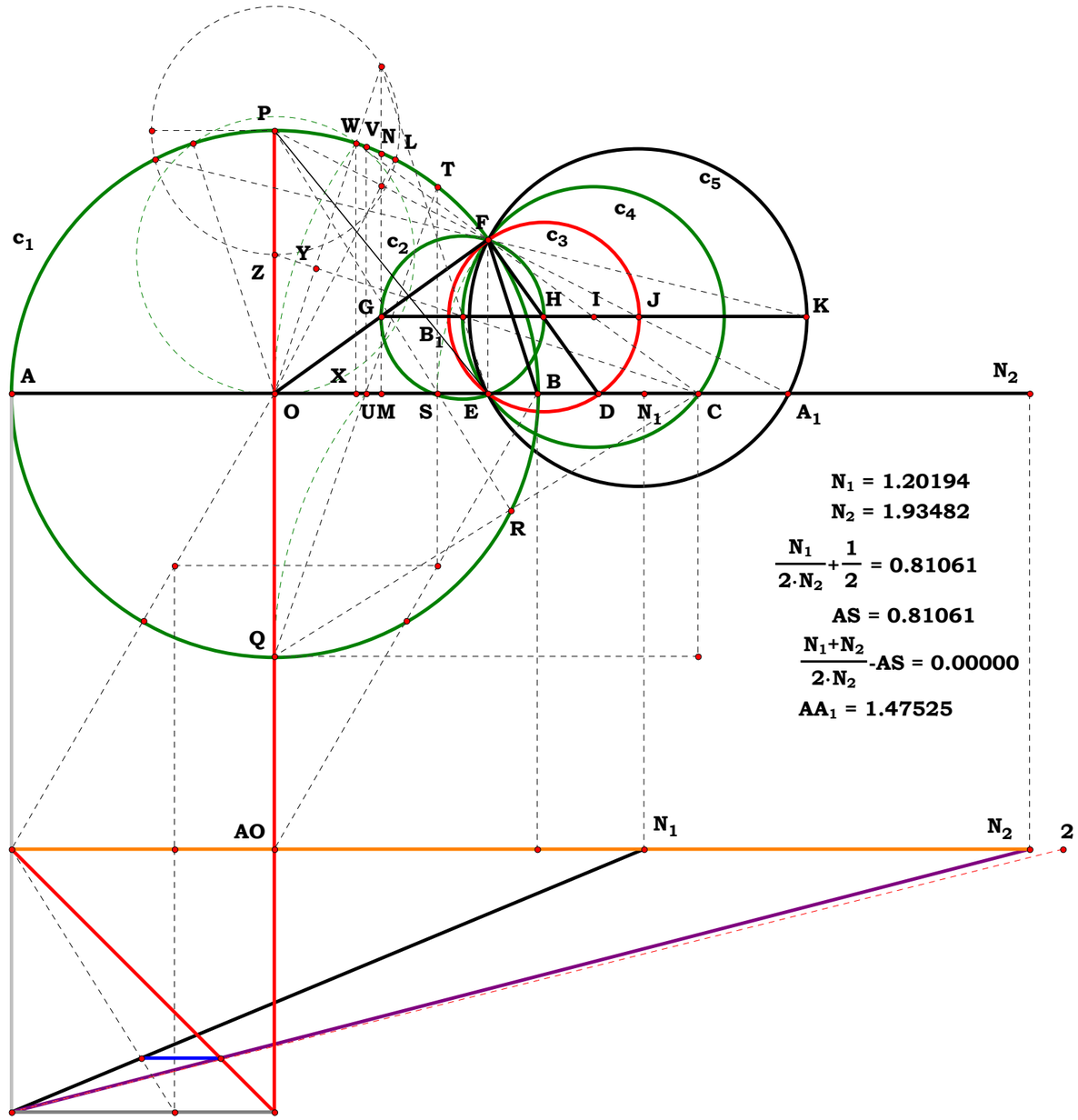
$GB_1 := \frac{GH}{2}$ $AA_1 := \frac{EO \cdot OP}{OP - EF} + AO$ $AA_1 = 1.475247$ $OA_1 := AA_1 - AO$

$PA_1 := \sqrt{OA_1^2 + OP^2}$ $EA_1 := OA_1 - EO$ $FA_1 := \frac{PA_1 \cdot EF}{OP}$ $FJ := \frac{FA_1}{2}$

$c_1 := AO$ $c_2 := GB_1$ $c_3 := FH$ $c_4 := CI$ $c_5 := FJ$

$c_1 = 0.5$ $c_2 = 0.153949$ $c_3 = 0.179725$

$c_4 = 0.247133$ $c_5 = 0.319863$



$c_1 = 0.5$ $c_2 = 0.153949$ $c_3 = 0.179725$

$c_4 = 0.247133$ $c_5 = 0.319863$

**Definitions.**

$$AS - \frac{N_1 + N_2}{2 \cdot N_2} = 0 \quad ST - \frac{\sqrt{(N_1 + N_2) \cdot (N_2 - N_1)}}{2 \cdot N_2} = 0 \quad AO - \frac{1}{2} = 0$$

$$OQ - \frac{1}{2} = 0 \quad OP - \frac{1}{2} = 0 \quad OS - \frac{N_1}{2 \cdot N_2} = 0 \quad PS - \frac{\sqrt{N_1^2 + N_2^2}}{2 \cdot N_2} = 0$$

$$CO - \frac{N_2}{2 \cdot N_1} = 0 \quad AC - \frac{N_1 + N_2}{2 \cdot N_1} = 0 \quad CW - \frac{N_2}{2 \cdot N_1} = 0$$

$$OW - \frac{1}{2} = 0 \quad OX - \frac{N_1}{4 \cdot N_2} = 0 \quad FW - \frac{N_1}{2 \cdot N_2} = 0$$

$$CF - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0 \quad CI - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$$

$$FO - \frac{1}{2} = 0 \quad EO - \frac{N_1 \cdot (3 \cdot N_2^2 - N_1^2)}{4 \cdot N_2^3} = 0 \quad BO - \frac{1}{2} = 0$$

$$BE - \frac{(N_1 + 2 \cdot N_2) \cdot (N_1 - N_2)^2}{4 \cdot N_2^3} = 0 \quad AE - \frac{(2 \cdot N_2 - N_1) \cdot (N_1 + N_2)^2}{4 \cdot N_2^3} = 0$$

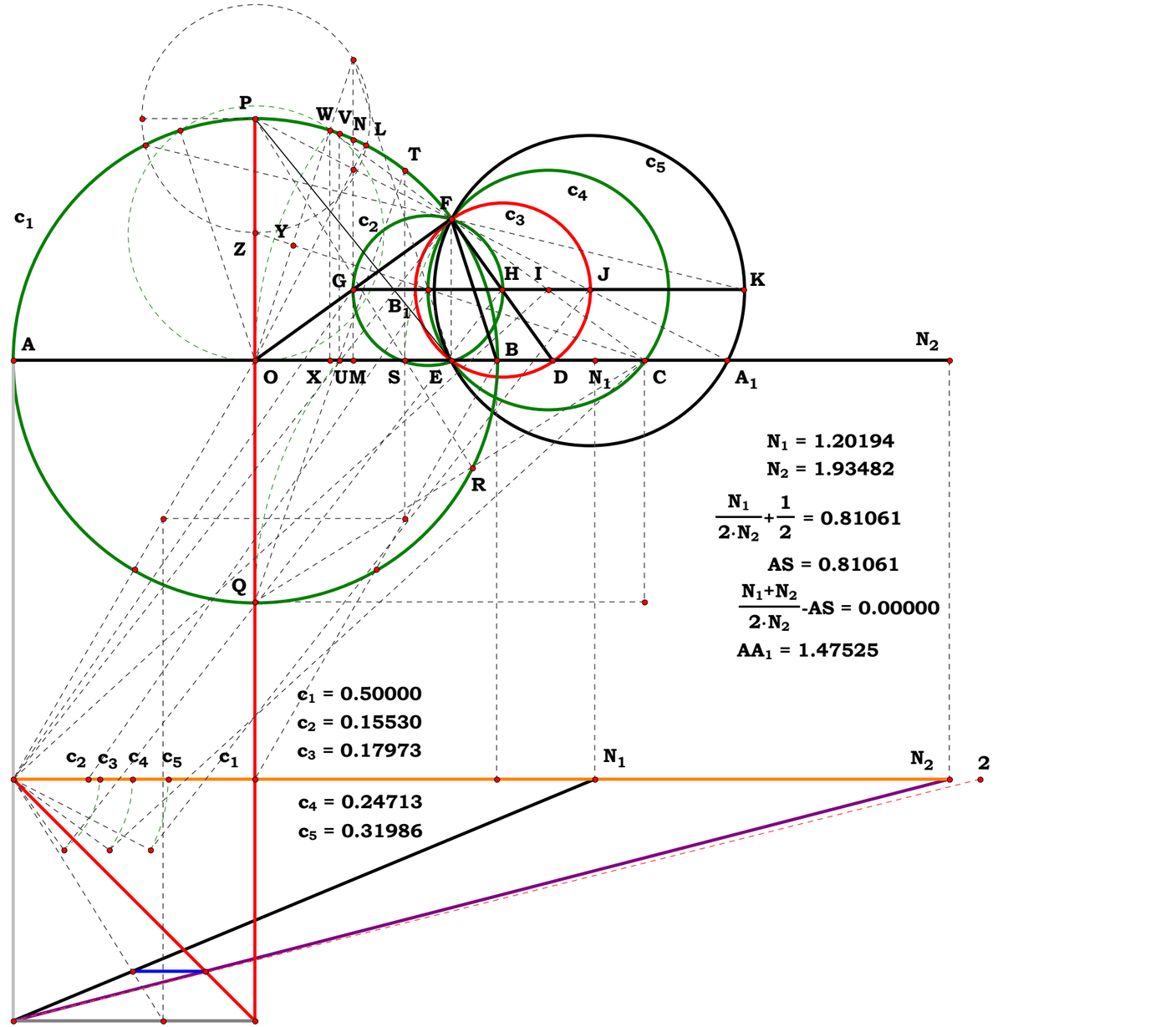
$$EF - \frac{(N_1 + N_2) \cdot (N_2 - N_1) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)}}{4 \cdot N_2^3} = 0$$

$$CE - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0$$

$$DF - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{2 \cdot N_1 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$

$$FH - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{4 \cdot N_1 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$

$$DE - \frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2 \cdot (N_1 - 2 \cdot N_2) \cdot (N_1 + 2 \cdot N_2)}{4 \cdot N_1 \cdot N_2^3 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$



$$DO - \left[\frac{N_2^3}{N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)}\right] = 0 \qquad GH - \frac{N_2^3}{2\cdot N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)} = 0$$

$$GB_1 - \frac{N_2^3}{4\cdot N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)} = 0$$

$$AA_1 - \frac{\left(N_1+N_2\right)\cdot\left[N_1\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - \left(N_1 - 2\cdot N_2\right)\cdot\left(N_1+N_2\right)\right]}{2\cdot\left(N_1^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} + 2\cdot N_2^3\right)} = 0$$

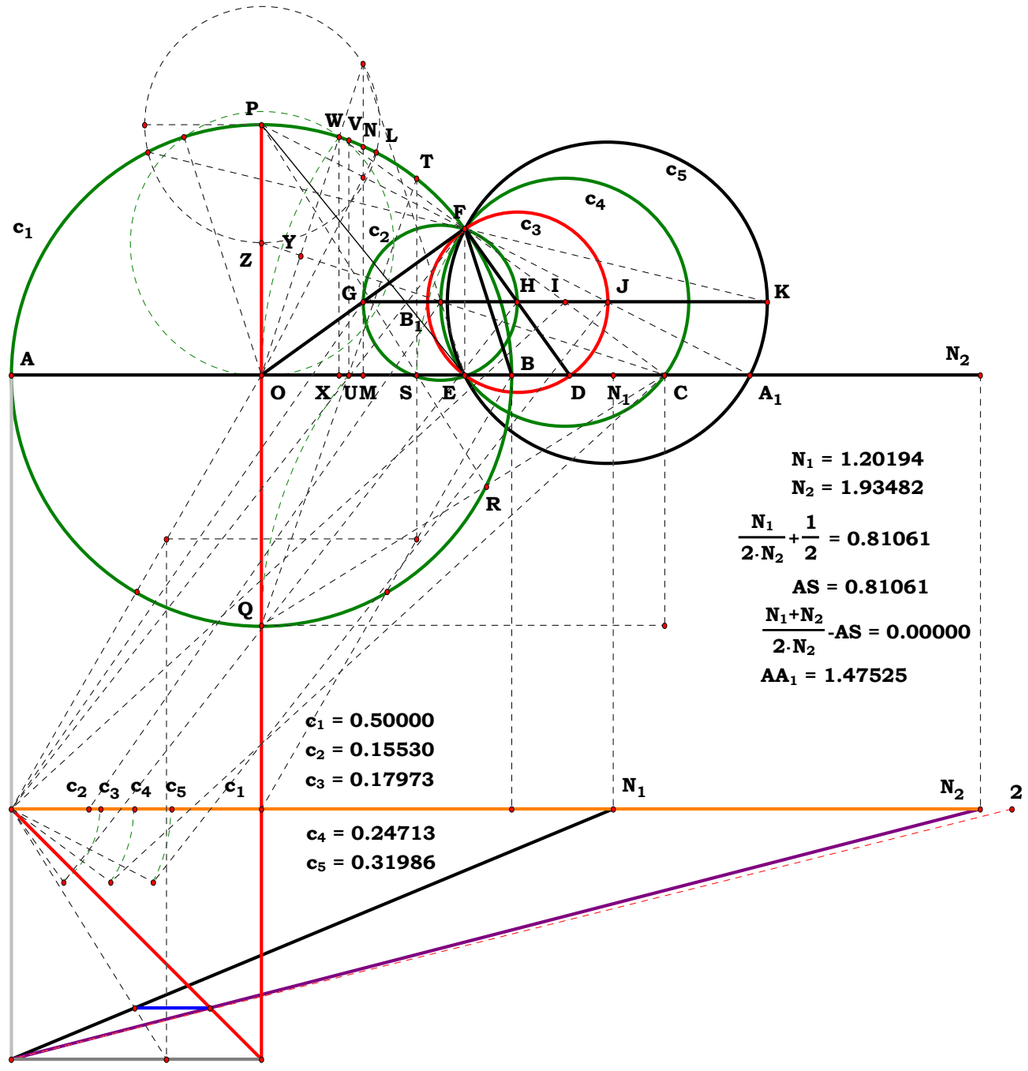
$$OA_1 - \frac{N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)}{2\cdot\left(N_1^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} + 2\cdot N_2^3\right)} = 0$$

$$PA_1 - \frac{\sqrt{N_2^3\cdot\left[\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right) + 2\cdot N_2^3\right]}}{\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot N_2^3\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right)}} = 0$$

$$EA_1 - \frac{\sqrt{4\cdot N_2^2 - N_1^2}\cdot N_1\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right)\cdot\left(N_1^2 - 3\cdot N_2^2\right)}{4\cdot N_2^3\cdot\left[\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right) + 2\cdot N_2^3\right]} = 0$$

$$FA_1 - \frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^3\cdot\left[2\cdot N_2^3 + \left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\right]}\cdot\sqrt{-\left(N_1 - 2\cdot N_2\right)\cdot\left(N_1 + 2\cdot N_2\right)\cdot\left(N_2 - N_1\right)}}{2\cdot N_2^3\cdot\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot N_2^3\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)}} = 0$$

$$FJ - \frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^3\cdot\left[2\cdot N_2^3 + \left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\right]}\cdot\sqrt{-\left(N_1 - 2\cdot N_2\right)\cdot\left(N_1 + 2\cdot N_2\right)\cdot\left(N_2 - N_1\right)}}{4\cdot N_2^3\cdot\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot N_2^3\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)}} = 0$$



Unit.

$BG := 1$

Given.

$N := 9 \quad AG := N$

051301

Descriptions.

$AB := AG - BG \quad BF := \frac{BG}{2} \quad AF := AB + BF$

$AN := AF \quad AK := AN \quad FK := BF$

$AE := \frac{AK^2 + AF^2 - FK^2}{2 \cdot AF} \quad AI := AE \quad IK := AK - AI$

$HI := IK \quad AH := AK - (HI + IK) \quad AC := \frac{AE \cdot AH}{AK}$

$BC := AC - AB \quad BE := AE - AB$

Definitions.

$N - 1 - AB = 0 \quad \frac{1}{2} - BF = 0 \quad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0$

$\frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0 \quad \frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0$

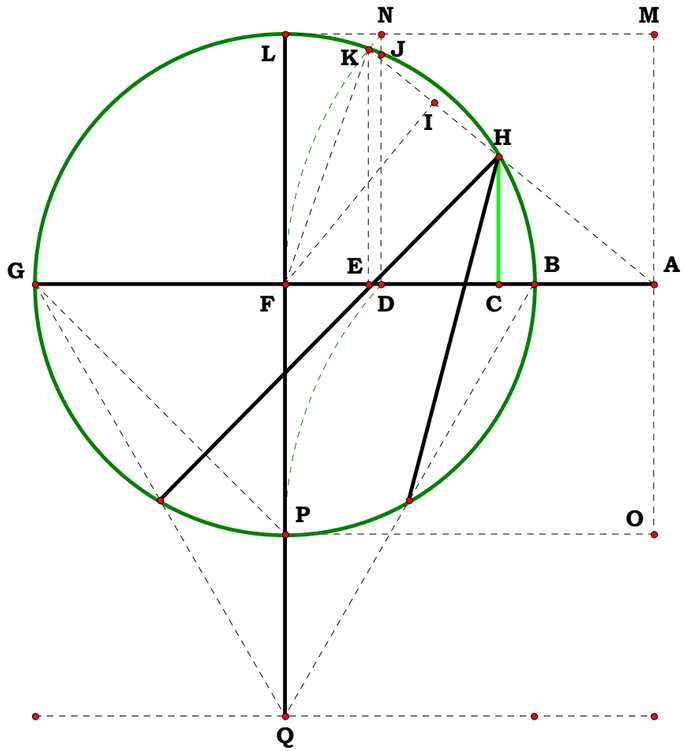
$2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \quad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \quad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

$\frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

## On Trisection

**For any given trisection what is the Algebraic names of BC and BE taking BG as unit?**



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

051401

Descriptions.

$BG := AG - AB \quad BO := \frac{BG}{2} \quad NO := BO \quad AO := AB + BO \quad AM := AO$

$AF := \frac{AM^2 + AO^2 - NO^2}{2 \cdot AO} \quad AK := AF \quad KM := AM - AF \quad JK := KM$

$AJ := AM - (JK + KM) \quad AD := \frac{AF \cdot AJ}{AM} \quad BD := AD - AB \quad DG := BG - BD$

$DJ := \sqrt{BD \cdot DG} \quad OT := DJ \quad OP := BO \quad DO := BO - BD \quad PT := OP - OT$

$OU := \frac{DO \cdot OP}{PT} \quad BU := OU - BO \quad AU := BU - AB \quad DU := AU + AD$

$PU := \sqrt{OU^2 + OP^2} \quad JU := \frac{PU \cdot DU}{OU} \quad JP := PU - JU \quad AP := \sqrt{AO^2 + OP^2}$

$AS := \frac{AO^2}{AP} \quad PS := AP - AS \quad HS := PS \quad AH := AP - (PS + HS) \quad CH := \frac{OP \cdot AH}{AP}$

$AC := \frac{AO \cdot AH}{AP} \quad BC := AC - AB \quad CU := BU + BC \quad HU := \sqrt{CU^2 + CH^2}$

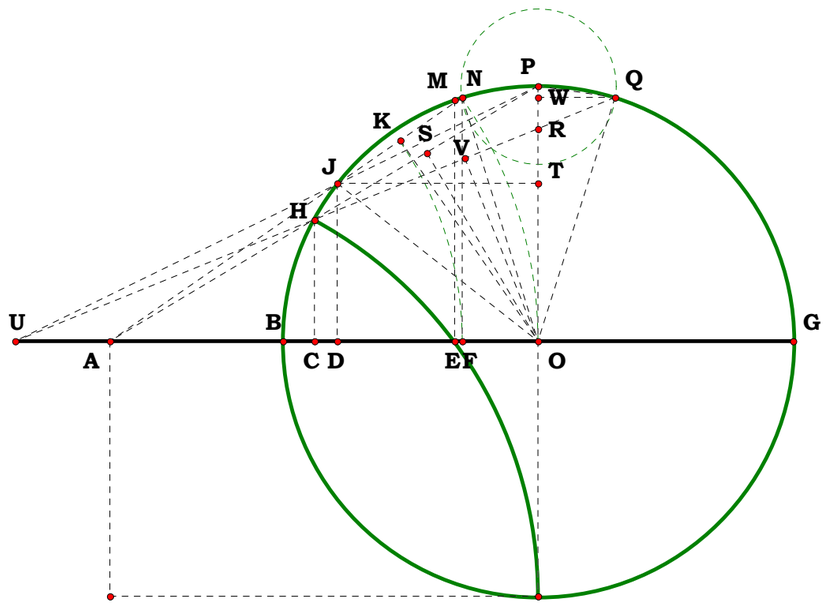
$UV := \frac{CU \cdot OU}{HU} \quad HV := UV - HU \quad QV := HV \quad QU := HU + (HV + QV)$

$OR := \frac{CH \cdot OU}{CU} \quad RU := \frac{HU \cdot OU}{CU} \quad QR := QU - RU \quad RW := \frac{CH \cdot QR}{HU}$

$QW := \frac{CU \cdot QR}{HU} \quad PW := OP - (OR + RW) \quad PQ := \sqrt{PW^2 + QW^2} \quad GU := BO + OU$

**For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?**

## On Trisection



**Definitions.**

$$BG-(N-1)=0 \quad BO-\frac{N-1}{2}=0 \quad NO-\frac{N-1}{2}=0 \quad AO-\frac{N+1}{2}=0 \quad AM-\frac{N+1}{2}=0$$

$$AF-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad AK-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad KM-\frac{(N-1)^2}{4\cdot(N+1)}=0 \quad JK-\frac{(N-1)^2}{4\cdot(N+1)}=0$$

$$AJ-\frac{2\cdot N}{N+1}=0 \quad AD-\frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3}=0 \quad BD-\frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3}=0 \quad DG-\frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3}=0$$

$$DJ-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OT-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OP-\frac{N-1}{2}=0$$

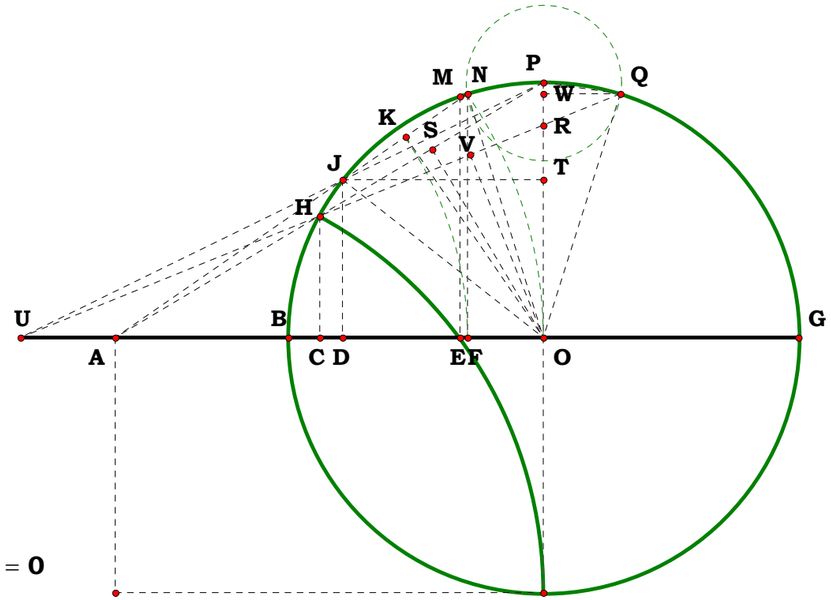
$$DO-\frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3}=0 \quad PT-\frac{(N-1)\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{2\cdot(N+1)^3}=0$$

$$OU-\frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}=0 \quad BU-\frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0$$

$$AU-\frac{N\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-6\cdot N-N^2+\sqrt{3\cdot N^2+10\cdot N+3}-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0 \quad DU-\frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$

$$PU-\frac{\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}}{\sqrt{2\cdot\left[N^6+6\cdot N^5+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)+1\right]-8\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}}=0$$

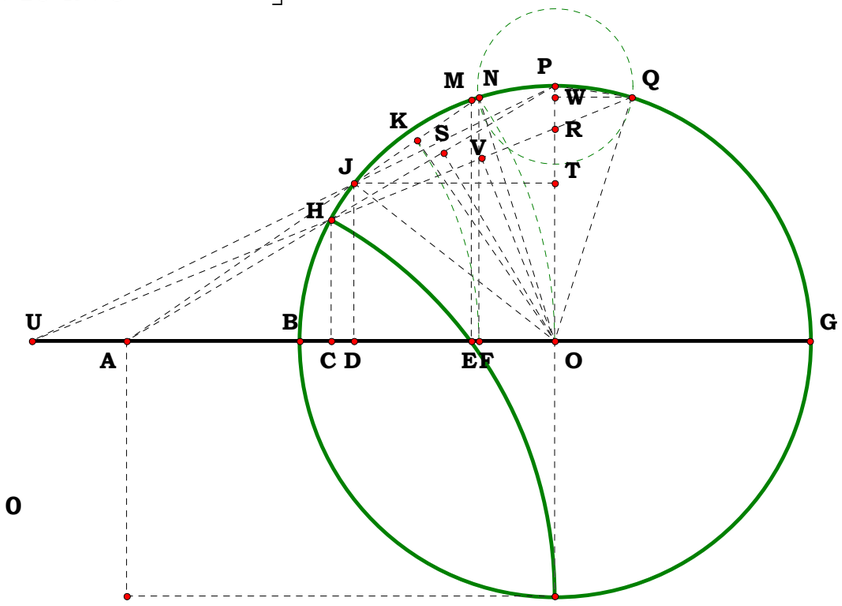
$$JU-\frac{\sqrt{2}\cdot N\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{(N+1)^3\cdot\sqrt{6\cdot N^5+N^6+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$



$$JP - \left[ \frac{\sqrt{2}\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}{2\cdot(N+1)^3\cdot\sqrt{\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}} \right] = 0$$

$$AP - \frac{\sqrt{N^2+1}}{\sqrt{2}} = 0 \qquad AS - \frac{\sqrt{2}\cdot(N+1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad PS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad HS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0$$

$$AH - \frac{\sqrt{2}\cdot N}{\sqrt{N^2+1}} = 0 \qquad CH - \frac{N\cdot(N-1)}{N^2+1} = 0 \qquad AC - \frac{N\cdot(N+1)}{N^2+1} = 0 \qquad BC - \frac{N-1}{N^2+1} = 0$$




$$CU - \frac{N\cdot(N-1)^2\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N+\sqrt{3\cdot N^2+10\cdot N+3}\right)}{\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$HU - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}}{\sqrt{\left(N^2+1\right)\cdot\left[N^6+\left(6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}} = 0$$

$$UV - \frac{(N-1)^3\cdot\sqrt{\left(N^2+1\right)\cdot\left[\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left[(N+1)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N\right]\cdot\left(N^2+4\cdot N+1\right)}{4\cdot\sqrt{(N+1)\cdot\left[(N+1)^3-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{10\cdot N+3\cdot N^2+3}+3\cdot N^2+N^3+1\right)^2} = 0$$

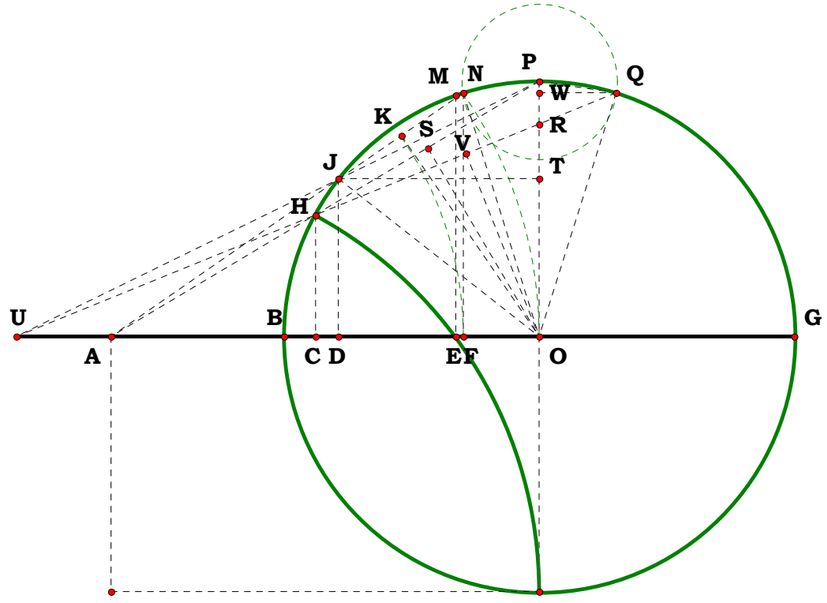
$$HV-(UV-HU)=0$$

$$QV-HV=0 \qquad QU-[HU+(HV+QV)]=0$$

$$OR-\frac{CH\cdot OU}{CU}=0 \qquad RU-\frac{HU\cdot OU}{CU}=0$$

$$QR-(QU-RU)=0 \qquad RW-\frac{CH\cdot QR}{HU}=0 \qquad QW-\frac{CU\cdot QR}{HU}=0$$

$$PW-[OP-(OR+RW)]=0$$




$$PQ-\frac{(N-1)\cdot\sqrt{14\cdot N+32\cdot N^2+14\cdot N^3+2\cdot N^4}-(N+1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+6\cdot N+1\right)+2}{2\cdot\sqrt{(N+1)\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}}=0$$

$$GU-\left[\frac{N\cdot(N-1)\cdot\left(3\cdot N+N^2-\sqrt{3\cdot N^2+10\cdot N+3}\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}\right]=0$$

Unit.

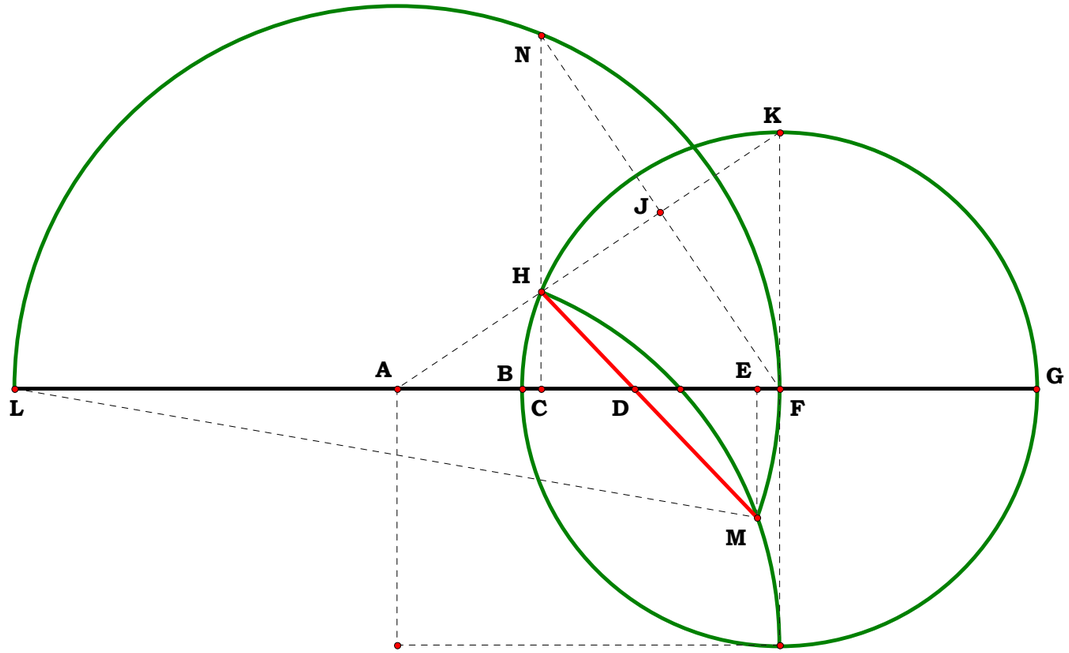
$AB := 1$

Given.

$N := 5 \quad AG := N$

## Segment DF And HM

**Given AB and AG, what is HM and DF?**




052201A

**Descriptions.**

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FK := BF \quad AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2} \quad AJ := \frac{AF^2}{AK} \quad JK := AK - AJ \quad HJ := JK$

$AH := AK - (JK + HJ) \quad AC := \frac{AF \cdot AH}{AK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot AF$

$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad AE := AF - EF \quad CE := AE - AC \quad CH := \frac{FK \cdot AH}{AK}$

$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$

**Definitions.**

$BG - (N - 1) = 0 \quad BF - \frac{N - 1}{2} = 0 \quad FK - \frac{N - 1}{2} = 0 \quad AF - \frac{N + 1}{2} = 0$

$AK - \frac{\sqrt{N^2 + 1}}{\sqrt{2}} = 0 \quad AJ - \frac{\sqrt{2} \cdot (N + 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad JK - \frac{\sqrt{2} \cdot (N - 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad HJ - \frac{\sqrt{2} \cdot (N - 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad AH - \frac{\sqrt{2} \cdot N}{\sqrt{N^2 + 1}} = 0 \quad AC - \frac{N \cdot (N + 1)}{N^2 + 1} = 0$

$EM - \frac{N - 1}{4} = 0 \quad FL - (N + 1) \quad EF - \frac{2 \cdot N - \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 2}{4} = 0 \quad AE - \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{4} = 0 \quad CE - \frac{\left(N^2 + 1\right) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - 4 \cdot N \cdot (N + 1)}{4 \cdot \left(N^2 + 1\right)} = 0$

$CH - \frac{N \cdot (N - 1)}{N^2 + 1} = 0 \quad HM - \frac{\sqrt{(N + 1) \cdot \left(3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1\right)}}{2 \cdot \sqrt{N^2 + 1}} = 0 \quad DE - \frac{\left(N^2 + 1\right) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \left(4 \cdot N^2 + 4 \cdot N\right)}{4 \cdot \left(N^2 + 4 \cdot N + 1\right)} = 0$

$DF - \frac{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1}{2 \cdot \left(N^2 + 4 \cdot N + 1\right)} = 0$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

052201B

Descriptions.

$$BG := AG - AB \quad BF := \frac{AB}{2} \quad FK := BF \quad GF := AG - BF$$

$$GK := \sqrt{GF^2 + FK^2} \quad GJ := \frac{GF^2}{GK} \quad JK := GK - GJ \quad HJ := JK$$

$$GH := GK - (JK + HJ) \quad CG := \frac{GF \cdot GH}{GK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot GF$$

$$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad GE := GF - EF \quad CE := GE - CG \quad CH := \frac{FK \cdot GH}{GK}$$

$$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$$

Definitions.

$$BG - (N - 1) = 0 \quad BF - \frac{1}{2} = 0 \quad FK - \frac{1}{2} = 0 \quad GF - \frac{2 \cdot N - 1}{2} = 0$$

$$GK - \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{\sqrt{2}} = 0 \quad GJ - \frac{\sqrt{2} \cdot (2 \cdot N - 1)^2}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad JK - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0$$

$$HJ - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad GH - \frac{\sqrt{2} \cdot N \cdot (N - 1)}{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad CG - \frac{N \cdot (N - 1) \cdot (2 \cdot N - 1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0$$

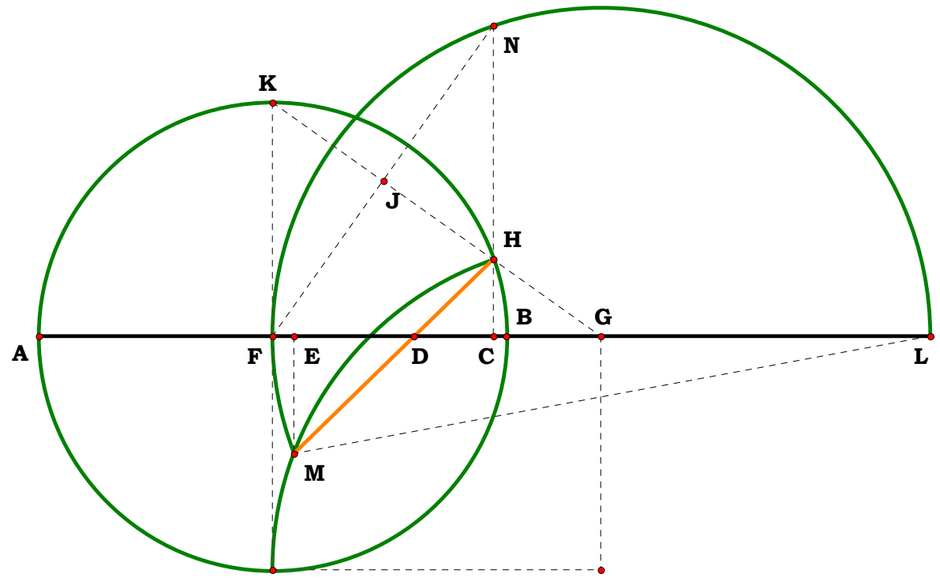
$$EM - \frac{1}{4} = 0 \quad FL - (2 \cdot N - 1) = 0 \quad EF - \frac{4 \cdot N - \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 2}{4} = 0 \quad GE - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0 \quad CE - \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 4 \cdot N \cdot (2 \cdot N - 1) \cdot (N - 1)}{4 \cdot \left(2 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$CH - \frac{N \cdot (N - 1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0 \quad HM - \frac{\sqrt{(2 \cdot N - 1) \cdot \left[\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3\right]}}{2 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad DE - \frac{4 \cdot N \cdot (2 \cdot N - 1) \cdot (1 - N) - \left(2 \cdot N - 2 \cdot N^2 - 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$$

$$DF - \frac{\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3}{2 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$$

## Segment DF And HM

**Given AB and AG, what is HM and DF?**



Unit.

$AB := 1$

## Point of Intersection

Given.

$N := 5 \qquad AH := N$

052701A

Descriptions.

$BH := AH - AB \qquad BG := \frac{BH}{2} \qquad GN := BG \qquad GM := BG$

$GP := BG \qquad AG := AB + BG \qquad AM := AG \qquad FG := \frac{GM^2 + AG^2 - AM^2}{2 \cdot AG}$

$AF := AG - FG \qquad AL := AF \qquad LM := AM - AL \qquad KL := LM$

$AK := AM - (LM + KL) \qquad AC := \frac{AF \cdot AK}{AM} \qquad CK := \sqrt{AK^2 - AC^2}$

$CG := AG - AC \qquad DG := \frac{CG \cdot GP}{(GP + CK)}$

Definitions.

$BH - (N - 1) = 0 \qquad BG - \frac{(N-1)}{2} = 0 \qquad GN - \frac{(N-1)}{2} = 0 \qquad GM - \frac{(N-1)}{2} = 0$

$GP - \frac{(N-1)}{2} = 0 \qquad AG - \frac{N+1}{2} = 0 \qquad AM - \frac{N+1}{2} = 0 \qquad FG - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

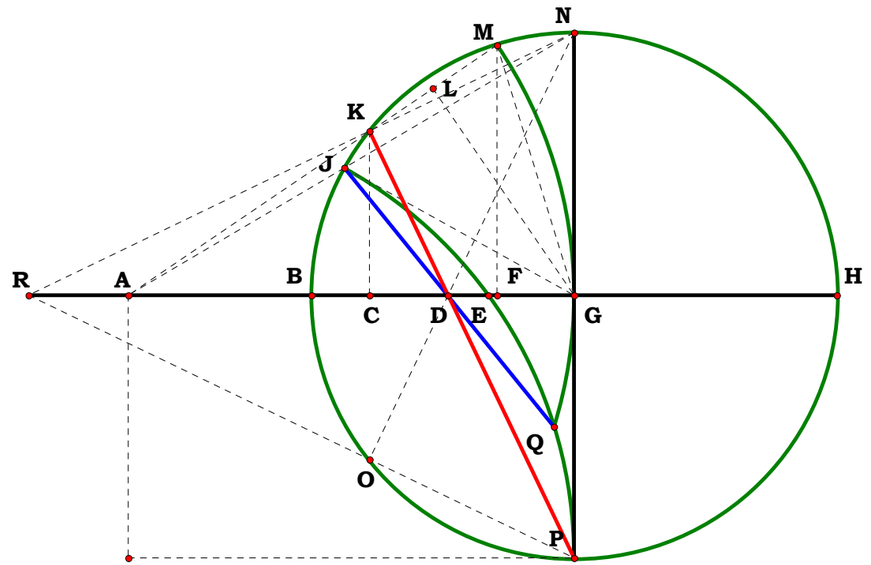
$AF - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad AL - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad LM - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

$KL - \frac{(N-1)^2}{4 \cdot (N+1)} = 0 \qquad AK - \frac{2 \cdot N}{N+1} = 0 \qquad AC - \frac{N \cdot \left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} = 0$

$CK - \frac{N \cdot (N-1) \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}}{(N+1)^3} = 0 \qquad CG - \frac{\left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{2 \cdot (N+1)^3} = 0$

$DG - \frac{(N-1)^2 \cdot \left(N^2 + 4 \cdot N + 1\right)}{2 \cdot \left[(N+1)^3 + 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}\right]} = 0$

Do KP and JQ intersect at D?




This is the other way to look at the figure I work on for trisection.

$m\angle ABC = 9.59307°$

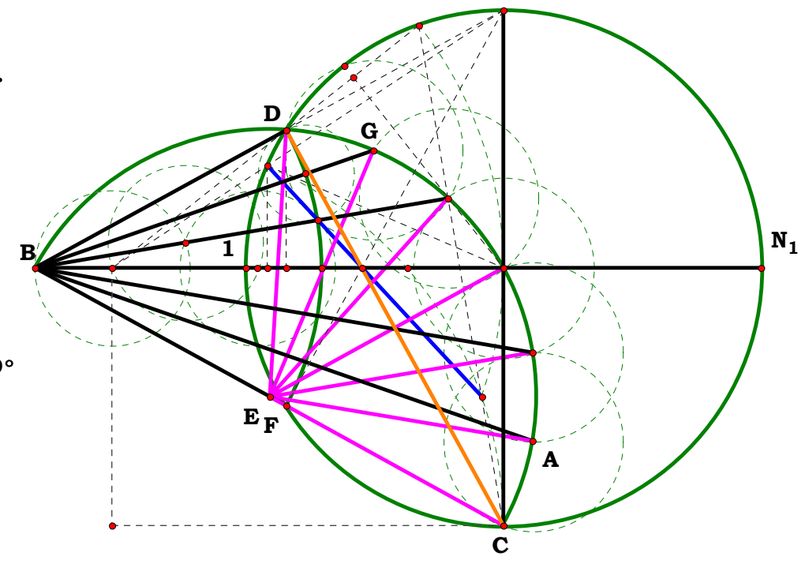
$m\angle DBF = 57.55844°$

$\frac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\frac{m\angle DEC}{m\angle DEG} = 6.00000$



**Unit.**

$AB := 1$

**Given.**

$N := 1.26810 \quad AH := N$

**052701A**

**Descriptions.**

$BH := AH - AB \quad BG := \frac{AB}{2} \quad GN := BG \quad GM := BG$

$GP := BG \quad GH := AH - BG \quad HM := GH \quad FG := \frac{GM^2 + GH^2 - HM^2}{2 \cdot GH}$

$FH := GH - FG \quad HL := FH \quad LM := HM - HL \quad KL := LM$

$HK := HM - (LM + KL) \quad CH := \frac{FH \cdot HK}{HM} \quad CK := \sqrt{HK^2 - CH^2}$

$CG := GH - CH \quad DG := \frac{CG \cdot GP}{(GP + CK)}$

**Definitions.**

$BH - (N - 1) = 0 \quad BG - \frac{1}{2} = 0 \quad GN - \frac{1}{2} = 0 \quad GM - \frac{1}{2} = 0$

$GP - \frac{1}{2} = 0 \quad GH - \frac{2 \cdot N - 1}{2} = 0 \quad HM - \frac{2 \cdot N - 1}{2} = 0$

$FG - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad FH - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HL - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$

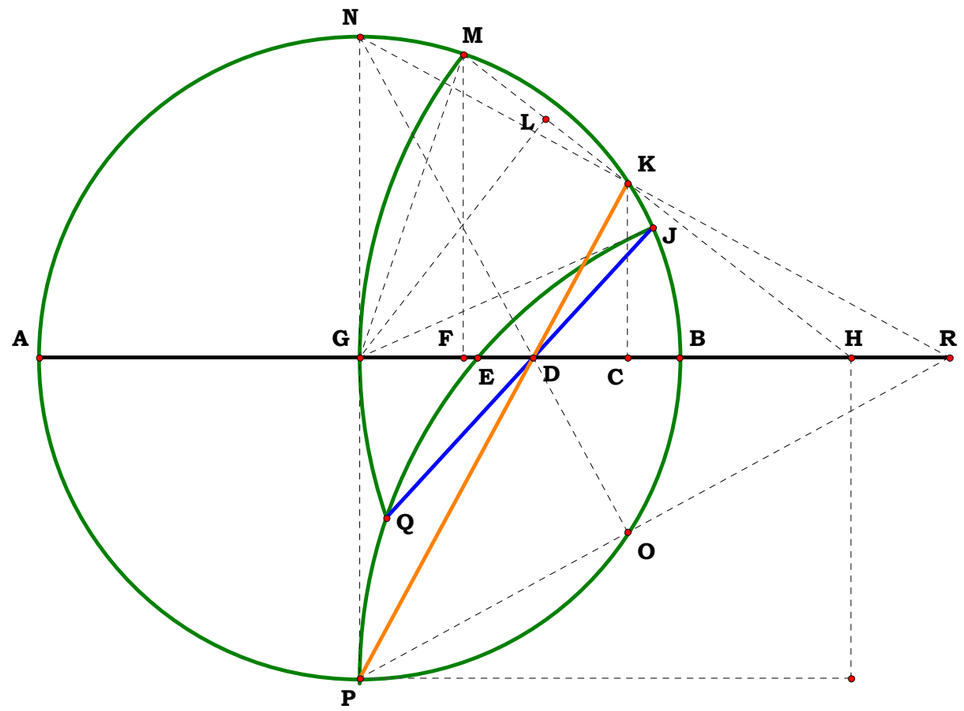
$LM - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad KL - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HK - \frac{2 \cdot N \cdot (N - 1)}{2 \cdot N - 1} = 0$

$CH - \frac{N \cdot (N - 1) \cdot \left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} = 0 \quad CK - \frac{N \cdot (N - 1) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{(2 \cdot N - 1)^3} = 0$

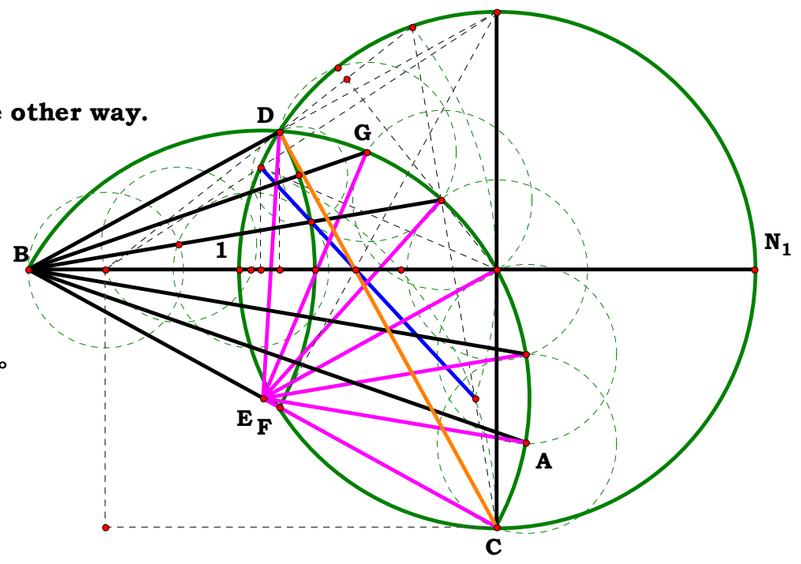
$CG - \frac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot (2 \cdot N - 1)^3} = 0 \quad DG - \frac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot \left[\left(2 \cdot N^2 - 2 \cdot N\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} + (2 \cdot N - 1)^3\right]} = 0$

## Point of Intersection

**Do KP and JQ intersect at D?**




**No, I am not drawing this the other way.**




$m\angle ABC = 9.59307°$

$m\angle DBF = 57.55844°$

$\frac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\frac{m\angle DEC}{m\angle DEG} = 6.00000$

**Unit.**

$AB := 1$

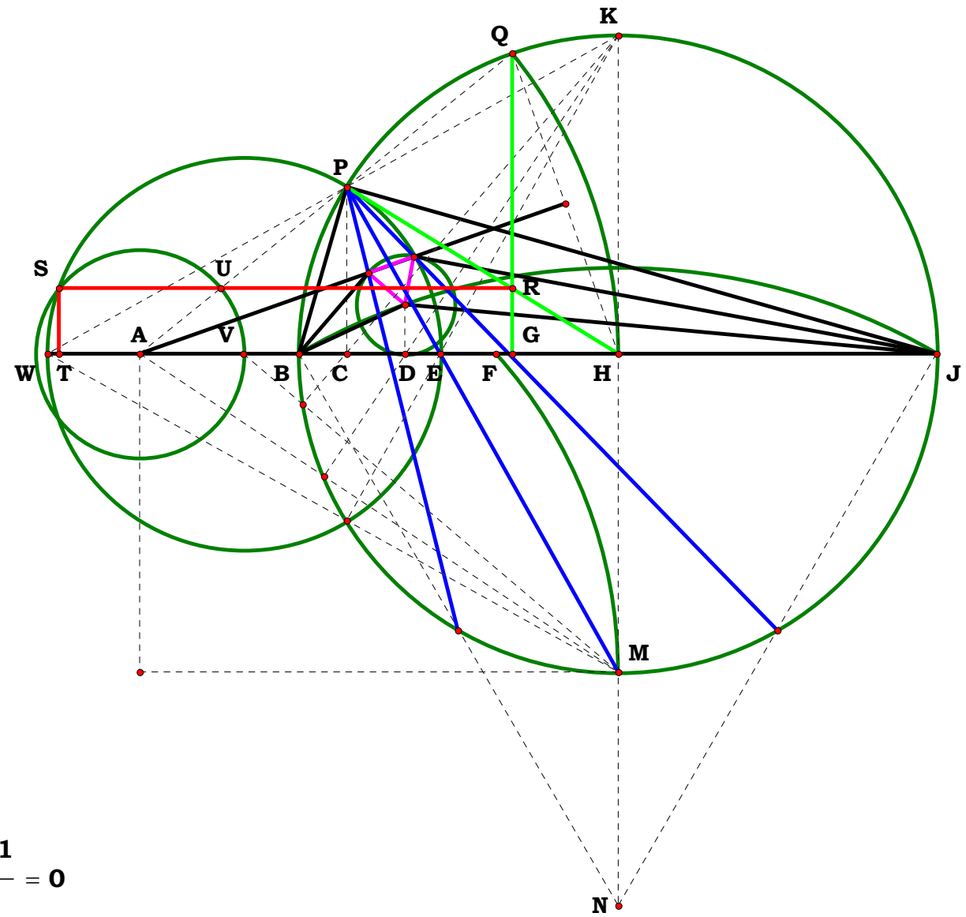
**Given.**

$N_1 := 5 \qquad AJ := N_1$

**0530011A**

## What is AV and ST?

**Given any angle BHP, the unit which defines it will lay between W and V.**




**Descriptions.**

$BJ := AJ - AB \qquad BH := \frac{BJ}{2} \qquad AH := AB + BH \qquad AQ := AH$

$PQ := \frac{BH^2}{AQ} \qquad HM := BH \qquad AP := AQ - PQ \qquad AC := \frac{AP^2 + AH^2 - BH^2}{2 \cdot AH}$

$CP := \sqrt{AP^2 - AC^2} \qquad HK := BH \qquad CH := AH - AC \qquad CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH} \qquad AV := AH - HV \qquad ST := \frac{CP \cdot AV}{AP}$

**Definitions.**

$AJ - N_1 = 0 \qquad BJ - (N_1 - 1) = 0 \qquad BH - \frac{N_1 - 1}{2} = 0 \qquad AH - \frac{N_1 + 1}{2} = 0$

$AQ - \frac{N_1 + 1}{2} = 0 \qquad PQ - \frac{(N_1 - 1)^2}{2 \cdot (N_1 + 1)} = 0 \qquad HM - \frac{N_1 - 1}{2} = 0 \qquad AP - \frac{2 \cdot N_1}{N_1 + 1} = 0$

$AC - \frac{N_1 \cdot (N_1^2 + 6 \cdot N_1 + 1)}{(N_1 + 1)^3} = 0 \qquad CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{(N_1 + 1)^3} = 0 \qquad HK - \frac{N_1 - 1}{2} = 0$

$CH - \frac{(N_1^2 + 4 \cdot N_1 + 1) \cdot (N_1 - 1)^2}{2 \cdot (N_1 + 1)^3} = 0 \qquad CK - \frac{(N_1 - 1) \cdot \sqrt{N_1^6 + 6 \cdot N_1^5 + 9 \cdot N_1^4 + 9 \cdot N_1^2 + 6 \cdot N_1 + 1}}{\sqrt{2} \cdot (N_1 + 1)^3} = 0$

$HV - \frac{(N_1 + 1)^3}{2 \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \qquad AV - \frac{N_1 \cdot (N_1 + 1)}{N_1^2 + 4 \cdot N_1 + 1} = 0 \qquad ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{2 \cdot (N_1 + 1) \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \qquad AV - \frac{(AB + BJ) \cdot (AB + BJ + AB)}{AB^2 + 2 \cdot AB \cdot BJ + 4 \cdot AB + BJ^2 + 4 \cdot BJ + AB} = 0$

**Unit.**

$AB := 1$

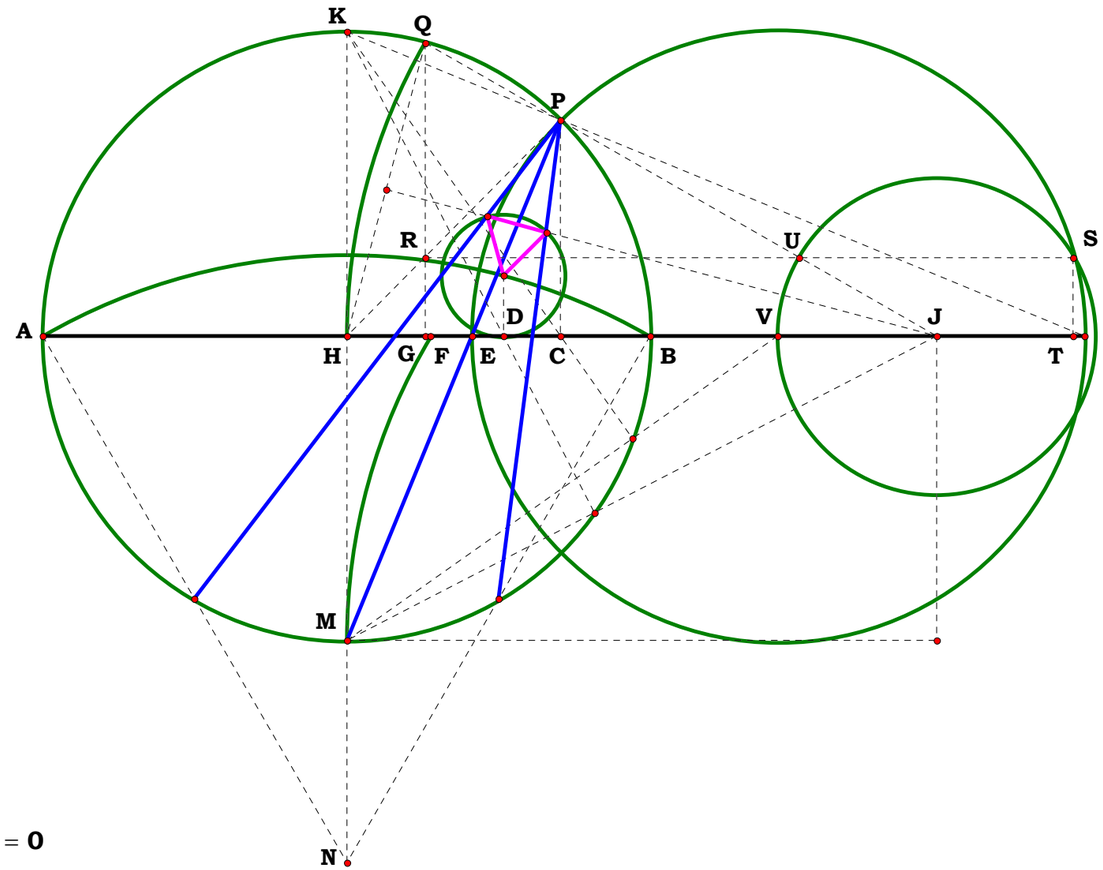
**Given.**

$N_1 := 1.47027$ $AJ := N_1$

**0530011B**

**What is AV and ST?**

**Given any angle BHP, the unit which defines it will lay between W and V.**




**Descriptions.**

$BJ := AJ - AB$ $BH := \frac{AB}{2}$ $JH := AJ - BH$ $JQ := JH$

$PQ := \frac{BH^2}{JQ}$ $HM := BH$ $JP := JQ - PQ$ $CJ := \frac{JP^2 + JH^2 - BH^2}{2 \cdot JH}$

$CP := \sqrt{JP^2 - CJ^2}$ $HK := BH$ $CH := JH - CJ$ $CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH}$ $JV := JH - HV$ $ST := \frac{CP \cdot JV}{JP}$

**Definitions.**

$AJ - N_1 = 0$ $BJ - (N_1 - 1) = 0$ $BH - \frac{1}{2} = 0$ $JH - \frac{2 \cdot N_1 - 1}{2}$

$JQ - \frac{2 \cdot N_1 - 1}{2} = 0$ $PQ - \frac{1}{2 \cdot (2 \cdot N_1 - 1)} = 0$ $HM - \frac{1}{2} = 0$ $JP - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$

$CJ - \frac{N_1 \cdot (N_1 - 1) \cdot (8 \cdot N_1^2 - 8 \cdot N_1 + 1)}{(2 \cdot N_1 - 1)^3} = 0$ $CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{(2 \cdot N_1 - 1)^3} = 0$

$HK - \frac{1}{2} = 0$ $CH - \frac{6 \cdot N_1^2 - 6 \cdot N_1 + 1}{2 \cdot (2 \cdot N_1 - 1)^3} = 0$ $CK - \frac{\sqrt{2 \cdot N_1 \cdot (N_1 - 1) \cdot [16 \cdot N_1^3 \cdot (N_1 - 2) + 37 \cdot N_1^2 - 21 \cdot N_1 + 6] + 1}}{\sqrt{2} \cdot (2 \cdot N_1 - 1)^3} = 0$

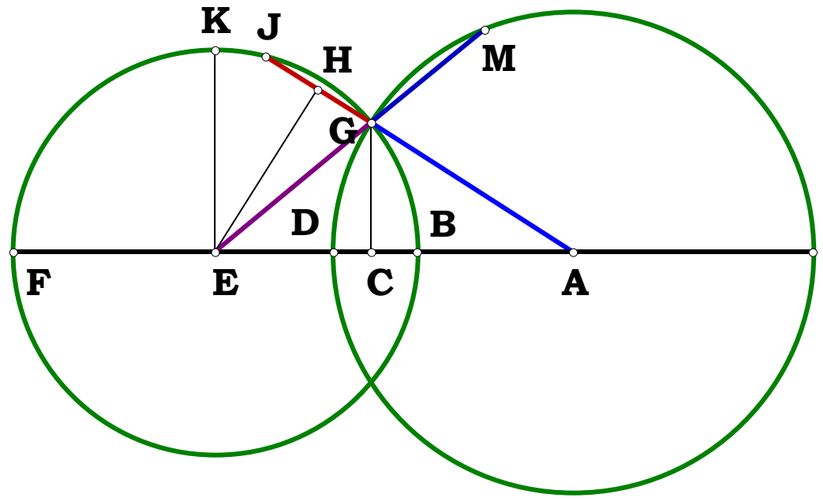
$HV - \frac{(2 \cdot N_1 - 1)^3}{2 \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0$ $JV - \frac{N_1 \cdot (2 \cdot N_1 - 1) \cdot (N_1 - 1)}{6 \cdot N_1^2 - 6 \cdot N_1 + 1} = 0$ $ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{2 \cdot (2 \cdot N_1 - 1) \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0$

Unit.
Given.
Descriptions.
Definitions.

060101

## A Small Extrapolation

**Given AE, AG, and EG, what is the Algebraic name of the segment GJ?**




$S_1 := 6.00604$ $S_2 := 4.02167$ $S_3 := 3.38667$

$AE := S_1$ $AG := S_2$ $EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2AE}$$

$$AH := \frac{AC \cdot AE}{AG}$$ $GH := AH - AG$

$HJ := GH$ $GJ := GH + HJ$

**Some Algebraic Names:**

$$\frac{S_2^2 + S_1^2 - S_3^2}{2S_1} - AC = 0$$

$$\frac{S_1^2 + S_2^2 - S_3^2}{2S_2} - AH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{2S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0$$

**Unit.**

$AB := 1$

**Given.**

$N := 5.727 \quad AG := N$

**060201A**

**Descriptions.**

$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

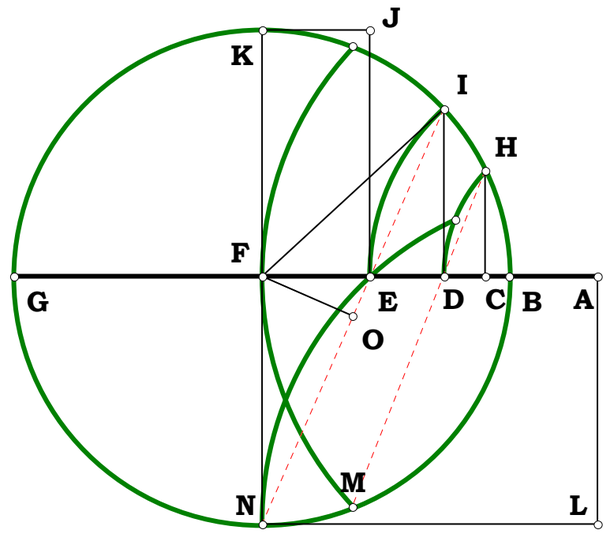
$FN := BF \qquad EF := BF - BE \qquad EN := \sqrt{FN^2 + EF^2}$

$NO := \frac{FN^2}{EN} \qquad NI := 2 \cdot NO \qquad EI := NI - EN$

$DE := \frac{EF \cdot EI}{EN}$

**Units From Both Sides**

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




**Definitions.**

$BG - (N - 1) = 0 \qquad BF - \frac{N - 1}{2} = 0 \qquad AF - \frac{N + 1}{2} = 0$

$AE - \sqrt{N} = 0 \qquad BE - \left(\sqrt{N} - 1\right) = 0 \qquad FN - \frac{N - 1}{2} = 0$

$EF - \frac{\left(\sqrt{N} - 1\right)^2}{2} = 0 \qquad EN - \frac{\sqrt{2} \cdot \sqrt{N + 1} \cdot \left(\sqrt{N} - 1\right)}{2} = 0 \qquad NO - \frac{\sqrt{2} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)^2}{4 \cdot \sqrt{N + 1}} = 0$

$NI - \frac{\sqrt{2} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)^2}{2 \cdot \sqrt{N + 1}} = 0 \qquad EI - \frac{\sqrt{2} \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N + 1}} = 0 \qquad DE - \frac{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2}{N + 1} = 0$

060201B

Unit.

$BG := 1$

Given.

$N := 2$

**Descriptions.**

$$BF := \frac{BG}{2} \qquad BE := \frac{BF}{N} \qquad EF := BF - BE$$

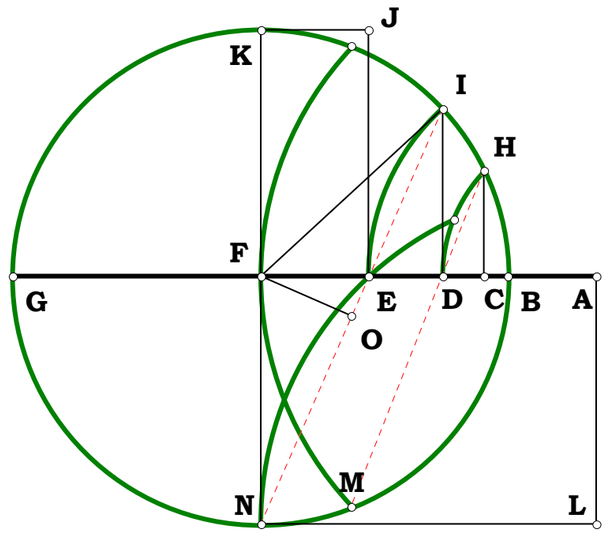
$$FN := BF \qquad EN := \sqrt{EF^2 + FN^2} \qquad NP := \frac{EN}{2}$$

$$LN := \frac{EN \cdot NP}{EF} \qquad AF := LN \qquad AB := AF - BF$$

$AB = 0.125$

**Units From Both Sides**

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




**Definitions.**

$$\frac{BG}{2} - BF = 0 \qquad \frac{BG}{(2 \cdot N)} - BE = 0 \qquad \frac{BG}{2} \cdot \frac{(N-1)}{N} - EF = 0$$

$$\frac{BG}{2} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - EN = 0 \qquad \frac{BG}{4} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - NP = 0$$

$$\frac{BG}{4} \cdot \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right)}{[N \cdot (N-1)]} - LN = 0 \qquad \frac{BG}{4 \cdot N \cdot (N-1)} - AB = 0$$

060201C

Unit.

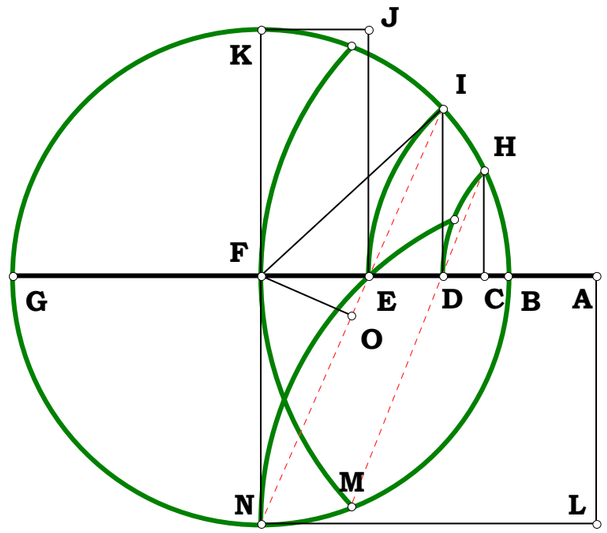
$BG := 1$

Given.

$N := 2$

**Units From Both Sides**

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




**Descriptions.**

$BF := \frac{BG}{2}$ $BD := \frac{BF}{N}$ $DG := BG - BD$

$DI := \sqrt{BD \cdot DG}$ $DF := BF - BD$ $FN := BF$

$EF := \frac{DF \cdot FN}{FN + DI}$ $EN := \sqrt{EF^2 + FN^2}$ $NP := \frac{EN}{2}$

$LN := \frac{EN \cdot NP}{EF}$ $AF := LN$ $AB := AF - BF$

$AB = 0.5$

**Definitions.**

$BD - \frac{1}{2} \cdot \frac{BG}{N} = 0$ $DG - \frac{1}{2} \cdot BG \cdot \frac{(2 \cdot N - 1)}{N} = 0$ $DI - \frac{1}{(2 \cdot N)} \cdot BG \cdot \sqrt{2 \cdot N - 1} = 0$

$DF - \frac{1}{2} \cdot BG \cdot \frac{(N - 1)}{N} = 0$ $EF - \frac{1}{2} \cdot BG \cdot \frac{(N - 1)}{\left(N + \sqrt{2 \cdot N - 1}\right)} = 0$ $NP - \frac{1}{4} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$

$EN - \frac{1}{2} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$ $LN - \frac{1}{2} \cdot BG \cdot \frac{N}{(N - 1)} = 0$ $AB - \frac{1}{2} \cdot \frac{BG}{(N - 1)} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$BD := \frac{BE}{2}$ $BC := \frac{BE}{N}$ $CE := BE - BC$

$CG := \sqrt{BC \cdot CE}$ $CD := BD - BC$ $AC := \frac{CG^2}{CD}$

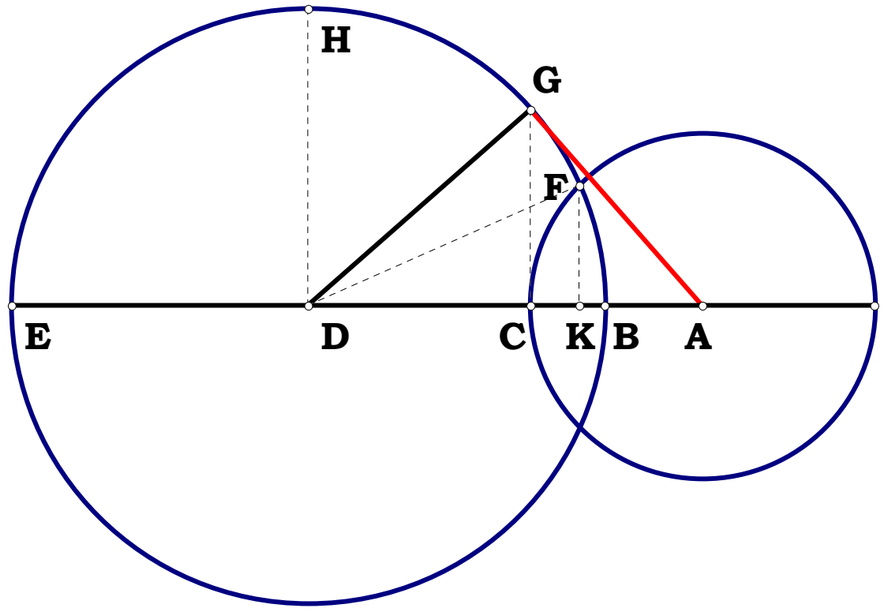
$AF := AC$ $AD := AC + CD$ $DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2AD}$ $BK := BD - DK$

$CK := BC - BK$

## Isolating A Problem

**If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?**




Definitions.

$\frac{N-1}{N} - CE = 0$ $\frac{\sqrt{N-1}}{N} - CG = 0$ $\frac{N-2}{2 \cdot N} - CD = 0$

$\frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0$ $\frac{N}{2 \cdot (N-2)} - AD = 0$ $\frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$\frac{3 \cdot N - 2}{N^3} - BK = 0$ $\frac{(N-1) \cdot (N-2)}{N^3} - CK = 0$ $\frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$BD := \frac{BE}{2}$ $BC := \frac{BD}{N}$ $AC := \frac{1}{2} \cdot \frac{BE}{N} \cdot \frac{(2 \cdot N - 1)}{(N - 1)}$

$CM := \frac{1}{4} \cdot BE \cdot (2 \cdot N - 1) \cdot \frac{(N - 1)}{N^3}$ $AM := AC - CM$

$AF := AC$ $FM := \sqrt{AF^2 - AM^2}$ $AD := \frac{1}{2} \cdot BE \cdot \frac{N}{(N - 1)}$

$FK := \frac{AD^2 - AC^2 - BD^2}{AC}$ $FK = 0.375$

Definitions.

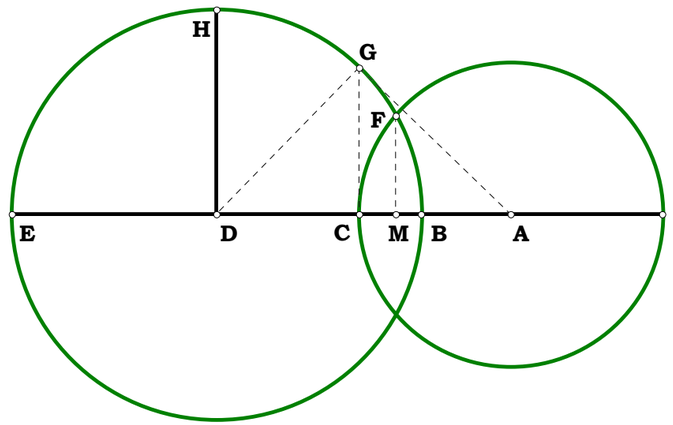
$BD - \frac{1}{2} = 0$ $BC - \frac{1}{2 \cdot N} = 0$ $AC - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0$

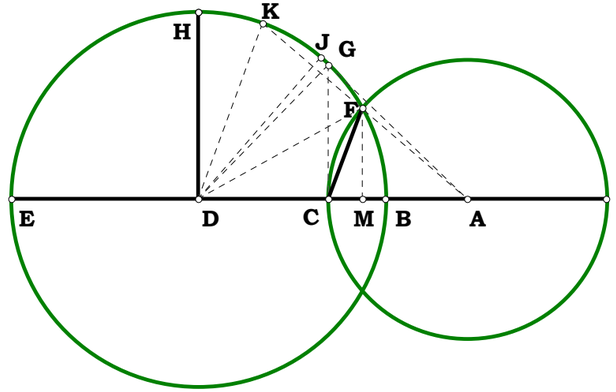
$CM - \frac{(N - 1) \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0$ $AM - \frac{(2 \cdot N - 1) \cdot \left(N^2 + 2 \cdot N - 1\right)}{4 \cdot N^3 \cdot (N - 1)} = 0$

$AF - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0$ $FM - \frac{\sqrt{(N + 1) \cdot (3 \cdot N - 1)} \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0$

$AD - \frac{N}{2 \cdot (N - 1)} = 0$ $FK - \frac{N - 1}{2 \cdot N} = 0$

**For any point C on BD, FCG is 1/3 of the angle FDH, will the Algebraic Name for DM remain constant if one 'steps back to it' from D?**



061001

Given.

$N_1 := 5$

$N_2 := 3$

**Descriptions.**

**For any $N_1 \cdot N_2$ what is DG?**

$AF := N_1 \cdot N_2 \quad BF := AF - N_2 \quad BE := \frac{BF}{2}$

$EK := BE \quad AE := N_2 + BE \quad DE := \frac{EK^2}{AE}$

$EF := BE \quad FM := BF \quad EM := \sqrt{FM^2 - EF^2}$

$GM := FM \quad GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2}$

$EQ := MQ - EM \quad DG := EQ$

**Definitions.**

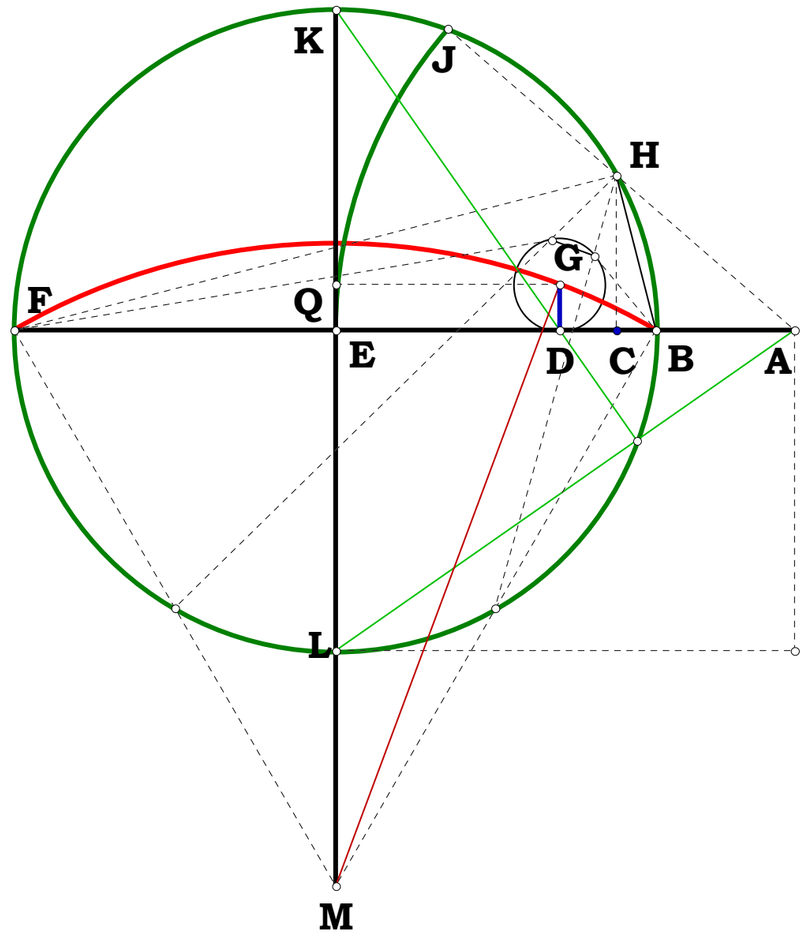
$N_2 \cdot (N_1 - 1) - BF = 0 \qquad \frac{N_2 \cdot (N_1 - 1)}{2} - BE = 0$

$\frac{1}{2} \cdot N_2 \cdot (N_1 + 1) - AE = 0 \qquad \frac{1}{2} \cdot N_2 \cdot \frac{(N_1 - 1)^2}{(N_1 + 1)} - DE = 0$

$\frac{1}{2} \cdot \sqrt{3} \cdot N_2 \cdot (N_1 - 1) - EM = 0 \qquad \frac{N_2 \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} \cdot (N_1 - 1)\right]}{2 \cdot (N_1 + 1)} - MQ = 0$

$\frac{N_2 \cdot (N_1 - 1) \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} - \sqrt{3} - \sqrt{3} \cdot N_1\right]}{2 \cdot (N_1 + 1)} - DG = 0$

**For Any $N_1 \cdot N_2$**



082601

**Unit.**

$AF := 1$

**Given.**

$N_1 := 4$

$N_2 := 3$

**Descriptions.**

$AC := \frac{AF}{2}$ $CJ := AC$ $BC := \frac{AC}{N_1}$

$AE := \frac{AF}{N_2}$ $EF := AF - AE$ $EH := \sqrt{AE \cdot EF}$

$EG := \frac{BC \cdot EH}{CJ}$ $CE := AE - AC$ $CG := \sqrt{CE^2 + EG^2}$

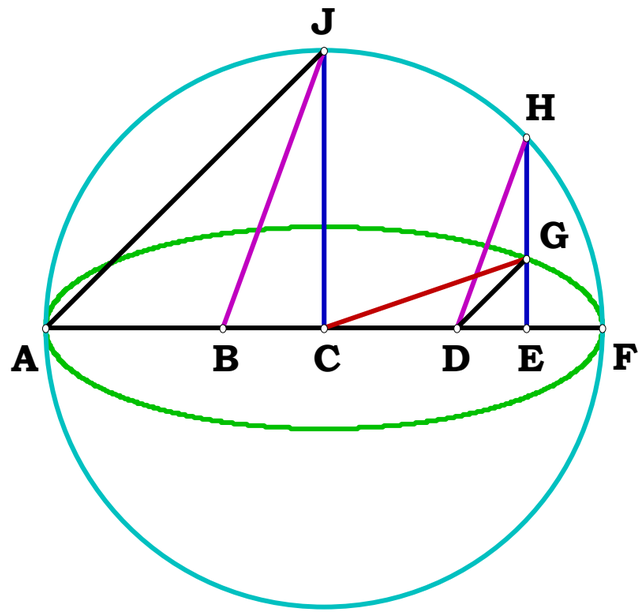
**Definitions.**

$BC - \frac{1}{(2 \cdot N_1)} = 0$ $AE - \frac{1}{N_2} = 0$ $EF - \left(1 - \frac{1}{N_2}\right) = 0$

$EH - \frac{\sqrt{N_2 - 1}}{N_2} = 0$ $EG - \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2} = 0$

$CE - \left(\frac{1}{N_2} - \frac{1}{2}\right) = 0$ $CG - \frac{1}{2} \cdot \frac{\sqrt{(N_2 - 2)^2 \cdot N_1^2 + 4 \cdot (N_2 - 1)}}{(N_1 \cdot N_2)} = 0$

**Elipse By Parallels**



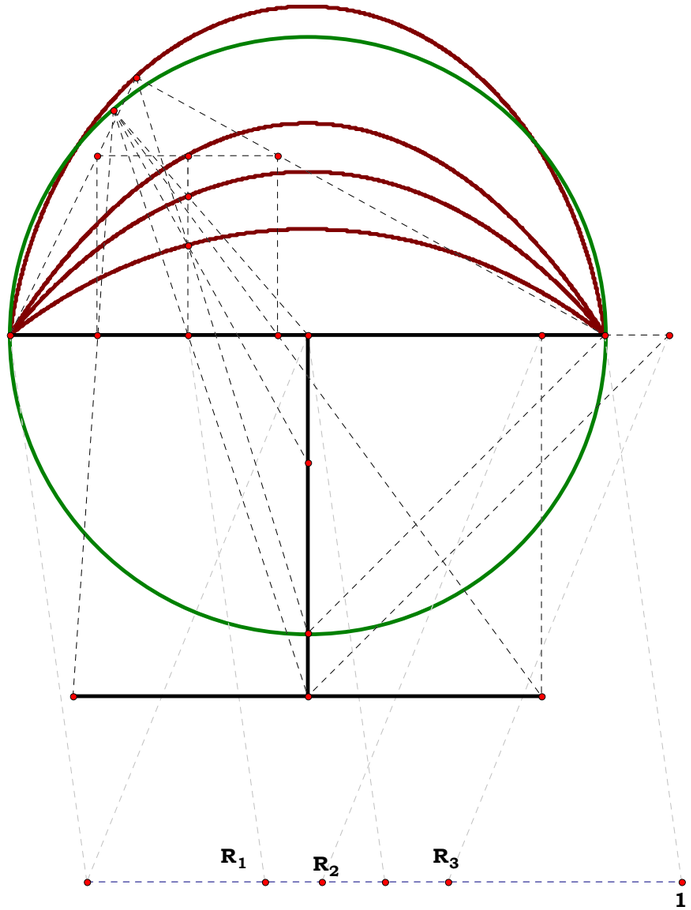
102201

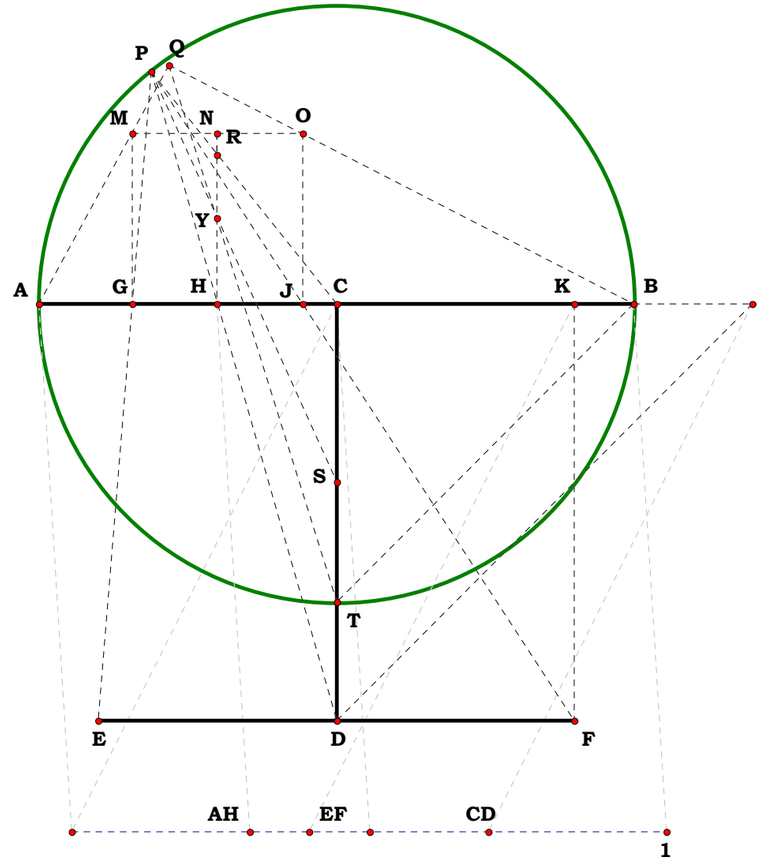
I had sketched this out over 18 years ago and have put off writing it up for some very good reasons, the most prominate is because of the way I wanted to write it up. Normally I aim for between one and four variables for this novel; this one I need six. For the work Basic Analog Mathematics that number of variables is not unusual, but here it is. For BAM, I set my limit on 8, which is twice that I set for this work.

I am so motivated to write this up that when I got to it in this Delian Quest revision, I put this project aside and did the projects OTOH (On The Other Hand), Alice Innocent Plays and Conducts Bach, and Sergio Vosh Goshen Inkscapes Durer, which took me over a month to do. I was curious about the state of ABC notation and Vector Graphics which have been on my mind for some 20 years now.

I also made some new Windows 95 and 98 virtual boxes which have all the software, and more, that I started these projects with.

**Four Curves and Procrastination.**




Unit = 1.00000
AH = 0.30000
$N_1$ = 6.00000
$N_2$ = 20.00000

EF = 0.80000
$N_3$ = 8.00000
$N_4$ = 10.00000

CD = 0.70000
$N_5$ = 7.00000
$N_6$ = 10.00000

I am not even going to go through all of this, just the major portion.

**Unit.**

$AB := 1$

**Given.**

$N_1 := 6 \quad N_2 := 20$

$N_3 := 8 \quad N_4 := 10$

$N_5 := 7 \quad N_6 := 11$

**Descriptions.**

$$AH := \frac{N_1}{N_2} \quad EF := \frac{N_3}{N_4} \quad CD := \frac{N_5}{N_6} \quad AC := \frac{AB}{2}$$

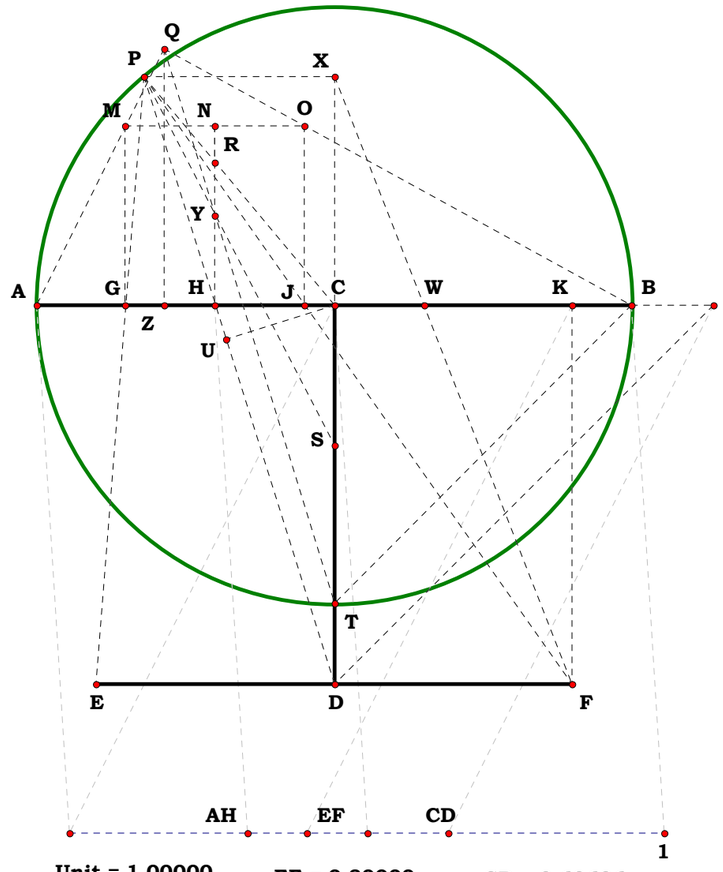
$$CP := AC \quad CH := \sqrt{(AC - AH)^2} \quad DH := \sqrt{CD^2 + CH^2}$$

$$CU := \frac{CD \cdot CH}{DH} \quad PU := \sqrt{CP^2 - CU^2} \quad HU := \frac{CH^2}{DH}$$

$$HP := PU - HU \quad DP := DH + HP \quad DX := \frac{CD \cdot DP}{DH}$$

$$DF := \frac{EF}{2} \quad CX := DX - CD \quad CW := \frac{DF \cdot CX}{DX}$$

$GJ := 2 \cdot CW$ **Etc.**




Unit = 1.00000
AH = 0.30000
$N_1$ = 6.00000
$N_2$ = 20.00000

EF = 0.80000
$N_3$ = 8.00000
$N_4$ = 10.00000

CD = 0.63636
$N_5$ = 7.00000
$N_6$ = 11.00000

**Definitions.**

$$AH - \frac{N_1}{N_2} = 0 \quad EF - \frac{N_3}{N_4} = 0 \quad CD - \frac{N_5}{N_6} = 0 \quad AC - \frac{1}{2}$$

$$CP - \frac{1}{2} = 0 \quad CH - \frac{\sqrt{(2 \cdot N_1 - N_2)^2}}{2 \cdot N_2} = 0 \quad DH - \frac{\sqrt{(4 \cdot N_5^2 + N_6^2) \cdot N_2^2 + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}}{2 \cdot N_2 \cdot N_6} = 0$$

$$CU - \frac{N_5 \cdot \sqrt{(N_2 - 2 \cdot N_1)^2}}{\sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$PU - \frac{\sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}}{2 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2}} = 0 \quad HU - \frac{N_6 \cdot (2 \cdot N_1 - N_2)^2}{2 \cdot N_2 \cdot \sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$HP - \frac{N_2 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} - N_6 \cdot (2 \cdot N_1 - N_2)^2}{2 \cdot N_2 \cdot \sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

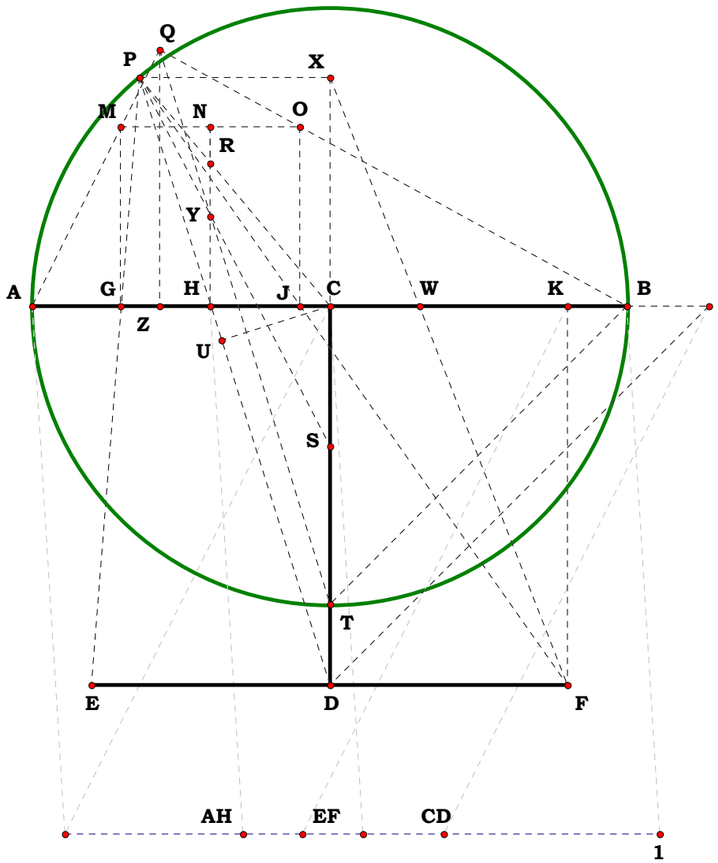
$$DP - \frac{4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}}{2 \cdot N_6 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2}} = 0$$

$$DX - \frac{N_2 \cdot N_5 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]}{N_6 \cdot \left[(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2\right]} = 0$$

$$DF - \frac{N_3}{2 \cdot N_4} = 0 \qquad CX - \frac{\left[\sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_5^2 \cdot N_1 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right] \cdot N_5}{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2} = 0$$

$$CW - \frac{N_3 \cdot N_6 \cdot \left[\sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right]}{2 \cdot N_2 \cdot N_4 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]} = 0$$

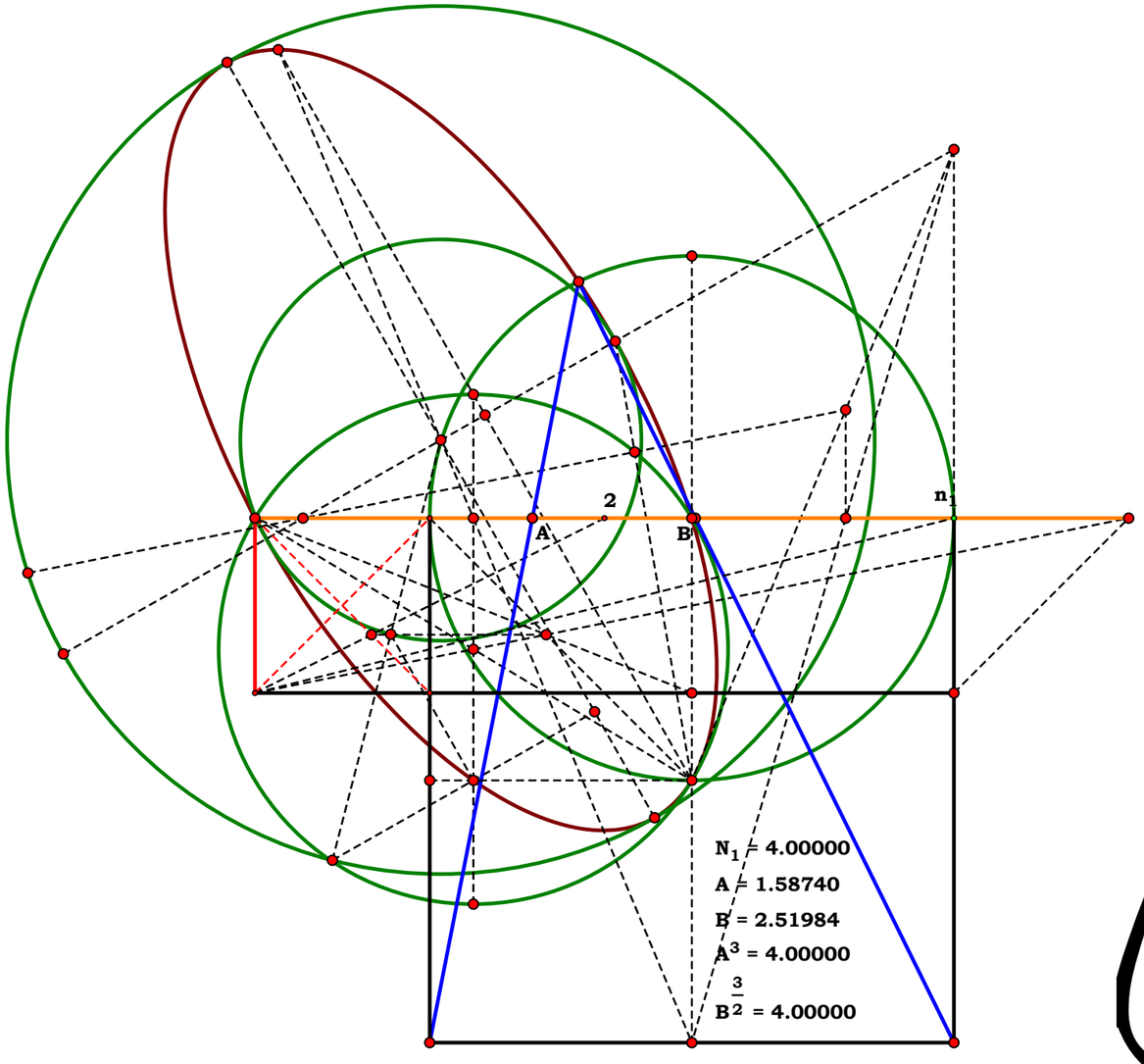
$$GJ - \frac{N_3 \cdot N_6 \cdot \left[\sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right]}{N_2 \cdot N_4 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]} = 0$$




Unit = 1.00000
AH = 0.30000
$N_1$ = 6.00000
$N_2$ = 20.00000

EF = 0.80000
$N_3$ = 8.00000
$N_4$ = 10.00000

CD = 0.63636
$N_5$ = 7.00000
$N_6$ = 11.00000




# The Delian Quest 2002

**John Clark**

# Just Another Proof Of Paper

**Unit.**

$AB := 1$

**Given.**

$N := 7 \qquad AF := N$

**010202**

**Descriptions.**

$BF := AF - AB \qquad BE := \frac{BF}{2} \qquad AE := BE + AB$

$AJ := AE \qquad EJ := BE \qquad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \qquad GJ := 2 \cdot Gb \qquad AG := AJ - GJ$

$Aa := AE - Ea \qquad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \qquad GU := \frac{Ja \cdot AG}{AJ}$

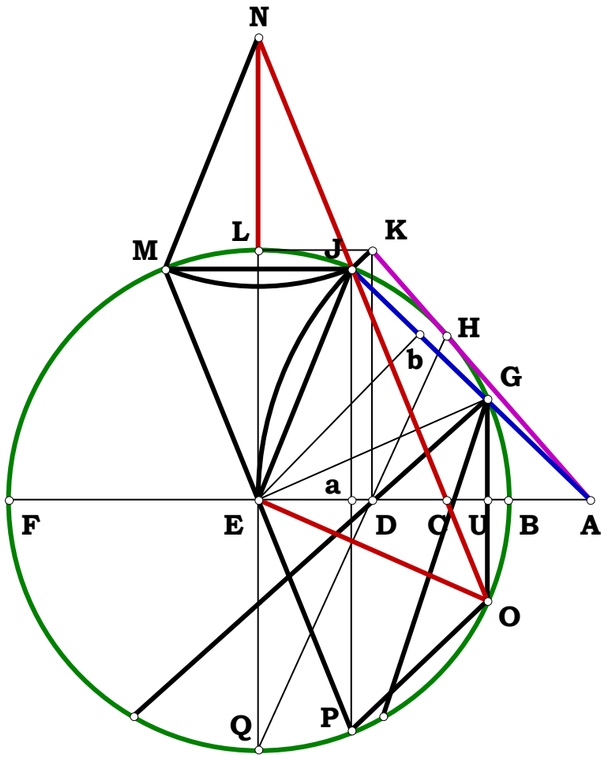
$Ua := Aa - AU \qquad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \qquad JN - BE = 0$

**From 4/29/94** $\qquad OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \qquad NO := JO + JN \qquad EU := Ua + Ea$

$EN := \sqrt{NO^2 - EU^2} \qquad EL := BE \qquad LN := EN - EL$



**Definitions.**

$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0 \qquad \frac{N+1}{2} - AE = 0$$

$$\frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0 \qquad \frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0 \qquad \frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

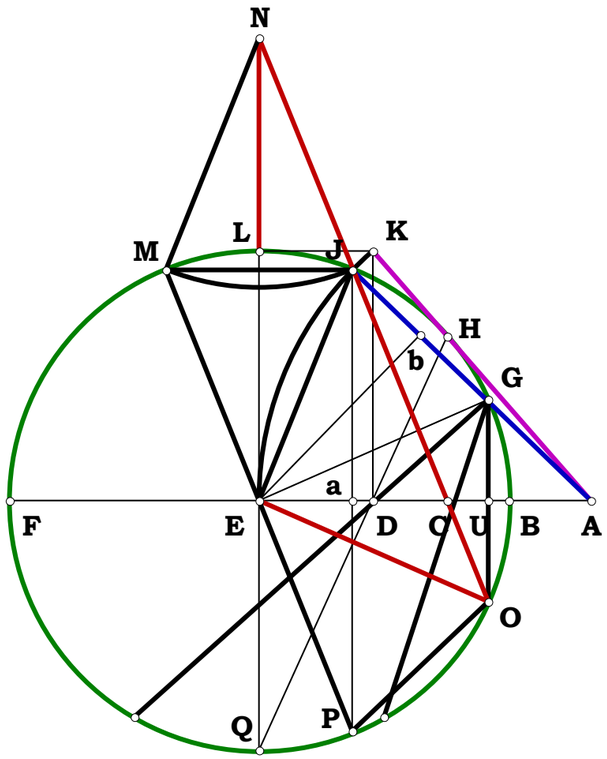
$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0 \qquad \frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0$$

$$\frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0 \qquad \frac{(N-1)^2}{2\cdot(N+1)} - OP = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$



062002A

Unit.

$AC := 1$

Given.

$N_1 := 4 \quad AF := N_1$

$N_2 := 3 \quad CE := N_2$

Descriptions.

$FH := AF \quad EH := CE \quad AD := CE \quad DF := AF - AD$

$EJ := CE \quad FJ := AF \quad DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$EG := \frac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF} \quad EM := \frac{DF \cdot EG}{EF} \quad CM := CE + EM$

$GM := \frac{DE \cdot EG}{EF} \quad CK := GM \quad GK := CM \quad BK := \frac{DF \cdot GK}{DE}$

$BC := CK + BK \quad BC - \frac{AC}{2} = 0$

Definitions.

$FH - N_1 = 0 \quad EH - N_2 = 0 \quad AD - N_2 = 0$

$DF - (N_1 - N_2) = 0 \quad EJ - N_2 = 0 \quad FJ - N_1 = 0$

$DE - 1 = 0 \quad EF - \sqrt{(N_1 - N_2)^2 + 1} = 0$

$EG - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot \sqrt{(N_1 - N_2)^2 + 1}} = 0 \quad EM - \frac{[1 - 2 \cdot N_2 \cdot (N_1 - N_2)] \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

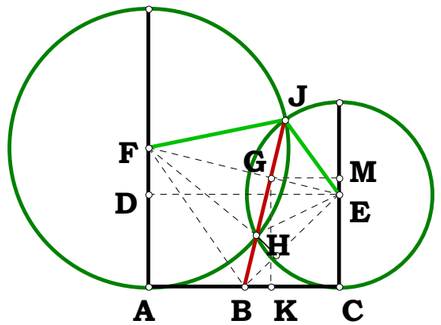
$CM - \frac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GM - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

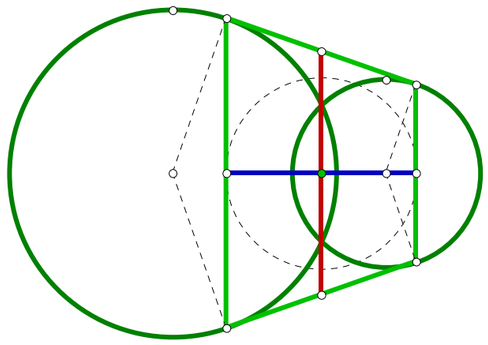
$CK - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GK - \frac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

$BK - \frac{(N_1 + N_2) \cdot (N_1 - N_2)}{2 \cdot (N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + 1)} = 0 \quad BC - \frac{1}{2} = 0$

## Easy Power-Line

**For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.**



Unit.

$AB := 1$

Given.

$N_1 := 7 \quad N_2 := 19$

$N_3 := 15 \quad N_4 := 17$

062002B

Descriptions.

$$AE := \frac{N_1}{N_2} \quad BF := \frac{N_3}{N_4} \quad BN := AE$$

$$FN := BF - BN \quad EF := \sqrt{AB^2 + FN^2}$$

$$EH := \frac{EF^2 + AE^2 - BF^2}{2 \cdot EF} \quad AO := \frac{AB \cdot EH}{EF}$$

$$EM := \frac{FN \cdot EH}{EF} \quad HO := AE + EM$$

$$KO := \frac{FN \cdot HO}{AB} \quad AB - (AO + KO) = 0.5$$

Definitions.

$$AE - \frac{N_1}{N_2} = 0 \quad BF - \frac{N_3}{N_4} = 0 \quad BN - \frac{N_1}{N_2} = 0$$

$$FN - \left(\frac{N_3}{N_4} - \frac{N_1}{N_2}\right) = 0 \quad FN - \frac{N_2 \cdot N_3 - N_1 \cdot N_4}{N_2 \cdot N_4} = 0$$

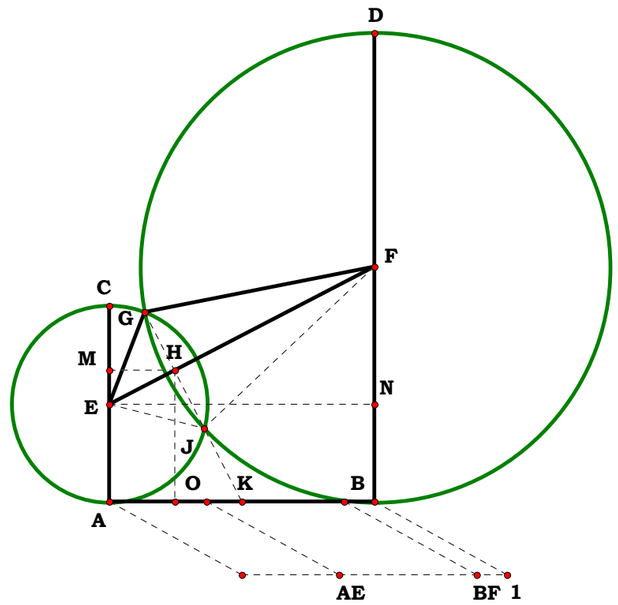
$$EF - \frac{\sqrt{\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}}{N_2 \cdot N_4} = 0$$

$$AO - \frac{N_4 \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$HO - \frac{N_2 \cdot N_4 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3\right)}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

## On the Other Hand

**Have you ever pondered a figure in terms not of the particulars but of the concept of parallel lines? What does parallel mean? Between any two objects there is one, and only one, difference. If this is not true, what does it mean for existence itself? Is it not a self-referential fallacy? And since this is obvious, what does it say for so called non-Euclidean Geometers? If one cannot master the first prnciple of reasoning, can one ever know when they are speaking and thinking gibberish?**




Unit = 1.00000
AB = 1.00000
AE = 0.36842
$N_1$ = 7.00000
$N_2$ = 19.00000

BF = 0.88235
$N_3$ = 15.00000
$N_4$ = 17.00000

**Between any two objects there is one, and only one, difference such that the operations equitable to both, become half the difference between them and this is called their power.**

$$EH - \frac{\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot N_4}{2 \cdot N_2 \cdot N_4 \cdot \sqrt{N_4^2 \cdot \left(N_1^2 + N_2^2\right) + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}} = 0$$

$$EM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right) \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot N_2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$KO - \frac{N_2^2 \cdot N_3^2 - \left(N_1 \cdot N_4\right)^2}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0 \quad AB - \frac{1}{2} = 0.5$$

071902

Descriptions.

Unit.

$AX := 1$

Given.

$N_1 := 9 \quad N_2 := 12$

$N_3 := 4 \quad N_4 := 11$

$$AB := \frac{N_1}{N_2} \quad AG := \frac{N_3}{N_4} \quad BG := \sqrt{AB^2 + AG^2}$$

I do not memorize equations; every time I have to use an equation from Pythagorus Revisited, I have to look it up.

$$BD := \frac{BG^2 + AB^2 - BG^2}{2 \cdot BG}$$ Or again; $$BD_1 := \frac{AB^2}{2 \cdot BG}$$

$$BE := \frac{AB \cdot BD}{BG} \quad DE := \frac{AG \cdot BE}{AB} \quad EF := \frac{AG \cdot DE}{AB}$$

$$BF := BE + EF \quad BF - \frac{AB}{2} = 0$$

Definitions.

$$AB - \frac{N_1}{N_2} = 0 \quad AG - \frac{N_3}{N_4} = 0$$

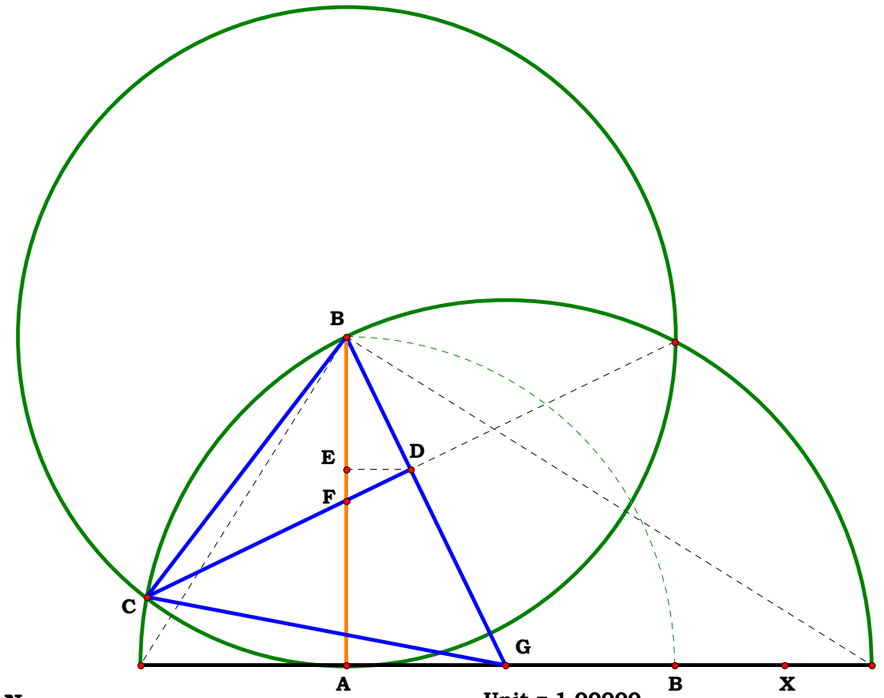
$$BG - \frac{\sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}}{N_2 \cdot N_4} = 0 \quad BD - \frac{N_1^2 \cdot N_4}{2 \cdot N_2 \cdot \sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}} = 0$$

$$BE - \frac{N_1^3 \cdot N_4^2}{2 \cdot N_2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \quad DE - \frac{N_1^2 \cdot N_3 \cdot N_4}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0$$

$$EF - \frac{N_1 \cdot N_2 \cdot N_3^2}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \quad BF - \frac{N_1}{2 \cdot N_2} = 0$$

## On Linear Division

**If G were at A, then one would have the simple textbook method, however, if one took any point G on the perpendicular to AB, the results would be the same.**




Unit = 1.00000
$x_X$ = 1.00000
AB = 0.75000
$N_1$ = 9.00000
$N_2$ = 12.00000

AG = 0.36364
$N_3$ = 4.00000
$N_4$ = 11.00000

090702A

Unit. $C1 := 1$

Given.

$X := 7$

$Y := 20$

Given X1 find AC

**Descriptions.**

$B1 := C1 \quad H1 := C1 \quad BC := 2 \cdot C1 \quad GH := BC \quad X1 := B1 \cdot \frac{X}{Y}$

$HX := \sqrt{X1^2 + H1^2} \quad HV := H1 \cdot \frac{GH}{HX} \quad N1 := X1 \cdot \frac{HV}{HX}$

$GN := \sqrt{C1^2 + N1^2} \quad A1 := \frac{C1^2}{N1} \quad AC := C1 + A1$

$AB := AC - BC \quad \frac{AC}{AB} = 4.313609$

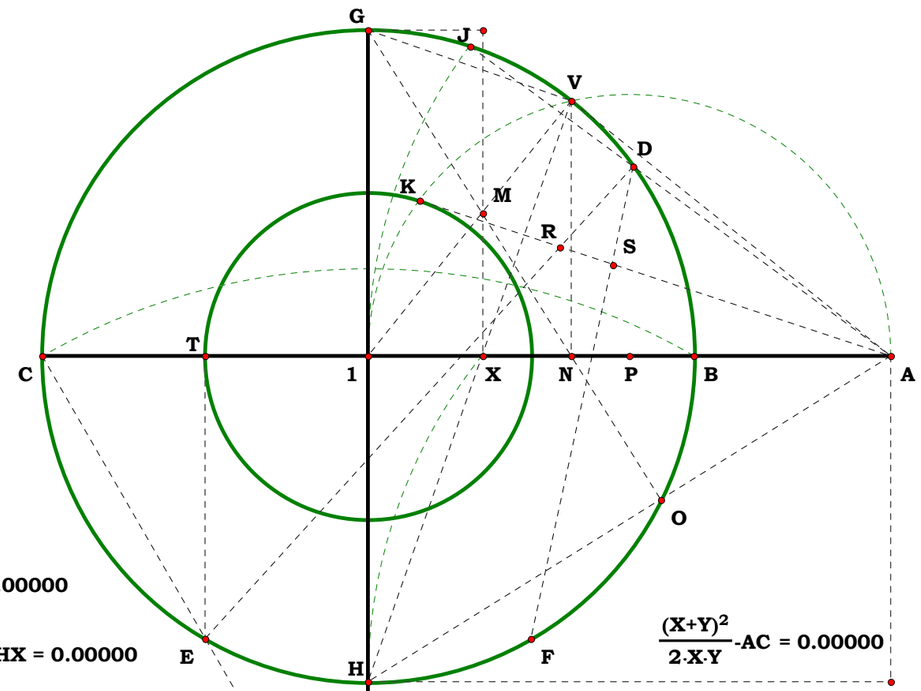
**Definitions.**

$B1 - 1 = 0 \quad H1 - 1 = 0 \quad BC - 2 = 0 \quad GH - 2 = 0 \quad X1 - \frac{X}{Y} = 0$

$HX - \frac{\sqrt{X^2 + Y^2}}{Y} = 0 \quad HV - \frac{2 \cdot Y}{\sqrt{X^2 + Y^2}} = 0 \quad N1 - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$

$GN - \frac{\sqrt{X^4 + 6 \cdot X^2 \cdot Y^2 + Y^4}}{\left(X^2 + Y^2\right)} = 0 \quad A1 - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0 \quad AC := \frac{(X + Y)^2}{2 \cdot X \cdot Y}$

$AB - \frac{(X - Y)^2}{2 \cdot X \cdot Y} = 0 \quad \frac{AC}{AB} - \frac{(X + Y)^2}{(X - Y)^2} = 0$




$\frac{X}{Y}$-X1 = 0.00000

$\frac{\sqrt{X^2+Y^2}}{Y}$-HX = 0.00000

$\frac{2 \cdot Y}{\sqrt{X^2+Y^2}}$-HV = 0.00000

$\frac{2 \cdot X \cdot Y}{X^2+Y^2}$-N1 = 0.00000

$\frac{\sqrt{X^4+6 \cdot X^2 \cdot Y^2+Y^4}}{X^2+Y^2}$-GN = 0.00000

$\frac{X^2+Y^2}{2 \cdot X \cdot Y}$-A1 = 0.00000

$\frac{(X+Y)^2}{2 \cdot X \cdot Y}$-AC = 0.00000

Unit = 1.00000

XY = 0.35000

X = 7.00000

Y = 20.00000

AC = 2.60357

m∠DB1 = 72.25197°

m∠DBS = 24.08399°

$\frac{m\angle DB1}{m\angle DBS}$ = 3.00000

m∠DC1 = 17.74803°

m∠DCR = 5.91601°

$\frac{m\angle DC1}{m\angle DCR}$ = 3.00000

090902

Unit. $AB := 1$

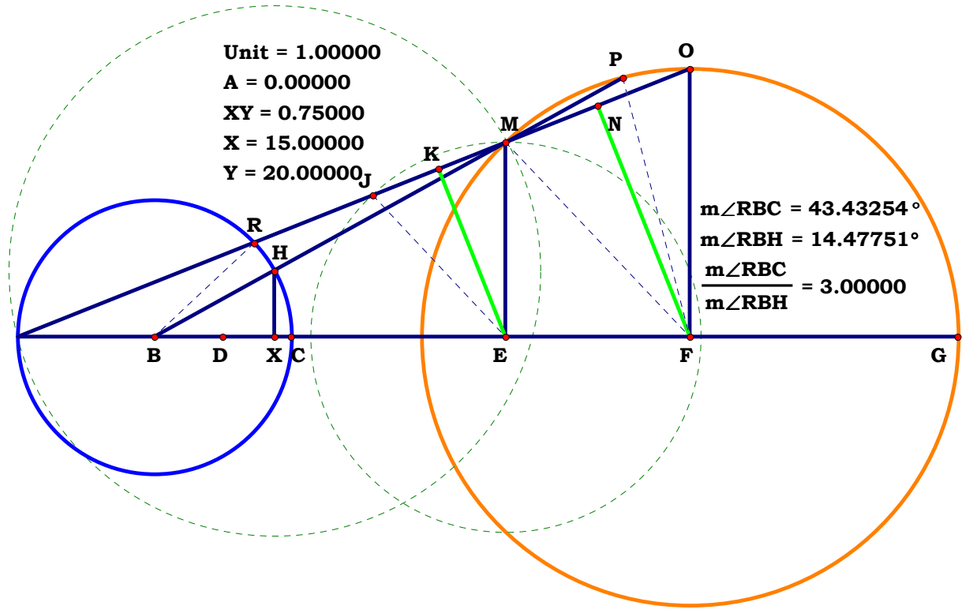
Given.

$X := 15$

$Y := 20$

## Trisection Illusion

**Basically, one is simply adding one half of angle CBH to it.**




**Description.**

$BC := AB \quad CD := \frac{BC}{2} \quad DX := CD \cdot \frac{X}{Y} \quad BX := CD + DX$

$AX := AB + CD + DX \quad HX := \sqrt{AX \cdot (2 \cdot AB - AX)}$

$HM := \sqrt{AX^2 + HX^2} \quad BM := AB + HM \quad EM := HX \cdot \frac{BM}{AB}$

$BE := BX \cdot \frac{EM}{HX} \quad AE := AB + BE \quad AM := \sqrt{AE^2 + EM^2}$

$AK := \frac{AE^2}{AM} \quad KM := AM - AK \quad AJ := AM - 2 \cdot KM$

$AF := \frac{AE \cdot AM}{AJ} \quad FO := AF \cdot \frac{EM}{AE} \quad FO = 1.957661$

**Definitions.**

$BC - 1 = 0 \quad CD - \frac{1}{2} = 0 \quad DX - \frac{X}{2 \cdot Y} = 0 \quad BX - \frac{X + Y}{2 \cdot Y} = 0$

$AX - \frac{X + 3 \cdot Y}{2 \cdot Y} = 0 \quad HX - \frac{\sqrt{(Y - X) \cdot (X + 3 \cdot Y)}}{2 \cdot Y} = 0 \quad HM - \frac{\sqrt{X + 3 \cdot Y}}{\sqrt{Y}} = 0$

$BM - \frac{\sqrt{Y} + \sqrt{X + 3 \cdot Y}}{\sqrt{Y}} = 0 \quad EM - \frac{\sqrt{(Y - X) \cdot (X + 3 \cdot Y)} \cdot \left(\sqrt{Y} + \sqrt{X + 3 \cdot Y}\right)}{2 \cdot Y^{\frac{3}{2}}} = 0 \quad BE - \frac{(X + Y) \cdot \left(\sqrt{Y} + \sqrt{X + 3 \cdot Y}\right)}{2 \cdot Y^{\frac{3}{2}}} = 0 \quad AE - \frac{X \cdot \sqrt{Y} + \sqrt{X + 3 \cdot Y} \cdot (X + Y) + 3 \cdot Y^{\frac{3}{2}}}{2 \cdot Y^{\frac{3}{2}}} = 0$

$$AM - \frac{\sqrt{\left(X^2 + 4 \cdot X \cdot Y + 3 \cdot Y^2\right) \cdot \sqrt{X + 3 \cdot Y} + 4 \cdot X \cdot Y^{\frac{3}{2}} - X \cdot (X + 3 \cdot Y)^{\frac{3}{2}} + Y \cdot (X + 3 \cdot Y)^{\frac{3}{2}} + 12 \cdot Y^{\frac{5}{2}}}}{\sqrt{2} \cdot \sqrt{Y^{\frac{5}{2}}}} = 0$$

$$\mathbf{AK}-\frac{\frac{\sqrt{2}}{4}\cdot\sqrt{\mathbf{Y}^{\frac{5}{2}}}\cdot\left(\mathbf{X}\cdot\sqrt{\mathbf{Y}}+\mathbf{X}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^{\frac{3}{2}}\right)^2}{\mathbf{Y}^3\cdot\sqrt{4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}-\mathbf{X}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+\mathbf{Y}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+12\cdot\mathbf{Y}^{\frac{5}{2}}+\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\cdot\left(\mathbf{X}^2+4\cdot\mathbf{X}\cdot\mathbf{Y}+3\cdot\mathbf{Y}^2\right)}}=0$$

$$\mathbf{KM}-\left[\frac{\left(\sqrt{\mathbf{Y}}\right)^5\cdot(\mathbf{Y}-\mathbf{X})\cdot\left[\sqrt{2}\cdot\mathbf{X}^2+12\cdot\sqrt{2}\cdot\mathbf{Y}^2+7\cdot\sqrt{2}\cdot\mathbf{X}\cdot\mathbf{Y}+2\cdot\sqrt{2}\cdot\sqrt{\mathbf{Y}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}\right]}{4\cdot\mathbf{Y}^3\cdot\sqrt{\mathbf{Y}^{\frac{5}{2}}}\cdot\sqrt{4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}-\mathbf{X}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+\mathbf{Y}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+12\cdot\mathbf{Y}^{\frac{5}{2}}+\mathbf{X}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+4\cdot\mathbf{X}\cdot\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}}}\right]=0$$

$$\mathbf{AJ}-\frac{\sqrt{2}\cdot\left[9\cdot\mathbf{X}\cdot\mathbf{Y}^2+6\cdot\mathbf{X}^2\cdot\mathbf{Y}+\mathbf{X}^3-\mathbf{Y}^{\frac{3}{2}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+3\cdot\mathbf{Y}^{\frac{5}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}\cdot\sqrt{\mathbf{Y}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}^2\cdot\sqrt{\mathbf{Y}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right]}{2\cdot\sqrt{\mathbf{Y}}\cdot\sqrt{\mathbf{Y}^{\frac{5}{2}}}\cdot\sqrt{4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}-\mathbf{X}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+\mathbf{Y}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+12\cdot\mathbf{Y}^{\frac{5}{2}}+\mathbf{X}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+4\cdot\mathbf{X}\cdot\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}}}=0$$

$$\mathbf{AF}-\frac{\left(\mathbf{X}\cdot\sqrt{\mathbf{Y}}+\mathbf{X}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^{\frac{3}{2}}\right)\cdot\left[4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}-\mathbf{X}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+\mathbf{Y}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+12\cdot\mathbf{Y}^{\frac{5}{2}}+\mathbf{X}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+4\cdot\mathbf{X}\cdot\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right]}{2\cdot\mathbf{Y}\cdot\left[9\cdot\mathbf{X}\cdot\mathbf{Y}^2+6\cdot\mathbf{X}^2\cdot\mathbf{Y}+\mathbf{X}^3-\mathbf{Y}^{\frac{3}{2}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+3\cdot\mathbf{Y}^{\frac{5}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}\cdot\sqrt{\mathbf{Y}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}^2\cdot\sqrt{\mathbf{Y}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right]}=0$$

$$\mathbf{FO}-\frac{\sqrt{(\mathbf{Y}-\mathbf{X})\cdot(\mathbf{X}+3\cdot\mathbf{Y})}\cdot\left(\sqrt{\mathbf{Y}}+\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right)\cdot\left[4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}-\mathbf{X}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+\mathbf{Y}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+12\cdot\mathbf{Y}^{\frac{5}{2}}+\mathbf{X}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+3\cdot\mathbf{Y}^2\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+4\cdot\mathbf{X}\cdot\mathbf{Y}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right]}{2\cdot\mathbf{Y}\cdot\left[9\cdot\mathbf{X}\cdot\mathbf{Y}^2+6\cdot\mathbf{X}^2\cdot\mathbf{Y}+\mathbf{X}^3-\mathbf{Y}^{\frac{3}{2}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+3\cdot\mathbf{Y}^{\frac{5}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}\cdot\sqrt{\mathbf{Y}}\cdot(\mathbf{X}+3\cdot\mathbf{Y})^{\frac{3}{2}}+4\cdot\mathbf{X}\cdot\mathbf{Y}^{\frac{3}{2}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}+\mathbf{X}^2\cdot\sqrt{\mathbf{Y}}\cdot\sqrt{\mathbf{X}+3\cdot\mathbf{Y}}\right]}=0$$

Unit. $C1 := 1$

Given.

$X := 11$

$Y := 20$

092102

Descriptions.

$BC := 2 \cdot C1 \quad B1 := C1 \quad X1 := B1 \cdot \frac{X}{Y} \quad MX := \sqrt{C1^2 + X1^2}$

$A1 := \frac{C1^2}{X1} \quad AC := A1 + C1 \quad HK := 2 \cdot BC$

$HS := BC + X1 \quad aS := \sqrt{HS \cdot (HK - HS)} \quad E1 := \sqrt{BC^2 - C1^2}$

$CD := E1 \quad d1 := \frac{C1^2}{CD} \quad aX := aS - E1$

$ad := \sqrt{(d1 + aX)^2 + X1^2}$

Definitions.

$BC - 2 = 0 \quad B1 - 1 = 0 \quad X1 - \frac{X}{Y} = 0$

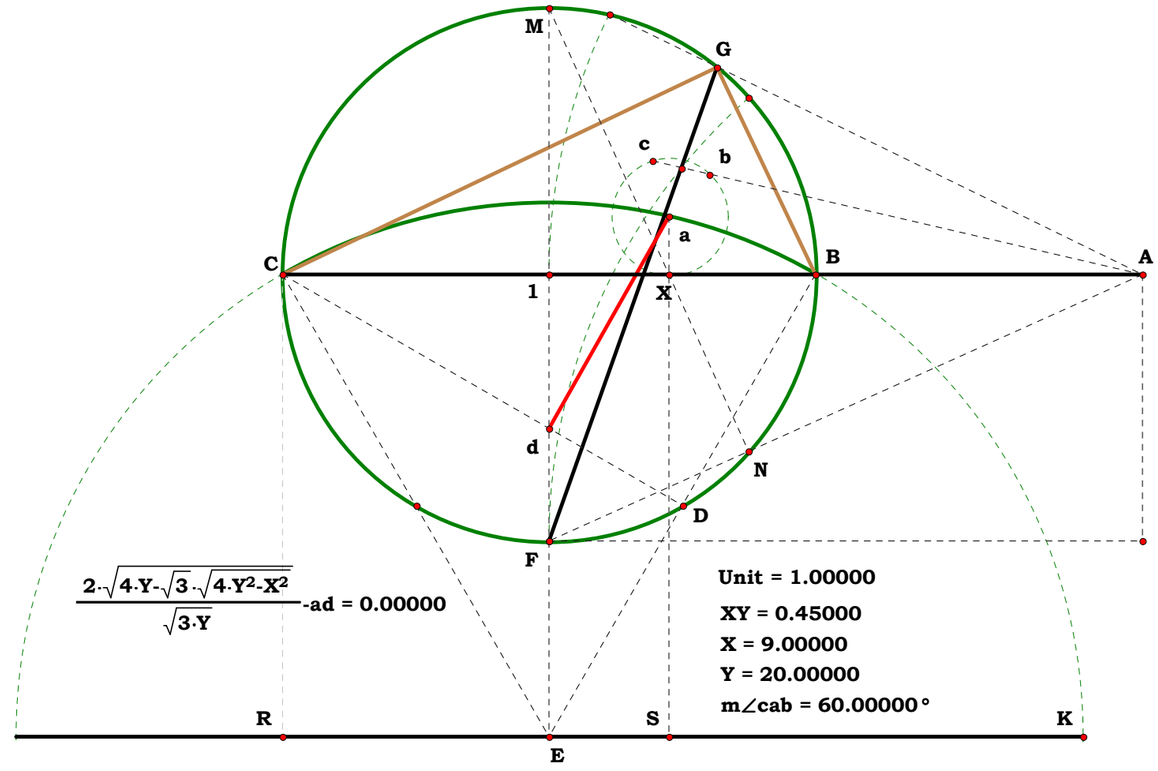
$MX - \frac{\sqrt{X^2 + Y^2}}{Y} = 0 \quad A1 - \frac{Y}{X} = 0 \quad AC - \frac{X + Y}{X} = 0$

$HK - 4 = 0 \quad HS - \frac{X + 2 \cdot Y}{Y} = 0 \quad aS - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{Y} = 0$

$E1 - \sqrt{3} = 0 \quad CD - \sqrt{3} = 0 \quad d1 - \frac{1}{\sqrt{3}} = 0 \quad aX - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$

$ad - \frac{2 \cdot \sqrt{4 \cdot Y - \sqrt{3} \cdot \sqrt{4 \cdot Y^2 - X^2}}}{\sqrt{3 \cdot Y}} = 0$

**Name the segement in red.**



092202

Unit. $C1 := 1$

Given. $X := 5$

$Y := 20$

**Descriptions.**

$BC := 2 \cdot C1$ $A1 := \frac{Y}{X}$ $AC := A1 + C1$ $X1 := C1 \cdot \frac{X}{Y}$

$a1 := \frac{C1}{2}$ $aA := \sqrt{A1^2 - a1^2}$ $b1 := \frac{a1^2}{A1}$ $ab := aA \cdot \frac{a1}{A1}$

$GK := 2 \cdot ab$ $G1 := 2 \cdot b1$ $AG := A1 - G1$

$AK := \sqrt{GK^2 + AG^2}$ $P1 := GK \cdot \frac{A1}{AK}$ $AP := \frac{AG \cdot P1}{GK}$

$PK := AK - AP$ $KN := 2 \cdot PK$ $AN := AK - KN$

$AF := AG \cdot \frac{AN}{AK}$ $FN := GK \cdot \frac{AN}{AK}$ $F1 := A1 - AF$ $FH := F1 + G1$

$Jc := GK - FN$ $DH := FH \cdot \frac{GK}{Jc}$ $D1 := DH - G1$ $Md := C1 - FN$ $E1 := F1 \cdot \frac{C1}{Md}$

$DE := D1 - E1$ $EO := GK \cdot \frac{DE}{DH}$ $DO := \sqrt{DE^2 + EO^2}$ $Ee := 2 \cdot DE$ $e1 := D1 - Ee$

$e1 - (DO + X1) = 0$

**Definitions.**

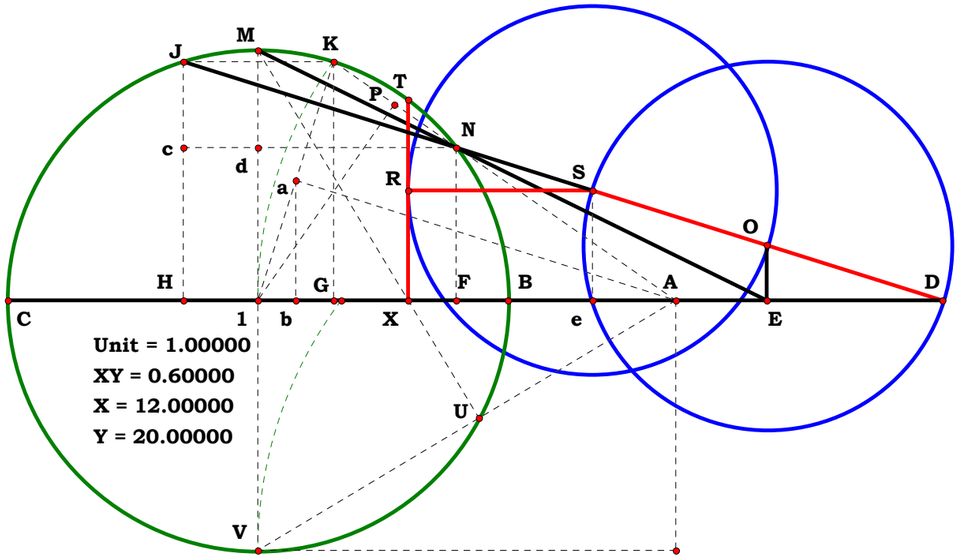
$BC - 2 = 0$ $A1 - \frac{Y}{X} = 0$ $AC - \frac{X + Y}{X} = 0$ $X1 - \frac{X}{Y} = 0$ $a1 - \frac{1}{2} = 0$

$aA - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot X} = 0$ $b1 - \frac{X}{4 \cdot Y} = 0$ $ab - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{4 \cdot Y} = 0$

$GK - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot Y} = 0$ $G1 - \frac{X}{2 \cdot Y} = 0$ $AG - \frac{2 \cdot Y^2 - X^2}{2 \cdot X \cdot Y} = 0$ $AK - \frac{Y}{X} = 0$

$P1 - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot Y} = 0$ $AP - \frac{2 \cdot Y^2 - X^2}{2 \cdot X \cdot Y} = 0$ $PK - \frac{X}{2 \cdot Y} = 0$ $KN - \frac{X}{Y} = 0$

## Pair of Blue Balls



$$AN - \frac{(Y-X)\cdot(X+Y)}{X\cdot Y} = 0 \qquad AF - \frac{\left(X^2-2\cdot Y^2\right)\cdot(X-Y)\cdot(X+Y)}{2\cdot X\cdot Y^3} = 0 \qquad F1 - \frac{X\cdot\left(3\cdot Y^2-X^2\right)}{2\cdot Y^3} = 0$$

$$FN - \frac{\sqrt{(2\cdot Y-X)\cdot(X+2\cdot Y)}\cdot(Y-X)\cdot(X+Y)}{2\cdot Y^3} = 0 \qquad FH - \frac{X\cdot(2\cdot Y-X)\cdot(X+2\cdot Y)}{2\cdot Y^3} = 0$$

$$Jc - \frac{X^2\cdot\sqrt{4\cdot Y^2-X^2}}{2\cdot Y^3} = 0 \qquad DH - \frac{\sqrt{(2\cdot Y-X)\cdot(X+2\cdot Y)}\cdot(2\cdot Y-X)\cdot(X+2\cdot Y)}{2\cdot X\cdot Y\cdot\sqrt{4\cdot Y^2-X^2}} = 0 \qquad D1 - \frac{2\cdot Y^2-X^2}{X\cdot Y} = 0$$

$$Md - \frac{X^2\cdot\sqrt{4\cdot Y^2-X^2}-Y^2\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3}{2\cdot Y^3} = 0 \qquad E1 - \frac{X\cdot\left(3\cdot Y^2-X^2\right)}{\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3} = 0$$

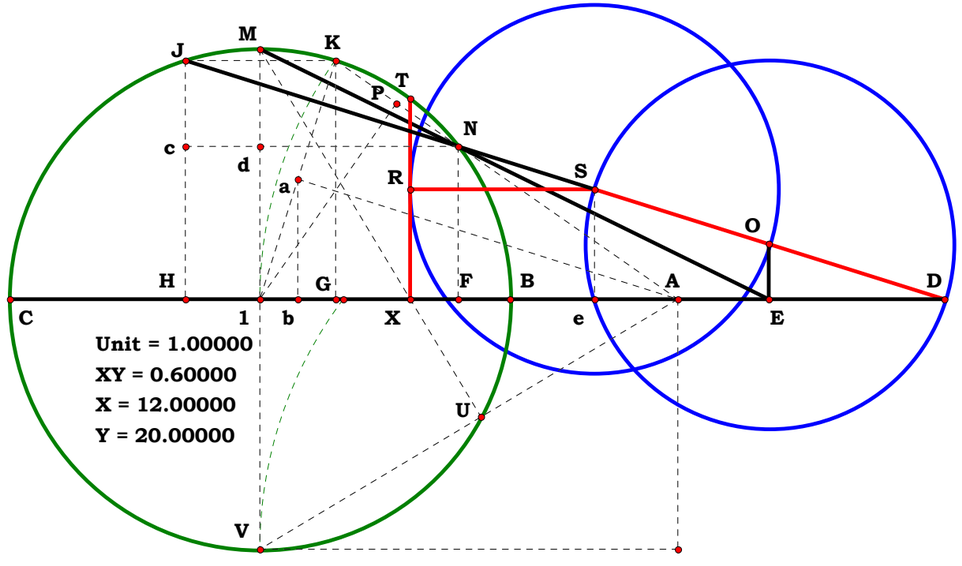
$$DE - \frac{(X-Y)\cdot(X+Y)\cdot\left[\left(2\cdot Y^2-X^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+X^2\cdot Y-4\cdot Y^3\right]}{X\cdot Y\cdot\left[\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3\right]} = 0 \qquad EO - \frac{(Y-X)\cdot(X-2\cdot Y)\cdot(X+2\cdot Y)\cdot(X+Y)\cdot\left(X^2-2\cdot Y^2+Y\cdot\sqrt{4\cdot Y^2-X^2}\right)}{Y\cdot(X-2\cdot Y)\cdot(X+2\cdot Y)\cdot\left[\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3\right]} = 0$$

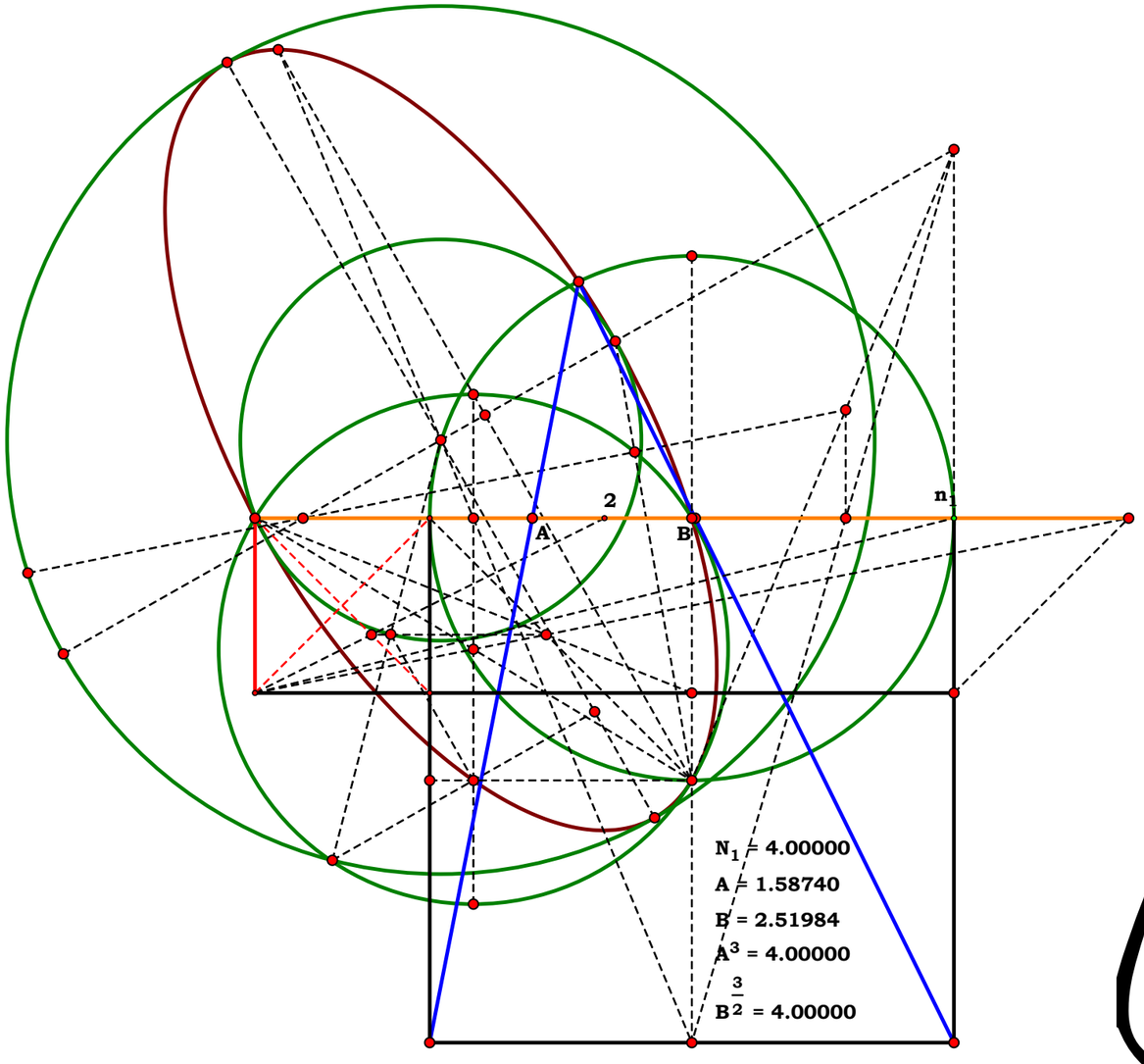
$$DO - \frac{2\cdot\sqrt{(X-Y)^2\cdot(X+Y)^2\cdot\left[\left(2\cdot X^2\cdot Y-4\cdot Y^3\right)\cdot\sqrt{4\cdot Y^2-X^2}+X^4-5\cdot X^2\cdot Y^2+8\cdot Y^4\right]}}{X\cdot\sqrt{\left(4\cdot X^2\cdot Y^3-4\cdot Y^5\right)\cdot\sqrt{4\cdot Y^2-X^2}+6\cdot X^4\cdot Y^2-X^6-9\cdot X^2\cdot Y^4+8\cdot Y^6}} = 0 \qquad Ee - \frac{2\cdot(X-Y)\cdot(X+Y)\cdot\left[\left(2\cdot Y^2-X^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+X^2\cdot Y-4\cdot Y^3\right]}{X\cdot Y\cdot\left[\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3\right]} = 0$$

$$e1 - \frac{\left(X^4-3\cdot X^2\cdot Y^2+2\cdot Y^4\right)\cdot\sqrt{4\cdot Y^2-X^2}+8\cdot X^2\cdot Y^3-2\cdot X^4\cdot Y-4\cdot Y^5}{X\cdot Y\cdot\left[\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3\right]} = 0$$

$$\frac{\dfrac{2\cdot\sqrt{(X-Y)^2\cdot(X+Y)^2\cdot\left[\left(2\cdot X^2\cdot Y-4\cdot Y^3\right)\cdot\sqrt{4\cdot Y^2-X^2}+X^4-5\cdot X^2\cdot Y^2+8\cdot Y^4\right]}}{X\cdot\sqrt{\left(4\cdot X^2\cdot Y^3-4\cdot Y^5\right)\cdot\sqrt{4\cdot Y^2-X^2}+6\cdot X^4\cdot Y^2-X^6-9\cdot X^2\cdot Y^4+8\cdot Y^6}} + \dfrac{X}{Y}}{\dfrac{\left(X^4-3\cdot X^2\cdot Y^2+2\cdot Y^4\right)\cdot\sqrt{4\cdot Y^2-X^2}+8\cdot X^2\cdot Y^3-2\cdot X^4\cdot Y-4\cdot Y^5}{X\cdot Y\cdot\left[\left(X^2-Y^2\right)\cdot\sqrt{4\cdot Y^2-X^2}+2\cdot Y^3\right]}} = 1$$

**Mathcad cannot reduce this equation. It can divide them and come up with the correct arithmetic, 1. It can subtract them and again come up with the correct arithmetic, but it cannot reduce them to either 0 or 1, which is a logical process.**






# The Delian Quest 2003

**John Clark**

021603

**Descriptions.**

**Unit.**

$AD := 1$

**Given.**

$N := 1.5$

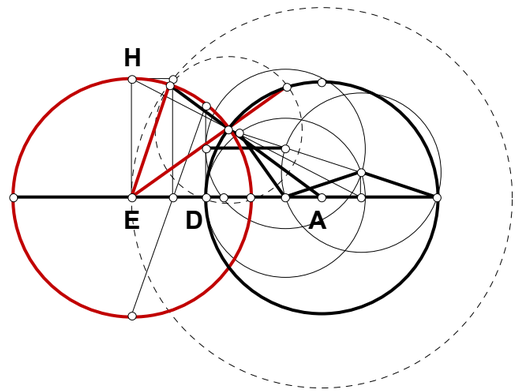
$$AB := \frac{AD}{2}$$

$$BD := AD - AB \qquad BC := \frac{BD}{N}$$

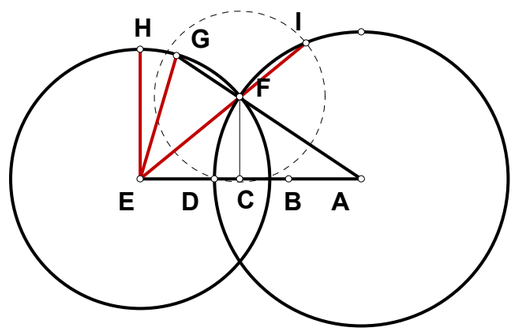
$$CD := BD - BC \qquad AC := AB + BC$$

$$AF := AD \qquad CF := \sqrt{AF^2 - AC^2}$$

$$FI := \sqrt{CD^2 + CF^2}$$




**From AD project a trisection to EH. Make AD the unit.**




$$DI := 2 \cdot CF \qquad AI := AD$$

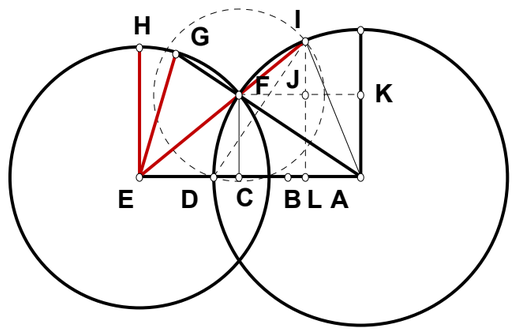
$$AL := \frac{\left(2AD^2 - DI^2\right)}{2 \cdot AD}$$

$$FJ := AC - AL \qquad IL := \sqrt{AI^2 - AL^2}$$

$$IJ := IL - CF \qquad EL := \frac{FJ \cdot IL}{IJ}$$

$$AE := AL + EL \qquad EI := \frac{FI \cdot IL}{IJ}$$

$$EF := EI - FI$$



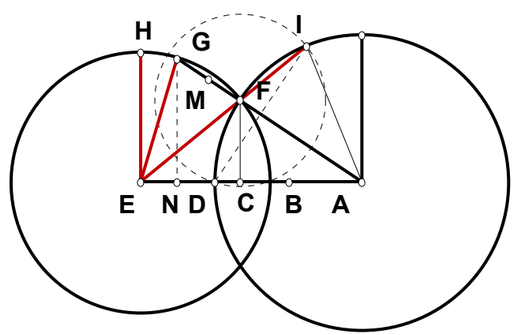
$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF}$$

$$AG := AF + FG$$

$$AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \qquad FM := \frac{FG}{2}$$

$$FM - EN = 0$$




**Definitions.** **In the following definitions, one should remember, N is not something which is a part of the figure, it is an assertion you make of the figure.**

$$AB - \frac{1}{2} = 0 \qquad BD - (N-1) = 0 \qquad BC - \frac{N-1}{N} = 0 \qquad CD - \frac{(N-1)^2}{N} = 0$$

$$AC - \frac{3 \cdot N - 2}{2 \cdot N} = 0 \qquad AF - 1 = 0 \qquad CF - \frac{\sqrt{(2-N) \cdot (5 \cdot N - 2)}}{2 \cdot N} = 0$$

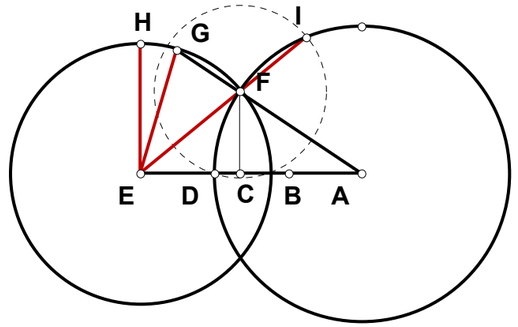
$$FI - \frac{\sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{2 \cdot \sqrt{N}} = 0 \qquad DI - \frac{\sqrt{(2-N) \cdot (5 \cdot N - 2)}}{N} = 0$$

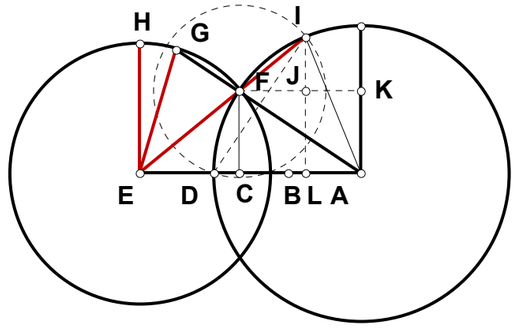
$$AI - 1 = 0 \qquad AL - \frac{7 \cdot N^2 - 12 \cdot N + 4}{2 \cdot N^2} = 0 \qquad FJ - \frac{(2 \cdot N - 1) \cdot (2 - N)}{N^2} = 0$$

$$IL - \frac{\sqrt{(5 \cdot N - 2) \cdot (2 - N)} \cdot (3 \cdot N - 2)}{2 \cdot N^2} = 0 \qquad IJ - \frac{\sqrt{12 \cdot N - 5 \cdot N^2 - 4} \cdot (N-1)}{N^2} = 0$$

$$EL - \frac{(2-N) \cdot (2 \cdot N - 1) \cdot (3 \cdot N - 2) \cdot \sqrt{(2-N) \cdot (5 \cdot N - 2)}}{2 \cdot N^2 \cdot (N-1) \cdot \sqrt{12 \cdot N - 5 \cdot N^2 - 4}} = 0 \qquad AE - \frac{N}{2 \cdot (N-1)} = 0$$

$$EI - \frac{(3 \cdot N - 2) \cdot \sqrt{-(N-2) \cdot (5 \cdot N - 2)} \cdot \sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{4 \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right) \cdot \sqrt{12 \cdot N - 5 \cdot N^2 - 4}} = 0 \qquad EF - \frac{\sqrt{N} \cdot \sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{4 \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)} = 0$$



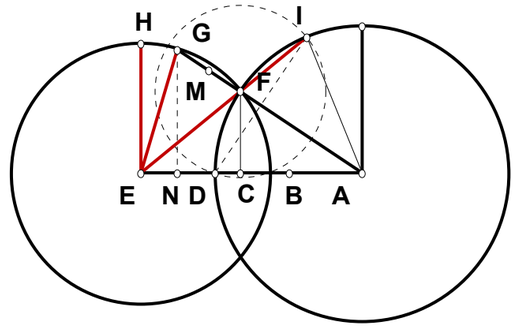



$$FG - \frac{\left(16\cdot N^3 - 4\cdot N^4 - 31\cdot N^2 + 36\cdot N - 16\right)}{16\cdot (N-1)^2} = 0$$

$$AG - \frac{N\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 7\cdot N + 4\right)}{16\cdot (N-1)^2} = 0$$

$$AN - \frac{(1-2\cdot N)\cdot (3\cdot N-2)\cdot \left(2\cdot N^2 - 7\cdot N + 4\right)}{32\cdot (N-1)^2} = 0$$

$$EN - \frac{12\cdot N^4 - 56\cdot N^3 + 93\cdot N^2 - 58\cdot N + 8}{32\cdot (N-1)^2} = 0$$




$$FM - \frac{16\cdot N^3 - 4\cdot N^4 - 31\cdot N^2 + 36\cdot N - 16}{32\cdot (N-1)^2} = 0$$

$$FM - EN = 0$$

$$\frac{(2\cdot N-3)\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 5\cdot N + 4\right)}{16\cdot (N-1)^2} = 0 \qquad 16\cdot (N-1)^2 = 4 \qquad (2\cdot N-3)\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 5\cdot N + 4\right) = 0$$

$$36\cdot N^3 - 8\cdot N^4 - 62\cdot N^2 + 47\cdot N - 12$$

$$36\cdot N^3 - 8\cdot N^4 - 62\cdot N^2 + 47\cdot N - 12 = 0$$

**Solve for N.**

$$\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{5}{4} + \frac{\sqrt{7}\cdot i}{4} \\ \frac{5}{4} - \frac{\sqrt{7}\cdot i}{4} \end{pmatrix}$$

**Two of these solutions are not even part of the grammar. If one finds anyone who defends them, as them, show me how complete induction and deduction of a unit cannot produce anything more than a recursion of that unit.**

021903

Unit. $AB := 1$

Given. $X := 11$

$Y := 20$

**Descriptions.**

$AC := 2 \cdot AB$ $AF := \frac{AB}{2}$ $CF := AC - AF$ $FX := CF \cdot \frac{X}{Y}$

$AX := AF + FX$ $CX := AC - AX$ $HX := \sqrt{AX \cdot CX}$

$CH := \sqrt{HX^2 + CX^2}$ $CT := \frac{CH}{2}$ $BT := HX \cdot \frac{CT}{CX}$

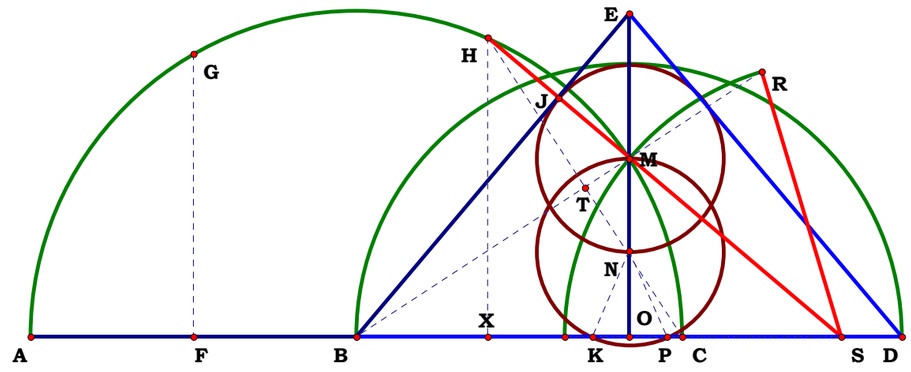
$BO := HX \cdot \frac{AB}{CH}$ $MO := CX \cdot \frac{AB}{CH}$ $BS := 2 \cdot BO$

$AO := AB + BO$ $OX := AO - AX$ $DX := OX \cdot \frac{HX}{HX - MO}$

$AD := AX + DX$ $HT := CT$ $MT := AB - BT$

$HM := \sqrt{HT^2 + MT^2}$ $JM := \frac{HM}{2}$

**Twin Circles in an Isosceles Triangle**




Unit = 1.00000

XY = 0.60000

X = 12.00000

Y = 20.00000

AX = 1.40000

$m\angle EDB = 49.81637°$

$m\angle RSH = 33.21091°$

$\frac{m\angle EDB}{m\angle RSH} = 1.50000$

$\frac{\sqrt{2 \cdot Y^{\frac{3}{2}} - 2 \cdot X \cdot \sqrt{Y} - \sqrt{Y - X} \cdot \sqrt{(2 \cdot X \cdot Y - 3 \cdot X^2) + Y^2}}}{2 \cdot \sqrt{\sqrt{Y} \cdot (Y - X)}} - JM = 0.00000$

**Definitions.**

**Due to the state of mathematics today, in order to insure that each of these equations is technically correct, I would have to repair everyone where Mathcad preferred the absurd construction resulting in imaginary numbers, which I only did on the last equation. As one can see, in Mathcad arranging by the alphabet trumps correct grammar and Mathcad will simply use a minus sign to justify the absurdity. I do some corrections along the way, but not all as I don't think anyone can live long enough having to correct automated stupidity. A computer will simply execute its programming commensurate with the wit of a programmer.**

$AC - 2 = 0$ $AF - \frac{1}{2} = 0$ $CF - \left(3 \cdot 2^{-1}\right) = 0$ $FX - \frac{3 \cdot X}{2 \cdot Y} = 0$ $AX - \frac{3 \cdot X + Y}{2 \cdot Y} = 0$ $CX - \frac{3 \cdot (Y - X)}{2 \cdot Y} = 0$ $HX - \frac{\sqrt{3 \cdot (3 \cdot X + Y) \cdot (Y - X)}}{2 \cdot Y} = 0$

$CH - \frac{\sqrt{3 \cdot (Y - X)}}{\sqrt{Y}} = 0$ $CT - \frac{\sqrt{3 \cdot (Y - X)}}{2 \cdot \sqrt{Y}} = 0$ $BT - \frac{\sqrt{3} \cdot \sqrt{(Y - X) \cdot (9 \cdot X + 3 \cdot Y)}}{6 \cdot \sqrt{Y} \cdot \sqrt{Y - X}} = 0$ $BO - \frac{\sqrt{3} \cdot \sqrt{-(X - Y) \cdot (9 \cdot X + 3 \cdot Y)}}{6 \cdot \sqrt{Y} \cdot \sqrt{Y - X}} = 0$ $MO - \frac{\sqrt{3} \cdot \sqrt{Y - X}}{2 \cdot \sqrt{Y}} = 0$

$BS - \frac{\sqrt{3} \cdot \sqrt{-(X - Y) \cdot (9 \cdot X + 3 \cdot Y)}}{3 \cdot \sqrt{Y} \cdot \sqrt{Y - X}} = 0$ $AO - \frac{6 \cdot \sqrt{Y} \cdot \sqrt{Y - X} + \sqrt{3} \cdot \sqrt{6 \cdot X \cdot Y - 9 \cdot X^2 + 3 \cdot Y^2}}{6 \cdot \sqrt{Y} \cdot \sqrt{Y - X}} = 0$ $OX - \frac{\sqrt{Y - X} \cdot (Y - 3 \cdot X) + \sqrt{Y} \cdot \sqrt{2 \cdot X \cdot Y - 3 \cdot X^2 + Y^2}}{2 \cdot Y \cdot \sqrt{Y - X}} = 0$

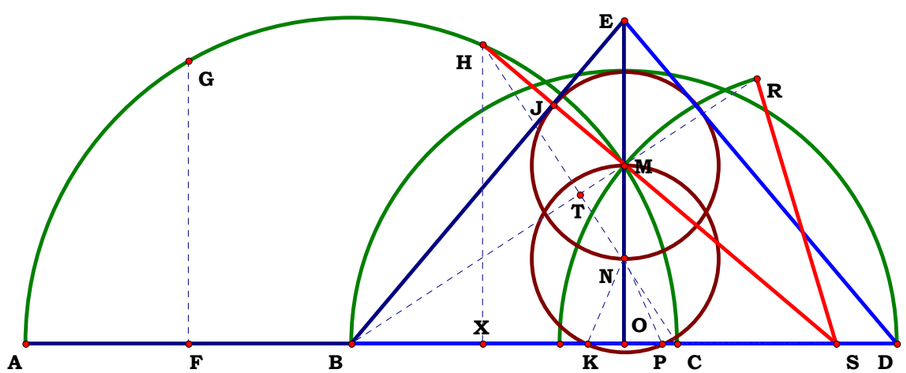
$$DX - \frac{\left(3\cdot X\cdot\sqrt{Y-X} - Y\cdot\sqrt{Y-X} - \sqrt{Y}\cdot\sqrt{2\cdot X\cdot Y - 3\cdot X^2 + Y^2}\right)\cdot\sqrt{((Y-X))\cdot(9\cdot X + 3\cdot Y)}}{2\cdot Y\cdot\sqrt{Y-X}\cdot\left[\sqrt{3}\cdot\sqrt{Y}\cdot\sqrt{Y-X} - \sqrt{((Y-X))\cdot(9\cdot X + 3\cdot Y)}\right]} = 0$$

$$AD - \frac{2\cdot\sqrt{Y}\cdot\sqrt{Y-X}\cdot\sqrt{2\cdot X\cdot Y - 3\cdot X^2 + Y^2}\cdot\sqrt{3}}{2\cdot\sqrt{Y}\cdot\left(\sqrt{Y-X}\cdot\sqrt{6\cdot X\cdot Y - 9\cdot X^2 + 3\cdot Y^2} - \sqrt{3}\cdot Y^{\frac{3}{2}} + \sqrt{3}\cdot X\cdot\sqrt{Y}\right)} = 0$$

$$HT - \frac{\sqrt{3\cdot(Y-X)}}{2\cdot\sqrt{Y}} = 0 \qquad MT - \frac{3\cdot\left(2\cdot\sqrt{Y}\cdot\sqrt{Y-X} - \sqrt{-3\cdot X^2 + 2\cdot X\cdot Y + Y^2}\right)}{6\cdot\sqrt{Y}\cdot\sqrt{Y-X}} = 0$$

$$HM - \frac{\sqrt{\sqrt{Y-X}\cdot\sqrt{2\cdot X\cdot Y - 3\cdot X^2 + Y^2} + 2\cdot X\cdot\sqrt{Y} - 2\cdot Y^{\frac{3}{2}}}}{\sqrt{\sqrt{Y}\cdot(X-Y)}} = 0$$

$$JM - \frac{\sqrt{2\cdot Y^{\frac{3}{2}} - 2\cdot X\cdot\sqrt{Y} - \sqrt{Y-X}\cdot\sqrt{2\cdot X\cdot Y - 3\cdot X^2 + Y^2}}}{2\cdot\sqrt{\sqrt{Y}\cdot(Y-X)}} = 0$$




Unit = 1.00000

XY = 0.60000

X = 12.00000

Y = 20.00000

AX = 1.40000

m∠EDB = 49.81637°

m∠RSH = 33.21091°

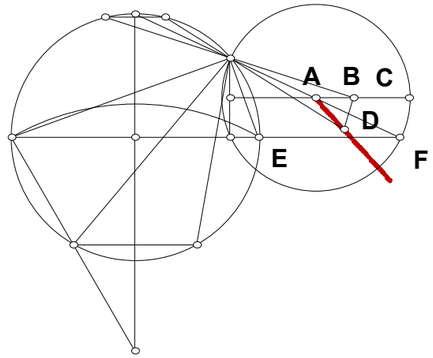
$\frac{m\angle EDB}{m\angle RSH} = 1.50000$

$$\frac{\sqrt{2\cdot Y^{\frac{3}{2}} - 2\cdot X\cdot\sqrt{Y} - \sqrt{Y-X}\cdot\sqrt{(2\cdot X\cdot Y - 3\cdot X^2) + Y^2}}}{2\cdot\sqrt{\sqrt{Y}\cdot(Y-X)}} - JM = 0.00000$$

## 022803

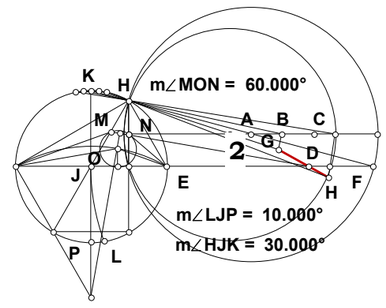
**If D were between EF, then it would be the point sought for angle trisection. When it is moved between A and C the locus AD is formed. This locus is not straight, but it is fairly straight.**

## Fair Pencil Construction

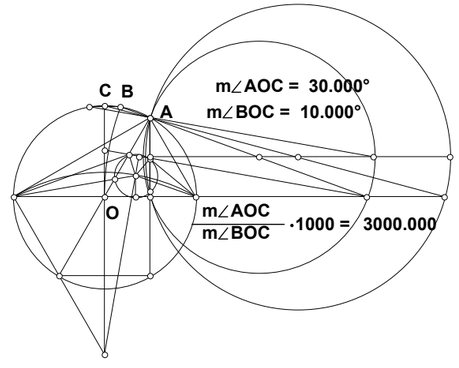



**Even if a large segment is used from two points on AD, say GH for an intercept, this drawing program claims that I have attained a trisection for any angle.**

**If a very small segment is taken, as below, tolerance is beyond the drawing program. What does it look like using Algebra? Trisection to within millionths, in some circumstances, may be tolerable.**




**As the different points of intersection is not actually viewable on the finer figure, the rougher figure will be used for drawings, but the equations will refer to the finer.**



**Descriptions.**

**Unit.**

$AC := 1$

**Given.**

$N_1 := 2$

$N_2 := 4$

$AO := \frac{AC}{2}$

$AB := \frac{AC}{N_2}$ $BC := AC - AB$

$BE := \sqrt{AB \cdot BC}$ $DE := \frac{BE}{2}$

$BO := AO - AB$ $GO := AO$

$DL := \frac{BO \cdot DE}{GO - BE}$ $EL := \sqrt{DL^2 + DE^2}$

$LN := EL$ $LM := \frac{LN}{N_1}$ $DM := DL + LM$ $EM := \sqrt{DE^2 + DM^2}$ $EP := \frac{EM \cdot BE}{DE}$

$BP := \frac{DM \cdot BE}{DE}$ $PO := BP + BO$

$EH := \frac{AO^2 + EP^2 - PO^2}{-EP}$

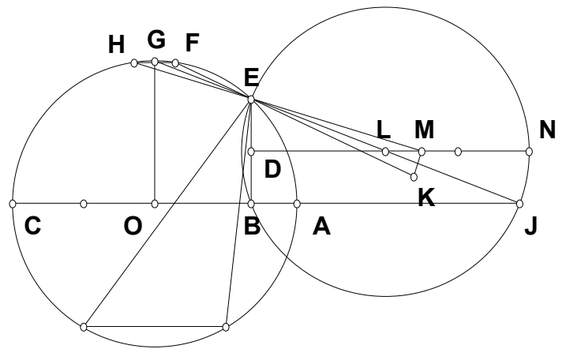
$HP := EP + EH$ $PR := \frac{BP \cdot HP}{EP}$

$OR := PR - PO$ $BQ := BO - OR$

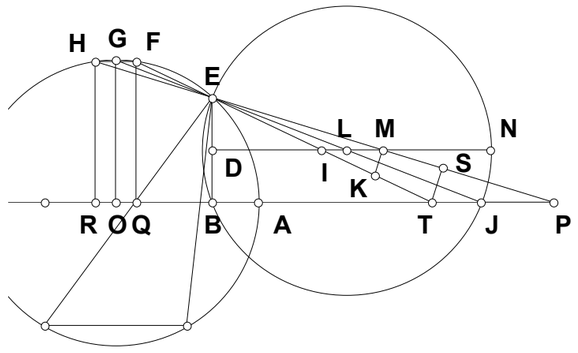
$HR := \frac{BE \cdot HP}{EP}$ $BT := \frac{BQ \cdot BE}{HR - BE}$

$ET := \sqrt{BE^2 + BT^2}$ $PT := BP - BT$ $PS := \frac{PT^2 + EP^2 - ET^2}{2 \cdot EP}$ $ES := EP - PS$ $ST := \sqrt{ET^2 - ES^2}$

$KM := \frac{ST \cdot EM}{ES}$ $EI := \frac{ET}{2}$ $EK := \frac{ET \cdot EM}{ES}$ $OT := BT + BO$




4




5

$$IW := EI DI := \sqrt{EI^2 - DE^2} \qquad DW := DI + IW$$

$$EW := \sqrt{DE^2 + DW^2} \qquad EZ := \frac{EW \cdot BE}{DE} \qquad BZ := \frac{DW \cdot BE}{DE}$$

$$OZ := BZ + BO \qquad TW := \sqrt{ET^2 - EW^2}$$

$$EX := \frac{AO^2 + EZ^2 - OZ^2}{-EZ} \qquad XZ := EZ + EX$$

$$Za := \frac{BZ \cdot XZ}{EZ} \qquad Oa := Za - OZ \qquad Bb := BO - Oa$$

$$Xa := \frac{BE \cdot XZ}{EZ} \qquad BU := \frac{Bb \cdot BE}{Xa - BE} \qquad EU := \sqrt{BE^2 + ]UZ\hat{}} := BZ - BU \qquad Zc := \frac{UZ^2 + EZ^2 - EU^2}{2 \cdot EZ}$$

$$Uc := \sqrt{UZ^2 - Zc^2} \qquad VW := \frac{Uc \cdot EW}{EZ - Zc} \qquad OU := BU + BO \qquad IM := DM - DI \quad IK := EK - EI$$

$$Ig := \frac{IK^2 + IM^2 - KM^2}{2 \cdot IM} \qquad EV := \frac{EU \cdot EW}{EZ - Zc} \qquad TU := OT - OU \quad UV := EV - EU$$

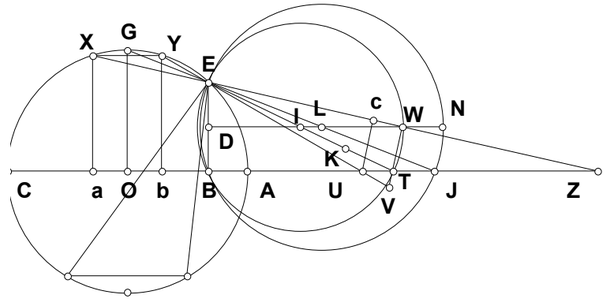
$$TV := VW - TW \qquad Uh := \frac{UV^2 + TU^2 - TV^2}{2 \cdot TU} \qquad Kg := \sqrt{IK^2 - Ig^2}$$

$$Ke := DE - Kg \qquad Vh := \sqrt{UV^2 - Uh^2} \qquad Dg := DI + Ig \qquad Bh := BU + Uh$$

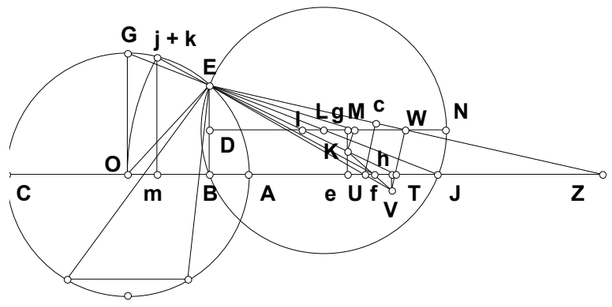
$$eh := Bh - Dg \qquad ef := \frac{eh \cdot Ke}{(Ke + Vh)} \qquad Bf := Dg + ef \qquad Of := Bf + BO$$

$$Om := \frac{AO^2}{2 \cdot Of} \quad Ej := \frac{AO^2}{Of} \qquad Om - \frac{Ej}{2} = 0 \qquad Ef := \sqrt{BE^2 + Bf^2} \qquad Ek := \frac{AO^2 + Ef^2 - Of^2}{-Ef}$$

$$Om - \frac{Ek}{2} = -2.639125 \times 10^{-12} \qquad \frac{Ek}{Ej} = 1$$




6




7

**And so the Trisection is accurate to within a few decimal places. 30 degree angle shown.**

**Unit.**

**Given.**

$\Delta := 22$

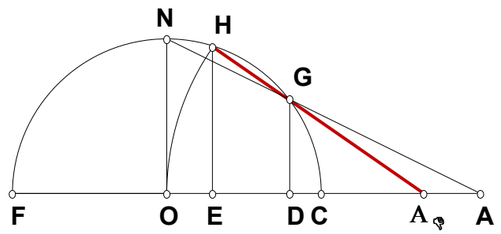
$\delta := 0 .. \Delta$

$N := 2$

**030503**

**Descriptions.**

# The Gravitating Answer.




**How many itterations to go beyond 15 decimal places precision in trisection? The itteration is from AO where GH determines a new A.**

$CF := 1 \quad CO := \frac{CF}{2}$

$CD := \frac{CO}{N} \quad DO := CO - CD$

$DF := DO + C \, DG := \sqrt{CD \cdot DF}$

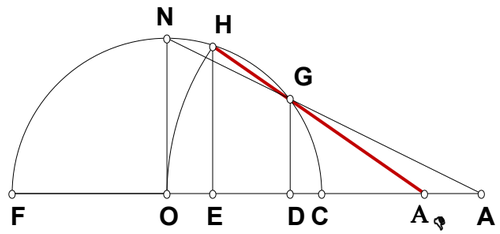
$AD_0 := \frac{DO \cdot DG}{CO - DG} \quad AO_0 := AD_0 + DO \quad EO_0 := \frac{CO^2}{2 \cdot (AD_0 + DO)} \quad DE_0 := DO - EO_0$

$EH_0 := \sqrt{(CO + EO_0) \cdot (CO - EO_0)}$

$$\begin{pmatrix} AD_{\delta+1} \\ AO_{\delta+1} \\ EO_{\delta+1} \\ DE_{\delta+1} \\ EF_{\delta+1} \\ EH_{\delta+1} \end{pmatrix} := \begin{bmatrix} \frac{DE_\delta \cdot DG}{EH_\delta - DG} \\ \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \\ \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ DO - \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ CO + \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ \sqrt{\left[ CO - \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right] \cdot \left[ CO + \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right]} \end{bmatrix}$$

$$AG := \sqrt{DG^2 + (AD_\Delta)^2}$$

$$GH := \frac{CO^2 + AG^2 - (AO_\Delta)^2}{-AG}$$




$$EO_\Delta - \frac{GH}{2} = 0$$

**The displayed precision is for 15 decimal places. Trisection is beyond that. Since the physical world is quantitized, physical trisection is possible.**

$$EO_\delta - \frac{GH}{2} =$$

| | $\delta =$ |
|---|---|
| -0.019836790725685 | 0 |
| -0.004498504834066 | 1 |
| -0.001019750164038 | 2 |
| -0.000231158851096 | 3 |
| -0.000052399460482 | 4 |
| -0.000011877993414 | 5 |
| -0.000002692522516 | 6 |
| -0.000000610345304 | 7 |
| -0.000000138354048 | 8 |
| -0.000000031362317 | 9 |
| -0.000000007109260 | 10 |
| -0.000000001611539 | 11 |
| -0.000000000365306 | 12 |
| ... | ... |

**One can see how rapidly each recursion increases precision. And so, for any required precision, one can trisect an angle grearter than that, relatively rapidly--especially if one combine yesterdays plate with today's.**

030803

Descriptions.

$$DO := \frac{DF}{2} \qquad Dx := \frac{DO}{N}$$

$$Ox := DO - Dx \qquad Fx := DF - Dx$$

$$Gx := \sqrt{Dx \cdot Fx} \qquad AO := \frac{Ox \cdot DO}{DO - Gx}$$

$$HO := DO \qquad LO := \frac{HO}{2}$$

$$Ex := \frac{Ox \cdot Gx}{Gx + DO} \qquad Ob := \frac{LO^2}{AO}$$

$$Lb := \sqrt{LO^2 - Ob^2} \qquad ab := \frac{Ex \cdot Lb}{Gx}$$

$$Aa := AO + ab - Ob$$

$$Ax := AO - Ox \qquad AE := Ax + Ex$$

$$Kc := \frac{Lb \cdot AE}{Aa} \qquad Cc := \frac{AO \cdot Kc}{DO}$$

$$Ec := \frac{ab \cdot AE}{Aa} \qquad CE := Cc + Ec \qquad AC := AE - CE$$

$$AB := \frac{AC}{2}$$

Unit.

$DF := 1$

Given.

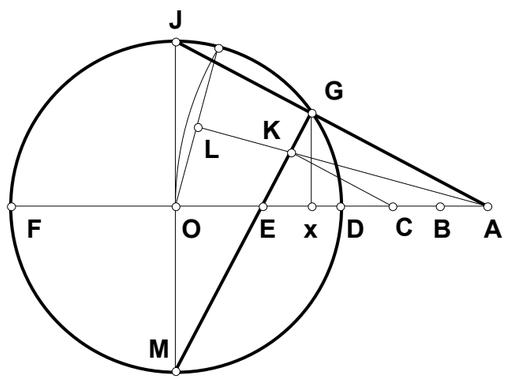
$N := 2$

$\Delta := 22$

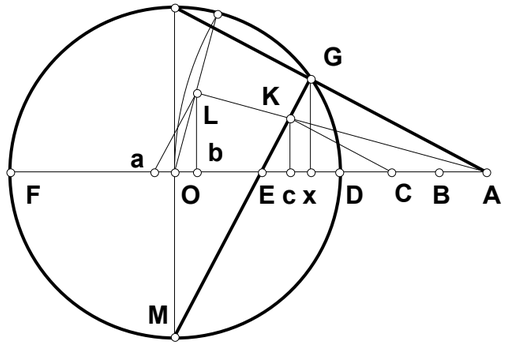
$\delta := 0 .. \Delta$

## Nothing Saved

**Is anything saved by starting from a much more precise point for itteration demonstrated in 0305?**




J

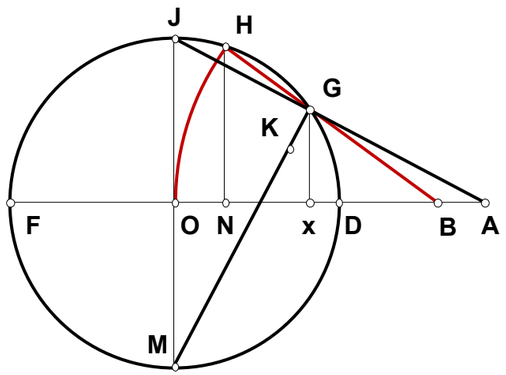


$$BO_0 := AO - AB$$

$$NO_0 := \frac{DO^2}{2\,BO_0}$$

$$Nx_0 := Ox - NO_0$$

$$HN_0 := \sqrt{(DO + NO_0)\cdot(DO - NO_0)}$$



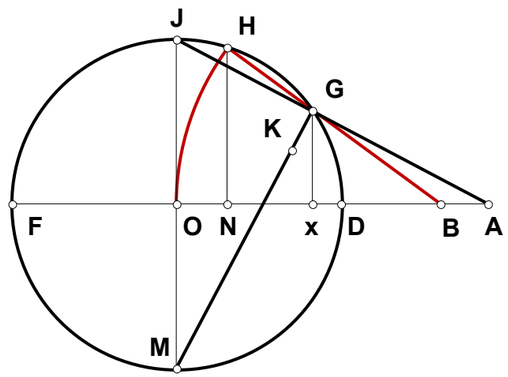

$$\begin{pmatrix} BO_{\delta+1} \\ NO_{\delta+1} \\ Nx_{\delta+1} \\ HN_{\delta+1} \end{pmatrix} := \begin{bmatrix} \frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta \\ \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ Ox - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ \sqrt{\left[DO + \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]\cdot\left[DO - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]} \end{bmatrix}$$

$$Bx_{\Delta} := BO_{\Delta} - Ox$$

$$BG := \sqrt{Gx^2 + (Bx_{\Delta})^2}$$

$$GH := \frac{DO^2 + BG^2 - (BO_{\Delta})^2}{-BG}$$

$$NO_{\Delta} - \frac{GH}{2} = 0$$




$NO_{\delta} - \frac{GH}{2} =$

| |
|---|
| -0.000733072475044 |
| -0.000166174134206 |
| -0.000037668619212 |
| -0.000008538782769 |
| -0.000001935584914 |
| -0.000000438761479 |
| -0.000000099459153 |
| -0.000000022545559 |
| -0.000000005110663 |
| -0.000000001158493 |
| -0.000000000262609 |
| -0.000000000059529 |
| -0.000000000013494 |
| ... |

$\delta =$

| |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| ... |

**Although one starts off in a more precise spot, not much in the way of steps for 15 decimal place precision is saved. The steps are a waste of time.**

**031503**

**Descriptions.**

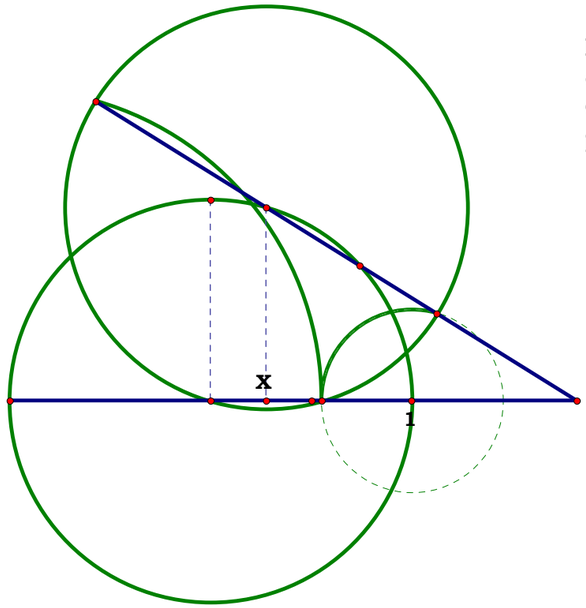
**This figure may appear to many to be very counter intuitive, so I will show the construction step by step. I think way back then is the only other time I drew this. I have lost count on how many ways one can actually draw a figure demonstrating trisection. Why the so called intellectuals still claim it is impossible is way beyond me and my talent.**

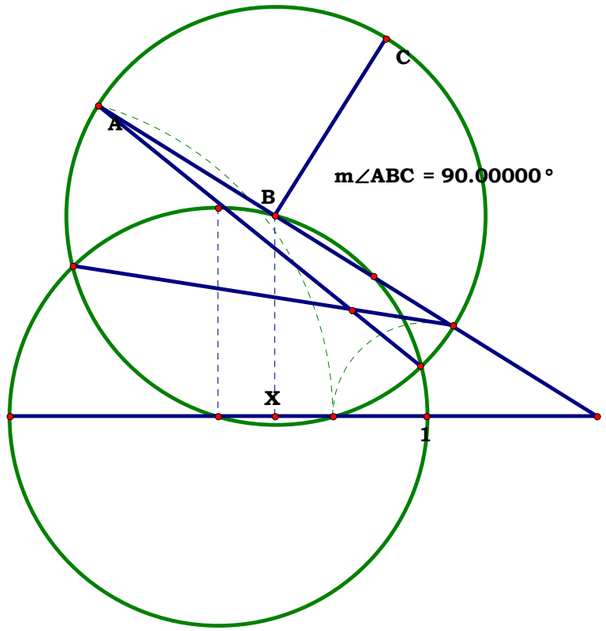
**X will range between the center and first half of that segment giving one 30 working degrees.**

**From that point I will draw the second circle to the center of the first which will produce a second point of intersection with the diameter. From that point I will draw two more circles to produce the line that terminates on the base like. I immediately have my square root point of the figure and I have the point**

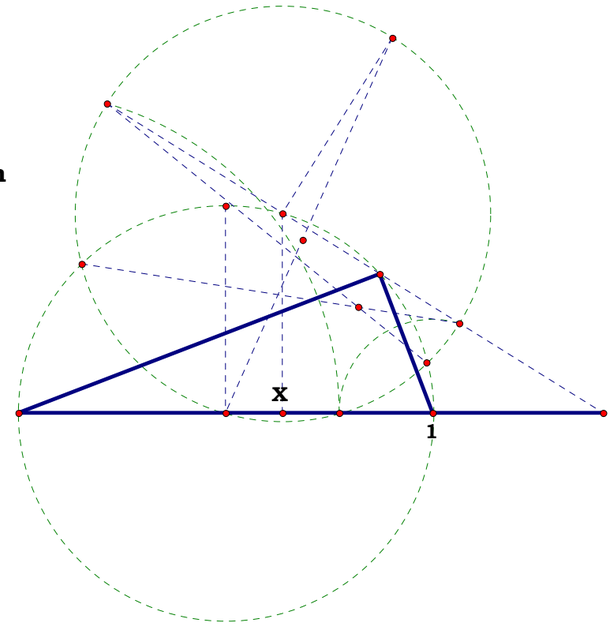
**I will construct two lines with opposing endpoints which will produce a point which is suspended in the air it seems.**

**Trisection and Square Roots**

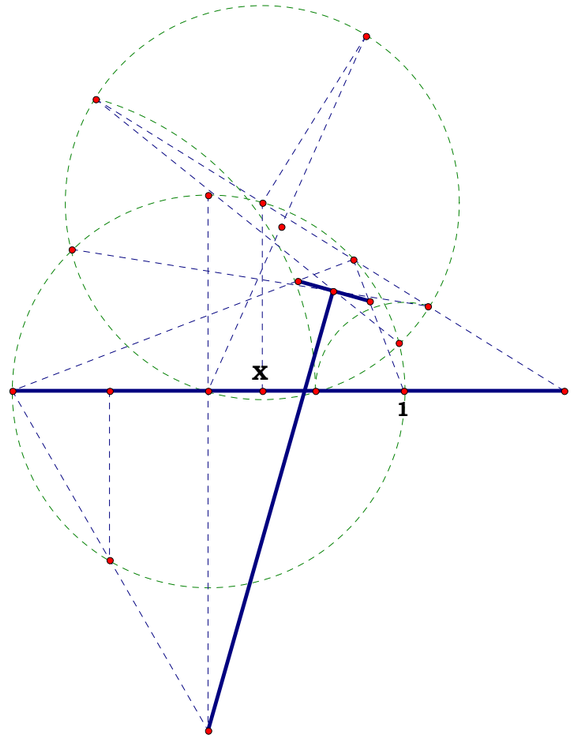



**I will use the second point of intersetion of top line to construct a right triangle. One might say a tent for my hanging point.**

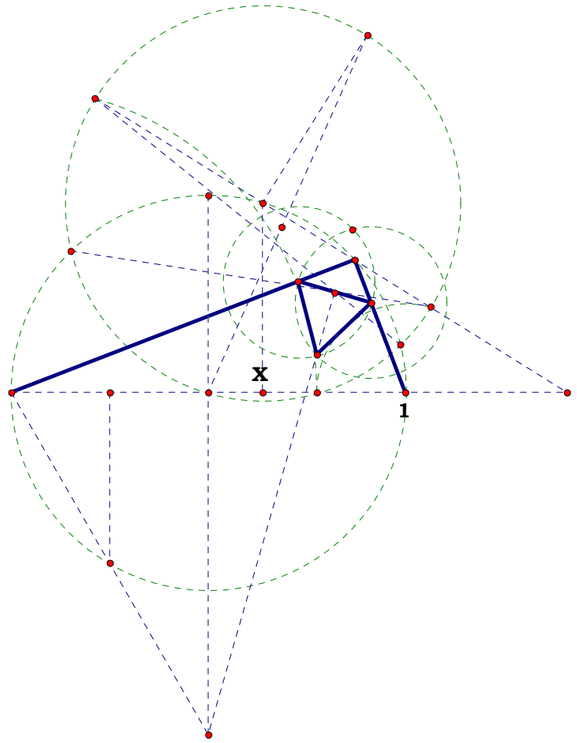



**From the point we draw a 60 degree arc from I am going to make a segment with our floating point and form a perpendicular limited by our tent so that we have a very skinny long T.**



**Descriptions.**

**With that skinny long T, we are going to make an equilateral triangle.**

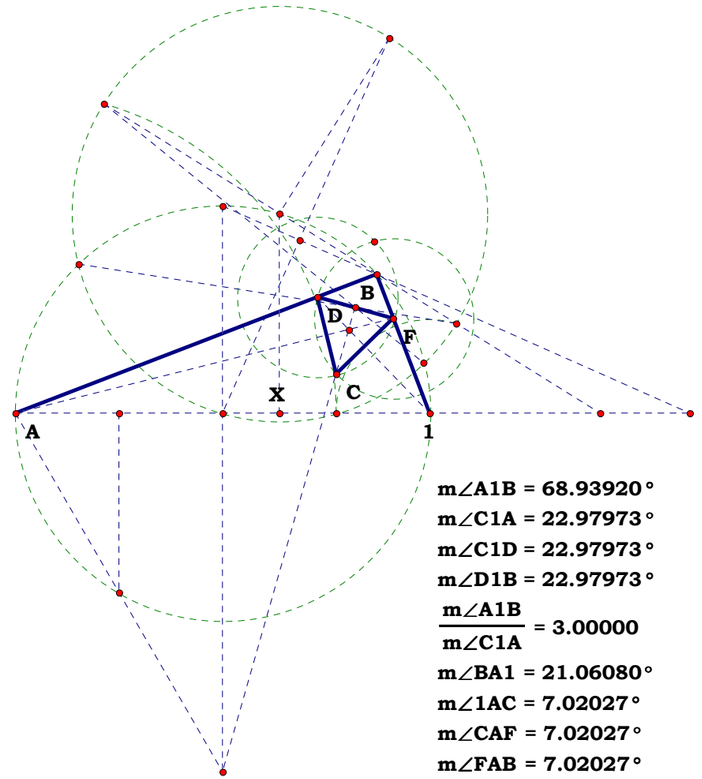



**That equilateral triangle is the triangle in a figure that demonstrates angle trisection.**

**And as you have seen for yourself, it just fell out of the sky.**

**I will not make another plate with all the dressing. One may say, this is just lunch to go.**

**And so, angle trisection is actually a well documented part of geometry, all one has to do is find the write ups at the end of their instrument of choice.**




$m\angle A1B = 68.93920°$
$m\angle C1A = 22.97973°$
$m\angle C1D = 22.97973°$
$m\angle D1B = 22.97973°$
$\frac{m\angle A1B}{m\angle C1A} = 3.00000$
$m\angle BA1 = 21.06080°$
$m\angle 1AC = 7.02027°$
$m\angle CAF = 7.02027°$
$m\angle FAB = 7.02027°$
$\frac{m\angle BA1}{m\angle 1AC} = 3.00000$

031503

**Descriptions.**

**Unit.**

$BO := 1$

**Given.**

$Y := 20$

$X := 11$

$AB := 2 \cdot BO$ $NO := \frac{X}{2 \cdot Y}$ $EO := BO$ $AO := BO$

$FO := \frac{EO}{2}$ $GO := \frac{NO}{2}$ $FG := \sqrt{FO^2 - GO^2}$

$CO := \frac{FO^2}{GO}$ $CO = 1.818182$ $AC := AO + CO$

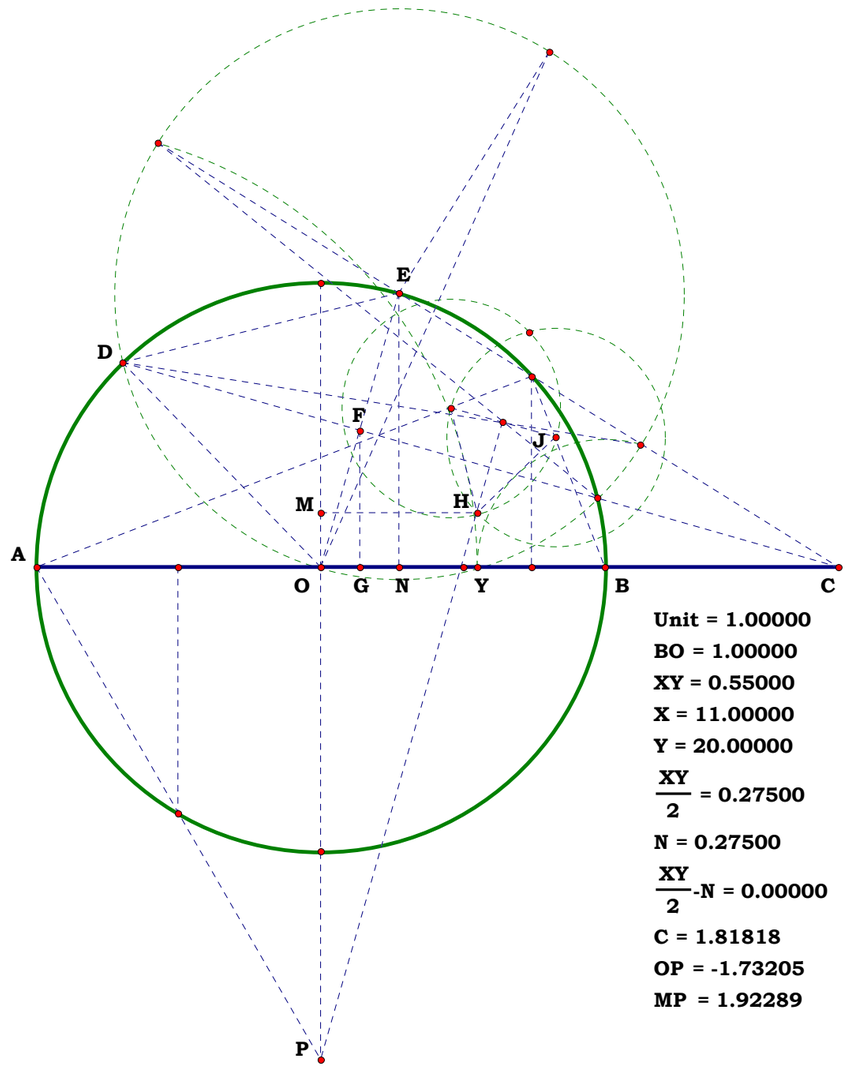
$OY := 2 \cdot NO$ $HP := AB$ $OP := \sqrt{AB^2 - BO^2}$

$OP = 1.732051$ $MP := \sqrt{AB^2 - OY^2}$ $MP = 1.922888$

$MO := MP - OP$ $MO = 0.190838$ $HJ := 2 \cdot MO$

$CY := CO - OY$ $\frac{AC}{CY} = 2.222222$ $AY := AO + OY$

$\frac{AC}{OY} = 5.123967$ $\frac{AB}{AY} = 1.290323$ etc.




Unit = 1.00000
BO = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000
$\frac{XY}{2} = 0.27500$
N = 0.27500
$\frac{XY}{2}$-N = 0.00000
C = 1.81818
OP = -1.73205
MP = 1.92289

**Definitions.**

$AB - 2 = 0$ $NO - \frac{X}{2 \cdot Y} = 0$ $EO - 1 = 0$ $AO - 1 = 0$ $FO - \frac{1}{2} = 0$

$GO - \frac{X}{4 \cdot Y} = 0$ $FG - \frac{\sqrt{4 \cdot Y^2 - X^2}}{4 \cdot Y} = 0$ $CO - \frac{Y}{X} = 0$ $AC - \frac{X + Y}{X} = 0$

$OY - \frac{X}{Y} = 0$ $HP - 2 = 0$ $OP - \sqrt{3} = 0$ $MP - \frac{\sqrt{4 \cdot Y^2 - X^2}}{Y} = 0$ $MO - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$ $HJ - 2 \cdot \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$

$CY - \frac{Y^2 - X^2}{X \cdot Y} = 0$ $AY - \frac{X + Y}{Y} = 0$ $\frac{AC}{CY} - \frac{Y}{(Y - X)} = 0$ $\frac{AC}{OY} - \frac{Y \cdot (X + Y)}{X^2} = 0$ $\frac{AB}{AY} - \frac{2 \cdot Y}{X + Y} = 0$

Unit.

$AB := 1$

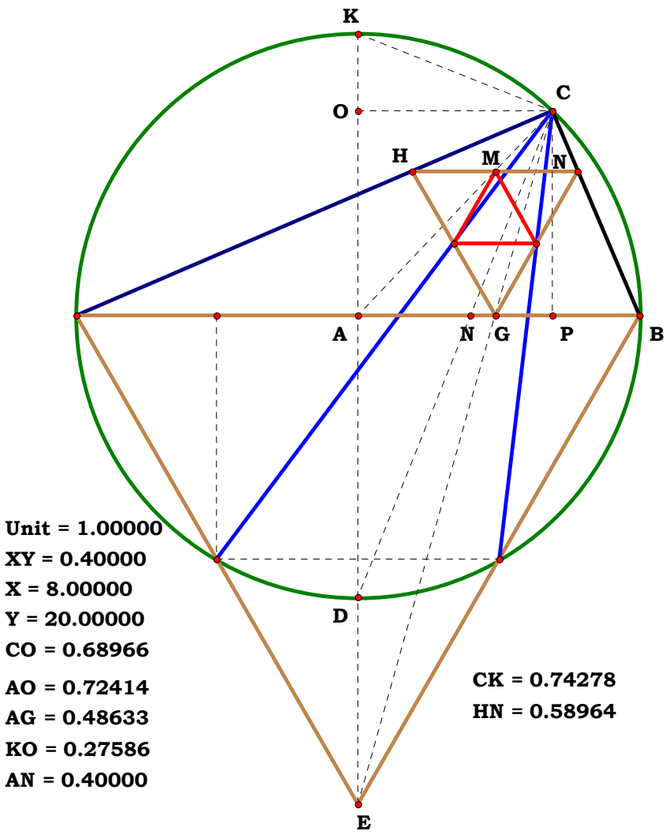
Given.

$Y := 20$

$X := 8$

032303

**Figure in a Figure.**




**Descriptions.**

$AN := \frac{X}{Y}$ $AD := AB$ $DN := \sqrt{AD^2 + AN^2}$ $CK := \frac{AN \cdot 2 \cdot AD}{DN}$

$KO := \frac{(AN \cdot CK)}{DN}$ $CO := \frac{AD \cdot KO}{AN}$ $AP := CO$ $AO := AB - KO$

$AK := \sqrt{3}$ $HN := \frac{2 \cdot AB \cdot AO}{AK + AO}$ $HN = 0.589644$

$AG := \frac{CO \cdot AK}{AK + AO}$ $AG = 0.48633$ etc.

**Definitions.**

$AN - \frac{X}{Y} = 0$ $AD - 1 = 0$ $DN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$ $CK - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$

$KO - \frac{2 \cdot X^2}{X^2 + Y^2} = 0$ $CO - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $AP - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$

$AO - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$ $AK - \sqrt{3} = 0$ $HN - \frac{2 \cdot (Y^2 - X^2)}{(X^2 + Y^2) \cdot (\sqrt{3} - 1) + 2 \cdot Y^2} = 0$

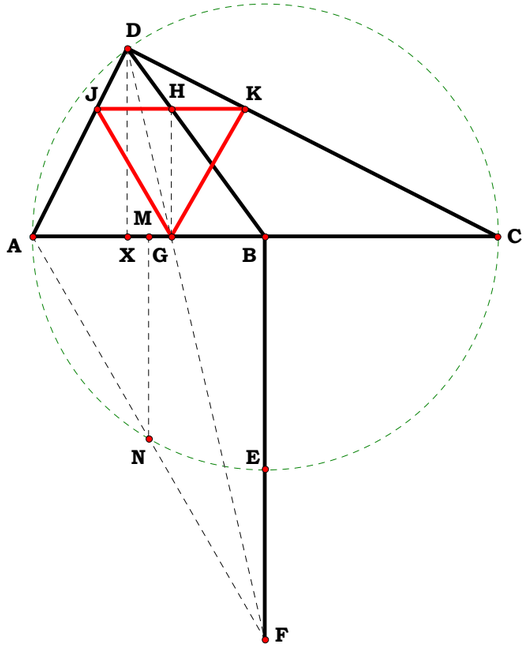
$AG - \frac{2 \cdot \sqrt{3} \cdot X \cdot Y}{(X^2 + Y^2) \cdot (\sqrt{3} - 1) + 2 \cdot Y^2} = 0$ etc.

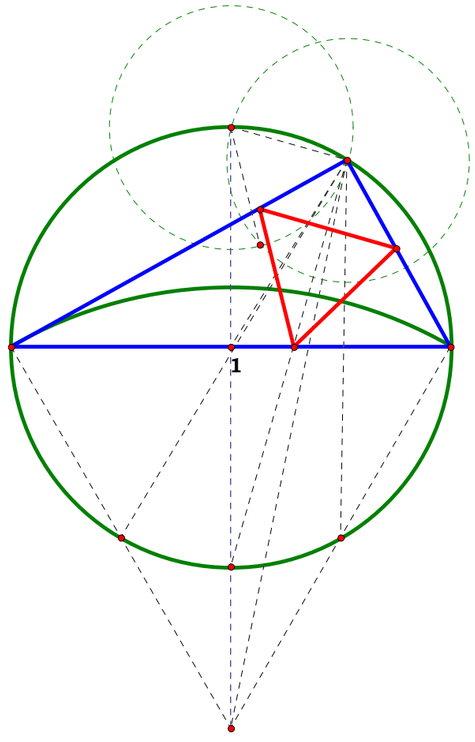
032403

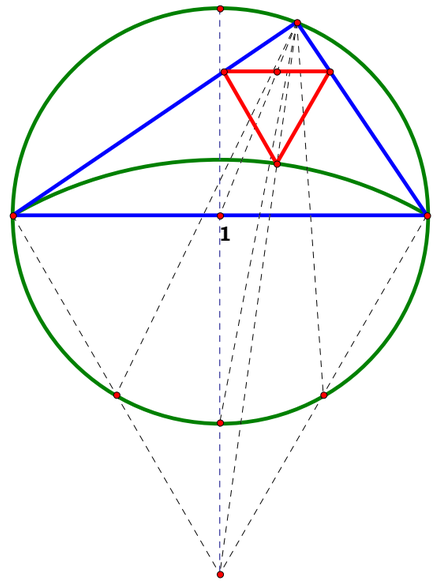
# Four Siblings

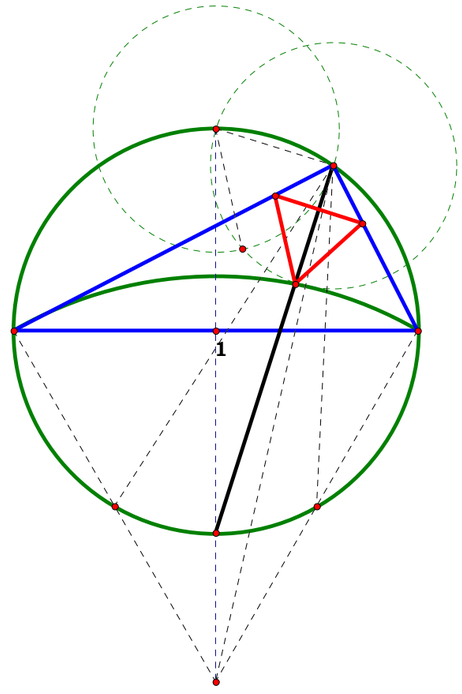
**When looking for the equalateral triangle which produces trisection in a right angle, we may come to meet its four lazy siblings.**

$$\frac{4\cdot Y\cdot\sqrt{X\cdot(Y-X)}}{Y\cdot\left(2\cdot\sqrt{X\cdot(Y-X)}+\sqrt{3}\cdot Y\right)}-JK = 0.00000$$



Unit. $AB := 1$

**Descriptions.**

$AD := \frac{AB}{2}$ $DF := \sqrt{AD \cdot (2 \cdot AB - AD)}$ $BH := AD \cdot \frac{AB}{DF}$

$AH := AB + BH$ $AE := \frac{AB^2}{AH}$ $BE := AB - AE$ $EG := AE$

$BG := \sqrt{EG^2 + BE^2}$

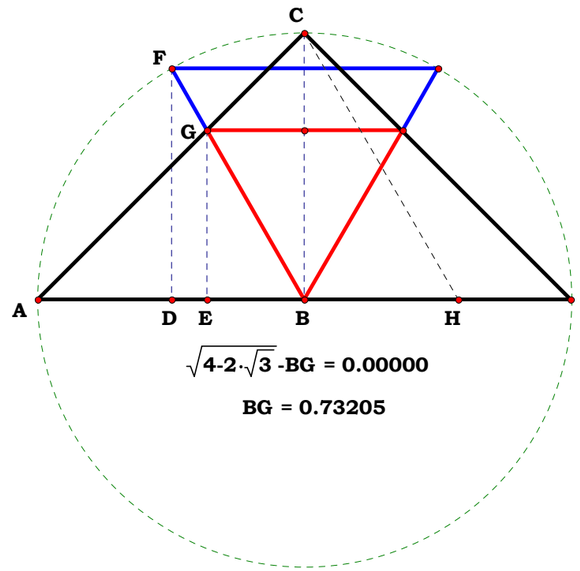
**Definitions.**

$AD - \frac{1}{2} = 0$ $DF - \frac{\sqrt{3}}{\sqrt{4}} = 0$ $BH - \frac{\sqrt{3}}{3} = 0$

$AH - \left(\frac{\sqrt{3}}{3} + 1\right) = 0$ $AE - \left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right) = 0$

$BE - \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 0$ $EG - \left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right) = 0$

$BG - \sqrt{4 - 2 \cdot \sqrt{3}} = 0$



032403A

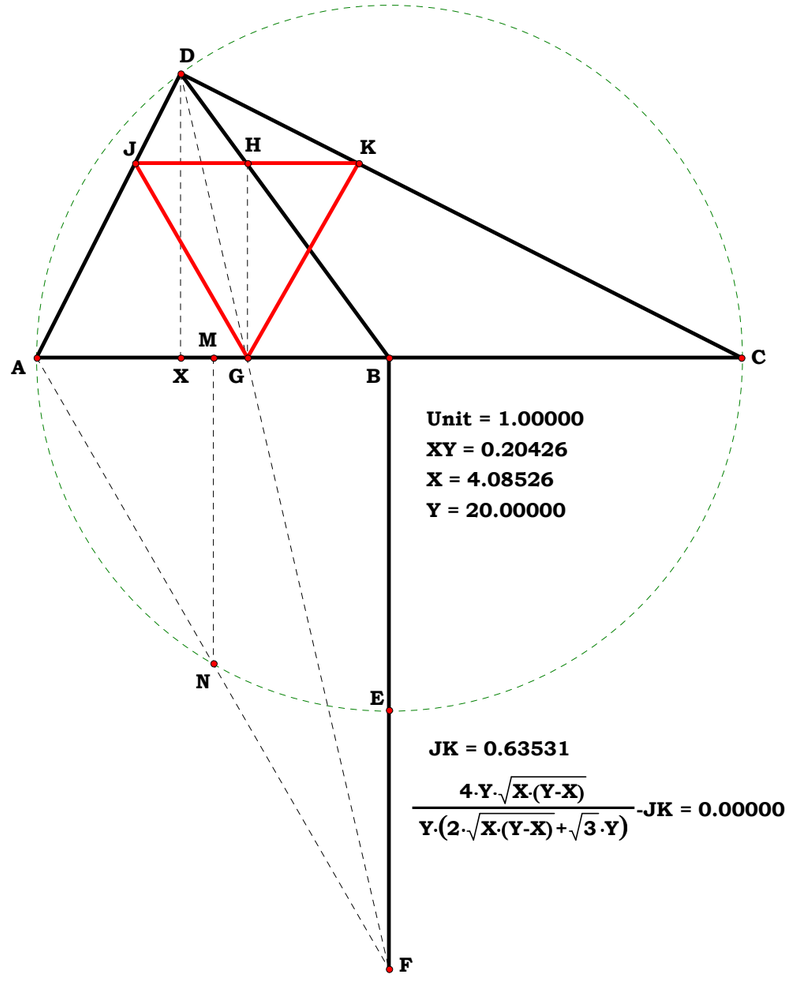
Unit. $AB := 1$

Given. $X := 15$

$Y := 20$

## Four Siblings Plate A

**When looking for the equalateral triangle which produces trisection in a right angle, we may come to meet its four lazy siblings.**




**Descriptions.**

$AC := 2 \cdot AB \quad AM := \frac{AB}{2} \quad CM := AC - AM \quad MN := \sqrt{AM \cdot CM}$

$BF := 2 \cdot MN \quad AX := AC \cdot \frac{X}{Y} \quad CX := AC - AX \quad DX := \sqrt{AX \cdot CX}$

$BX := AX - AB \quad BG := BX \cdot \frac{BF}{BF + DX} \quad GH := DX \cdot \frac{BG}{BX}$

$JK := AC \cdot \frac{DX - GH}{DX}$

**Definitions.**

$AC - 2 = 0 \quad AM - \frac{1}{2} = 0 \quad CM - 3 \cdot 2^{-1} = 0$

$MN - \sqrt{\left(3 \cdot 2^{-2}\right)} = 0 \quad BF - \sqrt{3} = 0 \quad AX - 2 \cdot \frac{X}{Y} = 0$

$CX - \frac{2 \cdot (Y - X)}{Y} = 0 \quad DX - \frac{2 \cdot \sqrt{X \cdot (Y - X)}}{Y} = 0 \quad BX - \frac{2 \cdot X - Y}{Y} = 0$

$BG - \frac{\sqrt{3} \cdot (2 \cdot X - Y)}{2 \cdot \sqrt{X \cdot (Y - X)} + \sqrt{3} \cdot Y} = 0 \quad GH - \frac{2 \cdot \sqrt{3} \cdot \sqrt{X \cdot (Y - X)}}{2 \cdot \sqrt{X \cdot (Y - X)} + \sqrt{3} \cdot Y} = 0$

$JK - \frac{4 \cdot Y \cdot \sqrt{X \cdot (Y - X)}}{Y \cdot \left[2 \cdot \sqrt{X \cdot (Y - X)} + \sqrt{3} \cdot Y\right]} = 0$

032403B

Unit. $AB := 1$

Given.

$X := 12$

$Y := 17$

# Four Siblings Plate B

**Descriptions.**

$AC := 2 \cdot AB \qquad AX := AC \cdot \frac{X}{Y} \qquad BX := AX - AB$

$GX := \sqrt{AB^2 + BX^2} \qquad DF := BX \cdot \frac{AC}{GX} \qquad FH := BX \cdot \frac{DF}{GX}$

$BH := AB - FH \qquad DX := \frac{GX \cdot BH}{AB}$

$JK := DX \cdot \sqrt{4 - 2 \cdot \sqrt{3}}$

**Definitions.**

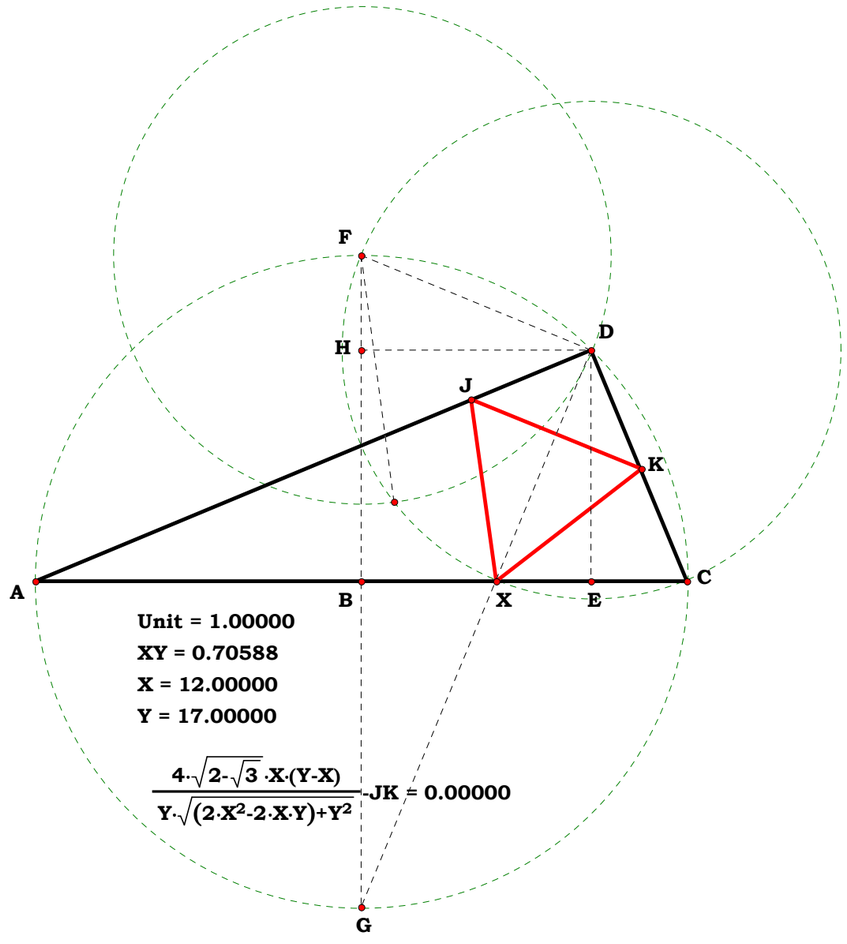
$AC - 2 = 0 \qquad AX - 2 \cdot \frac{X}{Y} = 0 \qquad BX - \frac{2 \cdot X - Y}{Y} = 0$

$GX - \frac{\sqrt{2 \cdot \left(2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2\right)}}{Y} = 0$

$DF - \frac{\sqrt{2} \cdot (2 \cdot X - Y)}{\sqrt{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2}} = 0 \qquad FH - \frac{(2 \cdot X - Y)^2}{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2} = 0$

$BH - \frac{2 \cdot X \cdot (Y - X)}{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2} = 0 \qquad DX - \frac{2 \cdot \sqrt{2} \cdot X \cdot (Y - X)}{Y \cdot \sqrt{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2}} = 0$

$JK - \frac{4 \cdot \sqrt{2 - \sqrt{3}} \cdot X \cdot (Y - X)}{Y \cdot \sqrt{2 \cdot X^2 - 2 \cdot X \cdot Y + Y^2}} = 0$



032403C

Unit. $AB := 1$

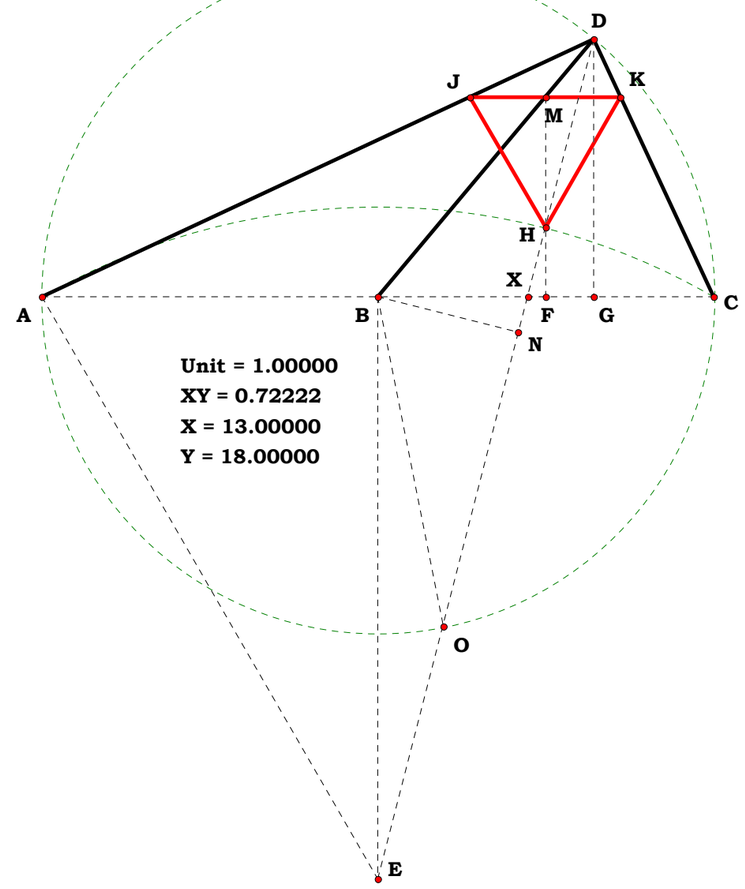
Given.

$X := 13$

$Y := 18$

## Four Siblings Plate C

$$2\cdot\frac{\sqrt{(5\cdot Y^2-4\cdot X^2)+3\cdot Y\cdot\sqrt{(8\cdot X\cdot Y-8\cdot X^2)+Y^2}+4\cdot X\cdot Y}-2\cdot\sqrt{2}\cdot\sqrt{(X^2-X\cdot Y)+Y^2}}{\sqrt{(5\cdot Y^2-4\cdot X^2)+3\cdot Y\cdot\sqrt{(8\cdot X\cdot Y-8\cdot X^2)+Y^2}+4\cdot X\cdot Y}}\text{-}JK = 0.00000$$




**Descriptions.**

$AC := 2\cdot AB$ $AE := AC$ $BE := \sqrt{3}$ $AX := AC\cdot\frac{X}{Y}$

$BX := AX - AB$ $EX := \sqrt{BE^2 + BX^2}$ $BN := BE\cdot\frac{BX}{EX}$

$NX := \frac{BX^2}{EX}$ $NO := \sqrt{AB^2 - BN^2}$ $DE := EX + NO - NX$

$DX := DE - EX$ $DG := BE\cdot\frac{DX}{EX}$ $BG := \sqrt{AB^2 - DG^2}$

$HX := AE - EX$ $FX := BX\cdot\frac{HX}{EX}$ $BF := BX + FX$

$BM := AB\cdot\frac{BF}{BG}$ $DM := AB - BM$ $JK := 2\cdot DM$

**Definitions.**

$AC - 2 = 0$ $AE - 2 = 0$ $BE - \sqrt{3} = 0$ $AX - 2\cdot\frac{X}{Y} = 0$

$BX - \frac{2\cdot X - Y}{Y} = 0$ $EX - \frac{2\cdot\sqrt{X^2 - X\cdot Y + Y^2}}{Y} = 0$ $BN - \frac{\sqrt{3}\cdot(2\cdot X - Y)}{2\cdot\sqrt{X^2 - X\cdot Y + Y^2}} = 0$

$NX - \frac{(2\cdot X - Y)^2}{2\cdot Y\cdot\sqrt{X^2 - X\cdot Y + Y^2}} = 0$ $NO - \frac{\sqrt{8\cdot X\cdot Y - 8\cdot X^2 + Y^2}}{2\cdot\sqrt{X^2 - X\cdot Y + Y^2}} = 0$

$DE - \frac{3\cdot Y + \sqrt{8\cdot X\cdot Y - 8\cdot X^2 + Y^2}}{2\cdot\sqrt{X^2 - X\cdot Y + Y^2}} = 0$ $DX - \frac{Y\cdot\sqrt{8\cdot X\cdot Y - 8\cdot X^2 + Y^2} - Y^2 - 4\cdot X^2 + 4\cdot X\cdot Y}{2\cdot Y\cdot\sqrt{X^2 - X\cdot Y + Y^2}} = 0$

$$DG - \frac{\sqrt{3}\cdot\left(Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}-Y^2-4\cdot X^2+4\cdot X\cdot Y\right)}{4\cdot\left(X^2-X\cdot Y+Y^2\right)}=0$$

$$HX - \frac{2\cdot\left(Y-\sqrt{X^2-X\cdot Y+Y^2}\right)}{Y}=0$$

$$BG - \frac{(2\cdot X-Y)\cdot\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}}{2\cdot\sqrt{2}\cdot\left(X^2-X\cdot Y+Y^2\right)}=0$$

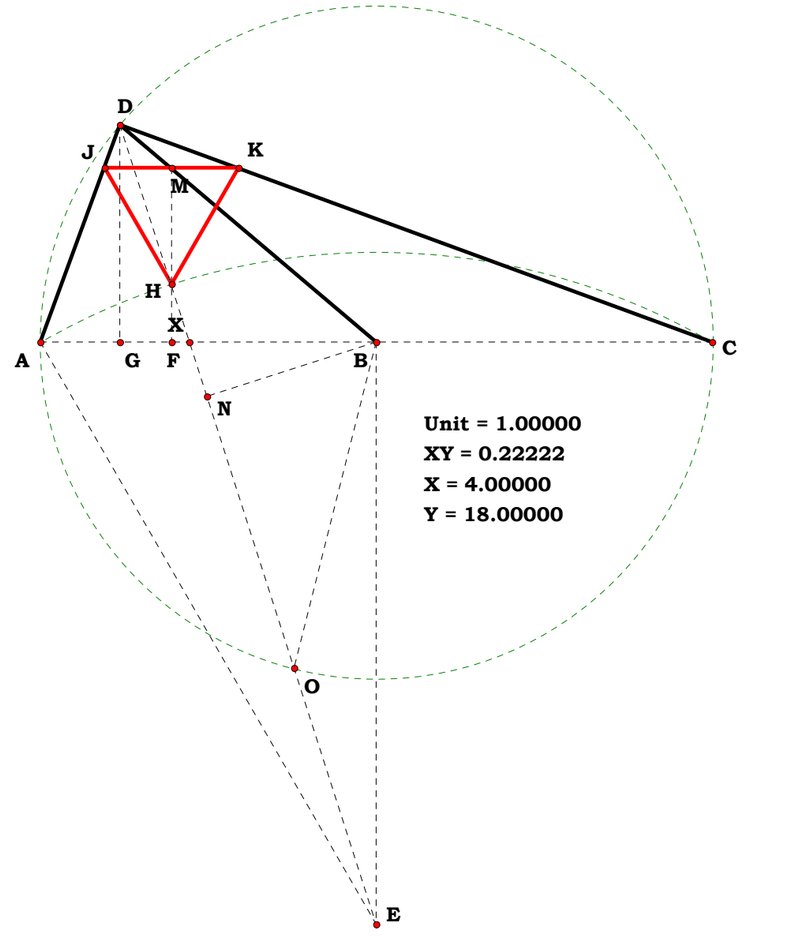
$$FX - \frac{(2\cdot X-Y)\cdot\left(Y-\sqrt{X^2-X\cdot Y+Y^2}\right)}{Y\cdot\sqrt{X^2-X\cdot Y+Y^2}}=0 \qquad BF - \frac{2\cdot X-Y}{\sqrt{X^2-X\cdot Y+Y^2}}=0$$

$$BM - \frac{2\cdot\sqrt{2}\cdot\sqrt{X^2-X\cdot Y+Y^2}}{\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}}=0$$

$$DM - \frac{\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}-2\cdot\sqrt{2}\cdot\sqrt{X^2-X\cdot Y+Y^2}}{\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}}=0$$

$$JK - 2\cdot\frac{\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}-2\cdot\sqrt{2}\cdot\sqrt{X^2-X\cdot Y+Y^2}}{\sqrt{5\cdot Y^2-4\cdot X^2+3\cdot Y\cdot\sqrt{8\cdot X\cdot Y-8\cdot X^2+Y^2}+4\cdot X\cdot Y}}=0$$

$$2\cdot\frac{\sqrt{(5\cdot Y^2-4\cdot X^2)+3\cdot Y\cdot\sqrt{(8\cdot X\cdot Y-8\cdot X^2)+Y^2}+4\cdot X\cdot Y}-2\cdot\sqrt{2}\cdot\sqrt{(X^2-X\cdot Y)+Y^2}}{\sqrt{(5\cdot Y^2-4\cdot X^2)+3\cdot Y\cdot\sqrt{(8\cdot X\cdot Y-8\cdot X^2)+Y^2}+4\cdot X\cdot Y}}\text{-JK = 0.00000}$$



**Unit.**
**Given.**

**040903**

**PacMan**
**Animation writeup**

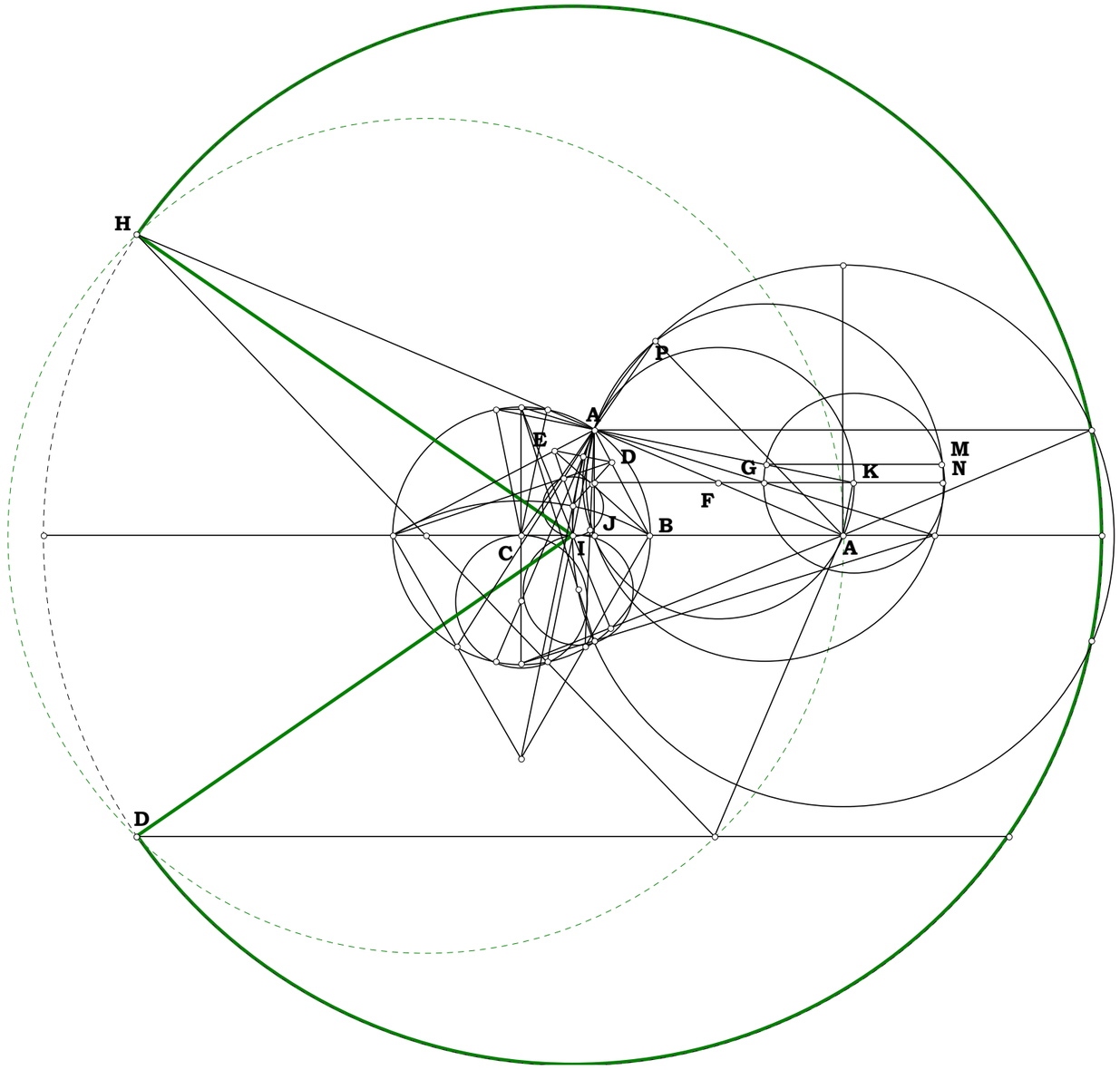
**Descriptions.**
**Definitions.**

**So called Fractal Geometry is concerned with the recursion of the perceptible, however it does not compare much with the recursion of an intelligible. And so, one may find, in the recursion a hidden message, from the impossible to packman, it is all just playing one tune, binary recursion is binary recursion.**

Animate

$m\angle FAK = 11.534°$
$m\angle ABC = 62.301°$
$m\angle AGJ = 34.601°$
$\frac{m\angle AGJ}{m\angle FAK} = 3.000$
$m\angle MKN = 11.534°$
$m\angle DBE = 20.767°$
$\frac{m\angle ABC}{m\angle DBE} = 3.000$
$m\angle HID = 69.203°$
$m\angle PAA = 23.068°$
$\frac{m\angle HID}{m\angle PAA} = 3.000$

**Eaten by HID or PakMan?**



**Unit.**
**Given.**
**Descriptions.**
**Definitions.**

**Lardner was wholly unaware that the problem actually descibes an ellipse which means that it is not indeterminate at all.**

## An Indeterminate Problem Reduced To An Equation

**Page 5 of *A Treatise on Algebraic Geometry* by Rev. Dionysius Lardner, 1831** **072903**

**Given the base AB, and the sum of the sides (AC and BC) of a triangle, to find the vertex (C).**
**Let AB = *a*, AC = *y*, and CB = *x*, and the the excess of the sum of the sides above the base be *d*.**

$\therefore\ y + x = a + b.$

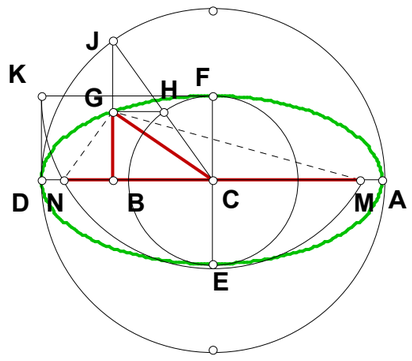
**Any values of *y* and *x*, which fulfill the conditions of this equation, represent the sides of the triangle, whose vertex solves the problem.**

---

**Perhaps this problem is indeterminate is because the author did not have a clue that a point (C) is not a magnitude. How does one find a non-magnitude from magnitues? Only by establishing a co-ordinate system.**

### 09/11/97 The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**




$N_1 := 3 \quad N_2 := 4$

$AD := 1 \quad EF := \frac{AD}{N_1} \quad AB := \frac{AD}{N_2}$

$BD := AD - AB \quad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \quad BC := AC - AB \quad CH := \frac{EF}{2}$

$CJ := AC \quad BG := \frac{BJ \cdot CH}{CJ}$

$CG := \sqrt{BG^2 + BC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$\frac{\sqrt{4 \cdot N_2 - 4 + N_2^2 \cdot N_1^2 - 4 \cdot N_2 \cdot N_1^2 + 4 \cdot N_1^2}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{\sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0 \qquad \frac{\sqrt{(N_1^2 - 1)}}{N_1} - MN = 0$$

120103A

**Unit.**

**Given.** $N := 1.454$

$AD := 3.073$

**Descriptions.**

$DH := AD \qquad BD := \frac{AD}{N} \qquad BH := \sqrt{BD^2 + DH^2} \qquad HI := \frac{BH}{2}$

$EH := \frac{BH \cdot HI}{DH} \qquad DE := DH - EH \qquad DF := 2 \cdot DE \qquad FH := DH - DF$

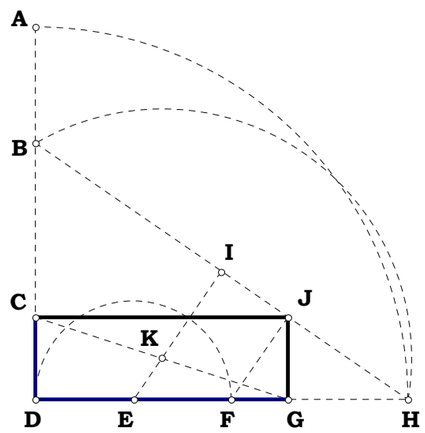
$HJ := \frac{DH \cdot FH}{BH} \qquad GJ := \frac{BD \cdot HJ}{BH} \qquad GH := \frac{DH \cdot HJ}{BH} \qquad DG := DH - GH$

$CD := GJ \qquad BC := BD - CD \qquad \frac{DG}{BC} - \frac{BC}{GH} = 0 \qquad \frac{BC}{GH} - \frac{GH}{GJ} = 0$

$\left(\frac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} - \frac{DG}{BC} = 0 \qquad \frac{AD}{N^3 + N} - GJ = 0 \qquad \frac{AD \cdot N^2}{N^2 + 1} - DG = 0$

$DG + DG^{\left(\frac{1}{3}\right)} \cdot GJ^{\left(\frac{2}{3}\right)} - AD = 0 \qquad \frac{DG \cdot N^2 + DG}{N^2} - AD = 0$

$N - \left(\frac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} = 0 \qquad \frac{AD}{N} = 2.11348 \qquad \frac{DG + DG^{\left(\frac{1}{3}\right)} \cdot GJ^{\left(\frac{2}{3}\right)}}{\left(\frac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)}} = 2.11348 \qquad DG^{\left(\frac{2}{3}\right)} \cdot GJ^{\left(\frac{1}{3}\right)} + GJ = 2.11348$




$AD = 2.45833$ in.

$AB = 0.76777$ in.

$DB = 1.69056$ in.

$\frac{AD}{DB} = 1.45415$

$BD = 1.69056$ in.

$EH = 1.81046$ in.

$HJ = 0.95793$ in.

$DG = 1.66903$ in.

$GJ = 0.54280$ in.

$\frac{DG}{GJ} = 3.07487$

120103B

**Unit.**

**Given.** $AD := 2.458$

$AB := .768$

**Descriptions.**

$DH := AD \quad BD := AD - AB$

$BH := \sqrt{BD^2 + DH^2} \quad HI := \frac{BH}{2} \quad EH := \frac{BH \cdot HI}{DH} \quad DE := DH - EH$

$DF := 2 \cdot DE \quad FH := DH - DF \quad HJ := \frac{DH \cdot FH}{BH} \quad GJ := \frac{BD \cdot HJ}{BH}$

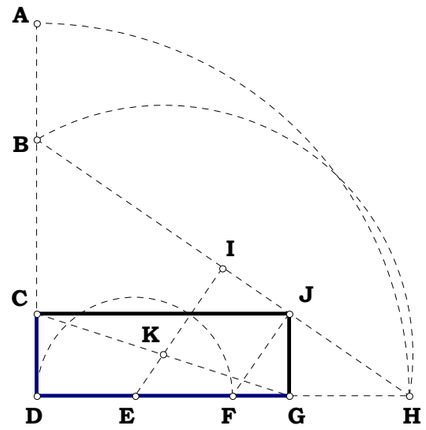
$GH := \frac{DH \cdot HJ}{BH} \quad DG := DH - GH \quad CD := GJ \quad BC := BD - CD$

$\frac{DG}{BC} - \frac{BC}{GH} = 0 \quad \frac{BC}{GH} - \frac{GH}{GJ} = 0 \quad \left(\frac{DG}{GJ}\right)^{\left(\frac{1}{3}\right)} - \frac{DG}{BC} = 0$

$\frac{AD^3}{\left(2 \cdot AD^2 - 2 \cdot AD \cdot AB + AB^2\right)} - DG = 0 \quad \left(\frac{AD}{BD}\right)^3 - \frac{DG}{CD} = 0$

$\frac{(AD - AB)^3}{AB^2 - 2 \cdot AB \cdot AD + 2 \cdot AD^2} - CD = 0 \quad \frac{AD^3}{(AD - AB)^3} = 3.076703$

$\frac{DG}{CD} = 3.076703 \quad \left(\frac{AD}{AD - AB}\right)^3 - \frac{DG}{CD} = 0$




$AD = 2.45833$ in.

$AB = 0.76777$ in.

$DB = 1.69056$ in.

$\frac{AD}{DB} = 1.45415$

$BD = 1.69056$ in.

$EH = 1.81046$ in.

$HJ = 0.95793$ in.

$DG = 1.66903$ in.

$GJ = 0.54280$ in.

$\frac{DG}{GJ} = 3.07487$

120103C

**Unit.**

**Given.** $X := 13$

$Y := 20$

**Descriptions.**

$AB := \frac{Y}{Y}$ $AC := \frac{X}{Y}$ $BC := \sqrt{AB^2 + AC^2}$ $BC = 1.192686$

$BD := \frac{BC}{2}$ $DE := \frac{AC \cdot BD}{AB}$ $BE := \frac{BC \cdot BD}{AB}$ $BE = 0.71125$

$AE := AB - BE$ $AF := 2 \cdot AE$ $BF := AB - AF$ $BG := \frac{AB \cdot BF}{BC}$

$BG = 0.354242$ $BH := \frac{AB \cdot BG}{BC}$ $BH = 0.297012$

$GH := \frac{AC \cdot BG}{BC}$ $GH = 0.193058$ $AH := AB - BH$

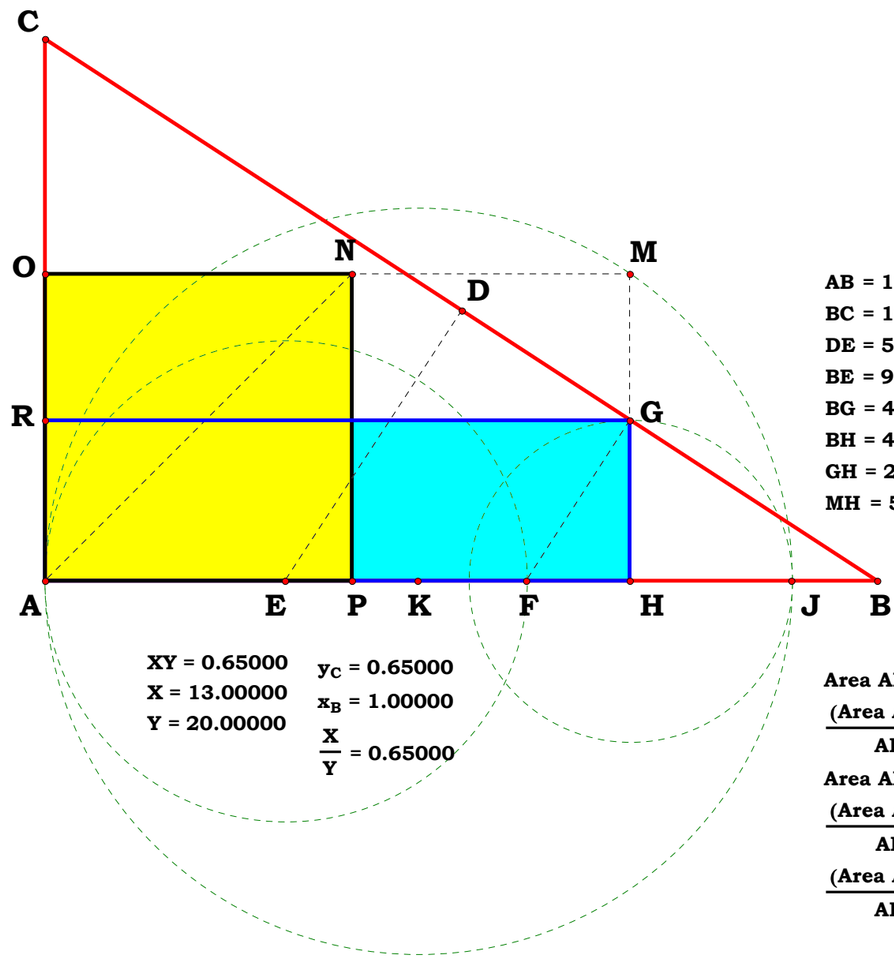
$AJ := AH + GH$ $HM := \sqrt{AH \cdot GH}$ $HM = 0.368398$

$AO := HM$ $AP := HM$ $NP := HM$ $AR := GH$ $GR := AH$

$APNO := AP^2$ $APNO = 0.135717$ $AHGR := AH \cdot GH$ $AHGR = 0.135717$

$APNO - AHGR = 0$ $CR := AC - AR$ $\frac{CR}{AH} = 0.65$ $\frac{GH}{BH} = 0.65$

$\left(\frac{GH}{AH}\right)^{\frac{1}{3}} - \frac{GH}{BH} = 0$ $\left(\frac{GH}{AP}\right)^2 - \frac{GH}{AH} = 0$




AB = 13.95400 cm
BC = 16.64274 cm
DE = 5.40889 cm
BE = 9.92478 cm
BG = 4.94310 cm
BH = 4.14451 cm
GH = 2.69393 cm
MH = 5.14063 cm

$\frac{BC}{AB} = 1.19269$
$\frac{DE}{AB} = 0.38762$
$\frac{BE}{AB} = 0.71125$
$\frac{BG}{AB} = 0.35424$
$\frac{BH}{AB} = 0.29701$
$\frac{GH}{AB} = 0.19306$
$\frac{MH}{AB} = 0.36840$

XY = 0.65000
X = 13.00000
Y = 20.00000

$y_C = 0.65000$
$x_B = 1.00000$
$\frac{X}{Y} = 0.65000$

Area APNO = 26.42609 $cm^2$
$\frac{(\text{Area APNO})}{AB^2} = 0.13572$
Area AHGR = 26.42609 $cm^2$
$\frac{(\text{Area AHGR})}{AB^2} = 0.13572$
$\frac{(\text{Area APNO})}{AB^2} - \frac{(\text{Area AHGR})}{AB^2} = 0.00000$

**Definitions.**

$$AB - 1 = 0 \quad AC - \frac{X}{Y} = 0 \quad BC - \frac{\sqrt{X^2+Y^2}}{Y} = 0 \quad BD - \frac{\sqrt{X^2+Y^2}}{2\cdot Y} = 0$$

$$DE - \frac{X\cdot\sqrt{X^2+Y^2}}{2\cdot Y^2} = 0 \quad BE - \frac{X^2+Y^2}{2\cdot Y^2} = 0 \quad AE - \frac{(Y-X)\cdot(X+Y)}{2\cdot Y^2} = 0$$

$$AF - \frac{(Y-X)\cdot(X+Y)}{Y^2} = 0 \quad BF - \frac{X^2}{Y^2} = 0 \quad BG - \frac{X^2}{Y\cdot\sqrt{X^2+Y^2}} = 0$$

$$BH - \frac{X^2}{X^2+Y^2} = 0 \quad GH - \frac{X^3}{Y\cdot\left(X^2+Y^2\right)} = 0 \quad AH - \frac{Y^2}{X^2+Y^2} = 0$$

$$AJ - \frac{(X+Y)\cdot\left(X^2-X\cdot Y+Y^2\right)}{Y\cdot\left(X^2+Y^2\right)} = 0 \quad HM - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$$

$$AO - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0 \quad AP - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0 \quad NP - \frac{\sqrt{X^3\cdot Y}}{\left(X^2+Y^2\right)} = 0$$

$$AR - \frac{X^3}{Y\cdot\left(X^2+Y^2\right)} = 0 \quad GR - \frac{Y^2}{X^2+Y^2} = 0 \quad APNO - \frac{X^3\cdot Y}{\left(X^2+Y^2\right)^2} = 0$$

$$AHGR - \frac{X^3\cdot Y}{\left(X^2+Y^2\right)^2} = 0 \quad CR - \frac{X\cdot Y}{X^2+Y^2} = 0 \quad \frac{CR}{AH} - \frac{X}{Y} = 0$$

$$\frac{GH}{BH} - \frac{X}{Y} = 0 \quad \left(\frac{GH}{AH}\right)^{\frac{1}{3}} - \frac{X}{Y} = 0 \quad \left(\frac{GH}{AP}\right)^2 - \frac{X^3}{Y^3} = 0$$

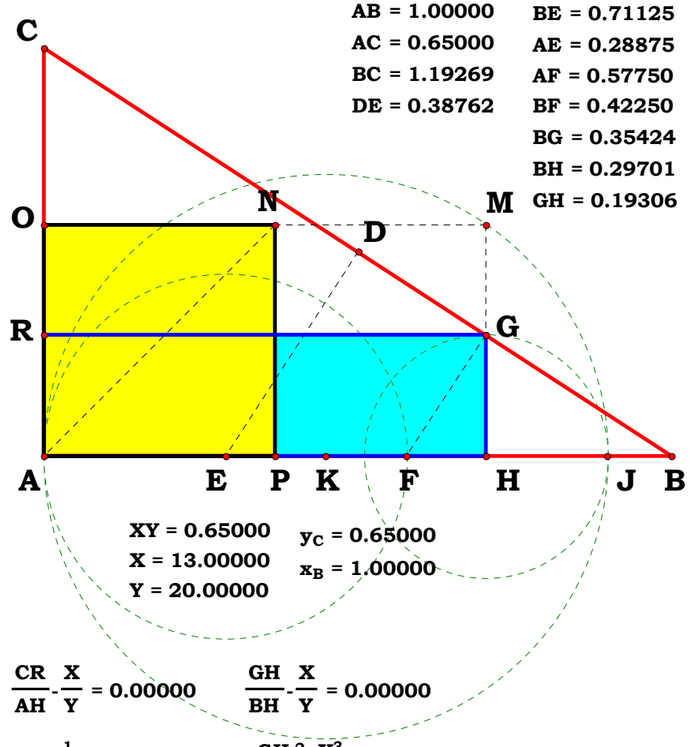
AB = 10.52500 cm
BC = 12.55302 cm
DE = 4.07973 cm
BE = 7.48591 cm
AE = 3.03909 cm

AF = 6.07819 cm
BF = 4.44681 cm
BG = 3.72840 cm
GH = 2.03194 cm
BH = 3.12605 cm

AH = 7.39895 cm
AJ = 9.43088 cm
HM = 3.87739 cm
AO = 3.87739 cm
AP = 3.87739 cm

NP = 3.87739 cm
AR = 2.03194 cm
GR = 7.39895 cm
Area APNO = 15.03418 cm²
Area AHGR = 15.03418 cm²

CR = 4.80931 cm
BH = 3.12605 cm
$\frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.36840$




AB = 1.00000
AC = 0.65000
BC = 1.19269
DE = 0.38762

BE = 0.71125
AE = 0.28875
AF = 0.57750
BF = 0.42250
BG = 0.35424
BH = 0.29701
GH = 0.19306

AH = 0.70299
AJ = 0.89605
HM = 0.36840
AO = 0.36840
AP = 0.36840
NP = 0.36840

AR = 0.19306
GR = 0.70299
APNO = 0.13572
AHGR = 0.13572
CR = 0.45694
BH = 0.29701

XY = 0.65000
X = 13.00000
Y = 20.00000

$y_C = 0.65000$
$x_B = 1.00000$

$\frac{CR}{AH} - \frac{X}{Y} = 0.00000$ $\quad \frac{GH}{BH} - \frac{X}{Y} = 0.00000$

$\frac{GH}{AH}^{\frac{1}{3}} - \frac{X}{Y} = 0.00000$ $\quad \frac{GH^2}{AP} - \frac{X^3}{Y^3} = 0.00000$

$AB - \frac{Y}{Y} = 0.00000$

$AC - \frac{X}{Y} = 0.00000$

$BC - \frac{\sqrt{X^2+Y^2}}{Y} = 0.00000$

$DE - \frac{X\cdot\sqrt{X^2+Y^2}}{2\cdot Y^2} = 0.00000$

$AO - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$BE - \frac{X^2+Y^2}{2\cdot Y^2} = 0.00000$

$HM - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$AE - \frac{(Y-X)\cdot(X+Y)}{2\cdot Y^2} = 0.00000$

$AP - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$AF - \frac{(Y-X)\cdot(X+Y)}{Y^2} = 0.00000$

$NP - \frac{\sqrt{X^3\cdot Y}}{X^2+Y^2} = 0.00000$

$BF - \frac{X^2}{Y^2} = 0.00000$

$AR - \frac{X^3}{Y\cdot(X^2+Y^2)} = 0.00000$

$BG - \frac{X^2}{Y\cdot\sqrt{X^2+Y^2}} = 0.00000$

$GR - \frac{Y^2}{X^2+Y^2} = 0.00000$

$BH - \frac{X^2}{X^2+Y^2} = 0.00000$

$APNO - \frac{X^3\cdot Y}{(X^2+Y^2)^2} = 0.00000$

$GH - \frac{X^3}{Y\cdot(X^2+Y^2)} = 0.00000$

$AHGR - \frac{X^3\cdot Y}{(X^2+Y^2)^2} = 0.00000$

$AH - \frac{Y^2}{X^2+Y^2} = 0.00000$

$CR - \frac{X\cdot Y}{X^2+Y^2} = 0.00000$

$AJ - \frac{(X+Y)\cdot((X^2-X\cdot Y)+Y^2)}{Y\cdot(X^2+Y^2)} = 0.00000$

120103D

**Unit.**

**Given.** $X := 11$

$Y := 20$

**Descriptions.**

$$AB := \frac{Y}{Y} \quad AX := \frac{X}{Y} \quad BX := \sqrt{AB^2 + AX^2} \quad BE := \frac{BX}{2} \quad BF := \frac{BX \cdot BE}{AB}$$

$$AG := \frac{AX \cdot AB}{BX} \quad GO := \frac{AG^2}{AX} \quad AO := \frac{AX \cdot AG}{BX} \quad BD := \frac{AB}{2} \quad DF := BF - BD$$

$$DO := BD - AO \quad DE := \frac{AX \cdot BD}{AB} \quad DP := \frac{DO \cdot DE}{GO} \quad AJ := \frac{DE \cdot DF}{DP + DF}$$

$$BN := \frac{AB \cdot AJ}{AX} \quad AN := AB - BN \quad JN := \sqrt{AJ^2 + AN^2} \quad JM := \frac{JN}{2}$$

**Definitions.**

$$AB - \frac{Y}{Y} = 0 \quad AX - \frac{X}{Y} = 0 \quad BX - \sqrt{\left(\frac{Y}{Y}\right)^2 + \left(\frac{X}{Y}\right)^2} = 0 \quad BE - \frac{\sqrt{\left(\frac{Y}{Y}\right)^2 + \left(\frac{X}{Y}\right)^2}}{2} = 0$$

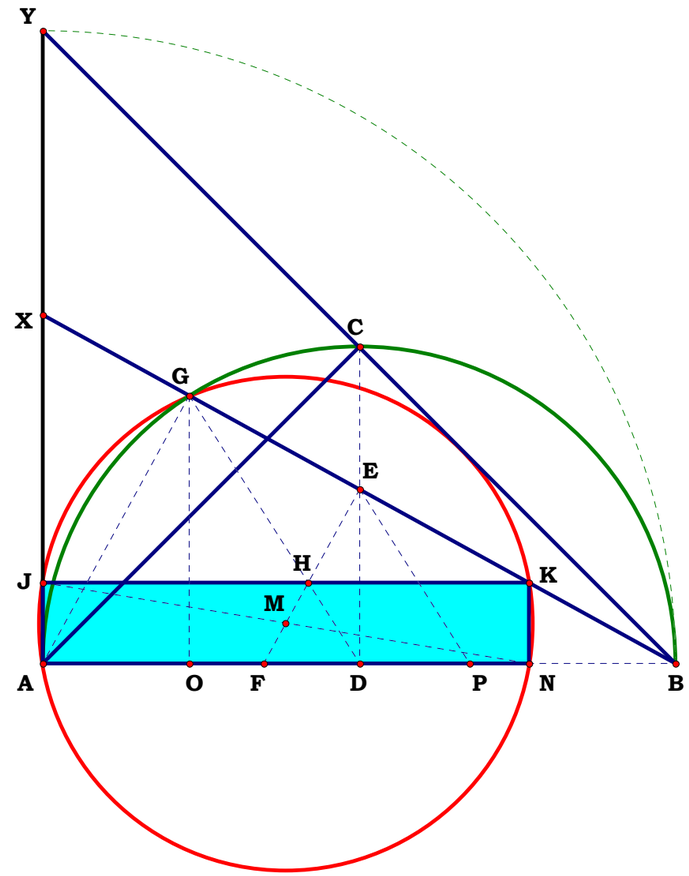
$$BF - \frac{X^2 + Y^2}{2 \cdot Y^2} = 0 \quad AG - \frac{X}{Y \cdot \sqrt{\frac{X^2}{Y^2} + 1}} = 0 \quad GO - \frac{X \cdot Y}{X^2 + Y^2} = 0 \quad AO - \frac{X^2}{X^2 + Y^2} = 0$$

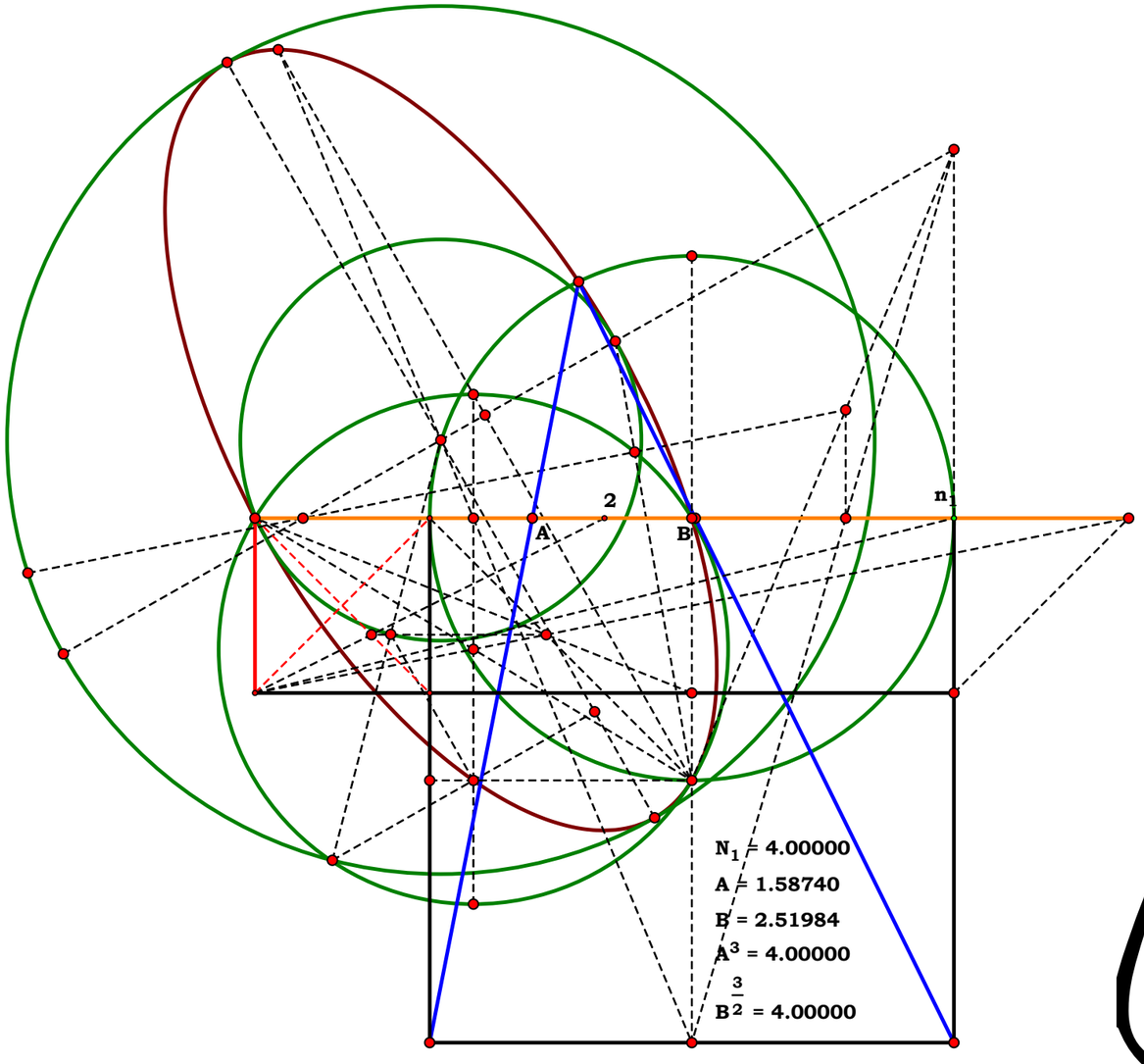
$$BD - \frac{1}{2} = 0 \quad DF - \frac{X^2}{2 \cdot Y^2} = 0 \quad DO - \frac{(Y - X) \cdot (X + Y)}{2 \cdot (X^2 + Y^2)} = 0 \quad DE - \frac{X}{2 \cdot Y} = 0$$

$$DP - \frac{(Y - X) \cdot (X + Y)}{4 \cdot Y^2} = 0 \quad AJ - \frac{X^3}{Y \cdot (X^2 + Y^2)} = 0 \quad BN - \frac{X^2}{X^2 + Y^2} = 0 \quad AN - \frac{Y^2}{X^2 + Y^2} = 0$$

$$JN - \sqrt{\frac{X^4 - X^2 \cdot Y^2 + Y^4}{Y^2 \cdot (X^2 + Y^2)}} = 0 \quad JM - \frac{\sqrt{\frac{X^4 - X^2 \cdot Y^2 + Y^4}{Y^2 \cdot (X^2 + Y^2)}}}{2} = 0$$

Unit = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000






# The Delian Quest 2004

**John Clark**

**Unit.**

$AD := 1$

**Given.**

**02**

**031604A**

**Descriptions.**

$AK := AD \quad AJ := AD \quad AH := \frac{\sqrt{2 \cdot AD^2}}{2}$

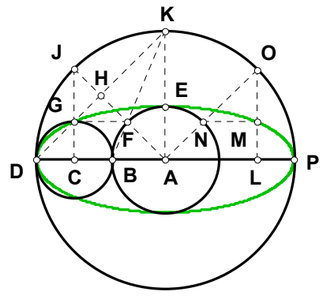
$HJ := AJ - AH \quad AC := AH$

$CD := AD - AC \quad FJ := 2 \cdot CD$

$AF := AJ - FJ \quad AB := AF$

$DK := \sqrt{2 \cdot AK^2} \quad AK + AB - DK = 0$

$\frac{AB}{CD} = 1.414214 \quad \frac{DK}{AD} = 1.414214 \sqrt{2} = 1.414214$

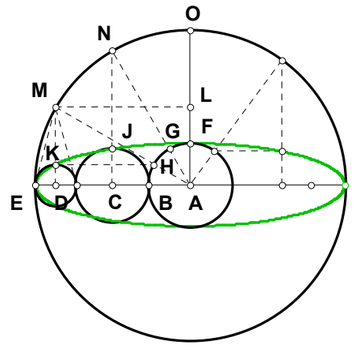


Unit.

$AE := 1$

Given.

03




031604B

Descriptions.

$AO := AE \quad AL := \frac{AO}{2}$

$AM := AE \quad ML := \sqrt{AM^2 - AL^2}$

$AD := ML \quad DE := AE - AD$

$DK := DE \quad DM := AL \quad AH := \frac{AM \cdot DK}{DM}$

$\frac{AH}{DE} = 2$

031704

**Unit.**

$AE := 1 \quad AG := AE$

**Given.**

$N := 3 \quad AC := N$

**Descriptions.**

$DE := AE \cdot 2 \quad CJ := AE$

$CE := \sqrt{AE^2 + AC^2}$

$CG := AE \quad FG := \frac{DE \cdot CG}{CE}$

**Definitions.**

$JL := FG \quad JM := \frac{JL^2}{2 \cdot CJ}$

$CM := CJ - JM \quad LM := \sqrt{JL^2 - JM^2} \quad NO := AC$

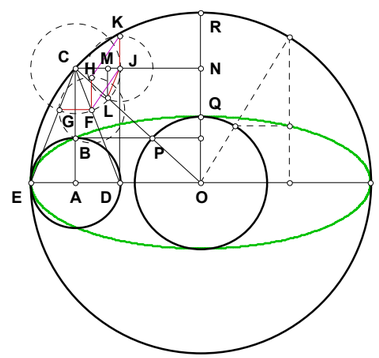
$CN := \frac{CM \cdot NO}{LM} \quad AO := CN \quad EO := AO + AE \quad OP := \frac{AE^2}{LM}$

$Major := EO \quad Minor := OP$

$Major - \frac{N^2 + 1}{2} = 0 \quad Minor - \frac{N^2 + 1}{2 \cdot N} = 0$

$\frac{Major}{Minor} - N = 0$

**Given AE, AG, AC**
**find the ellipse**




On given any AE, AC find the diameter of the Circle.

$Major = 5$

$Minor = 1.666667$

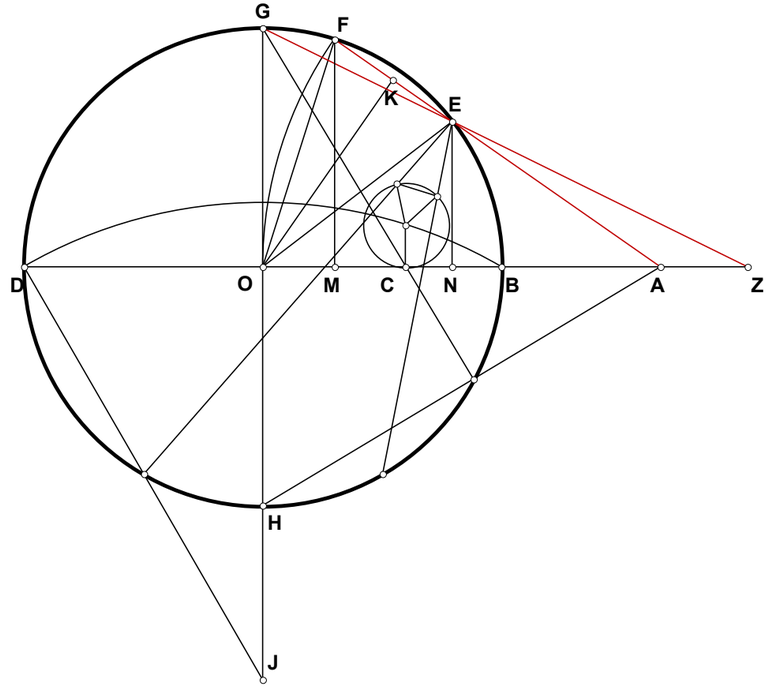
**Unit.**

$AB := 1$

**Given.**

$N := 4$

**032004**




**Descriptions.**

$AD := N \quad BD := AD - AB \quad BO := \frac{BD}{2} \quad AO := AB + BO$

$FO := BO \quad AF := AO \quad FK := \frac{FO^2}{2 \cdot AF} \quad EF := 2 \cdot FK$

$MO := FK \quad AM := AO - MO \quad FM := \sqrt{AF^2 - AM^2} \quad GO := BO$

$AE := AF - EF \quad EN := \frac{FM \cdot AE}{AF} \quad AN := \frac{AM \cdot AE}{AF} \quad NO := AO - AN$

$ZO := \frac{NO \cdot GO}{GO - EN} \quad AZ := ZO - AO \quad ZN := ZO - NO \quad GZ := \sqrt{ZO^2 + GO^2}$

$EG := \frac{GZ \cdot NO}{ZO} \quad EG = 1.323879$

**Definitions.**

$AD - N = 0 \quad BD - (N - 1) = 0 \quad BO - \frac{N - 1}{2} = 0 \quad AO - \frac{N + 1}{2} = 0 \quad FO - \frac{N - 1}{2} = 0 \quad AF - \frac{N + 1}{2} = 0 \quad FK - \frac{(N - 1)^2}{4 \cdot (N + 1)} = 0 \quad EF - \frac{(N - 1)^2}{2 \cdot (N + 1)} = 0$

$MO - \frac{(N - 1)^2}{4 \cdot (N + 1)} = 0 \quad AM - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N + 1)} = 0 \quad FM - \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)}{4 \cdot (N + 1)} = 0 \quad GO - \frac{N - 1}{2} = 0 \quad AE - \frac{2 \cdot N}{N + 1} = 0 \quad EN - \frac{N \cdot (N - 1) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(N + 1)^3} = 0$

$AN - \frac{N \cdot (N^2 + 6 \cdot N + 1)}{(N + 1)^3} = 0 \quad NO - \frac{(N^2 + 4 \cdot N + 1) \cdot (N - 1)^2}{2 \cdot (N + 1)^3} = 0 \quad ZO - \frac{(N - 1)^2 \cdot (N^2 + 4 \cdot N + 1)}{2 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} = 0 \quad AZ - \frac{N \cdot \left[(N + 1) \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} - (N^2 + 6 \cdot N + 1)\right]}{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1} = 0$

$ZN - \frac{\sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot (N^2 + 4 \cdot N + 1) \cdot (N - 1)^2}{(N + 1)^3 \cdot \left(3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1\right)} = 0 \quad GZ - \frac{\sqrt{(N - 1)^2 \cdot (N + 1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{[(N + 3) \cdot (3 \cdot N + 1)]} + 3 \cdot N^2 + N^3 + 1\right]}}{\sqrt{2 \cdot \left[N^6 + \left[27 \cdot (N^4 + N^2) + 6 \cdot (N^5 + N) + 60 \cdot N^3 + 1\right] - 4 \cdot N \cdot (N + 1)^3 \cdot \sqrt{[(N + 3) \cdot (3 \cdot N + 1)]}\right]}} = 0$

$EG - \frac{\sqrt{2} \cdot \sqrt{(N - 1)^2 \cdot (N + 1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]}{2 \cdot (N + 1)^3 \cdot \sqrt{27 \cdot (N^4 + N^2) + \left[6 \cdot (N^5 + N) + 60 \cdot N^3 + N^6 - 4 \cdot N \cdot (N + 1)^3 \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 1\right]}} = 0$

032304

**Descriptions.**

**Unit.**

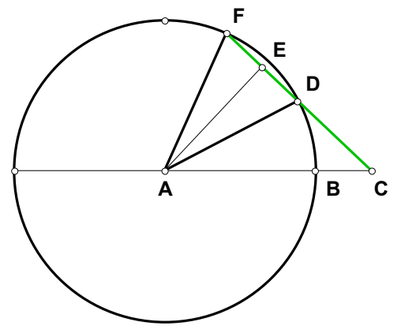
**Given.**

$N_1 := 1.708$ $AC := N_1$

$N_2 := 1.649$ $CF := N_2$

$N_3 := 1.24$ $AF := N_3$

**Given AC, AB and either point of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?**




$$DF_1 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

$CD := CF - DF_1$ $N_4 := CD$

$$DF_2 := \frac{N_3^2 + N_4^2 - N_1^2}{N_4}$$

$DF_1 = 0.812333$ $DF_2 = -0.812333$ $DF_1 + DF_2 = 0$

**Unit.**

$CE := 1$

**Given.**

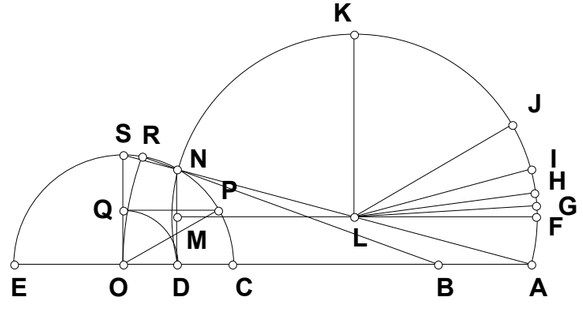
$N := 4$

**Definitions.**

**0405043**

**Descriptions.**

**All this extra work and I have lost accuracy from 022803!**



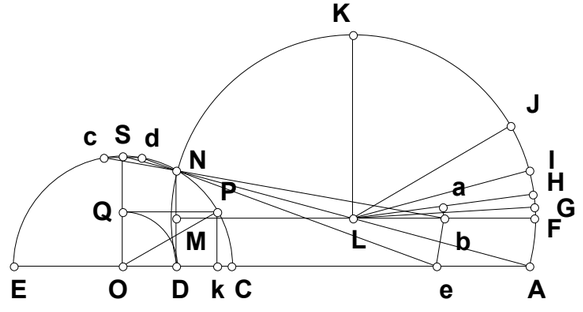

$CO := \frac{CE}{2}$ $CD := \frac{CE}{N}$

$DO := CO - CD$

$DN := \sqrt{(CO + DO) \cdot (CO - DO)}$

$AD := \frac{DO \cdot DN}{CO - DN}$ $DM := \frac{DN}{2}$

$AN := \sqrt{AD^2 + DN^2}$ $HL := \frac{AN}{2}$ $LA := \frac{HL}{2}$ $Ok := \sqrt{CO^2 - DO^2}$ $Ck := CO - Ok$




$Fm := \frac{Ck \cdot HL}{CO}$ $Jm := \frac{DO \cdot HL}{CO}$

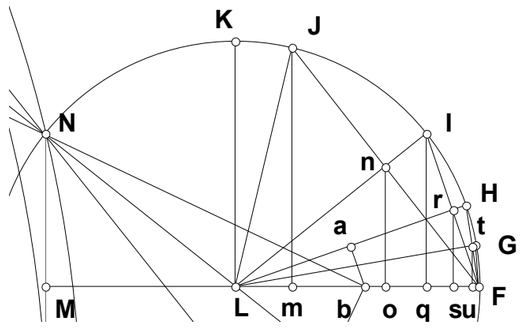
$JF := \sqrt{Fm^2 + Jm^2}$ $Fn := \frac{JF}{2}$

$Fo := \frac{Fm \cdot Fn}{JF}$ $Lo := HL - Fo$

$no := \frac{Jm}{2}$ $Ln := \sqrt{no^2 + Lo^2}$

$Iq := \frac{no \cdot HL}{Ln}$ $Lq := \frac{Lo \cdot HL}{Ln}$

$Fq := HL - Lq$ $FI := \sqrt{Iq^2 + Fq^2}$ $Fr := \frac{FI}{2}$




$Fs := \frac{Fq \cdot Fr}{FI}$ $Ls := HL - Fs$

$Lr := \sqrt{HL^2 - Fr^2}$ $Lb := \frac{HL^2}{2 \cdot Lr}$ $rs := \frac{Iq}{2}$

$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Hv := \frac{rs \cdot HL}{Lr}$$

$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Fv := HL - Lv$$

$$FH := \sqrt{Hv^2 + Fv^2} \qquad Ft := \frac{FH}{2}$$

$$Lf := \frac{HL}{2} \qquad Fu := \frac{Fv \cdot Ft}{FH} \qquad tu := \frac{Hv}{2}$$

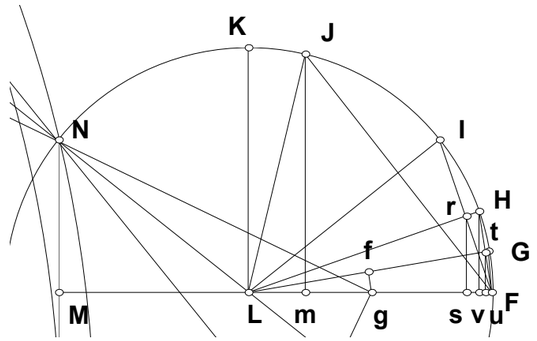
$$Lu := HL - Fu \qquad Lt := \sqrt{Lu^2 + tu^2}$$

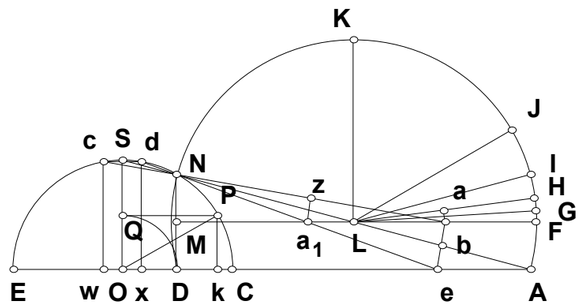
$$Lg := \frac{Lt \cdot Lf}{Lu} \qquad MN := \frac{DN}{2} \qquad NL := \frac{AN}{2}$$

$$ML := \sqrt{NL^2 - MN^2} \qquad Mb := ML + Lb$$

$$Nb := \sqrt{MN^2 + Mb^2}$$

$$Nc := \frac{CO^2 + (2 \cdot Nb)^2 - (2 \cdot Mb + DO)^2}{-2 \cdot Nb}$$




$$cw := \frac{MN \cdot (Nb + Nc)}{Nb} + MN \qquad Dw := \frac{2 \cdot Mb \cdot (2 \cdot Nb + Nc)}{2 \cdot Nb} - 2 \cdot Mb$$

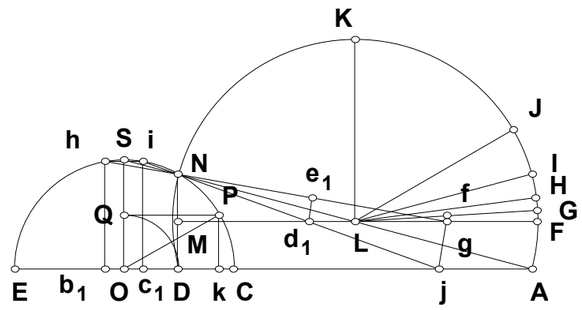
$$Ow := Dw - DO \qquad Dx := Dw - 2 \cdot Ow \qquad Ma_1 := \frac{Dx \cdot DN}{2 \cdot (cw - DN)}$$

$$Na_1 := \sqrt{MN^2 + (Ma_1)^2} \qquad ba_1 := Mb - Ma_1$$

$$Nz := \frac{(Na_1)^2 + Nb^2 - (ba_1)^2}{2 \cdot Nb} \qquad za_1 := \sqrt{(Na_1)^2 - Nz^2} \qquad be := \frac{za_1 \cdot Nb}{Nz}$$

$$Mg := ML + Lg \qquad Ng := \sqrt{MN^2 + Mg^2}$$

$$Nh := \frac{CO^2 + (2 \cdot Ng)^2 - (2 \cdot Mg + DO)^2}{-2 \cdot Ng}$$




$$hb_1 := \frac{MN \cdot (Ng + Nh)}{Ng} + MN \qquad Db_1 := \frac{2 \cdot Mg \cdot (2 \cdot Ng + Nh)}{2 \cdot Ng} - 2 \cdot Mg$$

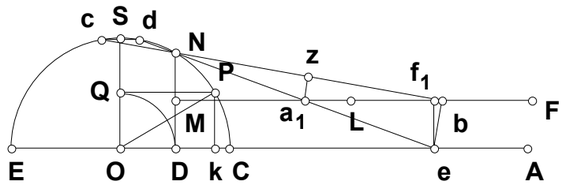
$$Ob_1 := Db_1 - DO \qquad Dc_1 := Db_1 - 2 \cdot Ob_1 \qquad Md_1 := \frac{Dc_1 \cdot DN}{2 \cdot (hb_1 - DN)}$$

$$Nd_1 := \sqrt{MN^2 + (Md_1)^2} \qquad gd_1 := Mg - Md_1 \qquad Ne_1 := \frac{(Nd_1)^2 + Ng^2 - (gd_1)^2}{2 \cdot Ng}$$

$$ed_1 := \sqrt{(Nd_1)^2 - (Ne_1)^2} \qquad gj := \frac{ed_1 \cdot Ng}{Ne_1} \qquad Ne := \frac{Na_1 \cdot Nb}{Nz}$$

$$ea_1 := Ne - Na_1$$

$$bf_1 := \frac{be^2 + ba_1^2 - ea_1^2}{2 \cdot ba_1}$$

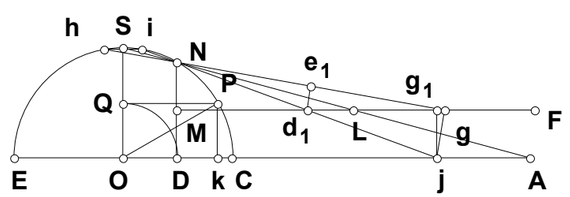


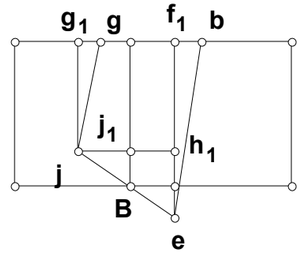

$$ef_1 := \sqrt{be^2 - bf_1^2}$$

$$Nj := \frac{Nd_1 \cdot Ng}{Ne_1} \qquad jd_1 := Nj - Nd_1$$

$$gg_1 := \frac{gj^2 + gd_1^2 - jd_1^2}{2 \cdot gd_1}$$

$$jg_1 := \sqrt{gj^2 - gg_1^2}$$






$$Mg_1 := Mg - gg_1 \qquad Mf_1 := Mb - bf_1$$

$$jh_1 := Mf_1 - Mg_1 \qquad eh_1 := ef_1 - jg_1$$

$$ej := \sqrt{jh_1^{\,2} + eh_1^{\,2}} \qquad Bj_1 := DM - jg_1$$

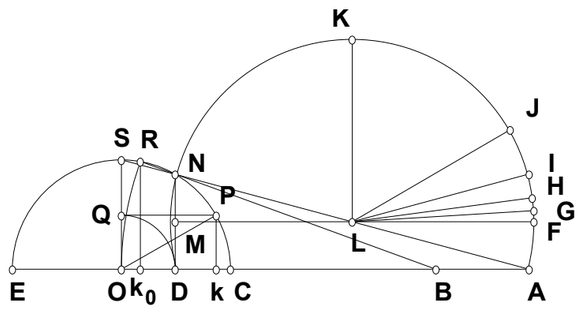
$$jj_1 := \frac{jh_1 \cdot Bj_1}{eh_1} \qquad BD := Mg_1 + jj_1 \qquad BO := BD + DO$$

$$BN := \sqrt{DN^2 + BD^2}$$

$$NR := \frac{CO^2 + BN^2 - BO^2}{-BN}$$

$$Bk_0 := \frac{BD \cdot (BN + NR)}{BN}$$

$$Ok_0 := BO - Bk_0$$




$$NR - 2 \cdot Ok_0 = -0.000000000019581$$

**Compared to 022803**

$$\frac{19591}{2639} = 7.423645$$

$$Om - \frac{Ek}{2} = -0.000000000002639$$

# The Ellipse

**041904A**

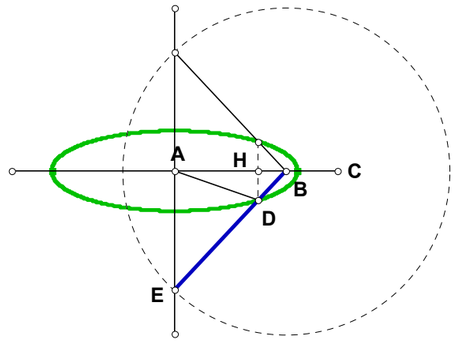
**I once worked on a project I called Eloi, which was about the different ways to construct an ellipse, the different equations one would have to use to make that figure, and the different ways one could solve for those ellipses. And this entertained me until I started thinking about the figure 8 locus. One can draw the figure eight as a locus between a straight line and a cirlce.**

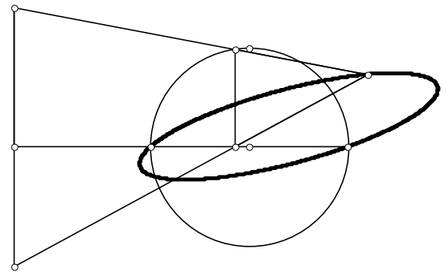
**All of this applies to science and mechanics when one is writing up equations to the motion of objects; How do you comprehend what you are seeing?**

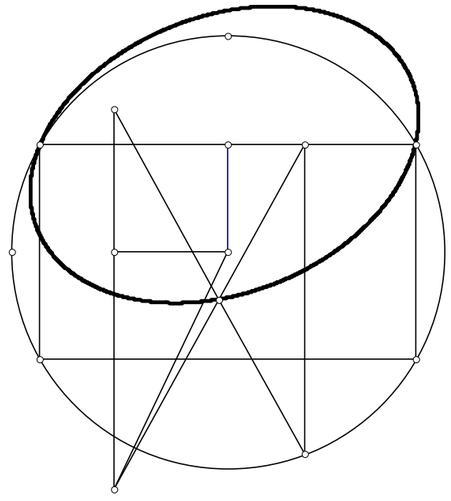
**This plate series is called the Straight Line Ellipse because it reduces the ellipse to a single linear action between X and Y axies. In other words, we draw what some would claim to be a trig function, which is actually a linear function, one just doess not see it. When every possible grammar is effected by complete induction and deduction of a unit, one should keep the unit in mind instead of obfuscating it with particular names which factually do not apply. One is always in danger of claiming that there are many different mathematics, yet the same single unit makes them all; this amounts to a thing is different from itself, and if one is stupid, it produces the modern mathematician.**

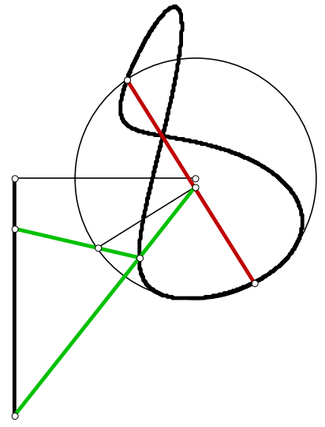
**As one can see by the last figure, the major and minor axis do not determine the resulting figure, meaning the shape is independent of the axis, but when it is shaped, or created is. The resulting figure, a photon, or a burst of energy, or one can say a pulse, is the product of a simple tic, toc, tic of two objects, an oscilation. At a certain point they interact, and release an elliptical signiture, or temporarily existing third object. This means one can spend a lot of time claiming that mechanics is wave mechanics, or quantum mechanics, or that these names both miss the point; how you write an object up does not create a new grammar, unless you are really stupid. Grammar is not a theory, and if you are teaching theory, you may as well elect a Pope.**

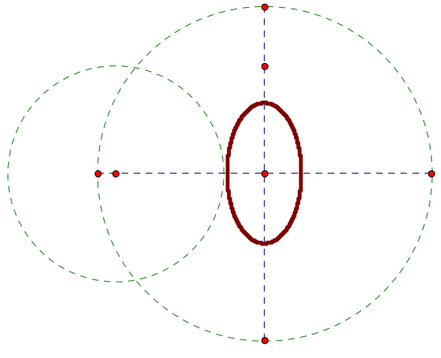
**What is more important, we start to get back to fundamentals in linguistic fact, between any two limits is one, and only one relative difference, which is not only called a unit, but even observed by Plato. We can obfuscate it, like all of the other ways we can produce an ellipse, but those figures do not express the fundamental ellipse.**









041904B

**Unit.**

$AC := 1$

**Given.**

$R_1 := 3$

$R_2 := 2$

**Descriptions.**

$BE := AC \qquad BD := \frac{BE}{R_1}$

$AB := \frac{AC}{R_2} \qquad AE := \sqrt{BE^2 - AB^2}$

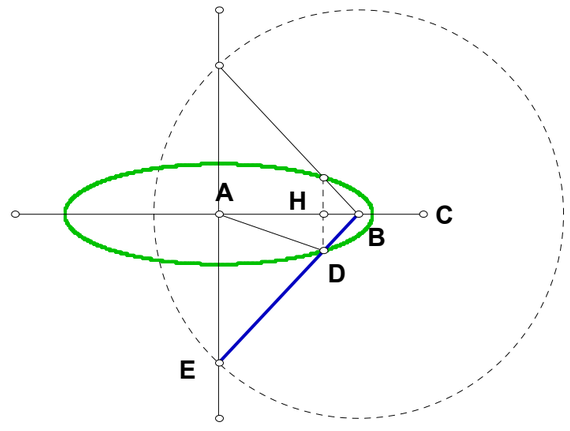
$BH := \frac{AB \cdot BD}{BE} \qquad DH := \frac{AE \cdot BD}{BE}$

$AH := AB - BH \qquad AD := \sqrt{AH^2 + DH^2}$

**Definitions.**

$$AD - \frac{\sqrt{R_1^2 - 2 \cdot R_1 + R_2^2}}{R_1 \cdot R_2} = 0$$

## Straight Line Ellipse: Cardinal



**Unit.**

$AC := 1$ $BE := AC$

**Given.**

$N_1 := .5$ $AB := N_1$

$N_2 := .3$ $BD := N_2$

**041904C**

**Descriptions.**

$$AE := \sqrt{BE^2 - AB^2}$$

$$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$$

$$AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$$

**Definitions.**

$$AE - \sqrt{(1 - N_1) \cdot (N_1 + 1)} = 0$$

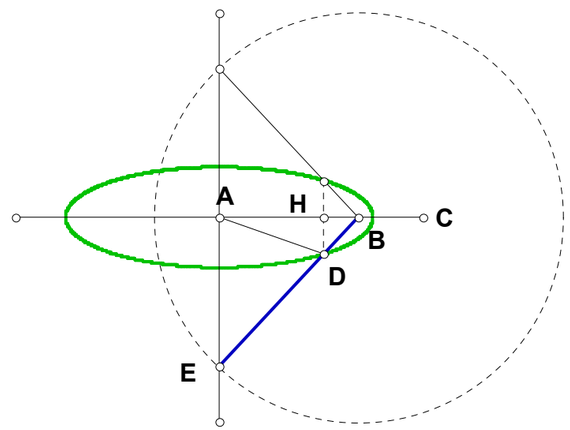
$$BH - N_1 \cdot N_2 = 0$$

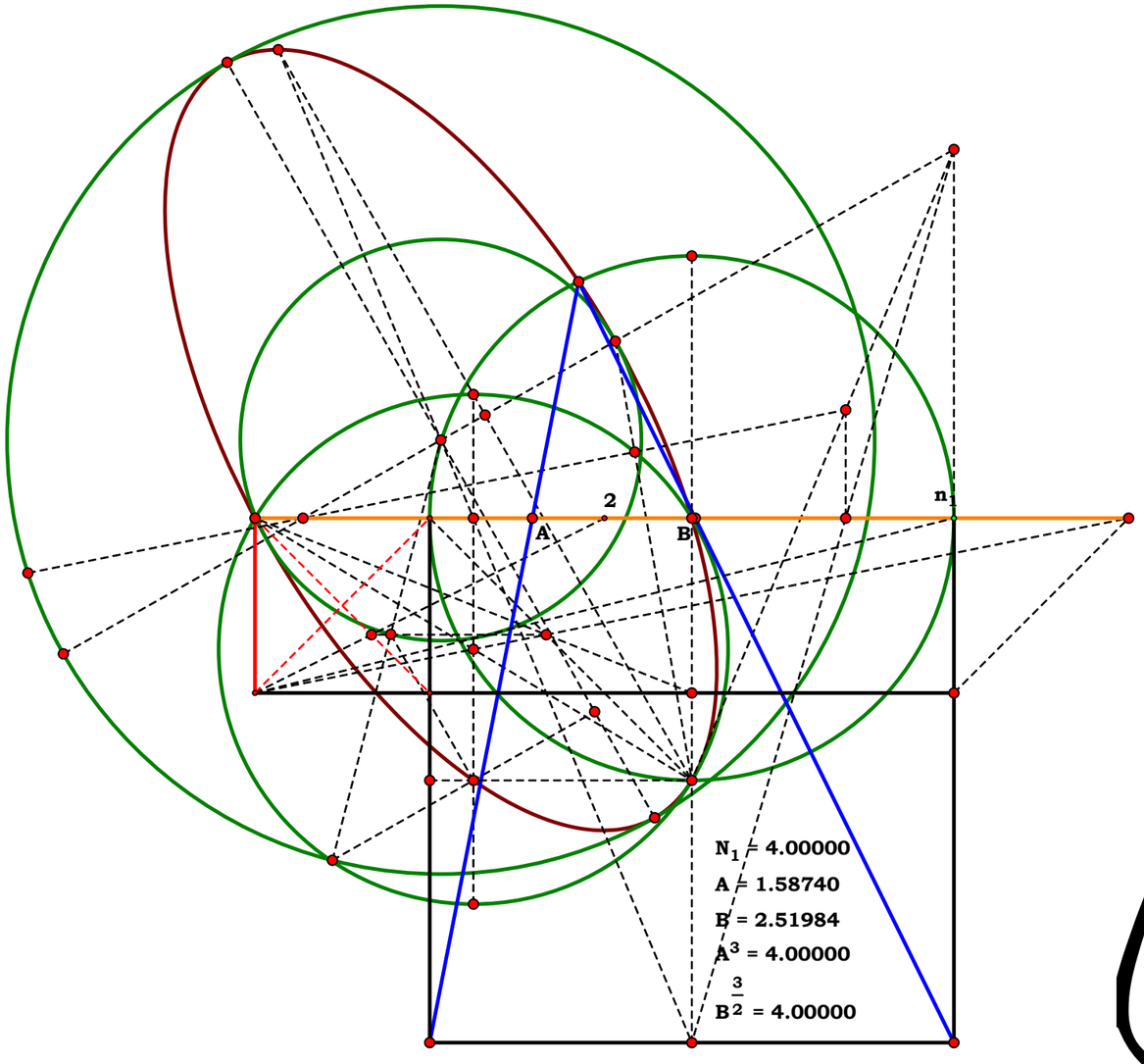
$$DH - \sqrt{(1 - N_1) \cdot (N_1 + 1)} \cdot N_2 = 0$$

$$AH - (N_1 - N_1 \cdot N_2) = 0$$

$$AD - \sqrt{N_1^2 - 2 \cdot N_1^2 \cdot N_2 + N_2^2} = 0$$

## Straight Line Ellipse: Ordinal






# The Delian Quest 2005

**John Clark**

Unit.

$AC := 1$

Given.

$N_1 := 3$

031405

Descriptions.

$AD := N_1 \qquad DV := AC$

$AV := \sqrt{AD^2 + DV^2} \qquad AF := 2 \cdot AC$

$VX := AF - AD \qquad AY := \frac{AV \cdot AC}{AC - VX}$

$AG := \frac{AD \cdot AY}{AV} \qquad BC := \frac{AC \cdot AC}{AC + AD}$

$CG := AG - AC \qquad BG := BC + CG$

$BE := \frac{BG}{2} \qquad CE := BE - BC \qquad ES := BE$

$CS := \sqrt{ES^2 - CE^2} \qquad CR := \frac{DV \cdot AC}{AD}$

$EU := \frac{ES \cdot CR}{CS} \qquad EZ := EU$

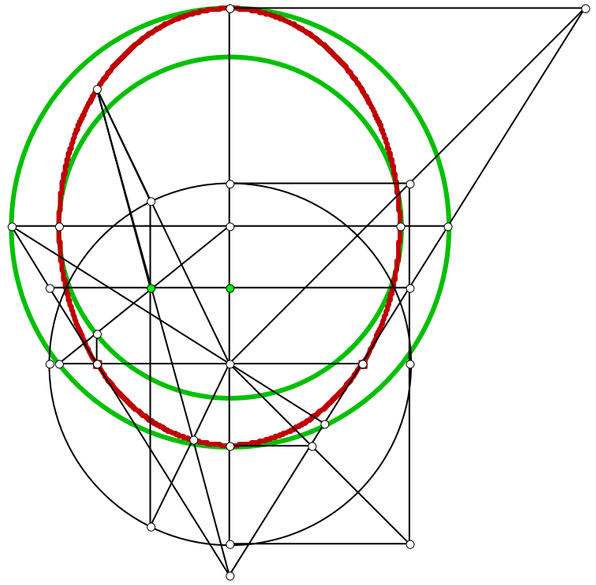
Definitions.

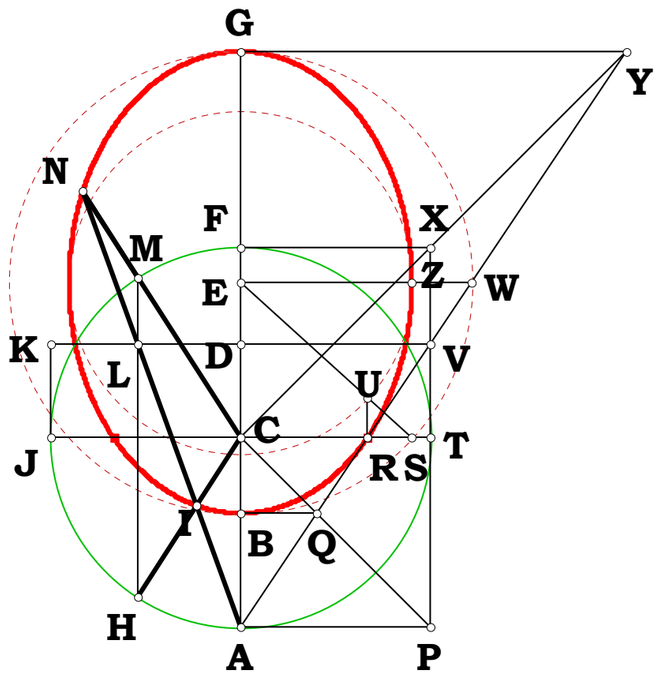
$$EZ - \frac{\sqrt{N_1{}^2 - 1}}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

$$BG - \frac{2 \cdot N_1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

## Another Ellipse

**The locus formed by N and I as determined by L provides an ellipse. Provide an Algebraic name for the Major and Minor Axis.**





0932305

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad BE := N_1$

$N_2 := .5 \quad BD := N_2$

$MN := 2 \cdot AB$

$DE := BE - BD \qquad AG := AB$

$EP := \frac{AG \cdot BE}{AB} \qquad EN := AB$

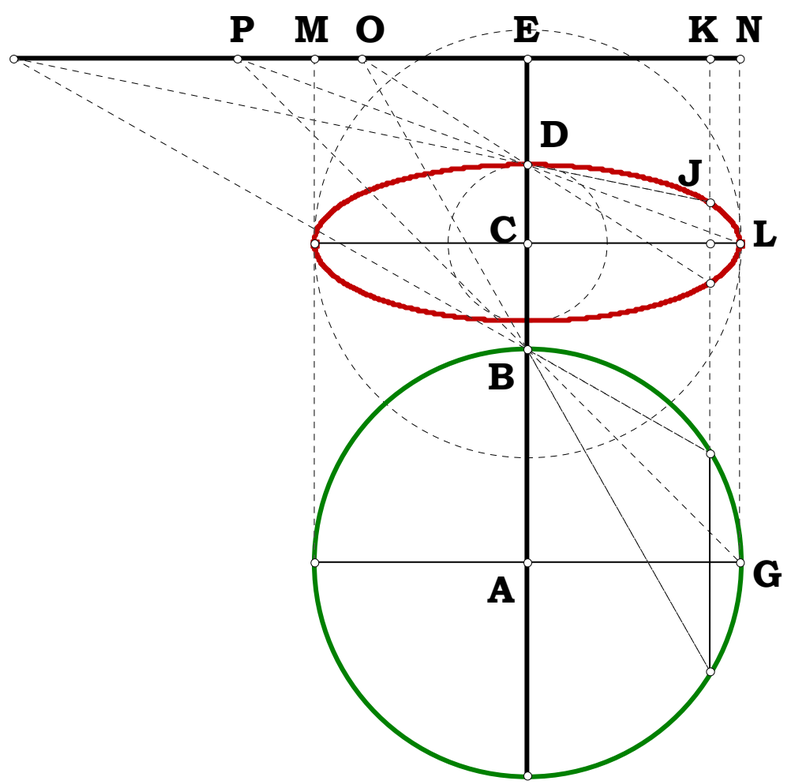
$NL := \frac{DE \cdot (EP + EN)}{EP}$

$CE := NL \qquad CD := CE - DE$

Definitions.

$\frac{(N_1 - N_2)}{N_1} - CD = 0$

An Ellipse



**032905**

**Descriptions.**

**Unit.**

**Given.**

$N_1 := 1.9167$ $AC := N_1$

$N_2 := .3244$ $CD := N_2$

$N_3 := .437$ $GI := N_3$

$CE := CD$ $CG := \sqrt{CE \cdot AC}$ $GH := 2 \cdot CG$

$CI := CG - GI$ $AD := AC - CD$ $AI := \sqrt{AC^2 + CI^2}$ $AM := \frac{AD}{2}$

$DM := AM$ $DU := \frac{CI \cdot AD}{AI}$ $AU := \frac{AC \cdot AD}{AI}$ $IU := AI - AU$ $AL := \frac{DU \cdot AI}{IU}$

$IL := \sqrt{AI^2 + AL^2}$ $DI := \sqrt{DU^2 + IU^2}$ $DL := IL - DI$ $DV := \frac{CD \cdot DL}{DI}$

$MV := DV - DM$ $AV := AM - MV$ $LV := \sqrt{AL^2 - AV^2}$ $MT := LV$

$AG := \sqrt{CG^2 + AC^2}$ $DW := \frac{CG \cdot AD}{AG}$ $AW := \frac{AC \cdot AD}{AG}$ $GW := AG - AW$

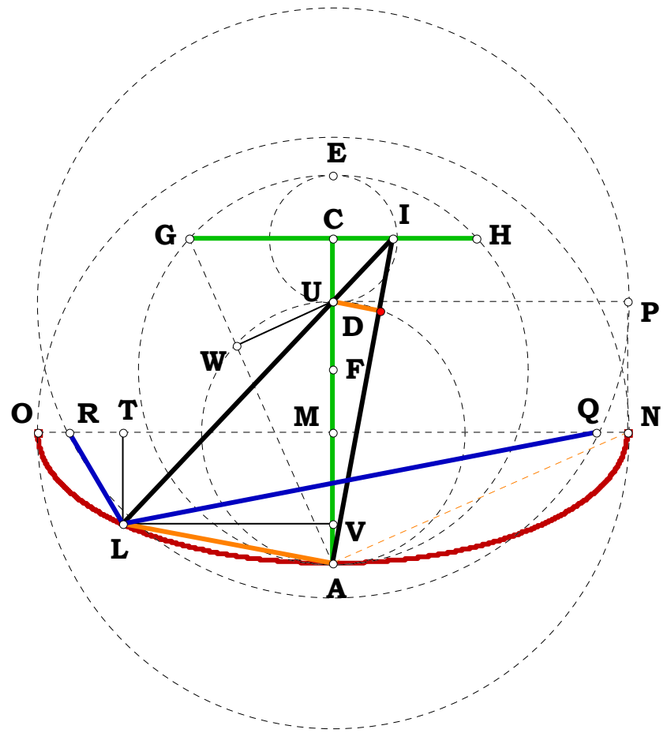
$AN := \frac{DW \cdot AG}{GW}$ $MN := \sqrt{AN^2 - AM^2}$ $ON := 2 \cdot MN$

**Definitions.**

$$ON - \sqrt{(N_2 - N_1)^2 \cdot \frac{N_1}{N_2}} = 0$$

## Elipse Projected From a Perpendicular.

**Let AC be some perpendicular on some line GH.**



**Parcing project for 032905b, using just straight lines.**

**Unit.** $BC := 1$

**Given.** $z := 7 \quad x := 6 \quad v := 12$

$y := 20 \quad w := 20 \quad u := 20$

**032905B**

**Descriptions.**

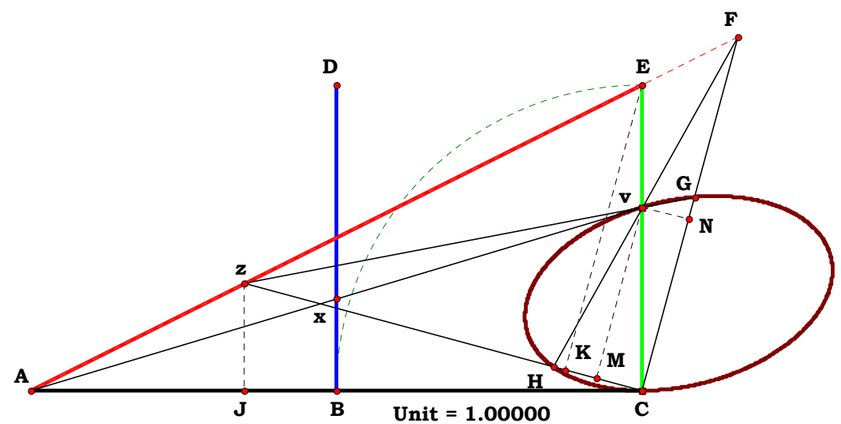
$$CE := BC \quad BD := BC \quad Cv := \frac{CE \cdot v}{u} \quad Bx := \frac{BD \cdot x}{w} \quad vx := \sqrt{BC^2 + (Cv - Bx)^2} \quad AC := \frac{BC \cdot Cv}{Cv - Bx}$$

$$AE := \sqrt{AC^2 + CE^2} \quad Az := AE \cdot \frac{z}{y} \quad AJ := \frac{AC \cdot Az}{AE} \quad Jz := \frac{CE \cdot AJ}{AC} \quad CJ := AC - AJ$$

$$Cz := \sqrt{Jz^2 + CJ^2} \quad Ez := AE - Az \quad Kz := \frac{Cz^2 + Ez^2 - CE^2}{2 \cdot Cz} \quad EK := \sqrt{Ez^2 - Kz^2}$$

$$CF := EK \cdot \frac{Cz}{Kz} \quad Fz := \frac{Ez \cdot Cz}{Kz} \quad CK := Cz - Kz \quad CM := \frac{CK \cdot Cv}{CE} \quad Mv := \frac{EK \cdot Cv}{CE} \quad CN := Mv$$

$$Nv := CM \quad FN := CF - CN \quad CH := \frac{Nv \cdot CF}{FN} \quad Mz := Cz - CM \quad vz := \sqrt{Mv^2 + Mz^2} \quad Gv := \frac{vz \cdot Nv}{Mz}$$




$z/y = 0.35000$ $\quad x/w = 0.30000$ $\quad v/C = 0.60000$

$z = 7.00000$ $\quad x = 6.00000$ $\quad v = 12.00000$

$y = 20.00000$ $\quad w = 20.00000$ $\quad u = 20.00000$

**Definitions.**

$$CE - 1 = 0 \quad BD - 1 = 0 \quad Cv - \frac{v}{u} = 0 \quad Bx - \frac{x}{w} = 0 \quad vx - \frac{\sqrt{u^2 \cdot w^2 + u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + v^2 \cdot w^2}}{u \cdot w} = 0 \quad AC - \frac{v \cdot w}{(v \cdot w - u \cdot x)} = 0 \quad AE - \frac{\sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2}}{(v \cdot w - u \cdot x)} = 0$$

$$Az - \frac{z \cdot \sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2}}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad AJ - \frac{v \cdot w \cdot z}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad Jz - \frac{z}{y} = 0 \quad CJ - \frac{v \cdot w \cdot (y - z)}{y \cdot (v \cdot w - u \cdot x)} = 0 \quad Ez - \frac{\sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2} \cdot (y - z)}{y \cdot (v \cdot w - u \cdot x)} = 0$$

$$Cz - \frac{\sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}{y \cdot (v \cdot w - u \cdot x)} = 0$$

$$Kz - \frac{2 \cdot u^2 \cdot x^2 \cdot y \cdot z - 2 \cdot u^2 \cdot x^2 \cdot z^2 - 4 \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z + 4 \cdot u \cdot v \cdot w \cdot x \cdot z^2 - 2 \cdot v^2 \cdot w^2 \cdot y^2 + 6 \cdot v^2 \cdot w^2 \cdot y \cdot z - 4 \cdot v^2 \cdot w^2 \cdot z^2}{2 \cdot y \cdot (u \cdot x - v \cdot w) \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$EK - \frac{v \cdot w \cdot (y - z)}{\sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0 \quad CF - \frac{v \cdot w \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}{v^2 \cdot w^2 \cdot y - u^2 \cdot x^2 \cdot z - 2 \cdot v^2 \cdot w^2 \cdot z + 2 \cdot u \cdot v \cdot w \cdot x \cdot z} = 0$$

$$\mathbf{Fz} - \frac{\sqrt{u^2 \cdot x^2 - 2 \cdot u \cdot v \cdot w \cdot x + 2 \cdot v^2 \cdot w^2} \cdot \left(u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2\right)}{y \cdot (u \cdot x - v \cdot w) \cdot \left(u^2 \cdot x^2 \cdot z - v^2 \cdot w^2 \cdot y + 2 \cdot v^2 \cdot w^2 \cdot z - 2 \cdot u \cdot v \cdot w \cdot x \cdot z\right)} = 0$$

$$\mathbf{CK} - \frac{z \cdot (v \cdot w - u \cdot x)}{\sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$\mathbf{CM} - \frac{v \cdot z \cdot (v \cdot w - u \cdot x)}{u \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$\mathbf{Nv} - \frac{v \cdot z \cdot (v \cdot w - u \cdot x)}{u \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$\mathbf{Mv} - \frac{v^2 \cdot w \cdot (y - z)}{u \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$\mathbf{CN} - \frac{v^2 \cdot w \cdot (y - z)}{u \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}} = 0$$

$$\mathbf{FN} - \left[\frac{\begin{array}{r} v \cdot w \cdot \left(u^3 \cdot x^2 - 2 \cdot u^2 \cdot v \cdot w \cdot x - u^2 \cdot v \cdot x^2 + 2 \cdot u \cdot v^2 \cdot w^2 + 2 \cdot u \cdot v^2 \cdot w \cdot x - 2 \cdot v^3 \cdot w^2\right) \cdot z^2 \ \ldots \\ + \ v \cdot w \cdot \left(y \cdot u^2 \cdot v \cdot x^2 - 2 \cdot y \cdot u \cdot v^2 \cdot w^2 - 2 \cdot y \cdot u \cdot v^2 \cdot w \cdot x + 3 \cdot y \cdot v^3 \cdot w^2\right) \cdot z - v \cdot w \cdot \left(v^3 \cdot w^2 \cdot y^2 - u \cdot v^2 \cdot w^2 \cdot y^2\right) \end{array}}{u \cdot \left(v^2 \cdot w^2 \cdot y - u^2 \cdot x^2 \cdot z - 2 \cdot v^2 \cdot w^2 \cdot z + 2 \cdot u \cdot v \cdot w \cdot x \cdot z\right) \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}\right] = 0$$

$$\mathbf{CH} - \frac{v \cdot z \cdot (u \cdot x - v \cdot w) \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}{\left(w^2 \cdot y^2 - 3 \cdot w^2 \cdot y \cdot z + 2 \cdot w^2 \cdot z^2\right) \cdot v^3 + \left(2 \cdot u \cdot w^2 \cdot y \cdot z - u \cdot w^2 \cdot y^2 - 2 \cdot u \cdot w^2 \cdot z^2 + 2 \cdot u \cdot x \cdot w \cdot y \cdot z - 2 \cdot u \cdot x \cdot w \cdot z^2\right) \cdot v^2 + \left(u^2 \cdot x^2 \cdot z^2 - y \cdot u^2 \cdot x^2 \cdot z + 2 \cdot w \cdot u^2 \cdot x \cdot z^2\right) \cdot v - u^3 \cdot x^2 \cdot z^2} = 0$$

$$\mathbf{Mz} - \left[\frac{u^3 \cdot x^2 \cdot z^2 - 2 \cdot u^2 \cdot v \cdot w \cdot x \cdot z^2 - u^2 \cdot v \cdot x^2 \cdot y \cdot z + u \cdot v^2 \cdot w^2 \cdot y^2 - 2 \cdot u \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot u \cdot v^2 \cdot w^2 \cdot z^2 + 2 \cdot u \cdot v^2 \cdot w \cdot x \cdot y \cdot z - v^3 \cdot w^2 \cdot y \cdot z}{u \cdot y \cdot (v \cdot w - u \cdot x) \cdot \sqrt{u^2 \cdot x^2 \cdot z^2 - 2 \cdot u \cdot v \cdot w \cdot x \cdot z^2 + v^2 \cdot w^2 \cdot y^2 - 2 \cdot v^2 \cdot w^2 \cdot y \cdot z + 2 \cdot v^2 \cdot w^2 \cdot z^2}}\right] = 0$$

$$\mathbf{vz} - \left[\frac{\sqrt{\left(u^2 \cdot v^2 \cdot y^2 - 2 \cdot u^2 \cdot v^2 \cdot y \cdot z + 2 \cdot u^2 \cdot v^2 \cdot z^2 - 2 \cdot u \cdot v^3 \cdot y \cdot z + v^4 \cdot y^2\right) \cdot w^2 + \left(4 \cdot x \cdot u^2 \cdot v^2 \cdot y \cdot z - 2 \cdot x \cdot u^3 \cdot v \cdot z^2 - 2 \cdot x \cdot u \cdot v^3 \cdot y^2\right) \cdot w + u^4 \cdot x^2 \cdot z^2 - 2 \cdot u^3 \cdot v \cdot x^2 \cdot y \cdot z + u^2 \cdot v^2 \cdot x^2 \cdot y^2}}{u \cdot y \cdot (v \cdot w - u \cdot x)}\right] = 0$$

$$\mathbf{Gv} - \frac{\begin{array}{r} v \cdot z \cdot (v \cdot w - u \cdot x) \cdot \sqrt{\left(u^2 \cdot v^2 \cdot y^2 - 2 \cdot u^2 \cdot v^2 \cdot y \cdot z + 2 \cdot u^2 \cdot v^2 \cdot z^2 - 2 \cdot u \cdot v^3 \cdot y \cdot z + v^4 \cdot y^2\right) \cdot w^2} \ \ldots \\ + \ \left(4 \cdot x \cdot u^2 \cdot v^2 \cdot y \cdot z - 2 \cdot x \cdot u^3 \cdot v \cdot z^2 - 2 \cdot x \cdot u \cdot v^3 \cdot y^2\right) \cdot w + u^4 \cdot x^2 \cdot z^2 - 2 \cdot u^3 \cdot v \cdot x^2 \cdot y \cdot z + u^2 \cdot v^2 \cdot x^2 \cdot y^2 \end{array}}{u \cdot \left[\left(u^3 \cdot x^2 - 2 \cdot u^2 \cdot v \cdot w \cdot x + 2 \cdot u \cdot v^2 \cdot w^2\right) \cdot z^2 + \left(2 \cdot y \cdot u \cdot v^2 \cdot w \cdot x - 2 \cdot y \cdot u \cdot v^2 \cdot w^2 - y \cdot u^2 \cdot v \cdot x^2 - y \cdot v^3 \cdot w^2\right) \cdot z + u \cdot v^2 \cdot w^2 \cdot y^2\right]} = 0$$

**Unit.**

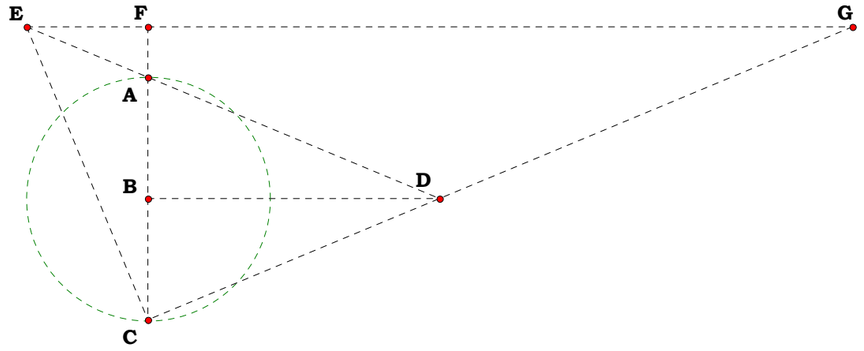
**Given.**

**Writeup project for 032905e**

**Descriptions.**

**Definitions.**

**Another way to do cube roots, one which the ancients were looking for, is accomplished by crossing an isosceles triangle with a right triangle. I put off investigating it for some future date. What it does say is that cube roots are not impossible, but perhaps difficult. I found this while examining the figure of the preceeding ellipse.**




**EF = 2.04153 cm**

**AF = 0.84843 cm**

**FG = 11.82053 cm**

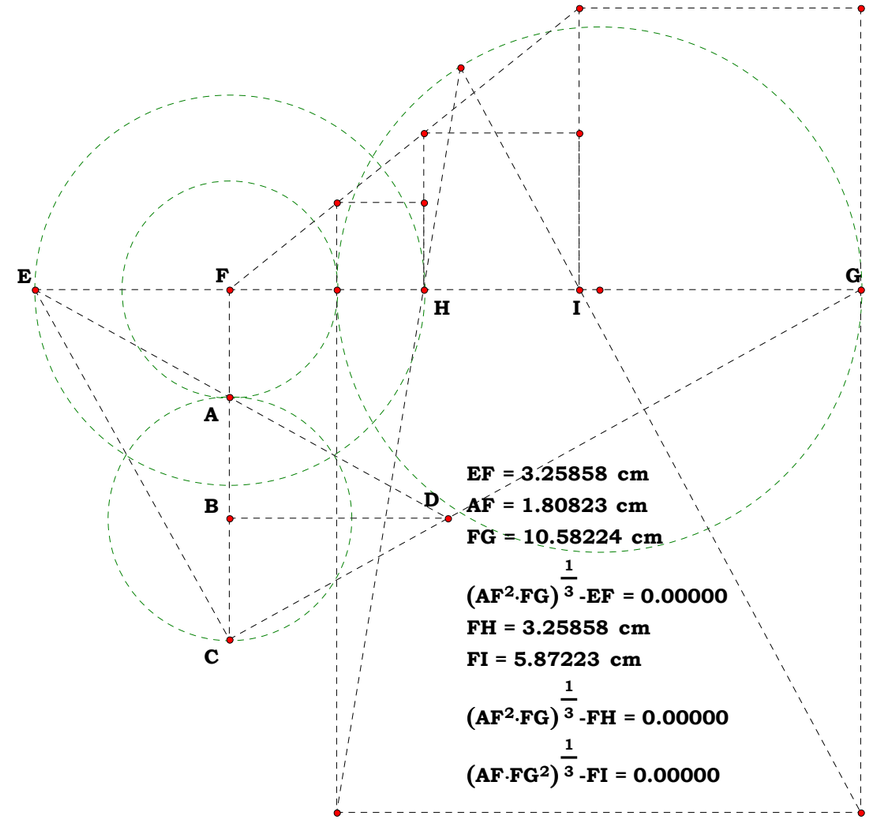
$$(AF^2 \cdot FG)^{\frac{1}{3}} - EF = 0.00000$$

**What can be said, however, is that the figure can be used to prove my original construction which would be much, much shorter than the original. However, my interest was no longer on the Delian Quest, but it was cooking to arrive at BAM, and this took time but once it did happen, what unfolded is a work which I probably have no time to finish, it consists of thousands of pages and covers all of how to use it for even logical operations.**

**The Delian Quest, essentially conquered, is making me comprehend that it is only the door to a much bigger place. The Delian Quest contains the search, the climax, and the after glow, but the results is contained in the volumes of BAM.**

**One has to remark, though, it is a very beautiful figure. Very simple, straightforward, and reasonable.**

**Filling in the rest of the figure, we see it.**




**It might be said that a elliptial construct proved the head of the figure, and another proved its hands.**

Unit.

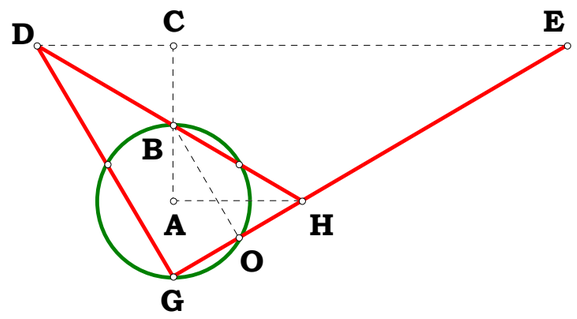
$AB := 1$

Given.

$N_1 := 2$ $\quad AH := N_1$

# 033005c

## $N^3$ and More




**Descriptions.**

$BH := \sqrt{AB^2 + AH^2}$ $\quad GO := \frac{AB \cdot 2 \cdot AB}{BH}$ $\quad HO := BH - GO$

$DH := \frac{BH \cdot BH}{HO}$ $\quad AC := \frac{AB \cdot DH}{BH}$ $\quad BO := \sqrt{BH^2 - HO^2}$

$DG := \frac{BO \cdot BH}{HO}$ $\quad DE := \frac{BH \cdot DG}{AB}$ $\quad CD := \frac{DG^2}{DE}$

$BC := AC - AB$ $\quad CE := DE - CD$ $\quad N_1^3 - \frac{CE}{BC} = 0$

**Definitions.**

$BH - \sqrt{N_1^2 + 1} = 0$ $\quad GO - \frac{2}{\sqrt{N_1^2 + 1}} = 0$ $\quad HO - \frac{(N_1 - 1) \cdot (N_1 + 1)}{\sqrt{N_1^2 + 1}} = 0$

$DH - \frac{\left(\sqrt{N_1^2 + 1}\right)^3}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad AC - \frac{N_1^2 + 1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad BO - \frac{2 \cdot N_1}{\sqrt{N_1^2 + 1}} = 0$

$DG - \frac{2 \cdot N_1 \cdot \sqrt{N_1^2 + 1}}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad DE - \frac{2 \cdot N_1 \cdot (N_1^2 + 1)}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad CD - \frac{2 \cdot N_1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$

$BC - \frac{2}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad CE - \frac{2 \cdot N_1^3}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad N_1^3 - N_1^3 = 0$

Given.

$N_2 := 4$

**Descriptions.**

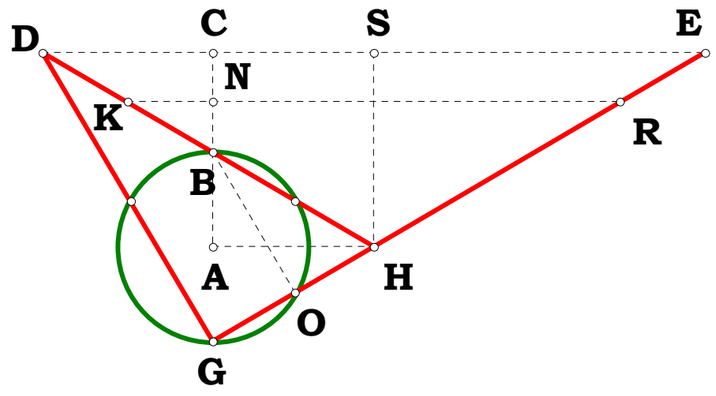
$BN := \frac{BC}{N_2}$ $KN := \frac{CD}{N_2}$

$CG := AC + AB$ $GN := 2 \cdot AB + \frac{BC}{N_2}$

$NR := \frac{CE \cdot GN}{CG}$

**Definitions.**

$\left[ {N_1}^3 \cdot N_2 - N_1 \cdot \left( N_2 - 1 \right) \right] - \frac{NR}{BN} = 0$




${N_1}^3 \cdot N_2 - \left( N_2 - 1 \right) \cdot N_1 = 26$ $\frac{NR}{BN} = 26$

$BN - \frac{2}{N_2 \cdot \left( N_1 - 1 \right) \cdot \left( N_1 + 1 \right)} = 0$ $KN - \frac{2 \cdot N_1}{N_2 \cdot \left( N_1 - 1 \right) \cdot \left( N_1 + 1 \right)} = 0$

$CG - \frac{2 \cdot {N_1}^2}{\left( N_1 - 1 \right) \cdot \left( N_1 + 1 \right)} = 0$ $GN - \left[ \frac{2 \cdot \left( N_2 \cdot {N_1}^2 - N_2 + 1 \right)}{N_2 \cdot \left( N_1 - 1 \right) \cdot \left( N_1 + 1 \right)} \right] = 0$

$NR - \frac{2 \cdot N_1 \cdot \left( N_2 \cdot {N_1}^2 - N_2 + 1 \right)}{N_2 \cdot \left( N_1 - 1 \right) \cdot \left( N_1 + 1 \right)} = 0$

033105

Unit.

$AB := 1$

Given.

$N_1 := .15$

$N_2 := .6$

Descriptions.

$CD := N_1$ $DE := 2 \cdot CD$ $AC := N_2$ $BC := AB - N_2$

$FG := DE + AB$ $AE := AC + CD$ $BD := BC + CD$

$AH := AE$ $BH := BD$

$AJ := \frac{AB^2 + AH^2 - BH^2}{2 \cdot AB}$ $HJ := \sqrt{AH^2 - AJ^2}$

Definitions.

$AE = 0.75$ $BD = 0.55$

$AJ = 0.63$ $HJ = 0.40694$

$CD - N_1 = 0$ $DE - 2 \cdot N_1 = 0$ $AC - N_2 = 0$ $BC - (1 - N_2) = 0$

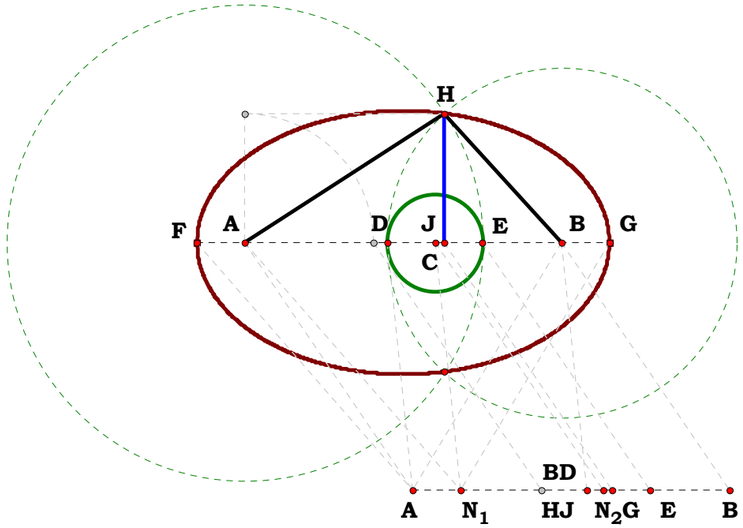
$FG - (2 \cdot N_1 + 1) = 0$ $AE - (N_2 + N_1) = 0$ $BD - (N_1 - N_2 + 1) = 0$

$AH - (N_2 + N_1) = 0$ $BH - (N_1 - N_2 + 1) = 0$

$AJ - (N_2 - N_1 + 2 \cdot N_1 \cdot N_2) = 0$ $HJ - \sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - N_2) \cdot (N_1 + 1)} = 0$

## Just Another Ellipse

**Given the difference between the foci and difference between the proportional radii, etc., etc.**




Unit = 1.00000

AB = 1.00000

$N_1$ = 0.15000

X = 3.00000

Y = 20.00000

$N_2$ = 0.60000

X = 12.00000

Y = 20.00000

AE = 0.75000

BD = 0.55000

AJ = 0.63000

HJ = 0.40694

041205A

**Unit.**

$CD := 1$

**Given.**

$N_1 := .278 \quad EF := N_1$

$N_2 := 3.095$

**Descriptions.**

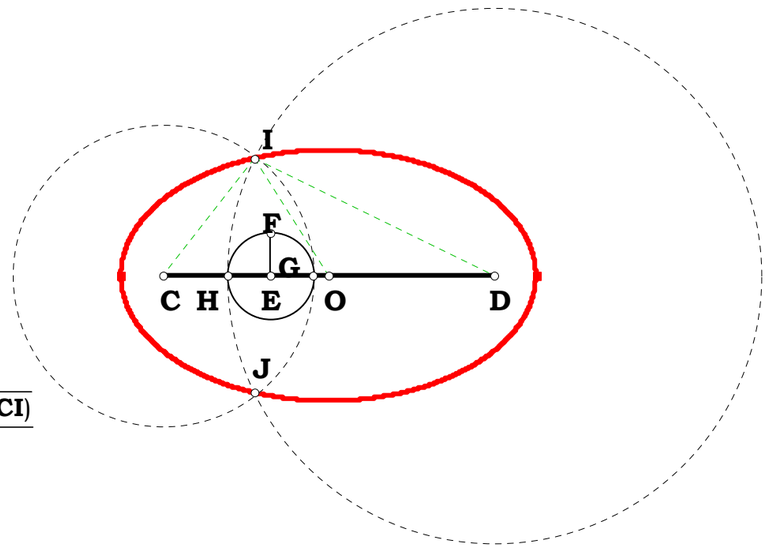
**Mixing methods of naming**

$$CO := \frac{CD}{2} \qquad CE := \frac{CD}{N_2} \qquad CG := CE + EF$$

$$DH := CD - CG + 2 \cdot EF \qquad CI := CG \qquad DI := DH$$

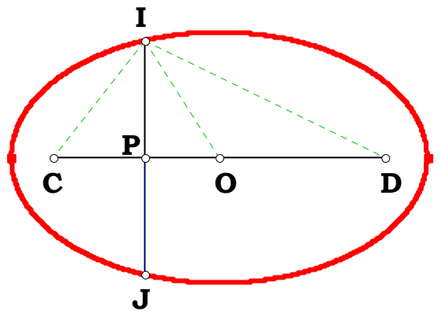
$$IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$




**Definitions.**

$$2 \cdot \frac{\sqrt{N_1 \cdot (1 + N_1) \cdot (N_2 - 1)}}{N_2} - IP = 0$$



# A trip to IO

**Unit.**

$CD := 1$

**Given.**

$N_1 := .278$

$N_2 := .3$

**041205B**

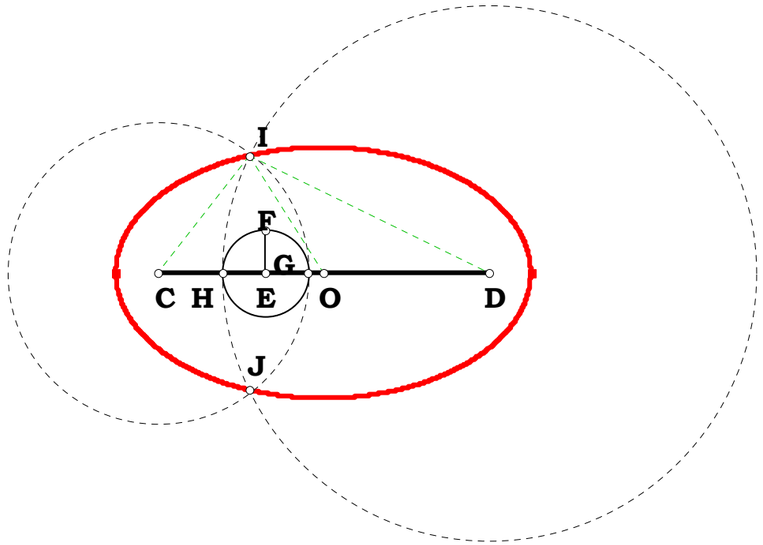
**Descriptions.**

$$CO := \frac{CD}{2} \qquad EF := N_1$$

$$CE := N_2 \qquad CG := CE + EF \qquad DH := CD - CG + 2 \cdot EF$$

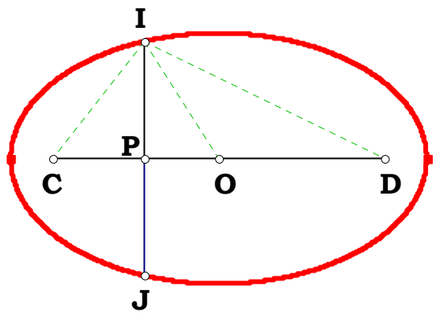
$$CI := CG \qquad DI := DH \qquad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$




**Definitions.**

$$2 \cdot \sqrt{\left[-N_1 \cdot N_2 \cdot (N_1 + 1) \cdot (N_2 - 1)\right]} - IP = 0$$



**Unit.**

$DH := 1$

**Given.**

$N_1 := 36 \qquad FH := N_1$

$\left(\sqrt{N_1}\right)^3$

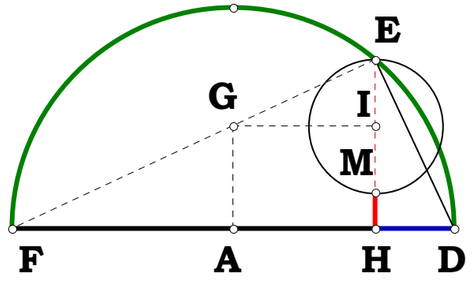
**041305b**

**Descriptions.**

$DF := DH + FH \qquad AD := \frac{DF}{2} \qquad EH := \sqrt{DH \cdot FH}$

$HI := \frac{EH \cdot AD}{FH} \qquad HM := EH - 2 \cdot (EH - HI)$

$DE := \sqrt{DH^2 + EH^2} \qquad EF := \sqrt{FH^2 + EH^2}$




**Definitions.**

$\frac{FH}{HM} = 216 \qquad \left(\frac{FH}{DH}\right)^{1.5} = 216 \qquad \frac{FH}{HM} - \left(\frac{DH}{HM}\right)^3 = 0 \qquad \frac{FH}{HM} - \left(\frac{EF}{DE}\right)^3 = 0$

$DF - (1 + N_1) = 0 \qquad AD - \frac{1 + N_1}{2} = 0 \qquad EH - \sqrt{N_1} = 0$

$HI - \frac{N_1 + 1}{2 \cdot \sqrt{N_1}} = 0 \qquad HM - \frac{1}{\sqrt{N_1}} = 0$

$DE - \sqrt{N_1 + 1} = 0 \qquad EF - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

$N_1^{\frac{3}{2}} = 216 \qquad (N_1)^{1.5} = 216 \qquad N_1^{\frac{3}{2}} - \left(\sqrt{N_1}\right)^3 = 0 \qquad N_1^{\frac{3}{2}} - \left[\frac{\sqrt{N_1 \cdot (N_1 + 1)}}{\sqrt{N_1 + 1}}\right]^3 = 0$

**Mathcad 15 will not reduce this last one.**

**One can see the taxing of logic in terms of precision on the computer.**

$N_1^{\frac{3}{2}} - \left(\frac{FH}{HM}\right) = 3.694822 \times 10^{-13} \qquad (N_1)^{1.5} - \left[\left(\frac{FH}{DH}\right)^{1.5}\right] = 0 \qquad N_1^{\frac{3}{2}} - \left(\frac{DH}{HM}\right)^3 = 1.13687 \times 10^{-12} \qquad N_1^{\frac{3}{2}} - \left(\frac{EF}{DE}\right)^3 = 0$

041405

Unit. $AB := 1$

Given.

$X := 4$

$Y := 20$

## Straight Cubes

**Historically, the search for a figure which expresses cubes and cube roots has gone as far back as Plato, this figure demonstrates that the solution never found is due to the unit as traditionally used in constructions has to be not linear, but planer, i.e. the unit is proportional to the figure. There are at least three different structures in this figure which expresses the same ratio, I simply write up the major one.**

**Descriptions.**

$$AC := 2 \cdot AB \qquad JX := AC \cdot \frac{X}{Y} \qquad KX := AC - JX \qquad MX := \sqrt{JX \cdot KX}$$

$$KM := \sqrt{MX^2 + KX^2} \qquad MT := \frac{KM}{2} \qquad GM := KM \cdot \frac{MT}{MX}$$

$$HM := 2 \cdot GM \qquad HX := HM - MX \qquad \left(\frac{MX}{JX}\right)^3 - \frac{HX}{JX} = 0$$

**Definitions.**

$$AC - 2 = 0 \qquad JX - 2 \cdot \frac{X}{Y} = 0 \qquad KX - \frac{2 \cdot (Y - X)}{Y} = 0$$

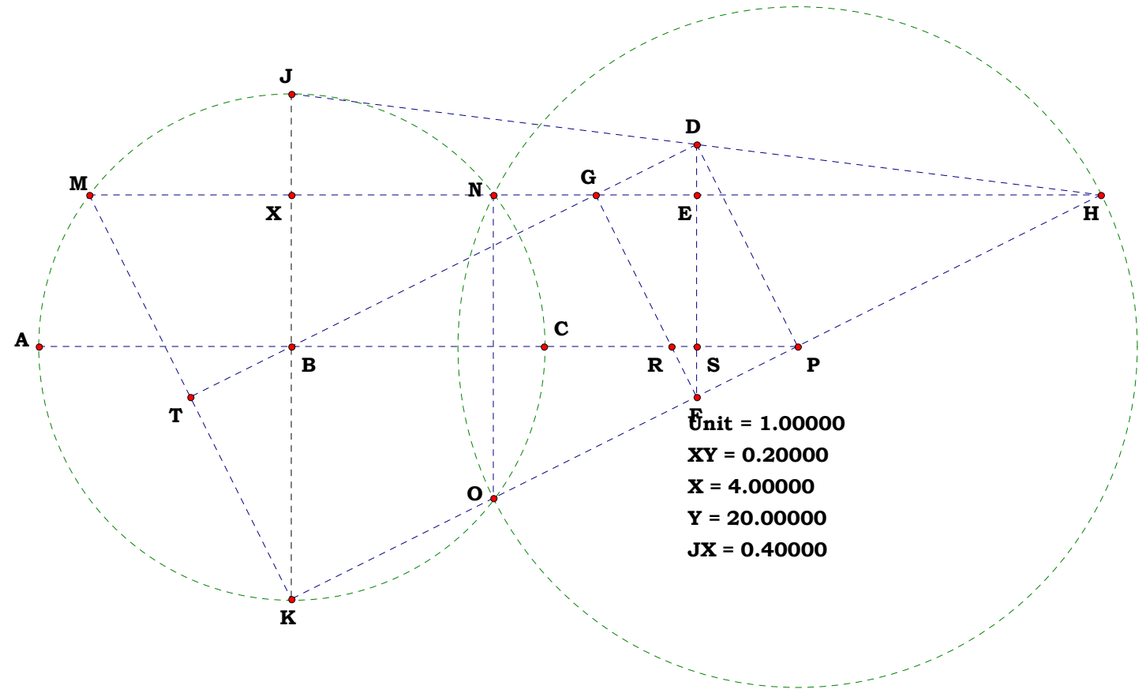
$$MX - \frac{\sqrt{4 \cdot X \cdot (Y - X)}}{Y} = 0 \qquad KM - \frac{\sqrt{4 \cdot (Y - X)}}{\sqrt{Y}} = 0$$

$$MT - \frac{\sqrt{Y - X}}{\sqrt{Y}} = 0 \qquad GM - \frac{Y - X}{\sqrt{X \cdot (Y - X)}} = 0 \qquad HM - \frac{2 \cdot (Y - X)}{\sqrt{X \cdot (Y - X)}} = 0$$

$$HX - \frac{2 \cdot (X - Y)^2}{Y \cdot \sqrt{X \cdot Y - X^2}} = 0 \qquad \frac{MX}{JX} - \frac{\sqrt{X \cdot Y - X^2}}{X} = 0 \qquad \frac{HX}{JX} - \frac{(X - Y)^2}{X \cdot \sqrt{X \cdot Y - X^2}} = 0$$

$$\left(\frac{\sqrt{X \cdot Y - X^2}}{X}\right)^3 = 8 \qquad \frac{(X - Y)^2}{X \cdot \sqrt{X \cdot Y - X^2}} = 8$$

$$\frac{(Y - X) \cdot \sqrt{X \cdot Y - X^2}}{X^2} - \frac{(X - Y)^2}{X \cdot \sqrt{X \cdot Y - X^2}} = 0$$



042205

Unit.

Given.

$N_1 := 2.604$ $DE := N_1$

$N_2 := 2.234$ $FI := N_2$

**Given the major axis and the difference between the two foci, whatis the minor axis?**

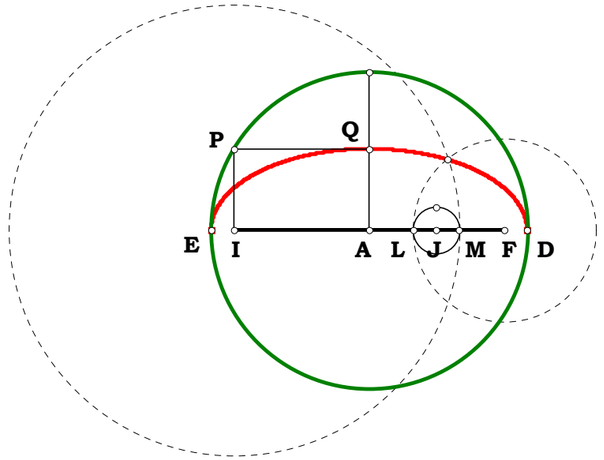
Descriptions.

$$EI := \frac{DE - FI}{2} \qquad DI := DE - EI$$

$$EP := \sqrt{EI \cdot DI} \qquad AQ := EP$$

Definitions.

$$AQ - \frac{1}{2} \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)} = 0$$




ED = 2.083 in.

FI = 1.787 in.

AQ = 0.535 in.

$$AQ - \frac{\sqrt{(ED+FI) \cdot (ED-FI)}}{2} = 0.000 \text{ in.}$$

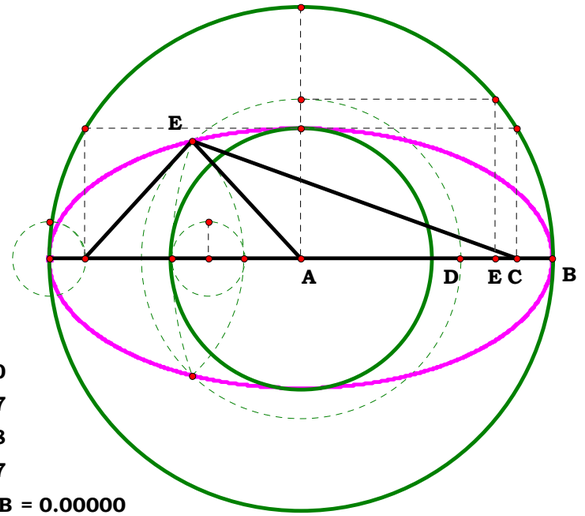
**Sum of Area**

**042305A**

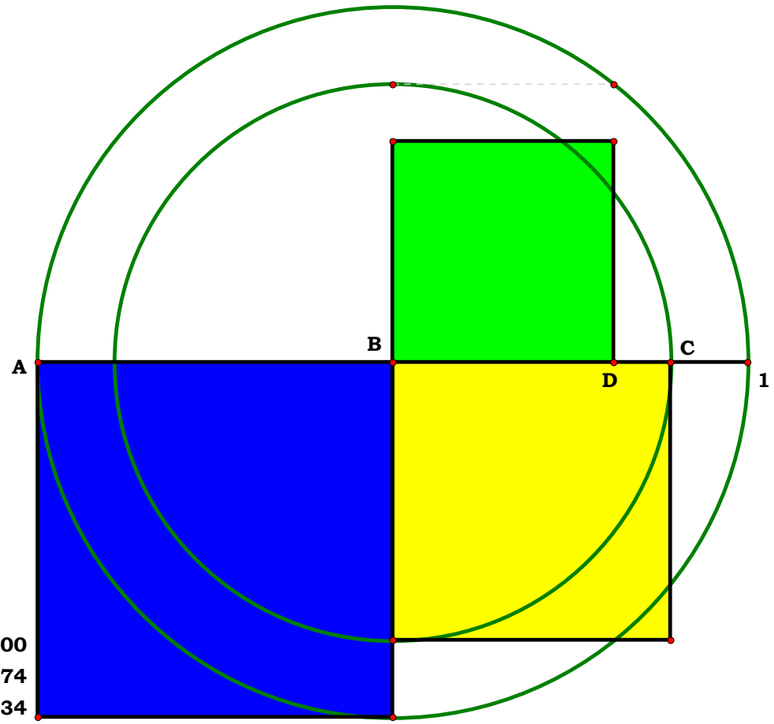
**Descriptions.**

**It seems that the bottom two figures came to mind while I was examining an ellipse. This consist of three different applications. This write-will be Plate A.**

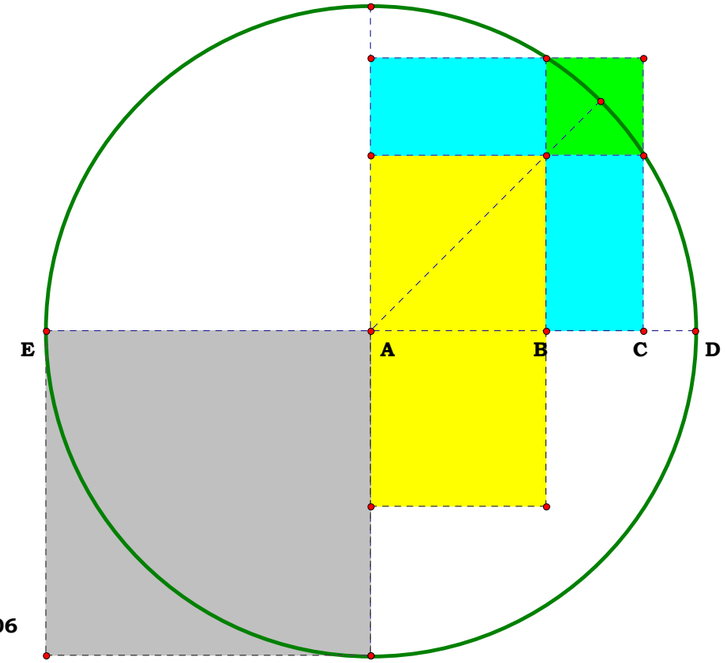
**Is there a perfect way to write-up an ellipse? This whole ellipse is wholly determined by the ratio AB to AC.**




**AB = 1.00000**
**AC = 0.85647**
**AD = 0.63408**
**AE = 0.77327**
$\sqrt{AD^2+AE^2}-AB = 0.00000$




**AB = 1.00000**
**AC = 0.78274**
**AD = 0.62234**
$AB^2-(AC^2+AD^2) = 0.00000$




**Unit = 1.00000**
**XY = 0.54037**
**X = 10.80742**
**Y = 20.00000**
**AB = 0.54037**
**AC = 0.84143**
**AD = 1.00000**
**AC-AB = 0.30106**
**BC = 0.30106**
$AD^2-(2\cdot AB^2+2\cdot AB\cdot BC+BC^2) = 0.00000$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 17$

Plate A.

042305A

Descriptions of ellipse

$AC := \frac{X}{Y}$ $AC = 0.85$ $BC := AB - AC$ $BP := 2 \cdot AB$

$AF := \sqrt{(BP - BC) \cdot BC}$ $AF = 0.526783$ $CO := 2 \cdot AC$

$NM := 2 \cdot BC$

Descriptions AE, either CM or NO has to be given.

Given NO: $NO := .63455$

$CM := CO - NO + NM$ $DO := NO$ $CE := CM$

$AD := \frac{\sqrt{2 \cdot DO^2 - CO^2 + 2 \cdot CE^2}}{2}$ (Pythagoras Revisted) $AD = 0.641135$

$DP := AB + AD$ $BD := BP - DP$ $AE := \sqrt{DP \cdot BD}$ $AE = 0.767428$

$\sqrt{AD^2 + AE^2} - AB = 0$

Definitions. The equation for AD is indifferent as to which side, DO or CD, of the elliptical triangle given.

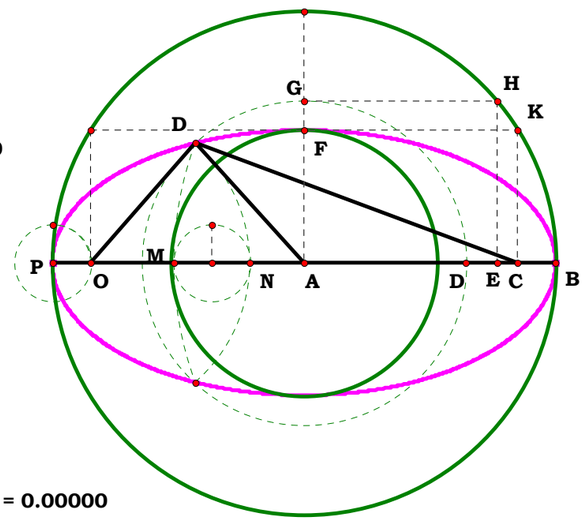
$AC - \frac{X}{Y} = 0$ $BC - \frac{Y - X}{Y} = 0$ $BP - 2 = 0$ $AF - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$ $CO - \frac{2 \cdot X}{Y} = 0$

$NM - \frac{2 \cdot (Y - X)}{Y} = 0$ $CM - (2 - NO) = 0$ $DO - NO = 0$ $CE - (2 - NO) = 0$ $AD - \frac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y} = 0$

$DP - \frac{Y + \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0$ $BD - \frac{Y - \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0$ $AE - \frac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y} = 0$

$\sqrt{\left[\frac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1$ $\sqrt{\left[\frac{\sqrt{CM \cdot Y^2 \cdot (CM - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot CM \cdot Y^2 - CM^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1$

Unit = 1.00000
XY = 0.85000
X = 17.00000
Y = 20.00000
AB = 1.00000
AC = 0.85000
AD = 0.64114
AE = 0.76743
AF = 0.52678
CM = 1.36545
NO = 0.63455
$\sqrt{AD^2+AE^2}-AB$ = 0.00000



042305B

Descriptions.

Unit.

$AB := 1$

$BE := AB$

Given.

$Y := 20$

$X := 12$

$BC := \frac{X}{Y}$ $AE := 2 \cdot AB$ $MN := AE$ $FN := AB + BC$

$FG := \sqrt{(MN - FN) \cdot FN}$ $BD := FG$

$AB^2 - \left(BC^2 + BD^2\right) = 0$

Definitions.

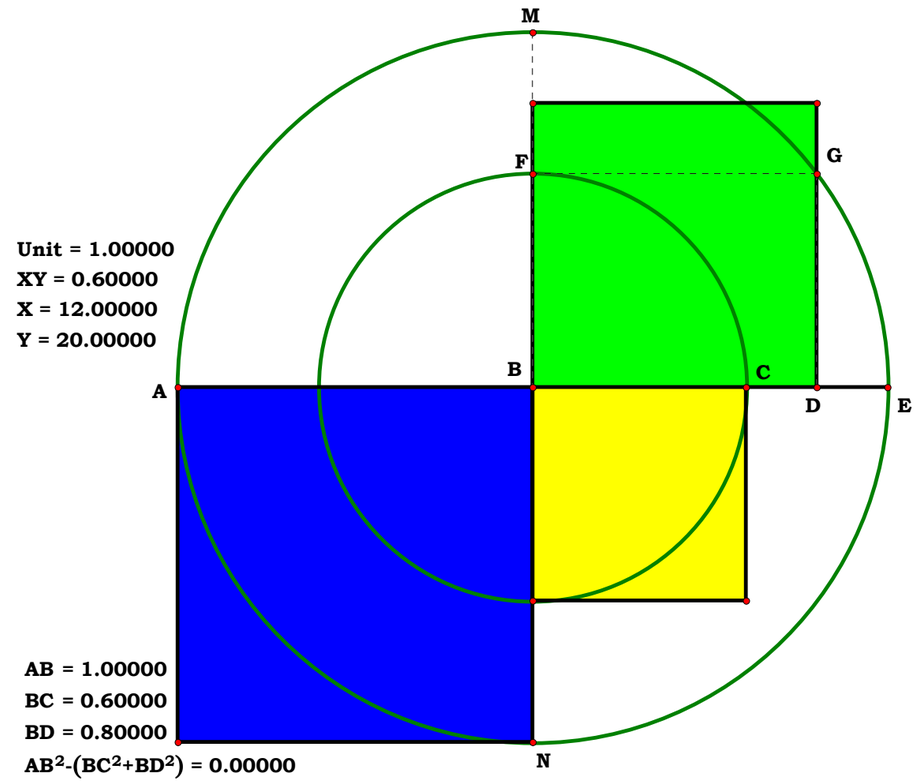
$BC - \frac{X}{Y} = 0$ $AE - 2 = 0$ $MN - 2 = 0$

$FN - \frac{X + Y}{Y} = 0$ $FG - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

$BD - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

## Sum of Area

Plate B.



Unit.

$AD := 1$

$AE := AD$

Given.

$Y := 20$

$X := 12$

**Sum of Area**

**Plate C.**

**Descriptions.**

$AB := \frac{X}{Y}$ $AB = 0.6$ $BE := AB + AE$ $DE := 2 \cdot AD$

$BD := DE - BE$ $AC := \sqrt{BE \cdot BD}$ $BC := AC - AB$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$

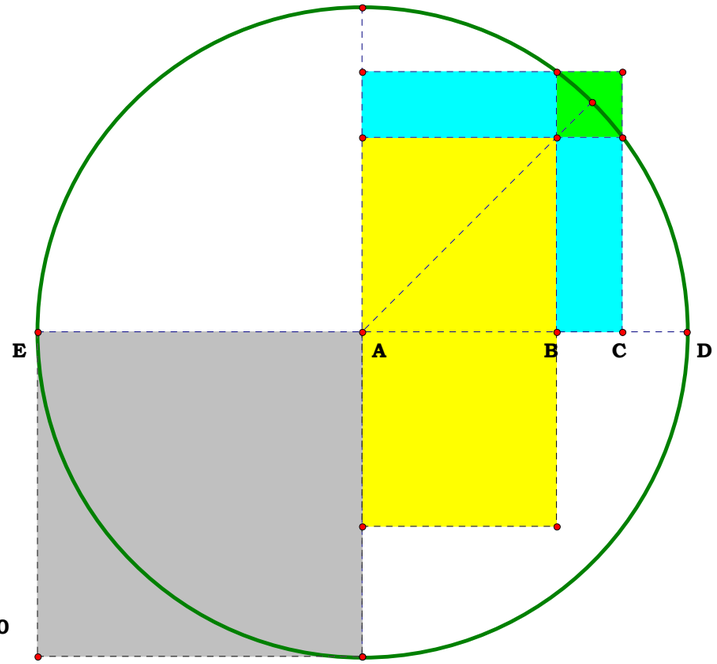
**Definitions.**

$AB - \frac{X}{Y} = 0$ $BE - \frac{X + Y}{Y} = 0$ $DE - 2 = 0$

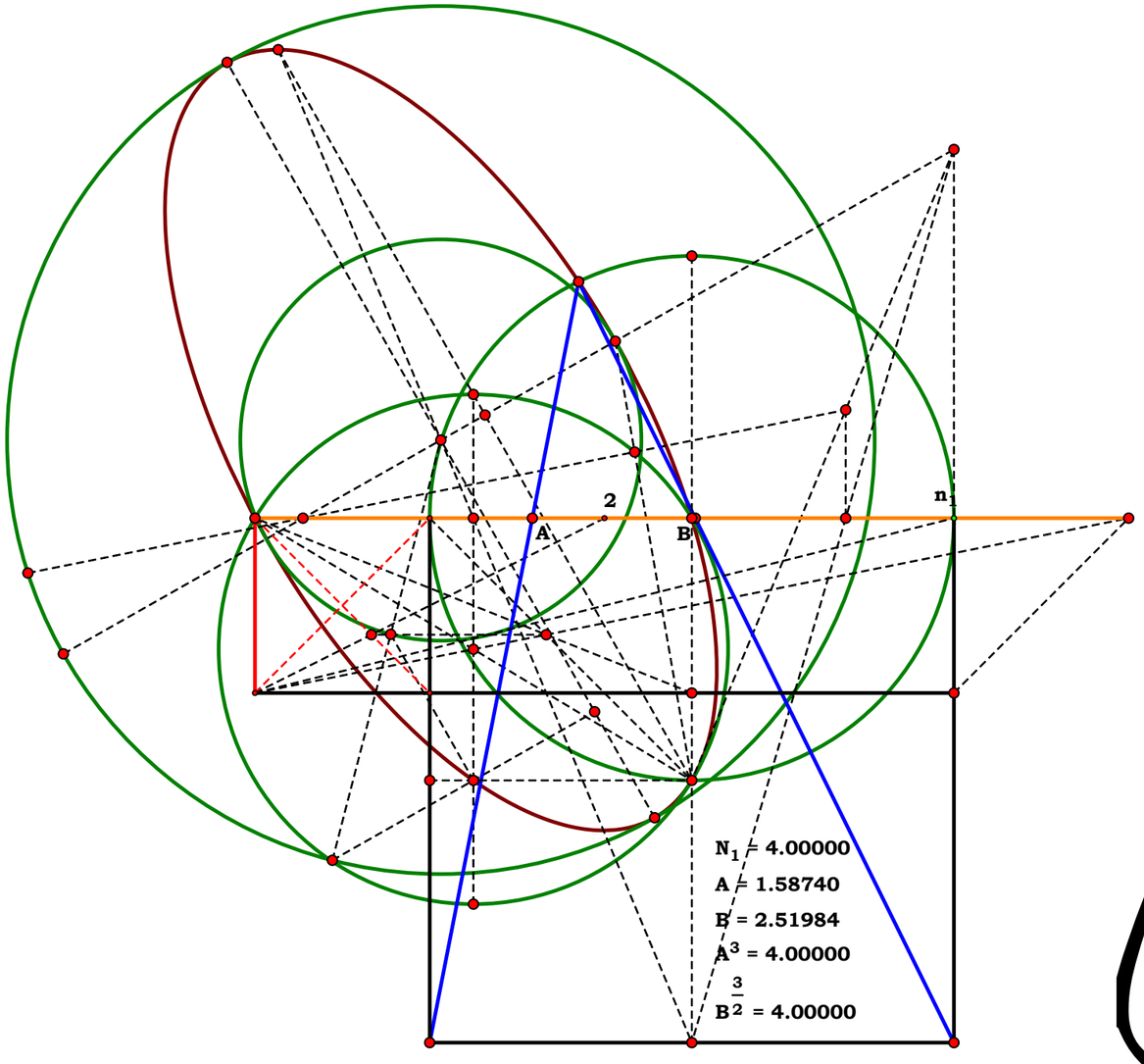
$BD - \frac{Y - X}{Y} = 0$ $AC - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

$BC - \frac{\sqrt{Y^2 - X^2} - X}{Y} = 0$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$




Unit = 1.00000
XY = 0.60000
X = 12.00000
Y = 20.00000
AB = 0.60000
AC = 0.80000
AD = 1.00000
AC-AB = 0.20000
BC = 0.20000
$AD^2-(2\cdot AB^2+2\cdot AB\cdot BC+BC^2)$ = 0.00000




# The Delian Quest 2006

**John Clark**

012306

# A January Ellipse

Unit. $AB := 1$

Given.

$x := 9 \qquad z := 15$

$w := 20 \qquad y := 10$

**Descriptions.**

$$Bx := \frac{x}{w} \quad Ax := AB - Bx \quad Bz := \frac{z}{y} \quad Hx := \sqrt{Ax \cdot (2 \cdot AB - Ax)}$$

$$Gx := \frac{Bx \cdot Hx}{Bz + Hx} \quad DG := \frac{Hx \cdot (Bx - Gx)}{Bx} \quad BF := \frac{Bx \cdot Bz}{(Bz - Hx)}$$

$$Fx := \left|BF - Bx\right| \quad EF := \frac{Bz \cdot Fx}{Bx}$$

**Definitions.**

$$Bx - \frac{x}{w} = 0 \quad Ax - \left(\frac{w - x}{w}\right) = 0 \quad Bz - \frac{z}{y} = 0$$

$$Hx - \frac{\sqrt{(w + x) \cdot (w - x)}}{w} = 0$$

$$Gx - \frac{x \cdot \sqrt{(w - x) \cdot (w + x)} \cdot y}{w \cdot \left[y \cdot \sqrt{(w - x) \cdot (w + x)} + w \cdot z\right]} = 0$$

$$DG - \frac{\sqrt{(w - x) \cdot (w + x)} \cdot z}{y \cdot \sqrt{(w - x) \cdot (w + x)} + w \cdot z} = 0$$

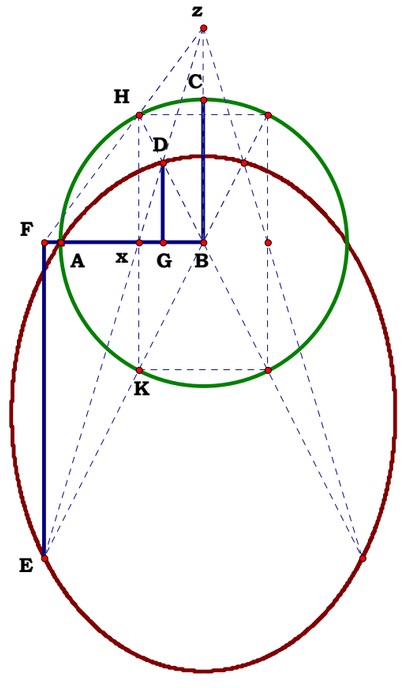
$$BF - \frac{x \cdot z}{w \cdot z - y \cdot \sqrt{(w - x) \cdot (w + x)}} = 0$$

$$Fx - \frac{\sqrt{w^2 - x^2} \cdot x \cdot y}{\left|w^2 \cdot z - w \cdot y \cdot \sqrt{w^2 - x^2}\right|} = 0$$

$$EF - \frac{w \cdot z \cdot \sqrt{w^2 - x^2}}{\left|w^2 \cdot z - w \cdot y \cdot \sqrt{w^2 - x^2}\right|} = 0$$

Unit = 1.00000
xw = 0.45000
x = 9.00000
w = 20.00000

zy = 1.50000
z = 15.00000
y = 10.00000
Hx = 0.89303




$\frac{w-x}{w}$-Ax = 0.00000

$\frac{z}{y}$-Bz = 0.00000

$\frac{\sqrt{(w+x)\cdot(w-x)}}{w}$-Hx = 0.00000

$\frac{x\cdot y\cdot\sqrt{(w-x)\cdot(w+x)}}{w\cdot\left(y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z\right)}$-Gx = 0.00000

$\frac{\sqrt{(w-x)\cdot(w+x)}\cdot z}{y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z}$-DG = 0.00000

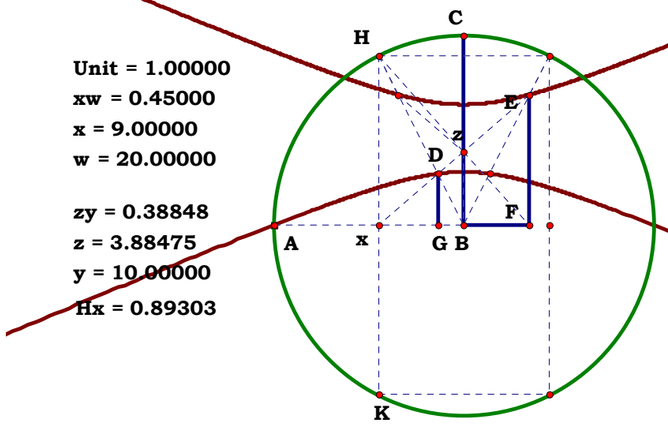
$\frac{x\cdot z}{w\cdot z-y\cdot\sqrt{(w-x)\cdot(w+x)}}$-BF = 0.00000

$\frac{x\cdot y\cdot\sqrt{w^2-x^2}}{\left|w^2\cdot z-w\cdot y\cdot\sqrt{w^2-x^2}\right|}$-Fx = 0.00000

$\frac{w\cdot z\cdot\sqrt{w^2-x^2}}{\left|w^2\cdot z-w\cdot y\cdot\sqrt{w^2-x^2}\right|}$-EF = 0.00000

Unit = 1.00000
xw = 0.45000
x = 9.00000
w = 20.00000

zy = 0.38848
z = 3.88475
y = 10.00000
Hx = 0.89303




$\frac{w-x}{w}$-Ax = 0.00000

$\frac{z}{y}$-Bz = 0.00000

$\frac{\sqrt{(w+x)\cdot(w-x)}}{w}$-Hx = 0.00000

$\frac{x\cdot y\cdot\sqrt{(w-x)\cdot(w+x)}}{w\cdot\left(y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z\right)}$-Gx = 0.00000

$\frac{\sqrt{(w-x)\cdot(w+x)}\cdot z}{y\cdot\sqrt{(w-x)\cdot(w+x)}+w\cdot z}$-DG = 0.00000

$\frac{x\cdot z}{w\cdot z-y\cdot\sqrt{(w-x)\cdot(w+x)}}$-BF = -0.69295

$\frac{x\cdot y\cdot\sqrt{w^2-x^2}}{\left|w^2\cdot z-w\cdot y\cdot\sqrt{w^2-x^2}\right|}$-Fx = 0.00000

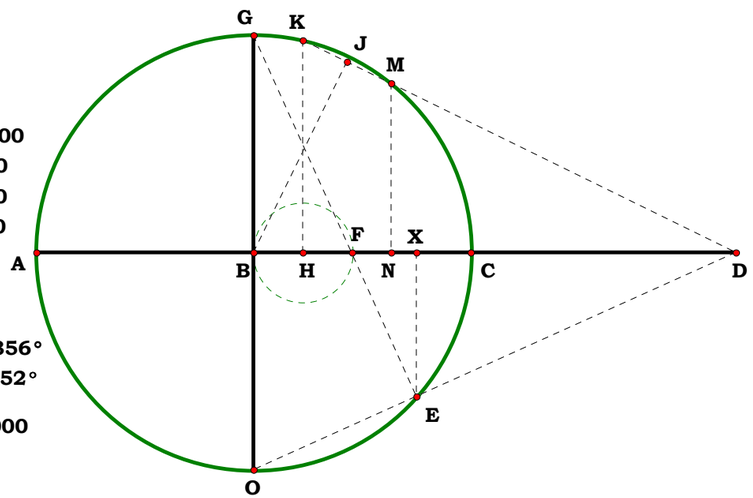
$\frac{w\cdot z\cdot\sqrt{w^2-x^2}}{\left|w^2\cdot z-w\cdot y\cdot\sqrt{w^2-x^2}\right|}$-EF = 0.00000

Unit. $AB := 1$

Given. $X := 15$

$Y := 20$

**Given CX what is CN?**




Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000

m∠MBG = 39.13356°
m∠KBG = 13.04452°
$\frac{m\angle MBG}{m\angle KBG}$ = 3.00000

012906

**Descriptions.**

$AC := 2 \cdot AB \quad BX := \frac{X}{Y} \quad AX := AB + BX \quad CX := AC - AX$

$EX := \sqrt{AX \cdot CX} \quad FX := \frac{BX \cdot EX}{AB + EX} \quad BD := EX \cdot \frac{AB}{FX}$

$BF := BX - FX \quad BH := \frac{BF}{2} \quad AH := AB + BH \quad CH := AC - AH$

$HK := \sqrt{AH \cdot CH} \quad DH := BD - BH \quad DK := \sqrt{HK^2 + DH^2}$

$DJ := DH \cdot \frac{DK}{BD} \quad JK := DK - DJ \quad DM := DK - 2 \cdot JK$

$DN := DH \cdot \frac{DM}{DK} \quad BN := BD - DN \quad CN := AB - BN$

**Definitions.**

$AC - 2 = 0 \quad BX - \frac{X}{Y} = 0 \quad AX - \frac{X+Y}{Y} = 0 \quad CX - \frac{Y-X}{Y} = 0 \quad EX - \frac{\sqrt{(Y-X)\cdot(X+Y)}}{Y} = 0 \quad FX - \frac{X\cdot\sqrt{(Y-X)\cdot(X+Y)}}{Y\cdot\left[Y+\sqrt{(Y-X)\cdot(X+Y)}\right]} = 0 \quad BD - \frac{Y+\sqrt{(Y-X)\cdot(X+Y)}}{X} = 0$

$BF - \frac{X}{Y+\sqrt{Y^2-X^2}} = 0 \quad BH - \frac{X}{2\cdot\left(Y+\sqrt{Y^2-X^2}\right)} = 0 \quad AH := \frac{X+2\cdot Y+2\cdot\sqrt{Y^2-X^2}}{2\cdot\left(Y+\sqrt{Y^2-X^2}\right)} \quad CH - \frac{2\cdot Y-X+2\cdot\sqrt{Y^2-X^2}}{2\cdot\left(Y+\sqrt{Y^2-X^2}\right)} = 0$

$HK - \frac{\sqrt{\left(X+2\cdot Y+2\cdot\sqrt{Y^2-X^2}\right)\cdot\left(2\cdot Y-X+2\cdot\sqrt{Y^2-X^2}\right)}}{2\cdot\left(Y+\sqrt{Y^2-X^2}\right)} = 0 \quad DH - \frac{4\cdot Y\cdot\sqrt{Y^2-X^2}-3\cdot X^2+4\cdot Y^2}{2\cdot X\cdot\left(Y+\sqrt{Y^2-X^2}\right)} = 0 \quad DK - \frac{\sqrt{8\cdot X^2\cdot Y^2-8\cdot Y^4-X^4-8\cdot Y^3\cdot\sqrt{Y^2-X^2}+4\cdot X^2\cdot Y\cdot\sqrt{Y^2-X^2}}}{X\cdot\sqrt{X^2-2\cdot Y\cdot\sqrt{Y^2-X^2}-2\cdot Y^2}} = 0$

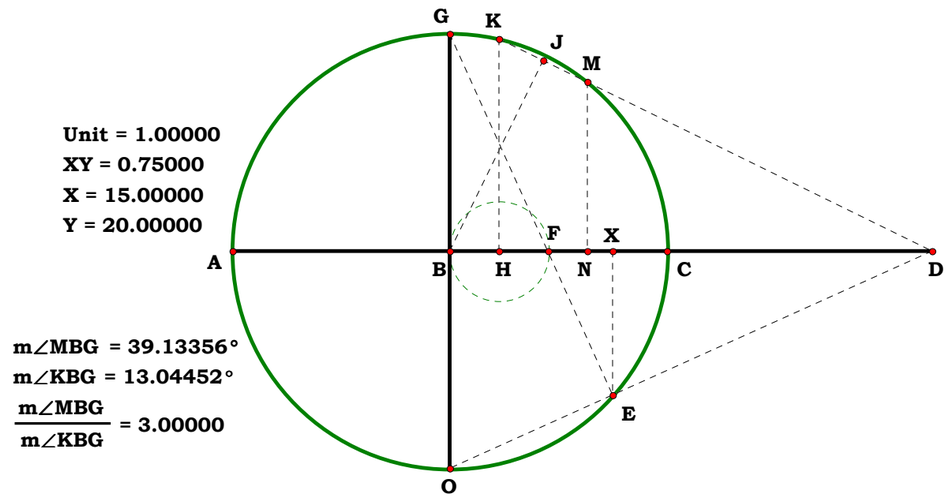
$DJ - \frac{\left(4\cdot Y\cdot\sqrt{Y^2-X^2}-3\cdot X^2+4\cdot Y^2\right)\cdot\sqrt{\left(4\cdot X^2\cdot Y-8\cdot Y^3\right)\cdot\sqrt{Y^2-X^2}+8\cdot X^2\cdot Y^2-X^4-8\cdot Y^4}}{2\cdot X\cdot\left(Y+\sqrt{Y^2-X^2}\right)\cdot\left[Y+\sqrt{-(X-Y)\cdot(X+Y)}\right]\cdot\sqrt{X^2-2\cdot Y\cdot\sqrt{Y^2-X^2}-2\cdot Y^2}} = 0 \quad JK - \frac{\sqrt{8\cdot X^2\cdot Y^2-8\cdot Y^4-X^4-8\cdot Y^3\cdot\sqrt{Y^2-X^2}+4\cdot X^2\cdot Y\cdot\sqrt{Y^2-X^2}}\cdot X}{2\cdot\sqrt{X^2-2\cdot Y\cdot\sqrt{Y^2-X^2}-2\cdot Y^2}\cdot\left(2\cdot Y\cdot\sqrt{Y^2-X^2}-X^2+2\cdot Y^2\right)} = 0$

$$DM - \frac{2\cdot\sqrt{\left(4\cdot X^2\cdot Y - 8\cdot Y^3\right)\cdot\sqrt{Y^2-X^2}+8\cdot X^2\cdot Y^2 - X^4 - 8\cdot Y^4}\cdot\left(X^2 - Y\cdot\sqrt{Y^2-X^2} - Y^2\right)}{X\cdot\left(X^2 - 2\cdot Y\cdot\sqrt{Y^2-X^2} - 2\cdot Y^2\right)^{\frac{3}{2}}} = 0$$

$$DN - \frac{\left(4\cdot Y\cdot\sqrt{Y^2-X^2} - 3\cdot X^2 + 4\cdot Y^2\right)\cdot\left(X^2 - Y\cdot\sqrt{Y^2-X^2} - Y^2\right)}{X\cdot\left(Y+\sqrt{Y^2-X^2}\right)\cdot\left(X^2 - 2\cdot Y\cdot\sqrt{Y^2-X^2} - 2\cdot Y^2\right)} = 0$$

$$BN - \frac{X\cdot\left(2\cdot X^2 - 3\cdot Y\cdot\sqrt{Y^2-X^2} - 3\cdot Y^2\right)}{3\cdot X^2\cdot Y - 4\cdot Y^3 + X^2\cdot\sqrt{Y^2-X^2} - 4\cdot Y^2\cdot\sqrt{Y^2-X^2}} = 0$$

$$CN - \frac{(Y-X)\cdot\left[(X+4\cdot Y)\cdot\sqrt{Y^2-X^2}+X\cdot Y - 2\cdot X^2 + 4\cdot Y^2\right]}{\left(4\cdot Y^2 - X^2\right)\cdot\sqrt{Y^2-X^2}+4\cdot Y^3 - 3\cdot X^2\cdot Y} = 0$$



Unit.

$AB := 1$

Given.

$Y := 20$

$X := 9$

102606A

Descriptions.

$AN := \frac{X}{Y}$ $AK := AB$ $AF := AB$ $KN := \sqrt{AN^2 + AK^2}$

$FG := \frac{AN \cdot 2 \cdot AB}{KN}$ $FG = 0.820729$

$GH := \frac{AB \cdot FG}{KN}$ $GH = 0.748441$ $GO := GH - AN$

$FH := \frac{AN \cdot FG}{KN}$ $AH := AF - FH$ $AH = 0.663202$

$CN := \frac{GO \cdot AB}{FH}$ $AC := CN + AN$ $AC = 1.336111$

$AJ := GH$ $CJ := AC - AJ$ $CE := CJ$ $CE = 0.58767$

$\sqrt{AC \cdot AJ} = 1$ Circles are to each other as their radii.

Definitions.

$AN - \frac{X}{Y} = 0$ $AK - 1 = 0$ $AF - 1 = 0$

$KN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$ $FG - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$

$GH - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $GO - \frac{X \cdot Y^2 - X^3}{Y \cdot (X^2 + Y^2)} = 0$ $FH - \frac{2 \cdot X^2}{X^2 + Y^2} = 0$ $AH - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$ $CN - \frac{Y^2 - X^2}{2 \cdot X \cdot Y} = 0$

$AC - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0$ $AJ - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $CJ - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot (X^2 + Y^2)} = 0$ $CE - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot (X^2 + Y^2)} = 0$

## Angles and Area Plate A

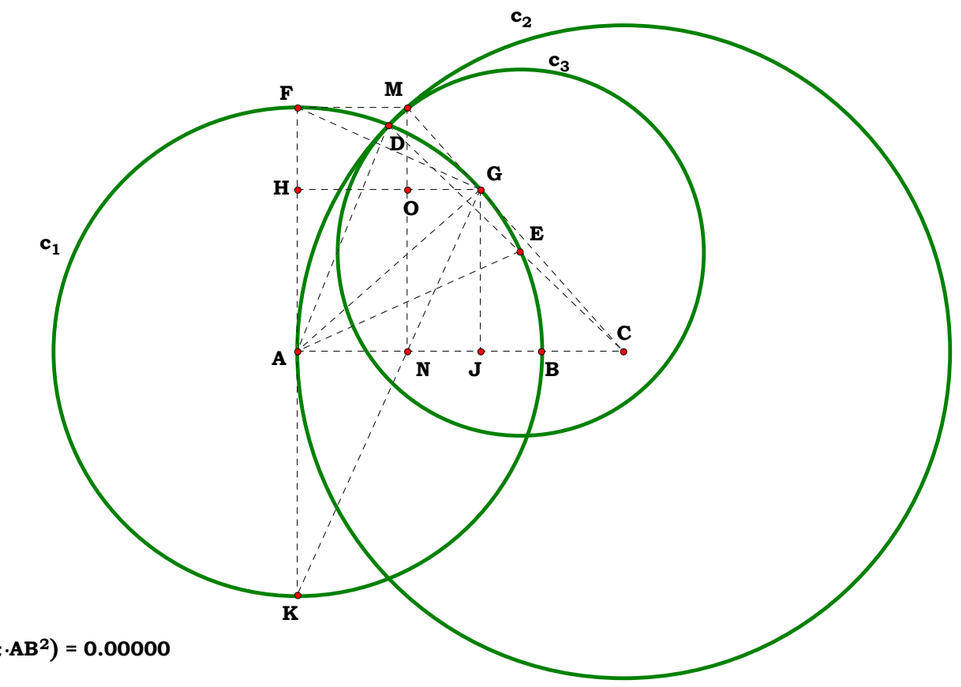
**While exploring trisection, I found something else actually worth exploring. An entirely different way to think of an angle. An angle is takes the so called Pythagorean down to its elements, and it is wholly independent of any particular angle.**

Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000
AB = 1.00000
AN = 0.45000
FG = 0.82073
GH = 0.74844
AJ = 0.74844
AH = 0.66320
CE = 0.58767
AC = 1.33611
$\sqrt{AC \cdot AJ}$ = 1.00000

$\frac{X}{Y}$ = 0.45000

$AN - \frac{X}{Y}$ = 0.00000

$\pi \cdot AB^2$ = 3.14159
$\pi \cdot AC^2$ = 5.60835
$\pi \cdot AJ^2$ = 1.75981
$\sqrt{(\pi \cdot AC^2) \cdot (\pi \cdot AJ^2)} - (\pi \cdot AB^2)$ = 0.00000




$\pi \cdot AB^2 = 3.141593$

$\pi \cdot AC^2 = 5.608349$

$\pi \cdot AJ^2 = 1.759806$

$\sqrt{\pi \cdot AJ^2 \cdot \pi \cdot AC^2} - \pi \cdot AB^2 = 0$

**Unit.**

$AB := 1$

**Given.**

$Y := 20$

$X := 9$

**102606B**

**Descriptions.**

$AC := \frac{X}{Y}$ $BC := AB - AC$

$BH := \frac{BC^2}{2 \cdot AB}$ **(Pythagoras Revisited)** $BH = 0.15125$

$BD := 2 \cdot BH$ $AD := AB - BD$ $FG := BD$

$\sqrt{AB \cdot BD} - BC = 0$

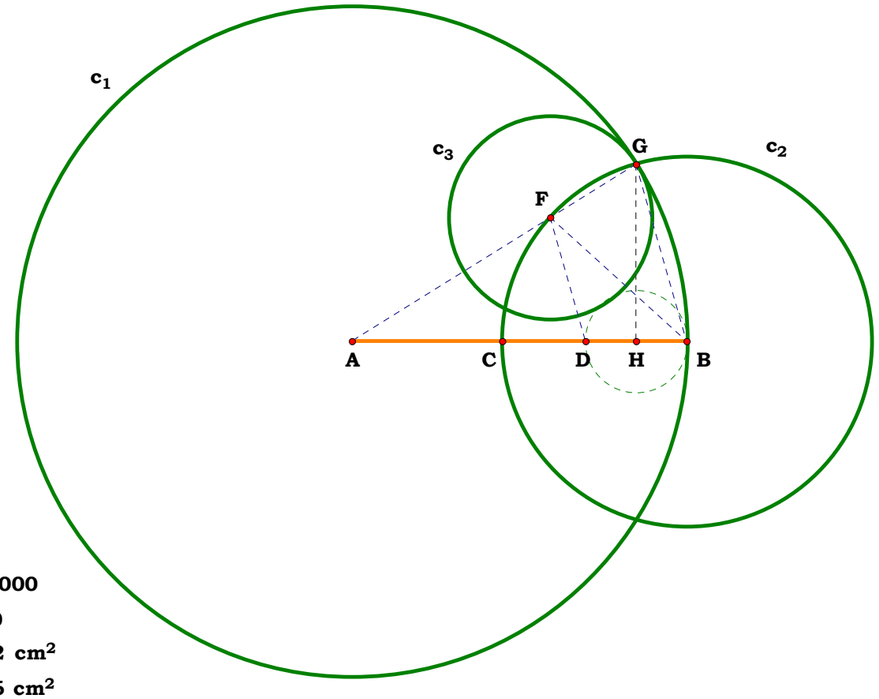
**Definitions.**

$AC - \frac{X}{Y} = 0$ $BC - \frac{Y - X}{Y} = 0$ $BH - \frac{(X - Y)^2}{2 \cdot Y^2} = 0$

$BD - \frac{(X - Y)^2}{Y^2} = 0$ $AD - \frac{X \cdot (2 \cdot Y - X)}{Y^2} = 0$

$FG - \frac{(X - Y)^2}{Y^2} = 0$

## Angles and Area Plate B




Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AB = 1.00000
AC = 0.45000
AD = 0.69750
BC = 0.55000
BD = 0.30250
BH = 0.15125

$\sqrt{AB \cdot BD}$-BC = 0.00000
$\sqrt{BD}$-BC = 0.00000
Area $c_1$ = 99.02532 $cm^2$
Area $c_2$ = 29.95516 $cm^2$
Area $c_3$ = 9.06144 $cm^2$
$\sqrt{(\text{Area } c_1) \cdot (\text{Area } c_3)}$-(Area $c_2$) = 0.00000 $cm^2$

110706 Sketchbook A

Unit.

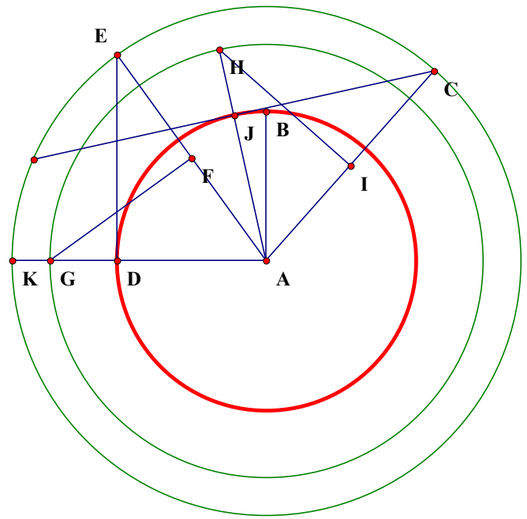
Given.

$N_1 := 3.12208$ $AD := N_1$

$N_2 := 5.31813$ $AC := N_2$

## Going around in a circle

**What is the tanget to AD from C? Although no one in their right mind would do this for a circle, it is essential as an introduction to finding the inverse ellilpse for any external point to it.**




Descriptions.

$AE := AC$

$DE := \sqrt{AE^2 - AD^2}$ $AF := \frac{AE}{2}$

$AG := \frac{AE \cdot AF}{AD}$ $AH := AG$ $AI := \frac{AC}{2}$

$AJ := \frac{AI \cdot AC}{AH}$ $AJ - N_1 = 0$

$CJ := \sqrt{AC^2 - AJ^2}$ $CJ = 4.305244$

Definitions.

$AE - N_2 = 0$

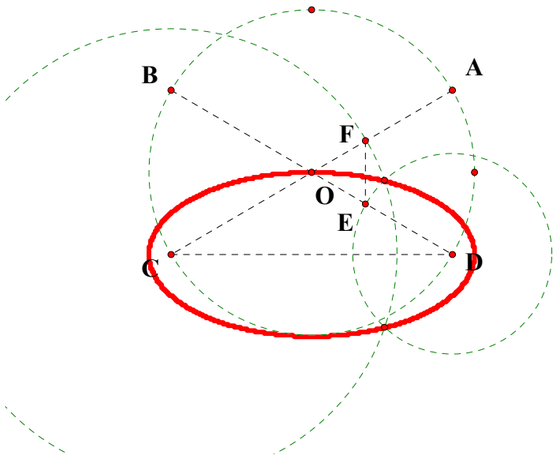
$DE - \sqrt{N_2^2 - N_1^2} = 0$ $AF - \frac{N_2}{2} = 0$

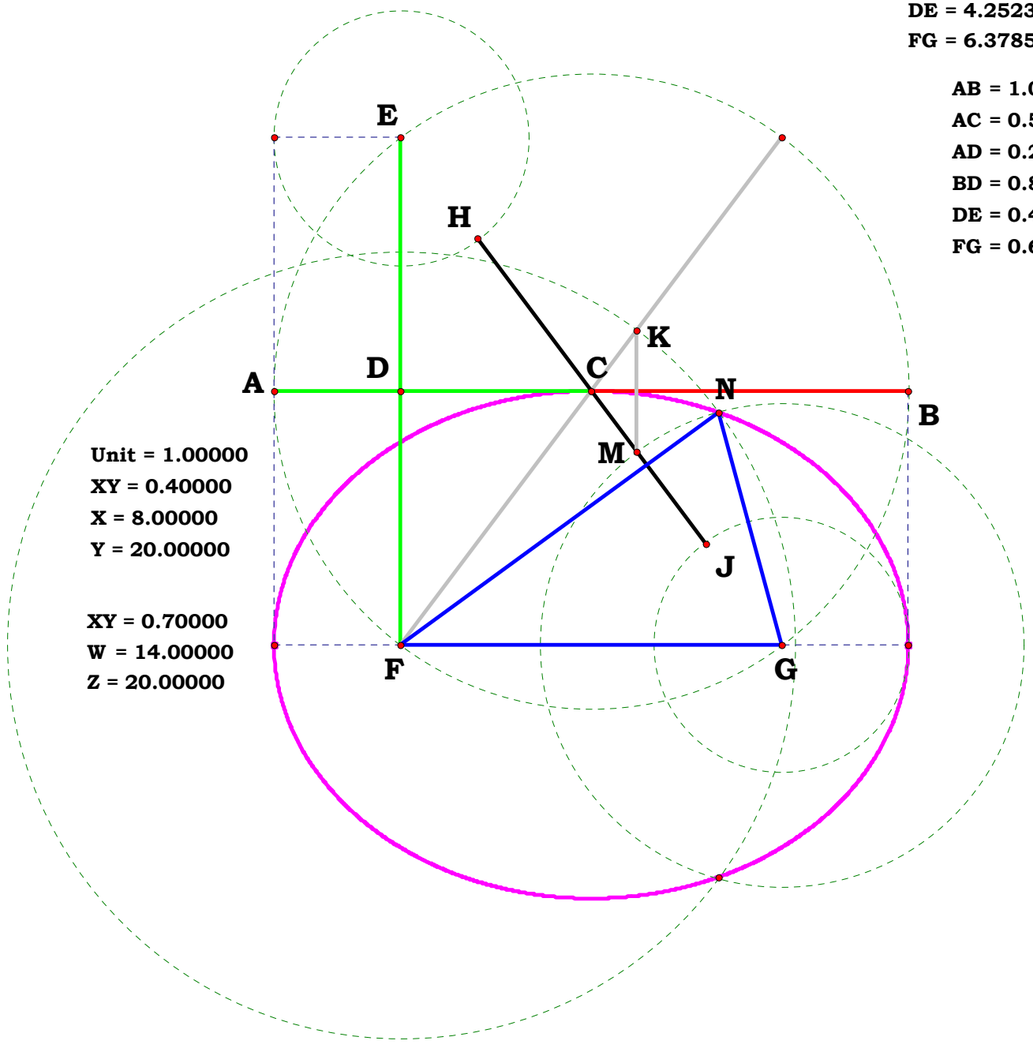
$AG - \frac{N_2^2}{2 \cdot N_1} = 0$ $AH - \frac{N_2^2}{2 \cdot N_1} = 0$ $AI - \frac{N_2}{2} = 0$

$AJ - N_1 = 0$ $CJ - \sqrt{N_2^2 - N_1^2} = 0$

**Really? Is this as far as you got on this project?**
**2008-0611**

**There are, as one can notice, a great deal of work left for some future time to do, this was one of them. These sketches are usually only complete to show what the finished project is aiming at, or to indicate the need for parsing to construct many individual projects.**




Unit = 1.00000
XY = 0.40000
X = 8.00000
Y = 20.00000

XY = 0.70000
W = 14.00000
Z = 20.00000

AB = 10.63083 cm
AC = 5.31542 cm
AD = 2.12617 cm
BD = 8.50467 cm
DE = 4.25233 cm
FG = 6.37850 cm
HJ = 6.37850 cm
HM = 4.46495 cm
FK = 6.59112 cm
FN = 6.59112 cm
GM = 4.03972 cm
GN = 4.03972 cm

AB = 1.00000
AC = 0.50000
AD = 0.20000
BD = 0.80000
DE = 0.40000
FG = 0.60000
HM = 0.42000
FK = 0.62000
FN = 0.62000
GM = 0.38000
GN = 0.38000

$AC-\frac{1}{2} = 0.00000$

$AD-\frac{X}{2 \cdot Y} = 0.00000$

$BD-\frac{2 \cdot Y-X}{2 \cdot Y} = 0.00000$

$DE-\frac{\sqrt{X \cdot (2 \cdot Y-X)}}{2 \cdot Y} = 0.00000$

$FG-\frac{Y-X}{Y} = 0.00000$

$HM-\frac{W \cdot (Y-X)}{Y \cdot Z} = 0.00000$

$FK-\frac{(2 \cdot W \cdot Y-2 \cdot W \cdot X)+X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$FN-\frac{(2 \cdot W \cdot Y-2 \cdot W \cdot X)+X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$GM-\frac{(2 \cdot W \cdot X-2 \cdot W \cdot Y-X \cdot Z)+2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

$GN-\frac{(2 \cdot W \cdot X-2 \cdot W \cdot Y-X \cdot Z)+2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$

110806

**Unit.** $AB := 1$

**Given.** $X := 8$ $W := 14$ $Y := 20$ $Z := 20$

**Descriptions.**

$$AC := \frac{AB}{2} \quad AD := \frac{X}{2 \cdot Y} \quad BD := AB - AD \quad DE := \sqrt{AD \cdot BD}$$

$$FG := AB - 2 \cdot AD \quad HJ := FG \quad HM := \frac{HJ \cdot W}{Z} \quad FK := HM + AD$$

$$FN := FK \quad GM := AB - FK \quad GN := GM$$

**Definitions.**

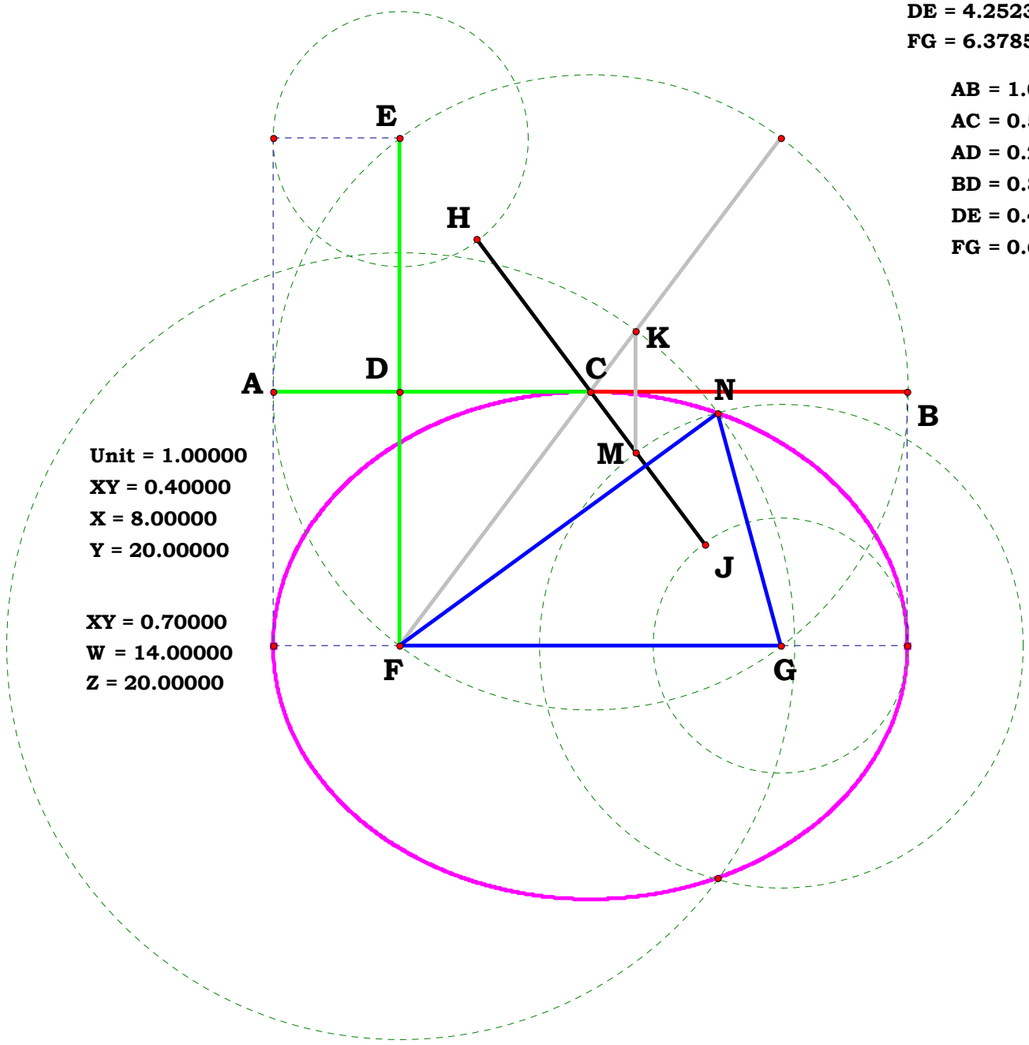
$$AC - \frac{1}{2} = 0 \quad AD - \frac{X}{2 \cdot Y} = 0 \quad BD - \frac{2 \cdot Y - X}{2 \cdot Y} = 0$$

$$DE - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{2 \cdot Y} = 0 \quad FG - \frac{Y - X}{Y} = 0 \quad HJ - \frac{Y - X}{Y} = 0$$

$$HM - \frac{W \cdot (Y - X)}{Y \cdot Z} = 0 \quad FK - \frac{2 \cdot W \cdot Y - 2 \cdot W \cdot X + X \cdot Z}{2 \cdot Y \cdot Z} = 0$$

$$FN - \frac{2 \cdot W \cdot Y - 2 \cdot W \cdot X + X \cdot Z}{2 \cdot Y \cdot Z} = 0$$

$$GM - \frac{2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0$$




Unit = 1.00000
XY = 0.40000
X = 8.00000
Y = 20.00000

XY = 0.70000
W = 14.00000
Z = 20.00000

AB = 10.63083 cm HJ = 6.37850 cm
AC = 5.31542 cm HM = 4.46495 cm
AD = 2.12617 cm FK = 6.59112 cm
BD = 8.50467 cm FN = 6.59112 cm
DE = 4.25233 cm GM = 4.03972 cm
FG = 6.37850 cm GN = 4.03972 cm

AB = 1.00000 HM = 0.42000
AC = 0.50000 FK = 0.62000
AD = 0.20000 FN = 0.62000
BD = 0.80000 GM = 0.38000
DE = 0.40000 GN = 0.38000
FG = 0.60000

$$AC - \frac{1}{2} = 0.00000$$

$$AD - \frac{X}{2 \cdot Y} = 0.00000$$

$$BD - \frac{2 \cdot Y - X}{2 \cdot Y} = 0.00000$$

$$DE - \frac{\sqrt{X \cdot (2 \cdot Y - X)}}{2 \cdot Y} = 0.00000$$

$$FG - \frac{Y - X}{Y} = 0.00000$$

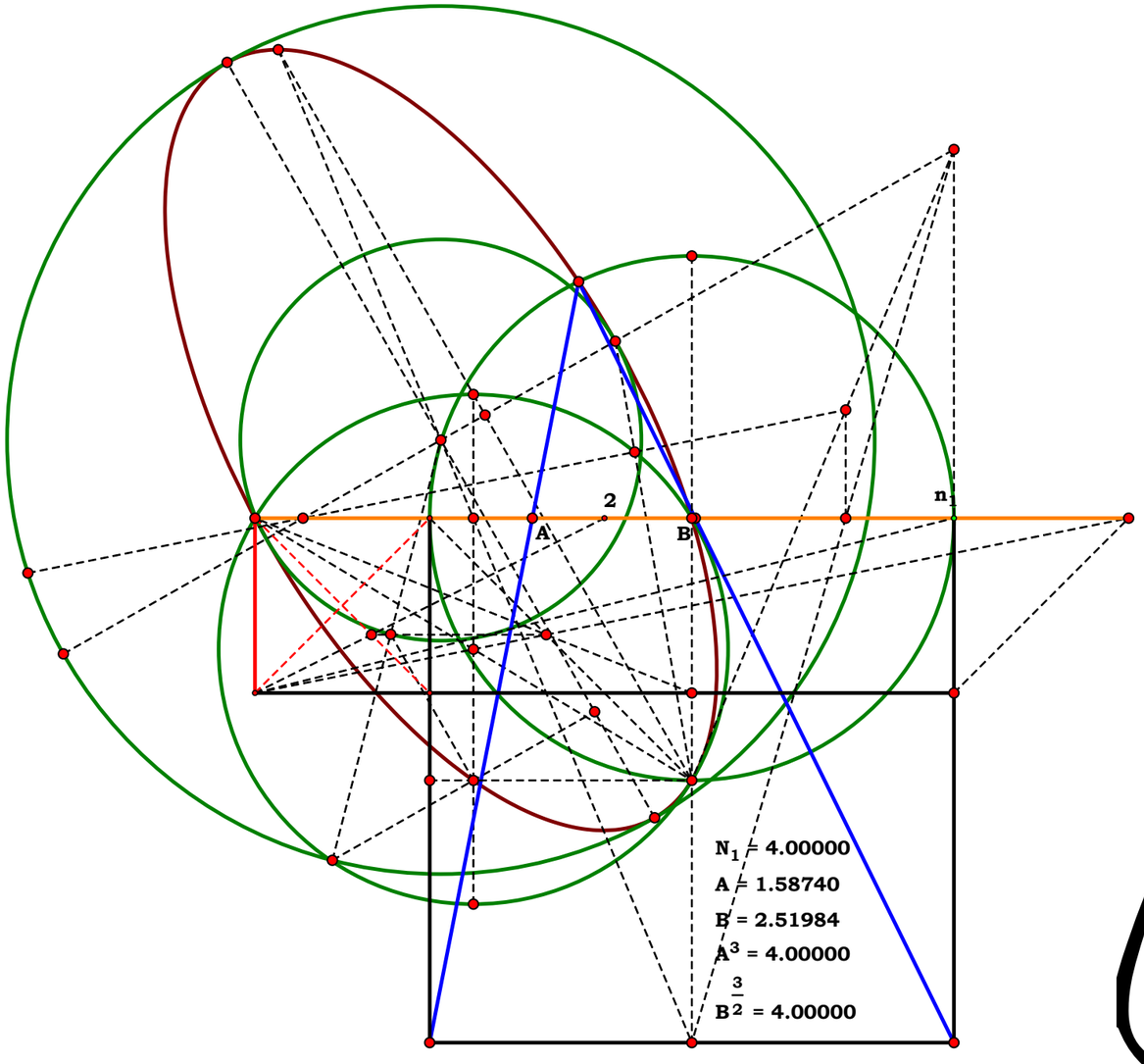
$$HM - \frac{W \cdot (Y - X)}{Y \cdot Z} = 0.00000$$

$$FK - \frac{(2 \cdot W \cdot Y - 2 \cdot W \cdot X) + X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$$

$$FN - \frac{(2 \cdot W \cdot Y - 2 \cdot W \cdot X) + X \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$$

$$GM - \frac{(2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z) + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$$

$$GN - \frac{(2 \cdot W \cdot X - 2 \cdot W \cdot Y - X \cdot Z) + 2 \cdot Y \cdot Z}{2 \cdot Y \cdot Z} = 0.00000$$




# The Delian Quest 2007

**John Clark**

**Unit.**

$DF := 1$

**Given.**

$N_1 := .49 \quad AI := N_1$

**060807A**

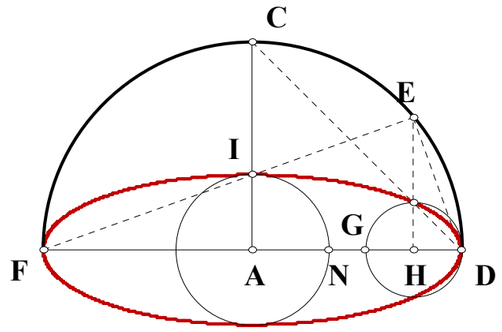
**Descriptions.**

$AF := \frac{DF}{2} \quad AN := AI \quad AD := AF$

$FI := \sqrt{AF^2 + AI^2} \quad DE := \frac{AI \cdot DF}{FI}$

$DH := \frac{DE^2}{DF} \quad GH := DH$

$GN := AD - (AN + GH + DH)$




**Definitions.**

$\frac{1}{2} - AF = 0 \qquad \frac{1}{2} \cdot \sqrt{1 + 4 \cdot N_1^2} - FI = 0$

$2 \cdot \frac{N_1}{\sqrt{1 + 4 N_1^2}} - DE = 0 \qquad 4 \cdot \frac{N_1^2}{\left(1 + 4 \cdot N_1^2\right)} - DH = 0$

$\frac{\left(2 \cdot N_1 + 1\right) \cdot \left(1 - 4 \cdot N_1 - 4 \cdot N_1^2\right)}{2 \cdot \left(4 \cdot N_1^2 + 1\right)} - GN = 0$

060807B

# Equation for an Ellipse

**Unit.**

**Given.**

$N_1 := 3.38667$ $\quad AB := N_1$

$N_2 := 2.05199$ $\quad AI := N_2$

$N_3 := 3.57039$ $\quad CE := N_3$

$N_4 := .94919$ $\quad CD := N_4$

**Descriptions.**

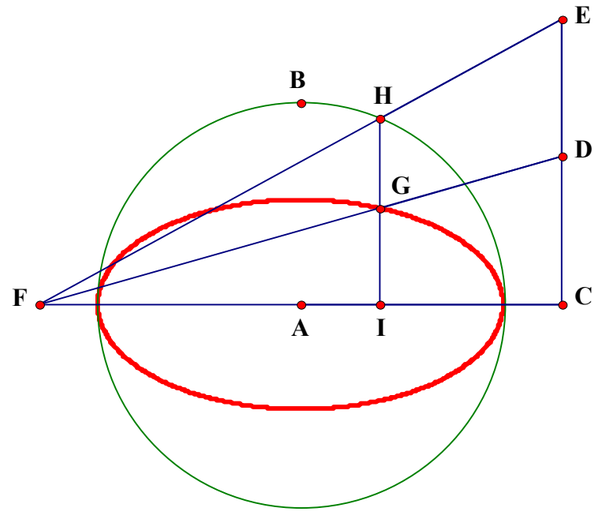
$$HI := \sqrt{(AB + AI) \cdot (AB - AI)}$$

$$GI := \frac{CD \cdot HI}{CE} \qquad AG := \sqrt{AI^2 + GI^2}$$

**Definitions.**

$$\frac{N_4 \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)}}{N_3} - GI = 0$$

$$\frac{1}{N_3} \cdot \sqrt{N_2^2 \cdot N_3^2 + N_4^2 \cdot N_1^2 - N_4^2 \cdot N_2^2} - AG = 0$$



061307

**Unit.**

**Given.**

**Descriptions.**

**Definitions.**

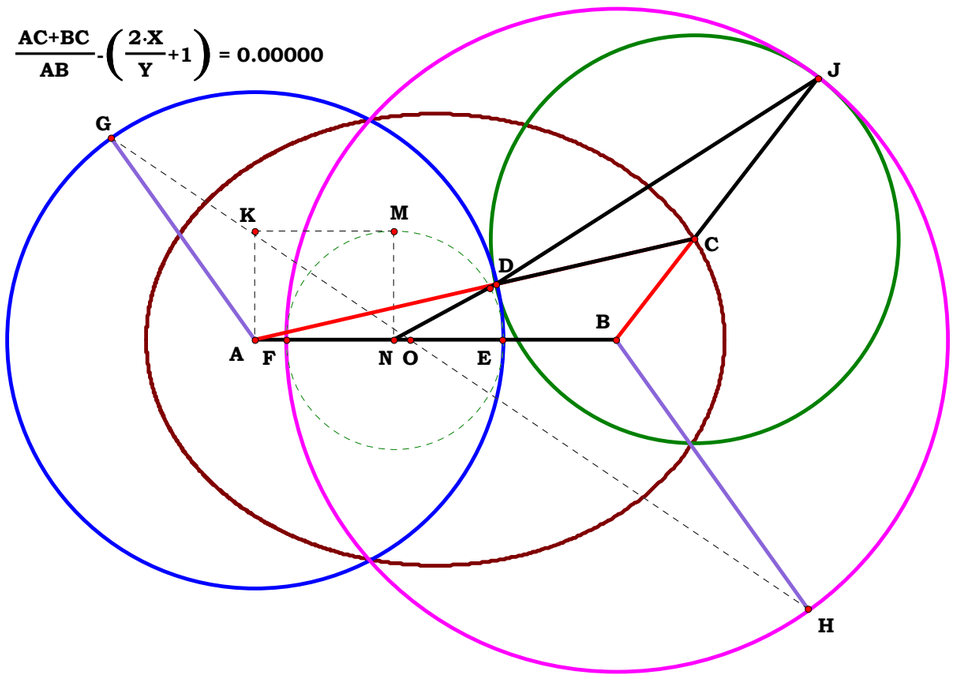
## Twin Cresent Ellipse

**The blue circle is always tangent to both and its center is always on the ellipse, Therefore, it always resides in a cresant and, it switches between inclusion and exclusion at the intersection of all three for each of the other circles. This means that it is aways excluded from an area included in the other two.**

Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000

$\frac{2 \cdot X}{Y}+1 = 1.60000$ $\frac{AC+BC}{AB} = 1.60000$ $\frac{AC^2 \cdot BC^2}{AB^2} = 7.10719\ cm^2$

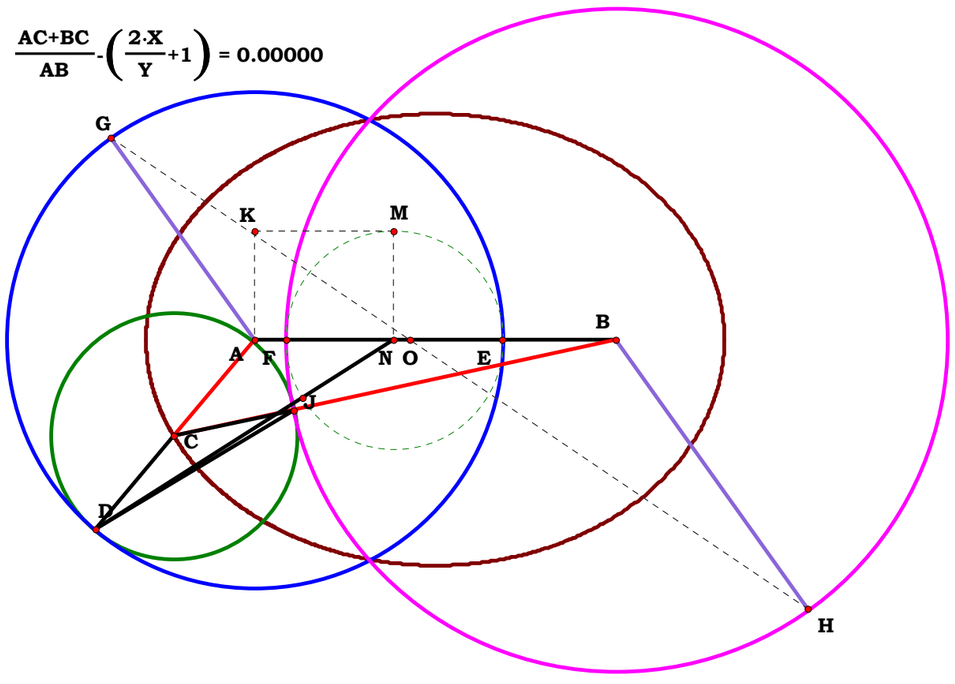
$\frac{AC+BC}{AB}-\left(\frac{2 \cdot X}{Y}+1\right) = 0.00000$




Unit = 1.00000
XY = 0.30000
X = 6.00000
Y = 20.00000

$\frac{2 \cdot X}{Y}+1 = 1.60000$ $\frac{AC+BC}{AB} = 1.60000$ $\frac{AC^2 \cdot BC^2}{AB^2} = 6.95837\ cm^2$

$\frac{AC+BC}{AB}-\left(\frac{2 \cdot X}{Y}+1\right) = 0.00000$




**In this ellipse, the foci, A and B, are the first thing drawn. Then one establishes a ratio with the unit. This affords one with a simple equation for the radii from the foci without even having to write it us, the arithmetic reveals the all. The ellipse is produced by the center of the circle CD. This is probably one of the simplest equations for an ellipse. One effectively has an ellipse produced by conjugate cresants by a circle always tangent to both. The figure might start you thinking about an alternate perspective to cosmology.**

062007A

**Unit.**

$AB := 1$

**Given.**

$X := 7 \quad Z := 12$

$W := 20 \quad Y := 19$

**Descriptions.**

$AC := 2 \cdot AB \quad BC := AB \quad BX := BC \cdot \frac{X}{W} \quad AX := AB + BX$

$BZ := BC \cdot \frac{Z}{Y} \quad AZ := AB + BZ \quad EZ := \sqrt{AZ \cdot (AC - AZ)}$

$GZ := BX \cdot \frac{EZ}{BC} \quad BJ := \frac{BC^2}{2 \cdot BZ} \quad BK := 2 \cdot BJ$

$KZ := BK - BZ \quad GK := \sqrt{GZ^2 + KZ^2}$

**Definitions.**

$AC - 2 = 0 \quad BC - 1 \quad BX - \frac{X}{W} = 0 \quad AX - \frac{W + X}{W} = 0$

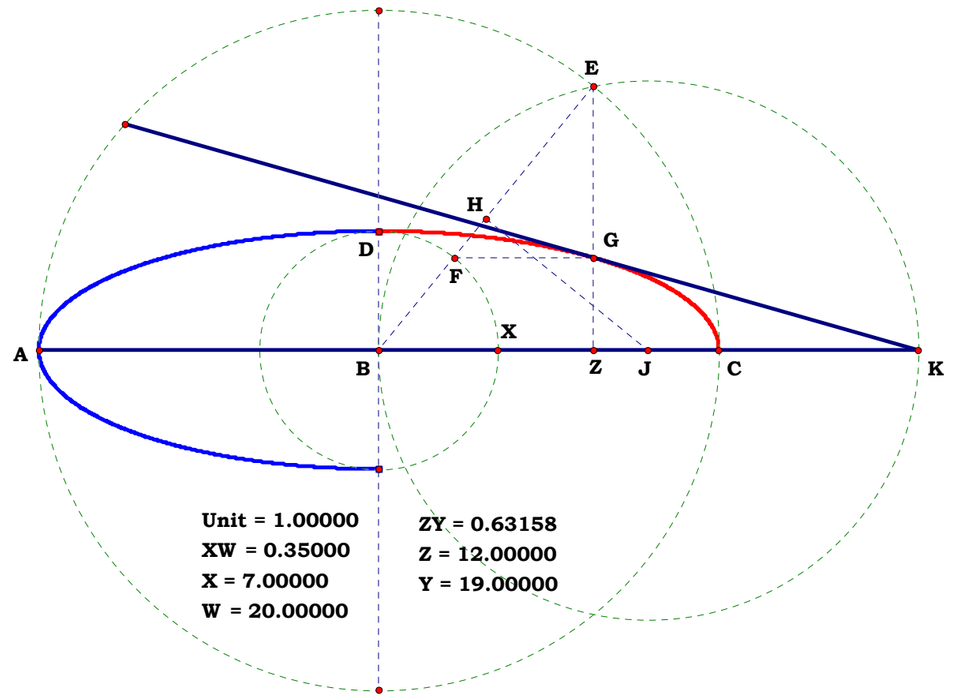
$BZ - \frac{Z}{Y} = 0 \quad AZ - \frac{Y + Z}{Y} = 0 \quad EZ - \frac{\sqrt{(Y - Z) \cdot (Y + Z)}}{Y} = 0$

$GZ - \frac{X \cdot \sqrt{(Y - Z) \cdot (Y + Z)}}{W \cdot Y} = 0 \quad BJ - \frac{Y}{2 \cdot Z} = 0 \quad BK - \frac{Y}{Z} = 0$

$KZ - \frac{(Y - Z) \cdot (Y + Z)}{Y \cdot Z} = 0$

$GK - \frac{\sqrt{(Y - Z) \cdot (Y + Z) \cdot \left(W^2 \cdot Y^2 - W^2 \cdot Z^2 + X^2 \cdot Z^2\right)}}{W \cdot Y \cdot Z} = 0$

## Tangent from Major Axis




Unit = 1.00000
XW = 0.35000
X = 7.00000
W = 20.00000

ZY = 0.63158
Z = 12.00000
Y = 19.00000

$\frac{(Y-Z)\cdot(Y+Z)}{Y\cdot Z}$-KZ = 0.00000

$\frac{X\cdot\sqrt{(Y-Z)\cdot(Y+Z)}}{W\cdot Y}$-GZ = 0.00000

$\frac{\sqrt{(Y-Z)\cdot(Y+Z)\cdot((W^2\cdot Y^2-W^2\cdot Z^2)+X^2\cdot Z^2)}}{W\cdot Y\cdot Z}$-GK = 0.00000

062007B

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := .39368$ $AI := N_1$

$N_2 := 1.25170$ $AD := N_2$

$AJ := \frac{AD}{2}$ From 080193

$EK := \frac{AB \cdot \sqrt{(2 \cdot AJ - AB) \cdot (2 \cdot AJ + AB)}}{2 \cdot AJ}$

$AK := \sqrt{AB^2 - EK^2}$ $DK := AD - AK$ $FK := \frac{AI \cdot EK}{AB}$

$DF := \sqrt{DK^2 + FK^2}$ $AG := \sqrt{AB^2 - AI^2}$ $AH := AG$ $HK := AH + AK$

$GK := AG - AK$ $FG := \sqrt{FK^2 + GK^2}$ $FM := FG$ $FH := \sqrt{HK^2 + FK^2}$

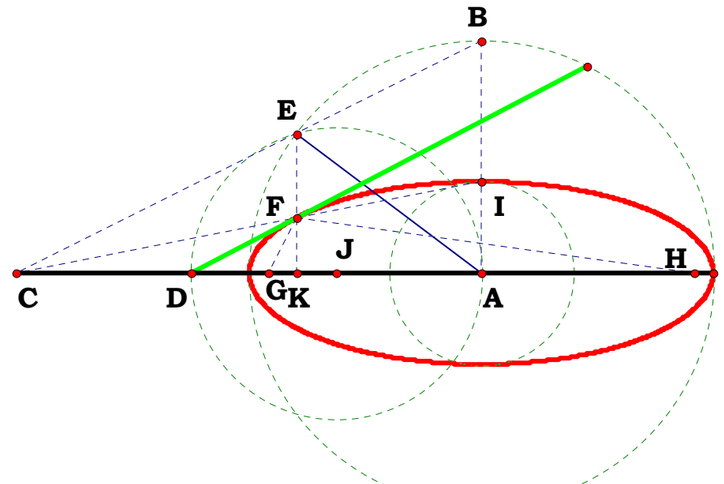
$HM := FH - FM$ $HO := \frac{HK \cdot HM}{FH}$ $GH := 2 \cdot AH$ $GO := GH - HO$

$MO := \frac{FK \cdot HM}{FH}$ $\frac{GO}{MO} - \frac{DK}{FK} = 0$

**Definitions.**

$N_1 \cdot \frac{\sqrt{(N_2 - 1) \cdot (N_2 + 1)}}{N_2} - FK = 0$

**Tangent from Major Axis**



Unit.

$AB := 1$

Given.

$X := 20$ $Z := 10$

$W := 6$ $Y := 4$

062007C

Descriptions.

$BE := AB$ $BX := \frac{X}{W}$ $AX := AB + BX$ $BZ := \frac{Z}{Y}$

$XZ := BX - BZ$ $CD := 2 \cdot BX$ $CF := XZ$

$DF := CD - CF$ $FG := \sqrt{CF \cdot DF}$ $BN := \frac{BE \cdot BX}{2 \cdot FG}$

$BP := 2 \cdot BN$ $FH := BE \cdot \frac{FG}{BX}$ $MP := BP - FH$

$BR := BZ \cdot \frac{BP}{MP}$ $PR := \sqrt{BP^2 + BR^2}$

Definitions.

$BE - 1 = 0$ $BX - \frac{X}{W} = 0$ $AX - \frac{W + X}{W} = 0$ $BZ - \frac{Z}{Y} = 0$

$XZ - \frac{X \cdot Y - W \cdot Z}{W \cdot Y} = 0$ $CD - 2 \cdot \frac{X}{W} = 0$ $CF - \frac{X \cdot Y - W \cdot Z}{W \cdot Y} = 0$

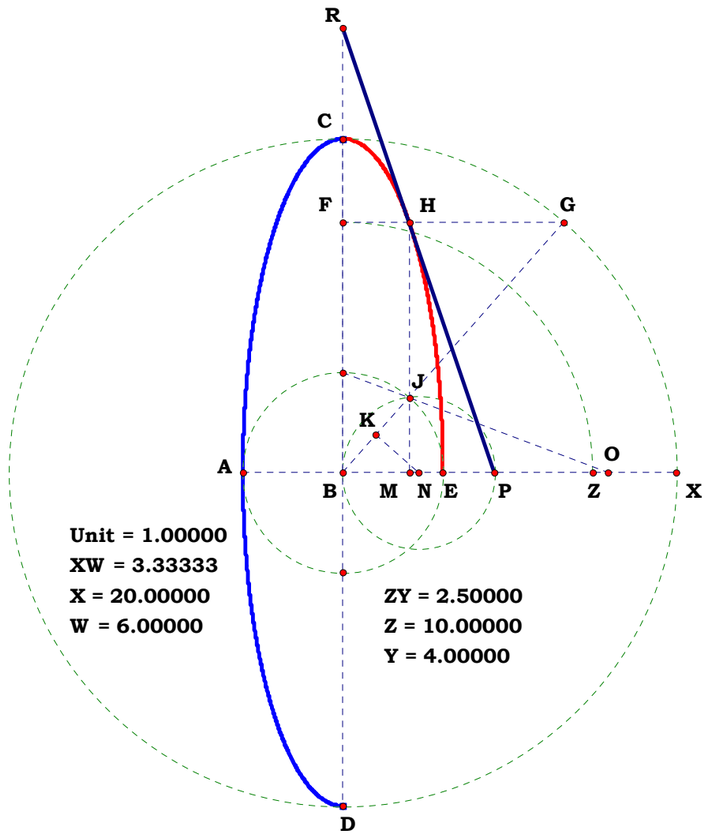
$DF - \frac{W \cdot Z + X \cdot Y}{W \cdot Y} = 0$ $FG - \frac{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}}{W \cdot Y} = 0$

$BN - \frac{X \cdot Y}{2 \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$ $BP - \frac{X \cdot Y}{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$

$FH - \frac{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}}{X \cdot Y} = 0$ $MP - \frac{W^2 \cdot Z^2}{X \cdot Y \cdot \sqrt{X^2 \cdot Y^2 - W^2 \cdot Z^2}} = 0$

$BR - \frac{X^2 \cdot Y \cdot \sqrt{X^2 \cdot Y^2 - W^2 \cdot Z^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$ $PR - \frac{X \cdot Y \cdot \sqrt{\left(W^4 \cdot Z^2 - W^2 \cdot X^2 \cdot Z^2 + X^4 \cdot Y^2\right)}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} = 0$

## Tangent from Minor Axis




$\frac{X \cdot Y}{\sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} - BP = 0.00000$

$\frac{X^2 \cdot Y \cdot \sqrt{X^2 \cdot Y^2 - W^2 \cdot Z^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} - BR = 0.00000$

$\frac{X \cdot Y \cdot \sqrt{\left(W^4 \cdot Z^2 - W^2 \cdot X^2 \cdot Z^2\right) + X^4 \cdot Y^2}}{W^2 \cdot Z \cdot \sqrt{(W \cdot Z + X \cdot Y) \cdot (X \cdot Y - W \cdot Z)}} - PR = 0.00000$

062007D

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := .40636$ $AI := N_1$

$N_2 := .60804$ $AD := N_2$

$AJ := \frac{AD}{2}$ $PK := \frac{AI \cdot \sqrt{(2 \cdot AJ - AI) \cdot (2 \cdot AJ + AI)}}{2 \cdot AJ}$ $AK := \sqrt{AI^2 - PK^2}$

$DK := AD - AK$ $FK := \frac{PK \cdot AB}{AI}$ $DF := \sqrt{DK^2 + FK^2}$ $AG := \sqrt{AB^2 - AI^2}$

$AH := AG$ $HR := AH + FK$ $GR := AG - FK$ $FG := \sqrt{AK^2 + GR^2}$ $FM := FG$

$FH := \sqrt{HR^2 + AK^2}$ $HM := FH - FM$ $HO := \frac{HR \cdot HM}{FH}$ $GH := 2 \cdot AH$

$AK \cdot HM$ $GO$ $FK$

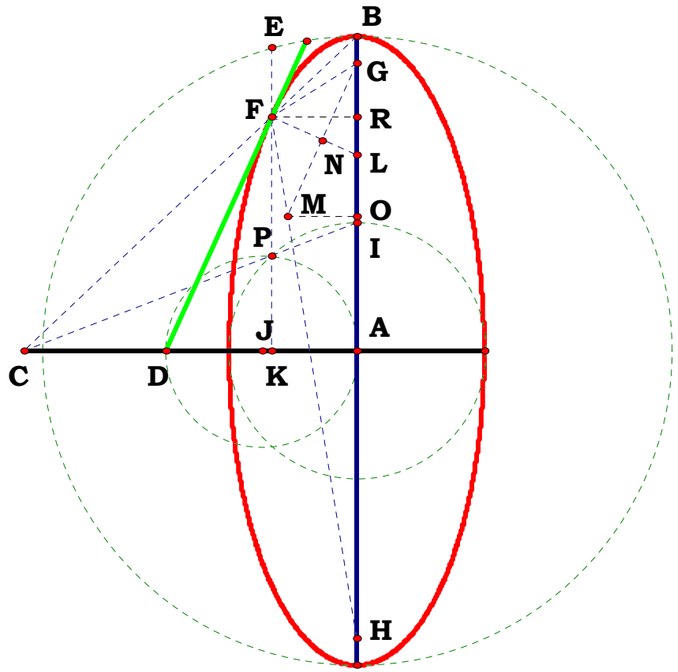
$GO := GH - HO$ $MO := \frac{AK \cdot HM}{FH}$ $\frac{GO}{MO} - \frac{FK}{DK} = 0$

Definitions.

$\frac{\sqrt{(N_2 - N_1) \cdot (N_2 + N_1)}}{N_2} - FK = 0$

$Major := N_1 \cdot \frac{\sqrt{(N_2 - 1) \cdot (N_2 + 1)}}{N_2}$

**Tangent from Minor Axis**



062407A

**Unit.**

$AB := 1$

**Given.**

$N_1 := .64776 \qquad AC := N_1$

$N_2 := .5444 \qquad EJ := N_2$

**Descriptions.**

$AJ := \sqrt{AB^2 - AC^2} \qquad FI := (AB - EJ) + AB \qquad DE := \sqrt{AJ^2 - (AB - EJ)^2}$

$AN := \dfrac{\sqrt{DE^2 \cdot (DE + AB) \cdot (-DE + AB)}}{DE} \qquad DN := AB - AN \qquad GJ := EJ - DN$

$HI := FI - DN \qquad GL := \dfrac{GJ \cdot 2 \cdot DE}{GJ + HI} \qquad HL := 2 \cdot DE - GL$

$JL := \sqrt{GL^2 + GJ^2} \qquad IL := \sqrt{HI^2 + HL^2} \qquad (JL + IL) - 2 \cdot AB = 0$

**Definitions:**

$AJ - \sqrt{\left(1 - N_1^2\right)} = 0 \qquad FI - \left(2 - N_2\right) = 0 \qquad DE - \sqrt{\left(2 \cdot N_2 - N_2^2 - N_1^2\right)} = 0$

$\sqrt{\left(N_1^2 + N_2^2 - 2 \cdot N_2 + 1\right)} - AN = 0 \qquad \left(\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} - N_2 + 1\right) - HI = 0$

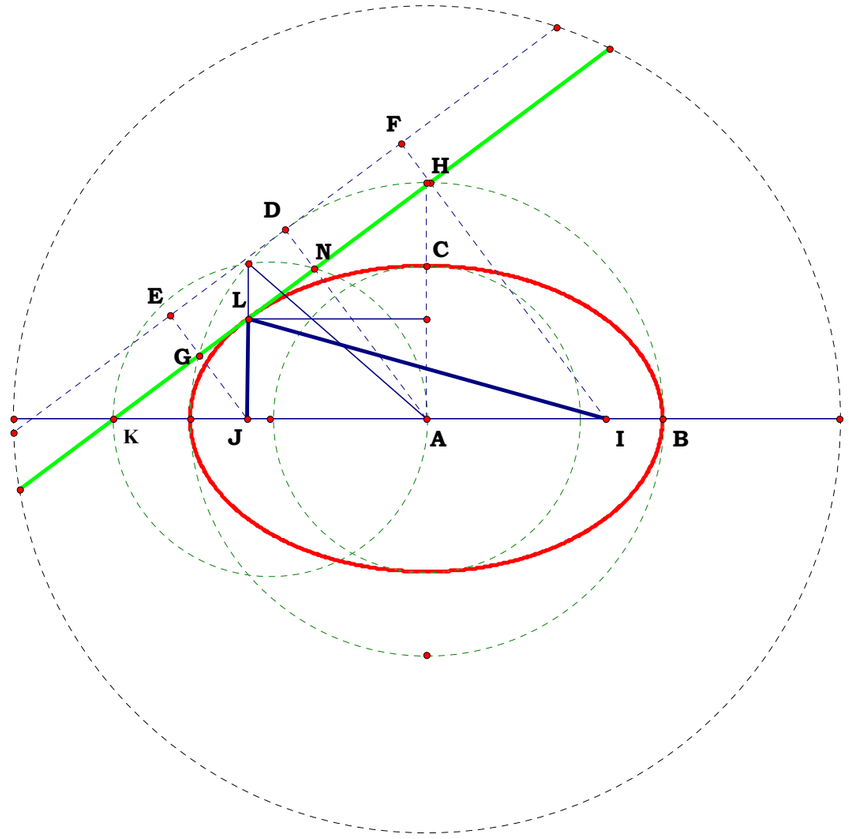
$1 - \sqrt{\left(N_1^2 + N_2^2 - 2 \cdot N_2 + 1\right)} - DN = 0 \qquad N_2 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} - 1 - GJ = 0$

$\dfrac{\sqrt{2 \cdot N_2 - N_2^2 - N_1^2} \cdot \left(N_2 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} - 1\right)}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1}} - GL = 0 \qquad \dfrac{\sqrt{2 \cdot N_2 - N_2^2 - N_1^2} \cdot \left(\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} - N_2 + 1\right)}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1}} - HL = 0$

$\dfrac{\sqrt{N_1^2 - 4 \cdot N_2 + 2 \cdot N_2^2 + 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1}} - JL = 0 \qquad \dfrac{\sqrt{N_1^2 - 4 \cdot N_2 + 2 \cdot N_2^2 - 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_2 + 1}} - IL = 0$

## Found on the Internet

**Found the construction, now I explore it with Algebra.**




AB = 3.95817 cm
AC = 2.56393 cm
EJ = 2.15482 cm
JL = 1.68103 cm
IL = 6.23530 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 0.64776$

$\frac{EJ}{AB} = 0.54440$

$\frac{JL}{AB} = 0.42470$

$\frac{IL}{AB} = 1.57530$

062407B

**Unit.**

$AB := 1$

**Given.**

$W := 20$ $Y := 20$

$X := 13$ $Z := 8$

**Descriptions.**

$AX := \frac{X}{W}$ $AZ := \frac{Z}{Y}$ $BX := AB - AX$ $AC := 2 \cdot AB$

$BH := BX$ $CZ := AC - AZ$ $MZ := \sqrt{AZ \cdot CZ}$

$GZ := BX \cdot \frac{MZ}{AB}$ $BZ := AB - AZ$ $BN := \frac{AB}{2}$ $BE := AB \cdot \frac{BN}{BZ}$

$BO := 2 \cdot BE$ $OZ := BO - BZ$ $GO := \sqrt{OZ^2 + GZ^2}$

**Definitions.**

$AX - \frac{X}{W} = 0$ $AZ - \frac{Z}{Y} = 0$ $BX - \frac{W - X}{W} = 0$ $AC - 2 = 0$

$BH - \frac{W - X}{W} = 0$ $CZ - \frac{2 \cdot Y - Z}{Y} = 0$ $MZ - \frac{\sqrt{Z \cdot (2 \cdot Y - Z)}}{Y} = 0$

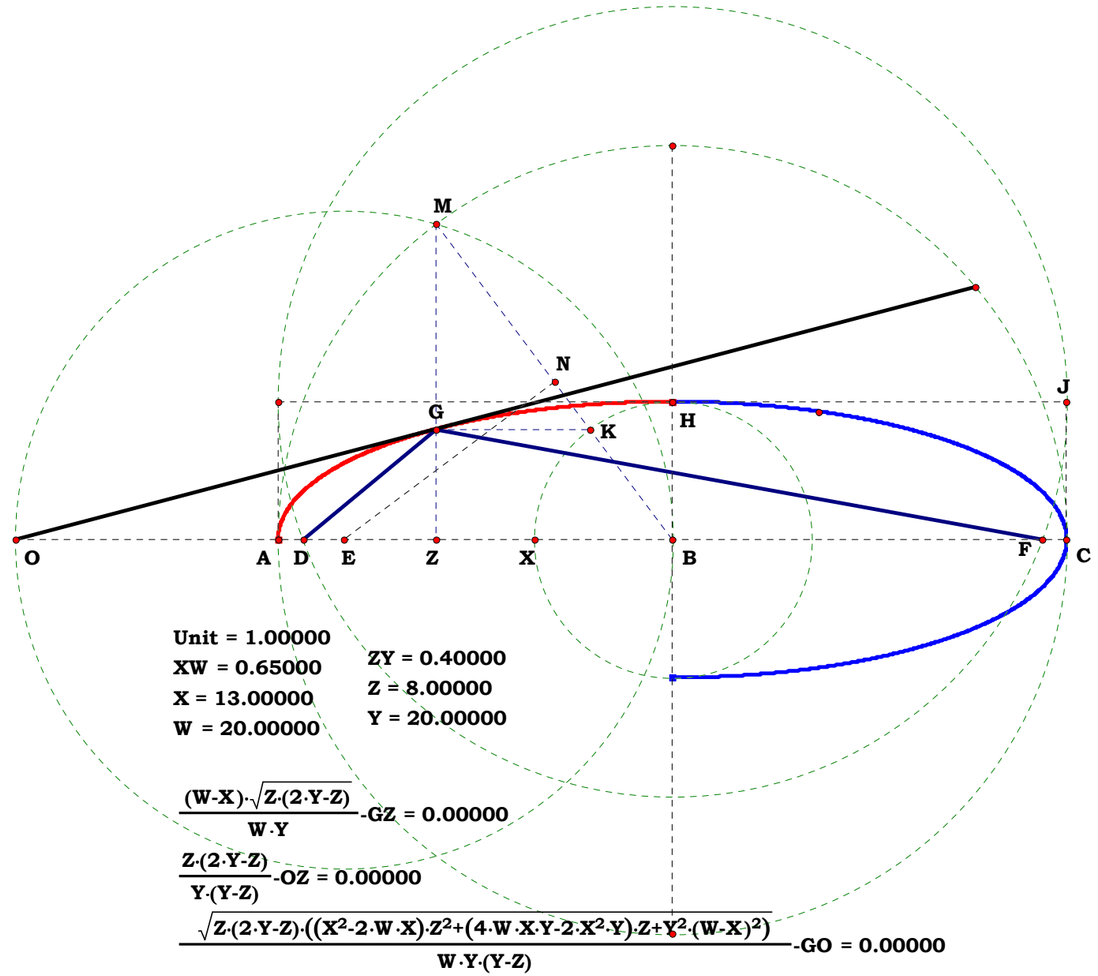
$GZ - \frac{(W - X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} = 0$ $BZ - \frac{Y - Z}{Y} = 0$ $BN - \frac{1}{2} = 0$

$BE - \frac{Y}{2 \cdot (Y - Z)} = 0$ $BO - \frac{Y}{Y - Z} = 0$ $OZ - \frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y - Z)} = 0$

$GO - \frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot \left[ \left( X^2 - 2 \cdot W \cdot X \right) \cdot Z^2 + \left( 4 \cdot W \cdot X \cdot Y - 2 \cdot X^2 \cdot Y \right) \cdot Z + Y^2 \cdot (W - X)^2 \right]}}{W \cdot Y \cdot (Y - Z)} = 0$

## Found on the Internet

**Writeup A and its figure were rather a bit bad and awkward.**



Unit.

$AB := 1$

Given.

$X := 8$ $Y := 20$

$W := 20$ $Z := 9$

062407C

Descriptions.

$AX := \frac{X}{W}$ $AZ := \frac{Z}{Y}$ $CZ := \sqrt{AZ \cdot (2AB - AZ)}$

$BX := AB - AX$ $BZ := AB - AZ$ $FZ := \frac{CZ^2}{BZ}$

$DZ := BX \cdot \frac{CZ}{AB}$ $DF := \sqrt{FZ^2 + DZ^2}$

Definitions.

$AX - \frac{X}{W} = 0$ $AZ - \frac{Z}{Y} = 0$ $CZ - \frac{\sqrt{Z \cdot (2 \cdot Y - Z)}}{Y} = 0$

$BX - \frac{W - X}{W} = 0$ $BZ - \frac{Y - Z}{Y} = 0$ $FZ - \frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y - Z)} = 0$

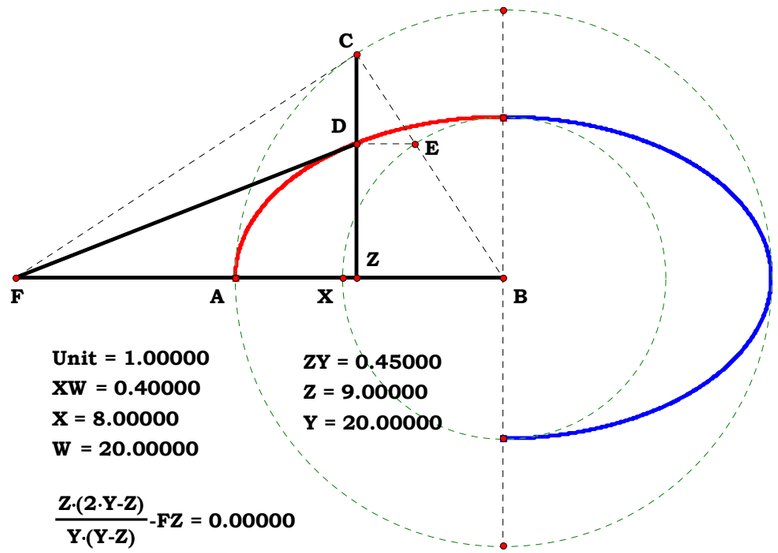
$DZ - \frac{(W - X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} = 0$

$DF - \frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot \left[Y^2 \cdot W^2 + \left(4 \cdot X \cdot Y \cdot Z - 2 \cdot X \cdot Y^2 - 2 \cdot X \cdot Z^2\right) \cdot W + X^2 \cdot (Y - Z)^2\right]}}{W \cdot Y \cdot (Y - Z)} = 0$

## Found on the Internet

**Found the construction, now I explore it with Algebra.**

**The raw construction was found on the internet. I added the structures required for writing it up in my usual algebraic method.**




Unit = 1.00000
XW = 0.40000
X = 8.00000
W = 20.00000

ZY = 0.45000
Z = 9.00000
Y = 20.00000

$\frac{Z \cdot (2 \cdot Y - Z)}{Y \cdot (Y - Z)} - FZ = 0.00000$

$\frac{(W - X) \cdot \sqrt{Z \cdot (2 \cdot Y - Z)}}{W \cdot Y} - DZ = 0.00000$

$\frac{\sqrt{Z \cdot (2 \cdot Y - Z) \cdot \left(Y^2 \cdot W^2 + \left(4 \cdot X \cdot Y \cdot Z - 2 \cdot X \cdot Y^2 - 2 \cdot X \cdot Z^2\right) \cdot W + X^2 \cdot (Y - Z)^2\right)}}{W \cdot Y \cdot (Y - Z)} - DF = 0.00000$

# My Name is John.

Hello. My name is John and I am going to explain how to multiply and divide a line by a line in Geometry. Now, if you are going to ask me if I am a geometer, I have to reply by myth. Explanation by myth is one the ancient Greek's methods of teaching by discourse.

Once upon a time, God created man; They created him male and female, in the image of God. Or one can say, male and female created They him, which is rather awkward, but it does have that ancient New England flair to it. At any rate, once upon a time is not this time. It came to pass as men multiplied on the earth that men started to work for a living and not being god's themselves needed a way to designate each other and so individuals, which are not, by definition man started calling each other by their craft. That is where we got Mr. Smith and Mr. Clark, etc. A vestige of this remains today. Not being man, we tend to think of each other by an assigned craft. I work in a factory, but my name is Clark. The conflict of course is why I spend the entirety of my wages in therapy.

Now this works to my advantage. I have learned that individuals calling themselves geometers (I am personally hoping for the day I become part of man) cannot multiply and divide a line by a line. So, I guess one could say, that a geometer is someone who cannot do the math, which is really a sign for some serious expenditure on therapy—and, if those in mind—field knew what they were doing, the outlay would be advantageous. Too bad they cannot define a man. Now a non-Euclidean Geometer is someone who not only cannot do the math, they demand, as part of their initiation rights, that one will never be able to do the math. So, in due respect to non-Euclidean Geometers, please stop reading and go back to your scribbling—and contradicting yourself. Doing geometry inside of or on the outside of a tennis ball, or a Frisbee, makes me think that one has spent way to many days on the court, spiking one's tea, and certainly missing the ball.

Now, if your like me, a factory worker, and someone were to give you two lines and say,

Hey, you (He is hairy and has a club). Here are two lines, show me how to multiply one by the other, and after that, show me how to divide one by the other.

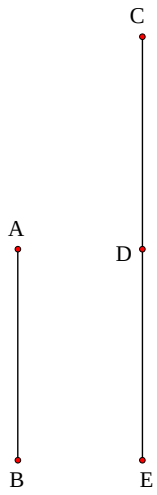
I would look at the man, think for a moment and draw a blank. What the heck does he mean? Then I would say, I am sorry, but I don't understand

what you mean. The man would leave off and I would go get another cup of coffee.

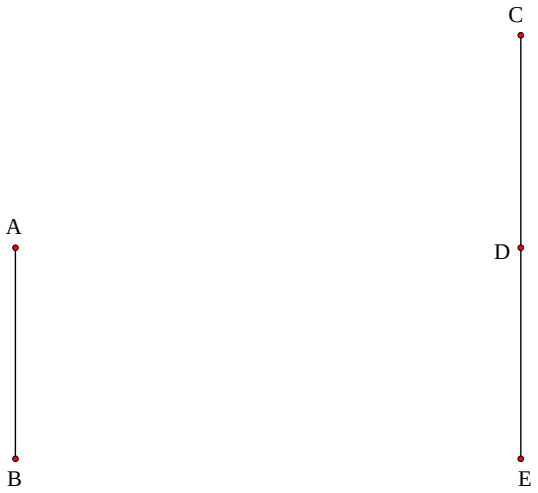
If I were a bit strange, I would consult Euclid's ***Elements*** and find to my dismay, the chap could do the math, but seems to have left this off for some reason, probably because it was too easy (so who don't lie for a friend?). Now, I happen to have in my possession a number of unpublished manuscripts which does have the answer in them and they are full of doing the math. I acquired them from the God's (and for those of you interested, the Delian Problem does have a solution—and it has something to do with Plato under extending himself). If it should be discovered that I am stealing a bit of fire, and giving it to man, please don't tell where you got it from. I have learned from first hand experience, you don't want to mess with Them—they be giants—really, really, big giants.

Now I am not going to explain this exactly as it was explained to me, as I have a poor memory. Please bear with me.

If I were given two lines, and asked to compare them, I would look at them and say;

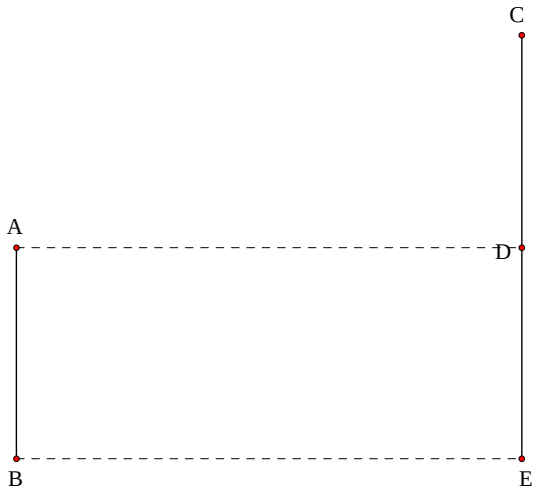



well, AB is shorter than CE. I mean, what can you do with two lines anyway. Reminds me of when I was a kid asking my mother what could I do with seven cents, realizing early on I was three cents short of a dime. If I were Euclid I would subtract one from the other and find that CE — AB = CD, or if you're a top down programmer, CE — AB = DE. If I move CE off a ways,

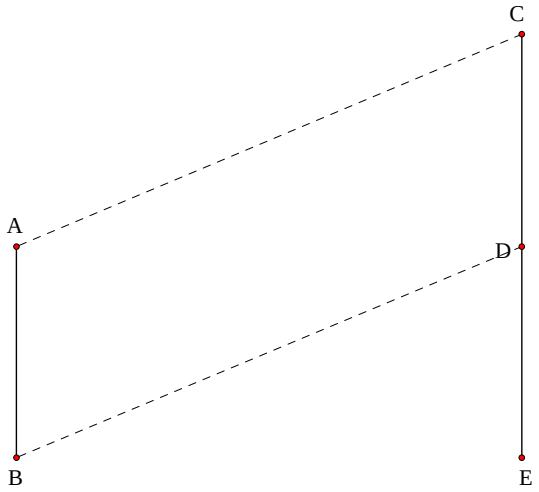



I would say that CE — AB = CD, or DE which ever you choose. Non-Euclidean Geometers, like Einstein, claim that this equality, this simultaneity, is not true and that at some point of moving AB and CE apart, as if it were part of the equation, does mysterious things to these segments. It amounts to a thief's logic—moving CE off sufficiently will make AB infinitely greater than CE 'cause we exact a kind of tribute on it and subtract that tribute as we go. It amounts to constructing a square say, of 25 square inches or so, and claiming if we repeat it enough, well, it just plain disappears—we wore it out. While on the other hand, there are those who claim that if I assert a point an infinite number of times, I can create a line. You know, like waving a knife in the air an infinite number of times an making a salad. This is the kind of mentality that makes credit card lenders rich. As I said, non-Euclidean Geometers are really crooked bankers in disguise—or really lousy cooks. A basic fact of abstraction, when you really know that a boundary is not the difference (a point is that which has not part), a form is in fact absolute, you know you can never attribute difference to that form, the form is applied as a boundary to any given difference—material. The cut is not the cutted! Wow, that was trashy!

Now if I had AB, and wanted to construct CE from it.



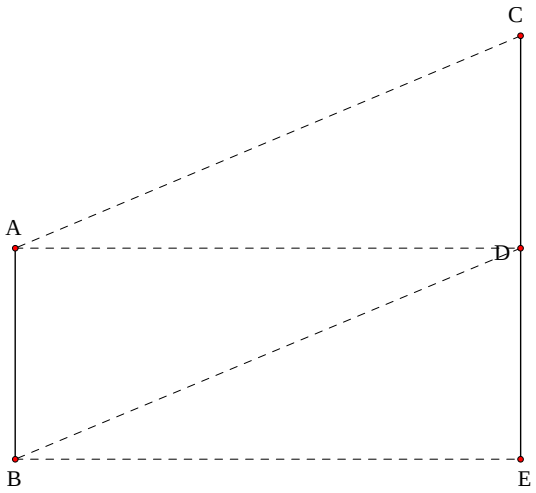

I could transfer one segment at a time



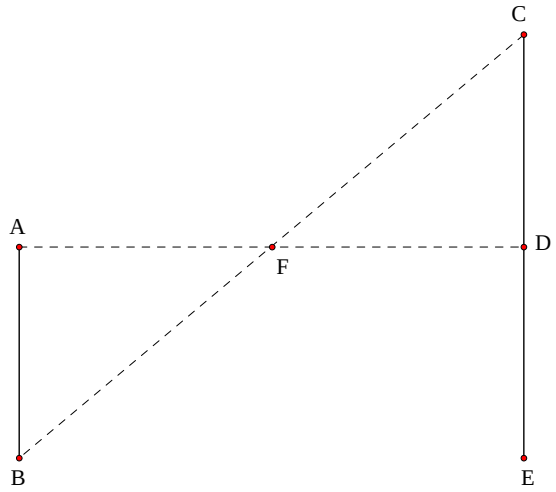

using parallel lines, but this is not multiplication, it is multiple processes, or simply addition. Parallel lines gives us the ability to do multiple additions, which is again not multiplication. One sign of that is that we have to assert each unit point in constructing CE. We have to assert each unit point just to do the parallels. Duh!

One of the things our ancient quibbling buddies, the Greeks, did tell us is that in order to multiply and divide, we have to have a unit. This is just part of plain simple Arithmetic. And they also said that when dealing with numbers in multiplication and division we were dealing with square and oblong (rectangular) numbers. Keep these ideas in mind. A square, an oblong, and a unit. Euclid drew a number of them. We will have need of them. For the moment let us learn what they did say about ratio,

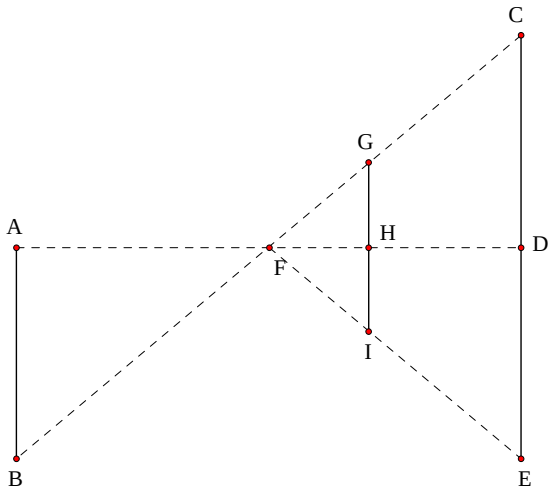
which we will also need. Now, if in constructing CE, we stayed up too late;—




and made a mistake in drawing—or were simply dyslexic;



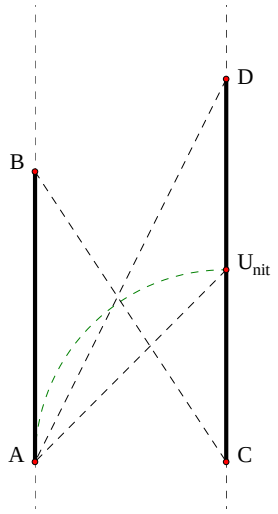

we would discover the ratio. As AB is to CD, so AF is to DF. And by George—(if you remember, he too was a hairy fellow and curious), One learns how to take any multiple and divide another segment of any length by the same multiple. From multiple addition, we have a kind of multiple division, but it is not division, it is still just a plain ratio, of anther segment.



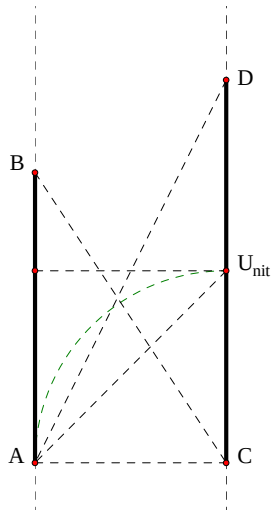

Now, as AB is to GH, so to DE is to HI, etc., etc. This is all fine and good, but, we still have not really learned how to multiply and divide. That is because these ratio's work regardless of the notion of unit, or square. Unless you are a crooked banker or a non-Euclidean Geometer, or a bad cook, this relationship is always true. There is one, and only one, difference between two points.

We are building our ideas up, one standard at a time. Intellectually, we fail, at the point we cannot abstract and use a standard—or what Plato called ***form*** because a boundary is not a difference and by definition (not a difference) always true. The divergence of language itself, starts with the inability to establish a standard even for a name. Many linguists call it the "growth" of language when meaning changes, but then they are non-Euclidean Geometers at heart also. What do they say of a government that has got its constitution saying exactly the opposite of what is written? If you want to reduce them to rubble, ask them outright, *Why can one word be or not be predicated of another*? Or again, if definition is conventional, and meaning can never be conventional, what in the heck does meaning have to do with definition? or even language? They will either get a funny look on their face mumbling to themselves, or start babbling non-sense to you. I have some books by the gods on that topic also. It is really simple, . . . but not here, not now.

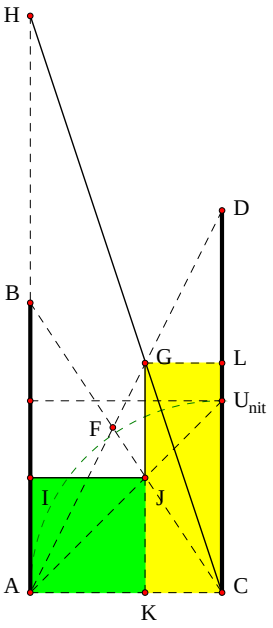
Multiplication and division rely on a standard in unit. So lets add that and see where we go.




At the outset the figure is very shy and unassuming. If you saw it laying in the street, you would hardly be pressed to pick it up. We have placed our segments the difference of our chosen unit apart, and we do have a square. No offence to Descartes who tried to find what I am doing, we don't have a number line, but a lined number. First time I ever seen a studious use of cross hairs actually miss the target.

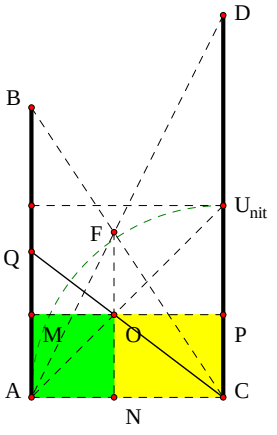



It don't look like much, but it can not only multiply and divide, one can use it to do much in the way of exponential manipulation as well. Let us take a closer look as to what the figure tells us.




$$\frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} - \frac{\text{(Area IJKA)}}{\text{(Area KGLC)}} = 0.00000$$

This is how we perform multiplication. Given AC as our unit, AB × CD = AH. In order to see this using the Arithmetic Grammar system, We divide AC by AC and get 1, our Unit. We then divide AB by AC which gives AB in terms of our unit. We then divide CE by AC and acquire that in units, and again for AH. We will find that by using the notion of Unit, Square and Oblong Numbers, which is incorporated in the idea of ratio, we can Multiply. And we can do what no binary calculator will ever do, we do it exactly. What about division?




$$\frac{\text{(Area MANO)}}{\text{(Area ONCP)}} - \frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} = 0.00000$$

Wouldn't you know it, there is a triplicate ratio in the figure! Right under our pencil. Didn't Euclid write that it was the hardest thing to do in geometry? Well, I have never taken geometry in school and set out to comprehend the triplicate ratio, guess I got somewhere. Going through our steps as before, we find that AB ÷ CD = AQ. Each of these steps is proven individually in Euclid. I suspect he was like Plato and wanted to see if his readers were smart enough to add and subtract ideas. And again, no binary computer will ever be up to Geometry, as Geometry is exact.

One can do a whole lot with this figure, through various projections. One can do a lot in the way of exponential manipulation. Try that with cross hairs! Some of the methods one will find in those unpublished books I was talking about. I don't know how long the gods will let me work on them, in fact, if it were not for Them, I would have been killed over thirty years ago. Imagine that, I am a walking contradiction, a living dead man. At any rate, I hope you have fun playing with the figure.

Now this is not the place to show the solution to the Delian Problem. My god, if one is just learning the simple four, by adding multiplication and division to our list of addition and subtraction, it may be too difficult realize a revolution in Euclidean Geometry based upon a standard long ago recognized but left unemployed—just like these. I will put the idea in the Geometer's Sketchpad file.

I hope I have made it clear that through multiple addition and subtraction, one leads into the understanding of ratio, just like Euclid did, but it is still a step away from multiplication and division. Those depend upon a respect for, and understanding of a standard in definition. We learn to add, and subtract. These teach us ratio—it is part of them. We learn about the units which is taught by them also. This then leads to multiplication and division and our primary four are thus established.

I do have some food for thought though. Using the facts of conventions in language, can you count the ways non-Euclidean geometries commit self-referential errors in simple logic? Apparently not, they are popular. Maybe it has something to do with linguist waving their knife in the air constructing sentences. What is prediction? Maybe I will read it to you sometime. The solution was once written on a Temple "Know Thyself." I will say this, as a sense system, the human mind is suppose to abstract form and create things with it. To deny form as the foundation for thought is simply a sign of dysfunction. I know, look at me.

# Multiplication And Division of Lines

**1. An unit is that by virtue of which each of the things that exist is called one.**
**Euclid's Elements**

312

**The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.**

**John Clark**

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Printed via import to MS Word.

# Following the Yellow Brick Road

**Multiplication**

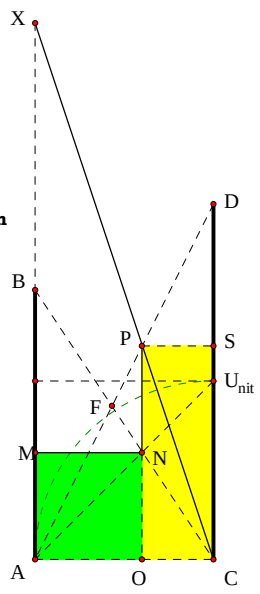
**Unit = 1.00000**

**AB = 1.51095**

**CD = 1.99270**

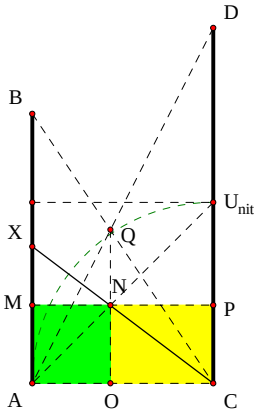
**AX = 3.01087**

**AB·CD-AX = 0.00000**




$$\frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} - \frac{\text{(Area MNOA)}}{\text{(Area OPSC)}} = 0.00000$$

**In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.**




$$\frac{\text{(Area MAON)}}{\text{(Area NOCP)}} - \frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} = 0.00000$$

**Division**

**Unit = 1.00000**

**AB = 1.48921**

**CD = 1.96403**

**AX = 0.75824**

$$\frac{\text{AB}}{\text{CD}} - \text{AX} = 0.00000$$

# Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

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| $\frac{1}{N_1} - N_3 = 0.00000$ | Link to 3 |
| $\frac{N_1}{N_2^2} - N_4 = 0.00000$ $\frac{N_1}{N_2^3} - N_5 = 0.00000$ | Link to 4 |
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Link to Introdution

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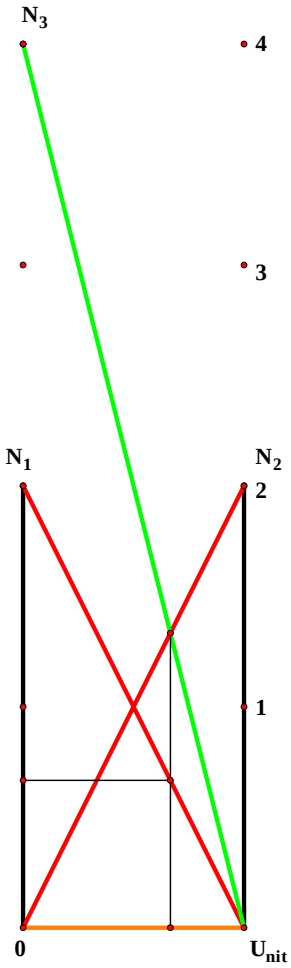
| $N_1^{\frac{2}{8}} - N_7 = 0.00000$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$ | | | |




$N_1 = 2.00000$

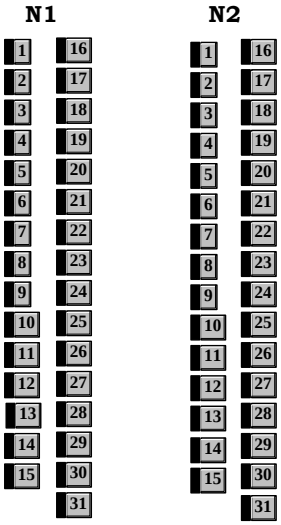
$N_2 = 2.00000$

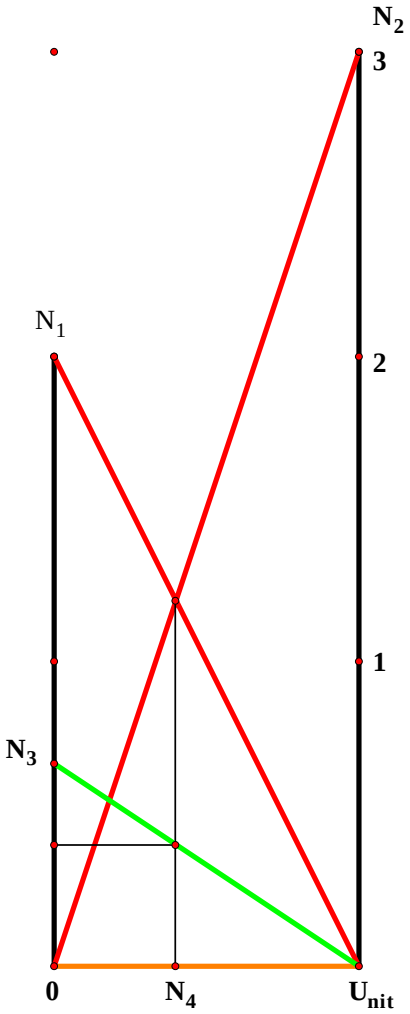
$N_1 \cdot N_2 = 4.00000$

$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

**Multiply N1 by N2**






$N_1 = 2.00000$

$N_2 = 3.00000$

$\frac{N_1}{N_2} = 0.66667$

$N_3 = 0.66667$

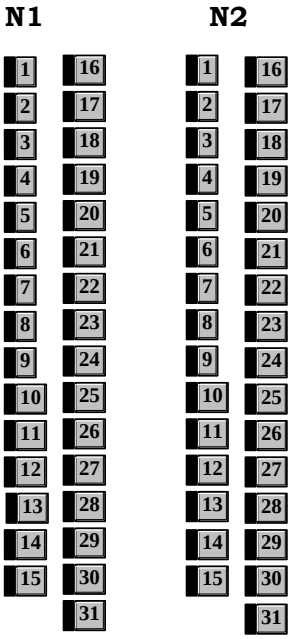
$\frac{U_{nit}}{0N_4} = 2.50000$

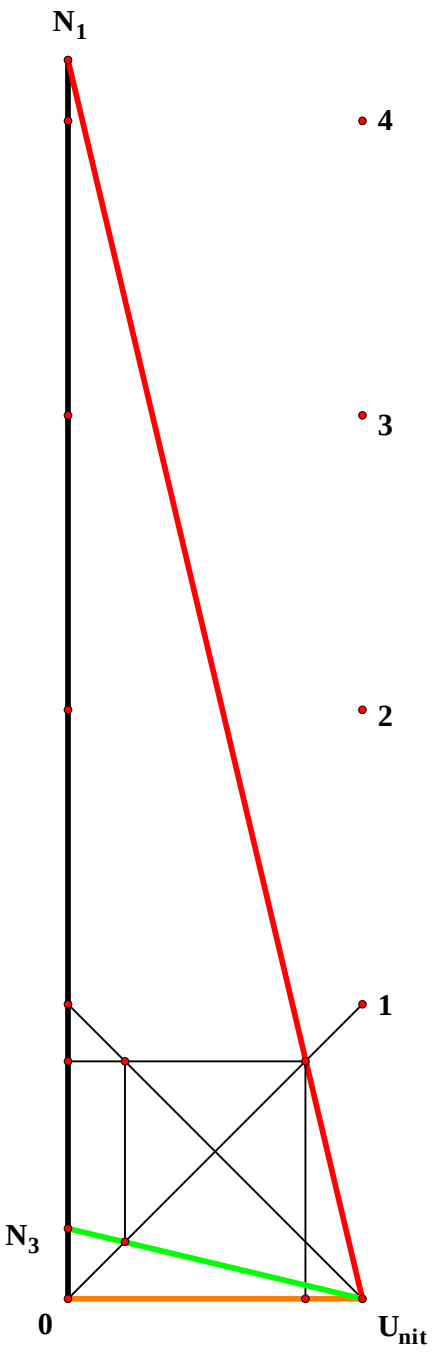
$\frac{N_1+N_2}{N_1} = 2.50000$

$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$

$\frac{N_1+N_2}{N_2} = 1.66667$

**Divide**
**N1 by N2**






**Find the Recprocal N1**

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

**N1**

| | |
|---|---|
| 1 | 16 |
| 2 | 17 |
| 3 | 18 |
| 4 | 19 |
| 5 | 20 |
| 6 | 21 |
| 7 | 22 |
| 8 | 23 |
| 9 | 24 |
| 10 | 25 |
| 11 | 26 |
| 12 | 27 |
| 13 | 28 |
| 14 | 29 |
| 15 | 30 |
| | 31 |

$N_1 = 3.00000$

$N_2 = 2.00000$

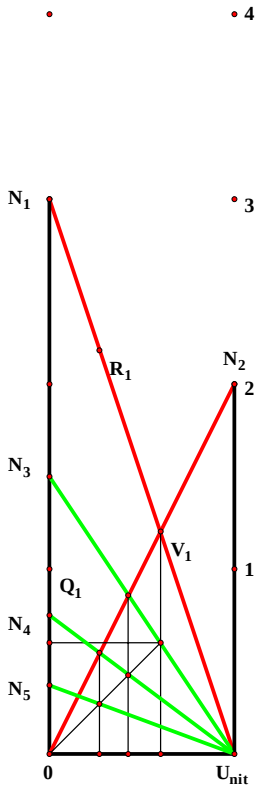
$\frac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

$N_4 = 0.75000$ $\quad \frac{N_1}{N_2^2} - N_4 = 0.00000$

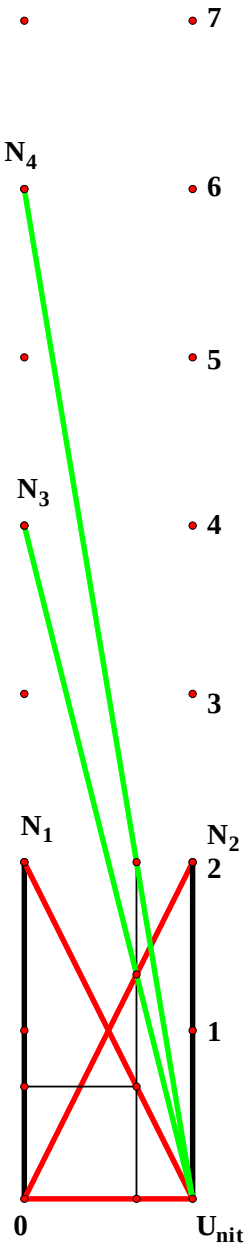
$N_5 = 0.37500$ $\quad \frac{N_1}{N_2^3} - N_5 = 0.00000$

**etc.**




**Divide N1 by a Power of N2**

| | N1 | N2 |
|---|---|---|
| 1 | 16 | 16 |
| 2 | 17 | 17 |
| 3 | 18 | 18 |
| 4 | 19 | 19 |
| 5 | 20 | 20 |
| 6 | 21 | 21 |
| 7 | 22 | 22 |
| 8 | 23 | 23 |
| 9 | 24 | 24 |
| 10 | 25 | 25 |
| 11 | 26 | 26 |
| 12 | 27 | 27 |
| 13 | 28 | 28 |
| 14 | 29 | 29 |
| 15 | 30 | 30 |
| | 31 | 31 |




$N_1 = 2.00000$

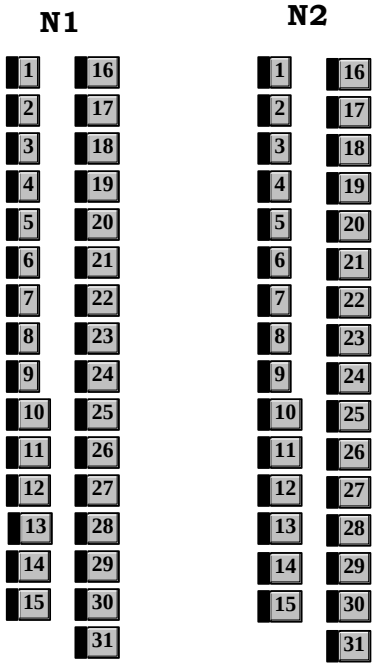
$N_2 = 2.00000$

$N_1 \cdot N_2 = 4.00000$

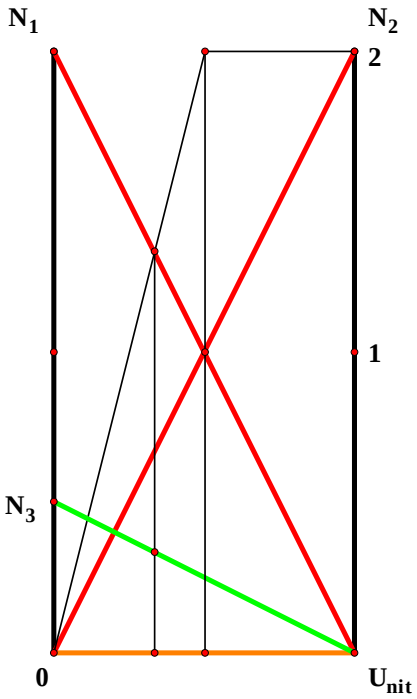
$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



3



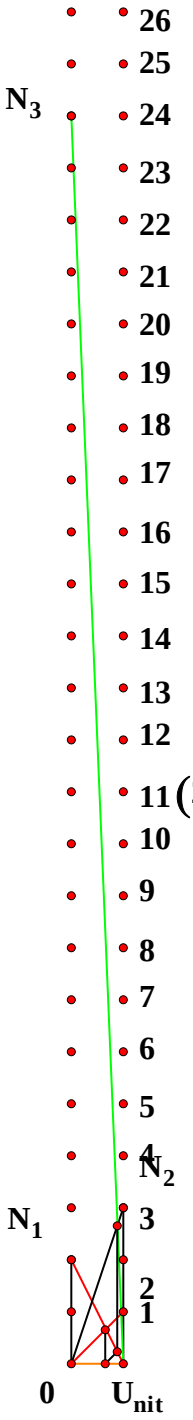

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$

| N1 | | N2 | |
|---|---|---|---|
| 1 | 16 | 1 | 16 |
| 2 | 17 | 2 | 17 |
| 3 | 18 | 3 | 18 |
| 4 | 19 | 4 | 19 |
| 5 | 20 | 5 | 20 |
| 6 | 21 | 6 | 21 |
| 7 | 22 | 7 | 22 |
| 8 | 23 | 8 | 23 |
| 9 | 24 | 9 | 24 |
| 10 | 25 | 10 | 25 |
| 11 | 26 | 11 | 26 |
| 12 | 27 | 12 | 27 |
| 13 | 28 | 13 | 28 |
| 14 | 29 | 14 | 29 |
| 15 | 30 | 15 | 30 |
| | 31 | | 31 |

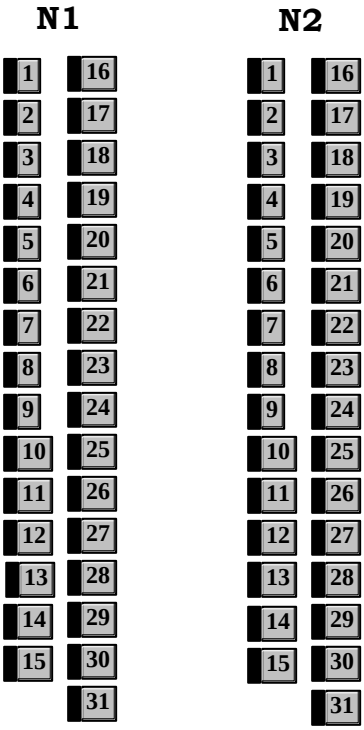



$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$



$\frac{BC}{BV_1} = 0.81650$ $\quad N_3^{-0.5} = 0.81650$

$N_3^{0} = 1.00000$

$\frac{BC}{BU_1} = 1.22474$ $\quad N_3^{0.5} = 1.22474$

$\frac{BC}{BP_1} = 1.50000$

$\frac{BC}{BR_1} = 2.25000$ $\quad N_3^{2} - \frac{BC}{BR_1} = 0.00000$

$\frac{BC}{BS_1} = 3.37500$ $\quad N_3^{3} - \frac{BC}{BS_1} = 0.00000$

$\frac{BC}{BT_1} = 5.06250$ $\quad N_3^{4} - \frac{BC}{BT_1} = 0.00000$

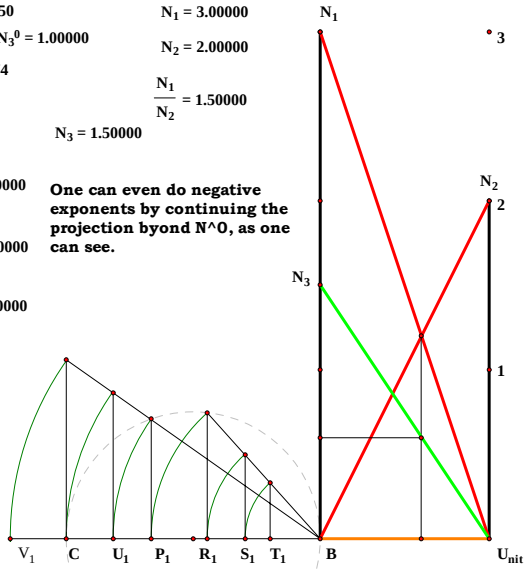
$N_1 = 3.00000$

$N_2 = 2.00000$

$\frac{N_1}{N_2} = 1.50000$

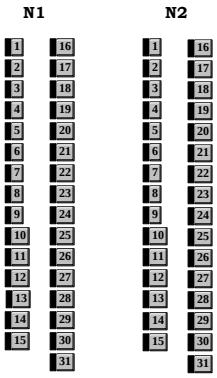
$N_3 = 1.50000$

**One can even do negative exponents by continuing the projection beyond N^0, as one can see.**




**Adding a figure to a figure.**

**I tossed this together, so it is not perfect--it just looks good**



**How to find a Proportional Unit for a given pair of Magnitudes**

$Unit_2 = 1.00000$

$N1_2 = 1.39420$

$N2_2 = 1.94380$

$N3_2 = 0.71726$

$N4_2 = 2.71004$

$\frac{N1_2}{N2_2} - N3_2 = 0.00000$

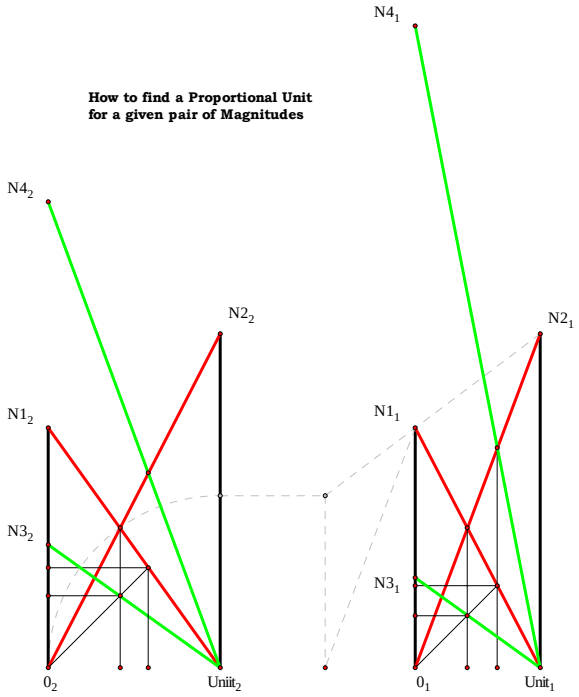
$N1_2 \cdot N2_2 - N4_2 = 0.00000$

$\frac{N4_2}{N2_2} = 1.39420$

$\frac{N2_2}{N1_2} = 1.39420$

$\frac{N1_2}{Unit_2} = 1.39420$

$\frac{Unit_2}{N3_2} = 1.39420$




$Unit_1 = 1.00000$

$N1_1 = 1.91861$

$N2_1 = 2.67492$

$N3_1 = 0.71726$

$N4_1 = 5.13213$

$\frac{N1_1}{N2_1} - N3_1 = 0.00000$

$N1_1 \cdot N2_1 - N4_1 = 0.00000$

$\frac{N4_1}{N2_1} = 1.91861$

$\frac{N2_1}{N1_1} = 1.39420$

$\frac{N1_1}{Unit_1} = 1.91861$

$\frac{Unit_1}{N3_1} = 1.39420$

**Unit = 1.00000**

$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 0.66667$

$N_4 = 6.00000$

$N_5 = 1.33333$

$N_6 = 0.88889$

$N_7 = 4.50000$

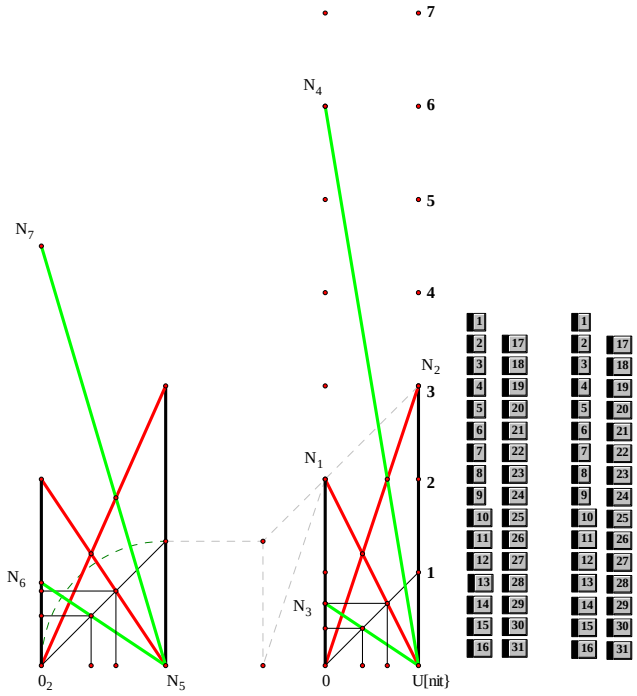
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1^2}{N_2} - N_5 = 0.00000$$

$$\frac{N_1^3}{N_2^2} - N_6 = 0.00000$$

$$\frac{N_2^2}{N_1} - N_7 = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.30952**

**$N_3$ = 0.86598**

**$N_4$ = 4.61905**

**$N_5$ = 0.74992**

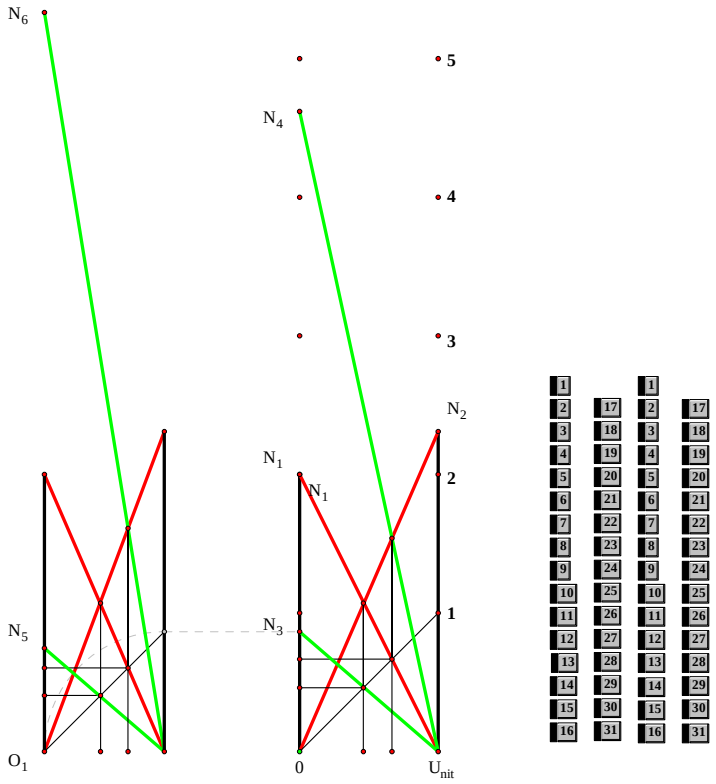
**$N_6$ = 5.33390**

$\frac{N_1}{N_2} - N_3 = 0.00000$

$N_1 \cdot N_2 - N_4 = 0.00000$

$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_2^2 - N_6 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.43939**

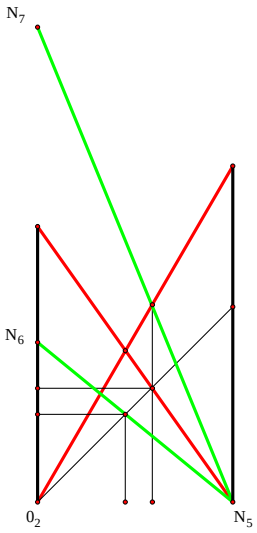
**$N_3$ = 0.81988** $\frac{N_1}{N_2}-N_3 = 0.00000$

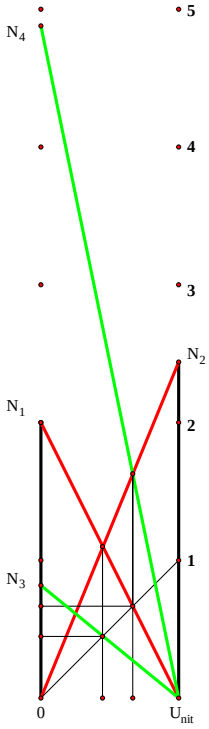
**$N_4$ = 4.87879** $N_1 \cdot N_2 - N_4 = 0.00000$

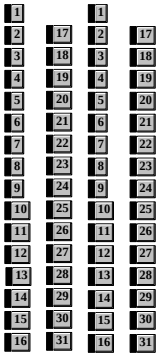
**$N_5$ = 1.41421** $\sqrt{2}-N_5 = 0.00000$

**$N_6$ = 1.15948** $\frac{\sqrt{2}\cdot N_1}{N_2}-N_6 = 0.00000$

**$N_7$ = 3.44982** $\frac{N_1 \cdot N_2}{\sqrt{2}}-N_7 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.00000** $\quad \frac{N_1}{N_2} - N_3 = 0.00000$

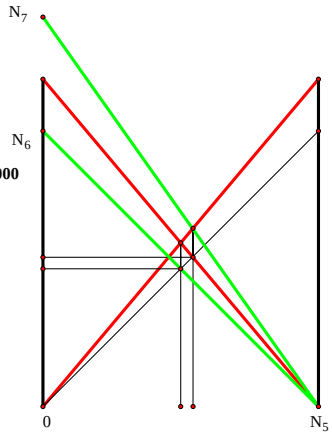
**$N_3$ = 1.00000**

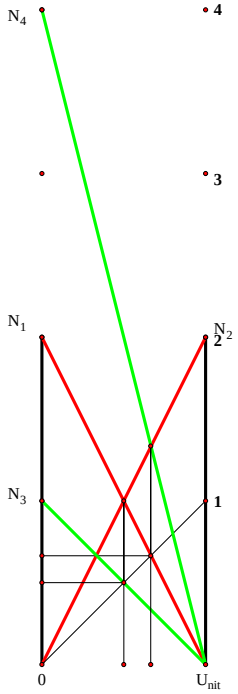
**$N_4$ = 4.00000** $\quad N_1 \cdot N_2 - N_4 = 0.00000$

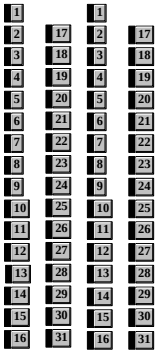
**$N_5$ = 1.68179** $\quad N_5 - 2^{0.75} = 0.00000$

**$N_6$ = 1.68179** $\quad \left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

**$N_7$ = 2.37841** $\quad \frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$



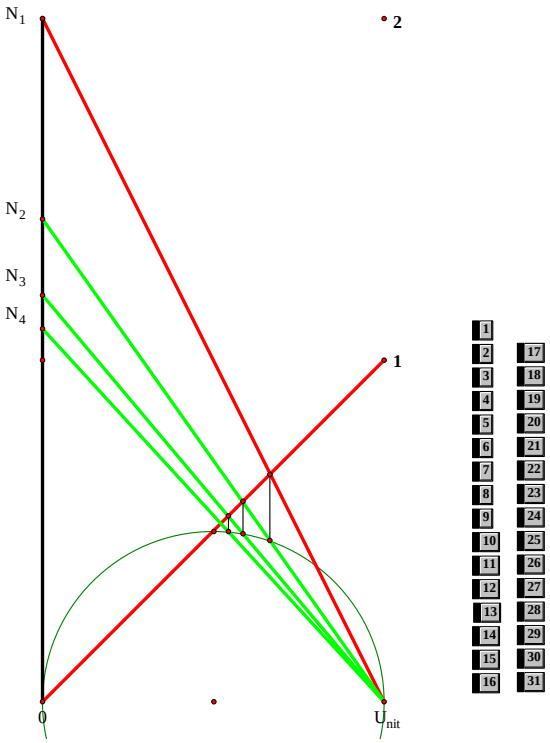
**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 1.41421** **$N_1^{0.5}$-$N_2$ = 0.00000**

**$N_3$ = 1.18921** **$N_1^{0.25}$-$N_3$ = 0.00000**

**$N_4$ = 1.09051** **$N_1^{0.125}$-$N_4$ = 0.00000**



**Unit = 1.00000**

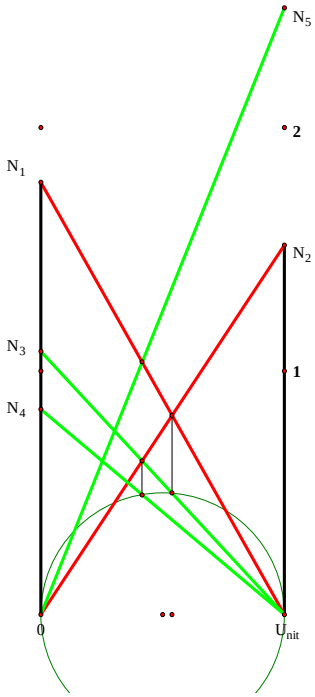
**$N_1$ = 1.77542**

**$N_2$ = 1.51695**

**$N_3$ = 1.08185** $\quad \frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

**$N_4$ = 0.84450** $\quad \frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

**$N_5$ = 2.48947** $\quad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$




| | | | |
|---|---|---|---|
| 1 | 17 | 1 | 17 |
| 2 | 18 | 2 | 18 |
| 3 | 19 | 3 | 19 |
| 4 | 20 | 4 | 20 |
| 5 | 21 | 5 | 21 |
| 6 | 22 | 6 | 22 |
| 7 | 23 | 7 | 23 |
| 8 | 24 | 8 | 24 |
| 9 | 25 | 9 | 25 |
| 10 | 26 | 10 | 26 |
| 11 | 27 | 11 | 27 |
| 12 | 28 | 12 | 28 |
| 13 | 29 | 13 | 29 |
| 14 | 30 | 14 | 30 |
| 15 | 31 | 15 | 31 |
| 16 | | 16 | |

**Unit = 1.00000**

**$N_1$ = 1.00000**

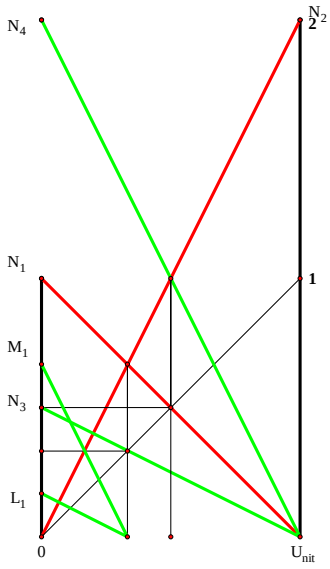
**$N_2$ = 2.00000** $\frac{N_1}{N_2}-N_3 = 0.00000$

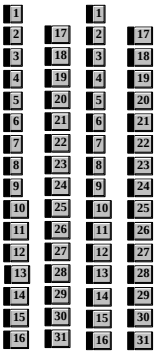
**$N_3$ = 0.50000**

**$N_4$ = 2.00000** $N_1 \cdot N_2 - N_4 = 0.00000$

**$L_1$ = 0.16667** $\frac{N_1^2}{(N_1+N_2)\cdot N_2}-L_1 = 0.00000$

**$M_1$ = 0.66667** $\frac{N_1^2 \cdot N_2}{N_1+N_2}-M_1 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

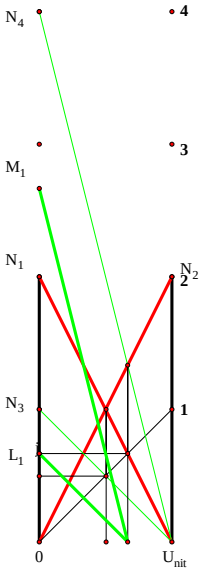
**$N_2$ = 2.00000** $\frac{N_1}{N_2}$**-$N_3$ = 0.00000**

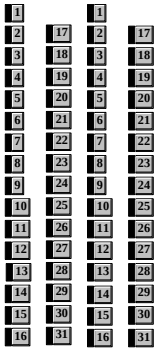
**$N_3$ = 1.00000**

**$N_4$ = 4.00000** **$N_1 \cdot N_2 - N_4 = 0.00000$**

**$L_1$ = 0.66667** $\frac{N_1^2}{N_2 \cdot (N_1+1)}$**-$L_1$ = 0.00000**

**$M_1$ = 2.66667** $\frac{N_1^2 \cdot N_2}{N_1+1}$**-$M_1$ = 0.00000**



$U_1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 1.00000$ $\quad \frac{N_1}{N_2} = 1.00000$

$N_4 = 4.00000$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$U[1] / U_2 = 0.25000$

$N_5 = 2.00000$

$N_6 = 2.00000$

$N_7 = 0.25000$ $\quad \frac{N_1^2}{N_2^4} - N_7 = 0.00000$

$N_8 = 16.00000$ $\quad N_2^4 - N_8 = 0.00000$

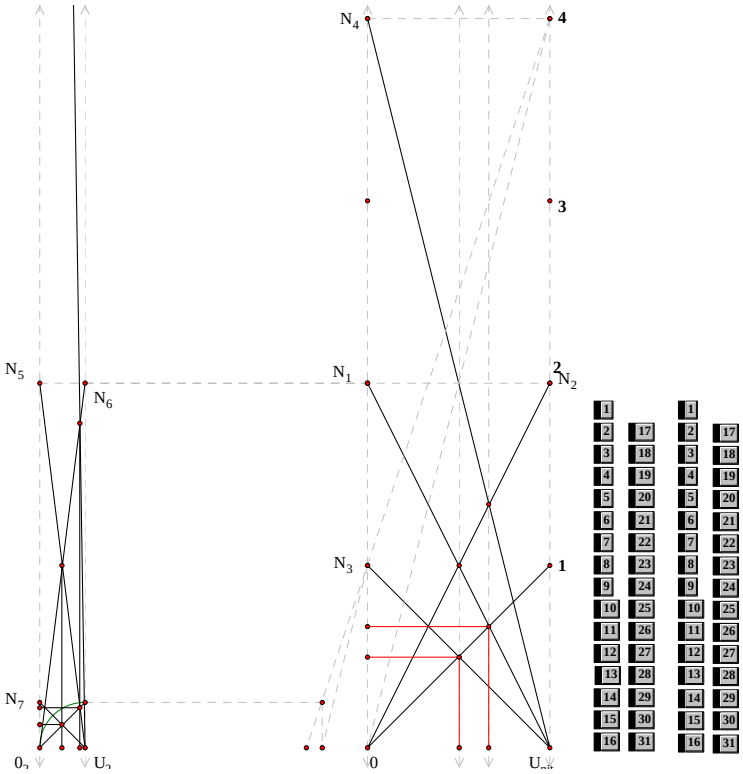
$U_2 = 1.00000$

$N_{25} = 8.00000$ $\quad N_2^3 - N_{25} = 0.00000$

$N_{26} = 8.00000$ $\quad \frac{N_2^4}{N_1} - N_{26} = 0.00000$

$N_{27} = 1.00000$ $\quad \frac{N_1}{N_2} - N_{27} = 0.00000$

$N_8 = 64.00000$ $\quad \frac{N_2^7}{N_1} - N_8 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 1.72159**

**$N_2$ = 2.12500**

**$N_3$ = 0.81016**

**$N_4$ = 3.65838**

**$N_5$ = 0.57386**

**$N_6$ = 1.14773**

**$N_7$ = 0.70833**

**$N_8$ = 1.41667**

**$N_9$ = 1.21946**

**$N_{10}$ = 2.43892**

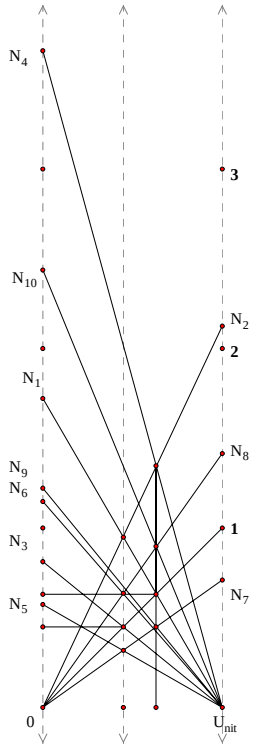
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

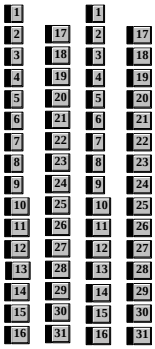
$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1}{N_2} = 0.81016$$

$$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$$N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 1.37056**

**$N_2$ = 1.73096**

**$N_3$ = 0.79179** $\frac{N_1}{N_2} = 0.79179$

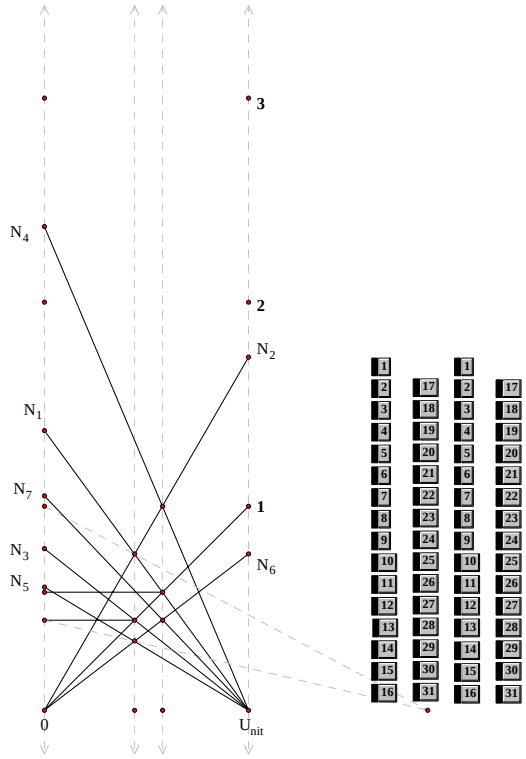
**$N_4$ = 2.37239** $N_1 \cdot N_2 = 2.37239$

$\frac{N_1}{N_2} = 0.79179$

**$N_5$ = 0.60565** $N_1 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000$

**$N_6$ = 0.76491** $N_1 \cdot \left( \frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000$

**$N_7$ = 1.04835** $N_1 \cdot N_2 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000$



**Unit = 1.00000**

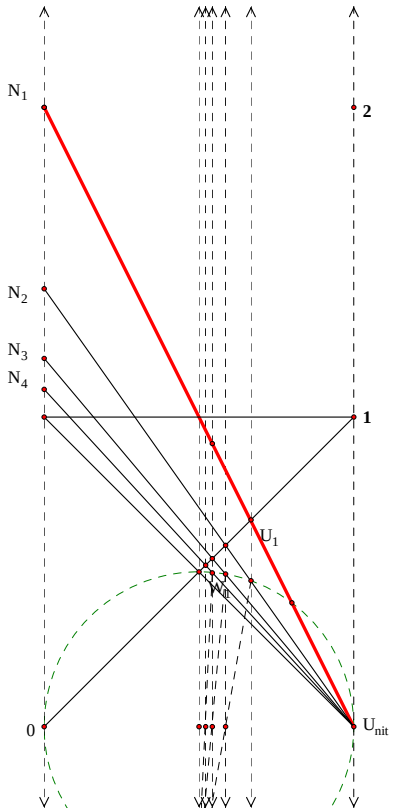
**$N_1$ = 2.00000**

**$N_2$ = 1.41421** $\quad N_1^{\frac{1}{2}} - N_2 = 0.00000$

**$N_3$ = 1.18921** $\quad N_1^{\frac{1}{4}} - N_3 = 0.00000$

**$N_4$ = 1.09051** $\quad N_1^{\frac{1}{8}} - N_4 = 0.00000$

**etc.**




1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

**Unit = 1.00000**

**Eight root series.**

**One can project as long as they like.**

$N_1 = 2.00000$ $N_1^{\frac{8}{8}} - N_1 = 0.00000$

$N_2 = 1.83401$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$

$N_3 = 1.68179$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$

$N_4 = 1.54221$ $N_1^{\frac{5}{8}} - N_4 = 0.00000$

$N_5 = 1.41421$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$

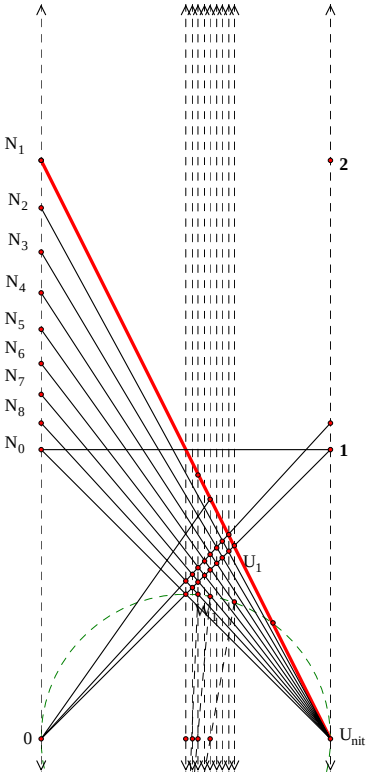
$N_6 = 1.29684$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$

$N_7 = 1.18921$ $N_1^{\frac{2}{8}} - N_7 = 0.00000$

$N_8 = 1.09051$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$

$N_0 = 1.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$

**etc.**





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31

The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

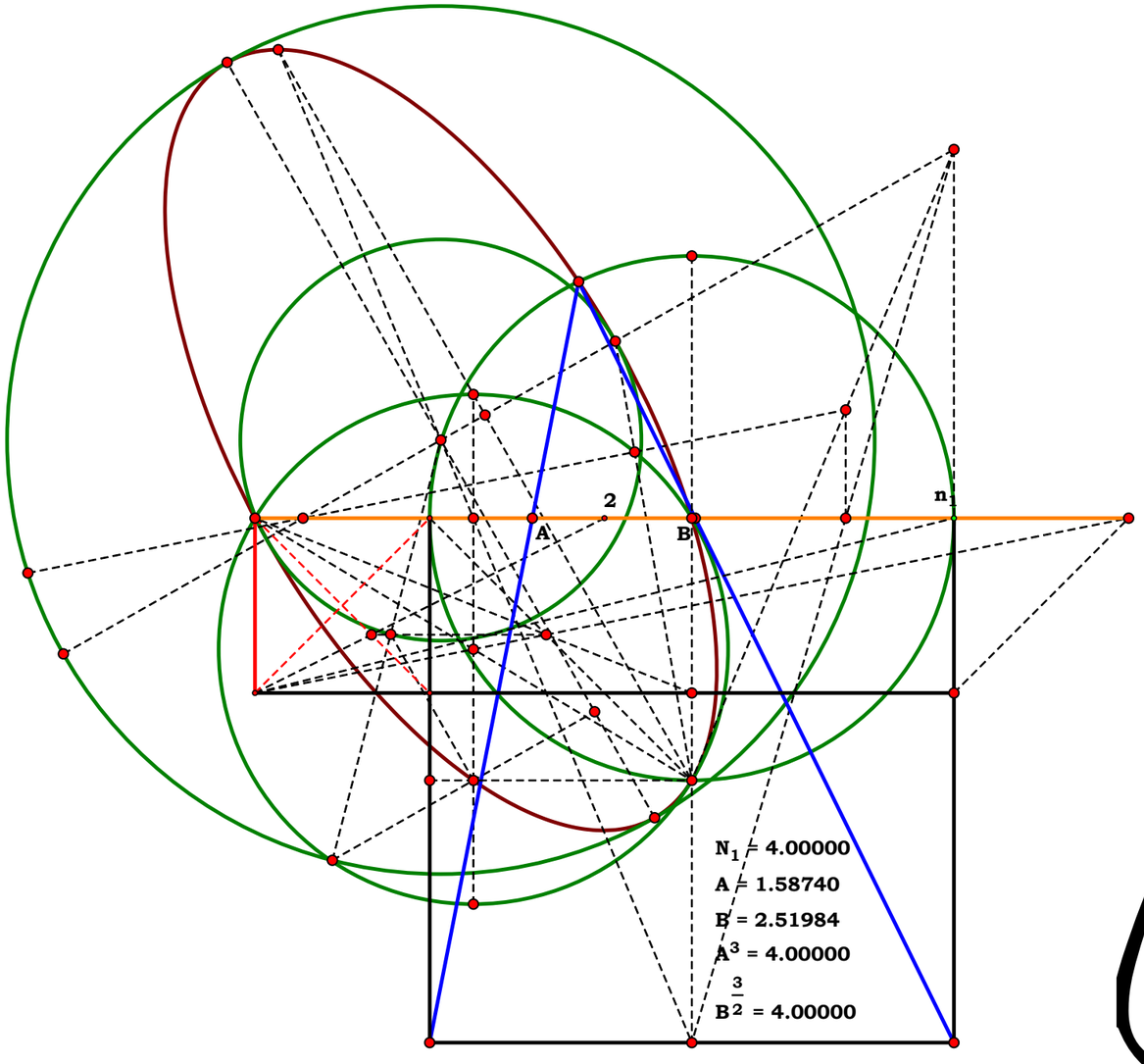
What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.




# The Delian Quest 2008

**John Clark**

Unit.

$BE := 1$

Given.

$N_1 := 3.86292 \quad AB := N_1$

$N_2 := .74482 \quad BD := N_2$

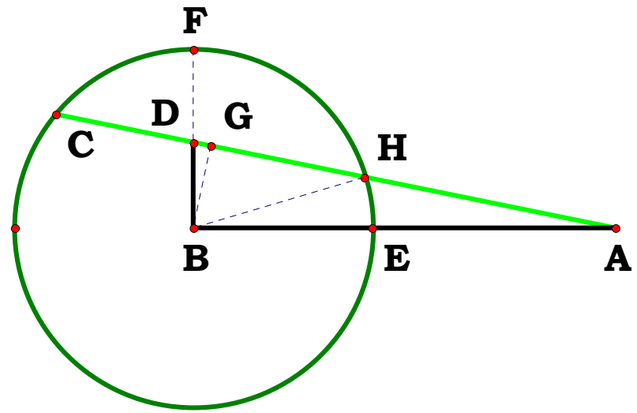
052108

Descriptions.

$AD := \sqrt{AB^2 + BD^2} \qquad DG := \frac{BD^2}{AD}$

$BH := BE \qquad BG := \frac{AB \cdot BD}{AD}$

$GH := \sqrt{BH^2 - BG^2} \qquad AC := AD + GH - DG$




Definitions.

$$\frac{\sqrt{AB^2 \cdot BE^2 + BD^2 \cdot BE^2 - AB^2 \cdot BD^2} + AB^2}{\sqrt{AB^2 + BD^2}} - AC = 0$$

$$AC - \frac{N_1^2 + \sqrt{N_1^2 - N_1^2 \cdot N_2^2 + N_2^2}}{\sqrt{N_1^2 + N_2^2}} = 0$$

052208

Unit.

Given.

$N_1 := 8.17825 \quad AB := N_1$

$N_2 := 2.36240 \quad BC := N_2$

$N_3 := .36912$

Descriptions.

$AD := N_1 - N_2 \quad DE := AD \cdot N_3 \quad AE := AD - DE$

$BE := AB - AE \quad AC := \sqrt{AB^2 + BC^2} \quad BH := \frac{AB \cdot BC}{AC}$

$BG := BE \quad CH := \frac{BC^2}{AC} \quad GH := \sqrt{BG^2 - BH^2} \quad FH := GH$

$AF := AC + FH - CH \quad AF = 11.753331$

Definitions.

$AD - (N_1 - N_2) = 0 \quad DE - (N_1 - N_2) \cdot N_3 = 0$

$AE - [(N_3 - 1) \cdot (N_2 - N_1)] = 0 \quad BE - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0$

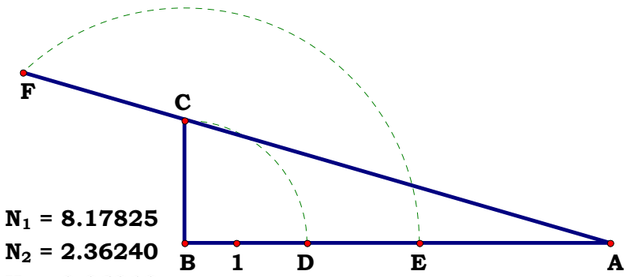
$AC - \sqrt{N_1^2 + N_2^2} = 0 \quad BH - \frac{N_1 \cdot N_2}{\sqrt{N_1^2 + N_2^2}} = 0$

$BG - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0 \quad CH - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

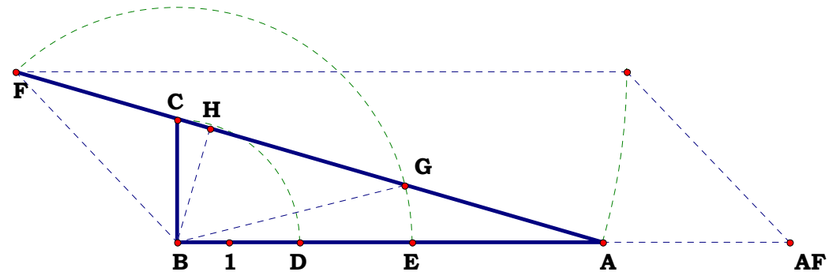
$GH - \frac{\sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$

$AF - \frac{N_1^2 + \sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$

**Given AB, BC and AE as a portion of AD, what is AF?**




$N_1 = 8.17825$
$N_2 = 2.36240$
$N_3 = 0.36912$
$X = 1.48192$
$Y = 4.01471$




$N_1 = 8.17825 \quad AF = 11.75336$
$N_2 = 2.36240$
$N_3 = 0.36912$
$X = 1.48192$
$Y = 4.01471$

060208A

**Unit.**

**Given.**

$N_1 := 1.40187$ $\quad AB := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := 1.13348$ $\quad CD := N_3$

## For a straight line ellipse and three givens.

### a: AB, AC, CD

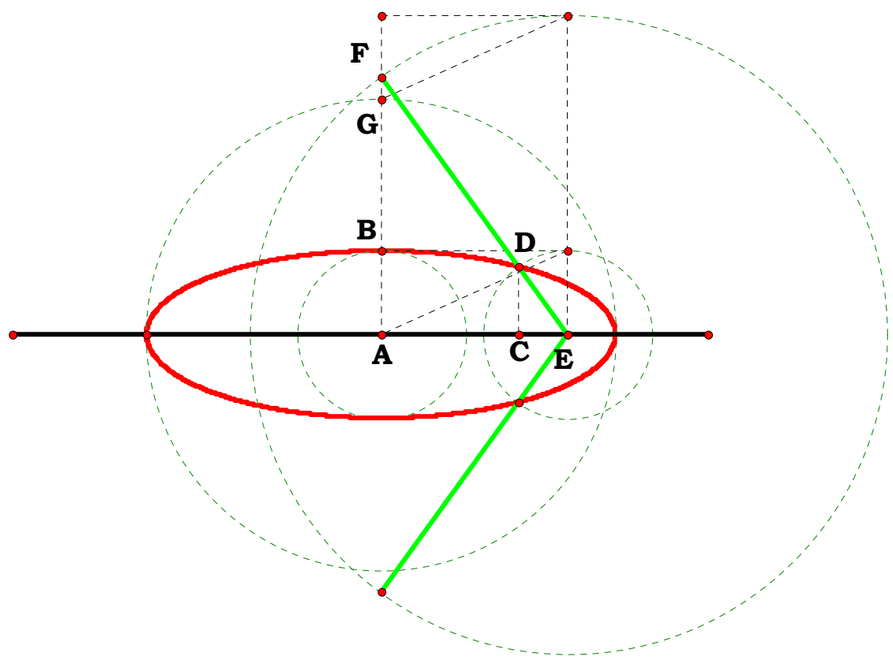
**Descriptions.**

$DE := AB \qquad CE := \sqrt{DE^2 - CD^2}$

$DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$

**Definitions.**

$$BG - N_1 \cdot \left( \frac{N_2}{\sqrt{N_1^2 - N_3^2}} - 1 \right) = 0$$



060208B

**Unit.**

**Given.**

$N_1 := .8249$ $\quad CE := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := 1.13348$ $\quad CD := N_3$

## For a straight line ellipse and three givens.

### b: CE, AC, CD.

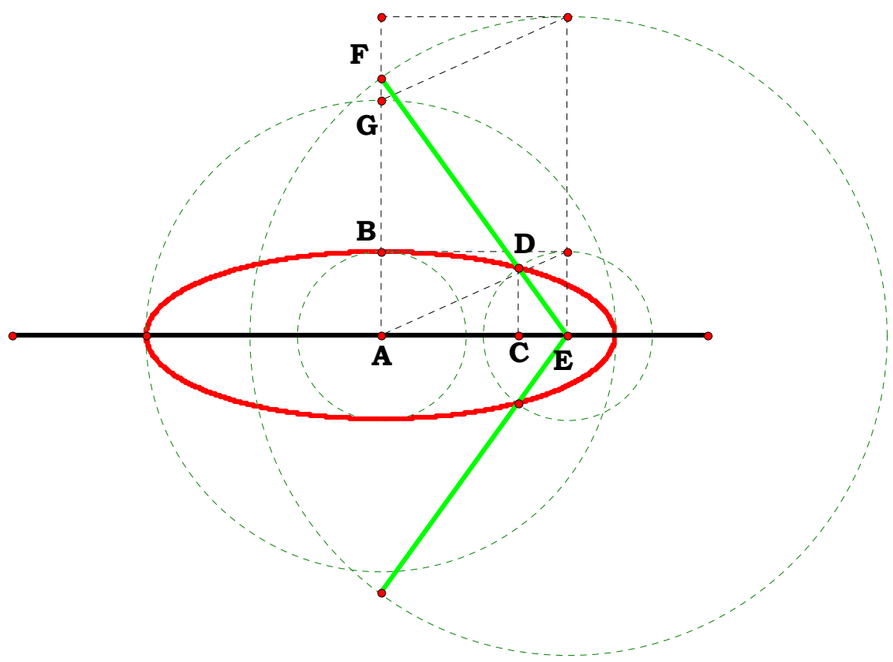
**Descriptions.**

$$AB := \sqrt{CD^2 + CE^2} \qquad DE := AB$$

$$DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$$

**Definitions.**

$$BG - \frac{\sqrt{N_1^2 + N_3^2} \cdot (N_2 - N_1)}{N_1} = 0$$



060208C

**Unit.**

**Given.**

$N_1 := 1.40187$ $\quad AB := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := .8249$ $\quad CE := N_3$

## For a straight line ellipse and three givens.

**Descriptions.**

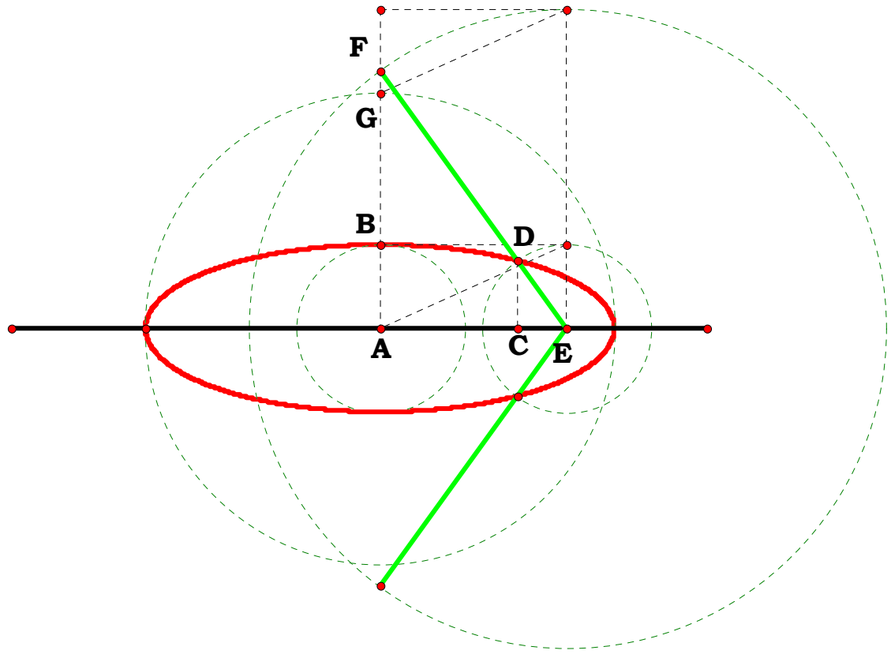
$DE := AB \qquad DF := \frac{DE \cdot AC}{CE}$

$BG := DF - AB$

**Definitions.**

$$BG - \frac{N_1 \cdot (N_2 - N_3)}{N_3} = 0$$

**c: AB, AC, CE**

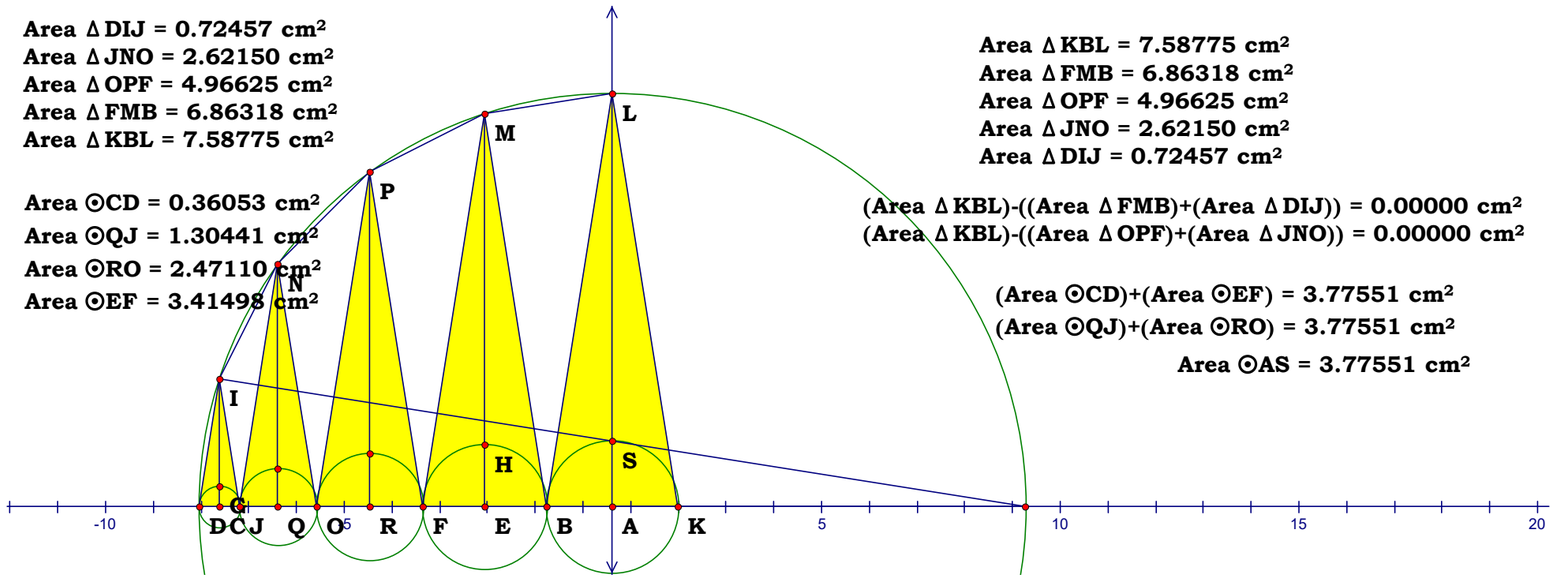


**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

**Procrastinated Write up for 060308**

**Angles are expressible as an elliptical progression, and they show very arithmetic properties to one another.**



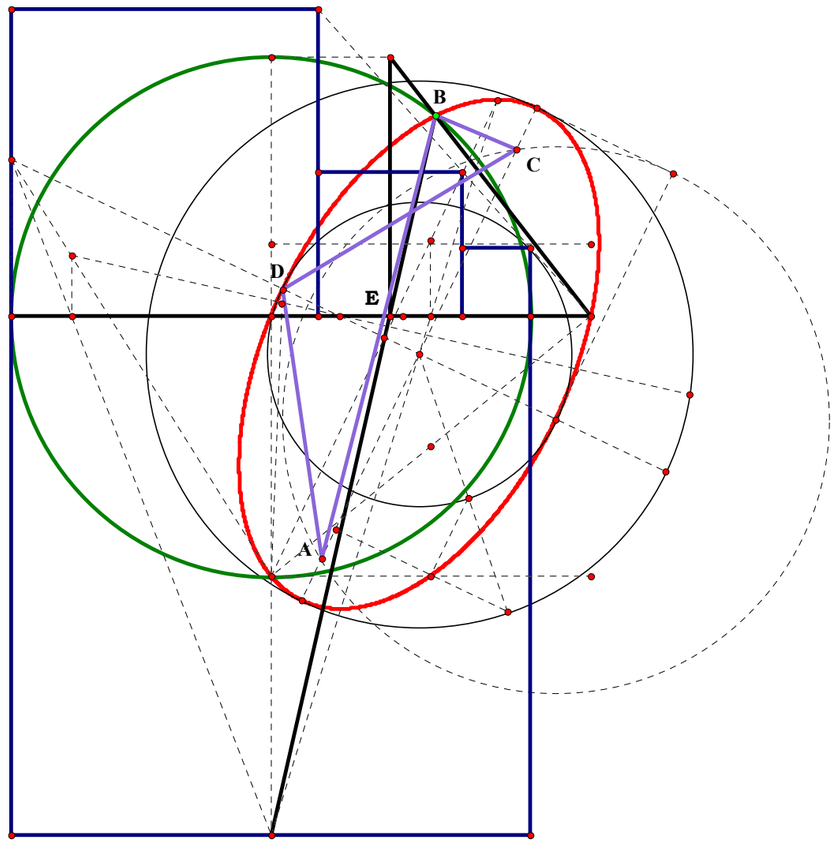
**Unit.**

**Given.** **Parcing project for 061308a**

**Descriptions.**

**Definitions.**

**See about writing up a proof of the figure using the fact that from the center of the two roots, point E, a simple construct will produce the intersection B for the figure. And chect to see how the point E moves during Gemini roots.**

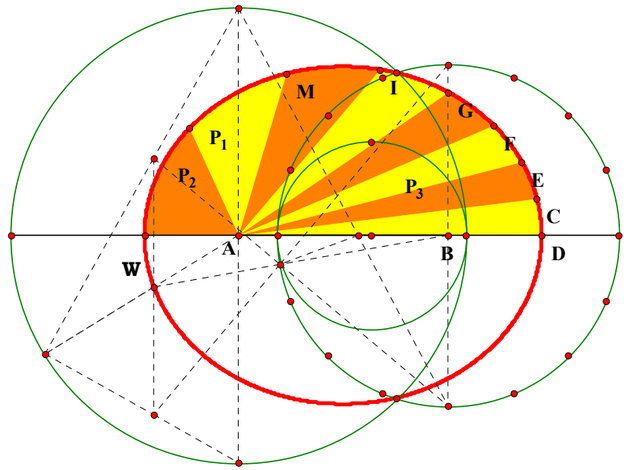


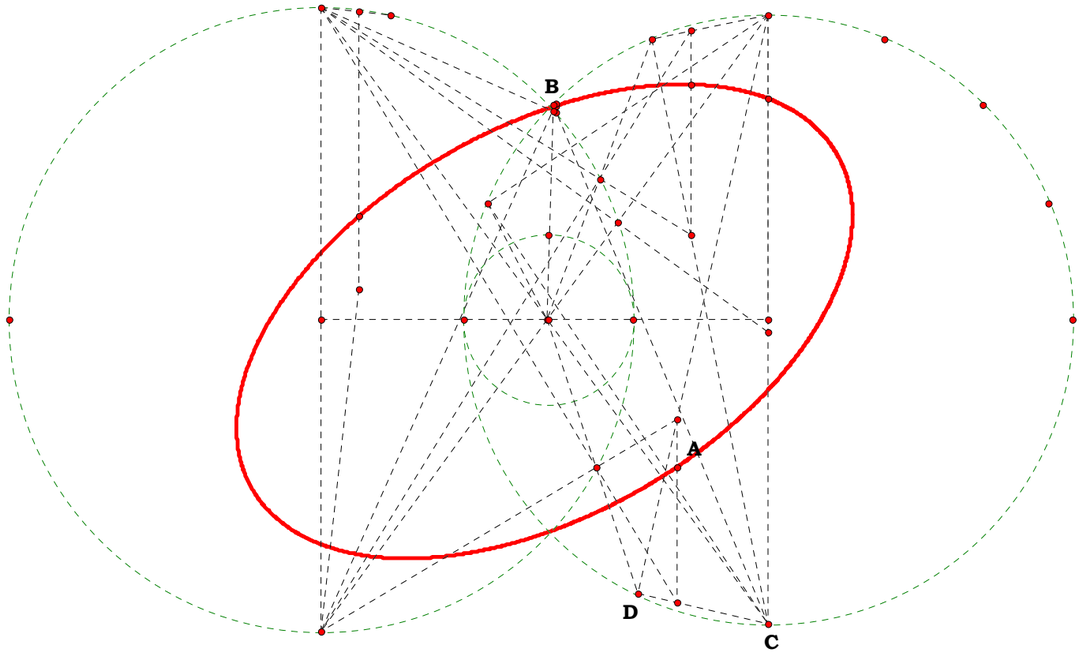
**Unit.**
**Given.**

**Parcing project for 061308b**

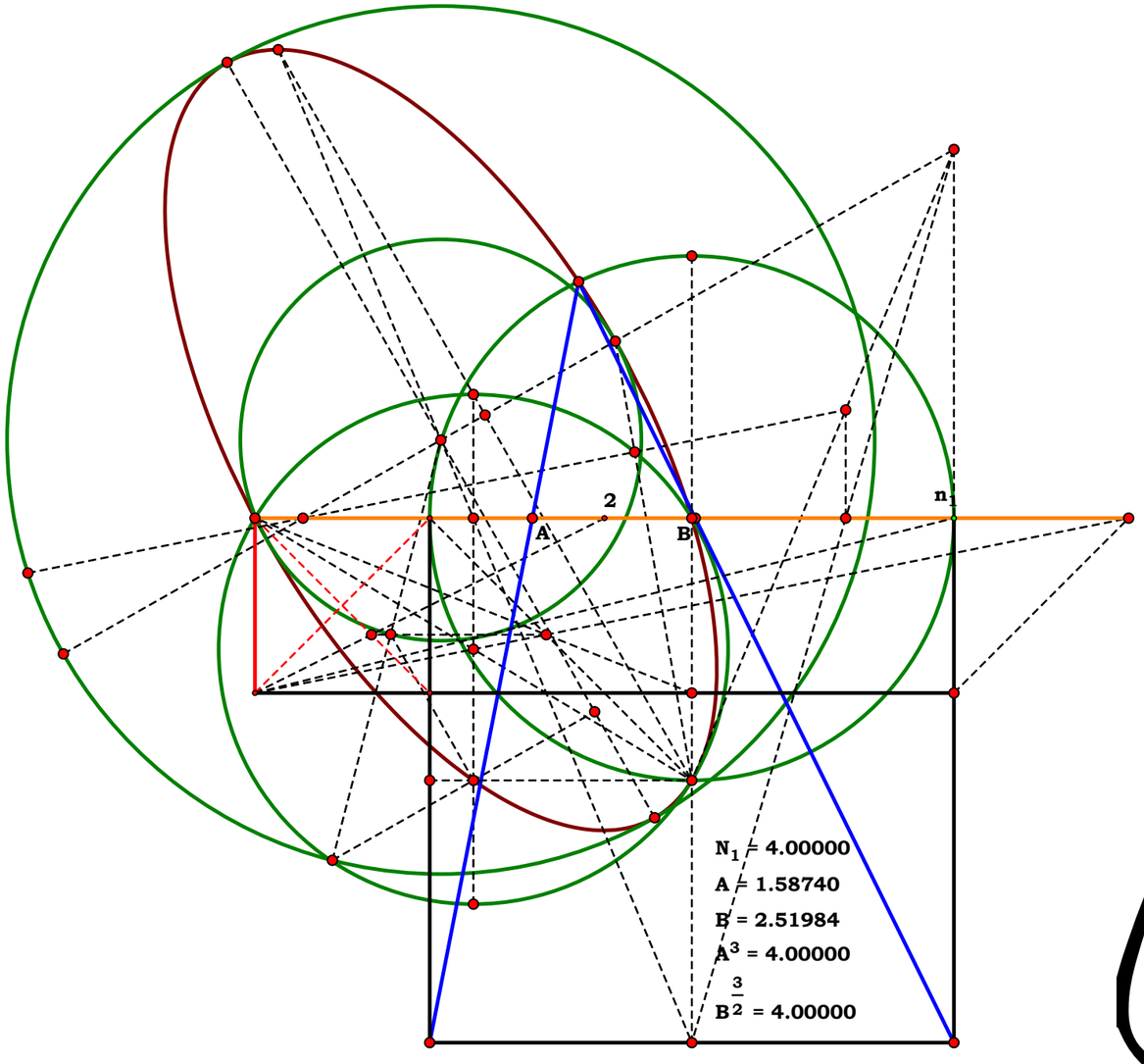
**Looking at mass in eliptical motions from a different point of view**

**Descriptions.**
**Definitions.**




**How you understand the ellipes determines how you think an object moving in that orbit ought to be comprehended and written up as a law of nature, however ponder this fact. The velocity of D is a constant; it determines the velocity of A, the object you can see orbiting say the sun. However, the velocity of A exhibits the same characteristics of a planet or asteroid, slingshot effect and all. The velocity is still due to the constant velocity of D. The current understanding of planitary interaction is not correct.**




# The Delian Quest 2010

**John Clark**

**Unit.**
**Given.**

**On angle trisection.**

**061110**

**Descriptions.**
**Definitions.**

**Parcing project. There is a whole series of plates here, write them up.**

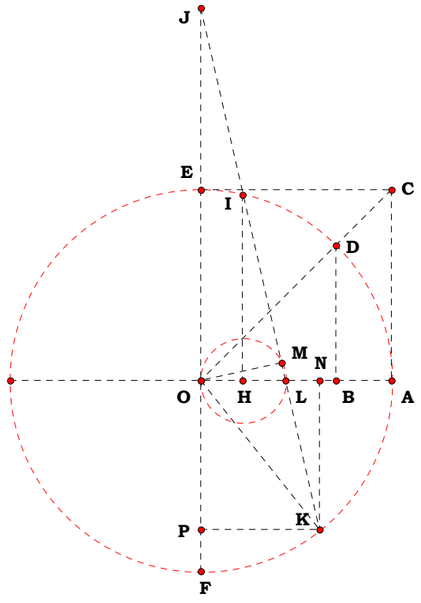
**AO = 3.19617 cm**
**CO = 4.52006 cm**
**BO = 2.26003 cm**

**HO = 0.70753 cm**

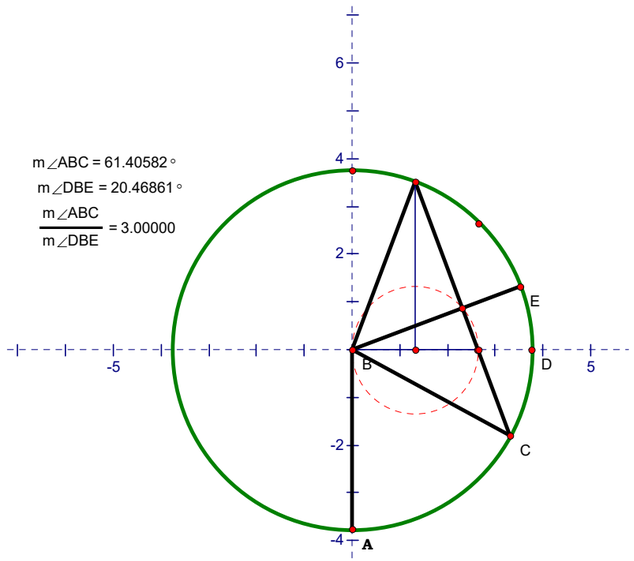
$\frac{BO}{HO}$ = 3.19427

**LM = 0.31325 cm**

**MO = 1.37995 cm**
**KM = 2.88292 cm**
**JK = 8.96201 cm**
**KP = 1.98389 cm**




m∠ABC = 61.40582°
m∠DBE = 20.46861°
$\frac{m\angle ABC}{m\angle DBE}$ = 3.00000




**m∠ABC = 55.69074°**

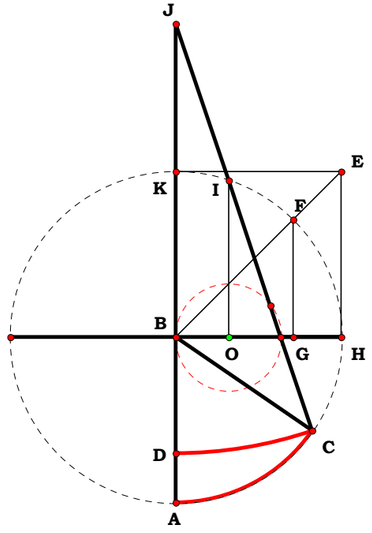
**m∠IBK = 18.56358°**

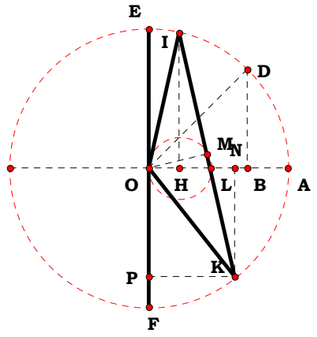
$\frac{m\angle ABC}{m\angle IBK}$ = 3.00000

**BH = 2.77283 cm**
**BG = 1.96069 cm**

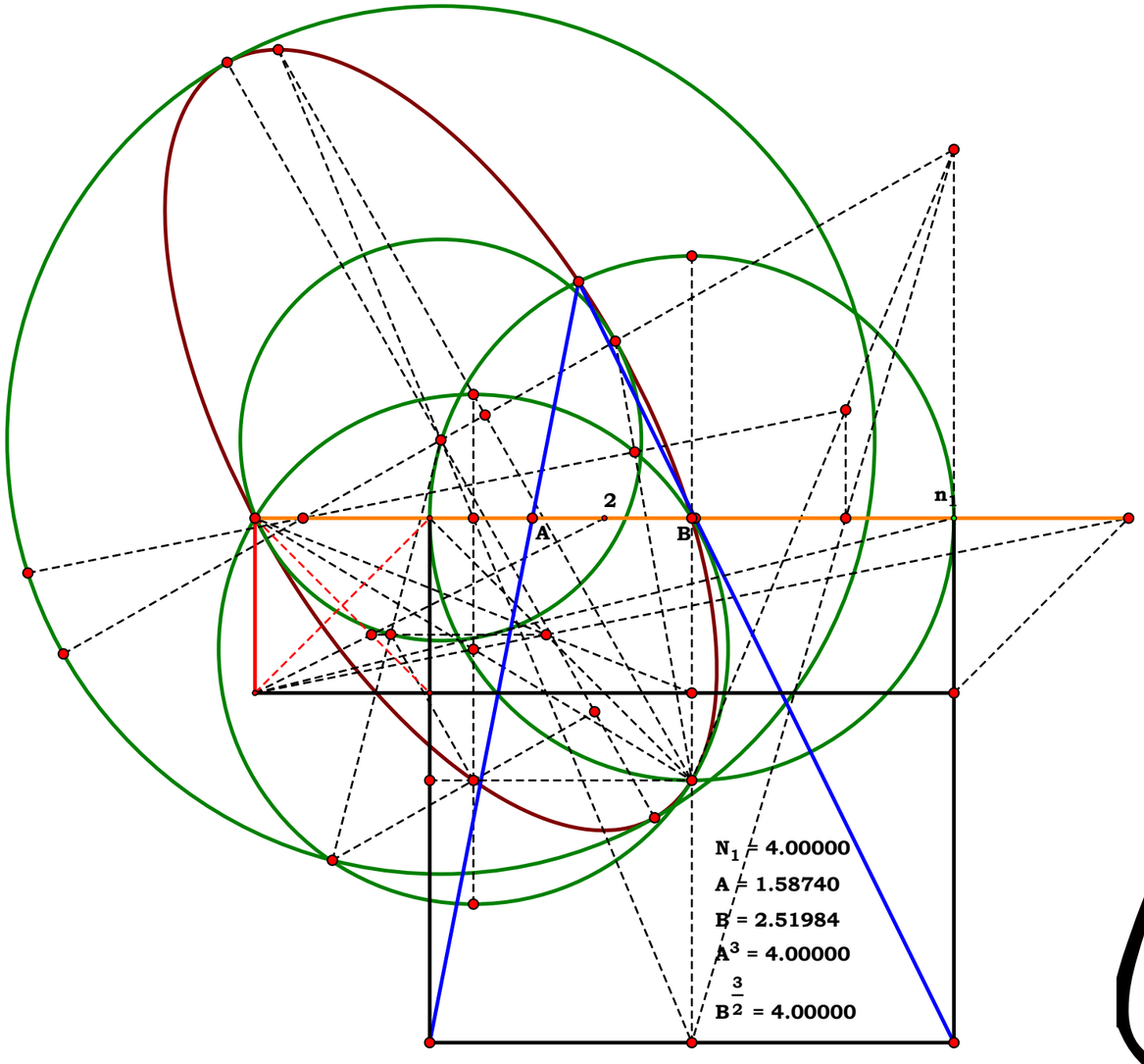
$\frac{BH}{BG}$ = 1.41421

$\sqrt{2}$ = 1.41421




$\frac{m\angle FOK}{m\angle EOI}$ = 3.00000




# The Delian Quest 2011

**John Clark**

020511

Descriptions.

**Rant**

**Percentages, ratio's, proportions, currency conversion, number conversion, etc. Let us say we have a zillion and one items we which to tanslate from one system of measure to another. How can we do all of the items at one and the same time? Well, if you are a non-Euclidean Geometer, you cannot, you are screwed. In fact, if you are a non-Euclidean Geometer, you are claiming that a unit differs from itself and are way too stupid to realize an obvios fact. So, no, I am not entrusting anything to them. And since they are supported by, if not every educational institution, then almost every-one, I had to drop out of school at an early age. The explicit and the tacit admission of their doctrines by our social structure even made me a social outcast. I say, they can argue with the foundation of their own psychology, Language.**

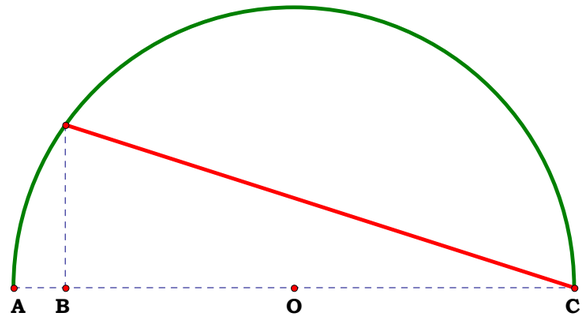
**Therefore, I do not believe that what is needed is a write-up demonstrating proportion, again. Maybe I was just entertaining a rant.**

**What do you think the sentence, As A is to B, so too, C is to D, means? And if you claim that is is not true, citing the mentally lame as authorities for that claim, why is it you prefer a moron over countless examples in your daily life?**

**Why do you suffer your religious leaders, your political leaders, your teachers and your schools to teach the lies?**

**It is not because you are mentally functional. All of my ranting can never change the fact that man is still being made, that his mind is still incapable of doing simple operations.**

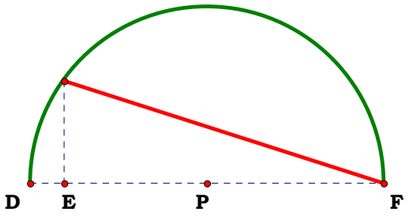
**Therefore, in order to put something constructive here, I will put a digitalization of The Science of Absolute Space, a title which is wholly indicative of someone who is illiterate.**




AC = 2.00000

AB = 0.18650

$\frac{AC}{AB}$ = 10.72378




DF = 5.91633 cm

DE = 0.55170 cm

$\frac{DF}{DE}$ = 10.72378

$\frac{AC}{AB} - \frac{DF}{DE}$ = 0.00000

# THE SCIENCE ABSOLUTE OF SPACE

**Bolyai János**

# THE SCIENCE ABSOLUTE OF SPACE

*Independent of the Truth or Falsity of Euclid's Axiom XI*

(*which can never be decided a priori*).

JOHN BOLYAI

TRANSLATED FROM THE LATIN

BY DR. GEORGE BRUCE HALSTED

PRESIDENT OF THE TEXAS ACADEMY OF SCIENCE

*FOURTH EDITION*.

VOLUME THREE OF THE NEOMONIC SERIES

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U. S. A. 1896

## TRANSLATOR'S INTRODUCTION.

The immortal *Elements* of Euclid was already in dim antiquity a classic, regarded as absolutely perfect, valid without restriction.

Elementary geometry was for two thousand years as stationary, as fixed, as peculiarly Greek, as the Parthenon. On this foundation pure science rose in Archimedes, in Apollonius, in Pappus; struggled in Theon, in Hypatia; declined in Proclus; fell into the long decadence of the Dark Ages.

The book that monkish Europe could no longer understand was then taught in Arabic by Saracen and Moor in the Universities of Bagdad and Cordova.

To bring the light, after weary, stupid centuries, to western Christendom, an Englishman, Adelhard of Bath, journeys, to learn Arabic, through Asia Minor, through Egypt, back to Spain. Disguised as a Mohammedan student, he got into Cordova about 1120, obtained a Moorish copy of Euclid's *Elements*, and made a translation from the Arabic into Latin.

The first printed edition of Euclid, published in Venice in 1482, was a Latin version from the Arabic. The translation into Latin from the Greek, made by Zamberti from a MS. of Theon's revision, was first published at Venice in 1505.

Twenty - eight years later appeared the *editio princeps* in Greek, published at Basle in 1533 by John Hervagius, edited by Simon Grynaeus. This was for a century and three-quarters the only printed Greek text of all the books, and from it the first English translation (1570) was made by "Henricus Billingsley," afterward Sir Henry Billingsley, Lord Mayor of London in 1591.

And even today, 1895, in the vast system of examinations carried out by the British Government, by Oxford, and by Cambridge, no proof of a theorem in geometry will be accepted which infringes Euclid's sequence of propositions.

Nor is the work unworthy of this extraordinary immortality.

Says Clifford : "This book has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state.

"The encouragement; for it contained a body of knowledge that was really known and could be relied on.

"The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained."

But Euclid stated his assumptions with the most painstaking candor, and would have smiled at the suggestion that he claimed for his conclusions any other truth than perfect deduction from assumed hypotheses. In favor of the external reality or truth of those assumptions he said no word.

Among Euclid's assumptions is one differing from the others in prolixity, whose place fluctuates in the manuscripts.

Peyrard, on the authority of the Vatican MS., puts it among the postulates, and it is often called the parallel-postulate. Heiberg, whose edition of the text is the latest and best (Leipzig, 1883–1888), gives it as the fifth postulate.

James Williamson, who published the closest translation of Euclid we have in English, indicating, by the use of italics, the words not in the original, gives this assumption as eleventh among the Common Notions.

Bolyai speaks of it as Euclid's Axiom XI. Todhunter has it as twelfth of the Axioms.

Clavius (1574) gives it as Axiom 13.

The Harpur Euclid separates it by forty-eight pages from the other axioms.

It is not used in the first twenty-eight propositions of Euclid. Moreover, when at length used, it appears as the inverse of a proposition already demonstrated, the seventeenth, and is only needed to prove the inverse of another proposition already demonstrated, the twenty-seventh.

Now the great Lambert expressly says that Proklus demanded a proof of this assumption because when inverted it is demonstrable.

All this suggested, at Europe's renaissance, not a doubt of the necessary external reality and exact applicability of the assumption, but the possibility of deducing it from the other assumptions and the twenty-eight propositions already proved by Euclid without it.

Euclid demonstrated things more axiomatic by far. He proves what every dog knows, that any two sides of a triangle are together greater than the third.

Yet after he has finished his demonstration, that straight lines making with a transversal equal alternate angles are parallel, in order to

prove the inverse, that parallels cut by a transversal make equal alternate angles, he brings in the unwieldy assumption thus translated by Williamson (Oxford, 1781) :

"11. And if a straight line meeting two straight lines make those angles which are inward and upon the same side of it less than two right angles, the two straight lines being produced indefinitely will meet each other on the side where the angles are less than two right angles."

As Staeckel says, "it requires a certain courage to declare such a requirement, alongside the other exceedingly simple assumptions and postulates." But was courage likely to fail the man who, asked by King Ptolemy if there were no shorter road in things geometric than through his *Elements*? answered, "To geometry there is no special way for kings!"

In the brilliant new light given by Bolyai and Lobachevski we now see that Euclid understood the crucial character of the question of parallels.

There are now for us no better proofs of the depth and systematic coherence of Euclid's masterpiece than the very things which, their cause unappreciated, seemed the most noticeable blots on his work.

Sir Henry Savile, in his Praelectiones on Euclid, Oxford, 1621, p. 140, says : "In pulcherrimo Geometriae corpore duo sunt naevi, duae labes . . . " etc., and these two blemishes are the theory of parallels and the doctrine of proportion; the very points in the Elements which now arouse our wondering admiration. But down to our very nineteenth century an ever renewing stream of mathematicians tried to wash away the first of these supposed stains from the most beauteous body of Geometry.

The year 1799 finds two extraordinary young men striving thus

"To gild refined gold, to paint the lily,
To cast a perfume o'er the violet."

At the end of that year Gauss from Braunschweig writes to Bolyai Farkas in Klausenburg (Kolozsvár) as follows : [Abhandlungen der Koeniglichen Gesellschaft der Wissenschaften zu Goettingen, Bd. 22, 1877.]

"I very much regret, that I did not make use of our former proximity, to find out *more* about your investigations in regard to the first grounds of geometry; I should certainly thereby have spared myself much vain labor, and would have become more restful than any one, such

as I, can be, so long as on such a subject there yet remains so much to be wished for.

In my own world thereon I myself have advanced far (though my other wholly heterogeneous employments leave me little time therefore) but *the* way, which I have hit upon, leads not so much to the goal, which one wishes, as much more to making doubtful the truth of geometry.

Indeed I have core upon much, which with most no doubt would pass for a proof, but which in my eyes proves as good as *nothing*.

For example, if one could prove, that a rectilineal triangle is possible, whose content may be greater, than any given surface, then I am in condition, to prove with perfect rigor all geometry.

Most would indeed let that pass as an axiom; I not; it might well be possible, that, how far apart soever one took the three vertices of the triangle in space, yet the content was always under a given limit.

I have more such theorems, but in none do I find anything satisfying."

From this letter we clearly see that in 1799 Gauss was still trying to prove that Euclid's is the only non-contradictory system of geometry,

and that it is the system regnant in the external space of our physical experience.

The first is false; the second can never be proven.

Before another quarter of a century, Bolyai János, then unborn, had created another possible universe; and, strangely enough, though nothing renders it impossible that the space of our physical experience may, this very year, be satisfactorily shown to belong to Bolyai János , yet the same is not true for Euclid.

To decide our space is Bolyai's, one need only show a single rectilineal triangle whose angle-sum measures less than a straight angle. And this could be shown to exist by imperfect measurements, such as human measurements must always be. For example, if our instruments for angular measurement could be brought to measure an angle to within one millionth of a second, then if the lack were as great as two millionths of a second, we could make certain its existence.

But to prove Euclid's system, we must show that a triangle's angle-sum is *exactly* a straight angle, which nothing human can ever do.

However this is anticipating, for in 1799 it seems that the mind of the elder Bolyai, Bolyai Farkas, was in precisely the same state as

that of his friend Gauss. Both were intensely trying to prove what now we know is indemonstrable. And perhaps Bolyai got nearer than Gauss to the unattainable. In his "Kurzer Grundriss eines Versuchs," etc., p. 46, we read : "Koennten jede 3 Punkte, die nicht in einer Geraden sind, in eine Sphaere fallen, so waere das Eucl. Ax. XI. bewiesen." Frischauf calls this "das anschaulichste Axiom." But in his Autobiography written in Magyar, of which my Life of Bolyai contains the first translation ever made, Bolyai Farkas says : "Yet I could not become satisfied with my different treatments of the question of parallels, which was ascribable to the long discontinuance of my studies, or more probably it was due to myself that I drove this problem to the point which robbed my rest, deprived me of tranquillity."

It is well-nigh certain that Euclid tried his own calm, immortal genius, and the genius of his race for perfection, against this self-same question. If so, the benign intellectual pride of the founder of the mathematical school of the greatest of universities, Alexandria, would not let the question cloak itself in the obscurities of the infinitely great or the infinitely small. He would say to himself : "Can I prove

this plain, straightforward, simple theorem : "those straights which are produced indefinitely from less than two right angles meet." [This is the form which occurs in the Greek of Eu. I. 29.]

Let us not underestimate the subtle power of that old Greek mind. We can produce no Venus of Milo. Euclid's own treatment of proportion is found as flawless in the chapter which Stolz devotes to it in 1885 as when through Newton it first gave us our present continuous number-system.

But what fortune had this genius in the fight with its self-chosen simple theorem? Was it found to be deducible from all the definitions, and the nine "Common Notions," and the five other Postulates of the immortal Elements? Not so. But meantime Euclid went ahead without it through twenty-eight propositions, more than half his first book. But at last came the practical pinch, then as now the triangle's angle-sum.

He gets it by his twenty-ninth theorem : "A straight falling upon two parallel straights makes the alternate angles equal."

But for the proof of this he needs that recalcitrant proposition which has how long been keeping him awake nights and waking

him up mornings? Now at last, true man of science, he acknowledges it indemonstrable by spreading it in all its ugly length among his postulates.

Since Schiaparelli has restored the astronomical system of Eudoxus, and Hultsch has published the writings of Autolycus, we see that Euclid knew surface-spherics, was familiar with triangles whose angle-sum is more than a straight angle. Did he ever think to carry out for himself the beautiful system of geometry which comes from the contradiction of his indemonstrable postulate; which exists if there be straights produced indefinitely from less than two right angles yet nowhere meeting; which is real if the triangle's angle-sum is less than a straight angle?

Of how naturally the three systems of geometry flow from just exactly the attempt we suppose Euclid to have made, the attempt to demonstrate his postulate fifth, we have a most romantic example in the work of the Italian priest, Saccheri, who died the twenty-fifth of October, 1733. He studied Euclid in the edition of Clavius, where the fifth postulate is given as Axiom 13. Saccheri says it should not be called an axiom, but ought to be demonstrated. He tries this seemingly simple

task; but his work swells to a quarto book of 101 pages.

Had he not been overawed by a conviction of the absolute necessity of Euclid's system, he might have anticipated Bolyai János, who ninety years later not only discovered the new world of mathematics but appreciated the transcendent import of his discovery.

Hitherto what was known of the Bolyais came wholly from the published works of the father Bolyai Farkas, and from a brief article by Architect Fr. Schmidt of Budapest "Aus dem Leben zweier ungarischer Mathematiker, Johann und Wolfgang Bolyai von Bolya." Grunert's Archiv, Bd. 48, 1868, p. 217.

In two communications sent me in September and October 1895, Herr Schmidt has very kindly and graciously put at my disposal the results of his subsequent researches, which I will here reproduce. But meantime I have from entirely another source come most unexpectedly into possession of original documents so extensive, so precious that I have determined to issue them in a separate volume devoted wholly to the life of the Bolyais; but these are not used in the sketch here given.

Bolyai Farkas was born February 9th, 1775, at Bolya, in that part of Transylvania

(Erdély) called Székelyföld. He studied first at Enyed, afterward at Klausenburg (Kolozsvár), then went with Baron Simon Kemény to Jena and afterward to Goettingen. Here he met Gauss, then in his 19th year, and the two formed a friendship which lasted for life.

The letters of Gauss to his friend were sent by Bolyai in 1855 to Professor Sartorius von Walterhausen, then working on his biography of Gauss. Everyone who met Bolyai felt that he was a profound thinker and a beautiful character.

Benzenberg said in a letter written in 1801 that Bolyai was one of the most extraordinary men he had ever known.

He returned home in 1799, and in January, 1804, was made professor of mathematics in the Reformed College of Maros-Vásárhely. Here for 47 years of active teaching he had for scholars nearly all the professors and nobility of the next generation in Erdély.

Sylvester has said that mathematics is poesy.

Bolyai's first published works were dramas.

His first published book on mathematics was an arithmetic :

Az arithmetica eleje. 8vo. i- xvi, 1–162 pp. The copy in the library of the Reformed College is enriched with notes by Bolyai János .

Next followed his chief work, to which he constantly refers in his later writings. It is in Latin, two volumes, 8vo, with title as - follows :

TENTAMEN | JUVENTUTEM STUDIOSAM IN ELEMENTA MATHESEOS PURAE, ELEMENTARIS AC | SUBLIMIORIS, METHODO INTUITIVA, EVIDENTIA— | QUE HUIC PROPRIA, INTRODUCENDI. |

CUM APPENDICE TRIPLICI. | Auctore Professore Matheseos et Physices Chemiaeque | Publ. Ordinario. | Tomus Primus. | *Maros Vasarhelyini*. 1832. | Typis Collegii Reformatorum per JOSEPHUM, et | SIMEONEM KALI de felsö Vist. | At the back of the title : Imprimatur. | M. Vásárhelyini Die | 12 Octobris, 1829. [ Paulus Horváth m. p. | Abbas, Parochus et Censor | Librorum.

Tomus Secundus. | *Maros Vasarhelyini*. 1833.

The first volume contains :

Preface of two pages : *Lectori salutem*.

A folio table : *Explicatio signorumn*.

*Index rerum* (I–XXXII). *Errata* (XXXIII–XXXVII).

*Pro tyronibus prima vice legentibus notanda sequentia* (XXXVIII–LII).

*Errores* (LIII–LXVI).

Scholion (LXVII - LXXIV).

Pluriium errorum haud animadversorum numerous minuitur (LXXV–LXXVI). Recensio per auctorem ipsum facta (LXXVII–LXXVIII).

Errores recentius detecti (L X X V–XCVIII).

Now comes the body of the text (pages 1–502).

Then, with special paging, and a new title page, comes the immortal Appendix, here given in English.

Professors Staeckel and Engel make a mistake in their "Parallellinien" in supposing that this Appendix is referred to in the title of " Tentamen." On page 241 they quote this title, including the words "Cum appendice triplici," and say : "In dem dritten Anhange, der nur 28 Seiten umfasst, hat Johann Bolyai seine neue Geometrie entwickelt."

It is not a third Appendix, nor is it referred to at all in the words "Cum appendice triplici."

These words, as explained in a prospectus in the Magyar language, issued by Bolyai Farkas, asking for subscribers, referred to a real triple Appendix, which appears, as it

should, at the end of the book Tomus Secundus, pp. 265 - 322.

The now world renowned Appendix by Bolyai János was an afterthought of the father, who prompted the son not "to occupy himself with the theory of parallels," as Staeckel says, but to translate from the German into Latin a condensation of his treatise, of which the principles were discovered and properly appreciated in 1823, and which was given in writing to Johann Walter von Eckwehr in 1825.

The father, without waiting for Vol. II, inserted this Latin translation, with separate paging (1 - 26), as an Appendix to his Vol. I, where, counting a page for the title and a page "Explicatio signorum," it has twenty-six numbered pages, followed by two unnumbered pages of Errata.

The treatise itself, therefore, contains only twenty-four pages—the most extraordinary two dozen pages in the whole history of thought! Milton received but a paltry £5 for his Paradise Lost; but it was at least plus £5.

Bolyai János, as we learn from Vol. II, p. 384, of "*Tentamen*," contributed for the

printing of his eternal twenty-six pages, 104 florins 50 kreuzers.

That this Appendix was finished considerably before the Vol. I, which it follows, is seen from the references in the text, breathing a just admiration for the Appendix and the genius of its author.

Thus the father says, p. 452 : Elegans est conceptus *similium*, quem J. B. *Appendicis Auctor* dedit. Again, p. 489 : *Appendicis Actor*, rem acumine singulari aggressus, Geometriam pro omni casu absolute veram posuit; quamvis e magna mole, tantum summe necessaria, in Appendice hujus tomi exhibuerit, multis (ut tetraedri resolutione generali, pluribusque aliis disquisitionibus elegantibus) brevitatis studio omissis.

And the volume ends as follows, p. 502 : Nec operae pretium est plura referre; quum res tota exaltiori contemplationis puncto, in ima penetranti oculo, tractetur in Appendice sequente, a quovis fideli veritatis purae alumno diagna legi.

The father gives a brief resumé of the results of his own determined, life-long, desperate efforts to do that at which Saccheri, J. H. Lambert, Gauss also had failed, to establish Euclid' theory of parallels *a priori*.

He says, p. 490 : "Tentamina idcirco quae olim feceram, breviter exponenda veniunt; ne saltem alius quis operam eandem perdat." He anticipates J. Delboeuf's "Prolégoménes philosophiques de la géométrie et solution des postulats," with the full consciousness in addition that it is *not* the solution,—that the final solution has crowned not his own intense efforts, but the genius of his son.

This son's Appendix which makes all preceding space only a special case, only a species under a genus, and so requiring a descriptive adjective, *Euclidean*, this wonderful production of pure genius, this strange Hungarian flower, was saved for the world after more than thirty-five years of oblivion, by the rare erudition of Professor Richard Baltzer of Dresden, afterward professor in the University of Giessen. He it was who first did justice publicly to the works of Lobachevski and Bolyai.

Incited by Baltzer, in 1866 J. Hoüel issued a French translation of Lobachevski's Theory of Parallels, and in a note to his Preface says : "M. Richard Baltzer, dans la seconde édition de ses excellents *Elenents de Geometrie*, a, le premier, introduit ces notions exactes à la place qu'elles doivent occuper," Honor to

Baltzer! But alas! father and son were already in their graves!

Fr. Schmidt in the article cited (1868) says : "It was nearly forty years before these profound views were rescued from oblivion, and Dr. R. Baltzer, of Dresden, has acquired imperishable titles to the gratitude of all friends of science as the first to draw attention to the works of Bolyai, in the second edition of his excellent *Elemente der Mathematik* (1866–67). Following the steps of Baltzer, Professor Hoüel, of Bordeaux, in a brochure entitled, Essai critique sur les principes fondamentaux de la Géométrie élémentaire, has given extracts from Bolyai's book, which will help in securing for these new ideas the justice they merit."

The father refers to the son's Appendix again in a subsequent book, Urtan elemei kezdöknek [Elements of the science of space for beginners] (1850 – 51), pp. 48. In the College are preserved three sets of figures for this book, two by the author and one by his grandson, a son of János.

The last work of Bolyai Farkas; the only one composed in German, is entitled,

Kurzer Grundriss eines Versuchs

I. Die Arithmetik, durch zvekmässig konstruirte

Begriffe, von eingebildeten und unendlich-kleinen Grössen gereinigt, anschaulich und logisch-streng darzustellen.

II. In der Geometrie, die Begriffe der geraden Linie, der Ebene, des Winkels allgemein, der winkellosen Formen, und der Krummen, der verschiedenen Arten der Gleichheit u. d. gl. nicht nur scharf zu bestimmen; sondern auch ihr Seyn im Raume zu beweisen : und da die Frage, *ob zwey von der dritten geschnittene Geraden, wenn die summe der inneren Winkel nicht = 2R, sich schneiden oder nicht?* neimand auf der Erde ohne ein Axiom (wie Euklid das XI) aufzustellen, beantworten wird; die davon unabhängige Geometrie abzusondern; und eine auf die *Ja*— Antwort, andere auf das *Nein* so zu bauen, das die Formeln der letzten, auf ein Wink auch in der ersten gültig seyen.

Nach ein lateinischen Werke von 1829, M. Vásérhely, und eben daselbst gedruckten ungrischen.

Maros Vásárhely 1851. 8vo. pp. 88.

In this book he says, referring to his son's Appendix : "Some copies of the work published here were sent at that time to Vienna, to Berlin, to Goettingen . . . . From Goettingen the giant of mathematics, who from

his pinnacle embraces in the same view the stars and the abysses, wrote that he was surprised to see accomplished what he had begun, only to leave it behind in his papers."

This refers to 1832. The only other record that Gauss ever mentioned the book is a letter from Gerling, written October 31st, 1851, to Wolfgang Boylai, on receipt of a copy of "Kurzer Grundriss." Gerling, a scholar of Gauss, had been from 1817 Professor of Astronomy at Marburg. He writes : "I do not mention my earlier occupation with the theory of parallels, for already in the year 1810–1812 with Gauss, as earlier 1809 with J. F. Pfaff I had learned to perceive how all previous attempts to prove the Euclidean axiom had miscarried. I had then also obtained preliminary knowledge of your works, and so, when I first [1820] had to print something of my view thereon, I wrote it exactly as it yet stands to read on page 187 of the latest edition.

''We had about this time [1819] here a law professor, Schweikart, who was formerly in Charkov, and had attained to similar ideas, since without help of the Euclidean axiom he developed in its beginnings a geometry which he called Astralgeometry. What he communicated to me thereon I sent to Gauss, who

then informed me how much farther already had been attained on this way, and later also expressed himself about the great acquisition, which is offered to the few expert judges in the Appendix to your book."

The "latest edition" mentioned appeared in 1851, and the passage referred to is : "This proof [of the parallel-axiom] has been sought in manifold ways by acute mathematicians, but yet until now not found with complete sufficiency. So long as it fails, the theorem, as all founded on it, remains a hypothesis, whose validity for our life indeed is sufficiently proven by *experience*, whose *general*, *necessary exactness*, however, could be doubted without absurdity."

Alas! that this feeble utterance should have seemed sufficient for more than thirty years to the associate of Gauss and Schweikart, the latter certainly one of the independent discoverers of the non-Euclidean geometry. But then, since neither of these sufficiently realized the transcendent importance of the matter to publish any of their thoughts on the subject, a more adequate conception of the issues at stake could scarcely be expected of the scholar and colleague. How different with Bolyai János and Lobachévski, who claimed

at once, unflinchingly, that their discovery marked an epoch in human thought so momentous as to be unsurpassed by anything recorded in the history of philosophy or of science, demonstrating as had never been proven before the supremacy of pure reason at the very moment of overthrowing what had forever seemed its surest possession, the axioms of geometry.

On the 9th of March, 1832, Bolyai Farkas was made corresponding member in the mathematics section of the Magyar Academy.

As professor he exercised a powerful influence in his country.

In his private life he was a type of true originality. He wore roomy black Hungarian pants, a white flannel jacket, high boots, and a broad hat like an old-time planter's. The smoke-stained wall of his antique domicile was adorned by pictures of his friend Gauss, of Schiller, and of Shakespeare, whom he loved to call the child of nature. His violin was his constant solace.

He died November 20th, 1856. It was his wish that his grave should bear no mark. The mother of Bolyai János , née, Arkosi Benkö Zsuzsanna, was beautiful, fascinating,

of extraordinary mental capacity, but always nervous.

János , a lively, spirited boy, was taught mathematics by his father. His progress was marvelous. He required no explanation of theorems propounded, and made his own demonstrations for them, always wishing his father to go on. "Like a demon, he always pushed me on to tell him more."

At 12, having passed the six classes of the Latin school, he entered the philosophic-curriculum, which he passed in two years with great distinction.

When about 13, his father, prevented from meeting his classes, sent his son in his stead. The students said they liked the lectures of the son better than those of the father. He already played exceedingly well on the violin.

In his fifteenth year he went to Vienna to K. K. Ingenieur-Akademie.

In August, 1823, he was appointed "souslieutenant" and sent to Temesvár, where he was to present himself on the 2nd of September.

From Temesvár, on November 3rd, 1823, János wrote to his father a letter in Magyar, of which a French translation was sent me by Professor Koncz József on February 14th,

1895. This will be given in full in my life of Bolyai; but here an extract will suffice :

'"*My Dear and Good Father*. "I have so much to write about my new inventions that it is impossible for the moment to enter into great details, so I write you only on one-fourth of a sheet. I await your answer to my letter of two sheets; and perhaps I would not have written you before receiving it, if 1 had not wished to address to you the letter I am writing to the Baroness, which letter I pray you to send her.

"First of all I reply to you in regard to the binominal.

* * * * * * * * * *

"Now to something else, so far as space permits. I intend to write, as soon as I have put it into order, and when possible to publish, a work on parallels.

"At this moment it is not yet finished, but the way which I have followed promises me with certainty the attainment of the goal, if it in general is attainable. It is not yet attained, but I have discovered such magnificent things that I am myself astonished at them.

"It would be damage eternal if they were

lost. When you see them, my father, you yourself will acknowledge it. Now I can not say more, only so much : *that from nothing I have created another wholly new world*. All that I have hitherto sent you compares to this only as a house of cards to a castle.

"P. S.—I dare to judge absolutely and with conviction of these works of my spirit before you, my father; I do not fear from you any false interpretation (that certainly I would not merit), which signifies that, in certain regards, I consider you as a second self."

Prom the Bolyai MSS., now the property of the College at Maros-Vásárhely, Fr. Schmidt has extracted the following statement by János :

"First in the year 1823 have I pierced through the problem in its essence, though also afterwards completions yet were added.

"I communicated in the year 1825 to my former teacher, Herr Johann Walter von Eckwehr (later k. k. General) [in the Austrian Army], a written treatise, which is still in his hands.

"On the prompting of my father I translated my treatise into the Latin language, and

it appeared as *Appendix* to the *Tentamen*, 1832."

The profound mathematical ability of Bolyai János showed itself physically not only in his handling of the violin, where he was a master, but also of arms, where he was unapproachable.

It was this skill, combined with his haughty temper, which caused his being retired as Captain on June 16th, 1833, though it saved him from the fate of a kindred spirit, the lamented Galois, killed in a duel when only 19. Bolyai, when in garrison with cavalry officers, was provoked by thirteen of them and accepted all their challenges on condition that he be permitted after each duel to play a bit on his violin. He came out victor from his thirteen duels, leaving his thirteen adversaries on the square.

He projected a universal language for speech as we have it for music and for mathematics.

He left parts of a book entitled : Principia doctrinae novae quantitatum imaginariarum perfectae uniceque satisfacientis, aliaeque disquisitiones analyticae et analytico-geometricae cardinales gravissimaeque; auctore

Johan. Bolyai de eadem, C. R. austriaco castrensium captaneo pensionato.

Vindobonae vel Maros Vásárhelyini, 1853.

Bolyai Farkas was a student at Goettingen from 1796 to 1799.

In 1799 he returned to Kolozsvár, where Bolyai János was born December 18th, 1802.

He died January 27th, 1860, four years after his father.

In 1894 a monumental stone was erected on his long-neglected grave in Maros-Vásárhely by the Hungarian Mathematico-Physical Society.

# APPENDIX.

SCIENTIAM SPATII *absolute veram* exhibens :

*a veritate aut falsitate Axiomatis* XI *Euclidei*

(*a priori haud unquam decidenda*)

*independentemn*. adjecta ad casum falsitatis,

quadratura circuli

geometrica.

Auctore JOHANNE BOLYAI de eadem, Geometrarum

in Exercitu Caesareo Regio Austriaco

Castrensium Capitaneo.

## EXPLANATION OF SIGNS.

*The straight AB* means the aggregate of all points situated in the same straight line with A and B.

*The sect AB* means that piece of the straight AB between the points A and B.

*The ray AB* means that half of the straight AB which commences at the point A and contains the point B.

*The plane ABC* means the aggregate of all points situated in the same plane as the three points (not in a straight) A, B, C.

*The hemi-plane ABC* means that half of the plane *ABC* which starts from the straight AB and contains the point C.

*ABC* means the smaller of the pieces into which the plane ABC is parted by the rays BA, BC, or the non-reflex angle of which the sides are the rays BA, BC.

ABCD (the point D being situated within ∠ABC, and the straights BA, CD not intersecting) means the portion of ∠ABC comprised between ray BA, sect BC, ray CD; while BACD designates the portion of the plane ABC comprised between the straights AB and CD.

⊥ is the sign of perpendicularity.

∥ is the sign of parallelism.

∠ means angle.

rt.∠ is right angle.

st.∠ is straight angle.

≅ is the sign of congruence, indicating that two magnitudes are superposable.

AB Π CD means ∠CAB = ∠ACD.

$x$ Y $a$ means $x$ converges toward the limit $a$.

Δ is triangle.

⊙$r$ means the [circumference of the] circle of radius r.

area ⊙$r$ means the area of the surface of the circle of radius $r$.

[Not Mentioned: □ , ∞]

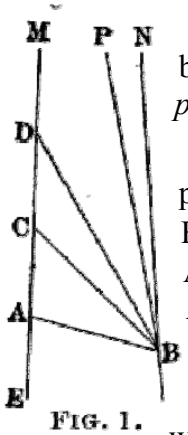

FIG. 1.

§1. If the ray AM is not cut by the ray [3] BN, situated in the same plane, but is cut by every ray BP comprised in the angle ABN, we will call ray BN *parallel* to ray AM; this is designated by BN ‖ AM;

It is evident that there *is one such ray BN*, *and only one*, passing through any point B (taken outside of the straight AM), and that the sum of the angles BAM, ABN can not exceed a st. ∠; for in moving BC around B until BAM + ABC = st. ∠, somewhere ray BC *first* does not cut ray AM, and it is then BC ‖ AM. It is clear that BN ‖ EM, wherever the point E be taken on the straight AM (supposing in all such cases AM > AE).

If while the point C goes away to infinity on ray AM, always CD = CB, we will have constantly CDB = (CBD < NBC); but NBC Y 0; and so also ADB Y 0.

§ 2. If BN ‖ AM, we will have also CN ‖ AM. For take D anywhere in MACN. If C is on ray BN, ray BD cuts ray AM, since BN ‖ AM, and so also ray CD cuts ray AM. But if C is on ray BR take BQ ‖ CD; BQ falls within the ∠ABN (§1), and cuts ray AM; and so also ray CD cuts ray AM. Therefore every ray CD (in ACN) cuts, in each case, the ray AM, without CN itself cutting ray AM. Therefore always CN ‖ AM.

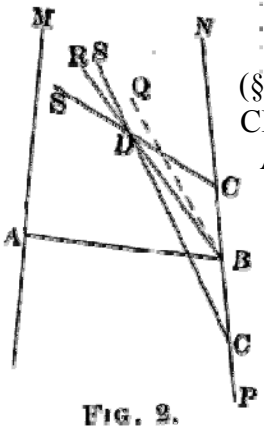

FIG. 2.

§ 3. (Fig. 2.) If BR and CS and each ‖ AM, and C is not on the ray BR, then ray BR and ray CS do not intersect. For if ray BR and ray CS had a common point D, then (§ 2) DR and DS would be each ‖ AM, and ray DS (§ 1) would fall on ray DR, and C on the ray BR (contrary to the hypothesis).

§ 4. If MAN > MAB, we will have for every point B of ray AB, a point C of ray AM, such that BCM = NAM.

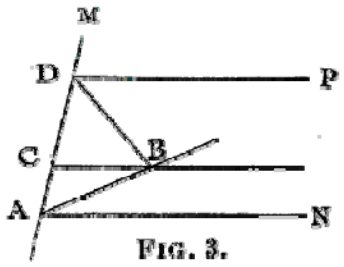

FIG. 3.

For (by § 1) is granted BDM > NAM, and so that MDP = MAN, and B falls in

NADP. If therefore NAM is carried along AM until ray AN arrives on ray DP, ray AN will somewhere have necessarily passed through B, and some BCM = NAM.

§ 5. If BN ‖ AM, there is on the straight [4] AM a point F such that FM Π BN. For by § 1 is granted BCM > CBN; and if CE = CB, and so EC Π BC; evidently BEM < EBN. The point P is moved on EC, the angle BPM always being called *u*, and the angle PBN always *v*, evidently *u* is at first less than the corresponding *v*, but afterwards greater. Indeed *u* increases *continuously* from BEM to BCM; since (by ~ 4) there exists no angle > BEM and < BCM, to which u does not at some time become equal. Likewise v decreases continuously from EBN to CBN. There is therefore on EC a point F such that BFM = FBN.

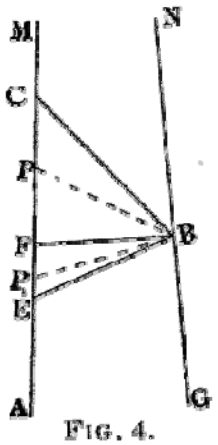

FIG. 4.

§ 6. If BN ‖ AM and E anywhere in the straight AM, and G in the straight BN; then GN ‖ EM and EM ‖ GN. For (by § 1) BN ‖ EM, whence (by § 2) GN ‖ EM. If moreover FM Π BN (§ 5); then MFBN ≅ NBFM, and consequently (since BN ‖ FM) also FM ‖ BN, and (by what precedes) EM ‖ GN.

§ 7. If BN and CP are each ‖ AM, and C not on the straight BN; also BN ‖ CP. For the rays BN and CP do not intersect (§ 3); but AM, BN and CP either are or are not in the same plane; and in the first case, AM either is or is not within BNCP.

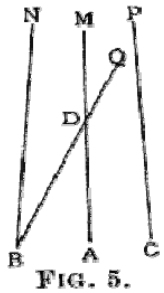

FIG. 5.

If AM, BN, CP are coplanar, and AM falls within BNCP; then every ray BQ (in NBC) cuts the ray AM in some point D (since BN ‖ AM); moreover, since DM ‖ CP (§ 6), the ray DQ will cut the ray CP, and so BN ‖ CP.

But if BN and CP are on the same side of AM; then one of them, for example CP, falls between the two other straights BN, AM : but every ray BQ (in NBA) cuts the ray AM, and so also the straight CP. Therefore BN ‖ CP.

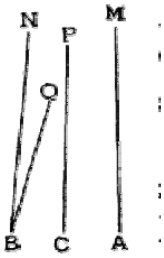

FIG. 6.

If the planes MAB, MAC make *an angle*; then CBN and ABN have in common nothing but the ray BN, while the ray AM (in ABN) and the ray BN, and so also NBC and the ray AM have nothing in common.

But hemi-plane BCD, drawn through any ray BD (in NBA), cuts the ray AM, since ray

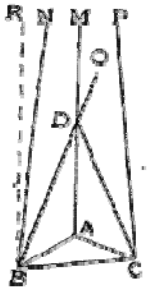

Fig. 7.

BQ cuts ray AM (as BN ‖ AM). Therefore in revolving the hemi-plane BCD around BC until it *begins* to leave the ray AM, the hemi-plane BCD at last will fall upon the hemi-plane BCN. For the same reason this same will fall upon hemi-plane BCP. Therefore BN falls in BCP. Moreover, if BR ‖ CP; then (because also AM ‖ CP) by like reasoning, BR falls in BAM, and also (since BR ‖ CP) in BCP. Therefore the straight BR, being common to the two planes MAB, PCB, of course is the straight BN, and hence BN ‖ CP.*

If therefore CP ‖ AM, and B exterior to the plane CAM; then the intersection BN of the planes BAM, BCP is ‖ as well to AM as to CP.

§ 8. If BN ‖ and Π CP (or more briefly BN ‖ Π CP), and AM (in NBCP) bisects ⊥ the sect BC; then BN ‖ AM.

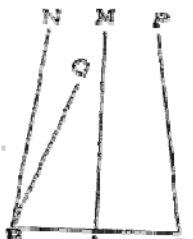

Fig. 8.

For if ray BN cut ray AM, also -ray CP would cut ray AM at the same point (because MABNB ≅ MACP), and this would be common to the rays BN, CP themselves,

* The third case being put before the other two, these can be demonstrated together with more brevity and elegance, like case 2 of ~10. [Author's note.]

although BN ‖ CP. But every ray BQ (in CBN) cuts ray CP; and so ray BQ cuts also ray AM. Consequently BN ‖ AN.

§ 9. If BN II AM, and MAP ⊥ MAB, and the ∠, which NBD makes with NBA (on that side of MABN, where MAP is) is < rt.∠; then MAP and NBD intersect.

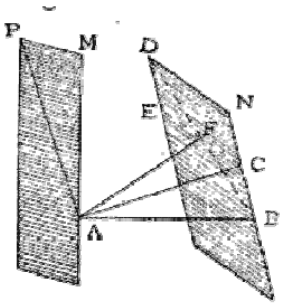

FIG. 9.

For let ∠BAM = rt.∠, and AC ⊥ BN (whether or not C falls on B), and CE ⊥ BN (in NBD); by hypothesis ∠ACE < rt.∠, and AF (⊥ CE) will fall in ACE.

Let ray AP be the intersection of the hemi-planes ABF, AMP (which have the point A common); since BAM ⊥ MAP, ∠BAP = ∠BAM = rt.∠.

If finally the hemi-plane ABF is placed upon the hemi-plane ABM (A and B remaining), ray AP will fall on ray AM; and since AC ⊥ BN, and sect AF < sect AC, evidently sect AF will terminate within ray BN, and so BF falls in ABN. But in *this* position, ray BF cuts ray AP (because BN ‖ AM); and so ray AP and ray BF intersect also *in the* original position; and the point of section is common to the hemi-planes MAP and NBD. Therefore the hemi-planes MAP and NBD intersect. Hence follows

easily that the hemi-planes MAP and NBD intersect if the sum of the interior angles which they make with MABN is < st.∠.

§ 10. If both BN and CP ‖ Π AM; also is BN ‖ Π CP.

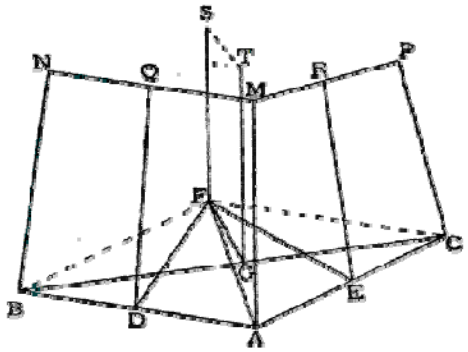

FIG. 10.

For either MAB and MAC make an angle, or they are in a plane.

If the first; let the hemi-plane QDF bisect ⊥ sect AB; then DQ ⊥ AB, and so DQ ‖ AM (§ 8); likewise if hemi-plane ERS bisects ⊥ sect AC, is ER ‖ AM; whence (§ 7) DQ ‖ ER.

Hence follows easily (by § 9), the hemi-planes QDF and ERS intersect, and have (§ 7) their intersection FS ‖ DQ, and (on account of BN ‖ DQ) also FS ‖ BN. Moreover (for any point of FS) FB = FA = FC, and the straight FS falls in the plane TGF, bisecting ⊥ sect BC. But (by § 7) (since FS ‖ BN) also GT ‖ BN. In the same way is proved GT ‖ CP. Meanwhile GT bisects ⊥ sect BC; and so TGBN ≅ TGCP (§ 1), and BN ‖ Π CP.

If BN, AM and CP are in a plane, let (falling without this plane) FS ‖ Π AM; then (from

what precedes) FS ‖ ⊥ both to BN and to CP, and so also BN ‖ ⊥ CP.

§ 11. Consider the aggregate of the point A, and *all* points of which any one B is such, that if BN ‖ AM, also BN Π AM; call it F; but the intersection of F with any plane containing the sect AM call L.

F has a point, and one only, on any straight ‖ AM; and evidently L is divided by ray AM into two congruent parts.

Call the ray AM *the axis* of L. Evidently also, in any plane containing the sect AM, there is for the *axis* ray AM a single L. Call any L of this sort the L of this ray AM (in the plane considered, being understood). Evidently by revolving L around AM we describe the F of which ray AM is called the axis, and in turn F *may be ascribed to the axis ray AM*.

§ 12. If B is anywhere on the L of ray AM, and BN ‖ Π AM (§ 11); then the L of ray AM and the L of ray BN *coincide*. For suppose, in distinction, L′ the L of ray BN. Let C be anywhere in L′, and CP ‖ Π BN (§ 11). Since BN ‖ Π AM, so CP ‖ Π AM (§ 10), and so C also will fall on L. And if C is anywhere on L, and CP ‖ Π AM; then CP ‖ Π BN (§ 10); and C also falls on L′ (§ 11). Thus L and L′ are the

same; and every ray BN is also axis of L, and between all axes of this L, is Π.

The same is evident in the same way of F.

§ 13. If BN ‖ AM, and CP ‖ DQ, and ∠BAM + ∠ABN = st.∠; then also ∠DCP + ∠CDQ = st.∠.

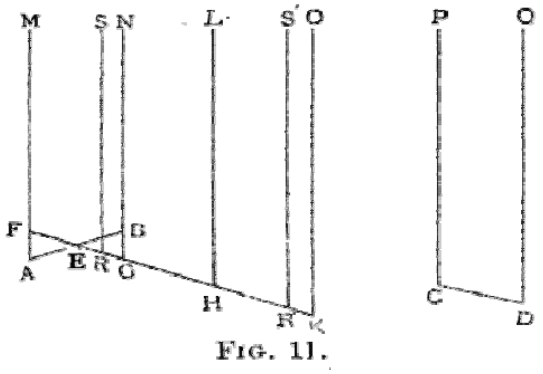

FIG. 11.

For let EA = EB, and EFM = DCP (§ 4). Since ∠BAM + ∠ABN = st. ∠ = ∠ABN + ∠ ABG, we have ∠EBG = ∠EAF; and so if also BG = AF, then ΔEBG ≅ ΔEAF, ∠BEG = ∠AEF and G will fall on the ray FE. Moreover ∠GFM + ∠FGN = st. ∠ (since ∠EGB = ∠EFA).

Also GN ‖ FM (§ 6).

Therefore if MFRS ≅ PCDQ, then RS ‖ GN (§ 7), and R falls within or without the sect FG (unless sect CD = sect FG, where the thing now is evident).

I. In the first case ∠FRS is not > (st. ∠ – ∠RFIM = ∠FGN), since RS ‖ FM. But as RS ‖ GN, also ∠FRS is not < ∠FGN; and so ∠FRS = ∠FGN, and ∠RFM + ∠FRS = ∠GFM +

$\angle FGN = \text{st.}\angle$. Therefore also $\angle DCP + \angle CDQ = \text{st.}\angle$.

II. If R falls without the sect FG; then $\angle NGR = \angle MFR$, and let $MFGN \cong NGHL \cong LHKO$, and so on, until FK = FR or begins to be > FR. Then KO ‖ HL ‖ FM (§7).

If K falls on R, then KO falls on RS (§ 1); and so $\angle RFM + \angle FRS = \angle KFM + \angle FKO = \angle KFM + \angle FGN = \text{st.}\angle$; but if R falls within the sect HK, then (by I) $\angle RHL + \angle KRS = \text{st.}\angle = \angle RFM + \angle FRS = \angle DCP + \angle CDQ$.

§ 14. If BN ‖ AM, and CP ‖ DQ, and $\angle BAM + \angle ABN < \text{st.}\angle$; then also $\angle DCP + \angle CDQ < \text{st.}\angle$.

For if $\angle DCP + \angle CDQ$ were not < st.∠, and so (by § 1) were = st.∠, then (by § 13) also $\angle BAM + \angle ABN = \text{st.}\angle$ (contra hyp.).

15. Weighing §§ 13 and 14, *the System of Geometry resting on the hypothesis of the truth of Euclid's Axiom XI is called Σ*; *and the system founded on the contrary hypothesis is S*.

*All things which are not expressly said to be in Σ or in S*, *it is understood are enunciated absolutely*, *that is are asserted true whether Σ or S is reality*.

§16. If AM is the axis of any L; then L, in Σ is a straight ⊥ AM.

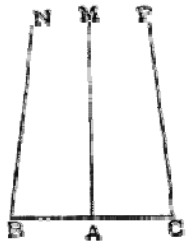

Fig. 12.

For suppose BN an axis from any point B of L; in Σ, ∠BAM + ∠ABN = st.∠, and so ∠BAM = rt.∠.

And if C is any point of the straight AB, and CP ‖ AM; then (by § 13) CP Π AM, and so C on L (§ 11 ).

But in S, no three points A, B, C on L or on F are in a straight. For some one of the axes AM, BN, CP (e.g. AM) falls between the two others; and then (by § 14) ∠BAM and ∠CAM are each < rt.∠.

§ 17. *L in S also is a line*, *and F a surface*. For (by § 11) any plane ⊥ to the axis ray AM (through any point of F) cuts F in [the circumference of] a circle, of which the plane (by § 14) is ⊥ to no other axis ray BN. If we revolve F about BN, any point of F (by § 12) will remain on F, and the section of F with a plane not ⊥ ray BN will describe a surface; and whatever be the points A, B taken on it, F can so be congruent to itself that A falls upon B (by § 12); therefore F is a *uniform surface*.

Hence evidently (by §§ 11 and 12) L is a uniform line.*

§ 18. *The intersection* with F of *any plane*, drawn through a point A of F obliquely to the axis AM, is, in S, *a circle*.

For take A, B, C, three points of this section, and BN, CP, axes; AMBN and AMCP make an angle, for otherwise the plane determined by A, B, C (from § 16) would contain AM, (contra hyp.). Therefore the planes bisecting ⊥ the sects AB, AC intersect (§ 10) in some *axis* ray FS (of F), and FB = FA = FC.

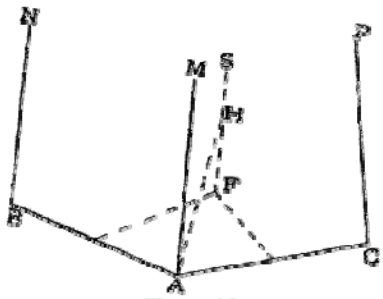

FIG. 13.

Make AH ⊥ FS, and revolve FAH about FS; A will describe a circle of radius HA, passing, through B and C, and situated *both* in F and in the plane ABC; nor have F and the plane ABC anything in common but ⊙ HA (§ 16).

It is also evident that in revolving the portion FA of the line L (as radius) in F around F, its extremity will describe ⊙ HA.

* It is not necessary to restrict the demonstration to the system S; since it may easily be so set forth, that it holds absolutely for S and for Σ.

§ 19. *The perpendicular* BT to the axis BN of L (falling in the plane of L) is, in S, N *tangent* to L. For L has in ray BT no point except B (§ 14), but if BQ falls in TBN, then the center of the section of the plane through BQ perpendicular to TBN with the F of ray BN (§ 18) is evidently located on ray BQ; and if sect BQ is a diameter, evidently ray BQ cuts in Q the line L of ray BN.

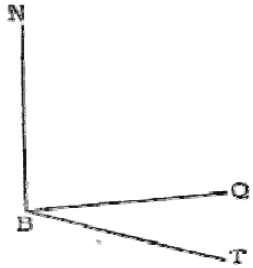

FIG. 14.

§ 20. Any two points of F determine a line L (§§ 11 and 18); and since (from §§ 16 and 19) L is ⊥ to all its axes, every ∠ of lines L in F is equal to the ∠ of the planes drawn through its sides perpendicular to F.

21. Two L form lines, ray AP and ray BD, in the same F, making with a third L form AB, a sum of interior angles < st.∠, intersect.

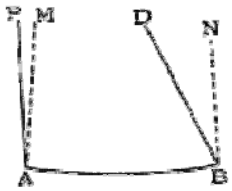

FIG. 15.

(By line AP in F, is to be understood the line L drawn through A and P, but by ray AP that half of this line beginning at A, in which P falls.)

For if AM, BN are axes of F, then the hemi-planes AMP, BND intersect (§ 9); and F cuts

their intersection (by §§ 7 and 11); and so also ray AP and ray BD intersect.

From this it is evident that Euclid's Axiom XI and all things which are claimed in geometry and plane trigonometry hold good *absolutely* in F, L lines being substituted in place of straights : therefore the trigonometric functions are taken here in the same sense as in Σ; and the circle of which the L form radius = $r$ in F, is $2\pi r$; and likewise area of $\odot r$ (in F) = $\pi r^2$ (by $\pi$ understanding $\frac{1}{2} \odot 1$ in F, or the known 3.1415926 . . . )

§ 22. If ray AB were the L of ray AM, and C on ray AM; and the ∠CAB (formed by the straight ray AM and the L form line ray AB), carried first along the ray AB, then along the ray BA, always forward to infinity : the path CD of C will be the line L of CM.

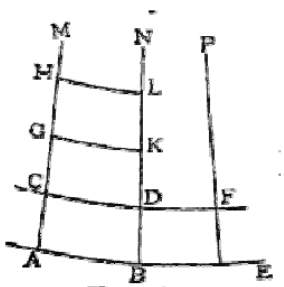

FIG. 16.

For let D be any point in line CD (called later L′, let DN be ‖ CM, and B the point of L falling on the straight DN. We shall have BN Π AM, and sect AC = sect BD, and so DN Π CM, consequently D in L′. But if D in L′ and DN ‖ CM, and B the point of L on the straight DN; we shall have AM Π BN and CM Π DN, whence manifestly sect BD = sect AC,

and D will fall on the path of the point C, and L′ and the line CD are the same. Such an L′ is designated by L′ 8 L.

§ 23. If the L form line CDF 8 ABE (§ 22), and AB = BE, and the rays AM, BN, EP are axes; manifestly CD = DF; and if any three points A, B, E are of line AB, and AB = n.CD, we shall also have AE = n.CF; and so (manifestly even for AB, AE, DC incommensurable), AB : CD = AE : CF, and AB : CD is *independent of AB*, *and completely determined by AC*.

This ratio AB : CD is designated by the capital letter (as X) corresponding to the small letter (as X) by which we represent the sect AC.

§ 24. Whatever be *x* and *y*, (§23), $Y = X^{\frac{x}{y}}$.

For, one of the quantities *x*, *y* is a multiple of the other (e. g. *y* of *x*), or it is not.

If *y* = n.*x*, take *x* = AC = CG = GH = &c., until we get AH = *y*.

Moreover, take CD 8 GK 8 HL.

We have ((§ 23) X = AB : CD – CD : GK = GK : HL; and so

$$\frac{AB}{HL} = \left(\frac{AB}{CD}\right)^n$$

or $Y = X^n = X^{\frac{y}{x}}$.

If *x*, *y* are multiples of *i*, suppose *x* = *mi*, and *y* = *ni*; (by the preceding) $X = I^m$, $Y = I^n$, consequently

$$Y = X^{\frac{n}{m}} = X^{\frac{y}{x}}$$

The same is easily extended to the case of the incommensurability of $x$ and $y$.

But if $q = y - x$, manifestly $Q = Y : X$. It is also manifest that in Σ, for any $x$, we have $X = 1$, but in S is X > 1, and for any AB and ABE there is such a CDF 8 AB, that CDF = AB, whence AMBN ≅ AMEP, though the first be any multiple of the second; which indeed is singular, but evidently does not prove the absurdity of S.

§ 25. *In any rectilineal triangle, the circles with radii equal to its sides are as the sines of the opposite angles*.

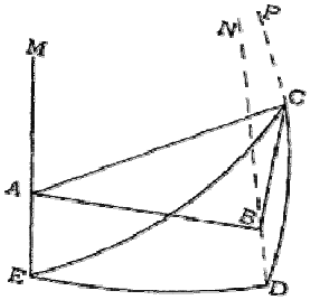

FIG. 17.

For take ∠ABC = rt.∠, and AM ⊥ BAC, and BN and CP ‖ AM; we shall have CAB ⊥ AMBN, and so (since CB ⊥ AMBN, consequently CPBN ⊥ AMBN.

Suppose the F of ray CP cuts the straights BN, AM respectively in D and E, and the bands CPBN, CPAM, BNAM along the L form lines CD, CE, DE. Then (§ 20) ∠CDE = the angle of NDC, NDE, and so = rt.∠; and by like reasoning ∠CED = CAB. But (by § 21) in the L line ΔCDE (supposing always here the radius = 1),

EC : DC = 1 : sin DEC = : sin CAB.

Also (by § 21)

EC : DC = ⊙EC : ⊙DC (in F) = ⊙AC : ⊙BC (§ 18); and so is also

⊙AC : ⊙BC - 1 : sin CAB;

whence the theorem is evident for any triangle.

§ 26. *In any spherical triangle*, *the sines of the sides are as the sines of the angles opposite*.

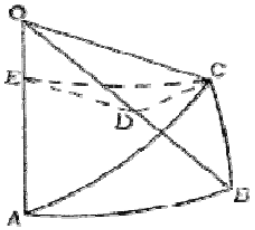

FIG. 18.

For take ∠ABC = rt.∠, and CED ⊥ to the radius OA of the sphere. We shall have CED ⊥ AOB, and (since also BOC ⊥ BOA), CD ⊥ OB. But in the triangles CEO, CDO (by § 25) ⊙EC : ⊙OC : ⊙DC = sin COE : 1 : sin COD = sin AC : 1 : sin BC; meanwhile also (§ 25) ⊙EC : ⊙DC = sin CDE : sin CED. Therefore, sin AC : sin BC = sin CDE : sin CED; but CDE = rt.∠ = CBA, and CED = CAB. Consequently

sin AC : sin BC = 1 : sin A.

*Spherical trigonometry*, *lowing from this*, *is thus established independently of Axiom XI*.

§ 27. If AC and BD are ⊥ AB, and CAB is carried along the straight AB; we shall have, designating by CD the path of the point C,

CD : AB = sin *u* : sin *v*.

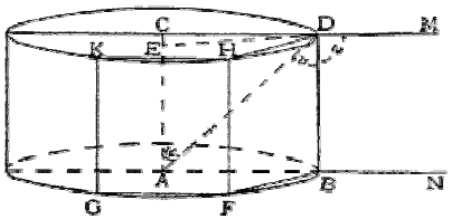

FIG. 19.

For take DE ⊥ CA; in the triangles ADE, ADB (by § 25) ⊙ED : ⊙AD : ⊙AB = sin *u* : 1 : sin *v*.

In revolving BACD about AC, B describes ⊙AB, and D describes ⊙ED; and designate here by *s*⊙CD the path of the said CD. Moreover, let there be any polygon BFG . . . inscribed in ⊙AB.

Passing through all the sides BF, FG, &c., planes ⊥ to ⊙AB we form also a polygonal figure of the same number of sides in *s*⊙CD, and we may demonstrate, as in § 23, that CD : AB = DH : BF = HK : FG, &c., and so

DH + HK &c. : BF + FG &c. : = CD : AB.

If each of the sides BF, FG . . . approaches the limit zero, manifestly

BF + FG + . . . Y⊙AB and

DH + HK + . . . ⊙ED.

Therefore also ⊙ED : ⊙AB = CD : AB. But we had ⊙ED : ⊙AB = sin *u* : sin *v*. Consequently

CD : AB = sin *u* : sin *v*.

If AC goes away from BD to infinity, CD : AB, and so also sin *u* : sin *v* remains *constant*; but *u* Y rt.∠ (§1), and if DM ‖ BN, *v* Y *z*; whence CD : AB = 1 : sin *z*.

The path called CD will be denoted by CD 8 AB.

§ 28. If BN ‖ Π AM, and C in ray AM, and AC = $x$. we shall have (§ 23)

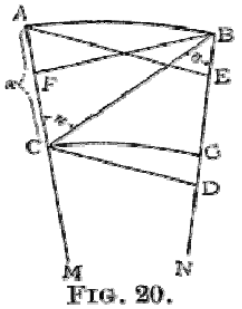

FIG. 20.

$$X = \sin u : \sin v.$$

For if CD and AE are ⊥ BN, and BF ⊥ AM; we shall have (as in § 27)

$$\odot BF : \odot DC = \sin u : \sin v.$$

But evidently BF = AE : therefore

$$\odot EA : \odot CD = \sin u : \sin v.$$

But in the F form surfaces of AM and CM (cutting AMBN in AB and CG) (by § 21)

$$\odot EA : \odot DC = AB : CG = X.$$

Therefore also

$$X = \sin u : \sin v.$$

§ 29. If ∠BAM – rt.∠, and sect AB = $y$, and BN ‖ AM, we shall have in S

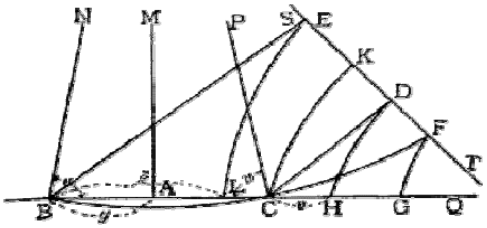

FIG. 21.

$$Y = \text{cotan } ½ u.$$

For, if sect AB = sect AC, and CP ‖ AM (and so BN ‖ CP), and ∠PCD = ∠QCD; there is given (§19) DS ⊥ ray CD, so that DS ‖ CP, and so (§ 1) DT ‖ CQ. Moreover, if BE ⊥ ray DS, then (§ 7) DS ‖ BN, and so (§ 6)

BN ‖ ES, and (since DT ‖ CG) BQ ‖ ET; consequently (§1) ∠EBN = ∠EBQ. Let BCF be an L-line of BN, and FG, DH, CK, EL, L form lines of FT, DT, CQ and ET; evidently (§ 22) HG = DF = DK = HC; therefore,

$$CG = 2CH = 2v.$$

Likewise it is evident $BG - 2BL = 2z$.

But BC = BG – CG; wherefore $y = z - v$, and so (§24) Y = Z : V.

Finally (§ 28)

Z = : sin ½ u,

and V = : sin (rt.∠ – ½ *u*),

consequently Y - cotan ½ *u*.

§ 30. However, it is easy to see (by § 25) that the solution of the problem of Plane Trigonometry, in S, requires the expression of the circle in terms of the radius; but this can by obtained by the rectification of L.

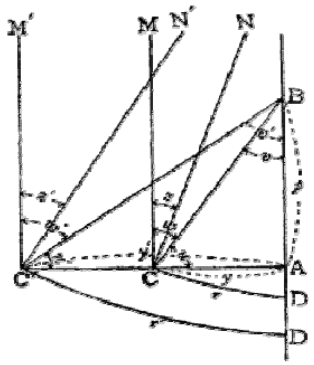


FIG. 22.

Let AB, CM, C'M′ be ⊥ ray AC, and B anywhere in ray AB; we shall have (§ 25)

$$\sin u : \sin v = \odot p : \odot y,$$

$$\text{and } \sin u' : \sin v' = \odot p' : \odot y';$$

$$\text{and so } \frac{\sin u}{\sin v} \cdot \odot y = \frac{\sin u'}{\sin v'} \cdot \odot y'.$$

But (by § 27) $\sin v : \sin v' = \cos u : \cos u'$;

consequently $\frac{\sin u}{\sin u} \cdot \odot y = \frac{\sin u'}{\sin u'} \cdot \odot y'$; or $\odot y : \odot y' = : \tan u' : \tan u = \tan w : \tan w'$.

Moreover, take CN and C'N′ ‖ AB, and CD, C'D' L-form lines ⊥ straight AB; we shall have also (§21)

$$\odot y : \odot y' = r : r', \text{ and so}$$

$$r : r' = \tan w : \tan w'.$$

Now let p beginning from A increase to infinity; then $w$ Y $z$, and $w'$ Y $z'$, whence also $r : r' = \tan z : \tan z'$.

Designate by $i$ the *constant*

$r : \tan z$ (independent of $r$);

whilst $y$ Y 0,

$\frac{r}{y} = \frac{i \tan z}{y}$ Y 1, and so

$\frac{y}{\tan z}$ Y $i$. From §29, tan z = ½ (Y – Y $^{-1}$);

therefore $\frac{2y}{\text{Y} - \text{Y}^{-1}}$ Y $i$,

or (§ 24). $\frac{2y \cdot \text{I}^{\frac{\text{y}}{\text{I}}}}{\text{I}^{\frac{2\text{y}}{\text{I}}} - 1}$ Y $i$.

But we know the limit of this expression (where $y$ Y 0) is.

$\frac{i}{\text{nat. log I}}$ Therefore

$$\frac{i}{\text{nat. log I}} = i, \text{ and}$$

$$\text{I} = e = 2.7182818 \ldots,$$

which noted quantity shines forth here also.

If obviously henceforth $i$ denote that sect of which the I = $e$, we shall have

$$r = i \tan z.$$

But (§ 21) $\odot y = 2\pi r$; therefore

$$\odot y = 2\pi i \tan z = \pi i \,(\text{Y} - \text{Y}^{-1}) = \pi i \left(e^{\frac{\text{y}}{\text{i}}} - e^{\frac{\text{y}}{-\text{i}}}\right)$$

$$= \frac{\pi y}{\text{nat. log Y}} (\text{Y} - \text{Y}^{-1}) \text{ (by § 24).}$$

§ 31. For the trigonometric solution of all right-angled rectilineal *triangles* (whence the resolution of all *triangles* is easy, in S, three equations suffice : indeed ($a$, $b$ denoting the sides, $c$ the hypothenuse, and $\alpha$, $\beta$ the angles opposite the sides) an equation expressing the relation

1st, between $a$, $c$, $\alpha$;

2d, between $a$, $\alpha$, $\beta$;

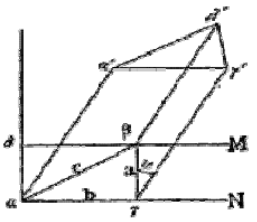

FIG. 23.

3d, between $a$, $b$, $c$;

of course from these equations emerge three others by elimination.

From §§ 25 and 30

$$1 : \sin a = (\text{C} - \text{C'}) : (\text{A} - \text{A}^{-1}) = \left(e^{\frac{\text{e}}{\text{i}}} - e^{\frac{-\text{e}}{\text{i}}}\right) : \left(e^{\frac{\text{a}}{\text{i}}} - e^{\frac{-\text{a}}{\text{i}}}\right)$$

(equation for $c$, $a$ and $\alpha$).

II. From § 27 follows (if $\beta$ M ‖ $\gamma$ N)

cos $\alpha$ : sin $\beta$ = 1 : sin $u$, but from § 29

$$1 : \sin u = \tfrac{1}{2}(A + A^{-1});$$

therefore cos $\alpha$ : sin $\beta = \tfrac{1}{2}(A + A^{-1}) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)$

(equation for $\alpha$, $\beta$ and $a$).

III. If $\alpha\alpha' \perp \beta\alpha\gamma$, and $\beta\beta'$ and $\gamma\gamma'$ ‖ $\alpha\alpha'$ (§ 27), and i $\beta'\alpha'\gamma' \perp \alpha\alpha'$; manifestly (as in § 27),

$$\frac{\beta\beta'}{\gamma\gamma'} = \frac{1}{\sin u} = \tfrac{1}{2}(A + A^{-1});$$

$$\frac{\gamma\gamma'}{\alpha\alpha'} = \tfrac{1}{2}(B + B^{-1});$$

$$\text{and } \frac{\beta\beta'}{\alpha\alpha'} = \tfrac{1}{2}(C + C^{-1}); \text{ consequently}$$

$$\tfrac{1}{2}(C + C^{-1}) = \tfrac{1}{2}(A + A^{-1}) \cdot \tfrac{1}{2}(B + B^{-1}), \text{ or}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right) \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

(equation for $a$, $b$ and $c$).

If $\gamma\alpha\delta$ = rt.∠, and $\beta\delta \perp \alpha\delta$;

$\odot c : \odot a = 1 : \sin \alpha$, and

$\odot c : \odot (d = \beta\delta) = 1 : \cos \alpha$,

and so (denoting by $\odot x^2$, for any $x$, the product $\odot x \cdot \odot x$) manifestly

$\odot a^2 + \odot d^2 - \odot c^2$.

But (by § 27 and II)

$$\odot d = \odot b \,.\, \tfrac{1}{2}(A + A^{-1}), \text{ consequently}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)^2 = \tfrac{1}{4}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2 \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)^2 + \left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2$$

*another* equation for $a$, $b$ and $c$ (the second

member of which may be easily reduced to a form *symmetric* or *invariable*).

Finally, from

$$\frac{\cos\alpha}{\sin\beta} = \tfrac{1}{2}(A + A^{-1}), \text{ and } \frac{\cos\beta}{\sin\alpha} = \tfrac{1}{2}(B + B^{-1}), \text{ we get}$$

(by III)

$$\cot\alpha \cot\beta = \tfrac{1}{2}\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)$$

(equation for $\alpha$, $\beta$, and $c$.)

§ 32. It still remains to show briefly the mode of resolving *problems* in S, which being accomplished (through the more obvious examples), finally will be candidly said what this theory shows.

I. Take AB a line in a plane, and $y = f(x)$ its equation in rectangular coordinates, call $dz$ any increment of $z$, and respectively $dx$, $dy$, $du$ the increments of $x$, of $y$, and of the area $u$, corresponding to this $dz$; take BH 8 CF, and express (from § 31) $\frac{BH}{dx}$ by means of $y$, and seek the *limit* of $\frac{dy}{dx}$ when $dx$ tends towards the limit zero (which is understood where a limit of this sort is sought) : then will become known also the limit of $\frac{dy}{BH}$ and so tan HBG; and

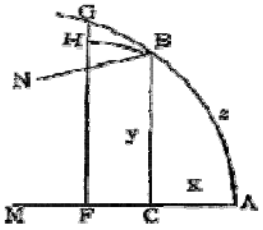

FIG. 24.

(since HBC manifestly is neither > nor < , and so = rt∠. ), the *tangent* at B of BG will be determined by *y*.

II. It can be demonstrated

$$\frac{dz^2}{dy^2 + \overline{\mathrm{BH}}^2}\,\mathrm{Y1}.$$

Hence is found the *limit* of $\frac{dz}{dx}$ and thence, by integration, *z* (expressed in terms of *x*.)

And of any line *given in the concrete*, the equation in *S* can be found; e. g., of L. For if ray AM be the axis of L; then any ray CB from ray AM cuts L [since (by § 19) any straight from A except the straight AM will cut L]; but (if BN is axis)

$$\mathrm{X} = 1 : \sin \mathrm{CBN}\ (\S 28),$$

and Y = cotan ½ CBN (§ 29), whence

$$\mathrm{Y} = \mathrm{X} + \sqrt{\mathrm{X}^2 - 1}.$$

or $e^{\frac{y}{i}} = e^{\frac{x}{i}} + \sqrt{e^{\frac{2x}{i}} - 1}$,

the equation sought.

Hence we get

$$\frac{dy}{dy}\,\mathrm{Y}\ \mathrm{X}(\mathrm{X}^2 - 1)^{\frac{-1}{2}};$$

and $\frac{\mathrm{BH}}{dx}$ Y1 : sin CBN = X; and so

$$\frac{dy}{\mathrm{BH}}\ \mathrm{Y}\ (\mathrm{X}^2 - 1)^{\frac{-1}{2}};$$

$$1 + \frac{dy^2}{BH^2} \text{ Y } X^2(X^2 - 1)^{-1},$$

$$\frac{dz^2}{BH^2} \text{ Y } X^2(X^2 - 1)^{-1},$$

and $$\frac{dz}{BH} \text{ Y } X (X^2 - 1)^{\frac{1}{2}} \text{ and}$$

$$\frac{dz}{dx} \text{ Y } X^2 (X^2 - 1)^{\frac{-1}{2}},$$ whence, by integration, we get (as in § 30)

$$z = i (X^2 - 1)^{\frac{1}{2}} = i \cot \text{CBN}.$$

III. Manfestly

$$\frac{du}{dx} \text{ Y } \frac{\text{HFCBH}}{dx}$$

which (unless given in $y$) now first is to be expressed in terms of $y$; whence we get $u$ by integrating.

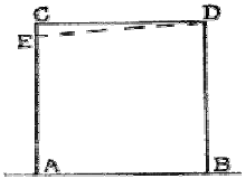

FIG. 25.

If AB = $p$, AC = $q$, CD = $r$, and CABDC = s; we might show (as in II) that

$$\frac{ds}{dq} \text{ Y } r, \text{ which } = -\tfrac{1}{2} p \left( e^{\frac{q}{i}} - e^{\frac{-q}{i}} \right), \text{ and, integrating, } s = \tfrac{1}{2} pi \left( e^{\frac{q}{i}} - e^{\frac{-q}{i}} \right).$$

This can also be deduced apart from integration.

For example, the equation of the circle (from § 31, III), of the straight (from § 31, II), of a conic (by what precedes), being expressed, the

areas bounded by these lines could also be expressed.

We know, that a surface *t*, 8 to a plane figure *p* (at the distance *q*), is to *p* in the ratio of the second powers of homologous lines, or as

$$¼\left(e^{\frac{q}{i}} - e^{\frac{-q}{i}}\right)^2 : 1.$$

It is easy to see, moreover, that the calculation of volume, treated in the same manner, requires two integrations (since the differential itself here is determined only by integration); and before all must be investigated the volume contained between *p* and *t*, and the aggregate of all the straights $\perp p$ and joining the boundaries of *p* and *t*.

We find for the volume of this solid (whether by integration or without it)

$$⅛\, pi\left(e^{\frac{2q}{i}} - e^{\frac{-2q}{i}}\right) + ½\, pq.$$

The surfaces of bodies may also be determined in S, as well as the *curvatures*, the *involutes*, and *evolutes* of any lines, etc.

As to curvature; this in S either is the curvature of L, or is determined either by the radius of a circle, or by the *distance* to a straight from the curve 8 to this straight; since from what precedes, it may easily be shown, that in a plane there are no uniform lines other than L-lines, circles and curves 8 to a straight.

IV. For the circle (as in III) $\frac{d \text{ area } \odot x}{dx}$ Y $\odot x$, whence (by § 29), integrating,

$$\text{area } \odot x = \pi i^2 \left( e^{\frac{x}{i}} - 2 + e^{\frac{-x}{i}} \right).$$

V. For the area CABDC = $u$ (inclosed by an L form line AB = $r$, the 8 to this, CD = $y$, and the sects AC = BD = $x$) $\frac{du}{dx}$ Y $y$; and (§ 24) $y = re^{\frac{-x}{i}}$, and so (integrating)

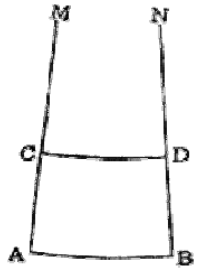

FIG. 26.

$$u = ri\left(1 - e^{\frac{x}{i}}\right).$$

If $x$ increases to infinity, then, in S, $e^{\frac{-x}{i}}$ Y 0, and so $u$ Y $ri$. By the *size* of MABN, in future this limit is understood.

In like manner is found, if $p$ is a figure on F, the space included by $p$ and the aggregate of axes drawn from the boundaries of $p$ is equal to ½ $pi$.

VI. If the angle at the center of a segment $z$ of a sphere is $2u$, and a great circle is $p$, and $x$ the arc FC (of the angle $u$); (§25)

$$1 : \sin u = p : \odot \text{BC},$$

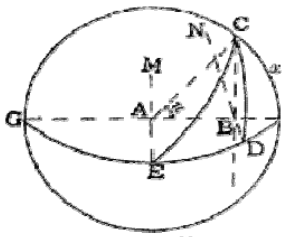

FIG. 27.

and hence $\odot \text{BC} = p \sin u$.

Meanwhile $x = \frac{pu}{2\pi}$, and $dx = \frac{pdu}{2\pi}$.

Moreover, $\frac{dz}{dx}$ Y ⊙ BC, and hence

$$\frac{dz}{du} \text{ Y } \frac{p^2}{2\pi} \sin u, \text{ whence (integrating)}$$

$$z = \frac{\text{ver sin } u}{2\pi} p^2.$$

The F may be conceived on which P falls (passing through the middle F of the segment); through AF and AC the planes FEM, CEM are placed, perpendicular to F and cutting F along FEG and CE; and consider the L form CD (from C ⊥ to FEG), and the L form CF; (§ 20) CEF = $u$, and (§ 21)

$$\frac{\text{FD}}{p} = \frac{\text{ver sin } u}{2\pi}, \text{ and so } z - \text{FD} \cdot p.$$

But (§ 21) $p = \pi \cdot$ FGD; therefore

$z = \pi \cdot$ FD $\cdot$ FDG. But (§ 21)

FD $\cdot$ FDG = FC $\cdot$ FC; consequently

$z = \pi \cdot$ FC $\cdot$ FC = area ⊙ FC, in F.

Now let BJ = CJ = $r$; (§ 30) $2r = i$ (Y – Y $^{-1}$), and so (§ 21)

area ⊙ $2r$ (in F) = $\pi i^2$(Y – Y $^{-1}$)$^2$.

Also (IV) area ⊙ $2y = \pi i^2$(Y$^2$ – 2 + Y $^{-2}$); therefore, area ⊙ $2r$ (in F) = area ⊙ $2y$, and so *the surface z of a segment of a sphere is equal to the surface of the circle described with the chord* FC *as a radius*.

Hence the whole surface of the sphere

$$= \text{area} \odot \text{FG} = -\text{FDG} \cdot p - \frac{p^2}{\pi},$$

*and the surfaces of spheres are to each other as the second powers of their great circles*.

VII. In like manner, in S, the volume of the sphere of radius $x$ is found

$$= \tfrac{1}{2}\pi i^3 (\text{X}^2 - \text{X}^{-2}) - 2\pi i^2 x;$$

the surface generated by the revolution of the line CD about AB

$$= \tfrac{1}{2}\pi i p (\text{Q}^2 - \text{Q}^{-2}),$$

and the body described by CABDC

$$= \tfrac{1}{4}\pi i^2 p (\text{Q}^2 - \text{Q}^{-2})^2.$$

*But in what manner all things treated from* (IV) *even to here*, *also may be reached apart from integration*, *for the sake of brevity is suppressed*.

*It can be demonstrated that the limit of every expression containing the letter i* (and so resting upon the hypothesis that *i* is given), *when i increases to infinity*, *expresses the quantity simply for* Σ (and so for the hypothesis of no *i*), if indeed the equations do not become identical.

But beware lest you understand to be supposed, that the system itself may be varied (for it is entirely determined in itself and by itself); but only *the hypothesis*, which may be

done successively, as long as we are not conducted to an absurdity. *Supposing* therefore that, in *such* an expression, the letter *i*, in case S is reality, designates that unique quantity whose I = *e*; but if Σ is actual, the said limit is supposed to be taken in place of the expression : manifestly *all the expressions originating from the hypothesis of the reality of* S (*in this sense*) *will be true absolutely*, a*lthough it be completely unknown whether or not* Σ *is reality*.

So e. g. from the expression obtained in § 30 easily (and as well by aid of differentiation as apart from it) emerges the known value in Σ,

$$\odot x = 2\pi x;$$

from I (§ 31) suitably treated, follows

$$1 : \sin\alpha = c : a,$$

but from II

$$\frac{\cos\alpha}{\sin\beta} = 1, \text{ and so}$$

$$\alpha + \beta = \text{rt.}\angle;$$

the first equation in III becomes identical, and so is true in Σ, although it there determines nothing; but from the second follows

$$c^2 = a^2 + b^2.$$

*These are the known fundamental equations of plane trigonometry in* Σ.

Moreover, we find (from § 32) in Σ, the area and the volume in III each = $pq$; from IV area $\odot x = \pi x^2$;

(from VII) the globe of radius $x$

$$= \tfrac{4}{3}\pi x^3, \text{ etc.}$$

The theorems enunciated at the end of VI are manifestly *true unconditionally*.

§ 33. It still remains to set forth (as promised in § 32) what this theory means.

I. Whether Σ or some one S is reality, remains undecided.

II. All things deduced from the hypothesis of the falsity of Axiom *XI* (always to be understood in the sense of § 32) are *absolutely true*, and so in this sense, depend upon no hypothesis.

There is therefore *a plane trigonometry a priori*, *in which the system alone really remains unknown*; and so where remain unknown solely the *absolute* magnitudes in the expressions, but where a *single* known case would manifestly fix the whole system. But spherical trigonometry is established absolutely in § 26.

(And we have, on F, a geometry wholly analogous to the plane geometry of Σ.)

III. If it were agreed that Σ exists, nothing more would be unknown in this respect; but

if it were *established* that Σ does not exist, then (§ 31), (e. g.) from the sides *x*, *y*, and the rectilineal angle they include being given in a special case, manifestly it would be impossible in itself and by itself to solve absolutely the triangle, that is, to determine *a priori* the other angles and *the ratio of the third side* to the two given; unless X, Y were determined, for which it would be necessary to have in concrete form a certain sect *a* whose A was known; and then *i* would be *the natural unit for length* (just as *e* is the base of *natural* logarithms).

If the existence of this *i* is determined, it will be evident how it could be constructed, at least very exactly, for practical use.

IV. In the sense explained (I and II), it is evident that all things in space can be solved by the modern analytic method (within just limits strongly to be praised).

V. Finally, to friendly readers will not be unacceptable; that for that case wherein not Σ but S is reality, a rectilineal figure is constructed equivalent to a circle.

§ 34. Through D we may draw DM ∥ AN in the following manner. From D drop DB ⊥ AN; from any point A of the straight AB erect AC ⊥ AN (in DBA), and let fall DC ⊥ AC. We

will have (§ 27) ⊙ CD : ⊙ AB – 1 : sin *z*, provided that DM ‖ BN. But sin *z* is not > 1; and so AB is not > DC. Therefore a quadrant described from the center A in BAC, with a radius = DC, will have a point B or O in common with ray BD. In the first case, manifestly $z = rt.\angle$; but in the second case (§ 25)

$$(\odot AO = \odot CD) : \odot AB = 1 \sin AOB,$$

and so *z* = AOB.

If therefore we take *z* = AOB, then DM will be ‖ BN.

§ 35. If S were reality; we may, as follows, draw a straight ⊥ to one arm of an acute angle, which is ‖ to the other.

Take AM ⊥ BC, and suppose AB = BC so small (by § 19), that if we draw BN ‖ AM (§ 34), ABN > the given angle.

Moreover draw CP ‖ AM (§ 34); and take NBG and PCD each equal to the given angle; rays BG and CD will cut; for if ray BG (falling *by construction* within NBC) cuts ray CP in E; we shall have (since BN Π CP), ∠EBC < ∠ECB, and so EC < EB. Take EF = EC, EFR

= ECD, and FS ‖ EP; then FS will fall within BFR. For since BN ‖ CP, and so BN ‖ EP, and BN ‖ FS; we shall have (§ 14)

$$\angle FBN + \angle BFS < (\text{st.} \angle = FBN + BFR);$$

therefore, BFS < BFR. Consequently, ray FR cuts ray EP, and so ray CD also cuts ray EG in some point D. Take now DG = DC and DGT = DCP = GBN; we shall have (since CD Π GD) BN Π GT Π CP. Let K (§ 19) be the point of the L-form line of BN falling in the ray BG, and KL the axis; we shall have BN Π KL, and so BKL = BGT = DCP; but also KL Π CP : therefore manifestly K fall on G, and GT ‖ BN. But if HO bisects ⊥ BG, we shall have constructed HO ‖ BN.

§36. Having given the ray CP and the plane MAB, take CB ⊥ the plane MAB, BN (in plane BCP) ⊥ BC, and CQ ‖ BN (§ 34); the intersection of ray CP (if this ray falls within BCQ) with ray BN (in the plane CBN), and so with the plane MAB is found. And if we are given the two planes PCQ, MAB, and we have CB ⊥ to plane MAB, CR ⊥ plane PCQ; and (in plane BCR) BN ⊥ BC, CS ⊥ CR, BN will fall in plane MAB, and CS in plane PCQ; and the

intersection of the straight BN with the straight CS (if there is one) having been found, the perpendicular drawn through this intersection, in PCQ, to the straight CS will manifestly be the intersection of plane MAB and plane PCQ.

§ 37. On the straight AM ‖ BN, is found such an A, that AM Π BN. If (by § 34) we construct outside of the plane NBM, GT ‖ BN, and make BG ⊥GT, GC = GB, and CP ‖ GT; and so place the hemi-plane TGD that it makes with hemi-plane TGB an angle equal to that which hemi-plane PCA makes with hemi-plane PCB; and is sought (by § 36) the intersection straight DQ of hemi-plane TGD with hemi-plane NBD; and BA is made ⊥ DQ.

We shall have indeed, on account of the similitude of the triangles of L lines produced on the F of BN (§ 21), manifestly DB = DA, and AM Π BN.

Hence easily appears (L-lines being given by their extremities alone) we may also find a fourth proportional, or a mean proportional, and execute in this way in F, apart from Axiom XI, all the geometric constructions made

on the plane in Σ. Thus e. g. a perigon can be geometrically divided into any special number of equal parts, if it is permitted to make this special partition in Σ.

§ 38. If we construct (by § 37) for example, NBQ = ⅓ rt.∠, and make (by. § 35), in S, AM ⊥ ray BQ and ‖ BN, and determine (by §37) IM Π BN; we shall have, if IA = *x*, (§ 28), X = 1 : sin ⅓ rt. ∠ = 2, and *x* will be constructed *geometrically*.

And NBQ may be so computed, that IA differs from *i* less than by anything given, which happens for sin NBQ = 1/*e*.

§ 39. If (in a plane) PQ and ST are 8 to the straight MN (§27), and AB, CD are equal perpendiculars to MN; manifestly ΔDEC ≅ ΔBEA; and so the angles (perhaps mixtilinear) ECP, EAT will fit, and EC = EA. If, moreover, CF = AG, then ΔACF ≅ ΔCAG, and each is half of the *quadrilateral* FAGC.

If FAGC, HAGK are two quadrilaterals of this sort on AG, between PQ and ST; their equivalence (as in Euclid) is evident, as also

the equivalence of the triangles AGC, AGH, standing on the same AG, and having their vertices on the line PQ. Moreover, ACF = CAG, GCQ = CGA, and ACF + ACG + GCQ = st.∠ (§ 32); and so also CAG + ACG + CGA = st.∠; therefore, in any triangle ACG of this sort, the sum of the three angles = st.∠. But whether the straight AG may have fallen upon AG (which 8 MN), or not; *the equivalence* of the rectilineal triangles AGC, AGH, as well of themselves, *as of the sums of their angles*, is evident.

§ 40. *Equivalent triangles* ABC, ABD, (henceforth rectilineal), *having one side equal*, *have the sums of their angles equal*. For let MN bisect AC and BC, and take (through C) PQ 8 MN; the point D will fall on line PQ.

For, if ray BD cuts the straight MN in the point E, and so (§ 39) the line PQ at the distance EF = EB; we shall have ΔABC = ΔABF, and so also ΔABD = ΔABF, whence D falls at F.

But if ray BD has not cut the straight MN, let C be the point, where the perpendicular bisecting the straight AB cuts the line PQ, and

let GS = HT, so, that the line ST meets the ray BD prolonged in a certain K (which it is evident can be made in a way like as in § 4); moreover take SR = SA, RO 8 ST, and O the intersection of ray BK with RO; then ΔABR = ΔABO (§39), and so ΔABC > ΔABD (contra hyp.).

§ 41. *Equivalent triangles ABC*, *DEF have the sums of their triangles equal*.

For let MN bisect AC and BC, and PQ bisect DF and FE; and take RS 8 MN, and TO 8 PQ; the perpendicular AG to RS will equal the perpendicular DH to TO, or one for example DH will be the greater.

In each case, the ⊙ DF, from center A, has with line-ray GS some point K in common, and (§ 39) ΔABK = ΔABC = ΔDEF. But the ΔAKB (by § 40) has the same angle-sum as ΔDFE, and (by § 39) as ΔABC. Therefore also the triangles ABC, DEF have each the same angle-sum.

In S the inverse of this theorem is true.

For take ABC, DEF two triangles having equal angle-sums, and ΔBAL = ΔDEF; these will have (by what precedes) equal angle-sums,

and so also will ΔABC and ΔABL, and hence manifestly

$$\text{BCL} + \text{BLC} + \text{CBL} = \text{st.}\angle.$$

However (by § 31), the angle-sum of any triangle, in S, is < st.∠.

Therefore L falls on C.

§ 42. Let $u$ be the supplement of the angle-sum of the ΔABC, but $v$ of ΔDEF; then is ΔABC : ΔDEF = $u$ : $v$. F For if $p$ be the area of each of the triangles ACG, GCH, HCB, DFK, KFE; and ΔABC = $m \cdot p$, and ΔDEF = $n \cdot p$; and $s$ the angle-sum of any triangle equivalent to $p$, manifestly

st. ∠ – $u$ = $m \cdot s$ – ($m$ – 1)st. ∠ = st. ∠ – $m$(st. ∠ – $s$); and $u$ = $m$(st.∠ – $s$); and in like manner $v$ = $n$(st.∠ – $s$).

Therefore ΔABC : ΔDEF = $m$ : $n$ = $u$ : $v$. It is evidently also easily extended to the case of the incommensurability of the triangles ABC, DEF.

In the same way is demonstrated that triangles on a sphere are as the excesses of the sums of their angles above a st. < .

If two angles of the spherical Δ are right, the third $z$ will be the said excess. But

(a great circle being called p) this Δ is manifestly

$$= \frac{z}{2\pi} \frac{p^2}{2\pi} \text{ (§ 32, VI);}$$

consequently, any triangle of whose angles the excess is z, is

$$= \frac{zp^2}{4\pi^2}.$$

§ 43. Now, in S, the area of a rectilineal Δ is expressed by means of the sum of its angles.

If AB increases to infinity; (§ 42) Δ ABC : (rt.∠ – *u* – *v*) will be constant. But Δ ABC Y BACN (§ 32, V), and rt.∠ – *u* – *v* Y *z* (§ 1); and so BACN : *z* = Δ ABC : (rt. ∠– *u* – *v*) = BAC'N' : *z*'.

Moreover, manifestly (§ 30) BDCN : BD'C'N' = *r* : *r*' tan *z* : tan *z*'.

But for *y*' Y 0, we have

$$\frac{\text{BD'C'N'}}{\text{BAC'N'}} \doteq 1 \text{ and also } \frac{\tan z'}{z'} \text{ Y } 1;$$

consequently,

$$\text{BDCN} : \text{BACN} = \tan z : z.$$

But (§ 32)

$$\text{BDCN} = r \cdot i = i^2 \tan z;$$

therefore,

$$\text{BACN} = z \cdot i^2.$$

Designating henceforth, for brevity, any triangle the supplement of whose angle-sum is $z$ by $\Delta$, we will therefore have $\Delta = z \cdot i^2$.

Hence it readily flows that, if OR ‖ AM and RO ‖ AB, the *area* comprehended between the straights OR, ST, BC (which is manifestly the absolute limit of the area of rectilineal triangles increasing without bound, or of $\Delta$ for $z$ Y st. $\angle$), is $= \pi i^2 =$ area $\odot i$, in F.

This limit being denoted by $\square$, moreover (by § 30) $\pi r^2 = \tan^2 z \cdot \square =$ area $\odot r$ in F (§ 21) = area $\odot s$ (by §32, VI) if the chord CD is called $s$.

If now, bisecting at right angles the given radius $s$ of the circle in a plane (or the L form radius of the circle in F), we construct (by § 34) DB ‖ Y CN; by dropping CA $\perp$ DB, and erecting CM $\perp$ CA, we shall get $z$; whence (by § 37), assuming at pleasure an L form radius for unity, $\tan^2 z$ *can be determined geometrically by means of two uniform lines of the same curvature* (which, their extremities alone being given and their axes

constructed, manifestly may be compared like straights, and in this respect considered equivalent to straights). Moreover, a quadrilateral, ex. gr. regular = □ is constructed as follows :

Take ABC = rt.∠, BAC = ½ rt. ∠, ACB = ¼ rt. ∠, and BC = $x$.

By mere square roots, X (from § 31, II) can be expressed and (by 37) constructed; and having X (by § 38 or also §§ 29 and 35), $x$ itself can be determined. And octuple Δ ABC is manifestly = □, and by this *a plane circle of radius s is geometrically squared by means of a rectilinear figure and uniform lines of the same species* (*equivalent to straights as to comparison inter se*); *but an F form circle is planified in the same manner*: *and we have either the Axiom* XI *of Euclid true or the geometric quadrature of the circle*, although thus far it has remained undecided, which of these two has place in reality.

Whenever $\tan^2 z$ is either a whole number, or a rational fraction, whose denominator (reduced to the simplest form) is either a prime number of the form $2^m + 1$ (of which is also $2 = 2^0 + 1$), or a product of however many prime numbers of this form, of which each (with the

exception of 2, which alone may occur any number of times) occurs only once as factor, we can, by the theory of polygons of the illustrious Gauss (remarkable invention of our, nay of every age) (and only for such values of $z$), construct a rectilineal figure = $\tan^2 z \, \square$ = area $\odot s$. For the division of $\square$ (the theorem of § 42 extending easily to any polygons) manifestly requires the partition of a st. $\angle$, which (as can be shown) can be achieved geometrically only under the said condition.

But in all such cases, what precedes conducts easily to the desired end. And any rectilineal figure can be converted geometrically into a regular polygon of $n$ sides, if $n$ falls under the Gaussian form. It remains, finally (that the thing may be completed in every respect), to demonstrate the impossibility (apart from any supposition), of deciding *a priori*, whether $\Sigma$, or some S (and which one) exists. This, however, is reserved for a more suitable occasion.

## APPENDIX I.

REMARKS ON THE PRECEDING TREATISE, BY BOLYAI FARKAS.

[From Vol. II of Tentamen, pp. 380 – 383.]

Finally it may be permitted to add something appertaining to the author of the *Appendix* in the first volume, who, however, may pardon me if something I have not touched with his acuteness.

The thing consists briefly in this : *the formulas of spherical trigonometry* (demonstrated in the said *Appendix* independently of Euclid's Axiom XI) *coincide with the formulas of plane trigonometry*, *if* (in a way provisionally speaking) *the sides of a spherical triangle are accepted as reals*, *but of a rectilineal triangle as imaginaries*; so that, as to trigonometric formulas, the plane may be considered as an imaginary sphere, if for real, that is accepted in which sin rt. $\angle$ = 1.

Doubtless, of the Euclidean axiom has been said in volume first enough and to spare : for

the case if it were not true, is demonstrated (Tom. I. App., p. 13), that there is given a certain *i*, for which the I there mentioned is = e (the base of natural logarithms), and for this case are established also (*ibidem*, p. 14) the formulas of plane trigonometry, and indeed so, that (by the side of p. 19, ibidem) the formulas are still valid for the case of the verity of the said axiom; indeed if the limits of the values are taken, supposing that *i* Y ∞; truly the Euclidean system is as if the limit of the anti-Euclidean (for *i* Y ∞).

Assume for the case of *i* existing, the unit = *i*, and extend the concepts sine and cosine also to imaginary arcs, so that, *p* designating an arc whether real or imaginary,

$$\frac{e^{p\sqrt{-1}} + e^{-p\sqrt{-1}}}{2}$$ is called the *cosine* of *p*, and

$$\frac{e^{p\sqrt{-1}} - e^{-p\sqrt{-1}}}{2\sqrt{-1}}$$ is called the *sine* of *p* (as Tom. I., p. 177).

Hence for *q* real

$$\frac{e^{q} - e^{-q}}{2\sqrt{-1}} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} - e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2\sqrt{-1}} = \sin(-q\sqrt{-1})$$

$$= -\sin(q\sqrt{-1}).$$

$$\frac{e^{q} + e^{-q}}{2} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} + e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2} = \cos(-q\sqrt{-1})$$

$$= \cos(q\sqrt{-1});$$

if of course also in the imaginary circle, the sine of a negative arc is the same as the sine of a positive arc otherwise equal to the first, except that it is negative, and the cosine of a positive arc and of a negative (if otherwise they be equal) the same.

In the said Appendix, § 25, is demonstrated absolutely, that is, independently of the said axiom; that, in any rectilineal triangle *the sines of the circles are as the circles of radii equal to the sides opposite*.

Moreover is demonstrated for the case of *i* existing, that the circle of radius *y* is

$$= \pi i \left( e^{\frac{y}{i}} - e^{\frac{-y}{i}} \right) \text{ which, for } i = 1, \text{ becomes}$$

$$\pi (e^{y} - e^{-y}).$$

Therefore (§ 31 *ibidem*), for a right-angled rectilineal triangle of which the sides are *a* and *b*, the hypothenuse *c*, and the angles opposite to the sides *a*, *b*, *c* are α, β, rt. ∠, (for i = 1), in I,

$$1 : \sin \alpha = \pi (e^{c} - e^{-c}) : \pi (e^{a} - e^{-a});$$

and so

$$1 : \sin \alpha = \frac{e^{c} - e^{-c}}{2\sqrt{-1}} : \frac{e^{a} - e^{-a}}{2\sqrt{-1}}.$$

Whence

$$1 : \sin \alpha = - \sin (c\sqrt{-1}) : - \sin (a\sqrt{-1}).$$

And hence

$$1 : \sin \alpha = \sin (c\sqrt{-1}) : \sin (a\sqrt{-1}).$$

In II becomes

$$\cos \alpha : \sin \beta = \cos (a\sqrt{-1}) : 1;$$

in III becomes

$$\cos (c\sqrt{-1}) = \cos (a\sqrt{-1}) \cdot \cos (b\sqrt{-1}).$$

These, as all the formulas of plane trigonometry deducible from them, coincide completely with the formulas of spherical trigonometry; except that if, ex. gr., also the sides and the angles opposite them of a right-angled spherical triangle and the hypothenuse bear the same names, the sides of the rectilineal triangle are to be divided by $\sqrt{-1}$ to obtain the formulas for the spherical triangle.

Obviously we get (clearly as Tom.,II., p. 252),

from I, $1 : \sin \alpha = \sin c : \sin a$;

from II, $1 : \cos a = \sin \beta : \cos \alpha$;

from III, $\cos c = \cos a \cos b$.

Though it be allowable to pass over other things; yet I have learned that the reader may be offended and impeded by the deduction omitted, (Tom. I., App., p. 19) [in § 32 at end] : it will not be irrelevant to show how, ex. gr., from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)\left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

follows

$$c^2 = a^2 + b^2.$$

(the theorem of Pythagoras for the Euclidean system); probably thus also the author deduced it, and the others also follow in the same manner.

Obviously we have, the powers of e being expressed by series (like Tom. I., p. 168),

$$e^{\frac{k}{i}} = 1 + \frac{k}{i} + \frac{\cancel{k^2}}{2i^2} + \frac{k^3}{2\cdot3\cdot i^3} + \frac{k^4}{2\cdot3\cdot4\cdot i^4} \cdot \cdot \cdot \cdot,$$

$$e^{\frac{-k}{i}} = 1 - \frac{k}{i} + \frac{\cancel{k^2}}{2i^2} - \frac{k^3}{2\cdot3\cdot i^3} + \frac{k^4}{2\cdot3\cdot4\cdot i^4} \cdot \cdot \cdot \cdot, \text{ and so}$$

$$e^{\frac{k}{i}} + e^{\frac{-k}{i}} = 2 + \frac{\cancel{k^2}}{i^2} + \frac{k^4}{3\cdot4\cdot i^4} + \frac{k^6}{3\cdot4\cdot5\cdot6\cdot i^6} \cdot \cdot \cdot \cdot,$$

$$= 2 + \frac{\cancel{k^2 + u}}{i^2}, \text{ (designating by}$$

$\frac{u}{i^2}$ the sum of all the terms after $\frac{k^2}{i^2}$); and we have $u$ Y 0, while $i$ Y $\infty$. For all the terms which follow $\frac{k^2}{i^2}$, are divided by $i^2$; the first term will be $\frac{k^4}{3\cdot4\cdot i^4}$; and any ratio $< \frac{k^2}{i^2}$; and though the ratio everywhere should remain this, the sum would be (Tom. I., p. 131),

$$\frac{k^4}{3\cdot4\cdot i^2} : \left(1 - \frac{k^2}{i^2}\right) = \frac{k^4}{3\cdot4\cdot(i^4 - k^2)}$$

which manifestly Y 0, while $i$ Y $\infty$.

And from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{(a+b)}{i}} + e^{\frac{-(a+b)}{i}} + e^{\frac{(a-b)}{i}} + e^{\frac{-(a-b)}{i}}\right)$$

follows (for $w$, $v$, $\lambda$ taken like $u$)

$$2 + \frac{\cancel{c^2 + w}}{i^2} = \frac{1}{2}\left(2 + \frac{\cancel{(a+b)^2 + v}}{i^2} + 2 + \frac{\cancel{(a+b)^2 + \lambda}}{i^2}\right)$$

And hence

$$c^2 = \frac{a^2 + 2ab + b^2 + a^2 - 2ab + b^2 + v + \lambda - w}{2}$$

which Y $a^2 + b^2$.

## APPENDIX II.

SOME POINTS IN JOHN BOLYAI'S

COMPARED WITH LOBACHEVSKI,

BY WOLFGANG BOLYAI.

[From *Kurzer Grundriss*, p. 82.]

Lobachevski and the author of the Appendix each consider two points A, B, of the sphere-limit, and the corresponding axes ray AM, ray BN (§ 23).

They demonstrate that, if α, β, γ designate the arcs of the circle limit AB, CD, HL, separated by segments of the axis AC = 1, AH = $x$, we have

$$\frac{\alpha}{\gamma} = \left(\frac{\alpha}{\beta}\right)^x .$$

Lobachevski represents the value of $\frac{\gamma}{\alpha}$ by $e^{-x}$, $e$ having some value > 1, dependent on the unit for length that we have chosen, and able to be supposed equal to the Naperian base.

The author of the Appendix is led directly to introduce the base of natural logarithms.

If we put $\frac{\alpha}{\beta} = \delta$, and $\gamma$, $\gamma'$ are arcs situated at the distances $y$, $i$ from α, we shall have

$$\frac{\alpha}{\gamma} = \delta^{y} = \mathrm{Y}, \frac{\alpha}{\gamma'} = \delta^{i} = \mathrm{I}, \text{ whence } \mathrm{Y} = \mathrm{I}^{\frac{y}{i}}.$$

He demonstrates afterward (§ 29) that, if $u$ is the angle which a straight makes with the perpendicular $y$ to its parallel, we have

$$\mathrm{Y} = \cot \tfrac{1}{2} u.$$

Therefore, if we put $z = \frac{\pi}{2} - u$, we have

$$\mathrm{Y} = \tan (z + \tfrac{1}{2} u) = \frac{\tan z + \tan - \tfrac{1}{2} u}{1 - \tan z \tan \tfrac{1}{2} u},$$

whence we get, having regard to the value of $\tan \frac{1}{2} - u = \mathrm{Y}^{-1}$,

$$\tan \mathrm{z} - \tfrac{1}{2} (\mathrm{Y} - \mathrm{Y}^{-1}) = \tfrac{1}{2} \left( \mathrm{I}^{\frac{y}{i}} - \mathrm{I}^{\frac{-y}{i}} \right) (\S 30).$$

If now $y$ is the semi-chord of the arc of circle-limit $2r$, we prove (§30) that $\frac{r}{\tan z} = \text{constant}$.

Representing this constant by $i$, and making $y$ tend toward zero, we have

$$\frac{2r}{2y} \mathrm{Y}\ 1, \text{ whence}$$

$$2y \mathrm{Y}\ 2i \tan z \mathrm{Y}\ i \frac{\mathrm{I}^{\frac{2y}{i}} - 1}{\mathrm{I}^{\frac{y}{i}}},$$

or putting $\frac{2y}{i} = k$, $\mathrm{I} = el$,

$$k \mathrm{I} \mathrm{I} \tfrac{\mathrm{y}}{\mathrm{i}} \mathrm{Y}\, e^{\mathrm{kl}} - 1 \mathrm{Y}\, kl\,(1 + \lambda),$$

$\lambda$ being infinitesimal at the same time as $k$. Therefore, for the limit, $1 = l$ and consequently $\mathrm{I} = e$.

The circle traced on the sphere-limit with the arc $r$ of the curve-limit for radius, has for length $2\pi r$. Therefore,

$$(\odot y = 2\pi r = 2\pi i \tan z = \pi i\,(\mathrm{Y} - \mathrm{Y}^{-1}).$$

In the rectilineal $\Delta$ where $\alpha$, $\beta$ designate the angles opposite the sides $a$, $b$, we have (§ 25)

$$\sin\alpha : \sin\beta = \odot\, a : \odot\, b = \pi\, i\,(\mathrm{A} - \mathrm{A}^{-1}) : \pi\, i\,(\mathrm{B} - \mathrm{B}^{-1})$$
$$= \sin(a\sqrt{-1}) : \sin(b\sqrt{-1}).$$

Thus in plane trigonometry as in spherical trigonometry, the sines of the angles are to each other as the sines of the opposite sides, only that on the sphere the sides are reals, and in the plane we must consider them as imaginaries, just as if the plane were an imaginary sphere.

We may arrive at this proposition without a preceding determination of the value of I.

If we designate the constant $\frac{r}{\tan z}$ by $q$, we shall have, as before

$$\odot\, y = \pi\, q\,(\mathrm{Y} - \mathrm{Y}^{-1}),$$

whence we deduce the same proportion as above, taking for *i* the distance for which the ratio I is equal to *e*.

If axiom XI is not true, there exists a determinate, which must be substituted in the formulas.

If, on the contrary, this axiom is true, we must make in the formulas $i = \infty$. Because, in this case, the quantity $\frac{\alpha}{\gamma} = Y$ is always = 1, the sphere-limit being a plane, and the axes being parallel in Euclid's sense.

The exponent $\frac{y}{i}$ must therefore be zero, and consequently $i = \infty$.

It is easy to see that Bolyai's formulas of plane trigonometry are in accord with those of Lobachevski.

Take for example the formula of § 37,

$$\tan \parallel (a) = \sin B \tan \parallel (p),$$

*a* being the hypothenuse of a right-angled triangle, *p* one side of the right angle, and B the angle opposite to this side.

Bolyai's formula of § 31, I, gives

$$1 : \sin B = (A - A^{-1}) : (P - P^{-1}).$$

Now, putting for brevity, $\frac{1}{2} \parallel (k) = k'$, we have $\tan 2p' : \tan 2a' = (\cot a' - \tan a') : (\cot p' - \tan p') = (A - A^{-1}) : (P - P^{-1}) : \sin B$.

## APPENDIX III.

LIGHT FROM NON-EUCLIDEAN SPACES ON THE

TEACHING OF ELEMENTARY GEOMETRY.

BY G. B. HALSTED.

As foreshadowed by Bolyai and Riemann, founded by Cayley, extended and interpreted for hyperbolic, parabolic, elliptic spaces by Klein, recast and applied to mechanics by Sir Robert Ball, projective metrics may be looked upon as characteristic of what is highest and most peculiarly modern in all the bewildering range of mathematical achievement. Mathematicians hold that number is wholly a creation of the human intellect, while on the contrary our space has an empirical element. Of possible geometries we can not say a priori which shall be that of our actual space, the space in which we move. Of course an advance so important, not only for mathematics but for philosophy, has had some metaphysical opponents, and as long ago as 1878 I mentioned in my Bibliography of Hyper-Space

and Non-Euclidean Geometry (American Journal of Mathematics, Vol. I, 1878, Vol. II, 1879) one of these, Schmitz-Dumont, as a sad paradoxer, and another, J. C. Becker, both of whom would ere this have shared the oblivion of still more antiquated fighters against the light, but that Dr. Schotten, praiseworthy for the very attempt at a comparative planimetry, happens to be himself a believer in the *a priori* founding of geometry, while his American reviewer, Mr. Ziwet, was then also an anti-non-Euclidean, though since converted.

He says, "we find that some of the best German text books do not try at all to define what is space, or what is a point, or even what is a straight line." Do any German geometries define space? I never remember to have met one that does.

In experience, what comes first is a bounded surface, with its boundaries, lines, and their boundaries, points. Are the points whose definitions are omitted anything different or better?

Dr. Schotten regards the two ideas "direction" and "distance" as intuitively given in the mind and as so simple as to not require definition.

When we read of two jockeys speeding

around a track in opposite directions, and also on page 87 of Richardson's Euclid, 1891, read, "The sides of the figure must be produced in the same direction of rotation; . . . going round the figure always in the same direction," we do not wonder that when Mr. Ziwet had written : "he therefore bases the definition of the straight line on these two ideas," he stops, modifies, and rubs that out as follows, "or rather recommends to elucidate the intuitive idea of the straight line possessed by any well-balanced mind by means of the still simpler ideas of direction" [in a circle] "and distance" [on a curve.

But when we come to geometry as a science, as foundation for work like that of Cayley and Ball, I think with Professor Chrystal : "It is essential to be careful with our definition of a *straight line*, for it will be found that virtually the properties of the straight line determine the nature of space.

"Our definition shall be that two points in *general* determine a straight line."

We presume that Mr. Ziwet glories in that unfortunate expression "a straight line is the shortest distance between two points," still occurring in Wentworth (New Plane Geometry, page 33), even after he has said, page 5,

"the length of the straight line is called the *distance* between two points." If the *length* of the one straight line between two points is the distance between those points, how can the straight line itself be the *shortest* distance? If there is only one distance, it is the longest as much as the shortest distance, and if it is the *length* of this shorto-longest distance which is the distance then it is not the straight line itself which is the longo-shortest distance. But Wentworth also says : "Of all lines joining two points the *shortest* is the straight line."

This general comparison involves the measurement of curves, which involves the theory of limits, to say nothing of ratio. The very ascription of length to a curve involves the idea of a limit. And then to introduce this general axiom, as does Wentworth, only to prove a very special case of itself, that two sides of a triangle are together greater than the third, is surely bad logic, bad pedagogy, bad mathematics.

This latter theorem, according to the first of Pascal's rules for demonstrations, should not be proved at all, since every dog knows it. But to this objection, as old as the sophists, Simson long ago answered for the science of

geometry, that the number of assumptions ought not to be increased without necessity; or as Dedekind has it : "*Was beweisbar ist*, *soll in der Wissenschaft nicht ohne Beweis geglaubt werden*."

Professor W. B. Smith (Ph. D., Goettingen), has written : "Nothing could be more unfortunate than the attempt to lay the notion of Direction at the bottom of Geometry."

Was it not this notion which led so good a mathematician as John Casey to give as a demonstration of a triangle's angle-sum the procedure called " a practical demonstration" on page 87 of Richardson's Euclid, and there described as "laying a 'straight edge' along one of the sides of the figure, and then turning it round so as to coincide with each side in turn."

This assumes that a segment of a straight line, a sect, may be translated without rotation, which assumption readily comes to view when you try the procedure in two-dimensional spherics. Though this fallacy was exposed by so eminent a geometer as Olaus Henrici in so public a place as the pages of 'Nature,' yet it has just been solemnly reproduced by Professor G. C. Edwards, of the University of California, in his Elements of Geometry : MacMillan, 1895.

It is of the greatest importance for every teacher to know and connect the commonest forms of assumption equivalent to Euclid's Axiom XI. If in a plane two straight lines perpendicular to a third nowhere meet, are there others, not both perpendicular to any third, which nowhere meet? Euclid' s Axiom XI is the assumption *No*. Playfair's answers *no* more simply. But the very same answer is given by the common assumption of our geometries, usually unnoticed, that a circle may be passed through any three points not costraight.

This equivalence was pointed out by Bolyai Parkas, who looks upon this as the simplest form of the assumption. Other equivalents are, the existence of any finite triangle whose angle-sum is a straight angle; or the existence of a plane rectangle; or that, in triangles, the angle-sum is constant.

One of Legendre's forms was that through every point within an angle a straight line may be drawn which cuts both arms.

But Legendre never saw through this matter because he had hot, as we have, the eyes of Bolyai and Lobachevski to see with. The same lack of their eyes has caused the author of the charming book " Euclid and His Modern

Rivals," to give us one more equivalent form : " In any circle, the inscribed equilateral tetragon is greater than any one of the segments which lie outside it." (A New Theory of Parallels by C. L. Dodgson, 3d. Ed., 1890.)

Any attempt to define a straight line by means of "direction" is simply a case of "argumentum in circulo." In all such attempts the loose word "direction" is used in a sense which presupposes the straight line. The directions from a point in Euclidean space are only the $\infty^2$ rays from that point.

Rays not costraight can be said to have the same direction only after a theory of parallels is presupposed, assumed.

Three of the exposures of Professor G. C. Edwards' fallacy are here reproduced. The first, already referred to, is from Nature, Vol. XXIX, p. 453, March 13, 1884.

"I select for discussion the 'quaternion proof" given by Sir William Hamilton. . . . Hamilton's proof consists in the following : "One side AB of the triangle ABC is turned about the point B till it lies in the continuation of BC; next, the line BC is made to slide along BC till B comes to C, and is then turned about C till it comes to lie in the continuation of AC.

" It is now again made to slide along CA till the point B comes to A, and is turned about A till it lies in the line AB. Hence it follows, *since rotation is independent of translation*, that the line has performed a whole revolution, that is, it has been turned through four right angles. But it has also described in succession the three exterior angles of the triangle, hence these are together equal to four right angles, and from this follows at once that the interior angles are equal to two right angles.

"To show how erroneous this reasoning is—in spite of Sir William Hamilton and in spite of quaternions—I need only point out that it holds exactly in the same manner for a triangle on the surface of the sphere, from which it would follow that the sum of the angles in a spherical triangle equals two right angles, whilst this sum is known to be always greater than two right angles. The proof depends only on the fact, that any line can be made to coincide with any other line, that two lines do so coincide when they have two points in common, and further, that a line may be turned about any point in it without leaving the surface. But if instead of the plane we take a spherical surface, and instead of a line a great

circle on the sphere, all these conditions are again satisfied.

"The reasoning employed must therefore be fallacious, and the error lies in the words printed in italics; for these words contain an assumption which has not been proved.

"O. HENRICI."

Perronet Thompson, of Queen's College, Cambridge, in a book of which the third edition is dated 1830, says:

"Professor Playfair, in the Notes to his 'Elements of Geometry' [1813], has proposed another demonstration, founded on a remarkable *non causa pro causa*.

"It purports to collect the fact [Eu. I., 32, Cor., 2] that (on the sides being successively prolonged to the same hand) the exterior angles of a rectilinear triangle are together equal to four right angles, from the circumstance that a straight line carried round the perimeter of a triangle by being applied to all the sides in succession, is brought into its old situation again; the argument being, that because this line has made the sort of somerset it would do by being turned through four right angles about a fixed point, the exterior

angles of the triangle have necessarily been equal to four right angles.

"The answer to which is, that there is no connexion between the things at all, and that the result will just as much take place where the exterior angles are avowedly not equal to four right angles.

"Take, for example, the plane triangle formed by three small arcs of the same or equal circles, as in the margin; and it is manifest that an arc of this circle may be carried round precisely in the way described and return to its old situation, and yet there be no pretense for inferring that the exterior angles were equal to four right angles.

"And if it is urged that these are curved lines and the statement made was of straight; then the answer is by demanding to know, what property of straight lines has been laid down or established, which determines that what is not true in the case of other lines is

true in theirs. It has been shown that, as a general proposition, the connexion between a line returning to its place and the exterior angles having been equal to four right angles, is a *non sequitur*; that it is a thing that may be or may not be; that the notion that it returns to its place *because* the exterior angles have been equal to four right angles, is a mistake. From which it is a legitimate conclusion, that if it had pleased nature to make the exterior angles of a triangle greater or less than four right angles, this would not have created the smallest impediment to the line's returning to its old situation after being carried round the sides; and consequently the line's returning is no evidence of the angles not being greater or less than four right angles."

Charles L. Dodgson, of Christ Church, Oxford, in his "Curiosa Mathematica," Part I, pp. 70 – 71, 3d Ed., 1890, says:

"Yet another process has been invented—quite fascinating in its brevity and its elegance—which, though involving the same fallacy as the Direction-Theory, proves Euc. I, 32, without even mentioning the dangerous word 'Direction.'

"We are told to take any triangle ABC; to produce CA to D; to make part of CD, viz., AD, revolve, about A, into the position ABE; then to make part of this line, viz., BE, revolve, about B, into the position BCF; and lastly to make part of this line, viz., CF, revolve, about C, till it lies along CD, of which it originally formed a part. We are then assured that it must have revolved through four right angles : from which it easily follows that the interior angles of the triangle are together equal to two right angles.

"The disproof of this fallacy is almost as brief and elegant as the fallacy itself. We first quote the general principle that we can not reasonably be told to make a line fulfill two conditions, either of which is enough by itself to fix its position : e. g., given three points X, Y, Z, we can not reasonably be told to draw a line from X which shall pass through Y and Z : we can make it pass through Y, but it must then take its chance of passing through Z; and *vice versa*.

" Now let us suppose that, while one part of

AE, viz., BE, revolves into the position BF, another little bit of it, viz., AG, revolves, through an equal angle, into the position AH; and that, while CF revolves into the position of lying along CD, AH revolves—and here comes the fallacy.

"You must not say 'revolves, through an equal angle, into the position of lying along AD,' for this would be to make AH *fulfill two conditions at once*.

"If you say that the one condition involves the other, you are virtually asserting that the lines CF, AH are equally inclined to CD—and this in *consequence* of AH having been so drawn that these same lines are equally inclined to AE.

"That is, you are asserting, 'A pair of lines which are equally inclined to a certain transversal, are so to any transversal.' [Deducible from Euc. I, 27, 28, 29.]"

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I found this in Chapter 2 of THE PARABOLA, ELLIPSE, AND HYPERBOLA, TREATED GEOMETRICALLY. BY
ROBERT WILLIAM GRIFFIN, A.M., LL.D.,
Now a Lemma is a given, but why? Could the author not show the construction? I remembered the figure from Heath's translation of The Elements by Euclid, but alas, the construction was not there either. So, I did a work up of the figure.



## LEMMA.

If a right line $AB$ be divided internally at $O$ in any ratio, and externally at $O'$ in the same ratio, and a circle be described on $OO'$ as diameter, the right lines joining any point $P$ on this circle with the extremities of the line $AB$ will have the same ratio.

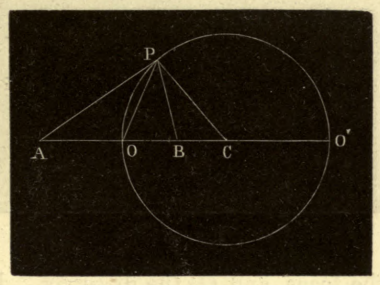



Fig. 1.

Bisect $OO'$ in $C$; join $CP$, $PO$.

Then $AO' : O'B = AO : OB$;

$\therefore AO' + AO : AO' - AO = O'B + OB : O'B - OB$;

$\therefore 2AC : 2OC = 2OC : 2BC$;

$\therefore AC : CP = CP : CB$;

$\therefore \Delta ACP$ is similar to $\Delta PCB$; (6 VI. Euclid.)

$\therefore \angle CPB = \angle CAP$;

but $\angle CPO = \angle COP$ (5 I. Euclid.)

$= \angle OAP + \angle OPA$ (32 I. Euclid.)

$= \angle CPB + \angle OPA$;

$\therefore \angle BPO = \angle OPA$;

$\therefore AP : PB = AO : OB$. (3 VI. Euclid.)

Pages from Heath's Translation of Euclid's Elements



and also, as $BD$ is to $DC$, so is $BA$ to $AE$: for $AD$ has been drawn parallel to $EC$, one of the sides of the triangle $BCE$: [VI. 2]

therefore also, as $BA$ is to $AC$, so is $BA$ to $AE$. [V. 11]

Therefore $AC$ is equal to $AE$, [V. 9]

so that the angle $AEC$ is also equal to the angle $ACE$. [I. 5]

But the angle $AEC$ is equal to the exterior angle $BAD$, [I. 29]

and the angle $ACE$ is equal to the alternate angle $CAD$; [*id.*]

therefore the angle $BAD$ is also equal to the angle $CAD$.

Therefore the angle $BAC$ has been bisected by the straight line $AD$.

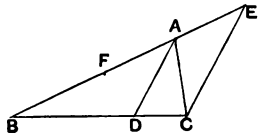
Therefore etc.

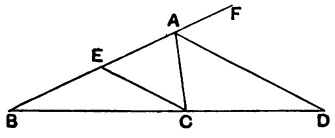
Q. E. D.

The demonstration assumes that $CE$ *will* meet $BA$ produced in some point $E$. This is proved in the same way as it is proved in VI. 4 that $BA$, $ED$ will meet if produced. The angles $ABD$, $BDA$ in the figure of VI. 3 are together less than two right angles, and the angle $BDA$ is equal to the angle $BCE$, since $DA$, $CE$ are parallel. Therefore the angles $ABC$, $BCE$ are together less than two right angles; and $BA$, $CE$ must meet, by I. Post. 5.

The corresponding proposition about the segments into which $BC$ is divided *externally* by the bisector of the *external angle* at $A$ when that bisector meets $BC$ produced (i.e. when the sides $AB$, $AC$ are not equal) is important. Simson gives it as a separate proposition, A, noting the fact that Pappus assumes the result without proof (Pappus, VII. p. 730, 24).

The best plan is however, as De Morgan says, to combine Props. 3 and A in one proposition, which may be enunciated thus: *If an angle of a triangle be bisected internally or externally by a straight line which cuts the opposite side or the opposite side produced, the segments of that side will have the same ratio as the other sides of the triangle; and, if a side of a triangle be divided internally or externally so that its segments have the same ratio as the other sides of the triangle, the straight line drawn from the point of section to the angular point which is opposite to the first mentioned side will bisect the interior or exterior angle at that angular point.*

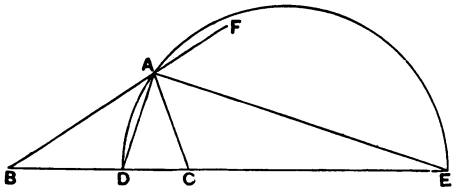



Let $AC$ be the smaller of the two sides $AB$, $AC$, so that the bisector $AD$ of the exterior angle at $A$ may meet $BC$ produced beyond $C$. Draw $CE$ through $C$ parallel to $DA$, meeting $BA$ in $E$.

Then, if $FAC$ is the exterior angle bisected by $AD$ in the case of external bisection, and if a point $F$ is taken on $AB$ in the figure of VI. 3, the proof of

VI. 3 can be used almost word for word for the other case. We have only to speak of the angle "$FAC$" for the angle "$BAC$," and of the angle "$FAD$" for the angle "$BAD$" wherever they occur, to say "let $BA$, or $BA$ produced, meet $CE$ in $E$," and to substitute "$BA$ or $BA$ produced" for "$BAE$" lower down.




If $AD$, $AE$ be the internal and external bisectors of the angle $A$ in a triangle of which the sides $AB$, $AC$ are unequal, $AC$ being the smaller, and if $AD$, $AE$ meet $BC$ and $BC$ produced in $D$, $E$ respectively,

the ratios of $BD$ to $DC$ and of $BE$ to $EC$ are alike equal to the ratio of $BA$ to $AC$.

Therefore $BE$ is to $EC$ as $BD$ to $DC$,

that is, $BE$ is to $EC$ as the difference between $BE$ and $ED$ is to the difference between $ED$ and $EC$,

whence $BE$, $ED$, $EC$ are in *harmonic progression*, or $DE$ is a *harmonic mean* between $BE$ and $EC$, or again $B$, $D$, $C$, $E$ is a *harmonic range*.

Since the angle $DAC$ is half of the angle $BAC$,

and the angle $CAE$ half of the angle $CAF$,

while the angles $BAC$, $CAF$ are equal to two right angles,

the angle $DAE$ is a right angle.

Hence the circle described on $DE$ as diameter passes through $A$.

Now, if the ratio of $BA$ to $AC$ is given, and if $BC$ is given, the points $D$, $E$ on $BC$ and $BC$ produced are given, and therefore so is the circle on $D$, $E$ as diameter. Hence *the locus of a point such that its distances from two given points are in a given ratio (not being a ratio of equality) is a circle.*

This locus was discussed by Apollonius in his *Plane Loci*, Book II., as we know from Pappus (VII. p. 666), who says that the book contained the theorem that, if from two given points straight lines inflected to another point are in a given ratio, the point in which they meet will lie on either a straight line or a circumference of a circle. The straight line is of course the locus when the ratio is one of equality. The other case is quoted in the following form by Eutocius (Apollonius, ed. Heiberg, II. pp. 180—4).

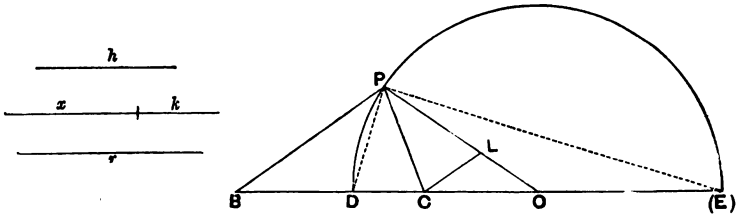
*Given two points in a plane and a proportion between unequal straight lines, it is possible to describe a circle in the plane so that the straight lines inflected from the given points to the circumference of the circle shall have a ratio the same as the given one.*

Apollonius' construction, as given by Eutocius, is remarkable because he makes no use of either of the points $D$, $E$. He finds $O$, the centre of the required circle, and the length of its radius directly from the data $BC$ and the given ratio which we will call $h : k$. But the construction was not discovered by Apollonius; it belongs to a much earlier date, since it appears in exactly

the same form in Aristotle, *Meteorologica* III. 5, 376 a 3 sqq. The analysis leading up to the construction is, as usual, not given either by Aristotle or Eutocius. We are told to take three straight lines $x$, $CO$ (a length measured along $BC$ produced beyond $C$, where $B$, $C$ are the points at which the greater and smaller of the inflected lines respectively terminate), and $r$, such that, if $h:k$ be the given ratio and $h > k$,

$$k : h = h : k + x, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha)$$

$$x : BC = k : CO = h : r. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta)$$




This determines the position of $O$, and the length of $r$, the radius of the required circle. The circle is then drawn, *any* point $P$ is taken on it and joined to $B$, $C$ respectively, and it is proved that

$$PB : PC = h : k.$$

We may conjecture that the analysis proceeded somewhat as follows.

It would be seen that $B$, $C$ are "conjugate points" with reference to the circle on $DE$ as diameter. (Cf. Apollonius, *Conics*, I. 36, where it is proved, in terms, for a circle as well as for an ellipse and a hyperbola, that, if the polar of $B$ meets the diameter $DE$ in $C$, then $EC : CD = EB : BD$.)

If $O$ be the middle point of $DE$, and therefore the centre of the circle, $D$, $E$ may be eliminated, as in the *Conics*, I. 37, thus.

Since $$EC : CD = EB : BD,$$

it follows that $$EC + CD : EC \sim CD = EB + BD : EB \sim BD,$$

or $$2OD : 2OC = 2OB : 2OD,$$

that is, $$BO \cdot OC = OD^2 = r^2, \text{ say.}$$

If therefore $P$ be any point on the circle with centre $O$ and radius $r$,

$$BO : OP = OP : OC,$$

so that $BOP$, $POC$ are similar triangles.

In addition, $h : k = BD : DC = BE : EC$

$$= BD + BE : DE = BO : r.$$

Hence we require that

$$BO : r = r : OC = BP : PC = h : k. \quad \ldots\ldots\ldots\ldots\ldots (\delta)$$

Therefore, alternately,

$$k : CO = h : r,$$

which is the second relation in ($\beta$) above.

Now assume a length $x$ such that each of the last ratios is equal to $x : BC$, as in ($\beta$).

Then $x : BC = k : CO = h : r.$

Therefore $x + k : BO = h : r,$

and, alternately, $x + k : h = BO : r$

$= h : k$, from ($\delta$) above;

and this is the relation ($\alpha$) which remained to be found.

Apollonius' proof of the construction is given by Eutocius, who begins by saying that it is manifest that $r$ is a mean proportional between $BO$ and $OC$. This is seen as follows.

From ($\beta$) we derive

$$x : BC = k : CO = h : r = (k + x) : BO,$$

whence $BO : r = (k + x) : h$

$= h : k$, by ($\alpha$),

$= r : CO$, by ($\beta$),

and therefore $r^2 = BO \cdot CO.$

But the triangles $BOP$, $POC$ have the angle at $O$ common, and, since $BO : OP = OP : OC$, the triangles are similar and the angles $OPC$, $OBP$ are equal.

[Up to this point Aristotle's proof is exactly the same; from this point it diverges slightly.]

If now $CL$ be drawn parallel to $BP$ meeting $OP$ in $L$, the angles $BPC$ $LCP$ are equal also.

Therefore the triangles $BPC$, $PCL$ are similar, and

$$BP : PC = PC : CL,$$

whence $BP^2 : PC^2 = BP : CL$

$= BO : OC$, by parallels,

$= BO^2 : OP^2$ (since $BO : OP = OP : OC$).

Therefore $BP : PC = BO : OP$

$= h : k$ (for $OP = r$).

[Aristotle infers this more directly from the similar triangles $POB$, $COP$. Since these triangles are similar,

$$OP : CP = OB : BP,$$

whence $BP : PC = BO : OP$

$= h : k.$]

Apollonius proves lastly, by *reductio ad absurdum*, that the last equation cannot be true with reference to any point $P$ which is not on the circle so described.

## Proposition 4.

*In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.*

**052211 Plate 1**

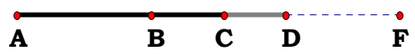
**Unit.**

**Given.**

$N_1 := 5 \quad AC := N_1$

$N_2 := 2 \quad CD := N_2$

**Given AC is not one half of AD, find AF, DF such that AC : CD :: AF : DF.**




**Descriptions.**

**The author of The PAKABOLA, ELLIPSE, AND HYPEKBOLA by R. W. Griffin, gave this figure as a Lemma, and Book VI Prop. 6 of Euclid only has it in the notes, but no construction.**

**Basically it is how to construct a circle with a given ratio, which is why it is probably the first thing in Chapt. 2 of Griffin's work. So, I will first demonstrate how to construct the figure from the internal ratio of AC : CD.**

$CK := CD$

$AK := AC - CK \qquad AG := \sqrt{2 \cdot AC^2}$

$AD := AC + CD \qquad AJ := \dfrac{AG \cdot AD}{AK}$

$AF := \dfrac{AC \cdot AJ}{AG} \qquad DF := AF - AD$

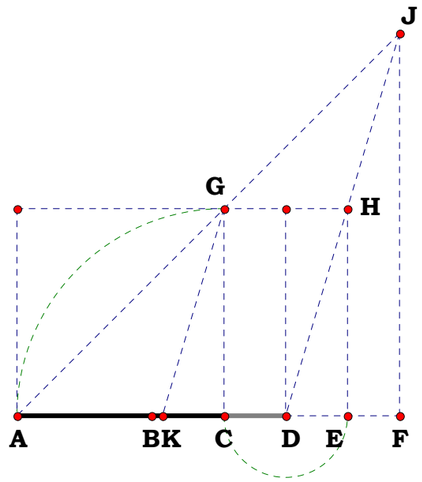
$CF := AF - AC$

**Definitions.**

$$AF - \left( N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2} \right) = 0 \qquad \frac{AC}{CD} - \frac{AF}{DF} = 0$$

$$DF - \left( N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2 \right) = 0$$

$$\left( \frac{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2}}{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2} \right) - \frac{N_1}{N_2} = 0$$



**Next demonstrate that from the circumference of CM, the proportion remains.**

$N_3 := 1$ $N_4 := 4$

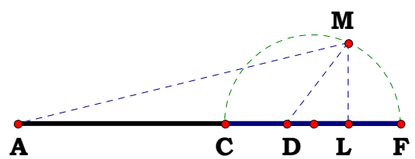
$CL := CF \cdot \frac{N_3}{N_4}$ $FL := CF - CL$

$AL := AC + CL$ $ML := \sqrt{CL \cdot FL}$

$AM := \sqrt{ML^2 + AL^2}$

$DL := CL - CD$

$DM := \sqrt{ML^2 + DL^2}$



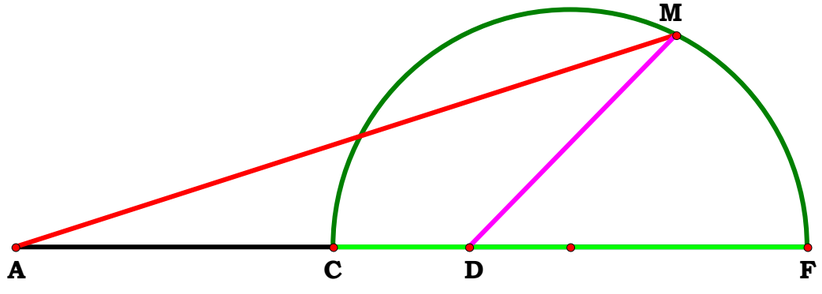

$\frac{AC}{CD} - \frac{AM}{DM} = 0$

$DM - \left[ \left| \frac{N_2}{(N_1 - N_2)} \cdot \left( \frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}} \right| \right] = 0$

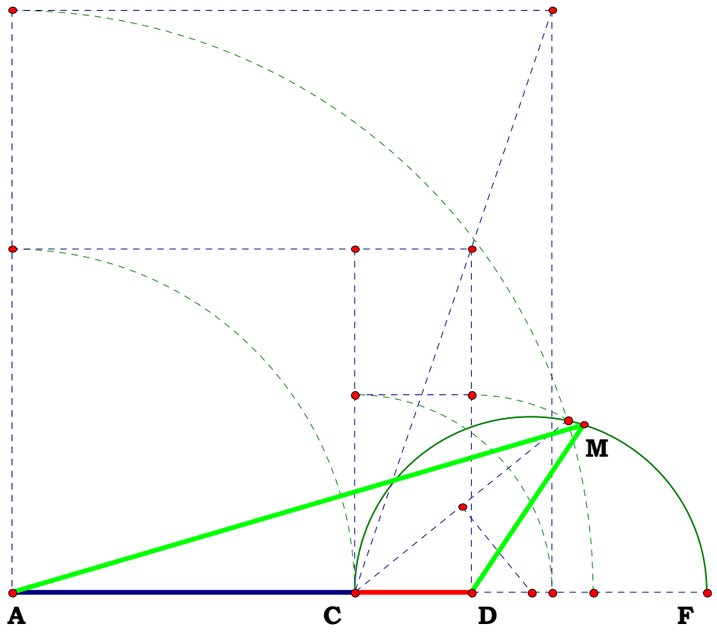
$AM - \left[ \left| \frac{N_1}{(N_1 - N_2)} \cdot \left( \frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}} \right| \right] = 0$

$$\left[ \frac{\frac{N_1}{(N_1 - N_2)} \cdot \left( \frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}}}{\frac{N_2}{(N_1 - N_2)} \cdot \left( \frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}}} \right] - \frac{N_1}{N_2} = 0$$

**Therefore the three ratios are equal. AC : CD :: AF : DF :: AM : DM, and one can construct a circle using a given ratio.**




***Alternate Construction***


M
A
C
D
F

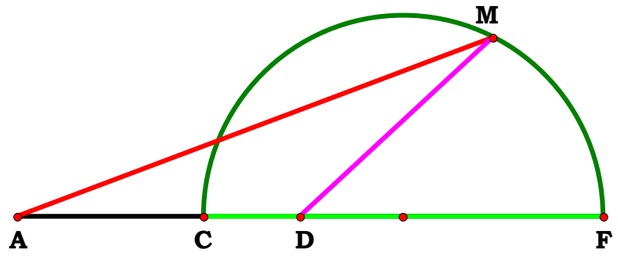
052211 Plate 2

Unit.

Given.

$N_1 := 3.22792$ $AC := N_1$

$N_2 := 8.65187$ $AF := N_2$




**Descriptions.**

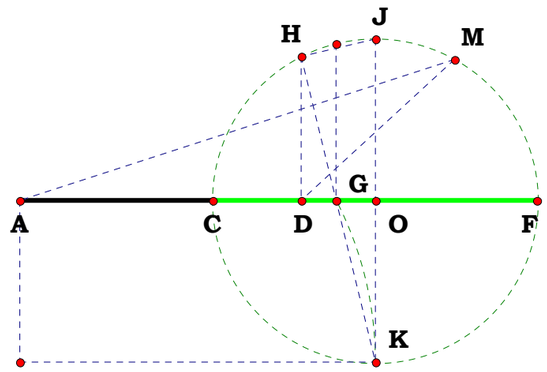
**For the given figure, find D if only AC and AF are given.**

$CF := AF - AC$

$AG := \sqrt{AC \cdot AF}$ $OK := \frac{CF}{2}$

$AO := AC + OK$

$GO := AO - AG$ $GK := \sqrt{OK^2 + GO^2}$



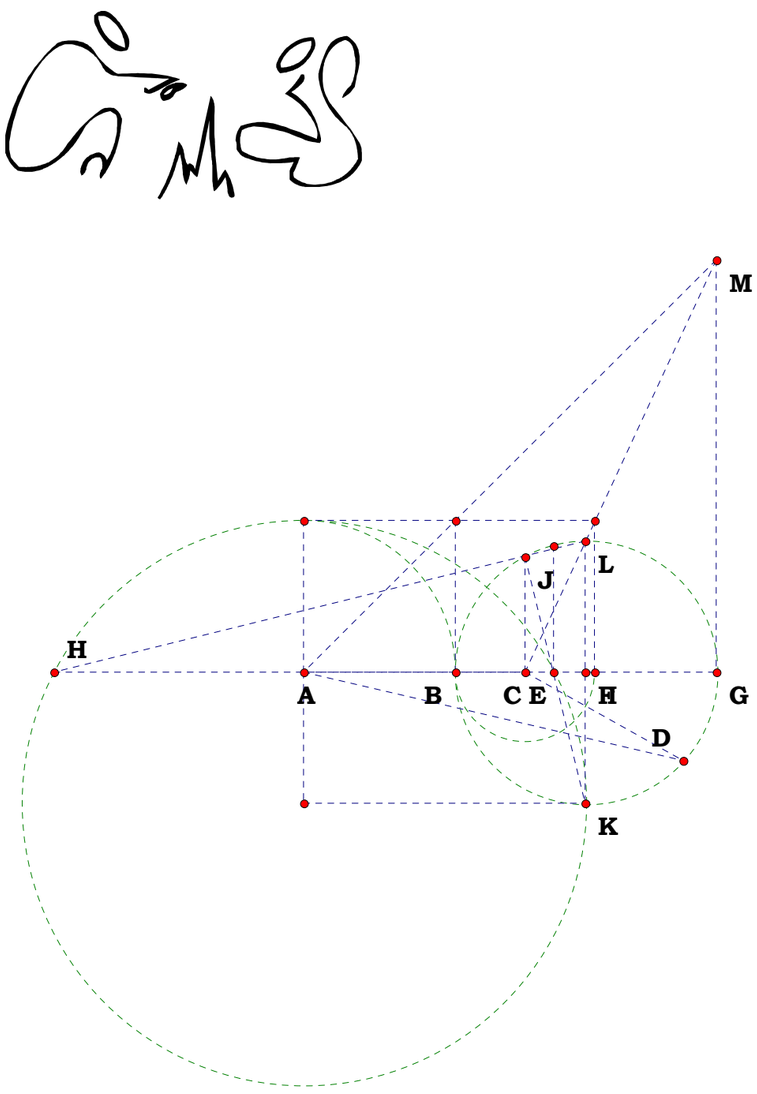

$HK := \frac{OK \cdot CF}{GK}$ $GH := HK - GK$ $DG := \frac{GO \cdot GH}{GK}$

$AD := AG - DG$ $DF := AF - AD$ $CD := AD - AC$

**Definitions.**

$\frac{AC}{CD} - \frac{AF}{DF} = 0$ $AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0$

$DF - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $CD - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$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**Unit.**

**Given.**

$N_1 := 3.86292$ $AB := N_1$

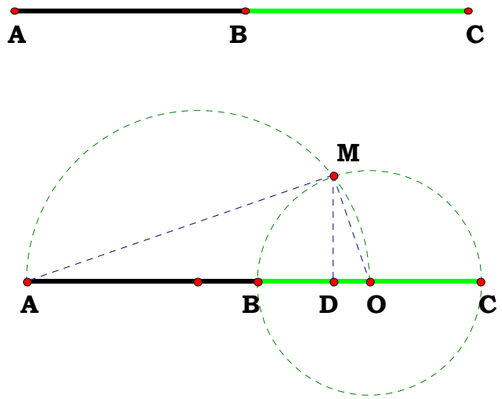
$N_2 := 7.59354$ $AC := N_2$

**052211**

**Descriptions.**

## Lemma Plate 3

**Simplify Plate 2.**




$BC := AC - AB$ $BO := \frac{BC}{2}$ $MO := BO$

$AO := AB + BO$ $AM := \sqrt{AO^2 - MO^2}$

$DO := \frac{MO^2}{AO}$ $BD := BO - DO$

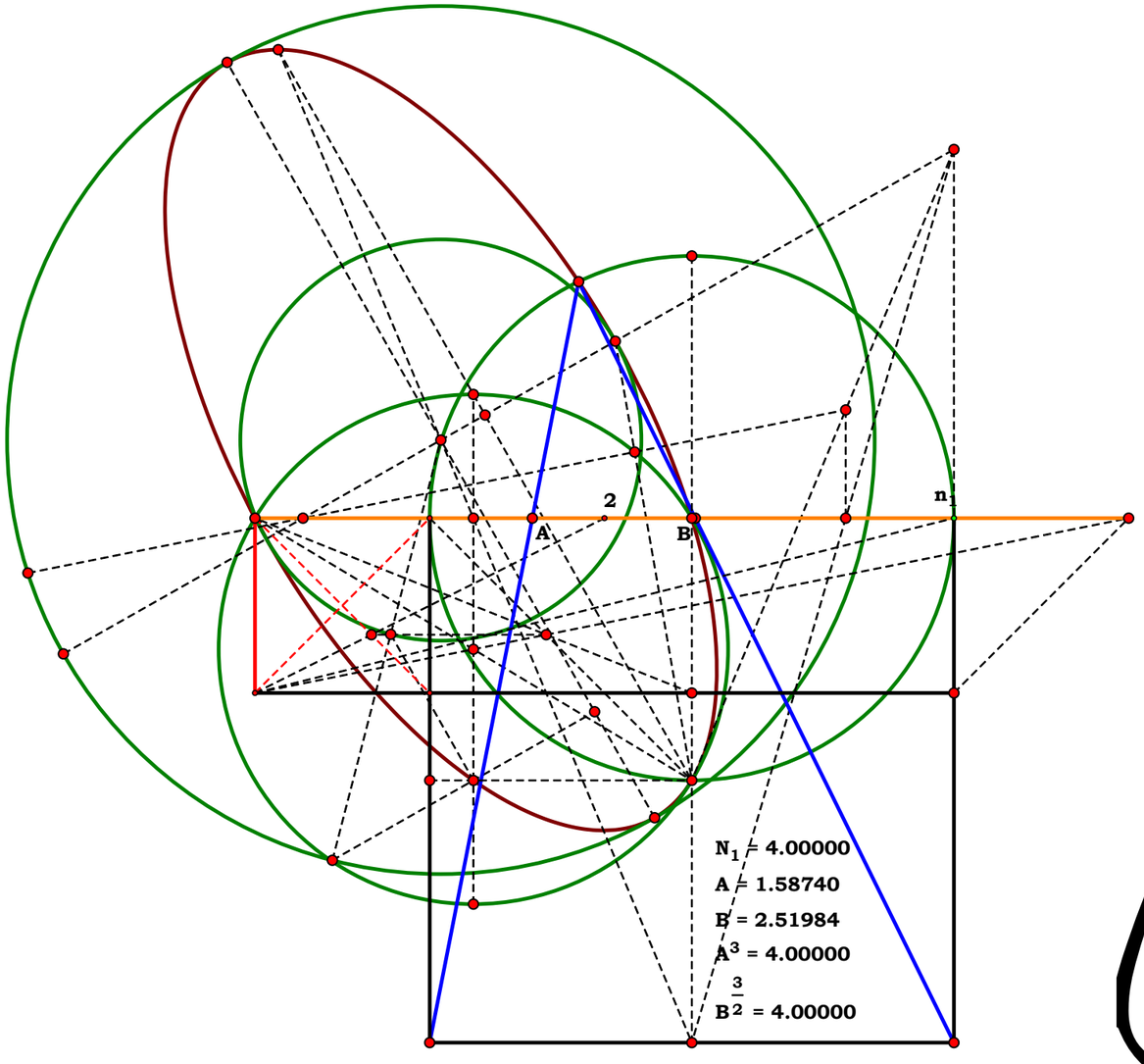
$CD := BC - BD$ $AD := AB + BD$




**Definitions.**

$\frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} - BD = 0$ $\frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} - CD = 0$

$\frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} - AD = 0$




# The Delian Quest 2015

**John Clark**

Unit.

Given.

$AB := 6.20183$

$AC := 4.89358$

$BC := 9.20468$

08092015

Descriptions.

$$GA := \frac{AC^2}{AB} \qquad HB := \frac{BC^2}{AB} \qquad GH := AB - (GA + HB) \qquad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \qquad CJ := \sqrt{AC^2 - JA^2} \qquad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

Definitions.

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0 \qquad JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0$$

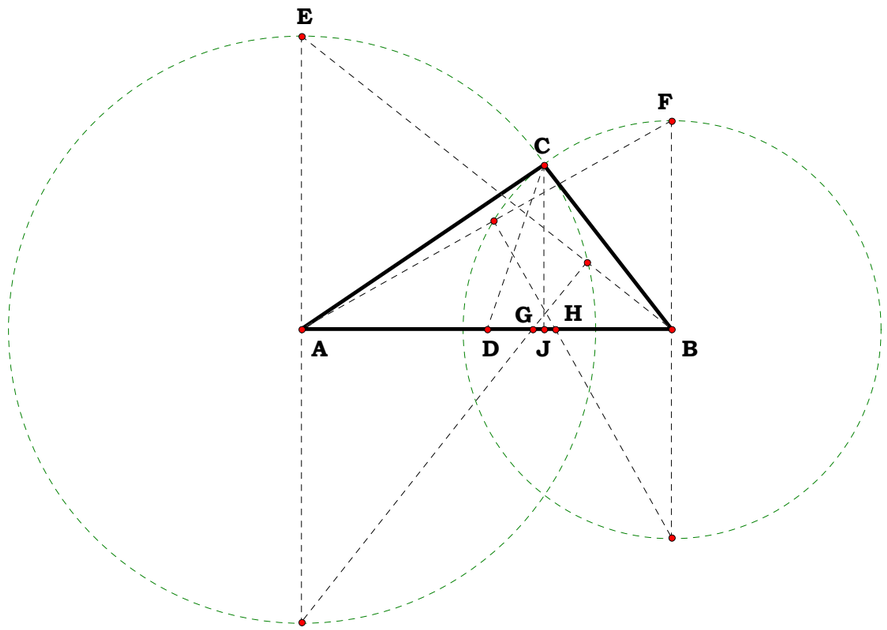
$$JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

**Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.**

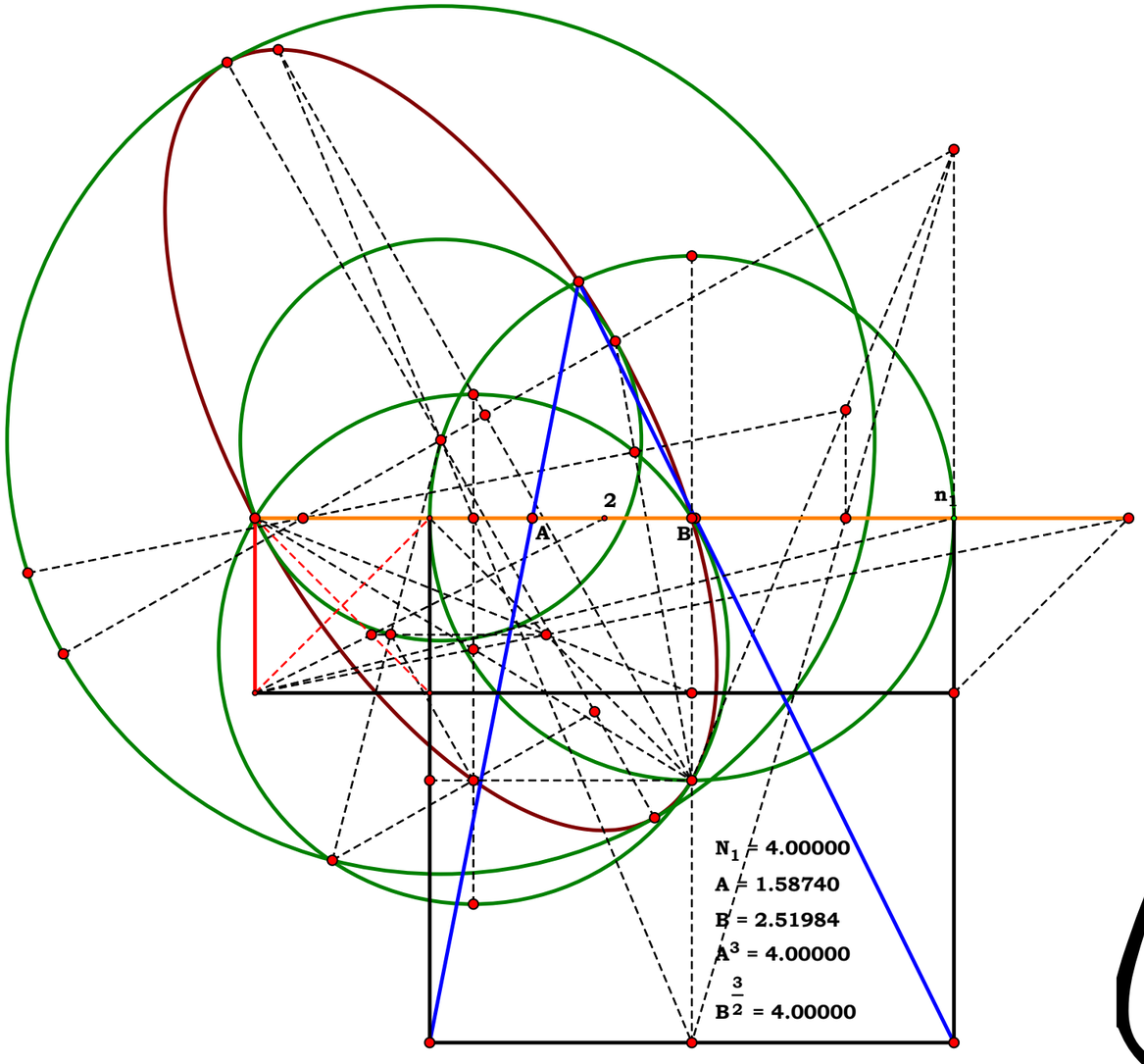
$$AJ := \sqrt{AC^2 - CJ^2} \qquad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0$$

$$BJ := \sqrt{BC^2 - CJ^2} \qquad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

**One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.**

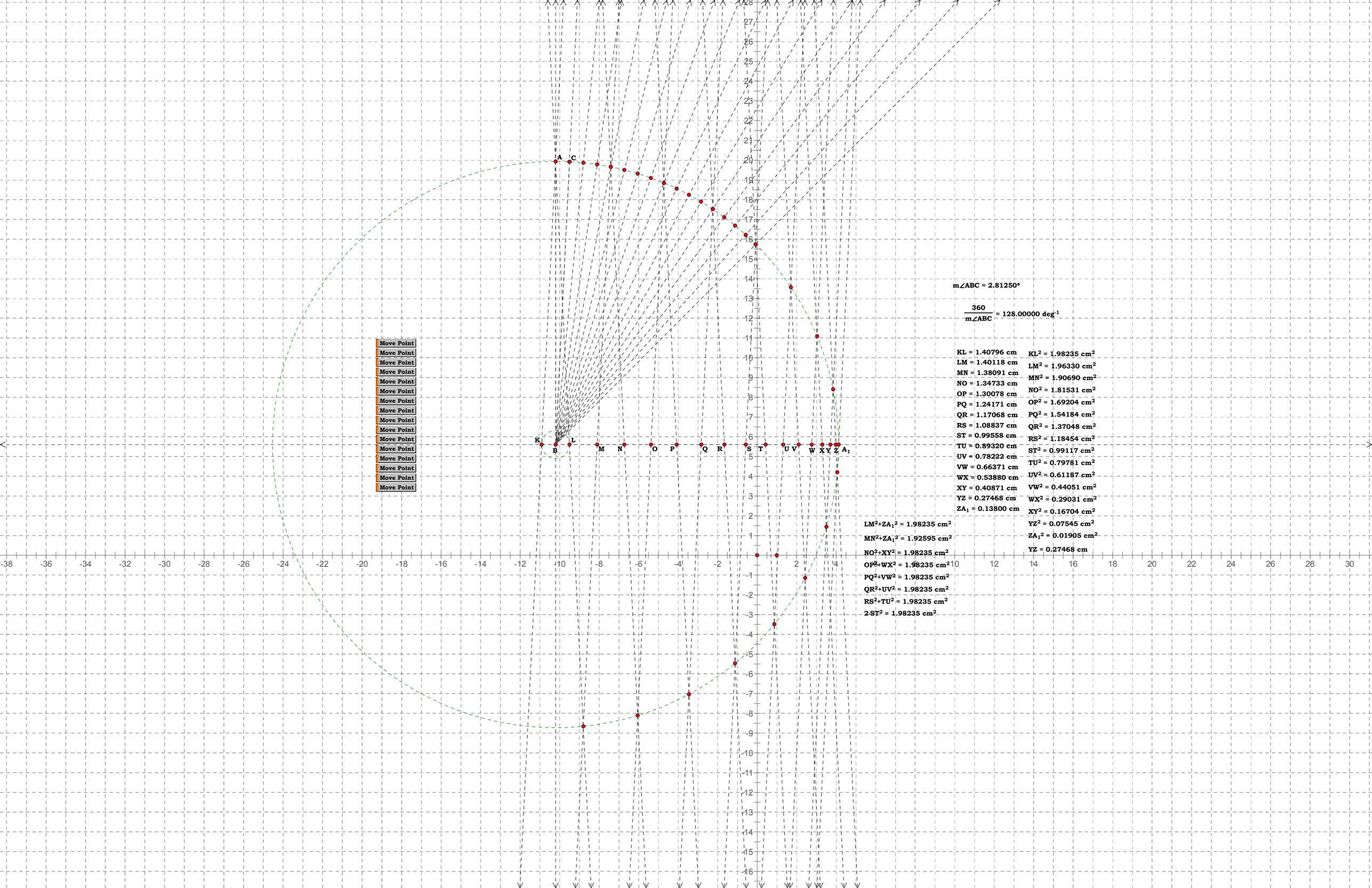



**Pythagoras Revisited Again!**




# The Delian Quest 2017

**John Clark**




$m\angle ABC = 2.81250°$

$\frac{360}{m\angle ABC} = 128.00000\ deg^{-1}$

| | |
|---|---|
| KL = 1.40796 cm | $KL^2 = 1.98235\ cm^2$ |
| LM = 1.40118 cm | $LM^2 = 1.96330\ cm^2$ |
| MN = 1.38091 cm | $MN^2 = 1.90690\ cm^2$ |
| NO = 1.34733 cm | $NO^2 = 1.81531\ cm^2$ |
| OP = 1.30078 cm | $OP^2 = 1.69204\ cm^2$ |
| PQ = 1.24171 cm | $PQ^2 = 1.54184\ cm^2$ |
| QR = 1.17068 cm | $QR^2 = 1.37048\ cm^2$ |
| RS = 1.08837 cm | $RS^2 = 1.18454\ cm^2$ |
| ST = 0.99558 cm | $ST^2 = 0.99117\ cm^2$ |
| TU = 0.89320 cm | $TU^2 = 0.79781\ cm^2$ |
| UV = 0.78222 cm | $UV^2 = 0.61187\ cm^2$ |
| VW = 0.66371 cm | $VW^2 = 0.44051\ cm^2$ |
| WX = 0.53880 cm | $WX^2 = 0.29031\ cm^2$ |
| XY = 0.40871 cm | $XY^2 = 0.16704\ cm^2$ |
| YZ = 0.27468 cm | $YZ^2 = 0.07545\ cm^2$ |
| $ZA_1 = 0.13800$ cm | $ZA_1^2 = 0.01905\ cm^2$ |
| | YZ = 0.27468 cm |

$LM^2+ZA_1^2 = 1.98235\ cm^2$

$MN^2+ZA_1^2 = 1.92595\ cm^2$

$NO^2+XY^2 = 1.98235\ cm^2$

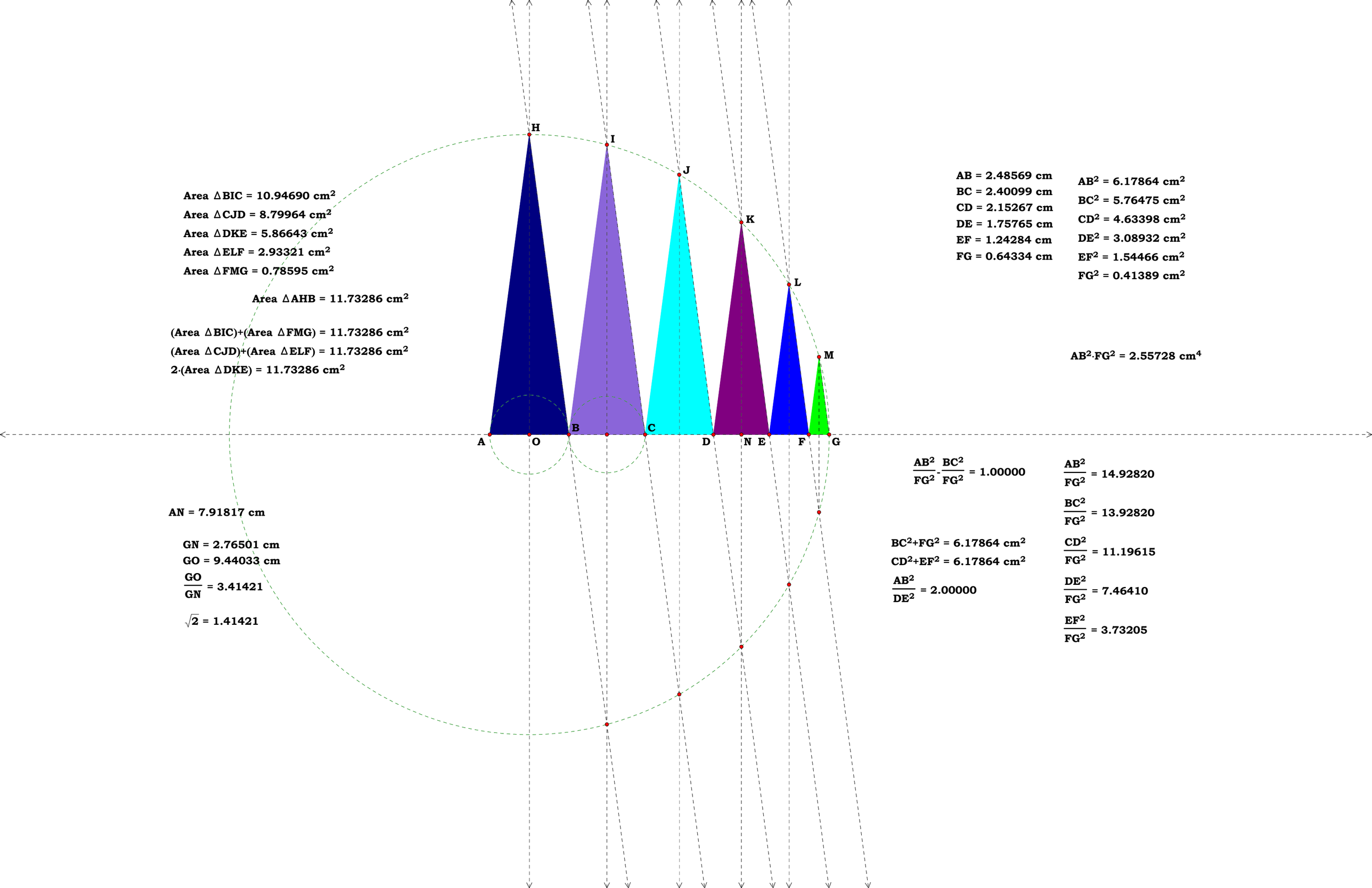
$OP^2+WX^2 = 1.98235\ cm^2$

$PQ^2+VW^2 = 1.98235\ cm^2$

$QR^2+UV^2 = 1.98235\ cm^2$

$RS^2+TU^2 = 1.98235\ cm^2$

$2 \cdot ST^2 = 1.98235\ cm^2$




Area △BIC = 10.94690 cm²

Area △CJD = 8.79964 cm²

Area △DKE = 5.86643 cm²

Area △ELF = 2.93321 cm²

Area △FMG = 0.78595 cm²

Area △AHB = 11.73286 cm²

(Area △BIC)+(Area △FMG) = 11.73286 cm²

(Area △CJD)+(Area △ELF) = 11.73286 cm²

2·(Area △DKE) = 11.73286 cm²

AB = 2.48569 cm

BC = 2.40099 cm

CD = 2.15267 cm

DE = 1.75765 cm

EF = 1.24284 cm

FG = 0.64334 cm

$AB^2$ = 6.17864 cm²

$BC^2$ = 5.76475 cm²

$CD^2$ = 4.63398 cm²

$DE^2$ = 3.08932 cm²

$EF^2$ = 1.54466 cm²

$FG^2$ = 0.41389 cm²

$AB^2 \cdot FG^2$ = 2.55728 cm⁴

AN = 7.91817 cm

GN = 2.76501 cm

GO = 9.44033 cm

$\frac{GO}{GN}$ = 3.41421

$\sqrt{2}$ = 1.41421

$\frac{AB^2}{FG^2} - \frac{BC^2}{FG^2}$ = 1.00000

$BC^2+FG^2$ = 6.17864 cm²

$CD^2+EF^2$ = 6.17864 cm²

$\frac{AB^2}{DE^2}$ = 2.00000

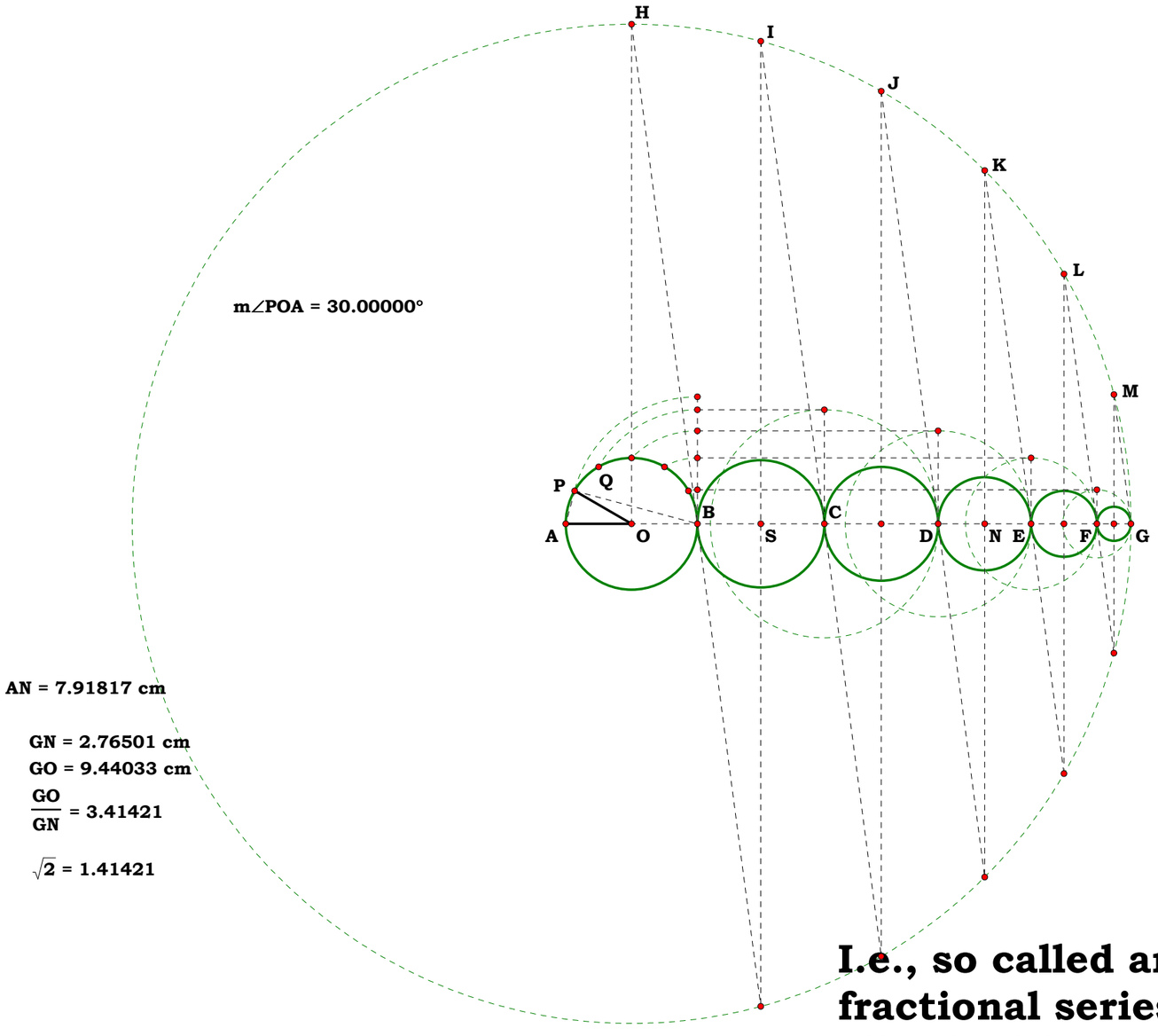
$\frac{AB^2}{FG^2}$ = 14.92820

$\frac{BC^2}{FG^2}$ = 13.92820

$\frac{CD^2}{FG^2}$ = 11.19615

$\frac{DE^2}{FG^2}$ = 7.46410

$\frac{EF^2}{FG^2}$ = 3.73205




m∠POA = 30.00000°

AN = 7.91817 cm

GN = 2.76501 cm
GO = 9.44033 cm
$\frac{GO}{GN}$ = 3.41421

$\sqrt{2}$ = 1.41421

AB = 2.48569 cm
BC = 2.40099 cm
CD = 2.15267 cm
DE = 1.75765 cm

EF = 1.24284 cm
QA = 1.24284 cm
FG = 0.64334 cm
PA = 0.64334 cm

$AB^2$ = 6.17864 $cm^2$
$BC^2$ = 5.76475 $cm^2$
$CD^2$ = 4.63398 $cm^2$
$DE^2$ = 3.08932 $cm^2$
$EF^2$ = 1.54466 $cm^2$
$FG^2$ = 0.41389 $cm^2$

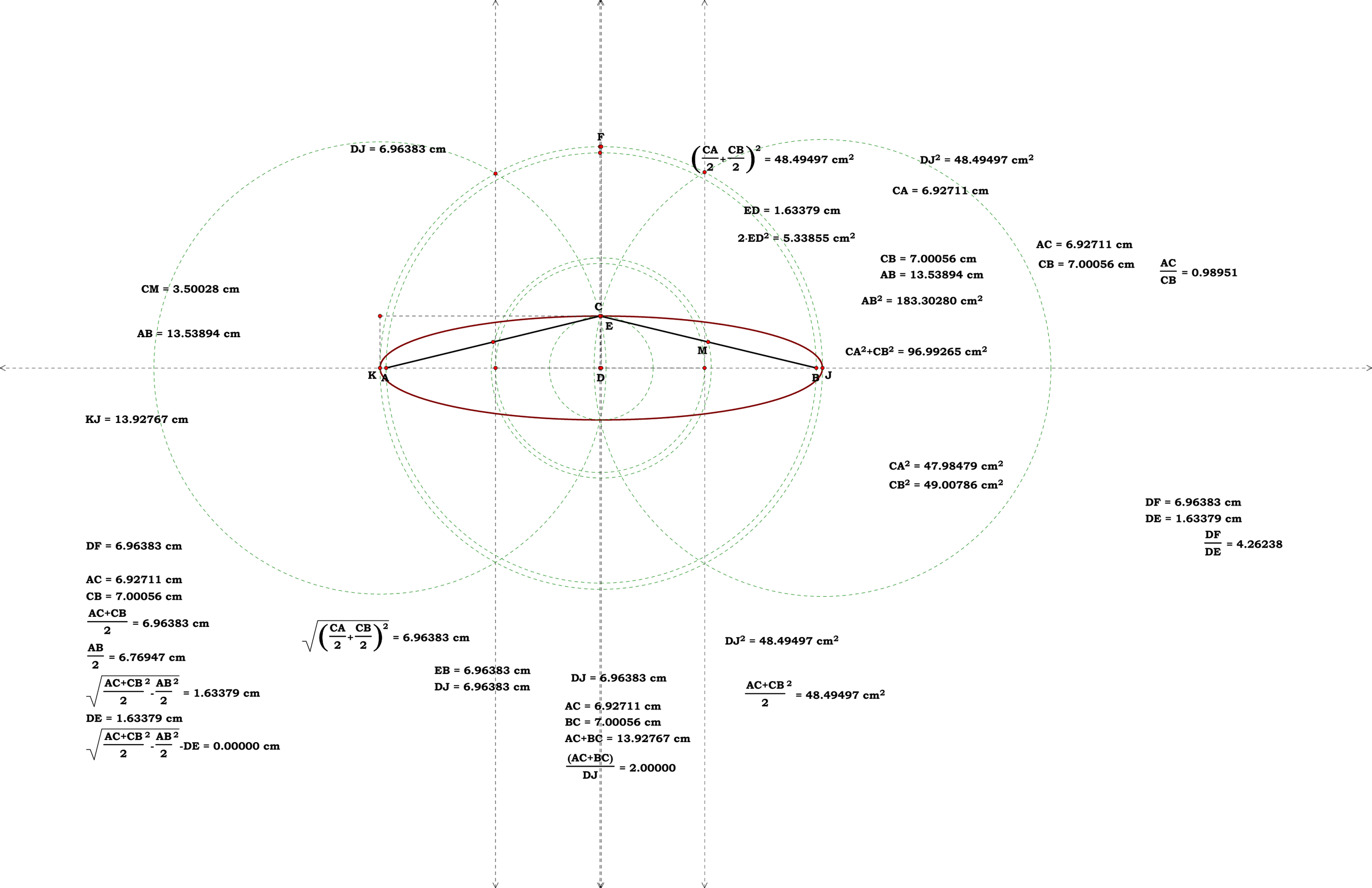
$AB^2 \cdot FG^2$ = 2.55728 $cm^4$

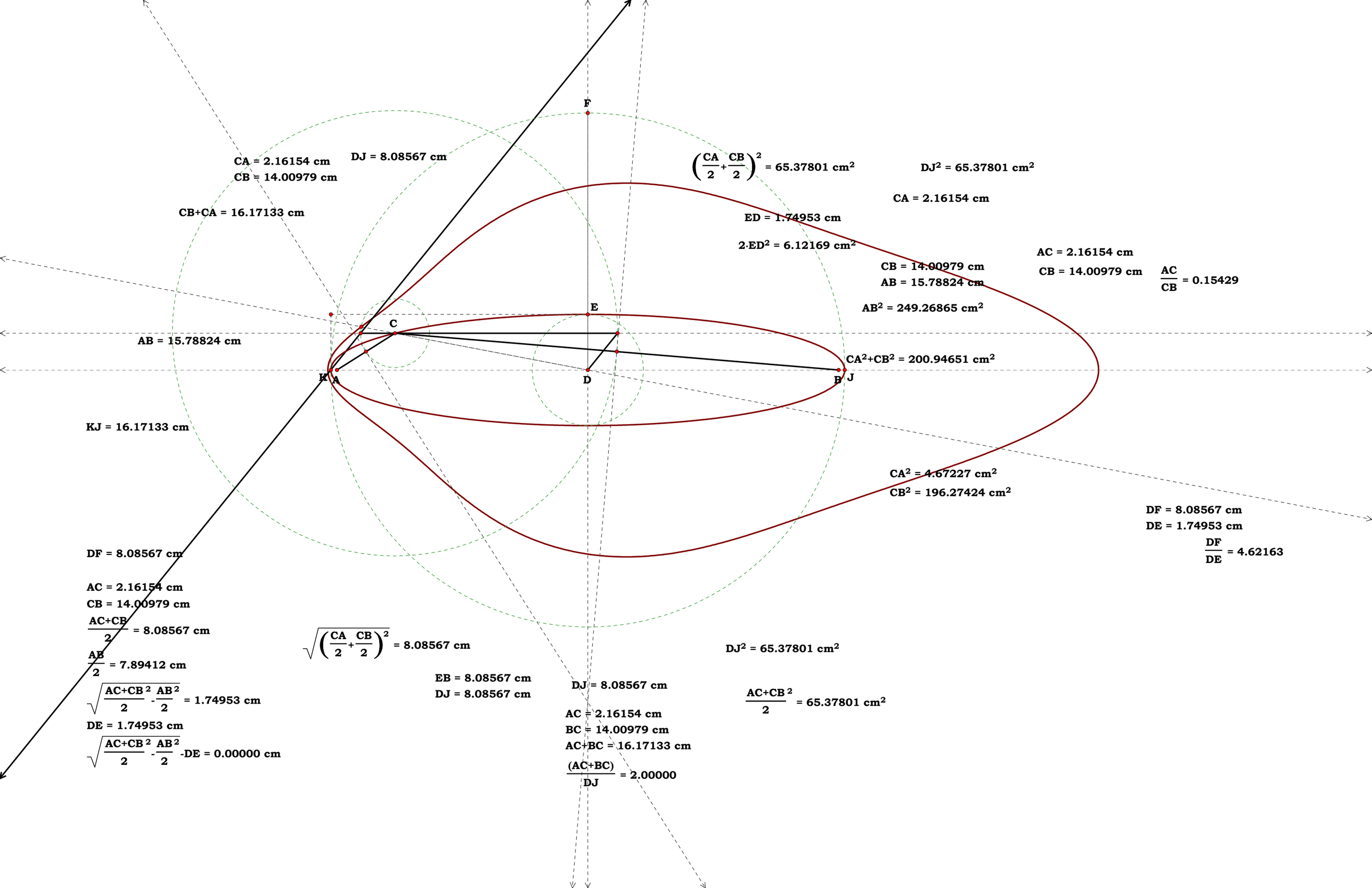
$\frac{AB^2}{FG^2} - \frac{BC^2}{FG^2}$ = 1.00000

$BC^2+FG^2$ = 6.17864 $cm^2$
$CD^2+EF^2$ = 6.17864 $cm^2$
$\frac{AB^2}{DE^2}$ = 2.00000

$\frac{AB^2}{FG^2}$ = 14.92820
$\frac{BC^2}{FG^2}$ = 13.92820
$\frac{CD^2}{FG^2}$ = 11.19615
$\frac{DE^2}{FG^2}$ = 7.46410
$\frac{EF^2}{FG^2}$ = 3.73205

**I.e., so called angular division is also a fractional series. One can call fractional series elliptical functions.**


F
C
E
M
K
A
D
B
J
DJ = 6.96383 cm
$\left(\frac{CA}{2}+\frac{CB}{2}\right)^2$ = 48.49497 cm²
DJ² = 48.49497 cm²
CA = 6.92711 cm
ED = 1.63379 cm
2·ED² = 5.33855 cm²
AC = 6.92711 cm
CB = 7.00056 cm
CB = 7.00056 cm
$\frac{AC}{CB}$ = 0.98951
AB = 13.53894 cm
AB² = 183.30280 cm²
CM = 3.50028 cm
AB = 13.53894 cm
CA²+CB² = 96.99265 cm²
KJ = 13.92767 cm
CA² = 47.98479 cm²
CB² = 49.00786 cm²
DF = 6.96383 cm
DE = 1.63379 cm
$\frac{DF}{DE}$ = 4.26238
DF = 6.96383 cm
AC = 6.92711 cm
CB = 7.00056 cm
$\frac{AC+CB}{2}$ = 6.96383 cm
$\frac{AB}{2}$ = 6.76947 cm
$\sqrt{\frac{AC+CB^2}{2}-\frac{AB^2}{2}}$ = 1.63379 cm
DE = 1.63379 cm
$\sqrt{\frac{AC+CB^2}{2}-\frac{AB^2}{2}}$ -DE = 0.00000 cm
$\sqrt{\left(\frac{CA}{2}+\frac{CB}{2}\right)^2}$ = 6.96383 cm
EB = 6.96383 cm
DJ = 6.96383 cm
DJ = 6.96383 cm
AC = 6.92711 cm
BC = 7.00056 cm
AC+BC = 13.92767 cm
$\frac{(AC+BC)}{DJ}$ = 2.00000
DJ² = 48.49497 cm²
$\frac{AC+CB^2}{2}$ = 48.49497 cm²


F
E
C
K
A
D
B
J
CA = 2.16154 cm
CB = 14.00979 cm
CB+CA = 16.17133 cm
DJ = 8.08567 cm
$\left(\frac{CA}{2}+\frac{CB}{2}\right)^2$ = 65.37801 cm²
DJ² = 65.37801 cm²
CA = 2.16154 cm
ED = 1.74953 cm
2·ED² = 6.12169 cm²
AC = 2.16154 cm
CB = 14.00979 cm
AB = 15.78824 cm
CB = 14.00979 cm
$\frac{AC}{CB}$ = 0.15429
AB² = 249.26865 cm²
AB = 15.78824 cm
CA²+CB² = 200.94651 cm²
KJ = 16.17133 cm
CA² = 4.67227 cm²
CB² = 196.27424 cm²
DF = 8.08567 cm
DE = 1.74953 cm
$\frac{DF}{DE}$ = 4.62163
DF = 8.08567 cm
AC = 2.16154 cm
CB = 14.00979 cm
$\frac{AC+CB}{2}$ = 8.08567 cm
$\frac{AB}{2}$ = 7.89412 cm
$\sqrt{\frac{AC+CB^2}{2}-\frac{AB^2}{2}}$ = 1.74953 cm
DE = 1.74953 cm
$\sqrt{\frac{AC+CB^2}{2}-\frac{AB^2}{2}}$ -DE = 0.00000 cm
$\sqrt{\left(\frac{CA}{2}+\frac{CB}{2}\right)^2}$ = 8.08567 cm
EB = 8.08567 cm
DJ = 8.08567 cm
DJ = 8.08567 cm
DJ² = 65.37801 cm²
AC = 2.16154 cm
BC = 14.00979 cm
AC+BC = 16.17133 cm
$\frac{(AC+BC)}{DJ}$ = 2.00000
$\frac{AC+CB^2}{2}$ = 65.37801 cm²

## Index is Base 0

**Indexs:** $I_{ndx} = 1$ $C_{indx} = 13.00000$

**Number of div. by difference at an index.**

$$\frac{(I_{ndx} \cdot N_2 - N_1 \cdot N_3) \cdot ((I_{ndx} \cdot N_2 + N_2) - N_1 \cdot N_3)}{N_1 \cdot N_2} = 105.00000$$

**Total number of fractions.**

$$\frac{N_1 \cdot (N_3 - 1)}{N_2} = 14.00000 \qquad \frac{N_1 \cdot N_3 - N_1}{N_2} = 14.00000$$

**Fraction at Index:**

**Num:** $N_1 \cdot N_3 - I_{ndx} \cdot N_2 = 30.00000$

**Den:** $N_1 = 4.00000$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} = 7.50000$$

**Fraction at Compliment:**

$$\frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 1.50000$$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} + \frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 9.00000$$

N[1] -> 0 … N[1] -> 11

N[2] -> 0 … N[2] -> 11

$N_1 = 4.00000$

$N_2 = 2.00000$

$N_3 = 8.00000$

Present 2 Actions

$$\frac{N_3}{N_3 + 1} = 0.88889$$

N[2] -> 0 … N[2] -> 11

$$\frac{N_1 \cdot N_3 - N_1 - I_{ndx} \cdot N_2}{N_2} = 13.00000$$

$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{H} = 4.00000$

$\frac{N_3}{A} = 7.50000$ $\frac{N_3}{I} = 3.50000$

$\frac{N_3}{B} = 7.00000$ $\frac{N_3}{J} = 3.00000$

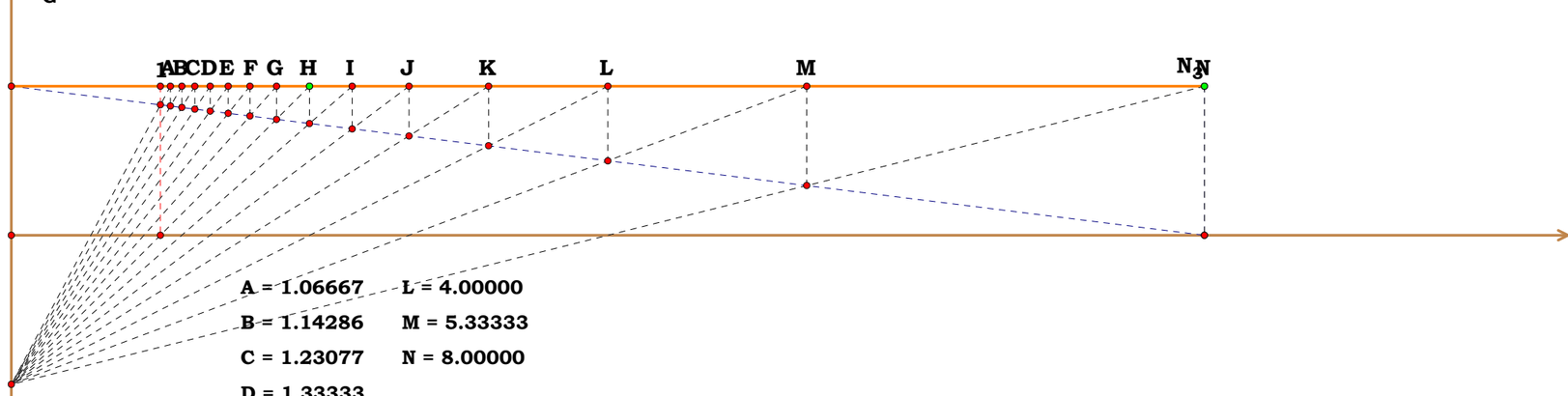
$\frac{N_3}{C} = 6.50000$ $\frac{N_3}{K} = 2.50000$

$\frac{N_3}{D} = 6.00000$ $\frac{N_3}{L} = 2.00000$

$\frac{N_3}{E} = 5.50000$ $\frac{N_3}{M} = 1.50000$

$\frac{N_3}{F} = 5.00000$ $\frac{N_3}{N} = 1.00000$

$\frac{N_3}{G} = 4.50000$




A = 1.06667 L = 4.00000

B = 1.14286 M = 5.33333

C = 1.23077 N = 8.00000

D = 1.33333

E = 1.45455

F = 1.60000

G = 1.77778

H = 2.00000

I = 2.28571

J = 2.66667

K = 3.20000

**Indexs:** Index = 0 $C_{indx} = 8.00$

## Number of div. by difference at an index.

$$\frac{(Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)\cdot((N_3+Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)-1)}{N_1\cdot N_2\cdot(N_3-1)} = 81.14286$$

**len of frac.** $\frac{N_1\cdot N_2\cdot N_3}{N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1)} = 1.00000$

## Total number of fractions.

$N_1\cdot N_2 = 8.00000$

## Fraction at Index:

**Num:** $N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1) = 64.00000$

**Den:** $N_1\cdot N_2 = 8.00000$

$$\frac{(N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1))}{(N_1\cdot N_2)} = 8.00000$$

## Fraction at Compliment:

$$\frac{N_1\cdot N_2\cdot N_3-C_{indx}\cdot(N_3-1)}{N_1\cdot N_2} = 1.00000$$

$$\frac{(N_3-C_{indx}\cdot N_3-Index\cdot N_3)+2\cdot N_1\cdot N_2\cdot N_3}{N_1\cdot N_2} = 9.00000$$

N[1] -> 0
N[1] -> 1
N[1] -> 2
N[1] -> 3
N[1] -> 4
N[1] -> 5
N[1] -> 6
N[1] -> 7
N[1] -> 8
N[1] -> 9
N[1] -> 10
N[1] -> 11

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$N_1 = 4.00000$
$N_2 = 2.00000$
$N_3 = 8.00000$

N[3] -> 0
N[3] -> 1
N[3] -> 2
N[3] -> 3
N[3] -> 4
N[3] -> 5
N[3] -> 6
N[3] -> 7
N[3] -> 8
N[3] -> 9
N[3] -> 10
N[3] -> 11

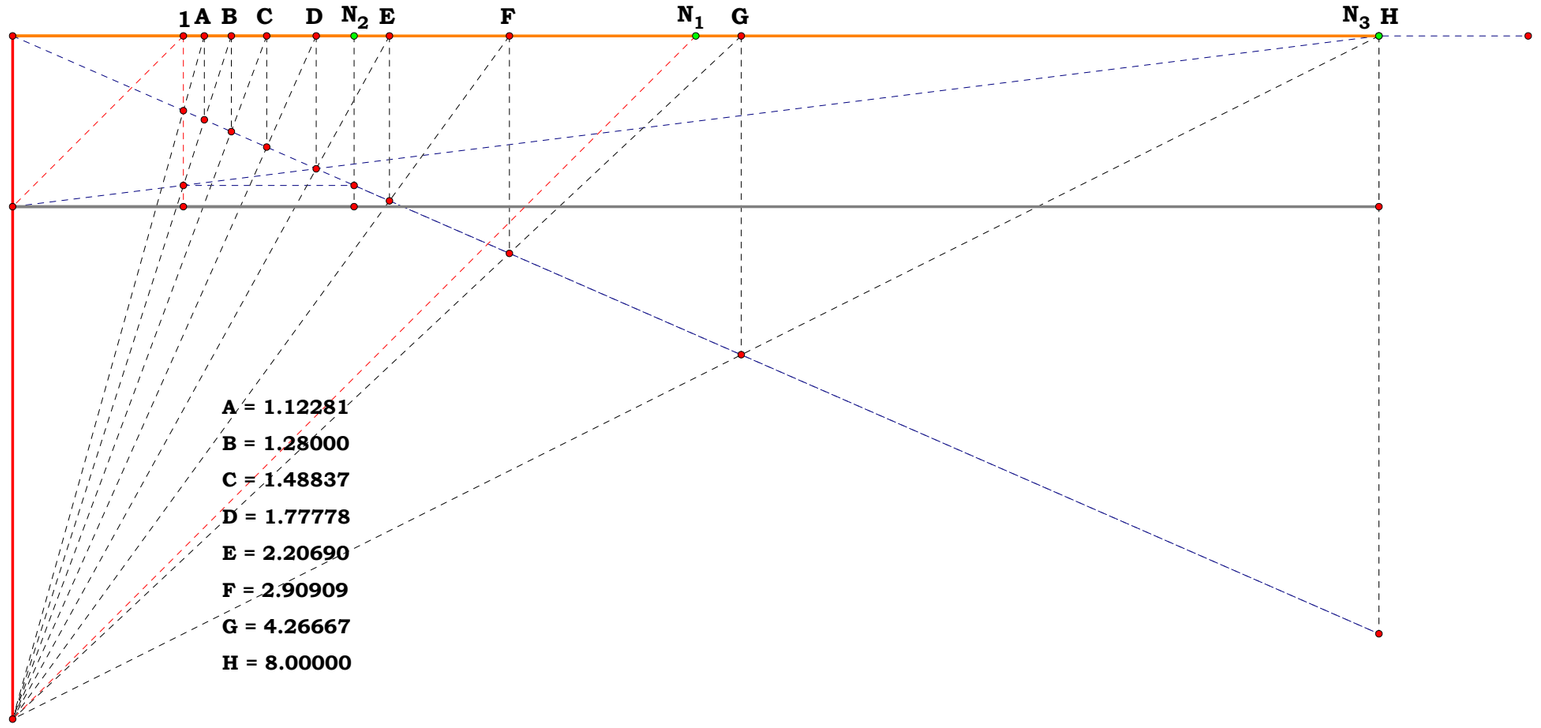
$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{E} = 3.62500$

$\frac{N_3}{A} = 7.12500$ $\frac{N_3}{F} = 2.75000$

$\frac{N_3}{B} = 6.25000$ $\frac{N_3}{G} = 1.87500$

$\frac{N_3}{C} = 5.37500$ $\frac{N_3}{H} = 1.00000$

$\frac{N_3}{D} = 4.50000$




A = 1.12281
B = 1.28000
C = 1.48837
D = 1.77778
E = 2.20690
F = 2.90909
G = 4.26667
H = 8.00000

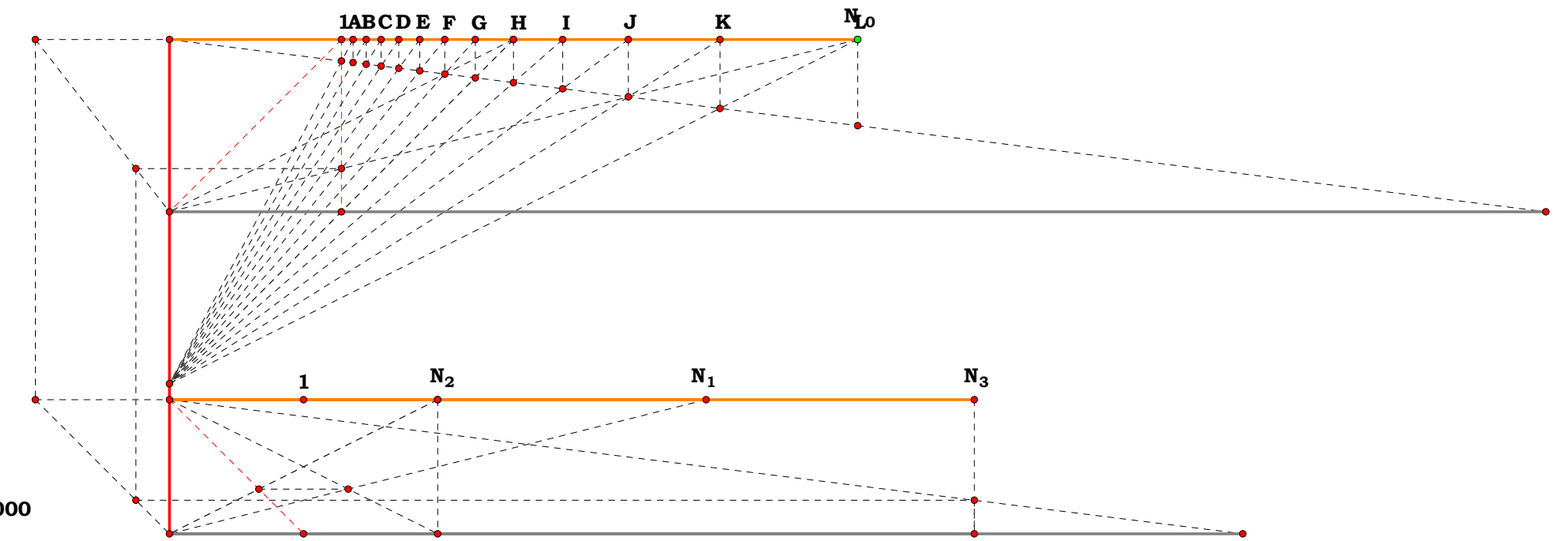
$$i_{dx} = 1$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)\cdot(((N_2-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)-N_0\cdot N_1\cdot N_3)}{N_1\cdot N_3\cdot(((N_2+i_{dx}\cdot(N_2-N_0\cdot N_2))-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)} = 60.00000$$

$$\frac{N_0\cdot N_1\cdot N_3}{i_{dx}\cdot(N_2-N_2\cdot N_0)+N_0\cdot N_1\cdot N_3} = 1.06667$$

$$\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 1.25000$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)}+\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 5.00000$$




## Total number of fractions.

$$\frac{N_1\cdot N_3}{N_2} = 12.00000$$

## Fraction at Index:

**Num:** $i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3 = 90.00000$

**Den:** $N_1\cdot N_3 = 24.00000$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)} = 3.75000$$

$N_0 = 4.00000$ $N_2 = 2.00000$

$N_1 = 4.00000$ $N_3 = 6.00000$ $N_1\cdot N_3 = 24.00000$

| | | | |
|---|---|---|---|
| N[1] -> 0 | N[2] -> 0 | N[3] -> 0 | N [0] -> 0 |
| N[1] -> 1 | N[2] -> 1 | N[3] -> 1 | N [0] -> 1 |
| N[1] -> 2 | N[2] -> 2 | N[3] -> 2 | N [0] -> 2 |
| N[1] -> 3 | N[2] -> 3 | N[3] -> 3 | N [0] -> 3 |
| N[1] -> 4 | N[2] -> 4 | N[3] -> 4 | N [0] -> 4 |
| N[1] -> 5 | N[2] -> 5 | N[3] -> 5 | N [0] -> 5 |
| N[1] -> 6 | N[2] -> 6 | N[3] -> 6 | N [0] -> 6 |
| N[1] -> 7 | N[2] -> 7 | N[3] -> 7 | N [0] -> 7 |
| N[1] -> 8 | N[2] -> 8 | N[3] -> 8 | N [0] -> 8 |
| N[1] -> 9 | N[2] -> 9 | N[3] -> 9 | N [0] -> 9 |
| N[1] -> 10 | N[2] -> 10 | N[3] -> 10 | N [0]-> 10 |
| N[1] -> 11 | N[2] -> 11 | N[3] -> 11 | N [0] -> 11 |

A = 1.06667
B = 1.14286
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667
K = 3.20000
L = 4.00000

$\frac{N_0}{A} = 3.75000$ $\frac{N_0}{G} = 2.25000$

$\frac{N_0}{B} = 3.50000$ $\frac{N_0}{H} = 2.00000$

$\frac{N_0}{C} = 3.25000$ $\frac{N_0}{I} = 1.75000$

$\frac{N_0}{D} = 3.00000$ $\frac{N_0}{J} = 1.50000$

$\frac{N_0}{E} = 2.75000$ $\frac{N_0}{K} = 1.25000$

$\frac{N_0}{F} = 2.50000$ $\frac{N_0}{L} = 1.00000$

N -> 0
N -> 1
N -> 2
N -> 3
N -> 4
N -> 5
N -> 6
N -> 7
N -> 8
N -> 9
N -> 10
N -> 11

N = 2.00000

Show Objects

$In_{dx}$ = 1

1 A B C D N E

$\frac{N^3}{(N^3-N)+1}$ = 1.14286

$\frac{N^3}{N^3-In_{dx}\cdot(N-1)}$ = 1.14286

$N^3$ = 8.00000

$N^3-In_{dx}\cdot(N-1)$ = 7.00000

$N^2$ = 4.00000

$x_A$ = 1.14286

$x_B$ = 1.33333

$x_C$ = 1.60000

$x_D$ = 2.00000

$x_E$ = 2.66667

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$N_1 = 5.00000$

$N_2 = 0.00000$

$n_a = 7.00000$

$n_b = 5.00000$

$r_{da} = 1.40000$

$\frac{n_a}{n_b} = 1.40000$

$N_1 \cdot \frac{n_a}{n_b} = 7.00000$

$n_c = 3.00000$

$n_d = 4.00000$

$r_{db} = 0.75000$

$\frac{n_c}{n_d} = 0.75000$

$\frac{n_c}{n_d} \cdot N_1 = 3.75000$

$In_{dx} = 0$

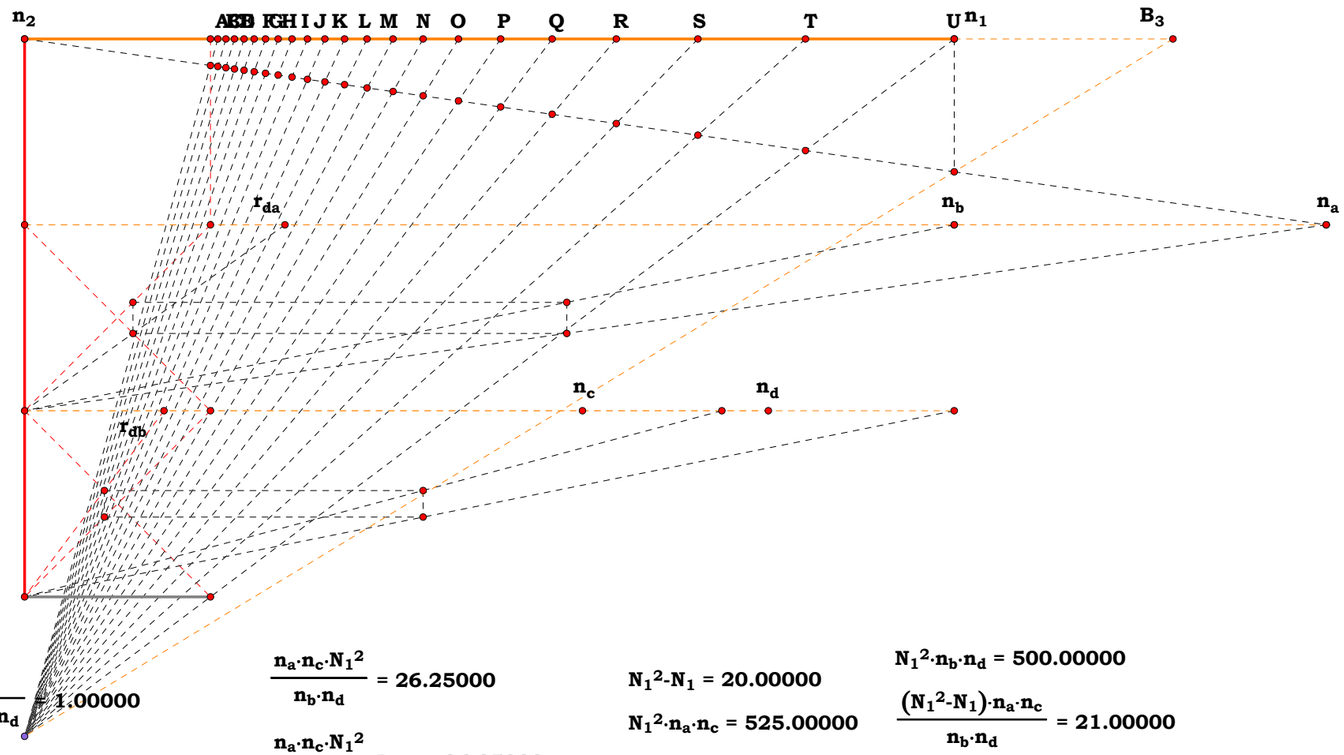
$\frac{26.25000}{26.25000} = 1.00000$

$B_3 = 6.17647$

$\frac{N_1}{B_3 - N_1} + 1 = 5.25000$

$\frac{7}{3.75} = 1.86667$

A = 1.03960
B = 1.08247
C = 1.12903
D = 1.17978
E = 1.23529
F = 1.29630
G = 1.36364
H = 1.43836
I = 1.52174
J = 1.61538
K = 1.72131
L = 1.84211
M = 1.98113
N = 2.14286
O = 2.33333
P = 2.56098
Q = 2.83784
R = 3.18182
S = 3.62069
T = 4.20000
U = 5.00000




$\frac{N_1^2 \cdot n_a \cdot n_c}{N_1^2 \cdot n_a \cdot n_c - In_{dx} \cdot n_b \cdot n_d} = 1.00000$

$\frac{n_a \cdot n_c \cdot N_1^2}{n_b \cdot n_d} = 26.25000$

$\frac{n_a \cdot n_c \cdot N_1^2}{n_b \cdot n_d} - In_{dx} = 26.25000$

$N_1^2 - N_1 = 20.00000$

$N_1^2 \cdot n_a \cdot n_c = 525.00000$

$N_1^2 \cdot n_b \cdot n_d = 500.00000$

$\frac{(N_1^2 - N_1) \cdot n_a \cdot n_c}{n_b \cdot n_d} = 21.00000$

Circle
Hide Lines
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8
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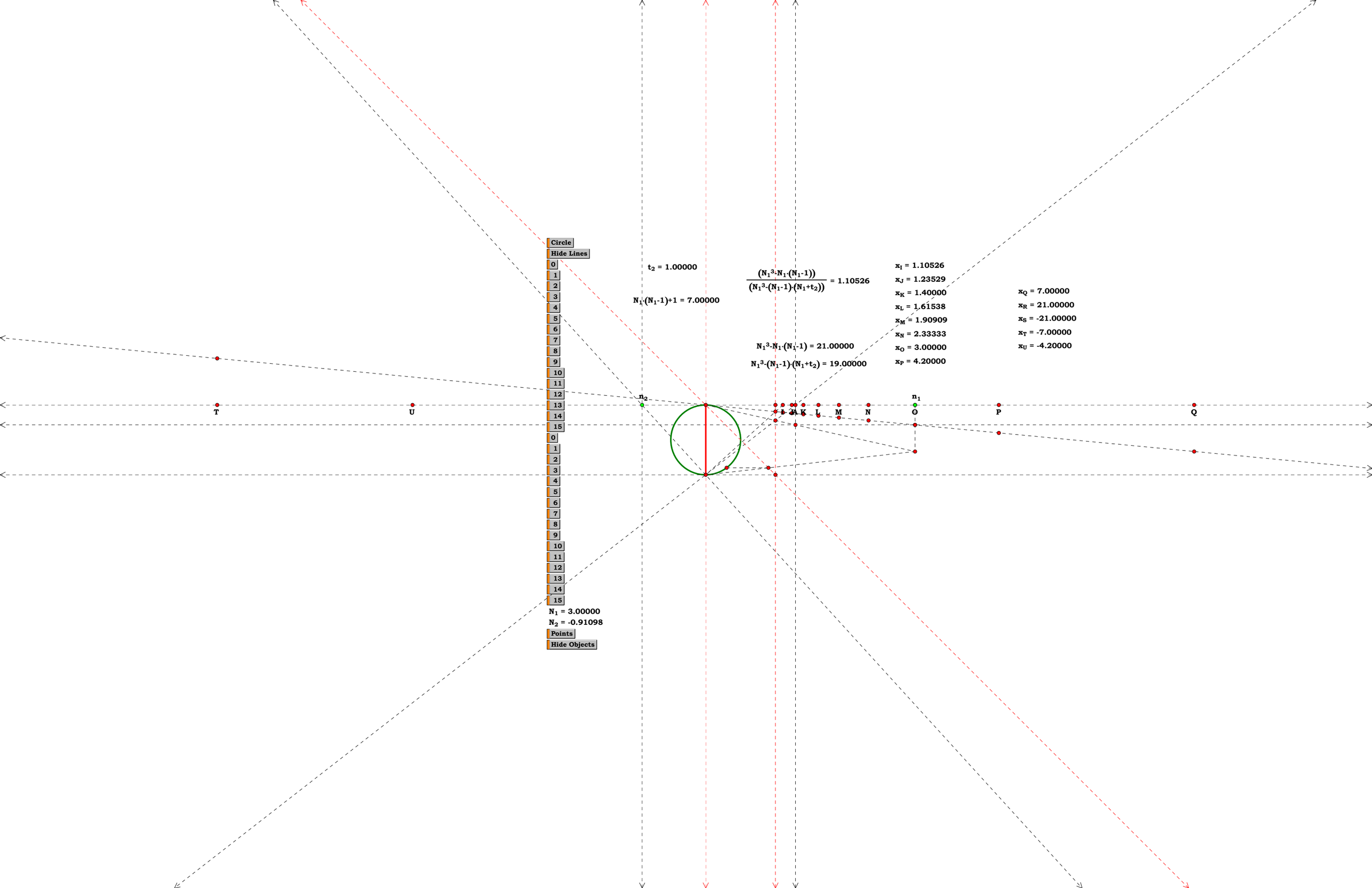
10
11
12
13
14
15
$N_1$ = 3.00000
$N_2$ = -0.91098
Points
Hide Objects

$n_2$
$n_1$
A
B
C

$x_A$ = 1.28571
$x_B$ = 1.80000
$x_C$ = 3.00000

$$\frac{N_1^2}{(N_1^2-N_1)+1} = 1.28571 \qquad t_1 = 2$$

$$\frac{N_1^2}{N_1^2-t_1\cdot(N_1-1)} = 1.80000$$


Circle
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8
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10
11
12
13
14
15
N1 = 3.00000
N2 = -0.91098
Points
Hide Objects
t2 = 1.00000
N1·(N1-1)+1 = 7.00000
(N1³-N1·(N1-1)) / (N1³-(N1-1)·(N1+t2)) = 1.10526
N1³-N1·(N1-1) = 21.00000
N1³-(N1-1)·(N1+t2) = 19.00000
xI = 1.10526
xJ = 1.23529
xK = 1.40000
xL = 1.61538
xM = 1.90909
xN = 2.33333
xO = 3.00000
xP = 4.20000
xQ = 7.00000
xR = 21.00000
xS = -21.00000
xT = -7.00000
xU = -4.20000
n2
n1
T
U
I
J
K
L
M
N
O
P
Q

## Index is Base 0

**Indexs:** $I_{ndx} = 1$ $C_{indx} = 13.00000$

**Number of div. by difference at an index.**

$$\frac{(I_{ndx} \cdot N_2 - N_1 \cdot N_3) \cdot ((I_{ndx} \cdot N_2 + N_2) - N_1 \cdot N_3)}{N_1 \cdot N_2} = 105.00000$$

**Total number of fractions.**

$$\frac{N_1 \cdot (N_3 - 1)}{N_2} = 14.00000 \qquad \frac{N_1 \cdot N_3 - N_1}{N_2} = 14.00000$$

**Fraction at Index:**

**Num:** $N_1 \cdot N_3 - I_{ndx} \cdot N_2 = 30.00000$

**Den:** $N_1 = 4.00000$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} = 7.50000$$

**Fraction at Compliment:**

$$\frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 1.50000$$

$$\frac{(N_1 \cdot N_3 - I_{ndx} \cdot N_2)}{N_1} + \frac{N_1 + N_2 \cdot I_{ndx}}{N_1} = 9.00000$$

N[1] -> 0 … N[1] -> 11

N[2] -> 0 … N[2] -> 11

$N_1 = 4.00000$

$N_2 = 2.00000$

$N_3 = 8.00000$

Present 2 Actions

$$\frac{N_3}{N_3 + 1} = 0.88889$$

N[2] -> 0 … N[2] -> 11

$$\frac{N_1 \cdot N_3 - N_1 - I_{ndx} \cdot N_2}{N_2} = 13.00000$$

$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{H} = 4.00000$

$\frac{N_3}{A} = 7.50000$ $\frac{N_3}{I} = 3.50000$

$\frac{N_3}{B} = 7.00000$ $\frac{N_3}{J} = 3.00000$

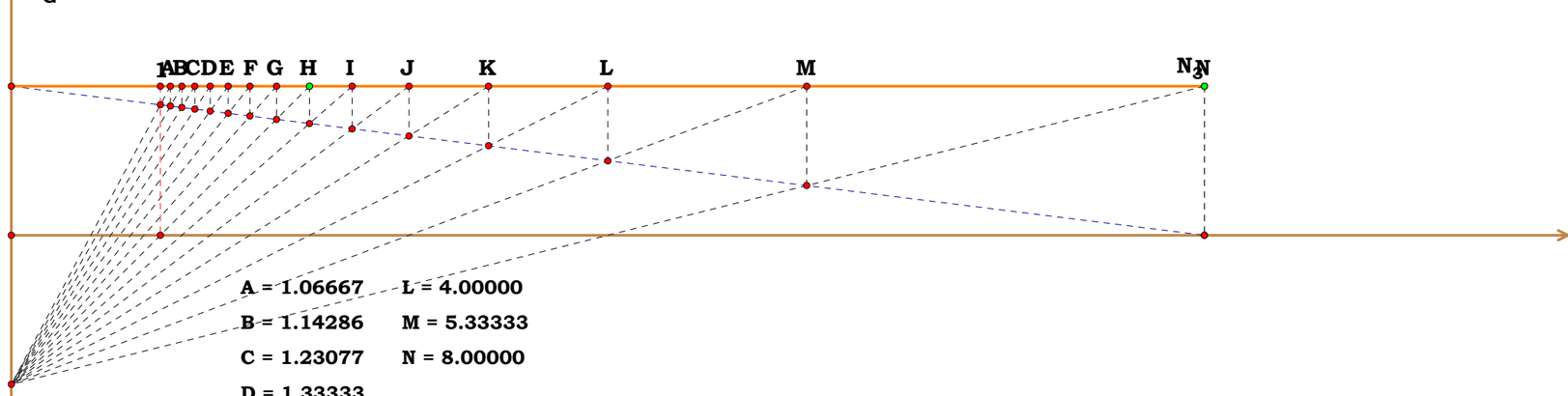
$\frac{N_3}{C} = 6.50000$ $\frac{N_3}{K} = 2.50000$

$\frac{N_3}{D} = 6.00000$ $\frac{N_3}{L} = 2.00000$

$\frac{N_3}{E} = 5.50000$ $\frac{N_3}{M} = 1.50000$

$\frac{N_3}{F} = 5.00000$ $\frac{N_3}{N} = 1.00000$

$\frac{N_3}{G} = 4.50000$




A = 1.06667 L = 4.00000

B = 1.14286 M = 5.33333

C = 1.23077 N = 8.00000

D = 1.33333

E = 1.45455

F = 1.60000

G = 1.77778

H = 2.00000

I = 2.28571

J = 2.66667

K = 3.20000

**Indexs:** Index = 0 $C_{indx} = 8.00$

## Number of div. by difference at an index.

$$\frac{(Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)\cdot((N_3+Index\cdot(1-N_3)+N_1\cdot N_2\cdot N_3)-1)}{N_1\cdot N_2\cdot(N_3-1)} = 81.14286$$

**len of frac.** $\frac{N_1\cdot N_2\cdot N_3}{N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1)} = 1.00000$

## Total number of fractions.

$N_1\cdot N_2 = 8.00000$

## Fraction at Index:

**Num:** $N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1) = 64.00000$

**Den:** $N_1\cdot N_2 = 8.00000$

$$\frac{(N_1\cdot N_2\cdot N_3-Index\cdot(N_3-1))}{(N_1\cdot N_2)} = 8.00000$$

## Fraction at Compliment:

$$\frac{N_1\cdot N_2\cdot N_3-C_{indx}\cdot(N_3-1)}{N_1\cdot N_2} = 1.00000$$

$$\frac{(N_3-C_{indx}\cdot N_3-Index\cdot N_3)+2\cdot N_1\cdot N_2\cdot N_3}{N_1\cdot N_2} = 9.00000$$

N[1] -> 0
N[1] -> 1
N[1] -> 2
N[1] -> 3
N[1] -> 4
N[1] -> 5
N[1] -> 6
N[1] -> 7
N[1] -> 8
N[1] -> 9
N[1] -> 10
N[1] -> 11

N[2] -> 0
N[2] -> 1
N[2] -> 2
N[2] -> 3
N[2] -> 4
N[2] -> 5
N[2] -> 6
N[2] -> 7
N[2] -> 8
N[2] -> 9
N[2] -> 10
N[2] -> 11

$N_1 = 4.00000$
$N_2 = 2.00000$
$N_3 = 8.00000$

N[3] -> 0
N[3] -> 1
N[3] -> 2
N[3] -> 3
N[3] -> 4
N[3] -> 5
N[3] -> 6
N[3] -> 7
N[3] -> 8
N[3] -> 9
N[3] -> 10
N[3] -> 11

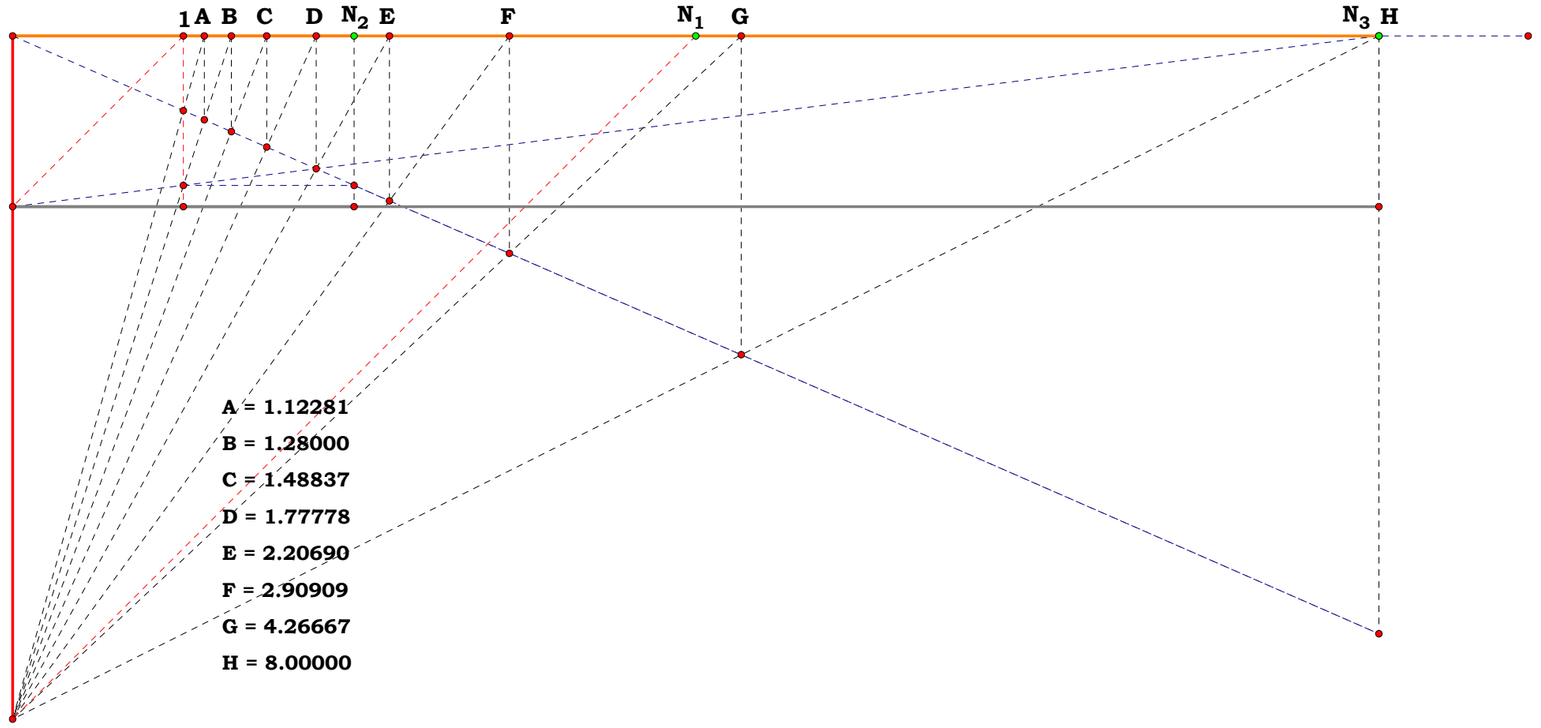
$\frac{N_3}{1} = 8.00000$ $\frac{N_3}{E} = 3.62500$

$\frac{N_3}{A} = 7.12500$ $\frac{N_3}{F} = 2.75000$

$\frac{N_3}{B} = 6.25000$ $\frac{N_3}{G} = 1.87500$

$\frac{N_3}{C} = 5.37500$ $\frac{N_3}{H} = 1.00000$

$\frac{N_3}{D} = 4.50000$




A = 1.12281
B = 1.28000
C = 1.48837
D = 1.77778
E = 2.20690
F = 2.90909
G = 4.26667
H = 8.00000

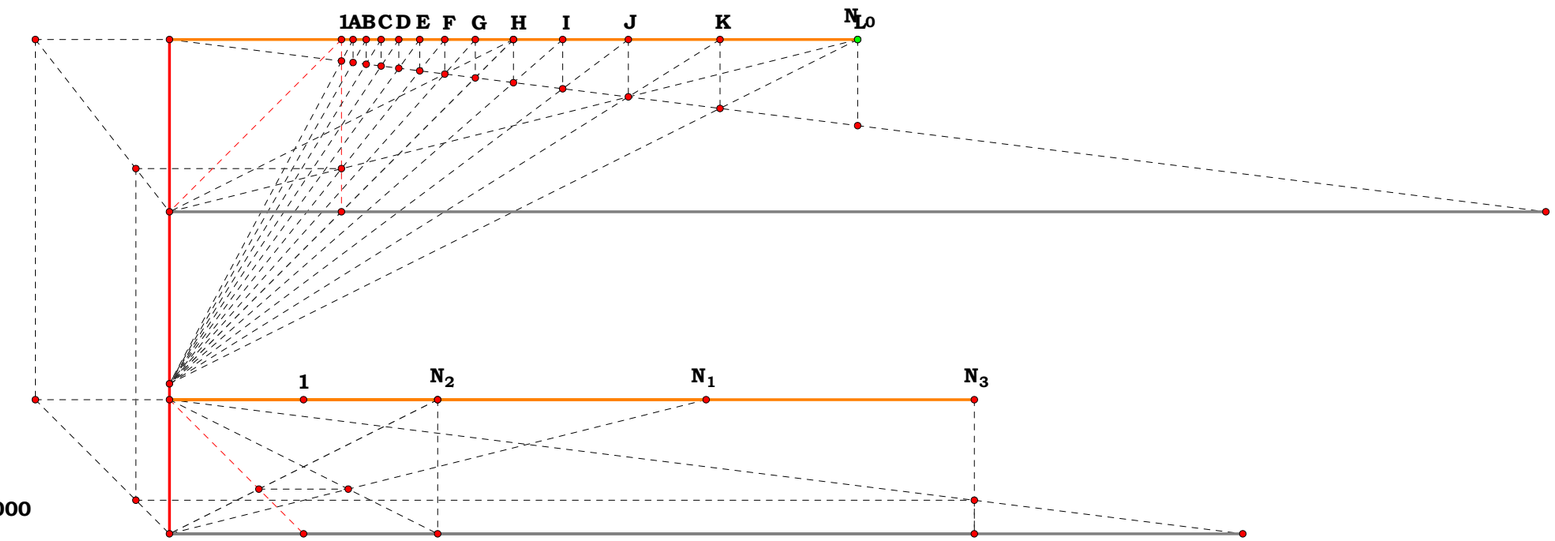
$$i_{dx} = 1$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)\cdot(((N_2-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)-N_0\cdot N_1\cdot N_3)}{N_1\cdot N_3\cdot(((N_2+i_{dx}\cdot(N_2-N_0\cdot N_2))-i_{dx}\cdot N_2-N_0\cdot N_2)+i_{dx}\cdot N_0\cdot N_2)} = 60.00000$$

$$\frac{N_0\cdot N_1\cdot N_3}{i_{dx}\cdot(N_2-N_2\cdot N_0)+N_0\cdot N_1\cdot N_3} = 1.06667$$

$$\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 1.25000$$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)}+\frac{N_1\cdot N_3-i_{dx}\cdot N_2\cdot(1-N_0)}{N_1\cdot N_3} = 5.00000$$




## Total number of fractions.

$$\frac{N_1\cdot N_3}{N_2} = 12.00000$$

## Fraction at Index:

**Num:** $i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3 = 90.00000$

**Den:** $N_1\cdot N_3 = 24.00000$

$$\frac{(i_{dx}\cdot(N_2-N_0\cdot N_2)+N_0\cdot N_1\cdot N_3)}{(N_1\cdot N_3)} = 3.75000$$

$N_0 = 4.00000$ $N_2 = 2.00000$

$N_1 = 4.00000$ $N_3 = 6.00000$ $N_1\cdot N_3 = 24.00000$

| | | | |
|---|---|---|---|
| N[1] -> 0 | N[2] -> 0 | N[3] -> 0 | N [0] -> 0 |
| N[1] -> 1 | N[2] -> 1 | N[3] -> 1 | N [0] -> 1 |
| N[1] -> 2 | N[2] -> 2 | N[3] -> 2 | N [0] -> 2 |
| N[1] -> 3 | N[2] -> 3 | N[3] -> 3 | N [0] -> 3 |
| N[1] -> 4 | N[2] -> 4 | N[3] -> 4 | N [0] -> 4 |
| N[1] -> 5 | N[2] -> 5 | N[3] -> 5 | N [0] -> 5 |
| N[1] -> 6 | N[2] -> 6 | N[3] -> 6 | N [0] -> 6 |
| N[1] -> 7 | N[2] -> 7 | N[3] -> 7 | N [0] -> 7 |
| N[1] -> 8 | N[2] -> 8 | N[3] -> 8 | N [0] -> 8 |
| N[1] -> 9 | N[2] -> 9 | N[3] -> 9 | N [0] -> 9 |
| N[1] -> 10 | N[2] -> 10 | N[3] -> 10 | N [0]-> 10 |
| N[1] -> 11 | N[2] -> 11 | N[3] -> 11 | N [0] -> 11 |

A = 1.06667
B = 1.14286
C = 1.23077
D = 1.33333
E = 1.45455
F = 1.60000
G = 1.77778
H = 2.00000
I = 2.28571
J = 2.66667
K = 3.20000
L = 4.00000

$\frac{N_0}{A} = 3.75000$ $\frac{N_0}{G} = 2.25000$

$\frac{N_0}{B} = 3.50000$ $\frac{N_0}{H} = 2.00000$

$\frac{N_0}{C} = 3.25000$ $\frac{N_0}{I} = 1.75000$

$\frac{N_0}{D} = 3.00000$ $\frac{N_0}{J} = 1.50000$

$\frac{N_0}{E} = 2.75000$ $\frac{N_0}{K} = 1.25000$

$\frac{N_0}{F} = 2.50000$ $\frac{N_0}{L} = 1.00000$

N -> 0
N -> 1
N -> 2
N -> 3
N -> 4
N -> 5
N -> 6
N -> 7
N -> 8
N -> 9
N -> 10
N -> 11

N = 2.00000

Show Objects

$In_{dx}$ = 1

1 A B C D N E

$\frac{N^3}{(N^3-N)+1}$ = 1.14286

$\frac{N^3}{N^3-In_{dx}\cdot(N-1)}$ = 1.14286

$N^3$ = 8.00000

$N^3-In_{dx}\cdot(N-1)$ = 7.00000

$N^2$ = 4.00000

$x_A$ = 1.14286

$x_B$ = 1.33333

$x_C$ = 1.60000

$x_D$ = 2.00000

$x_E$ = 2.66667

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle | Points
Show Lines | Hide Objects

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$N_1 = 5.00000$

$N_2 = 0.00000$

$n_a = 7.00000$

$n_b = 5.00000$

$r_{da} = 1.40000$

$\frac{n_a}{n_b} = 1.40000$

$N_1 \cdot \frac{n_a}{n_b} = 7.00000$

$n_c = 3.00000$

$n_d = 4.00000$

$r_{db} = 0.75000$

$\frac{n_c}{n_d} = 0.75000$

$\frac{n_c}{n_d} \cdot N_1 = 3.75000$

$In_{dx} = 0$

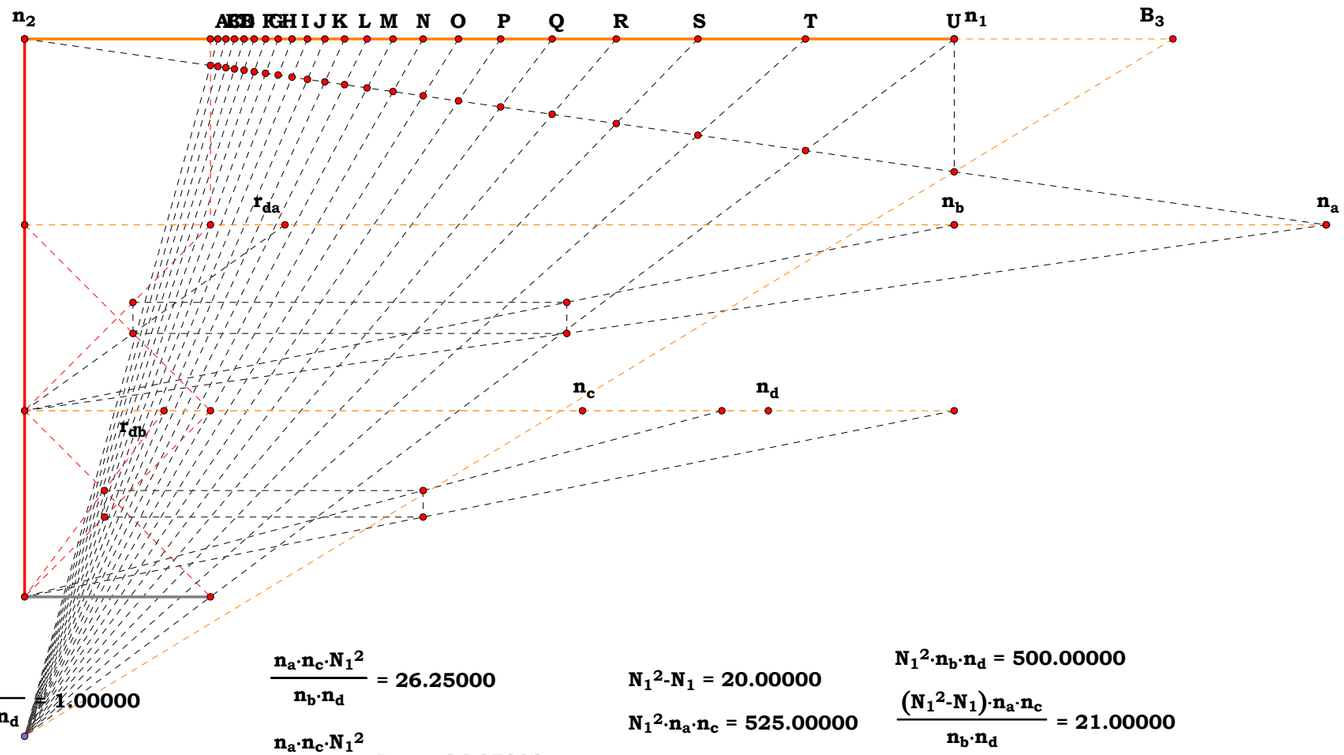
$\frac{26.25000}{26.25000} = 1.00000$

$B_3 = 6.17647$

$\frac{N_1}{B_3 - N_1} + 1 = 5.25000$

$\frac{7}{3.75} = 1.86667$

A = 1.03960
B = 1.08247
C = 1.12903
D = 1.17978
E = 1.23529
F = 1.29630
G = 1.36364
H = 1.43836
I = 1.52174
J = 1.61538
K = 1.72131
L = 1.84211
M = 1.98113
N = 2.14286
O = 2.33333
P = 2.56098
Q = 2.83784
R = 3.18182
S = 3.62069
T = 4.20000
U = 5.00000




$\frac{N_1^2 \cdot n_a \cdot n_c}{N_1^2 \cdot n_a \cdot n_c - In_{dx} \cdot n_b \cdot n_d} = 1.00000$

$\frac{n_a \cdot n_c \cdot N_1^2}{n_b \cdot n_d} = 26.25000$

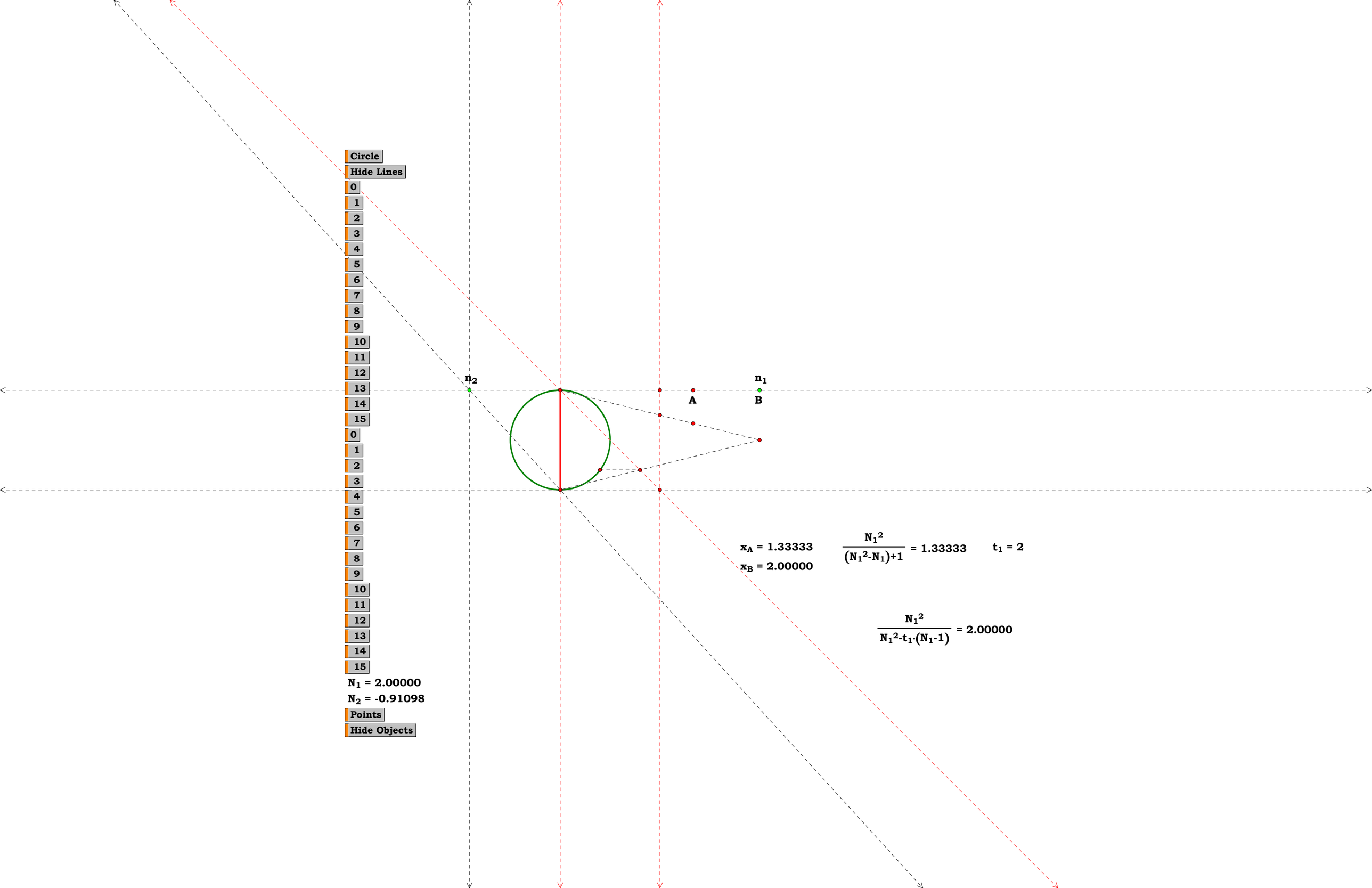
$\frac{n_a \cdot n_c \cdot N_1^2}{n_b \cdot n_d} - In_{dx} = 26.25000$

$N_1^2 - N_1 = 20.00000$

$N_1^2 \cdot n_a \cdot n_c = 525.00000$

$N_1^2 \cdot n_b \cdot n_d = 500.00000$

$\frac{(N_1^2 - N_1) \cdot n_a \cdot n_c}{n_b \cdot n_d} = 21.00000$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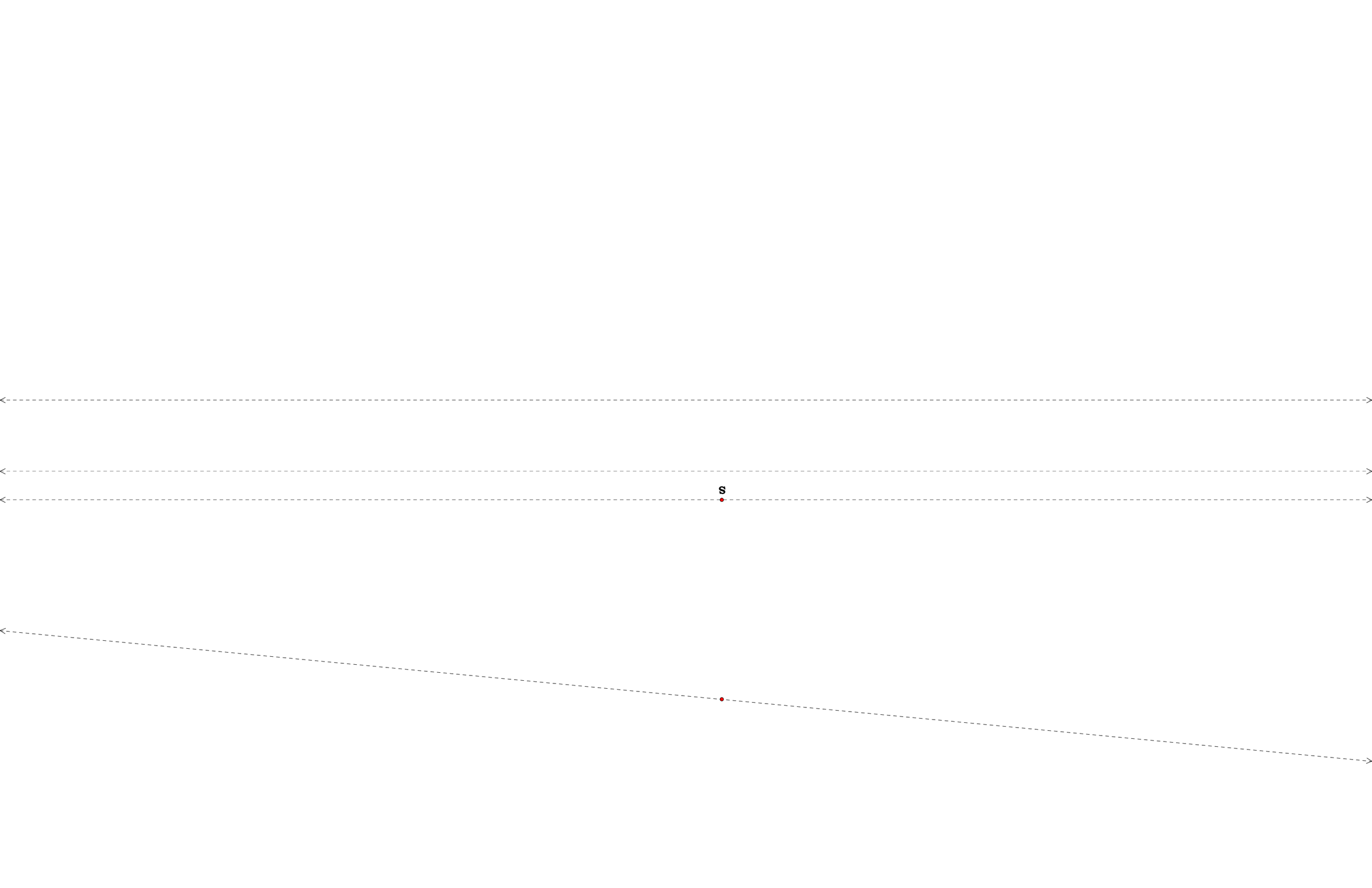
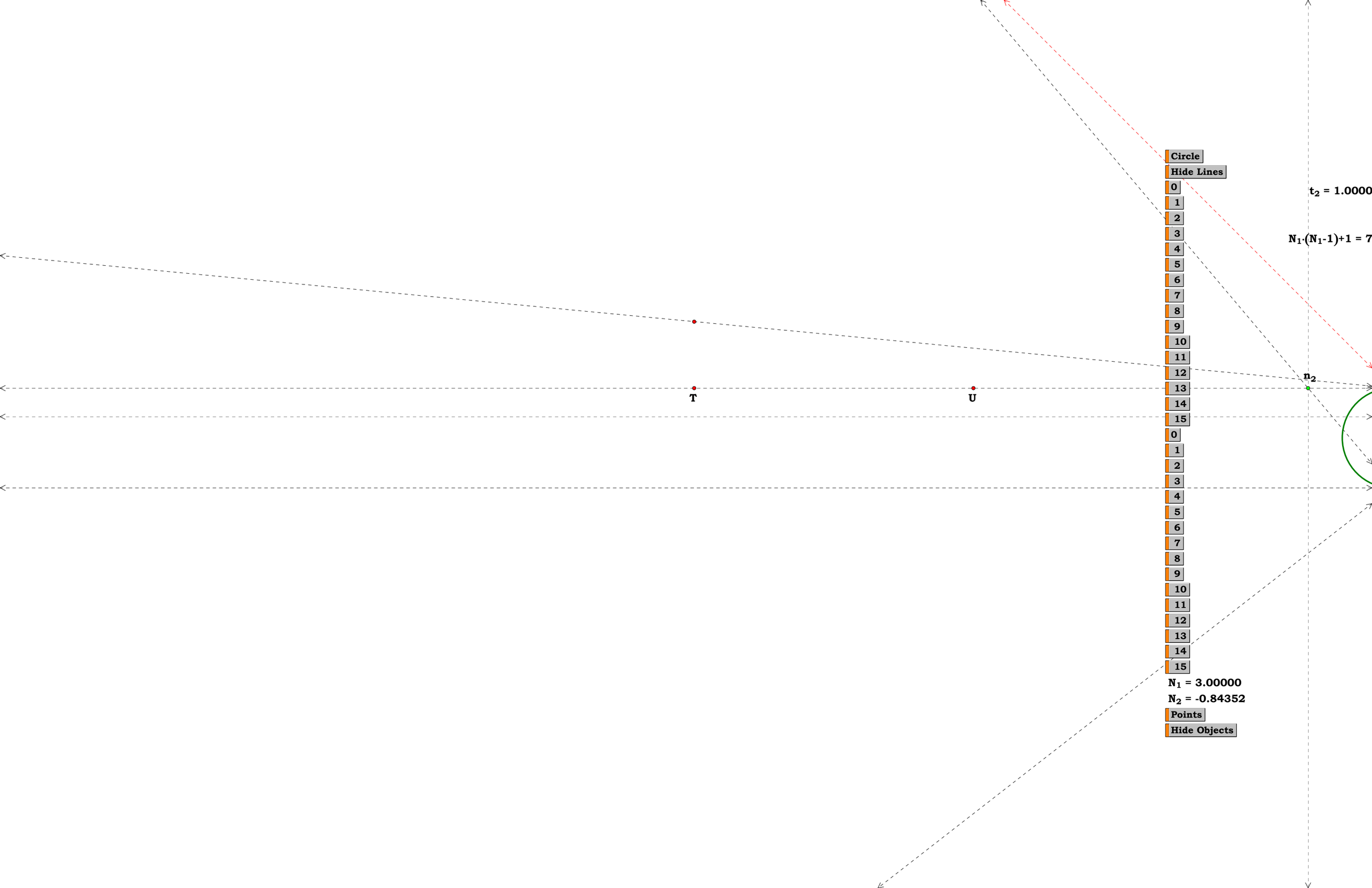
10
11
12
13
14
15
N1 = 2.00000
N2 = -0.91098
Points
Hide Objects
n2
n1
A
B
xA = 1.33333
xB = 2.00000
N1²/(N1²-N1)+1 = 1.33333
t1 = 2
N1²/N1²-t1·(N1-1) = 2.00000


S


T
U
n2
t2 = 1.0000
N1·(N1-1)+1 = 7
Circle
Hide Lines
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0
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15
N1 = 3.00000
N2 = -0.84352
Points
Hide Objects

0000

= 7.00000

$x_I$ = 1.10526

$x_J$ = 1.23529

$x_K$ = 1.40000

$x_L$ = 1.61538

$x_M$ = 1.90909

$x_N$ = 2.33333

$x_O$ = 3.00000

$x_P$ = 4.20000

$x_Q$ = 7.00000

$x_R$ = 21.00000

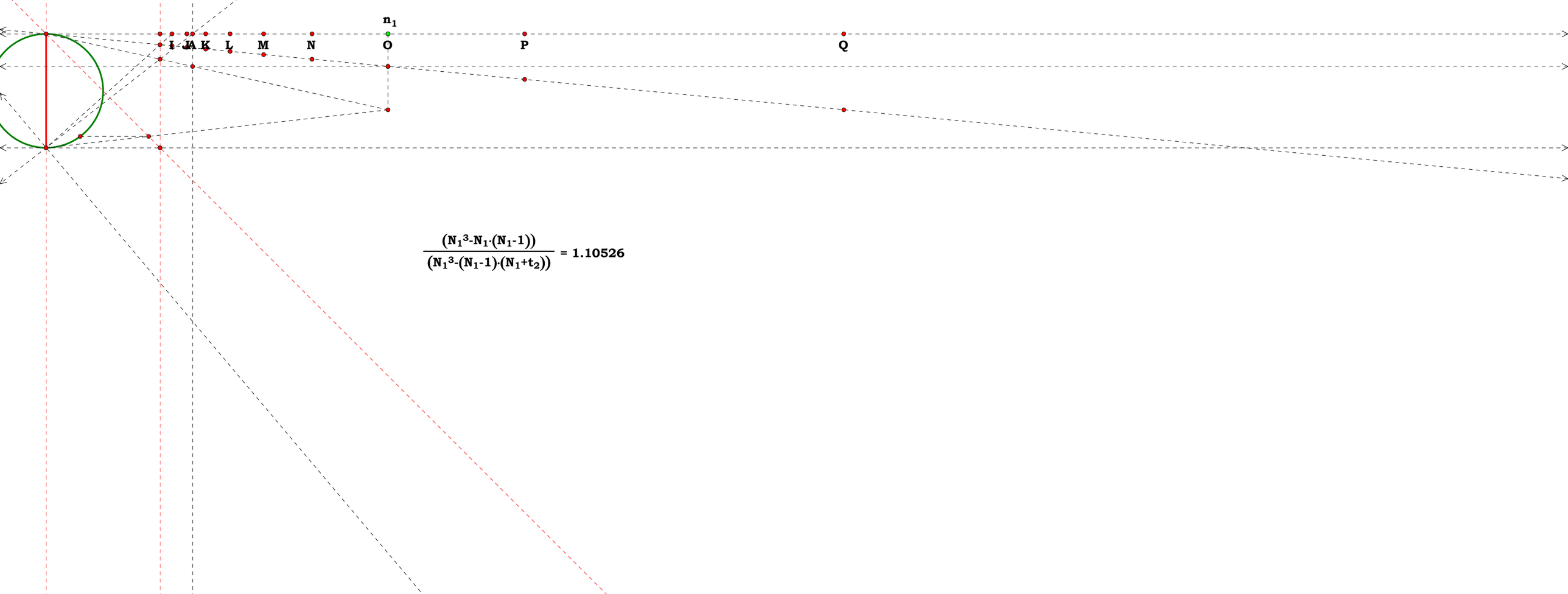
$x_S$ = -21.00000

$x_T$ = -7.00000

$x_U$ = -4.20000

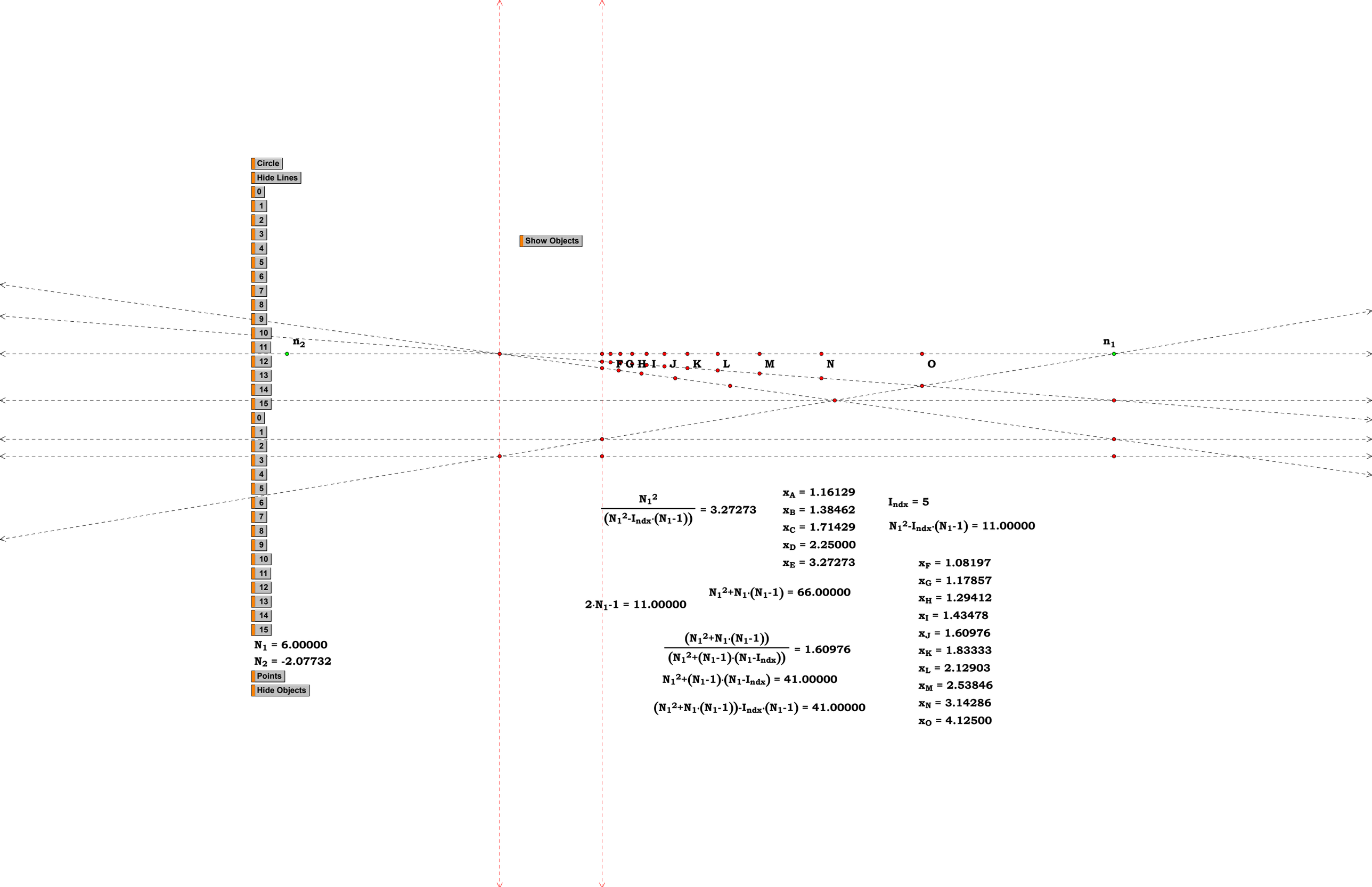
$N_1^3-N_1\cdot(N_1-1)$ = 21.00000

$N_1^3-(N_1-1)\cdot(N_1+t_2)$ = 19.00000




$$\frac{(N_1^3-N_1\cdot(N_1-1))}{(N_1^3-(N_1-1)\cdot(N_1+t_2))} = 1.10526$$

R




$N_1 = 6.00000$

$N_2 = -2.07732$

Points

Hide Objects

$$\frac{N_1^2}{(N_1^2 - I_{ndx}\cdot(N_1-1))} = 3.27273$$

$x_A = 1.16129$

$x_B = 1.38462$

$x_C = 1.71429$

$x_D = 2.25000$

$x_E = 3.27273$

$I_{ndx} = 5$

$N_1^2 - I_{ndx}\cdot(N_1-1) = 11.00000$

$N_1^2 + N_1\cdot(N_1-1) = 66.00000$

$2\cdot N_1 - 1 = 11.00000$

$$\frac{(N_1^2 + N_1\cdot(N_1-1))}{(N_1^2 + (N_1-1)\cdot(N_1-I_{ndx}))} = 1.60976$$

$N_1^2 + (N_1-1)\cdot(N_1-I_{ndx}) = 41.00000$

$(N_1^2 + N_1\cdot(N_1-1)) - I_{ndx}\cdot(N_1-1) = 41.00000$

$x_F = 1.08197$

$x_G = 1.17857$

$x_H = 1.29412$

$x_I = 1.43478$

$x_J = 1.60976$

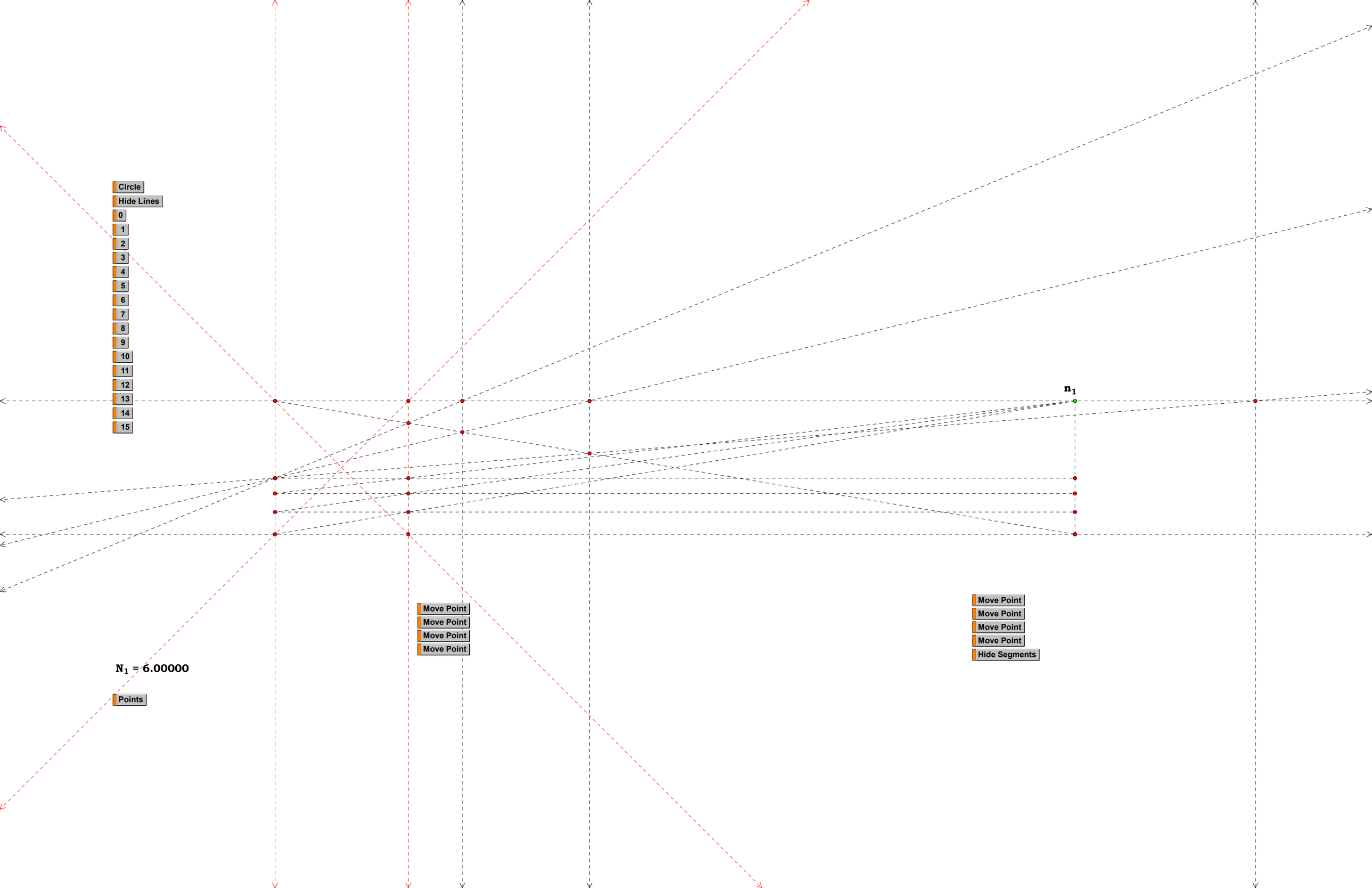
$x_K = 1.83333$

$x_L = 2.12903$

$x_M = 2.53846$

$x_N = 3.14286$

$x_O = 4.12500$


Circle
Hide Lines
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15
$n_1$
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Move Point
Hide Segments
$N_1$ = 6.00000
Points

Circle
Show Lines
0
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15

$N_1 = 5.00000$

Points

Circle
Show Lines
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15

Points

$I_{ndx} = 3$

$n_3 = 3.00000$

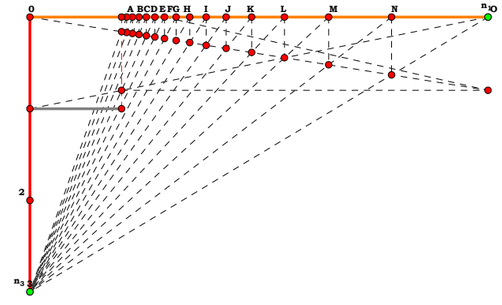
$N_1^2 \cdot n_3 = 75.00000$

$N_1^2 \cdot n_3 \cdot I_{ndx} \cdot (N_1 - 1) = 63.00000$

$$\frac{(N_1^2 \cdot n_3)}{(N_1^2 \cdot n_3 \cdot I_{ndx} \cdot (N_1 - 1))} = 1.19048$$

$N_1 \cdot n_3 = 15.00000$

$x_A = 1.05634$
$x_B = 1.11940$
$x_C = 1.19048$
$x_D = 1.27119$
$x_E = 1.36364$
$x_F = 1.47059$
$x_G = 1.59574$
$x_H = 1.74419$
$x_I = 1.92308$
$x_J = 2.14286$
$x_K = 2.41935$
$x_L = 2.77778$
$x_M = 3.26087$
$x_N = 3.94737$
$x_O = 5.00000$

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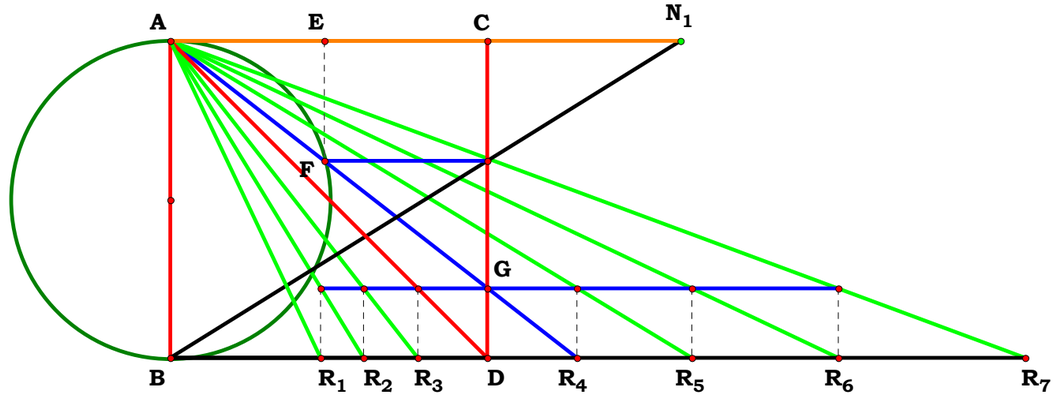
Circles-Plate 2

$AB := 1$

$N_1 := 1.60907$

$N_1 = 1.60907$
$R_1 = 0.47533$
$R_2 = 0.60907$
$R_3 = 0.78043$
$R_4 = 1.28135$
$R_5 = 1.64185$
$R_6 = 2.10379$
$R_7 = 2.69569$




$EF := 1 - \frac{1}{N_1}$ $AE := \sqrt{EF \cdot (1 - EF)}$

$R_4 := \frac{AE}{EF}$ $CG := \frac{1}{R_4}$ $R_3 := CG$ $R_2 := R_3 \cdot CG$

$R_1 := R_2 \cdot CG$ $R_5 := \frac{R_4}{CG}$ $R_6 := \frac{R_5}{CG}$ $R_7 := \frac{R_6}{CG}$

$R_1 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-3} = 0$ $R_2 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-2} = 0$ $R_3 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-1} = 0$ $1 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{0} = 0$

$R_4 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{1} = 0$ $R_5 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{2} = 0$ $R_6 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{3} = 0$ $R_7 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{4} = 0$

$R_1 = 0.475336$
$R_2 = 0.60907$
$R_3 = 0.780429$
$R_4 = 1.281346$
$R_5 = 1.641847$
$R_6 = 2.103775$
$R_7 = 2.695663$

$R_1 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-3} = 0.00000$

$R_2 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-2} = 0.00000$

$R_3 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{-1} = 0.00000$

$R_4 - \left(\frac{1}{\sqrt{N_1 - 1}}\right) = 0.00000$

$R_5 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{2} = 0.00000$

$R_6 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{3} = 0.00000$

$R_7 - \left(\frac{1}{\sqrt{N_1 - 1}}\right)^{4} = 0.00000$

$\frac{1}{\sqrt{N_1 - 1}} = 1.37048$

$N_1 = 1.53242$

$R_1 = 0.38849$

$R_2 = 0.53242$

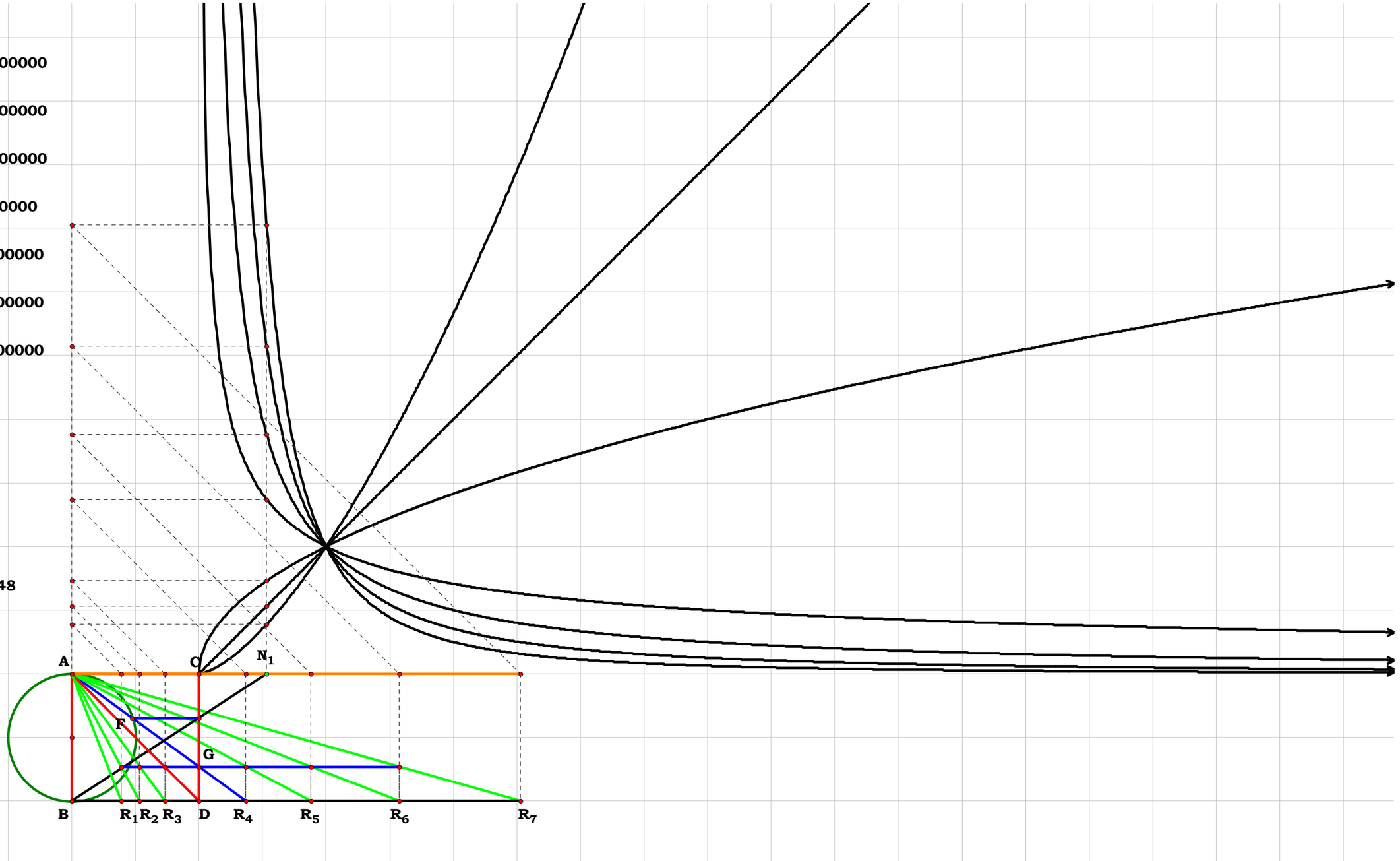
$R_3 = 0.72967$

$R_4 = 1.37048$

$R_5 = 1.87822$

$R_6 = 2.57408$

$R_7 = 3.52773$

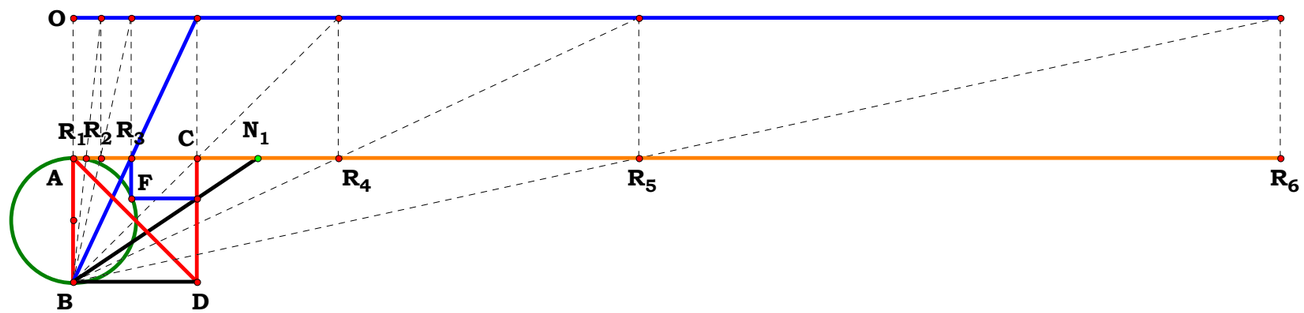


Circles-Plate 3

$AB := 1$

$N_1 := 1.48165$

$N_1 = 1.48165$
$R_1 = 0.10277$
$R_2 = 0.21940$
$R_3 = 0.46840$
$R_4 = 2.13491$
$R_5 = 4.55784$
$R_6 = 9.73059$




$$RF := \frac{N_1 - 1}{N_1} \qquad R_3 := \sqrt{RF \cdot (1 - RF)}$$

$$R_3 - \frac{\sqrt{N_1 - 1}}{N_1} = 0$$

$$R_2 := R_3^{\,2} \quad R_1 := R_3^{\,3} \quad R_4 := R_3^{\,-1} \quad R_5 := R_3^{\,-2} \quad R_6 := R_3^{\,-3}$$

$$R_1 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-3} = 0 \quad R_2 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-2} = 0 \quad R_3 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{-1} = 0$$

$$AB - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{0} = 0 \quad R_4 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{1} = 0 \quad R_5 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{2} = 0 \quad R_6 - \left(\frac{N_1}{\sqrt{N_1 - 1}}\right)^{3} = 0$$

$R_1 = 0.102769$
$R_2 = 0.219402$
$R_3 = 0.468404$
$R_4 = 2.134911$
$R_5 = 4.557846$
$R_6 = 9.730598$

$R_1\text{-}\frac{N_1^{\,-3}}{\sqrt{N_1\text{-}1}} = 0.00000$

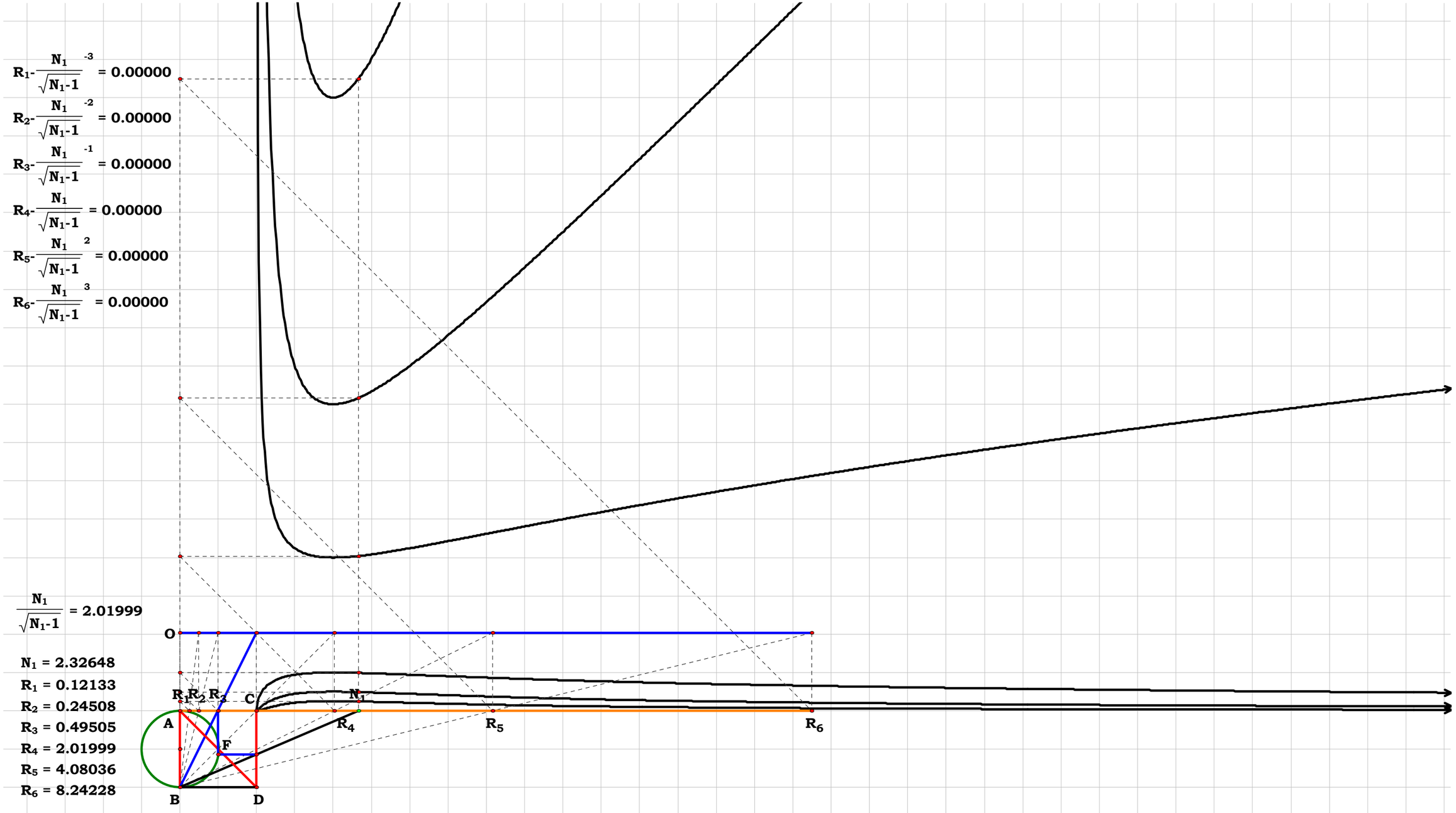
$R_2\text{-}\frac{N_1^{\,-2}}{\sqrt{N_1\text{-}1}} = 0.00000$

$R_3\text{-}\frac{N_1^{\,-1}}{\sqrt{N_1\text{-}1}} = 0.00000$

$R_4\text{-}\frac{N_1}{\sqrt{N_1\text{-}1}} = 0.00000$

$R_5\text{-}\frac{N_1^{\,2}}{\sqrt{N_1\text{-}1}} = 0.00000$

$R_6\text{-}\frac{N_1^{\,3}}{\sqrt{N_1\text{-}1}} = 0.00000$




$\frac{N_1}{\sqrt{N_1\text{-}1}} = 2.01999$

$N_1 = 2.32648$

$R_1 = 0.12133$

$R_2 = 0.24508$

$R_3 = 0.49505$

$R_4 = 2.01999$

$R_5 = 4.08036$

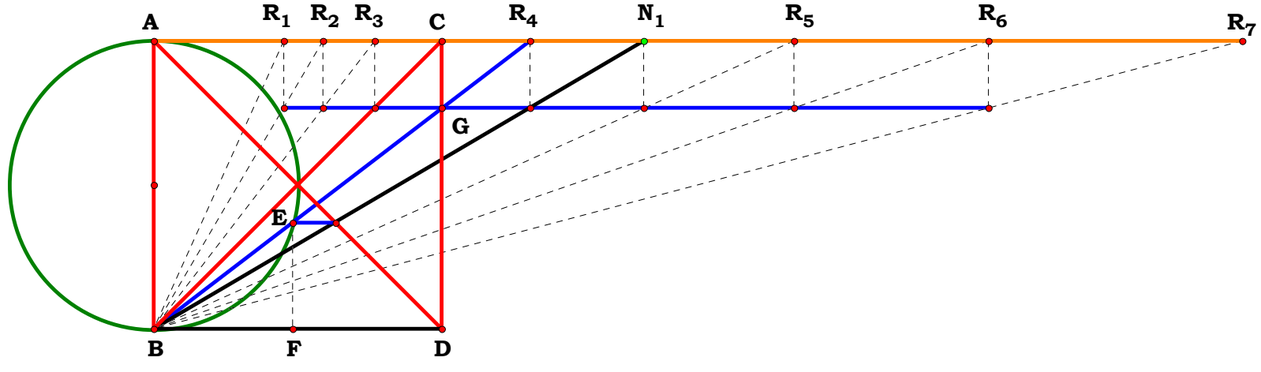
$R_6 = 8.24228$

Circles-Plate 4

$AB := 1$

$N_1 := 1.70189$

$N_1 = 1.70189$
$R_1 = 0.45041$
$R_2 = 0.58758$
$R_3 = 0.76654$
$R_4 = 1.30456$
$R_5 = 2.22022$
$R_6 = 2.89641$
$R_7 = 3.77855$




$EF := \frac{1}{N_1 + 1}$ $BF := \sqrt{EF \cdot (1 - EF)}$

$DG := \frac{EF}{BF}$ $R_4 := \frac{1}{DG}$ $R_4 - \frac{\sqrt{N_1} \cdot (N_1 + 1)}{\sqrt{(N_1 + 1)^2}} = 0$ $R_4 - \sqrt{N_1} = 0$

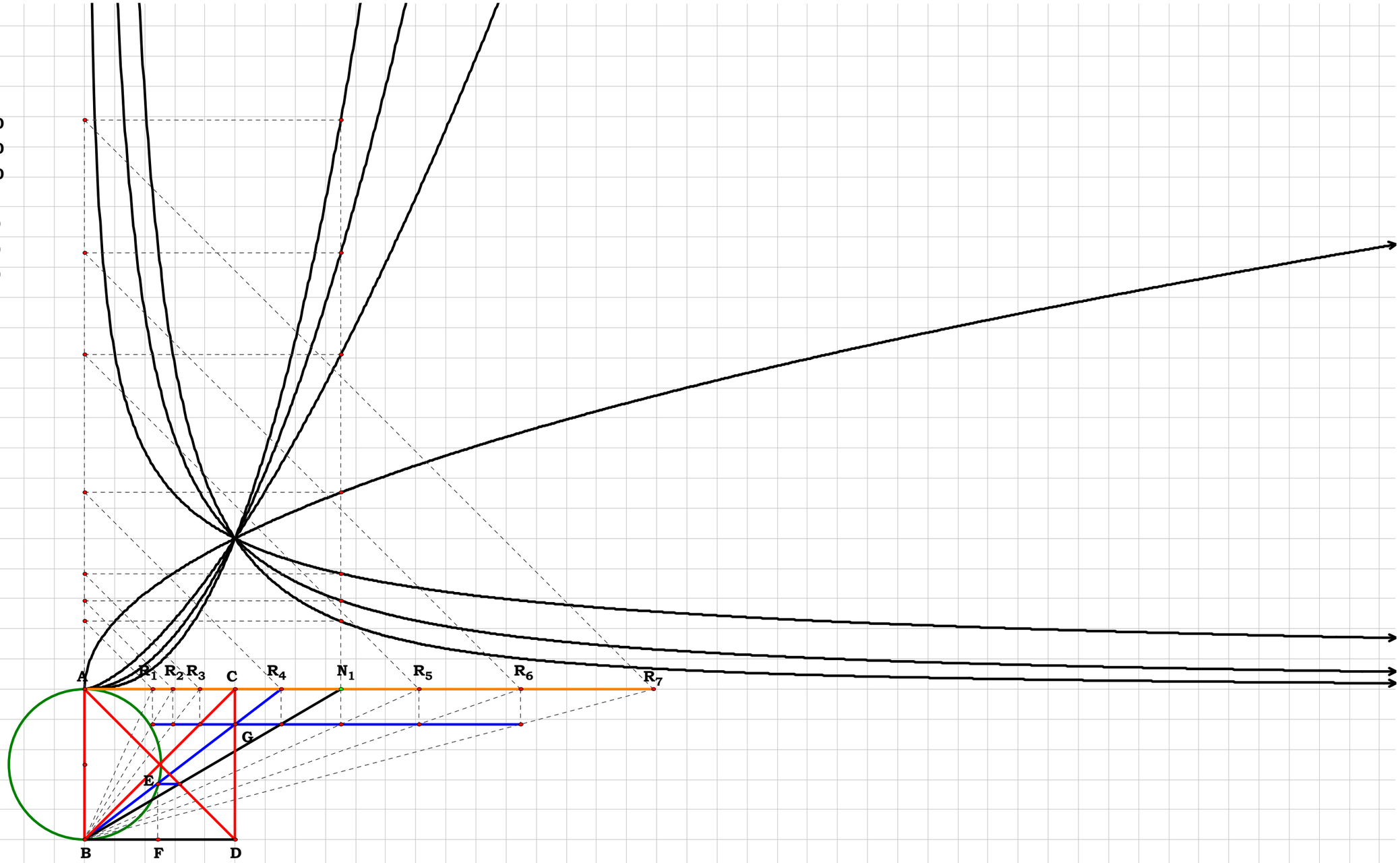
$R_3 := \frac{1}{R_4}$ $R_2 := \frac{1}{R_4^2}$ $R_1 := \frac{1}{R_4^3}$ $N_1 - R_4^2 = 0$ $R_5 := R_4^3$ $R_6 := R_4^4$ $R_7 := R_4^5$

$R_1 - \left(\sqrt{N_1}\right)^{-3} = 0$ $R_2 - \left(\sqrt{N_1}\right)^{-2} = 0$ $R_3 - \left(\sqrt{N_1}\right)^{-1} = 0$ $AB - \left(\sqrt{N_1}\right)^0 = 0$ $R_4 - \left(\sqrt{N_1}\right)^1 = 0$

$N_1 - \left(\sqrt{N_1}\right)^2 = 0$ $R_5 - \left(\sqrt{N_1}\right)^3 = 0$ $R_6 - \left(\sqrt{N_1}\right)^4 = 0$ $R_7 - \left(\sqrt{N_1}\right)^5 = 0$

$R_1 = 0.450405$
$R_2 = 0.587582$
$R_3 = 0.766539$
$R_4 = 1.304565$
$R_5 = 2.220226$
$R_6 = 2.89643$
$R_7 = 3.778581$

$R_1\text{-}\sqrt{N_1}^{-3} = 0.00000$
$R_2\text{-}\sqrt{N_1}^{-2} = 0.00000$
$R_3\text{-}\sqrt{N_1}^{-1} = 0.00000$
$R_4\text{-}\sqrt{N_1} = 0.00000$
$R_5\text{-}\sqrt{N_1}^{3} = 0.00000$
$R_6\text{-}\sqrt{N_1}^{4} = 0.00000$
$R_7\text{-}\sqrt{N_1}^{5} = 0.00000$




$\sqrt{N_1} = 1.30456$

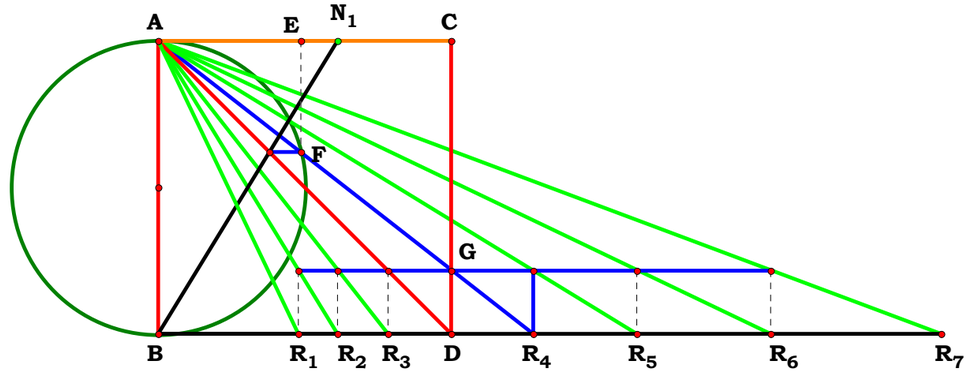
$N_1 = 1.70189$
$R_1 = 0.45041$
$R_2 = 0.58758$
$R_3 = 0.76654$
$R_4 = 1.30456$
$R_5 = 2.22022$
$R_6 = 2.89641$
$R_7 = 3.77855$

Circles-Plate 5

$AB := 1$

$N_1 := .61225$

$N_1 = 0.61225$
$R_1 = 0.47906$
$R_2 = 0.61225$
$R_3 = 0.78246$
$R_4 = 1.27802$
$R_5 = 1.63333$
$R_6 = 2.08742$
$R_7 = 2.66776$




$EF := \frac{N_1}{N_1 + 1}$ $AE := \sqrt{EF \cdot (1 - EF)}$

$R_4 := \frac{AE}{EF}$ $CG := \frac{1}{R_4}$ $R_3 := CG$

$R_2 := R_3 \cdot CG$ $R_1 := R_2 \cdot CG$ $R_5 := \frac{R_4}{CG}$ $R_6 := \frac{R_5}{CG}$ $R_7 := \frac{R_6}{CG}$

$R_4 - \frac{N_1 + 1}{\sqrt{N_1} \cdot \sqrt{(N_1 + 1)^2}} = 0$ $R_4 - \frac{1}{\sqrt{N_1}} = 0$

$R_1 - \left(\frac{1}{\sqrt{N_1}}\right)^{-3} = 0$ $R_2 - \left(\frac{1}{\sqrt{N_1}}\right)^{-2} = 0$ $R_3 - \left(\frac{1}{\sqrt{N_1}}\right)^{-1} = 0$ $AB - \left(\frac{1}{\sqrt{N_1}}\right)^{0} = 0$

$R_4 - \left(\frac{1}{\sqrt{N_1}}\right)^{1} = 0$ $R_5 - \left(\frac{1}{\sqrt{N_1}}\right)^{2} = 0$ $R_6 - \left(\frac{1}{\sqrt{N_1}}\right)^{3} = 0$ $R_7 - \left(\frac{1}{\sqrt{N_1}}\right)^{4} = 0$

$R_1 = 0.479064$
$R_2 = 0.61225$
$R_3 = 0.782464$
$R_4 = 1.278014$
$R_5 = 1.63332$
$R_6 = 2.087405$
$R_7 = 2.667733$

$R_1\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{-3} = 0.00000$

$R_2\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{-2} = 0.00000$

$R_3\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{-1} = 0.00000$

$R_4\text{-}\left(\frac{1}{\sqrt{N_1}}\right) = 0.00000$

$R_5\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{2} = 0.00000$

$R_6\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{3} = 0.00000$

$R_7\text{-}\left(\frac{1}{\sqrt{N_1}}\right)^{4} = 0.00000$

$\frac{1}{\sqrt{N_1}} = 1.19296$

$N_1 = 0.70267$

$R_1 = 0.58901$

$R_2 = 0.70267$

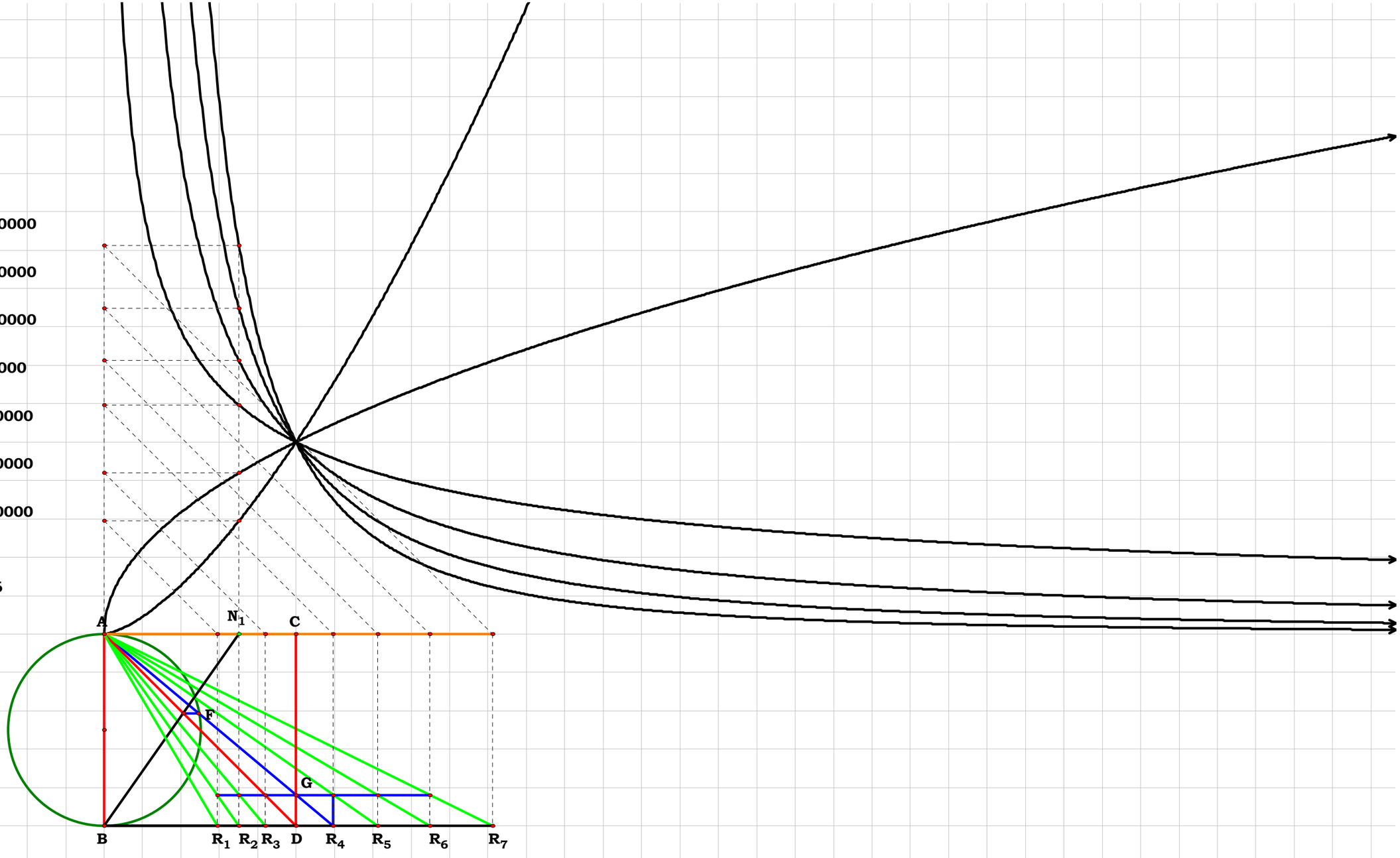
$R_3 = 0.83825$

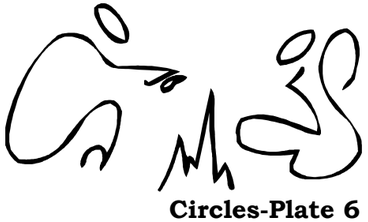
$R_4 = 1.19296$

$R_5 = 1.42315$

$R_6 = 1.69775$

$R_7 = 2.02534$



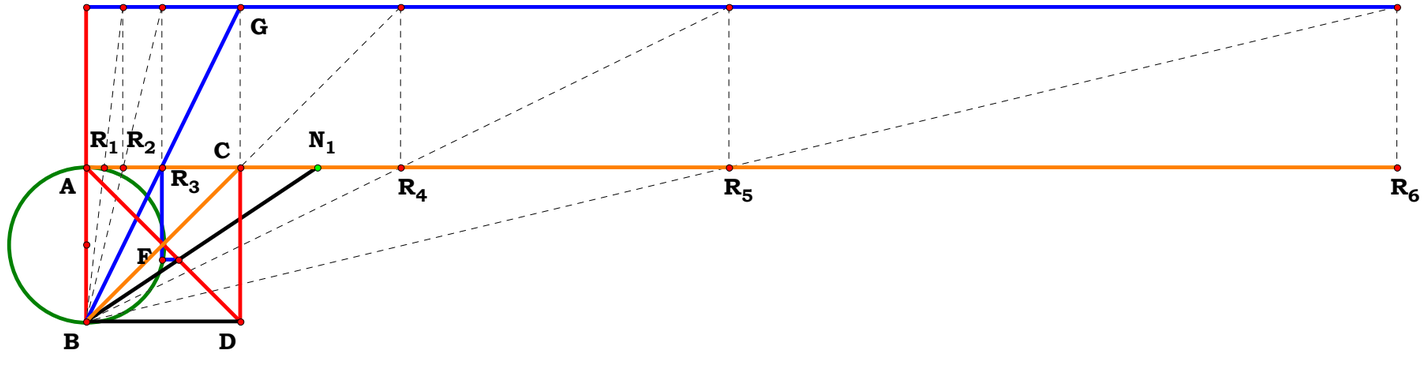


Circles-Plate 6

$AB := 1$

$N_1 := 1.49352$

$N_1 = 1.49352$
$R_1 = 0.11773$
$R_2 = 0.24021$
$R_3 = 0.49011$
$R_4 = 2.04036$
$R_5 = 4.16308$
$R_6 = 8.49419$




$FR := \frac{N_1}{N_1 + 1}$ $R_3 := \sqrt{FR \cdot (1 - FR)}$

$R_2 := R_3^{\,2}$ $R_1 := R_3^{\,3}$ $AB - \frac{R_3}{R_3} = 0$ $R_4 := \frac{R_3}{R_3^{\,2}}$ $R_5 := \frac{R_3}{R_3^{\,3}}$ $R_6 := \frac{R_3}{R_3^{\,4}}$

$R_3 - \sqrt{\frac{N_1}{(N_1 + 1)^2}} = 0$

$R_1 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-3} = 0$ $R_2 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-2} = 0$ $R_3 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{-1} = 0$

$R_4 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right) = 0$ $R_5 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{2} = 0$ $R_6 - \left(\frac{N_1 + 1}{\sqrt{N_1}}\right)^{3} = 0$

$R_1$-$\frac{N_1+1^{-3}}{\sqrt{N_1}}$ = 0.00000

$R_2$-$\frac{N_1+1^{-2}}{\sqrt{N_1}}$ = 0.00000

$R_3$-$\frac{N_1+1^{-1}}{\sqrt{N_1}}$ = 0.00000

$R_4$-$\frac{N_1+1}{\sqrt{N_1}}$ = 0.00000

$R_5$-$\frac{N_1+1^{2}}{\sqrt{N_1}}$ = 0.00000

$R_6$-$\frac{N_1+1^{3}}{\sqrt{N_1}}$ = 0.00000

$\frac{N_1+1}{\sqrt{N_1}}$ = 2.01533

$N_1$ = 1.28083

$R_1$ = 0.12217

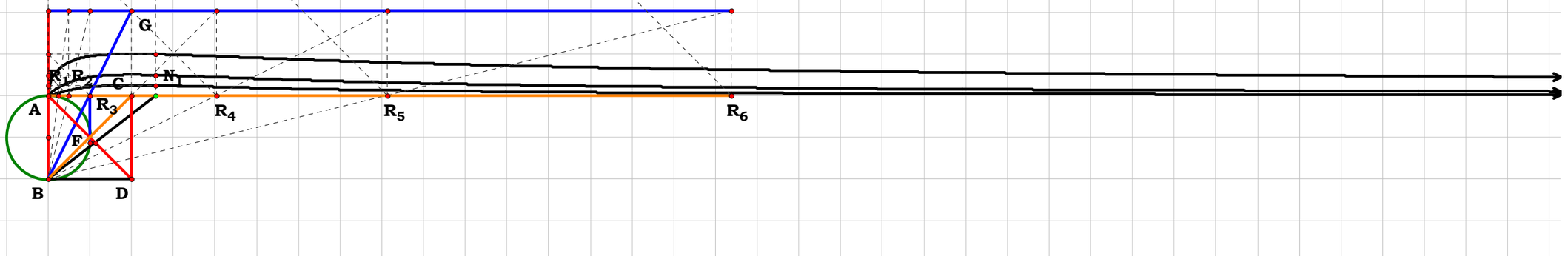
$R_2$ = 0.24621

$R_3$ = 0.49620

$R_4$ = 2.01533

$R_5$ = 4.06157

$R_6$ = 8.18543

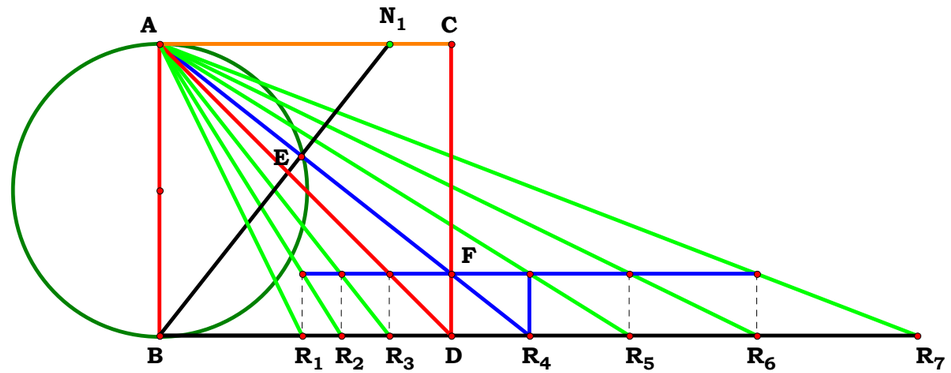


Circles-Plate 7

$AB := 1$

$N_1 := .78798$

$N_1 = 0.78798$
$R_1 = 0.48926$
$R_2 = 0.62091$
$R_3 = 0.78798$
$R_4 = 1.26907$
$R_5 = 1.61054$
$R_6 = 2.04389$
$R_7 = 2.59384$




$R_4 := \frac{1}{N_1}$

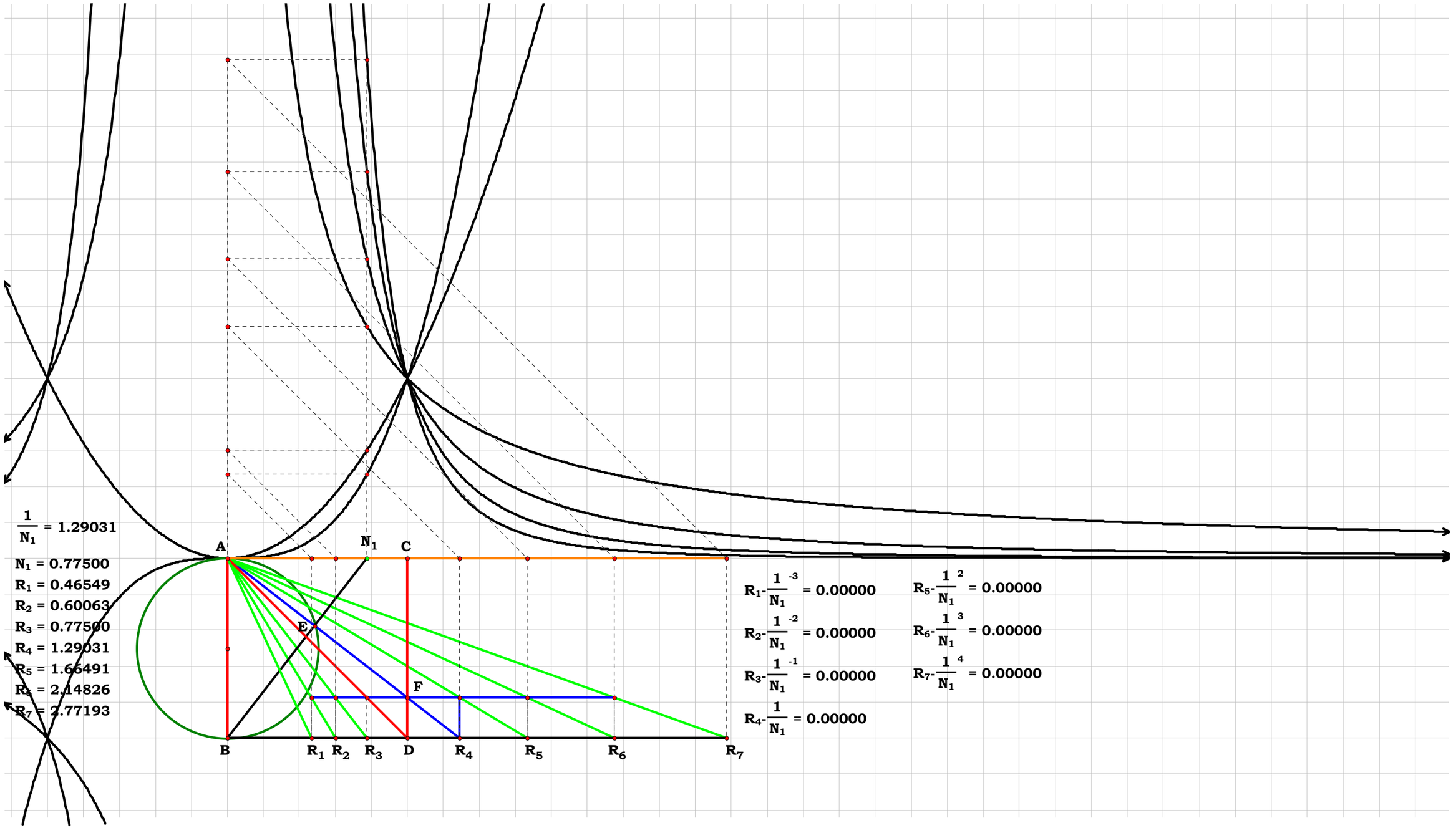
$BN_1 := \sqrt{1^2 + N_1^2}$ $\quad EN_1 := \frac{N_1^2}{BN_1}$ $\quad BE := BN_1 - EN_1$

$R_4 := \frac{BN_1 \cdot BE}{N_1}$ $\quad R_4 - \frac{1}{N_1} = 0$

$R_1 := R_4^{-3}$ $\quad R_2 := R_4^{-2}$ $\quad R_3 := R_4^{-1}$ $\quad R_5 := R_4^{2}$ $\quad R_6 := R_4^{3}$ $\quad R_7 := R_4^{4}$

$R_1 - \left(\frac{1}{N_1}\right)^{-3} = 0$ $\quad R_2 - \left(\frac{1}{N_1}\right)^{-2} = 0$ $\quad R_3 - \left(\frac{1}{N_1}\right)^{-1} = 0$ $\quad AB - \left(\frac{1}{N_1}\right)^{0} = 0$

$R_4 - \left(\frac{1}{N_1}\right)^{1} = 0$ $\quad R_5 - \left(\frac{1}{N_1}\right)^{2} = 0$ $\quad R_6 - \left(\frac{1}{N_1}\right)^{3} = 0$ $\quad R_7 - \left(\frac{1}{N_1}\right)^{4} = 0$

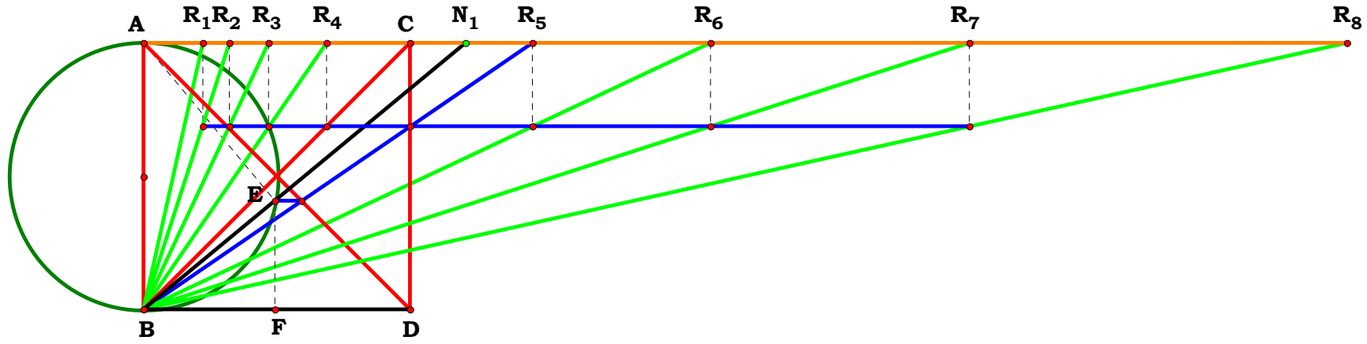

$\frac{1}{N_1}$ = 1.29031
N1 = 0.77500
R1 = 0.46549
R2 = 0.60063
R3 = 0.77500
R4 = 1.29031
R5 = 1.66491
R6 = 2.14826
R7 = 2.77193
A
N1
C
E
F
B
R1
R2
R3
D
R4
R5
R6
R7
R1-1/N1 -3 = 0.00000
R2-1/N1 -2 = 0.00000
R3-1/N1 -1 = 0.00000
R4-1/N1 = 0.00000
R5-1/N1 2 = 0.00000
R6-1/N1 3 = 0.00000
R7-1/N1 4 = 0.00000

Circles-Plate 8

$AB := 1$

$N_1 := 1.20725$

$N_1 = 1.20725$
$R_1 = 0.22162$
$R_2 = 0.32301$
$R_3 = 0.47077$
$R_4 = 0.68613$
$R_5 = 1.45746$
$R_6 = 2.12418$
$R_7 = 3.09591$
$R_8 = 4.51215$



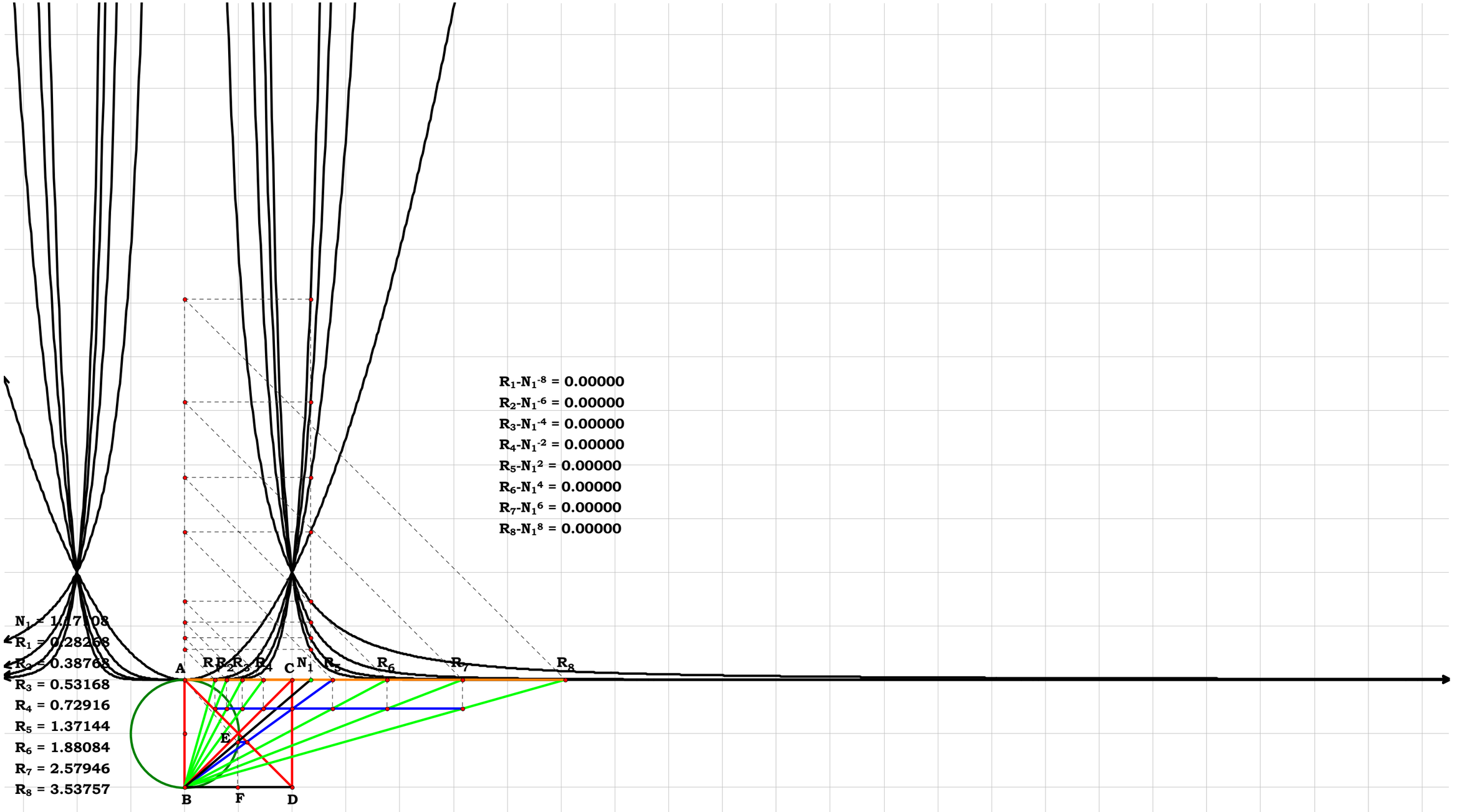

$EF := \frac{1}{N_1^2 + 1}$ $R_5 := \frac{1 - EF}{EF}$ $R_5 - N_1^2 = 0$

$R_4 := \frac{1}{R_5}$ $R_3 := R_4^2$ $R_2 := R_4^3$ $R_1 := R_4^4$

$R_6 := R_5^2$ $R_7 := R_5^3$ $R_8 := R_5^4$

$R_1 - N_1^{-8} = 0$ $R_2 - N_1^{-6} = 0$ $R_3 - N_1^{-4} = 0$ $R_4 - N_1^{-2} = 0$

$R_5 - N_1^2 = 0$ $R_6 - N_1^4 = 0$ $R_7 - N_1^6 = 0$ $R_8 - N_1^8 = 0$

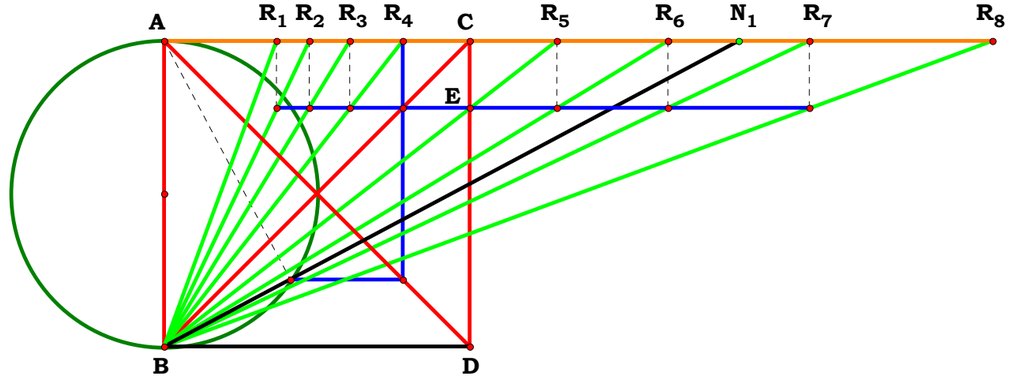

$R_1$-$N_1^{-8}$ = 0.00000
$R_2$-$N_1^{-6}$ = 0.00000
$R_3$-$N_1^{-4}$ = 0.00000
$R_4$-$N_1^{-2}$ = 0.00000
$R_5$-$N_1^{2}$ = 0.00000
$R_6$-$N_1^{4}$ = 0.00000
$R_7$-$N_1^{6}$ = 0.00000
$R_8$-$N_1^{8}$ = 0.00000
$N_1$ = 1.17708
$R_1$ = 0.28258
$R_2$ = 0.38763
$R_3$ = 0.53168
$R_4$ = 0.72916
$R_5$ = 1.37144
$R_6$ = 1.88084
$R_7$ = 2.57946
$R_8$ = 3.53757
A
B
C
D
E
F
$N_1$
$R_1$
$R_2$
$R_3$
$R_4$
$R_5$
$R_6$
$R_7$
$R_8$

Circles-Plate 9

$AB := 1$

$N_1 := 1.88007$

$N_1 = 1.88007$
$R_1 = 0.36916$
$R_2 = 0.47360$
$R_3 = 0.60758$
$R_4 = 0.77948$
$R_5 = 1.28291$
$R_6 = 1.64587$
$R_7 = 2.11150$
$R_8 = 2.70888$



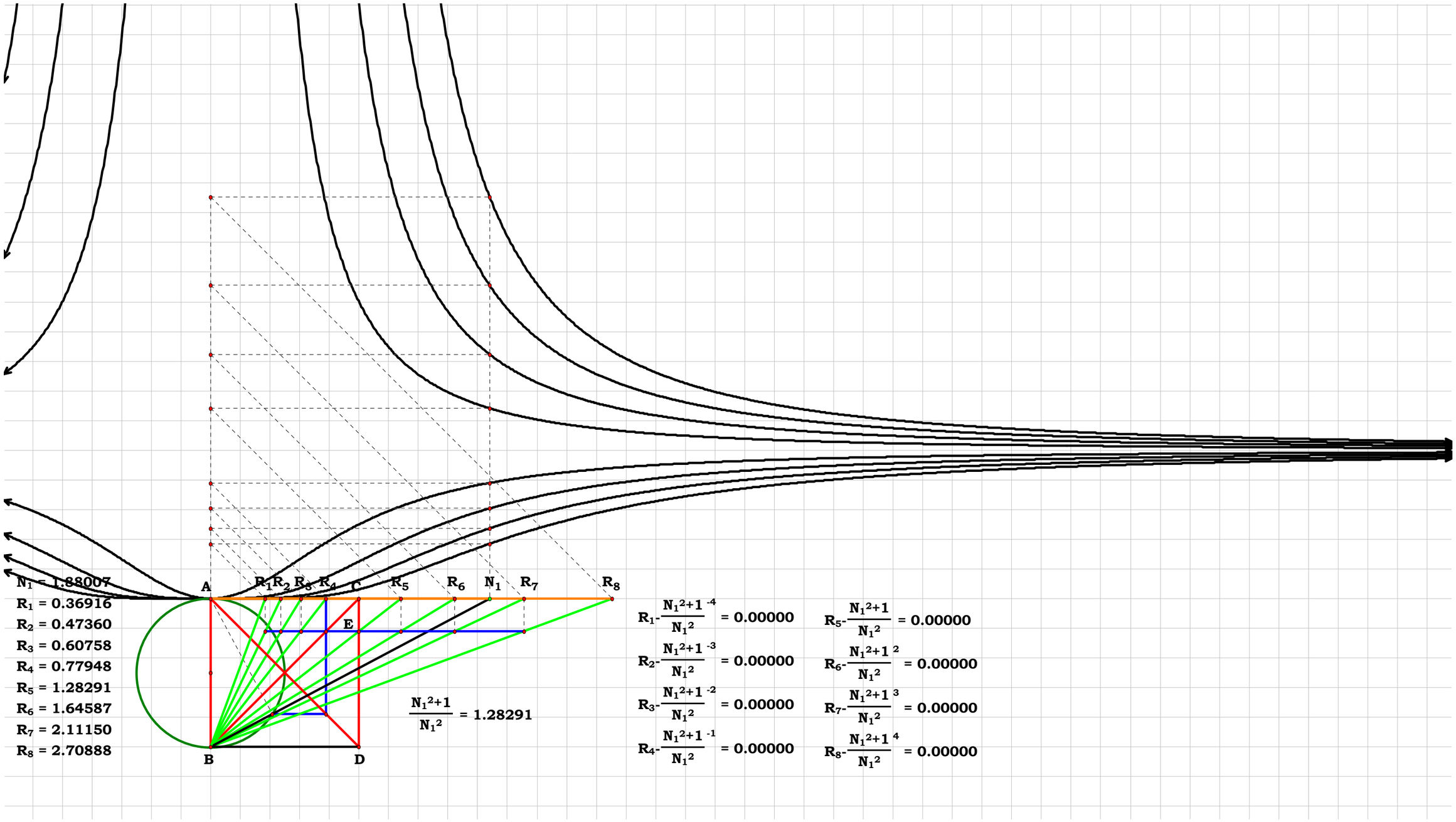

$$CE := \frac{1}{N_1^2 + 1} \qquad R_4 := 1 - CE \qquad R_4 - \frac{N_1^2}{N_1^2 + 1} = 0$$

$$R_3 := R_4^2 \qquad R_2 := R_4^3 \qquad R_1 := R_4^4 \qquad R_5 := R_4^{-1}$$

$$R_6 := R_4^{-2} \qquad R_7 := R_4^{-3} \qquad R_8 := R_4^{-4}$$

$$R_1 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{-4} = 0 \qquad R_2 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{-3} = 0 \qquad R_3 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{-2} = 0 \qquad R_4 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{-1} = 0$$

$$R_5 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right] = 0 \qquad R_6 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{2} = 0 \qquad R_7 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{3} = 0 \qquad R_8 - \left[\frac{\left(N_1^2 + 1\right)}{N_1^2}\right]^{4} = 0$$




$N_1$ = 1.88007
$R_1$ = 0.36916
$R_2$ = 0.47360
$R_3$ = 0.60758
$R_4$ = 0.77948
$R_5$ = 1.28291
$R_6$ = 1.64587
$R_7$ = 2.11150
$R_8$ = 2.70888

$$\frac{N_1^2+1}{N_1^2} = 1.28291$$

$$R_1\text{-}\frac{N_1^2+1}{N_1^2}^{-4} = 0.00000 \qquad R_5\text{-}\frac{N_1^2+1}{N_1^2} = 0.00000$$

$$R_2\text{-}\frac{N_1^2+1}{N_1^2}^{-3} = 0.00000 \qquad R_6\text{-}\frac{N_1^2+1}{N_1^2}^{2} = 0.00000$$

$$R_3\text{-}\frac{N_1^2+1}{N_1^2}^{-2} = 0.00000 \qquad R_7\text{-}\frac{N_1^2+1}{N_1^2}^{3} = 0.00000$$

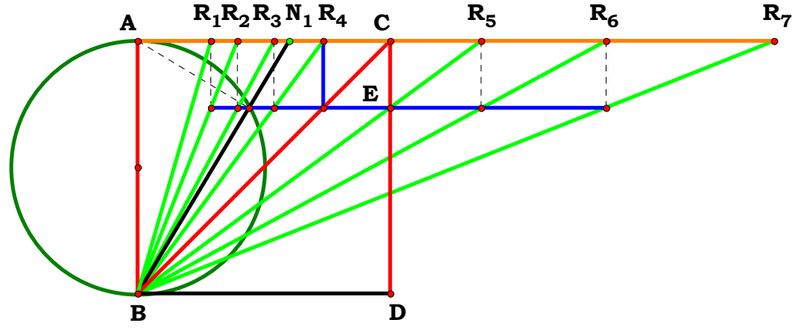
$$R_4\text{-}\frac{N_1^2+1}{N_1^2}^{-1} = 0.00000 \qquad R_8\text{-}\frac{N_1^2+1}{N_1^2}^{4} = 0.00000$$

Circles-Plate 10

$AB := 1$

$N_1 := .60053$

$N_1 = 0.60053$
$R_1 = 0.29177$
$R_2 = 0.39699$
$R_3 = 0.54016$
$R_4 = 0.73495$
$R_5 = 1.36063$
$R_6 = 1.85131$
$R_7 = 2.51896$



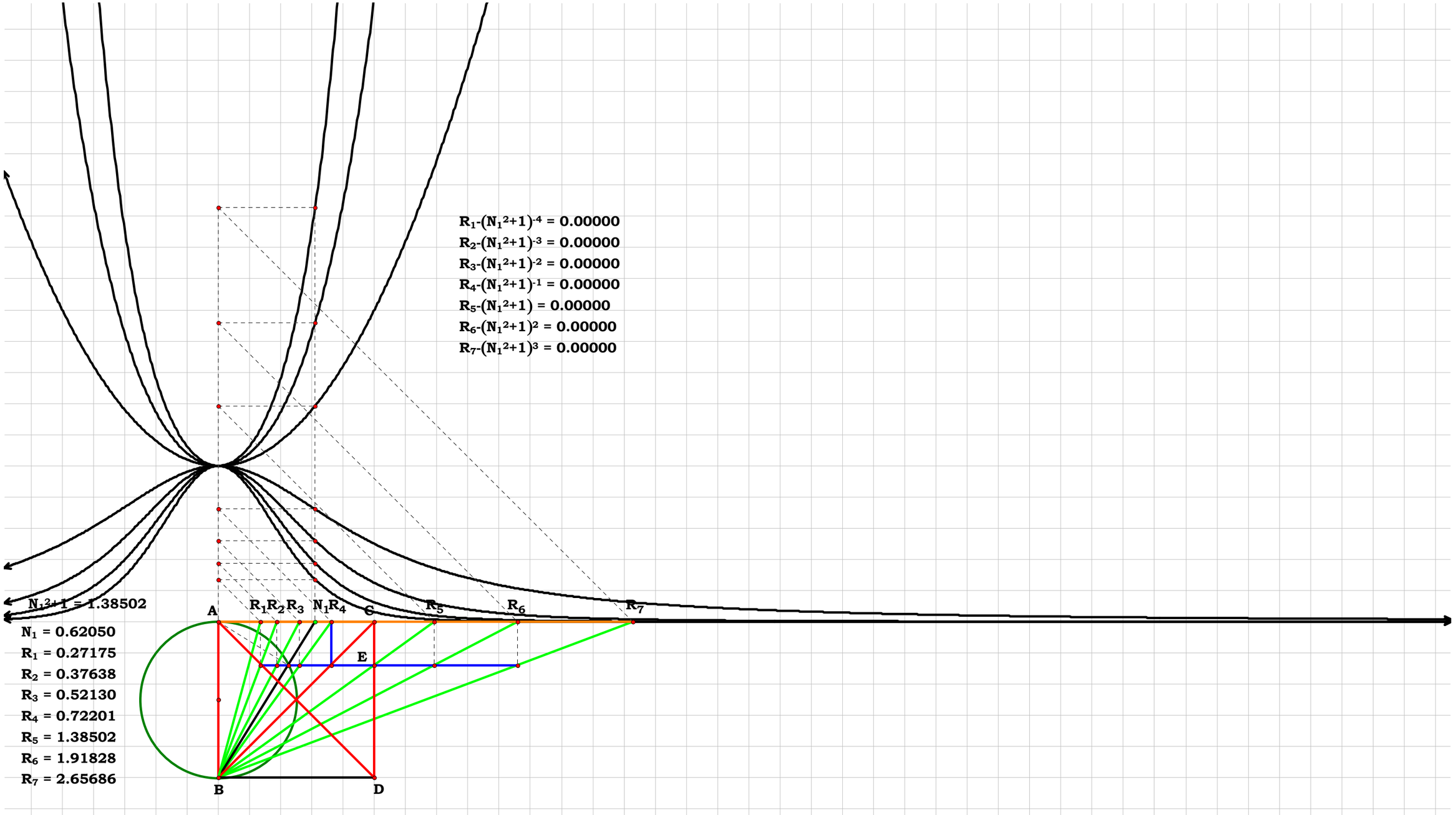

$DE := \frac{1}{N_1^2 + 1}$ $R_4 := DE$ $R_3 := R_4^2$ $R_2 := R_4^3$

$R_1 := R_4^4$ $R_5 := R_4^{-1}$ $R_6 := R_4^{-2}$ $R_7 := R_4^{-3}$

$R_1 - \left(N_1^2 + 1\right)^{-4} = 0$ $R_2 - \left(N_1^2 + 1\right)^{-3} = 0$ $R_3 - \left(N_1^2 + 1\right)^{-2} = 0$ $R_4 - \left(N_1^2 + 1\right)^{-1} = 0$

$R_5 - \left(N_1^2 + 1\right) = 0$ $R_6 - \left(N_1^2 + 1\right)^2 = 0$ $R_7 - \left(N_1^2 + 1\right)^3 = 0$

$R_1 = 0.291764$
$R_2 = 0.396985$
$R_3 = 0.540152$
$R_4 = 0.73495$
$R_5 = 1.360636$
$R_6 = 1.851331$
$R_7 = 2.518988$

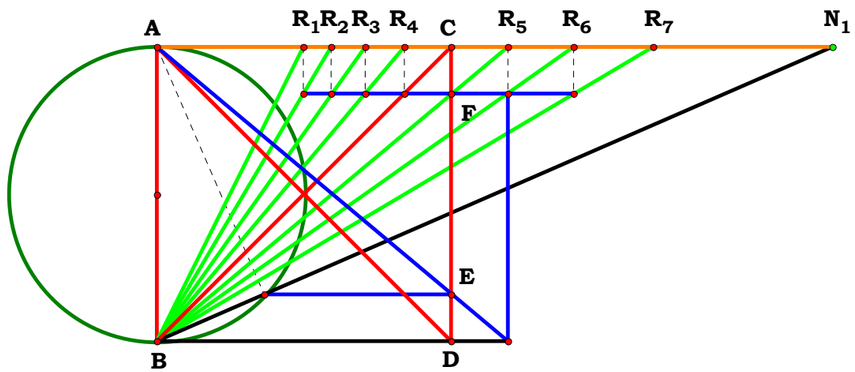

$R_1$-$(N_1^2+1)^{-4}$ = 0.00000
$R_2$-$(N_1^2+1)^{-3}$ = 0.00000
$R_3$-$(N_1^2+1)^{-2}$ = 0.00000
$R_4$-$(N_1^2+1)^{-1}$ = 0.00000
$R_5$-$(N_1^2+1)$ = 0.00000
$R_6$-$(N_1^2+1)^2$ = 0.00000
$R_7$-$(N_1^2+1)^3$ = 0.00000
$N_1^2+1$ = 1.38502
$N_1$ = 0.62050
$R_1$ = 0.27175
$R_2$ = 0.37638
$R_3$ = 0.52130
$R_4$ = 0.72201
$R_5$ = 1.38502
$R_6$ = 1.91828
$R_7$ = 2.65686
A
$R_1$ $R_2$ $R_3$ $N_1$ $R_4$
C
$R_5$
$R_6$
$R_7$
E
B
D

Circles-Plate 12

$AB := 1$

$N_1 := 2.29403$

$N_1 = 2.29403$
$R_1 = 0.49863$
$R_2 = 0.59339$
$R_3 = 0.70614$
$R_4 = 0.84032$
$R_5 = 1.19002$
$R_6 = 1.41615$
$R_7 = 1.68525$



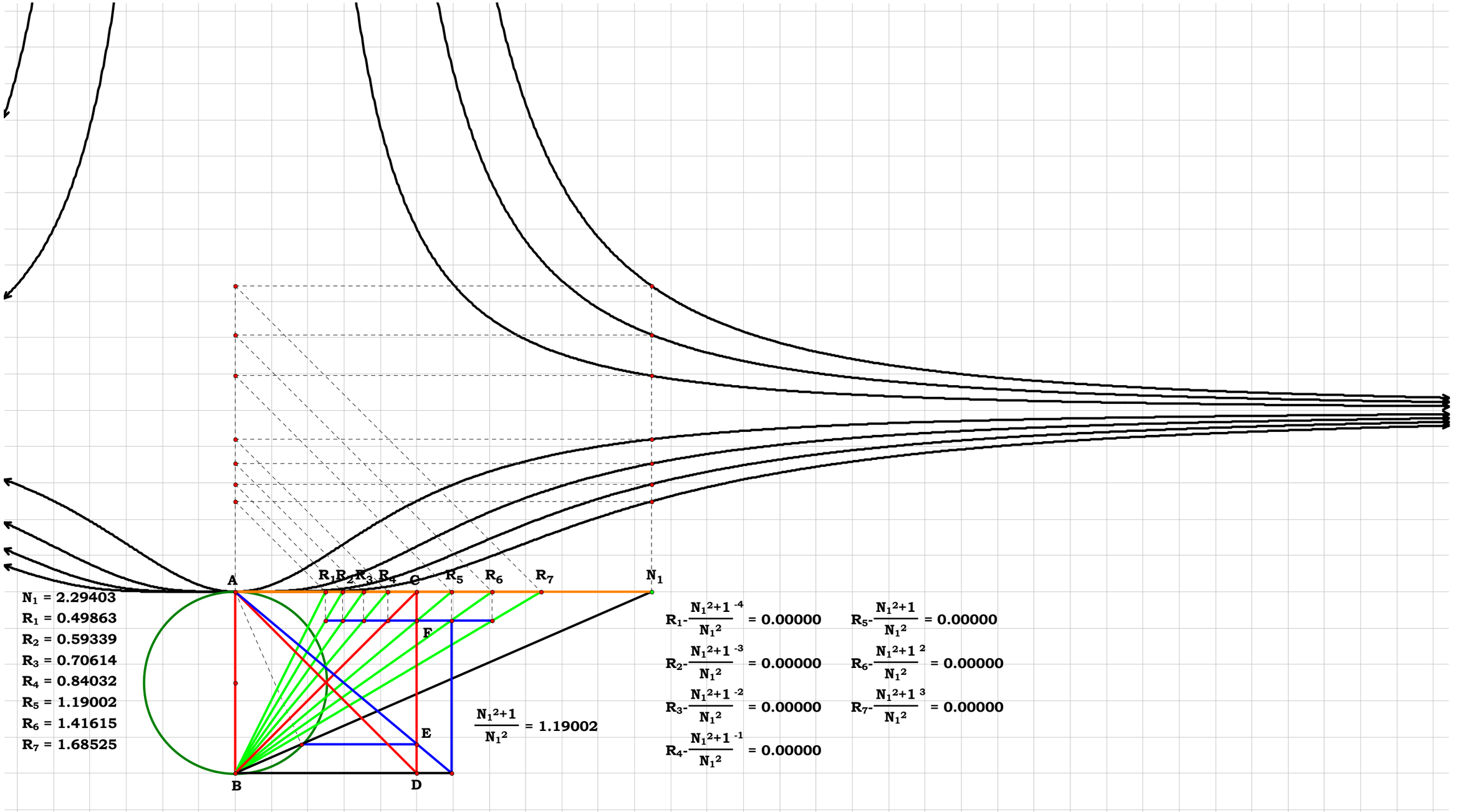

$DE := \frac{1}{N_1^{\,2} + 1}$ $R_5 := \frac{1}{1 - DE}$

$R_4 := R_5^{\,-1}$ $R_3 := R_4^{\,2}$ $R_2 := R_4^{\,3}$ $R_1 := R_4^{\,4}$

$R_6 := R_5^{\,2}$ $R_7 := R_5^{\,3}$

$R_1 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{-4} = 0$ $R_2 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{-3} = 0$ $R_3 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{-2} = 0$ $R_4 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{-1} = 0$

$R_5 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{1} = 0$ $R_6 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{2} = 0$ $R_7 - \left(\frac{N_1^{\,2} + 1}{N_1^{\,2}}\right)^{3} = 0$

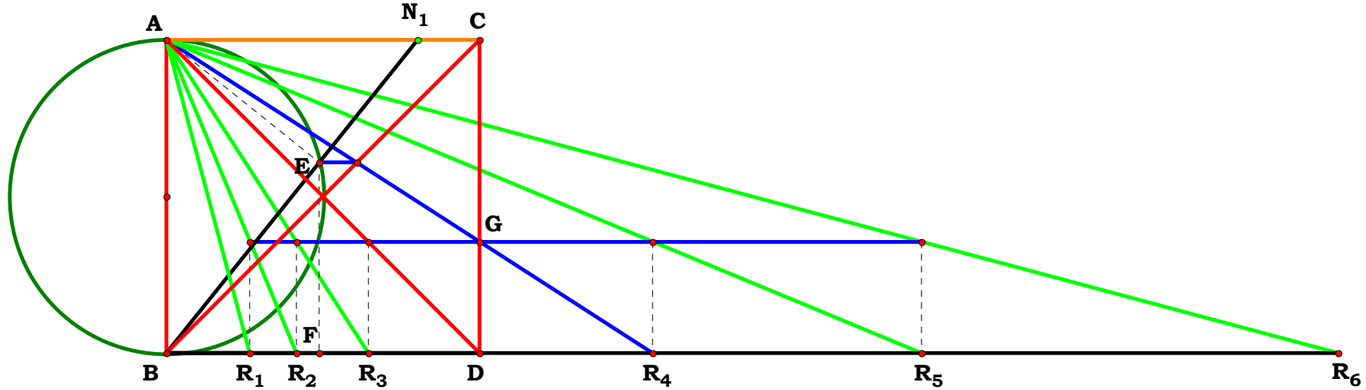

A
B
C
D
E
F
$R_1$ $R_2$ $R_3$ $R_4$ $R_5$ $R_6$ $R_7$
$N_1$
$N_1$ = 2.29403
$R_1$ = 0.49863
$R_2$ = 0.59339
$R_3$ = 0.70614
$R_4$ = 0.84032
$R_5$ = 1.19002
$R_6$ = 1.41615
$R_7$ = 1.68525
$\frac{N_1^2+1}{N_1^2}$ = 1.19002
$R_1-\frac{N_1^2+1}{N_1^2}^{-4}$ = 0.00000
$R_2-\frac{N_1^2+1}{N_1^2}^{-3}$ = 0.00000
$R_3-\frac{N_1^2+1}{N_1^2}^{-2}$ = 0.00000
$R_4-\frac{N_1^2+1}{N_1^2}^{-1}$ = 0.00000
$R_5-\frac{N_1^2+1}{N_1^2}$ = 0.00000
$R_6-\frac{N_1^2+1}{N_1^2}^{2}$ = 0.00000
$R_7-\frac{N_1^2+1}{N_1^2}^{3}$ = 0.00000

Circles-Plate 13

$AB := 1$

$N_1 := .80275$

$N_1 = 0.80275$
$R_1 = 0.26761$
$R_2 = 0.41527$
$R_3 = 0.64441$
$R_4 = 1.55180$
$R_5 = 2.40807$
$R_6 = 3.73684$



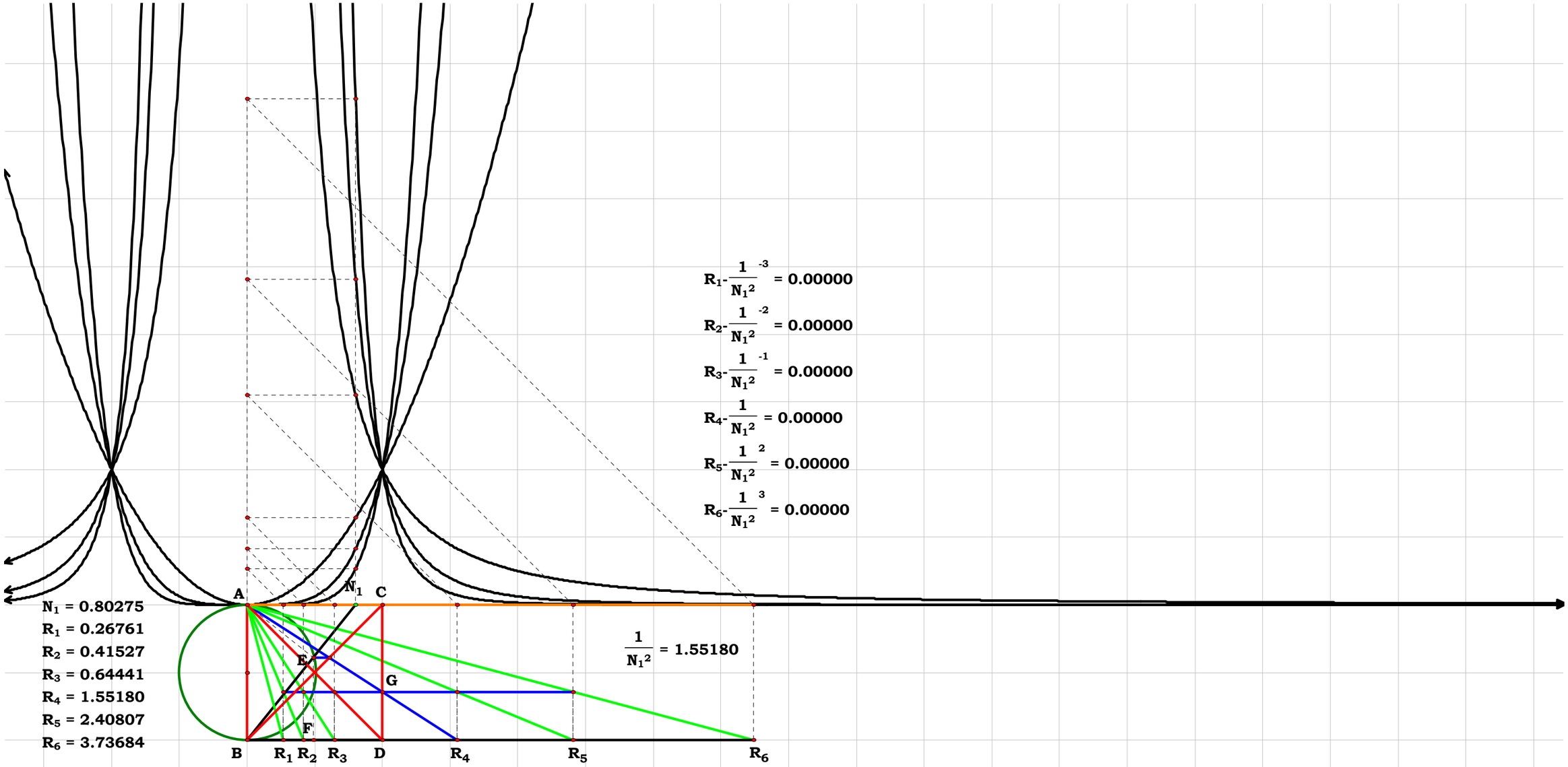

$EF := \frac{1}{N_1^2 + 1}$ $R_4 := \frac{EF}{1 - EF}$ $CG := \frac{1}{R_4}$

$R_3 := R_4^{-1}$ $R_2 := R_4^{-2}$ $R_1 := R_4^{-3}$ $R_5 := R_4^{2}$ $R_6 := R_4^{3}$

$R_1 - \left(\frac{1}{N_1^2}\right)^{-3} = 0$ $R_2 - \left(\frac{1}{N_1^2}\right)^{-2} = 0$ $R_3 - \left(\frac{1}{N_1^2}\right)^{-1} = 0$

$R_4 - \left(\frac{1}{N_1^2}\right) = 0$ $R_5 - \left(\frac{1}{N_1^2}\right)^{2} = 0$ $R_6 - \left(\frac{1}{N_1^2}\right)^{3} = 0$

$R_1 = 0.267597$
$R_2 = 0.415261$
$R_3 = 0.644408$
$R_4 = 1.551813$
$R_5 = 2.408123$
$R_6 = 3.736957$

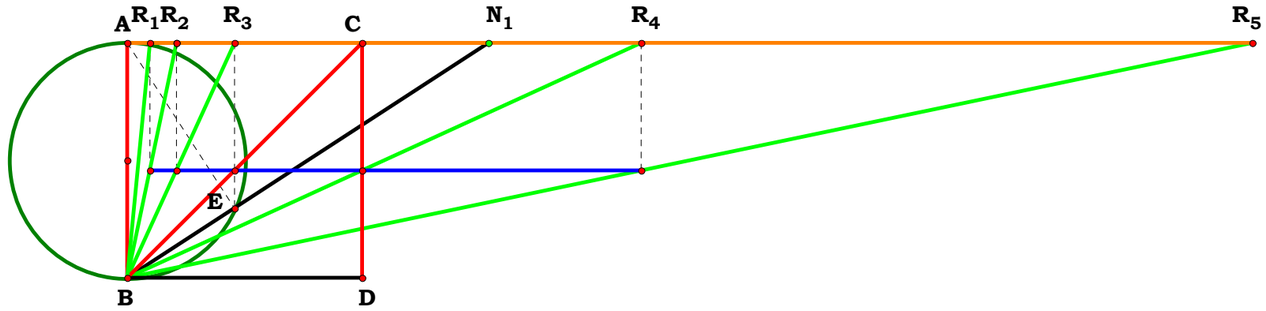

$R_1\text{-}\frac{1}{N_1^2}^{-3} = 0.00000$
$R_2\text{-}\frac{1}{N_1^2}^{-2} = 0.00000$
$R_3\text{-}\frac{1}{N_1^2}^{-1} = 0.00000$
$R_4\text{-}\frac{1}{N_1^2} = 0.00000$
$R_5\text{-}\frac{1}{N_1^2}^{2} = 0.00000$
$R_6\text{-}\frac{1}{N_1^2}^{3} = 0.00000$
$N_1 = 0.80275$
$R_1 = 0.26761$
$R_2 = 0.41527$
$R_3 = 0.64441$
$R_4 = 1.55180$
$R_5 = 2.40807$
$R_6 = 3.73684$
$\frac{1}{N_1^2} = 1.55180$
A
B
C
D
E
F
G
$N_1$
$R_1$
$R_2$
$R_3$
$R_4$
$R_5$
$R_6$

Circles-Plate 14

$AB := 1$

$N_1 := 1.53604$

$N_1 = 1.53604$
$R_1 = 0.09559$
$R_2 = 0.20906$
$R_3 = 0.45723$
$R_4 = 2.18707$
$R_5 = 4.78326$

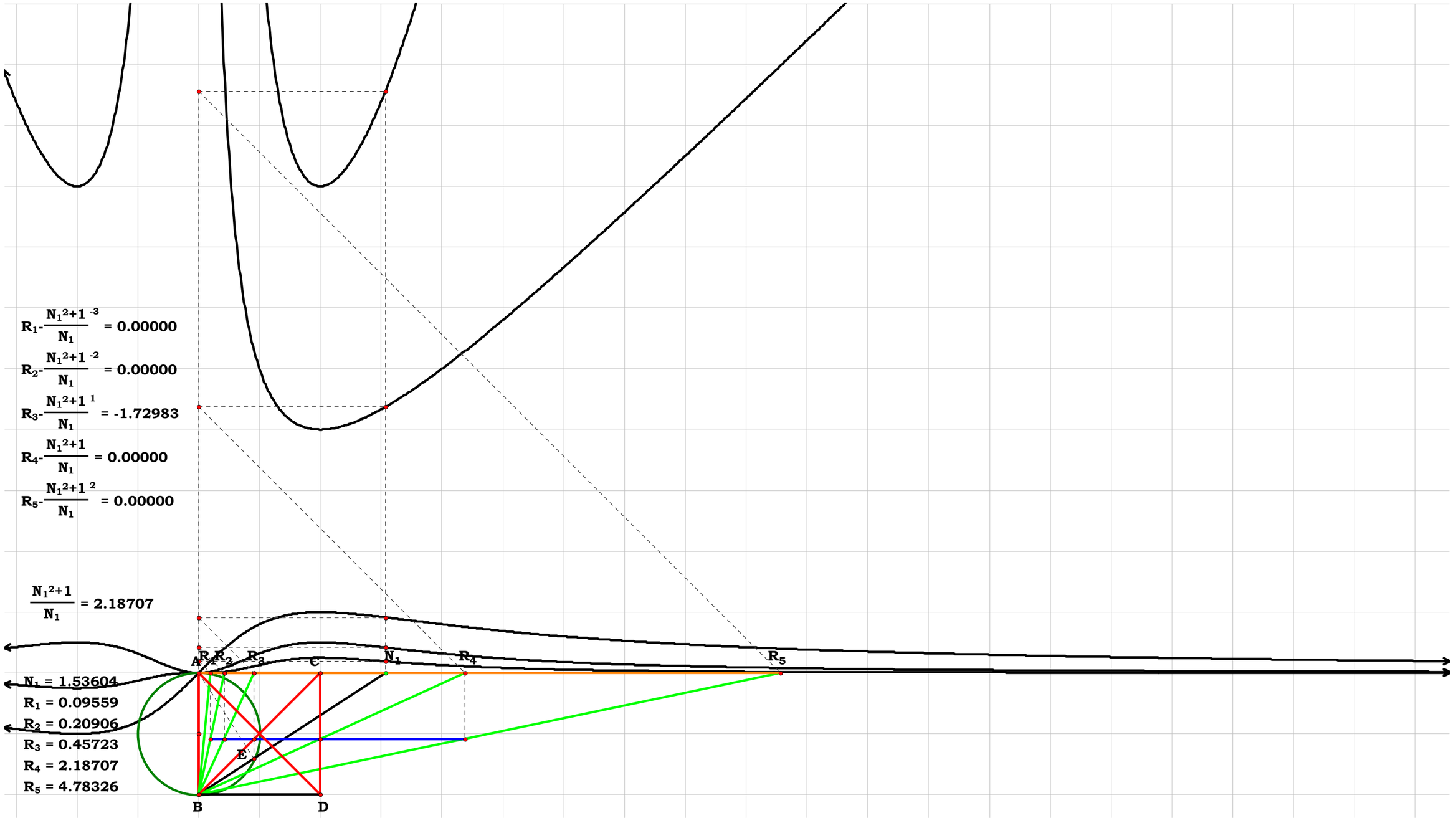


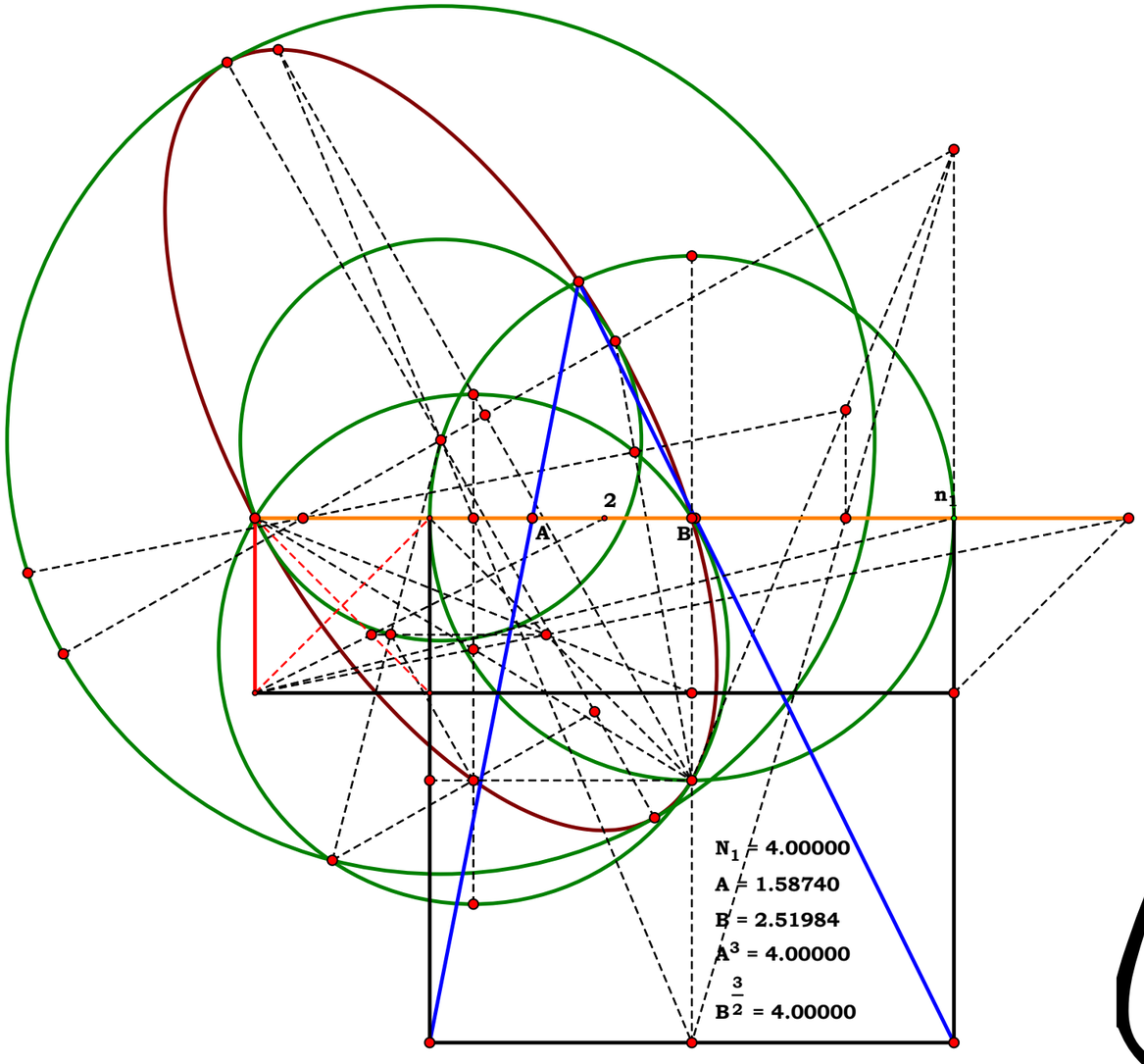

$R_3 := \frac{N_1}{N_1^2 + 1}$ $R_2 := R_3^2$ $R_1 := R_3^3$

$R_4 := R_3^{-1}$ $R_5 := R_3^{-2}$

$R_1 - \left[\frac{\left(N_1^2 + 1\right)}{N_1}\right]^{-3} = 0$ $R_2 - \left[\frac{\left(N_1^2 + 1\right)}{N_1}\right]^{-2} = 0$ $R_3 - \left[\frac{\left(N_1^2 + 1\right)}{N_1}\right]^{-1} = 0$

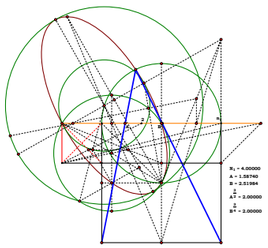
$R_4 - \left[\frac{\left(N_1^2 + 1\right)}{N_1}\right] = 0$ $R_5 - \left[\frac{\left(N_1^2 + 1\right)}{N_1}\right]^{2} = 0$


$R_1 - \frac{N_1^2+1}{N_1}^{-3} = 0.00000$
$R_2 - \frac{N_1^2+1}{N_1}^{-2} = 0.00000$
$R_3 - \frac{N_1^2+1}{N_1}^{1} = -1.72983$
$R_4 - \frac{N_1^2+1}{N_1} = 0.00000$
$R_5 - \frac{N_1^2+1}{N_1}^{2} = 0.00000$
$\frac{N_1^2+1}{N_1} = 2.18707$
A
B
C
D
E
$R_1$
$R_2$
$R_3$
$N_1$
$R_4$
$R_5$
$N_1 = 1.53604$
$R_1 = 0.09559$
$R_2 = 0.20906$
$R_3 = 0.45723$
$R_4 = 2.18707$
$R_5 = 4.78326$




# The Delian Quest 2019

**John Clark**

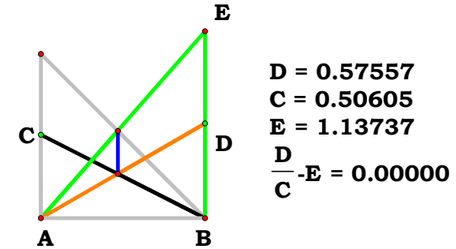


# Two Triangles

Saturday, April 6, 2019

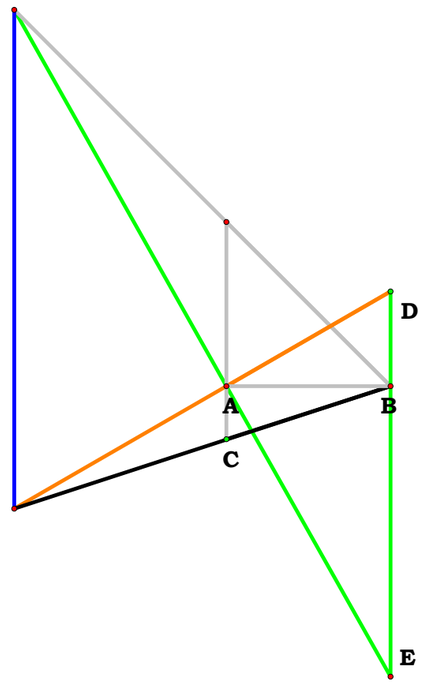
Just the first book or two of Euclid's *Elements* should give one enough information to discover *Basic Analog Mathematics*. However, it apparently did not happen. The reason being is that it is a whole lot easier to repeat and memorize perceptible information than to see the intelligible being expressed.

In this little essay, I am not going to say much, I am just going to present a little figure which I call Two Triangles. Just imagine what you can do with two right angles on the same base. I can call one ACB and the other ADB. I will simply present a series of plates which only differ in so far as a point go.

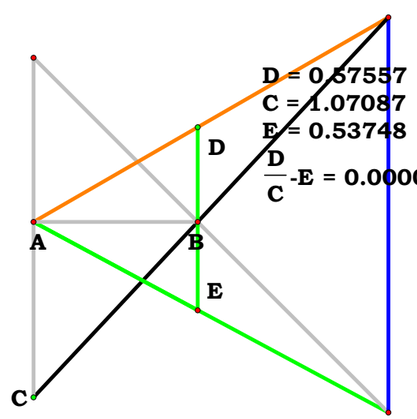



Let us call AB the standard unit by which C and D and their produce E is named. We simply have a right triangle to perform our operation. Call it AB. The intersection of the two triangles perpendicular to the base AB fall on what we can call the segment known to be the square root of 2. We are not, however, interested where it intersects the segment, but only that it intersects the line which contains the segment. We are interested in it only insofar as it expresses and projects a ratio.

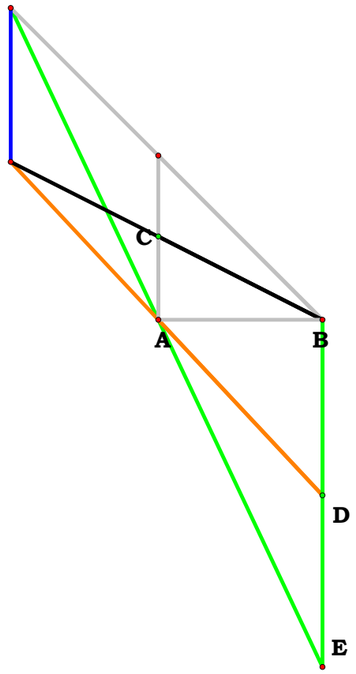
Now, we can give C and D any values we like. We can imagine them as triangles ACB and ADB, however, we have no wish to think or speak in terms of the mystical angle. We adhere to the notion that a two-dimensional plane is expressible as a ratio between two units.



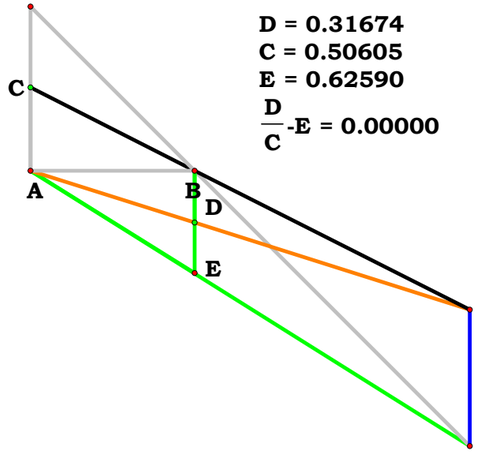

D = 0.57557
C = 0.32472
E = 1.77253
$\frac{D}{C}$-E = 0.00000



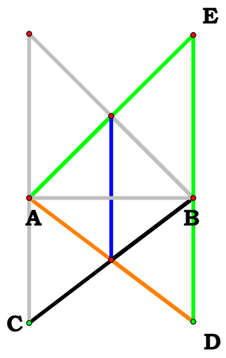

D = 0.57557
C = 1.07087
E = 0.53748
$\frac{D}{C}$-E = 0.00000




D = 1.07058
C = 0.50605
E = 2.11556
$\frac{D}{C}$-E = 0.00000




D = 0.31674
C = 0.50605
E = 0.62590
$\frac{D}{C}$-E = 0.00000




D = 0.75520
C = 0.76318
E = 0.98955
$\frac{D}{C}$-E = 0.00000

One of the things which BAM helps one with, or one of the things Geometry helps one with, is not to set the standard of understanding a figure based on the perceptible, which can be very confusing, but on the intelligible content established by standards. The standard references what is intersected while the mind is looking at where.

One may notice, even in the *Elements* as we have them today, propositions written up by a weaker mind writes up the same proposition in terms of cases based on perceptible location. One can see here, if they were done correctly, the equation never changes. Notice also that any and every other type of triangle can be found using in the figure. I am not interested in obtuse, acute, or any other name one can give to any other expression of a triangle. I am only interested in the fact that a two-dimensional matrix can produce results using an unit whatsoever when compared to another and I do not need Cartesian Geometry, Trigonometry, or Calculus to do it.

The ability to equate an analog to its logical name is not, in any wise apparent to the eye. One has to find and use standards to express it, and comprehend it in the mind. You can call an analog an isosceles right triangle in gray, or a method of dividing two given things of the same relative difference in accordance with a standard unit. All of the other so called triangles are simply parts of a much bigger and better ordered universe.

Let us take our little figure, Lay it on its side and imagine that C and D are on two parallels and AB is just a unit.

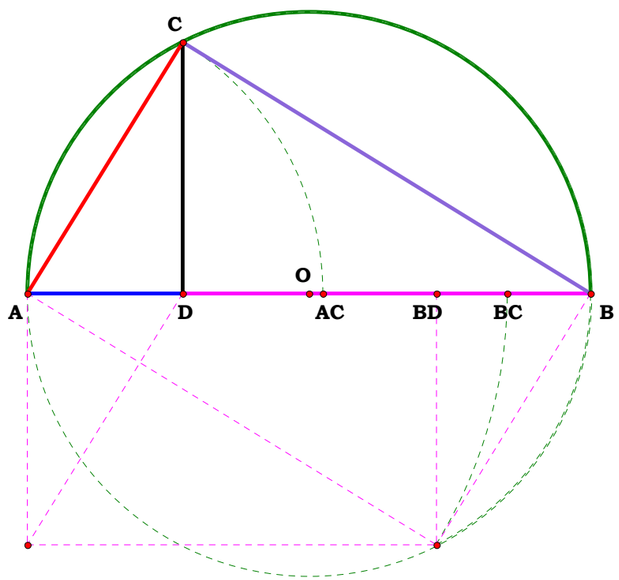
111619

**Descriptions.**

**Basic Givens for a right triangle. Although it is common to recite only one of these results, errantly called the Pythagorean Theorem, it helps to at least suspect the entire set attempting to write up a figure. One should consider a right triangle to consit of six distinct segments.**

**And if one muse somewhat further, one can claim that it is a fractile, which simply means the recurson of a unit behavior on a figure which produces proportional results.**

## Right Triangle Basic Math




$\frac{CD^2}{AD}$-BD = 0.00000
$\frac{CD^2}{BD}$-AD = 0.00000
$\frac{AC^2}{AB}$-AD = 0.00000
$\frac{AC^2}{AD}$-AB = 0.00000
$\frac{BC^2}{AB}$-BD = 0.00000
$\frac{BC^2}{BD}$-AB = 0.00000

AB = 2.00000
AC = 1.04881
BC = 1.70294
AD = 0.55000
BD = 1.45000
CD = 0.89303

$AB^2$ = 4.00000
$AC^2$ = 1.10000
$AD^2$ = 0.30250
$BC^2$ = 2.90000
$BD^2$ = 2.10250
$CD^2$ = 0.79750

$(AC^2+BC^2)-AB^2$ = 0.00000
$(AD^2+CD^2)-AC^2$ = 0.00000
$(CD^2+BD^2)-BC^2$ = 0.00000

Hide Point A
Hide Point B
Show Base Line Points (20)

X0 X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15 X16 X17 X18 X19 X20

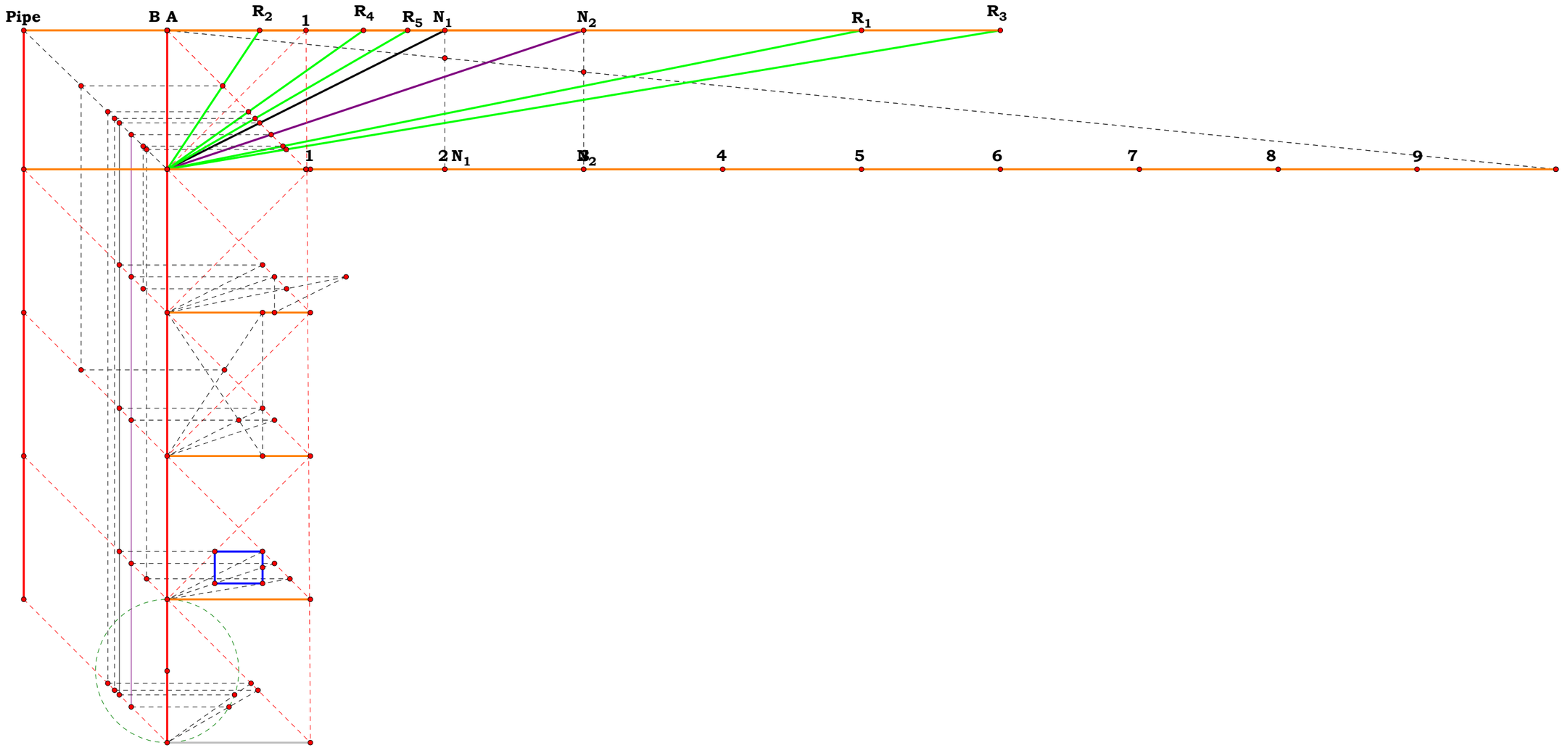
Hide Intersections
1 2 3 4 5 6 7 8 9 10 0

Hide Intersections
1 2 3 4 5 6 7 8 9 10 0 Pipe

$A = 0.00000$
$N_1 = 2.00000$
$N_2 = 3.00000$
$R_1 = 5.00000$
$R_2 = 0.66667$
$R_3 = 6.00000$
$R_4 = 1.41421$
$R_5 = 1.73205$

$N_1+N_2 = 5.00000$
$\frac{N_1}{N_2} = 0.66667$
$N_1 \cdot N_2 = 6.00000$
$\sqrt{N_1} = 1.41421$
$\sqrt{N_2} = 1.73205$

Y0 Y1 Y2 Y3 Y4 Y5 Y6 Y7 Y8 Y9 Y10 Y11 Y12 Y13 Y14 Y15 Y16 Y17 Y18 Y19 Y20



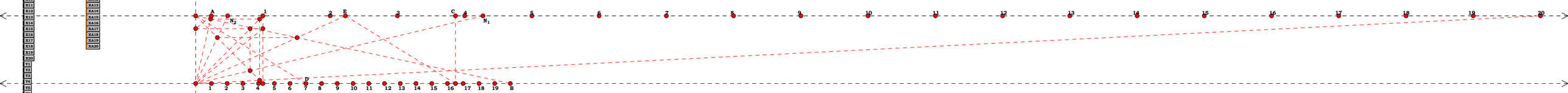

$A = 0.00000$
$B = 0.00000$

Hide Point N[1]
Hide Point N[2]
Hide Base Line Points [20]
X1
X2
X3
X4
X5
X6
X7
X8
X9
X10
X11
X12
X13
X14
X15
X16
X17
X18
X19
X20
Y1
Y2
Y3
Y4
Y5
Y6
Y7
Y8
Y9
Y10
Y11
Y12
Y13
Y14
Y15
Y16
Y17
Y18
Y19
Y20

Hide Points

Y0
XA1
XA2
XA3
XA4
XA5
XA6
XA7
XA8
XA9
XA10
XA11
XA12
XA13
XA14
XA15
XA16
XA17
XA18
XA19
XA20

**Constructing a numbered line which is adjustable in steps of 20. N1 will set the range, and N2 one can select the working domain. This will go up to 20 squared.**




$N_1 = 4.27256$
$N_2 = 0.47553$
$A = 0.23405$
$B = 4.68103$
$C = 3.86434$
$D = 1.63836$
$E = 2.22598$

$N_1 \cdot 20 = 85.45125$

$\frac{N_2 \cdot B}{A} = 9.51062$

$\frac{C}{A} = 16.51062$

$\frac{D}{A} = 7.00000$

$\frac{20}{A} = 85.45125$

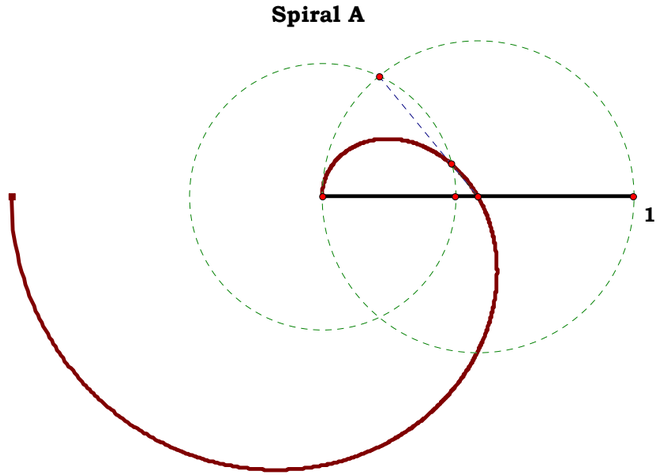
$\frac{B}{A} = 20.00000$

$\frac{E}{A} = 9.51062$

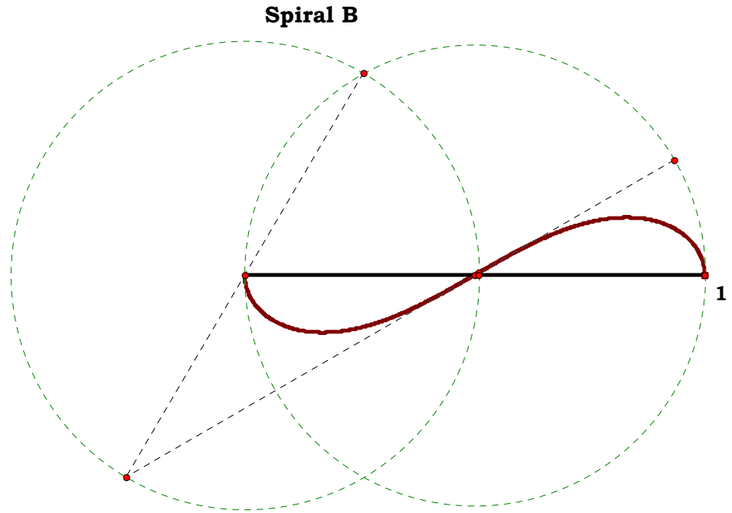
120119

# Simple Spirals

Spiral A




Spiral B



Unit. $AB := 1$

Given. $X := 12$

$Y := 20$

**120119A Spiral**

**Descriptions.**

$$AC := \frac{AB}{2} \quad CE := AC \quad AD := \frac{X}{Y} \quad AE := AD \quad AH := \frac{AE^2}{AB}$$

$$BD := AB - AD \quad BH := AB - AH \quad EH := \sqrt{(AH \cdot BH)}$$

$$CH := \sqrt{AC^2 - EH^2} \quad EK := \frac{AE}{2} \quad EJ := \frac{EK \cdot AD}{AC}$$

$$EF := 2 \cdot EJ \quad CF := |EF - CE| \quad CG := \frac{CH \cdot CF}{CE}$$

$$FG := \frac{EH \cdot CF}{CE} \cdot \frac{|AC - AD|}{AC - AD}$$

**Definitions.**

$$AC - \frac{1}{2} = 0 \quad CE - \frac{1}{2} = 0 \quad AD - \frac{X}{Y} = 0 \quad AE - \frac{X}{Y} = 0 \quad AH - \left(\frac{X}{Y}\right)^2 = 0$$

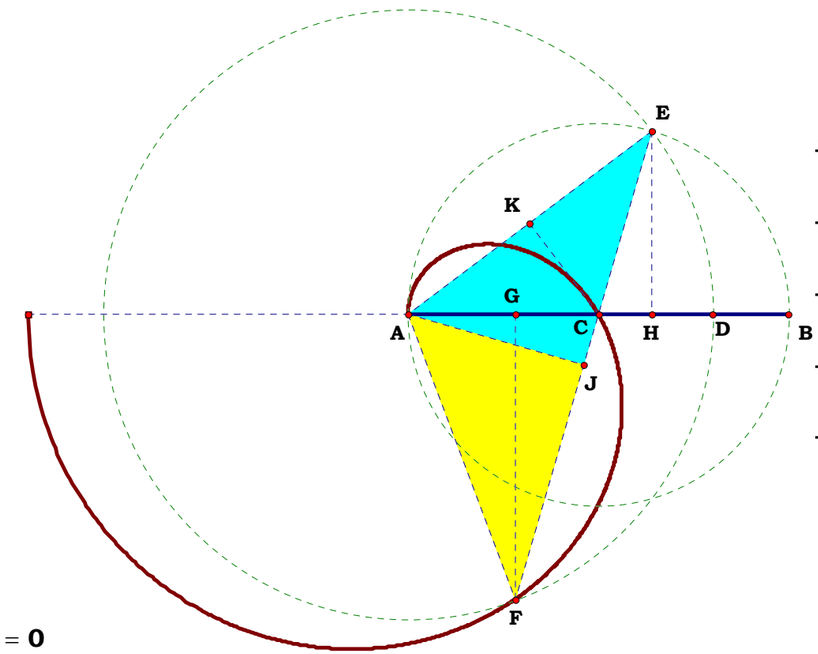
$$BD - \frac{Y - X}{Y} = 0 \quad BH - \frac{(Y - X) \cdot (X + Y)}{Y^2} = 0 \quad EH - \frac{X \cdot \sqrt{(Y - X) \cdot (X + Y)}}{Y^2} = 0$$

$$CH - \frac{|2 \cdot X^2 - Y^2|}{2 \cdot Y^2} = 0 \quad EK - \frac{X}{2 \cdot Y} = 0 \quad EJ - \frac{X^2}{Y^2} = 0 \quad EF - \frac{2 \cdot X^2}{Y^2} = 0$$

$$CF - \frac{|(2 \cdot X - Y) \cdot (2 \cdot X + Y)|}{2 \cdot Y^2} = 0 \quad CG - \frac{|(Y - 2 \cdot X) \cdot (2 \cdot X + Y)| \cdot |2 \cdot X^2 - Y^2|}{2 \cdot Y^4} = 0$$

$$FG - \frac{X \cdot |(Y - 2 \cdot X) \cdot (2 \cdot X + Y)| \cdot |Y - 2 \cdot X| \cdot \sqrt{(Y - X) \cdot (X + Y)}}{Y^4 \cdot (Y - 2 \cdot X)} = 0$$

## Spiral A




Unit = 1.00000

XY = 0.80000

X = 16.00000

Y = 20.00000

$\frac{X \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^2}$-EH = 0.00000

$\frac{|2 \cdot X^2 - Y^2|}{2 \cdot Y^2}$-CH = 0.00000

$\frac{X \cdot |(Y-2 \cdot X) \cdot (2 \cdot X+Y)| \cdot |Y-2 \cdot X| \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^4 \cdot (Y-2 \cdot X)}$-FG = 0.00000

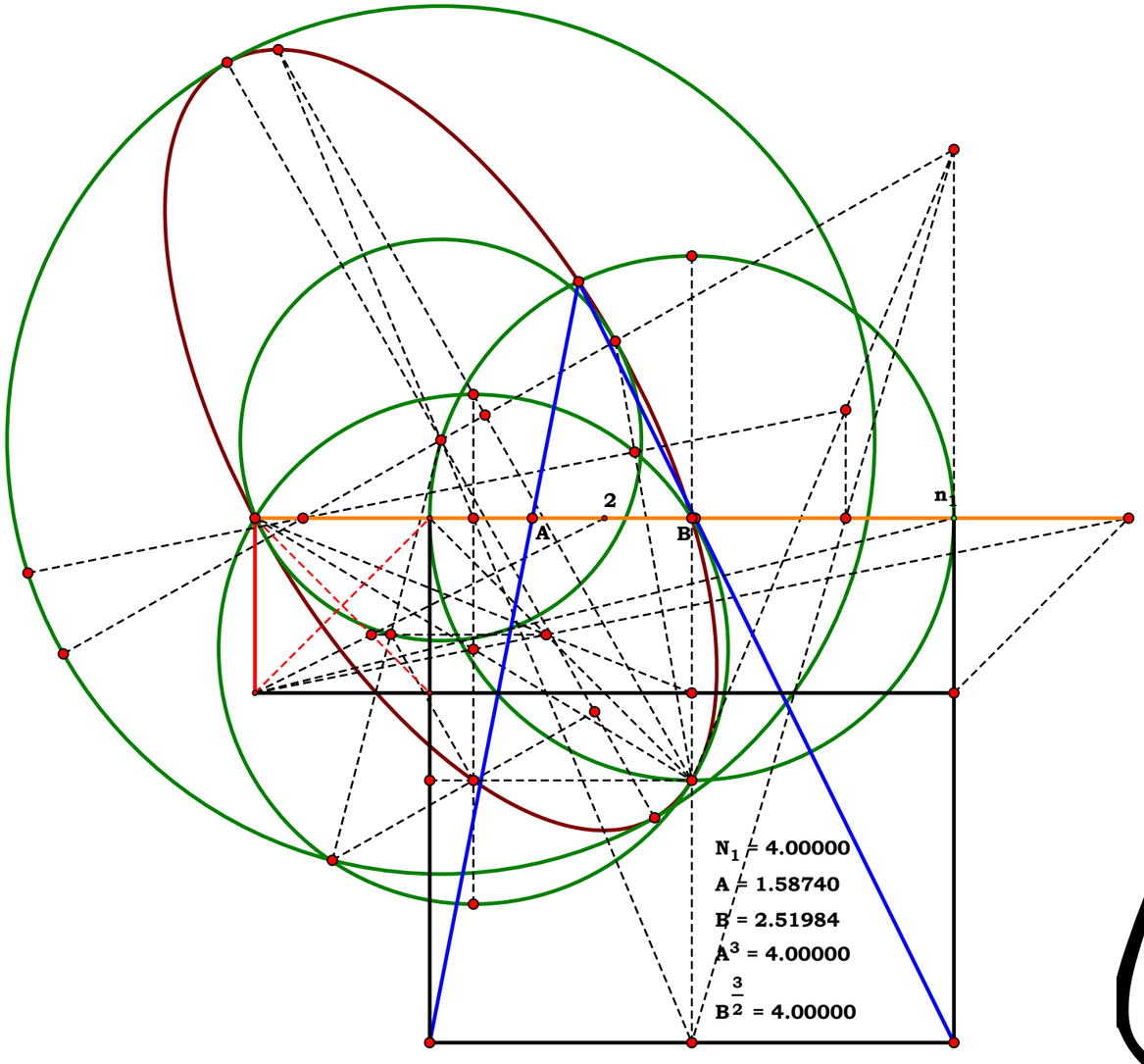
$\frac{|2 \cdot X^2 - Y^2| \cdot |(2 \cdot X-Y) \cdot (2 \cdot X+Y)|}{2 \cdot Y^4}$-CG = 0.00000

$\frac{X \cdot |(Y-2 \cdot X) \cdot (2 \cdot X+Y)| \cdot |Y-2 \cdot X| \cdot \sqrt{(Y-X) \cdot (X+Y)}}{Y^4 \cdot (Y-2 \cdot X)}$-FG = 0.00000

$\frac{(\text{Area } \triangle AFJ)}{(\text{Area } \triangle AJE)}$ = 1.00000

$\frac{FG}{CG}$ = 3.42857

$\frac{HE}{CH}$-$\frac{FG}{CG}$ = 0.00000




# The Delian Quest 2020

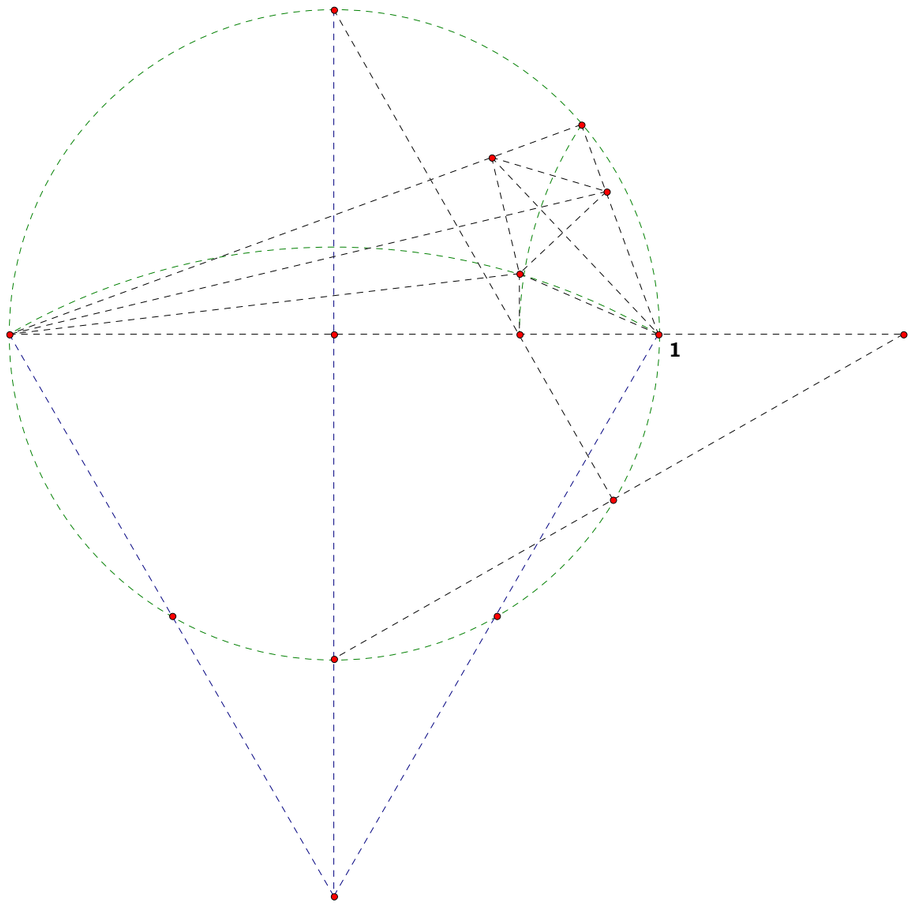
**John Clark**

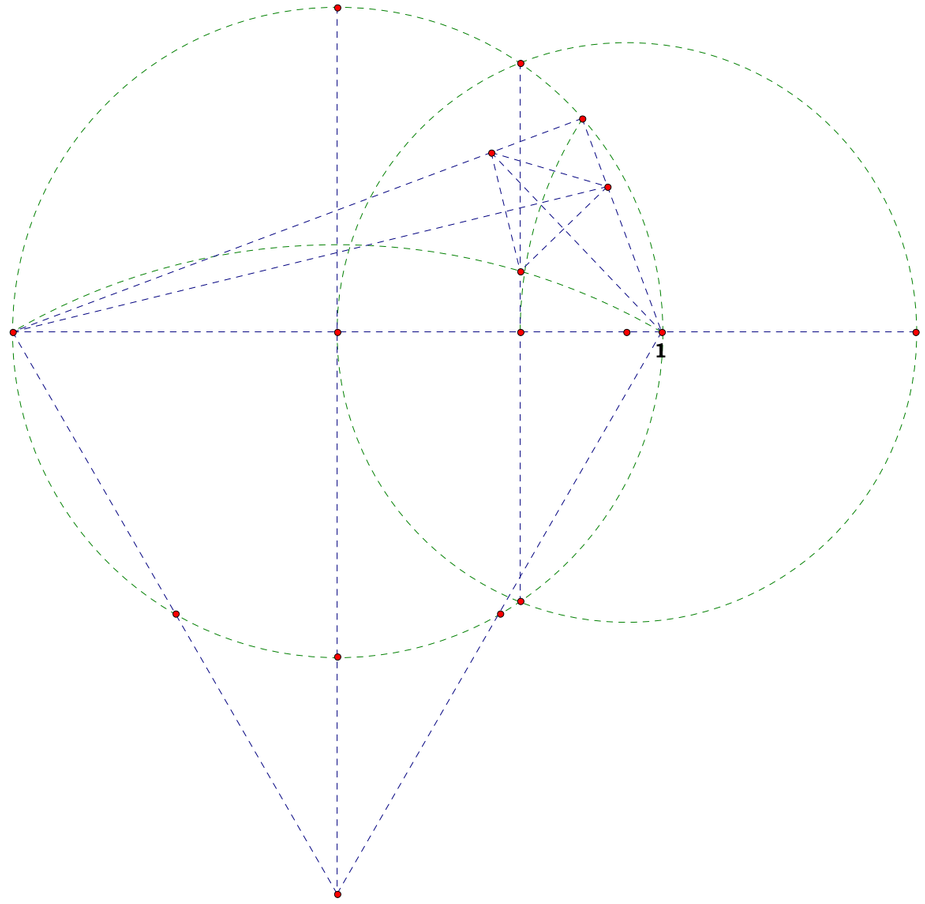
012220

**Redo 080400**

**Construction variation.**

**Trisecting any angle in Geometry is only possible when you know the difference between the perceptible and the intelleligible.**



Unit.

$AB := 1$

Given.

$X := 10$

$Y := 20$

012220

Descriptions.

What is the ratio of DO to OV?

$AO := \frac{AB}{2}$ $AD := AO + AO \cdot \frac{X}{Y}$ $AR := \frac{AO}{2}$ $RJ := \sqrt{AR \cdot (AB - AR)}$

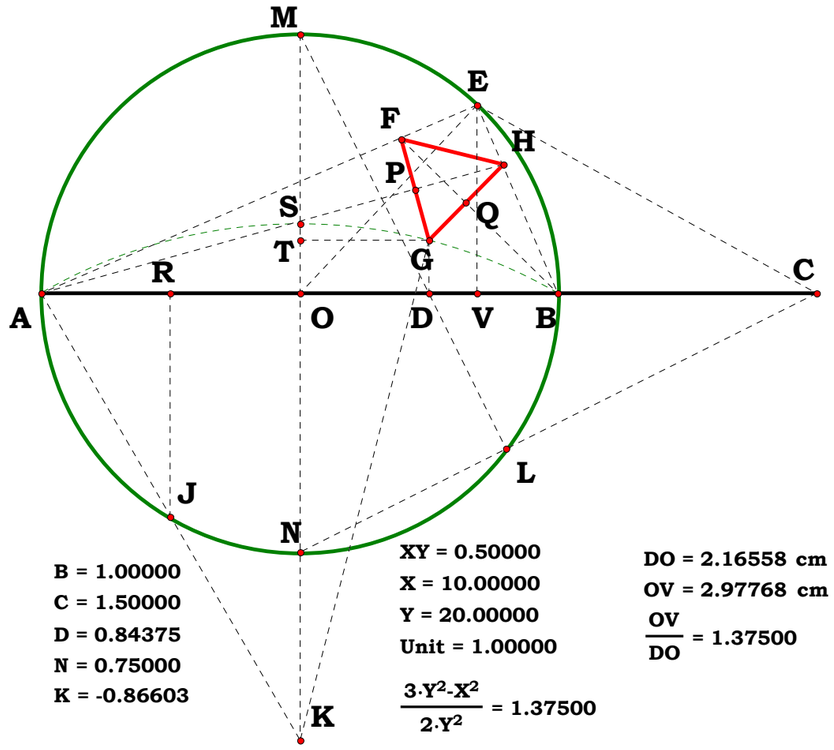
$KO := 2 \cdot RJ$ $DO := AD - AO$ $AJ := AO$ $AK := 2 \cdot AJ$ $EO := AO$

$KS := AK$ $GK := AK$ $KT := \sqrt{GK^2 - DO^2}$ $TO := KT - KO$

$DG := TO$ $MN := AB$ $MO := AO$ $DM := \sqrt{MO^2 + DO^2}$

$LN := \frac{DO \cdot MN}{DM}$ $CO := \frac{MO^2}{DO}$ $CD := CO - DO$ $CE := CD$

$OV := \frac{CO^2 + EO^2 - CE^2}{2 \cdot CO}$ $\frac{OV}{DO} = 1.375$ $\frac{DO}{OV} = 0.727273$




Definitions.

$AO - \frac{1}{2} = 0$ $AD - \frac{X + Y}{2 \cdot Y} = 0$ $AR - \frac{1}{4} = 0$ $RJ - \frac{\sqrt{3}}{4} = 0$

$KO - \frac{\sqrt{3}}{2} = 0$ $DO - \frac{X}{2 \cdot Y} = 0$ $AJ - \frac{1}{2} = 0$ $AK - 1 = 0$

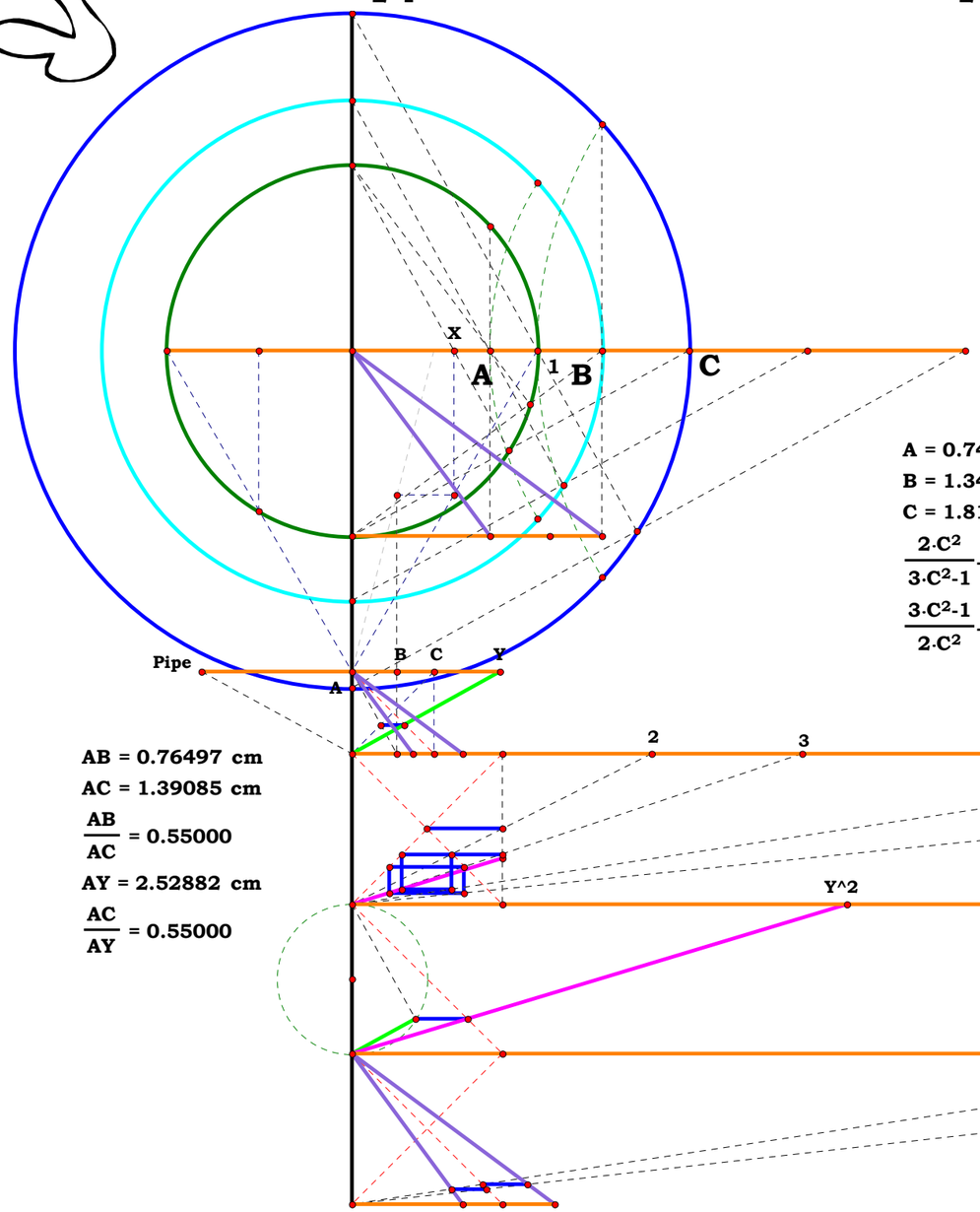
$EO - \frac{1}{2} = 0$ $KS - 1 = 0$ $GK - 1 = 0$ $KT - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot Y} = 0$ $TO - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$

$DG - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$ $MN - 1 = 0$ $MO - \frac{1}{2} = 0$ $DM - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$ $LN - \frac{X}{\sqrt{X^2 + Y^2}} = 0$

$CO - \frac{Y}{2 \cdot X} = 0$ $CD - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $CE - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $OV - \frac{X \cdot \left(3 \cdot Y^2 - X^2\right)}{4 \cdot Y^3} = 0$ $\frac{OV}{DO} - \frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} = 0$ $\frac{DO}{OV} - \frac{2 \cdot Y^2}{\left(3 \cdot Y^2 - X^2\right)} = 0$

One can say now that the length of the sides of an equalateral triangle in a right triangle used in trisection is $\frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y}$

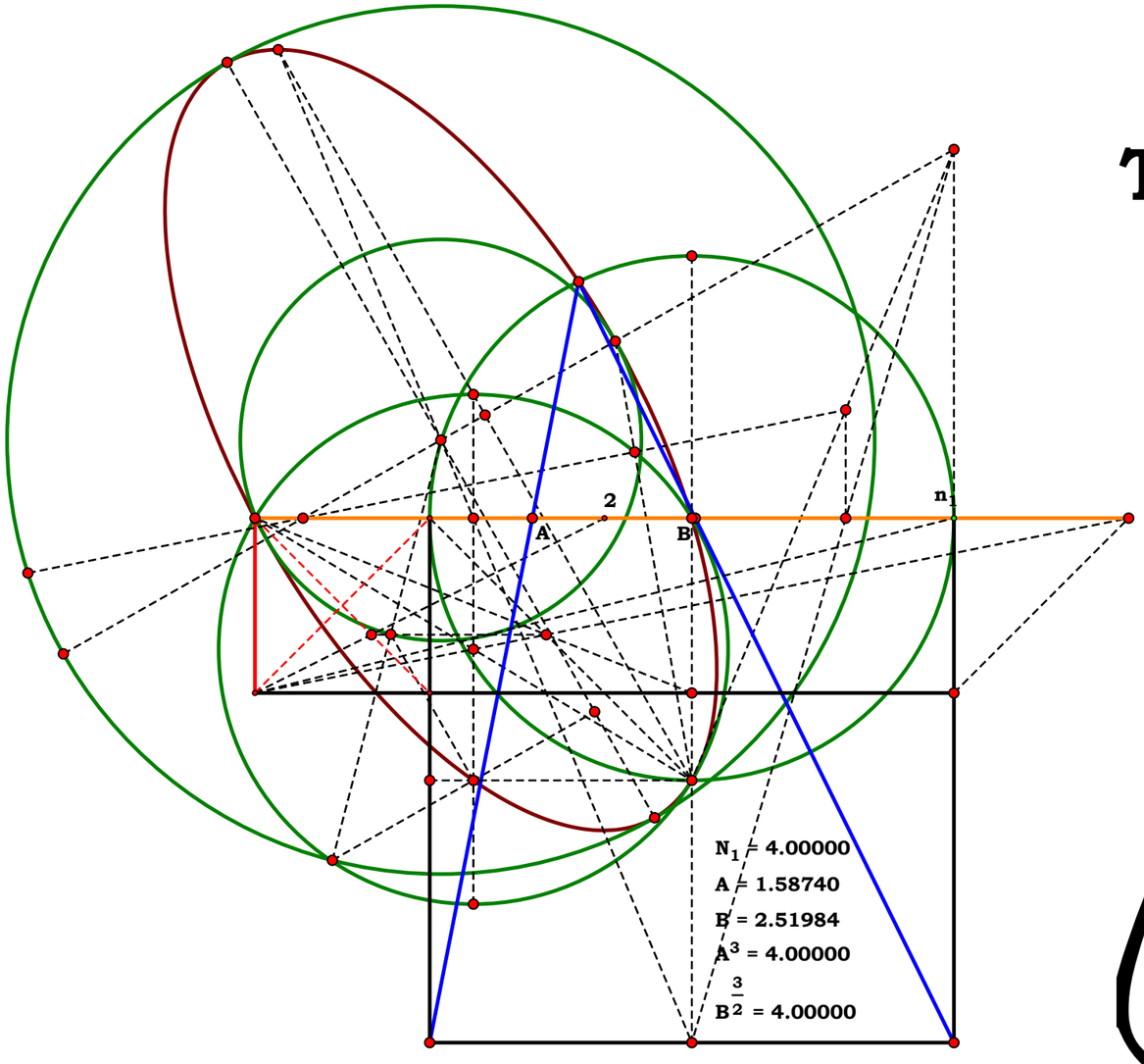
$\frac{X}{Y} = 0.5$ $\frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} = 1.375$ $OV = 0.34375$ $DO \cdot \frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} - OV = 0$ $DO = 0.25$ $OV \cdot \frac{2 \cdot Y^2}{\left(3 \cdot Y^2 - X^2\right)} - DO = 0$




Unit = 1.00000
$x_1$ = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000

A = 0.74143
B = 1.34875
C = 1.81818
$\frac{2 \cdot C^2}{3 \cdot C^2 - 1} - A = 0.00000$
$\frac{3 \cdot C^2 - 1}{2 \cdot C^2} - B = 0.00000$

AB = 0.76497 cm
AC = 1.39085 cm
$\frac{AB}{AC} = 0.55000$
AY = 2.52882 cm
$\frac{AC}{AY} = 0.55000$




# The Delian Quest Side Jobs

**John Clark**

**Descriptions.** **020320 Easy Cube**

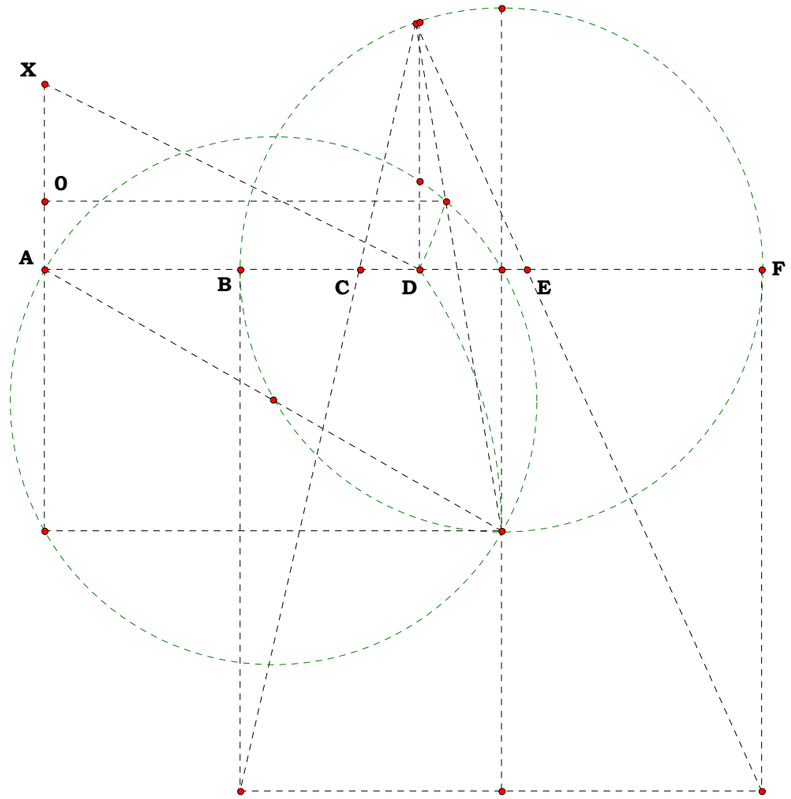
**Often, one would like to create an easy plate for cube roots, instead of doing the whole figure. Here is the simplest and most accurate way to achive it.**

**Draw X to A anywhere and then construct 0 parallel to AF. Have your macro make X seek 0.**
**You might believe that simple geometry can out Calculus Calculus, maybe in these plates you will change your mind. Calculus is a work frought with grammatical contradictions, Cartesian Geometry, Calculus, Trigonometry are not even grammatically correct. They are not derived from a correct concept of grammar as any possible grammar is afforded by complete induction and deduction of a simple binary unit.**

Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.46101
C = 1.60738
D = 1.91330

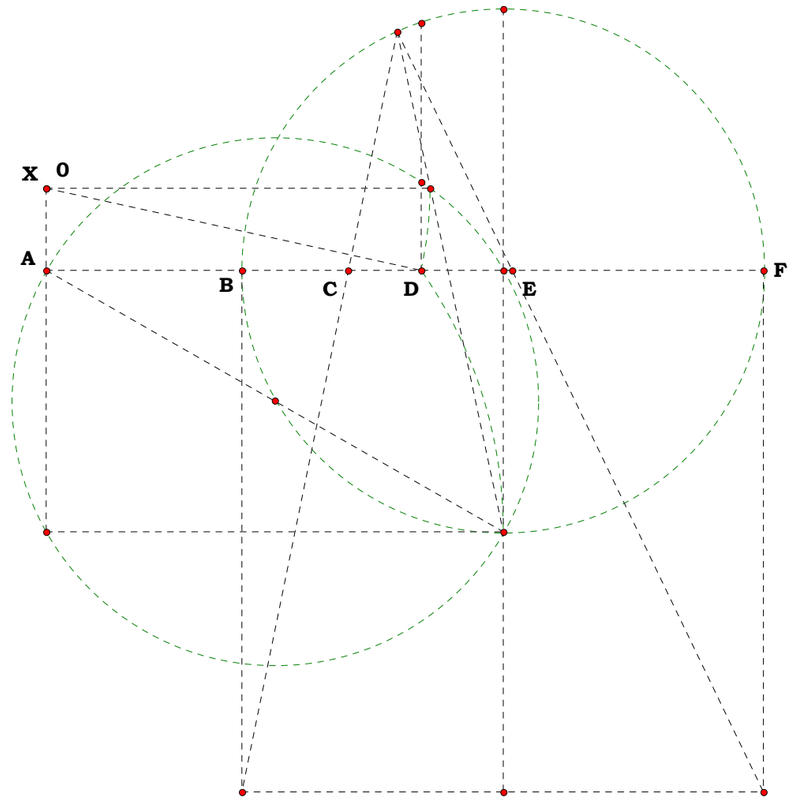
$F^{\frac{1}{2}}$-D = 0.00000
$F^{\frac{1}{3}}$-C = -0.06619
$F^{\frac{2}{3}}$-E = -0.08575




Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.37526
C = 1.54119
D = 1.91330

$F^{\frac{1}{2}}$-D = 0.00000
$F^{\frac{1}{3}}$-C = 0.00000
$F^{\frac{2}{3}}$-E = 0.00000

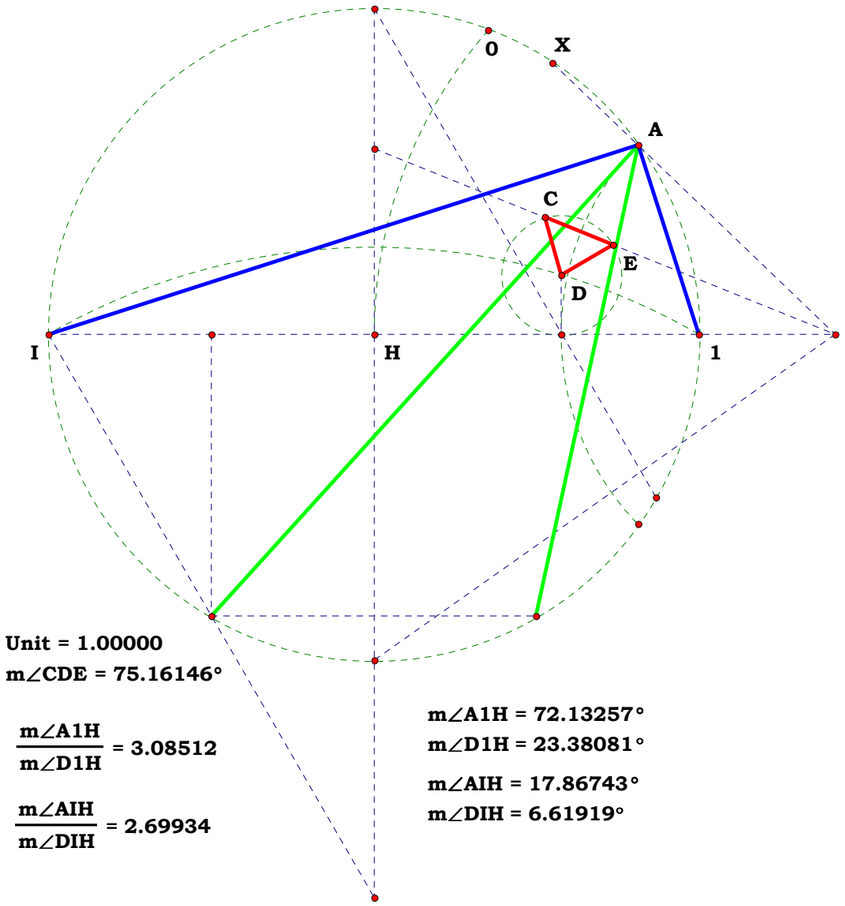


## 020320 Easy Trisection

**Descriptions.**

**Sometimes one is want to trisect some particular angle the easy way. Here it is. Draw it up with X anywhere and project 0. Have your macro make X seek 0. It is a whole lot neater than sliding a piece of paper.**

**By knowing the geometric end results, one can write algorithms which are a whole lot more accurate and efficient than by just using the traditional methods.**

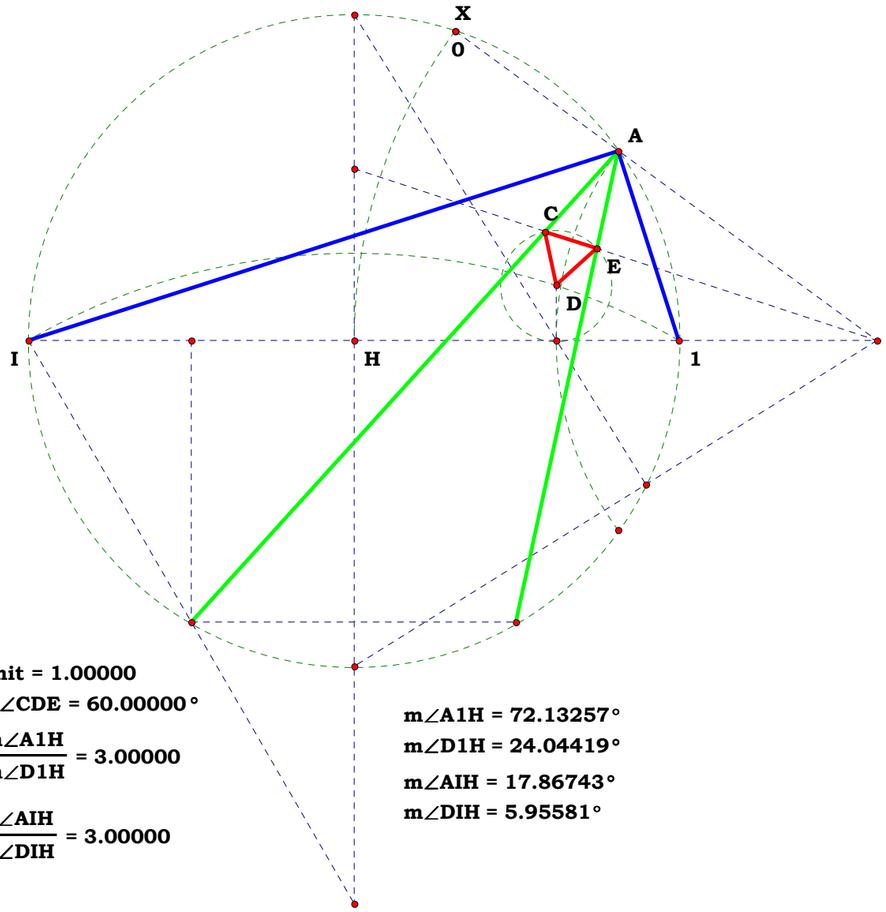



**Unit = 1.00000**
**m∠CDE = 75.16146°**

$$\frac{m\angle A1H}{m\angle D1H} = 3.08512$$

$$\frac{m\angle AIH}{m\angle DIH} = 2.69934$$

**m∠A1H = 72.13257°**
**m∠D1H = 23.38081°**
**m∠AIH = 17.86743°**
**m∠DIH = 6.61919°**




**Unit = 1.00000**
**m∠CDE = 60.00000°**

$$\frac{m\angle A1H}{m\angle D1H} = 3.00000$$

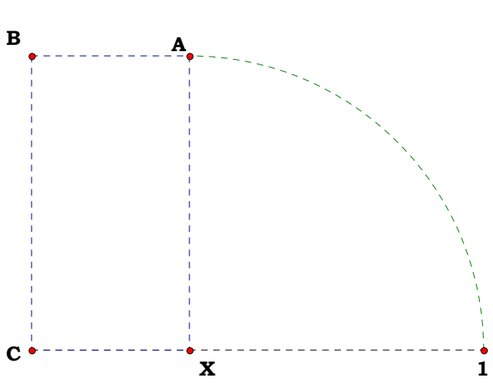
$$\frac{m\angle AIH}{m\angle DIH} = 3.00000$$

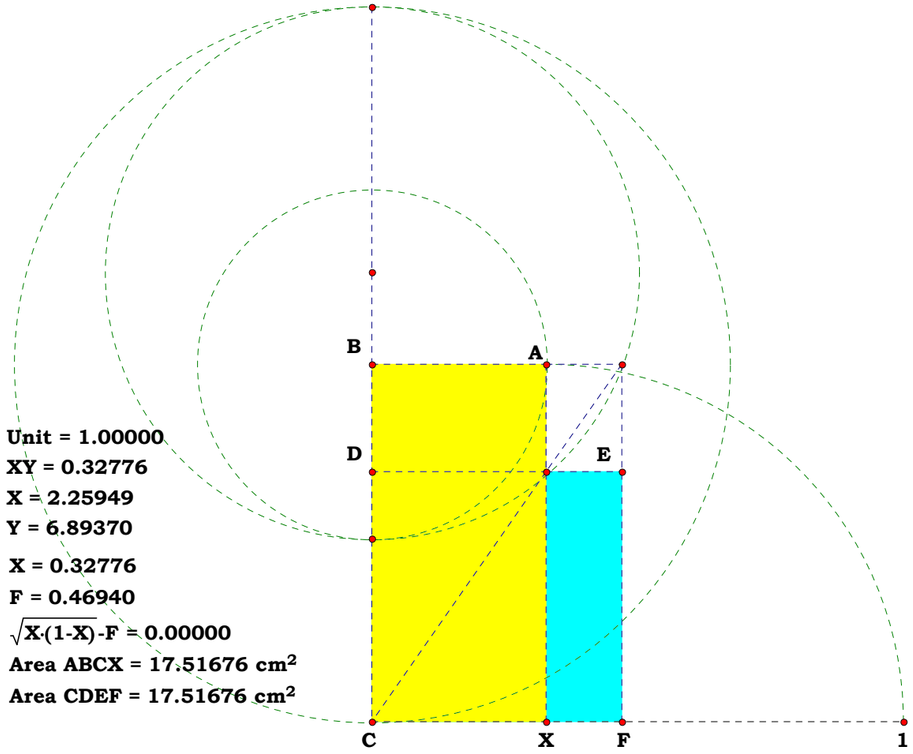
**m∠A1H = 72.13257°**
**m∠D1H = 24.04419°**
**m∠AIH = 17.86743°**
**m∠DIH = 5.95581°**

**Descriptions.**

**This is just another way to study rectangles, complements and square roots.**

# 020320 Squaring a Rectangle



# The Holy Grail

Sunday, February 23, 2020

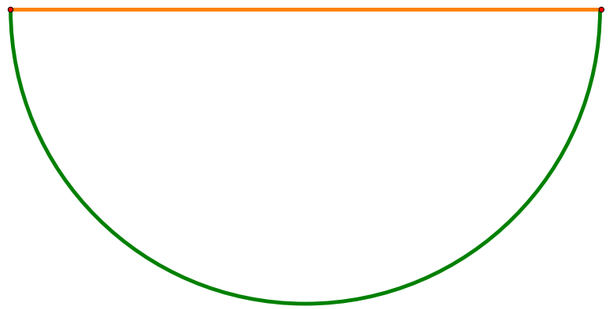
▸ noun

(the Grail or the Holy Grail) (in medieval legend) the cup or platter used by Christ at the Last Supper, and in which Joseph of Arimathea received Christ's blood at the Cross. Quests for it undertaken by medieval knights are described in versions of the Arthurian legends written from the early 13th century onward.

A thing which is eagerly pursued or sought after: the enterprise society where profit at any cost has become the holy grail.

– origin from Old French graal, from medieval Latin gradalis ‘dish’.

Most people, around the world believe that the Holy Grail, if they believe in the Grail, has been lost to history, but this is decidedly not true:—



There it is, it has also been called the Bowl of Siddhartha and even the Philosopher’s Stone. It is certainly not lost; it is mankind that is lost. Here is the mystery people do not comprehend; perhaps no one has explained it to them. Before I get into that, I need to dispel other myths men tell each other, especially about the Bible and the science of their own evolution.

Did you know that it is written, in several places of the Bible, that man cannot even read that Book until after a certain time in history? That man is still being made and until he reaches a certain point in his making, he will only dream that he is a man, that he has understanding. It is also written that man will be in this condition until a pure language is introduced to him. Today, even scholars still do not know the relationship between Language and Grammar. I can put that relationship into grammar, but your ability to comprehend what I say is determined by how much of a man you are, how much of you is complete, as a man.

A man is measured by his distinction from other forms of life on this planet. That distinction resides in his ability to clearly see Language, which nothing in all of creation can speak. It is an intelligible. Language is a biological inheritance. Every form of life is made from it, and every form of life expresses its comprehension of it, from the most primitive forms of life to the most complex. Language is Universal and Intelligible. Grammar, which is a physical recursion of language, is Particular and Perceptible. A species can only formulate their systems of grammar to the degree that they comprehend language. As scholars, even the current scholars, have and are still, expounding their confusion and lack of comprehension in each of these, all I can do is explain it to you, your own state of creation determines your ability to comprehend what I say. Suffice it to say, both science and religion today are still lost as to what man is, why he is, and what his purpose is in this life, even though everything was long ago put into simple words, words which are decidedly very provable.

Every life support system of a living organism is designed for the salvation of the life of that organism; every one of them. However, that salvation is particular to that part of the environment that life support system can process. Each of them is particular, and thus, do not have the ability to process time, itself. The mind is one such life support

system, and it is the most powerful life support system possible. It is the most powerful possible, because it is designed to process the intelligible which is over every possible relative difference, even time, itself. When functional, man will even conquer death. A mind is a symbolic information processor constructed to predict the results of every action, every relative difference, and every thing. It achieves this from the Universal of Language, also called the Word of God, which simple minds believe is just some book, but it is factually the Word of Creation itself. Since nothing but God can speak in language, man has to speak in grammar systems which are in the image of that language. Language is, in a metaphor, such as Adam and Eve, A Conjugate Binary Pair which affords even reality itself, complete induction and deduction of every thing.

Today, all one has to do is meditate on their computer to realize that all of information is processed using binary; however, it takes any species a long time to evolve to the stage where it can comprehend this fact with a mind.

One is also, in metaphor, informed that there are exactly four systems of grammar derived from binary, arrived at by simple binary recursion. They are also informed in metaphor not only the fact that every possible grammar is metaphorical, able to use the binary unit for complete induction and deduction with that grammar, but also that of the four grammars three are logical, and the last is analogical, this last grammar can be used to metaphorically to illustrate every possible line of reasoning in every possible grammar, it is called Geometry. The first three grammars are all logical, Common Grammar, Arithmetic, and Algebra.

So, in this little section of my work, I will show you how to understand the philosophers stone, the cup of Siddhartha, the Holy Grail, of the life of mind and body. You use an image of the Cup, like this:—

You turn your world upside down, turn from illiteracy to literacy. Literacy gives a species the ability to turn the past into a future and to bring that future to pass, which, deliberately, is the solution to the name of the Beast, 666. It is a puzzle that those with eyes can easily solve.

**List of Plates to use for the story.**

The plates used in the outline.

# Homind's Quest for The Holy Grail

**Story project: this is simply an outline of that story. How pedantic can one be and still hold the storyline?**

**The whole idea is an ordered progression, unlike our real thrashing aboout. This ordering, howevever, is to be implied in the story, and never mentioned. It is to follow step by step the available moves with straightedge and compass with what the figure offers. One can even turn Hominid into several characters over several generations. This project should be done.**

**022020**

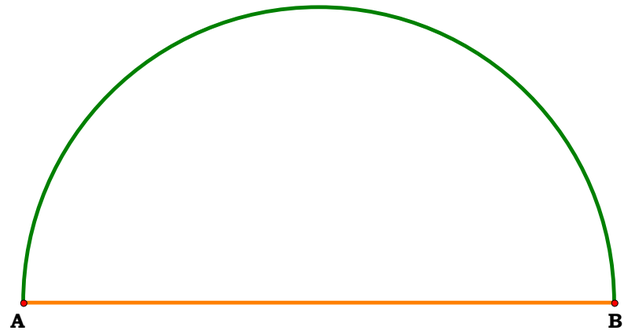
**Descriptions.**

**Once upon a time, Hominid took time to ponder the common stick which he found laying on the ground and as he wanted to think about it long and hard, he preserved that stick in memory. He learnt how to draw in the ground with his stick and to follow those drawings in memory.**

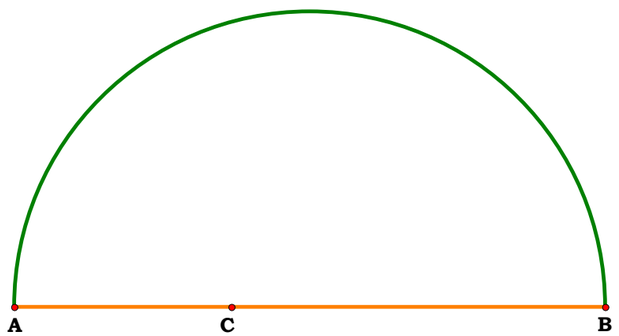


**He found, that in order to ponder this stick, that it might be advantageous to name it, and so he did. He named when it came, A, and when it left, B.**

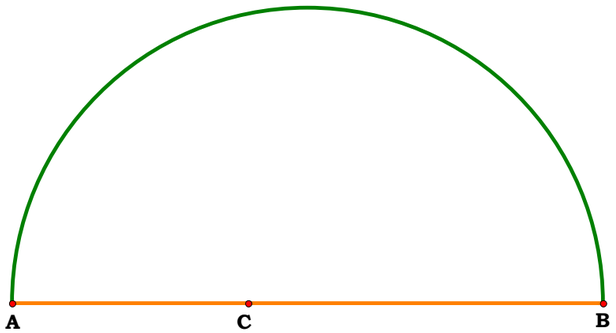
**One day, Hominid decided to do for the stick as he had done for himself, build it a shelter.**




**This shelter reminded him of his own home and on another day, while meditating on his stick, in its home, it seemed that this stick and home might feel better if it had at least one occupant. And so Hominid placed in his home an occupant like himself.**



**In order to meditate on his little home, with its littlel man, Hominid decided that it might be advantageous to increase his understanding of names. Eventually, Hominid learnt meditation about C and where he was at through a process he called arithmetic. Suddenly, Homid realize that this gave him a lot to think about.**




**Given.**

$AB := 20$

$AC := 8$

$BC := AB - AC \quad BC = 12$

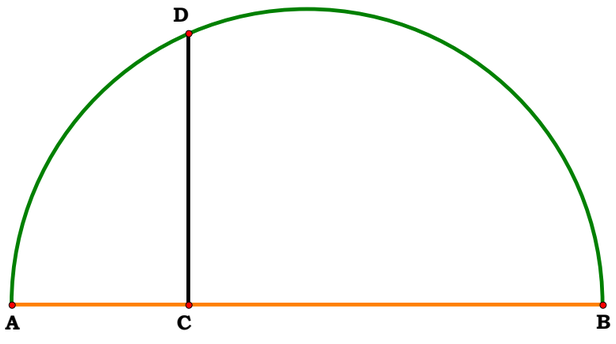
C = 8.00000
B = 20.00000

$\frac{AB}{AC} = 2.5 \quad \frac{AB}{BC} = 1.666667 \quad \frac{AC}{AB} = 0.4 \quad \frac{AC}{AB} = 0.4 \quad \frac{AC}{BC} = 0.666667 \quad \frac{BC}{AC} = 1.5 \quad AC \cdot BC = 96$

**All of the places, his arithmetic told him, were in the home of C, except one. This last result troubled Hominid. 96 could not possibly be in the home of C. What is the meaning of this? He began to wonder about the future home of C and about things C could never see.**

**One day, Hominid wanted C to be more like him, and ponder the sky dome and so, he noted it as the following.**

**Then Hominid began to wonder, again, what would there be to ponder in CD, in itself, by itself? How would one clime up to heaven, to D to learn?**




C = 6.00000
B = 20.00000

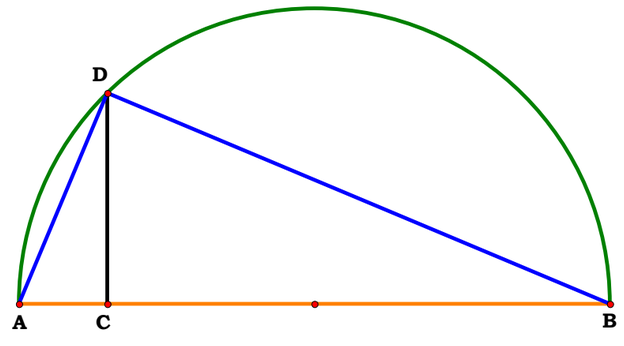
**Hominid built C ladders up into heaven to D and over time, he learnt from his arithmetic and his stick, his shelter, its occupant.**

$$CD := \sqrt{AC \cdot BC}$$

$$AD := \sqrt{AC^2 + CD^2} \qquad BD := \sqrt{BC^2 + CD^2}$$

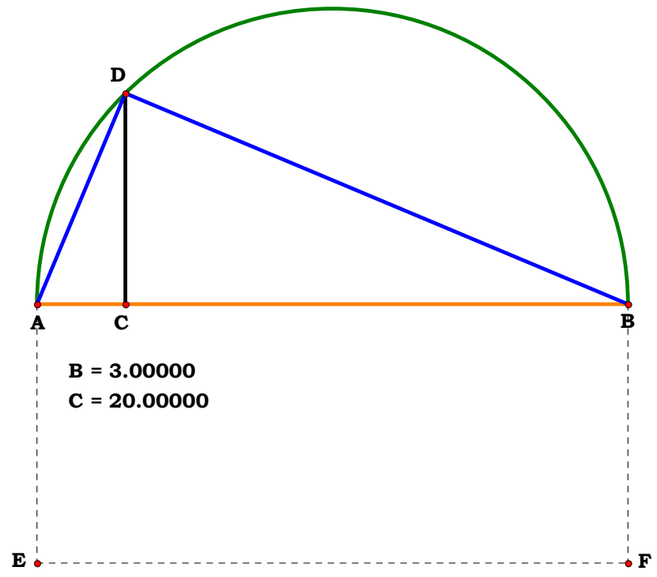
$$\frac{CD^2}{AC} - BC = 0 \qquad \frac{CD^2}{BC} - AC = 0$$

$$\frac{AD^2}{AC} - AB = 0 \qquad \frac{BD^2}{BC} - AB = 0$$

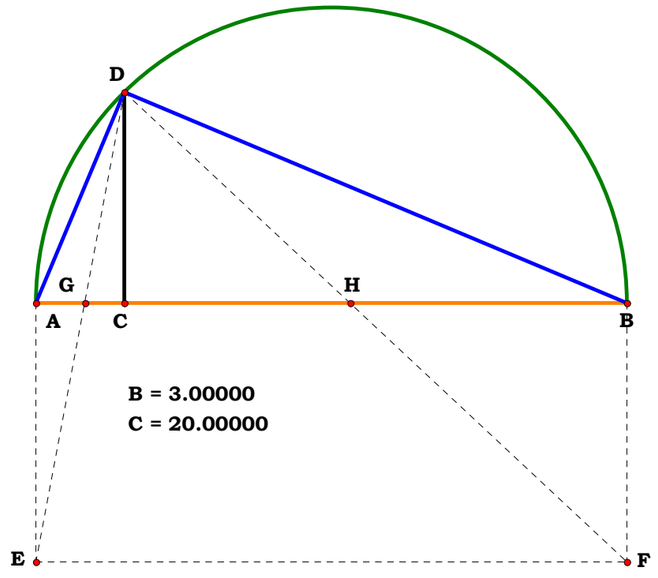



B = 3.00000
C = 20.00000

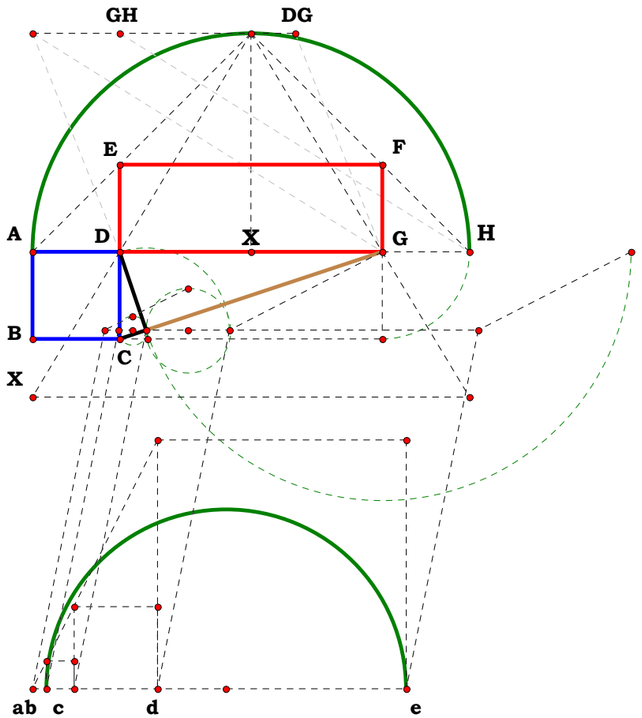
**And so on. As the number of things which Homid learnt grew, Homid thought that it was time that C had a basement to put all of his stuff in, the home was getting crowded, which to him was an odd thing to think, as crowd simply means to group into one with the added connotation, a bit much for one place. But, he built his basement anyway.**
**For a long time Homid and his little man played with all the toys they had found together in the room in his mind using a simple stick.**



**In our little story, Hominid starts pondering this issue, which will eventually lead to the following.**



**First and second basement archives. Here, X is fixed midpoint. The finishing touch, is in the next plate, to move it and learn the final ratio's.**




$\frac{DG}{DE} = 3.00000$

AD = 0.20000
GH = 0.20000
DE = 0.20000
DG = 0.60000

DEFG = 0.12000
ABCD = 0.04000
$\frac{DEFG}{ABCD} = 3.00000$

$x_b = 0.03162$
$x_c = 0.09487$
$x_d = 0.28460$
$x_e = 0.85381$

a = 1.00000
b = 3.00000
c = 9.00000
d = 27.00000

$\frac{d}{b} = 9.00000$

$d^{\frac{1}{3}} - b = 0.00000$
$d^{\frac{2}{3}} - c = 0.00000$
$b^3 - d = 0.00000$
$b^2 - c = 0.00000$
$\frac{b}{d} = 0.11111$

$\frac{Z}{Y}^2 = 9.00000$
$\frac{Z}{Y}^3 = 27.00000$

$\frac{Z}{Y} = 3.00000$

$\frac{x_e}{x_d} = 3.00000$
$\frac{x_d}{x_c} = 3.00000$
$\frac{x_c}{x_b} = 3.00000$

**Final basement archive.**

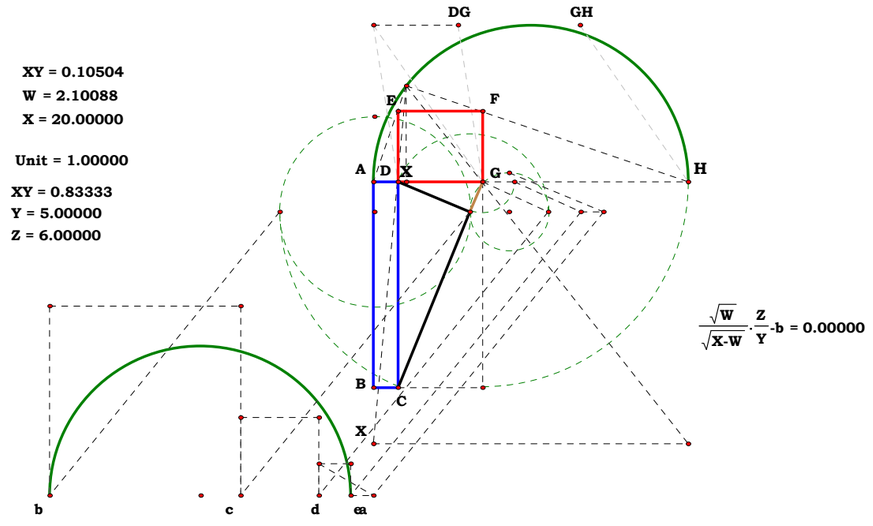
**In the above, the aviary, exponential manipuloation and how to locate any bird in the sky has to be incorporated. This story might take some time.**

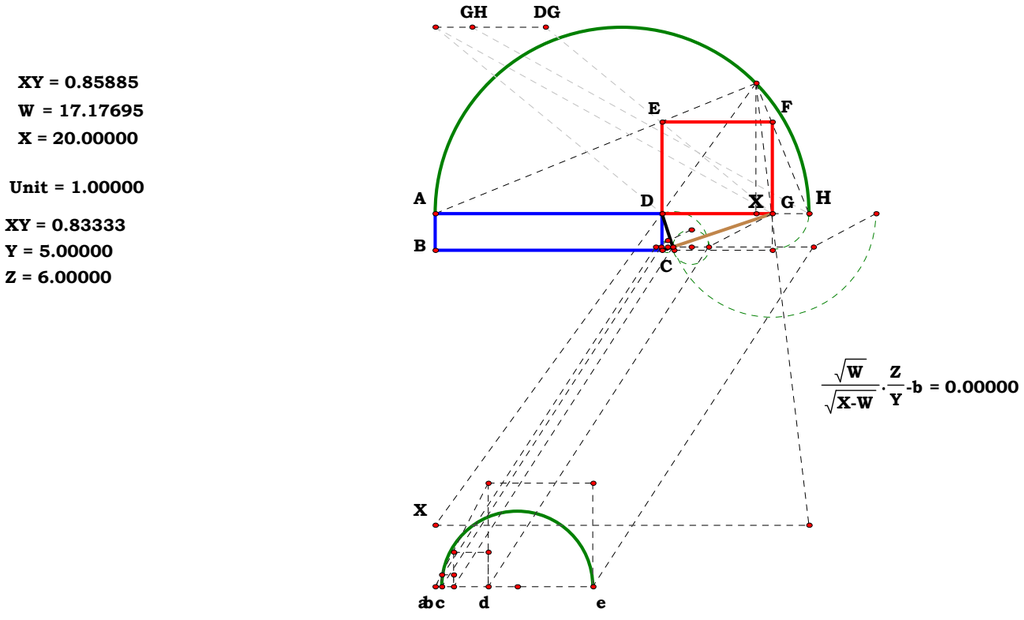
**One has to add in induction and deduction, in Hominid's home and how he goes beyond it.**

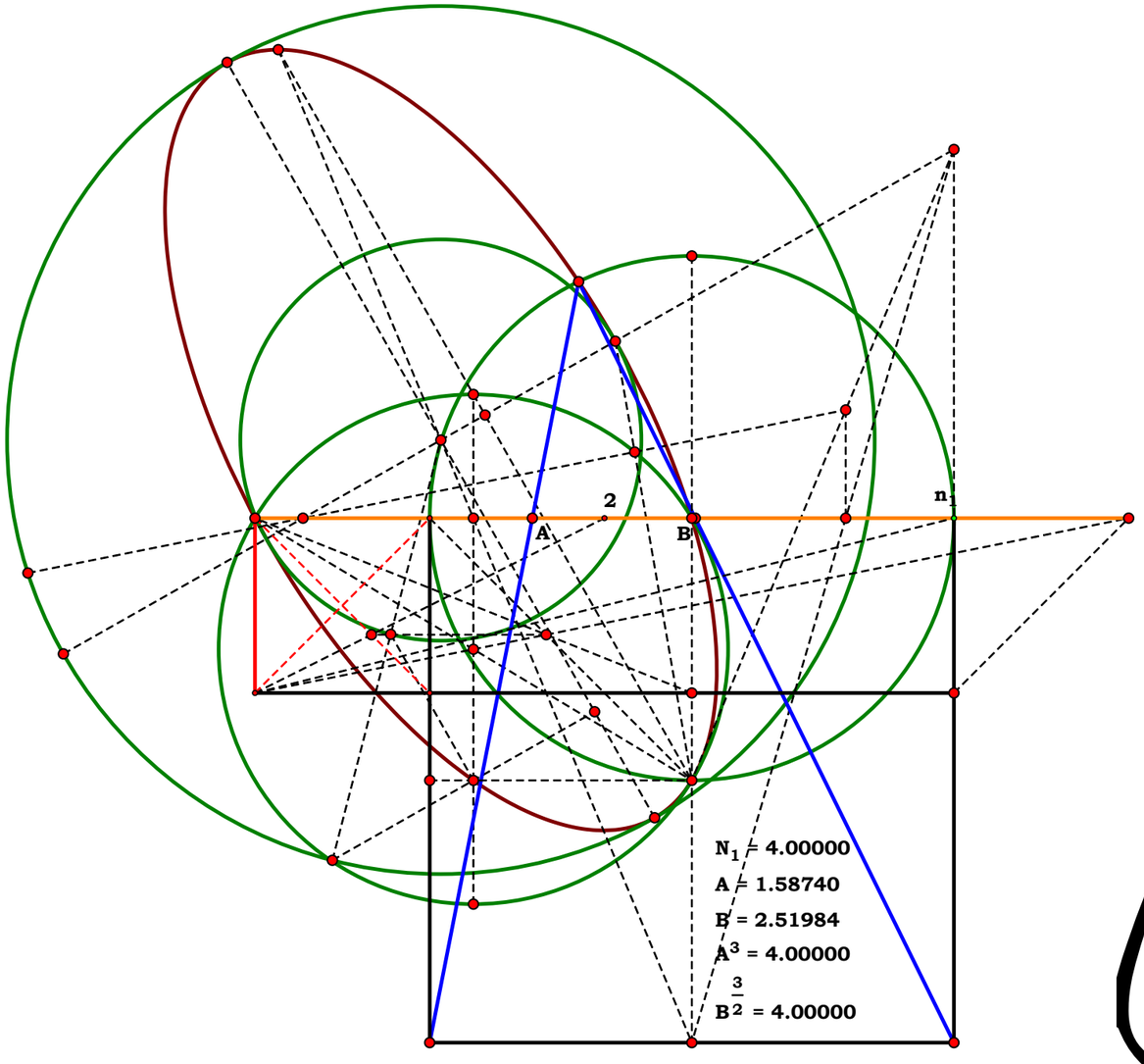
**This last plate demonstrates the whole of matematics in a single unit. All the simple relationships formed by it. It all resolves down to simple arithmetic.**

**Induction and deduction in every grammar system does not change the fact that recursion can never change simple arithmetic or binary progression. The Holy Grail has always been the image of 1.**

**Nothing like a single equation for the whole of grammar.**






# The Delian Quest
# Food on My Plate

**John Clark**

**Work to at least attempt to do.**

110194C

Given.

$N_1 := 4.55192$

$N_2 := 2.63187$

**Descriptions.**

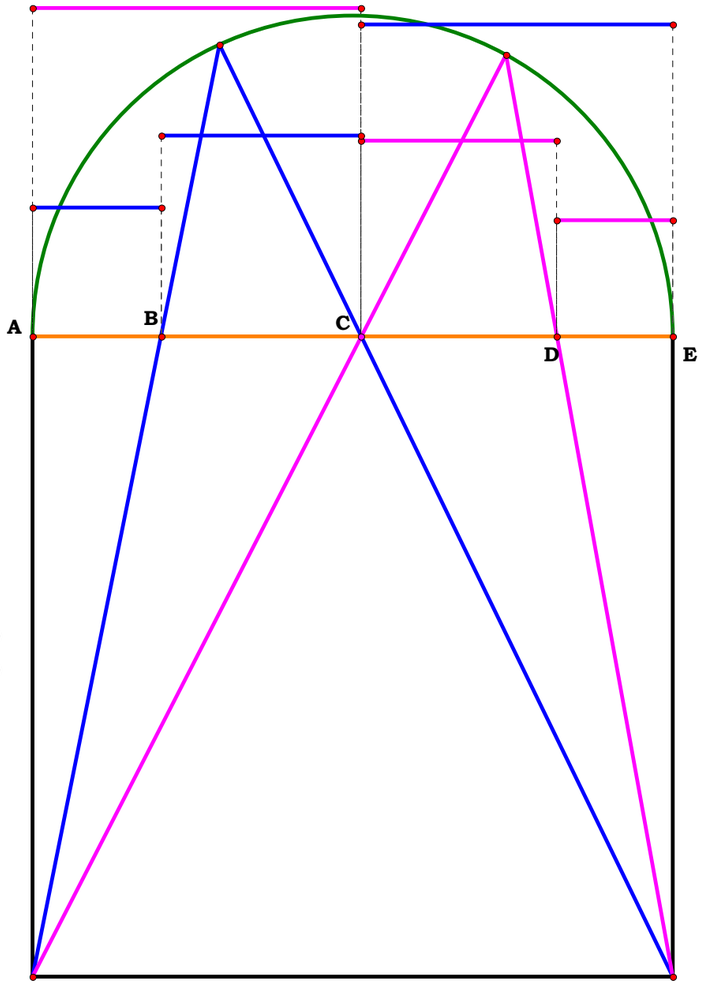
**Maybe I am remembering badly, but there was something about trying to find a way to construct duplicate ratios? How hard can it be? one can find duplicate ratios on any segment given any point, two of them in fact.**

**I do not think I need to keep writing the figure up, but maybe later.**

**Definitions.**

## Duplicate Ratios

**Procrastinated write-up?**




A = 0.00000
AB = 0.20070
AC = 0.51326
AD = 0.81848
AE = 1.00000

BC = 0.31255
CE = 0.48674
CD = 0.30523
DE = 0.18152

$\sqrt{AB \cdot CE} - BC = 0.00000$
$\sqrt{AC \cdot DE} - CD = 0.00000$

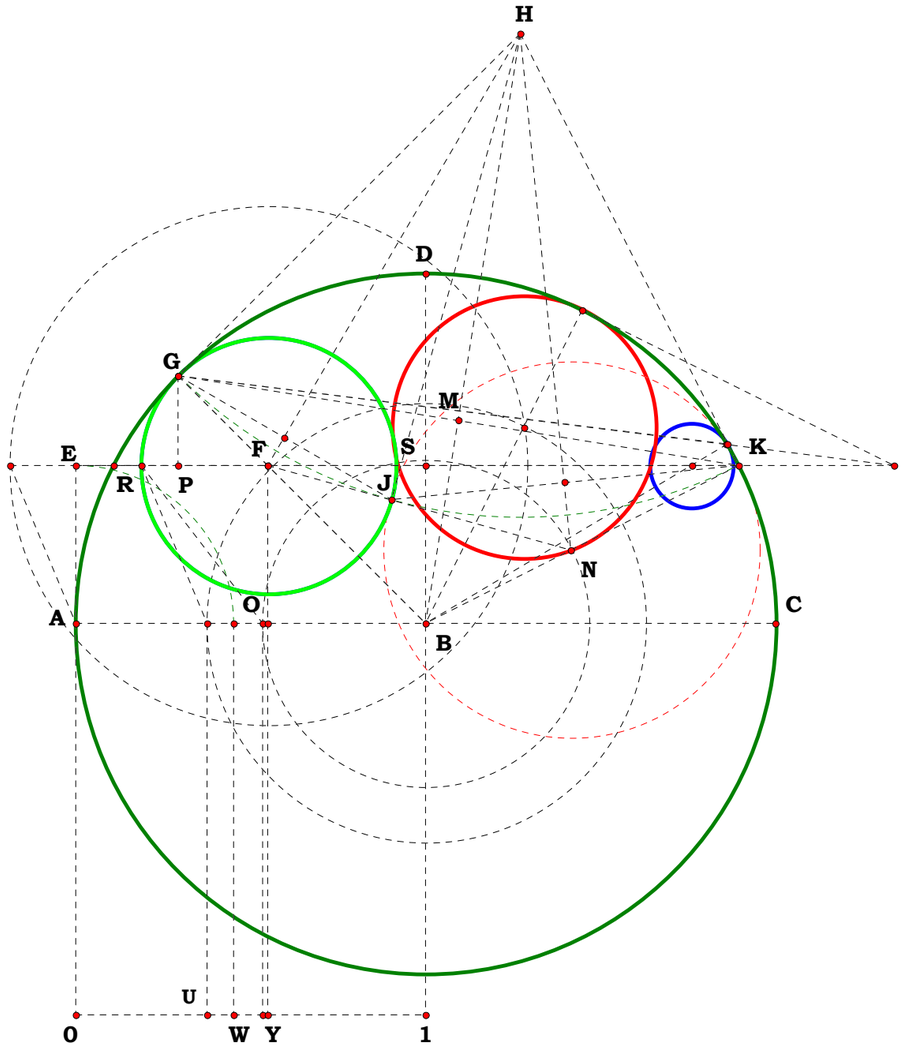
042296B

**Unit.**
**Given.**

**Definitions.**

**Place EF and GH and find JK.**

**I do not think I have drawn this figure correctly since the first time I drew it. Every write up of it after the drawing has been in error. I may get around to writing it up now that I disected and redrew it correctly.**




Unit = 1.00000
W/X = 0.45000
W = 9.00000
X = 20.00000

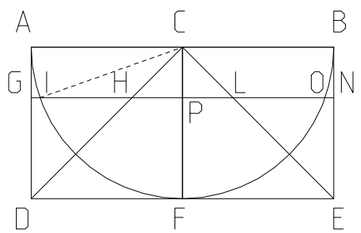
Y/Z = 0.55000
Y = 11.00000
Z = 20.00000

XY = 0.70000
U = 14.00000
V = 20.00000

Imagine a cylinder about the axis CF. The area of the circular ribbon GI is always equal to the area of the circle HL cut by the cone CDF. Galileo uses this to "prove" that a point is equal to a circle. He will first states that the volume of the cone, CDE is equal to volume of the bowl ADF. Because they both degenerate, one to a point, C, the other to a circle with radius AC, the point must be equal to the circle. He further states, since this is the case, all circles are equal to each other.

"Now, since as these solids diminish equality is maintained between them up to the very last, we are justified in saying that, at the extreme and final end of this diminution, they are still equal and that one is not infinitely greater than the other. It appears therefore that we may equate the circumference of a large circle to a single point. And this which is true of the solids is true also of the surfaces"

Galileo took this example from "twelfth proposition of the second book of *De centro gravitatis solidorem* by the Archimedes of our age, Luca Valerio."




$t := 0 .. 1 \quad N := 1 \cdot 10^{153}$

$AD := 5 \quad CI := AD \quad CP_0 := 0 \quad CP_1 := \frac{AD}{N} \quad CI := AD \quad GP := AD$

$IP_t := \sqrt{CI^2 - (CP_t)^2} \quad HP_t := CP_t \quad GP^2 - \left[(IP_t)^2 + (HP_t)^2\right] =$

| |
|---|
| 0 |
| 0 |

$GN := 2 \cdot AD \quad IO_t := 2 \cdot IP_t \quad HL_t := 2 \cdot HP_t \quad GN^2 - \left[(IO_t)^2 + (HL_t)^2\right] \cdot \pi \cdot GN^2 - \left[\pi \cdot (IO_t)^2 + \pi \cdot (HL_t)^2\right] =$

| |
|---|
| 0 |
| 0 |

| |
|---|
| 0 |
| 0 |

$\left[\pi \cdot GN^2 - \pi \cdot (IO_t)^2\right] =$

| |
|---|
| 0 |
| 0 |

$\pi \cdot (HL_t)^2 =$

| |
|---|
| 0 |
| $3.142 \cdot 10^{-304}$ |

$\left[\pi \cdot GN^2 - \pi \cdot (IO_t)^2\right] - \pi \cdot (HL_t)^2 =$

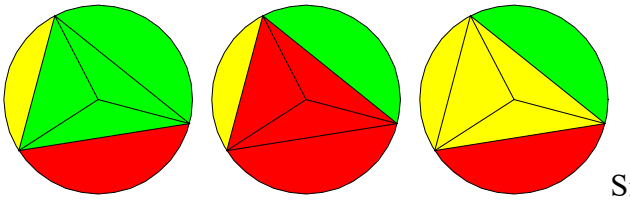
| |
|---|
| 0 |
| $-3.142 \cdot 10^{-304}$ |

I will completely agree with his analysis, providing that he does not change the subject. They are both still equal in volume, in his original statement, or area in his proof, which is 0. He does not see what he has done, he has in fact exonerated Euclid. The boundary is not part of the figure. By the looks of the text, Galileo was completely awestruck by this simple little miracle. Galileo was not the first to change coordinate systems of reference to make it appear that he won a bad argument. Plato used it to stun many audiences in his dialogues. Einstein used it to bend space. Galileo still had a "ball bearing" metaphor for geometry.

[+] [$] [#] **The Non-Euclidean Critique**

1. I have often observed the wonder that the sum of angles in a triangle is equal to 180º as if it were some big mystery. Have they not read Euclid? Do they not understand? Perhaps if we restate the observation as "The sum of angles in a triangle is equal to one half the angle of a circle, what would be said? Whatever is done to the triangle, must now be done to the circle!

2. Since the three terms of every triangle lie on the circumference of a circle, the angles all cut off a portion of that circumference. When one thinks of a triangle, they should automatically call to mind the circle.



3. Since the angle at the circumference of a circle is one half the angle described from the center of the circle then the total angularity of the triangle is one half that of the circle. It does not matter what it is measured by. Where is the mystery in simply dividing a thing in half? In other words, if one look at a circle as angle, then one must see the triangle as half a circle.

4. Now let someone claim that they have a triangle with a different sum of angles, and I now ask, show me its double. Some quick minded individuals would soon realize that the claimed geometry is now also bereft of a circle. And if the simple arc is missing, so too must any claim of the more complex. As "4" does not precede "2," a complex arc does not precede the circle. The relationship is symmetrical, show me the symmetry in any other claim.

5. And as far as parallel lines, that is incredibly simple. A plane is a two relation. Being a two relation, concepts must be predicated deductively from a binary set (study Aristotle.) If I deny one relation as a premise, no grammatical manipulation can possible reinstate it. In other words, given two one-relation propositions in a two-relation, one can predicate between these one-relation propositions the second relation or not (there ain't no middle!) If I predicate the second relation, the predicate is called "angle," if I do not, "not-angle," I have predicated the first or parallel. If I premise the

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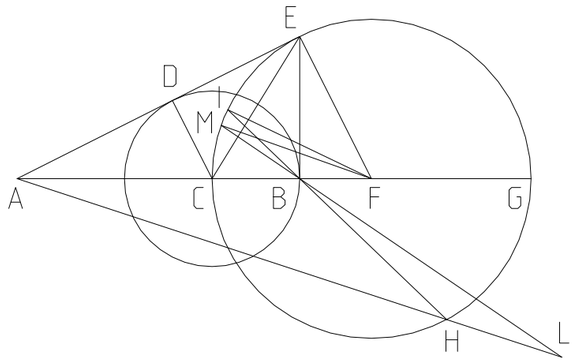
[+] A:009
[$] NONEUCLID
[#] NONEUCLID

second, the binary set still has the first to be predicated, and *vice versâ*. Boole did not invent binary logic.

Given a straight line divided into unequal parts which bear to each other any ratio whatever, to describe a circle such that two straight lines drawn from the ends of the given line to any point on the circumference will bear to each other the same ratio as the two parts of the given line, thus making those lines which are drawn from the same terminal points homologous.




$$AB := 100 \qquad CB := \frac{AB}{2 \cdot 2} \qquad CB < \frac{AB}{2} = 1$$

$$AC := AB - CB \qquad CD := CB$$

$$AD := \sqrt{AC^2 - CD^2}$$

$$BE := \frac{CD \cdot AB}{AD} \qquad AE := \frac{AC \cdot AB}{AD}$$

$$AF := \frac{AC \cdot AE}{AD} \qquad FE := \frac{CD \cdot AF}{AC}$$

$$FC := AF - AC \qquad FE = FC = 0$$

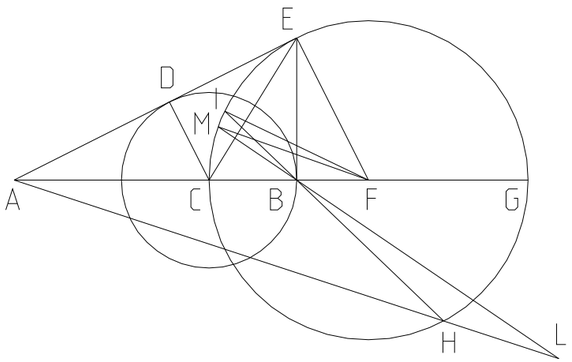
$$FE = FC = 0 \qquad FE = AF - AC = 0 \qquad FE = AC \cdot \frac{AE}{AD} - AC = (FE = AC^2 \cdot \frac{AB}{AD^2} - AC = 0$$

$$FE = -AC \cdot \frac{\left(AC \cdot AB - AC^2 + CD^2\right)}{\left(-AC^2 + CD^2\right)} = FE = -AC \cdot \frac{\left(AC \cdot AB - AC^2 + CB^2\right)}{\left(-AC^2 + CB^2\right)} = FE = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 0$$

$$CD \cdot \frac{AF}{AC} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 0 \qquad CD \cdot \frac{AE}{AD} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 0 \qquad CD \cdot AC \cdot \frac{AB}{AD^2} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$$

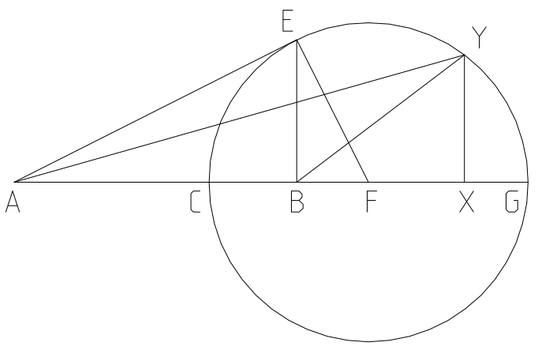
$$CD \cdot AC \cdot \frac{AB}{\left(AC^2 - CD^2\right)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1 \qquad CB \cdot AC \cdot \frac{AB}{\left(AC^2 - CB^2\right)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$$

$$CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$$




$$\frac{AC}{CB} = \frac{AE}{BE} = 0 \qquad \frac{AE}{BE} = AC \cdot \frac{AB}{(AD \cdot BE)} = 0$$

$$AC \cdot \frac{AB}{(AD \cdot BE)} = \frac{AC}{CD} = 1 \qquad \frac{AC}{CD} = \frac{AC}{CB} = 1$$




$$CG := 2 \cdot FC \qquad CX := \frac{CG}{.3} \qquad GX := CG - CX$$

$$XY := \sqrt{GX \cdot CX} \qquad AX := AC + CX$$

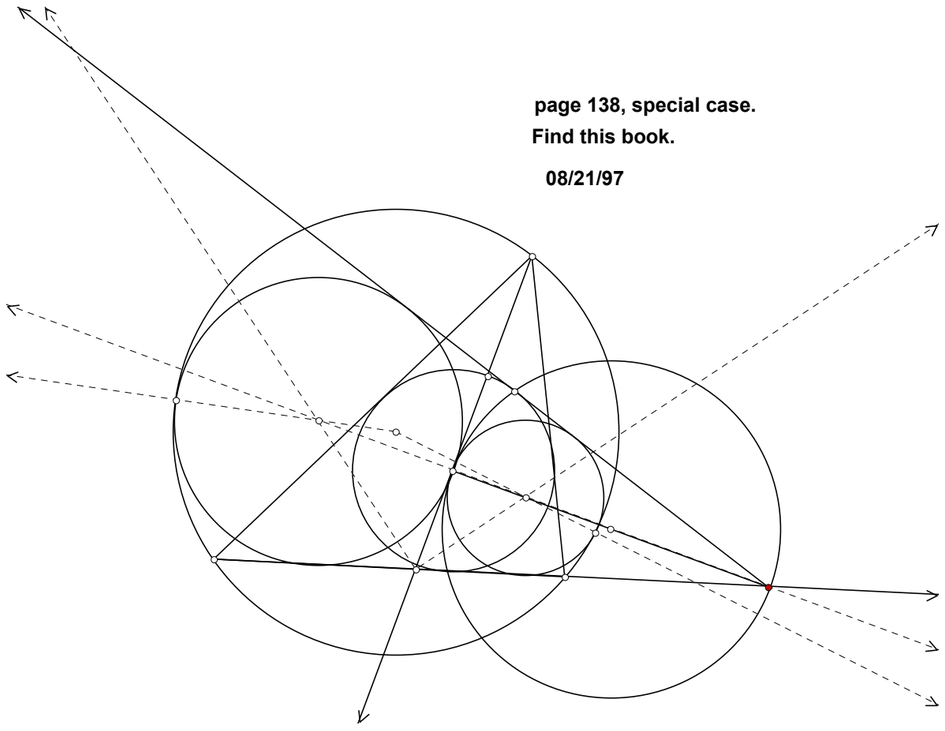
$$BX := CX - CB \qquad AY := \sqrt{AX^2 + XY^2}$$

$$BY := \sqrt{BX^2 + XY^2}$$

$$\frac{AC}{CB} = \frac{AY}{BY} = 0$$

082197

In my files I have the following plate, from what book I do not recall, however, the figure as it was written up there was listed as a "special case," of what I do not recall either. But I want to write it up because it is not a special case of anything, it is actually a plate showing that one can treat every triangle as an eight circle problem, simply add the remaining two sides by recursion of the first. I suspect now that if someone thought this was a special case of something, then they did not comprehend the actual relationships, they are easily found by compass.
The project would start with the equations from 062793 and 040694. So, this is a project I am interested in doing and have been for a long time. I might even find the book it came from. Might be interesting to find the equations for all eight circles. Maybe some day.

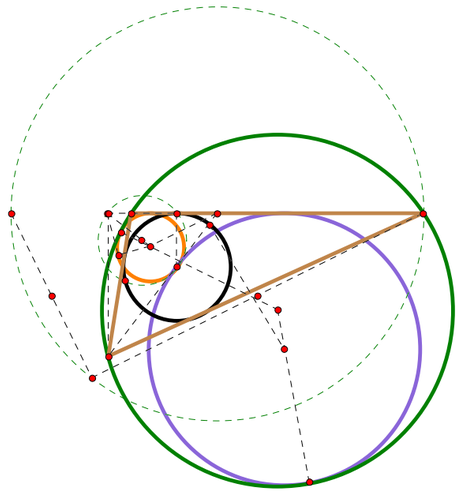


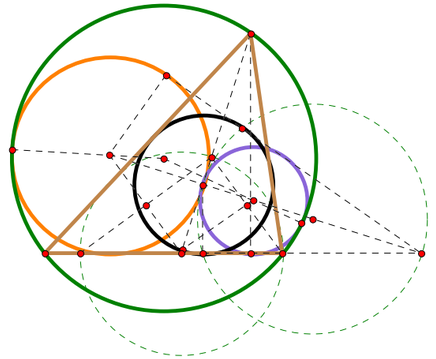
page 138, special case.
Find this book.

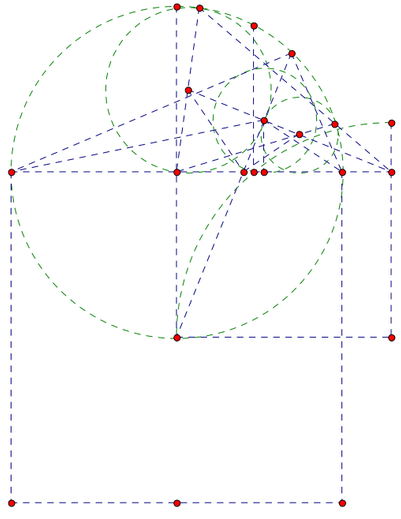
08/21/97

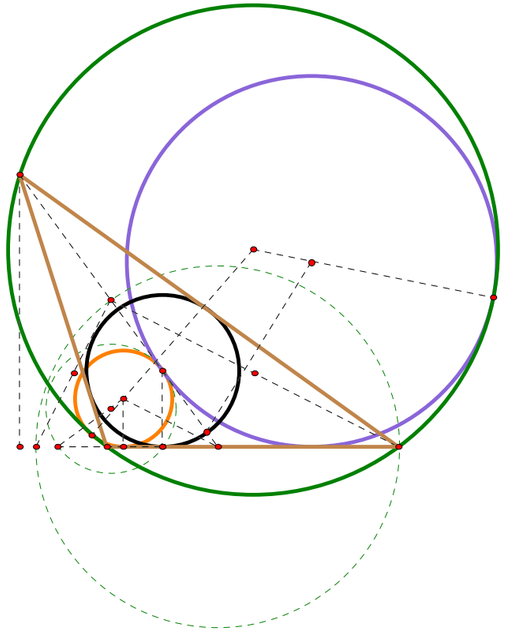
## Eight Circles and a Forgotten Book.

**The book is Computing in Euclidean Geometry Du and Hwang.**





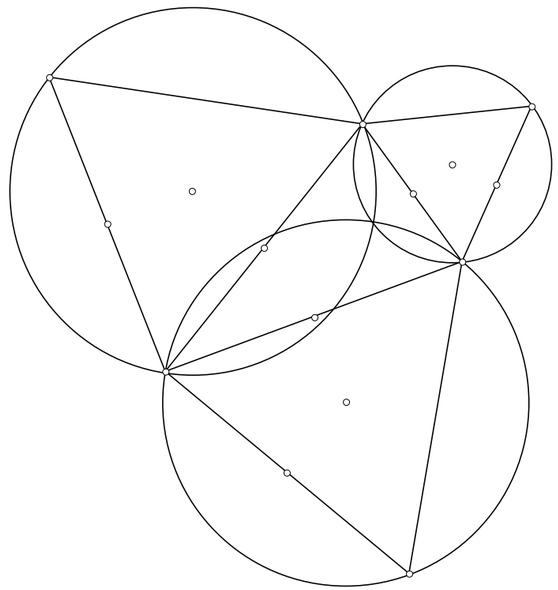




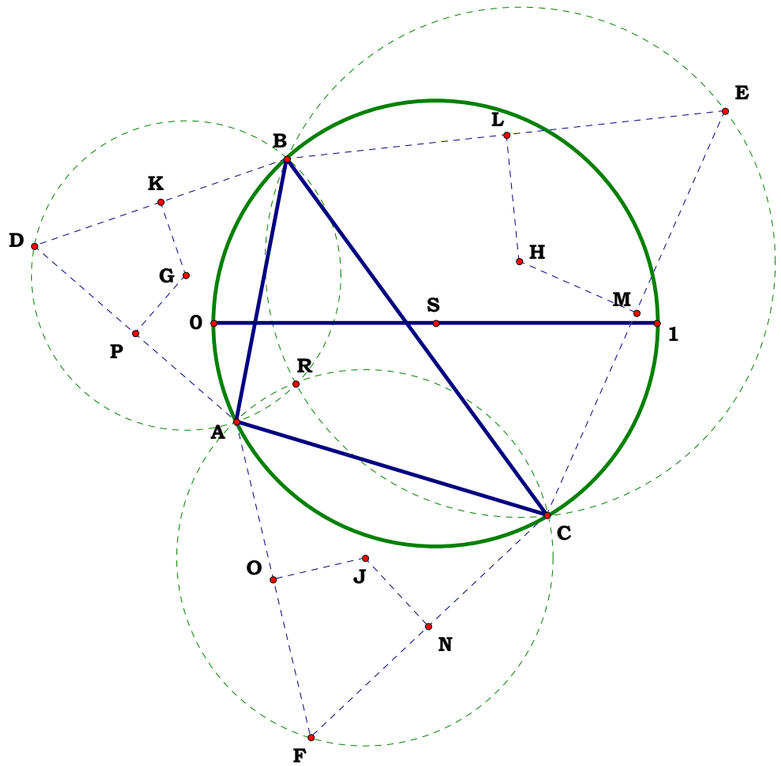
**Unit.**
**Given.**

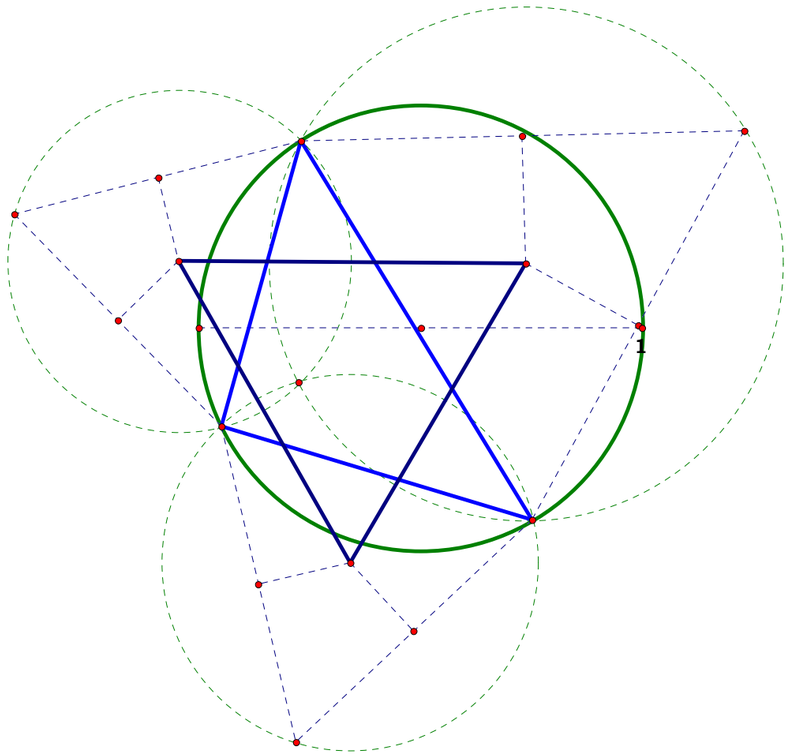
**Steiner Point**

**082297**
**Descriptions.**
**Definitions.**



**Steiner point solution (a) page 361 Computing in Euclidean Geometry Du and Hwang 1985**



100299

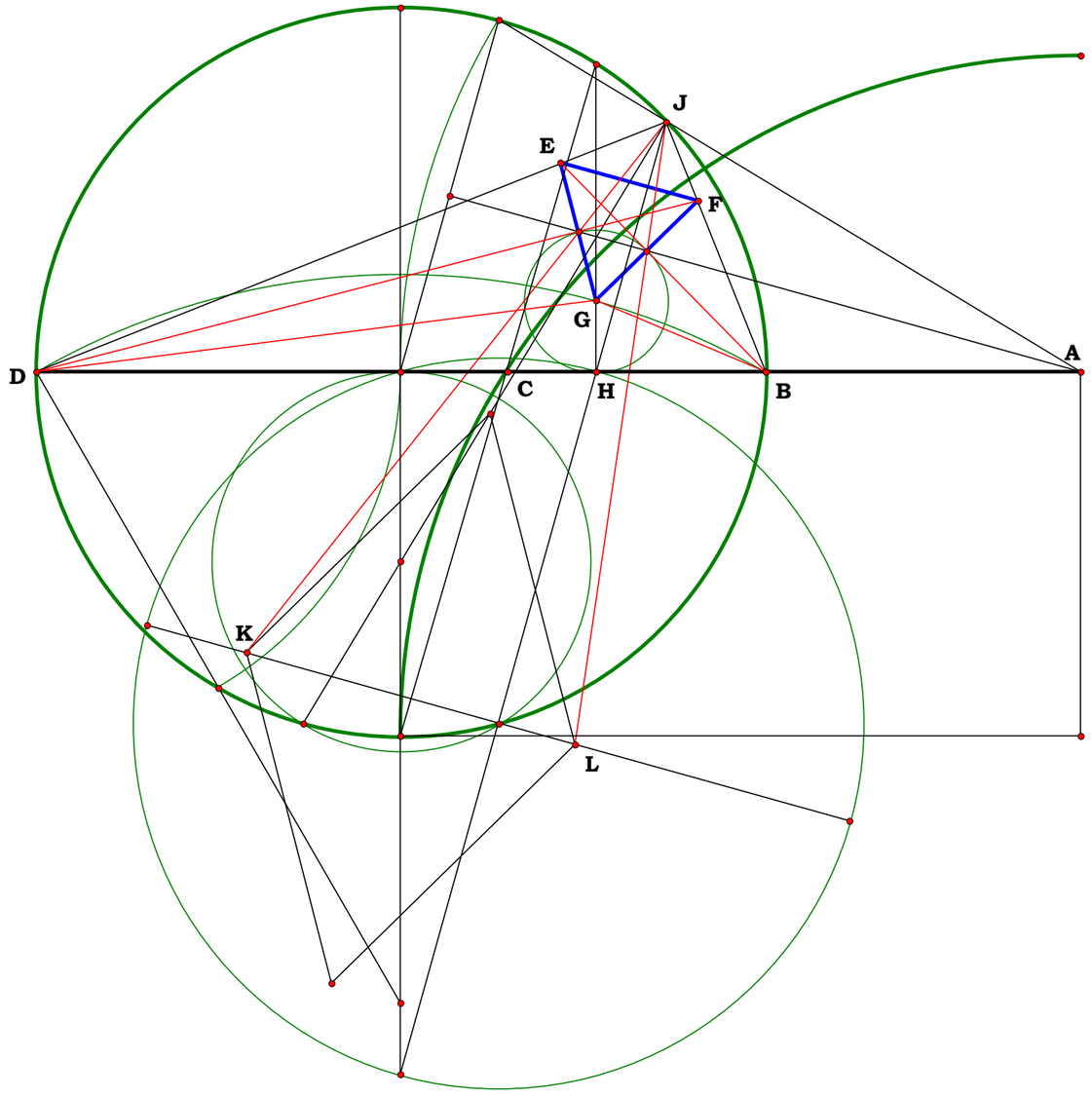
**Enough here to keep one busy for a while.**

**The Circle and Segment, which if one can see it is fundamental to Jacob's Ladder which I use for BAM and BAG. They represent induction and deduction, arithmetic and geometric processing. This is why so many of my plates use it.**

**Again we see that trisection is directly related to square roots. However, there is just a lot of beauty seeing all of the interactions in the figure.**

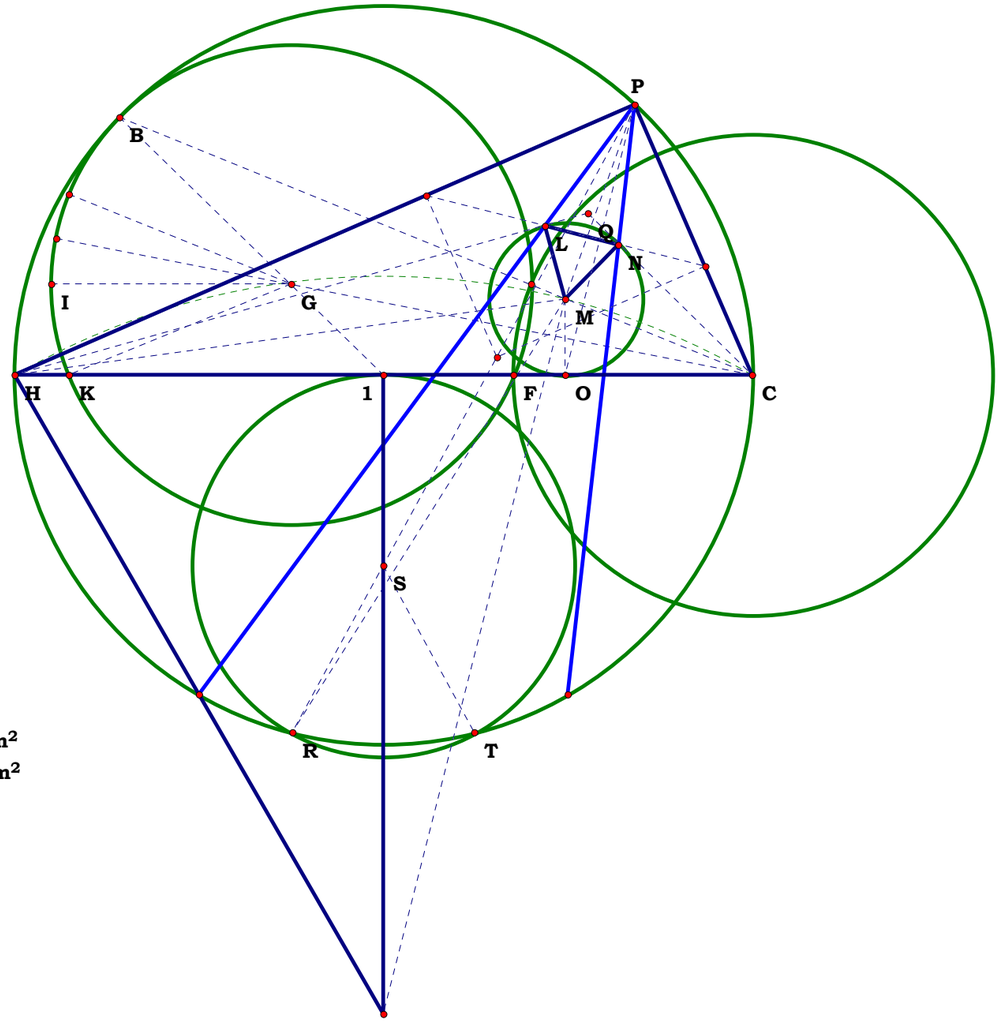
AB = 5.42961 cm
AC = 9.87931 cm
AD = 17.97565 cm
$\sqrt{AB \cdot AD}$-AC = 0.00000 cm
m∠DJB = 90.00000°
m∠KJL = 30.00000°
$\frac{m\angle DJB}{m\angle KJL}$ = 3.00000
m∠JBH = 68.31951°
m∠JBE = 22.77317°
m∠EBG = 22.77317°
m∠GBH = 22.77317°
$\frac{m\angle JBH}{m\angle JBE}$ = 3.00000
m∠JDH = 21.68049°
m∠JDF = 7.22683°
m∠FDG = 7.22683°
m∠GDH = 7.22683°
$\frac{m\angle JDH}{m\angle JDF}$ = 3.00000

Animate Point



100499

**Parcing project**




m∠KGB = 66.41899°
m∠KGI = 22.13966°
$\frac{m\angle KGB}{m\angle KGI}$ = 3.00000
m∠LMN = 60.00000°
m∠MNL = 60.00000°
m∠NLM = 60.00000°

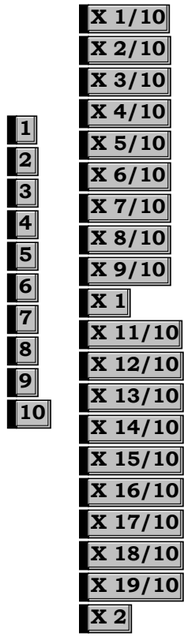
m∠PC1 = 66.41899°
m∠PCQ = 22.13966°
$\frac{m\angle PC1}{m\angle PCQ}$ = 3.00000

m∠PH1 = 23.58101°
m∠PHL = 7.86034°
$\frac{m\angle PH1}{m\angle PHL}$ = 3.00000

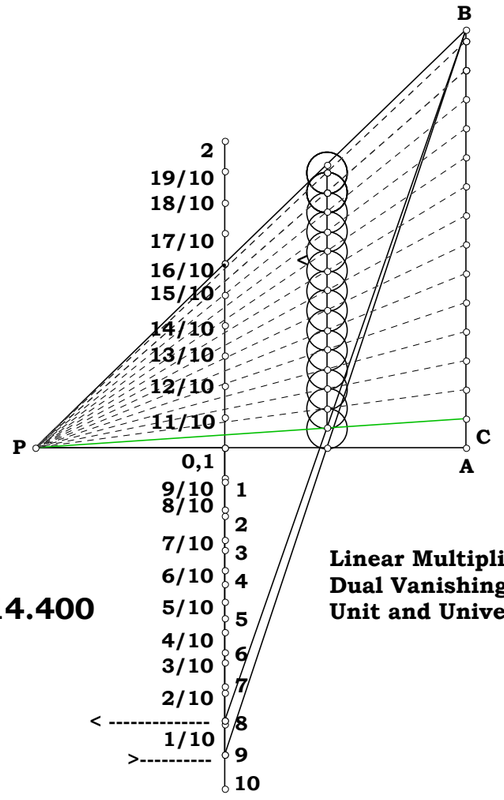
Area ⊙CF = 50.70109 cm$^2$
Area ⊙GB = 50.70109 cm$^2$

m∠RST = 57.11732°

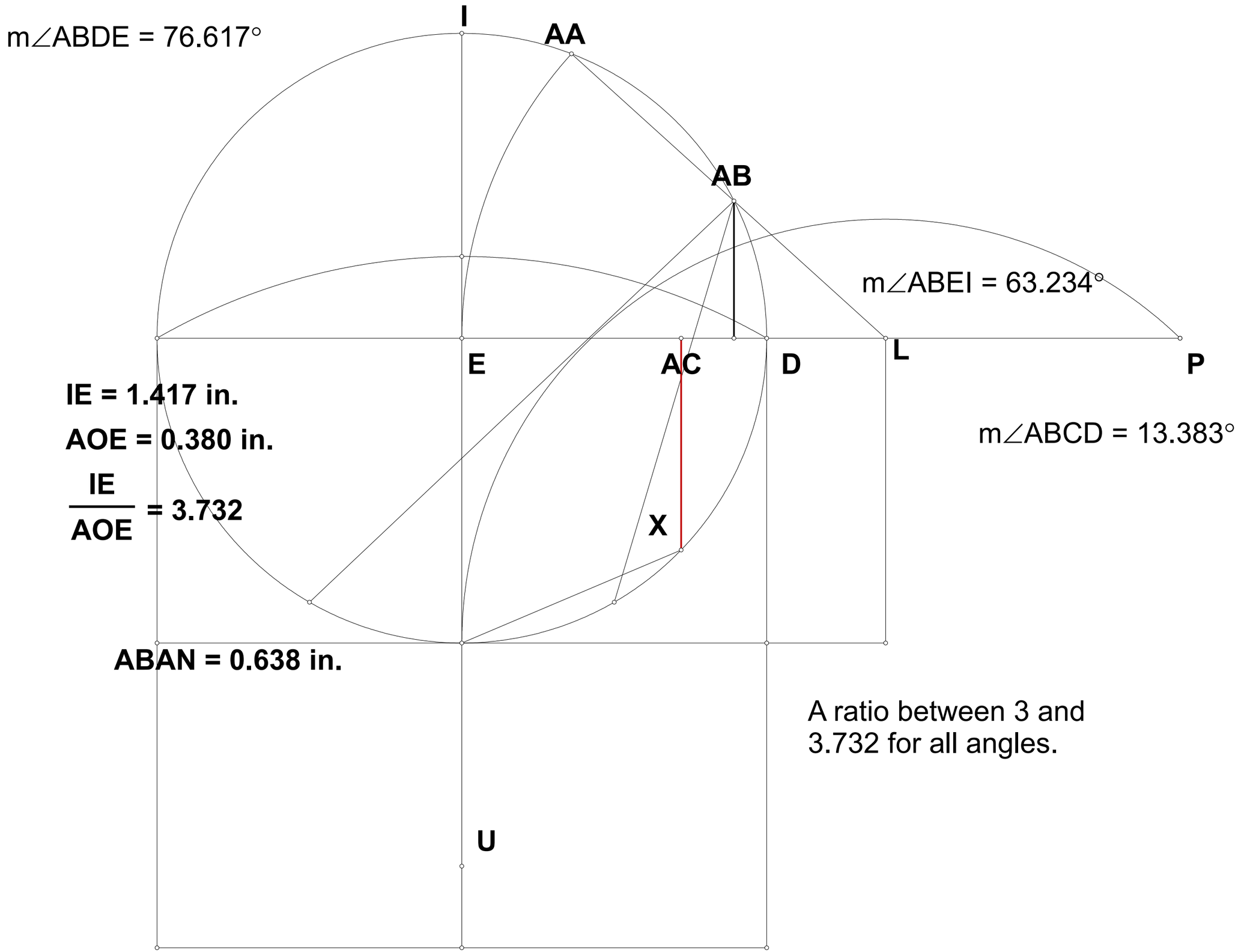
100799




$$\frac{AB}{AC} = 14.400$$




**Linear Multiplication and Division,**
**Dual Vanishing Point Method**
**Unit and Universe of Discourse.**


m∠ABDE = 76.617°
I
AA
AB
m∠ABEI = 63.234°
E
AC
D
L
P
IE = 1.417 in.
AOE = 0.380 in.
IE / AOE = 3.732
X
ABAN = 0.638 in.
U
m∠ABCD = 13.383°
A ratio between 3 and 3.732 for all angles.

090702B

**Unit.** $C1 := 1$

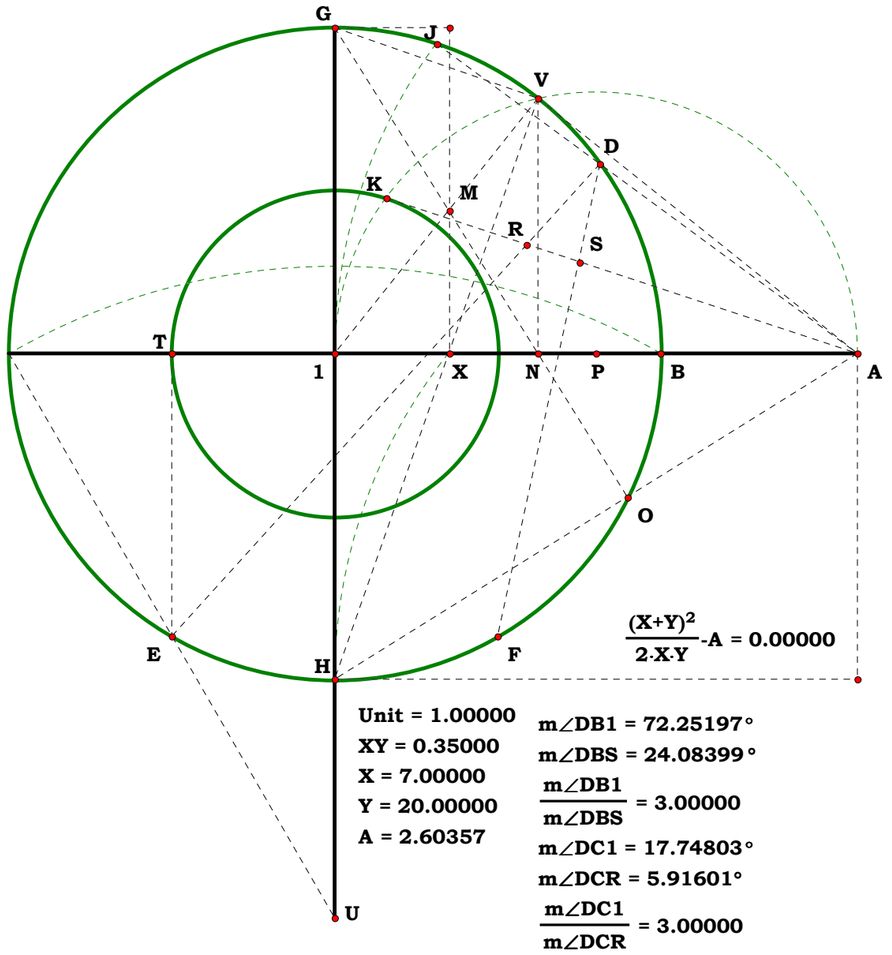
**Given.**

$X := 7$

$Y := 20$

**Descriptions.**

**Definitions.**

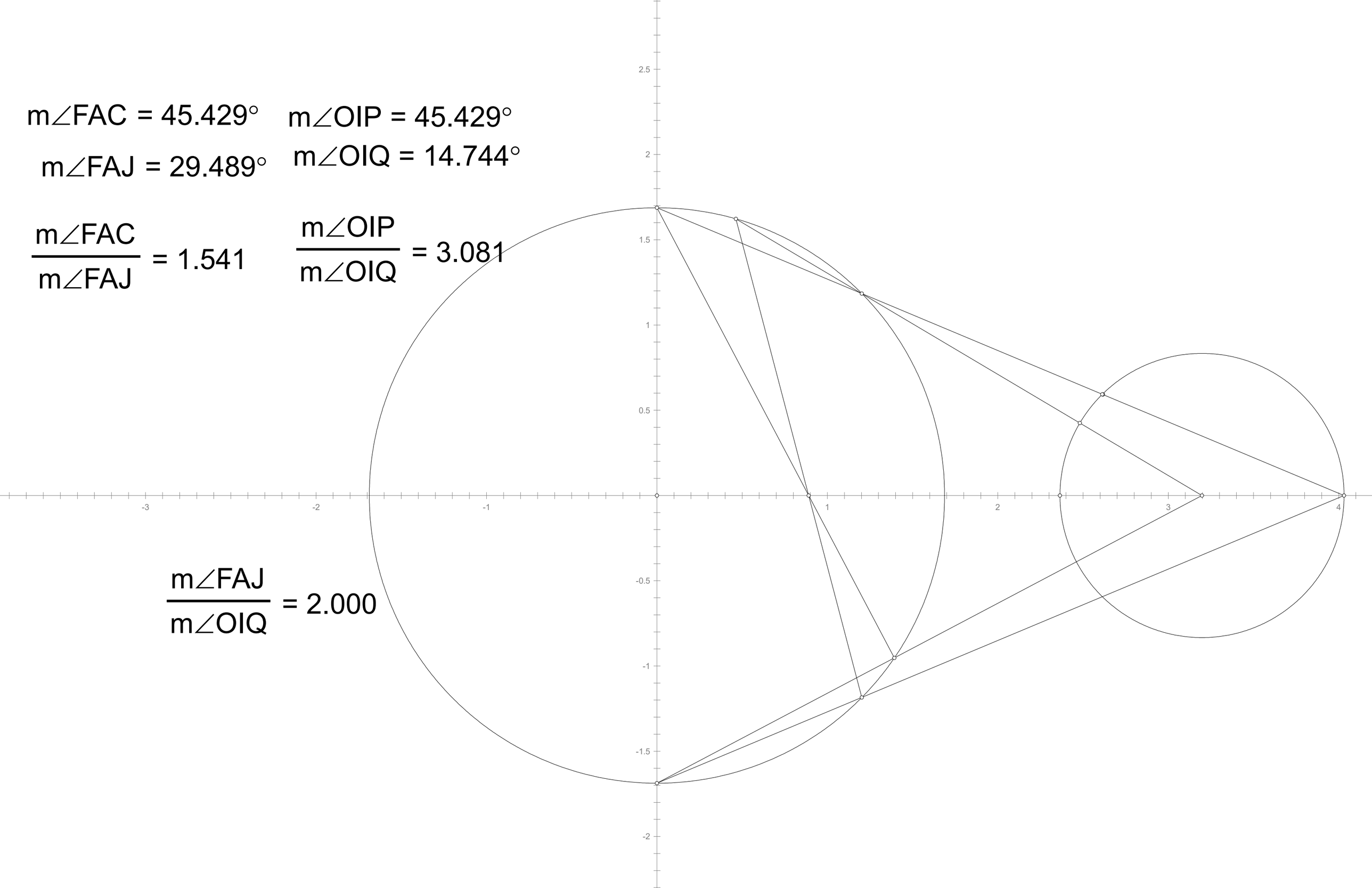


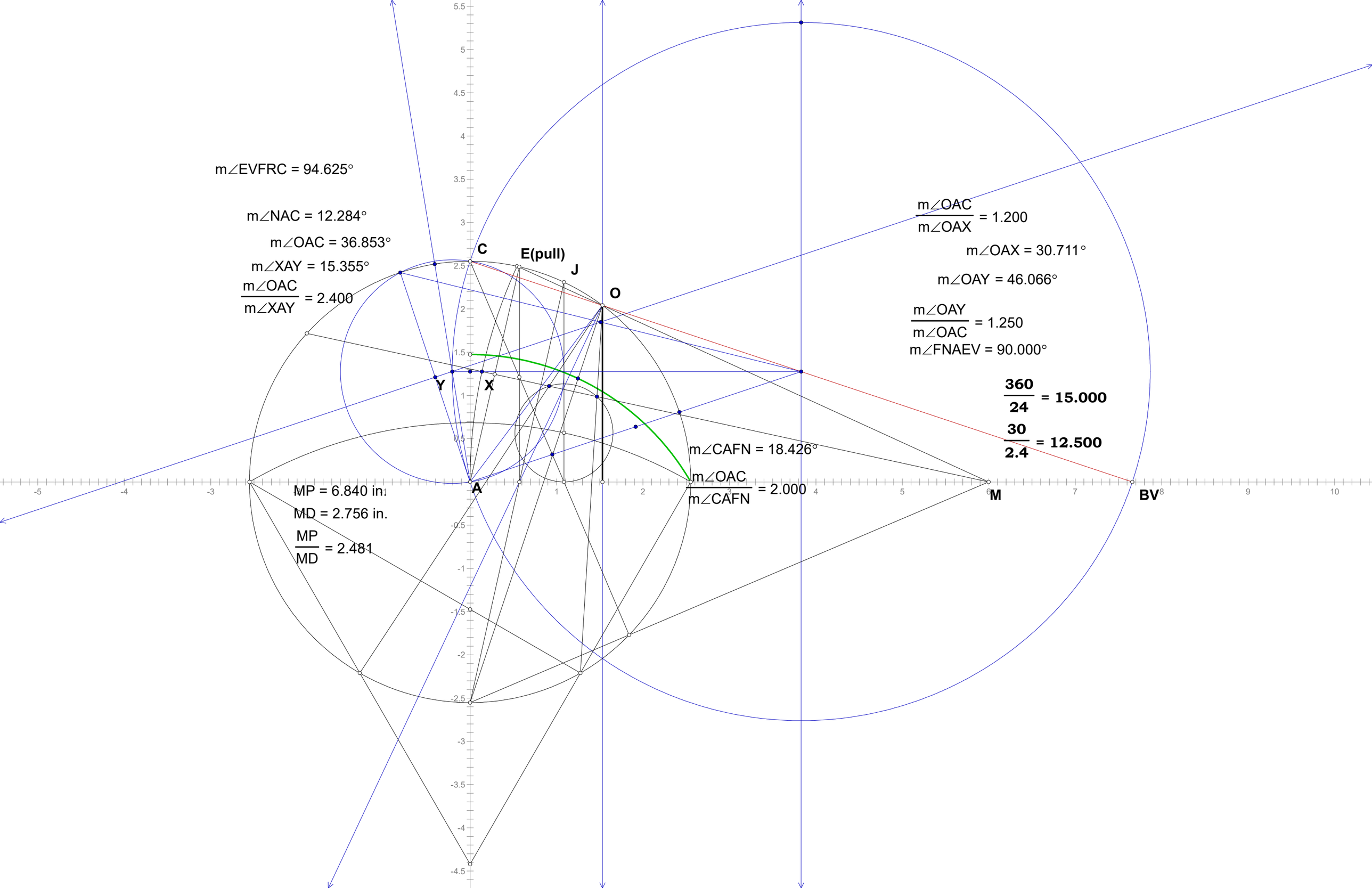
$m\angle FAC = 45.429°$ $m\angle OIP = 45.429°$

$m\angle FAJ = 29.489°$ $m\angle OIQ = 14.744°$

$$\frac{m\angle FAC}{m\angle FAJ} = 1.541 \qquad \frac{m\angle OIP}{m\angle OIQ} = 3.081$$

$$\frac{m\angle FAJ}{m\angle OIQ} = 2.000$$



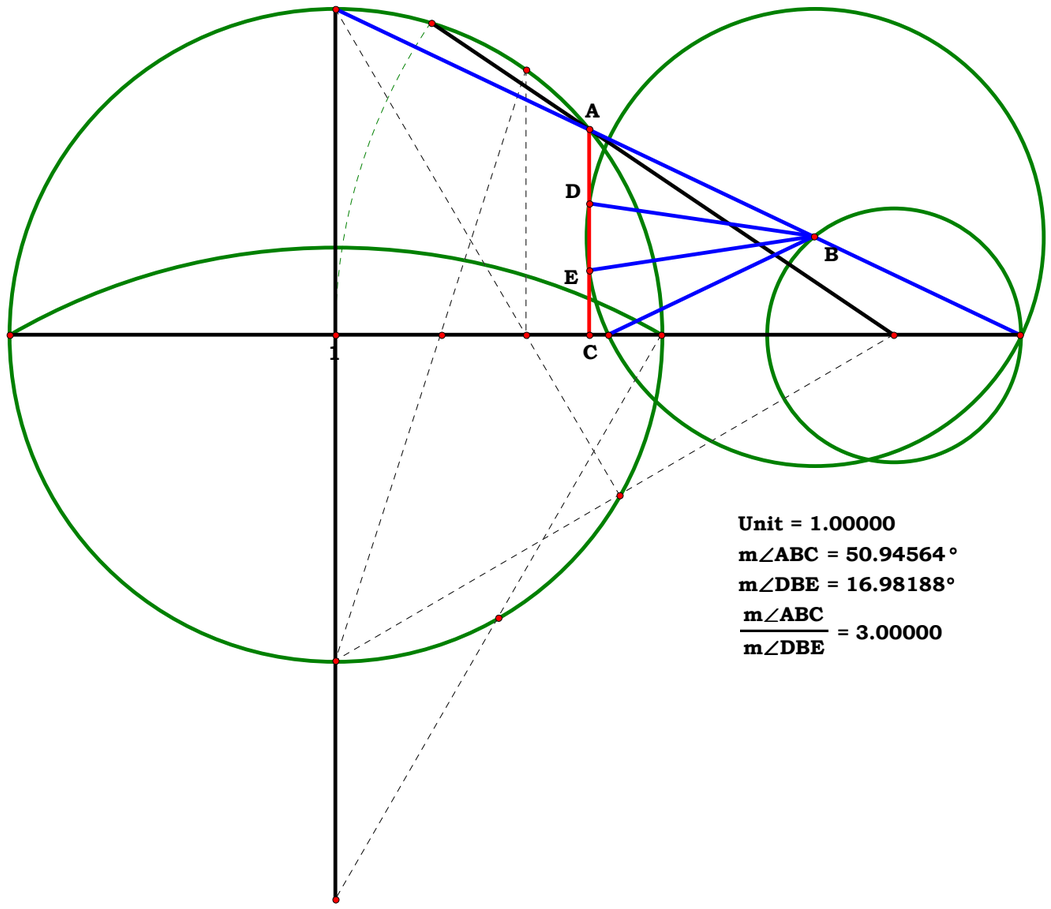

m∠EVFRC = 94.625°
m∠NAC = 12.284°
m∠OAC = 36.853°
m∠XAY = 15.355°
m∠OAC / m∠XAY = 2.400
C
E(pull)
J
O
Y
X
A
MP = 6.840 in
MD = 2.756 in.
MP / MD = 2.481
m∠CAFN = 18.426°
m∠OAC / m∠CAFN = 2.000
m∠OAC / m∠OAX = 1.200
m∠OAX = 30.711°
m∠OAY = 46.066°
m∠OAY / m∠OAC = 1.250
m∠FNAEV = 90.000°
360 / 24 = 15.000
30 / 2.4 = 12.500
M
BV

093002

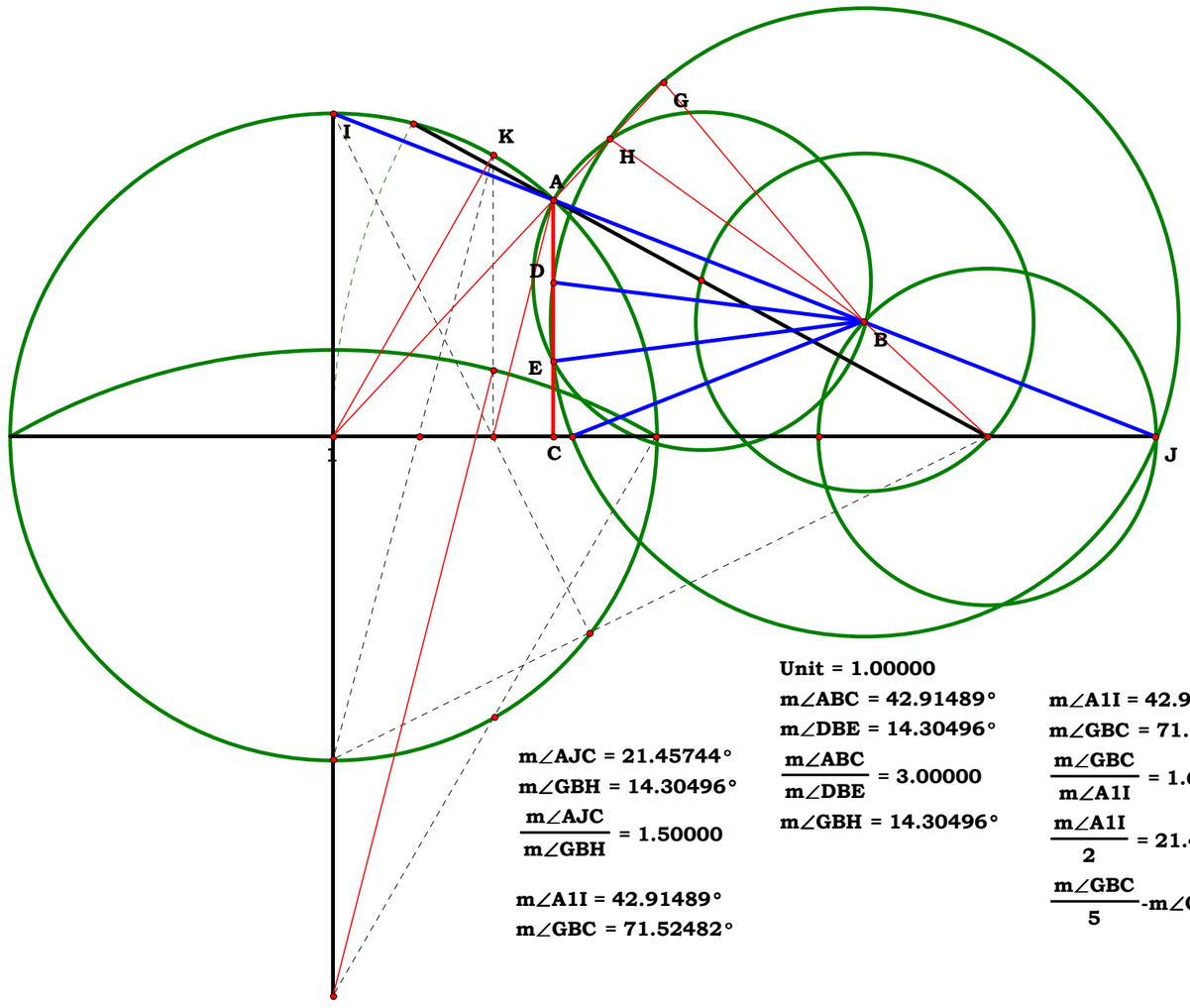
**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

## Trisection by Pole




**Unit = 1.00000**
**m∠ABC = 50.94564°**
**m∠DBE = 16.98188°**
$\frac{\text{m}\angle\text{ABC}}{\text{m}\angle\text{DBE}} = 3.00000$




Unit = 1.00000

$m\angle ABC = 42.91489°$

$m\angle DBE = 14.30496°$

$\frac{m\angle ABC}{m\angle DBE} = 3.00000$

$m\angle GBH = 14.30496°$

$m\angle AJC = 21.45744°$

$m\angle GBH = 14.30496°$

$\frac{m\angle AJC}{m\angle GBH} = 1.50000$

$m\angle A1I = 42.91489°$

$m\angle GBC = 71.52482°$

$m\angle A1I = 42.91489°$

$m\angle GBC = 71.52482°$

$\frac{m\angle GBC}{m\angle A1I} = 1.66667$

$\frac{m\angle A1I}{2} = 21.45744°$

$\frac{m\angle GBC}{5} - m\angle GBH = 0.00000°$

**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

# Parcing project 100402

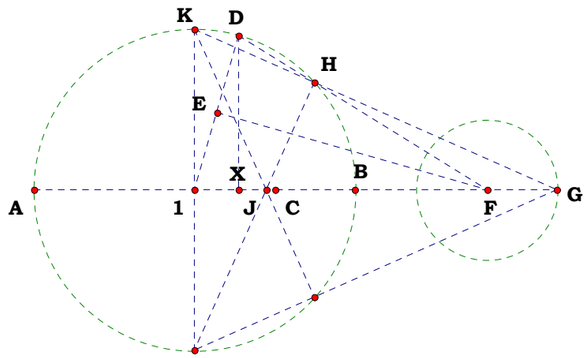



**Plate A**

**Unit = 1.00000**
**XY = 0.55000**
**X = 11.00000**
**Y = 20.00000**

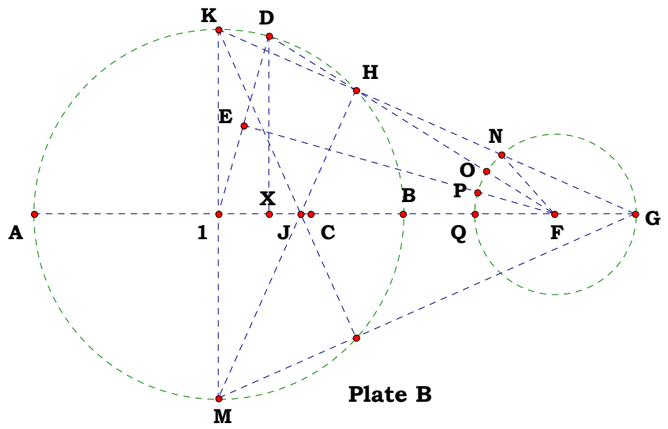



**Plate B**

**Unit = 1.00000**
**XY = 0.55000**
**X = 11.00000**
**Y = 20.00000**

**m∠NFQ = 47.88604°**
**m∠NFO = 15.96201°**
**m∠OFP = 15.96201°**
**m∠PFQ = 15.96201°**

$$\frac{m\angle NFQ}{m\angle NFO} = 3.00000$$

## Adding 042398

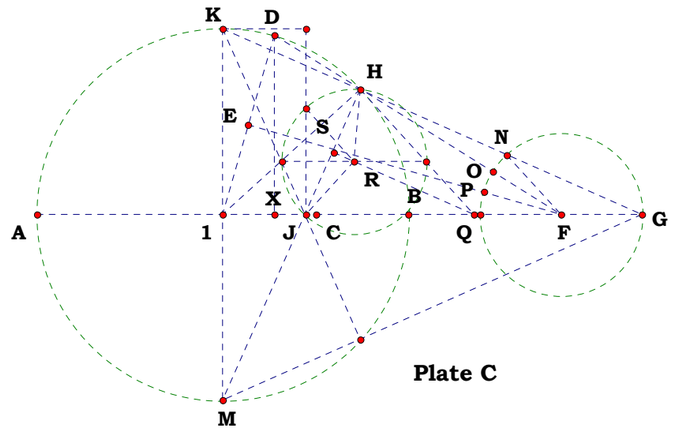



**Plate C**

**Unit = 1.00000**
**XY = 0.55000**
**X = 11.00000**
**Y = 20.00000**

**m∠NFQ = 47.88604°**
**m∠NFO = 15.96201°**
**m∠OFP = 15.96201°**
**m∠PFQ = 15.96201°**

$$\frac{m\angle NFQ}{m\angle NFO} = 3.00000$$

**m∠HRJ = 143.65813°**
**m∠HRS = 47.88604°**

$$\frac{m\angle HRJ}{m\angle HRS} = 3.00000$$

$$\frac{m\angle HRJ}{m\angle NFO} = 9.00000$$

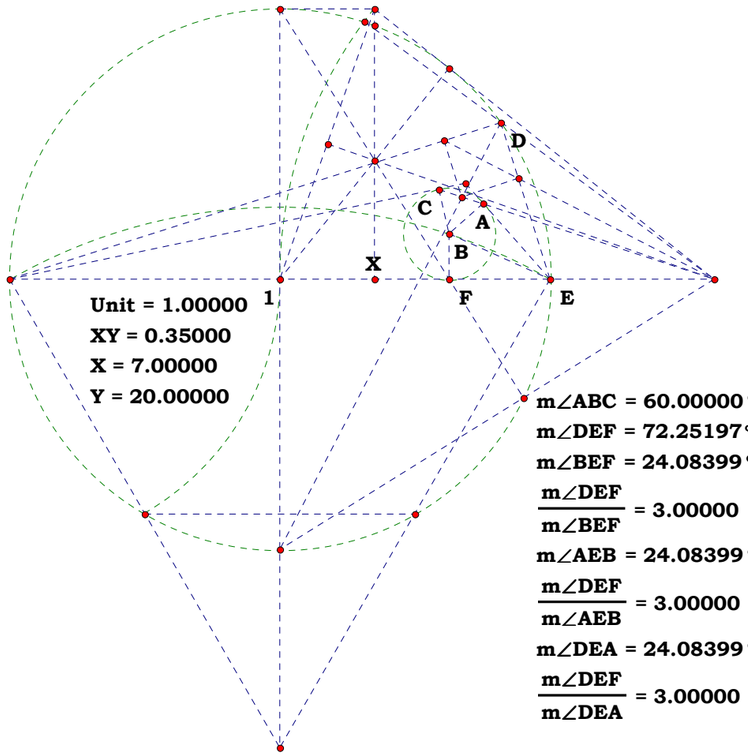
**One can add all the developments if one likes and see how they all relate.**

**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

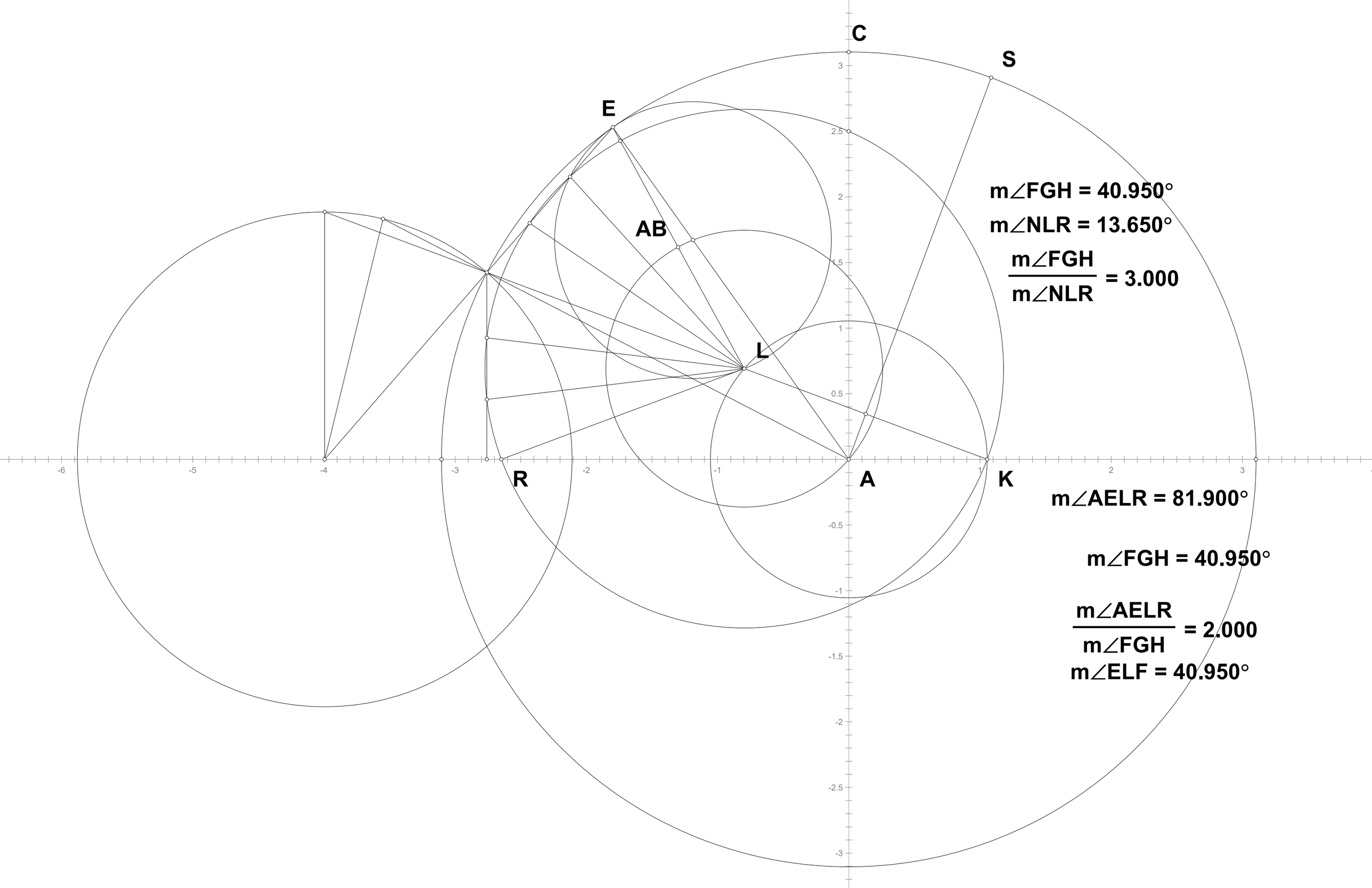
# Parcing project 101402

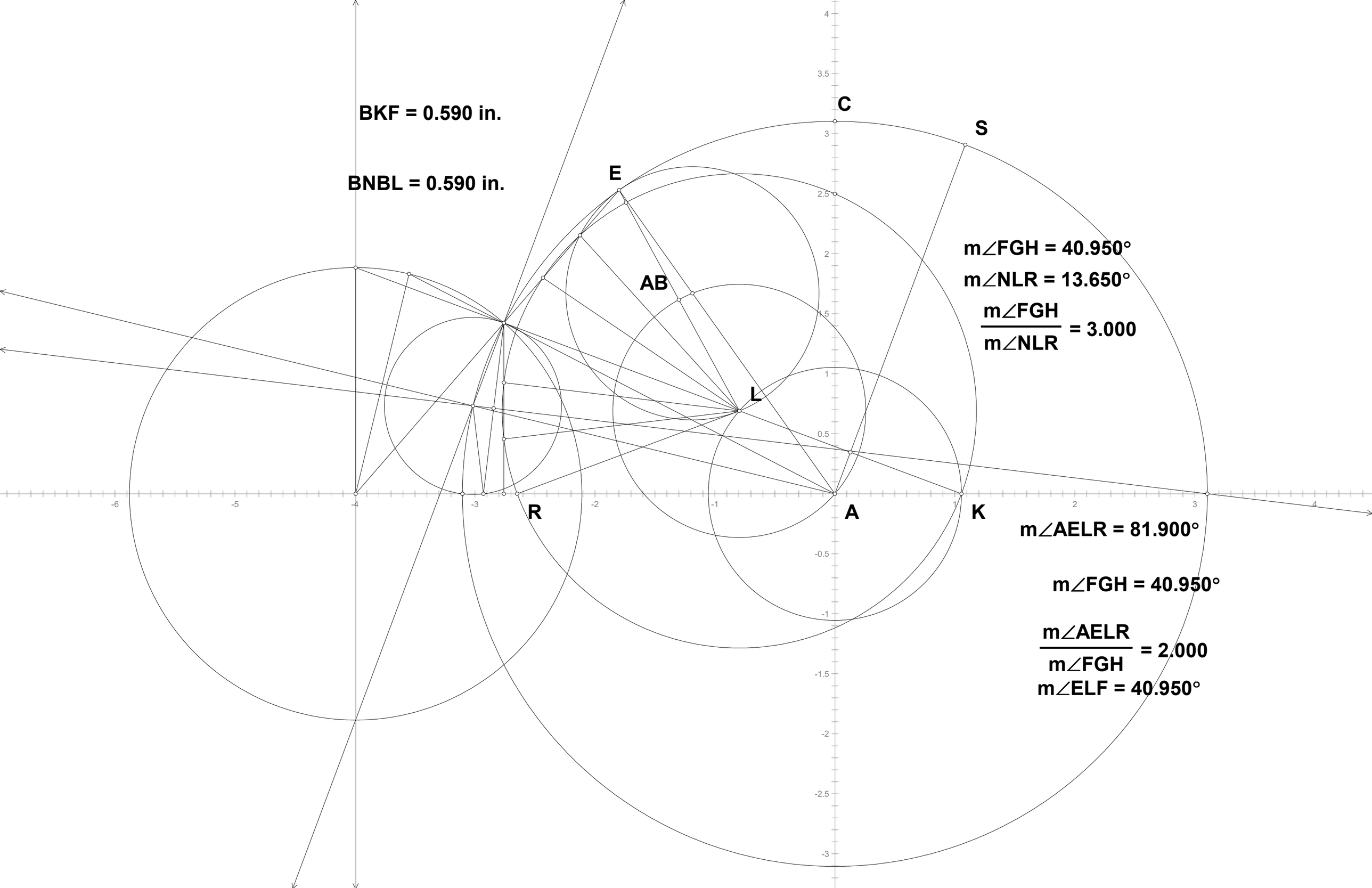
## Square, rectangle and complements.




Unit = 1.00000
XY = 0.35000
X = 7.00000
Y = 20.00000

$m\angle ABC = 60.00000°$
$m\angle DEF = 72.25197°$
$m\angle BEF = 24.08399°$
$\frac{m\angle DEF}{m\angle BEF} = 3.00000$
$m\angle AEB = 24.08399°$
$\frac{m\angle DEF}{m\angle AEB} = 3.00000$
$m\angle DEA = 24.08399°$
$\frac{m\angle DEF}{m\angle DEA} = 3.00000$


C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

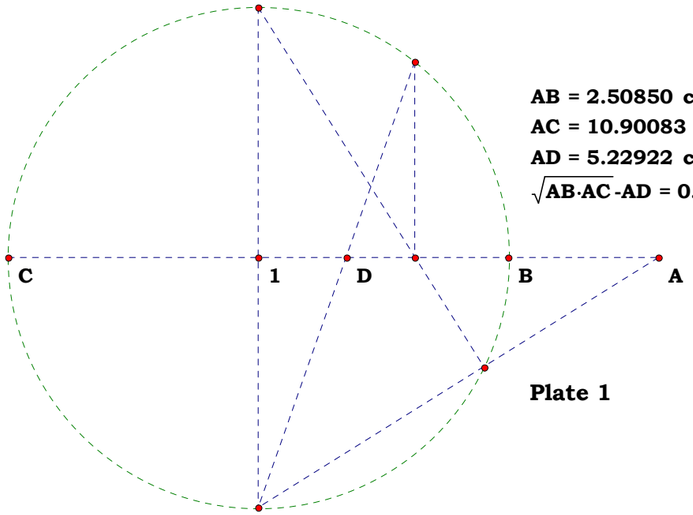

BKF = 0.590 in.
BNBL = 0.590 in.
C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

111402A

**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

## Parcing project for 111402




**AB = 2.50850 cm**
**AC = 10.90083 cm**
**AD = 5.22922 cm**
**$\sqrt{AB \cdot AC}$ - AD = 0.00000 cm**

**Plate 1**

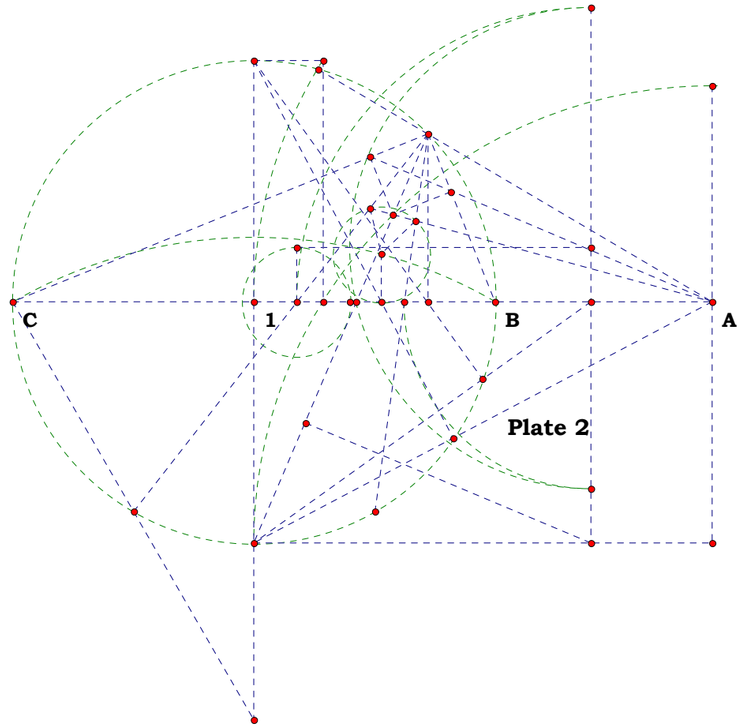


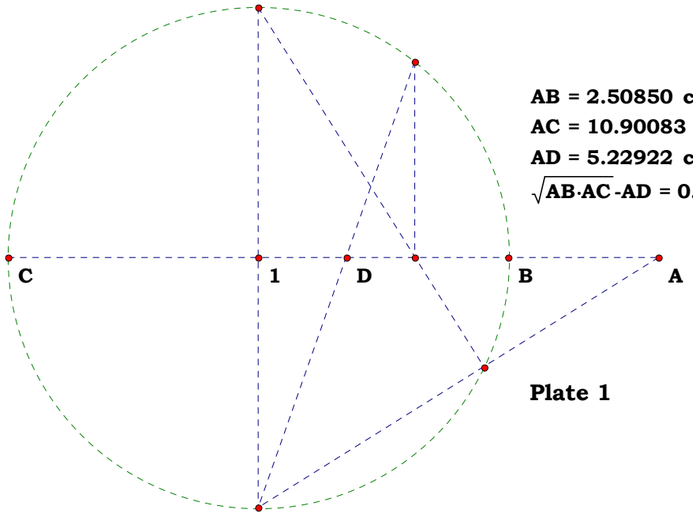

**Plate 2**

111402

**Unit.**
**Given.**

**Parcing project for 111402**

**Descriptions.**
**Definitions.**




**AB = 2.50850 cm**
**AC = 10.90083 cm**
**AD = 5.22922 cm**
**$\sqrt{AB \cdot AC}$ - AD = 0.00000 cm**

**Plate 1**

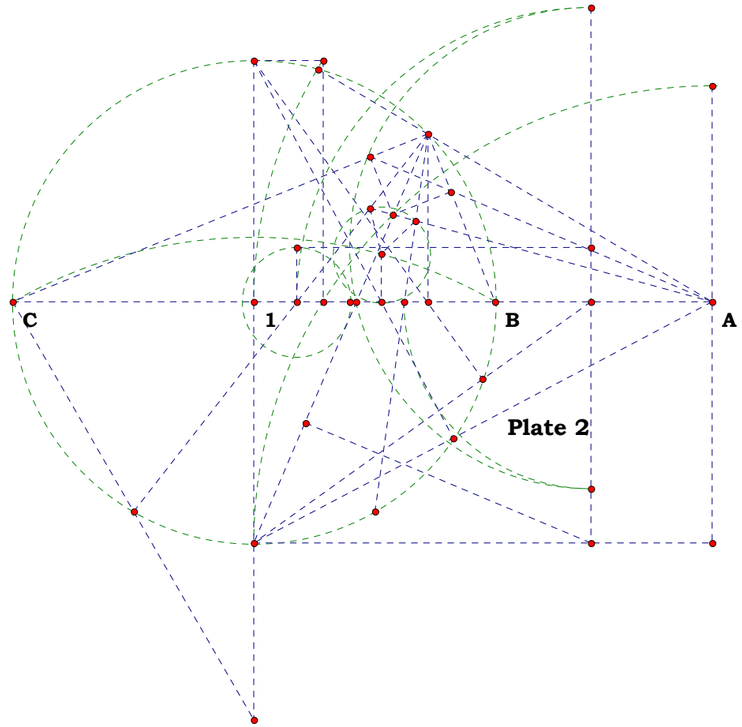



**Plate 2**

032403D

# Four Siblings Plate D

Unit. $AB := 1$

Given.

$X := 11$

$Y := 17$

Not done, in process

Error in figure, not correct E and F do not terminate at D.

**Descriptions.**

$AC := 2 \cdot AB$ $AX := AC \cdot \frac{X}{Y}$ $BX := AX - AB$ $EX := \sqrt{AB^2 + BX^2}$

$GX := \frac{BX^2}{EX}$ $EG := EX - GX$ $DE := 2 \cdot EG$ $BF := \sqrt{3}$

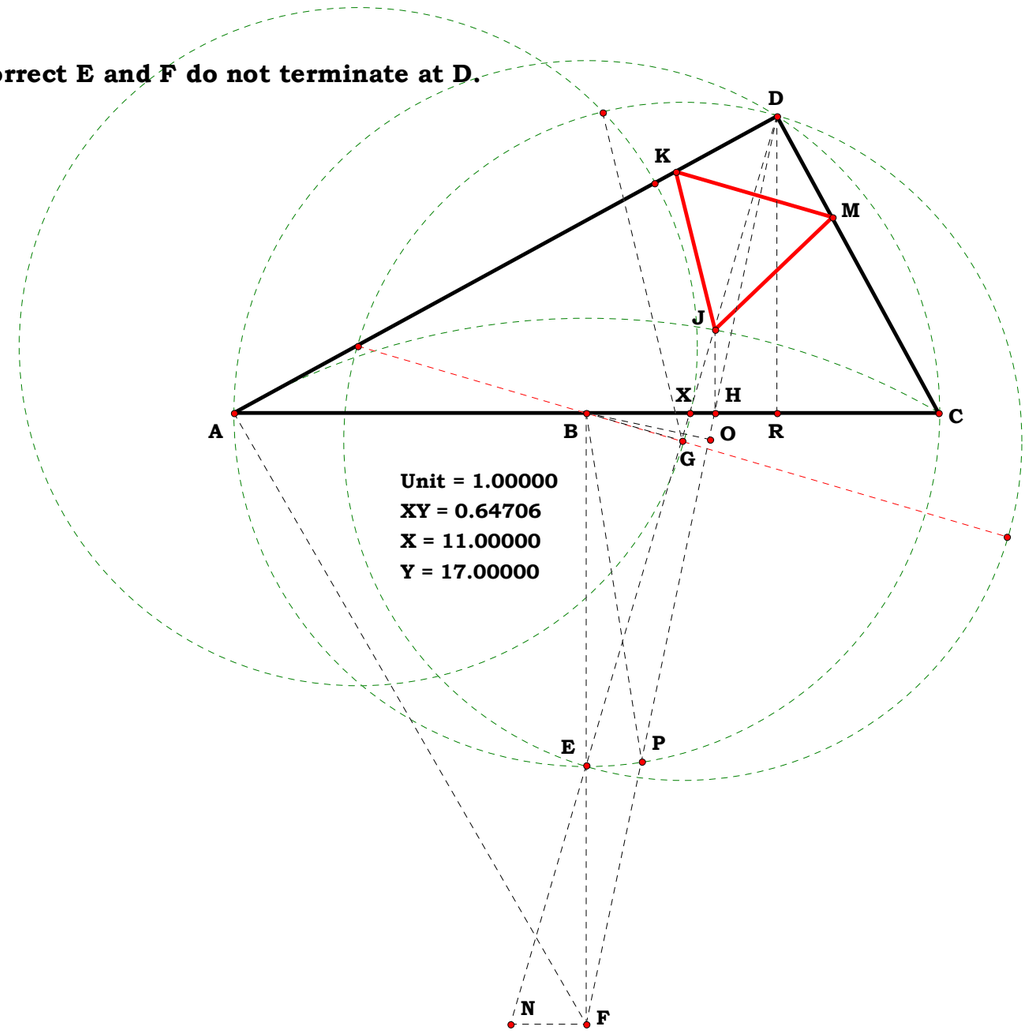
$EF := BF - AB$ $DX := DE - EX$ $EN := EX \cdot \frac{EF}{AB}$ $DR := AB \cdot \frac{DX}{EX}$

$BR := \sqrt{AB^2 - DR^2}$ $BH := BR \cdot \frac{BF}{BF + DR}$ $HX := BH - BX$

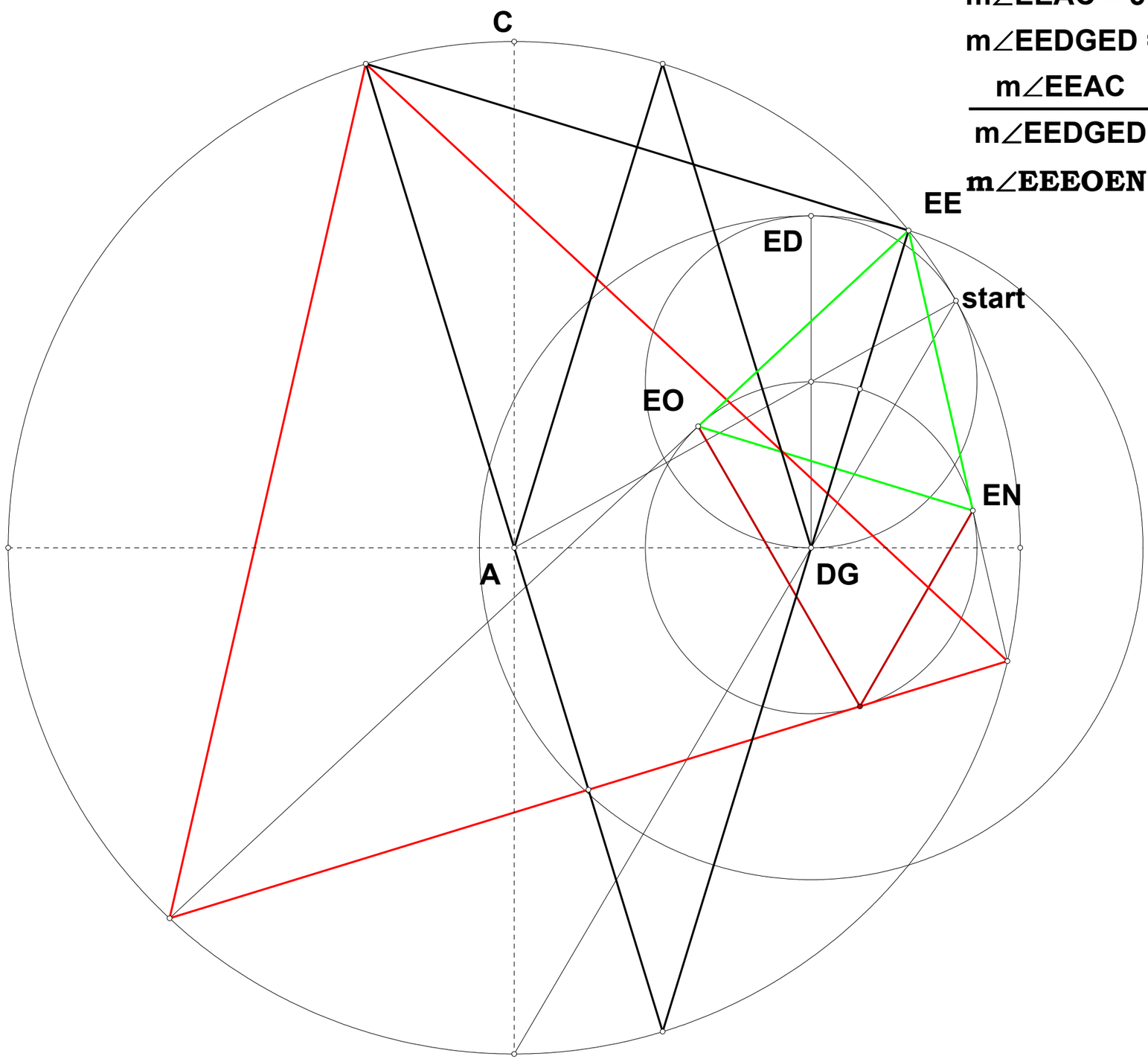
$RX := BR - BX$ $HR := BR - BH$ $JH := DR \cdot \frac{HX}{RX}$ $JH = 0.239225$

**Definitions.**

$DX = 0.876375$



**Animate Point**




$m\angle EEAC = 51.180°$

$m\angle EEDGED = 17.060°$

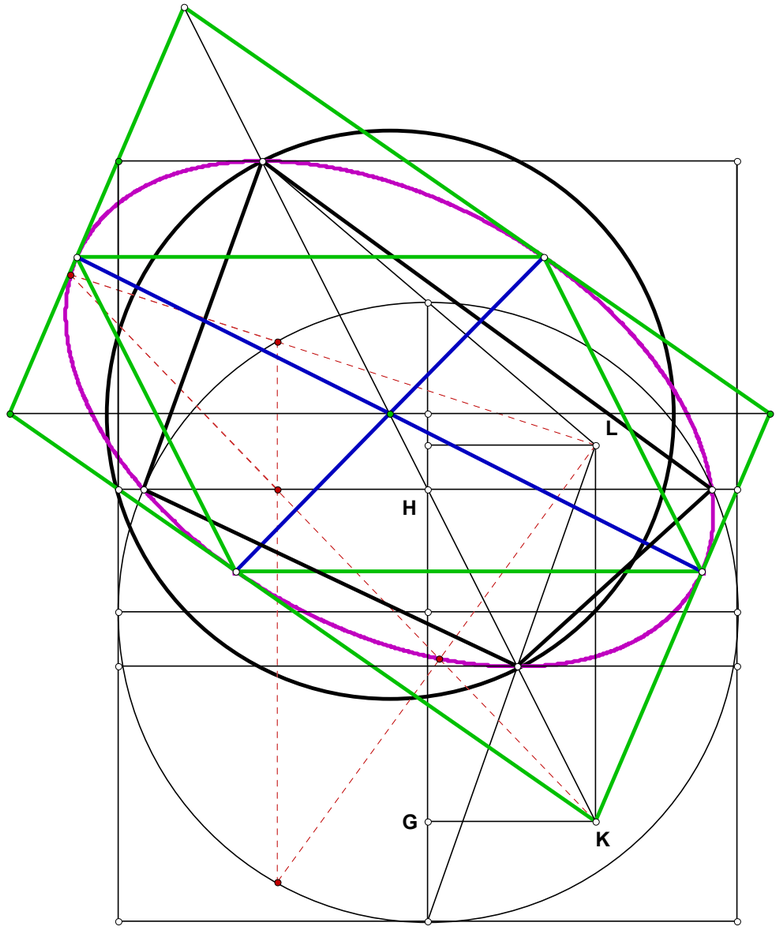
$\frac{m\angle EEAC}{m\angle EEDGED} = 3.000$

$m\angle EEEOEN = 60.000°$

**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

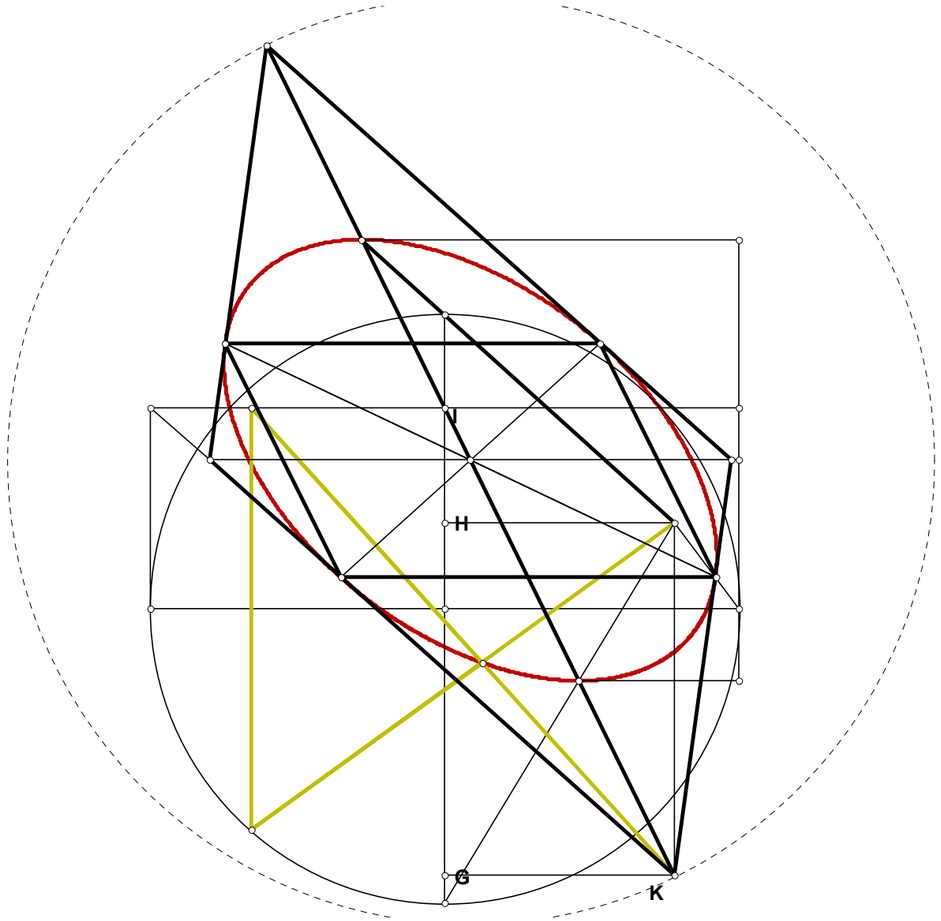
**Parcing project for 031605**



**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

**Parcing project for 031705**

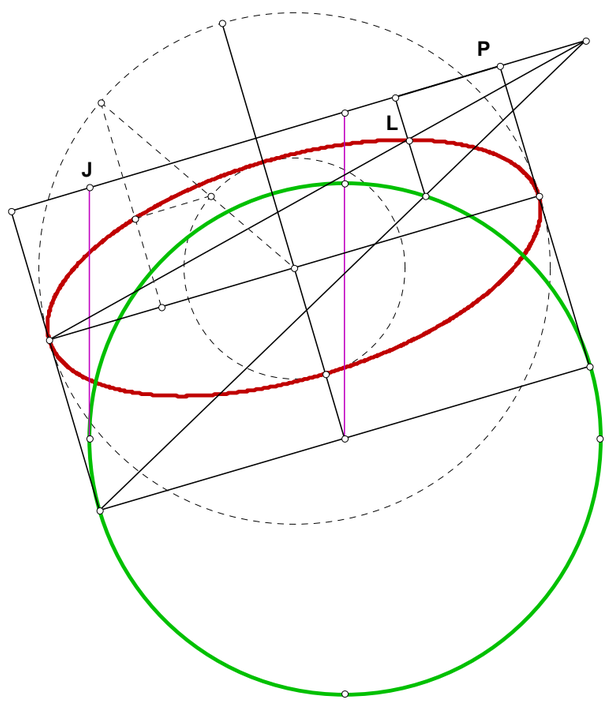


**Unit.**
**Given.**

**Parcing project for 032405**

**Descriptions.**
**Definitions.**

032405a.gsp

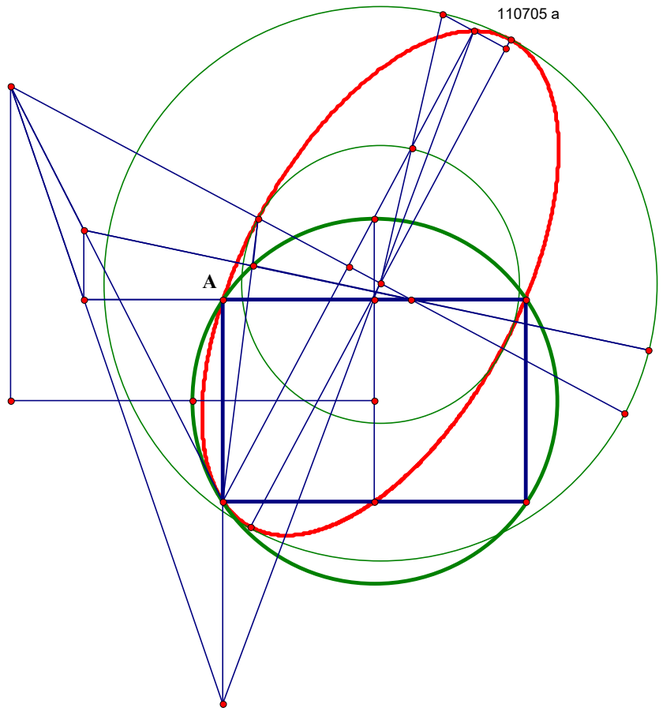


**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

**Procrastinated Writeup for 110705**

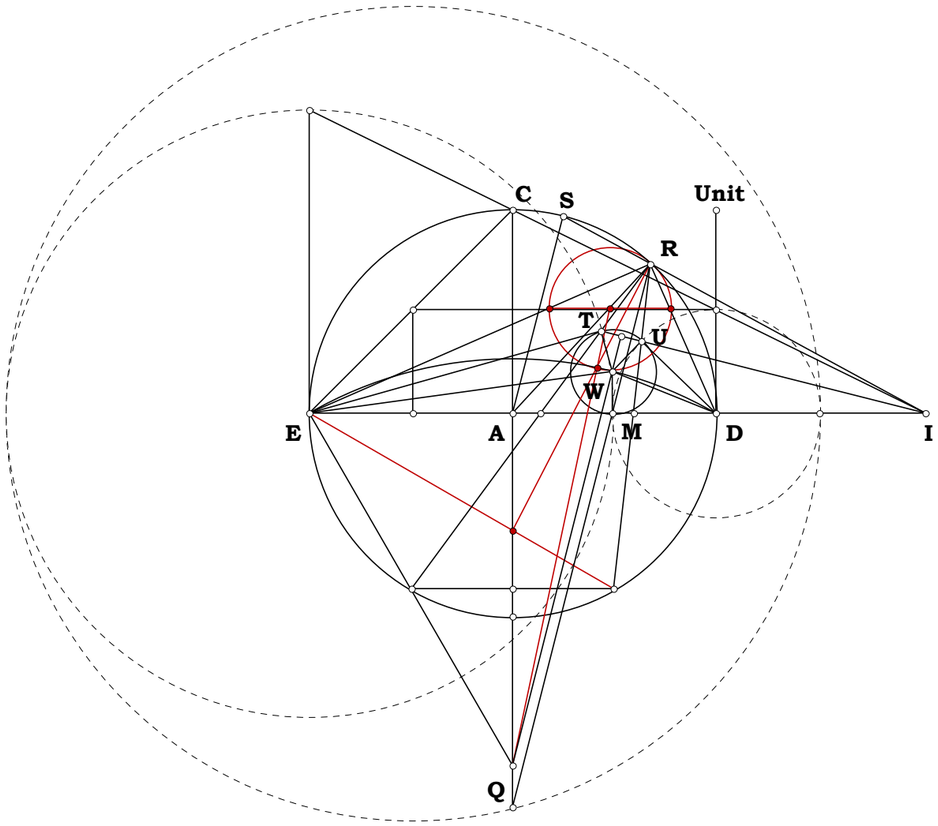
**And by looking at it, maybe another twenty years it get done.**



**110706 Sketchbook D**

**Descriptions.**
**Definitions.**

**Unit.**
**Given.**

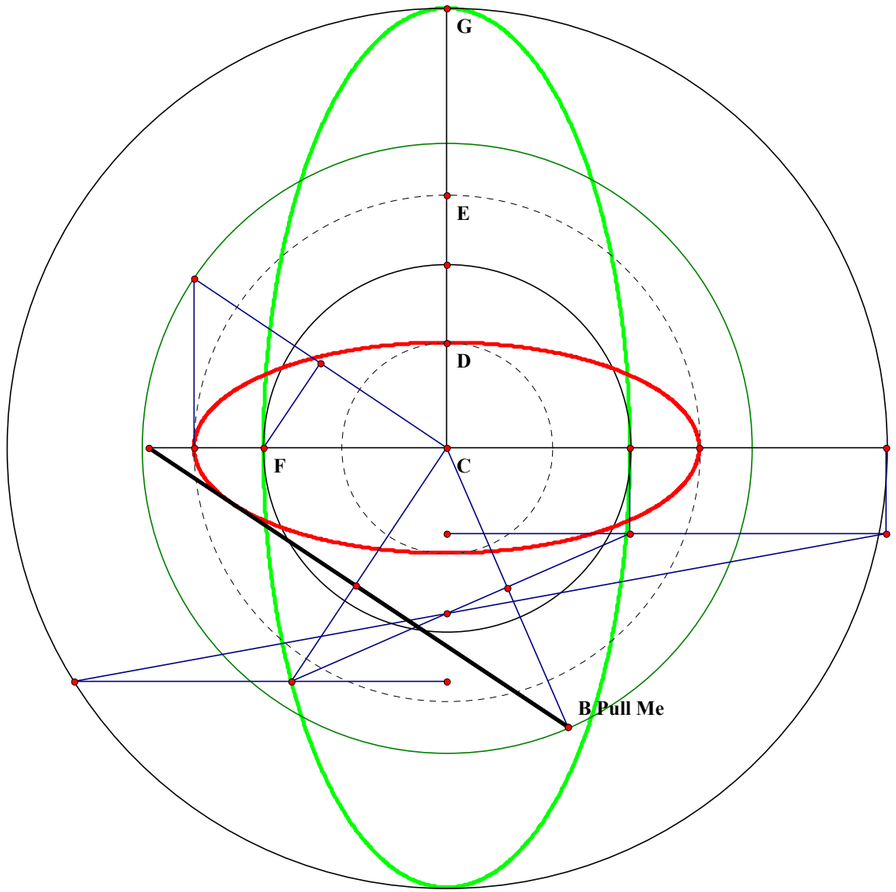


**Unit.**
**Given.**

**110706 Sketchbook B**

**Descriptions.**
**Definitions.**

**From any point B construct a tangent to any ellipse (red).**



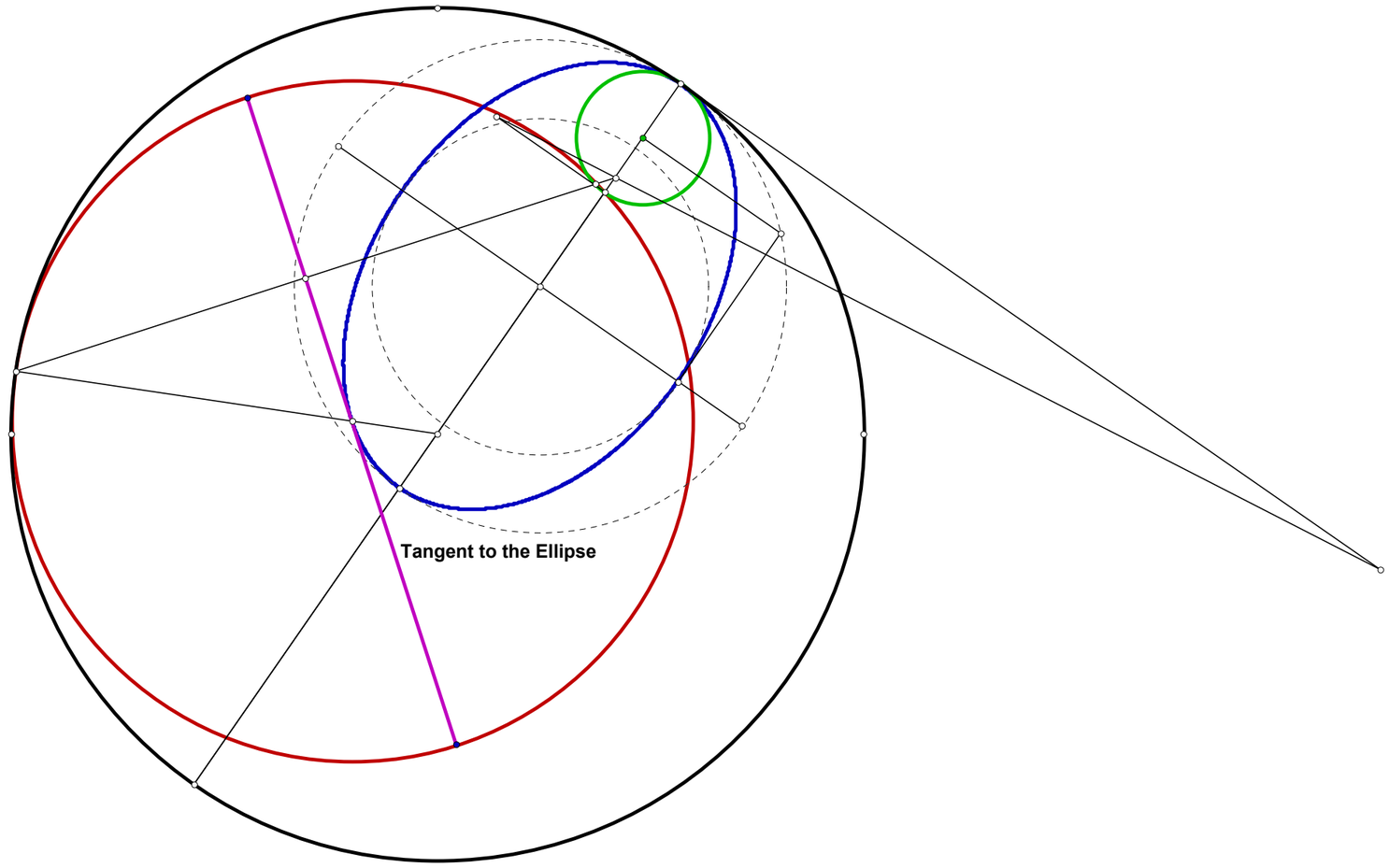

**Project the inverse ellipse (green) and solve.**

$$\frac{CE}{CD}\text{-}\frac{CG}{CF} = 0.00000$$

**Unit.**
**Given.**

**110706 Sketchbook C**

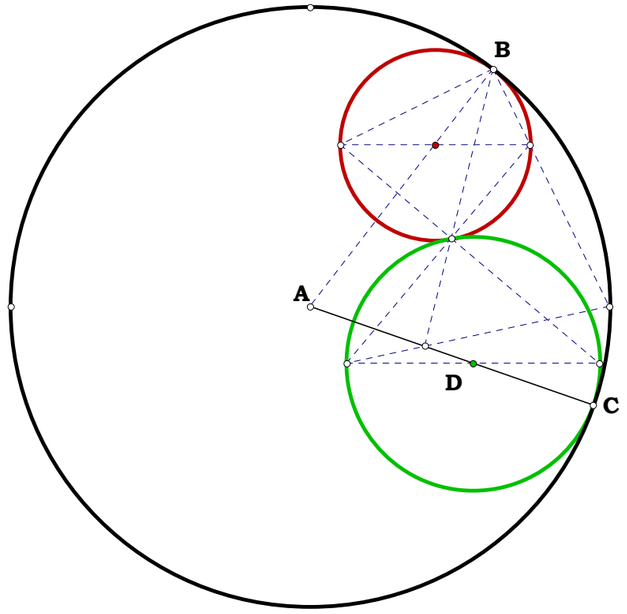
**Descriptions.**
**Definitions.**



**Unit.**
**Given.**

**110706 Sketchbook E**

**Descriptions.**
**Definitions.**



**Unit.**

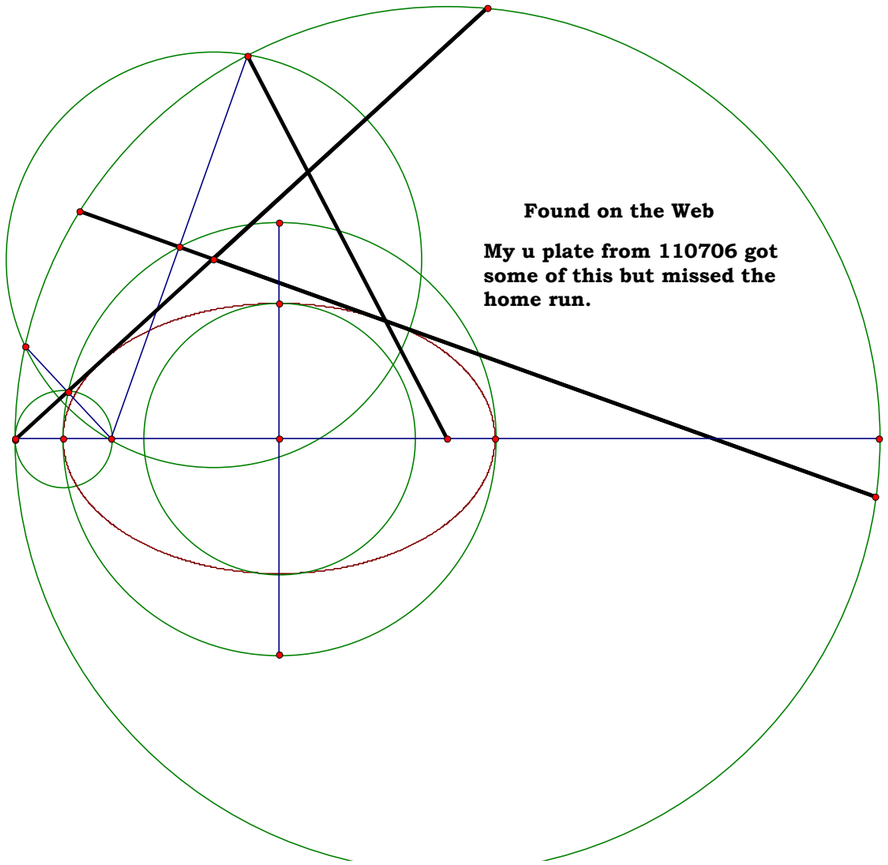
**Given.**

**Parcing project for 062507**

**This is the second plate on the web I found for tangents, and it is quite good.**

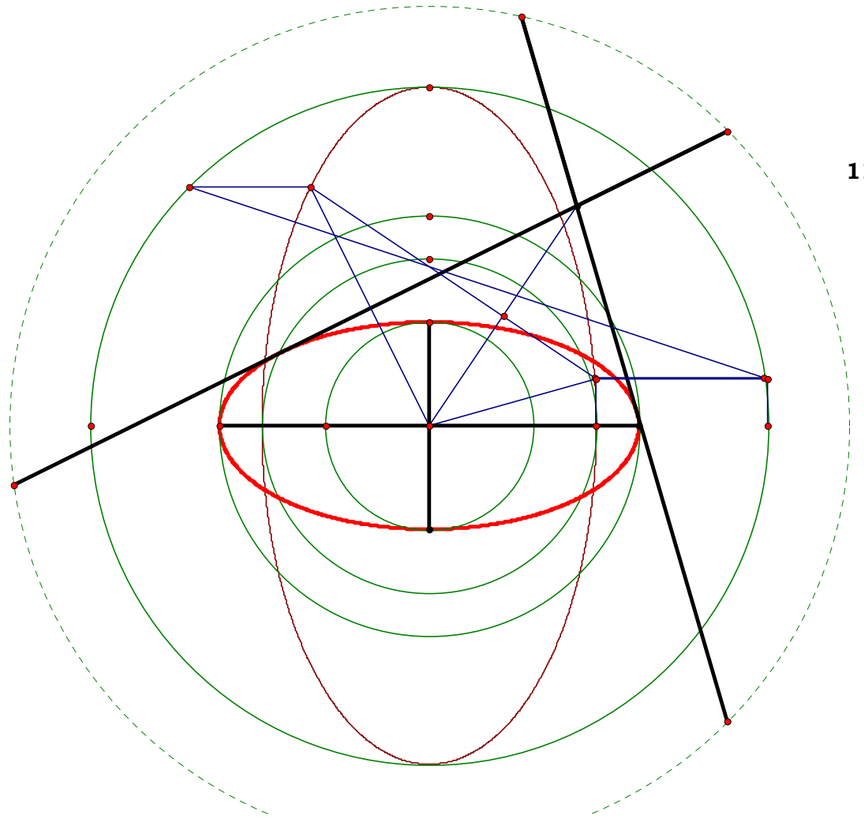
**Descriptions.**

**Definitions.**




**I did find a cleaner plate for 110706 here**

**I call it the inverse ellipse method. I should at least example the stps in construction.**



**Unit.**
**Given.**

**Descriptions.**
**Definitions.**

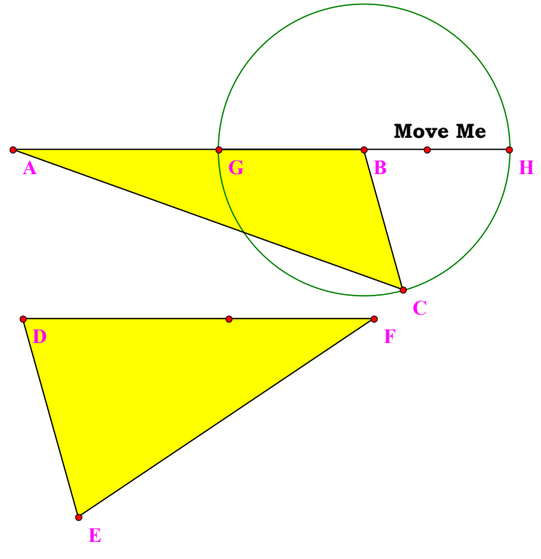
## Parcing project for 072707

Area △ABC = 6.89734 cm$^2$
Area △DEF = 9.76888 cm$^2$
$\frac{\text{(Area △DEF)}}{\text{(Area △ABC)}}$ = 1.41633

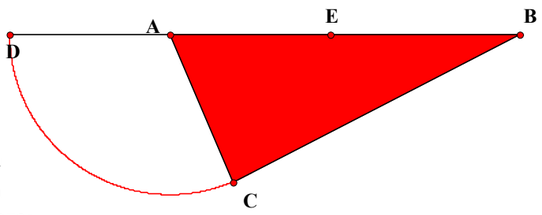
AB = 5.88433 cm
BG = 2.43524 cm

$\frac{AB}{BG}$ = 2.41633

m∠FDE = 74.29322°
m∠HBC = 74.29322°

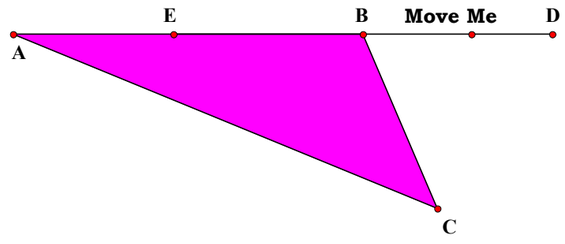


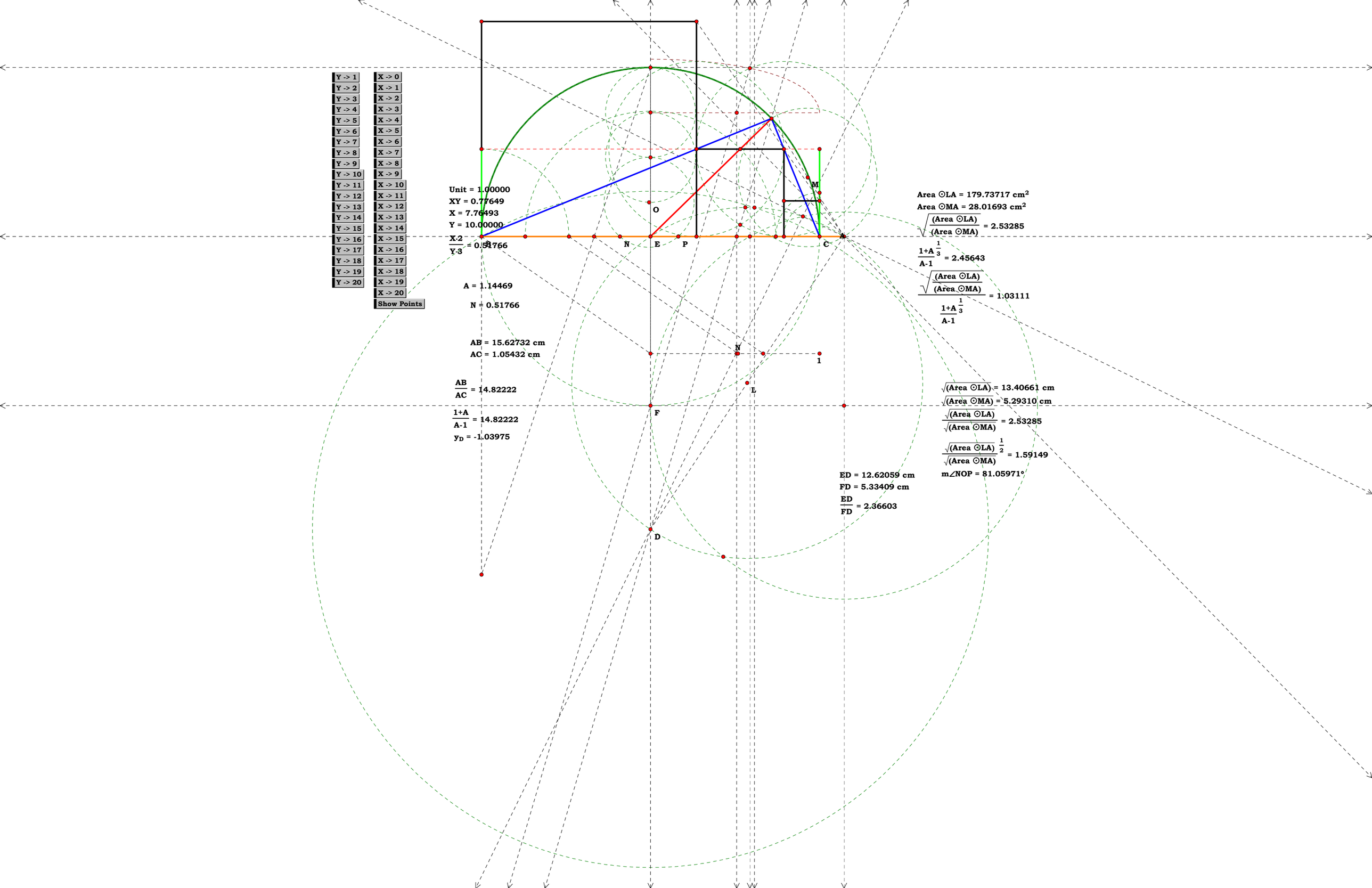

**A series of plates exploring the relationship between an angle common to two figures which maintain a constant ratio in area.**

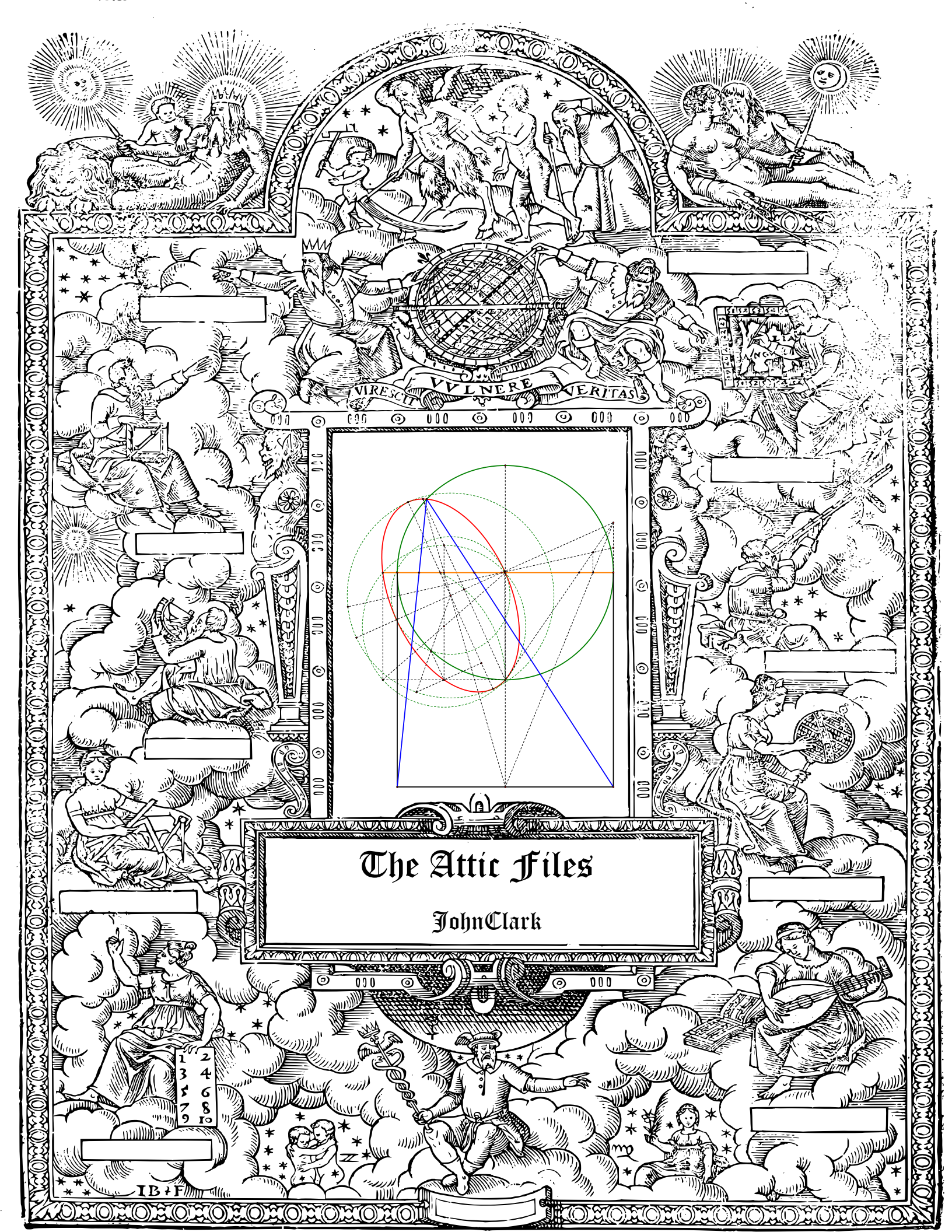



AB = 5.86317 cm
BE = 3.17743 cm
m∠BAC = 66.89978°
m∠DBC = 66.89978°
$\frac{m\angle BAC}{m\angle DBC}$ = 1.00000

Area △ACB = 7.24215 cm$^2$
Area △ABC = 8.56804 cm$^2$
$\frac{\text{(Area △ACB)}}{\text{(Area △ABC)}}$ = 0.84525




Y -> 1
Y -> 2
Y -> 3
Y -> 4
Y -> 5
Y -> 6
Y -> 7
Y -> 8
Y -> 9
Y -> 10
Y -> 11
Y -> 12
Y -> 13
Y -> 14
Y -> 15
Y -> 16
Y -> 17
Y -> 18
Y -> 19
Y -> 20
X -> 0
X -> 1
X -> 2
X -> 3
X -> 4
X -> 5
X -> 6
X -> 7
X -> 8
X -> 9
X -> 10
X -> 11
X -> 12
X -> 13
X -> 14
X -> 15
X -> 16
X -> 17
X -> 18
X -> 19
X -> 20
Show Points
Unit = 1.00000
XY = 0.77649
X = 7.76493
Y = 10.00000
X·2 / Y·3 = 0.51766
A = 1.14469
N = 0.51766
AB = 15.62732 cm
AC = 1.05432 cm
AB / AC = 14.82222
(1+A) / (A-1) = 14.82222
y_D = -1.03975
ED = 12.62059 cm
FD = 5.33409 cm
ED / FD = 2.36603
Area ⊙LA = 179.73717 cm²
Area ⊙MA = 28.01693 cm²
√(Area ⊙LA) / (Area ⊙MA) = 2.53285
(1+A^1/3) / (A-1) = 2.45643
√(Area ⊙LA) / (Area ⊙MA) / (1+A^1/3) / (A-1) = 1.03111
√(Area ⊙LA) = 13.40661 cm
√(Area ⊙MA) = 5.29310 cm
√(Area ⊙LA) / √(Area ⊙MA) = 2.53285
√(Area ⊙LA)^1/2 / √(Area ⊙MA) = 1.59149
m∠NOP = 81.05971°
B
N
E
P
C
O
M
N
1
L
F
D




# The Attic Files

JohnClark

# The Attic Files

My Logo is called John 312, which is a line count for each figure, it is also a reference in the *Book*. This might seem arrogant, however, I am alive today because of what some may call a miracle: I view this miracle as other worldly intervention. During my life time something has had an interest in me, which appears to have disappeared a long time ago when I finally understood why. Once I was on track, I have been apparently left to my own devices. I was asked to express the evolution of the mind of man, an evolution which is based on a Universal called Language, which cannot be seen or taught, it is an intelligible. Language is Universal and Intelligible. Intelligence along uses the paradigm of Language to construct Grammar. As Language is the Universal Intelligible of a Binary, which is the definition of any and every thing, it produces exactly four categories of Grammar. Currently the human race is illiterate, and tosses words around without any knowledge of order or their foundation. Mankind, like every madman, is elevated in its own eyes, when in fact, we are still fools. It is the manipulation of memory which helps bring us back to reality, not the burning fires of hallucinating dribble with pasted together words. This is why I have my own intelligible attic. Perception determines conception, conception determines will. We learn from reality to make real things. A sane person does not go about dribbling words like theory of this or that. As Plato said, we can only name the parts of things which means there is not one damned theoretical part of that at all. We can either effect and learn binary recursion, or we can spout gibberish calling it science with all of its hilariously ridiculous jargon and *factual* gibberish.

The attic is where old and sometimes young people put things in storage. This is a collection of such things, the evolution of the *Delian Quest* and the foundation of my other works in geometry such as *Basic Analog Mathematics* and the *BAM Sample Dictionary*. There are things I may want to take out of this storage and rework for the *Delian Quest*, and I may want to meditate on figures here and see things I should write-up. I may seem to be slow with this work, however, it is by far not the only work I do as a search for me under johnclark8659 or Phil8659 on the Internet Archive will show.

As far as my logo goes, a truck driver once called it the Happy Campers, John, the fire and a mythical Lady of the Lotus. I have always imagined that this mystical LOL would be the love of my life. I never met her, which means that I am not, at least yet, a happy camper.

Preface

Part of this work has been previously published in some shareware circles using, Replica and then Envoy. Neither kept pace with me. I was hoping that someone would give me some feedback, perhaps a figure or two to help educate me. In the years it has been out there, not so much as a peep. It was labeled “Meditations,” but it is actually an illustrated work of fiction. The main characters are One Circle, One Square, and One Line. The consultant to the trio is a psychologist called E. Algebra. From time to time the trio would collar me and inform me of their progress, what I remember I have recorded. I enjoyed the stories of this trio, mainly because I had never heard them before.

Contents

1989 The Delian Solution: Sometime before this I had made my mind up that I needed another long term goal. Finding the solution to the Delian problem seem to fill the bill, how much longer can one get than the impossible? Not only is the problem considered impossible, but I know spit about Geometry. But here I was a little arrogant, as I had solved a couple of problems before considered impossible. I have learned though that it is sometimes not the answer that is impossible, but that the ears to hear the answer are plugged. This plug is often a guardian, an astute guardian wary of who passes by and stopping any alleged intruder.

I wrote this green little paper when I found something I had not seen before, which is not saying much. It was originally hand typed, hand drawn, copied and sent out and it returned again to me. My next phase was “big” drawings which, considering mercy, I present none.

06_20_92 A Triplicate Ratio: It was some time before I acquired a computer system to write in. And some time before I acquired a good drawing program. And some time before I was offered Mathcad to explore figures with. This paper was written up in Word for Windows.

08_12_92 Cubing a Sphere, a Rusty Construction: I had originally done this prior to the 1989 paper and worked it with a hand calculator.

01_08_93 Pythagoras Revisited: I have not yet acquired my math program, but I did write a macro for Word that sent parameters to QBasic and QBasic passed answers back to Word.

I would have thought that by now, the Pythagorean Theorem would have been stated in it’s generalization for all triangles. I have never seen it’s generalization, so I wrote one.

04_19_93 Not knowing how to solve the figure, or of all the richness that the figure contained, time and time again I will attempt Euclidean calculus.

05_21_93 Personal insights on the need to understand thinking in general.

05_26_93 Another estimation.

06_03_93 Exploring the properties of the curve CJ: I had gone through several figures to estimate the solution, many of them did not include graphics. I had a time with graphics. Some methods were completely unacceptable. I finally settled on a little translator for Word to get them into Mathcad while I drew them in Tommy, which I still do. It is still the best way. I had tried to generalize this figure, but I did not know just what I was looking for and at this time abandoned it's generalization.

06_07_93 For All Triangles Find DC:

06_09_93 Actually I wrote this file just to test out if I could write an acceptable Euclidean logical filter. I had no idea that Descartes presented the same idea in the most obscure way that he could.

06_21_93 A Pyramid of Ratios. I started this series because once looking at the figure, I could see the pattern of ratios. In most of this series I did not use the symbolic processor to find the equation, as it seemed fairly outright. Once I started using it, though, I was hooked.

06_23_93 I am just exploring the figure.

06_27_C3 Inscribe a Circle about a Triangle. I wanted to find the formula for the fun of it.

07_01_93 Just exploring.

07_10_93 From Maxima andMinima: Just anotherguesstimate.

07_15_93 Another Pyramid of Ratios:

07_17_93 Pyramid of Ratios III:

07_18_E3 The Pythagorean Completion: This is actually a remake of Pythagoras Revisited. I have nothing against Heron's formula for finding the area of a triangle given just the length of the three sides, except I have never seen it solely in terms of the givens. I just cannot understand that formula. If a thing is solved, the solution is presented only in the terms of the givens.

07_25_93 Pyramid of Ratio Series IV Working the Curve: These series come out so simple and nice. If they were complicated I could not do them.

10_05_93 Exploring.

11_06_93 Gruntwork I on the Delian Solution: From time to time, I will log constants of the figure. Somewhere along the line, I will stop using the title Gruntwork.

11_09_93 Solve for Cube Placement: I have never seen any exponential problems in Euclidean geometry but I have not seen much Euclidean Geometry. The books I buy hoping to find something are very naïve. I will write a few problems on exponentiation and present the solutions.

11_10_E3 Gruntwork II on the Delian Solution.

11_11_B3 The Archamedian Paper Trisector: I have seen a simple proof of the figure using trigonometry, but I hate trig. I will use trig about one time in the future to set sample points. If I get the ambition I will rewrite it out of that paper.

11_12_93 To Square A Circle: Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. Finding it would bring back some productivity to a prison term. It is so simple, I am amazed that it was never rediscovered.

11_18_93 One of the methods for projecting to the similarity point for a cube relationship with a given.

11_22_93 Using iteration to perform cube root abstraction. Not as good as 96 process.

11_24_93 POR Series IV Figure: Generalization of the Series IV figure.

11_25_93 Using iteration to trisect any given angle.

11_29_93 Exploring.

11_29_A3 Ditto.

12_02_93 POR Roots and Powers.

12_04_B3 Two Prime Exponential Series.

12_06_93 Alternate methods; Square Root. Actually I write up several ways to do a square root.

12_06_B3 Gruntwork 4 on the Delian Solution.

12_07_93 This was also in the 89 document.

12_07_B3 Exploring the ratio of squares.

12_11_93 Logging some constants.

12_12_93 The Square Root

12_12_B3 The Square Root figure generalized.

12_16_93 Euclidean Exponential Series. After 48 pages of sieve work I figure out the exponential series. Of course it was embarrassing to look at.

04_06_B4 Inscribing a Circle in a given Triangle: I thought that since I did one on the outside, I may as well do one on the inside.

04_21_B4 The Cradle: Exploring a similarity point.

04_26_94 Tangents and Similarity Points: I just like figuring them out.

04_27_94 The Chordal or Power Line of two Circles: I got the idea from a book.

04_28_B4 Power Point: Another idea from a book.

04_30_94 Division, $A^2$:

05_01_94 Two Circles and a Parallel: My answer to “Two Men and a Baby.”

05_04_94 Two Circles, given a tangent on one.

05_06_F4 Research on trisection.

05_07_94 Research on trisection.

05_16_94 Just some circles:

05_23_94 Looking at a graph. There are some who think that if an answer is given in an arithmetic range that it is not good, however since arithmetic is a logical system naming the same thing the Algebra and Geometry does it often indicates that one can reduce the process to a very nice Algebraic formula.

07_26_94 A fair pencil construction.

07_27_B4 Ditto:

07_30_94 Just another fair Pencil Construction for Cube.

10_27_94 Trivial method for doing Square Roots.

10_28_94 Ditto

10_31_94 Given AB & BE, divide BE such that . .

11_01_94 Cube problem and solution.

11_04_94 Ditto.

11_17_94 Looking at a relationship.

12_24_94 Power Line At square Root.

12_25_94 Two prime exponential series developed through power line progression.

12_26_94 Logging a constant.

01_06_95 Alternate method for Quad Roots:

01_12_95 Archamedian Trisection Revisited: This is still in work, but it looks promising.

03_28_95 Exploring the cube: I should call this the gas gauge.

04_01_95 Exponential Progressions. One can take any ratio and raise it to any whole power. An arithmetician would say “number” to any whole power.

04_12_95 English. I finally put this file in it's own directory. It had been kicked around a lot. It has seen some versions of Word. Basically it's an explanation of why I have never passed an English class. One would assume that since all of our languages are logic systems, English grammar would have come out of the closet.

04_22_95 About the Laws of Exponents and Ratios:

04_24_95 Unit and Universe, induction and deduction.

09_13_95 A Study in Placement. One of my fellow workers had a problem his daughter brought home from a trig class, about the height of a ball on some contraption. Asked me if I could solve it. I hate trig, I gave him the solution using Euclidean Geometry. He assured me my answer was wrong, until his daughter corrected him. Perhaps this figure is a hangover from that?

10_14_95 Trivial Method: Square Root:

10_20_95 Given AB and BD divide BD such that . . .

11_01_95 A Modification of a Square Root Figure, Gemini Roots: Well I do not know what else to call them, after all I don't know spit about plane geometry.

11_05_95 Short Method Gemini Roots.

12_01_95 Method for Equals. Just some circles.

12_07_95 Euler's Line: Scholia. If we tag someone's name onto every little thing done, why we would be walking phone books. I'm still trying to find the biography on the guy called Irrational.

12_16_95 Descartes gave a fragment of a figure, I thought I would develop it. When viewing one of his works, I get the impression that he was mechanically inclined.

12_18_95 Another piece of scholia on Descartes. I don't have much in the way of geometry books to learn from. This one soon leaves me in the dust with conic sections. I keep hoping to find some books on construction.

12_20_95 Just for fun.

12_21_95 Pascal's triangle with exponential division.

12_21_B5 Dividing an exponentiated integer by an exponentiated integer of the same power. (And some other goodies.)

12_29_95 Given AC and CD find BC when BC =. . .

01_04_96 The Archamedian Paper Trisector Without the Numbers. This paper is a clue to logical processes that far transcend any ability to proof them with arithmetic. They rely solely upon some notions of set theory.

01_07_96 A rusty Compass construction for the duplication of the cube.

01_08_96 Alternate Method, Power Line.

01_08_B6 Looking at some ratios.

01_09_96 The Algebraist. A Word essay.

01_10_96 To Prove and not to Proof? A Word essay.

01_11_96 On Logic Systems: I thought that this year I would attempt some essays before I forget all that I am thinking.

01_11_M6 Just exploring.

01_13_96 Pyramid of Rations, Moving the Point: Thought I was done with these? I do one when I get too bored with myself.

01_14_96 Looking at the equation. Some algebraist forget themselves and believe that they are arithmeticians. I suppose if cars could think some Cads would think they are roller blades.

01_15_96 Just exploring.

01_16_96 Looking at some more ratios.

01_17_96 A right angle duplication of a linear ratio.

01_17_B6 Dividing a right triangle in a given linear ratio to a similarity point.

01_18_96 I’m getting around to trying to generalize the curve CJ.

01_21_96 Just exploring.

01_22_96 Trivial Method: Square Root.

01_25_A6 Projecting to the point of cube root similarity.

01_29_96 Linear division . . .

01_31_96 Hitting JG from any BN.

02_02_96 Given BF and BC such that . . .

02_04_96 Psychology: A personal hypothesis. There are so many so called head workers that are not satisfied unless they can type you from a book. The “you too!” syndrome.

02_11_96 The Trinity: a brief manifesto: I always did like that word, but could not find one for me, so I wrote one.

02_14_96 Use iteration to find any Gemini roots. Since the Delian Problem was stated in the material, I find this figure acceptable.

02_24_96 Making the Point: Word essay.

03_01_96 Does every $n^{th}$ root series have at least one square root pair? I don’t know if there is a point to being so obvious, but it works some times. Ask a college freshman if they can multiply a cup by a cup watch the general confusion.

04_14_96 Method for Unequals: I had this sketched out in 95, but if I put it back there, I would have a bunch of links to redo in this document.

04_15_96 On Gemini Roots:

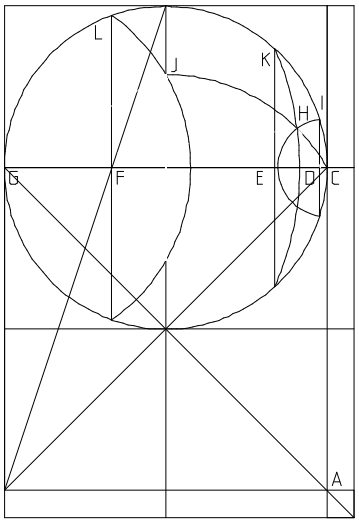
04_16_96 Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO.

04_17_96 Given a point of tangency, draw a circle in a crescent tangent to the other side.

04_22_96 Place EF and GH and find JK.

04_23_96 Does CF always equal EP?

04_24_96 Given a Circle, place the next on the diameter.



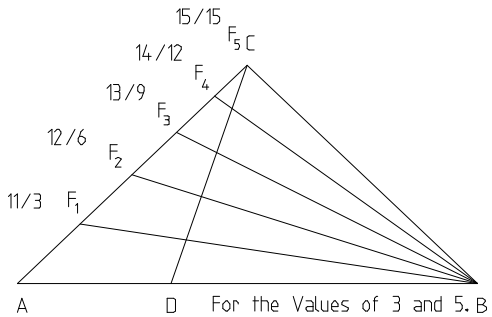

# One Circle

**By John Clark**

# One Square

# One Line

## And the Delian Quest




# 1989

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

# THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

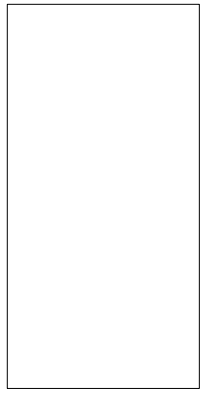


Plate 1

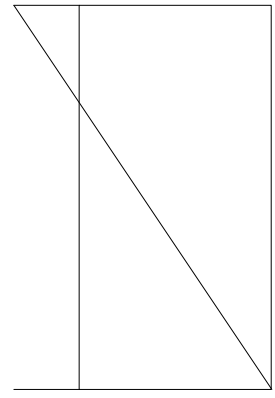


Plate 2

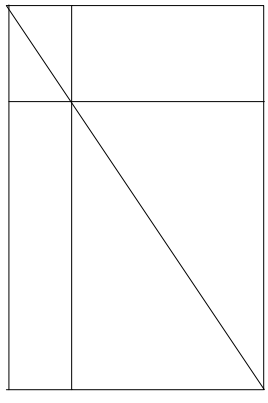


Plate 3

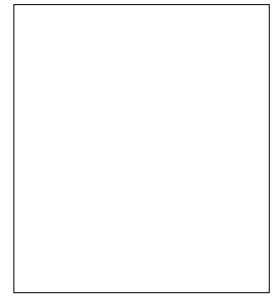


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

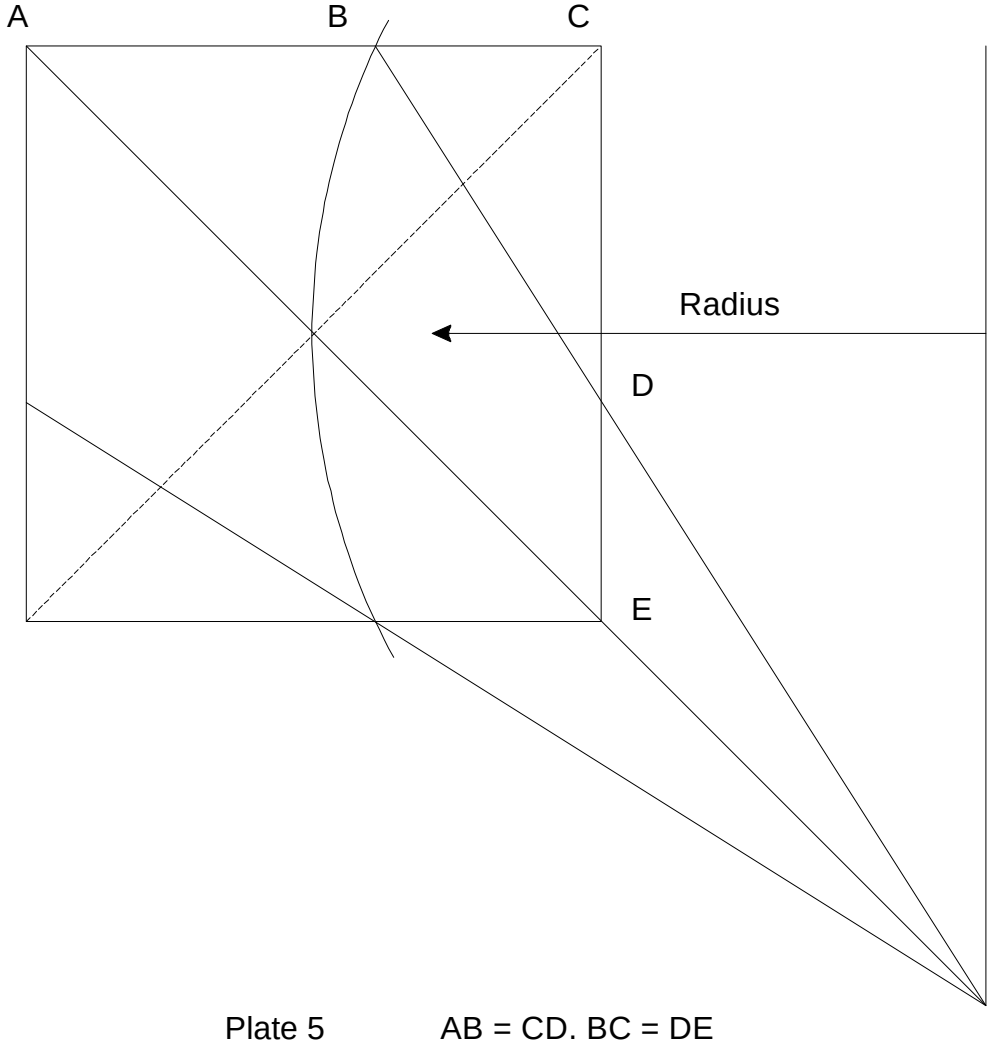
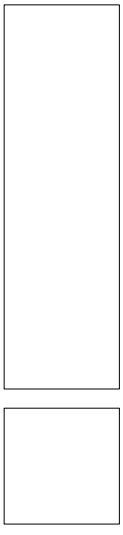
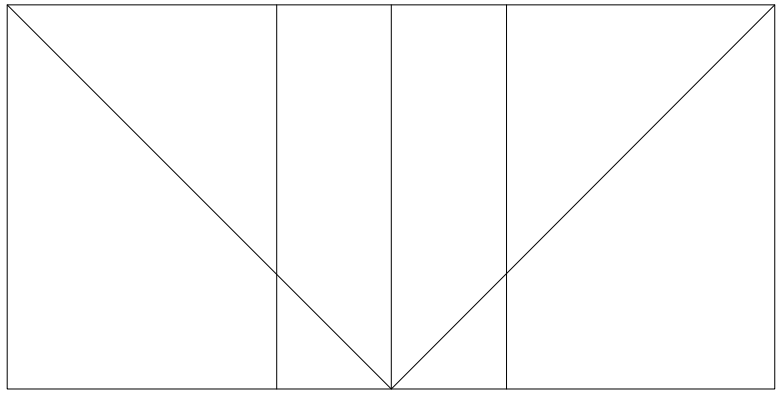



Plate 5 AB = CD. BC = DE

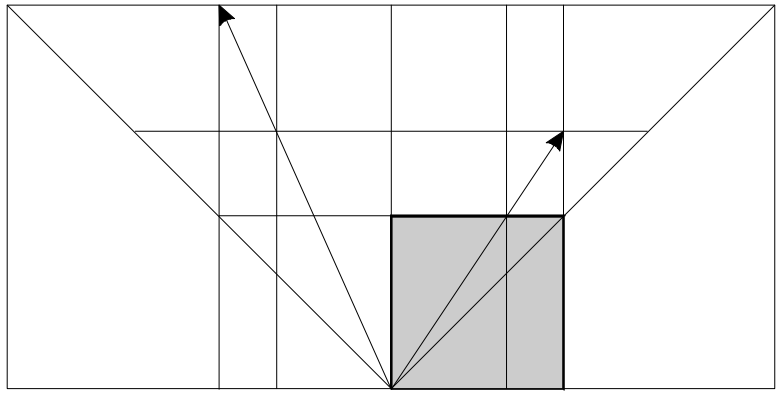
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



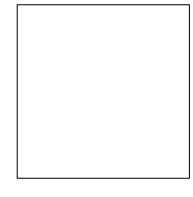
P.6



P.7

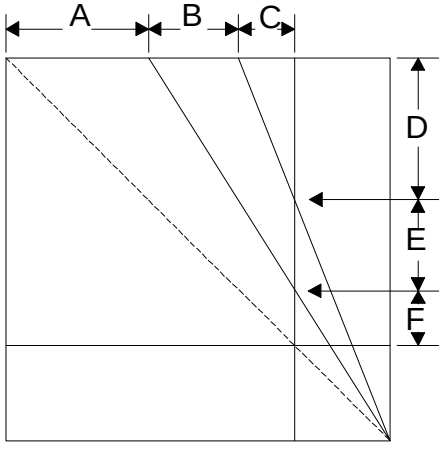


P.8

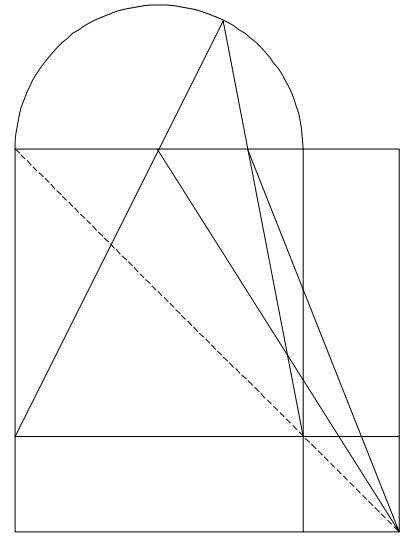


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

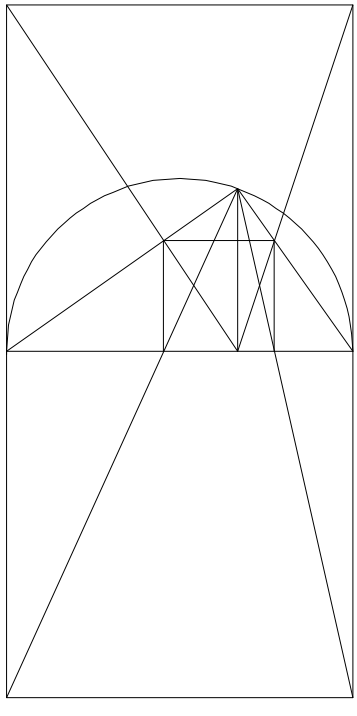



P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

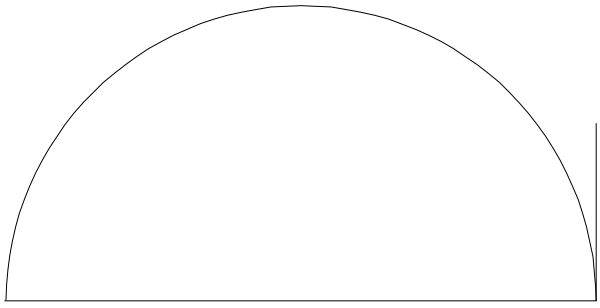


Plate 13

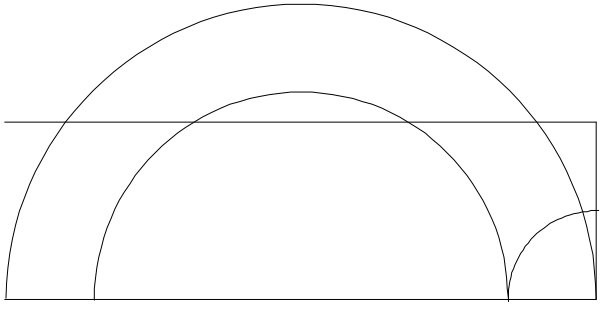


Plate 14

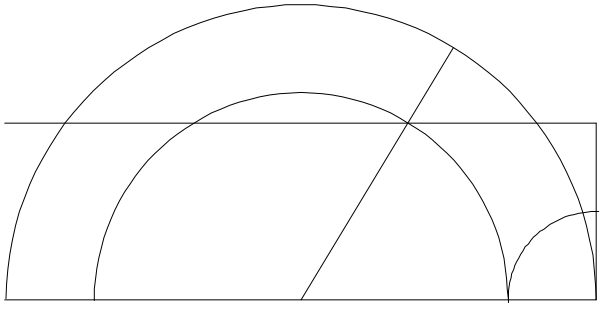


Plate 15

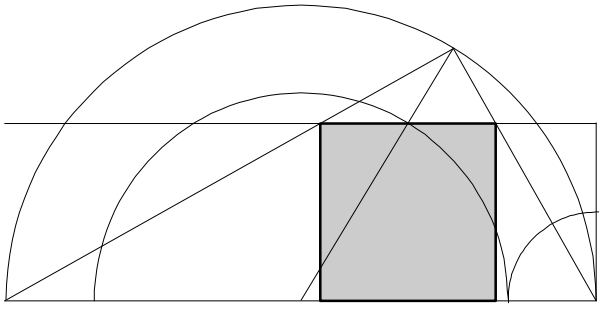
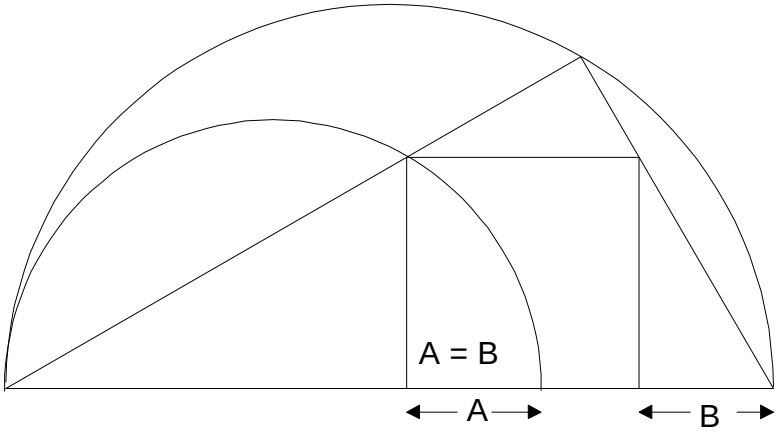
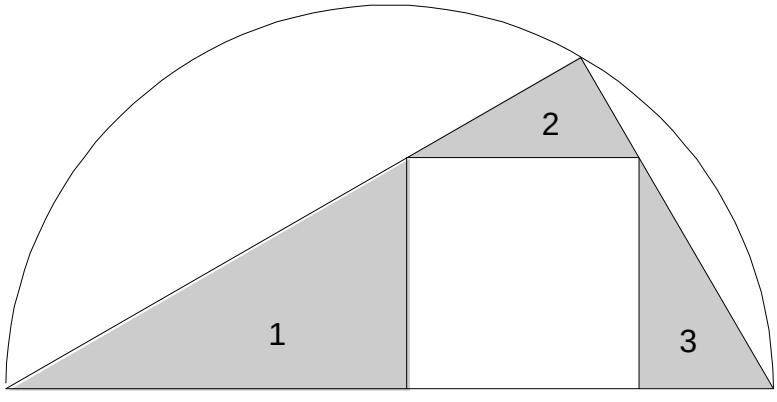


Plate 16

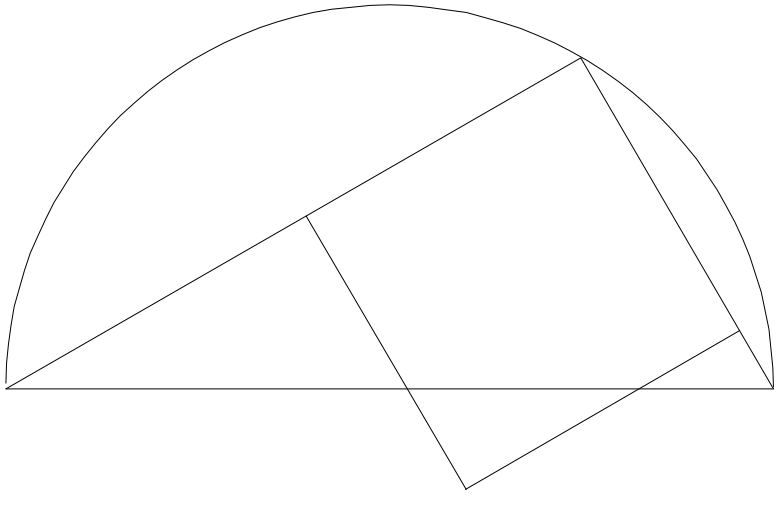
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.




In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"




P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

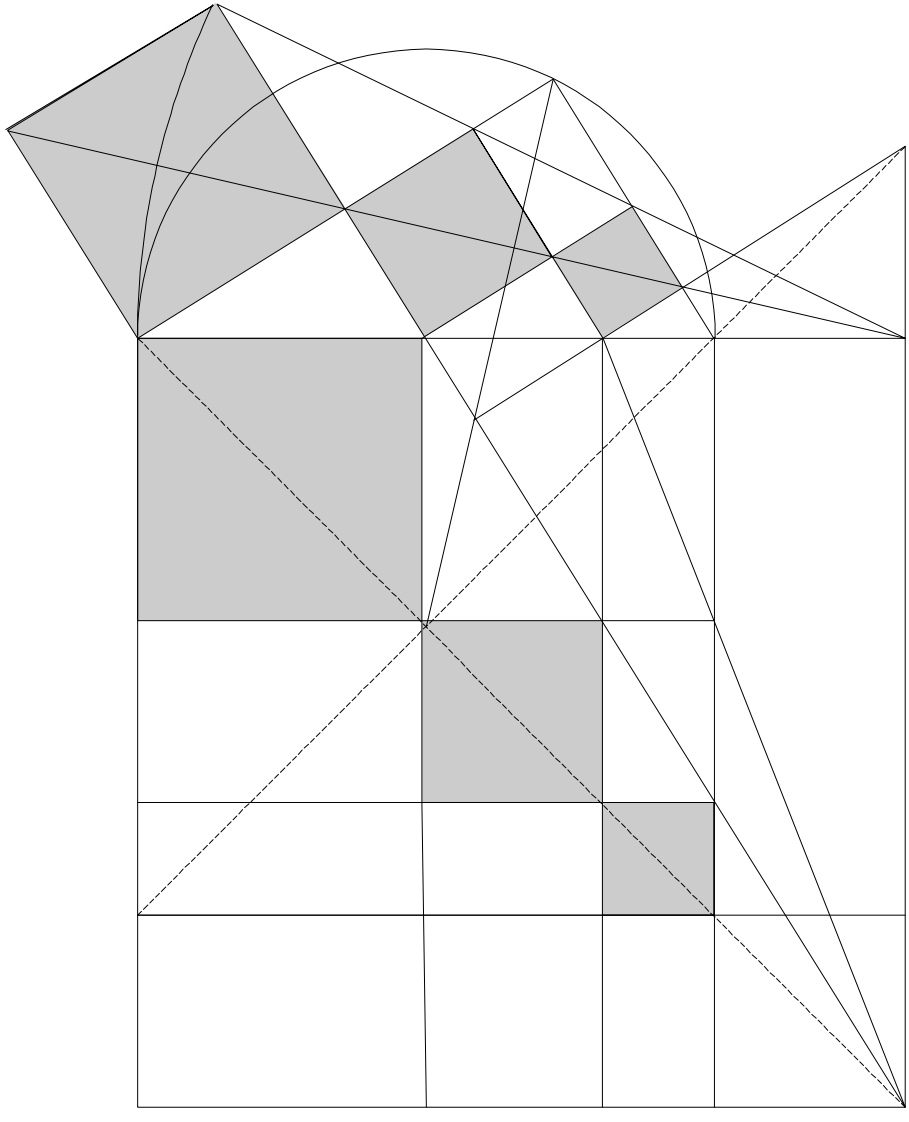
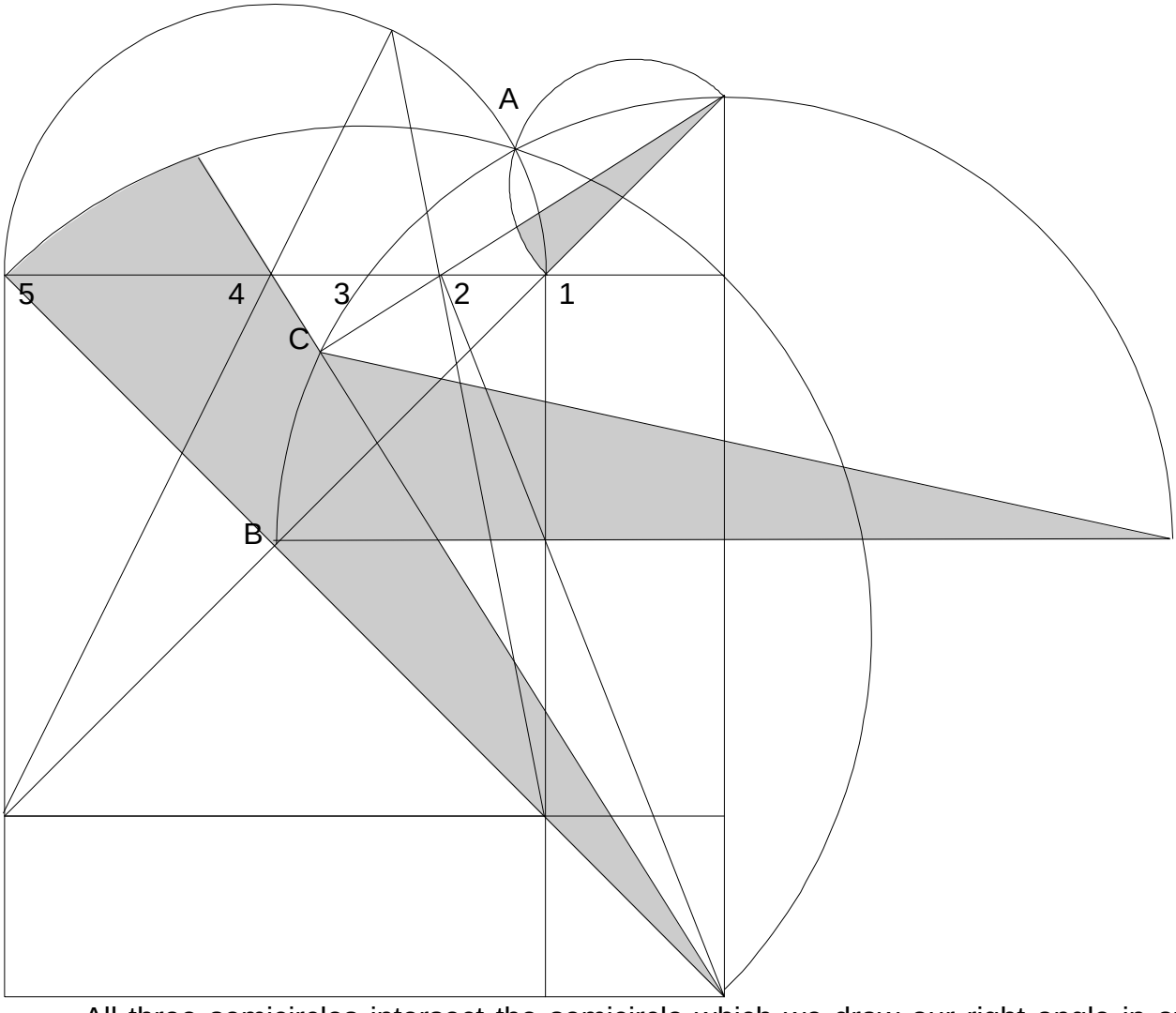


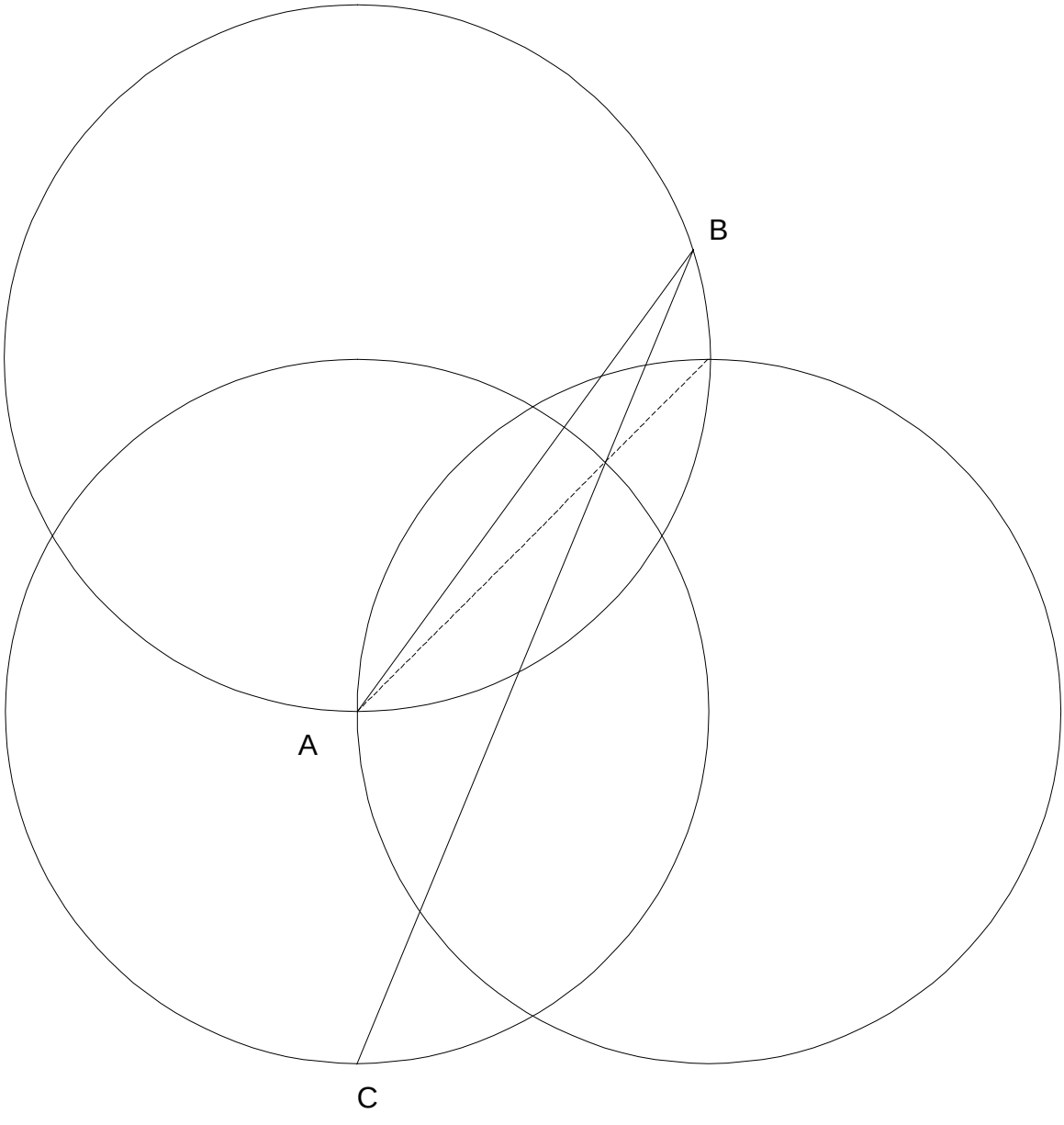
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.




All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.




How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

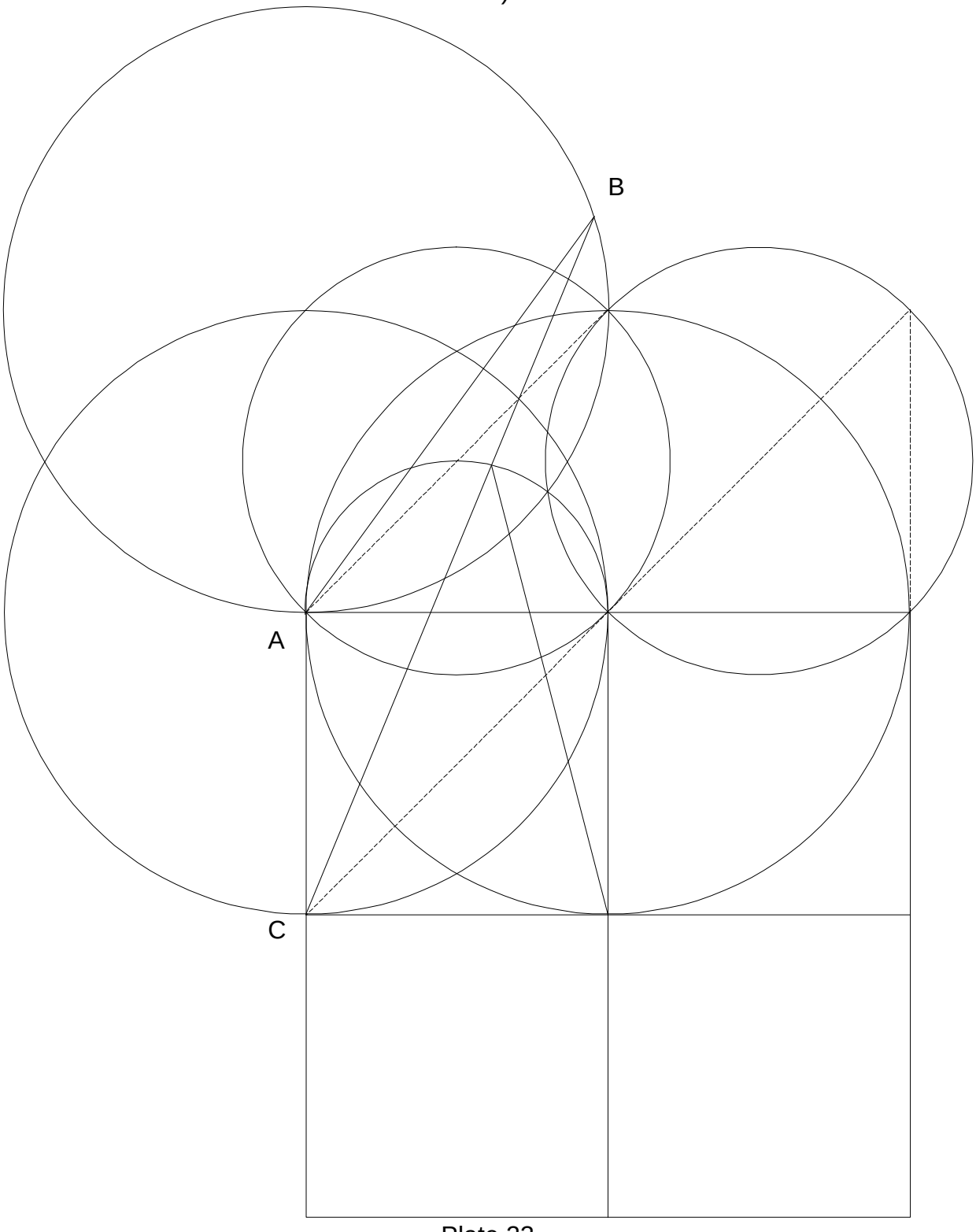
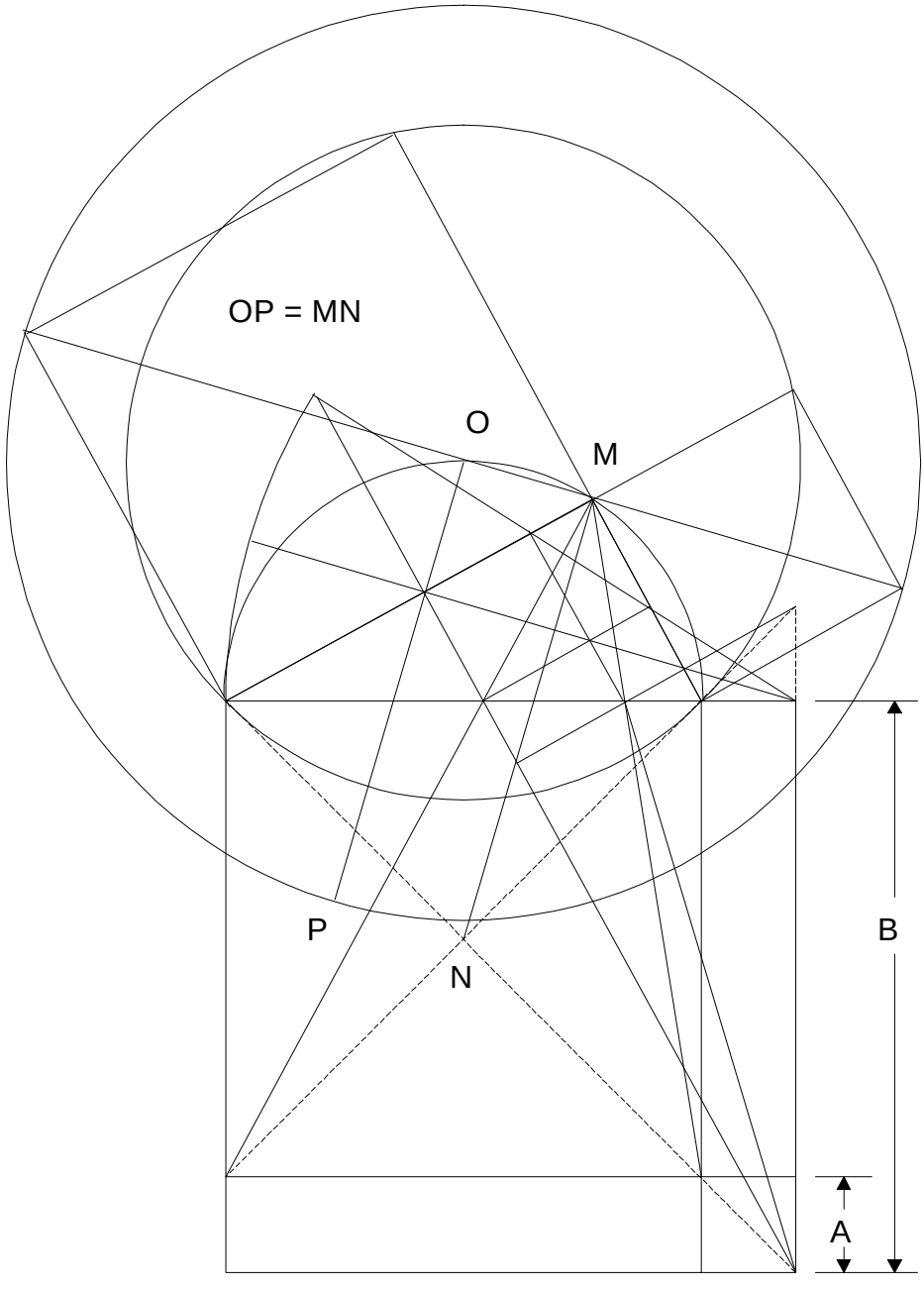



Plate 23

On Plate 24 the radius for the circle OP is given by MN.




One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of $B^2A$ (if you have missed it, the figure gives both roots, $A^2B$ and $B^2A$) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

# GEOMETRIAE DEDICATA

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3508 TA Utrecht
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Telephone: (0)30-531519

Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

# American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA  Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

**Location:**
201 Charles **Street**
Providence, **RI**
02904


December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

*Serving the mathematical community for over 100 years*

# American Mathematical Society

Department of Mathematics
**Roger E. Howe**
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

# JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

**société mathématique defrance**

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

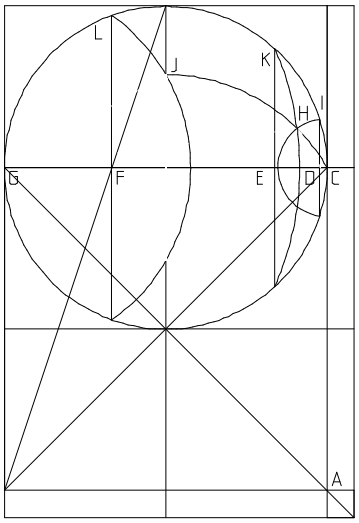
the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

P.J. : Manuscrit



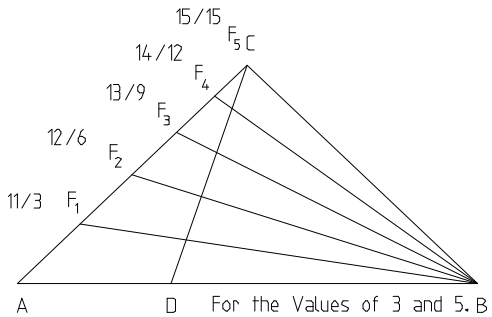

# One Circle

**By John Clark**

# One Square

# One Line

## And the Delian Quest




# 1992

# A Triplicate Ratio

Given any point on the diagonal external to a square and any length of line less than the side of the square, extend from that point two rays such that it cuts two sides of that square into triplicate ratio's, one of which is the length of the given line.

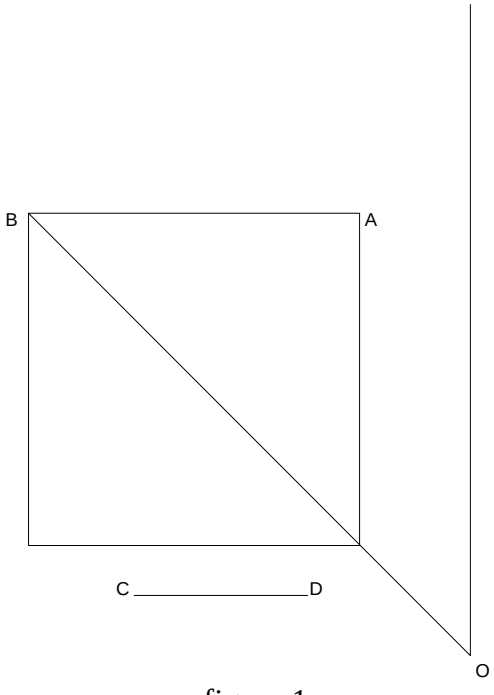



figure 1

Let AB be the given square, CD the given line and O the given point on the external diagonal.

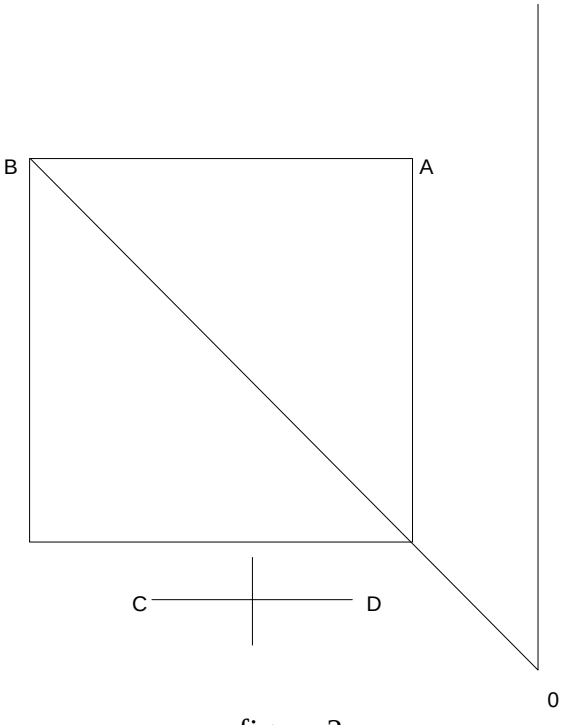



figure 2

Divide CD.

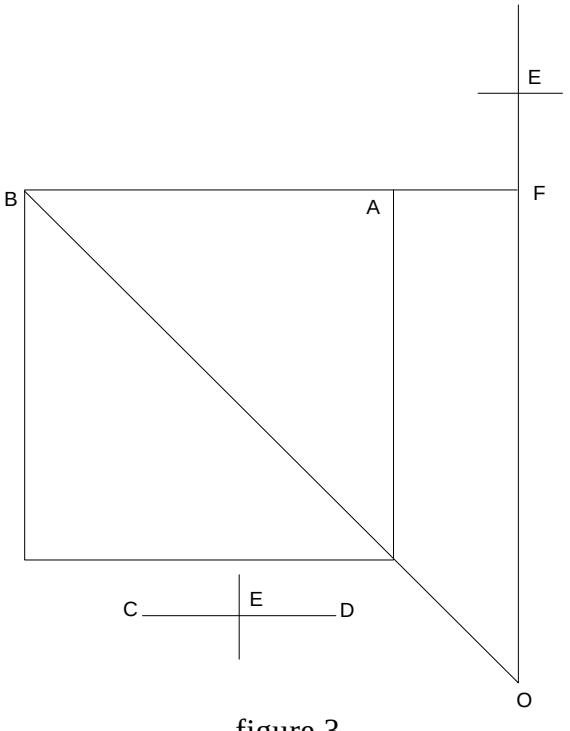



figure 3

Copy $\frac{1}{2}$ CD to and asEF.

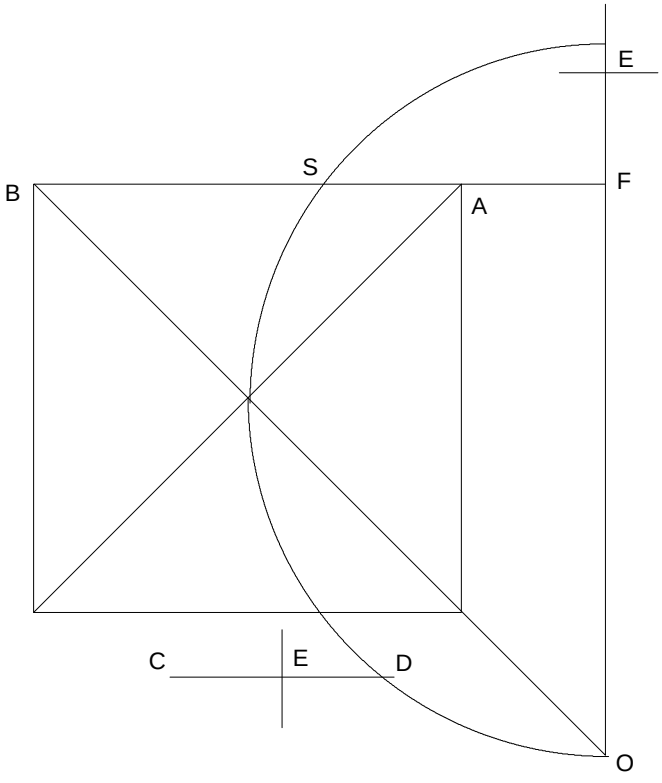



figure 4

S is the square root of $FA \cdot FB$

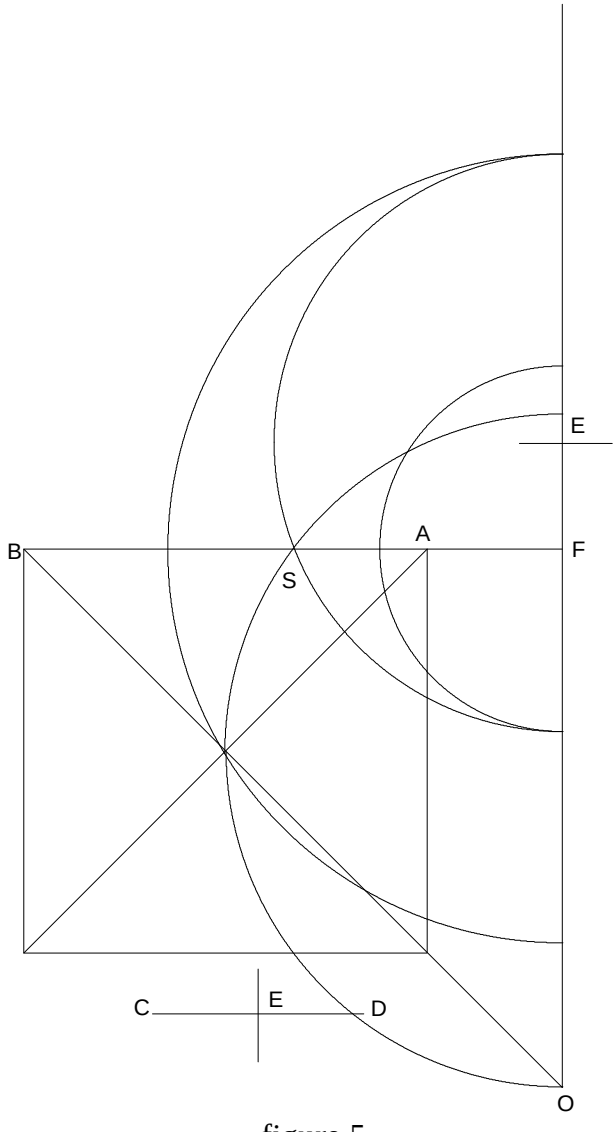



figure 5

This maneuver may look easy, but it gave my poor ignorant head a workout.

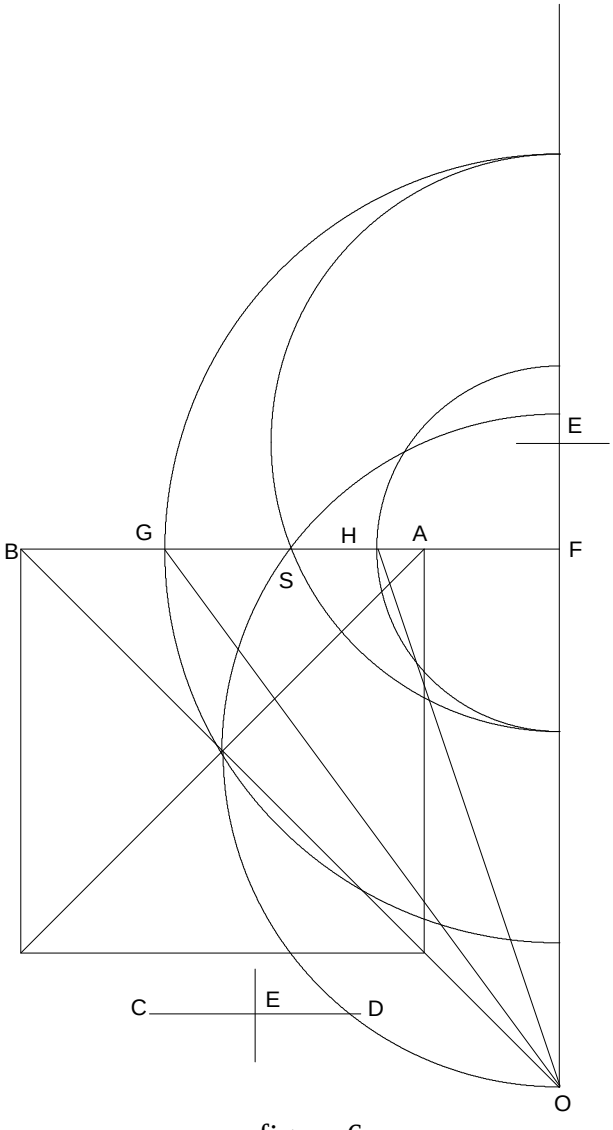
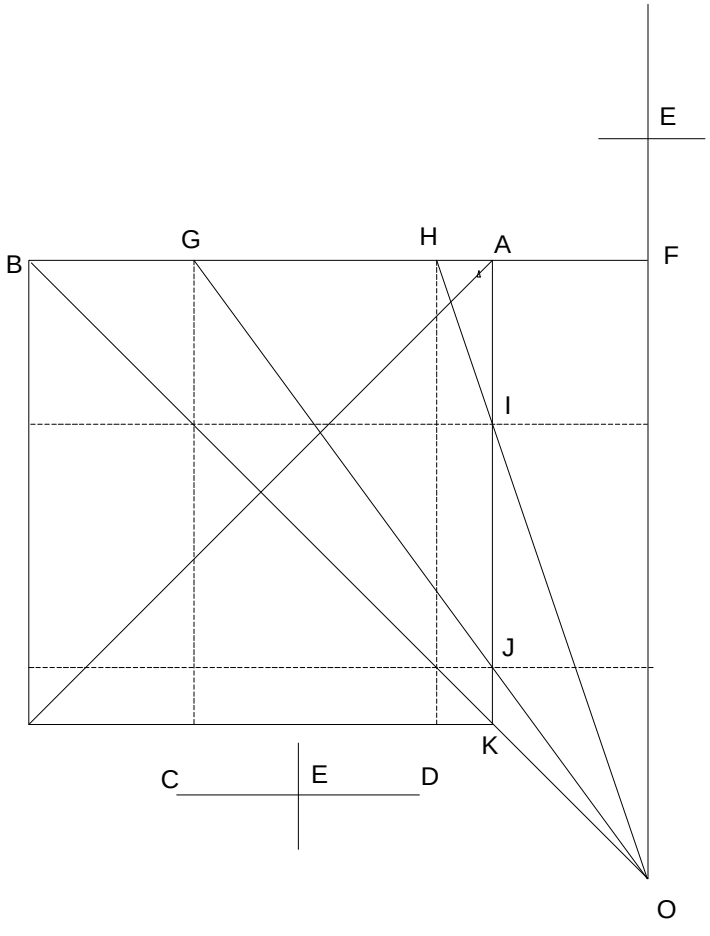


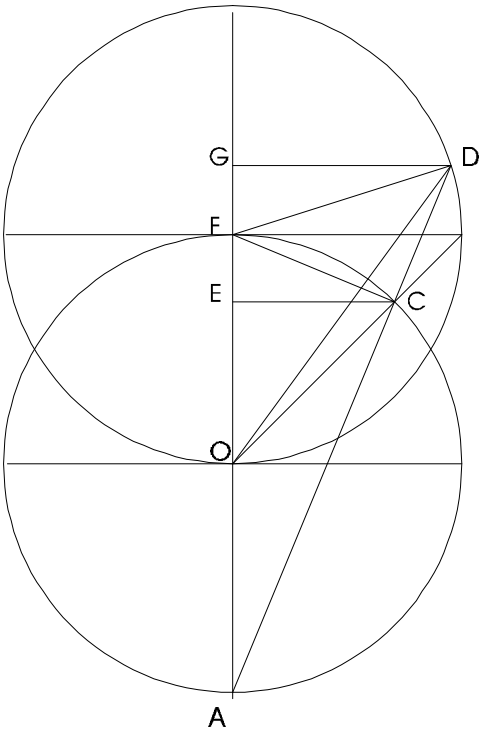

figure 6

We can now place in our two rays from O and proceed to clean up the drawing.




BG = AI, GH = IJ, HA = JK, and both GH and IJ =CD, which is what I had set out to do.

# Cubing a Sphere, a Rusty Construction.




Cubing a sphere off the base of a right triangle. I later found out that someone had squared a circle off the base of a right triangle but the figure was lost. This figure is in the 1989 Document on the Delian Solution.

$R := 10$

$AO := R \quad CO := R \quad DF := R \quad CE := \sqrt{\frac{CO^2}{2}}$

$AE := R + CE \quad AC := \sqrt{CE^2 + AE^2} \quad AF := 2 \cdot R$

$CF := \sqrt{AF^2 - AC^2} \quad CD := \sqrt{DF^2 - CF^2} \quad AD := AC + CD$

$DG := \frac{CE \cdot AD}{AC} \quad AG := \frac{AE \cdot AD}{AC} \quad GO := AG - R$

$DO := \sqrt{DG^2 + GO^2}$

$DO = 16.135$

$CUBE\_ROOT = 16.12$

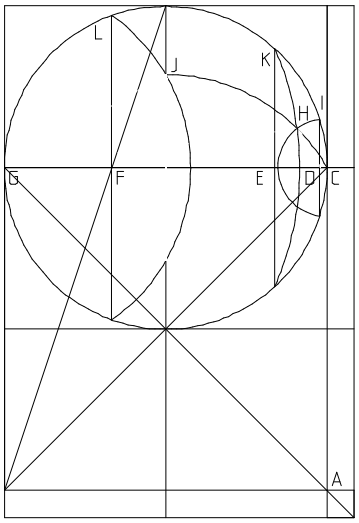
$CUBE\_ROOT := \left(\frac{4}{3} \cdot \pi \cdot R^3\right)^{\frac{1}{3}}$

$PI := \frac{DO^3}{\frac{4}{3} \cdot R^3}$ $\quad PI = 3.15049$

$\pi = 3.14159$

$TOLERANCE := \frac{DO}{CUBE\_ROOT}$

$TOLERANCE - 1 = 0.0009429$



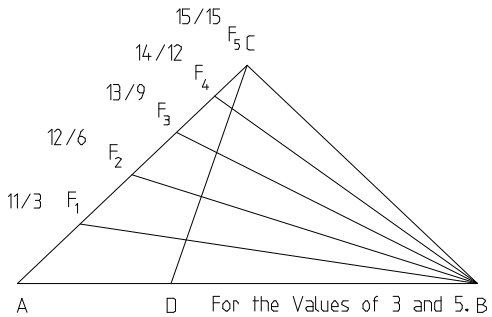

# One Circle

**By John Clark**

# One Square

# One Line

## And the Delian Quest



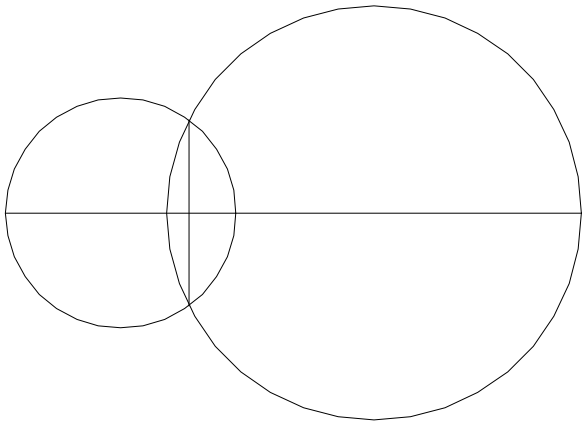

# 1993

# Pythagoras Revisited

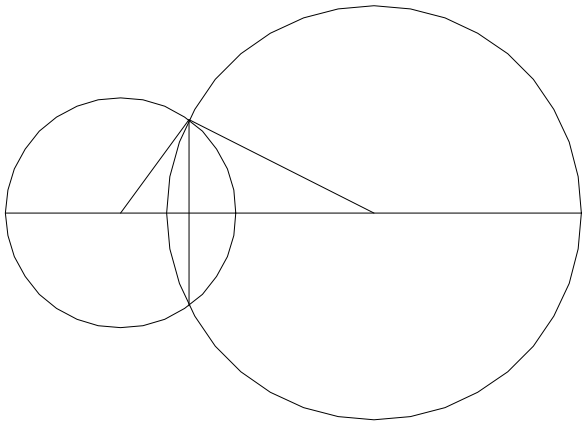
I had set aside a Euclidean figure that abstracts any cube root that I had developed to work on a full expression of the Archimedean paper angle trisector. While triangulating the tolerability of a particular compass move, I ran across the following problem, and solution:

The Pythagorean Theorem is itself a special case statement regarding the area of the squares on the sides of a triangle as compared to the area of the square on the base of the triangle. The special case, is of course, the right triangle. Primarily what will be done here is threefold, 1) demonstrate, in a completely Euclidean construction, how to find the area of any triangle given only the length of the three sides; 2) Present a data sheet regarding the relationship between the squares on a triangle; 3) Note that the process perhaps extends trigonometric analysis to all triangles. In order to establish some order to the way in which we view the common triangle, I refer to the base always as the side with the longest segment.

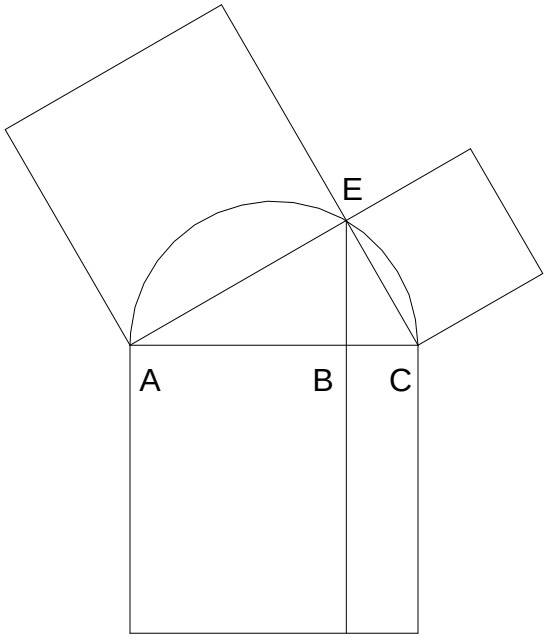
I became interested in the problem when I needed to find the perpendicular bisector formed by the intersection of two circles.



In this situation we are given both radii and the difference between them. Literally we are given only the lengths of the common triangle.

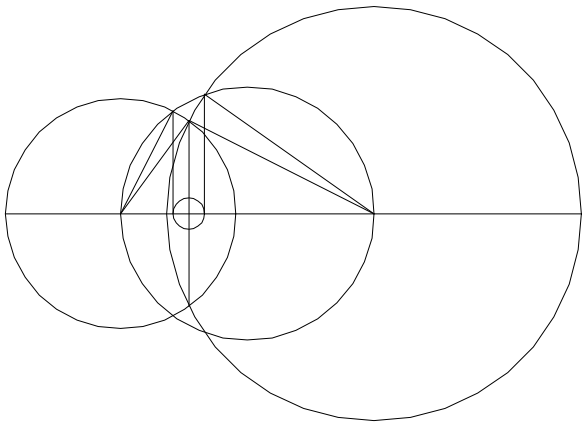


In order to solve the problem, we go back to basics. Given a right triangle how do we find the perpendicular bisector?

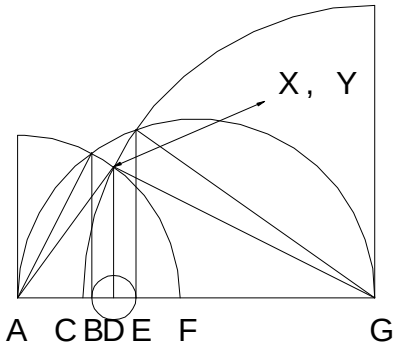



The ratio of the square under a right triangle to the squares on its sides is quite direct. When one views a right triangle they should immediately see three square roots. $\sqrt{(AB)(BC)}$ = BE, $\frac{AE^2}{AC}$ = AB, $\frac{CE^2}{AC}$ = BC. Thus, for a right triangle the perpendicular bisector is quite well known and its area is easy to find from just its three segments. If one will recall the area of a triangle is $\frac{1}{2}$ bh, and h is the perpendicular.

The process for a common triangle is a tad bit more involved. What is done for common triangles is quite simple, one simply completes simultaneous right triangles (like completing squares).



After finding the perpendicular bisectors for the completed triangles it will be found that the perpendicular bisector that one seeks is exactly $\frac{1}{2}$ the difference between the previous two. This result is quite enticing to me, it seems to indicate that a ratio process for angles may be achievable based on the relation of the lengths of the sides alone. The perpendicular bisector now being known as a numerical function of the lengths of the sides, the computation of the area of the triangle follows. The only other method for finding the area of a common triangle based solely on the lengths of its sides is of course Heron's which resulted in a non-Euclidean formula. The following process can be used on the following figure.




Given

$AF = a,\ CG = b,\ CF = c \quad CF \le |CG - AF|$

Then

$$AG = AF + CG - CF,\ AB = \frac{AF^2}{AG},\ EG = \frac{CG^2}{AG},\ BE = AG - AB - EG$$

$$DE = \frac{BE}{2},\ BD = DE,\ AD = AB + BD,\ DG = AG - AD$$

$$DY = \sqrt{CG^2 - DG^2},\ DX = \sqrt{AF^2 - AD^2}$$

Therefore

$$DY - DX = 0$$

There are some who will not appreciate this particular form, but when using computation, it states the conclusion.

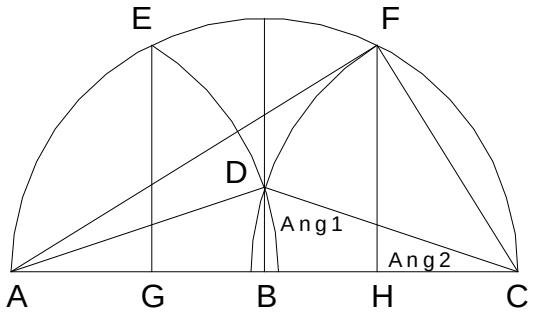
Table of values for the above expressions,

$a = 20,\ b = 30$

| c = | DY = | DX = |
|---|---|---|
| 20 | 18.856 | 18.856 |
| 19 | 18.566 | 18.566 |
| 18 | 18.247 | 18.247 |
| 17 | 17.899 | 17.899 |
| 16 | 17.52 | 17.52 |
| 15 | 17.109 | 17.109 |
| 14 | 16.667 | 16.667 |
| 13 | 16.189 | 16.189 |
| 12 | 15.675 | 15.675 |
| 11 | 15.121 | 15.121 |
| 10 | 14.524 | 14.524 |
| 9 | 13.877 | 13.877 |
| 8 | 13.175 | 13.175 |
| 7 | 12.408 | 12.408 |
| 6 | 11.564 | 11.564 |
| 5 | 10.625 | 10.625 |
| 4 | 9.564 | 9.564 |
| 3 | 8.334 | 8.334 |
| 2 | 6.846 | 6.846 |
| 1 | 4.87 | 4.87 |
| 0 | 0 | 0 |

The table relates to Book One of Euclid, *the endpoints of two segments of a triangle cannot meet in two different places*

In order to start investigating the relationship of the squares on the sides of the triangle to the base, one would start with isosceles triangles with a standard base.




Ang1 = a deg, Ang2 = 90 deg - Ang1, Ab = 3, BC = AB, F = tan(Ang2)

BD = (BC)F, CD = $\sqrt{BC^2 + BD^2}$, AC = 2AB, CF = CD, AF = $\sqrt{AC^2 - CF^2}$

CH = $\frac{CD^2}{AC}$, BH = BC - CH, GH = 2BH

SOSOS = 2CD2 Sum Of Squares On Sides

AOCR = (GH)(AC) Area Of Center Rectangle

AOER = 2(CH)(AC) Area of Extreme Rectangles

Ratio1 = $\frac{AOER}{AOCR}$

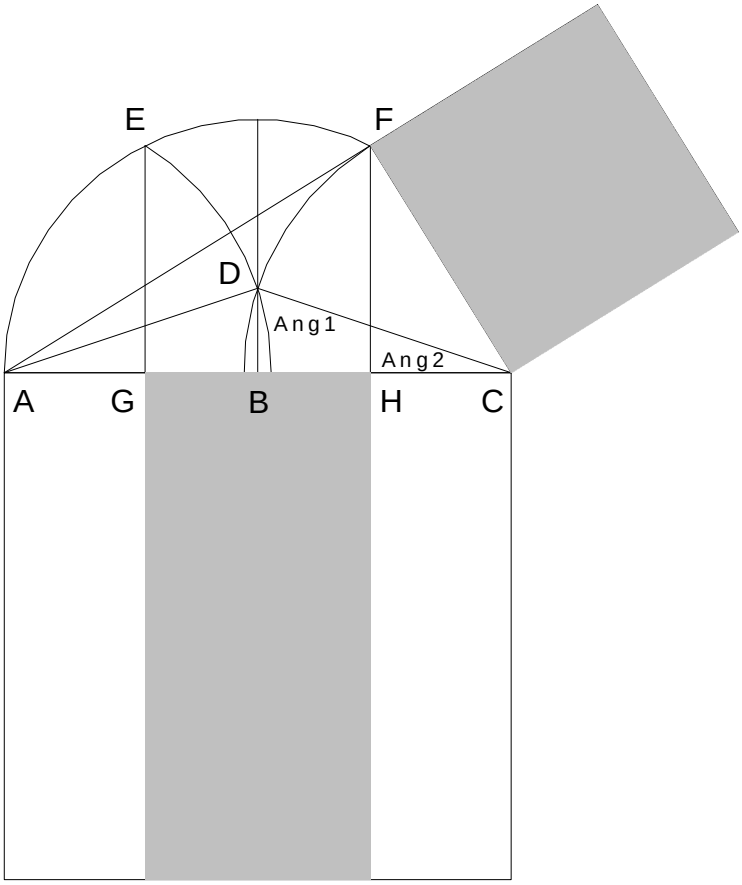
Ratio2 = $\frac{AOCR}{AOER}$

Table of Ratios in One Degree Increments for a base of 36

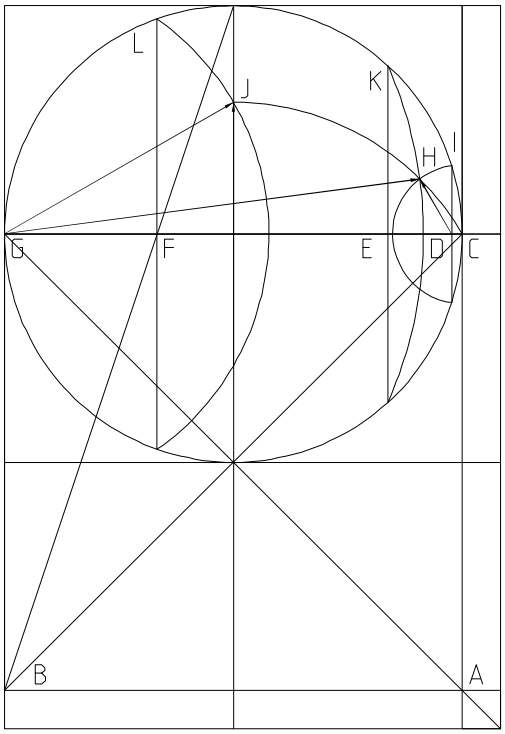
| Ang1 (Total ang. = 2Ang1) | Ang2 | Sum SOSOS | Cent AOCR | Extrm AOER | Ratio1 $\frac{AOER}{AOCR}$ | Ratio2 $\frac{AOCR}{AOER}$ | Ratio3 $\frac{AC^2}{AOER}$ | Ratio4 $\frac{AC^2}{AOCR}$ |
|---|---|---|---|---|---|---|---|---|
| 90 | 0 | 18 | 18 | 18 | 1 | 1 | 2 | 2 |
| 89 | 1 | 18.005 | 17.995 | 18.005 | 1.001 | 0.999 | 1.999 | 2.001 |
| 88 | 2 | 18.022 | 17.978 | 18.022 | 1.002 | 0.998 | 1.998 | 2,002 |
| 87 | 3 | 18.049 | 17.951 | 18.049 | 1.006 | 0.995 | 1.995 | 2.006 |
| 86 | 4 | 18.088 | 17.912 | 18.088 | 1.01 | 0.99 | 1.99 | 2.01 |
| 85 | 5 | 18.138 | 17.862 | 18.138 | 1.015 | 0.985 | 1.985 | 2.015 |
| 84 | 6 | 18.199 | 17.801 | 18.199 | 1.022 | 0.978 | 1.978 | 2.022 |
| 83 | 7 | 18.271 | 17.729 | 18.271 | 1.031 | 0.97 | 1.97 | 2.031 |
| 82 | 8 | 18.356 | 17.644 | 18.356 | 1.04 | 0.961 | 1.961 | 2.04 |
| 81 | 9 | 18.452 | 17.548 | 18.452 | 1.051 | 0.951 | 1.951 | 2.051 |
| 80 | 10 | 18.56 | 17.44 | 18.56 | 1.064 | 0.94 | 1.94 | 2.064 |
| 79 | 11 | 18.68 | 17.32 | 18.68 | 1.079 | 0.927 | 1.927 | 2.079 |
| 78 | 12 | 18.813 | 17.187 | 18.813 | 1.095 | 0.914 | 1.914 | 2.095 |
| 77 | 13 | 18.959 | 17.041 | 18.059 | 1.113 | 0.899 | 1.899 | 2.113 |
| 76 | 14 | 19.119 | 16.881 | 19.119 | 1.133 | 0.883 | 1.883 | 2.133 |
| 75 | 15 | 19.292 | 16.708 | 19.292 | 1.155 | 0.866 | 1.866 | 2.155 |
| 74 | 16 | 19.48 | 16.52 | 19.48 | 1.179 | 0.848 | 1.848 | 2.179 |
| 73 | 17 | 19.682 | 16.318 | 19.682 | 1.206 | 0.829 | 1.829 | 2.206 |

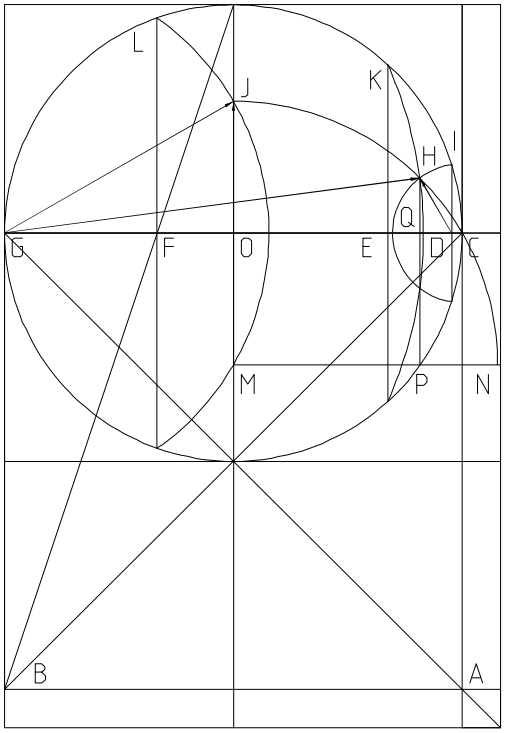
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 72 | 18 | 19.9 | 16.1 | 19.9 | 1.236 | 0.809 | 1.809 | 2.236 |
| 71 | 19 | 20.134 | 15.866 | 20.134 | 1.269 | 0.788 | 1.788 | 2.269 |
| 70 | 20 | 20.385 | 15.615 | 20.385 | 1.305 | 0.766 | 1.766 | 2.305 |
| 69 | 21 | 20.652 | 15.348 | 20.652 | 1.346 | 0.743 | 1.743 | 2.346 |
| 68 | 22 | 20.938 | 15.062 | 20.938 | 1.39 | 0.719 | 1.719 | 2.39 |
| 67 | 23 | 21.243 | 14.757 | 21.243 | 1.44 | 0.695 | 1.695 | 2.44 |
| 66 | 24 | 21.568 | 14.432 | 21.568 | 1.494 | 0.669 | 1.669 | 2.494 |
| 65 | 25 | 21.914 | 14.086 | 21.914 | 1.556 | 0.643 | 1.643 | 2.556 |
| 64 | 26 | 22.282 | 13.718 | 22.282 | 1.624 | 0.616 | 1.616 | 2.624 |
| 63 | 27 | 22.673 | 13.327 | 22.673 | 1.701 | 0.588 | 1.588 | 2.701 |
| 62 | 28 | 23.089 | 12.911 | 23.089 | 1.788 | 0.559 | 1.559 | 2.788 |
| 61 | 29 | 23.531 | 12.469 | 23.531 | 1.887 | 0.53 | 1.53 | 2.887 |
| 60 | 30 | 24 | 12 | 24 | 2 | 0.5 | 1.5 | 3 |
| 59 | 31 | 24.499 | 11.501 | 24.499 | 2.13 | 0.469 | 1.469 | 3.13 |
| 58 | 32 | 25.028 | 10.972 | 25.028 | 2.281 | 0.438 | 1.438 | 3.281 |
| 57 | 33 | 25.591 | 10.409 | 25.591 | 2.459 | 0.407 | 1.407 | 3.459 |
| 56 | 34 | 26.189 | 9.811 | 26.189 | 2.669 | 0.375 | 1.375 | 3.669 |
| 55 | 35 | 26.825 | 9.175 | 26.825 | 2.924 | 0.342 | 1.342 | 3.924 |
| 54 | 36 | 27.502 | 8.498 | 27.502 | 3.236 | 0.309 | 1.309 | 4.236 |
| 53 | 37 | 28.221 | 7.779 | 28.221 | 3.628 | 0.276 | 1.276 | 4.628 |
| 52 | 38 | 28.987 | 7.013 | 28.987 | 4.134 | 0.242 | 1.242 | 5.134 |
| 51 | 39 | 29.804 | 6.196 | 28.804 | 4.81 | 0.208 | 1.208 | 5.81 |
| 50 | 40 | 30.674 | 5.326 | 30.674 | 5.759 | 0.174 | 1.174 | 6.759 |
| 49 | 41 | 31.602 | 4.398 | 31.602 | 7.185 | 0.139 | 1.139 | 8.185 |
| 48 | 42 | 32.593 | 3.407 | 32.597 | 9.567 | 0.105 | 1.105 | 10.567 |
| 47 | 43 | 33.653 | 2.347 | 33.653 | 14.336 | 0.07 | 1.07 | 15.336 |
| 46 | 44 | 34.786 | 1.214 | 34.786 | 28.654 | 0.035 | 1.035 | 29.654 |
| 45 | 45 | 36 | 0 | 36 | 0 | 0 | 1 | 0 |
| 44 | 46 | 37.302 | - 1.302 | 37.302 | - 28.654 | - 0.035 | 0.965 | -27.654 |
| 43 | 47 | 38.7 | - 2.7 | 38.7 | - 14,336 | - 0. 07 | 0.93 | - 13.336 |
| 42 | 48 | 40.202 | - 4.202 | 40.202 | - 9.567 | - 0.105 | 0.895 | - 8.567 |
| 41 | 49 | 41.82 | - 5.82 | 41.82 | - 7.185 | - 0.139 | 0.861 | - 6.185 |
| 40 | 50 | 43.565 | - 7.565 | 43.565 | - 5.759 | - 0.174 | 0.826 | - 4.759 |
| 39 | 51 | 45.449 | - 9.449 | 45.449 | - 4.81 | - 0.208 | 0.792 | - 3.81 |
| 38 | 52 | 47.489 | - 11.489 | 47.489 | - 4.134 | - 0.242 | 0.758 | - 3.134 |
| 37 | 53 | 49.699 | - 13.699 | 49.699 | - 3.628 | - 0.276 | 0.724 | -2.628 |
| 36 | 54 | 52.1 | - 16.1 | 52.1 | - 3.236 | - 0.309 | 0.691 | - 2.236 |
| 35 | 55 | 54.713 | - 18.713 | 54.713 | - 2.924 | - 0.342 | 0.658 | -1.924 |
| 34 | 56 | 57.564 | - 21.564 | 57.564 | - 2.669 | - 0.375 | 0.625 | - 1.669 |
| 33 | 57 | 60.681 | - 24.681 | 60.681 | - 2.459 | - 0.407 | 0.593 | - 1.459 |
| 32 | 58 | 64.099 | - 28.099 | 64.099 | - 2.281 | - 0.438 | 0.562 | - 1.281 |
| 31 | 59 | 67.858 | - 31.857 | 67.857 | - 2.13 | - 0.469 | 0.531 | -1.13 |
| 30 | 60 | 72 | -36 | 72 | - 2 | - 0.5 | 0.5 | - 1 |

As the table demonstrates, before one reaches an apex angle of 90°, the difference between the completed perpendiculars is subtracted from the base, after one reaches 90° the difference is added to the base.




There are, of course, more tables to do in order to study the figure.




# Exploring the properties of the curve CJ.

06_03_93.MCD

Choosing any arbitrary point on CJ does the projection DE equal the square root of CD · EG? This possibility was suggested by my work on finding the geometrical solution to the problem : Given only the lengths of the three sides of any triangle, find its area. That work suggests that DE will remain constant over a range if CJ had the same center as circle CLG. An essential portion of that solution will be used in this work for finding the difference and position of DE.

$CG := 10 \quad FG := \frac{CG}{3} \quad FO := \frac{CG}{6} \quad LO := \frac{CG}{2}$

$FL := \sqrt{LO^2 - FO^2} \quad GL := \sqrt{FG^2 + FL^2}$

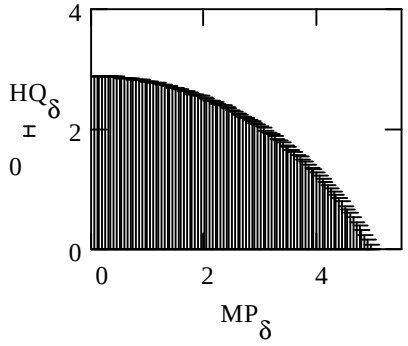
$GO := \frac{CG}{2} \quad GJ := GL \quad JO := \sqrt{GJ^2 - GO^2}$

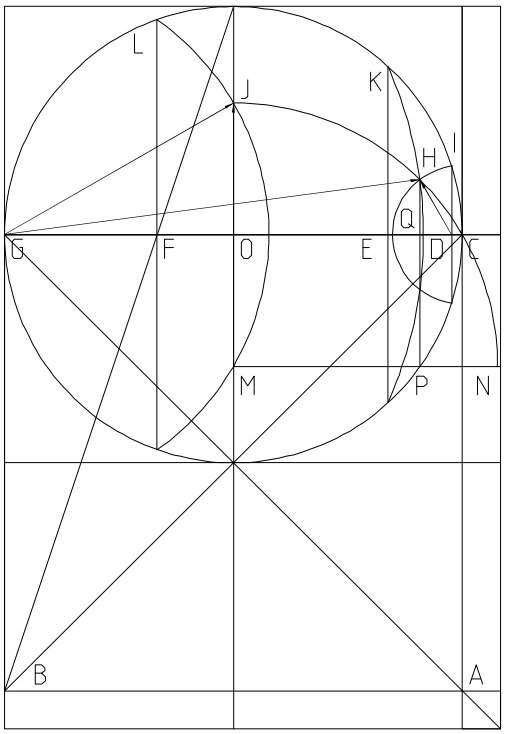
$JM := JO \cdot 2 \quad CO := GO \quad MN := CO$

$\delta := 1..100 \quad MP_\delta := \frac{\delta}{100} \cdot MN \quad HM := JM$

$HP_\delta := \sqrt{HM^2 - \left(MP_\delta\right)^2} \quad MO := JO \quad PQ := MO$

$HQ_\delta := HP_\delta - PQ \quad QO_\delta := MP_\delta$





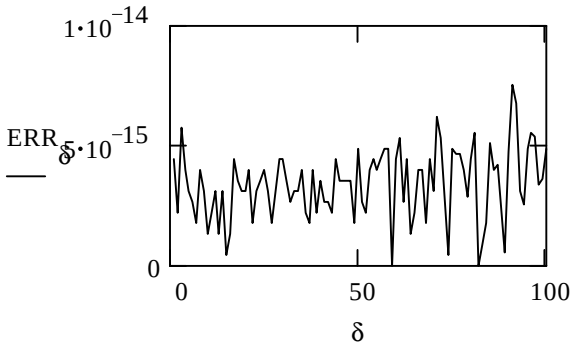

$$GQ_\delta := GO + QO_\delta \qquad GH_\delta := \sqrt{(GQ_\delta)^2 + (HQ_\delta)^2}$$

$$GK_\delta := GH_\delta \qquad EG_\delta := \frac{(GK_\delta)^2}{CG} \qquad CQ_\delta := CO - QO_\delta$$

$$CH_\delta := \sqrt{(CQ_\delta)^2 + (HQ_\delta)^2} \qquad CI_\delta := CH_\delta$$

$$CD_\delta := \frac{(CI_\delta)^2}{CG} \qquad DE_\delta := CG - EG_\delta - CD_\delta$$

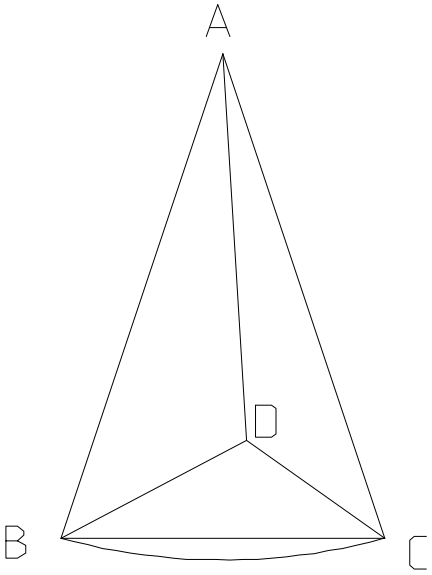
$$ERR_\delta := \sqrt{EG_\delta \cdot CD_\delta} - DE_\delta$$




As can be seen by the graph, the only error is the limits contained in the program itself! So CJ is part of our cube root figure, as the square in a right triangle is equal to the square of the remaining two segments, and all three squares taken to the reduced ratio functions in a cube root relationship.

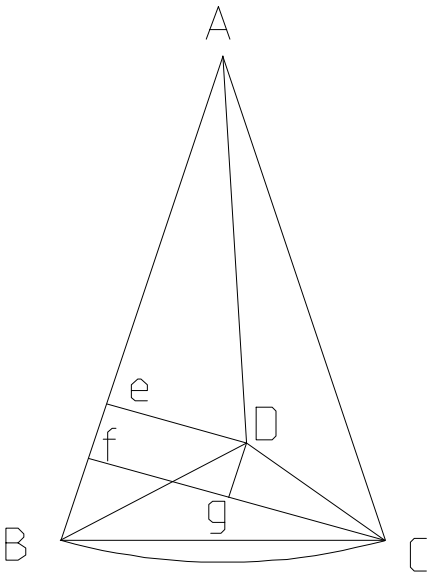
# For All Triangles Find DC

06_07_C3.MCD



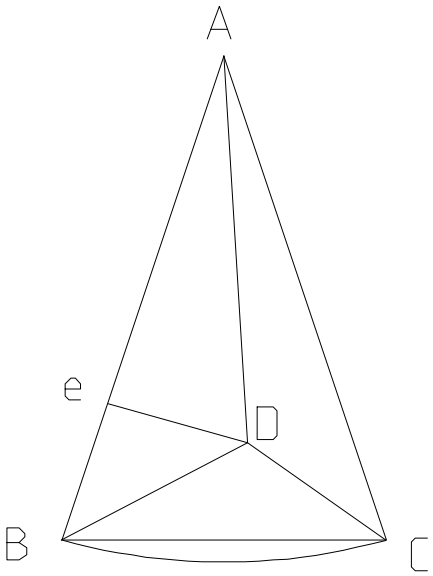

The first of these series ran the process through for equalaterals. I expanded in this mod for all triangles, I will use the process later to find the Euler line, given only the length of three sides of a triangle.

Given: AB, AC, AD, BC, BD
Find: DC




# Process Summary

Construct: De
Construct: Cf
ef = Be - Bf
Dg = ef
CD = $\sqrt{Cg^2 + Dg^2}$




Given:

$AB := 7.18 \quad AC := 9.02 \quad AD := 8.09$

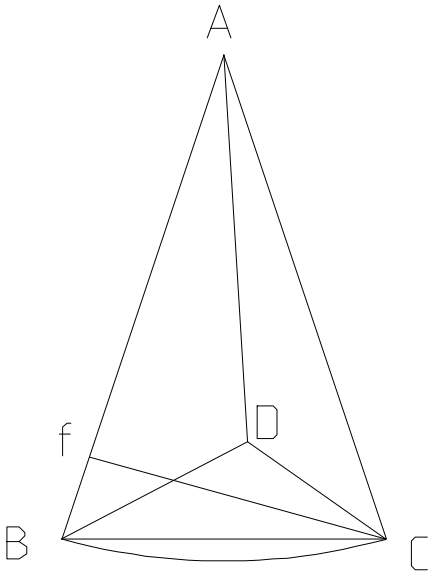
$BD := 7.28 \quad BC := 3.85$

Find CD.

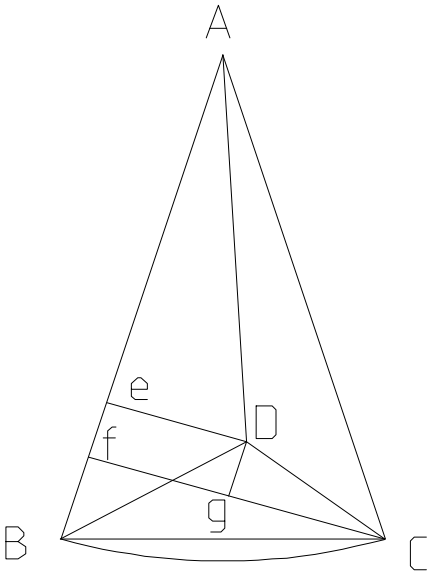
Find: Be

$$Be := \frac{1}{2}\cdot\frac{BD^2}{AB} + \frac{1}{2}\cdot AB - \frac{1}{2}\cdot\frac{AD^2}{AB}$$

$Be = 2.723$



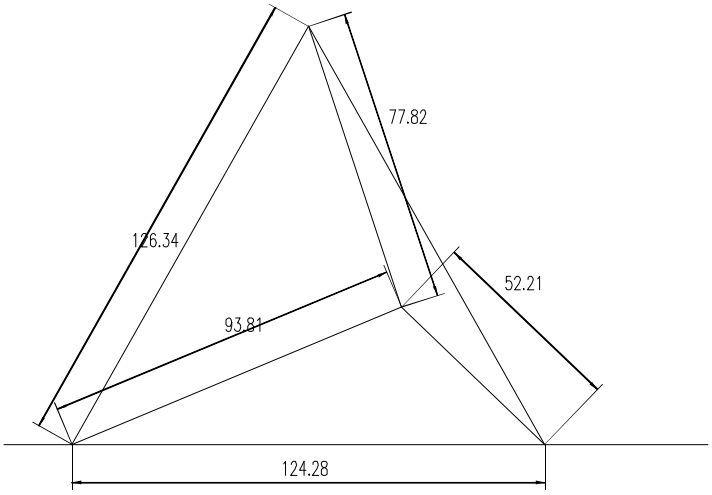

$$Bf := \frac{1}{2}\cdot\frac{BC^2}{AB} + \frac{1}{2}\cdot AB - \frac{1}{2}\cdot\frac{AC^2}{AB}$$



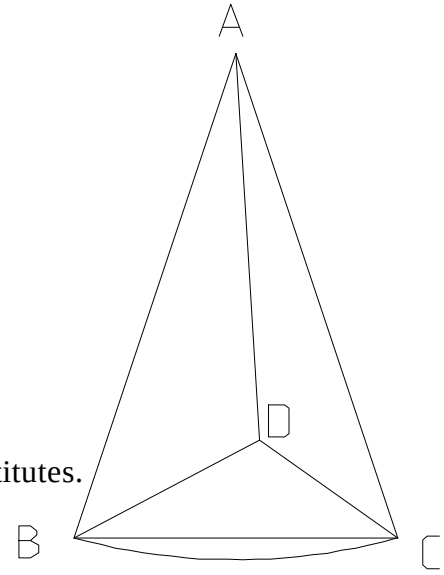

$ef := Be - Bf \quad Dg := ef \quad De := \sqrt{BD^2 - Be^2}$

$Cf := \sqrt{BC^2 - Bf^2} \quad gf := De \quad Cg := Cf - gf$

$DC := \sqrt{Cg^2 + Dg^2} \quad DC = 4.844$




$DC = 4.844$

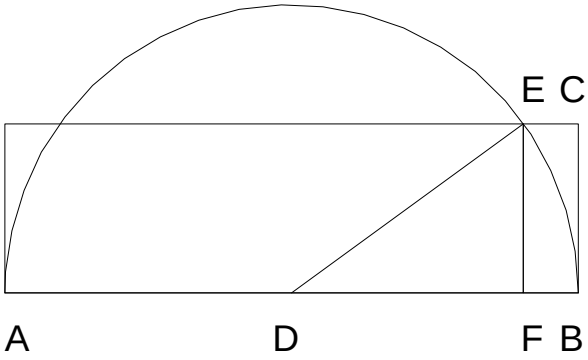



Due to the inability to format the equation I had to make some substitutes.

$A := AB \quad B := BC \quad C := AC \quad D := BD \quad E := AD$

$$F := \sqrt{2}\cdot\frac{\sqrt{\begin{aligned}&B^2\cdot A^2 \ldots\\&+\sqrt{C + A - B}\cdot\sqrt{C - A + B}\cdot\sqrt{C + A + B}\cdot\sqrt{C - A - B}\cdot\sqrt{D + E - A}\cdot\sqrt{D - E - A}\cdot\sqrt{D + A - E}\cdot\sqrt{D + A + E}\ldots\\&+ D^2\cdot A^2 - D^2\cdot B^2 + D^2\cdot C^2 - A^4 + A^2\cdot C^2 + E^2\cdot B^2 + E^2\cdot A^2 - E^2\cdot C^2\end{aligned}}}{2\cdot A}$$

$F = 4.844$




Given length AB and length CB which is 1/2 of AB or less, place CB such that it is the square root of the two constructed segments. The second line in the equations will not permit BC to exceed Euclidean specifications.

$AB := 100 \quad BC := 43$

$BC := if\left(BC \geq 0, if\left(BC \leq \frac{AB}{2}, BC, 0\right), 0\right)$

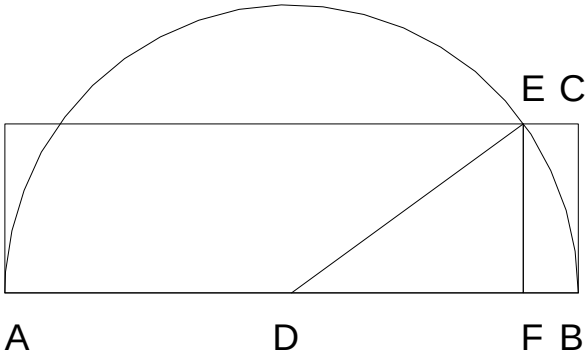
$AD := \frac{AB}{2} \qquad DE := AD$

$FE := BC \quad DB := AD \quad DF := \sqrt{DE^2 - FE^2} \quad AF := AD + DF \quad BF := DB - DF$

$AF = 75.515 \qquad BF = 24.485 \qquad AF + BF = 100 \qquad \sqrt{BF \cdot AF} - BC = 0$

$AF := \frac{1}{2} \cdot AB + \frac{1}{2} \cdot \sqrt{AB^2 - 4 \cdot BC^2} \qquad BF := \frac{1}{2} \cdot AB - \frac{1}{2} \cdot \sqrt{AB^2 - 4 \cdot BC^2}$

$AF = 75.515 \qquad BF = 24.485$




Now if we eliminate the Euclidean qualification we bring the figure up to the 20th century with imaginary numbers.

$AB := 100 \quad BC := 60$

$AD := \frac{AB}{2} \qquad DE := AD$

$FE := BC \quad DB := AD \quad DF := \sqrt{DE^2 - FE^2} \quad AF := AD + DF \quad BF := DB - DF$

$AF = 50 + 33.166i \qquad BF = 50 - 33.166i \qquad AF + BF = 100 \qquad \sqrt{BF \cdot AF} - BC = 0$

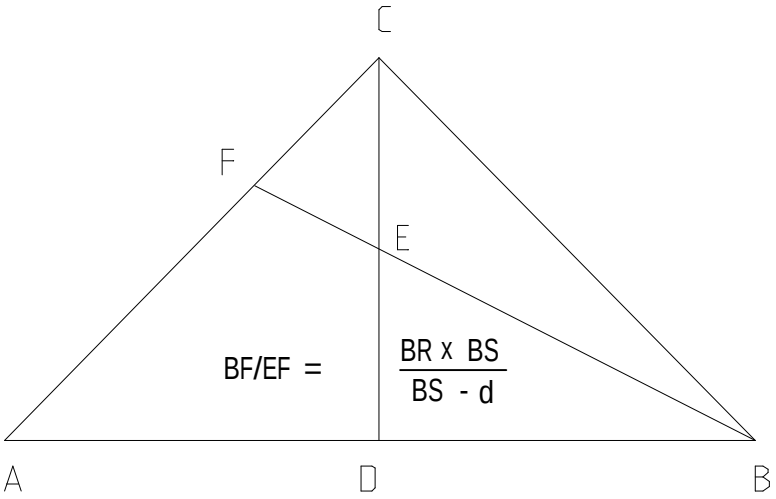
$AF := \frac{1}{2} \cdot AB + \frac{1}{2} \cdot \sqrt{AB^2 - 4 \cdot BC^2} \qquad BF := \frac{1}{2} \cdot AB - \frac{1}{2} \cdot \sqrt{AB^2 - 4 \cdot BC^2}$

$AF = 50 + 33.166i \qquad BF = 50 - 33.166i$

# A Pyramid of Ratios

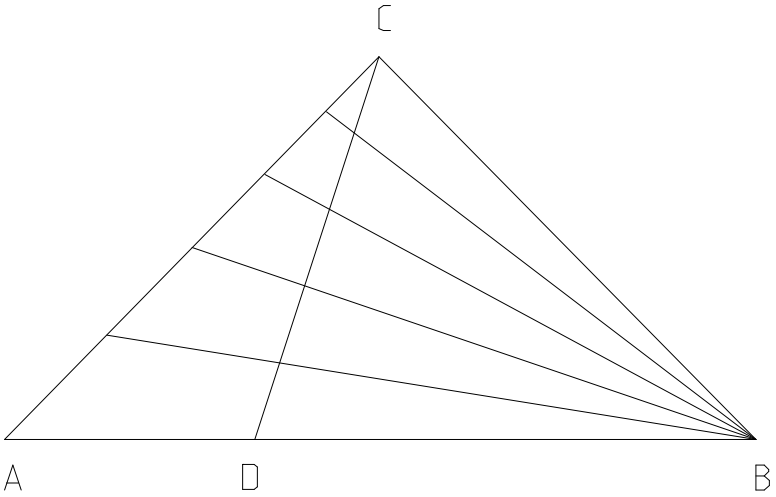
06_21_93.MCD

The typical teaching concerning the division of a line in Euclidean geometry is quite straight forward, this work is an extension of the application.

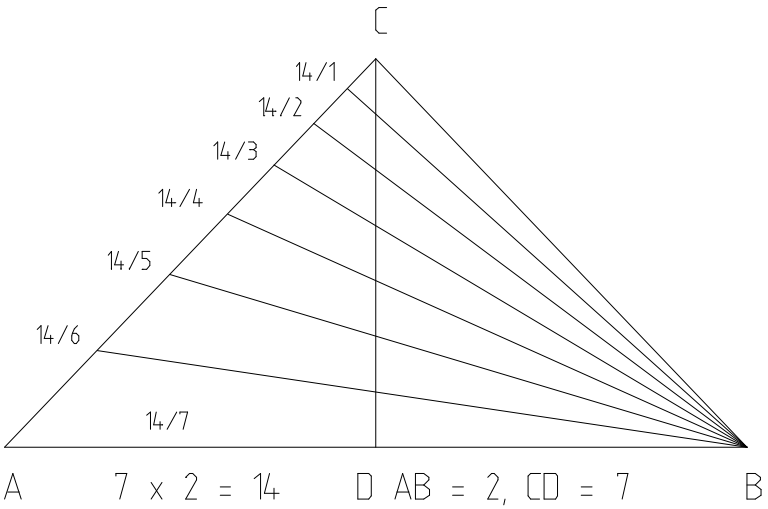



A pyramid of ratios is the expression of the relationships between the base of the right triangle and some bisectors. There is more than one model.

How does this one work? I wish to know the ratio between EF:BF. As you can see the base of the triangle has been divided into two equal parts, and so has the bisector. AB = 2 and CD = 2. $2 \times 2 = 4$, therefore segment EF will be 1/4 of segment BF. Now let us turn to a more developed expression of the figure.

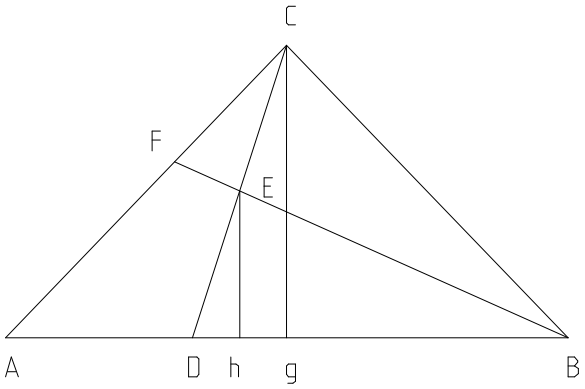



The base can also be divided into any ratio, it can even be part of a larger figure. One must keep in mind the distinction between numerical measure, which will be used to compute lengths, and the concept of ratio.




The ratio here is 2 to 7, so that we are working with the number 14. In descending order, 1/14, 2/14, 3/14, 4/14, 5/14, 6/14 and 7/14.

In this work I will be exploring the figure to see what else it will yield. With Mathcad we shall be able to explore drawings that would be too tedious to draw and actually be quite an engineering feat if one could.




One can plug in numbers for the first three variables at the end of the document.

$AB := BASE\_LENGTH$

$BR := BASE\_RATIO$

$BS := BISEC\_SEG$

$Cg := \frac{AB}{2} \quad Ag := Cg \quad Bg := Cg$

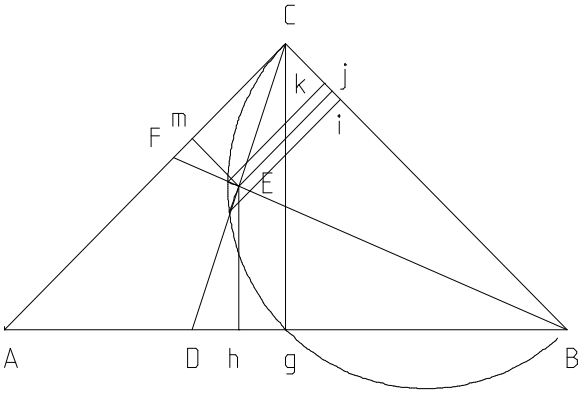
Number that has been defined equals $BS \times BR$.

$BS \cdot BR = 0.45 \quad AD := \frac{AB}{BR}$

$\delta := 0 .. BS \quad Dg := Ag - AD \quad CD := \sqrt{Dg^2 + Cg^2} \quad DE_\delta := \frac{CD}{BS} \cdot \delta \quad Dh_\delta := \frac{Dg \cdot DE_\delta}{CD}$

$Eh_\delta := \sqrt{(DE_\delta)^2 - (Dh_\delta)^2} \quad gh_\delta := Dg - Dh_\delta \quad Bh_\delta := Bg + gh_\delta \quad BE_\delta := \sqrt{(Bh_\delta)^2 + (Eh_\delta)^2}$

$CE_\delta := CD - DE_\delta$




$BC := \sqrt{2 \cdot Bg^2} \quad Bi_\delta := \frac{(BE_\delta)^2}{BC} \quad Ck_\delta := \frac{(CE_\delta)^2}{BC}$

$ik_\delta := BC - Bi_\delta - Ck_\delta \quad ij_\delta := \frac{ik_\delta}{2}$

$Bj_\delta := Bi_\delta + ij_\delta \quad Cj_\delta := BC - Bj_\delta$

The algorithm, completely Euclidean, can be used to find the area of any triangle knowing just the length of the three sides. Here it is used, even when the sides exceed the base in length.

$Em_\delta := Cj_\delta$

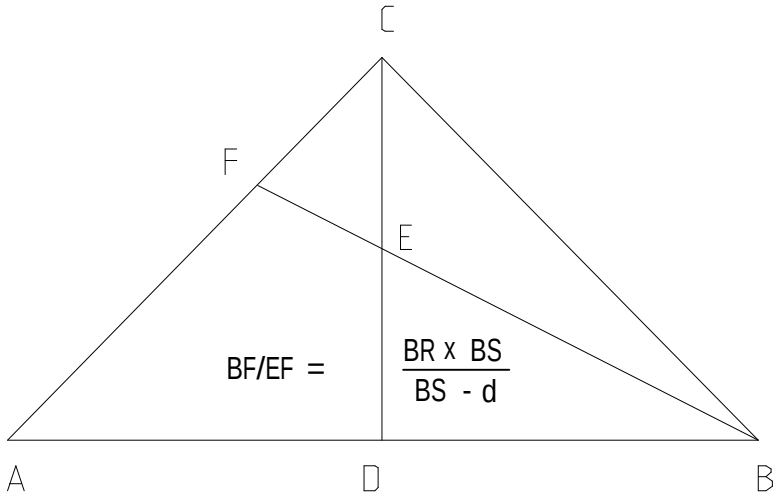
$EF_\delta := \frac{BE_\delta \cdot Em_\delta}{Bj_\delta} \quad BF_\delta := BE_\delta + EF_\delta$

$$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$$

| |
|---|
| 0.05 |
| 0.056 |
| 0.064 |
| 0.075 |
| 0.09 |
| 0.112 |
| 0.15 |
| 0.225 |
| 0.45 |
| 0 |

$$\text{if}\left(BS - \delta, \frac{BR \cdot BS}{BS - \delta}, 0\right)$$

| |
|---|
| 0.05 |
| 0.056 |
| 0.064 |
| 0.075 |
| 0.09 |
| 0.113 |
| 0.15 |
| 0.225 |
| 0.45 |
| 0 |




**Plug in values here!**

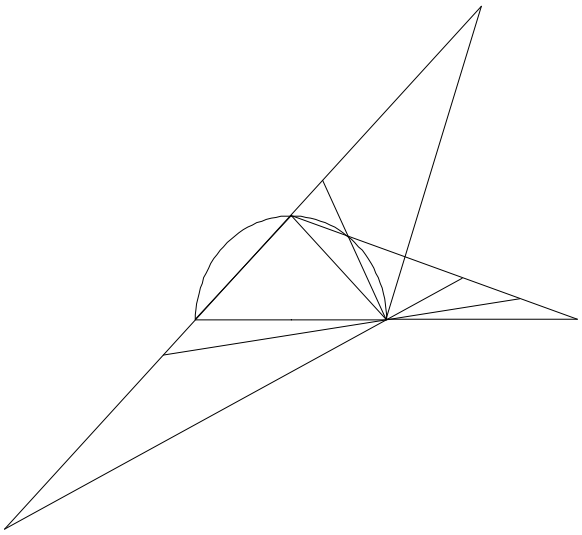
BASE_LENGTH ≡ 21

(BS) = BISEC_SEG ≡ 9

(BR) = BASE_RATIO ≡ .05

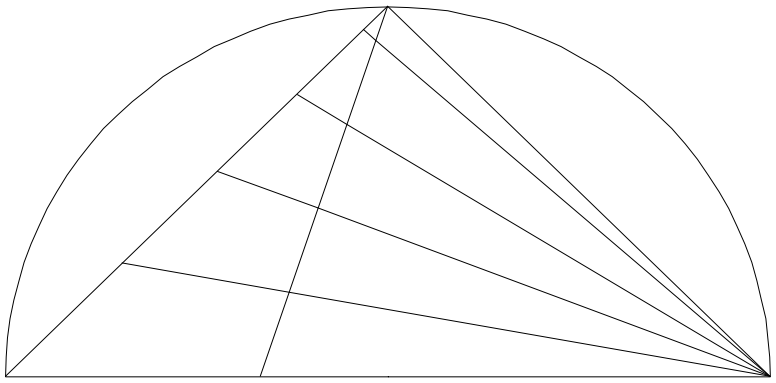
A basic Euclidean divisional / multiplicative series.

Try the values of 5 for the vertical number of divisions and .5 (1/2) for the base divisions



One can use the numbers generated by the program to discover what a particular set of ratios would draw out to be.

A fractional number for the base means that the entire base is the fractional part of a larger base.

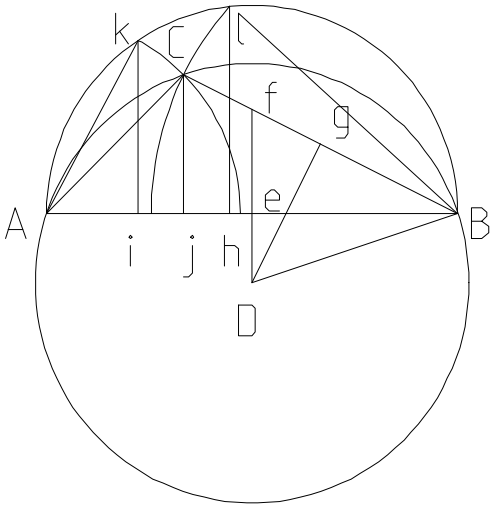


The algorithm places the remainder at the top for fractional divisions.

# Inscribe a Circle about a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

$\Delta := (AB + AC > BC)\cdot(AB + BC > AC)\cdot(BC + AC > AB)$ $\quad NOT(X) := X = 0$




$Ae := \frac{AB}{2}$ $\quad Ak := AC$ $\quad Bl := BC$ $\quad \delta := 0..2$

$Ai := \frac{Ak^2}{AB}$ $\quad Bh := \frac{Bl^2}{AB}$ $\quad Ah := AB - Bh$

$hi := Ah - Ai$ $\quad Aj := Ai + \frac{hi}{2}$ $\quad Cj := \sqrt{AC^2 - Aj^2}$

$Be := Ae$ $\quad Bj := AB - Aj$ $\quad Bg := \frac{BC}{2}$

$Bf := \frac{BC\cdot Be}{Bj}$ $\quad fg := Bf - Bg$ $\quad Dg := if\left(Cj, \frac{Bj\cdot fg}{Cj}, 0\right)$

$BD := if\left(Dg, \sqrt{Dg^2 + Bg^2}, \infty\right)$

$radius := if(\Delta, BD, 0)$ $\quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

radius = 3.007

imaginary_radius = 0

$\Delta = 1$

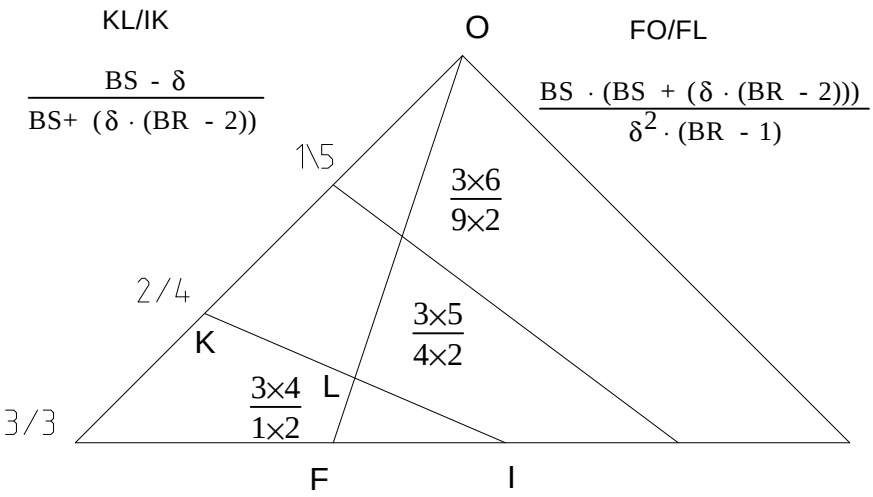
$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix}$ $\quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix}$ $\quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix}$

Reducing to one equation, $\quad AB \equiv 3$ $\quad AC \equiv 5$ $\quad BC \equiv 6$

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

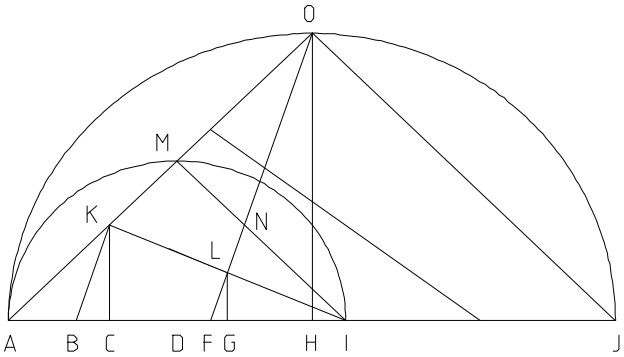
$R^T = (\ 3.007 \quad 3.007 \quad 3.007\ )$

# Another Pyramid of Ratios 07_15_93




Although this is a simpler figure to proof, the ratios involved are quite interesting. The base is divided by the perpendicular bisector into a ratio. The remainder of the base, and the opposite side are then divided into equal ratios. The resultant progressions are very nice.

The two ratios to find are KL:IK and FL:FO.




$AJ := 55 \quad AH := \frac{AJ}{2} \quad HJ := AH$

$BR := BASE\_RATIO$

$BS := BISEC\_SEG$

$AF := \frac{AJ}{BR} \quad FJ := AJ - AF \quad \delta := 0..BS \quad FI_\delta := \frac{FJ}{BS} \cdot \delta \quad AI_\delta := AF + FI_\delta$

$AD_\delta := \frac{AI_\delta}{2} \quad AO := \sqrt{\frac{AJ^2}{2}} \quad AK_\delta := \frac{AO}{BS} \cdot \delta \quad FH := AH - AF \quad HO := AH$

$FO := \sqrt{HO^2 + FH^2} \quad CK_\delta := \sqrt{\frac{(AK_\delta)^2}{2}} \quad AC_\delta := CK_\delta \quad CI_\delta := AI_\delta - AC_\delta$

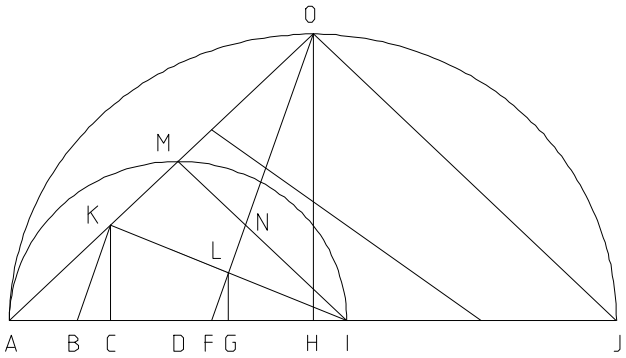
$$IK_\delta := \sqrt{\left(CK_\delta\right)^2 + \left(CI_\delta\right)^2}$$

$$BC_\delta := \frac{FH \cdot CK_\delta}{HO} \qquad BI_\delta := BC_\delta + CI_\delta$$

$$BK_\delta := \sqrt{\left(CK_\delta\right)^2 + \left(BC_\delta\right)^2}$$

$$FL_\delta := \frac{BK_\delta \cdot FI_\delta}{BI_\delta}$$

$$IL_\delta := \frac{IK_\delta \cdot FL_\delta}{BK_\delta} \qquad KL_\delta := IK_\delta - IL_\delta$$



(BR) = BASE_RATIO ≡ 3

(BS) = BISEC_SEG ≡ 5

← Insert Values Here.

| $\delta$ |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

| $BS - \delta$ |
|---|
| 5 |
| 4 |
| 3 |
| 2 |
| 1 |
| 0 |

| $BS + (\delta \cdot (BR - 2))$ |
|---|
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |

| $\frac{KL_\delta}{IK_\delta}$ |
|---|
| 1 |
| 0.667 |
| 0.429 |
| 0.25 |
| 0.111 |
| 0 |

| $\frac{BS - \delta}{BS + (\delta \cdot (BR - 2))}$ |
|---|
| 1 |
| 0.667 |
| 0.429 |
| 0.25 |
| 0.111 |
| 0 |

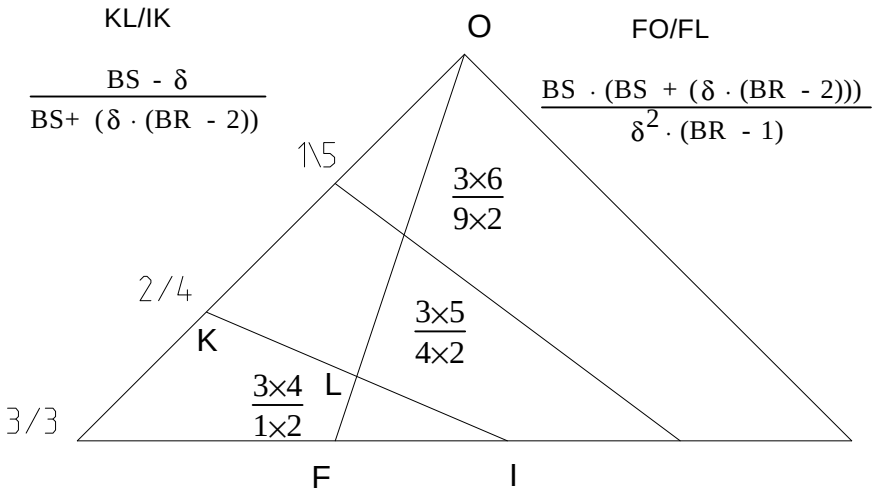
| $\delta^2 \cdot (BR - 1)$ |
|---|
| 0 |
| 2 |
| 8 |
| 18 |
| 32 |
| 50 |

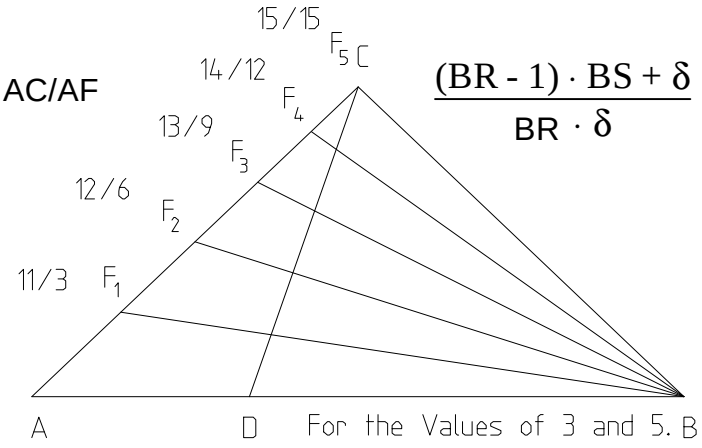
| $\text{if}\left(\delta, \frac{FO}{FL_\delta}, 0\right)$ |
|---|
| 0 |
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

| $\text{if}\left[\delta, \frac{BS \cdot (BS + (\delta \cdot (BR - 2)))}{\delta^2 \cdot (BR - 1)}, 0\right]$ |
|---|
| 0 |
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

| $FL_\delta$ |
|---|
| 0 |
| 1.933 |
| 6.626 |
| 13.044 |
| 20.613 |
| 28.988 |

| $\text{if}\left[\delta, \frac{FO}{\frac{BS \cdot (BS + (\delta \cdot (BR - 2)))}{\delta^2 \cdot (BR - 1)}}, 0\right]$ |
|---|
| 0 |
| 1.933 |
| 6.626 |
| 13.044 |
| 20.613 |
| 28.988 |

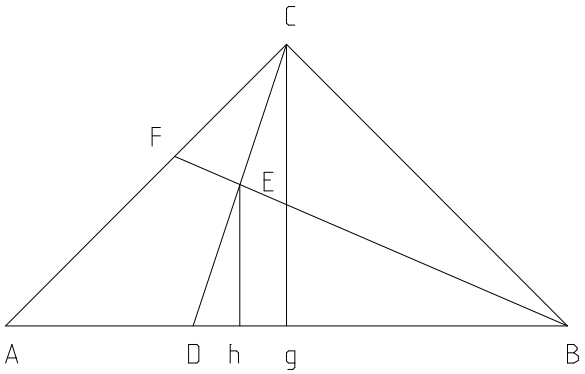





$$\frac{(BR - 1)\cdot BS + \delta}{BR \cdot \delta}$$

# Pyramid of Ratios III

Another set of ratios to explore is AC/AF Again one can plug in numbers for the first three variables at the end of the document. In most of these early works, I did not use the symbolic processor to find the formula as they seeem rather obvious. I do not think it could have gotten by my switch for AF anyway.




$AB := BASE\_LENGTH$

$BR := BASE\_RATIO$

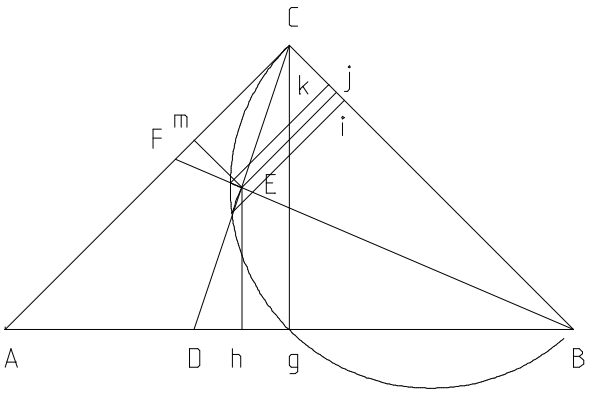
$BS := BISEC\_SEG$

$Cg := \frac{AB}{2} \quad Ag := Cg \quad Bg := Cg$

$AD := \frac{AB}{BR} \quad \delta := 0..BS$

$Dg := Ag - AD \quad CD := \sqrt{Dg^2 + Cg^2} \quad DE_\delta := \frac{CD}{BS}\cdot\delta$

$Dh_\delta := \frac{Dg\cdot DE_\delta}{CD} \quad Eh_\delta := \sqrt{(DE_\delta)^2 - (Dh_\delta)^2} \quad gh_\delta := Dg - Dh_\delta \quad Bh_\delta := Bg + gh_\delta$

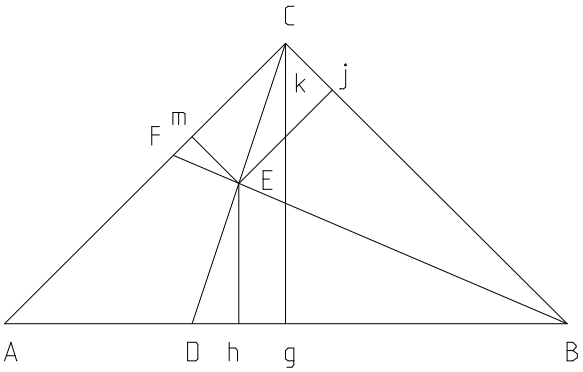



$BE_\delta := \sqrt{(Bh_\delta)^2 + (Eh_\delta)^2} \quad CE_\delta := CD - DE_\delta$

$BC := \sqrt{2\cdot Bg^2} \quad Bi_\delta := \frac{(BE_\delta)^2}{BC} \quad Ck_\delta := \frac{(CE_\delta)^2}{BC}$

$ik_\delta := BC - Bi_\delta - Ck_\delta \quad ij_\delta := \frac{ik_\delta}{2}$

$Bj_\delta := Bi_\delta + ij_\delta \quad Cj_\delta := BC - Bj_\delta \quad AC := BC$

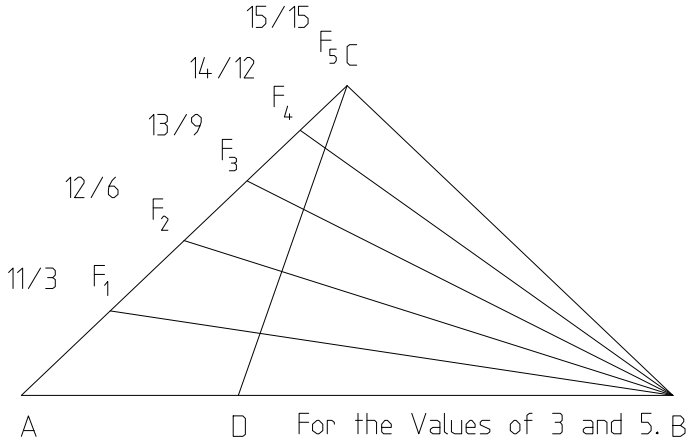



$$Em_\delta := Cj_\delta \qquad EF_\delta := \frac{BE_\delta \cdot Em_\delta}{Bj_\delta}$$

$$BF_\delta := BE_\delta + EF_\delta \qquad CF_\delta := \sqrt{\left(BF_\delta\right)^2 - BC^2}$$

If AF exceeds the figure it becomes part of AC.

$$AF_\delta := \text{if}\left(BR > 1, AC - CF, \text{if}\left(BF_\delta < 0, AC - CF, AC + CF\right)\right)_\delta$$




**Plug in values here!**

(AB) ⇐BASE_LENGTH≡ 22

(BS) ⇐BISEC_SEG≡ 5

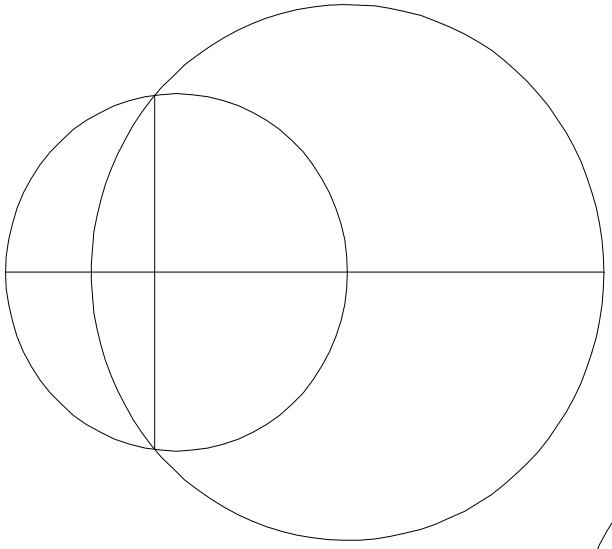
(BR) ⇐BASE_RATIO≡ 3

| $\text{if}\left(\delta, \frac{AC}{AF_\delta}, 0\right)$ | $\text{if}\left[\delta, \frac{(BR-1)\cdot BS + \delta}{BR\cdot\delta}, 0\right]$ | $\delta$ |
|---|---|---|
| 0 | 0 | 0 |
| 3.667 | 3.667 | 1 |
| 2 | 2 | 2 |
| 1.444 | 1.444 | 3 |
| 1.167 | 1.167 | 4 |
| 1 | 1 | 5 |

# The Pythagorean Completion.

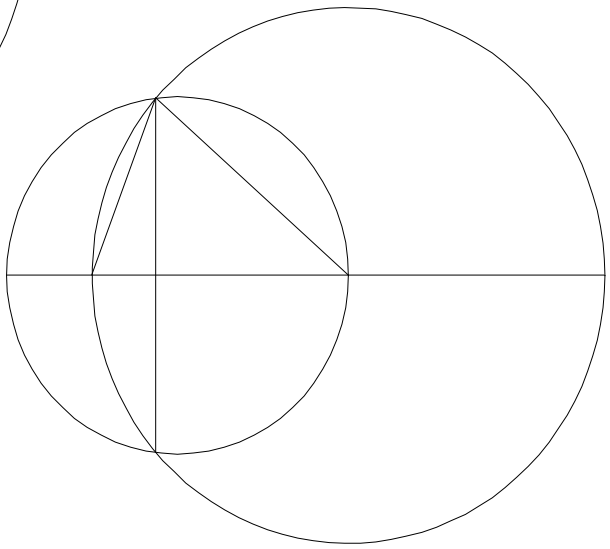
**A triangle theorem.**

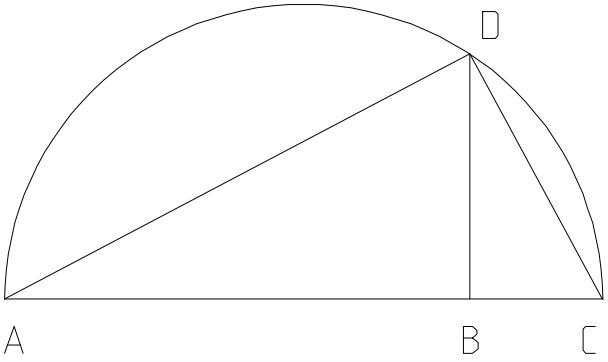
07_18_E3.MCD



Given the distance between non-equal intersecting circles and their radii, find the height and placement of the perpendicular bisector that they form.

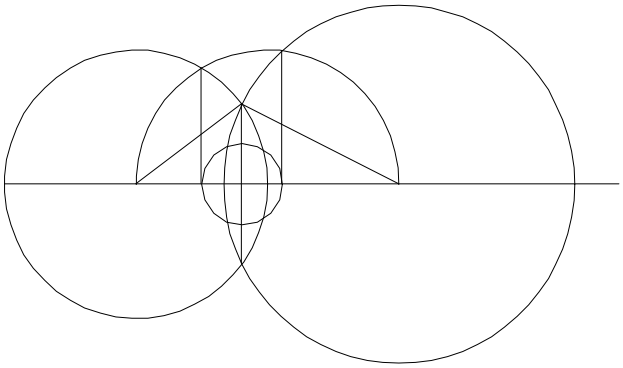
This problem can be restated as, given a triangle knowing only the length of the three sides, find its area.



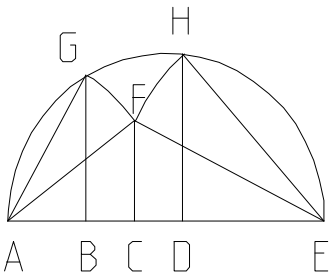



When one looks at a right triangle, one should see a square root and two square derivatives. The root, BD, is the root of AB × BC.
The two square derivatives are AB = AD²/AC and of course BC = CD²/AC. The square derivatives allow one to find the perpendicular bisector of the right triangle. These last two can be found with plain division instead of a root function. Since the area of a triangle is 1/2bh, finding the area of a right triangle is easy knowing just the length of two sides.

The process for finding the perpendicular for any other triangle is by completing simultaneous right triangles. The two bisectors created by these triangles will place the bisector of our starting triangle exactly in the center of them. I will use the following algorithm extensively when working in Euclidean geometry.



al•go•rithm n.
1. Math. a) any systematic method of solving a certain kind of problem. Ref. Lib.




In this version, AC has been "formulated" into the lowest terms. I have nothing against Heron's formula, but a correct reduction has only the input constants showing.

$AE := 6 \quad AF := 5 \quad EF := 4$ ← Plug your values in here.

*The sum of any two sides of a triangle is greater than the third.* Euclid.

$$Is\_This\_a\_Triangle := \frac{1}{(AE + AF \geq EF)\cdot(AE + EF \geq AF)\cdot(AF + EF \geq AE)} \quad AG := AF \quad EH := EF$$

$$AB := \frac{AG^2}{AE} \quad DE := \frac{EH^2}{AE} \quad AD := AE - DE \quad BD := AD - AB \quad AC := \frac{1}{2}\cdot\frac{AF^2}{AE} + \frac{1}{2}\cdot AE - \frac{1}{2}\cdot\frac{EF^2}{AE}$$

The perpendicular bisector from AC is given as

$AC = 3.75$

$$CE := AE - AC \quad CF_1 := \sqrt{EF^2 - CE^2} \quad CF_2 := \sqrt{AF^2 - AC^2} \quad CF_1 = 3.307$$

*The endpoints of two sides of a triangle cannot meet in two different places.* Euclid.

$CF_1 - CF_2 = 0$

$$Area := \frac{AE}{2}\cdot CF_1 \quad Area = 9.922 \qquad S_1 := AE \quad S_2 := AF \quad S_3 := EF$$

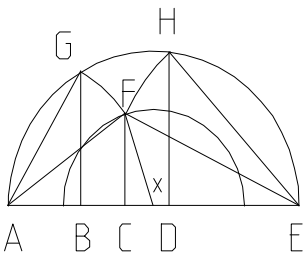
$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{4} = 9.922$$

The following result could lead to a more general rule regarding the squares on two sides of any triangle as related to the third side.

$$(AE^2 - (BD \cdot AE)) - (AF^2 + EF^2) = 0$$

$AE^2 - (BD \cdot AE) = 41$ $\quad$ $EF^2 + AF^2 = 41$

If we agree to call BD the right angle defect, then*the squares on two sides of a triangle are equal to the remaining side squared minus its right angle defect multiplied by that side.* Or if one wants to be less obtuse,




$$Ax := \frac{AE}{2} \quad Cx := Ax - AC \quad Fx := \sqrt{(CF_1)^2 + Cx^2}$$

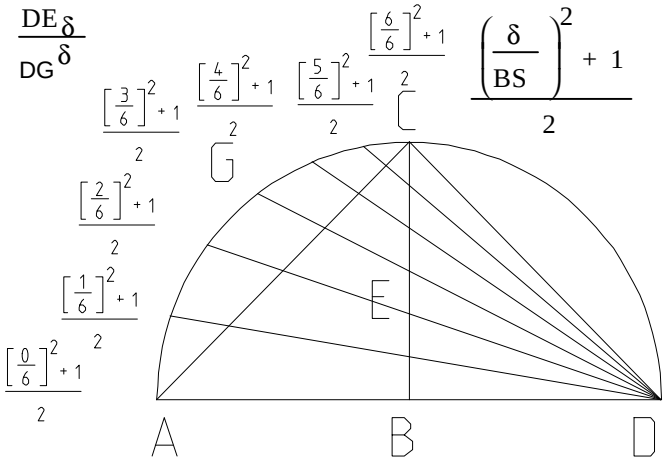
$$\frac{AE^2}{2} + 2 \cdot Fx^2 = 41 \qquad AF^2 + EF^2 = 41$$

Calling Fx the radial to the bisector,*The squares on any two sides of a triangle are equal to half of the square on the remaining side added to twice the square of its radial bisector*.

$$(AF^2 + EF^2) - \left(\frac{AE^2}{2} + 2 \cdot Fx^2\right) = 0$$

$$Fx - \sqrt{\frac{(AF^2 + EF^2) - \frac{AE^2}{2}}{2}} = 0 \qquad Fx - \frac{1}{2} \cdot \sqrt{2 \cdot AF^2 + 2 \cdot EF^2 - AE^2} = 0$$

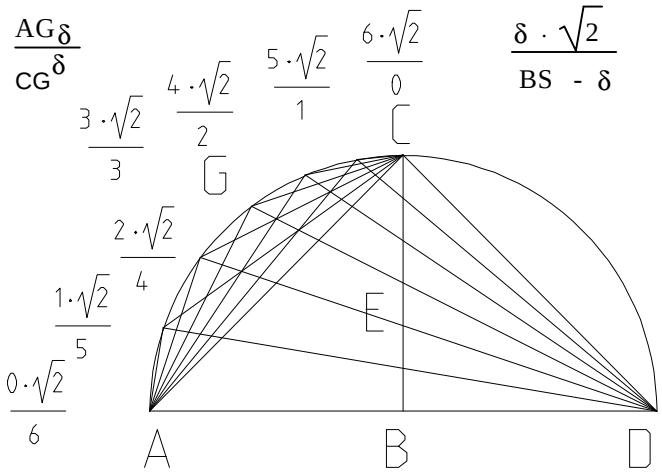
It is shown that there are at least two formulas which relate the squares of any two sides of a triangle to the third, this makes the Pythagorean theorem a statement of a single case. This single case is, however, a fundamental geometrical tool.

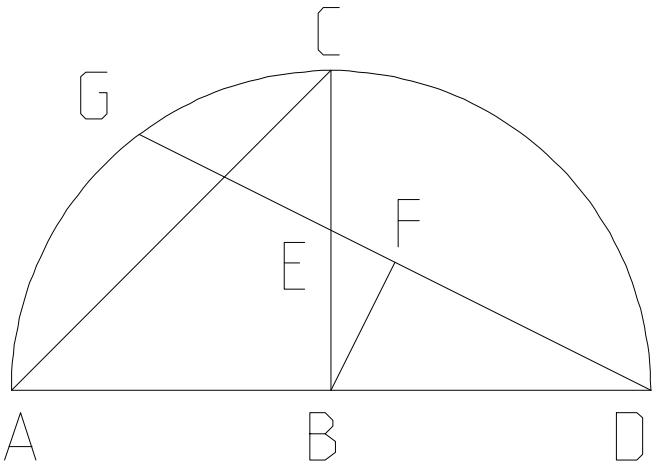



07_25_93.MCD

# Pyramid Of Ratio Series IV
# Working the Curve

This the ratios will be working against a curve, instead of a straight line intercept. The ratios found are DE:DG and AG:GC. This series is not yet completed, as the ratio AB:AD remains fixed.
At the end of this paper, $\pi$ will be calculated to any number of given segments by summing all the segments AG.




$$AD := 1 \quad \delta := 0..BS \quad AB := \frac{AD}{2}$$

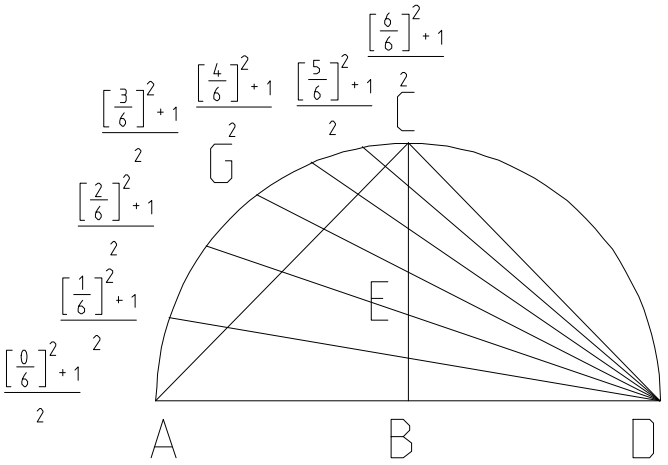
The following will keep the tables from getting out of hand.

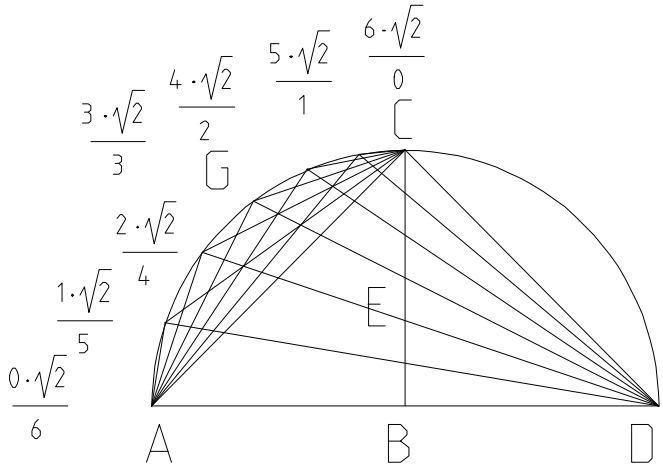
$$BC := AB \quad BD := AB \quad BE_\delta := \frac{BC}{BS}\cdot\delta$$

$$DE_\delta := \sqrt{BD^2 + (BE_\delta)^2} \quad DF_\delta := \frac{BD^2}{DE_\delta}$$

$$DG_\delta := 2\cdot DF_\delta$$

| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{BS}\right)^2 + 1}{2}$ |
|---|---|
| 0.5 | 0.5 |
| 0.514 | 0.514 |
| 0.556 | 0.556 |
| 0.625 | 0.625 |
| 0.722 | 0.722 |
| 0.847 | 0.847 |
| 1 | 1 |



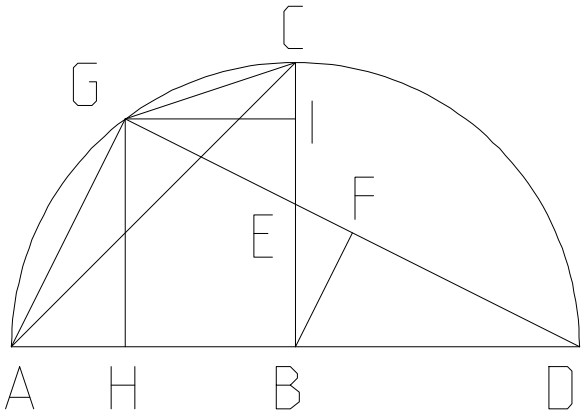

The series formed in these papers are somewhat attractive for their simplicity.

$$DH_\delta := \frac{(DG_\delta)^2}{AD} \quad BH_\delta := DH_\delta - BD \quad GH_\delta := \sqrt{(DG_\delta)^2 - (DH_\delta)^2} \quad BI_\delta := GH_\delta \quad CI_\delta := BC - BI_\delta$$

$GI_\delta := BH_\delta \quad AH_\delta := AB - BH_\delta$

$CG_\delta := \sqrt{\left(GI_\delta\right)^2 + \left(CI_\delta\right)^2}$

$AG_\delta := \sqrt{\left(AH_\delta\right)^2 + \left(GH_\delta\right)^2}$

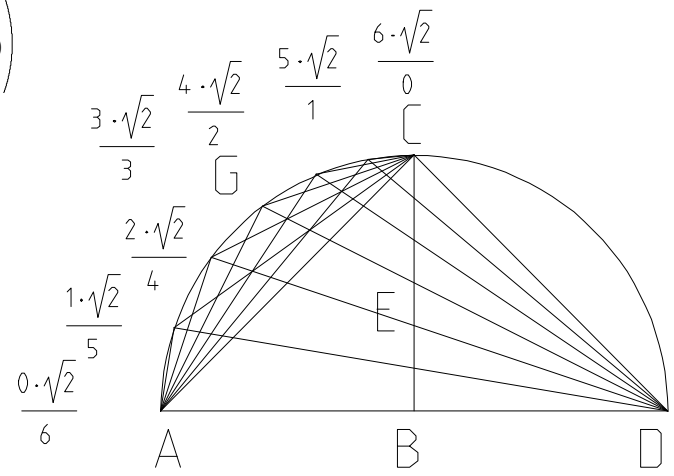


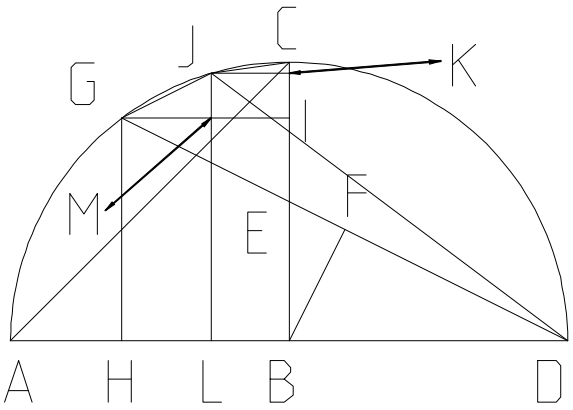

$\text{if}\left(BS - \delta, \frac{AG_\delta}{CG_\delta}, 0\right)$

| |
|---|
| 0 |
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |

$\text{if}\left(BS - \delta, \frac{\delta \cdot \sqrt{2}}{BS - \delta}, 0\right)$

| |
|---|
| 0 |
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |






Finding the value of PI to any given number of unequal segments.

$\chi := 1 .. BS$

$JK_\chi := GI_\chi \quad GM_\chi := GI_{\chi - 1} - JK_\chi$

$CK_\chi := CI_\chi \quad IK_\chi := CI_{\chi - 1} - CK_\chi$

$GJ_\delta := \sqrt{\left(GM_\delta\right)^2 + \left(IK_\delta\right)^2}$

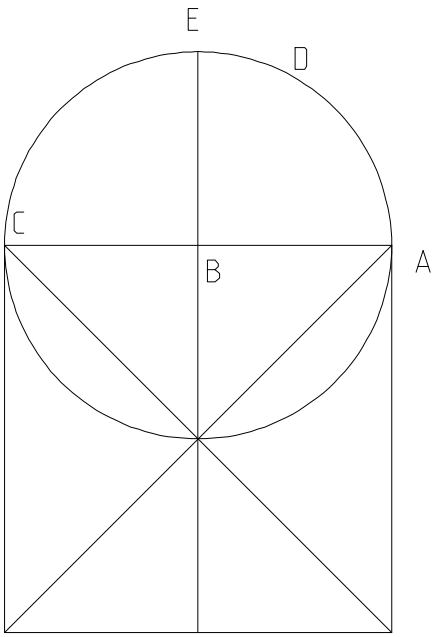
$BS \equiv 6 \qquad \sum_\delta GJ_\delta \cdot 4 = 3.13154608512668$

Computed for 5000 segments $BS \equiv 5000 \qquad \sum_\delta GJ_\delta \cdot 4 = 3.141592639069159$

$\pi = 3.141592653589793$

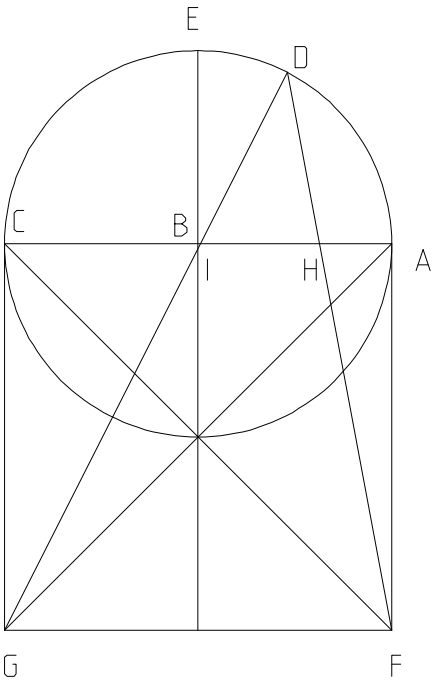
# Gruntwork I on the Delian Solution

11_06_93.mcd

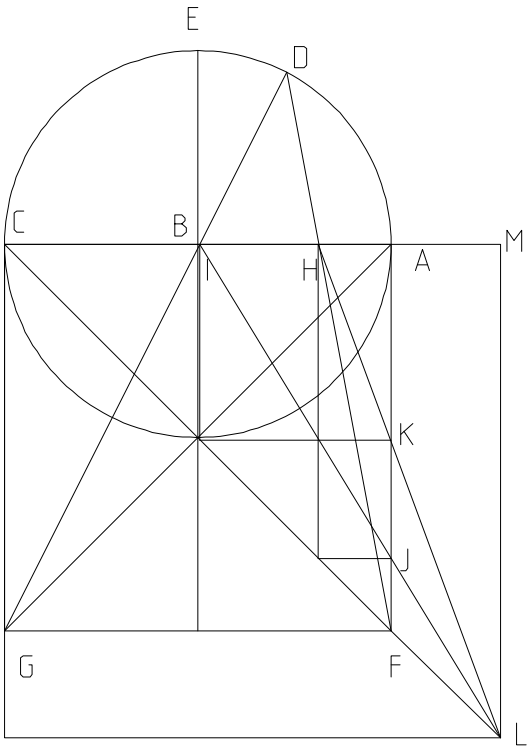



According to number theory, it is not possible to develop a cube root relationship using only linear and square root operations. Watch.

Given any circle ABE and any point, D, between AE.

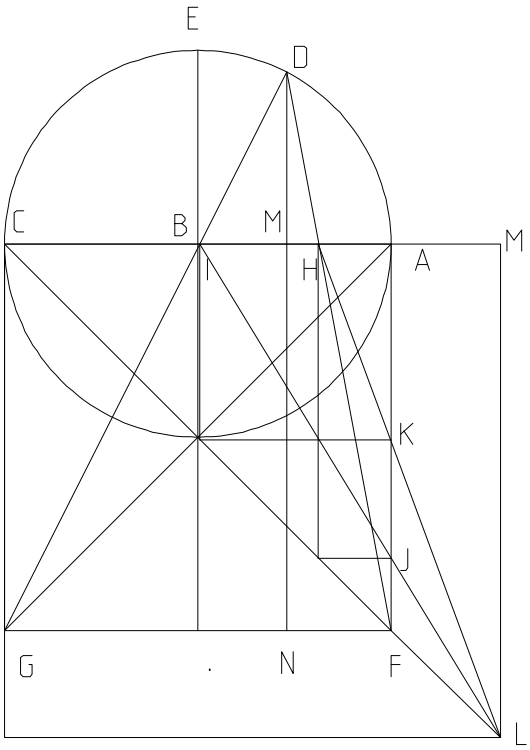



Describe FDG.




Describe AK, JK, FJ such that AK = BC, JK = HI, FJ = AH.

Do IJ and HK and CF meet at one and only one point L?




$AC := 10 \quad AB := \frac{AC}{2} \quad BC := AB \quad INC := 500$

$MN := AC \quad BD := AB \quad \delta := 0 .. INC - 1$

$AM_\delta := \frac{AB}{INC} \cdot \delta \quad BM_\delta := AB - AM_\delta \quad FN_\delta := AM_\delta$

$DM_\delta := \sqrt{BD^2 - (BM_\delta)^2} \quad DN_\delta := MN + DM_\delta$

$GN_\delta := BC + BM_\delta \quad DG_\delta := \sqrt{(GN_\delta)^2 + (DN_\delta)^2}$

$IM_\delta := \frac{GN_\delta \cdot DM_\delta}{DN_\delta} \quad DF_\delta := \sqrt{(FN_\delta)^2 + (DN_\delta)^2}$

$HM_\delta := \frac{FN_\delta \cdot DM_\delta}{DN_\delta} \quad AH_\delta := AM_\delta - HM_\delta$

Construct BQ and HP parallel with AO.
KT and JU parallel with CF.
RT and SU parallel with AF.

$AI_\delta := AM_\delta + IM_\delta \quad CI_\delta := AC - AI_\delta \quad CQ_\delta := \sqrt{\frac{(CI_\delta)^2}{2}}$

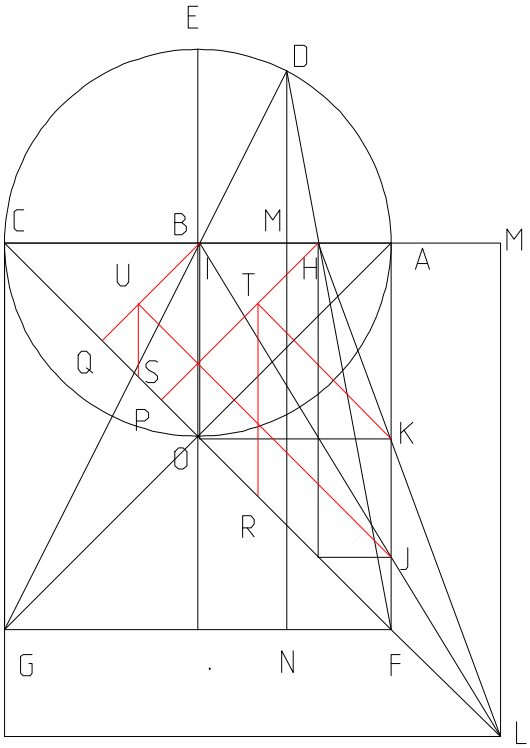
$CH_\delta := AC - AH_\delta \quad CP_\delta := \sqrt{\frac{(CH_\delta)^2}{2}} \quad CO := \sqrt{\frac{AC^2}{2}}$

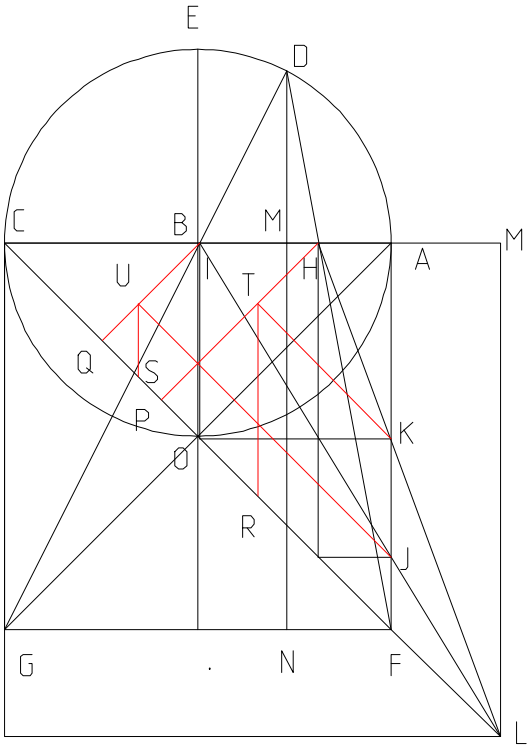
$AK_\delta := CI_\delta \quad AJ_\delta := CH_\delta \quad FJ_\delta := AH_\delta \quad US_\delta := FJ_\delta$

$FK_\delta := AI_\delta \quad RT_\delta := FK_\delta \quad IJ_\delta := \sqrt{(AI_\delta)^2 + (AJ_\delta)^2}$

$HK_\delta := \sqrt{(AH_\delta)^2 + (AK_\delta)^2} \quad QU_\delta := \sqrt{\frac{(US_\delta)^2}{2}}$

$PT_\delta := \sqrt{\frac{(RT_\delta)^2}{2}} \quad IQ_\delta := CQ_\delta \quad HP_\delta := CP_\delta$






$IU_\delta := IQ_\delta - QU_\delta \quad HT_\delta := HP_\delta - PT_\delta$

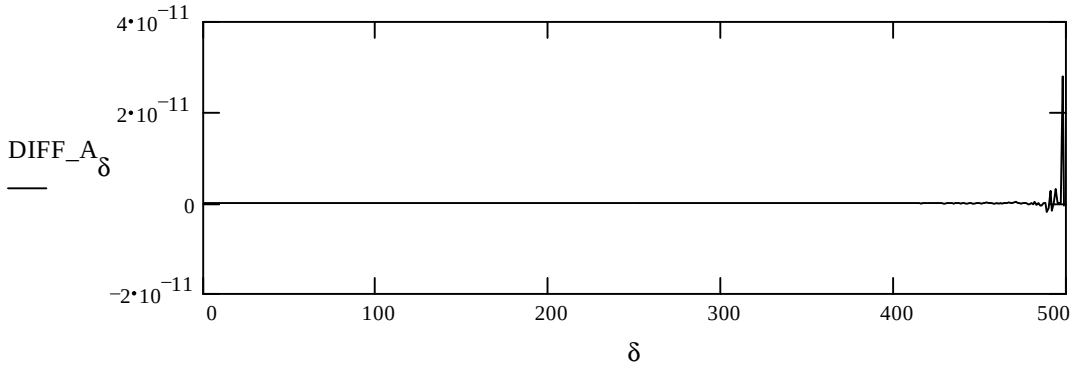
$KT_\delta := \sqrt{(HK_\delta)^2 - (HT_\delta)^2} \quad JU_\delta := \sqrt{(IJ_\delta)^2 - (IU_\delta)^2}$

$LQ_\delta := \frac{JU_\delta \cdot IQ_\delta}{IU_\delta} \quad LP_\delta := \frac{KT_\delta \cdot HP_\delta}{HT_\delta} \quad OQ_\delta := CO - CQ_\delta$

$OP_\delta := CO - CP_\delta \quad LO1_\delta := LQ_\delta - OQ_\delta$

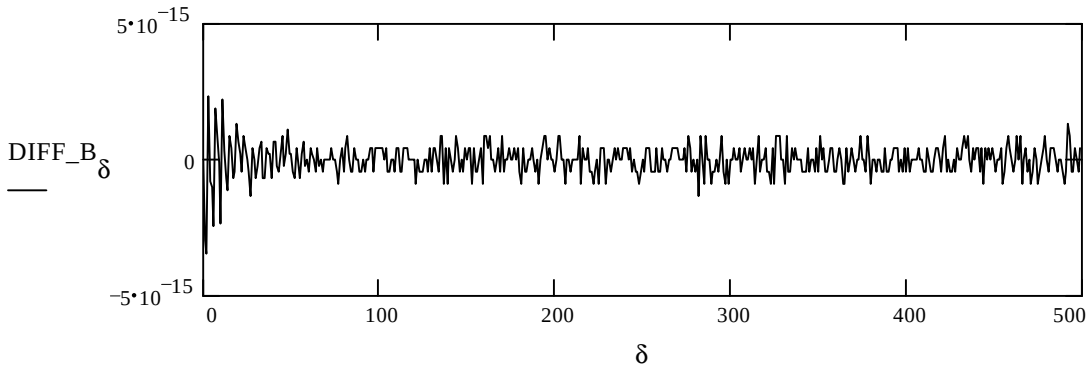
$LO2_\delta := LP_\delta - OP_\delta$

$DIFF\_A_\delta := LO1_\delta - LO2_\delta$

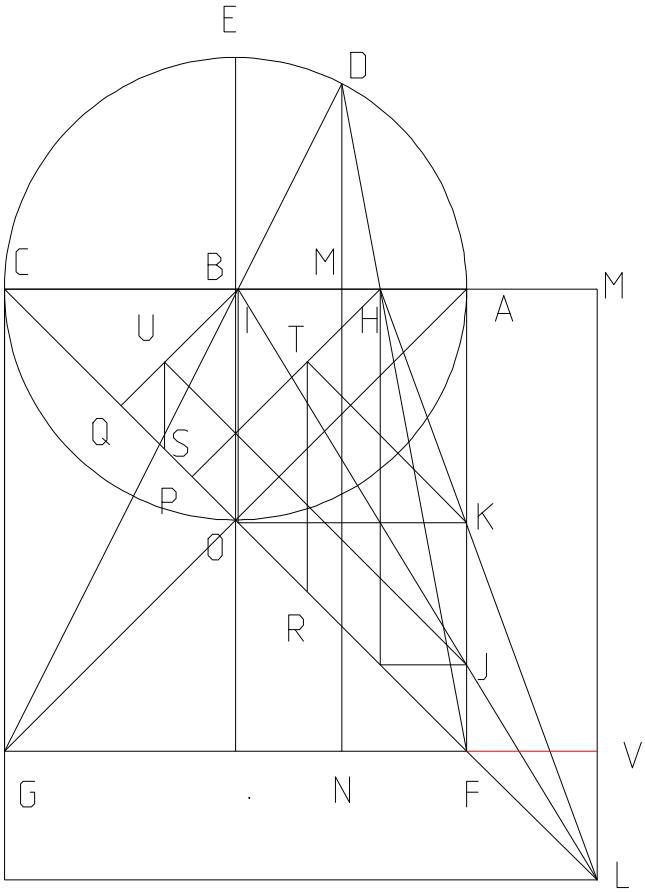



IJ, HK, and CF meet in one point L

$HI_\delta := HM_\delta + IM_\delta$ $DIFF\_B_\delta := HI_\delta - \sqrt{AH_\delta \cdot CI_\delta}$




The segment HI is the square root of the remaining segments AH, CI.



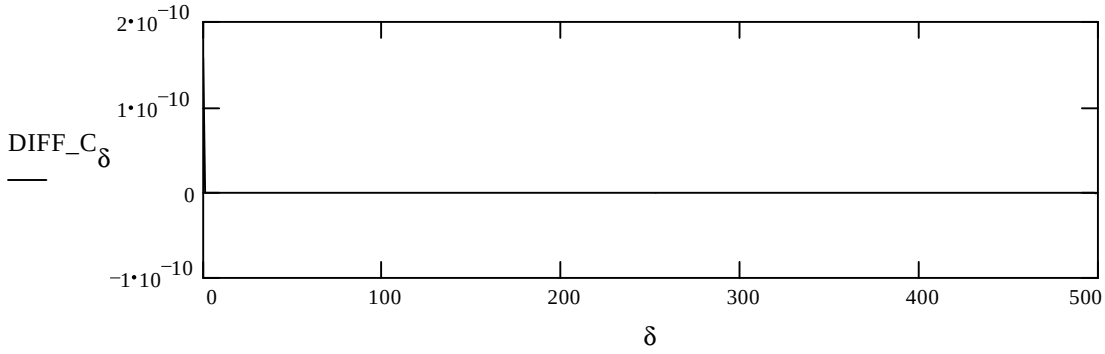

$FO := CO$ $FL_\delta := LO1_\delta - FO$

$FV_\delta := \sqrt{\frac{(FL_\delta)^2}{2}}$ $AM_\delta := FV_\delta$

$HM_\delta := AM_\delta + AH_\delta$ $IM_\delta := HM_\delta + HI_\delta$

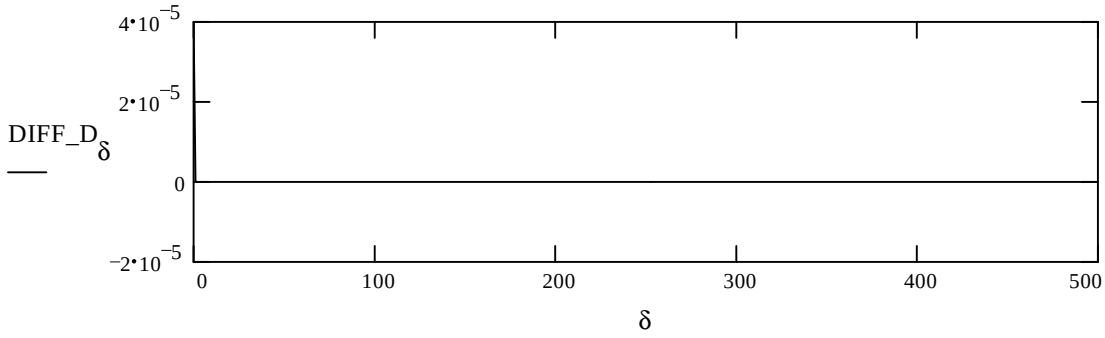
$CM_\delta := AM_\delta + AC$

$$DIFF\_C_\delta := \left[ \left( AM_\delta \right)^2 \cdot CM_\delta \right]^{\frac{1}{3}} - HM_\delta$$




HM is the cube root of $\left( AM_\delta \right)^2 \cdot CM_\delta$

$$DIFF\_D_\delta := \left[ AM_\delta \cdot \left( CM_\delta \right)^2 \right]^{\frac{1}{3}} - IM_\delta$$




IM is the cube root of $AM_\delta \cdot \left( CM_\delta \right)^2$

# Solve for Cube Placement

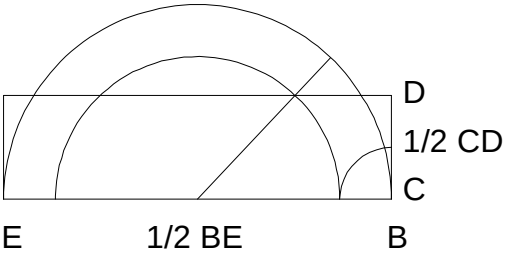
With straight edge and compass only, solve the given problem.
BE is the difference between the segments AE and AB.
CD is the difference between the cube root of AB squared by AE and the cube root of AE squared by AB. Find AB.

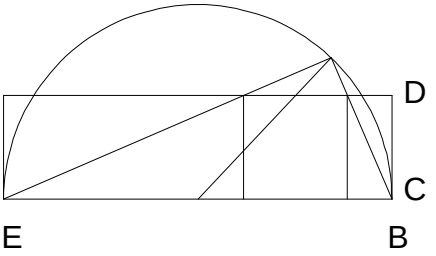
$BE = AE - AB$

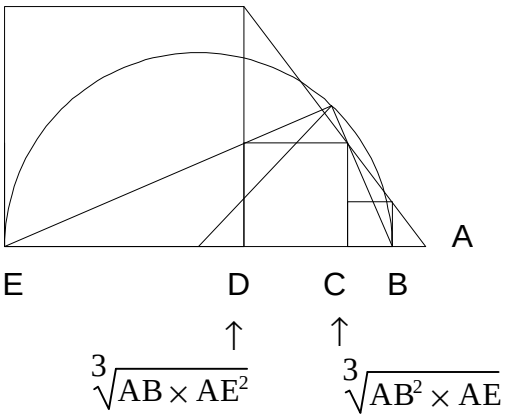
$CD = (AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}$

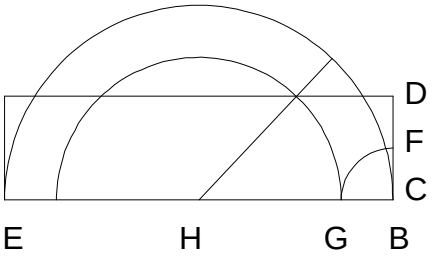
$AB = ?$

## Process Summary



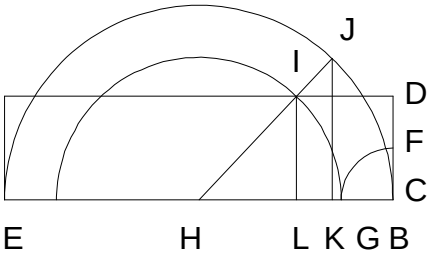






$\Delta := 500 \qquad \delta := 1 .. \Delta \qquad BE := 100$

$CD_\delta := \frac{BE}{3 \cdot \Delta} \cdot \delta \qquad CF_\delta := \frac{CD_\delta}{2} \qquad BG_\delta := CF_\delta$

$BH := \frac{BE}{2} \qquad EH := BH \qquad GH_\delta := BH - BG_\delta$



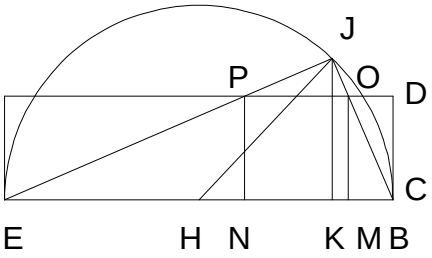

$HI_\delta := GH_\delta \qquad IL_\delta := CD_\delta \qquad HJ := BH$

$HL_\delta := \sqrt{(HI_\delta)^2 - (IL_\delta)^2} \qquad HK_\delta := \frac{HL_\delta \cdot HJ}{HI_\delta}$

$JK_\delta := \sqrt{HJ^2 - (HK_\delta)^2} \qquad BK_\delta := BH - HK_\delta$

$EK_\delta := EH + HK_\delta$

$CJ_\delta := \sqrt{(JK_\delta)^2 + (BK_\delta)^2} \qquad MO_\delta := CD_\delta$



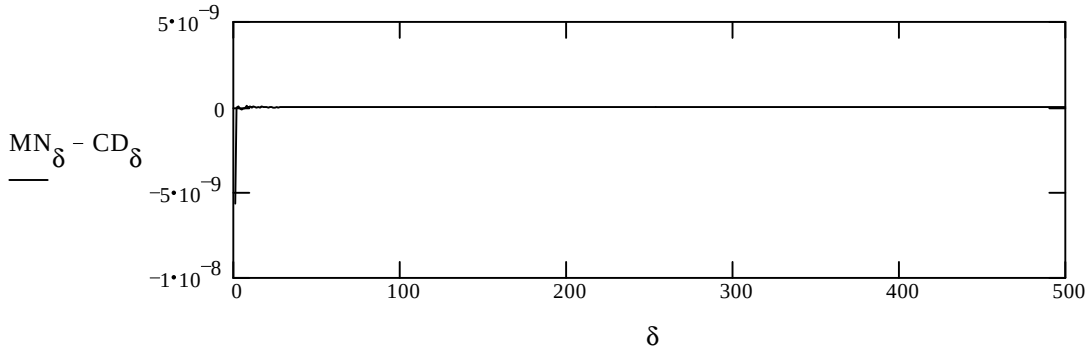

$EJ_\delta := \sqrt{(EK_\delta)^2 + (JK_\delta)^2} \qquad NP_\delta := CD_\delta$

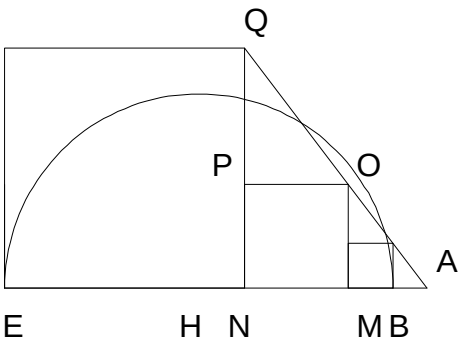
$EN_\delta := \frac{EK_\delta \cdot NP_\delta}{JK_\delta} \qquad BN_\delta := BE - EN_\delta$

$BM_\delta := \frac{BK_\delta \cdot MO_\delta}{JK_\delta} \qquad MN_\delta := BN_\delta - BM_\delta$

$OP_\delta := MN_\delta$

It appears that MN = CD.






$$NQ_\delta := EN_\delta \quad PQ_\delta := NQ_\delta - NP_\delta$$

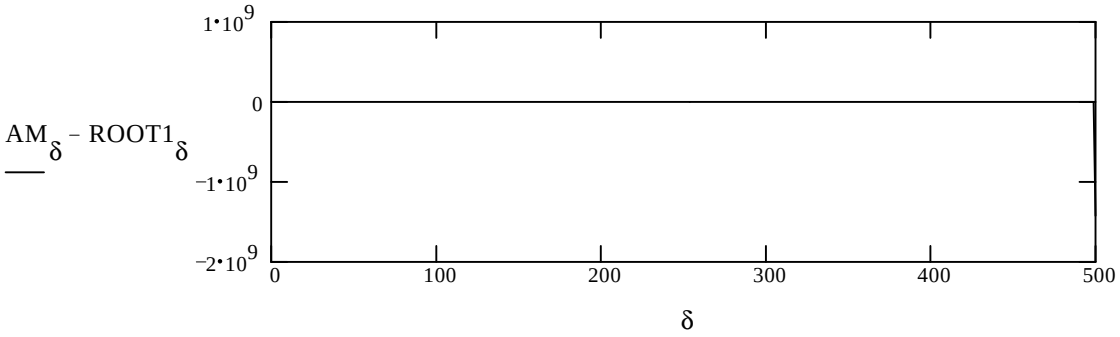
$$AN_\delta := \frac{OP_\delta \cdot NQ_\delta}{PQ_\delta} \quad AB_\delta := AN_\delta - BN_\delta$$

$$AM_\delta := AB_\delta + BM_\delta \quad AN_\delta := AB_\delta + BN_\delta$$

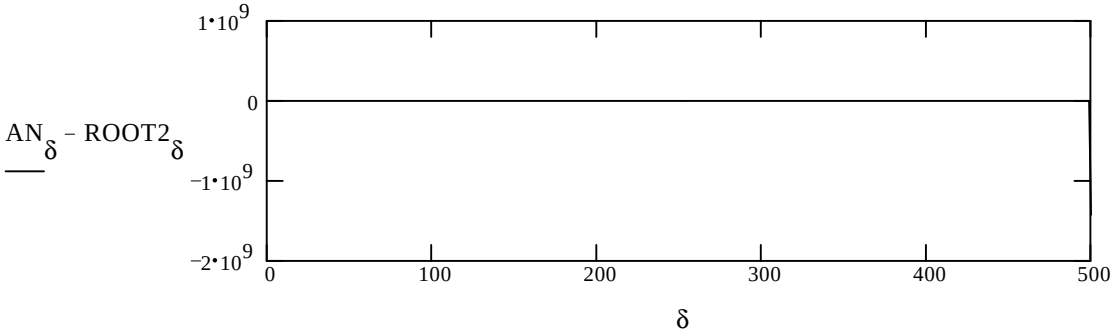
$$AE_\delta := AB_\delta + BE$$

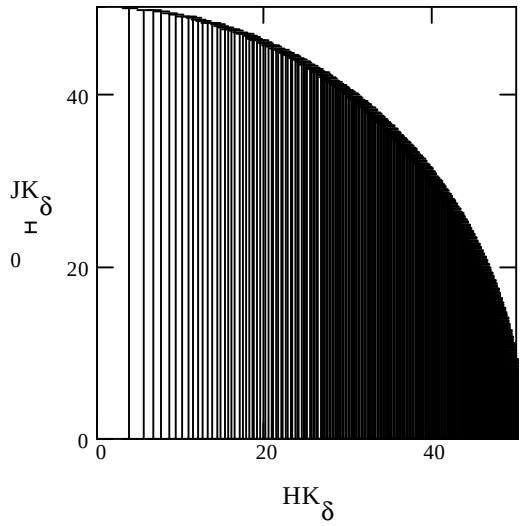
$$ROOT1_\delta := \left[ \left( AB_\delta \right)^2 \cdot AE_\delta \right]^{\frac{1}{3}}$$

$$ROOT2_\delta := \left[ AB_\delta \cdot \left( AE_\delta \right)^2 \right]^{\frac{1}{3}}$$




We seem to have a cube root relationship

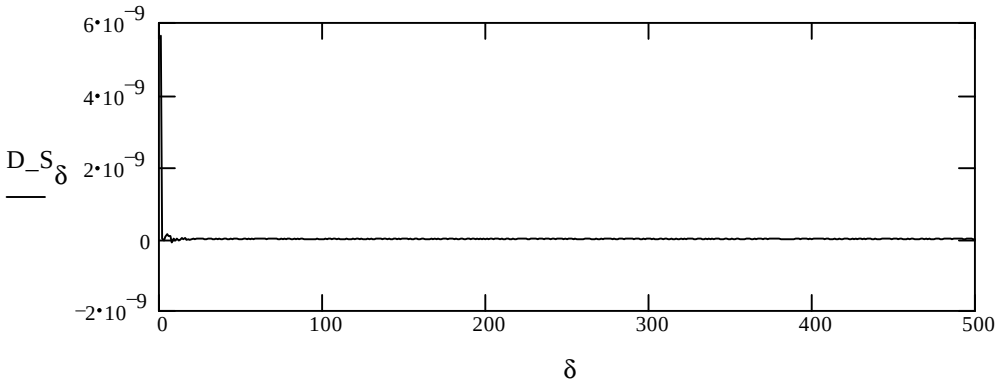




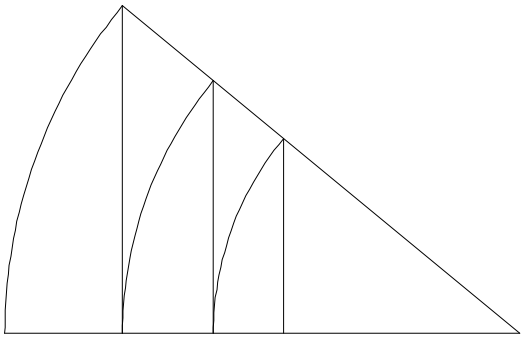

Sample Points along the arc.

Last ratio. $\frac{BE}{CD_{\Delta}} = 3$

$D\_S_{\delta} := \sqrt{EN_{\delta} \cdot BM_{\delta}} - MN_{\delta}$




The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship. Later we will come to realize that a perpendicular to the rectangle in a right triangle on the hypotenuse is equal to the square root of the two external segments of the rectangle. These two will be called Gemini roots as I do not know the proper term.

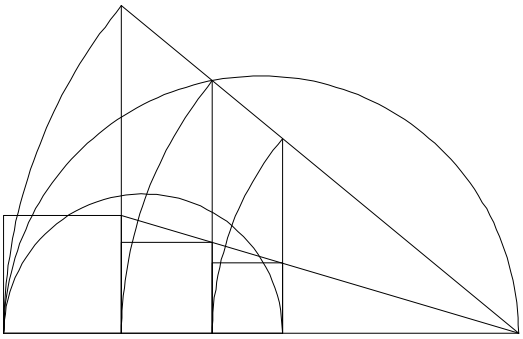


11_10_E3.MCD

# Gruntwork II on the Delian Solution

Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?

Process Summary

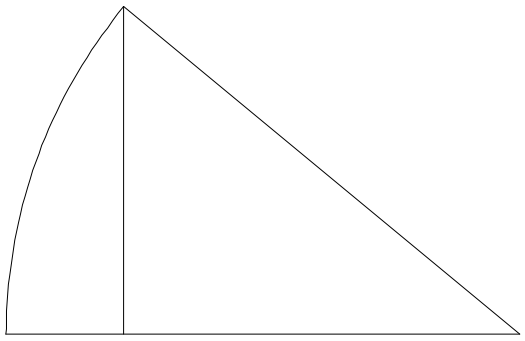


The figure indicates a possible cubic relationship. This is what will be tested along with some of the constants involved.

$D := 10 \quad \Delta := 90 \quad \delta := 1 .. \Delta \cdot D$

$\Delta$ will give the the range as 90 degrees.
D will further divide each degree into smaller segments.

F



A

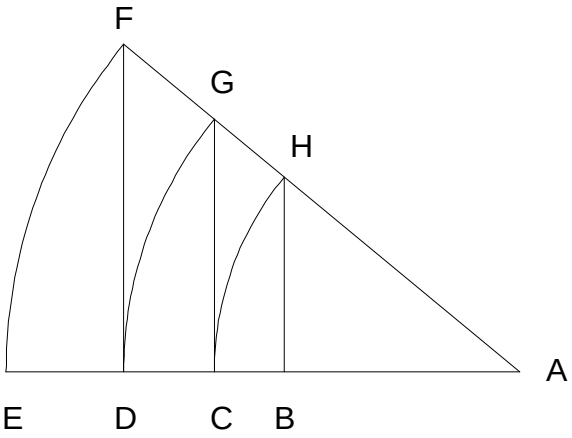
E D

$AE := 100$

$AF := AE \quad \angle A_\delta := \frac{\delta \cdot deg}{D}$

$DF_\delta := AF \cdot \left( \sin\left( \angle A_\delta \right) \right)$

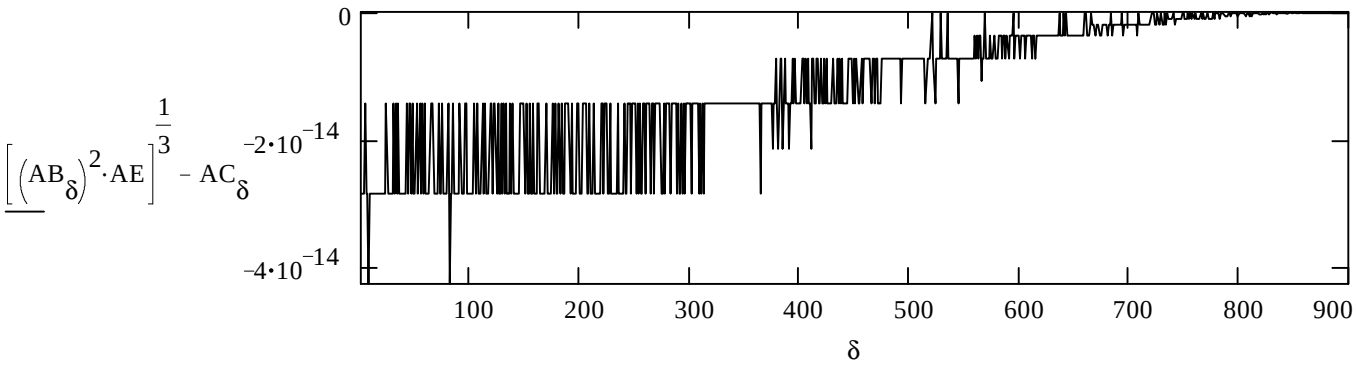
$AD_\delta := \sqrt{AF^2 - \left( DF_\delta \right)^2}$

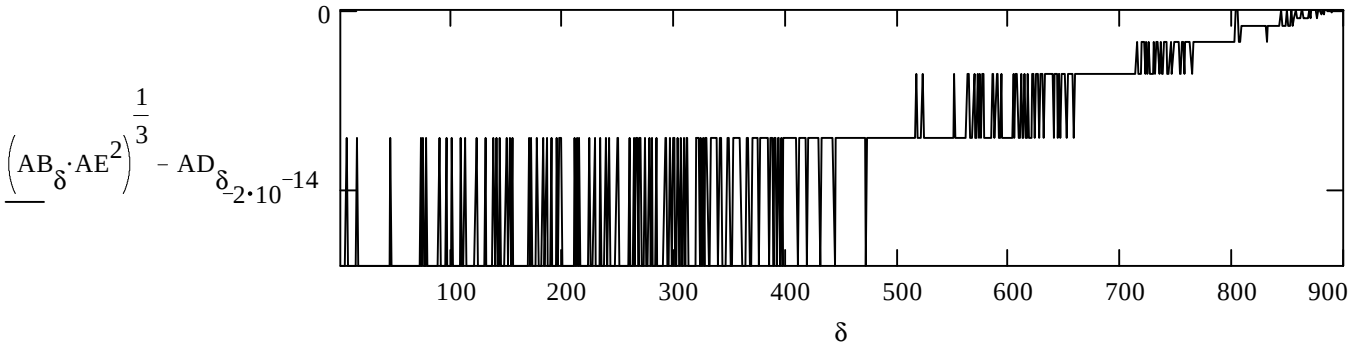



$$AG_\delta := AD_\delta \qquad AC_\delta := \frac{AD_\delta \cdot AG_\delta}{AF}$$

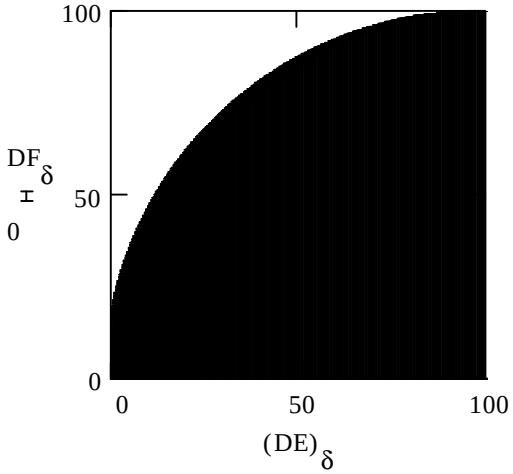
$$AH_\delta := AC_\delta \qquad AB_\delta := \frac{AD_\delta \cdot AH_\delta}{AF}$$

$$DE_\delta := AE - AD_\delta$$



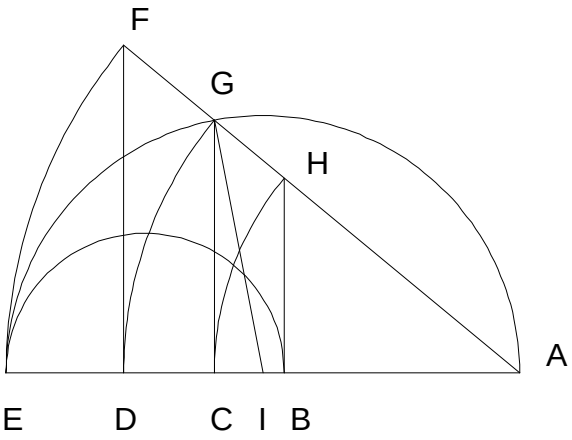




The simple figure yields a cube root relationship.




Sample points taken along DE.

There are $D \cdot \Delta = 900$ figures represented.

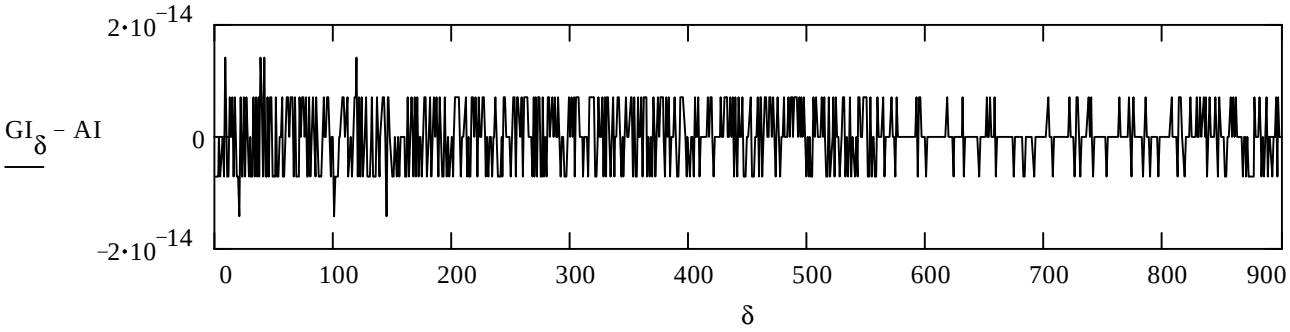



Does CG always meet at the circumference? (AGE)

$$CG_\delta := \sqrt{(AG_\delta)^2 - (AC_\delta)^2}$$

$$AI := \frac{AE}{2} \qquad CI_\delta := AC_\delta - AI$$

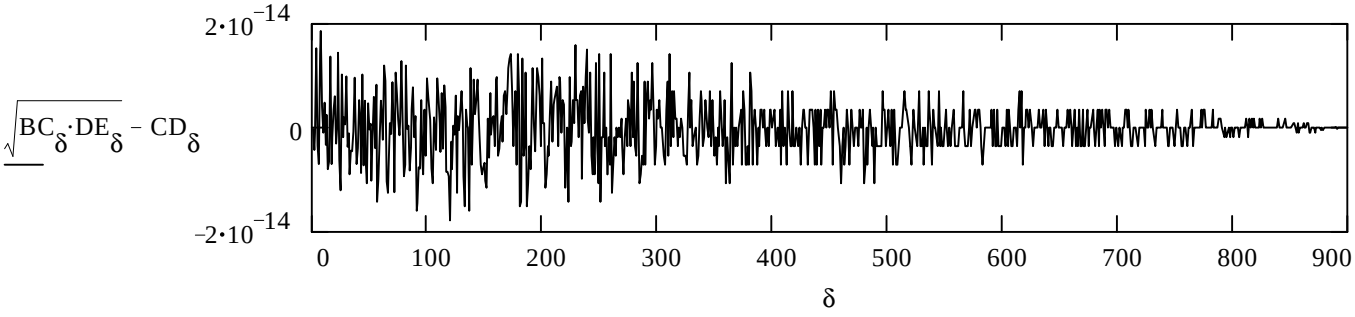
$$GI_\delta := \sqrt{(CG_\delta)^2 + (CI_\delta)^2}$$



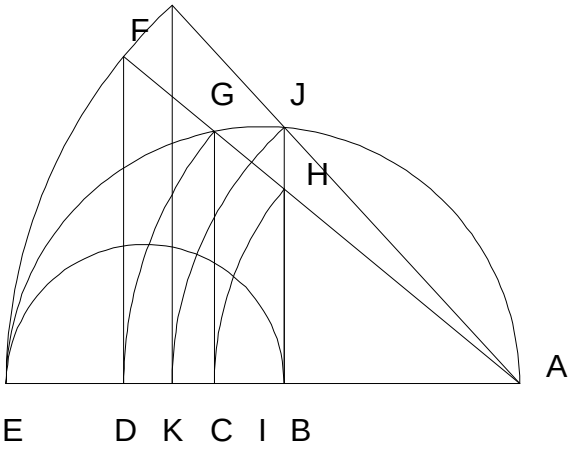

I would interpret this to mean yes.

Is then, CD the square root of BC and DE?

$$BC_\delta := AC_\delta - AB_\delta \qquad CD_\delta := AD_\delta - AC_\delta \qquad DE_\delta := AE - AD_\delta$$




I would interpret this to mean yes also.



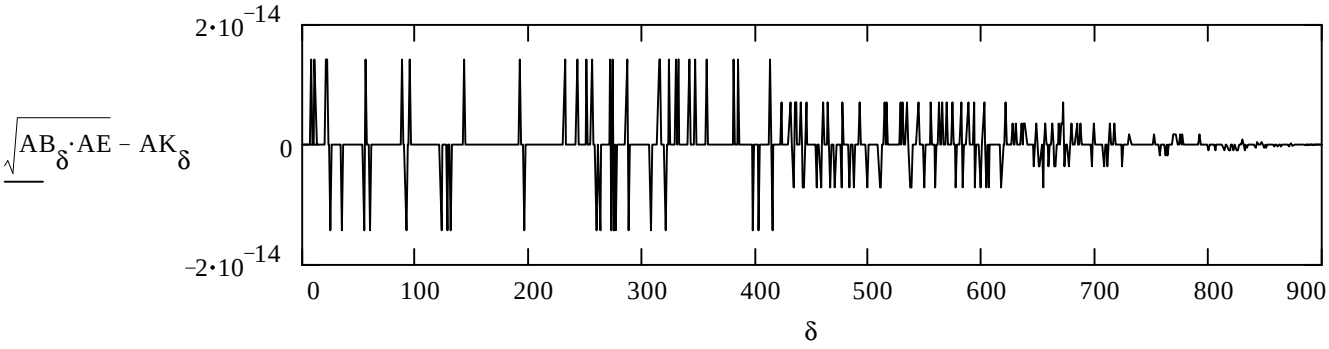

Is AK the square root of AB AE?

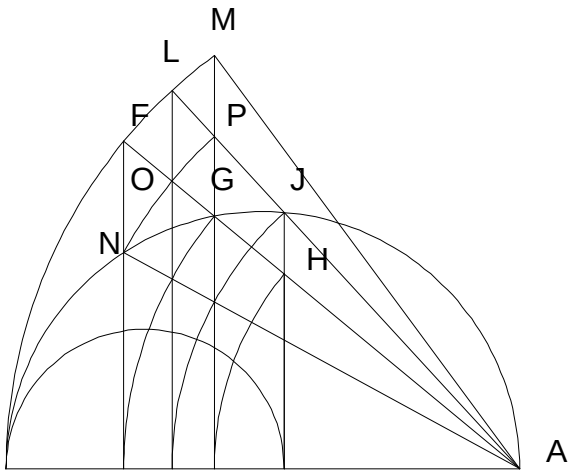
$$BE_\delta := AE - AB_\delta \qquad BJ_\delta := \sqrt{AB_\delta \cdot BE_\delta}$$

$$AJ_\delta := \sqrt{\left(AB_\delta\right)^2 + \left(BJ_\delta\right)^2}$$

$$AK_\delta := AJ_\delta$$

Apparently so.




Do N, O, and P lie on the same arc?

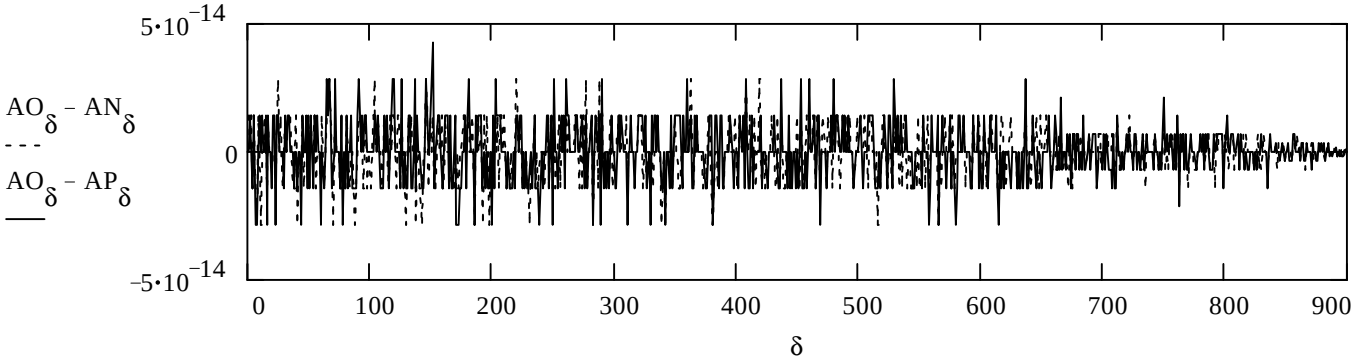
$$DN_\delta := \sqrt{AD_\delta \cdot DE_\delta}$$

$$AN_\delta := \sqrt{\left(AD_\delta\right)^2 + \left(DN_\delta\right)^2}$$

$$AO_\delta := \frac{AG_\delta \cdot AK_\delta}{AC_\delta}$$

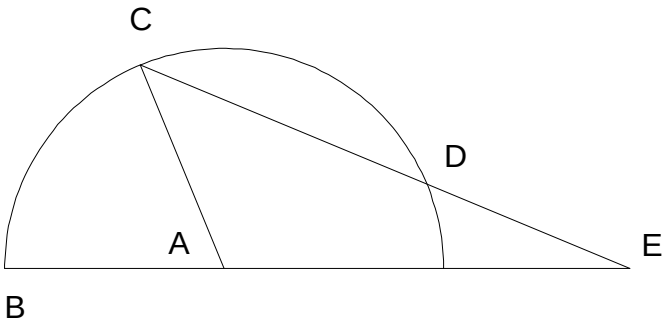
$$AP_\delta := \frac{AJ_\delta \cdot AC_\delta}{AB_\delta}$$

Another affirmative.



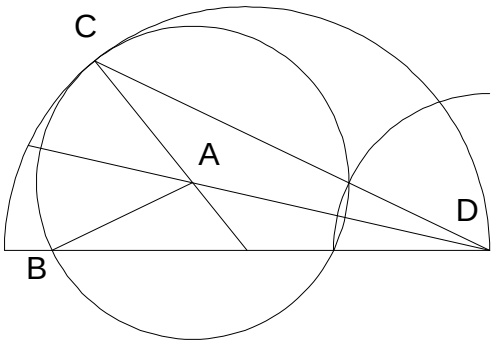
# The Archamedian Paper Trisector

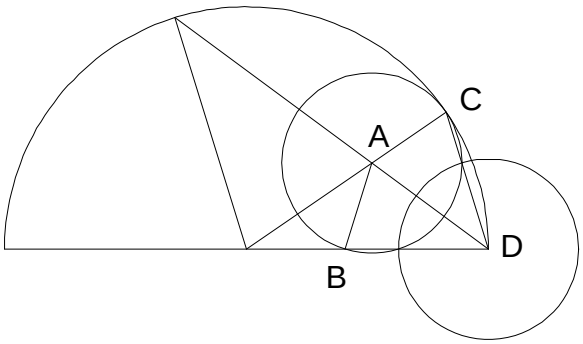
11_11_B3.MCD




I have never seen the developed figure from which the Archamedian Pager Trisector derives. DE = AC, and by using a piece of marked paper, CDE can be somewhat aligned producing the trisection CEB = 1/3 CAB.

Process Summary

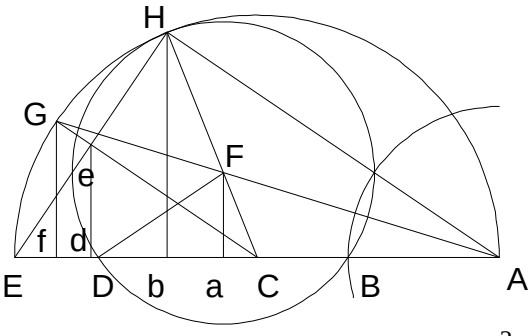



In both cases CDB is 1/3 of CAB. It appears that trisection might occur for the entire circle.

$$AE := 100000 \quad \delta := 0..\Delta$$

$$Ab_\delta := \text{if}\left[ View, \frac{AE}{View} - \left(\frac{\frac{AE}{View}}{\Delta}\cdot\delta\right), AE - \left(\frac{AE}{\Delta}\cdot\delta\right)\right]$$



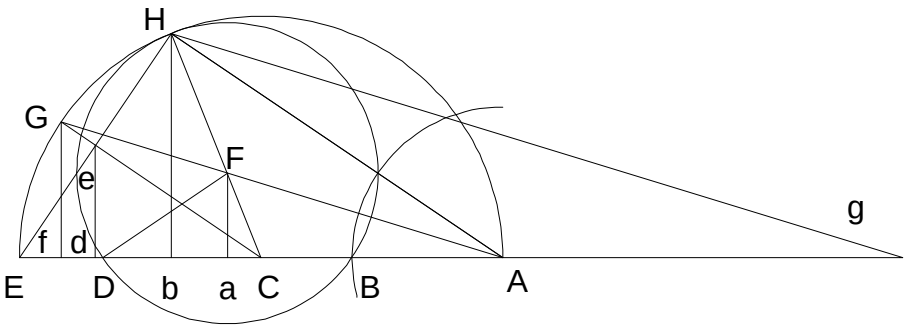

$$Eb_\delta := AE - Ab_\delta$$

$$Hb_\delta := \sqrt{Ab_\delta \cdot Eb_\delta} \quad EH_\delta := \sqrt{\left(Hb_\delta\right)^2 + \left(Eb_\delta\right)^2}$$

$$AC := \frac{AE}{2} \quad CE := AC \quad Ee_\delta := \frac{EH_\delta}{2}$$

$$Ce_\delta := \sqrt{CE^2 - \left(Ee_\delta\right)^2} \quad Cd_\delta := \frac{\left(Ce_\delta\right)^2}{CE} \quad CG := AC \quad Cf_\delta := \frac{Cd_\delta \cdot CG}{Ce_\delta}$$

$$de_\delta := \sqrt{\left(Ce_\delta\right)^2 - \left(Cd_\delta\right)^2} \quad Gf_\delta := \frac{de_\delta \cdot Cf_\delta}{Cd_\delta} \quad Af_\delta := AC + Cf_\delta \quad AG_\delta := \sqrt{\left(Af_\delta\right)^2 + \left(Gf_\delta\right)^2}$$



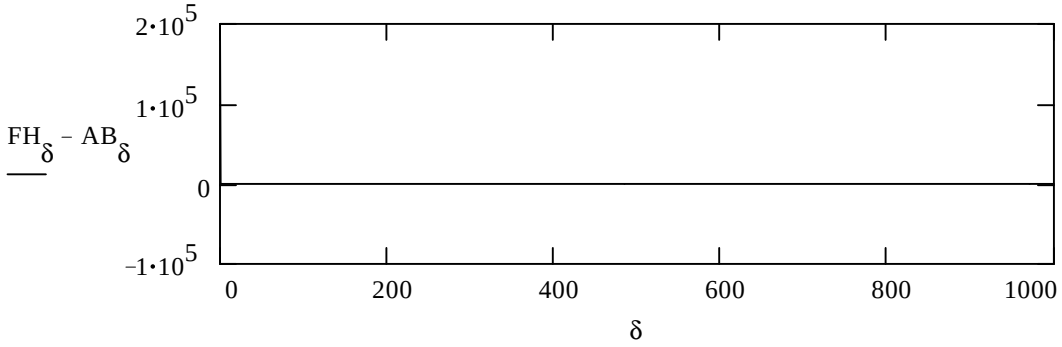

$CH := AC$

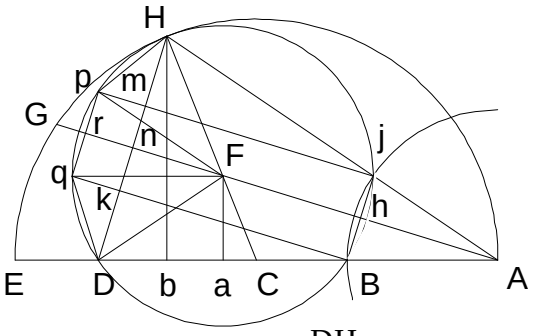
$bg_\delta := \frac{Af_\delta \cdot Hb_\delta}{Gf_\delta}$ $Cb_\delta := Ab_\delta - AC$ $Cg_\delta := bg_\delta - Cb_\delta$ $CF_\delta := \frac{CH \cdot AC}{Cg_\delta}$

$FH_\delta := CH - CF_\delta$ $Ca_\delta := \frac{Cb_\delta \cdot CF_\delta}{CH}$ $Fa_\delta := \sqrt{(CF_\delta)^2 - (Ca_\delta)^2}$ $DF_\delta := FH_\delta$

$Da_\delta := \sqrt{(DF_\delta)^2 - (Fa_\delta)^2}$ $Aa_\delta := AC + Ca_\delta$ $Ba_\delta := Da_\delta$ $AB_\delta := Aa_\delta - Ba_\delta$

By what is known about trisection, AB must equal FH.




$AD_\delta := AB_\delta + (2 \cdot Da_\delta)$ $Db_\delta := AD_\delta - Ab_\delta$

$DH_\delta := \sqrt{(Hb_\delta)^2 + (Db_\delta)^2}$ $Bj_\delta := \frac{DH_\delta \cdot AB_\delta}{AD_\delta}$

$Bh_\delta := \frac{Bj_\delta}{2}$ $Ah_\delta := \sqrt{(AB_\delta)^2 - (Bh_\delta)^2}$

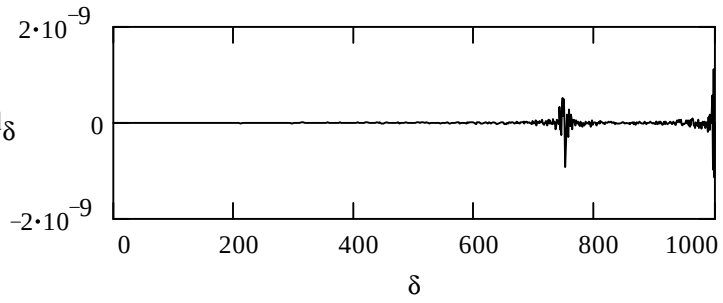
$Fr_\delta := Ah_\delta$ $Hn_\delta := \frac{DH_\delta}{2}$ $pq_\delta := Bj_\delta$ $mk_\delta := Bj_\delta$

$Fn_\delta := \text{if}\left[AD_\delta > AC, \sqrt{(FH_\delta)^2 - (Hn_\delta)^2}, -\sqrt{(FH_\delta)^2 - (Hn_\delta)^2}\right]$ $nr_\delta := Fr_\delta - Fn_\delta$

$Hm_\delta := \frac{DH_\delta - mk_\delta}{2}$ $mp_\delta := nr_\delta$ $Hp_\delta := \sqrt{(mp_\delta)^2 + (Hm_\delta)^2}$ $HE_\delta := \sqrt{(Hb_\delta)^2 + (Eb_\delta)^2}$

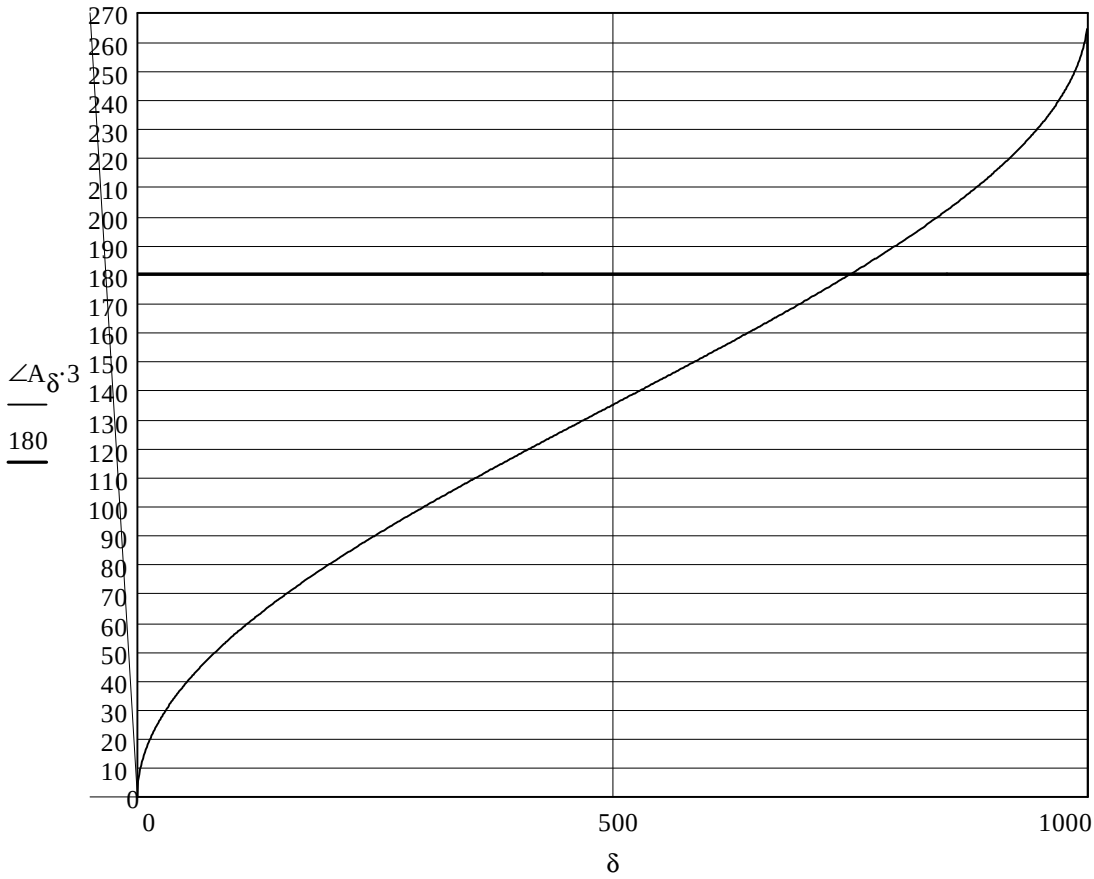
.

Does Hp = pq?

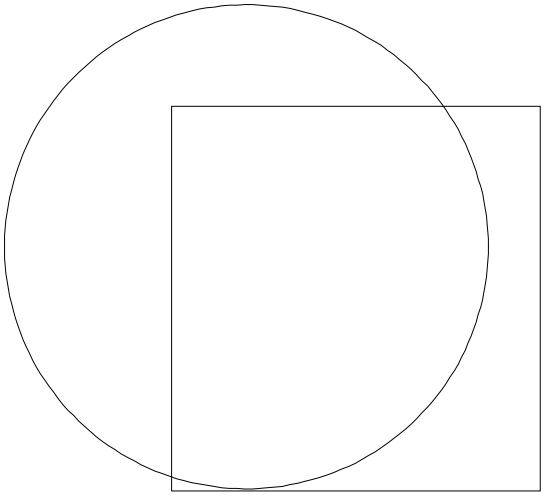



$\angle A_\delta := \text{asin}\left(\frac{Bh_\delta}{AB_\delta}\right) \cdot \frac{2}{\text{deg}}$

Plug resolution in here $\Delta \equiv 1000$ $\angle A_{\Delta - 1} \cdot 3 = 264.5635$ $View \equiv |0|$




One can safely say that the limit of the Archamedian paper trisector is 270 degrees.



# To Square A Circle

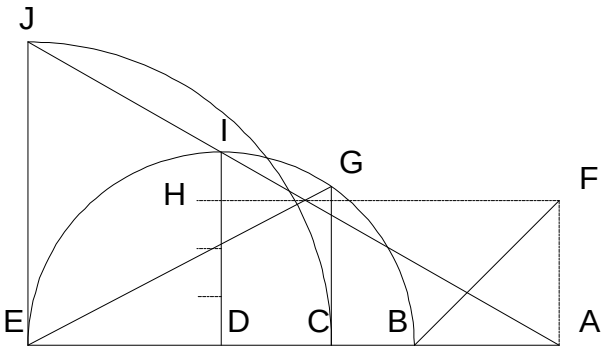
11_12_93.MCD

Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?

Using the approximation, $\pi = 22/7$, square the circle off the base of a right triangle.

## Process Summary

The most primitive formula for Area is A = 1/2bh. We will not use its transformation to $\pi r^2$.




Special Value for BE, try 4, then look at the corresponding value for EJ.

$BE := 4 \quad BD := \frac{BE}{2} \quad DE := BD$

$DI := BD \quad DH := \frac{3}{4} \cdot DI$

$AB := DH \quad AD := AB + BD$

$AE := AB + BE \quad EJ := \frac{DI \cdot AE}{AD}$

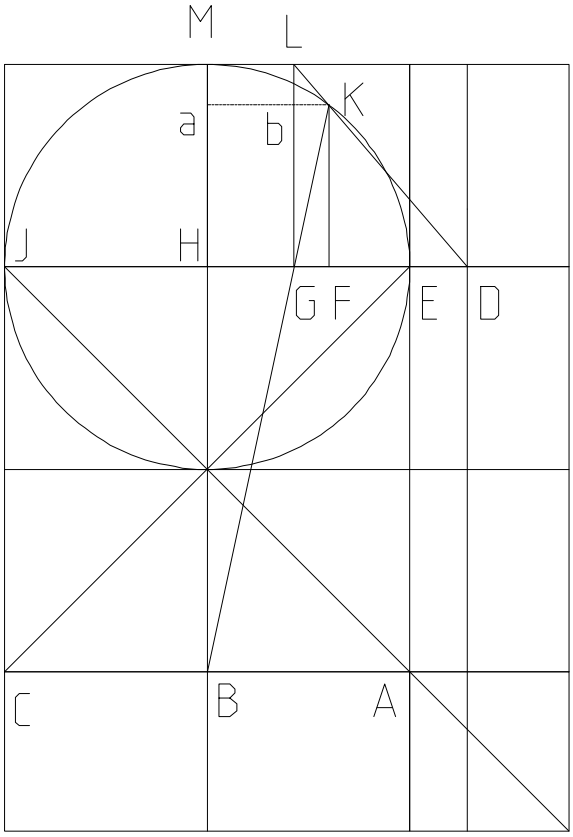
$CE := EJ \quad BC := BE - CE$

$CG := \sqrt{BC \cdot CE} \quad EG := \sqrt{CE^2 + CG^2} \quad \pi\_A := \frac{EG^2}{BD^2}$

$\pi\_A = 3.14285714 \qquad EJ = 3.14285714$

$\frac{22}{7} = 3.14285714 \qquad \pi\_A - \frac{22}{7} = 0$

Basically we have taken one half of the circle for 1/2b and crudely projected the arc EI for the height.




11_18_93.MCD

For any point K, construct GL parallel to HM. LK then projects to the point from which the figure forms the cubic.

$EJ := 100 \quad \Delta := 1000 \quad \delta := 0 .. \Delta - 1$

$EH := \frac{EJ}{2} \quad EF_\delta := \frac{EH}{\Delta} \cdot \delta \quad FJ_\delta := EJ - EF_\delta \quad BH := EJ$

$FK_\delta := \sqrt{EF_\delta \cdot FJ_\delta} \quad FH_\delta := EH - EF_\delta \quad Ka_\delta := FH_\delta \quad Ha_\delta := FK_\delta$

$Ba_\delta := BH + Ha_\delta \quad GH_\delta := \frac{Ka_\delta \cdot BH}{Ba_\delta} \quad FG_\delta := FH_\delta - GH_\delta$

$Kb_\delta := FG_\delta \quad GL := EH \quad Gb_\delta := FK_\delta$

$Lb_\delta := GL - Gb_\delta \quad DG_\delta := if\left(GH_\delta, \frac{Kb_\delta \cdot GL}{Lb_\delta}, 0\right) \quad DF_\delta := DG_\delta - FG_\delta \quad DE_\delta := DG_\delta - \left(FG_\delta + EF_\delta\right)$

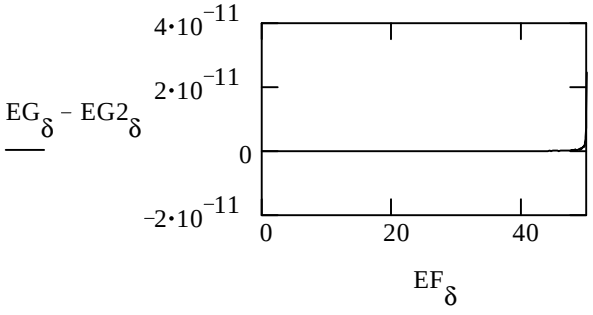
$DJ_\delta := EJ + DE_\delta \quad EG_\delta := EF_\delta + FG_\delta$

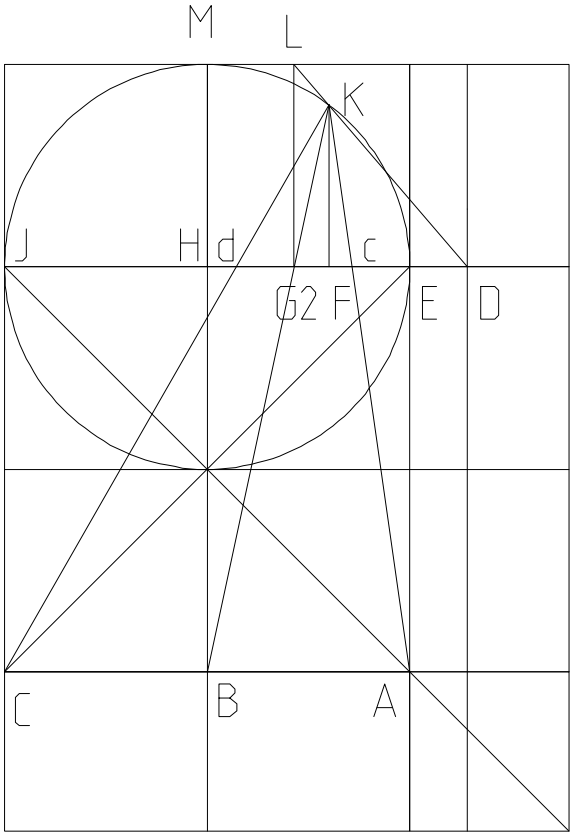
$Dc_\delta := if\left[GH_\delta, \left[\left(DE_\delta\right)^2 \cdot DJ_\delta\right]^{\frac{1}{3}}, 0\right]$

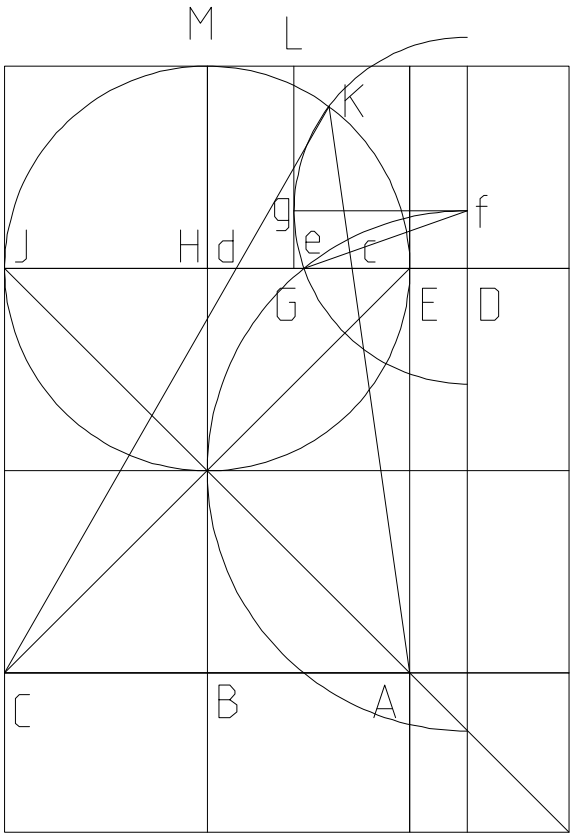
$Dd_\delta := if\left[GH_\delta, \left[DE_\delta \cdot \left(DJ_\delta\right)^2\right]^{\frac{1}{3}}, 0\right]$

$cd_\delta := Dd_\delta - Dc_\delta \quad G2c_\delta := \frac{cd_\delta}{2}$

$Ec_\delta := Dc_\delta - DE_\delta \quad EG2_\delta := Ec_\delta + G2c_\delta$



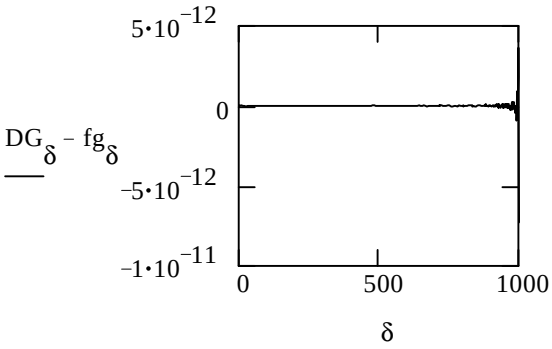



I should say that fg is equal to DG.

$Gc_\delta := G2c_\delta$

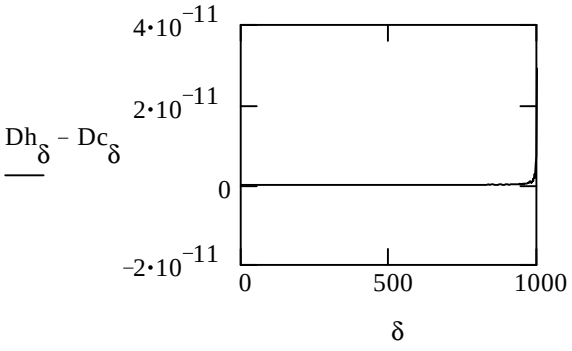
$Df_\delta := Gc_\delta De_\delta := \sqrt{DE_\delta \cdot DJ_\delta}$

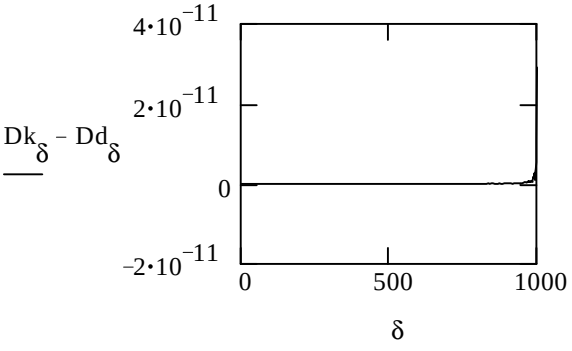
$ef_\delta := \sqrt{(Df_\delta)^2 + (De_\delta)^2}$ $fg_\delta := ef_\delta$

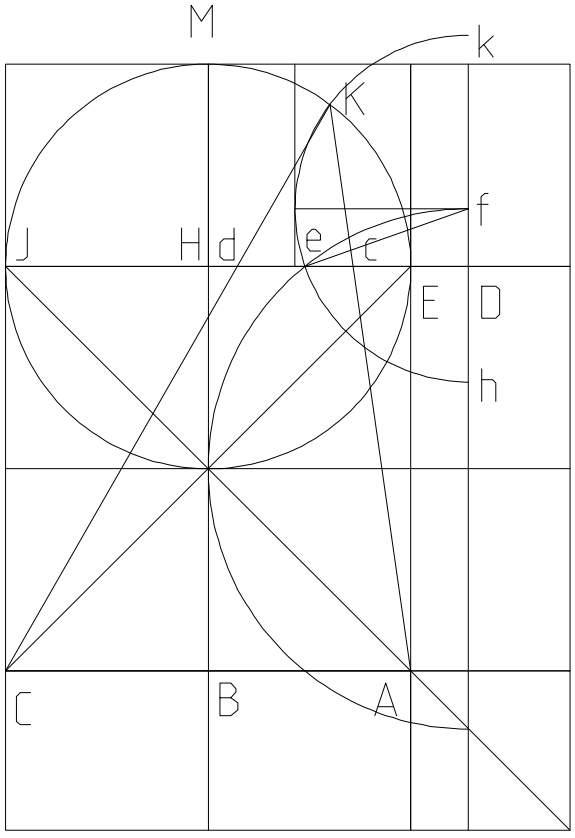


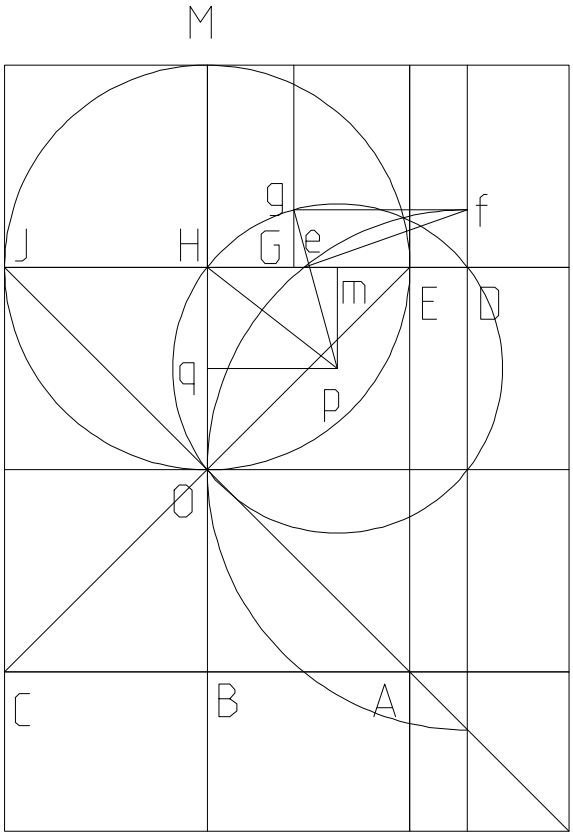

I should say that Dh is equal to Dc and that Dk is equal to Dd.

$Dh_\delta := ef_\delta - Df_\delta$ $Dk_\delta := ef_\delta + Df_\delta$





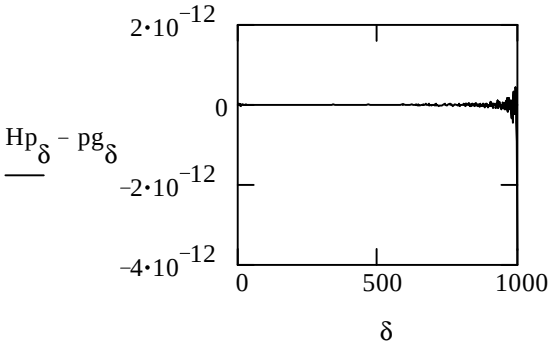

I should say that the circle pDH passes through the point g.

$DH_\delta := DE_\delta + EH \quad Hm_\delta := \frac{DH_\delta}{2} \quad HO := EH$

$Hq := \frac{HO}{2} \quad Hp_\delta := \sqrt{Hq^2 + (Hm_\delta)^2}$

$Gg_\delta := Df_\delta \quad mp := Hq \quad Gm_\delta := Hm_\delta - GH_\delta$

$pg_\delta := \sqrt{(Gg_\delta + mp)^2 + (Gm_\delta)^2}$




$HJ := EH \quad HP := \frac{HJ}{3}$

$JP := \frac{2 \cdot HJ}{3} \quad EP := EH + HP \quad PQ := \sqrt{EP \cdot JP}$

$JQ := \sqrt{PQ^2 + JP^2} \quad JR := JQ \quad HR := \sqrt{JR^2 - HJ^2}$

$RS := JQ \quad GS_\delta := \sqrt{RS^2 - (GH_\delta)^2} - HR$

$ES_\delta := \sqrt{(EG_\delta)^2 + (GS_\delta)^2} \quad GJ_\delta := HJ + GH_\delta$

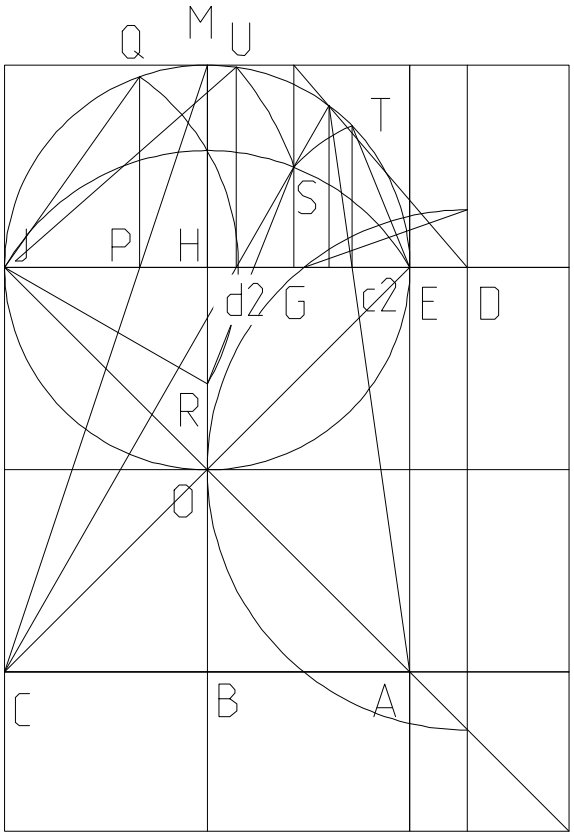
$JS_\delta := \sqrt{(GJ_\delta)^2 + (GS_\delta)^2} \quad ET_\delta := ES_\delta \quad JU_\delta := JS_\delta$

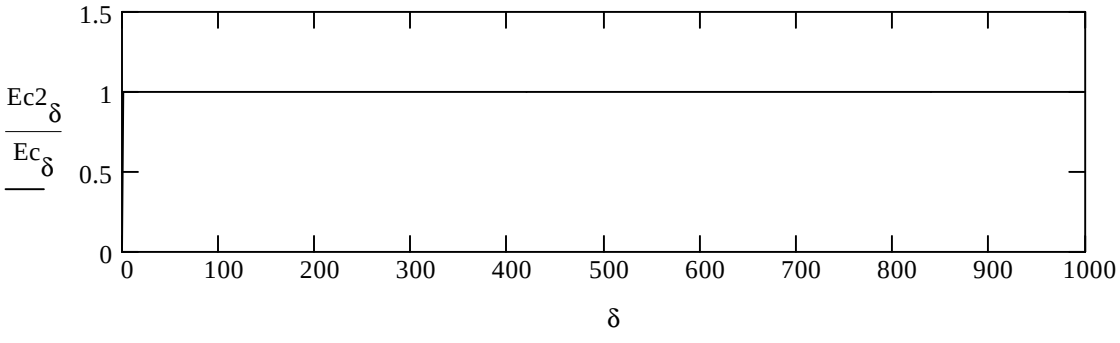
$Ed2_\delta := EJ - \frac{(JU_\delta)^2}{EJ} \quad Ec2_\delta := \frac{(ET_\delta)^2}{EJ}$

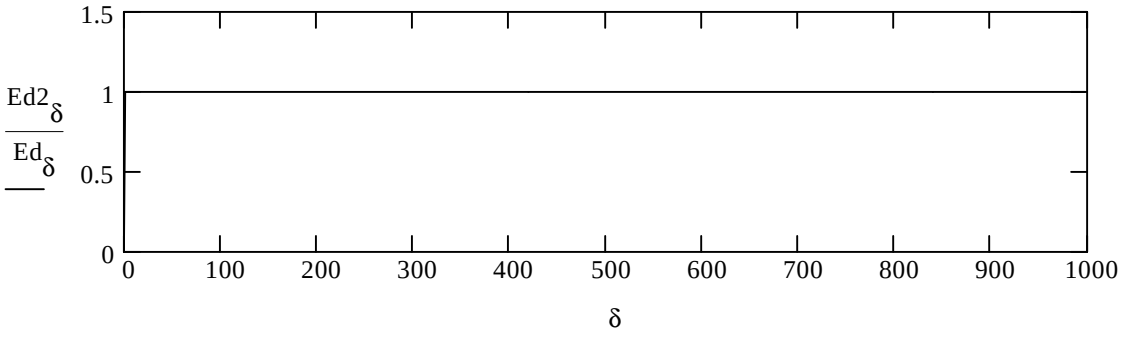
$Ec_\delta := Dc_\delta - DE_\delta \quad Ed_\delta := Dd_\delta - DE_\delta$

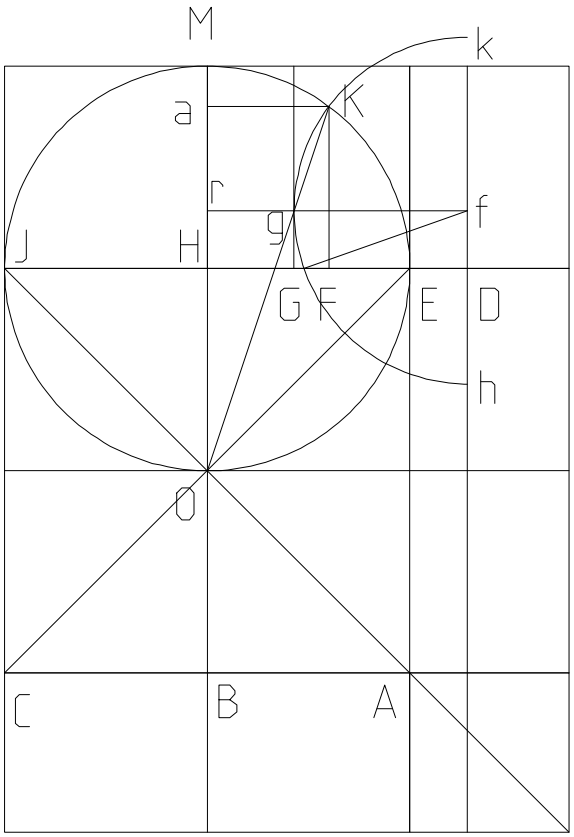
$Ed_{\Delta - 1} = 66.633327772$

$Ed2_{\Delta - 1} = 66.633327772$





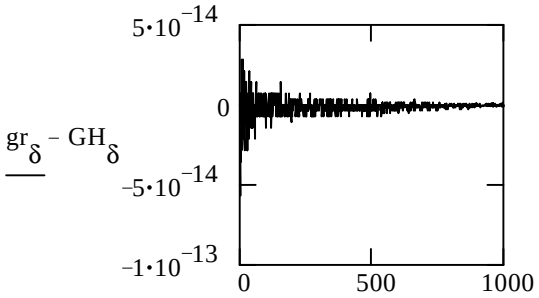



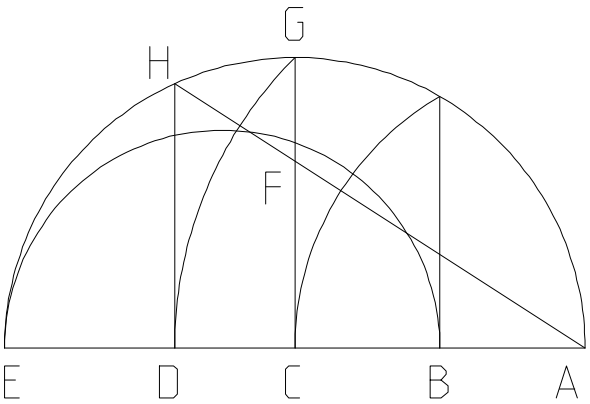




OK passes through g.

$Oa_\delta := HO + Ha_\delta \quad Hr_\delta := Gg_\delta \quad Or_\delta := HO + Hr_\delta$

$$gr_\delta := \frac{Ka_\delta \cdot Or_\delta}{Oa_\delta}$$






Using iteration to perform a geometric convergent series for cube root abstraction.

$A := if(X<Y, if(X<Z, X, Z), if(Y<Z, Y, Z))$

$B := if(X>Y, if(X>Z, X, Z), if(Y>Z, Y, Z))$

$W := X + Y + Z \quad C := W - (A + B)$

$AB := \sqrt{A \cdot C} \quad BE := B - AB \quad AE := AB + BE$

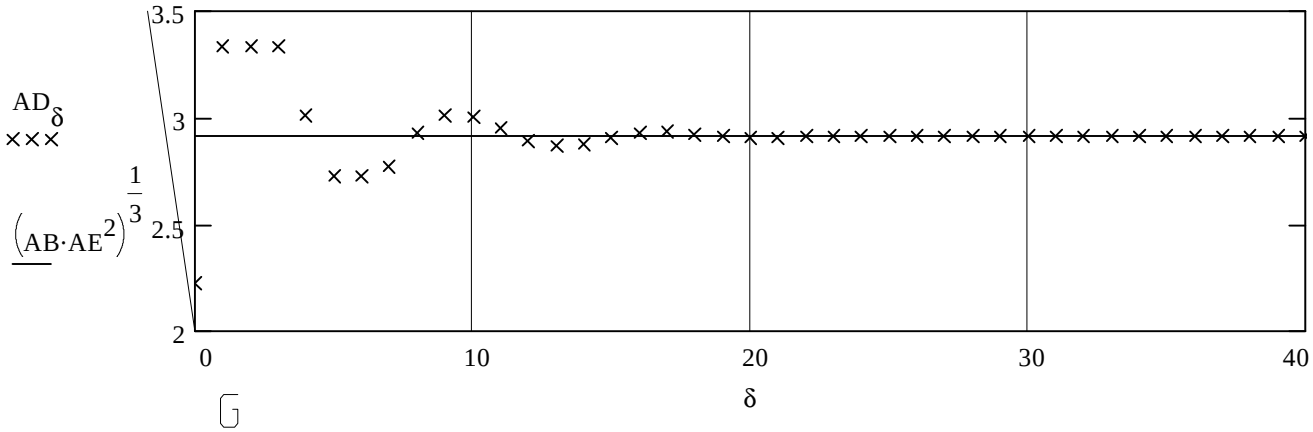
$\delta := 0..\Delta \quad AC := \sqrt{AE \cdot AB} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

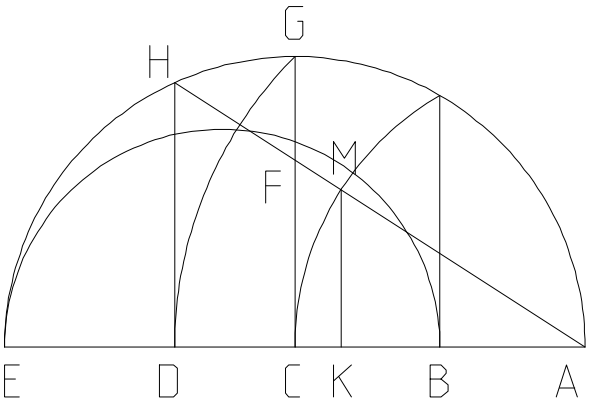
$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AC \\ AE - AC \\ CG \\ CG \\ AG \end{bmatrix} \qquad \begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$

$\Delta \equiv 40$

$X \equiv 5 \quad Y \equiv .9900 \quad Z \equiv Y$

$A = 0.99 \quad C = 0.99 \quad B = 5$



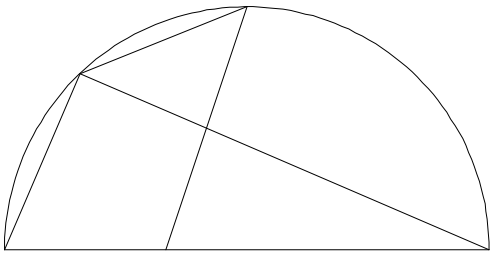




$AH_\Delta := \sqrt{(AD_\Delta)^2 + (DH_\Delta)^2} \quad AM := AC \quad AK := \dfrac{AD_\Delta \cdot AM}{AH_\Delta}$

$AK = 1.6986 \quad AK^3 = 4.9008 \quad X \cdot Y \cdot Z = 4.9005$

$(AB \cdot AE^2)^{\frac{1}{3}} = 2.9142$

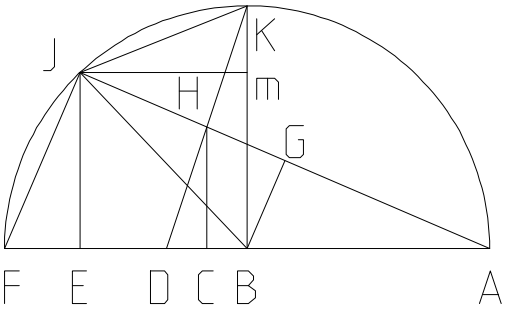
$AD_\Delta = 2.9143$



POR Series IV Figure 11_24_C3.MCD

Generalize the work of 07_25_93 to include the variable base ratio. This particular modification of the file has seen the use of the symbolic processor to figure out the last ratio that I have been unable to putz. The two ratio's found are FJ/JK and AH/AJ. I put the first in the graphic at left, the other would not fit.

$$\frac{FJ_\delta}{JK_\delta} = \frac{\sqrt{2}\cdot BR\cdot \delta}{(BS-\delta)\cdot(BR-1)\cdot 2}$$




$AF := 1 \quad BR := BASE\_RATIO$

$BS := BISECTOR\_SEGMENTS$

$DF := \frac{AF}{BR} \quad AD := AF - DF \quad \delta := 1..BS$

$AB := \frac{AF}{2} \quad BF := AB \quad BD := BF - DF$

$BK := AB \quad DK := \sqrt{BD^2 + BK^2}$

$DH_\delta := \frac{DK}{BS}\cdot\delta \quad CD_\delta := \frac{BD}{BS}\cdot\delta \quad CH_\delta := \sqrt{(DH_\delta)^2 - (CD_\delta)^2} \quad BC_\delta := BD - CD_\delta$

$AC_\delta := AB + BC_\delta \quad AH_\delta := \sqrt{(AC_\delta)^2 + (CH_\delta)^2} \quad AG_\delta := \frac{AC_\delta\cdot AB}{AH_\delta} \quad AJ_\delta := AG_\delta\cdot 2$

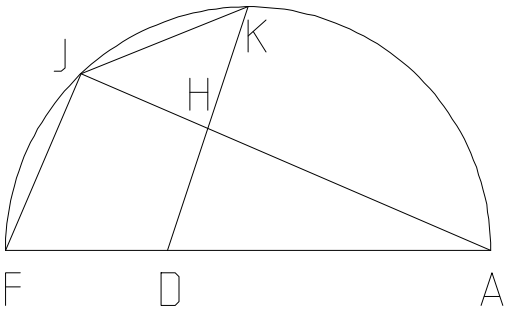
$AE_\delta := \frac{(AJ_\delta)^2}{AF} \quad EF_\delta := AF - AE_\delta \quad EJ_\delta := \sqrt{EF_\delta\cdot AE_\delta} \quad FJ_\delta := \sqrt{(EF_\delta)^2 + (EJ_\delta)^2}$

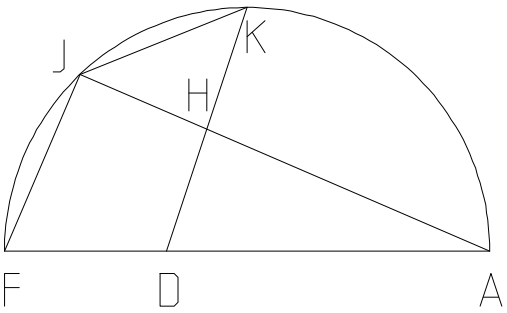
$Bm_\delta := EJ_\delta \quad BE_\delta := AE_\delta - AB$

$Jm_\delta := BE_\delta$

$Km_\delta := BK - Bm_\delta$

$JK_\delta := \sqrt{(Jm_\delta)^2 + (Km_\delta)^2}$






$$\text{if}\left[BS - \delta, \frac{\frac{\delta\cdot\sqrt{2}}{BS - \delta}}{\frac{FJ_\delta}{JK_\delta}}, 0\right] \qquad \frac{BR - 2}{BR} + 1 = 1.333$$

| |
|---|
| 1.333 |
| 1.333 |
| 1.333 |
| 1.333 |
| 0 |

$$\text{if}\left(JK_\delta, \frac{FJ_\delta}{JK_\delta}, 0\right)$$

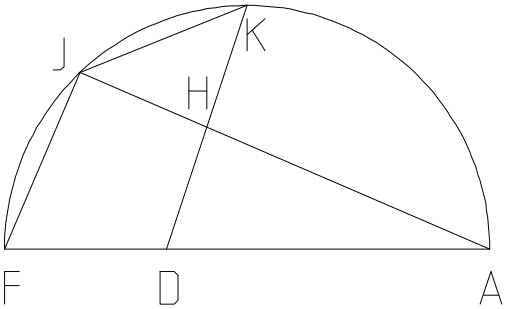
| |
|---|
| 0.265 |
| 0.707 |
| 1.591 |
| 4.243 |
| $6.369\cdot10^{15}$ |

$$\text{if}\left(BS - \delta, \frac{\delta\cdot\sqrt{2}}{BS - \delta}, 0\right)$$

| |
|---|
| 0.354 |
| 0.943 |
| 2.121 |
| 5.657 |
| 0 |

$$\text{if}\left[BS - \delta, \frac{\sqrt{2}\cdot BR\cdot\delta}{(BS - \delta)\cdot(BR - 1)\cdot 2}, 0\right]$$

| |
|---|
| 0.265 |
| 0.707 |
| 1.591 |
| 4.243 |
| 0 |




I broke down and used the symbolic processor.
Two versions of the formula are given. It is apparent why I could not get this one myself, I'd never 'ave thunk it. From this point on I will use the symbolic processor to find formula's more often.

$$\frac{\left(1-\frac{1}{BR}-\frac{\frac{1}{2}-\frac{1}{BR}}{BS}\cdot\delta\right)^2+\frac{\left(\frac{1}{2}-\frac{1}{BR}\right)^2+\frac{1}{4}}{BS^2}\cdot\delta^2-\frac{\left(\frac{1}{2}-\frac{1}{BR}\right)^2}{BS^2}\cdot\delta^2}{1-\frac{1}{BR}-\frac{\frac{1}{2}-\frac{1}{BR}}{BS}\cdot\delta}$$

$\frac{AH_\delta}{AJ_\delta}$

| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |

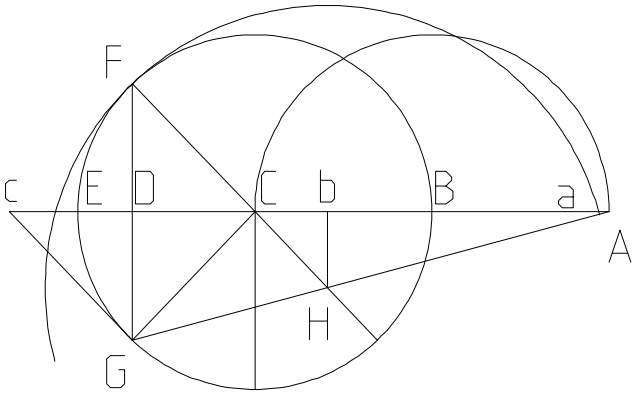
| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |

$$\frac{-2\cdot BR^2\cdot BS^2+4\cdot BR\cdot BS^2+2\cdot BR^2\cdot BS\cdot\delta\ \ldots}{+-6\cdot BR\cdot BS\cdot\delta-2\cdot BS^2+4\cdot BS\cdot\delta-\delta^2\cdot BR^2+2\cdot\delta^2\cdot BR-2\cdot\delta^2}$$
$$BR\cdot(BS\cdot(-2\cdot BR\cdot BS+2\cdot BS+\delta\cdot BR-2\cdot\delta))$$

BASE_RATIO ≡ 3

BISECTOR_SEGMENTS ≡ 5

| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |




11_25_93.MCD

Use iteration to trisect any given angle.

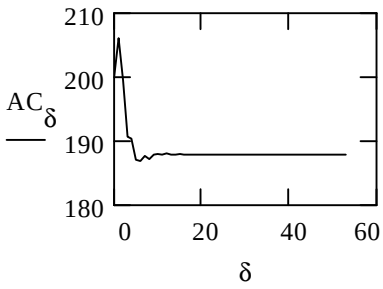
$BE := 200 \quad BC := \frac{BE}{2} \quad CE := BC$

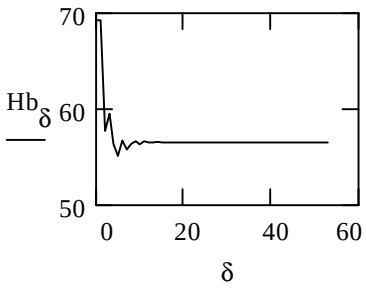
$CD := if(CDI > 0, if(CDI < BC, CDI, 100), 0)$

$CF := CE \quad BD := BC + CD \quad DF := \sqrt{CF^2 - CD^2}$

$DG := DF \quad Gc := CF \quad Dc := CD \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} Ac_0 \\ CH_0 \\ FH_0 \\ AC_0 \\ bc_0 \\ Hb_0 \end{bmatrix} := \begin{bmatrix} BE + Dc + CD \\ \frac{Gc \cdot BE}{BE + Dc} \\ CF + \frac{Gc \cdot BE}{BE + Dc} \\ BE \\ \frac{Dc \cdot BE}{BE + Dc} \\ \sqrt{\left(\frac{Gc \cdot BE}{BE + Dc}\right)^2 - \left(\frac{Dc \cdot BE}{BE + Dc}\right)^2} \end{bmatrix} \begin{bmatrix} AC_{\delta+1} \\ Ac_{\delta+1} \\ CH_{\delta+1} \\ FH_{\delta+1} \\ bc_{\delta+1} \\ Hb_{\delta+1} \end{bmatrix} := \begin{bmatrix} \sqrt{(FH_\delta)^2 - (Hb_\delta)^2} + bc_\delta \\ AC_\delta + Dc + CD \\ \frac{CF \cdot AC_\delta}{Ac_\delta} \\ CF + CH_\delta \\ \frac{Dc \cdot AC_\delta}{Ac_\delta} \\ \sqrt{(CH_\delta)^2 - (bc_\delta)^2} \end{bmatrix}$$



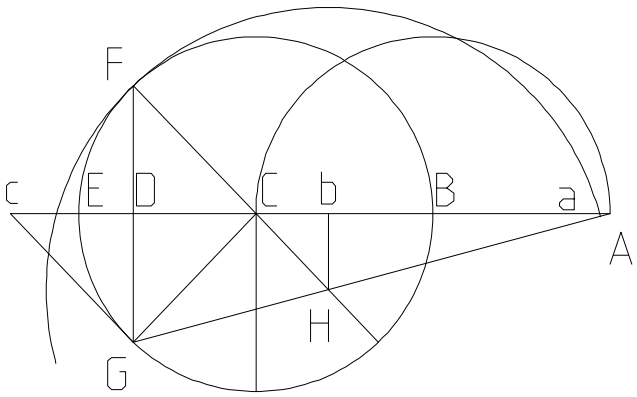

$$\angle KCD := \frac{\operatorname{asin}\left(\frac{DF}{CF}\right)}{deg}$$

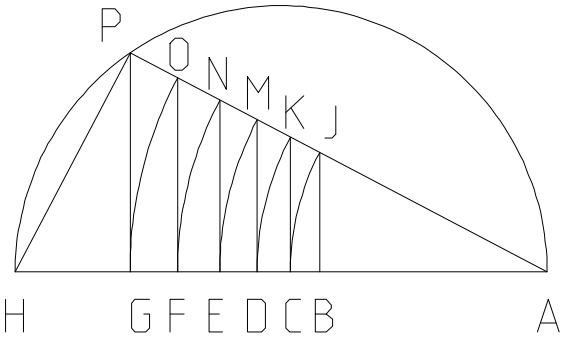
$$\angle BAI := \frac{\operatorname{asin}\left(\frac{Hb_\Delta}{FH_\Delta}\right)}{deg}$$

$\Delta \equiv 53$ $\angle KCD = 60$ $\frac{\angle KCD}{\angle BAI} = 3$

$CDI \equiv 50$ $\angle BAI = 20$

Range: >0 and <100






# 12_02_93.MCD POR Roots and Powers
## (Pyramid of Ratio Series V)

Is the progression noticed in 11_29_93 a continuous phenomenon?

$AH := 17 \qquad \delta := 1 .. AH \qquad AP_\delta := \frac{AH}{\delta}$

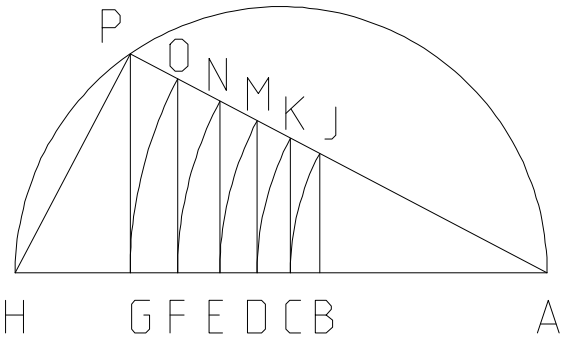
$AG_\delta := \frac{(AP_\delta)^2}{AH} \qquad AO_\delta := AG_\delta$

$AF_\delta := \frac{(AG_\delta)^2}{AP_\delta} \qquad AE_\delta := \frac{(AF_\delta)^2}{AO_\delta} \qquad AN_\delta := AF_\delta \qquad AD_\delta := \frac{(AE_\delta)^2}{AN_\delta} \qquad AM_\delta := AE_\delta$

$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta} \qquad AK_\delta := AD_\delta \qquad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta} \qquad A := 1 \qquad \Delta := A .. 5$

| $\frac{AH}{AP_\Delta}$ | $\left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}}$ | $\left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}}$ | $\left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |

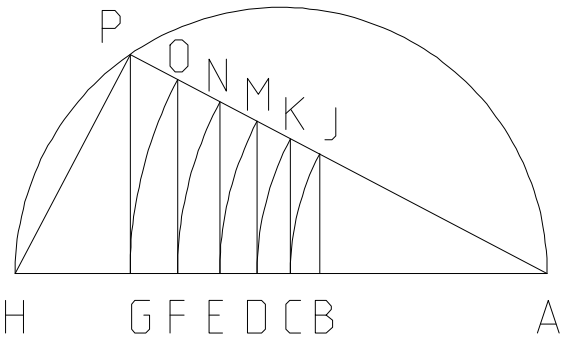
| $\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}}$ | $\left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}}$ | $\frac{AC_\Delta}{AB_\Delta}$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 |



$$\begin{bmatrix} \frac{AH}{1}, \frac{AH}{1^2}, \frac{AH}{1^3}, \frac{AH}{1^4}, \text{etc} \\ \frac{AH}{2}, \frac{AH}{2^2}, \frac{AH}{2^3}, \frac{AH}{2^4}, \text{etc} \\ \frac{AH}{3}, \frac{AH}{3^2}, \frac{AH}{3^3}, \frac{AH}{3^4}, \text{etc} \\ \frac{AH}{4}, \frac{AH}{4^2}, \frac{AH}{4^3}, \frac{AH}{4^4}, \text{etc} \\ \frac{AH}{5}, \frac{AH}{5^2}, \frac{AH}{5^3}, \frac{AH}{5^4}, \text{etc} \end{bmatrix}$$

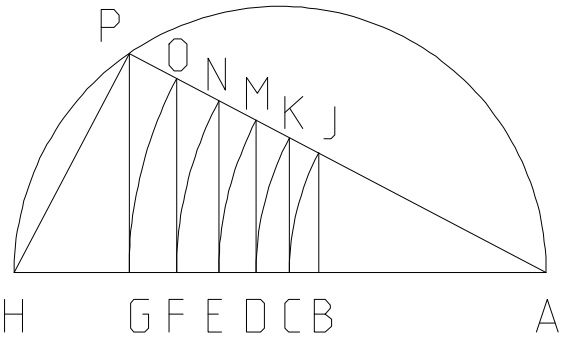
| $AP_\Delta$ | $AG_\Delta$ | $AF_\Delta$ | $AE_\Delta$ | $AD_\Delta$ | $AC_\Delta$ | $AB_\Delta$ |
|---|---|---|---|---|---|---|
| 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 8.5 | 4.25 | 2.125 | 1.063 | 0.531 | 0.266 | 0.133 |
| 5.667 | 1.889 | 0.63 | 0.21 | 0.07 | 0.023 | 0.008 |
| 4.25 | 1.063 | 0.266 | 0.066 | 0.017 | 0.004 | 0.001 |
| 3.4 | 0.68 | 0.136 | 0.027 | 0.005 | 0.001 | $2.176 \cdot 10^{-4}$ |

$$\frac{AH}{5^7} = 2.176 \cdot 10^{-4}$$




| $AP_\Delta$ | $AG_\Delta \cdot \frac{AH}{AP_\Delta}$ | $AF_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^2$ | $AE_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^3$ |
|---|---|---|---|
| 17 | 17 | 17 | 17 |
| 8.5 | 8.5 | 8.5 | 8.5 |
| 5.667 | 5.667 | 5.667 | 5.667 |
| 4.25 | 4.25 | 4.25 | 4.25 |
| 3.4 | 3.4 | 3.4 | 3.4 |

| $AD_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^4$ | $AC_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^5$ | $AB_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^6$ |
|---|---|---|
| 17 | 17 | 17 |
| 8.5 | 8.5 | 8.5 |
| 5.667 | 5.667 | 5.667 |
| 4.25 | 4.25 | 4.25 |
| 3.4 | 3.4 | 3.4 |




| $\frac{AH^2}{AP_\Delta}$ | $\left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}}$ | $\left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}}$ |
|---|---|---|---|---|---|
| 17 | 17 | 17 | 17 | 17 | 17 |
| 34 | 34 | 34 | 34 | 34 | 34 |
| 51 | 51 | 51 | 51 | 51 | 51 |
| 68 | 68 | 68 | 68 | 68 | 68 |
| 85 | 85 | 85 | 85 | 85 | 85 |

| $\left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}}$ | $\left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH$ |
|---|---|
| 17 | 17 |
| 34 | 34 |
| 51 | 51 |
| 68 | 68 |
| 85 | 85 |

$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}}$ $\left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}}$ $\left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}}$ $\left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}}$ $\left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}}$ $\left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}}$ $\left(\frac{AH}{AP_\Delta}\right)^{3}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 64 | 64 | 64 | 64 | 64 | 64 | 64 |
| 125 | 125 | 125 | 125 | 125 | 125 | 125 |

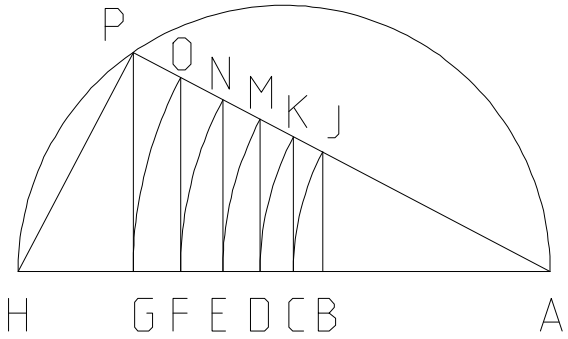
$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}}$ $\left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}}$ $\left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}}$ $\left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}}$ $\left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}}$ $\left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}}$ $\left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 128 | 128 | 128 | 128 | 128 | 128 | 128 |
| $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ |
| $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ |
| $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.812 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ |

$GH_\delta := AH - AG_\delta$ $GP_\delta := \sqrt{AG_\delta \cdot GH_\delta}$ $FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta}$ $EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta}$ $DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta}$

$CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta}$ $BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$

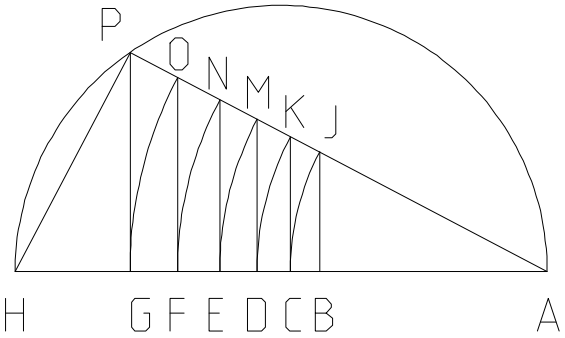
| $GP_\Delta$ | $FO_\Delta$ | $EN_\Delta$ | $DM_\Delta$ | $CK_\Delta$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 7.361 | 3.681 | 1.84 | 0.92 | 0.46 | 0.23 |
| 5.343 | 1.781 | 0.594 | 0.198 | 0.066 | 0.022 |
| 4.115 | 1.029 | 0.257 | 0.064 | 0.016 | 0.004 |
| 3.331 | 0.666 | 0.133 | 0.027 | 0.005 | 0.001 |




| $\frac{GP_\Delta}{\Delta^5}$ | $\frac{FO_\Delta}{\Delta^4}$ | $\frac{EN_\Delta}{\Delta^3}$ | $\frac{DM_\Delta}{\Delta^2}$ | $\frac{CK_\Delta}{\Delta}$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

| $\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5}$ | $\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4}$ | $\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3}$ | $\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2}$ | $\frac{CK_\Delta}{\frac{AH}{AP_\Delta}}$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

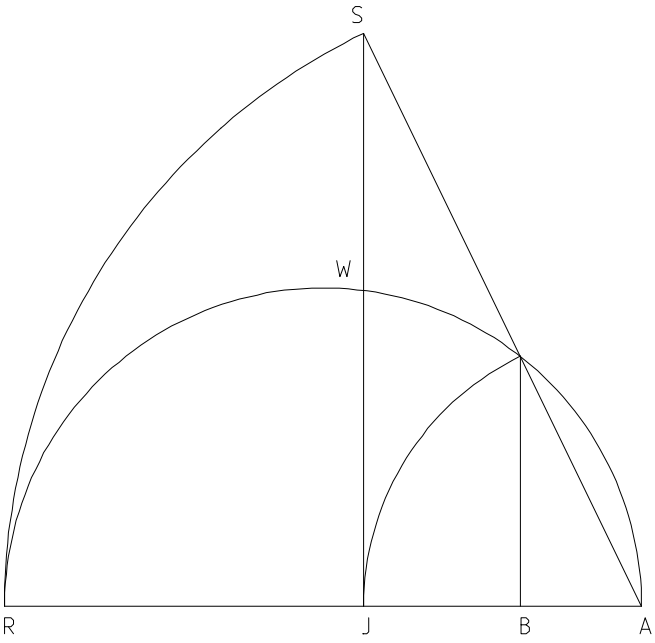
| $GP_\Delta$ | $FO_\Delta$ | $EN_\Delta$ | $DM_\Delta$ | $CK_\Delta$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 7.361 | 3.681 | 1.84 | 0.92 | 0.46 | 0.23 |
| 5.343 | 1.781 | 0.594 | 0.198 | 0.066 | 0.022 |
| 4.115 | 1.029 | 0.257 | 0.064 | 0.016 | 0.004 |
| 3.331 | 0.666 | 0.133 | 0.027 | 0.005 | 0.001 |




$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = 3.681 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = 1.84 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = 0.92$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = 1.781 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = 0.594 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = 0.198$$

# Two Prime Exponential Series



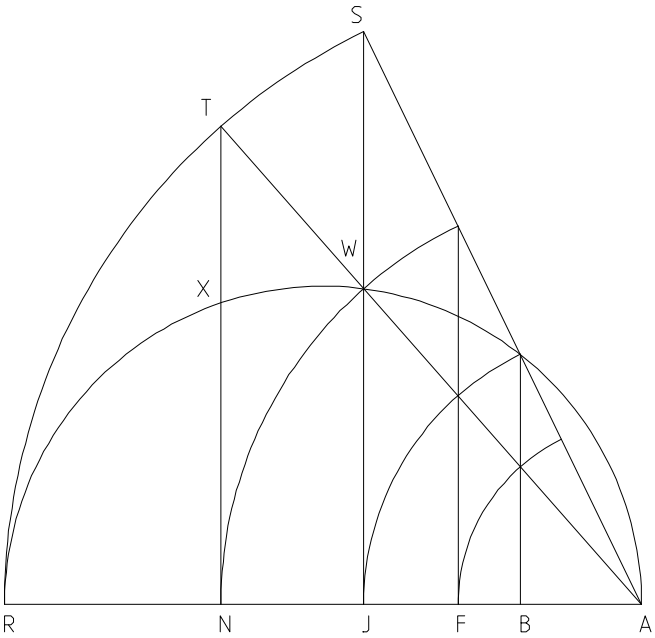

$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ | $AJ_\delta$ |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

I think that one is in the position that they are in when dividing an angle, twice never gets to thrice, at least that is what I decided some time ago. Let me double check anyway. An obvious pattern should emerge.

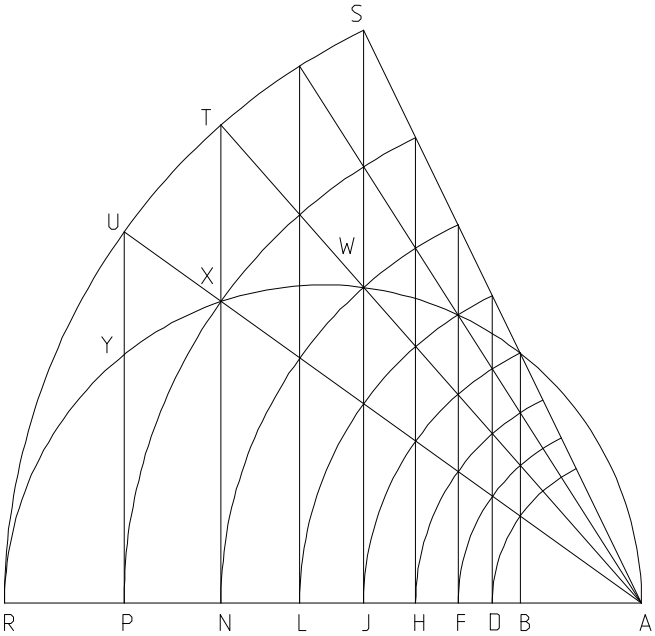



$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ | $AF_\delta$ | $AJ_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |



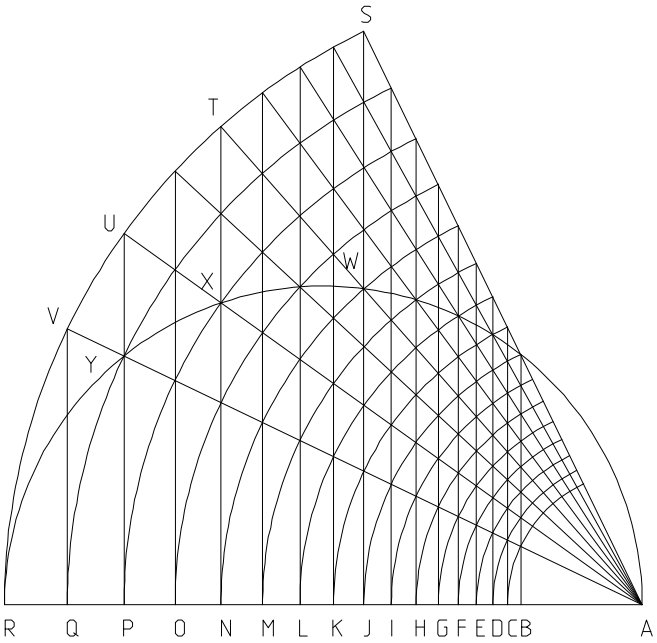

$$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$$

$$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$$

$$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$$

$$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$$

| $AB_\delta$ | $AD_\delta$ | $AF_\delta$ | $AH_\delta$ | $AJ_\delta$ | $AL_\delta$ | $AN_\delta$ | $AP_\delta$ |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |




$$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$$

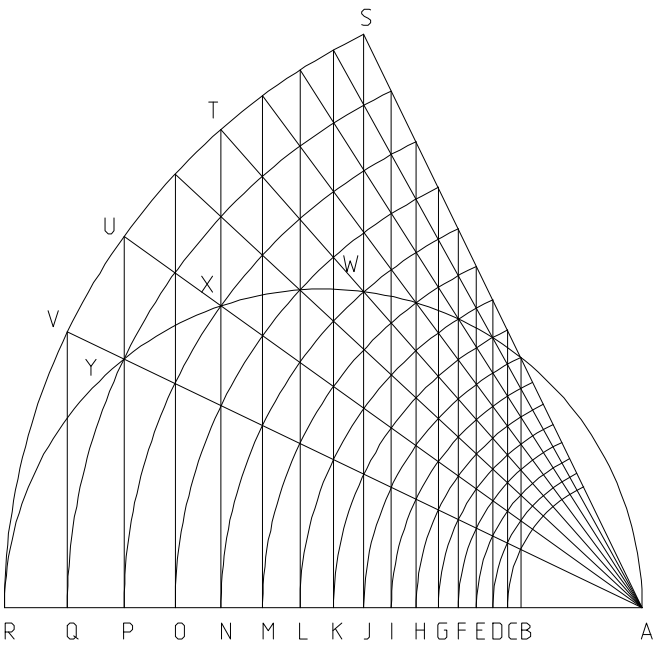
$$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$$

$$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$$

| $AB_\delta$ | $AC_\delta$ | $AD_\delta$ | $AE_\delta$ | $AF_\delta$ |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |

| $AG_\delta$ | $AH_\delta$ | $AI_\delta$ | $AJ_\delta$ | $AK_\delta$ | $AL_\delta$ | $AM_\delta$ | $AN_\delta$ | $AO_\delta$ | $AP_\delta$ | $AQ_\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |




$\frac{AR}{AB_\delta}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AC_\delta}\right)^{\frac{16}{15}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AD_\delta}\right)^{\frac{16}{14}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AE_\delta}\right)^{\frac{16}{13}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AF_\delta}\right)^{\frac{16}{12}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AG_\delta}\right)^{\frac{16}{11}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AH_\delta}\right)^{\frac{16}{10}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AI_\delta}\right)^{\frac{16}{9}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AJ_\delta}\right)^{\frac{16}{8}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AK_\delta}\right)^{\frac{16}{7}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AL_\delta}\right)^{\frac{16}{6}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AM_\delta}\right)^{\frac{16}{5}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AN_\delta}\right)^{\frac{16}{4}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AO_\delta}\right)^{\frac{16}{3}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AP_\delta}\right)^{\frac{16}{2}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AQ_\delta}\right)^{\frac{16}{1}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AQ_\delta}\right)^{8}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AP_\delta}\right)^{\frac{8}{2}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AO_\delta}\right)^{\frac{8}{3}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AN_\delta}\right)^{\frac{8}{4}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AM_\delta}\right)^{\frac{8}{5}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AL_\delta}\right)^{\frac{8}{6}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AK_\delta}\right)^{\frac{8}{7}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AJ_\delta}\right)$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AI_\delta}\right)^{\frac{8}{9}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AH_\delta}\right)^{\frac{8}{10}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AG_\delta}\right)^{\frac{8}{11}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AF_\delta}\right)^{\frac{8}{12}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AE_\delta}\right)^{\frac{8}{13}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AD_\delta}\right)^{\frac{8}{14}}$

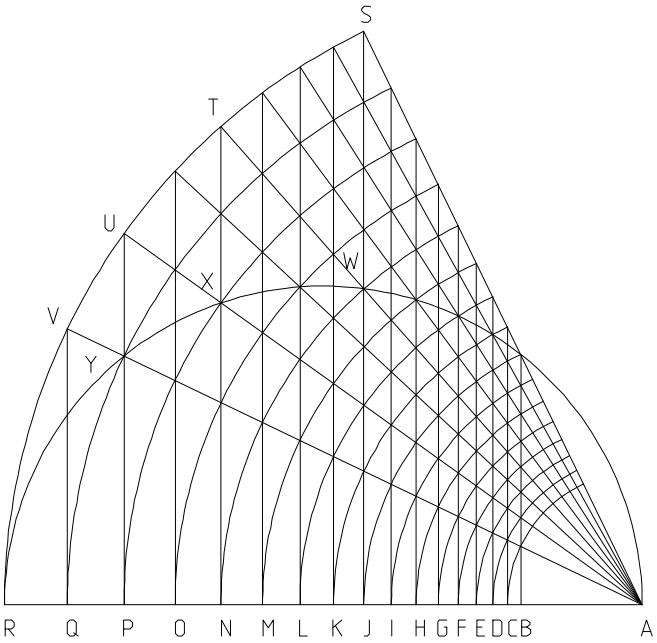
| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AC_\delta}\right)^{\frac{8}{15}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AB_\delta}\right)^{\frac{8}{16}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |




$$\left(AB_{\delta}\cdot AR\right)^{\frac{1}{2}} - AJ_{\delta} \qquad \left[\left(AB_{\delta}\right)^{3}\cdot AR\right]^{\frac{1}{4}} - AF_{\delta}$$

$$\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$\left[\left(AB_{\delta}\right)^{7}\cdot AR\right]^{\frac{1}{8}} - AD_{\delta} \qquad \left[\left(AB_{\delta}\right)^{15}\cdot AR\right]^{\frac{1}{16}} - AC_{\delta}$$

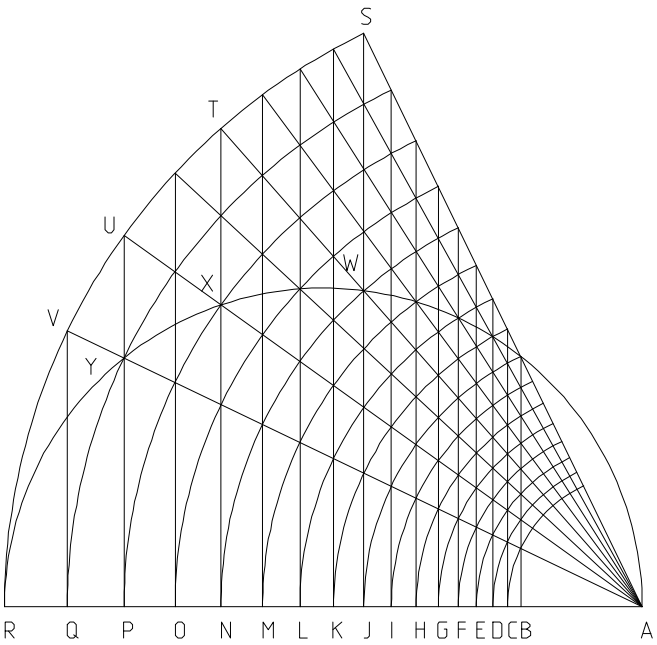
$$\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$\left(AB_{\delta}\cdot AR^{3}\right)^{\frac{1}{4}} - AN_{\delta} \qquad \left(AB_{\delta}\cdot AR^{7}\right)^{\frac{1}{8}} - AP_{\delta} \qquad \left(AB_{\delta}\cdot AR^{15}\right)^{\frac{1}{16}} - AQ_{\delta}$$

$$\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

$$\left[\left(AC_{\delta}\right)^{2}\cdot AR\right]^{\frac{1}{3}} - AH_{\delta} \qquad \left[\left(AC_{\delta}\right)^{4}\cdot AR\right]^{\frac{1}{5}} - AF_{\delta} \qquad \left[\left(AC_{\delta}\right)^{8}\cdot AR\right]^{\frac{1}{9}}$$

$$\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}5.612\\4.003\\3.15\\2.615\\2.247\end{bmatrix}$$




| $AQ_\delta$ | $AP_\delta$ | $AO_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 9.576 | 9.17 | 8.781 | 8.409 |
| 9.336 | 8.717 | 8.138 | 7.598 |
| 9.17 | 8.409 | 7.711 | 7.071 |
| 9.043 | 8.178 | 7.395 | 6.687 |
| 8.941 | 7.993 | 7.147 | 6.389 |

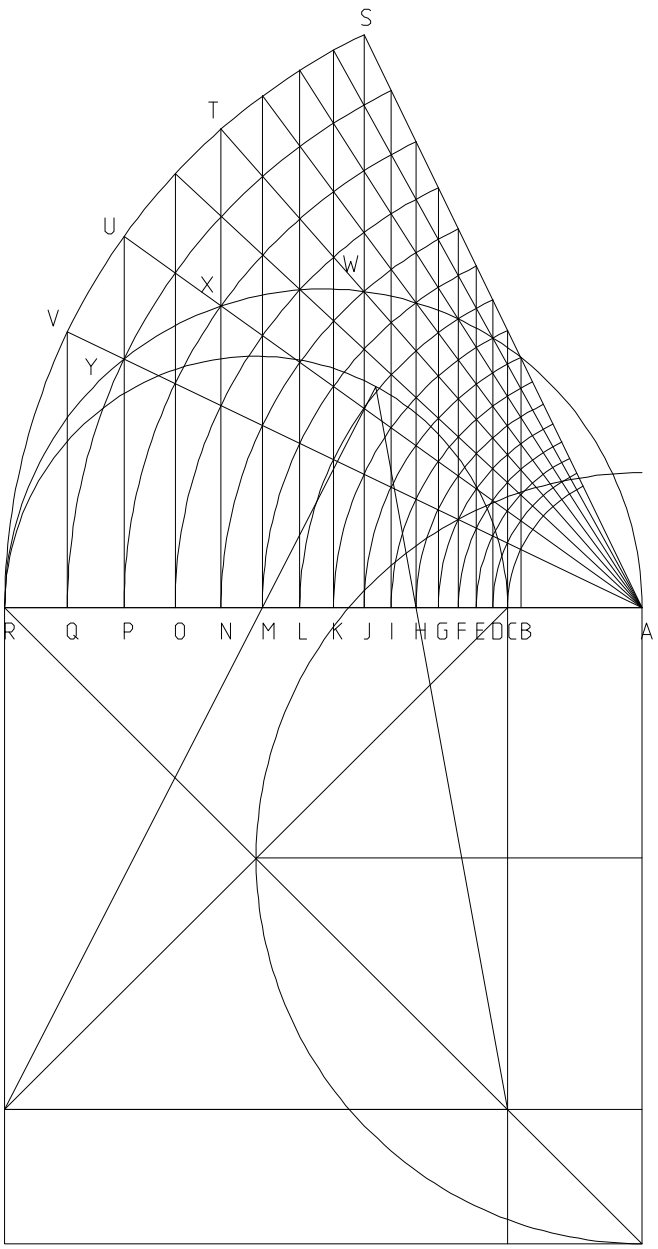
| $AM_\delta$ | $AL_\delta$ | $AK_\delta$ | $AJ_\delta$ |
|---|---|---|---|
| 8.052 | 7.711 | 7.384 | 7.071 |
| 7.094 | 6.623 | 6.184 | 5.774 |
| 6.484 | 5.946 | 5.453 | 5 |
| 6.047 | 5.469 | 4.945 | 4.472 |
| 5.713 | 5.107 | 4.566 | 4.082 |

| $AI_\delta$ | $AH_\delta$ | $AG_\delta$ | $AF_\delta$ | $AE_\delta$ | $AD_\delta$ | $AC_\delta$ | $AB_\delta$ |
|---|---|---|---|---|---|---|---|
| 6.771 | 6.484 | 6.209 | 5.946 | 5.694 | 5.453 | 5.221 | 5 |
| 5.39 | 5.033 | 4.699 | 4.387 | 4.096 | 3.824 | 3.57 | 3.333 |
| 4.585 | 4.204 | 3.856 | 3.536 | 3.242 | 2.973 | 2.726 | 2.5 |
| 4.044 | 3.657 | 3.307 | 2.991 | 2.704 | 2.446 | 2.212 | 2 |
| 3.65 | 3.263 | 2.918 | 2.608 | 2.332 | 2.085 | 1.864 | 1.667 |

| $\left[\left(AB_\delta\right)\cdot AR^2\right]^{\frac{1}{3}}$ | $\left(AC_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AD_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AE_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AF_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AG_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|---|---|
| 7.937 | 8.052 | 8.17 | 8.288 | 8.409 | 8.531 |
| 6.934 | 7.094 | 7.258 | 7.426 | 7.598 | 7.774 |
| 6.3 | 6.484 | 6.674 | 6.87 | 7.071 | 7.278 |
| 5.848 | 6.047 | 6.254 | 6.467 | 6.687 | 6.915 |
| 5.503 | 5.713 | 5.93 | 6.155 | 6.389 | 6.632 |

| $\left(AH_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AI_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AJ_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AK_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AL_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AM_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|---|---|
| 8.655 | 8.781 | 8.909 | 9.039 | 9.17 | 9.303 |
| 7.954 | 8.138 | 8.327 | 8.52 | 8.717 | 8.919 |
| 7.492 | 7.711 | 7.937 | 8.17 | 8.409 | 8.655 |
| 7.151 | 7.395 | 7.647 | 7.908 | 8.178 | 8.456 |
| 6.885 | 7.147 | 7.418 | 7.701 | 7.993 | 8.297 |

| $\left(AN_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AO_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AP_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AQ_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|
| 9.439 | 9.576 | 9.715 | 9.857 |
| 9.125 | 9.336 | 9.553 | 9.774 |
| 8.909 | 9.17 | 9.439 | 9.715 |
| 8.745 | 9.043 | 9.351 | 9.67 |
| 8.613 | 8.941 | 9.281 | 9.634 |




$\left[\left(AC_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$\left(AC_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$AH_\delta$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$AM_\delta$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$\left[\left(AF_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$\left(AF_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$AJ_\delta$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$AN_\delta$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$\frac{16}{3} = 5.333$

There are five found cubics between the givens.

$\left[\left(AI_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$\left(AI_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$\left[\left(AL_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$\left(AL_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$\left[\left(AO_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$\left(AO_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$AL_\delta$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$AO_\delta$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$AN_\delta$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$AP_\delta$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$AP_\delta$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$AQ_\delta$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$S1_\delta := \sqrt{AH_\delta \cdot AM_\delta}$ $S2_\delta := \sqrt{AJ_\delta \cdot AN_\delta}$ $S3_\delta := \sqrt{AL_\delta \cdot AO_\delta}$ $S4_\delta := \sqrt{AN_\delta \cdot AP_\delta}$ $S5_\delta := \sqrt{AP_\delta \cdot AQ_\delta}$

| $AJ_\delta$ |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

| $S1_\delta$ |
|---|
| 7.226 |
| 5.975 |
| 5.221 |
| 4.703 |
| 4.318 |

| $S2_\delta$ | $AL_\delta$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

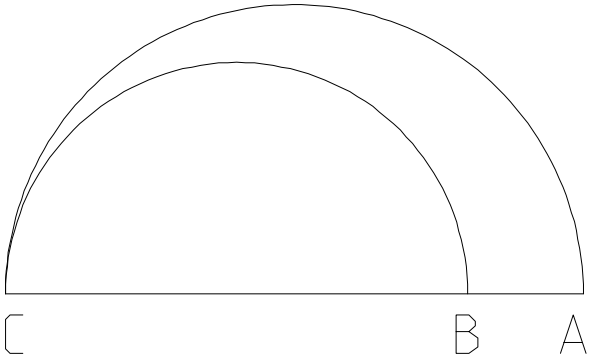
| $S3_\delta$ |
|---|
| 8.229 |
| 7.342 |
| 6.771 |
| 6.359 |
| 6.042 |

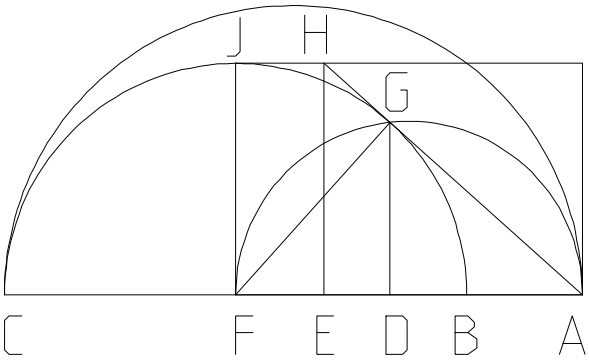
| $S4_\delta$ | $AO_\delta$ |
|---|---|
| 8.781 | 8.781 |
| 8.138 | 8.138 |
| 7.711 | 7.711 |
| 7.395 | 7.395 |
| 7.147 | 7.147 |

| $S5_\delta$ |
|---|
| 9.371 |
| 9.021 |
| 8.781 |
| 8.599 |
| 8.454 |

12_06_93

# Alternate methods; Square Root .




Demonstrate that the square root of two segments can never be more than one half of the difference between them

Let AB and AC be the two segments and BC the difference between them.

$BC := 10 \quad \delta := 1..100 \quad BF := \frac{BC}{2}$

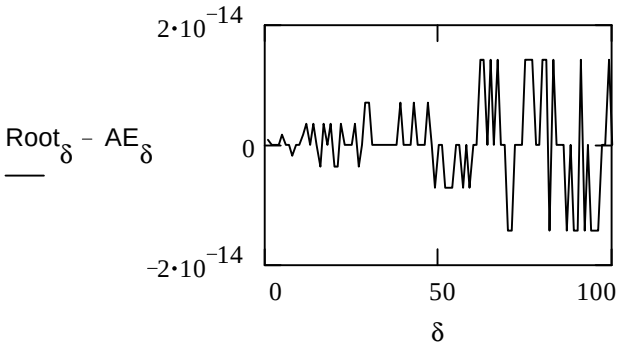
$AB_\delta := \delta \quad AF_\delta := AB_\delta + BF \quad FG := BF$

$DF_\delta := \frac{FG^2}{AF_\delta} \quad FJ := BF \quad AD_\delta := AF_\delta - DF_\delta$

$DG_\delta := \sqrt{AD_\delta \cdot DF_\delta} \quad EH := BF$

$Root_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

$AE_\delta := \frac{AD_\delta \cdot EH}{DG_\delta} \quad AC_\delta := AB_\delta + BC$

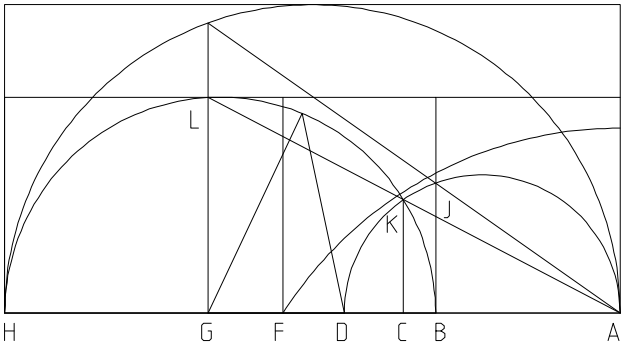



$$\sum_\delta \left(Root_\delta - AE_\delta\right) = 0$$

Since AG can never exceed AF, the result is obvious.

Loging the constant relationship K and J on the cubic.

# Gruntwork 4 on the Delian Solution



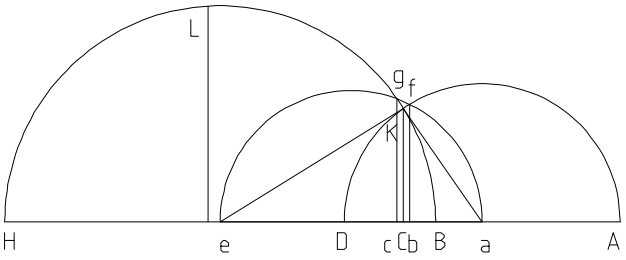

$BH := 10 \quad \delta := 1 .. 1000 \quad AB_\delta := \delta$

$AH_\delta := BH + AB_\delta \quad AF_\delta := \sqrt{AB_\delta \cdot AH_\delta}$

$AG_\delta := \left[ \left(AH_\delta\right)^2 \cdot AB_\delta \right]^{\frac{1}{3}}$

$AD_\delta := \left[ AH_\delta \cdot \left(AB_\delta\right)^2 \right]^{\frac{1}{3}}$

$BG_\delta := AG_\delta - AB_\delta \quad GH_\delta := BH - BG_\delta \quad GL_\delta := \sqrt{GH_\delta \cdot BG_\delta}$



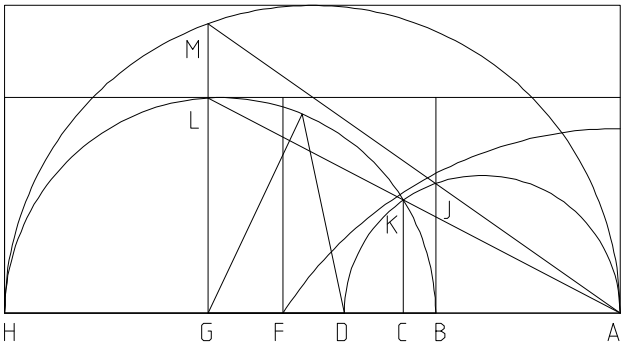

$Aa_\delta := \frac{AD_\delta}{2} \quad Be := \frac{BH}{2} \quad Ka_\delta := Aa_\delta$

$Ke := Be \quad eg := Ke \quad af_\delta := Ka_\delta$

$Ae_\delta := AB_\delta + Be \quad ae_\delta := Ae_\delta - Aa_\delta$

$ce_\delta := \frac{eg^2}{ae_\delta} \quad ab_\delta := \frac{\left(af_\delta\right)^2}{ae_\delta} \quad ac_\delta := ae_\delta - ce_\delta \quad bc_\delta := ac_\delta - ab_\delta \quad Cb_\delta := \frac{bc_\delta}{2}$

$Ca_\delta := ab_\delta + Cb_\delta \quad AC_\delta := Aa_\delta + Ca_\delta \quad Ce_\delta := ae_\delta - Ca_\delta \quad CK_\delta := \sqrt{Ke^2 - \left(Ce_\delta\right)^2}$




$BD_\delta := AD_\delta - AB_\delta$

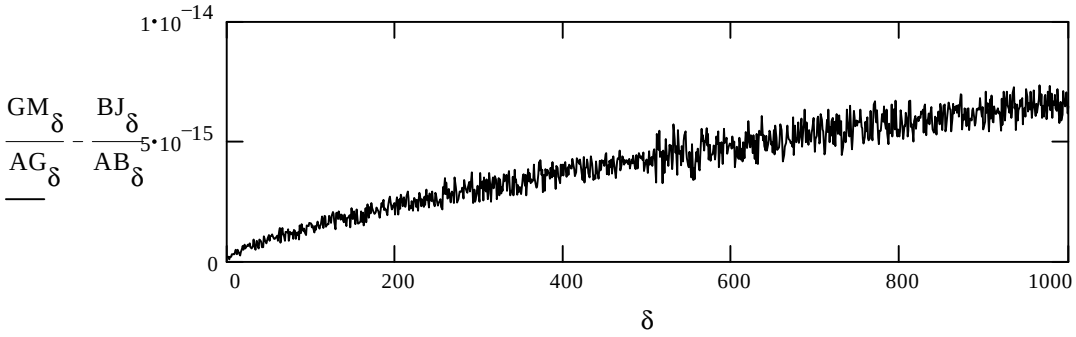
$BJ_\delta := \sqrt{BD_\delta \cdot AB_\delta}$ $GM_\delta := \sqrt{AG_\delta \cdot GH_\delta}$

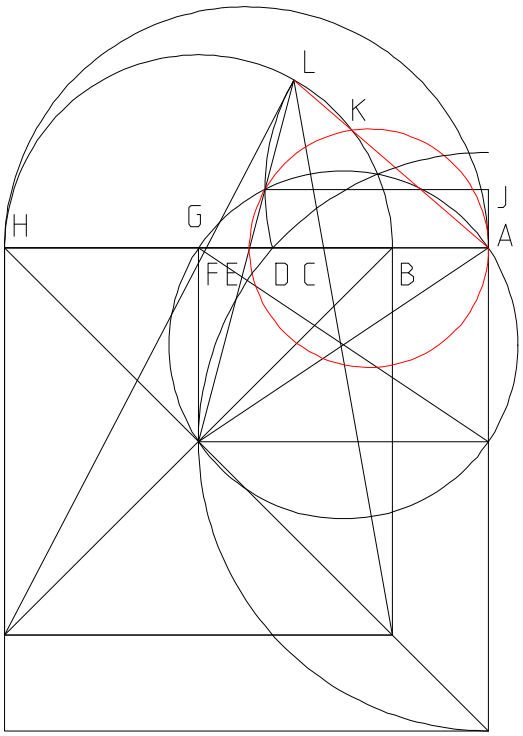
Is K on CK colinear with AL?

$$\sum_\delta \left( \frac{GL_\delta}{AG_\delta} - \frac{CK_\delta}{AC_\delta} \right) = 2.35 \cdot 10^{-13}$$

Is J on BJ colinear with AM?

$$\sum_\delta \left( \frac{GM_\delta}{AG_\delta} - \frac{BJ_\delta}{AB_\delta} \right) = 4.063 \cdot 10^{-12}$$






The structure in red appears to be a constant.

$BH := 5000 \quad \delta := S..F \quad AB_\delta := \delta^5 \cdot 10^{-8}$

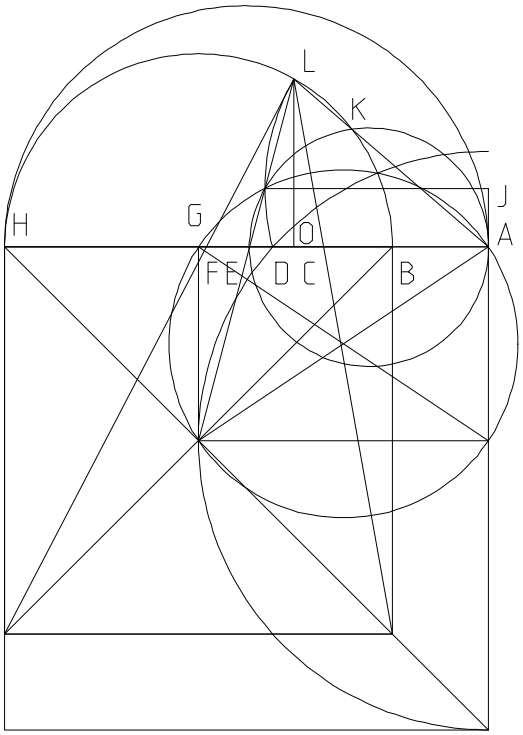
$BG := \frac{BH}{2} \quad AH_\delta := BH + AB_\delta$

$AC_\delta := \left[ \left( AB_\delta \right)^2 \cdot AH_\delta \right]^{\frac{1}{3}}$

$AF_\delta := \left[ AB_\delta \cdot \left( AH_\delta \right)^2 \right]^{\frac{1}{3}}$

$AJ_\delta := \frac{AF_\delta - AC_\delta}{2} \quad BC_\delta := AC_\delta - AB_\delta$

$AD_\delta := \sqrt{AB_\delta \cdot AH_\delta} \quad BF_\delta := AF_\delta - AB_\delta$



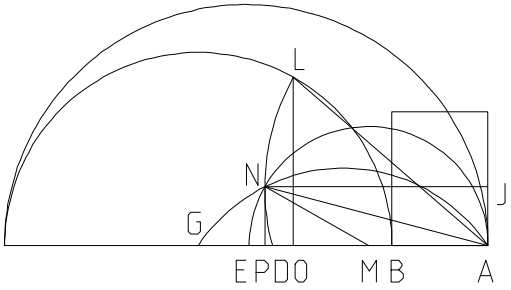

$CF_\delta := AF_\delta - AC_\delta \quad FH_\delta := AH_\delta - AF_\delta$

$CO_\delta := \frac{BC_\delta \cdot CF_\delta}{FH_\delta + BC_\delta} \quad BO_\delta := BC_\delta + CO_\delta$

$GO_\delta := BG - BO_\delta \quad GL := BG$

$LO_\delta := \sqrt{GL^2 - \left( GO_\delta \right)^2}$

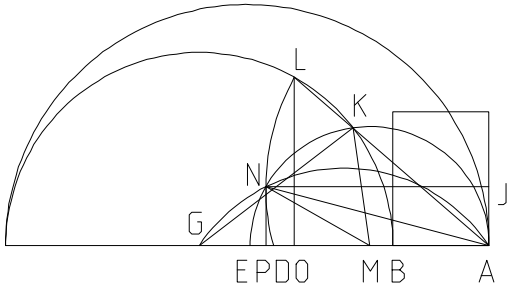
$AO_\delta := AB_\delta + BO_\delta$




$$JD_\delta := \sqrt{(AD_\delta)^2 + (AJ_\delta)^2} \quad JN_\delta := JD_\delta$$

$$NP_\delta := AJ_\delta \quad AN_\delta := \sqrt{(AJ_\delta)^2 + (JN_\delta)^2}$$

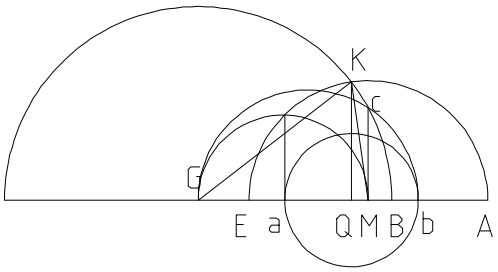
$$AP_\delta := JN_\delta \quad AE_\delta := \frac{(AN_\delta)^2}{AP_\delta}$$




$$AM_\delta := \frac{AE_\delta}{2} \quad MK_\delta := AM_\delta \quad GK := BG$$

$$AG_\delta := AB_\delta + BG$$

$$GM_\delta := AG_\delta - AM_\delta \quad Ma_\delta := \frac{(MK_\delta)^2}{GM_\delta}$$

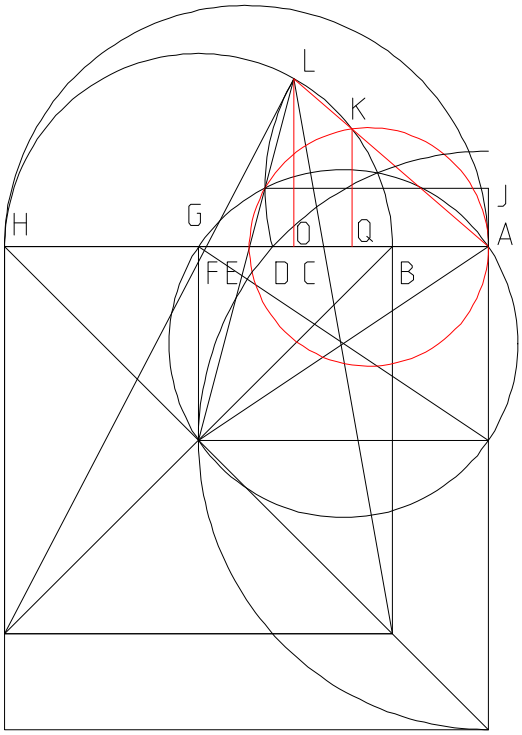



$$Gb_\delta := \frac{(GK)^2}{GM_\delta} \quad Ga_\delta := GM_\delta - Ma_\delta$$

$$ab_\delta := Gb_\delta - Ga_\delta \quad Qa_\delta := \frac{ab_\delta}{2}$$

$$GQ_\delta := Ga_\delta + Qa_\delta \quad AQ_\delta := AG_\delta - GQ_\delta$$

$$KQ_\delta := \sqrt{GK^2 - (GQ_\delta)^2}$$




$\left(AB_S\right)^2 \cdot AH_S = 4.886 \cdot 10^4$ $BO_S = 36.27$

$AB_S \cdot \left(AH_S\right)^2 = 7.822 \cdot 10^7$ $BO_F = 109.49532$

$\left(AB_F\right)^2 \cdot AH_F = 1.417 \cdot 10^6$ $AB_S = 3.125$

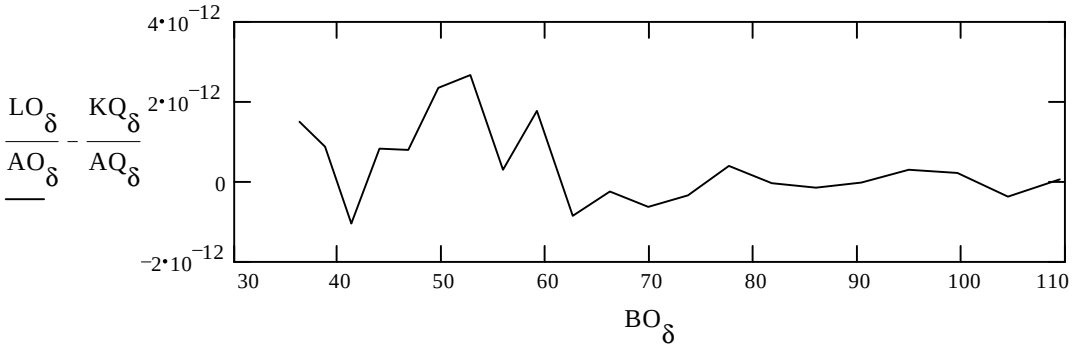
$AB_F \cdot \left(AH_F\right)^2 = 4.23 \cdot 10^8$ $AB_F = 16.807$

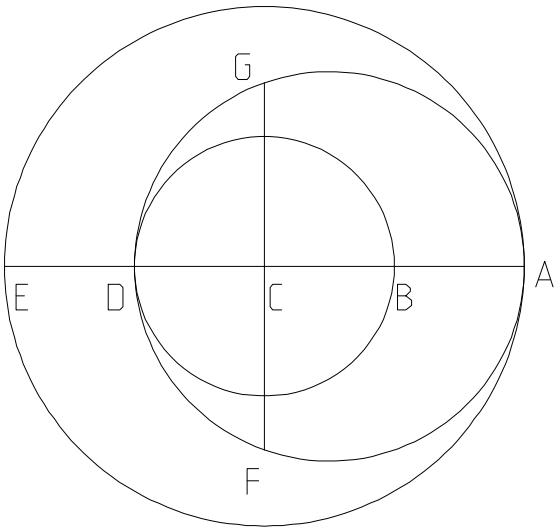
$LO_S = 424.30457$ $KQ_S = 36.53305$

$LO_F = 731.77$ $KQ_F = 111.891$

$$\sum_{\delta}\left(\frac{LO_\delta}{AO_\delta} - \frac{KQ_\delta}{AQ_\delta}\right) = 8.477 \cdot 10^{-12}$$

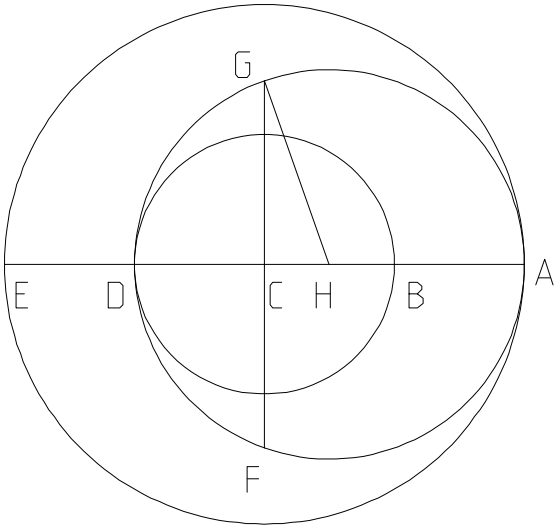
$S \equiv 50$ $F \equiv 70$






# The Square Root

It may be noticed that I use the adjacent figure in my work for doing square roots. I believe that it is the primary figure for doing square roots. Given segment AE and segment BD, segment FG is their square root. It has a major advantage of being able to square larger figures on paper, not to mention makes something of the development of exponential series.




$AE := 100 \quad \delta := 1 .. AE \quad BD_\delta := \delta$

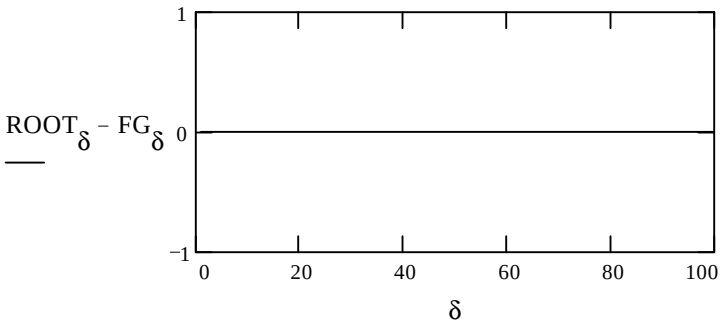
$AC := \frac{AE}{2} \quad BC_\delta := \frac{BD_\delta}{2} \quad AB_\delta := AC - BC_\delta$

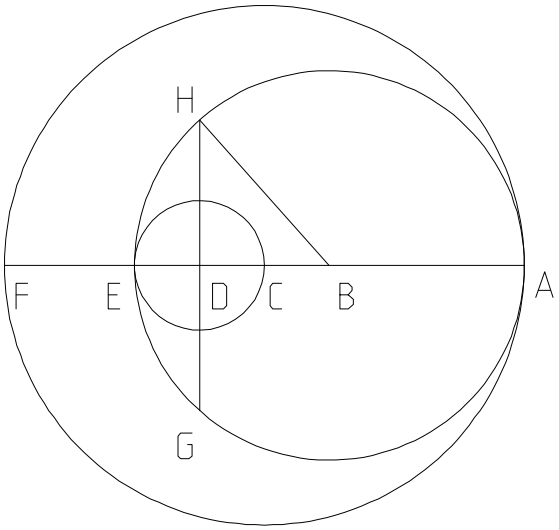
$AD_\delta := AC + BC_\delta \quad AH_\delta := \frac{AD_\delta}{2}$

$GH_\delta := AH_\delta \quad CH_\delta := AC - AH_\delta$

$FG_\delta := 2 \cdot \sqrt{\left(GH_\delta\right)^2 - \left(CH_\delta\right)^2}$

$ROOT_\delta := \sqrt{\left(BD_\delta\right) \cdot AE}$






12_12_B3.MCD

Completely generalize the square root figure.

$AF := 7 \qquad DF := \frac{AF}{BR} \qquad AD := AF - DF$

$\delta := 1 .. LBR \qquad DE_\delta := \frac{DF}{LBR} \cdot \delta$

$CE_\delta := 2 \cdot DE_\delta \qquad AE_\delta := AD + DE_\delta$

$AB_\delta := \frac{AE_\delta}{2} \qquad BD_\delta := AD - AB_\delta$

$BH_\delta := AB_\delta \qquad GH_\delta := 2 \cdot \sqrt{\left| (BH_\delta)^2 - (BD_\delta)^2 \right|}$

Set AF to unity so that it may be eliminated. Setting BR to 2 will yeild the familiar square root. BR may even take fractional values.

**Plug in values here.**

BR=BASE RATIO,
LBR=LITTLE BASE RATIO

$BR \equiv 3 \qquad LBR \equiv 5$

$\left| \frac{(2 \cdot BR) - 2}{BR} \right| = 1.333$

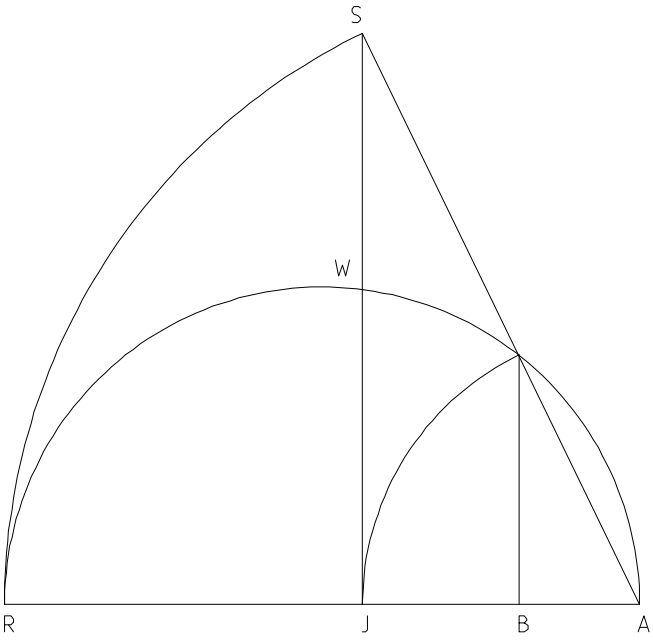
The equation below simplifies to the next.

$GH_\delta \qquad \left[ \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot \left| \frac{(2 \cdot BR) - 2}{BR} \right| \right]^{\frac{1}{2}} \cdot AF \qquad \frac{\sqrt{2} \cdot \sqrt{\left| (2 \cdot BR) - 2 \right|}}{BR \cdot \sqrt{LBR}} \cdot \sqrt{\delta} \cdot AF$

| | | |
|---|---|---|
| 2.951 | 2.951 | 2.951 |
| 4.174 | 4.174 | 4.174 |
| 5.112 | 5.112 | 5.112 |
| 5.903 | 5.903 | 5.903 |
| 6.6 | 6.6 | 6.6 |

$CE_\delta \qquad \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot AF$

| | |
|---|---|
| 0.933 | 0.933 |
| 1.867 | 1.867 |
| 2.8 | 2.8 |
| 3.733 | 3.733 |
| 4.667 | 4.667 |




12_16_93, using 12_04_93.MCD

$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

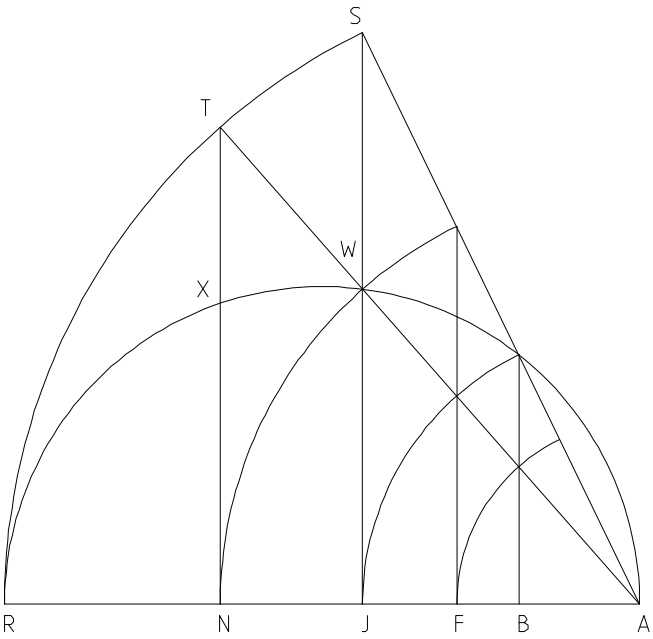
$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ | $AJ_\delta$ |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.

# Euclidean Exponential Series



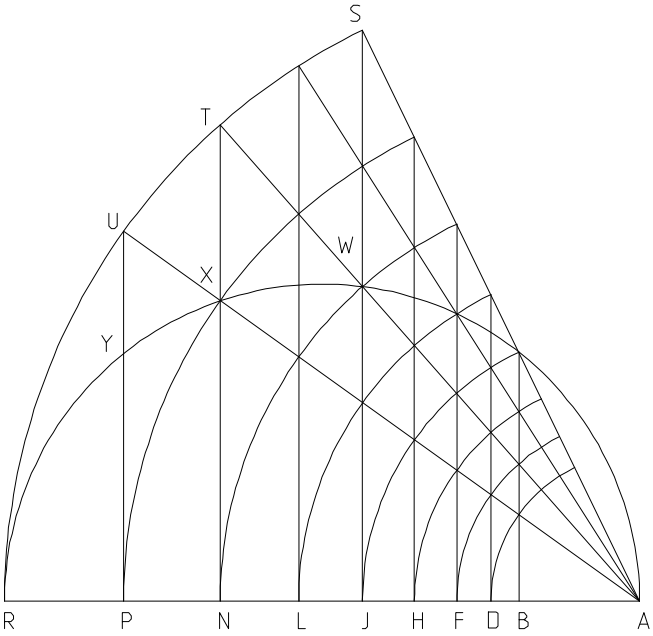

$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ | $AF_\delta$ | $AJ_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |

What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.



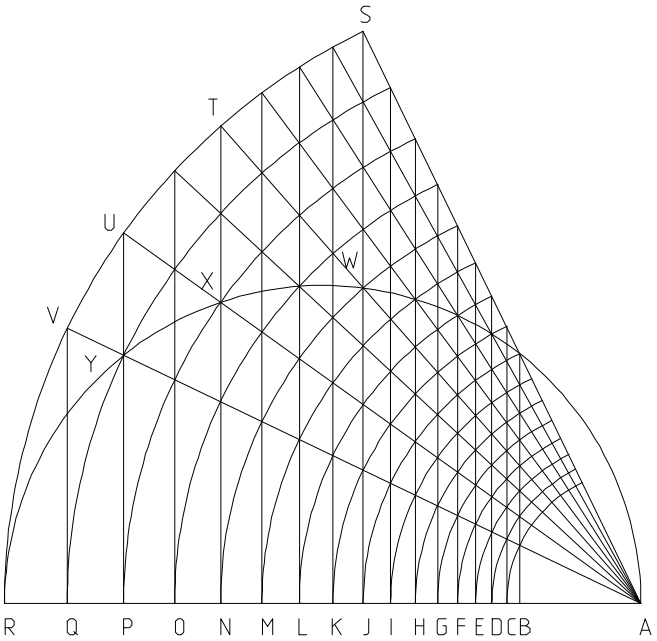

$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$

$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$

$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$

$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$

| $AB_\delta$ | $AD_\delta$ | $AF_\delta$ | $AH_\delta$ | $AJ_\delta$ | $AL_\delta$ | $AN_\delta$ | $AP_\delta$ |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |




$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$

$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$

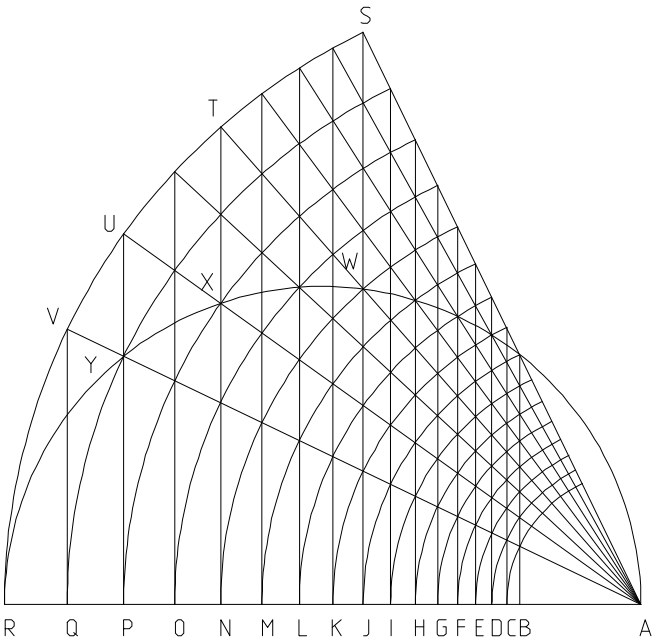
$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$

$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$

| $AB_\delta$ | $AC_\delta$ | $AD_\delta$ | $AE_\delta$ | $AF_\delta$ |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |

| $AG_\delta$ | $AH_\delta$ | $AI_\delta$ | $AJ_\delta$ | $AK_\delta$ | $AL_\delta$ | $AM_\delta$ | $AN_\delta$ | $AO_\delta$ | $AP_\delta$ | $AQ_\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |

Values found by the investigator of 12_14_93




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AN_\delta$ | $\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

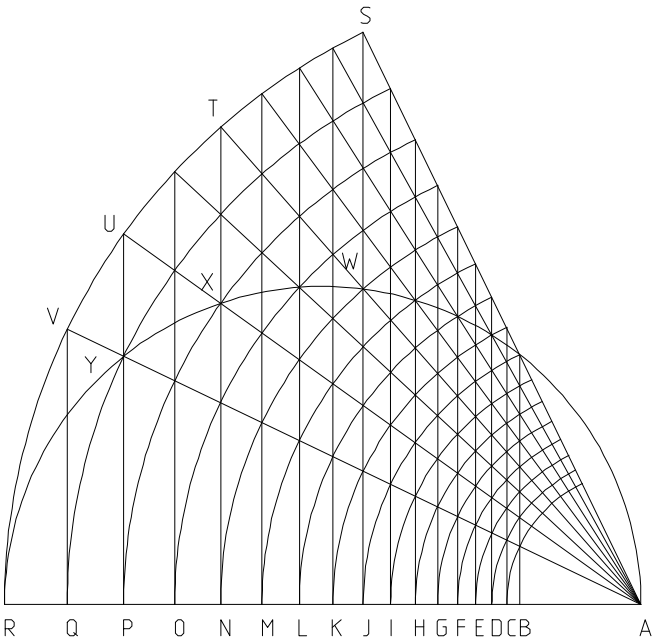
| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{7} \cdot AR\right]^{\frac{1}{8}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left[\left(AB_\delta\right)^2 \cdot AR^{14}\right]^{\frac{1}{16}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AO_\delta$ | $\left[\left(AB_\delta\right)^3 \cdot AR^{13}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.781 | 8.781 |
| 8.138 | 8.138 |
| 7.711 | 7.711 |
| 7.395 | 7.395 |
| 7.147 | 7.147 |

| $AN_\delta$ | $\left[\left(AB_\delta\right)^4 \cdot AR^{12}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AM_\delta$ | $\left[\left(AB_\delta\right)^5 \cdot AR^{11}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.052 | 8.052 |
| 7.094 | 7.094 |
| 6.484 | 6.484 |
| 6.047 | 6.047 |
| 5.713 | 5.713 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^6 \cdot AR^{10}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AK_\delta$ | $\left[\left(AB_\delta\right)^7 \cdot AR^{9}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.384 | 7.384 |
| 6.184 | 6.184 |
| 5.453 | 5.453 |
| 4.945 | 4.945 |
| 4.566 | 4.566 |

| $AJ_\delta$ | $\left[\left(AB_\delta\right)^8 \cdot AR^{8}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.071 | 7.071 |
| 5.774 | 5.774 |
| 5 | 5 |
| 4.472 | 4.472 |
| 4.082 | 4.082 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^9 \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{10} \cdot AR^{6}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{12} \cdot AR^{4}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{14} \cdot AR^{2}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |

| $AB_\delta$ | $\left[\left(AB_\delta\right)^{16} \cdot AR^{0}\right]^{\frac{1}{16}}$ |
|---|---|
| 5 | 5 |
| 3.333 | 3.333 |
| 2.5 | 2.5 |
| 2 | 2 |
| 1.667 | 1.667 |

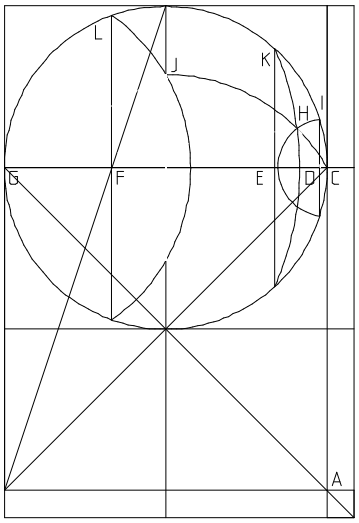
$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

Resultant Equation

depending on direction of transcription.



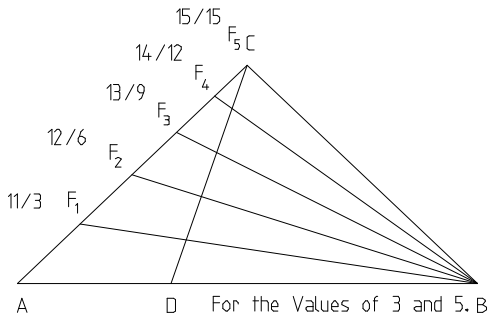

# One Circle

**By John Clark**

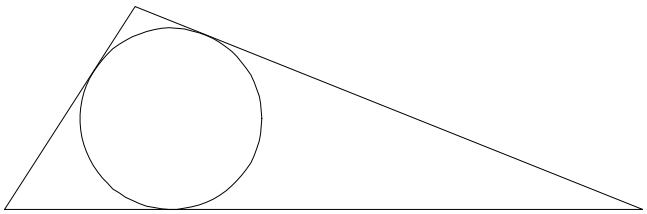
# One Square

# One Line

## And the Delian Quest

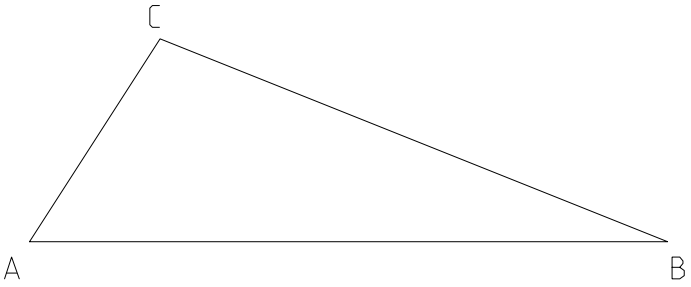



# 1994



# Inscribing a Circle in a given Triangle.

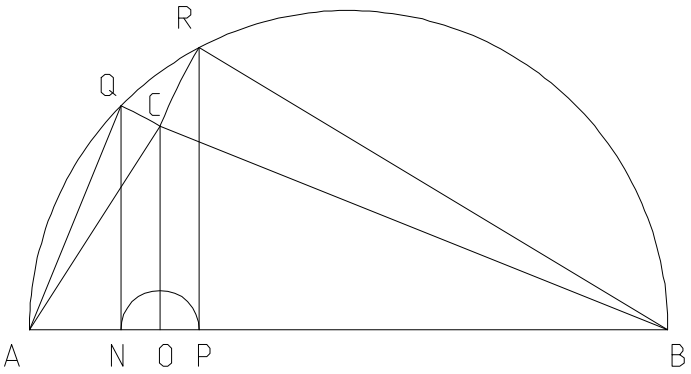
Place the length for the sides of the triangle at the end of the document.




$$AB := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad AC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix}$$

$$BC := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix} \quad \delta := 0..2$$

$$Is\_This\_A\_Triangle := \left(AB_0 + AC_0 \geq BC_0\right) \cdot \left(AB_0 + BC_0 \geq AC_0\right) \cdot \left(AC_0 + BC_0 \geq AB_0\right) \quad AQ_\delta := AC_\delta$$




$$BR_\delta := BC_\delta \quad AN_\delta := \frac{\left(AQ_\delta\right)^2}{AB_\delta}$$

$$BP_\delta := \frac{\left(BR_\delta\right)^2}{AB_\delta} \quad AP_\delta := AB_\delta - BP_\delta$$

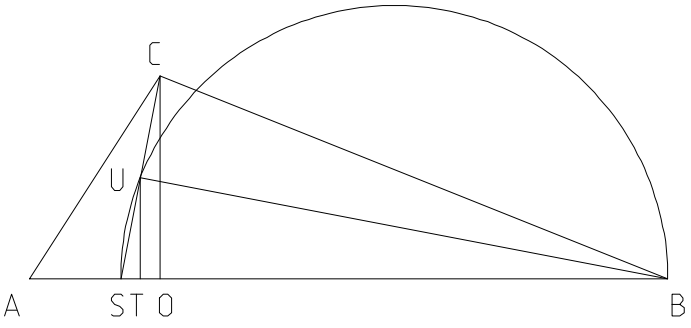
$$NP_\delta := AP_\delta - AN_\delta \quad NO_\delta := \frac{NP_\delta}{2}$$

$$AO := AN + NO \quad BO_\delta := AB_\delta - AO_\delta$$

$$CO_\delta := \sqrt{\left(BC_\delta\right)^2 - \left(BO_\delta\right)^2}$$

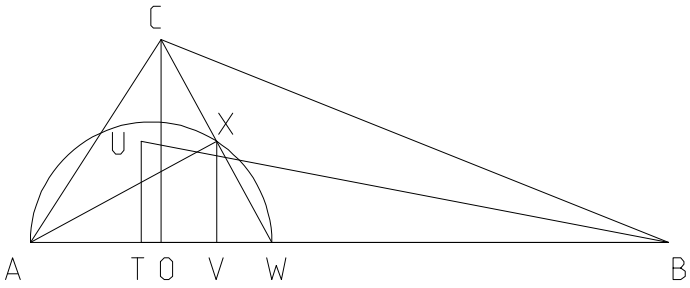
$$BS_\delta := BC_\delta \quad SO_\delta := BS_\delta - BO_\delta$$

$$CS_\delta := \sqrt{\left(SO_\delta\right)^2 + \left(CO_\delta\right)^2} \quad SU_\delta := \frac{CS_\delta}{2}$$




$$BU_\delta := \sqrt{\left(BS_\delta\right)^2 - \left(SU_\delta\right)^2}$$

$$ST_\delta := \frac{\left(SU_\delta\right)^2}{BS_\delta} \quad TU_\delta := \sqrt{\left(SU_\delta\right)^2 - \left(ST_\delta\right)^2}$$

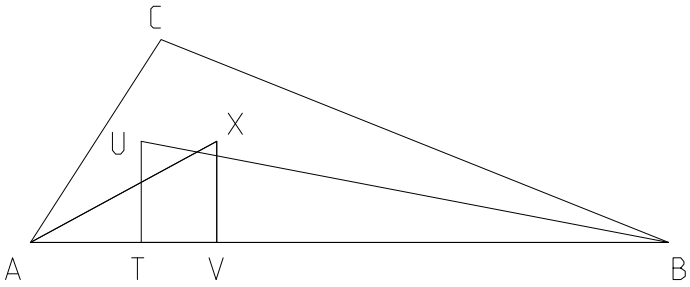



$$AW_\delta := AC_\delta \quad WO_\delta := AW_\delta - AO_\delta$$

$$CW_\delta := \sqrt{(WO_\delta)^2 + (CO_\delta)^2}$$

$$WX_\delta := \frac{CW_\delta}{2}$$

$$AX_\delta := \sqrt{(AW_\delta)^2 - (WX_\delta)^2}$$

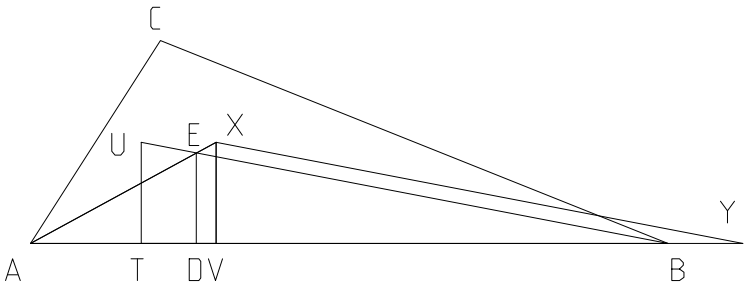



$$WV_\delta := \frac{(WX_\delta)^2}{AW_\delta}$$

$$VX_\delta := \sqrt{(WX_\delta)^2 - (WV_\delta)^2}$$

$$WV_\delta := \sqrt{(WX_\delta)^2 - (VX_\delta)^2}$$

$$AV_\delta := AW_\delta - WV_\delta \quad BV_\delta := AB_\delta - AV_\delta \quad XY_\delta := \frac{BU_\delta \cdot VX_\delta}{TU_\delta} \quad VY_\delta := \sqrt{(XY_\delta)^2 - (VX_\delta)^2}$$




$$AY_\delta := AV_\delta + VY_\delta$$

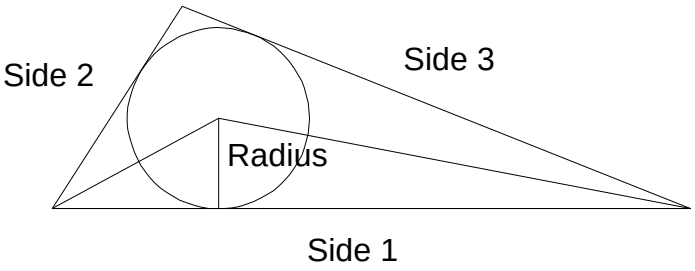
$$AD_\delta := \frac{AV_\delta \cdot AB_\delta}{AY_\delta} \quad AE_\delta := \frac{AX_\delta \cdot AD_\delta}{AV_\delta}$$

$$DE_\delta := \sqrt{(AE_\delta)^2 - (AD_\delta)^2}$$

Plug Side Values In Here  Side_1 ≡ 20  Side_2 ≡ 26  Side_3 ≡ 21

$AB^T = (\ 20 \quad 26 \quad 21\ )$  Is_This_A_Triangle = 1

Given 3 lengths, no matter what order they are entered, DE should remain a constant.

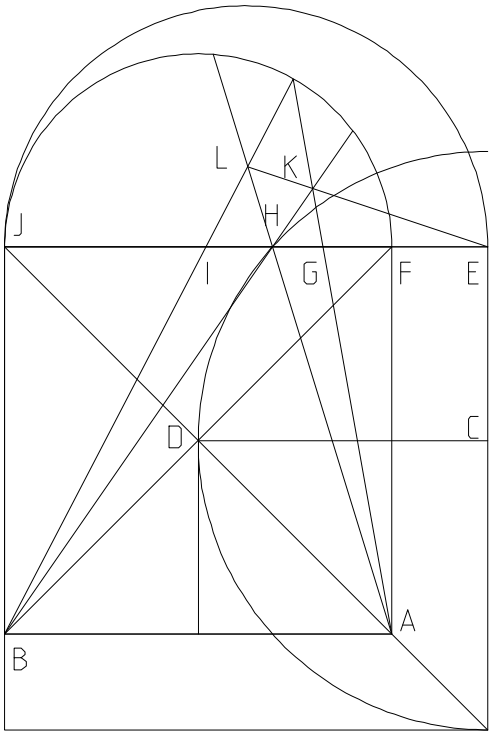



$S_1 := AB \quad S_2 := AC \quad S_3 := BC$

You will note that the formula derived from the process is more consistent with imaginaries.

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}{2\cdot\sqrt{S1_\delta + S2_\delta + S3_\delta}}$$

$$Radius = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix} \qquad DE = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix}$$




# The Cradle

Is EL and EK always collinear?

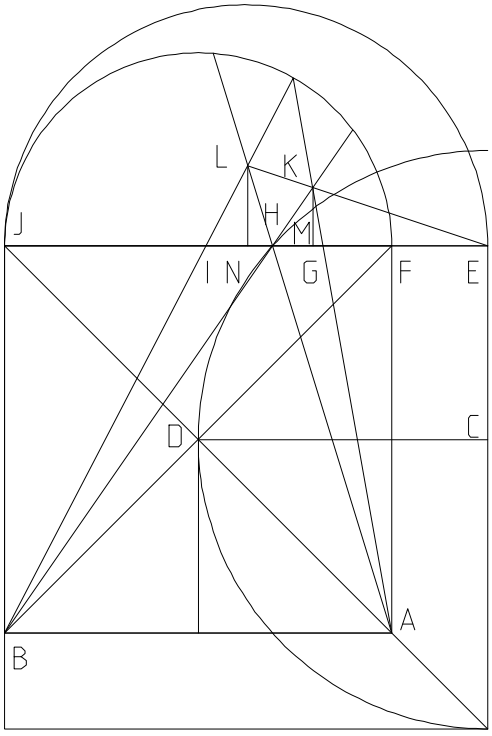
$l := 100 \qquad \delta := 1..l \qquad EF_\delta := \delta^5 \cdot 10^{-8}$

$FJ := 10 \qquad EJ_\delta := FJ + EF_\delta$

$EH_\delta := \sqrt{EF_\delta \cdot EJ_\delta} \qquad FH_\delta := EH_\delta - EF_\delta$

$EG_\delta := \left[ \left( EF_\delta \right)^2 \cdot EJ_\delta \right]^{\frac{1}{3}} \qquad FG_\delta := EG_\delta - EF_\delta$

$EI_\delta := \left[ \left( EJ_\delta \right)^2 \cdot EF_\delta \right]^{\frac{1}{3}} \qquad FI_\delta := EI_\delta - EF_\delta$



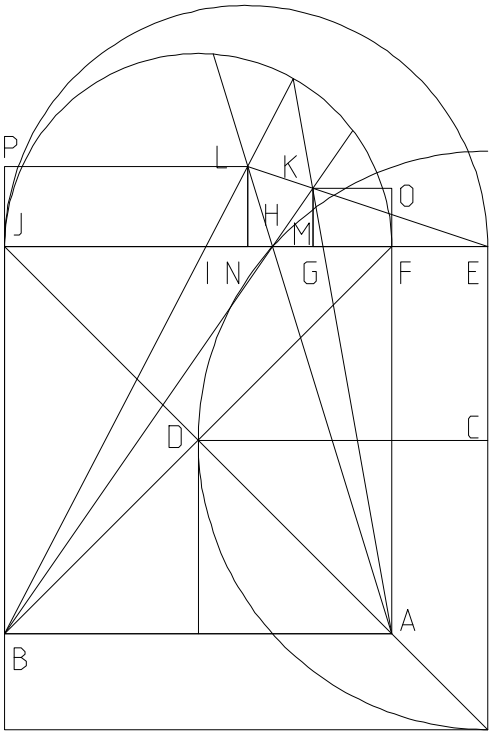

Basically it is demonstrated that the two triangles EKM and ELN are proportional, which is sufficient.

$IJ_\delta := FJ - FI_\delta$

$HJ_\delta := FJ - FH_\delta$

$FN_\delta := \frac{FH_\delta \cdot FJ}{\left( FH_\delta + IJ_\delta \right)} \qquad EN_\delta := FN_\delta + EF_\delta$

$FM_\delta := \frac{FG_\delta \cdot FJ}{\left( FG_\delta + HJ_\delta \right)} \qquad EM_\delta := FM_\delta + EF_\delta$




$$JN_\delta := FJ - FN_\delta \quad LP_\delta := JN_\delta$$

$$BJ := FJ \quad BP_\delta := \frac{BJ \cdot LP_\delta}{IJ_\delta}$$

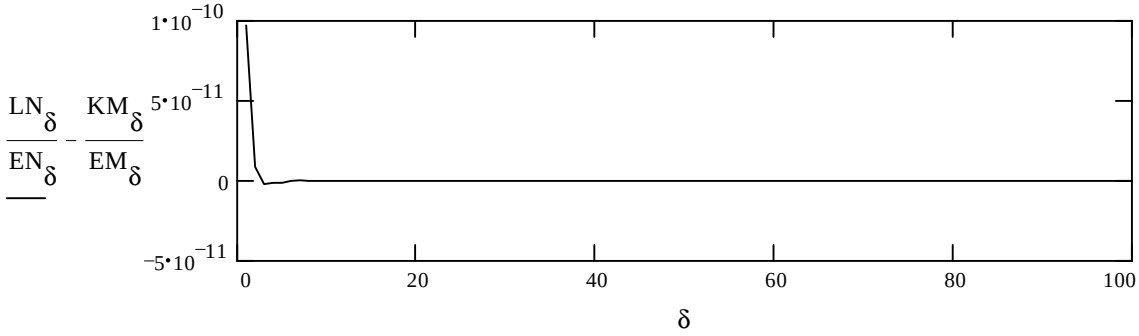
$$JP_\delta := BP_\delta - BJ \quad LN_\delta := JP_\delta$$

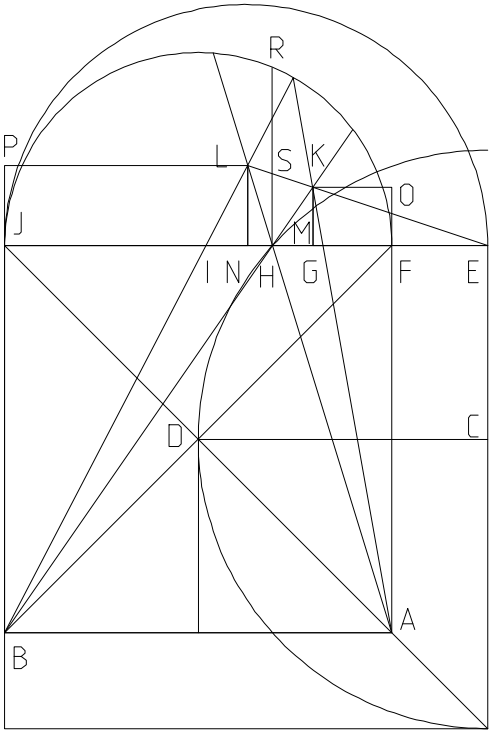
$$KO_\delta := FM_\delta \quad AF := FJ$$

$$AO_\delta := \frac{AF \cdot KO_\delta}{FG_\delta} \quad FO_\delta := AO_\delta - AF$$

$$KM_\delta := FO_\delta \qquad \sum_\delta \left( \frac{LN_\delta}{EN_\delta} - \frac{KM_\delta}{EM_\delta} \right) = 1.015 \cdot 10^{-10}$$

They are two lines with identical slopes, terminating at the same point.

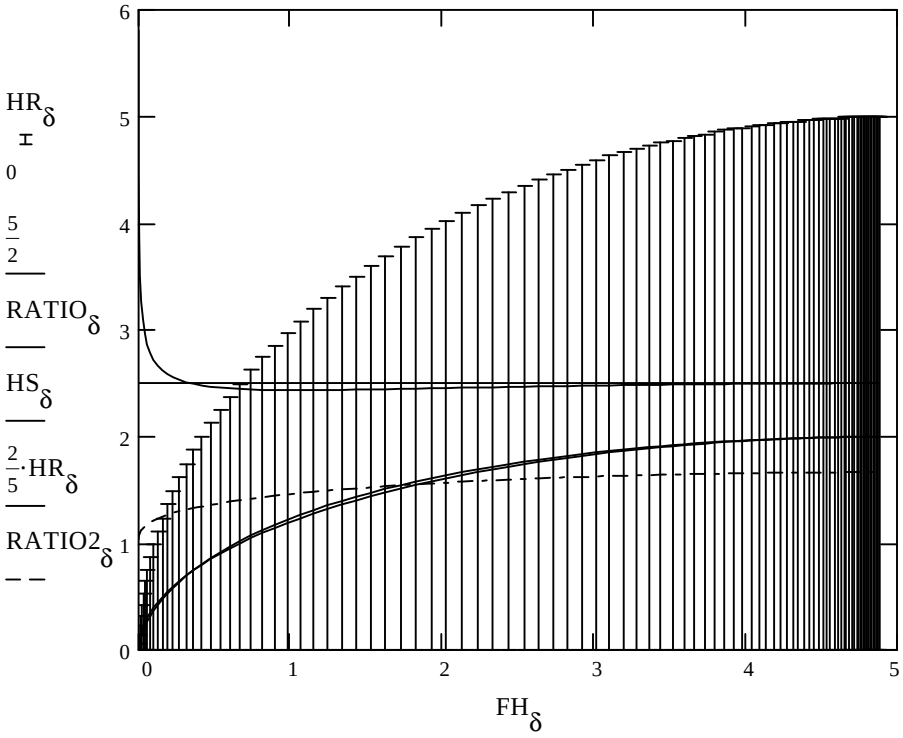





$$HR_\delta := \sqrt{FH_\delta \cdot HJ_\delta} \qquad HS_\delta := \frac{LN_\delta \cdot EH_\delta}{EN_\delta}$$

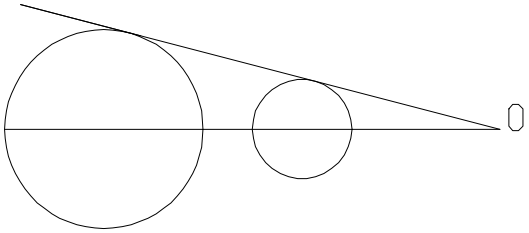
$$RATIO_\delta := \frac{HR_\delta}{HS_\delta} \qquad GI_\delta := EI_\delta - EG_\delta$$

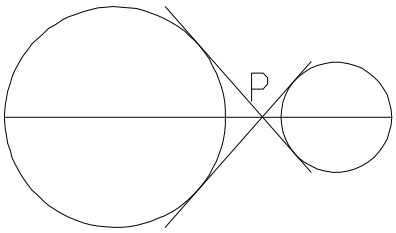
$$RATIO2_\delta := \frac{GI_\delta}{HS_\delta}$$

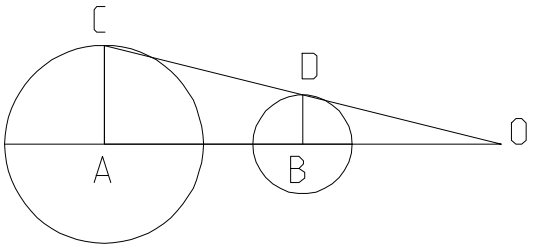


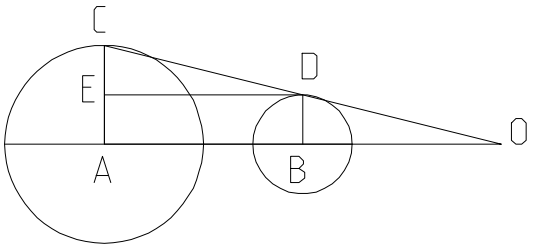
# Tangents and Similarity Points

04_26_94.MCD




O and P are points of origin for the ratio of the two circles that can also have a tangent ray to both circles. Develop formulas that would locate the particular points given using just the radius of the two circles and the difference between them.

O and P are called the similarity points (sp) of the two circles. O is the external similarity point and P is the internal similarity point.

I will work with point O first.

Given $R_L$ = large radius

$R_S$ = small radius

D = difference between origins.

$R_L := 4 \quad R_S := 1 \quad D := 8$

$AC := R_L \quad BD := R_S \quad AB := D$

If the difference between the circles is less than $R_L$-$R_S$, than one of course has an imaginary situation for the external similarity point, $R_L + R_S$ for the internal. At $R_L$-$R_S$ the smaller is in the larger and they touch at one point, $R_L + R_S$ they are external to one another and touching.

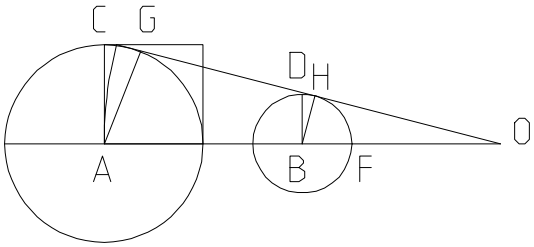
$DE := AB \quad AE := BD \quad CE := AC - AE$

$$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$$

$EOR_L$ "External similarity point Origin to center of Radius Large"

$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right)$$

$EOR_L = 10.667$




What is the length of line (OG) tangent to both circles?

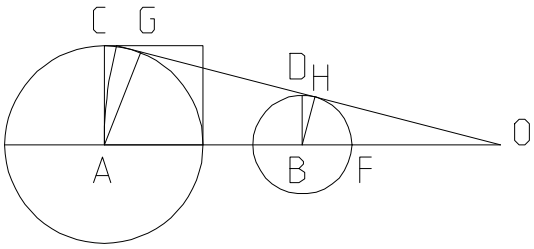
$AG := AC \quad GO := \sqrt{AO^2 - AG^2}$

$GO = 9.888$

And what is the formula?
$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"

$$EOT_{LR} := R_L \cdot \frac{\sqrt{(R_L - R_S + D)\cdot(-R_L + R_S + D)}}{R_L - R_S}$$

$EOT_{LR} = 9.888$




What is the length of the line tangent to the least circle (HO)?

$BH := BD \quad BO := AO - AB \qquad BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

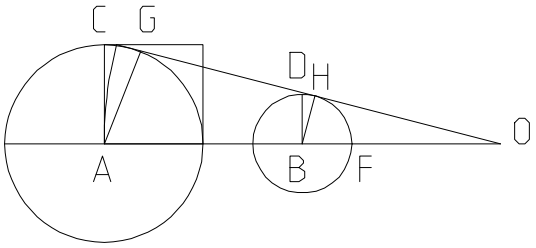
$HO = 2.472$

And what is the formula?

$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S}$$

$EOT_{SR} = 2.472$




Lastly what is the length of line from tangent to tangent of these circles?

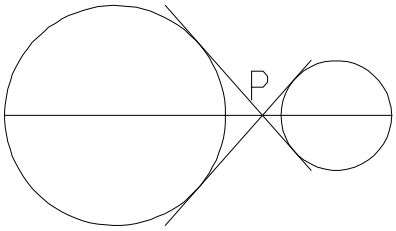
$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

And what is the formula?
ETT "Tangent to Tangent"

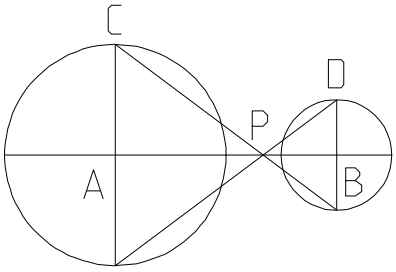
$$ETT := \sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}$$

$ETT = 7.416$




I will now turn my attention to the point P, the internal similarity point.

$$AP := \frac{AB\cdot AC}{AC + BD} \qquad AP = 6.4$$



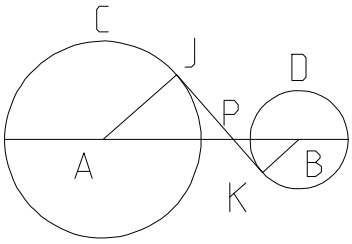

$IOR_L$ "Internal similarity point to center of Radius Large"

$$IOR_L := D\cdot\frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

$IOR_S$ "Internal similarity point to center of Radius Small"

$$IOR_S := D\cdot\frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$



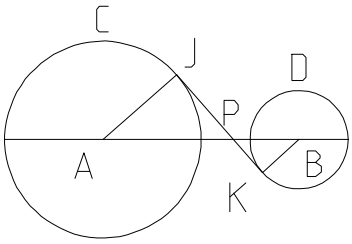

$AJ := AC \quad BK := BD \quad JP := \sqrt{AP^2 - AJ^2}$

$JP = 4.996$

$IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"

$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996$

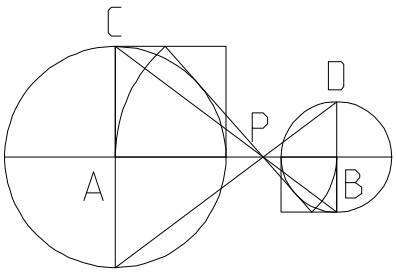



$KP := \sqrt{BP^2 - BK^2} \quad KP = 1.249$

$IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249$



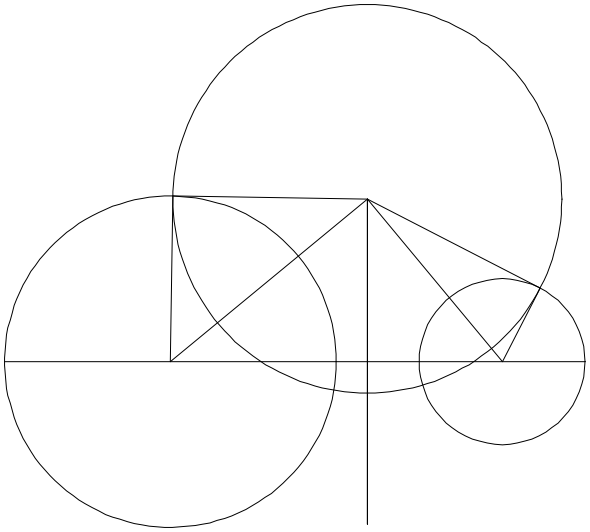

$JK := JP + KP \quad JK = 6.245$

ITT "Internal similarity point Tangent to Tangent"

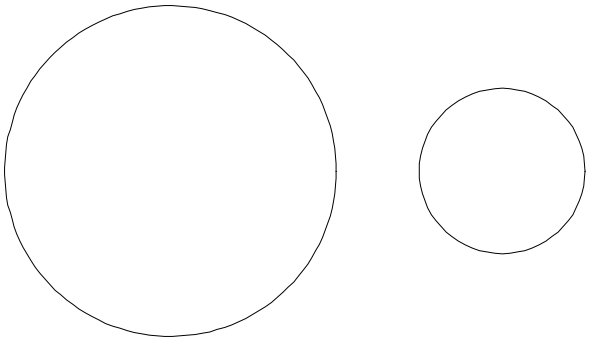
$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}$$

$ITT = 6.245$

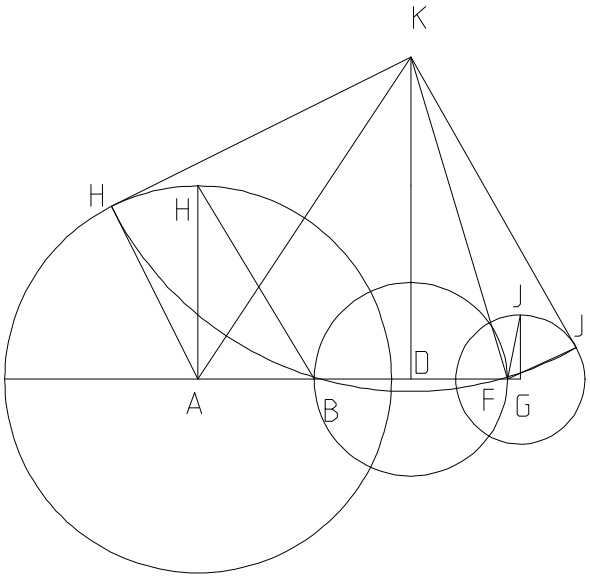
# The Chordal or Power Line of two Circles 04_27_94.MCD



The solution for finding the chordal as outlined in ***100 Great Problems of Elementary Mathematics Their History and Solution*** by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.
The figure I work with is a transformation of the one on the left.



Given two circles find their chordal or power line given just their radius and difference between their centers, and reduce the tautological chains to formulas.




$R_1 := 3 \quad R_2 := 1 \quad D := 1$

$AH := R_1 \quad GJ := R_2 \quad AG := D$

$AB := \frac{AH^2}{AG} \quad FG := \frac{GJ^2}{AG}$

$BF := AG - AB - FG \quad BD := \frac{BF}{2}$

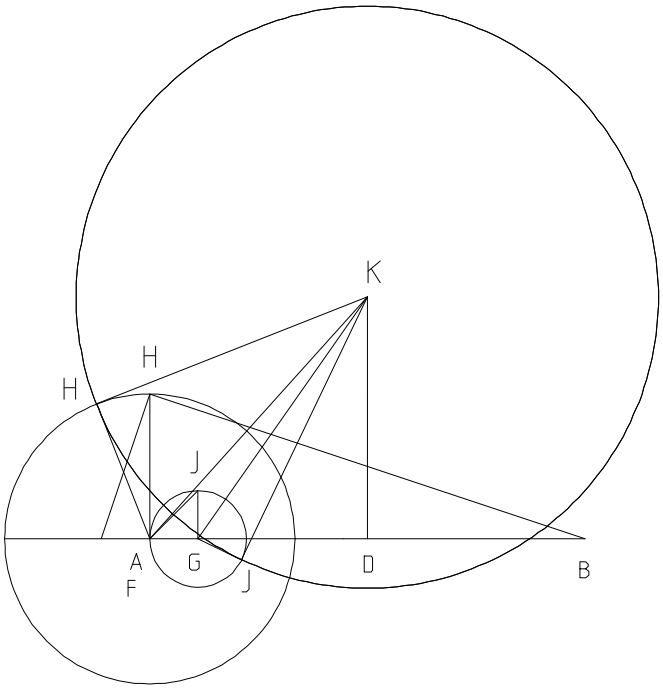
$AD := AB + BD \quad DF := BD$

$DG := DF + FG$

CR "Center to Radius"

$$CR_1 := \frac{1}{2}\cdot\frac{\left(R_1^2 - R_2^2 + D^2\right)}{D} \qquad CR_1 = 4.5 \qquad AD = 4.5$$

$$CR_2 := \frac{1}{2}\cdot\frac{\left(R_2^2 - R_1^2 + D^2\right)}{D} \qquad CR_2 = -3.5 \qquad DG = -3.5$$




**Locate length K here** $DK := 5$

$$AK := \sqrt{DK^2 + AD^2} \qquad GK := \sqrt{DK^2 + DG^2}$$

$$HK := \sqrt{AK^2 - AH^2} \qquad JK := \sqrt{GK^2 - GJ^2}$$

$HK = 6.021$

LP "Length of Power line"

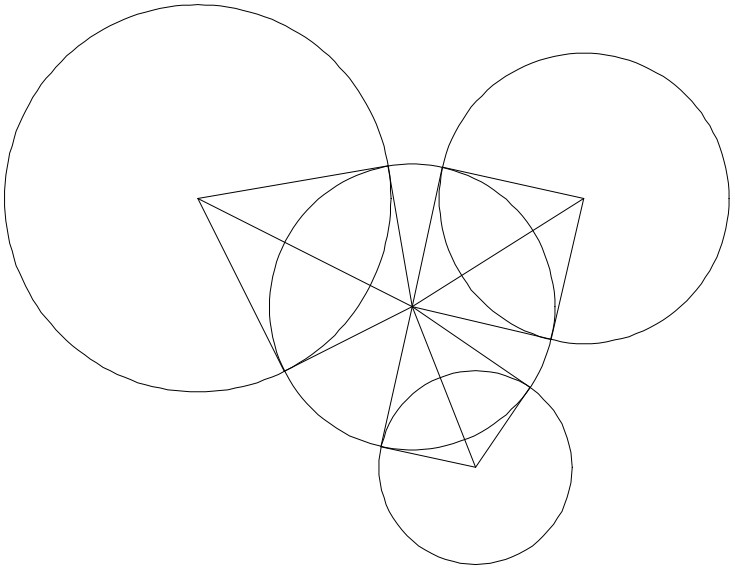
$LP := DK$

LT "Length of radial Tangent" LP is the variable chosen for circle center on the power line.

$$LT := \frac{1}{2}\cdot\left|\frac{\sqrt{4\cdot LP^2\cdot D^2 + R_1^4 - 2\cdot R_1^2\cdot D^2 - 2\cdot R_1^2\cdot R_2^2 + D^4 - 2\cdot D^2\cdot R_2^2 + R_2^4}}{D}\right|$$

$LT = 6.021$ $JK = 6.021$

The process does not seem to recognize any special cases.



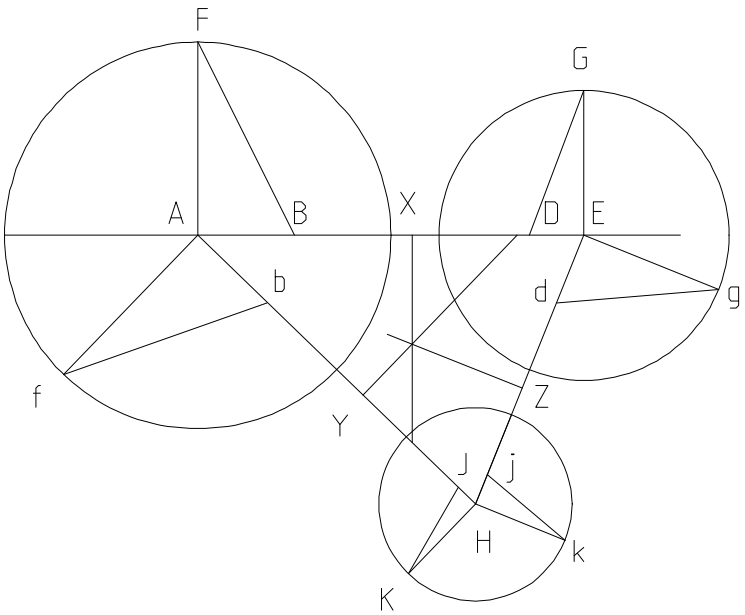
# Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the formula for the Power Point and the Length of the resultant Tangent.

The distance between each set of circles is given as $D_1$, $D_2$, and $D_3$. Naturally they must form a triangle.

$D_1 := 4 \quad D_2 := 5 \quad D_3 := 2$

$\Delta := \left(D_1 + D_2 \geq D_3\right) \cdot \left(D_2 + D_3 \geq D_1\right) \cdot \left(D_1 + D_3 \geq D_2\right)$




$\Delta = 1$ $\Delta$ "Is this a Triangle?

$R_1 := 3 \quad R_2 := 2 \quad R_3 := 4$

$AE := D_1 \quad AH := D_2 \quad EH := D_3$

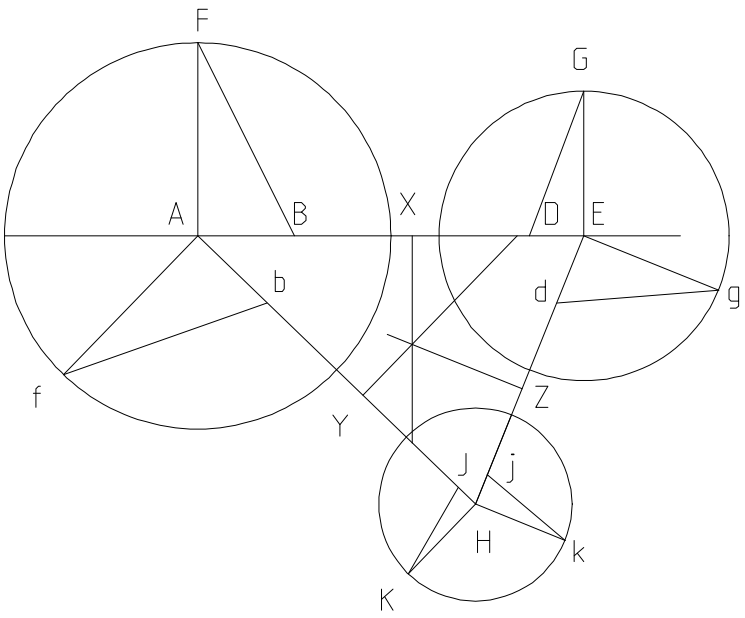
$AF := R_1 \quad HK := R_2 \quad EG := R_3$

$Af := AF \quad Hk := HK \quad Eg := EG$

$AB := \frac{AF^2}{AE} \quad DE := \frac{EG^2}{AE}$

$Ab := \frac{Af^2}{AH} \quad HJ := \frac{HK^2}{AH}$

$Hj := \frac{Hk^2}{EH} \quad Ed := \frac{Eg^2}{EH}$




$BD := AE - AB - DE \quad BX := \frac{BD}{2}$

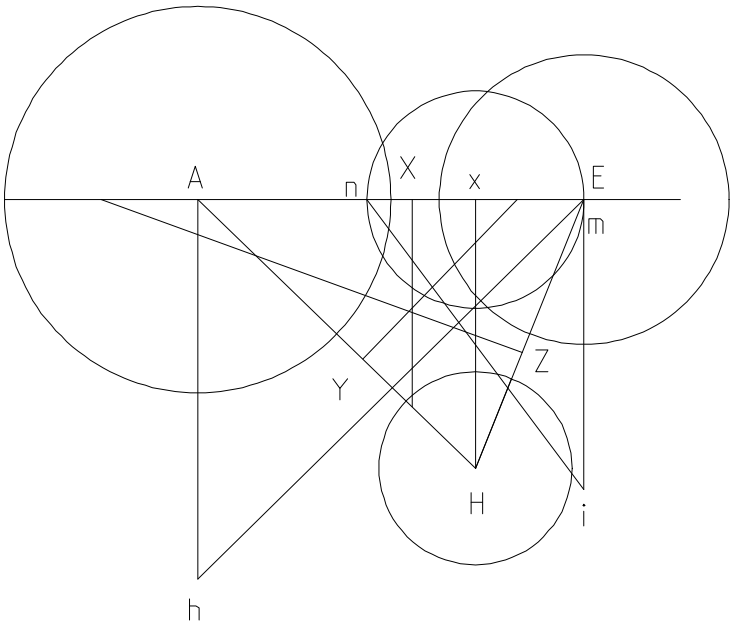
$bJ := AH - Ab - HJ \quad bY := \frac{bJ}{2}$

$dj := EH - Ed - Hj \quad dZ := \frac{dj}{2}$

$AX := AB + BX \quad AX = 1.125$

$AY := Ab + bY$

$EZ := Ed + dZ \quad EZ = 4$



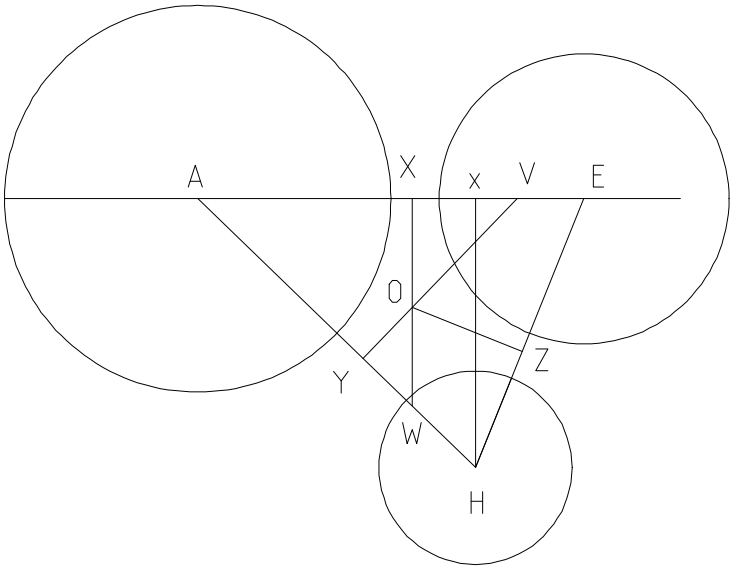

$Ah := AH \quad Ei := EH$

$Am := \frac{Ah^2}{AE} \quad En := \frac{Ei^2}{AE}$

$An := AE - En \quad mn := Am - An$

$nx := \frac{mn}{2} \quad Ax := An + nx$

$Hx := \sqrt{AH^2 - Ax^2}$




$$WX := \frac{Hx \cdot AX}{Ax} \qquad WX = 0.462$$

$$VY := \frac{Hx \cdot AY}{Ax} \qquad VY = 1.232$$

$$AV := \frac{AH \cdot AY}{Ax} \qquad AV = 3.243$$

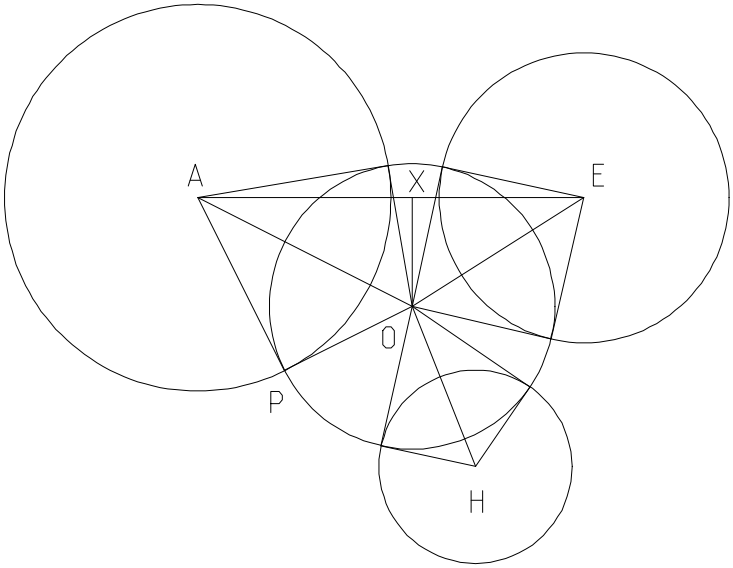
$$VX := AV - AX$$

$$OX := \frac{AY \cdot VX}{VY} \qquad OX = 5.157$$

PP "Power Point"

$$PP := \frac{1}{2} \cdot \frac{\left( \begin{array}{l} D_1^2 \cdot R_1^2 + D_1^2 \cdot D_2^2 - 2 \cdot D_1^2 \cdot R_2^2 + R_1^2 \cdot D_3^2 - R_1^2 \cdot D_2^2 - D_1^4 + D_1^2 \cdot D_3^2 \ldots \\ + R_3^2 \cdot D_1^2 - R_3^2 \cdot D_3^2 + R_3^2 \cdot D_2^2 \end{array} \right)}{\left( D_1 \cdot \sqrt{\left| -2 \cdot D_1^2 \cdot D_2^2 + D_1^4 - 2 \cdot D_1^2 \cdot D_3^2 + D_3^4 - 2 \cdot D_2^2 \cdot D_3^2 + D_2^4 \right|} \right)}$$

$$PP = 5.157$$




$EX := AE - AX$

$AO := \sqrt{AX^2 + OX^2}$ $\quad EO := \sqrt{EX^2 + OX^2}$

$AP := AF$ $\quad EP := EG$

$OP_A := \sqrt{AO^2 - AP^2}$ $\quad OP_E := \sqrt{EO^2 - EP^2}$

$OP_A = 4.342$ $\quad OP_E = 4.342$

LT "Length of Tangent"

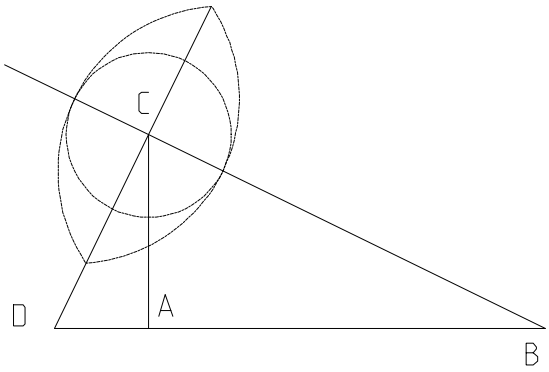
$$LT := \frac{\sqrt{\begin{aligned}&-R_1^4\cdot D_3^2 - R_1^2\cdot R_3^2\cdot D_1^2 - R_1^2\cdot D_3^4 + D_1^2\cdot R_1^2\cdot D_3^2 - D_1^2\cdot D_3^2\cdot D_2^2 \ldots\\&+ R_2^2\cdot R_1^2\cdot D_3^2 - R_2^2\cdot R_1^2\cdot D_2^2 - R_2^2\cdot R_3^2\cdot D_3^2 + R_2^2\cdot R_3^2\cdot D_2^2 \ldots\\&+ R_3^2\cdot D_1^2\cdot D_2^2 - R_3^2\cdot D_2^4 - R_3^4\cdot D_2^2 + D_1^2\cdot R_1^2\cdot R_2^2 + D_1^2\cdot D_2^2\cdot R_2^2 \ldots\\&+ D_1^2\cdot R_2^2\cdot D_3^2 + D_1^2\cdot R_2^2\cdot R_3^2 + R_1^2\cdot R_3^2\cdot D_3^2 + R_3^2\cdot D_3^2\cdot D_2^2 - D_1^2\cdot R_2^4 \ldots\\&+ -D_1^4\cdot R_2^2 + R_1^2\cdot D_3^2\cdot D_2^2 + R_1^2\cdot R_3^2\cdot D_2^2\end{aligned}}}{\sqrt{-2\cdot D_1^2\cdot D_2^2 + D_1^4 - 2\cdot D_1^2\cdot D_3^2 + D_3^4 - 2\cdot D_3^2\cdot D_2^2 + D_2^4}}$$

$LT = 4.342$

# Division, $A^2$

04_30_94.MCD

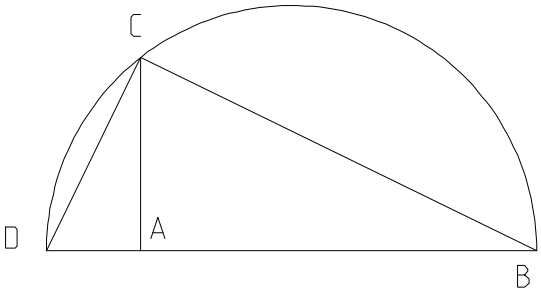
One does not work with geometry often, so it may be that one does not keep basics in mind when trying to work a figure. This little paper is about a basic move. I bring this to light, as I have seen a ratio often given as division. In geometry, so far as I know, one cannot divide a line by a line, but can form a series of the nature $A^N : B^N$ as a linear figure. Given A and B one can raise them to any whole power simultaneously with a couple of simple moves based on the figure immediately below. See work done in 1995.

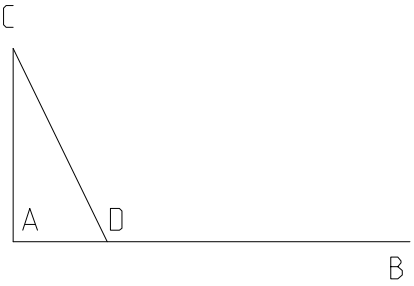



Divide $AC^2$ by AB.

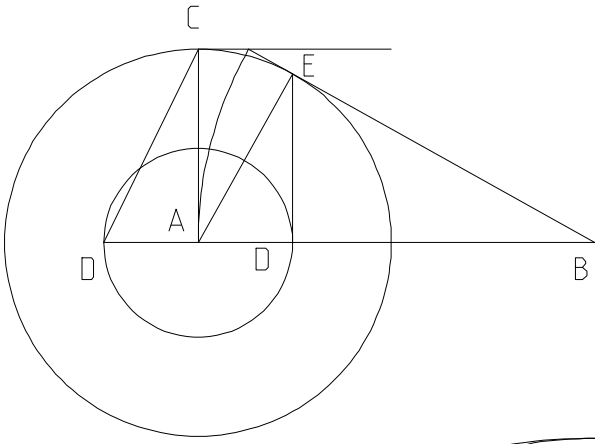
Process Summary

I have noticed that my solutions depend upon this basic move.

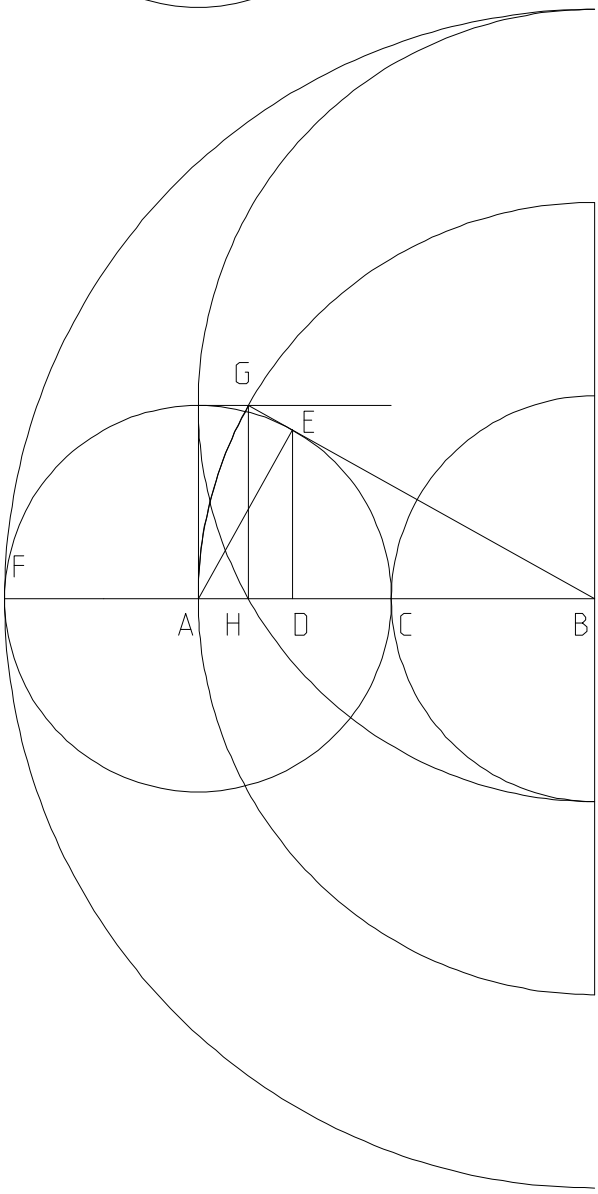



In some works I have represent it simply as the figure on the left. One may realize that in my Pythagorean Completion I used the circular form and it is often expressed as a pole and polar arrangement. A terminology that does not seem fit for the processes that they represent, physical and not mathematical.




The jargon is that B is called a Pole and D is on a polar, a segment of which is DE. But it can easily be seen that the figure is a transformation. It is another way of dividing $AC^2$ by AB. The figure now raises a question for me. I had thought that I answered it previously, but I cannot find it in my files. Is BE always collinear with BG? This paper is helping me mediate on poles and polars, which names do not help me understand the true ratio involved. Mathematicians seem to like a proliferation of names. And, god forbid, B and D are called conjugate in respect to each other. I have a hard enough time remembering my own name, that is why I keep my id. (in the Freudian sense) close at hand.




$AC := 5 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CF := 2 \cdot AC \quad AG := AC \quad BF_\delta := CF + BC_\delta$

$BH_\delta := \sqrt{BF_\delta \cdot BC_\delta} \quad GH := AC$

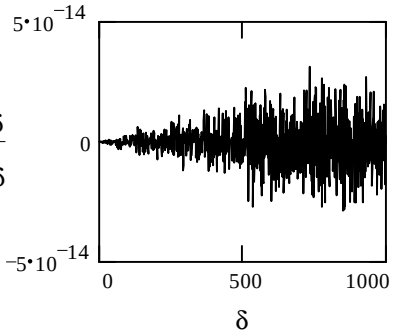
$BG_\delta := \sqrt{(BH_\delta)^2 + GH^2} \quad AE := AC$

$AB_\delta := AC + BC_\delta \quad AD_\delta := \frac{AE^2}{AB_\delta}$

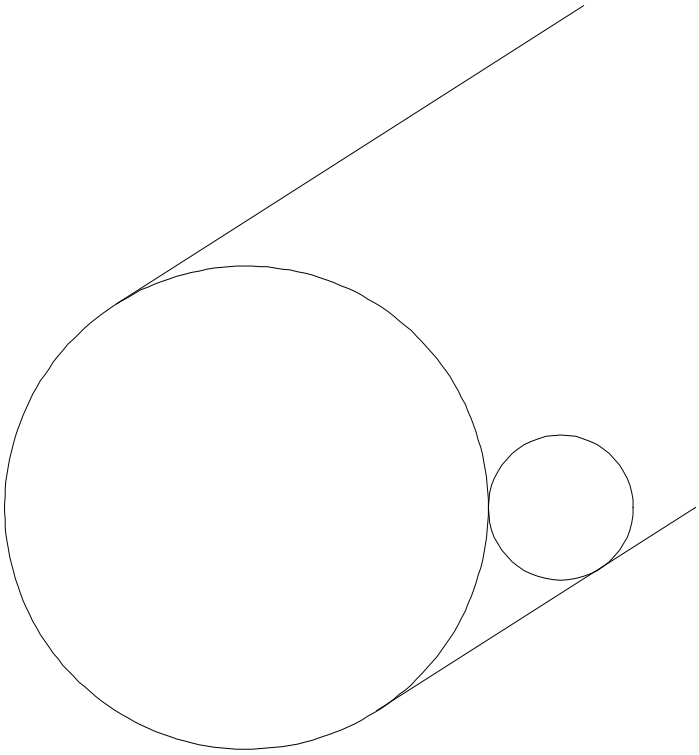
$BE_\delta := \sqrt{(AB_\delta)^2 - AE^2} \quad BD_\delta := AB_\delta - AD_\delta$

$DE_\delta := \sqrt{(BE_\delta)^2 - (BD_\delta)^2}$

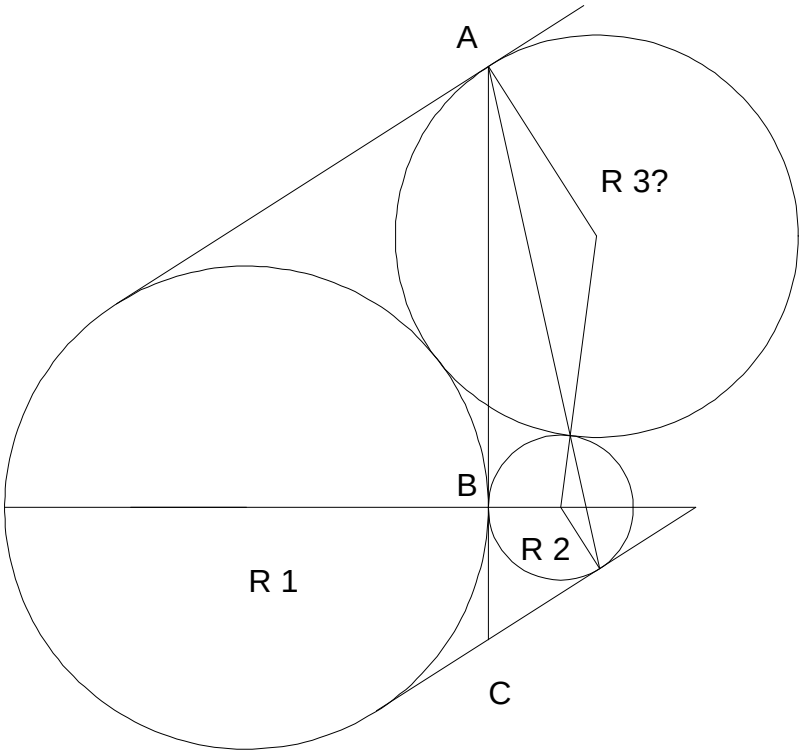
$\frac{GH}{BH_\delta} - \frac{DE_\delta}{BD_\delta}$



# Two Circles and a Parallel

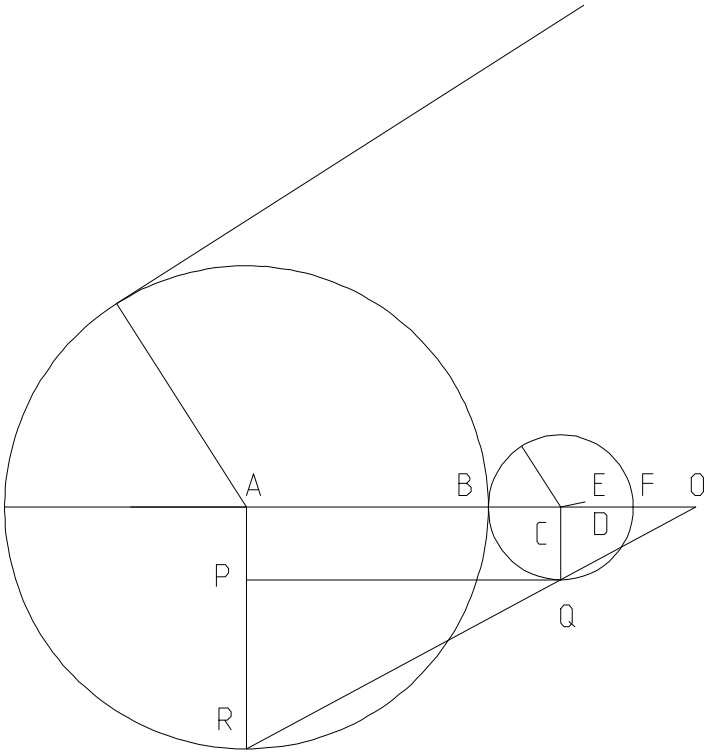


What I would like to do is to is drop in a circle that is tangent to the given two circles that are already tangent and tangent to the line from the similarity point and also have this circle tangent to the parallel of the similarity line that lies tangent to the first circle. The formula derived for my process tells me that I will do it the hard way. It will ***predict*** an easier method.




Process Summary

Given the radius of the two circles, what is the radius of the third? Attempt to develop a formula for the resultant radius. And also, of the power line between parallels, what is the ratio of AC:BC in terms of the given radius' ?




Find the Similarity Point.

$AB := R_1 \quad CF := R_2 \quad BC := CF$

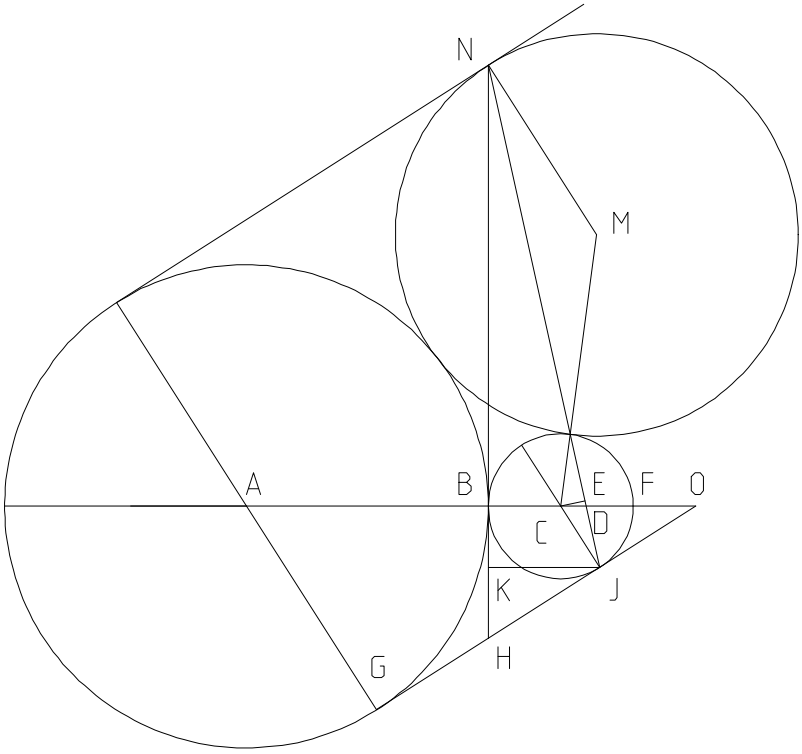
$CB := CF \quad AC := AB + BC$

$PQ := AC \quad AR := AB$

$CQ := CF \quad AP := CQ$

$PR := AR - AP$

$AO := \frac{PQ \cdot AR}{PR}$




Find the segment of the power line (HN) between parallels.

$AG := AB \quad GO := \sqrt{AO^2 - AG^2}$

$BO := AO - AB$

$BH := \frac{AG \cdot BO}{GO} \quad BN := \frac{BH \cdot AB}{BC}$

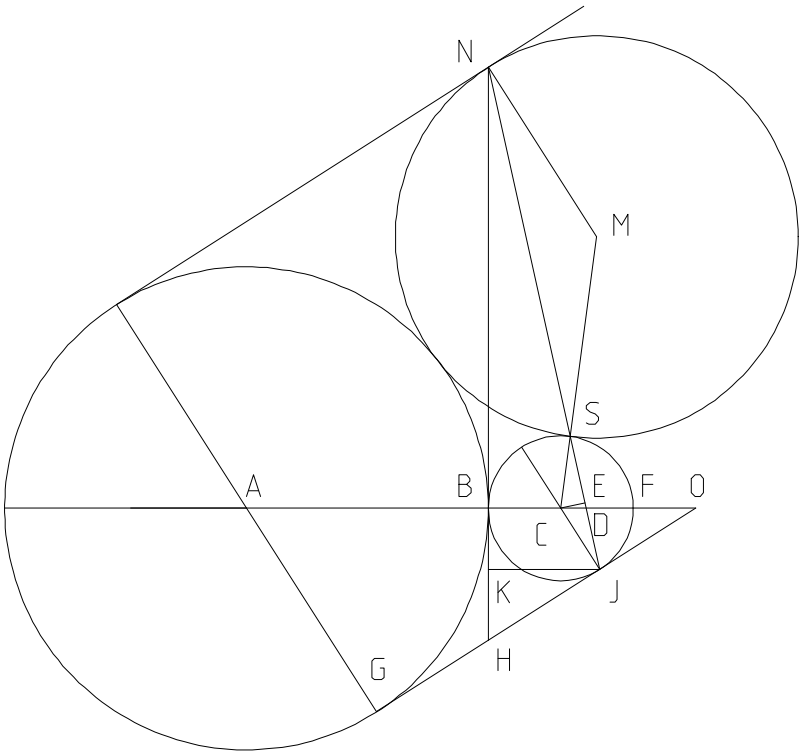
$HO := \frac{AO \cdot BH}{AG} \quad CO := BO - BC$

$JO := \frac{GO \cdot CO}{AO} \quad HN := BH + BN$

Find JN

$HJ := HO - JO \quad KH := \frac{AG \cdot HJ}{AO}$

$KN := HN - KH$




$KJ := \sqrt{HJ^2 - KH^2}$

$JN := \sqrt{KN^2 + \text{Find MS}}$

$BD := \frac{KJ \cdot BN}{KN}$ $CD := BD - BC$

$CE := \frac{KN \cdot CD}{JN}$ $CJ := BC$

$EJ := \sqrt{CJ^2 - CE^2}$ $CS := 2 \cdot EJ$

$NS := JN - JS$ $CS := BC$

$MS := \frac{CS \cdot NS}{JS}$

## Plug Values in Here

$R_1 \equiv 8$ $R_2 \equiv 6$

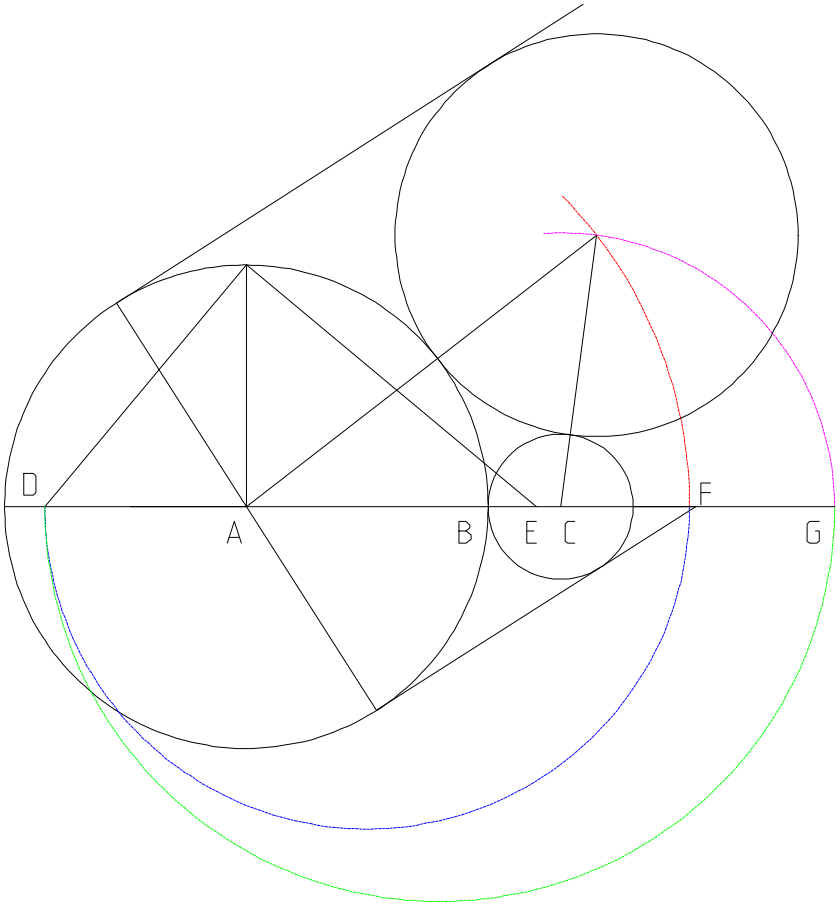
There was too much work here for the symbolic processor to reduce all the equations to one, easily. It took me three days to nurse the processor through it, and this is a short work. The formula does not have the "sour" spot. (Place both $R_1$ and $R_2$ to the same value to see it.) When $R_2$ is 1/4 of $R_1$, there are six tangents.

$$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad R_3 = 2.667 \qquad MS = 2.667$$

The formula tells me at least two things, 1) There is a second method to solve the problem, 2) the process is a rather baroque method of dividing a square.

$$\frac{HN}{BN} = 1.75 \qquad \frac{R_1 + R_2}{R_1} = 1.75 \qquad \frac{HN}{BH} = 2.333 \qquad \frac{R_1 + R_2}{R_2} = 2.333$$

The results for the ratios of the power line segment are very nice also.




What is the construction suggested by the found formula of

$$\frac{R_1^2}{4 \cdot R_2} \quad ?$$

$AB := R_1 \quad BC := R_2$

$AE := 4 \cdot BC$

$AD := \frac{AB^2}{AE}$

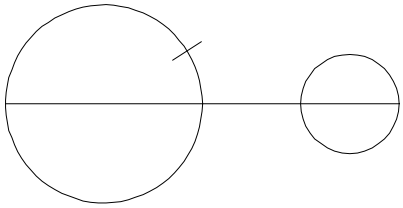
$AF := AB + AD$

$CG := BC + AD$

$R_3 = 2.667 \qquad AD = 2.667$

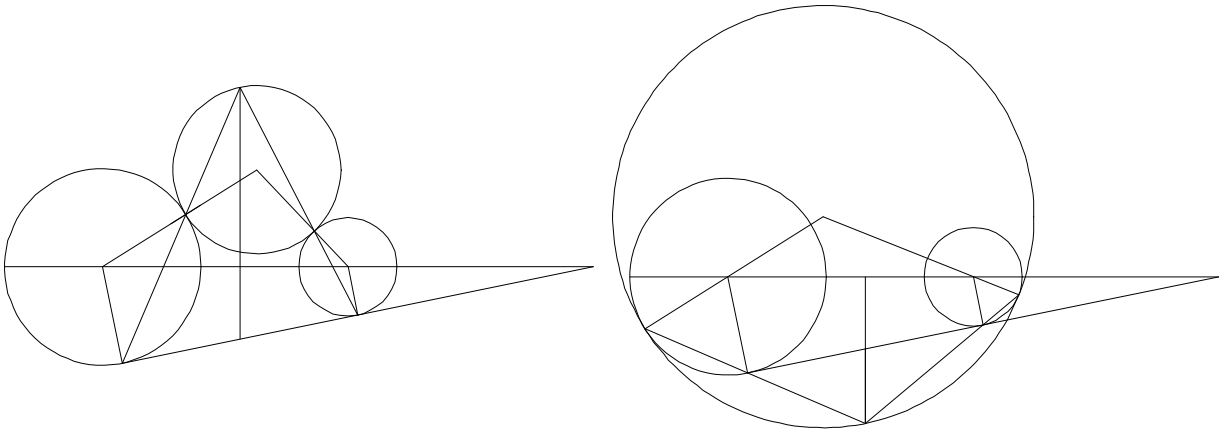
For those who may become confused as to the dashed lines, the segment AD is added to the radius of the both circles their intersection is the center of the circle sought.

# Two Circles, given a tangent on one.

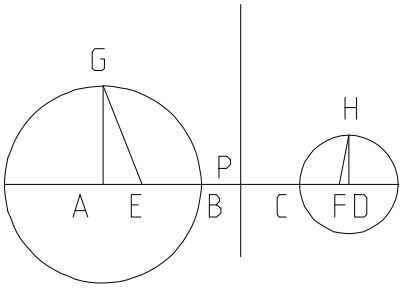


Given two circles and a point that is on the circumference of one, find a circle tangent to the circle at that point and also tangent to the other circle. The convention for this point will be "from the power line".

Process Summary



Find the power line.



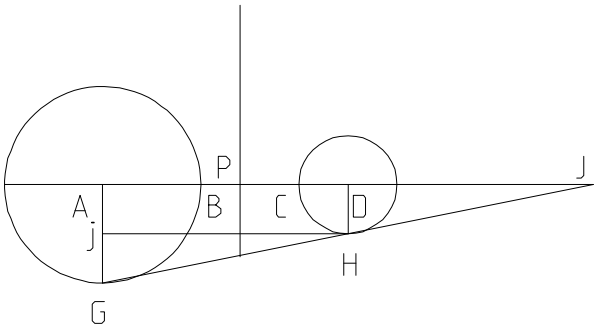

$AB := R_1 \quad CD := R_2 \quad AD := D$

$AG := AB \quad DH := CD \quad AE := \frac{AG^2}{AD}$

$DF := \frac{DH^2}{AD} \quad EF := AD - AE - DF$

$EP := \frac{EF}{2} \quad AP := AE + EP \quad DP := DF + EP$

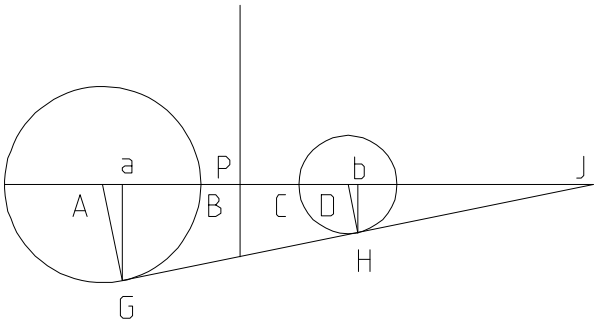
$BP := AP - AB$




$Hj := AD \quad Aj := DH$

$Gj := AG - DH \quad AJ := \frac{Hj \cdot AG}{Gj}$

$DJ := AJ - AD \quad HJ := \sqrt{DJ^2 - DH^2}$




$GJ := \sqrt{AJ^2 - AG^2} \quad Aa := \frac{AG^2}{AJ}$

$Ga := \sqrt{AG^2 - Aa^2} \quad Db := \frac{DH^2}{DJ}$

$Hb := \sqrt{DH^2 - Db^2}$

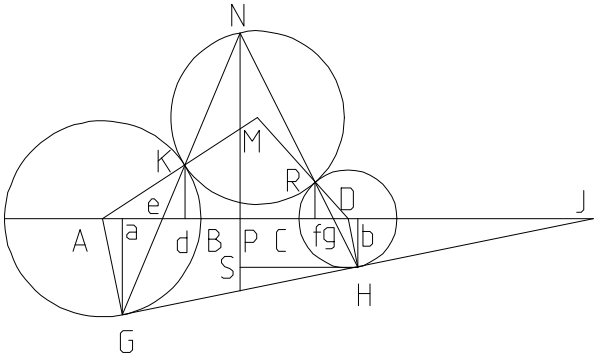
$Ba := AB - Aa$

$Pb := DP + Db$

$P := \left| \text{if}(P \le 2 \cdot AB, P, 0) \right|$

$Bd := P$

$AK := AB \quad ad := Ba - Bd$




$Ad := AB - Bd \quad Kd := \sqrt{AK^2 - Ad^2}$

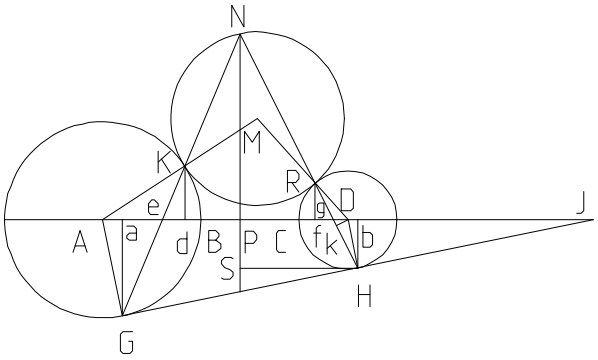
$de := \frac{ad \cdot Kd}{Kd + Ga} \quad Pe := BP + Bd + de$

$NP := \frac{Kd \cdot Pe}{de} \quad HS := Pb \quad PS := Hb$

$NS := NP + PS \quad Pg := \frac{HS \cdot NP}{NS}$

$Dg := DP - Pg \quad bg := Dg + Db$

$Hg := \sqrt{bg^2 + Hb^2}$




To save clutter, see 07_18_93.MCD Mod C.

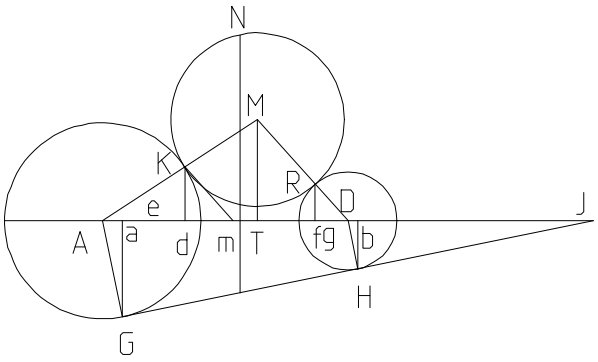
$$gk := \frac{1}{2}\cdot\frac{Dg^2}{Hg} + \frac{1}{2}\cdot Hg - \frac{1}{2}\cdot\frac{DH^2}{Hg}$$

$Dk := \sqrt{Dg^2 - gk^2} \quad Hk := \sqrt{DH^2 - Dk^2}$

$Rg := Hk - gk$

$$Df := \frac{1}{2}\cdot\frac{CD^2}{Dg} + \frac{1}{2}\cdot Dg - \frac{1}{2}\cdot\frac{Rg^2}{Dg}$$

$DR := CD \quad Rf := \sqrt{DR^2 - Df^2}$




$$dm := \frac{Df\cdot Kd}{Rf} \quad Am := Ad + dm$$

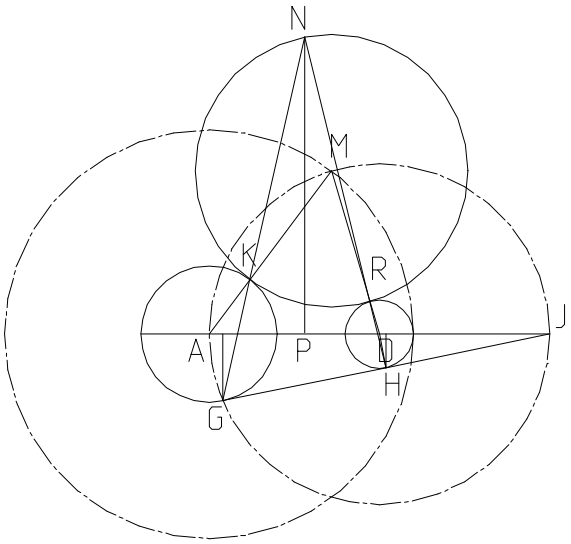
$$MT := \frac{Kd\cdot AD}{Am} \quad AM := \frac{AK\cdot MT}{Kd}$$

$KM := AM - AK$

$R_3 := |KM| \quad P = 19$

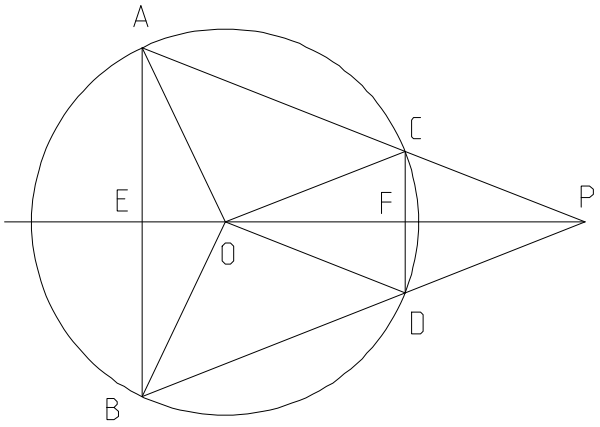
Plug Values in Here. Results is in $P_3$.

$R_1 \equiv 10 \; R_2 \equiv 3 \quad D \equiv 16 \quad P \equiv 19 \quad R_3 = 14.836$



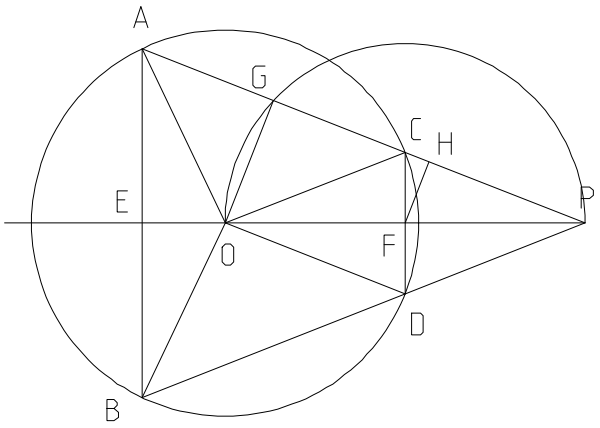

Let us say that instead of choosing a point of tangency upon one of the circles, I wish to place a given circle tangent to both. Derive the name of the length of the power line indicated and find th points of tangency.

Work in progress.




Given that CP = CO what is the relationship between AE and CF?

In order to derive a resonable answer, I have constrained the figure to a maximum of 180º. You will recall that the figure is capable of 270º.




$$CF_\delta := \frac{\frac{1}{2}\cdot\sqrt{2}\cdot CP}{\delta} \qquad FP_\delta := \sqrt{CP^2 - \left(CF_\delta\right)^2}$$

$$CH_\delta := \frac{\left(CF_\delta\right)^2}{CP} \qquad FH_\delta := \sqrt{\left(CF_\delta\right)^2 - \left(CH_\delta\right)^2}$$

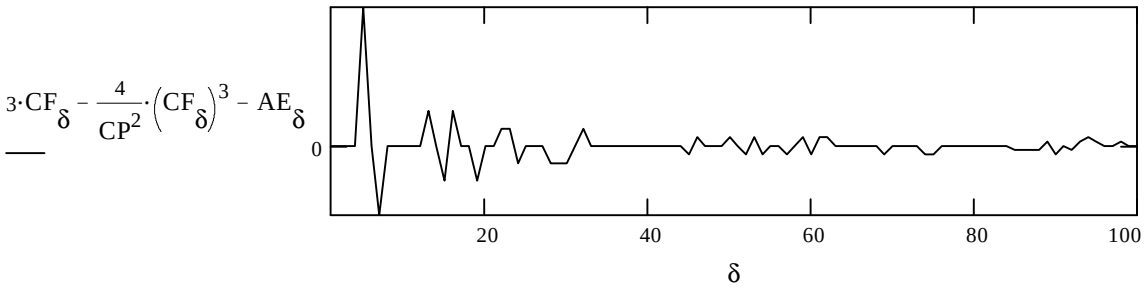
$$OP_\delta := 2\cdot FP_\delta \qquad GO_\delta := \frac{FH_\delta\cdot OP_\delta}{FP_\delta} \qquad CO := CP$$

$$CG_\delta := \sqrt{CO^2 - \left(GO_\delta\right)^2} \qquad AC_\delta := 2\cdot CG_\delta$$

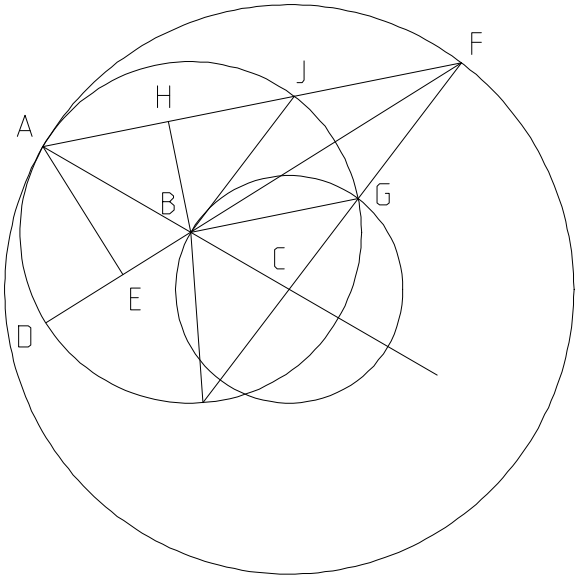
$$AP_\delta := AC_\delta + CP \qquad AE_\delta := \frac{CF_\delta\cdot AP_\delta}{CP}$$

CF is adjusted through D and cannot be less than 1 for the ratio to hold. This constrains the answer to between 0º and 180º.

$$CP \equiv 10 \qquad \delta \equiv 1 \,..\, 100 \qquad 3\cdot CF - \frac{4}{CP^2}\cdot CF^3 - AE = 0$$




The resultant equation seems to support my earlier statement that the same tool used on the cube root figure could also be used on the trisector. To eliminate the radius, simply set it equal to 1.




What is the relationship of DE to BC? $D := 4$

$AB := 99 \quad BC := \frac{AB}{2}\cdot D \quad BG := AB$

$AC := AB + BC \quad AF := \frac{BG\cdot AC}{BC} \quad FJ := AB$

$AJ := AF - FJ \quad HJ := \frac{AJ}{2} \quad BJ := AB$

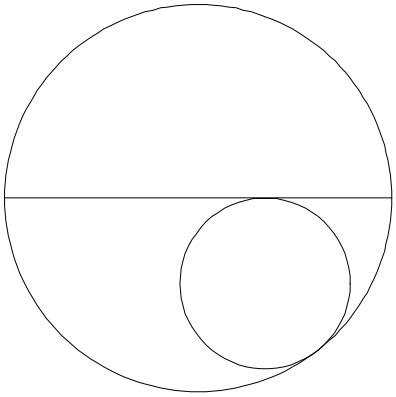
$BH := \sqrt{BJ^2 - HJ^2} \quad FH := HJ + FJ$

$BF := \sqrt{BH^2 + FH^2} \quad BD := AB \quad DF := BF + BD$

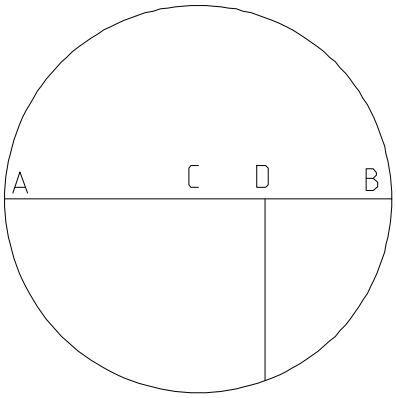
$EF := \frac{FH\cdot AF}{BF} \quad DE := DF - EF \quad DE = 138.133$

$$\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}} + AB - \left[\frac{1}{2}\cdot AB\cdot\frac{(AB + BC)}{BC} + \frac{1}{2}\cdot AB\right]\cdot AB\cdot\frac{(AB + BC)}{\left[BC\cdot\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}}\right]} = 138.133$$

$$\frac{-1}{2}\cdot AB\cdot\frac{\left[-\sqrt{2\cdot BC + AB}\cdot BC - 2\cdot BC^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{2\cdot BC + AB}\right]}{BC^{\left(\frac{3}{2}\right)}} = 138.133$$



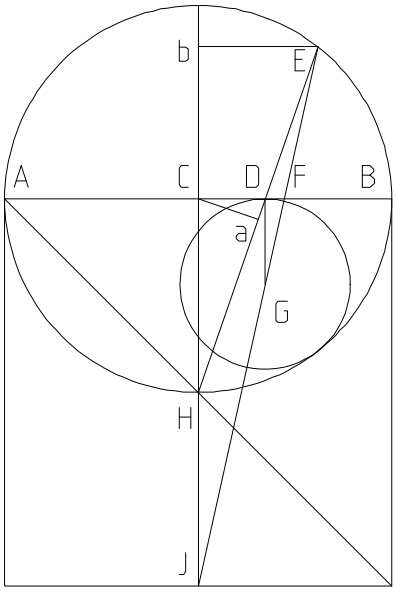
The smaller circle is tangent to the diameter of the larger and also tangent to circumference of the larger. Given the point of tangency on the diameter, what is the radius that will make it tangent to the circumference?




Given a point on the diameter, what is the radius of the inner tangent circle?

$$AB := 100 \quad AC := \frac{AB}{2} \quad CB := AC$$

$$\Delta := 100 \quad \delta := 1 .. \Delta \quad CD_\delta := \frac{CB}{\Delta} \cdot \delta$$




$$CH := AC \quad DH_\delta := \sqrt{\left(CD_\delta\right)^2 + CH^2}$$

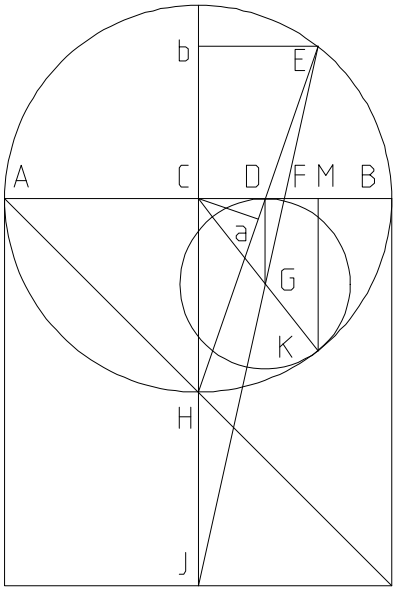
$$Ha_\delta := \frac{CH^2}{DH_\delta} \quad EH_\delta := 2 \cdot Ha_\delta \quad HJ := AC$$

$$Eb_\delta := \frac{CD_\delta \cdot EH_\delta}{DH_\delta} \quad Hb_\delta := \sqrt{\left(EH_\delta\right)^2 - \left(Eb_\delta\right)^2}$$

$$Cb_\delta := Hb_\delta - CH \quad CJ := CH + HJ$$

$$Jb_\delta := CJ + Cb_\delta \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

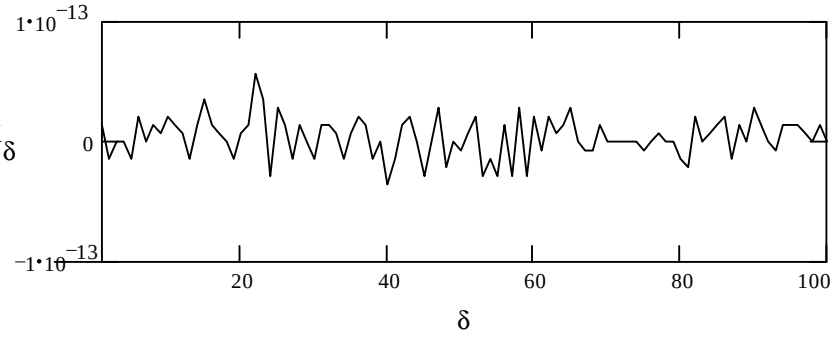


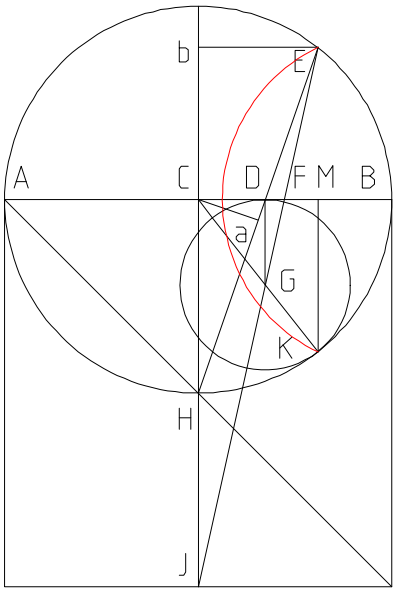

Does CG + GK = AC ?

$CG_\delta := \sqrt{(CD_\delta)^2 + (DG_\delta)^2}$ $GK_\delta := DG_\delta$

$CK_\delta := CG_\delta + GK_\delta$ $\sum_\delta (AC - CK_\delta) = 3.908 \cdot 10^{-13}$

$AC - CK_\delta$

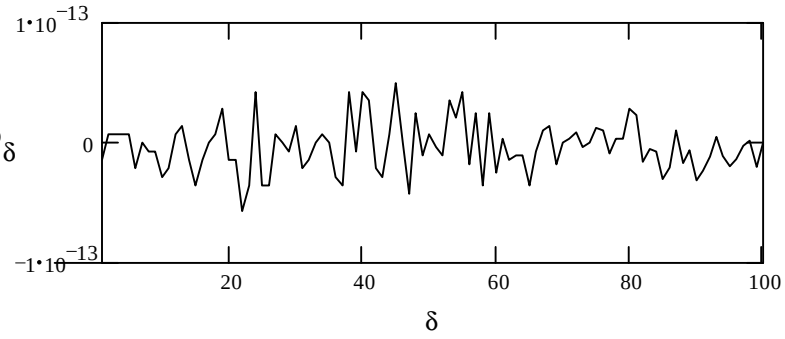


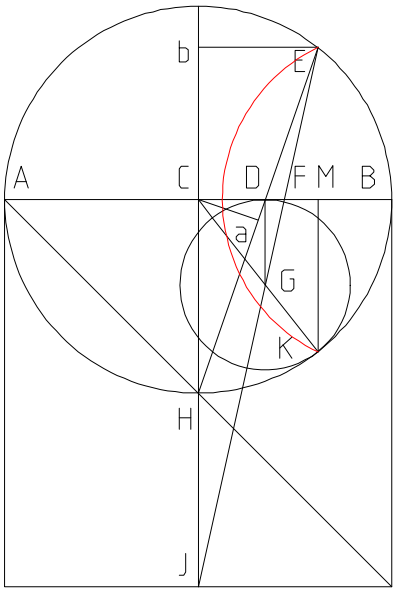




Does Cb = KM?

$KM_\delta := \frac{DG_\delta \cdot CK_\delta}{CG_\delta}$ $\sum_\delta (KM_\delta - Cb_\delta) = -3.524 \cdot 10^{-13}$

$KM_\delta - Cb_\delta$

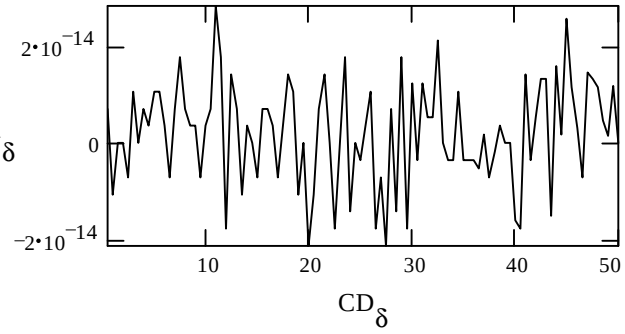


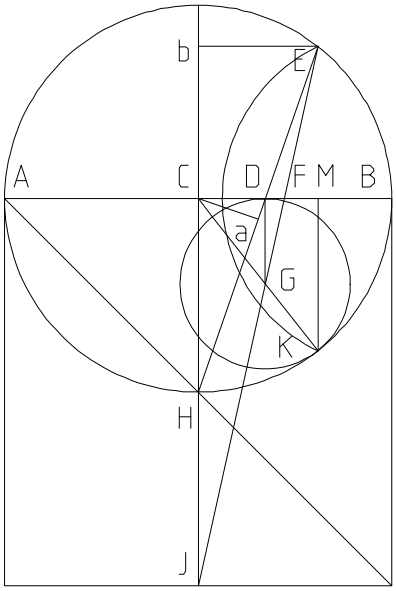



What is the formula for DG when given CD?

$$\sum_{\delta}\left[\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}\right] = 2.072\cdot 10^{-13}$$

$$\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}$$




Given the point on the radius, find DG.

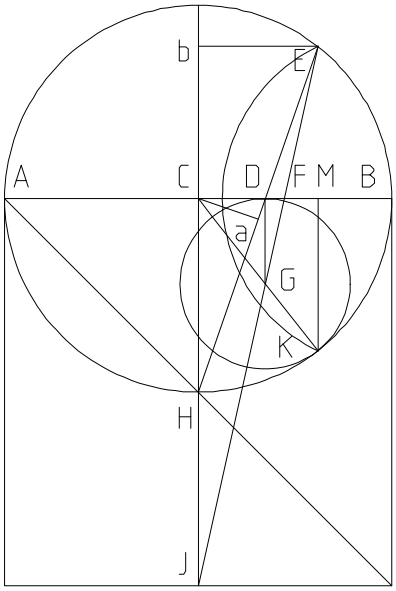
$AB := 4 \quad AC := \frac{AB}{2} \quad CB := AC \quad CM_{\delta} := \frac{CB}{\Delta}\cdot\delta$

$AM_{\delta} := AC + CM_{\delta} \quad BM_{\delta} := CB - CM_{\delta}$

$KM_{\delta} := \sqrt{AM_{\delta}\cdot BM_{\delta}} \quad Cb_{\delta} := KM_{\delta}$

$CH := AC \quad HJ := AC \quad CJ := CH + HJ$

$Jb_{\delta} := CJ + Cb_{\delta} \quad Hb_{\delta} := CH + Cb_{\delta}$



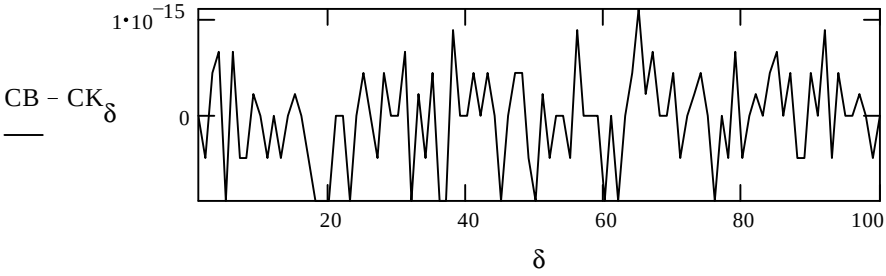

$$Eb_\delta := CM_\delta \quad CD_\delta := \frac{Eb_\delta \cdot CH}{Hb_\delta} \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

Does CK = CB? Make sure there is no typo.

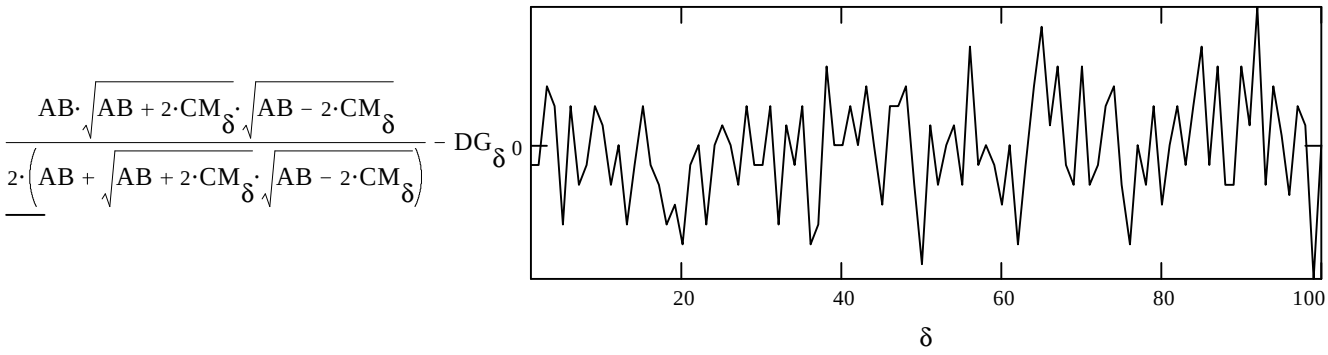
$$CG_\delta := \sqrt{\left(CD_\delta\right)^2 + \left(DG_\delta\right)^2} \quad GK_\delta := DG_\delta$$

$$CK_\delta := CG_\delta + GK_\delta \quad \sum_\delta \left(CB - CK_\delta\right) = -3.331 \cdot 10^{-15}$$

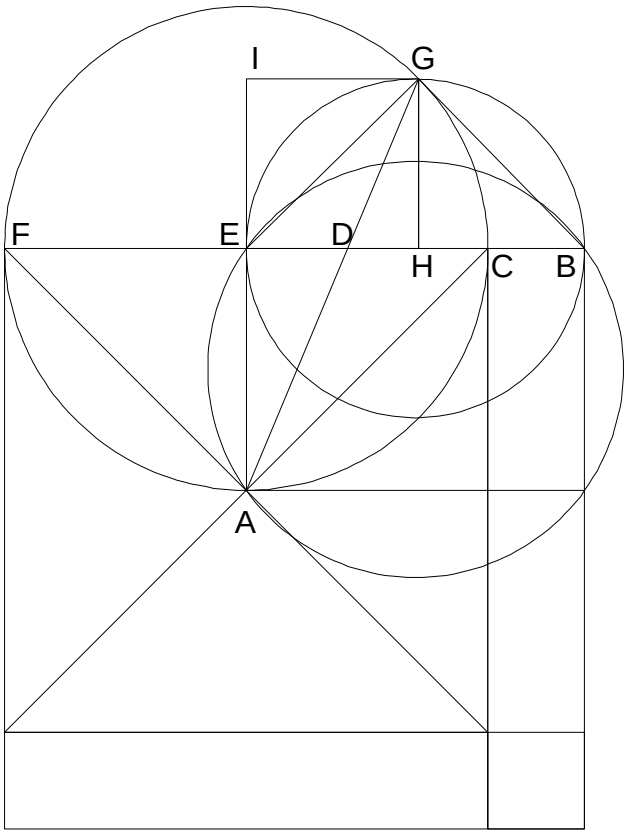



What is the formula for DG, given CM, the perpendicular to the point on the circumference?

$$\sum_\delta \left[ \frac{AB \cdot \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta}}{2 \cdot \left( AB + \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta} \right)} - DG_\delta \right] = -1.749 \cdot 10^{-15}$$



Trivial method for doing Square Roots.




If I draw a circle BE, then a line AG, BD is the square root of $BC \times BF$.

$CF := 1000 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CE := \frac{CF}{2} \quad BE_\delta := BC_\delta + CE$

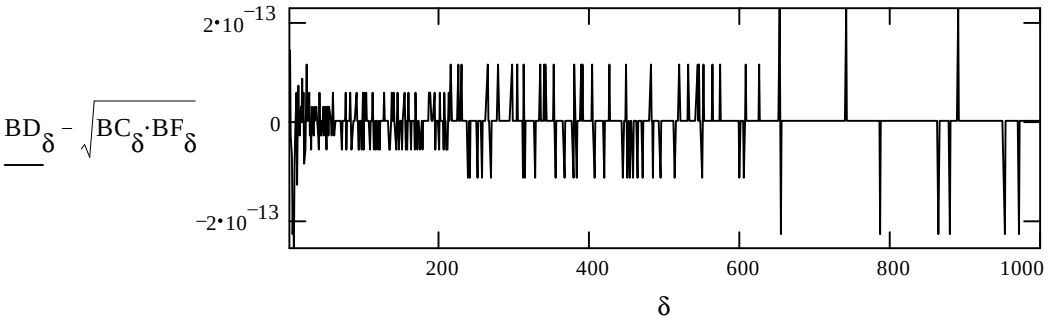
$EG := CE \quad BG_\delta := \sqrt{(BE_\delta)^2 - EG^2}$

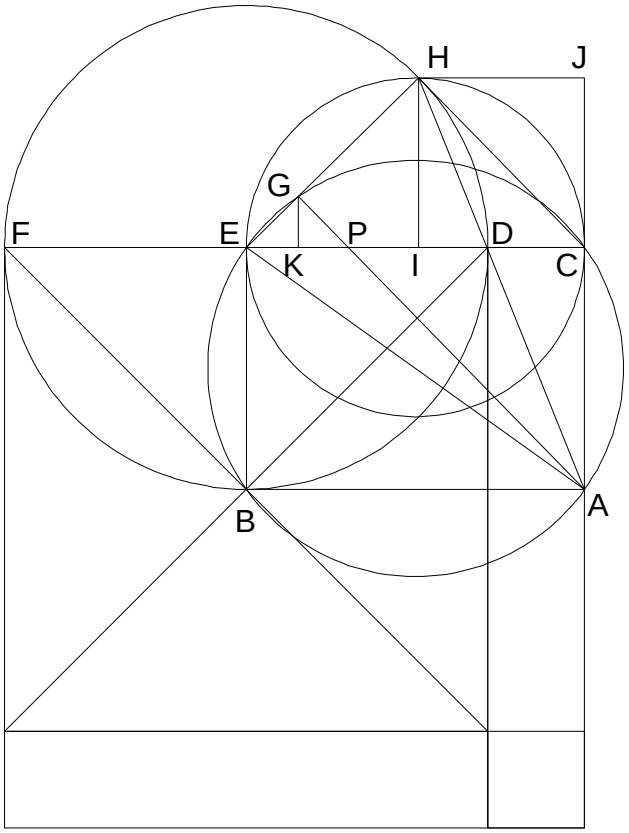
$EH_\delta := \frac{EG^2}{BE_\delta} \quad GH_\delta := \sqrt{EG^2 - (EH_\delta)^2}$

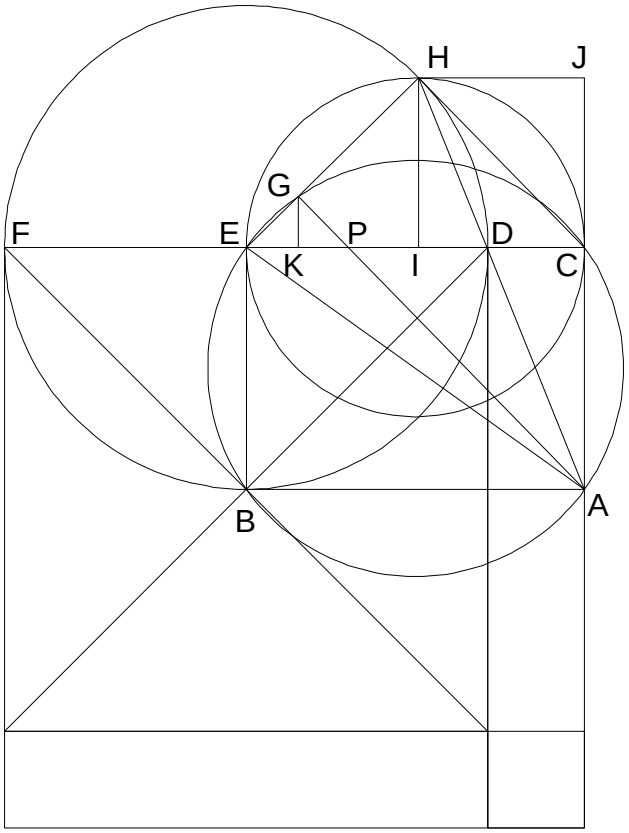
$GI_\delta := EH_\delta \quad EI_\delta := GH_\delta \quad AE := CE$

$AI_\delta := AE + EI_\delta \quad DE_\delta := \frac{GI_\delta \cdot AE}{AI_\delta}$

$BD_\delta := BE_\delta - DE_\delta \quad BF_\delta := BC_\delta + CF$






10_28_94.MCD

Trivial method for doing Square Root.

If I drop in a circle CHE, another ABE, and a right triangle, CHE, then AG provides CP which is the square root of $CD \times CF$.

$DF := 10 \quad \delta := 1 .. 100 \quad CD_\delta := \delta$

$DE := \frac{DF}{2} \quad AC := DE \quad CE_\delta := DE + CD_\delta$

$EH := DE \quad AE_\delta := \sqrt{AC^2 + (CE_\delta)^2}$

$CH_\delta := \sqrt{(CE_\delta)^2 - EH^2} \quad EI_\delta := \frac{EH^2}{CE_\delta}$

$HI_\delta := \sqrt{EH^2 - (EI_\delta)^2} \quad CJ_\delta := HI_\delta$

$CI_\delta := CE_\delta - EI_\delta \quad HJ_\delta := CI_\delta$

$AJ_\delta := AC + CJ_\delta \quad AH_\delta := \sqrt{(AJ_\delta)^2 + (HJ_\delta)^2}$

$HG_\delta := \frac{1}{2} \cdot \frac{(AH_\delta)^2}{EH} + \frac{1}{2} \cdot EH - \frac{1}{2} \cdot \frac{(AE_\delta)^2}{EH}$

$EG_\delta := EH - HG_\delta$

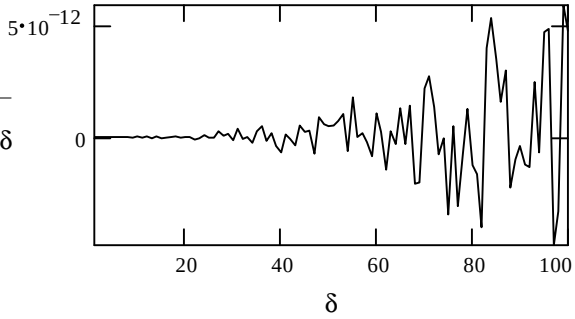
$AG_\delta := \sqrt{(AE_\delta)^2 - (EG_\delta)^2}$

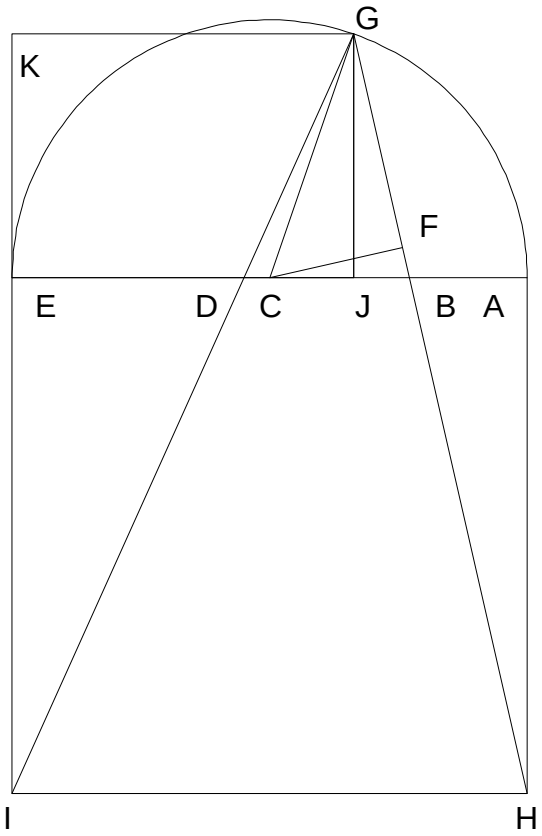
$GK_\delta := \frac{HI_\delta \cdot EG_\delta}{EH} \quad GP_\delta := \frac{AG_\delta \cdot GK_\delta}{AC + GK_\delta}$

$EP_\delta := \sqrt{(EG_\delta)^2 + (GP_\delta)^2}$

$CP_\delta := CE_\delta - EP_\delta \qquad CF_\delta := CD_\delta + DF$

$\underline{CP_\delta} - \sqrt{CD_\delta \cdot CF_\delta}$






10_31_94.MCD

Given AB & BE, divide BE such that BD + DE = BE and $BD = \sqrt{AB \cdot DE}$

**The square in a right triangle on the hypotenuse is equal to the square of the remaining two segments** (and all three squares taken to the point of similarity form a cubic relationship).

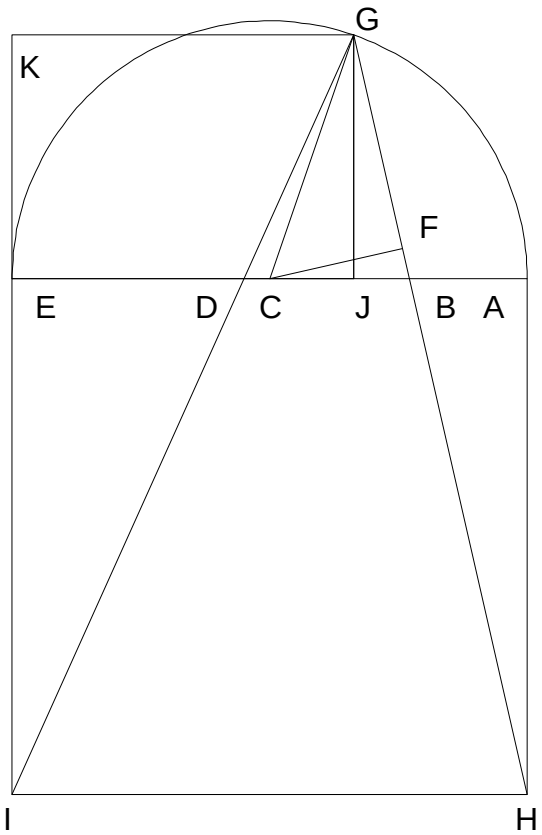
$AE := AB + BE \quad AC := \frac{AE}{2}$

$AH := AE \quad EI := AE \quad BH := \sqrt{AB^2 + AH^2}$

$CG := AC \quad BC := AC - AB \quad BF := \frac{AB \cdot BC}{BH}$

$CF := \frac{AH \cdot BC}{BH} \quad FG := \sqrt{CG^2 - CF^2}$

$BG := BF + FG \quad BJ := \frac{AB \cdot BG}{BH}$



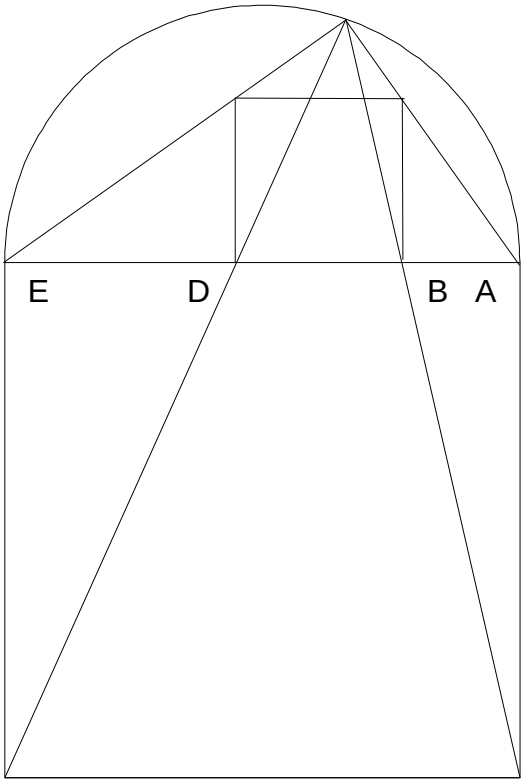

$GJ := \frac{AH \cdot BJ}{AB} \quad AJ := AB + BJ$

$EJ := AE - AJ \quad GK := EJ \quad EK := GJ$

$IK := EI + EK \quad DE := \frac{GK \cdot EI}{IK}$

$BD := BE - DE$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB \cdot DE} = 0$




Reducing the previous tautological chain to single equations for BD and DE.

$AB \equiv 20 \quad BE \equiv 12$

$$BD := \frac{\sqrt{AB}\cdot\left[-AB^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{AB + 4\cdot BE} + \sqrt{AB}\cdot BE \ldots + BE\cdot\sqrt{AB + 4\cdot BE}\right]}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

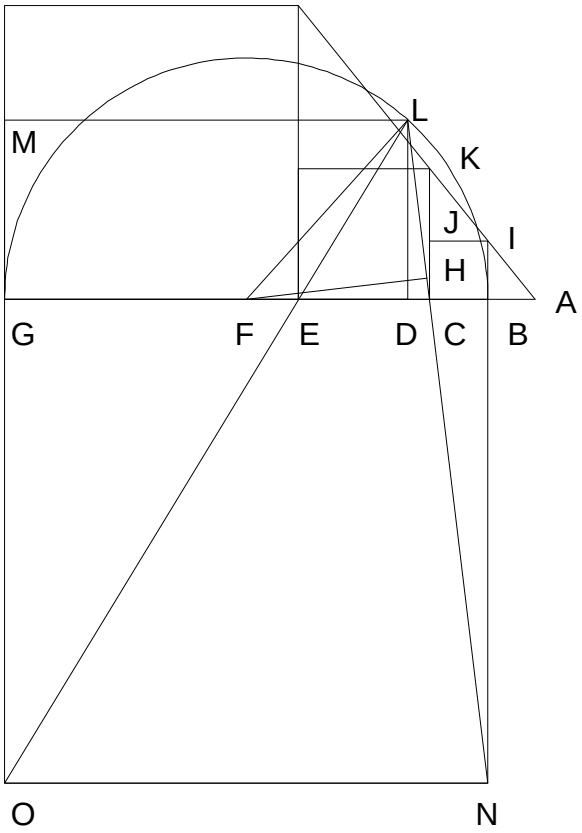
$BD = 8.439$

$$DE := \frac{AB^2 + 2\cdot AB\cdot BE - AB^{\left(\frac{3}{2}\right)}\cdot\sqrt{AB + 4\cdot BE} + 2\cdot BE^2}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

$DE = 3.561$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB\cdot DE} = 0$

Given AB & BE, BE has been divided such that BD + DE = BE and $BD = \sqrt{AB\cdot DE}$




11_01_94.MCD

Given AG - AB = BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BG > 3 BC.

$BG := 6 \quad BC := 1.9 \quad BN := BG$

$BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN}$

$FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN}$

$GM := DL \quad BD := BC + CD \quad DG := BG - BD$

$LM := DG \quad GO := BG \quad MO := GO + GM$

$EG := \frac{LM \cdot GO}{MO} \quad CG := BG - BC$

$CE := CG - EG \quad CJ := BC \quad CK := CE$

$IJ := BC \quad JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2}$

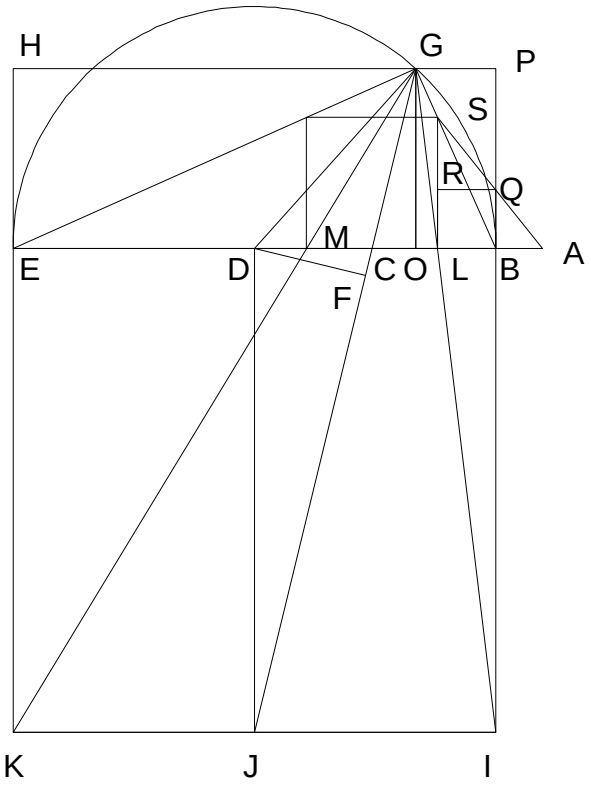
$BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE$

$AB = 36.723 \quad AG = 42.723 \quad AE = 40.621$

$\left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

Given AG - AB =BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, found was AB, AG, and $(AB \cdot AG^2)^{1/3}$.




Given AE - AB = BE and

$$\frac{(AB\cdot AE^2)^{\frac{1}{3}} - (AB^2\cdot AE)^{\frac{1}{3}}}{2} + (AB^2\cdot AE)^{\frac{1}{3}} - AE = BC,$$

find AB.

$BE := 70 \quad BC := 34 \quad BD := \frac{BE}{2} \quad BI := BE$

$DJ := BE \quad EK := BE \quad CD := BD - BC$

$CJ := \sqrt{DJ^2 + CD^2} \quad DF := \frac{DJ\cdot CD}{CJ}$

$DG := BD \quad FG := \sqrt{DG^2 - DF^2} \quad CF := \frac{CD\cdot DF}{DJ}$

$CG := FG - CF \quad CO := \frac{CD\cdot CG}{CJ}$

$DO := CD + CO$

$BO := BD - DO \quad GP := BO \quad GO := \frac{DJ\cdot CG}{CJ}$

$BP := GO \quad IP := BI + BP \quad BL := \frac{GP\cdot BI}{IP}$

$EH := GO \quad HK := EK + EH \quad EO := BE - BO$

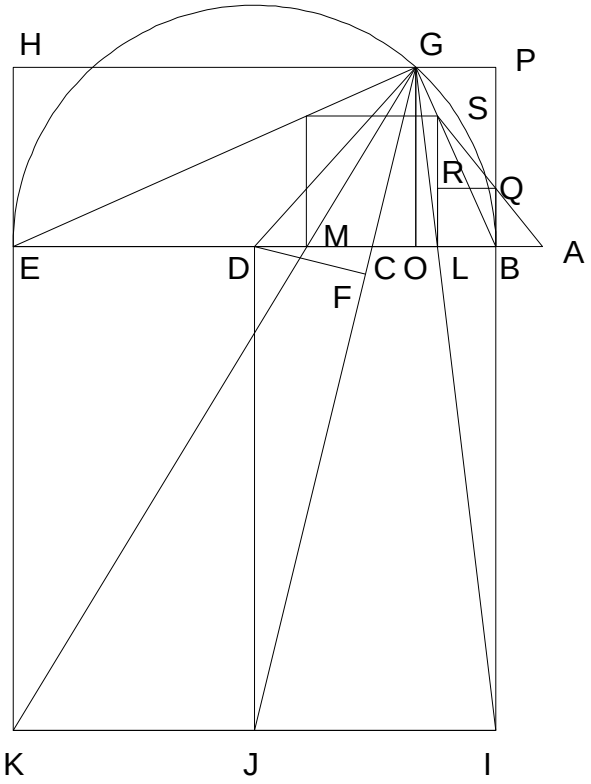
$GH := EO \quad EM := \frac{GH\cdot EK}{HK} \quad BM := BE - EM$

$BQ := BL \quad LR := BL \quad QR := BL$

$LM := BM - BL$

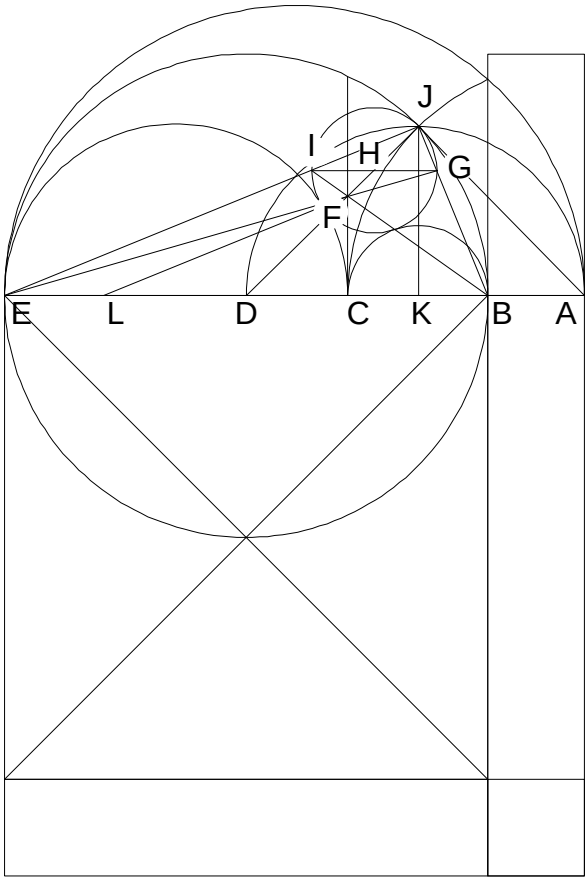
$LS := LM \quad RS := LS - LR \quad AB := \frac{QR\cdot BQ}{RS}$

$AE := AB + BE$




$AB = 510.028 \quad AE = 580.028 \quad BC = 34$

$$\frac{(AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}}{2} + (AB^2 \cdot AE)^{\frac{1}{3}} - AB = 34$$




# Power Line At Square Root.

$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BE := 100 \quad AE_\delta := AB_\delta + BE$

$AC_\delta := \sqrt{AB_\delta \cdot AE_\delta} \quad BD := \frac{BE}{2} \quad DE := BD \quad DJ := BD$

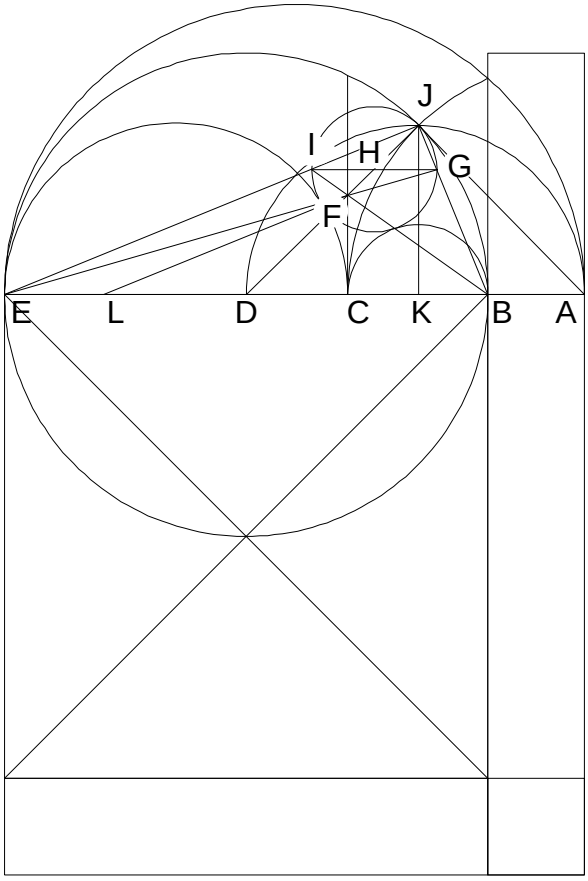
$AD_\delta := AB_\delta + BD \quad DK_\delta := \frac{DJ^2}{AD_\delta} \quad AK_\delta := AD_\delta - DK_\delta$

$BC_\delta := AC_\delta - AB_\delta \quad JK_\delta := \sqrt{AK_\delta \cdot DK_\delta} \quad CD_\delta := BD - BC_\delta$

$CF_\delta := \frac{JK_\delta \cdot CD_\delta}{DK_\delta} \quad BK_\delta := AK_\delta - AB_\delta \quad EK_\delta := BE - BK_\delta$

$CL_\delta := \frac{EK_\delta \cdot CF_\delta}{JK_\delta} \quad BL_\delta := BC_\delta + CL_\delta$

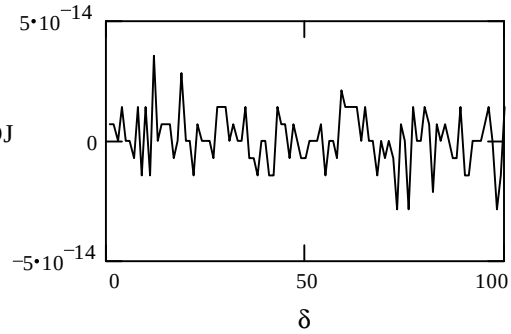
$BF_\delta := \sqrt{(BC_\delta)^2 + (CF_\delta)^2}$

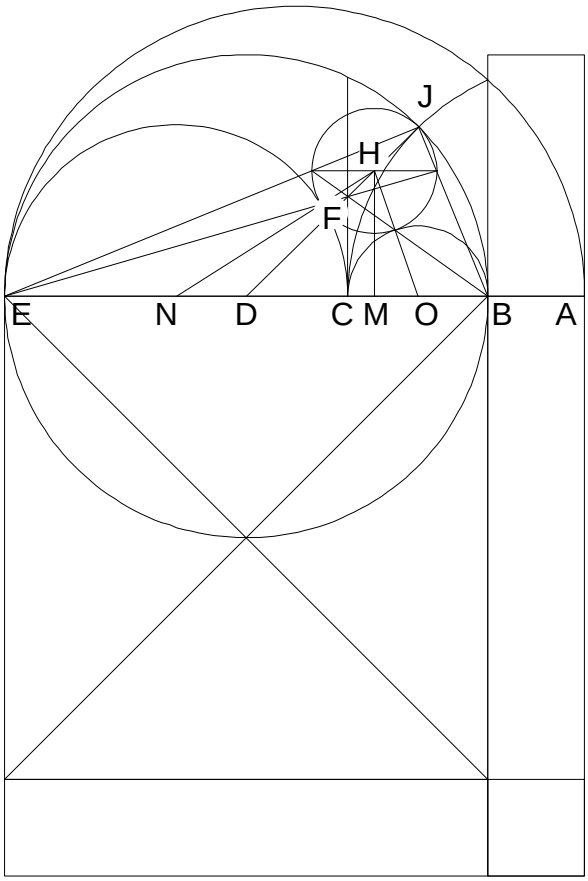



$BI_\delta := \frac{BF_\delta \cdot BE}{BL_\delta} \quad FI_\delta := BI_\delta - BF_\delta \quad GI_\delta := \frac{BE \cdot FI_\delta}{BF_\delta}$

$HJ_\delta := \frac{GI_\delta}{2} \quad DF_\delta := \sqrt{(CD_\delta)^2 + (CF_\delta)^2} \quad FH_\delta := \frac{DF_\delta \cdot GI_\delta}{BE}$

$(DF_\delta + FH_\delta + HJ_\delta) - DJ$



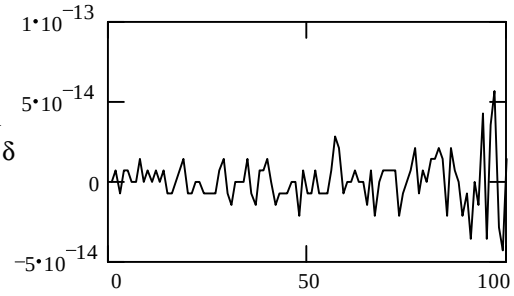



$DH_\delta := DF_\delta + FH_\delta \quad DM_\delta := \frac{CD_\delta \cdot DH_\delta}{DF_\delta} \quad CE_\delta := DE + CD_\delta$

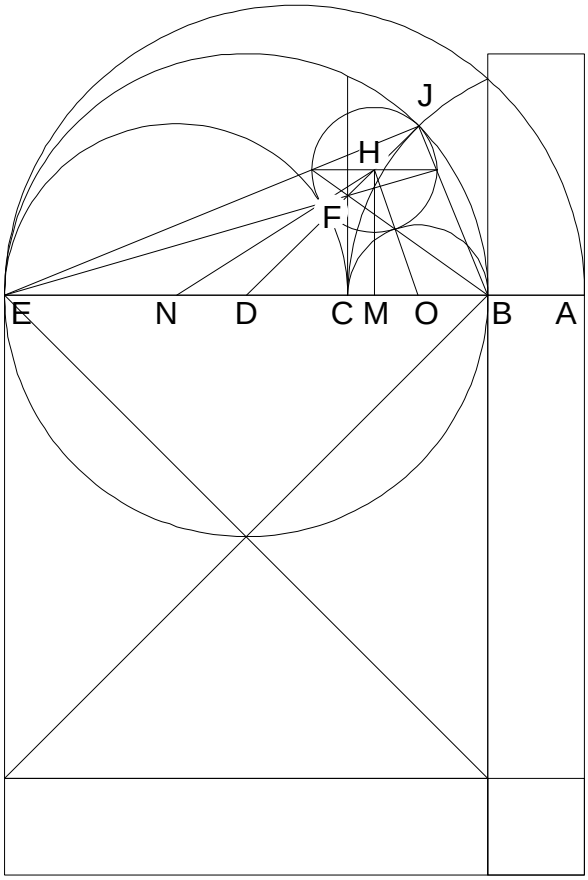
$CM_\delta := DM_\delta - CD_\delta \quad CN_\delta := \frac{CE_\delta}{2} \quad MN_\delta := CN_\delta + CM_\delta$

$HM_\delta := \frac{CF_\delta \cdot DH_\delta}{DF_\delta} \quad HN_\delta := \sqrt{(MN_\delta)^2 + (HM_\delta)^2}$

$\underline{(CN_\delta + HJ_\delta) - HN_\delta}$



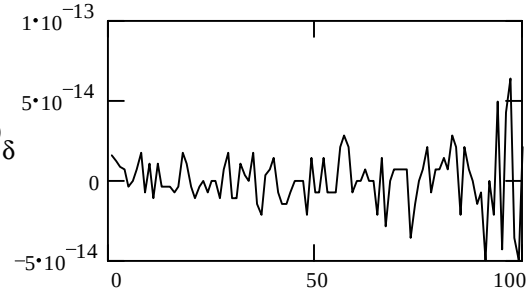

— Is Tangent?




$CO_\delta := \frac{BC_\delta}{2} \quad MO_\delta := CO_\delta - CM_\delta$

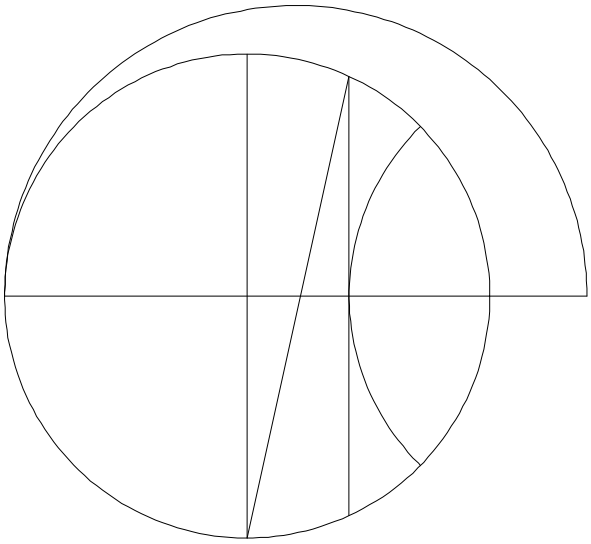
$HO_\delta := \sqrt{(HM_\delta)^2 + (MO_\delta)^2}$

$\underline{(CO_\delta + HJ_\delta) - HO_\delta}$

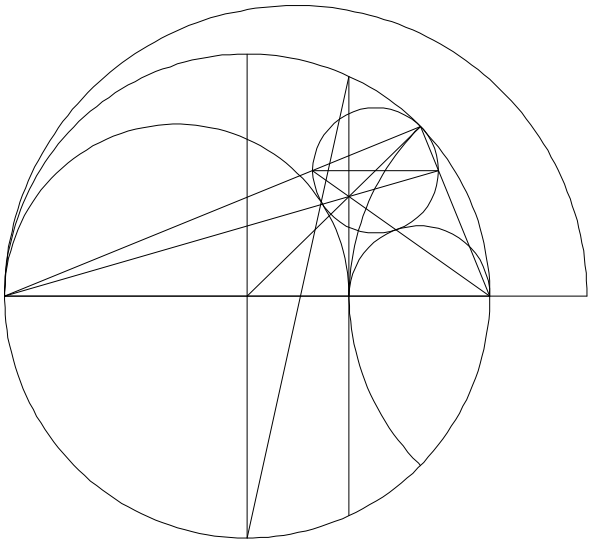


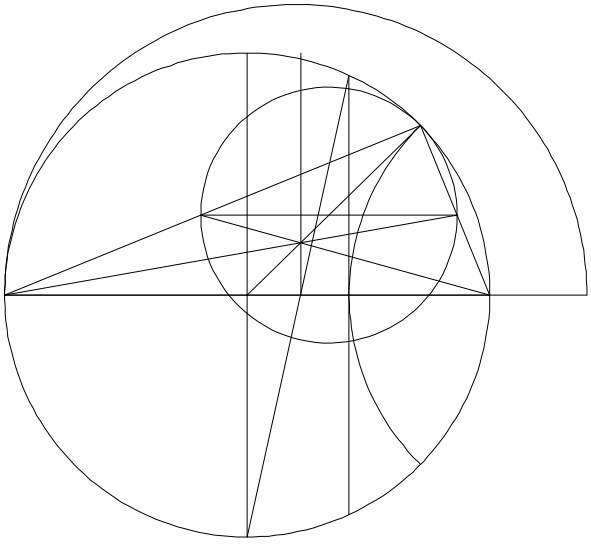

— Is Tangent?

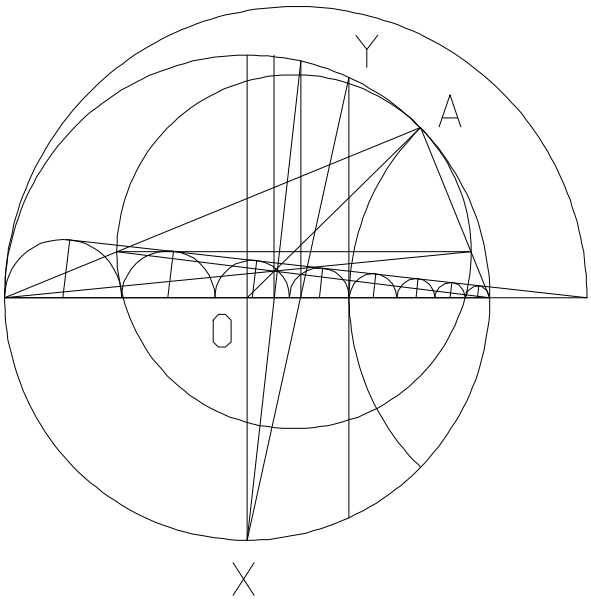
# Two prime exponential series developed through power line progression.



I will present a series of plates to explain the process. The process can be infinitly repeated, supposing you had the tools to do it with.

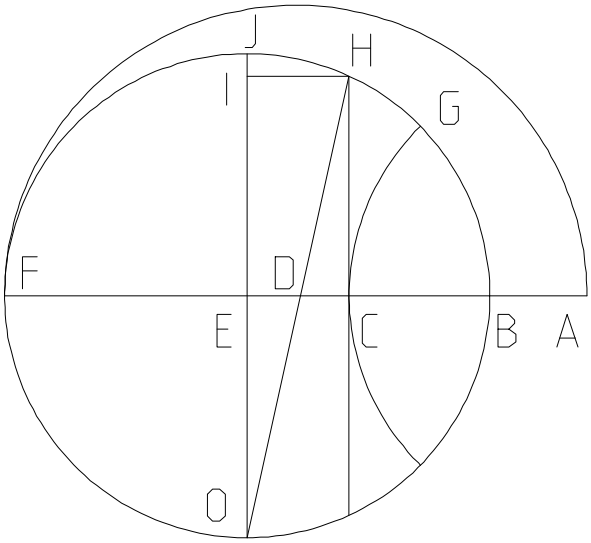







It is clear how OA uses the power line XY to provide a 2 prime exponential series.

Possible Problem: From a similarity point outside of a circle, place some 2 prine sequence of smaller circles on the larger circles diameter, all tangent in sequence.



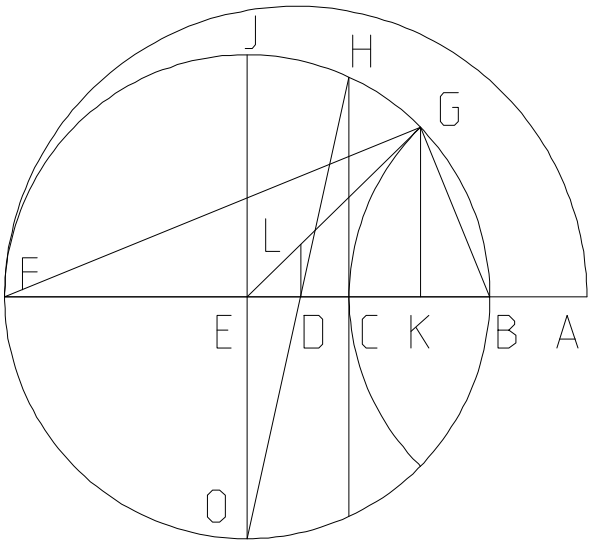

$AB := 1 \quad BF := 5 \quad AF := AB + BF$

$AC := \sqrt{AB \cdot AF} \qquad BE := \frac{BF}{2} \quad EO := BE$

$BC := AC - AB \qquad CF := BF - BC$

$CH := \sqrt{BC \cdot CF} \quad EI := CH \quad CE := BE - BC$

$HI := CE \quad IO := EO + EI \quad DE := \frac{HI \cdot EO}{IO}$

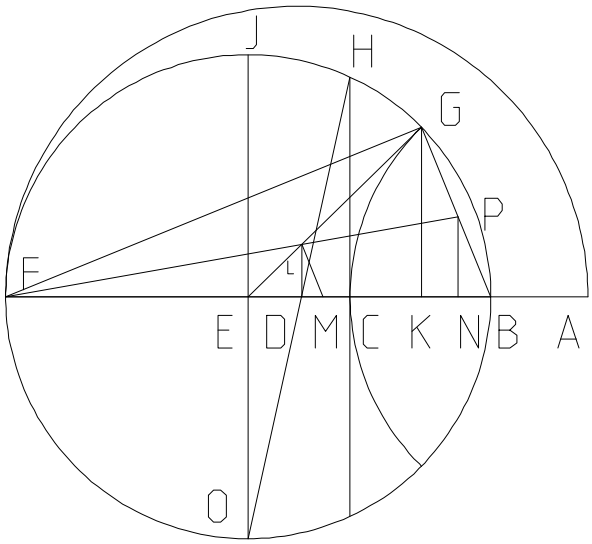



See 12_26_94.MCD for next equation.

$$GK := \frac{BF \cdot \left(\sqrt{AB} \cdot \sqrt{AB + BF}\right)}{(2 \cdot AB + BF)}$$

$EG := BE \quad EK := \sqrt{EG^2 - GK^2}$

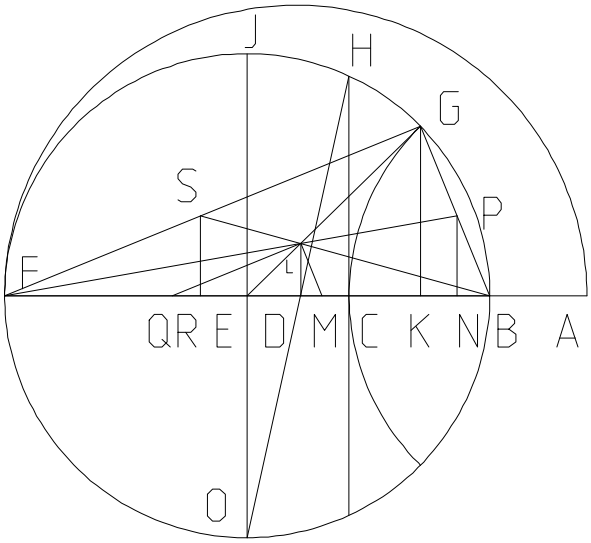
$DL := \frac{GK \cdot DE}{EK}$




$KN := BE - EK \quad DM := \frac{KN \cdot DE}{EK}$

$EF := BE \quad FM := EF + DM + DE$

$BN := \frac{DM \cdot BF}{FM} \quad NP := \frac{DL \cdot BF}{FM}$



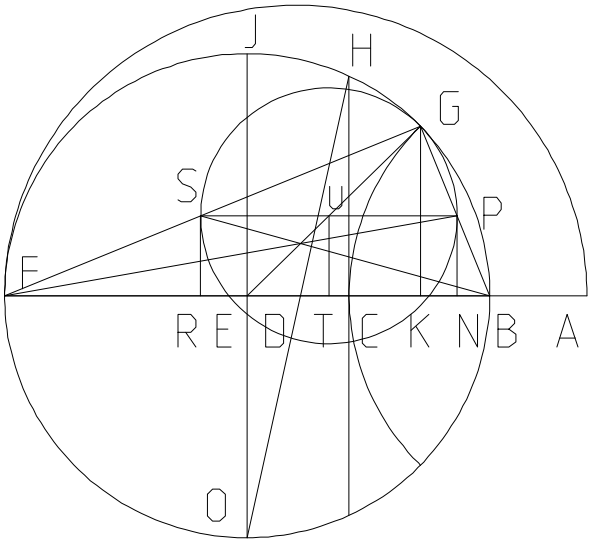

$KF := EK + EF \quad DQ := \frac{KF \cdot DL}{GK}$

$BD := BE - DE \quad BQ := BD + DQ$

$BR := \frac{BD \cdot BF}{BQ} \quad RS := \frac{DL \cdot BR}{BD}$

Are RS and NP equal?

$RS - NP = 0$




$TU := NP \quad ET := \frac{EK \cdot TU}{GK} \quad NR := BR - BN$

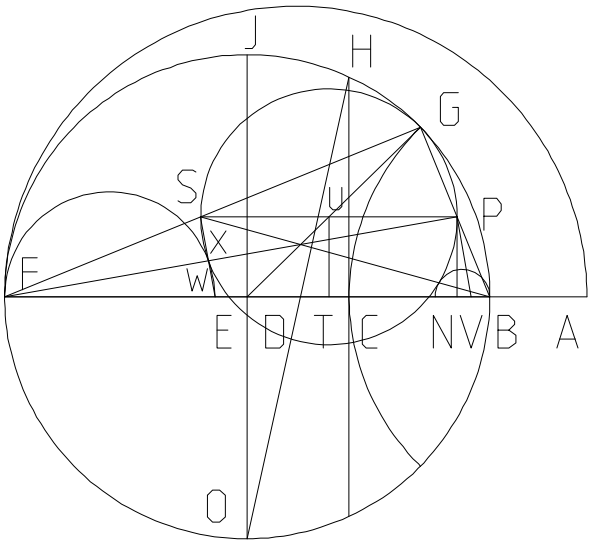
$EN := BE - BN \quad NT := EN - ET$

$PS := NR \quad PU := NT \quad EU := \sqrt{ET^2 + TU^2}$

Is NT half of NR? $\frac{NR}{NT} = 2$

Does GU = PU? $GU := EG - EU$

$GU - PU = 0$



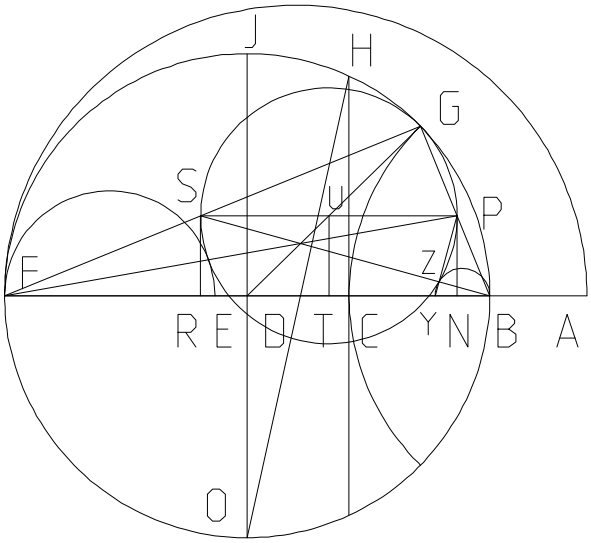

$BT := BN + NT \quad FN := BF - BN \quad FP := \sqrt{NP^2 + FN^2}$

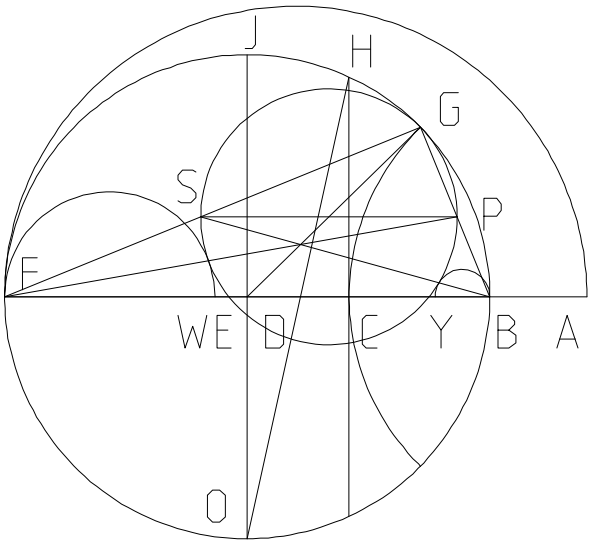
$FV := \frac{FP^2}{FN} \quad PX := \frac{FP \cdot PS}{FV} \quad FX := FP - PX$

$FW := \frac{FV \cdot FX}{FP} \quad BW := BF - FW \quad AW := AB + BW$

Is AW a quarter root of AB, AF?

$\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AW = 0$



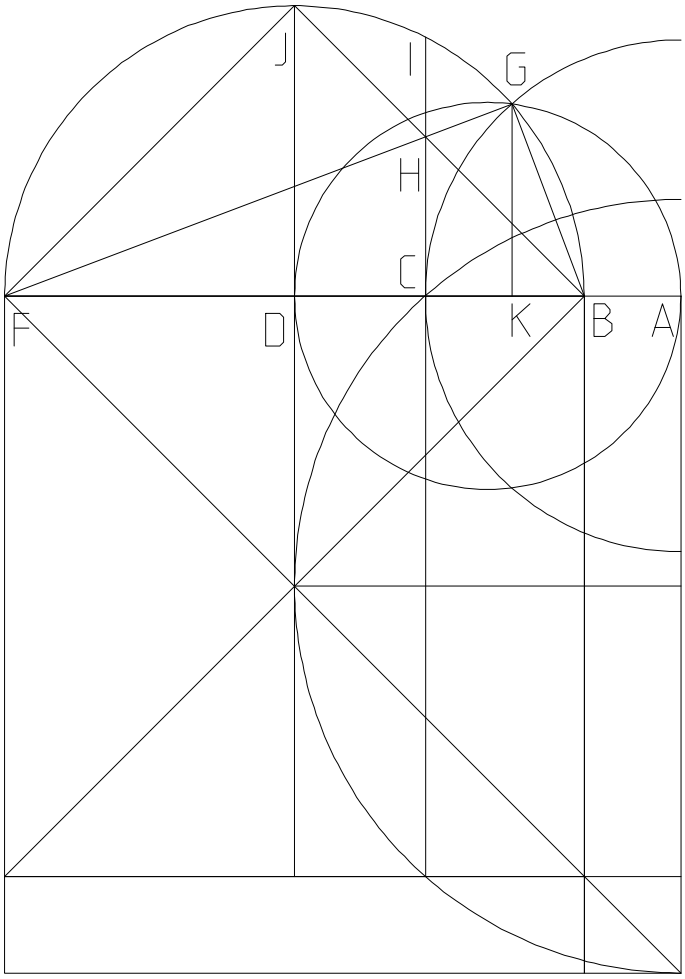

$BS := \sqrt{BR^2 + RS^2}$ $SZ := \frac{BR \cdot PS}{BS}$

$BZ := BS - SZ$ $BY := \frac{PS \cdot BZ}{SZ}$

$AY := AB + BY$

Is AY a quarter root of AB, AF?

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AY = 0$




Does H and G have a constant relationship?

$\delta := 1 .. 1000 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$BF := 6 \quad BD := \frac{BF}{2} \quad AF_\delta := AB_\delta + BF$

$AC_\delta := \sqrt{AB_\delta \cdot AF_\delta} \quad DJ := BD \quad DF := BD$

$BC_\delta := AC_\delta - AB_\delta \quad CD_\delta := BD - BC_\delta$

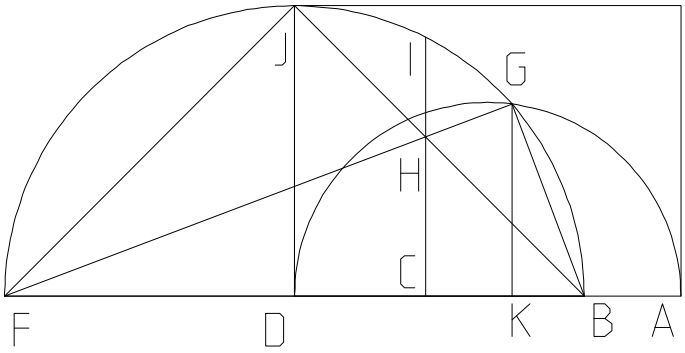
$CH_\delta := \frac{DJ \cdot BC_\delta}{BD} \quad AG_\delta := AC_\delta$

$AD_\delta := AB_\delta + BD \quad AK_\delta := \frac{(AG_\delta)^2}{AD_\delta}$

$GK_\delta := \sqrt{(AG_\delta)^2 - (AK_\delta)^2}$

$BK_\delta := AK_\delta - AB_\delta \quad CK_\delta := BC_\delta - BK_\delta$

$KF_\delta := CK_\delta + CD_\delta + DF$

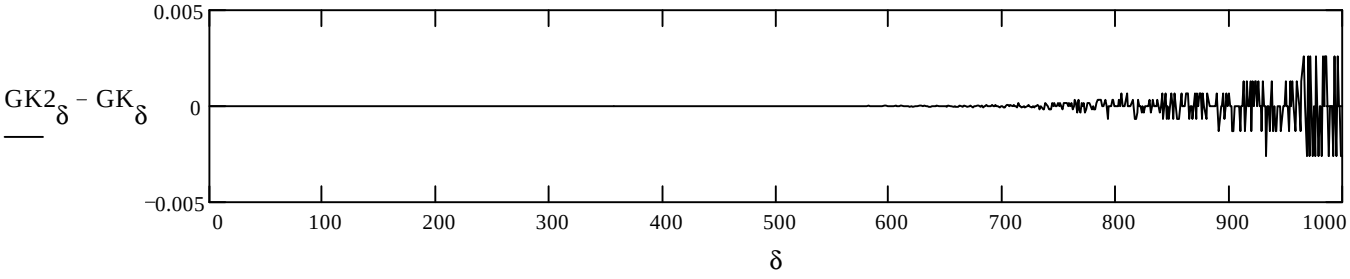


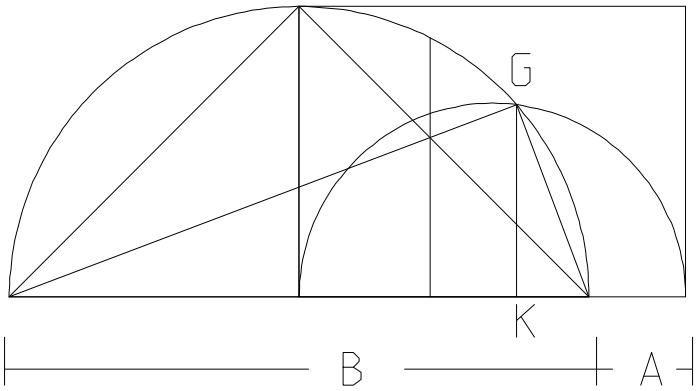

Does FH and FG have identical slopes?

$CF_\delta := CD_\delta + DF$

$GK2_\delta := \frac{CH_\delta \cdot KF_\delta}{CF_\delta}$

Therefore G and H are constantly co-linear.





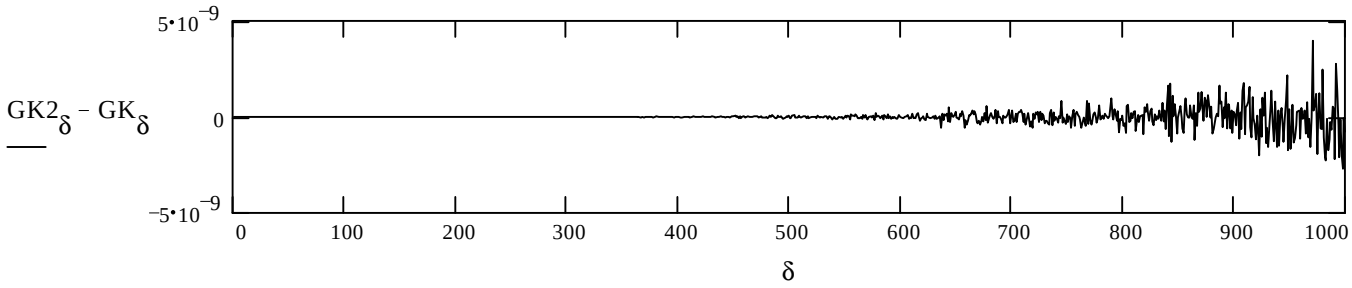

Thus this file can be redone as: "Given BC = $\sqrt{AB \cdot AF}$ and BF, find AB."

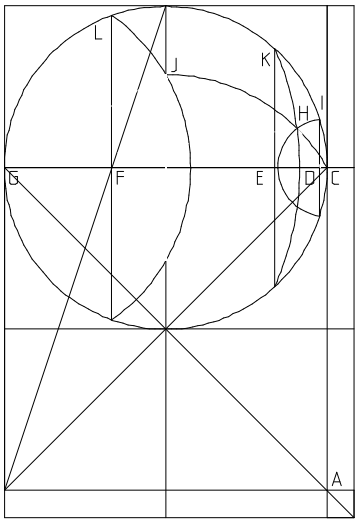
The Formula for GK vs. GK2 demonstrates that the symbolic processor cannot always resolve to simplest form. GK2 is the processors final attempt. An attempt with Mathcad 6 gives the same result.

$A_\delta := AB_\delta \quad B := BF$

$$GK2_\delta := B \cdot \frac{\left[ \left(A_\delta\right)^{\left(\frac{3}{2}\right)} \cdot \sqrt{A_\delta + B} - \left(A_\delta\right)^2 + B \cdot \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} - B \cdot A_\delta \right]}{\left[ \left(2 \cdot A_\delta + B\right) \cdot \left(B - \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} + A_\delta\right) \right]}$$

$$GK_\delta := \frac{B \cdot \left(\sqrt{A_\delta} \cdot \sqrt{A_\delta + B}\right)}{\left(2 \cdot A_\delta + B\right)}$$





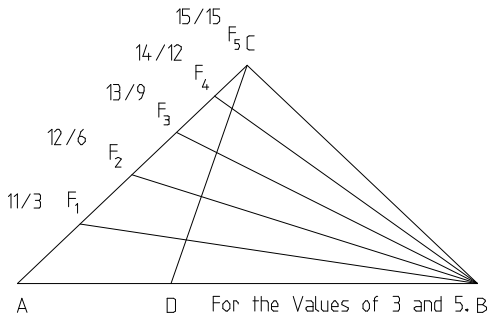

# One Circle

**By John Clark**

# One Square

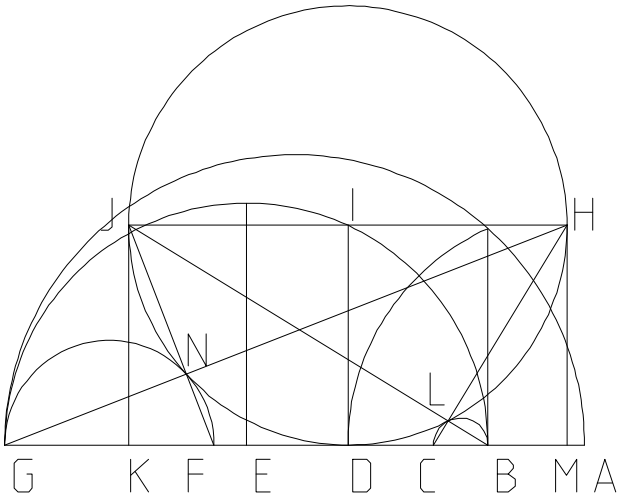
# One Line

## And the Delian Quest




# 1995

# Alternate method for Quad Roots




$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BG := 10 \quad AG_\delta := AB_\delta + BG$

$AD_\delta := \sqrt{AB_\delta \cdot AG_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

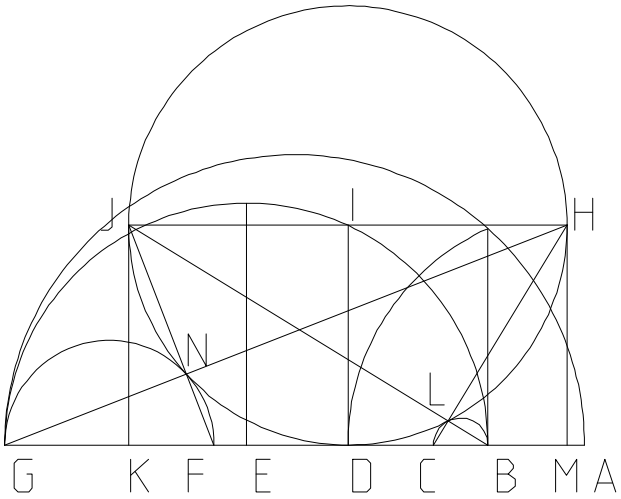
$DG_\delta := BG - BD_\delta \quad DI_\delta := \sqrt{BD_\delta \cdot DG_\delta}$

$HI_\delta := DI_\delta \quad IJ_\delta := DI_\delta \quad HJ_\delta := HI_\delta + IJ_\delta$

$DK_\delta := DI_\delta \quad JK_\delta := DI_\delta \quad BK_\delta := BD_\delta + DK_\delta$

$BJ_\delta := \sqrt{\left(BK_\delta\right)^2 + \left(JK_\delta\right)^2} \quad JL_\delta := \frac{BK_\delta \cdot HJ_\delta}{BJ_\delta}$

$BL_\delta := BJ_\delta - JL_\delta \quad BC_\delta := \frac{HJ_\delta \cdot BL_\delta}{JL_\delta}$




$HM_\delta := DI_\delta \quad DM_\delta := DI_\delta \quad MG_\delta := DM_\delta + DG_\delta$

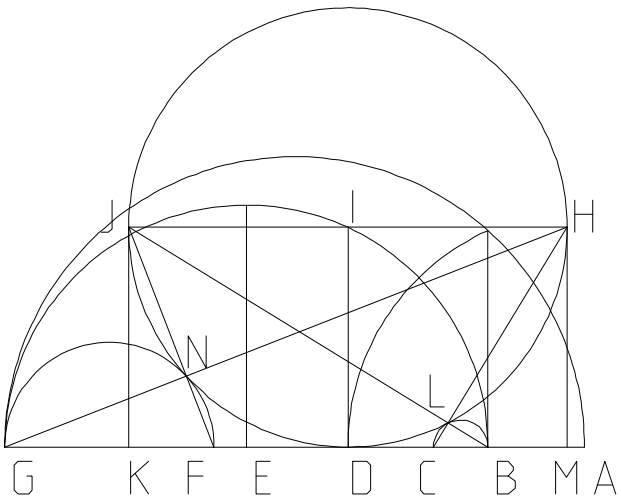
$GH_\delta := \sqrt{\left(MG_\delta\right)^2 + \left(HM_\delta\right)^2} \quad HN_\delta := \frac{MG_\delta \cdot HJ_\delta}{GH_\delta}$

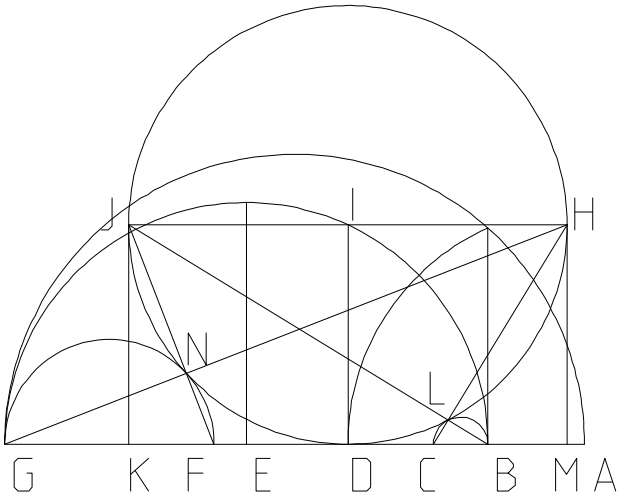
$GN_\delta := GH_\delta - HN_\delta \quad FG_\delta := \frac{HJ_\delta \cdot GN_\delta}{HN_\delta}$

$BF_\delta := BG - FG_\delta$

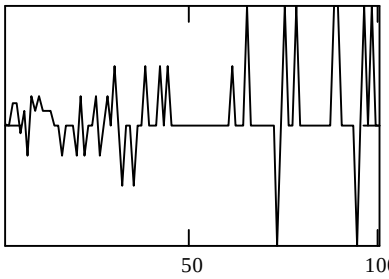
$AF_\delta := AB_\delta + BF_\delta$

$AC_\delta := AB_\delta + BC_\delta$



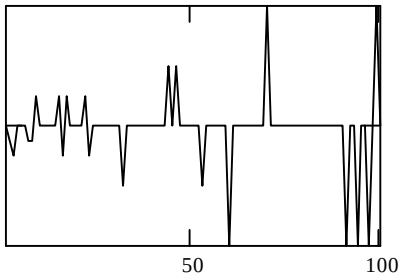



$$\left[ AC_{\delta} - \left[ \left( AB_{\delta} \right)^{3} \cdot AG_{\delta} \right]^{\frac{1}{4}} \right]_{0}$$



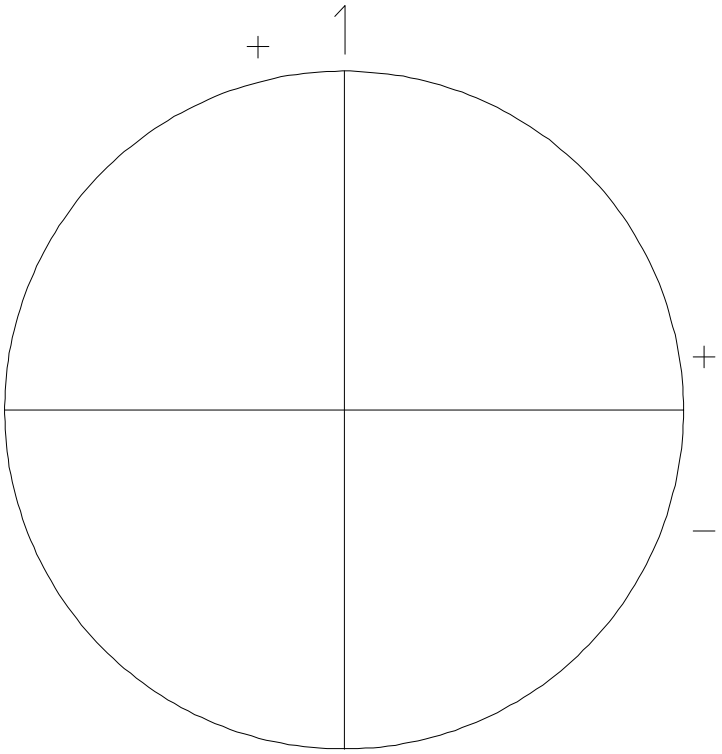

δ

$$\left[ AF_{\delta} - \left[ AB_{\delta} \cdot \left( AG_{\delta} \right)^{3} \right]^{\frac{1}{4}} \right]_{0}$$




δ

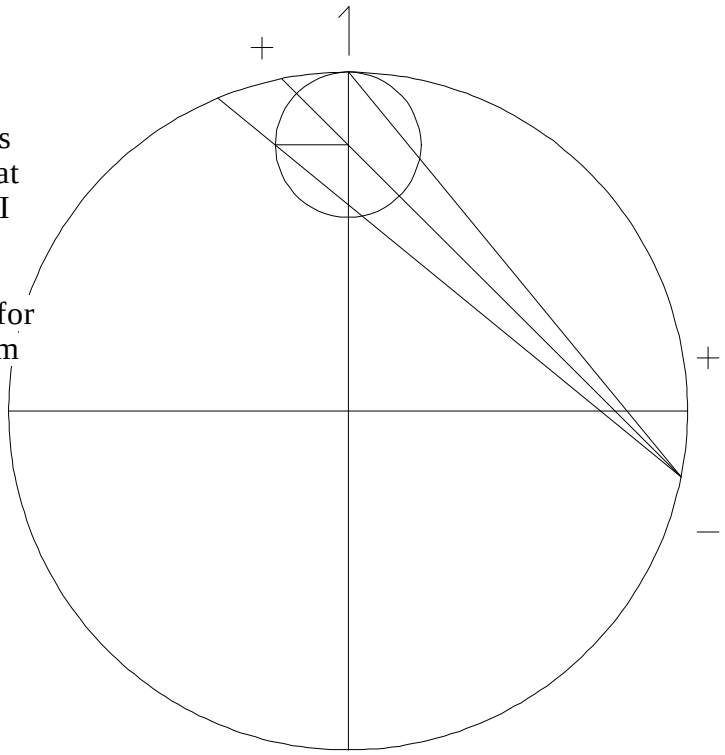
The symbolic processor on my computer could not reduce the chain to the final equations.




Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

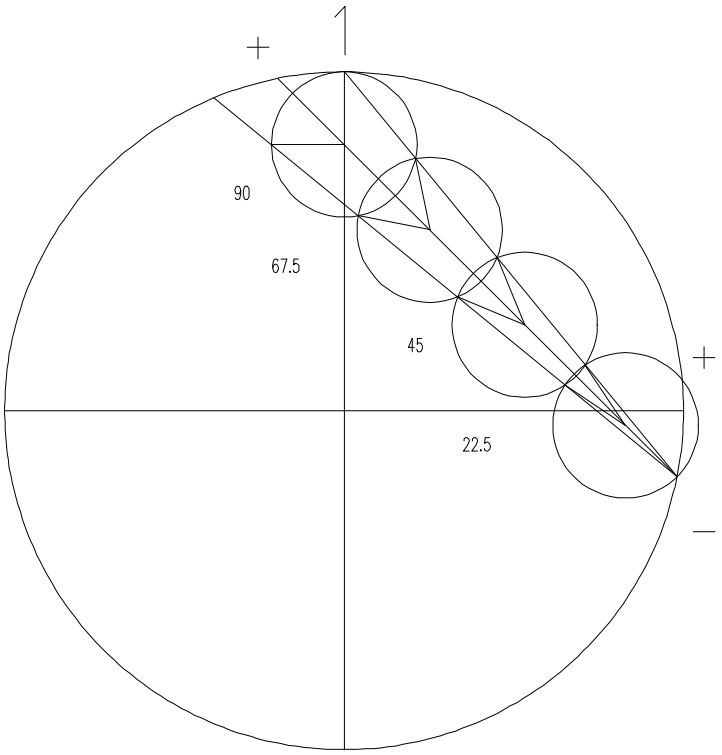
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$




$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

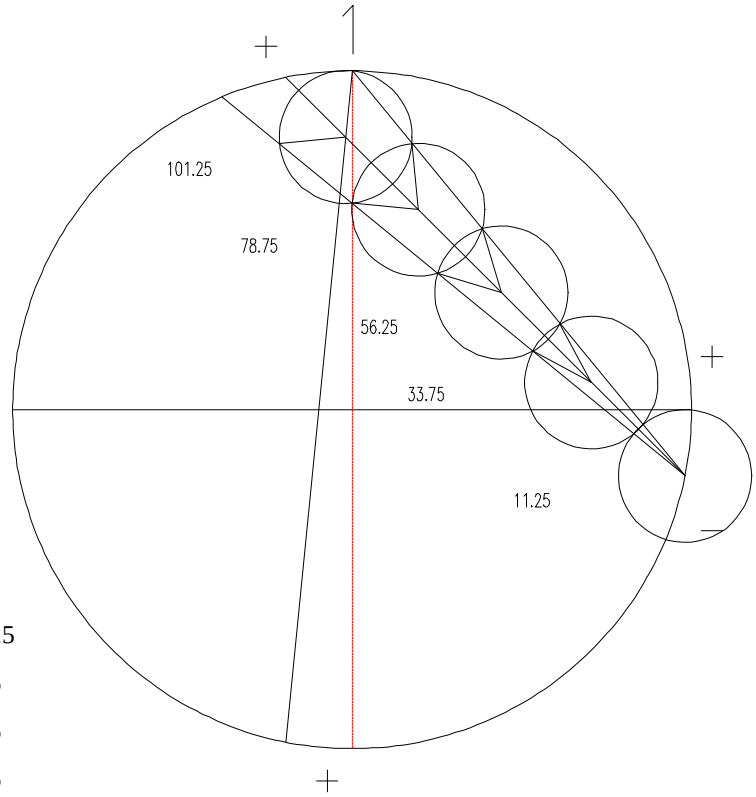
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

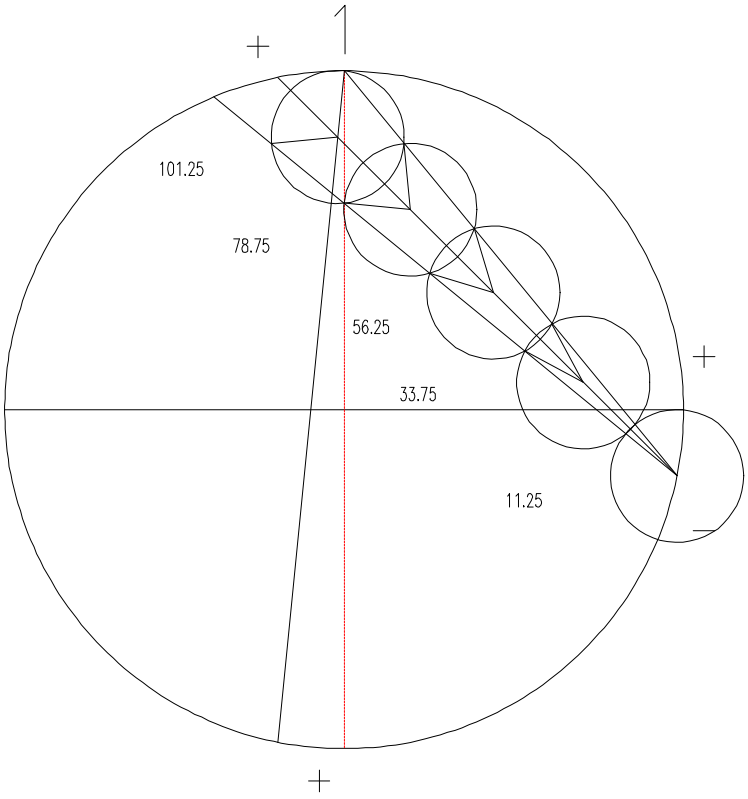
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$




| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

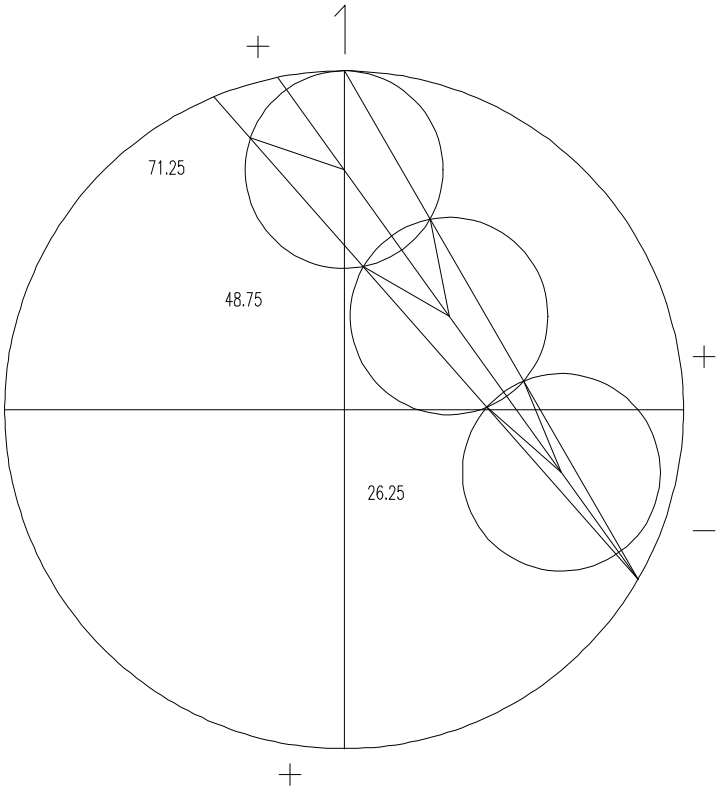
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

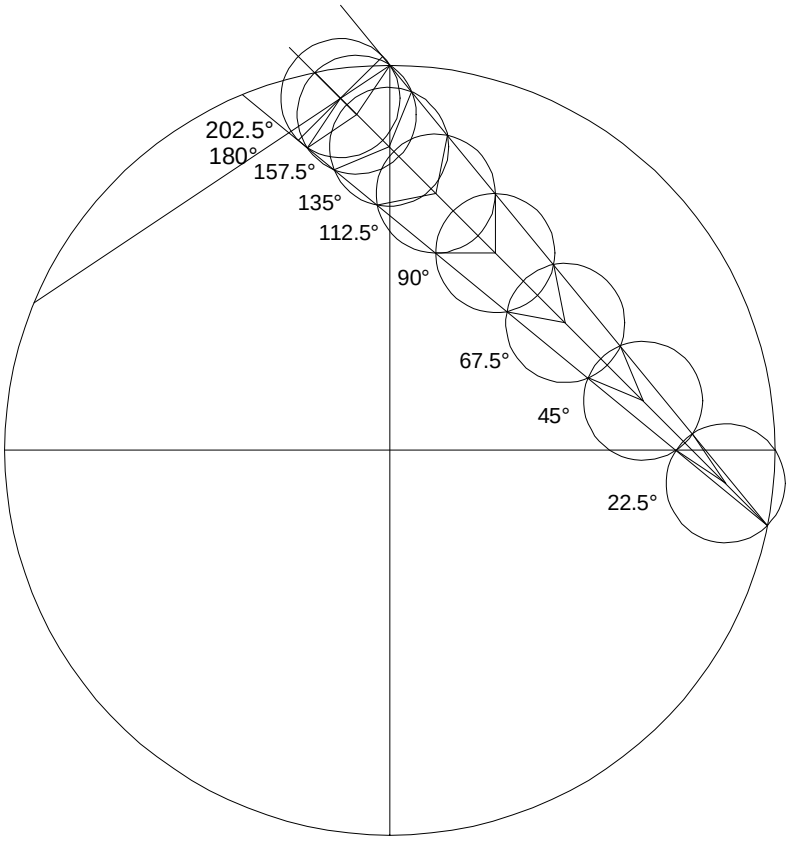
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

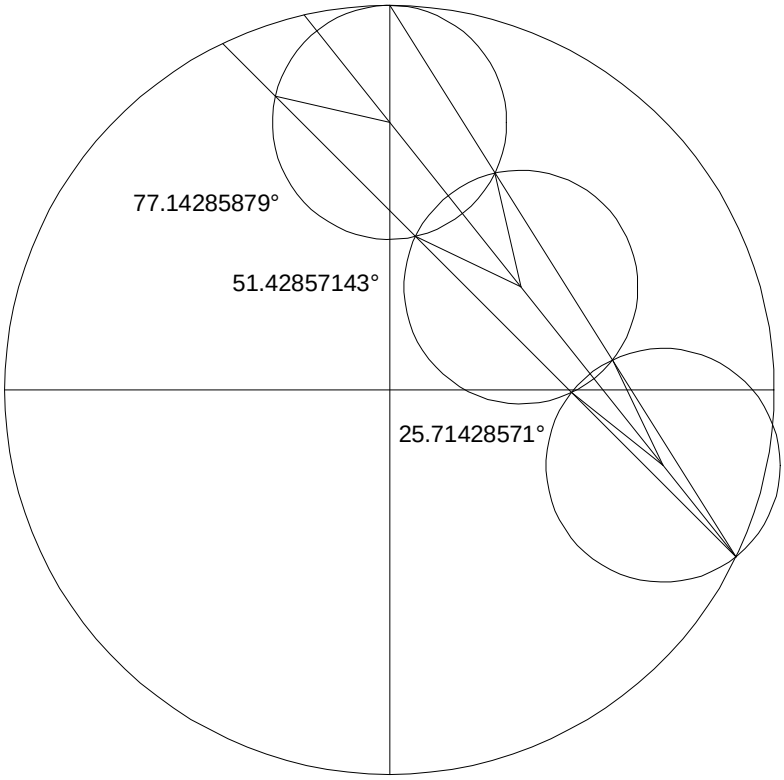
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

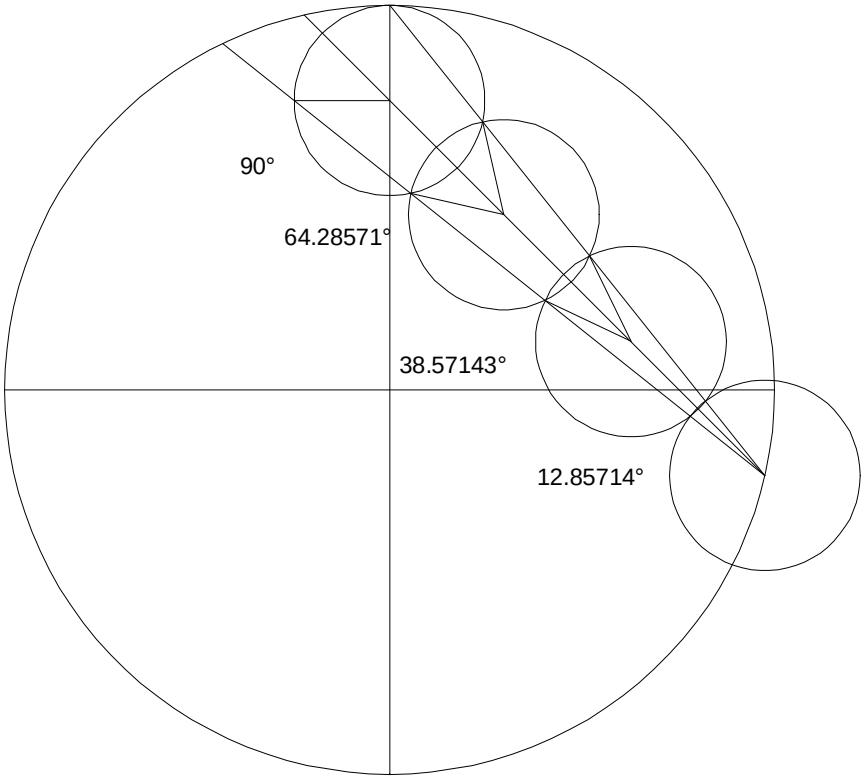
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

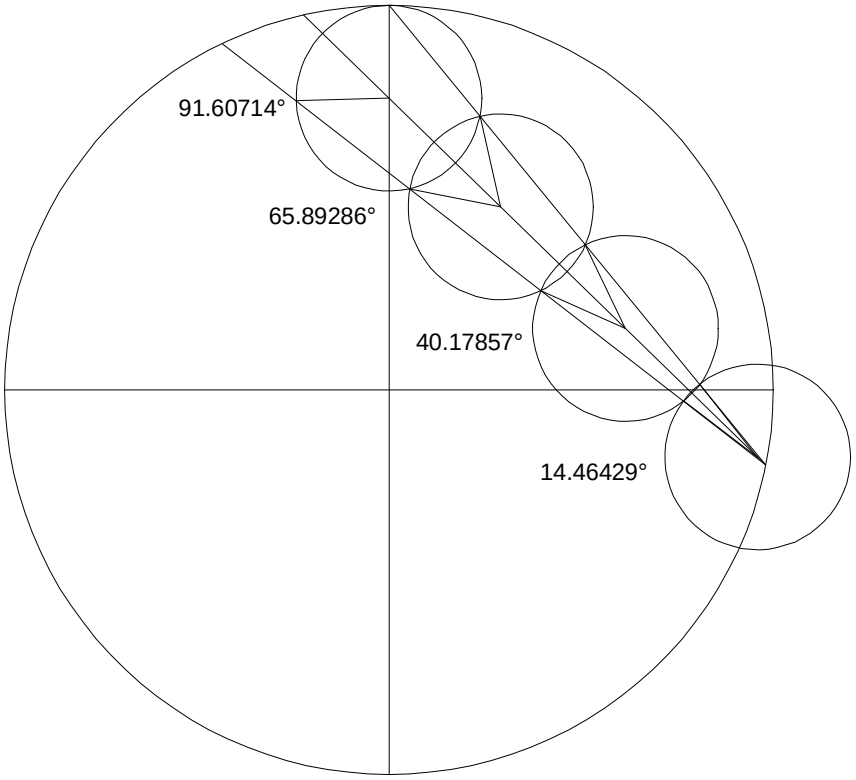
$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

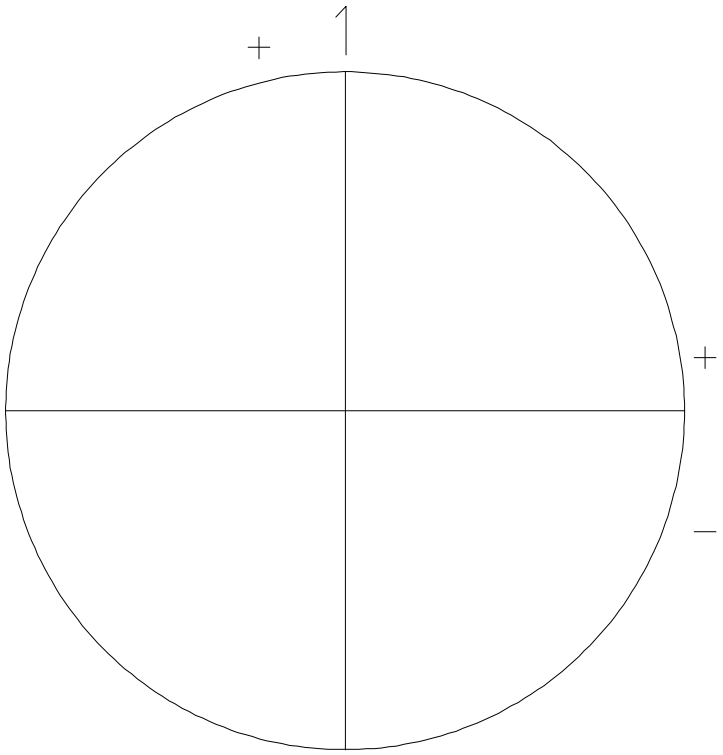
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

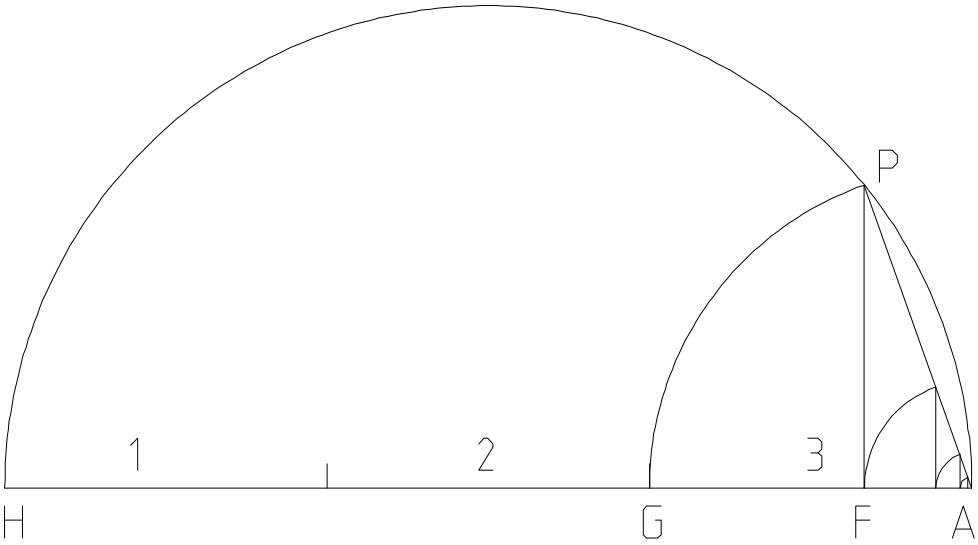
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




Work in progress.




Exponential Progressions. 04_01_95

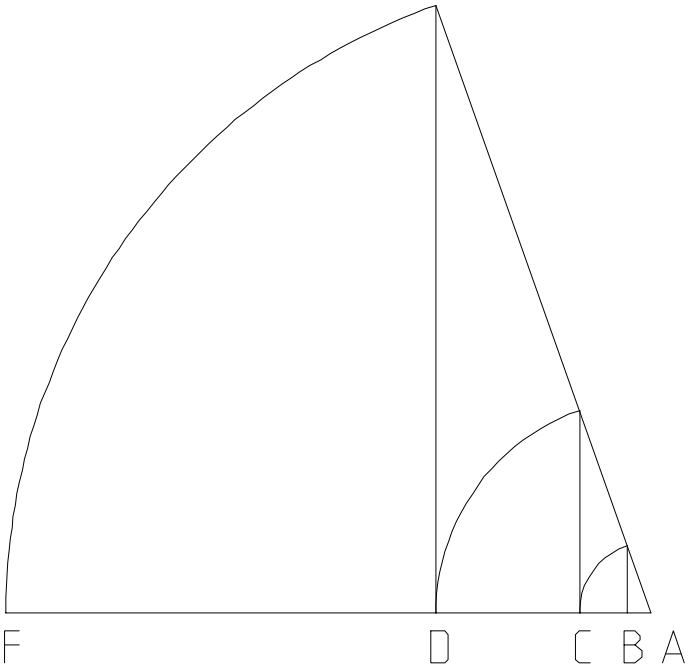
If I want to multiply any number by any power, this is the a geometric process for doing so.

The given figure is drawn for the third power of 3.

$AH := 10 \qquad \delta := 0..10 \qquad BS := 8$

The third division between A and F is very hard to see. BS = Base Segments

$AP := \frac{AH}{BS} \qquad AP = 1.25$




$AG_0 := \frac{(AP)^2}{AH} \qquad AF_0 := AG_0 \qquad \Delta := 2..10$

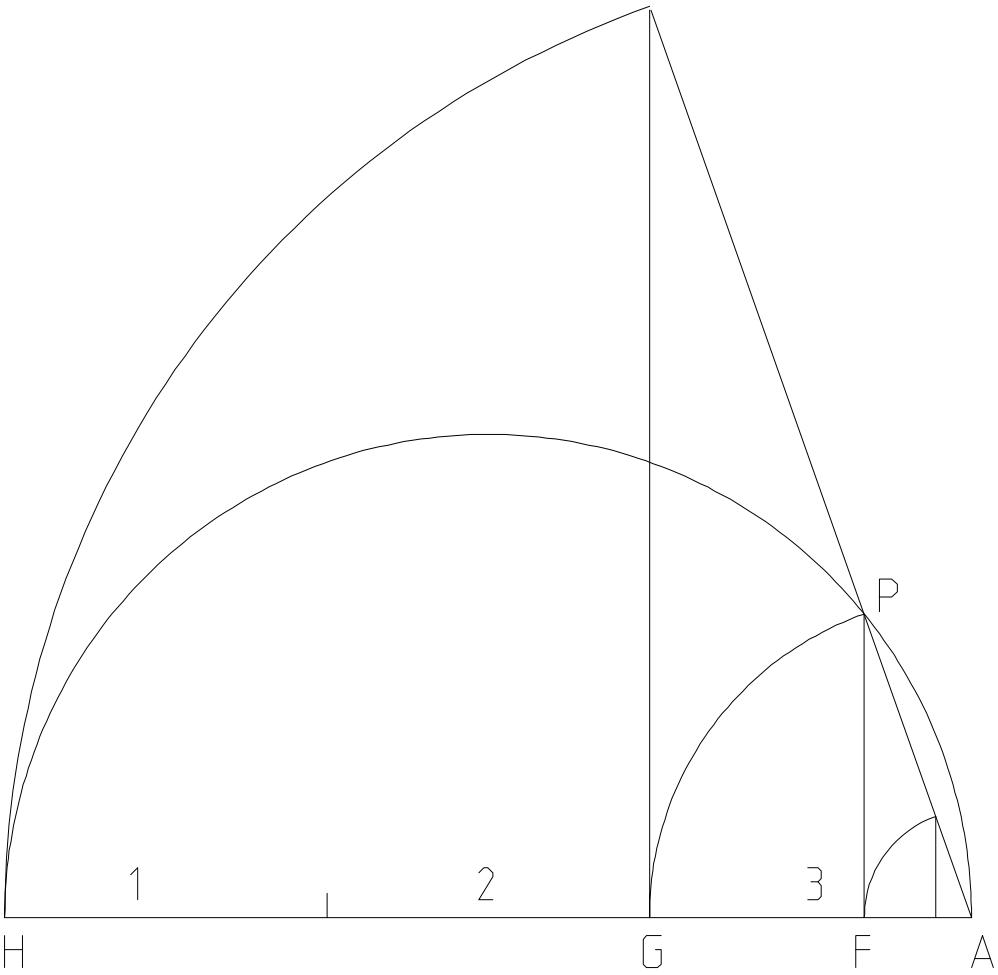
$\begin{pmatrix} AF_{\delta+1} \\ AG_{\delta+1} \end{pmatrix} := \begin{bmatrix} AG_\delta \\ \frac{AP \cdot AG_\delta}{AH} \end{bmatrix} \qquad AB_\delta := \frac{AG_1}{AF_\delta}$

| $\delta$ | $AB_\Delta$ | $BS^\delta$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 8 | 8 |
| 2 | 64 | 64 |
| 3 | 512 | 512 |
| 4 | 4096 | 4096 |
| 5 | 32768 | 32768 |
| 6 | 262144 | 262144 |
| 7 | 2097152 | 2097152 |
| 8 | 16777216 | 16777216 |
| 9 | | 134217728 |
| 10 | | $1.07374182 \cdot 10^9$ |

Making the number of divisions 3, provides 3 cube result. AB divides AF 27 times. Etc. It can be seen that using a normal straight edge and compass one needs a very large piece of paper to work this.

You will notice that I took only one of the possible two divisions from which to project from. The other would be 2/3. At 2/3 my unit divisions would still be 27, but now AB would take up 2 cube of them, or AB would be 8 units.

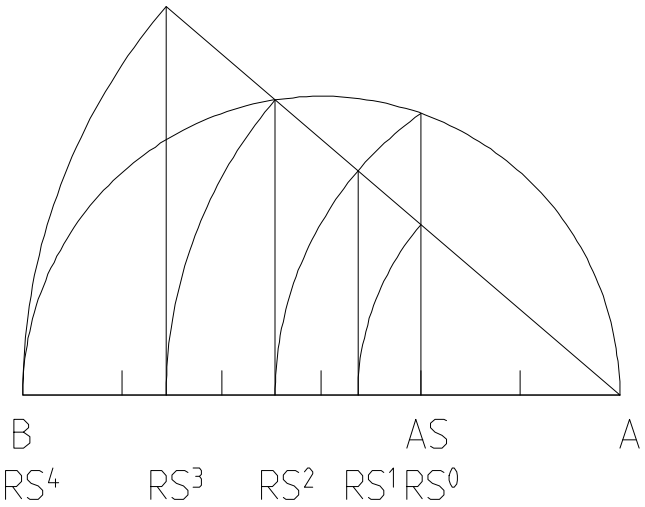
For an 8 cube series then, the value for AB would be 1 of 512, 8 of 512, 27 of 512, 64 of 512, 125 of 512, 216 of 512, 343 of 512




Since the slope is established, one can take the first two divisions and utilizes the whole figure.

$E := 1 .. BS$ $ROOT := 3$

| E | $\frac{BS^{ROOT}}{E^{ROOT}}$ | $E^{ROOT}$ |
|---|---|---|
| 1 | 512 | 1 |
| 2 | 64 | 8 |
| 3 | 18.96296296 | 27 |
| 4 | 8 | 64 |
| 5 | 4.096 | 125 |
| 6 | 2.37037037 | 216 |
| 7 | 1.49271137 | 343 |
| 8 | 1 | 512 |




# About The Laws of Exponents and Ratios

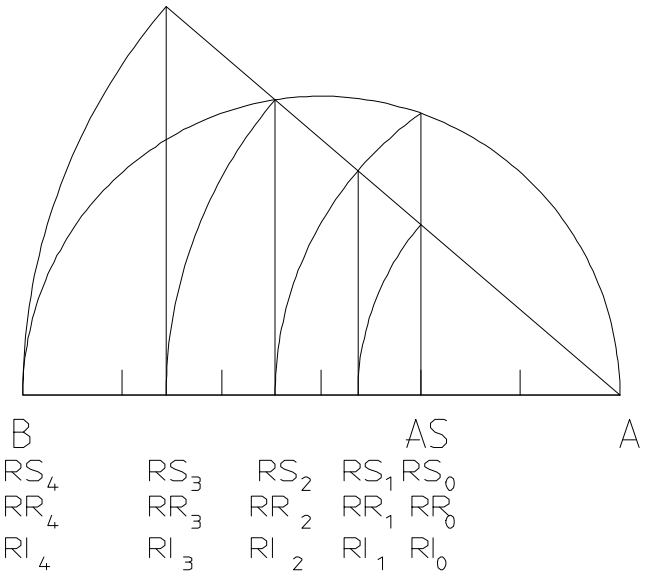
04_22_95.MCD

$\Delta := 22 \quad \delta := 1 .. \Delta \quad AB := 7$

Base Segments = BS $BS := 99$

Base Index = BI $BI := 13$

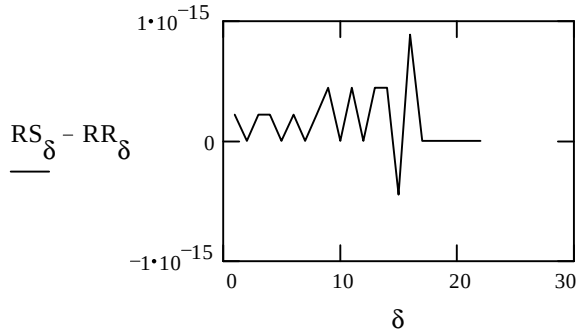
Root Series = RS $RS_\delta := \left[ \left( \frac{BI}{BS} \cdot AB \right)^{\Delta - \delta} \cdot AB^{\delta} \right]^{\frac{1}{\Delta}}$



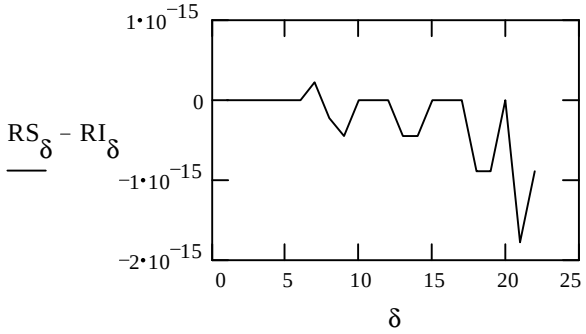

Root Series By Ratio = RR $RR_\delta := \left( \frac{BI}{BS} \right)^{\frac{\Delta - \delta}{\Delta}} \cdot AB$

Root Series By Inverse Ratio = RI

$RI_\delta := \left( \frac{BS}{BI} \right)^{\frac{\delta}{\Delta}} \cdot \left( \frac{BI}{BS} \cdot AB \right)$




$\sum_\delta \left( RS_\delta - RR_\delta \right) = 0$

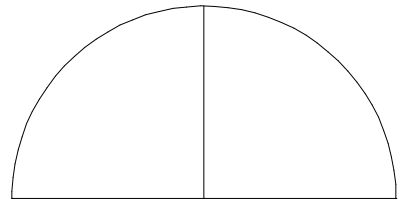



$\sum_\delta \left( RS_\delta - RI_\delta \right) = 0$

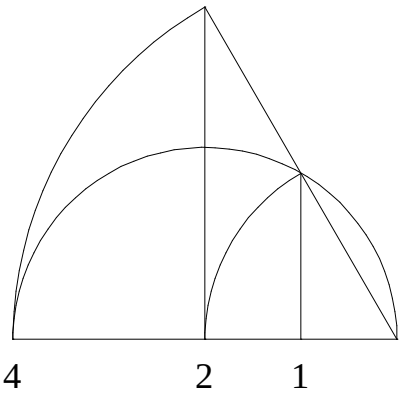
On the concept of unit and universe of discourse:

Euclidean exponentiation provides a good example with which to demonstrate the distinction between the concepts of unit and the universe of discourse.

Taking 2 for the *universe* of discourse would be represented graphically as:

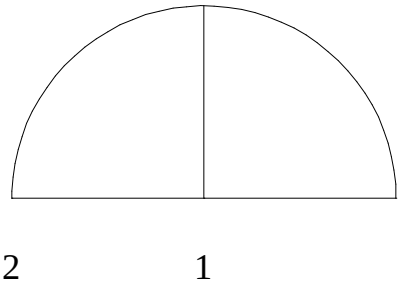



To represent $2^2$ within this universe of discourse one would draw:

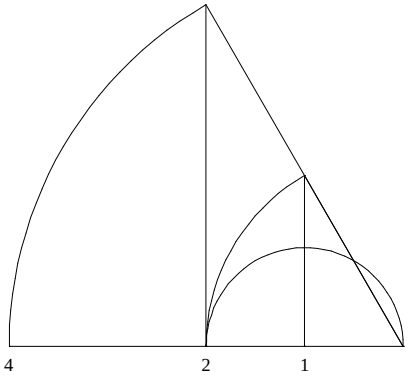



Our original 2 is divided into a $4^{th}$ segment.

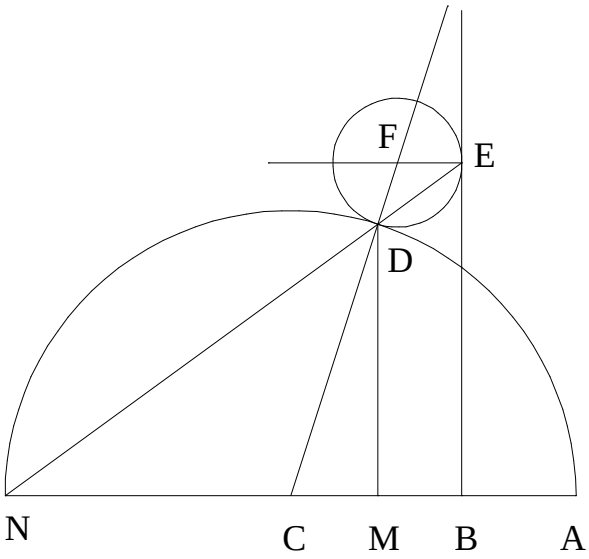
Now if 2 is taken as the *unit* of discourse we may still represent it as;



however to represent $2^3$ now would be drawn as




One will notice that in example 1, the unit changed while the universe remained, in example 2 the unit remained while the universe changed. One could call example 1 an example of deduction and example 2 an example of induction.




09_13_95.MCD

A Study in Placement.

Given AC, AB, AM find EF.

Segment A.

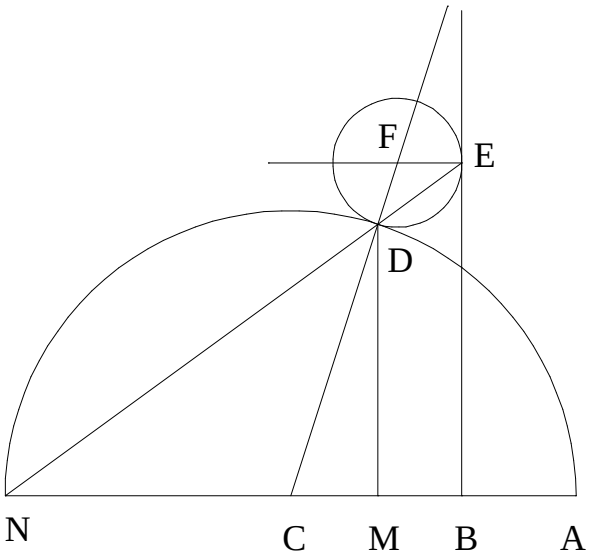
$AC := 3.125$ $AB := 4.25$ $AM := 5.34857$

$CD := AC$ $CN := AC$ $CM := AC - AM$ $DM := \sqrt{CD^2 - CM^2}$

$MN := CM + CN$ $BC := AC - AB$ $BN := CN + BC$ $DN := \sqrt{DM^2 + MN^2}$

$EN := \frac{DN \cdot BN}{MN}$ $ED := EN - DN$ $EF := \frac{AC \cdot ED}{DN}$ $EF = 3.80843$

The resultant equation in terms of the givens is $EF := \frac{AC \cdot (AM - AB)}{2 \cdot AC - AM}$ $EF = 3.80843$




Segment B.

Given AC, AB, DN, find EF.

$DN := 2.37359$

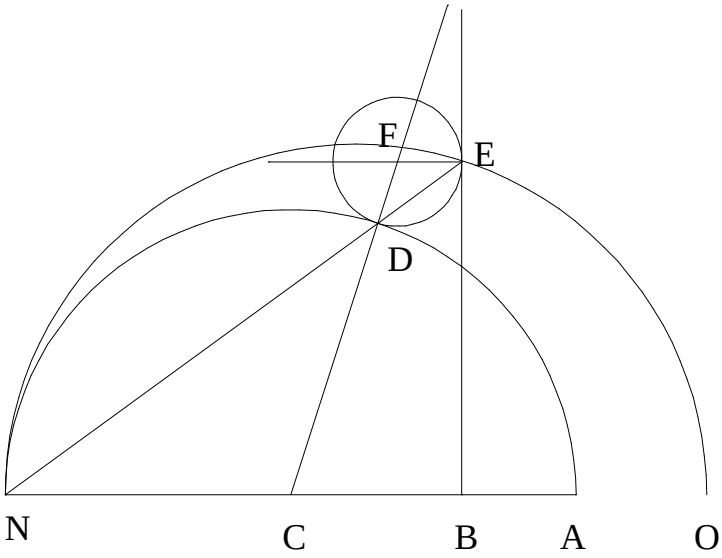
$AN := 2 \cdot AC$ $\quad MN := \frac{DN^2}{AN}$ $\quad CN := AC$ $\quad BC := AC - AB$ $\quad BN := CN + BC$

$EN := \frac{DN \cdot BN}{MN}$ $\quad ED := EN - DN$ $\quad EF := \frac{AC \cdot ED}{DN}$ $\quad EF = 3.80844$

The equation for this process is

$$EF := \frac{AC \cdot (4 \cdot AC^2 - 2 \cdot AC \cdot AB - DN^2)}{DN^2}$$

$EF = 3.80844$




Segment C.

Given AC, AB, BE, find EF.

$BE := 4.87173$

$AN := 2 \cdot AC$ $\quad BN := AN - AB$ $\quad EN := \sqrt{BE^2 + BN^2}$ $\quad ON := \frac{EN^2}{BN}$

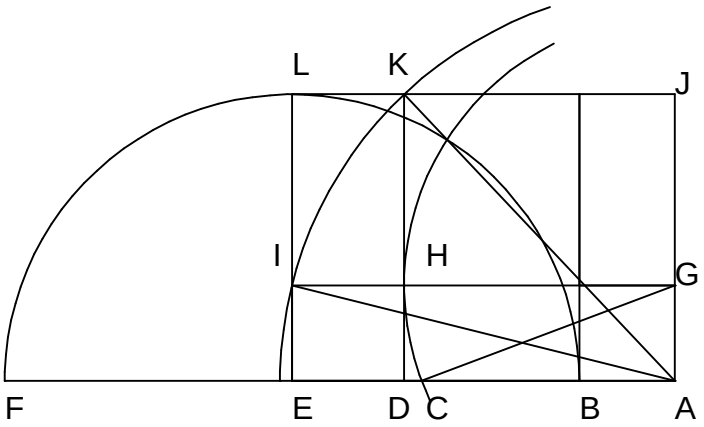
$DN := \frac{EN \cdot AN}{ON}$ $\quad DE := EN - DN$ $\quad EF := \frac{AC \cdot DE}{DN}$ $\quad EF = 3.80844$

The equation for this process is $EF := \frac{BE^2 - 2 \cdot AC \cdot AB + AB^2}{2 \cdot (2 \cdot AC - AB)}$ $\quad EF = 3.80844$

10_14_5C.MCD

Trivial Method: Square Root

Generalize the figure of 10_14_95.MCD




Starting at any point G, between A and J, the square root of AB·AF can always be projected to point C. Such a progression can be used on the cube root figure.

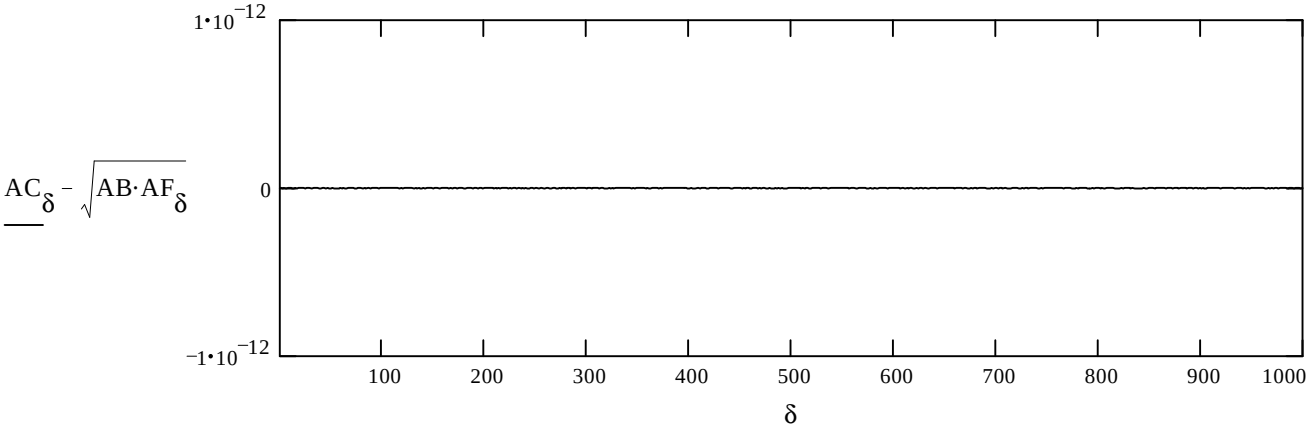
$\delta := 1 .. 1000 \quad AB := 10 \quad BF := 10 \quad BE := \frac{BF}{2}$

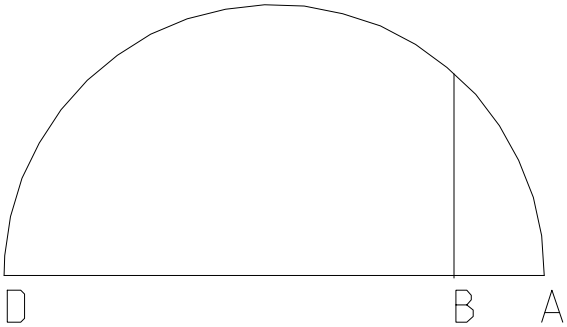
$AE := BE + AB \quad AJ := BE \quad AG_\delta := \frac{AJ}{\delta}$

$EI_\delta := AG_\delta \quad AI_\delta := \sqrt{(AE)^2 + (EI_\delta)^2}$

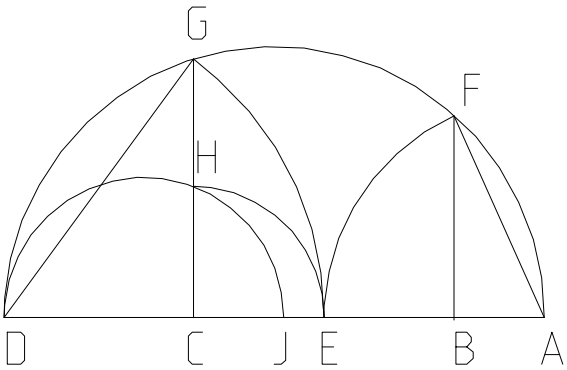
$AK_\delta := AI_\delta \quad DK := AJ \quad AD_\delta := \sqrt{(AK_\delta)^2 - DK^2} \quad GH_\delta := AD_\delta \quad CG_\delta := GH_\delta$

$AF_\delta := BF + AB \quad AC_\delta := \sqrt{(CG_\delta)^2 - (AG_\delta)^2}$






Given AB and BD divide BD such that $AB \cdot CD = \frac{BC^2}{4}$. And what is the reltionship of AC to AB and BD? Now the date on this file is not exact as I sketched this out on a piece of paper and forgot to date it.




$AB := 5 \quad BD := 1 \quad AD := AB + BD$

$BF := \sqrt{AB \cdot BD} \quad AE := \sqrt{AB^2 + BF^2}$

$DE := AD - AE \quad DG := DE \quad CD := \frac{DG^2}{AD}$

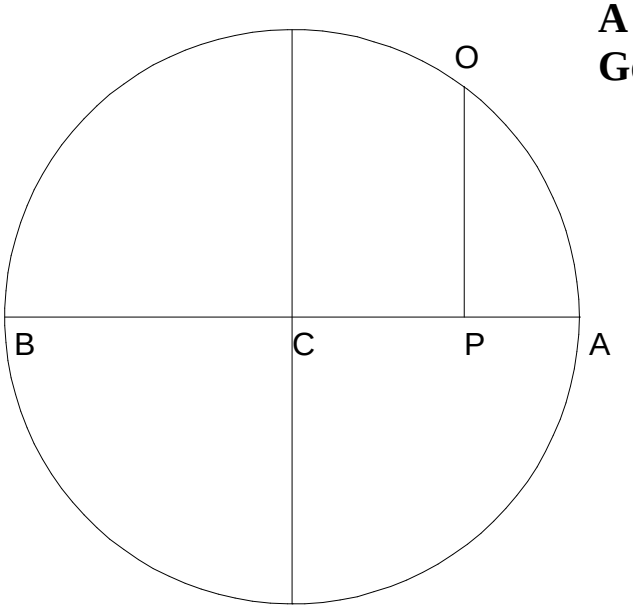
$CD = 0.046$

$BC := BD - CD \quad AB \cdot CD - \frac{BC^2}{4} = 0$

$CE := DE - CD \quad CH := CE \quad CJ := \frac{CH^2}{CD} \quad AB - CJ = 0$

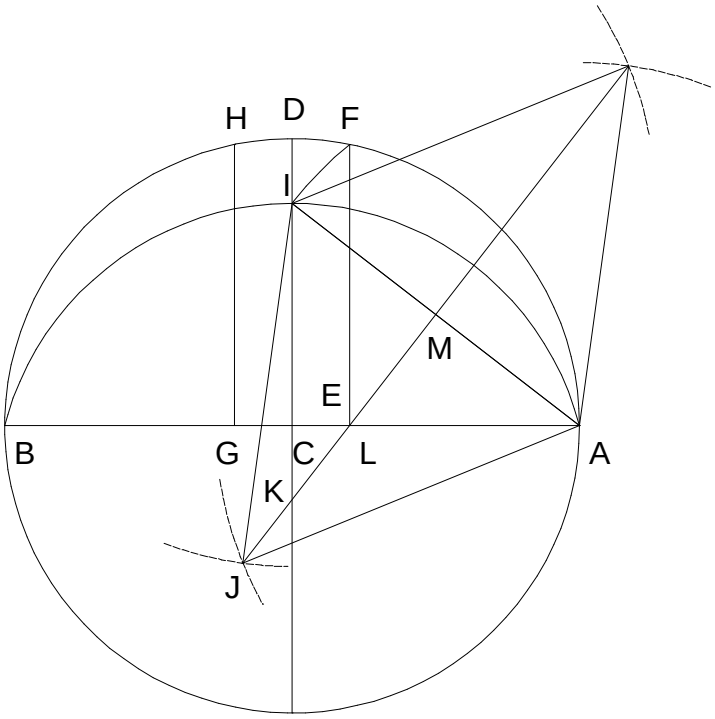
$AC := AB + BC \quad AC = 5.954$

$$AC - \left[ 2 \cdot \frac{AB^{\left(\frac{3}{2}\right)}}{\sqrt{AB + BD}} + 2 \cdot \frac{\sqrt{AB}}{\sqrt{AB + BD}} \cdot BD - AB \right] = 0$$




# A Modification of a Square Root Figure. Gemini Roots

One of the square root figures displays a one to one ratio between what could be called the vertical segment OP and the root of the two horizontal segments AP and BP. With a slight modification however, one can demonstrate a many to one relationship between three base segments.




GL has a ratio to the root of $AL \cdot BG$. Developing the arc AIC from it will give a means of keeping that ratio.

BS = Base Segments, set at end of doc.

$AB := 10 \quad AC := \frac{AB}{2} \quad EG := \frac{AB}{BS}$

$BC := AC \quad CD := AC \quad CE := \frac{EG}{2}$

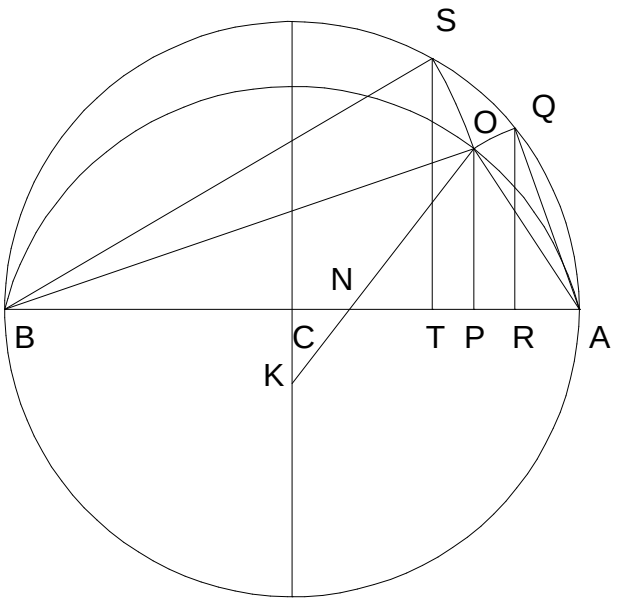
$AE := AC - CE \quad BE := BC + CE$

$EF := \sqrt{AE \cdot BE} \quad AF := \sqrt{AE^2 + EF^2}$

$AI := AF \quad AM := \frac{AI}{2} \quad CI := \sqrt{AI^2 - AC^2}$

$AL := \frac{AI \cdot AM}{AC} \quad CL := AC - AL$

$CK := \frac{AC \cdot CE}{CI} \quad IK := CI + CK$




$$\delta := 1 .. \Delta \qquad AN_\delta := \frac{AC}{\Delta}\cdot\delta$$

$$CN_\delta := AC - AN_\delta \qquad KO := IK$$

$$KN_\delta := \sqrt{CK^2 + \left(CN_\delta\right)^2} \qquad NO_\delta := KO - KN_\delta$$

$$NP_\delta := \frac{CN_\delta \cdot NO_\delta}{KN_\delta} \qquad AP_\delta := AC - CN_\delta - NP_\delta$$

$$OP_\delta := \frac{CK \cdot NO_\delta}{KN_\delta} \qquad AO_\delta := \sqrt{\left(AP_\delta\right)^2 + \left(OP_\delta\right)^2}$$

$$AQ_\delta := AO_\delta \qquad AR_\delta := \frac{\left(AQ_\delta\right)^2}{AB}$$

$$BP_\delta := BC + CN_\delta + NP_\delta$$

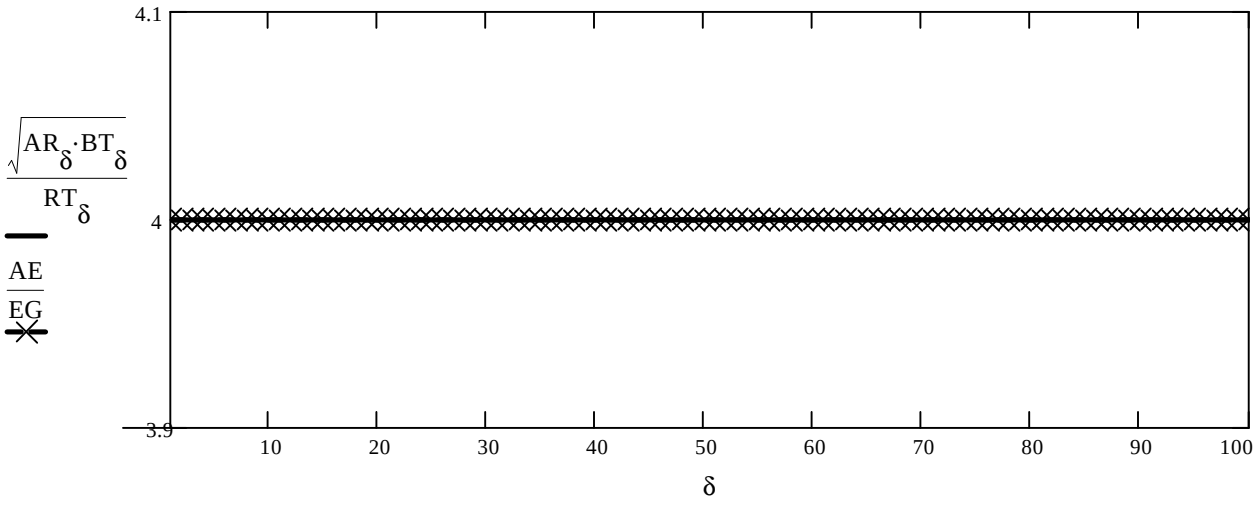
$$BO_\delta := \sqrt{\left(BP_\delta\right)^2 + \left(OP_\delta\right)^2} \qquad BS_\delta := BO_\delta \qquad BT_\delta := \frac{\left(BS_\delta\right)^2}{AB} \qquad AT_\delta := AB - BT_\delta \qquad RT_\delta := AT_\delta - AR_\delta$$

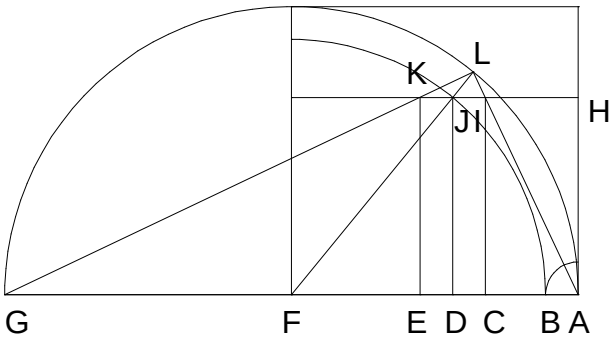
Set the number of Base Segments here and see if a constant relationship is expressed in the graph.

$BS \equiv 9$

Set the number of drawings in $\Delta$.

$\Delta \equiv 100$





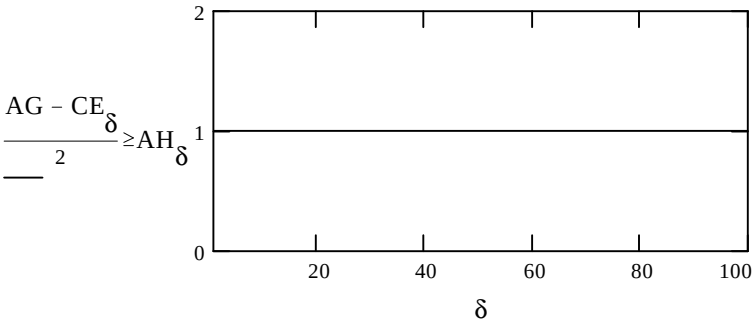

# Short Method Gemini Roots.

Given AG, CE, AH, place CE so that CE:AH as CE:CI. Or more simply that CI $= \sqrt{AC_\delta \cdot EG_\delta} = AH$.

$\delta := 1 .. 100$ $\Delta := 8$

$AG := \Delta \cdot 2 + 1$ $CE_\delta := \frac{1}{\delta}$ $AH_\delta := \Delta \cdot CE_\delta$

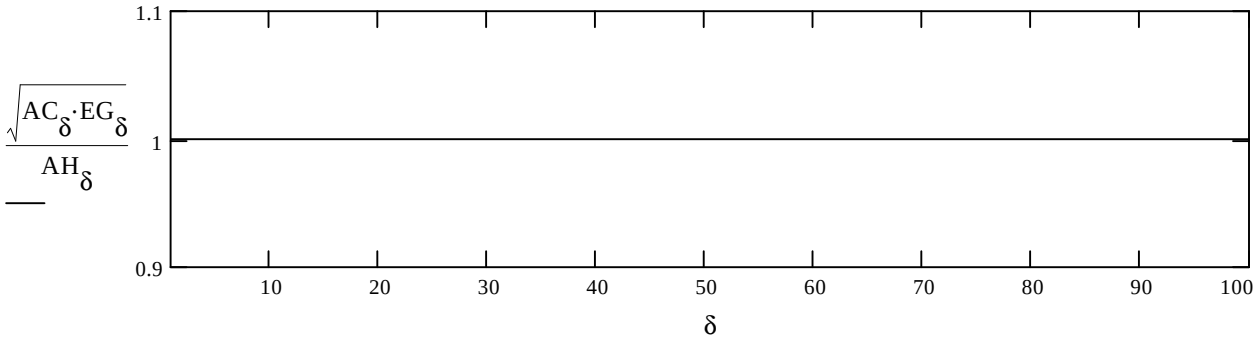
With the values given is the constuction possible? (1 for yes and 0 for no.)

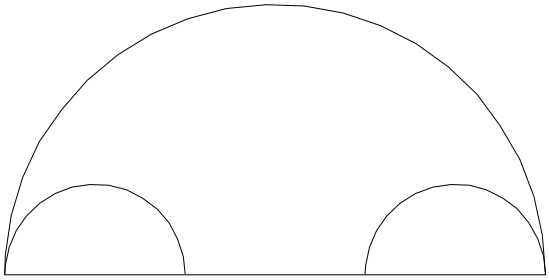



$DJ_\delta := AH_\delta$ $AB_\delta := \frac{CE_\delta}{2}$ $AF := \frac{AG}{2}$ $FG := AF$ $BF_\delta := AF - AB_\delta$ $FJ_\delta := BF_\delta$

$FD_\delta := \sqrt{(FJ_\delta)^2 - (DJ_\delta)^2}$ $DC_\delta := AB_\delta$ $DE_\delta := AB_\delta$ $AD_\delta := AF - FD_\delta$

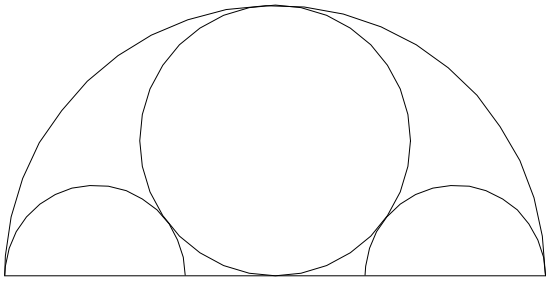
$AC_\delta := AD_\delta - DC_\delta$ $AE_\delta := AD_\delta + DE_\delta$ $EF_\delta := AF - AE_\delta$ $EG_\delta := EF_\delta + FG$

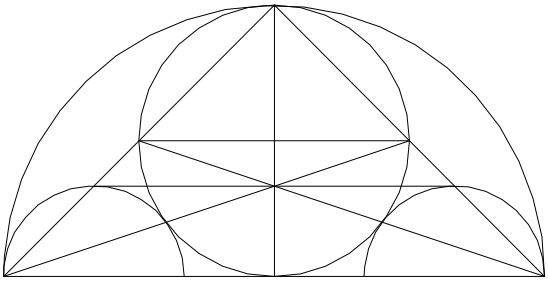


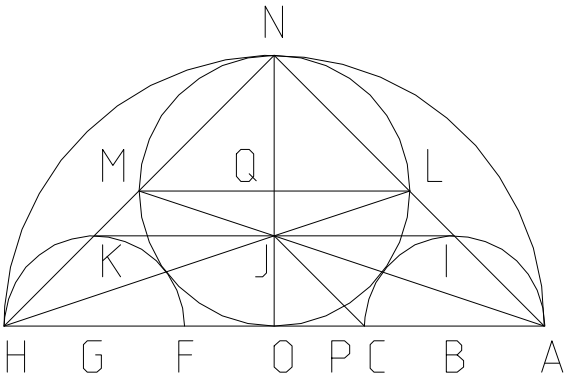


# Method for Equals.

At the inner extremities of a great circle I have two equal smaller ones. Find the circle tangent to all three








$AH := 10 \quad AC := 3 \quad AO := \frac{AH}{2} \quad AB := \frac{AC}{2}$

$JO := AB \quad OP := JO \quad HO := AO$

$JP := \sqrt{JO^2 + OP^2} \quad HP := HO + OP$

$AL := \frac{JP \cdot AH}{HP} \quad NO := AO \quad AN := \sqrt{AO^2 + NO^2}$

$LN := AN - AL \quad LQ := \frac{AO \cdot LN}{AN} \quad LQ = 2.692$

Reducing LQ as an expression of the two givens, $LQ_F := \frac{AH \cdot (AH - AC)}{2 \cdot (AH + AC)} \quad LQ_F - LQ = 0$

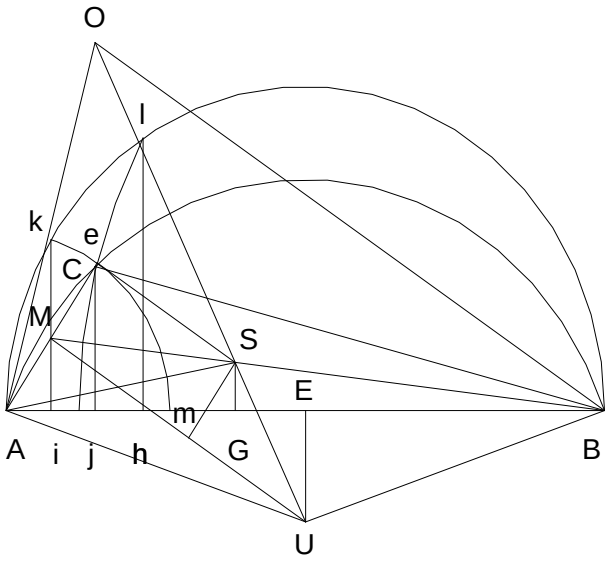
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

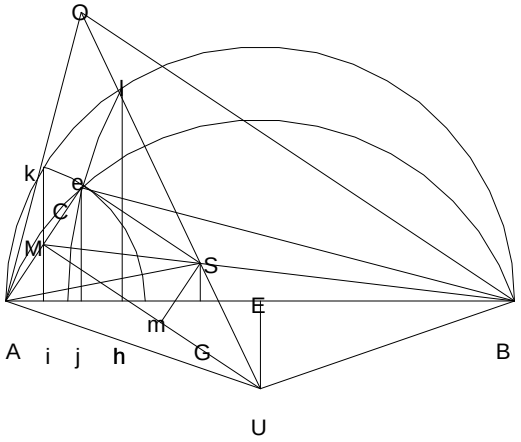
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

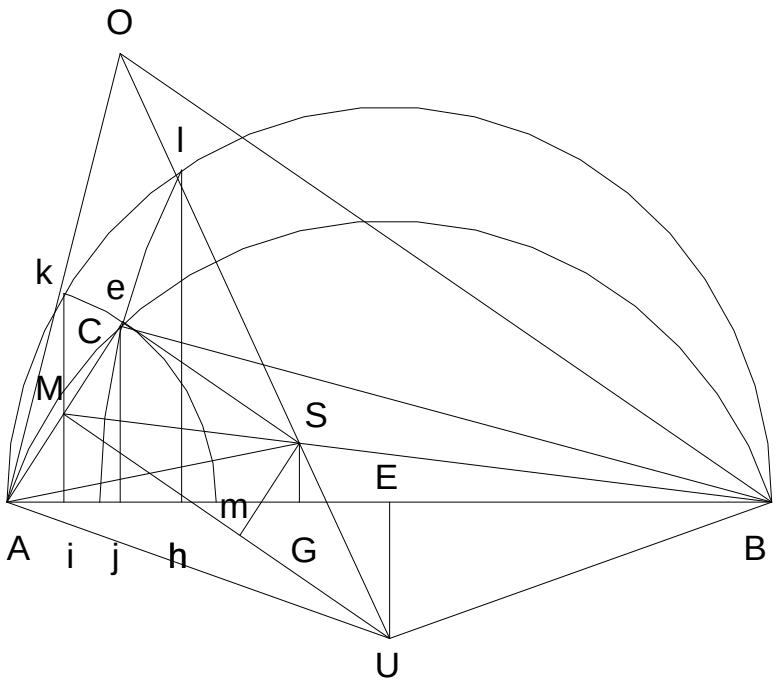
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

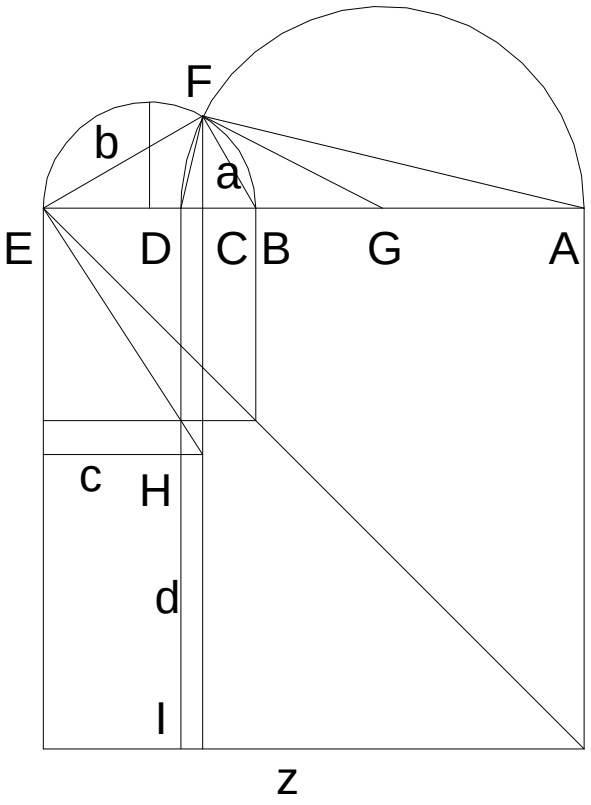



$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.




12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \frac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \frac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.622$

$c := DE$ $\quad c = 0.622$

$d := HI$ $\quad d = 6.186$

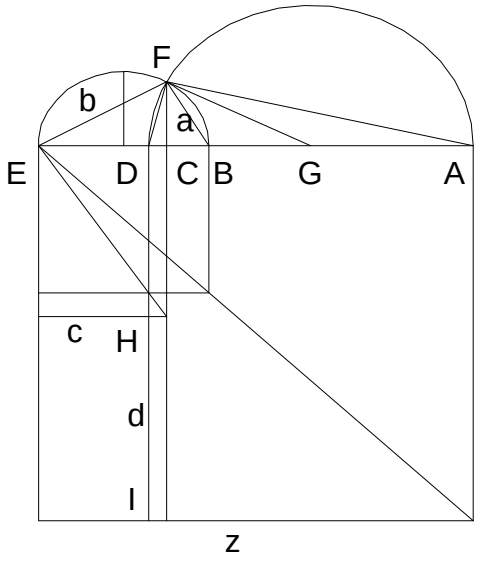
$$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

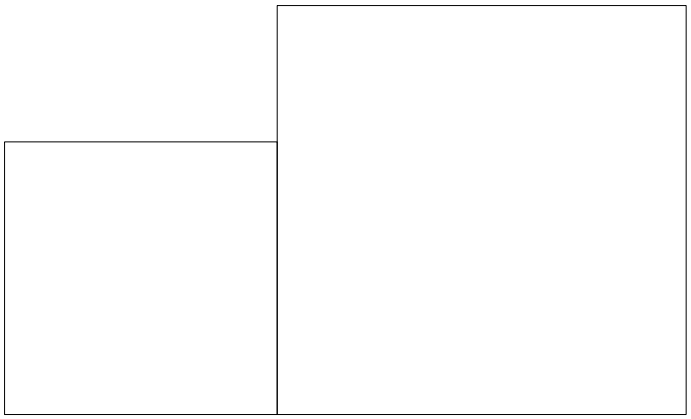
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

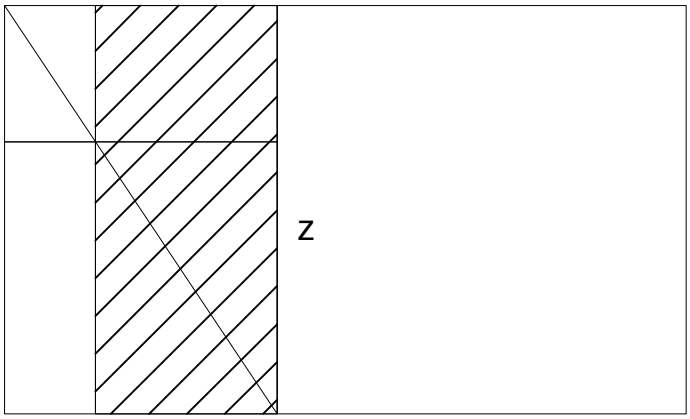
$$(c\cdot d)-p=0$$

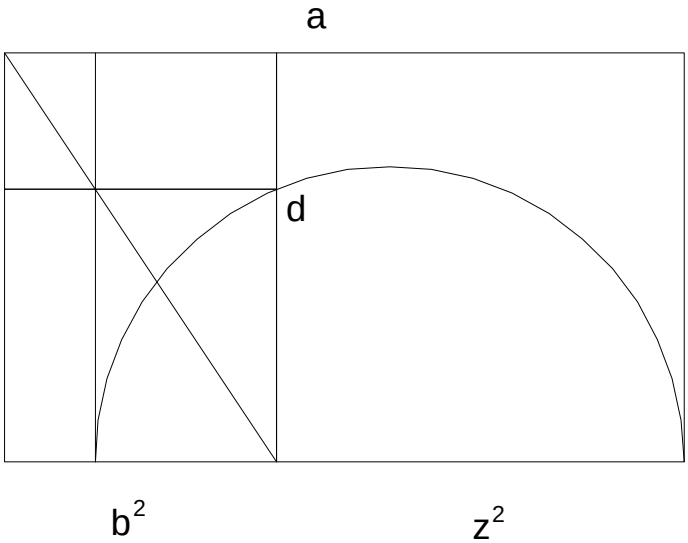
$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$




12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

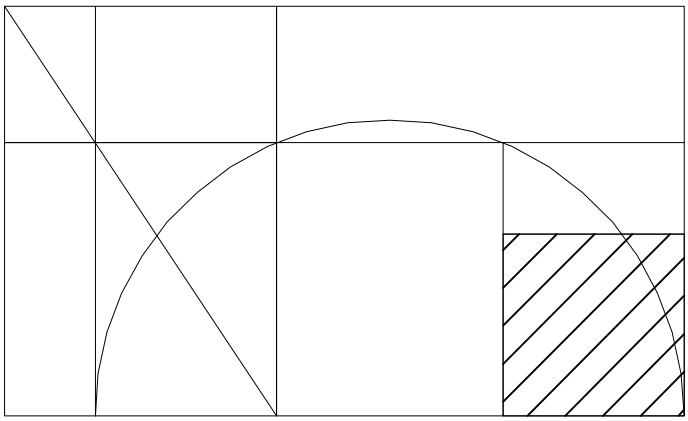
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

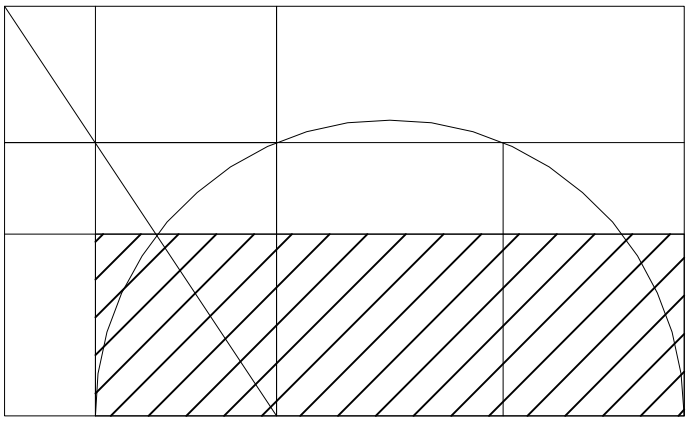
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

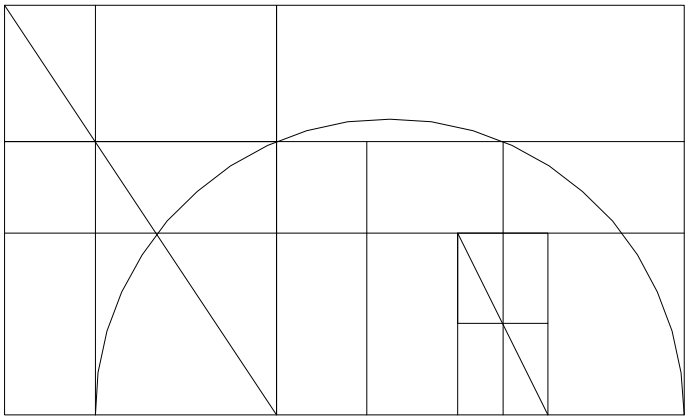
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



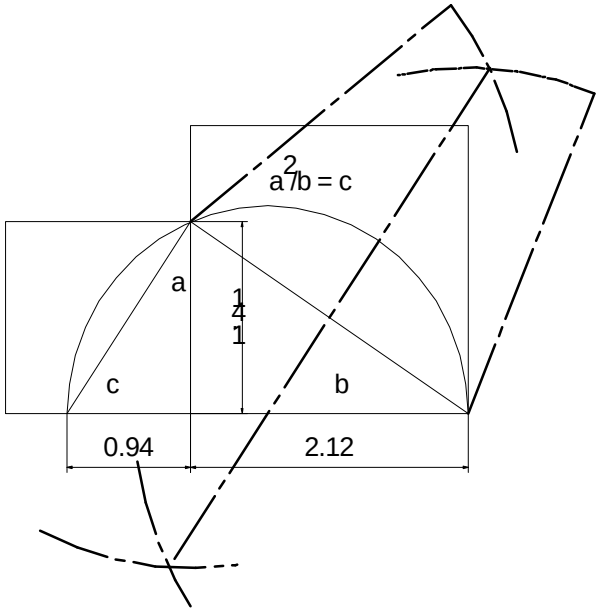
$b^2$ $z^2$



$b^2$ $z^2$

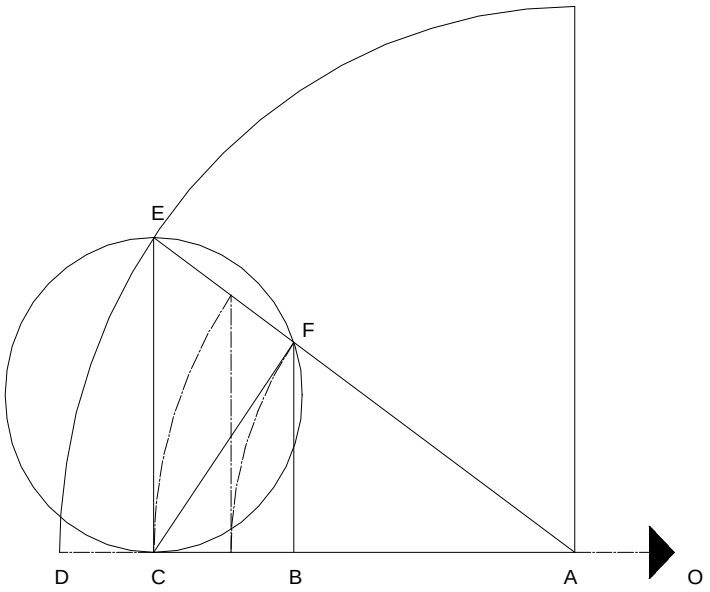


$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

## Just for fun.




$N := 1.3$

$AD := 42 \qquad AC := \frac{AD}{N} \qquad CO := AD + AC$

$CD := AD - AC \qquad CE := \sqrt{CD \cdot CO}$

$AF := \frac{AC^2}{AD} \qquad AB := \frac{AF \cdot AC}{AD}$

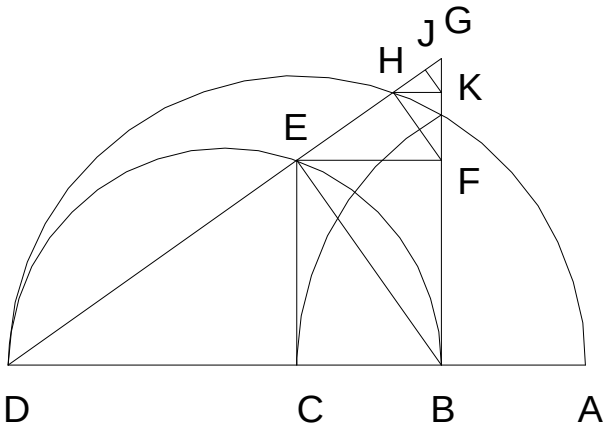
$AC^3 - \left(AD^2 \cdot AB\right) = 0$

$AB = 19.117$

$AC - \left(AD^2 \cdot AB\right)^{\frac{1}{3}} = 0$

The ratio of BC to CE is common to another cube root expression, which?

Pascal's triangle with exponential division.




$AB := 3 \qquad AD := 5$

$BD := AD - AB \qquad AC := \sqrt{AB \cdot AD}$

$CD := AD - AC \qquad BC := BD - CD$

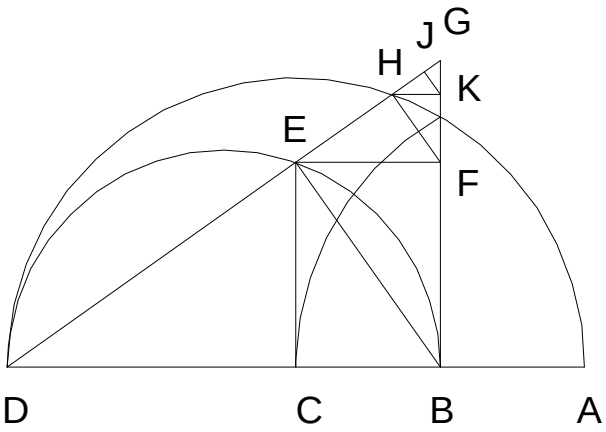
$CD := BD - BC \qquad CE := \sqrt{BC \cdot CD}$

$BE := \sqrt{BC^2 + CE^2} \qquad DE := \sqrt{CD^2 + CE^2}$

$BG := \frac{BD \cdot BE}{DE} \qquad FG := \frac{BC \cdot BG}{BD}$

$EG := \frac{BE \cdot BG}{BD} \qquad DG := DE + EG$

$GH := \frac{BC \cdot EG}{BD} \qquad GJ := \frac{BC \cdot GH}{BD}$




$\frac{BD}{BC} = 2.291 \qquad \frac{DG}{GH} = 5.249 \qquad \frac{DG}{GJ} = 12.025$

$N := 1 .. 3 \qquad \frac{\left(\sqrt{AD} + \sqrt{AB}\right)^N}{\sqrt{AB^N}}$

| |
|---|
| 2.291 |
| 5.249 |
| 12.025 |

$\sqrt{AD} + \sqrt{AB}$

$\frac{A + B}{A}$

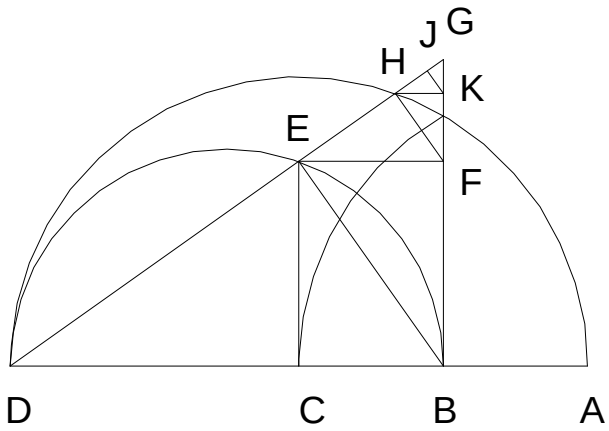
$\left(\sqrt{AD}\right)^2 + 2 \cdot \sqrt{AB} \cdot \sqrt{AD} + \left(\sqrt{AB}\right)^2$

$\frac{A^2 + 2AB + B^2}{A^2}$

$\left(\sqrt{AD}\right)^3 + 3 \cdot AD \cdot \sqrt{AB} + 3 \cdot \sqrt{AD} \cdot AB + \left(\sqrt{AB}\right)^3$

$\frac{A^3 + 3A^2 \cdot B + 3A \cdot B^2 + B^3}{A^3}$

Dividing an exponentiated integer by an exponentiated integer of the same power, straight edge and compass construction. Followed by who knows what!




$CD := 1.5 \qquad BC := .75 \qquad BD := BC + CD$

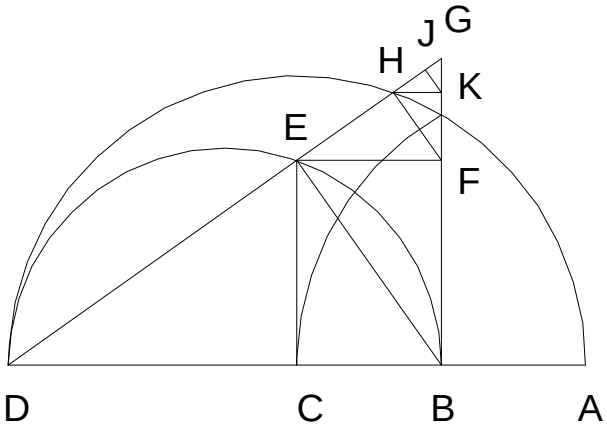
$CE := \sqrt{BC \cdot CD} \qquad BE := \sqrt{BC^2 + CE^2}$

$EF := BC \qquad FG := \frac{BC \cdot EF}{CE} \qquad FG = 0.53$

$DE := \sqrt{CD^2 + CE^2} \qquad EG := \frac{BD \cdot EF}{DE}$

$GH := \frac{BC \cdot EF}{DE} \qquad DG := DE + EG$

$JK := \frac{FG \cdot GH}{EG} \qquad GJ := \frac{BC \cdot GH}{BD} \qquad FH := \frac{CE \cdot EG}{BD} \qquad \frac{BD}{BC} = 3 \qquad \frac{DG}{GH} = 9 \qquad \frac{DG}{GJ} = 27$



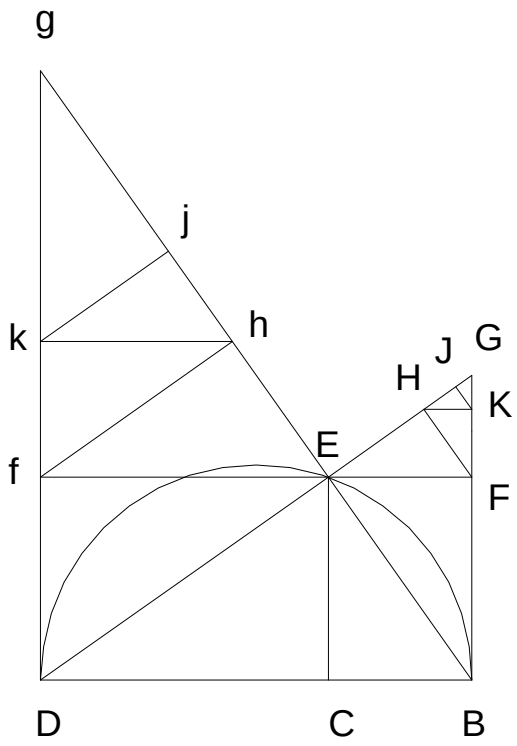

$n := 1..3$

| $\frac{a^n}{b^n}$ | $\frac{BD^n}{BC^n}$ |
|---|---|
| | 3 |
| | 9 |
| | 27 |

$BC + CD = 2.25$ $\sqrt{DG \cdot DE} = 2.25$

$\frac{\sqrt{BC + CD} \cdot BC}{\sqrt{CD}} = 0.919$ $\sqrt{DG \cdot GH} = 0.919$

$\frac{\left(\sqrt{BC}\right)^3}{\sqrt{CD}} = 0.53$ $\sqrt{DG \cdot GJ} = 0.53$




$Ef := CD$ $Eg := \frac{BD \cdot Ef}{BE}$ $fh := \frac{CE \cdot Ef}{BE}$

$hk := \frac{CE \cdot fh}{BE}$ $hj := \frac{BC \cdot hk}{BE}$ $Eg := \frac{BD \cdot Ef}{BE}$

$Bg := BE + Eg$ $hg := \frac{CD \cdot Ef}{BE}$ $jg := \frac{CD \cdot hk}{BE}$

$\frac{BD}{CD} = 1.5$ $\frac{Bg}{hg} = 2.25$ $\frac{Bg}{jg} = 3.375$ $\left(\frac{BD}{CD}\right)^n$

| |
|---|
| 1.5 |
| 2.25 |
| 3.375 |

$\frac{Bg}{EG} = 4.243$ $\frac{Bg}{GH} = 12.728$ $\frac{Bg}{GJ} = 38.184$ $\sqrt{CD} \cdot \frac{(BC + CD)^n}{\left(\sqrt{BC}\right)^{2 \cdot n + 1}}$

| |
|---|
| 4.243 |
| 12.728 |
| 38.184 |

$n := 1 .. 4$

$\sqrt{Bg \cdot GJ} = 0.631$

$\sqrt{Bg \cdot GH} = 1.092$

$\sqrt{Bg \cdot EG} = 1.892$

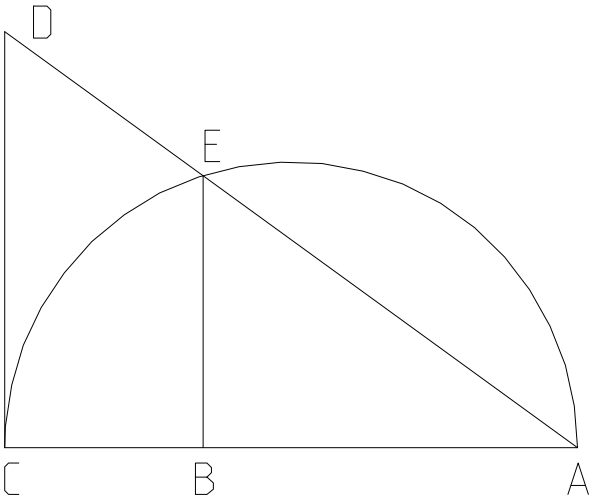
$\sqrt{Bg \cdot DG} = 3.277$

$\frac{BC^{\left(\frac{7}{4}\right)}}{CD^{\left(\frac{1}{4}\right)} \cdot \sqrt{BC + CD}} \cdot \frac{\left(\sqrt{BC + CD}\right)^n}{\left(\sqrt{BC}\right)^n}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |

$\frac{BC^{\left(\frac{7}{4} - \frac{1}{2} \cdot n\right)}}{CD^{\left(\frac{1}{4}\right)}} \cdot (BC + CD)^{\left(\frac{-1}{2} + \frac{1}{2} \cdot n\right)}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |




Given AC and CD find BC when it is equal to $\frac{AC \cdot CD^2}{AC^2 + CD^2}$.

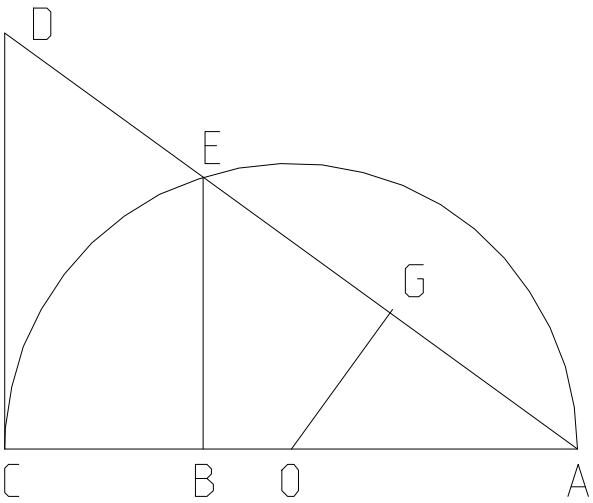
$AC := 15 \quad CD := 5 \quad AO := \frac{AC}{2}$

$AD := \sqrt{AC^2 + CD^2}$

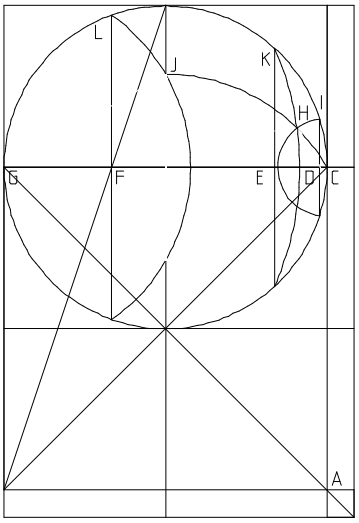
$AG := \frac{AC \cdot AO}{AD} \quad AE := 2 \cdot AG$

$AB := \frac{AC \cdot AE}{AD} \quad BC := AC - AB \quad BC = 1.5$

$BC := \frac{AC \cdot CD^2}{\left(AC^2 + CD^2\right)} \quad BC = 1.5$




A cube divided by the sum of two squares.




# One Circle

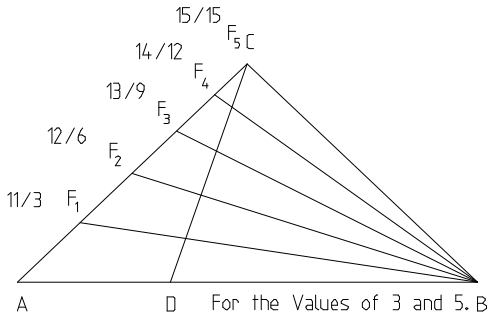
**By John Clark**

# One Square

# One Line

## And the Delian Quest

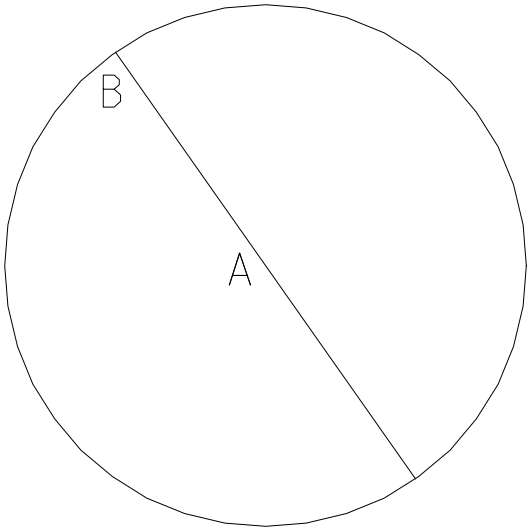
# 1996



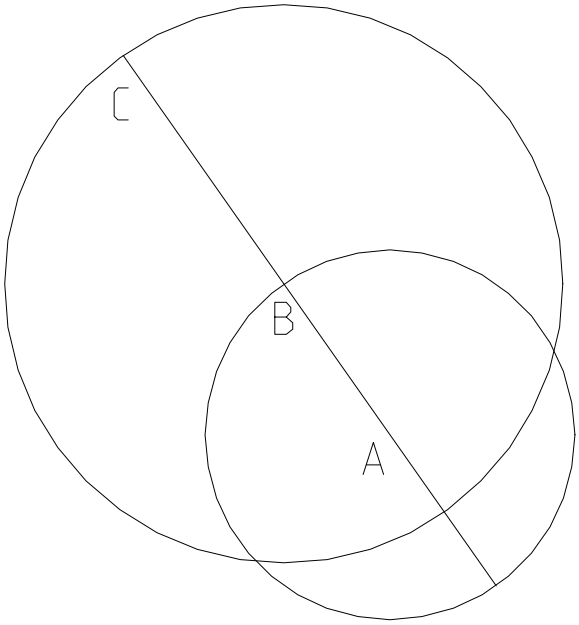
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

## The Archamedian Paper Trisector- Without the Numbers.

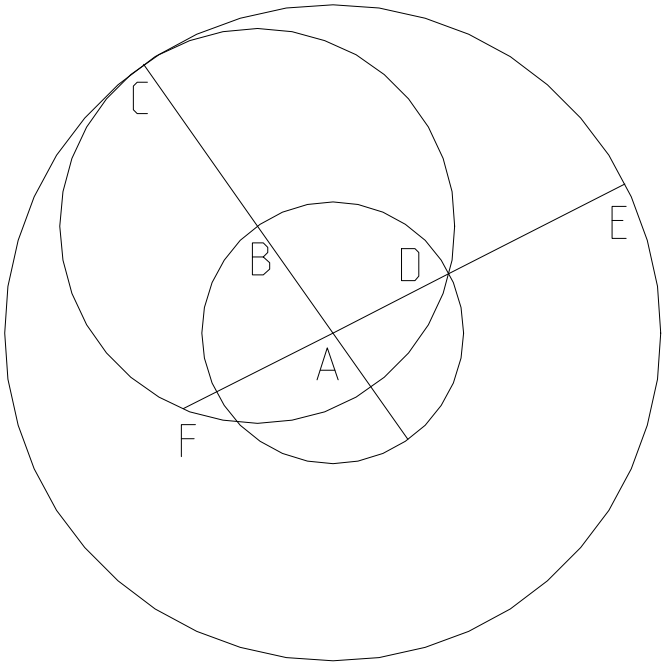
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.




Given any circle AB

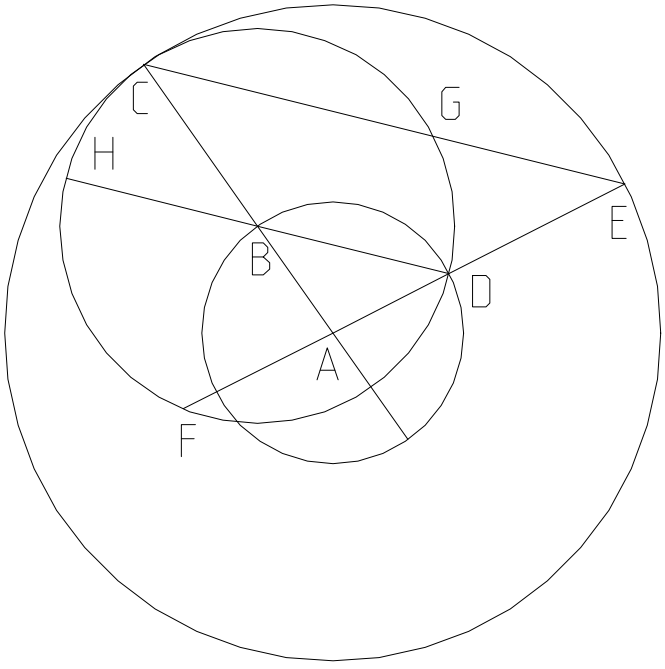



Given any circle BC such that BC≤ 2AB.

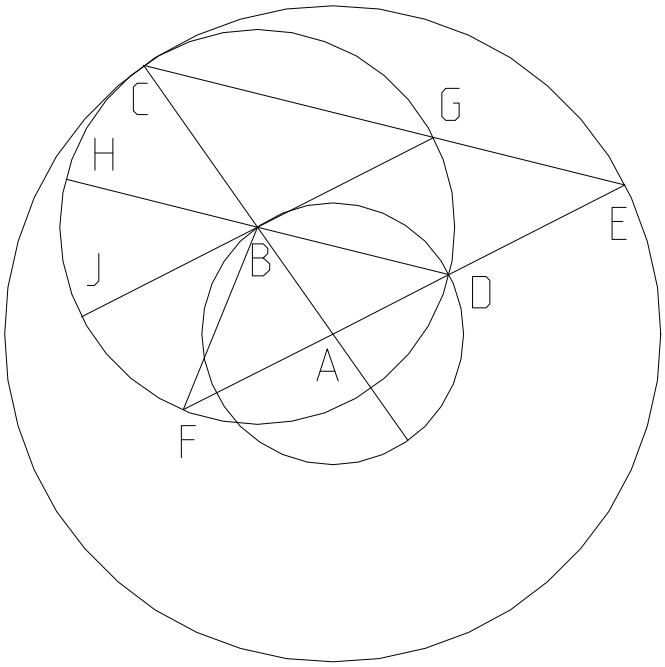



Construct AE such that AE = AC.

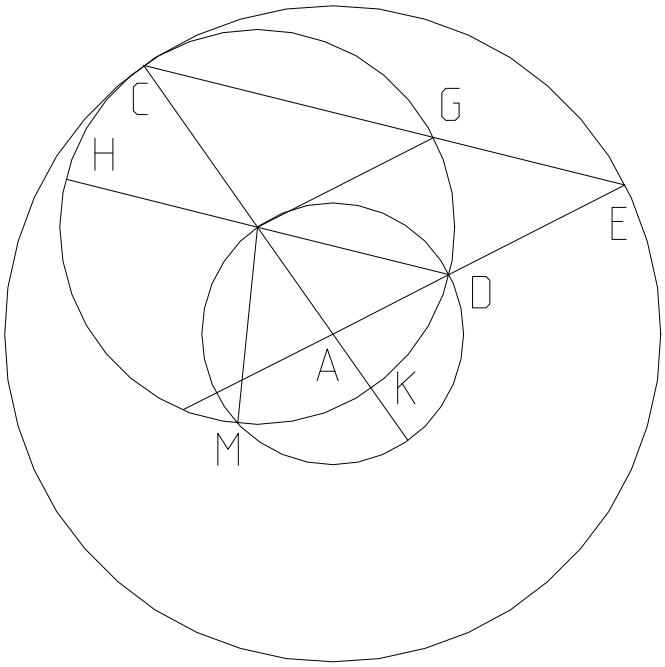
Since AC = AB+ BC and AD = AB, DE = BC.




Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



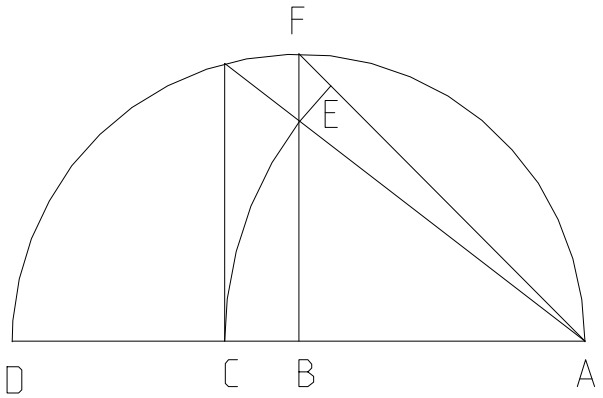

Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.




By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

A rusty Compass construction for the duplication of the cube.

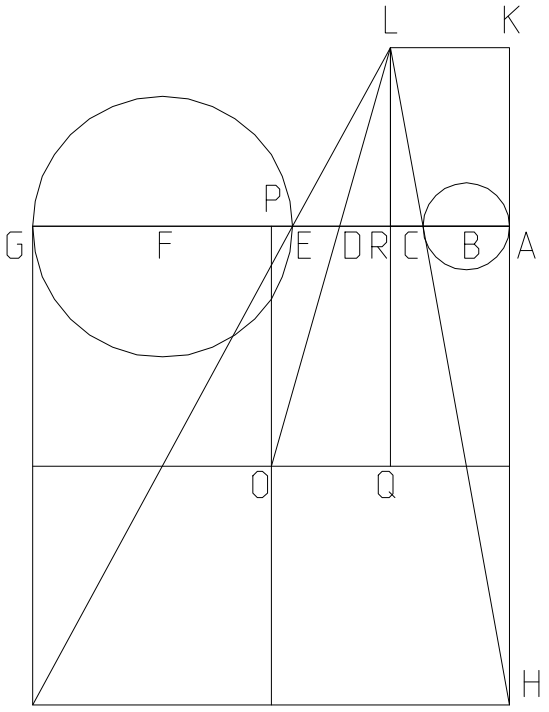



$AD := 2 \quad AB := \frac{AD}{2} \quad AF := \sqrt{2 \cdot AB^2} \quad AE := \frac{AF}{9} \cdot 8$

$AC := AE \quad AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26 \qquad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.




# Alternate Method, Power Line.

Given AB, EF, BF, find the power line intersection on BF. Looking back to 94, it seems I never derived a formula for it either.

$AB := R_1 \quad BF := D \quad EF := R_2$

$BC := AB \quad AC := AB + BC \quad FG := EF$

$EG := EF + FG \quad AG := AB + BF + FG \quad AH := AG$

$GJ := AG \quad AP := \frac{AG}{2} \quad CE := BF - (BC + EF)$

$LR := \frac{AH \cdot CE}{AC + EG} \quad PO := AP \quad AK := LR$

$KL := \frac{AC \cdot (AH + AK)}{AH} \quad AR := KL$

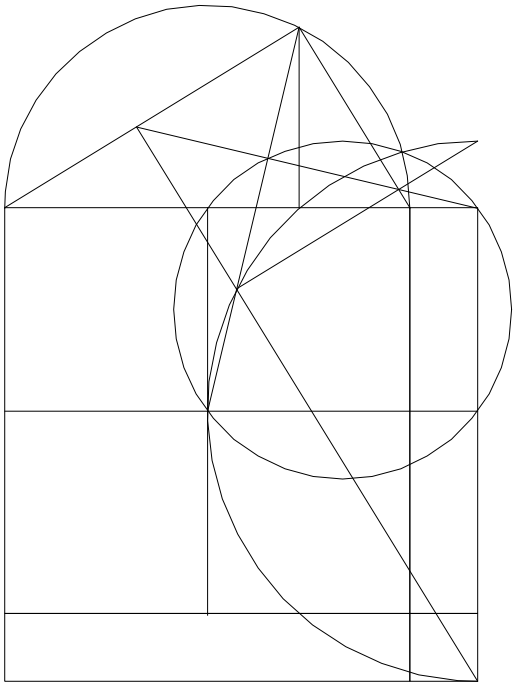
$PR := AP - AR \quad OQ := PR \quad QR := PO$

$LQ := QR + LR \quad DR := \frac{OQ \cdot LR}{LQ} \quad AD := AR + DR$

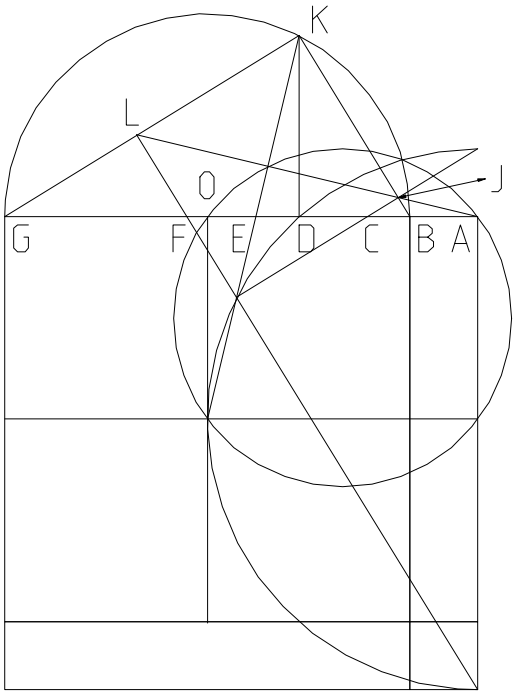
$AD = 25.333$ Plug values in below.

$R_1 \equiv 9 \quad R_2 \equiv 1 \quad D \equiv 30$

$$\frac{(R_1 + D + R_2) \cdot (D - R_2 + R_1)}{2 \cdot D} = 25.333$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.




$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

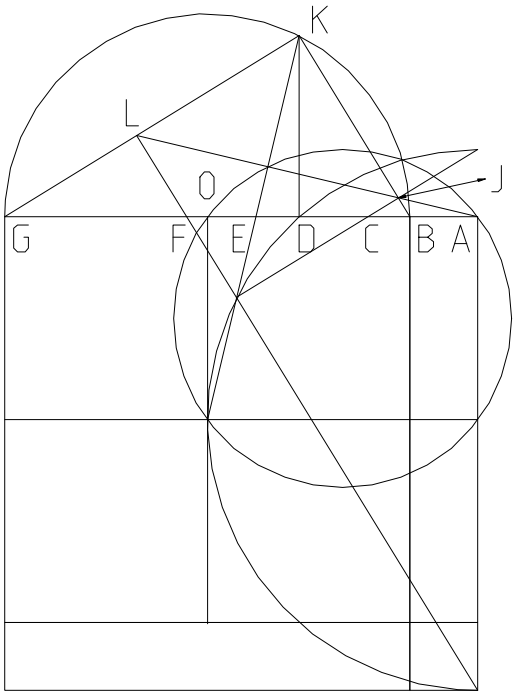
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



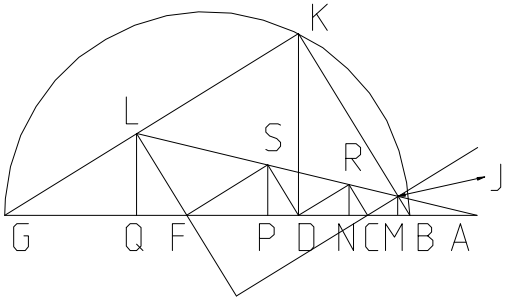

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$




$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$

$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

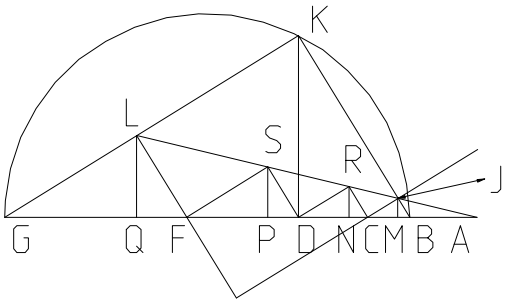
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$

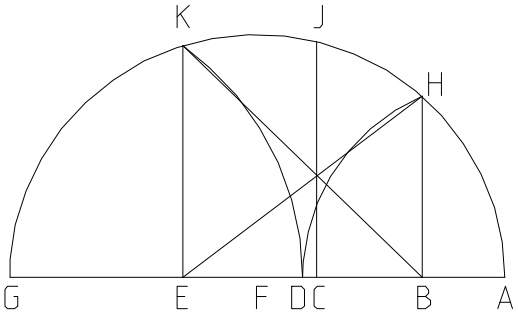



$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

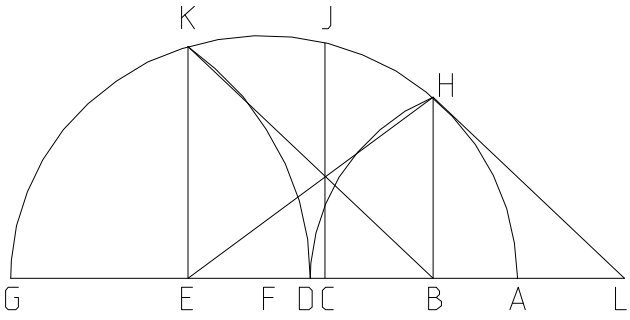
$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$




If the figure was drawn differently, XC would be $\sqrt{XB \cdot XE}$, irregardless of how XB and XE were placed, however that would require part of the figure that is not given here.




$$AG := N \quad AB := \frac{AG}{N}$$

$$BG := AG - AB \quad BH := \sqrt{AB \cdot BG}$$

$$AH := \sqrt{AB^2 + BH^2} \quad AD := AH \quad DG := AG - AD$$

$$GK := DG \quad GE := \frac{GK^2}{AG} \quad AE := AG - GE$$

$$BE := AE - AB$$

$$EG := AG - AE \quad EK := \sqrt{AE \cdot EG} \quad BL := \frac{BE \cdot BH}{EK}$$

$$EL := BE + BL \quad BC := \frac{BL \cdot BE}{EL} \quad AC := AB + BC$$

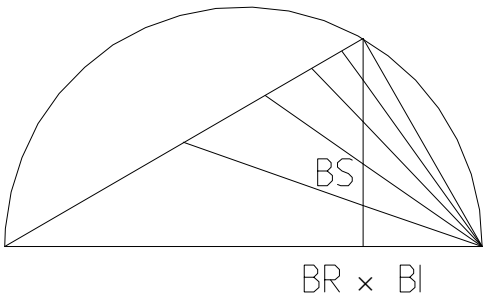
$N \equiv 4$

Make N any number and watch the equations, then make it equal to 1 and see what happens. Now this is strange work, for the formula is an identity with AC, so what happens at 1? This is an example of Binary contradiction.

$AC = 2$

$$\frac{\begin{bmatrix} AB^{\left(\frac{5}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{7}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + 2 \cdot AB \cdot \sqrt{BG} \cdot (AB + BG)^{\left(\frac{3}{2}\right)} \ldots \\ + - AB^{\left(\frac{5}{2}\right)} \cdot \sqrt{BG} - AB^{\left(\frac{3}{2}\right)} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}}{\begin{bmatrix} AB^{\left(\frac{1}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{3}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + AB^{\left(\frac{3}{2}\right)} \cdot \sqrt{BG} + \sqrt{AB} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}} = 2$$

# Pyramid of Ratios, Moving the Point



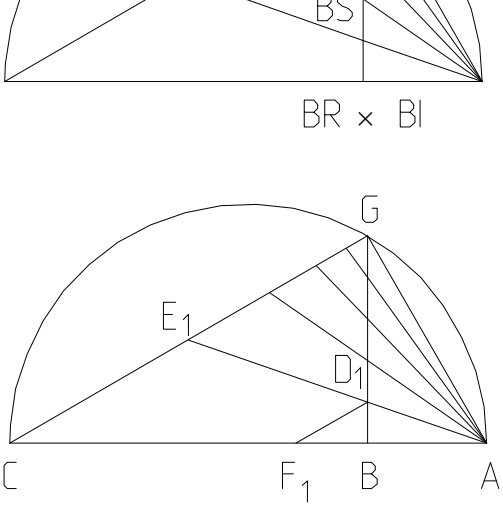

BR = Base Ratio, BS = Bisector Segments, BI = Base Index.

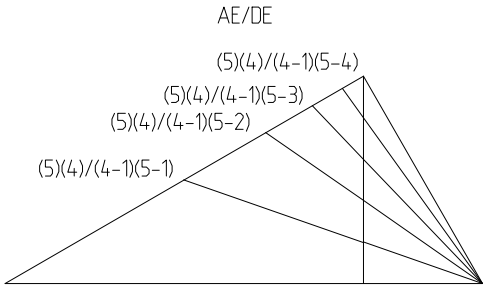
$BR \equiv 4 \quad BS \equiv 5 \quad BI := 2 \quad AC := BR \quad AB := \frac{AC}{BR} \cdot BI$

$BC := AC - AB \quad BG := \sqrt{AB \cdot BC} \quad \delta := 1 .. BS - 1$

$BD_\delta := \frac{BG}{BS} \cdot \delta \quad BF_\delta := \frac{BC \cdot BD_\delta}{BG} \quad AD_\delta := \sqrt{AB^2 + (BD_\delta)^2}$

$AE_\delta := \frac{AD_\delta \cdot AC}{AB + BF_\delta} \quad DE_\delta := AE_\delta - AD_\delta$



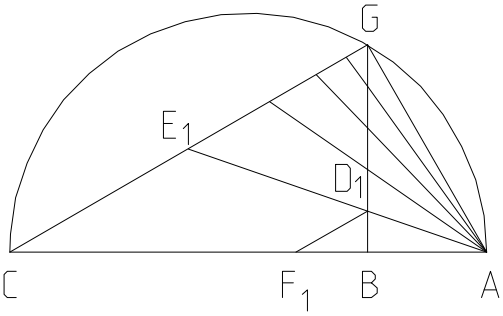




$\frac{AE_\delta}{DE_\delta}$

| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |

$\frac{BS \cdot BR}{(BR - BI) \cdot (BS - \delta)}$

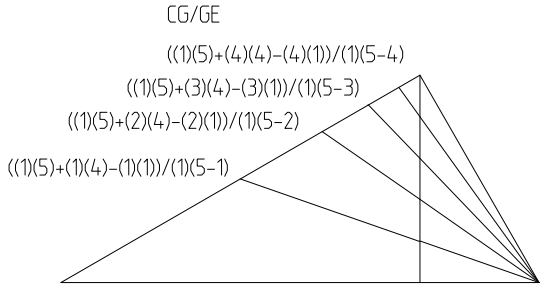
| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |




$AF_\delta := AB + BF_\delta \quad EJ_\delta := \frac{BD_\delta \cdot AC}{AF_\delta}$

$AJ_\delta := \sqrt{(AE_\delta)^2 - (EJ_\delta)^2} \quad CJ_\delta := AC - AJ_\delta$

$CE_\delta := \sqrt{(CJ_\delta)^2 + (EJ_\delta)^2} \quad CG := \sqrt{BG^2 + BC^2}$




$GE_\delta := CG - CE_\delta$

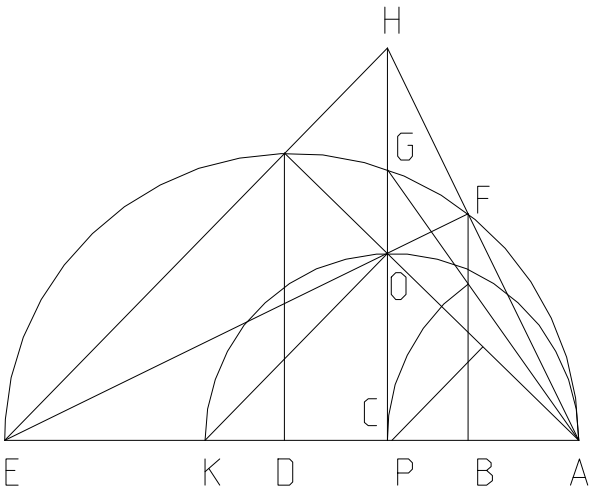
$\frac{CG}{GE_\delta}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

$\frac{(BI \cdot BS + \delta \cdot BR - \delta \cdot BI)}{(BI \cdot (BS - \delta))}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

What is AD. What is BD to $\sqrt{AB \cdot DE}$ ?




$AE := 5.5 \quad AB := 1.05 \quad BE := AE - AB$

$AC := \left(AB^2 \cdot AE\right)^{\frac{1}{3}} \quad CE := AE - AC$

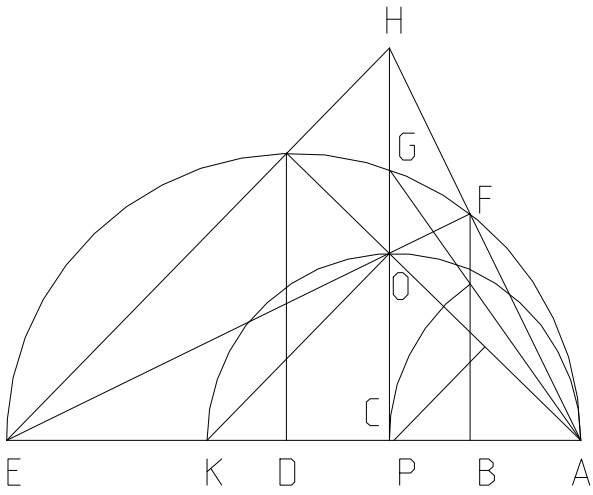
$BF := \sqrt{AB \cdot BE} \quad CO := \frac{BF \cdot CE}{BE} \quad AO := \sqrt{AC^2 + CO^2}$

$AP := \frac{1}{2} \cdot \frac{AO^2}{AC} \quad AK := 2 \cdot AP \quad AD := \frac{AC \cdot AE}{AK}$

$DE := AE - AD \quad BD := AD - AB$

$AD = 2.807$

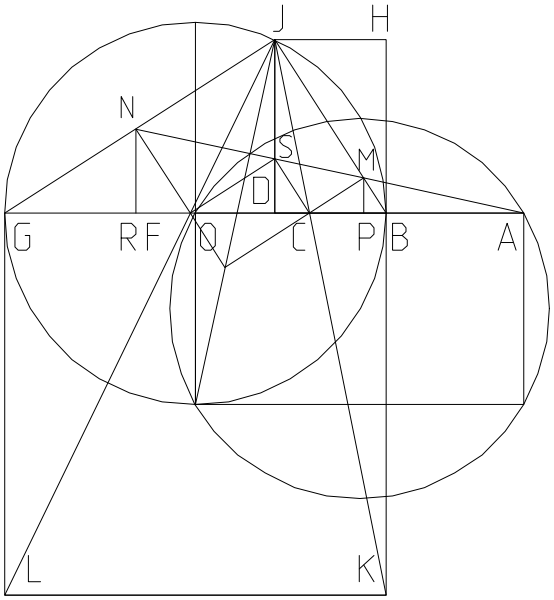
$$\frac{AE^{\frac{3}{3}} \cdot AB^{\frac{1}{3}} + AE^{\frac{2}{3}} \cdot AB^{\frac{2}{3}} + AE^{\frac{1}{3}} \cdot AB^{\frac{3}{3}}}{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}} = 2.807$$




$$\frac{\sqrt{AB \cdot DE}}{BD} = 0.957$$

$$\frac{AB^{\frac{1}{6}} \cdot AE^{\frac{1}{6}} \cdot \sqrt{AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} + AE - AE^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} \cdot \sqrt{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}}}{AE + AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} - 2 \cdot AB} = 0.957$$

The figure cuts the base in Cube Roots and provides some interesting ratios.




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

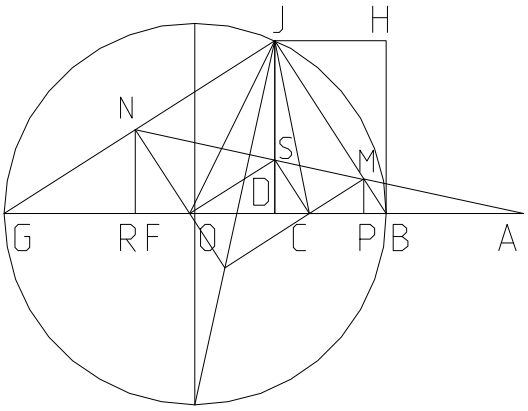
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

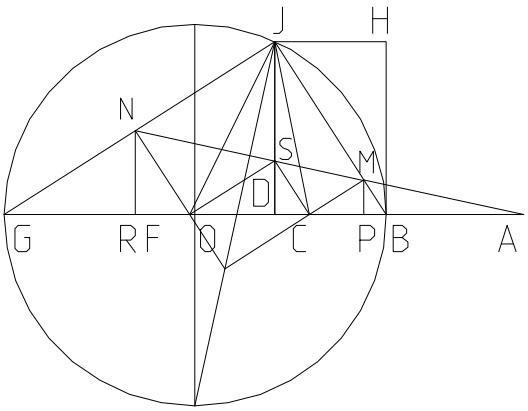
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

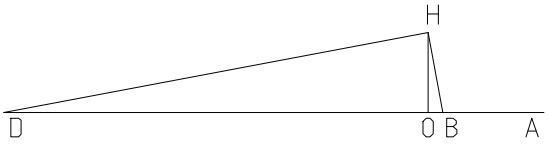
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$




$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

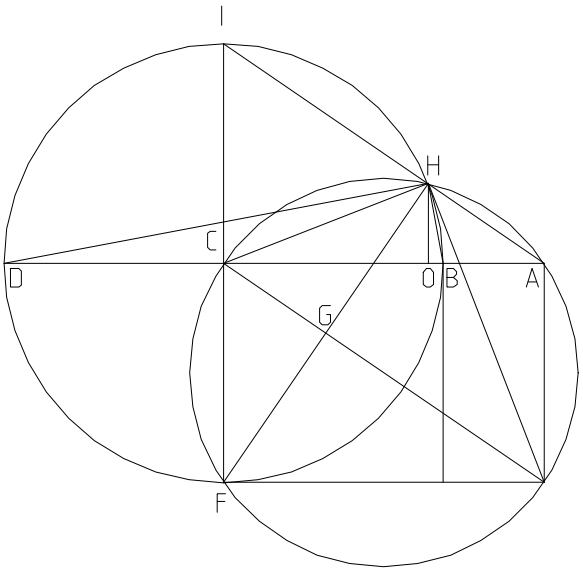
$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$




Given AD and AB on AD, place a right triangle on BD as base such that the opposite sides are in the ratio of AB to AD.

$BD := 8 \quad AB := 2 \quad AD := AB + BD$

$BC := \frac{BD}{2} \quad AC := AB + BC \quad CF := BC$




$CI := BC \quad CH := BC \quad AE := BC$

$CE := \sqrt{AC^2 + AE^2} \quad CG := \frac{CH^2}{CE}$

$GH := \sqrt{CH^2 - CG^2} \quad FH := 2 \cdot GH \quad FI := CF + CI$

$HI := \sqrt{FI^2 - FH^2} \quad AI := \sqrt{AC^2 + CI^2}$

$AH := AI - HI \quad AO := \frac{AC \cdot AH}{AI} \quad HO := \frac{CI \cdot AO}{AC}$

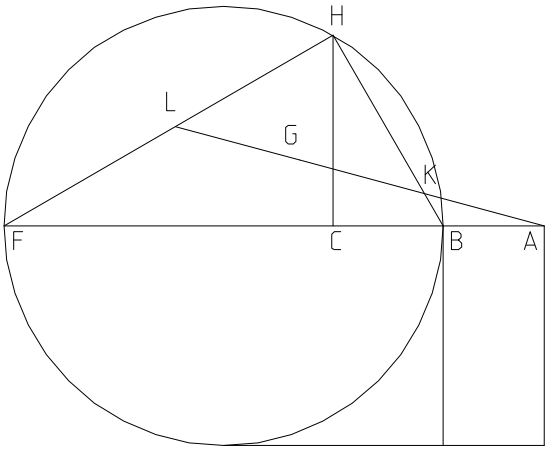
$BO := AO - AB \quad DO := BD - BO$

$DH := \sqrt{DO^2 + HO^2} \quad BH := \sqrt{BO^2 + HO^2}$

$\frac{DH}{BH} = 5 \qquad \frac{AD}{AB} = 5$

Given a straight edge and compass, AB and BD find the sum of six cubes divided by the sum of five squares.

$$\frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BD + AB \cdot BD^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BD + BD^2} = 2.308 \qquad AO = 2.308$$




Given AF and AB on AF and a right triangle on BF divide the sides of the triangle such that a section on one side is to the other as AB is to AF.

Now it can be realized that there are stipulations as to possible placements of the given triangle.

$AB := 3 \quad BF := 10 \quad BC := 1$

$AC := AB + BC \quad BE := \frac{BF}{2} \quad AE := AB + BE$

$$DOABLE := \frac{2\cdot AB^3 + 3\cdot AB^2\cdot BF + AB\cdot BF^2}{2\cdot AB^2 + 2\cdot AB\cdot BF + BF^2} \le AC < AE$$

$DOABLE = 1$

$AF := AB + BF$

$CF := BF - BC \quad CH := \sqrt{BC\cdot CF} \quad EJ := BE$

$CE := BE - BC \quad JH := \sqrt{CE^2 + (CH + EJ)^2}$

$DE := \frac{CE\cdot EJ}{EJ + CH} \quad AD := AE - DE$

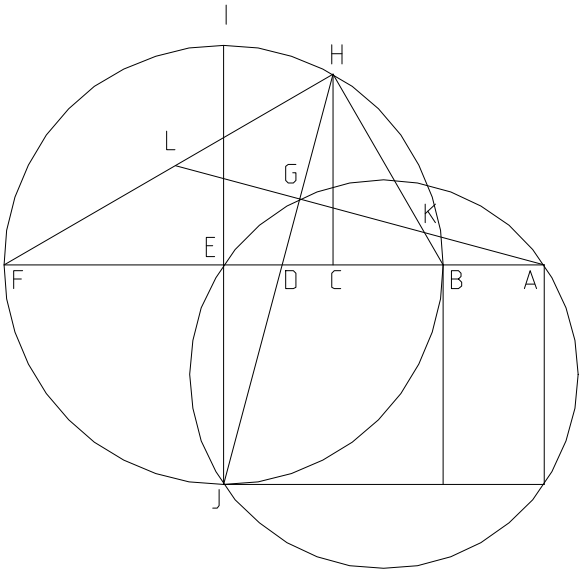
$JD := \sqrt{DE^2 + EJ^2} \quad DG := \frac{DE\cdot AD}{JD}$

$AG := \sqrt{AD^2 - DG^2} \quad GH := JH - (JD + DG)$

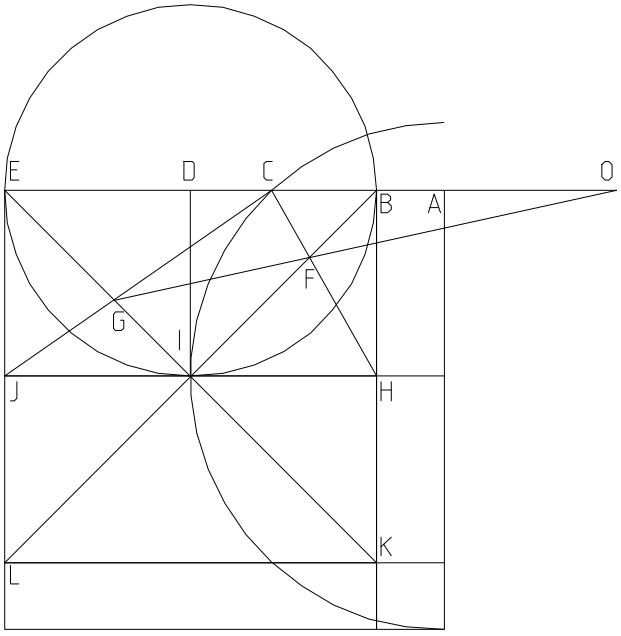
$HK := \sqrt{2\cdot GH^2} \quad BH := \sqrt{BC^2 + CH^2} \quad HL := HK$

$BK := BH - HK \quad FH := \sqrt{CF^2 + CH^2}$

$FL := FH - HL$




$\frac{FL}{BK} = 4.333 \qquad \frac{AF}{AB} = 4.333$




Projecting from KL or HJ is productive, can I find any other productive points?

$AB := 5 \quad BE := 3 \quad BK := BE \quad AE := AB + BE$

$BD := \frac{BE}{2} \quad AC := \sqrt{AB \cdot AE} \quad HI := BD \quad IJ := BD$

$BC := AC - AB \quad CE := BE - BC$

$\Delta := 2 \quad \delta := 1 .. \Delta$

$BH_\delta := \frac{BK}{\Delta} \cdot \delta \quad Ha_\delta := \frac{BH_\delta \cdot HI}{BC} \quad EJ_\delta := BH_\delta$

$Ba_\delta := Ha_\delta + BD \quad Bb_\delta := \frac{(BH_\delta)^2}{Ba_\delta} \quad Jc_\delta := \frac{EJ_\delta \cdot IJ}{CE}$

$Ec_\delta := Jc_\delta + BD \quad Ed_\delta := \frac{(EJ_\delta)^2}{Ec_\delta} \quad Ef_\delta := Bb_\delta$

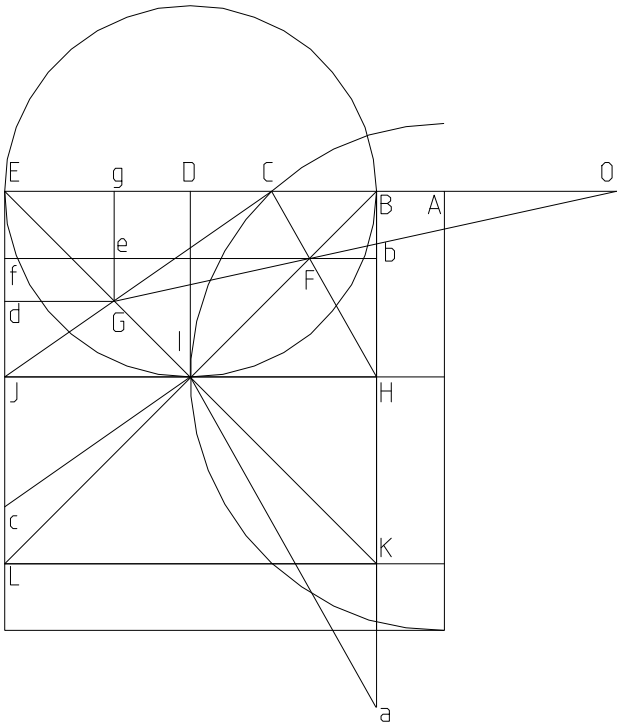
$df_\delta := Ed_\delta - Ef_\delta \quad Ge_\delta := df_\delta \quad Fb_\delta := \frac{HI \cdot BH_\delta}{Ba_\delta}$

$Gd_\delta := \frac{IJ \cdot EJ_\delta}{Ec_\delta} \quad ef_\delta := Gd_\delta \quad Fe_\delta := BE - (ef_\delta + Fb_\delta)$

$Gg_\delta := Ed_\delta \quad Og_\delta := \frac{Fe_\delta \cdot Gg_\delta}{Ge_\delta} \quad Eg_\delta := ef_\delta$

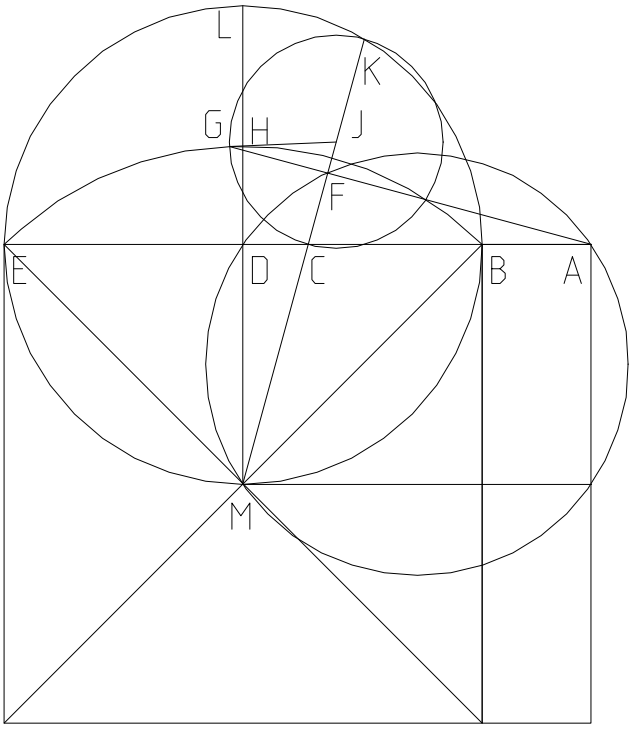
$EO_\delta := Og_\delta + Eg_\delta \quad BO_\delta := EO_\delta - BE$

| $\left(\frac{EO_\delta}{BO_\delta}\right)$ |
|---|
| 1.2649 |
| 1.6 |




$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} = 1.2649 \qquad \frac{AE}{AB} = 1.6$

I have not found any.




The power line for cube root abstraction is developed off from a simple curve.

$$AB := 33 \quad BE := 11 \quad BD := \frac{BE}{2} \quad AE := AB + BE$$

$$R_1 := \frac{\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AB}{2} \quad R_2 := \frac{AE - \left(AB \cdot AE^2\right)^{\frac{1}{3}}}{2}$$

$$D := \left[\left(AB \cdot AE^2\right)^{\frac{1}{3}} - \left(AB^2 \cdot AE\right)^{\frac{1}{3}}\right] + R_1 + R_2$$

$$BC := \frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D}$$

The formula for the power line (BC) was given in 01_08_96.MCD

$$CD := BD - BC \quad AC := BC + AB \quad DM := BD$$

$$DL := BD \quad CM := \sqrt{CD^2 + DM^2} \quad ML := DM + DL$$

$$MK := \frac{DM \cdot ML}{CM} \quad CK := MK - CM \quad CJ := \frac{CK}{2}$$

$$JG := CJ \quad JM := CM + CJ \quad BM := \sqrt{2 \cdot BD^2}$$

$$GM := BM \quad FJ := \frac{JG^2}{JM} \quad FM := JM - FJ$$

$$AD := BD + AB \quad HM := \frac{CM \cdot FM}{DM}$$

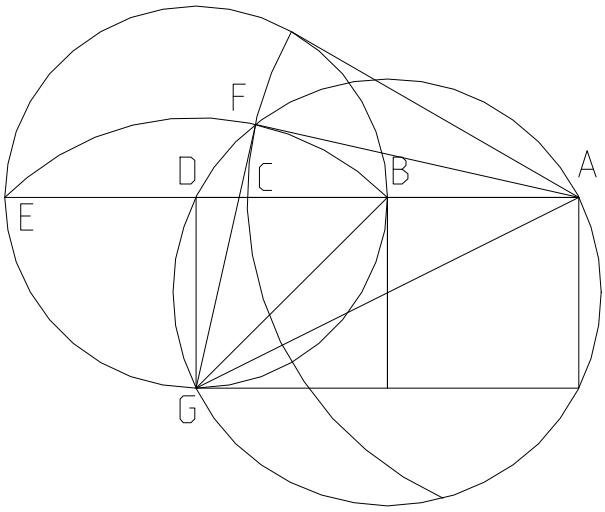
$$DH := HM - DM \quad AH := \sqrt{DH^2 + AD^2}$$

$$CF := FM - CM \quad FH := \frac{CD \cdot HM}{CM}$$

$$AF_1 := AH - FH \quad AF_2 := \sqrt{AC^2 - CF^2}$$

$$AF_1 - AF_2 = 0$$

Trivial Method; Square Root




$N := 9003$

$AE := N \quad AB := \frac{AE}{N} \quad BE := AE - AB \quad BD := \frac{BE}{2}$

$DG := BD \; AD := AB + BD \quad AG := \sqrt{AD^2 + DG^2}$

$BG := \sqrt{BD^2 + DG^2} \quad FG := BG \quad AF := \sqrt{AG^2 - FG^2}$

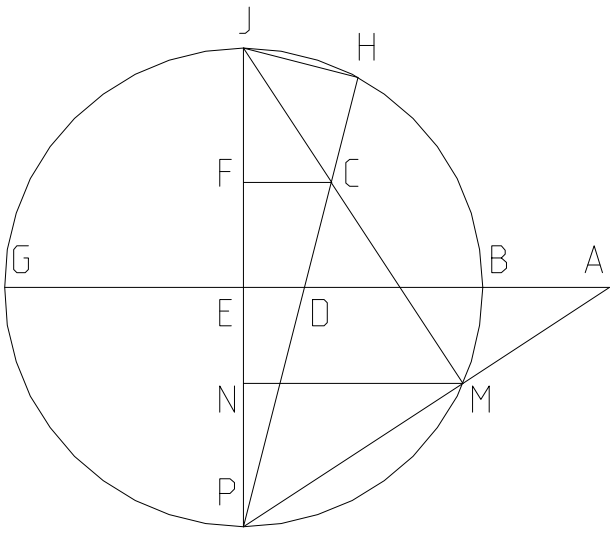
$AC := \sqrt{AB \cdot AE}$

$AC - AF = 0$

$AF = 94.884$

$AC = 94.884$

Given a point on BG, project to the point of cubic similarity.




$BG := 100 \quad BD := 49 \qquad BE := \frac{BG}{2}$

$EG := BE \quad DE := BE - BD \quad EP := BE$

$EJ := BE \quad DP := \sqrt{DE^2 + EP^2} \quad JP := EP + EJ$

$HP := \frac{EP \cdot JP}{DP} \quad DH := HP - DP \quad CD := \frac{DH}{2}$

$CP := DP + CD \quad CF := \frac{DE \cdot CP}{DP} \quad FP := \frac{EP \cdot CP}{DP}$

$EF := FP - EP \quad FJ := EJ - EF \quad CJ := \sqrt{CF^2 + FJ^2}$

$JM := \frac{FJ \cdot JP}{CJ} \quad JN := \frac{FJ \cdot JM}{CJ} \quad NP := JP - JN$

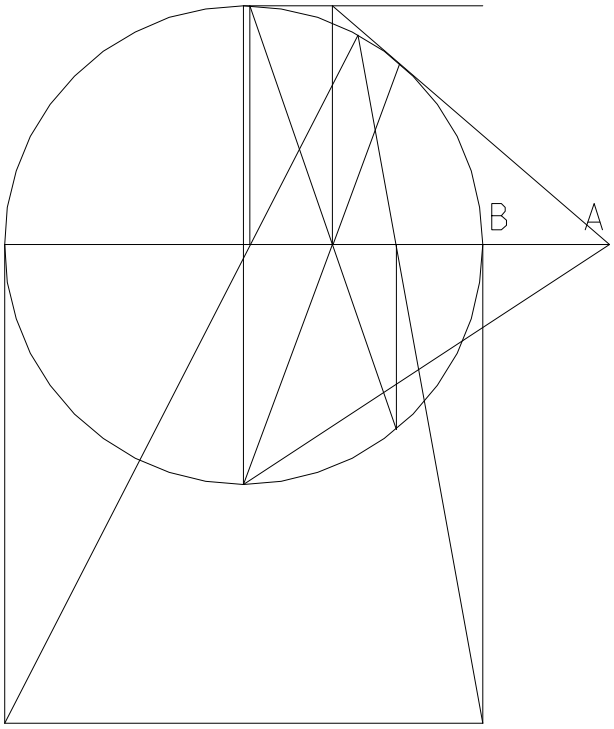
$MP := \frac{CF \cdot JP}{CJ} \quad AP := \frac{MP \cdot EP}{NP} \quad MN := \frac{CF \cdot JM}{CJ}$

$$\frac{BG^4 - 3 \cdot BG^3 \cdot BD + 3 \cdot BG^2 \cdot BD^2 - BD^3 \cdot BG}{BG^3 - 3 \cdot BG^2 \cdot BD + 3 \cdot BG \cdot BD^2 - 2 \cdot BD^3} = 884.222$$

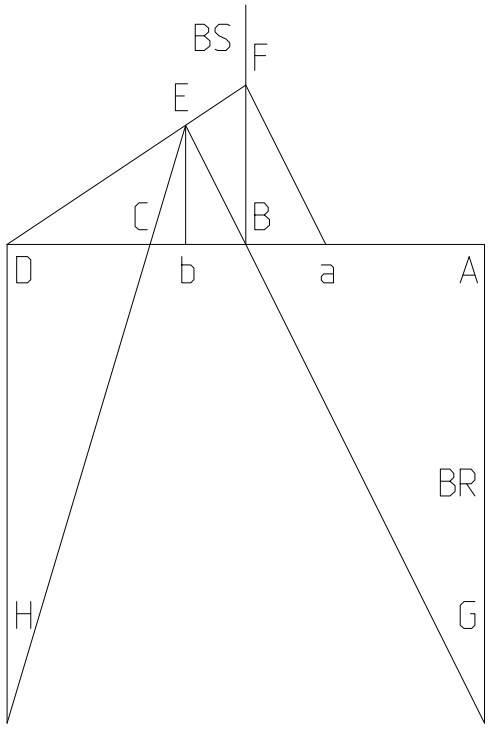
$AE := \frac{MN \cdot AP}{MP} \quad AG := AE + EG$

$$AG = 884.222$$

$$\frac{BG \cdot \left[ (AG - BG)^{\frac{2}{3}} \cdot AG^{\frac{2}{3}} + AG^{\frac{1}{3}} \cdot BG - AG^{\frac{4}{3}} - BG \cdot (AG - BG)^{\frac{1}{3}} + (AG - BG)^{\frac{1}{3}} \cdot AG \right]}{(AG - BG)^{\frac{1}{3}} \cdot (2 \cdot AG - BG)} - BD = 0$$




One may be tempted to state the obvious, $\frac{A^{N+1}}{A^{N}} := A_{\tau}$, but what is not so obvious at first blush is that the processes themselves are assigned dimensional values. This has significance when using mathematics to theorize dimensions beyond three. Dimensions are so generally defined that processes are legitimate dimensional differences, but it is also impossible to defend mathematical theory about dimensions as objective. It becomes a point of Philosophical Mystic contemplation to realize that relationships concerning a single dimensional object and several processes adding dimensionally to the whole, is true of a multidimensional object without those processes!




Linear division $\frac{2\cdot(A + B)}{A}$

$BR := \frac{1}{4}$ $BS := 3$

$AD := \frac{BR}{BR}$ $AG := AD\cdot BR$ $AB := \frac{AD}{2}$

$BF := AB\cdot BS$ $Ba := \frac{AB\cdot BF}{AG}$ $BD := AB$

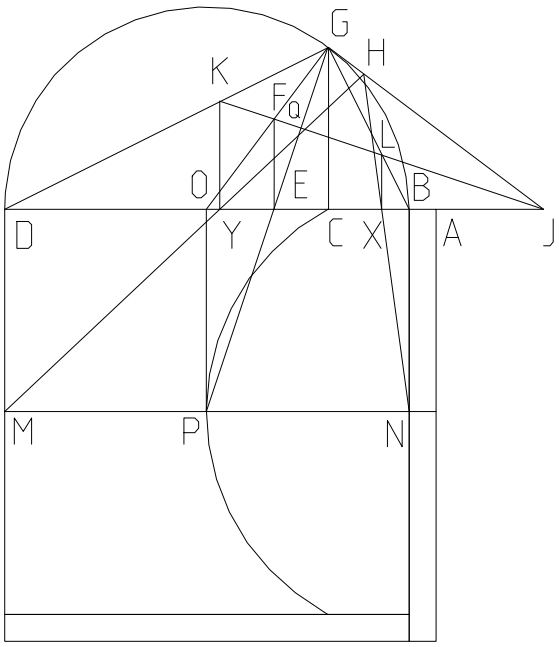
$Da := BD + Ba$ $Bb := \frac{Ba\cdot BD}{Da}$ $Db := BD - Bb$

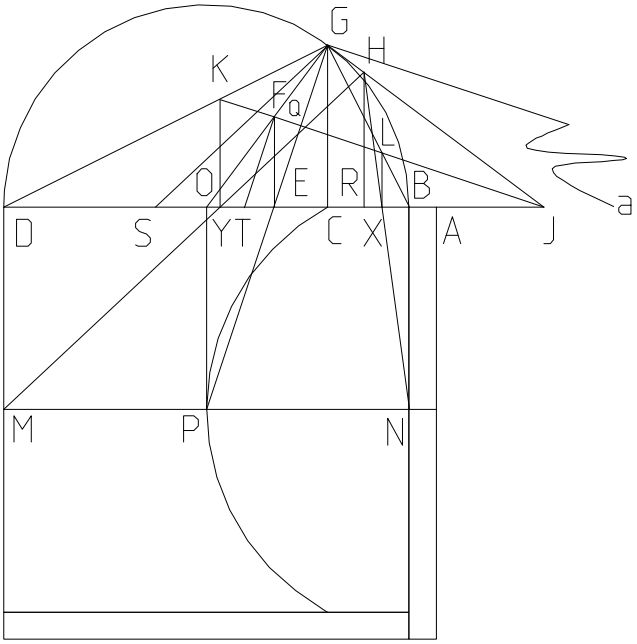
$Eb := \frac{BF\cdot Db}{BD}$ $DH := AG$

$DC := \frac{Db\cdot DH}{DH + Eb}$ $\frac{AD}{DC} = 26$

$\frac{2\cdot(BR + BS)}{BR} = 26$

Hitting JG from any BN while maintaining complimentary roots.




$AB := 2 \quad BD := 5 \quad BO := \frac{BD}{2} \quad AD := AB + BD$

$DO := BO \quad AC := \sqrt{AB \cdot AD} \quad BC := AC - AB$

$CD := BD - BC \quad CG := \sqrt{BC \cdot CD} \quad CO := BO - BC$

$N := 7 \quad OP := BO \cdot N \quad BN := OP \quad DM := OP$

$NP := BO \quad MP := BO \quad EO := \frac{CO \cdot OP}{OP + CG}$

$CE := CO - EO \quad EF := \frac{OP \cdot CE}{CO} \quad GO := BO$

$CJ := \frac{CG^2}{CO} \quad EJ := CJ + CE \quad Ca := \frac{EJ \cdot CG}{EF}$

$DJ := CD + CJ \quad Da := CD + Ca \quad JY := \frac{Ca \cdot DJ}{Da}$

$KY := \frac{EF \cdot JY}{EJ} \quad JK := \sqrt{JY^2 + KY^2} \quad JG := \sqrt{CJ^2 + CG^2}$

$GP := \sqrt{CO^2 + (OP + CG)^2} \quad EP := \sqrt{EO^2 + OP^2}$

$ET := \frac{EO \cdot EF}{OP} \quad JT := EJ + ET \quad FT := \sqrt{ET^2 + EF^2}$

$EG := GP - EP \quad EQ := \frac{FT \cdot EJ}{JT} \quad GQ := EG - EQ$

$KL := 2 \cdot GQ \quad JL := JK - KL$

$DY := DJ - JY \quad CS := \frac{DY \cdot CG}{DM} \quad JS := CS + CJ$

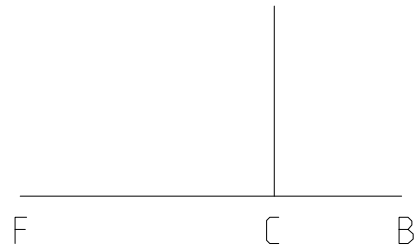
$YR := \frac{CS \cdot JY}{JS} \quad JR := JY - YR \quad HR := \frac{CG \cdot JR}{CJ}$

$AJ := CJ - AC \quad JX := \frac{JY \cdot JL}{JK}$

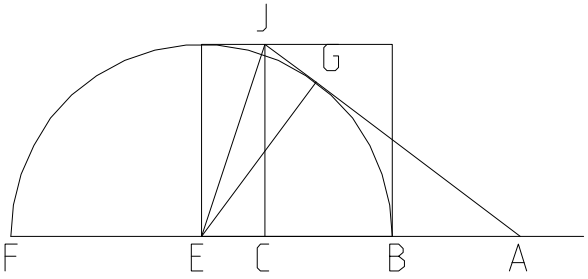
$BR_1 := JR - (AJ + AB) \quad BX := JX - (AJ + AB)$

$BR_2 := \frac{BX \cdot (BN + HR)}{BN}$

$BR_2 - BR_1 = 0$




Given BF and BC such that $\sqrt{(AB + BF)\cdot AB} = AB + BC$, find AB. It is obvious from the construction that answers are obtainable when BC is less than 1/2 of BF.

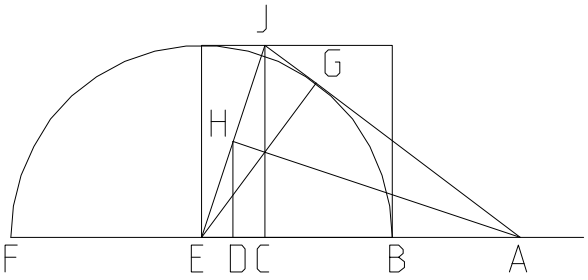



$\Delta := 5 \qquad \delta := 1..\Delta - 1 \quad BE := 5 \qquad BF := BE\cdot 2$

$BC_\delta := \frac{BE}{\Delta}\cdot\delta \qquad CE_\delta := BE - BC_\delta \qquad EG := BE$

$CJ := BE \quad EJ_\delta := \sqrt{\left(CE_\delta\right)^2 + CJ^2} \qquad DH := \frac{CJ}{2}$

$DE_\delta := \frac{CE_\delta}{2} \qquad AD_\delta := \frac{DH^2}{DE_\delta} \qquad AE_\delta := AD_\delta + DE_\delta$




$AC_\delta := AE_\delta - CE_\delta \qquad AJ_\delta := \sqrt{\left(AC_\delta\right)^2 + CJ^2}$

$GJ_\delta := \sqrt{\left(EJ_\delta\right)^2 - EG^2} \qquad AB_\delta := AE_\delta - BE$

$AF_\delta := AB_\delta + BF$

$\left[\frac{1}{2}\cdot\frac{BE}{\Delta}\cdot\delta\cdot\frac{(-2\cdot\Delta + \delta)}{(-\Delta + \delta)}\right] - AC_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$GJ_\delta - CE_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

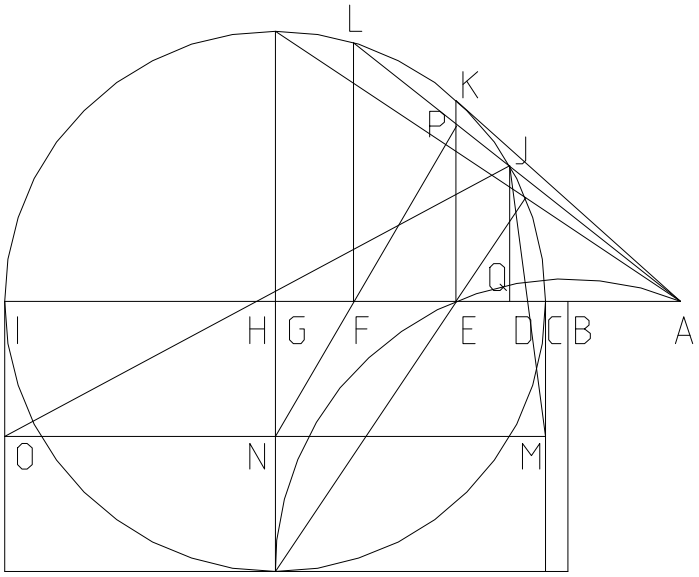
$AB_\delta + BC_\delta - \sqrt{AB_\delta\cdot(AB + BF)_\delta}$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$A := \frac{BF}{2} \qquad B := BC_2 \qquad \frac{2\cdot A\cdot B - B^2}{2\cdot A - 2\cdot B} - \left(AB_2 + BC_2\right) = 0$

$\sqrt{\left(AB_2 + BF\right)\cdot AB_2} - \left(AB_2 + BC_2\right) = 0$

$\frac{BF\cdot BC_2 - \left(BC_2\right)^2}{BF - 2\cdot BC_2} - \left(AB_2 + BC_2\right) = 0$ A rectangle minus a square divided by a line yeilds a line.




Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

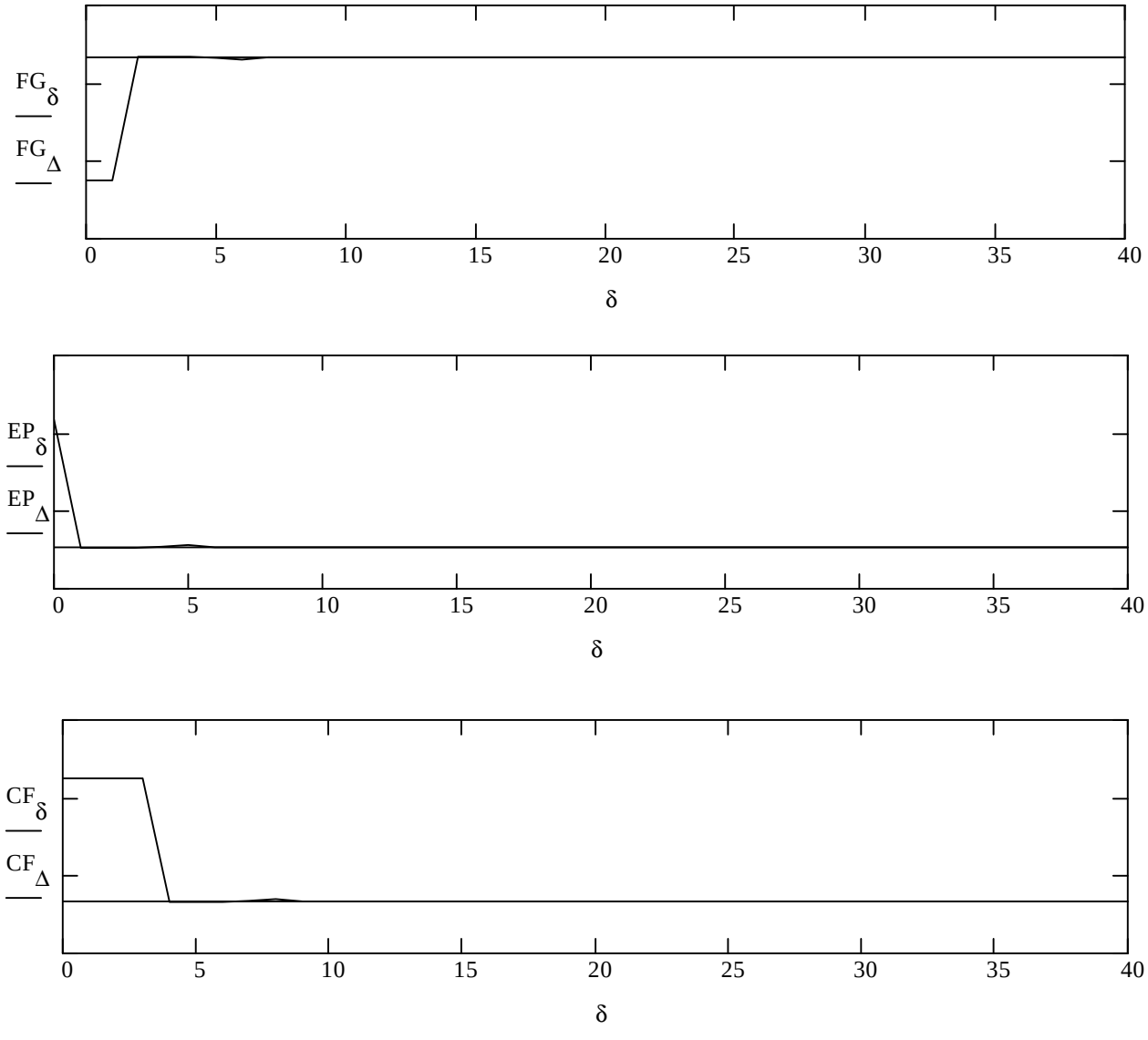
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

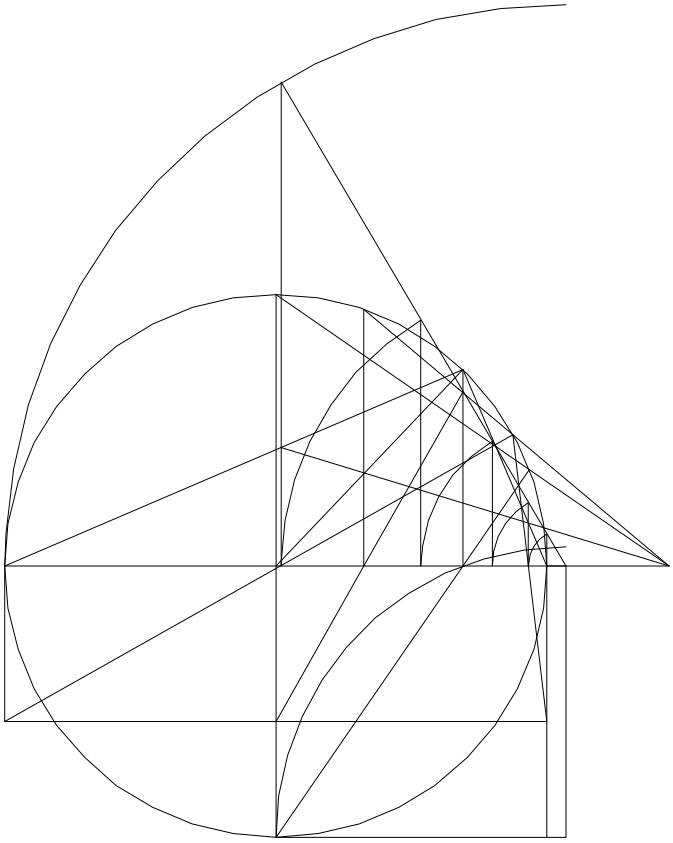
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.



Because of it's long projection, the last vertices is not drawn.

Does every $n^{th}$ root series have at least one square root pair?

$n := 5 \qquad \delta := 0..\frac{n}{2}$

$A := 3 \quad B := 10$

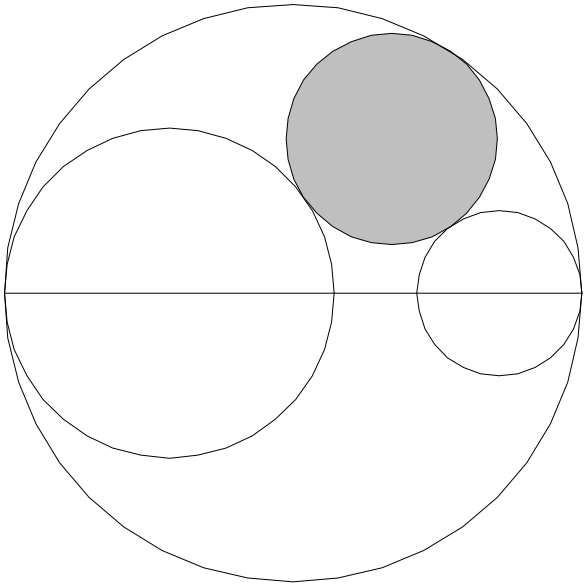
$$\sqrt{\left(A^{n-1}\cdot B^{1}\right)^{\frac{1}{n}}\cdot\left(A^{1}\cdot B^{n-1}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B} = 0$$

$$\sqrt{\left(A^{n-\delta}\cdot B^{\delta}\right)^{\frac{1}{n}}\cdot\left(A^{\delta}\cdot B^{n-\delta}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B}$$

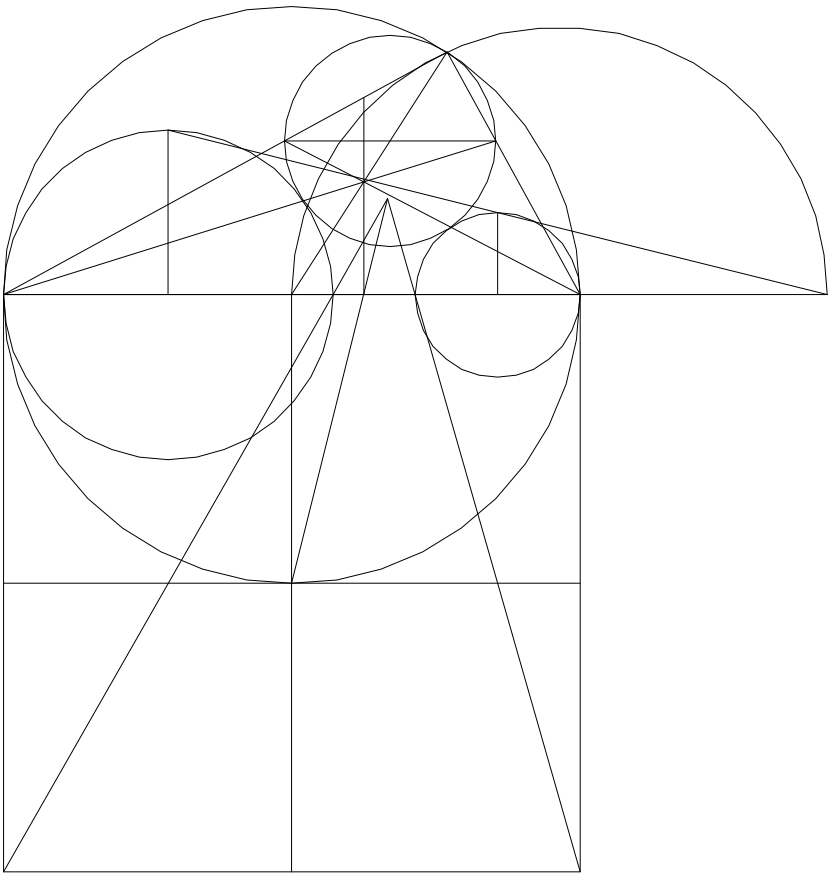
| |
|---|
| 0 |
| 0 |
| 0 |

A root series has as many vertices on a circle as it has square root pairs, and it has the greater whole of n/2 vertices where n is the root series denominator.

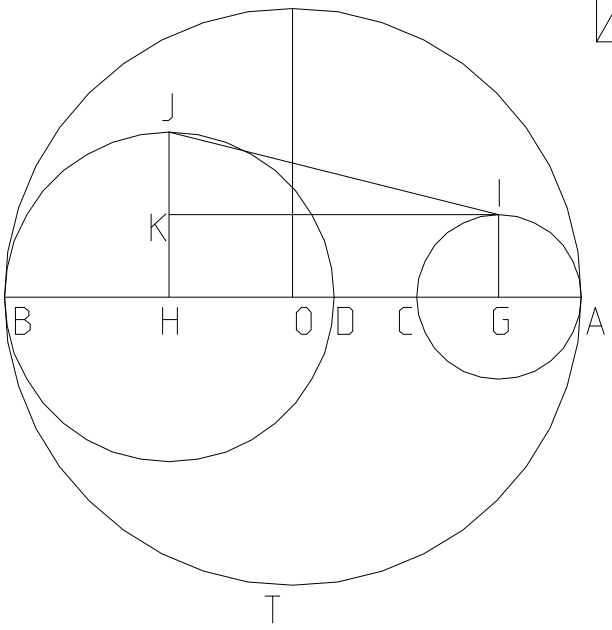
# Method for Unequals



Given three circles in the said configuration, find the fourth.
I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."



Process Summary.

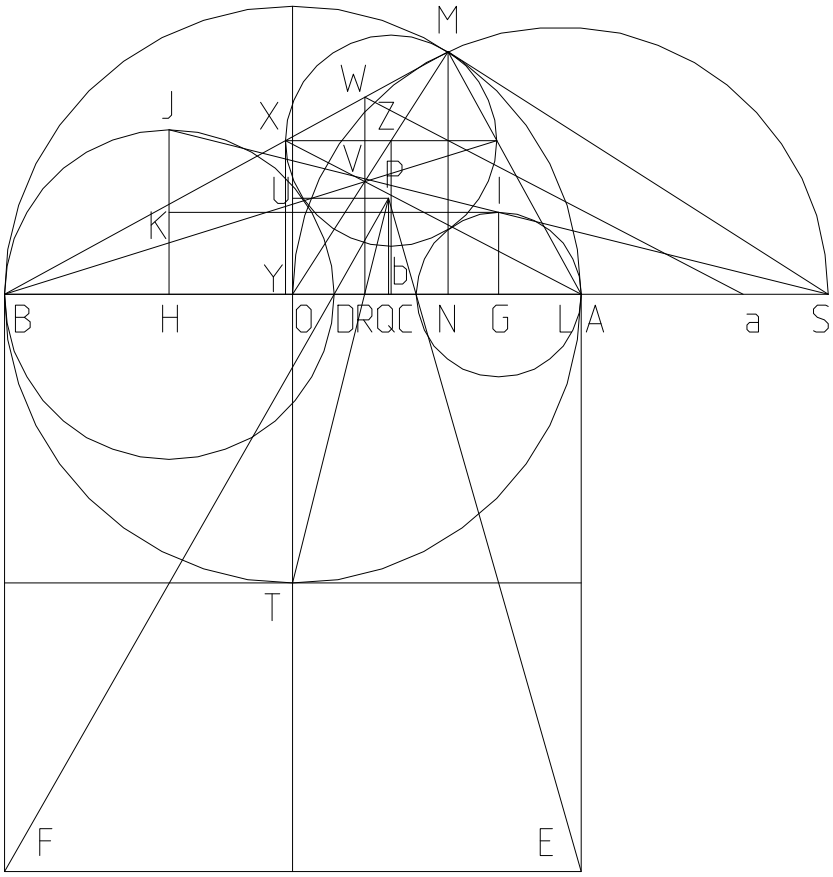


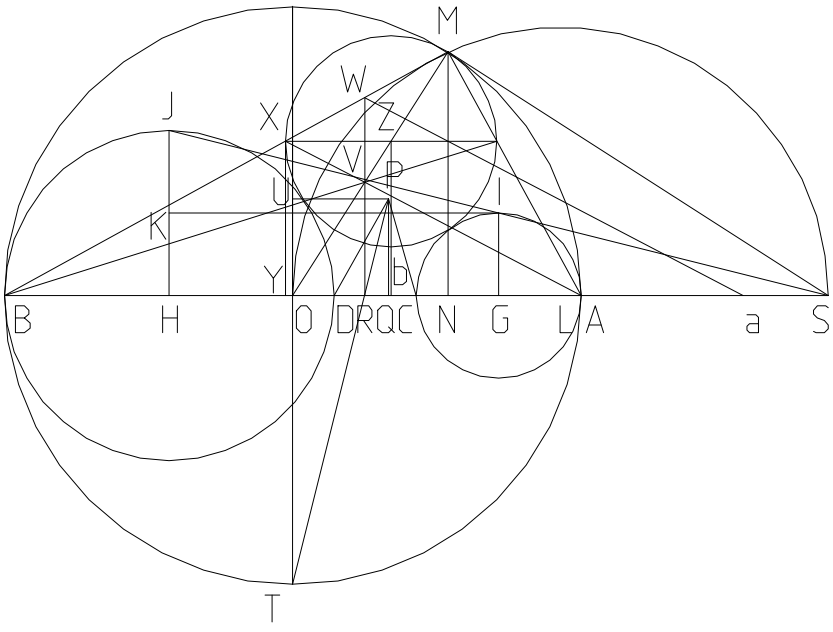

AO := 5 AG := 1 BH := 3 AB := 2·AO

BO := AO CG := AG GI := AG HJ := BH

DH := BH GH := AB – (AG + BH) HK := GI

JK := HJ – HK




$HS := \frac{GH \cdot HJ}{JK}$ $AH := AB - BH$

$AS := HS - AH$ $OS := AO + AS$

$SL := \frac{OS}{2}$ $MO := AO$ $MS := \sqrt{OS^2 - MO^2}$

$MN := \frac{MO \cdot MS}{OS}$ $NS := \frac{MS \cdot MN}{MO}$

$AN := NS - AS$ $ON := AO - AN$

$BN := AB - AN$ $AM := \sqrt{MN^2 + AN^2}$

$AE := AB$ $BF := AB$ $AC := 2 \cdot AG$

$BD := 2 \cdot BH$ $CD := AB - (AC + BD)$

$PQ := \frac{AE \cdot CD}{(AC + BD)}$ $OU := PQ$

$CQ := \frac{AC \cdot PQ}{AE}$ $AQ := AC + CQ$

$OQ := AO - AQ$ $OT := AO$

$TU := OT + OU$ $OR := \frac{OQ \cdot OT}{TU}$

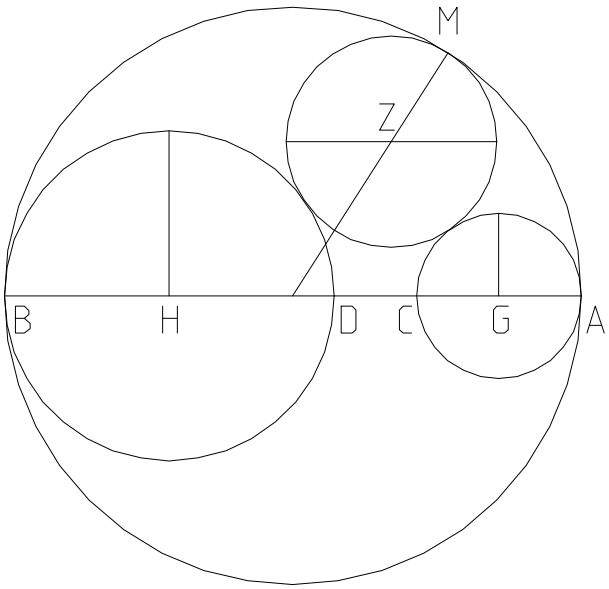
$RV := \frac{MN \cdot OR}{ON}$ $BR := BO + OR$

$RW := \frac{MN \cdot BR}{BN}$ $AR := AO - OR$

$Ra := \frac{AR \cdot RW}{RV}$ $XY := \frac{RW \cdot AB}{BR + Ra}$

$Zb := XY$ $OZ := \frac{MO \cdot Zb}{MN}$

$MZ := MO - OZ$ $MZ = 1.818$

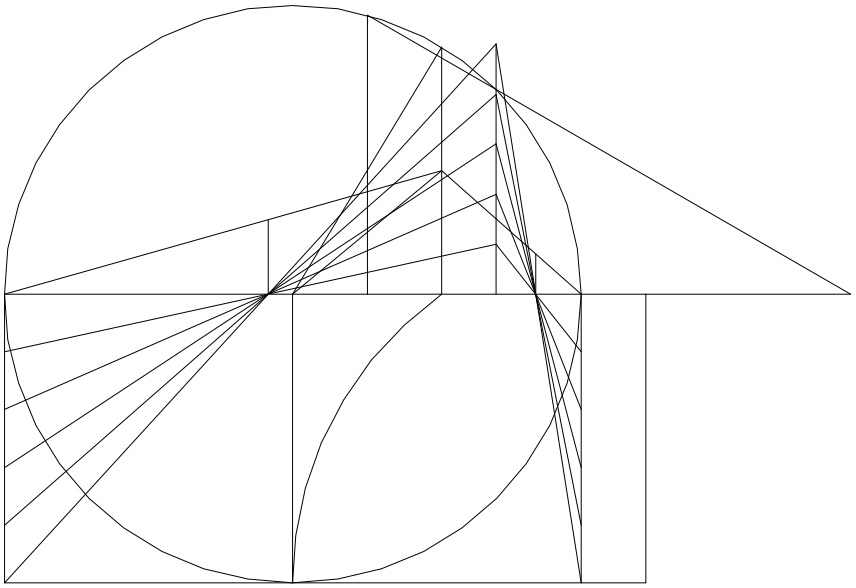


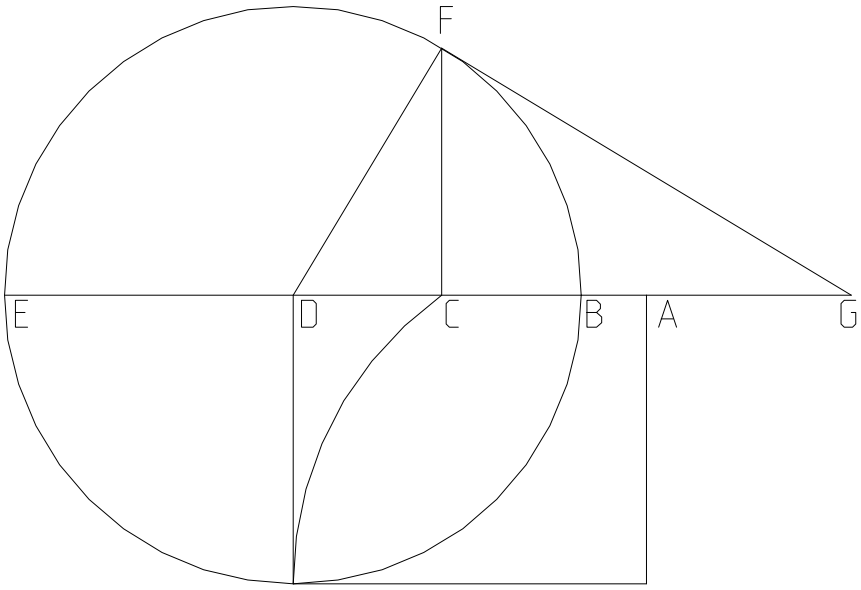

$$MZ = 1.818$$

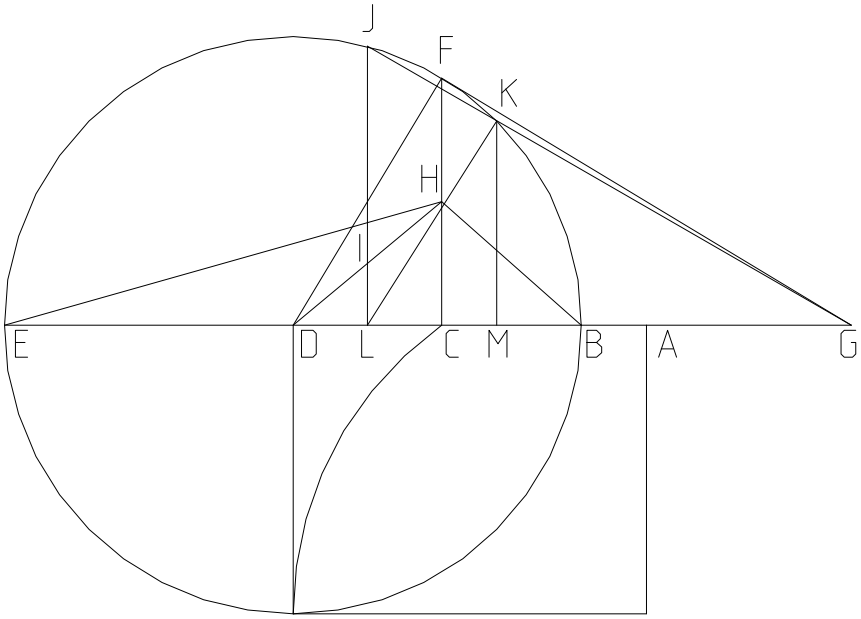
$$\frac{AB^3 - 2\cdot AB^2\cdot AG - 2\cdot AB^2\cdot BH + 4\cdot AB\cdot BH\cdot AG}{2\cdot AB^2 - 8\cdot BH\cdot AG} = 1.818$$

Neither in the process, nor in the Algebraic name. is the order of AG and BH recognized. Neither does it matter if they intersect.

# On Gemini Roots






$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

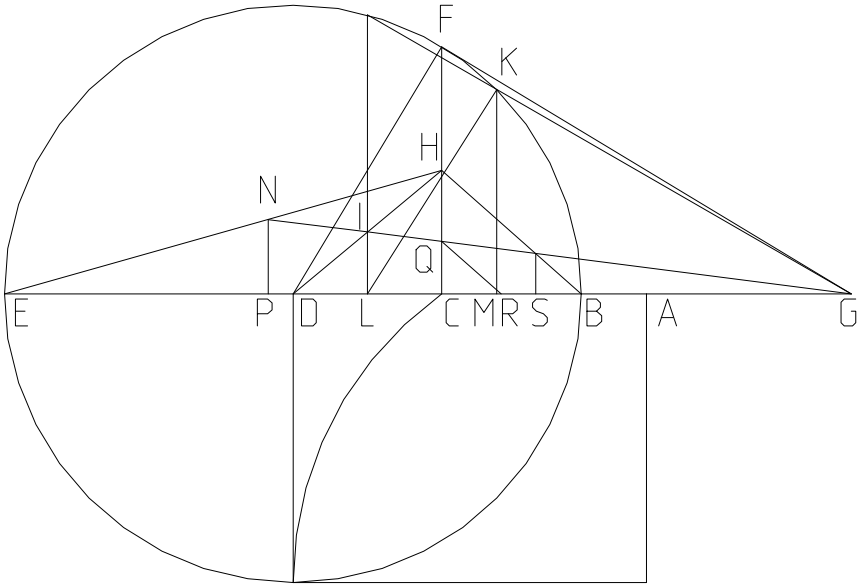
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

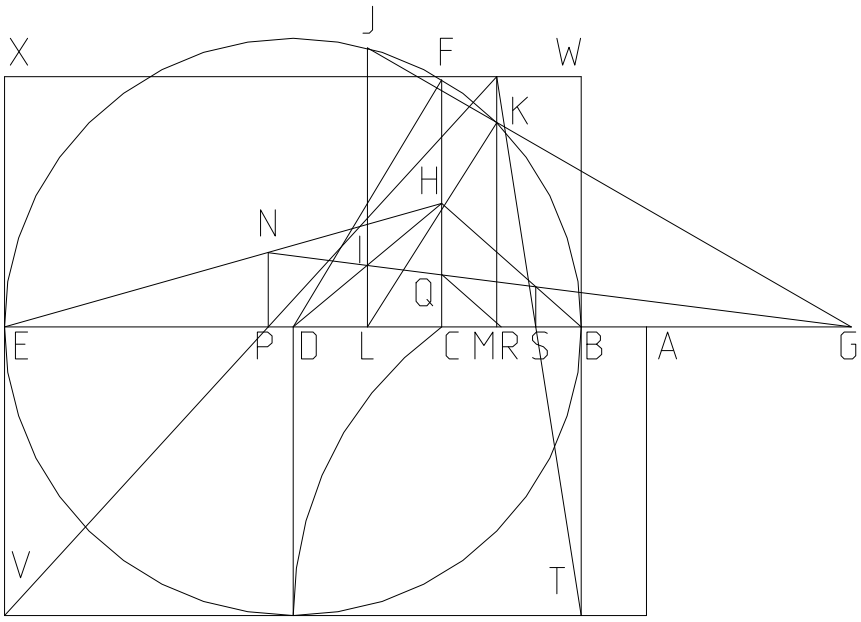



$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

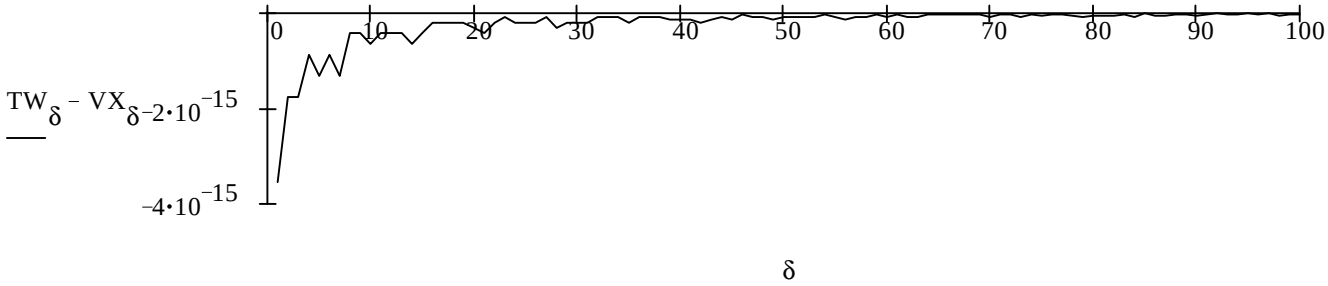
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$

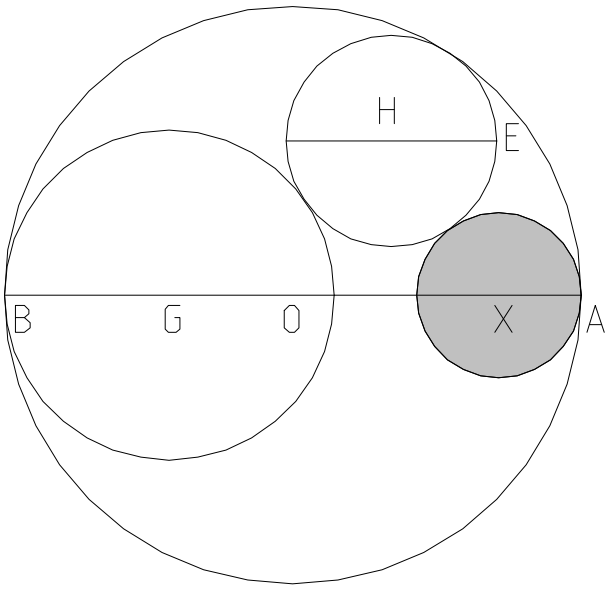



$$\delta := 1..100$$

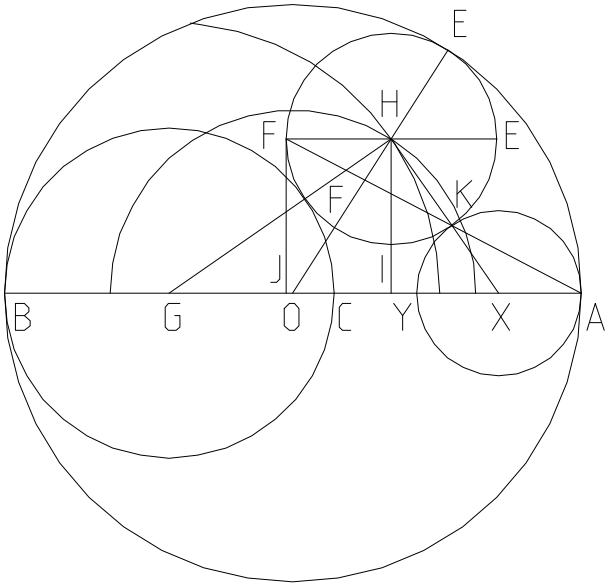
$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$






Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO. This of course means that if the sum of BG and EH is equal to AO, we have no result.




$AO := 5 \quad BG := 2.5 \quad EH := 1.5$

$AB := 2 \cdot AO \quad BC := 2 \cdot BG \quad EF := 2 \cdot EH$

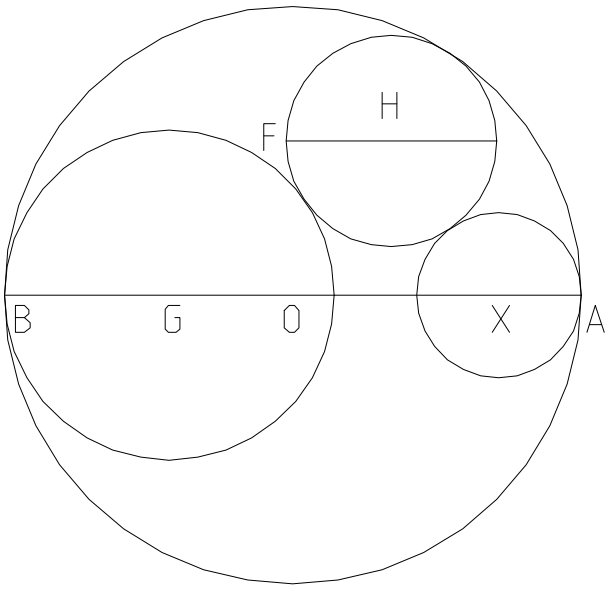
$GH := BG + EH \quad OH := AO - EH \quad GO := AO - BG$

$$GI := \frac{GH^2 + GO^2 - OH^2}{2 \cdot GO} \quad HI := \sqrt{GH^2 - GI^2}$$

$AG := AB - BG \quad AI := AG - GI \quad IJ := EH$

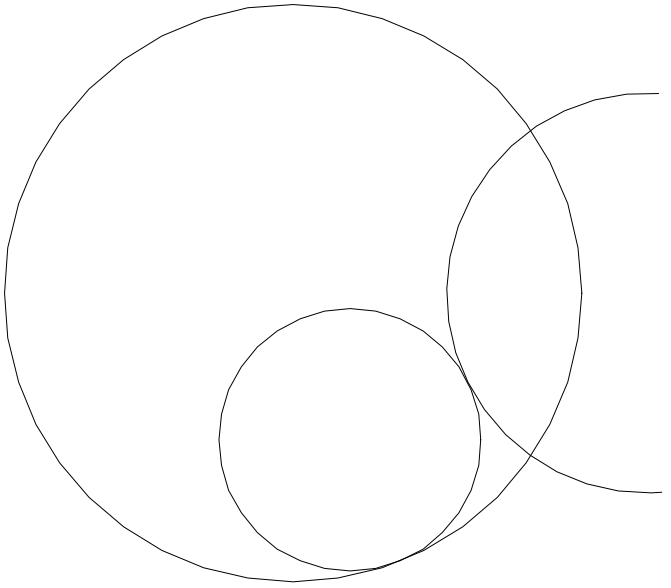
$$AJ := AI + IJ \quad FJ := HI \quad AF := \sqrt{AJ^2 + FJ^2}$$

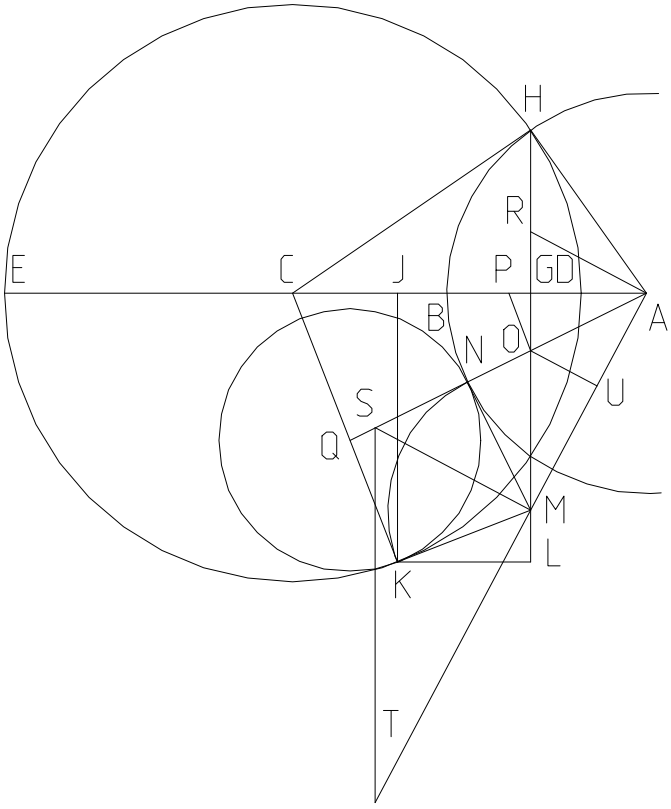
$$FK := \frac{AJ \cdot EF}{AF} \quad AK := AF - FK \quad AY := \frac{AF \cdot AK}{AJ}$$




$$AX := \frac{AY}{2} \qquad AX = 2.857$$

$$\frac{AO^3 - AO^2 \cdot EH - AO^2 \cdot BG}{AO^2 - AO \cdot BG - EH \cdot BG} = 2.857$$






Given a point of tangency, draw a circle in a crescent tangent to the other side. This figure is given for the tangent on the exterior of the crescent, the other will become obvious.

$AB := Concave\_Radius$

$CD := Convex\_Radius \quad AC := Center\_Difference$

$DE := 2 \cdot CD \quad BF := 2 \cdot AB \quad CE := CD$

$AE := AC + CE \quad BE := AE - AB$

$$CG := \frac{CD^2 + AC^2 - AB^2}{2 \cdot AC} \quad AG := AC - CG$$

$GE := AE - AG$

$GJ := Power\_Line\_Tangent \cdot GE$

$EJ := GE - GJ$

$DJ := DE - EJ \quad JK := \sqrt{DJ \cdot EJ}$

$CJ := CG - GJ \quad CK := CD \quad KL := GJ$

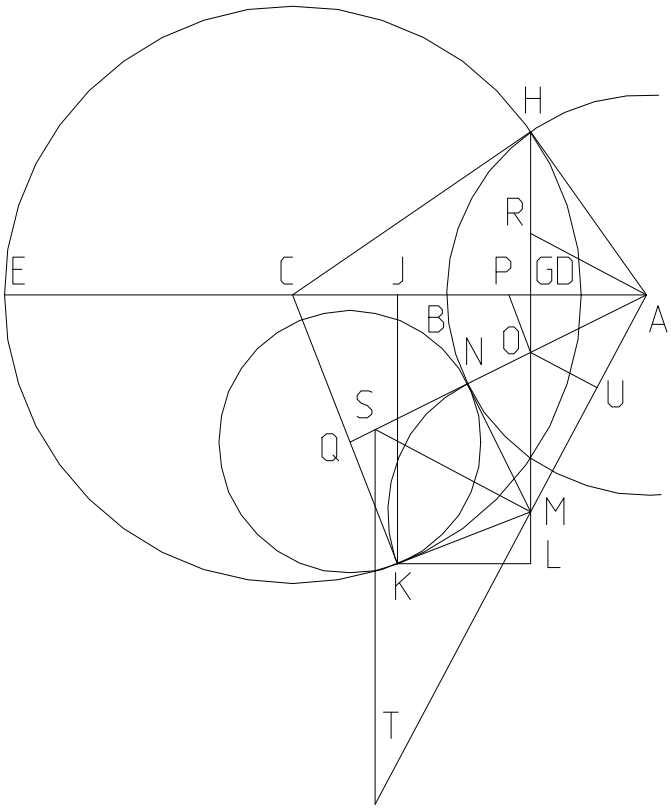
$$LM := \frac{CJ \cdot KL}{JK} \quad GL := JK \quad GM := GL - LM$$

$$AM := \sqrt{AG^2 + GM^2} \quad AN := AB$$

$$AS := \frac{AM^2}{AN} \quad MR := \frac{AM^2}{GM}$$

$$MS := \sqrt{AS^2 - AM^2} \quad AR := \sqrt{MR^2 - AM^2}$$

$$ST := \frac{MR \cdot MS}{AR} \quad MT := \frac{AM \cdot MS}{AR}$$




$$MO := \frac{ST \cdot AM}{AM + MT}$$

$$GO := GM - MO \quad GP := \frac{CJ \cdot GO}{JK}$$

$$AP := AG + GP \quad OP := \frac{CK \cdot GO}{JK}$$

$$CQ := \frac{OP \cdot AC}{AP} \quad QK := CK - CQ$$

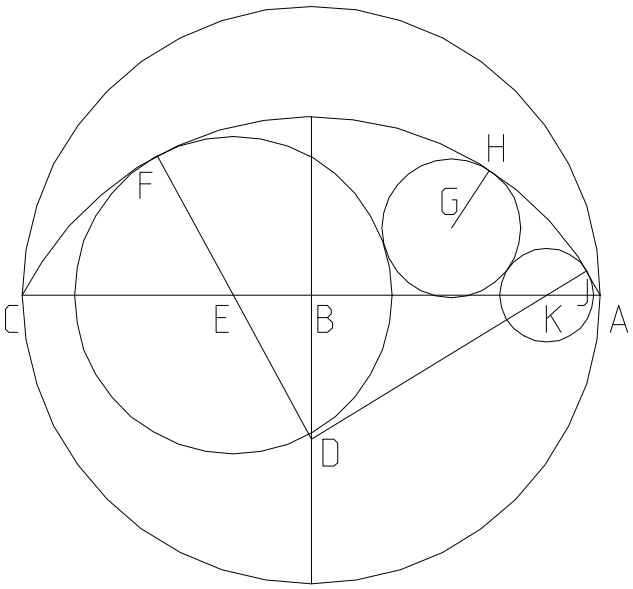
$$QK = 0.206$$

$$Concave\_Radius \equiv 2.37$$

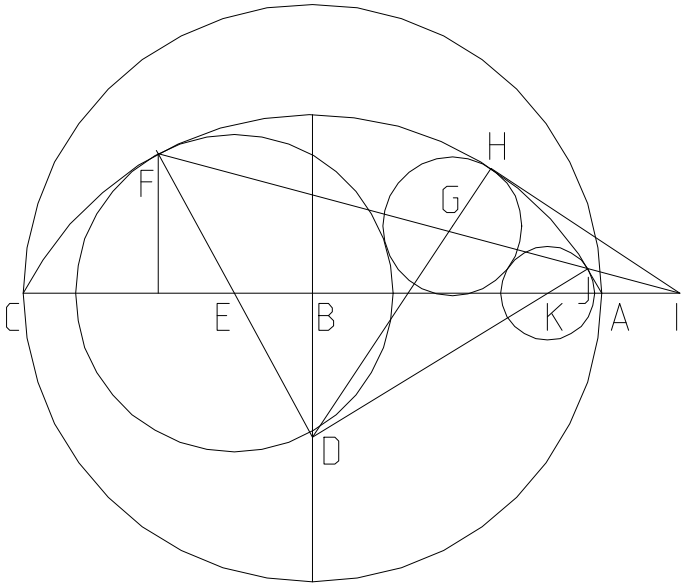
$$Convex\_Radius \equiv 1.5$$

$$Center\_Difference \equiv 1.84$$

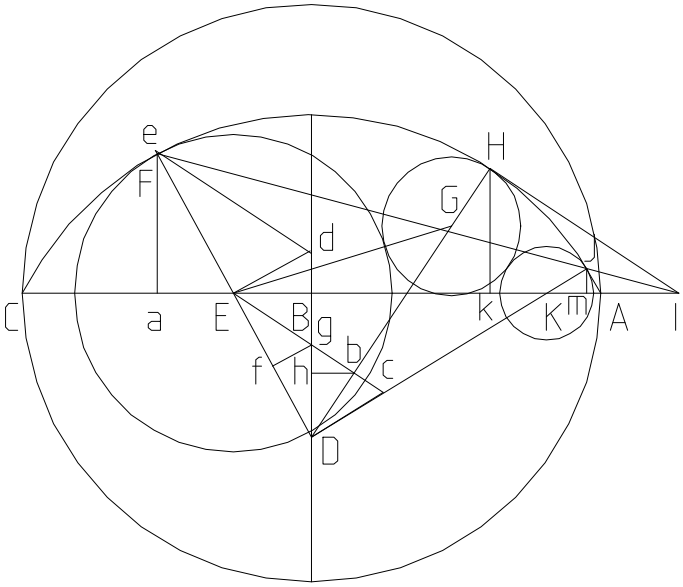
$$Power\_Line\_Tangent \equiv \frac{1}{3} \quad \text{Given as Fraction} < 1.$$




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

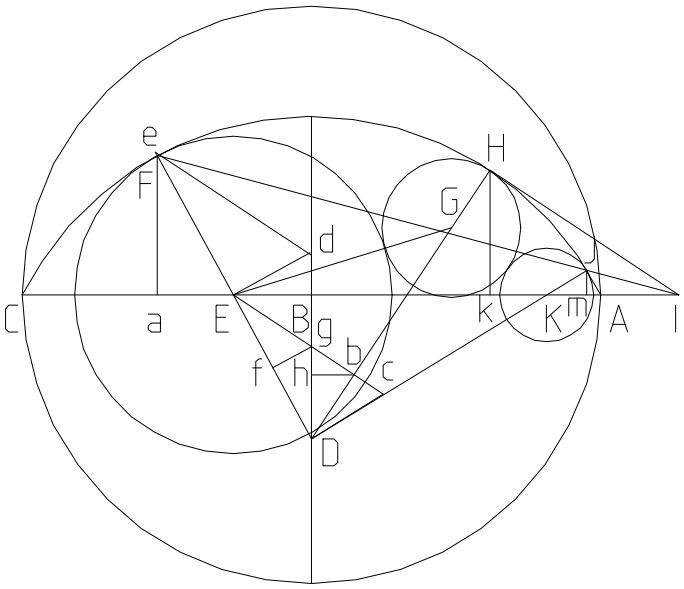
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

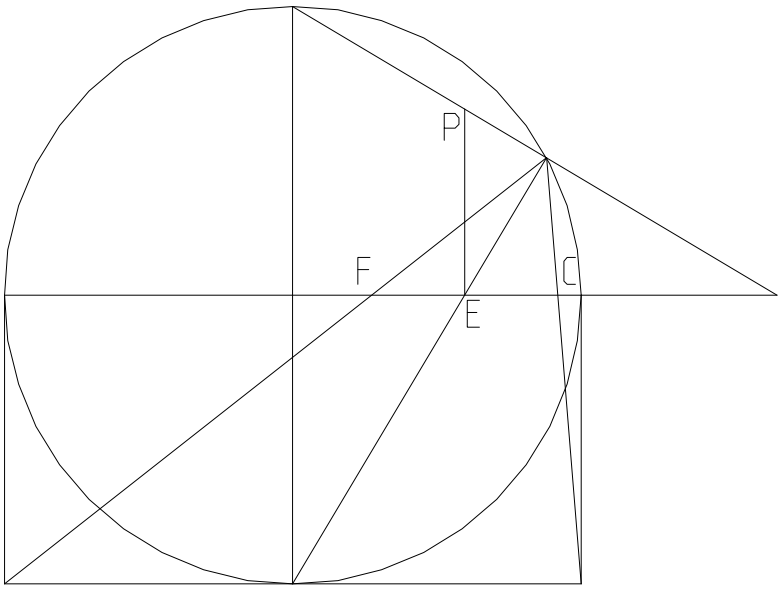
$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

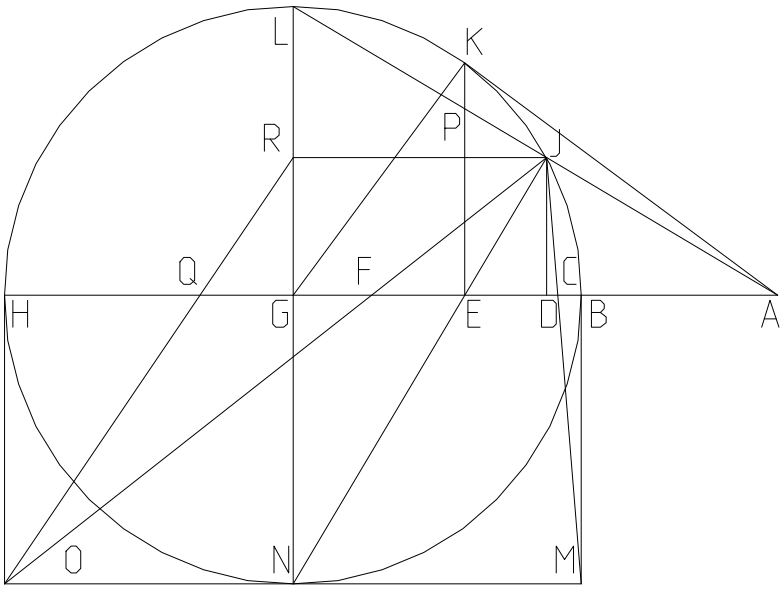
$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.




I found this little sketch in my notebook and have no idea of when I did it or why.

Does CF always equal EP?




$\Delta := 100 \quad \delta := 1..\Delta - 1 \quad BG := 10 \quad GH := BG$

$GN := BG \quad BD_\delta := \frac{BG}{\Delta}\cdot\delta \quad DG_\delta := BG - BD_\delta$

$DH_\delta := GH + DG_\delta \quad DJ_\delta := \sqrt{BD_\delta \cdot DH_\delta}$

$EG_\delta := \frac{DG_\delta \cdot GN}{GN + DJ_\delta} \quad BE_\delta := BG - EG_\delta$

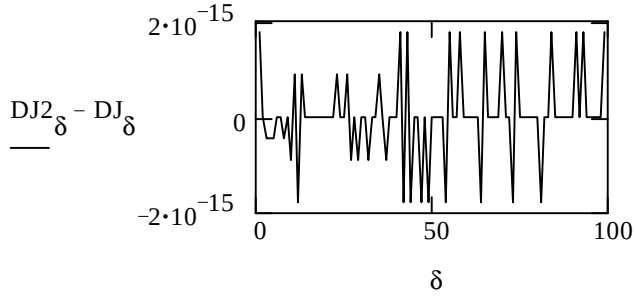
$EH_\delta := EG_\delta + GH \quad EK_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

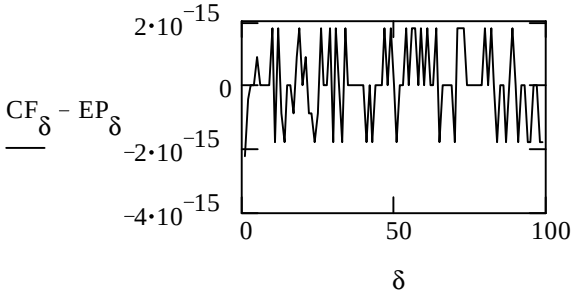
$AE_\delta := \frac{(EK_\delta)^2}{EG_\delta} \quad AB_\delta := AE_\delta - BE_\delta$

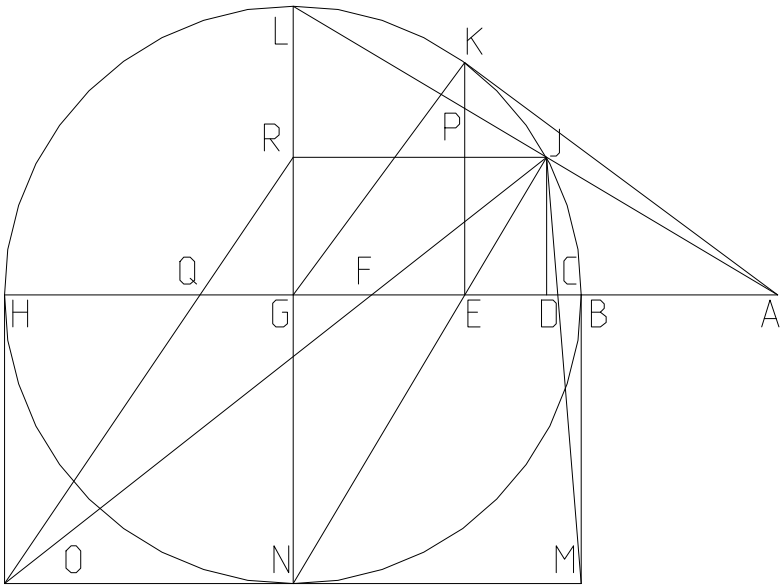
$GL := BG \qquad AD_\delta := AB_\delta + BD_\delta$

$AG_\delta := AB_\delta + BG \qquad DJ2_\delta := \frac{GL \cdot AD_\delta}{AG_\delta}$

$EP_\delta := \frac{GL \cdot AE_\delta}{AG_\delta} \quad GR_\delta := DJ_\delta \quad NO := BG \quad NR_\delta := GN + GR_\delta \quad GQ_\delta := \frac{NO \cdot GR_\delta}{NR_\delta} \quad CF_\delta := 2 \cdot GQ_\delta$






Reducing both by the symbolic processor leaves a little.

$$BG := 10 \qquad BD := \frac{BG}{13}$$

$$CF := 2\cdot BG\cdot\sqrt{BD}\cdot\frac{\sqrt{2\cdot BG - BD}}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)}$$

$$EP := -2\cdot BG\cdot\frac{\left(-BG\cdot\sqrt{BD}\cdot\sqrt{2\cdot BG - BD} - 2\cdot BG\cdot BD + BD^2\right)}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)^2}$$

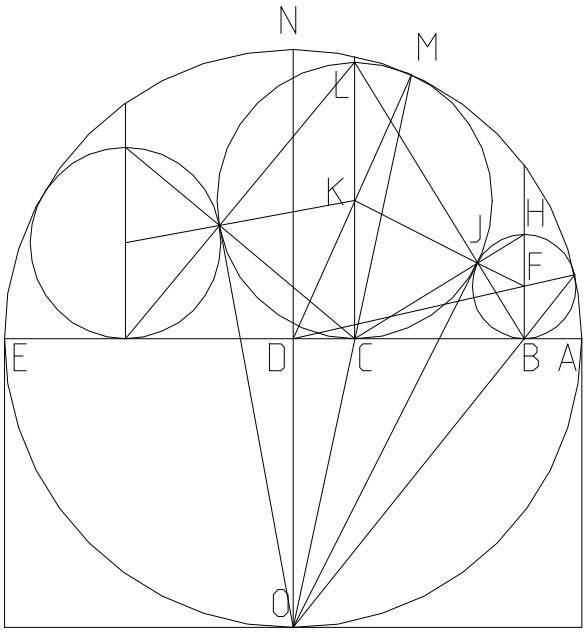
$EP = 5.556 \qquad EP - CF = 0$

And if I divide one by the other and reduce; $\dfrac{\left[BG\cdot\sqrt{2\cdot BG - BD} + 2\cdot BG\cdot\sqrt{BD} - BD^{\left(\frac{3}{2}\right)}\right]}{\left[\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)\cdot\sqrt{2\cdot BG - BD}\right]} = 1$

This is another figure that I had sketched out last year but never got around to writing down.

Given a Circle, place the next on the diameter.

I tried to reduce this series with the symbolic processor, but it is having trouble, at some point it switches AC for EC and I get the other circle.



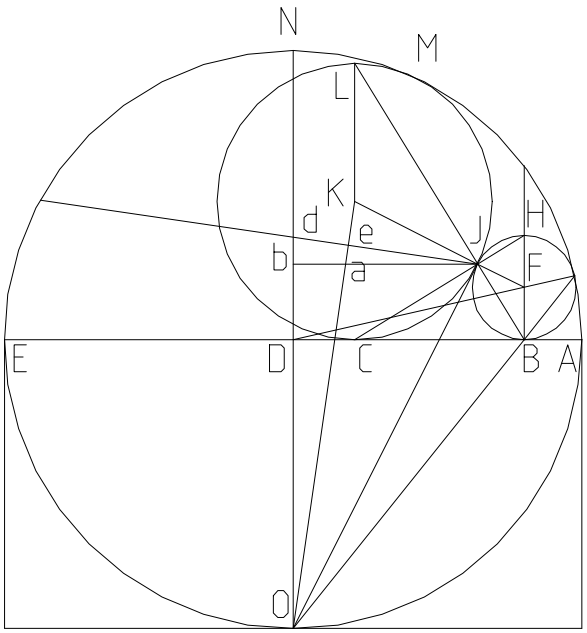

$AD := Radius \quad AE := 2 \cdot AD \quad AC := \frac{AE}{N}$

$CD := AD - AC \quad DO := AD \quad CO := \sqrt{DO^2 + CD^2}$

$NO := AE \quad MO := \frac{DO \cdot NO}{CO} \quad CM := MO - CO$

$CK := \frac{DO \cdot CM}{MO} \quad KO := \sqrt{CD^2 + (DO + CK)^2}$

$JK := CK$

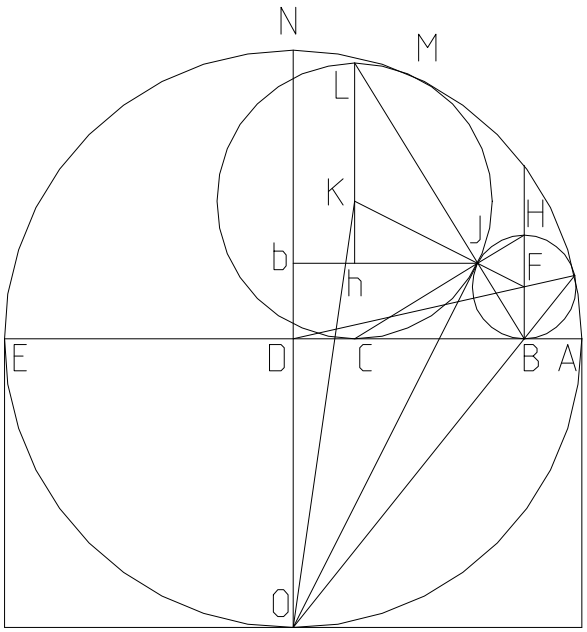



$Ke := \frac{JK^2}{KO} \quad Oe := KO - Ke \quad de := \frac{CD \cdot Oe}{DO + CK}$

$Je := \sqrt{JK^2 - Ke^2}$

$Jd := de + Je \quad bd := \frac{CD \cdot Jd}{KO}$

$Od := \frac{KO \cdot de}{CD} \quad Db := Od - DO - bd$




$Kh := CK - Db$

$Lh := CK + Kh$ $\quad FJ := \frac{JK \cdot Db}{Lh}$

$N \equiv \frac{10}{1}$ $\quad FJ = 3.965$

$Radius \equiv 108$ $\quad CK = 19.44$

And from the other side;

$N \equiv \frac{10}{9}$ $\quad FJ = 51.53$

$Radius \equiv 108$ $\quad CK = 19.44$

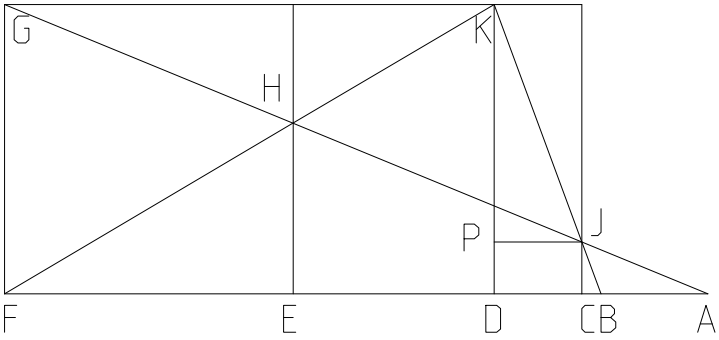
This figure might be recognized as the similarity point for Gemini root projection.

Given AC, CF, and that

$$AC = -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

find BC.

N can be any value whatever, except 0.




$CF := 216 \quad AC := 47.29 \quad N := 100000$

$CE := \frac{CF}{2} \quad AF := AC + CF \quad AE := AC + CE$

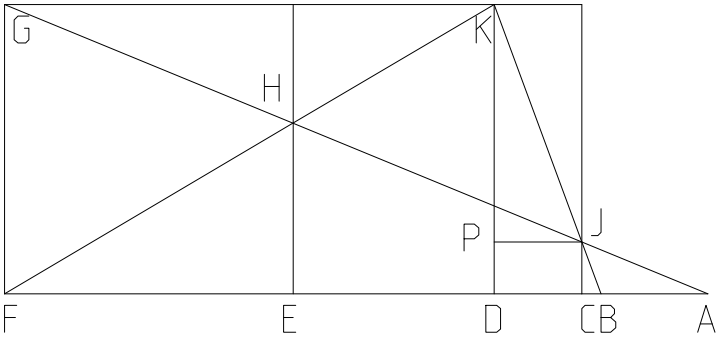
$FG := N \quad EH := \frac{FG\cdot AE}{AF} \quad EF := CE$

$DF := \frac{EF\cdot FG}{EH} \quad CJ := \frac{FG\cdot AC}{AF} \quad DP := CJ$

$CD := CF - DF \quad DK := FG \quad KP := DK - DP$

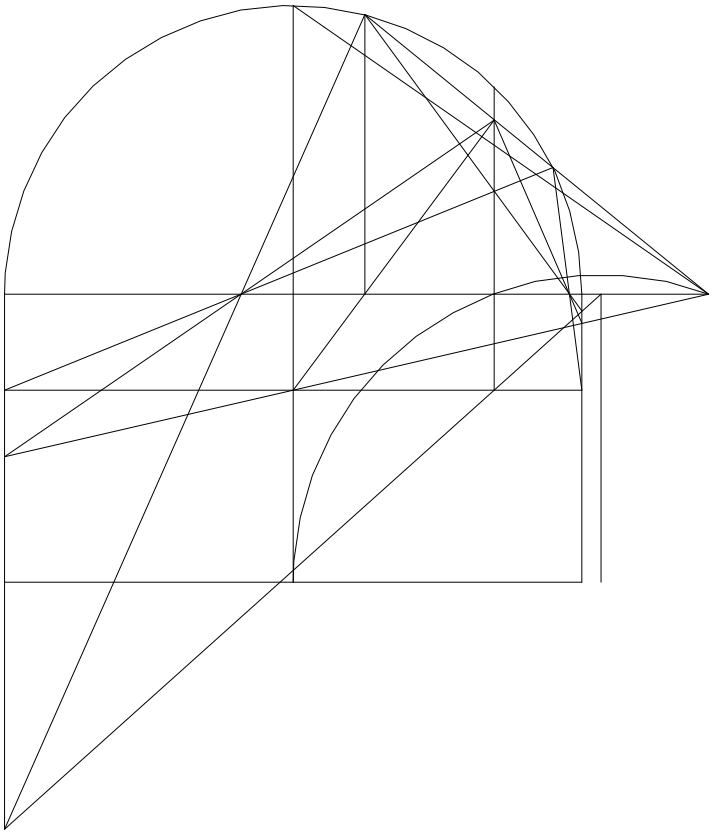
$BD := \frac{CD\cdot DK}{KP} \quad BD = 40.089$

$BC := BD - CD \quad BC = 7.201$



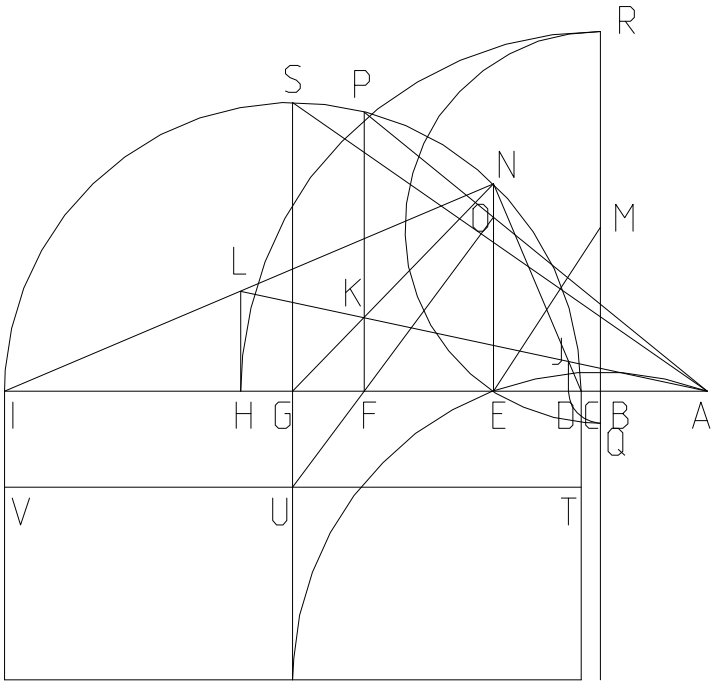

$$AC_f := -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

$$\frac{AC_f}{AC} = 1$$



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

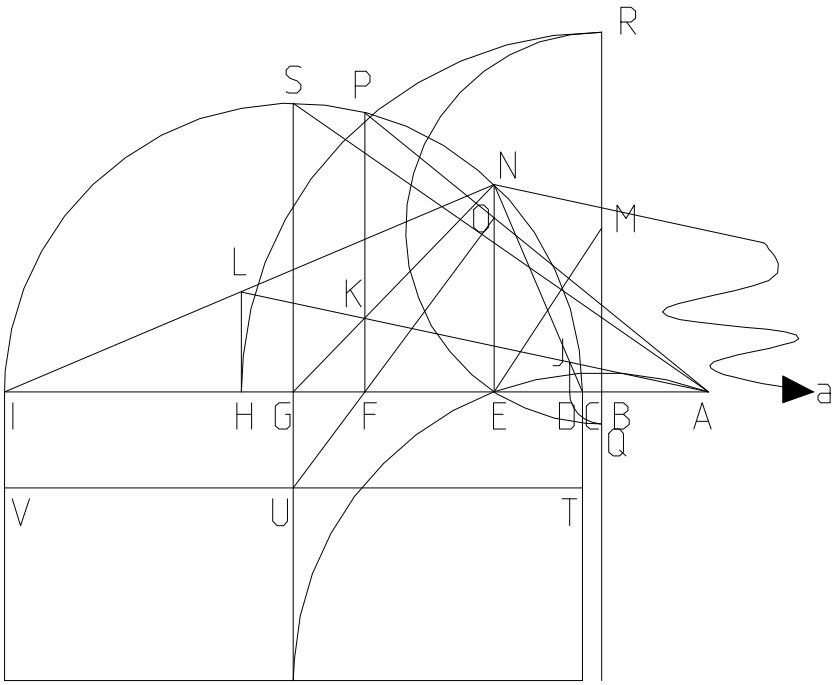
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



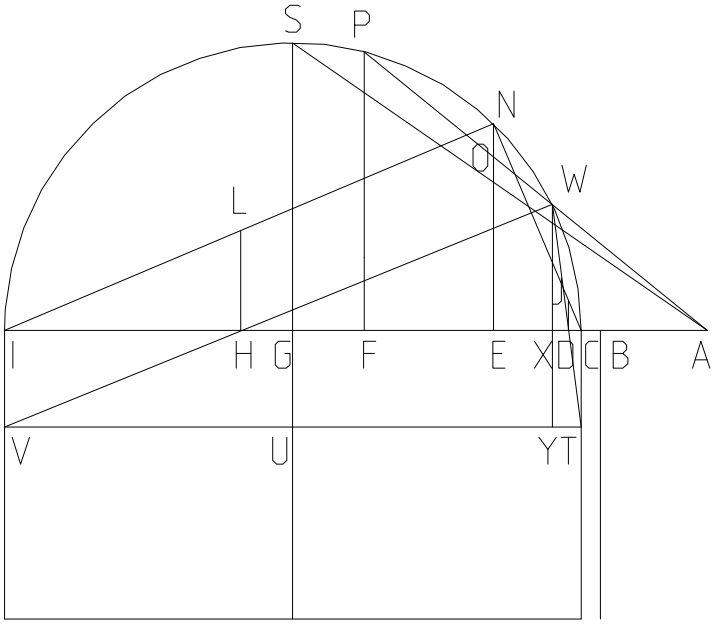

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

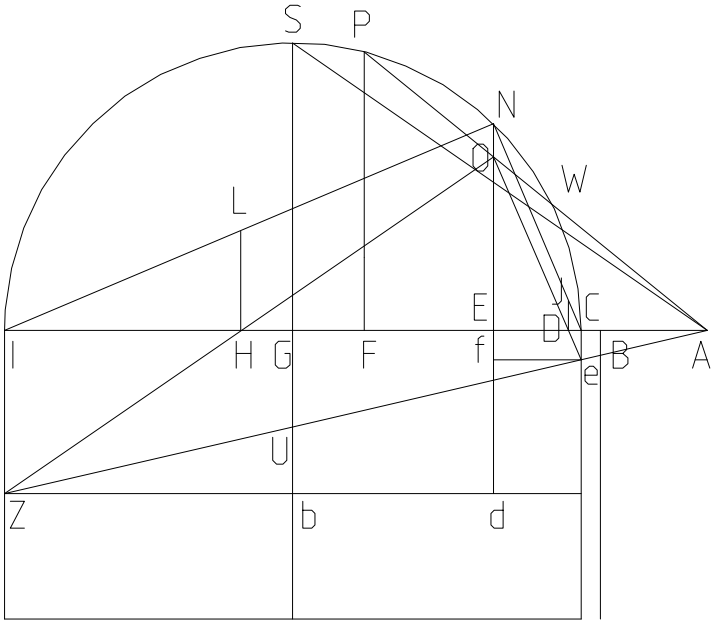
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

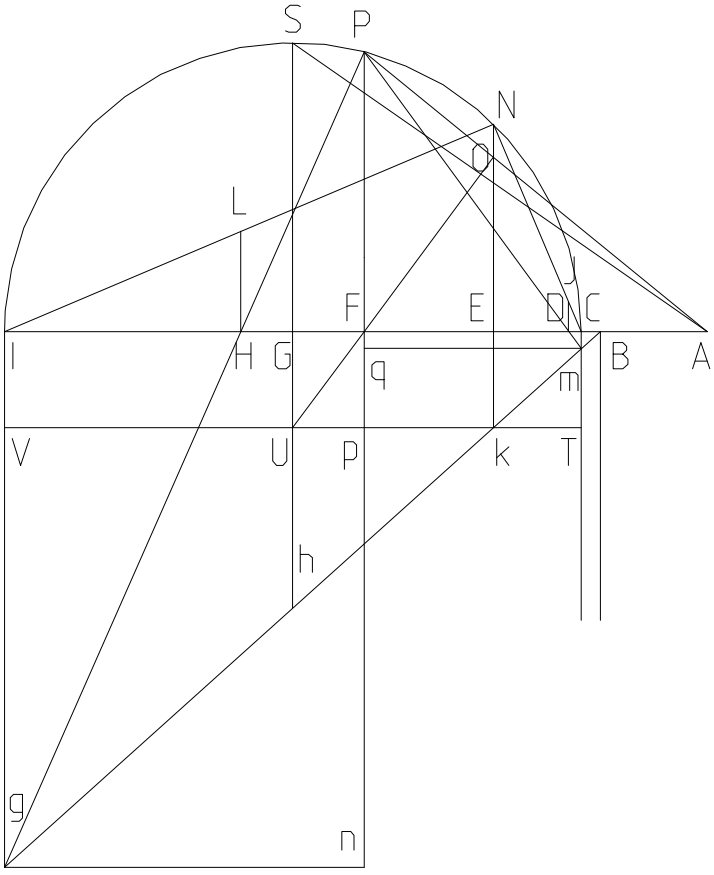
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



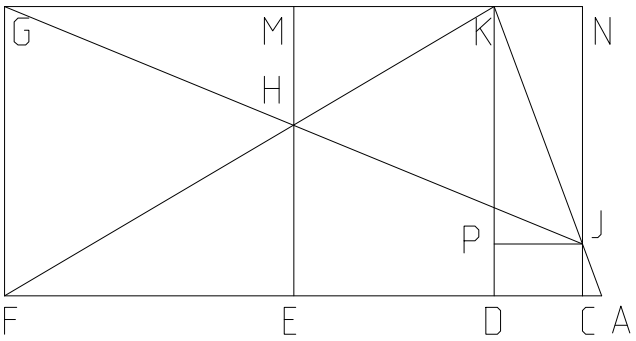

$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

Given CF and CD such that CD = $\sqrt{AF \cdot AC} - AC$ find AC.




$CF := 216 \quad CD := 32.89 \quad CE := \frac{CF}{2} \quad EF := CE$

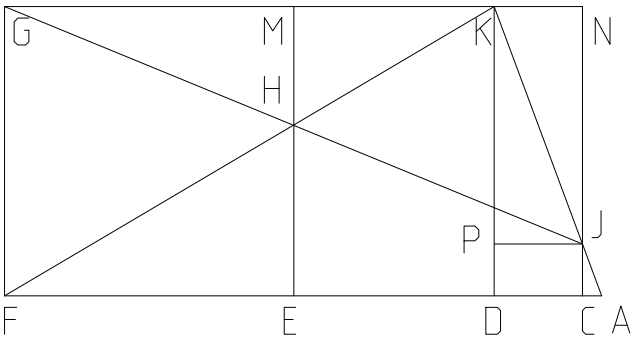
Except for 0, N can be any value.

$N := 108 \quad FG := N \quad DK := N \quad DF := CF - CD$

$EH := \frac{DK \cdot EF}{DF} \quad EM := FG \quad HM := EM - EH$

$CN := FG \quad HM := EM - EH \quad GN := CF$

$GM := EF \quad JN := \frac{HM \cdot GN}{GM} \quad JC := CN - JN$




$KP := JN \quad JP := CD \quad AD := \frac{JP \cdot DK}{KP}$

$AC := AD - CD \quad AC = 7.201$

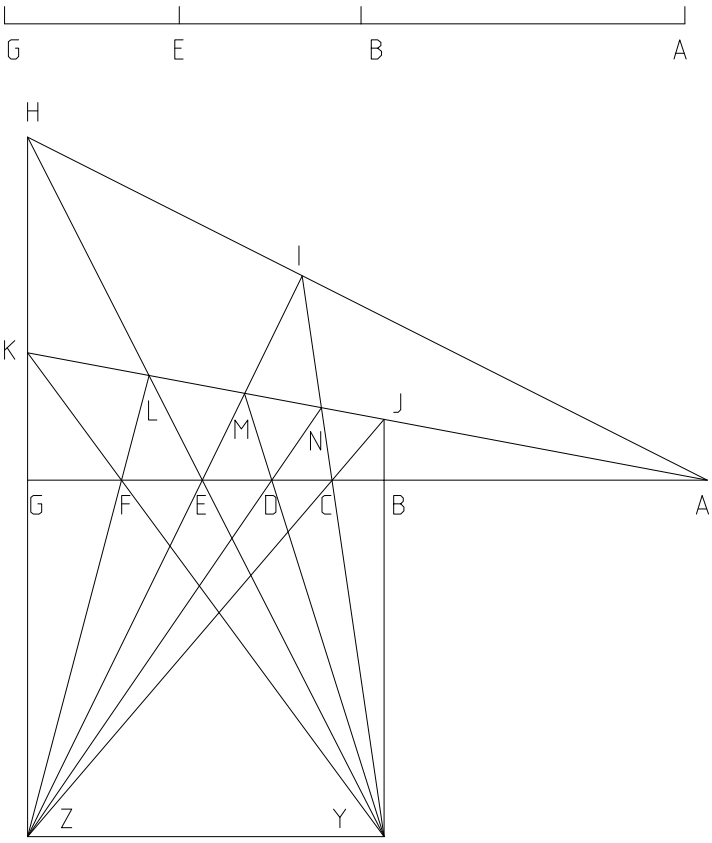
$AF := AC + CF$

$\frac{\sqrt{AF \cdot AC} - AC}{CD} = 1 \qquad \frac{\sqrt{AF \cdot AC}}{AD} = 1$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use $5^{th}$ root series for example.




$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

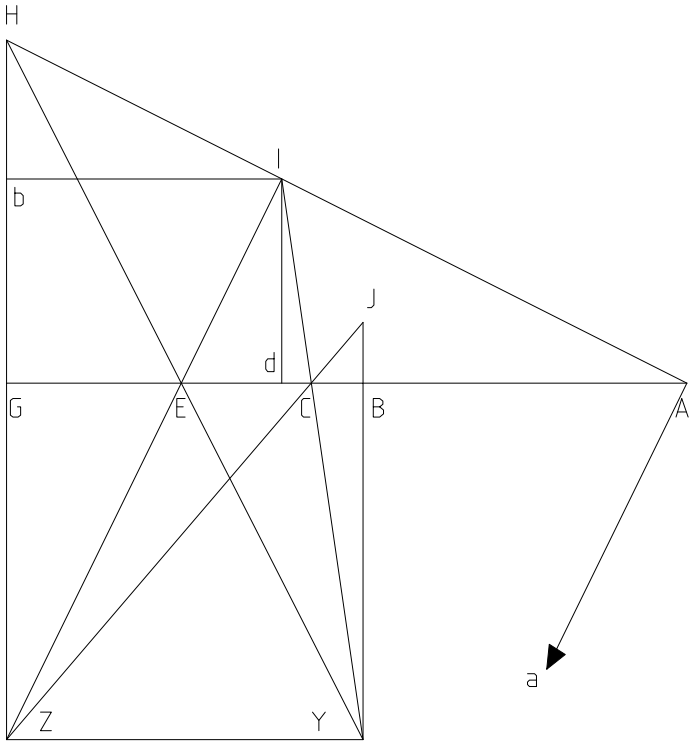
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

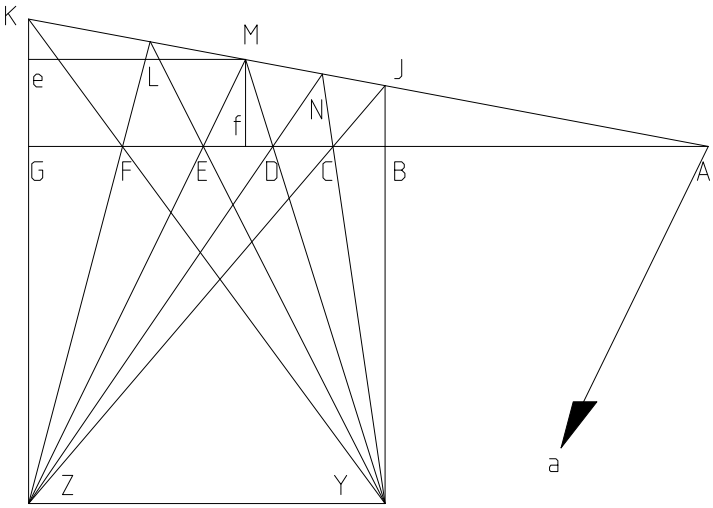
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



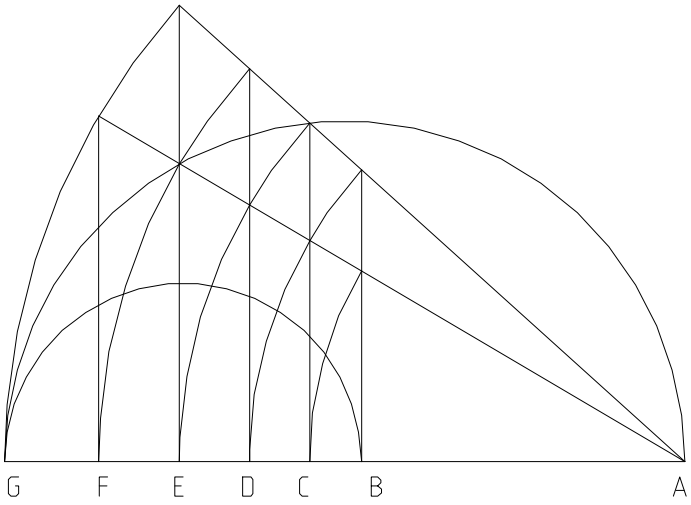



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



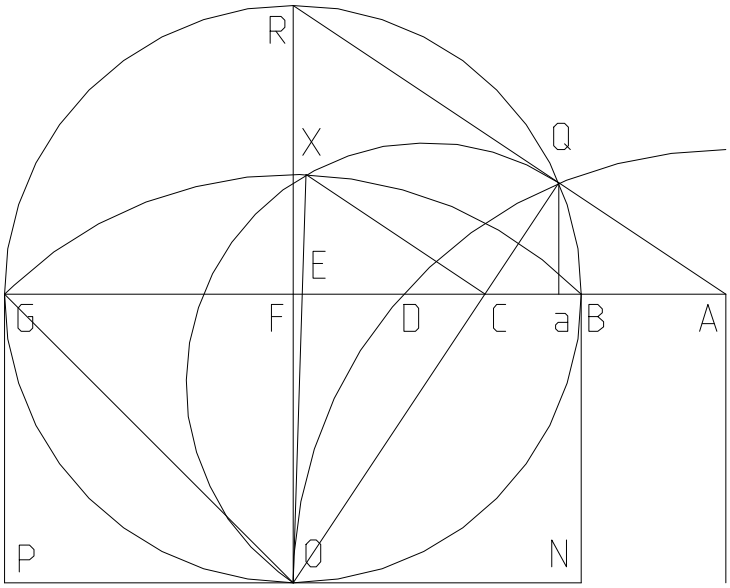

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

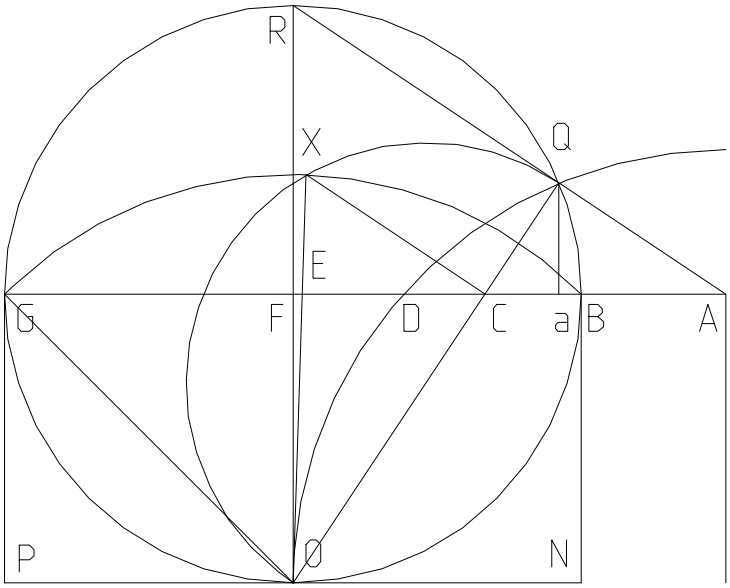
$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.




Is CX a constant?

I have had so much back work to catch up on I post dated a couple.

$AB := 54 \quad AG := 270$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FO := BF$

$AF := AB + BF \quad FR := BF$

$AR := \sqrt{AF^2 + FR^2} \quad AQ := \frac{AB \cdot AG}{AR}$

$Aa := \frac{AF \cdot AQ}{AR} \quad Qa := \frac{FR \cdot AQ}{AR}$

$Fa := AF - Aa \quad OQ := \sqrt{Fa^2 + (FO + Qa)^2}$

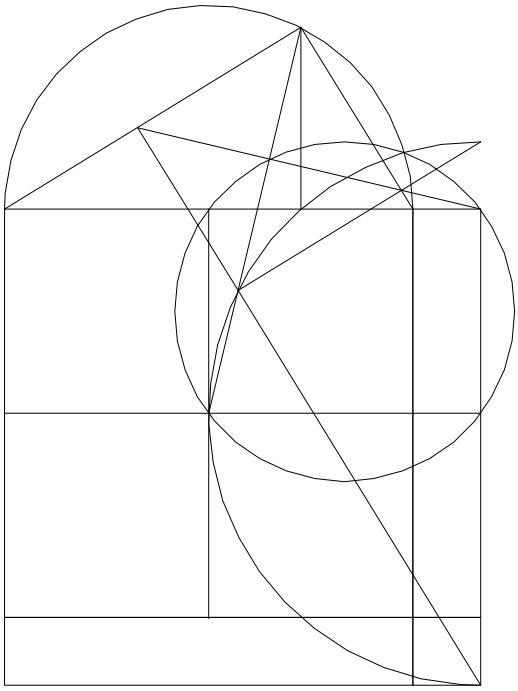
$CQ := \frac{OQ \cdot Qa}{FO + Qa} \quad OX := \sqrt{BF^2 + FO^2}$

$CO := OQ - CQ$

$\frac{\frac{OX^2}{OQ}}{CO} = 1$

Both expressions reduce to,

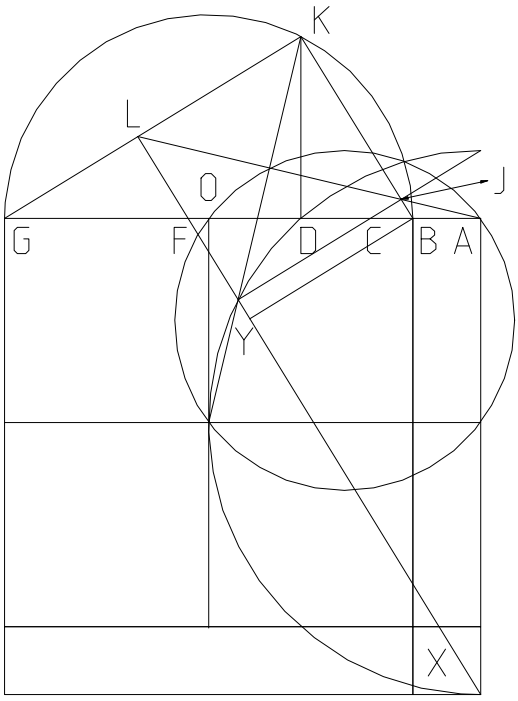
$CQ = 49.923 \qquad OQ - \frac{OX^2}{OQ} = 49.923 \qquad \frac{AG - AB}{AG + AB} \cdot \frac{\sqrt{2 \cdot (AG \cdot AB)}}{\sqrt{AB^2 + AG^2}} = 49.923$



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.




$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

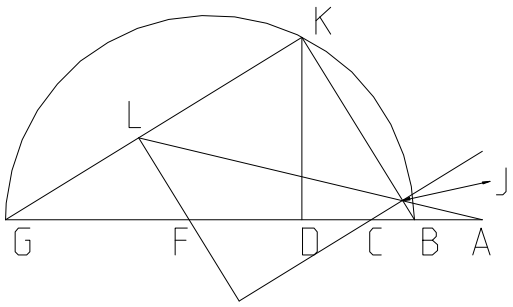
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



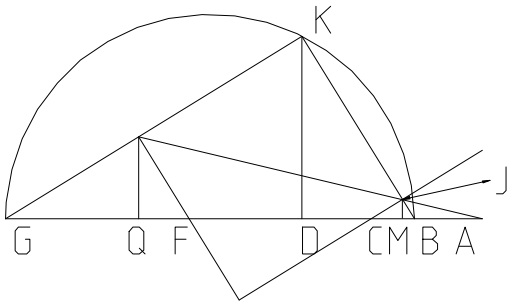

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

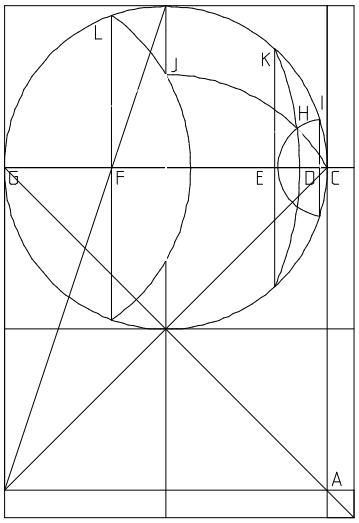
$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

On the left is the first and last of the series, on the right is the entire series.

| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$



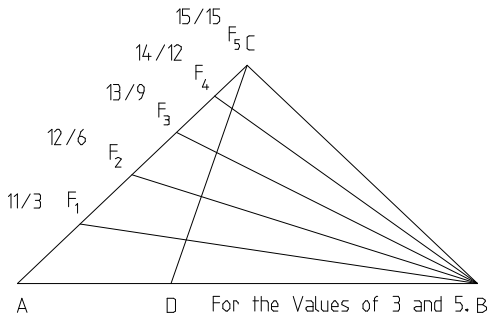

# One Circle

**By John Clark**

# One Square

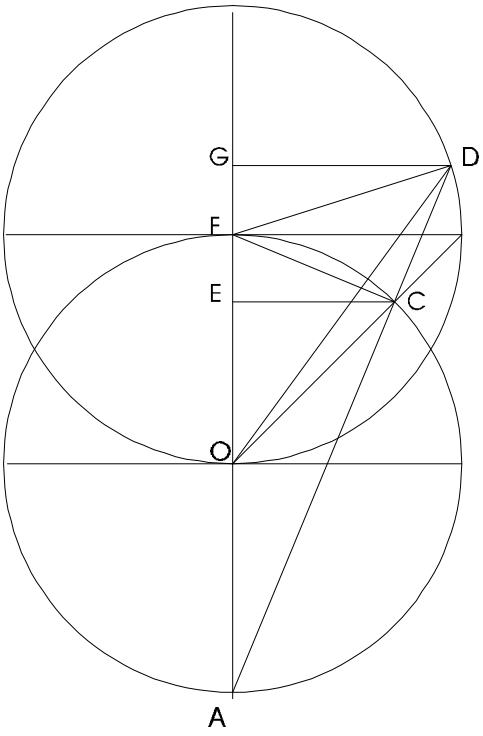
# One Line

## And the Delian Quest




# 1992

# Cubing a Sphere, a Rusty Construction.




Cubing a sphere off the base of a right triangle. I later found out that someone had squared a circle off the base of a right triangle but the figure was lost. This figure is in the 1989 Document on the Delian Solution.

$R := 10$

$AO := R \quad CO := R \quad DF := R \quad CE := \sqrt{\frac{CO^2}{2}}$

$AE := R + CE \quad AC := \sqrt{CE^2 + AE^2} \quad AF := 2 \cdot R$

$CF := \sqrt{AF^2 - AC^2} \quad CD := \sqrt{DF^2 - CF^2} \quad AD := AC + CD$

$DG := \frac{CE \cdot AD}{AC} \quad AG := \frac{AE \cdot AD}{AC} \quad GO := AG - R$

$DO := \sqrt{DG^2 + GO^2}$

$DO = 16.135$

$CUBE\_ROOT = 16.12$

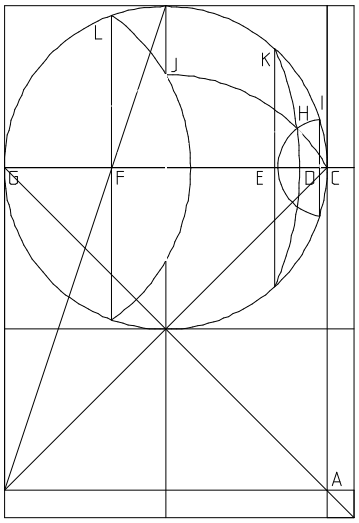
$CUBE\_ROOT := \left(\frac{4}{3} \cdot \pi \cdot R^3\right)^{\frac{1}{3}}$

$PI := \frac{DO^3}{\frac{4}{3} \cdot R^3}$ $\quad PI = 3.15049$

$\pi = 3.14159$

$TOLERANCE := \frac{DO}{CUBE\_ROOT}$

$TOLERANCE - 1 = 0.0009429$



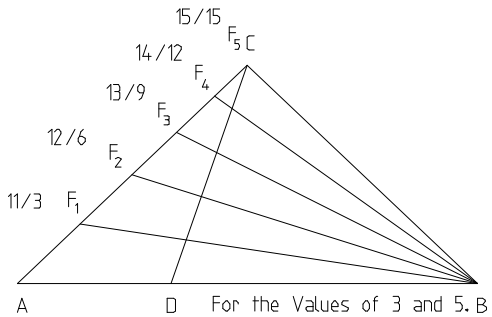

# One Circle

**By John Clark**

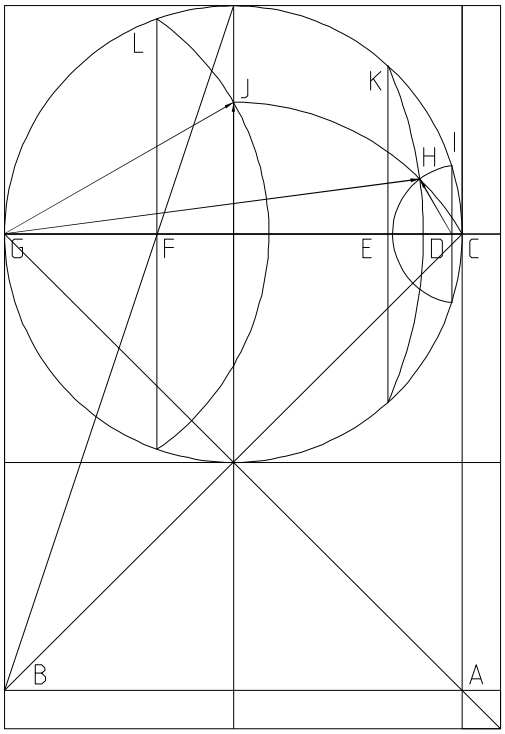
# One Square

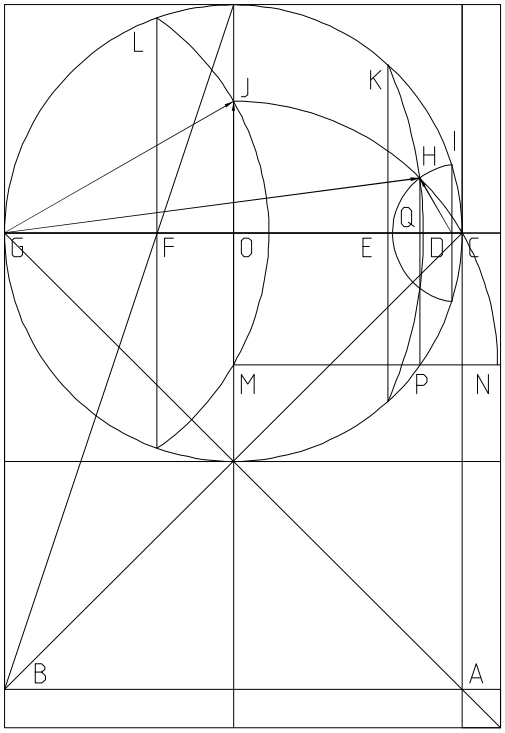
# One Line

## And the Delian Quest




# 1993




# Exploring the properties of the curve CJ.

06_03_93.MCD

Choosing any arbitrary point on CJ does the projection DE equal the square root of CD · EG? This possibility was suggested by my work on finding the geometrical solution to the problem : Given only the lengths of the three sides of any triangle, find its area. That work suggests that DE will remain constant over a range if CJ had the same center as circle CLG. An essential portion of that solution will be used in this work for finding the difference and position of DE.

$CG := 10 \quad FG := \frac{CG}{3} \quad FO := \frac{CG}{6} \quad LO := \frac{CG}{2}$

$FL := \sqrt{LO^2 - FO^2} \quad GL := \sqrt{FG^2 + FL^2}$

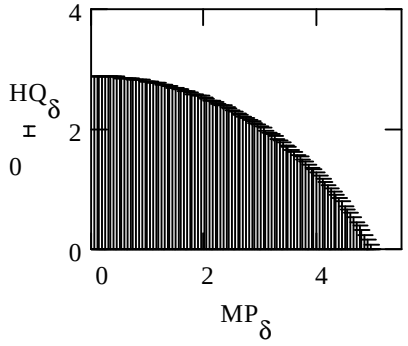
$GO := \frac{CG}{2} \quad GJ := GL \quad JO := \sqrt{GJ^2 - GO^2}$

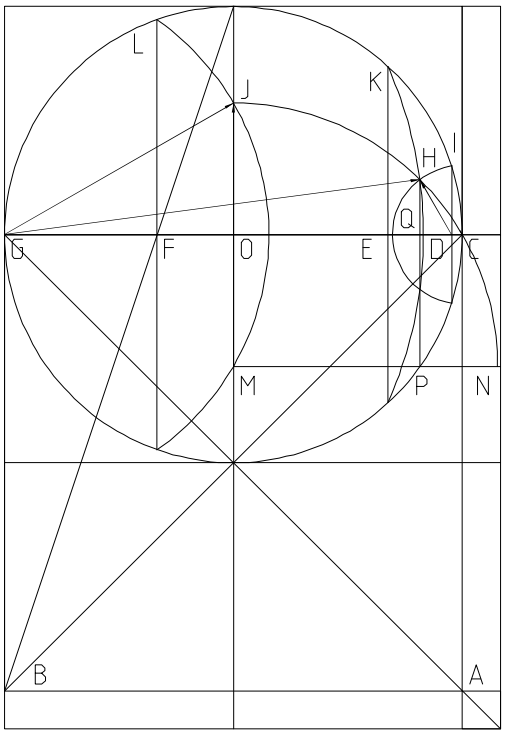
$JM := JO \cdot 2 \quad CO := GO \quad MN := CO$

$\delta := 1..100 \quad MP_\delta := \frac{\delta}{100} \cdot MN \quad HM := JM$

$HP_\delta := \sqrt{HM^2 - \left(MP_\delta\right)^2} \quad MO := JO \quad PQ := MO$

$HQ_\delta := HP_\delta - PQ \quad QO_\delta := MP_\delta$





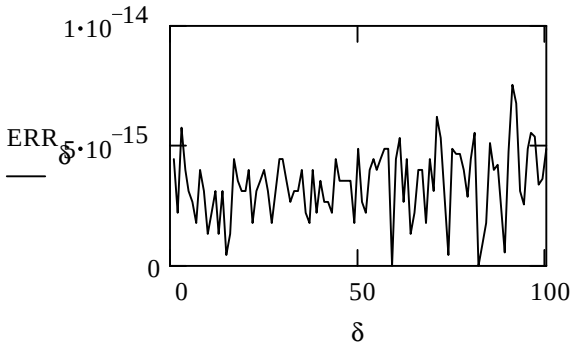

$$GQ_\delta := GO + QO_\delta \qquad GH_\delta := \sqrt{(GQ_\delta)^2 + (HQ_\delta)^2}$$

$$GK_\delta := GH_\delta \qquad EG_\delta := \frac{(GK_\delta)^2}{CG} \qquad CQ_\delta := CO - QO_\delta$$

$$CH_\delta := \sqrt{(CQ_\delta)^2 + (HQ_\delta)^2} \qquad CI_\delta := CH_\delta$$

$$CD_\delta := \frac{(CI_\delta)^2}{CG} \qquad DE_\delta := CG - EG_\delta - CD_\delta$$

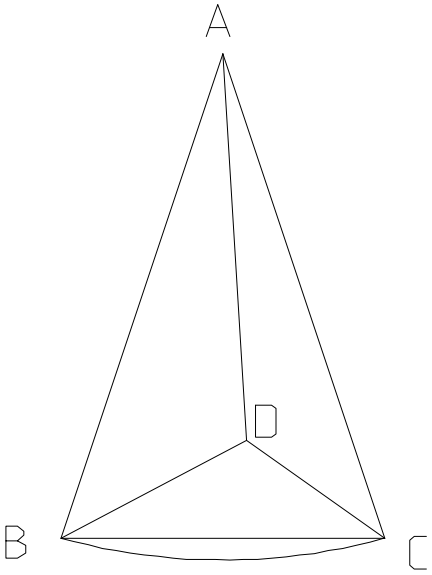
$$ERR_\delta := \sqrt{EG_\delta \cdot CD_\delta} - DE_\delta$$




As can be seen by the graph, the only error is the limits contained in the program itself! So CJ is part of our cube root figure, as the square in a right triangle is equal to the square of the remaining two segments, and all three squares taken to the reduced ratio functions in a cube root relationship.

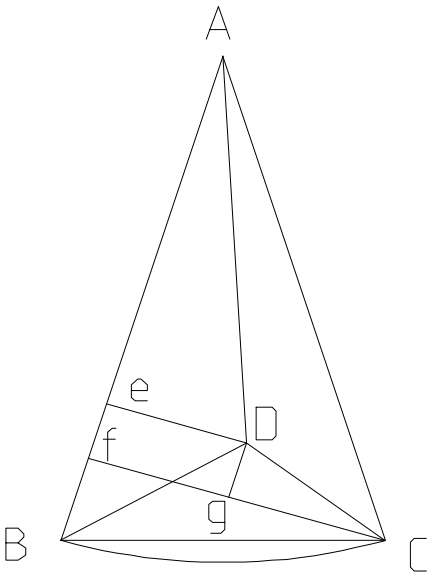
# For All Triangles Find DC

06_07_C3.MCD



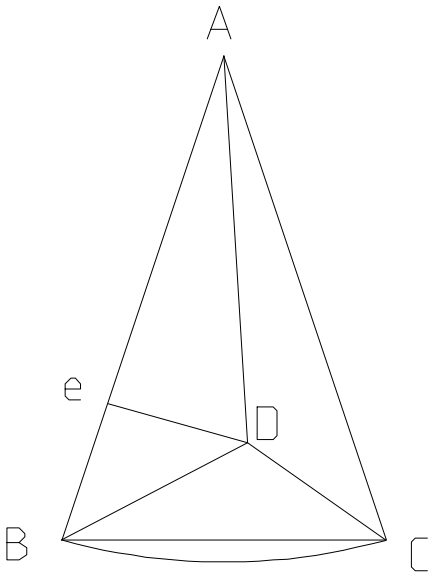

The first of these series ran the process through for equalaterals. I expanded in this mod for all triangles, I will use the process later to fi the Euler line, given only the length of three sides of a triangle.

Given: AB, AC, AD, BC, BD
Find: DC




# Process Summary

Construct: De
Construct: Cf
ef = Be - Bf
Dg = ef
CD = $\sqrt{Cg^2 + Dg^2}$




Given:

$AB := 7.18 \quad AC := 9.02 \quad AD := 8.09$

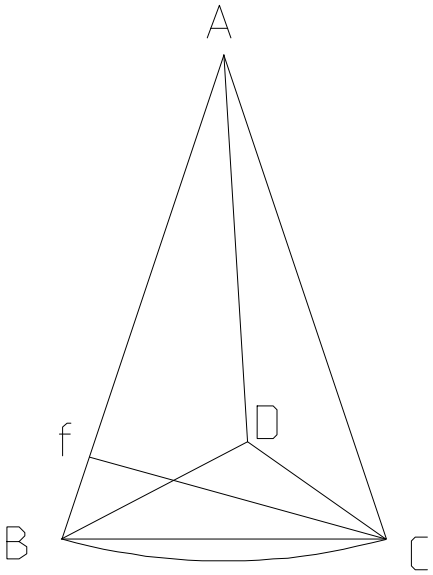
$BD := 7.28 \quad BC := 3.85$

Find CD.

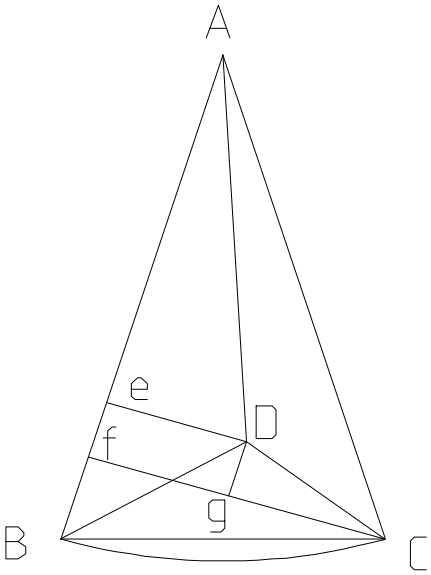
Find: Be

$$Be := \frac{1}{2}\cdot\frac{BD^2}{AB} + \frac{1}{2}\cdot AB - \frac{1}{2}\cdot\frac{AD^2}{AB}$$

$Be = 2.723$



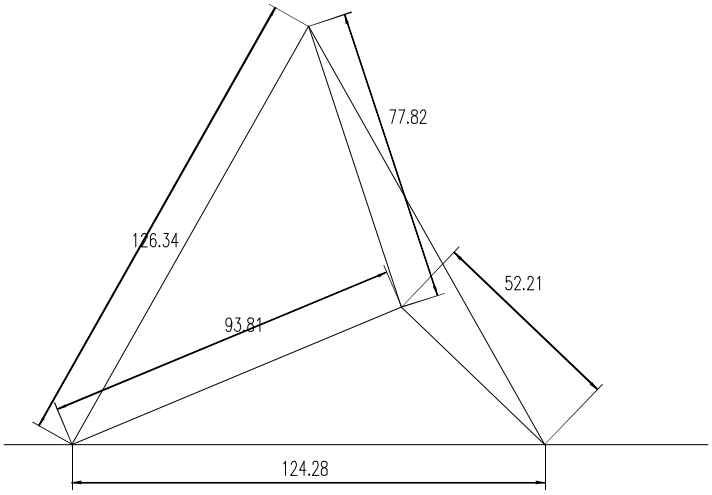

$$Bf := \frac{1}{2}\cdot\frac{BC^2}{AB} + \frac{1}{2}\cdot AB - \frac{1}{2}\cdot\frac{AC^2}{AB}$$



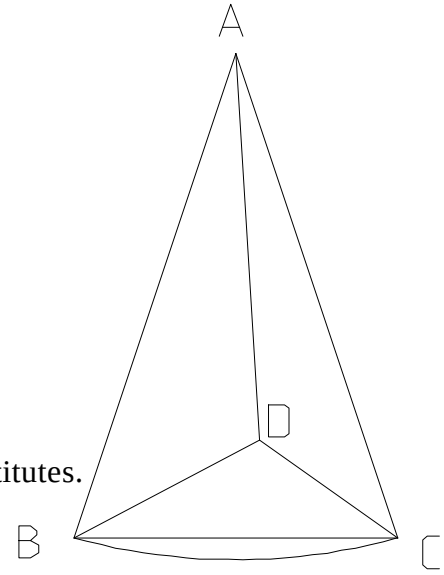

$ef := Be - Bf \quad Dg := ef \quad De := \sqrt{BD^2 - Be^2}$

$Cf := \sqrt{BC^2 - Bf^2} \quad gf := De \quad Cg := Cf - gf$

$DC := \sqrt{Cg^2 + Dg^2} \quad DC = 4.844$




$DC = 4.844$

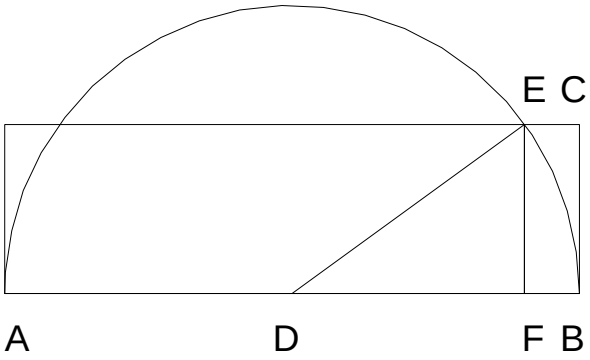



Due to the inability to format the equation I had to make some substitutes.

$A := AB \quad B := BC \quad C := AC \quad D := BD \quad E := AD$

$$F := \sqrt{2}\cdot\frac{\sqrt{\begin{array}{l}B^2\cdot A^2 \ldots \\ +\sqrt{C+A-B}\cdot\sqrt{C-A+B}\cdot\sqrt{C+A+B}\cdot\sqrt{C-A-B}\cdot\sqrt{D+E-A}\cdot\sqrt{D-E-A}\cdot\sqrt{D+A-E}\cdot\sqrt{D+A+E}\ldots \\ +D^2\cdot A^2 - D^2\cdot B^2 + D^2\cdot C^2 - A^4 + A^2\cdot C^2 + E^2\cdot B^2 + E^2\cdot A^2 - E^2\cdot C^2\end{array}}}{2\cdot A}$$

$F = 4.844$




Given length AB and length CB which is 1/2 of AB or less, place CB such that it is the square root of the two constructed segments. The second line in the equations will not permit BC to exceed Euclidean specifications.

$AB := 10 \quad BC := 4$

$$BC := \text{if}\left(BC \geq 0, \text{if}\left(BC \leq \frac{AB}{2}, BC, 0\right), 0\right)$$

$$AD := \frac{AB}{2} \quad DE := AD$$

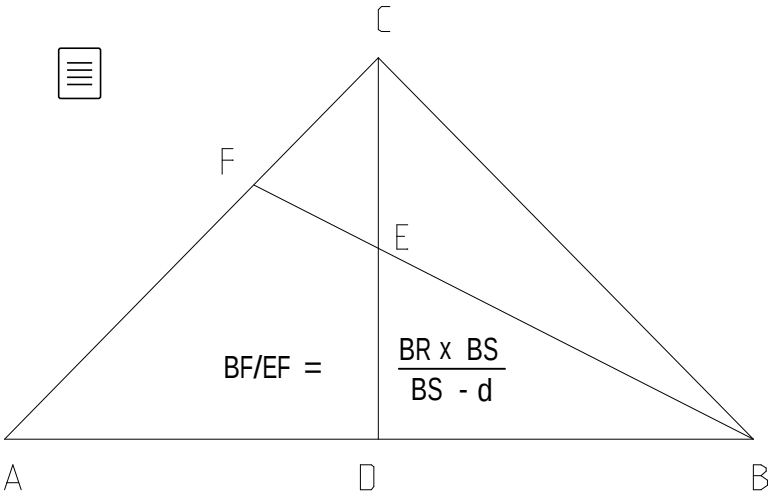
$FE := BC \quad DB := AD \quad DF := \sqrt{DE^2 - FE^2} \quad AF := AD + DB \quad BF := DB - DF \quad ROOT := \sqrt{AF \cdot BF}$

$BC = 4 \qquad ROOT = 4 \qquad AF = 8$

# A Pyramid of Ratios

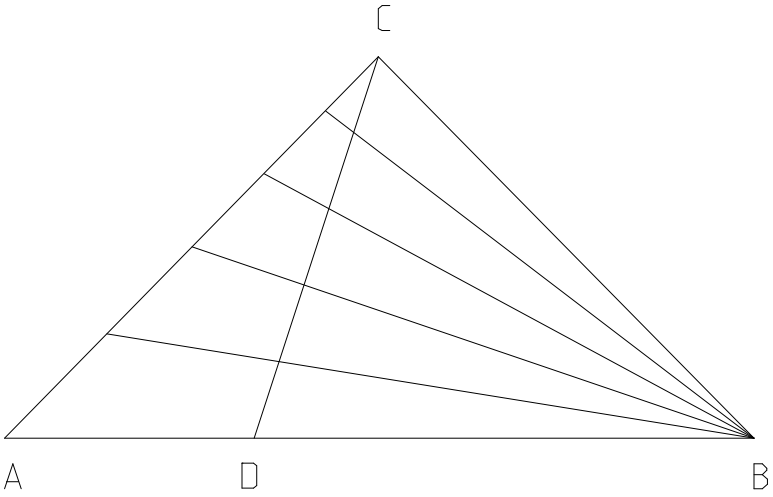
06_21_93.MCD

The typical teaching concerning the division of a line in Euclidean geometry is quite straight forward, this work is an extension of the application.

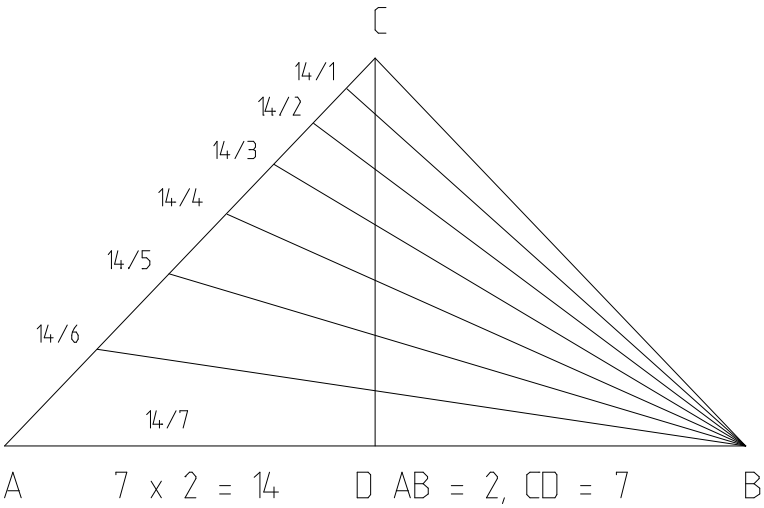



A pyramid of ratios is the expression of the relationships between the base of the right triangle and some bisectors. There is more than one model.

How does this one work? I wish to know the ratio between EF:BF. As you can see the base of the triangle has been divided into two equal parts, and so has the bisector. AB = 2 and CD = 2. $2 \times 2 = 4$, therefore segment EF will be 1/4 of segment BF. Now let us turn to a more developed expression of the figure.

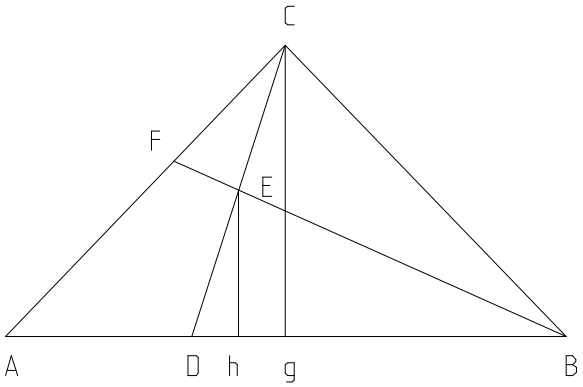



The base can also be divided into any ratio, it can even be part of a larger figure. One must keep in mind the distinction between numerical measure, which will be used to compute lengths, and the concept of ratio.




The ratio here is 2 to 7, so that we are working with the number 14. In descending order, 1/14, 2/14, 3/14, 4/14, 5/14 and 6/14.

In this work I will be exploring the figure to see what else it will yield. With Mathcad we shall be able to explore drawings that would be too tedious to draw and actually be quite an engineering feat if one could.




One can plug in numbers for the first three variables at the end of the document.

$AB := BASE\_LENGTH$

$BR := BASE\_RATIO$

$BS := BISEC\_SEG$

$Cg := \frac{AB}{2} \quad Ag := Cg \quad Bg := Cg$

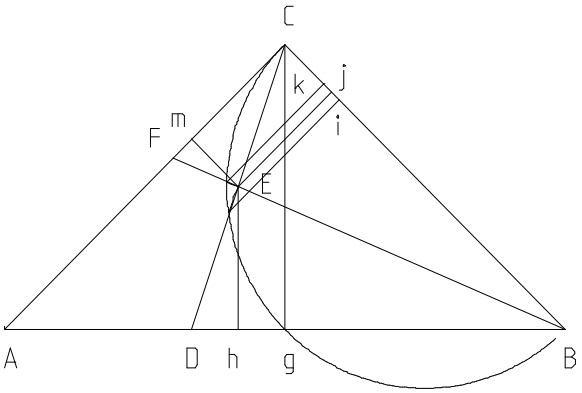
Number that has been defined equals $BS \times BR$.

$BS \cdot BR = 0.45 \quad AD := \frac{AB}{BR}$

$\delta := 0 .. BS \quad Dg := Ag - AD \quad CD := \sqrt{Dg^2 + Cg^2} \quad DE_\delta := \frac{CD}{BS} \cdot \delta \quad Dh_\delta := \frac{Dg \cdot DE_\delta}{CD}$

$Eh_\delta := \sqrt{(DE_\delta)^2 - (Dh_\delta)^2} \quad gh_\delta := Dg - Dh_\delta \quad Bh_\delta := Bg + gh_\delta \quad BE_\delta := \sqrt{(Bh_\delta)^2 + (Eh_\delta)^2}$

$CE_\delta := CD - DE_\delta$




$BC := \sqrt{2 \cdot Bg^2} \quad Bi_\delta := \frac{(BE_\delta)^2}{BC} \quad Ck_\delta := \frac{(CE_\delta)^2}{BC}$

$ik_\delta := BC - Bi_\delta - Ck_\delta \quad ij_\delta := \frac{ik_\delta}{2}$

$Bj_\delta := Bi_\delta + ij_\delta \quad Cj_\delta := BC - Bj_\delta$

The algorithm, completely Euclidean, can be used to find the area of any triangle knowing just the length of the three sides. Here it is used, even when the sides exceed the base in length.

$Em_\delta := Cj_\delta$

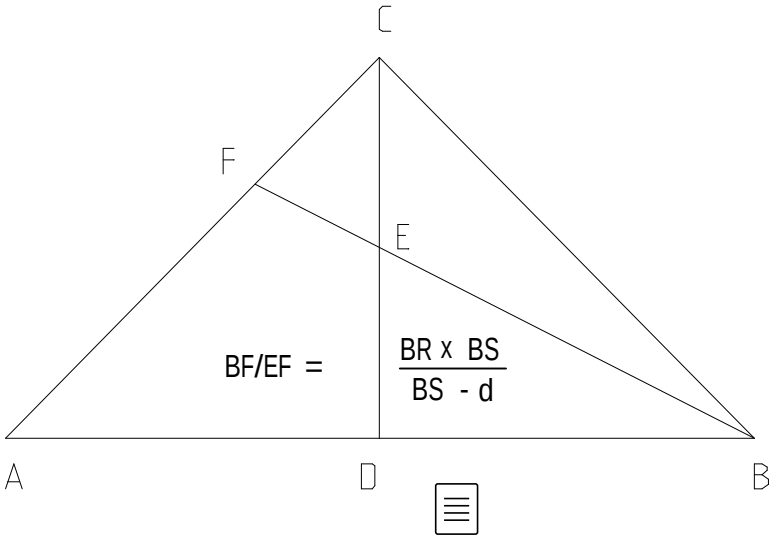
$EF_\delta := \frac{BE_\delta \cdot Em_\delta}{Bj_\delta} \quad BF_\delta := BE_\delta + EF_\delta$

$$\text{if}\left(EF_{\delta}, \frac{BF_{\delta}}{EF_{\delta}}, 0\right)$$

| |
|---|
| 0.05 |
| 0.056 |
| 0.064 |
| 0.075 |
| 0.09 |
| 0.112 |
| 0.15 |
| 0.225 |
| 0.45 |
| 0 |

$$\text{if}\left(BS - \delta, \frac{BR \cdot BS}{BS - \delta}, 0\right)$$

| |
|---|
| 0.05 |
| 0.056 |
| 0.064 |
| 0.075 |
| 0.09 |
| 0.113 |
| 0.15 |
| 0.225 |
| 0.45 |
| 0 |




**Plug in values here!**

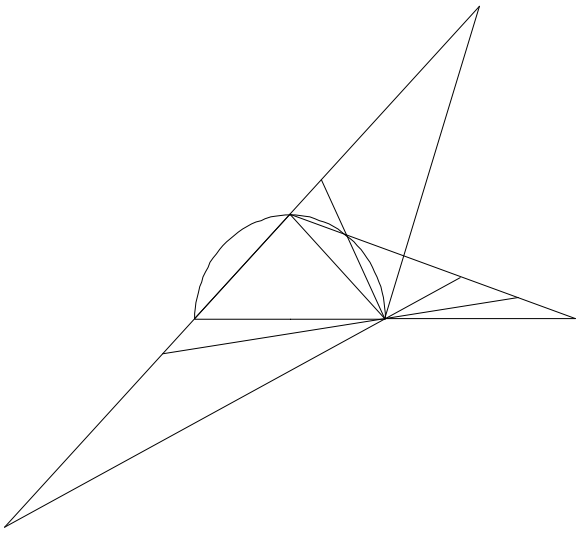
BASE_LENGTH ≡ 21

(BS) = BISEC_SEG ≡ 9

(BR) = BASE_RATIO ≡ .05

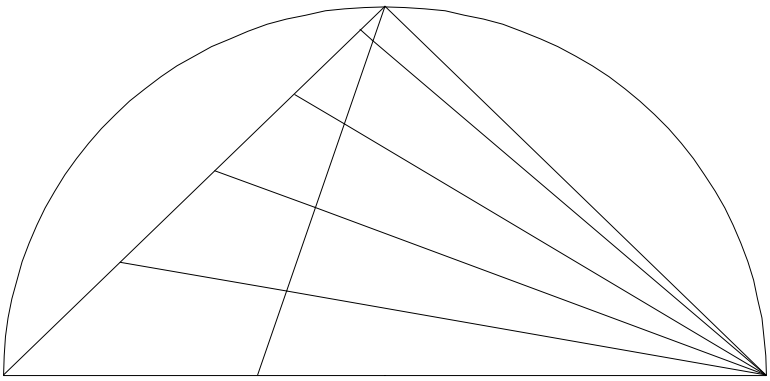
A basic Euclidean divisional / multiplicative series.

Try the values of 5 for the vertical number of divisions and .5 (1/2) for the base divisions



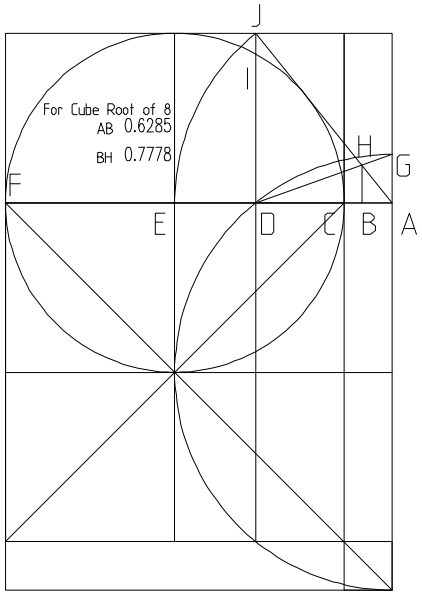
One can use the numbers generated by the program to discover what a particular set of ratios would draw out to be.

A fractional number for the base means that the entire base is the fractional part of a larger base.



The algorithm places the remainder at the top for fractional divisions.

Determine the relationships involved between the elements of the figure. 06_23_93.mcd




$AC := 1$

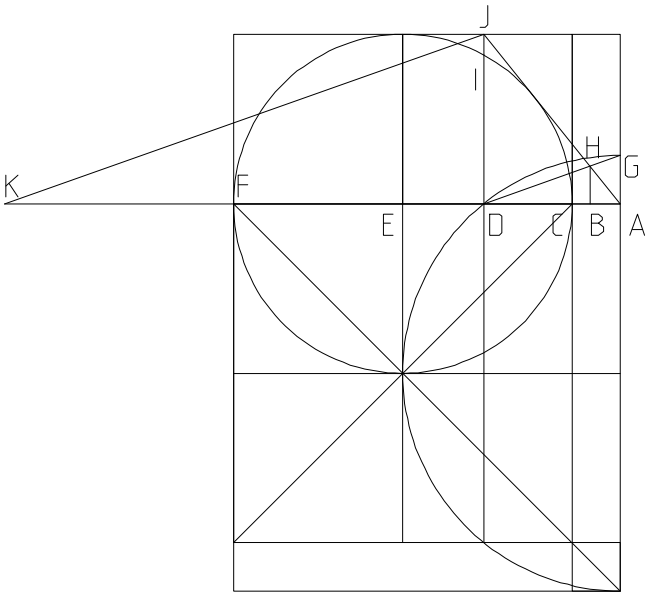
$\delta := 1, 100 .. 1000 \quad CF_\delta := \delta \quad AF_\delta := AC + CF_\delta$

$CE_\delta := \frac{CF_\delta}{2} \quad DJ_\delta := CE_\delta \quad AD_\delta := \sqrt{AC \cdot AF_\delta}$

$CD_\delta := AD_\delta - AC \quad DF_\delta := CF_\delta - CD_\delta$

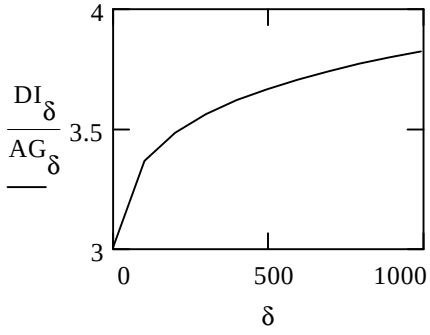
$DI_\delta := \sqrt{CD_\delta \cdot DF_\delta} \quad DE_\delta := CE_\delta - CD_\delta$

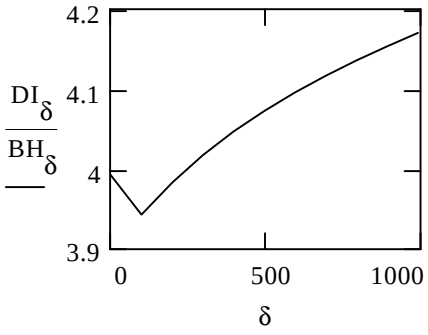
$$AG_\delta := \frac{\left[ \left( AF_\delta \right)^2 \cdot AC \right]^{\frac{1}{3}} - \left[ (AC)^2 \cdot AF_\delta \right]^{\frac{1}{3}}}{2}$$




$$DK_\delta := \frac{AD_\delta \cdot DJ_\delta}{AG_\delta} \quad AK_\delta := DK_\delta + AD_\delta$$

$$AB_\delta := \frac{\left( AD_\delta \right)^2}{AK_\delta} \quad BH_\delta := \frac{DJ_\delta \cdot AB_\delta}{AD_\delta}$$

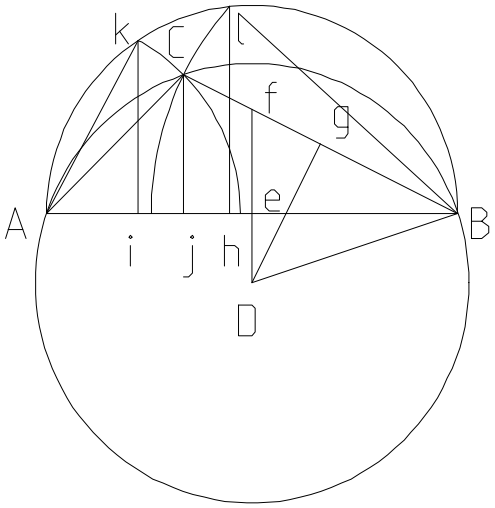


# Inscribe a Circle about a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

$\Delta := (AB + AC > BC)\cdot(AB + BC > AC)\cdot(BC + AC > AB)$ $\quad NOT(X) := X = 0$




$Ae := \frac{AB}{2}$ $\quad Ak := AC$ $\quad Bl := BC$ $\quad \delta := 0..2$

$Ai := \frac{Ak^2}{AB}$ $\quad Bh := \frac{Bl^2}{AB}$ $\quad Ah := AB - Bh$

$hi := Ah - Ai$ $\quad Aj := Ai + \frac{hi}{2}$ $\quad Cj := \sqrt{AC^2 - Aj^2}$

$Be := Ae$ $\quad Bj := AB - Aj$ $\quad Bg := \frac{BC}{2}$

$Bf := \frac{BC\cdot Be}{Bj}$ $\quad fg := Bf - Bg$ $\quad Dg := if\left(Cj, \frac{Bj\cdot fg}{Cj}, 0\right)$

$BD := if\left(Dg, \sqrt{Dg^2 + Bg^2}, \infty\right)$

$radius := if(\Delta, BD, 0)$ $\quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

radius = 3.007

imaginary_radius = 0

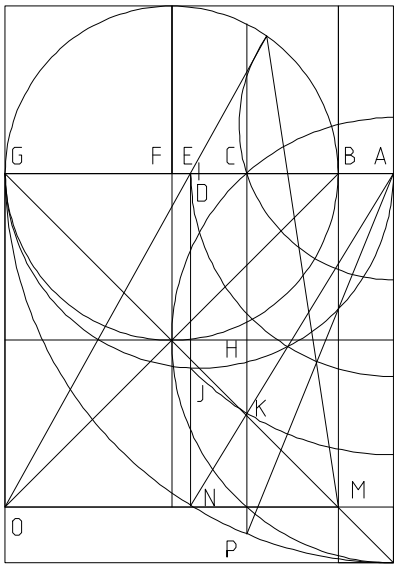
$\Delta = 1$

$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix}$ $\quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix}$ $\quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix}$

Reducing to one equation, $\quad AB \equiv 3$ $\quad AC \equiv 5$ $\quad BC \equiv 6$

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = (\ 3.007 \quad 3.007 \quad 3.007\ )$




Determine the ratio range of HK:HP

$BG := 10 \quad \delta := 1 .. 100 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$AG_\delta := AB_\delta + BG \quad AC_\delta := \sqrt{AG_\delta \cdot AB_\delta}$

$AP_\delta := AG_\delta \quad AN_\delta := AG_\delta \quad BF := \frac{BG}{2}$

$CP_\delta := \sqrt{(AP_\delta)^2 - (AC_\delta)^2} \quad FH := BF$

$AF_\delta := AB_\delta + BF \quad AD_\delta := \frac{AG_\delta}{2}$

$CD_\delta := AD_\delta - AC_\delta \quad DH_\delta := AD_\delta$

$CH_\delta := \sqrt{(DH_\delta)^2 - (CD_\delta)^2} \quad HP_\delta := CP_\delta - CH_\delta$

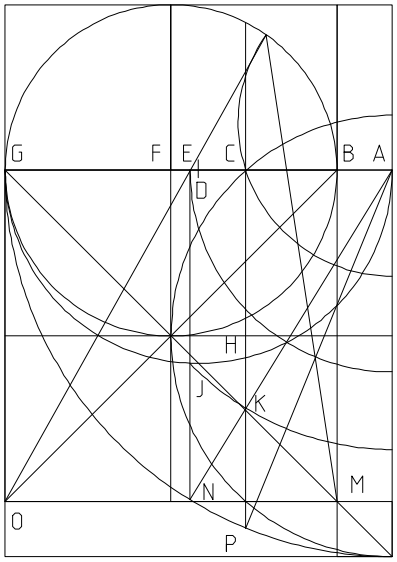
$CUBE_\delta := (AG_\delta)^2 \cdot AB_\delta \quad ROOT_\delta := (CUBE_\delta)^{\frac{1}{3}}$

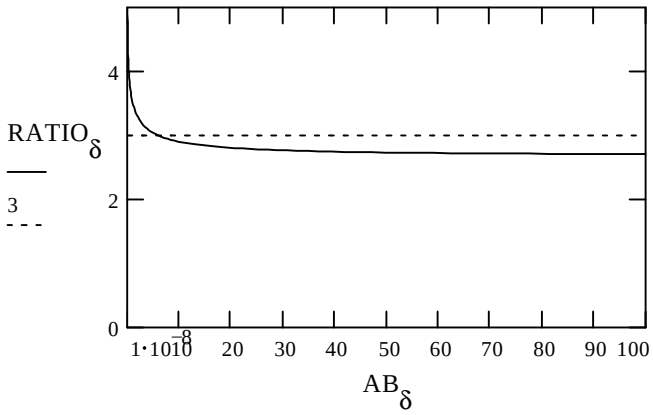
$EN_\delta := \sqrt{(AN_\delta)^2 - (ROOT_\delta)^2} \quad CK_\delta := \frac{EN_\delta \cdot AC_\delta}{ROOT_\delta}$

$KP_\delta := CP_\delta - CK_\delta \quad HK_\delta := CK_\delta - CH_\delta$

$RATIO_\delta := \frac{HP_\delta}{HK_\delta}$

$\frac{HP_\delta}{HK_\delta}$



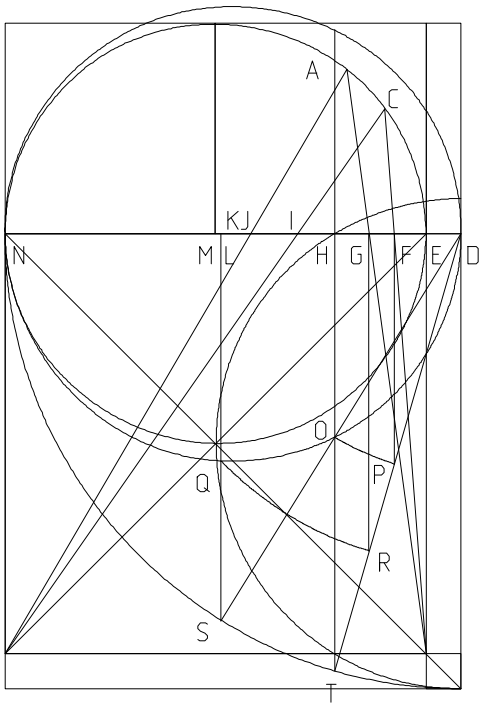

$RATIO_1 = 38.246$
$AB_1 = 1 \cdot 10^{-8}$

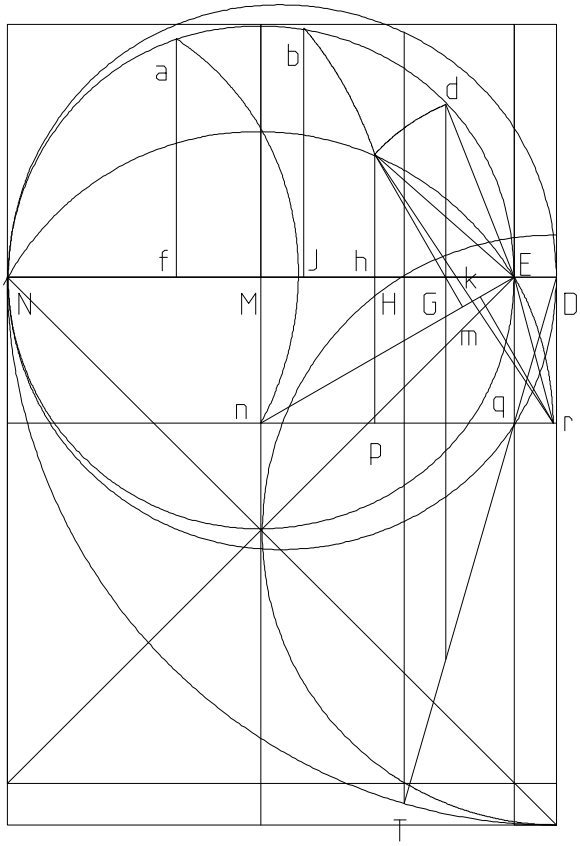
$RATIO_{20} = 5.164$
$AB_{20} = 0.032$

$RATIO_{30} = 4.113$
$AB_{30} = 0.243$

$RATIO_{100} = 2.707$
$AB_{100} = 100$

# From Maxima and Minima




Given

$EN := 10 \qquad \delta := 1..100 \qquad DE_\delta := \delta^5 \cdot 10^{-8}$

Find lengths GJ and FI.

$DN_\delta := DE_\delta + EN \qquad DH_\delta := \sqrt{DE_\delta \cdot DN_\delta}$

K is the center of DN

$DT_\delta := DN_\delta \qquad HT_\delta := \sqrt{(DT_\delta)^2 - (DH_\delta)^2} \qquad DK_\delta := \frac{DN_\delta}{2}$

$HK_\delta := DK_\delta - DH_\delta \qquad KO_\delta := DK_\delta \qquad HO_\delta := \sqrt{(KO_\delta)^2 - (HK_\delta)^2}$

$DS_\delta := DN_\delta \qquad DO_\delta := \sqrt{(DH_\delta)^2 + (HO_\delta)^2} \qquad LS_\delta := \frac{HO_\delta \cdot DS_\delta}{DO_\delta}$

$DL_\delta := \sqrt{(DS_\delta)^2 - (LS_\delta)^2} \qquad KL_\delta := DK_\delta - DL_\delta \qquad KQ_\delta := DK_\delta$

$LQ_\delta := \sqrt{(KQ_\delta)^2 - (KL_\delta)^2} \qquad DQ_\delta := \sqrt{(DL_\delta)^2 + (LQ_\delta)^2} \qquad DR_\delta := DQ_\delta$

$DG_\delta := \frac{DH_\delta \cdot DR_\delta}{DT_\delta} \qquad DP_\delta := DO_\delta \qquad DF_\delta := \frac{DH_\delta \cdot DP_\delta}{DT_\delta} \qquad EM := \frac{EN}{2}$

$Mf := \frac{EN}{6} \qquad Ma := EM \qquad af := \sqrt{Ma^2 - Mf^2} \qquad Nf := \frac{EN}{3} \qquad Na := \sqrt{af^2 + Nf^2}$

$Nn := Na \qquad MN := EM \qquad Mn := \sqrt{Nn^2 - MN^2} \qquad nr := Na \qquad nq := EM$

$qr := nr - nq \qquad Eq := Mn \qquad Er := \sqrt{Eq^2 + qr^2} \qquad En := Na \qquad Ek := \frac{Er^2}{2 \cdot En}$

$EG_\delta := DG_\delta - DE_\delta \qquad GN_\delta := EN - EG_\delta \qquad Gd_\delta := \sqrt{EG_\delta \cdot GN_\delta}$

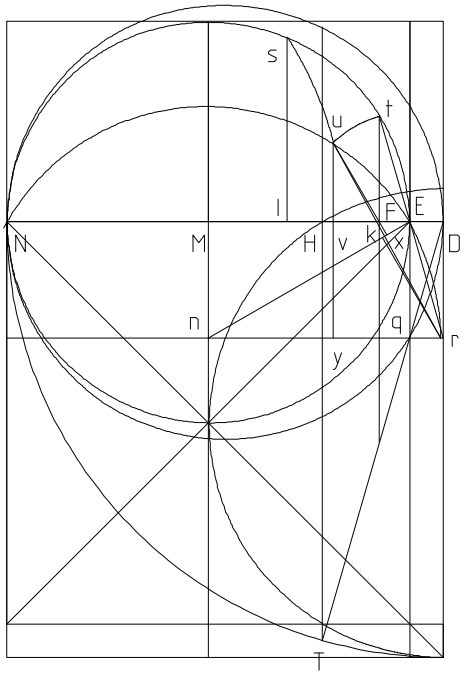
$Ed_\delta := \sqrt{(EG_\delta)^2 + (Gd_\delta)^2} \qquad Ee_\delta := Ed_\delta \qquad Em_\delta := \frac{(Ee_\delta)^2}{2 \cdot En} \qquad km_\delta := Em_\delta - Ek$

$em_\delta := \sqrt{(Ee_\delta)^2 - Em_\delta} \qquad kr_\delta := \sqrt{Er^2 - Ek^2} \qquad er_\delta := \sqrt{(em_\delta + kr_\delta)^2 + (km_\delta)^2}$

$pr_\delta := \frac{(er_\delta)^2}{2 \cdot nr} \qquad pq_\delta := pr_\delta - qr \qquad Eh_\delta := pq_\delta \qquad ep_\delta := \sqrt{(er_\delta)^2 - (pr_\delta)^2}$

$hp := Mn \qquad eh_\delta := ep_\delta - hp \qquad Nh_\delta := EN - Eh_\delta \qquad Ne_\delta := \sqrt{(Nh_\delta)^2 + (eh_\delta)^2}$

$Nb_\delta := Ne_\delta \qquad NJ_\delta := \frac{(Nb_\delta)^2}{EN} \qquad EJ_\delta := EN - NJ_\delta \qquad GJ_\delta := EJ_\delta - EG_\delta$




$EF_\delta := DF_\delta - DE_\delta \quad FN_\delta := EN - EF_\delta$

$Ft_\delta := \sqrt{EF_\delta \cdot FN_\delta} \quad Et_\delta := \sqrt{(EF_\delta)^2 + (Ft_\delta)^2}$

$Eu_\delta := Et_\delta \quad Ex_\delta := \frac{(Eu_\delta)^2}{2 \cdot En} \quad kx_\delta := Ek - Ex_\delta$

$ux_\delta := \sqrt{(Eu_\delta)^2 - (Ex_\delta)^2} \quad ur_\delta := \sqrt{(ux_\delta + kr_\delta)^2 + kx_\delta}$

$ry_\delta := \frac{(ur_\delta)^2}{2 \cdot nr} \quad qy_\delta := ry_\delta - qr \quad Ev_\delta := qy_\delta$

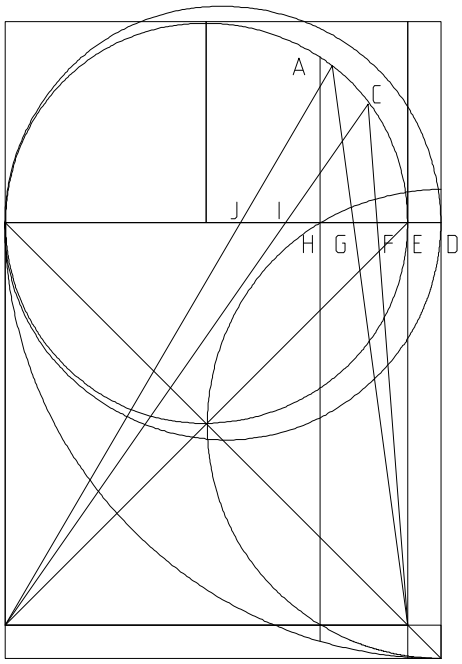
$uy_\delta := \sqrt{(ur_\delta)^2 - (ry_\delta)^2} \quad vy := Mn \quad uv_\delta := uy_\delta - vy$

$Nv_\delta := EN - Ev_\delta \quad Nu_\delta := \sqrt{(Nv_\delta)^2 + (uv_\delta)^2}$

$Ns_\delta := Nu_\delta \quad NI_\delta := \frac{(Ns_\delta)^2}{EN} \quad EI_\delta := EN - NI_\delta$

$FI_\delta := EI_\delta - EF_\delta$

$CUBE\_S_\delta := (DE_\delta)^2 \cdot DN_\delta \quad RT\_S_\delta := (CUBE\_S_\delta)^{\frac{1}{3}} \quad CUBE\_L_\delta := DE_\delta \cdot (DN_\delta)^2 \quad RT\_L_\delta := (CUBE\_L_\delta)^{\frac{1}{3}}$




$BASE\_C_\delta := RT\_L_\delta - RT\_S_\delta$

$BS\_PJ1_\delta := \sqrt{GJ_\delta + FI_\delta} \quad BS\_PJ2_\delta := \frac{GJ_\delta + FI_\delta}{2}$

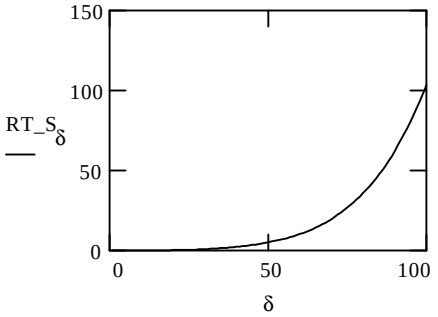
$DIF1_\delta := BASE\_C_\delta - BS\_PJ1_\delta$

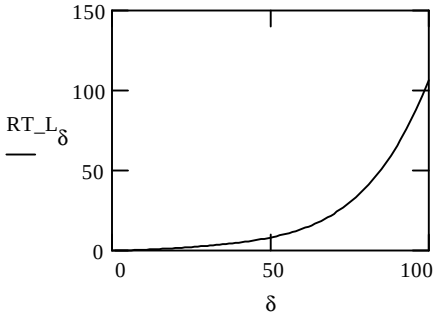
$DIF2_\delta := BASE\_C_\delta - BS\_PJ2_\delta$

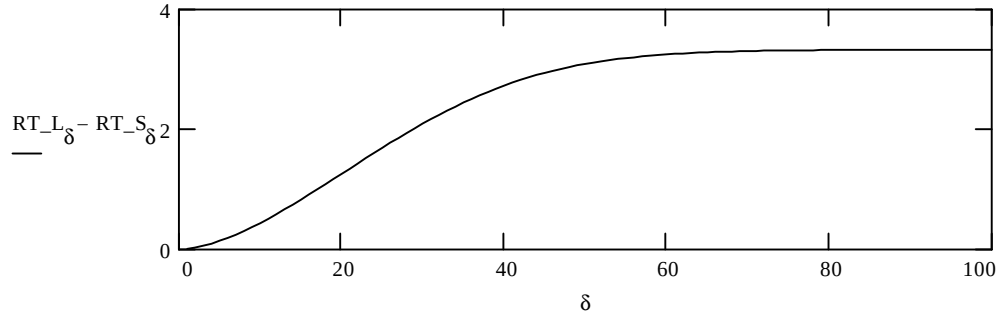
$E\_L_\delta := DG_\delta - RT\_S_\delta \quad E\_S_\delta := RT\_S_\delta - DF_\delta$

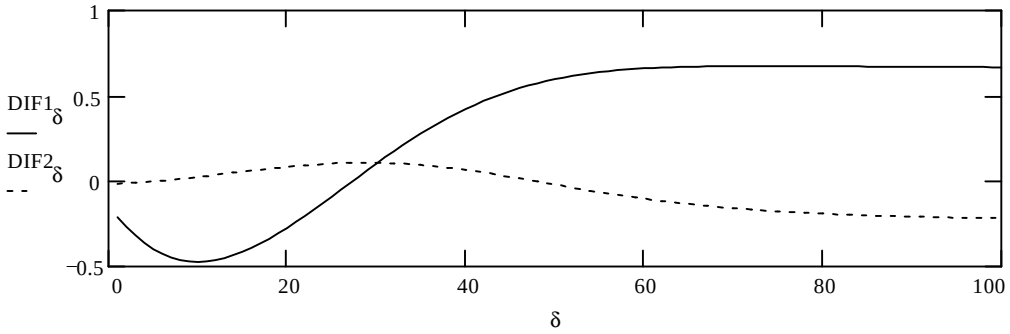
$RATIO_\delta := \frac{E\_L_\delta}{E\_S_\delta} \quad A := 1..25 \quad B := 26..50$

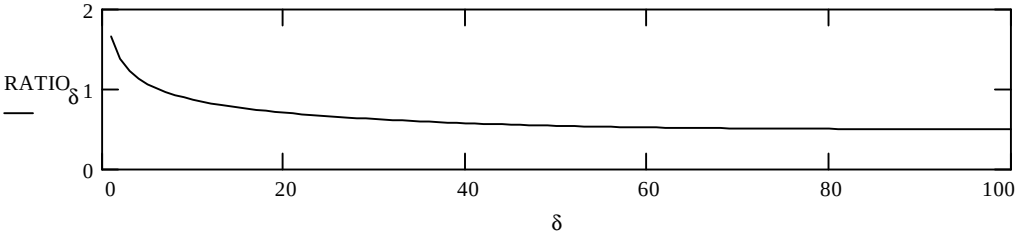
$C := 51..75 \quad D := 76..100$


150
100
50
0
RT_S_δ
0
50
100
δ

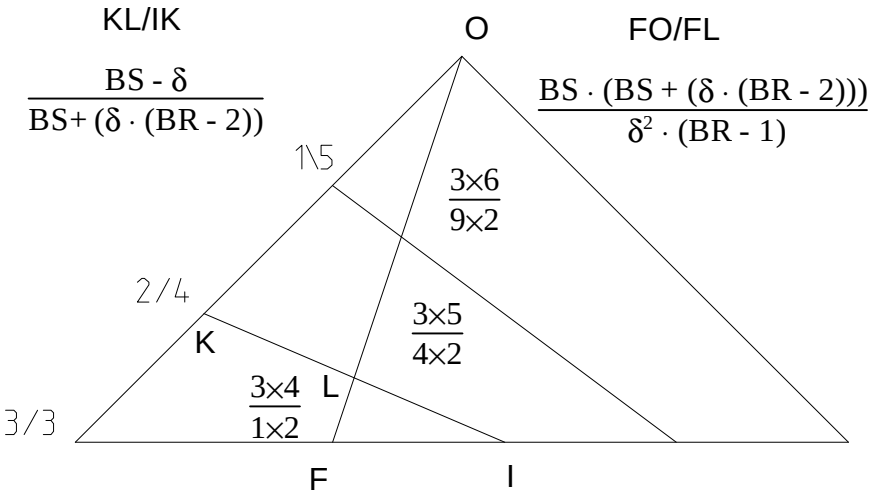

150
100
50
0
RT_L_δ
0
50
100
δ


4
2
0
RT_L_δ − RT_S_δ
0
20
40
60
80
100
δ


1
0.5
0
−0.5
DIF1_δ
DIF2_δ
0
20
40
60
80
100
δ

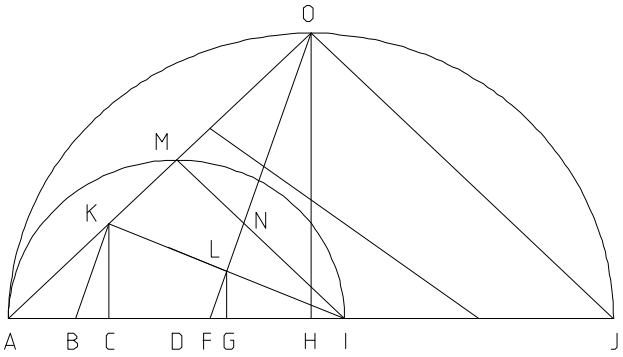

2
1
0
RATIO_δ
0
20
40
60
80
100
δ

# Another Pyramid of Ratios 07_15_93




Although this is a simpler figure to proof, the ratios involved are quite interesting. The base is divided by the perpendicular bisector into a ratio. The remainder of the base, and the opposite side are then divided into equal ratios. The resultant progressions are very nice.

The two ratios to find are KL:IK and FL:FO.




$AJ := 55 \quad AH := \frac{AJ}{2} \quad HJ := AH$

$BR := BASE\_RATIO$

$BS := BISEC\_SEG$

$AF := \frac{AJ}{BR} \quad FJ := AJ - AF \quad \delta := 0 .. BS \quad FI_\delta := \frac{FJ}{BS} \cdot \delta \quad AI_\delta := AF + FI_\delta$

$AD_\delta := \frac{AI_\delta}{2} \quad AO := \sqrt{\frac{AJ^2}{2}} \quad AK_\delta := \frac{AO}{BS} \cdot \delta \quad FH := AH - AF \quad HO := AH$

$FO := \sqrt{HO^2 + FH^2} \quad CK_\delta := \sqrt{\frac{(AK_\delta)^2}{2}} \quad AC_\delta := CK_\delta \quad CI_\delta := AI_\delta - AC_\delta$

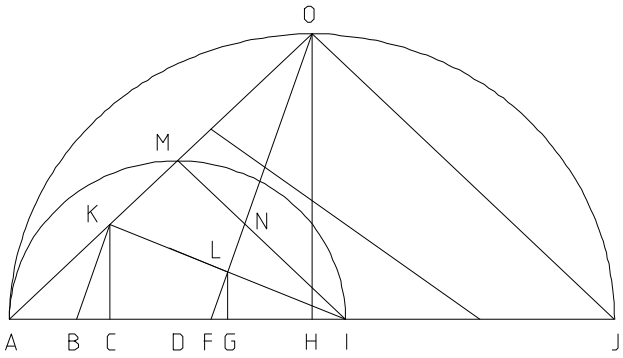
$$IK_\delta := \sqrt{\left(CK_\delta\right)^2 + \left(CI_\delta\right)^2}$$

$$BC_\delta := \frac{FH \cdot CK_\delta}{HO} \qquad BI_\delta := BC_\delta + CI_\delta$$

$$BK_\delta := \sqrt{\left(CK_\delta\right)^2 + \left(BC_\delta\right)^2}$$

$$FL_\delta := \frac{BK_\delta \cdot FI_\delta}{BI_\delta}$$

$$IL_\delta := \frac{IK_\delta \cdot FL_\delta}{BK_\delta} \qquad KL_\delta := IK_\delta - IL_\delta$$



(BR) = BASE_RATIO ≡ 3

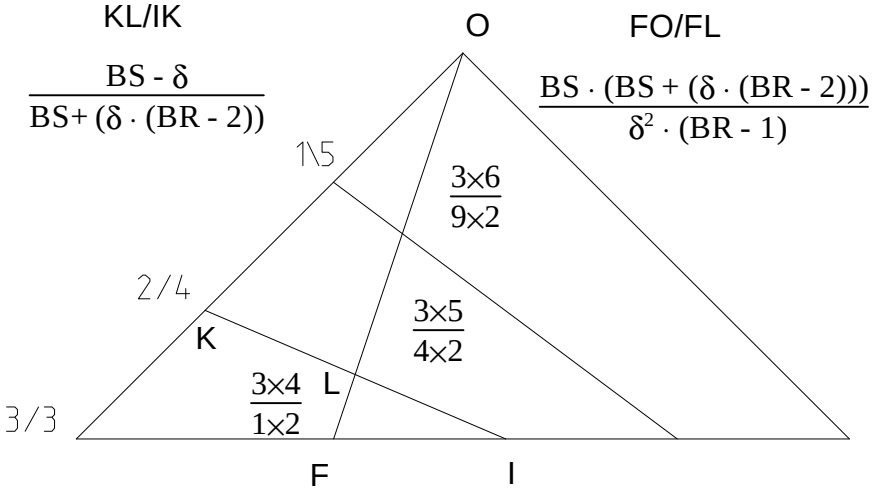
(BS) = BISEC_SEG ≡ 5

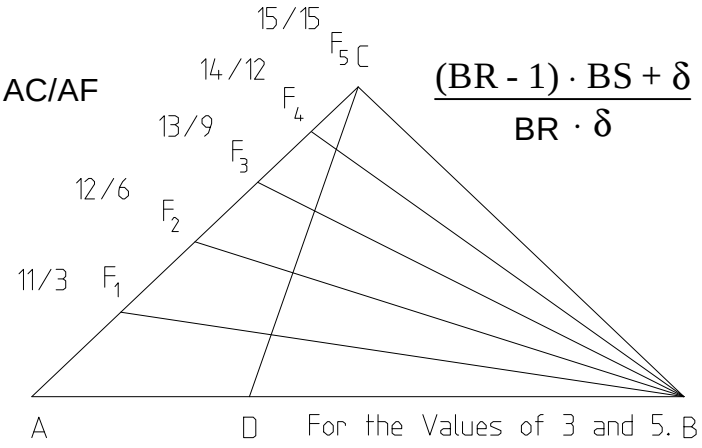
← Insert Values Here.

| $\delta$ | $BS - \delta$ | $BS + (\delta \cdot (BR - 2))$ |
|---|---|---|
| 0 | 5 | 5 |
| 1 | 4 | 6 |
| 2 | 3 | 7 |
| 3 | 2 | 8 |
| 4 | 1 | 9 |
| 5 | 0 | 10 |

| $\frac{KL_\delta}{IK_\delta}$ | $\frac{BS - \delta}{BS + (\delta \cdot (BR - 2))}$ | $\delta^2 \cdot (BR - 1)$ |
|---|---|---|
| 1 | 1 | 0 |
| 0.667 | 0.667 | 2 |
| 0.429 | 0.429 | 8 |
| 0.25 | 0.25 | 18 |
| 0.111 | 0.111 | 32 |
| 0 | 0 | 50 |

| $if\left(\delta, \frac{FO}{FL_\delta}, 0\right)$ | $if\left[\delta, \frac{BS \cdot (BS + (\delta \cdot (BR - 2)))}{\delta^2 \cdot (BR - 1)}, 0\right]$ | $FL_\delta$ | $if\left[\delta, \frac{FO}{\frac{BS \cdot (BS + (\delta \cdot (BR - 2)))}{\delta^2 \cdot (BR - 1)}}, 0\right]$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 15 | 15 | 1.933 | 1.933 |
| 4.375 | 4.375 | 6.626 | 6.626 |
| 2.222 | 2.222 | 13.044 | 13.044 |
| 1.406 | 1.406 | 20.613 | 20.613 |
| 1 | 1 | 28.988 | 28.988 |

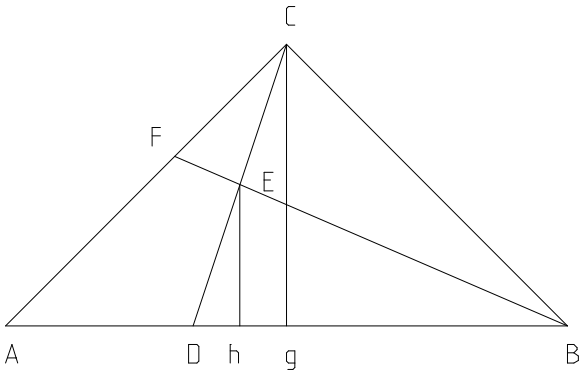





$$\frac{(BR - 1)\cdot BS + \delta}{BR \cdot \delta}$$

# Pyramid of Ratios III

Another set of ratios to explore is AC/AF Again one can plug in numbers for the first three variables at the end of the document. In most of these early works, I did not use the symbolic processor to find the formula as they seeem rather obvious. I do not think it could have gotten by my switch for AF anyway.




$AB := BASE\_LENGTH$

$BR := BASE\_RATIO$

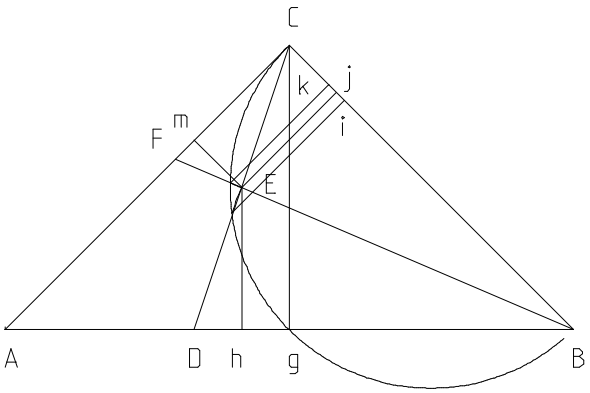
$BS := BISEC\_SEG$

$Cg := \frac{AB}{2} \quad Ag := Cg \quad Bg := Cg$

$AD := \frac{AB}{BR} \quad \delta := 0..BS$

$Dg := Ag - AD \quad CD := \sqrt{Dg^2 + Cg^2} \quad DE_\delta := \frac{CD}{BS}\cdot\delta$

$Dh_\delta := \frac{Dg\cdot DE_\delta}{CD} \quad Eh_\delta := \sqrt{(DE_\delta)^2 - (Dh_\delta)^2} \quad gh_\delta := Dg - Dh_\delta \quad Bh_\delta := Bg + gh_\delta$

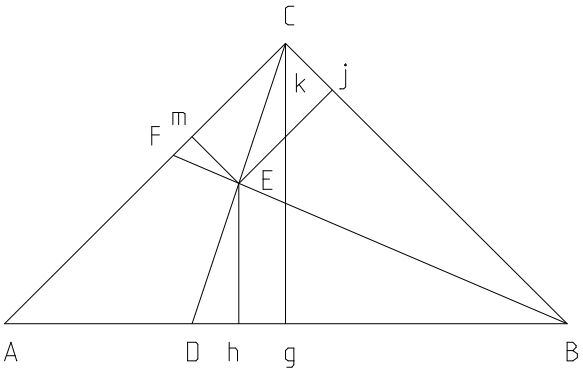



$BE_\delta := \sqrt{(Bh_\delta)^2 + (Eh_\delta)^2} \quad CE_\delta := CD - DE_\delta$

$BC := \sqrt{2\cdot Bg^2} \quad Bi_\delta := \frac{(BE_\delta)^2}{BC} \quad Ck_\delta := \frac{(CE_\delta)^2}{BC}$

$ik_\delta := BC - Bi_\delta - Ck_\delta \quad ij_\delta := \frac{ik_\delta}{2}$

$Bj_\delta := Bi_\delta + ij_\delta \quad Cj_\delta := BC - Bj_\delta \quad AC := BC$

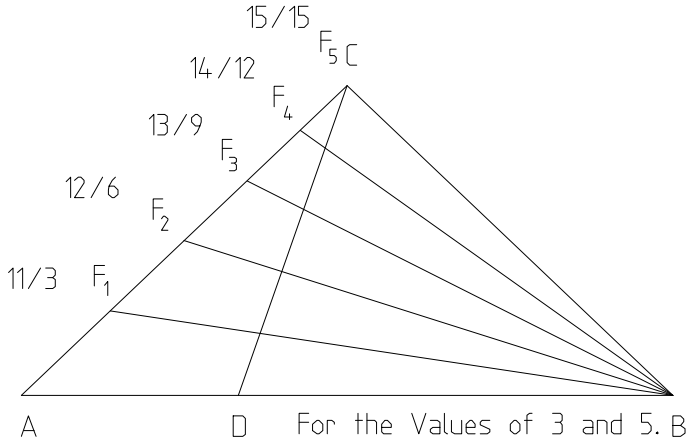



$$Em_\delta := Cj_\delta \qquad EF_\delta := \frac{BE_\delta \cdot Em_\delta}{Bj_\delta}$$

$$BF_\delta := BE_\delta + EF_\delta \qquad CF_\delta := \sqrt{\left(BF_\delta\right)^2 - BC^2}$$

If AF exceeds the figure it becomes part of AC.

$$AF_\delta := \text{if}\left(BR > 1, AC - CF, \text{if}\left(BF_\delta < 0, AC - CF, AC + CF\right)\right)_\delta$$




**Plug in values here!**

(AB) ⇐BASE_LENGTH≡ 22

(BS) ⇐BISEC_SEG≡ 5

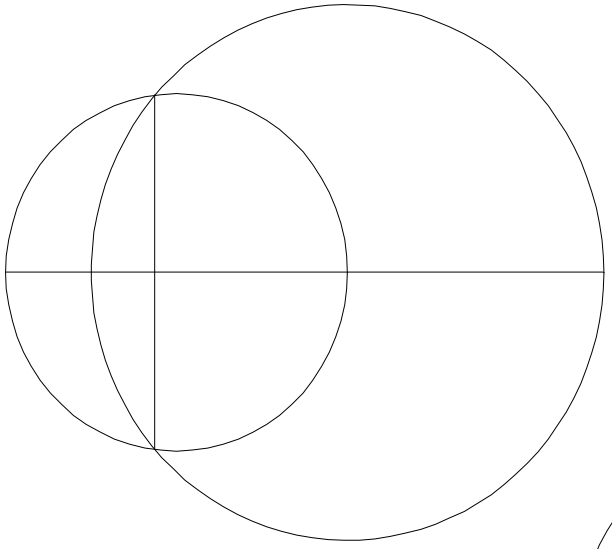
(BR) ⇐BASE_RATIO≡ 3

| $\text{if}\left(\delta, \frac{AC}{AF_\delta}, 0\right)$ | $\text{if}\left[\delta, \frac{(BR-1)\cdot BS + \delta}{BR\cdot\delta}, 0\right]$ | $\delta$ |
|---|---|---|
| 0 | 0 | 0 |
| 3.667 | 3.667 | 1 |
| 2 | 2 | 2 |
| 1.444 | 1.444 | 3 |
| 1.167 | 1.167 | 4 |
| 1 | 1 | 5 |

# The Pythagorean Completion.

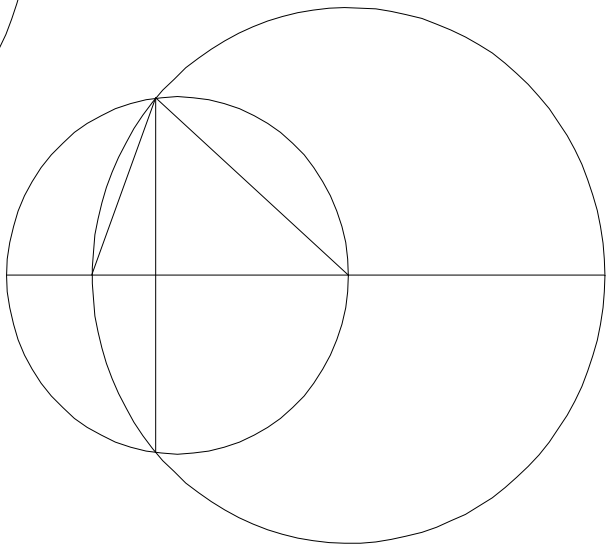
**A triangle theorem.**

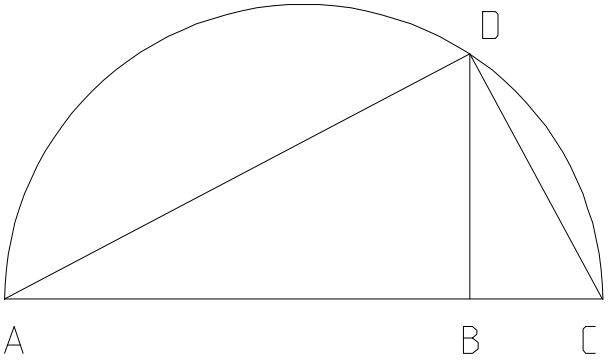
07_18_E3.MCD



Given the distance between non-equal intersecting circles and their radii, find the height and placement of the perpendicular bisector that they form.

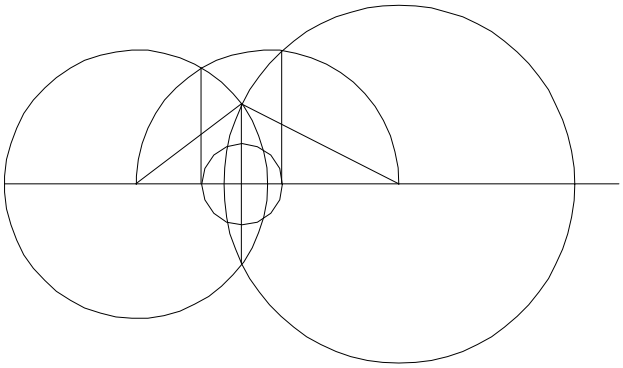
This problem can be restated as, given a triangle knowing only the length of the three sides, find its area.



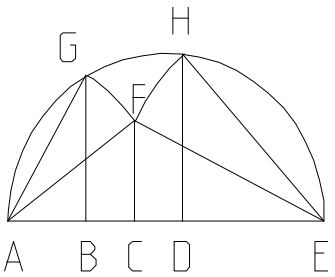



When one looks at a right triangle, one should see a square root and two square derivatives. The root, BD, is the root of AB × BC.
The two square derivatives are AB = AD²/AC and of course BC = CD²/AC. The square derivatives allow one to find the perpendicular bisector of the right triangle. These last two can be found with plain division instead of a root function. Since the area of a triangle is 1/2bh, finding the area of a right triangle is easy knowing just the length of two sides.

The process for finding the perpendicular for any other triangle is by completing simultaneous right triangles. The two bisectors created by these triangles will place the bisector of our starting triangle exactly in the center of them. I will use the following algorithm extensively when working in Euclidean geometry.



al•go•rithm n.
1. Math. a) any systematic method of solving a certain kind of problem. Ref. Lib.




In this version, AC has been "formulated" into the lowest terms. I have nothing against Heron's formula, but a correct reduction has only the input constants showing.

$AE := 6 \quad AF := 5 \quad EF := 4$ ← Plug your values in here.

*The sum of any two sides of a triangle is greater than the third.* Euclid.

$$Is\_This\_a\_Triangle := \frac{1}{(AE + AF \geq EF)\cdot(AE + EF \geq AF)\cdot(AF + EF \geq AE)} \quad AG := AF \quad EH := EF$$

$$AB := \frac{AG^2}{AE} \quad DE := \frac{EH^2}{AE} \quad AD := AE - DE \quad BD := AD - AB \quad AC := \frac{1}{2}\cdot\frac{AF^2}{AE} + \frac{1}{2}\cdot AE - \frac{1}{2}\cdot\frac{EF^2}{AE}$$

The perpendicular bisector from AC is given as

$AC = 3.75$

$$CE := AE - AC \quad CF_1 := \sqrt{EF^2 - CE^2} \quad CF_2 := \sqrt{AF^2 - AC^2} \quad CF_1 = 3.307$$

*The endpoints of two sides of a triangle cannot meet in two different places.* Euclid.

$CF_1 - CF_2 = 0$

$$Area := \frac{AE}{2}\cdot CF_1 \quad Area = 9.922 \qquad S_1 := AE \quad S_2 := AF \quad S_3 := EF$$

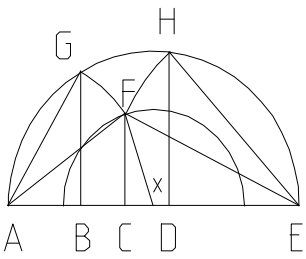
$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{4} = 9.922$$

The following result could lead to a more general rule regarding the squares on two sides of any triangle as related to the third side.

$$(AE^2 - (BD \cdot AE)) - (AF^2 + EF^2) = 0$$

$AE^2 - (BD \cdot AE) = 41$ $\quad$ $EF^2 + AF^2 = 41$

If we agree to call BD the right angle defect, then*the squares on two sides of a triangle are equal to the remaining side squared minus its right angle defect multiplied by that side.* Or if one wants to be less obtuse,




$$Ax := \frac{AE}{2} \quad Cx := Ax - AC \quad Fx := \sqrt{(CF_1)^2 + Cx^2}$$

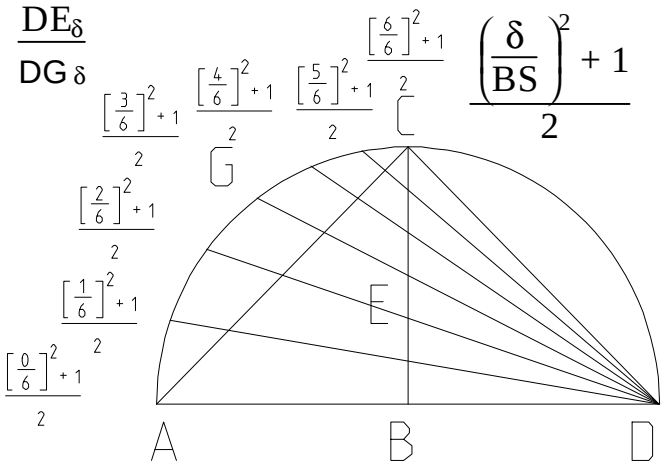
$$\frac{AE^2}{2} + 2 \cdot Fx^2 = 41 \qquad AF^2 + EF^2 = 41$$

Calling Fx the radial to the bisector,*The squares on any two sides of a triangle are equal to half of the square on the remaining side added to twice the square of its radial bisector*.

$$(AF^2 + EF^2) - \left(\frac{AE^2}{2} + 2 \cdot Fx^2\right) = 0$$

$$Fx - \sqrt{\frac{(AF^2 + EF^2) - \frac{AE^2}{2}}{2}} = 0 \qquad Fx - \frac{1}{2} \cdot \sqrt{2 \cdot AF^2 + 2 \cdot EF^2 - AE^2} = 0$$

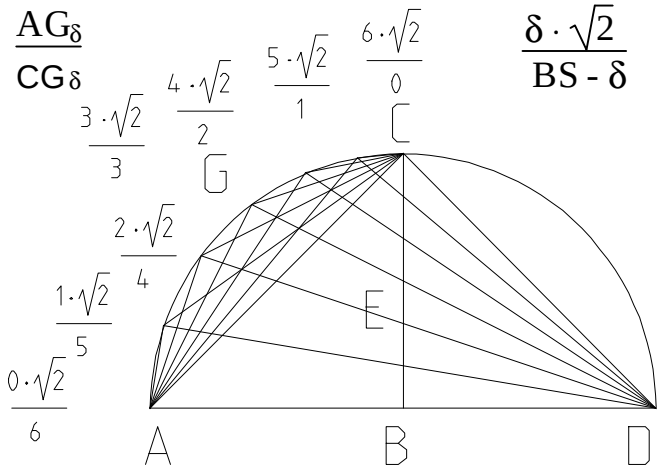
It is shown that there are at least two formulas which relate the squares of any two sides of a triangle to the third, this makes the Pythagorean theorem a statement of a single case. This single case is, however, a fundamental geometrical tool.

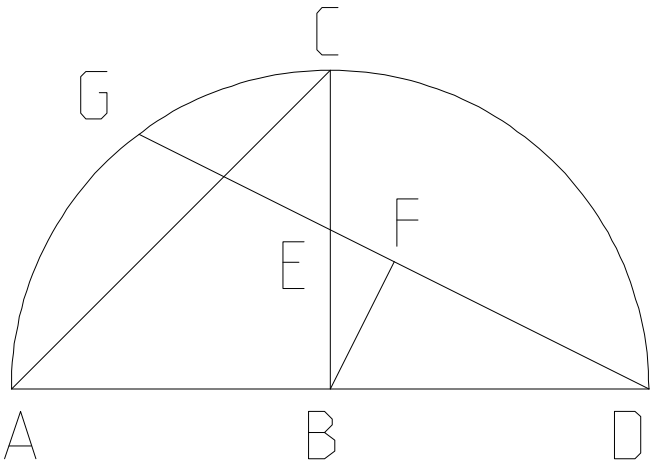



07_25_93.MCD

# Pyramid Of Ratio Series IV
# Working the Curve

This the ratios will be working against a curve, instead of a straight line intercept. The ratios found are DE:DG and AG:GC. This series is not yet completed, as the ratio AB:AD remains fixed.
At the end of this paper, $\pi$ will be calculated to any number of given segments by summing all the segments AG.




$$AD := 1 \quad \delta := 0 .. BS \quad AB := \frac{AD}{2}$$

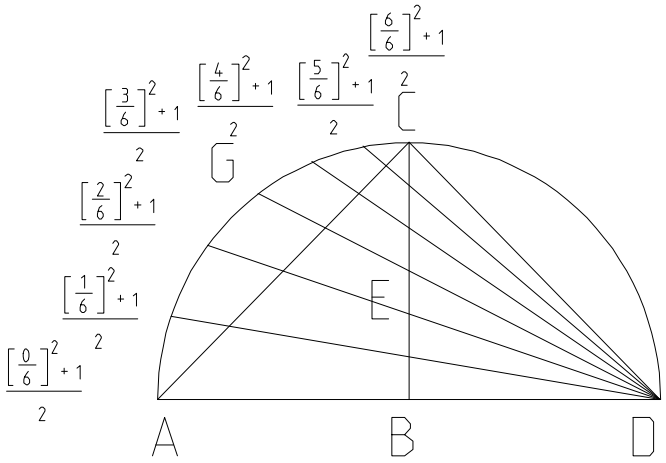
The following will keep the tables from getting out of hand.

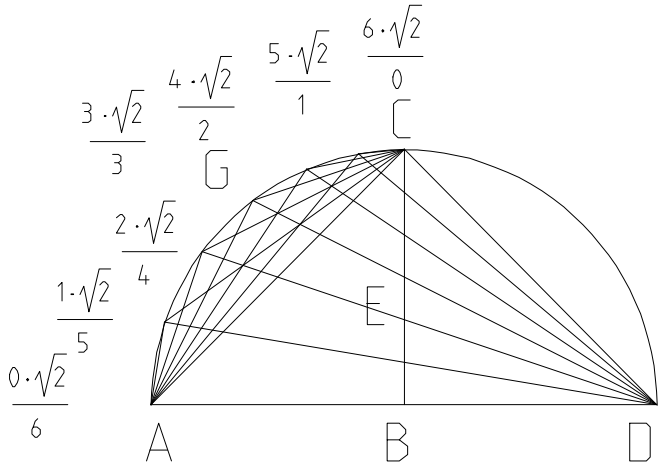
$$BC := AB \quad BD := AB \quad BE_\delta := \frac{BC}{BS} \cdot \delta$$

$$DE_\delta := \sqrt{BD^2 + \left(BE_\delta\right)^2} \quad DF_\delta := \frac{BD^2}{DE_\delta}$$

$$DG_\delta := 2 \cdot DF_\delta$$

| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{BS}\right)^2 + 1}{2}$ |
|---|---|
| 0.5 | 0.5 |
| 0.514 | 0.514 |
| 0.556 | 0.556 |
| 0.625 | 0.625 |
| 0.722 | 0.722 |
| 0.847 | 0.847 |
| 1 | 1 |



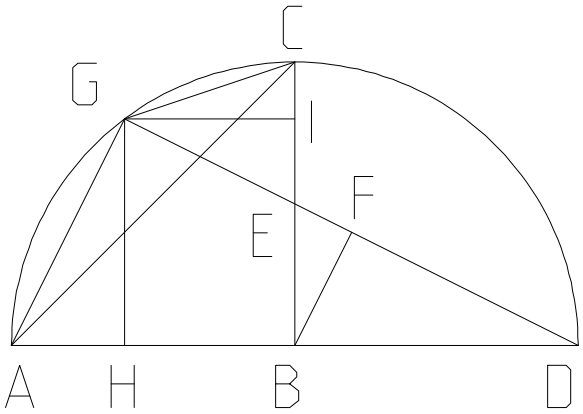

The series formed in these papers are somewhat attractive for their simplicity.

$$DH_\delta := \frac{(DG_\delta)^2}{AD} \quad BH_\delta := DH_\delta - BD \quad GH_\delta := \sqrt{(DG_\delta)^2 - (DH_\delta)^2} \quad BI_\delta := GH_\delta \quad CI_\delta := BC - BI_\delta$$

$GI_\delta := BH_\delta \quad AH_\delta := AB - BH_\delta$

$CG_\delta := \sqrt{\left(GI_\delta\right)^2 + \left(CI_\delta\right)^2}$

$AG_\delta := \sqrt{\left(AH_\delta\right)^2 + \left(GH_\delta\right)^2}$

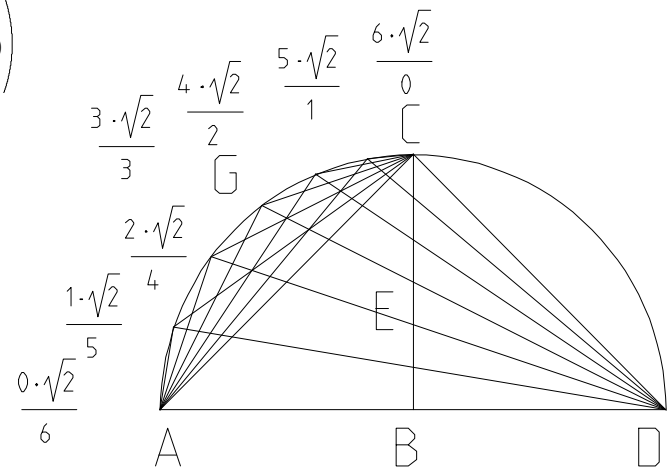


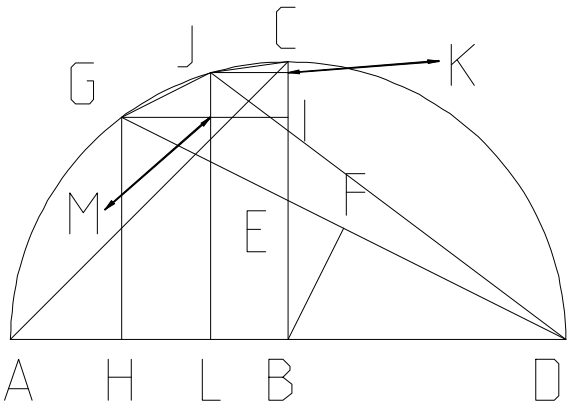

$\text{if}\left(BS - \delta, \frac{AG_\delta}{CG_\delta}, 0\right)$

| |
|---|
| 0 |
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |

$\text{if}\left(BS - \delta, \frac{\delta \cdot \sqrt{2}}{BS - \delta}, 0\right)$

| |
|---|
| 0 |
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |






Finding the value of PI to any given number of unequal segments.

$\chi := 1 .. BS$

$JK_\chi := GI_\chi \quad GM_\chi := GI_{\chi - 1} - JK_\chi$

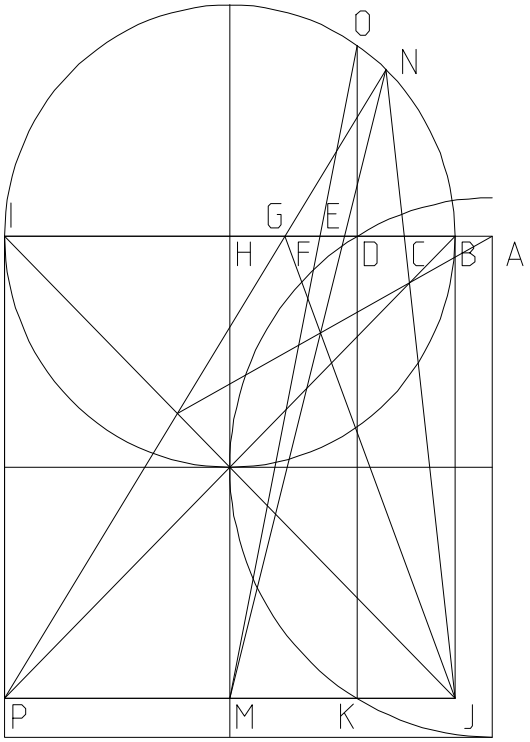
$CK_\chi := CI_\chi \quad IK_\chi := CI_{\chi - 1} - CK_\chi$

$GJ_\delta := \sqrt{\left(GM_\delta\right)^2 + \left(IK_\delta\right)^2}$

$BS \equiv 6 \qquad \sum_\delta GJ_\delta \cdot 4 = 3.13154608512668$

Computed for 5000 segments $BS \equiv 5000 \qquad \sum_\delta GJ_\delta \cdot 4 = 3.141592639069159$

$\pi = 3.141592653589793$




What is the relationship of DE:DF?

$BI := 10 \quad BH := \frac{BI}{2} \quad HI := BH$

$z := 100 \qquad \delta := 1 .. z \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$AI_\delta := AB_\delta + BI \quad AD_\delta := \sqrt{AB_\delta \cdot AI_\delta}$

$JM := BH \quad BD_\delta := AD_\delta - AB_\delta$

$DH_\delta := BH - BD_\delta \quad MK_\delta := DH_\delta$

$HO := BH \quad DO_\delta := \sqrt{HO^2 - \left(DH_\delta\right)^2}$

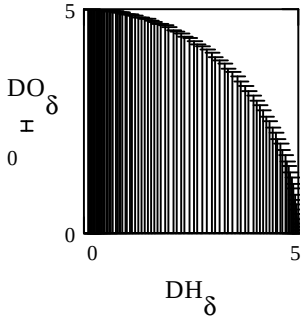
$DK := BI \quad KO_\delta := DK + DO_\delta$

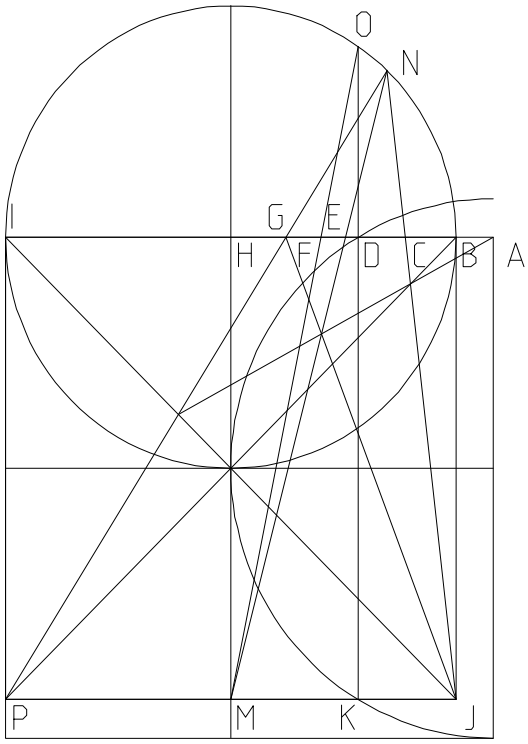
$DF_\delta := \frac{MK_\delta \cdot DO_\delta}{KO_\delta} \quad AG_\delta := \left[ AB_\delta \cdot \left(AI_\delta\right)^2 \right]^{\frac{1}{3}}$

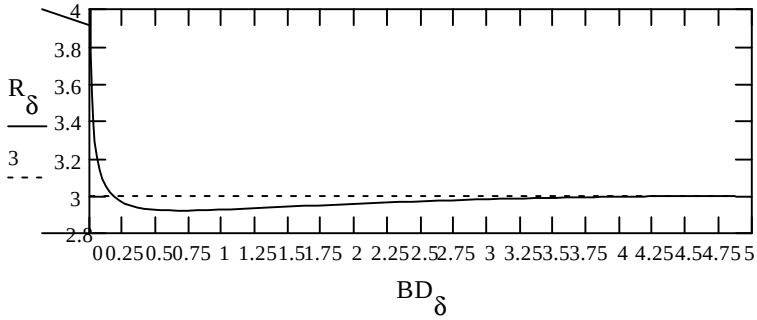
$AC_\delta := \left[ \left(AB_\delta\right)^2 \cdot AI_\delta \right]^{\frac{1}{3}} \quad AE_\delta := \frac{AG_\delta + AC_\delta}{2}$

$DE_\delta := AE_\delta - AD_\delta \qquad R_\delta := \frac{DF_\delta}{DE_\delta}$

Sample points.



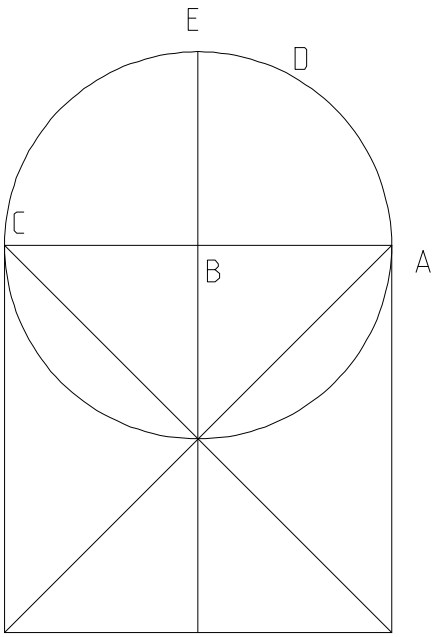




Apparently not a constant.

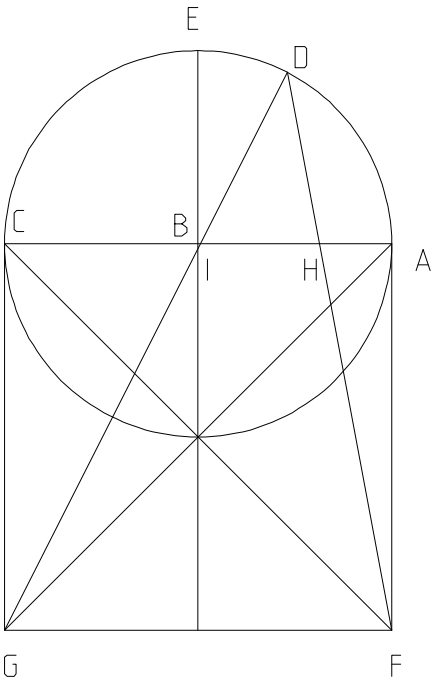
# Gruntwork I on the Delian Solution

11_06_93.mcd

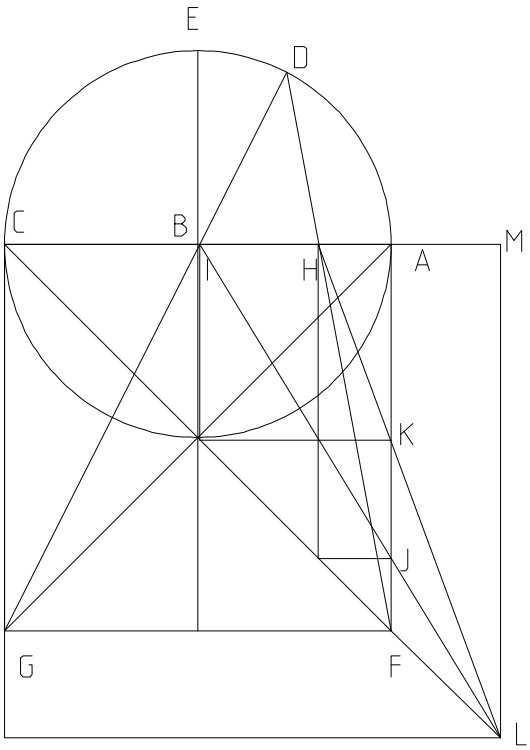



According to number theory, it is not possible to develop a cube root relationship using only linear and square root operations. Watch.

Given any circle ABE and any point, D, between AE.

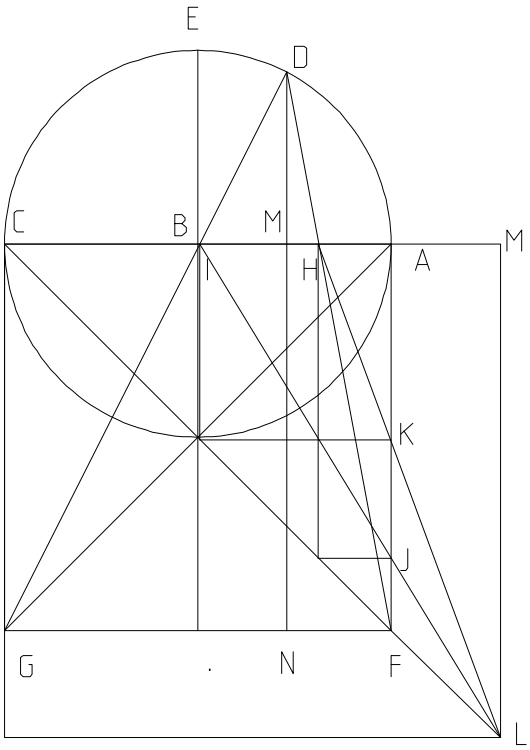



Describe FDG.




Describe AK, JK, FJ such that AK = BC, JK = HI, FJ = AH.

Do IJ and HK and CF meet at one and only one point L?




$AC := 10 \quad AB := \frac{AC}{2} \quad BC := AB \quad INC := 500$

$MN := AC \quad BD := AB \quad \delta := 0 .. INC - 1$

$AM_\delta := \frac{AB}{INC} \cdot \delta \quad BM_\delta := AB - AM_\delta \quad FN_\delta := AM_\delta$

$DM_\delta := \sqrt{BD^2 - (BM_\delta)^2} \quad DN_\delta := MN + DM_\delta$

$GN_\delta := BC + BM_\delta \quad DG_\delta := \sqrt{(GN_\delta)^2 + (DN_\delta)^2}$

$IM_\delta := \frac{GN_\delta \cdot DM_\delta}{DN_\delta} \quad DF_\delta := \sqrt{(FN_\delta)^2 + (DN_\delta)^2}$

$HM_\delta := \frac{FN_\delta \cdot DM_\delta}{DN_\delta} \quad AH_\delta := AM_\delta - HM_\delta$

Construct BQ and HP parallel with AO.
KT and JU parallel with CF.
RT and SU parallel with AF.

$AI_\delta := AM_\delta + IM_\delta \quad CI_\delta := AC - AI_\delta \quad CQ_\delta := \sqrt{\frac{(CI_\delta)^2}{2}}$

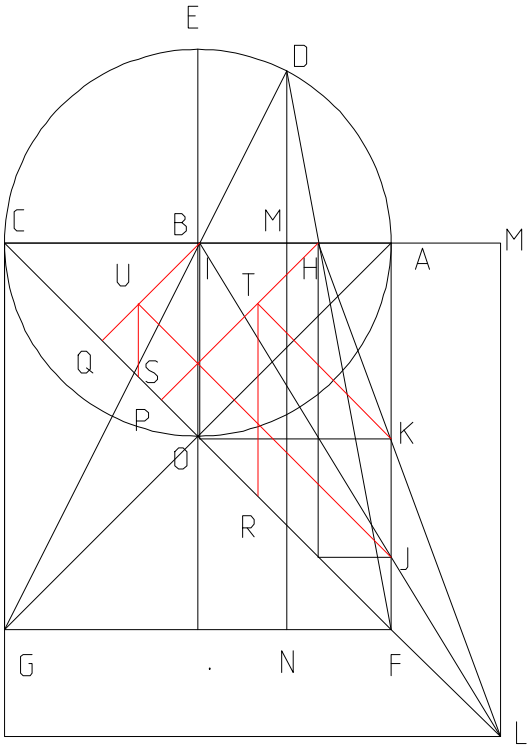
$CH_\delta := AC - AH_\delta \quad CP_\delta := \sqrt{\frac{(CH_\delta)^2}{2}} \quad CO := \sqrt{\frac{AC^2}{2}}$

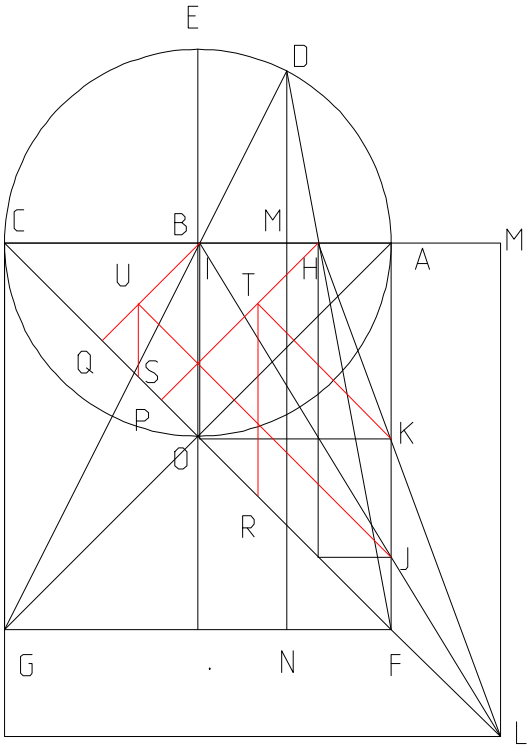
$AK_\delta := CI_\delta \quad AJ_\delta := CH_\delta \quad FJ_\delta := AH_\delta \quad US_\delta := FJ_\delta$

$FK_\delta := AI_\delta \quad RT_\delta := FK_\delta \quad IJ_\delta := \sqrt{(AI_\delta)^2 + (AJ_\delta)^2}$

$HK_\delta := \sqrt{(AH_\delta)^2 + (AK_\delta)^2} \quad QU_\delta := \sqrt{\frac{(US_\delta)^2}{2}}$

$PT_\delta := \sqrt{\frac{(RT_\delta)^2}{2}} \quad IQ_\delta := CQ_\delta \quad HP_\delta := CP_\delta$






$IU_\delta := IQ_\delta - QU_\delta \quad HT_\delta := HP_\delta - PT_\delta$

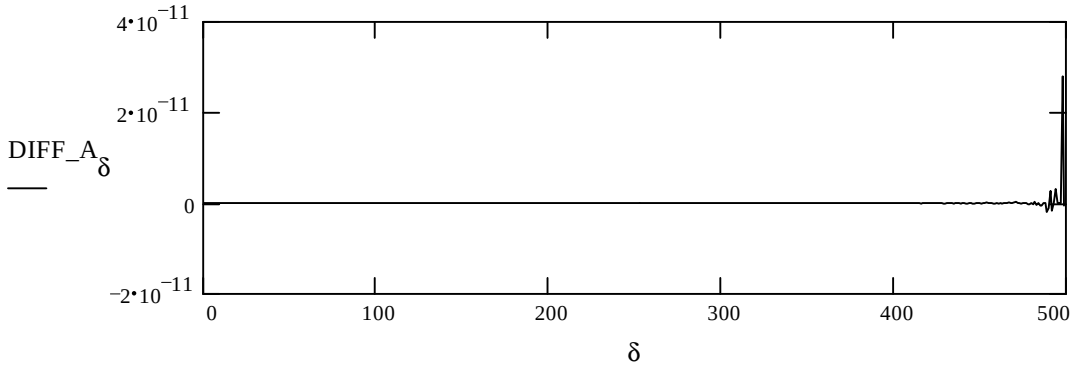
$KT_\delta := \sqrt{(HK_\delta)^2 - (HT_\delta)^2} \quad JU_\delta := \sqrt{(IJ_\delta)^2 - (IU_\delta)^2}$

$LQ_\delta := \frac{JU_\delta \cdot IQ_\delta}{IU_\delta} \quad LP_\delta := \frac{KT_\delta \cdot HP_\delta}{HT_\delta} \quad OQ_\delta := CO - CQ_\delta$

$OP_\delta := CO - CP_\delta \quad LO1_\delta := LQ_\delta - OQ_\delta$

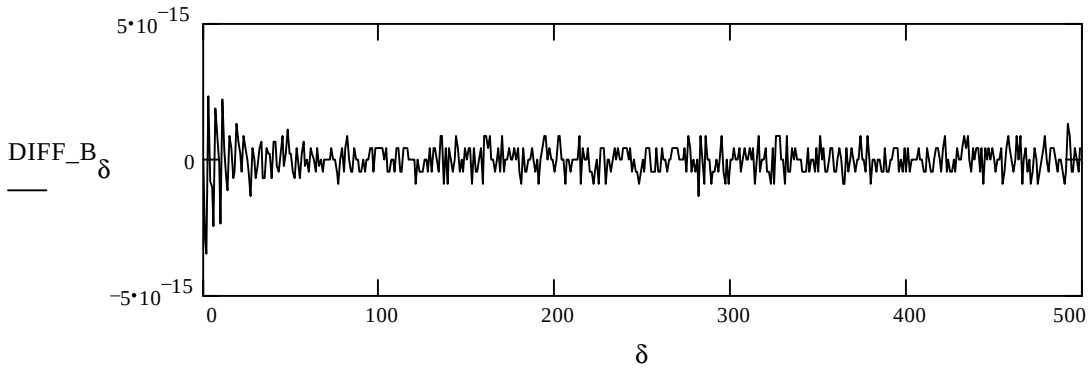
$LO2_\delta := LP_\delta - OP_\delta$

$DIFF\_A_\delta := LO1_\delta - LO2_\delta$

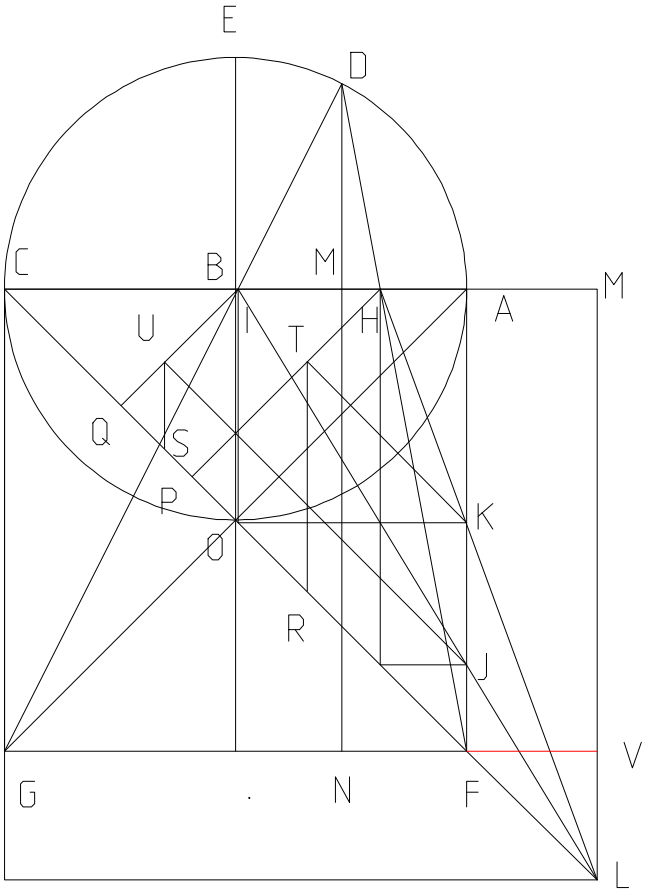



IJ, HK, and CF meet in one point L

$HI_\delta := HM_\delta + IM_\delta$ $DIFF\_B_\delta := HI_\delta - \sqrt{AH_\delta \cdot CI_\delta}$




The segment HI is the square root of the remaining segments AH, CI.



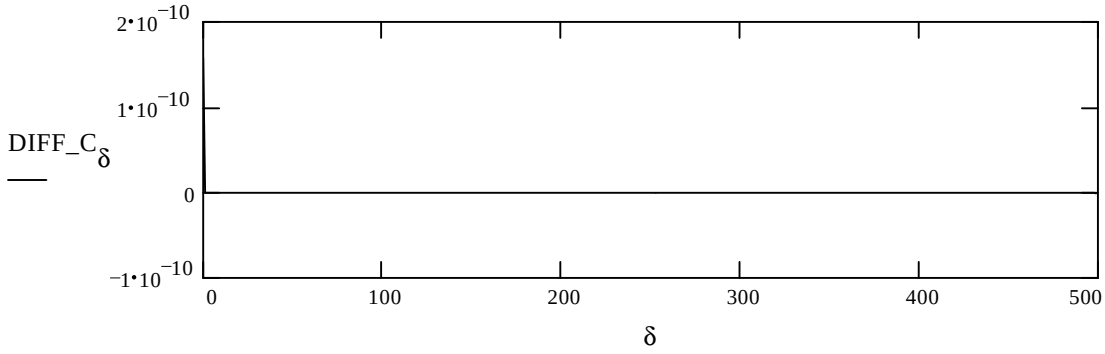

$FO := CO$ $FL_\delta := LO1_\delta - FO$

$FV_\delta := \sqrt{\frac{(FL_\delta)^2}{2}}$ $AM_\delta := FV_\delta$

$HM_\delta := AM_\delta + AH_\delta$ $IM_\delta := HM_\delta + HI_\delta$

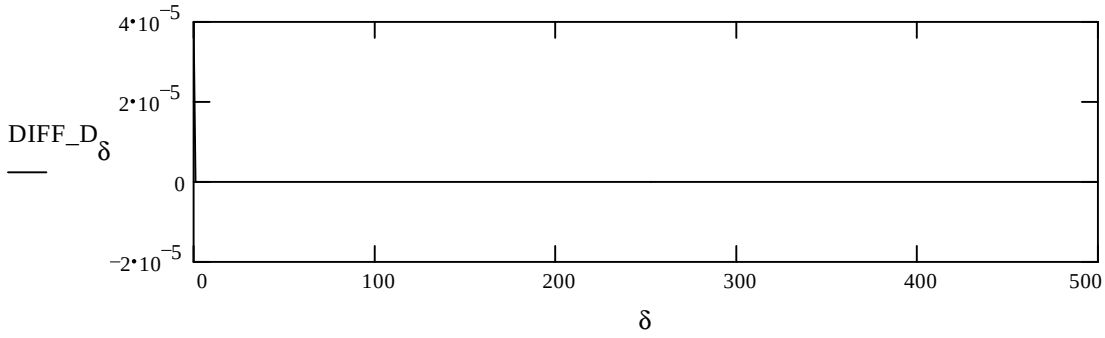
$CM_\delta := AM_\delta + AC$

$$DIFF\_C_\delta := \left[ \left( AM_\delta \right)^2 \cdot CM_\delta \right]^{\frac{1}{3}} - HM_\delta$$




HM is the cube root of $\left( AM_\delta \right)^2 \cdot CM_\delta$

$$DIFF\_D_\delta := \left[ AM_\delta \cdot \left( CM_\delta \right)^2 \right]^{\frac{1}{3}} - IM_\delta$$




IM is the cube root of $AM_\delta \cdot \left( CM_\delta \right)^2$

# Solve for Cube Placement

With straight edge and compass only, solve the given problem.
BE is the difference between the segments AE and AB.
CD is the difference between the cube root of AB squared by AE and the cube root of AE squared by AB. Find AB.

E B

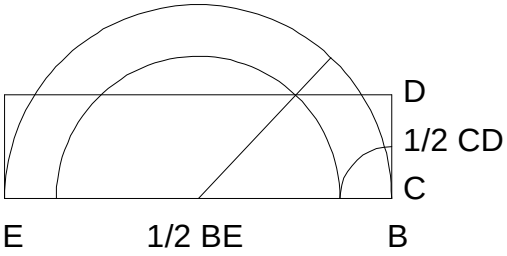
D C

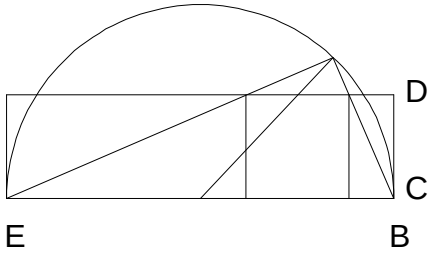
$BE = AE - AB$

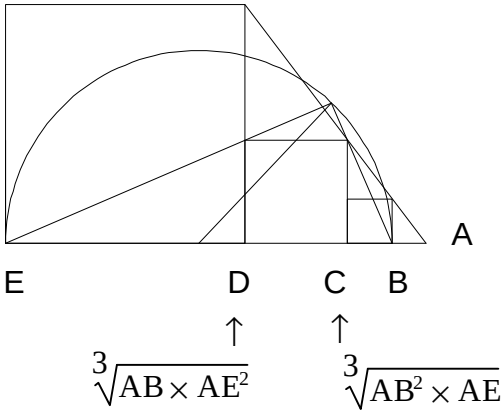
$CD = (AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}$

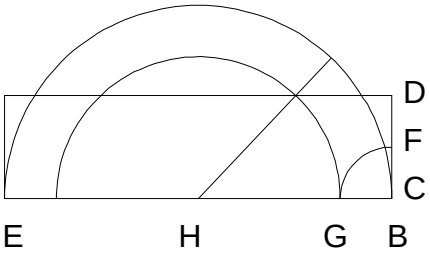
$AB = ?$

## Process Summary



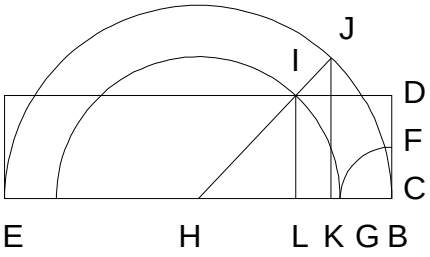






$\Delta := 500 \qquad \delta := 1 .. \Delta \qquad BE := 100$

$CD_\delta := \frac{BE}{3 \cdot \Delta} \cdot \delta \qquad CF_\delta := \frac{CD_\delta}{2} \qquad BG_\delta := CF_\delta$

$BH := \frac{BE}{2} \qquad EH := BH \qquad GH_\delta := BH - BG_\delta$

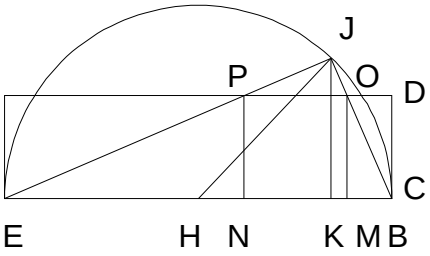



$HI_\delta := GH_\delta \qquad IL_\delta := CD_\delta \qquad HJ := BH$

$HL_\delta := \sqrt{(HI_\delta)^2 - (IL_\delta)^2} \qquad HK_\delta := \frac{HL_\delta \cdot HJ}{HI_\delta}$

$JK_\delta := \sqrt{HJ^2 - (HK_\delta)^2} \qquad BK_\delta := BH - HK_\delta$

$EK_\delta := EH + HK_\delta$




$CJ_\delta := \sqrt{(JK_\delta)^2 + (BK_\delta)^2} \qquad MO_\delta := CD_\delta$

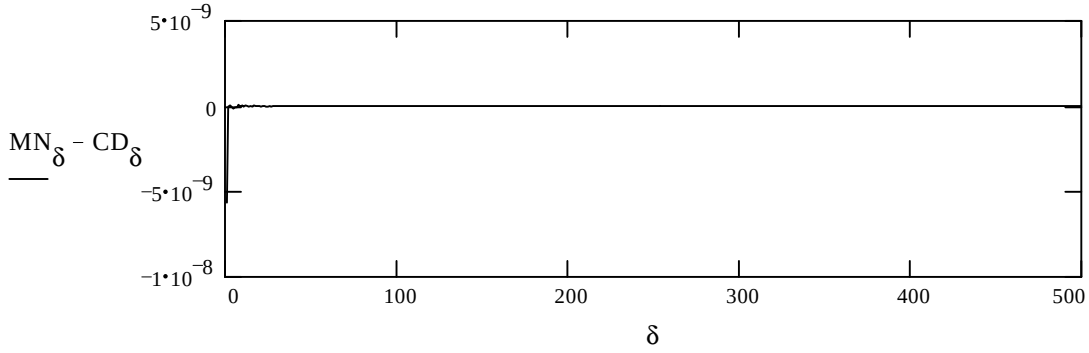
$EJ_\delta := \sqrt{(EK_\delta)^2 + (JK_\delta)^2} \qquad NP_\delta := CD_\delta$

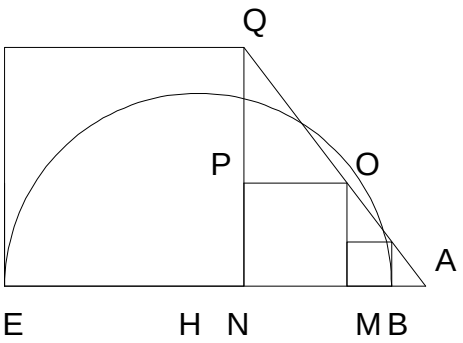
$EN_\delta := \frac{EK_\delta \cdot NP_\delta}{JK_\delta} \qquad BN_\delta := BE - EN_\delta$

$BM_\delta := \frac{BK_\delta \cdot MO_\delta}{JK_\delta} \qquad MN_\delta := BN_\delta - BM_\delta$

$OP_\delta := MN_\delta$

It appears that MN = CD.






$$NQ_\delta := EN_\delta \quad PQ_\delta := NQ_\delta - NP_\delta$$

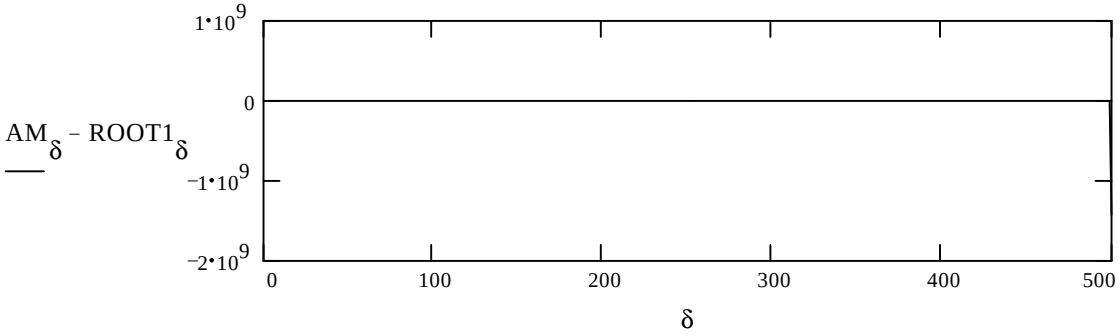
$$AN_\delta := \frac{OP_\delta \cdot NQ_\delta}{PQ_\delta} \quad AB_\delta := AN_\delta - BN_\delta$$

$$AM_\delta := AB_\delta + BM_\delta \quad AN_\delta := AB_\delta + BN_\delta$$

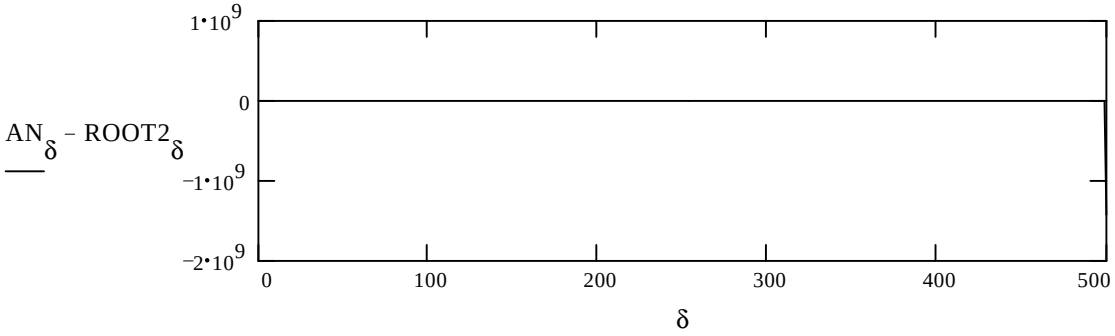
$$AE_\delta := AB_\delta + BE$$

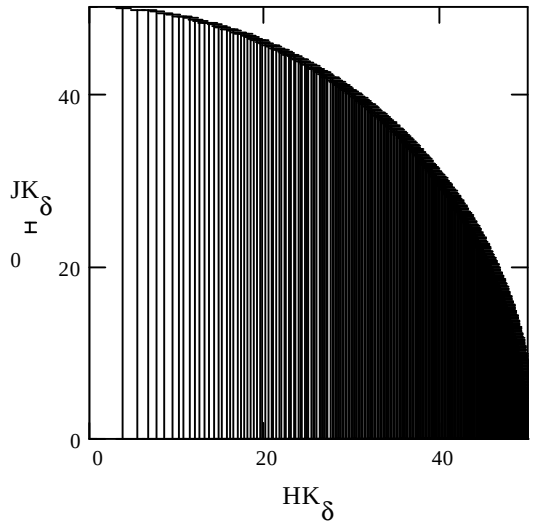
$$ROOT1_\delta := \left[ \left(AB_\delta\right)^2 \cdot AE_\delta \right]^{\frac{1}{3}}$$

$$ROOT2_\delta := \left[ AB_\delta \cdot \left(AE_\delta\right)^2 \right]^{\frac{1}{3}}$$




We seem to have a cube root relationship

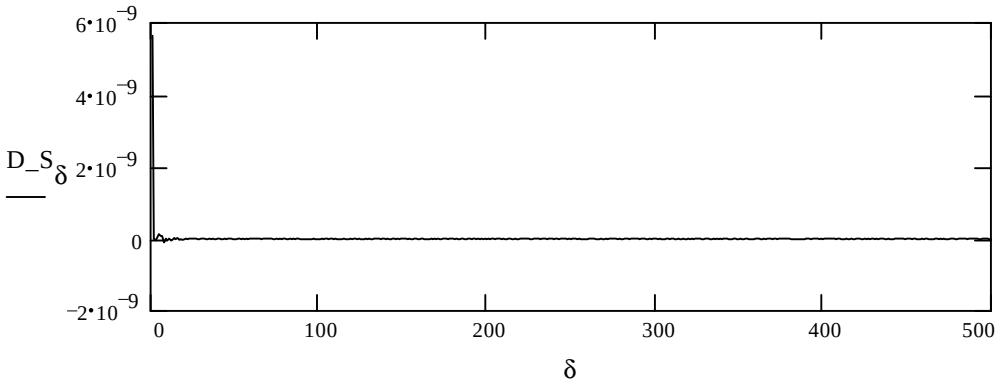




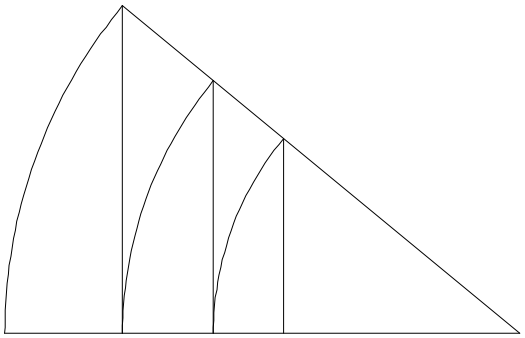

Sample Points along the arc.

Last ratio. $\frac{BE}{CD_{\Delta}} = 3$

$D\_S_{\delta} := \sqrt{EN_{\delta} \cdot BM_{\delta}} - MN_{\delta}$




The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

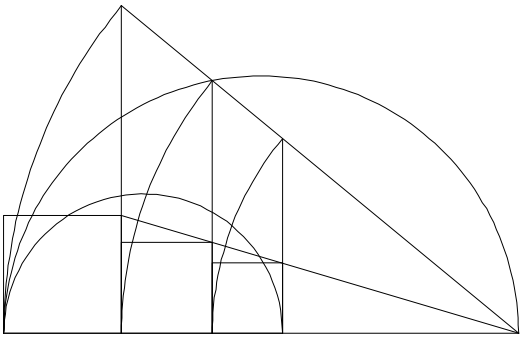


11_10_E3.MCD

# Gruntwork II on the Delian Solution

Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?

Process Summary

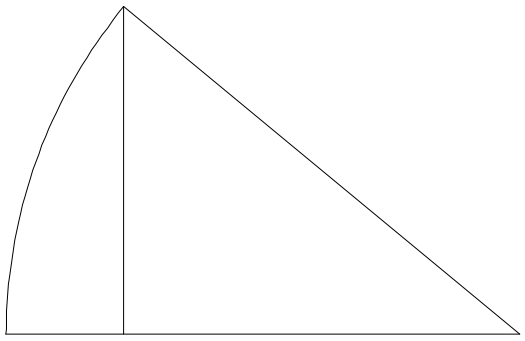


The figure indicates a possible cubic relationship. This is what will be tested along with some of the constants involved.

$D := 10 \quad \Delta := 90 \quad \delta := 1 .. \Delta \cdot D$

$\Delta$ will give the the range as 90 degrees.
D will further divide each degree into smaller segments.

F



A

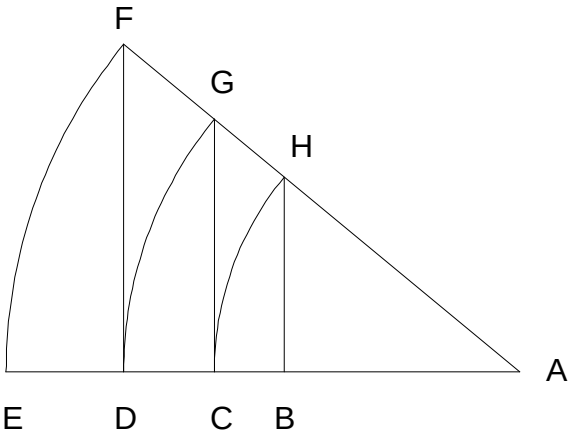
E D

$AE := 100$

$AF := AE \quad \angle A_\delta := \frac{\delta \cdot deg}{D}$

$DF_\delta := AF \cdot \left( \sin\left( \angle A_\delta \right) \right)$

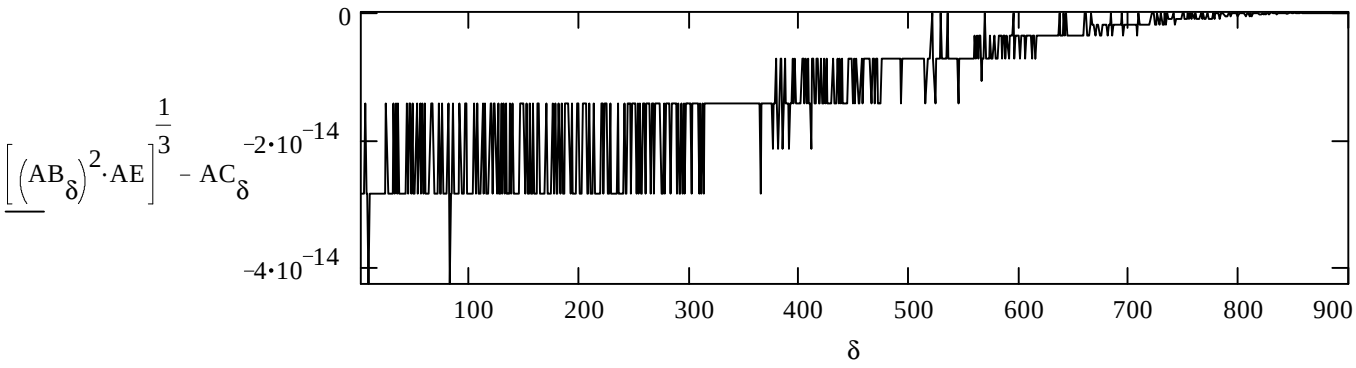
$AD_\delta := \sqrt{AF^2 - \left( DF_\delta \right)^2}$

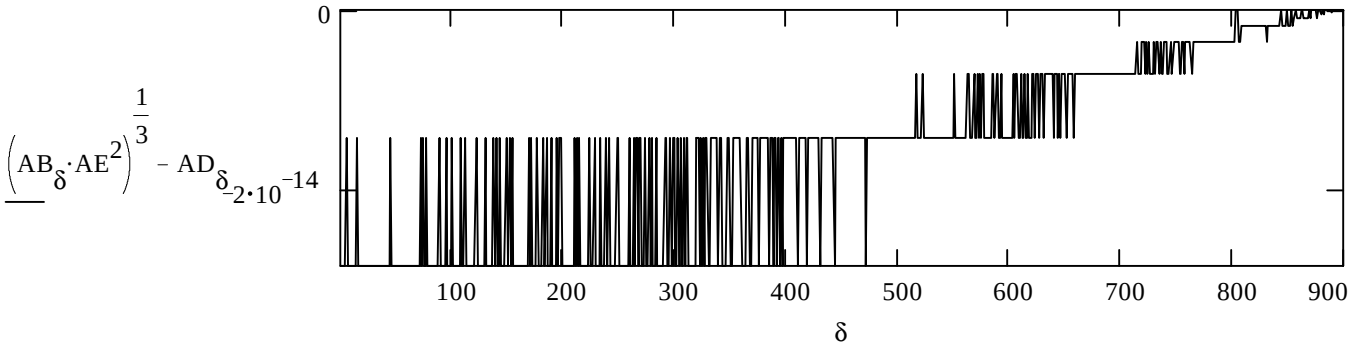



$$AG_\delta := AD_\delta \qquad AC_\delta := \frac{AD_\delta \cdot AG_\delta}{AF}$$

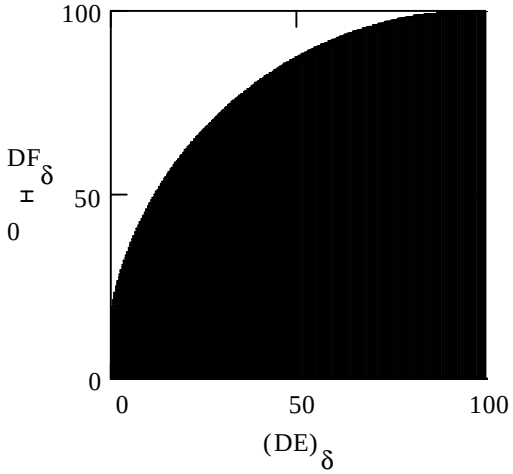
$$AH_\delta := AC_\delta \qquad AB_\delta := \frac{AD_\delta \cdot AH_\delta}{AF}$$

$$DE_\delta := AE - AD_\delta$$



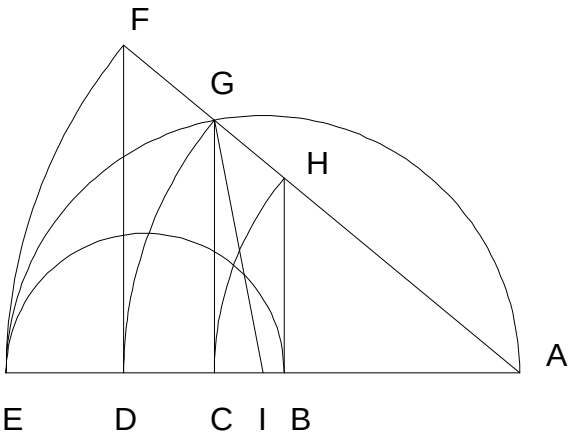




The simple figure yields a cube root relationship.




Sample points taken along DE.

There are $D \cdot \Delta = 900$ figures represented.

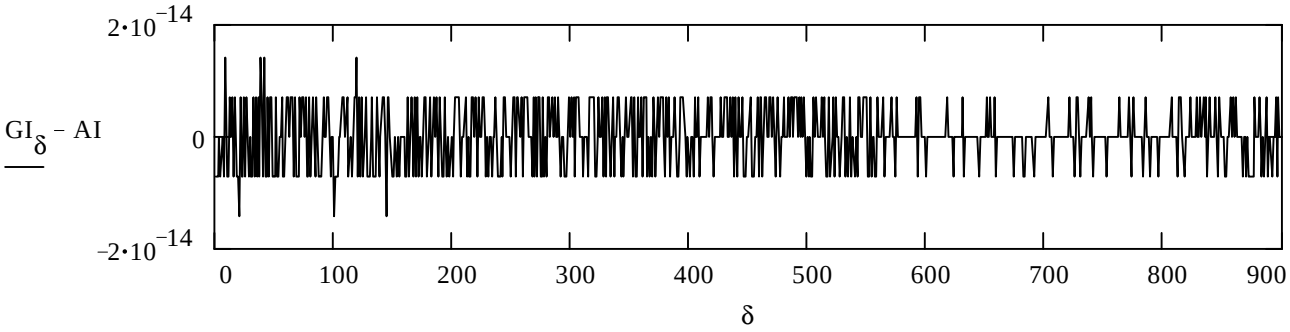



Does CG always meet at the circumference? (AGE)

$$CG_\delta := \sqrt{(AG_\delta)^2 - (AC_\delta)^2}$$

$$AI := \frac{AE}{2} \qquad CI_\delta := AC_\delta - AI$$

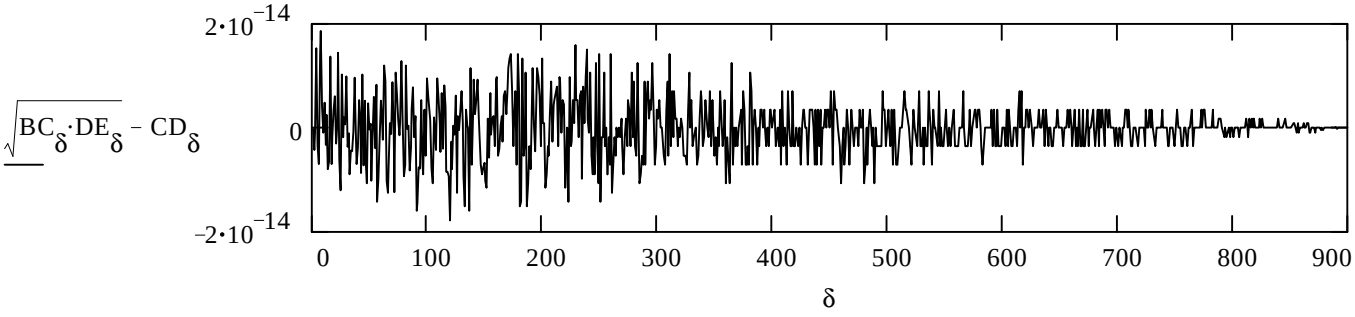
$$GI_\delta := \sqrt{(CG_\delta)^2 + (CI_\delta)^2}$$



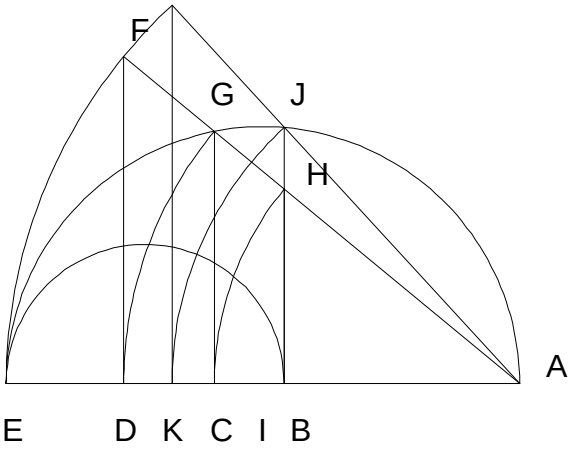

I would interpret this to mean yes.

Is then, CD the square root of BC and DE?

$$BC_\delta := AC_\delta - AB_\delta \qquad CD_\delta := AD_\delta - AC_\delta \qquad DE_\delta := AE - AD_\delta$$




I would interpret this to mean yes also.



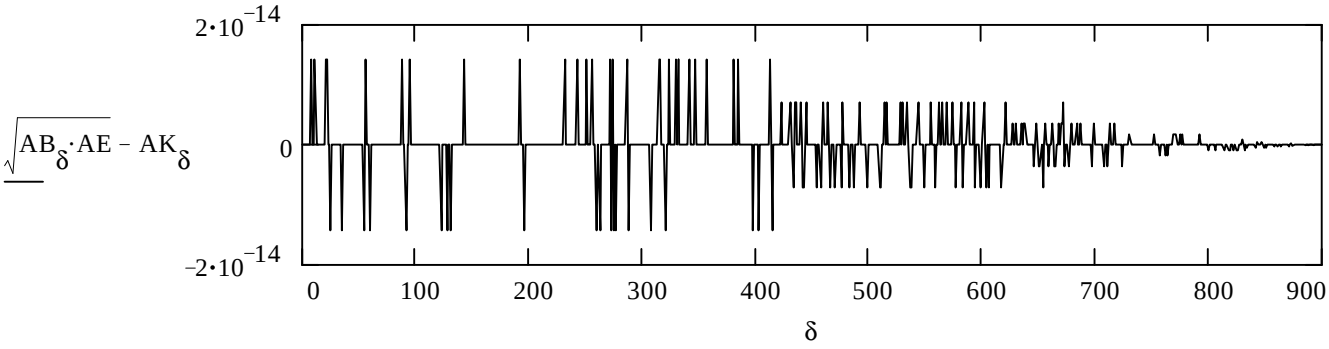

Is AK the square root of AB AE?

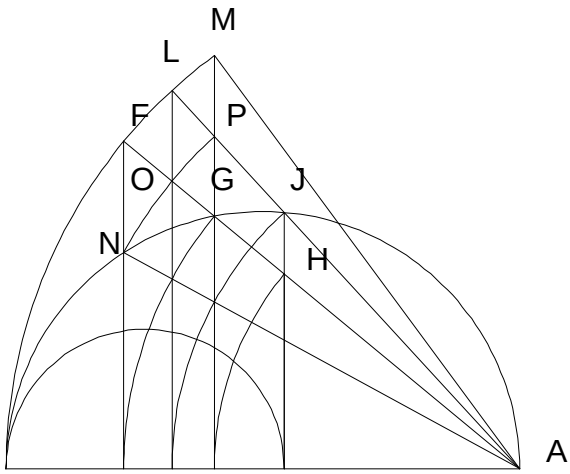
$$BE_\delta := AE - AB_\delta \qquad BJ_\delta := \sqrt{AB_\delta \cdot BE_\delta}$$

$$AJ_\delta := \sqrt{\left(AB_\delta\right)^2 + \left(BJ_\delta\right)^2}$$

$$AK_\delta := AJ_\delta$$

Apparently so.




Do N, O, and P lie on the same arc?

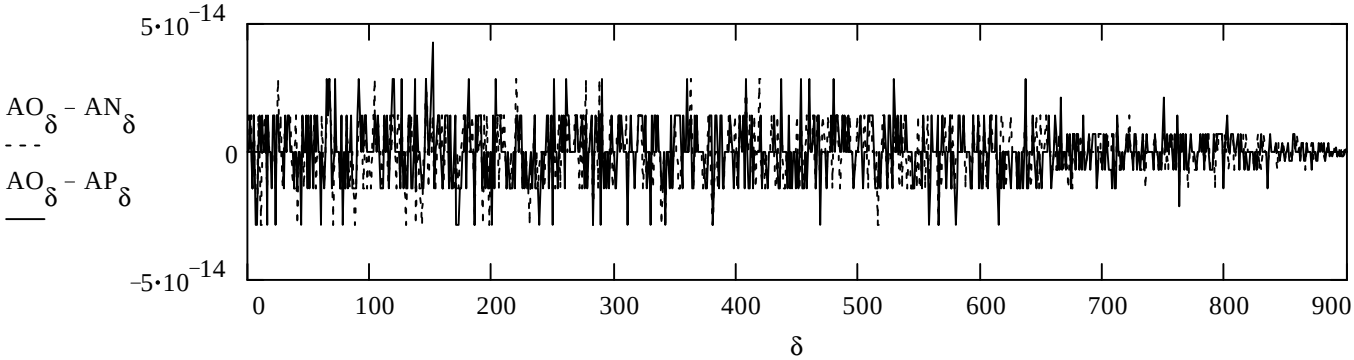
$$DN_\delta := \sqrt{AD_\delta \cdot DE_\delta}$$

$$AN_\delta := \sqrt{\left(AD_\delta\right)^2 + \left(DN_\delta\right)^2}$$

$$AO_\delta := \frac{AG_\delta \cdot AK_\delta}{AC_\delta}$$

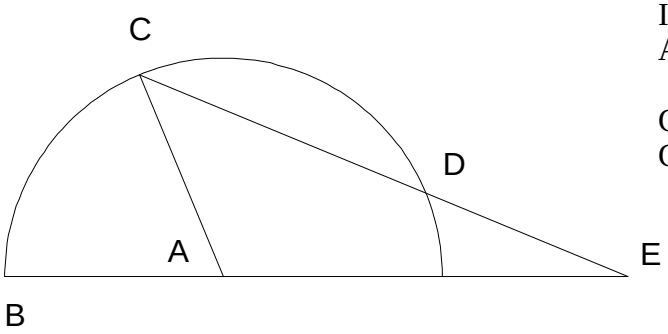
$$AP_\delta := \frac{AJ_\delta \cdot AC_\delta}{AB_\delta}$$

Another affirmative.



# The Archamedian Paper Trisector

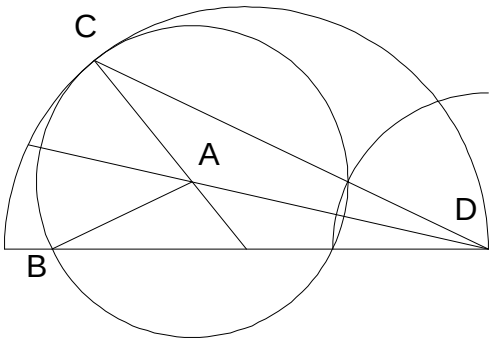
11_11_B3.MCD

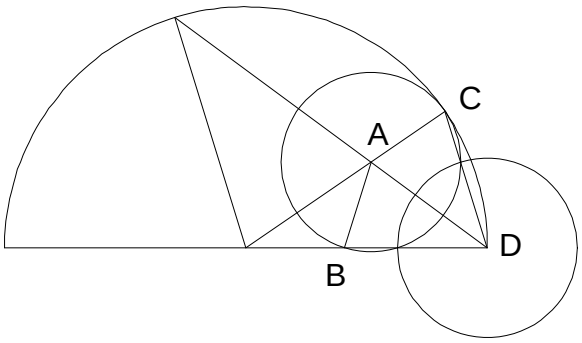



I have never seen the developed figure from which the ARchamedian Pager Trisector derives.

DE = AC, and by using a piece of marked paper, CDE can be somewhat aligned producing the trisection CEB = 1/3 CAB.

Process Summary

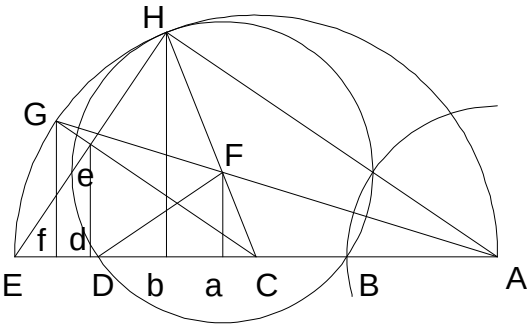



In both cases CDB is 1/3 of CAB. It appears that trisection might occur for the entire circle.

$$AE := 100000 \quad \delta := 0..\Delta$$

$$Ab_\delta := \text{if}\left[ View, \frac{AE}{View} - \left(\frac{\frac{AE}{View}}{\Delta}\cdot\delta\right), AE - \left(\frac{AE}{\Delta}\cdot\delta\right)\right]$$



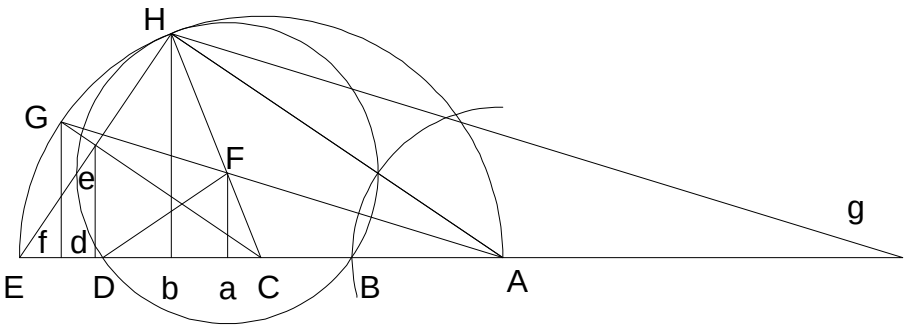

$$Eb_\delta := AE - Ab_\delta$$

$$Hb_\delta := \sqrt{Ab_\delta \cdot Eb_\delta} \quad EH_\delta := \sqrt{\left(Hb_\delta\right)^2 + \left(Eb_\delta\right)^2}$$

$$AC := \frac{AE}{2} \quad CE := AC \quad Ee_\delta := \frac{EH_\delta}{2}$$

$$Ce_\delta := \sqrt{CE^2 - \left(Ee_\delta\right)^2} \quad Cd_\delta := \frac{\left(Ce_\delta\right)^2}{CE} \quad CG := AC \quad Cf_\delta := \frac{Cd_\delta \cdot CG}{Ce_\delta}$$

$$de_\delta := \sqrt{\left(Ce_\delta\right)^2 - \left(Cd_\delta\right)^2} \quad Gf_\delta := \frac{de_\delta \cdot Cf_\delta}{Cd_\delta} \quad Af_\delta := AC + Cf_\delta \quad AG_\delta := \sqrt{\left(Af_\delta\right)^2 + \left(Gf_\delta\right)^2}$$



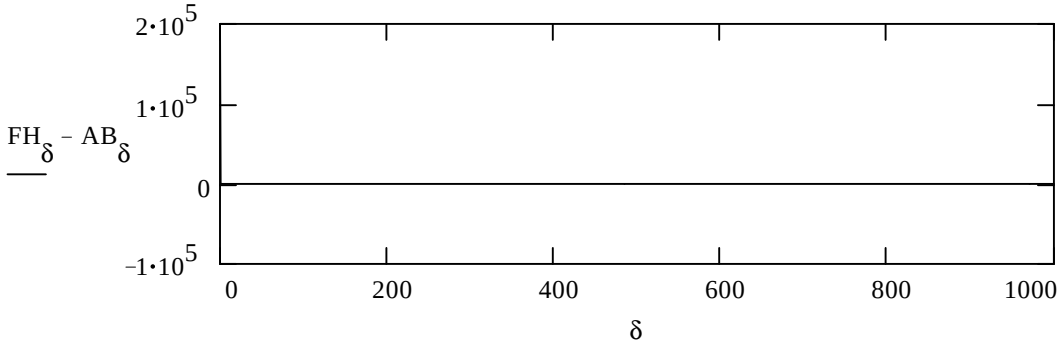

$CH := AC$

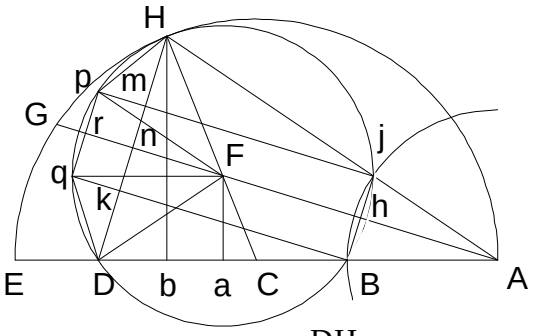
$bg_\delta := \frac{Af_\delta \cdot Hb_\delta}{Gf_\delta}$ $Cb_\delta := Ab_\delta - AC$ $Cg_\delta := bg_\delta - Cb_\delta$ $CF_\delta := \frac{CH \cdot AC}{Cg_\delta}$

$FH_\delta := CH - CF_\delta$ $Ca_\delta := \frac{Cb_\delta \cdot CF_\delta}{CH}$ $Fa_\delta := \sqrt{(CF_\delta)^2 - (Ca_\delta)^2}$ $DF_\delta := FH_\delta$

$Da_\delta := \sqrt{(DF_\delta)^2 - (Fa_\delta)^2}$ $Aa_\delta := AC + Ca_\delta$ $Ba_\delta := Da_\delta$ $AB_\delta := Aa_\delta - Ba_\delta$

By what is known about trisection, AB must equal FH.




$AD_\delta := AB_\delta + (2 \cdot Da_\delta)$ $Db_\delta := AD_\delta - Ab_\delta$

$DH_\delta := \sqrt{(Hb_\delta)^2 + (Db_\delta)^2}$ $Bj_\delta := \frac{DH_\delta \cdot AB_\delta}{AD_\delta}$

$Bh_\delta := \frac{Bj_\delta}{2}$ $Ah_\delta := \sqrt{(AB_\delta)^2 - (Bh_\delta)^2}$

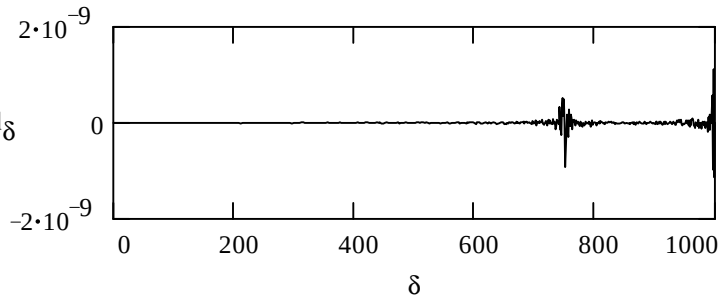
$Fr_\delta := Ah_\delta$ $Hn_\delta := \frac{DH_\delta}{2}$ $pq_\delta := Bj_\delta$ $mk_\delta := Bj_\delta$

$Fn_\delta := \text{if}\left[AD_\delta > AC, \sqrt{(FH_\delta)^2 - (Hn_\delta)^2}, -\sqrt{(FH_\delta)^2 - (Hn_\delta)^2}\right]$ $nr_\delta := Fr_\delta - Fn_\delta$

$Hm_\delta := \frac{DH_\delta - mk_\delta}{2}$ $mp_\delta := nr_\delta$ $Hp_\delta := \sqrt{(mp_\delta)^2 + (Hm_\delta)^2}$ $HE_\delta := \sqrt{(Hb_\delta)^2 + (Eb_\delta)^2}$

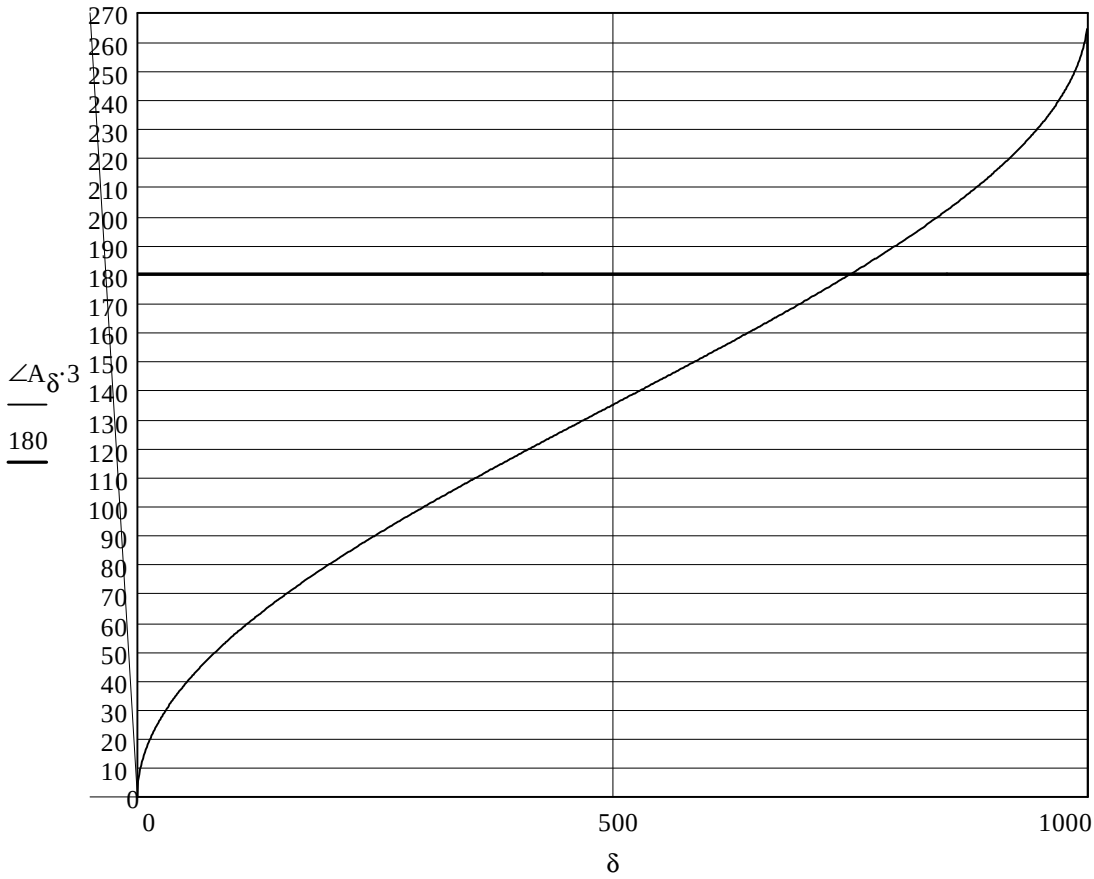
.

Does Hp = pq?

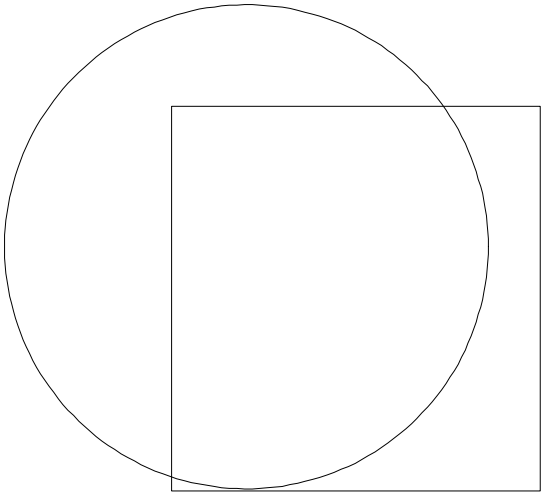



$\angle A_\delta := \text{asin}\left(\frac{Bh_\delta}{AB_\delta}\right) \cdot \frac{2}{\text{deg}}$

Plug resolution in here $\Delta \equiv 1000$ $\angle A_{\Delta - 1} \cdot 3 = 264.5635$ $View \equiv |0|$




One can safely say that the limit of the Archamedian paper trisector is 270 degrees.



# To Square A Circle

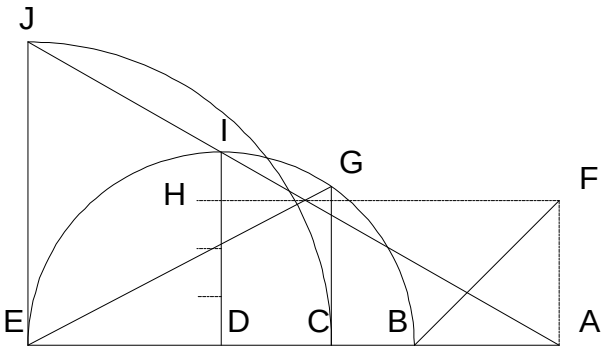
11_12_93.MCD

Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?

Using the approximation, $\pi = 22/7$, square the circle off the base of a right triangle.

## Process Summary

The most primitive formula for Area is A = 1/2bh. We will not use its transformation to $\pi r^2$.




Special Value for BE, try 4, then look at the corresponding value for EJ.

$BE := 4 \quad BD := \frac{BE}{2} \quad DE := BD$

$DI := BD \quad DH := \frac{3}{4} \cdot DI$

$AB := DH \quad AD := AB + BD$

$AE := AB + BE \quad EJ := \frac{DI \cdot AE}{AD}$

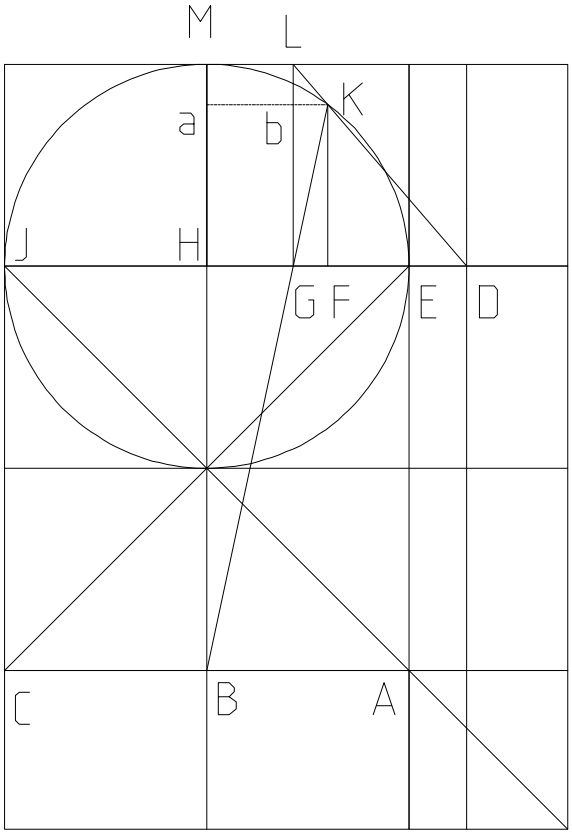
$CE := EJ \quad BC := BE - CE$

$CG := \sqrt{BC \cdot CE} \quad EG := \sqrt{CE^2 + CG^2} \quad \pi\_A := \frac{EG^2}{BD^2}$

$\pi\_A = 3.14285714 \qquad EJ = 3.14285714$

$\frac{22}{7} = 3.14285714 \qquad \pi\_A - \frac{22}{7} = 0$

Basically we have taken one half of the circle for 1/2b and crudely projected the arc EI for the height.




11_18_93.MCD

For any point K, construct GL parallel to HM. LK then projects to the point from which the figure forms the cubic.

$EJ := 100 \quad \Delta := 1000 \quad \delta := 0 .. \Delta - 1$

$EH := \frac{EJ}{2} \quad EF_\delta := \frac{EH}{\Delta} \cdot \delta \quad FJ_\delta := EJ - EF_\delta \quad BH := EJ$

$FK_\delta := \sqrt{EF_\delta \cdot FJ_\delta} \quad FH_\delta := EH - EF_\delta \quad Ka_\delta := FH_\delta \quad Ha_\delta := FK_\delta$

$Ba_\delta := BH + Ha_\delta \quad GH_\delta := \frac{Ka_\delta \cdot BH}{Ba_\delta} \quad FG_\delta := FH_\delta - GH_\delta$

$Kb_\delta := FG_\delta \quad GL := EH \quad Gb_\delta := FK_\delta$

$Lb_\delta := GL - Gb_\delta \quad DG_\delta := \text{if}\left(GH_\delta, \frac{Kb_\delta \cdot GL}{Lb_\delta}, 0\right) \quad DF_\delta := DG_\delta - FG_\delta \quad DE_\delta := DG_\delta - \left(FG_\delta + EF_\delta\right)$

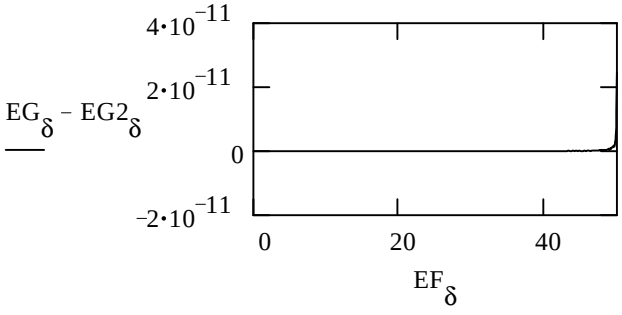
$DJ_\delta := EJ + DE_\delta \quad EG_\delta := EF_\delta + FG_\delta$

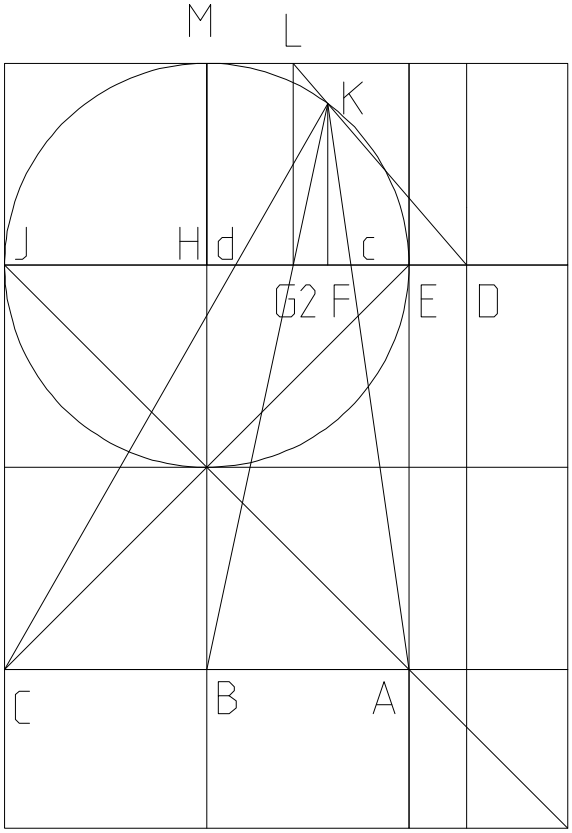
$Dc_\delta := \text{if}\left[GH_\delta, \left[\left(DE_\delta\right)^2 \cdot DJ_\delta\right]^{\frac{1}{3}}, 0\right]$

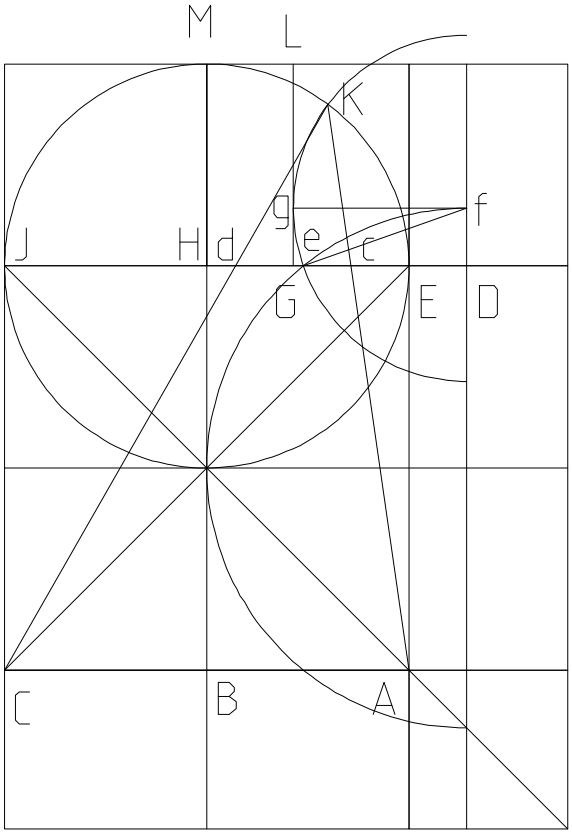
$Dd_\delta := \text{if}\left[GH_\delta, \left[DE_\delta \cdot \left(DJ_\delta\right)^2\right]^{\frac{1}{3}}, 0\right]$

$cd_\delta := Dd_\delta - Dc_\delta \quad G2c_\delta := \frac{cd_\delta}{2}$

$Ec_\delta := Dc_\delta - DE_\delta \quad EG2_\delta := Ec_\delta + G2c_\delta$



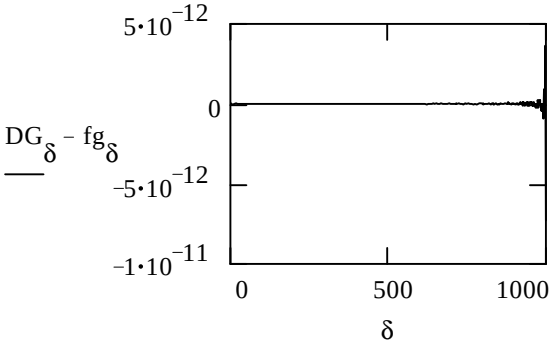



I should say that fg is equal to DG.

$Gc_\delta := G2c_\delta$

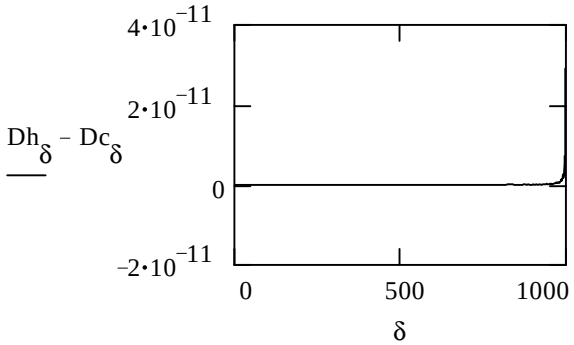
$Df_\delta := Gc_\delta De_\delta := \sqrt{DE_\delta \cdot DJ_\delta}$

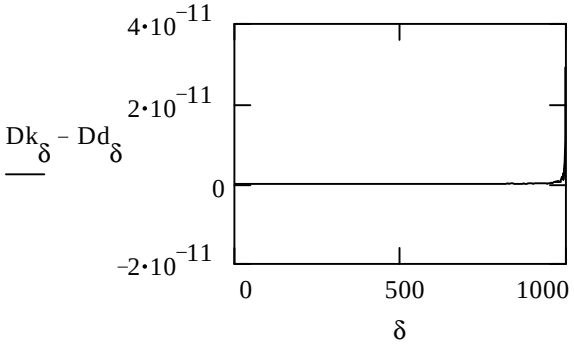
$ef_\delta := \sqrt{\left(Df_\delta\right)^2 + \left(Def_\delta\right)^2} := ef_\delta$

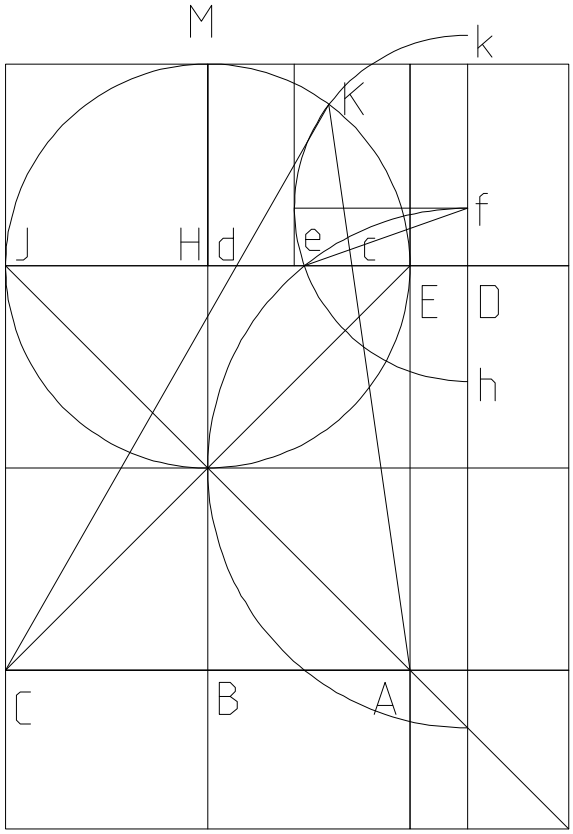


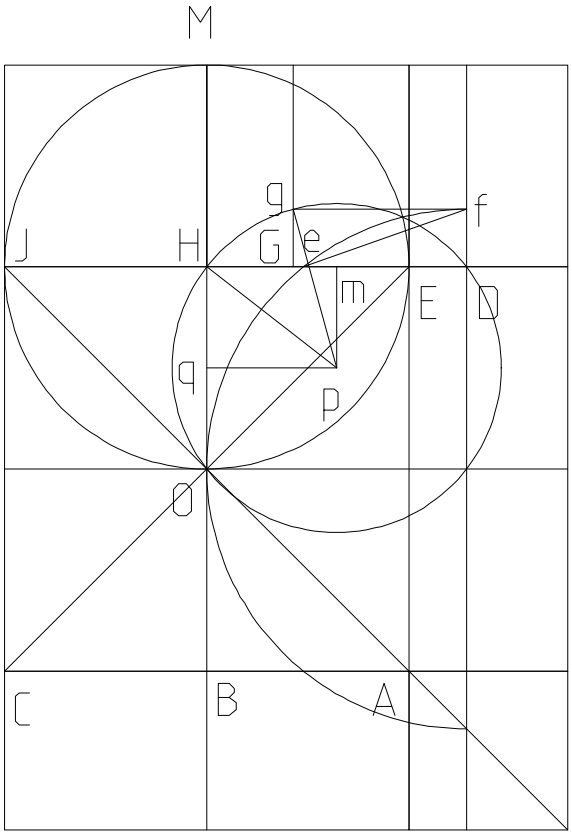

I should say that Dh is equal to Dc and that Dk is equal to Dd.

$Dh_\delta := ef_\delta - Df_\delta$ $Dk_\delta := ef_\delta + Df_\delta$





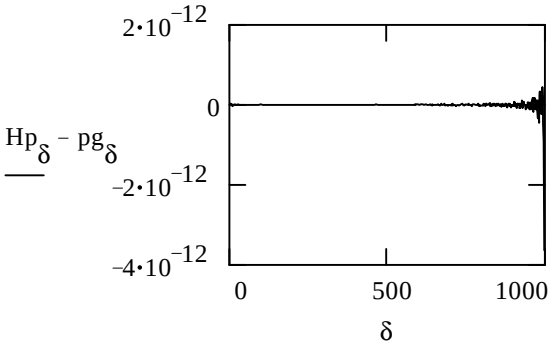

I should say that the circle pDH passes through the point g.

$DH_\delta := DE_\delta + EH \quad Hm_\delta := \frac{DH_\delta}{2} \quad HO := EH$

$Hq := \frac{HO}{2} \quad Hp_\delta := \sqrt{Hq^2 + (Hm_\delta)^2}$

$Gg_\delta := Df_\delta \quad mp := Hq \quad Gm_\delta := Hm_\delta - GH_\delta$

$pg_\delta := \sqrt{(Gg_\delta + mp)^2 + (Gm_\delta)^2}$




$HJ := EH \quad HP := \frac{HJ}{3}$

$JP := \frac{2 \cdot HJ}{3} \quad EP := EH + HP \quad PQ := \sqrt{EP \cdot JP}$

$JQ := \sqrt{PQ^2 + JP^2} \quad JR := JQ \quad HR := \sqrt{JR^2 - HJ^2}$

$RS := JQ \quad GS_\delta := \sqrt{RS^2 - (GH_\delta)^2} - HR$

$ES_\delta := \sqrt{(EG_\delta)^2 + (GS_\delta)^2} \quad GJ_\delta := HJ + GH_\delta$

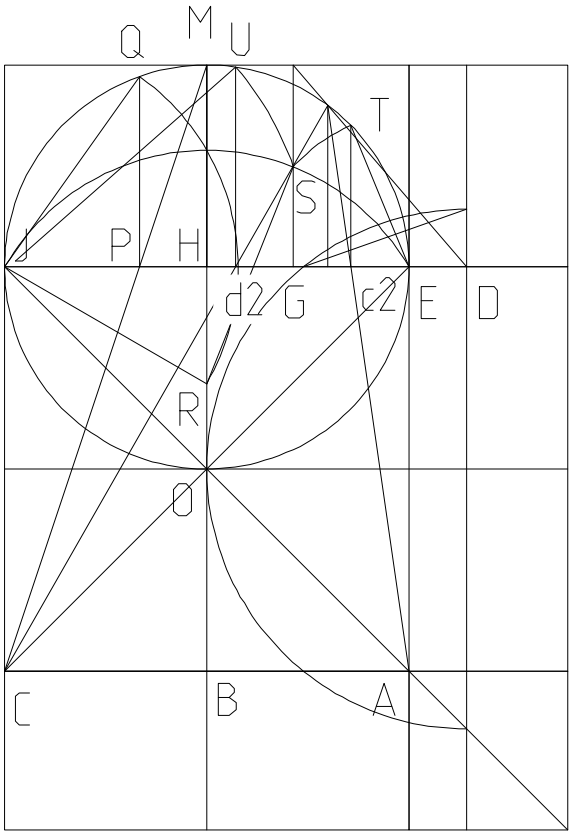
$JS_\delta := \sqrt{(GJ_\delta)^2 + (GS_\delta)^2} \quad ET_\delta := ES_\delta \quad JU_\delta := JS_\delta$

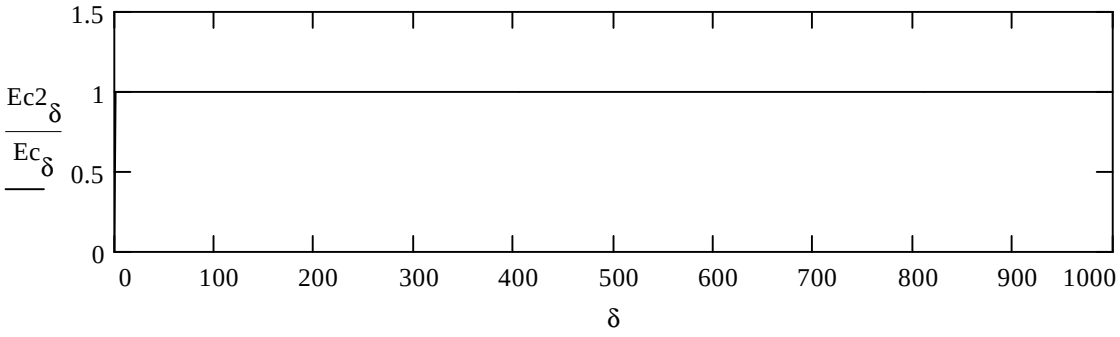
$Ed2_\delta := EJ - \frac{(JU_\delta)^2}{EJ} \quad Ec2_\delta := \frac{(ET_\delta)^2}{EJ}$

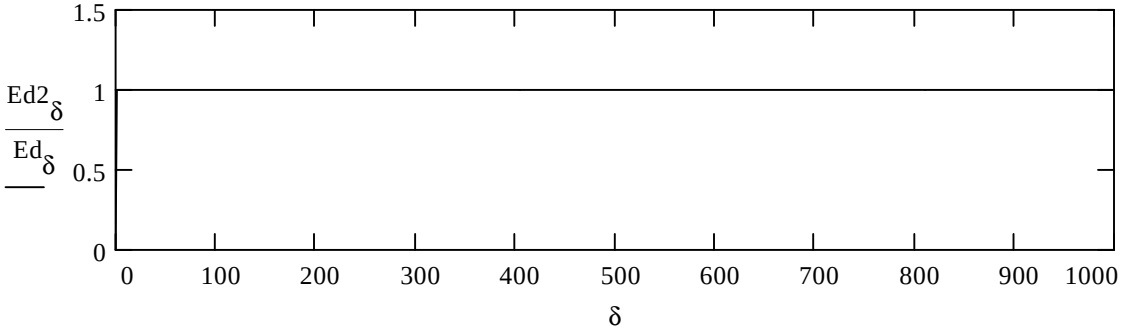
$Ec_\delta := Dc_\delta - DE_\delta \quad Ed_\delta := Dd_\delta - DE_\delta$

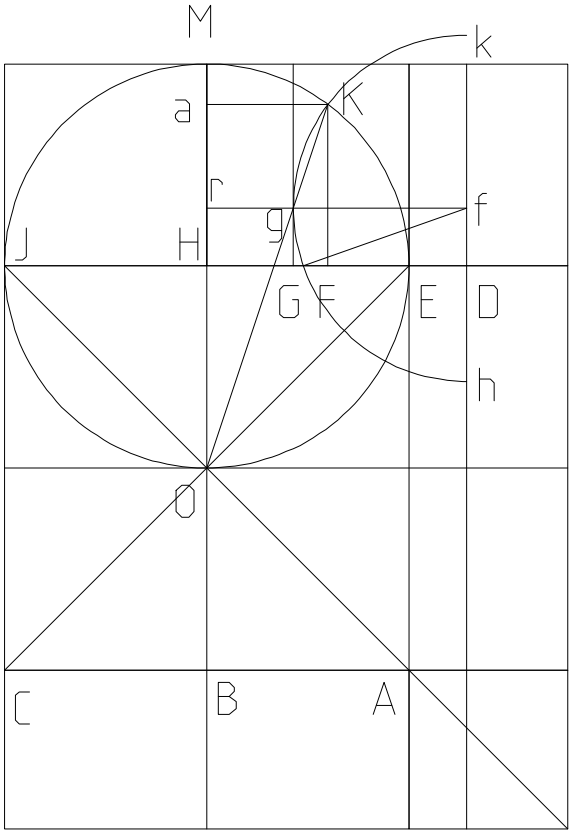
$Ed_{\Delta - 1} = 66.633327772$

$Ed2_{\Delta - 1} = 66.633327772$





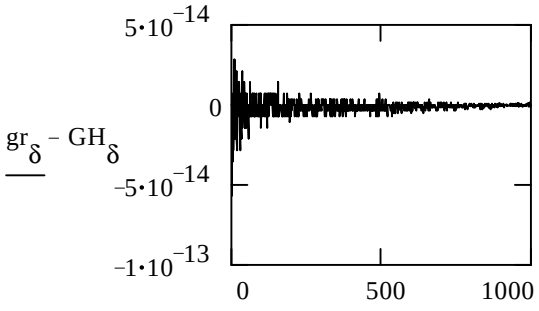



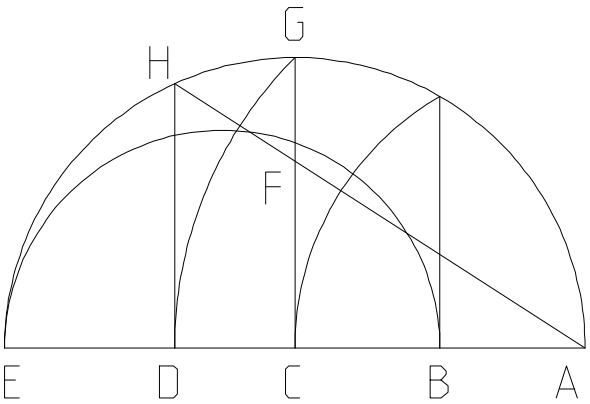




OK passes through g.

$Oa_\delta := HO + Ha_\delta \quad Hr_\delta := Gg_\delta \quad Or_\delta := HO + Hr_\delta$

$$gr_\delta := \frac{Ka_\delta \cdot Or_\delta}{Oa_\delta}$$






Using iteration to perform a geometric convergent series for cube root abstraction.

$A := if(X<Y, if(X<Z, X, Z), if(Y<Z, Y, Z))$

$B := if(X>Y, if(X>Z, X, Z), if(Y>Z, Y, Z))$

$W := X + Y + Z \quad C := W - (A + B)$

$AB := \sqrt{A \cdot C} \quad BE := B - AB \quad AE := AB + BE$

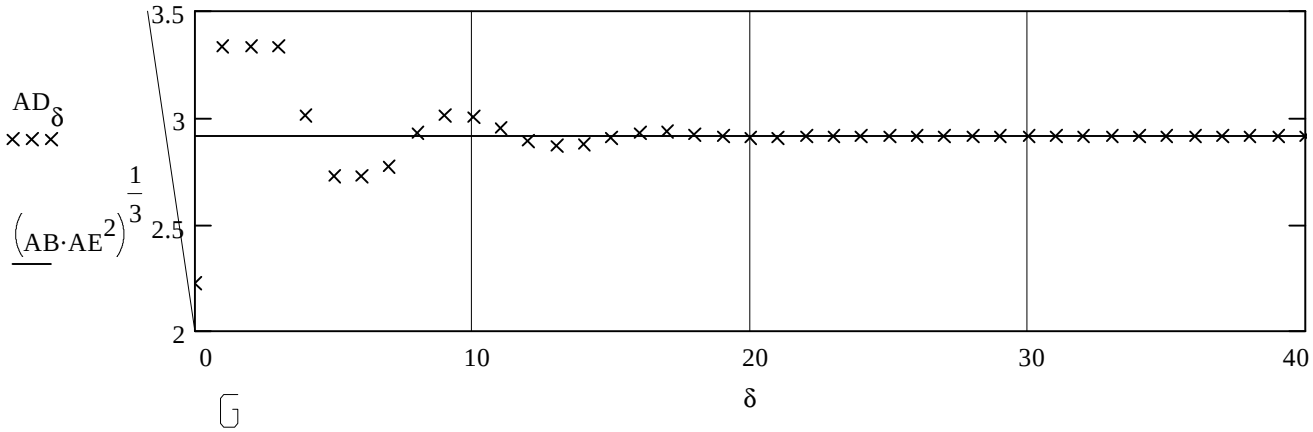
$\delta := 0..\Delta \quad AC := \sqrt{AE \cdot AB} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

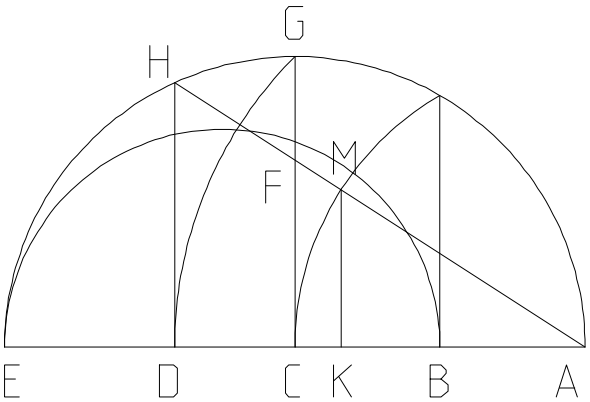
$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AC \\ AE - AC \\ CG \\ CG \\ AG \end{bmatrix} \qquad \begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix}$$

$\Delta \equiv 40$

$X \equiv 5 \quad Y \equiv .9900 \quad Z \equiv Y$

$A = 0.99 \quad C = 0.99 \quad B = 5$



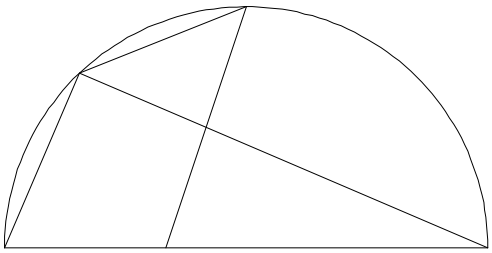




$AH_\Delta := \sqrt{(AD_\Delta)^2 + (DH_\Delta)^2} \quad AM := AC \quad AK := \dfrac{AD_\Delta \cdot AM}{AH_\Delta}$

$AK = 1.6986 \quad AK^3 = 4.9008 \quad X \cdot Y \cdot Z = 4.9005$

$(AB \cdot AE^2)^{\frac{1}{3}} = 2.9142$

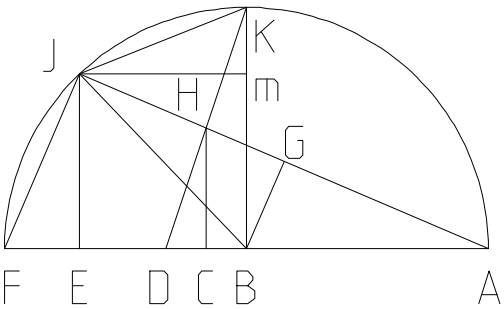
$AD_\Delta = 2.9143$



POR Series IV Figure 11_24_C3.MCD

Generalize the work of 07_25_93 to include the variable base ratio. This particular modification of the file has seen the use of the symbolic processor to figure out the last ratio that I have been unable to putz. The two ratio's found are FJ/JK and AH/AJ. I put the first in the graphic at left, the other would not fit.

$$\frac{FJ_\delta}{JK_\delta} = \frac{\sqrt{2}\cdot BR\cdot \delta}{(BS-\delta)\cdot(BR-1)\cdot 2}$$




$AF := 1 \quad BR := BASE\_RATIO$

$BS := BISECTOR\_SEGMENTS$

$DF := \frac{AF}{BR} \quad AD := AF - DF \quad \delta := 1..BS$

$AB := \frac{AF}{2} \quad BF := AB \quad BD := BF - DF$

$BK := AB \quad DK := \sqrt{BD^2 + BK^2}$

$DH_\delta := \frac{DK}{BS}\cdot\delta \quad CD_\delta := \frac{BD}{BS}\cdot\delta \quad CH_\delta := \sqrt{(DH_\delta)^2 - (CD_\delta)^2} \quad BC_\delta := BD - CD_\delta$

$AC_\delta := AB + BC_\delta \quad AH_\delta := \sqrt{(AC_\delta)^2 + (CH_\delta)^2} \quad AG_\delta := \frac{AC_\delta\cdot AB}{AH_\delta} \quad AJ_\delta := AG_\delta\cdot 2$

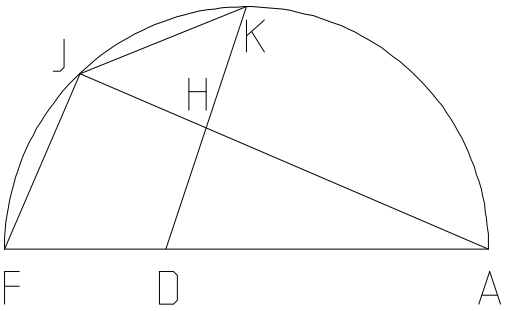
$AE_\delta := \frac{(AJ_\delta)^2}{AF} \quad EF_\delta := AF - AE_\delta \quad EJ_\delta := \sqrt{EF_\delta\cdot AE_\delta} \quad FJ_\delta := \sqrt{(EF_\delta)^2 + (EJ_\delta)^2}$

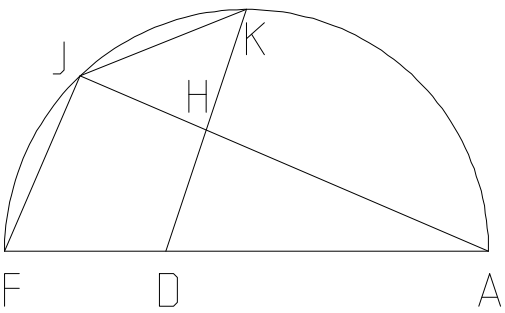
$Bm_\delta := EJ_\delta \quad BE_\delta := AE_\delta - AB$

$Jm_\delta := BE_\delta$

$Km_\delta := BK - Bm_\delta$

$JK_\delta := \sqrt{(Jm_\delta)^2 + (Km_\delta)^2}$



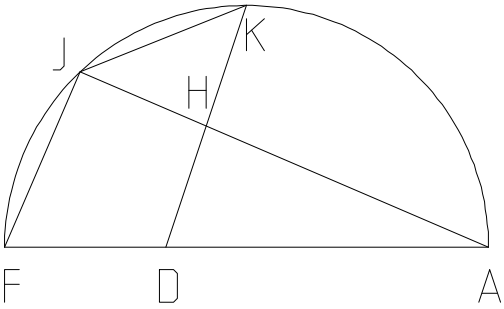



$$\text{if}\left[BS - \delta, \frac{\frac{\delta\cdot\sqrt{2}}{BS - \delta}}{\frac{FJ_\delta}{JK_\delta}}, 0\right] \qquad \frac{BR - 2}{BR} + 1 = 1.333$$

| |
|---|
| 1.333 |
| 1.333 |
| 1.333 |
| 1.333 |
| 0 |

$$\text{if}\left(JK_\delta, \frac{FJ_\delta}{JK_\delta}, 0\right) \text{if}\left(BS - \delta, \frac{\delta\cdot\sqrt{2}}{BS - \delta}, 0\right) \qquad \text{if}\left[BS - \delta, \frac{\sqrt{2}\cdot BR\cdot\delta}{(BS - \delta)\cdot(BR - 1)\cdot 2}, 0\right]$$

| | | |
|---|---|---|
| 0.265 | 0.354 | 0.265 |
| 0.707 | 0.943 | 0.707 |
| 1.591 | 2.121 | 1.591 |
| 4.243 | 5.657 | 4.243 |
| $6.369\cdot 10^{15}$ | 0 | 0 |




I broke down and used the symbolic processor.
Two versions of the formula are given. It is apparent why I could not get this one myself, I'd never 'ave thunk it. From this point on I will use the symbolic processor to find formula's more often.

$$\frac{\left(1-\frac{1}{BR}-\frac{\frac{1}{2}-\frac{1}{BR}}{BS}\cdot\delta\right)^2+\frac{\left(\frac{1}{2}-\frac{1}{BR}\right)^2+\frac{1}{4}}{BS^2}\cdot\delta^2-\frac{\left(\frac{1}{2}-\frac{1}{BR}\right)^2}{BS^2}\cdot\delta^2}{1-\frac{1}{BR}-\frac{\frac{1}{2}-\frac{1}{BR}}{BS}\cdot\delta}$$

$\frac{AH_\delta}{AJ_\delta}$

| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |

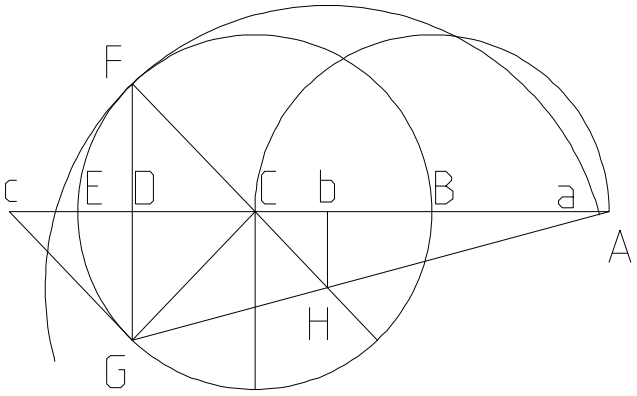
| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |

$$\frac{-2\cdot BR^2\cdot BS^2+4\cdot BR\cdot BS^2+2\cdot BR^2\cdot BS\cdot\delta\ \ldots}{+-6\cdot BR\cdot BS\cdot\delta-2\cdot BS^2+4\cdot BS\cdot\delta-\delta^2\cdot BR^2+2\cdot\delta^2\cdot BR-2\cdot\delta^2}{BR\cdot(BS\cdot(-2\cdot BR\cdot BS+2\cdot BS+\delta\cdot BR-2\cdot\delta))}$$

BASE_RATIO ≡ 3

BISECTOR_SEGMENTS ≡ 5

| |
|---|
| 0.649 |
| 0.667 |
| 0.725 |
| 0.833 |
| 1 |




11_25_93.MCD

Use iteration to trisect any given angle.

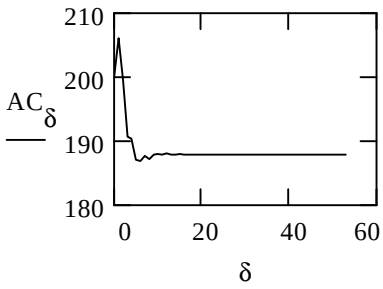
$BE := 200 \quad BC := \frac{BE}{2} \quad CE := BC$

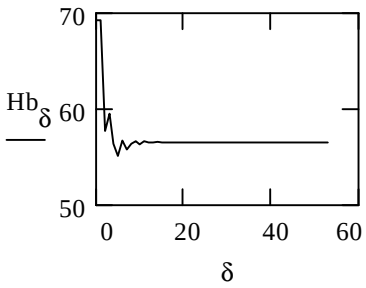
$CD := if(CDI > 0, if(CDI < BC, CDI, 100), 0)$

$CF := CE \quad BD := BC + CD \quad DF := \sqrt{CF^2 - CD^2}$

$DG := DF \quad Gc := CF \quad Dc := CD \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} Ac_0 \\ CH_0 \\ FH_0 \\ AC_0 \\ bc_0 \\ Hb_0 \end{bmatrix} := \begin{bmatrix} BE + Dc + CD \\ \frac{Gc \cdot BE}{BE + Dc} \\ CF + \frac{Gc \cdot BE}{BE + Dc} \\ BE \\ \frac{Dc \cdot BE}{BE + Dc} \\ \sqrt{\left(\frac{Gc \cdot BE}{BE + Dc}\right)^2 - \left(\frac{Dc \cdot BE}{BE + Dc}\right)^2} \end{bmatrix} \begin{bmatrix} AC_{\delta+1} \\ Ac_{\delta+1} \\ CH_{\delta+1} \\ FH_{\delta+1} \\ bc_{\delta+1} \\ Hb_{\delta+1} \end{bmatrix} := \begin{bmatrix} \sqrt{(FH_\delta)^2 - (Hb_\delta)^2} + bc_\delta \\ AC_\delta + Dc + CD \\ \frac{CF \cdot AC_\delta}{Ac_\delta} \\ CF + CH_\delta \\ \frac{Dc \cdot AC_\delta}{Ac_\delta} \\ \sqrt{(CH_\delta)^2 - (bc_\delta)^2} \end{bmatrix}$$



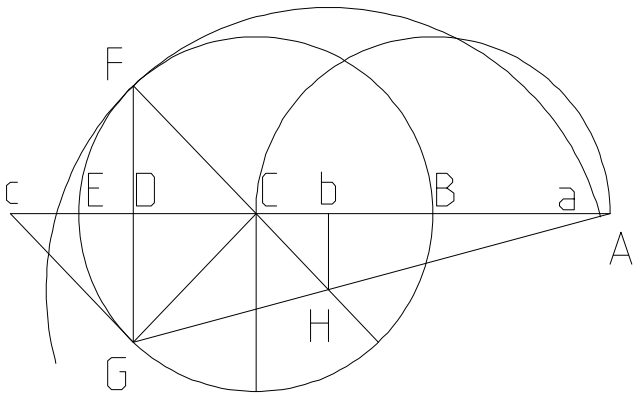

$$\angle KCD := \frac{\operatorname{asin}\left(\frac{DF}{CF}\right)}{deg}$$

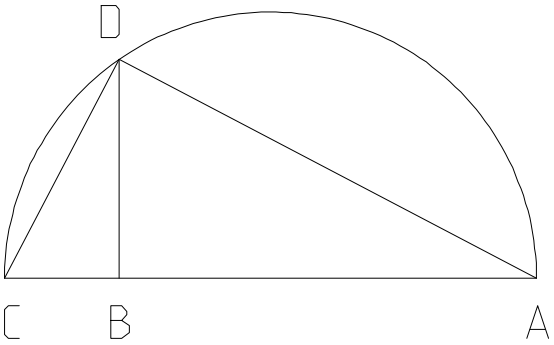
$$\angle BAI := \frac{\operatorname{asin}\left(\frac{Hb_\Delta}{FH_\Delta}\right)}{deg}$$

$\Delta \equiv 53$ $\angle KCD = 60$ $\frac{\angle KCD}{\angle BAI} = 3$

$CDI \equiv 50$ $\angle BAI = 20$

Range: >0 and <100

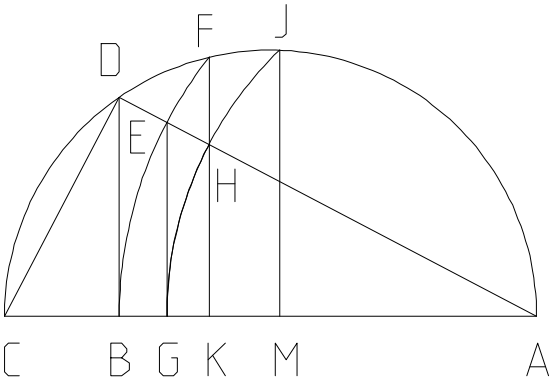





11_29_93.MCD

$AC := 10 \quad \delta := 1 .. AC \cdot p \quad AD_\delta := \frac{\delta}{p}$

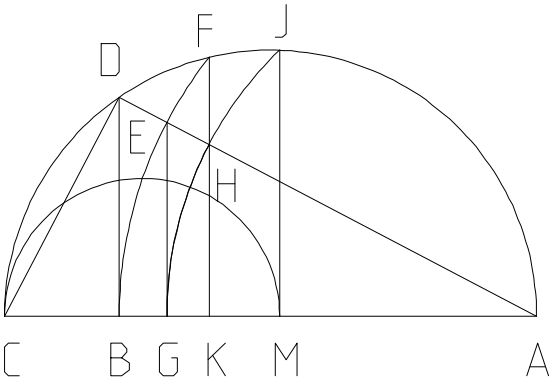
$AB_\delta := \frac{(AD_\delta)^2}{AC}$




$AE_\delta := AB_\delta \quad AG_\delta := \frac{(AB_\delta)^2}{AD_\delta}$

$AK_\delta := \frac{(AG_\delta)^2}{AE_\delta} \quad AJ_\delta := AG_\delta$

$AM_\delta := \frac{(AJ_\delta)^2}{AC} \quad CM_\delta := AC - AM_\delta$

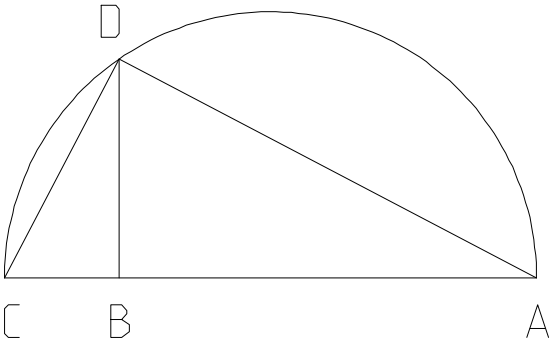



$$\sum_\delta \left[ \left[ (AM_\delta)^2 \cdot AC \right]^{\frac{1}{3}} - AK_\delta \right] = -3.539 \cdot 10^{-15}$$

$$\sum_\delta \left[ \left( AM_\delta \cdot AC^2 \right)^{\frac{1}{3}} - AB_\delta \right] = -1.471 \cdot 10^{-15}$$

$A := 0 \quad \Delta := A .. A + 10 \quad p \equiv 1$

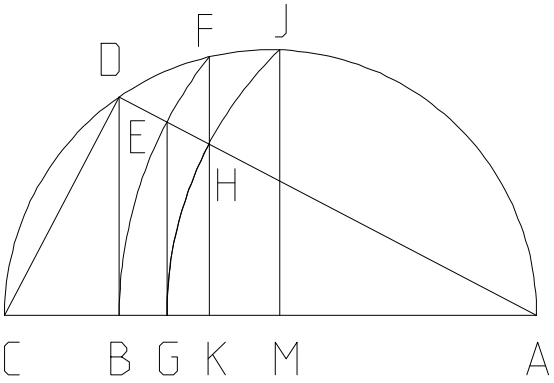
| $AK_\Delta$ | $(AM_\Delta)^2 \cdot AC$ | $AM_\Delta \cdot AC^2$ | $AB_\Delta$ | $AG_\Delta$ | $\frac{AB_\Delta}{AG_\Delta}$ | $CM_\Delta$ | $AM_\Delta$ | $\frac{AM_\Delta}{if(CM_\Delta, CM_\Delta, 1)}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.001 | $1 \cdot 10^{-9}$ | 0.001 | 0.1 | 0.01 | 10 | 10 | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-6}$ |
| 0.016 | $4.096 \cdot 10^{-6}$ | 0.064 | 0.4 | 0.08 | 5 | 9.999 | $6.4 \cdot 10^{-4}$ | $6.4 \cdot 10^{-5}$ |
| 0.081 | $5.314 \cdot 10^{-4}$ | 0.729 | 0.9 | 0.27 | 3.333 | 9.993 | 0.007 | $7.295 \cdot 10^{-4}$ |
| 0.256 | 0.017 | 4.096 | 1.6 | 0.64 | 2.5 | 9.959 | 0.041 | 0.004 |
| 0.625 | 0.244 | 15.625 | 2.5 | 1.25 | 2 | 9.844 | 0.156 | 0.016 |
| 1.296 | 2.177 | 46.656 | 3.6 | 2.16 | 1.667 | 9.533 | 0.467 | 0.049 |
| 2.401 | 13.841 | 117.649 | 4.9 | 3.43 | 1.429 | 8.824 | 1.176 | 0.133 |
| 4.096 | 68.719 | 262.144 | 6.4 | 5.12 | 1.25 | 7.379 | 2.621 | 0.355 |
| 6.561 | 282.43 | 531.441 | 8.1 | 7.29 | 1.111 | 4.686 | 5.314 | 1.134 |
| 10 | $1 \cdot 10^3$ | $1 \cdot 10^3$ | 10 | 10 | 1 | 0 | 10 | 10 |




$$AC := 10 \quad d := 1 .. AC \cdot p \quad AB_d := AC - \frac{d}{p}$$

$$BC_d := AC - AB_d \quad BD_d := \sqrt{AB_d \cdot BC_d}$$

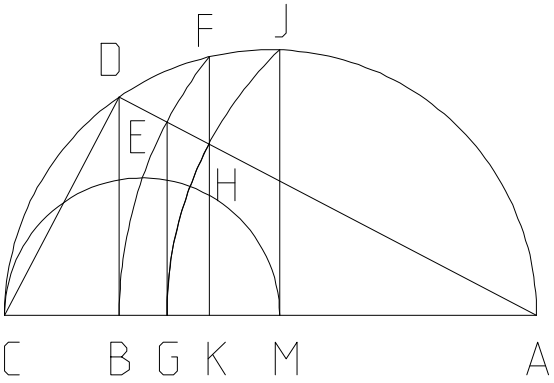
$$AD_d := \sqrt{\left(AB_d\right)^2 + \left(BD_d\right)^2}$$




$$AE_d := AB_d \quad AG_d := \frac{\left(AB_d\right)^2}{AD_d}$$

$$AK_d := \frac{\left(AG_d\right)^2}{AE_d} \quad AJ_d := AG_d$$

$$AM_d := \frac{\left(AJ_d\right)^2}{AC} \quad CM_d := AC - AM_d$$



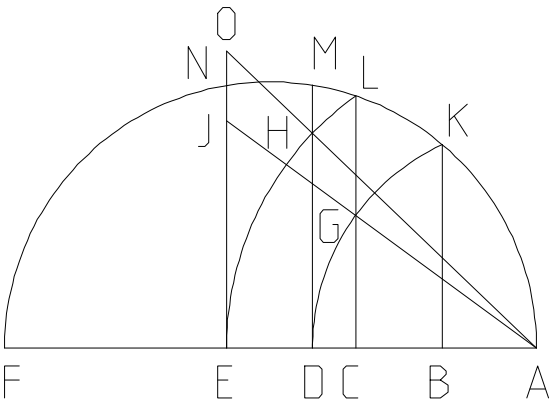

$$\sum_d \left[ \left[ \left(AM_d\right)^2 \cdot AC \right]^{\frac{1}{3}} - AK_d \right] = -3.126 \cdot 10^{-13}$$

$$\sum_d \left[ \left(AM_d \cdot AC^2\right)^{\frac{1}{3}} - AB_d \right] = -5.083 \cdot 10^{-13}$$

$A := 50 \quad \Delta := A .. A + 10 \quad p \equiv 100$

| $AK_\Delta$ | $\left(AM_\Delta\right)^2 \cdot AC$ | $AM_\Delta \cdot AC^2$ | $AB_\Delta$ | $AG_\Delta$ | $\frac{AB_\Delta}{AG_\Delta}$ | $CM_\Delta$ | $AM_\Delta$ | $\frac{AM_\Delta}{CM_\Delta}$ |
|---|---|---|---|---|---|---|---|---|
| 9.025 | 735.092 | 857.375 | 9.5 | 9.259 | 1.026 | 1.426 | 8.574 | 6.011 |
| 9.006 | 730.461 | 854.67 | 9.49 | 9.245 | 1.027 | 1.453 | 8.547 | 5.881 |
| 8.987 | 725.855 | 851.971 | 9.48 | 9.23 | 1.027 | 1.48 | 8.52 | 5.755 |
| 8.968 | 721.273 | 849.278 | 9.47 | 9.216 | 1.028 | 1.507 | 8.493 | 5.635 |
| 8.949 | 716.716 | 846.591 | 9.46 | 9.201 | 1.028 | 1.534 | 8.466 | 5.519 |
| 8.93 | 712.182 | 843.909 | 9.45 | 9.186 | 1.029 | 1.561 | 8.439 | 5.407 |
| 8.911 | 707.672 | 841.232 | 9.44 | 9.172 | 1.029 | 1.588 | 8.412 | 5.299 |
| 8.892 | 703.186 | 838.562 | 9.43 | 9.157 | 1.03 | 1.614 | 8.386 | 5.194 |
| 8.874 | 698.724 | 835.897 | 9.42 | 9.143 | 1.03 | 1.641 | 8.359 | 5.094 |
| 8.855 | 694.285 | 833.238 | 9.41 | 9.128 | 1.031 | 1.668 | 8.332 | 4.997 |
| 8.836 | 689.87 | 830.584 | 9.4 | 9.114 | 1.031 | 1.694 | 8.306 | 4.903 |

11_29_3A.MCD



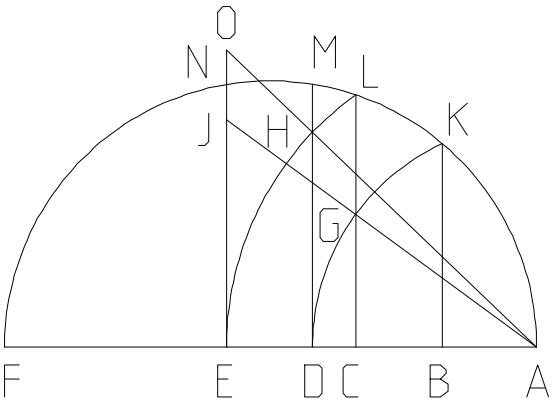

The ray that performs the cube root abstraction must pass through the window OJ, take a look at it.

$AF := 10 \quad \delta := 0 .. 1000 \quad AB_\delta := \frac{\delta}{100}$

$BF_\delta := AF - AB_\delta \quad AD_\delta := \sqrt{AB_\delta \cdot AF}$

$AK_\delta := AD_\delta \quad DF_\delta := AF - AD_\delta$

$DM_\delta := \sqrt{AD_\delta \cdot DF_\delta} \quad BK_\delta := \sqrt{AB_\delta \cdot BF_\delta}$

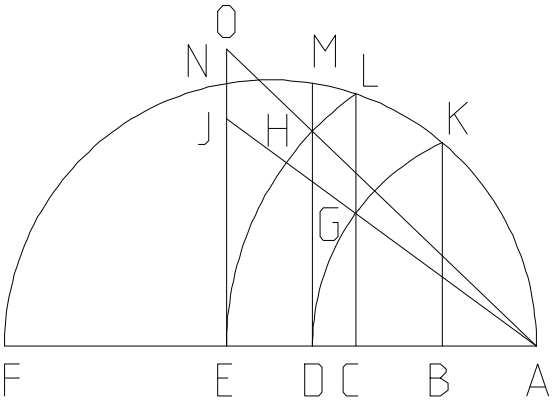



$BD_\delta := AD_\delta - AB_\delta \quad CD_\delta := \frac{BD_\delta}{3}$

$AC_\delta := AD_\delta - CD_\delta \quad CF_\delta := AF - AC_\delta$

$CL_\delta := \sqrt{AC_\delta \cdot CF_\delta}$

$AL_\delta := \sqrt{(AC_\delta)^2 + (CL_\delta)^2} \quad AE_\delta := AL_\delta$



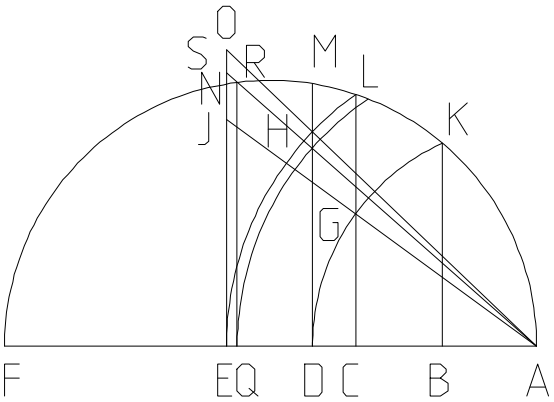

$AH_\delta := AL_\delta \quad AG_\delta := AK_\delta$

$CG_\delta := \sqrt{(AG_\delta)^2 - (AC_\delta)^2}$

$DH_\delta := \sqrt{(AH_\delta)^2 - (AD_\delta)^2}$

$EO_\delta := \frac{DH_\delta \cdot AE_\delta}{AD_\delta} \quad EJ_\delta := \frac{CG_\delta \cdot AE_\delta}{AC_\delta}$

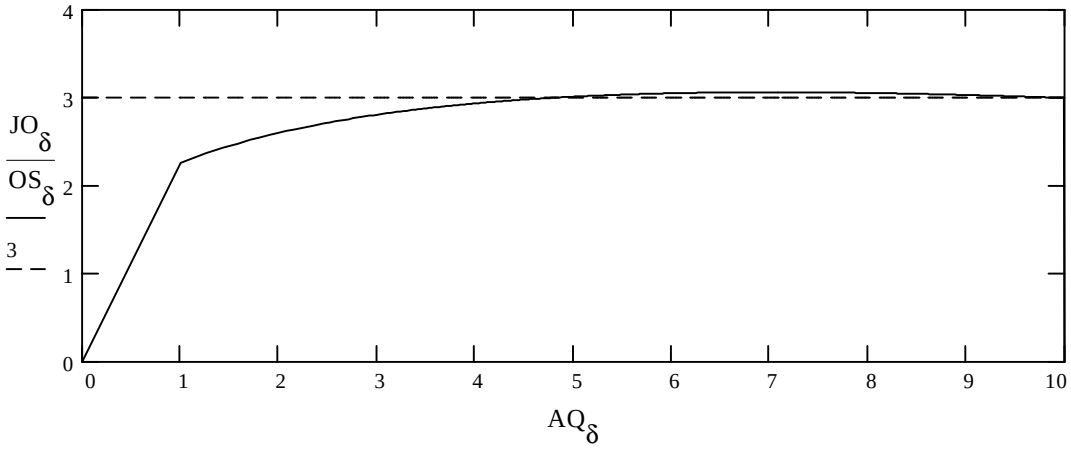
$JO_\delta := EO_\delta - EJ_\delta$

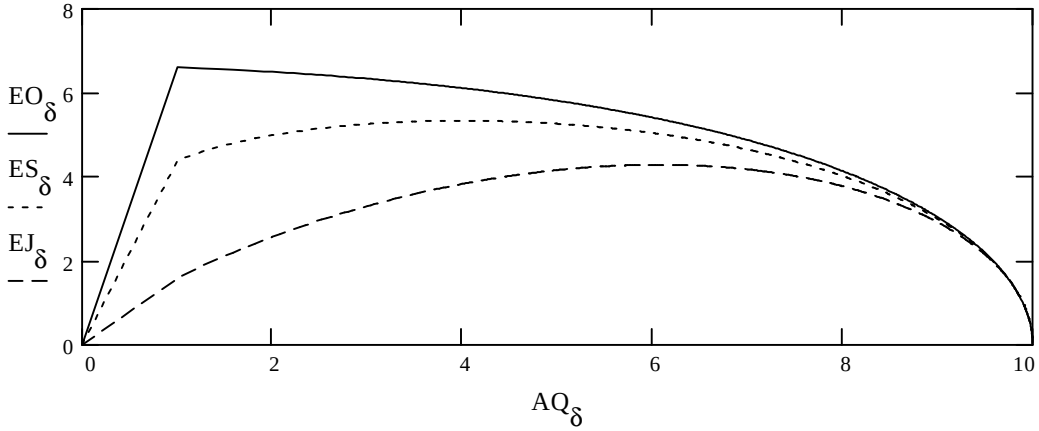


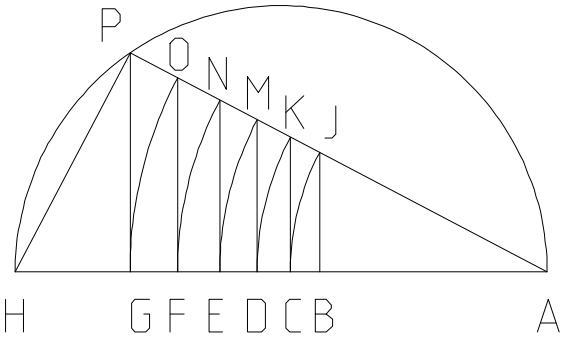

$$AQ_\delta := \left(AB_\delta \cdot AF^2\right)^{\frac{1}{3}} \qquad FQ_\delta := AF - AQ_\delta$$

$$QR_\delta := \sqrt{AQ_\delta \cdot FQ_\delta} \qquad ES_\delta := \frac{QR_\delta \cdot AE_\delta}{AQ_\delta}$$

$$OS_\delta := EO_\delta - ES_\delta$$






# 12_02_93.MCD POR Roots and Powers
## (Pyramid of Ratio Series V)

Is the progression noticed in 11_29_93 a continuous phenomenon?

$AH := 17 \qquad \delta := 1 .. AH \qquad AP_\delta := \frac{AH}{\delta}$

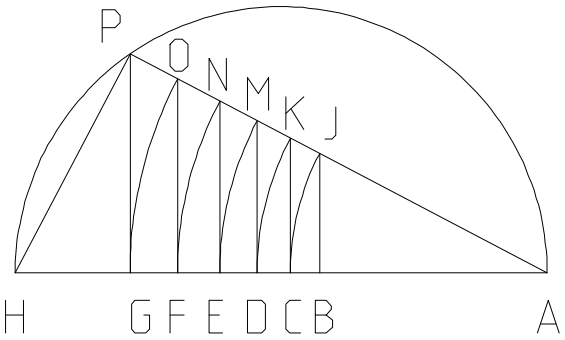
$AG_\delta := \frac{(AP_\delta)^2}{AH} \qquad AO_\delta := AG_\delta$

$AF_\delta := \frac{(AG_\delta)^2}{AP_\delta} \qquad AE_\delta := \frac{(AF_\delta)^2}{AO_\delta} \qquad AN_\delta := AF_\delta \qquad AD_\delta := \frac{(AE_\delta)^2}{AN_\delta} \qquad AM_\delta := AE_\delta$

$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta} \qquad AK_\delta := AD_\delta \qquad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta} \qquad A := 1 \qquad \Delta := A .. 5$

| $\frac{AH}{AP_\Delta}$ | $\left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}}$ | $\left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}}$ | $\left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}}$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |

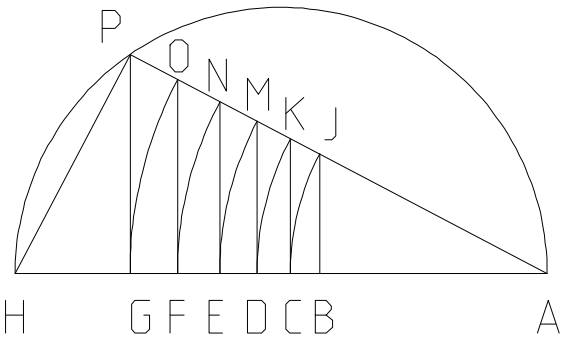
| $\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}}$ | $\left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}}$ | $\frac{AC_\Delta}{AB_\Delta}$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 |



$$\begin{bmatrix} \frac{AH}{1}, \frac{AH}{1^2}, \frac{AH}{1^3}, \frac{AH}{1^4}, \text{etc} \\ \frac{AH}{2}, \frac{AH}{2^2}, \frac{AH}{2^3}, \frac{AH}{2^4}, \text{etc} \\ \frac{AH}{3}, \frac{AH}{3^2}, \frac{AH}{3^3}, \frac{AH}{3^4}, \text{etc} \\ \frac{AH}{4}, \frac{AH}{4^2}, \frac{AH}{4^3}, \frac{AH}{4^4}, \text{etc} \\ \frac{AH}{5}, \frac{AH}{5^2}, \frac{AH}{5^3}, \frac{AH}{5^4}, \text{etc} \end{bmatrix}$$

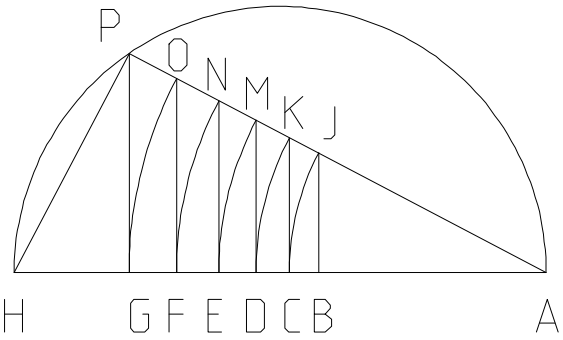
| $AP_\Delta$ | $AG_\Delta$ | $AF_\Delta$ | $AE_\Delta$ | $AD_\Delta$ | $AC_\Delta$ | $AB_\Delta$ |
|---|---|---|---|---|---|---|
| 17 | 17 | 17 | 17 | 17 | 17 | 17 |
| 8.5 | 4.25 | 2.125 | 1.063 | 0.531 | 0.266 | 0.133 |
| 5.667 | 1.889 | 0.63 | 0.21 | 0.07 | 0.023 | 0.008 |
| 4.25 | 1.063 | 0.266 | 0.066 | 0.017 | 0.004 | 0.001 |
| 3.4 | 0.68 | 0.136 | 0.027 | 0.005 | 0.001 | $2.176 \cdot 10^{-4}$ |

$$\frac{AH}{5^7} = 2.176 \cdot 10^{-4}$$




| $AP_\Delta$ | $AG_\Delta \cdot \frac{AH}{AP_\Delta}$ | $AF_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^2$ | $AE_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^3$ |
|---|---|---|---|
| 17 | 17 | 17 | 17 |
| 8.5 | 8.5 | 8.5 | 8.5 |
| 5.667 | 5.667 | 5.667 | 5.667 |
| 4.25 | 4.25 | 4.25 | 4.25 |
| 3.4 | 3.4 | 3.4 | 3.4 |

| $AD_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^4$ | $AC_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^5$ | $AB_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^6$ |
|---|---|---|
| 17 | 17 | 17 |
| 8.5 | 8.5 | 8.5 |
| 5.667 | 5.667 | 5.667 |
| 4.25 | 4.25 | 4.25 |
| 3.4 | 3.4 | 3.4 |




| $\frac{AH^2}{AP_\Delta}$ | $\left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}}$ | $\left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}}$ | $\left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}}$ | $\left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}}$ | $\left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}}$ |
|---|---|---|---|---|---|
| 17 | 17 | 17 | 17 | 17 | 17 |
| 34 | 34 | 34 | 34 | 34 | 34 |
| 51 | 51 | 51 | 51 | 51 | 51 |
| 68 | 68 | 68 | 68 | 68 | 68 |
| 85 | 85 | 85 | 85 | 85 | 85 |

| $\left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}}$ | $\left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH$ |
|---|---|
| 17 | 17 |
| 34 | 34 |
| 51 | 51 |
| 68 | 68 |
| 85 | 85 |

$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}}$ $\left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}}$ $\left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}}$ $\left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}}$ $\left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}}$ $\left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}}$ $\left(\frac{AH}{AP_\Delta}\right)^{3}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 64 | 64 | 64 | 64 | 64 | 64 | 64 |
| 125 | 125 | 125 | 125 | 125 | 125 | 125 |

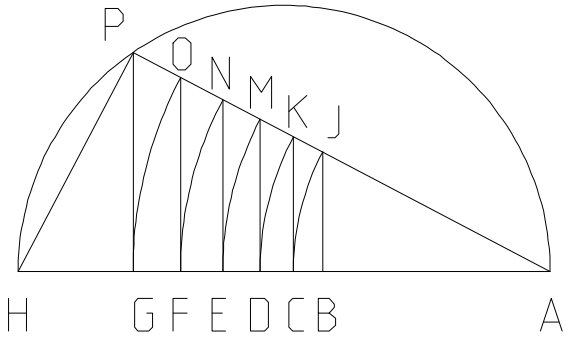
$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}}$ $\left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}}$ $\left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}}$ $\left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}}$ $\left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}}$ $\left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}}$ $\left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}}$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 128 | 128 | 128 | 128 | 128 | 128 | 128 |
| $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ |
| $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ |
| $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.812 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ |

$GH_\delta := AH - AG_\delta$ $GP_\delta := \sqrt{AG_\delta \cdot GH_\delta}$ $FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta}$ $EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta}$ $DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta}$

$CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta}$ $BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$

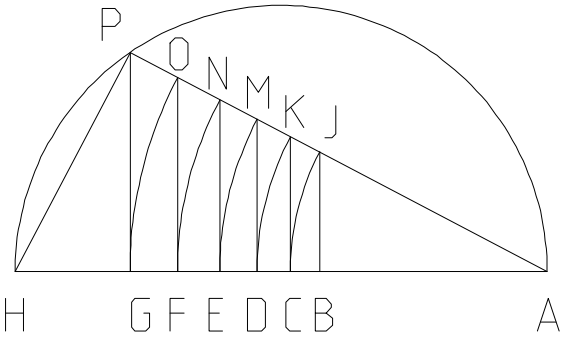
| $GP_\Delta$ | $FO_\Delta$ | $EN_\Delta$ | $DM_\Delta$ | $CK_\Delta$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 7.361 | 3.681 | 1.84 | 0.92 | 0.46 | 0.23 |
| 5.343 | 1.781 | 0.594 | 0.198 | 0.066 | 0.022 |
| 4.115 | 1.029 | 0.257 | 0.064 | 0.016 | 0.004 |
| 3.331 | 0.666 | 0.133 | 0.027 | 0.005 | 0.001 |




| $\frac{GP_\Delta}{\Delta^5}$ | $\frac{FO_\Delta}{\Delta^4}$ | $\frac{EN_\Delta}{\Delta^3}$ | $\frac{DM_\Delta}{\Delta^2}$ | $\frac{CK_\Delta}{\Delta}$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

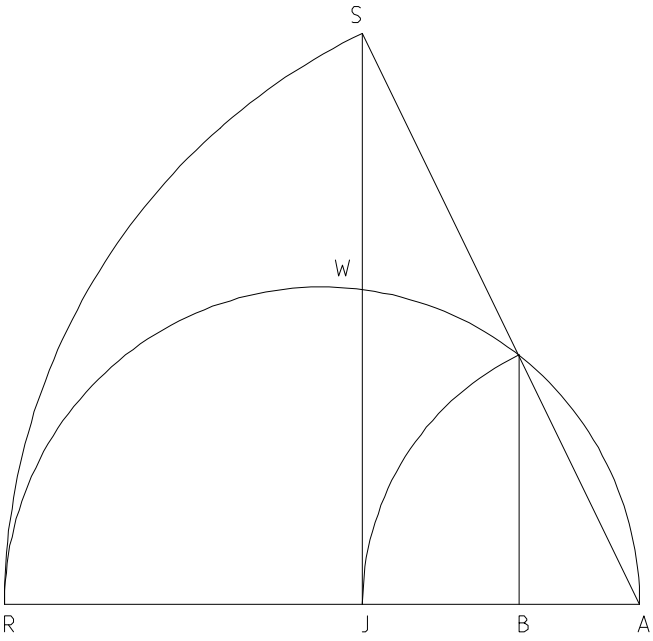
| $\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5}$ | $\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4}$ | $\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3}$ | $\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2}$ | $\frac{CK_\Delta}{\frac{AH}{AP_\Delta}}$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| 0.022 | 0.022 | 0.022 | 0.022 | 0.022 | 0.022 |
| 0.004 | 0.004 | 0.004 | 0.004 | 0.004 | 0.004 |
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

| $GP_\Delta$ | $FO_\Delta$ | $EN_\Delta$ | $DM_\Delta$ | $CK_\Delta$ | $BJ_\Delta$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 7.361 | 3.681 | 1.84 | 0.92 | 0.46 | 0.23 |
| 5.343 | 1.781 | 0.594 | 0.198 | 0.066 | 0.022 |
| 4.115 | 1.029 | 0.257 | 0.064 | 0.016 | 0.004 |
| 3.331 | 0.666 | 0.133 | 0.027 | 0.005 | 0.001 |




$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = 3.681 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = 1.84 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = 0.92$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = 1.781 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = 0.594 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = 0.198$$




12_04_B3.MCD $2 \cdot 2^N$ Exponential Ratio.

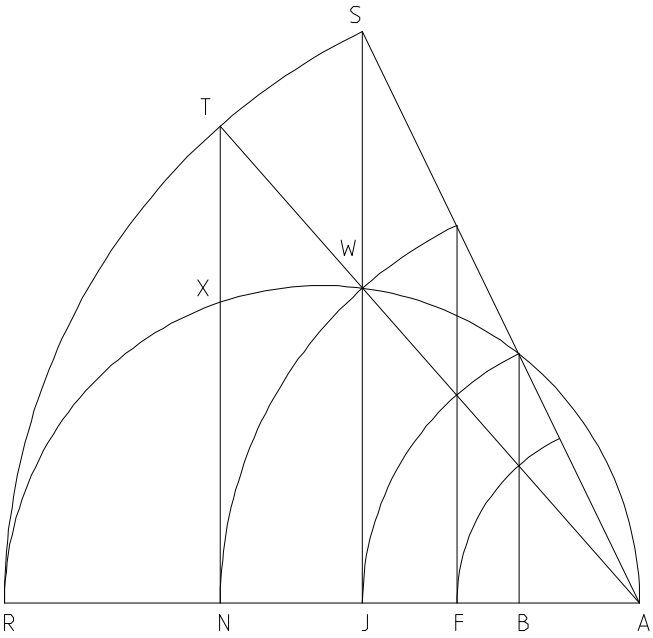
$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ | $AJ_\delta$ |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

I think that one is in the position that they are in when dividing an angle, twice never gets to thrice, at least that is what I decided some time ago. Let me double check anyway. An obvious pattern should emerge.

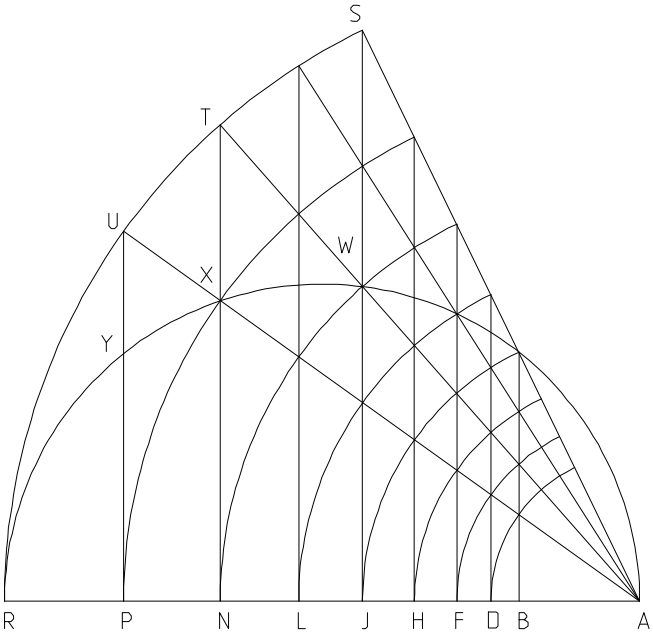



$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ | $AF_\delta$ | $AJ_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |



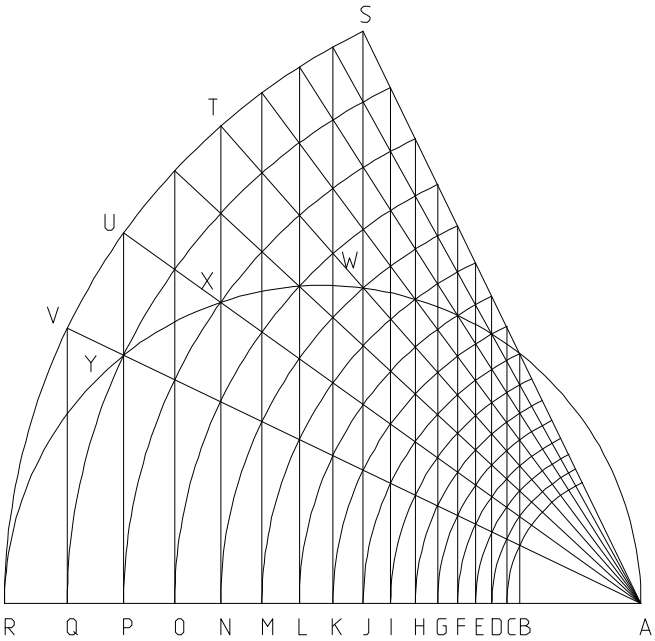

$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$

$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$

$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$

$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$

| $AB_\delta$ | $AD_\delta$ | $AF_\delta$ | $AH_\delta$ | $AJ_\delta$ | $AL_\delta$ | $AN_\delta$ | $AP_\delta$ |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |




$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$

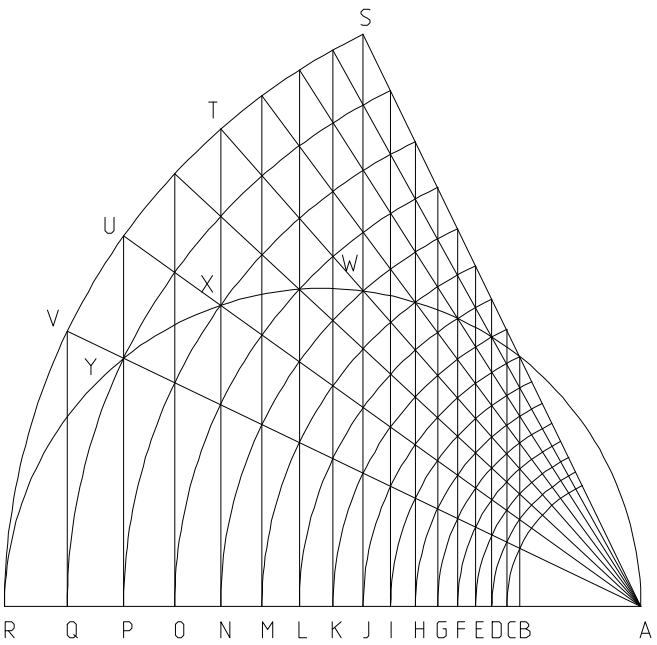
$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$

$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$

$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$

| $AB_\delta$ | $AC_\delta$ | $AD_\delta$ | $AE_\delta$ | $AF_\delta$ |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |

| $AG_\delta$ | $AH_\delta$ | $AI_\delta$ | $AJ_\delta$ | $AK_\delta$ | $AL_\delta$ | $AM_\delta$ | $AN_\delta$ | $AO_\delta$ | $AP_\delta$ | $AQ_\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |




$\frac{AR}{AB_\delta}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AC_\delta}\right)^{\frac{16}{15}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AD_\delta}\right)^{\frac{16}{14}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AE_\delta}\right)^{\frac{16}{13}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AF_\delta}\right)^{\frac{16}{12}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AG_\delta}\right)^{\frac{16}{11}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AH_\delta}\right)^{\frac{16}{10}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AI_\delta}\right)^{\frac{16}{9}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AJ_\delta}\right)^{\frac{16}{8}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AK_\delta}\right)^{\frac{16}{7}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AL_\delta}\right)^{\frac{16}{6}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AM_\delta}\right)^{\frac{16}{5}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AN_\delta}\right)^{\frac{16}{4}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AO_\delta}\right)^{\frac{16}{3}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AP_\delta}\right)^{\frac{16}{2}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AQ_\delta}\right)^{\frac{16}{1}}$

| |
|---|
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |

$\left(\frac{AR}{AQ_\delta}\right)^{8}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AP_\delta}\right)^{\frac{8}{2}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AO_\delta}\right)^{\frac{8}{3}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AN_\delta}\right)^{\frac{8}{4}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AM_\delta}\right)^{\frac{8}{5}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AL_\delta}\right)^{\frac{8}{6}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AK_\delta}\right)^{\frac{8}{7}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AJ_\delta}\right)$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AI_\delta}\right)^{\frac{8}{9}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AH_\delta}\right)^{\frac{8}{10}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AG_\delta}\right)^{\frac{8}{11}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AF_\delta}\right)^{\frac{8}{12}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AE_\delta}\right)^{\frac{8}{13}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AD_\delta}\right)^{\frac{8}{14}}$

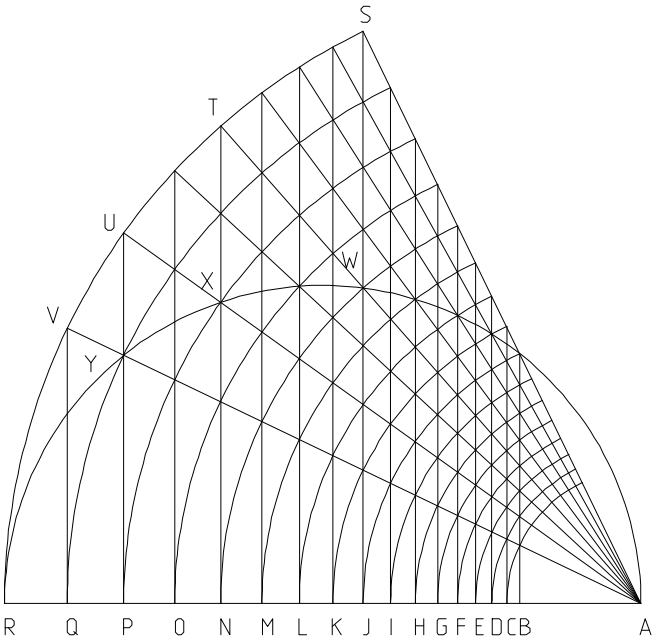
| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AC_\delta}\right)^{\frac{8}{15}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |

$\left(\frac{AR}{AB_\delta}\right)^{\frac{8}{16}}$

| |
|---|
| 1.414 |
| 1.732 |
| 2 |
| 2.236 |
| 2.449 |




$$\left(AB_{\delta}\cdot AR\right)^{\frac{1}{2}} - AJ_{\delta} \qquad \left[\left(AB_{\delta}\right)^{3}\cdot AR\right]^{\frac{1}{4}} - AF_{\delta}$$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

$$\left[\left(AB_{\delta}\right)^{7}\cdot AR\right]^{\frac{1}{8}} - AD_{\delta} \qquad \left[\left(AB_{\delta}\right)^{15}\cdot AR\right]^{\frac{1}{16}} - AC_{\delta}$$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

$$\left(AB_{\delta}\cdot AR^{3}\right)^{\frac{1}{4}} - AN_{\delta} \qquad \left(AB_{\delta}\cdot AR^{7}\right)^{\frac{1}{8}} - AP_{\delta} \qquad \left(AB_{\delta}\cdot AR^{15}\right)^{\frac{1}{16}} - AQ_{\delta}$$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

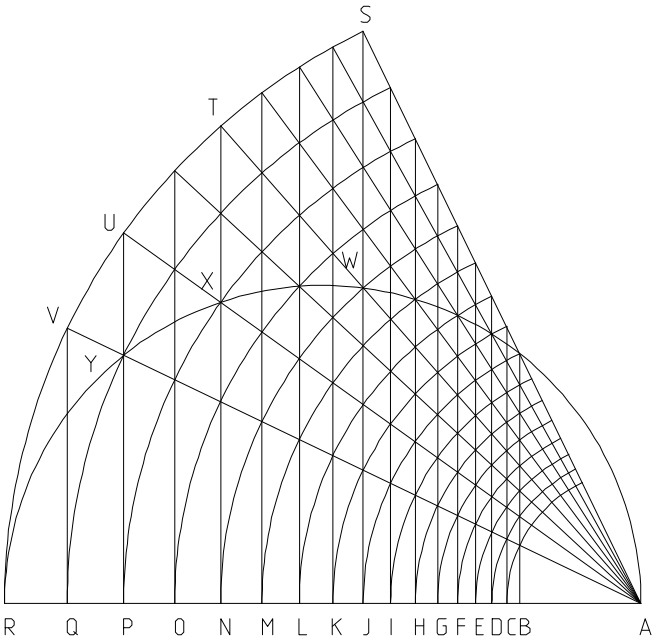
| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

$$\left[\left(AC_{\delta}\right)^{2}\cdot AR\right]^{\frac{1}{3}} - AH_{\delta} \qquad \left[\left(AC_{\delta}\right)^{4}\cdot AR\right]^{\frac{1}{5}} - AF_{\delta} \qquad \left[\left(AC_{\delta}\right)^{8}\cdot AR\right]^{\frac{1}{9}}$$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

| |
|---|
| 5.612 |
| 4.003 |
| 3.15 |
| 2.615 |
| 2.247 |




| $AQ_\delta$ | $AP_\delta$ | $AO_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 9.576 | 9.17 | 8.781 | 8.409 |
| 9.336 | 8.717 | 8.138 | 7.598 |
| 9.17 | 8.409 | 7.711 | 7.071 |
| 9.043 | 8.178 | 7.395 | 6.687 |
| 8.941 | 7.993 | 7.147 | 6.389 |

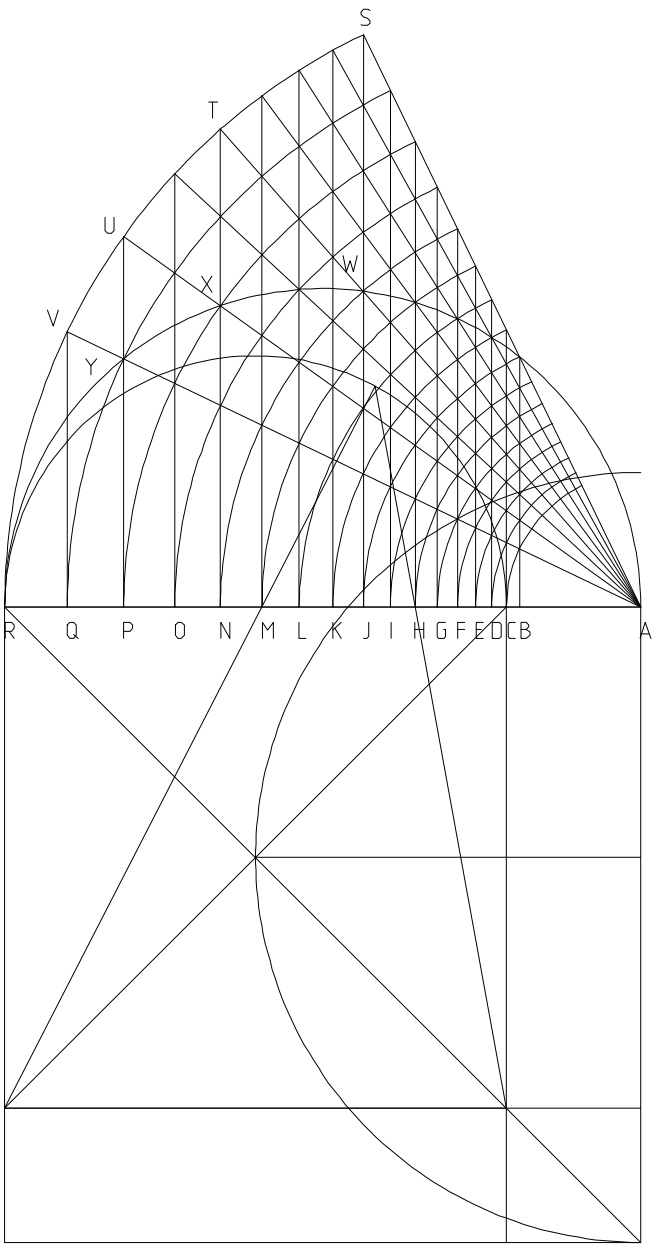
| $AM_\delta$ | $AL_\delta$ | $AK_\delta$ | $AJ_\delta$ |
|---|---|---|---|
| 8.052 | 7.711 | 7.384 | 7.071 |
| 7.094 | 6.623 | 6.184 | 5.774 |
| 6.484 | 5.946 | 5.453 | 5 |
| 6.047 | 5.469 | 4.945 | 4.472 |
| 5.713 | 5.107 | 4.566 | 4.082 |

| $AI_\delta$ | $AH_\delta$ | $AG_\delta$ | $AF_\delta$ | $AE_\delta$ | $AD_\delta$ | $AC_\delta$ | $AB_\delta$ |
|---|---|---|---|---|---|---|---|
| 6.771 | 6.484 | 6.209 | 5.946 | 5.694 | 5.453 | 5.221 | 5 |
| 5.39 | 5.033 | 4.699 | 4.387 | 4.096 | 3.824 | 3.57 | 3.333 |
| 4.585 | 4.204 | 3.856 | 3.536 | 3.242 | 2.973 | 2.726 | 2.5 |
| 4.044 | 3.657 | 3.307 | 2.991 | 2.704 | 2.446 | 2.212 | 2 |
| 3.65 | 3.263 | 2.918 | 2.608 | 2.332 | 2.085 | 1.864 | 1.667 |

| $\left[\left(AB_\delta\right)\cdot AR^2\right]^{\frac{1}{3}}$ | $\left(AC_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AD_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AE_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AF_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AG_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|---|---|
| 7.937 | 8.052 | 8.17 | 8.288 | 8.409 | 8.531 |
| 6.934 | 7.094 | 7.258 | 7.426 | 7.598 | 7.774 |
| 6.3 | 6.484 | 6.674 | 6.87 | 7.071 | 7.278 |
| 5.848 | 6.047 | 6.254 | 6.467 | 6.687 | 6.915 |
| 5.503 | 5.713 | 5.93 | 6.155 | 6.389 | 6.632 |

| $\left(AH_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AI_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AJ_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AK_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AL_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AM_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|---|---|
| 8.655 | 8.781 | 8.909 | 9.039 | 9.17 | 9.303 |
| 7.954 | 8.138 | 8.327 | 8.52 | 8.717 | 8.919 |
| 7.492 | 7.711 | 7.937 | 8.17 | 8.409 | 8.655 |
| 7.151 | 7.395 | 7.647 | 7.908 | 8.178 | 8.456 |
| 6.885 | 7.147 | 7.418 | 7.701 | 7.993 | 8.297 |

| $\left(AN_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AO_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AP_\delta\cdot AR^2\right)^{\frac{1}{3}}$ | $\left(AQ_\delta\cdot AR^2\right)^{\frac{1}{3}}$ |
|---|---|---|---|
| 9.439 | 9.576 | 9.715 | 9.857 |
| 9.125 | 9.336 | 9.553 | 9.774 |
| 8.909 | 9.17 | 9.439 | 9.715 |
| 8.745 | 9.043 | 9.351 | 9.67 |
| 8.613 | 8.941 | 9.281 | 9.634 |




$\left[\left(AC_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$\left(AC_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$AH_\delta$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$AM_\delta$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$\left[\left(AF_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$\left(AF_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$AJ_\delta$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$AN_\delta$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$\frac{16}{3} = 5.333$

There are five found cubics between the givens.

$\left[\left(AI_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$\left(AI_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$\left[\left(AL_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$\left(AL_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$\left[\left(AO_\delta\right)^2 \cdot AR\right]^{\frac{1}{3}}$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$\left(AO_\delta \cdot AR^2\right)^{\frac{1}{3}}$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$AL_\delta$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$AO_\delta$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$AN_\delta$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$AP_\delta$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$AP_\delta$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$AQ_\delta$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$S1_\delta := \sqrt{AH_\delta \cdot AM_\delta}$ $S2_\delta := \sqrt{AJ_\delta \cdot AN_\delta}$ $S3_\delta := \sqrt{AL_\delta \cdot AO_\delta}$ $S4_\delta := \sqrt{AN_\delta \cdot AP_\delta}$ $S5_\delta := \sqrt{AP_\delta \cdot AQ_\delta}$

| $AJ_\delta$ |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

| $S1_\delta$ |
|---|
| 7.226 |
| 5.975 |
| 5.221 |
| 4.703 |
| 4.318 |

| $S2_\delta$ | $AL_\delta$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

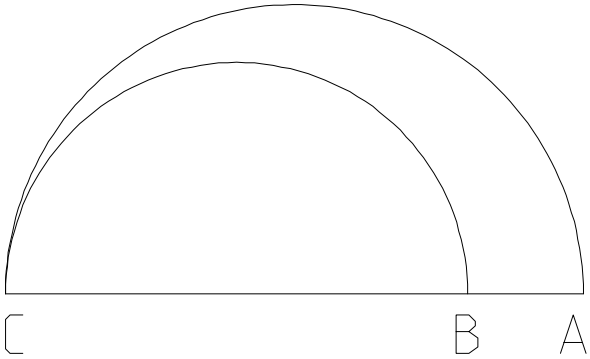
| $S3_\delta$ |
|---|
| 8.229 |
| 7.342 |
| 6.771 |
| 6.359 |
| 6.042 |

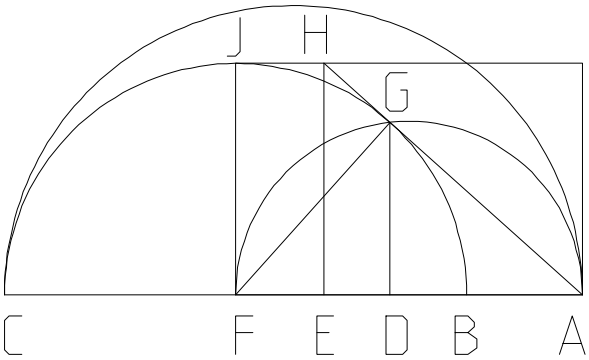
| $S4_\delta$ | $AO_\delta$ |
|---|---|
| 8.781 | 8.781 |
| 8.138 | 8.138 |
| 7.711 | 7.711 |
| 7.395 | 7.395 |
| 7.147 | 7.147 |

| $S5_\delta$ |
|---|
| 9.371 |
| 9.021 |
| 8.781 |
| 8.599 |
| 8.454 |

12_06_93

# Alternate methods; Square Root .




Demonstrate that the square root of two segments can never be more than one half of the difference between them

Let AB and AC be the two segments and BC the difference between them.

$BC := 10 \quad \delta := 1..100 \quad BF := \frac{BC}{2}$

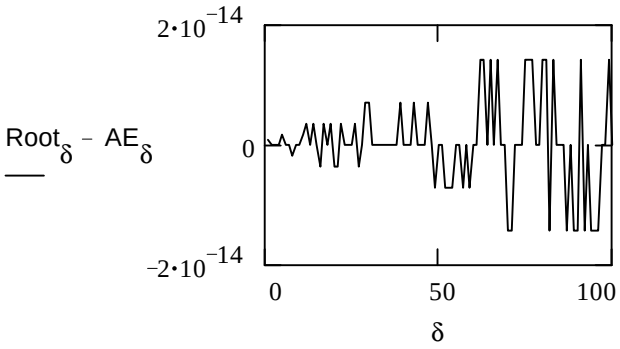
$AB_\delta := \delta \quad AF_\delta := AB_\delta + BF \quad FG := BF$

$DF_\delta := \frac{FG^2}{AF_\delta} \quad FJ := BF \quad AD_\delta := AF_\delta - DF_\delta$

$DG_\delta := \sqrt{AD_\delta \cdot DF_\delta} \quad EH := BF$

$Root_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

$AE_\delta := \frac{AD_\delta \cdot EH}{DG_\delta} \quad AC_\delta := AB_\delta + BC$

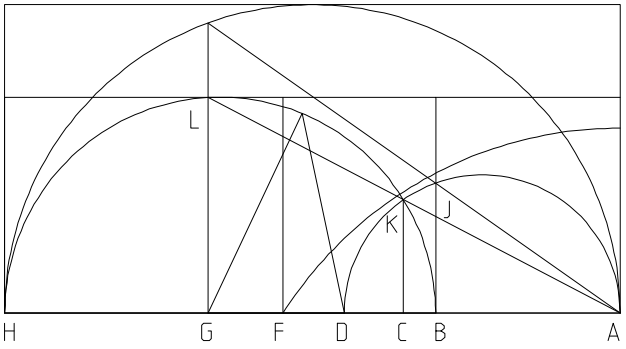



$$\sum_\delta \left(Root_\delta - AE_\delta\right) = 0$$

Since AG can never exceed AF, the result is obvious.

Loging the constant relationship K and J on the cubic.

# Gruntwork 4 on the Delian Solution



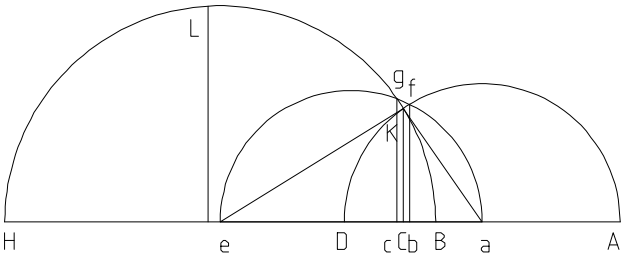

$BH := 10 \quad \delta := 1 .. 1000 \quad AB_\delta := \delta$

$AH_\delta := BH + AB_\delta \quad AF_\delta := \sqrt{AB_\delta \cdot AH_\delta}$

$AG_\delta := \left[ \left(AH_\delta\right)^2 \cdot AB_\delta \right]^{\frac{1}{3}}$

$AD_\delta := \left[ AH_\delta \cdot \left(AB_\delta\right)^2 \right]^{\frac{1}{3}}$

$BG_\delta := AG_\delta - AB_\delta \quad GH_\delta := BH - BG_\delta \quad GL_\delta := \sqrt{GH_\delta \cdot BG_\delta}$



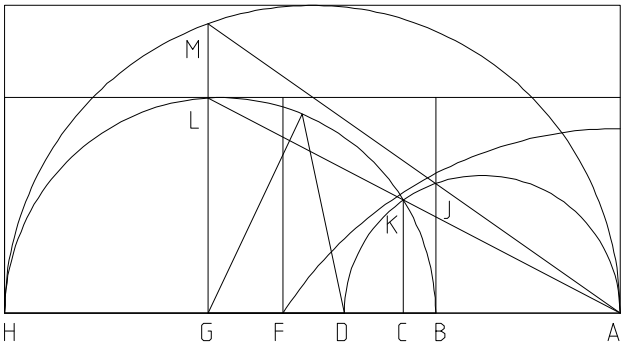

$Aa_\delta := \frac{AD_\delta}{2} \quad Be := \frac{BH}{2} \quad Ka_\delta := Aa_\delta$

$Ke := Be \quad eg := Ke \quad af_\delta := Ka_\delta$

$Ae_\delta := AB_\delta + Be \quad ae_\delta := Ae_\delta - Aa_\delta$

$ce_\delta := \frac{eg^2}{ae_\delta} \quad ab_\delta := \frac{\left(af_\delta\right)^2}{ae_\delta} \quad ac_\delta := ae_\delta - ce_\delta \quad bc_\delta := ac_\delta - ab_\delta \quad Cb_\delta := \frac{bc_\delta}{2}$

$Ca_\delta := ab_\delta + Cb_\delta \quad AC_\delta := Aa_\delta + Ca_\delta \quad Ce_\delta := ae_\delta - Ca_\delta \quad CK_\delta := \sqrt{Ke^2 - \left(Ce_\delta\right)^2}$




$BD_\delta := AD_\delta - AB_\delta$

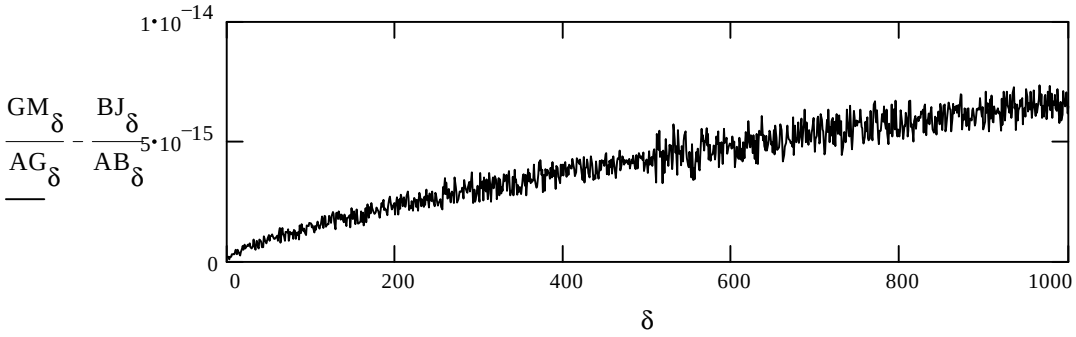
$BJ_\delta := \sqrt{BD_\delta \cdot AB_\delta}$ $GM_\delta := \sqrt{AG_\delta \cdot GH_\delta}$

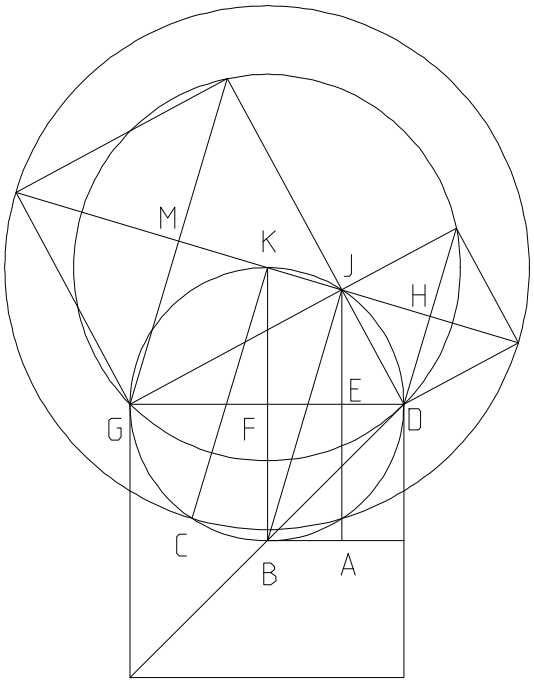
Is K on CK colinear with AL?

$$\sum_\delta \left( \frac{GL_\delta}{AG_\delta} - \frac{CK_\delta}{AC_\delta} \right) = 2.35 \cdot 10^{-13}$$

Is J on BJ colinear with AM?

$$\sum_\delta \left( \frac{GM_\delta}{AG_\delta} - \frac{BJ_\delta}{AB_\delta} \right) = 4.063 \cdot 10^{-12}$$






12_07_93.MCD

A few years back I seen a relationship that I had promised myself to take a closer look at one day; BJ = CK = HM .

$DG := 10 \quad DF := \frac{DG}{2} \quad \Delta := 1000$

$\delta := 0..\Delta \quad EF_\delta := \frac{DF \cdot \delta}{\Delta} \quad AB_\delta := EF_\delta$

$BF := DF \quad AE := DF \quad DE_\delta := DF - EF_\delta$

$FG := DF \quad EG_\delta := FG + EF_\delta$

$EJ_\delta := \sqrt{DE_\delta \cdot EG_\delta} \quad AJ_\delta := AE + EJ_\delta$

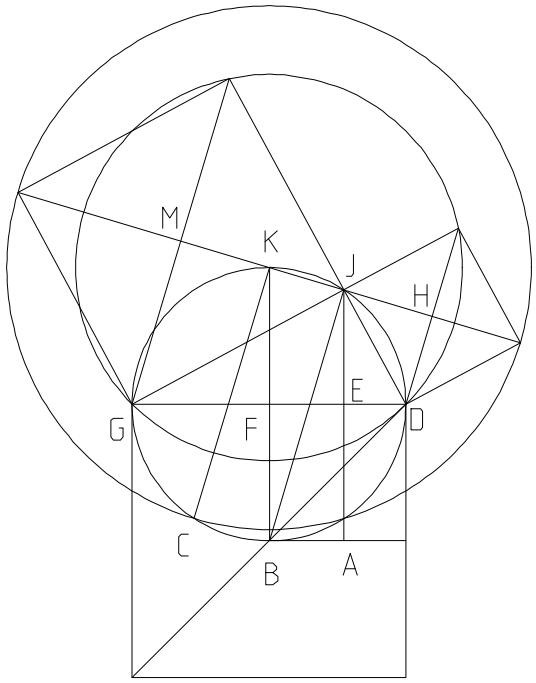
$BJ_\delta := \sqrt{(AB_\delta)^2 + (AJ_\delta)^2}$

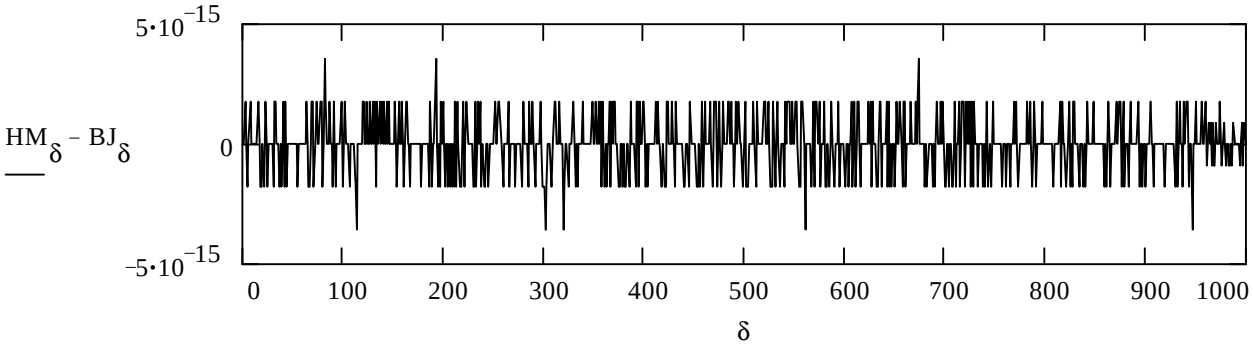
$DJ_\delta := \sqrt{(DE_\delta)^2 + (EJ_\delta)^2} \quad HJ_\delta := \frac{\sqrt{2 \cdot (DJ_\delta)^2}}{2}$

$GJ_\delta := \sqrt{(EG_\delta)^2 + (EJ_\delta)^2} \quad JM_\delta := \frac{\sqrt{2 \cdot (GJ_\delta)^2}}{2}$

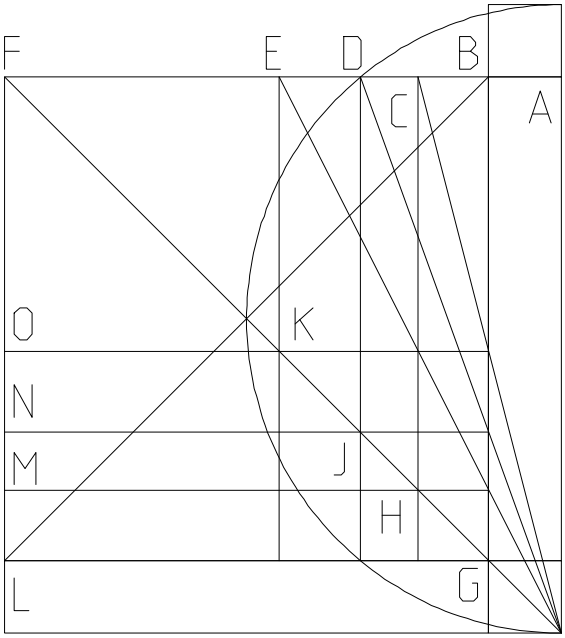
$HM_\delta := HJ_\delta + JM_\delta$

$\sum_\delta (HM_\delta - BJ_\delta) = -1.776 \cdot 10^{-15}$



12_07_B3Exploring the ratio of squares.




$BF := 1000 \quad \Delta := 400 \quad \delta := 1..\Delta \quad AB_\delta := \delta^5 \cdot 10^{-9}$

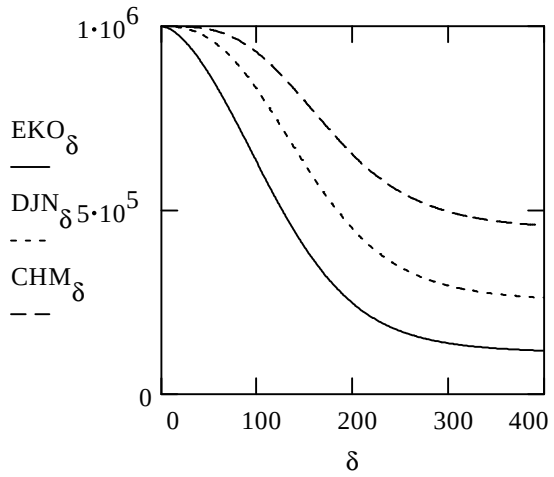
$AF_\delta := AB_\delta + BF \quad AD_\delta := \sqrt{AB_\delta \cdot AF_\delta}$

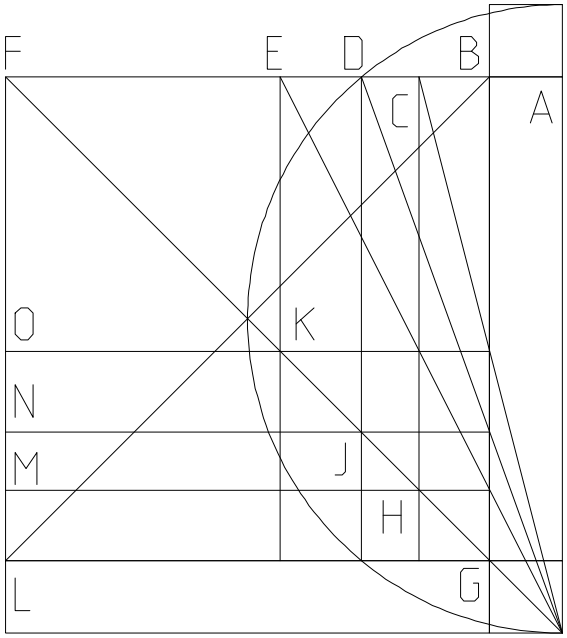
$AE_\delta := \left[ AB_\delta \cdot \left( AF_\delta \right)^2 \right]^{\frac{1}{3}} \quad AC_\delta := \left[ \left( AB_\delta \right)^2 \cdot AF_\delta \right]^{\frac{1}{3}}$

$EF_\delta := AF_\delta - AE_\delta \quad DF_\delta := AF_\delta - AD_\delta$

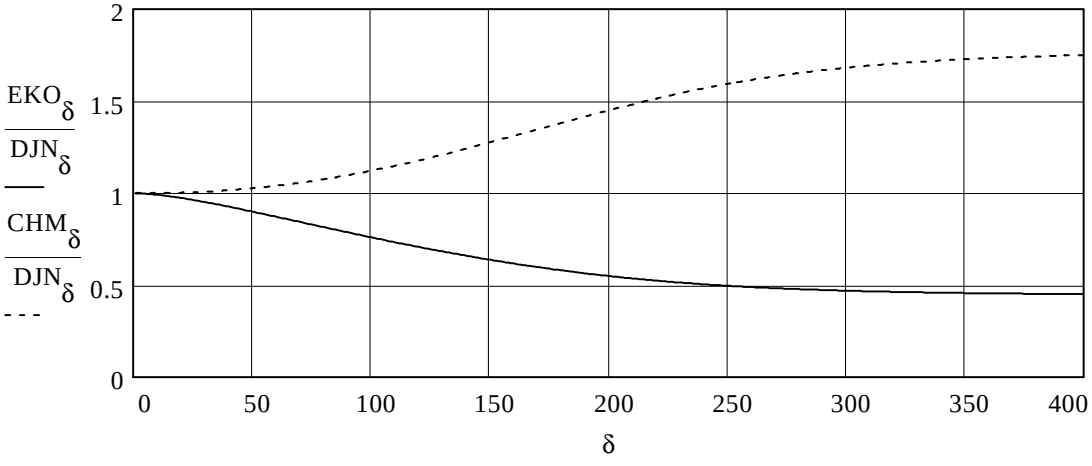
$CF_\delta := AF_\delta - AC_\delta \quad EKO_\delta := \left( EF_\delta \right)^2$

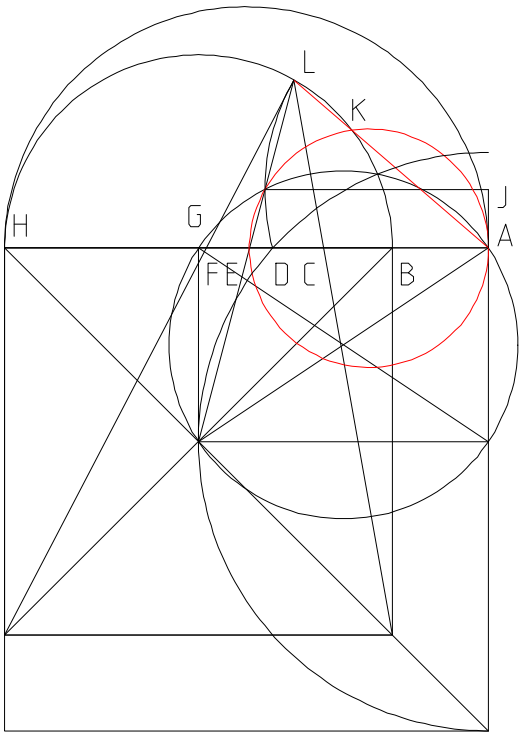
$DJN_\delta := \left( DF_\delta \right)^2 \quad CHM_\delta := \left( CF_\delta \right)^2$




$AB_1 = 1 \cdot 10^{-9} \quad AB_{25} = 0.01 \quad AB_{100} = 10$






The structure in red appears to be a constant.

$BH := 5000 \quad \delta := S..F \quad AB_\delta := \delta^5 \cdot 10^{-8}$

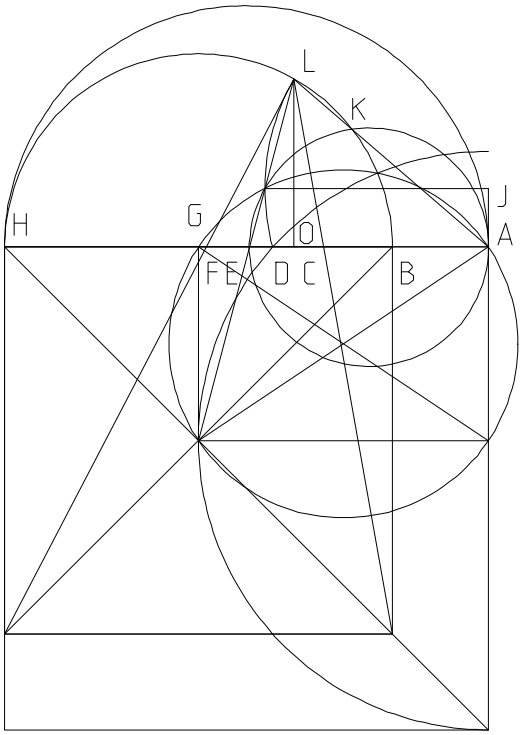
$BG := \frac{BH}{2} \quad AH_\delta := BH + AB_\delta$

$AC_\delta := \left[ \left( AB_\delta \right)^2 \cdot AH_\delta \right]^{\frac{1}{3}}$

$AF_\delta := \left[ AB_\delta \cdot \left( AH_\delta \right)^2 \right]^{\frac{1}{3}}$

$AJ_\delta := \frac{AF_\delta - AC_\delta}{2} \quad BC_\delta := AC_\delta - AB_\delta$

$AD_\delta := \sqrt{AB_\delta \cdot AH_\delta} \quad BF_\delta := AF_\delta - AB_\delta$



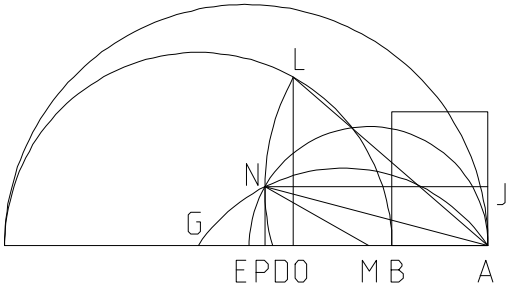

$CF_\delta := AF_\delta - AC_\delta \quad FH_\delta := AH_\delta - AF_\delta$

$CO_\delta := \frac{BC_\delta \cdot CF_\delta}{FH_\delta + BC_\delta} \quad BO_\delta := BC_\delta + CO_\delta$

$GO_\delta := BG - BO_\delta \quad GL := BG$

$LO_\delta := \sqrt{GL^2 - \left( GO_\delta \right)^2}$

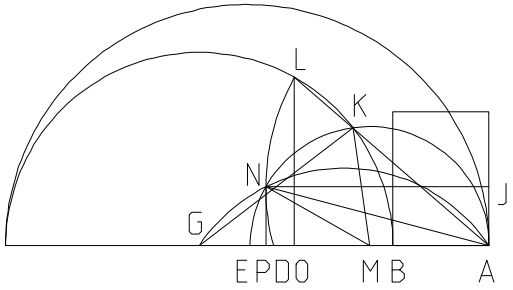
$AO_\delta := AB_\delta + BO_\delta$




$$JD_\delta := \sqrt{(AD_\delta)^2 + (AJ_\delta)^2} \quad JN_\delta := JD_\delta$$

$$NP_\delta := AJ_\delta \quad AN_\delta := \sqrt{(AJ_\delta)^2 + (JN_\delta)^2}$$

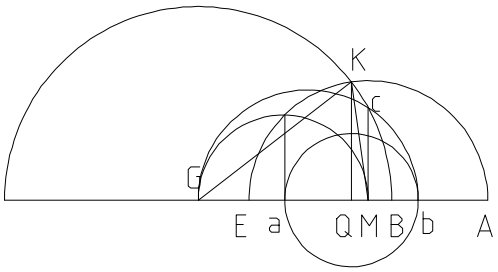
$$AP_\delta := JN_\delta \quad AE_\delta := \frac{(AN_\delta)^2}{AP_\delta}$$




$$AM_\delta := \frac{AE_\delta}{2} \quad MK_\delta := AM_\delta \quad GK := BG$$

$$AG_\delta := AB_\delta + BG$$

$$GM_\delta := AG_\delta - AM_\delta \quad Ma_\delta := \frac{(MK_\delta)^2}{GM_\delta}$$

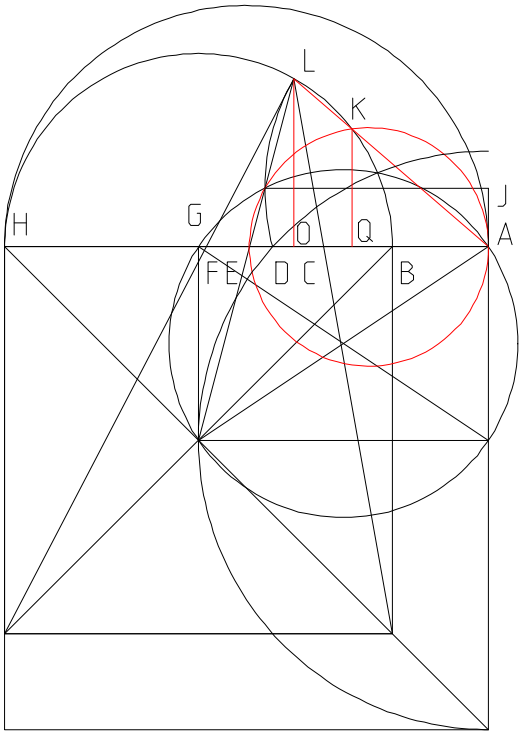



$$Gb_\delta := \frac{(GK)^2}{GM_\delta} \quad Ga_\delta := GM_\delta - Ma_\delta$$

$$ab_\delta := Gb_\delta - Ga_\delta \quad Qa_\delta := \frac{ab_\delta}{2}$$

$$GQ_\delta := Ga_\delta + Qa_\delta \quad AQ_\delta := AG_\delta - GQ_\delta$$

$$KQ_\delta := \sqrt{GK^2 - (GQ_\delta)^2}$$




$\left(AB_S\right)^2 \cdot AH_S = 4.886 \cdot 10^4$ $BO_S = 36.27$

$AB_S \cdot \left(AH_S\right)^2 = 7.822 \cdot 10^7$ $BO_F = 109.49532$

$\left(AB_F\right)^2 \cdot AH_F = 1.417 \cdot 10^6$ $AB_S = 3.125$

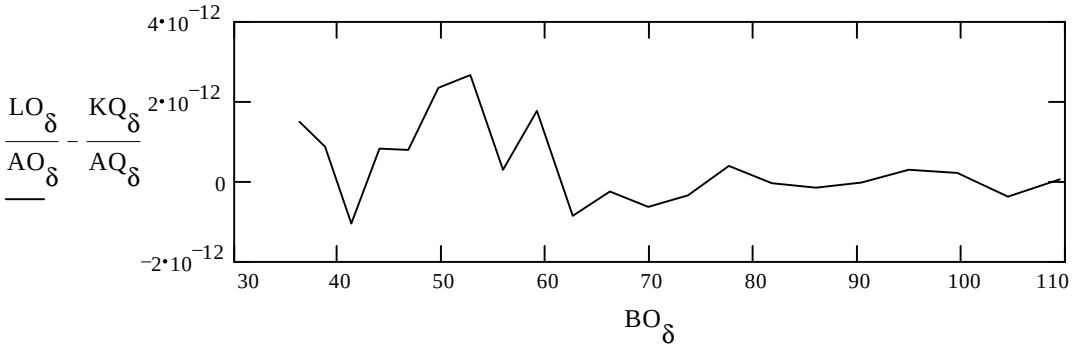
$AB_F \cdot \left(AH_F\right)^2 = 4.23 \cdot 10^8$ $AB_F = 16.807$

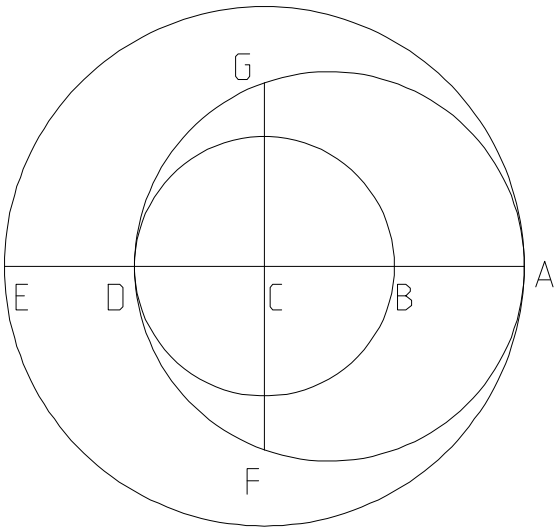
$LO_S = 424.30457$ $KQ_S = 36.53305$

$LO_F = 731.77$ $KQ_F = 111.891$

$$\sum_{\delta}\left(\frac{LO_\delta}{AO_\delta} - \frac{KQ_\delta}{AQ_\delta}\right) = 8.477 \cdot 10^{-12}$$

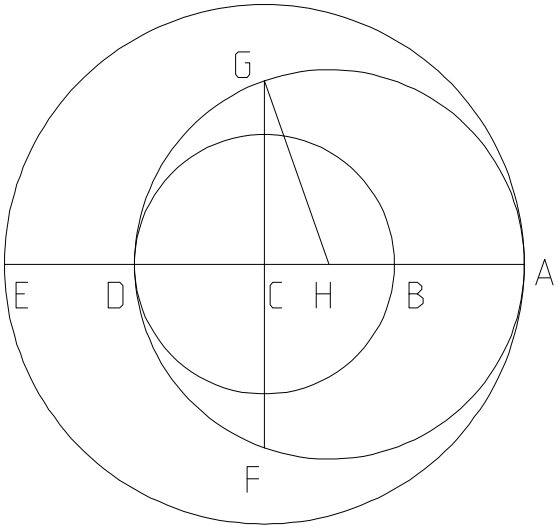
$S \equiv 50$ $F \equiv 70$






# The Square Root

It may be noticed that I use the adjacent figure in my work for doing square roots. I believe that it is the primary figure for doing square roots. Given segment AE and segment BD, segment FG is their square root. It has a major advantage of being able to square larger figures on paper, not to mention makes something of the development of exponential series.




$AE := 100 \quad \delta := 1 .. AE \quad BD_\delta := \delta$

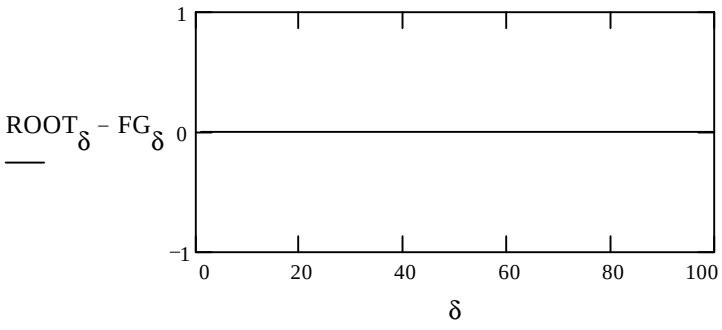
$AC := \frac{AE}{2} \quad BC_\delta := \frac{BD_\delta}{2} \quad AB_\delta := AC - BC_\delta$

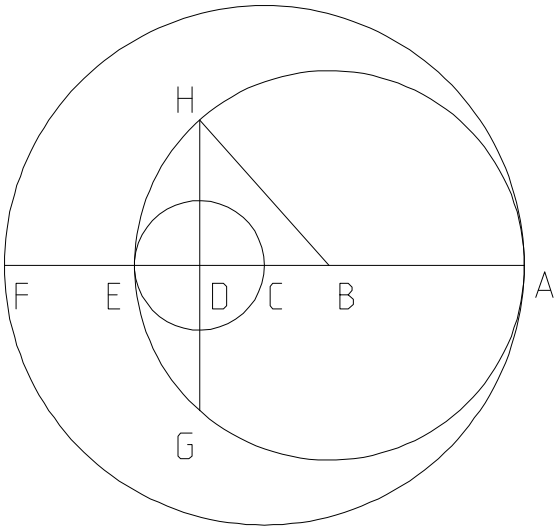
$AD_\delta := AC + BC_\delta \quad AH_\delta := \frac{AD_\delta}{2}$

$GH_\delta := AH_\delta \quad CH_\delta := AC - AH_\delta$

$FG_\delta := 2 \cdot \sqrt{\left(GH_\delta\right)^2 - \left(CH_\delta\right)^2}$

$ROOT_\delta := \sqrt{\left(BD_\delta\right) \cdot AE}$






12_12_B3.MCD

Completely generalize the square root figure.

$AF := 7 \qquad DF := \frac{AF}{BR} \qquad AD := AF - DF$

$\delta := 1 .. LBR \qquad DE_\delta := \frac{DF}{LBR} \cdot \delta$

$CE_\delta := 2 \cdot DE_\delta \qquad AE_\delta := AD + DE_\delta$

$AB_\delta := \frac{AE_\delta}{2} \qquad BD_\delta := AD - AB_\delta$

$BH_\delta := AB_\delta \qquad GH_\delta := 2 \cdot \sqrt{\left| (BH_\delta)^2 - (BD_\delta)^2 \right|}$

Set AF to unity so that it may be eliminated. Setting BR to 2 will yeild the familiar square root. BR may even take fractional values.

**Plug in values here.**

BR=BASE RATIO,
LBR=LITTLE BASE RATIO

$BR \equiv 3 \qquad LBR \equiv 5$

$\left| \frac{(2 \cdot BR) - 2}{BR} \right| = 1.333$

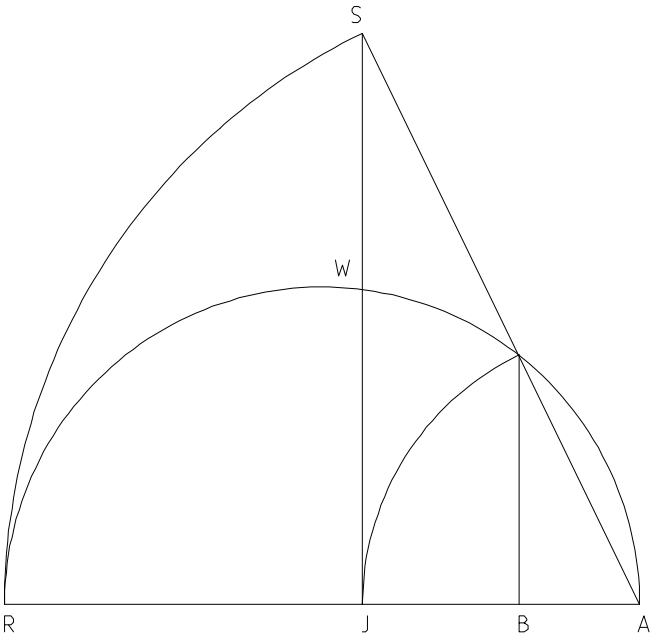
The equation below simplifies to the next.

$GH_\delta \qquad \left[ \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot \left| \frac{(2 \cdot BR) - 2}{BR} \right| \right]^{\frac{1}{2}} \cdot AF \qquad \frac{\sqrt{2} \cdot \sqrt{\left| (2 \cdot BR) - 2 \right|}}{BR \cdot \sqrt{LBR}} \cdot \sqrt{\delta} \cdot AF$

| | | |
|---|---|---|
| 2.951 | 2.951 | 2.951 |
| 4.174 | 4.174 | 4.174 |
| 5.112 | 5.112 | 5.112 |
| 5.903 | 5.903 | 5.903 |
| 6.6 | 6.6 | 6.6 |

$CE_\delta \qquad \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot AF$

| | |
|---|---|
| 0.933 | 0.933 |
| 1.867 | 1.867 |
| 2.8 | 2.8 |
| 3.733 | 3.733 |
| 4.667 | 4.667 |




12_16_93, using 12_04_93.MCD

$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

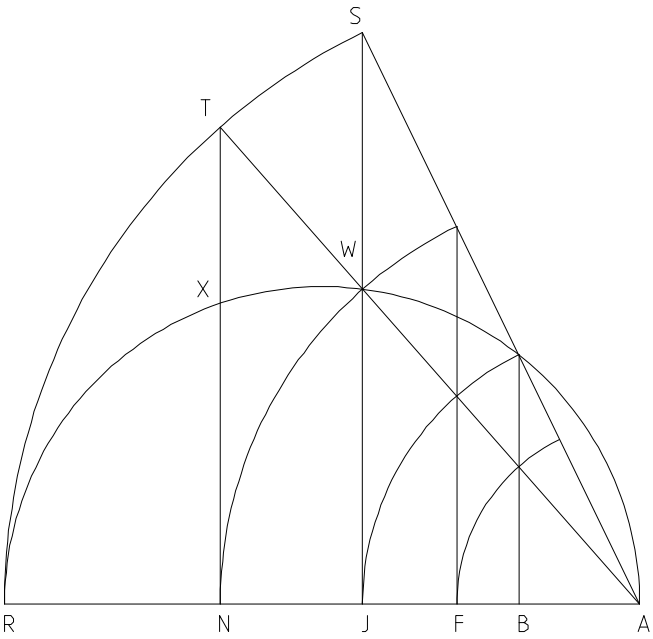
$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ | $AJ_\delta$ |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.

# Euclidean Exponential Series



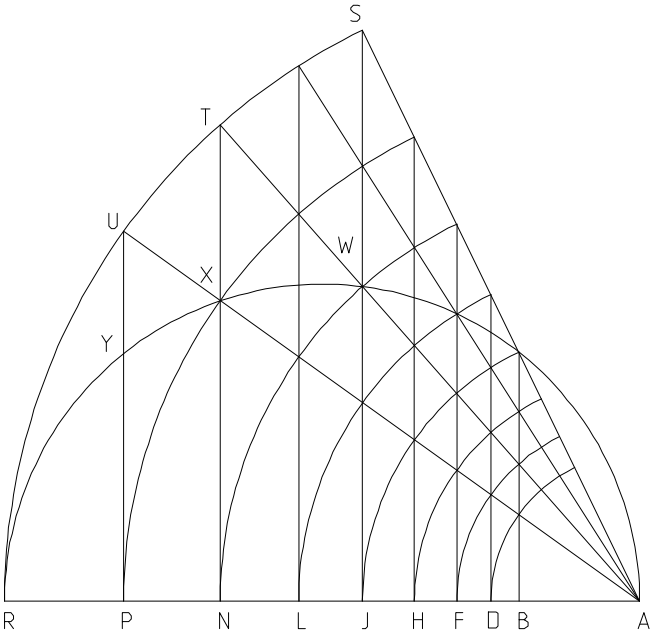

$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ | $AF_\delta$ | $AJ_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |

What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.



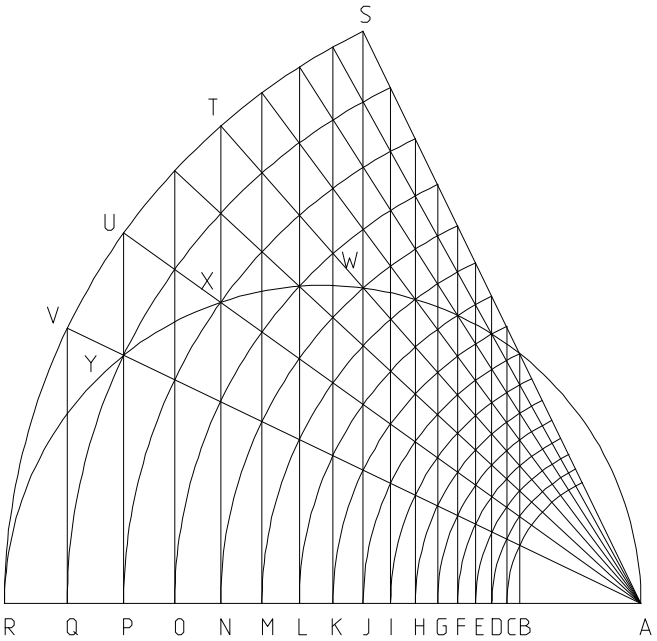

$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$

$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$

$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$

$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$

| $AB_\delta$ | $AD_\delta$ | $AF_\delta$ | $AH_\delta$ | $AJ_\delta$ | $AL_\delta$ | $AN_\delta$ | $AP_\delta$ |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |




$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$

$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$

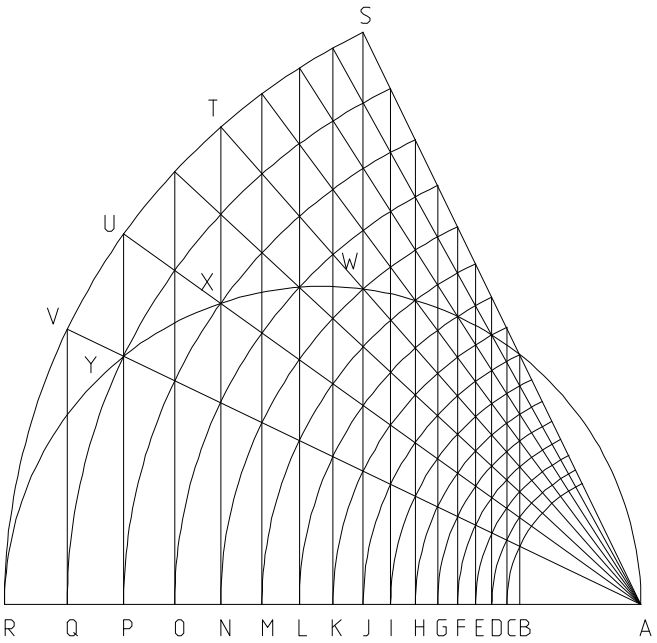
$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$

$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$

| $AB_\delta$ | $AC_\delta$ | $AD_\delta$ | $AE_\delta$ | $AF_\delta$ |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |

| $AG_\delta$ | $AH_\delta$ | $AI_\delta$ | $AJ_\delta$ | $AK_\delta$ | $AL_\delta$ | $AM_\delta$ | $AN_\delta$ | $AO_\delta$ | $AP_\delta$ | $AQ_\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |

Values found by the investigator of 12_14_93




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AN_\delta$ | $\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

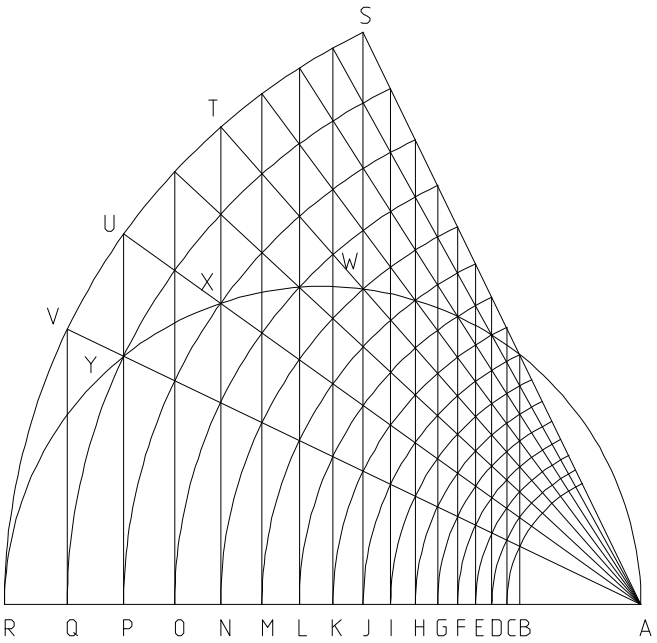
| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{7} \cdot AR\right]^{\frac{1}{8}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left[\left(AB_\delta\right)^2 \cdot AR^{14}\right]^{\frac{1}{16}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AO_\delta$ | $\left[\left(AB_\delta\right)^3 \cdot AR^{13}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.781 | 8.781 |
| 8.138 | 8.138 |
| 7.711 | 7.711 |
| 7.395 | 7.395 |
| 7.147 | 7.147 |

| $AN_\delta$ | $\left[\left(AB_\delta\right)^4 \cdot AR^{12}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AM_\delta$ | $\left[\left(AB_\delta\right)^5 \cdot AR^{11}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.052 | 8.052 |
| 7.094 | 7.094 |
| 6.484 | 6.484 |
| 6.047 | 6.047 |
| 5.713 | 5.713 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^6 \cdot AR^{10}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AK_\delta$ | $\left[\left(AB_\delta\right)^7 \cdot AR^{9}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.384 | 7.384 |
| 6.184 | 6.184 |
| 5.453 | 5.453 |
| 4.945 | 4.945 |
| 4.566 | 4.566 |

| $AJ_\delta$ | $\left[\left(AB_\delta\right)^8 \cdot AR^{8}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.071 | 7.071 |
| 5.774 | 5.774 |
| 5 | 5 |
| 4.472 | 4.472 |
| 4.082 | 4.082 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^9 \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{10} \cdot AR^{6}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{12} \cdot AR^{4}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{14} \cdot AR^{2}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |

| $AB_\delta$ | $\left[\left(AB_\delta\right)^{16} \cdot AR^{0}\right]^{\frac{1}{16}}$ |
|---|---|
| 5 | 5 |
| 3.333 | 3.333 |
| 2.5 | 2.5 |
| 2 | 2 |
| 1.667 | 1.667 |

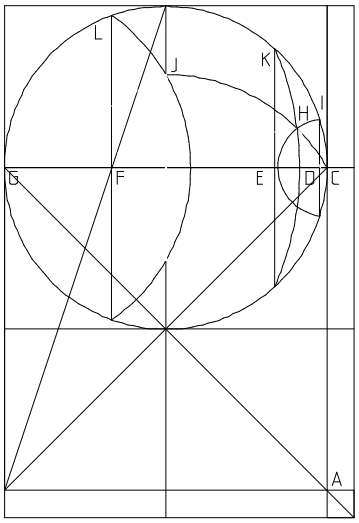
$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

Resultant Equation

depending on direction of transcription.



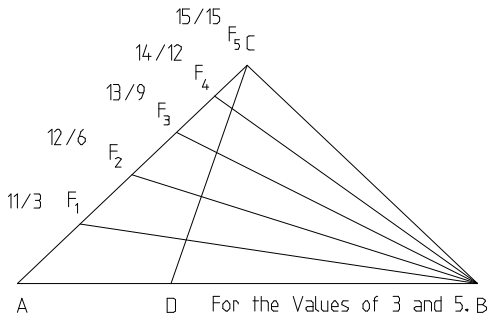

# One Circle

**By John Clark**

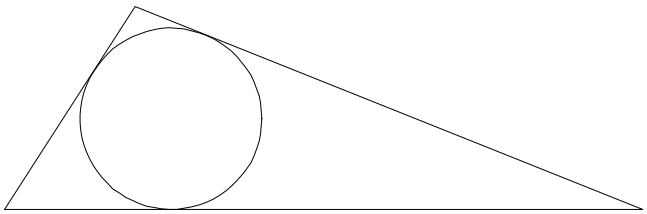
# One Square

# One Line

## And the Delian Quest

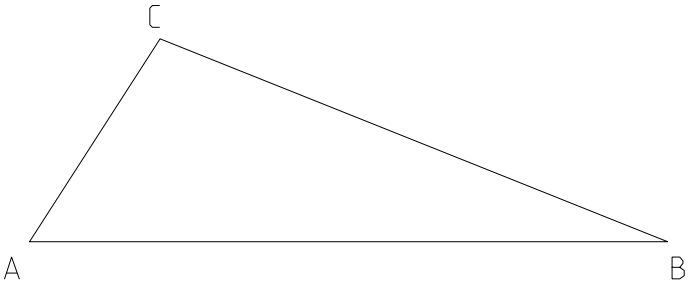



# 1994



# Inscribing a Circle in a given Triangle.

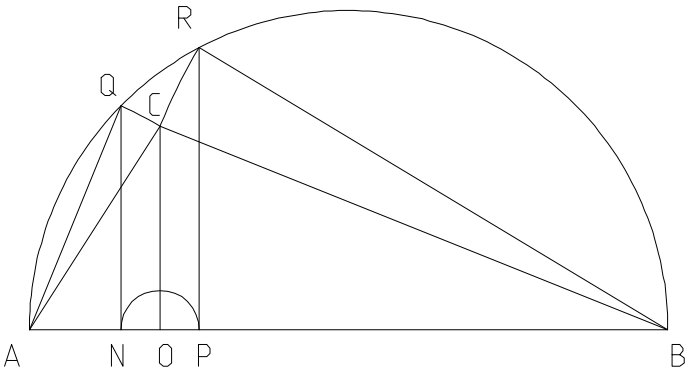
Place the length for the sides of the triangle at the end of the document.




$$AB := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad AC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix}$$

$$BC := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix} \quad \delta := 0..2$$

$$Is\_This\_A\_Triangle := \left(AB_0 + AC_0 \geq BC_0\right) \cdot \left(AB_0 + BC_0 \geq AC_0\right) \cdot \left(AC_0 + BC_0 \geq AB_0\right) \quad AQ_\delta := AC_\delta$$




$$BR_\delta := BC_\delta \quad AN_\delta := \frac{\left(AQ_\delta\right)^2}{AB_\delta}$$

$$BP_\delta := \frac{\left(BR_\delta\right)^2}{AB_\delta} \quad AP_\delta := AB_\delta - BP_\delta$$

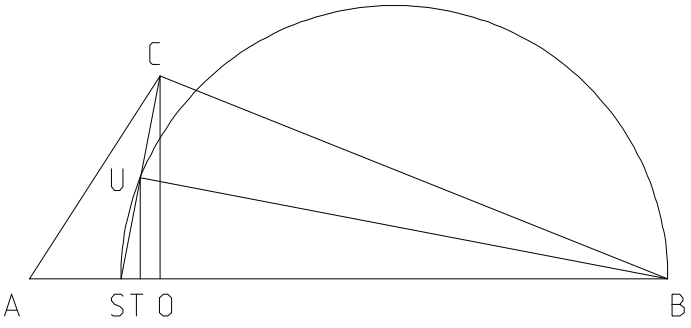
$$NP_\delta := AP_\delta - AN_\delta \quad NO_\delta := \frac{NP_\delta}{2}$$

$$AO := AN + NO \quad BO_\delta := AB_\delta - AO_\delta$$

$$CO_\delta := \sqrt{\left(BC_\delta\right)^2 - \left(BO_\delta\right)^2}$$

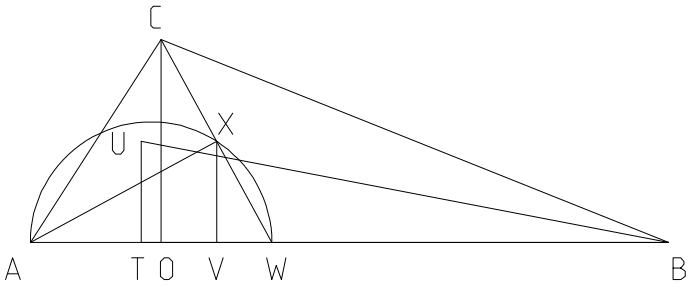
$$BS_\delta := BC_\delta \quad SO_\delta := BS_\delta - BO_\delta$$

$$CS_\delta := \sqrt{\left(SO_\delta\right)^2 + \left(CO_\delta\right)^2} \quad SU_\delta := \frac{CS_\delta}{2}$$




$$BU_\delta := \sqrt{\left(BS_\delta\right)^2 - \left(SU_\delta\right)^2}$$

$$ST_\delta := \frac{\left(SU_\delta\right)^2}{BS_\delta} \quad TU_\delta := \sqrt{\left(SU_\delta\right)^2 - \left(ST_\delta\right)^2}$$

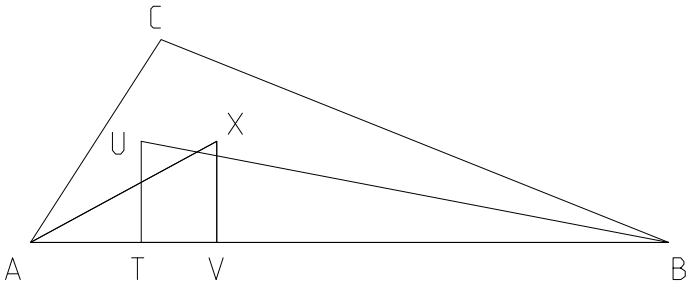



$$AW_\delta := AC_\delta \quad WO_\delta := AW_\delta - AO_\delta$$

$$CW_\delta := \sqrt{(WO_\delta)^2 + (CO_\delta)^2}$$

$$WX_\delta := \frac{CW_\delta}{2}$$

$$AX_\delta := \sqrt{(AW_\delta)^2 - (WX_\delta)^2}$$

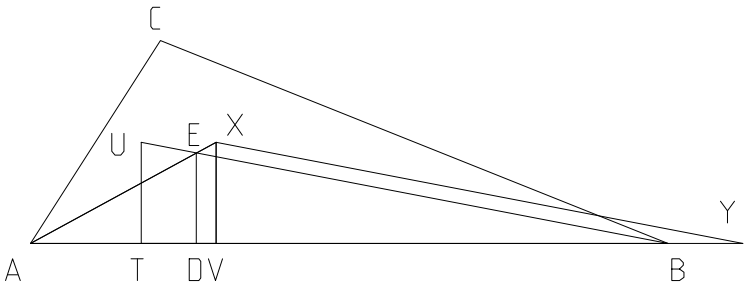



$$WV_\delta := \frac{(WX_\delta)^2}{AW_\delta}$$

$$VX_\delta := \sqrt{(WX_\delta)^2 - (WV_\delta)^2}$$

$$WV_\delta := \sqrt{(WX_\delta)^2 - (VX_\delta)^2}$$

$$AV_\delta := AW_\delta - WV_\delta \quad BV_\delta := AB_\delta - AV_\delta \quad XY_\delta := \frac{BU_\delta \cdot VX_\delta}{TU_\delta} \quad VY_\delta := \sqrt{(XY_\delta)^2 - (VX_\delta)^2}$$




$$AY_\delta := AV_\delta + VY_\delta$$

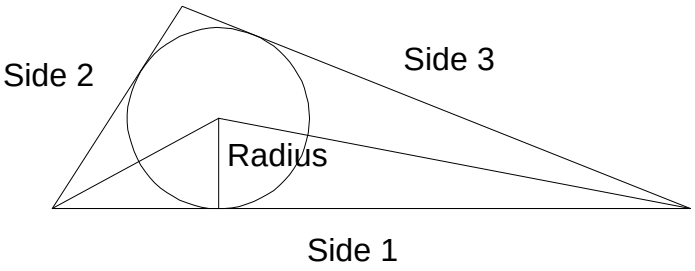
$$AD_\delta := \frac{AV_\delta \cdot AB_\delta}{AY_\delta} \quad AE_\delta := \frac{AX_\delta \cdot AD_\delta}{AV_\delta}$$

$$DE_\delta := \sqrt{(AE_\delta)^2 - (AD_\delta)^2}$$

Plug Side Values In Here  Side_1 ≡ 20  Side_2 ≡ 26  Side_3 ≡ 21

$AB^T = (\ 20 \quad 26 \quad 21\ )$  Is_This_A_Triangle = 1

Given 3 lengths, no matter what order they are entered, DE should remain a constant.

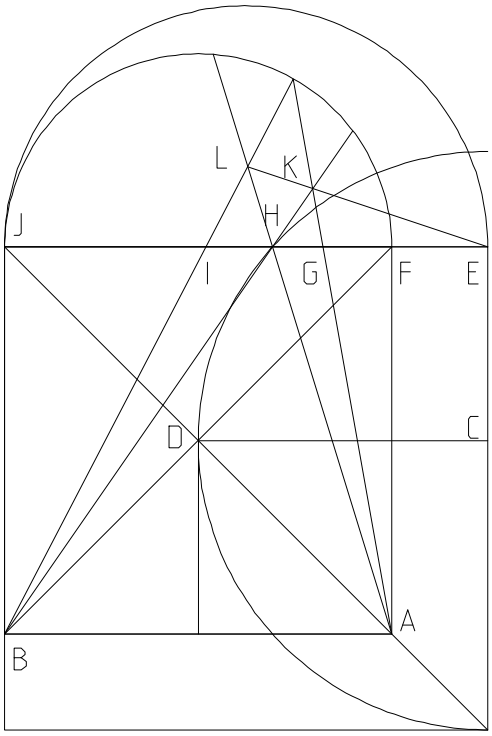



$S_1 := AB \quad S_2 := AC \quad S_3 := BC$

You will note that the formula derived from the process is more consistent with imaginaries.

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}{2\cdot\sqrt{S1_\delta + S2_\delta + S3_\delta}}$$

$$Radius = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix} \qquad DE = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix}$$




# The Cradle

Is EL and EK always collinear?

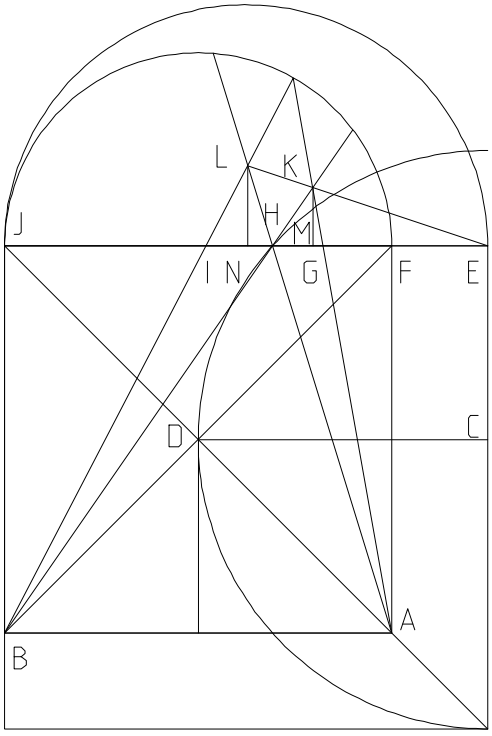
$l := 100 \qquad \delta := 1..l \qquad EF_\delta := \delta^5 \cdot 10^{-8}$

$FJ := 10 \qquad EJ_\delta := FJ + EF_\delta$

$EH_\delta := \sqrt{EF_\delta \cdot EJ_\delta} \qquad FH_\delta := EH_\delta - EF_\delta$

$EG_\delta := \left[ \left( EF_\delta \right)^2 \cdot EJ_\delta \right]^{\frac{1}{3}} \qquad FG_\delta := EG_\delta - EF_\delta$

$EI_\delta := \left[ \left( EJ_\delta \right)^2 \cdot EF_\delta \right]^{\frac{1}{3}} \qquad FI_\delta := EI_\delta - EF_\delta$



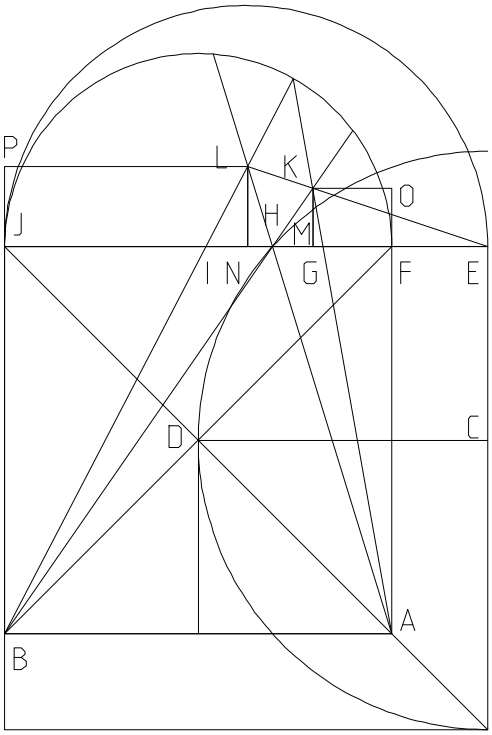

Basically it is demonstrated that the two triangles EKM and ELN are proportional, which is sufficient.

$IJ_\delta := FJ - FI_\delta$

$HJ_\delta := FJ - FH_\delta$

$FN_\delta := \frac{FH_\delta \cdot FJ}{\left( FH_\delta + IJ_\delta \right)} \qquad EN_\delta := FN_\delta + EF_\delta$

$FM_\delta := \frac{FG_\delta \cdot FJ}{\left( FG_\delta + HJ_\delta \right)} \qquad EM_\delta := FM_\delta + EF_\delta$




$$JN_\delta := FJ - FN_\delta \quad LP_\delta := JN_\delta$$

$$BJ := FJ \quad BP_\delta := \frac{BJ \cdot LP_\delta}{IJ_\delta}$$

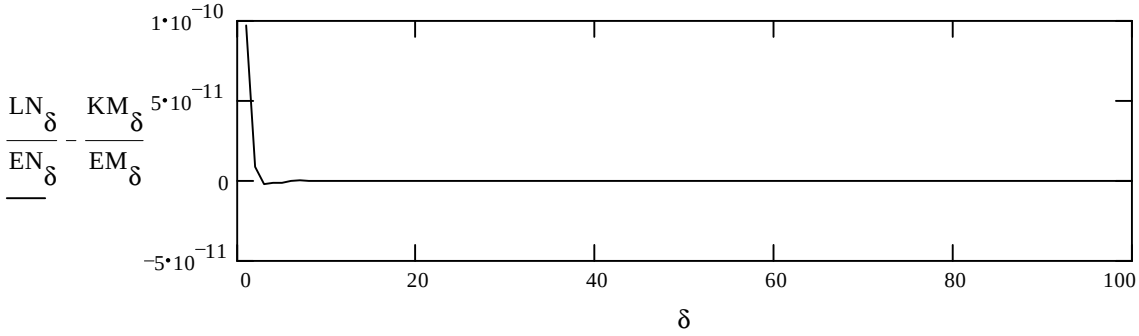
$$JP_\delta := BP_\delta - BJ \quad LN_\delta := JP_\delta$$

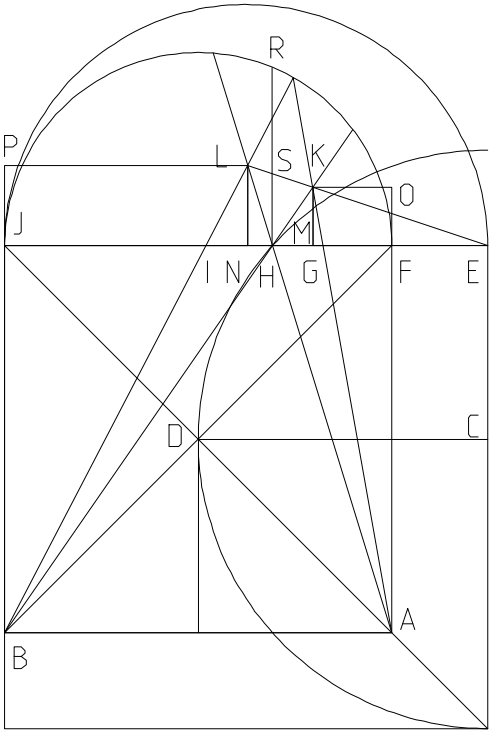
$$KO_\delta := FM_\delta \quad AF := FJ$$

$$AO_\delta := \frac{AF \cdot KO_\delta}{FG_\delta} \quad FO_\delta := AO_\delta - AF$$

$$KM_\delta := FO_\delta \qquad \sum_\delta \left( \frac{LN_\delta}{EN_\delta} - \frac{KM_\delta}{EM_\delta} \right) = 1.015 \cdot 10^{-10}$$

They are two lines with identical slopes, terminating at the same point.

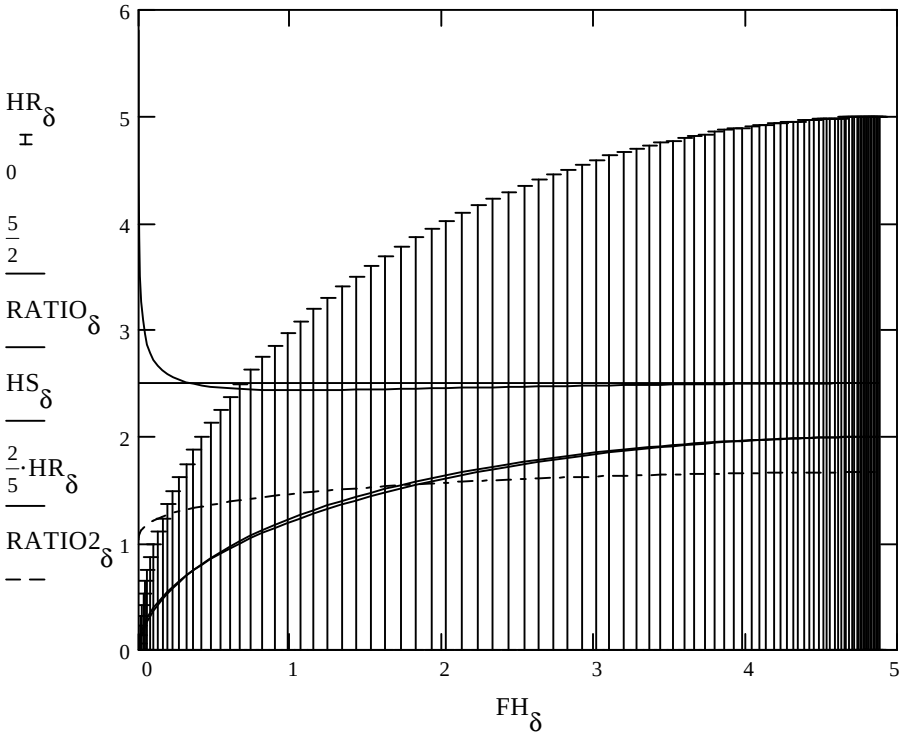





$$HR_\delta := \sqrt{FH_\delta \cdot HJ_\delta} \qquad HS_\delta := \frac{LN_\delta \cdot EH_\delta}{EN_\delta}$$

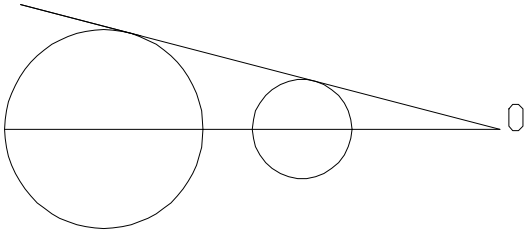
$$RATIO_\delta := \frac{HR_\delta}{HS_\delta} \qquad GI_\delta := EI_\delta - EG_\delta$$

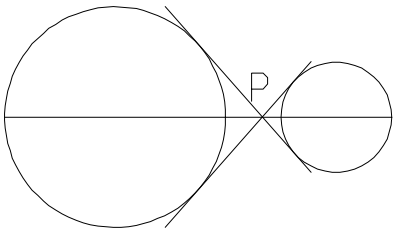
$$RATIO2_\delta := \frac{GI_\delta}{HS_\delta}$$

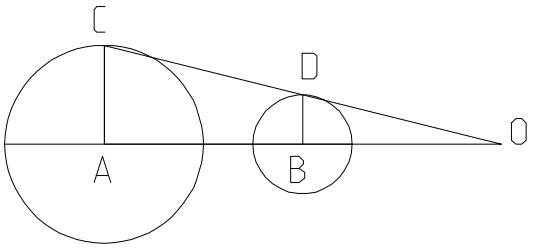


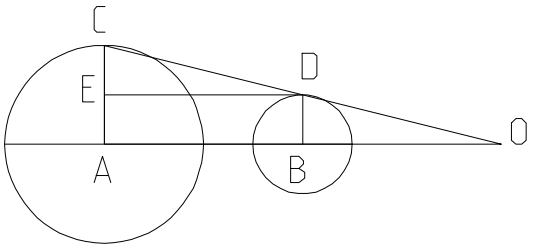
# Tangents and Similarity Points

04_26_94.MCD




O and P are points of origin for the ratio of the two circles that can also have a tangent ray to both circles. Develop formulas that would locate the particular points given using just the radius of the two circles and the difference between them.

O and P are called the similarity points (sp) of the two circles. O is the external similarity point and P is the internal similarity point.

I will work with point O first.

Given $R_L$ = large radius

$R_S$ = small radius

D = difference between origins.

$R_L := 4 \quad R_S := 1 \quad D := 8$

$AC := R_L \quad BD := R_S \quad AB := D$

If the difference between the circles is less than $R_L$-$R_S$, than one of course has an imaginary situation for the external similarity point, $R_L + R_S$ for the internal. At $R_L$-$R_S$ the smaller is in the larger and they touch at one point, $R_L + R_S$ they are external to one another and touching.

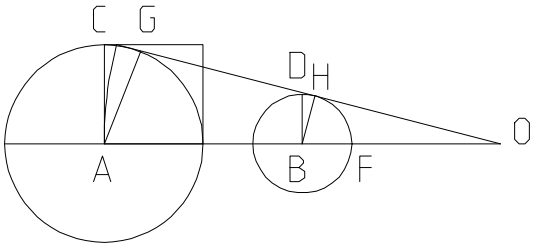
$DE := AB \quad AE := BD \quad CE := AC - AE$

$$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$$

$EOR_L$ "External similarity point Origin to center of Radius Large"

$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right)$$

$EOR_L = 10.667$




What is the length of line (OG) tangent to both circles?

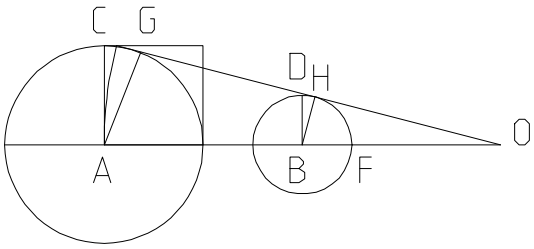
$AG := AC \quad GO := \sqrt{AO^2 - AG^2}$

$GO = 9.888$

And what is the formula?
$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"

$$EOT_{LR} := R_L \cdot \frac{\sqrt{(R_L - R_S + D)\cdot(-R_L + R_S + D)}}{R_L - R_S}$$

$EOT_{LR} = 9.888$




What is the length of the line tangent to the least circle (HO)?

$BH := BD \quad BO := AO - AB \qquad BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

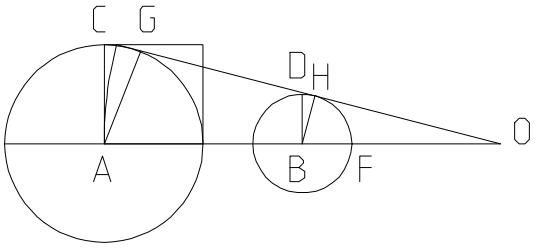
$HO = 2.472$

And what is the formula?

$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S}$$

$EOT_{SR} = 2.472$




Lastly what is the length of line from tangent to tangent of these circles?

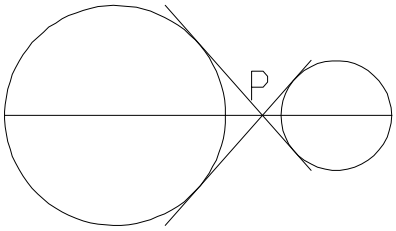
$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

And what is the formula?
ETT "Tangent to Tangent"

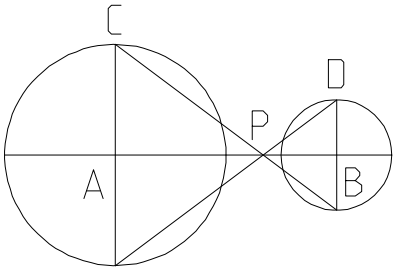
$$ETT := \sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}$$

$ETT = 7.416$




I will now turn my attention to the point P, the internal similarity point.

$$AP := \frac{AB\cdot AC}{AC + BD} \qquad AP = 6.4$$



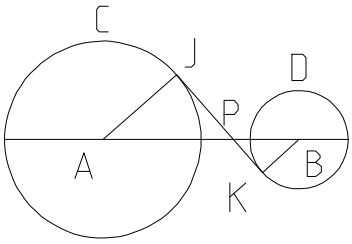

$IOR_L$ "Internal similarity point to center of Radius Large"

$$IOR_L := D\cdot\frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

$IOR_S$ "Internal similarity point to center of Radius Small"

$$IOR_S := D\cdot\frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$



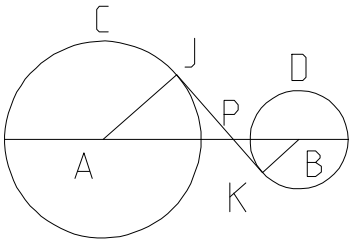

$AJ := AC \quad BK := BD \quad JP := \sqrt{AP^2 - AJ^2}$

$JP = 4.996$

$IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"

$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996$

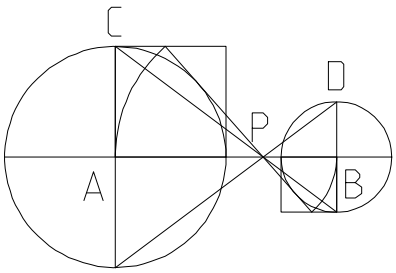



$KP := \sqrt{BP^2 - BK^2} \quad KP = 1.249$

$IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249$



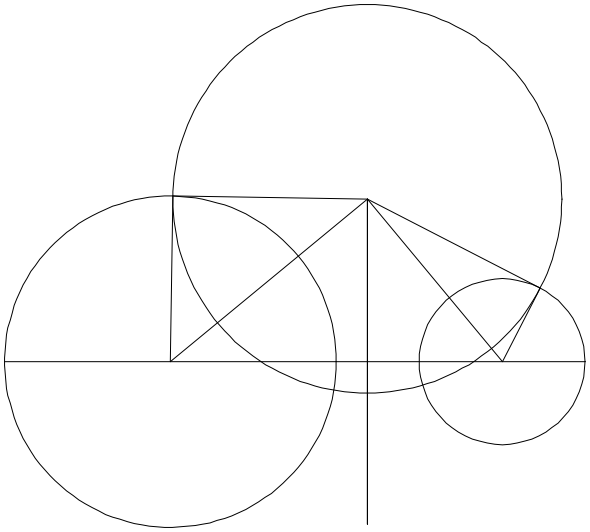

$JK := JP + KP \quad JK = 6.245$

ITT "Internal similarity point Tangent to Tangent"

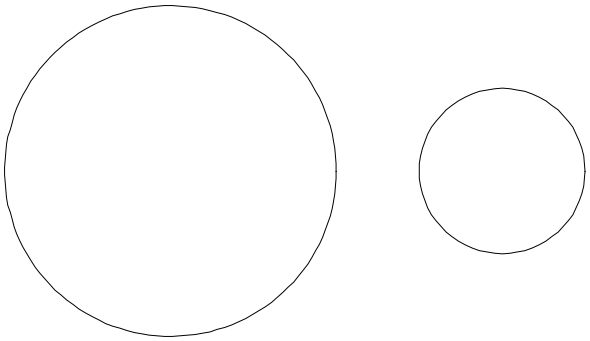
$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}$$

$ITT = 6.245$

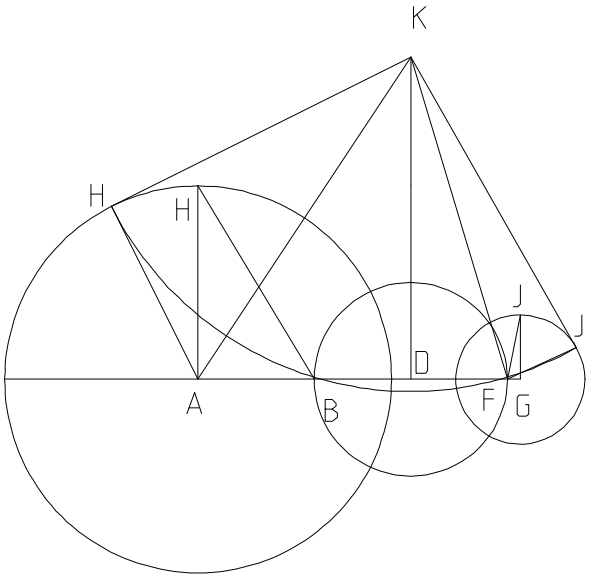
# The Chordal or Power Line of two Circles 04_27_94.MCD



The solution for finding the chordal as outlined in ***100 Great Problems of Elementary Mathematics Their History and Solution*** by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.
The figure I work with is a transformation of the one on the left.



Given two circles find their chordal or power line given just their radius and difference between their centers, and reduce the tautological chains to formulas.




$R_1 := 3 \quad R_2 := 1 \quad D := 1$

$AH := R_1 \quad GJ := R_2 \quad AG := D$

$AB := \frac{AH^2}{AG} \quad FG := \frac{GJ^2}{AG}$

$BF := AG - AB - FG \quad BD := \frac{BF}{2}$

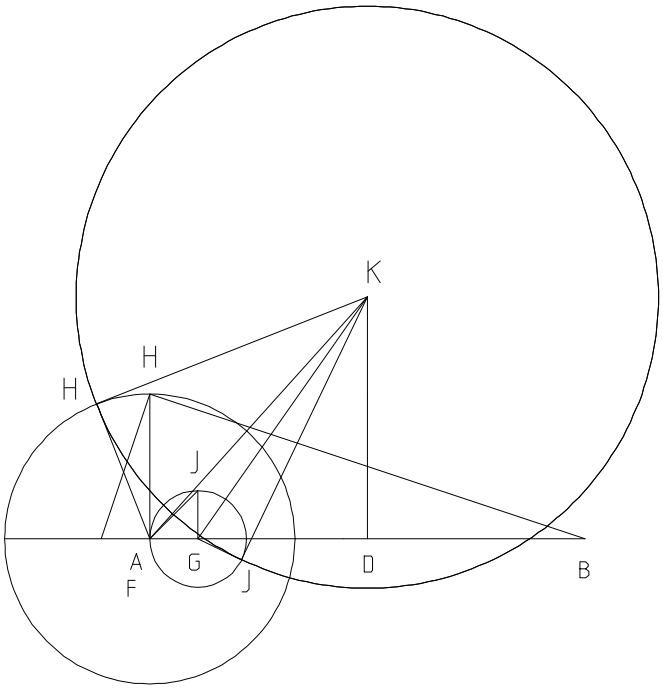
$AD := AB + BD \quad DF := BD$

$DG := DF + FG$

CR "Center to Radius"

$$CR_1 := \frac{1}{2}\cdot\frac{\left(R_1^2 - R_2^2 + D^2\right)}{D} \qquad CR_1 = 4.5 \qquad AD = 4.5$$

$$CR_2 := \frac{1}{2}\cdot\frac{\left(R_2^2 - R_1^2 + D^2\right)}{D} \qquad CR_2 = -3.5 \qquad DG = -3.5$$




**Locate length K here** $DK := 5$

$$AK := \sqrt{DK^2 + AD^2} \qquad GK := \sqrt{DK^2 + DG^2}$$

$$HK := \sqrt{AK^2 - AH^2} \qquad JK := \sqrt{GK^2 - GJ^2}$$

$HK = 6.021$

LP "Length of Power line"

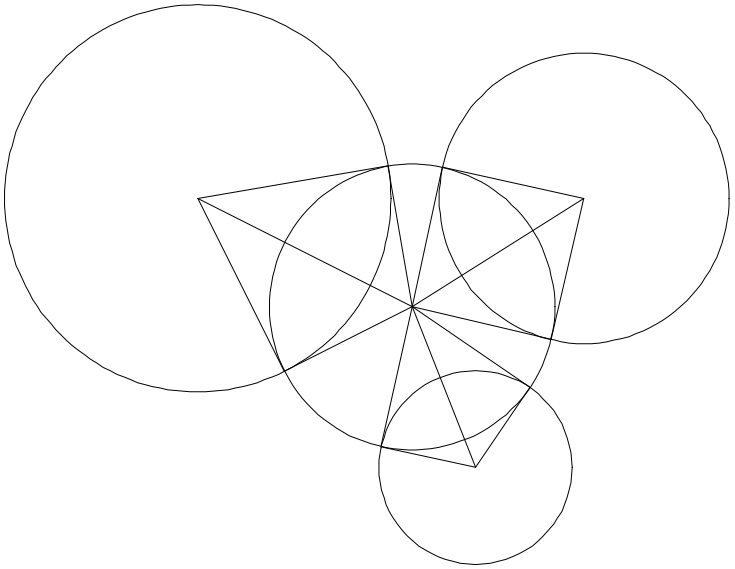
$LP := DK$

LT "Length of radial Tangent" LP is the variable chosen for circle center on the power line.

$$LT := \frac{1}{2}\cdot\left|\frac{\sqrt{4\cdot LP^2\cdot D^2 + R_1^4 - 2\cdot R_1^2\cdot D^2 - 2\cdot R_1^2\cdot R_2^2 + D^4 - 2\cdot D^2\cdot R_2^2 + R_2^4}}{D}\right|$$

$LT = 6.021$ $JK = 6.021$

The process does not seem to recognize any special cases.



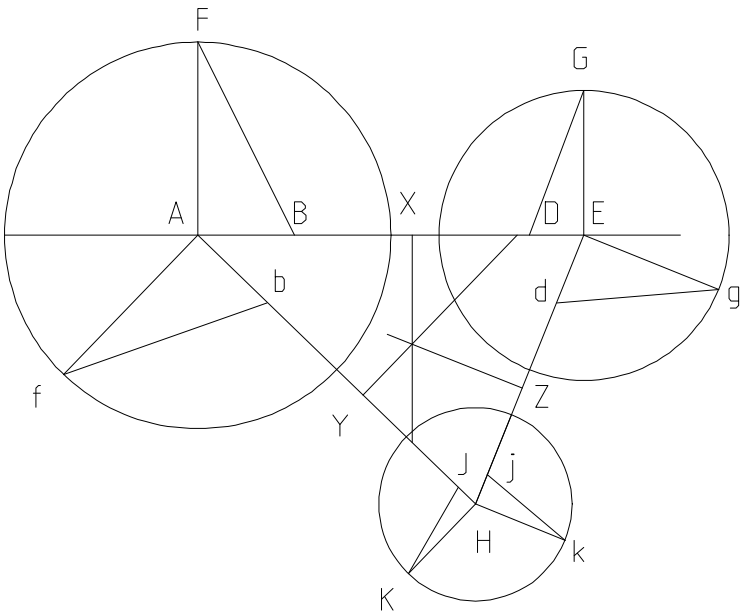
# Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the formula for the Power Point and the Length of the resultant Tangent.

The distance between each set of circles is given as $D_1$, $D_2$, and $D_3$. Naturally they must form a triangle.

$D_1 := 4 \quad D_2 := 5 \quad D_3 := 2$

$\Delta := \left(D_1 + D_2 \geq D_3\right) \cdot \left(D_2 + D_3 \geq D_1\right) \cdot \left(D_1 + D_3 \geq D_2\right)$




$\Delta = 1$ $\Delta$ "Is this a Triangle?

$R_1 := 3 \quad R_2 := 2 \quad R_3 := 4$

$AE := D_1 \quad AH := D_2 \quad EH := D_3$

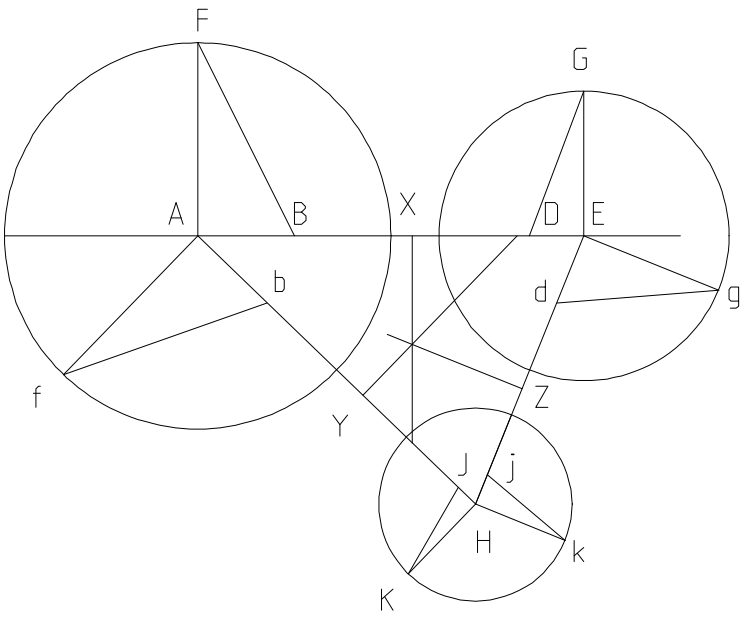
$AF := R_1 \quad HK := R_2 \quad EG := R_3$

$Af := AF \quad Hk := HK \quad Eg := EG$

$AB := \frac{AF^2}{AE} \quad DE := \frac{EG^2}{AE}$

$Ab := \frac{Af^2}{AH} \quad HJ := \frac{HK^2}{AH}$

$Hj := \frac{Hk^2}{EH} \quad Ed := \frac{Eg^2}{EH}$




$BD := AE - AB - DE \quad BX := \frac{BD}{2}$

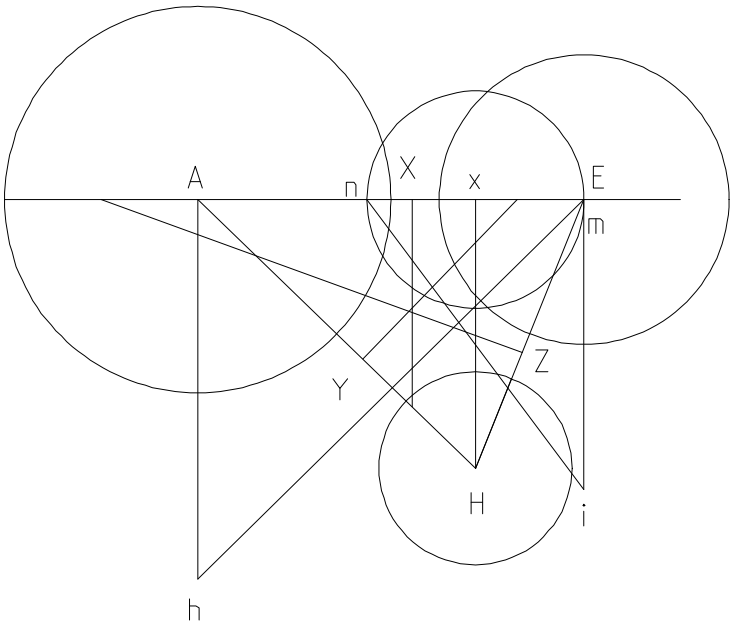
$bJ := AH - Ab - HJ \quad bY := \frac{bJ}{2}$

$dj := EH - Ed - Hj \quad dZ := \frac{dj}{2}$

$AX := AB + BX \quad AX = 1.125$

$AY := Ab + bY$

$EZ := Ed + dZ \quad EZ = 4$



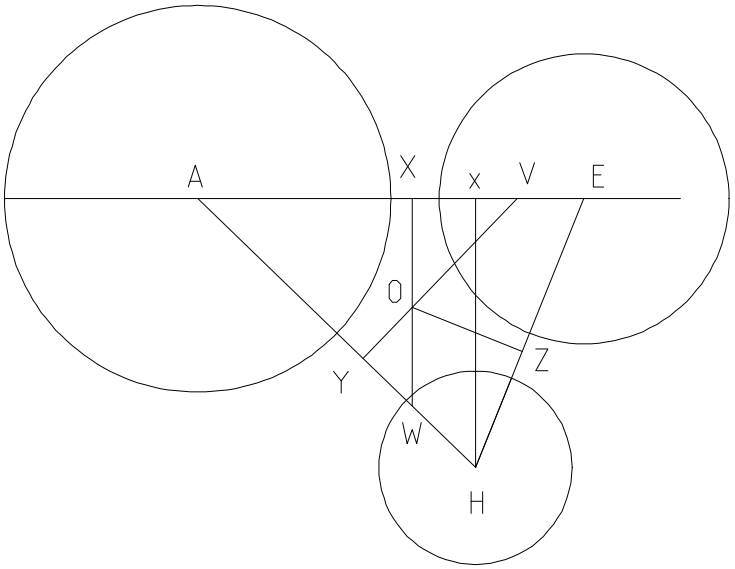

$Ah := AH \quad Ei := EH$

$Am := \frac{Ah^2}{AE} \quad En := \frac{Ei^2}{AE}$

$An := AE - En \quad mn := Am - An$

$nx := \frac{mn}{2} \quad Ax := An + nx$

$Hx := \sqrt{AH^2 - Ax^2}$




$$WX := \frac{Hx \cdot AX}{Ax} \qquad WX = 0.462$$

$$VY := \frac{Hx \cdot AY}{Ax} \qquad VY = 1.232$$

$$AV := \frac{AH \cdot AY}{Ax} \qquad AV = 3.243$$

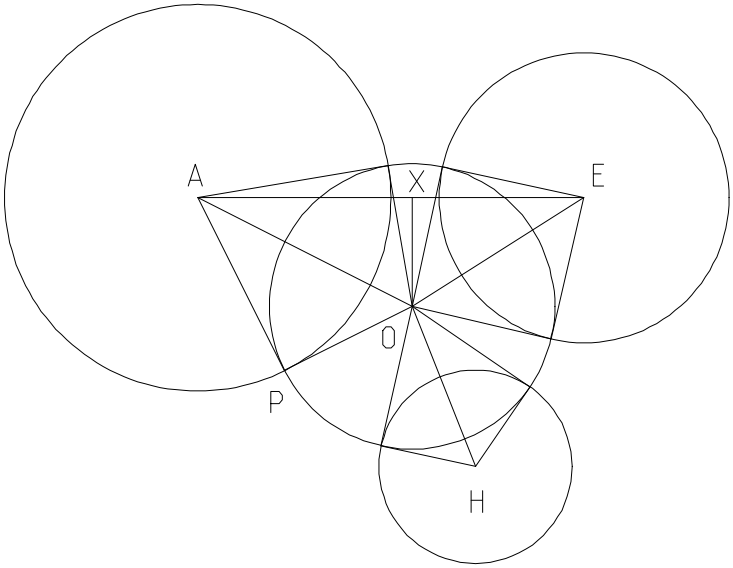
$$VX := AV - AX$$

$$OX := \frac{AY \cdot VX}{VY} \qquad OX = 5.157$$

PP "Power Point"

$$PP := \frac{1}{2} \cdot \frac{\left( \begin{array}{l} D_1^2 \cdot R_1^2 + D_1^2 \cdot D_2^2 - 2 \cdot D_1^2 \cdot R_2^2 + R_1^2 \cdot D_3^2 - R_1^2 \cdot D_2^2 - D_1^4 + D_1^2 \cdot D_3^2 \ldots \\ + R_3^2 \cdot D_1^2 - R_3^2 \cdot D_3^2 + R_3^2 \cdot D_2^2 \end{array} \right)}{\left( D_1 \cdot \sqrt{\left| -2 \cdot D_1^2 \cdot D_2^2 + D_1^4 - 2 \cdot D_1^2 \cdot D_3^2 + D_3^4 - 2 \cdot D_2^2 \cdot D_3^2 + D_2^4 \right|} \right)}$$

$$PP = 5.157$$




$EX := AE - AX$

$AO := \sqrt{AX^2 + OX^2}$ $\quad EO := \sqrt{EX^2 + OX^2}$

$AP := AF$ $\quad EP := EG$

$OP_A := \sqrt{AO^2 - AP^2}$ $\quad OP_E := \sqrt{EO^2 - EP^2}$

$OP_A = 4.342$ $\quad OP_E = 4.342$

LT "Length of Tangent"

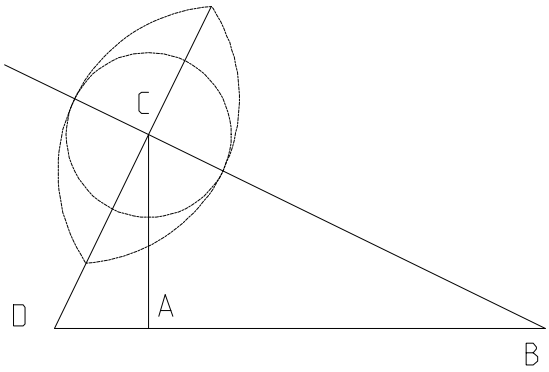
$$LT := \frac{\sqrt{\begin{aligned}&-R_1^4\cdot D_3^2 - R_1^2\cdot R_3^2\cdot D_1^2 - R_1^2\cdot D_3^4 + D_1^2\cdot R_1^2\cdot D_3^2 - D_1^2\cdot D_3^2\cdot D_2^2 \ldots\\&+ R_2^2\cdot R_1^2\cdot D_3^2 - R_2^2\cdot R_1^2\cdot D_2^2 - R_2^2\cdot R_3^2\cdot D_3^2 + R_2^2\cdot R_3^2\cdot D_2^2 \ldots\\&+ R_3^2\cdot D_1^2\cdot D_2^2 - R_3^2\cdot D_2^4 - R_3^4\cdot D_2^2 + D_1^2\cdot R_1^2\cdot R_2^2 + D_1^2\cdot D_2^2\cdot R_2^2 \ldots\\&+ D_1^2\cdot R_2^2\cdot D_3^2 + D_1^2\cdot R_2^2\cdot R_3^2 + R_1^2\cdot R_3^2\cdot D_3^2 + R_3^2\cdot D_3^2\cdot D_2^2 - D_1^2\cdot R_2^4 \ldots\\&+ -D_1^4\cdot R_2^2 + R_1^2\cdot D_3^2\cdot D_2^2 + R_1^2\cdot R_3^2\cdot D_2^2\end{aligned}}}{\sqrt{-2\cdot D_1^2\cdot D_2^2 + D_1^4 - 2\cdot D_1^2\cdot D_3^2 + D_3^4 - 2\cdot D_3^2\cdot D_2^2 + D_2^4}}$$

$LT = 4.342$

# Division, $A^2$

04_30_94.MCD

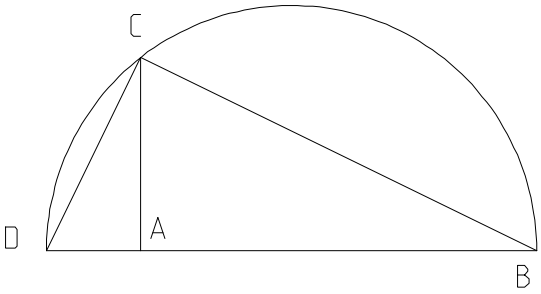
One does not work with geometry often, so it may be that one does not keep basics in mind when trying to work a figure. This little paper is about a basic move. I bring this to light, as I have seen a ratio often given as division. In geometry, so far as I know, one cannot divide a line by a line, but can form a series of the nature $A^N : B^N$ as a linear figure. Given A and B one can raise them to any whole power simultaneously with a couple of simple moves based on the figure immediately below. See work done in 1995.

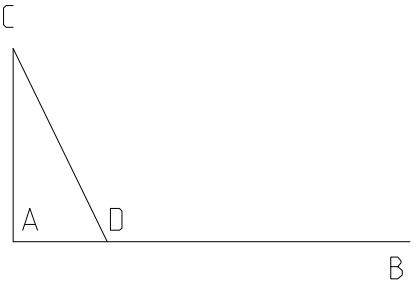



Divide $AC^2$ by AB.

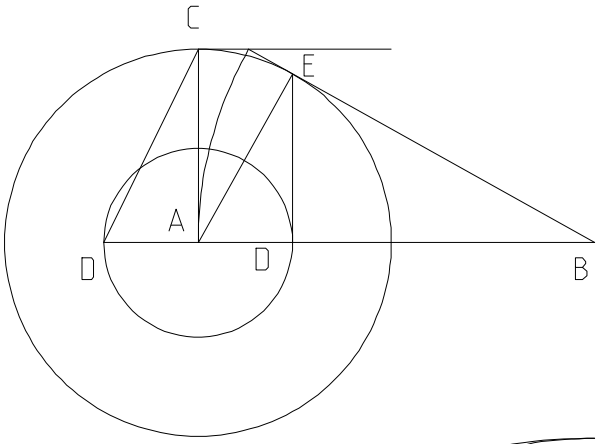
Process Summary

I have noticed that my solutions depend upon this basic move.

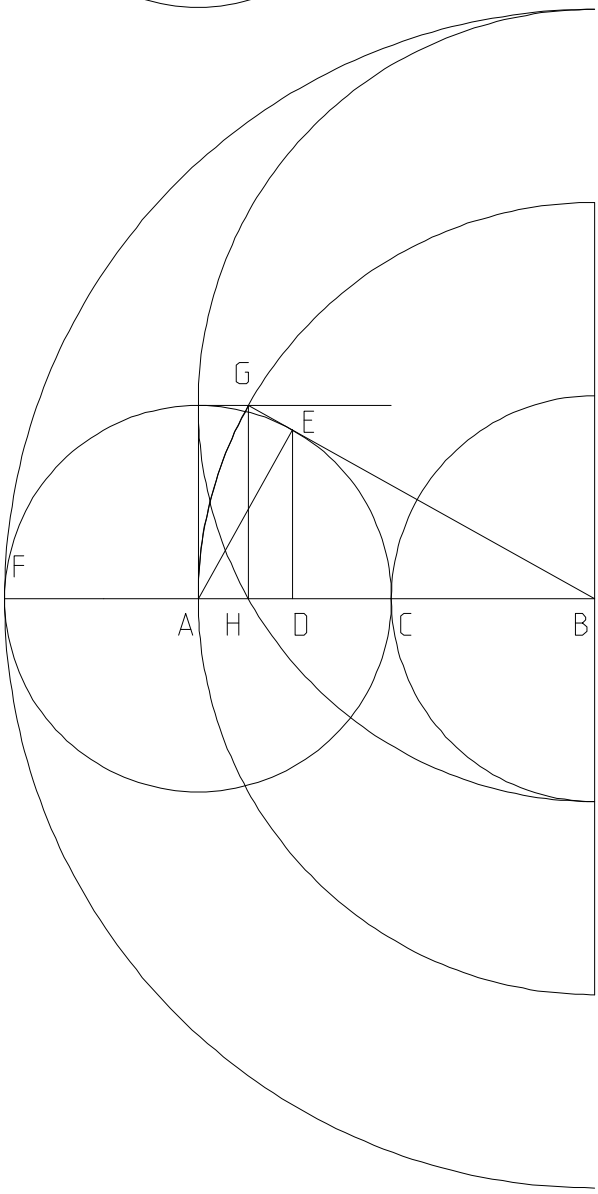



In some works I have represent it simply as the figure on the left. One may realize that in my Pythagorean Completion I used the circular form and it is often expressed as a pole and polar arrangement. A terminology that does not seem fit for the processes that they represent, physical and not mathematical.




The jargon is that B is called a Pole and D is on a polar, a segment of which is DE. But it can easily be seen that the figure is a transformation. It is another way of dividing $AC^2$ by AB. The figure now raises a question for me. I had thought that I answered it previously, but I cannot find it in my files. Is BE always collinear with BG? This paper is helping me mediate on poles and polars, which names do not help me understand the true ratio involved. Mathematicians seem to like a proliferation of names. And, god forbid, B and D are called conjugate in respect to each other. I have a hard enough time remembering my own name, that is why I keep my id. (in the Freudian sense) close at hand.




$AC := 5 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CF := 2 \cdot AC \quad AG := AC \quad BF_\delta := CF + BC_\delta$

$BH_\delta := \sqrt{BF_\delta \cdot BC_\delta} \quad GH := AC$

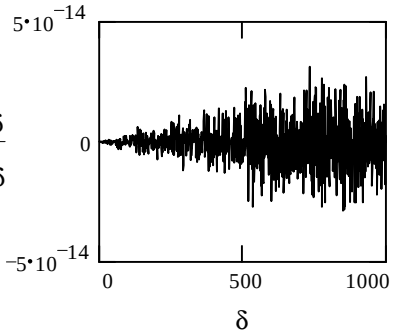
$BG_\delta := \sqrt{(BH_\delta)^2 + GH^2} \quad AE := AC$

$AB_\delta := AC + BC_\delta \quad AD_\delta := \frac{AE^2}{AB_\delta}$

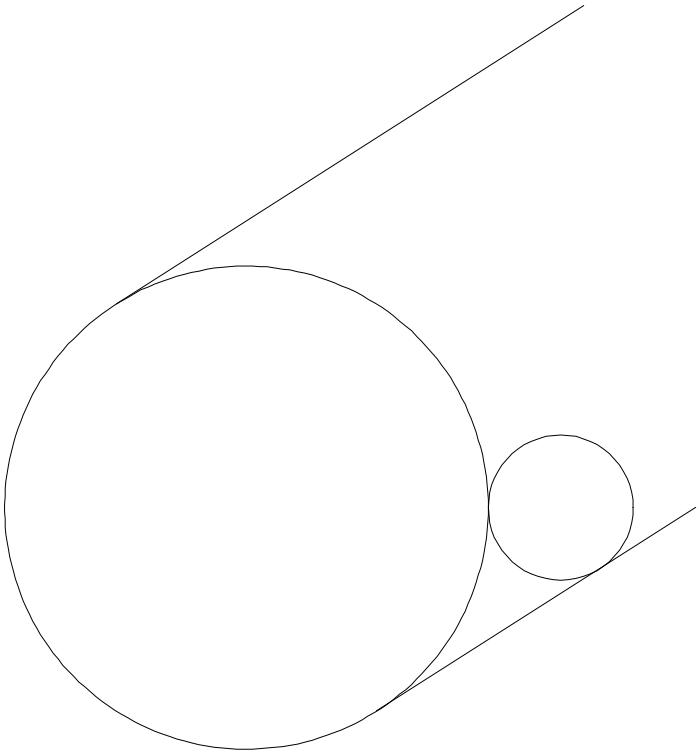
$BE_\delta := \sqrt{(AB_\delta)^2 - AE^2} \quad BD_\delta := AB_\delta - AD_\delta$

$DE_\delta := \sqrt{(BE_\delta)^2 - (BD_\delta)^2}$

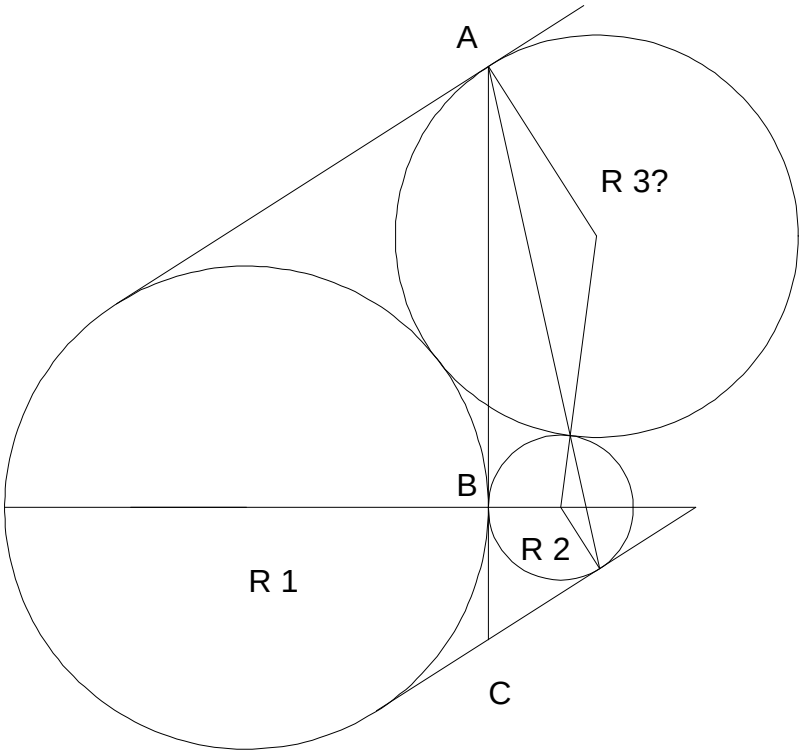
$\frac{GH}{BH_\delta} - \frac{DE_\delta}{BD_\delta}$



# Two Circles and a Parallel

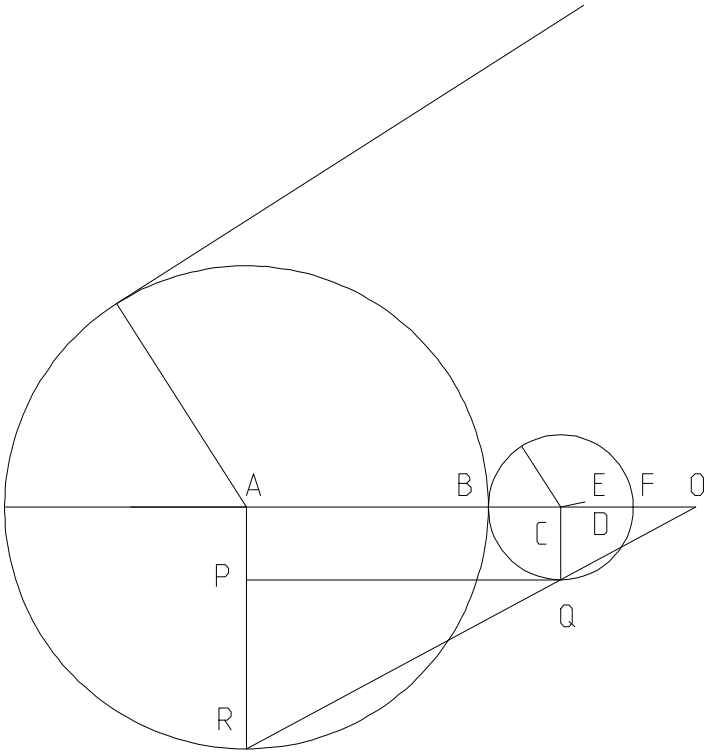


What I would like to do is to is drop in a circle that is tangent to the given two circles that are already tangent and tangent to the line from the similarity point and also have this circle tangent to the parallel of the similarity line that lies tangent to the first circle. The formula derived for my process tells me that I will do it the hard way. It will ***predict*** an easier method.




Process Summary

Given the radius of the two circles, what is the radius of the third? Attempt to develop a formula for the resultant radius. And also, of the power line between parallels, what is the ratio of AC:BC in terms of the given radius' ?




Find the Similarity Point.

$AB := R_1 \quad CF := R_2 \quad BC := CF$

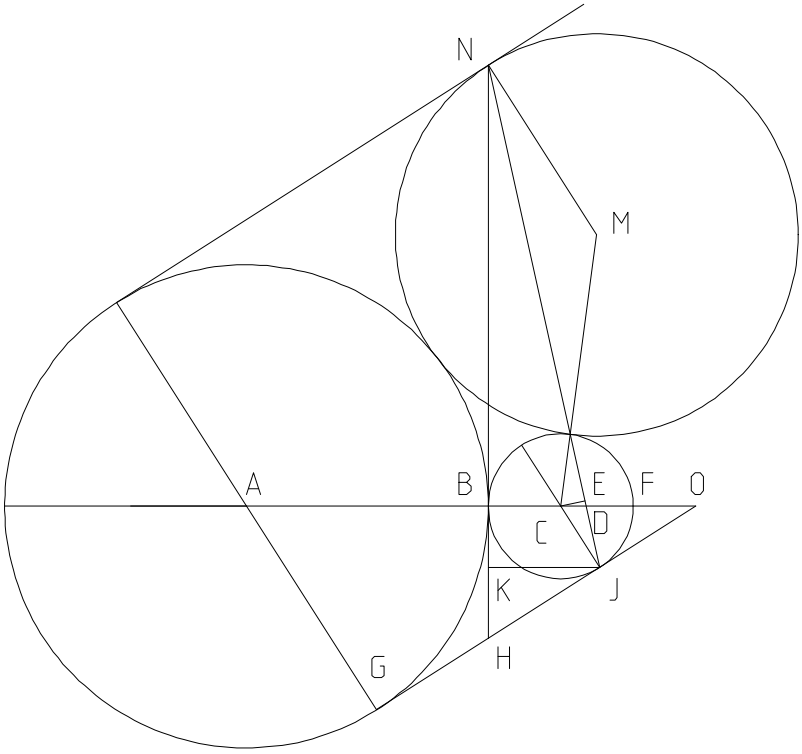
$CB := CF \quad AC := AB + BC$

$PQ := AC \quad AR := AB$

$CQ := CF \quad AP := CQ$

$PR := AR - AP$

$AO := \frac{PQ \cdot AR}{PR}$




Find the segment of the power line (HN) between parallels.

$AG := AB \quad GO := \sqrt{AO^2 - AG^2}$

$BO := AO - AB$

$BH := \frac{AG \cdot BO}{GO} \quad BN := \frac{BH \cdot AB}{BC}$

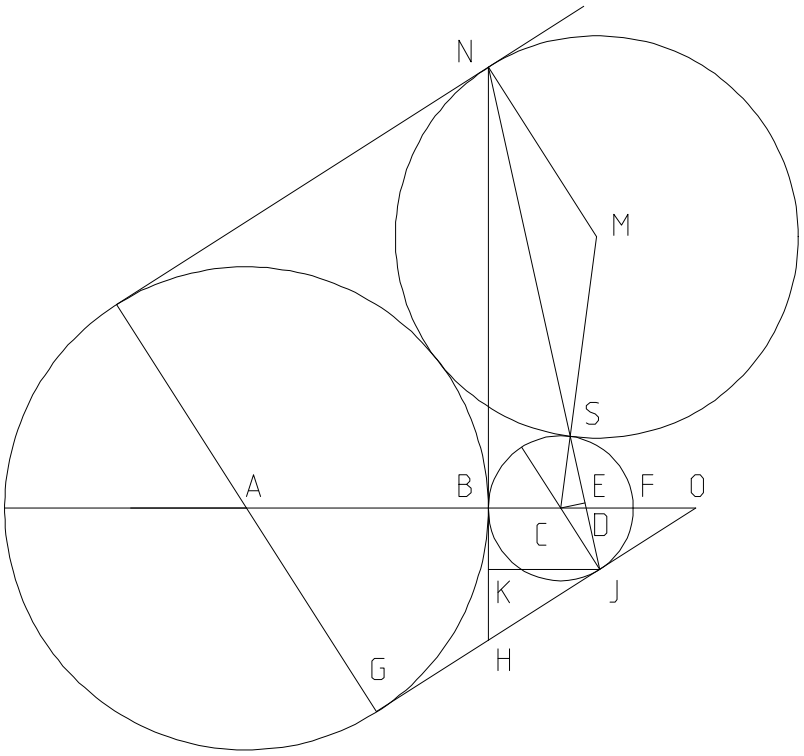
$HO := \frac{AO \cdot BH}{AG} \quad CO := BO - BC$

$JO := \frac{GO \cdot CO}{AO} \quad HN := BH + BN$

Find JN

$HJ := HO - JO \quad KH := \frac{AG \cdot HJ}{AO}$

$KN := HN - KH$




$KJ := \sqrt{HJ^2 - KH^2}$

$JN := \sqrt{KN^2 + \text{Find MS}}$

$BD := \frac{KJ \cdot BN}{KN}$ $CD := BD - BC$

$CE := \frac{KN \cdot CD}{JN}$ $CJ := BC$

$EJ := \sqrt{CJ^2 - CE^2}$ $CS := 2 \cdot EJ$

$NS := JN - JS$ $CS := BC$

$MS := \frac{CS \cdot NS}{JS}$

## Plug Values in Here

$R_1 \equiv 8$ $R_2 \equiv 6$

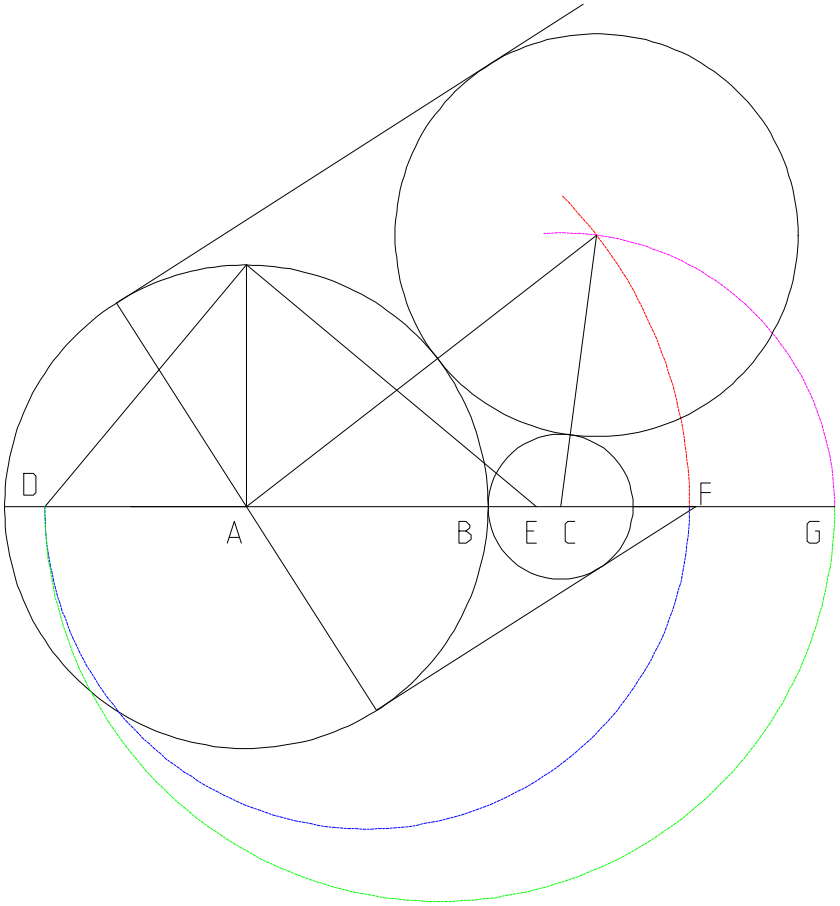
There was too much work here for the symbolic processor to reduce all the equations to one, easily. It took me three days to nurse the processor through it, and this is a short work. The formula does not have the "sour" spot. (Place both $R_1$ and $R_2$ to the same value to see it.) When $R_2$ is 1/4 of $R_1$, there are six tangents.

$$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad R_3 = 2.667 \qquad MS = 2.667$$

The formula tells me at least two things, 1) There is a second method to solve the problem, 2) the process is a rather baroque method of dividing a square.

$$\frac{HN}{BN} = 1.75 \qquad \frac{R_1 + R_2}{R_1} = 1.75 \qquad \frac{HN}{BH} = 2.333 \qquad \frac{R_1 + R_2}{R_2} = 2.333$$

The results for the ratios of the power line segment are very nice also.




What is the construction suggested by the found formula of

$$\frac{R_1^2}{4 \cdot R_2} \quad ?$$

$AB := R_1 \quad BC := R_2$

$AE := 4 \cdot BC$

$AD := \frac{AB^2}{AE}$

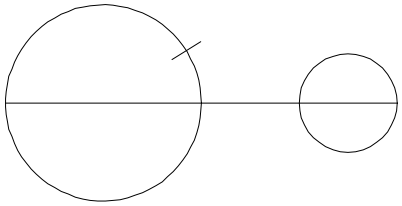
$AF := AB + AD$

$CG := BC + AD$

$R_3 = 2.667 \qquad AD = 2.667$

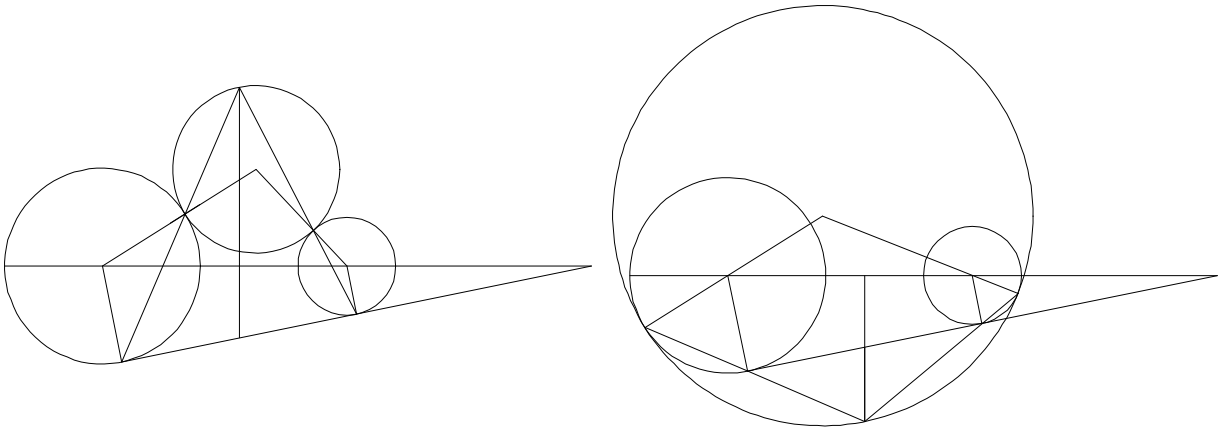
For those who may become confused as to the dashed lines, the segment AD is added to the radius of the both circles their intersection is the center of the circle sought.

# Two Circles, given a tangent on one.

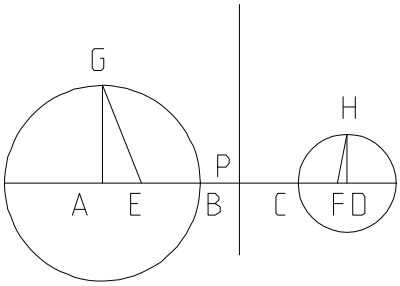


Given two circles and a point that is on the circumference of one, find a circle tangent to the circle at that point and also tangent to the other circle. The convention for this point will be "from the power line".

Process Summary



Find the power line.



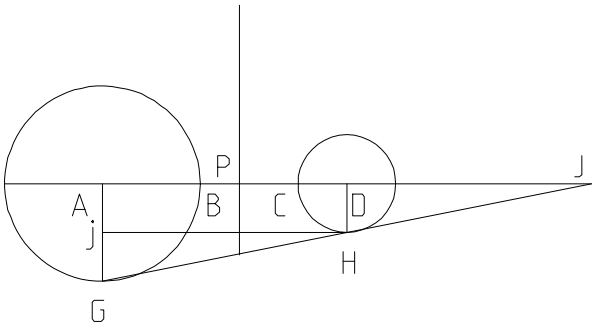

$AB := R_1$ $CD := R_2$ $AD := D$

$AG := AB$ $DH := CD$ $AE := \frac{AG^2}{AD}$

$DF := \frac{DH^2}{AD}$ $EF := AD - AE - DF$

$EP := \frac{EF}{2}$ $AP := AE + EP$ $DP := DF + EP$

$BP := AP - AB$




$Hj := AD$ $Aj := DH$

$Gj := AG - DH$ $AJ := \frac{Hj \cdot AG}{Gj}$

$DJ := AJ - AD$ $HJ := \sqrt{DJ^2 - DH^2}$

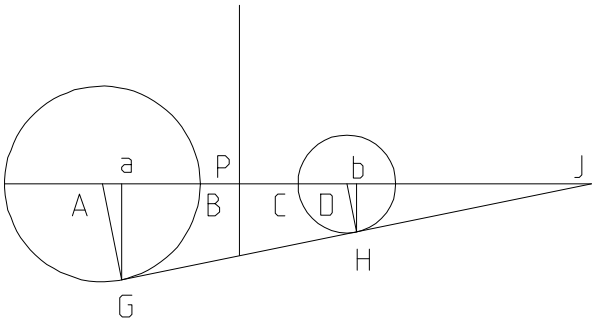
$GJ := \sqrt{AJ^2 - AG^2}$ $Aa := \frac{AG^2}{AJ}$

$Ga := \sqrt{AG^2 - Aa^2}$ $Db := \frac{DH^2}{DJ}$

$Hb := \sqrt{DH^2 - Db^2}$

$Ba := AB - Aa$

$Pb := DP + Db$




$P := \left| \text{if}(P \le 2 \cdot AB, P, 0) \right|$

$Bd := P$

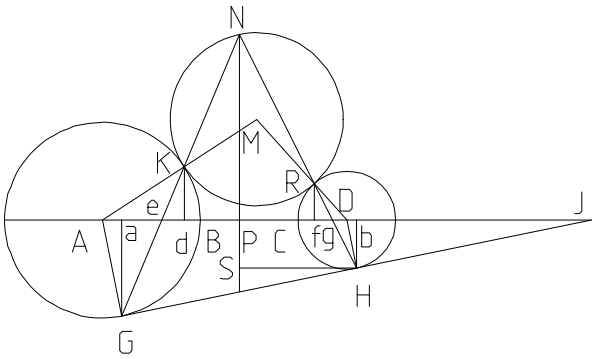
$AK := AB$ $ad := Ba - Bd$

$Ad := AB - Bd$ $Kd := \sqrt{AK^2 - Ad^2}$

$de := \frac{ad \cdot Kd}{Kd + Ga}$ $Pe := BP + Bd + de$

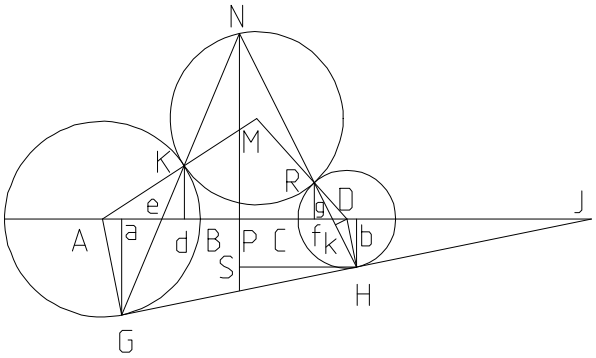
$NP := \frac{Kd \cdot Pe}{de}$ $HS := Pb$ $PS := Hb$

$NS := NP + PS$ $Pg := \frac{HS \cdot NP}{NS}$



$Dg := DP - Pg \qquad bg := Dg + Db$

$Hg := \sqrt{bg^2 + Hb^2}$




To save clutter, see 07_18_93.MCD Mod C.

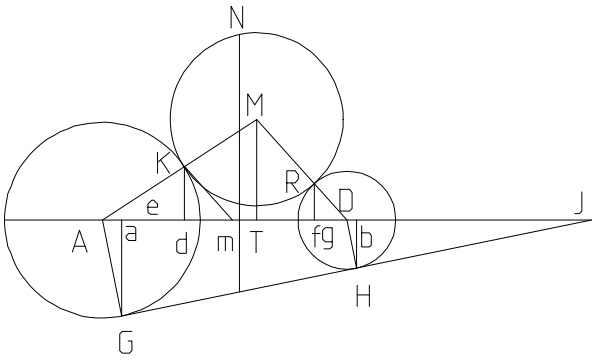
$$gk := \frac{1}{2}\cdot\frac{Dg^2}{Hg} + \frac{1}{2}\cdot Hg - \frac{1}{2}\cdot\frac{DH^2}{Hg}$$

$Dk := \sqrt{Dg^2 - gk^2} \qquad Hk := \sqrt{DH^2 - Dk^2}$

$Rg := Hk - gk$

$$Df := \frac{1}{2}\cdot\frac{CD^2}{Dg} + \frac{1}{2}\cdot Dg - \frac{1}{2}\cdot\frac{Rg^2}{Dg}$$

$DR := CD \qquad Rf := \sqrt{DR^2 - Df^2}$




$$dm := \frac{Df\cdot Kd}{Rf} \qquad Am := Ad + dm$$

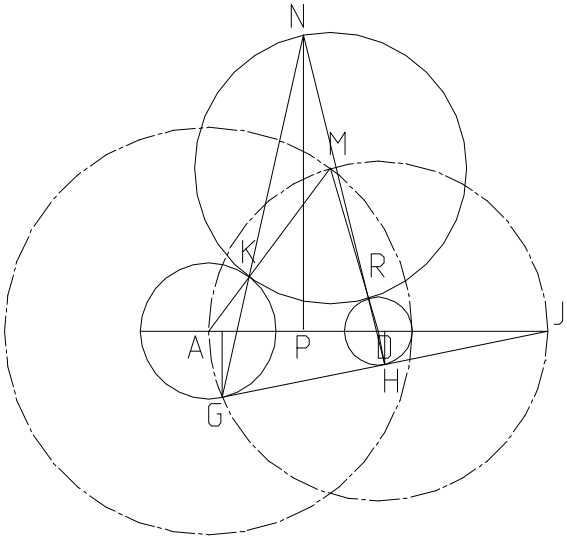
$$MT := \frac{Kd\cdot AD}{Am} \qquad AM := \frac{AK\cdot MT}{Kd}$$

$KM := AM - AK$

$R_3 := |KM| \qquad P = 19$

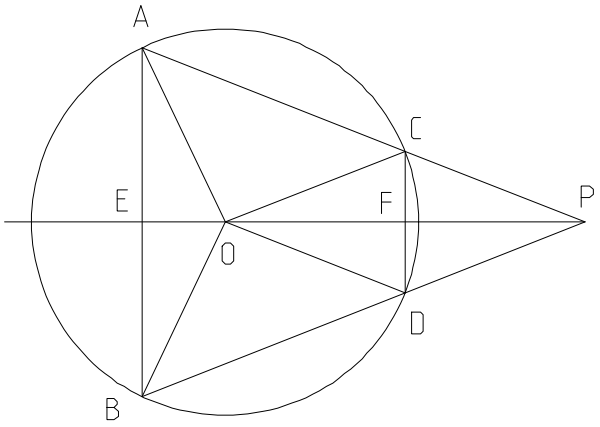
Plug Values in Here. Results is in $P_3$.

$R_1 \equiv 10 \quad R_2 \equiv 3 \quad D \equiv 16 \quad P \equiv 19 \quad R_3 = 14.836$



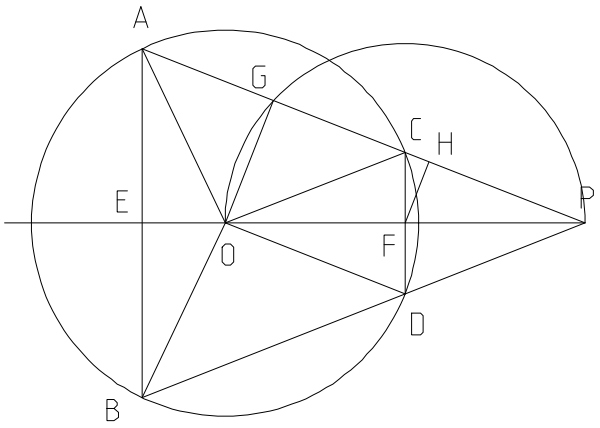

Let us say that instead of choosing a point of tangency upon one of the circles, I wish to place a given circle tangent to both. Derive the name of the length of the power line indicated and find th points of tangency.

Work in progress.




Given that CP = CO what is the relationship between AE and CF?

In order to derive a resonable answer, I have constrained the figure to a maximum of 180º. You will recall that the figure is capable of 270º.




$$CF_\delta := \frac{\frac{1}{2}\cdot\sqrt{2}\cdot CP}{\delta} \qquad FP_\delta := \sqrt{CP^2 - \left(CF_\delta\right)^2}$$

$$CH_\delta := \frac{\left(CF_\delta\right)^2}{CP} \qquad FH_\delta := \sqrt{\left(CF_\delta\right)^2 - \left(CH_\delta\right)^2}$$

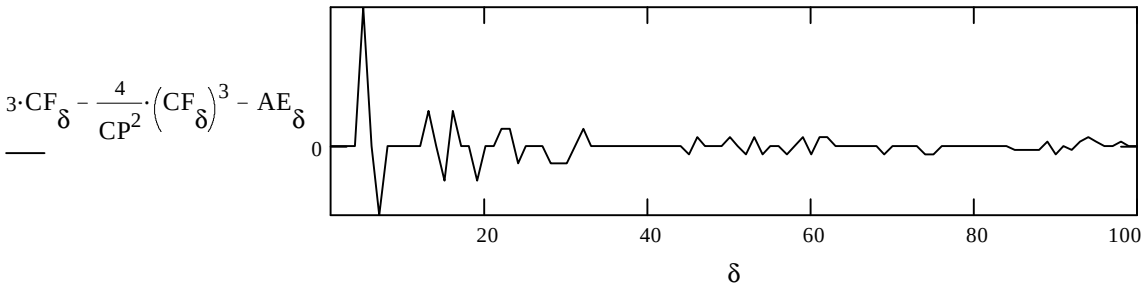
$$OP_\delta := 2\cdot FP_\delta \qquad GO_\delta := \frac{FH_\delta\cdot OP_\delta}{FP_\delta} \qquad CO := CP$$

$$CG_\delta := \sqrt{CO^2 - \left(GO_\delta\right)^2} \qquad AC_\delta := 2\cdot CG_\delta$$

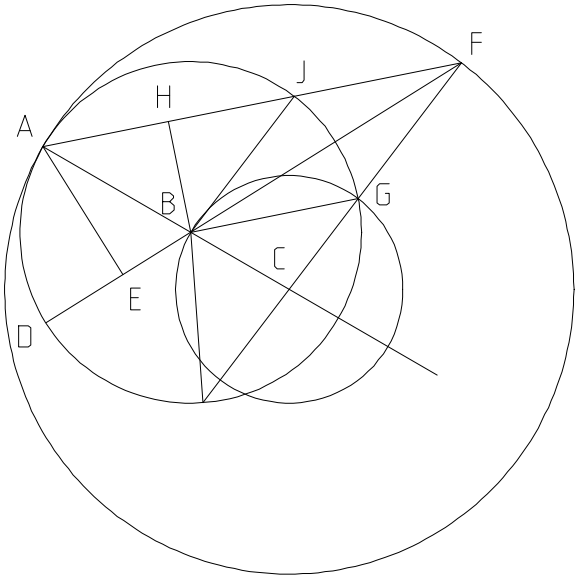
$$AP_\delta := AC_\delta + CP \qquad AE_\delta := \frac{CF_\delta\cdot AP_\delta}{CP}$$

CF is adjusted through D and cannot be less than 1 for the ratio to hold. This constrains the answer to between 0º and 180º.

$$CP \equiv 10 \qquad \delta \equiv 1 \,..\, 100 \qquad 3\cdot CF - \frac{4}{CP^2}\cdot CF^3 - AE = 0$$




The resultant equation seems to support my earlier statement that the same tool used on the cube root figure could also be used on the trisector. To eliminate the radius, simply set it equal to 1.




What is the relationship of DE to BC? $D := 4$

$AB := 99 \quad BC := \frac{AB}{2}\cdot D \quad BG := AB$

$AC := AB + BC \quad AF := \frac{BG\cdot AC}{BC} \quad FJ := AB$

$AJ := AF - FJ \quad HJ := \frac{AJ}{2} \quad BJ := AB$

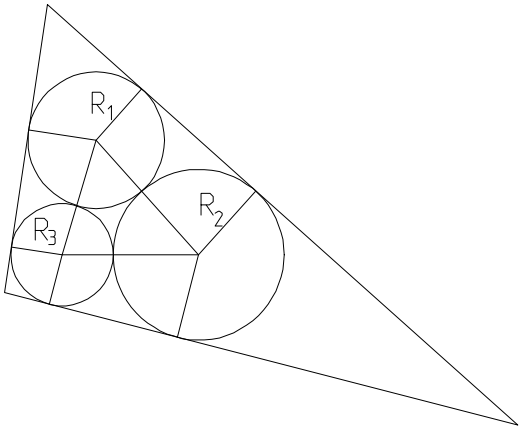
$BH := \sqrt{BJ^2 - HJ^2} \quad FH := HJ + FJ$

$BF := \sqrt{BH^2 + FH^2} \quad BD := AB \quad DF := BF + BD$

$EF := \frac{FH\cdot AF}{BF} \quad DE := DF - EF \quad DE = 138.133$

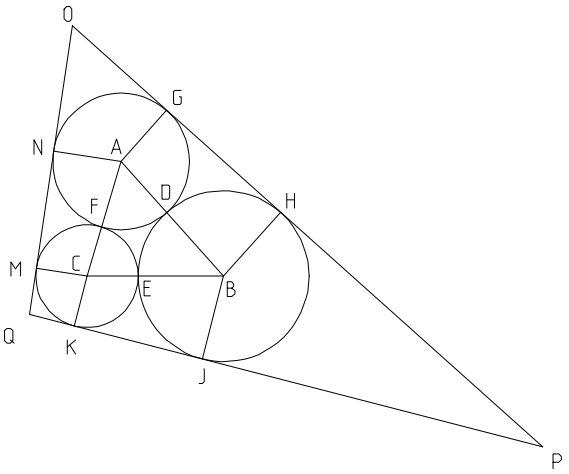
$$\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}} + AB - \left[\frac{1}{2}\cdot AB\cdot\frac{(AB + BC)}{BC} + \frac{1}{2}\cdot AB\right]\cdot AB\cdot\frac{(AB + BC)}{\left[BC\cdot\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}}\right]} = 138.133$$

$$\frac{-1}{2}\cdot AB\cdot\frac{\left[-\sqrt{2\cdot BC + AB}\cdot BC - 2\cdot BC^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{2\cdot BC + AB}\right]}{BC^{\left(\frac{3}{2}\right)}} = 138.133$$




Given the radius of three tangent circles, what is the length of the sides of the triangle that contains them?

$R_1 := 1 \quad R_2 := 2 \quad R_3 := 3$




$AF := R_1 \quad AD := R_1 \quad AF := R_1$

$AN := R_1$

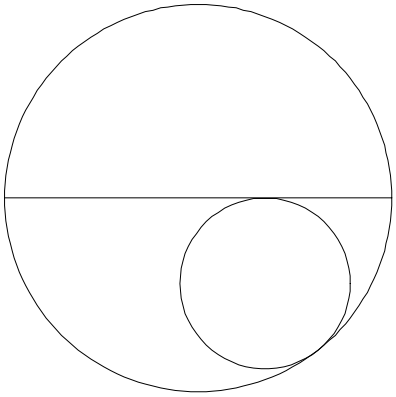
$BD := R_2 \quad BH := R_2 \quad BJ := R_2$

$BE := R_2 \quad AB := AD + BD$

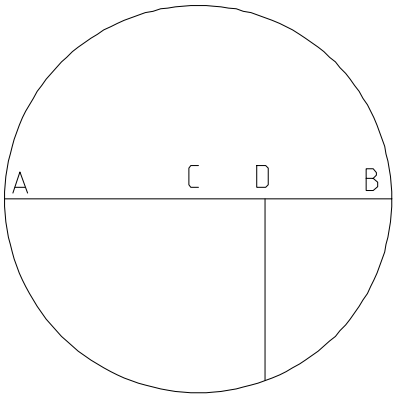
$CE := R_3 \quad CF := R_3 \quad CK := R_3$

$CM := R_3 \quad BC := BE + CE$

$AC := AF + CF$



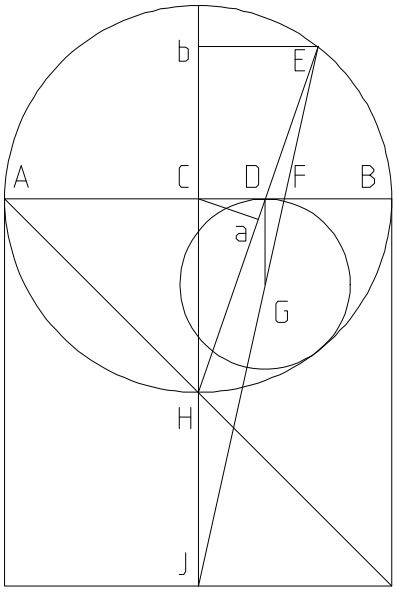
The smaller circle is tangent to the diameter of the larger and also tangent to circumference of the larger. Given the point of tangency on the diameter, what is the radius that will make it tangent to the circumference?




Given a point on the diameter, what is the radius of the inner tangent circle?

$$AB := 100 \quad AC := \frac{AB}{2} \quad CB := AC$$

$$\Delta := 100 \quad \delta := 1 .. \Delta \quad CD_\delta := \frac{CB}{\Delta} \cdot \delta$$




$$CH := AC \quad DH_\delta := \sqrt{\left(CD_\delta\right)^2 + CH^2}$$

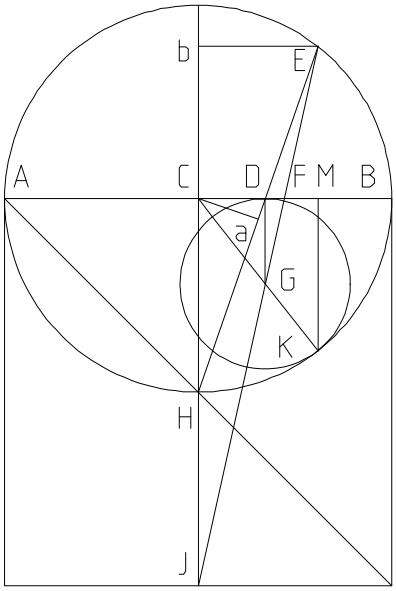
$$Ha_\delta := \frac{CH^2}{DH_\delta} \quad EH_\delta := 2 \cdot Ha_\delta \quad HJ := AC$$

$$Eb_\delta := \frac{CD_\delta \cdot EH_\delta}{DH_\delta} \quad Hb_\delta := \sqrt{\left(EH_\delta\right)^2 - \left(Eb_\delta\right)^2}$$

$$Cb_\delta := Hb_\delta - CH \quad CJ := CH + HJ$$

$$Jb_\delta := CJ + Cb_\delta \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

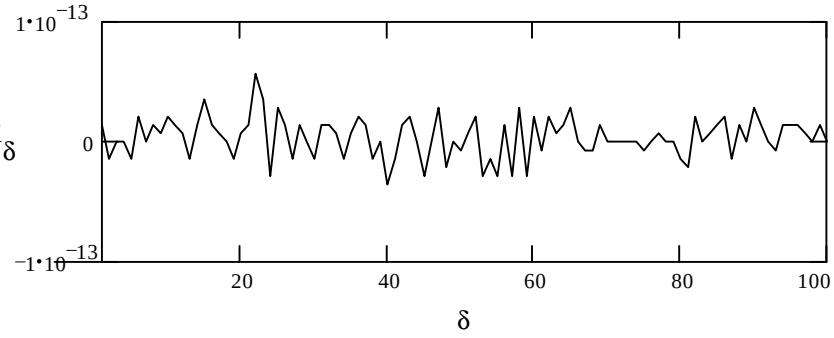


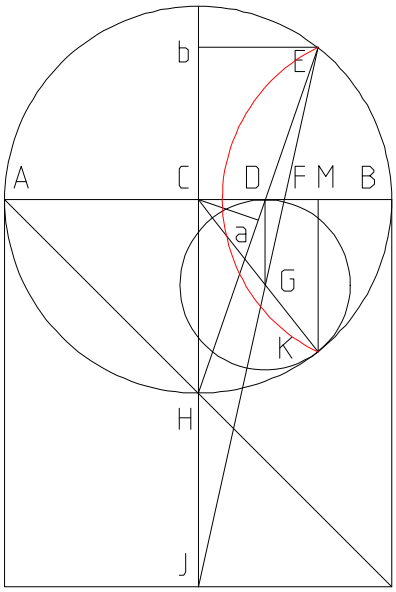

Does CG + GK = AC ?

$CG_\delta := \sqrt{(CD_\delta)^2 + (DG_\delta)^2}$ $GK_\delta := DG_\delta$

$CK_\delta := CG_\delta + GK_\delta$ $\sum_\delta (AC - CK_\delta) = 3.908 \cdot 10^{-13}$

$AC - CK_\delta$

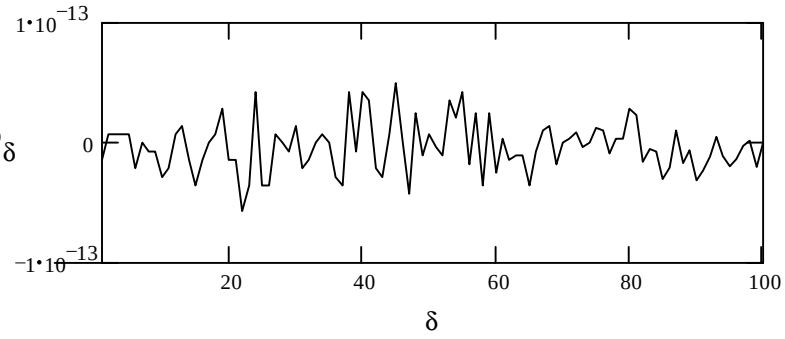


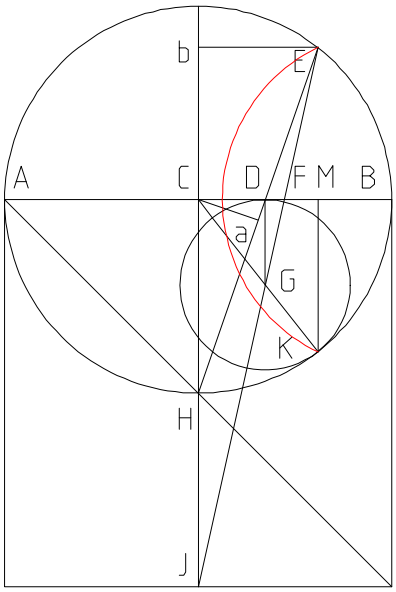




Does Cb = KM?

$KM_\delta := \frac{DG_\delta \cdot CK_\delta}{CG_\delta}$ $\sum_\delta (KM_\delta - Cb_\delta) = -3.524 \cdot 10^{-13}$

$KM_\delta - Cb_\delta$

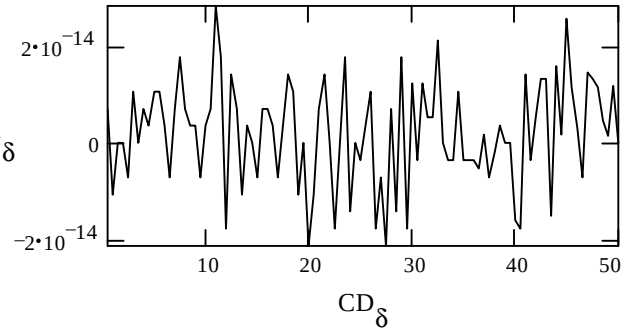


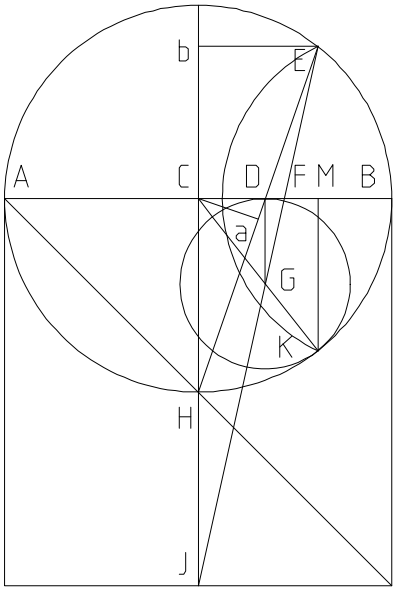



What is the formula for DG when given CD?

$$\sum_{\delta}\left[\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}\right] = 2.072\cdot 10^{-13}$$

$$\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}$$




Given the point on the radius, find DG.

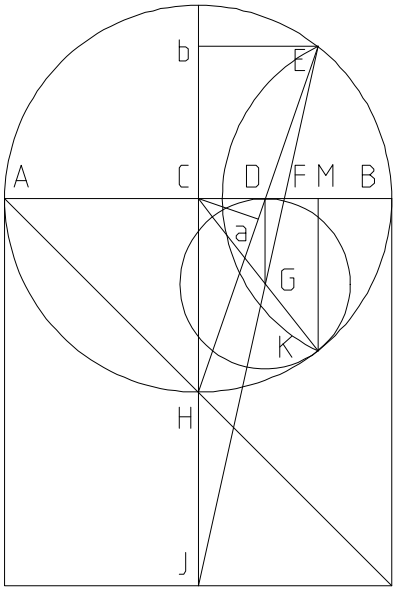
$AB := 4 \quad AC := \frac{AB}{2} \quad CB := AC \quad CM_{\delta} := \frac{CB}{\Delta}\cdot\delta$

$AM_{\delta} := AC + CM_{\delta} \quad BM_{\delta} := CB - CM_{\delta}$

$KM_{\delta} := \sqrt{AM_{\delta}\cdot BM_{\delta}} \quad Cb_{\delta} := KM_{\delta}$

$CH := AC \quad HJ := AC \quad CJ := CH + HJ$

$Jb_{\delta} := CJ + Cb_{\delta} \quad Hb_{\delta} := CH + Cb_{\delta}$



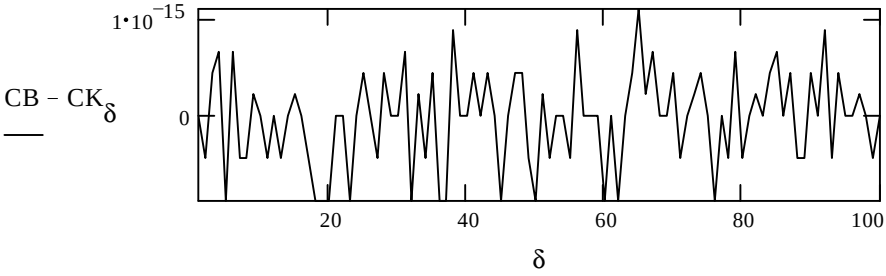

$$Eb_\delta := CM_\delta \quad CD_\delta := \frac{Eb_\delta \cdot CH}{Hb_\delta} \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

Does CK = CB? Make sure there is no typo.

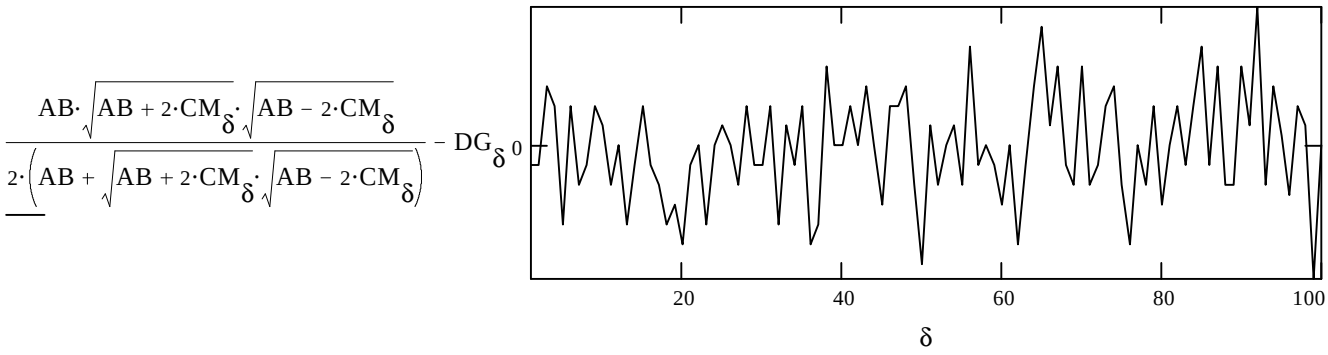
$$CG_\delta := \sqrt{\left(CD_\delta\right)^2 + \left(DG_\delta\right)^2} \quad GK_\delta := DG_\delta$$

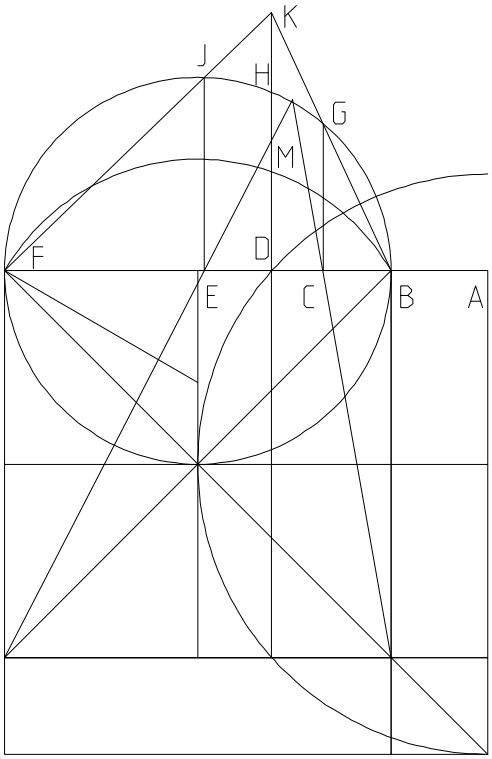
$$CK_\delta := CG_\delta + GK_\delta \quad \sum_\delta \left(CB - CK_\delta\right) = -3.331 \cdot 10^{-15}$$




What is the formula for DG, given CM, the perpendicular to the point on the circumference?

$$\sum_\delta \left[ \frac{AB \cdot \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta}}{2 \cdot \left( AB + \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta} \right)} - DG_\delta \right] = -1.749 \cdot 10^{-15}$$






HM:HK is not a 1:1 ratio, but what does the graph of the ratio look like?

$BF := 3 \qquad \Delta := 500 \qquad \delta := 1 .. \Delta$

$AB_\delta := \delta^5 \cdot 10^{-12} \qquad AF_\delta := AB_\delta + BF$

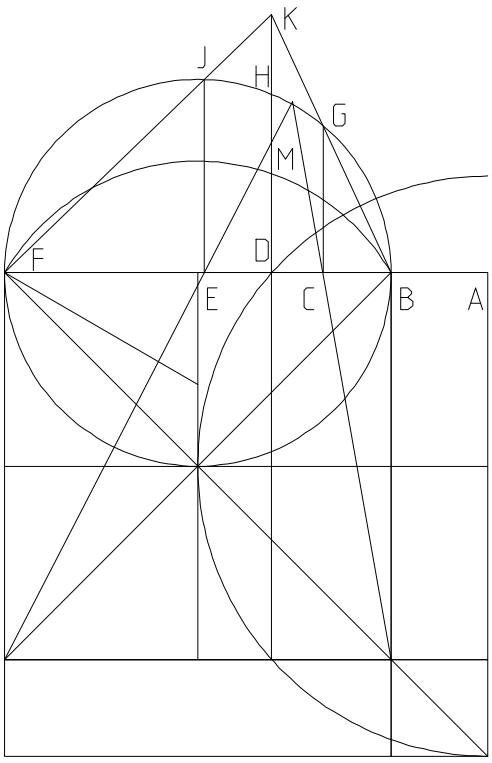
$AD_\delta := \sqrt{AB_\delta \cdot AF_\delta}$

$AE_\delta := \left[ \left( AF_\delta \right)^2 \cdot AB_\delta \right]^{\frac{1}{3}} \qquad BE_\delta := AE_\delta - AB_\delta$

$EF_\delta := BF - BE_\delta \qquad EJ_\delta := \sqrt{BE_\delta \cdot EF_\delta}$

$AC_\delta := \left[ \left( AB_\delta \right)^2 \cdot AF_\delta \right]^{\frac{1}{3}} \qquad BC_\delta := AC_\delta - AB_\delta$

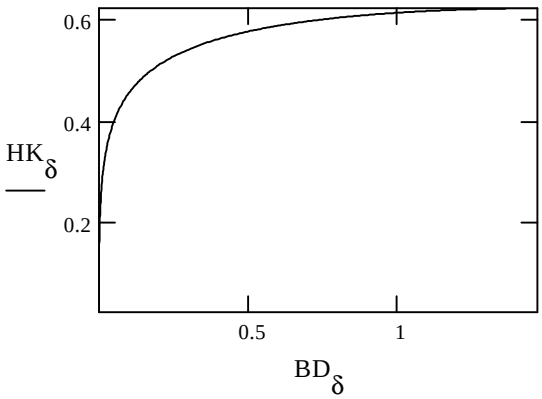
$CF_\delta := BF - BC_\delta \qquad CG_\delta := \sqrt{BC_\delta \cdot CF_\delta}$

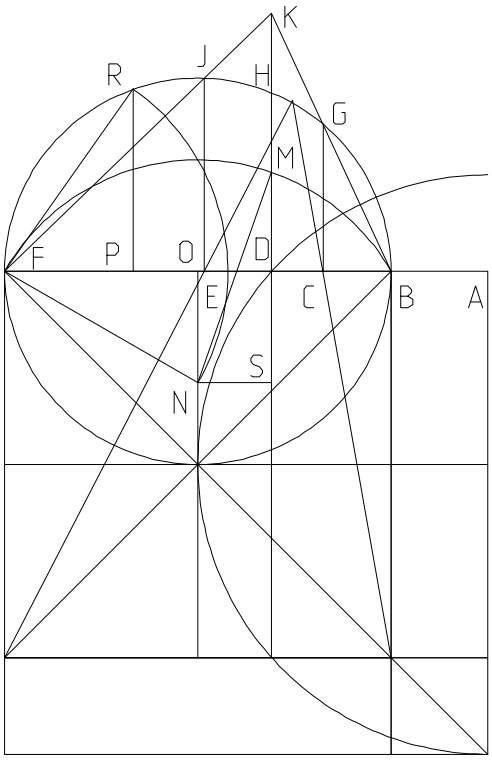



$DF_\delta := AF_\delta - AD_\delta \qquad DK_\delta := \frac{EJ_\delta \cdot DF_\delta}{EF_\delta}$

$BD_\delta := AD_\delta - AB_\delta \qquad DH_\delta := \sqrt{BD_\delta \cdot DF_\delta}$

$HK_\delta := DK_\delta - DH_\delta$






$$FP := \frac{BF}{3} \quad BP := BF - FP \quad PR := \sqrt{BP \cdot FP}$$

$$FR := \sqrt{FP^2 + PR^2} \quad FN := FR \quad FO := \frac{BF}{2}$$

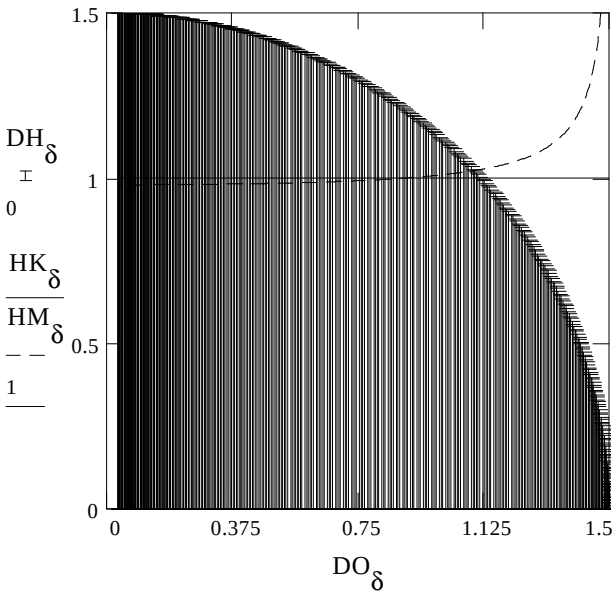
$$EN := \sqrt{FN^2 - FO^2} \quad BO := FO$$

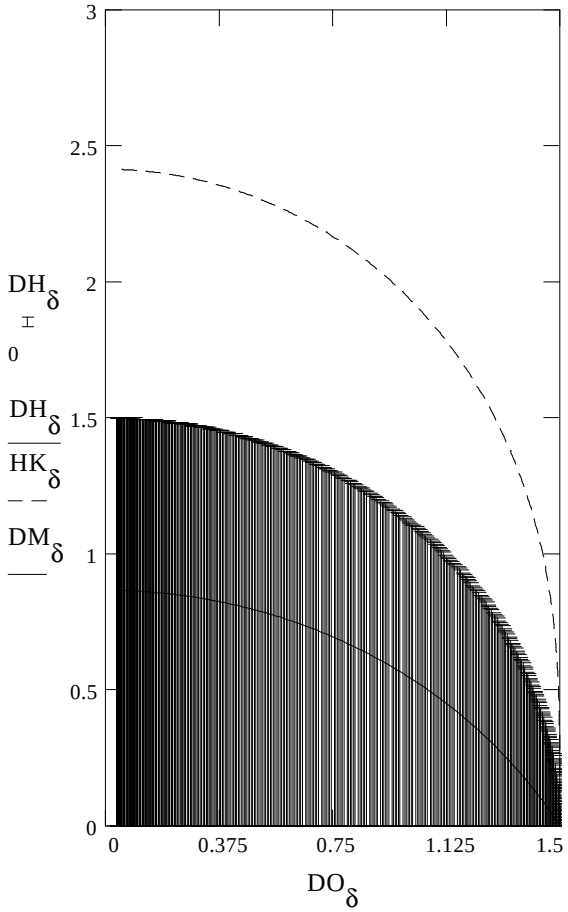
$$DO_\delta := BO - BD_\delta \quad NS_\delta := DO_\delta \quad MN := FR$$

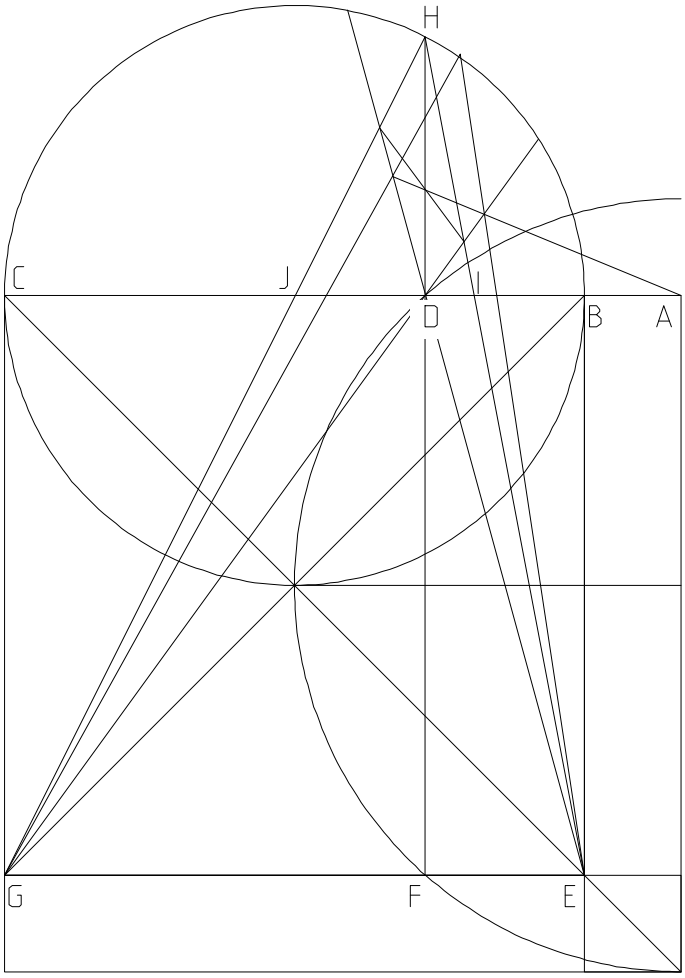
$$MS_\delta := \sqrt{MN^2 - \left(NS_\delta\right)^2} \quad DS := EN$$

$$DM_\delta := MS_\delta - DS \quad HM_\delta := DH_\delta - DM_\delta$$

Sampling distribution






07_26_94.MCD

A fair pencil construction.

$\delta := 1 .. 100 \quad AB_\delta := \delta^5 \cdot 10^{-8} \quad BC := 2$

$AC_\delta := BC + AB_\delta \quad AD_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

$BD_\delta := AD_\delta - AB_\delta \quad CD_\delta := BC - BD_\delta$

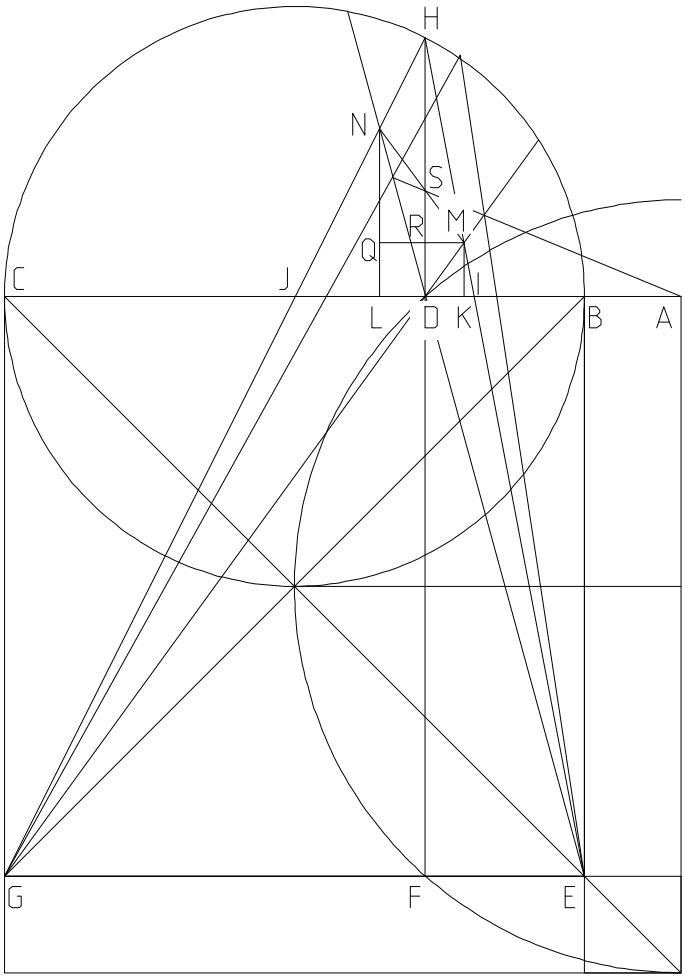
$DH_\delta := \sqrt{BD_\delta \cdot CD_\delta} \quad EF_\delta := BD_\delta$

$FD := BC \quad FH_\delta := FD + DH_\delta$

$FG_\delta := CD_\delta \quad FH_\delta := FD + DH_\delta$

$DI_\delta := \frac{EF_\delta \cdot DH_\delta}{FH_\delta} \quad DJ_\delta := \frac{FG_\delta \cdot DH_\delta}{FH_\delta}$

$BI_\delta := BD_\delta - DI_\delta$




$BJ_\delta := BD_\delta + DJ_\delta \quad CG := BC$

$CJ_\delta := BC - BJ_\delta \quad BE := BC$

$LN_\delta := \frac{CG \cdot DJ_\delta}{CJ_\delta + BD_\delta} \quad DL_\delta := \frac{BD_\delta \cdot LN_\delta}{BE}$

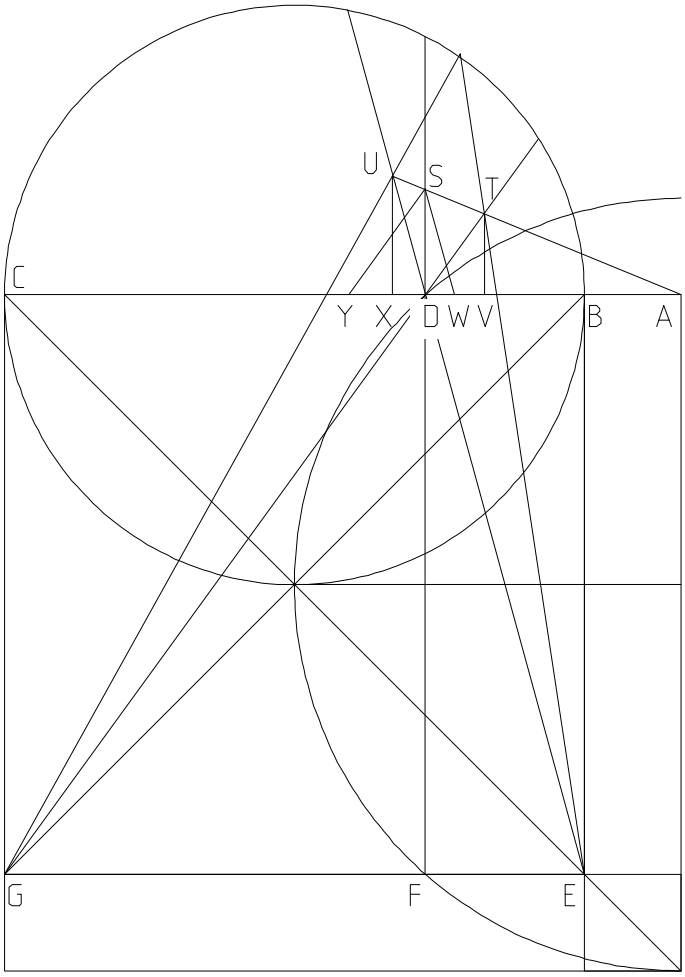
$KM_\delta := \frac{BE \cdot DI_\delta}{CD_\delta + BI_\delta} \quad DK_\delta := \frac{CD_\delta \cdot KM_\delta}{CG}$

$DR_\delta := KM_\delta \quad LQ_\delta := KM_\delta \quad NQ_\delta := LN_\delta - LQ_\delta$

$KL_\delta := DL_\delta + DK_\delta \quad MQ_\delta := KL_\delta$

$MR_\delta := DK_\delta \quad RS_\delta := \frac{NQ_\delta \cdot MR_\delta}{MQ_\delta}$

$DS_\delta := DR_\delta + RS_\delta$




$$DY_\delta := \frac{CD_\delta \cdot DS_\delta}{CG} \quad DW_\delta := \frac{BD_\delta \cdot DS_\delta}{BE}$$

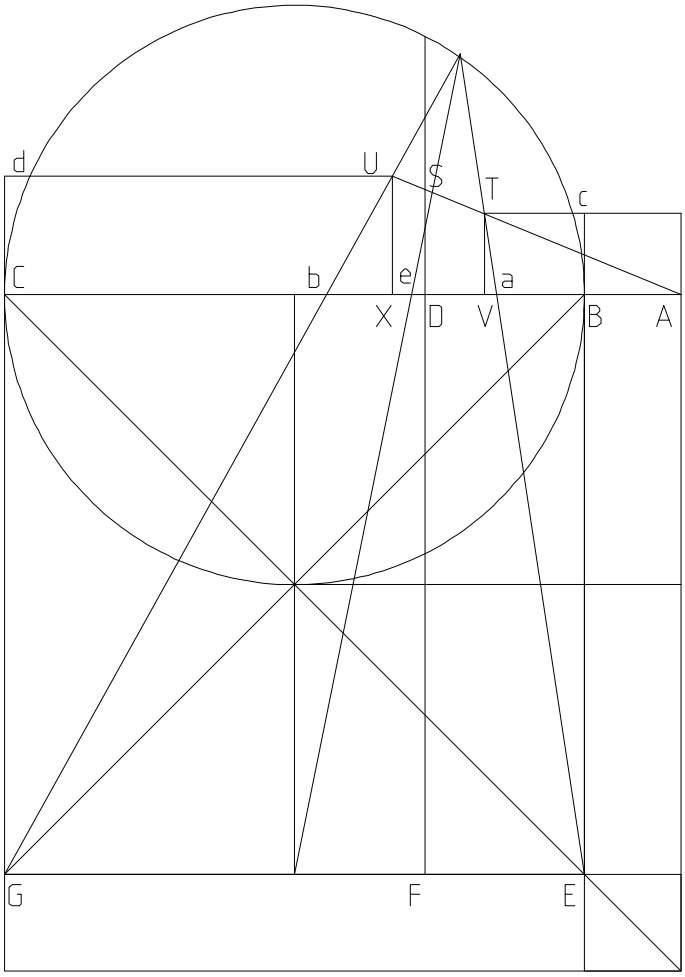
$$AW_\delta := AD_\delta - DW_\delta \quad AS_\delta := \sqrt{(DS_\delta)^2 + (AD_\delta)^2}$$

$$AU_\delta := \frac{AS_\delta \cdot AD_\delta}{AW_\delta} \quad AX_\delta := \frac{AD_\delta \cdot AU_\delta}{AS_\delta}$$

$$AY_\delta := AD_\delta + DY_\delta \quad AT_\delta := \frac{AS_\delta \cdot AD_\delta}{AY_\delta}$$

$$AV_\delta := \frac{AD_\delta \cdot AT_\delta}{AS_\delta} \quad TV_\delta := \frac{DS_\delta \cdot AV_\delta}{AD_\delta}$$

$$UX_\delta := \frac{DS_\delta \cdot AX_\delta}{AD_\delta}$$




$$Cd_\delta := UX_\delta \quad Ac_\delta := TV_\delta \quad CX_\delta := AC_\delta - AX_\delta$$

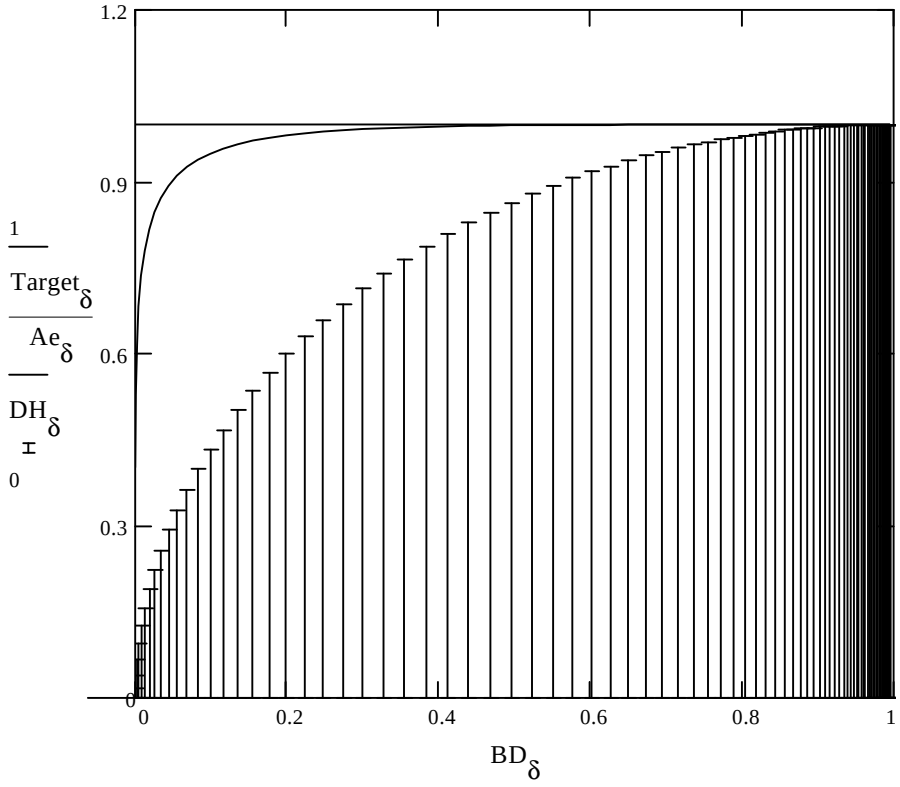
$$Ud_\delta := CX_\delta \quad Tc_\delta := AV_\delta - AB_\delta$$

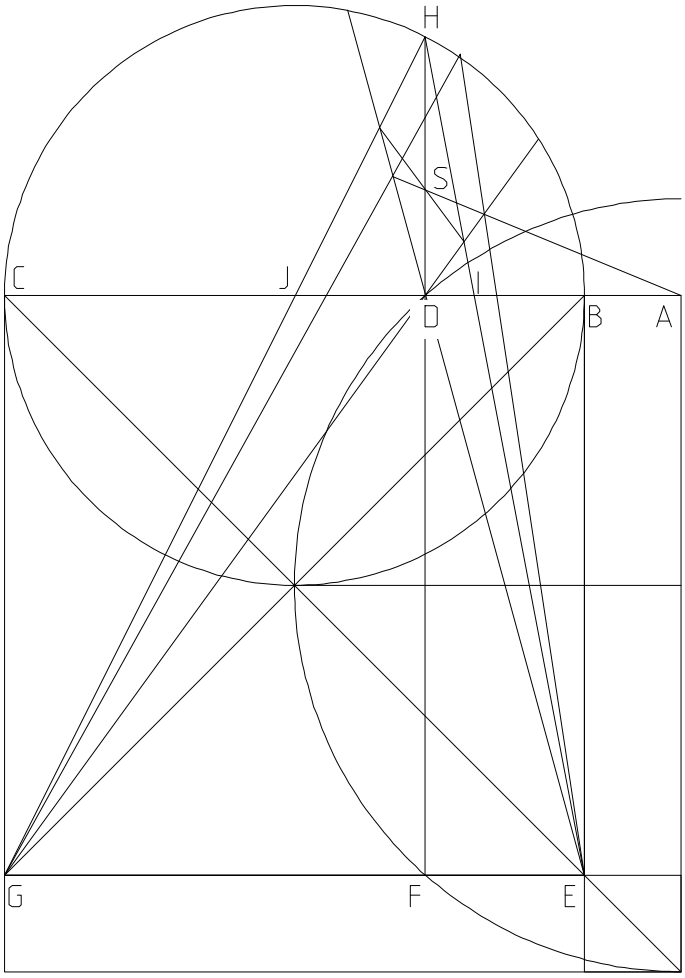
$$Xb_\delta := \frac{Ud_\delta \cdot UX_\delta}{CG + Cd_\delta} \quad Va_\delta := \frac{Tc_\delta \cdot TV_\delta}{BE + Ac_\delta}$$

$$Ab_\delta := AX_\delta + Xb_\delta \quad Aa_\delta := AV_\delta - Va_\delta$$

$$Ae_\delta := \frac{Ab_\delta + Aa_\delta}{2}$$

$$Target_\delta := \frac{\left[(AB_\delta)^2 \cdot AC_\delta\right]^{\frac{1}{3}} + \left[AB_\delta \cdot (AC_\delta)^2\right]^{\frac{1}{3}}}{2}$$


1.2
0.9
0.6
0.3
0
1
Target_δ
Ae_δ
DH_δ
τ
0
0
0.2
0.4
0.6
0.8
1
BD_δ




07_27_B4.MCD

A fair pencil construction. Taking S as a projecton point for AS, with which to develope the roots.

$\delta := 1 .. 100 \quad AB_\delta := \delta^5 \cdot 10^{-8} \quad BC := 2$

$AC_\delta := BC + AB_\delta \quad AD_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

$BD_\delta := AD_\delta - AB_\delta \quad CD_\delta := BC - BD_\delta$

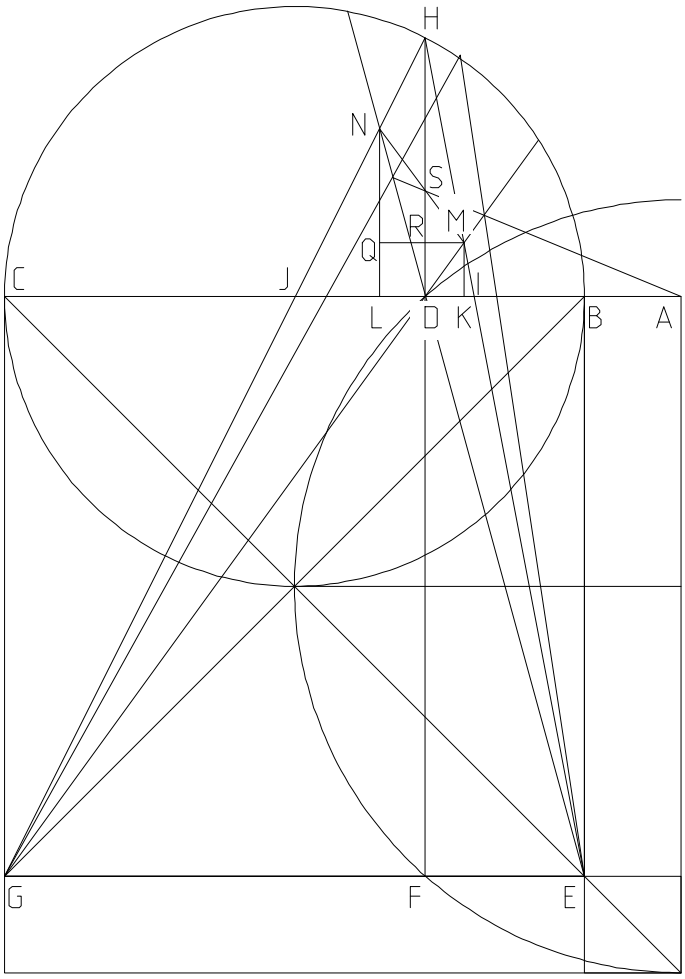
$DH_\delta := \sqrt{BD_\delta \cdot CD_\delta} \quad EF_\delta := BD_\delta$

$FD := BC \quad FH_\delta := FD + DH_\delta$

$FG_\delta := CD_\delta \quad FH_\delta := FD + DH_\delta$

$DI_\delta := \frac{EF_\delta \cdot DH_\delta}{FH_\delta} \quad DJ_\delta := \frac{FG_\delta \cdot DH_\delta}{FH_\delta}$

$BI_\delta := BD_\delta - DI_\delta$




$BJ_\delta := BD_\delta + DJ_\delta \quad CG := BC$

$CJ_\delta := BC - BJ_\delta \quad BE := BC$

$LN_\delta := \frac{CG \cdot DJ_\delta}{CJ_\delta + BD_\delta} \quad DL_\delta := \frac{BD_\delta \cdot LN_\delta}{BE}$

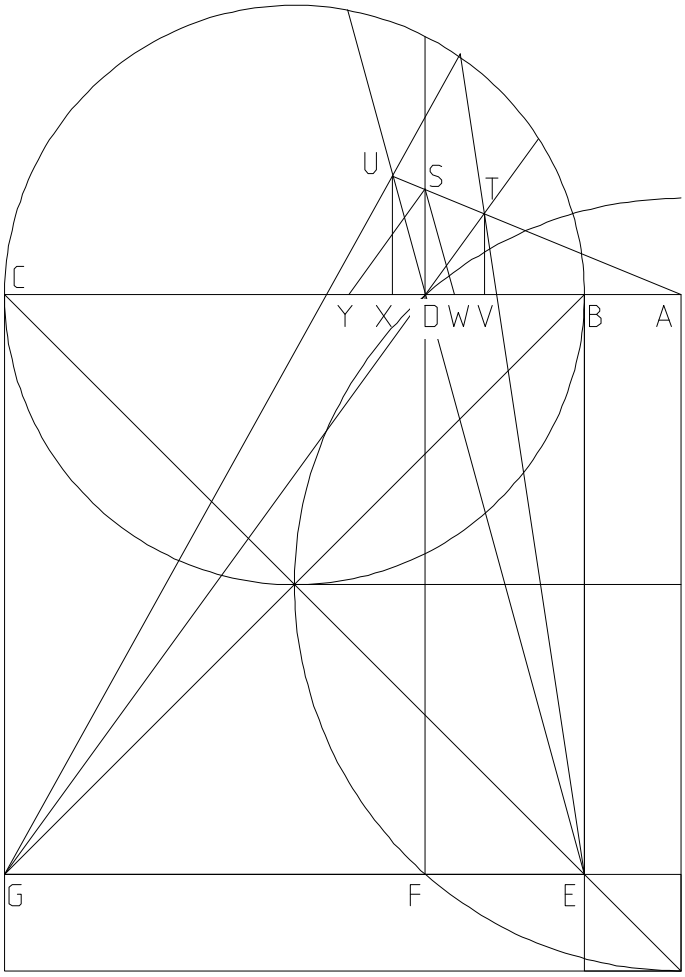
$KM_\delta := \frac{BE \cdot DI_\delta}{CD_\delta + BI_\delta} \quad DK_\delta := \frac{CD_\delta \cdot KM_\delta}{CG}$

$DR_\delta := KM_\delta \quad LQ_\delta := KM_\delta \quad NQ_\delta := LN_\delta - LQ_\delta$

$KL_\delta := DL_\delta + DK_\delta \quad MQ_\delta := KL_\delta$

$MR_\delta := DK_\delta \quad RS_\delta := \frac{NQ_\delta \cdot MR_\delta}{MQ_\delta}$

$DS_\delta := DR_\delta + RS_\delta \quad DS1_\delta := DS_\delta$




$$DY_\delta := \frac{CD_\delta \cdot DS_\delta}{CG} \quad DW_\delta := \frac{BD_\delta \cdot DS_\delta}{BE}$$

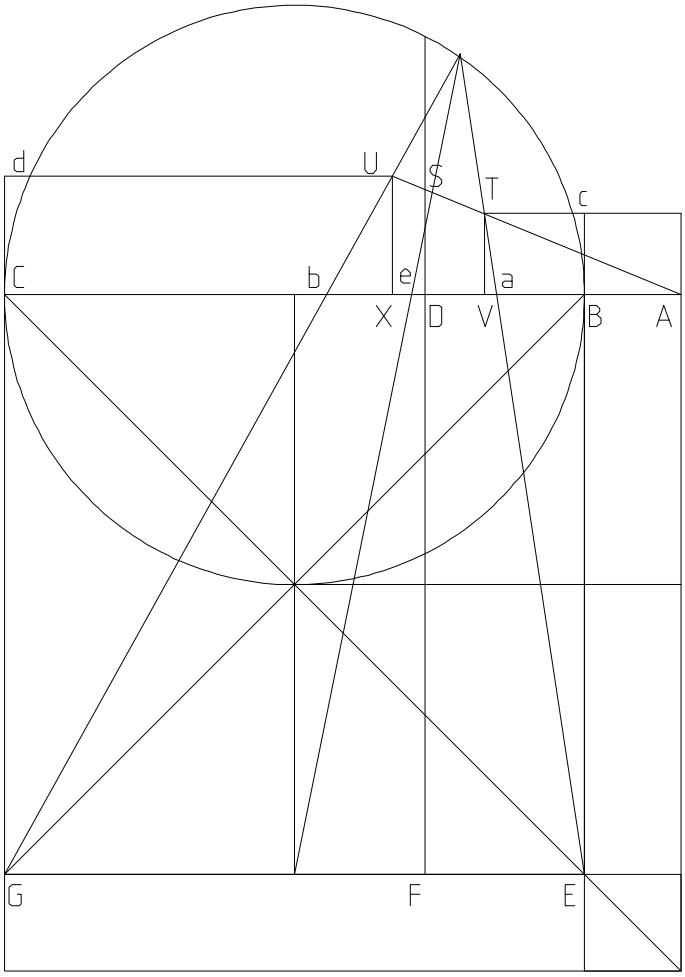
$$AW_\delta := AD_\delta - DW_\delta \quad AS_\delta := \sqrt{(DS_\delta)^2 + (AD_\delta)^2}$$

$$AU_\delta := \frac{AS_\delta \cdot AD_\delta}{AW_\delta} \quad AX_\delta := \frac{AD_\delta \cdot AU_\delta}{AS_\delta}$$

$$AY_\delta := AD_\delta + DY_\delta \quad AT_\delta := \frac{AS_\delta \cdot AD_\delta}{AY_\delta}$$

$$AV_\delta := \frac{AD_\delta \cdot AT_\delta}{AS_\delta} \quad TV_\delta := \frac{DS_\delta \cdot AV_\delta}{AD_\delta}$$

$$UX_\delta := \frac{DS_\delta \cdot AX_\delta}{AD_\delta}$$



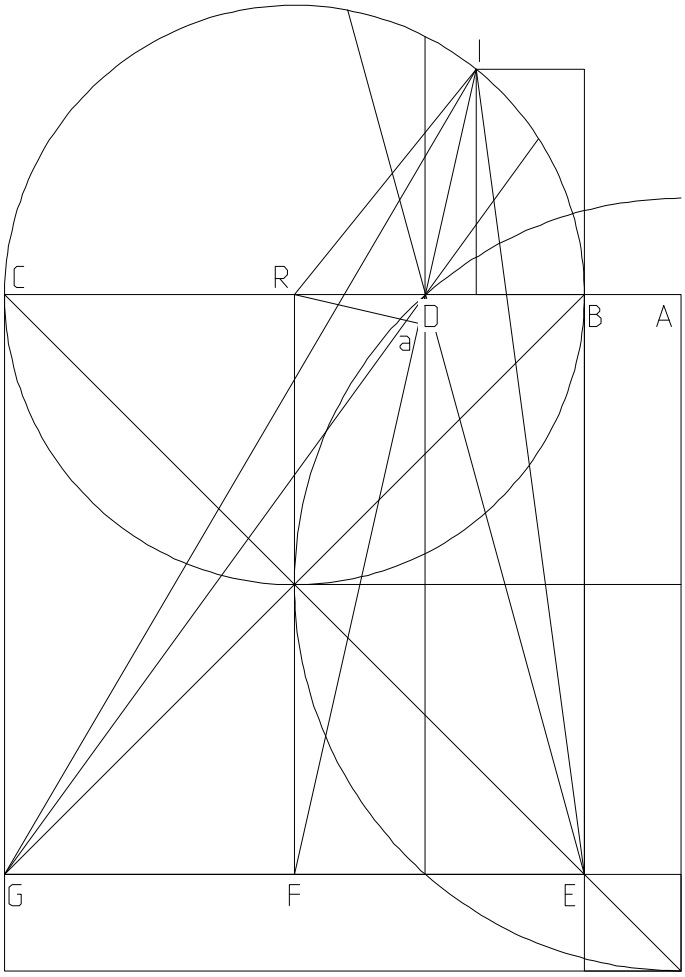

$$Cd_\delta := UX_\delta \quad Ac_\delta := TV_\delta \quad CX_\delta := AC_\delta - AX_\delta$$

$$Ud_\delta := CX_\delta \quad Tc_\delta := AV_\delta - AB_\delta$$

$$Xb_\delta := \frac{Ud_\delta \cdot UX_\delta}{CG + Cd_\delta} \quad Va_\delta := \frac{Tc_\delta \cdot TV_\delta}{BE + Ac_\delta}$$

$$Ab_\delta := AX_\delta + Xb_\delta \quad Aa_\delta := AV_\delta - Va_\delta$$

$$Ae1_\delta := \frac{Ab_\delta + Aa_\delta}{2}$$




Second fair pencil.

$AC_\delta := AB_\delta + BC \quad CG := BC \quad FR := BC$

$BE := BC \quad BR := \frac{BC}{2} \quad CR := BR \quad RI := BR$

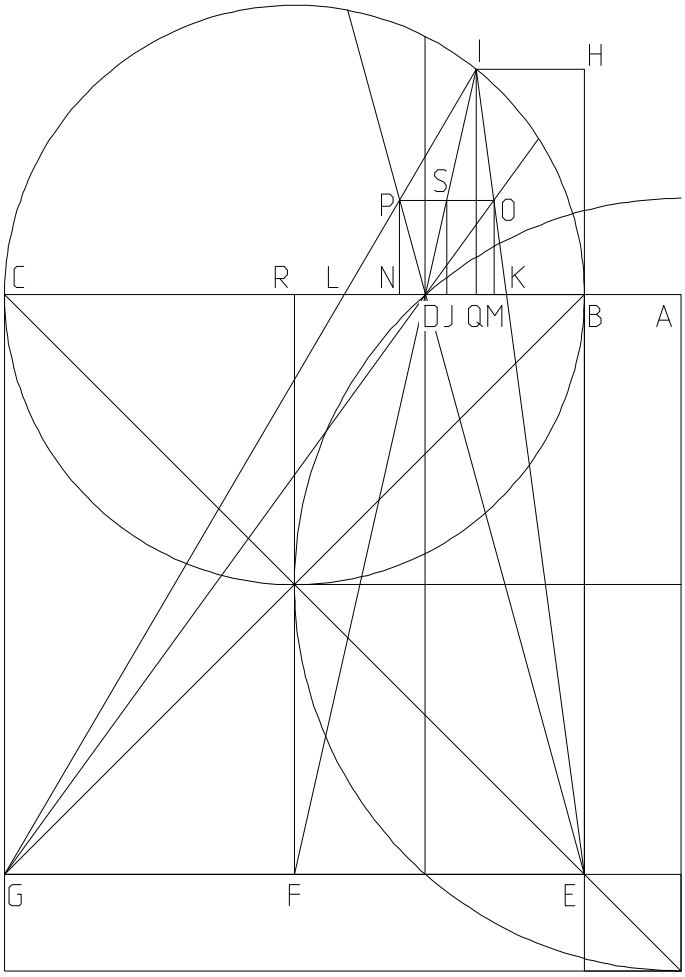
$AD_\delta := \sqrt{AB_\delta \cdot AC_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

$DR_\delta := BR - BD_\delta \quad DF_\delta := \sqrt{FR^2 + (DR_\delta)^2}$

$Da_\delta := \frac{(DR_\delta)^2}{DF_\delta} \quad Ra_\delta := \sqrt{(DR_\delta)^2 - (Da_\delta)^2}$

$Ia_\delta := \sqrt{RI^2 - (Ra_\delta)^2} \quad DI_\delta := Ia_\delta - Da_\delta$

$CD_\delta := BC - BD_\delta$




$IQ_\delta := \frac{FR \cdot DI_\delta}{DF_\delta} \quad BH_\delta := IQ_\delta \quad DQ_\delta := \frac{DR_\delta \cdot DI_\delta}{DF_\delta}$

$BQ_\delta := BD_\delta - DQ_\delta \quad HI_\delta := BQ_\delta$

$EH_\delta := BE + BH_\delta \quad BK_\delta := \frac{HI_\delta \cdot BE}{EH_\delta}$

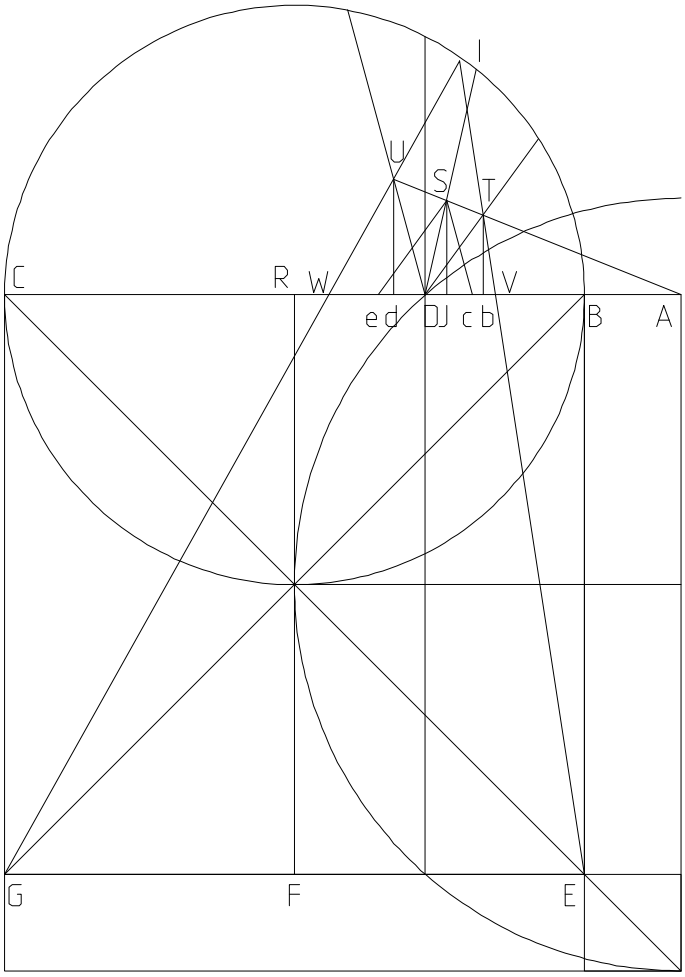
$DK_\delta := BD_\delta - BK_\delta \quad DL_\delta := DK_\delta$

$BL_\delta := BD_\delta + DL_\delta \quad CL_\delta := BC - BL_\delta$

$NP_\delta := \frac{CG \cdot DL_\delta}{CL_\delta + BD_\delta} \quad MO_\delta := \frac{BE \cdot DK_\delta}{CD_\delta + BK_\delta}$

The sum of NP and MO should be 0, ie. parallel to BC.

$\sum_\delta (NP_\delta - MO_\delta) = 0 \qquad JS_\delta := MO_\delta$




$$DJ_\delta := \frac{DR_\delta \cdot JS_\delta}{FR} \qquad AJ_\delta := AD_\delta - DJ_\delta$$

$$AS_\delta := \sqrt{\left(AJ_\delta\right)^2 + \left(JS_\delta\right)^2}$$

$$Jc_\delta := \frac{BD_\delta \cdot JS_\delta}{BE} \qquad Dc_\delta := DJ_\delta + Jc_\delta$$

$$Ac_\delta := AD_\delta - Dc_\delta \qquad Ud_\delta := \frac{JS_\delta \cdot AD_\delta}{Ac_\delta}$$

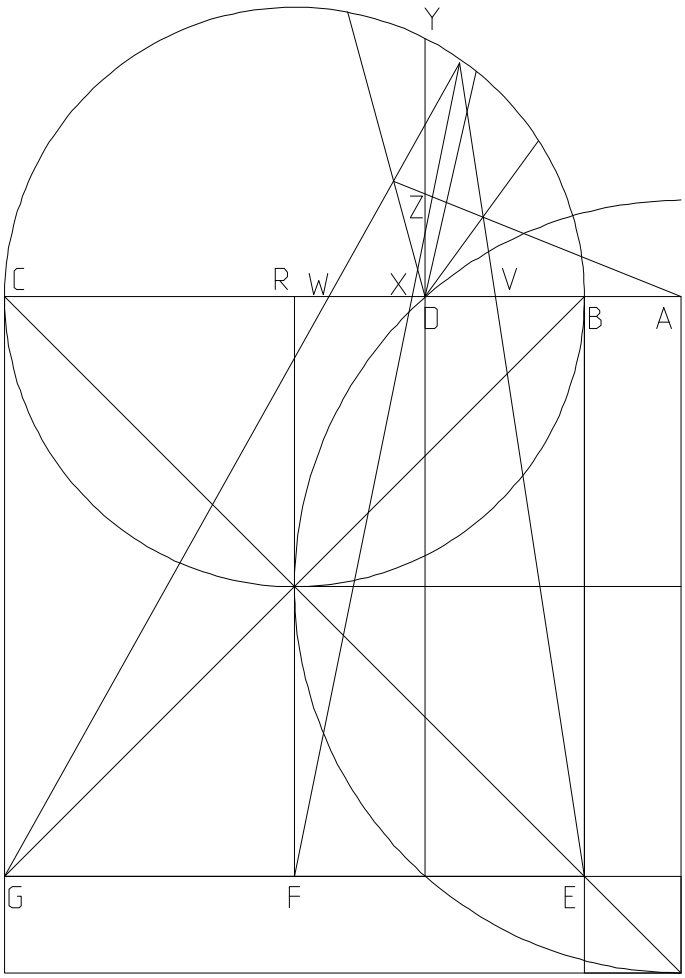
$$Je_\delta := \frac{CD_\delta \cdot JS_\delta}{CG} \qquad Ae_\delta := AJ_\delta + Je_\delta$$

$$Tb_\delta := \frac{JS_\delta \cdot AD_\delta}{Ae_\delta} \qquad Ad_\delta := \frac{AJ_\delta \cdot Ud_\delta}{JS_\delta}$$

$$Ab_\delta := \frac{AJ_\delta \cdot Tb_\delta}{JS_\delta} \qquad Bb_\delta := Ab_\delta - AB_\delta$$

$$Cd_\delta := AC_\delta - Ad_\delta \qquad BV_\delta := \frac{Bb_\delta \cdot BE}{BE + Tb_\delta}$$

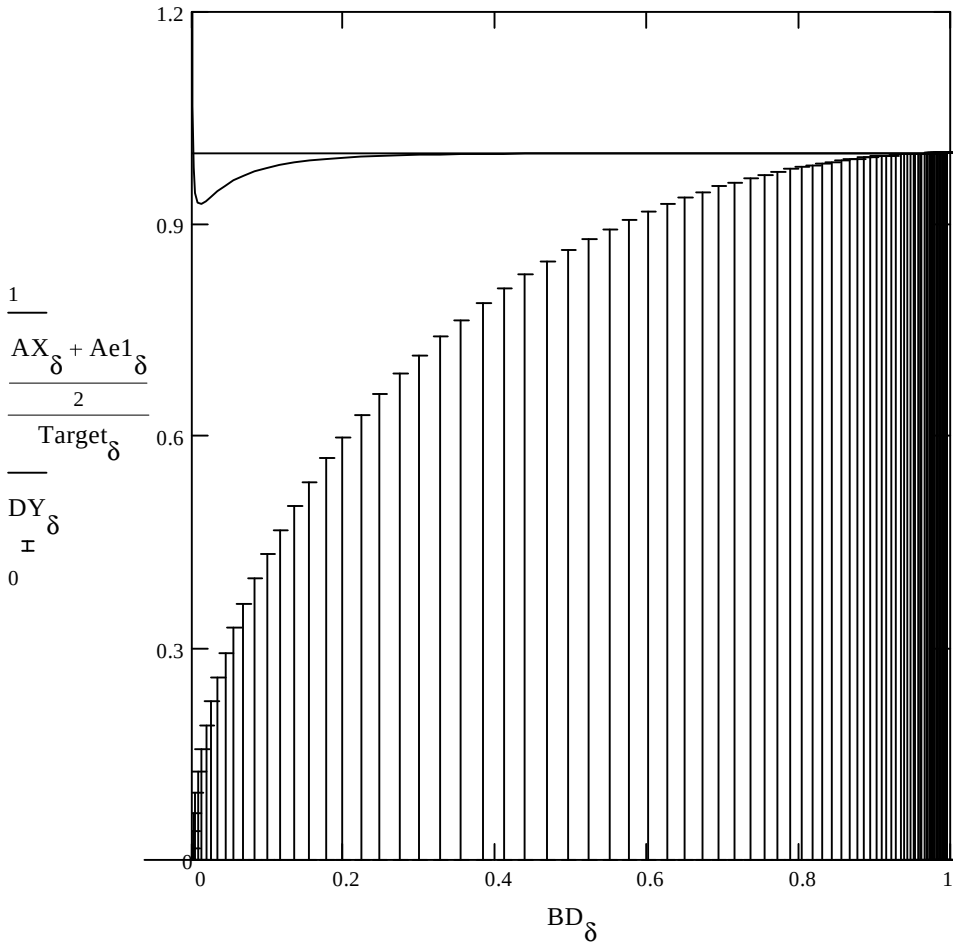
$$CW_\delta := \frac{Cd_\delta \cdot CG}{CG + Ud_\delta}$$

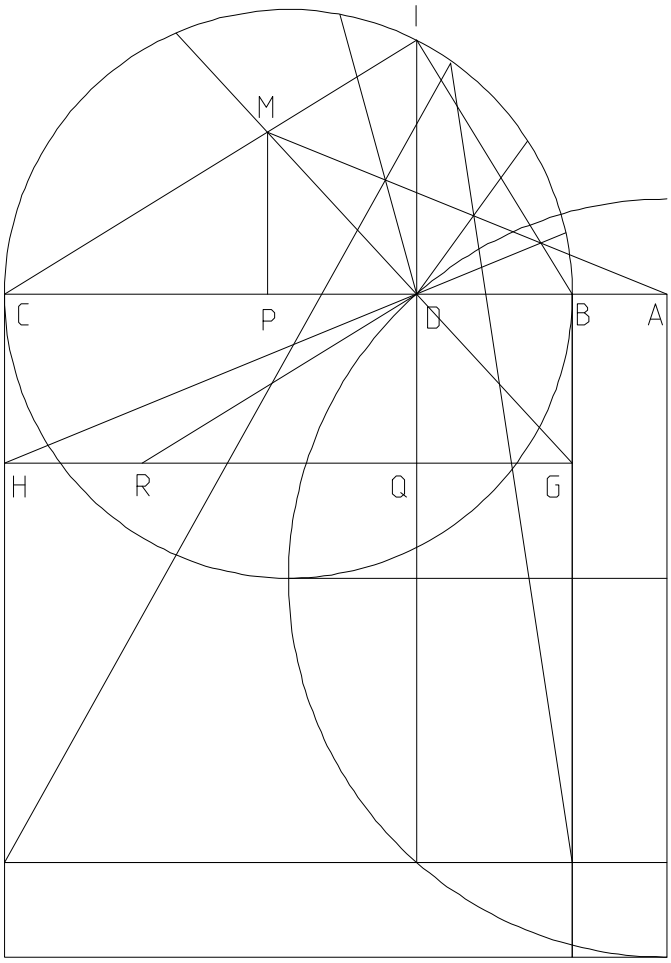



$$AV_\delta := AB_\delta + BV_\delta \qquad AW_\delta := AC_\delta - CW_\delta$$

$$AX_\delta := \frac{AV_\delta + AW_\delta}{2} \qquad DY_\delta := \sqrt{BD_\delta \cdot CD_\delta}$$

$$Target_\delta := \frac{\left[\left(AB_\delta\right)^2 \cdot AC_\delta\right]^{\frac{1}{3}} + \left[AB_\delta \cdot \left(AC_\delta\right)^2\right]^{\frac{1}{3}}}{2}$$


1.2
0.9
0.6
0.3
0
0
0.2
0.4
0.6
0.8
1
BD δ
1
AX δ + Ae1 δ
2
Target δ
DY δ
0




Just another fair Pencil Construction for Cube. Taking 2/3 of DI provides the points of projection for G and H.

$\delta := 0..800 \quad AB_\delta := \delta^5 \cdot 10^{-10} \quad BC := 2$

$AC_\delta := AB_\delta + BC \quad AD_\delta := \sqrt{AB_\delta \cdot AC_\delta}$

$BD_\delta := AD_\delta - AB_\delta \quad CD_\delta := BC - BD_\delta$

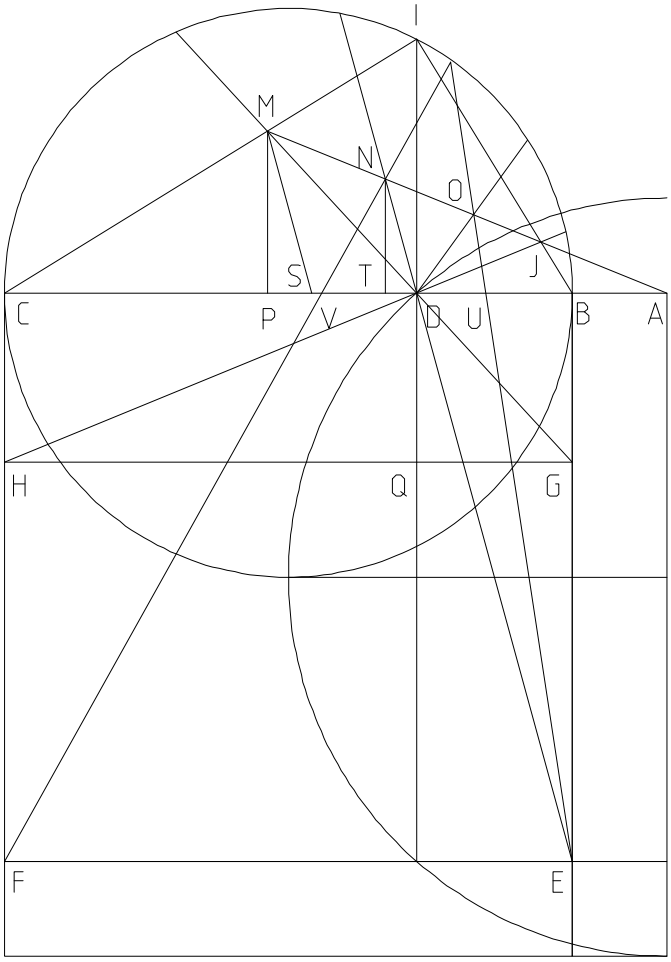
$DI_\delta := \sqrt{BD_\delta \cdot CD_\delta} \quad BG_\delta := \frac{DI_\delta \cdot 2}{3}$

$CH_\delta := BG_\delta \quad DQ_\delta := BG_\delta \quad GQ_\delta := BD_\delta$

$QR_\delta := \frac{CD_\delta \cdot DQ_\delta}{DI_\delta} \quad GR_\delta := QR_\delta + GQ_\delta$

$CP_\delta := \frac{QR_\delta \cdot CD_\delta}{GR_\delta} \quad MP_\delta := \frac{DQ_\delta \cdot CP_\delta}{QR_\delta}$

$AP_\delta := AC_\delta - CP_\delta$



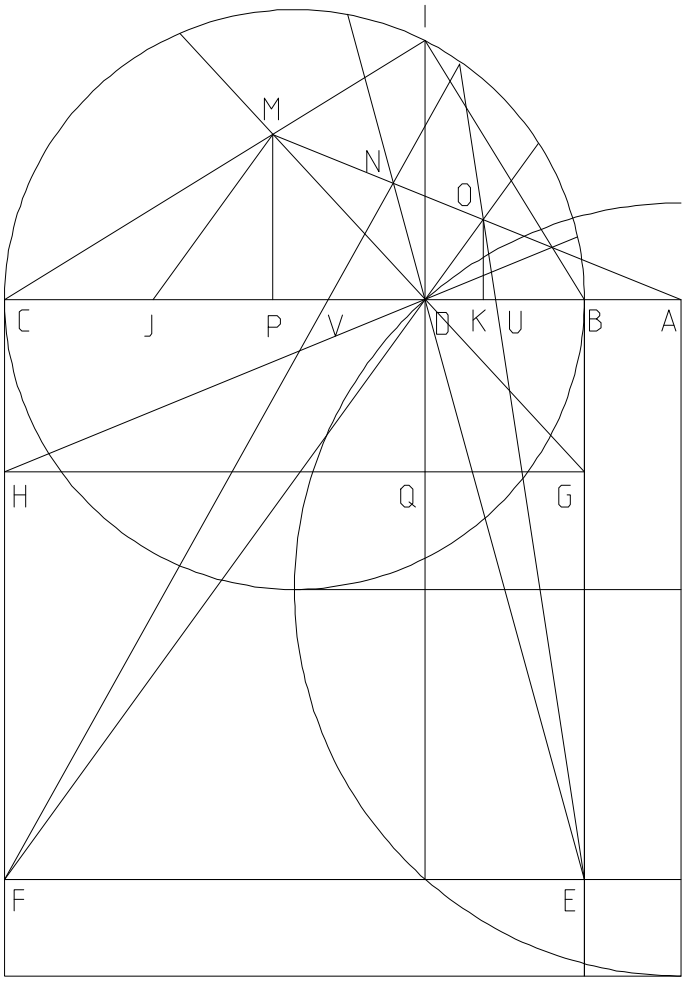

$BE := BC \quad CF := BC \quad PS_\delta := \frac{BD_\delta \cdot MP_\delta}{BE}$

$AM_\delta := \sqrt{(AP_\delta)^2 + (MP_\delta)^2} \quad AS_\delta := AP_\delta - PS_\delta$

$AN_\delta := \frac{AM_\delta \cdot AD_\delta}{AS_\delta} \quad AT_\delta := \frac{AP_\delta \cdot AN_\delta}{AM_\delta}$

$NT_\delta := \frac{MP_\delta \cdot AN_\delta}{AM_\delta} \quad CT_\delta := AC_\delta - AT_\delta$

$TV_\delta := \frac{CT_\delta \cdot NT_\delta}{CF + NT_\delta} \quad AV_\delta := AT_\delta + TV_\delta$




$$JP_\delta := \frac{CD_\delta \cdot MP_\delta}{CF} \qquad AJ_\delta := AP_\delta + JP_\delta$$

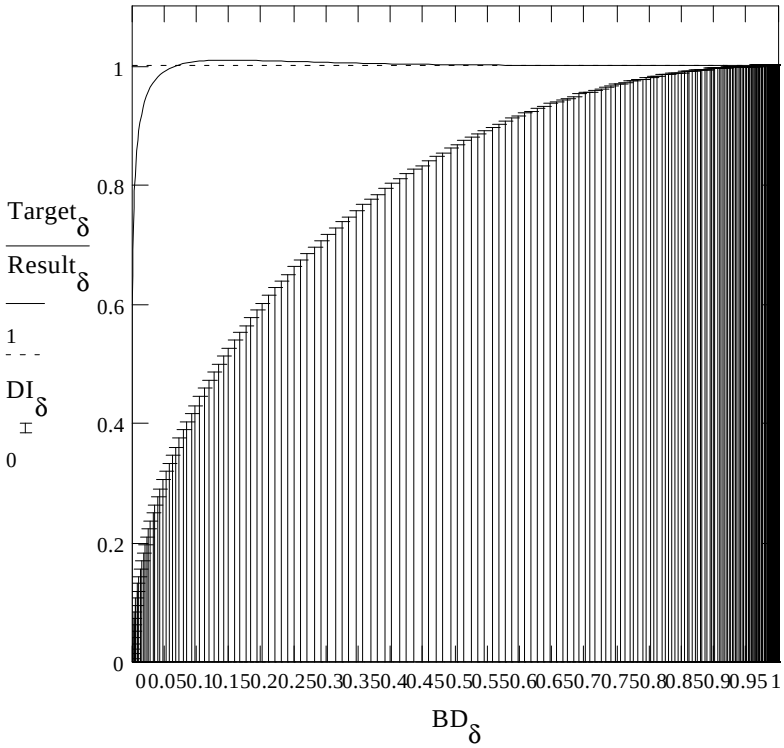
$$DK_\delta := \frac{JP_\delta \cdot AD_\delta}{AJ_\delta} \qquad KO_\delta := \frac{MP_\delta \cdot DK_\delta}{JP_\delta}$$

$$BK_\delta := BD_\delta - DK_\delta \qquad KU_\delta := \frac{BK_\delta \cdot KO_\delta}{BE + KO_\delta}$$

$$BU_\delta := BK_\delta - KU_\delta \qquad AU_\delta := AB_\delta + BU_\delta$$

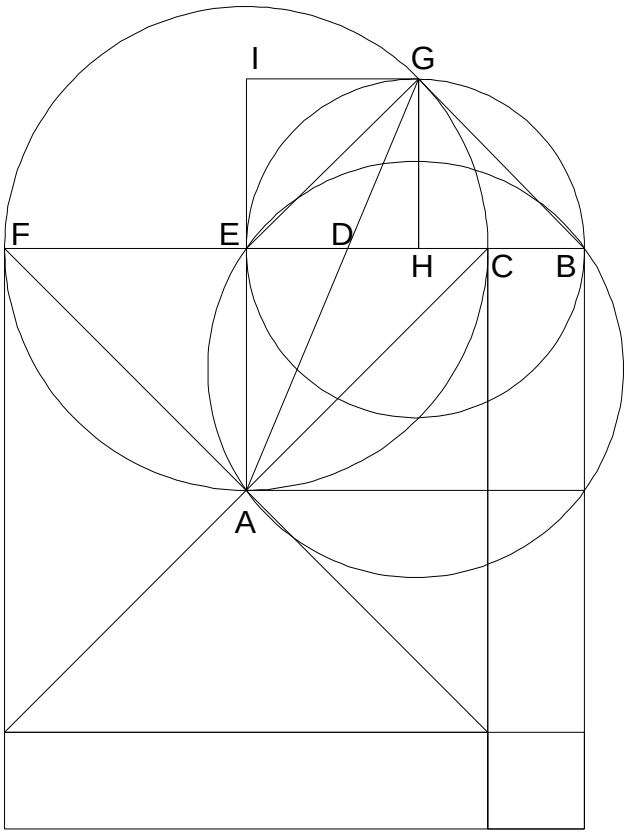
$$Target_\delta := \frac{\left[\left(AB_\delta\right)^2 \cdot AC_\delta\right]^{\frac{1}{3}} + \left[AB_\delta \cdot \left(AC_\delta\right)^2\right]^{\frac{1}{3}}}{2}$$

$$Result_\delta := \frac{AU_\delta + AV_\delta}{2}$$




DI represents sample points.

Trivial method for doing Square Roots.




If I draw a circle BE, then a line AG, BD is the square root of $BC \times BF$.

$CF := 1000 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CE := \frac{CF}{2} \quad BE_\delta := BC_\delta + CE$

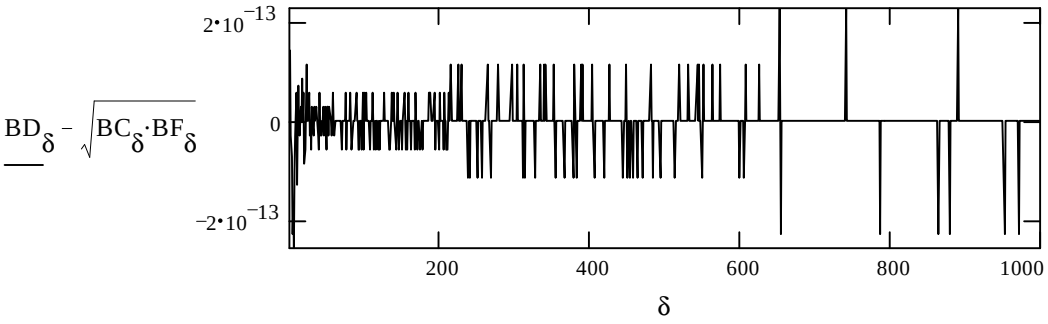
$EG := CE \quad BG_\delta := \sqrt{(BE_\delta)^2 - EG^2}$

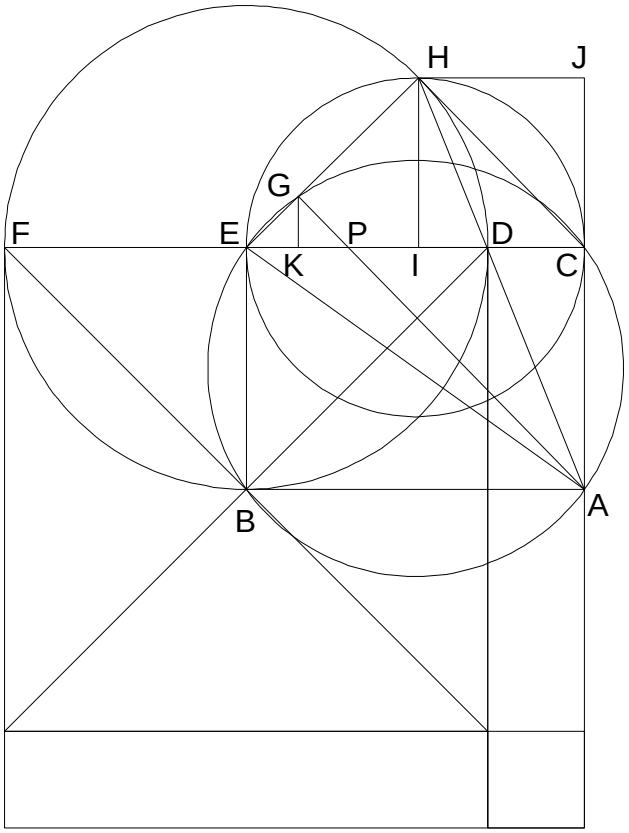
$EH_\delta := \frac{EG^2}{BE_\delta} \quad GH_\delta := \sqrt{EG^2 - (EH_\delta)^2}$

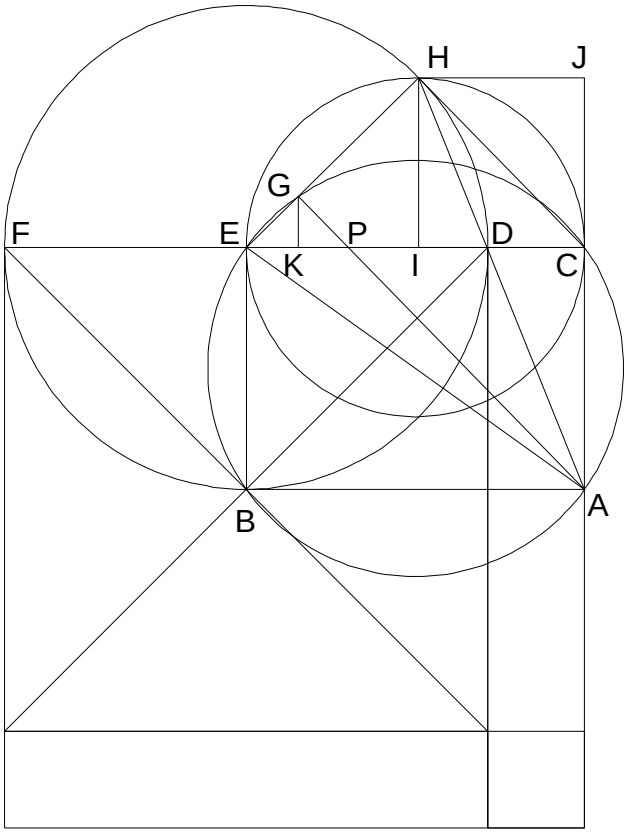
$GI_\delta := EH_\delta \quad EI_\delta := GH_\delta \quad AE := CE$

$AI_\delta := AE + EI_\delta \quad DE_\delta := \frac{GI_\delta \cdot AE}{AI_\delta}$

$BD_\delta := BE_\delta - DE_\delta \quad BF_\delta := BC_\delta + CF$






10_28_94.MCD

Trivial method for doing Square Root.

If I drop in a circle CHE, another ABE, and a right triangle, CHE, then AG provides CP which is the square root of $CD \times CF$.

$DF := 10 \quad \delta := 1 .. 100 \quad CD_\delta := \delta$

$DE := \frac{DF}{2} \quad AC := DE \quad CE_\delta := DE + CD_\delta$

$EH := DE \quad AE_\delta := \sqrt{AC^2 + (CE_\delta)^2}$

$CH_\delta := \sqrt{(CE_\delta)^2 - EH^2} \quad EI_\delta := \frac{EH^2}{CE_\delta}$

$HI_\delta := \sqrt{EH^2 - (EI_\delta)^2} \quad CJ_\delta := HI_\delta$

$CI_\delta := CE_\delta - EI_\delta \quad HJ_\delta := CI_\delta$

$AJ_\delta := AC + CJ_\delta \quad AH_\delta := \sqrt{(AJ_\delta)^2 + (HJ_\delta)^2}$

$HG_\delta := \frac{1}{2} \cdot \frac{(AH_\delta)^2}{EH} + \frac{1}{2} \cdot EH - \frac{1}{2} \cdot \frac{(AE_\delta)^2}{EH}$

$EG_\delta := EH - HG_\delta$

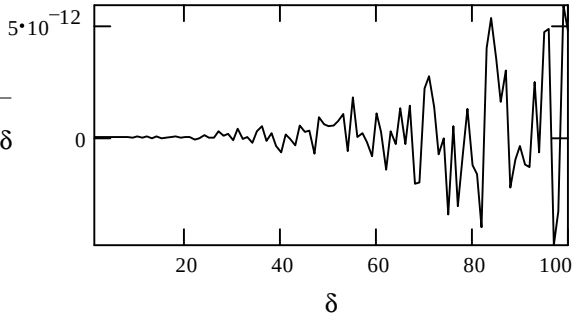
$AG_\delta := \sqrt{(AE_\delta)^2 - (EG_\delta)^2}$

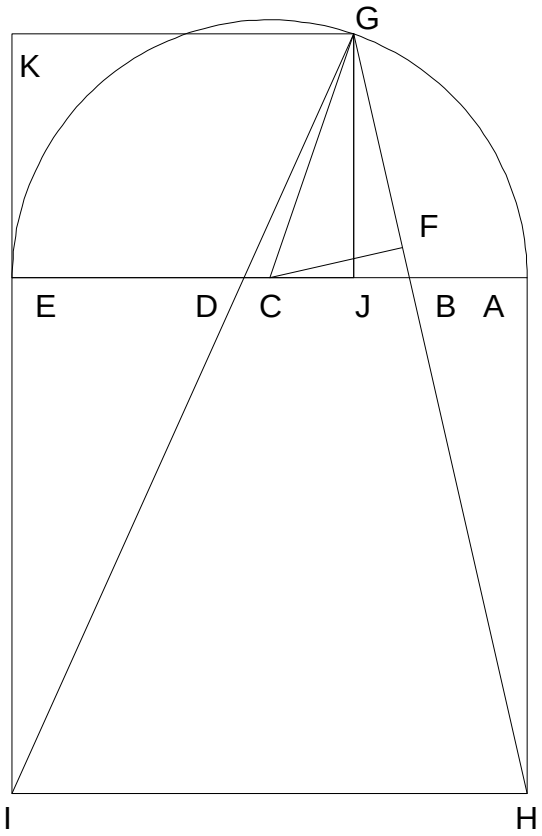
$GK_\delta := \frac{HI_\delta \cdot EG_\delta}{EH} \quad GP_\delta := \frac{AG_\delta \cdot GK_\delta}{AC + GK_\delta}$

$EP_\delta := \sqrt{(EG_\delta)^2 + (GP_\delta)^2}$

$CP_\delta := CE_\delta - EP_\delta \qquad CF_\delta := CD_\delta + DF$

$\underline{CP_\delta} - \sqrt{CD_\delta \cdot CF_\delta}$






10_31_94.MCD

Given AB & BE, divide BE such that BD + DE = BE and $BD = \sqrt{AB \cdot DE}$

**The square in a right triangle on the hypotenuse is equal to the square of the remaining two segments** (and all three squares taken to the point of similarity form a cubic relationship).

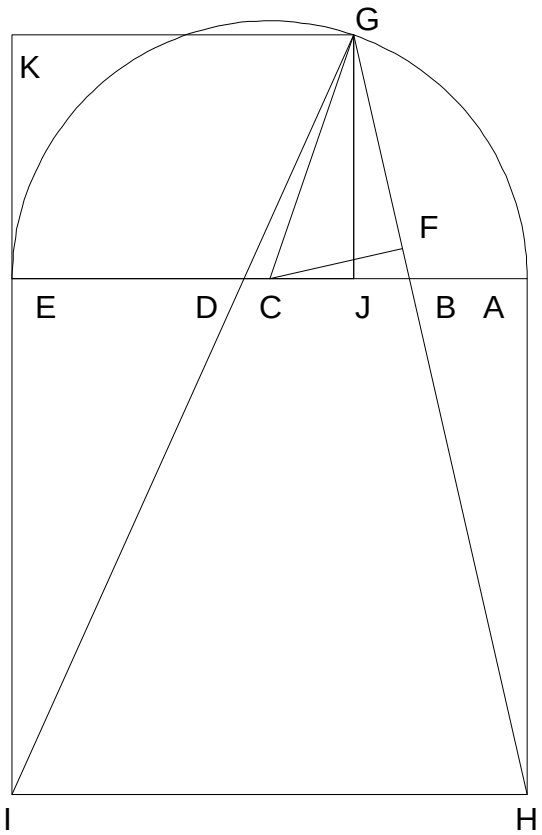
$AE := AB + BE \quad AC := \frac{AE}{2}$

$AH := AE \quad EI := AE \quad BH := \sqrt{AB^2 + AH^2}$

$CG := AC \quad BC := AC - AB \quad BF := \frac{AB \cdot BC}{BH}$

$CF := \frac{AH \cdot BC}{BH} \quad FG := \sqrt{CG^2 - CF^2}$

$BG := BF + FG \quad BJ := \frac{AB \cdot BG}{BH}$



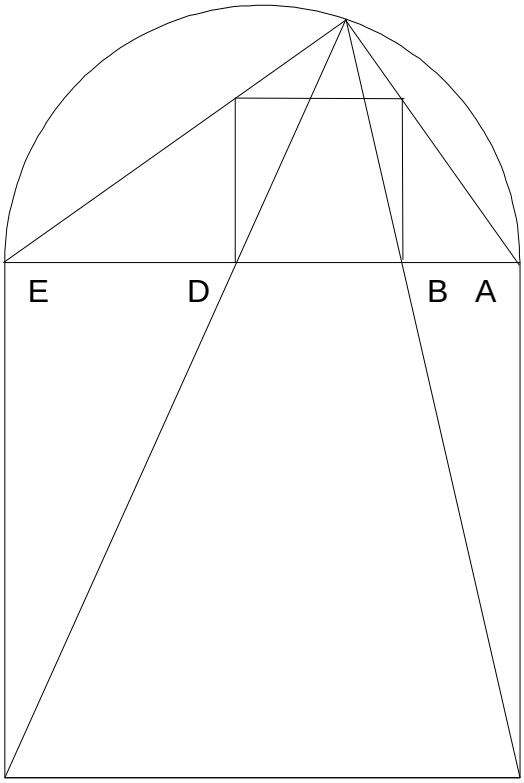

$GJ := \frac{AH \cdot BJ}{AB} \quad AJ := AB + BJ$

$EJ := AE - AJ \quad GK := EJ \quad EK := GJ$

$IK := EI + EK \quad DE := \frac{GK \cdot EI}{IK}$

$BD := BE - DE$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB \cdot DE} = 0$




Reducing the previous tautological chain to single equations for BD and DE.

$AB \equiv 20 \quad BE \equiv 12$

$$BD := \frac{\sqrt{AB}\cdot\left[-AB^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{AB + 4\cdot BE} + \sqrt{AB}\cdot BE \ldots + BE\cdot\sqrt{AB + 4\cdot BE}\right]}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

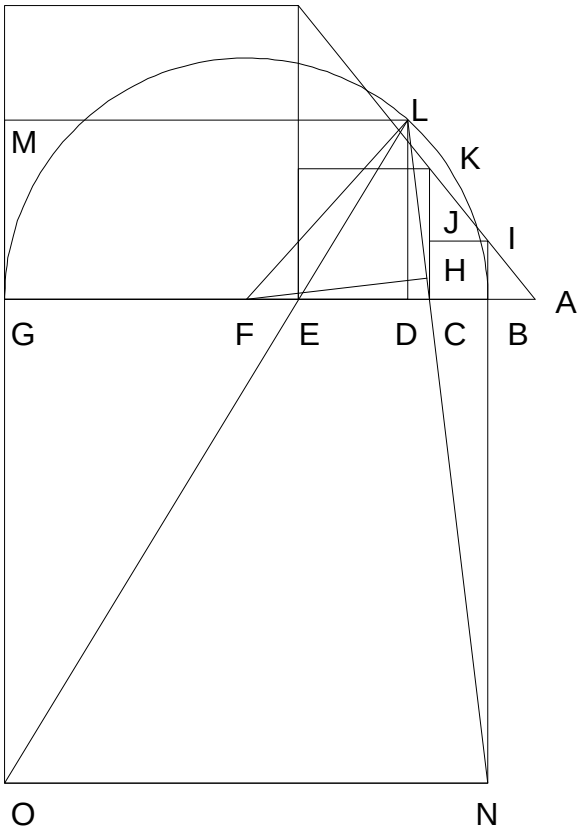
$BD = 8.439$

$$DE := \frac{AB^2 + 2\cdot AB\cdot BE - AB^{\left(\frac{3}{2}\right)}\cdot\sqrt{AB + 4\cdot BE} + 2\cdot BE^2}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

$DE = 3.561$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB\cdot DE} = 0$

Given AB & BE, BE has been divided such that BD + DE = BE and $BD = \sqrt{AB\cdot DE}$




11_01_94.MCD

Given AG - AB = BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BG > 3 BC.

$BG := 6 \quad BC := 1.9 \quad BN := BG$

$BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN}$

$FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN}$

$GM := DL \quad BD := BC + CD \quad DG := BG - BD$

$LM := DG \quad GO := BG \quad MO := GO + GM$

$EG := \frac{LM \cdot GO}{MO} \quad CG := BG - BC$

$CE := CG - EG \quad CJ := BC \quad CK := CE$

$IJ := BC \quad JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2}$

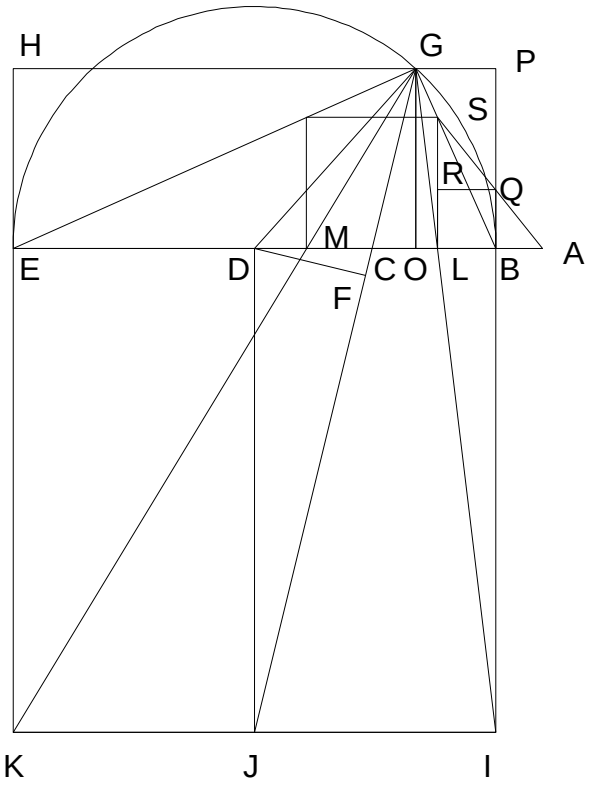
$BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE$

$AB = 36.723 \quad AG = 42.723 \quad AE = 40.621$

$\left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

Given AG - AB =BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, found was AB, AG, and $(AB \cdot AG^2)^{1/3}$.




Given AE - AB = BE and

$$\frac{(AB\cdot AE^2)^{\frac{1}{3}} - (AB^2\cdot AE)^{\frac{1}{3}}}{2} + (AB^2\cdot AE)^{\frac{1}{3}} - AE = BC,$$

find AB.

$BE := 70 \quad BC := 34 \quad BD := \frac{BE}{2} \quad BI := BE$

$DJ := BE \quad EK := BE \quad CD := BD - BC$

$CJ := \sqrt{DJ^2 + CD^2} \quad DF := \frac{DJ\cdot CD}{CJ}$

$DG := BD \quad FG := \sqrt{DG^2 - DF^2} \quad CF := \frac{CD\cdot DF}{DJ}$

$CG := FG - CF \quad CO := \frac{CD\cdot CG}{CJ}$

$DO := CD + CO$

$BO := BD - DO \quad GP := BO \quad GO := \frac{DJ\cdot CG}{CJ}$

$BP := GO \quad IP := BI + BP \quad BL := \frac{GP\cdot BI}{IP}$

$EH := GO \quad HK := EK + EH \quad EO := BE - BO$

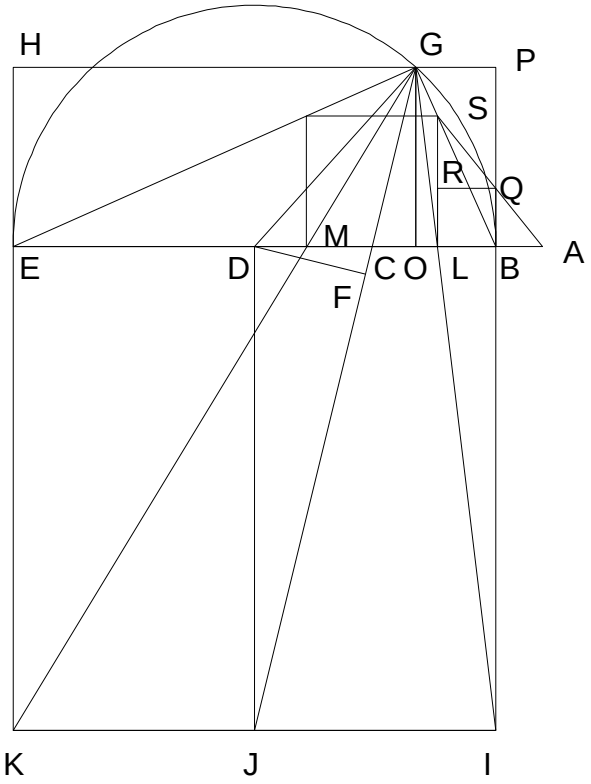
$GH := EO \quad EM := \frac{GH\cdot EK}{HK} \quad BM := BE - EM$

$BQ := BL \quad LR := BL \quad QR := BL$

$LM := BM - BL$

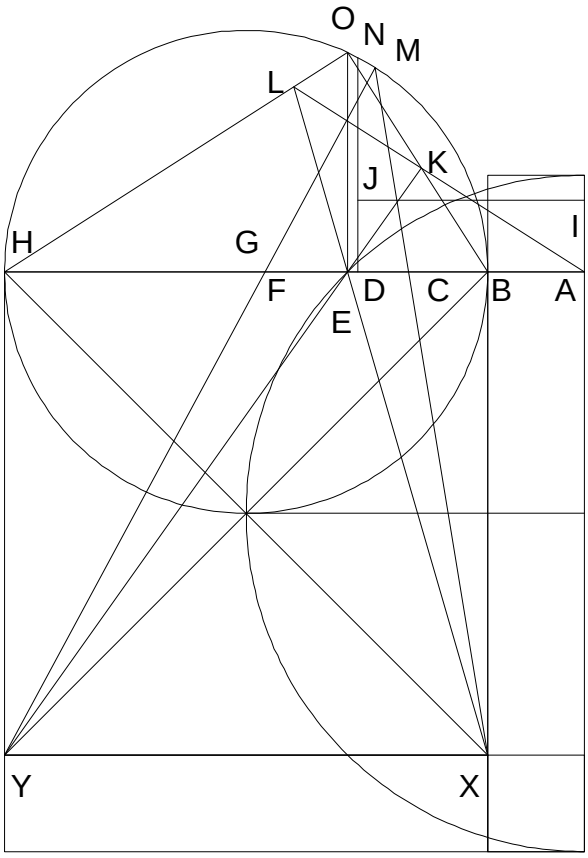
$LS := LM \quad RS := LS - LR \quad AB := \frac{QR\cdot BQ}{RS}$

$AE := AB + BE$




$AB = 510.028 \quad AE = 580.028 \quad BC = 34$

$$\frac{(AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}}{2} + (AB^2 \cdot AE)^{\frac{1}{3}} - AB = 34$$




11_17_94.MCD

How close is DN/3 to CF/2? Graph it.

Given $\delta := 1 .. 1000$ $BH := 1000$

$BX := BH$ $HY := BH$ $AB_\delta := \delta$

$AH_\delta := AB_\delta + BH$ $AE_\delta := \sqrt{AB_\delta \cdot AH_\delta}$

$AC_\delta := \left[ \left(AB_\delta\right)^2 \cdot AH_\delta \right]^{\frac{1}{3}}$

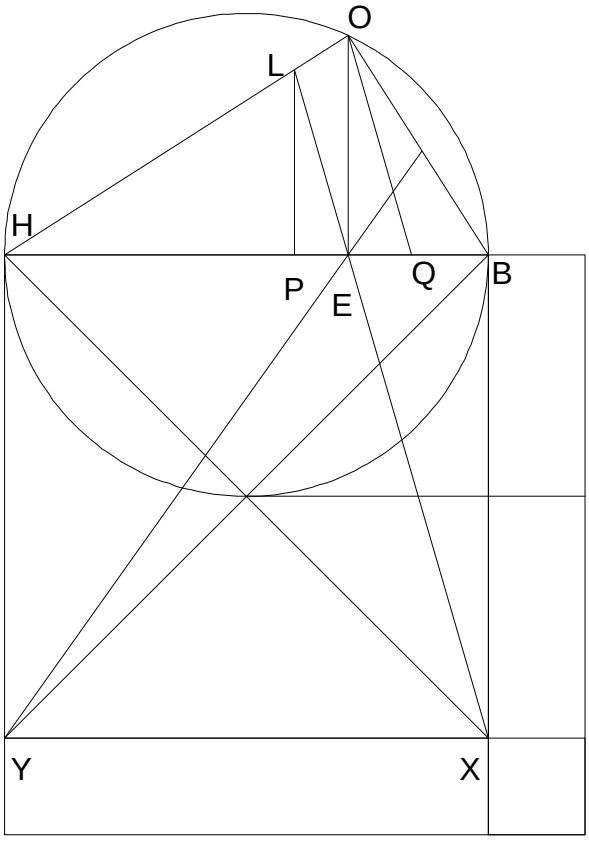
$AF_\delta := \left[ AB_\delta \cdot \left(AH_\delta\right)^2 \right]^{\frac{1}{3}}$

$CF_\delta := AF_\delta - AC_\delta$ $BE_\delta := AE_\delta - AB_\delta$

$EH_\delta := BH - BE_\delta$ $EO_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

$BO_\delta := \sqrt{\left(BE_\delta\right)^2 + \left(EO_\delta\right)^2}$

$HO_\delta := \sqrt{\left(EH_\delta\right)^2 + \left(EO_\delta\right)^2}$




$EQ_\delta := \frac{BE_\delta \cdot EH_\delta}{BX}$ $HQ_\delta := EH_\delta + EQ_\delta$

$EP_\delta := \frac{EQ_\delta \cdot EH_\delta}{HQ_\delta}$ $BP_\delta := BE_\delta + EP_\delta$

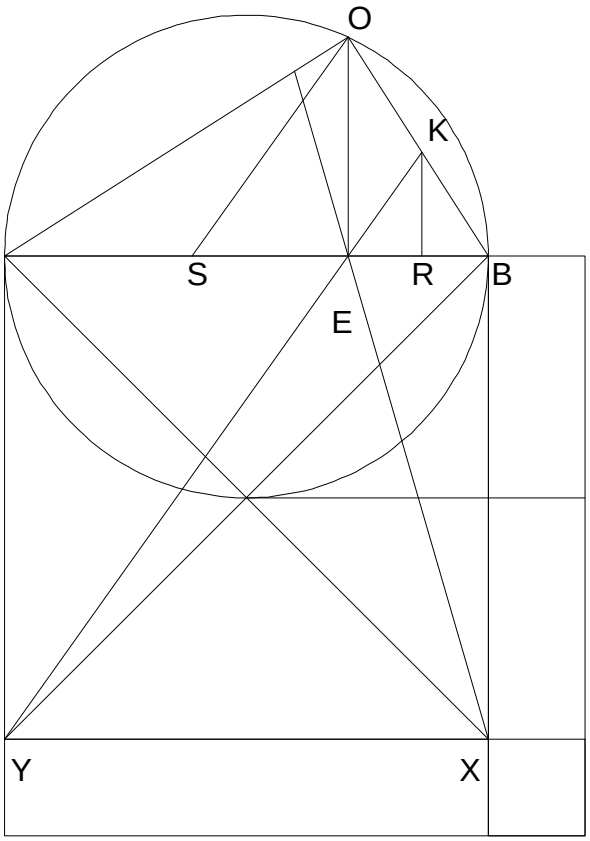
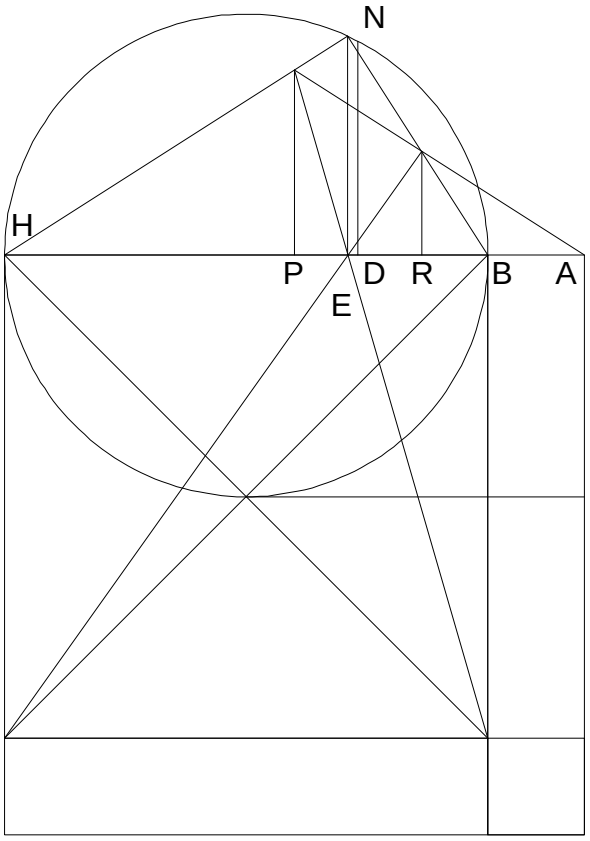



Plate 3

$$ES_\delta := \frac{EH_\delta \cdot EO_\delta}{HY} \qquad BS_\delta := BE_\delta + ES_\delta$$

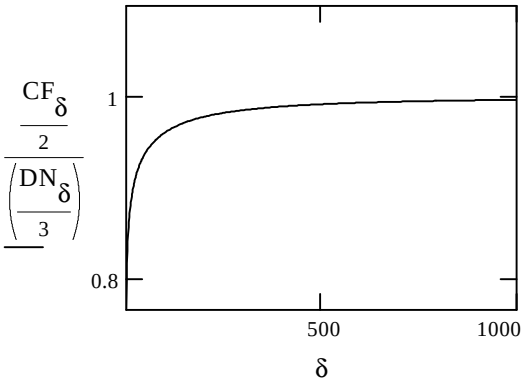
$$BR_\delta := \frac{BE_\delta \cdot BE_\delta}{BS_\delta}$$

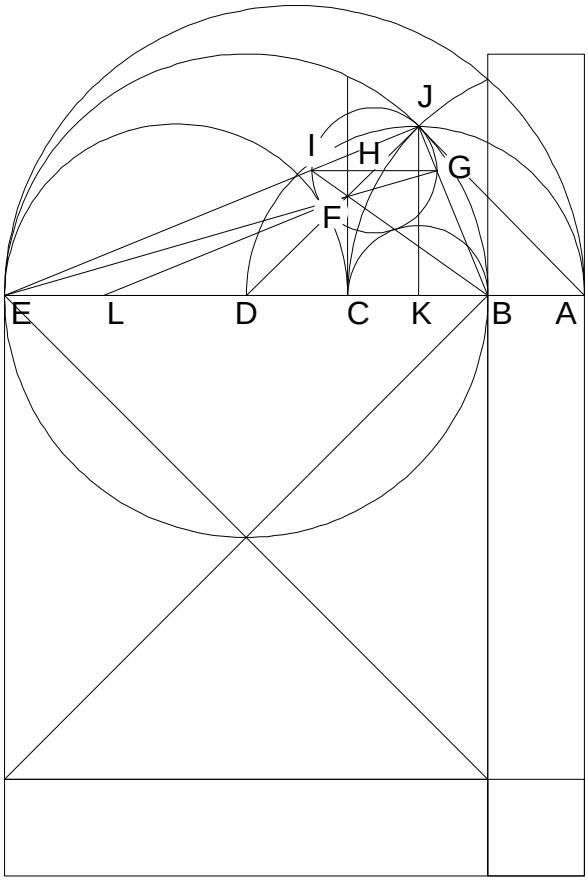



$$PR_\delta := BP_\delta - BR_\delta$$

$$DR_\delta := \frac{PR_\delta}{2} \qquad BD_\delta := BR_\delta + DR_\delta$$

$$DH_\delta := BH - BD_\delta \qquad DN_\delta := \sqrt{BD_\delta \cdot DH_\delta}$$






# Power Line At Square Root.

$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BE := 100 \quad AE_\delta := AB_\delta + BE$

$AC_\delta := \sqrt{AB_\delta \cdot AE_\delta} \quad BD := \frac{BE}{2} \quad DE := BD \quad DJ := BD$

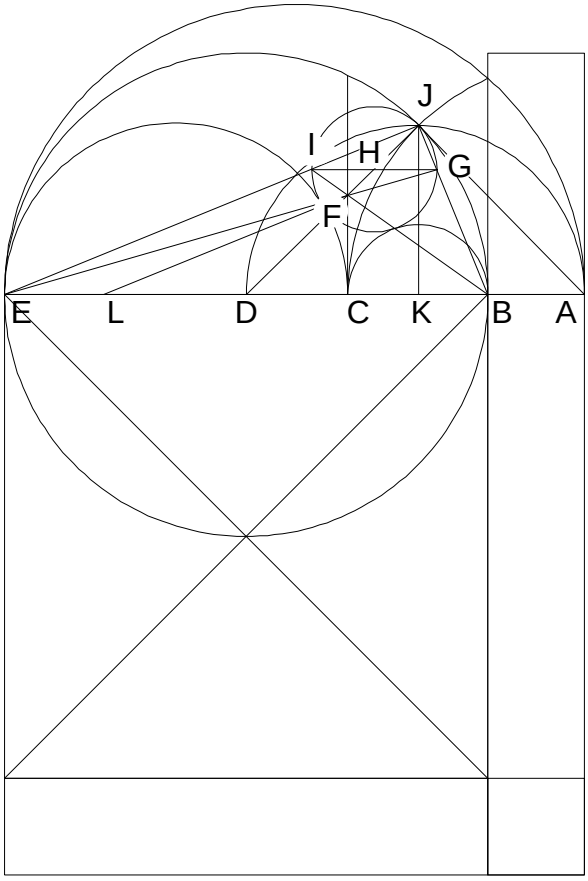
$AD_\delta := AB_\delta + BD \quad DK_\delta := \frac{DJ^2}{AD_\delta} \quad AK_\delta := AD_\delta - DK_\delta$

$BC_\delta := AC_\delta - AB_\delta \quad JK_\delta := \sqrt{AK_\delta \cdot DK_\delta} \quad CD_\delta := BD - BC_\delta$

$CF_\delta := \frac{JK_\delta \cdot CD_\delta}{DK_\delta} \quad BK_\delta := AK_\delta - AB_\delta \quad EK_\delta := BE - BK_\delta$

$CL_\delta := \frac{EK_\delta \cdot CF_\delta}{JK_\delta} \quad BL_\delta := BC_\delta + CL_\delta$

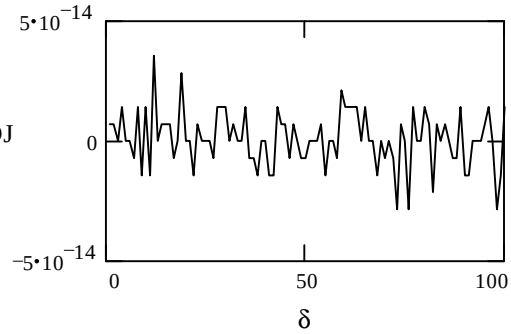
$BF_\delta := \sqrt{(BC_\delta)^2 + (CF_\delta)^2}$

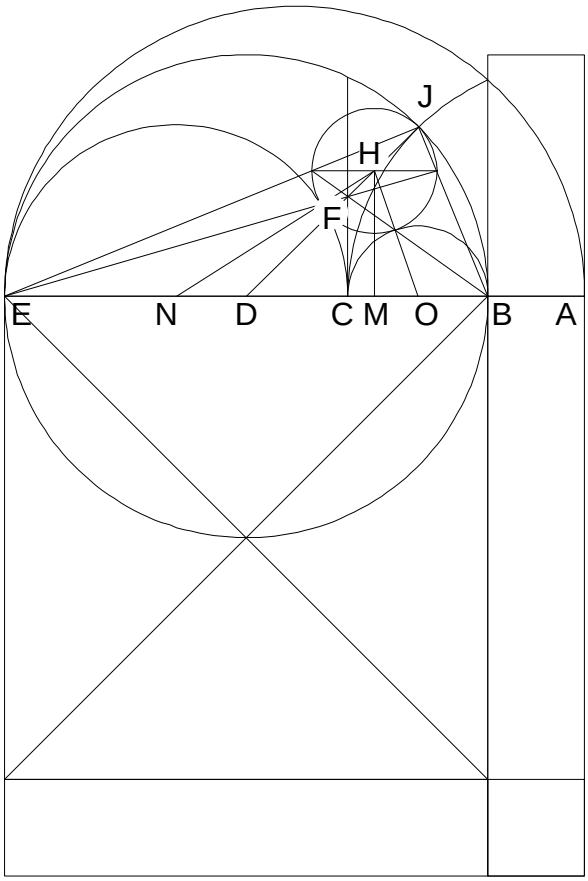



$BI_\delta := \frac{BF_\delta \cdot BE}{BL_\delta} \quad FI_\delta := BI_\delta - BF_\delta \quad GI_\delta := \frac{BE \cdot FI_\delta}{BF_\delta}$

$HJ_\delta := \frac{GI_\delta}{2} \quad DF_\delta := \sqrt{(CD_\delta)^2 + (CF_\delta)^2} \quad FH_\delta := \frac{DF_\delta \cdot GI_\delta}{BE}$

$(DF_\delta + FH_\delta + HJ_\delta) - DJ$



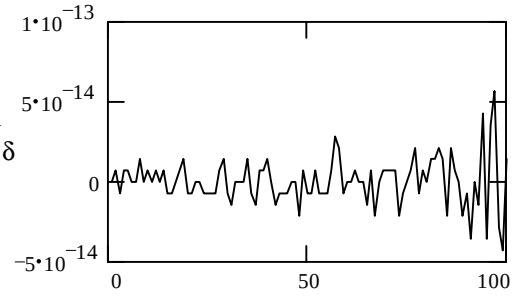



$DH_\delta := DF_\delta + FH_\delta \quad DM_\delta := \frac{CD_\delta \cdot DH_\delta}{DF_\delta} \quad CE_\delta := DE + CD_\delta$

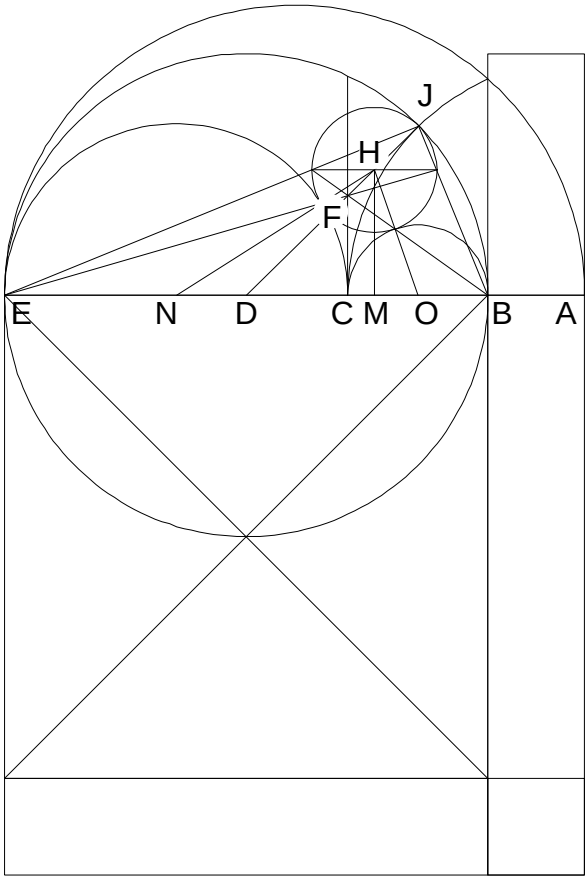
$CM_\delta := DM_\delta - CD_\delta \quad CN_\delta := \frac{CE_\delta}{2} \quad MN_\delta := CN_\delta + CM_\delta$

$HM_\delta := \frac{CF_\delta \cdot DH_\delta}{DF_\delta} \quad HN_\delta := \sqrt{(MN_\delta)^2 + (HM_\delta)^2}$

$\underline{(CN_\delta + HJ_\delta) - HN_\delta}$



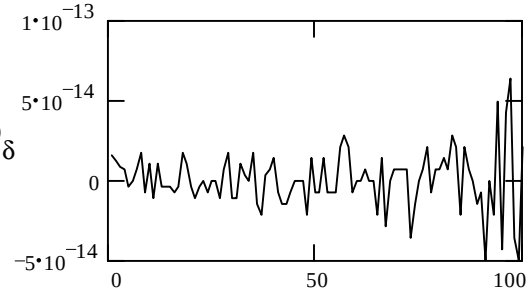

— Is Tangent?




$CO_\delta := \frac{BC_\delta}{2} \quad MO_\delta := CO_\delta - CM_\delta$

$HO_\delta := \sqrt{(HM_\delta)^2 + (MO_\delta)^2}$

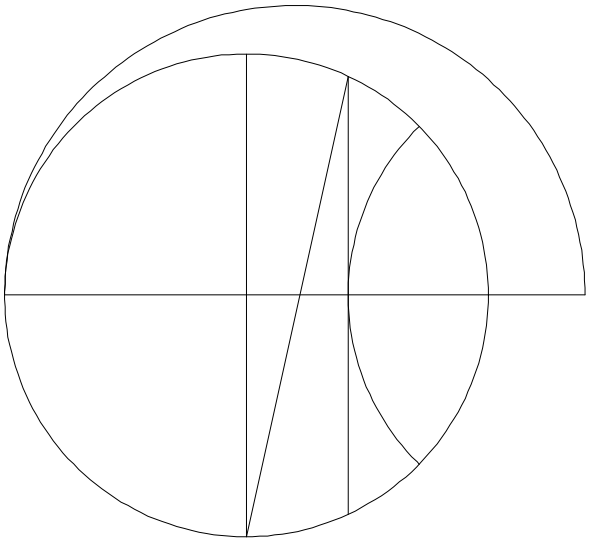
$\underline{(CO_\delta + HJ_\delta) - HO_\delta}$

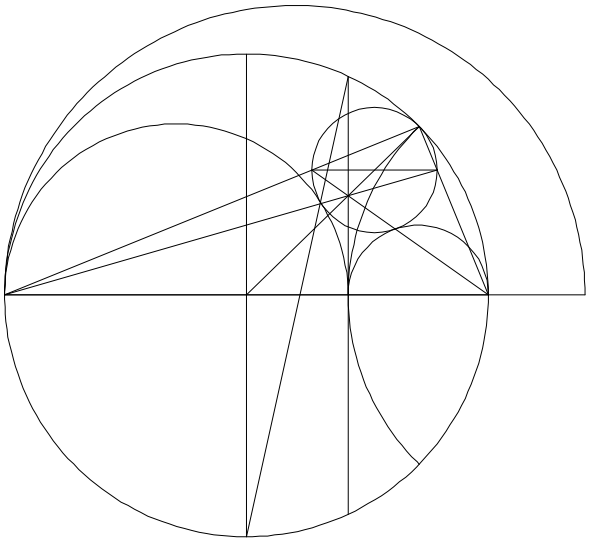


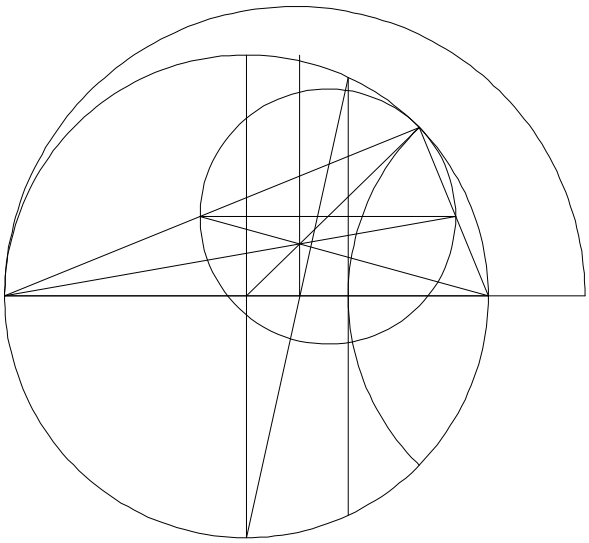

— Is Tangent?

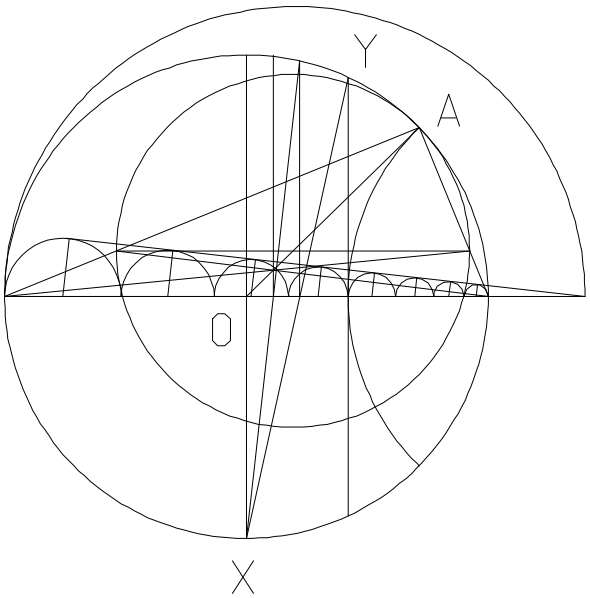
# Two prime exponential series developed through power line progression.

I will present a series of plates to explain the process. The process can be infinitly repeated, supposing you had the tools to do it with.



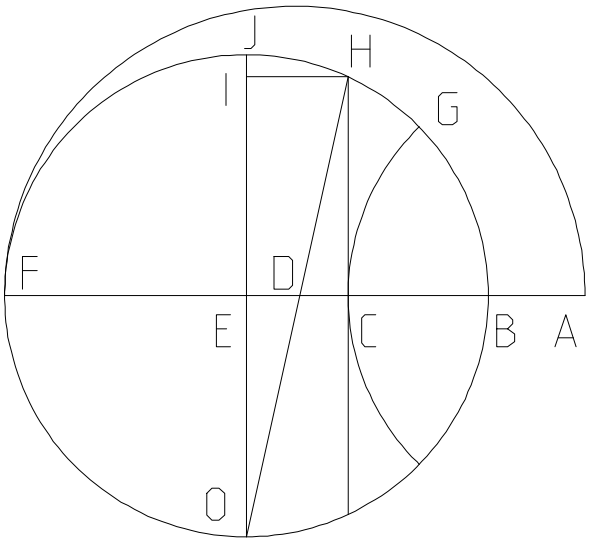







It is clear how OA uses the power line XY to provide a 2 prime exponential series.

Possible Problem: From a similarity point outside of a circle, place some 2 prine sequence of smaller circles on the larger circles diameter, all tangent in sequence.



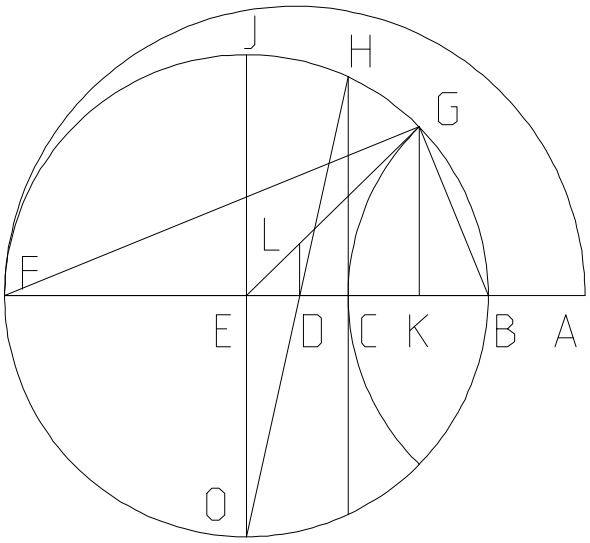

$AB := 1 \quad BF := 5 \quad AF := AB + BF$

$AC := \sqrt{AB \cdot AF} \qquad BE := \frac{BF}{2} \quad EO := BE$

$BC := AC - AB \qquad CF := BF - BC$

$CH := \sqrt{BC \cdot CF} \quad EI := CH \quad CE := BE - BC$

$HI := CE \quad IO := EO + EI \quad DE := \frac{HI \cdot EO}{IO}$

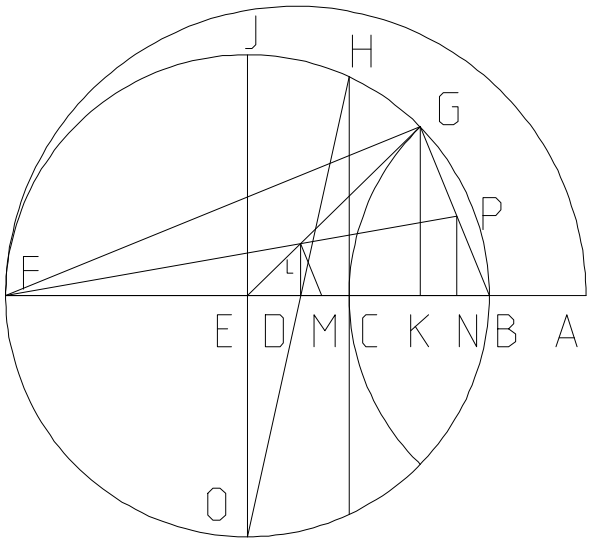



See 12_26_94.MCD for next equation.

$$GK := \frac{BF \cdot \left(\sqrt{AB} \cdot \sqrt{AB + BF}\right)}{(2 \cdot AB + BF)}$$

$EG := BE \quad EK := \sqrt{EG^2 - GK^2}$

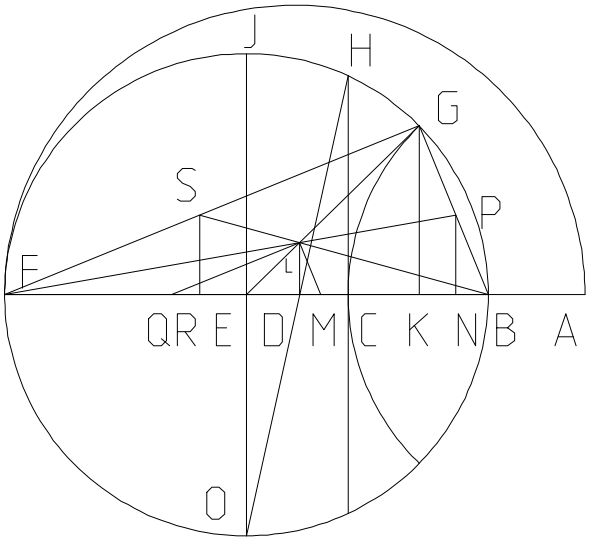
$DL := \frac{GK \cdot DE}{EK}$




$KN := BE - EK \quad DM := \frac{KN \cdot DE}{EK}$

$EF := BE \quad FM := EF + DM + DE$

$BN := \frac{DM \cdot BF}{FM} \quad NP := \frac{DL \cdot BF}{FM}$



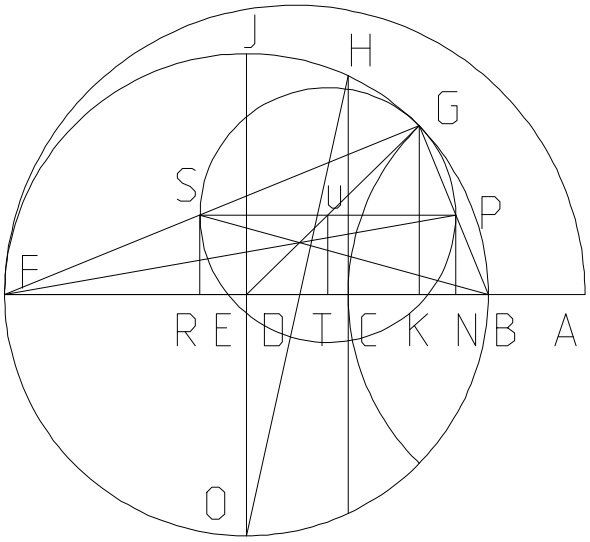

$KF := EK + EF \quad DQ := \frac{KF \cdot DL}{GK}$

$BD := BE - DE \quad BQ := BD + DQ$

$BR := \frac{BD \cdot BF}{BQ} \quad RS := \frac{DL \cdot BR}{BD}$

Are RS and NP equal?

$RS - NP = 0$




$TU := NP \quad ET := \frac{EK \cdot TU}{GK} \quad NR := BR - BN$

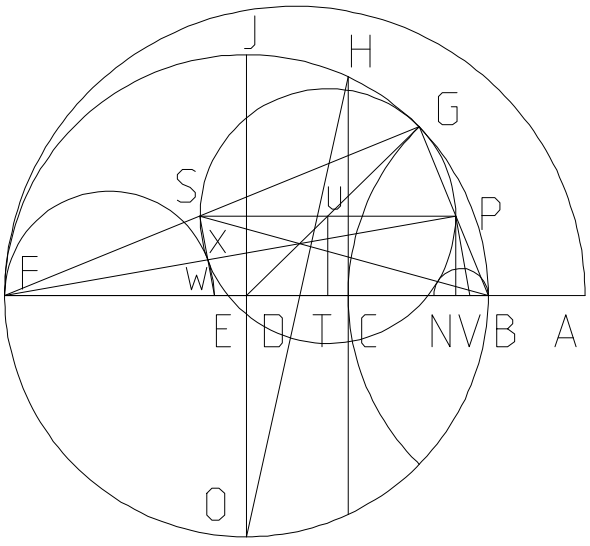
$EN := BE - BN \quad NT := EN - ET$

$PS := NR \quad PU := NT \quad EU := \sqrt{ET^2 + TU^2}$

Is NT half of NR? $\frac{NR}{NT} = 2$

Does GU = PU? $GU := EG - EU$

$GU - PU = 0$



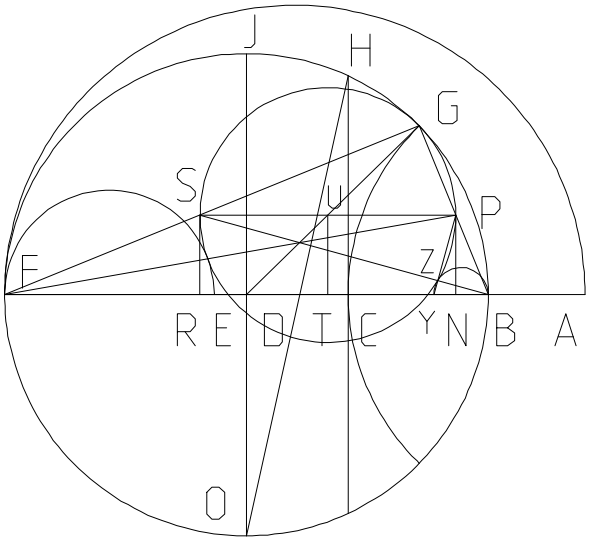

$BT := BN + NT \quad FN := BF - BN \quad FP := \sqrt{NP^2 + FN^2}$

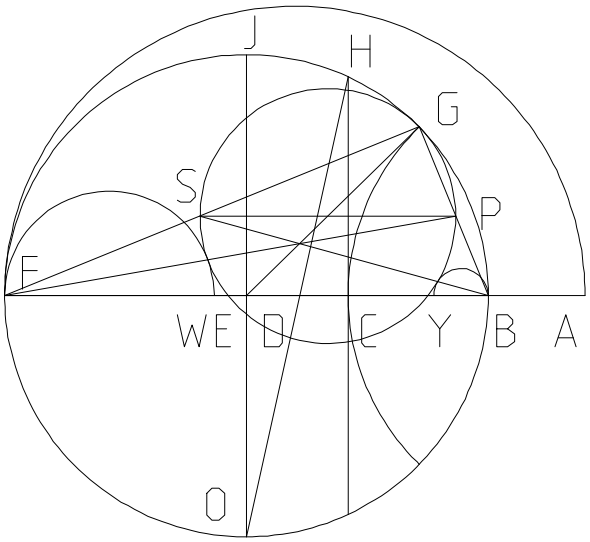
$FV := \frac{FP^2}{FN} \quad PX := \frac{FP \cdot PS}{FV} \quad FX := FP - PX$

$FW := \frac{FV \cdot FX}{FP} \quad BW := BF - FW \quad AW := AB + BW$

Is AW a quarter root of AB, AF?

$(AB \cdot AF^3)^{\frac{1}{4}} - AW = 0$



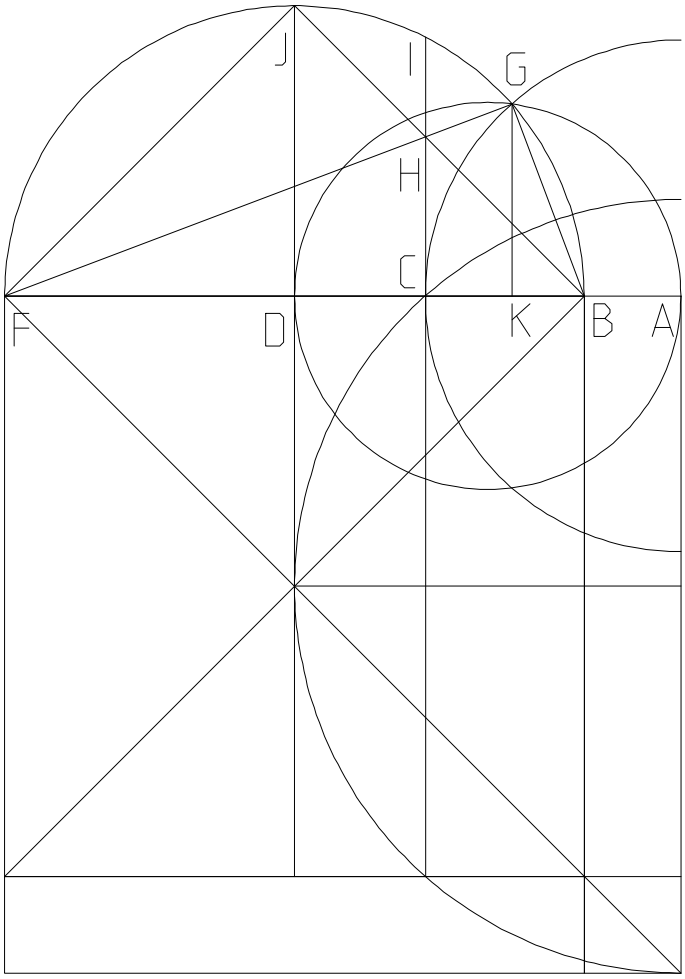

$BS := \sqrt{BR^2 + RS^2}$ $SZ := \frac{BR \cdot PS}{BS}$

$BZ := BS - SZ$ $BY := \frac{PS \cdot BZ}{SZ}$

$AY := AB + BY$

Is AY a quarter root of AB, AF?

$(AB^3 \cdot AF)^{\frac{1}{4}} - AY = 0$




Does H and G have a constant relationship?

$\delta := 1 .. 1000 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$BF := 6 \quad BD := \frac{BF}{2} \quad AF_\delta := AB_\delta + BF$

$AC_\delta := \sqrt{AB_\delta \cdot AF_\delta} \quad DJ := BD \quad DF := BD$

$BC_\delta := AC_\delta - AB_\delta \quad CD_\delta := BD - BC_\delta$

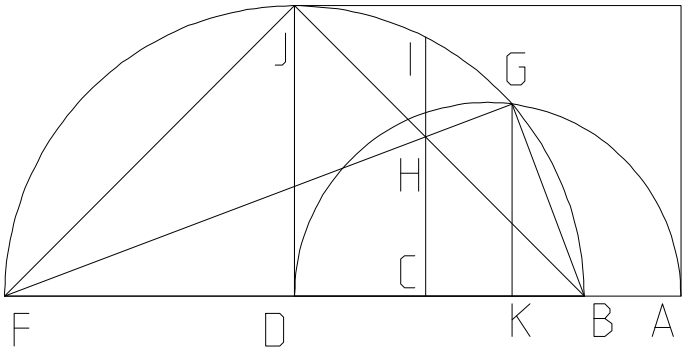
$CH_\delta := \frac{DJ \cdot BC_\delta}{BD} \quad AG_\delta := AC_\delta$

$AD_\delta := AB_\delta + BD \quad AK_\delta := \frac{(AG_\delta)^2}{AD_\delta}$

$GK_\delta := \sqrt{(AG_\delta)^2 - (AK_\delta)^2}$

$BK_\delta := AK_\delta - AB_\delta \quad CK_\delta := BC_\delta - BK_\delta$

$KF_\delta := CK_\delta + CD_\delta + DF$

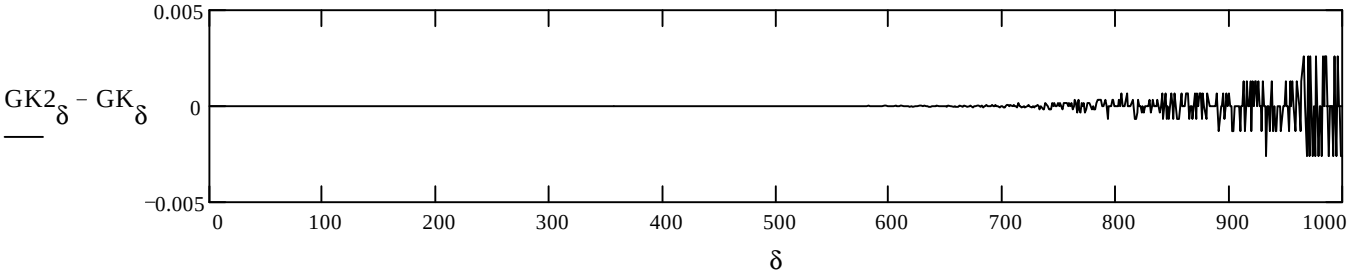


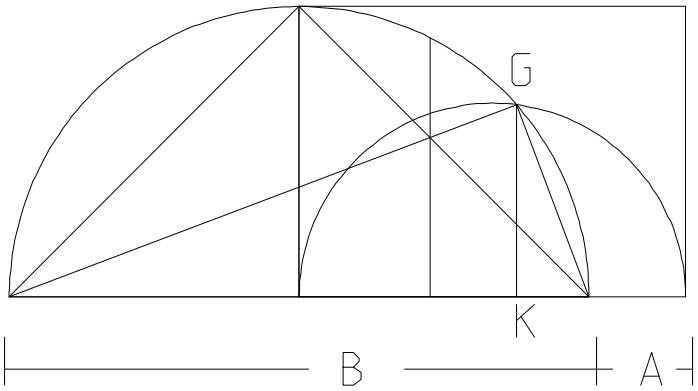

Does FH and FG have identical slopes?

$CF_\delta := CD_\delta + DF$

$GK2_\delta := \frac{CH_\delta \cdot KF_\delta}{CF_\delta}$

Therefore G and H are constantly co-linear.





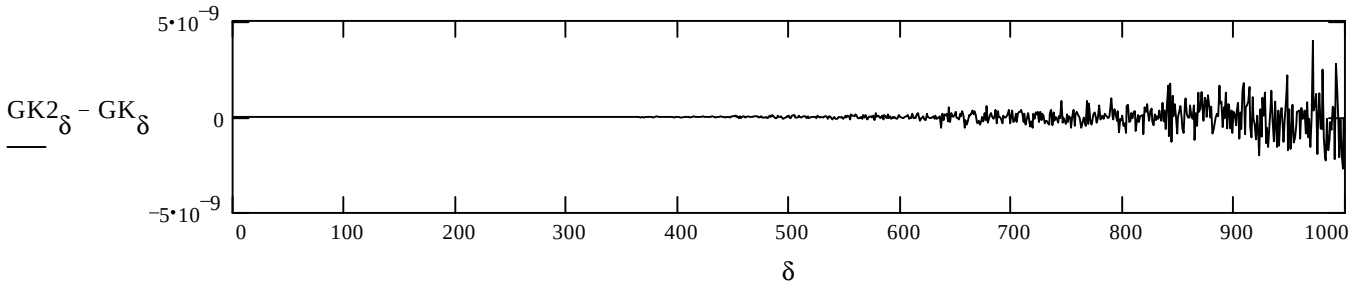

Thus this file can be redone as: "Given BC = $\sqrt{AB \cdot AF}$ and BF, find AB."

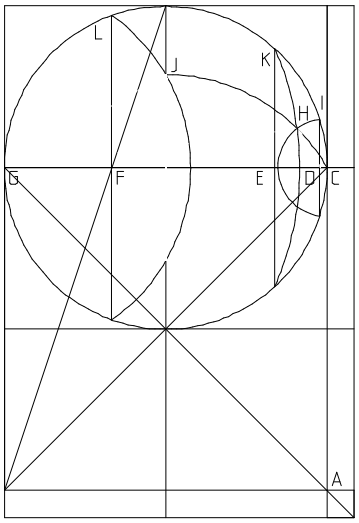
The Formula for GK vs. GK2 demonstrates that the symbolic processor cannot always resolve to simplest form. GK2 is the processors final attempt. An attempt with Mathcad 6 gives the same result.

$A_\delta := AB_\delta \quad B := BF$

$$GK2_\delta := B \cdot \frac{\left[ \left(A_\delta\right)^{\left(\frac{3}{2}\right)} \cdot \sqrt{A_\delta + B} - \left(A_\delta\right)^2 + B \cdot \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} - B \cdot A_\delta \right]}{\left[ \left(2 \cdot A_\delta + B\right) \cdot \left(B - \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} + A_\delta\right) \right]}$$

$$GK_\delta := \frac{B \cdot \left(\sqrt{A_\delta} \cdot \sqrt{A_\delta + B}\right)}{\left(2 \cdot A_\delta + B\right)}$$





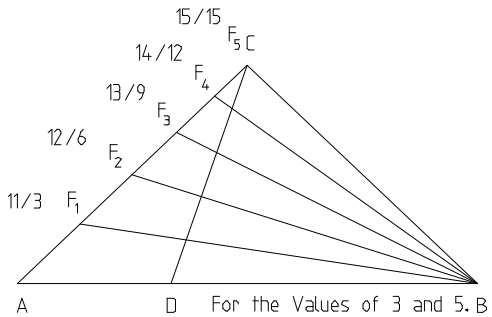

# One Circle

**By John Clark**

# One Square

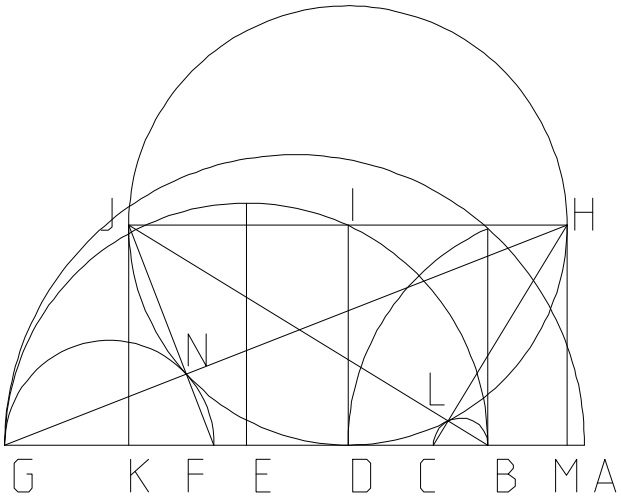
# One Line

## And the Delian Quest




# 1995

# Alternate method for Quad Roots




$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BG := 10 \quad AG_\delta := AB_\delta + BG$

$AD_\delta := \sqrt{AB_\delta \cdot AG_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

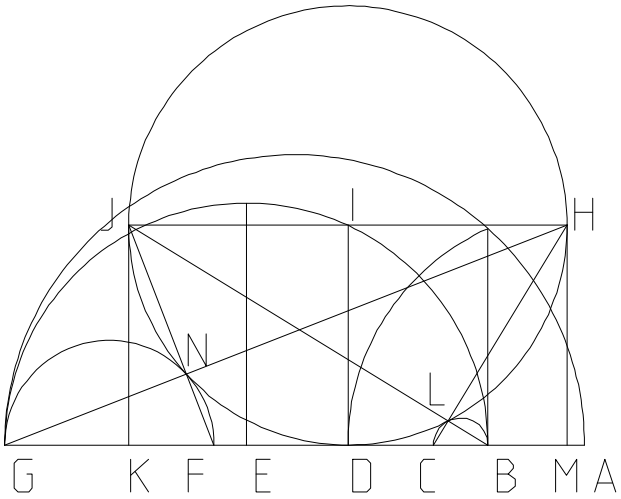
$DG_\delta := BG - BD_\delta \quad DI_\delta := \sqrt{BD_\delta \cdot DG_\delta}$

$HI_\delta := DI_\delta \quad IJ_\delta := DI_\delta \quad HJ_\delta := HI_\delta + IJ_\delta$

$DK_\delta := DI_\delta \quad JK_\delta := DI_\delta \quad BK_\delta := BD_\delta + DK_\delta$

$BJ_\delta := \sqrt{\left(BK_\delta\right)^2 + \left(JK_\delta\right)^2} \quad JL_\delta := \frac{BK_\delta \cdot HJ_\delta}{BJ_\delta}$

$BL_\delta := BJ_\delta - JL_\delta \quad BC_\delta := \frac{HJ_\delta \cdot BL_\delta}{JL_\delta}$




$HM_\delta := DI_\delta \quad DM_\delta := DI_\delta \quad MG_\delta := DM_\delta + DG_\delta$

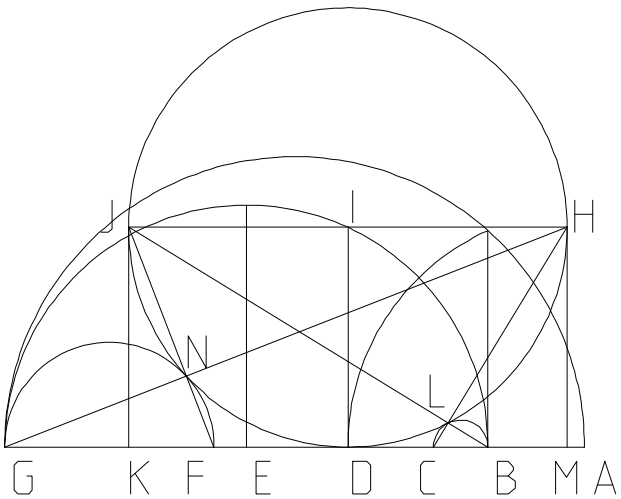
$GH_\delta := \sqrt{\left(MG_\delta\right)^2 + \left(HM_\delta\right)^2} \quad HN_\delta := \frac{MG_\delta \cdot HJ_\delta}{GH_\delta}$

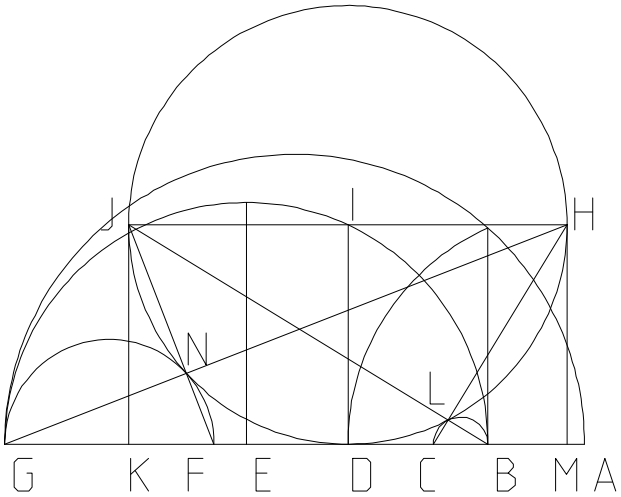
$GN_\delta := GH_\delta - HN_\delta \quad FG_\delta := \frac{HJ_\delta \cdot GN_\delta}{HN_\delta}$

$BF_\delta := BG - FG_\delta$

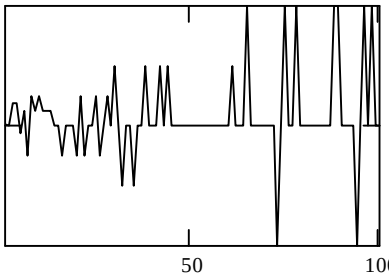
$AF_\delta := AB_\delta + BF_\delta$

$AC_\delta := AB_\delta + BC_\delta$



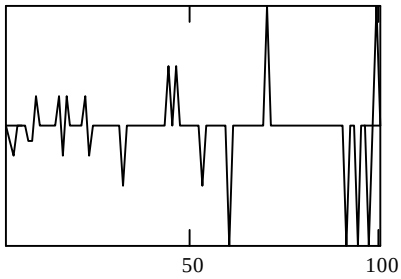



$$\left[ AC_{\delta} - \left[ \left( AB_{\delta} \right)^{3} \cdot AG_{\delta} \right]^{\frac{1}{4}} \right]_{0}$$



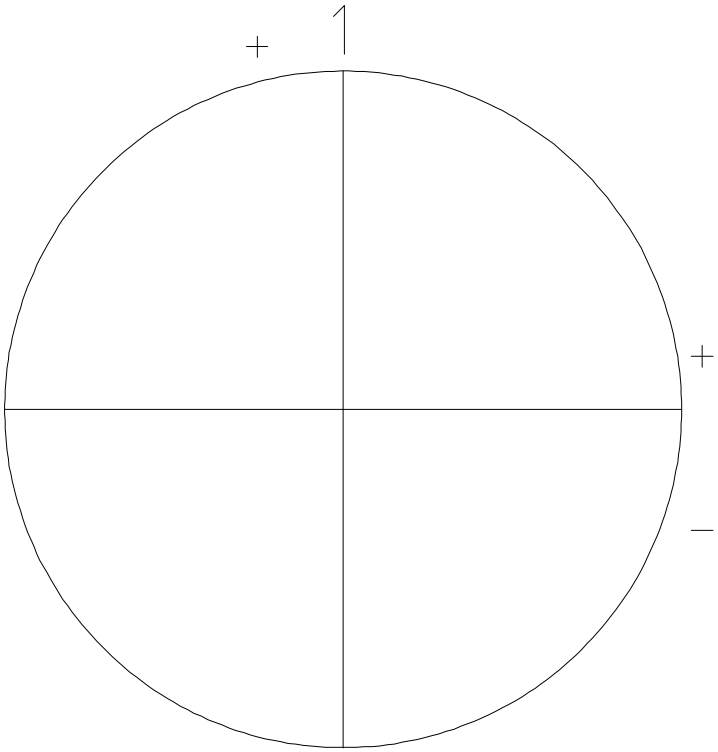

δ

$$\left[ AF_{\delta} - \left[ AB_{\delta} \cdot \left( AG_{\delta} \right)^{3} \right]^{\frac{1}{4}} \right]_{0}$$




δ

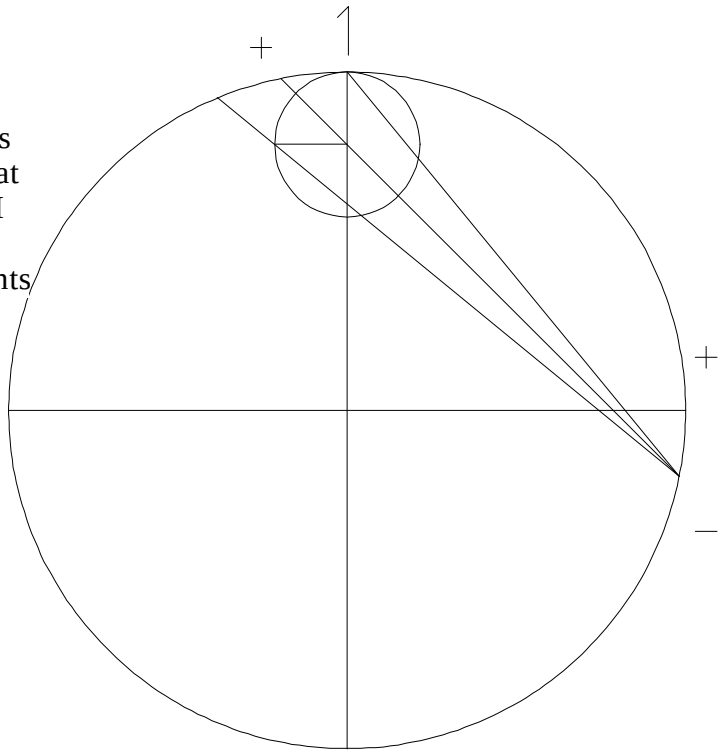
The symbolic processor on my computer could not reduce the chain to the final equations.




Archamedian Trisector Revisited.

I am curious as to why the Archamedian trisector is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

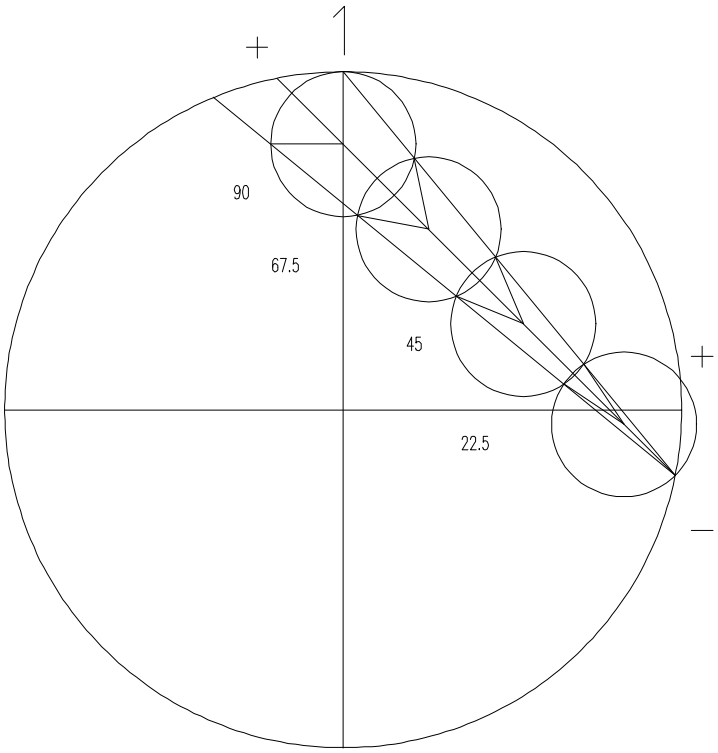
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimiter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the agle I am working with. I have maked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$




$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

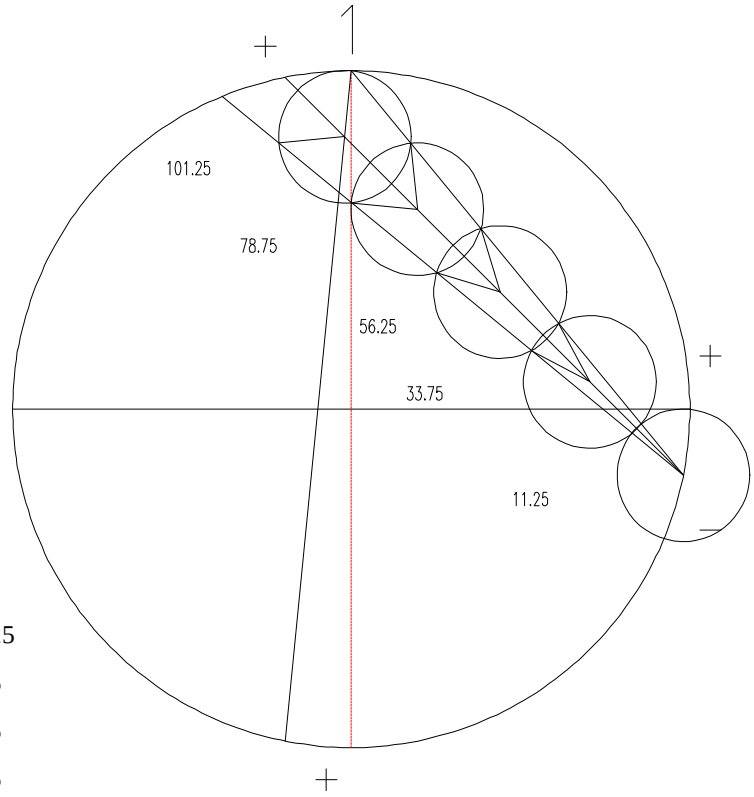
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

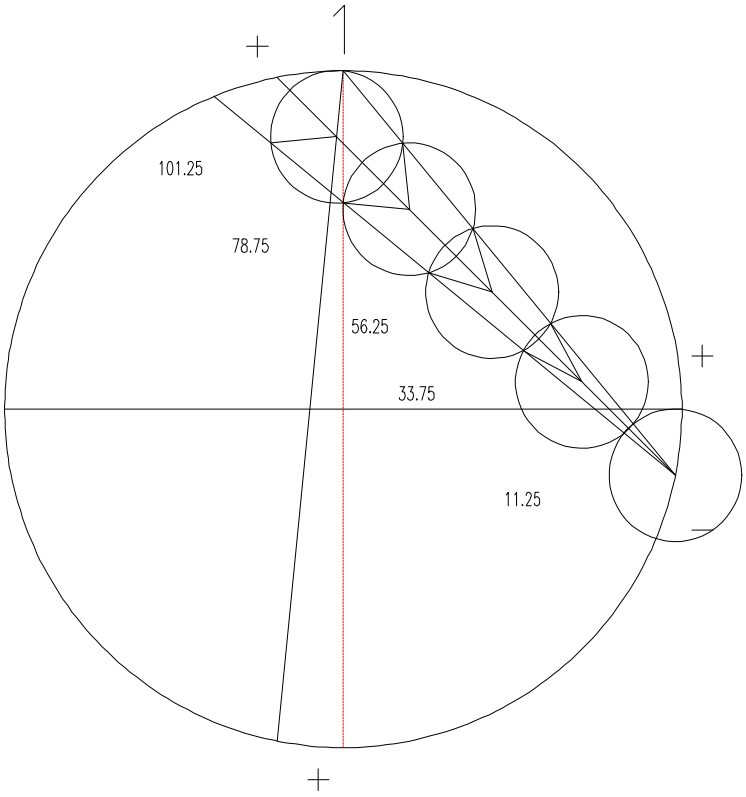
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$




| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

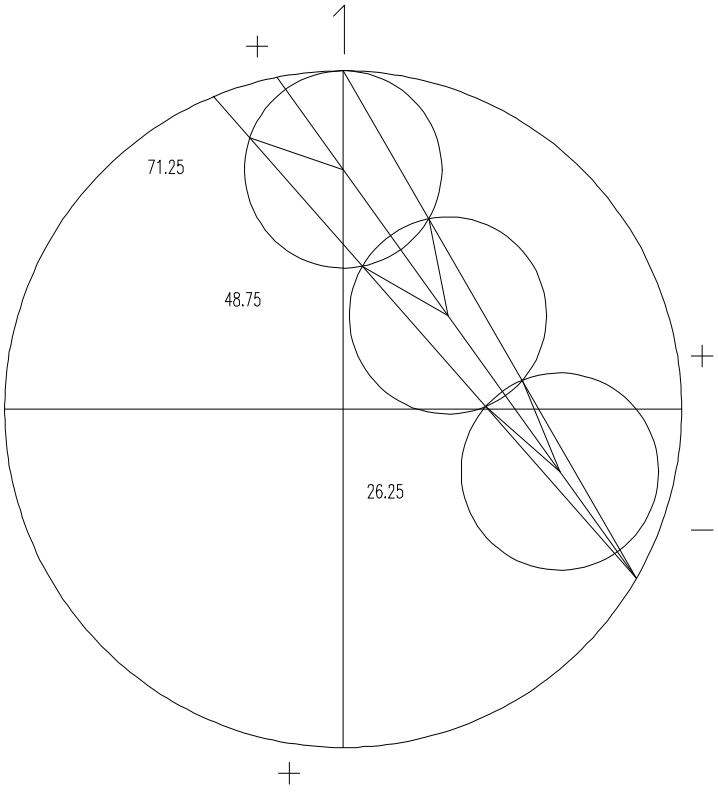
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.79166667$ $\frac{19}{24} = 0.79166667$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.74952631$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.24990526$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.74998579$

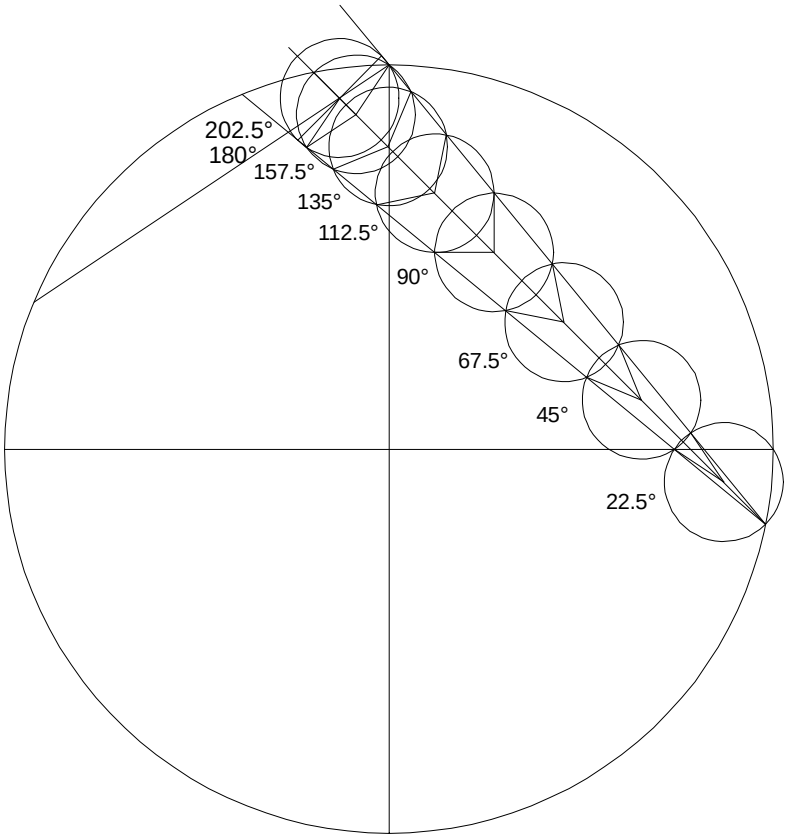
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

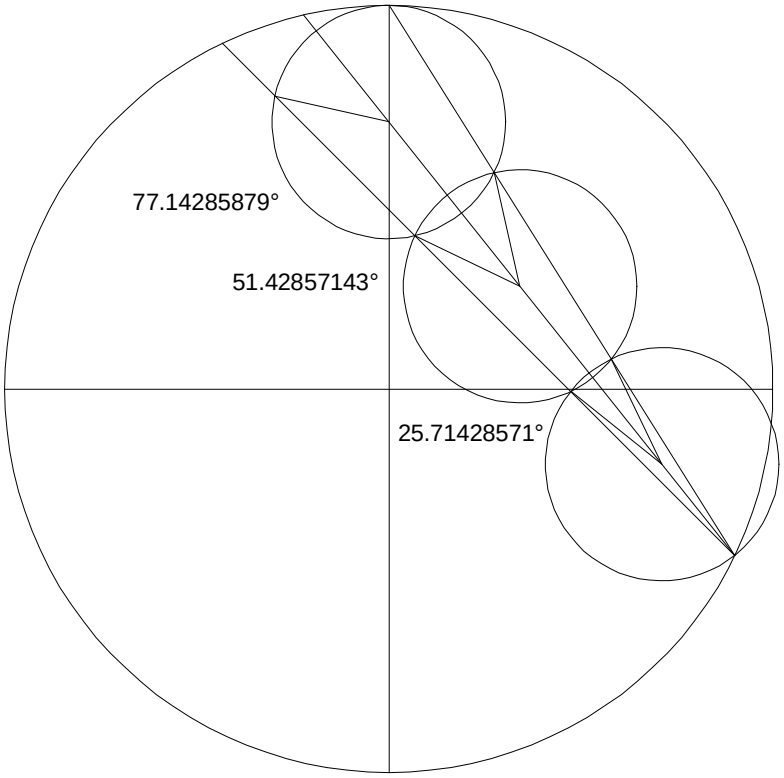
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.85714286$ $\frac{6}{7} = 0.85714286$

$\frac{B \cdot 6}{6} \cdot 90 = 77.14285714$ $\frac{B \cdot 4}{6} \cdot 90 = 51.42857143$

$\frac{B \cdot 2}{6} \cdot 90 = 25.71428571$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.14285714$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.42857143$

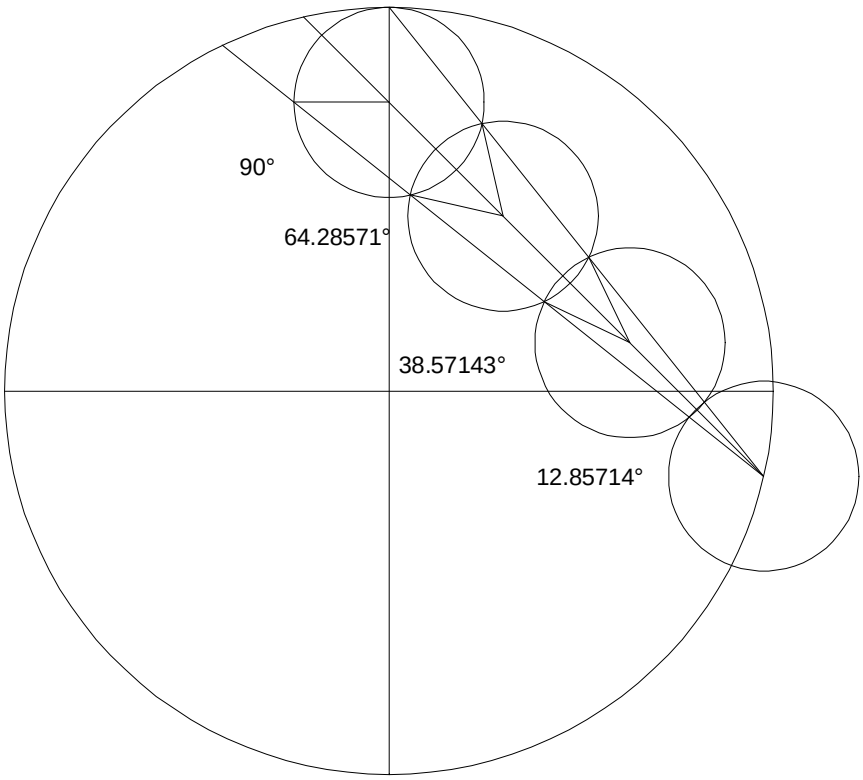
$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.71428571$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.28571429$

$\frac{B \cdot 3}{7} \cdot 90 = 38.57142857$ $\frac{B \cdot 1}{7} \cdot 90 = 12.85714286$



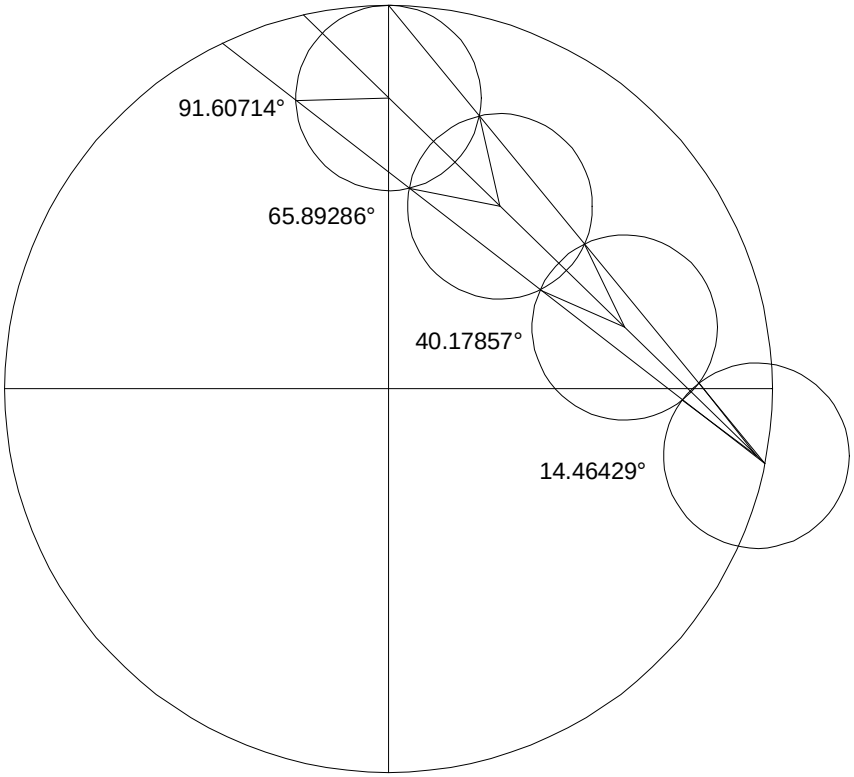

$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.28571429$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.57142857$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.85714286$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.01785714$

$\frac{B \cdot 57}{57} \cdot 90 = 91.60714286$ $\frac{B \cdot 41}{57} \cdot 90 = 65.89285714$

$\frac{B \cdot 25}{57} \cdot 90 = 40.17857143$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.60714286$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.89285714$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.17857143$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.46428571$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.28571429$

$\frac{B \cdot 3}{7} \cdot 90 = 38.57142857$ $\frac{B \cdot 1}{7} \cdot 90 = 12.85714286$

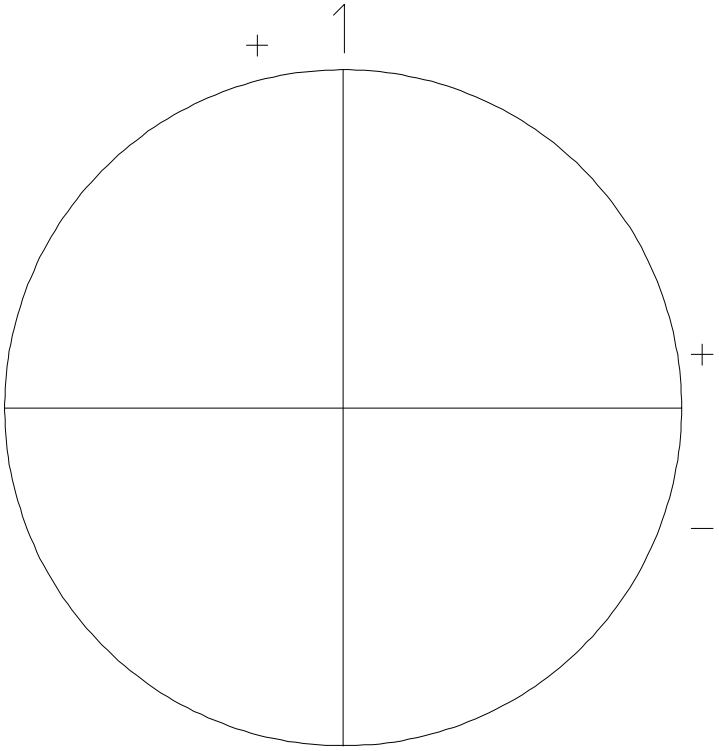
$7 + 1 - 1 = 7$

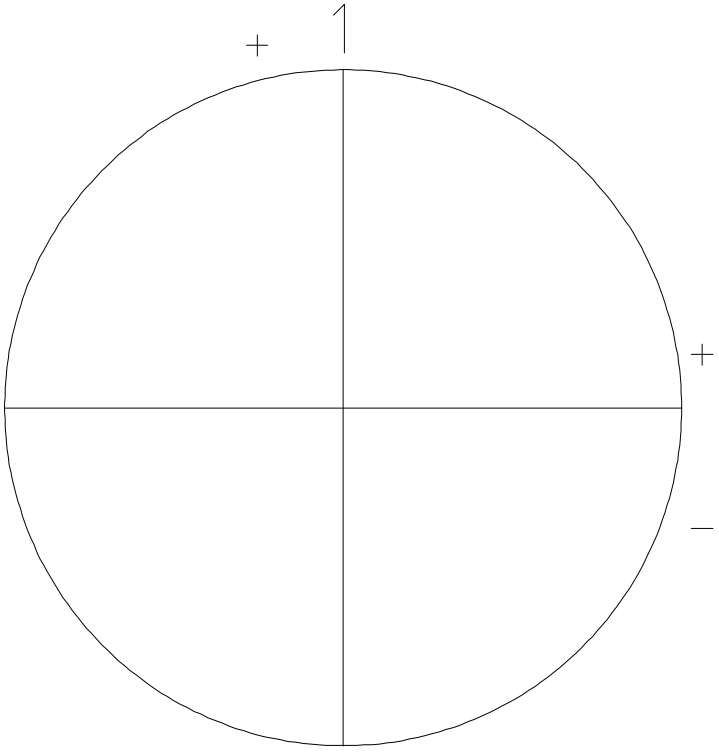
$7 + 1 - 1 - (1 \cdot 2) = 5$

$7 + 1 - 1 - (2 \cdot 2) = 3$

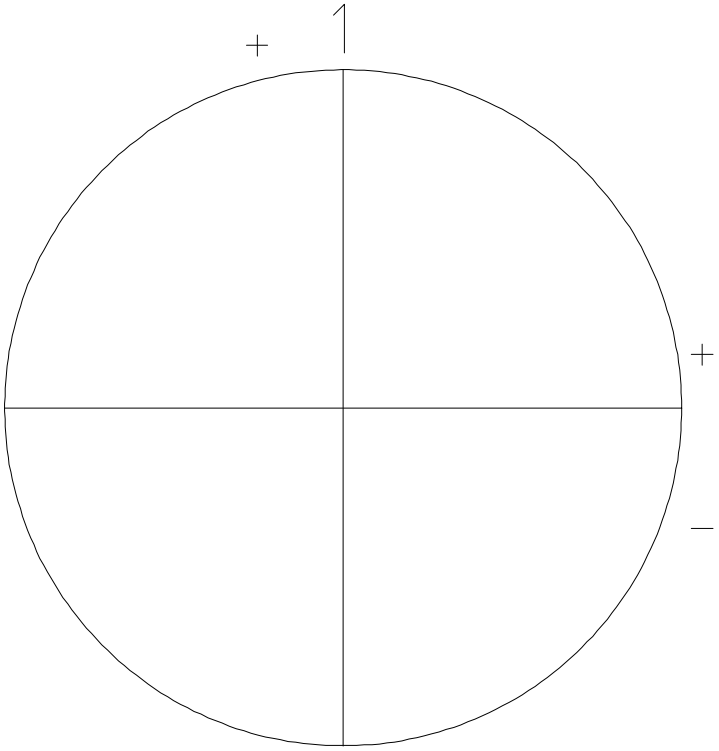
$7 + 1 - 1 - 2 - 2 - 2 = 1$

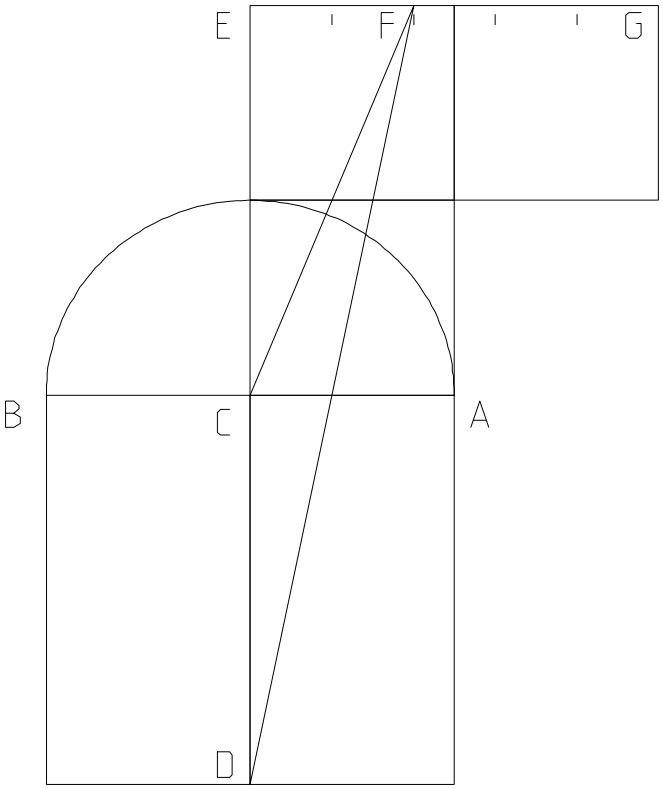
$\mathrm{mod}(7 + 1 - 1, 2) = 1$



$$\frac{91.607}{12.857} \cdot 8 = 57.00054445$$






Exploring the cube.

03_28_95.MCD

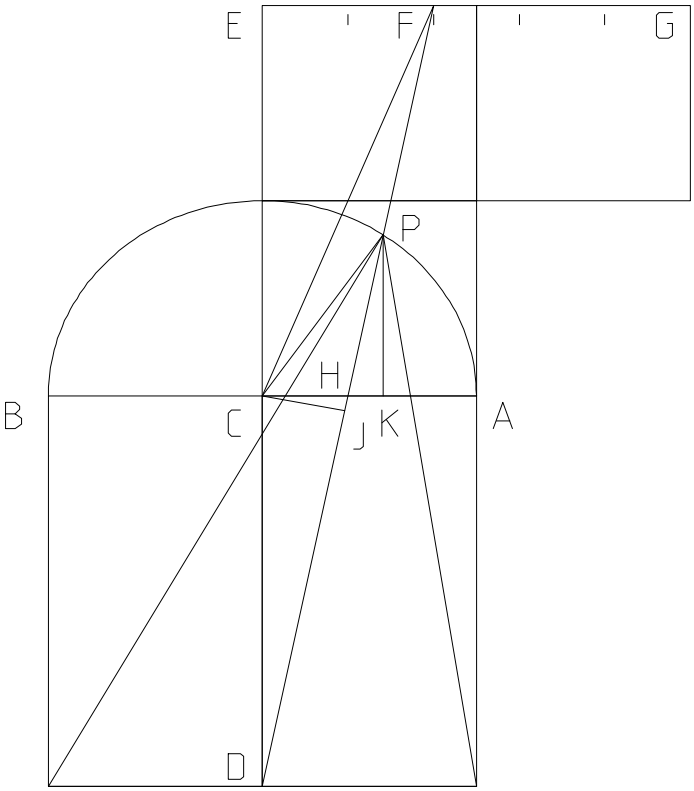
I have noticed a relationship among perfect cubes for the curve listed in previous work as CJ. Because of the nature of the projection the curve is not actually needed here to demonstrate.

Using perfect cube numbers (N), the point F seems to fall exactly on some unit value of N - 1. It also appears to fall on sequencial multiples of N - 1 in its own sequence.

$\delta := 1 .. \Delta - 1 \quad AB := 1$

$EG := AB \quad CD := AB \quad CE := AB$

$EF_\delta := \frac{EG}{\Delta - 1} \cdot \delta \quad AC := \frac{AB}{2}$




$DE := CD + CE \quad CH_\delta := \frac{EF_\delta \cdot CD}{DE}$

$DF_\delta := \sqrt{DE^2 + (EF_\delta)^2}$

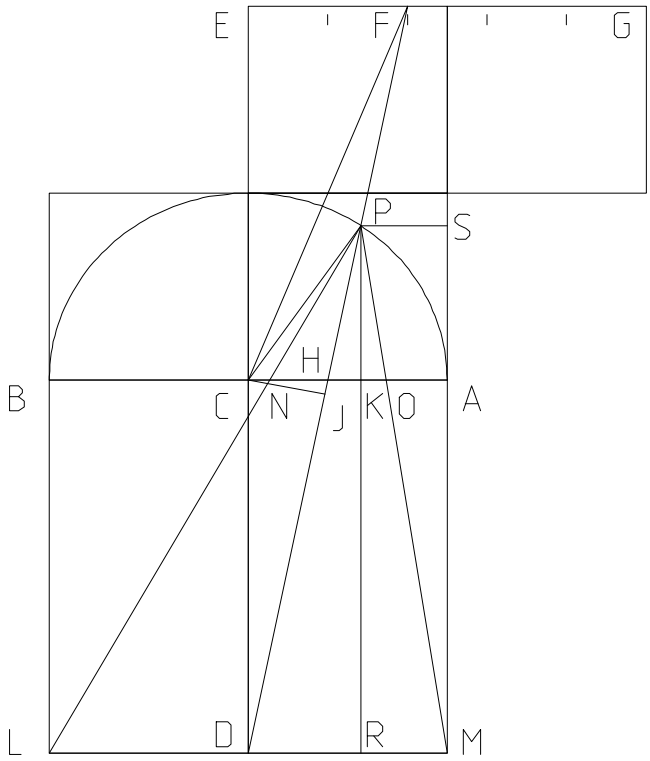
$CJ_\delta := \frac{DE \cdot CH_\delta}{DF_\delta}$

$HJ_\delta := \sqrt{(CH_\delta)^2 - (CJ_\delta)^2}$

$CP := AC \quad JP_\delta := \sqrt{CP^2 - (CJ_\delta)^2}$

$HP_\delta := JP_\delta - HJ_\delta$

$KP_\delta := \frac{DE \cdot HP_\delta}{DF_\delta}$




$$KR := CD \quad PR_\delta := KR + KP_\delta$$

$$LM := AB \quad AS_\delta := KP_\delta \quad BT_\delta := KP_\delta$$

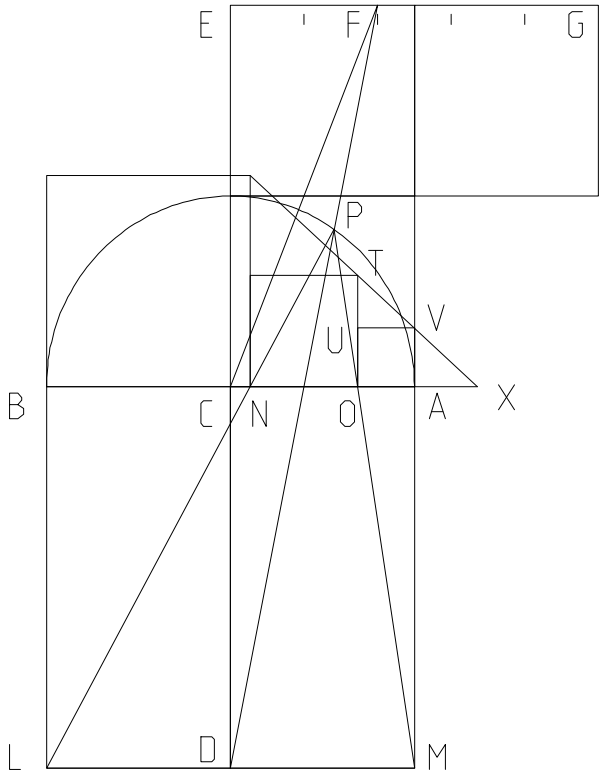
$$NO_\delta := \frac{LM \cdot KP_\delta}{PR_\delta}$$

$$HK_\delta := \frac{EF_\delta \cdot KP_\delta}{DE} \quad CK_\delta := CH_\delta + HK_\delta$$

$$AK_\delta := AC - CK_\delta \quad PS_\delta := AK_\delta$$

$$AM := AB \quad MS_\delta := AM + AS_\delta$$

$$AO_\delta := \frac{PS_\delta \cdot AM}{MS_\delta} \quad AN_\delta := AO_\delta + NO_\delta$$




$$OT_\delta := NO_\delta \quad OU_\delta := AO_\delta$$

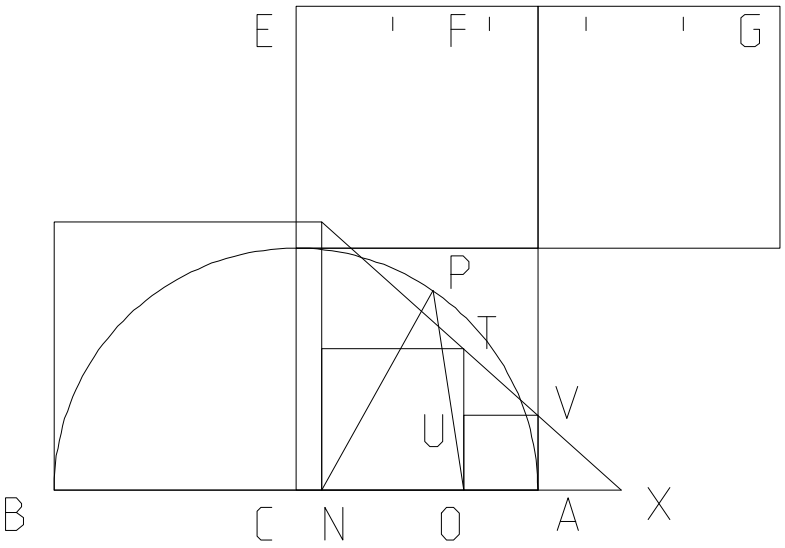
$$TU_\delta := OT_\delta - OU_\delta \quad UV_\delta := AO_\delta$$

$$OX_\delta := \text{if}\left(TU_\delta, \frac{UV_\delta \cdot OT_\delta}{TU_\delta}, 0\right)$$

$$AX_\delta := OX_\delta - AO_\delta$$

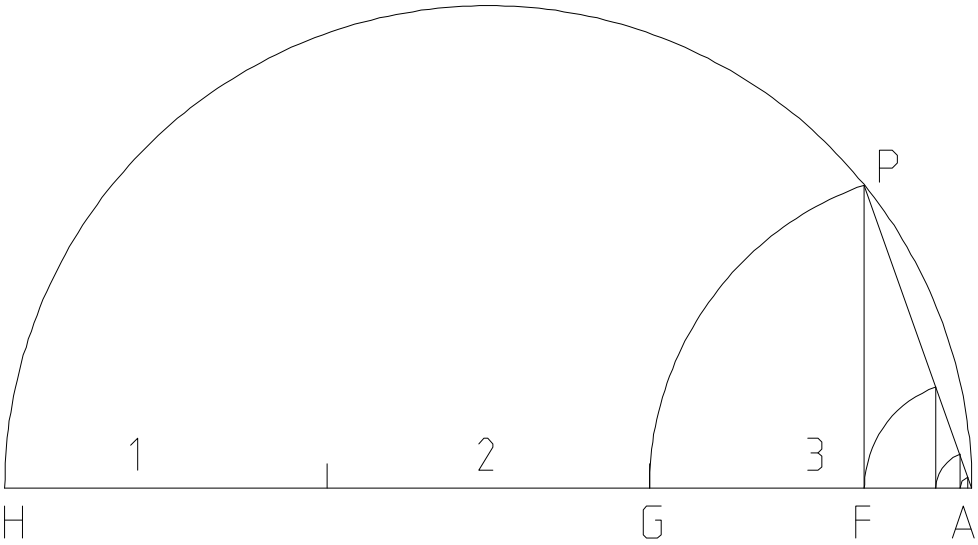
$$DIVS_\delta := \text{if}\left(AX_\delta, \frac{AB + AX_\delta}{AX_\delta}, 0\right)$$

$$TABLE := augment(DIVS, (augment(AX, augment(EF, DIVS))))^{T}$$




$\Delta \equiv 8$

$$TABLE = \begin{bmatrix} 0 & 1.911411471 & 3.770673353 & 8 & 19.557606074 & 64 & 454.830372231 & 0 \\ 0 & 1.097199269 & 0.360923094 & 0.142857143 & 0.053886261 & 0.015873016 & 0.002203466 & 0 \\ 0 & 0.142857143 & 0.285714286 & 0.428571429 & 0.571428571 & 0.714285714 & 0.857142857 & 1 \\ 0 & 1.911411471 & 3.770673353 & 8 & 19.557606074 & 64 & 454.830372231 & 0 \end{bmatrix}$$




Exponential Progressions. 04_01_95

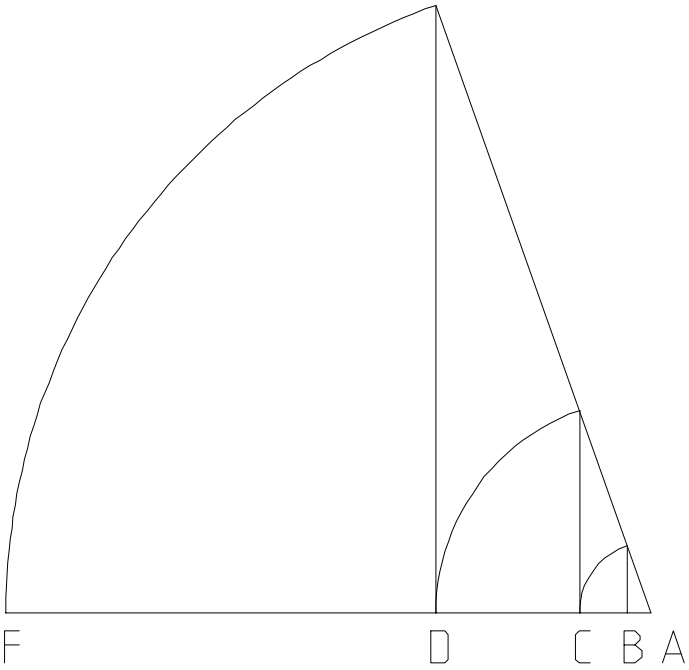
If I want to multiply any number by any power, this is the a geometric process for doing so.

The given figure is drawn for the third power of 3.

$AH := 10 \qquad \delta := 0..10 \qquad BS := 8$

The third division between A and F is very hard to see. BS = Base Segments

$AP := \frac{AH}{BS} \qquad AP = 1.25$




$AG_0 := \frac{(AP)^2}{AH} \qquad AF_0 := AG_0 \qquad \Delta := 2..10$

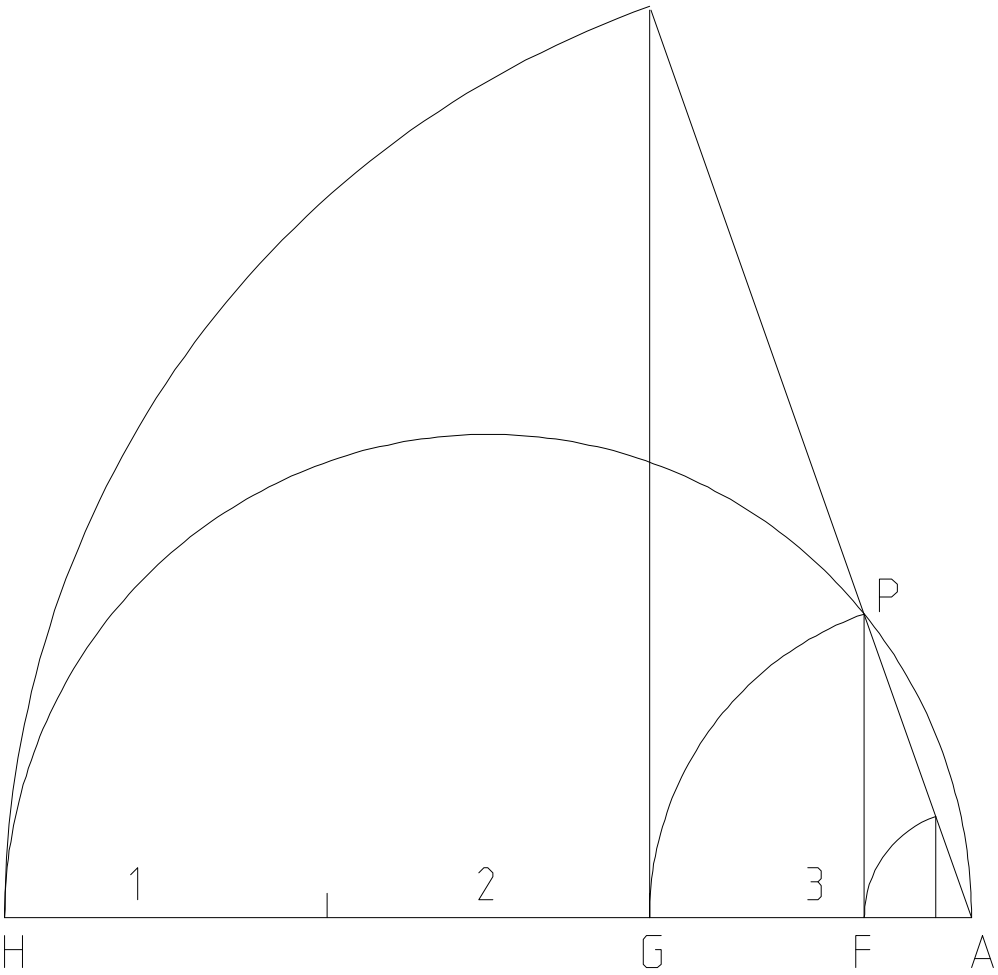
$\begin{pmatrix} AF_{\delta+1} \\ AG_{\delta+1} \end{pmatrix} := \begin{bmatrix} AG_\delta \\ \frac{AP \cdot AG_\delta}{AH} \end{bmatrix} \qquad AB_\delta := \frac{AG_1}{AF_\delta}$

| $\delta$ | $AB_\Delta$ | $BS^\delta$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 8 | 8 |
| 2 | 64 | 64 |
| 3 | 512 | 512 |
| 4 | 4096 | 4096 |
| 5 | 32768 | 32768 |
| 6 | 262144 | 262144 |
| 7 | 2097152 | 2097152 |
| 8 | 16777216 | 16777216 |
| 9 | | 134217728 |
| 10 | | $1.07374182 \cdot 10^9$ |

Making the number of divisions 3, provides 3 cube result. AB divides AF 27 times. Etc. It can be seen that using a normal straight edge and compass one needs a very large piece of paper to work this.

You will notice that I took only one of the possible two divisions from which to project from. The other would be 2/3. At 2/3 my unit divisions would still be 27, but now AB would take up 2 cube of them, or AB would be 8 units.

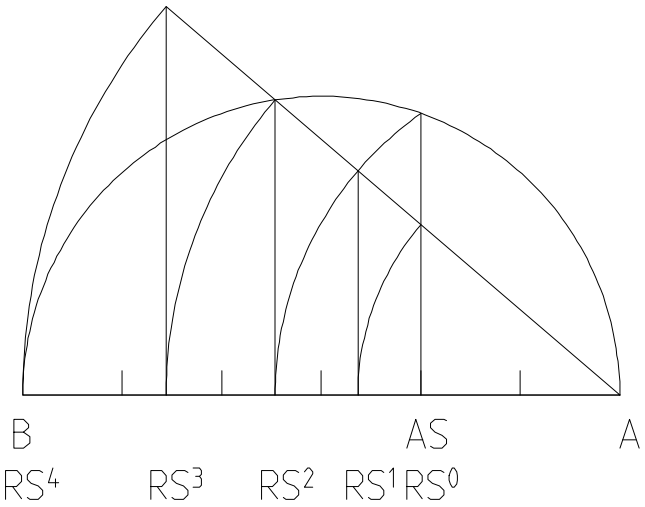
For an 8 cube series then, the value for AB would be 1 of 512, 8 of 512, 27 of 512, 64 of 512, 125 of 512, 216 of 512, 343 of 512




Since the slope is established, one can take the first two divisions and utilizes the whole figure.

$E := 1 .. BS$ $ROOT := 3$

| E | $\frac{BS^{ROOT}}{E^{ROOT}}$ | $E^{ROOT}$ |
|---|---|---|
| 1 | 512 | 1 |
| 2 | 64 | 8 |
| 3 | 18.96296296 | 27 |
| 4 | 8 | 64 |
| 5 | 4.096 | 125 |
| 6 | 2.37037037 | 216 |
| 7 | 1.49271137 | 343 |
| 8 | 1 | 512 |




# About The Laws of Exponents and Ratios

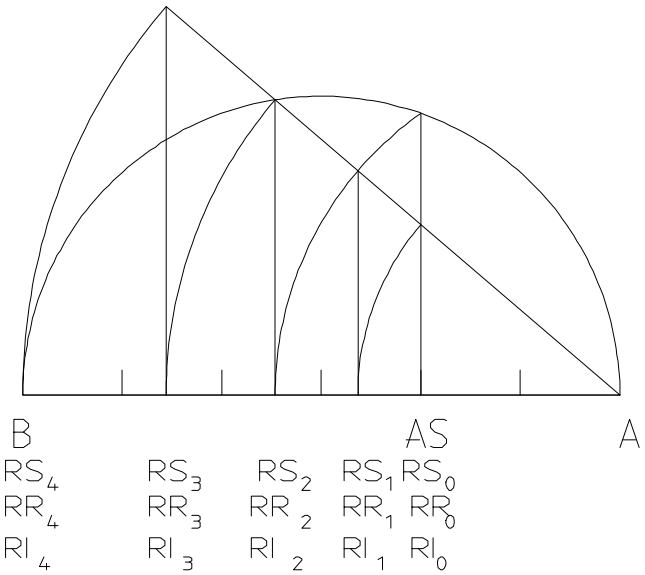
04_22_95.MCD

$\Delta := 22 \quad \delta := 1 .. \Delta \quad AB := 7$

Base Segments = BS $BS := 99$

Base Index = BI $BI := 13$

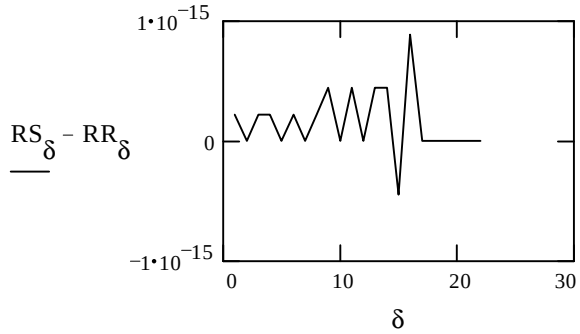
Root Series = RS $RS_\delta := \left[ \left( \frac{BI}{BS} \cdot AB \right)^{\Delta - \delta} \cdot AB^{\delta} \right]^{\frac{1}{\Delta}}$



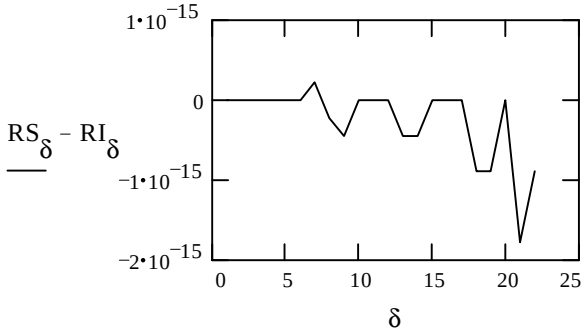

Root Series By Ratio = RR $RR_\delta := \left( \frac{BI}{BS} \right)^{\frac{\Delta - \delta}{\Delta}} \cdot AB$

Root Series By Inverse Ratio = RI

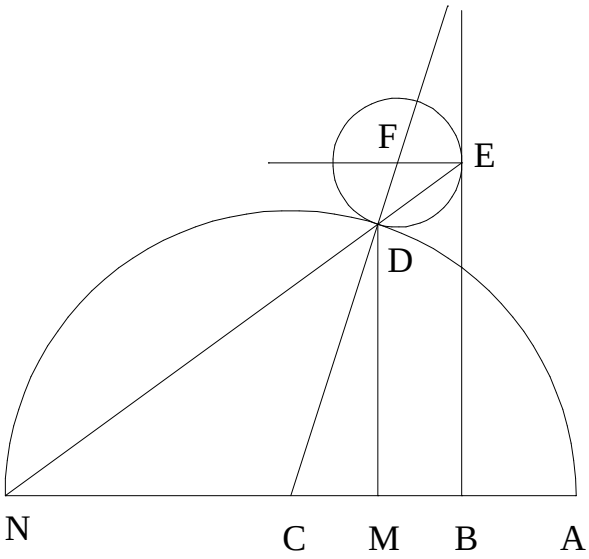
$RI_\delta := \left( \frac{BS}{BI} \right)^{\frac{\delta}{\Delta}} \cdot \left( \frac{BI}{BS} \cdot AB \right)$




$\sum_\delta \left( RS_\delta - RR_\delta \right) = 0$




$\sum_\delta \left( RS_\delta - RI_\delta \right) = 0$




09_13_95.MCD

A Study in Placement.

Given AC, AB, AM find EF.

Segment A.

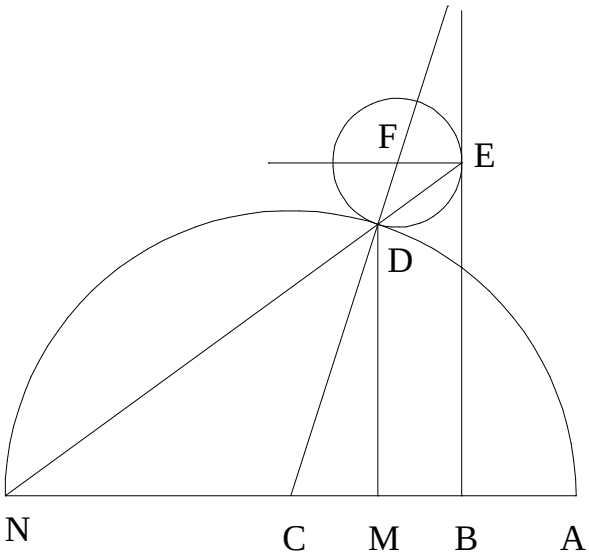
$AC := 3.125$ $AB := 4.25$ $AM := 5.34857$

$CD := AC$ $CN := AC$ $CM := AC - AM$ $DM := \sqrt{CD^2 - CM^2}$

$MN := CM + CN$ $BC := AC - AB$ $BN := CN + BC$ $DN := \sqrt{DM^2 + MN^2}$

$EN := \frac{DN \cdot BN}{MN}$ $ED := EN - DN$ $EF := \frac{AC \cdot ED}{DN}$ $EF = 3.80843$

The resultant equation in terms of the givens is $EF := \frac{AC \cdot (AM - AB)}{2 \cdot AC - AM}$ $EF = 3.80843$




Segment B.

Given AC, AB, DN, find EF.

$DN := 2.37359$

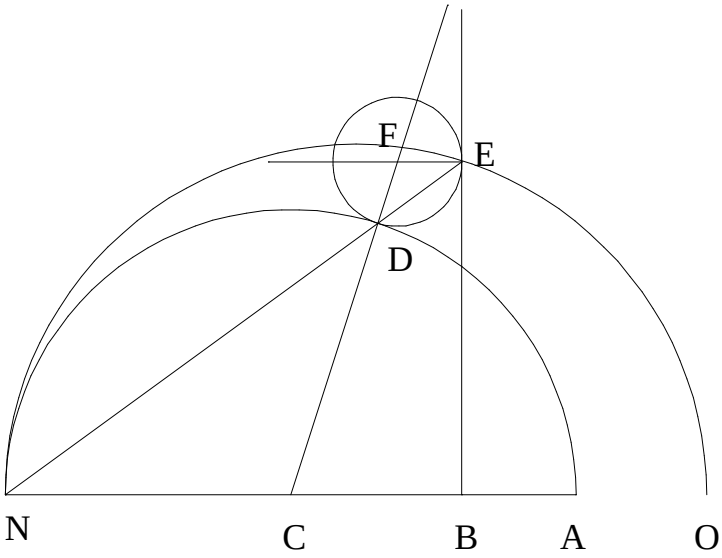
$AN := 2 \cdot AC$ $\quad MN := \frac{DN^2}{AN}$ $\quad CN := AC$ $\quad BC := AC - AB$ $\quad BN := CN + BC$

$EN := \frac{DN \cdot BN}{MN}$ $\quad ED := EN - DN$ $\quad EF := \frac{AC \cdot ED}{DN}$ $\quad EF = 3.80844$

The equation for this process is

$$EF := \frac{AC \cdot (4 \cdot AC^2 - 2 \cdot AC \cdot AB - DN^2)}{DN^2}$$

$EF = 3.80844$




Segment C.

Given AC, AB, BE, find EF.

$BE := 4.87173$

$AN := 2 \cdot AC$ $\quad BN := AN - AB$ $\quad EN := \sqrt{BE^2 + BN^2}$ $\quad ON := \frac{EN^2}{BN}$

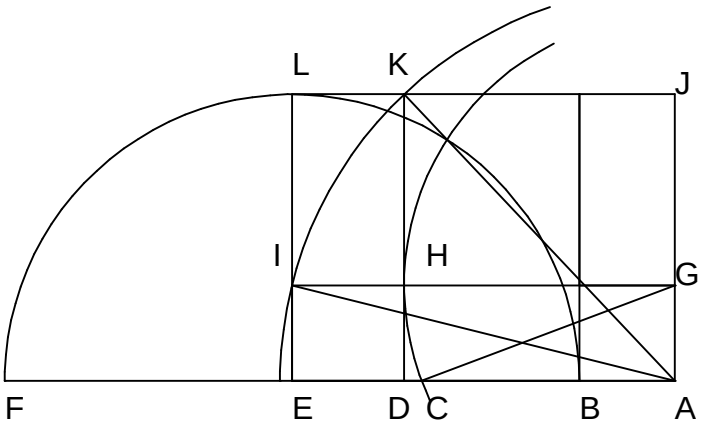
$DN := \frac{EN \cdot AN}{ON}$ $\quad DE := EN - DN$ $\quad EF := \frac{AC \cdot DE}{DN}$ $\quad EF = 3.80844$

The equation for this process is $EF := \frac{BE^2 - 2 \cdot AC \cdot AB + AB^2}{2 \cdot (2 \cdot AC - AB)}$ $\quad EF = 3.80844$

10_14_5C.MCD

Trivial Method: Square Root

Generalize the figure of 10_14_95.MCD




Starting at any point G, between A and J, the square root of AB·AF can always be projected to point C. Such a progression can be used on the cube root figure.

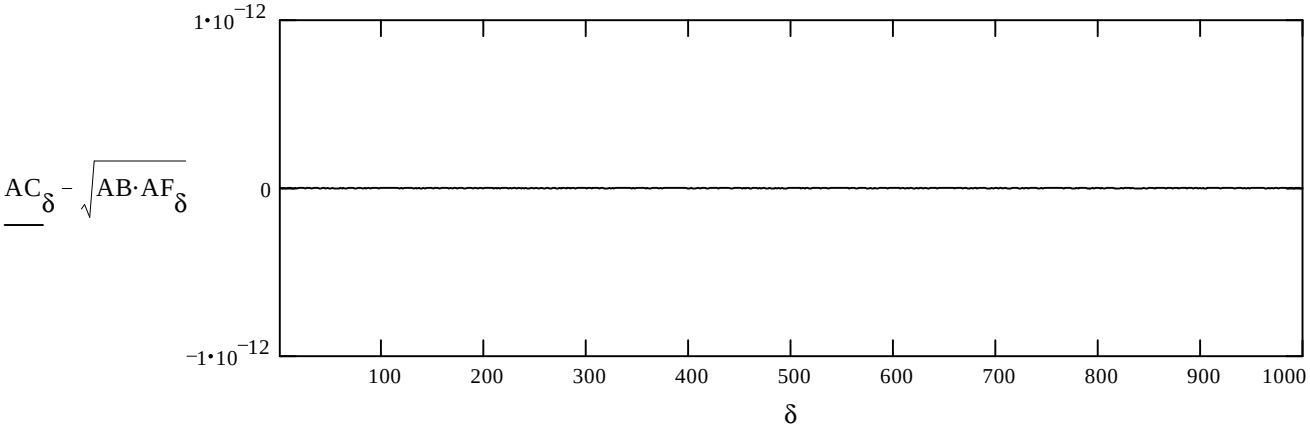
$\delta := 1 .. 1000 \quad AB := 10 \quad BF := 10 \quad BE := \frac{BF}{2}$

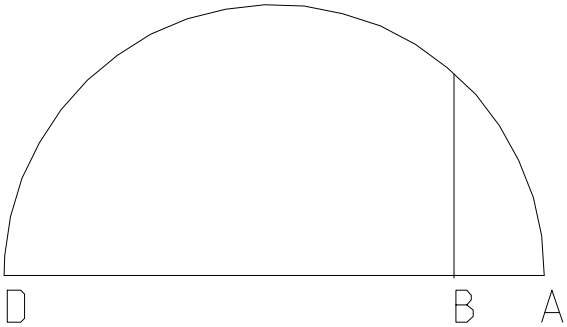
$AE := BE + AB \quad AJ := BE \quad AG_\delta := \frac{AJ}{\delta}$

$EI_\delta := AG_\delta \quad AI_\delta := \sqrt{(AE)^2 + (EI_\delta)^2}$

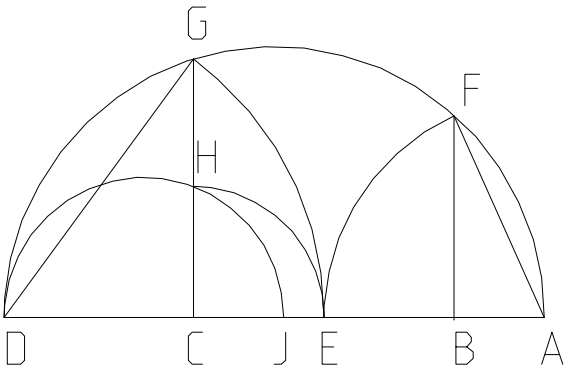
$AK_\delta := AI_\delta \quad DK := AJ \quad AD_\delta := \sqrt{(AK_\delta)^2 - DK^2} \quad GH_\delta := AD_\delta \quad CG_\delta := GH_\delta$

$AF_\delta := BF + AB \quad AC_\delta := \sqrt{(CG_\delta)^2 - (AG_\delta)^2}$






Given AB and BD divide BD such that $AB \cdot CD = \frac{BC^2}{4}$. And what is the reltionship of AC to AB and BD? Now the date on this file is not exact as I sketched this out on a piece of paper and forgot to date it.




$AB := 5 \quad BD := 1 \quad AD := AB + BD$

$BF := \sqrt{AB \cdot BD} \quad AE := \sqrt{AB^2 + BF^2}$

$DE := AD - AE \quad DG := DE \quad CD := \frac{DG^2}{AD}$

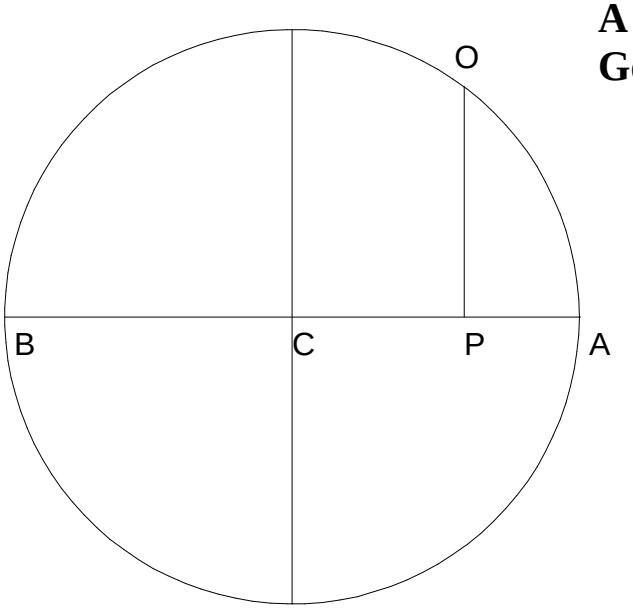
$CD = 0.046$

$BC := BD - CD \quad AB \cdot CD - \frac{BC^2}{4} = 0$

$CE := DE - CD \quad CH := CE \quad CJ := \frac{CH^2}{CD} \quad AB - CJ = 0$

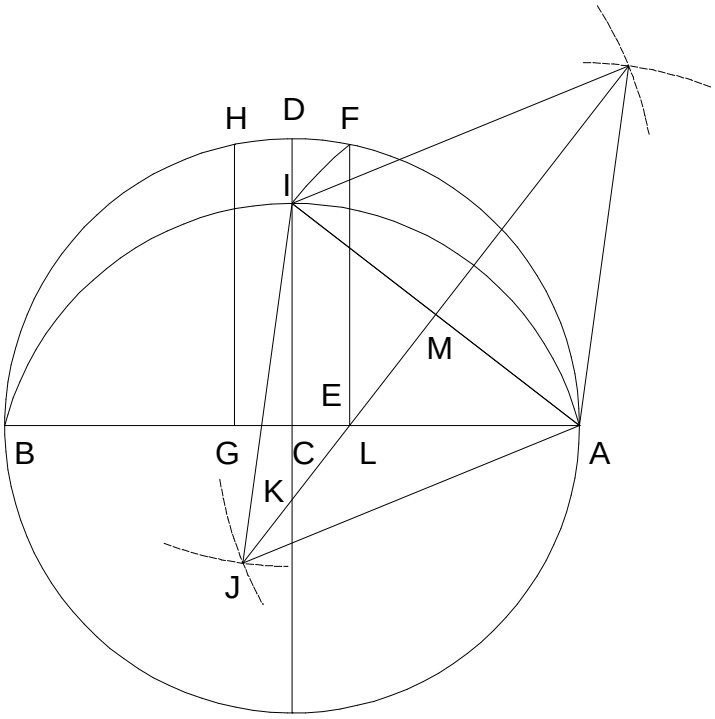
$AC := AB + BC \quad AC = 5.954$

$$AC - \left[ 2 \cdot \frac{AB^{\left(\frac{3}{2}\right)}}{\sqrt{AB + BD}} + 2 \cdot \frac{\sqrt{AB}}{\sqrt{AB + BD}} \cdot BD - AB \right] = 0$$




# A Modification of a Square Root Figure. Gemini Roots

One of the square root figures displays a one to one ratio between what could be called the vertical segment OP and the root of the two horizontal segments AP and BP. With a slight modification however, one can demonstrate a many to one relationship between three base segments.




GL has a ratio to the root of $AL \cdot BG$. Developing the arc AIC from it will give a means of keeping that ratio.

BS = Base Segments, set at end of doc.

$AB := 10 \quad AC := \frac{AB}{2} \quad EG := \frac{AB}{BS}$

$BC := AC \quad CD := AC \quad CE := \frac{EG}{2}$

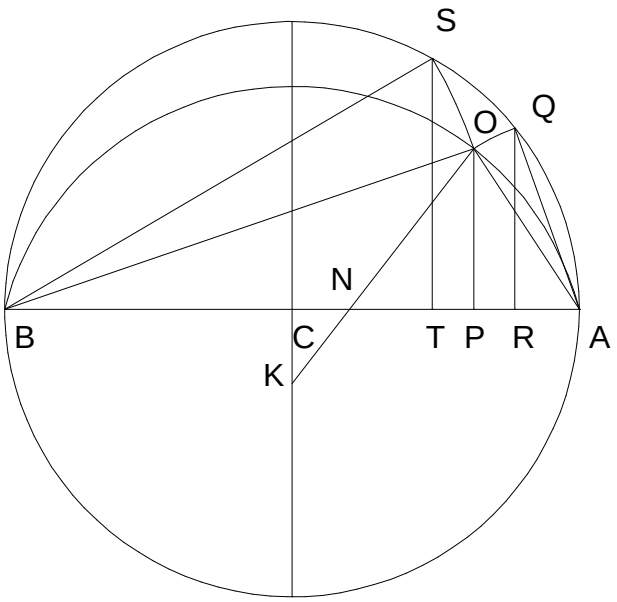
$AE := AC - CE \quad BE := BC + CE$

$EF := \sqrt{AE \cdot BE} \quad AF := \sqrt{AE^2 + EF^2}$

$AI := AF \quad AM := \frac{AI}{2} \quad CI := \sqrt{AI^2 - AC^2}$

$AL := \frac{AI \cdot AM}{AC} \quad CL := AC - AL$

$CK := \frac{AC \cdot CE}{CI} \quad IK := CI + CK$




$$\delta := 1 .. \Delta \qquad AN_\delta := \frac{AC}{\Delta}\cdot\delta$$

$$CN_\delta := AC - AN_\delta \qquad KO := IK$$

$$KN_\delta := \sqrt{CK^2 + \left(CN_\delta\right)^2} \qquad NO_\delta := KO - KN_\delta$$

$$NP_\delta := \frac{CN_\delta \cdot NO_\delta}{KN_\delta} \qquad AP_\delta := AC - CN_\delta - NP_\delta$$

$$OP_\delta := \frac{CK \cdot NO_\delta}{KN_\delta} \qquad AO_\delta := \sqrt{\left(AP_\delta\right)^2 + \left(OP_\delta\right)^2}$$

$$AQ_\delta := AO_\delta \qquad AR_\delta := \frac{\left(AQ_\delta\right)^2}{AB}$$

$$BP_\delta := BC + CN_\delta + NP_\delta$$

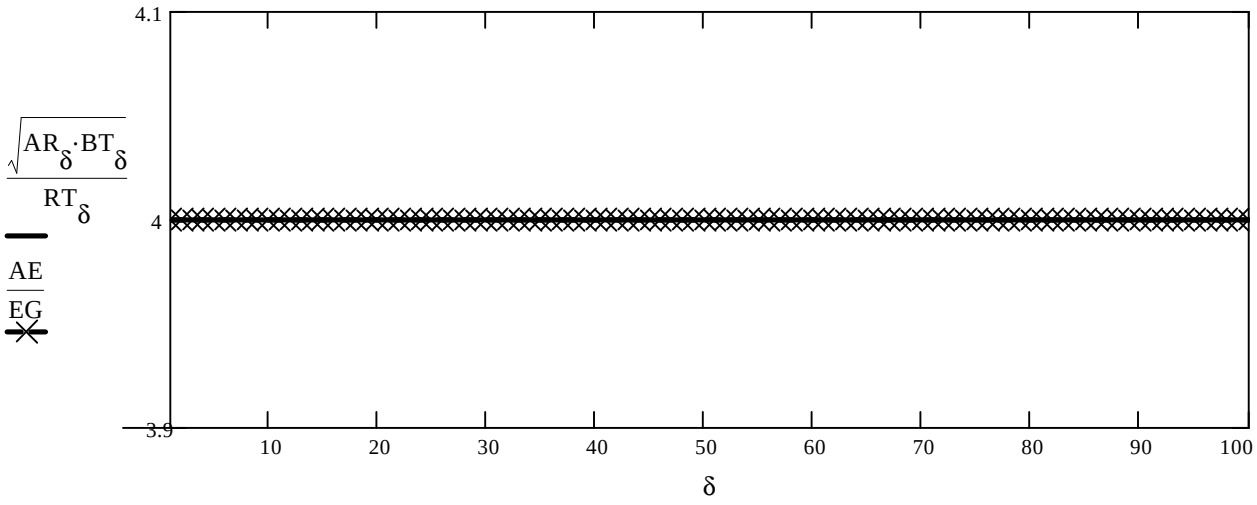
$$BO_\delta := \sqrt{\left(BP_\delta\right)^2 + \left(OP_\delta\right)^2} \qquad BS_\delta := BO_\delta \qquad BT_\delta := \frac{\left(BS_\delta\right)^2}{AB} \qquad AT_\delta := AB - BT_\delta \qquad RT_\delta := AT_\delta - AR_\delta$$

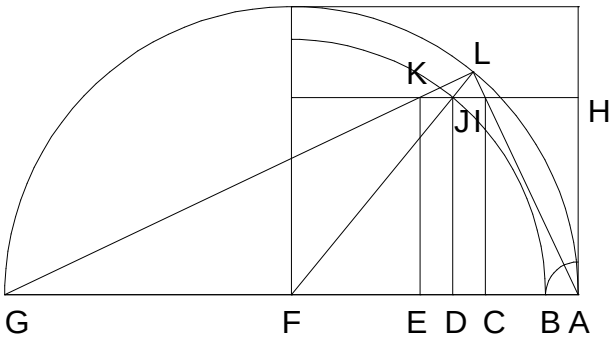
Set the number of Base Segments here and see if a constant relationship is expressed in the graph.

$BS \equiv 9$

Set the number of drawings in $\Delta$.

$\Delta \equiv 100$





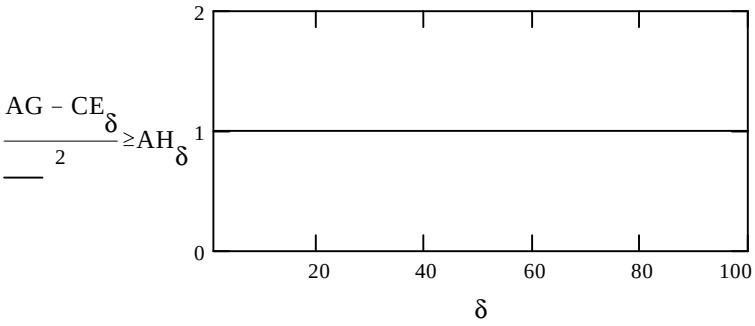

# Short Method Gemini Roots.

Given AG, CE, AH, place CE so that CE:AH as CE:CI. Or more simply that CI $= \sqrt{AC_\delta \cdot EG_\delta} = AH$.

$\delta := 1 .. 100$ $\Delta := 8$

$AG := \Delta \cdot 2 + 1$ $CE_\delta := \frac{1}{\delta}$ $AH_\delta := \Delta \cdot CE_\delta$

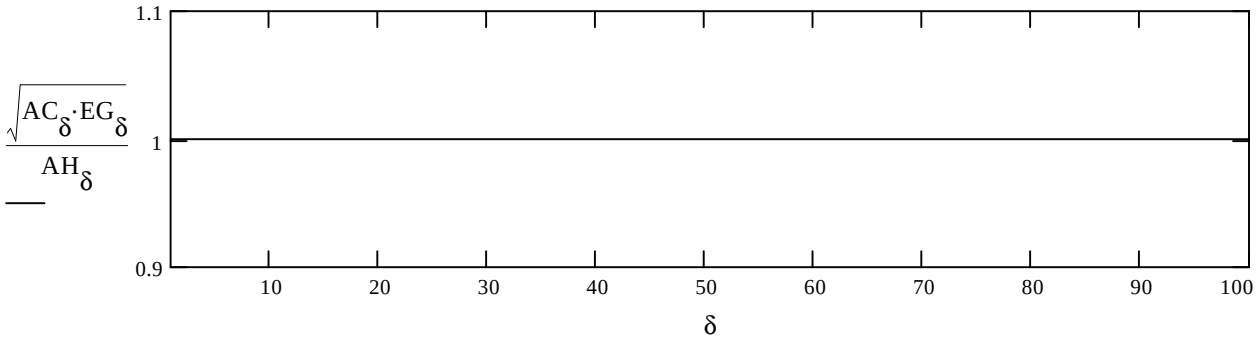
With the values given is the constuction possible? (1 for yes and 0 for no.)

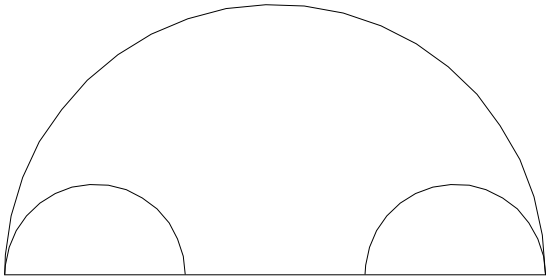



$DJ_\delta := AH_\delta$ $AB_\delta := \frac{CE_\delta}{2}$ $AF := \frac{AG}{2}$ $FG := AF$ $BF_\delta := AF - AB_\delta$ $FJ_\delta := BF_\delta$

$FD_\delta := \sqrt{(FJ_\delta)^2 - (DJ_\delta)^2}$ $DC_\delta := AB_\delta$ $DE_\delta := AB_\delta$ $AD_\delta := AF - FD_\delta$

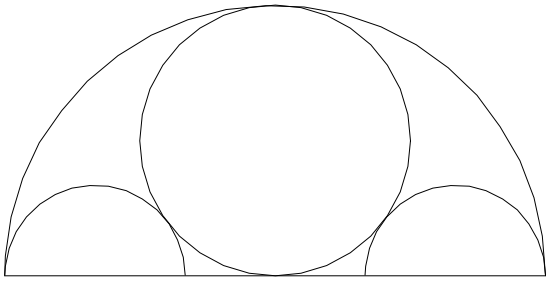
$AC_\delta := AD_\delta - DC_\delta$ $AE_\delta := AD_\delta + DE_\delta$ $EF_\delta := AF - AE_\delta$ $EG_\delta := EF_\delta + FG$

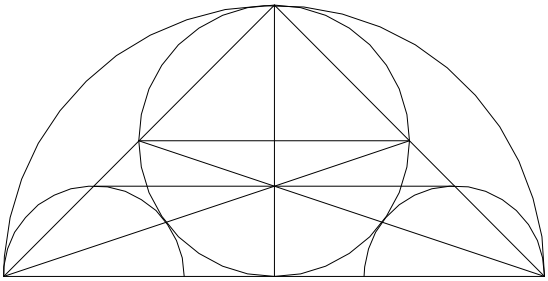


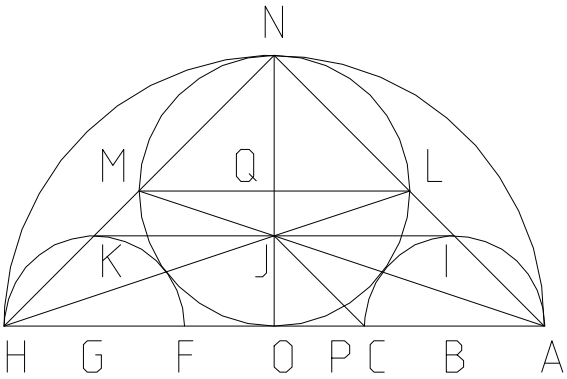


# Method for Equals.

At the inner extremities of a great circle I have two equal smaller ones. Find the circle tangent to all three








$AH := 10 \quad AC := 3 \quad AO := \frac{AH}{2} \quad AB := \frac{AC}{2}$

$JO := AB \quad OP := JO \quad HO := AO$

$JP := \sqrt{JO^2 + OP^2} \quad HP := HO + OP$

$AL := \frac{JP \cdot AH}{HP} \quad NO := AO \quad AN := \sqrt{AO^2 + NO^2}$

$LN := AN - AL \quad LQ := \frac{AO \cdot LN}{AN} \quad LQ = 2.692$

Reducing LQ as an expression of the two givens, $LQ_F := \frac{AH \cdot (AH - AC)}{2 \cdot (AH + AC)} \quad LQ_F - LQ = 0$

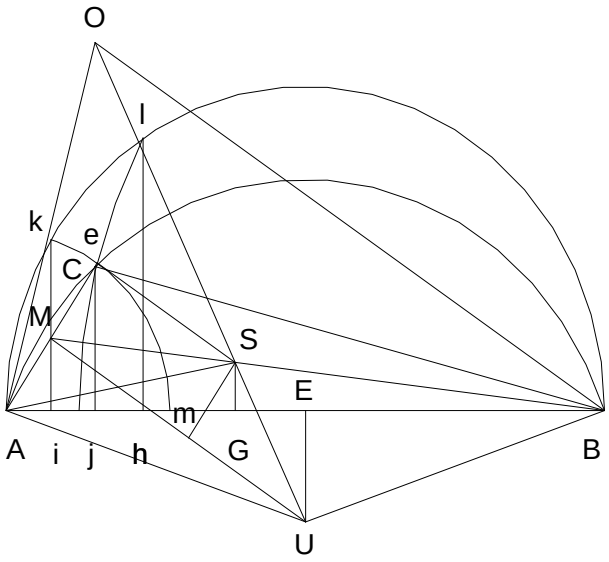
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

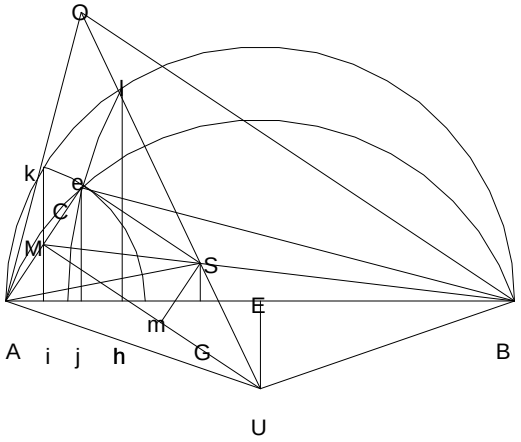
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

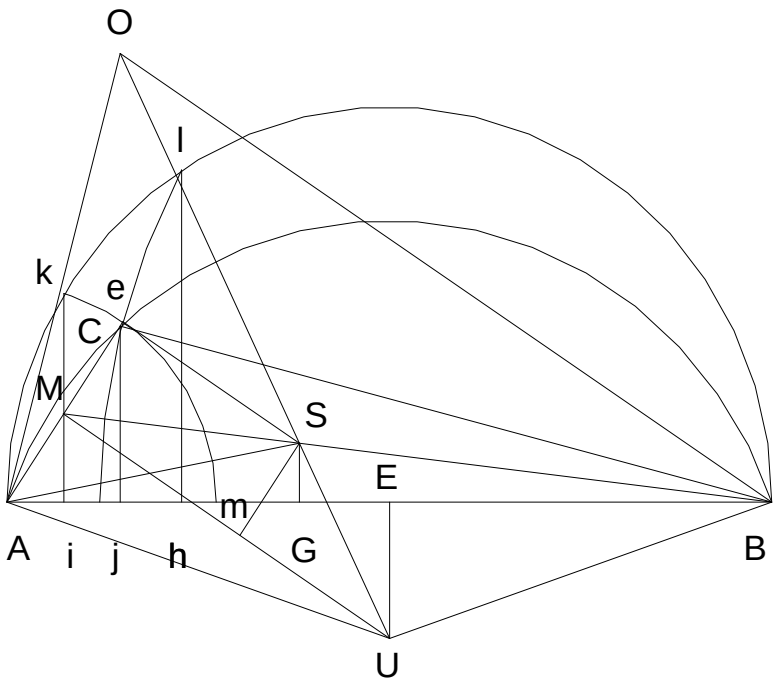
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75



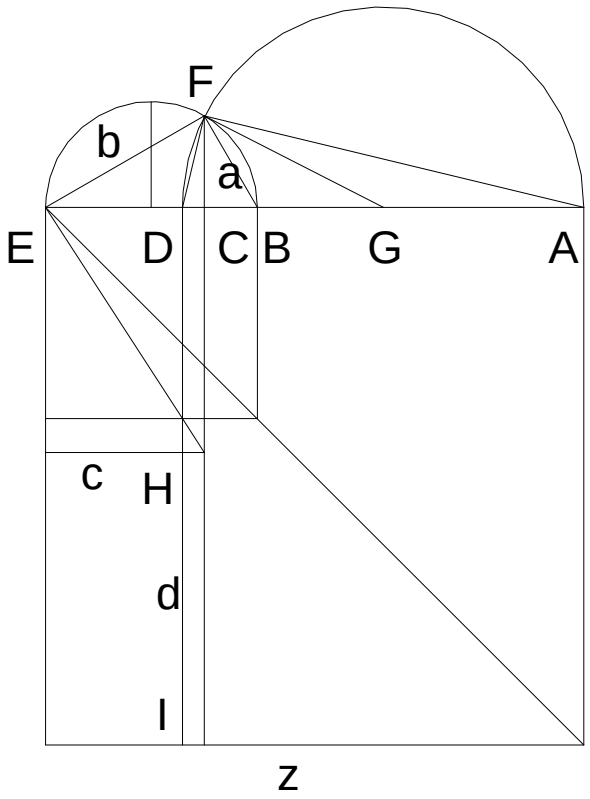

$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of th unknowns as a function of the three givens.

12_16_95.MCD




Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a \qquad BE := \sqrt{a^2 + b^2} \qquad BC := \frac{a^2}{BE}$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2} \qquad CG := \sqrt{FG^2 - CF^2}$

$AG := FG \qquad AC := AG + CG$

$BG := CG - BC \qquad DG := FG$

$BD := DG - BG \qquad AB := AG + BG$

$AE := AB + BE \qquad DE := BE - BD$

$DH := \frac{b^2}{DE} \qquad DI := AE \qquad HI := DI - DH$

$z := AE \qquad z = 12.622$

$c := DE \qquad c = 0.622$

$d := HI \qquad d = 6.186$

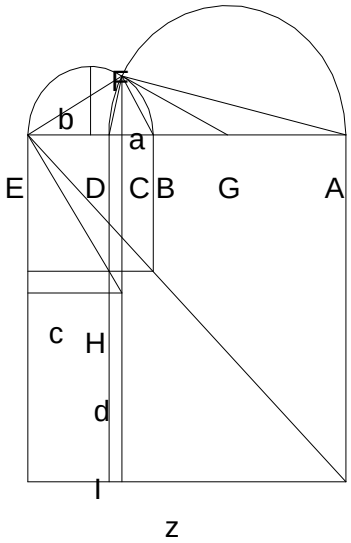
$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

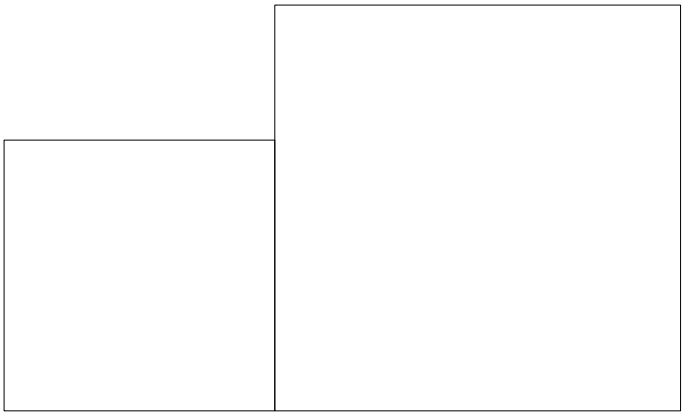
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$

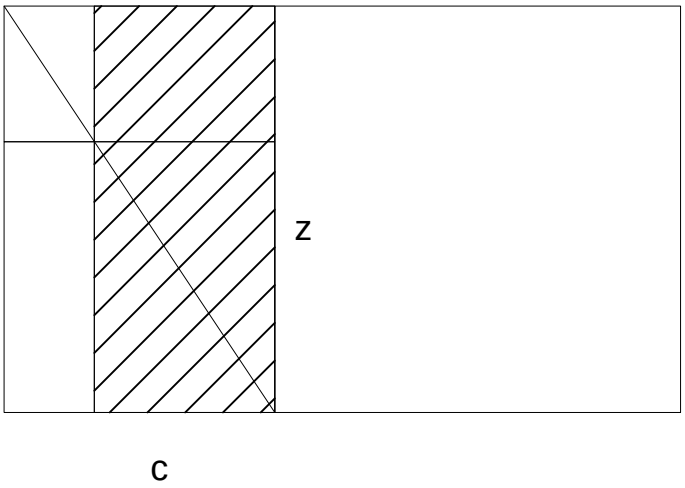



Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

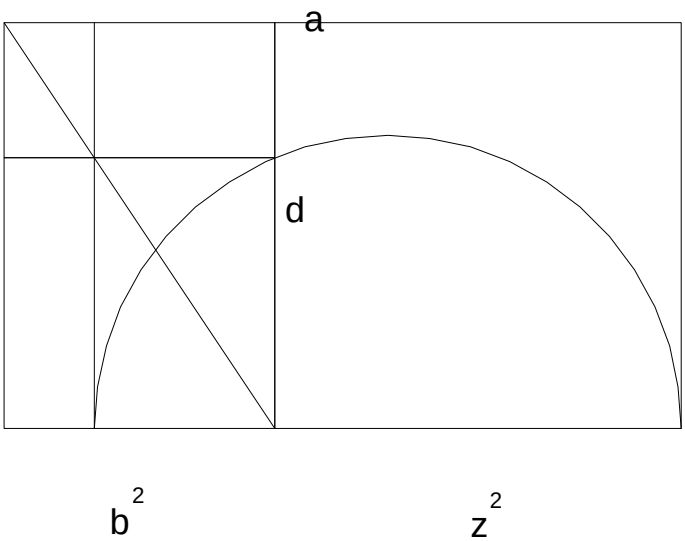
$b := 2.12 \qquad z := 1.41$




Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.




$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

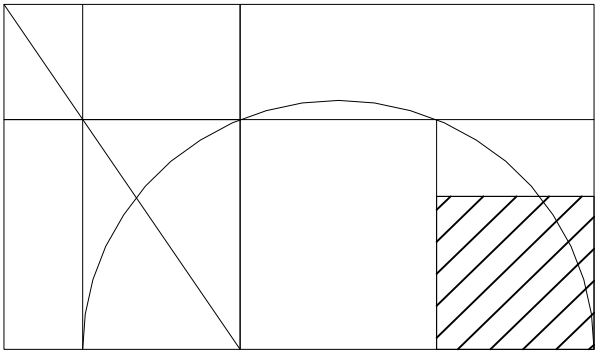
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

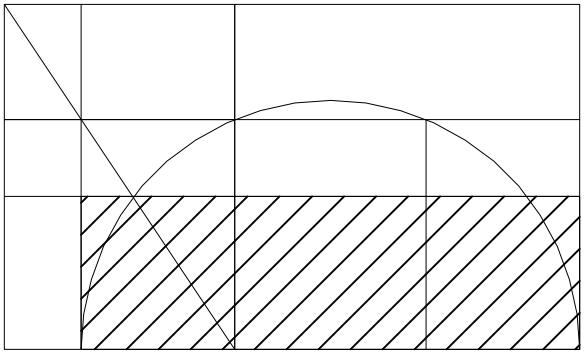
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

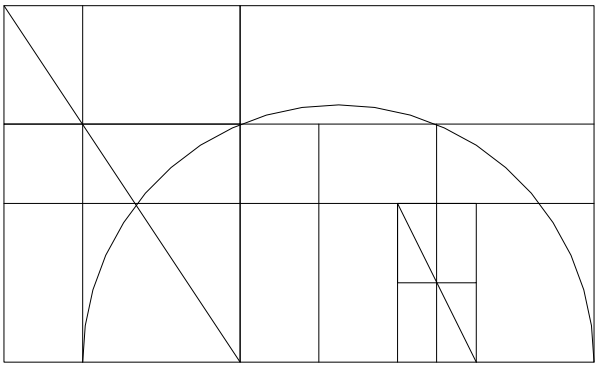
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



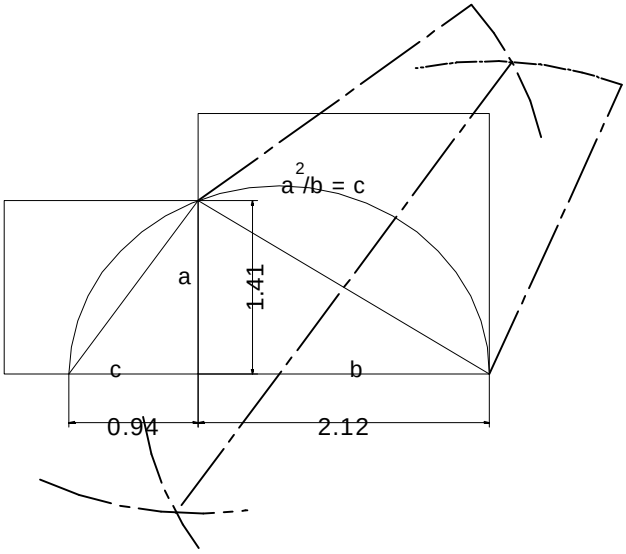
$b^2$ $z^2$



$b^2$ $z^2$

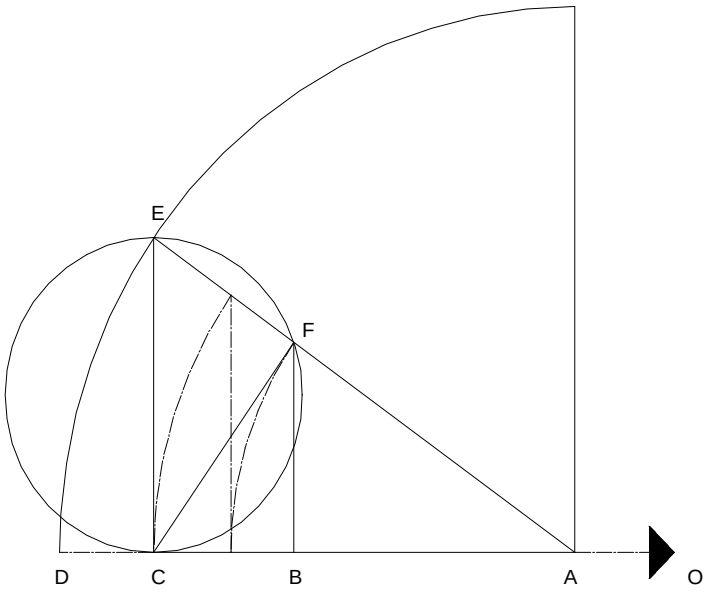


$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

Just for fun.




$N := 9$

$AD := 1 \qquad AC := \frac{AD}{N} \qquad CO := AD + AC$

$CD := AD - AC \qquad CE := \sqrt{CD \cdot CO}$

$AF := \frac{AC^2}{AD} \qquad AB := \frac{AF \cdot AC}{AD}$

$AC^3 - \left(AD^2 \cdot AB\right) = 0$

$AC - \left(AD^3 \cdot AB\right)^{\frac{1}{3}} = 0$

The ratio of BC to CE is common to another cube root expression, which?

Pascal's triangle with exponential division.

$AB := 3 \qquad AD := 5$

$BD := AD - AB \qquad AC := \sqrt{AB \cdot AD}$

$CD := AD - AC \qquad BC := BD - CD$

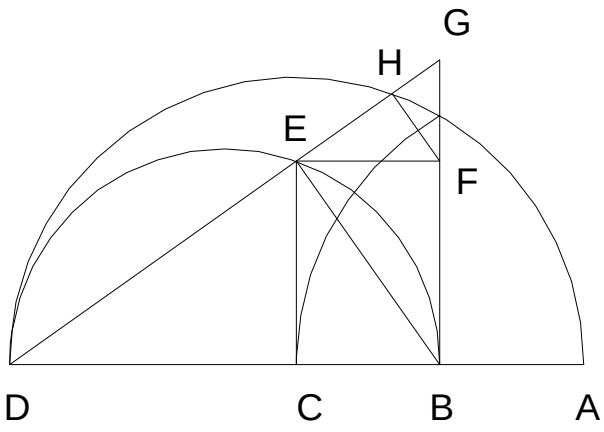
$CD := BD - BC \qquad CE := \sqrt{BC \cdot CD}$

$BE := \sqrt{BC^2 + CE^2} \qquad DE := \sqrt{CD^2 + CE^2}$

$BG := \frac{BD \cdot BE}{DE} \qquad FG := \frac{BC \cdot BG}{BD}$

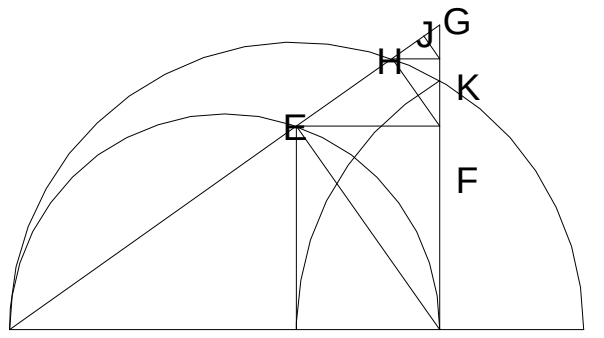
$EG := \frac{BE \cdot BG}{BD} \qquad DG := DE + EG$

$GH := \frac{BC \cdot EG}{BD} \qquad GJ := \frac{BC \cdot GH}{BD}$




$\frac{BD}{BC} = 2.291 \qquad \frac{DG}{GH} = 5.249 \qquad \frac{DG}{GJ} = 12.025$

$N := 1 \qquad \frac{\left(\sqrt{AD} + \sqrt{AB}\right)^N}{\sqrt{AB^N}} = 2.291$




$\frac{A + B}{A}$

$\sqrt{AD} + \sqrt{AB}$

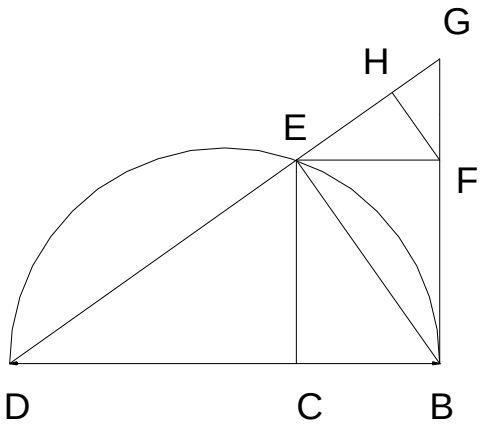
$\frac{A^2 + 2AB + B^2}{A^2}$

$\left(\sqrt{AD}\right)^2 + 2 \cdot \sqrt{AB} \cdot \sqrt{AD} + \left(\sqrt{AB}\right)^2$

$\frac{A^3 + 3A^2 \cdot B + 3A \cdot B^2 + B^3}{A^3}$

$\left(\sqrt{AD}\right)^3 + 3 \cdot AD \cdot \sqrt{AB} + 3 \cdot \sqrt{AD} \cdot AB + \left(\sqrt{AB}\right)^3$

Dividing an exponentiated integer by an exponentiated integer of the same power, straight edge and compass construction.




$CD := 1.5 \quad BC := .75 \quad BD := BC + CD$

$CE := \sqrt{BC \cdot CD} \quad BE := \sqrt{BC^2 + CE^2}$

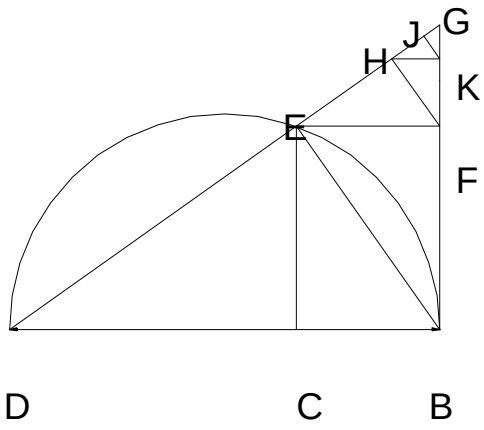
$EF := BC \quad FG := \frac{BC \cdot EF}{CE} \quad FG = 0.53$

$DE := \sqrt{CD^2 + CE^2} \quad EG := \frac{BD \cdot EF}{DE}$

$GH := \frac{BC \cdot EF}{DE} \quad DG := DE + EG$

$JK := \frac{FG \cdot GH}{EG} \quad GJ := \frac{BC \cdot GH}{BD} \quad FH := \frac{CE \cdot EG}{BD} \quad \frac{BD}{BC} = 3 \quad \frac{DG}{GH} = 9 \quad \frac{DG}{GJ} = 27$

A formula for the figure is then,




$\sqrt{BD \cdot BC} = 1.299$

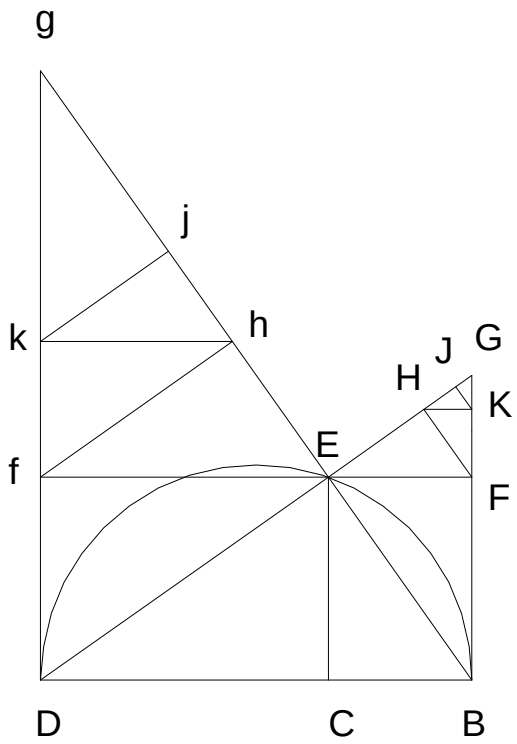
$\sqrt{DG \cdot GH} = 0.919$

$\sqrt{DG \cdot GJ} = 0.53 \quad \frac{\sqrt{BC + CD}}{\sqrt{CD}} \cdot \frac{\sqrt{BC}}{\sqrt{BD}} \cdot \sqrt{BD \cdot BC} = 0.919$

$\frac{\sqrt{BC + CD}}{\sqrt{CD}} \cdot BC = 0.919$

$\frac{\left(\sqrt{BC}\right)^3}{\sqrt{CD}} = 0.53 \quad \frac{BC}{\left(\sqrt{CD} \cdot \sqrt{BD}\right)} \cdot \sqrt{BD \cdot BC} = 0.53$

$\frac{a^n}{b^n} \quad n := 3 \quad \frac{BD^n}{BC^n} = 27$




$Ef := CD \qquad Eg := \frac{BD \cdot Ef}{BE} \qquad fh := \frac{CE \cdot Ef}{BE}$

$hk := \frac{CE \cdot fh}{BE} \qquad hj := \frac{BC \cdot hk}{BE} \qquad Eg := \frac{BD \cdot Ef}{BE}$

$Bg := BE + Eg \qquad hg := \frac{CD \cdot Ef}{BE} \qquad jg := \frac{CD \cdot hk}{BE}$

$\frac{BD}{CD} = 1.5 \qquad \frac{Bg}{hg} = 2.25 \qquad \frac{Bg}{jg} = 3.375$

$n := 4 \qquad \left(\frac{BD}{CD}\right)^n = 5.063$

$\frac{Bg}{EG} = 4.243 \qquad \frac{Bg}{GH} = 12.728 \qquad \frac{Bg}{GJ} = 38.184 \qquad \sqrt{CD} \cdot \frac{(BC + CD)^n}{\left(\sqrt{BC}\right)^{2 \cdot n + 1}} = 114.551$

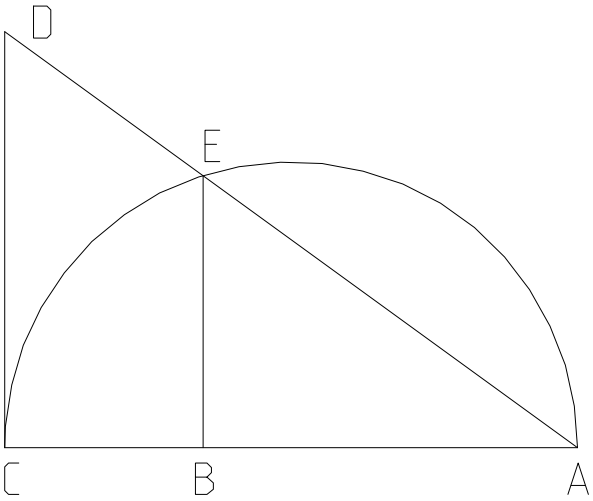
$\sqrt{Bg \cdot GJ} = 0.631$

$\sqrt{Bg \cdot GH} = 1.092$

$\sqrt{Bg \cdot EG} = 1.892$

$\sqrt{Bg \cdot DG} = 3.277$

$\frac{BC^{\left(\frac{7}{4}\right)}}{CD^{\left(\frac{1}{4}\right)} \cdot \sqrt{BC + CD}} \cdot \frac{\left(\sqrt{BC + CD}\right)^n}{\left(\sqrt{BC}\right)^n} = 3.277$




Just curious on how BC works out between AC and CD.

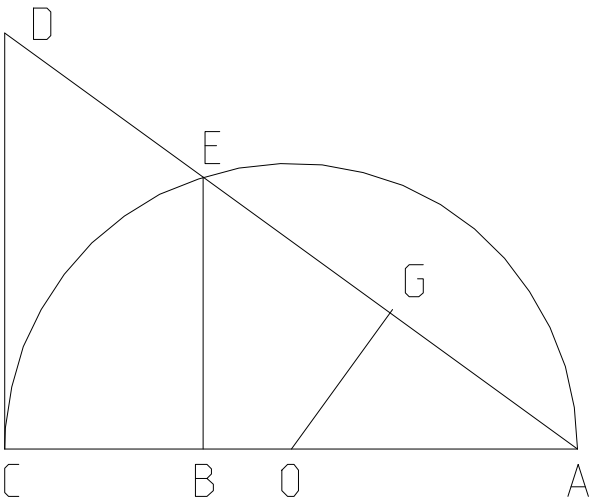
$AC := 15 \quad CD := 5 \quad AO := \frac{AC}{2}$

$AD := \sqrt{AC^2 + CD^2}$

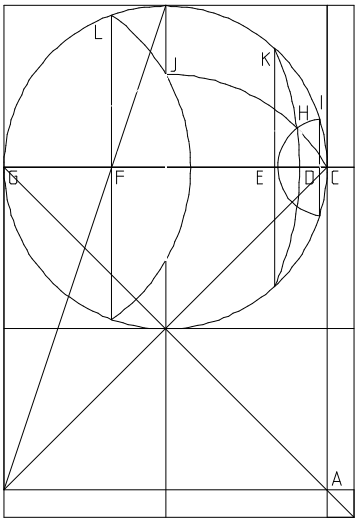
$AG := \frac{AC \cdot AO}{AD} \quad AE := 2 \cdot AG$

$AB := \frac{AC \cdot AE}{AD} \quad BC := AC - AB \quad BC = 1.5$

$BC := \frac{AC \cdot CD^2}{\left(AC^2 + CD^2\right)} \quad BC = 1.5$




A cube divided by the sum of two squares.




# One Circle

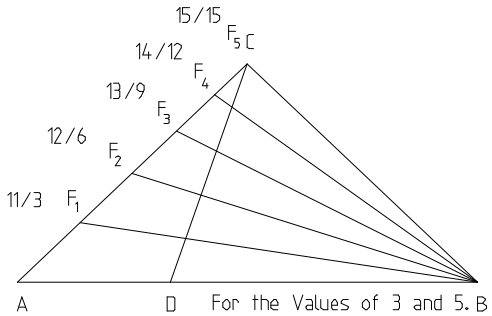
**By John Clark**

# One Square

# One Line

## And the Delian Quest

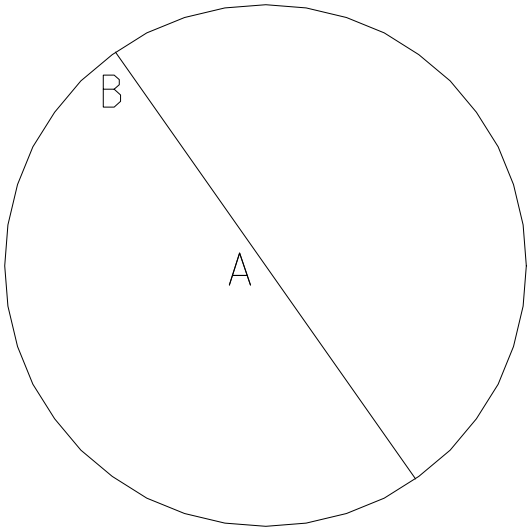
# 1996



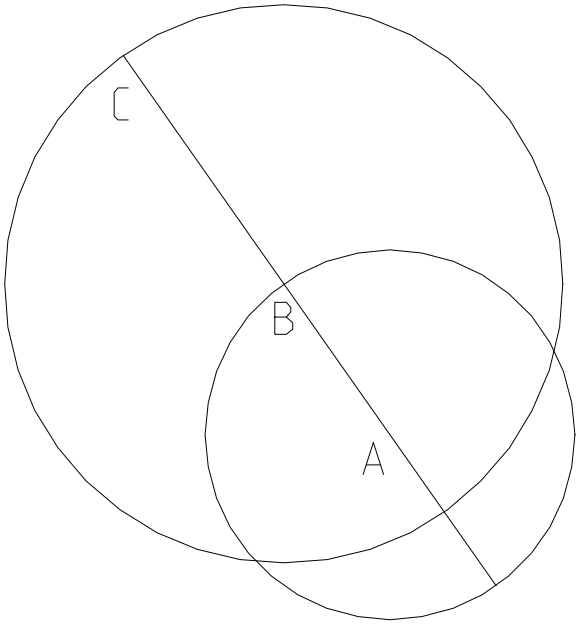
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

## The Archamedian Paper Trisector- Without the Numbers.

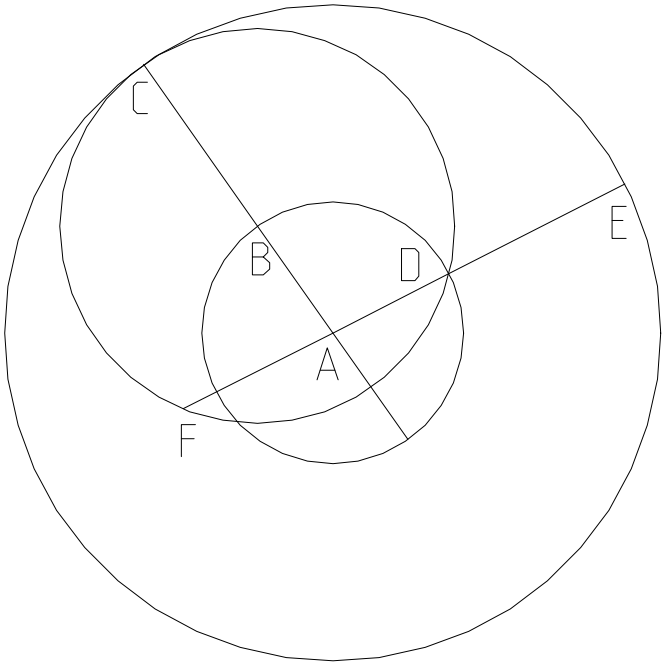
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of figure is not assumed, but done.




Given any circle AB

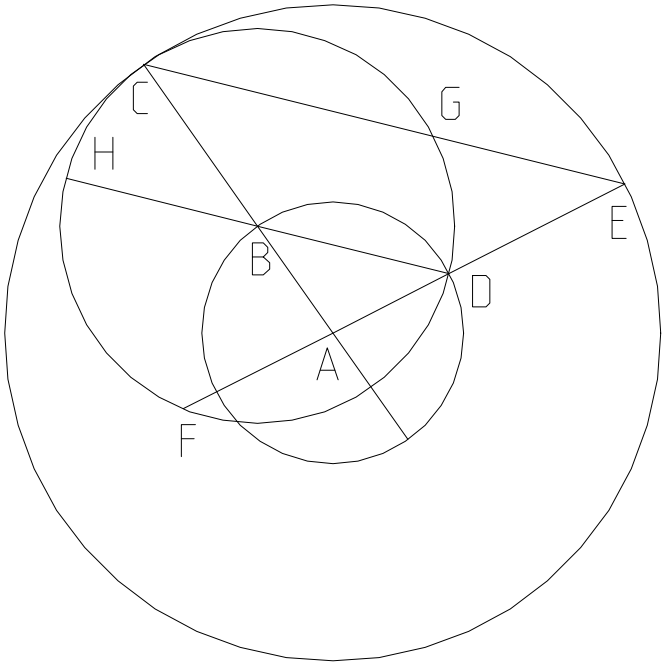



Given any circle BC such that BC≤ 2AB.

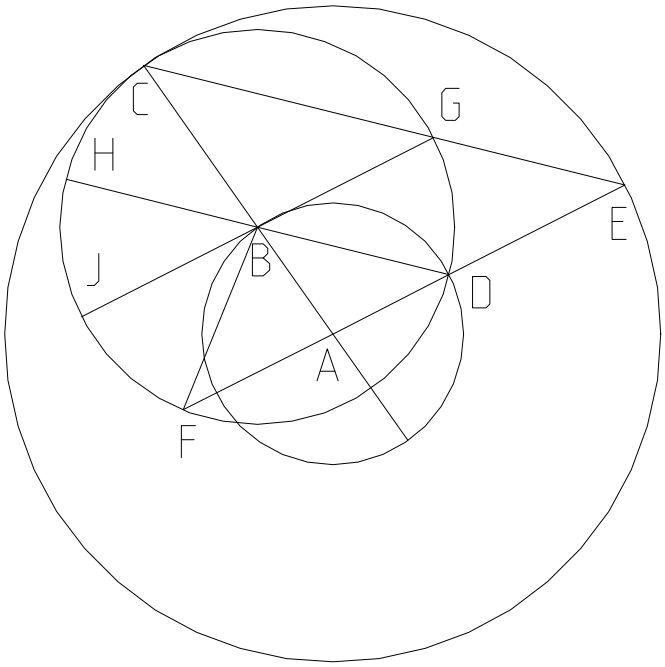



Construct AE such that AE = AC.

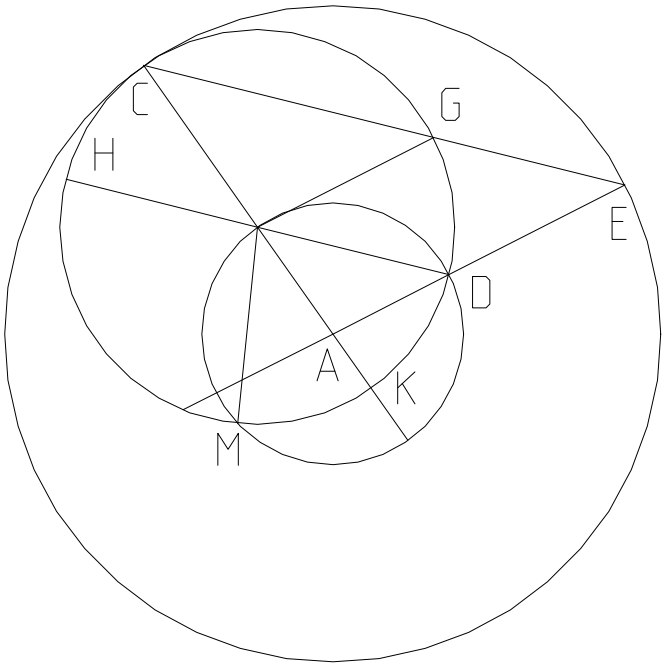
Since AE = AC + BE and AD = AB, DE = BC.




Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths. First the more dubious one.



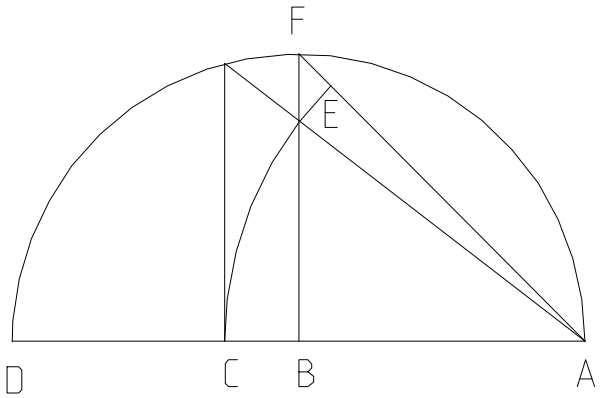

Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF. The construction of GJ makes this more dubious than the next, which has no faults.




By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG. It would not take much then to arrive at the last construction with the elimination of any doubt. But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

A rusty Compass construction.

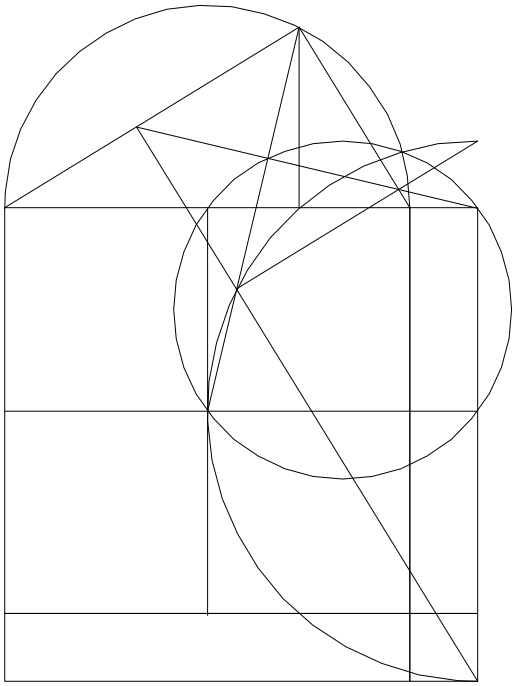



$AD := 2 \quad AB := \frac{AD}{2} \quad AF := \sqrt{2 \cdot AB^2} \quad AE := \frac{AF}{9} \cdot 8$

$AC := AE \quad AC = 1.257$

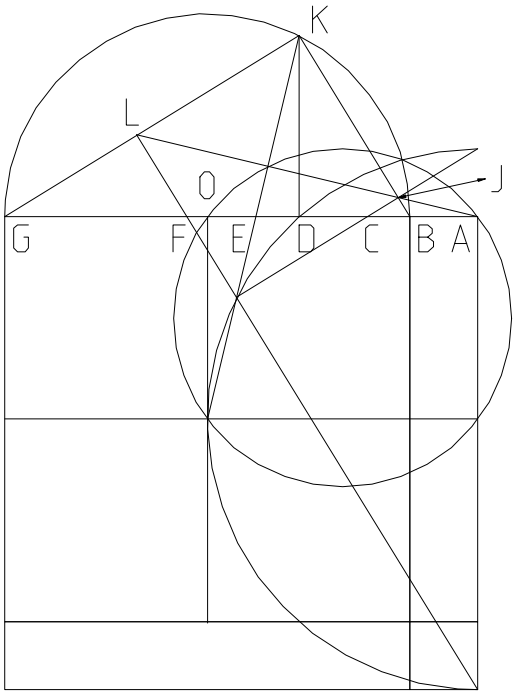
$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26 \qquad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.



01_08_96.MCD

The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.




$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

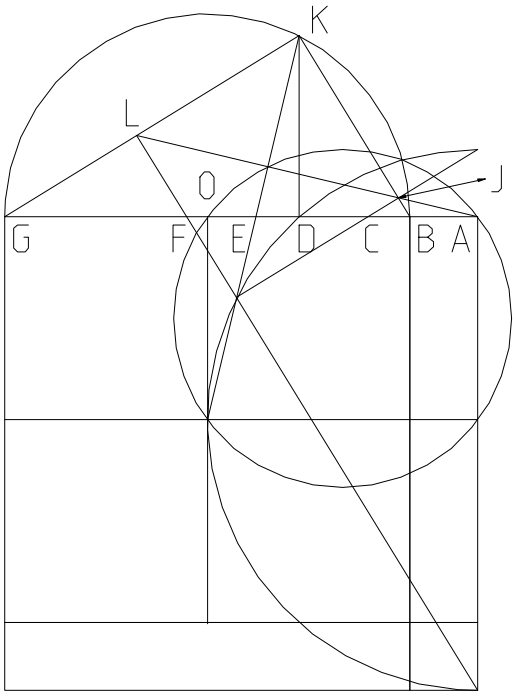
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



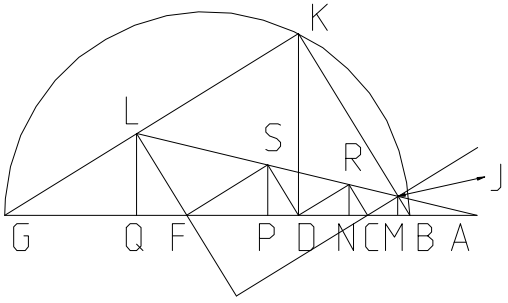

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$




$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$

$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

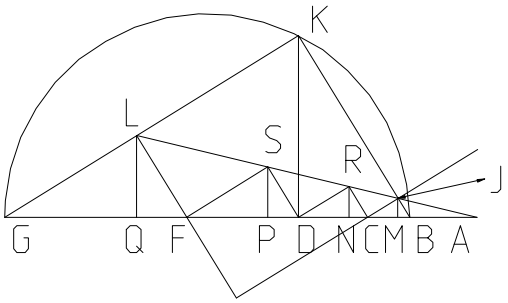
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$

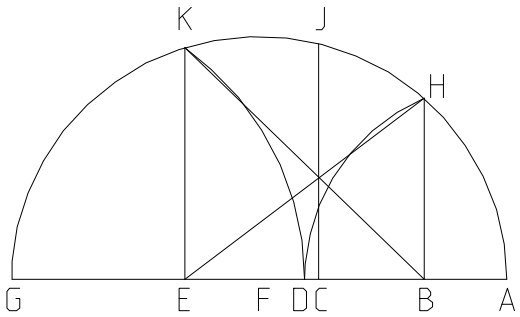



$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

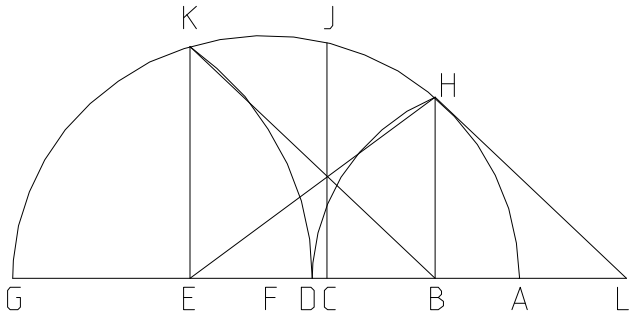
$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$




If the figure was drawn differently, XC would be $\sqrt{XB \cdot XE}$, irregardless of how XB and XE were placed, however that would require part of the figure that is not given here.




$$AG := N \quad AB := \frac{AG}{N}$$

$$BG := AG - AB \quad BH := \sqrt{AB \cdot BG}$$

$$AH := \sqrt{AB^2 + BH^2} \quad AD := AH \quad DG := AG - AD$$

$$GK := DG \quad GE := \frac{GK^2}{AG} \quad AE := AG - GE$$

$$BE := AE - AB$$

$$EG := AG - AE \quad EK := \sqrt{AE \cdot EG} \quad BL := \frac{BE \cdot BH}{EK}$$

$$EL := BE + BL \quad BC := \frac{BL \cdot BE}{EL} \quad AC := AB + BC$$

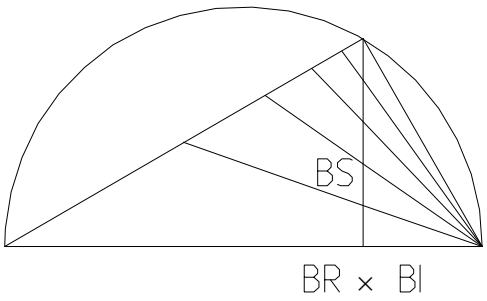
Make N any number and watch the equations, then make it equal to 1 and see what happens. Now this is strange work, for the formula is an identity with AC, so what happens at 1? This is an example of Binary contradiction.

$N \equiv 4$

$AC = 2$

$$\frac{\left[\begin{array}{l} AB^{\left(\frac{5}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{7}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + 2 \cdot AB \cdot \sqrt{BG} \cdot (AB + BG)^{\left(\frac{3}{2}\right)} \ldots \\ + - AB^{\left(\frac{5}{2}\right)} \cdot \sqrt{BG} - AB^{\left(\frac{3}{2}\right)} \cdot BG^{\left(\frac{3}{2}\right)} \end{array}\right]}{\left[\begin{array}{l} AB^{\left(\frac{1}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{3}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + AB^{\left(\frac{3}{2}\right)} \cdot \sqrt{BG} + \sqrt{AB} \cdot BG^{\left(\frac{3}{2}\right)} \end{array}\right]} = 2$$

# Pyramid of Ratios, Moving the Point



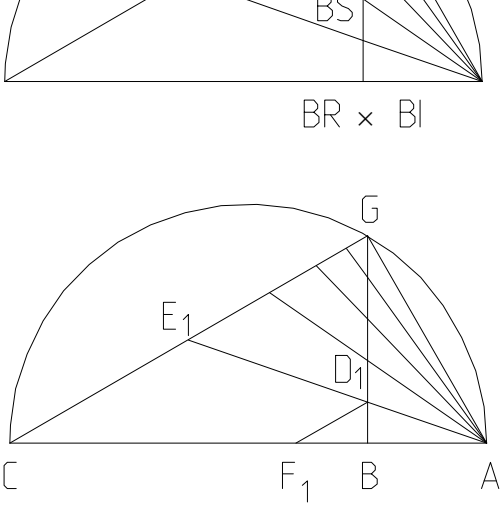

BR = Base Ratio, BS = Bisector Segments, BI = Base Index.

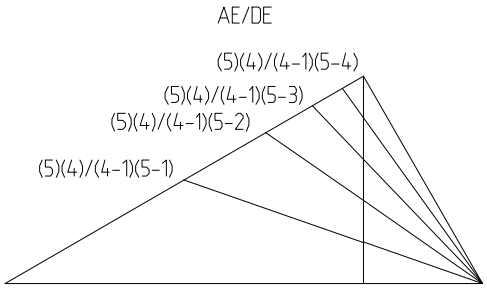
$BR \equiv 4 \quad BS \equiv 5 \quad BI := 2 \quad AC := BR \quad AB := \frac{AC}{BR} \cdot BI$

$BC := AC - AB \quad BG := \sqrt{AB \cdot BC} \quad \delta := 1 .. BS - 1$

$BD_\delta := \frac{BG}{BS} \cdot \delta \quad BF_\delta := \frac{BC \cdot BD_\delta}{BG} \quad AD_\delta := \sqrt{AB^2 + (BD_\delta)^2}$

$AE_\delta := \frac{AD_\delta \cdot AC}{AB + BF_\delta} \quad DE_\delta := AE_\delta - AD_\delta$



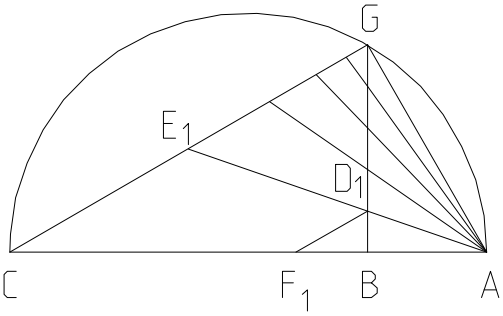




$\frac{AE_\delta}{DE_\delta}$

| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |

$\frac{BS \cdot BR}{(BR - BI) \cdot (BS - \delta)}$

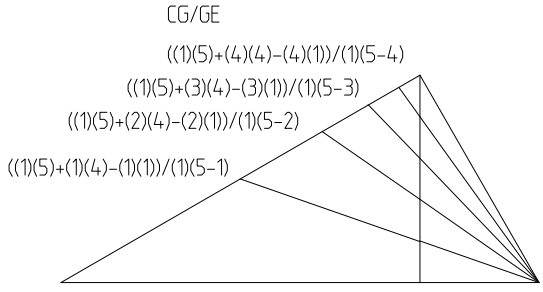
| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |




$AF_\delta := AB + BF_\delta \quad EJ_\delta := \frac{BD_\delta \cdot AC}{AF_\delta}$

$AJ_\delta := \sqrt{(AE_\delta)^2 - (EJ_\delta)^2} \quad CJ_\delta := AC - AJ_\delta$

$CE_\delta := \sqrt{(CJ_\delta)^2 + (EJ_\delta)^2} \quad CG := \sqrt{BG^2 + BC^2}$




$GE_\delta := CG - CE_\delta$

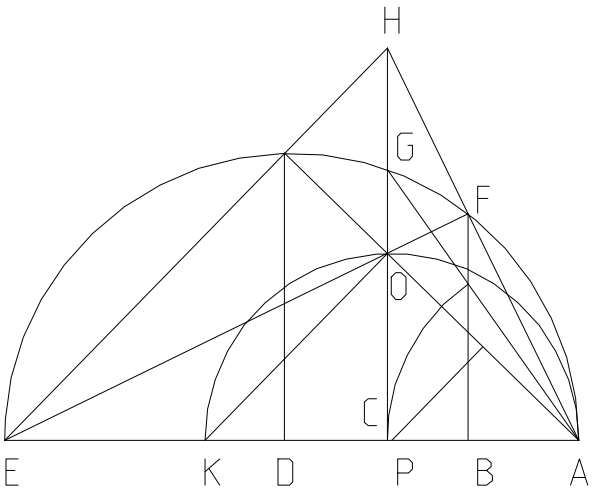
$\frac{CG}{GE_\delta}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

$\frac{(BI \cdot BS + \delta \cdot BR - \delta \cdot BI)}{(BI \cdot (BS - \delta))}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

What is AD. What is BD to $\sqrt{AB \cdot DE}$ ?




$AE := 5.5 \quad AB := 1.05 \quad BE := AE - AB$

$AC := \left(AB^2 \cdot AE\right)^{\frac{1}{3}} \quad CE := AE - AC$

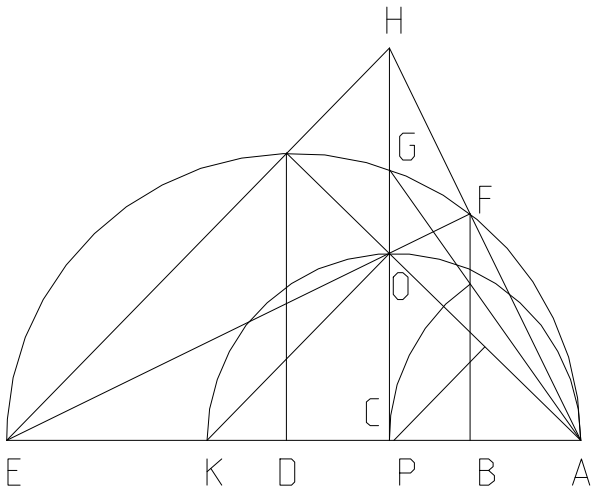
$BF := \sqrt{AB \cdot BE} \quad CO := \frac{BF \cdot CE}{BE} \quad AO := \sqrt{AC^2 + CO^2}$

$AP := \frac{1}{2} \cdot \frac{AO^2}{AC} \quad AK := 2 \cdot AP \quad AD := \frac{AC \cdot AE}{AK}$

$DE := AE - AD \quad BD := AD - AB$

$AD = 2.807$

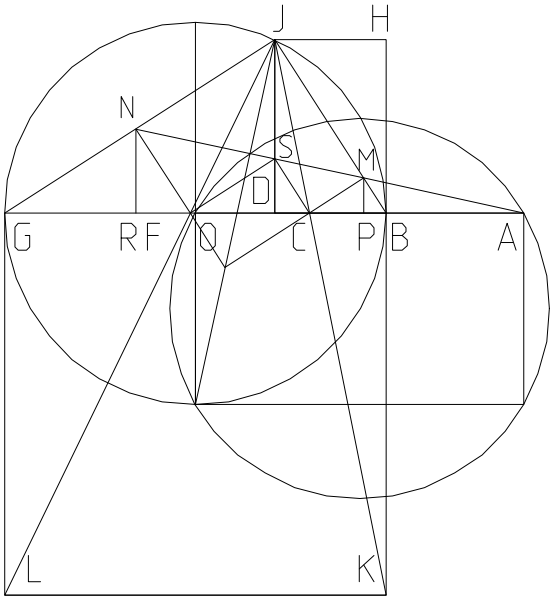
$$\frac{AE^{\frac{3}{3}} \cdot AB^{\frac{1}{3}} + AE^{\frac{2}{3}} \cdot AB^{\frac{2}{3}} + AE^{\frac{1}{3}} \cdot AB^{\frac{3}{3}}}{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}} = 2.807$$




$$\frac{\sqrt{AB \cdot DE}}{BD} = 0.957$$

$$\frac{AB^{\frac{1}{6}} \cdot AE^{\frac{1}{6}} \cdot \sqrt{AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} + AE - AE^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} \cdot \sqrt{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}}}{AE + AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} - 2 \cdot AB} = 0.957$$

The figure cuts the base in Cube Roots and provides some interesting ratios.




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

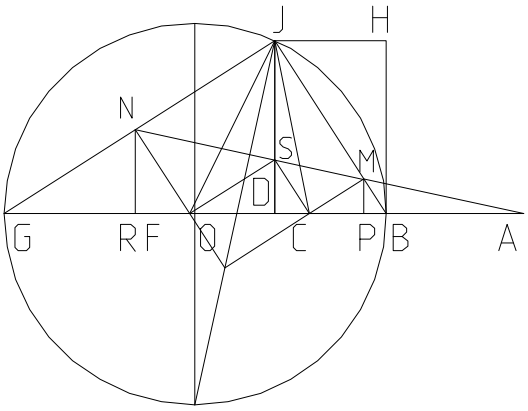
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

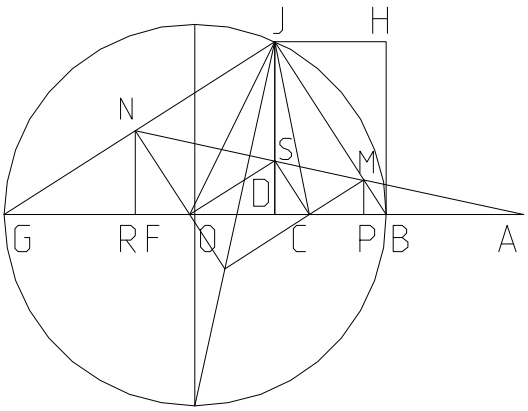
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

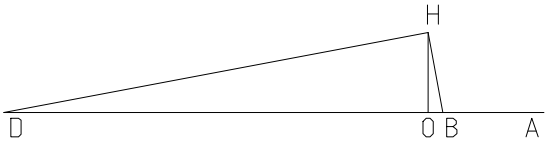
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$




$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

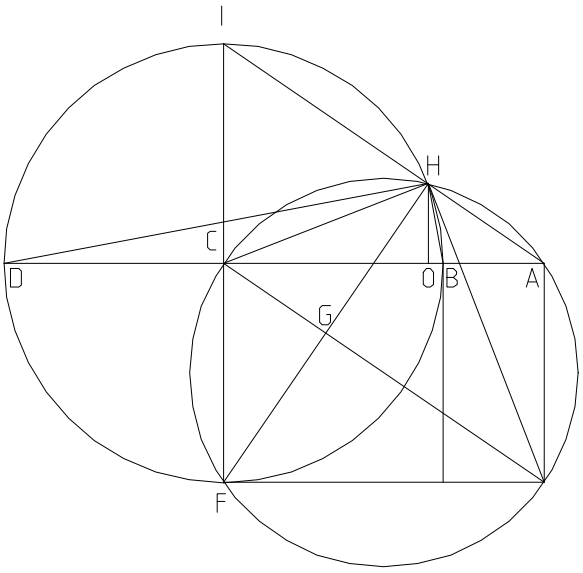
$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$




Given AD and AB on AD, place a right triangle on BD as base such that the opposite sides are in the ratio of AB to AD.

$BD := 8 \quad AB := 2 \quad AD := AB + BD$

$BC := \frac{BD}{2} \quad AC := AB + BC \quad CF := BC$




$CI := BC \quad CH := BC \quad AE := BC$

$CE := \sqrt{AC^2 + AE^2} \quad CG := \frac{CH^2}{CE}$

$GH := \sqrt{CH^2 - CG^2} \quad FH := 2 \cdot GH \quad FI := CF + CI$

$HI := \sqrt{FI^2 - FH^2} \quad AI := \sqrt{AC^2 + CI^2}$

$AH := AI - HI \quad AO := \frac{AC \cdot AH}{AI} \quad HO := \frac{CI \cdot AO}{AC}$

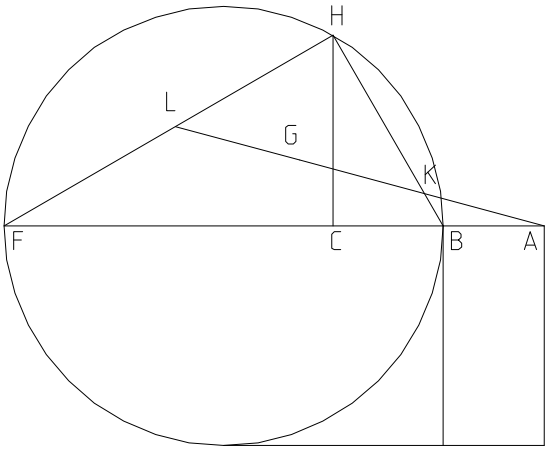
$BO := AO - AB \quad DO := BD - BO$

$DH := \sqrt{DO^2 + HO^2} \quad BH := \sqrt{BO^2 + HO^2}$

$\frac{DH}{BH} = 5 \qquad \frac{AD}{AB} = 5$

Given a straight edge and compass, AB and BD find the sum of six cubes divided by the sum of five squares.

$$\frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BD + AB \cdot BD^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BD + BD^2} = 2.308 \qquad AO = 2.308$$




Given AF and AB on AF and a right triangle on BF divide the sides of the triangle such that a section on one side is to the other as AB is to AF.

Now it can be realized that there are stipulations as to possible placements of the given triangle.

$AB := 3 \quad BF := 10 \quad BC := 1$

$AC := AB + BC \quad BE := \frac{BF}{2} \quad AE := AB + BE$

$$DOABLE := \frac{2\cdot AB^3 + 3\cdot AB^2\cdot BF + AB\cdot BF^2}{2\cdot AB^2 + 2\cdot AB\cdot BF + BF^2} \le AC < AE$$

$DOABLE = 1$

$AF := AB + BF$

$CF := BF - BC \quad CH := \sqrt{BC\cdot CF} \quad EJ := BE$

$CE := BE - BC \quad JH := \sqrt{CE^2 + (CH + EJ)^2}$

$DE := \frac{CE\cdot EJ}{EJ + CH} \quad AD := AE - DE$

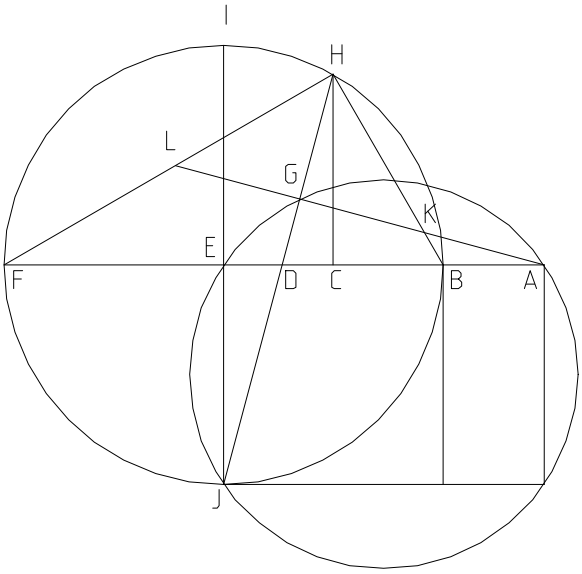
$JD := \sqrt{DE^2 + EJ^2} \quad DG := \frac{DE\cdot AD}{JD}$

$AG := \sqrt{AD^2 - DG^2} \quad GH := JH - (JD + DG)$

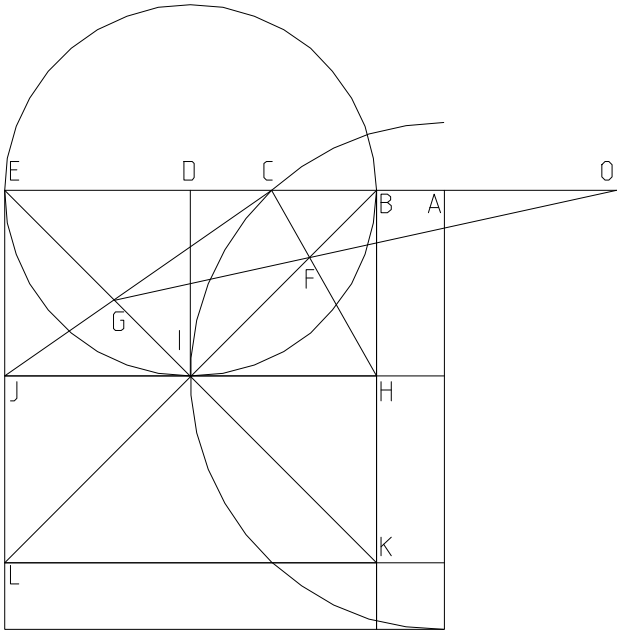
$HK := \sqrt{2\cdot GH^2} \quad BH := \sqrt{BC^2 + CH^2} \quad HL := HK$

$BK := BH - HK \quad FH := \sqrt{CF^2 + CH^2}$

$FL := FH - HL$




$\frac{FL}{BK} = 4.333 \qquad \frac{AF}{AB} = 4.333$




Projecting from KL or HJ is productive, can I find any other productive points?

$AB := 5 \quad BE := 3 \quad BK := BE \quad AE := AB + BE$

$BD := \frac{BE}{2} \quad AC := \sqrt{AB \cdot AE} \quad HI := BD \quad IJ := BD$

$BC := AC - AB \quad CE := BE - BC$

$\Delta := 2 \quad \delta := 1 .. \Delta$

$BH_\delta := \frac{BK}{\Delta} \cdot \delta \quad Ha_\delta := \frac{BH_\delta \cdot HI}{BC} \quad EJ_\delta := BH_\delta$

$Ba_\delta := Ha_\delta + BD \quad Bb_\delta := \frac{(BH_\delta)^2}{Ba_\delta} \quad Jc_\delta := \frac{EJ_\delta \cdot IJ}{CE}$

$Ec_\delta := Jc_\delta + BD \quad Ed_\delta := \frac{(EJ_\delta)^2}{Ec_\delta} \quad Ef_\delta := Bb_\delta$

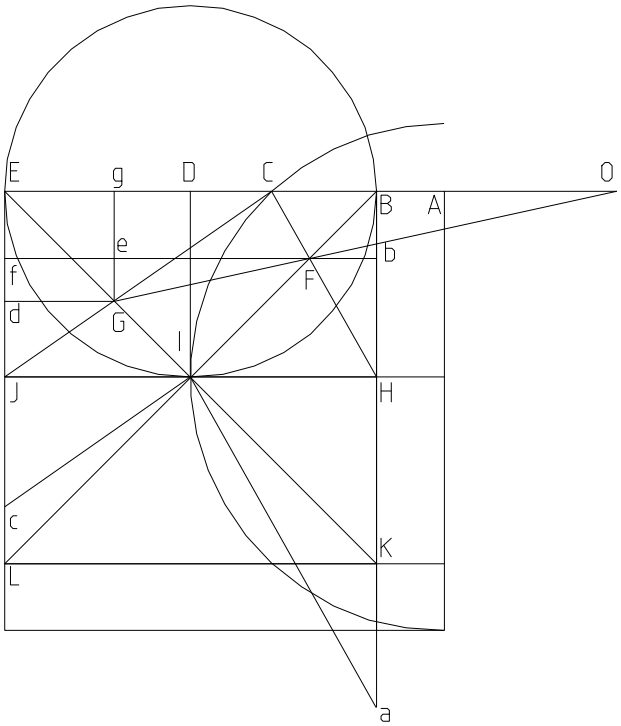
$df_\delta := Ed_\delta - Ef_\delta \quad Ge_\delta := df_\delta \quad Fb_\delta := \frac{HI \cdot BH_\delta}{Ba_\delta}$

$Gd_\delta := \frac{IJ \cdot EJ_\delta}{Ec_\delta} \quad ef_\delta := Gd_\delta \quad Fe_\delta := BE - (ef_\delta + Fb_\delta)$

$Gg_\delta := Ed_\delta \quad Og_\delta := \frac{Fe_\delta \cdot Gg_\delta}{Ge_\delta} \quad Eg_\delta := ef_\delta$

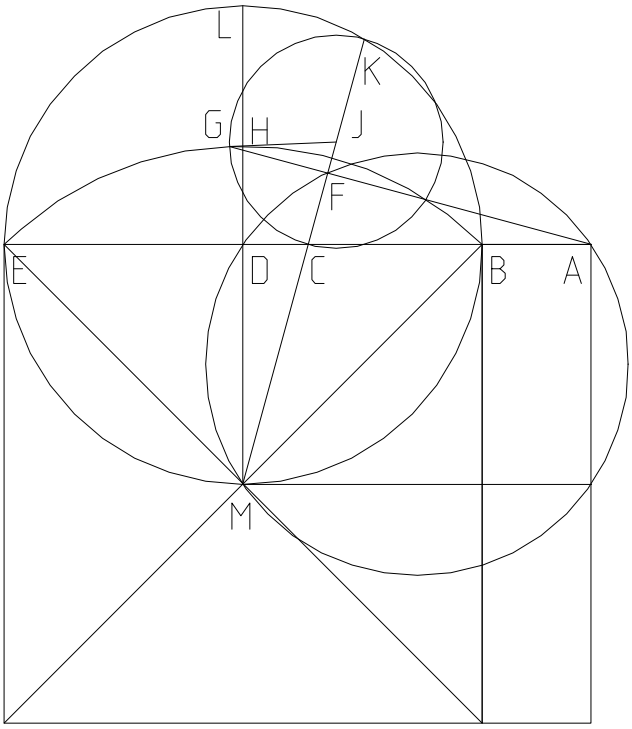
$EO_\delta := Og_\delta + Eg_\delta \quad BO_\delta := EO_\delta - BE$

| $\left(\frac{EO_\delta}{BO_\delta}\right)$ |
|---|
| 1.2649 |
| 1.6 |




$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} = 1.2649 \qquad \frac{AE}{AB} = 1.6$

I have not found any.




The power line for cube root abstraction is developed off from a simple curve.

$$AB := 33 \quad BE := 11 \quad BD := \frac{BE}{2} \quad AE := AB + BE$$

$$R_1 := \frac{\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AB}{2} \quad R_2 := \frac{AE - \left(AB \cdot AE^2\right)^{\frac{1}{3}}}{2}$$

$$D := \left[\left(AB \cdot AE^2\right)^{\frac{1}{3}} - \left(AB^2 \cdot AE\right)^{\frac{1}{3}}\right] + R_1 + R_2$$

$$BC := \frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D}$$

The formula for the power line (BC) was given in 01_08_96.MCD

$$CD := BD - BC \quad AC := BC + AB \quad DM := BD$$

$$DL := BD \quad CM := \sqrt{CD^2 + DM^2} \quad ML := DM + DL$$

$$MK := \frac{DM \cdot ML}{CM} \quad CK := MK - CM \quad CJ := \frac{CK}{2}$$

$$JG := CJ \quad JM := CM + CJ \quad BM := \sqrt{2 \cdot BD^2}$$

$$GM := BM \quad FJ := \frac{JG^2}{JM} \quad FM := JM - FJ$$

$$AD := BD + AB \quad HM := \frac{CM \cdot FM}{DM}$$

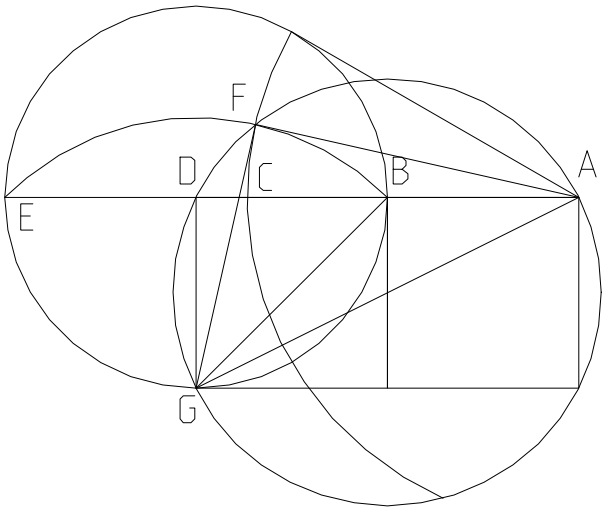
$$DH := HM - DM \quad AH := \sqrt{DH^2 + AD^2}$$

$$CF := FM - CM \quad FH := \frac{CD \cdot HM}{CM}$$

$$AF_1 := AH - FH \quad AF_2 := \sqrt{AC^2 - CF^2}$$

$$AF_1 - AF_2 = 0$$

Trivial Method; Square Root




$N := 9003$

$AE := N \quad AB := \frac{AE}{N} \quad BE := AE - AB \quad BD := \frac{BE}{2}$

$DG := BD \; AD := AB + BD \quad AG := \sqrt{AD^2 + DG^2}$

$BG := \sqrt{BD^2 + DG^2} \quad FG := BG \quad AF := \sqrt{AG^2 - FG^2}$

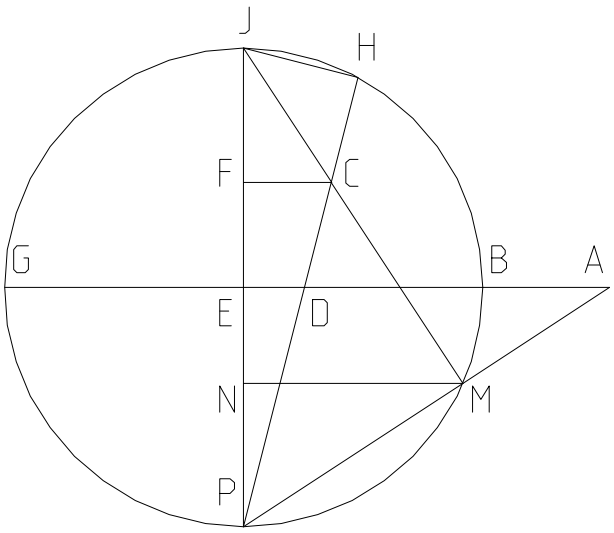
$AC := \sqrt{AB \cdot AE}$

$AC - AF = 0$

$AF = 94.884$

$AC = 94.884$

Given a point on BG, project to the point of cubic similarity.




$BG := 100 \quad BD := 49 \qquad BE := \frac{BG}{2}$

$EG := BE \quad DE := BE - BD \quad EP := BE$

$EJ := BE \quad DP := \sqrt{DE^2 + EP^2} \quad JP := EP + EJ$

$HP := \frac{EP \cdot JP}{DP} \quad DH := HP - DP \quad CD := \frac{DH}{2}$

$CP := DP + CD \quad CF := \frac{DE \cdot CP}{DP} \quad FP := \frac{EP \cdot CP}{DP}$

$EF := FP - EP \quad FJ := EJ - EF \quad CJ := \sqrt{CF^2 + FJ^2}$

$JM := \frac{FJ \cdot JP}{CJ} \quad JN := \frac{FJ \cdot JM}{CJ} \quad NP := JP - JN$

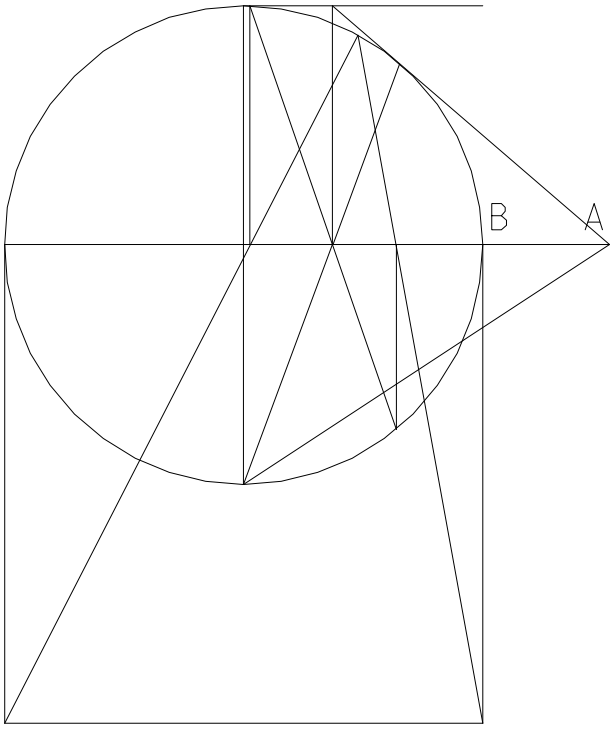
$MP := \frac{CF \cdot JP}{CJ} \quad AP := \frac{MP \cdot EP}{NP} \quad MN := \frac{CF \cdot JM}{CJ}$

$$\frac{BG^4 - 3 \cdot BG^3 \cdot BD + 3 \cdot BG^2 \cdot BD^2 - BD^3 \cdot BG}{BG^3 - 3 \cdot BG^2 \cdot BD + 3 \cdot BG \cdot BD^2 - 2 \cdot BD^3} = 884.222$$

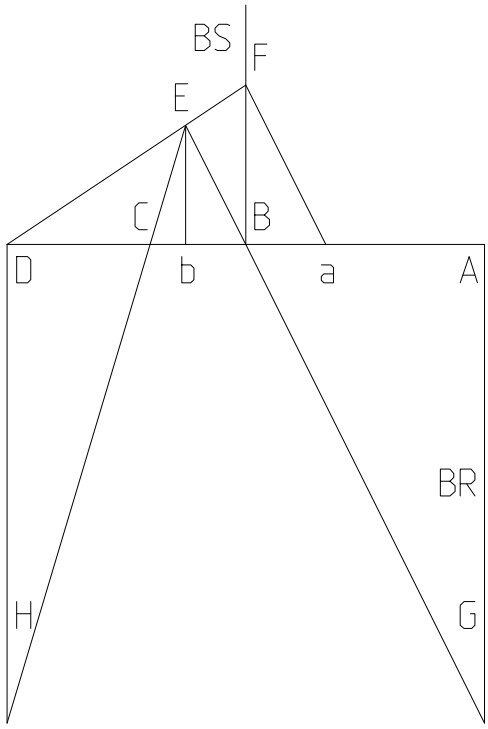
$AE := \frac{MN \cdot AP}{MP} \quad AG := AE + EG$

$$AG = 884.222$$

$$\frac{BG \cdot \left[ (AG - BG)^{\frac{2}{3}} \cdot AG^{\frac{2}{3}} + AG^{\frac{1}{3}} \cdot BG - AG^{\frac{4}{3}} - BG \cdot (AG - BG)^{\frac{1}{3}} + (AG - BG)^{\frac{1}{3}} \cdot AG \right]}{(AG - BG)^{\frac{1}{3}} \cdot (2 \cdot AG - BG)} - BD = 0$$




One may be tempted to state the obvious, $\frac{A^{N+1}}{A^{N}} := A_{\tau}$, but what is not so obvious at first blush is that the processes themselves are assigned dimensional values. This has significance when using mathematics to theorize dimensions beyond three. Dimensions are so generally defined that processes are legitimate dimensional differences, but it is also impossible to defend mathematical theory about dimensions as objective. It becomes a point of Philosophical Mystic contemplation to realize that relationships concerning a single dimensional object and several processes adding dimensionally to the whole, is true of a multidimensional object without those processes!




Linear division $\frac{2\cdot(A + B)}{A}$

$BR := \frac{1}{4}$ $BS := 3$

$AD := \frac{BR}{BR}$ $AG := AD\cdot BR$ $AB := \frac{AD}{2}$

$BF := AB\cdot BS$ $Ba := \frac{AB\cdot BF}{AG}$ $BD := AB$

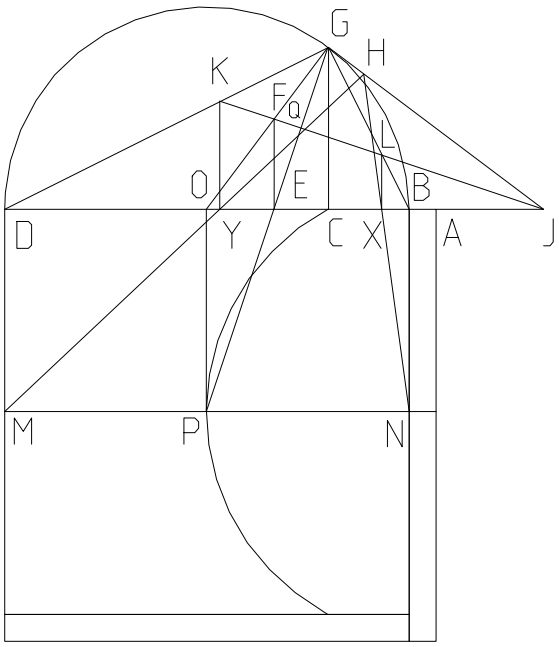
$Da := BD + Ba$ $Bb := \frac{Ba\cdot BD}{Da}$ $Db := BD - Bb$

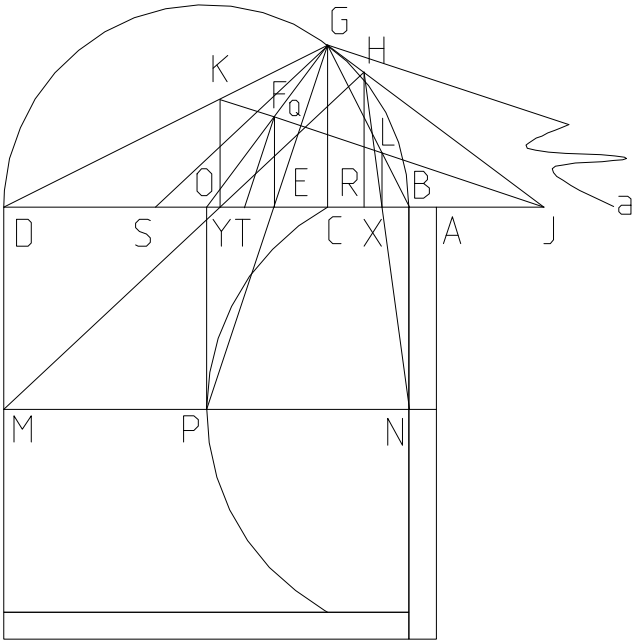
$Eb := \frac{BF\cdot Db}{BD}$ $DH := AG$

$DC := \frac{Db\cdot DH}{DH + Eb}$ $\frac{AD}{DC} = 26$

$\frac{2\cdot(BR + BS)}{BR} = 26$

Hitting JG from any BN while maintaining complimentary roots.




$AB := 2 \quad BD := 5 \quad BO := \frac{BD}{2} \quad AD := AB + BD$

$DO := BO \quad AC := \sqrt{AB \cdot AD} \quad BC := AC - AB$

$CD := BD - BC \quad CG := \sqrt{BC \cdot CD} \quad CO := BO - BC$

$N := 7 \quad OP := BO \cdot N \quad BN := OP \quad DM := OP$

$NP := BO \quad MP := BO \quad EO := \frac{CO \cdot OP}{OP + CG}$

$CE := CO - EO \quad EF := \frac{OP \cdot CE}{CO} \quad GO := BO$

$CJ := \frac{CG^2}{CO} \quad EJ := CJ + CE \quad Ca := \frac{EJ \cdot CG}{EF}$

$DJ := CD + CJ \quad Da := CD + Ca \quad JY := \frac{Ca \cdot DJ}{Da}$

$KY := \frac{EF \cdot JY}{EJ} \quad JK := \sqrt{JY^2 + KY^2} \quad JG := \sqrt{CJ^2 + CG^2}$

$GP := \sqrt{CO^2 + (OP + CG)^2} \quad EP := \sqrt{EO^2 + OP^2}$

$ET := \frac{EO \cdot EF}{OP} \quad JT := EJ + ET \quad FT := \sqrt{ET^2 + EF^2}$

$EG := GP - EP \quad EQ := \frac{FT \cdot EJ}{JT} \quad GQ := EG - EQ$

$KL := 2 \cdot GQ \quad JL := JK - KL$

$DY := DJ - JY \quad CS := \frac{DY \cdot CG}{DM} \quad JS := CS + CJ$

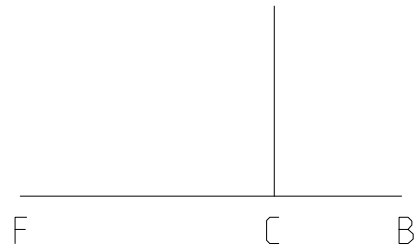
$YR := \frac{CS \cdot JY}{JS} \quad JR := JY - YR \quad HR := \frac{CG \cdot JR}{CJ}$

$AJ := CJ - AC \quad JX := \frac{JY \cdot JL}{JK}$

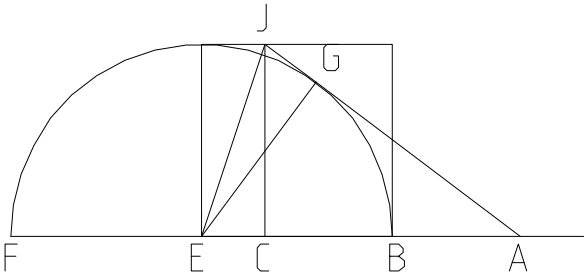
$BR_1 := JR - (AJ + AB) \quad BX := JX - (AJ + AB)$

$BR_2 := \frac{BX \cdot (BN + HR)}{BN}$

$BR_2 - BR_1 = 0$




Given BF and BC such that $\sqrt{(AB + BF)\cdot AB} = AB + BC$, find AB. It is obvious from the construction that answers are obtainable when BC is less than 1/2 of BF.

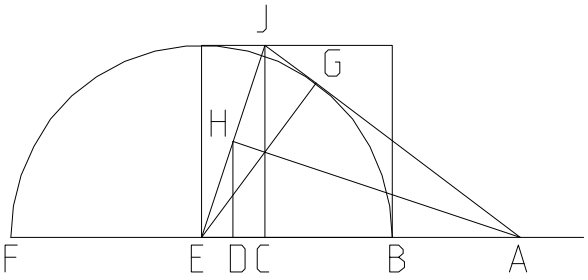



$\Delta := 5 \qquad \delta := 1..\Delta - 1 \quad BE := 5 \qquad BF := BE\cdot 2$

$BC_\delta := \frac{BE}{\Delta}\cdot\delta \qquad CE_\delta := BE - BC_\delta \qquad EG := BE$

$CJ := BE \quad EJ_\delta := \sqrt{\left(CE_\delta\right)^2 + CJ^2} \qquad DH := \frac{CJ}{2}$

$DE_\delta := \frac{CE_\delta}{2} \qquad AD_\delta := \frac{DH^2}{DE_\delta} \qquad AE_\delta := AD_\delta + DE_\delta$




$AC_\delta := AE_\delta - CE_\delta \qquad AJ_\delta := \sqrt{\left(AC_\delta\right)^2 + CJ^2}$

$GJ_\delta := \sqrt{\left(EJ_\delta\right)^2 - EG^2} \qquad AB_\delta := AE_\delta - BE$

$AF_\delta := AB_\delta + BF$

$\left[\frac{1}{2}\cdot\frac{BE}{\Delta}\cdot\delta\cdot\frac{(-2\cdot\Delta + \delta)}{(-\Delta + \delta)}\right] - AC_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$GJ_\delta - CE_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

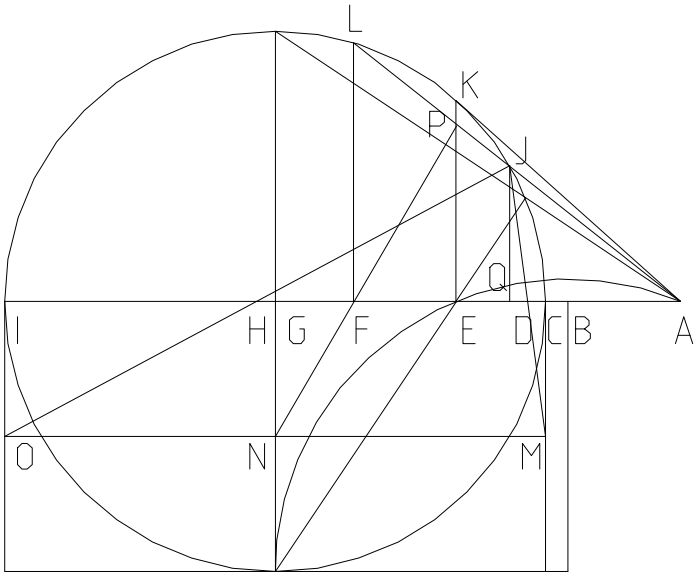
$AB_\delta + BC_\delta - \sqrt{AB_\delta\cdot(AB + BF)_\delta}$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$A := \frac{BF}{2} \qquad B := BC_2 \qquad \frac{2\cdot A\cdot B - B^2}{2\cdot A - 2\cdot B} - \left(AB_2 + BC_2\right) = 0$

$\sqrt{\left(AB_2 + BF\right)\cdot AB_2} - \left(AB_2 + BC_2\right) = 0$

$\frac{BF\cdot BC_2 - \left(BC_2\right)^2}{BF - 2\cdot BC_2} - \left(AB_2 + BC_2\right) = 0$ A rectangle minus a square divided by a line yeilds a line.




Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

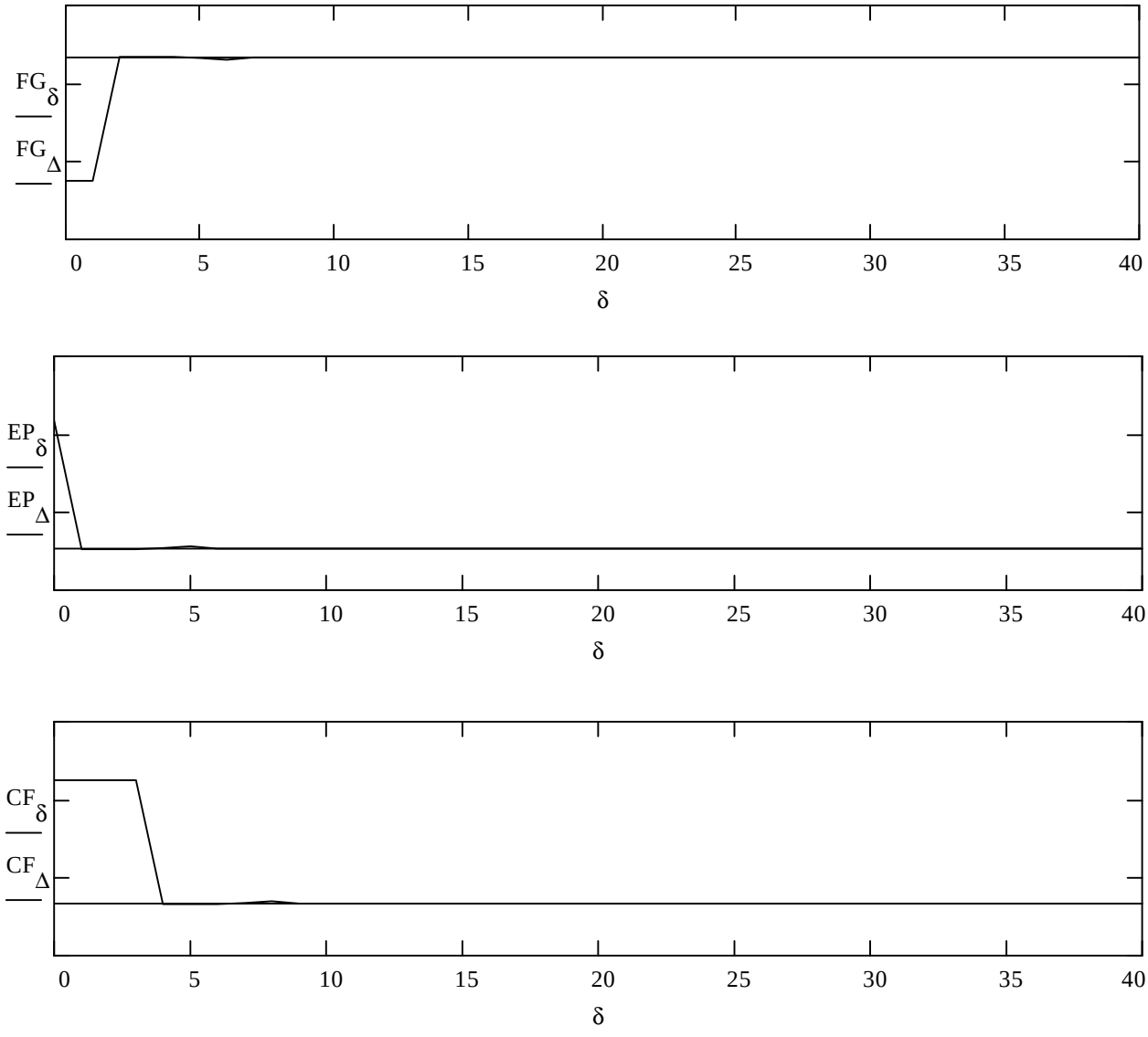
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

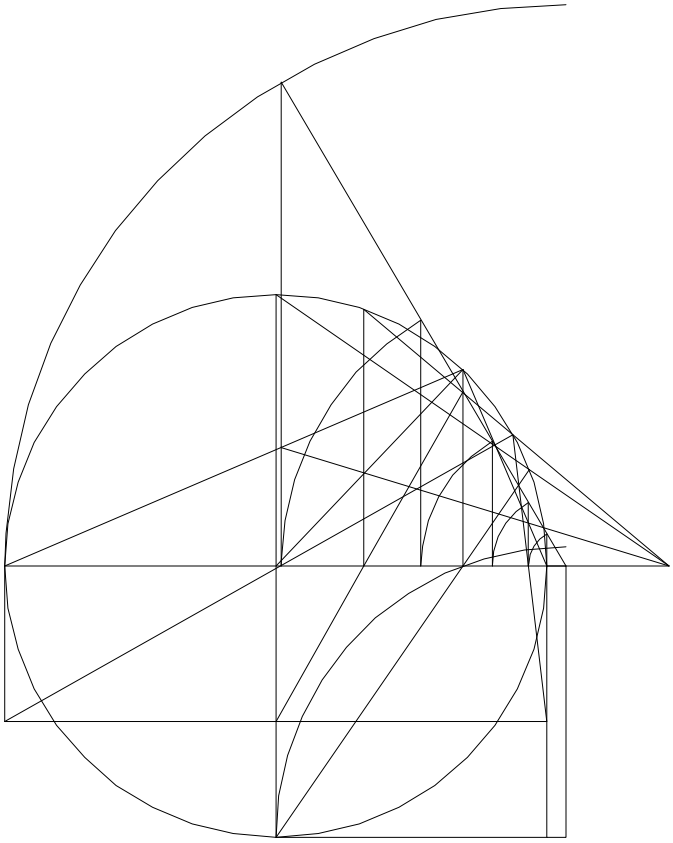
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. This solution alone will conquer contiguous domains and will satisfy the purist.



Because of it's long projection, the last vertices is not drawn.

Does every $n^{th}$ root series have at least one square root pair?

$n := 5 \qquad \delta := 0..\frac{n}{2}$

$A := 3 \quad B := 10$

$$\sqrt{\left(A^{n-1}\cdot B^{1}\right)^{\frac{1}{n}}\cdot\left(A^{1}\cdot B^{n-1}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B} = 0$$

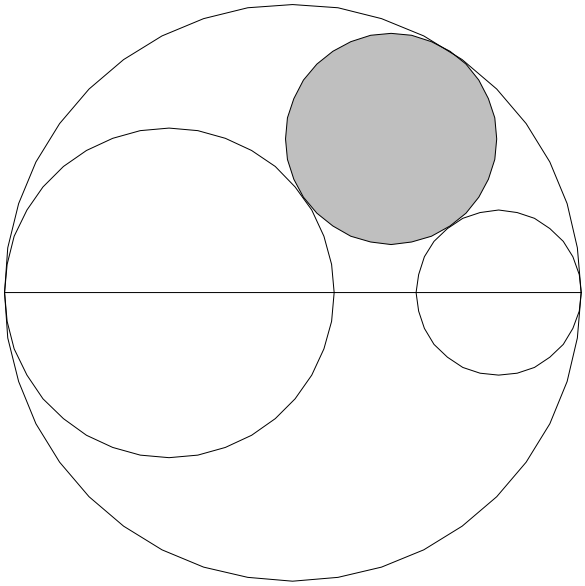
$$\sqrt{\left(A^{n-\delta}\cdot B^{\delta}\right)^{\frac{1}{n}}\cdot\left(A^{\delta}\cdot B^{n-\delta}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B}$$

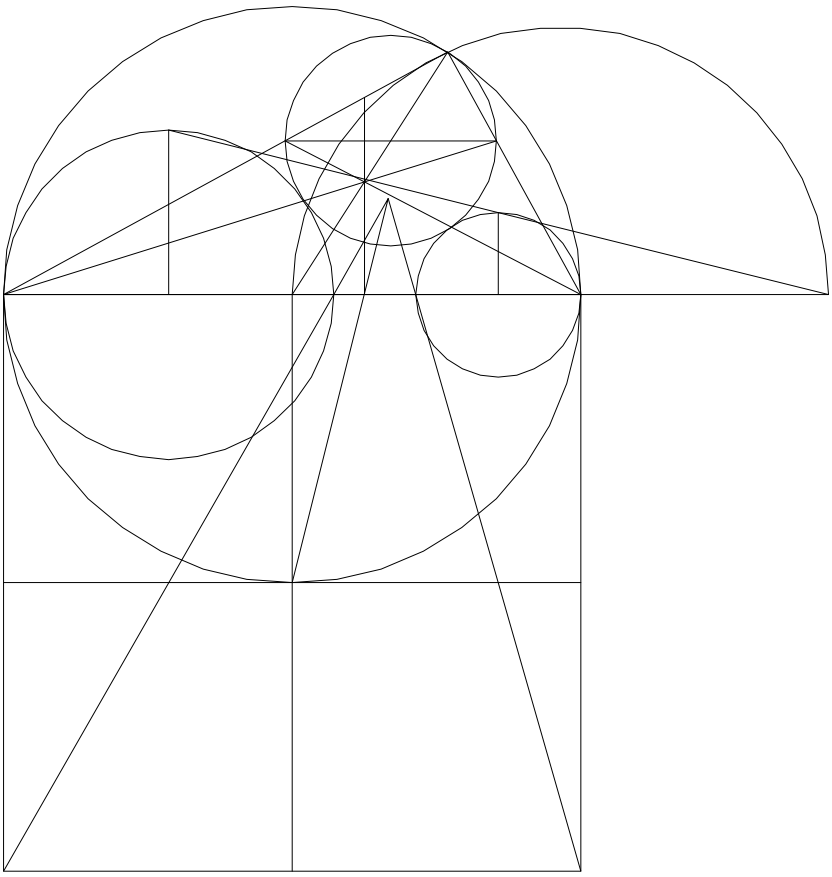
| |
|---|
| 0 |
| 0 |
| 0 |

A root series has as many vertices on a circle as it has square root pairs, and it has the greater whole of n/2 vertices where n is the root series denominator.

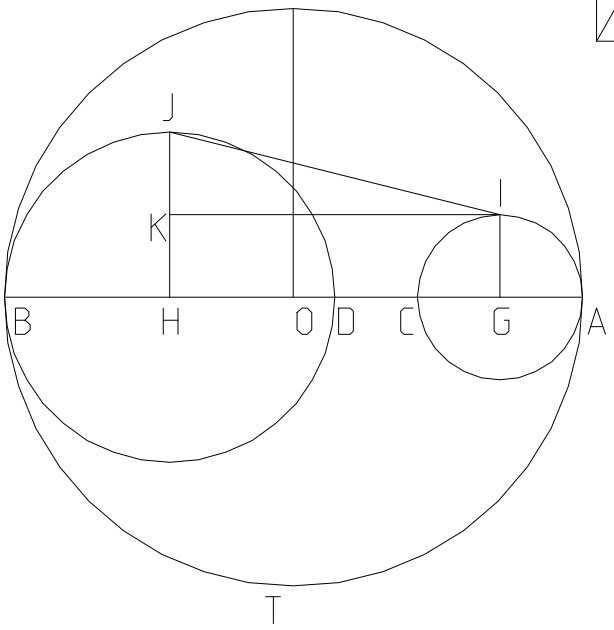
# Method for Unequals

Given three circles in the said configuration, find the fourth. I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."





Process Summary.

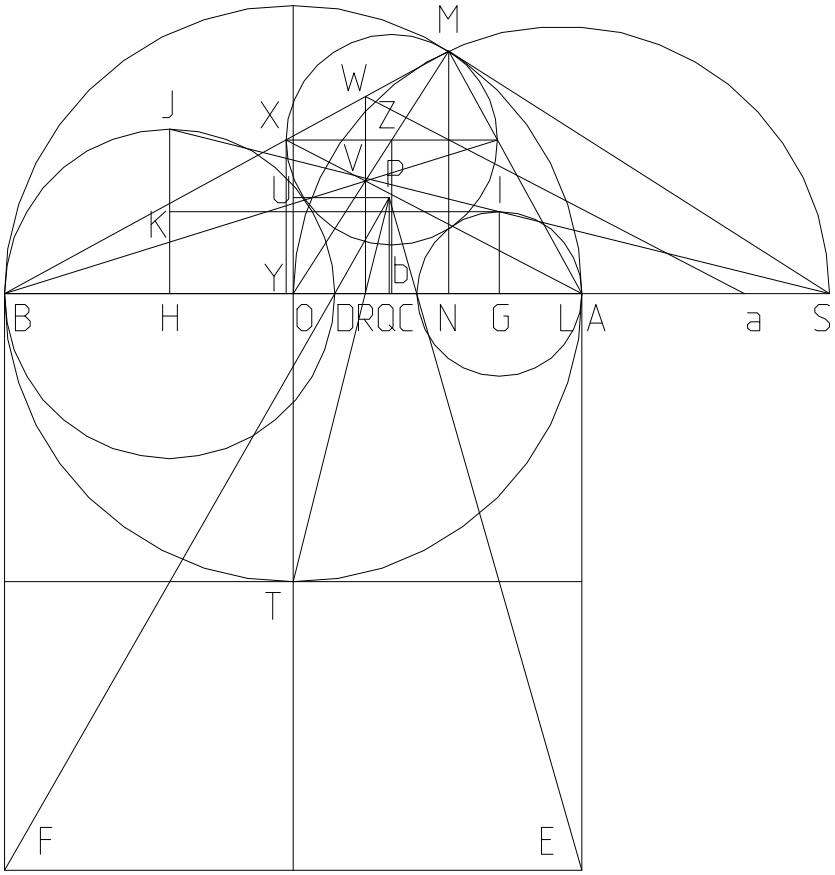


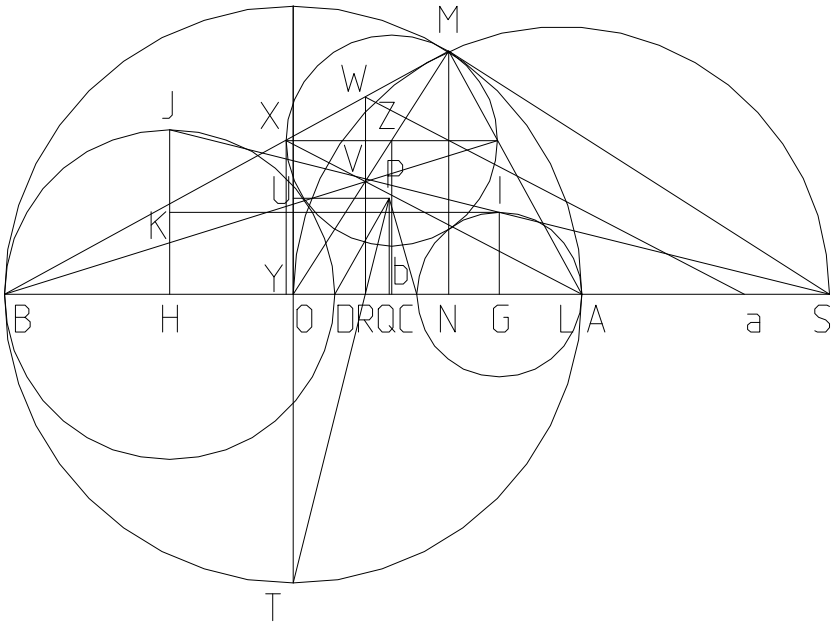

AO := 5 AG := 1 BH := 3 AB := 2·AO

BO := AO CG := AG GI := AG HJ := BH

DH := BH GH := AB − (AG + BH) HK := GI

JK := HJ − HK




$$HS := \frac{GH \cdot HJ}{JK} \quad AH := AB - BH$$

$$AS := HS - AH \quad OS := AO + AS$$

$$SL := \frac{OS}{2} \quad MO := AO \quad MS := \sqrt{OS^2 - MO^2}$$

$$MN := \frac{MO \cdot MS}{OS} \quad NS := \frac{MS \cdot MN}{MO}$$

$$AN := NS - AS \quad ON := AO - AN$$

$$BN := AB - AN \quad AM := \sqrt{MN^2 + AN^2}$$

$$AE := AB \quad BF := AB \quad AC := 2 \cdot AG$$

$$BD := 2 \cdot BH \quad CD := AB - (AC + BD)$$

$$PQ := \frac{AE \cdot CD}{(AC + BD)} \quad OU := PQ$$

$$CQ := \frac{AC \cdot PQ}{AE} \quad AQ := AC + CQ$$

$$OQ := AO - AQ \quad OT := AO$$

$$TU := OT + OU \quad OR := \frac{OQ \cdot OT}{TU}$$

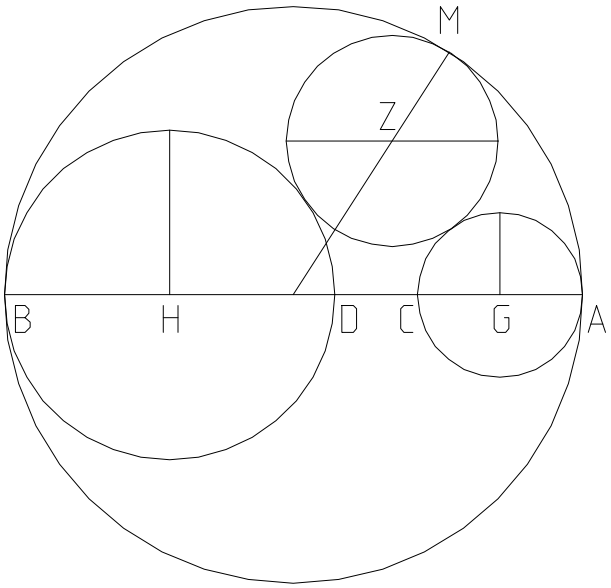
$$RV := \frac{MN \cdot OR}{ON} \quad BR := BO + OR$$

$$RW := \frac{MN \cdot BR}{BN} \quad AR := AO - OR$$

$$Ra := \frac{AR \cdot RW}{RV} \quad XY := \frac{RW \cdot AB}{BR + Ra}$$

$$Zb := XY \quad OZ := \frac{MO \cdot Zb}{MN}$$

$$MZ := MO - OZ \quad MZ = 1.818$$

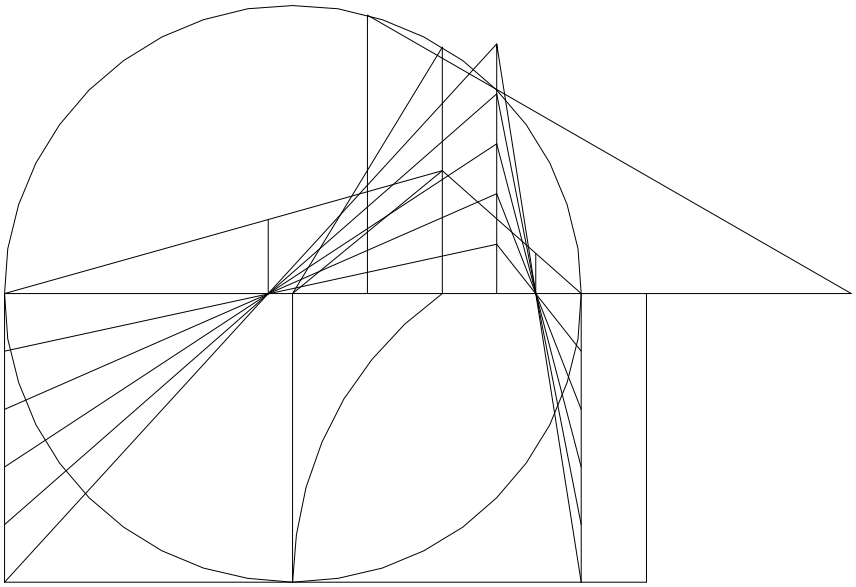


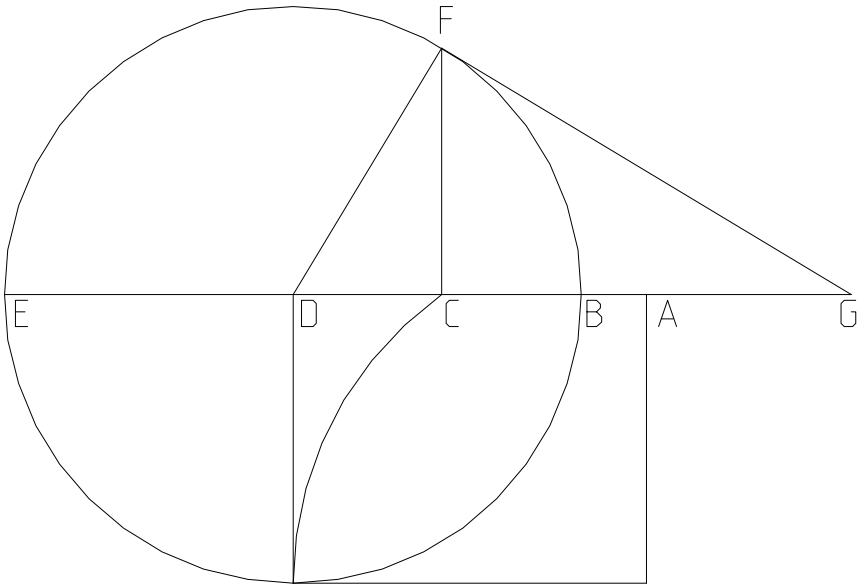

$$MZ = 1.818$$

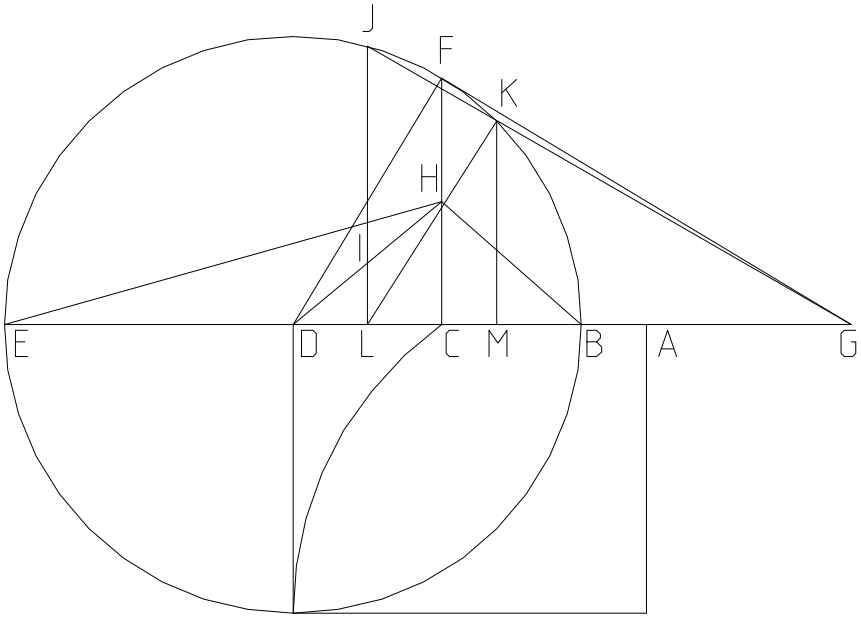
$$\frac{AB^3 - 2\cdot AB^2\cdot AG - 2\cdot AB^2\cdot BH + 4\cdot AB\cdot BH\cdot AG}{2\cdot AB^2 - 8\cdot BH\cdot AG} = 1.818$$

Neither in the process, nor in the Algebraic name. is the order of AG and BH recognized. Neither does it matter if they intersect.

# On Gemini Roots






$AB := 1 \qquad BE := 10 \qquad BD := \frac{BE}{2}$

$AE := AB + BE \qquad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \qquad CE := BE - BC$

$DF := BD \qquad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \qquad CG := \frac{CF^2}{CD}$

$BG := CG - BC \qquad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \qquad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \qquad DL := \frac{CD \cdot DI}{DH}$

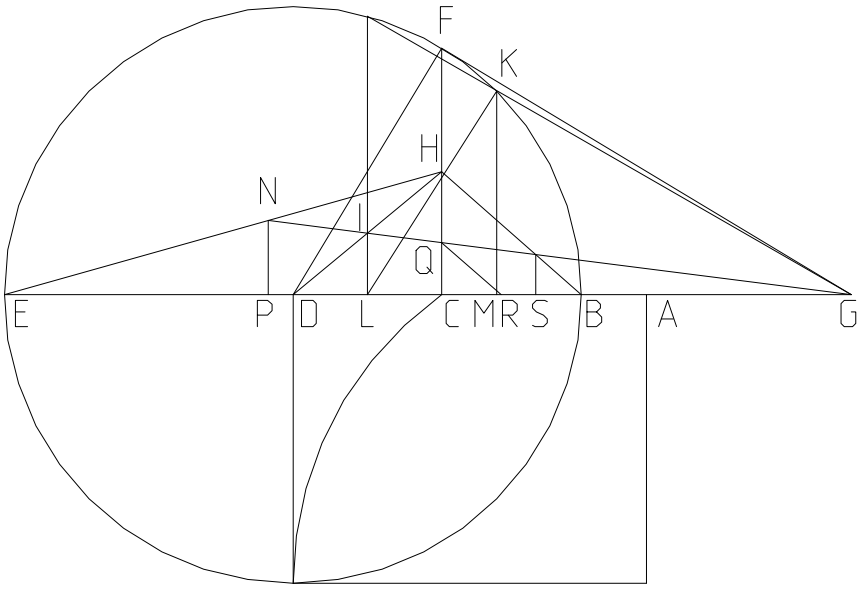
$BL := BD - DL \qquad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \qquad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \qquad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \qquad BM := GM - BG$

$EM := BE - BM$

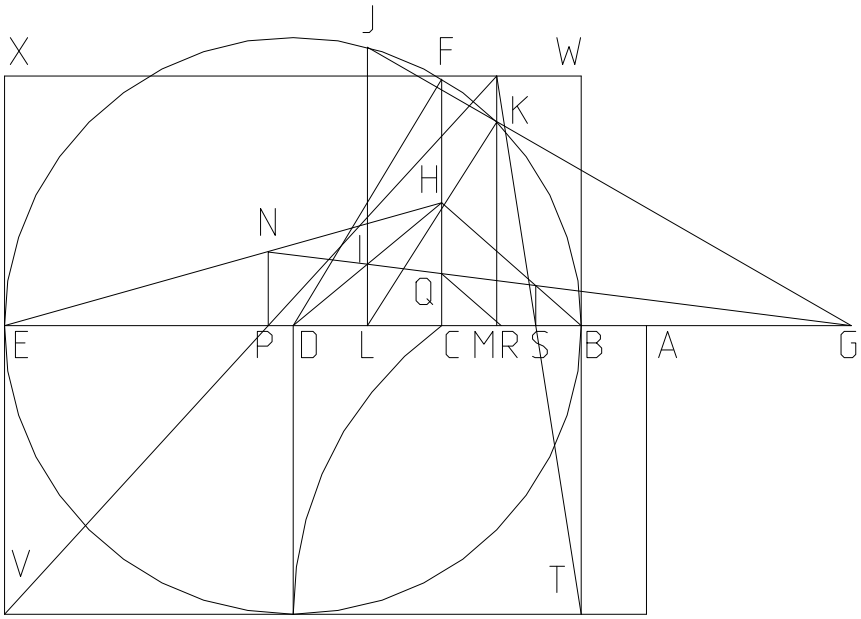



$$IL := \sqrt{DI^2 - DL^2} \qquad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \qquad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \qquad CR := \frac{BC \cdot CQ}{CH}$$

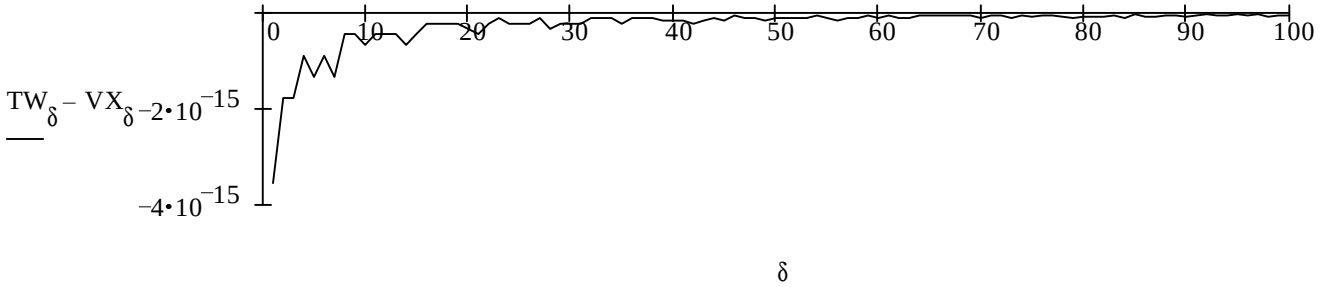
$$GR := CG - CR \qquad BS := \frac{CR \cdot BG}{GR}$$

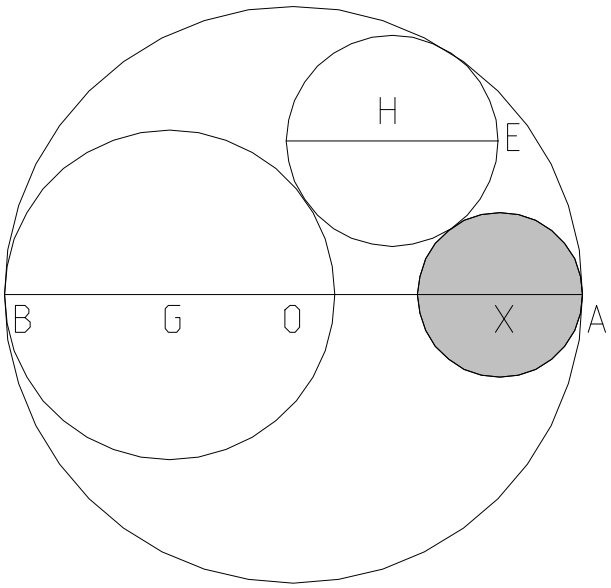



$$\delta := 1..100$$

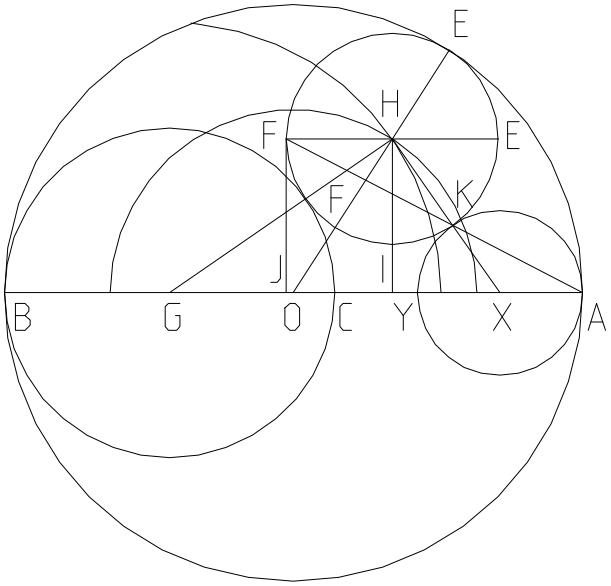
$$E_\delta := \frac{BE}{\delta} \qquad BT_\delta := E_\delta \qquad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$






Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO. This of course means that if the sum of BG and EH is equal to AO, we have no result.




$AO := 5 \qquad BG := 2.5 \quad EH := 1.5$

$AB := 2 \cdot AO \quad BC := 2 \cdot BG \quad EF := 2 \cdot EH$

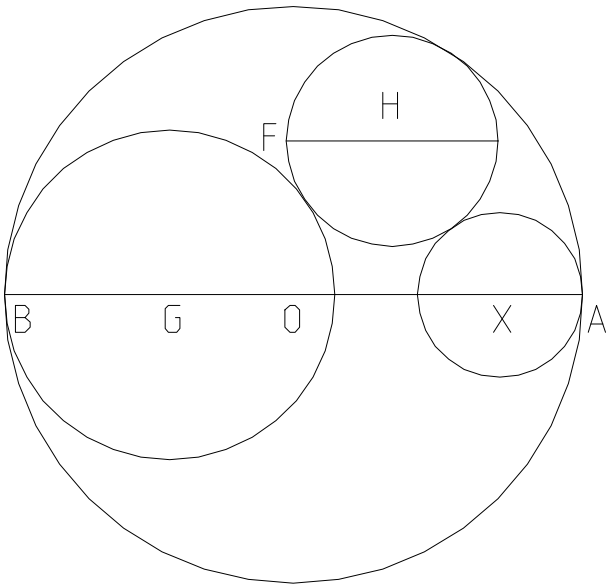
$GH := BG + EH \quad OH := AO - EH \quad GO := AO - BG$

$$GI := \frac{GH^2 + GO^2 - OH^2}{2 \cdot GO} \qquad HI := \sqrt{GH^2 - GI^2}$$

$AG := AB - BG \quad AI := AG - GI \quad IJ := EH$

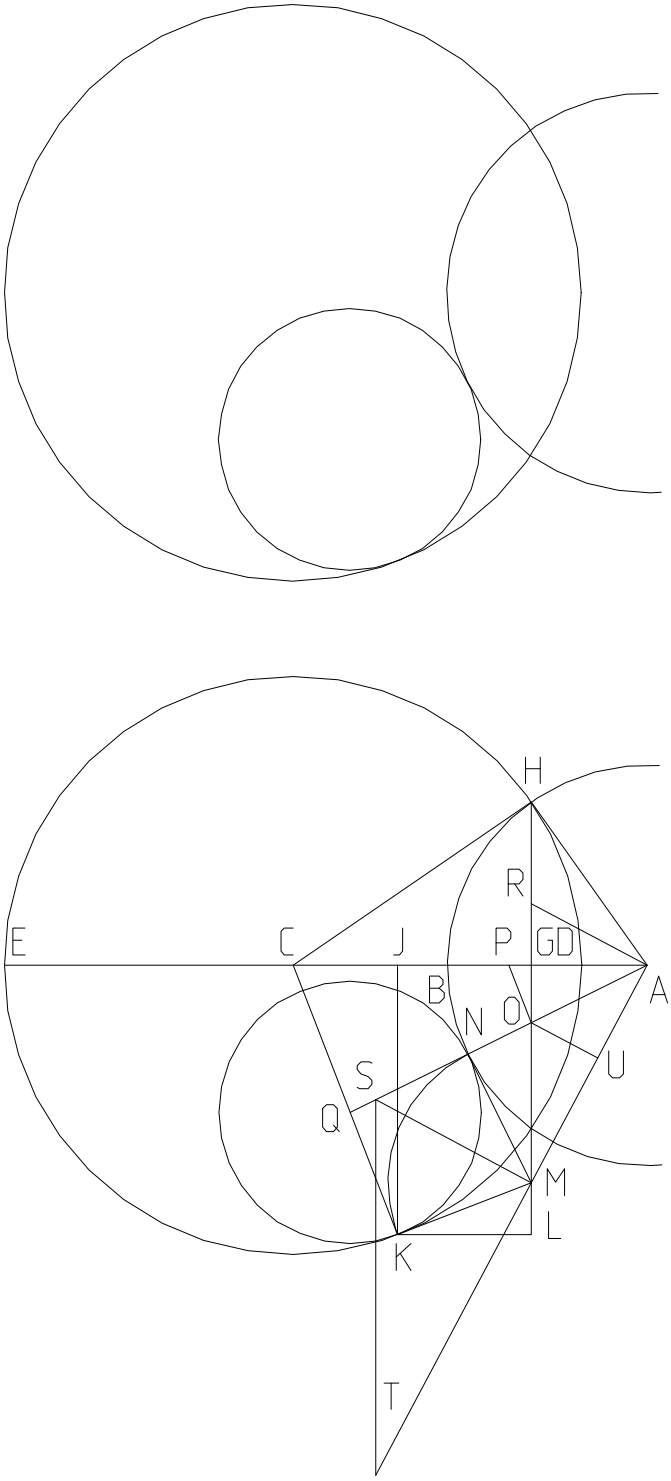
$AJ := AI + IJ \quad FJ := HI \quad AF := \sqrt{AJ^2 + FJ^2}$

$$FK := \frac{AJ \cdot EF}{AF} \qquad AK := AF - FK \qquad AY := \frac{AF \cdot AK}{AJ}$$




$$AX := \frac{AY}{2} \qquad AX = 2.857$$

$$\frac{AO^3 - AO^2 \cdot EH - AO^2 \cdot BG}{AO^2 - AO \cdot BG - EH \cdot BG} = 2.857$$




Given a point of tangency, draw a circle in a crescent tangent to the other side. This figure is given for the tangent on the exterior of the crescent, the other will become obvious.

$AB := Concave\_Radius$

$CD := Convex\_Radius \quad AC := Center\_Difference$

$DE := 2 \cdot CD \quad BF := 2 \cdot AB \quad CE := CD$

$AE := AC + CE \quad BE := AE - AB$

$$CG := \frac{CD^2 + AC^2 - AB^2}{2 \cdot AC} \quad AG := AC - CG$$

$GE := AE - AG$

$GJ := Power\_Line\_Tangent\,GE$

$EJ := GE - GJ$

$DJ := DE - EJ \quad JK := \sqrt{DJ \cdot EJ}$

$CJ := CG - GJ \quad CK := CD \quad KL := GJ$

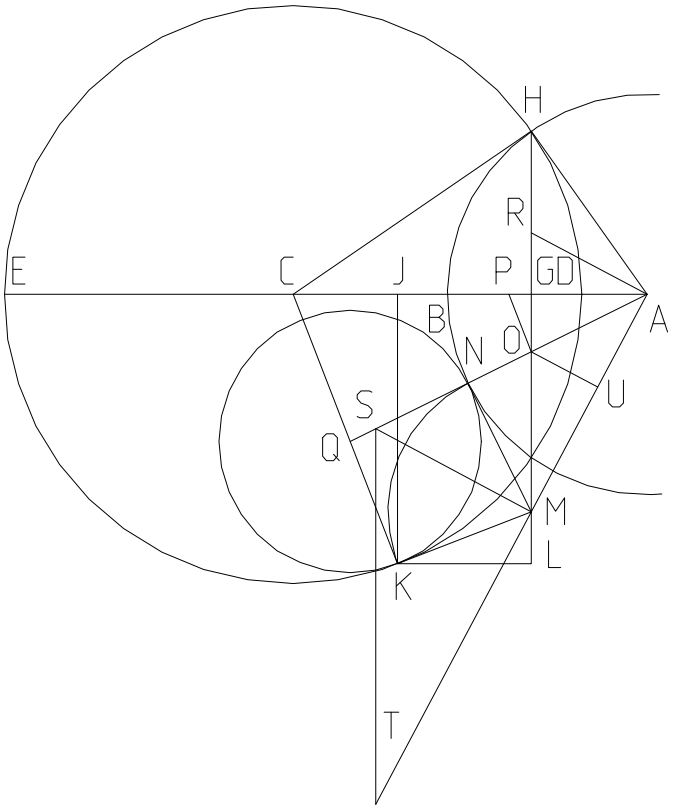
$$LM := \frac{CJ \cdot KL}{JK} \quad GL := JK \quad GM := GL - LM$$

$$AM := \sqrt{AG^2 + GM^2} \quad AN := AB$$

$$AS := \frac{AM^2}{AN} \quad MR := \frac{AM^2}{GM}$$

$$MS := \sqrt{AS^2 - AM^2} \quad AR := \sqrt{MR^2 - AM^2}$$

$$ST := \frac{MR \cdot MS}{AR} \quad MT := \frac{AM \cdot MS}{AR}$$




$$MO := \frac{ST \cdot AM}{AM + MT}$$

$$GO := GM - MO \qquad GP := \frac{CJ \cdot GO}{JK}$$

$$AP := AG + GP \qquad OP := \frac{CK \cdot GO}{JK}$$

$$CQ := \frac{OP \cdot AC}{AP} \qquad QK := CK - CQ$$

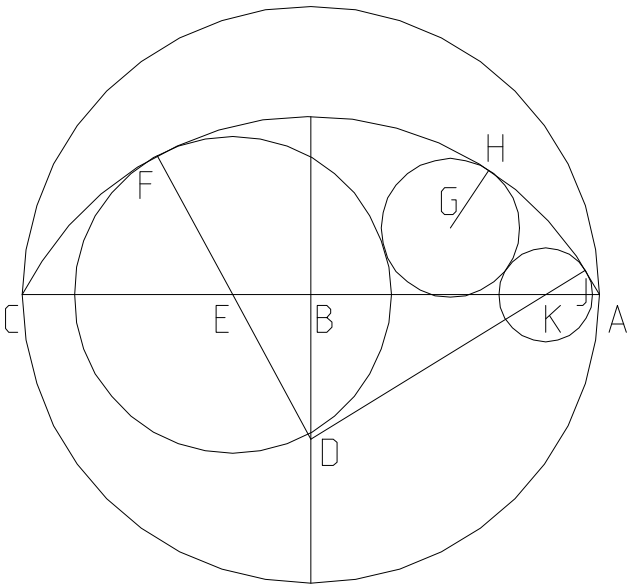
$QK = 0.206$

Concave_Radius $\equiv 2.37$

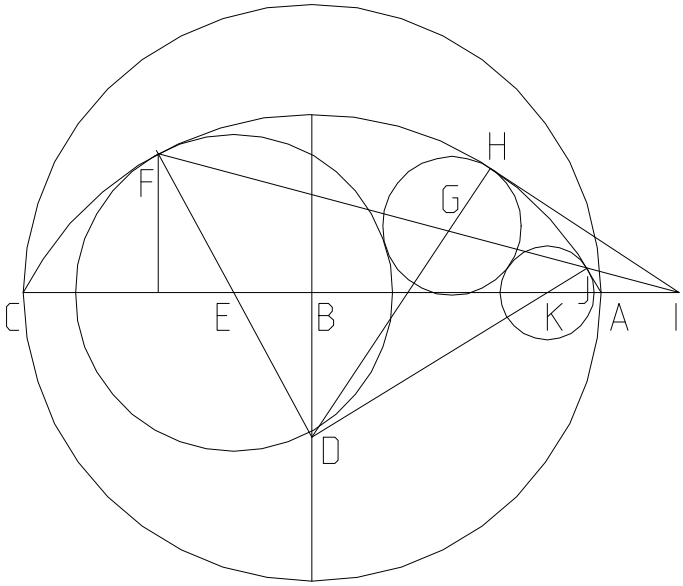
Convex_Radius $\equiv 1.5$

Center_Difference $\equiv 1.84$

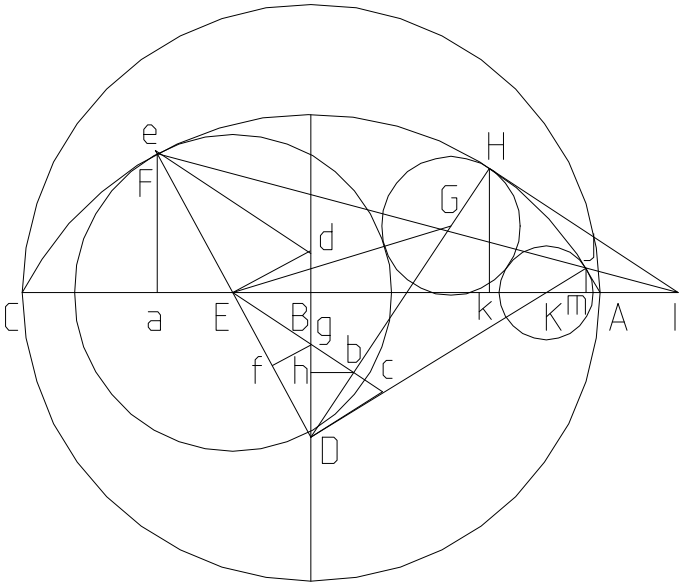
Power_Line_Tangent $\equiv \frac{1}{3}$ Given as Fraction < 1.




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

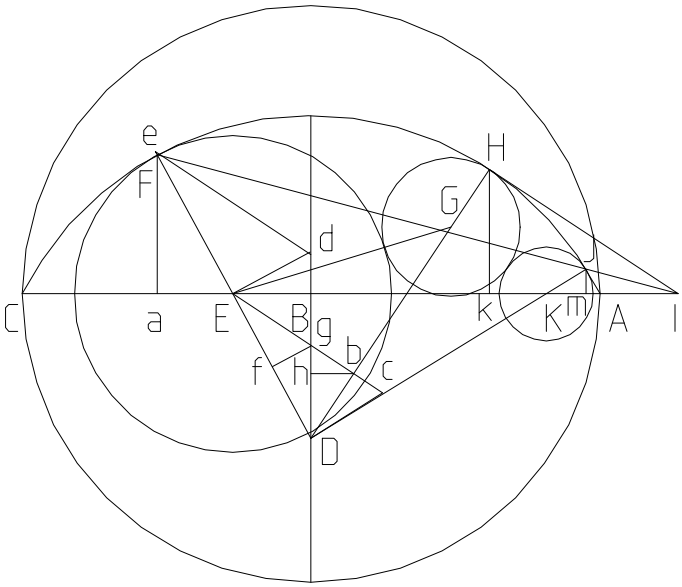
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \qquad Dc := \frac{Db \cdot DE}{Eb} \qquad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \qquad Ee := \frac{DE \cdot Ed}{Dc} \qquad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \qquad bg := Eb - Eg \qquad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \qquad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \qquad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \qquad Ik := \frac{Hk^2}{Mk} \qquad HI := \sqrt{Hk^2 + Ik^2}$$

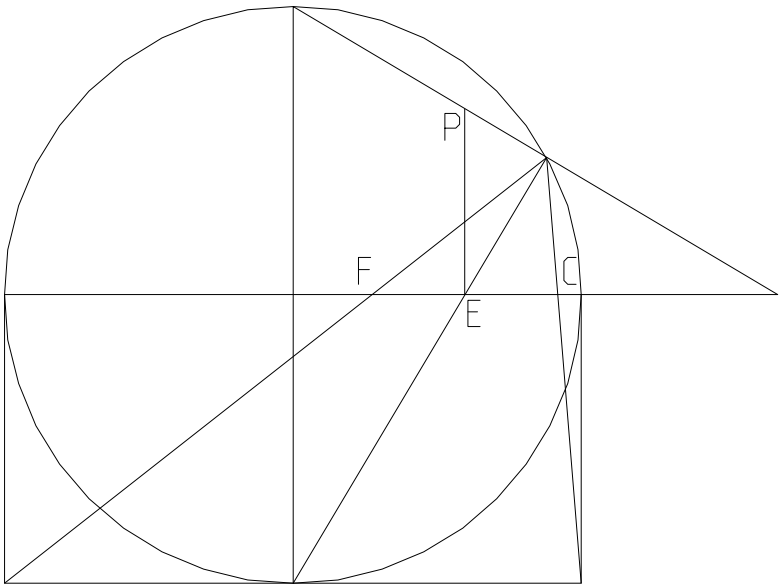
$$Ea := \frac{BE \cdot EF}{DE} \qquad Ba := BE + Ea \qquad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \qquad FI := \sqrt{Ia^2 + Fa^2} \qquad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \qquad JK := \frac{CD \cdot Jm}{BD + Jm}$$

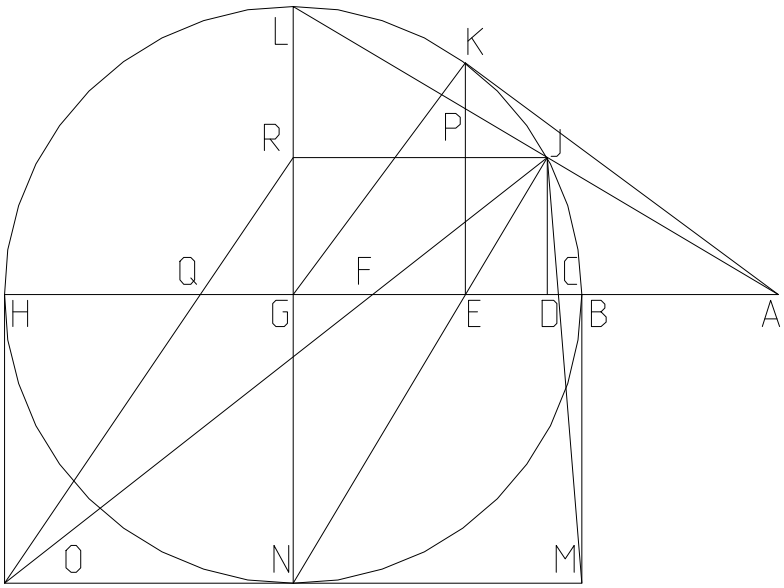
$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.




I found this little sketch in my notebook and have no idea of when I did it or why.

Does CF always equal EP?




$\Delta := 100 \quad \delta := 1..\Delta - 1 \quad BG := 10 \quad GH := BG$

$GN := BG \quad BD_\delta := \frac{BG}{\Delta}\cdot\delta \quad DG_\delta := BG - BD_\delta$

$DH_\delta := GH + DG_\delta \quad DJ_\delta := \sqrt{BD_\delta \cdot DH_\delta}$

$EG_\delta := \frac{DG_\delta \cdot GN}{GN + DJ_\delta} \quad BE_\delta := BG - EG_\delta$

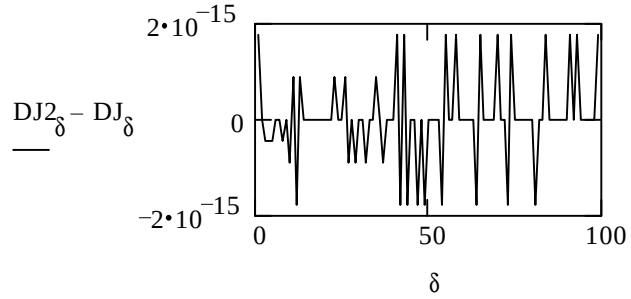
$EH_\delta := EG_\delta + GH \quad EK_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

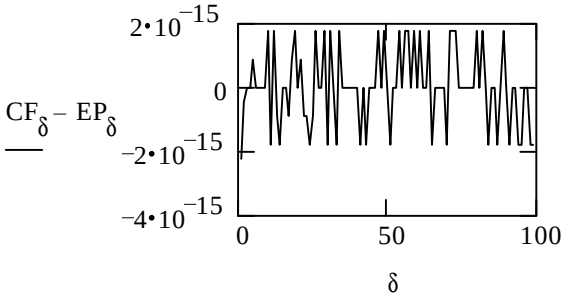
$AE_\delta := \frac{(EK_\delta)^2}{EG_\delta} \quad AB_\delta := AE_\delta - BE_\delta$

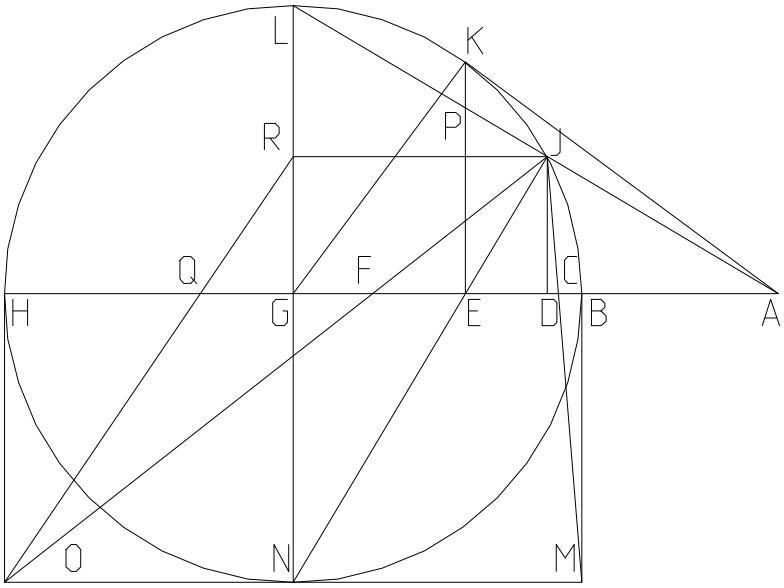
$GL := BG \quad AD_\delta := AB_\delta + BD_\delta$

$AG_\delta := AB_\delta + BG \quad DJ2_\delta := \frac{GL \cdot AD_\delta}{AG_\delta}$

$EP_\delta := \frac{GL \cdot AE_\delta}{AG_\delta} \quad GR_\delta := DJ_\delta \quad NO := BG \quad NR_\delta := GN + GR_\delta \quad GQ_\delta := \frac{NO \cdot GR_\delta}{NR_\delta} \quad CF_\delta := 2 \cdot GQ_\delta$






Reducing both by the symbolic processor leaves a little.

$$BG := 10 \qquad BD := \frac{BG}{13}$$

$$CF := 2\cdot BG\cdot\sqrt{BD}\cdot\frac{\sqrt{2\cdot BG - BD}}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)}$$

$$EP := -2\cdot BG\cdot\frac{\left(-BG\cdot\sqrt{BD}\cdot\sqrt{2\cdot BG - BD} - 2\cdot BG\cdot BD + BD^2\right)}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)^2}$$

$EP = 5.556 \qquad EP - CF = 0$

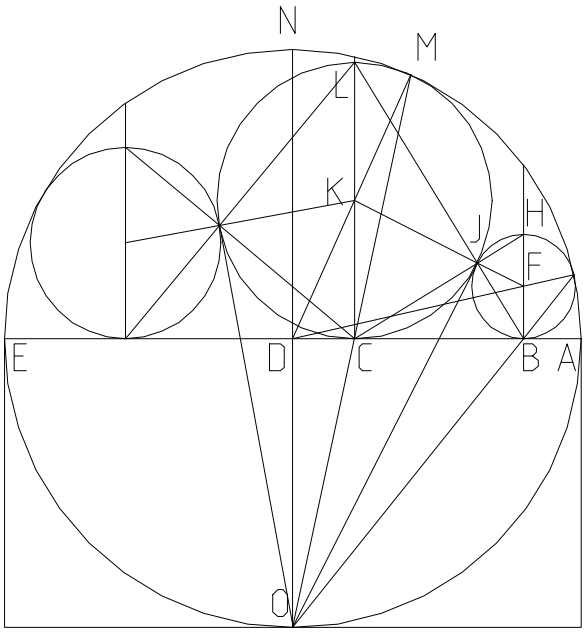
And if I divide one by the other and reduce;

$$\frac{\left[BG\cdot\sqrt{2\cdot BG - BD} + 2\cdot BG\cdot\sqrt{BD} - BD^{\left(\frac{3}{2}\right)}\right]}{\left[\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)\cdot\sqrt{2\cdot BG - BD}\right]} = 1$$

This is another figure that I had sketched out last year but never got around to writing down.

Given a Circle, place the next on the diameter.

I tried to reduce this series with the symbolic processor, but it is having trouble, at some point it switches AC for EC and I get the other circle.



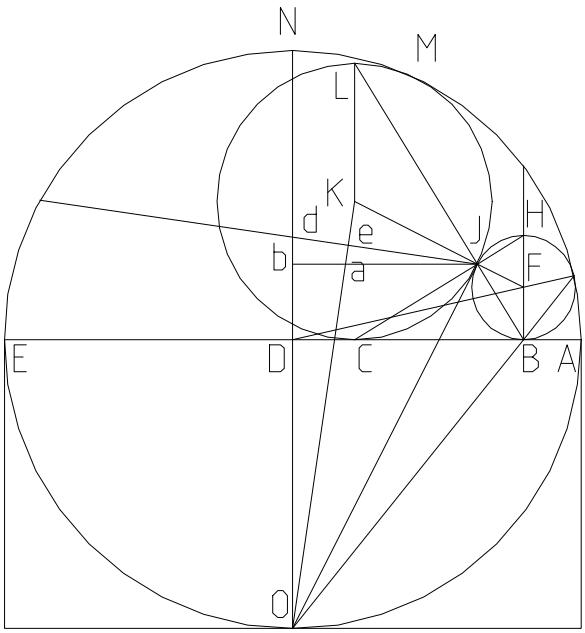

$$AD := Radius \quad AE := 2 \cdot AD \quad AC := \frac{AE}{N}$$

$$CD := AD - AC \quad DO := AD \quad CO := \sqrt{DO^2 + CD^2}$$

$$NO := AE \quad MO := \frac{DO \cdot NO}{CO} \quad CM := MO - CO$$

$$CK := \frac{DO \cdot CM}{MO} \quad KO := \sqrt{CD^2 + (DO + CK)^2}$$

$$JK := CK$$

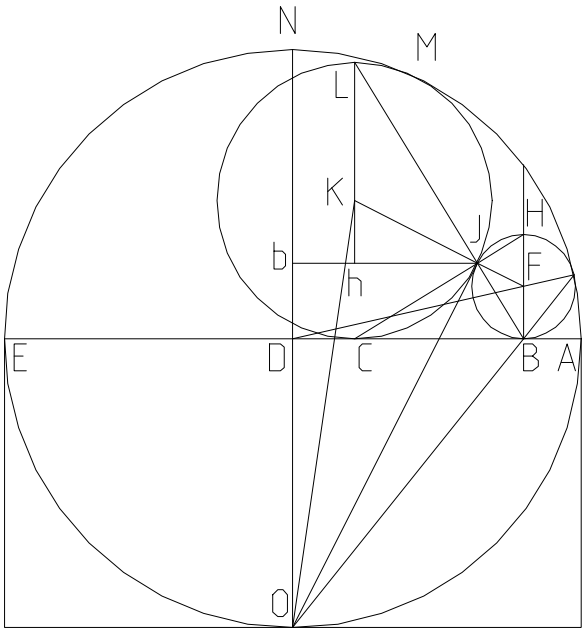



$$Ke := \frac{JK^2}{KO} \quad Oe := KO - Ke \quad de := \frac{CD \cdot Oe}{DO + CK}$$

$$Je := \sqrt{JK^2 - Ke^2}$$

$$Jd := de + Je \quad bd := \frac{CD \cdot Jd}{KO}$$

$$Od := \frac{KO \cdot de}{CD} \quad Db := Od - DO - bd$$




$Kh := CK - Db$

$Lh := CK + Kh$ $\quad FJ := \frac{JK \cdot Db}{Lh}$

$N \equiv \frac{10}{1}$ $\quad FJ = 3.965$

$Radius \equiv 108$ $\quad CK = 19.44$

And from the other side;

$N \equiv \frac{10}{9}$ $\quad FJ = 51.53$

$Radius \equiv 108$ $\quad CK = 19.44$

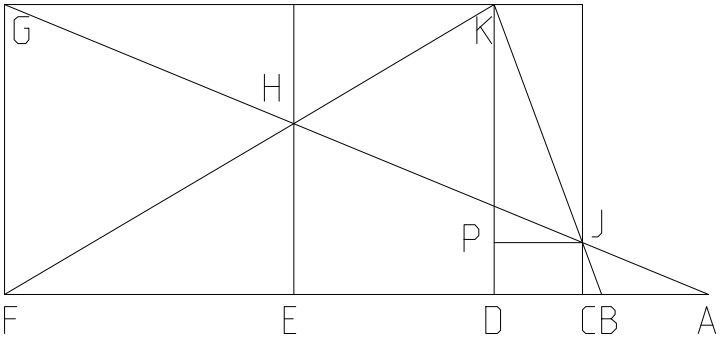
This figure might be recognized as the similarity point for Gemini root projection.

Given AC, CF, and that

$$AC = -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC+CF}-BC\right)}{\left(-CF+2\cdot\sqrt{BC}\cdot\sqrt{BC+CF}-2\cdot BC\right)}$$

find BC.

N can be any value whatever, except 0.




$CF := 216 \quad AC := 47.29 \quad N := 100000$

$CE := \frac{CF}{2} \quad AF := AC + CF \quad AE := AC + CE$

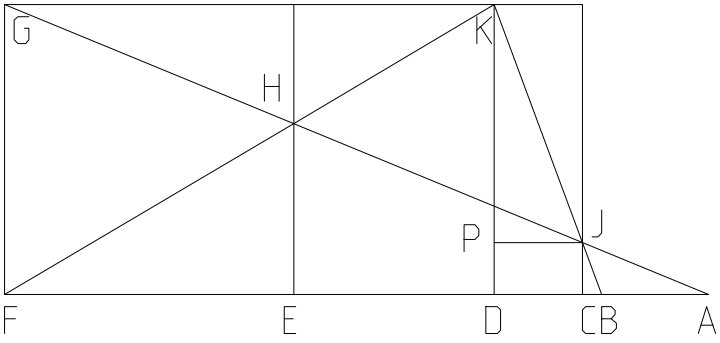
$FG := N \quad EH := \frac{FG\cdot AE}{AF} \quad EF := CE$

$DF := \frac{EF\cdot FG}{EH} \quad CJ := \frac{FG\cdot AC}{AF} \quad DP := CJ$

$CD := CF - DF \quad DK := FG \quad KP := DK - DP$

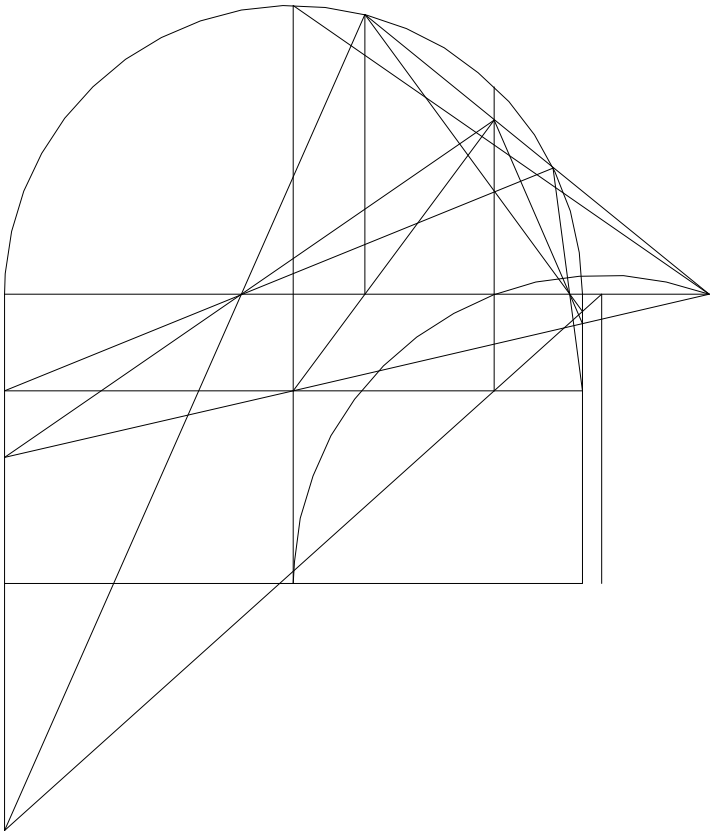
$BD := \frac{CD\cdot DK}{KP} \quad BD = 40.089$

$BC := BD - CD \quad BC = 7.201$



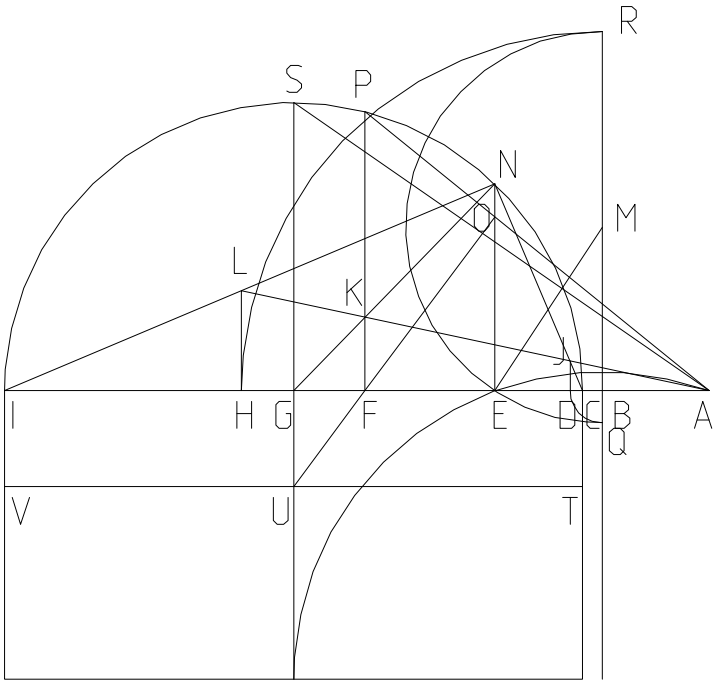

$$AC_f := -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC+CF}-BC\right)}{\left(-CF+2\cdot\sqrt{BC}\cdot\sqrt{BC+CF}-2\cdot BC\right)}$$

$$\frac{AC_f}{AC} = 1$$



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

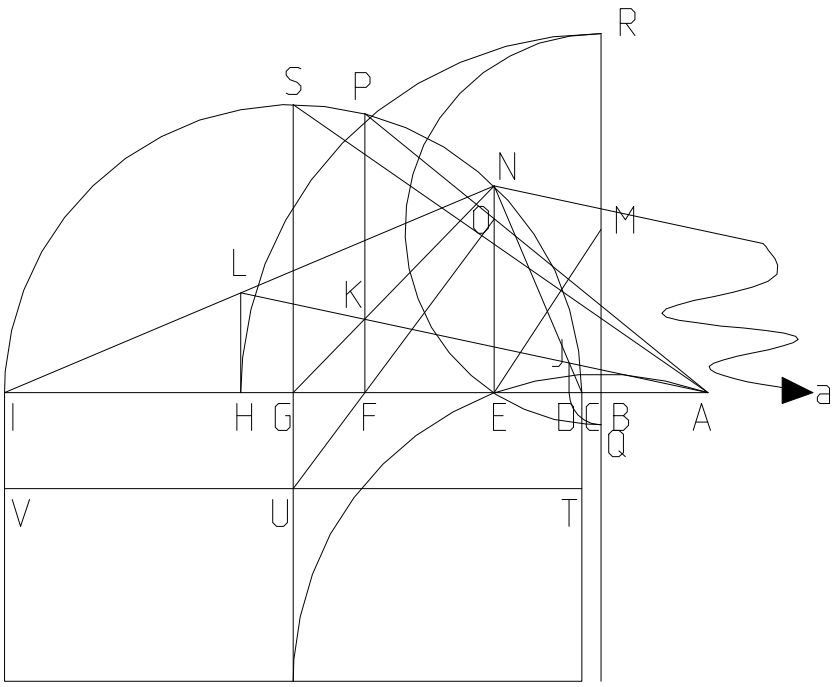
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



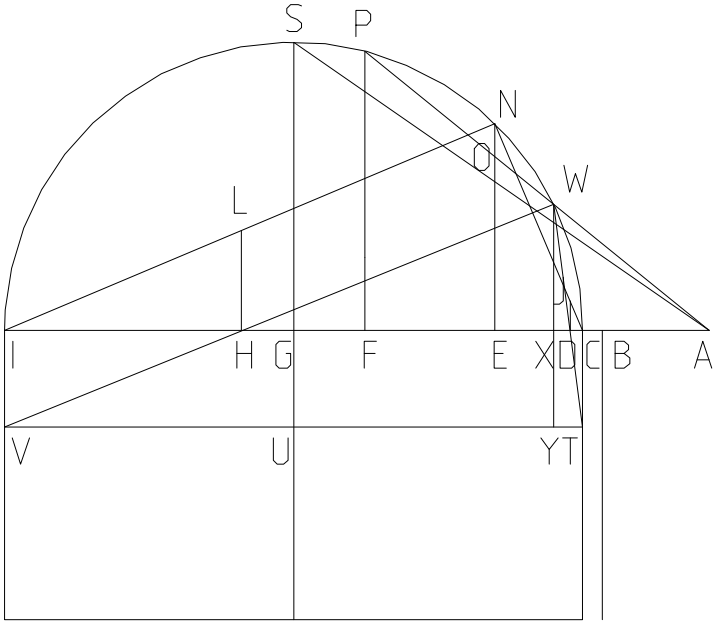

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

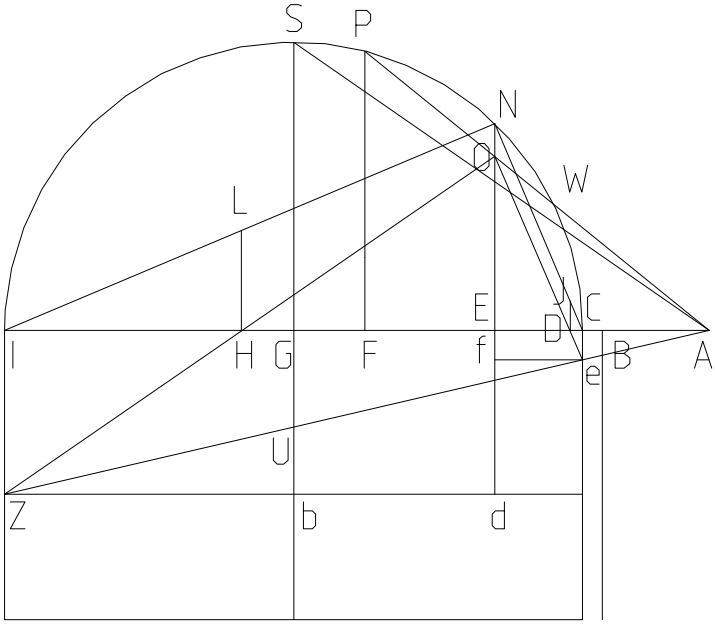
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

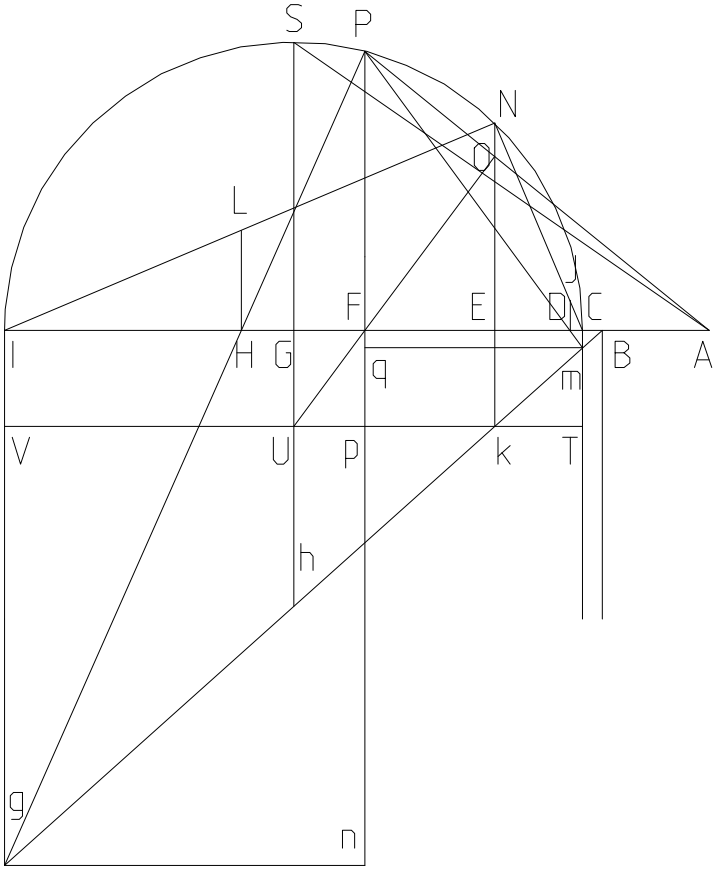
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



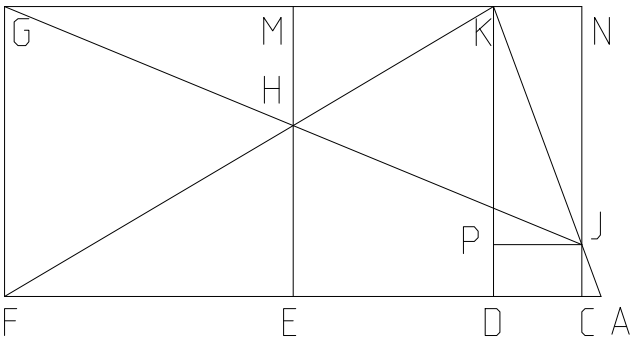

$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

Given CF and CD such that $CD = \sqrt{AF \cdot AC} - AC$ find AC.




$CF := 216 \quad CD := 32.89 \quad CE := \frac{CF}{2} \quad EF := CE$

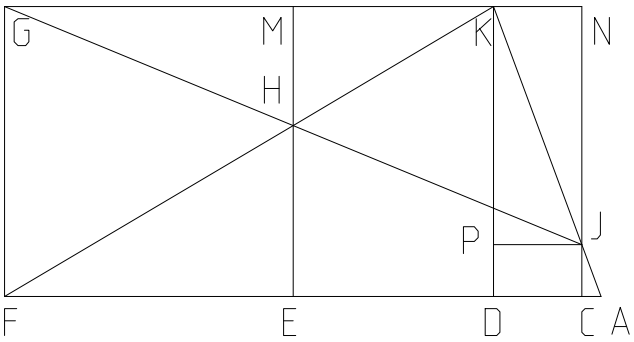
Except for 0, N can be any value.

$N := 108 \quad FG := N \quad DK := N \quad DF := CF - CD$

$EH := \frac{DK \cdot EF}{DF} \quad EM := FG \quad HM := EM - EH$

$CN := FG \quad HM := EM - EH \quad GN := CF$

$GM := EF \quad JN := \frac{HM \cdot GN}{GM} \quad JC := CN - JN$




$KP := JN \quad JP := CD \quad AD := \frac{JP \cdot DK}{KP}$

$AC := AD - CD \quad AC = 7.201$

$AF := AC + CF$

$\frac{\sqrt{AF \cdot AC} - AC}{CD} = 1 \qquad \frac{\sqrt{AF \cdot AC}}{AD} = 1$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

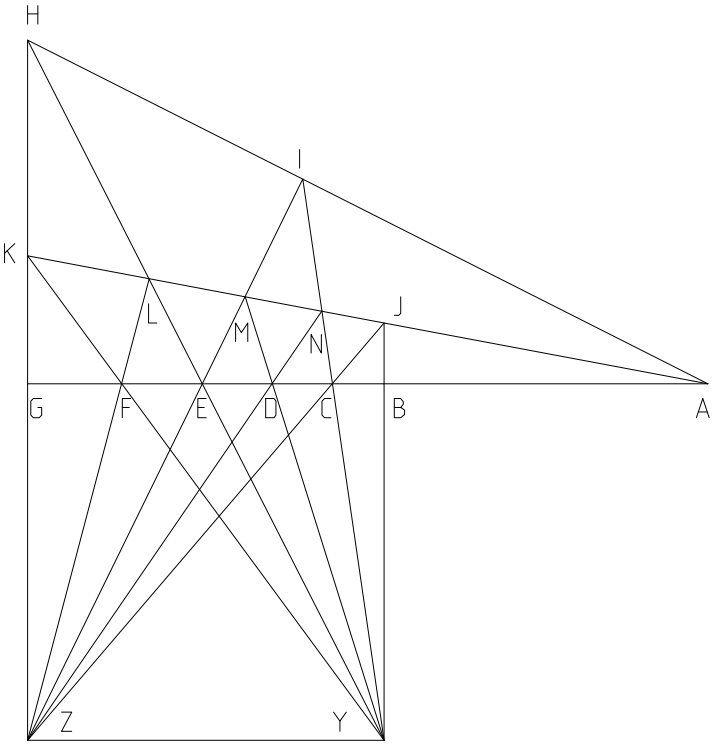
Process Summary will use $\underline{th}$ root series for example.

$AG := 3^5$ $AB := 1$ $AE := 3^3$

$BG := AG - AB$ $GZ := BG$ $YZ := BG$

$BY := BG$ $BE := AE - AB$ $EG := BG - BE$

$$GH := \frac{BY \cdot EG}{BE}$$



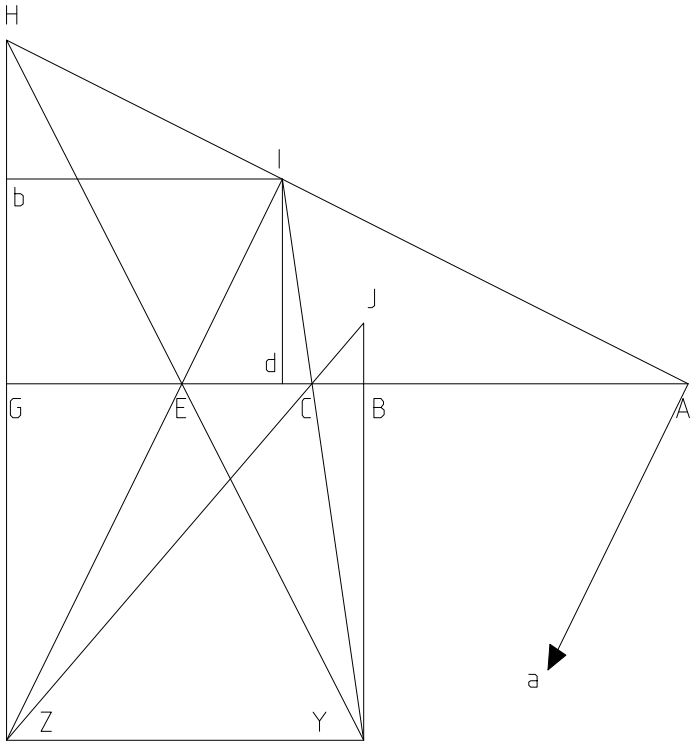

$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

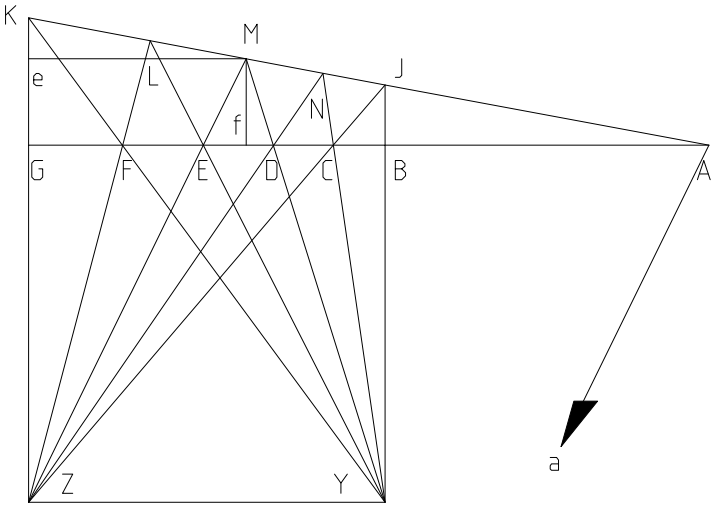
$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$



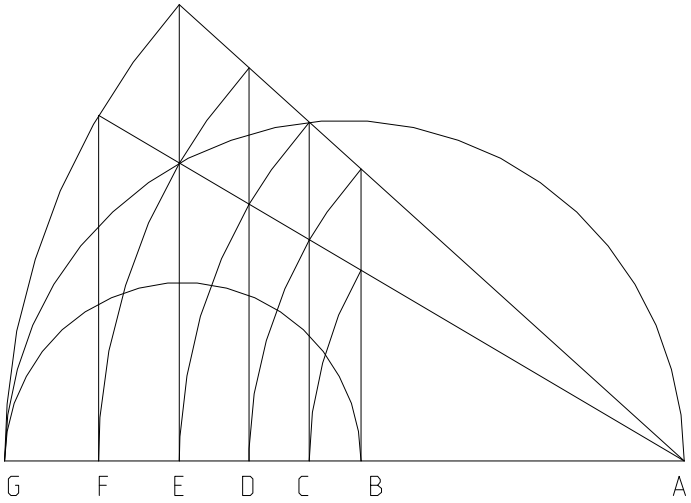



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



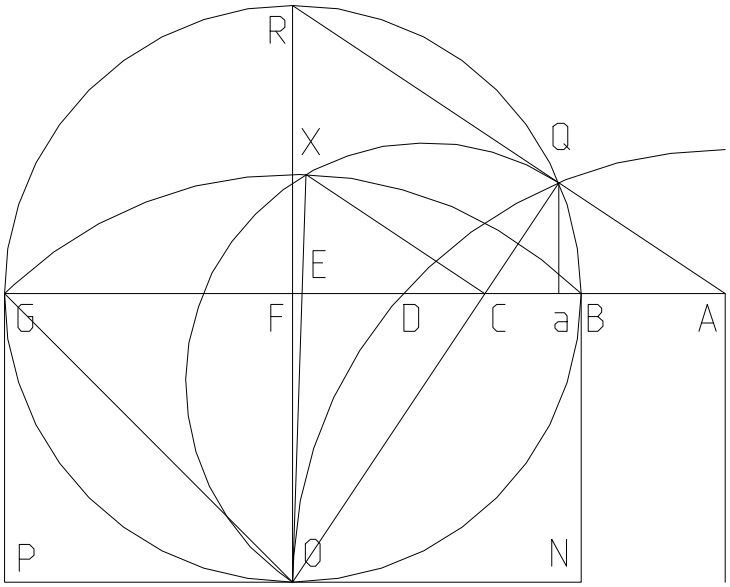

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

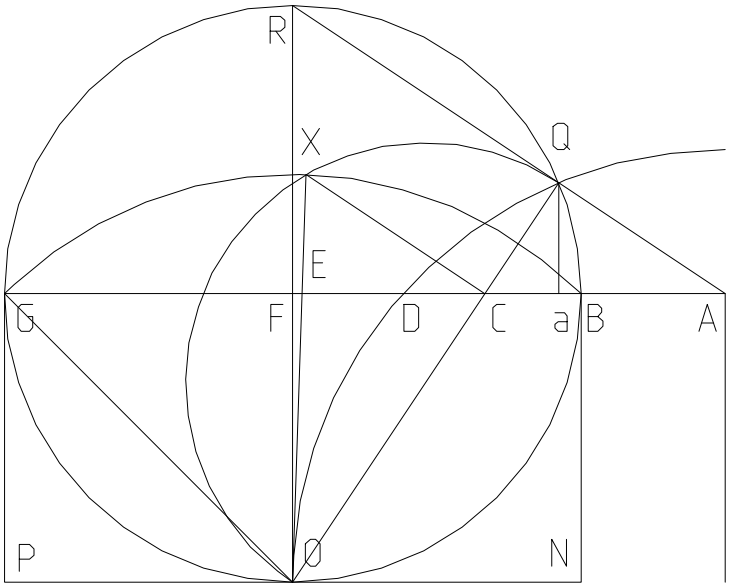
$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.




04_29_96.MCD

What is CX?

I have had so much back work to catch up on I post dated a couple.

$AB := 54 \quad AG := 270$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FO := BF$

$AF := AB + BF \quad FR := BF$

$AR := \sqrt{AF^2 + FR^2} \quad AQ := \frac{AB \cdot AG}{AR}$

$Aa := \frac{AF \cdot AQ}{AR} \quad Qa := \frac{FR \cdot AQ}{AR}$

$Fa := AF - Aa \quad OQ := \sqrt{Fa^2 + (FO + Qa)^2}$

$CQ := \frac{OQ \cdot Qa}{FO + Qa} \quad OX := \sqrt{BF^2 + FO^2}$

$CO := OQ - CQ \qquad CQ = 49.923$

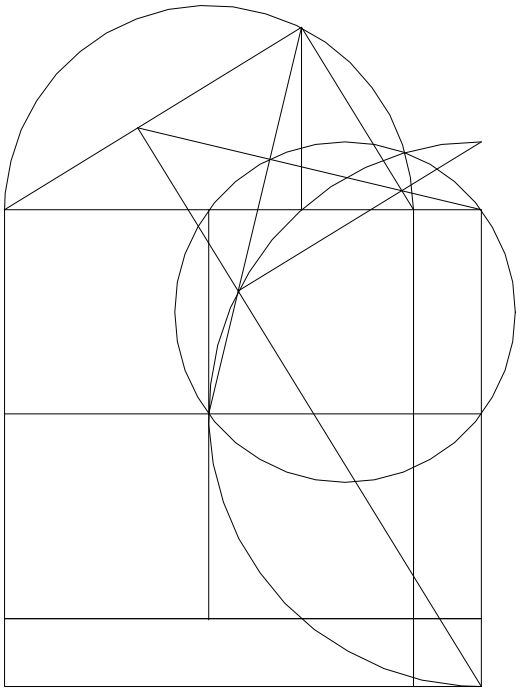
$\frac{AG - AB}{AG + AB} \cdot \frac{\sqrt{2} \cdot (AG \cdot AB)}{\sqrt{AB^2 + AG^2}} = 49.923$

$CX := \sqrt{CQ \cdot CO} \qquad CX = 80.498$

$CX := \sqrt{AB} \cdot \sqrt{AG} \cdot \frac{(AG - AB)}{(AB + AG)}$

$CX2 := AB^3 \cdot \frac{AG}{(AB + AG)^2} - 2 \cdot AB^2 \cdot \frac{AG^2}{(AB + AG)^2} + AB \cdot \frac{AG^3}{(AB + AG)^2} \qquad \sqrt{CX2} = 80.498$

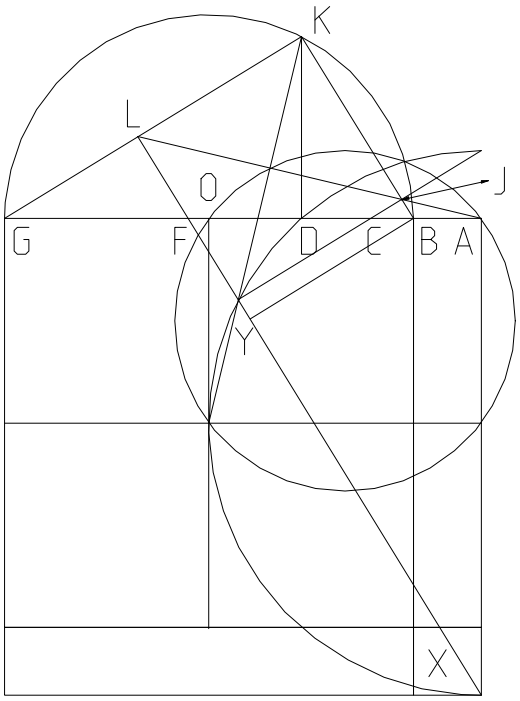
It is the square root of the intersection of two cubic equations. I wonder what the geometry of those equations would look like?



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.




$N = 4$ $Root = 4$ $M = 1$ $BG := N$ $AB := M$

$AG := AB + BG$ $BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}}$ $AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

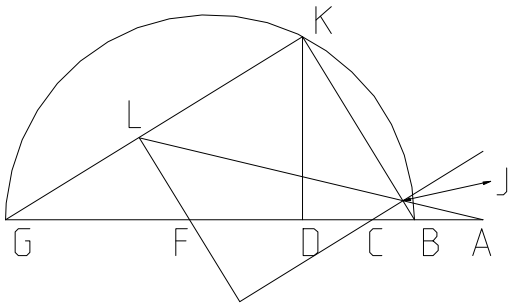
$BC := AC - AB$ $FG := AG - AF$ $FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX}$ $BD := \frac{FY \cdot BG}{FX}$ $AD := BD + AB$

$DG := AG - AD$ $DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2}$ $GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG}$ $GL := \frac{GK \cdot FG}{BG}$



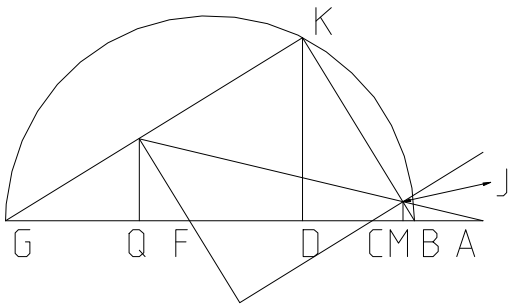

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

On the left is the first and last of the series, on the right is the entire series.

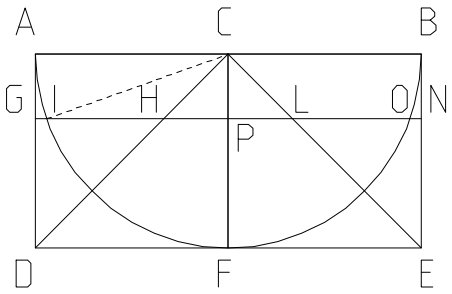
| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$

Imagine a cylinder about the axis CF. The area of the circular ribbon GI is always equal to the area of the circle HL cut by the cone CDF. Galileo uses this to "prove" that a point is equal to a circle. He will first states that the volume of the cone, CDE is equal to volume of the bowl ADF. Because they both degenerate, one to a point, C, the other to a circle with radius AC, the point must be equal to the circle. He further states, since this is the case, all circles are equal to each other.

"Now, since as these solids diminish equality is maintained between them up to the very last, we are justified in saying that, at the extreme and final end of this diminution, they are still equal and th one is not infinitely greater than the other. It appears therefore that we may equate the circumference of a large circle to a single point. And this which is true of the solids is true also of the surfaces"

Galileo took this example from "twelfth proposition of the second book o*De centro gravitatis solidorem* by the Archimedes of our age, Luca Valerio."




$t := 0..1 \qquad N := 1 \cdot 10^{153}$

$AD := 5 \quad CI := AD \quad CP_0 := 0 \quad CP_1 := \frac{AD}{N} \quad CI := AD \quad GP := AD$

$IP_t := \sqrt{CI^2 - (CP_t)^2} \quad HP_t := CP_t \quad GP^2 - \left[ (IP_t)^2 + (HP_t)^2 \right]$

$\begin{matrix} 0 \\ 0 \end{matrix}$

$GN := 2 \cdot AD \quad IO_t := 2 \cdot IP_t \quad HL_t := 2 \cdot HP_t \quad GN^2 - \left[ (IO_t)^2 + (HL_t)^2 \right] \quad \pi \cdot GN^2 - \left[ \pi \cdot (IO_t)^2 + \pi \cdot (HL_t)^2 \right]$

$\begin{matrix} 0 \\ 0 \end{matrix} \qquad \begin{matrix} 0 \\ 0 \end{matrix}$

$\left[ \pi \cdot GN^2 - \pi \cdot (IO_t)^2 \right]$

$\begin{matrix} 0 \\ 0 \end{matrix}$

$\pi \cdot (HL_t)^2$

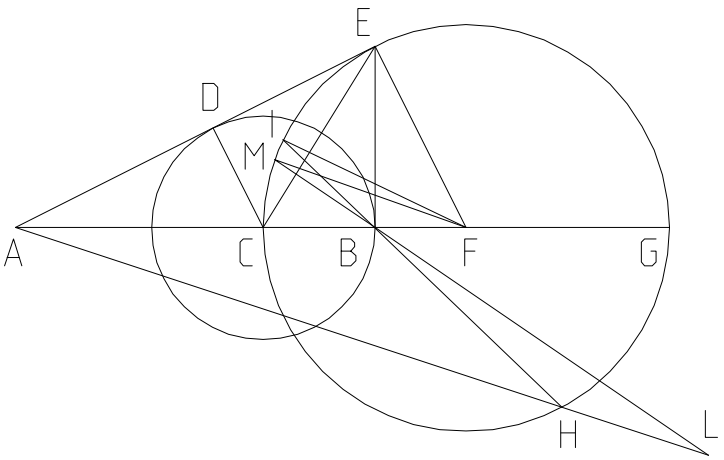
$\begin{matrix} 0 \\ 3.142 \cdot 10^{-304} \end{matrix}$

$\left[ \pi \cdot GN^2 - \pi \cdot (IO_t)^2 \right] - \pi \cdot (HL_t)^2$

$\begin{matrix} 0 \\ -3.142 \cdot 10^{-304} \end{matrix}$

I will completely agree with his analysis, providing that he does not change the subject. They are both still equal in volume, in his original statement, or area in his proof, which is 0. He does not se what he has done, he has in fact exonerated Euclid. The boundary is not part of the figure. By the lool of the text, Galileo was completely awestruck by this simple little miracle. Galileo was not the first to change coordinate systems of reference to make it appear that he won a bad argument. Plato used it to stun many audiences in his dialogues. Einstein used it to bend space. Galileo still had a "ball bearing" metaphor for geometry.

Given a straight line divided into unequal parts which bear to each other any ratio whatever, to describe a circle such that two straight lines drawn from the ends of the given line to any point on the circumference will be each other the same ratio as the two parts of the given line, thus making those lines which are drawn from the same terminal points homologous.




$AB := 100 \qquad CB := \frac{AB}{2 \cdot 2} \qquad CB < \frac{AB}{2} = 1$

$AC := AB - CB \qquad CD := CB$

$AD := \sqrt{AC^2 - CD^2}$

$BE := \frac{CD \cdot AB}{AD} \qquad AE := \frac{AC \cdot AB}{AD}$

$AF := \frac{AC \cdot AE}{AD} \qquad FE := \frac{CD \cdot AF}{AC}$

$FC := AF - AC \qquad FE = FC = 1$

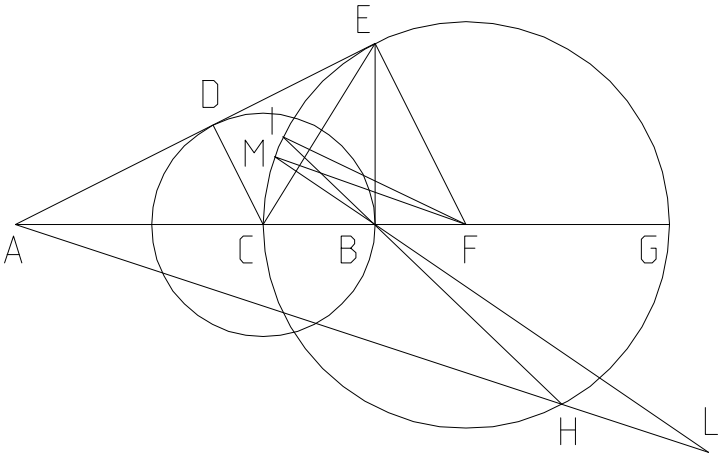
$FE = FC = 1 \qquad FE = AF - AC = 1 \qquad FE = AC \cdot \frac{AE}{AD} - AC = 1 \qquad FE = AC^2 \cdot \frac{AB}{AD^2} - AC = 1$

$FE = -AC \cdot \frac{\left(AC \cdot AB - AC^2 + CD^2\right)}{\left(-AC^2 + CD^2\right)} = 1 \qquad FE = -AC \cdot \frac{\left(AC \cdot AB - AC^2 + CB^2\right)}{\left(-AC^2 + CB^2\right)} = 1 \qquad FE = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$

$CD \cdot \frac{AF}{AC} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1 \qquad CD \cdot \frac{AE}{AD} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1 \qquad CD \cdot AC \cdot \frac{AB}{AD^2} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$

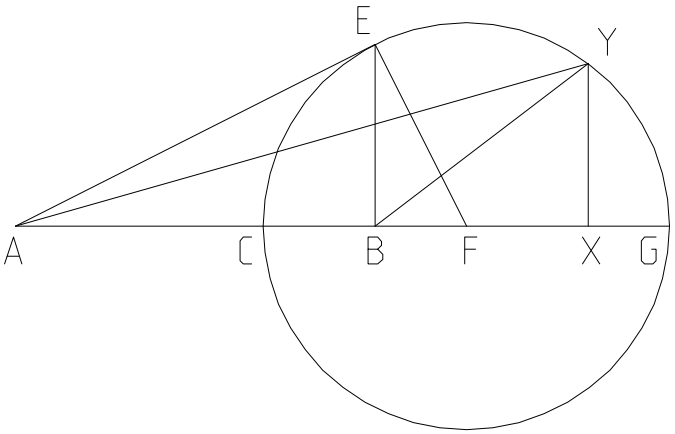
$CD \cdot AC \cdot \frac{AB}{\left(AC^2 - CD^2\right)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1 \qquad CB \cdot AC \cdot \frac{AB}{\left(AC^2 - CB^2\right)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$

$CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = CB \cdot \frac{(AB - CB)}{(AB - 2 \cdot CB)} = 1$




$$\frac{AC}{CB}=\frac{AE}{BE}=1 \qquad \frac{AE}{BE}=AC\cdot\frac{AB}{(AD\cdot BE)}=1$$

$$AC\cdot\frac{AB}{(AD\cdot BE)}=\frac{AC}{CD}=1 \qquad \frac{AC}{CD}=\frac{AC}{CB}=1$$



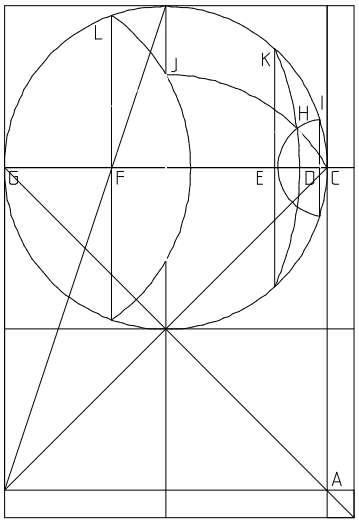

$CG := 2\cdot FC \qquad CX := \frac{CG}{.3} \qquad GX := CG - CX$

$XY := \sqrt{GX\cdot CX} \qquad AX := AC + CX$

$BX := CX - CB \qquad AY := \sqrt{AX^2 + XY^2}$

$BY := \sqrt{BX^2 + XY^2}$

$$\frac{AC}{CB}=\frac{AY}{BY}=1$$



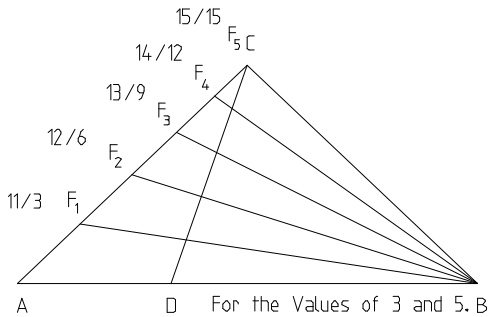

# One Circle

**By John Clark**

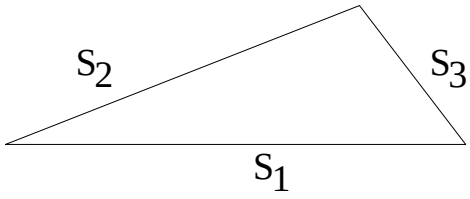
# One Square

# One Line

## And the Delian Quest



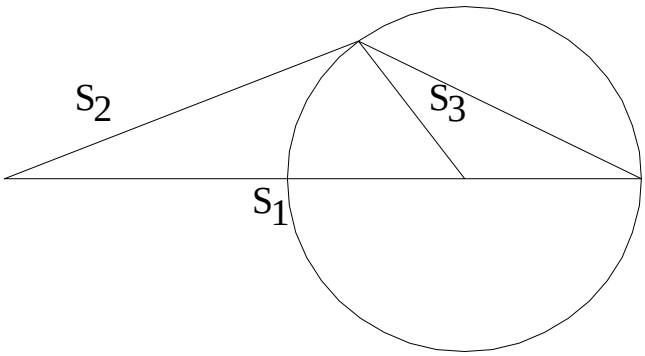

# 1997




Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?

$n := 1 .. 3$

$S_1 := \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad S_2 := \begin{bmatrix} b \\ c \\ a \end{bmatrix} \quad S_3 := \begin{bmatrix} c \\ a \\ b \end{bmatrix}$




$Is\_This\_a\_Triangle := \left(S_{1_1} + S_{2_1} > S_{3_1}\right) \cdot \left(S_{1_1} + S_{3_1} > S_{2_1}\right) \cdot \left(S_{2_1} + S_{3_1} > S_{1_1}\right)$

As was learned in school, the area of a triagle is given $\frac{1}{2} \cdot B \cdot H$.

From 04_02_97.MCD I show that

$$H_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

And since $B := S_1$

Area is defined as $A_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{4}$

| $\frac{1 \cdot B_n \cdot H_n}{2} - A_n$ | $A_n$ | $H_n$ |
|---|---|---|
| 0 | 2.905 | 2.905 |
| 0 | 2.905 | 1.936 |
| 0 | 2.905 | 1.452 |

What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.

$Acute_n := if\left[\sqrt{\left(S\,1_n\right)^2 + \left(S\,2_n\right)^2} > S\,3_n, 1, 0\right]$ $Acute_n$

| |
|---|
| 0 |
| 1 |
| 1 |

$Acute2_n := if\left[\sqrt{\left(S\,1_n\right)^2 + \left(S\,3_n\right)^2} > S\,2_n, 1, 0\right]$ $Acute2_n$

| |
|---|
| 1 |
| 0 |
| 1 |

| $H_n$ | $S\,1_n$ | $S\,2_n$ | $S\,3_n$ |
|---|---|---|---|
| 2.905 | 2 | 3 | 4 |
| 1.936 | 3 | 4 | 2 |
| 1.452 | 4 | 2 | 3 |

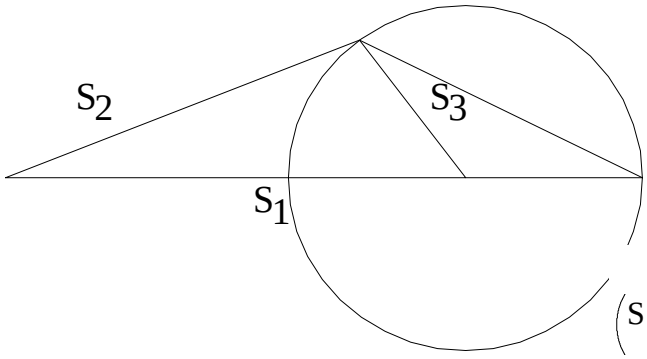
if we halve $\angle S\,1\,S\,2$ $\frac{\left(S\,1_n + S\,2_n\right)\cdot H_n}{2} - A_n$

| |
|---|
| 4.357 |
| 3.873 |
| 1.452 |

if we halve $\angle S\,1\,S\,3$ $\frac{\left(S\,1_n + S\,3_n\right)\cdot H_n}{2} - A_n$

| |
|---|
| 5.809 |
| 1.936 |
| 2.179 |

Is_This_a_Triangle= 1




Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.

$\left(S\,2_n > S\,3_n\right) - \left[\frac{\left(B_n + S\,2_n\right)\cdot H_n}{2} - A_n > \frac{\left(B_n + S\,3_n\right)\cdot H_n}{2} - A_n\right]$

| |
|---|
| 0 |
| 0 |
| 0 |

Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.

$$H_n = \frac{\sqrt{S1_n + S2_n + S3_n} \cdot \sqrt{-S1_n + S2_n + S3_n} \cdot \sqrt{S1_n - S2_n + S3_n} \cdot \sqrt{S1_n + S2_n - S3_n}}{2 \cdot S1_n}$$

$S_1, S_2$ and $\sqrt{S_1^2 + S_2^2} > S_3$ find $S_3$

$Acute_n$ $Is\_This\_a\_Triangle = 1$

| $Acute_n$ |
|---|
| 0 |
| 1 |
| 1 |

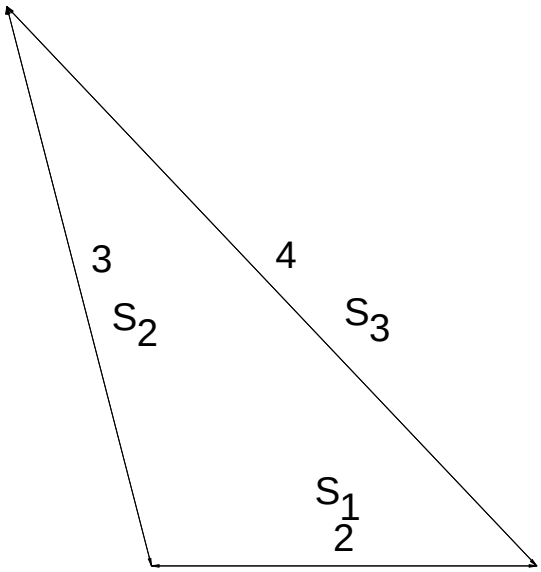
$$S4_n := \sqrt{(S2_n)^2 - (H_n)^2}$$

$$SX_n := if(Acute_n, S1_n - S4_n, S1_n + S4_n)$$

$a \equiv 2$ $b \equiv 3$ $c \equiv 4$ ← Plug your values in here.

$$S3_n := \sqrt{(H_n)^2 + (SX_n)^2}$$

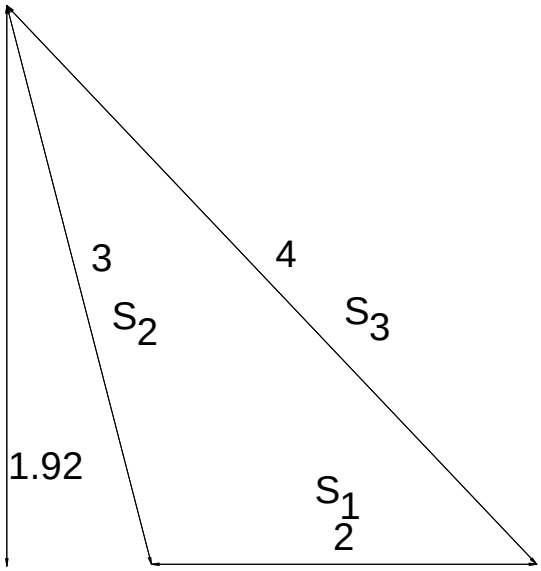
| $S1_n$ | $S2_n$ | $S3_n$ |
|---|---|---|
| 2 | 3 | 4 |
| 3 | 4 | 2 |
| 4 | 2 | 3 |



Given the base, one side and the height of a triangle, find the remaining side.

$S_1 := 3 \quad S_2 := 4 \qquad H := 1$

$$H = \frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1}$$




Just try asking Mathcad Plus 6 to solve for S3 in the previous equation.

$S_4 := \sqrt{S_2^2 - H^2} \qquad S_{Xa} := S_1 + S_4 \quad S_{Xb} := S_1 - S_4$

$S_4 = 3.873$

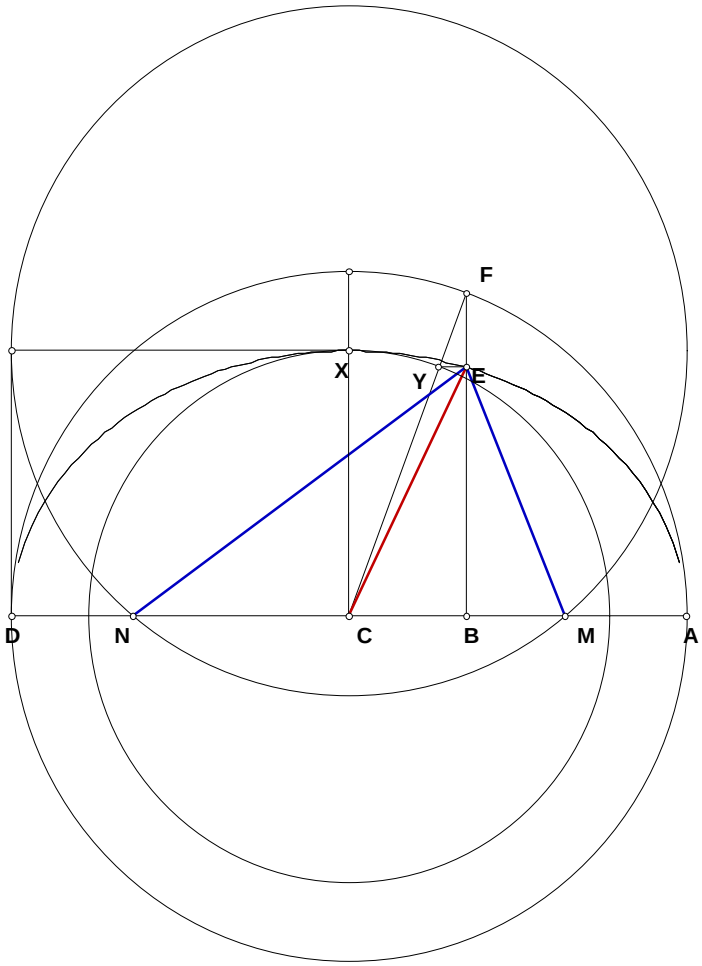
$S_{3a} := \sqrt{H^2 + S_{Xa}^2} \qquad S_{3a} = 6.945$

$S_{3b} := \sqrt{H^2 + S_{Xb}^2} \qquad S_{3b} = 1.327$

$S_4 < S_1 = 0$

# The Ellipse 09_11_97

Given that the major axis is AD and the minor axis is twice CX, derive the formula for the radius CE, the height BE, and the foci axis MN.




$$AD := 10 \quad AC := \frac{AD}{2} \quad R := 7 \quad \delta := 1 .. R - 1$$

$$AB_\delta := \delta \cdot \frac{AD}{R} \quad BD_\delta := AD - AB_\delta$$

$$CX := \frac{AC \cdot 3}{4} \quad CY := CX \quad CF := AC$$

$$BF_\delta := \sqrt{AB_\delta \cdot BD_\delta} \quad BC_\delta := \sqrt{CF^2 - \left(BF_\delta\right)^2}$$

$$BE_\delta := \frac{BF_\delta \cdot CY}{CF} \quad CE_\delta := \sqrt{\left(BE_\delta\right)^2 + \left(BC_\delta\right)^2}$$

$$CE := \frac{\sqrt{16 \cdot AB \cdot BD \cdot CX^2 + AD^4 - 4 \cdot AB \cdot BD \cdot AD^2}}{2 \cdot AD}$$

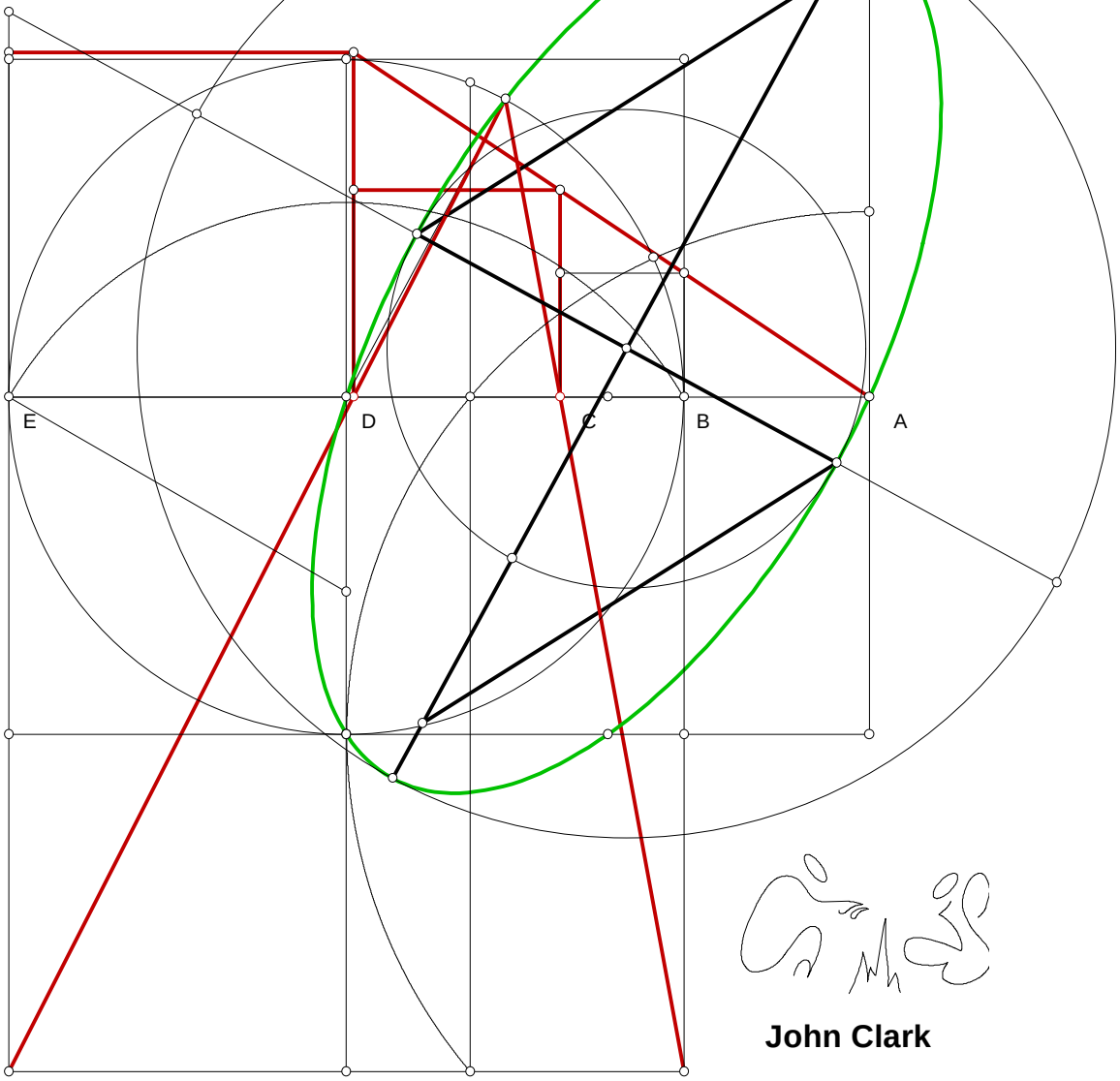
$$MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CX^2} \quad BE := 2 \cdot \sqrt{AB} \cdot \sqrt{BD} \cdot \frac{CX}{AD}$$

The length of the radius is given in CE, the height in BE, and the foci axis in MN.

# The Delian Quest

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$






**John Clark**

# The Delian Quest

## Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. The smattering of English is of no great consequence. The figure is Geometric Grammar, the equations are Algebraic Grammar.

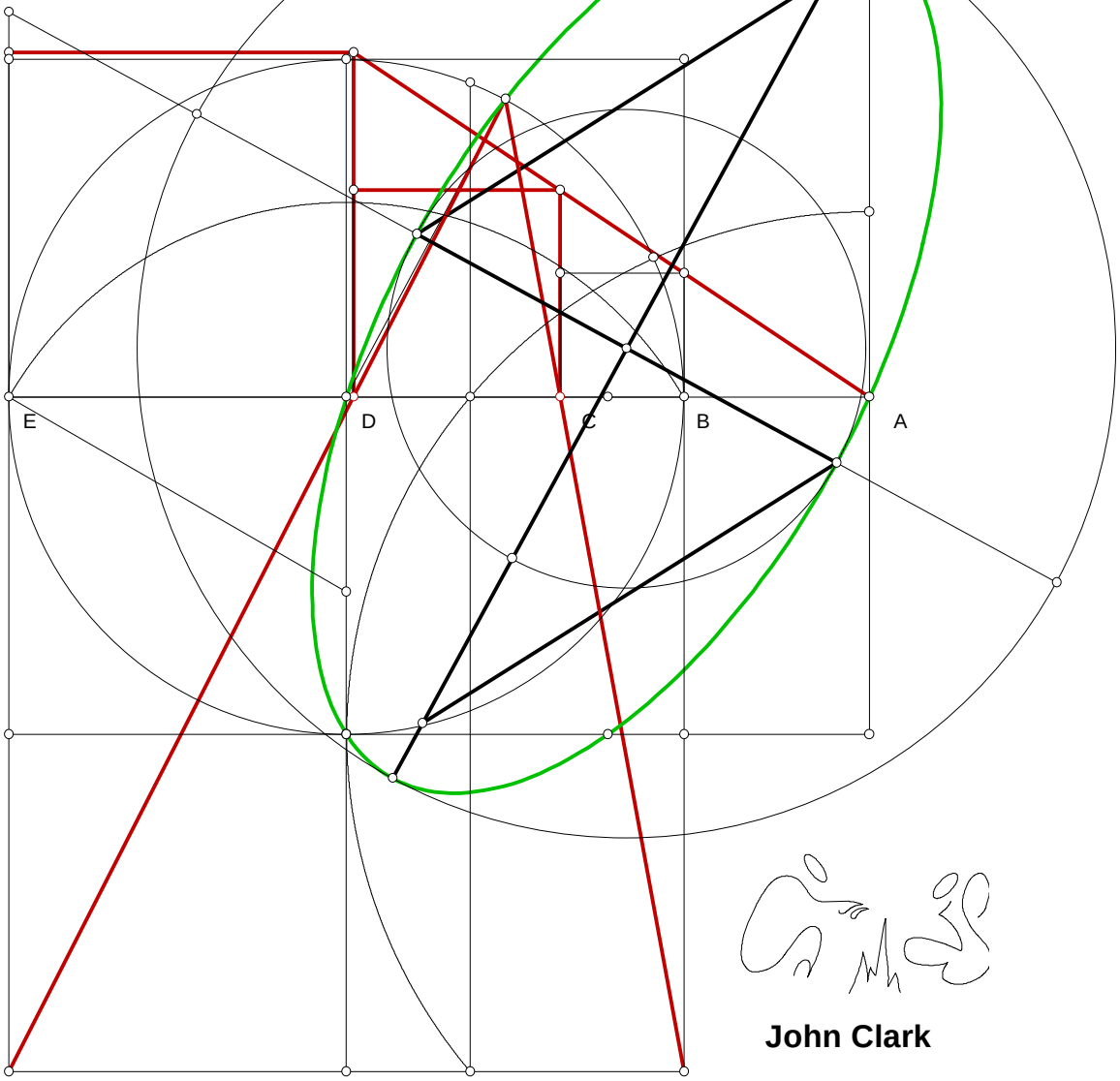
Place Introduction to logic here.

# The Delian Quest

## 1989

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$






John Clark

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

# THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

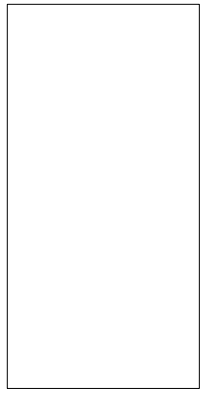


Plate 1

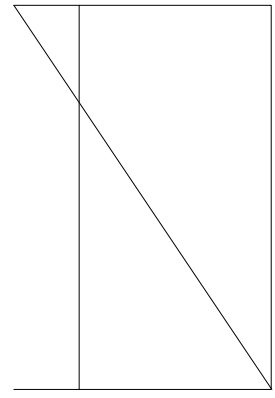


Plate 2

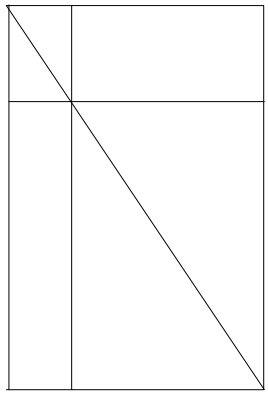


Plate 3

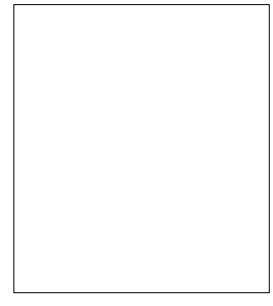


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

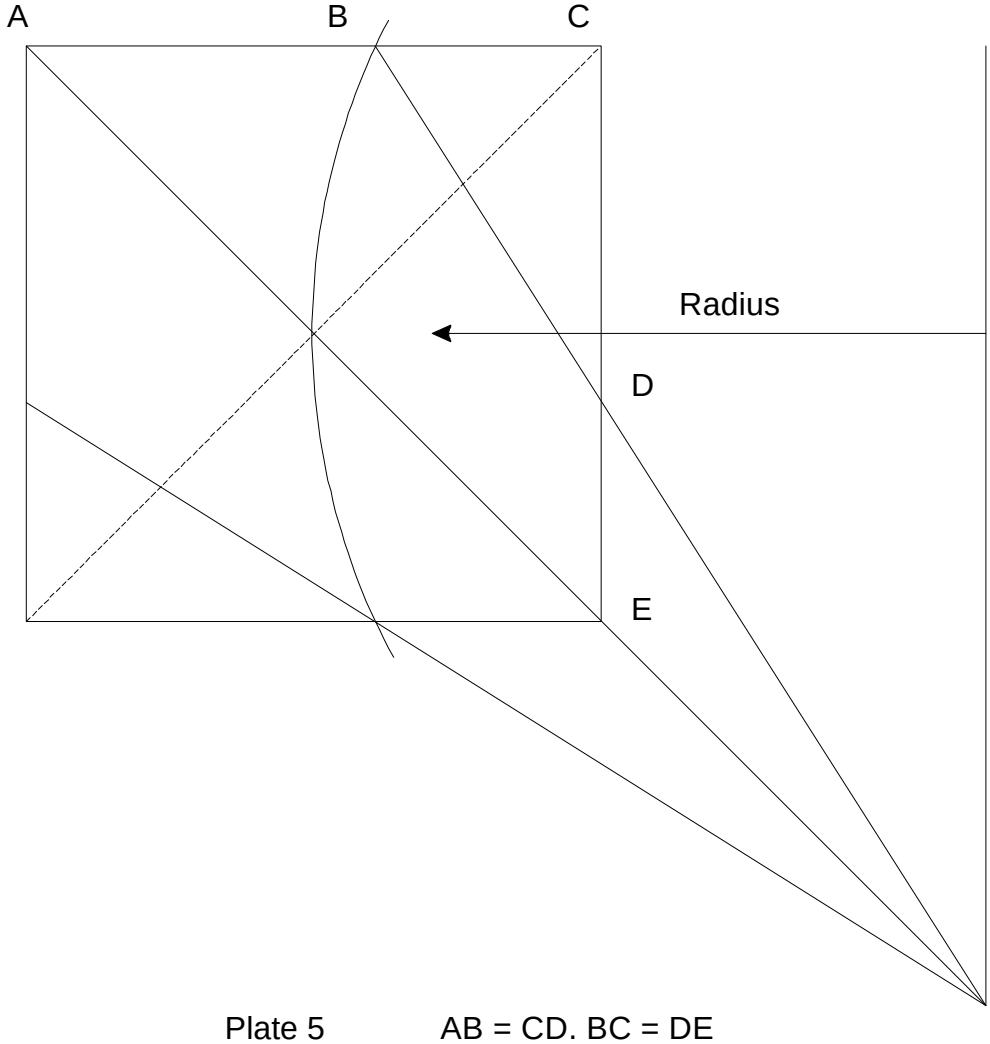
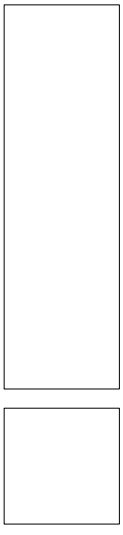
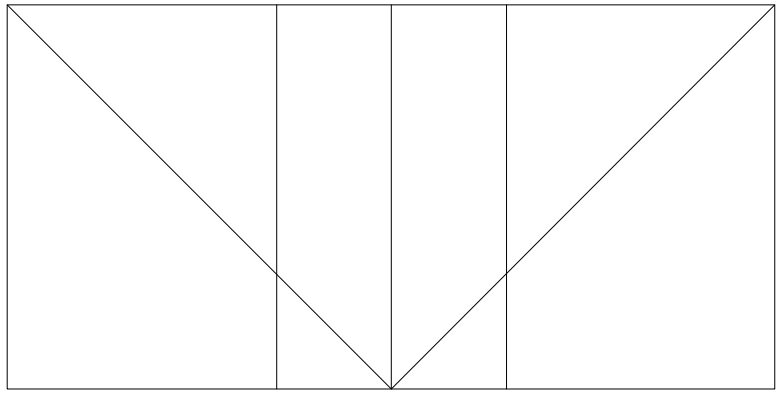



Plate 5 AB = CD. BC = DE

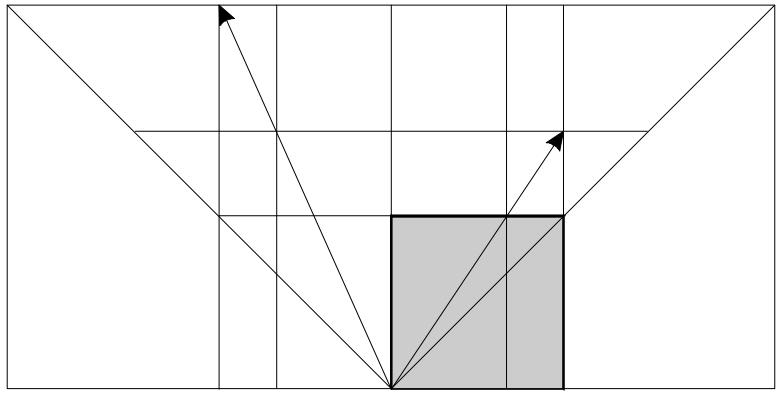
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



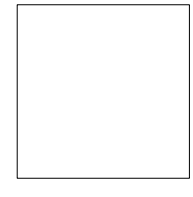
P.6



P.7

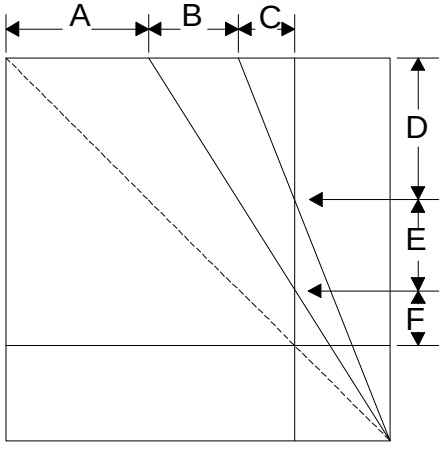


P.8

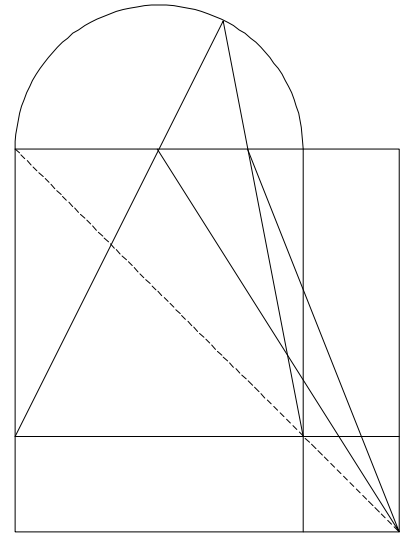


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

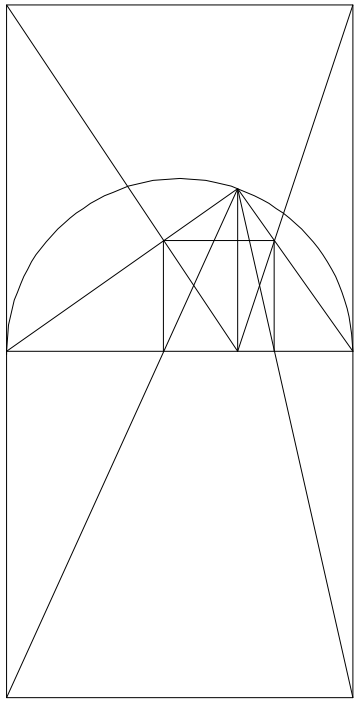



P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

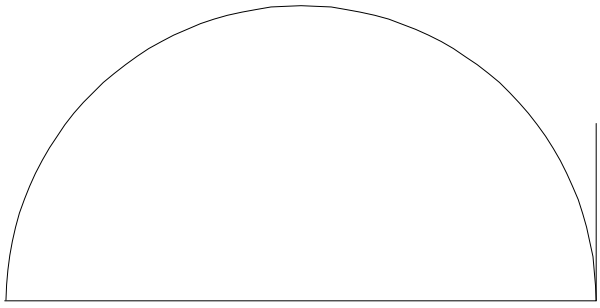


Plate 13

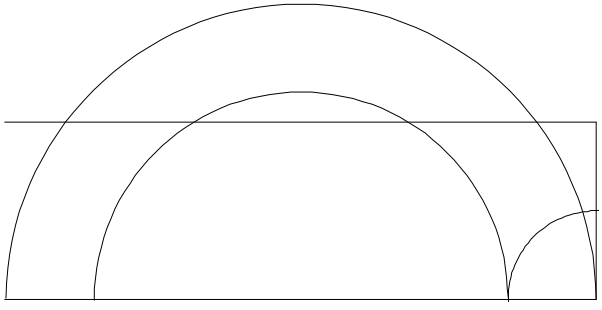


Plate 14

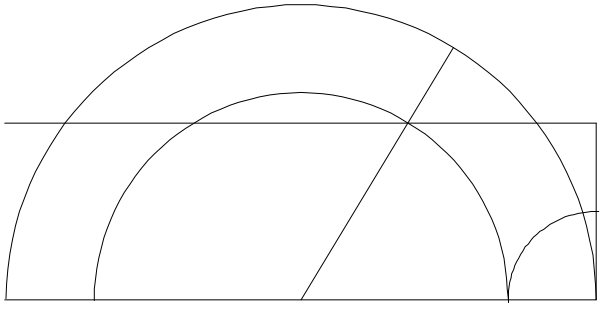


Plate 15

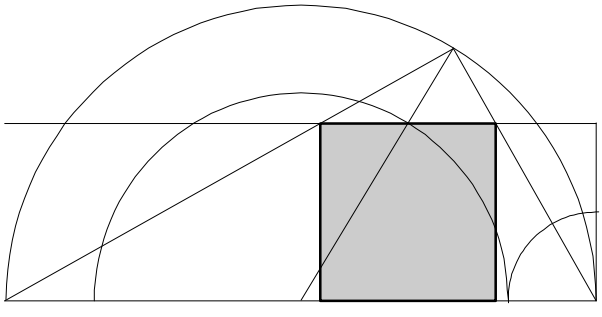
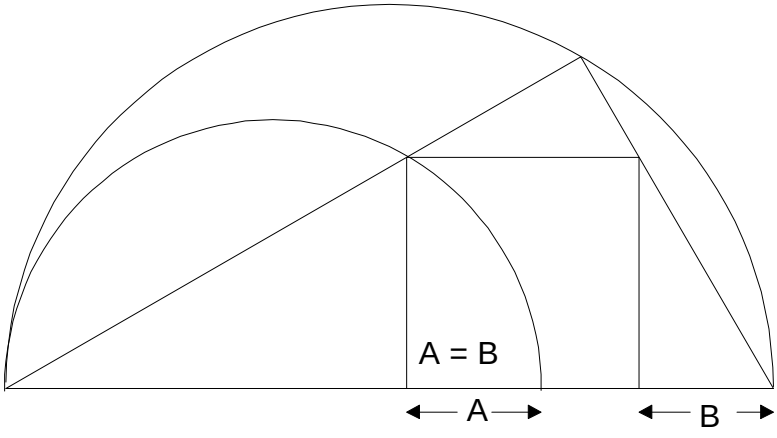
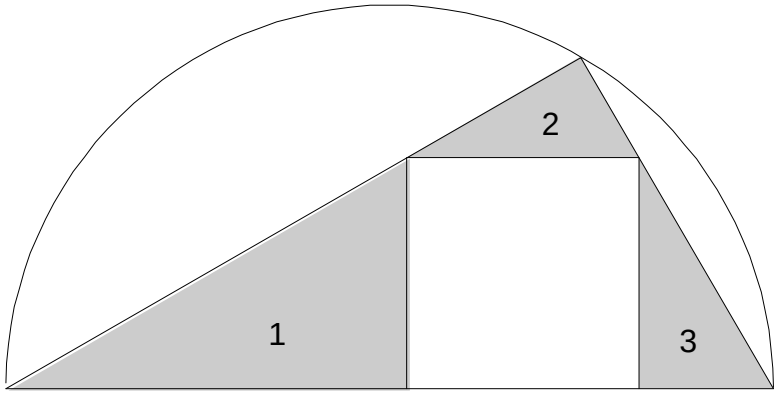


Plate 16

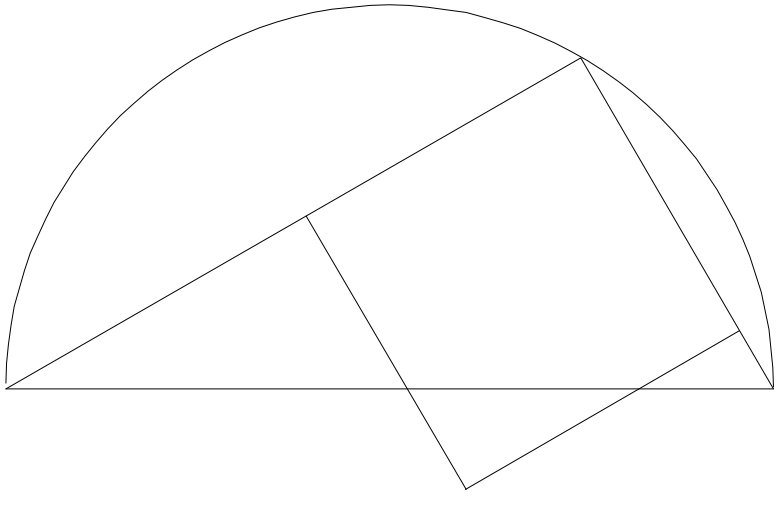
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.




In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"




P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

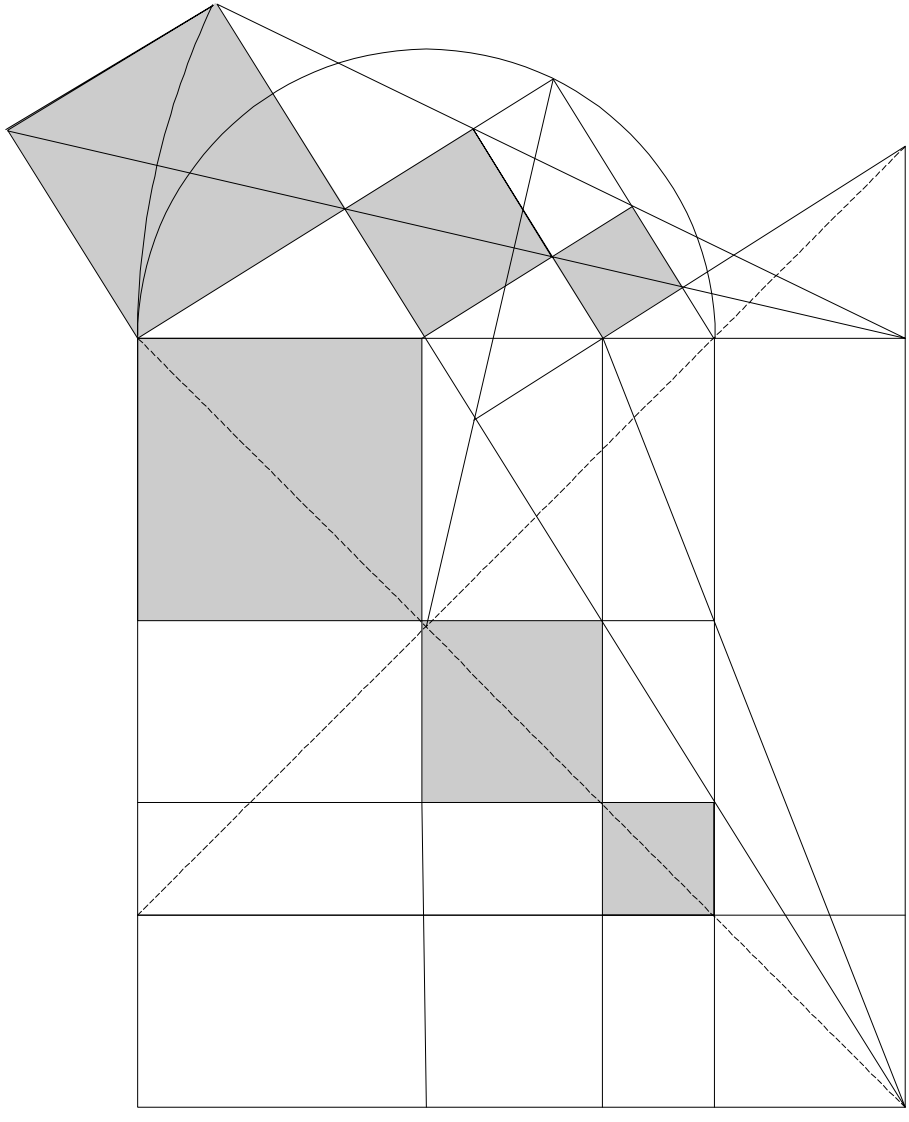
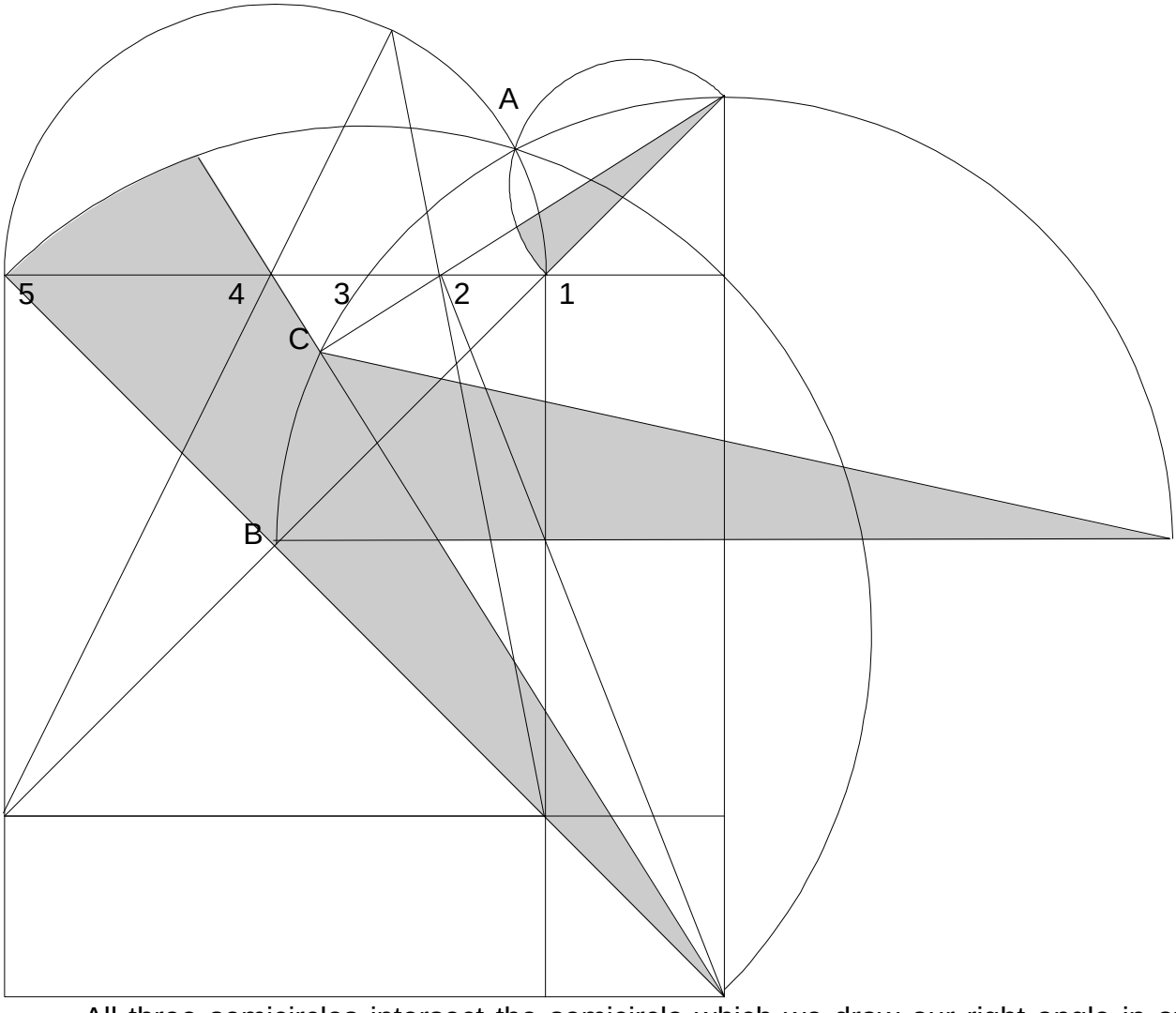


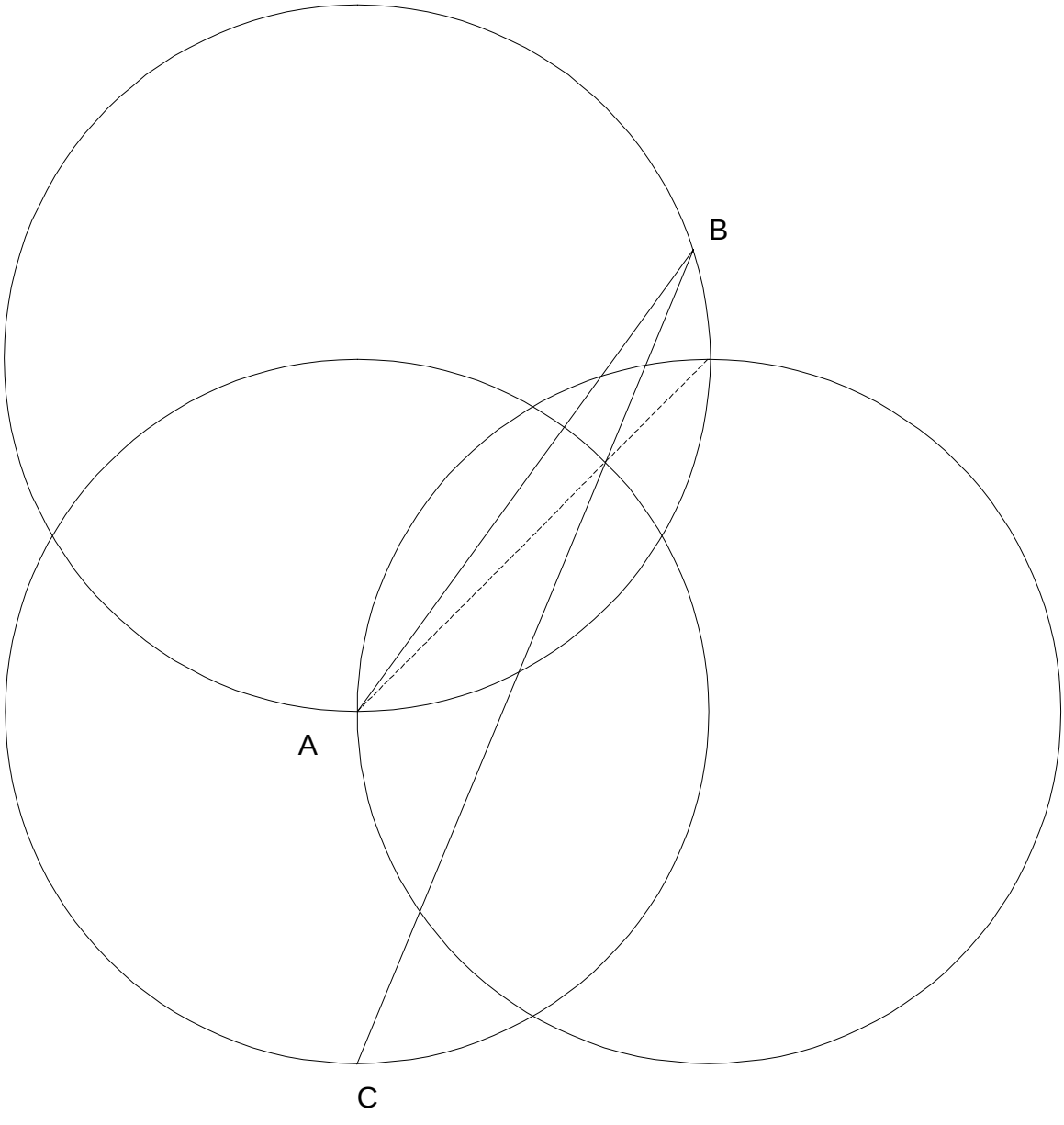
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.




All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.




How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

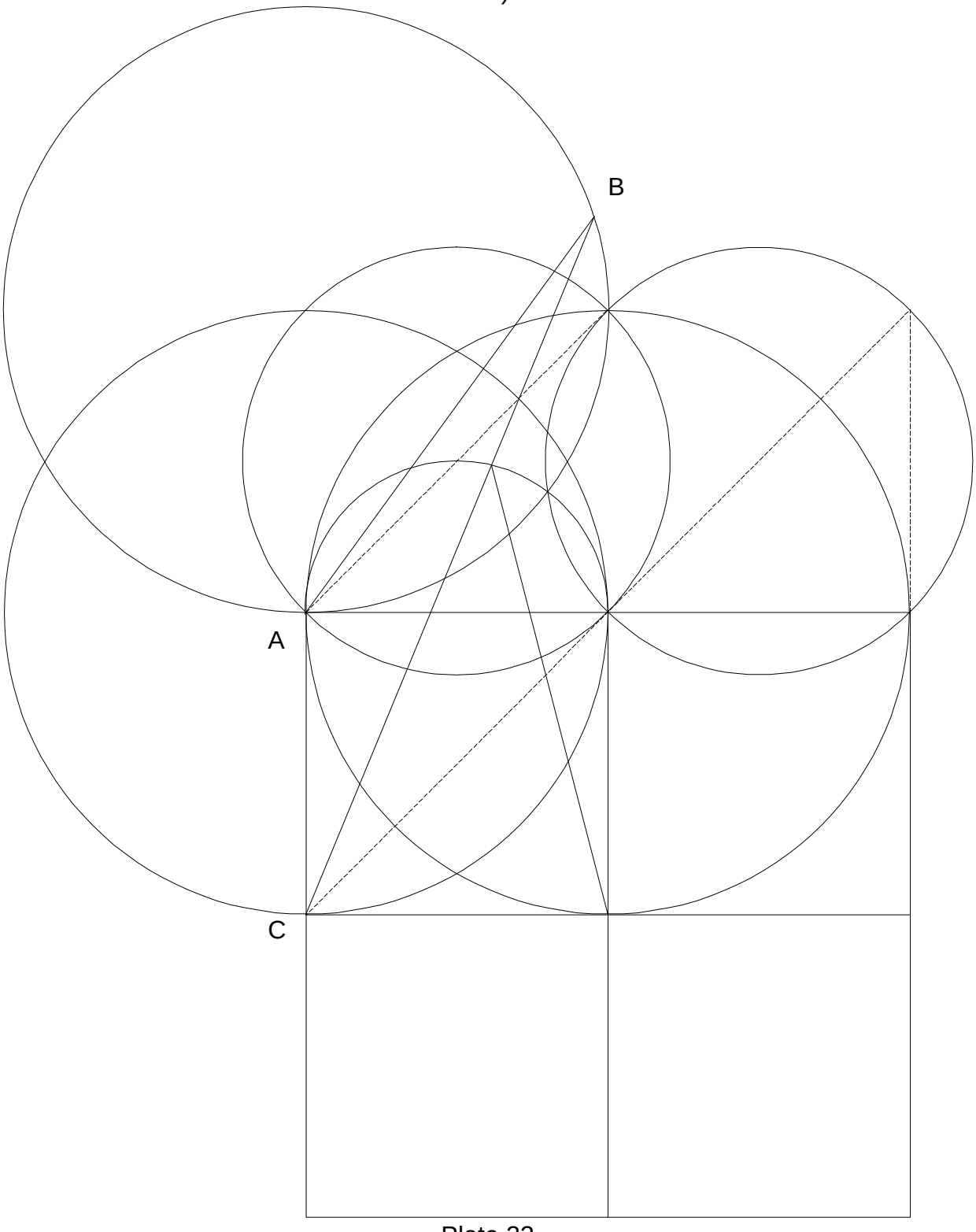
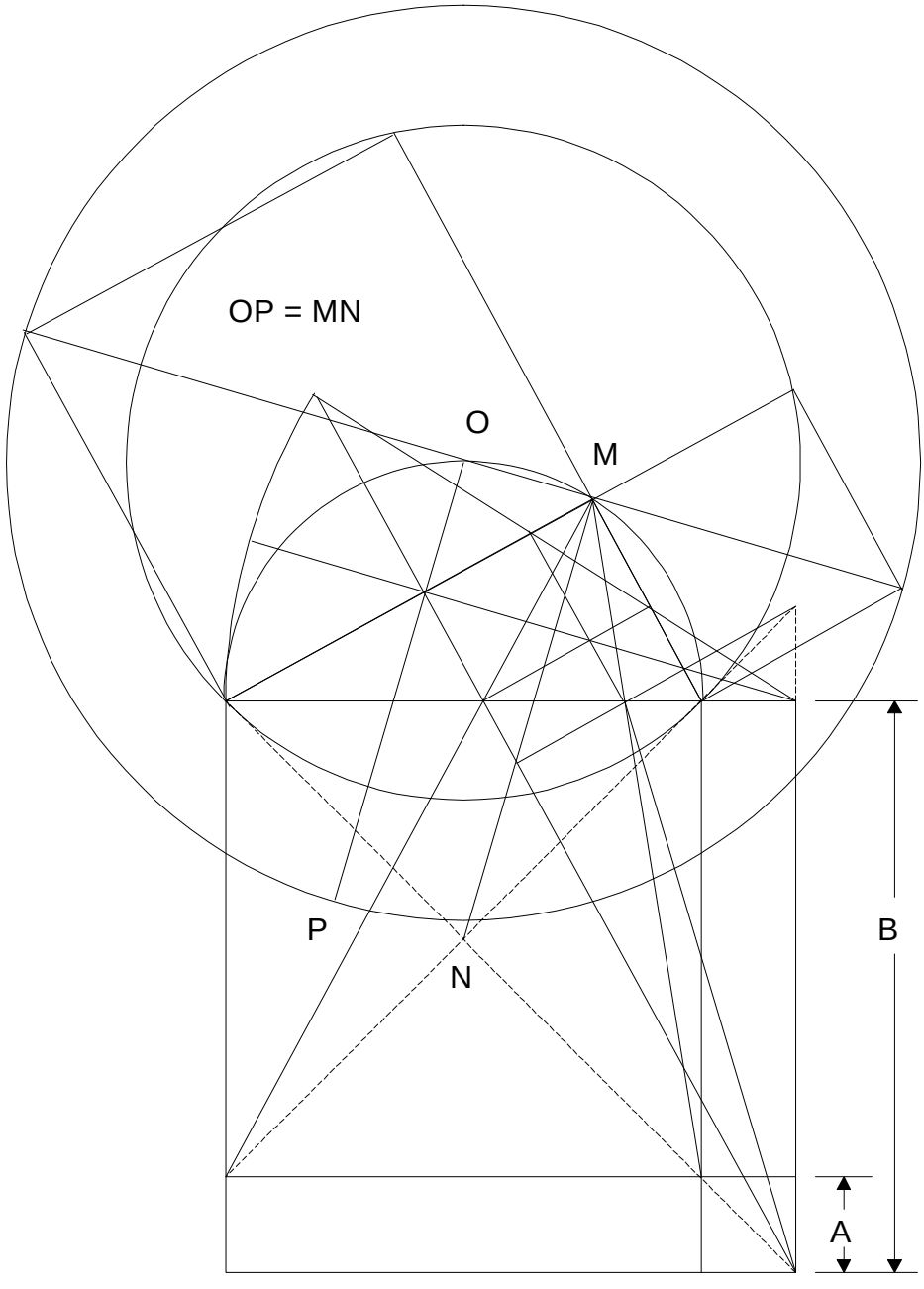



Plate 23

On Plate 24 the radius for the circle OP is given by MN.




One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of $B^2A$ (if you have missed it, the figure gives both roots, $A^2B$ and $B^2A$) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

# GEOMETRIAE DEDICATA

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3508 TA Utrecht
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Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

# American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA  Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

**Location:**
201 Charles **Street**
Providence, **RI**
02904


December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

*Serving the mathematical community for over 100 years*

# American Mathematical Society

Department of Mathematics
**Roger E. Howe**
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

# JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

**société mathématique defrance**

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

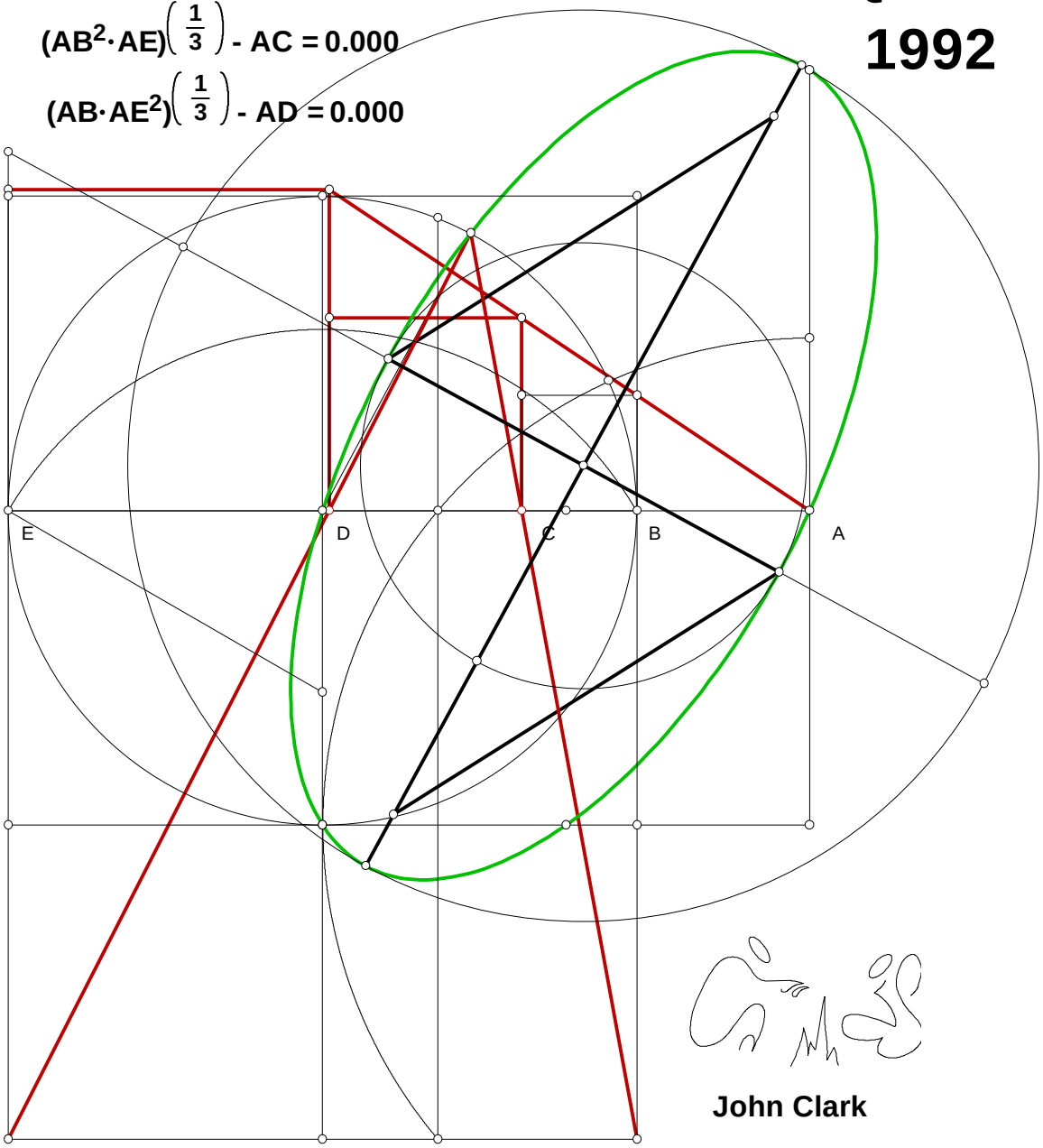
P.J. : Manuscrit

# The Delian Quest

## 1992

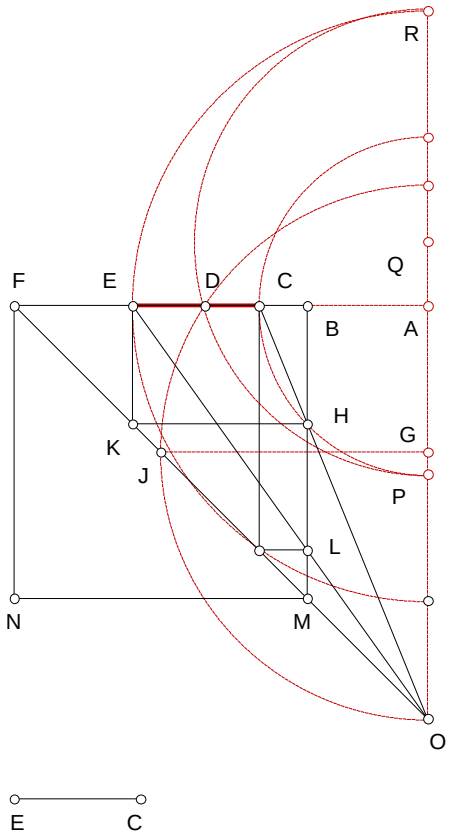
$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$






John Clark







# A Triplicate Ratio 06/20/92

**Given some point O place CE on BF such that O is the point of similarity.**

$N := 1.52 \quad N_2 := .89 \quad N_3 := .66$

$BF := N \quad MO := N_2 \quad CE := N_3$

$FM := \sqrt{2 \cdot BF^2} \quad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF} \quad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \quad QR := DQ \quad QP := DQ$

$AP := QP - AQ \quad AR := QR + AQ \quad AC := AP$

$AE := AR \quad AO := AF \quad BC := AC - AB$

$EF := AF - AE \quad EK := EF \quad BH := EK$

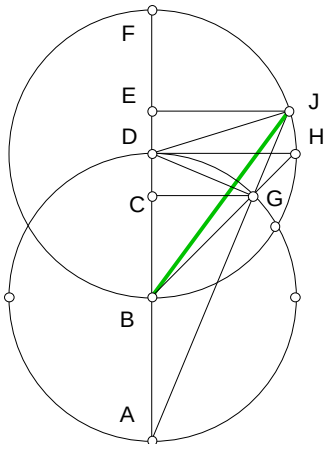
$\frac{BC}{BH} - \frac{AC}{AO} = 0$

**The last ratio can be tediously proved by reducing each term to the givens.**

**Edit 062800**

# 08/12/92 Rusty Cube of a Sphere

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**




$N := 9$ $\quad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

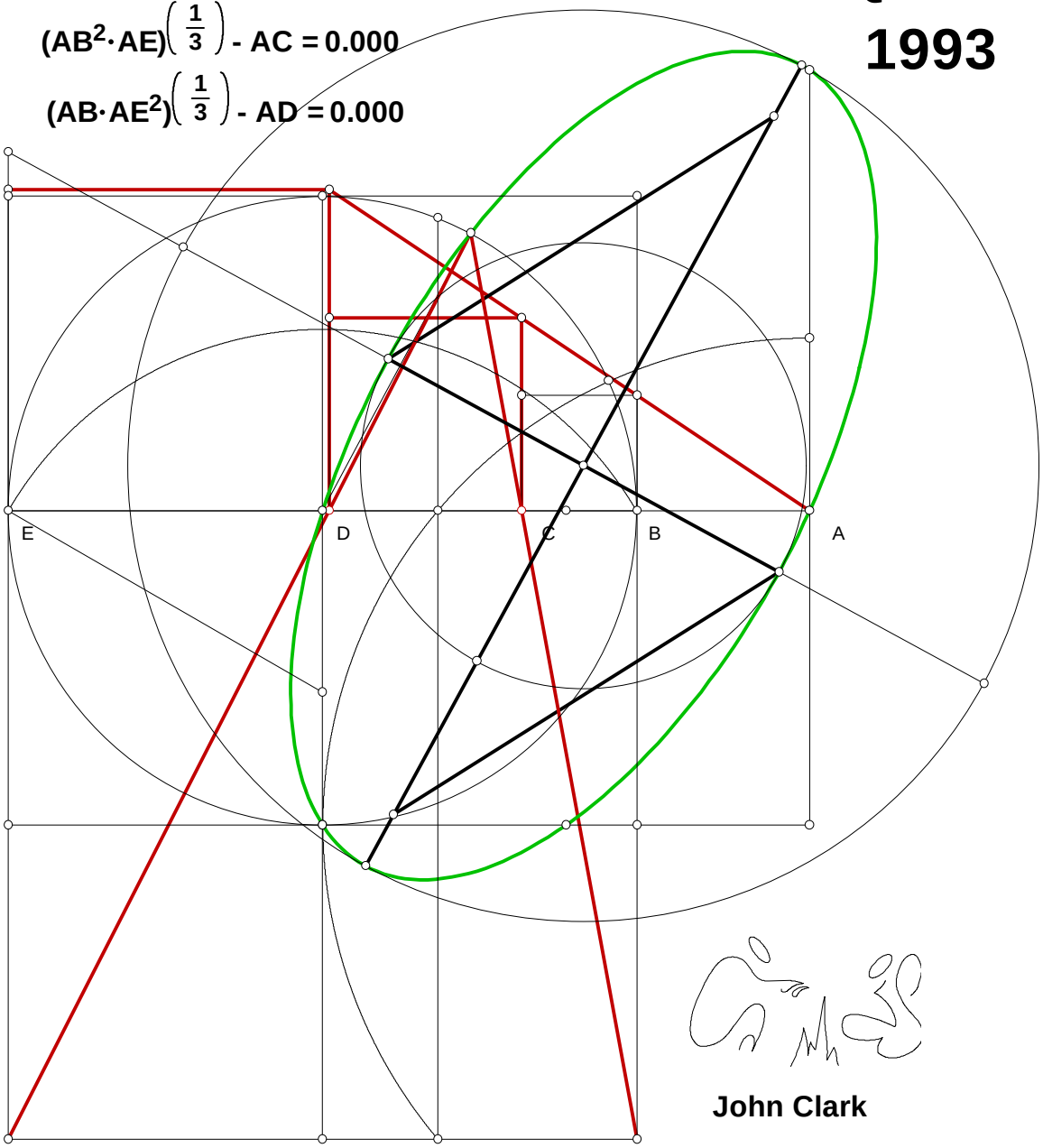
$\frac{CUBE\_ROOT}{BJ} = 0.99906$

# The Delian Quest

## 1993

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

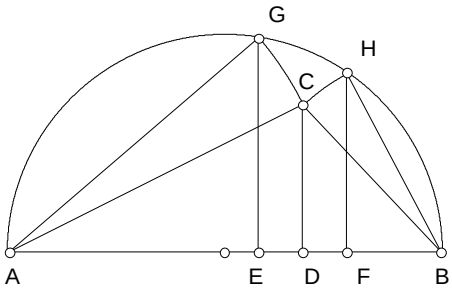





John Clark

## 01/08/93 Pythagoras Revisited

**Given AB, BC, AC, what is CD and CJ?**




$S_1 := 2.24 \quad S_2 := 1.046 \quad S_3 := 1.7$

$S_1 := S_1 \quad S_2 := S_2 \quad S_3 := S_3$

$AB := S_1 \quad BC := S_2 \quad AC := S_3$

$AG := AC \quad BH := BC \quad AE := \frac{AG^2}{AB}$

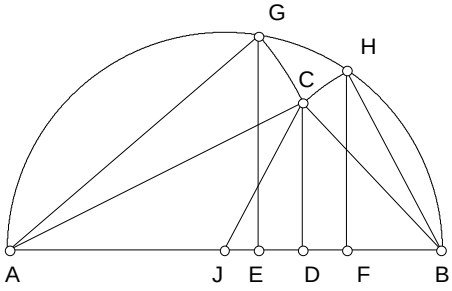
$BF := \frac{BH^2}{AB} \quad EF := AB - (AE + BF)$

$DE := \frac{EF}{2} \quad AD := AE + DE \quad AD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$BD := BF + DE \quad BD := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CD := \sqrt{AC^2 - AD^2}$

$CD := \frac{\sqrt{(-S_1 + S_2 - S_3) \cdot (S_1 + S_2 + S_3) \cdot (S_1 - S_2 - S_3) \cdot (S_1 + S_2 - S_3)}}{2 \cdot S_1}$



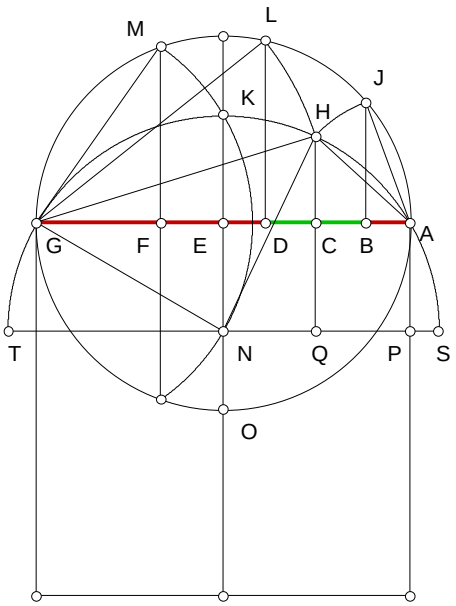

$AJ := \frac{AB}{2} \quad JD := AD - AJ \quad CJ := \sqrt{JD^2 + CD^2}$

$CJ := \frac{1}{2} \cdot \sqrt{2 \cdot S_3^2 - S_1^2 + 2 \cdot S_2^2}$

$(AC^2 + BC^2) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

**The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).**

# 06/03/93.MCD Exploring The Curve CJ




**Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?**

$N := 2 \qquad N_2 := 4$

$AG := N \qquad AC := \frac{AG}{N_2} \qquad GF := \frac{AG}{3} \qquad AE := \frac{AG}{2}$

$EG := AE \qquad AF := AG - GF \qquad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \qquad GN := GM \qquad EN := \sqrt{GN^2 - EG^2}$

$NH := GM \qquad NS := GM \qquad PN := AE \qquad PS := NS - PN$

$ST := 2 \cdot GM \qquad SQ := AC + PS \qquad QT := ST - SQ$

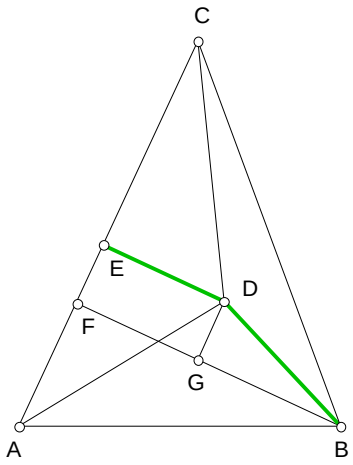
$QH := \sqrt{SQ \cdot QT} \qquad CQ := EN \qquad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \qquad CG := AG - AC \qquad GH := \sqrt{CG^2 + CH^2} \qquad AJ := AH \qquad AB := \frac{AJ^2}{AG}$

$GL := GH \qquad DG := \frac{GL^2}{AG} \qquad BD := AG - (AB + DG) \qquad \sqrt{AB \cdot DG} - BD = 0$

$$\left( \frac{-N}{3} + \frac{N}{3} \cdot \frac{\sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2} \right) - BD = 0$$

# 06/07/93 For All Triangles Find BD




**Given AB, BC, AC, CD, AD, find BD**

**To simplify use line names found in 010893**

$S_1 := 5.55 \quad S_2 := 4.61 \quad S_3 := 1.5$

$S_5 := 4.5 \quad S_6 := 5.48$

$AB := S_5 \quad BC := S_6 \quad AC := S_1 \quad CD := S_3 \quad AD := S_2$

$$DE := \frac{\sqrt{\left(-S_1 + S_2 - S_3\right)\cdot\left(S_1 + S_2 + S_3\right)\cdot\left(S_1 - S_2 - S_3\right)\cdot\left(S_1 + S_2 - S_3\right)}}{2\cdot S_1}$$

$$BF := \frac{\sqrt{\left(-S_1 + S_5 - S_6\right)\cdot\left(S_1 + S_5 + S_6\right)\cdot\left(S_1 - S_5 - S_6\right)\cdot\left(S_1 + S_5 - S_6\right)}}{2\cdot S_1}$$

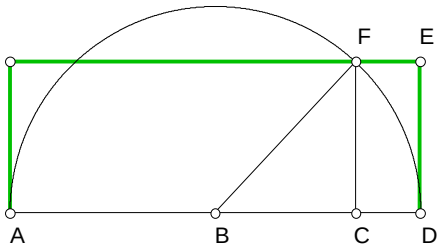
$CE := \sqrt{CD^2 - DE^2} \quad CF := \sqrt{BC^2 - BF^2} \quad EF := CF - CE \quad GF := DE$

$BG := BF - GF \quad DG := EF \quad BD := \sqrt{DG^2 + BG^2} \quad BD = 3.983$

**OR**

$BG_2 := BF + GF \quad DG_2 := EF \quad BD_2 := \sqrt{DG^2 + BG_2^2} \quad BD_2 = 5.757$

# 06/09/93 Rectangular Roots




**Given AD and DE divide AD into the rectangular roots of DE.**

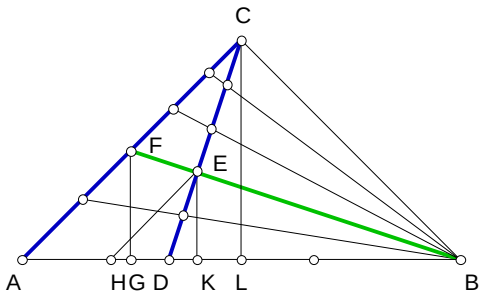
$N := 5 \qquad N_2 := 2$

$AD := N \quad DE := N_2 \quad CF := DE \quad BF := \frac{AD}{2}$

$AB := BF \quad BC := \sqrt{BF^2 - CF^2} \quad AC := AB + BC \quad CD := AD - AC \quad \sqrt{CD \cdot AC} - DE = 0$

$AC := \frac{1}{2} \cdot N + \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2} \qquad CD := \frac{1}{2} \cdot N - \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2}$

# 06/21/93 Pyramid of Ratios I




**Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?**

$N1 := 3 \qquad N2 := 5 \qquad \delta := 1 .. N2$

$AB := 1 \qquad AD := \frac{AB}{N1} \qquad AL := \frac{AB}{2}$

$DL := AL - AD \qquad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \qquad CD := \sqrt{DL^2 + CL^2}$

| $\delta$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

$DE_\delta := \frac{CD \cdot \delta}{N2} \qquad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \qquad AK_\delta := AD + DK_\delta \qquad BK_\delta := AB - AK_\delta \qquad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \qquad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \qquad BH_\delta := BK_\delta + HK_\delta \qquad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \qquad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \qquad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\text{if}\left(N2 - \delta, \frac{N1 \cdot N2}{N2 - \delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\frac{AC}{AF_\delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

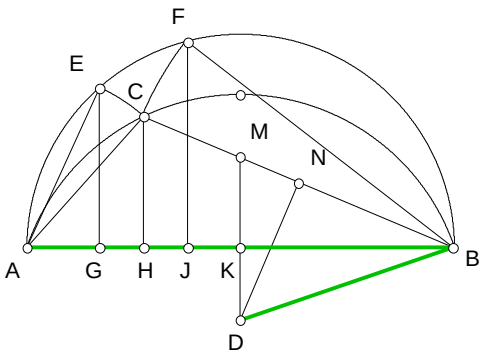
$\frac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

# 06/27/93 Describe A Circle About a Triangle

## Given the difference between three non-collinear points, find the radius of the circle that circumscribes them

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $\quad NOT(X) := X = 0$ $\quad \delta := 0 .. 2$




$BK := \frac{AB}{2}$ $\quad AE := AC$ $\quad BF := BC$

$AG := \frac{AE^2}{AB}$ $\quad BJ := \frac{BF^2}{AB}$ $\quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $\quad BH := BJ + HJ$ $\quad CH := \sqrt{BC^2 - BH^2}$

$BN := \frac{BC}{2}$ $\quad BM := \frac{BC \cdot BK}{BH}$ $\quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $\quad BD := \sqrt{BN^2 + DN^2}$

$radius := if(\Delta, BD, 0)$ $\quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary\_radius = 0$

$\Delta = 1$

$S_1 := \begin{bmatrix} AB \\ AC \\ BC \end{bmatrix}$ $\quad S_2 := \begin{bmatrix} AC \\ BC \\ AB \end{bmatrix}$ $\quad S_3 := \begin{bmatrix} BC \\ AB \\ AC \end{bmatrix}$

$AB \equiv 3$ $\quad AC \equiv 4$ $\quad BC \equiv 6$

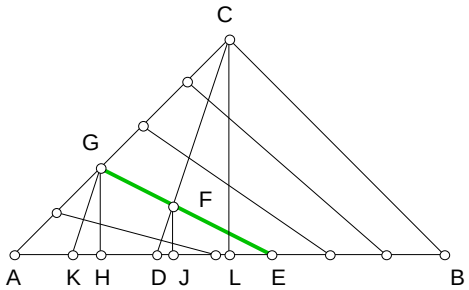
## The name of the Radius as a proportion of the given names

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = [\, 3.375 \quad 3.375 \quad 3.375 \,]$

# 07/15/93 Pyramid of Ratios II

**AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?**




$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

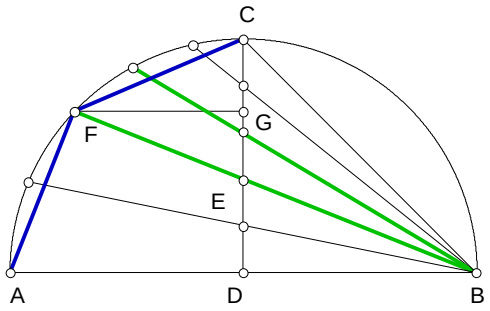
| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

# 07/25/93 Pyramid of Ratios III

## Dividing DC by an number provides wht in terms of BE/BF and AF/CF?




$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

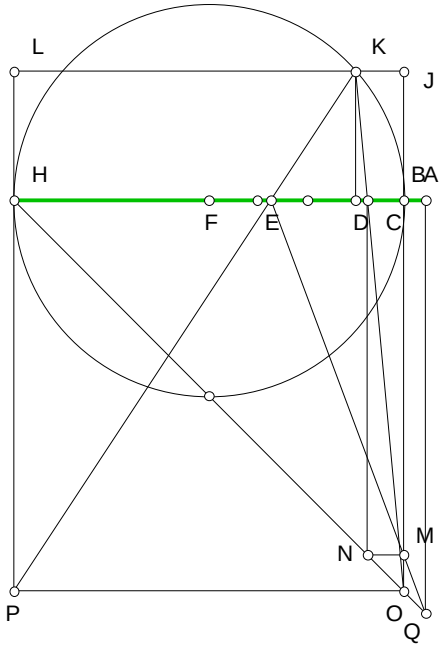
$DG := DE + EG \qquad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$ | $\frac{BE_\delta}{BF_\delta}$ | $\frac{BF_\delta}{BE_\delta}$ | $\frac{2}{\left(\frac{\delta}{N}\right)^2 + 1}$ |
|---|---|---|---|---|
| 0.514 | 0.514 | 0.514 | 1.946 | 1.946 |
| 0.556 | 0.556 | 0.556 | 1.8 | 1.8 |
| 0.625 | 0.625 | 0.625 | 1.6 | 1.6 |
| 0.722 | 0.722 | 0.722 | 1.385 | 1.385 |
| 0.847 | 0.847 | 0.847 | 1.18 | 1.18 |
| 1 | 1 | 1 | 1 | 1 |

| $\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$ | $\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$ |
|---|---|
| 0.283 | 0.283 |
| 0.707 | 0.707 |
| 1.414 | 1.414 |
| 2.828 | 2.828 |
| 7.071 | 7.071 |
| 0 | 0 |

# 11/06/93 Gruntwork I on the Delian Solution




**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?**

$N := 4$ $BH := 1$

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $BJ := DK$ $BO := BH$

$JO := BJ + BO$ $JK := BD$ $CD := \frac{JK \cdot DK}{JO}$

$KL := DH$ $LP := JO$ $DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD$ $CE := CD + DE$ $MN := BC$

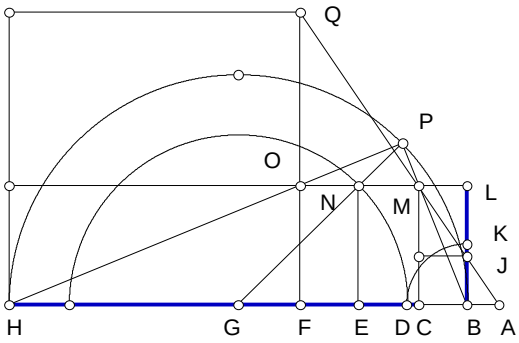
$GH := MN$ $CH := BH - BC$ $HN := \sqrt{2 \cdot CH^2}$ $GM := HN$ $EH := CH - CE$ $EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG}$ $HO := \sqrt{2 \cdot BH^2}$ $OQ := HQ - HO$ $OR := \sqrt{\frac{OQ^2}{2}}$ $AB := OR$ $AC := AB + BC$

$AE := AB + BD + DE$ $AH := AB + BH$ $\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$

## 11/09/93 Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem.**
**BH is the difference between the segments AH and AB.**
**CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**



$N := 2 \qquad BH := N - 1$

$BG := \frac{BH}{2} \qquad CF := N^{\frac{2}{3}} - N^{\frac{1}{3}}$

$BL := CF \qquad GP := BG \qquad BK := \frac{BL}{2}$

$BD := BK \qquad NP := BD \qquad GN := GP - NP$

$EN := BL \qquad GE := \sqrt{GN^2 - EN^2}$

$CE := BD \qquad BC := BG - (GE + CE)$

$GH := BG \qquad EF := BD$

$FH := GH + GE - EF \qquad FQ := FH \qquad FO := BL \qquad OQ := FQ - FO \qquad MO := CF \qquad AF := \frac{MO \cdot FQ}{OQ}$

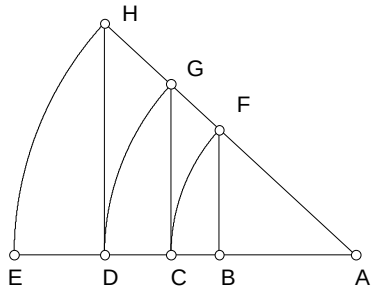
$AC := AF - CF \qquad AH := AF + FH \qquad AB := AH - BH$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 2$$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

## 11/10/93 Gruntwork II on the Delian Solution

**Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?**




$N := 1.1$ $AE := 10$

$DE := \frac{AE}{N}$ $AD := AE - DE$ $AH := AE$

$AG := AD$ $AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

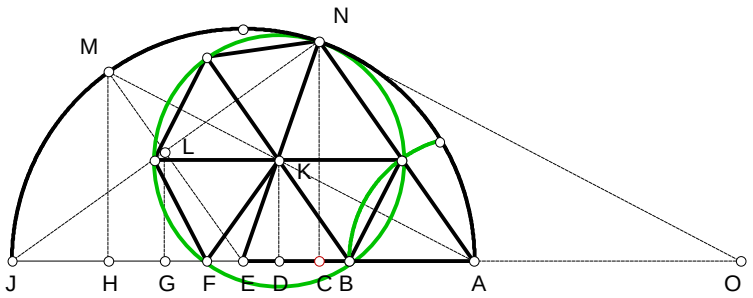
$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 1331$ $\frac{AD}{AB} = 121$ $\frac{AC}{AB} = 11$

## 11/11/93 The Archamedian Paper Trisector

**If one accepts the facts of the original figure, one only need prove that BK = AB.**
**If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?**




$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN \cdot JL}{JN}$ $GJ := \frac{CJ \cdot JL}{JN}$

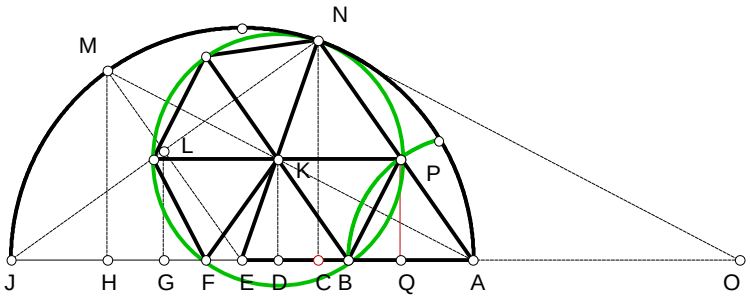
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

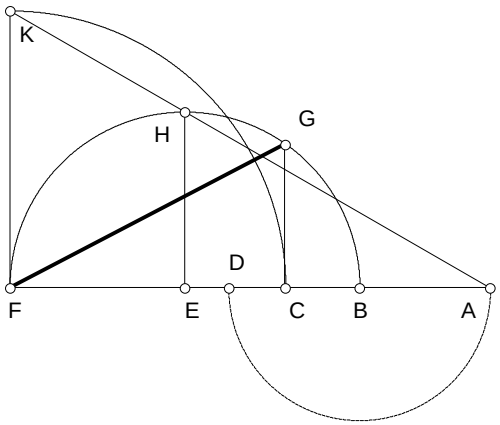
$AB - BK = 0$ $AB = 0.25$

**If PK is parallel to AJ, then . . . .**




$AN := \sqrt{AC^2 + CN^2}$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

**11/12/93 To Square A Circle Off The Base Of A Right Triangle.**




**Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?**

**Using the approximation, $\pi$ = 22/7, square the circle off the base of a right triangle.**

$$BF := 1 \qquad BE := \frac{BF}{2} \qquad EF := BE \qquad EH := BE \qquad BD := \frac{3}{4}\cdot BE \qquad AB := BD$$

$$AE := AB + BE \qquad AF := AB + BF \qquad FK := \frac{EH \cdot AF}{AE} \qquad CF := FK \qquad BC := BF - CF$$

$$CG := \sqrt{BC \cdot CF} \qquad FG := \sqrt{CF^2 + CG^2} \qquad \pi\_A := \frac{FG^2}{BE^2}$$

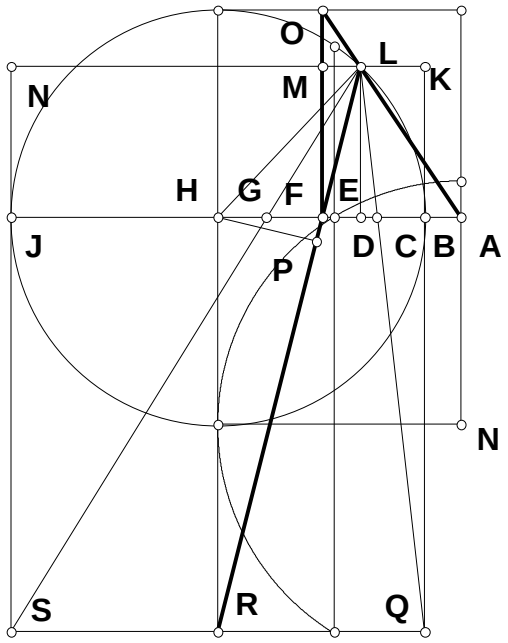
$$\pi = 3.14159265359$$

$$\pi\_A = 3.142857142857$$

$$\frac{\pi}{\pi\_A} = 0.999597662505843$$

# 11/18/93 Exploring Cube Roots Plate A

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.**




$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad MO := FO - DL \quad LM := DF$

$AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF \quad BQ := BJ$

$BK := DL \quad BD := BH - (FH + DF)$

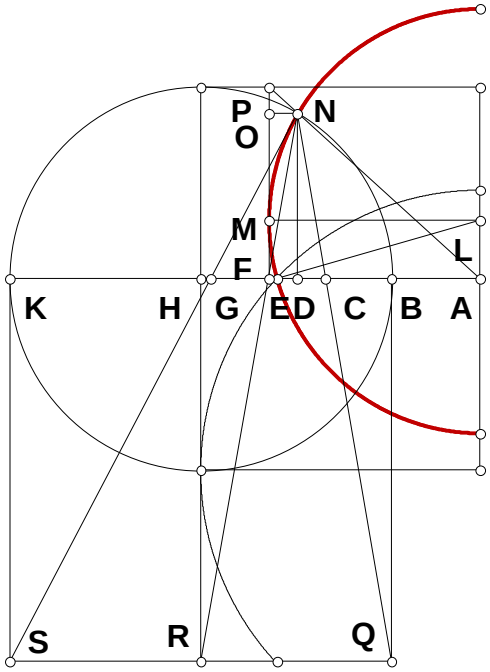
$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad HJ := BH \quad DJ := HJ + FH + DF \quad LN := DJ$

$JS := BJ \quad JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC$

$AG := AB + BG \quad AJ := AB + BJ \quad \left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

# 11/18/93 Exploring Cube Roots Plate B

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**




$N := 1.2$ $BK := 1$

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

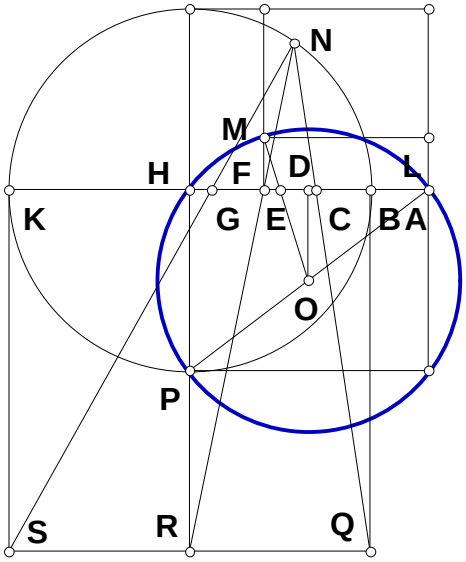
$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$ $AB = 1.523$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

# 11/18/93 Exploring Cube Roots Plate C

## The circle AO passes through point M.




$N := 1$

$BK := 1$ $\quad AB := \frac{BK}{N}$ $\quad AK := AB + BK$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}}$ $\quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$ $\quad \frac{AK}{AB} = 2$

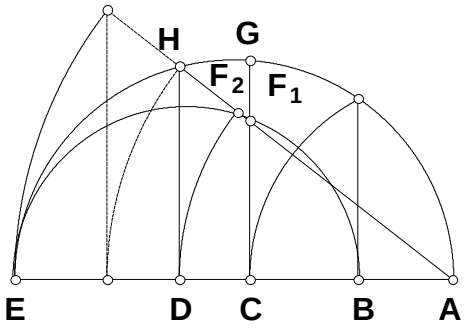
$CG := AG - AC$ $\quad CF := \frac{CG}{2}$ $\quad BH := \frac{BK}{2}$

$AH := AB + BH$ $\quad HP := BH$ $\quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2}$ $\quad DO := \frac{HP}{2}$ $\quad AF := AC + CF$ $\quad AD := \frac{AH}{2}$

$DF := AF - AD$ $\quad FM := CF$ $\quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$




## 11/22/93 Cube by Iteration

**When $F_1$ and $F_2$ are the same point on C, then a sixth root series has been constructed. Use iteration to place $F_2$ on $F_1$.**

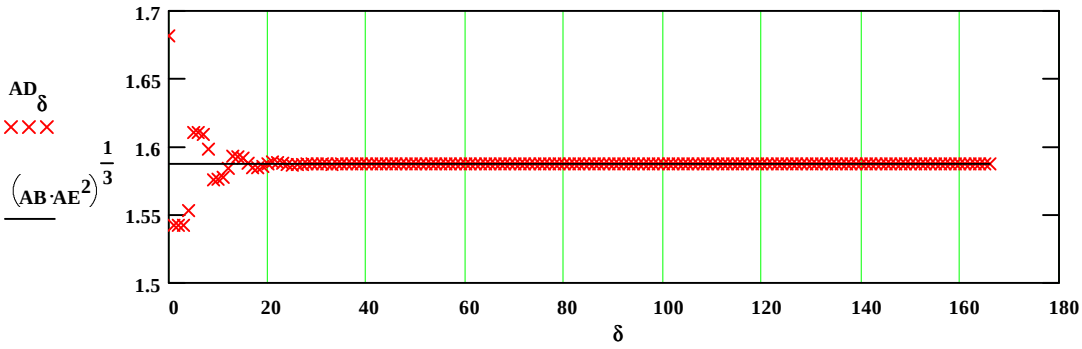
$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \right]^2} \end{bmatrix}$$

$$\begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + \left(CF_\delta\right)^2} \end{bmatrix}$$

$\Delta \equiv 166$

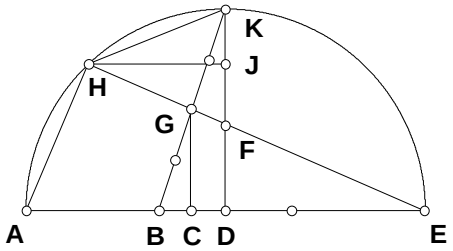



$\left(AB^2 \cdot AE^4\right)^{\frac{1}{6}} - AD_\Delta = 0$

$\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD_\Delta = 0$

# 11/24/93 POR Series IV

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**




$N_1 := 3 \qquad N_2 := 5 \qquad AE := 1$

$\alpha := 1 .. N_1 - 1 \qquad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\frac{AH}{HK} = 0.265 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

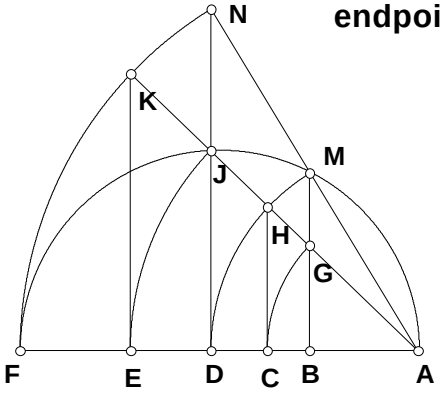
$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$

## 12/04/93 Exponential Series M^(1/2^N)

**Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.**




$N := 8$

$AF := N \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

$AN := AF \quad AD := \frac{AB \cdot AN}{AM} \qquad DF := AF - AD$

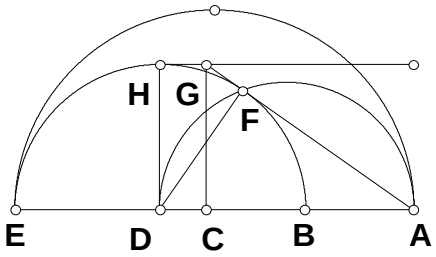
$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

$AK := AF \quad AE := \frac{AD \cdot AK}{AJ} \qquad AH := AD \quad AC := \frac{AD \cdot AH}{AJ}$

$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$N^{\frac{1}{4}} - AC = 0 \qquad N^{\frac{2}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AE = 0$

# 12/06/93 Alternate Method: Square Root
# Common Segment Common Endpoint




$N := 6 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2}$

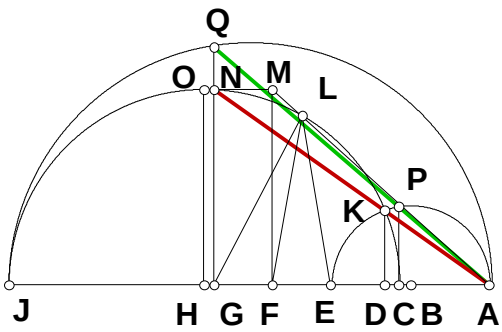
$DF := BD \qquad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$

## 12/06/93B Gruntwork IV on the Delian Solution

### Are APQ colinear? Are AKN colinear?




$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$AE := \left(AC^2 \cdot AJ\right)^{\left(\frac{1}{3}\right)} \qquad AG := \left(AC \cdot AJ^2\right)^{\left(\frac{1}{3}\right)}$

$CG := AG - AC \qquad CJ := AJ - AC$

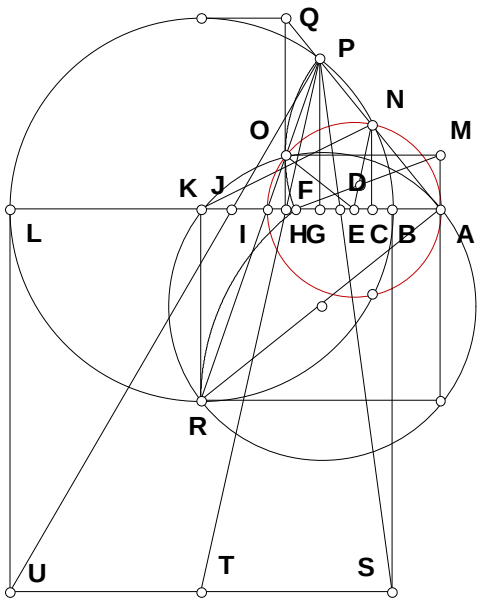
$GJ := CJ - CG \qquad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \qquad CE := AE - AC \qquad CH := \frac{CJ}{2}$

$BK := \frac{AE}{2} \qquad HK := CH \qquad HJ := CH \qquad AH := AJ - HJ \qquad BH := AH - AB \qquad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$DE := AE - (AB + BD) \qquad AD := AE - DE \qquad DK := \sqrt{AD \cdot DE} \qquad \frac{AG}{GN} - \frac{AD}{DK} = 0$

$GQ := \sqrt{AG \cdot GJ} \qquad CP := \sqrt{AC \cdot CE} \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$




## 12/11/93

**The structure in red appears to be a constant.**

$N := 6$ $AB := 1$ $AL := AB \cdot N$

$BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$

$AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$ $BE := AE - AB$ $BJ := AJ - AB$

$JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$

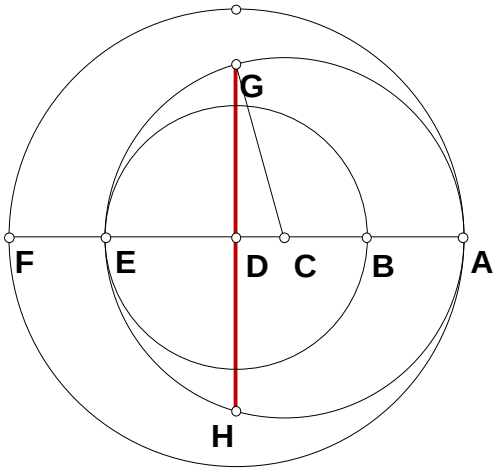
$KR := BK$ $KL := BK$ $FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$

$AD := \frac{AI}{2}$ $KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

# 12/12/93 The Square Root




**Square root by common segment common midpoint. Given AF BE is GH their root?**

$N := 5$ $BE := 1$ $AF := BE \cdot N$

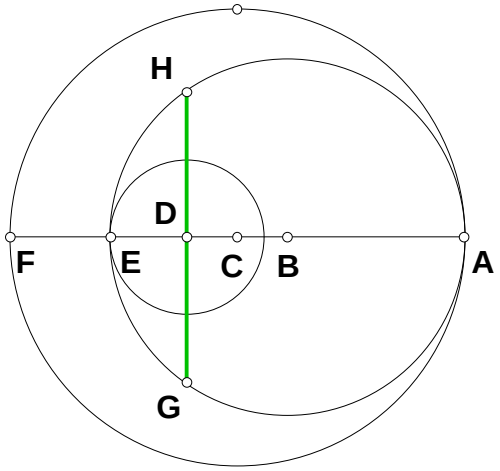
$AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$ $AB := AD - BD$

$AE := BE + AB$ $AC := \frac{AE}{2}$ $CG := AC$

$CD := AD - AC$ $GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$

# 12/12/93 Generalize The Previous Square Root Figure




$N_1 := 1 \qquad N_2 := 3 \qquad N_3 := 2$

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

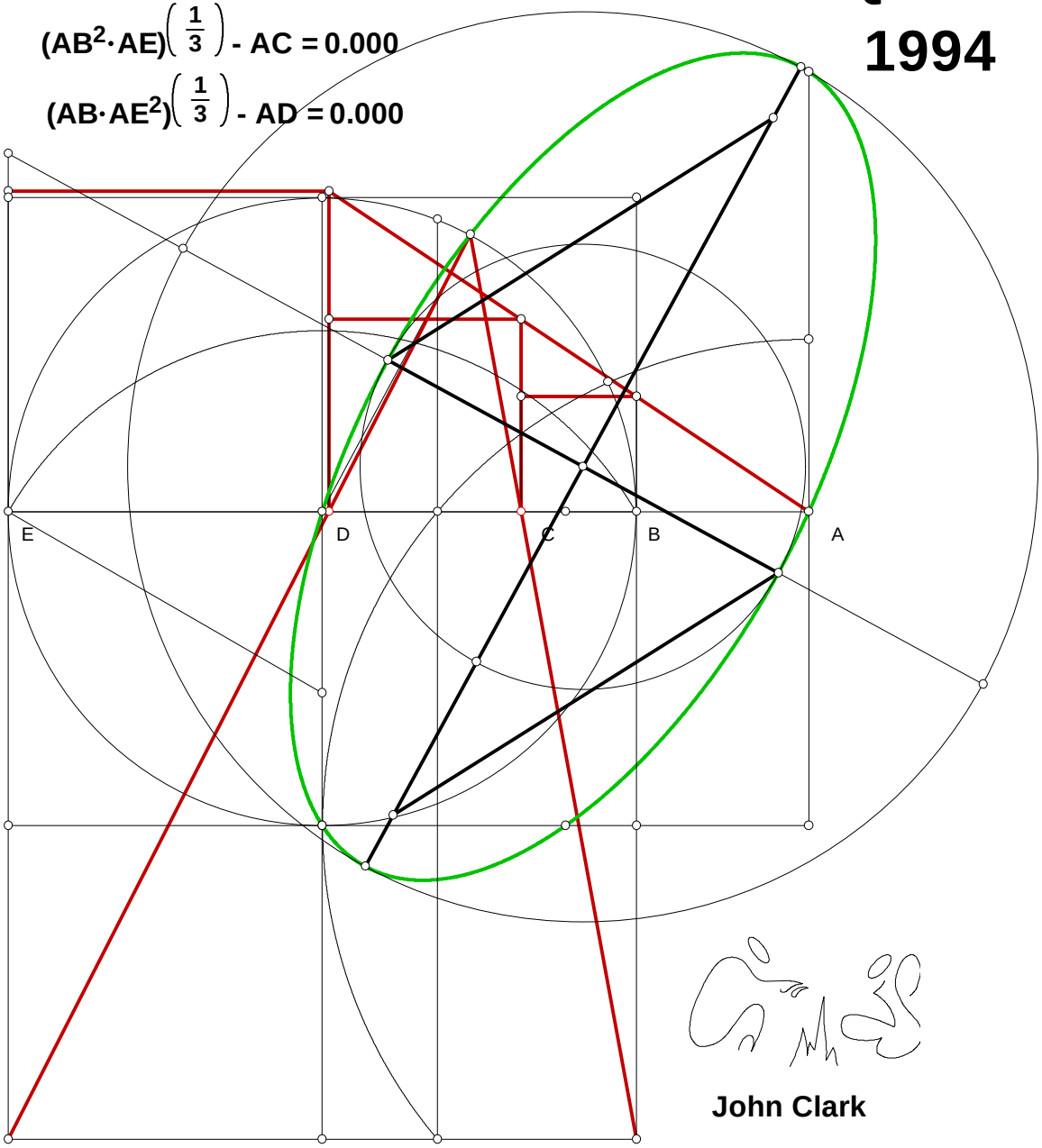
$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$

# The Delian Quest

## 1994

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

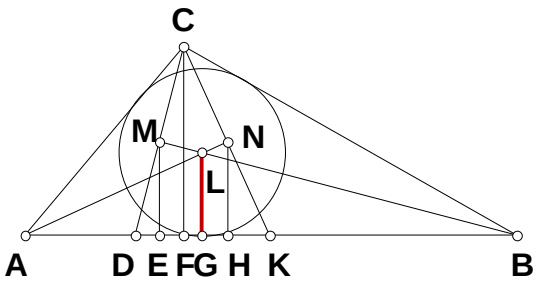





John Clark

# 04/06/94 Inscribing A Circle In A Given Triangle

## Given three sides of a triangle, what is the length of the radius?




$AB := 3 \qquad BC := 4 \qquad AC := 5$

$AK := AC \quad BD := BC \quad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$

$FK := AK - AF \quad CF := \sqrt{AC^2 - AF^2}$

$CK := \sqrt{FK^2 + CF^2} \qquad AN := \frac{CF \cdot AK}{CK}$

$AH := \frac{CF \cdot AN}{CK} \quad HN := \frac{FK \cdot AN}{CK} \quad BF := AB - AF$

$DF := BD - BF \quad CD := \sqrt{CF^2 + DF^2} \quad BM := \frac{CF \cdot BD}{CD} \quad BE := \frac{CF \cdot BM}{CD} \quad GL := \frac{HN \cdot AB}{AH + BE}$

$GL = 1$

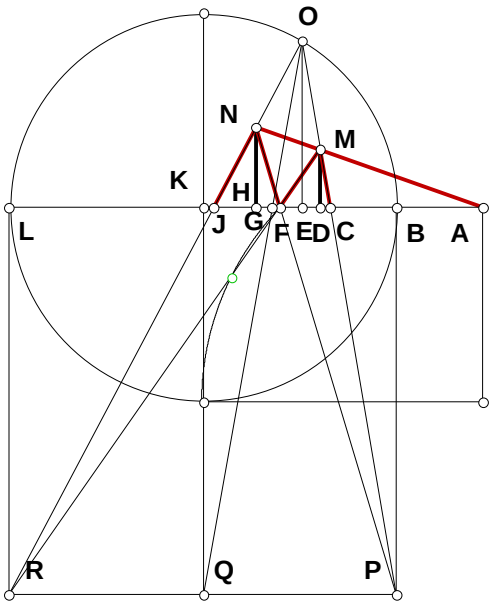
$S_1 := \begin{bmatrix} AB \\ BC \\ AC \end{bmatrix} \qquad S_2 := \begin{bmatrix} BC \\ AC \\ AB \end{bmatrix} \qquad S_3 := \begin{bmatrix} AC \\ AB \\ BC \end{bmatrix} \qquad \delta := 0 .. 2$

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}}$$

$$Radius = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

# 04/21/94 The Cradle

## Are AMN colinear?




$N := 5 \qquad AB := 1 \qquad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \qquad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

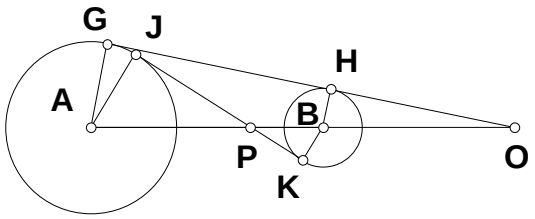
$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

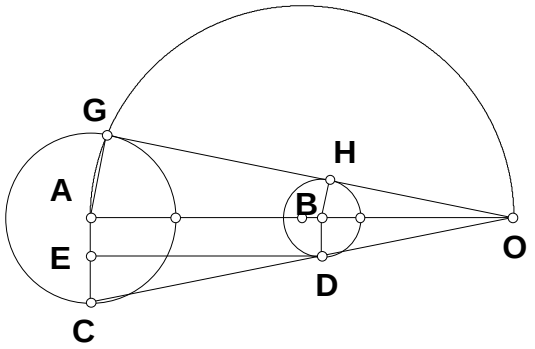
$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$

# 04/26/94 Tangents and Similarity Points




**What is the Algebraic names of the similarity points O and P in relation to the radius of each circle and the difference between their centers?**

**I will work with point O first.**

**Given $R_L$ = large radius**
**$R_S$ = small radius**
**D = difference between origins.**

$R_L := 4 \qquad R_S := 1 \qquad D := 8$

$AC := R_L \qquad BD := R_S \qquad AB := D$

$DE := AB \qquad AE := BD \qquad CE := AC - AE$

$$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$$

**$EOR_L$ "External similarity point Origin to center of Radius Large"**

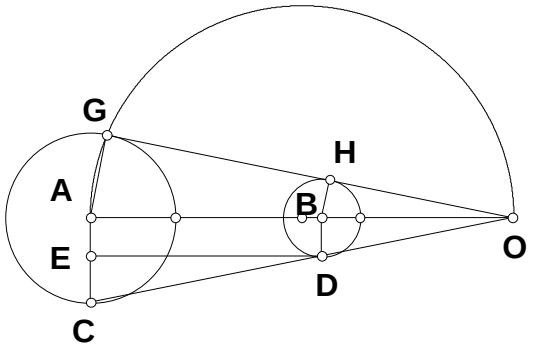
$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right) \qquad EOR_L = 10.667$$

**What is the length of line (OG) tangent to both circles?**

$AG := AC \qquad GO := \sqrt{AO^2 - AG^2} \qquad GO = 9.888$

**And what is the formula?**
**$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"**

$$EOT_{LR} := R_L \cdot \frac{\sqrt{\left(R_L - R_S + D\right) \cdot \left(-R_L + R_S + D\right)}}{R_L - R_S} \qquad EOT_{LR} = 9.888$$




**What is the length of the line tangent to the least circle (HO)?**

$BH := BD$ $BO := AO - AB$ $BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

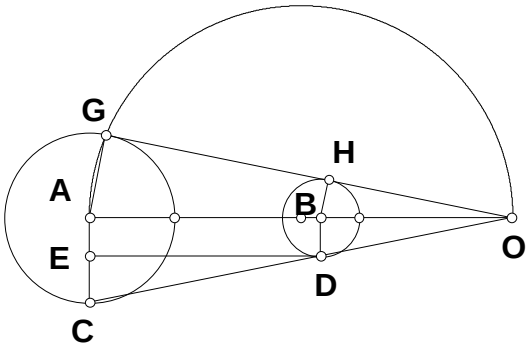
$HO = 2.472$

**And what is the formula?**

**$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"**

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S} \qquad EOT_{SR} = 2.472$$

**Lastly what is the length of line from tangent to tangent of these circles?**



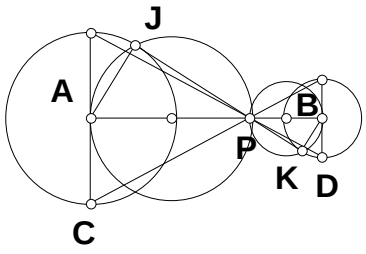

$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

**And what is the formula?**
**ETT "Tangent to Tangent"**

$$ETT := \sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}$$

$ETT = 7.416$




**I will now turn my attention to the point P, the internal similarity point.**

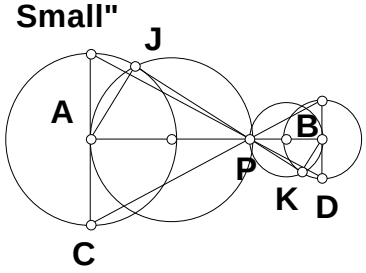
$$AP := \frac{AB \cdot AC}{AC + BD} \qquad AP = 6.4$$

**$IOR_L$ "Internal similarity point to center of Radius Large"**

$$IOR_L := D \cdot \frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

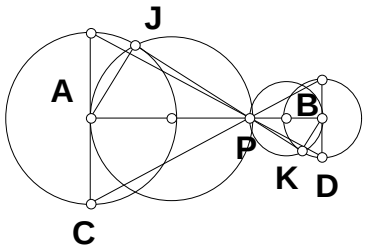
**$IOR_S$ "Internal similarity point to center of Radius Small"**




$$IOR_S := D \cdot \frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$

$AJ := AC \qquad BK := BD \qquad JP := \sqrt{AP^2 - AJ^2} \qquad JP = 4.996$

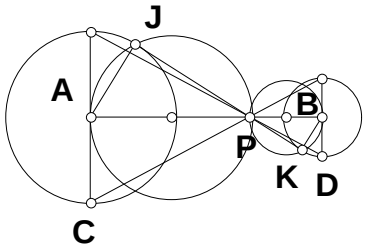
**$IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"**




$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996 \qquad KP := \sqrt{BP^2 - BK^2} \qquad KP = 1.249$

**$IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"**



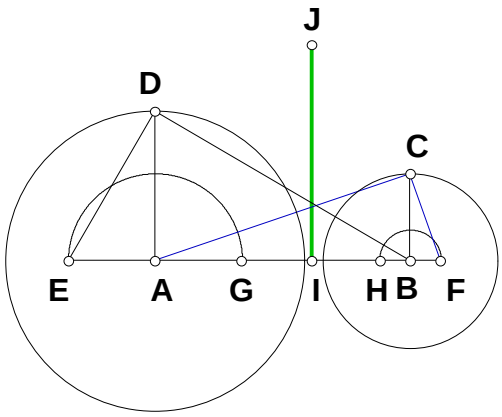

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249 \qquad JK := JP + KP \qquad JK = 6.245$

**ITT "Internal similarity point Tangent to Tangent"**

$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)} \qquad ITT = 6.245$$

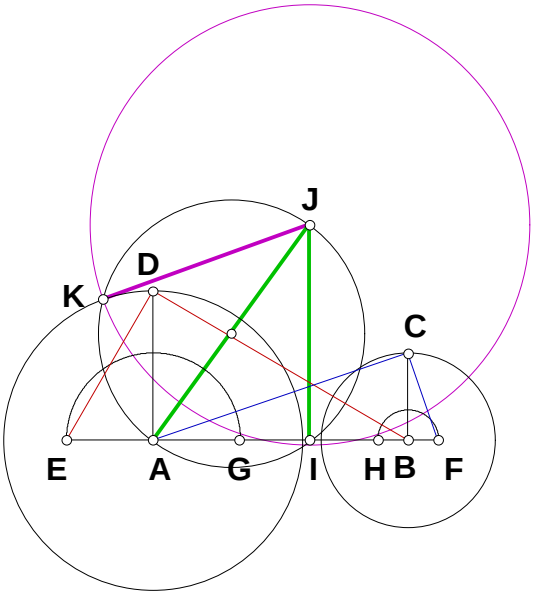
# 04/27/94 The Chordal or Power Line of two Circles




**The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**
**Given two circles find their chordal or power line given just their radius and difference between their centers.**

$AB := 1.323 \quad AD := .771 \quad BC := .448 \quad AE := \frac{AD^2}{AB} \quad BF := \frac{BC^2}{AB} \quad GH := AB - (AE + BF)$

$GI := \frac{GH}{2} \quad AI := AE + GI \quad BI := BF + GI$




$D := AB \quad R_1 := AD \quad R_2 := BC$

$$AI - \frac{\left(R_1^2 + D^2 - R_2^2\right)}{2 \cdot D} = 0$$

$$BI - \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D} = 0$$

**If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.**

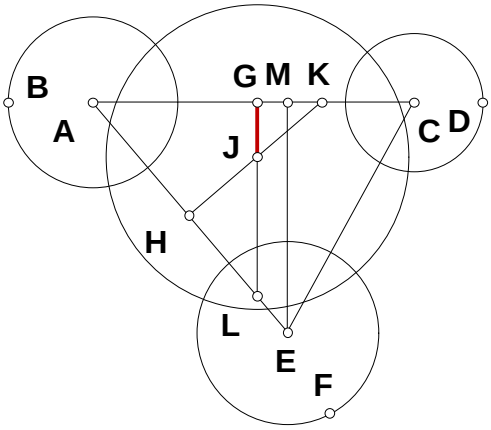
$IJ := 1.112 \quad AJ := \sqrt{AI^2 + IJ^2} \quad AK := AD$

$JK := \sqrt{AJ^2 - AK^2} \quad P := IJ$

$$JK - \frac{\sqrt{R_1^4 - 2 \cdot R_1^2 \cdot D^2 - 2 \cdot R_1^2 \cdot R_2^2 + D^4 - 2 \cdot R_2^2 \cdot D^2 + R_2^4 + 4 \cdot P^2 \cdot D^2}}{2 \cdot D} = 0$$

# 04/28/94 Power Point

**Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the formula for the Power Point and the Length of the resultant Tangent.**




$AB := .438 \quad CD := .354 \quad EF := .471$

$AC := 1.667 \quad AE := 1.559 \quad CE := 1.357$

$R_1 := AB \quad R_2 := CD \quad R_3 := EF$

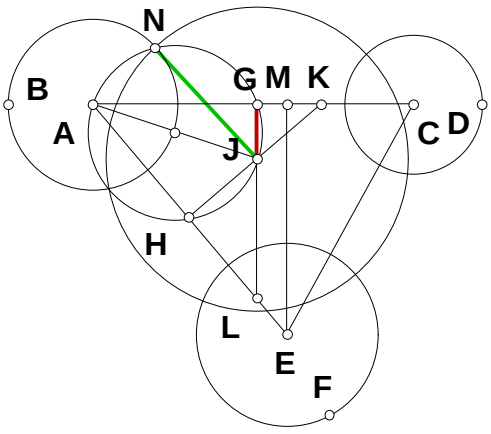
$D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{R_1^2 + D_1^2 - R_2^2}{2 \cdot D_1}$$

$$AH := \frac{R_1^2 + D_2^2 - R_3^2}{2 \cdot D_2}$$

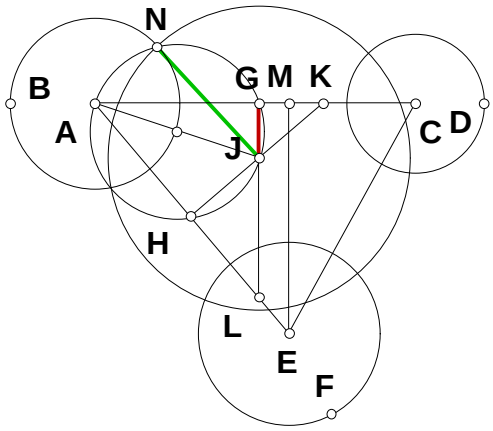
$$AM := \frac{D_2^2 + D_1^2 - D_3^2}{2 \cdot D_1} \qquad EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{1}{2} \cdot \frac{\left[\begin{array}{l}\left(D_2^2 \cdot D_1^2 - D_2^2 \cdot R_1^2 + D_2^2 \cdot R_2^2 + D_3^2 \cdot R_1^2 - 2 \cdot D_1^2 \cdot R_3^2\right) \ldots \\ + R_2^2 \cdot D_1^2 - D_3^2 \cdot R_2^2 - D_1^4 + D_3^2 \cdot D_1^2 + D_1^2 \cdot R_1^2\end{array}\right]}{D_1 \cdot \sqrt{-\left(D_2 + D_1 - D_3\right) \cdot \left(D_2 + D_1 + D_3\right) \cdot \left(D_2 - D_1 - D_3\right) \cdot \left(D_2 - D_1 + D_3\right)}} = 0$$



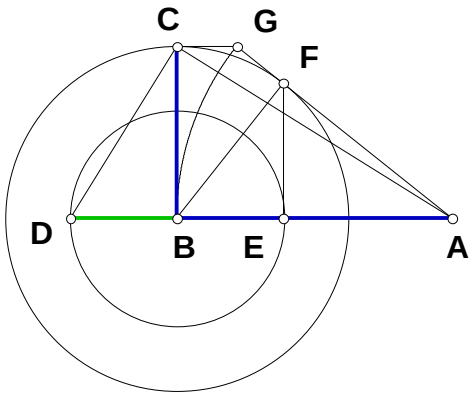

$$AJ := \sqrt{AG^2 + GJ^2} \qquad AN := AB \qquad JN := \sqrt{AJ^2 - AN^2}$$

$JN = 0.786$




$$JN - \sqrt{\frac{-\left[\begin{array}{l}\left(-D_3^2 \cdot R_1^2 \cdot D_1^2 + R_1^2 \cdot D_3^4 + R_2^4 \cdot D_2^2 + D_1^2 \cdot D_3^2 \cdot D_2^2\right) \ldots \\ + \left(R_1^4 \cdot D_3^2 - R_2^2 \cdot D_1^2 \cdot D_2^2 + R_2^2 \cdot D_2^4 + R_1^2 \cdot R_2^2 \cdot D_1^2\right) \ldots \\ + \left(-D_2^2 \cdot D_1^2 \cdot R_3^2 - D_1^2 \cdot R_3^2 \cdot R_1^2 - D_1^2 \cdot R_3^2 \cdot D_3^2 - D_1^2 \cdot R_3^2 \cdot R_2^2\right) \ldots \\ + \left(-D_2^2 \cdot R_1^2 \cdot D_3^2 + R_3^2 \cdot D_3^2 \cdot R_2^2 - R_1^2 \cdot R_2^2 \cdot D_2^2 + D_1^4 \cdot R_3^2\right) \ldots \\ + \left(-R_1^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot R_3^2 - D_3^2 \cdot R_1^2 \cdot R_3^2\right) \ldots \\ + D_2^2 \cdot R_1^2 \cdot R_3^2 + D_1^2 \cdot R_3^4\end{array}\right]}{\left(D_2 - D_3 + D_1\right) \cdot \left(D_2 + D_3 + D_1\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(-D_2 - D_3 + D_1\right)}} = 0$$

# 04/30/94 Division $N^2$




$N_1 := 5$ $N_2 := 2$

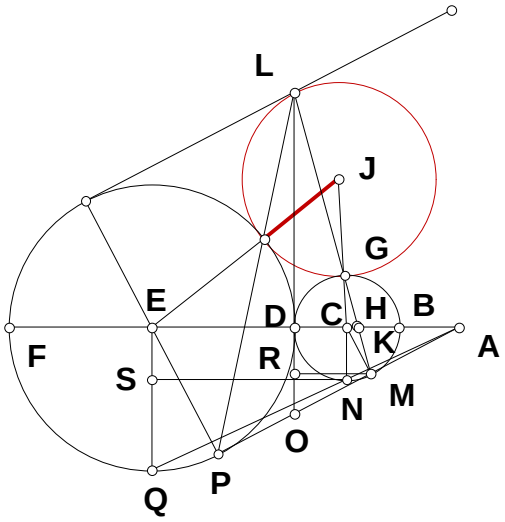
$AB := N_1$ $BC := N_2$

$AC := \sqrt{AB^2 + BC^2}$ $CD := \frac{BC \cdot AC}{AB}$

$BD_1 := \sqrt{CD^2 - BC^2}$

$BD_2 := \frac{BC^2}{AB}$ $BD_1 - BD_2 = 0$

## 05/01/94 Two Circles And A Parallel




**Given the radius of two tangent circles find the radius of the third that is tangent two the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.**

$R_1 := 3 \qquad R_2 := 2$

$DE := R_1 \qquad BC := R_2 \qquad CN := BC$

$EQ := DE \qquad CD := BC \qquad CE := CD + DE$

$ES := CN \qquad NS := CE \qquad SQ := EQ - ES$

$AE := \frac{NS \cdot EQ}{SQ} \qquad AD := AE - DE \qquad EP := DE$

$AP := \sqrt{AE^2 - EP^2} \qquad DO := \frac{EP \cdot AD}{AP}$

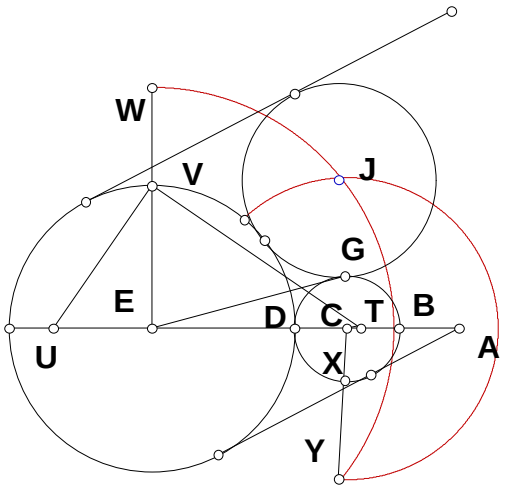
$DL := \frac{DO \cdot DE}{CD} \qquad AC := AD - CD \qquad AM := \frac{AP \cdot AC}{AE} \qquad AO := \frac{AE \cdot AD}{AP} \qquad MO := AO - AM$

$MR := \frac{AD \cdot MO}{AO} \qquad RO := \frac{DO \cdot MR}{AD} \qquad DR := DO - RO \qquad LR := DR + DL \qquad ML := \sqrt{MR^2 + LR^2}$

$DK := \frac{MR \cdot DL}{LR} \qquad CK := DK - CD \qquad CH := \frac{LR \cdot CK}{ML} \qquad CM := BC \qquad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \qquad GL := ML - MG \qquad GJ := \frac{CM \cdot GL}{MG}$

**The Algebraic name for GJ suggests a simpler method of construction.**




$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad GJ - R_3 = 0$

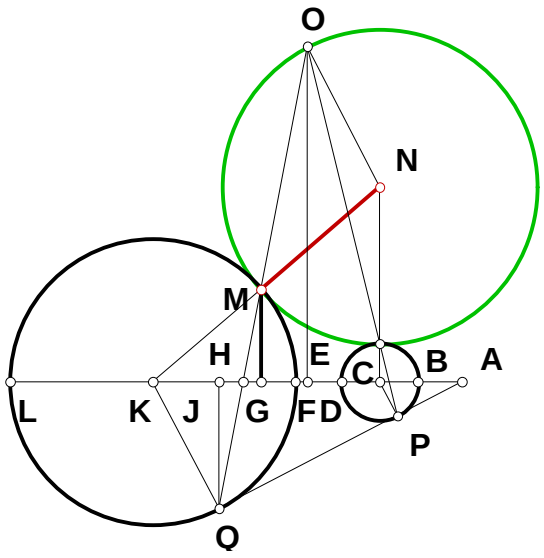
$ET := 4 \cdot BC \qquad EV := DE \qquad EU := \frac{DE^2}{ET}$

$VW := EU \qquad XY := EU \qquad EV := DE \qquad CX := BC$

$EW := EV + VW \qquad CY := CX + XY$

$GJ_2 := EU \qquad GJ - GJ_2 = 0$

# 05/04/94 Two Circles And A Tangent




**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**

$R_1 := 5 \qquad R_2 := 4 \qquad D := 3$

$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK$

$$AK := \frac{D \cdot R_1}{R_1 - R_2} \qquad EK := \frac{R_1^2 + D^2 - R_2^2}{2 \cdot D}$$

$$AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$$

$N_1 := 4 \qquad N_2 := 8 \qquad FG := FL \cdot \frac{N_1}{N_2} \quad GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK}$

$AF := AK - FK \quad FJ := AJ - AF \quad JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG$

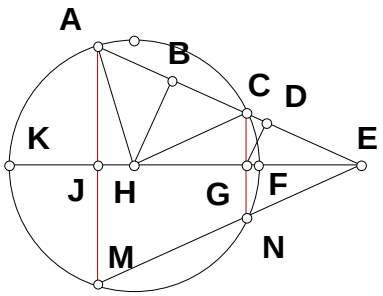
$$QM := \sqrt{(JQ + GM)^2 + GJ^2} \qquad GH := \frac{GJ \cdot GM}{JQ + GM} \qquad HM := \frac{QM \cdot GM}{JQ + GM} \qquad EF := EK - FK$$

$$EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \qquad MO := |HO - HM| \quad KM := FK \quad MN := \frac{KM \cdot MO}{QM}$$

**The Algebraic name of the radius of the tangent circle,**

$$MN - \left| \frac{N_2 \cdot R_2^2 + 2 \cdot N_2 \cdot R_1 \cdot D - N_2 \cdot R_1^2 - N_2 \cdot D^2 - 4 \cdot R_1 \cdot N_1 \cdot D}{2 \cdot (-N_2 \cdot R_1 + N_2 \cdot R_2 + D \cdot N_2 - 2 \cdot N_1 \cdot D)} \right| = 0$$

# 05/06/94 A Ratio In Trisection




## What is AK to CG?

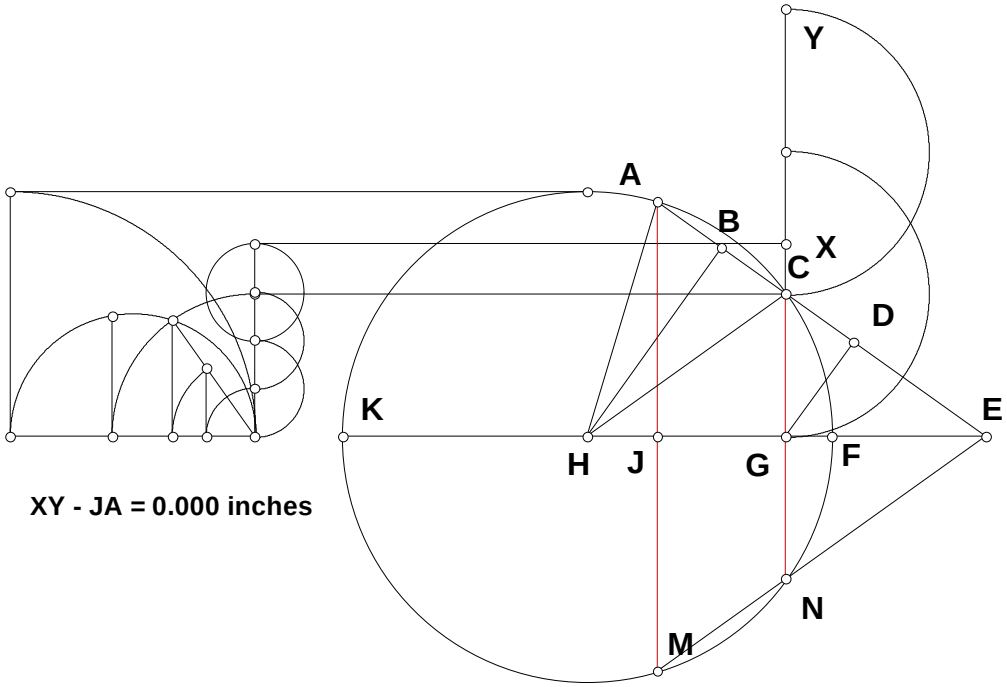
$N_1 := 3 \qquad N_2 := 8 \qquad FH := 1 \qquad CE := FH$

$CG := \frac{N_1}{2 \cdot N_2} \cdot \sqrt{2} \cdot CE \qquad EG := \sqrt{CE^2 - CG^2} \qquad CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2} \qquad EH := 2 \cdot EG \qquad BH := \frac{DG \cdot EH}{EG} \qquad CH := FH$

$BC := \sqrt{CH^2 - BH^2} \qquad AC := 2 \cdot BC \qquad AE := AC + CE \qquad AJ := \frac{CG \cdot AE}{CE} \qquad 3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0$

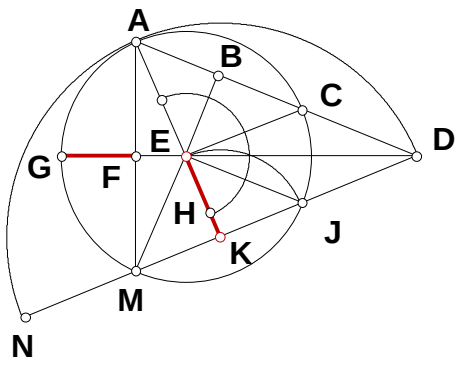
$$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$$




XY - JA = 0.000 inches

# 05/07/94 A Trisection Ratio

## In trisection, what is the ratio of FG/EK?




$N := 2$

$AE := 3 \quad EH := \frac{AE}{2} \quad HK := AE \cdot N \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

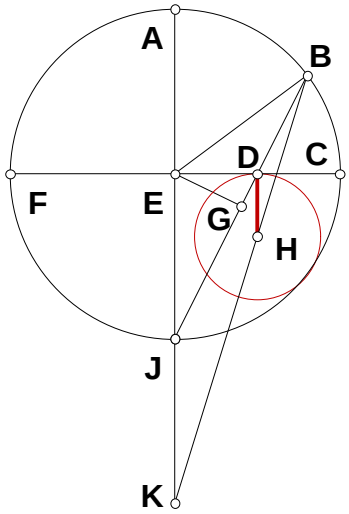
### Algebraic Names,

$$FG - \left[ AE + \frac{AE \cdot (N+1) \cdot (2 \cdot N - 1)}{(2N+1) \cdot \sqrt{(N+1) \cdot (2N+1)}} \right] = 0 \qquad EK - \left( \frac{1}{2} \cdot AE + AE \cdot N \right) = 0$$

$$\frac{FG}{EK} - 2 \cdot \frac{\left[ (2N+1) \cdot \sqrt{(N+1) \cdot (2N+1)} + (N+1) \cdot (2 \cdot N - 1) \right]}{\left[ (2N+1)^2 \cdot \sqrt{(N+1) \cdot (2N+1)} \right]} = 0$$

## 05/16/94A Tangent Diameter and Circles

**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.**




$$CF := 1 \qquad CE := \frac{CF}{2} \qquad N_1 := 4 \qquad N_2 := 8 \qquad N_3 := 1$$

$$DE := CE \cdot \frac{N_1}{N_2} \qquad EJ := CF \cdot N_3 \qquad DJ := \sqrt{DE^2 + EJ^2}$$

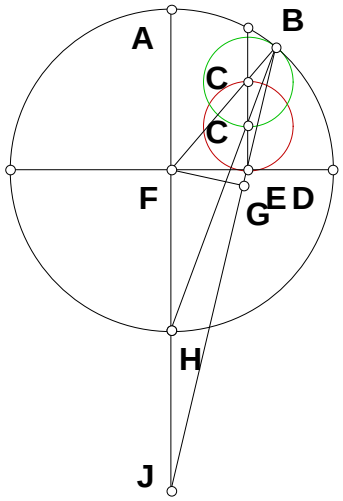
$$JG := \frac{EJ^2}{DJ} \qquad BE := CE \qquad EG := \sqrt{EJ^2 - JG^2}$$

$$BG := \sqrt{BE^2 - EG^2} \qquad BJ := BG + JG \qquad JK := CE$$

$$BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$$

$$DH - \frac{\sqrt{N_1^2 + 4 \cdot N_3^2 \cdot N_2^2 - 4 \cdot N_3^2 \cdot N_1^2} \cdot N_2 - N_1^2}{2 \cdot N_2 \cdot \left( \sqrt{N_1^2 + 4 \cdot N_3^2 \cdot N_2^2 - 4 \cdot N_3^2 \cdot N_1^2} + 4 \cdot N_3^2 \cdot N_2 \right)} = 0$$

# 05/16/94B Tangent Diameter and Circles




**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.**

$N_1 := 4 \quad N_2 := 8 \quad N_3 := 2 \quad AH := 1 \quad AF := \frac{AH}{2}$

$DF := AF \quad DE := DF \cdot \frac{N_1}{N_2} \quad AJ := AH \cdot N_3$

$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$

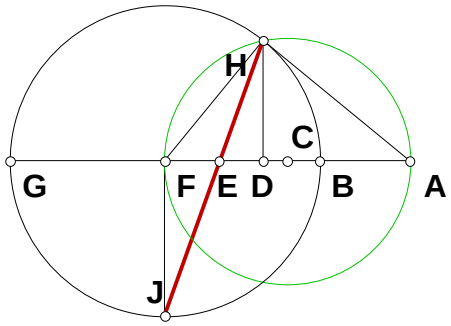
$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad CE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot CE\right]^2} \quad BC - CE = 0 \quad CE = 0.10286$

$$CE - \frac{-\left(-\sqrt{N_2^2 + 8 \cdot N_3^2 \cdot N_2 \cdot N_1 - 4 \cdot N_3^2 \cdot N_1^2 - 8 \cdot N_3 \cdot N_2 \cdot N_1 + 4 \cdot N_3 \cdot N_1^2} \cdot N_2 \ldots + N_2^2 - 2 \cdot N_2 \cdot N_1 + N_1^2\right)}{\left[2 \cdot N_2 \cdot \left(\sqrt{N_2^2 + 8 \cdot N_3^2 \cdot N_2 \cdot N_1 - 4 \cdot N_3^2 \cdot N_1^2 - 8 \cdot N_3 \cdot N_2 \cdot N_1 + 4 \cdot N_3 \cdot N_1^2} \ldots + 4 \cdot N_3^2 \cdot N_2 - 4 \cdot N_3 \cdot N_2 + N_2\right)\right]} = 0$$

# 10/27/94 Trivial Method Square Root

## AE is the square root of AB x AG.




$N := 5$ $AB := 1$

$AG := AB \cdot N$ $BG := AG - AB$

$BF := \frac{BG}{2}$ $AF := AB + BF$ $FH := BF$ $FD := \frac{FH^2}{AF}$

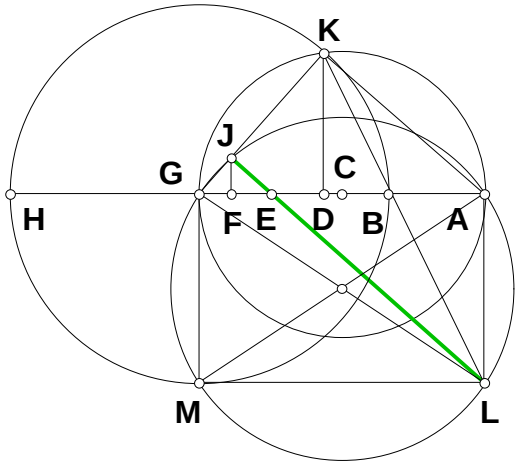
$AD := AF - FD$ $DF := AF - AD$ $BD := AD - AB$

$DG := BG - BD$ $FJ := BF$ $DH := \sqrt{BD \cdot DG}$

$DE := \frac{DF \cdot DH}{DH + FJ}$ $AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$

# 10/28/94 Trivial Method Square Root




$N := 4.917 \quad AB := .5 \quad AH := AB \cdot N \quad BH := AH - AB$

$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

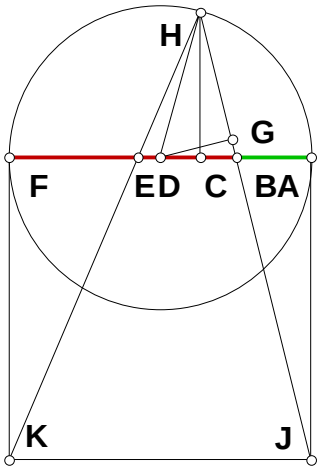
$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad JL := \sqrt{GL^2 - GJ^2}$

$FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$

# 10/31/94 Square Root of a Segment

**Given a unit take the square root of a segment.**




$N := 11 \qquad AB := 1$

$AF := AB \cdot N \qquad BF := AF - AB \qquad AD := \frac{AF}{2}$

$AJ := AF \qquad FK := AF \qquad BD := AD - AB \qquad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \qquad DH := AD \qquad DG := \frac{AJ \cdot BD}{BJ} \qquad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \qquad BC := \frac{AB \cdot (BG + GH)}{BJ} \qquad AC := AB + BC$

$CF := AF - AC \qquad CH := \sqrt{AC \cdot CF} \qquad CE := \frac{CF \cdot CH}{(CH + FK)}$

$EF := CF - CE \qquad BE := BC + CE \qquad \sqrt{EF \cdot AB} - BE = 0$

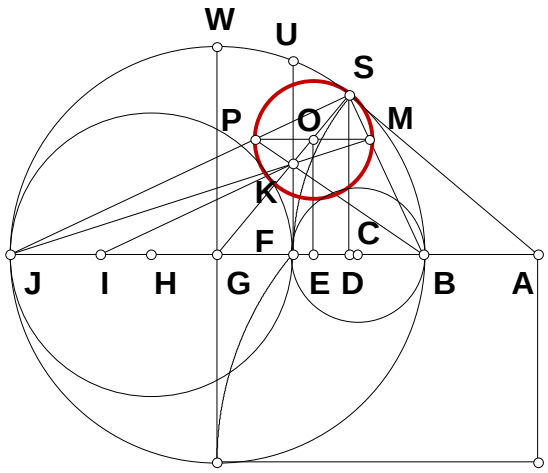
$DF := BF - BE$

$$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$$

$$DF - \frac{2\,N^2 - 2 \cdot N + 1 - \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$$

# 12/24/94 Power Line At Square Root

**In this square root figure, what is the Algebraic name of the tangent circle in red?**




$N := 5 \qquad AB := 1 \qquad AJ := AB \cdot N$

$AF := \sqrt{AB \cdot AJ} \qquad BJ := AJ - AB \qquad BG := \frac{BJ}{2}$

$AG := AB + BG \qquad GS := BG \qquad DG := \frac{GS^2}{AG}$

$FG := AG - AF \qquad BD := BG - DG \qquad DJ := BJ - BD$

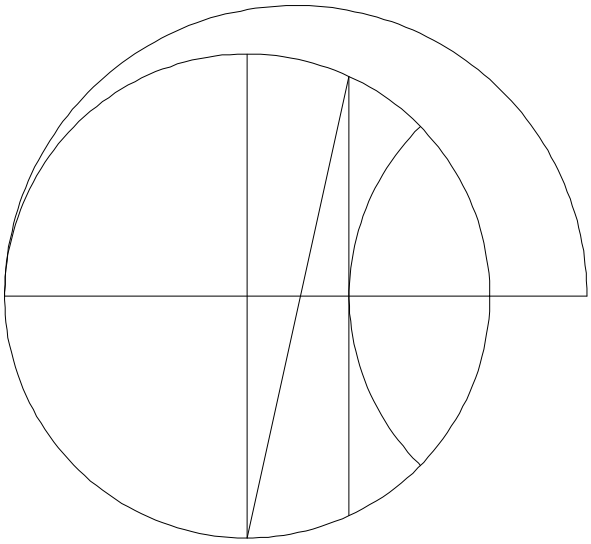
$DS := \sqrt{BD \cdot DJ} \qquad FK := \frac{DS \cdot FG}{DG} \qquad BF := AF - AB$

$BK := \sqrt{BF^2 + FK^2} \qquad FI := \frac{DJ \cdot FK}{DS} \qquad BI := FI + BF$

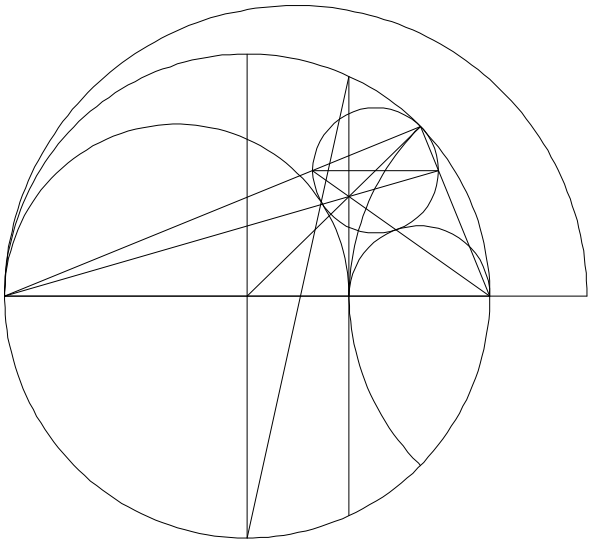
$BP := \frac{BK \cdot BJ}{BI} \qquad KP := BP - BK \qquad MP := \frac{BJ \cdot KP}{BK} \qquad OS := \frac{MP}{2}$

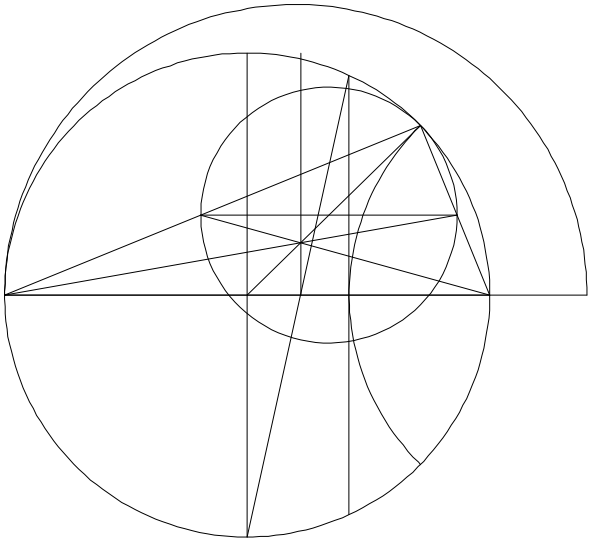
$$OS - \frac{-2 \cdot N^2 + 2 \cdot N - \sqrt{N} + N^{\left(\frac{5}{2}\right)}}{2 \cdot \left[N^2 - \sqrt{N} - N^{\left(\frac{3}{2}\right)} + 1\right]} = 0$$

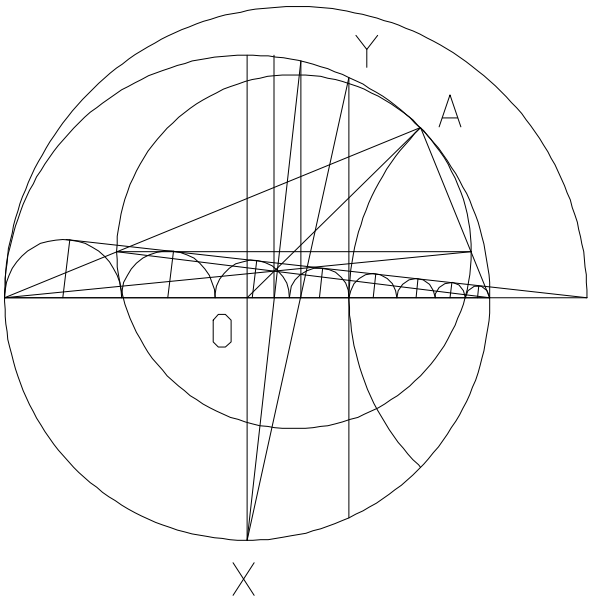
# Two prime exponential series developed through power line progression.



I will present a series of plates to explain the process. The process can be infinitly repeated, supposing you had the tools to do it with.

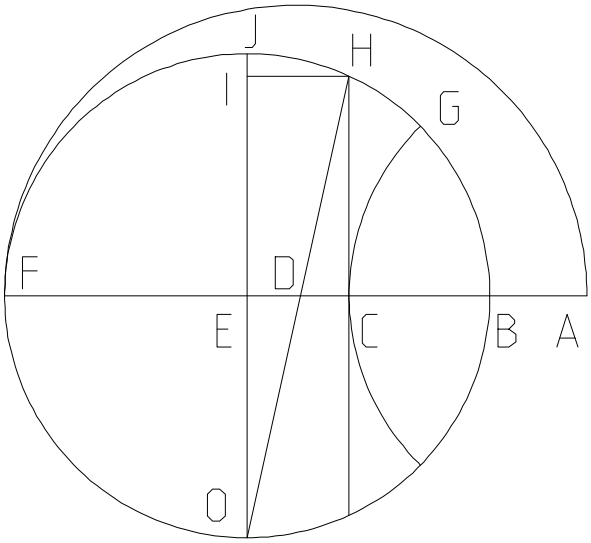







It is clear how OA uses the power line XY to provide a 2 prime exponential series.

Possible Problem: From a similarity point outside of a circle, place some 2 prine sequence of smaller circles on the larger circles diameter, all tangent in sequence.



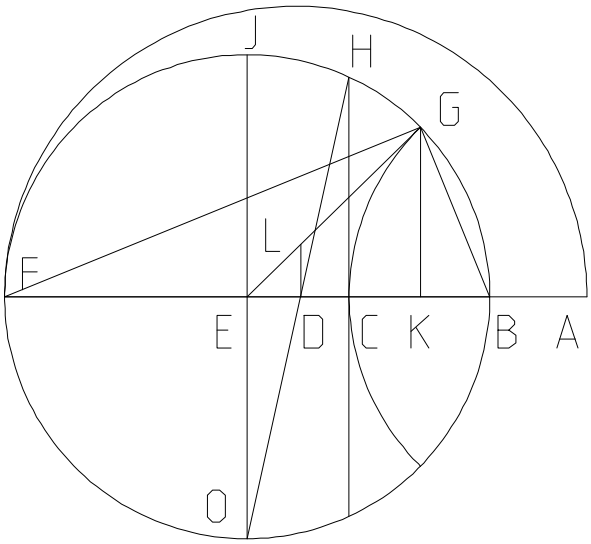

$AB := 1 \quad BF := 5 \quad AF := AB + BF$

$AC := \sqrt{AB \cdot AF} \qquad BE := \frac{BF}{2} \quad EO := BE$

$BC := AC - AB \qquad CF := BF - BC$

$CH := \sqrt{BC \cdot CF} \quad EI := CH \quad CE := BE - BC$

$HI := CE \quad IO := EO + EI \quad DE := \frac{HI \cdot EO}{IO}$

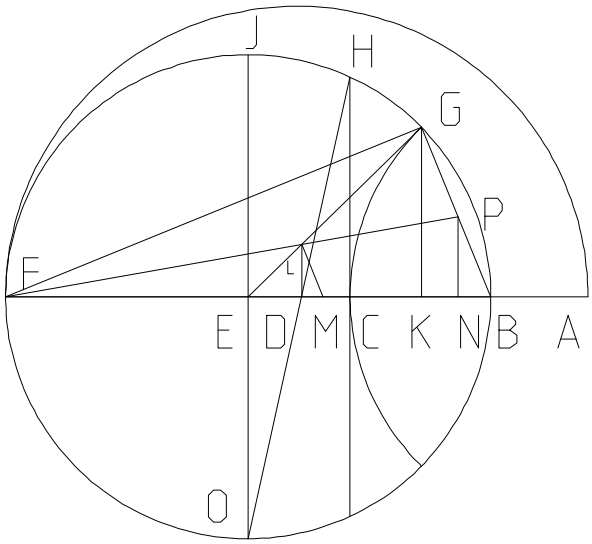



See 12_26_94.MCD for next equation.

$$GK := \frac{BF \cdot \left(\sqrt{AB} \cdot \sqrt{AB + BF}\right)}{(2 \cdot AB + BF)}$$

$EG := BE \quad EK := \sqrt{EG^2 - GK^2}$

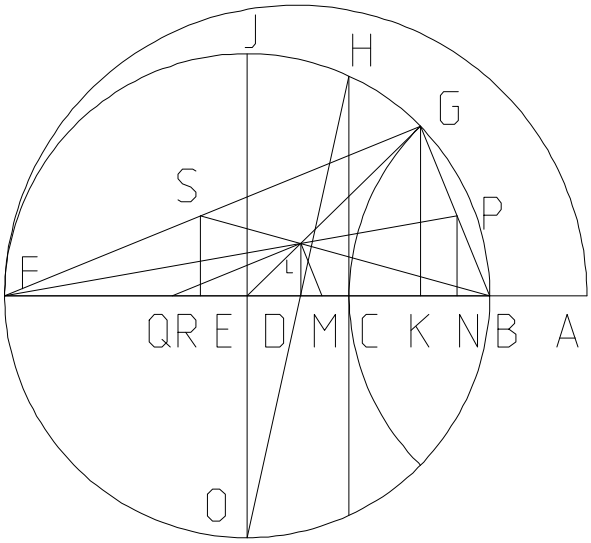
$DL := \frac{GK \cdot DE}{EK}$




$KN := BE - EK \quad DM := \frac{KN \cdot DE}{EK}$

$EF := BE \quad FM := EF + DM + DE$

$BN := \frac{DM \cdot BF}{FM} \quad NP := \frac{DL \cdot BF}{FM}$



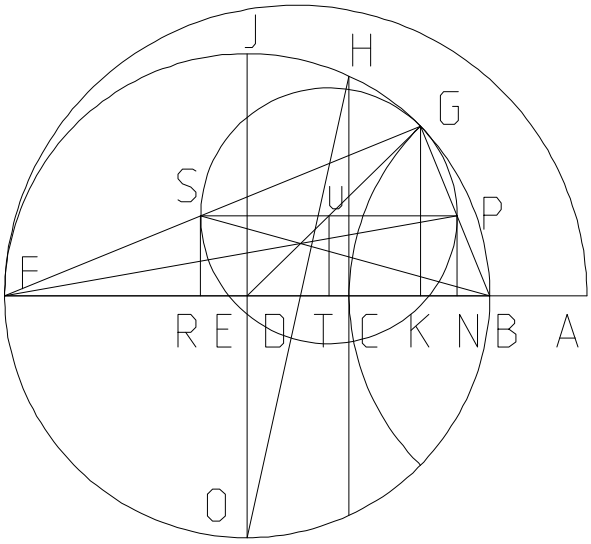

$KF := EK + EF \quad DQ := \frac{KF \cdot DL}{GK}$

$BD := BE - DE \quad BQ := BD + DQ$

$BR := \frac{BD \cdot BF}{BQ} \quad RS := \frac{DL \cdot BR}{BD}$

Are RS and NP equal?

$RS - NP = 0$




$TU := NP \quad ET := \frac{EK \cdot TU}{GK} \quad NR := BR - BN$

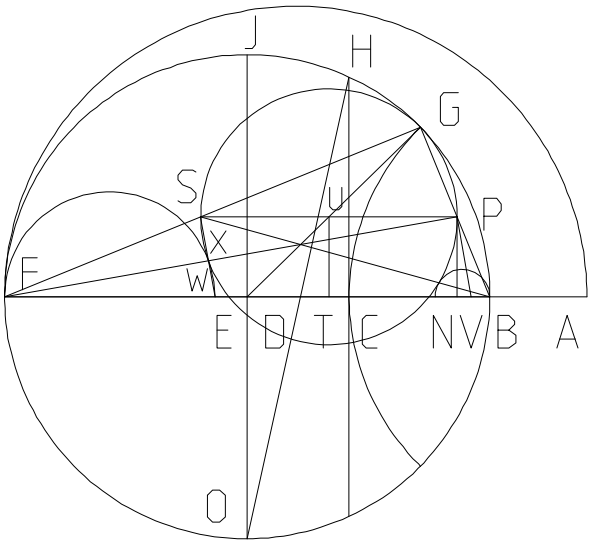
$EN := BE - BN \quad NT := EN - ET$

$PS := NR \quad PU := NT \quad EU := \sqrt{ET^2 + TU^2}$

Is NT half of NR? $\frac{NR}{NT} = 2$

Does GU = PU? $GU := EG - EU$

$GU - PU = 0$



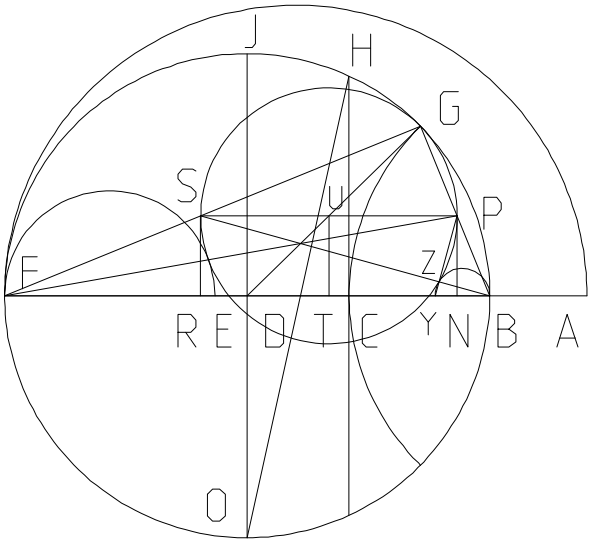

$BT := BN + NT \quad FN := BF - BN \quad FP := \sqrt{NP^2 + FN^2}$

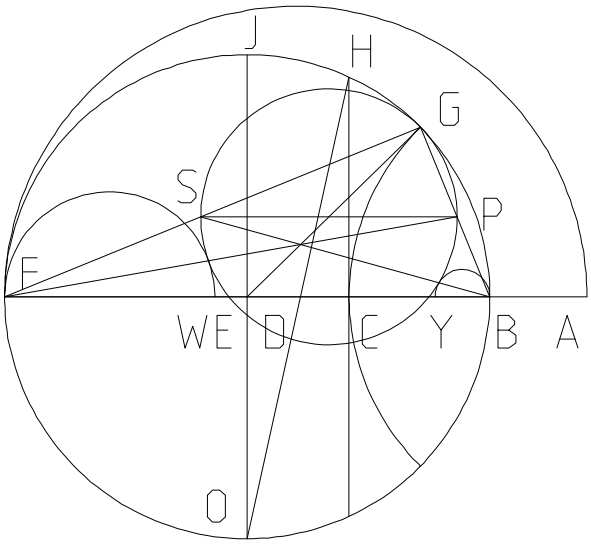
$FV := \frac{FP^2}{FN} \quad PX := \frac{FP \cdot PS}{FV} \quad FX := FP - PX$

$FW := \frac{FV \cdot FX}{FP} \quad BW := BF - FW \quad AW := AB + BW$

Is AW a quarter root of AB, AF?

$\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AW = 0$



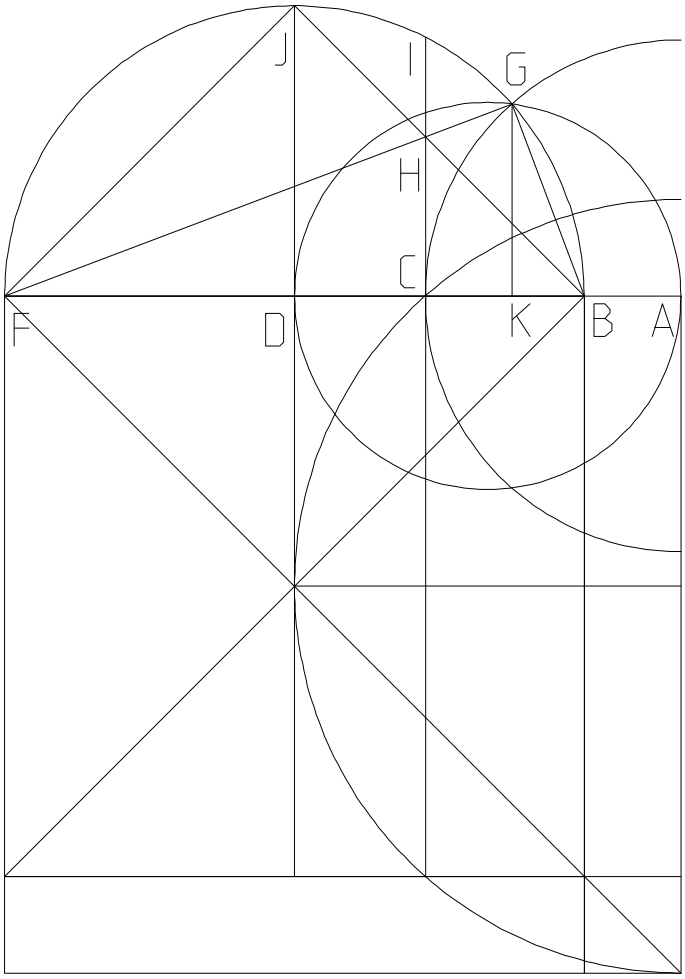

$BS := \sqrt{BR^2 + RS^2}$ $SZ := \frac{BR \cdot PS}{BS}$

$BZ := BS - SZ$ $BY := \frac{PS \cdot BZ}{SZ}$

$AY := AB + BY$

Is AY a quarter root of AB, AF?

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AY = 0$




Does H and G have a constant relationship?

$\delta := 1 .. 1000 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$BF := 6 \quad BD := \frac{BF}{2} \quad AF_\delta := AB_\delta + BF$

$AC_\delta := \sqrt{AB_\delta \cdot AF_\delta} \quad DJ := BD \quad DF := BD$

$BC_\delta := AC_\delta - AB_\delta \quad CD_\delta := BD - BC_\delta$

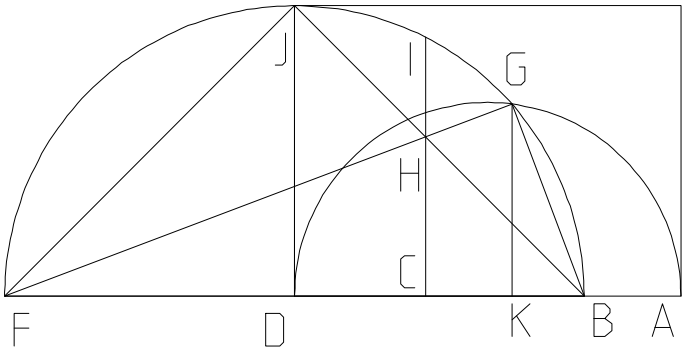
$CH_\delta := \frac{DJ \cdot BC_\delta}{BD} \quad AG_\delta := AC_\delta$

$AD_\delta := AB_\delta + BD \quad AK_\delta := \frac{(AG_\delta)^2}{AD_\delta}$

$GK_\delta := \sqrt{(AG_\delta)^2 - (AK_\delta)^2}$

$BK_\delta := AK_\delta - AB_\delta \quad CK_\delta := BC_\delta - BK_\delta$

$KF_\delta := CK_\delta + CD_\delta + DF$

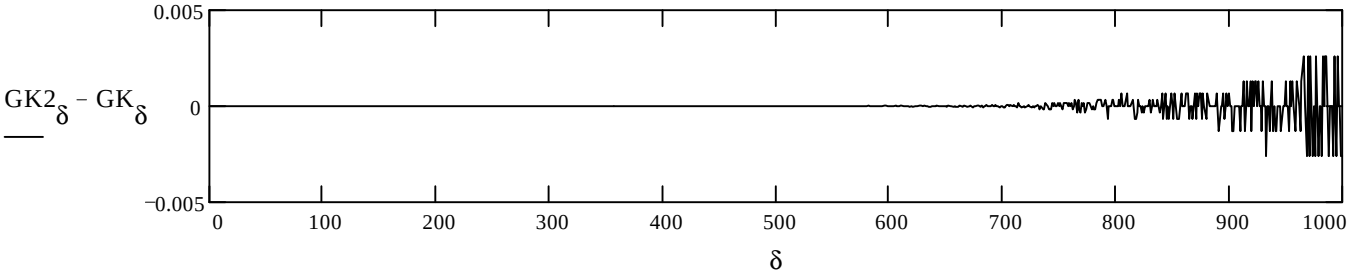


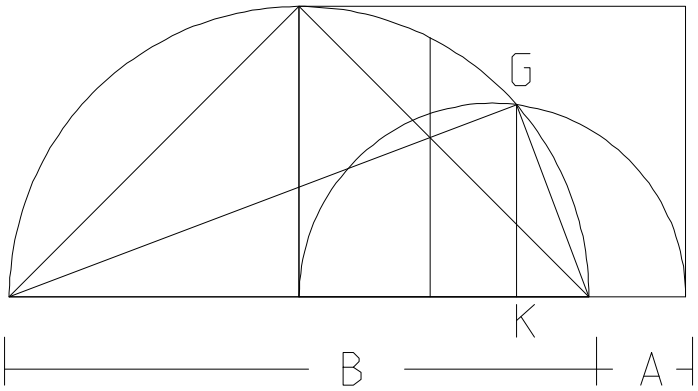

Does FH and FG have identical slopes?

$CF_\delta := CD_\delta + DF$

$GK2_\delta := \frac{CH_\delta \cdot KF_\delta}{CF_\delta}$

Therefore G and H are constantly co-linear.





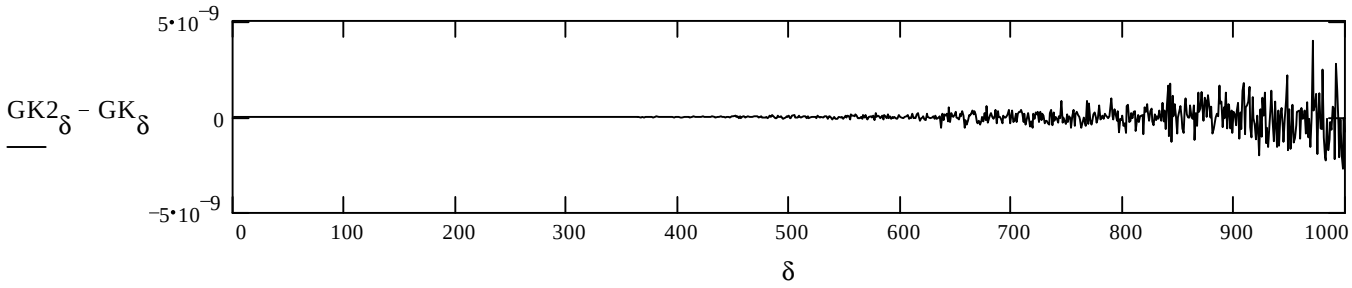

Thus this file can be redone as: "Given BC = $\sqrt{AB \cdot AF}$ and BF, find AB."

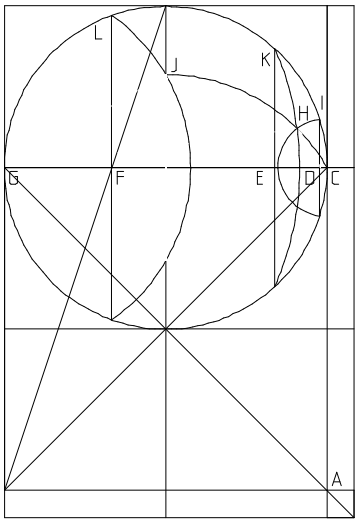
The Formula for GK vs. GK2 demonstrates that the symbolic processor cannot always resolve to simplest form. GK2 is the processors final attempt. An attempt with Mathcad 6 gives the same result.

$A_\delta := AB_\delta \quad B := BF$

$$GK2_\delta := B \cdot \frac{\left[ \left(A_\delta\right)^{\left(\frac{3}{2}\right)} \cdot \sqrt{A_\delta + B} - \left(A_\delta\right)^2 + B \cdot \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} - B \cdot A_\delta \right]}{\left[ \left(2 \cdot A_\delta + B\right) \cdot \left(B - \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} + A_\delta\right) \right]}$$

$$GK_\delta := \frac{B \cdot \left(\sqrt{A_\delta} \cdot \sqrt{A_\delta + B}\right)}{\left(2 \cdot A_\delta + B\right)}$$





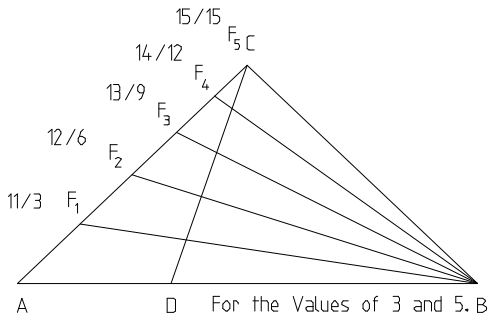

# One Circle

**By John Clark**

# One Square

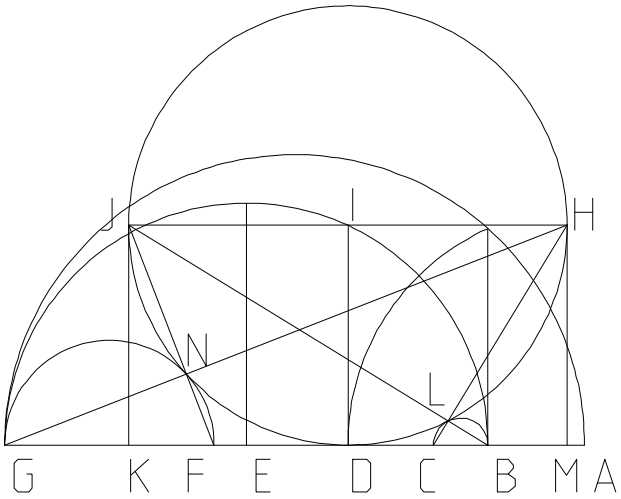
# One Line

## And the Delian Quest




# 1995

# Alternate method for Quad Roots




$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BG := 10 \quad AG_\delta := AB_\delta + BG$

$AD_\delta := \sqrt{AB_\delta \cdot AG_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

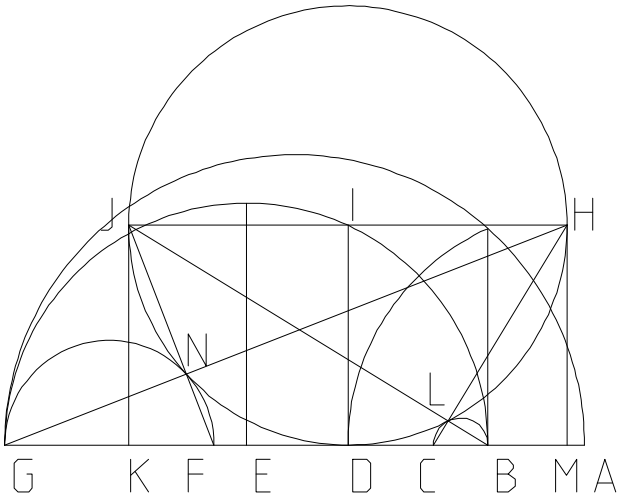
$DG_\delta := BG - BD_\delta \quad DI_\delta := \sqrt{BD_\delta \cdot DG_\delta}$

$HI_\delta := DI_\delta \quad IJ_\delta := DI_\delta \quad HJ_\delta := HI_\delta + IJ_\delta$

$DK_\delta := DI_\delta \quad JK_\delta := DI_\delta \quad BK_\delta := BD_\delta + DK_\delta$

$BJ_\delta := \sqrt{\left(BK_\delta\right)^2 + \left(JK_\delta\right)^2} \quad JL_\delta := \frac{BK_\delta \cdot HJ_\delta}{BJ_\delta}$

$BL_\delta := BJ_\delta - JL_\delta \quad BC_\delta := \frac{HJ_\delta \cdot BL_\delta}{JL_\delta}$




$HM_\delta := DI_\delta \quad DM_\delta := DI_\delta \quad MG_\delta := DM_\delta + DG_\delta$

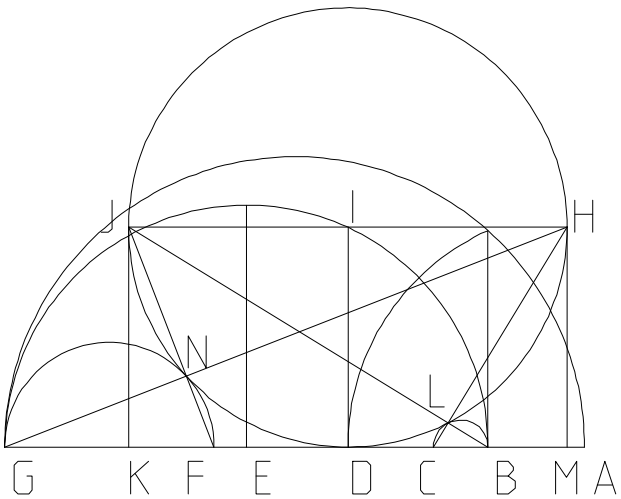
$GH_\delta := \sqrt{\left(MG_\delta\right)^2 + \left(HM_\delta\right)^2} \quad HN_\delta := \frac{MG_\delta \cdot HJ_\delta}{GH_\delta}$

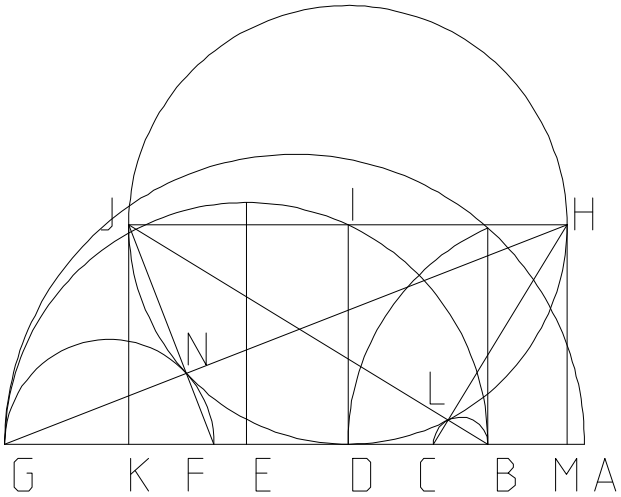
$GN_\delta := GH_\delta - HN_\delta \quad FG_\delta := \frac{HJ_\delta \cdot GN_\delta}{HN_\delta}$

$BF_\delta := BG - FG_\delta$

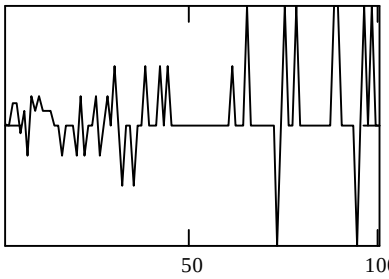
$AF_\delta := AB_\delta + BF_\delta$

$AC_\delta := AB_\delta + BC_\delta$



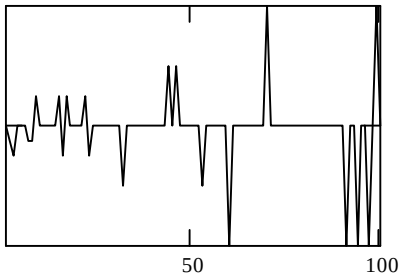



$$\left[ AC_{\delta} - \left[ \left( AB_{\delta} \right)^{3} \cdot AG_{\delta} \right]^{\frac{1}{4}} \right]_{0}$$



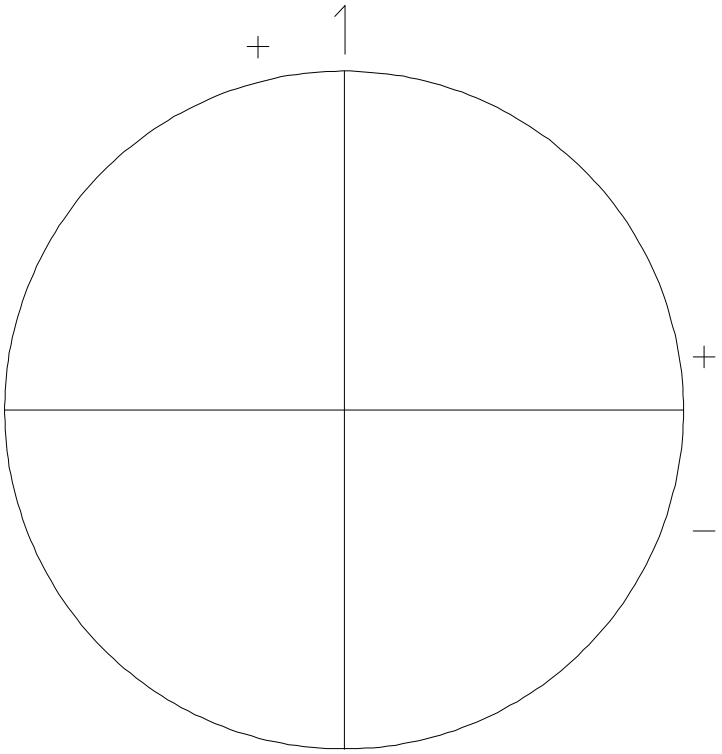

δ

$$\left[ AF_{\delta} - \left[ AB_{\delta} \cdot \left( AG_{\delta} \right)^{3} \right]^{\frac{1}{4}} \right]_{0}$$




δ

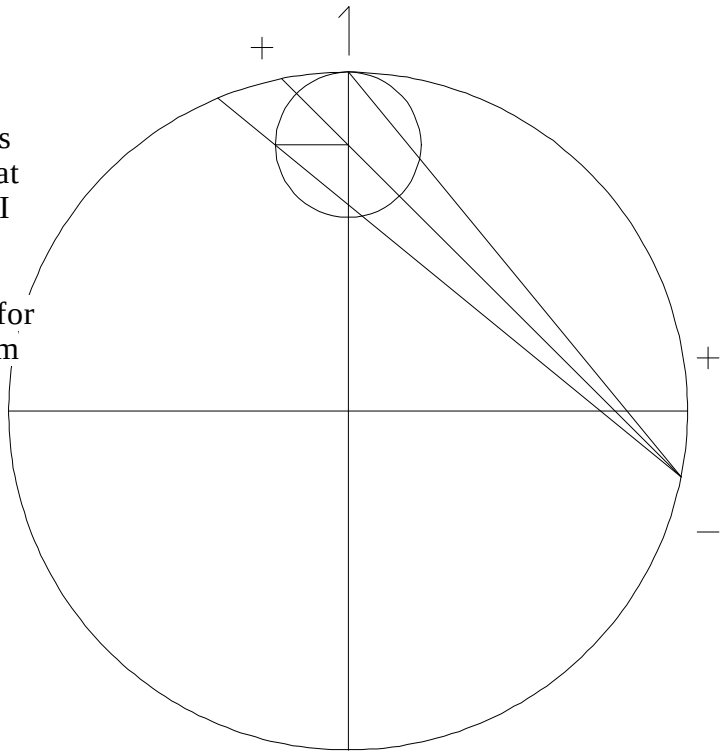
The symbolic processor on my computer could not reduce the chain to the final equations.




Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

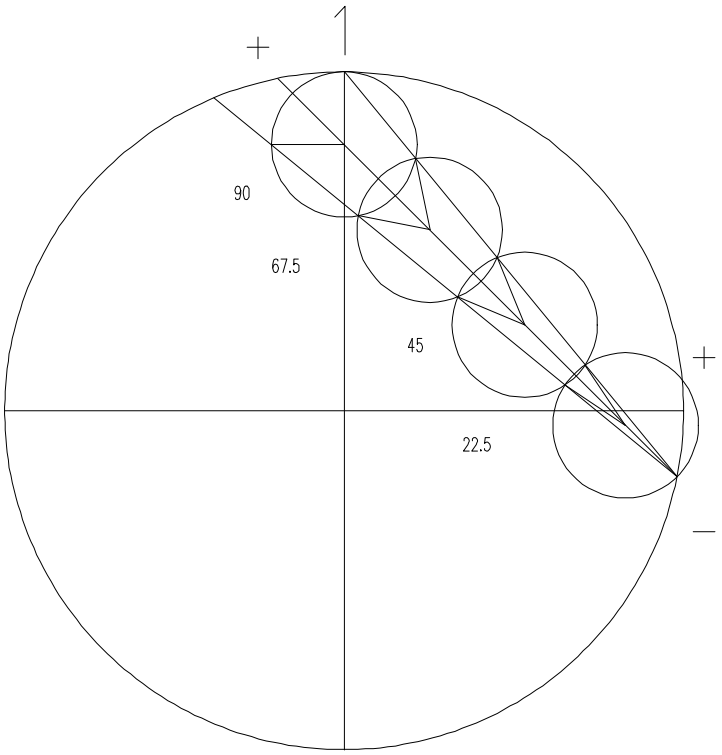
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$




$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

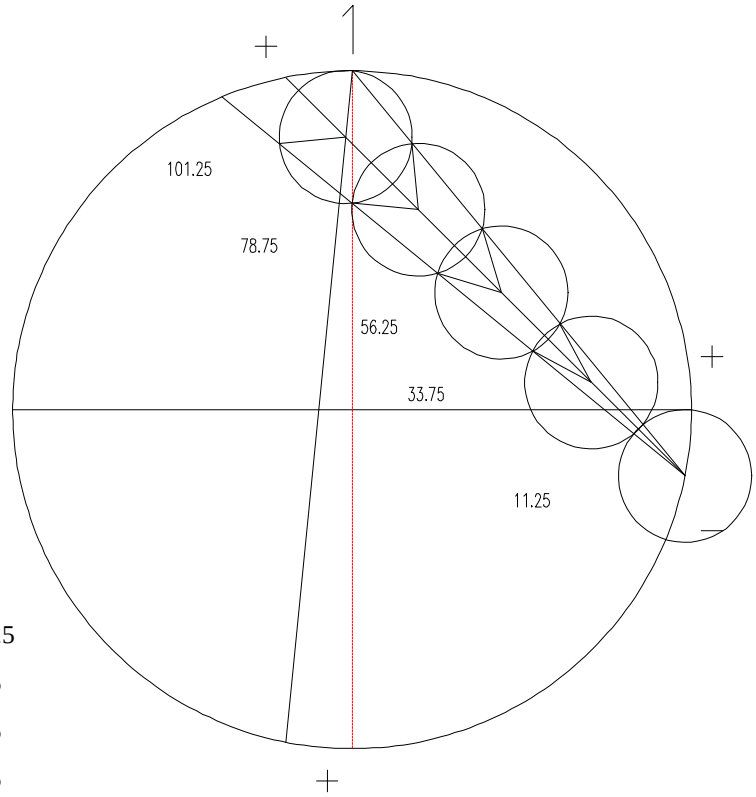
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

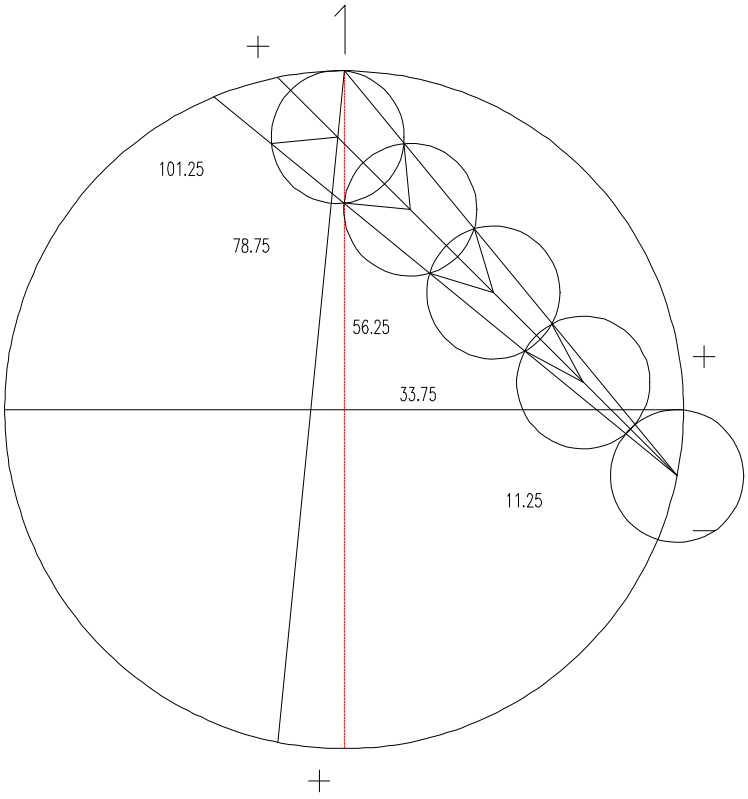
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$




| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

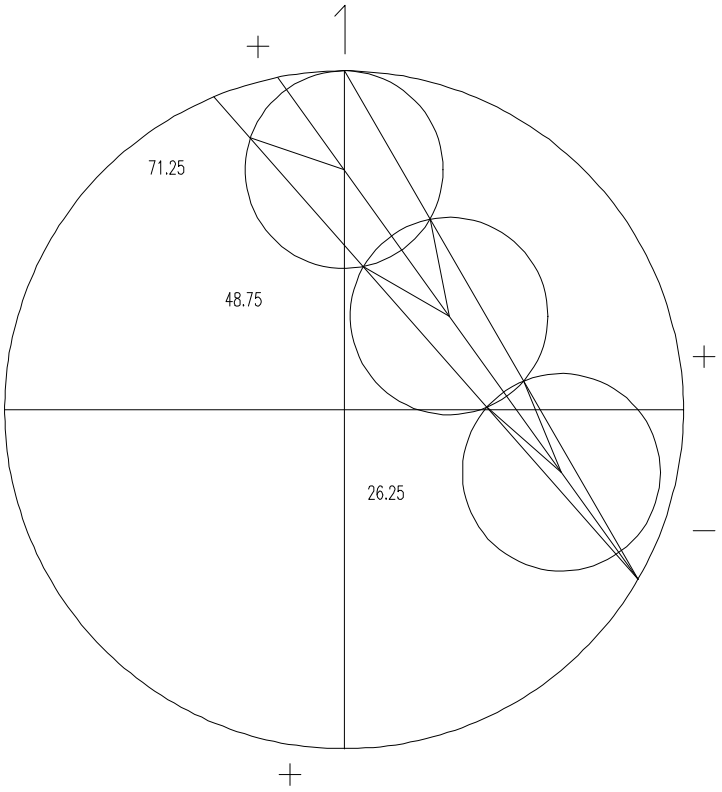
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

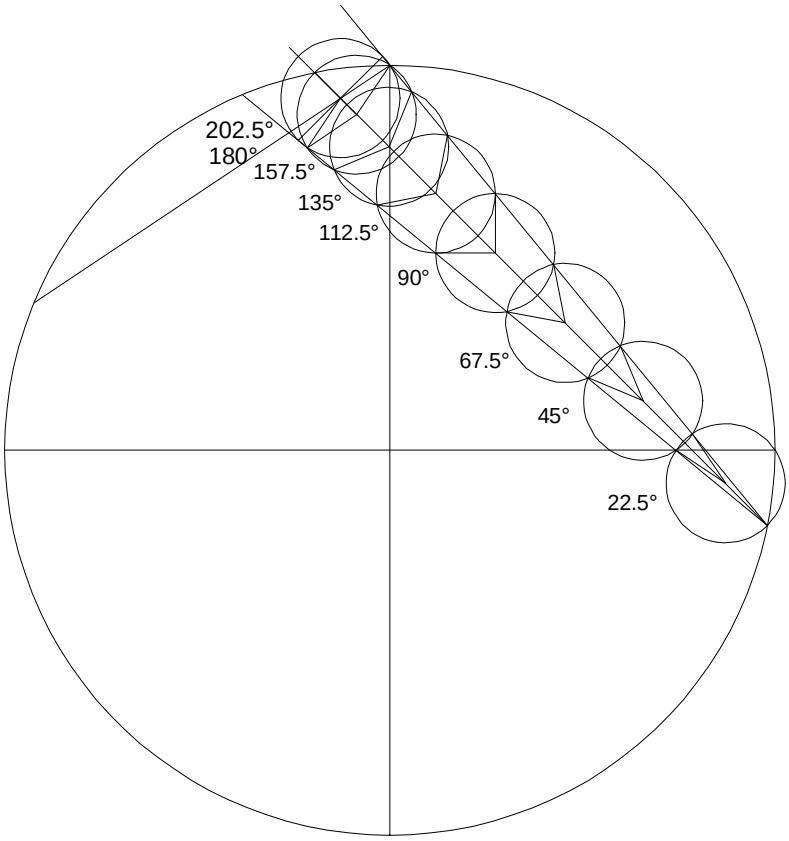
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

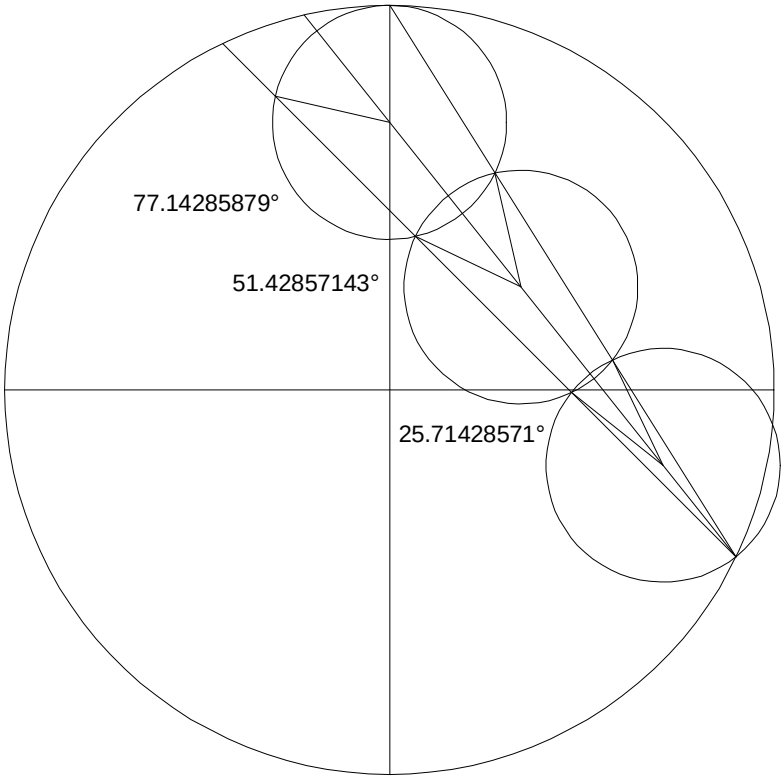
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

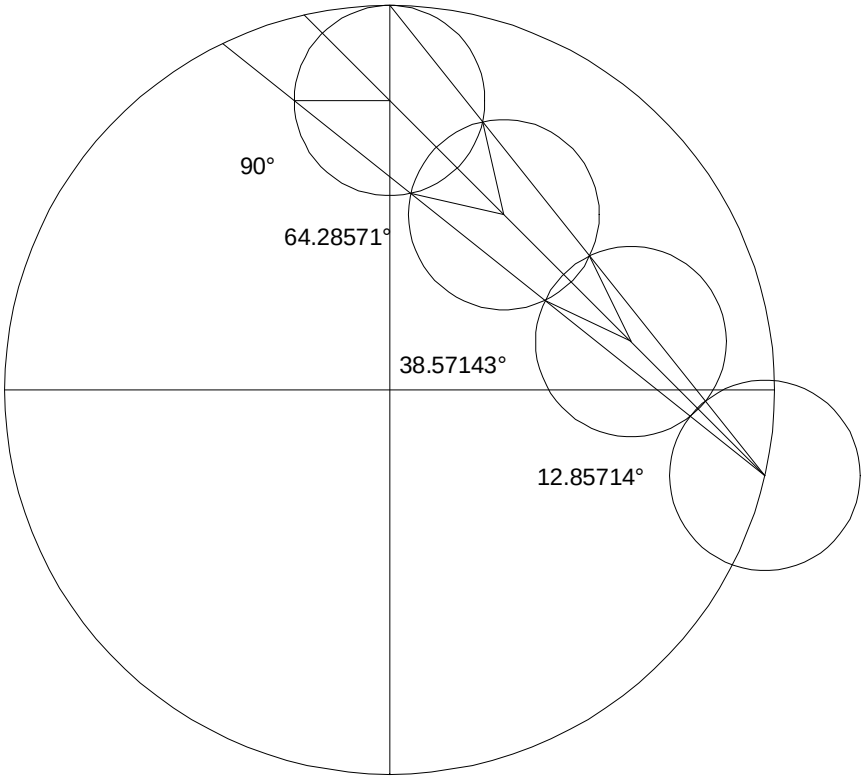
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

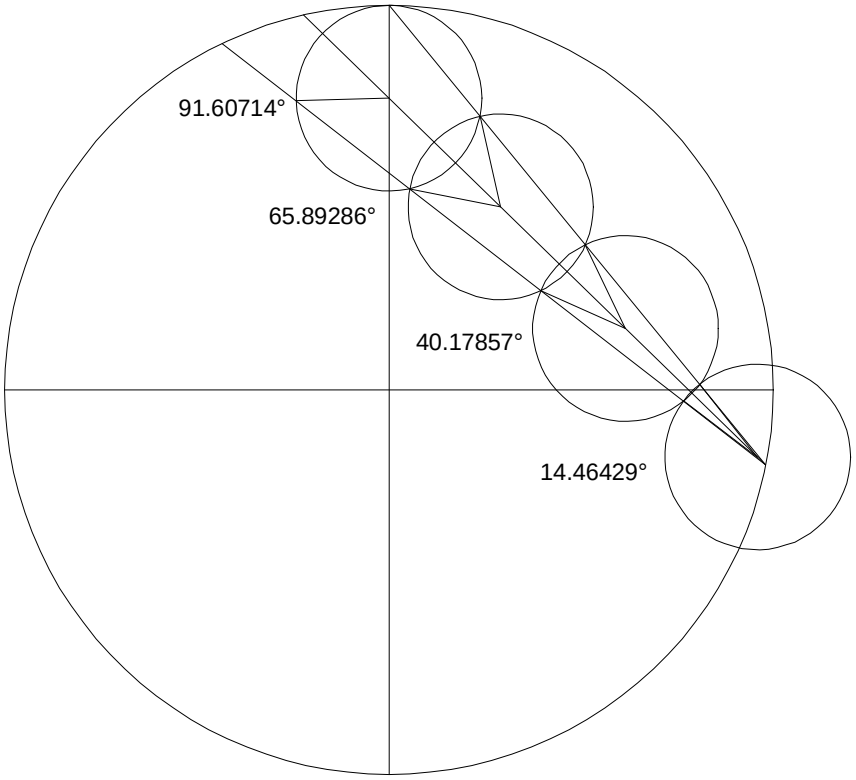
$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

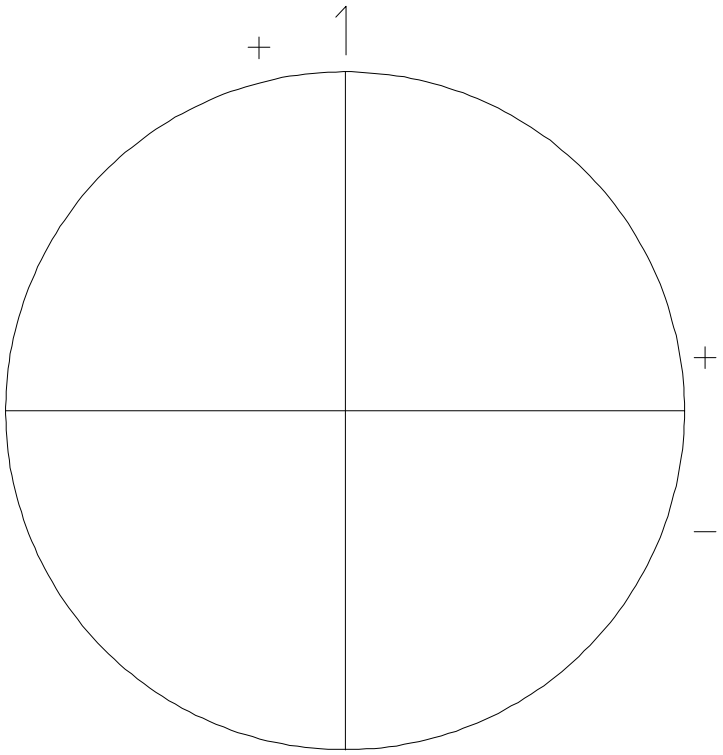
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

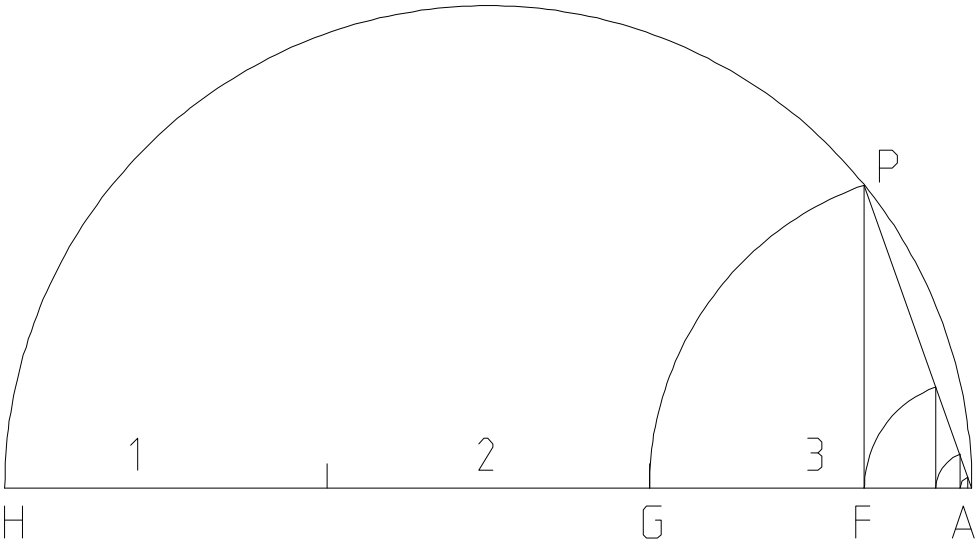
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




Work in progress.




Exponential Progressions. 04_01_95

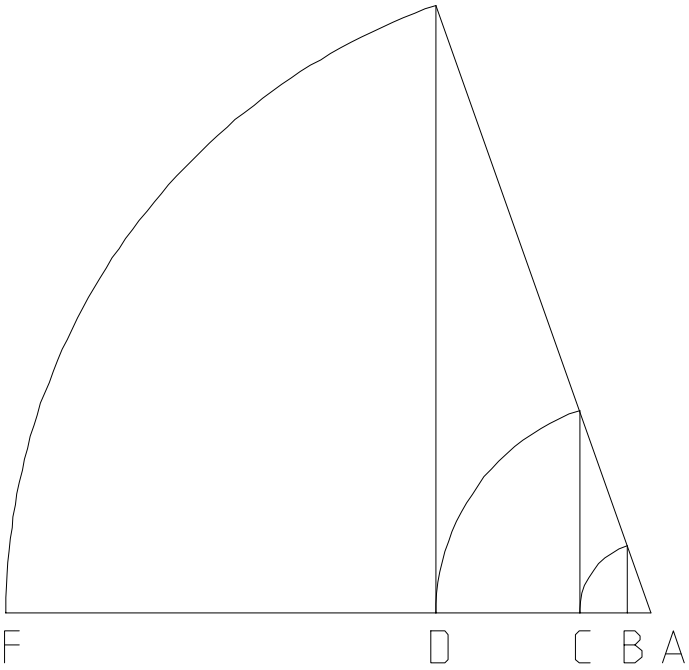
If I want to multiply any number by any power, this is the a geometric process for doing so.

The given figure is drawn for the third power of 3.

$AH := 10 \qquad \delta := 0..10 \qquad BS := 8$

The third division between A and F is very hard to see. BS = Base Segments

$AP := \frac{AH}{BS} \qquad AP = 1.25$




$AG_0 := \frac{(AP)^2}{AH} \qquad AF_0 := AG_0 \qquad \Delta := 2..10$

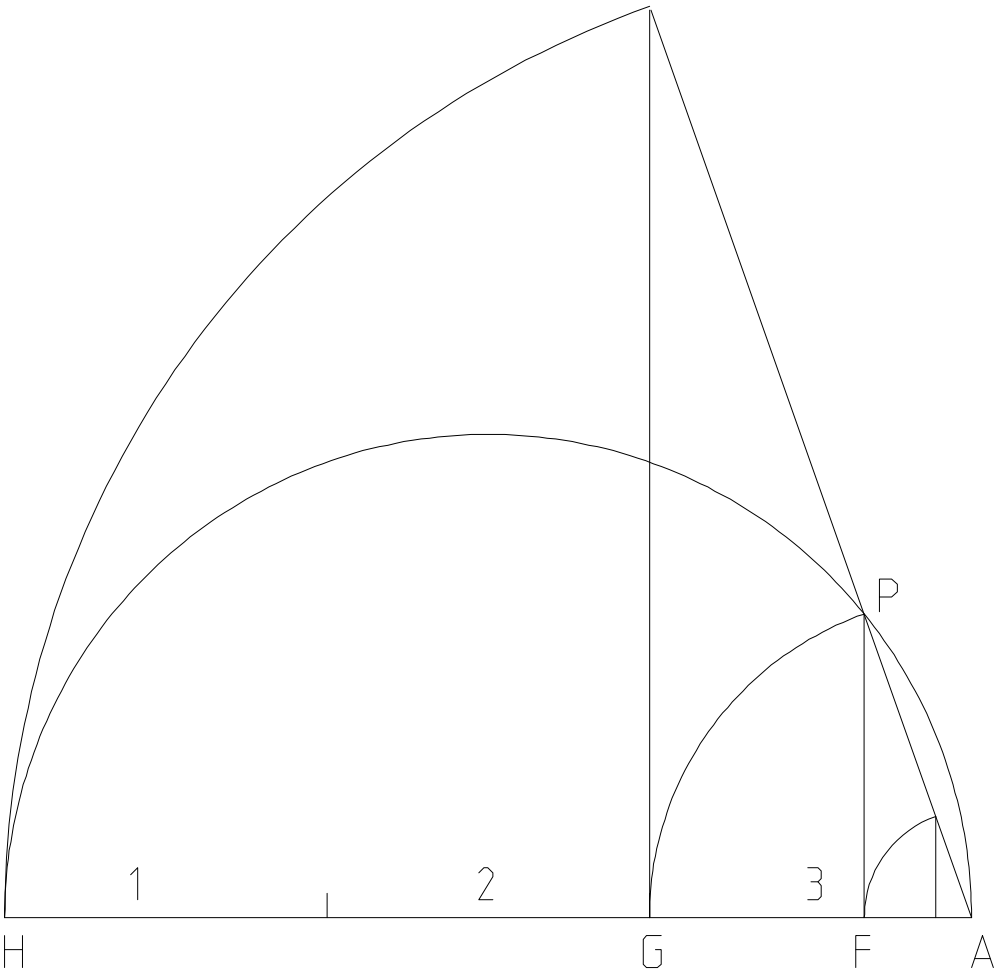
$\begin{pmatrix} AF_{\delta+1} \\ AG_{\delta+1} \end{pmatrix} := \begin{bmatrix} AG_\delta \\ \frac{AP \cdot AG_\delta}{AH} \end{bmatrix} \qquad AB_\delta := \frac{AG_1}{AF_\delta}$

| $\delta$ | $AB_\Delta$ | $BS^\delta$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 8 | 8 |
| 2 | 64 | 64 |
| 3 | 512 | 512 |
| 4 | 4096 | 4096 |
| 5 | 32768 | 32768 |
| 6 | 262144 | 262144 |
| 7 | 2097152 | 2097152 |
| 8 | 16777216 | 16777216 |
| 9 | | 134217728 |
| 10 | | $1.07374182 \cdot 10^9$ |

Making the number of divisions 3, provides 3 cube result. AB divides AF 27 times. Etc. It can be seen that using a normal straight edge and compass one needs a very large piece of paper to work this.

You will notice that I took only one of the possible two divisions from which to project from. The other would be 2/3. At 2/3 my unit divisions would still be 27, but now AB would take up 2 cube of them, or AB would be 8 units.

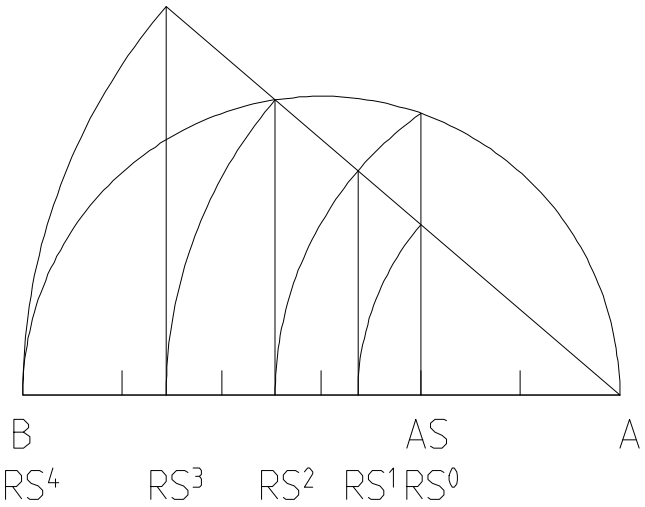
For an 8 cube series then, the value for AB would be 1 of 512, 8 of 512, 27 of 512, 64 of 512, 125 of 512, 216 of 512, 343 of 512




Since the slope is established, one can take the first two divisions and utilizes the whole figure.

$E := 1 .. BS$ $ROOT := 3$

| E | $\frac{BS^{ROOT}}{E^{ROOT}}$ | $E^{ROOT}$ |
|---|---|---|
| 1 | 512 | 1 |
| 2 | 64 | 8 |
| 3 | 18.96296296 | 27 |
| 4 | 8 | 64 |
| 5 | 4.096 | 125 |
| 6 | 2.37037037 | 216 |
| 7 | 1.49271137 | 343 |
| 8 | 1 | 512 |




# About The Laws of Exponents and Ratios

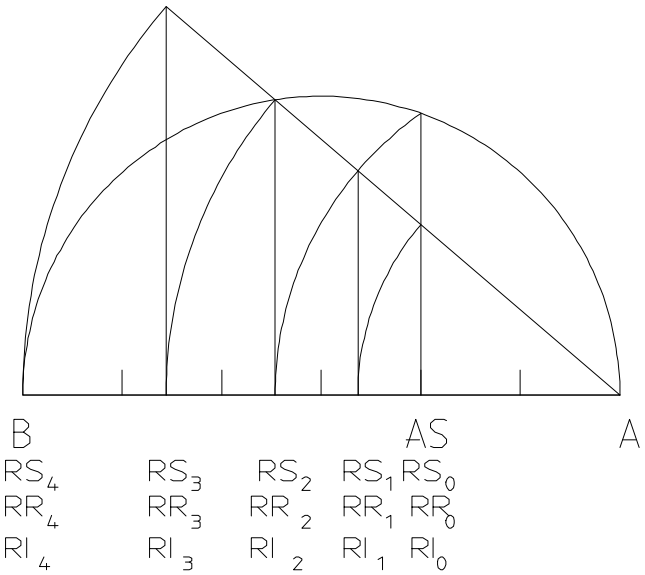
04_22_95.MCD

$\Delta := 22 \quad \delta := 1 .. \Delta \quad AB := 7$

Base Segments = BS $BS := 99$

Base Index = BI $BI := 13$

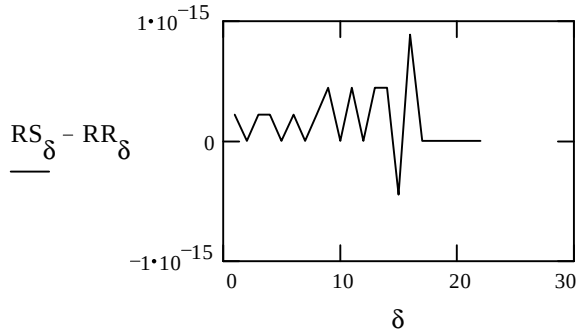
Root Series = RS $RS_\delta := \left[ \left( \frac{BI}{BS} \cdot AB \right)^{\Delta - \delta} \cdot AB^{\delta} \right]^{\frac{1}{\Delta}}$



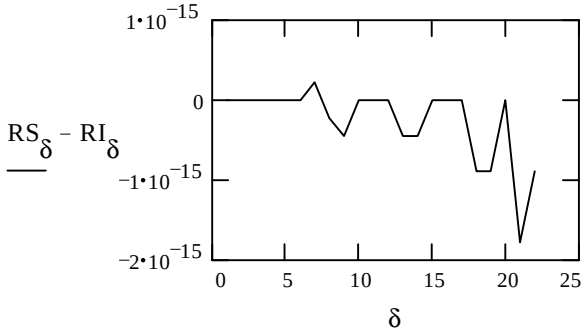

Root Series By Ratio = RR $RR_\delta := \left( \frac{BI}{BS} \right)^{\frac{\Delta - \delta}{\Delta}} \cdot AB$

Root Series By Inverse Ratio = RI

$RI_\delta := \left( \frac{BS}{BI} \right)^{\frac{\delta}{\Delta}} \cdot \left( \frac{BI}{BS} \cdot AB \right)$




$\sum_\delta \left( RS_\delta - RR_\delta \right) = 0$

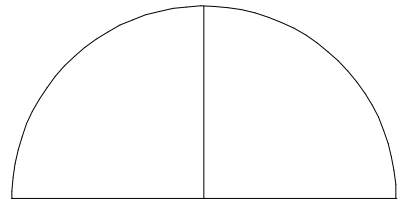



$\sum_\delta \left( RS_\delta - RI_\delta \right) = 0$

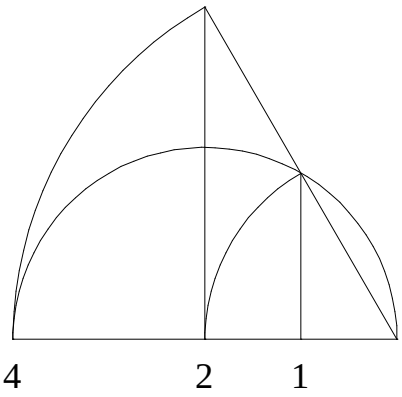
On the concept of unit and universe of discourse:

Euclidean exponentiation provides a good example with which to demonstrate the distinction between the concepts of unit and the universe of discourse.

Taking 2 for the *universe* of discourse would be represented graphically as:

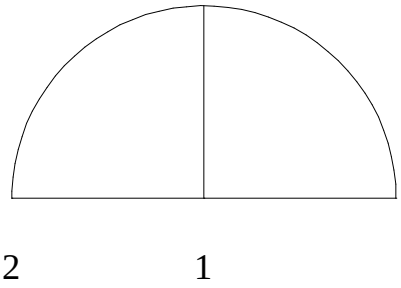



To represent $2^2$ within this universe of discourse one would draw:

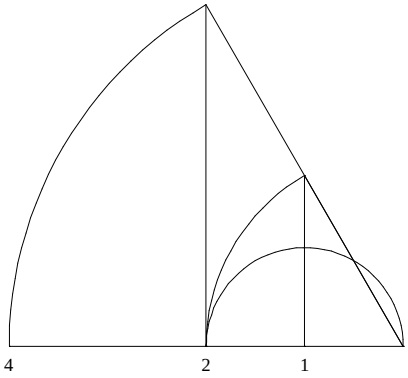



Our original 2 is divided into a $4^{th}$ segment.

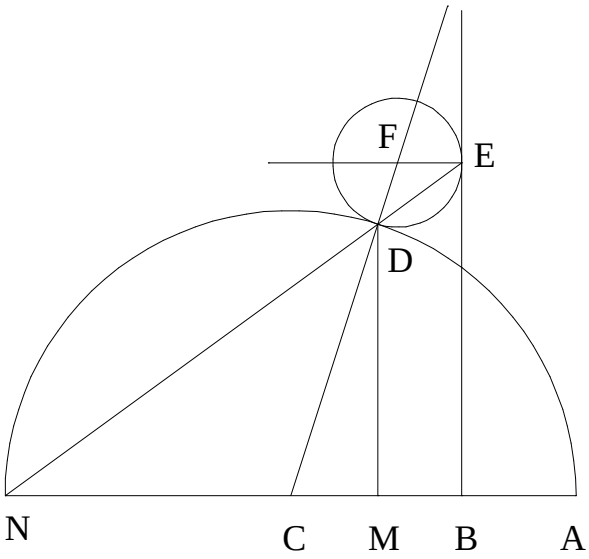
Now if 2 is taken as the *unit* of discourse we may still represent it as;



however to represent $2^3$ now would be drawn as




One will notice that in example 1, the unit changed while the universe remained, in example 2 the unit remained while the universe changed. One could call example 1 an example of deduction and example 2 an example of induction.




09_13_95.MCD

A Study in Placement.

Given AC, AB, AM find EF.

Segment A.

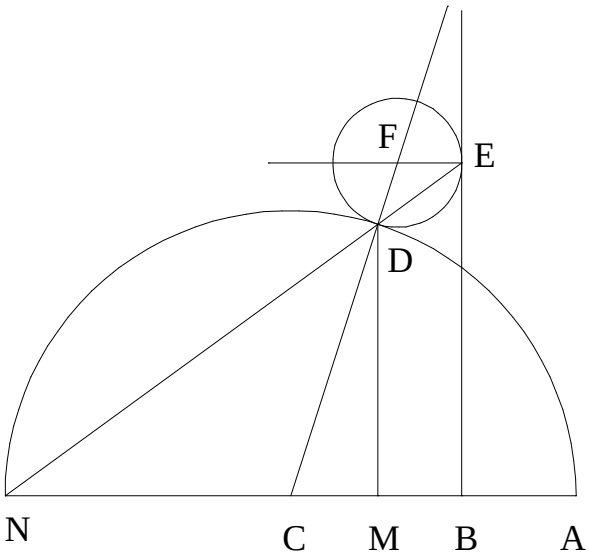
$AC := 3.125$ $AB := 4.25$ $AM := 5.34857$

$CD := AC$ $CN := AC$ $CM := AC - AM$ $DM := \sqrt{CD^2 - CM^2}$

$MN := CM + CN$ $BC := AC - AB$ $BN := CN + BC$ $DN := \sqrt{DM^2 + MN^2}$

$EN := \frac{DN \cdot BN}{MN}$ $ED := EN - DN$ $EF := \frac{AC \cdot ED}{DN}$ $EF = 3.80843$

The resultant equation in terms of the givens is $EF := \frac{AC \cdot (AM - AB)}{2 \cdot AC - AM}$ $EF = 3.80843$




Segment B.

Given AC, AB, DN, find EF.

$DN := 2.37359$

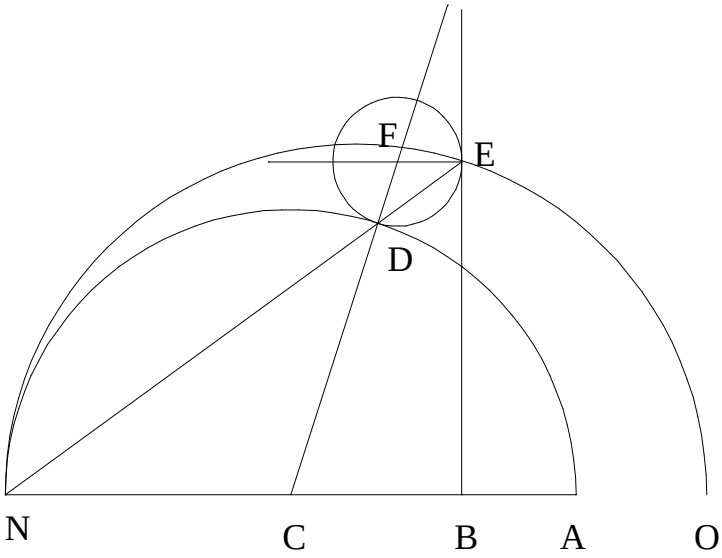
$AN := 2 \cdot AC$ $\quad MN := \frac{DN^2}{AN}$ $\quad CN := AC$ $\quad BC := AC - AB$ $\quad BN := CN + BC$

$EN := \frac{DN \cdot BN}{MN}$ $\quad ED := EN - DN$ $\quad EF := \frac{AC \cdot ED}{DN}$ $\quad EF = 3.80844$

The equation for this process is

$$EF := \frac{AC \cdot (4 \cdot AC^2 - 2 \cdot AC \cdot AB - DN^2)}{DN^2}$$

$EF = 3.80844$




Segment C.

Given AC, AB, BE, find EF.

$BE := 4.87173$

$AN := 2 \cdot AC$ $\quad BN := AN - AB$ $\quad EN := \sqrt{BE^2 + BN^2}$ $\quad ON := \frac{EN^2}{BN}$

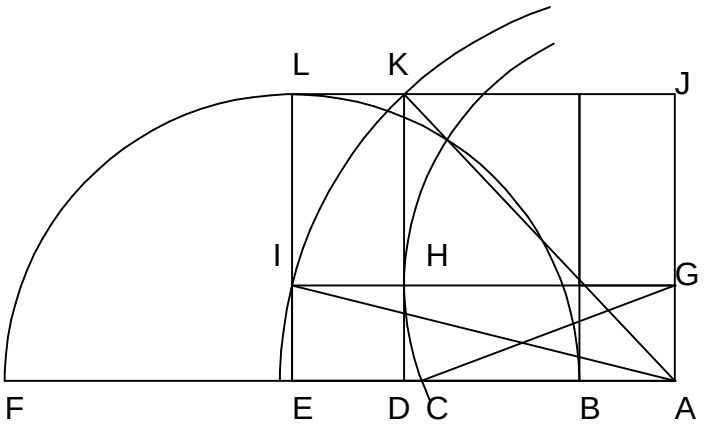
$DN := \frac{EN \cdot AN}{ON}$ $\quad DE := EN - DN$ $\quad EF := \frac{AC \cdot DE}{DN}$ $\quad EF = 3.80844$

The equation for this process is $EF := \frac{BE^2 - 2 \cdot AC \cdot AB + AB^2}{2 \cdot (2 \cdot AC - AB)}$ $\quad EF = 3.80844$

10_14_5C.MCD

Trivial Method: Square Root

Generalize the figure of 10_14_95.MCD




Starting at any point G, between A and J, the square root of AB·AF can always be projected to point C. Such a progression can be used on the cube root figure.

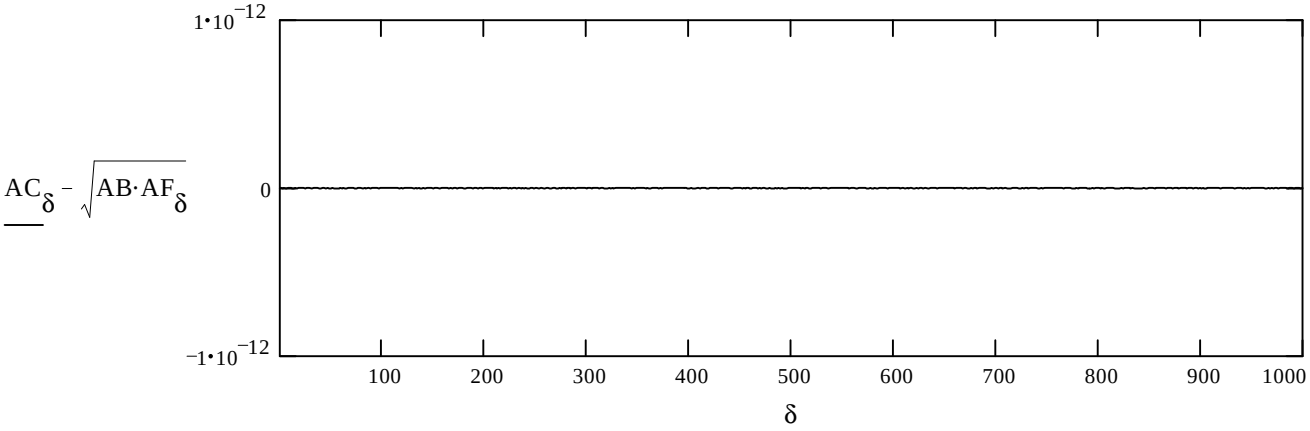
$\delta := 1 .. 1000 \quad AB := 10 \quad BF := 10 \quad BE := \frac{BF}{2}$

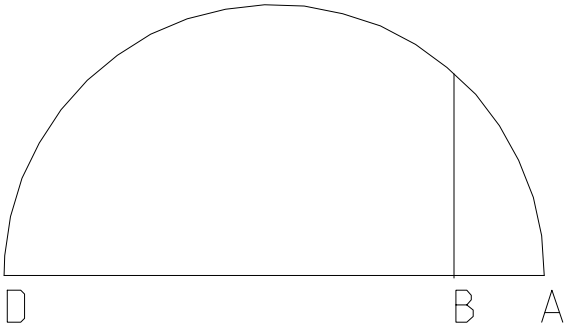
$AE := BE + AB \quad AJ := BE \quad AG_\delta := \frac{AJ}{\delta}$

$EI_\delta := AG_\delta \quad AI_\delta := \sqrt{(AE)^2 + (EI_\delta)^2}$

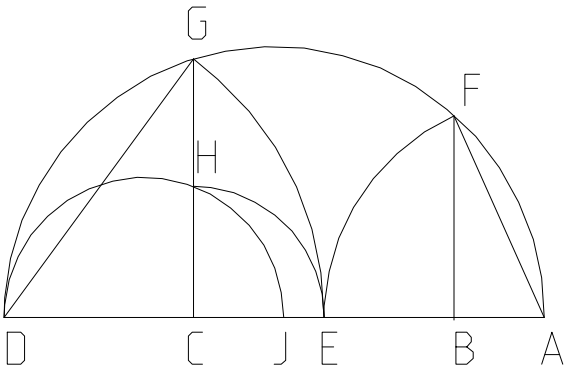
$AK_\delta := AI_\delta \quad DK := AJ \quad AD_\delta := \sqrt{(AK_\delta)^2 - DK^2} \quad GH_\delta := AD_\delta \quad CG_\delta := GH_\delta$

$AF_\delta := BF + AB \quad AC_\delta := \sqrt{(CG_\delta)^2 - (AG_\delta)^2}$






Given AB and BD divide BD such that $AB \cdot CD = \frac{BC^2}{4}$. And what is the reltionship of AC to AB and BD? Now the date on this file is not exact as I sketched this out on a piece of paper and forgot to date it.




$AB := 5 \quad BD := 1 \quad AD := AB + BD$

$BF := \sqrt{AB \cdot BD} \quad AE := \sqrt{AB^2 + BF^2}$

$DE := AD - AE \quad DG := DE \quad CD := \frac{DG^2}{AD}$

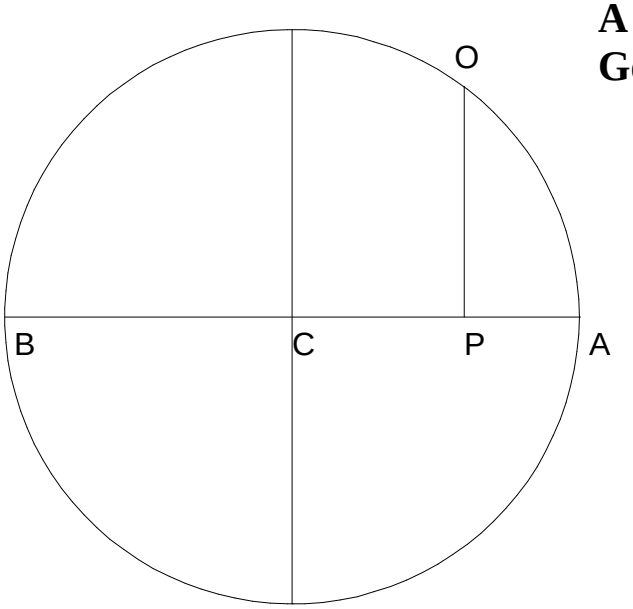
$CD = 0.046$

$BC := BD - CD \quad AB \cdot CD - \frac{BC^2}{4} = 0$

$CE := DE - CD \quad CH := CE \quad CJ := \frac{CH^2}{CD} \quad AB - CJ = 0$

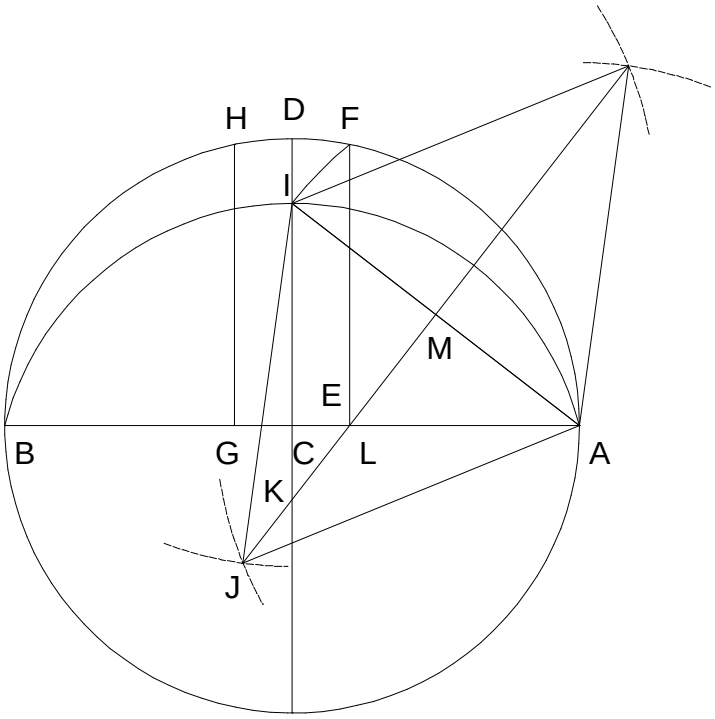
$AC := AB + BC \quad AC = 5.954$

$$AC - \left[ 2 \cdot \frac{AB^{\left(\frac{3}{2}\right)}}{\sqrt{AB + BD}} + 2 \cdot \frac{\sqrt{AB}}{\sqrt{AB + BD}} \cdot BD - AB \right] = 0$$




# A Modification of a Square Root Figure. Gemini Roots

One of the square root figures displays a one to one ratio between what could be called the vertical segment OP and the root of the two horizontal segments AP and BP. With a slight modification however, one can demonstrate a many to one relationship between three base segments.




GL has a ratio to the root of $AL \cdot BG$. Developing the arc AIC from it will give a means of keeping that ratio.

BS = Base Segments, set at end of doc.

$AB := 10 \quad AC := \frac{AB}{2} \quad EG := \frac{AB}{BS}$

$BC := AC \quad CD := AC \quad CE := \frac{EG}{2}$

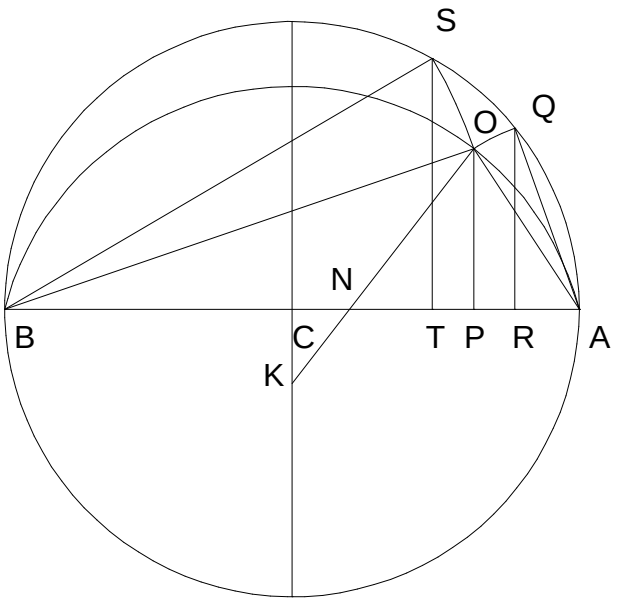
$AE := AC - CE \quad BE := BC + CE$

$EF := \sqrt{AE \cdot BE} \quad AF := \sqrt{AE^2 + EF^2}$

$AI := AF \quad AM := \frac{AI}{2} \quad CI := \sqrt{AI^2 - AC^2}$

$AL := \frac{AI \cdot AM}{AC} \quad CL := AC - AL$

$CK := \frac{AC \cdot CE}{CI} \quad IK := CI + CK$




$$\delta := 1 .. \Delta \qquad AN_\delta := \frac{AC}{\Delta}\cdot\delta$$

$$CN_\delta := AC - AN_\delta \qquad KO := IK$$

$$KN_\delta := \sqrt{CK^2 + \left(CN_\delta\right)^2} \qquad NO_\delta := KO - KN_\delta$$

$$NP_\delta := \frac{CN_\delta \cdot NO_\delta}{KN_\delta} \qquad AP_\delta := AC - CN_\delta - NP_\delta$$

$$OP_\delta := \frac{CK \cdot NO_\delta}{KN_\delta} \qquad AO_\delta := \sqrt{\left(AP_\delta\right)^2 + \left(OP_\delta\right)^2}$$

$$AQ_\delta := AO_\delta \qquad AR_\delta := \frac{\left(AQ_\delta\right)^2}{AB}$$

$$BP_\delta := BC + CN_\delta + NP_\delta$$

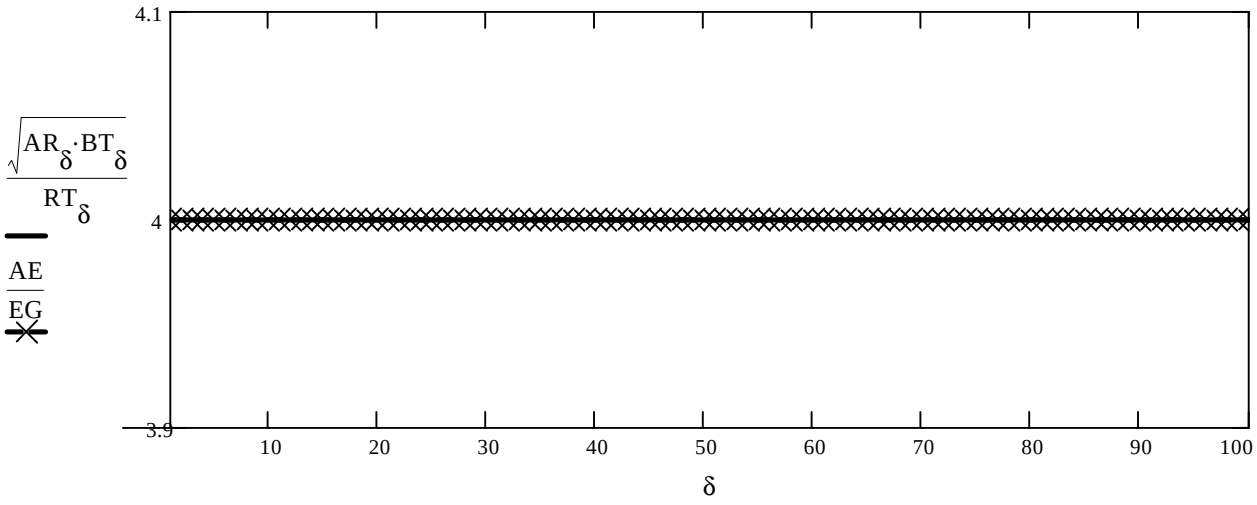
$$BO_\delta := \sqrt{\left(BP_\delta\right)^2 + \left(OP_\delta\right)^2} \qquad BS_\delta := BO_\delta \qquad BT_\delta := \frac{\left(BS_\delta\right)^2}{AB} \qquad AT_\delta := AB - BT_\delta \qquad RT_\delta := AT_\delta - AR_\delta$$

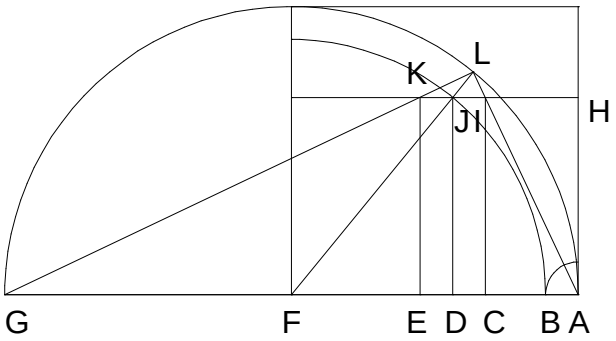
Set the number of Base Segments here and see if a constant relationship is expressed in the graph.

$BS \equiv 9$

Set the number of drawings in $\Delta$.

$\Delta \equiv 100$





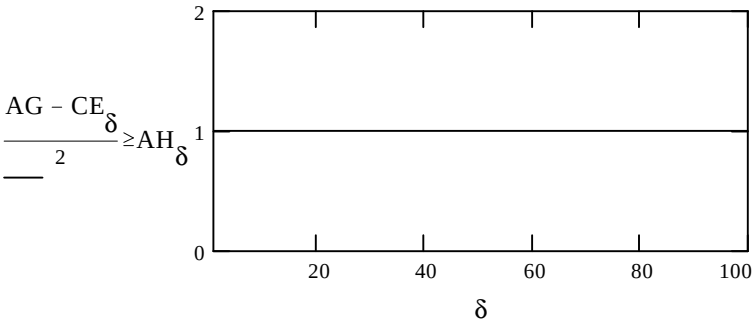

# Short Method Gemini Roots.

Given AG, CE, AH, place CE so that CE:AH as CE:CI. Or more simply that CI $= \sqrt{AC_\delta \cdot EG_\delta} = AH$.

$\delta := 1 .. 100$ $\Delta := 8$

$AG := \Delta \cdot 2 + 1$ $CE_\delta := \frac{1}{\delta}$ $AH_\delta := \Delta \cdot CE_\delta$

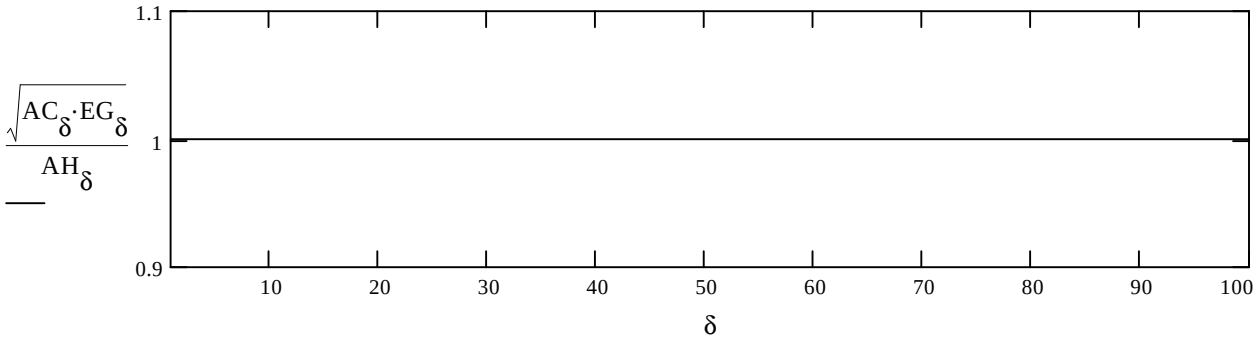
With the values given is the constuction possible? (1 for yes and 0 for no.)

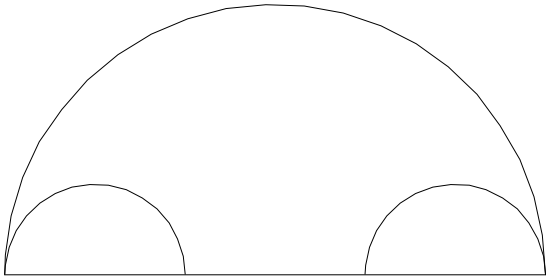



$DJ_\delta := AH_\delta$ $AB_\delta := \frac{CE_\delta}{2}$ $AF := \frac{AG}{2}$ $FG := AF$ $BF_\delta := AF - AB_\delta$ $FJ_\delta := BF_\delta$

$FD_\delta := \sqrt{(FJ_\delta)^2 - (DJ_\delta)^2}$ $DC_\delta := AB_\delta$ $DE_\delta := AB_\delta$ $AD_\delta := AF - FD_\delta$

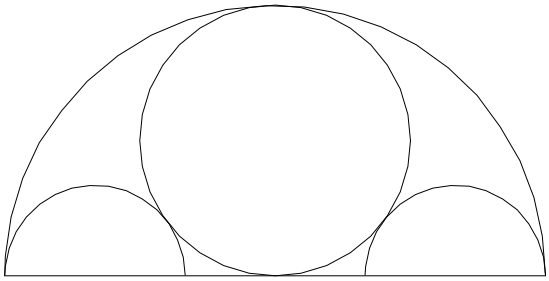
$AC_\delta := AD_\delta - DC_\delta$ $AE_\delta := AD_\delta + DE_\delta$ $EF_\delta := AF - AE_\delta$ $EG_\delta := EF_\delta + FG$

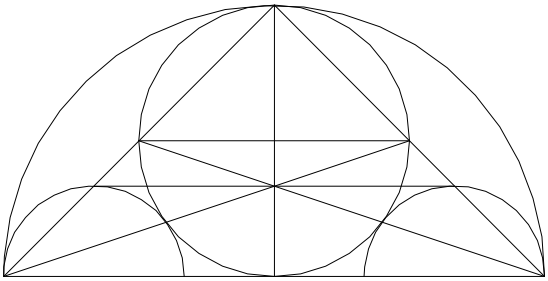


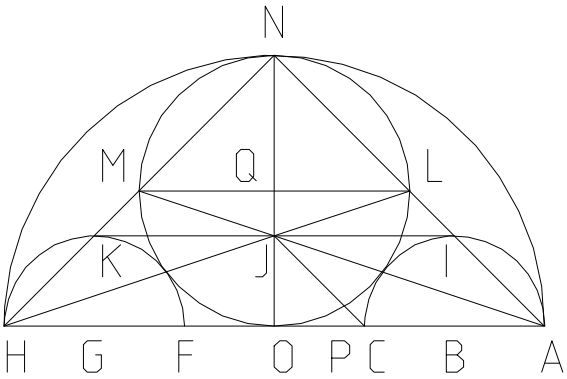


# Method for Equals.

At the inner extremities of a great circle I have two equal smaller ones. Find the circle tangent to all three








$AH := 10 \quad AC := 3 \quad AO := \frac{AH}{2} \quad AB := \frac{AC}{2}$

$JO := AB \quad OP := JO \quad HO := AO$

$JP := \sqrt{JO^2 + OP^2} \quad HP := HO + OP$

$AL := \frac{JP \cdot AH}{HP} \quad NO := AO \quad AN := \sqrt{AO^2 + NO^2}$

$LN := AN - AL \quad LQ := \frac{AO \cdot LN}{AN} \quad LQ = 2.692$

Reducing LQ as an expression of the two givens, $LQ_F := \frac{AH \cdot (AH - AC)}{2 \cdot (AH + AC)} \quad LQ_F - LQ = 0$

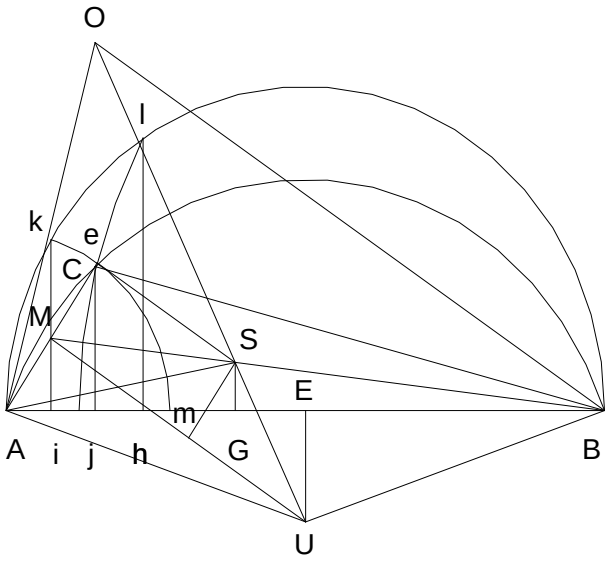
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

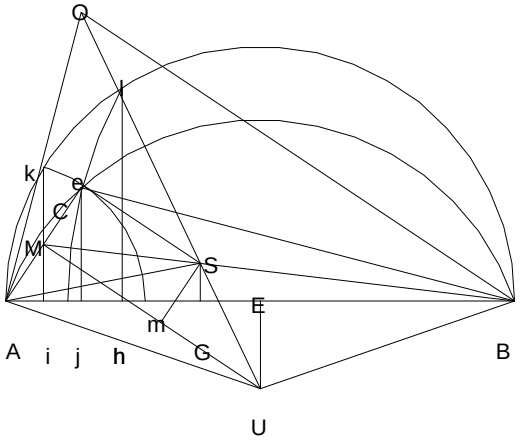
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

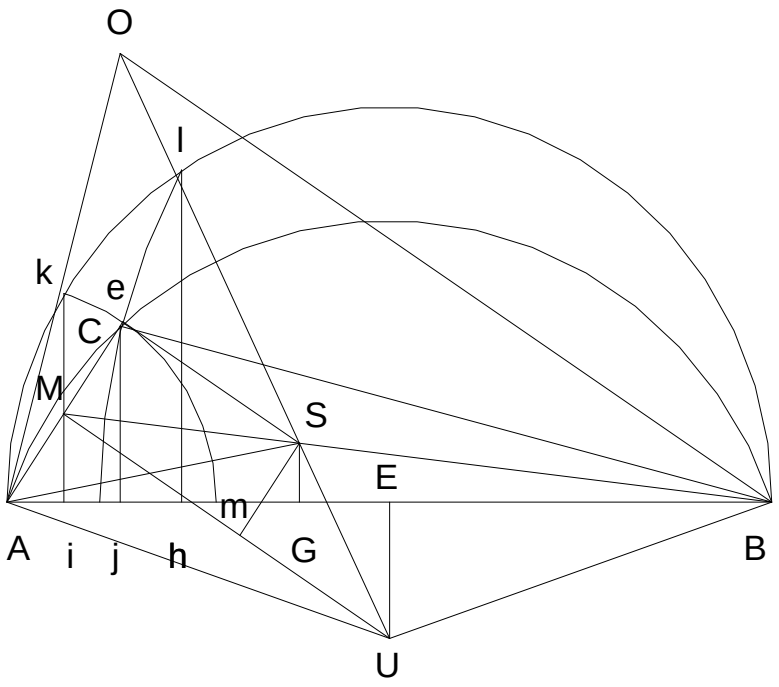
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

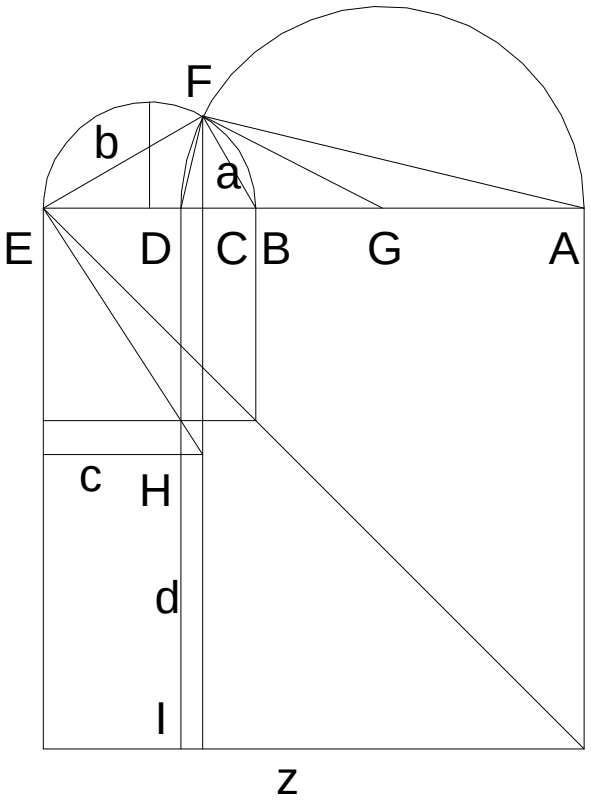



$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.




12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \frac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \frac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.622$

$c := DE$ $\quad c = 0.622$

$d := HI$ $\quad d = 6.186$

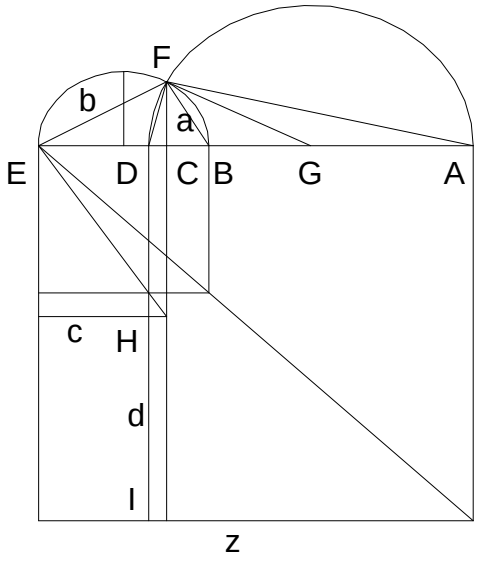
$$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

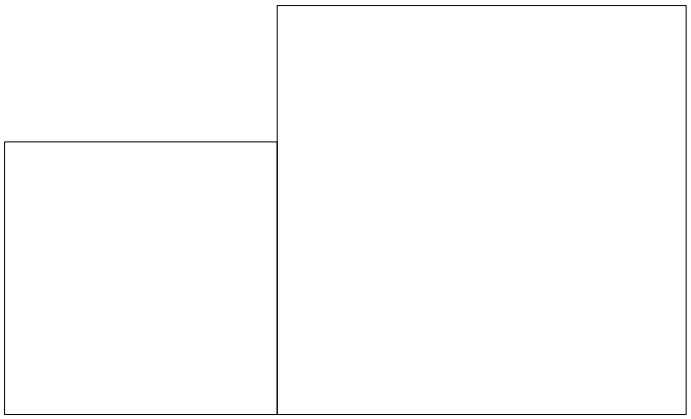
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

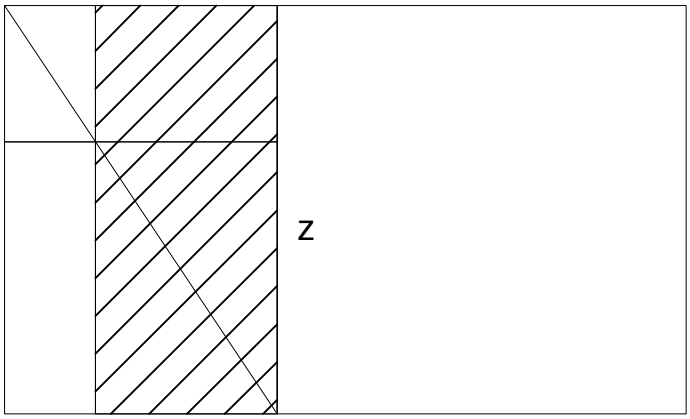
$$(c\cdot d)-p=0$$

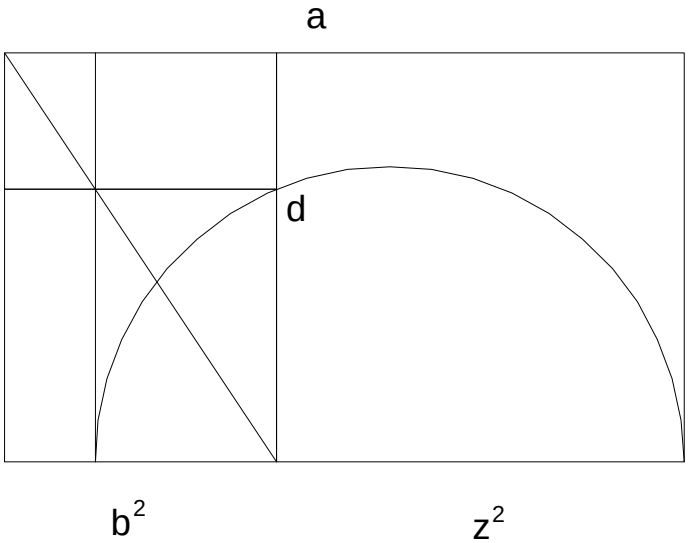
$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$




12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

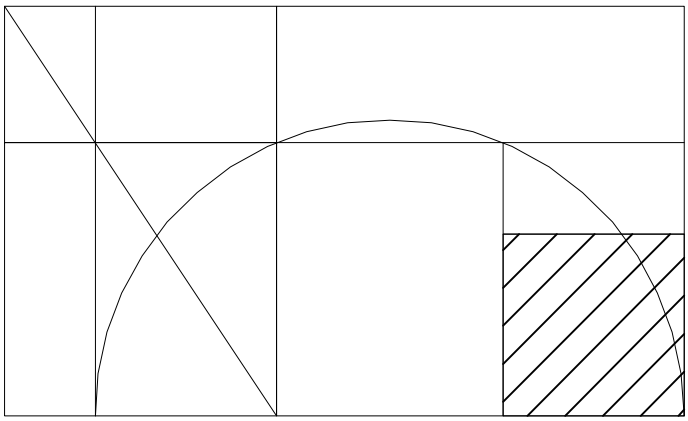
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

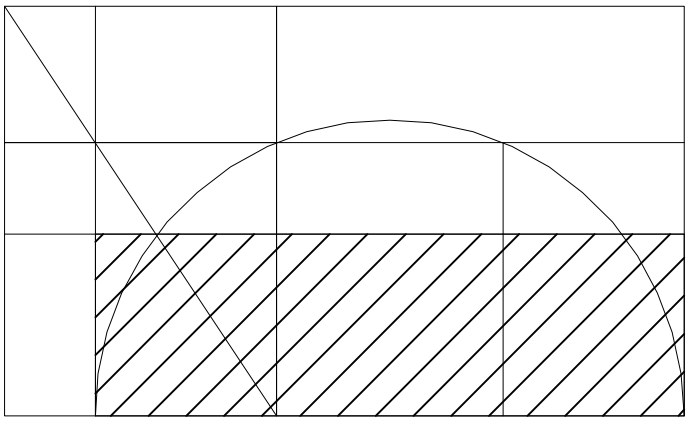
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

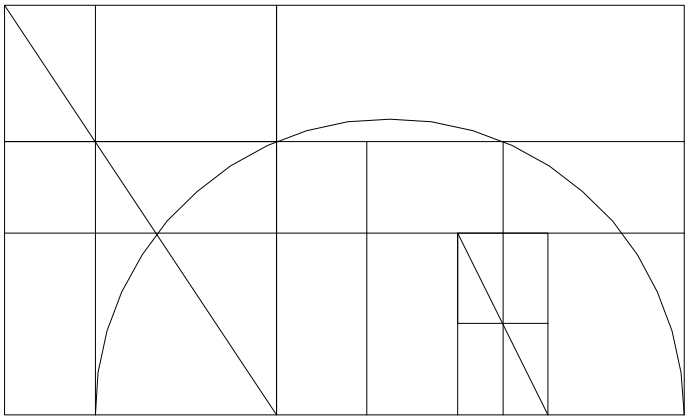
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



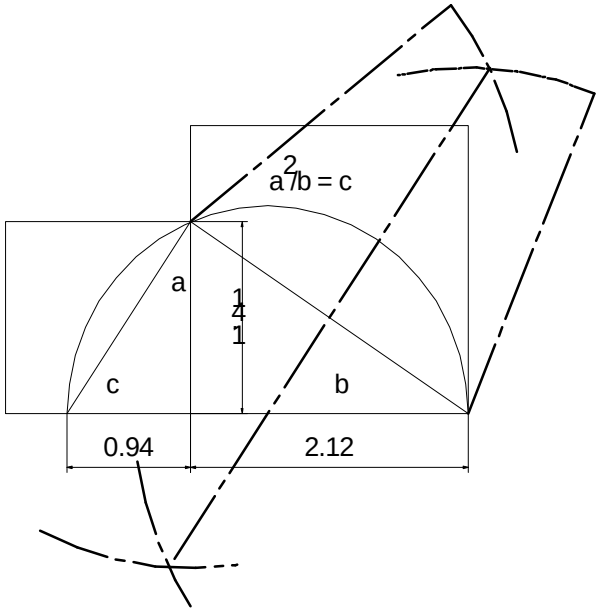
$b^2$ $z^2$



$b^2$ $z^2$

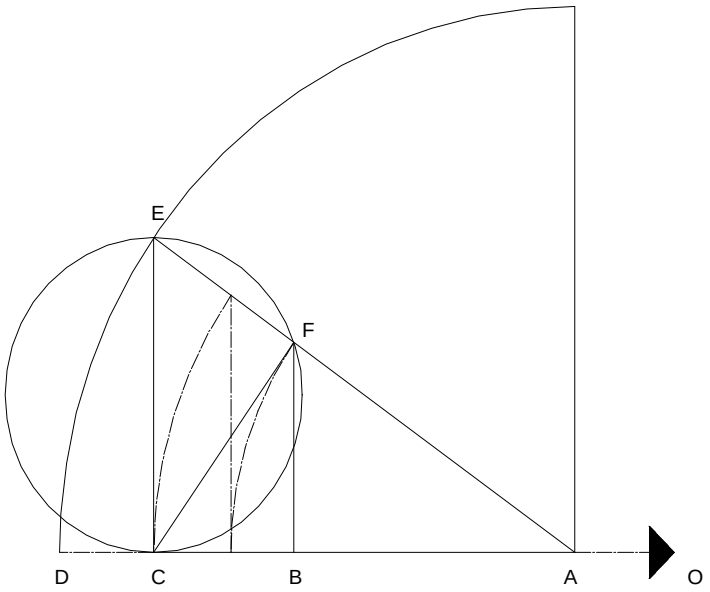


$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

## Just for fun.




$N := 1.3$

$AD := 42 \qquad AC := \frac{AD}{N} \qquad CO := AD + AC$

$CD := AD - AC \qquad CE := \sqrt{CD \cdot CO}$

$AF := \frac{AC^2}{AD} \qquad AB := \frac{AF \cdot AC}{AD}$

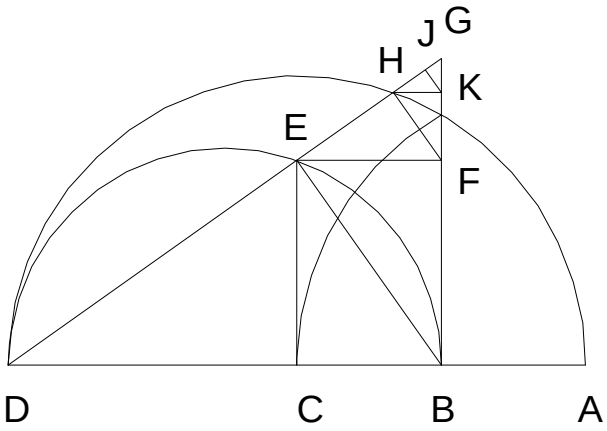
$AC^3 - \left(AD^2 \cdot AB\right) = 0$

$AB = 19.117$

$AC - \left(AD^2 \cdot AB\right)^{\frac{1}{3}} = 0$

The ratio of BC to CE is common to another cube root expression, which?

Pascal's triangle with exponential division.




$AB := 3 \qquad AD := 5$

$BD := AD - AB \qquad AC := \sqrt{AB \cdot AD}$

$CD := AD - AC \qquad BC := BD - CD$

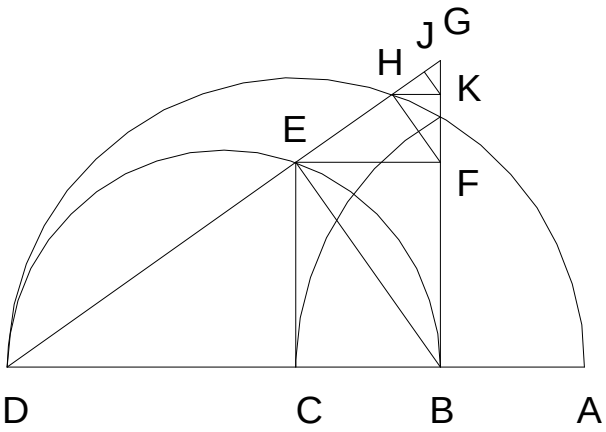
$CD := BD - BC \qquad CE := \sqrt{BC \cdot CD}$

$BE := \sqrt{BC^2 + CE^2} \qquad DE := \sqrt{CD^2 + CE^2}$

$BG := \frac{BD \cdot BE}{DE} \qquad FG := \frac{BC \cdot BG}{BD}$

$EG := \frac{BE \cdot BG}{BD} \qquad DG := DE + EG$

$GH := \frac{BC \cdot EG}{BD} \qquad GJ := \frac{BC \cdot GH}{BD}$




$\frac{BD}{BC} = 2.291 \qquad \frac{DG}{GH} = 5.249 \qquad \frac{DG}{GJ} = 12.025$

$N := 1 .. 3 \qquad \frac{\left(\sqrt{AD} + \sqrt{AB}\right)^N}{\sqrt{AB^N}}$

| |
|---|
| 2.291 |
| 5.249 |
| 12.025 |

$\sqrt{AD} + \sqrt{AB}$

$\frac{A + B}{A}$

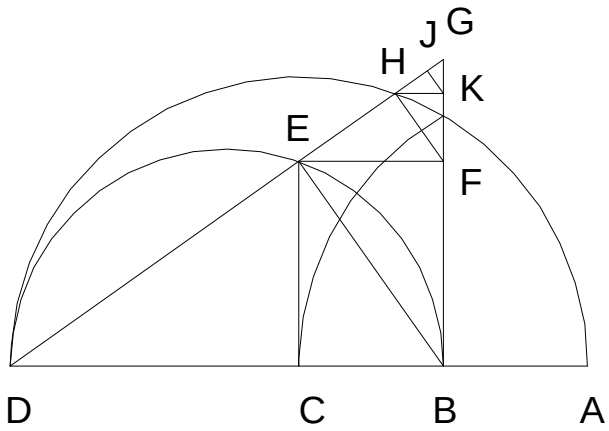
$\left(\sqrt{AD}\right)^2 + 2 \cdot \sqrt{AB} \cdot \sqrt{AD} + \left(\sqrt{AB}\right)^2$

$\frac{A^2 + 2AB + B^2}{A^2}$

$\left(\sqrt{AD}\right)^3 + 3 \cdot AD \cdot \sqrt{AB} + 3 \cdot \sqrt{AD} \cdot AB + \left(\sqrt{AB}\right)^3$

$\frac{A^3 + 3A^2 \cdot B + 3A \cdot B^2 + B^3}{A^3}$

Dividing an exponentiated integer by an exponentiated integer of the same power, straight edge and compass construction. Followed by who knows what!




$CD := 1.5 \qquad BC := .75 \qquad BD := BC + CD$

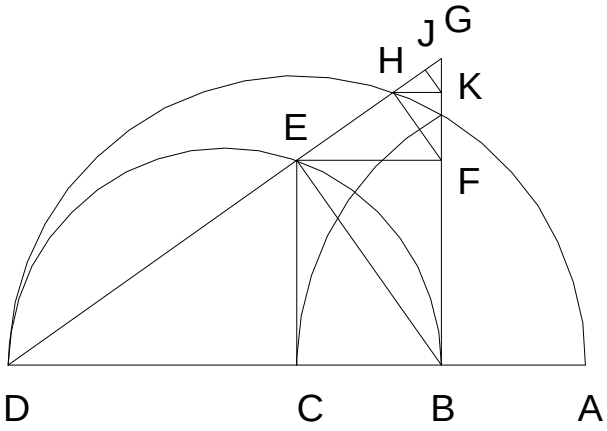
$CE := \sqrt{BC \cdot CD} \qquad BE := \sqrt{BC^2 + CE^2}$

$EF := BC \qquad FG := \frac{BC \cdot EF}{CE} \qquad FG = 0.53$

$DE := \sqrt{CD^2 + CE^2} \qquad EG := \frac{BD \cdot EF}{DE}$

$GH := \frac{BC \cdot EF}{DE} \qquad DG := DE + EG$

$JK := \frac{FG \cdot GH}{EG} \qquad GJ := \frac{BC \cdot GH}{BD} \qquad FH := \frac{CE \cdot EG}{BD} \qquad \frac{BD}{BC} = 3 \qquad \frac{DG}{GH} = 9 \qquad \frac{DG}{GJ} = 27$



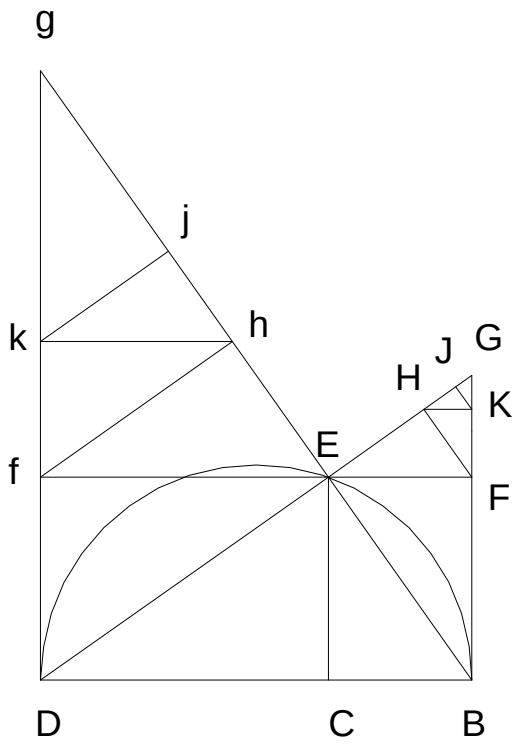

$n := 1..3$

| $\frac{a^n}{b^n}$ | $\frac{BD^n}{BC^n}$ |
|---|---|
| | 3 |
| | 9 |
| | 27 |

$BC + CD = 2.25$ $\sqrt{DG \cdot DE} = 2.25$

$\frac{\sqrt{BC + CD} \cdot BC}{\sqrt{CD}} = 0.919$ $\sqrt{DG \cdot GH} = 0.919$

$\frac{\left(\sqrt{BC}\right)^3}{\sqrt{CD}} = 0.53$ $\sqrt{DG \cdot GJ} = 0.53$




$Ef := CD$ $Eg := \frac{BD \cdot Ef}{BE}$ $fh := \frac{CE \cdot Ef}{BE}$

$hk := \frac{CE \cdot fh}{BE}$ $hj := \frac{BC \cdot hk}{BE}$ $Eg := \frac{BD \cdot Ef}{BE}$

$Bg := BE + Eg$ $hg := \frac{CD \cdot Ef}{BE}$ $jg := \frac{CD \cdot hk}{BE}$

$\frac{BD}{CD} = 1.5$ $\frac{Bg}{hg} = 2.25$ $\frac{Bg}{jg} = 3.375$ $\left(\frac{BD}{CD}\right)^n$

| |
|---|
| 1.5 |
| 2.25 |
| 3.375 |

$\frac{Bg}{EG} = 4.243$ $\frac{Bg}{GH} = 12.728$ $\frac{Bg}{GJ} = 38.184$ $\sqrt{CD} \cdot \frac{(BC + CD)^n}{\left(\sqrt{BC}\right)^{2 \cdot n + 1}}$

| |
|---|
| 4.243 |
| 12.728 |
| 38.184 |

$n := 1 .. 4$

$\sqrt{Bg \cdot GJ} = 0.631$

$\sqrt{Bg \cdot GH} = 1.092$

$\sqrt{Bg \cdot EG} = 1.892$

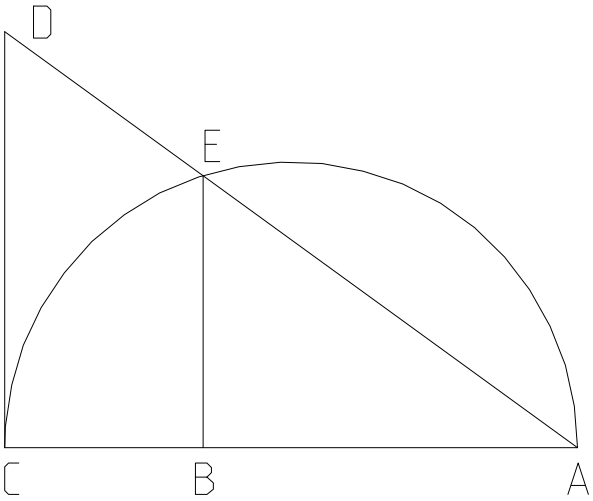
$\sqrt{Bg \cdot DG} = 3.277$

$\frac{BC^{\left(\frac{7}{4}\right)}}{CD^{\left(\frac{1}{4}\right)} \cdot \sqrt{BC + CD}} \cdot \frac{\left(\sqrt{BC + CD}\right)^n}{\left(\sqrt{BC}\right)^n}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |

$\frac{BC^{\left(\frac{7}{4} - \frac{1}{2} \cdot n\right)}}{CD^{\left(\frac{1}{4}\right)}} \cdot (BC + CD)^{\left(\frac{-1}{2} + \frac{1}{2} \cdot n\right)}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |




Given AC and CD find BC when it is equal to $\frac{AC \cdot CD^2}{AC^2 + CD^2}$.

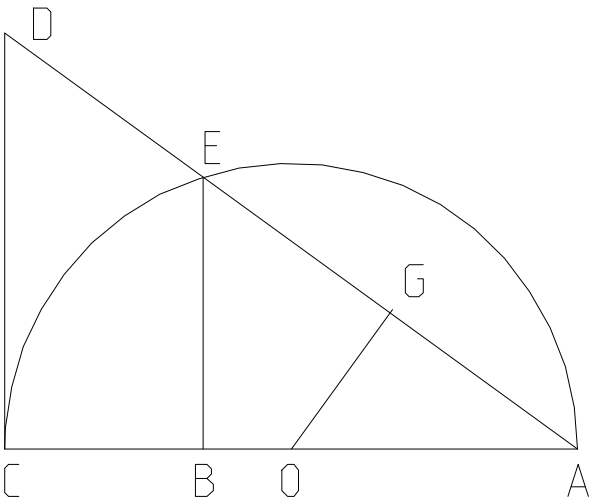
$AC := 15 \quad CD := 5 \quad AO := \frac{AC}{2}$

$AD := \sqrt{AC^2 + CD^2}$

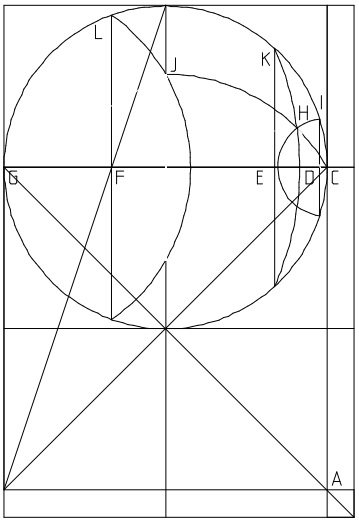
$AG := \frac{AC \cdot AO}{AD} \quad AE := 2 \cdot AG$

$AB := \frac{AC \cdot AE}{AD} \quad BC := AC - AB \quad BC = 1.5$

$BC := \frac{AC \cdot CD^2}{\left(AC^2 + CD^2\right)} \quad BC = 1.5$




A cube divided by the sum of two squares.




# One Circle

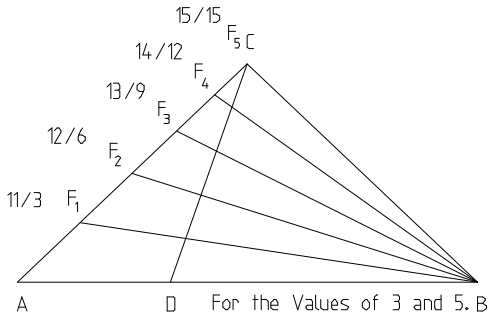
**By John Clark**

# One Square

# One Line

## And the Delian Quest

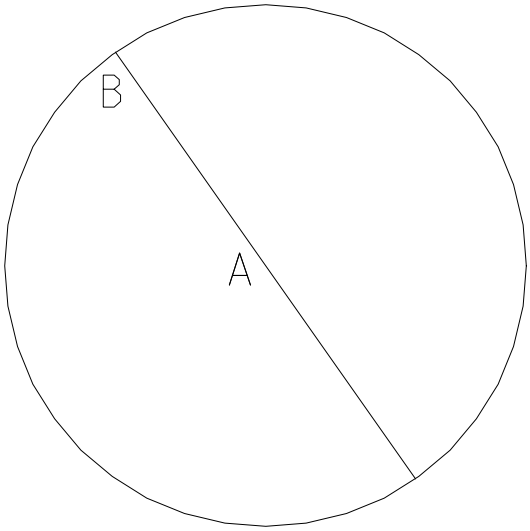
# 1996



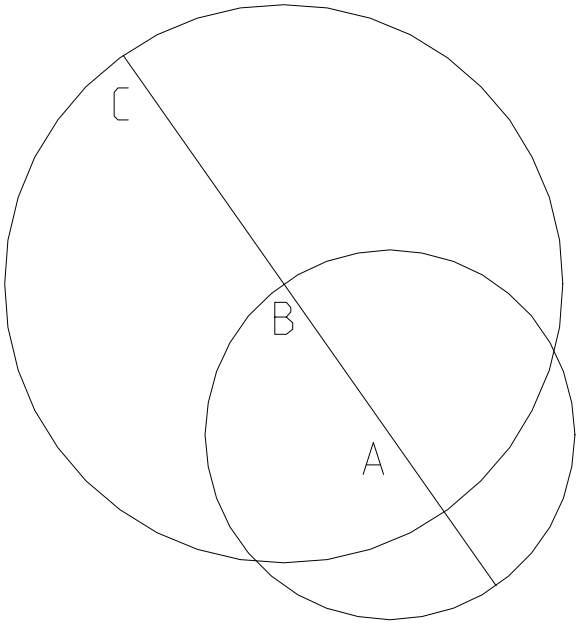
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

## The Archamedian Paper Trisector- Without the Numbers.

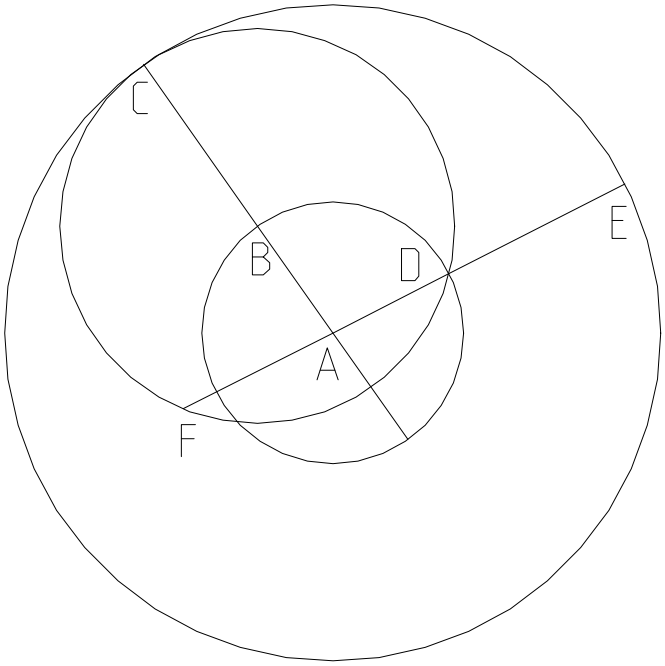
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.




Given any circle AB

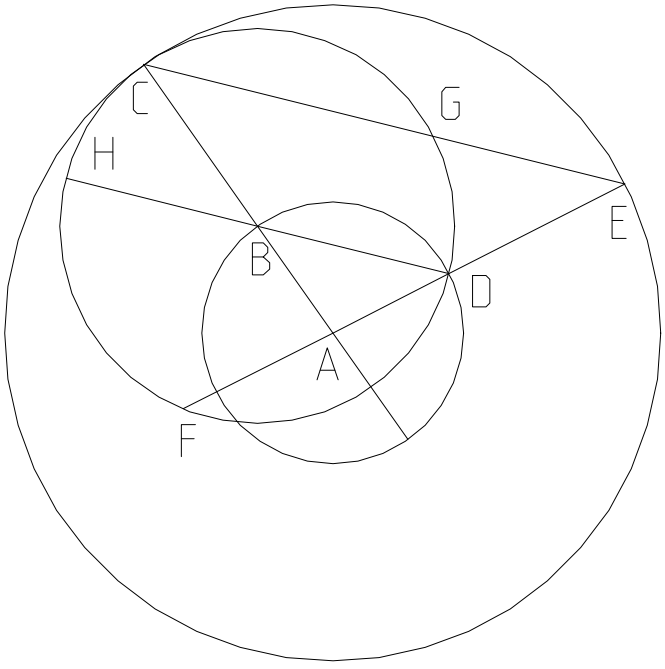



Given any circle BC such that BC≤ 2AB.

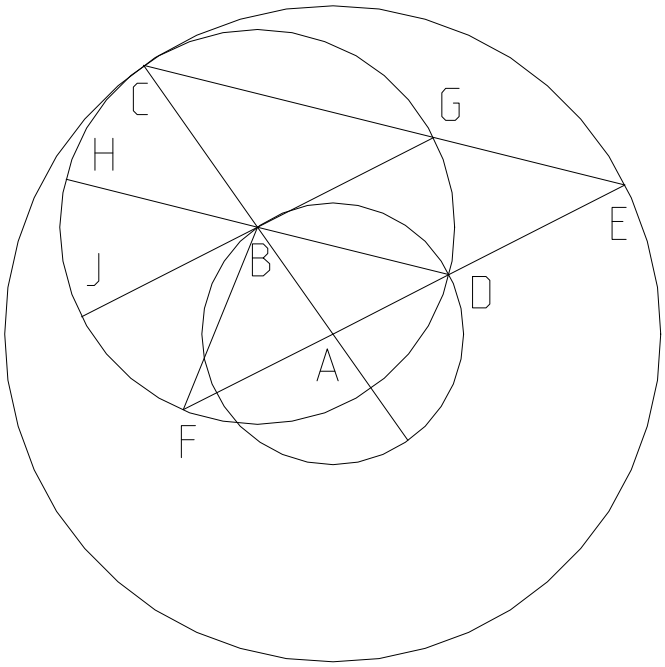



Construct AE such that AE = AC.

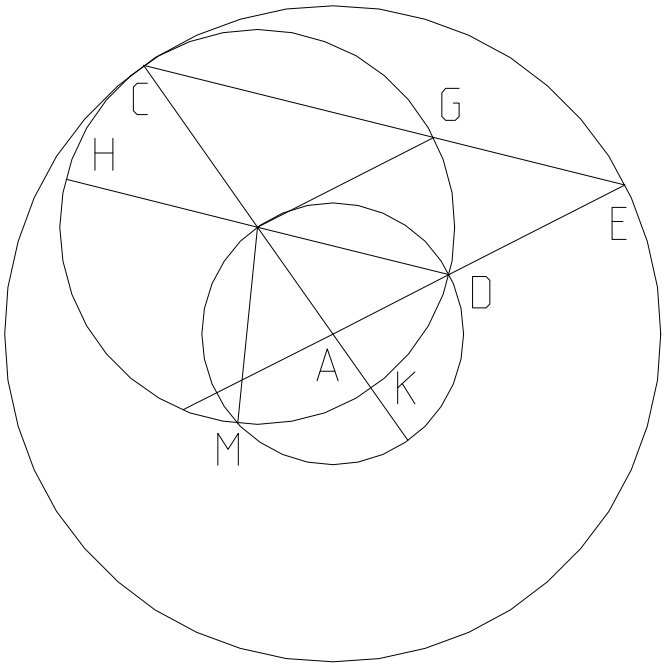
Since AC = AB+ BC and AD = AB, DE = BC.




Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



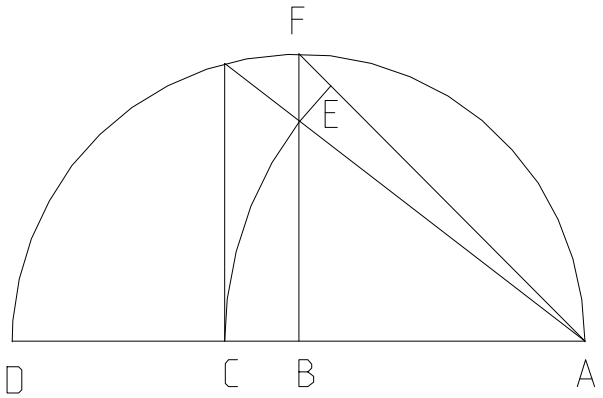

Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.




By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

A rusty Compass construction for the duplication of the cube.

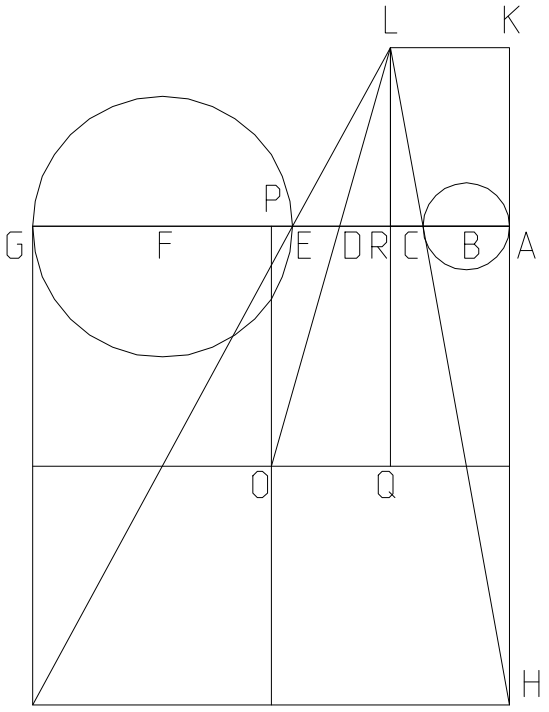



$AD := 2 \quad AB := \frac{AD}{2} \quad AF := \sqrt{2 \cdot AB^2} \quad AE := \frac{AF}{9} \cdot 8$

$AC := AE \quad AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26 \qquad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.




# Alternate Method, Power Line.

Given AB, EF, BF, find the power line intersection on BF. Looking back to 94, it seems I never derived a formula for it either.

$AB := R_1 \quad BF := D \quad EF := R_2$

$BC := AB \quad AC := AB + BC \quad FG := EF$

$EG := EF + FG \quad AG := AB + BF + FG \quad AH := AG$

$GJ := AG \quad AP := \frac{AG}{2} \quad CE := BF - (BC + EF)$

$LR := \frac{AH \cdot CE}{AC + EG} \quad PO := AP \quad AK := LR$

$KL := \frac{AC \cdot (AH + AK)}{AH} \quad AR := KL$

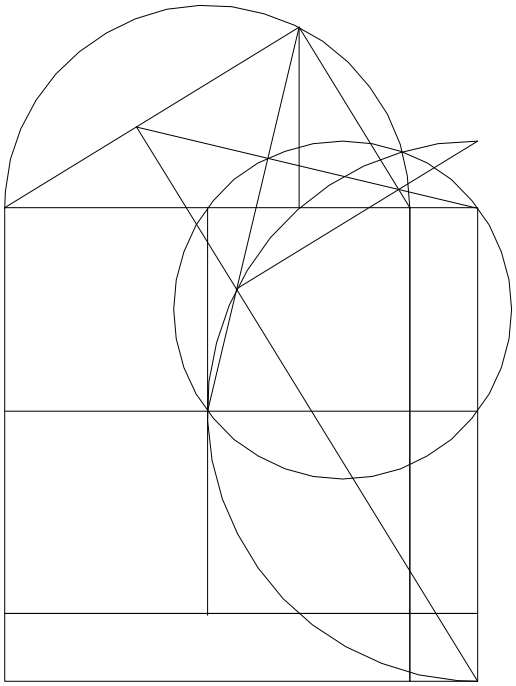
$PR := AP - AR \quad OQ := PR \quad QR := PO$

$LQ := QR + LR \quad DR := \frac{OQ \cdot LR}{LQ} \quad AD := AR + DR$

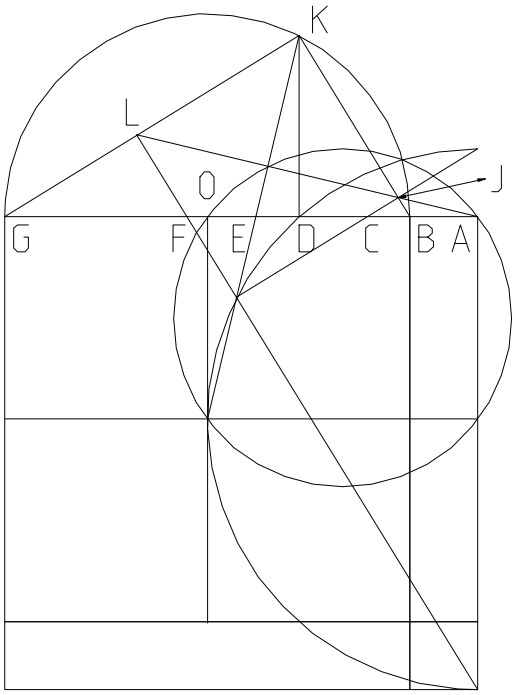
$AD = 25.333$ Plug values in below.

$R_1 \equiv 9 \quad R_2 \equiv 1 \quad D \equiv 30$

$$\frac{(R_1 + D + R_2) \cdot (D - R_2 + R_1)}{2 \cdot D} = 25.333$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.




$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

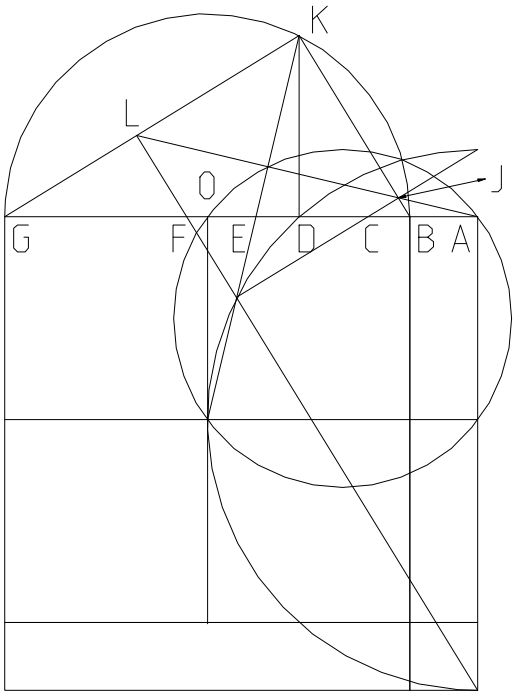
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



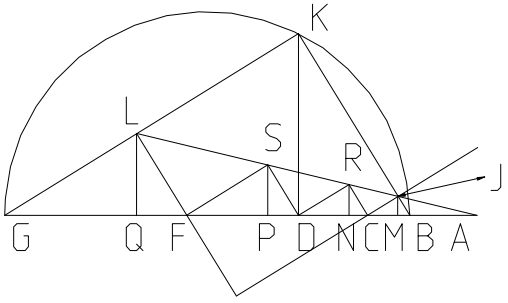

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$




$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$

$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

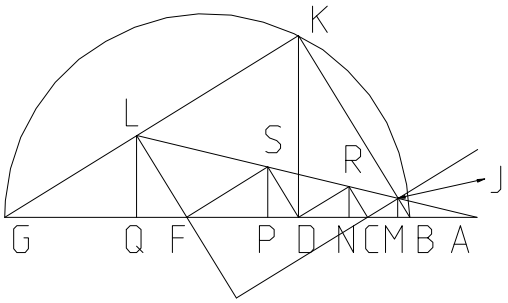
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$

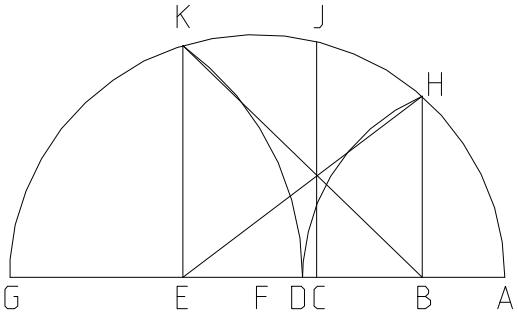



$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

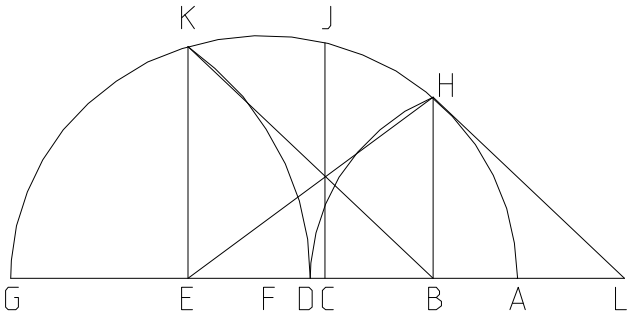
$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$




If the figure was drawn differently, XC would be $\sqrt{XB \cdot XE}$, irregardless of how XB and XE were placed, however that would require part of the figure that is not given here.




$$AG := N \quad AB := \frac{AG}{N}$$

$$BG := AG - AB \quad BH := \sqrt{AB \cdot BG}$$

$$AH := \sqrt{AB^2 + BH^2} \quad AD := AH \quad DG := AG - AD$$

$$GK := DG \quad GE := \frac{GK^2}{AG} \quad AE := AG - GE$$

$$BE := AE - AB$$

$$EG := AG - AE \quad EK := \sqrt{AE \cdot EG} \quad BL := \frac{BE \cdot BH}{EK}$$

$$EL := BE + BL \quad BC := \frac{BL \cdot BE}{EL} \quad AC := AB + BC$$

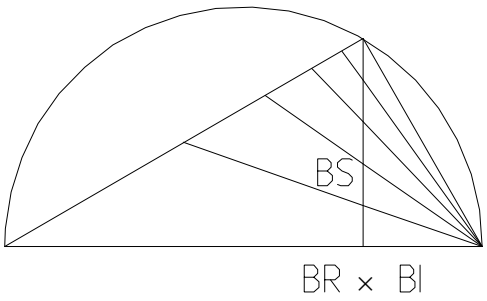
$N \equiv 4$

Make N any number and watch the equations, then make it equal to 1 and see what happens. Now this is strange work, for the formula is an identity with AC, so what happens at 1? This is an example of Binary contradiction.

$AC = 2$

$$\frac{\begin{bmatrix} AB^{\left(\frac{5}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{7}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + 2 \cdot AB \cdot \sqrt{BG} \cdot (AB + BG)^{\left(\frac{3}{2}\right)} \ldots \\ + - AB^{\left(\frac{5}{2}\right)} \cdot \sqrt{BG} - AB^{\left(\frac{3}{2}\right)} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}}{\begin{bmatrix} AB^{\left(\frac{1}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{3}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + AB^{\left(\frac{3}{2}\right)} \cdot \sqrt{BG} + \sqrt{AB} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}} = 2$$

# Pyramid of Ratios, Moving the Point



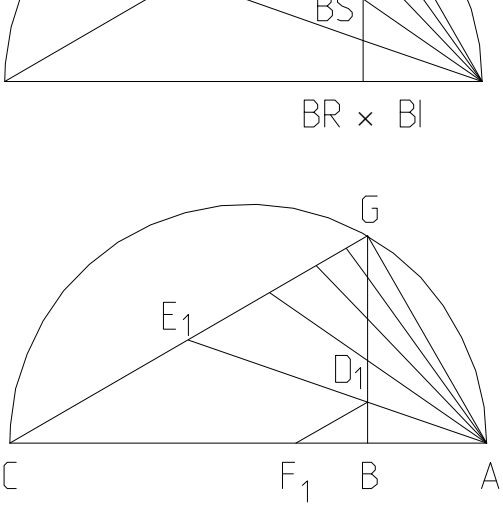

BR = Base Ratio, BS = Bisector Segments, BI = Base Index.

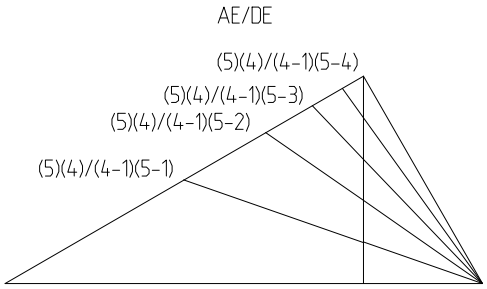
$BR \equiv 4 \quad BS \equiv 5 \quad BI := 2 \quad AC := BR \quad AB := \frac{AC}{BR} \cdot BI$

$BC := AC - AB \quad BG := \sqrt{AB \cdot BC} \quad \delta := 1 .. BS - 1$

$BD_\delta := \frac{BG}{BS} \cdot \delta \quad BF_\delta := \frac{BC \cdot BD_\delta}{BG} \quad AD_\delta := \sqrt{AB^2 + (BD_\delta)^2}$

$AE_\delta := \frac{AD_\delta \cdot AC}{AB + BF_\delta} \quad DE_\delta := AE_\delta - AD_\delta$



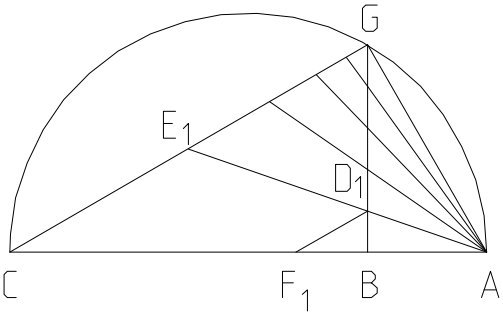




$\frac{AE_\delta}{DE_\delta}$

| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |

$\frac{BS \cdot BR}{(BR - BI) \cdot (BS - \delta)}$

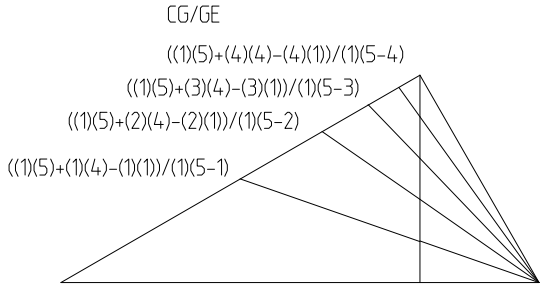
| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |




$AF_\delta := AB + BF_\delta \quad EJ_\delta := \frac{BD_\delta \cdot AC}{AF_\delta}$

$AJ_\delta := \sqrt{(AE_\delta)^2 - (EJ_\delta)^2} \quad CJ_\delta := AC - AJ_\delta$

$CE_\delta := \sqrt{(CJ_\delta)^2 + (EJ_\delta)^2} \quad CG := \sqrt{BG^2 + BC^2}$




$GE_\delta := CG - CE_\delta$

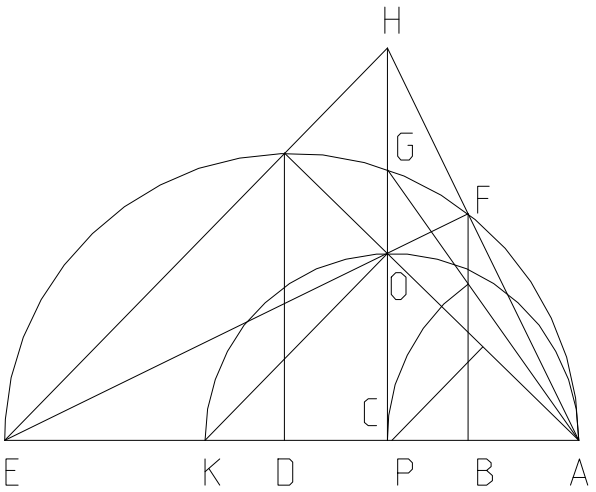
$\frac{CG}{GE_\delta}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

$\frac{(BI \cdot BS + \delta \cdot BR - \delta \cdot BI)}{(BI \cdot (BS - \delta))}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

What is AD. What is BD to $\sqrt{AB \cdot DE}$ ?




$AE := 5.5 \quad AB := 1.05 \quad BE := AE - AB$

$AC := \left(AB^2 \cdot AE\right)^{\frac{1}{3}} \quad CE := AE - AC$

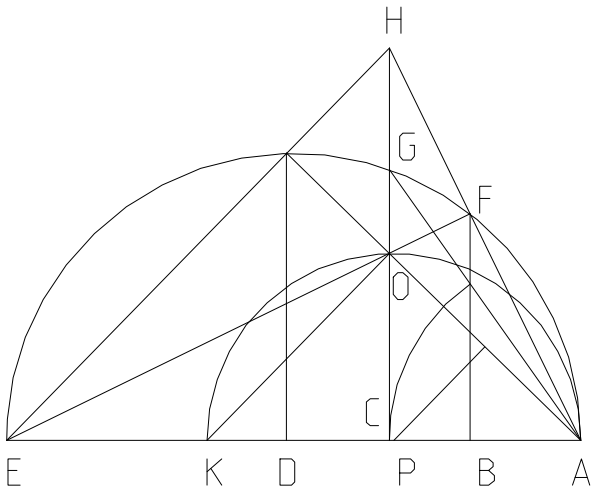
$BF := \sqrt{AB \cdot BE} \quad CO := \frac{BF \cdot CE}{BE} \quad AO := \sqrt{AC^2 + CO^2}$

$AP := \frac{1}{2} \cdot \frac{AO^2}{AC} \quad AK := 2 \cdot AP \quad AD := \frac{AC \cdot AE}{AK}$

$DE := AE - AD \quad BD := AD - AB$

$AD = 2.807$

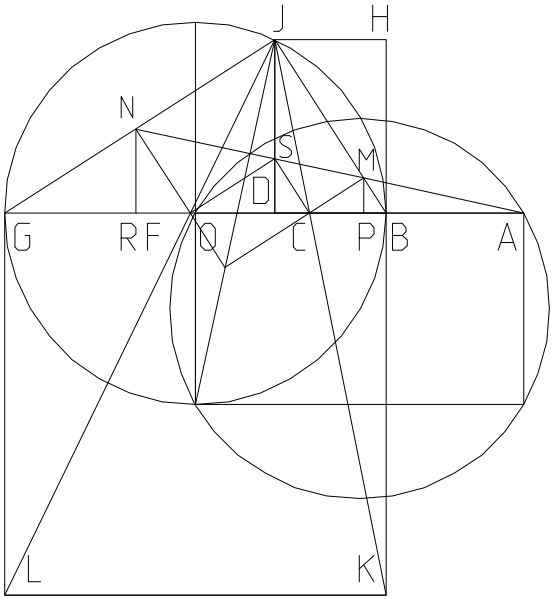
$$\frac{AE^{\frac{3}{3}} \cdot AB^{\frac{1}{3}} + AE^{\frac{2}{3}} \cdot AB^{\frac{2}{3}} + AE^{\frac{1}{3}} \cdot AB^{\frac{3}{3}}}{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}} = 2.807$$




$$\frac{\sqrt{AB \cdot DE}}{BD} = 0.957$$

$$\frac{AB^{\frac{1}{6}} \cdot AE^{\frac{1}{6}} \cdot \sqrt{AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} + AE - AE^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} \cdot \sqrt{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}}}{AE + AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} - 2 \cdot AB} = 0.957$$

The figure cuts the base in Cube Roots and provides some interesting ratios.




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

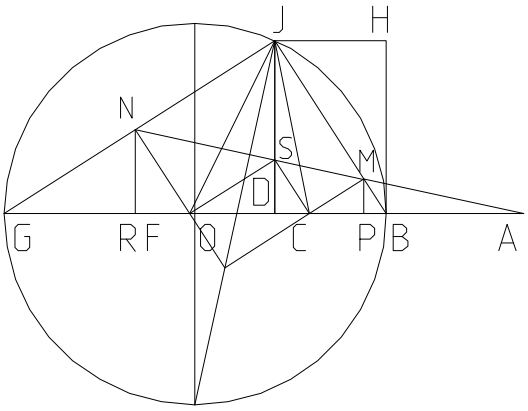
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

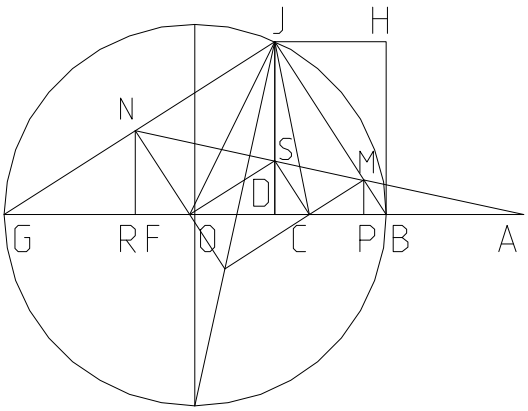
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

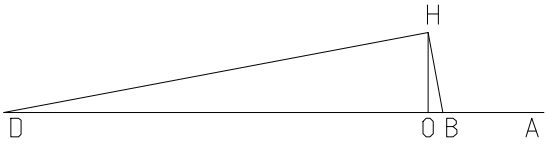
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$




$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

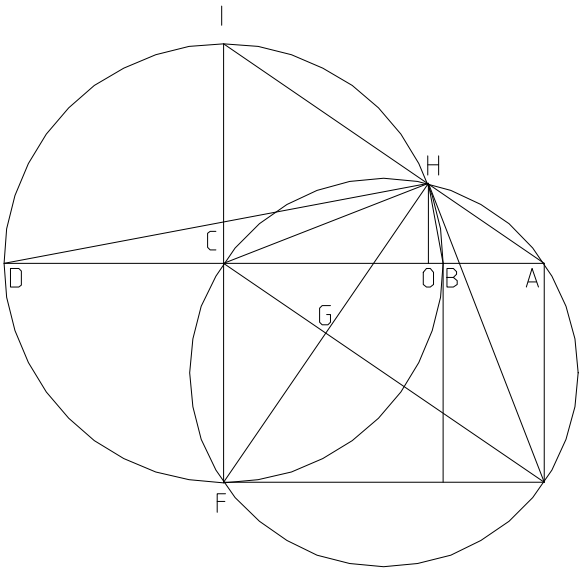
$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$




Given AD and AB on AD, place a right triangle on BD as base such that the opposite sides are in the ratio of AB to AD.

$BD := 8 \quad AB := 2 \quad AD := AB + BD$

$BC := \frac{BD}{2} \quad AC := AB + BC \quad CF := BC$




$CI := BC \quad CH := BC \quad AE := BC$

$CE := \sqrt{AC^2 + AE^2} \quad CG := \frac{CH^2}{CE}$

$GH := \sqrt{CH^2 - CG^2} \quad FH := 2 \cdot GH \quad FI := CF + CI$

$HI := \sqrt{FI^2 - FH^2} \quad AI := \sqrt{AC^2 + CI^2}$

$AH := AI - HI \quad AO := \frac{AC \cdot AH}{AI} \quad HO := \frac{CI \cdot AO}{AC}$

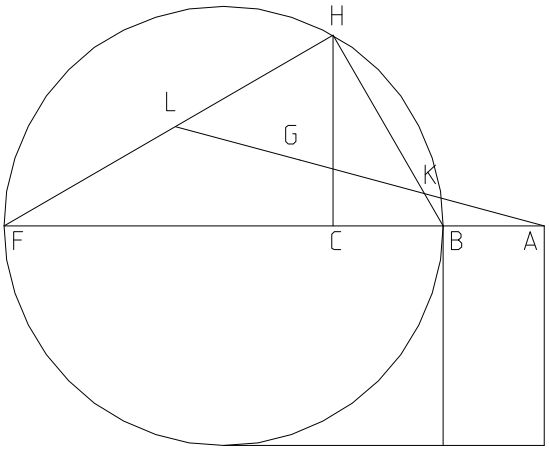
$BO := AO - AB \quad DO := BD - BO$

$DH := \sqrt{DO^2 + HO^2} \quad BH := \sqrt{BO^2 + HO^2}$

$\frac{DH}{BH} = 5 \qquad \frac{AD}{AB} = 5$

Given a straight edge and compass, AB and BD find the sum of six cubes divided by the sum of five squares.

$$\frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BD + AB \cdot BD^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BD + BD^2} = 2.308 \qquad AO = 2.308$$




Given AF and AB on AF and a right triangle on BF divide the sides of the triangle such that a section on one side is to the other as AB is to AF.

Now it can be realized that there are stipulations as to possible placements of the given triangle.

$AB := 3 \quad BF := 10 \quad BC := 1$

$AC := AB + BC \quad BE := \frac{BF}{2} \quad AE := AB + BE$

$$DOABLE := \frac{2\cdot AB^3 + 3\cdot AB^2\cdot BF + AB\cdot BF^2}{2\cdot AB^2 + 2\cdot AB\cdot BF + BF^2} \le AC < AE$$

$DOABLE = 1$

$AF := AB + BF$

$CF := BF - BC \quad CH := \sqrt{BC\cdot CF} \quad EJ := BE$

$CE := BE - BC \quad JH := \sqrt{CE^2 + (CH + EJ)^2}$

$DE := \frac{CE\cdot EJ}{EJ + CH} \quad AD := AE - DE$

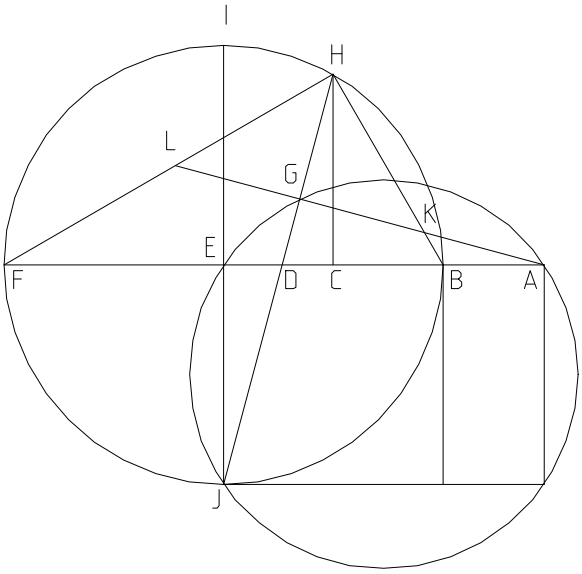
$JD := \sqrt{DE^2 + EJ^2} \quad DG := \frac{DE\cdot AD}{JD}$

$AG := \sqrt{AD^2 - DG^2} \quad GH := JH - (JD + DG)$

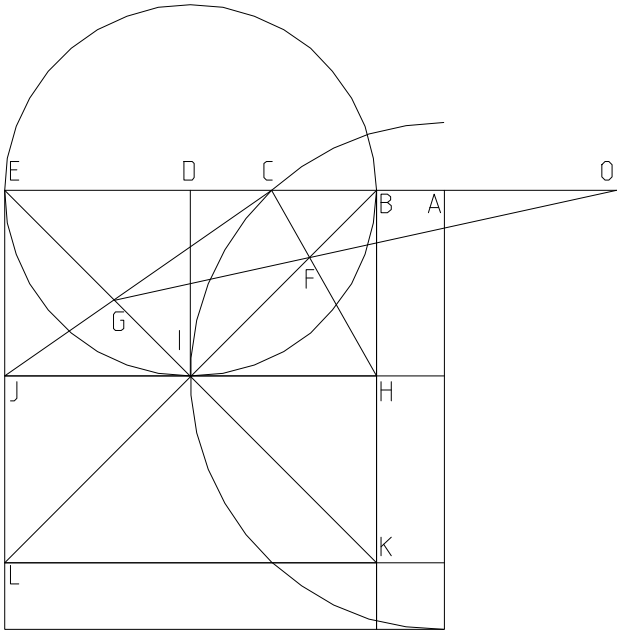
$HK := \sqrt{2\cdot GH^2} \quad BH := \sqrt{BC^2 + CH^2} \quad HL := HK$

$BK := BH - HK \quad FH := \sqrt{CF^2 + CH^2}$

$FL := FH - HL$




$\frac{FL}{BK} = 4.333 \qquad \frac{AF}{AB} = 4.333$




Projecting from KL or HJ is productive, can I find any other productive points?

$AB := 5 \quad BE := 3 \quad BK := BE \quad AE := AB + BE$

$BD := \frac{BE}{2} \quad AC := \sqrt{AB \cdot AE} \quad HI := BD \quad IJ := BD$

$BC := AC - AB \quad CE := BE - BC$

$\Delta := 2 \quad \delta := 1 .. \Delta$

$BH_\delta := \frac{BK}{\Delta} \cdot \delta \quad Ha_\delta := \frac{BH_\delta \cdot HI}{BC} \quad EJ_\delta := BH_\delta$

$Ba_\delta := Ha_\delta + BD \quad Bb_\delta := \frac{(BH_\delta)^2}{Ba_\delta} \quad Jc_\delta := \frac{EJ_\delta \cdot IJ}{CE}$

$Ec_\delta := Jc_\delta + BD \quad Ed_\delta := \frac{(EJ_\delta)^2}{Ec_\delta} \quad Ef_\delta := Bb_\delta$

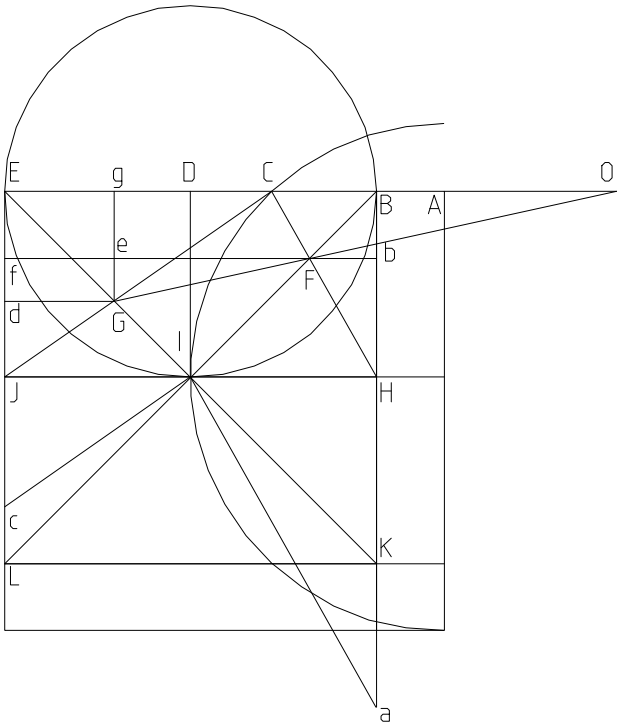
$df_\delta := Ed_\delta - Ef_\delta \quad Ge_\delta := df_\delta \quad Fb_\delta := \frac{HI \cdot BH_\delta}{Ba_\delta}$

$Gd_\delta := \frac{IJ \cdot EJ_\delta}{Ec_\delta} \quad ef_\delta := Gd_\delta \quad Fe_\delta := BE - (ef_\delta + Fb_\delta)$

$Gg_\delta := Ed_\delta \quad Og_\delta := \frac{Fe_\delta \cdot Gg_\delta}{Ge_\delta} \quad Eg_\delta := ef_\delta$

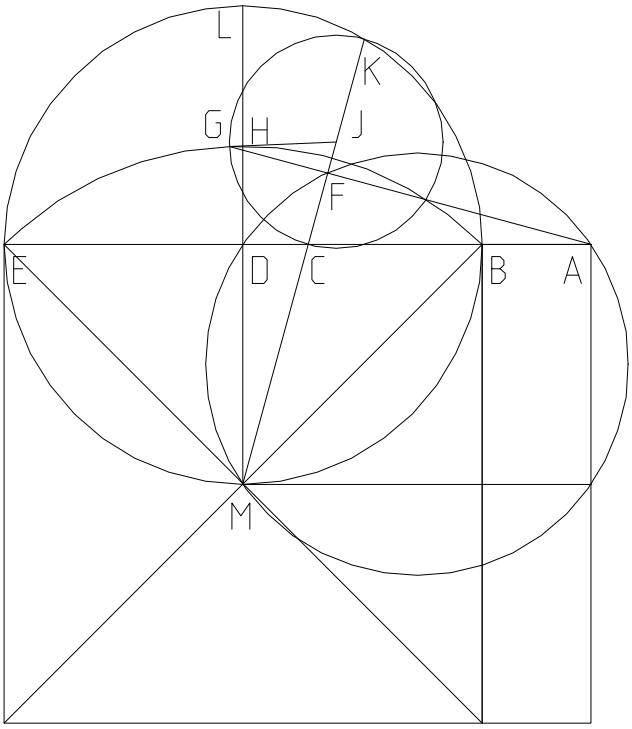
$EO_\delta := Og_\delta + Eg_\delta \quad BO_\delta := EO_\delta - BE$

| $\left(\frac{EO_\delta}{BO_\delta}\right)$ |
|---|
| 1.2649 |
| 1.6 |




$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} = 1.2649 \qquad \frac{AE}{AB} = 1.6$

I have not found any.




The power line for cube root abstraction is developed off from a simple curve.

$$AB := 33 \quad BE := 11 \quad BD := \frac{BE}{2} \quad AE := AB + BE$$

$$R_1 := \frac{\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AB}{2} \quad R_2 := \frac{AE - \left(AB \cdot AE^2\right)^{\frac{1}{3}}}{2}$$

$$D := \left[\left(AB \cdot AE^2\right)^{\frac{1}{3}} - \left(AB^2 \cdot AE\right)^{\frac{1}{3}}\right] + R_1 + R_2$$

$$BC := \frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D}$$

The formula for the power line (BC) was given in 01_08_96.MCD

$$CD := BD - BC \quad AC := BC + AB \quad DM := BD$$

$$DL := BD \quad CM := \sqrt{CD^2 + DM^2} \quad ML := DM + DL$$

$$MK := \frac{DM \cdot ML}{CM} \quad CK := MK - CM \quad CJ := \frac{CK}{2}$$

$$JG := CJ \quad JM := CM + CJ \quad BM := \sqrt{2 \cdot BD^2}$$

$$GM := BM \quad FJ := \frac{JG^2}{JM} \quad FM := JM - FJ$$

$$AD := BD + AB \quad HM := \frac{CM \cdot FM}{DM}$$

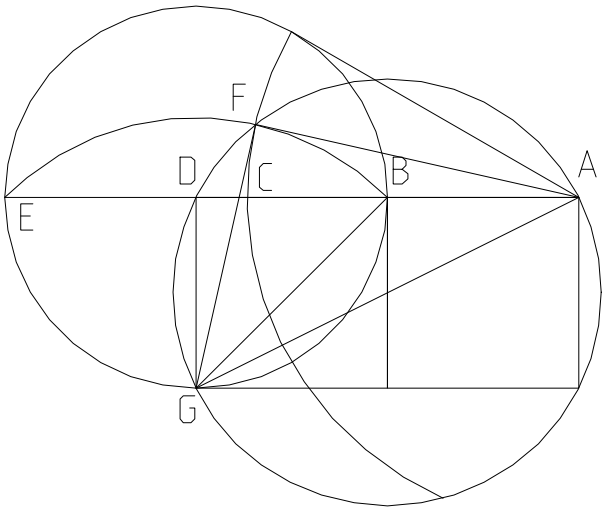
$$DH := HM - DM \quad AH := \sqrt{DH^2 + AD^2}$$

$$CF := FM - CM \quad FH := \frac{CD \cdot HM}{CM}$$

$$AF_1 := AH - FH \quad AF_2 := \sqrt{AC^2 - CF^2}$$

$$AF_1 - AF_2 = 0$$

Trivial Method; Square Root




$N := 9003$

$AE := N \quad AB := \frac{AE}{N} \quad BE := AE - AB \quad BD := \frac{BE}{2}$

$DG := BD \; AD := AB + BD \quad AG := \sqrt{AD^2 + DG^2}$

$BG := \sqrt{BD^2 + DG^2} \quad FG := BG \quad AF := \sqrt{AG^2 - FG^2}$

$AC := \sqrt{AB \cdot AE}$

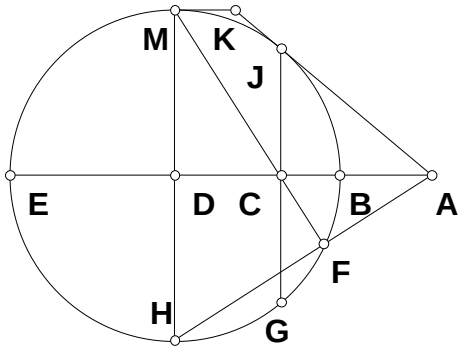
$AC - AF = 0$

$AF = 94.884$

$AC = 94.884$

## 01/24/96 Tangent

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**




$N := 5 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2} \qquad AD := AB + BD$

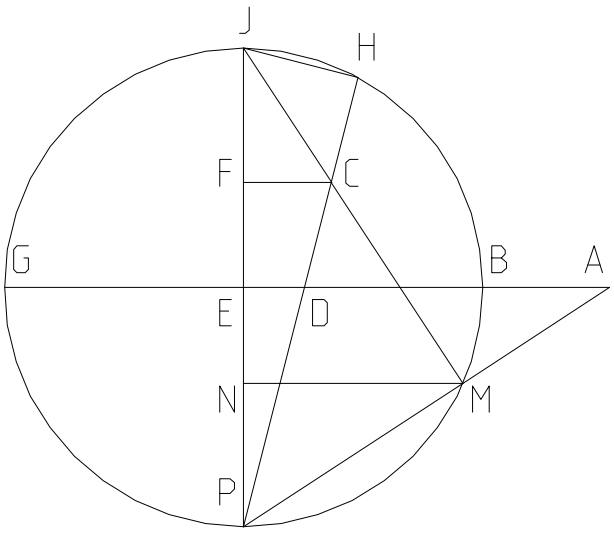
$DM := BD \qquad DH := BD \qquad CD := \frac{DH \cdot DM}{AD}$

$BC := BD - CD \qquad CE := BE - BC$

$CJ := \sqrt{BC \cdot CE} \qquad AJ := \sqrt{(AB + BC)^2 + CJ^2}$

$$\sqrt{(AB + BC)^2 + CJ^2} - \sqrt{AB \cdot AE} = 0$$

Given a point on BG, project to the point of cubic similarity.




$BG := 100 \quad BD := 49 \qquad BE := \frac{BG}{2}$

$EG := BE \quad DE := BE - BD \quad EP := BE$

$EJ := BE \quad DP := \sqrt{DE^2 + EP^2} \quad JP := EP + EJ$

$HP := \frac{EP \cdot JP}{DP} \quad DH := HP - DP \quad CD := \frac{DH}{2}$

$CP := DP + CD \quad CF := \frac{DE \cdot CP}{DP} \quad FP := \frac{EP \cdot CP}{DP}$

$EF := FP - EP \quad FJ := EJ - EF \quad CJ := \sqrt{CF^2 + FJ^2}$

$JM := \frac{FJ \cdot JP}{CJ} \quad JN := \frac{FJ \cdot JM}{CJ} \quad NP := JP - JN$

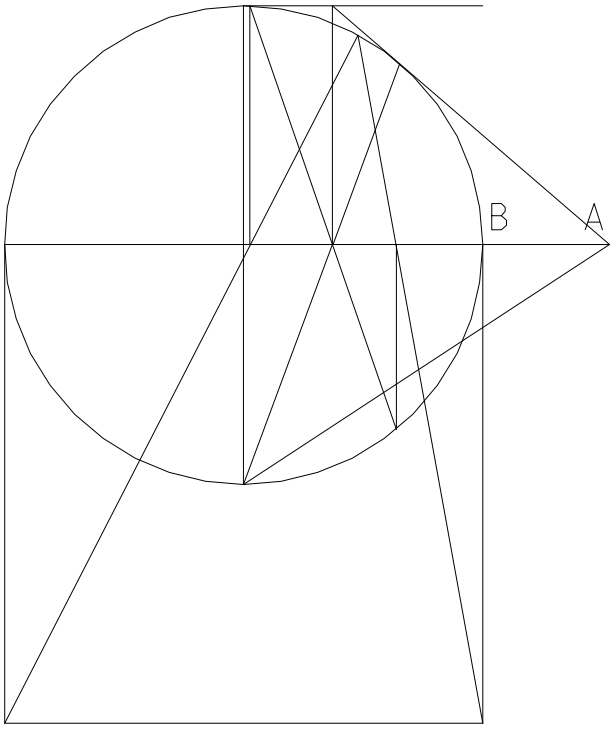
$MP := \frac{CF \cdot JP}{CJ} \quad AP := \frac{MP \cdot EP}{NP} \quad MN := \frac{CF \cdot JM}{CJ}$

$$\frac{BG^4 - 3 \cdot BG^3 \cdot BD + 3 \cdot BG^2 \cdot BD^2 - BD^3 \cdot BG}{BG^3 - 3 \cdot BG^2 \cdot BD + 3 \cdot BG \cdot BD^2 - 2 \cdot BD^3} = 884.222$$

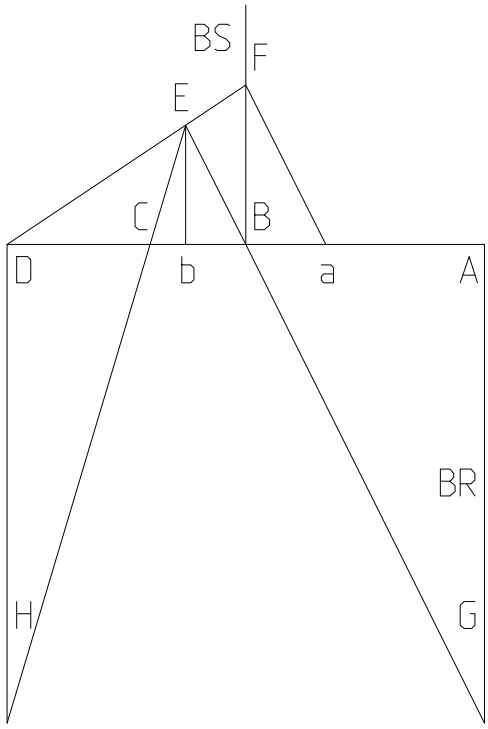
$AE := \frac{MN \cdot AP}{MP} \quad AG := AE + EG$

$$AG = 884.222$$

$$\frac{BG \cdot \left[ (AG - BG)^{\frac{2}{3}} \cdot AG^{\frac{2}{3}} + AG^{\frac{1}{3}} \cdot BG - AG^{\frac{4}{3}} - BG \cdot (AG - BG)^{\frac{1}{3}} + (AG - BG)^{\frac{1}{3}} \cdot AG \right]}{(AG - BG)^{\frac{1}{3}} \cdot (2 \cdot AG - BG)} - BD = 0$$




One may be tempted to state the obvious, $\frac{A^{N+1}}{A^{N}} := A_{\tau}$, but what is not so obvious at first blush is that the processes themselves are assigned dimensional values. This has significance when using mathematics to theorize dimensions beyond three. Dimensions are so generally defined that processes are legitimate dimensional differences, but it is also impossible to defend mathematical theory about dimensions as objective. It becomes a point of Philosophical Mystic contemplation to realize that relationships concerning a single dimensional object and several processes adding dimensionally to the whole, is true of a multidimensional object without those processes!




Linear division $\frac{2\cdot(A + B)}{A}$

$BR := \frac{1}{4}$ $BS := 3$

$AD := \frac{BR}{BR}$ $AG := AD\cdot BR$ $AB := \frac{AD}{2}$

$BF := AB\cdot BS$ $Ba := \frac{AB\cdot BF}{AG}$ $BD := AB$

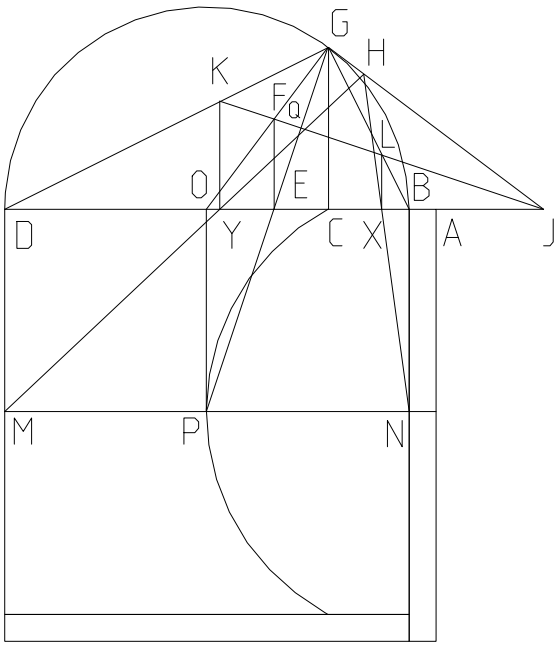
$Da := BD + Ba$ $Bb := \frac{Ba\cdot BD}{Da}$ $Db := BD - Bb$

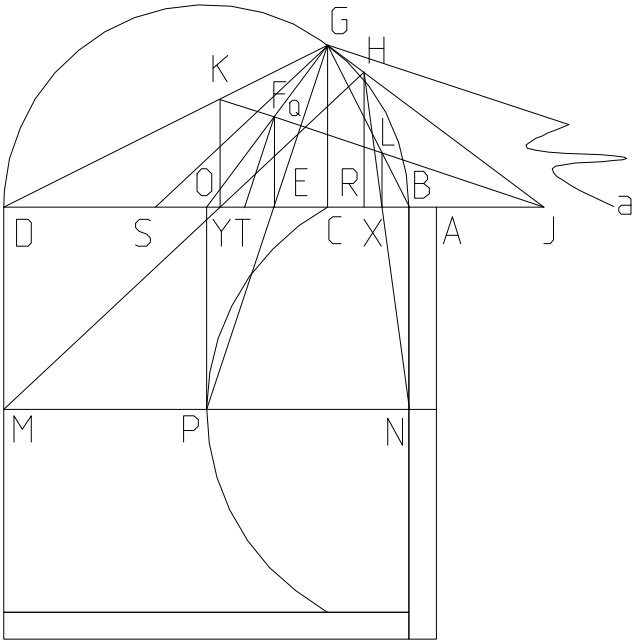
$Eb := \frac{BF\cdot Db}{BD}$ $DH := AG$

$DC := \frac{Db\cdot DH}{DH + Eb}$ $\frac{AD}{DC} = 26$

$\frac{2\cdot(BR + BS)}{BR} = 26$

Hitting JG from any BN while maintaining complimentary roots.




$AB := 2 \quad BD := 5 \quad BO := \frac{BD}{2} \quad AD := AB + BD$

$DO := BO \quad AC := \sqrt{AB \cdot AD} \quad BC := AC - AB$

$CD := BD - BC \quad CG := \sqrt{BC \cdot CD} \quad CO := BO - BC$

$N := 7 \quad OP := BO \cdot N \quad BN := OP \quad DM := OP$

$NP := BO \quad MP := BO \quad EO := \frac{CO \cdot OP}{OP + CG}$

$CE := CO - EO \quad EF := \frac{OP \cdot CE}{CO} \quad GO := BO$

$CJ := \frac{CG^2}{CO} \quad EJ := CJ + CE \quad Ca := \frac{EJ \cdot CG}{EF}$

$DJ := CD + CJ \quad Da := CD + Ca \quad JY := \frac{Ca \cdot DJ}{Da}$

$KY := \frac{EF \cdot JY}{EJ} \quad JK := \sqrt{JY^2 + KY^2} \quad JG := \sqrt{CJ^2 + CG^2}$

$GP := \sqrt{CO^2 + (OP + CG)^2} \quad EP := \sqrt{EO^2 + OP^2}$

$ET := \frac{EO \cdot EF}{OP} \quad JT := EJ + ET \quad FT := \sqrt{ET^2 + EF^2}$

$EG := GP - EP \quad EQ := \frac{FT \cdot EJ}{JT} \quad GQ := EG - EQ$

$KL := 2 \cdot GQ \quad JL := JK - KL$

$DY := DJ - JY \quad CS := \frac{DY \cdot CG}{DM} \quad JS := CS + CJ$

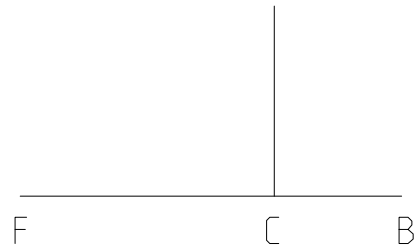
$YR := \frac{CS \cdot JY}{JS} \quad JR := JY - YR \quad HR := \frac{CG \cdot JR}{CJ}$

$AJ := CJ - AC \quad JX := \frac{JY \cdot JL}{JK}$

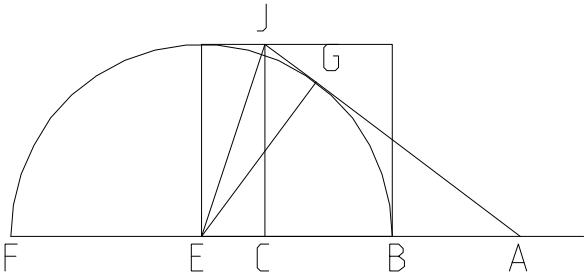
$BR_1 := JR - (AJ + AB) \quad BX := JX - (AJ + AB)$

$BR_2 := \frac{BX \cdot (BN + HR)}{BN}$

$BR_2 - BR_1 = 0$




Given BF and BC such that $\sqrt{(AB + BF)\cdot AB} = AB + BC$, find AB. It is obvious from the construction that answers are obtainable when BC is less than 1/2 of BF.

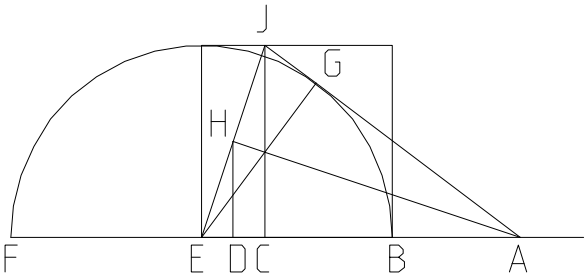



$\Delta := 5 \qquad \delta := 1..\Delta - 1 \quad BE := 5 \qquad BF := BE\cdot 2$

$BC_\delta := \frac{BE}{\Delta}\cdot\delta \qquad CE_\delta := BE - BC_\delta \qquad EG := BE$

$CJ := BE \quad EJ_\delta := \sqrt{\left(CE_\delta\right)^2 + CJ^2} \qquad DH := \frac{CJ}{2}$

$DE_\delta := \frac{CE_\delta}{2} \qquad AD_\delta := \frac{DH^2}{DE_\delta} \qquad AE_\delta := AD_\delta + DE_\delta$




$AC_\delta := AE_\delta - CE_\delta \qquad AJ_\delta := \sqrt{\left(AC_\delta\right)^2 + CJ^2}$

$GJ_\delta := \sqrt{\left(EJ_\delta\right)^2 - EG^2} \qquad AB_\delta := AE_\delta - BE$

$AF_\delta := AB_\delta + BF$

$\left[\frac{1}{2}\cdot\frac{BE}{\Delta}\cdot\delta\cdot\frac{(-2\cdot\Delta + \delta)}{(-\Delta + \delta)}\right] - AC_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$GJ_\delta - CE_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

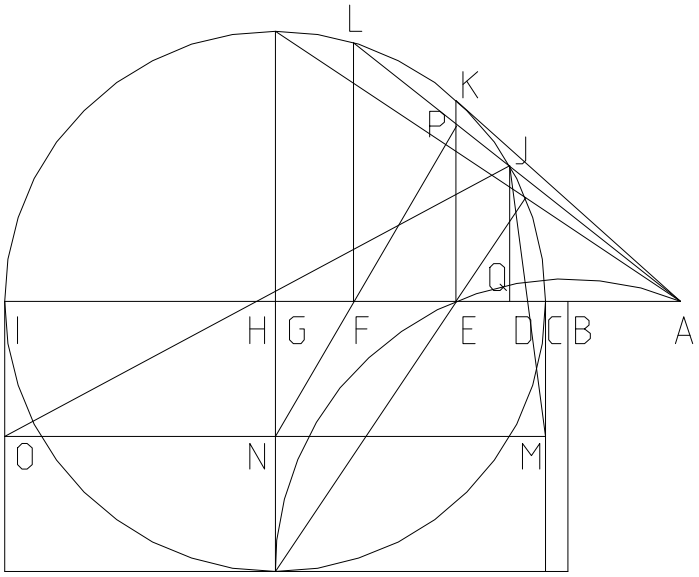
$AB_\delta + BC_\delta - \sqrt{AB_\delta\cdot(AB + BF)_\delta}$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$A := \frac{BF}{2} \qquad B := BC_2 \qquad \frac{2\cdot A\cdot B - B^2}{2\cdot A - 2\cdot B} - \left(AB_2 + BC_2\right) = 0$

$\sqrt{\left(AB_2 + BF\right)\cdot AB_2} - \left(AB_2 + BC_2\right) = 0$

$\frac{BF\cdot BC_2 - \left(BC_2\right)^2}{BF - 2\cdot BC_2} - \left(AB_2 + BC_2\right) = 0$ A rectangle minus a square divided by a line yeilds a line.




Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

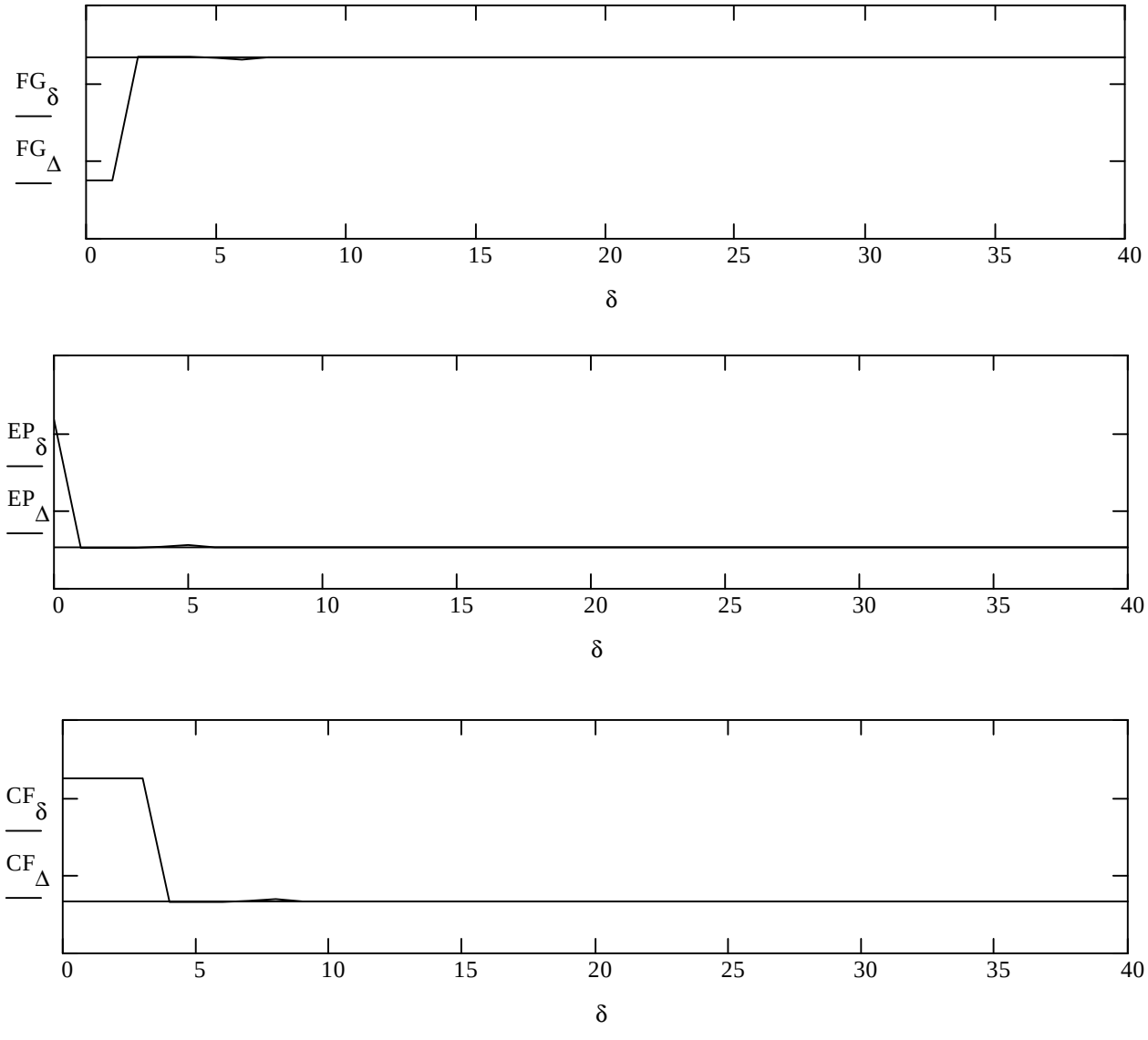
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

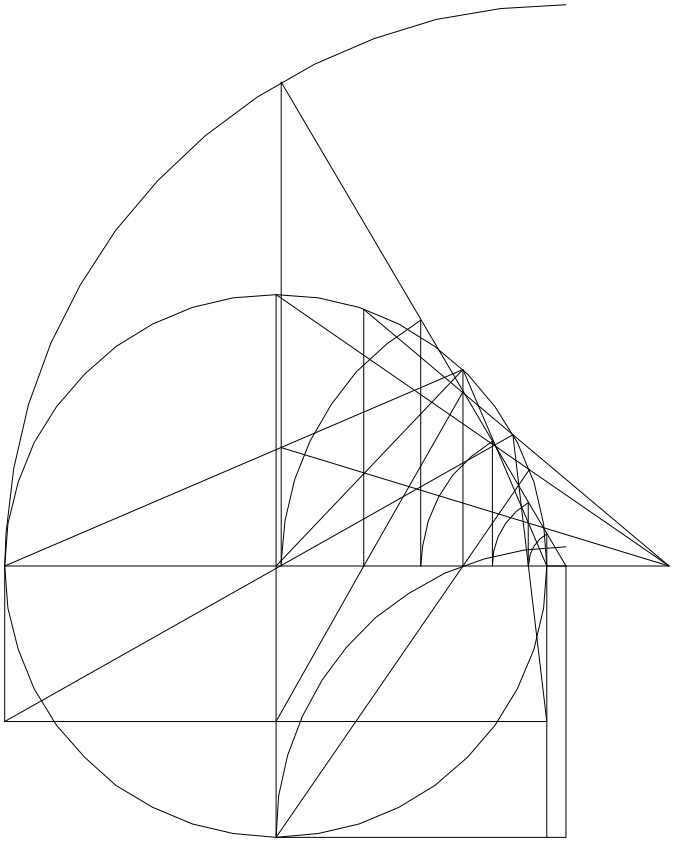
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.



Because of it's long projection, the last vertices is not drawn.

Does every $n^{th}$ root series have at least one square root pair?

$n := 5 \qquad \delta := 0..\frac{n}{2}$

$A := 3 \quad B := 10$

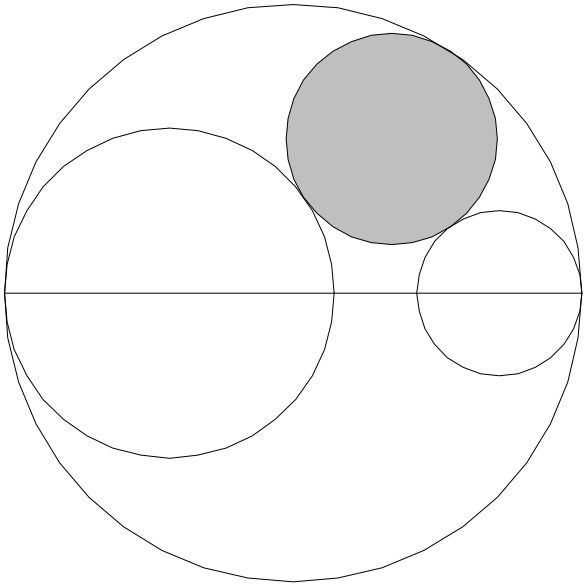
$$\sqrt{\left(A^{n-1}\cdot B^{1}\right)^{\frac{1}{n}}\cdot\left(A^{1}\cdot B^{n-1}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B} = 0$$

$$\sqrt{\left(A^{n-\delta}\cdot B^{\delta}\right)^{\frac{1}{n}}\cdot\left(A^{\delta}\cdot B^{n-\delta}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B}$$

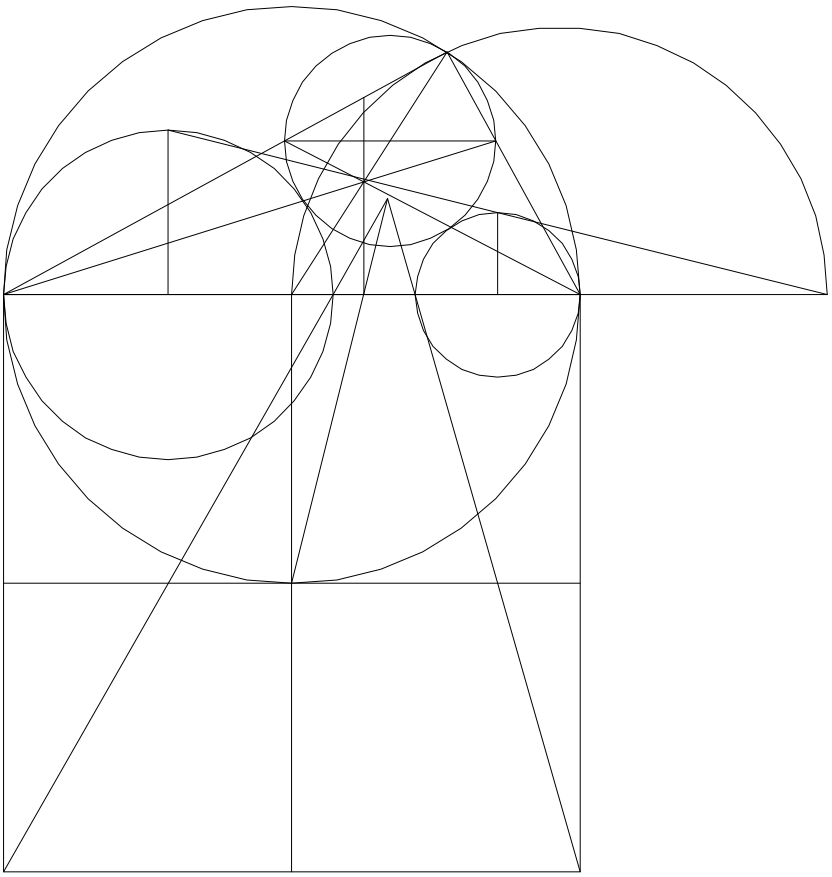
| |
|---|
| 0 |
| 0 |
| 0 |

A root series has as many vertices on a circle as it has square root pairs, and it has the greater whole of n/2 vertices where n is the root series denominator.

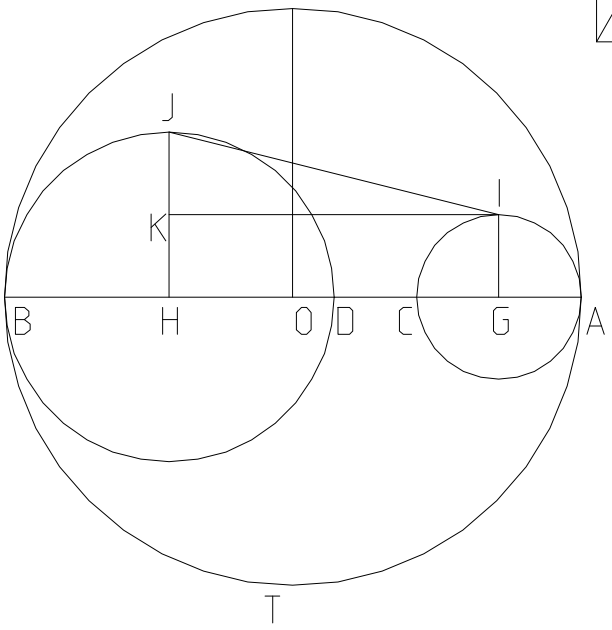
# Method for Unequals



Given three circles in the said configuration, find the fourth.
I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."



Process Summary.

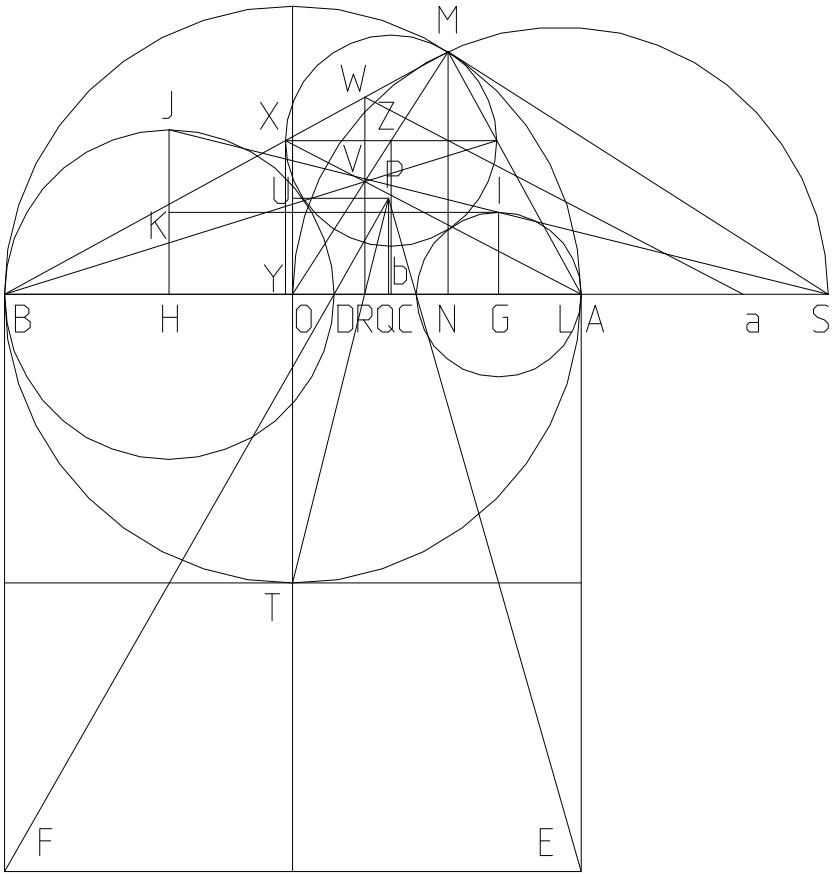


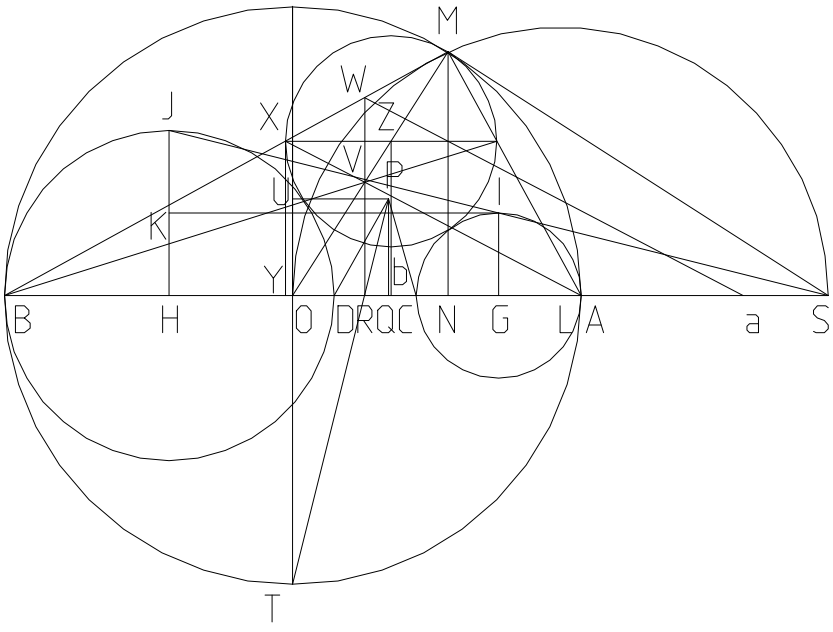

AO := 5 AG := 1 BH := 3 AB := 2·AO

BO := AO CG := AG GI := AG HJ := BH

DH := BH GH := AB – (AG + BH) HK := GI

JK := HJ – HK




$HS := \frac{GH \cdot HJ}{JK}$ $AH := AB - BH$

$AS := HS - AH$ $OS := AO + AS$

$SL := \frac{OS}{2}$ $MO := AO$ $MS := \sqrt{OS^2 - MO^2}$

$MN := \frac{MO \cdot MS}{OS}$ $NS := \frac{MS \cdot MN}{MO}$

$AN := NS - AS$ $ON := AO - AN$

$BN := AB - AN$ $AM := \sqrt{MN^2 + AN^2}$

$AE := AB$ $BF := AB$ $AC := 2 \cdot AG$

$BD := 2 \cdot BH$ $CD := AB - (AC + BD)$

$PQ := \frac{AE \cdot CD}{(AC + BD)}$ $OU := PQ$

$CQ := \frac{AC \cdot PQ}{AE}$ $AQ := AC + CQ$

$OQ := AO - AQ$ $OT := AO$

$TU := OT + OU$ $OR := \frac{OQ \cdot OT}{TU}$

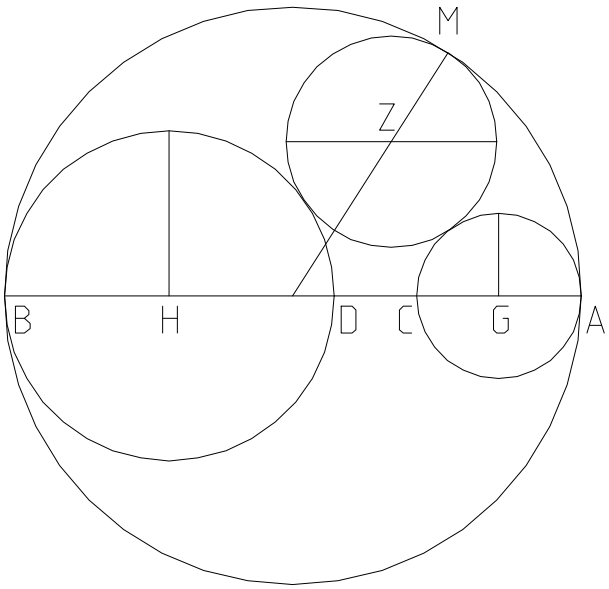
$RV := \frac{MN \cdot OR}{ON}$ $BR := BO + OR$

$RW := \frac{MN \cdot BR}{BN}$ $AR := AO - OR$

$Ra := \frac{AR \cdot RW}{RV}$ $XY := \frac{RW \cdot AB}{BR + Ra}$

$Zb := XY$ $OZ := \frac{MO \cdot Zb}{MN}$

$MZ := MO - OZ$ $MZ = 1.818$

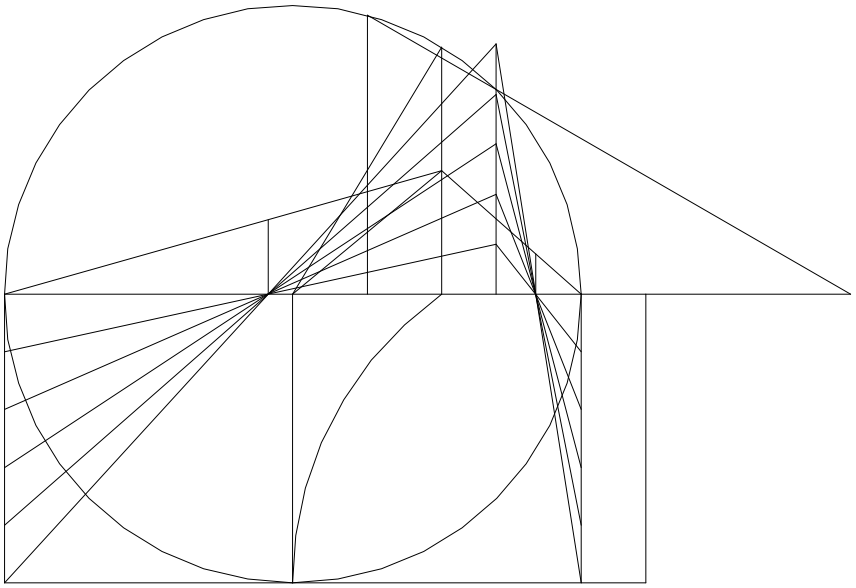


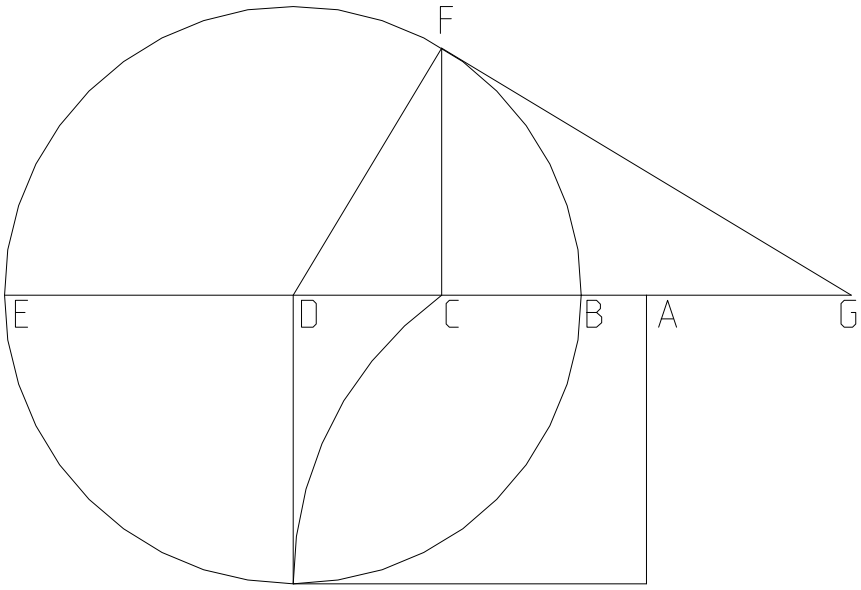

$$MZ = 1.818$$

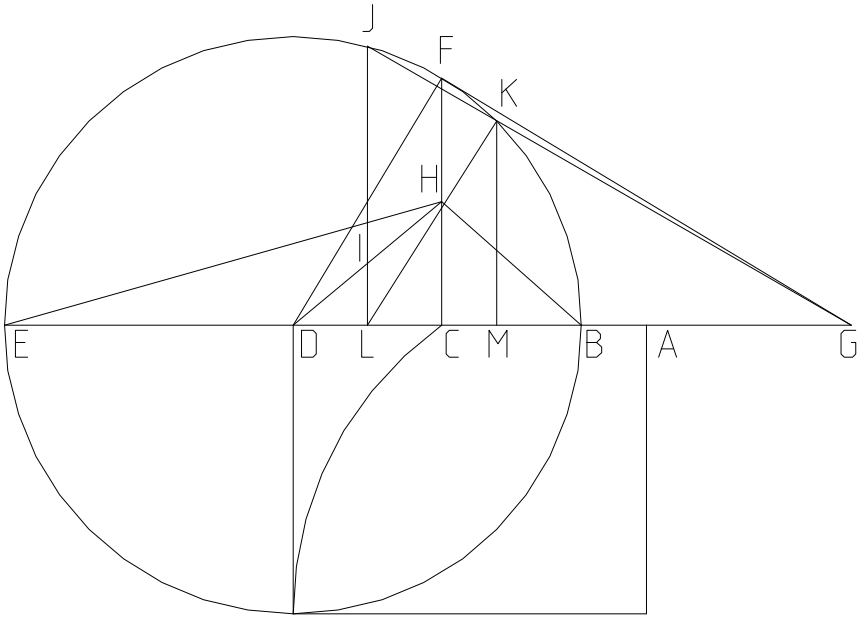
$$\frac{AB^3 - 2\cdot AB^2\cdot AG - 2\cdot AB^2\cdot BH + 4\cdot AB\cdot BH\cdot AG}{2\cdot AB^2 - 8\cdot BH\cdot AG} = 1.818$$

Neither in the process, nor in the Algebraic name. is the order of AG and BH recognized. Neither does it matter if they intersect.

# On Gemini Roots






$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

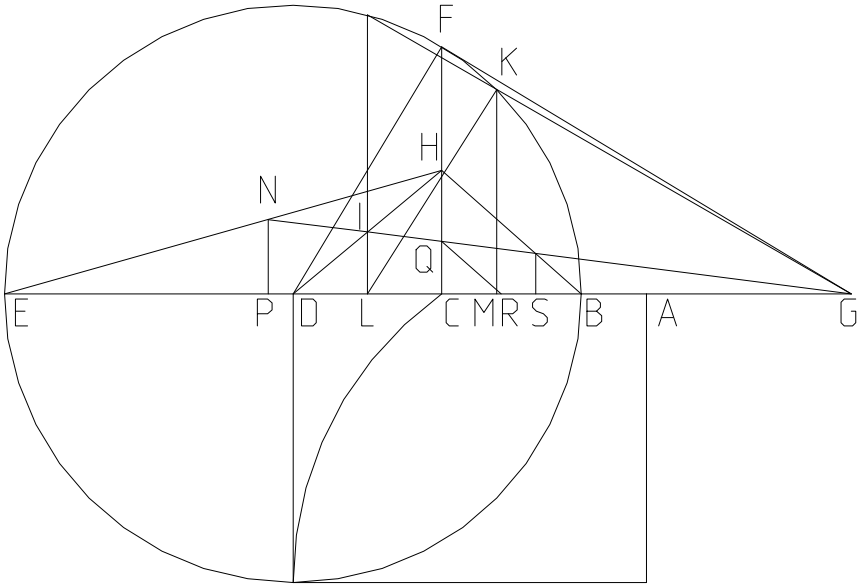
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

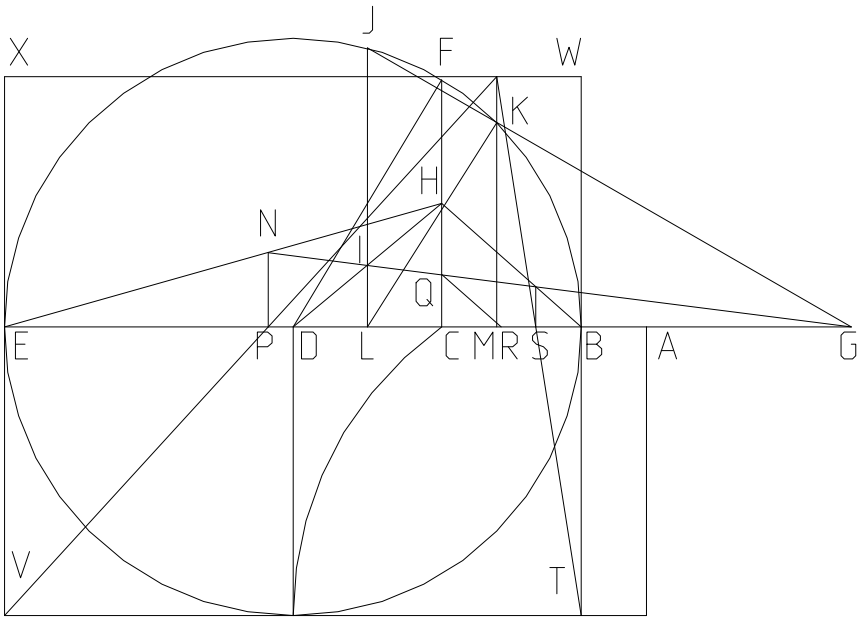



$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

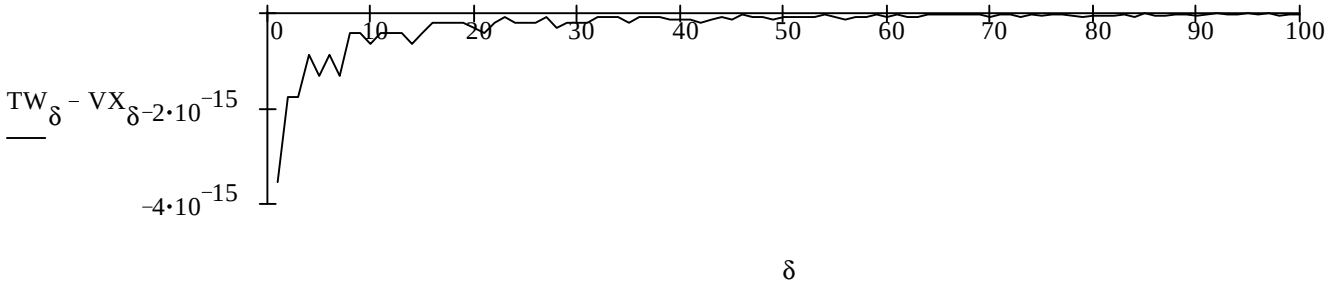
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$

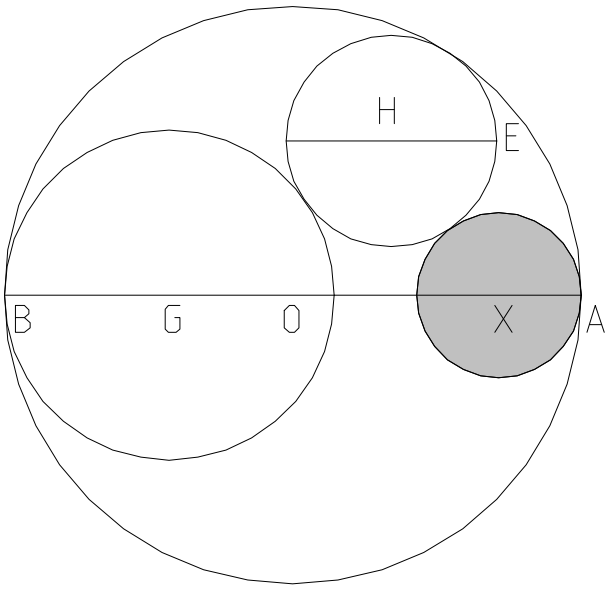



$$\delta := 1..100$$

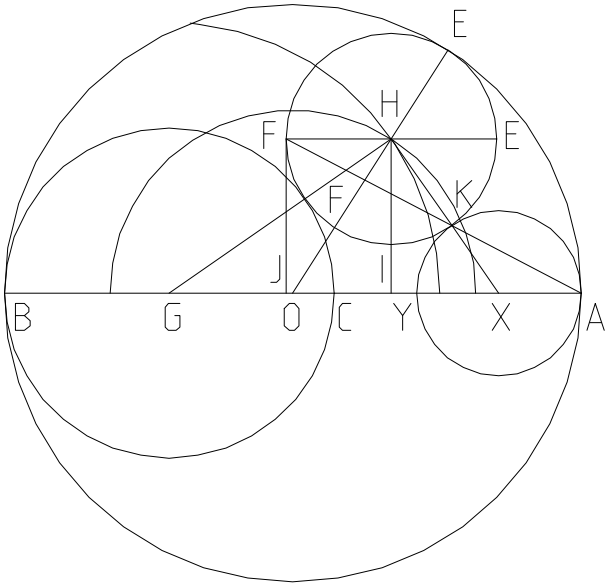
$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$






Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO. This of course means that if the sum of BG and EH is equal to AO, we have no result.




$AO := 5 \quad BG := 2.5 \quad EH := 1.5$

$AB := 2 \cdot AO \quad BC := 2 \cdot BG \quad EF := 2 \cdot EH$

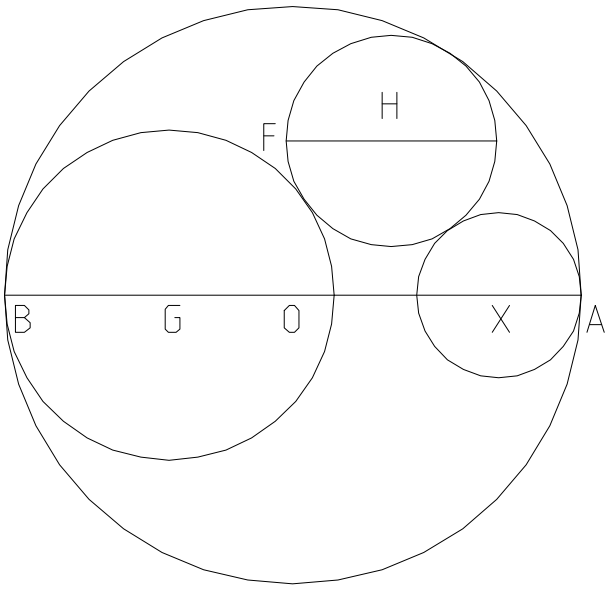
$GH := BG + EH \quad OH := AO - EH \quad GO := AO - BG$

$$GI := \frac{GH^2 + GO^2 - OH^2}{2 \cdot GO} \quad HI := \sqrt{GH^2 - GI^2}$$

$AG := AB - BG \quad AI := AG - GI \quad IJ := EH$

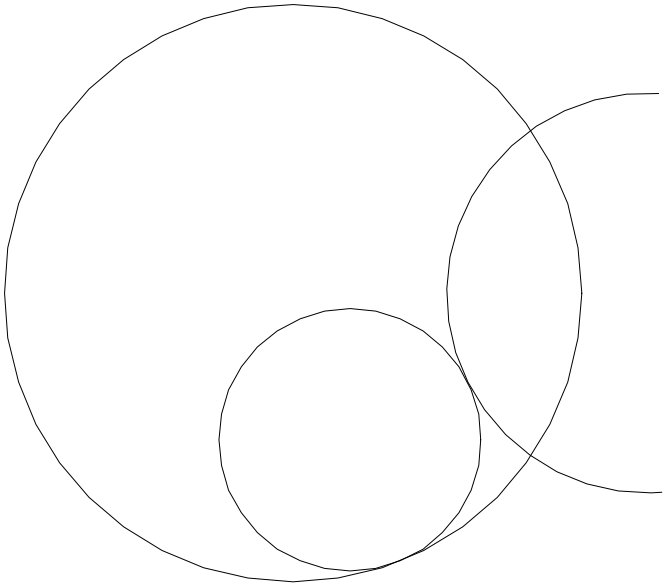
$$AJ := AI + IJ \quad FJ := HI \quad AF := \sqrt{AJ^2 + FJ^2}$$

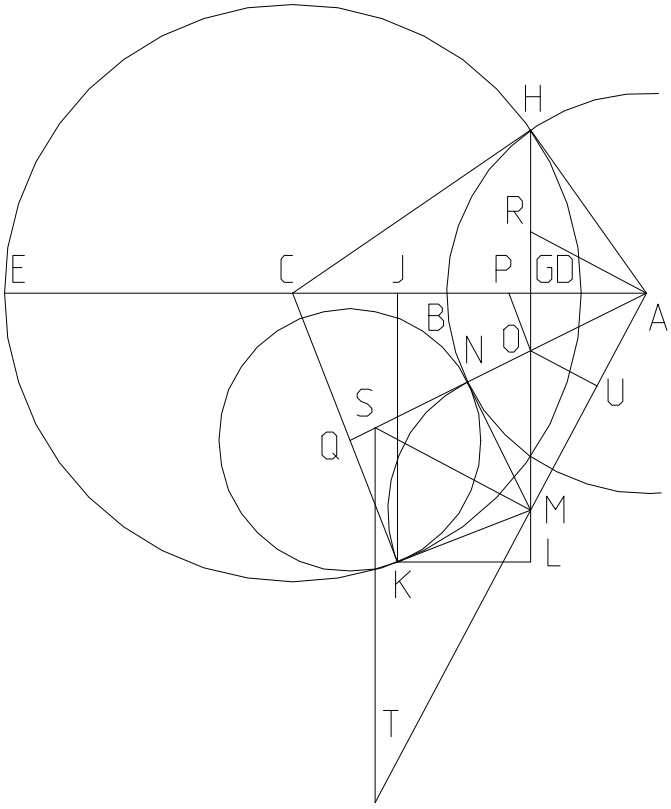
$$FK := \frac{AJ \cdot EF}{AF} \quad AK := AF - FK \quad AY := \frac{AF \cdot AK}{AJ}$$




$$AX := \frac{AY}{2} \qquad AX = 2.857$$

$$\frac{AO^3 - AO^2 \cdot EH - AO^2 \cdot BG}{AO^2 - AO \cdot BG - EH \cdot BG} = 2.857$$






Given a point of tangency, draw a circle in a crescent tangent to the other side. This figure is given for the tangent on the exterior of the crescent, the other will become obvious.

$AB := Concave\_Radius$

$CD := Convex\_Radius \quad AC := Center\_Difference$

$DE := 2 \cdot CD \quad BF := 2 \cdot AB \quad CE := CD$

$AE := AC + CE \quad BE := AE - AB$

$$CG := \frac{CD^2 + AC^2 - AB^2}{2 \cdot AC} \quad AG := AC - CG$$

$GE := AE - AG$

$GJ := Power\_Line\_Tangent \cdot GE$

$EJ := GE - GJ$

$DJ := DE - EJ \quad JK := \sqrt{DJ \cdot EJ}$

$CJ := CG - GJ \quad CK := CD \quad KL := GJ$

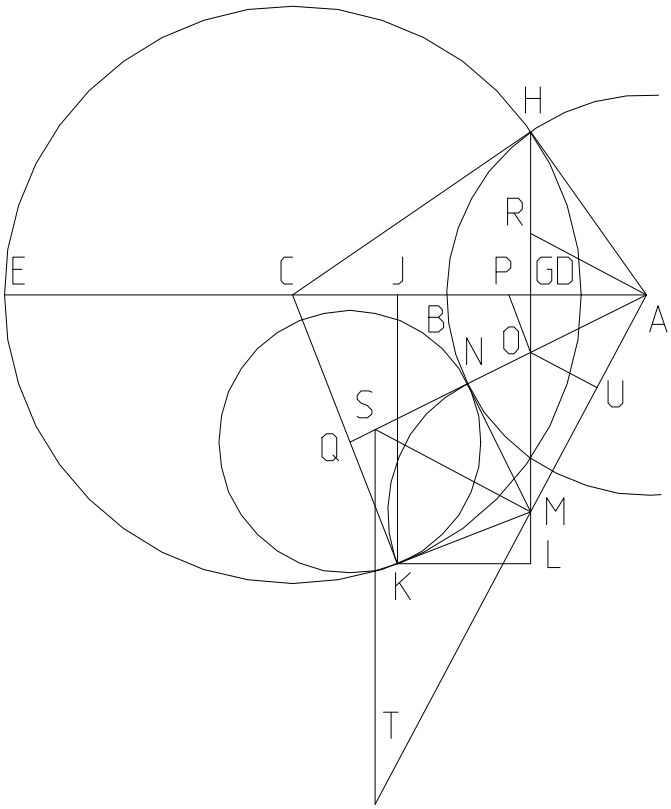
$$LM := \frac{CJ \cdot KL}{JK} \quad GL := JK \quad GM := GL - LM$$

$$AM := \sqrt{AG^2 + GM^2} \quad AN := AB$$

$$AS := \frac{AM^2}{AN} \quad MR := \frac{AM^2}{GM}$$

$$MS := \sqrt{AS^2 - AM^2} \quad AR := \sqrt{MR^2 - AM^2}$$

$$ST := \frac{MR \cdot MS}{AR} \quad MT := \frac{AM \cdot MS}{AR}$$




$$MO := \frac{ST \cdot AM}{AM + MT}$$

$$GO := GM - MO \quad GP := \frac{CJ \cdot GO}{JK}$$

$$AP := AG + GP \quad OP := \frac{CK \cdot GO}{JK}$$

$$CQ := \frac{OP \cdot AC}{AP} \quad QK := CK - CQ$$

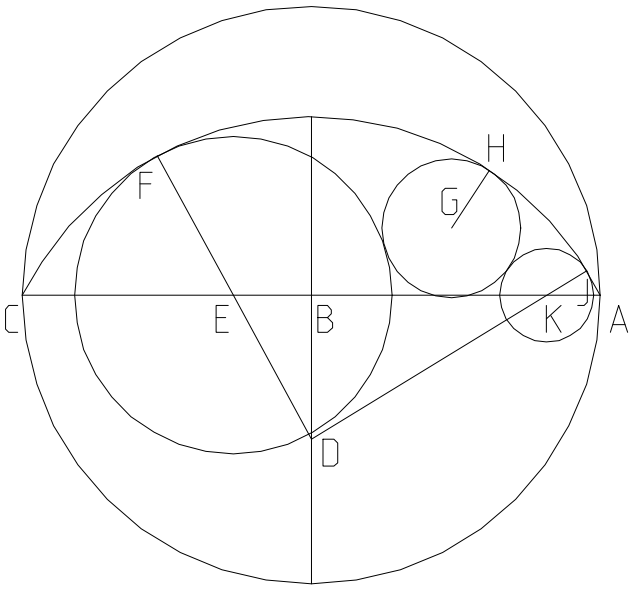
$$QK = 0.206$$

$$Concave\_Radius \equiv 2.37$$

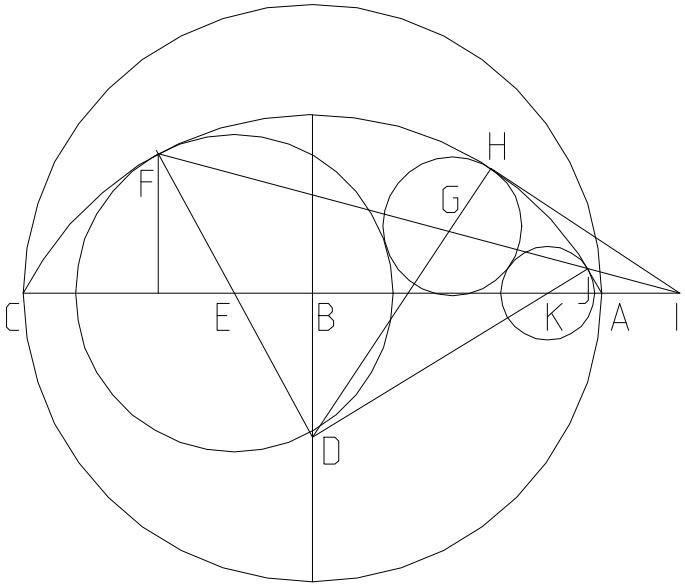
$$Convex\_Radius \equiv 1.5$$

$$Center\_Difference \equiv 1.84$$

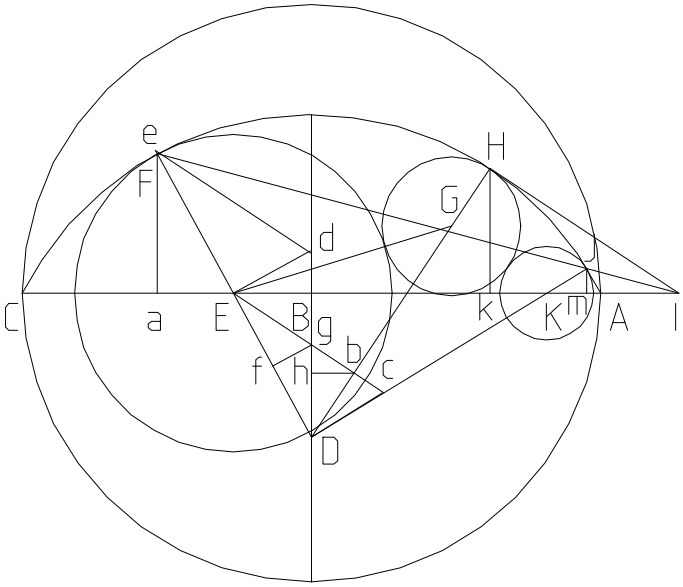
$$Power\_Line\_Tangent \equiv \frac{1}{3} \quad \text{Given as Fraction} < 1.$$




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

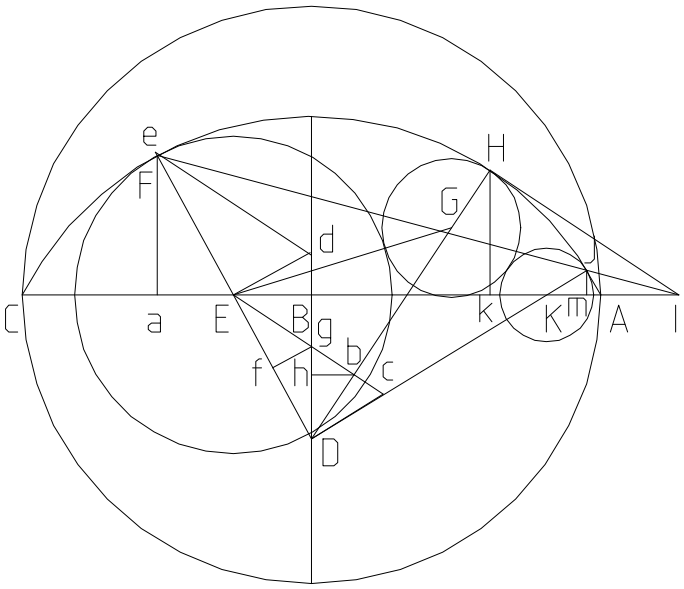
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

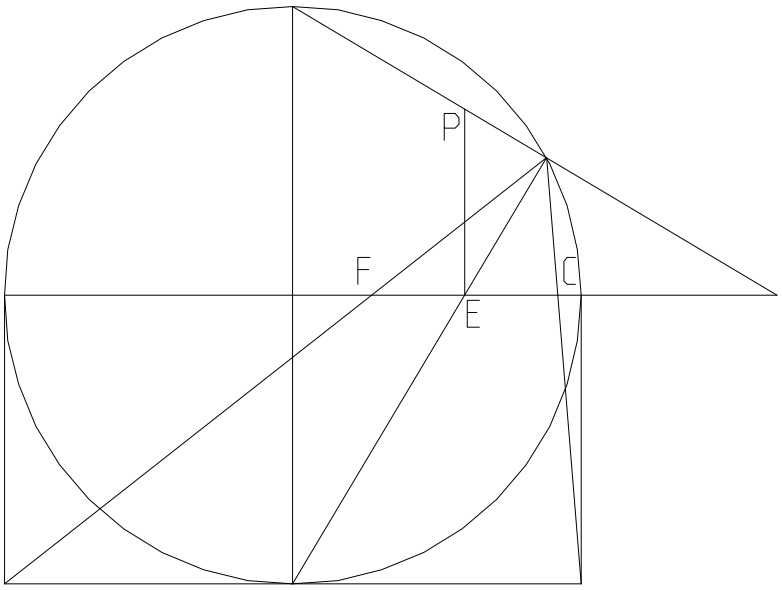
$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

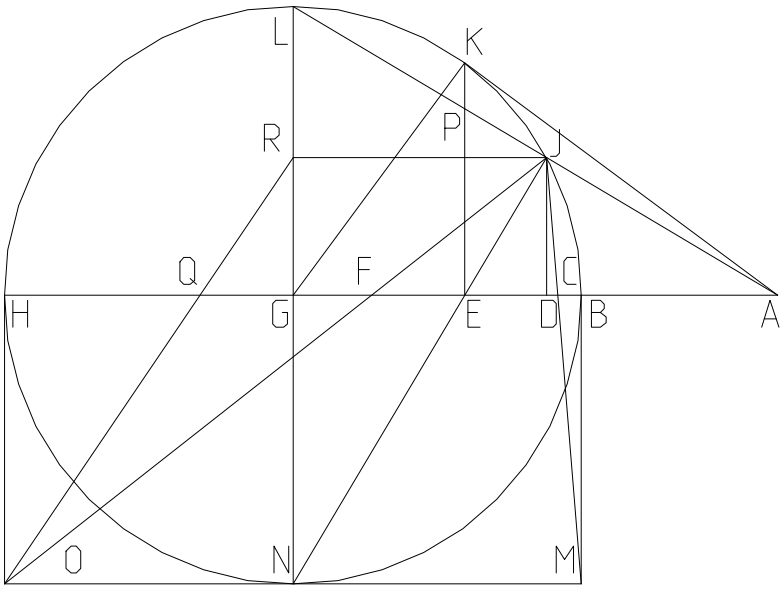
$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.




I found this little sketch in my notebook and have no idea of when I did it or why.

Does CF always equal EP?




$\Delta := 100 \quad \delta := 1..\Delta - 1 \quad BG := 10 \quad GH := BG$

$GN := BG \quad BD_\delta := \frac{BG}{\Delta}\cdot\delta \quad DG_\delta := BG - BD_\delta$

$DH_\delta := GH + DG_\delta \quad DJ_\delta := \sqrt{BD_\delta \cdot DH_\delta}$

$EG_\delta := \frac{DG_\delta \cdot GN}{GN + DJ_\delta} \quad BE_\delta := BG - EG_\delta$

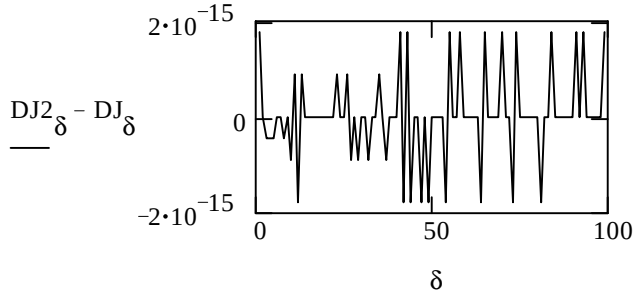
$EH_\delta := EG_\delta + GH \quad EK_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

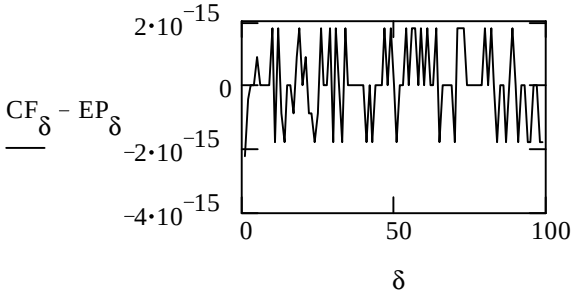
$AE_\delta := \frac{(EK_\delta)^2}{EG_\delta} \quad AB_\delta := AE_\delta - BE_\delta$

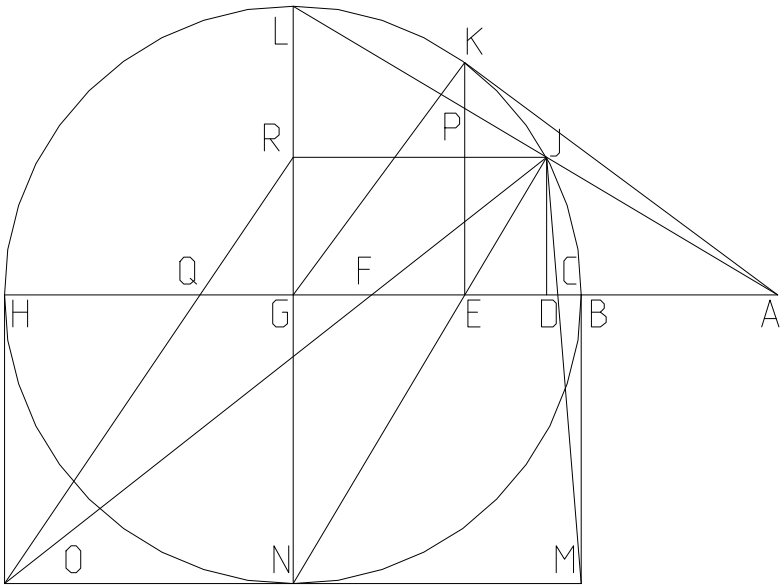
$GL := BG \qquad AD_\delta := AB_\delta + BD_\delta$

$AG_\delta := AB_\delta + BG \qquad DJ2_\delta := \frac{GL \cdot AD_\delta}{AG_\delta}$

$EP_\delta := \frac{GL \cdot AE_\delta}{AG_\delta} \quad GR_\delta := DJ_\delta \quad NO := BG \quad NR_\delta := GN + GR_\delta \quad GQ_\delta := \frac{NO \cdot GR_\delta}{NR_\delta} \quad CF_\delta := 2 \cdot GQ_\delta$






Reducing both by the symbolic processor leaves a little.

$$BG := 10 \qquad BD := \frac{BG}{13}$$

$$CF := 2\cdot BG\cdot\sqrt{BD}\cdot\frac{\sqrt{2\cdot BG - BD}}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)}$$

$$EP := -2\cdot BG\cdot\frac{\left(-BG\cdot\sqrt{BD}\cdot\sqrt{2\cdot BG - BD} - 2\cdot BG\cdot BD + BD^2\right)}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)^2}$$

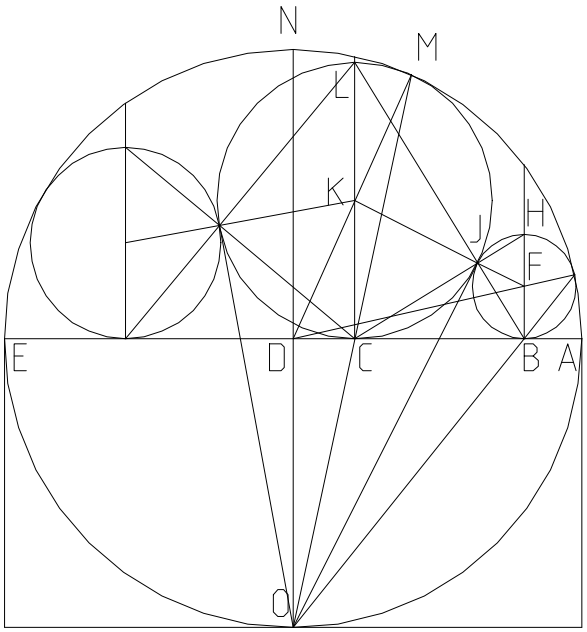
$EP = 5.556 \qquad EP - CF = 0$

And if I divide one by the other and reduce; $\dfrac{\left[BG\cdot\sqrt{2\cdot BG - BD} + 2\cdot BG\cdot\sqrt{BD} - BD^{\left(\frac{3}{2}\right)}\right]}{\left[\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)\cdot\sqrt{2\cdot BG - BD}\right]} = 1$

This is another figure that I had sketched out last year but never got around to writing down.

Given a Circle, place the next on the diameter.

I tried to reduce this series with the symbolic processor, but it is having trouble, at some point it switches AC for EC and I get the other circle.



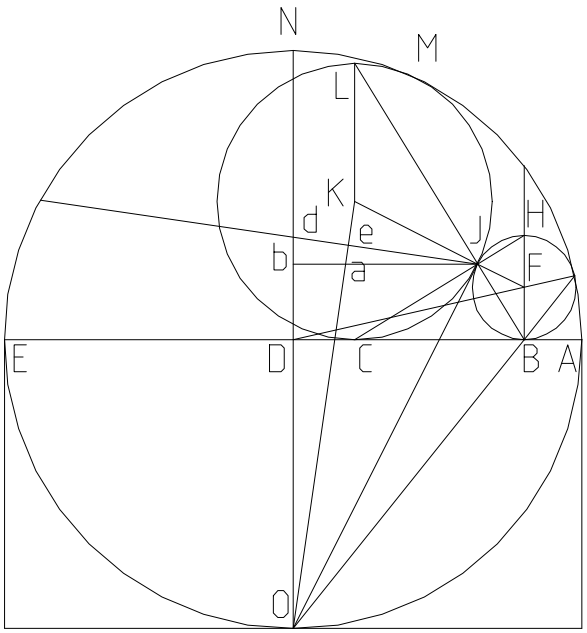

$AD := Radius \quad AE := 2 \cdot AD \quad AC := \frac{AE}{N}$

$CD := AD - AC \quad DO := AD \quad CO := \sqrt{DO^2 + CD^2}$

$NO := AE \quad MO := \frac{DO \cdot NO}{CO} \quad CM := MO - CO$

$CK := \frac{DO \cdot CM}{MO} \quad KO := \sqrt{CD^2 + (DO + CK)^2}$

$JK := CK$

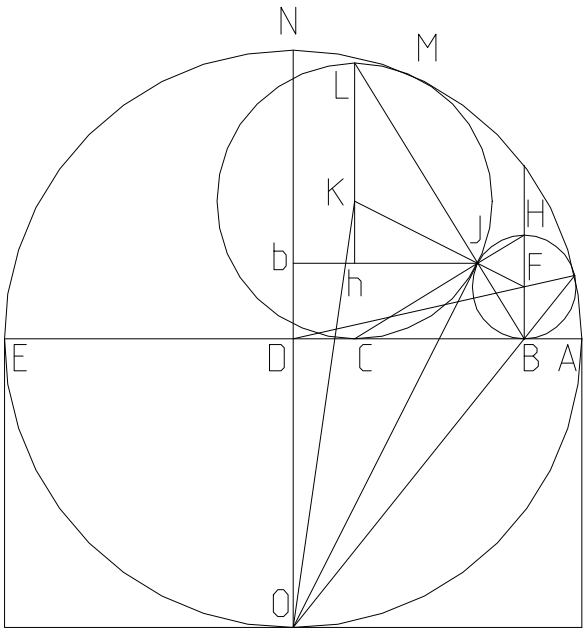



$Ke := \frac{JK^2}{KO} \quad Oe := KO - Ke \quad de := \frac{CD \cdot Oe}{DO + CK}$

$Je := \sqrt{JK^2 - Ke^2}$

$Jd := de + Je \quad bd := \frac{CD \cdot Jd}{KO}$

$Od := \frac{KO \cdot de}{CD} \quad Db := Od - DO - bd$




$Kh := CK - Db$

$Lh := CK + Kh$ $\quad FJ := \frac{JK \cdot Db}{Lh}$

$N \equiv \frac{10}{1}$ $\quad FJ = 3.965$

$Radius \equiv 108$ $\quad CK = 19.44$

And from the other side;

$N \equiv \frac{10}{9}$ $\quad FJ = 51.53$

$Radius \equiv 108$ $\quad CK = 19.44$

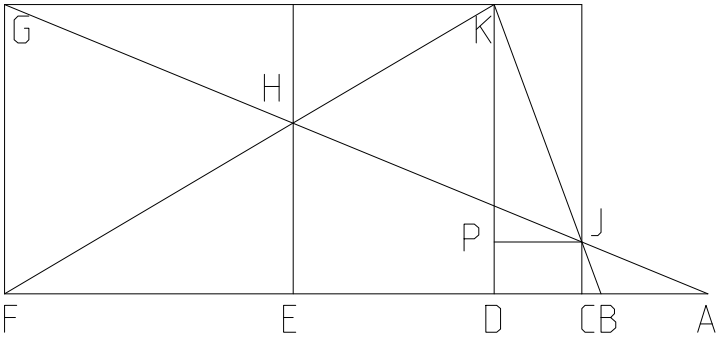
This figure might be recognized as the similarity point for Gemini root projection.

Given AC, CF, and that

$$AC = -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

find BC.

N can be any value whatever, except 0.




$CF := 216 \quad AC := 47.29 \quad N := 100000$

$CE := \frac{CF}{2} \quad AF := AC + CF \quad AE := AC + CE$

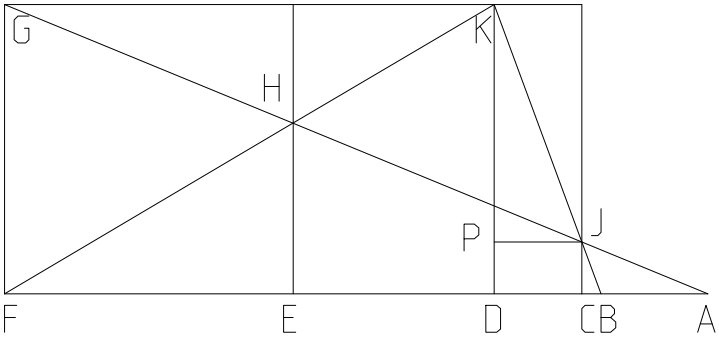
$FG := N \quad EH := \frac{FG\cdot AE}{AF} \quad EF := CE$

$DF := \frac{EF\cdot FG}{EH} \quad CJ := \frac{FG\cdot AC}{AF} \quad DP := CJ$

$CD := CF - DF \quad DK := FG \quad KP := DK - DP$

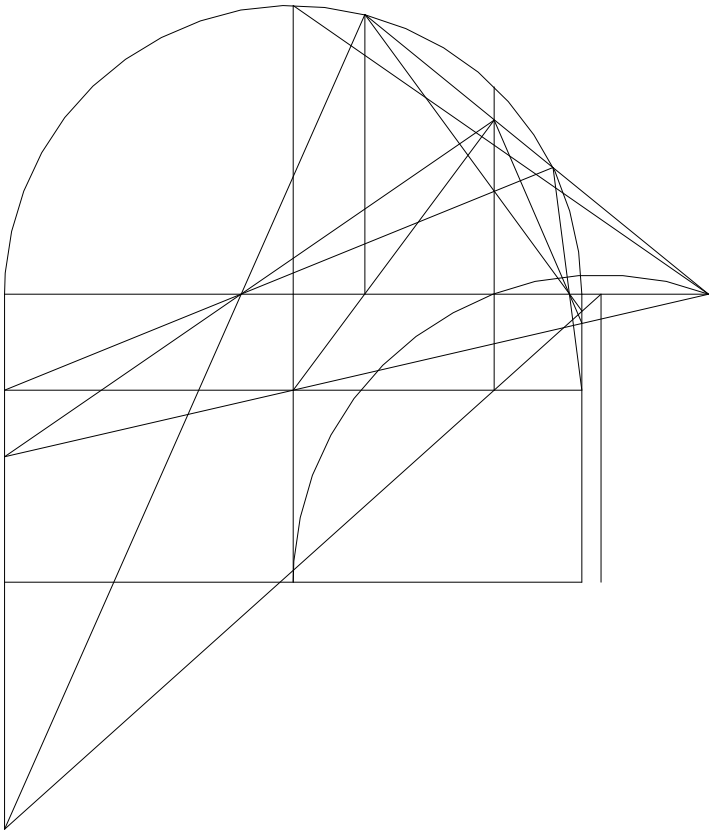
$BD := \frac{CD\cdot DK}{KP} \quad BD = 40.089$

$BC := BD - CD \quad BC = 7.201$



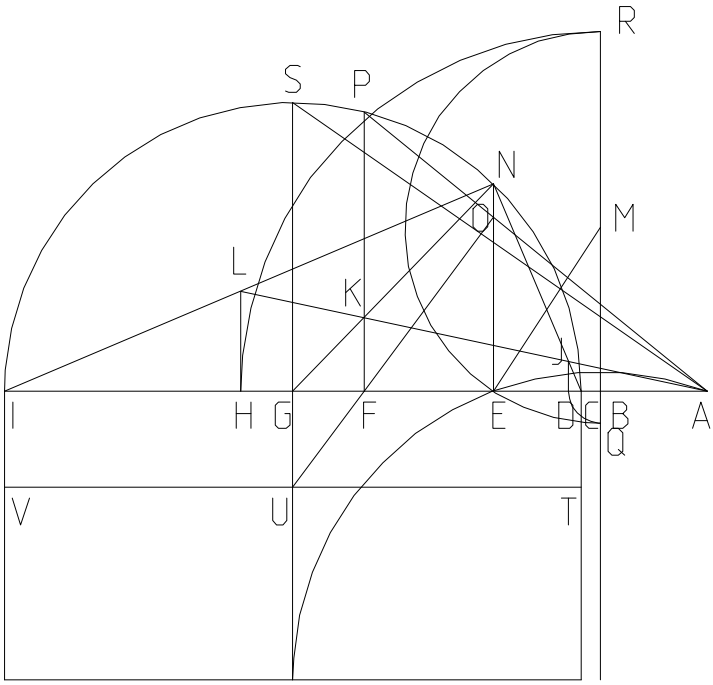

$$AC_f := -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

$$\frac{AC_f}{AC} = 1$$



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

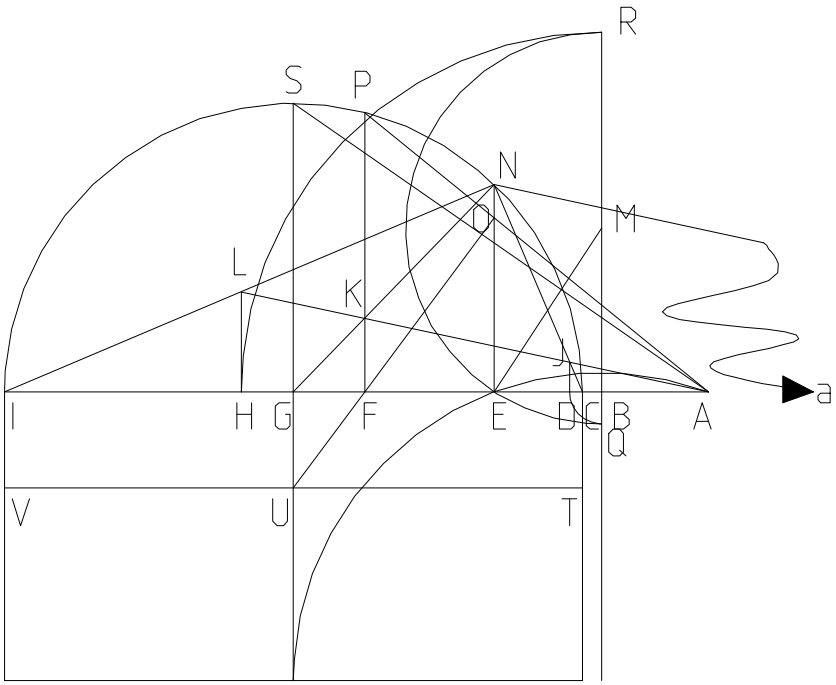
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



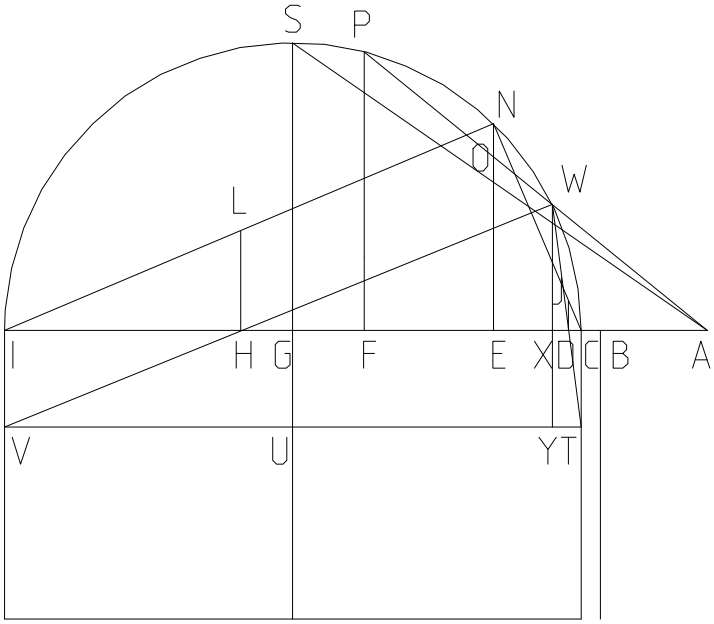

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

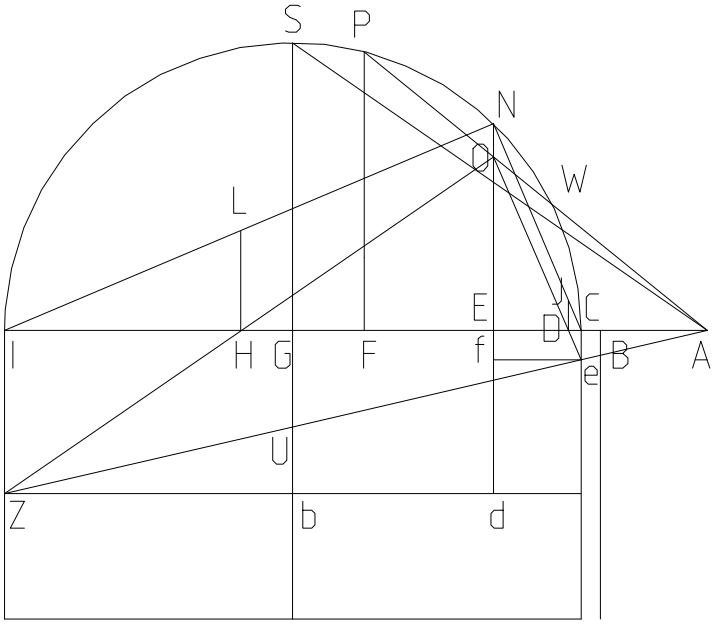
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

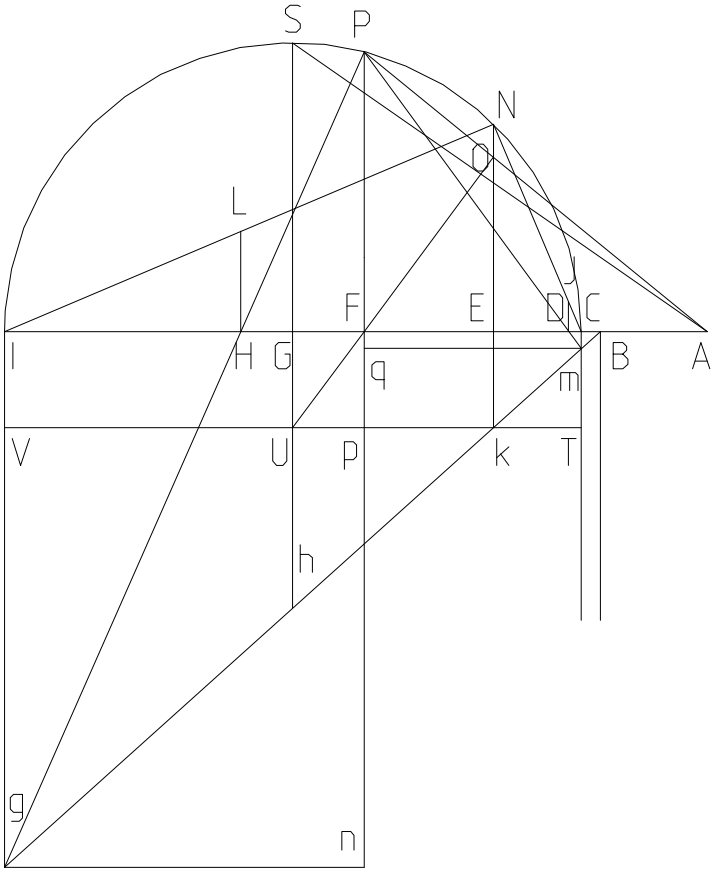
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



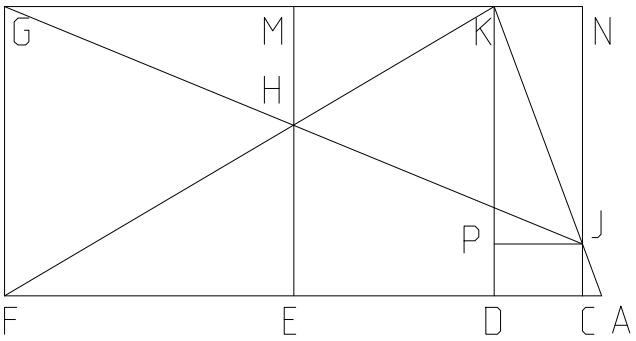

$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

Given CF and CD such that CD = $\sqrt{AF \cdot AC} - AC$ find AC.




$CF := 216 \quad CD := 32.89 \quad CE := \frac{CF}{2} \quad EF := CE$

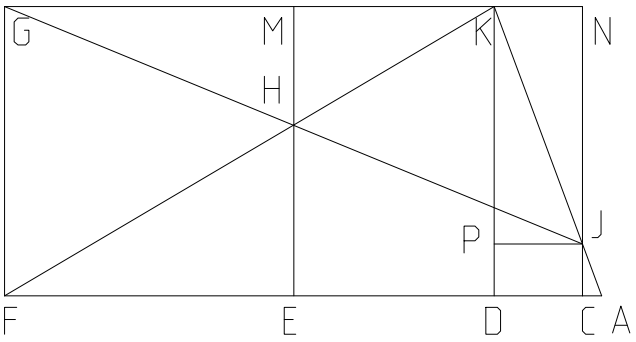
Except for 0, N can be any value.

$N := 108 \quad FG := N \quad DK := N \quad DF := CF - CD$

$EH := \frac{DK \cdot EF}{DF} \quad EM := FG \quad HM := EM - EH$

$CN := FG \quad HM := EM - EH \quad GN := CF$

$GM := EF \quad JN := \frac{HM \cdot GN}{GM} \quad JC := CN - JN$




$KP := JN \quad JP := CD \quad AD := \frac{JP \cdot DK}{KP}$

$AC := AD - CD \quad AC = 7.201$

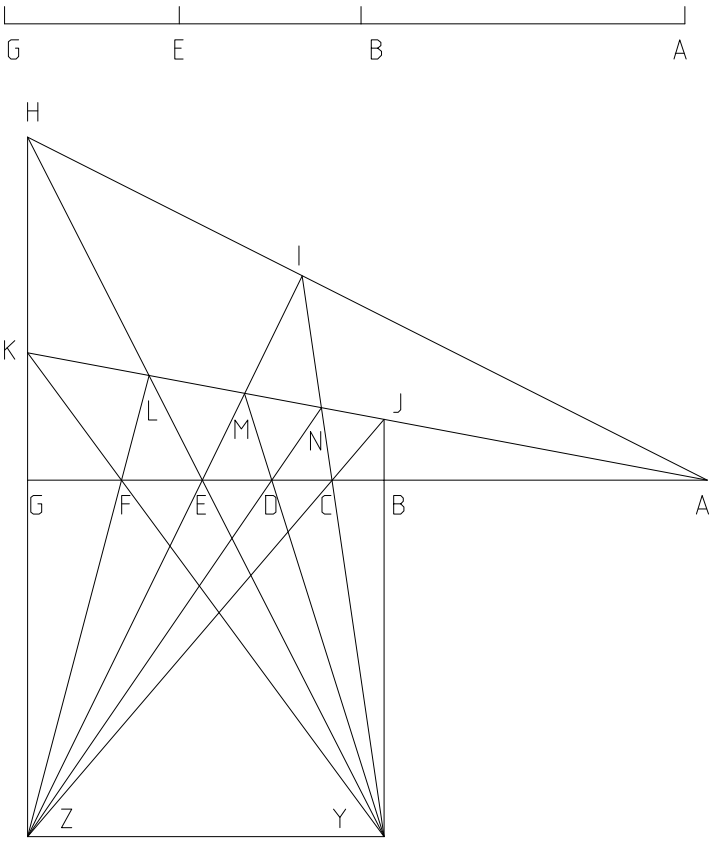
$AF := AC + CF$

$\frac{\sqrt{AF \cdot AC} - AC}{CD} = 1 \qquad \frac{\sqrt{AF \cdot AC}}{AD} = 1$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use $5^{th}$ root series for example.




$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

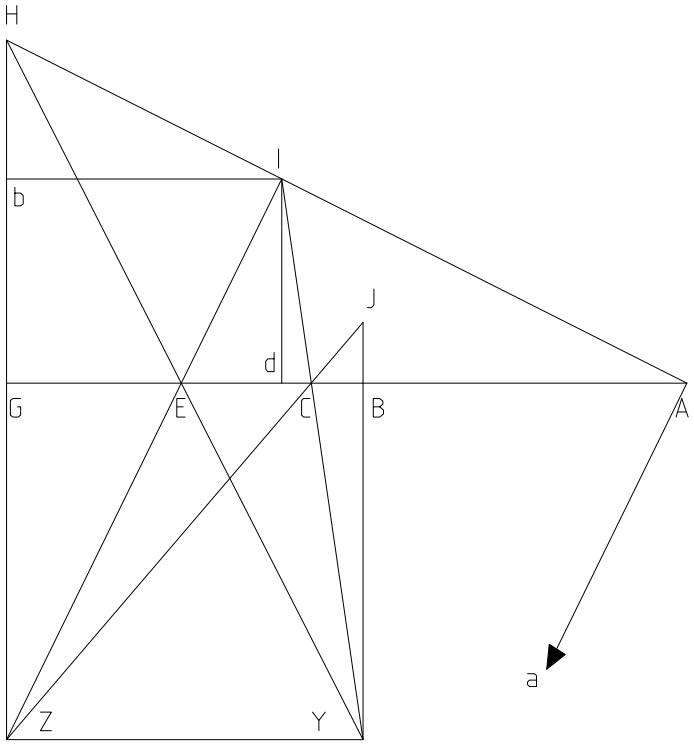
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

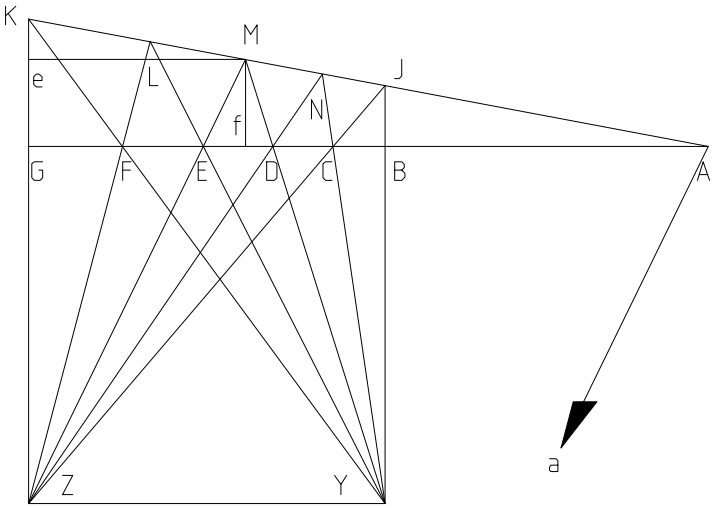
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



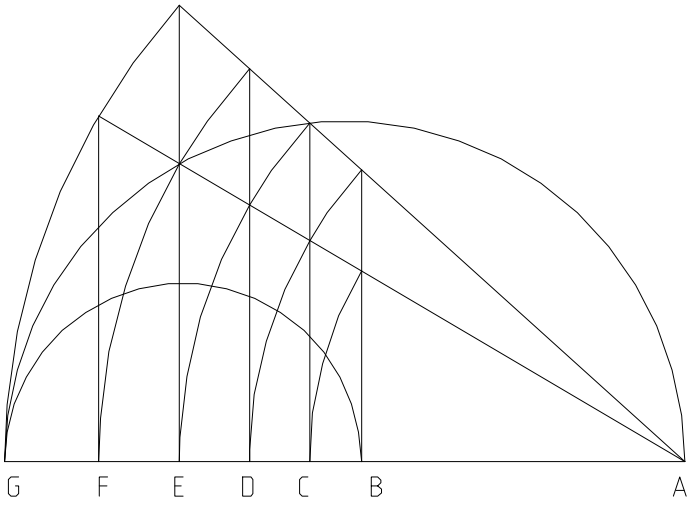



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



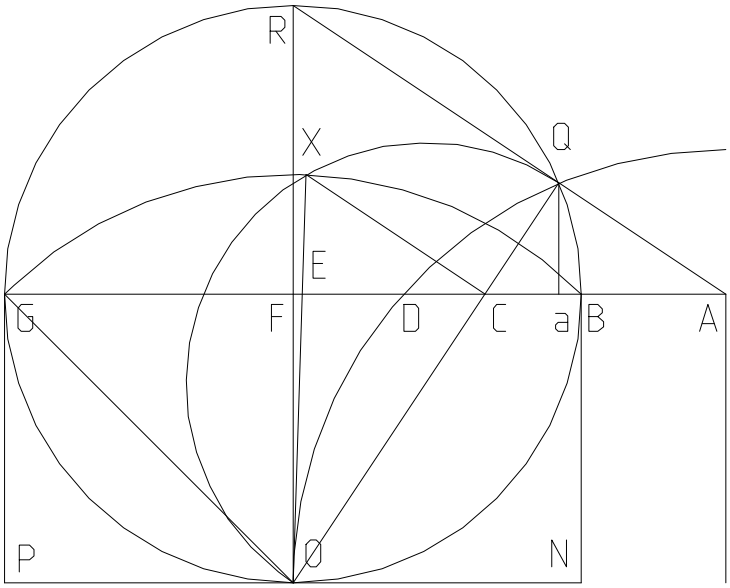

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

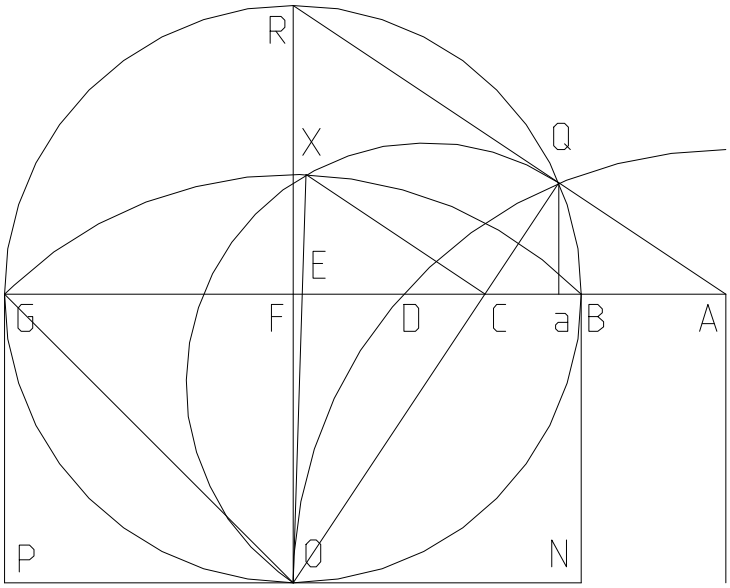
$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.




Is CX a constant?

I have had so much back work to catch up on I post dated a couple.

$AB := 54 \quad AG := 270$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FO := BF$

$AF := AB + BF \quad FR := BF$

$AR := \sqrt{AF^2 + FR^2} \quad AQ := \frac{AB \cdot AG}{AR}$

$Aa := \frac{AF \cdot AQ}{AR} \quad Qa := \frac{FR \cdot AQ}{AR}$

$Fa := AF - Aa \quad OQ := \sqrt{Fa^2 + (FO + Qa)^2}$

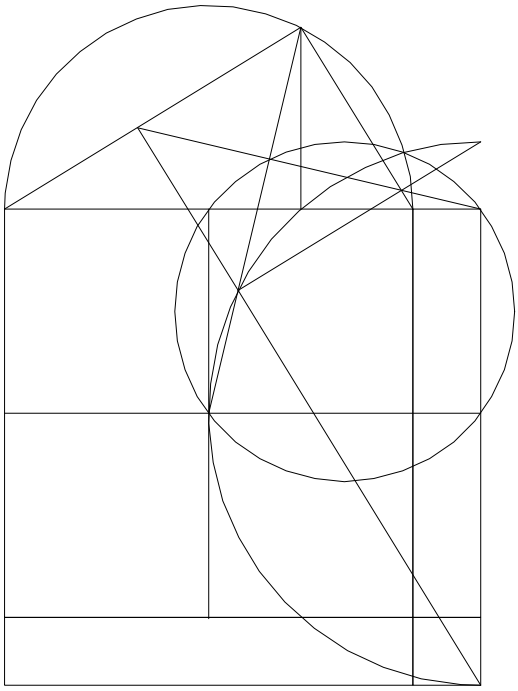
$CQ := \frac{OQ \cdot Qa}{FO + Qa} \quad OX := \sqrt{BF^2 + FO^2}$

$CO := OQ - CQ$

$\frac{\frac{OX^2}{OQ}}{CO} = 1$

Both expressions reduce to,

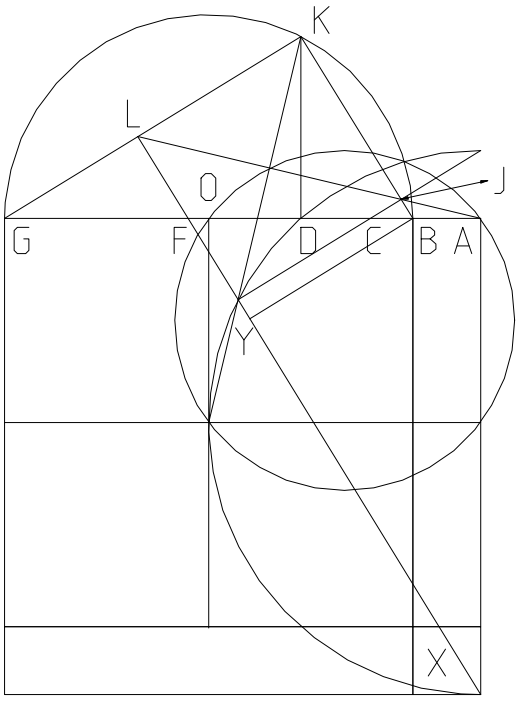
$CQ = 49.923 \qquad OQ - \frac{OX^2}{OQ} = 49.923 \qquad \frac{AG - AB}{AG + AB} \cdot \frac{\sqrt{2 \cdot (AG \cdot AB)}}{\sqrt{AB^2 + AG^2}} = 49.923$



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.




$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

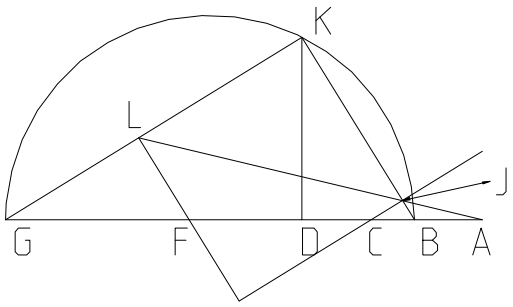
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



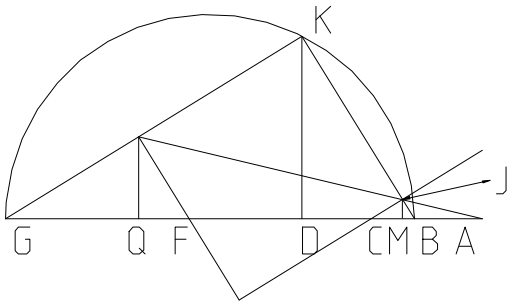

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

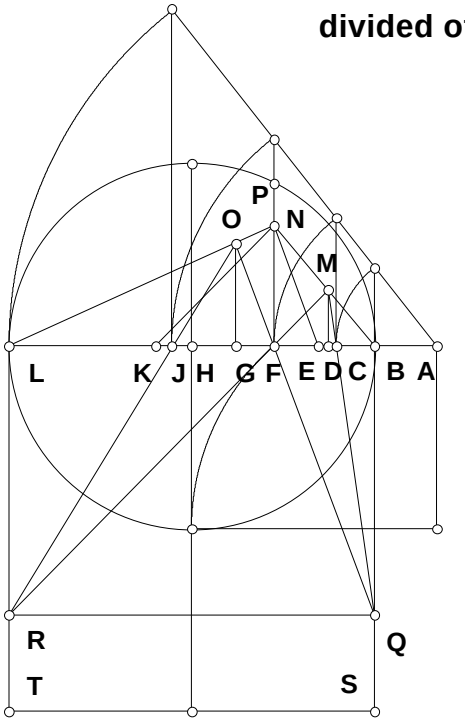
On the left is the first and last of the series, on the right is the entire series.

| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$

# 12/20/96 Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off as shown in the figure.**




$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \qquad BS := BL \qquad LT := BL$

$BH := \frac{BL}{2} \qquad HL := BH \qquad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \qquad FL := AL - AF \qquad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \qquad FN := \frac{BQ \cdot FP}{BS} \qquad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \qquad FG := \frac{EF \cdot FL}{EL} \qquad GO := \frac{FN \cdot FG}{EF}$

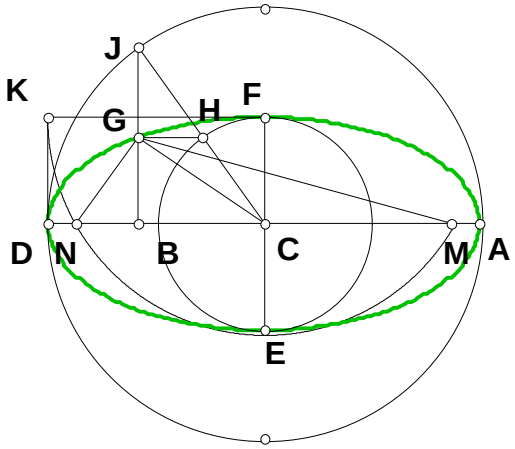
$GL := FL - FG \qquad LR := BQ \qquad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$

## 09/11/97 The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**




$N_1 := 5 \qquad N_2 := 1 \qquad N_3 := 3$

$N_4 := 8$

$AD := N_1 \qquad EF := N_2 \qquad AB := AD \cdot \frac{N_3}{N_4}$

$BD := AD - AB \qquad BJ := \sqrt{AB \cdot BD}$

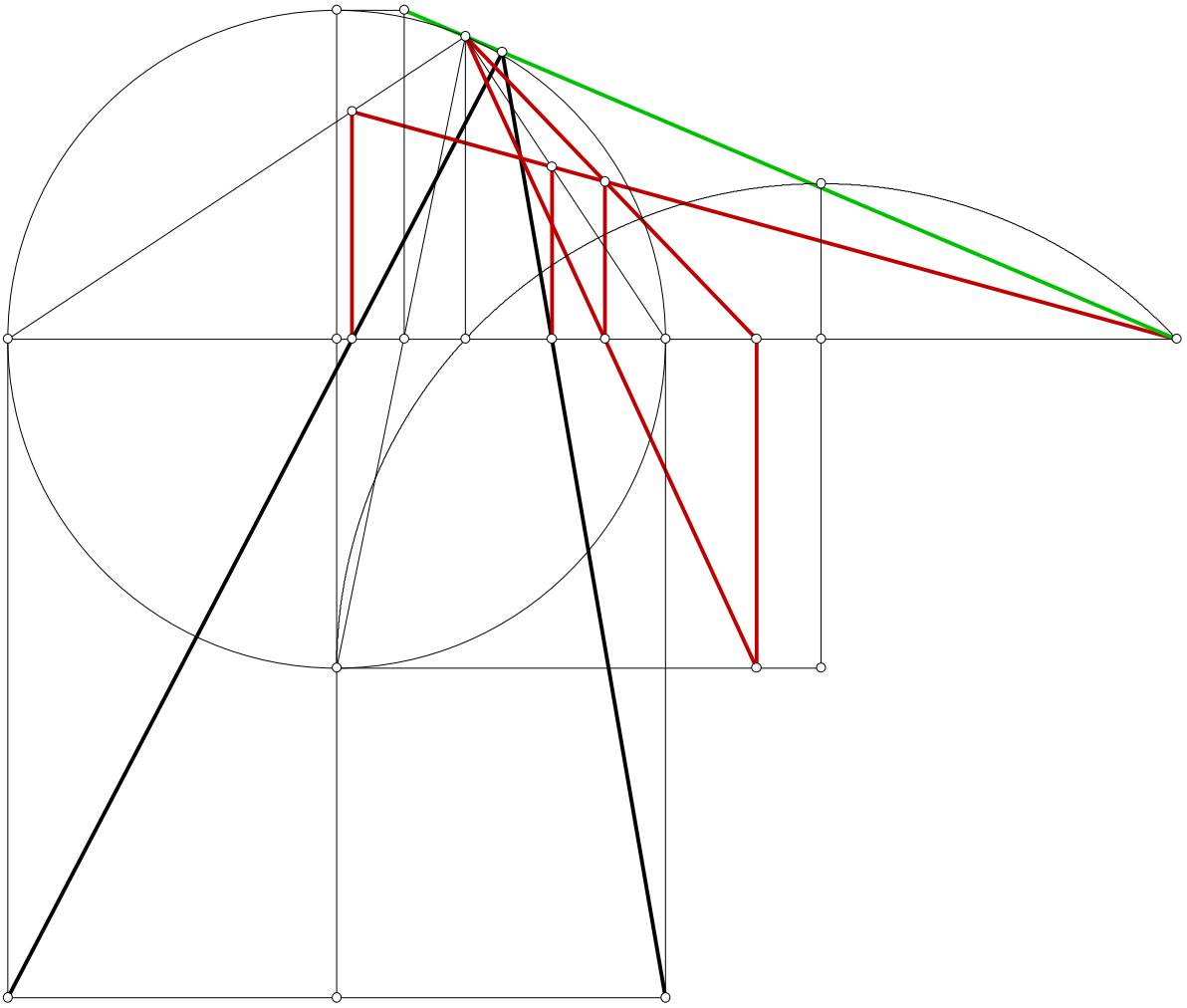
$AC := \frac{AD}{2} \qquad BC := AC - AB \qquad CH := \frac{EF}{2}$

$CJ := AC \qquad BG := \frac{BJ \cdot CH}{CJ}$

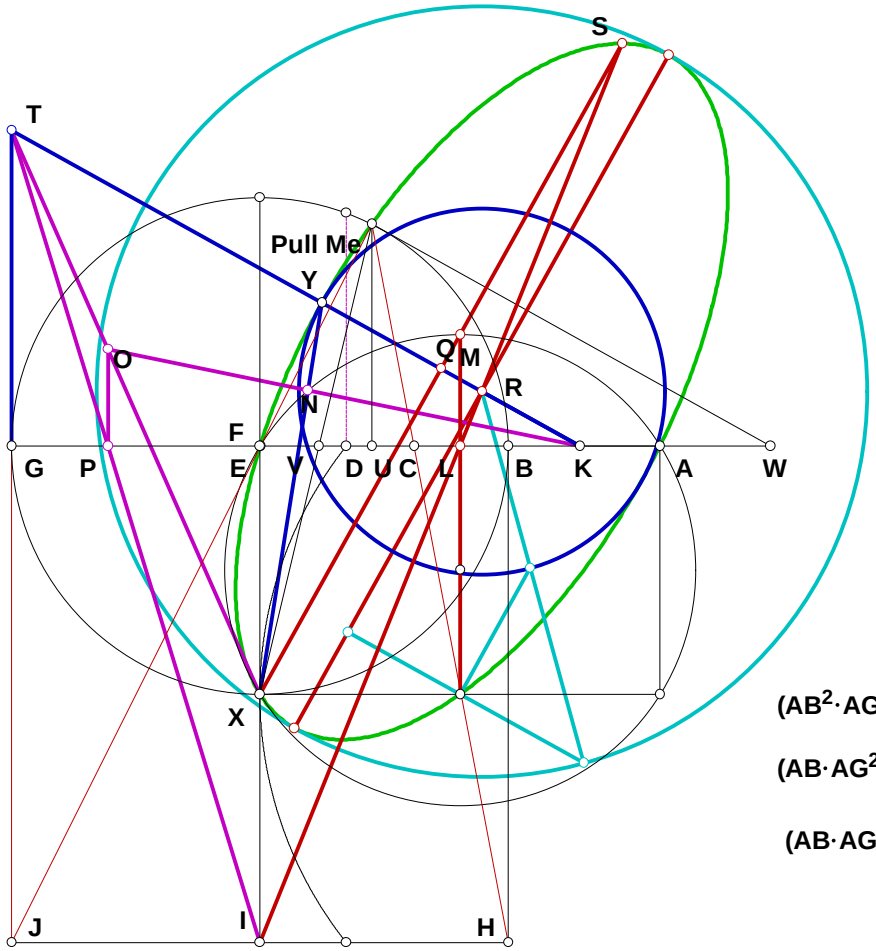
$CG := \sqrt{BG^2 + BC^2} \qquad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$CG - \frac{1}{2} \cdot \sqrt{\frac{-\left(-4 \cdot N_3 \cdot N_2{}^2 \cdot N_4 + 4 \cdot N_3{}^2 \cdot N_2{}^2 - N_1{}^2 \cdot N_4{}^2 + 4 \cdot N_1{}^2 \cdot N_3 \cdot N_4 - 4 \cdot N_1{}^2 \cdot N_3{}^2\right)}{N_4{}^2}} = 0$$

$$MN - \sqrt{(N_1 - N_2) \cdot (N_1 + N_2)} = 0 \qquad BG - \sqrt{\frac{\left(N_3 \cdot N_4 - N_3{}^2\right)}{N_4{}^2}} \cdot N_2 = 0$$



For the delian quest
John Clark 08/11/99
jclark8659@wwnet.net




··· Clear
··· Ellipse
··· Step One Center of Ellipse
··· Step Two
··· Step Three
··· Step Four Minimum
··· Step Five Maximum

$(AB^2 \cdot AG)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AG^2)^{\left(\frac{1}{3}\right)} - AE = 0.000$

$(AB \cdot AG)^{\left(\frac{1}{2}\right)} - AD = 0.000$

# a two Dimensional Solution to the Delian Problem.

PD = 0.601 inches

PC = 5.299 inches

$\frac{PC}{PD}$ = 8.821

ARAS = 1.323 inches

QR = 1.323 inches

ARAS - QR = 0.000 inches

PD = 0.601 inches

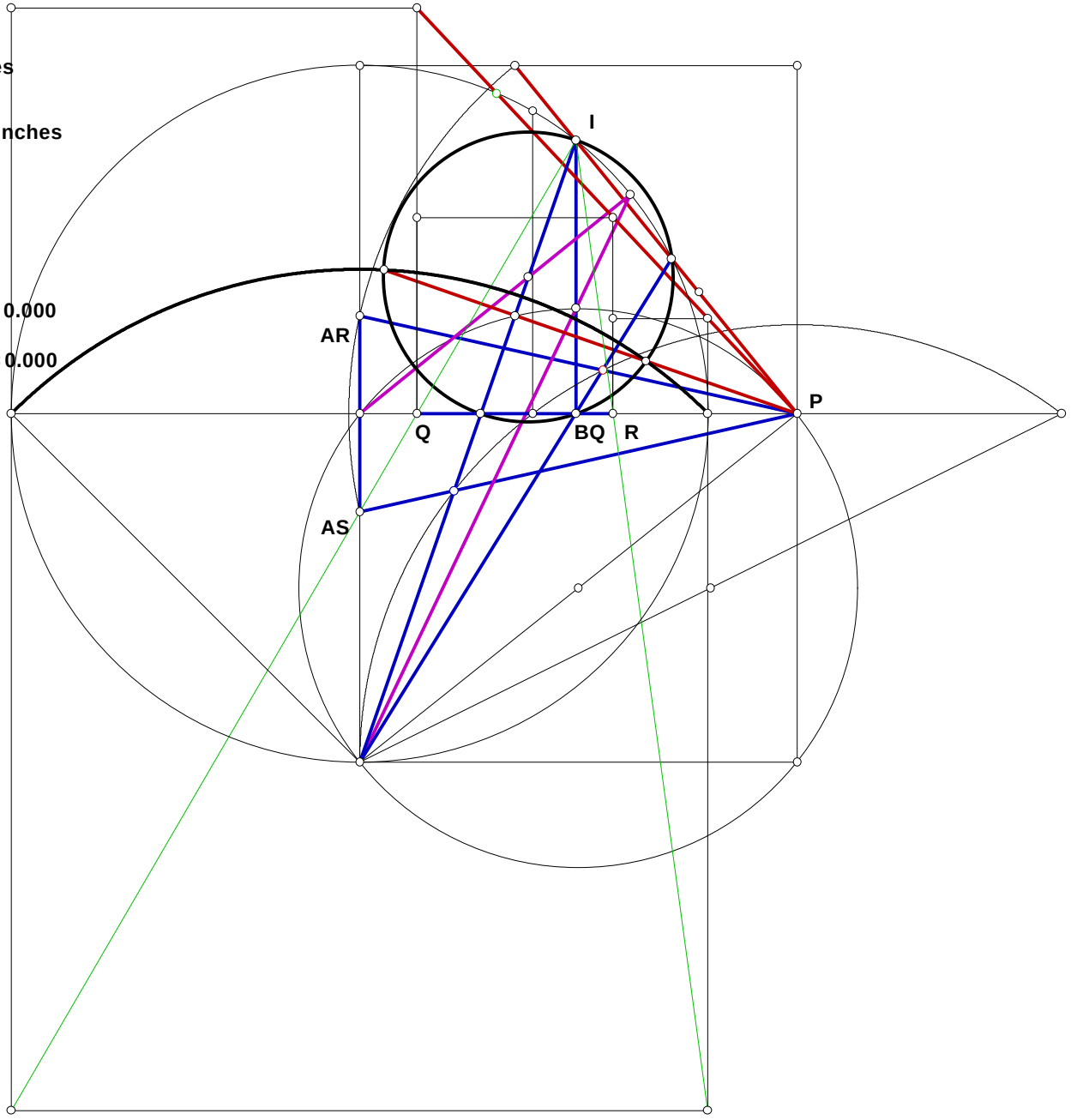
PC = 5.299 inches

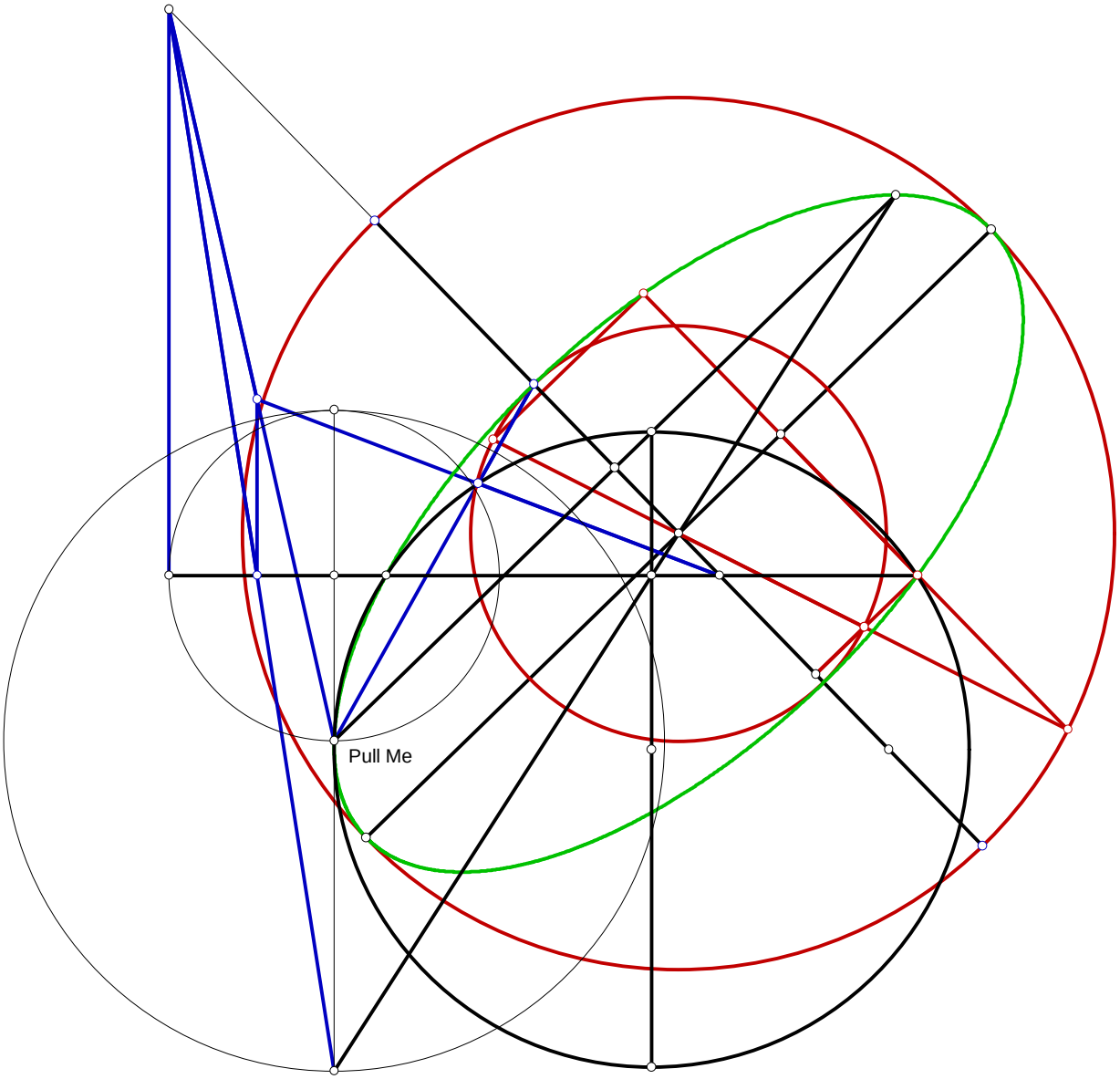
PR = 1.241 inches

PQ = 2.564 inches

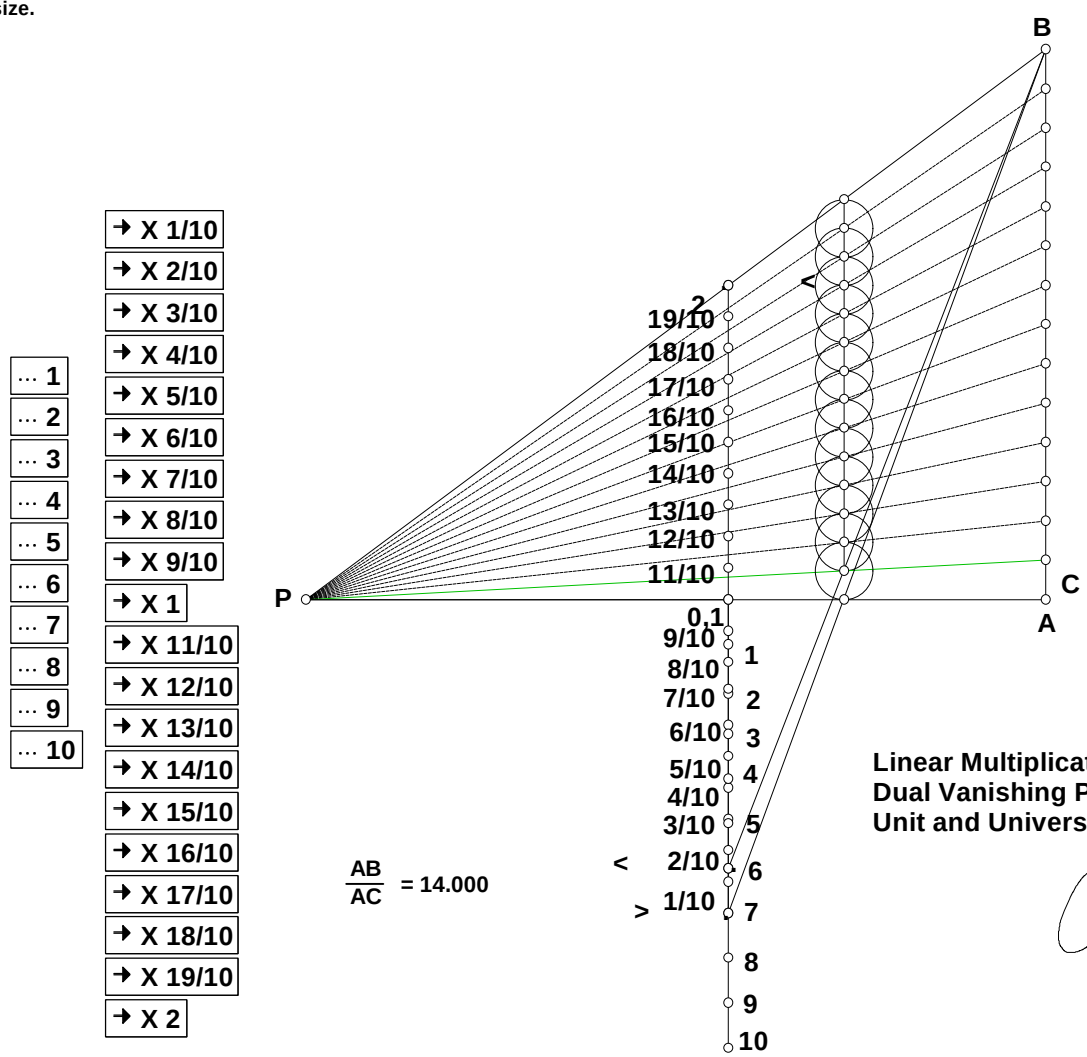
$(PD^2 \cdot PC)\left(\frac{1}{3}\right)$ - PR = 0.000

$(PD \cdot PC^2)\left(\frac{1}{3}\right)$ - PQ = 0.000




Pull Me

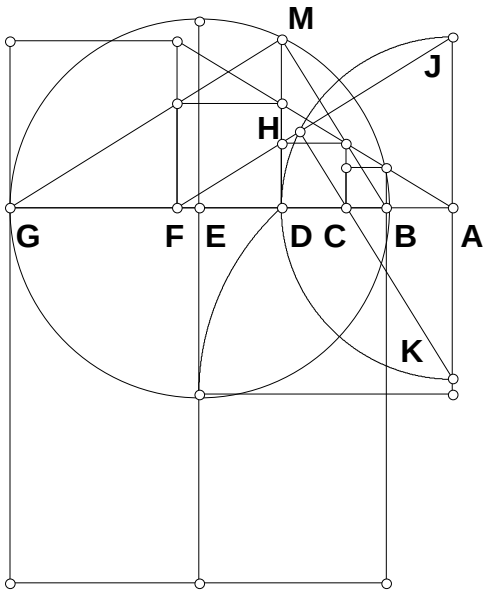
**Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.**




$\frac{AB}{AC}$ = 14.000

**Linear Multiplication and Division,**
**Dual Vanishing Point Method**
**Unit and Universe of Discourse.**

# 07/09/00 Alternate Method Quad Roots




$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

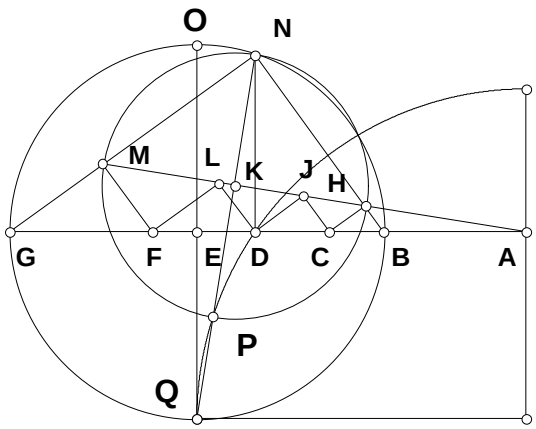
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

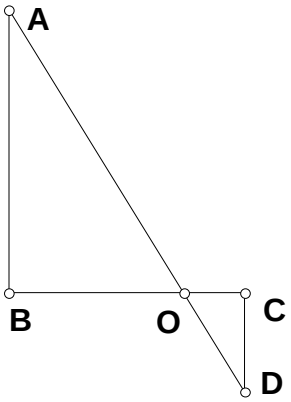
$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

# 08/01/00 Alternate Method Quad Roots



## 08/07/00 Proportion Series II

**Two unknowns have the same proportion as two givens and the sum of the unknowns are known. Find the two unknowns.**




$AB := 9 \qquad CD := 3 \qquad BC := 5$

$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

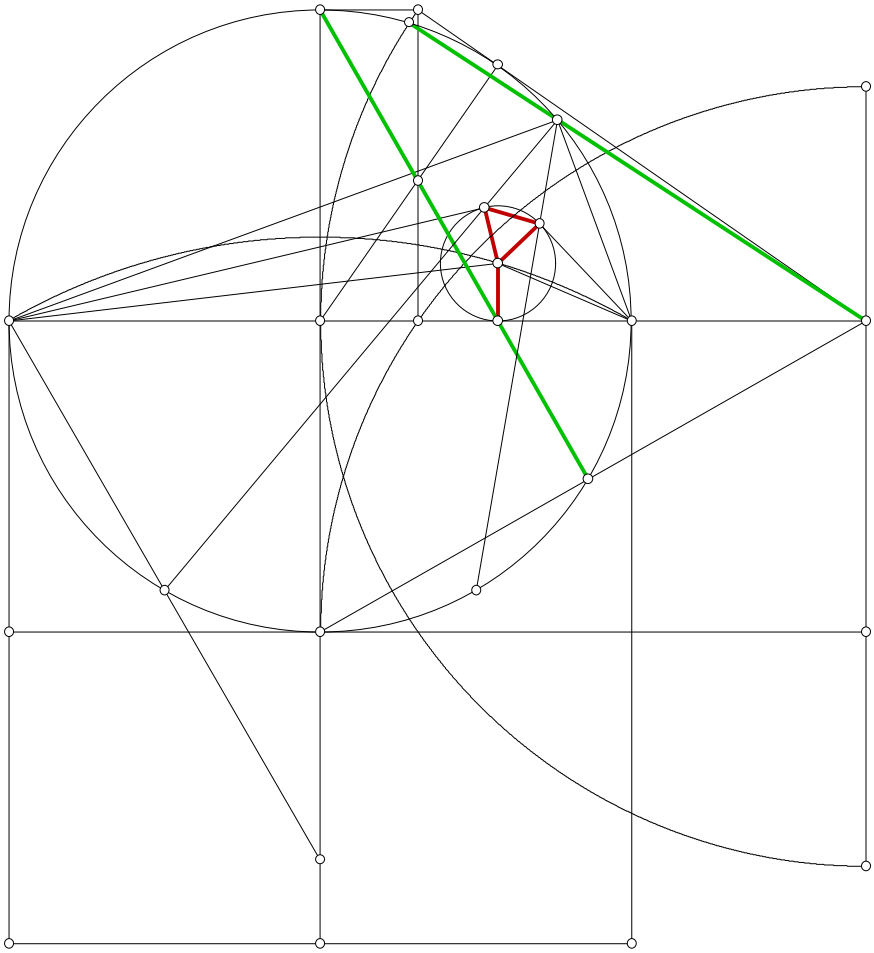
$\frac{AB}{CD} - \frac{BO}{CO} = 0$

m∠UDG = 69.867°

m∠UDCI = 23.289°

$\frac{m\angle UDG}{m\angle UDCI} = 3.000$

# Trisection and the square root figure.

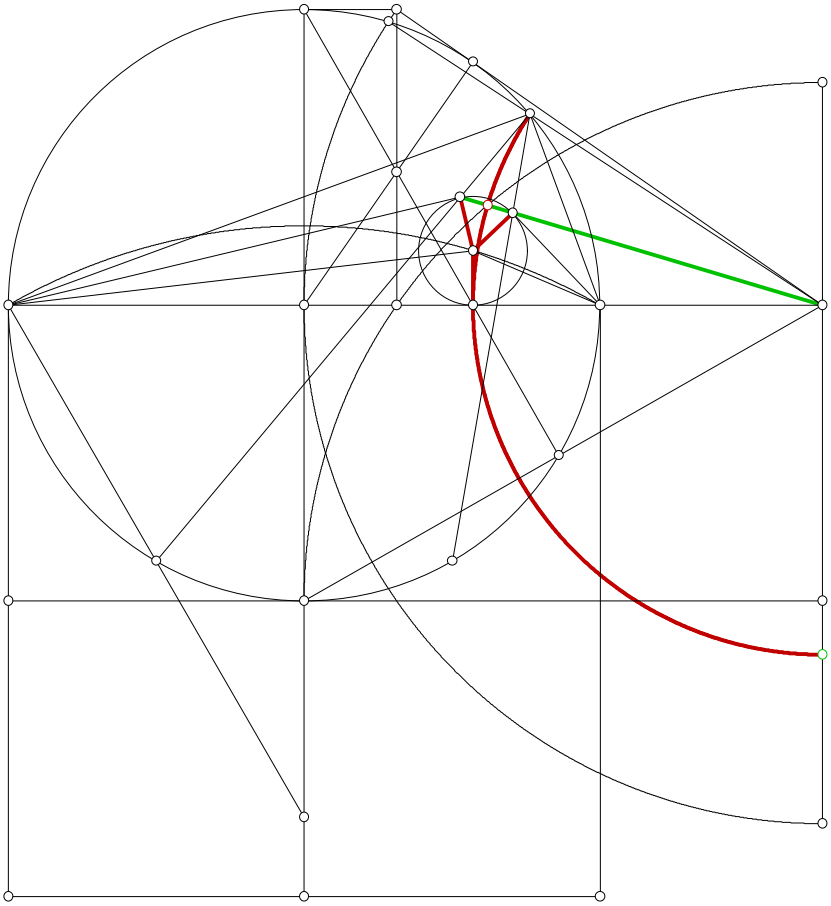


m∠UDG = 69.867°

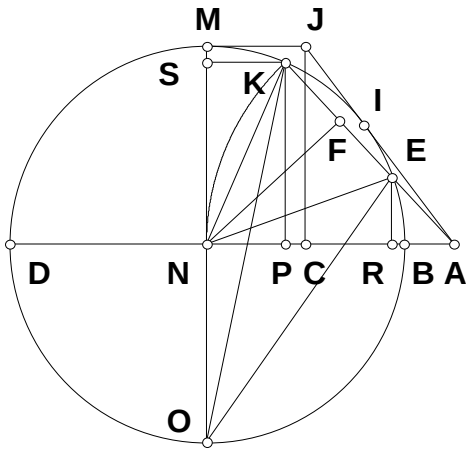
m∠UDCI = 23.289°

$$\frac{m\angle UDG}{m\angle UDCI} = 3.000$$

# Trisection and the square root figure.



# 08/23/00 Trisection In A Square Root Figure




$N := 5 \qquad AB := 4 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN \qquad CJ := BN$

$AJ := \sqrt{AC^2 + CJ^2} \qquad AK := AJ$

$AN := AB + BN \qquad AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$

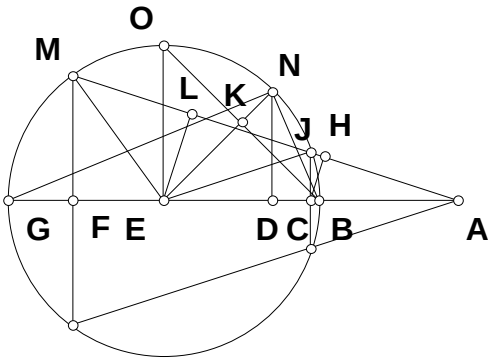
$AF := \frac{AP \cdot AN}{AK} \qquad FK := AK - AF$

$EF := FK \qquad AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK}$

$$AB \cdot (AB + BD) \cdot \frac{\left(8 \cdot AB^2 + 8 \cdot AB \cdot BD + BD^2\right)}{(2 \cdot AB + BD)^3} - AR = 0$$

$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$

# 09/03/00 A Ratio In Trisection




## How does BF vary with BC?

$N_1 := 1 \quad N_2 := 8$

$BE := 1 \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2} \quad EN := BE$

$EK := \frac{BE \cdot BE}{BO} \quad KN := EN - EK \quad BK := \frac{BO}{2} \quad BG := 2 \cdot BE$

$BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE$

$AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC \quad AE := AB + BE$

$JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB$

$$BF - \frac{1}{4} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{14 \cdot N_1^3 \cdot \sqrt{2} - 24 \cdot N_1^2 \cdot \sqrt{2} \cdot N_2 + 36 \cdot N_1^2 \cdot N_2 + 9 \cdot N_1 \cdot \sqrt{2} \cdot N_2^2 - 18 \cdot N_1 \cdot N_2^2 + 4 \cdot N_2^3 - 20 \cdot N_1^3}{2 \cdot N_2^3}$$

# The Delian Quest

> "Socrates: And I, Meno, like what I am saying. Some things I have said of which I am not altogether confident. But that we shall be better and braver and less helpless if we think that we ought to inquire, than we should have been if we indulged in the idle fancy that there was no knowing and no use in seeking to know what we do not know;—that is a theme upon which I am ready to fight, in word and deed, to the utmost of my power." ***Meno***, by Plato

## Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. The smattering of English is of no great consequence. The figure is Geometric Grammar, the equations are Algebraic Grammar. I take exception to any teaching that does not recognize the geometric figure as ***the*** Grammar of Geometry. Much of what is called geometry is apparently stated by those who have no clue. The Geometric Grammar system is a relatiologic and Algebraic Grammar is a tautologic–together they make a proposition–and together and only together can knowledge be imparted.

## The Delian Problem

From what I remember, the Delian Problem acquired its name from the circumstances of the problems inception. In order to stop a plague in ancient Greece an oracle at Delos suggested–supposedly by direction of a *God*–that the alter of Apollo be doubled. The alter of Apollo was a cube. What it boiled down to was the abstraction of the cube root of 2–the cube root of twice its current volume. Now if in only geometry can one precisely abstract the square root of 2, it seemed reasonable that only in geometry could one abstract the cube root of 2 as well. What this means is that an act of will can be performed for which no Arithmetic name is possible. The conventions of Arithmetic Grammar cannot provide an Arithmetic name for the square root of 2. Algebraic grammar overcomes this problem of irrationality in Arithmetic Grammar by incorporating operands as part of the Algebraic name. The problem in Geometric Grammar was, no one knew how to abstract cube roots. No one knew how to assert that act of will. In time, the general response to the Delian Problem was that it is an impossible problem to solve. No name could be provided in Arithmetic Grammar, and none in the Grammar of Geometry, while in Algebraic grammar, a name could be simply had, but no clue as to how to render the fact.

## A Class Idea

Those things that are grouped in accordance by a common characteristic all have the same name by that measure. Some people may recognize this notion as Set Theory, some may see it as the founding principle of all Grammars because it is the foundation upon which the concept of words are based, a fact of the craft of measurement itself, some may realize that it is the Sensor Model of reality, however one comprehends it, it is derived from A = A and A – A = 0 which is derived from knowing and not knowing. This has a very important application for if there is no difference between two things, then there is no knowledge of 'two-ness' under the class definition–therefore both items are regarded as the same–treated the same, respected the same. By definition, it is not possible to act

differently in any case or any instance toward members of the same set. Action is a reaction from cause, but one has stated that all members are the same, no difference or cause can then exist between them under that class definition.

In regard to the Delian problem one might apply it in this manner; take the straightedge and compass and see if they have a common characteristic which would group them into one class. To my knowledge such an application of the foundation of logic has never been proposed. If one finds that common characteristic then one can look around and see if they can abstract other tools that have that same characteristic. Perhaps these tools are what is needed to solve some problems. It would be the class characteristic that would define a Two Dimensional Geometers Writing Instrument–this is the true custom, the same notion that determines any language that one speaks. Logically, reasonably, factually, one cannot state that Geometry is to be written in only two tools of a class of tools if that class is well defined, for there is no way to distinguish these tools under the class concept, the definition of class membership–providing of course one can abstract the class concept that the tools reside in. Aristotle quite rightly determined that a definition renders both ***substance*** and ***form***–every bit of knowledge is known that way, ***substance*** and ***form***. Even in our mind, to think of ***substance*** we supply a generic ***form*** to it, and to think of ***form*** we assert a generic ***substance*** for it–otherwisde we could not conceive of either. What is the ***substance*** and the ***form*** of the Geometric Writing Instruments we are familiar with?

A straight edge provides one with one and only one difference (***substance***) between two points (***form***). A compass provides us with one and only one difference (***substance***) between two points (***form***). One may comprehend the straightedge however as providing the Unit of discourse, while the compass the Universe. Has anyone ever comprehended the ellipse as a figure that is produced as one with one difference (***substance***)–a sum–between two points (***form***)? Is it possible that the *Gods* gave a problem, just like the problem of understanding the Bible, whereby the only solution is to think by one measure–class concepts? The only possible path to a solution is when men start to reason by respecting what the Bible calls a Holy understanding, the Law of Reciprocity? ***A*** = ***A***. I don't know. I do know that the tool that constructs an ellipse is in the same class as the straightedge and compass in regard to the same definition of ***substance*** and ***form***, and also that the ellipse can solve the Delian Problem. I would strongly suspect that the problems given by the *Gods* all have a common solution which involves a growth in men's understanding. I did not suspect it when I started my search, I started the search because either no *God* gave the Delian Problem, or if given it would seem reasonable that a *God* would only ask for the possible and if it were possible, then it was possible for me to find it.

## Proof

What is ***proof***? What ***proof*** is seems to have taken a bad turn in history. Currently what is called proof is a mess of non-sense. Since all knowledge is in propositional form, that being ***substance*** constrained by ***form***, which produces the first division in logic, that of tautologics and relatiologics, proof is not different in concept from all knowledge. ***Proof*** is not different from the concept of ***definition***. ***Proof*** is circumscribing ***substance*** with ***form***. Tautologics are based on ***form***, Relatiologics on ***substance***, proof is saying in a

tautologic what is said in a relatiologic. In this text, I used Geometric Grammar (figures–***substance***) for the relatiologic and for the tautologic I use Algebraic Grammar (***form***). When the grammar is correct in each, and when the ***form*** is the ***form*** of the ***substance*** and the ***substance*** is the ***substance*** of the ***form***, then and only then does one have proof and truth.

It does not matter if we construct a ‘word’, a ‘sentence’, a ‘definition’, a ‘proof’ these all reside under one concept. That concept can be called Set Theory or propositional form, or ‘to know’ all of these words are based on the definition of body sense,

‘That which acquires something from the environment (***substance***), processes it (re-***forms***), and the product is used to sustain and promote life.’

The idea that we are composed of seven senses, all of which acquire something to build that one group, the one proposition, of human life called ‘me’ is fundamental and undeniable of our own psychology and physiology.

## Proposition

Even the process of group construction (propositional construction) follows the definition of proposition itself. One method is particular (the ***substance***), each member is enumerated, the other method is universal (the ***form***), each member has the same definition. Each method renders a judgment as to inclusion and exclusion of a member into a class. As Plato pointed out, only by comprehending all the details of both methods can one be said to have understanding. The methods themselves follow propositional form in another respect–the particular method is not effected through understanding but the method of forms (definition) is–this too Plato pointed out and is why he championed the method. Using class definition to define Geometric Writing Instruments is an idea that has not been promoted or conceived, and if enacted many mistakes will be made, but nevertheless, we may not be altogether certain as to what we are doing, but we can be certain that life is in that quest for knowing. Perception determines conception, conception determines will.

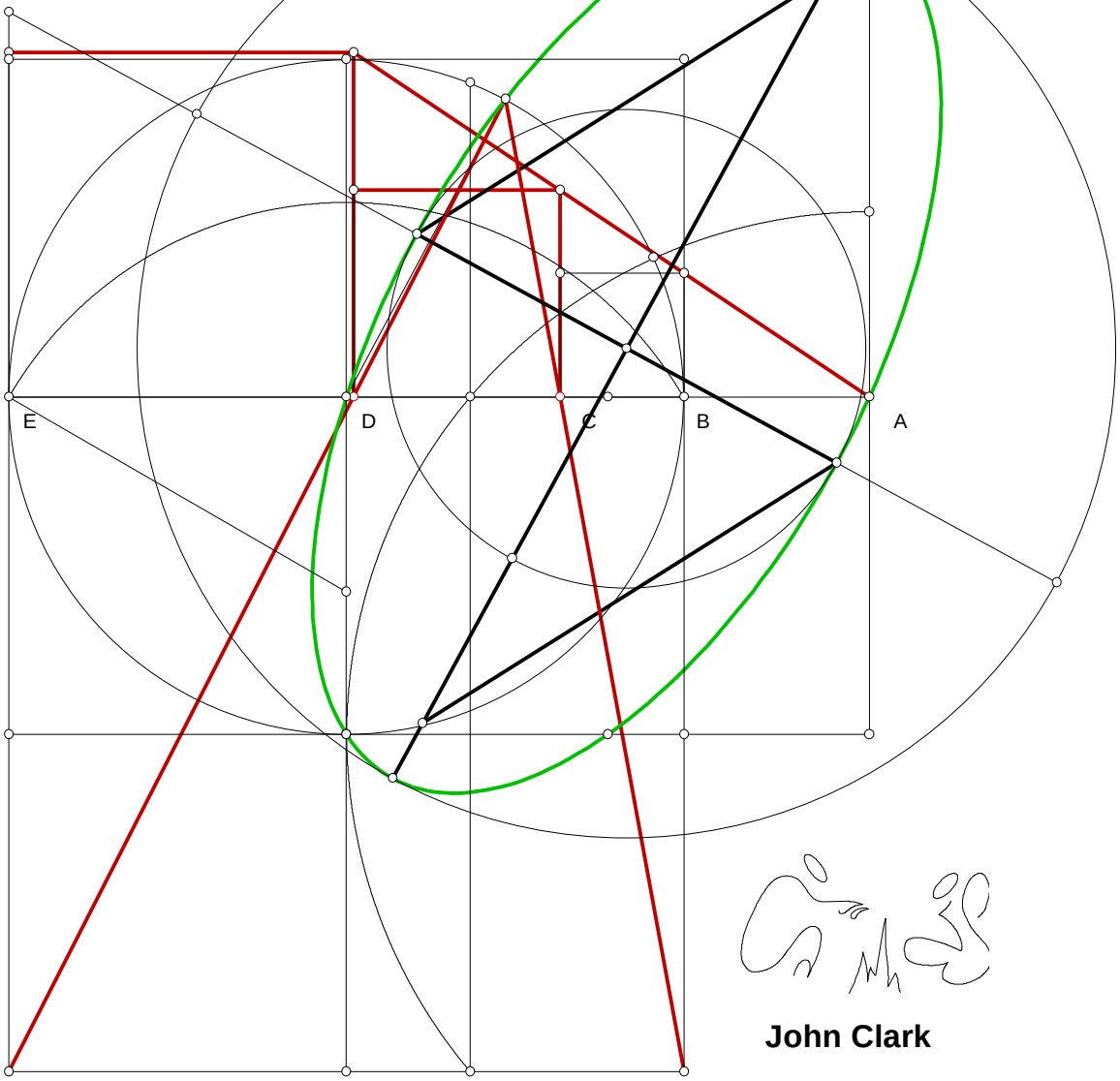
## Reading

Since perception determines conception and conception determines will, learning to ‘say’ what we ‘see’ can be improved by practice–being fully aware of what we are doing and why. One must read the figure and the equations together in this work. j.c.

# The Delian Quest

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$






**John Clark**

# The Delian Quest

## Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. The smattering of English is of no great consequence. The figure is Geometric Grammar, the equations are Algebraic Grammar.

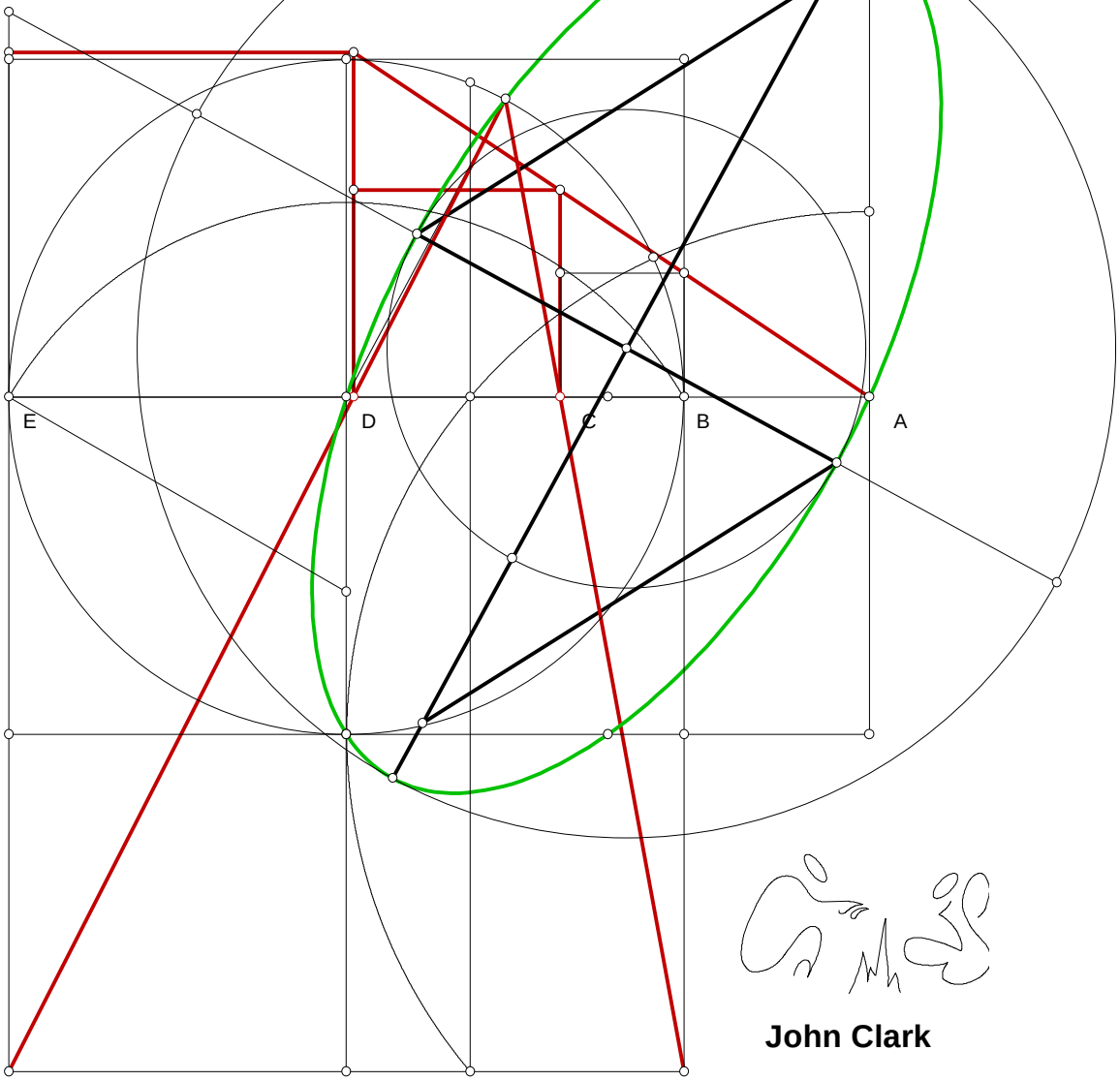
Place Introduction to logic here.

# The Delian Quest

## 1989

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$






John Clark

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

# THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

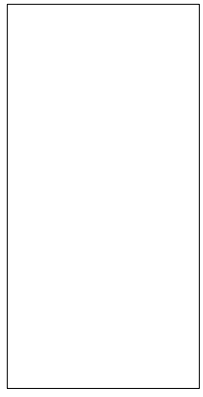


Plate 1

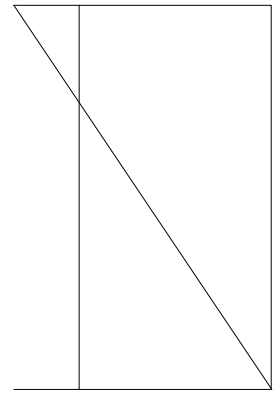


Plate 2

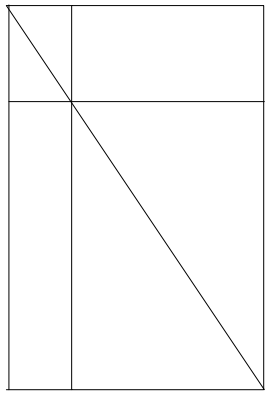


Plate 3

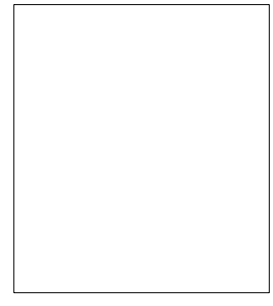


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

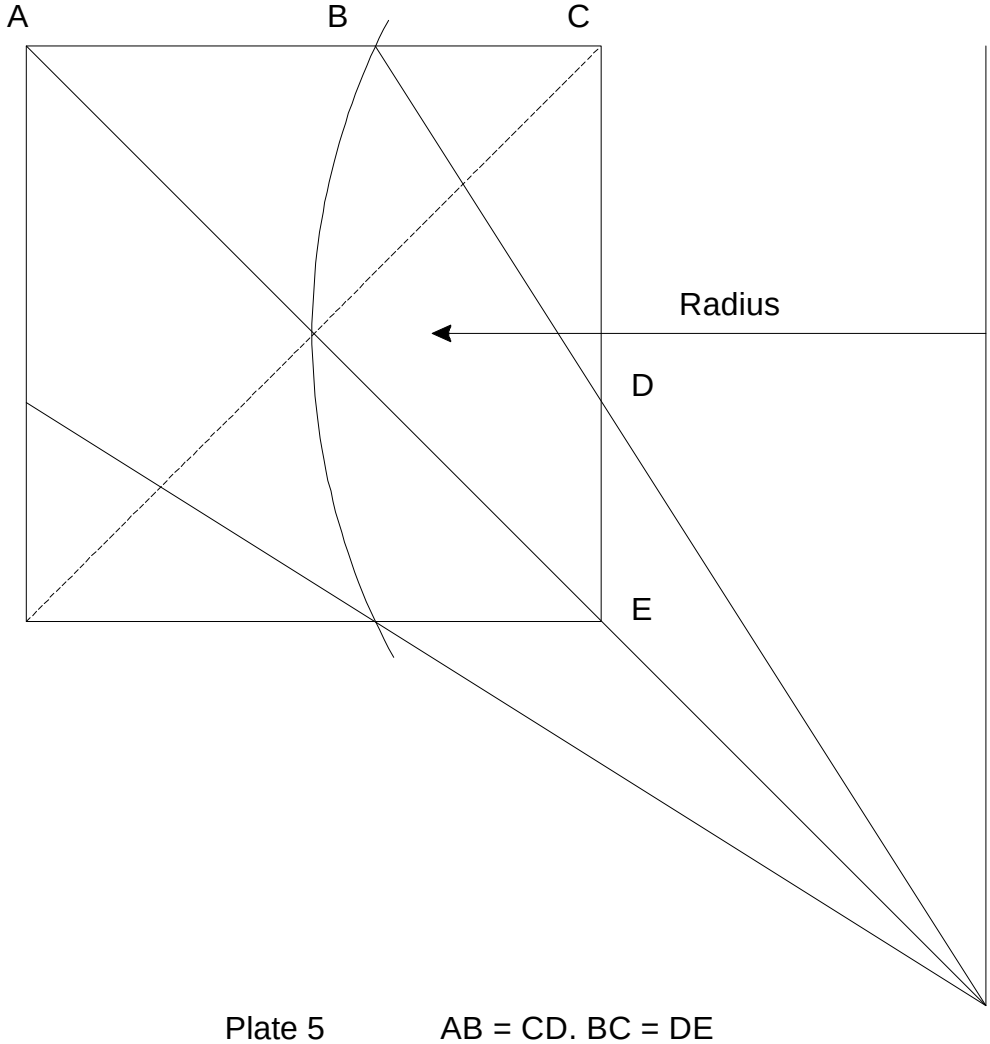
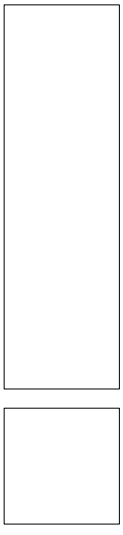
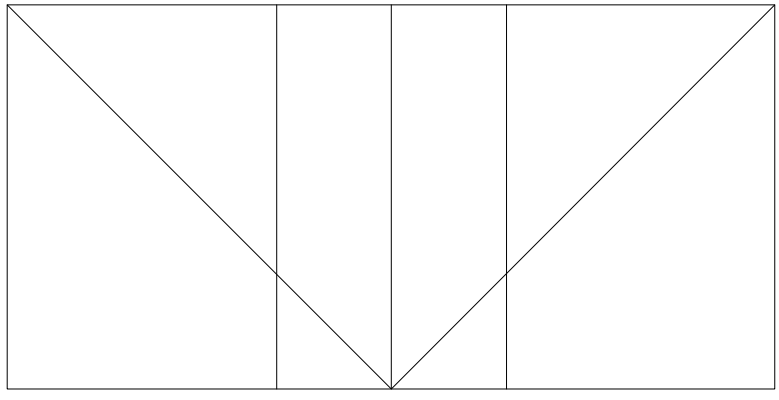



Plate 5 AB = CD. BC = DE

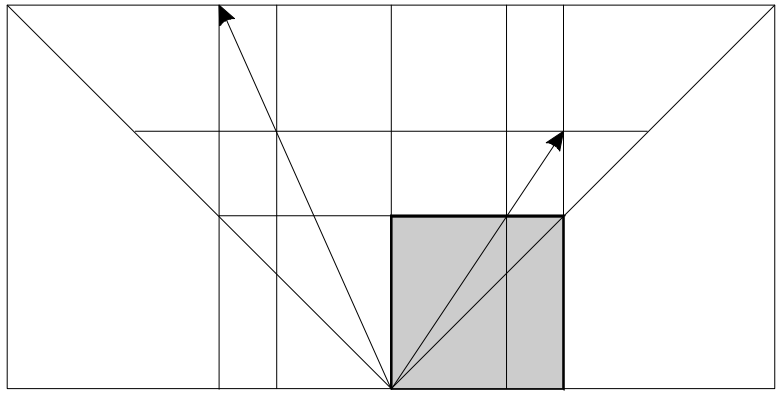
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



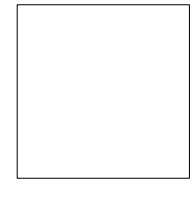
P.6



P.7

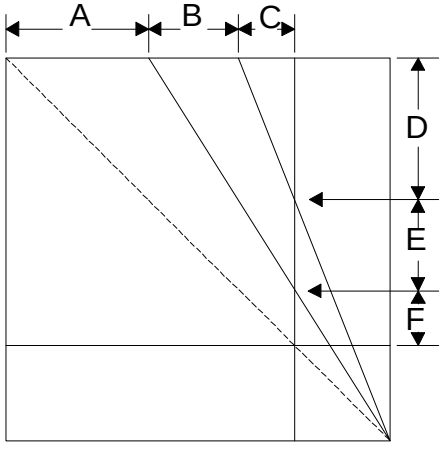


P.8

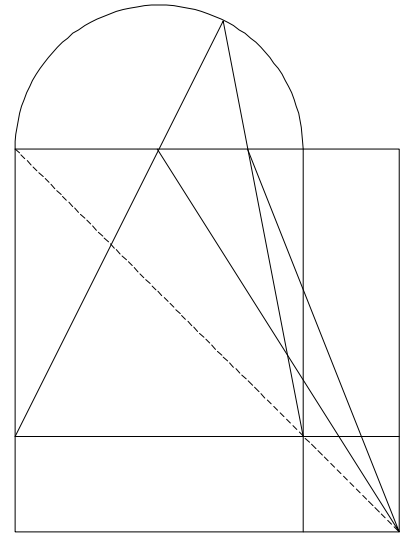


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

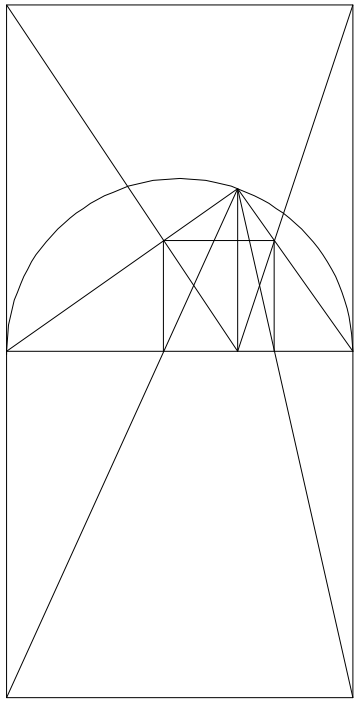



P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

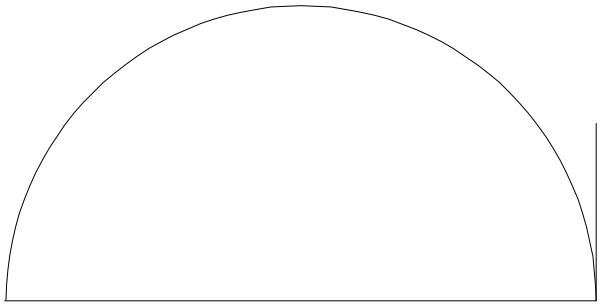


Plate 13

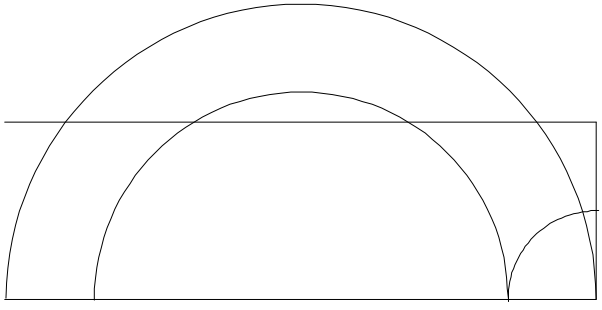


Plate 14

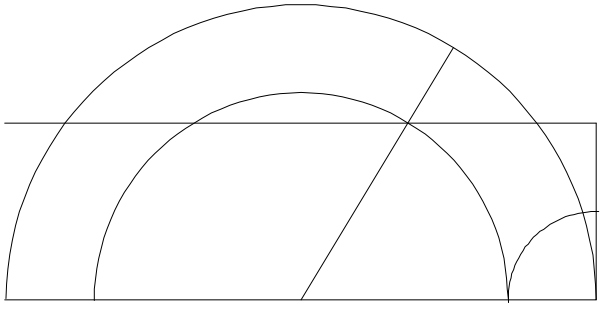


Plate 15

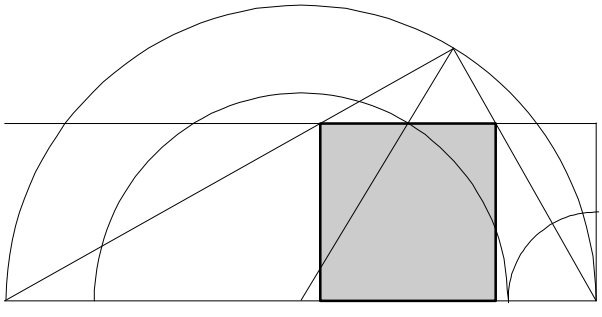
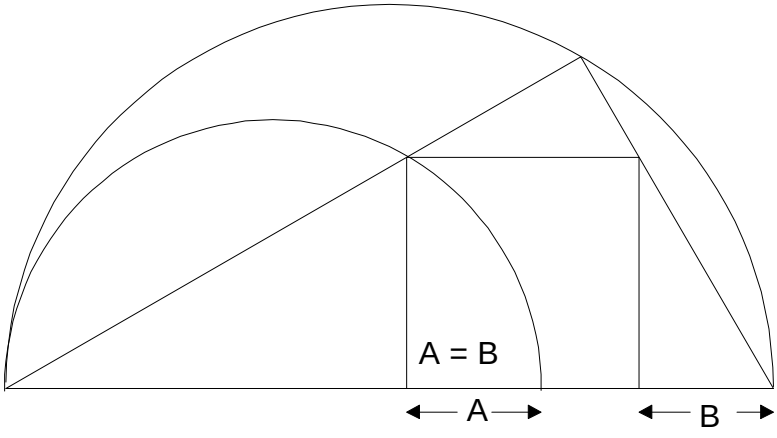
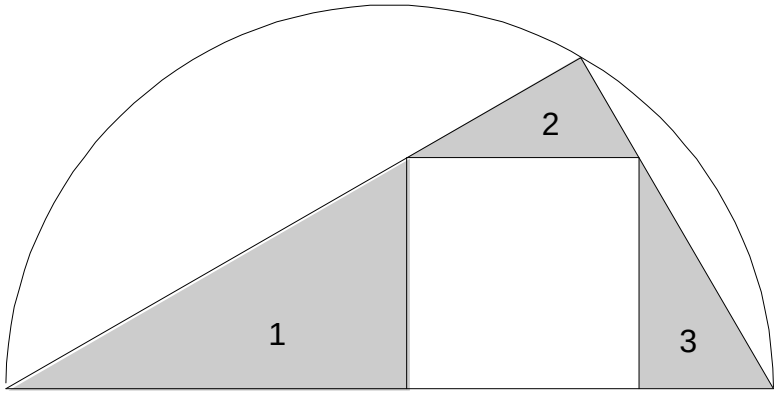


Plate 16

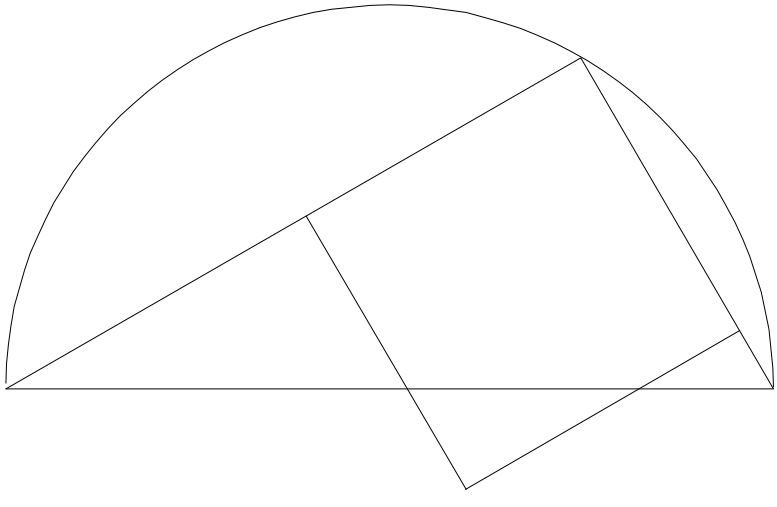
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.




In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"




P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

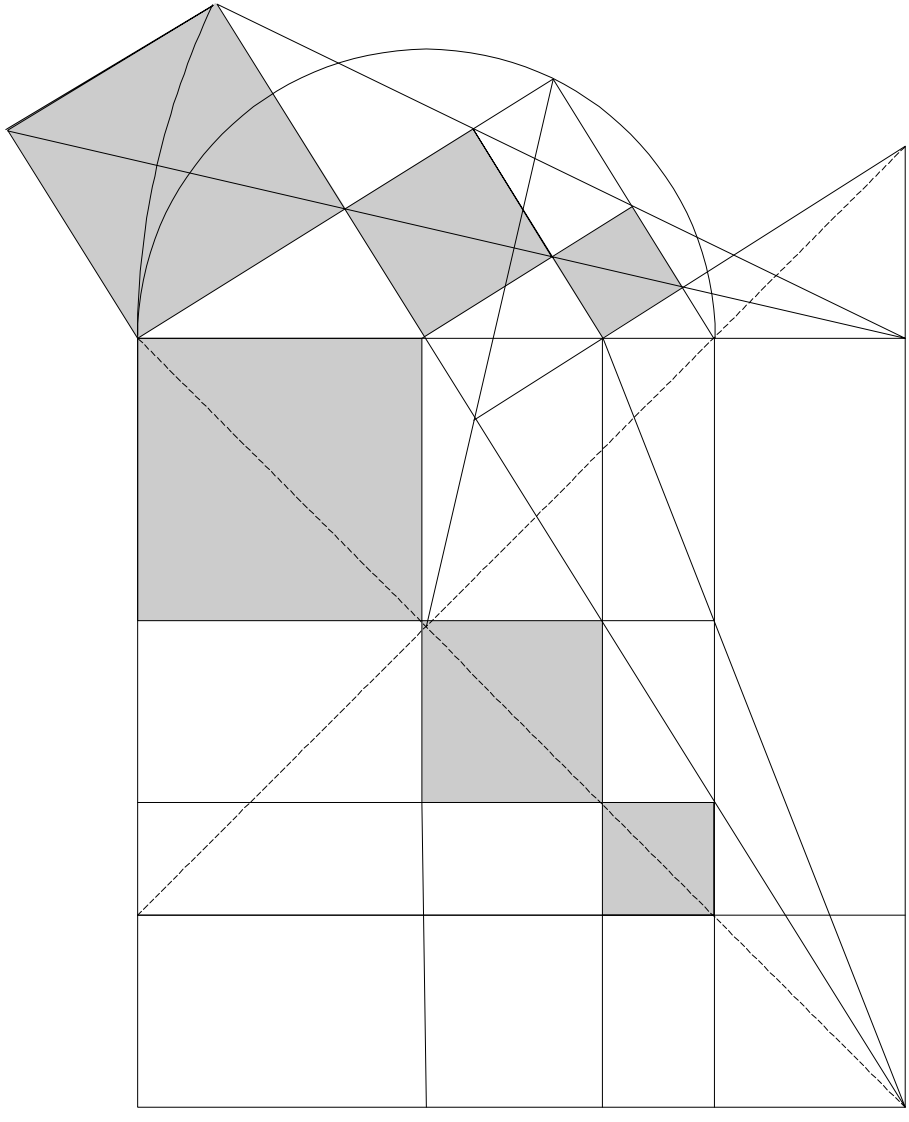
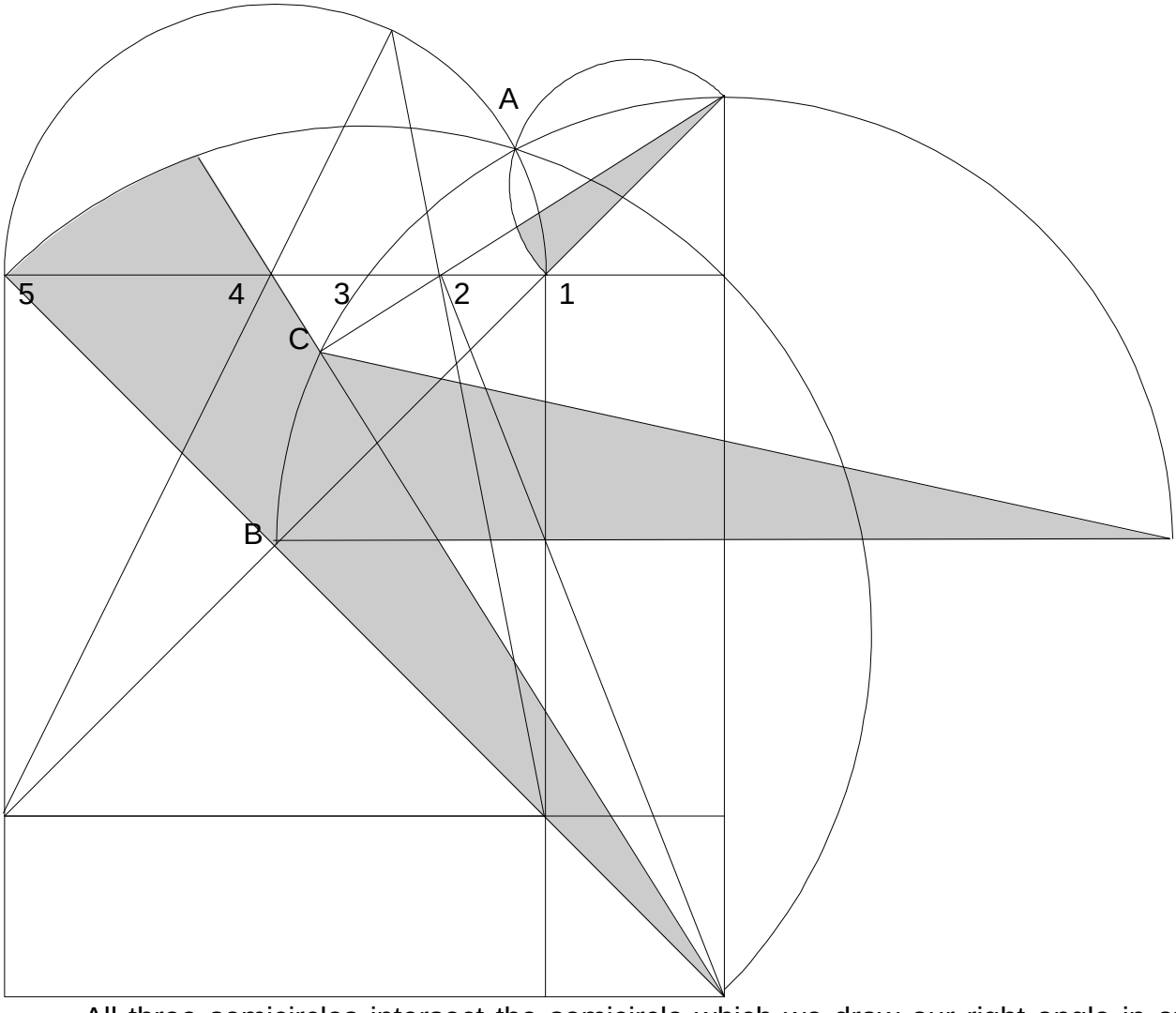


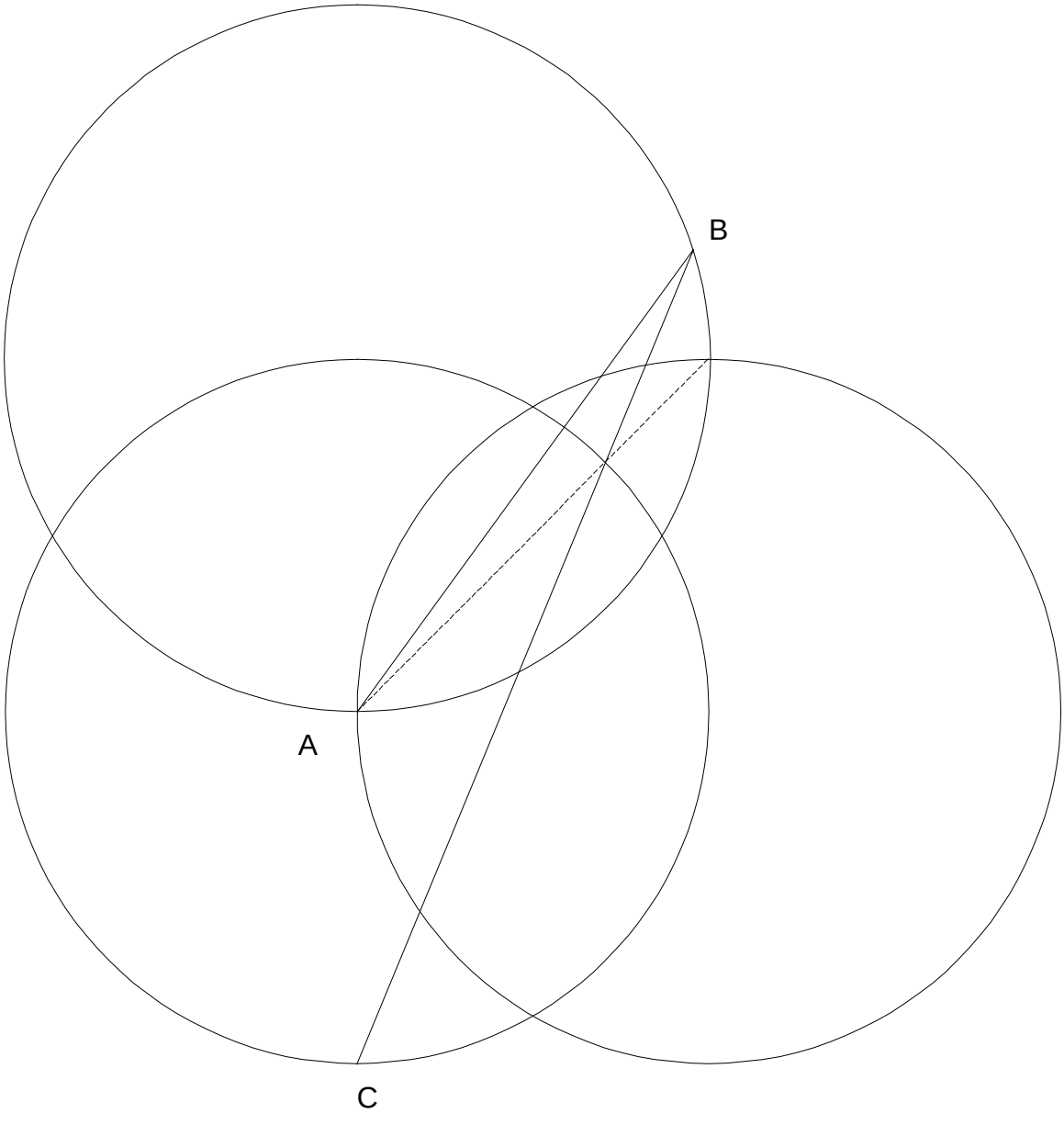
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.




All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.




How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

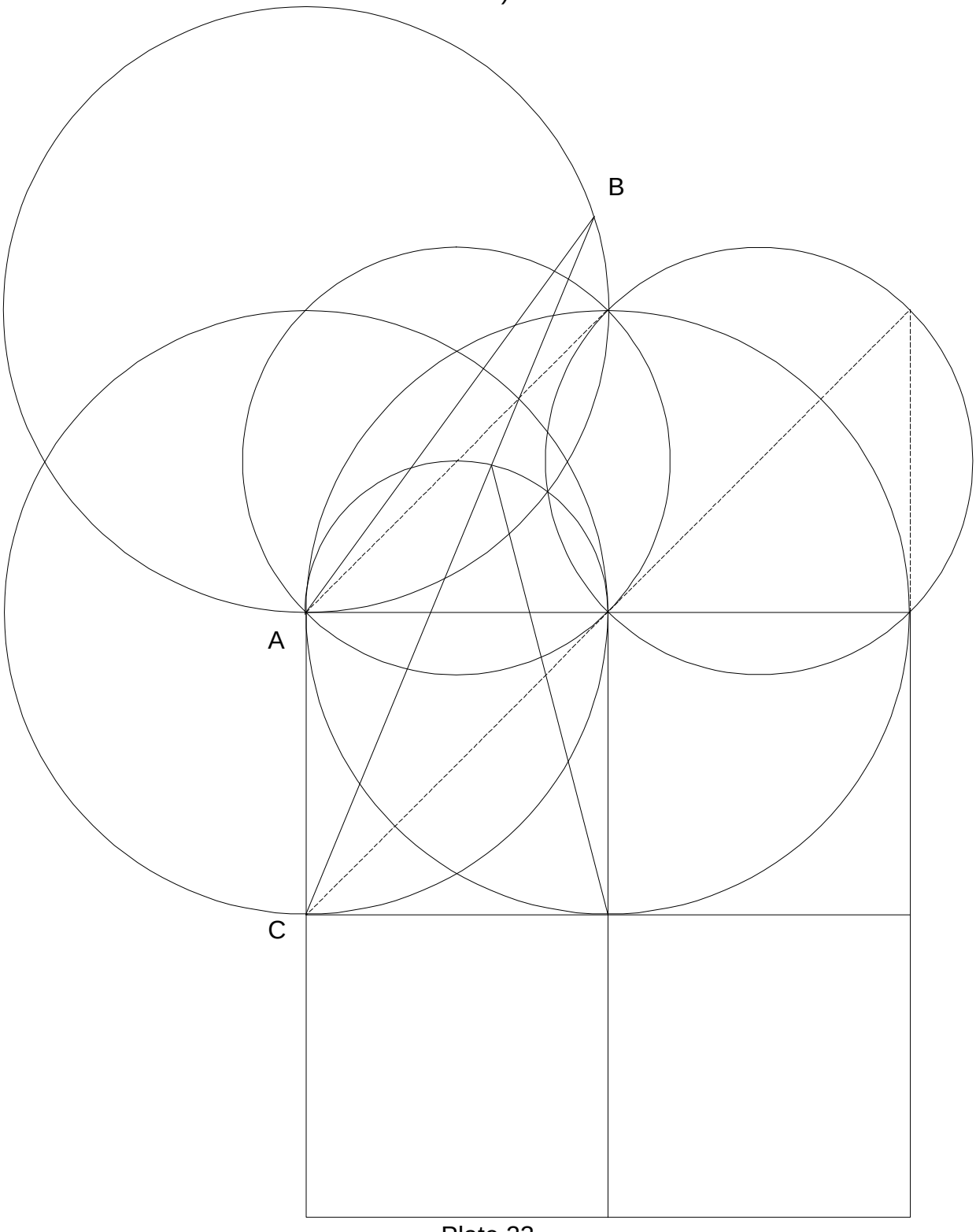
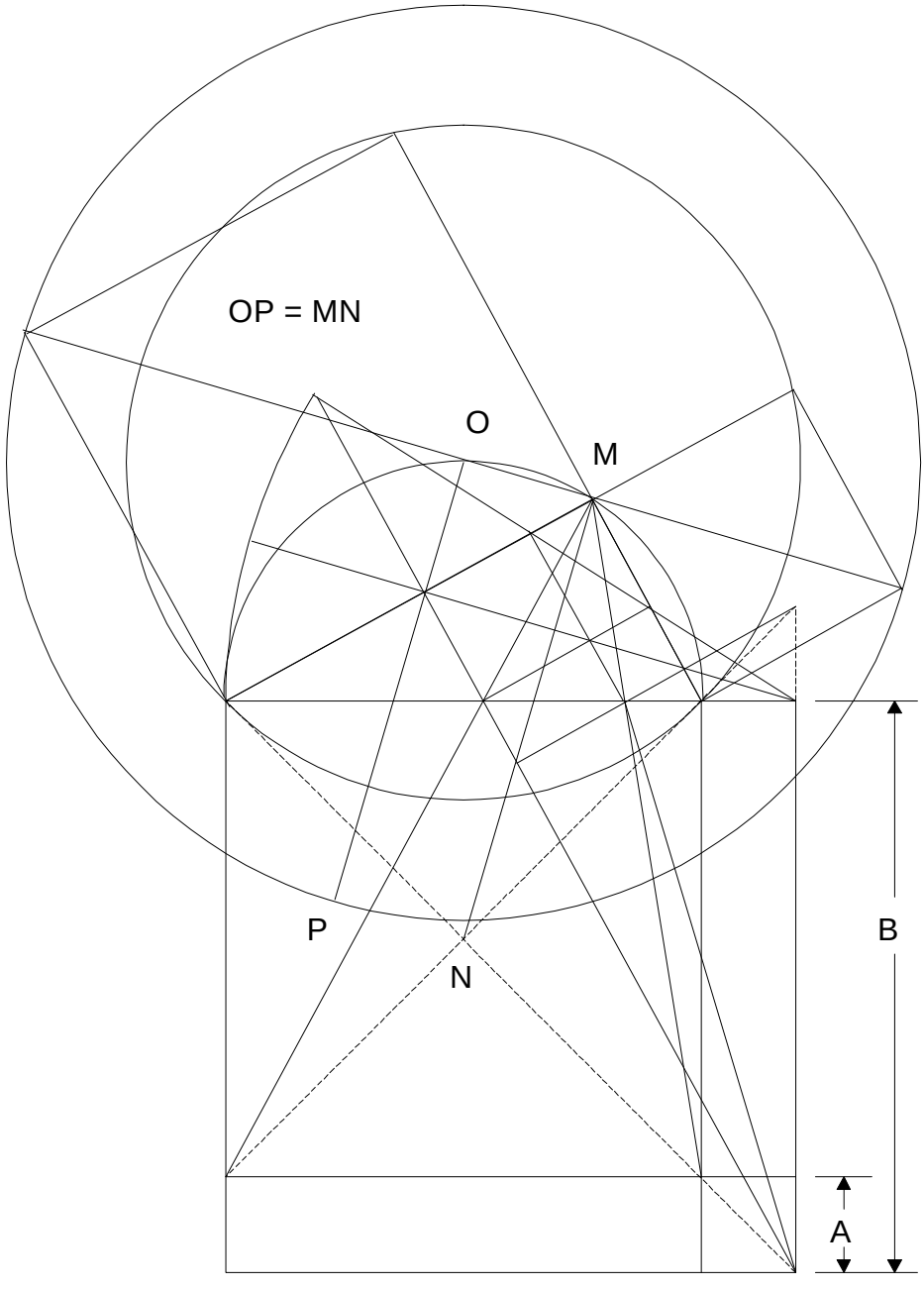



Plate 23

On Plate 24 the radius for the circle OP is given by MN.




One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of $B^2A$ (if you have missed it, the figure gives both roots, $A^2B$ and $B^2A$) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

# GEOMETRIAE DEDICATA

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3508 TA Utrecht
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Telephone: (0)30-531519

Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

# American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA  Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

**Location:**
201 Charles **Street**
Providence, **RI**
02904


December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

*Serving the mathematical community for over 100 years*

# American Mathematical Society

Department of Mathematics
**Roger E. Howe**
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

# JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

**société mathématique defrance**

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

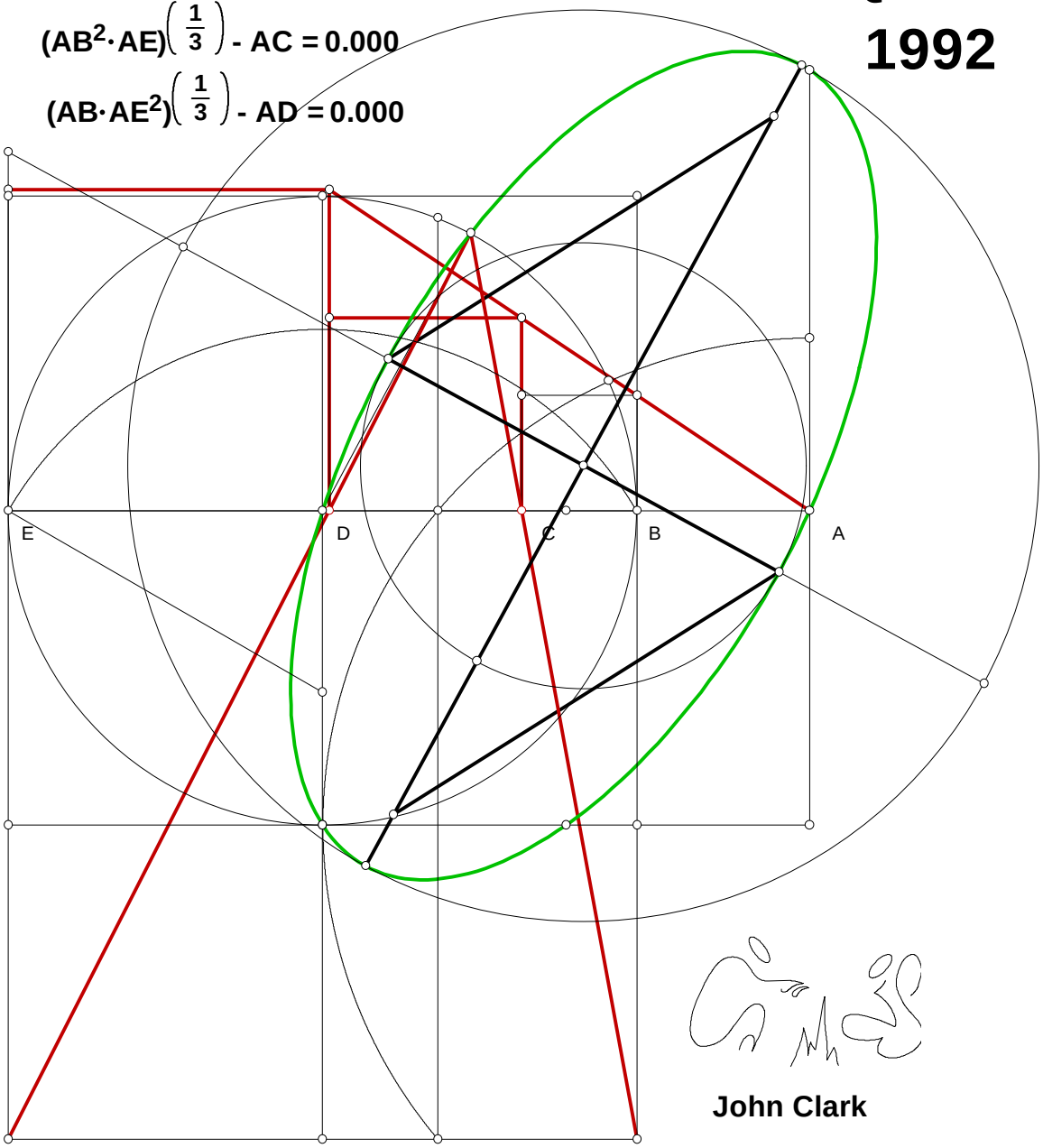
P.J. : Manuscrit

# The Delian Quest

## 1992

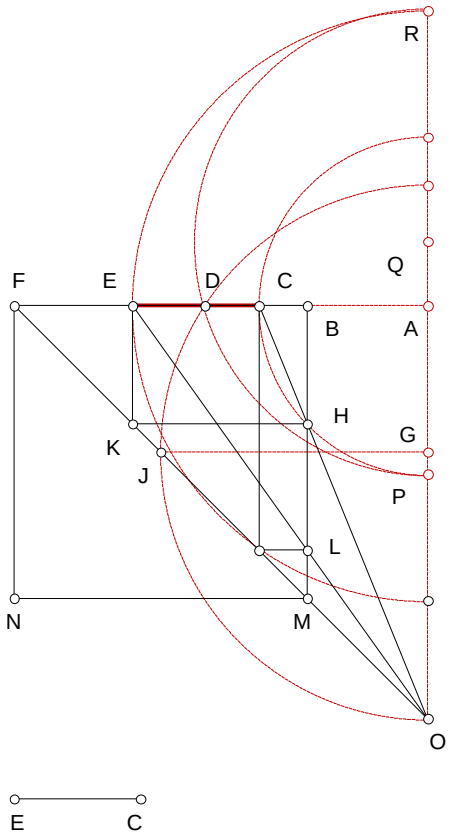
$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$






John Clark







# A Triplicate Ratio 06/20/92

**Given some point O place CE on BF such that O is the point of similarity.**

$N := 1.52 \quad N_2 := .89 \quad N_3 := .66$

$BF := N \quad MO := N_2 \quad CE := N_3$

$FM := \sqrt{2 \cdot BF^2} \quad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF} \quad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \quad QR := DQ \quad QP := DQ$

$AP := QP - AQ \quad AR := QR + AQ \quad AC := AP$

$AE := AR \quad AO := AF \quad BC := AC - AB$

$EF := AF - AE \quad EK := EF \quad BH := EK$

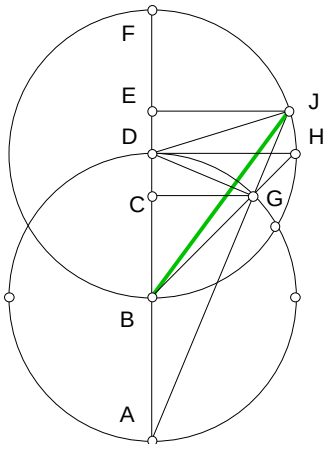
$\frac{BC}{BH} - \frac{AC}{AO} = 0$

**The last ratio can be tediously proved by reducing each term to the givens.**

**Edit 062800**

# 08/12/92 Rusty Cube of a Sphere

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**




$N := 9$ $\quad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

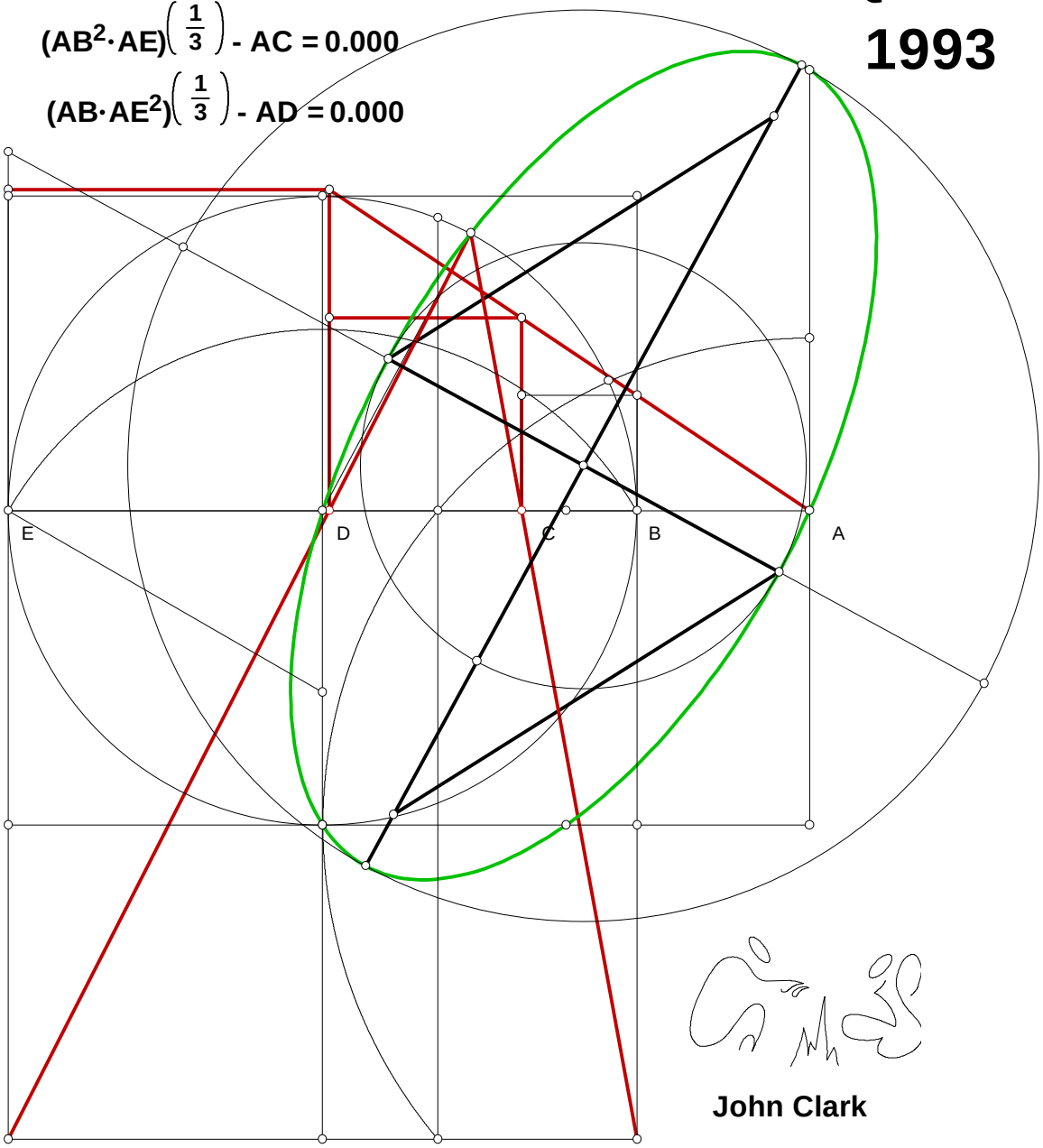
$\frac{CUBE\_ROOT}{BJ} = 0.99906$

# The Delian Quest

## 1993

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

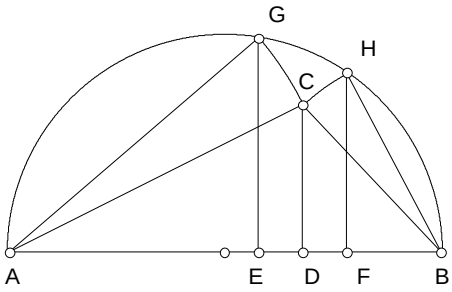





John Clark

## 01/08/93 Pythagoras Revisited

**Given AB, BC, AC, what is CD and CJ?**




$S_1 := 2.24 \quad S_2 := 1.046 \quad S_3 := 1.7$

$S_1 := S_1 \quad S_2 := S_2 \quad S_3 := S_3$

$AB := S_1 \quad BC := S_2 \quad AC := S_3$

$AG := AC \quad BH := BC \quad AE := \frac{AG^2}{AB}$

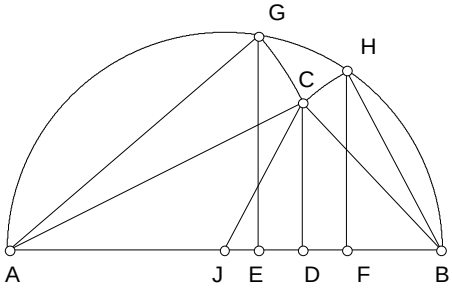
$BF := \frac{BH^2}{AB} \quad EF := AB - (AE + BF)$

$DE := \frac{EF}{2} \quad AD := AE + DE \quad AD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$BD := BF + DE \quad BD := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CD := \sqrt{AC^2 - AD^2}$

$CD := \frac{\sqrt{(-S_1 + S_2 - S_3) \cdot (S_1 + S_2 + S_3) \cdot (S_1 - S_2 - S_3) \cdot (S_1 + S_2 - S_3)}}{2 \cdot S_1}$



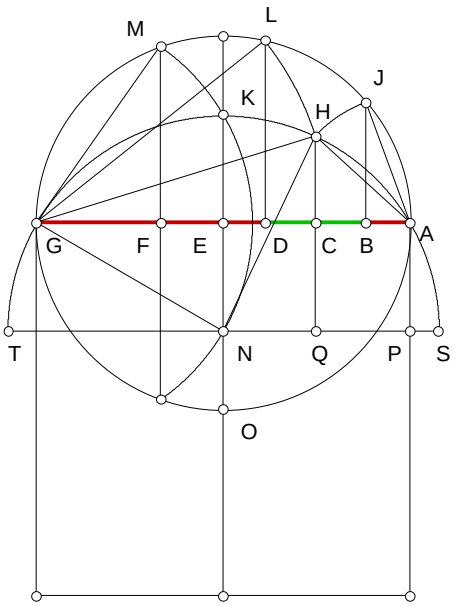

$AJ := \frac{AB}{2} \quad JD := AD - AJ \quad CJ := \sqrt{JD^2 + CD^2}$

$CJ := \frac{1}{2} \cdot \sqrt{2 \cdot S_3^2 - S_1^2 + 2 \cdot S_2^2}$

$(AC^2 + BC^2) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

**The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).**

# 06/03/93.MCD Exploring The Curve CJ




**Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?**

$N := 2 \qquad N_2 := 4$

$AG := N \qquad AC := \frac{AG}{N_2} \qquad GF := \frac{AG}{3} \qquad AE := \frac{AG}{2}$

$EG := AE \qquad AF := AG - GF \qquad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \qquad GN := GM \qquad EN := \sqrt{GN^2 - EG^2}$

$NH := GM \qquad NS := GM \qquad PN := AE \qquad PS := NS - PN$

$ST := 2 \cdot GM \qquad SQ := AC + PS \qquad QT := ST - SQ$

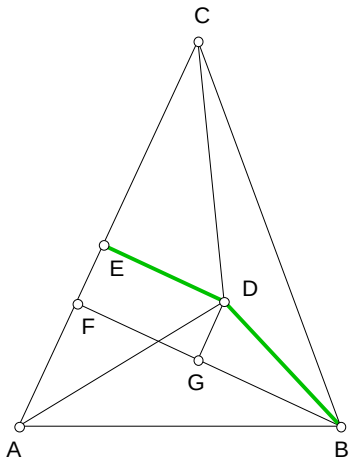
$QH := \sqrt{SQ \cdot QT} \qquad CQ := EN \qquad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \qquad CG := AG - AC \qquad GH := \sqrt{CG^2 + CH^2} \qquad AJ := AH \qquad AB := \frac{AJ^2}{AG}$

$GL := GH \qquad DG := \frac{GL^2}{AG} \qquad BD := AG - (AB + DG) \qquad \sqrt{AB \cdot DG} - BD = 0$

$$\left( \frac{-N}{3} + \frac{N}{3} \cdot \frac{\sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2} \right) - BD = 0$$

# 06/07/93 For All Triangles Find BD




**Given AB, BC, AC, CD, AD, find BD**

**To simplify use line names found in 010893**

$S_1 := 5.55 \quad S_2 := 4.61 \quad S_3 := 1.5$

$S_5 := 4.5 \quad S_6 := 5.48$

$AB := S_5 \quad BC := S_6 \quad AC := S_1 \quad CD := S_3 \quad AD := S_2$

$$DE := \frac{\sqrt{\left(-S_1 + S_2 - S_3\right)\cdot\left(S_1 + S_2 + S_3\right)\cdot\left(S_1 - S_2 - S_3\right)\cdot\left(S_1 + S_2 - S_3\right)}}{2\cdot S_1}$$

$$BF := \frac{\sqrt{\left(-S_1 + S_5 - S_6\right)\cdot\left(S_1 + S_5 + S_6\right)\cdot\left(S_1 - S_5 - S_6\right)\cdot\left(S_1 + S_5 - S_6\right)}}{2\cdot S_1}$$

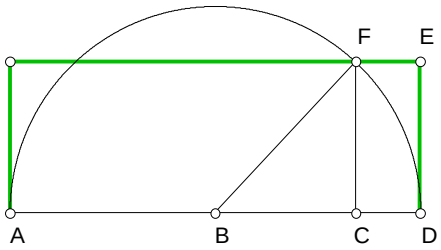
$CE := \sqrt{CD^2 - DE^2} \quad CF := \sqrt{BC^2 - BF^2} \quad EF := CF - CE \quad GF := DE$

$BG := BF - GF \quad DG := EF \quad BD := \sqrt{DG^2 + BG^2} \quad BD = 3.983$

**OR**

$BG_2 := BF + GF \quad DG_2 := EF \quad BD_2 := \sqrt{DG^2 + BG_2^2} \quad BD_2 = 5.757$

# 06/09/93 Rectangular Roots




**Given AD and DE divide AD into the rectangular roots of DE.**

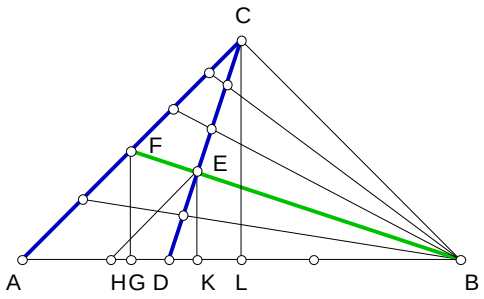
$N := 5 \qquad N_2 := 2$

$AD := N \quad DE := N_2 \quad CF := DE \quad BF := \frac{AD}{2}$

$AB := BF \quad BC := \sqrt{BF^2 - CF^2} \quad AC := AB + BC \quad CD := AD - AC \quad \sqrt{CD \cdot AC} - DE = 0$

$AC := \frac{1}{2} \cdot N + \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2} \qquad CD := \frac{1}{2} \cdot N - \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2}$

# 06/21/93 Pyramid of Ratios I




**Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?**

$N1 := 3 \qquad N2 := 5 \qquad \delta := 1 .. N2$

$AB := 1 \qquad AD := \frac{AB}{N1} \qquad AL := \frac{AB}{2}$

$DL := AL - AD \qquad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \qquad CD := \sqrt{DL^2 + CL^2}$

| $\delta$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

$DE_\delta := \frac{CD \cdot \delta}{N2} \qquad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \qquad AK_\delta := AD + DK_\delta \qquad BK_\delta := AB - AK_\delta \qquad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \qquad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \qquad BH_\delta := BK_\delta + HK_\delta \qquad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \qquad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \qquad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\text{if}\left(N2 - \delta, \frac{N1 \cdot N2}{N2 - \delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\frac{AC}{AF_\delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

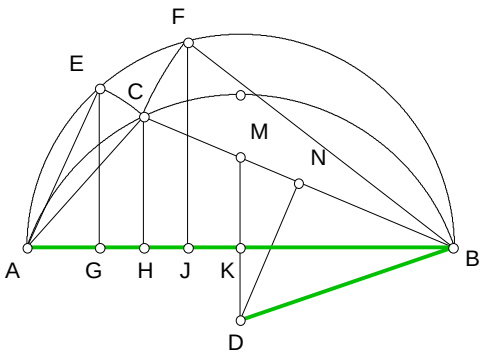
$\frac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

# 06/27/93 Describe A Circle About a Triangle

## Given the difference between three non-collinear points, find the radius of the circle that circumscribes them

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $\quad NOT(X) := X = 0$ $\quad \delta := 0 .. 2$




$BK := \frac{AB}{2}$ $\quad AE := AC$ $\quad BF := BC$

$AG := \frac{AE^2}{AB}$ $\quad BJ := \frac{BF^2}{AB}$ $\quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $\quad BH := BJ + HJ$ $\quad CH := \sqrt{BC^2 - BH^2}$

$BN := \frac{BC}{2}$ $\quad BM := \frac{BC \cdot BK}{BH}$ $\quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $\quad BD := \sqrt{BN^2 + DN^2}$

$radius := if(\Delta, BD, 0)$ $\quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary\_radius = 0$

$\Delta = 1$

$S_1 := \begin{bmatrix} AB \\ AC \\ BC \end{bmatrix}$ $\quad S_2 := \begin{bmatrix} AC \\ BC \\ AB \end{bmatrix}$ $\quad S_3 := \begin{bmatrix} BC \\ AB \\ AC \end{bmatrix}$

$AB \equiv 3$ $\quad AC \equiv 4$ $\quad BC \equiv 6$

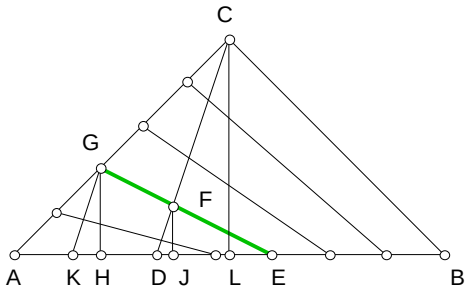
## The name of the Radius as a proportion of the given names

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = [\, 3.375 \quad 3.375 \quad 3.375 \,]$

# 07/15/93 Pyramid of Ratios II

**AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?**




$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

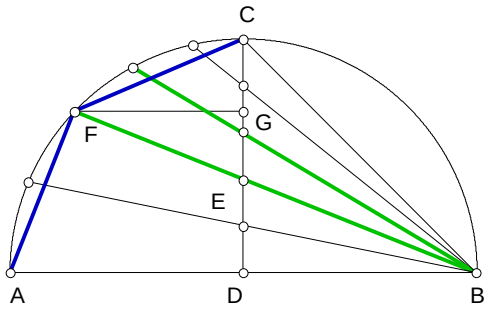
| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

# 07/25/93 Pyramid of Ratios III

## Dividing DC by an number provides wht in terms of BE/BF and AF/CF?




$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

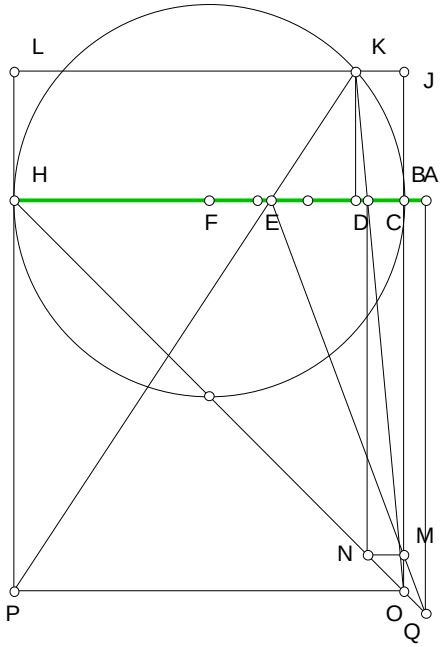
$DG := DE + EG \qquad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$ | $\frac{BE_\delta}{BF_\delta}$ | $\frac{BF_\delta}{BE_\delta}$ | $\frac{2}{\left(\frac{\delta}{N}\right)^2 + 1}$ |
|---|---|---|---|---|
| 0.514 | 0.514 | 0.514 | 1.946 | 1.946 |
| 0.556 | 0.556 | 0.556 | 1.8 | 1.8 |
| 0.625 | 0.625 | 0.625 | 1.6 | 1.6 |
| 0.722 | 0.722 | 0.722 | 1.385 | 1.385 |
| 0.847 | 0.847 | 0.847 | 1.18 | 1.18 |
| 1 | 1 | 1 | 1 | 1 |

| $\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$ | $\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$ |
|---|---|
| 0.283 | 0.283 |
| 0.707 | 0.707 |
| 1.414 | 1.414 |
| 2.828 | 2.828 |
| 7.071 | 7.071 |
| 0 | 0 |

# 11/06/93 Gruntwork I on the Delian Solution




**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?**

$N := 4$ $BH := 1$

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $BJ := DK$ $BO := BH$

$JO := BJ + BO$ $JK := BD$ $CD := \frac{JK \cdot DK}{JO}$

$KL := DH$ $LP := JO$ $DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD$ $CE := CD + DE$ $MN := BC$

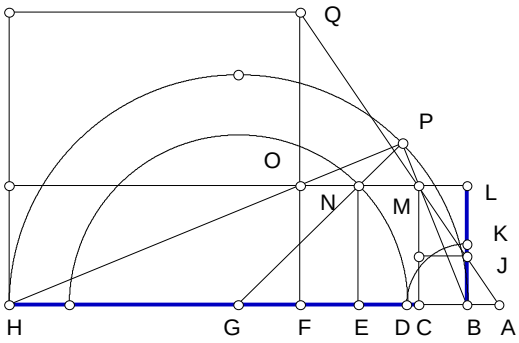
$GH := MN$ $CH := BH - BC$ $HN := \sqrt{2 \cdot CH^2}$ $GM := HN$ $EH := CH - CE$ $EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG}$ $HO := \sqrt{2 \cdot BH^2}$ $OQ := HQ - HO$ $OR := \sqrt{\frac{OQ^2}{2}}$ $AB := OR$ $AC := AB + BC$

$AE := AB + BD + DE$ $AH := AB + BH$ $\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$

## 11/09/93 Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem.**
**BH is the difference between the segments AH and AB.**
**CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**



$N := 2 \qquad BH := N - 1$

$BG := \frac{BH}{2} \qquad CF := N^{\frac{2}{3}} - N^{\frac{1}{3}}$

$BL := CF \qquad GP := BG \qquad BK := \frac{BL}{2}$

$BD := BK \qquad NP := BD \qquad GN := GP - NP$

$EN := BL \qquad GE := \sqrt{GN^2 - EN^2}$

$CE := BD \qquad BC := BG - (GE + CE)$

$GH := BG \qquad EF := BD$

$FH := GH + GE - EF \qquad FQ := FH \qquad FO := BL \qquad OQ := FQ - FO \qquad MO := CF \qquad AF := \frac{MO \cdot FQ}{OQ}$

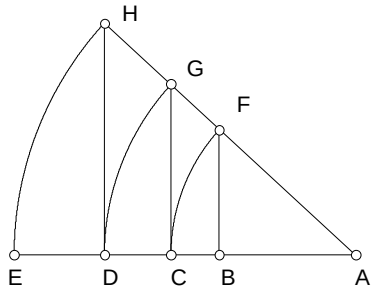
$AC := AF - CF \qquad AH := AF + FH \qquad AB := AH - BH$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 2$$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

## 11/10/93 Gruntwork II on the Delian Solution

**Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?**




$N := 1.1$ $AE := 10$

$DE := \frac{AE}{N}$ $AD := AE - DE$ $AH := AE$

$AG := AD$ $AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

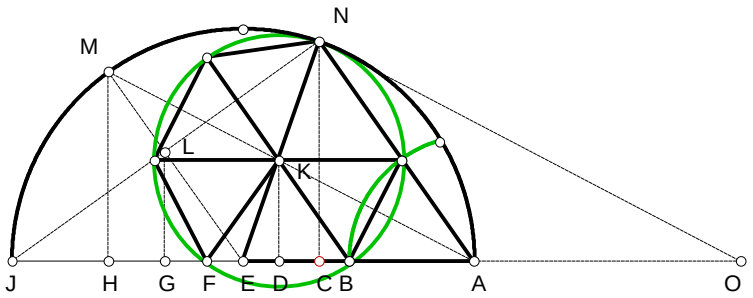
$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 1331$ $\frac{AD}{AB} = 121$ $\frac{AC}{AB} = 11$

## 11/11/93 The Archamedian Paper Trisector

**If one accepts the facts of the original figure, one only need prove that BK = AB.**
**If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?**




$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN \cdot JL}{JN}$ $GJ := \frac{CJ \cdot JL}{JN}$

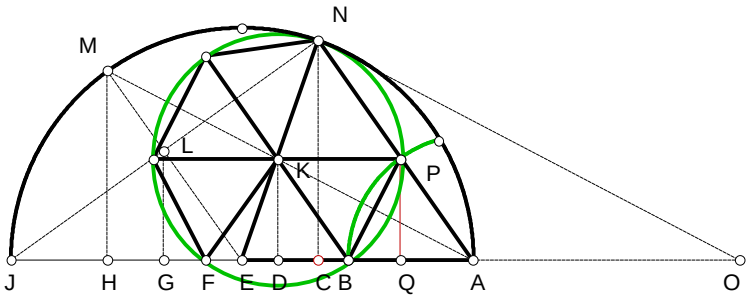
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

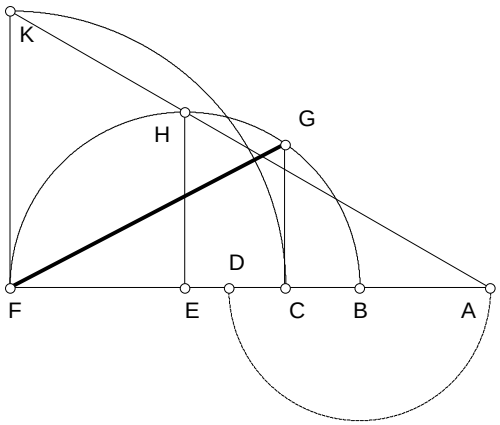
$AB - BK = 0$ $AB = 0.25$

**If PK is parallel to AJ, then . . . .**




$AN := \sqrt{AC^2 + CN^2}$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

**11/12/93 To Square A Circle Off The Base Of A Right Triangle.**




**Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?**

**Using the approximation, $\pi$ = 22/7, square the circle off the base of a right triangle.**

$$BF := 1 \qquad BE := \frac{BF}{2} \qquad EF := BE \qquad EH := BE \qquad BD := \frac{3}{4}\cdot BE \qquad AB := BD$$

$$AE := AB + BE \qquad AF := AB + BF \qquad FK := \frac{EH \cdot AF}{AE} \qquad CF := FK \qquad BC := BF - CF$$

$$CG := \sqrt{BC \cdot CF} \qquad FG := \sqrt{CF^2 + CG^2} \qquad \pi\_A := \frac{FG^2}{BE^2}$$

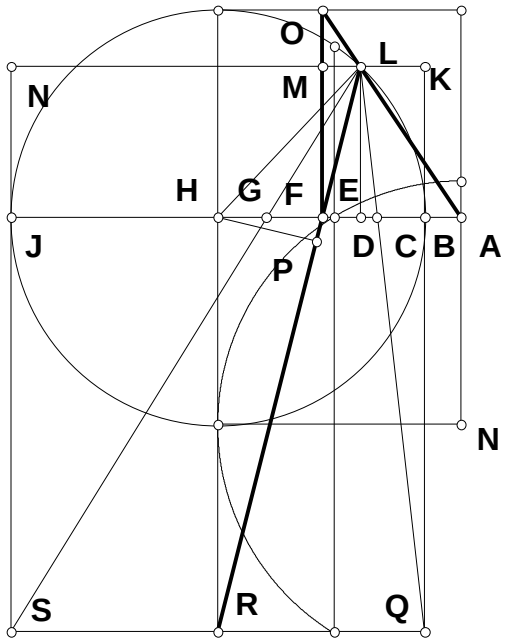
$$\pi = 3.14159265359$$

$$\pi\_A = 3.142857142857$$

$$\frac{\pi}{\pi\_A} = 0.999597662505843$$

# 11/18/93 Exploring Cube Roots Plate A

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.**




$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad MO := FO - DL \quad LM := DF$

$AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF \quad BQ := BJ$

$BK := DL \quad BD := BH - (FH + DF)$

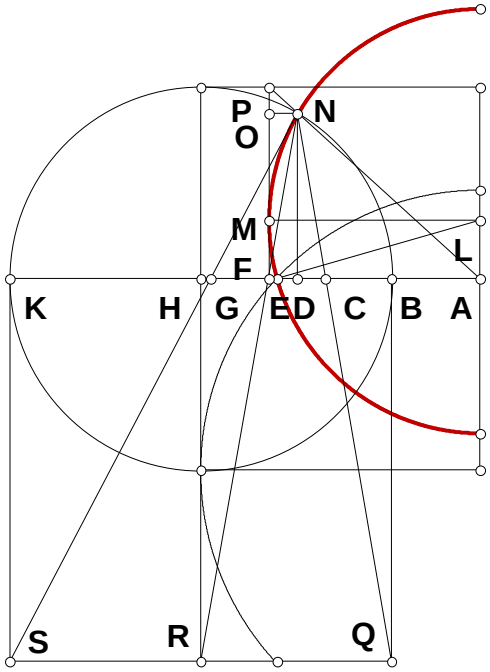
$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad HJ := BH \quad DJ := HJ + FH + DF \quad LN := DJ$

$JS := BJ \quad JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC$

$AG := AB + BG \quad AJ := AB + BJ \quad \left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

# 11/18/93 Exploring Cube Roots Plate B

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**




$N := 1.2$ $BK := 1$

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

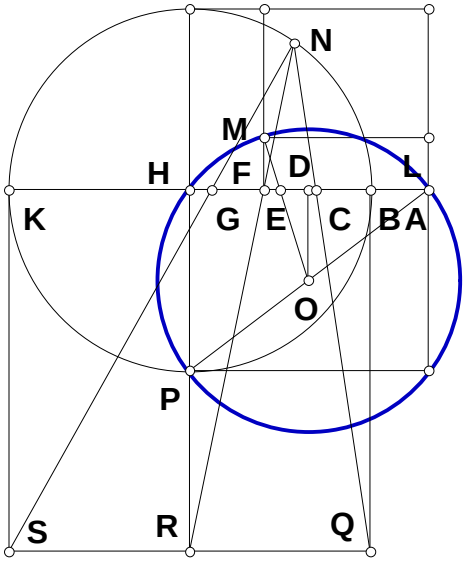
$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$ $AB = 1.523$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

# 11/18/93 Exploring Cube Roots Plate C

## The circle AO passes through point M.




$N := 1$

$BK := 1$ $\quad AB := \frac{BK}{N}$ $\quad AK := AB + BK$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}}$ $\quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$ $\quad \frac{AK}{AB} = 2$

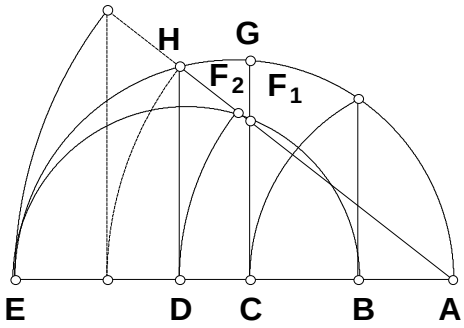
$CG := AG - AC$ $\quad CF := \frac{CG}{2}$ $\quad BH := \frac{BK}{2}$

$AH := AB + BH$ $\quad HP := BH$ $\quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2}$ $\quad DO := \frac{HP}{2}$ $\quad AF := AC + CF$ $\quad AD := \frac{AH}{2}$

$DF := AF - AD$ $\quad FM := CF$ $\quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$




## 11/22/93 Cube by Iteration

**When $F_1$ and $F_2$ are the same point on C, then a sixth root series has been constructed. Use iteration to place $F_2$ on $F_1$.**

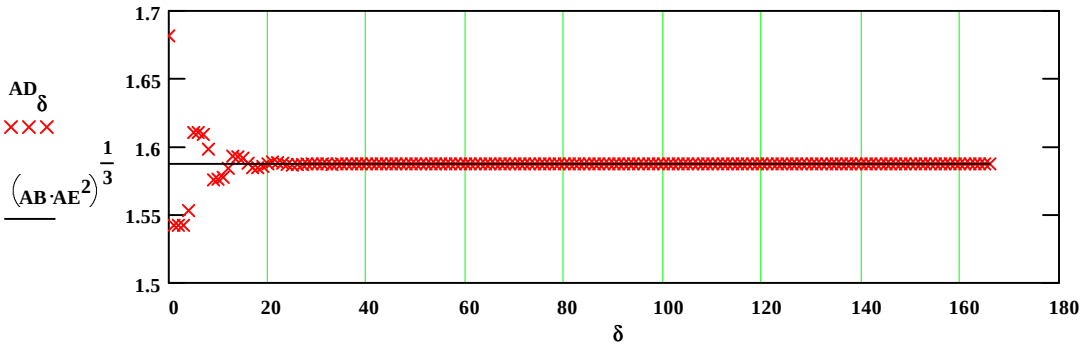
$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \right]^2} \end{bmatrix}$$

$$\begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + \left(CF_\delta\right)^2} \end{bmatrix}$$

$\Delta \equiv 166$

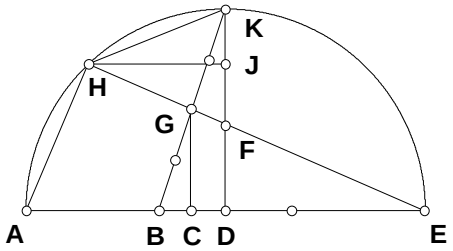



$\left(AB^2 \cdot AE^4\right)^{\frac{1}{6}} - AD_\Delta = 0$

$\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD_\Delta = 0$

# 11/24/93 POR Series IV

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**




$N_1 := 3 \qquad N_2 := 5 \qquad AE := 1$

$\alpha := 1 .. N_1 - 1 \qquad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\frac{AH}{HK} = 0.265 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

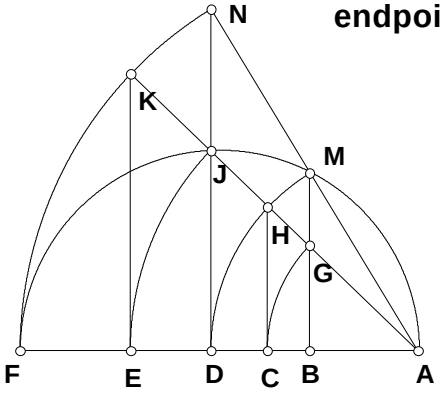
$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$

## 12/04/93 Exponential Series M^(1/2^N)

**Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.**




$N := 8$

$AF := N \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

$AN := AF \quad AD := \frac{AB \cdot AN}{AM} \qquad DF := AF - AD$

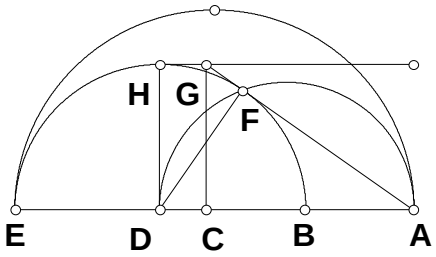
$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

$AK := AF \quad AE := \frac{AD \cdot AK}{AJ} \qquad AH := AD \quad AC := \frac{AD \cdot AH}{AJ}$

$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$N^{\frac{1}{4}} - AC = 0 \qquad N^{\frac{2}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AE = 0$

# 12/06/93 Alternate Method: Square Root
# Common Segment Common Endpoint




$N := 6 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2}$

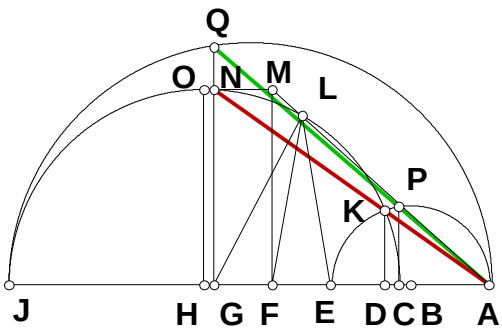
$DF := BD \qquad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$

## 12/06/93B Gruntwork IV on the Delian Solution

### Are APQ colinear? Are AKN colinear?




$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$AE := \left(AC^2 \cdot AJ\right)^{\left(\frac{1}{3}\right)} \qquad AG := \left(AC \cdot AJ^2\right)^{\left(\frac{1}{3}\right)}$

$CG := AG - AC \qquad CJ := AJ - AC$

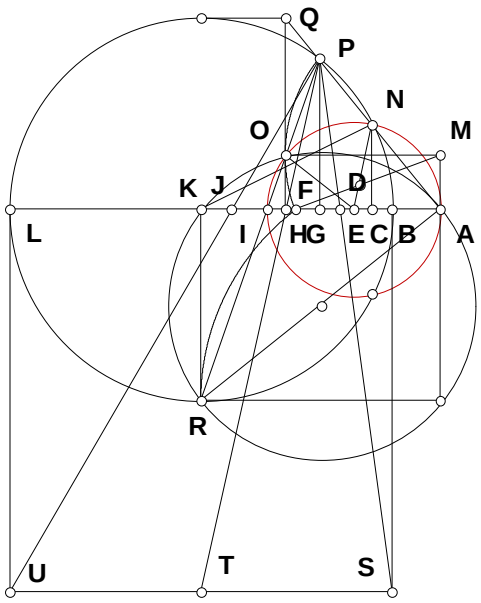
$GJ := CJ - CG \qquad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \qquad CE := AE - AC \qquad CH := \frac{CJ}{2}$

$BK := \frac{AE}{2} \qquad HK := CH \qquad HJ := CH \qquad AH := AJ - HJ \qquad BH := AH - AB \qquad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$DE := AE - (AB + BD) \qquad AD := AE - DE \qquad DK := \sqrt{AD \cdot DE} \qquad \frac{AG}{GN} - \frac{AD}{DK} = 0$

$GQ := \sqrt{AG \cdot GJ} \qquad CP := \sqrt{AC \cdot CE} \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$




## 12/11/93

**The structure in red appears to be a constant.**

$N := 6$ $AB := 1$ $AL := AB \cdot N$

$BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$

$AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$ $BE := AE - AB$ $BJ := AJ - AB$

$JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

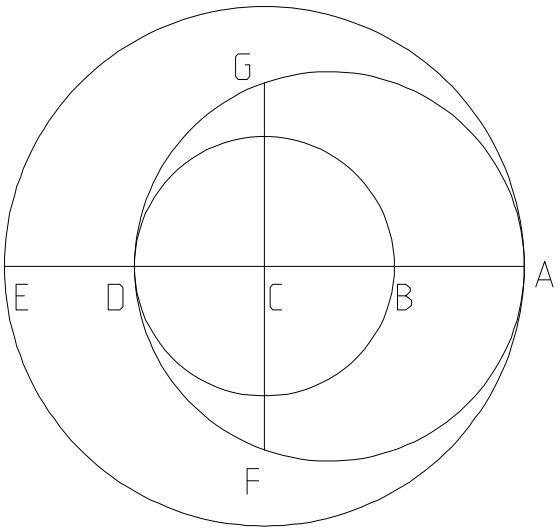
$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$

$KR := BK$ $KL := BK$ $FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$

$AD := \frac{AI}{2}$ $KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

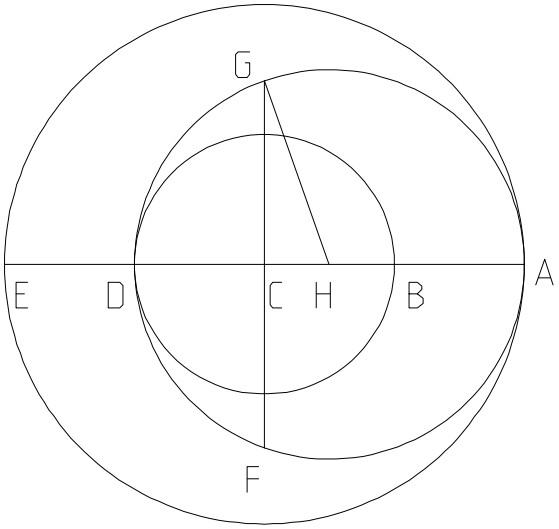
$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$




# The Square Root

It may be noticed that I use the adjacent figure in my work for doing square roots. I believe that it is the primary figure for doing square roots. Given segment AE and segment BD, segment FG is their square root. It has a major advantage of being able to square larger figures on paper, not to mention makes something of the development of exponential series.




$AE := 100 \quad \delta := 1 .. AE \quad BD_\delta := \delta$

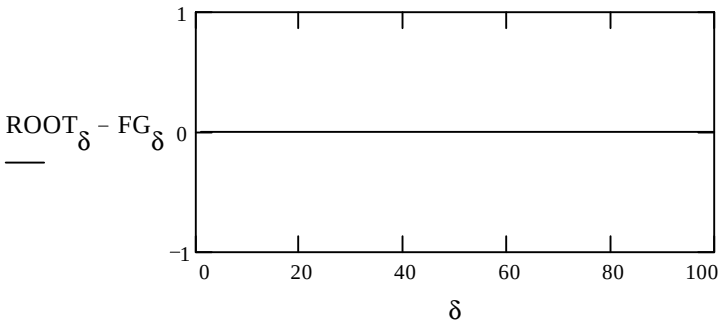
$AC := \frac{AE}{2} \quad BC_\delta := \frac{BD_\delta}{2} \quad AB_\delta := AC - BC_\delta$

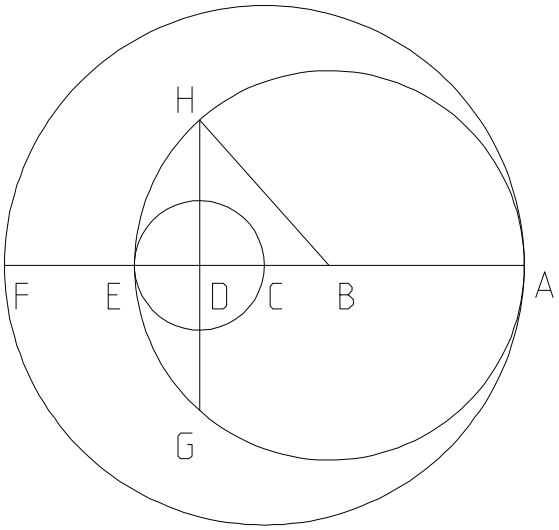
$AD_\delta := AC + BC_\delta \quad AH_\delta := \frac{AD_\delta}{2}$

$GH_\delta := AH_\delta \quad CH_\delta := AC - AH_\delta$

$FG_\delta := 2 \cdot \sqrt{\left(GH_\delta\right)^2 - \left(CH_\delta\right)^2}$

$ROOT_\delta := \sqrt{\left(BD_\delta\right) \cdot AE}$






12_12_B3.MCD

Completely generalize the square root figure.

$AF := 7 \qquad DF := \frac{AF}{BR} \qquad AD := AF - DF$

$\delta := 1 .. LBR \qquad DE_\delta := \frac{DF}{LBR} \cdot \delta$

$CE_\delta := 2 \cdot DE_\delta \qquad AE_\delta := AD + DE_\delta$

$AB_\delta := \frac{AE_\delta}{2} \qquad BD_\delta := AD - AB_\delta$

$BH_\delta := AB_\delta \qquad GH_\delta := 2 \cdot \sqrt{\left| (BH_\delta)^2 - (BD_\delta)^2 \right|}$

Set AF to unity so that it may be eliminated. Setting BR to 2 will yeild the familiar square root. BR may even take fractional values.

**Plug in values here.**

BR=BASE RATIO,
LBR=LITTLE BASE RATIO

$BR \equiv 3 \qquad LBR \equiv 5$

$\left| \frac{(2 \cdot BR) - 2}{BR} \right| = 1.333$

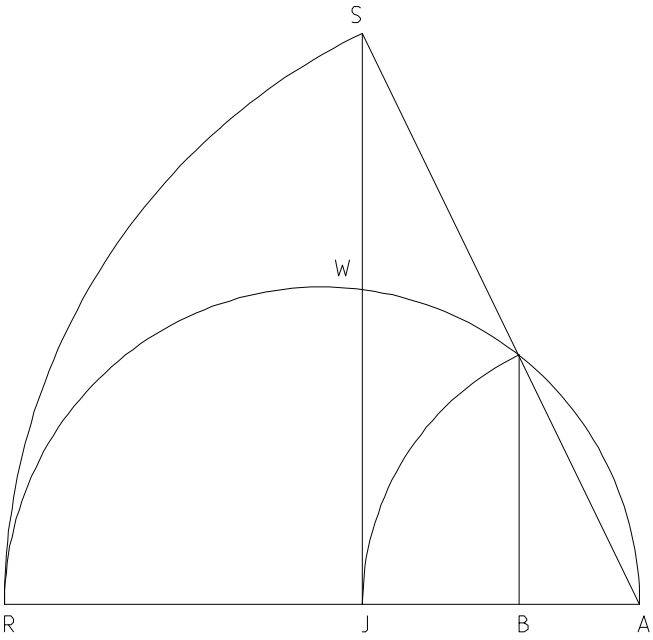
The equation below simplifies to the next.

$GH_\delta \qquad \left[ \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot \left| \frac{(2 \cdot BR) - 2}{BR} \right| \right]^{\frac{1}{2}} \cdot AF \qquad \frac{\sqrt{2} \cdot \sqrt{\left| (2 \cdot BR) - 2 \right|}}{BR \cdot \sqrt{LBR}} \cdot \sqrt{\delta} \cdot AF$

| | | |
|---|---|---|
| 2.951 | 2.951 | 2.951 |
| 4.174 | 4.174 | 4.174 |
| 5.112 | 5.112 | 5.112 |
| 5.903 | 5.903 | 5.903 |
| 6.6 | 6.6 | 6.6 |

$CE_\delta \qquad \frac{BR - (BR - 2)}{BR \cdot LBR} \cdot \delta \cdot AF$

| | |
|---|---|
| 0.933 | 0.933 |
| 1.867 | 1.867 |
| 2.8 | 2.8 |
| 3.733 | 3.733 |
| 4.667 | 4.667 |




12_16_93, using 12_04_93.MCD

$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

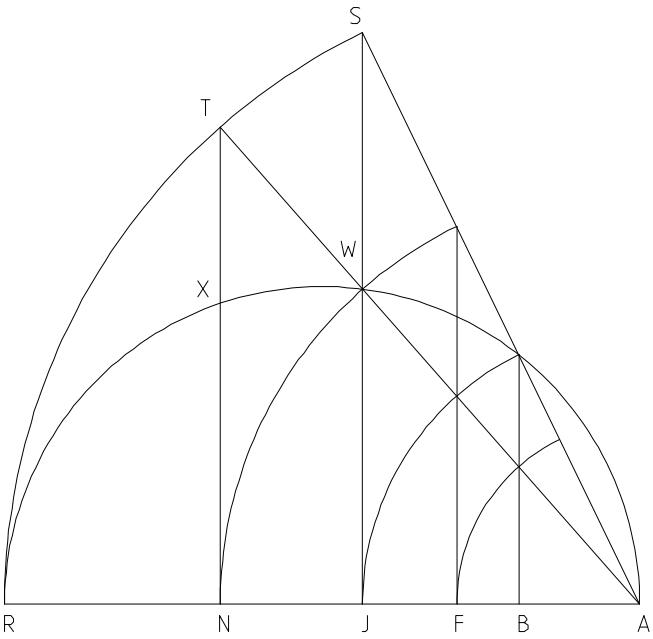
$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ | $AJ_\delta$ |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.

# Euclidean Exponential Series



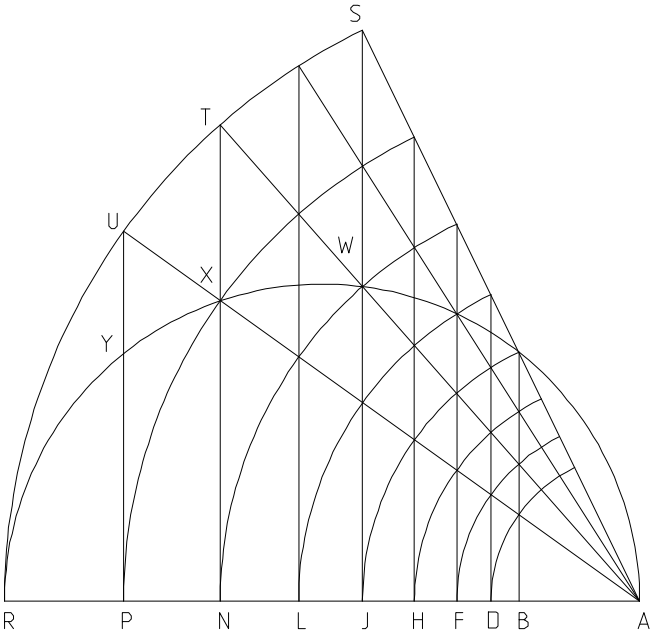

$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ | $AF_\delta$ | $AJ_\delta$ | $AN_\delta$ |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |

What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.



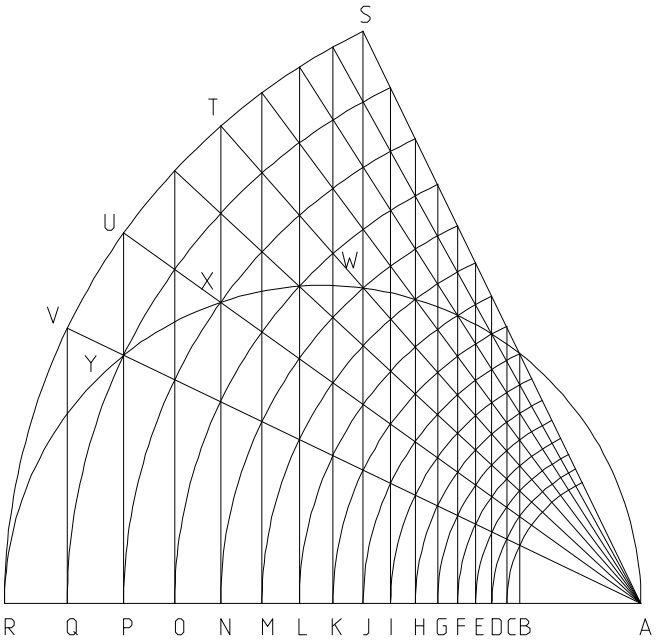

$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta}$

$AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$

$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta}$

$AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$

| $AB_\delta$ | $AD_\delta$ | $AF_\delta$ | $AH_\delta$ | $AJ_\delta$ | $AL_\delta$ | $AN_\delta$ | $AP_\delta$ |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |




$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta}$

$AM_\delta := \frac{(AN_\delta)^2}{AO_\delta} \quad AK_\delta := \frac{(AL_\delta)^2}{AM_\delta}$

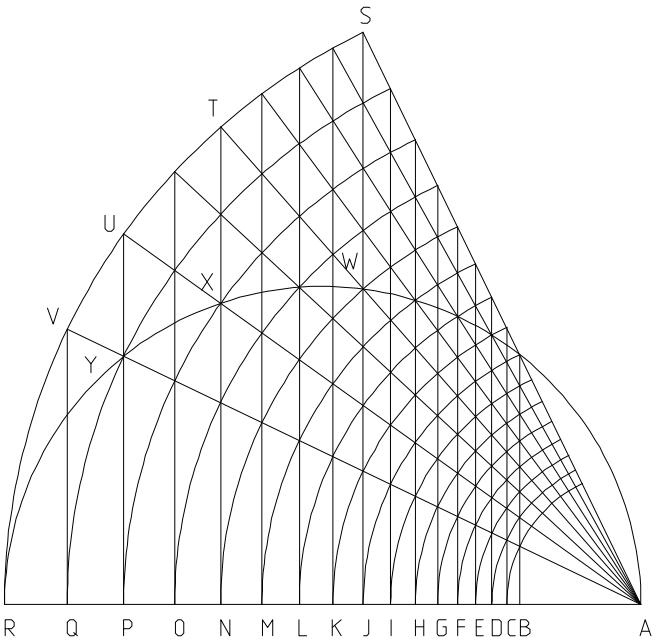
$AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$

$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$

| $AB_\delta$ | $AC_\delta$ | $AD_\delta$ | $AE_\delta$ | $AF_\delta$ |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |

| $AG_\delta$ | $AH_\delta$ | $AI_\delta$ | $AJ_\delta$ | $AK_\delta$ | $AL_\delta$ | $AM_\delta$ | $AN_\delta$ | $AO_\delta$ | $AP_\delta$ | $AQ_\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |

Values found by the investigator of 12_14_93




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AN_\delta$ | $\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

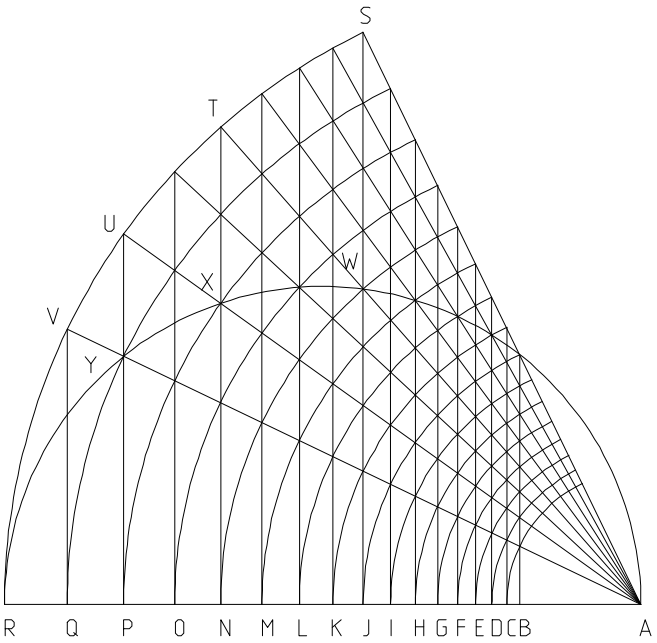
| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{7} \cdot AR\right]^{\frac{1}{8}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |




| $AQ_\delta$ | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ |
|---|---|
| 9.576 | 9.576 |
| 9.336 | 9.336 |
| 9.17 | 9.17 |
| 9.043 | 9.043 |
| 8.941 | 8.941 |

| $AP_\delta$ | $\left[\left(AB_\delta\right)^2 \cdot AR^{14}\right]^{\frac{1}{16}}$ |
|---|---|
| 9.17 | 9.17 |
| 8.717 | 8.717 |
| 8.409 | 8.409 |
| 8.178 | 8.178 |
| 7.993 | 7.993 |

| $AO_\delta$ | $\left[\left(AB_\delta\right)^3 \cdot AR^{13}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.781 | 8.781 |
| 8.138 | 8.138 |
| 7.711 | 7.711 |
| 7.395 | 7.395 |
| 7.147 | 7.147 |

| $AN_\delta$ | $\left[\left(AB_\delta\right)^4 \cdot AR^{12}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.409 | 8.409 |
| 7.598 | 7.598 |
| 7.071 | 7.071 |
| 6.687 | 6.687 |
| 6.389 | 6.389 |

| $AM_\delta$ | $\left[\left(AB_\delta\right)^5 \cdot AR^{11}\right]^{\frac{1}{16}}$ |
|---|---|
| 8.052 | 8.052 |
| 7.094 | 7.094 |
| 6.484 | 6.484 |
| 6.047 | 6.047 |
| 5.713 | 5.713 |

| $AL_\delta$ | $\left[\left(AB_\delta\right)^6 \cdot AR^{10}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.711 | 7.711 |
| 6.623 | 6.623 |
| 5.946 | 5.946 |
| 5.469 | 5.469 |
| 5.107 | 5.107 |

| $AK_\delta$ | $\left[\left(AB_\delta\right)^7 \cdot AR^{9}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.384 | 7.384 |
| 6.184 | 6.184 |
| 5.453 | 5.453 |
| 4.945 | 4.945 |
| 4.566 | 4.566 |

| $AJ_\delta$ | $\left[\left(AB_\delta\right)^8 \cdot AR^{8}\right]^{\frac{1}{16}}$ |
|---|---|
| 7.071 | 7.071 |
| 5.774 | 5.774 |
| 5 | 5 |
| 4.472 | 4.472 |
| 4.082 | 4.082 |

| $AI_\delta$ | $\left[\left(AB_\delta\right)^9 \cdot AR^{7}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.771 | 6.771 |
| 5.39 | 5.39 |
| 4.585 | 4.585 |
| 4.044 | 4.044 |
| 3.65 | 3.65 |

| $AH_\delta$ | $\left[\left(AB_\delta\right)^{10} \cdot AR^{6}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.484 | 6.484 |
| 5.033 | 5.033 |
| 4.204 | 4.204 |
| 3.657 | 3.657 |
| 3.263 | 3.263 |

| $AG_\delta$ | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ |
|---|---|
| 6.209 | 6.209 |
| 4.699 | 4.699 |
| 3.856 | 3.856 |
| 3.307 | 3.307 |
| 2.918 | 2.918 |

| $AF_\delta$ | $\left[\left(AB_\delta\right)^{12} \cdot AR^{4}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.946 | 5.946 |
| 4.387 | 4.387 |
| 3.536 | 3.536 |
| 2.991 | 2.991 |
| 2.608 | 2.608 |

| $AE_\delta$ | $\left[\left(AB_\delta\right)^{13} \cdot AR^{3}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.694 | 5.694 |
| 4.096 | 4.096 |
| 3.242 | 3.242 |
| 2.704 | 2.704 |
| 2.332 | 2.332 |

| $AD_\delta$ | $\left[\left(AB_\delta\right)^{14} \cdot AR^{2}\right]^{\frac{1}{16}}$ |
|---|---|
| 5.453 | 5.453 |
| 3.824 | 3.824 |
| 2.973 | 2.973 |
| 2.446 | 2.446 |
| 2.085 | 2.085 |

| $AC_\delta$ | $\left[\left(AB_\delta\right)^{15} \cdot AR\right]^{\frac{1}{16}}$ |
|---|---|
| 5.221 | 5.221 |
| 3.57 | 3.57 |
| 2.726 | 2.726 |
| 2.212 | 2.212 |
| 1.864 | 1.864 |

| $AB_\delta$ | $\left[\left(AB_\delta\right)^{16} \cdot AR^{0}\right]^{\frac{1}{16}}$ |
|---|---|
| 5 | 5 |
| 3.333 | 3.333 |
| 2.5 | 2.5 |
| 2 | 2 |
| 1.667 | 1.667 |

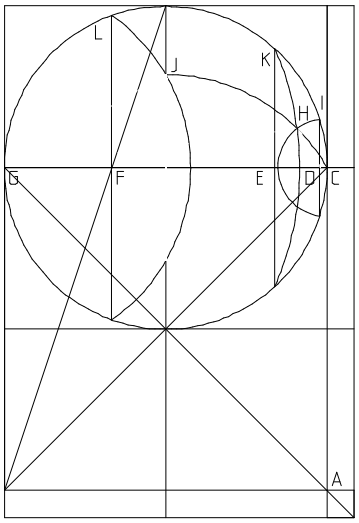
$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

Resultant Equation

depending on direction of transcription.



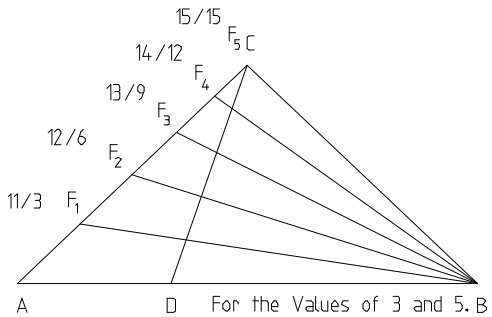

# One Circle

**By John Clark**

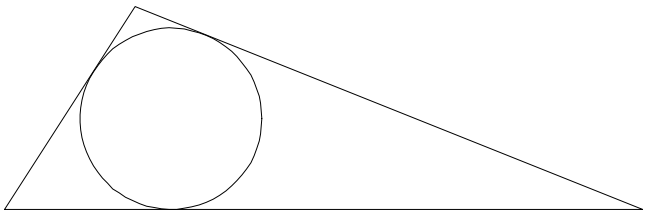
# One Square

# One Line

## And the Delian Quest

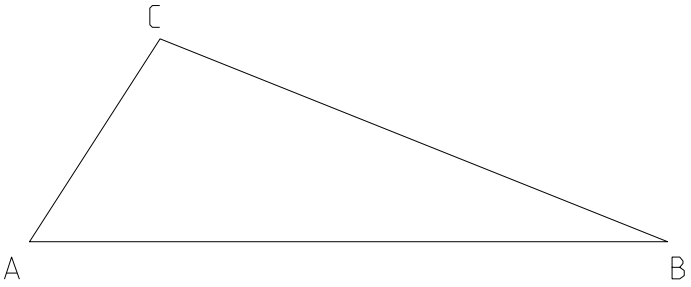



# 1994



# Inscribing a Circle in a given Triangle.

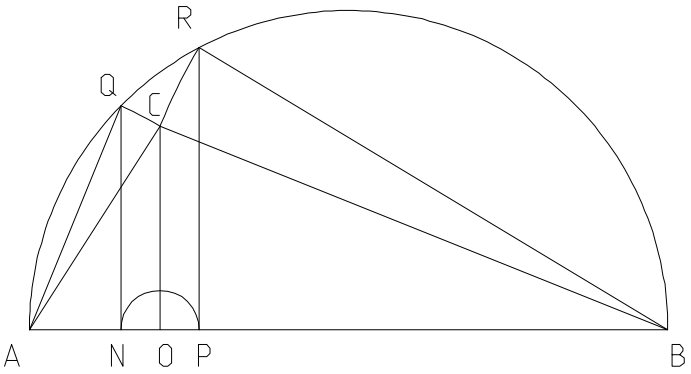
Place the length for the sides of the triangle at the end of the document.




$$AB := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad AC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix}$$

$$BC := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix} \quad \delta := 0..2$$

$$Is\_This\_A\_Triangle := \left(AB_0 + AC_0 \geq BC_0\right) \cdot \left(AB_0 + BC_0 \geq AC_0\right) \cdot \left(AC_0 + BC_0 \geq AB_0\right) \quad AQ_\delta := AC_\delta$$




$$BR_\delta := BC_\delta \quad AN_\delta := \frac{\left(AQ_\delta\right)^2}{AB_\delta}$$

$$BP_\delta := \frac{\left(BR_\delta\right)^2}{AB_\delta} \quad AP_\delta := AB_\delta - BP_\delta$$

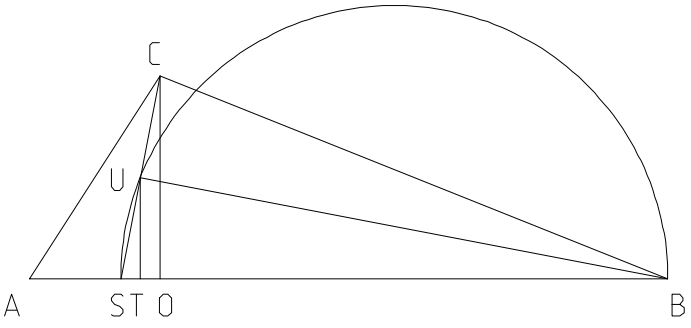
$$NP_\delta := AP_\delta - AN_\delta \quad NO_\delta := \frac{NP_\delta}{2}$$

$$AO := AN + NO \quad BO_\delta := AB_\delta - AO_\delta$$

$$CO_\delta := \sqrt{\left(BC_\delta\right)^2 - \left(BO_\delta\right)^2}$$

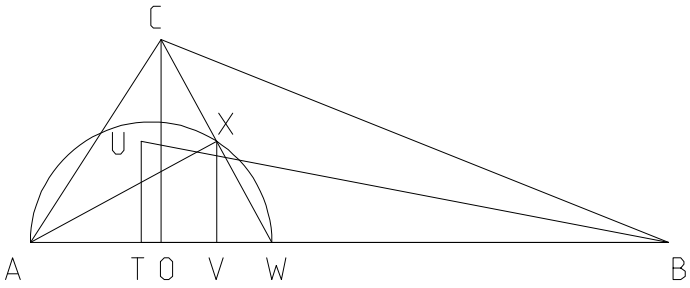
$$BS_\delta := BC_\delta \quad SO_\delta := BS_\delta - BO_\delta$$

$$CS_\delta := \sqrt{\left(SO_\delta\right)^2 + \left(CO_\delta\right)^2} \quad SU_\delta := \frac{CS_\delta}{2}$$




$$BU_\delta := \sqrt{\left(BS_\delta\right)^2 - \left(SU_\delta\right)^2}$$

$$ST_\delta := \frac{\left(SU_\delta\right)^2}{BS_\delta} \quad TU_\delta := \sqrt{\left(SU_\delta\right)^2 - \left(ST_\delta\right)^2}$$

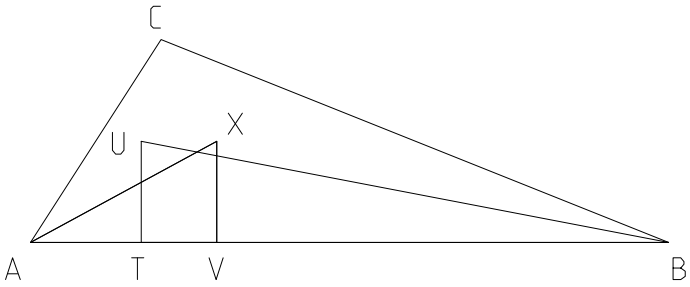



$$AW_\delta := AC_\delta \quad WO_\delta := AW_\delta - AO_\delta$$

$$CW_\delta := \sqrt{(WO_\delta)^2 + (CO_\delta)^2}$$

$$WX_\delta := \frac{CW_\delta}{2}$$

$$AX_\delta := \sqrt{(AW_\delta)^2 - (WX_\delta)^2}$$

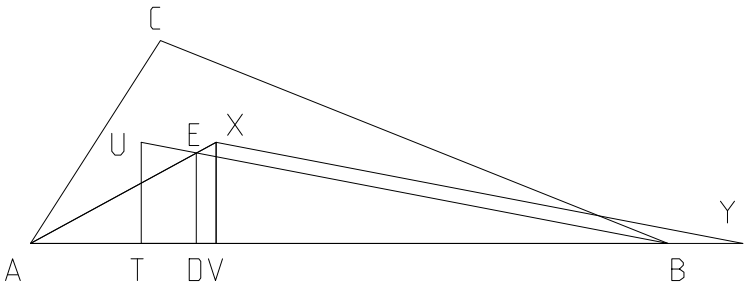



$$WV_\delta := \frac{(WX_\delta)^2}{AW_\delta}$$

$$VX_\delta := \sqrt{(WX_\delta)^2 - (WV_\delta)^2}$$

$$WV_\delta := \sqrt{(WX_\delta)^2 - (VX_\delta)^2}$$

$$AV_\delta := AW_\delta - WV_\delta \quad BV_\delta := AB_\delta - AV_\delta \quad XY_\delta := \frac{BU_\delta \cdot VX_\delta}{TU_\delta} \quad VY_\delta := \sqrt{(XY_\delta)^2 - (VX_\delta)^2}$$




$$AY_\delta := AV_\delta + VY_\delta$$

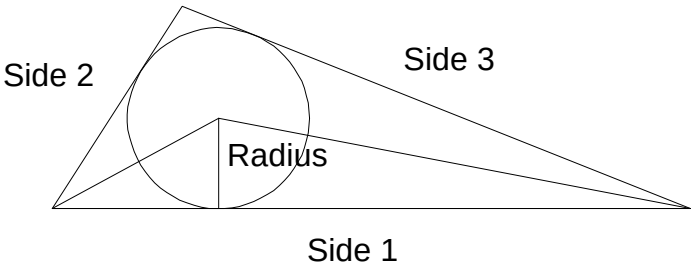
$$AD_\delta := \frac{AV_\delta \cdot AB_\delta}{AY_\delta} \quad AE_\delta := \frac{AX_\delta \cdot AD_\delta}{AV_\delta}$$

$$DE_\delta := \sqrt{(AE_\delta)^2 - (AD_\delta)^2}$$

Plug Side Values In Here  Side_1 ≡ 20  Side_2 ≡ 26  Side_3 ≡ 21

$AB^T = (\ 20 \quad 26 \quad 21\ )$  Is_This_A_Triangle = 1

Given 3 lengths, no matter what order they are entered, DE should remain a constant.

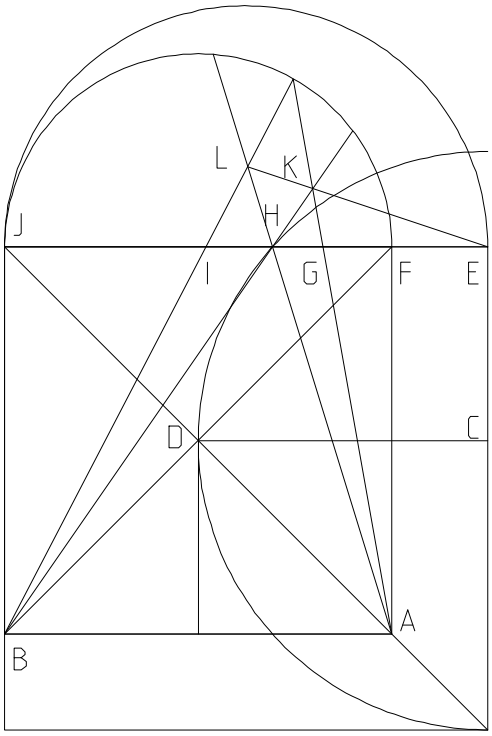



$S_1 := AB \quad S_2 := AC \quad S_3 := BC$

You will note that the formula derived from the process is more consistent with imaginaries.

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta - S2_\delta + S3_\delta}\cdot\sqrt{S1_\delta + S2_\delta - S3_\delta}}{2\cdot\sqrt{S1_\delta + S2_\delta + S3_\delta}}$$

$$Radius = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix} \qquad DE = \begin{pmatrix} 6.147 \\ 6.147 \\ 6.147 \end{pmatrix}$$




# The Cradle

Is EL and EK always collinear?

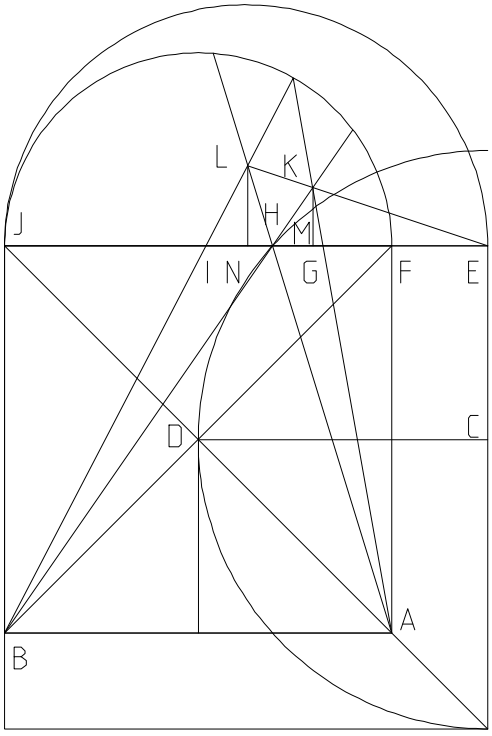
$l := 100 \qquad \delta := 1..l \qquad EF_\delta := \delta^5 \cdot 10^{-8}$

$FJ := 10 \qquad EJ_\delta := FJ + EF_\delta$

$EH_\delta := \sqrt{EF_\delta \cdot EJ_\delta} \qquad FH_\delta := EH_\delta - EF_\delta$

$EG_\delta := \left[ \left( EF_\delta \right)^2 \cdot EJ_\delta \right]^{\frac{1}{3}} \qquad FG_\delta := EG_\delta - EF_\delta$

$EI_\delta := \left[ \left( EJ_\delta \right)^2 \cdot EF_\delta \right]^{\frac{1}{3}} \qquad FI_\delta := EI_\delta - EF_\delta$



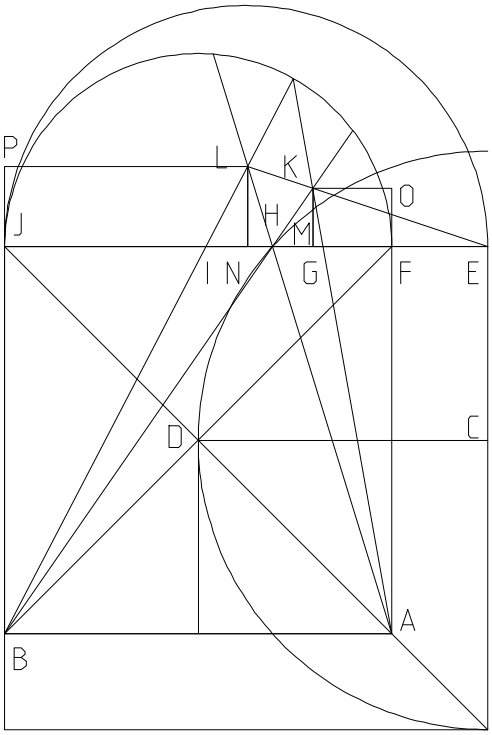

Basically it is demonstrated that the two triangles EKM and ELN are proportional, which is sufficient.

$IJ_\delta := FJ - FI_\delta$

$HJ_\delta := FJ - FH_\delta$

$FN_\delta := \frac{FH_\delta \cdot FJ}{\left( FH_\delta + IJ_\delta \right)} \qquad EN_\delta := FN_\delta + EF_\delta$

$FM_\delta := \frac{FG_\delta \cdot FJ}{\left( FG_\delta + HJ_\delta \right)} \qquad EM_\delta := FM_\delta + EF_\delta$




$$JN_\delta := FJ - FN_\delta \quad LP_\delta := JN_\delta$$

$$BJ := FJ \quad BP_\delta := \frac{BJ \cdot LP_\delta}{IJ_\delta}$$

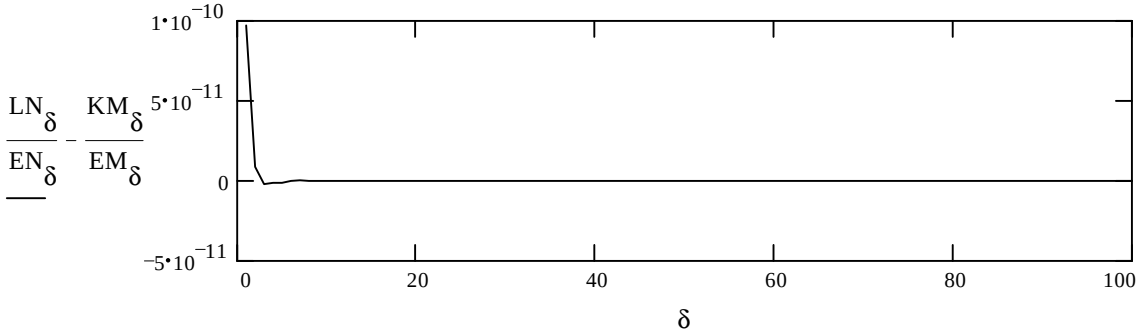
$$JP_\delta := BP_\delta - BJ \quad LN_\delta := JP_\delta$$

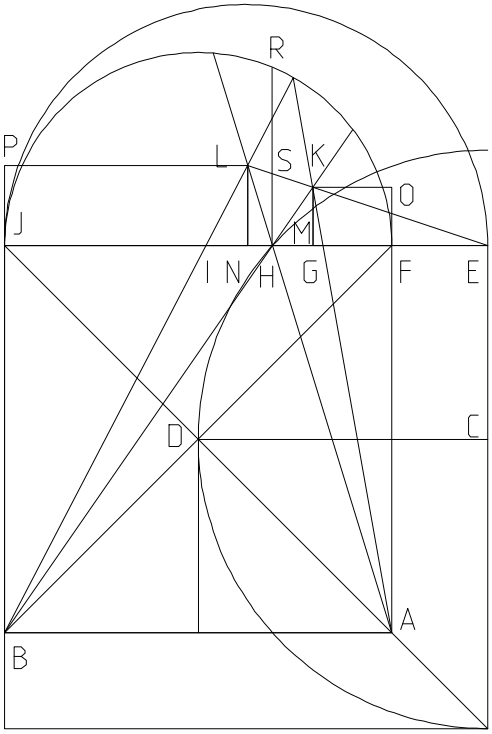
$$KO_\delta := FM_\delta \quad AF := FJ$$

$$AO_\delta := \frac{AF \cdot KO_\delta}{FG_\delta} \quad FO_\delta := AO_\delta - AF$$

$$KM_\delta := FO_\delta \qquad \sum_\delta \left( \frac{LN_\delta}{EN_\delta} - \frac{KM_\delta}{EM_\delta} \right) = 1.015 \cdot 10^{-10}$$

They are two lines with identical slopes, terminating at the same point.

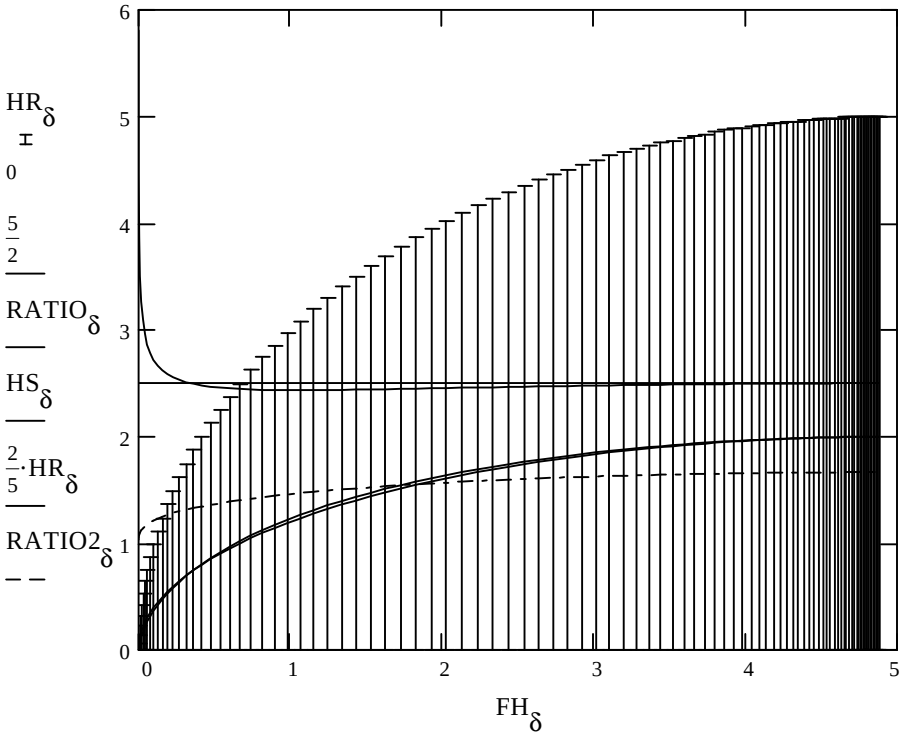





$$HR_\delta := \sqrt{FH_\delta \cdot HJ_\delta} \qquad HS_\delta := \frac{LN_\delta \cdot EH_\delta}{EN_\delta}$$

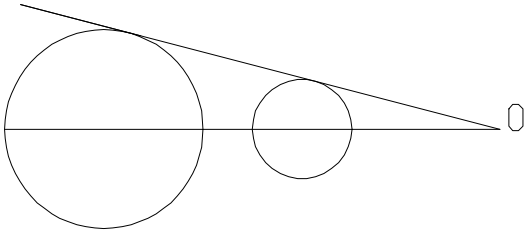
$$RATIO_\delta := \frac{HR_\delta}{HS_\delta} \qquad GI_\delta := EI_\delta - EG_\delta$$

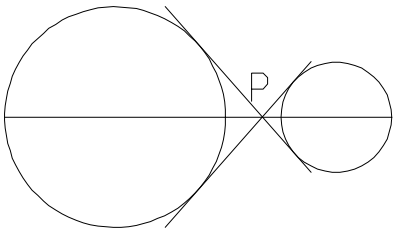
$$RATIO2_\delta := \frac{GI_\delta}{HS_\delta}$$

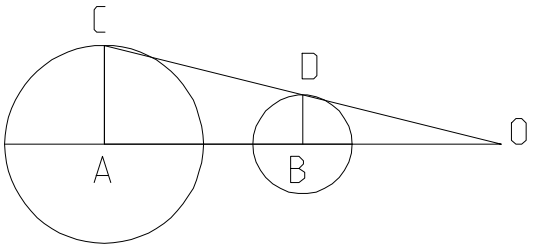


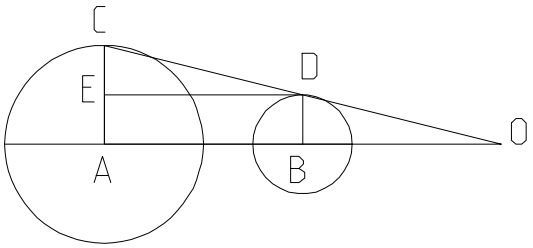
# Tangents and Similarity Points

04_26_94.MCD




O and P are points of origin for the ratio of the two circles that can also have a tangent ray to both circles. Develop formulas that would locate the particular points given using just the radius of the two circles and the difference between them.

O and P are called the similarity points (sp) of the two circles. O is the external similarity point and P is the internal similarity point.

I will work with point O first.

Given $R_L$ = large radius

$R_S$ = small radius

D = difference between origins.

$R_L := 4 \quad R_S := 1 \quad D := 8$

$AC := R_L \quad BD := R_S \quad AB := D$

If the difference between the circles is less than $R_L$-$R_S$, than one of course has an imaginary situation for the external similarity point, $R_L + R_S$ for the internal. At $R_L$-$R_S$ the smaller is in the larger and they touch at one point, $R_L + R_S$ they are external to one another and touching.

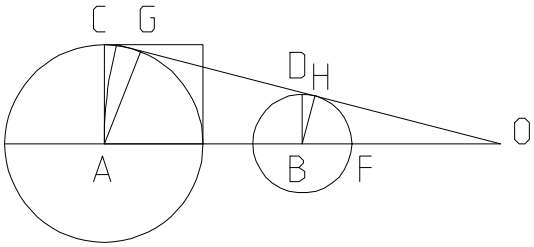
$DE := AB \quad AE := BD \quad CE := AC - AE$

$$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$$

$EOR_L$ "External similarity point Origin to center of Radius Large"

$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right)$$

$EOR_L = 10.667$




What is the length of line (OG) tangent to both circles?

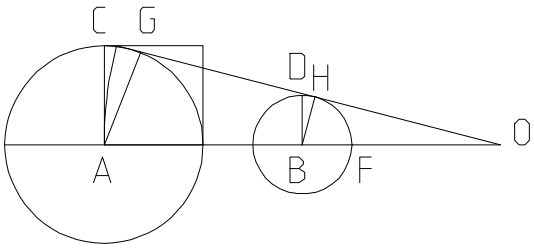
$AG := AC \quad GO := \sqrt{AO^2 - AG^2}$

$GO = 9.888$

And what is the formula?
$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"

$$EOT_{LR} := R_L \cdot \frac{\sqrt{(R_L - R_S + D)\cdot(-R_L + R_S + D)}}{R_L - R_S}$$

$EOT_{LR} = 9.888$




What is the length of the line tangent to the least circle (HO)?

$BH := BD \quad BO := AO - AB \qquad BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

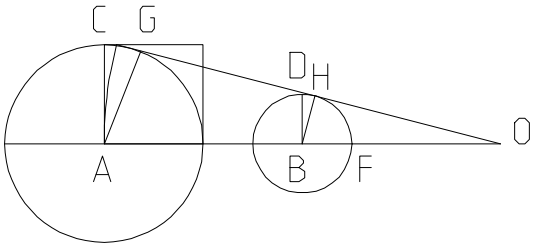
$HO = 2.472$

And what is the formula?

$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S}$$

$EOT_{SR} = 2.472$




Lastly what is the length of line from tangent to tangent of these circles?

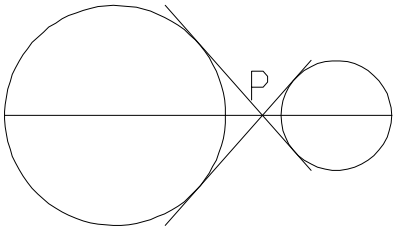
$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

And what is the formula?
ETT "Tangent to Tangent"

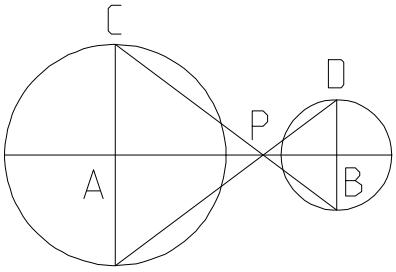
$$ETT := \sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}$$

$ETT = 7.416$




I will now turn my attention to the point P, the internal similarity point.

$$AP := \frac{AB\cdot AC}{AC + BD} \qquad AP = 6.4$$



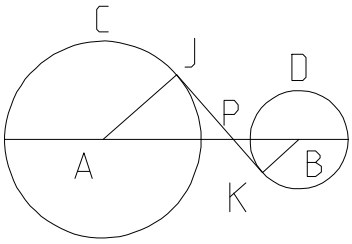

$IOR_L$ "Internal similarity point to center of Radius Large"

$$IOR_L := D\cdot\frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

$IOR_S$ "Internal similarity point to center of Radius Small"

$$IOR_S := D\cdot\frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$



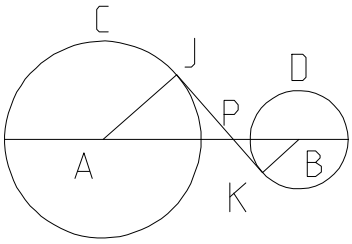

$AJ := AC \quad BK := BD \quad JP := \sqrt{AP^2 - AJ^2}$

$JP = 4.996$

$IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"

$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996$

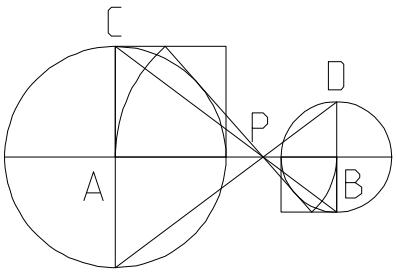



$KP := \sqrt{BP^2 - BK^2} \quad KP = 1.249$

$IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249$



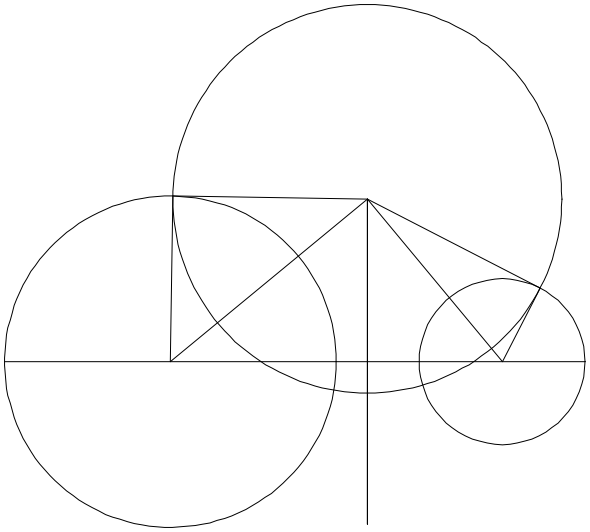

$JK := JP + KP \quad JK = 6.245$

ITT "Internal similarity point Tangent to Tangent"

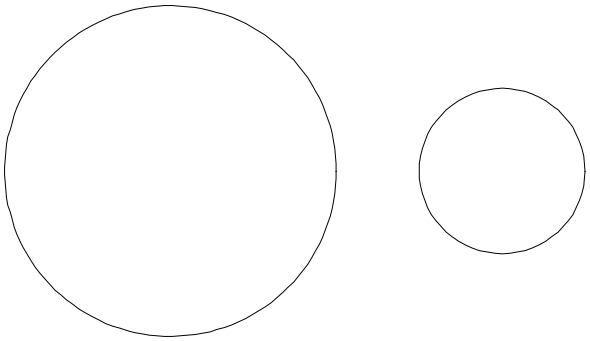
$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}$$

$ITT = 6.245$

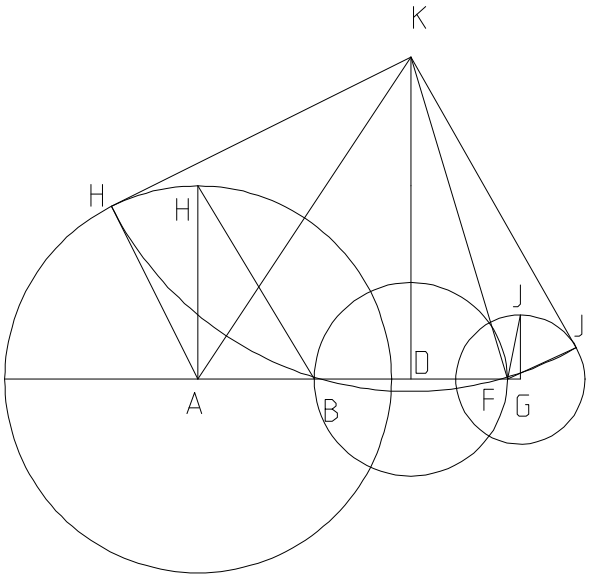
# The Chordal or Power Line of two Circles 04_27_94.MCD



The solution for finding the chordal as outlined in ***100 Great Problems of Elementary Mathematics Their History and Solution*** by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.
The figure I work with is a transformation of the one on the left.



Given two circles find their chordal or power line given just their radius and difference between their centers, and reduce the tautological chains to formulas.




$R_1 := 3 \quad R_2 := 1 \quad D := 1$

$AH := R_1 \quad GJ := R_2 \quad AG := D$

$AB := \frac{AH^2}{AG} \quad FG := \frac{GJ^2}{AG}$

$BF := AG - AB - FG \quad BD := \frac{BF}{2}$

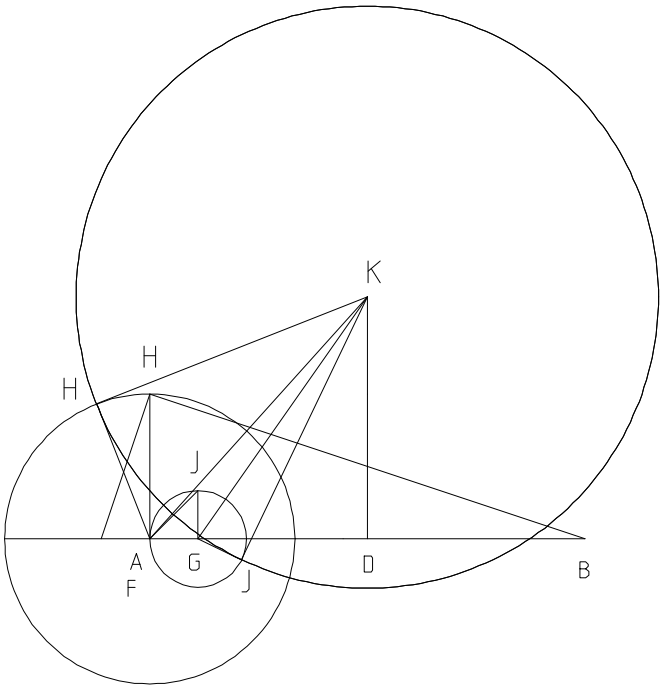
$AD := AB + BD \quad DF := BD$

$DG := DF + FG$

CR "Center to Radius"

$$CR_1 := \frac{1}{2}\cdot\frac{\left(R_1^2 - R_2^2 + D^2\right)}{D} \qquad CR_1 = 4.5 \qquad AD = 4.5$$

$$CR_2 := \frac{1}{2}\cdot\frac{\left(R_2^2 - R_1^2 + D^2\right)}{D} \qquad CR_2 = -3.5 \qquad DG = -3.5$$




**Locate length K here** $DK := 5$

$$AK := \sqrt{DK^2 + AD^2} \qquad GK := \sqrt{DK^2 + DG^2}$$

$$HK := \sqrt{AK^2 - AH^2} \qquad JK := \sqrt{GK^2 - GJ^2}$$

$HK = 6.021$

LP "Length of Power line"

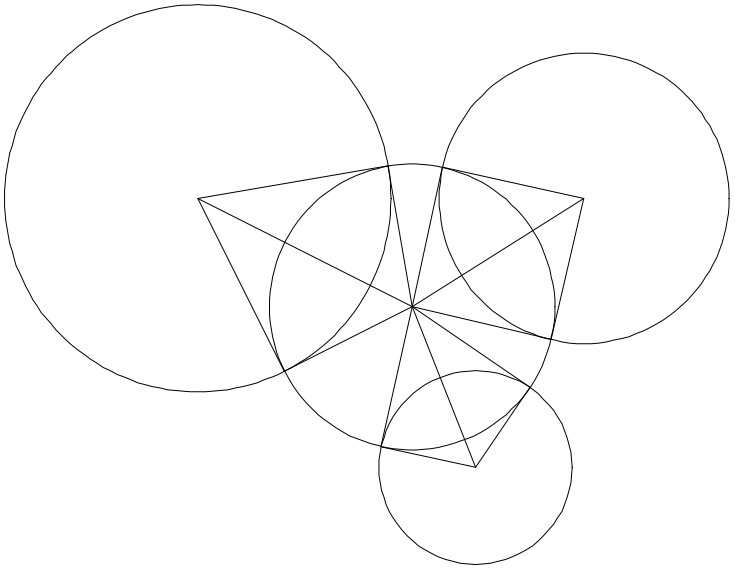
$LP := DK$

LT "Length of radial Tangent" LP is the variable chosen for circle center on the power line.

$$LT := \frac{1}{2}\cdot\left|\frac{\sqrt{4\cdot LP^2\cdot D^2 + R_1^4 - 2\cdot R_1^2\cdot D^2 - 2\cdot R_1^2\cdot R_2^2 + D^4 - 2\cdot D^2\cdot R_2^2 + R_2^4}}{D}\right|$$

$LT = 6.021$ $JK = 6.021$

The process does not seem to recognize any special cases.



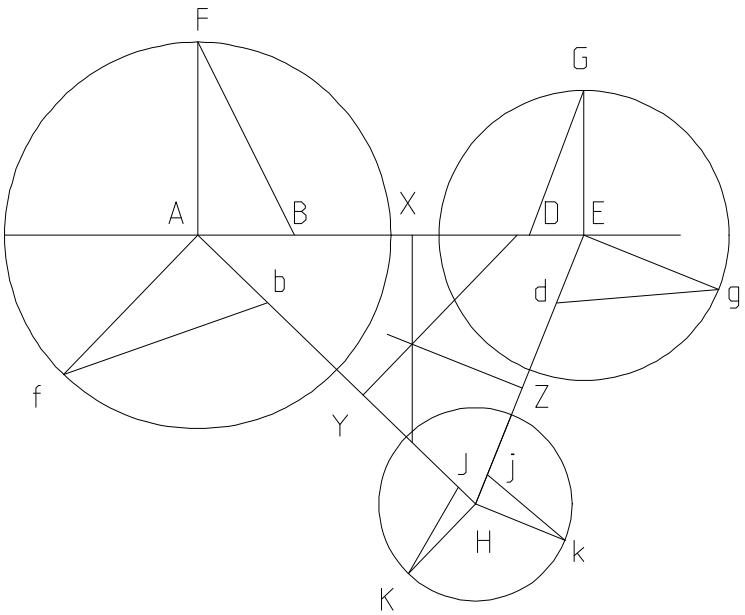
# Power Point

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the formula for the Power Point and the Length of the resultant Tangent.

The distance between each set of circles is given as $D_1$, $D_2$, and $D_3$. Naturally they must form a triangle.

$D_1 := 4 \quad D_2 := 5 \quad D_3 := 2$

$\Delta := \left(D_1 + D_2 \geq D_3\right) \cdot \left(D_2 + D_3 \geq D_1\right) \cdot \left(D_1 + D_3 \geq D_2\right)$




$\Delta = 1$ $\Delta$ "Is this a Triangle?

$R_1 := 3 \quad R_2 := 2 \quad R_3 := 4$

$AE := D_1 \quad AH := D_2 \quad EH := D_3$

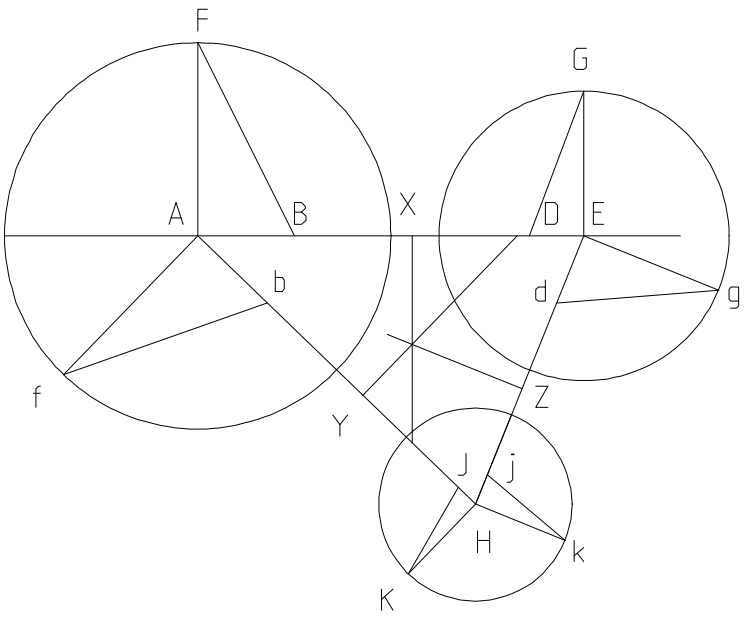
$AF := R_1 \quad HK := R_2 \quad EG := R_3$

$Af := AF \quad Hk := HK \quad Eg := EG$

$AB := \frac{AF^2}{AE} \quad DE := \frac{EG^2}{AE}$

$Ab := \frac{Af^2}{AH} \quad HJ := \frac{HK^2}{AH}$

$Hj := \frac{Hk^2}{EH} \quad Ed := \frac{Eg^2}{EH}$




$BD := AE - AB - DE \quad BX := \frac{BD}{2}$

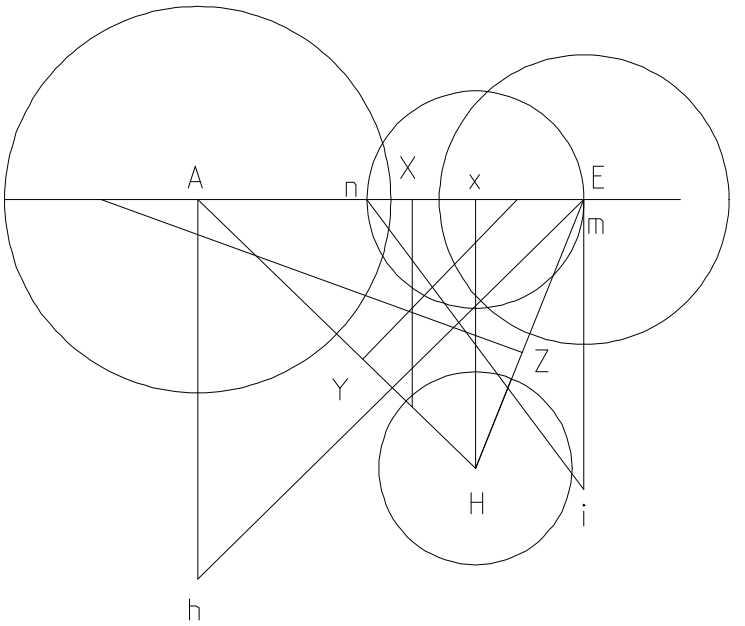
$bJ := AH - Ab - HJ \quad bY := \frac{bJ}{2}$

$dj := EH - Ed - Hj \quad dZ := \frac{dj}{2}$

$AX := AB + BX \quad AX = 1.125$

$AY := Ab + bY$

$EZ := Ed + dZ \quad EZ = 4$



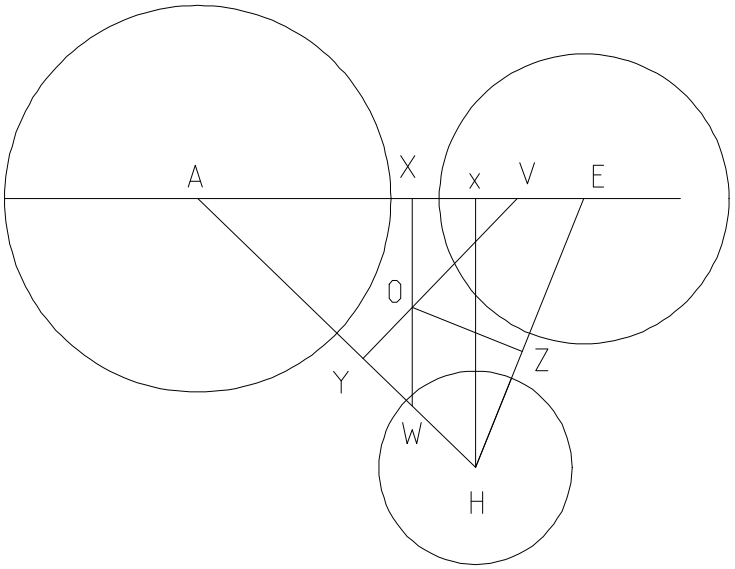

$Ah := AH \quad Ei := EH$

$Am := \frac{Ah^2}{AE} \quad En := \frac{Ei^2}{AE}$

$An := AE - En \quad mn := Am - An$

$nx := \frac{mn}{2} \quad Ax := An + nx$

$Hx := \sqrt{AH^2 - Ax^2}$




$$WX := \frac{Hx \cdot AX}{Ax} \qquad WX = 0.462$$

$$VY := \frac{Hx \cdot AY}{Ax} \qquad VY = 1.232$$

$$AV := \frac{AH \cdot AY}{Ax} \qquad AV = 3.243$$

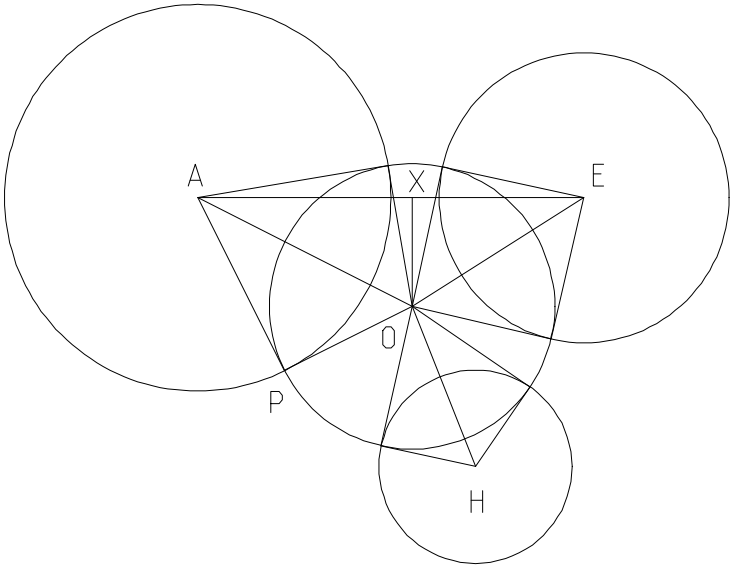
$$VX := AV - AX$$

$$OX := \frac{AY \cdot VX}{VY} \qquad OX = 5.157$$

PP "Power Point"

$$PP := \frac{1}{2} \cdot \frac{\left( \begin{array}{l} D_1^2 \cdot R_1^2 + D_1^2 \cdot D_2^2 - 2 \cdot D_1^2 \cdot R_2^2 + R_1^2 \cdot D_3^2 - R_1^2 \cdot D_2^2 - D_1^4 + D_1^2 \cdot D_3^2 \ldots \\ + R_3^2 \cdot D_1^2 - R_3^2 \cdot D_3^2 + R_3^2 \cdot D_2^2 \end{array} \right)}{\left( D_1 \cdot \sqrt{\left| -2 \cdot D_1^2 \cdot D_2^2 + D_1^4 - 2 \cdot D_1^2 \cdot D_3^2 + D_3^4 - 2 \cdot D_2^2 \cdot D_3^2 + D_2^4 \right|} \right)}$$

$$PP = 5.157$$




$EX := AE - AX$

$AO := \sqrt{AX^2 + OX^2}$ $\quad EO := \sqrt{EX^2 + OX^2}$

$AP := AF$ $\quad EP := EG$

$OP_A := \sqrt{AO^2 - AP^2}$ $\quad OP_E := \sqrt{EO^2 - EP^2}$

$OP_A = 4.342$ $\quad OP_E = 4.342$

LT "Length of Tangent"

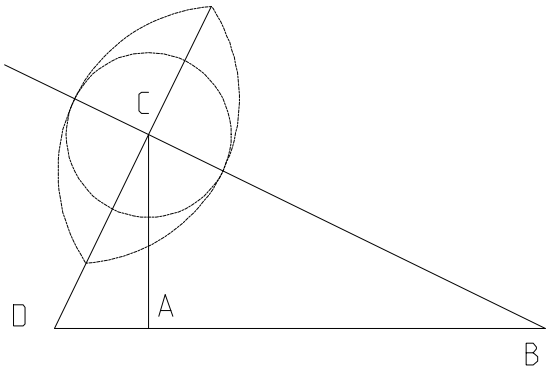
$$LT := \frac{\sqrt{\begin{aligned}&-R_1^4\cdot D_3^2 - R_1^2\cdot R_3^2\cdot D_1^2 - R_1^2\cdot D_3^4 + D_1^2\cdot R_1^2\cdot D_3^2 - D_1^2\cdot D_3^2\cdot D_2^2 \ldots\\&+ R_2^2\cdot R_1^2\cdot D_3^2 - R_2^2\cdot R_1^2\cdot D_2^2 - R_2^2\cdot R_3^2\cdot D_3^2 + R_2^2\cdot R_3^2\cdot D_2^2 \ldots\\&+ R_3^2\cdot D_1^2\cdot D_2^2 - R_3^2\cdot D_2^4 - R_3^4\cdot D_2^2 + D_1^2\cdot R_1^2\cdot R_2^2 + D_1^2\cdot D_2^2\cdot R_2^2 \ldots\\&+ D_1^2\cdot R_2^2\cdot D_3^2 + D_1^2\cdot R_2^2\cdot R_3^2 + R_1^2\cdot R_3^2\cdot D_3^2 + R_3^2\cdot D_3^2\cdot D_2^2 - D_1^2\cdot R_2^4 \ldots\\&+ -D_1^4\cdot R_2^2 + R_1^2\cdot D_3^2\cdot D_2^2 + R_1^2\cdot R_3^2\cdot D_2^2\end{aligned}}}{\sqrt{-2\cdot D_1^2\cdot D_2^2 + D_1^4 - 2\cdot D_1^2\cdot D_3^2 + D_3^4 - 2\cdot D_3^2\cdot D_2^2 + D_2^4}}$$

$LT = 4.342$

# Division, $A^2$

04_30_94.MCD

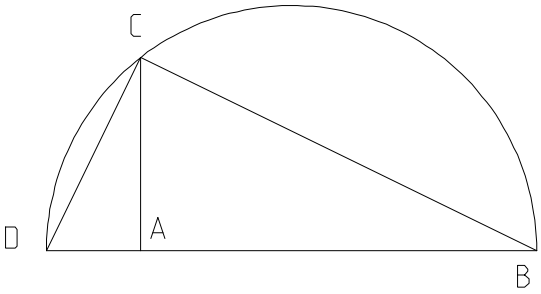
One does not work with geometry often, so it may be that one does not keep basics in mind when trying to work a figure. This little paper is about a basic move. I bring this to light, as I have seen a ratio often given as division. In geometry, so far as I know, one cannot divide a line by a line, but can form a series of the nature $A^N : B^N$ as a linear figure. Given A and B one can raise them to any whole power simultaneously with a couple of simple moves based on the figure immediately below. See work done in 1995.

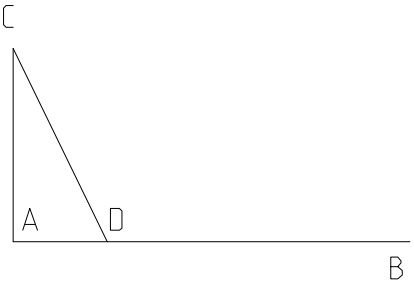



Divide $AC^2$ by AB.

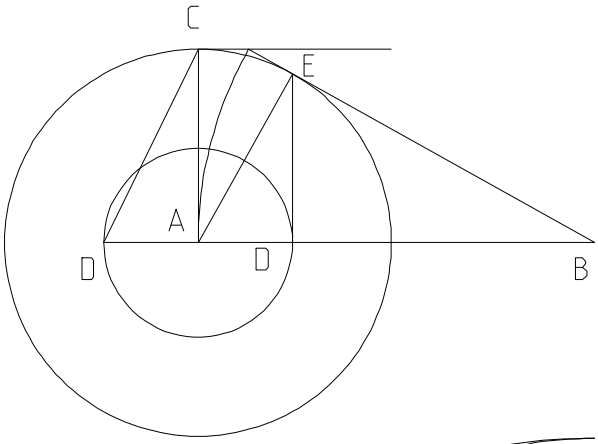
Process Summary

I have noticed that my solutions depend upon this basic move.

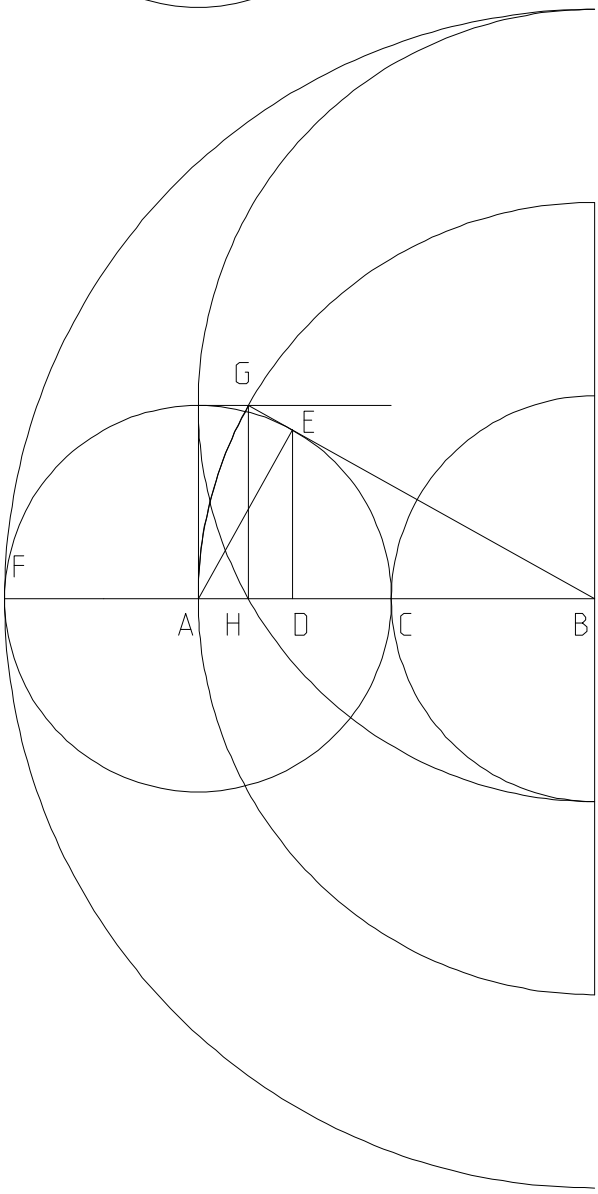



In some works I have represent it simply as the figure on the left. One may realize that in my Pythagorean Completion I used the circular form and it is often expressed as a pole and polar arrangement. A terminology that does not seem fit for the processes that they represent, physical and not mathematical.




The jargon is that B is called a Pole and D is on a polar, a segment of which is DE. But it can easily be seen that the figure is a transformation. It is another way of dividing $AC^2$ by AB. The figure now raises a question for me. I had thought that I answered it previously, but I cannot find it in my files. Is BE always collinear with BG? This paper is helping me mediate on poles and polars, which names do not help me understand the true ratio involved. Mathematicians seem to like a proliferation of names. And, god forbid, B and D are called conjugate in respect to each other. I have a hard enough time remembering my own name, that is why I keep my id. (in the Freudian sense) close at hand.




$AC := 5 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CF := 2 \cdot AC \quad AG := AC \quad BF_\delta := CF + BC_\delta$

$BH_\delta := \sqrt{BF_\delta \cdot BC_\delta} \quad GH := AC$

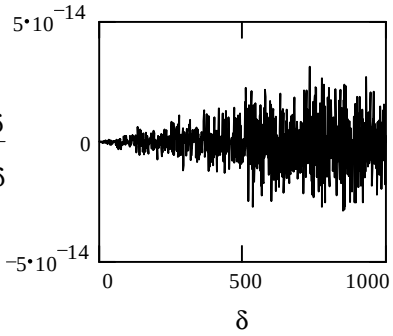
$BG_\delta := \sqrt{(BH_\delta)^2 + GH^2} \quad AE := AC$

$AB_\delta := AC + BC_\delta \quad AD_\delta := \frac{AE^2}{AB_\delta}$

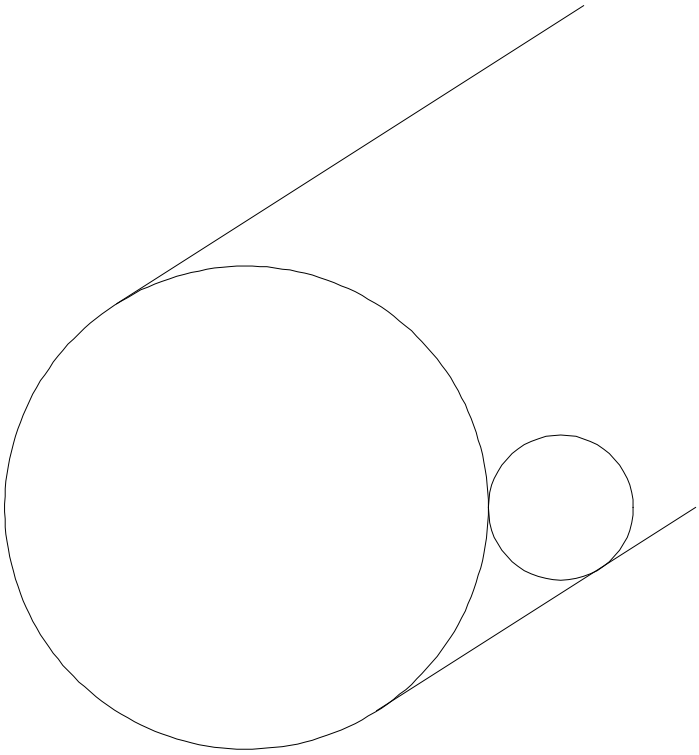
$BE_\delta := \sqrt{(AB_\delta)^2 - AE^2} \quad BD_\delta := AB_\delta - AD_\delta$

$DE_\delta := \sqrt{(BE_\delta)^2 - (BD_\delta)^2}$

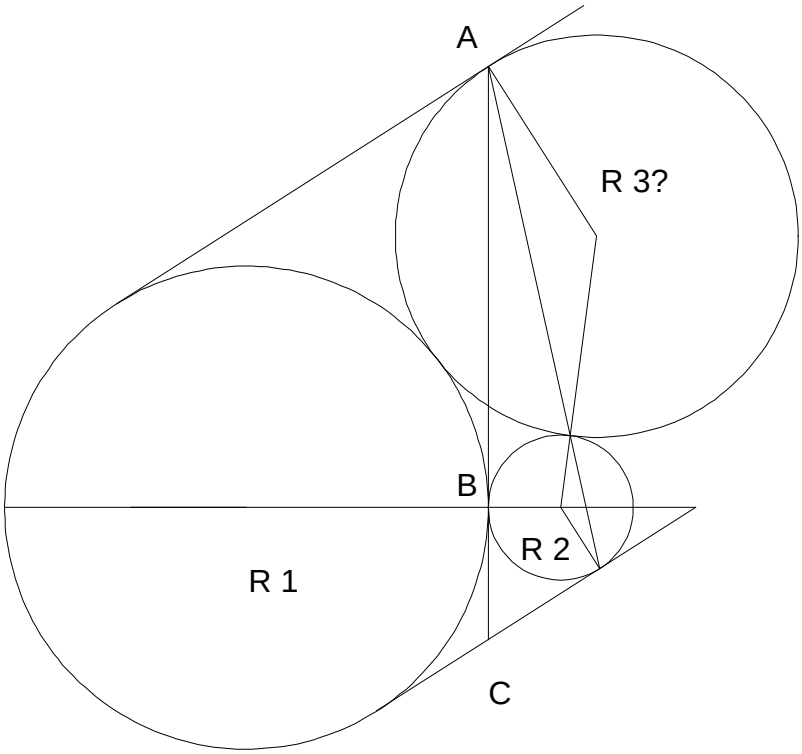
$\frac{GH}{BH_\delta} - \frac{DE_\delta}{BD_\delta}$



# Two Circles and a Parallel

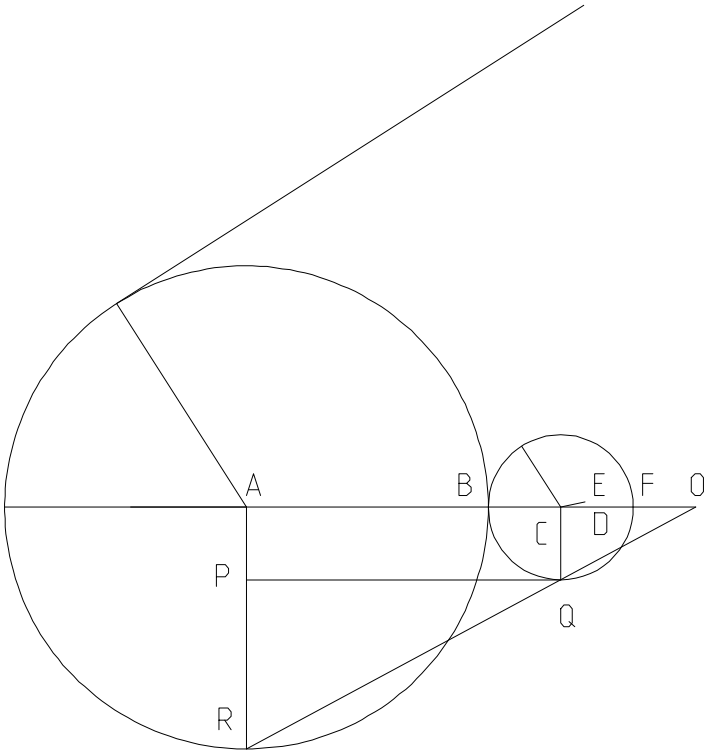


What I would like to do is to is drop in a circle that is tangent to the given two circles that are already tangent and tangent to the line from the similarity point and also have this circle tangent to the parallel of the similarity line that lies tangent to the first circle. The formula derived for my process tells me that I will do it the hard way. It will ***predict*** an easier method.




Process Summary

Given the radius of the two circles, what is the radius of the third? Attempt to develop a formula for the resultant radius. And also, of the power line between parallels, what is the ratio of AC:BC in terms of the given radius' ?




Find the Similarity Point.

$AB := R_1 \quad CF := R_2 \quad BC := CF$

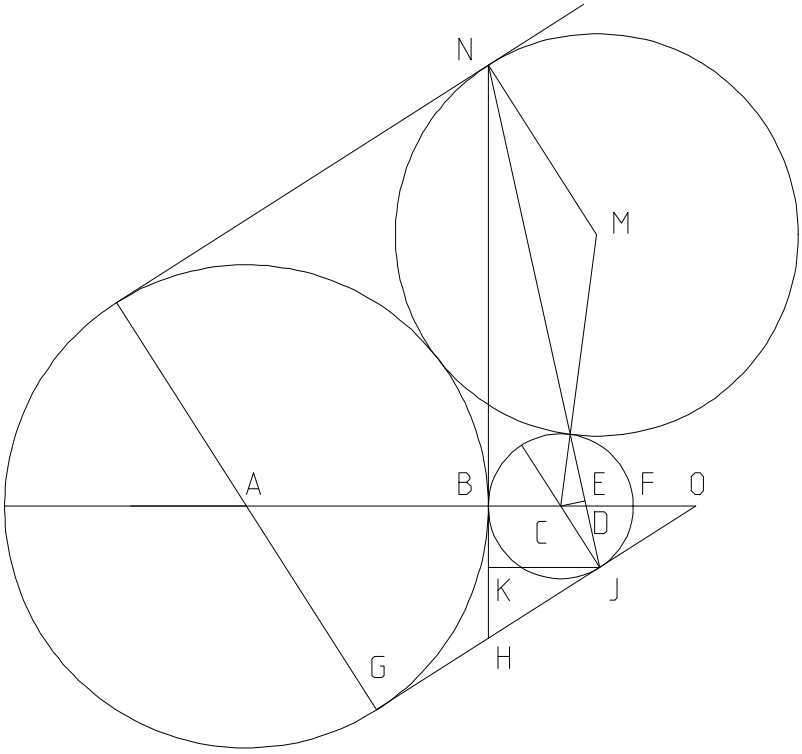
$CB := CF \quad AC := AB + BC$

$PQ := AC \quad AR := AB$

$CQ := CF \quad AP := CQ$

$PR := AR - AP$

$AO := \frac{PQ \cdot AR}{PR}$




Find the segment of the power line (HN) between parallels.

$AG := AB \quad GO := \sqrt{AO^2 - AG^2}$

$BO := AO - AB$

$BH := \frac{AG \cdot BO}{GO} \quad BN := \frac{BH \cdot AB}{BC}$

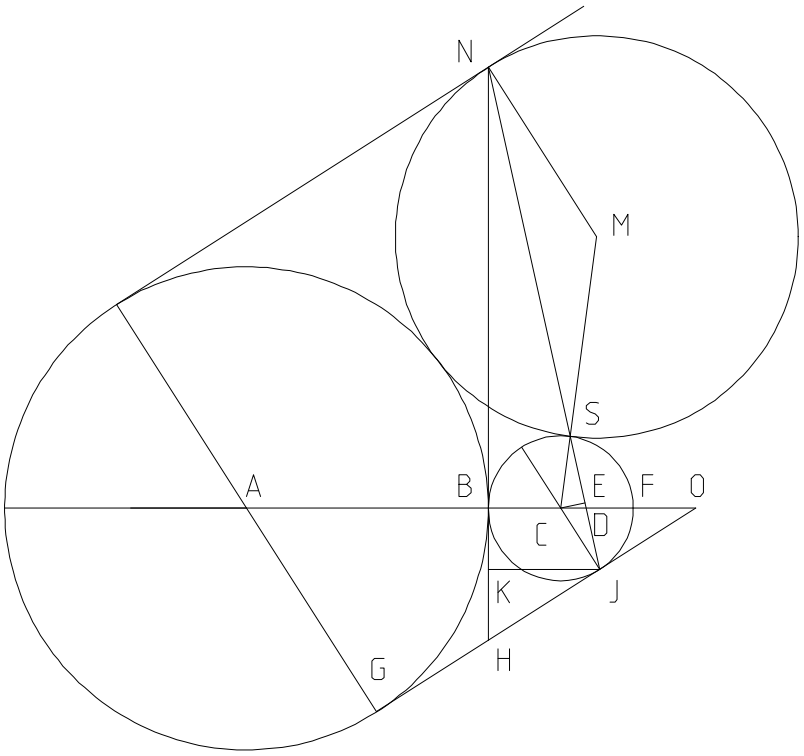
$HO := \frac{AO \cdot BH}{AG} \quad CO := BO - BC$

$JO := \frac{GO \cdot CO}{AO} \quad HN := BH + BN$

Find JN

$HJ := HO - JO \quad KH := \frac{AG \cdot HJ}{AO}$

$KN := HN - KH$




$KJ := \sqrt{HJ^2 - KH^2}$

$JN := \sqrt{KN^2 + \text{Find MS}}$

$BD := \frac{KJ \cdot BN}{KN}$ $CD := BD - BC$

$CE := \frac{KN \cdot CD}{JN}$ $CJ := BC$

$EJ := \sqrt{CJ^2 - CE^2}$ $CS := 2 \cdot EJ$

$NS := JN - JS$ $CS := BC$

$MS := \frac{CS \cdot NS}{JS}$

## Plug Values in Here

$R_1 \equiv 8$ $R_2 \equiv 6$

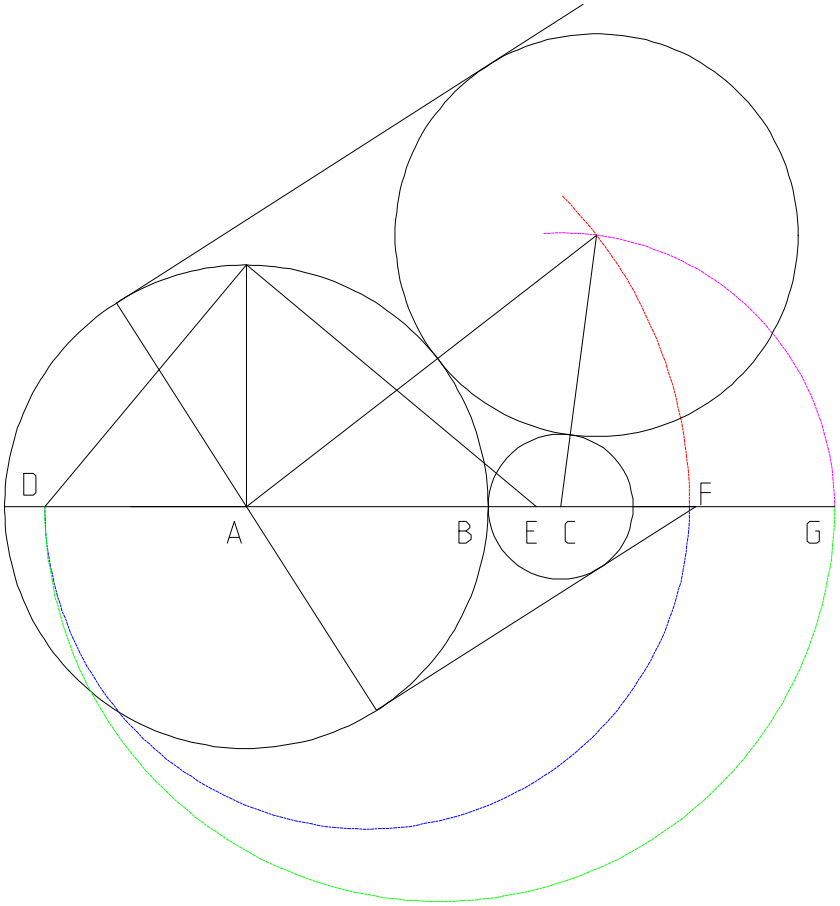
There was too much work here for the symbolic processor to reduce all the equations to one, easily. It took me three days to nurse the processor through it, and this is a short work. The formula does not have the "sour" spot. (Place both $R_1$ and $R_2$ to the same value to see it.) When $R_2$ is 1/4 of $R_1$, there are six tangents.

$$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad R_3 = 2.667 \qquad MS = 2.667$$

The formula tells me at least two things, 1) There is a second method to solve the problem, 2) the process is a rather baroque method of dividing a square.

$$\frac{HN}{BN} = 1.75 \qquad \frac{R_1 + R_2}{R_1} = 1.75 \qquad \frac{HN}{BH} = 2.333 \qquad \frac{R_1 + R_2}{R_2} = 2.333$$

The results for the ratios of the power line segment are very nice also.




What is the construction suggested by the found formula of

$$\frac{R_1^2}{4 \cdot R_2} \quad ?$$

$AB := R_1 \quad BC := R_2$

$AE := 4 \cdot BC$

$AD := \frac{AB^2}{AE}$

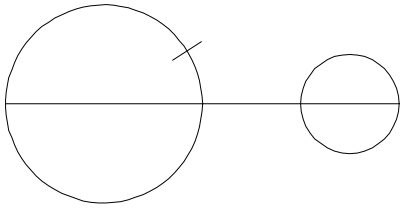
$AF := AB + AD$

$CG := BC + AD$

$R_3 = 2.667 \qquad AD = 2.667$

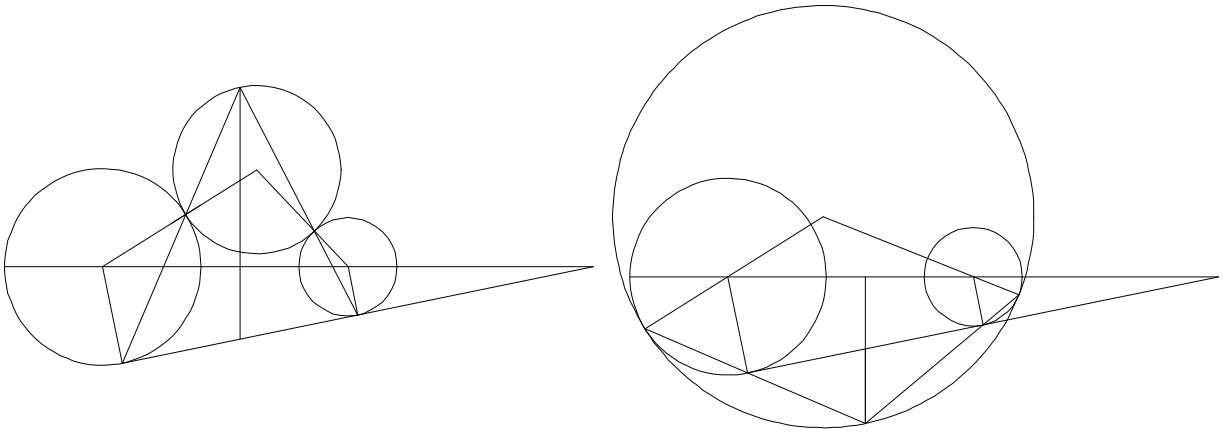
For those who may become confused as to the dashed lines, the segment AD is added to the radius of the both circles their intersection is the center of the circle sought.

# Two Circles, given a tangent on one.

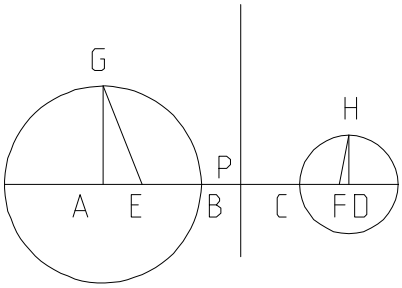


Given two circles and a point that is on the circumference of one, find a circle tangent to the circle at that point and also tangent to the other circle. The convention for this point will be "from the power line".

Process Summary



Find the power line.



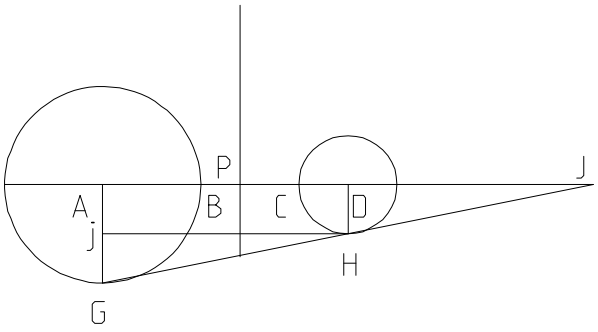

$AB := R_1 \quad CD := R_2 \quad AD := D$

$AG := AB \quad DH := CD \quad AE := \frac{AG^2}{AD}$

$DF := \frac{DH^2}{AD} \quad EF := AD - AE - DF$

$EP := \frac{EF}{2} \quad AP := AE + EP \quad DP := DF + EP$

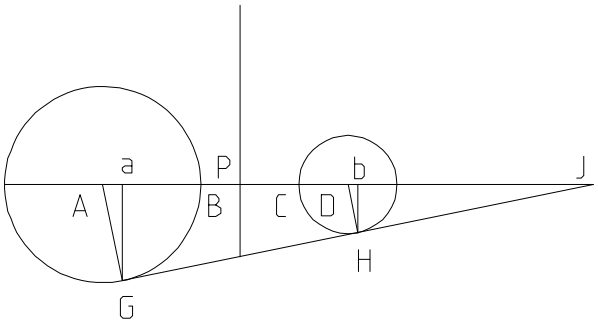
$BP := AP - AB$




$Hj := AD \quad Aj := DH$

$Gj := AG - DH \quad AJ := \frac{Hj \cdot AG}{Gj}$

$DJ := AJ - AD \quad HJ := \sqrt{DJ^2 - DH^2}$




$GJ := \sqrt{AJ^2 - AG^2} \quad Aa := \frac{AG^2}{AJ}$

$Ga := \sqrt{AG^2 - Aa^2} \quad Db := \frac{DH^2}{DJ}$

$Hb := \sqrt{DH^2 - Db^2}$

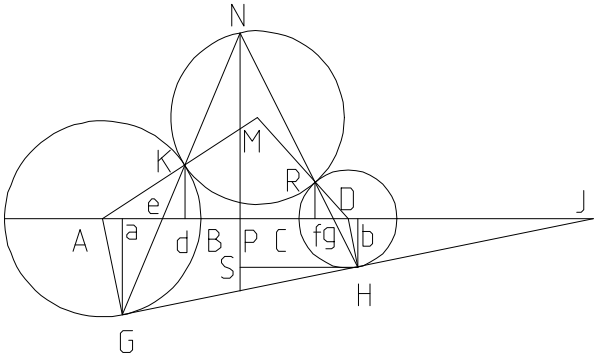
$Ba := AB - Aa$

$Pb := DP + Db$

$P := \left| \text{if}(P \le 2 \cdot AB, P, 0) \right|$

$Bd := P$

$AK := AB \quad ad := Ba - Bd$




$Ad := AB - Bd \quad Kd := \sqrt{AK^2 - Ad^2}$

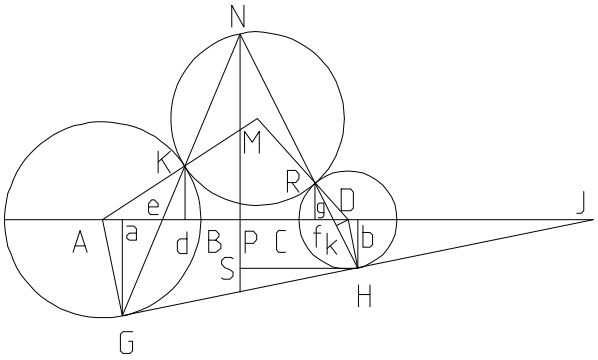
$de := \frac{ad \cdot Kd}{Kd + Ga} \quad Pe := BP + Bd + de$

$NP := \frac{Kd \cdot Pe}{de} \quad HS := Pb \quad PS := Hb$

$NS := NP + PS \quad Pg := \frac{HS \cdot NP}{NS}$

$Dg := DP - Pg \quad bg := Dg + Db$

$Hg := \sqrt{bg^2 + Hb^2}$




To save clutter, see 07_18_93.MCD Mod C.

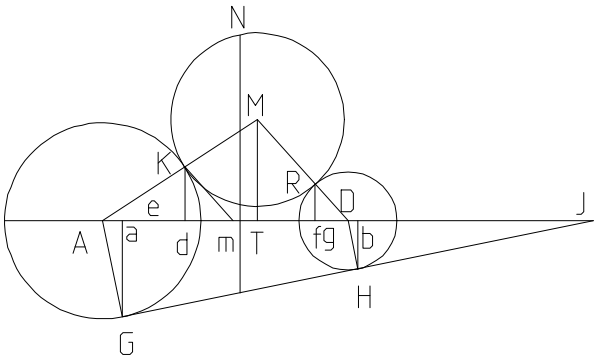
$$gk := \frac{1}{2}\cdot\frac{Dg^2}{Hg} + \frac{1}{2}\cdot Hg - \frac{1}{2}\cdot\frac{DH^2}{Hg}$$

$Dk := \sqrt{Dg^2 - gk^2} \quad Hk := \sqrt{DH^2 - Dk^2}$

$Rg := Hk - gk$

$$Df := \frac{1}{2}\cdot\frac{CD^2}{Dg} + \frac{1}{2}\cdot Dg - \frac{1}{2}\cdot\frac{Rg^2}{Dg}$$

$DR := CD \quad Rf := \sqrt{DR^2 - Df^2}$




$$dm := \frac{Df\cdot Kd}{Rf} \quad Am := Ad + dm$$

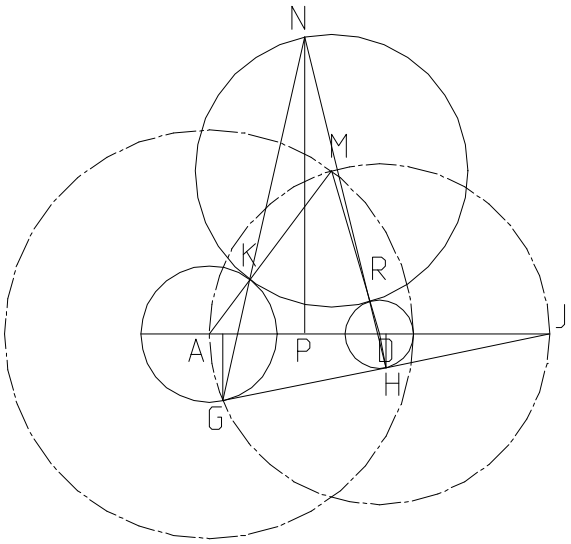
$$MT := \frac{Kd\cdot AD}{Am} \quad AM := \frac{AK\cdot MT}{Kd}$$

$KM := AM - AK$

$R_3 := |KM| \quad P = 19$

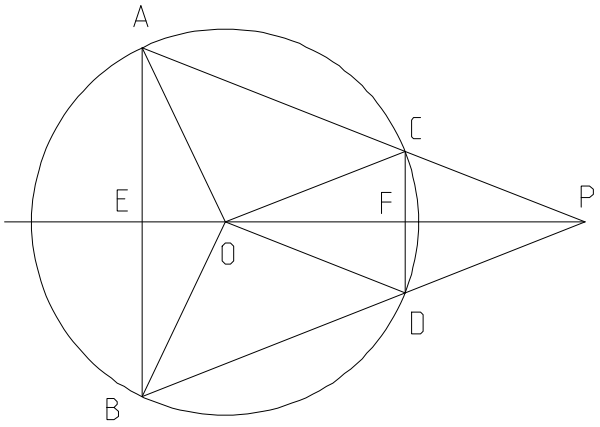
Plug Values in Here. Results is in $P_3$.

$R_1 \equiv 10 \; R_2 \equiv 3 \quad D \equiv 16 \quad P \equiv 19 \quad R_3 = 14.836$



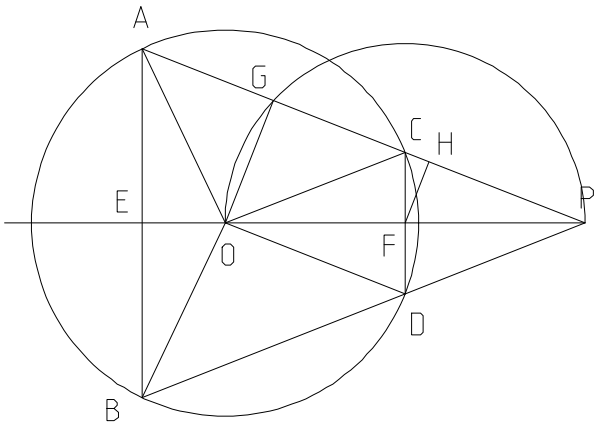

Let us say that instead of choosing a point of tangency upon one of the circles, I wish to place a given circle tangent to both. Derive the name of the length of the power line indicated and find th points of tangency.

Work in progress.




Given that CP = CO what is the relationship between AE and CF?

In order to derive a resonable answer, I have constrained the figure to a maximum of 180º. You will recall that the figure is capable of 270º.




$$CF_\delta := \frac{\frac{1}{2}\cdot\sqrt{2}\cdot CP}{\delta} \qquad FP_\delta := \sqrt{CP^2 - \left(CF_\delta\right)^2}$$

$$CH_\delta := \frac{\left(CF_\delta\right)^2}{CP} \qquad FH_\delta := \sqrt{\left(CF_\delta\right)^2 - \left(CH_\delta\right)^2}$$

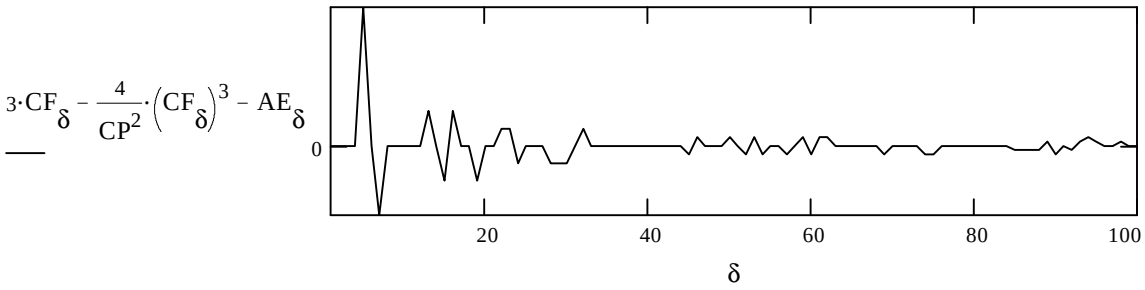
$$OP_\delta := 2\cdot FP_\delta \qquad GO_\delta := \frac{FH_\delta\cdot OP_\delta}{FP_\delta} \qquad CO := CP$$

$$CG_\delta := \sqrt{CO^2 - \left(GO_\delta\right)^2} \qquad AC_\delta := 2\cdot CG_\delta$$

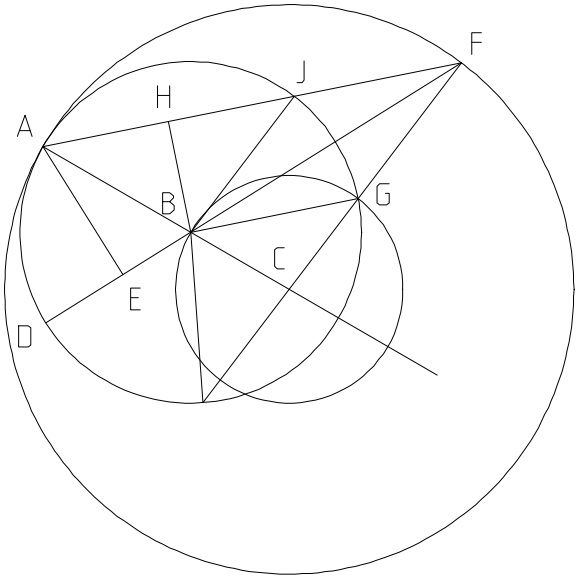
$$AP_\delta := AC_\delta + CP \qquad AE_\delta := \frac{CF_\delta\cdot AP_\delta}{CP}$$

CF is adjusted through D and cannot be less than 1 for the ratio to hold. This constrains the answer to between 0º and 180º.

$$CP \equiv 10 \qquad \delta \equiv 1 \,..\, 100 \qquad 3\cdot CF - \frac{4}{CP^2}\cdot CF^3 - AE = 0$$




The resultant equation seems to support my earlier statement that the same tool used on the cube root figure could also be used on the trisector. To eliminate the radius, simply set it equal to 1.




What is the relationship of DE to BC? $D := 4$

$AB := 99 \quad BC := \frac{AB}{2}\cdot D \quad BG := AB$

$AC := AB + BC \quad AF := \frac{BG\cdot AC}{BC} \quad FJ := AB$

$AJ := AF - FJ \quad HJ := \frac{AJ}{2} \quad BJ := AB$

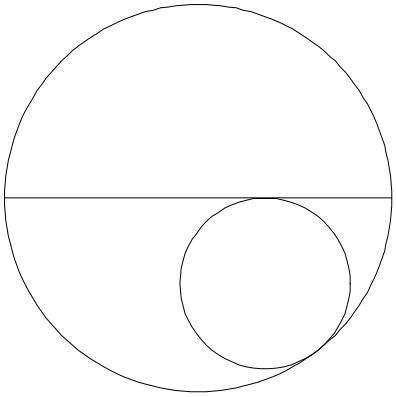
$BH := \sqrt{BJ^2 - HJ^2} \quad FH := HJ + FJ$

$BF := \sqrt{BH^2 + FH^2} \quad BD := AB \quad DF := BF + BD$

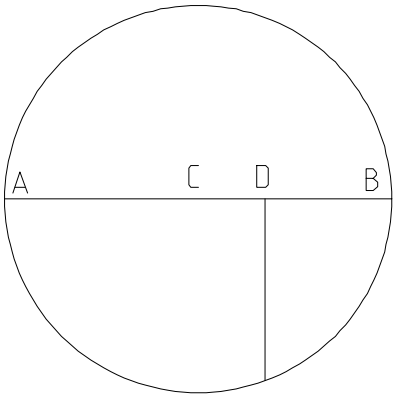
$EF := \frac{FH\cdot AF}{BF} \quad DE := DF - EF \quad DE = 138.133$

$$\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}} + AB - \left[\frac{1}{2}\cdot AB\cdot\frac{(AB + BC)}{BC} + \frac{1}{2}\cdot AB\right]\cdot AB\cdot\frac{(AB + BC)}{\left[BC\cdot\sqrt{AB^2 + AB^2\cdot\frac{(AB + BC)}{BC}}\right]} = 138.133$$

$$\frac{-1}{2}\cdot AB\cdot\frac{\left[-\sqrt{2\cdot BC + AB}\cdot BC - 2\cdot BC^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{2\cdot BC + AB}\right]}{BC^{\left(\frac{3}{2}\right)}} = 138.133$$



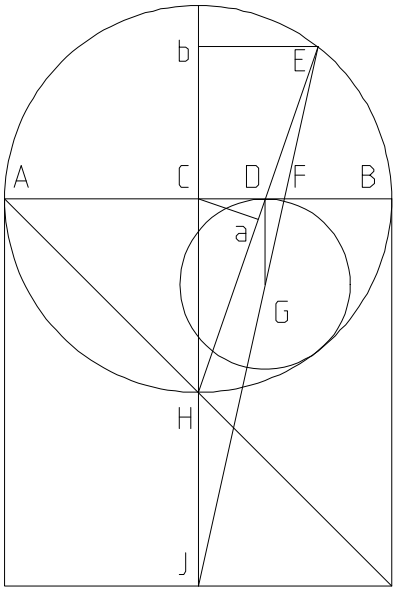
The smaller circle is tangent to the diameter of the larger and also tangent to circumference of the larger. Given the point of tangency on the diameter, what is the radius that will make it tangent to the circumference?




Given a point on the diameter, what is the radius of the inner tangent circle?

$$AB := 100 \quad AC := \frac{AB}{2} \quad CB := AC$$

$$\Delta := 100 \quad \delta := 1 .. \Delta \quad CD_\delta := \frac{CB}{\Delta} \cdot \delta$$




$$CH := AC \quad DH_\delta := \sqrt{\left(CD_\delta\right)^2 + CH^2}$$

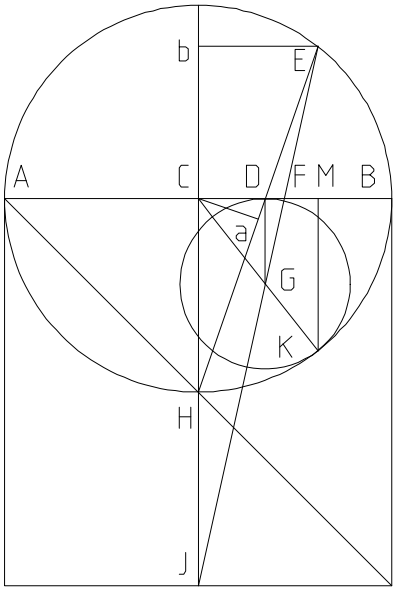
$$Ha_\delta := \frac{CH^2}{DH_\delta} \quad EH_\delta := 2 \cdot Ha_\delta \quad HJ := AC$$

$$Eb_\delta := \frac{CD_\delta \cdot EH_\delta}{DH_\delta} \quad Hb_\delta := \sqrt{\left(EH_\delta\right)^2 - \left(Eb_\delta\right)^2}$$

$$Cb_\delta := Hb_\delta - CH \quad CJ := CH + HJ$$

$$Jb_\delta := CJ + Cb_\delta \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

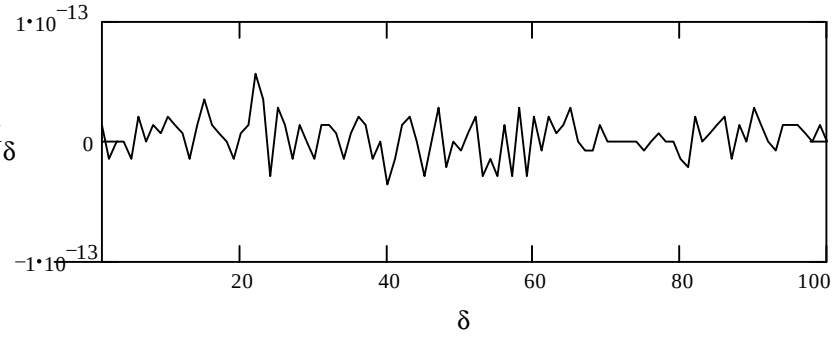


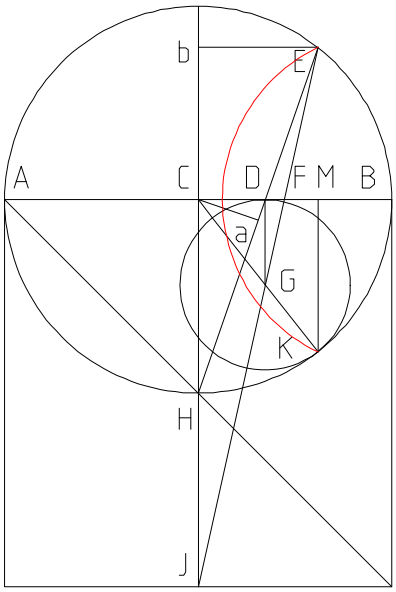

Does CG + GK = AC ?

$CG_\delta := \sqrt{(CD_\delta)^2 + (DG_\delta)^2}$ $GK_\delta := DG_\delta$

$CK_\delta := CG_\delta + GK_\delta$ $\sum_\delta (AC - CK_\delta) = 3.908 \cdot 10^{-13}$

$AC - CK_\delta$

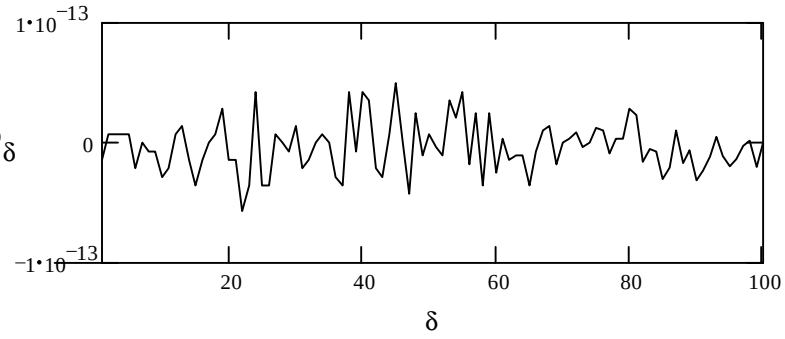


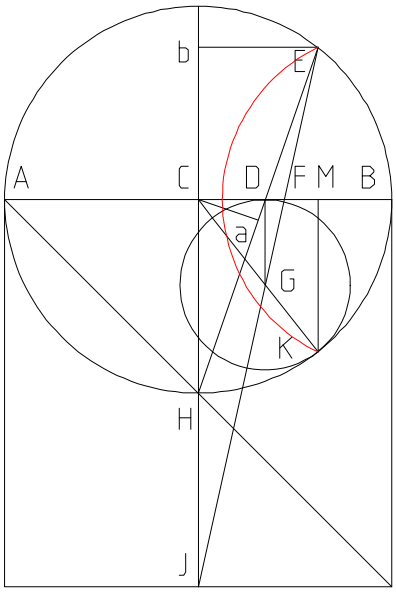




Does Cb = KM?

$KM_\delta := \frac{DG_\delta \cdot CK_\delta}{CG_\delta}$ $\sum_\delta (KM_\delta - Cb_\delta) = -3.524 \cdot 10^{-13}$

$KM_\delta - Cb_\delta$

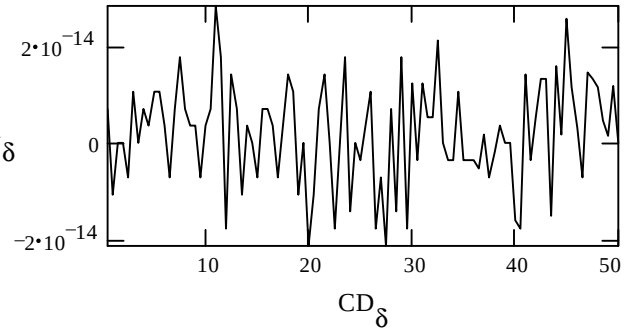


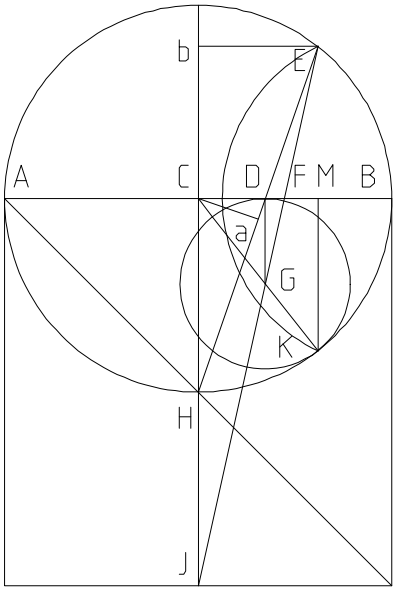



What is the formula for DG when given CD?

$$\sum_{\delta}\left[\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}\right] = 2.072\cdot 10^{-13}$$

$$\frac{AB^2 - 4\cdot\left(CD_{\delta}\right)^2}{4\cdot AB} - DG_{\delta}$$




Given the point on the radius, find DG.

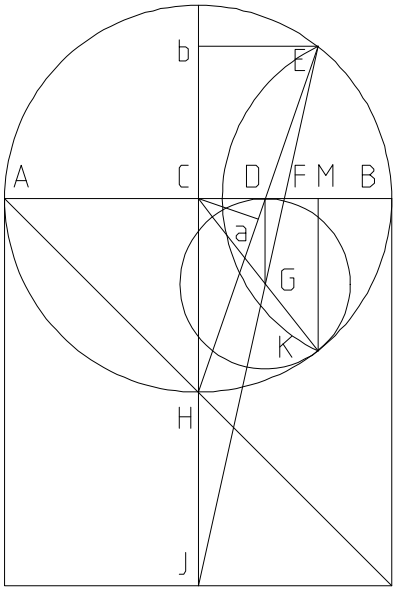
$AB := 4 \quad AC := \frac{AB}{2} \quad CB := AC \quad CM_{\delta} := \frac{CB}{\Delta}\cdot\delta$

$AM_{\delta} := AC + CM_{\delta} \quad BM_{\delta} := CB - CM_{\delta}$

$KM_{\delta} := \sqrt{AM_{\delta}\cdot BM_{\delta}} \quad Cb_{\delta} := KM_{\delta}$

$CH := AC \quad HJ := AC \quad CJ := CH + HJ$

$Jb_{\delta} := CJ + Cb_{\delta} \quad Hb_{\delta} := CH + Cb_{\delta}$



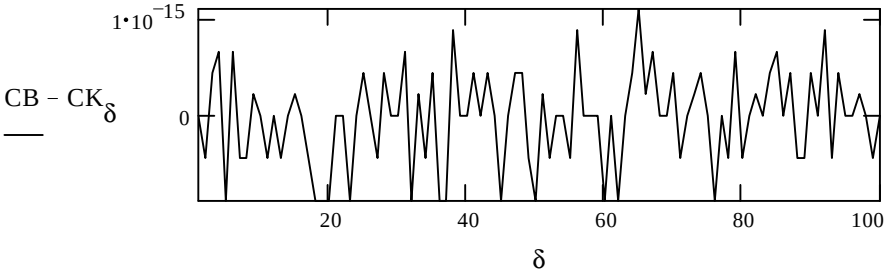

$$Eb_\delta := CM_\delta \quad CD_\delta := \frac{Eb_\delta \cdot CH}{Hb_\delta} \quad CF_\delta := \frac{Eb_\delta \cdot CJ}{Jb_\delta}$$

$$DF_\delta := CF_\delta - CD_\delta \quad DG_\delta := \frac{CJ \cdot DF_\delta}{CF_\delta}$$

Does CK = CB? Make sure there is no typo.

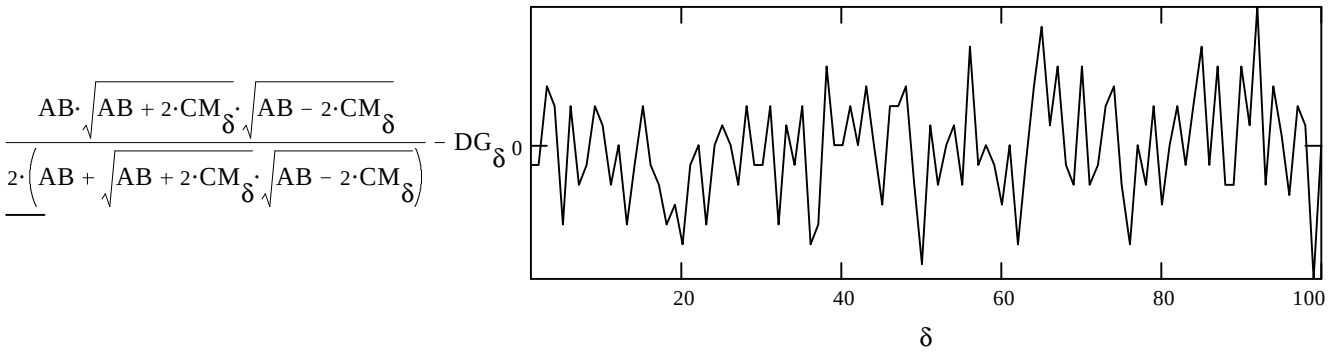
$$CG_\delta := \sqrt{\left(CD_\delta\right)^2 + \left(DG_\delta\right)^2} \quad GK_\delta := DG_\delta$$

$$CK_\delta := CG_\delta + GK_\delta \quad \sum_\delta \left(CB - CK_\delta\right) = -3.331 \cdot 10^{-15}$$

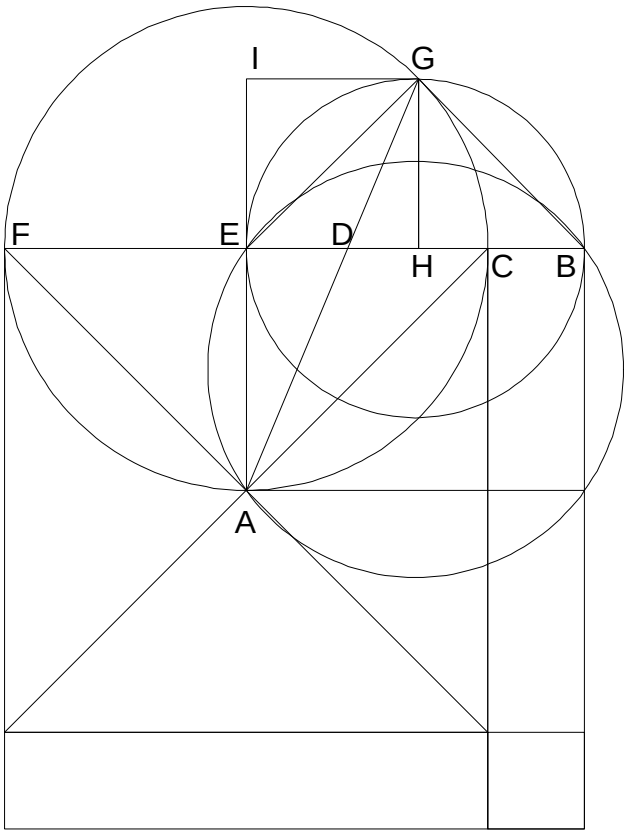



What is the formula for DG, given CM, the perpendicular to the point on the circumference?

$$\sum_\delta \left[ \frac{AB \cdot \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta}}{2 \cdot \left( AB + \sqrt{AB + 2 \cdot CM_\delta} \cdot \sqrt{AB - 2 \cdot CM_\delta} \right)} - DG_\delta \right] = -1.749 \cdot 10^{-15}$$



Trivial method for doing Square Roots.




If I draw a circle BE, then a line AG, BD is the square root of $BC \times BF$.

$CF := 1000 \quad \delta := 1 .. 1000 \quad BC_\delta := \delta$

$CE := \frac{CF}{2} \quad BE_\delta := BC_\delta + CE$

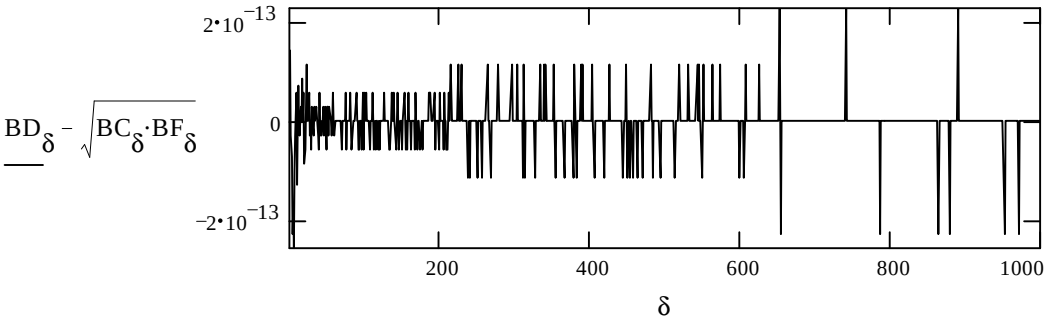
$EG := CE \quad BG_\delta := \sqrt{(BE_\delta)^2 - EG^2}$

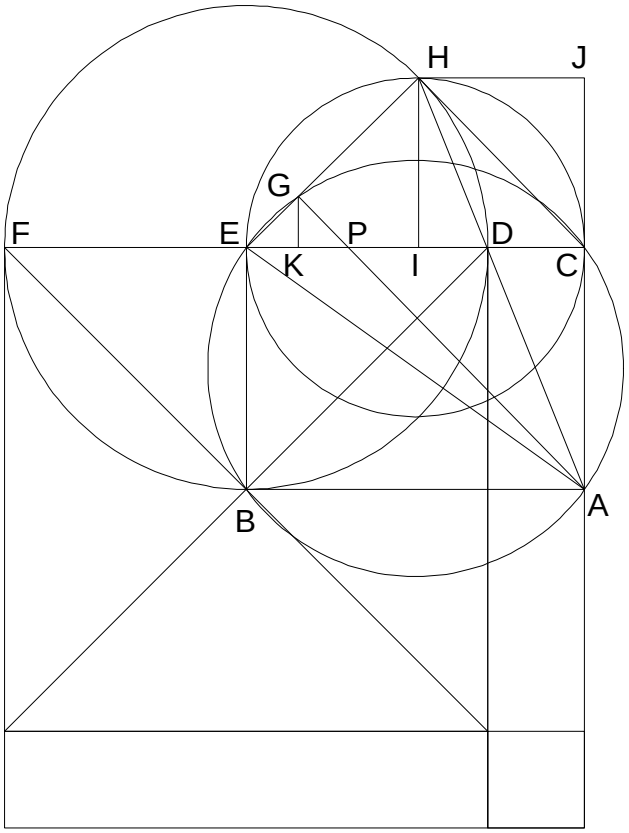
$EH_\delta := \frac{EG^2}{BE_\delta} \quad GH_\delta := \sqrt{EG^2 - (EH_\delta)^2}$

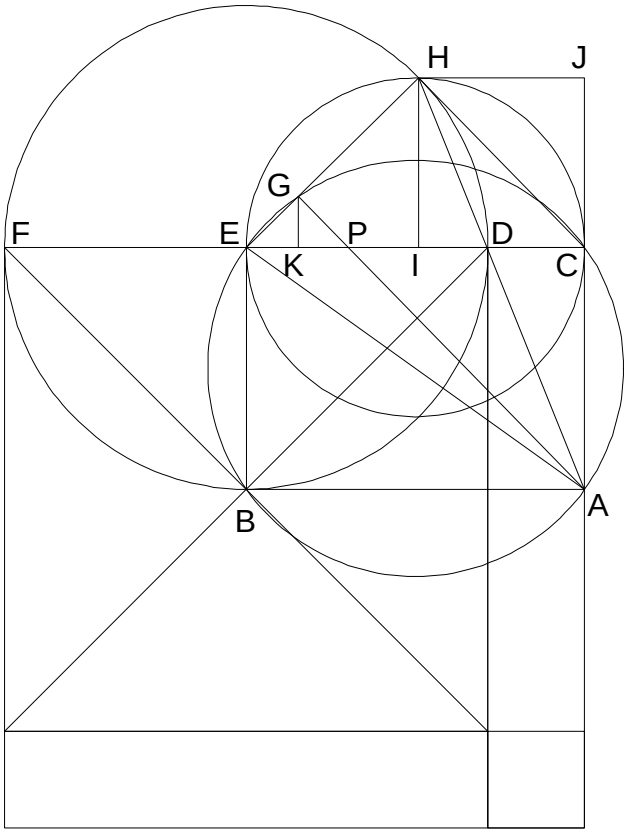
$GI_\delta := EH_\delta \quad EI_\delta := GH_\delta \quad AE := CE$

$AI_\delta := AE + EI_\delta \quad DE_\delta := \frac{GI_\delta \cdot AE}{AI_\delta}$

$BD_\delta := BE_\delta - DE_\delta \quad BF_\delta := BC_\delta + CF$






10_28_94.MCD

Trivial method for doing Square Root.

If I drop in a circle CHE, another ABE, and a right triangle, CHE, then AG provides CP which is the square root of $CD \times CF$.

$DF := 10 \quad \delta := 1 .. 100 \quad CD_\delta := \delta$

$DE := \frac{DF}{2} \quad AC := DE \quad CE_\delta := DE + CD_\delta$

$EH := DE \quad AE_\delta := \sqrt{AC^2 + (CE_\delta)^2}$

$CH_\delta := \sqrt{(CE_\delta)^2 - EH^2} \quad EI_\delta := \frac{EH^2}{CE_\delta}$

$HI_\delta := \sqrt{EH^2 - (EI_\delta)^2} \quad CJ_\delta := HI_\delta$

$CI_\delta := CE_\delta - EI_\delta \quad HJ_\delta := CI_\delta$

$AJ_\delta := AC + CJ_\delta \quad AH_\delta := \sqrt{(AJ_\delta)^2 + (HJ_\delta)^2}$

$HG_\delta := \frac{1}{2} \cdot \frac{(AH_\delta)^2}{EH} + \frac{1}{2} \cdot EH - \frac{1}{2} \cdot \frac{(AE_\delta)^2}{EH}$

$EG_\delta := EH - HG_\delta$

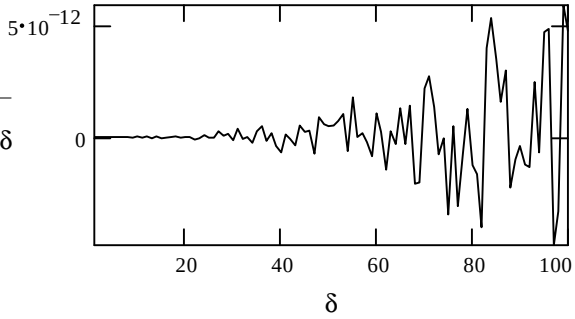
$AG_\delta := \sqrt{(AE_\delta)^2 - (EG_\delta)^2}$

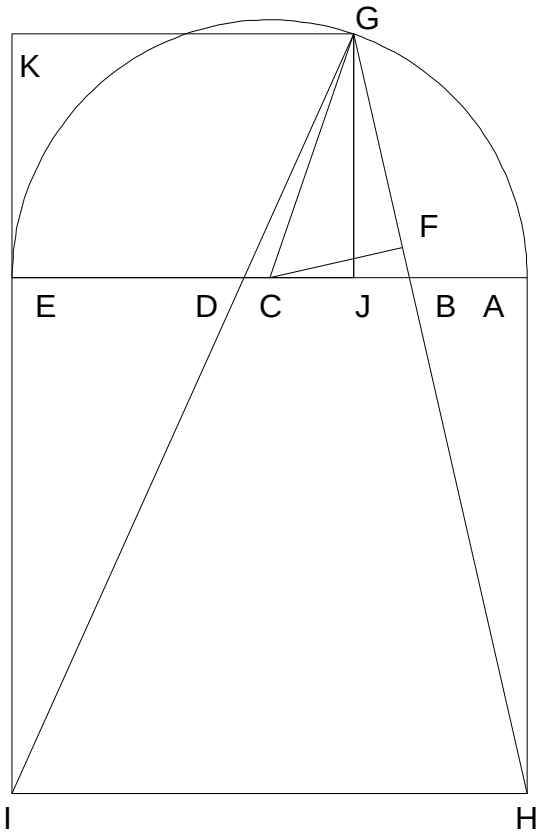
$GK_\delta := \frac{HI_\delta \cdot EG_\delta}{EH} \quad GP_\delta := \frac{AG_\delta \cdot GK_\delta}{AC + GK_\delta}$

$EP_\delta := \sqrt{(EG_\delta)^2 + (GP_\delta)^2}$

$CP_\delta := CE_\delta - EP_\delta \qquad CF_\delta := CD_\delta + DF$

$\underline{CP_\delta} - \sqrt{CD_\delta \cdot CF_\delta}$






10_31_94.MCD

Given AB & BE, divide BE such that BD + DE = BE and $BD = \sqrt{AB \cdot DE}$

**The square in a right triangle on the hypotenuse is equal to the square of the remaining two segments** (and all three squares taken to the point of similarity form a cubic relationship).

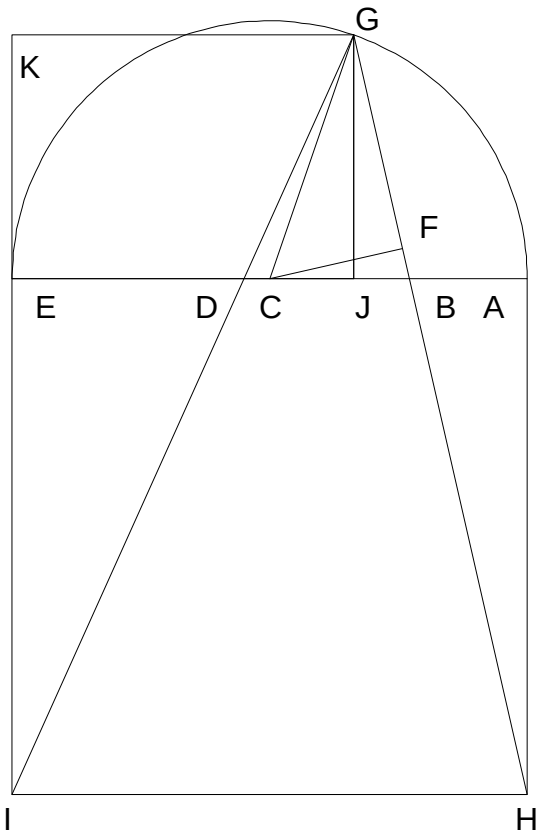
$AE := AB + BE \quad AC := \frac{AE}{2}$

$AH := AE \quad EI := AE \quad BH := \sqrt{AB^2 + AH^2}$

$CG := AC \quad BC := AC - AB \quad BF := \frac{AB \cdot BC}{BH}$

$CF := \frac{AH \cdot BC}{BH} \quad FG := \sqrt{CG^2 - CF^2}$

$BG := BF + FG \quad BJ := \frac{AB \cdot BG}{BH}$



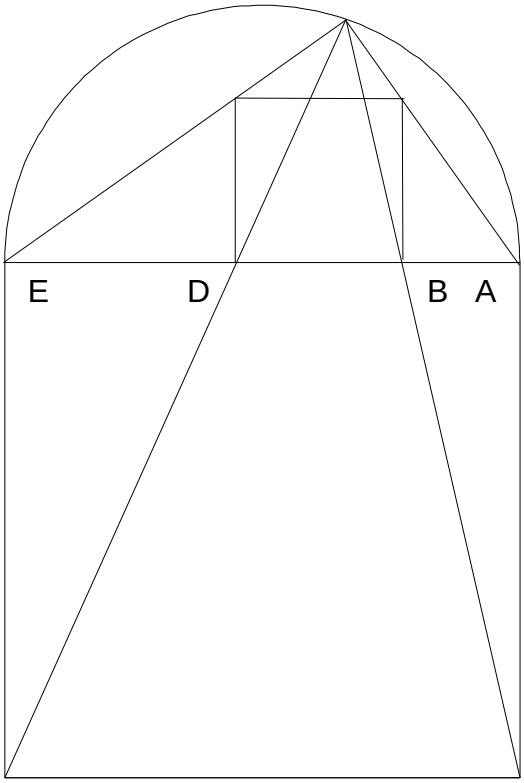

$GJ := \frac{AH \cdot BJ}{AB} \quad AJ := AB + BJ$

$EJ := AE - AJ \quad GK := EJ \quad EK := GJ$

$IK := EI + EK \quad DE := \frac{GK \cdot EI}{IK}$

$BD := BE - DE$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB \cdot DE} = 0$




Reducing the previous tautological chain to single equations for BD and DE.

$AB \equiv 20 \quad BE \equiv 12$

$$BD := \frac{\sqrt{AB}\cdot\left[-AB^{\left(\frac{3}{2}\right)} + AB\cdot\sqrt{AB + 4\cdot BE} + \sqrt{AB}\cdot BE \ldots + BE\cdot\sqrt{AB + 4\cdot BE}\right]}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

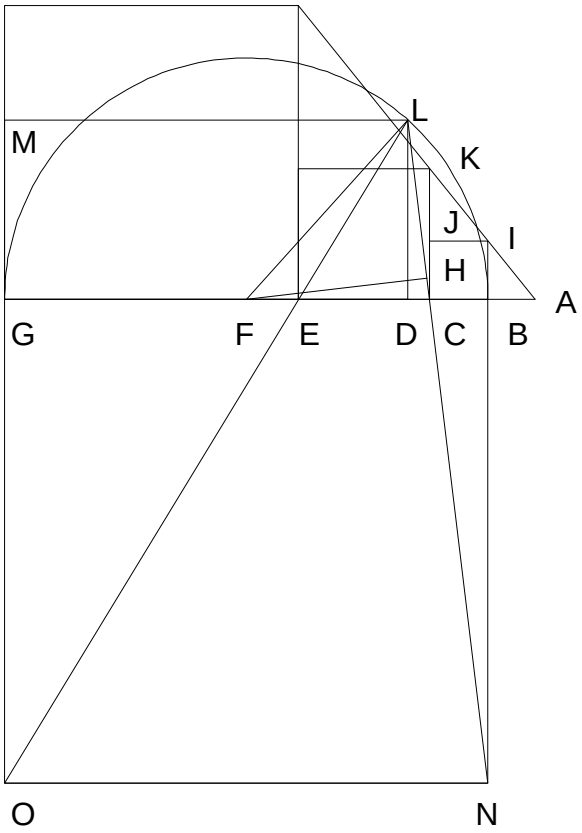
$BD = 8.439$

$$DE := \frac{AB^2 + 2\cdot AB\cdot BE - AB^{\left(\frac{3}{2}\right)}\cdot\sqrt{AB + 4\cdot BE} + 2\cdot BE^2}{3\cdot AB + 2\cdot BE + \sqrt{AB}\cdot\sqrt{AB + 4\cdot BE}}$$

$DE = 3.561$

$BE - (BD + DE) = 0 \qquad BD - \sqrt{AB\cdot DE} = 0$

Given AB & BE, BE has been divided such that BD + DE = BE and $BD = \sqrt{AB\cdot DE}$




11_01_94.MCD

Given AG - AB = BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BG > 3 BC.

$BG := 6 \quad BC := 1.9 \quad BN := BG$

$BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN}$

$FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN}$

$GM := DL \quad BD := BC + CD \quad DG := BG - BD$

$LM := DG \quad GO := BG \quad MO := GO + GM$

$EG := \frac{LM \cdot GO}{MO} \quad CG := BG - BC$

$CE := CG - EG \quad CJ := BC \quad CK := CE$

$IJ := BC \quad JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2}$

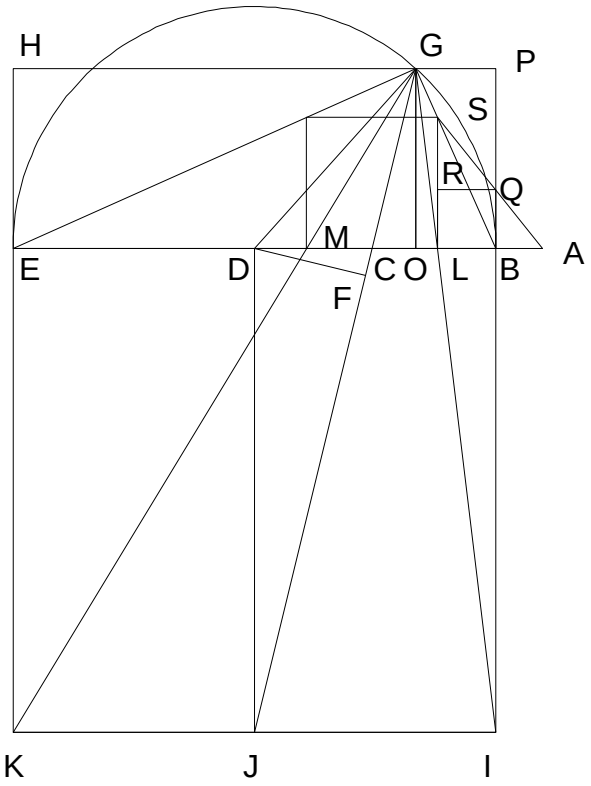
$BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE$

$AB = 36.723 \quad AG = 42.723 \quad AE = 40.621$

$\left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

Given AG - AB =BG and $(AE^2 \cdot AG)^{1/3}$ - AB = BC, found was AB, AG, and $(AB \cdot AG^2)^{1/3}$.




Given AE - AB = BE and

$$\frac{(AB\cdot AE^2)^{\frac{1}{3}} - (AB^2\cdot AE)^{\frac{1}{3}}}{2} + (AB^2\cdot AE)^{\frac{1}{3}} - AE = BC,$$

find AB.

$BE := 70 \quad BC := 34 \quad BD := \frac{BE}{2} \quad BI := BE$

$DJ := BE \quad EK := BE \quad CD := BD - BC$

$CJ := \sqrt{DJ^2 + CD^2} \quad DF := \frac{DJ\cdot CD}{CJ}$

$DG := BD \quad FG := \sqrt{DG^2 - DF^2} \quad CF := \frac{CD\cdot DF}{DJ}$

$CG := FG - CF \quad CO := \frac{CD\cdot CG}{CJ}$

$DO := CD + CO$

$BO := BD - DO \quad GP := BO \quad GO := \frac{DJ\cdot CG}{CJ}$

$BP := GO \quad IP := BI + BP \quad BL := \frac{GP\cdot BI}{IP}$

$EH := GO \quad HK := EK + EH \quad EO := BE - BO$

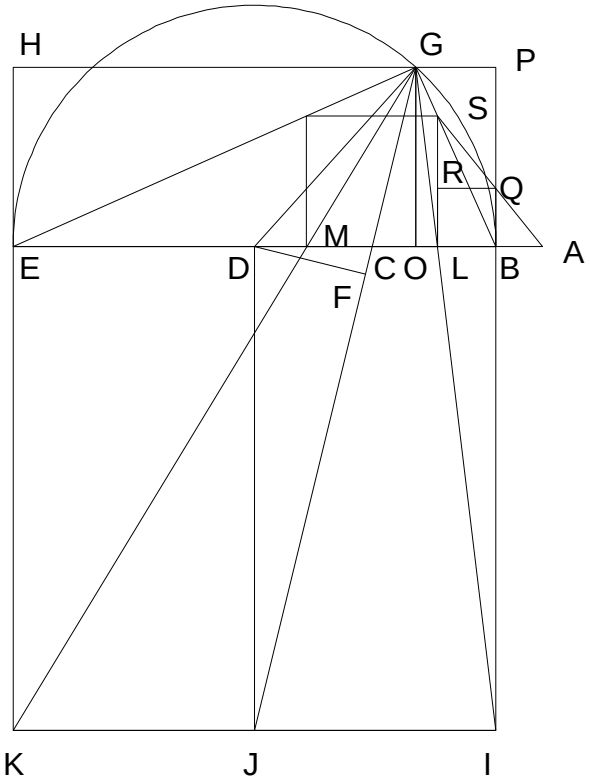
$GH := EO \quad EM := \frac{GH\cdot EK}{HK} \quad BM := BE - EM$

$BQ := BL \quad LR := BL \quad QR := BL$

$LM := BM - BL$

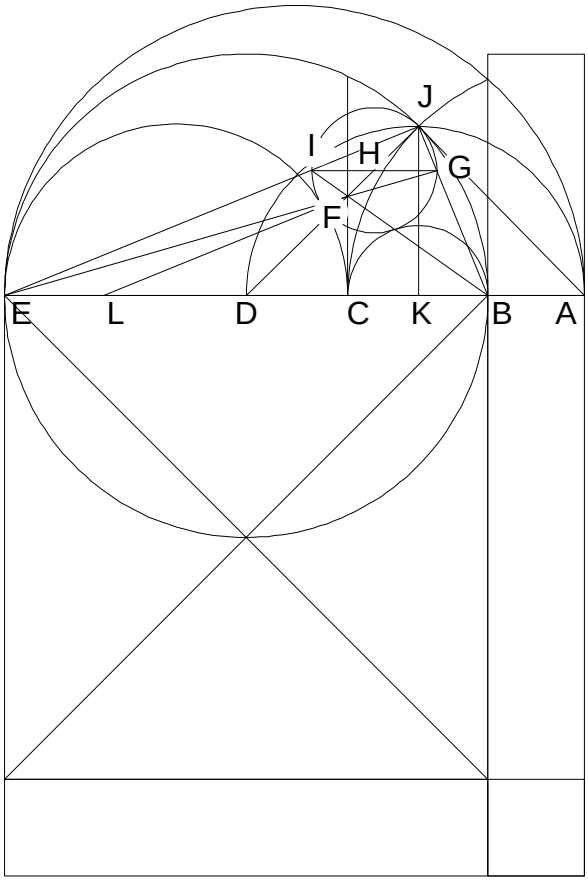
$LS := LM \quad RS := LS - LR \quad AB := \frac{QR\cdot BQ}{RS}$

$AE := AB + BE$




$AB = 510.028 \quad AE = 580.028 \quad BC = 34$

$$\frac{(AB \cdot AE^2)^{\frac{1}{3}} - (AB^2 \cdot AE)^{\frac{1}{3}}}{2} + (AB^2 \cdot AE)^{\frac{1}{3}} - AB = 34$$




# Power Line At Square Root.

$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BE := 100 \quad AE_\delta := AB_\delta + BE$

$AC_\delta := \sqrt{AB_\delta \cdot AE_\delta} \quad BD := \frac{BE}{2} \quad DE := BD \quad DJ := BD$

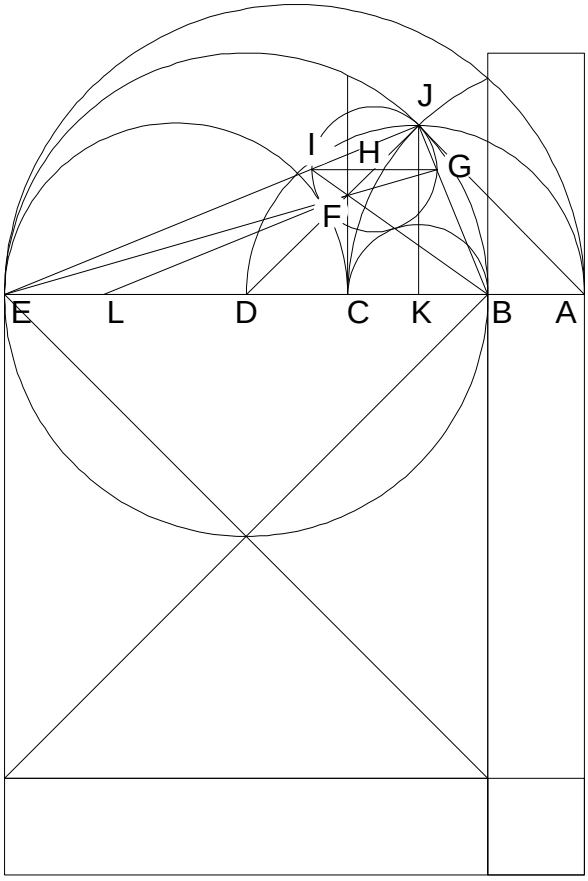
$AD_\delta := AB_\delta + BD \quad DK_\delta := \frac{DJ^2}{AD_\delta} \quad AK_\delta := AD_\delta - DK_\delta$

$BC_\delta := AC_\delta - AB_\delta \quad JK_\delta := \sqrt{AK_\delta \cdot DK_\delta} \quad CD_\delta := BD - BC_\delta$

$CF_\delta := \frac{JK_\delta \cdot CD_\delta}{DK_\delta} \quad BK_\delta := AK_\delta - AB_\delta \quad EK_\delta := BE - BK_\delta$

$CL_\delta := \frac{EK_\delta \cdot CF_\delta}{JK_\delta} \quad BL_\delta := BC_\delta + CL_\delta$

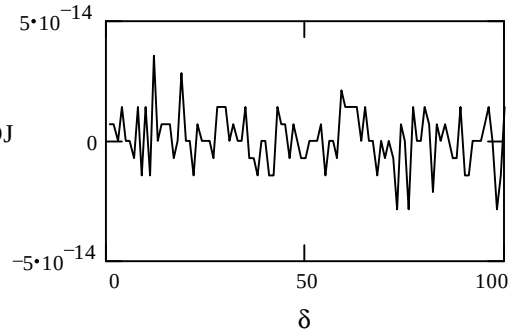
$BF_\delta := \sqrt{(BC_\delta)^2 + (CF_\delta)^2}$

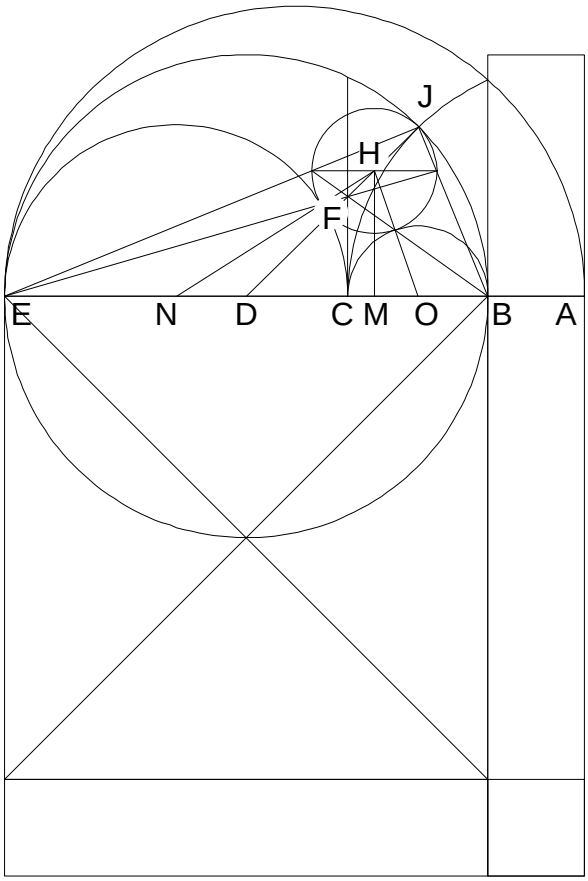



$BI_\delta := \frac{BF_\delta \cdot BE}{BL_\delta} \quad FI_\delta := BI_\delta - BF_\delta \quad GI_\delta := \frac{BE \cdot FI_\delta}{BF_\delta}$

$HJ_\delta := \frac{GI_\delta}{2} \quad DF_\delta := \sqrt{(CD_\delta)^2 + (CF_\delta)^2} \quad FH_\delta := \frac{DF_\delta \cdot GI_\delta}{BE}$

$(DF_\delta + FH_\delta + HJ_\delta) - DJ$



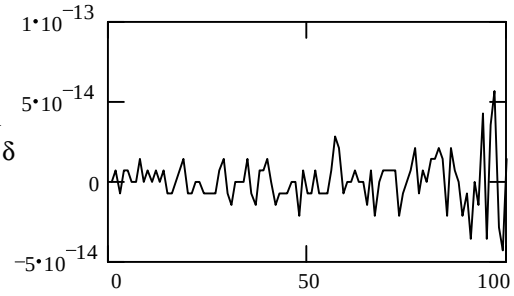



$DH_\delta := DF_\delta + FH_\delta \quad DM_\delta := \frac{CD_\delta \cdot DH_\delta}{DF_\delta} \quad CE_\delta := DE + CD_\delta$

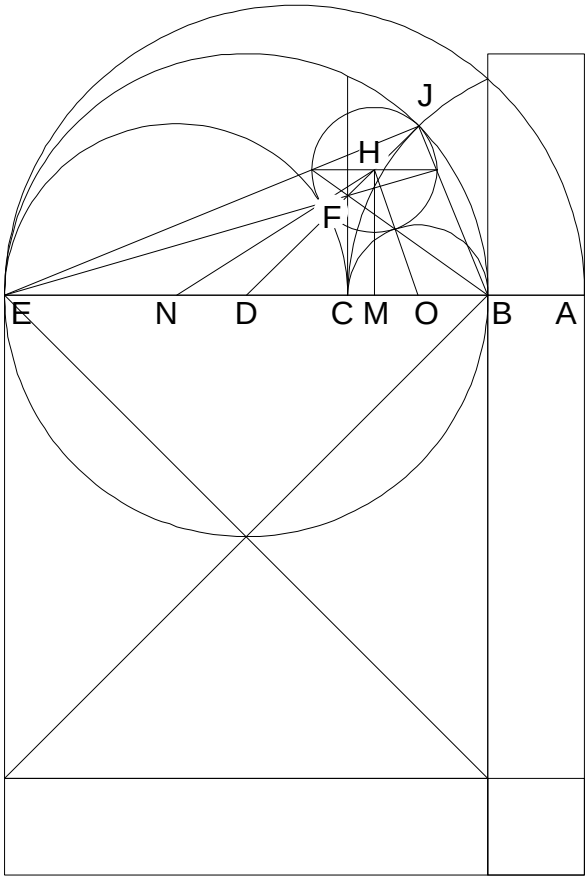
$CM_\delta := DM_\delta - CD_\delta \quad CN_\delta := \frac{CE_\delta}{2} \quad MN_\delta := CN_\delta + CM_\delta$

$HM_\delta := \frac{CF_\delta \cdot DH_\delta}{DF_\delta} \quad HN_\delta := \sqrt{(MN_\delta)^2 + (HM_\delta)^2}$

$\underline{(CN_\delta + HJ_\delta) - HN_\delta}$



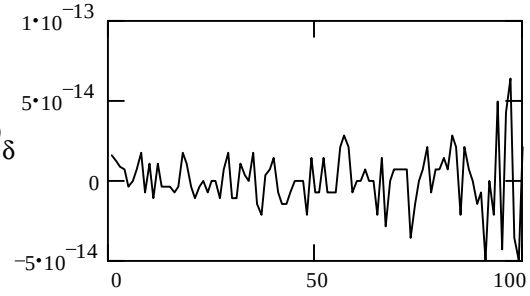

— Is Tangent?




$CO_\delta := \frac{BC_\delta}{2} \quad MO_\delta := CO_\delta - CM_\delta$

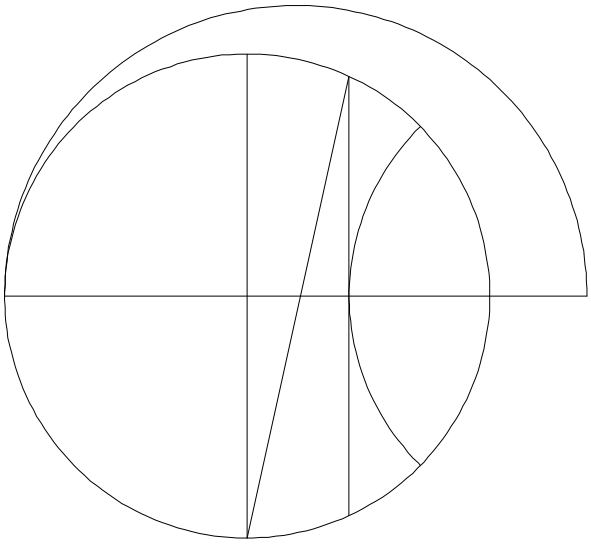
$HO_\delta := \sqrt{(HM_\delta)^2 + (MO_\delta)^2}$

$\underline{(CO_\delta + HJ_\delta) - HO_\delta}$

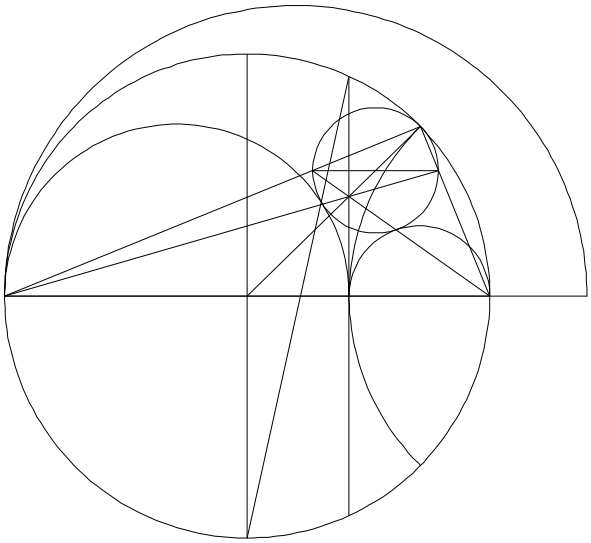


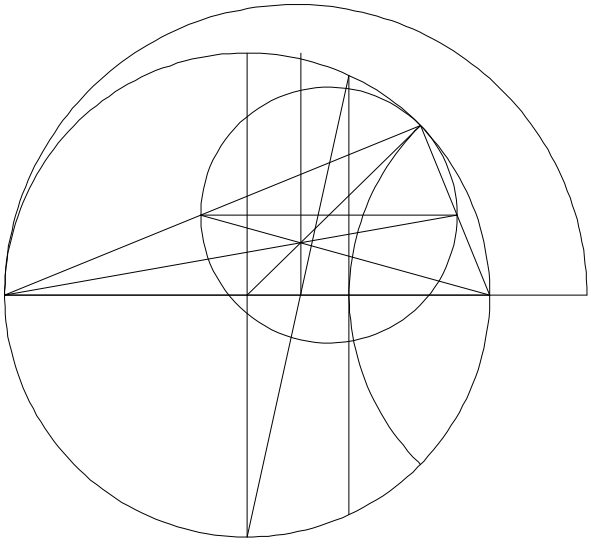

— Is Tangent?

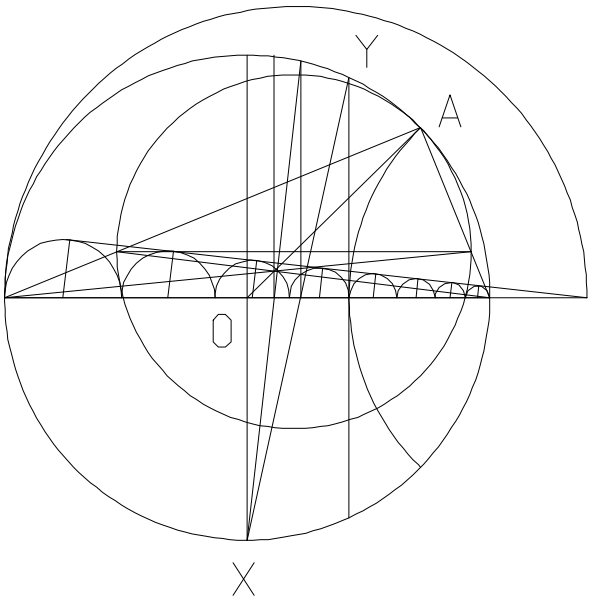
# Two prime exponential series developed through power line progression.



I will present a series of plates to explain the process. The process can be infinitly repeated, supposing you had the tools to do it with.

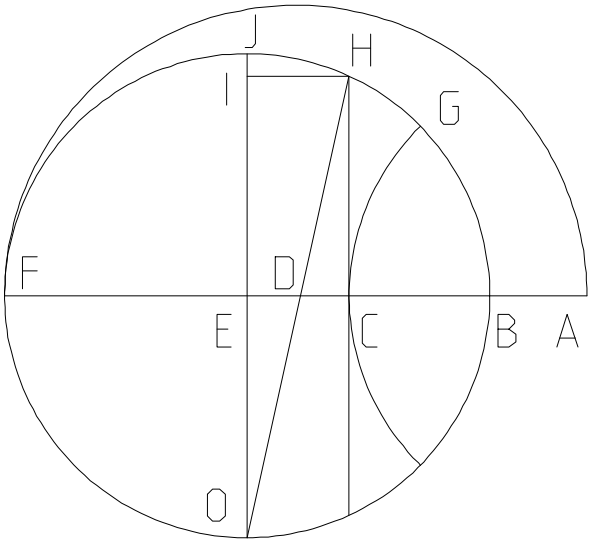







It is clear how OA uses the power line XY to provide a 2 prime exponential series.

Possible Problem: From a similarity point outside of a circle, place some 2 prine sequence of smaller circles on the larger circles diameter, all tangent in sequence.



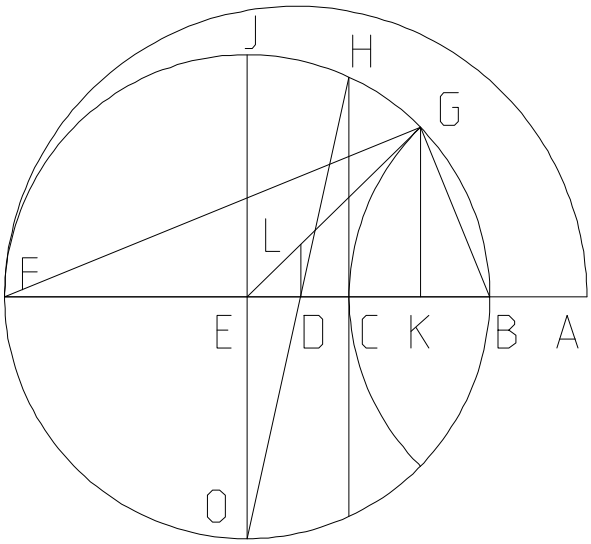

$AB := 1 \quad BF := 5 \quad AF := AB + BF$

$AC := \sqrt{AB \cdot AF} \qquad BE := \frac{BF}{2} \quad EO := BE$

$BC := AC - AB \qquad CF := BF - BC$

$CH := \sqrt{BC \cdot CF} \quad EI := CH \quad CE := BE - BC$

$HI := CE \quad IO := EO + EI \quad DE := \frac{HI \cdot EO}{IO}$

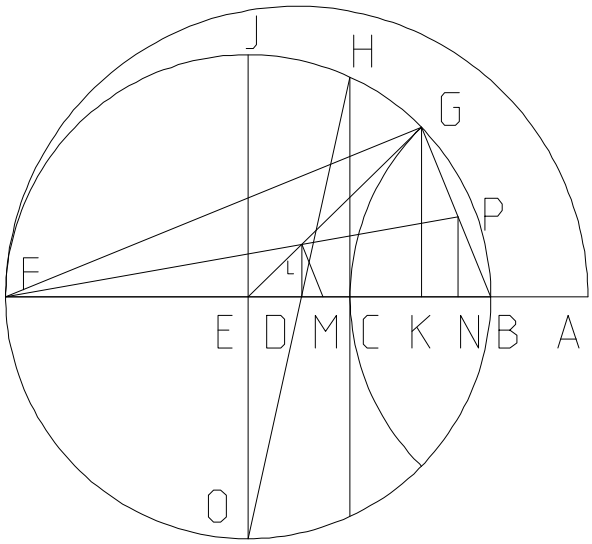



See 12_26_94.MCD for next equation.

$$GK := \frac{BF \cdot \left(\sqrt{AB} \cdot \sqrt{AB + BF}\right)}{(2 \cdot AB + BF)}$$

$EG := BE \quad EK := \sqrt{EG^2 - GK^2}$

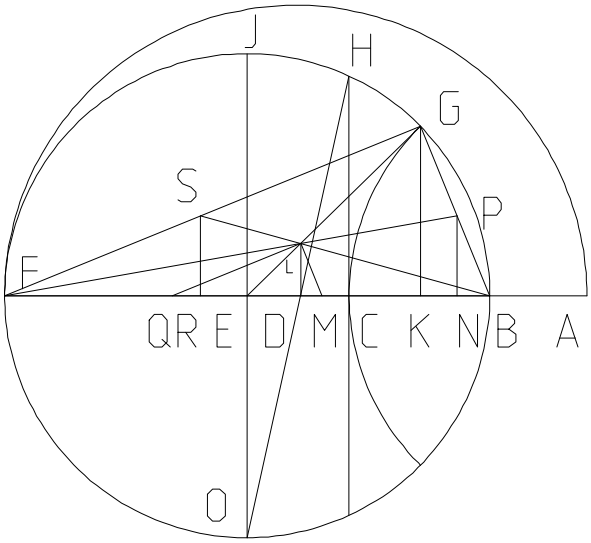
$DL := \frac{GK \cdot DE}{EK}$




$KN := BE - EK \quad DM := \frac{KN \cdot DE}{EK}$

$EF := BE \quad FM := EF + DM + DE$

$BN := \frac{DM \cdot BF}{FM} \quad NP := \frac{DL \cdot BF}{FM}$



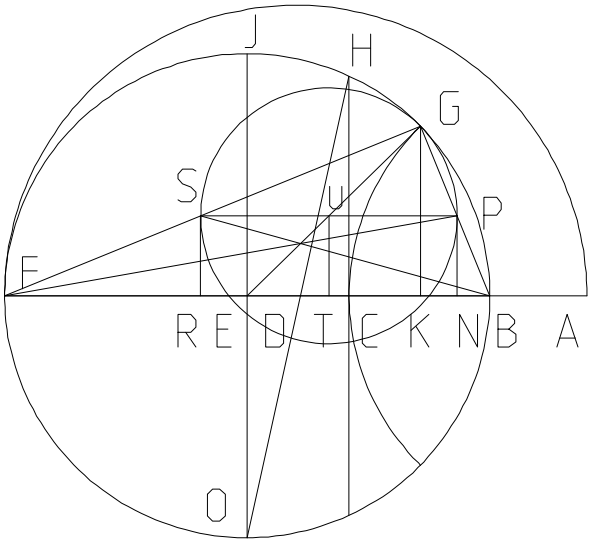

$KF := EK + EF \quad DQ := \frac{KF \cdot DL}{GK}$

$BD := BE - DE \quad BQ := BD + DQ$

$BR := \frac{BD \cdot BF}{BQ} \quad RS := \frac{DL \cdot BR}{BD}$

Are RS and NP equal?

$RS - NP = 0$




$TU := NP \quad ET := \frac{EK \cdot TU}{GK} \quad NR := BR - BN$

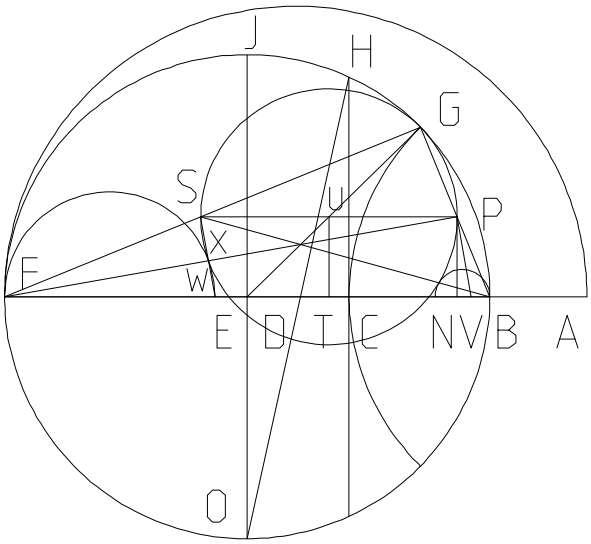
$EN := BE - BN \quad NT := EN - ET$

$PS := NR \quad PU := NT \quad EU := \sqrt{ET^2 + TU^2}$

Is NT half of NR? $\frac{NR}{NT} = 2$

Does GU = PU? $GU := EG - EU$

$GU - PU = 0$



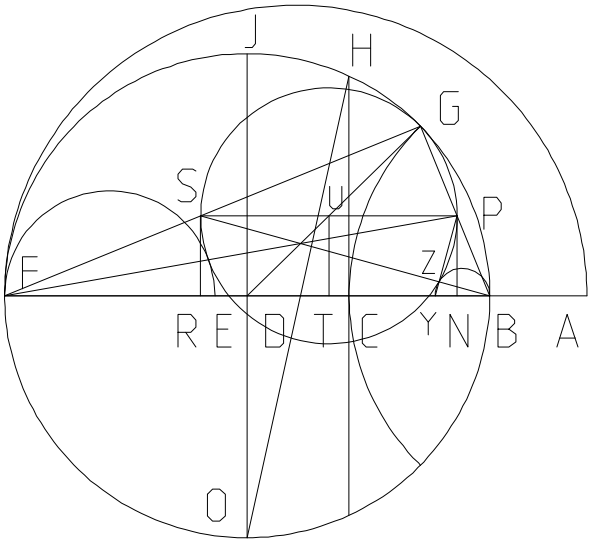

$BT := BN + NT \quad FN := BF - BN \quad FP := \sqrt{NP^2 + FN^2}$

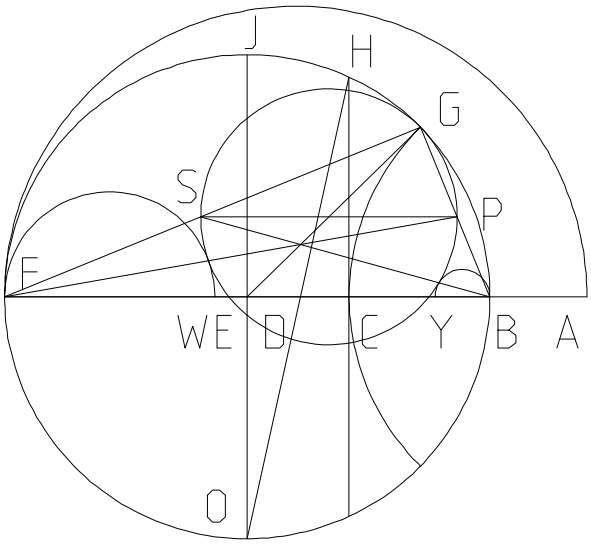
$FV := \frac{FP^2}{FN} \quad PX := \frac{FP \cdot PS}{FV} \quad FX := FP - PX$

$FW := \frac{FV \cdot FX}{FP} \quad BW := BF - FW \quad AW := AB + BW$

Is AW a quarter root of AB, AF?

$\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AW = 0$



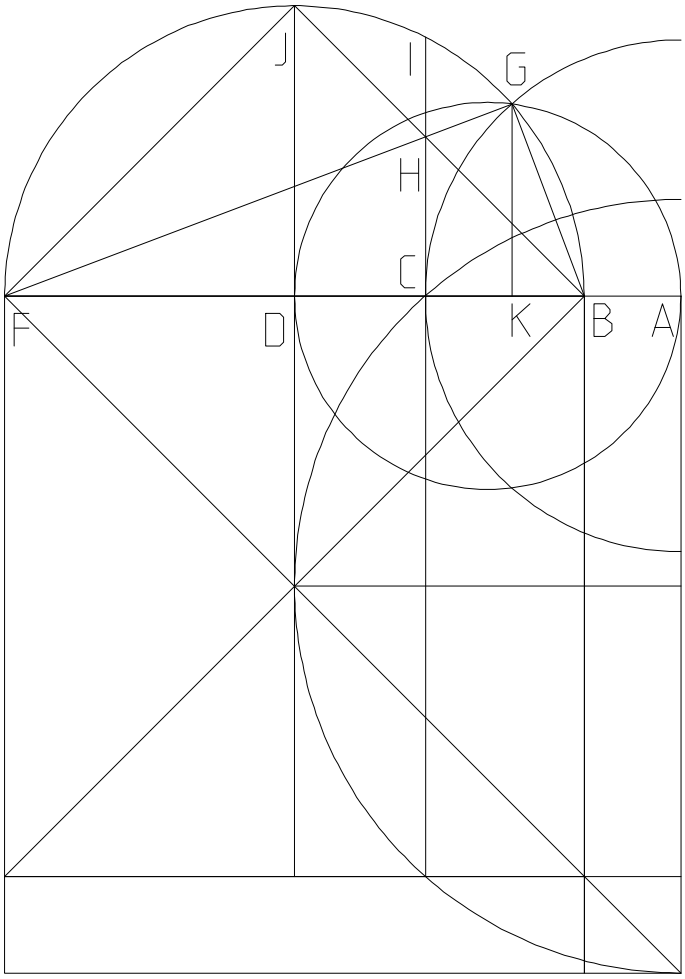

$BS := \sqrt{BR^2 + RS^2}$ $SZ := \frac{BR \cdot PS}{BS}$

$BZ := BS - SZ$ $BY := \frac{PS \cdot BZ}{SZ}$

$AY := AB + BY$

Is AY a quarter root of AB, AF?

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AY = 0$




Does H and G have a constant relationship?

$\delta := 1 .. 1000 \quad AB_\delta := \delta^5 \cdot 10^{-8}$

$BF := 6 \quad BD := \frac{BF}{2} \quad AF_\delta := AB_\delta + BF$

$AC_\delta := \sqrt{AB_\delta \cdot AF_\delta} \quad DJ := BD \quad DF := BD$

$BC_\delta := AC_\delta - AB_\delta \quad CD_\delta := BD - BC_\delta$

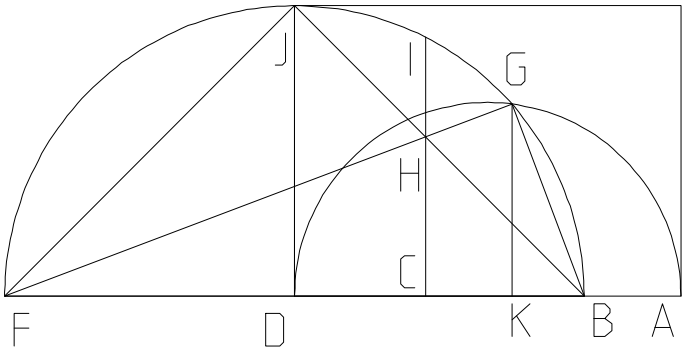
$CH_\delta := \frac{DJ \cdot BC_\delta}{BD} \quad AG_\delta := AC_\delta$

$AD_\delta := AB_\delta + BD \quad AK_\delta := \frac{(AG_\delta)^2}{AD_\delta}$

$GK_\delta := \sqrt{(AG_\delta)^2 - (AK_\delta)^2}$

$BK_\delta := AK_\delta - AB_\delta \quad CK_\delta := BC_\delta - BK_\delta$

$KF_\delta := CK_\delta + CD_\delta + DF$

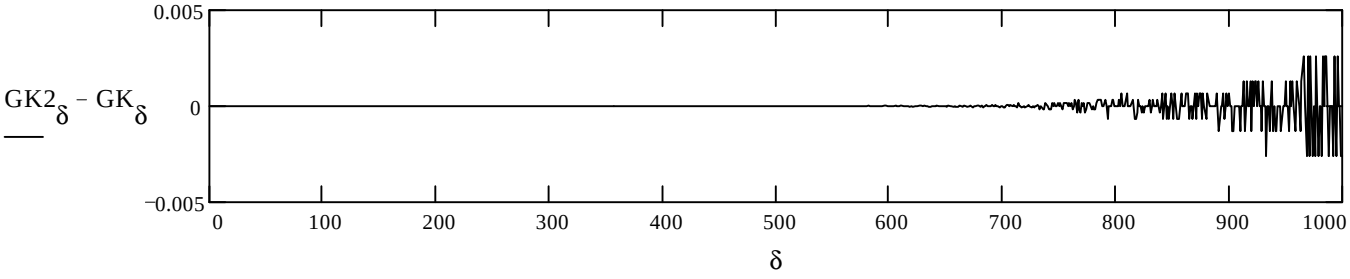


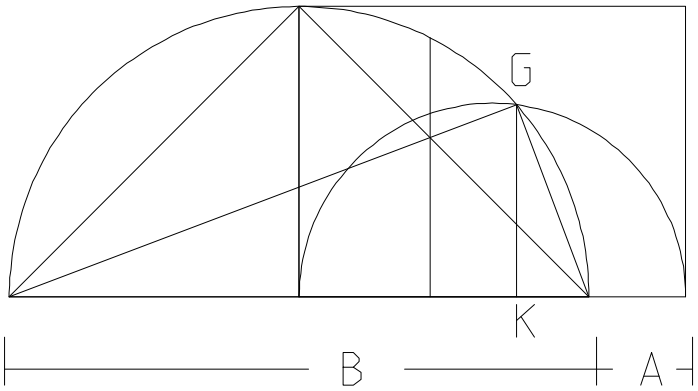

Does FH and FG have identical slopes?

$CF_\delta := CD_\delta + DF$

$GK2_\delta := \frac{CH_\delta \cdot KF_\delta}{CF_\delta}$

Therefore G and H are constantly co-linear.





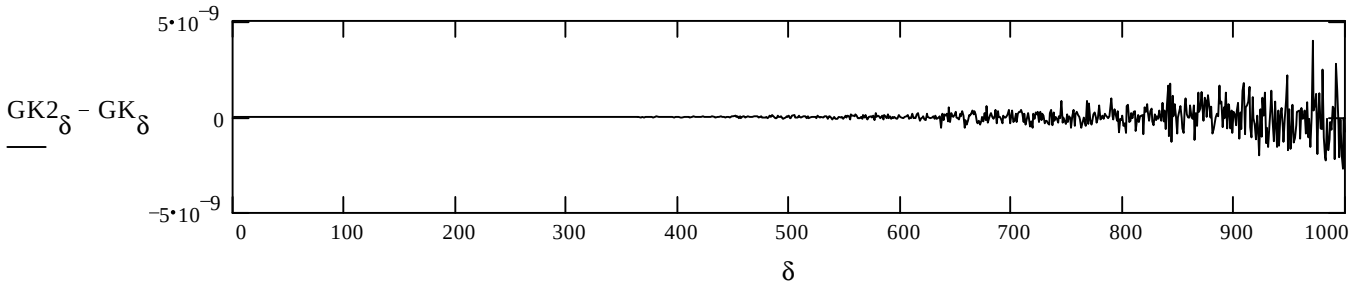

Thus this file can be redone as: "Given BC = $\sqrt{AB \cdot AF}$ and BF, find AB."

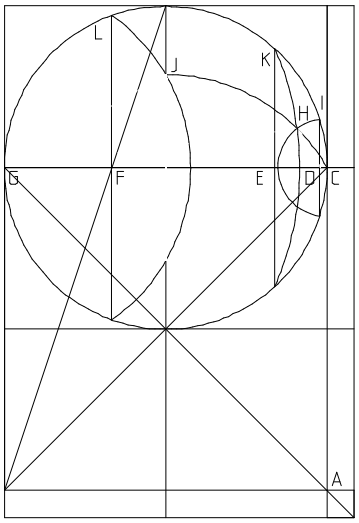
The Formula for GK vs. GK2 demonstrates that the symbolic processor cannot always resolve to simplest form. GK2 is the processors final attempt. An attempt with Mathcad 6 gives the same result.

$A_\delta := AB_\delta \quad B := BF$

$$GK2_\delta := B \cdot \frac{\left[ \left(A_\delta\right)^{\left(\frac{3}{2}\right)} \cdot \sqrt{A_\delta + B} - \left(A_\delta\right)^2 + B \cdot \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} - B \cdot A_\delta \right]}{\left[ \left(2 \cdot A_\delta + B\right) \cdot \left(B - \sqrt{A_\delta} \cdot \sqrt{A_\delta + B} + A_\delta\right) \right]}$$

$$GK_\delta := \frac{B \cdot \left(\sqrt{A_\delta} \cdot \sqrt{A_\delta + B}\right)}{\left(2 \cdot A_\delta + B\right)}$$





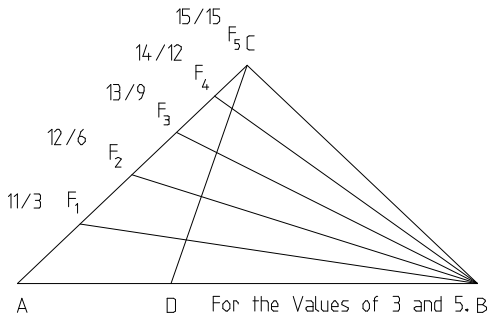

# One Circle

**By John Clark**

# One Square

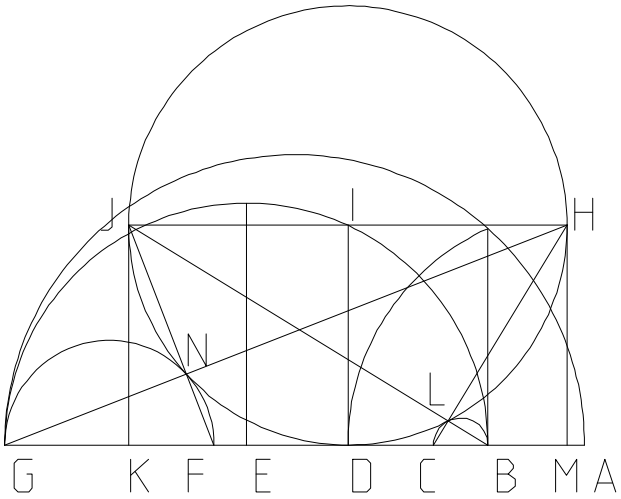
# One Line

## And the Delian Quest




# 1995

# Alternate method for Quad Roots




$\delta := 1 .. 100 \quad AB_\delta := \delta \quad BG := 10 \quad AG_\delta := AB_\delta + BG$

$AD_\delta := \sqrt{AB_\delta \cdot AG_\delta} \quad BD_\delta := AD_\delta - AB_\delta$

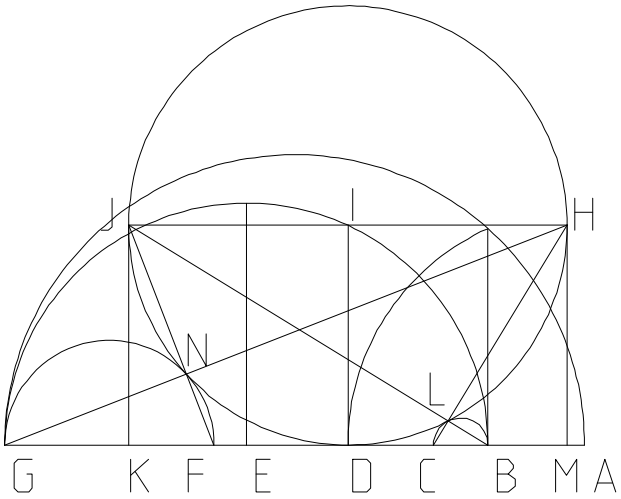
$DG_\delta := BG - BD_\delta \quad DI_\delta := \sqrt{BD_\delta \cdot DG_\delta}$

$HI_\delta := DI_\delta \quad IJ_\delta := DI_\delta \quad HJ_\delta := HI_\delta + IJ_\delta$

$DK_\delta := DI_\delta \quad JK_\delta := DI_\delta \quad BK_\delta := BD_\delta + DK_\delta$

$BJ_\delta := \sqrt{\left(BK_\delta\right)^2 + \left(JK_\delta\right)^2} \quad JL_\delta := \frac{BK_\delta \cdot HJ_\delta}{BJ_\delta}$

$BL_\delta := BJ_\delta - JL_\delta \quad BC_\delta := \frac{HJ_\delta \cdot BL_\delta}{JL_\delta}$




$HM_\delta := DI_\delta \quad DM_\delta := DI_\delta \quad MG_\delta := DM_\delta + DG_\delta$

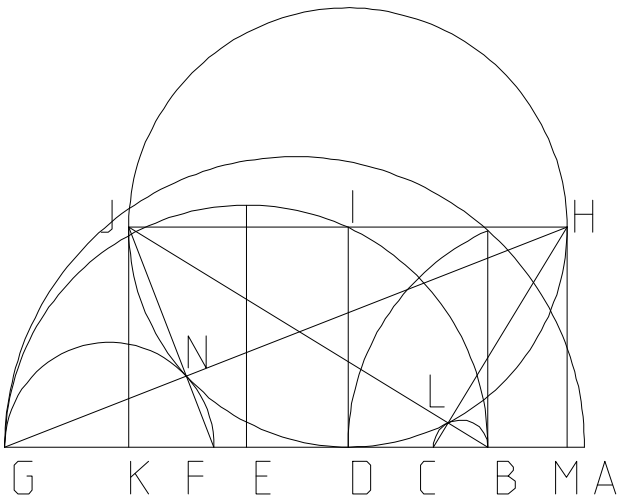
$GH_\delta := \sqrt{\left(MG_\delta\right)^2 + \left(HM_\delta\right)^2} \quad HN_\delta := \frac{MG_\delta \cdot HJ_\delta}{GH_\delta}$

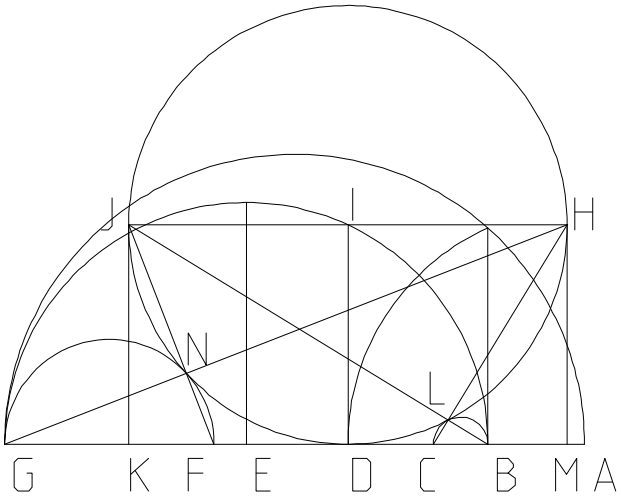
$GN_\delta := GH_\delta - HN_\delta \quad FG_\delta := \frac{HJ_\delta \cdot GN_\delta}{HN_\delta}$

$BF_\delta := BG - FG_\delta$

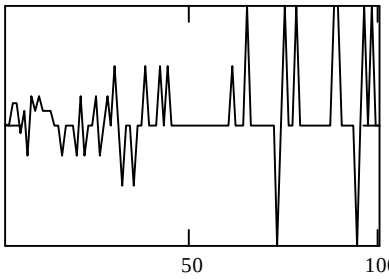
$AF_\delta := AB_\delta + BF_\delta$

$AC_\delta := AB_\delta + BC_\delta$



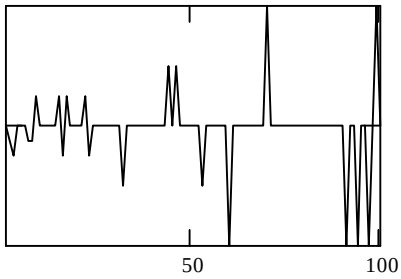



$$\left[ AC_{\delta} - \left[ \left( AB_{\delta} \right)^{3} \cdot AG_{\delta} \right]^{\frac{1}{4}} \right]_{0}$$



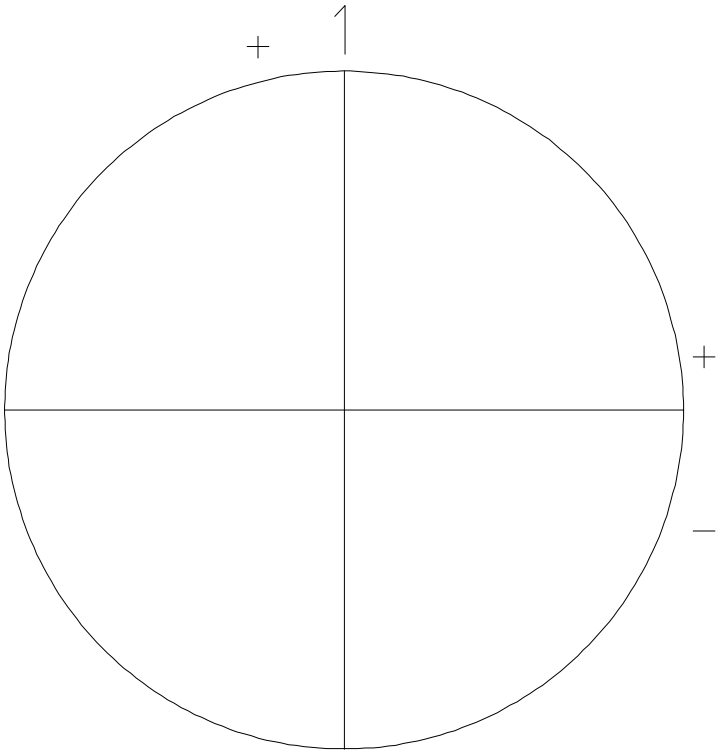

δ

$$\left[ AF_{\delta} - \left[ AB_{\delta} \cdot \left( AG_{\delta} \right)^{3} \right]^{\frac{1}{4}} \right]_{0}$$




δ

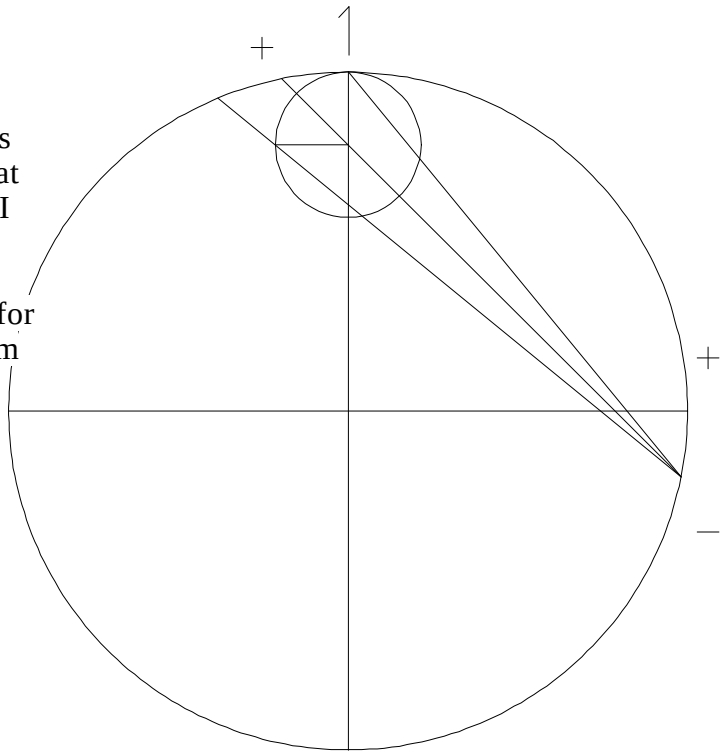
The symbolic processor on my computer could not reduce the chain to the final equations.




Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

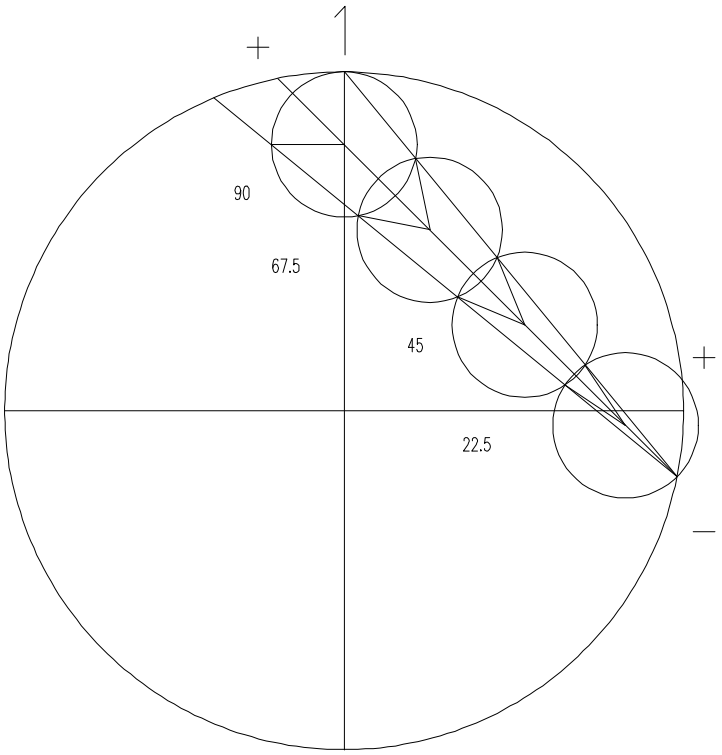
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$




$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

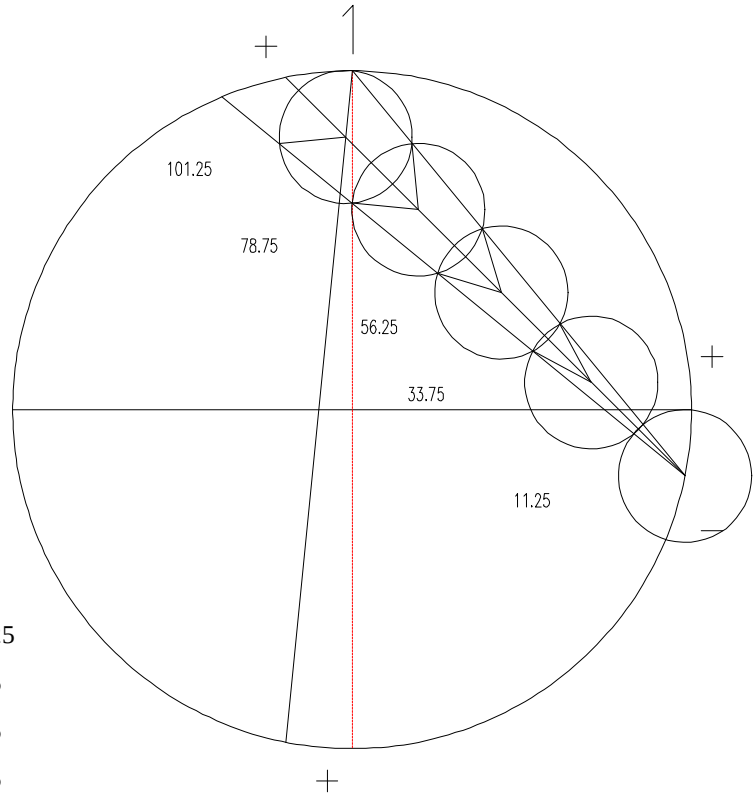
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

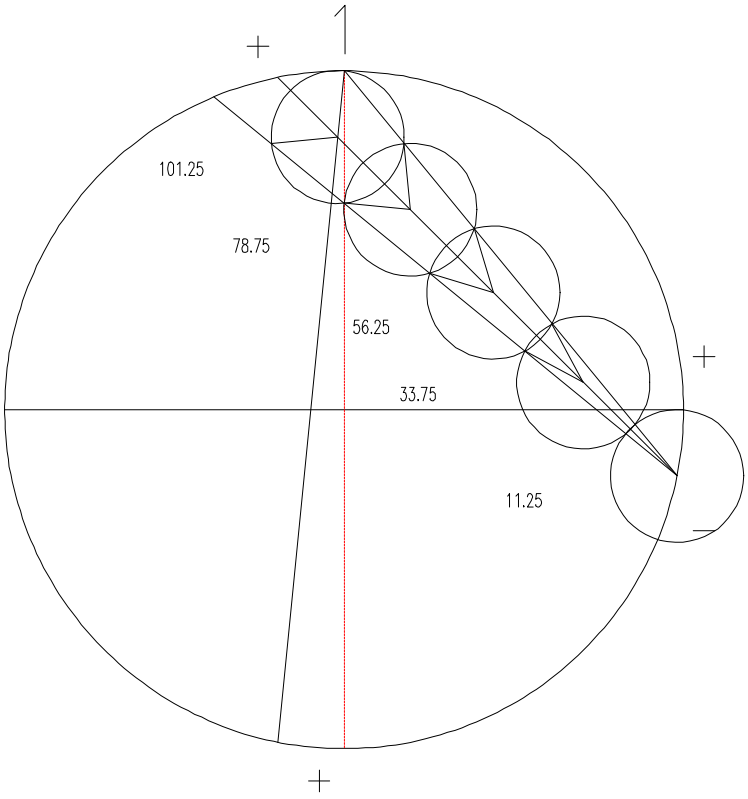
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$




| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

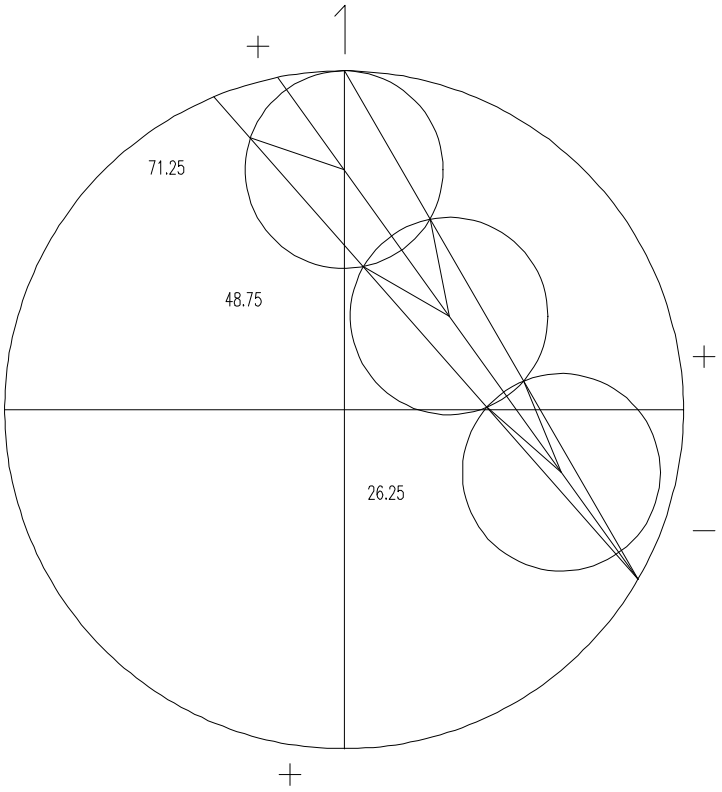
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

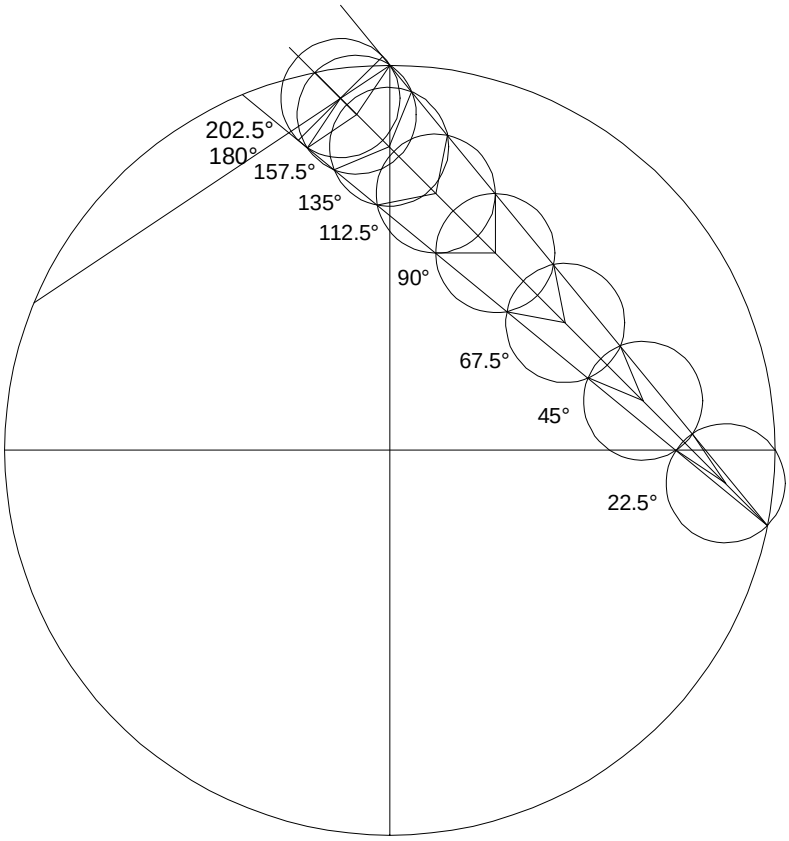
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

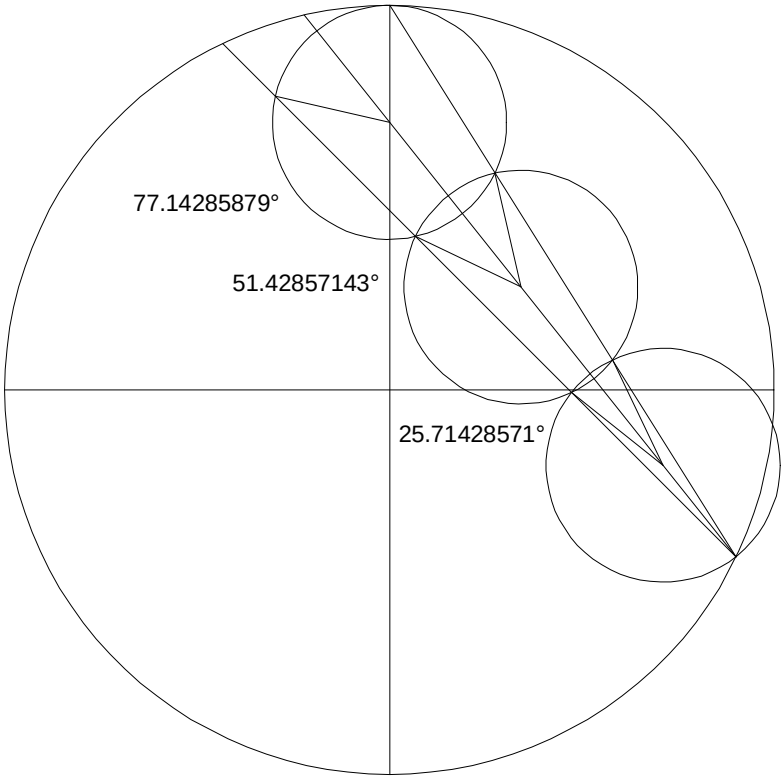
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

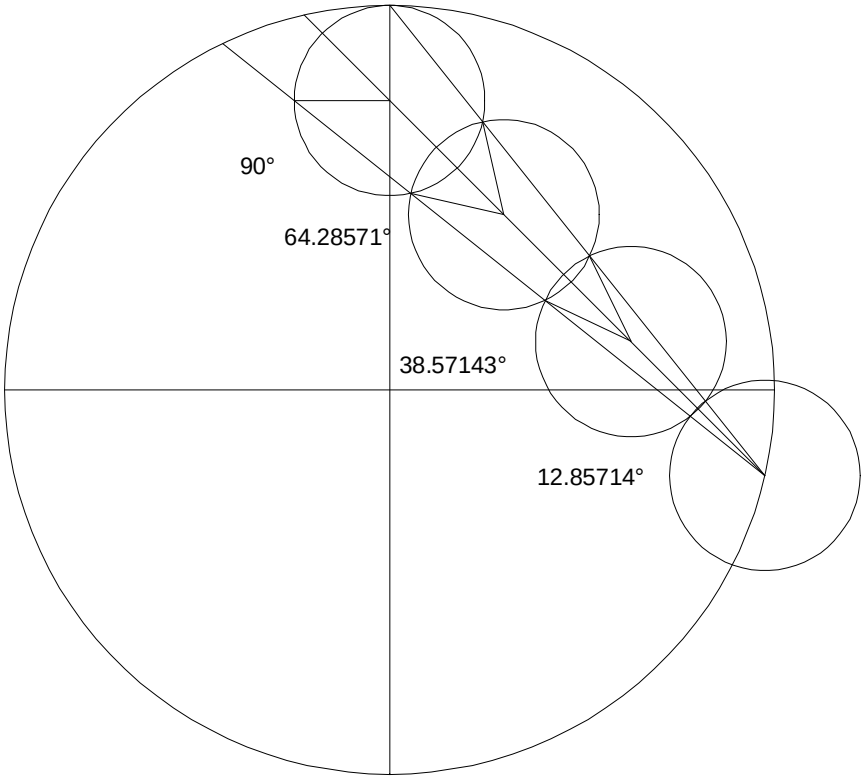
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

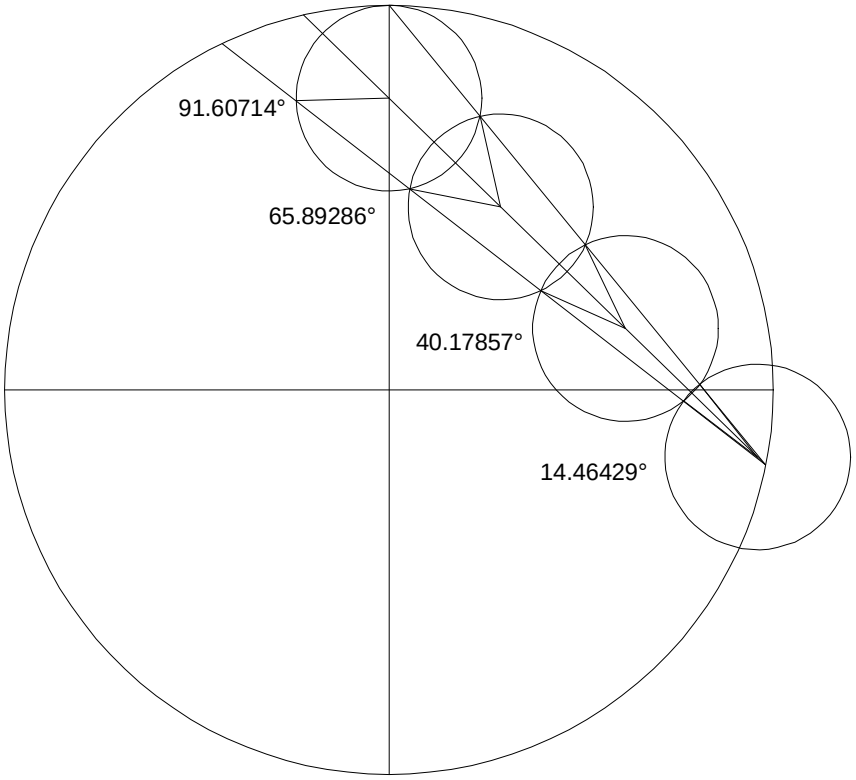
$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

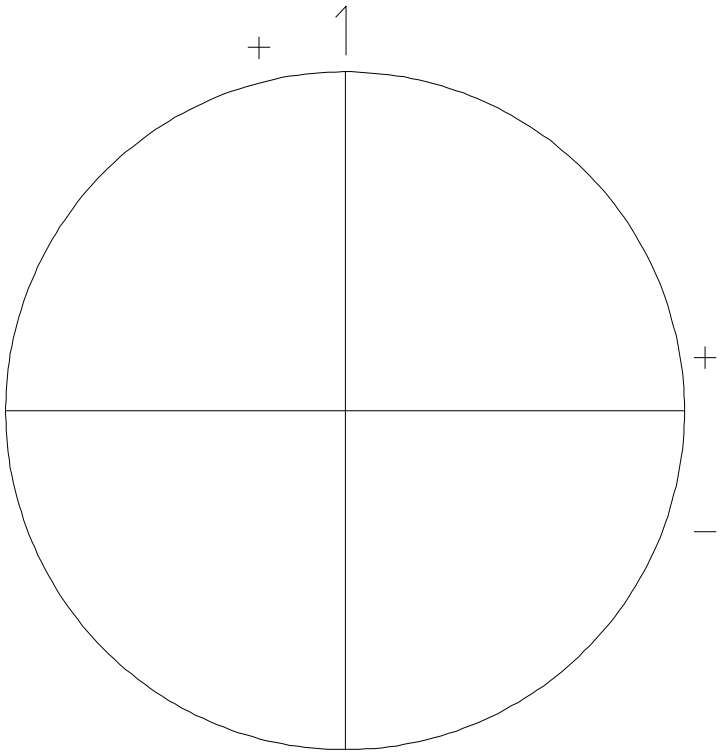
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

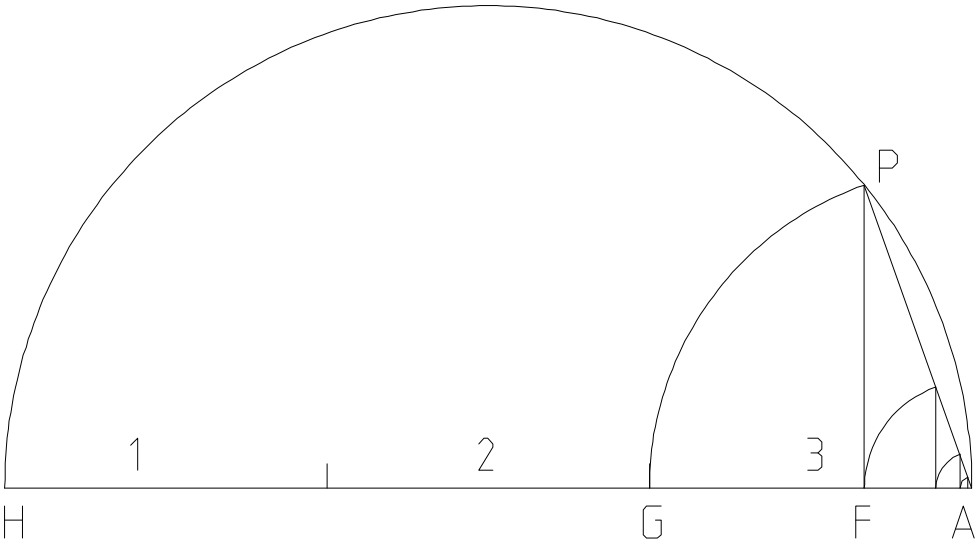
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




Work in progress.




Exponential Progressions. 04_01_95

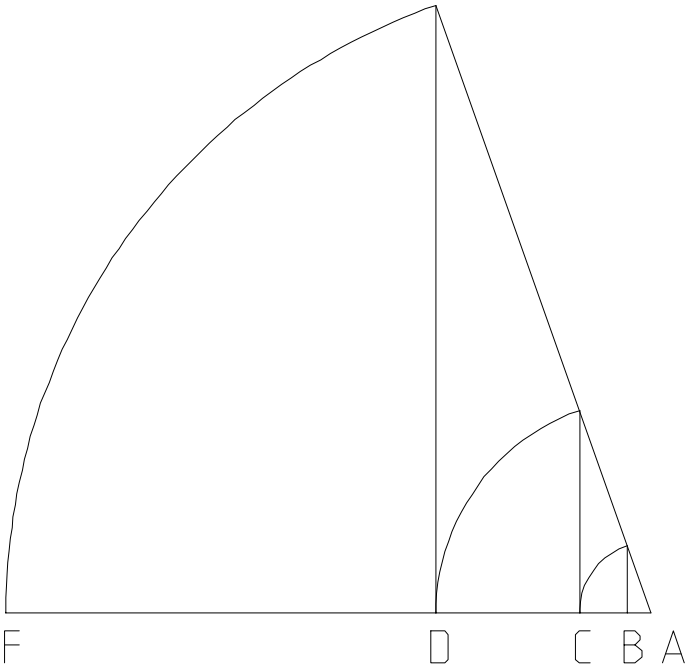
If I want to multiply any number by any power, this is the a geometric process for doing so.

The given figure is drawn for the third power of 3.

$AH := 10 \qquad \delta := 0..10 \qquad BS := 8$

The third division between A and F is very hard to see. BS = Base Segments

$AP := \frac{AH}{BS} \qquad AP = 1.25$




$AG_0 := \frac{(AP)^2}{AH} \qquad AF_0 := AG_0 \qquad \Delta := 2..10$

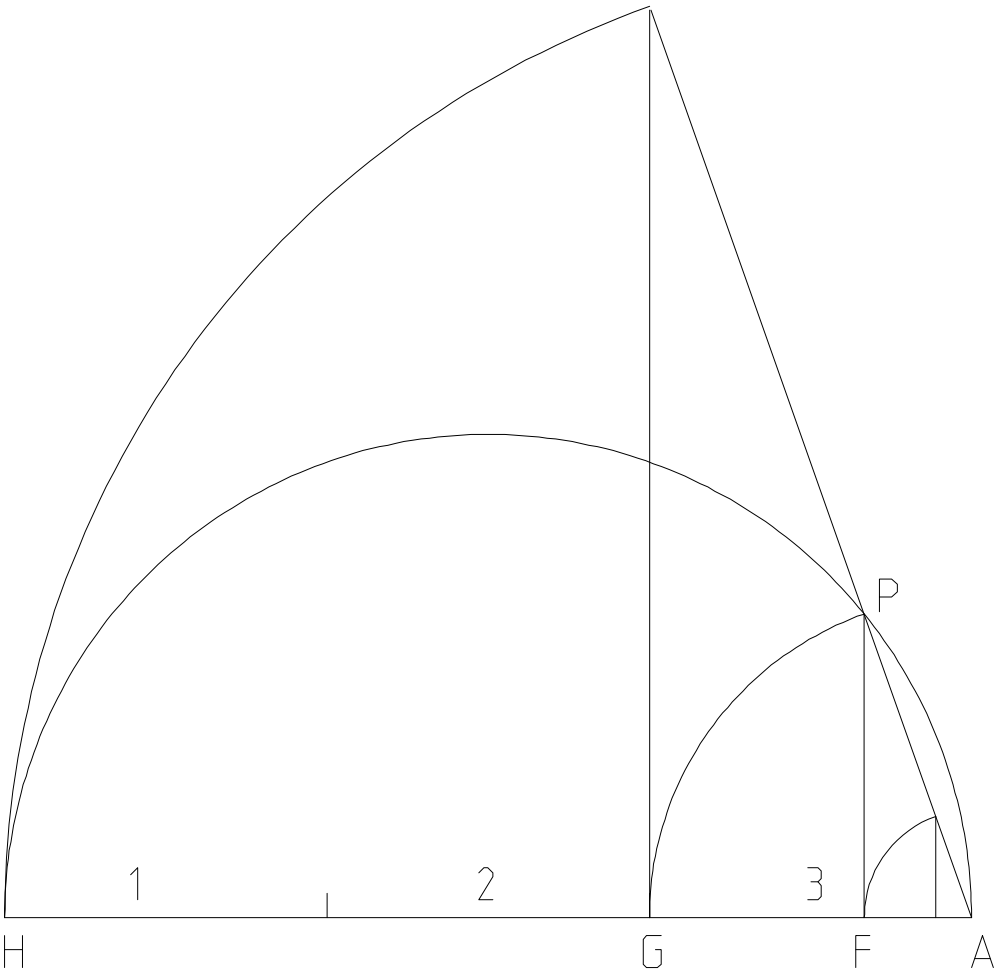
$\begin{pmatrix} AF_{\delta+1} \\ AG_{\delta+1} \end{pmatrix} := \begin{bmatrix} AG_\delta \\ \frac{AP \cdot AG_\delta}{AH} \end{bmatrix} \qquad AB_\delta := \frac{AG_1}{AF_\delta}$

| $\delta$ | $AB_\Delta$ | $BS^\delta$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 8 | 8 |
| 2 | 64 | 64 |
| 3 | 512 | 512 |
| 4 | 4096 | 4096 |
| 5 | 32768 | 32768 |
| 6 | 262144 | 262144 |
| 7 | 2097152 | 2097152 |
| 8 | 16777216 | 16777216 |
| 9 | | 134217728 |
| 10 | | $1.07374182 \cdot 10^9$ |

Making the number of divisions 3, provides 3 cube result. AB divides AF 27 times. Etc. It can be seen that using a normal straight edge and compass one needs a very large piece of paper to work this.

You will notice that I took only one of the possible two divisions from which to project from. The other would be 2/3. At 2/3 my unit divisions would still be 27, but now AB would take up 2 cube of them, or AB would be 8 units.

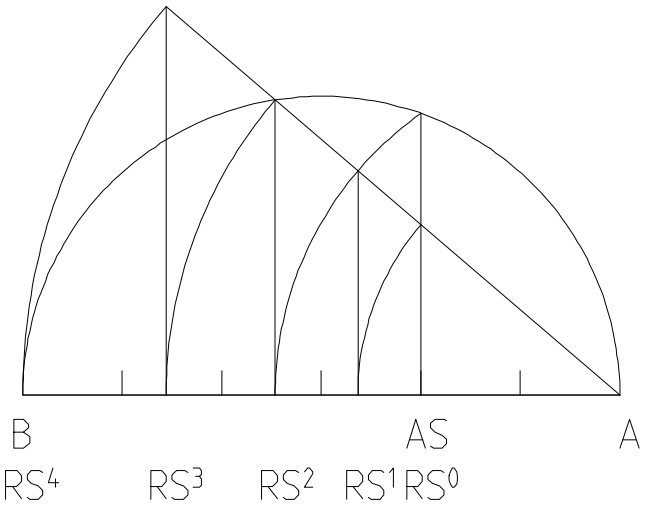
For an 8 cube series then, the value for AB would be 1 of 512, 8 of 512, 27 of 512, 64 of 512, 125 of 512, 216 of 512, 343 of 512




Since the slope is established, one can take the first two divisions and utilizes the whole figure.

$E := 1 .. BS$ $ROOT := 3$

| E | $\frac{BS^{ROOT}}{E^{ROOT}}$ | $E^{ROOT}$ |
|---|---|---|
| 1 | 512 | 1 |
| 2 | 64 | 8 |
| 3 | 18.96296296 | 27 |
| 4 | 8 | 64 |
| 5 | 4.096 | 125 |
| 6 | 2.37037037 | 216 |
| 7 | 1.49271137 | 343 |
| 8 | 1 | 512 |




# About The Laws of Exponents and Ratios

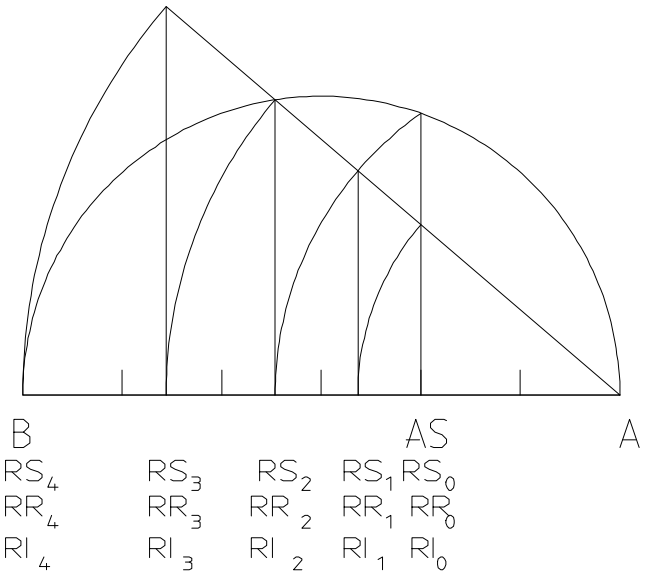
04_22_95.MCD

$\Delta := 22 \quad \delta := 1 .. \Delta \quad AB := 7$

Base Segments = BS $BS := 99$

Base Index = BI $BI := 13$

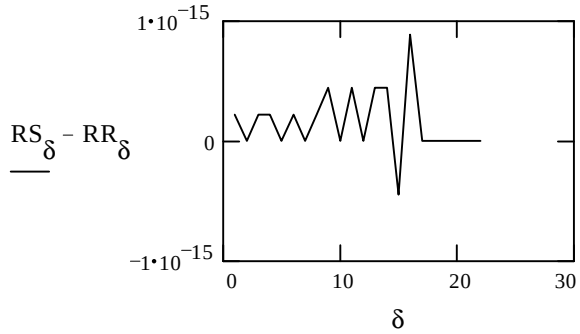
Root Series = RS $RS_\delta := \left[ \left( \frac{BI}{BS} \cdot AB \right)^{\Delta - \delta} \cdot AB^{\delta} \right]^{\frac{1}{\Delta}}$



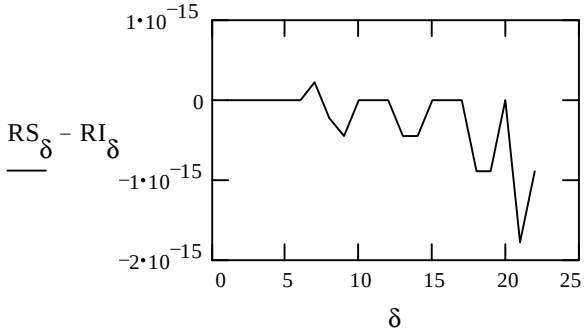

Root Series By Ratio = RR $RR_\delta := \left( \frac{BI}{BS} \right)^{\frac{\Delta - \delta}{\Delta}} \cdot AB$

Root Series By Inverse Ratio = RI

$RI_\delta := \left( \frac{BS}{BI} \right)^{\frac{\delta}{\Delta}} \cdot \left( \frac{BI}{BS} \cdot AB \right)$




$\sum_\delta \left( RS_\delta - RR_\delta \right) = 0$

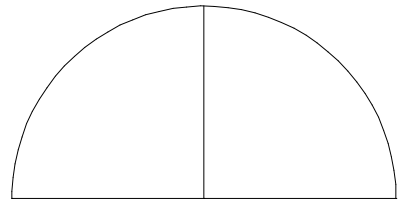



$\sum_\delta \left( RS_\delta - RI_\delta \right) = 0$

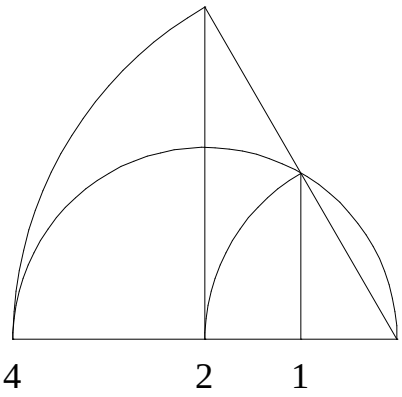
On the concept of unit and universe of discourse:

Euclidean exponentiation provides a good example with which to demonstrate the distinction between the concepts of unit and the universe of discourse.

Taking 2 for the *universe* of discourse would be represented graphically as:

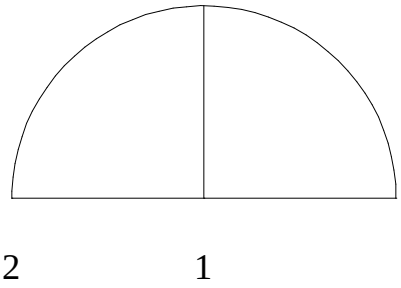



To represent $2^2$ within this universe of discourse one would draw:

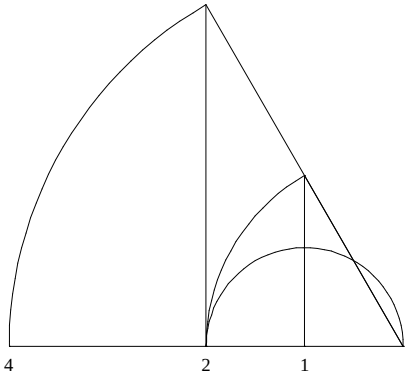



Our original 2 is divided into a $4^{th}$ segment.

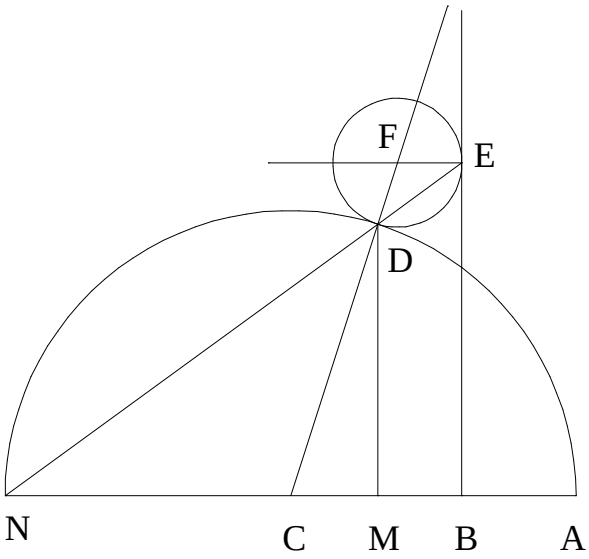
Now if 2 is taken as the *unit* of discourse we may still represent it as;



however to represent $2^3$ now would be drawn as




One will notice that in example 1, the unit changed while the universe remained, in example 2 the unit remained while the universe changed. One could call example 1 an example of deduction and example 2 an example of induction.




09_13_95.MCD

A Study in Placement.

Given AC, AB, AM find EF.

Segment A.

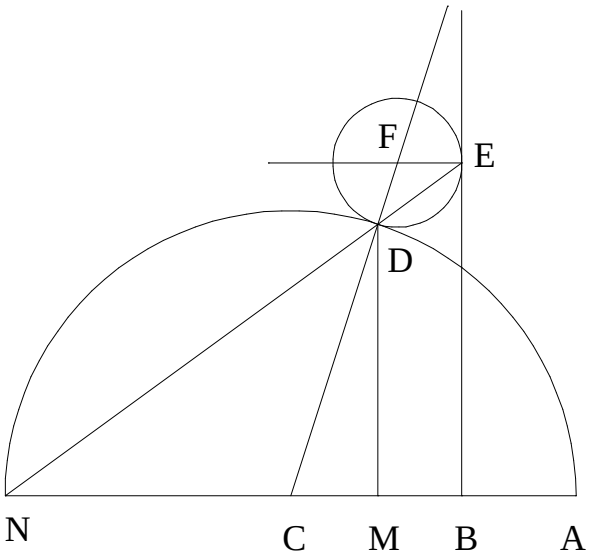
$AC := 3.125$ $AB := 4.25$ $AM := 5.34857$

$CD := AC$ $CN := AC$ $CM := AC - AM$ $DM := \sqrt{CD^2 - CM^2}$

$MN := CM + CN$ $BC := AC - AB$ $BN := CN + BC$ $DN := \sqrt{DM^2 + MN^2}$

$EN := \frac{DN \cdot BN}{MN}$ $ED := EN - DN$ $EF := \frac{AC \cdot ED}{DN}$ $EF = 3.80843$

The resultant equation in terms of the givens is $EF := \frac{AC \cdot (AM - AB)}{2 \cdot AC - AM}$ $EF = 3.80843$




Segment B.

Given AC, AB, DN, find EF.

$DN := 2.37359$

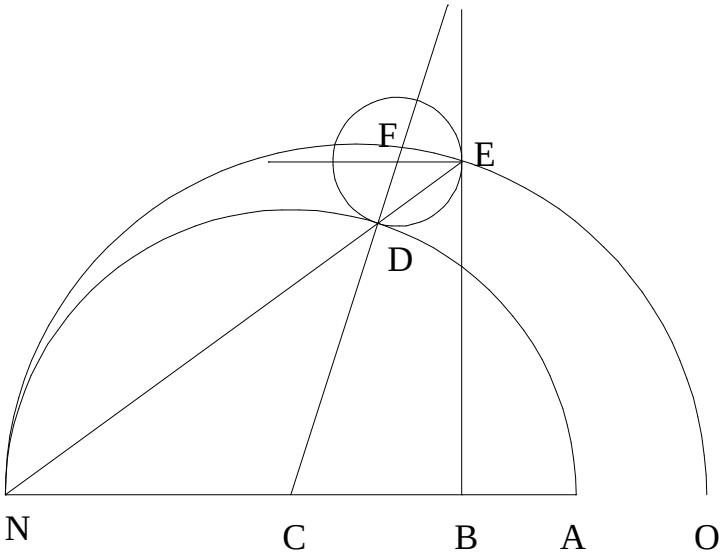
$AN := 2 \cdot AC$ $\quad MN := \frac{DN^2}{AN}$ $\quad CN := AC$ $\quad BC := AC - AB$ $\quad BN := CN + BC$

$EN := \frac{DN \cdot BN}{MN}$ $\quad ED := EN - DN$ $\quad EF := \frac{AC \cdot ED}{DN}$ $\quad EF = 3.80844$

The equation for this process is

$$EF := \frac{AC \cdot (4 \cdot AC^2 - 2 \cdot AC \cdot AB - DN^2)}{DN^2}$$

$EF = 3.80844$




Segment C.

Given AC, AB, BE, find EF.

$BE := 4.87173$

$AN := 2 \cdot AC$ $\quad BN := AN - AB$ $\quad EN := \sqrt{BE^2 + BN^2}$ $\quad ON := \frac{EN^2}{BN}$

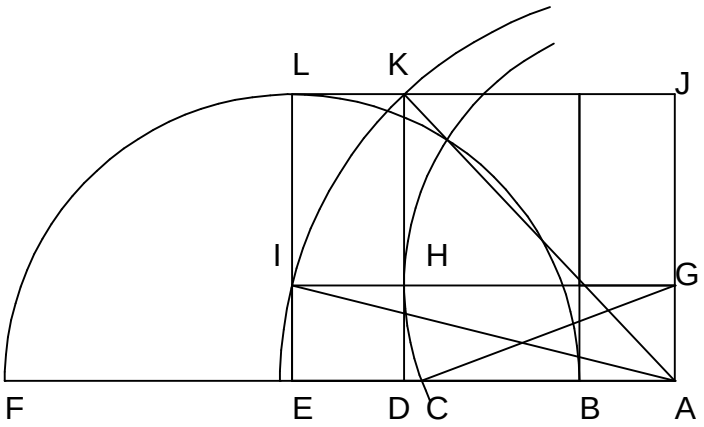
$DN := \frac{EN \cdot AN}{ON}$ $\quad DE := EN - DN$ $\quad EF := \frac{AC \cdot DE}{DN}$ $\quad EF = 3.80844$

The equation for this process is $EF := \frac{BE^2 - 2 \cdot AC \cdot AB + AB^2}{2 \cdot (2 \cdot AC - AB)}$ $\quad EF = 3.80844$

10_14_5C.MCD

Trivial Method: Square Root

Generalize the figure of 10_14_95.MCD




Starting at any point G, between A and J, the square root of AB·AF can always be projected to point C. Such a progression can be used on the cube root figure.

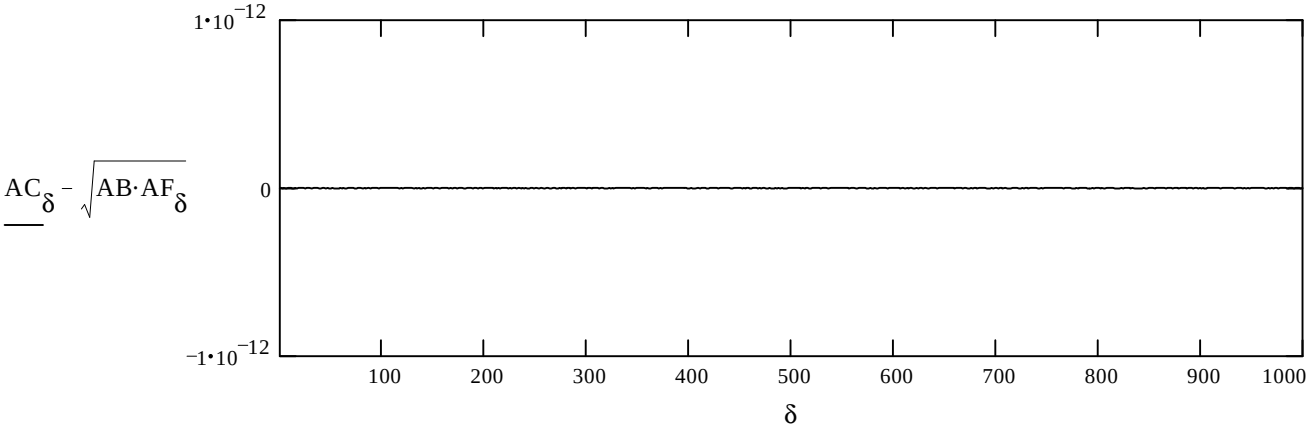
$\delta := 1 .. 1000 \quad AB := 10 \quad BF := 10 \quad BE := \frac{BF}{2}$

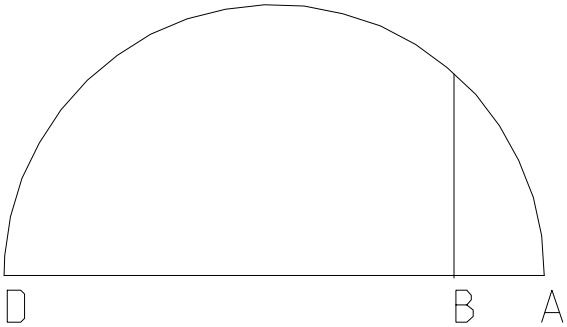
$AE := BE + AB \quad AJ := BE \quad AG_\delta := \frac{AJ}{\delta}$

$EI_\delta := AG_\delta \quad AI_\delta := \sqrt{(AE)^2 + (EI_\delta)^2}$

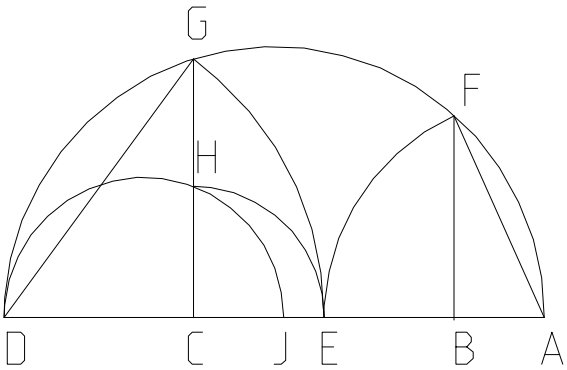
$AK_\delta := AI_\delta \quad DK := AJ \quad AD_\delta := \sqrt{(AK_\delta)^2 - DK^2} \quad GH_\delta := AD_\delta \quad CG_\delta := GH_\delta$

$AF_\delta := BF + AB \quad AC_\delta := \sqrt{(CG_\delta)^2 - (AG_\delta)^2}$






Given AB and BD divide BD such that $AB \cdot CD = \frac{BC^2}{4}$. And what is the reltionship of AC to AB and BD? Now the date on this file is not exact as I sketched this out on a piece of paper and forgot to date it.




$AB := 5 \quad BD := 1 \quad AD := AB + BD$

$BF := \sqrt{AB \cdot BD} \quad AE := \sqrt{AB^2 + BF^2}$

$DE := AD - AE \quad DG := DE \quad CD := \frac{DG^2}{AD}$

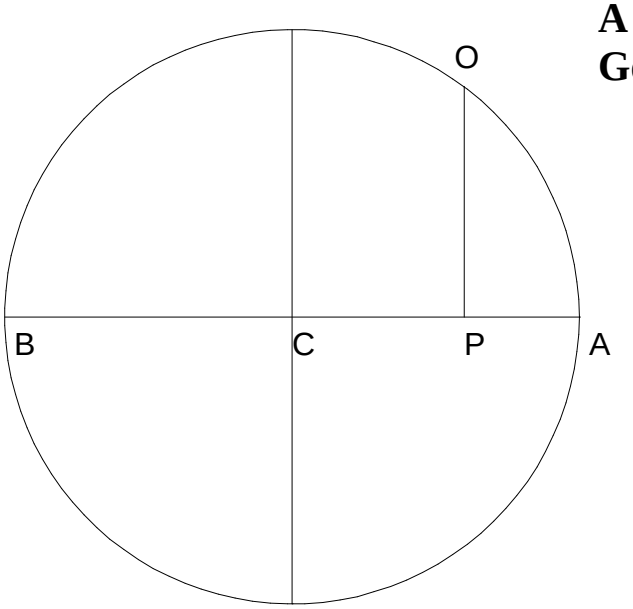
$CD = 0.046$

$BC := BD - CD \quad AB \cdot CD - \frac{BC^2}{4} = 0$

$CE := DE - CD \quad CH := CE \quad CJ := \frac{CH^2}{CD} \quad AB - CJ = 0$

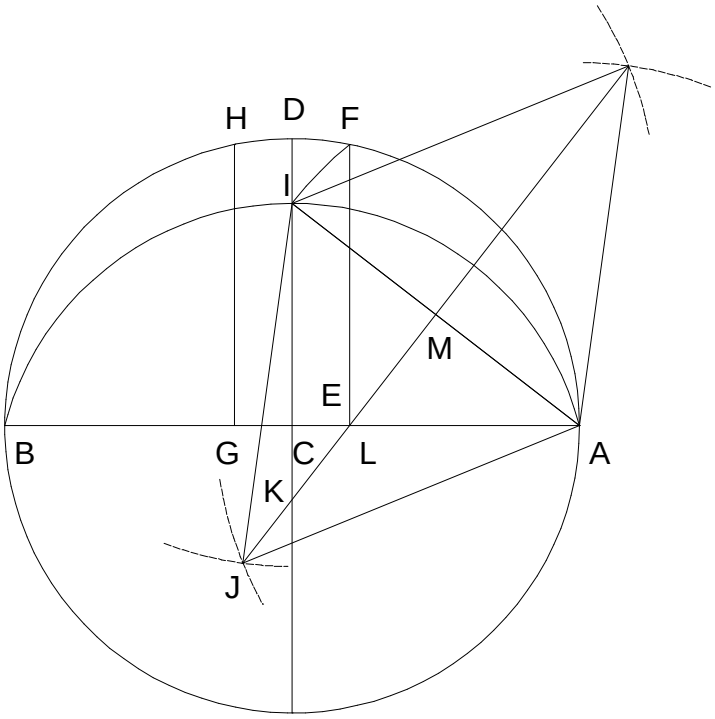
$AC := AB + BC \quad AC = 5.954$

$$AC - \left[ 2 \cdot \frac{AB^{\left(\frac{3}{2}\right)}}{\sqrt{AB + BD}} + 2 \cdot \frac{\sqrt{AB}}{\sqrt{AB + BD}} \cdot BD - AB \right] = 0$$




# A Modification of a Square Root Figure. Gemini Roots

One of the square root figures displays a one to one ratio between what could be called the vertical segment OP and the root of the two horizontal segments AP and BP. With a slight modification however, one can demonstrate a many to one relationship between three base segments.




GL has a ratio to the root of $AL \cdot BG$. Developing the arc AIC from it will give a means of keeping that ratio.

BS = Base Segments, set at end of doc.

$AB := 10 \quad AC := \frac{AB}{2} \quad EG := \frac{AB}{BS}$

$BC := AC \quad CD := AC \quad CE := \frac{EG}{2}$

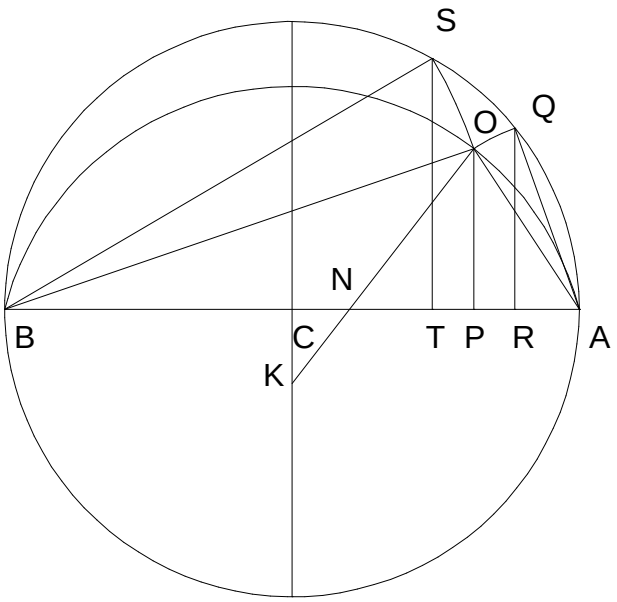
$AE := AC - CE \quad BE := BC + CE$

$EF := \sqrt{AE \cdot BE} \quad AF := \sqrt{AE^2 + EF^2}$

$AI := AF \quad AM := \frac{AI}{2} \quad CI := \sqrt{AI^2 - AC^2}$

$AL := \frac{AI \cdot AM}{AC} \quad CL := AC - AL$

$CK := \frac{AC \cdot CE}{CI} \quad IK := CI + CK$




$$\delta := 1 .. \Delta \qquad AN_\delta := \frac{AC}{\Delta}\cdot\delta$$

$$CN_\delta := AC - AN_\delta \qquad KO := IK$$

$$KN_\delta := \sqrt{CK^2 + \left(CN_\delta\right)^2} \qquad NO_\delta := KO - KN_\delta$$

$$NP_\delta := \frac{CN_\delta \cdot NO_\delta}{KN_\delta} \qquad AP_\delta := AC - CN_\delta - NP_\delta$$

$$OP_\delta := \frac{CK \cdot NO_\delta}{KN_\delta} \qquad AO_\delta := \sqrt{\left(AP_\delta\right)^2 + \left(OP_\delta\right)^2}$$

$$AQ_\delta := AO_\delta \qquad AR_\delta := \frac{\left(AQ_\delta\right)^2}{AB}$$

$$BP_\delta := BC + CN_\delta + NP_\delta$$

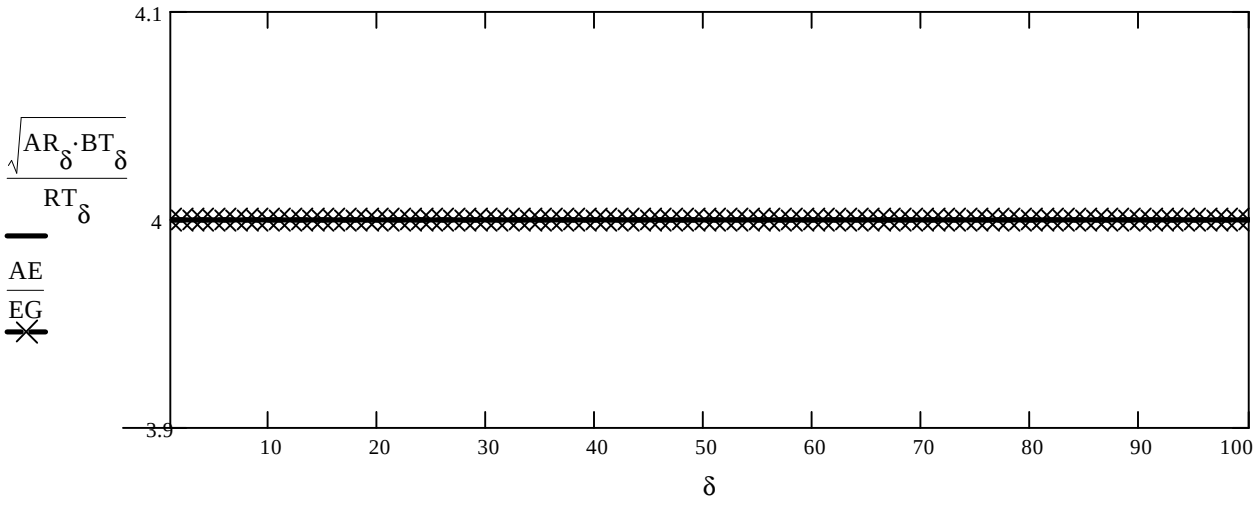
$$BO_\delta := \sqrt{\left(BP_\delta\right)^2 + \left(OP_\delta\right)^2} \qquad BS_\delta := BO_\delta \qquad BT_\delta := \frac{\left(BS_\delta\right)^2}{AB} \qquad AT_\delta := AB - BT_\delta \qquad RT_\delta := AT_\delta - AR_\delta$$

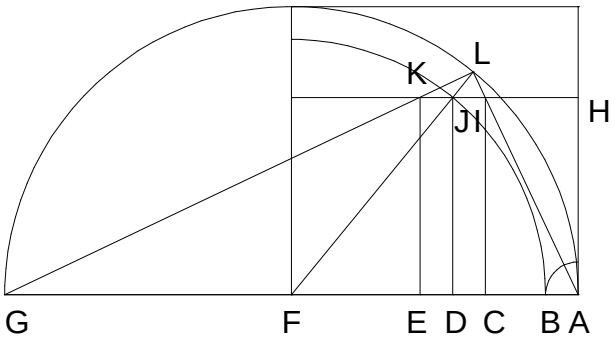
Set the number of Base Segments here and see if a constant relationship is expressed in the graph.

$BS \equiv 9$

Set the number of drawings in $\Delta$.

$\Delta \equiv 100$





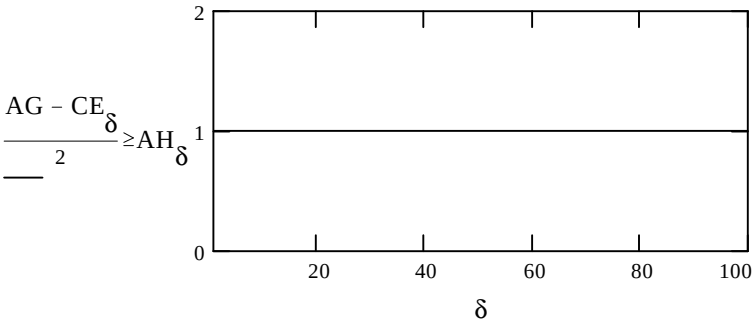

# Short Method Gemini Roots.

Given AG, CE, AH, place CE so that CE:AH as CE:CI. Or more simply that CI $= \sqrt{AC_\delta \cdot EG_\delta} = AH$.

$\delta := 1 .. 100$ $\Delta := 8$

$AG := \Delta \cdot 2 + 1$ $CE_\delta := \frac{1}{\delta}$ $AH_\delta := \Delta \cdot CE_\delta$

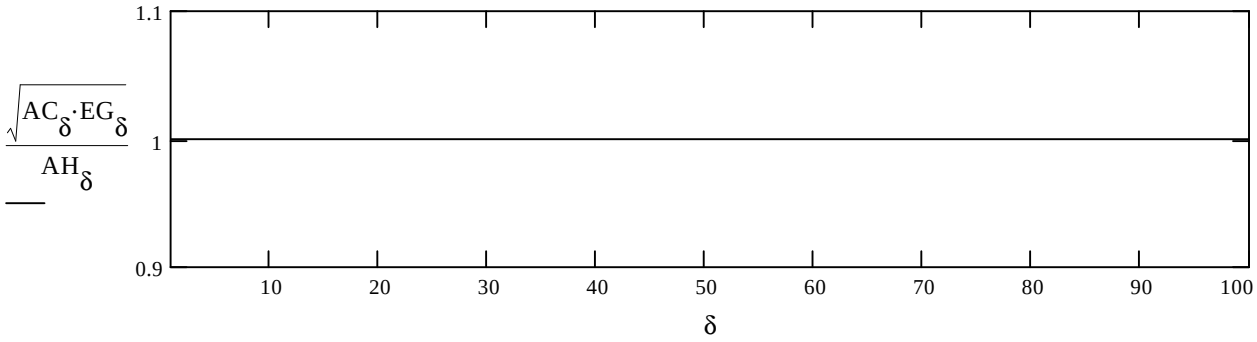
With the values given is the constuction possible? (1 for yes and 0 for no.)

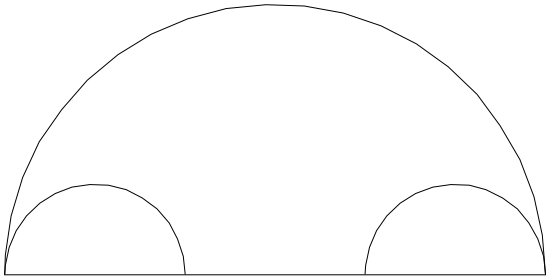



$DJ_\delta := AH_\delta$ $AB_\delta := \frac{CE_\delta}{2}$ $AF := \frac{AG}{2}$ $FG := AF$ $BF_\delta := AF - AB_\delta$ $FJ_\delta := BF_\delta$

$FD_\delta := \sqrt{(FJ_\delta)^2 - (DJ_\delta)^2}$ $DC_\delta := AB_\delta$ $DE_\delta := AB_\delta$ $AD_\delta := AF - FD_\delta$

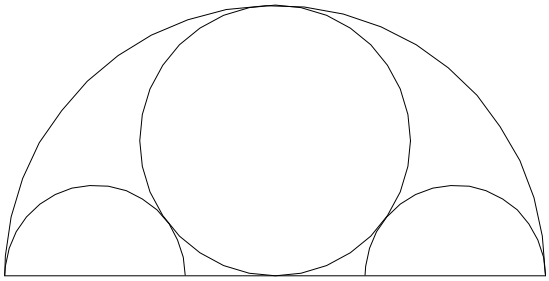
$AC_\delta := AD_\delta - DC_\delta$ $AE_\delta := AD_\delta + DE_\delta$ $EF_\delta := AF - AE_\delta$ $EG_\delta := EF_\delta + FG$

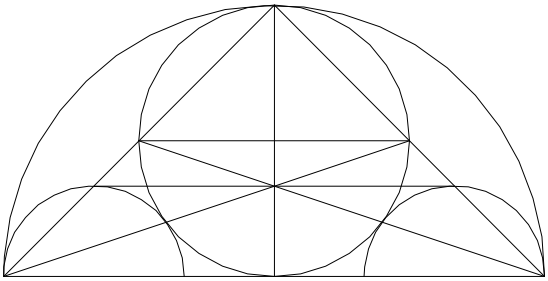


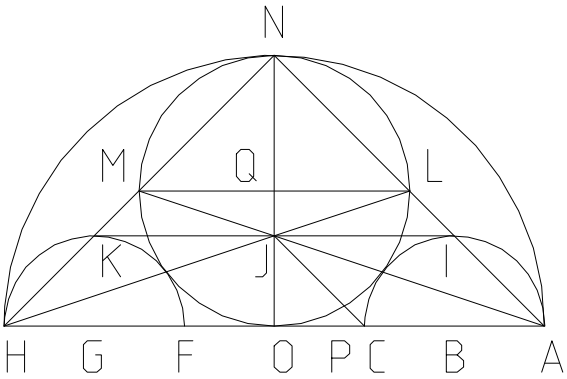


# Method for Equals.

At the inner extremities of a great circle I have two equal smaller ones. Find the circle tangent to all three








$AH := 10 \quad AC := 3 \quad AO := \frac{AH}{2} \quad AB := \frac{AC}{2}$

$JO := AB \quad OP := JO \quad HO := AO$

$JP := \sqrt{JO^2 + OP^2} \quad HP := HO + OP$

$AL := \frac{JP \cdot AH}{HP} \quad NO := AO \quad AN := \sqrt{AO^2 + NO^2}$

$LN := AN - AL \quad LQ := \frac{AO \cdot LN}{AN} \quad LQ = 2.692$

Reducing LQ as an expression of the two givens, $LQ_F := \frac{AH \cdot (AH - AC)}{2 \cdot (AH + AC)} \quad LQ_F - LQ = 0$

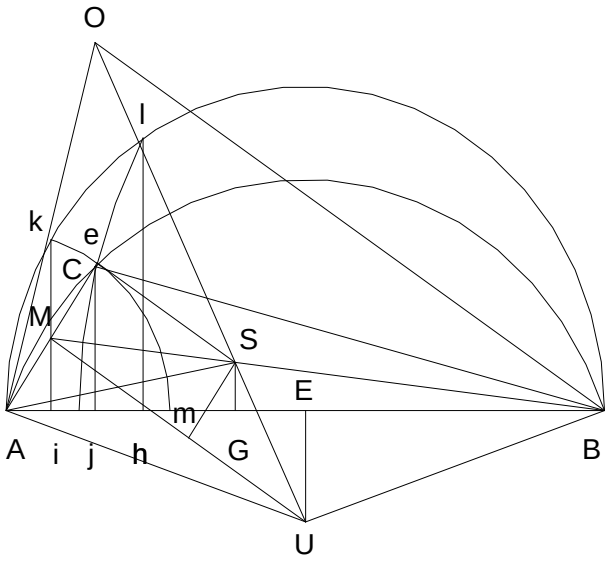
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

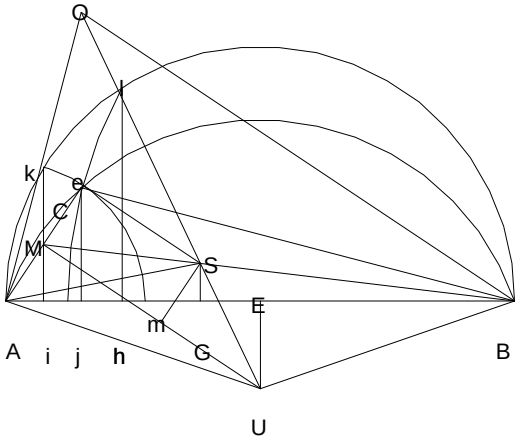
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

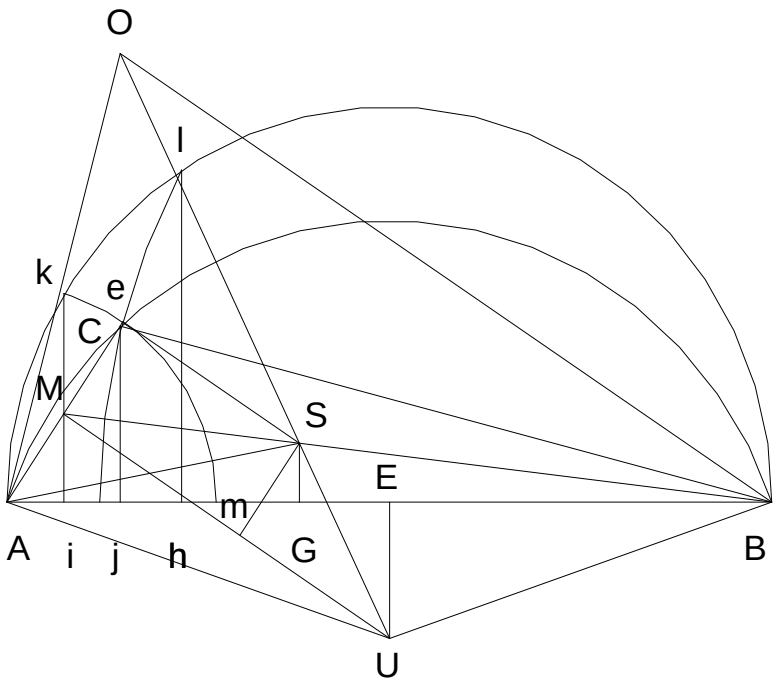
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

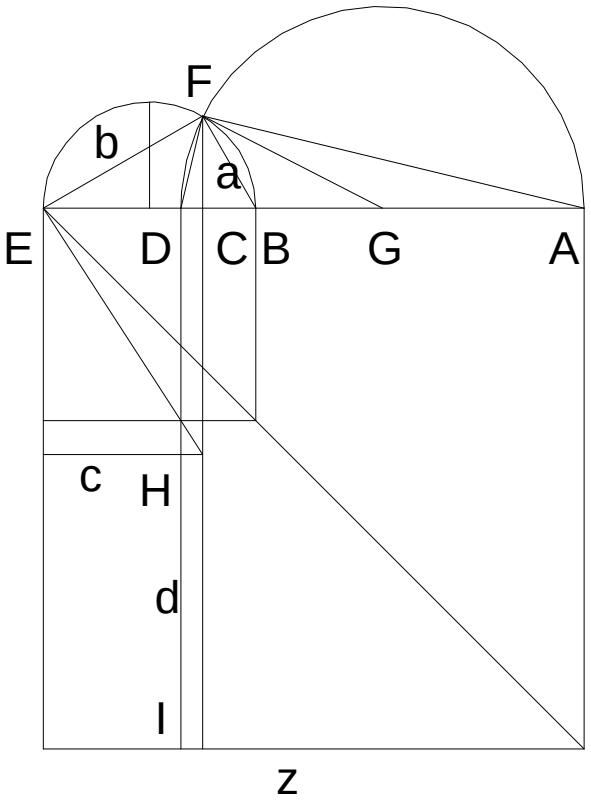



$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.




12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \frac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \frac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.622$

$c := DE$ $\quad c = 0.622$

$d := HI$ $\quad d = 6.186$

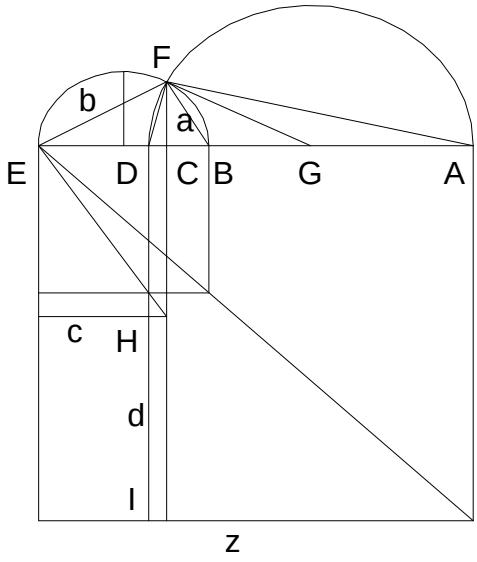
$$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

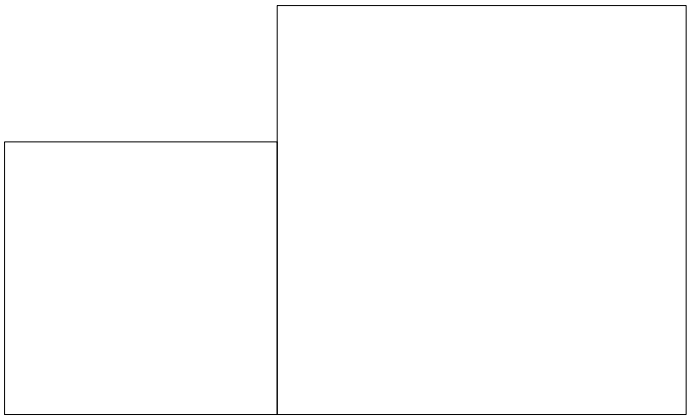
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

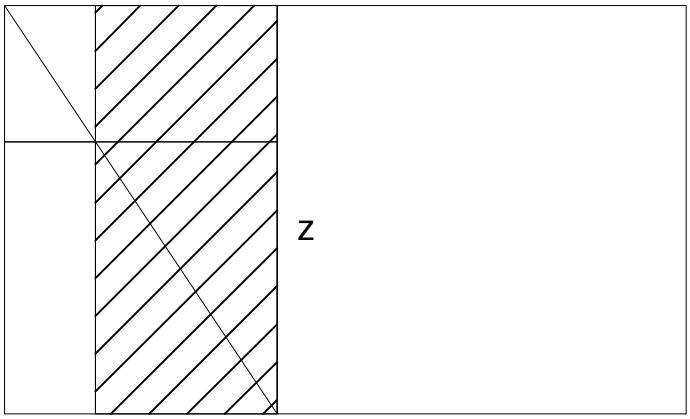
$$(c\cdot d)-p=0$$

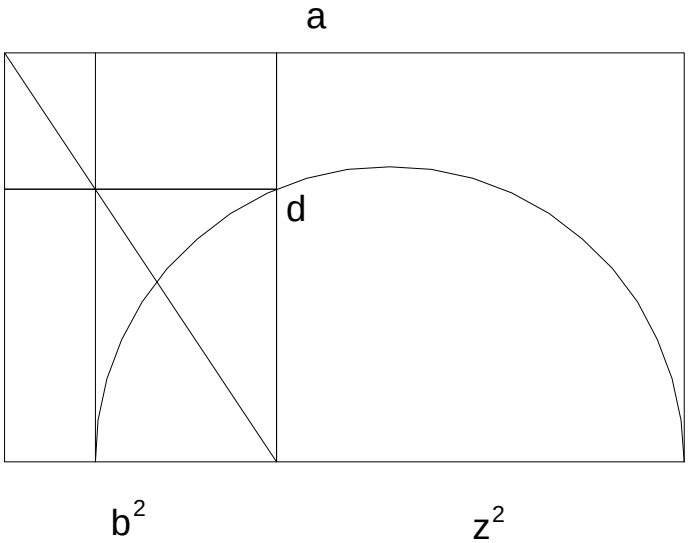
$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$




12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

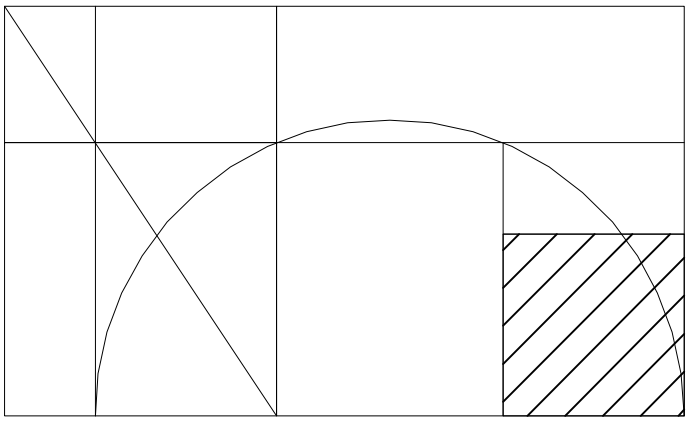
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

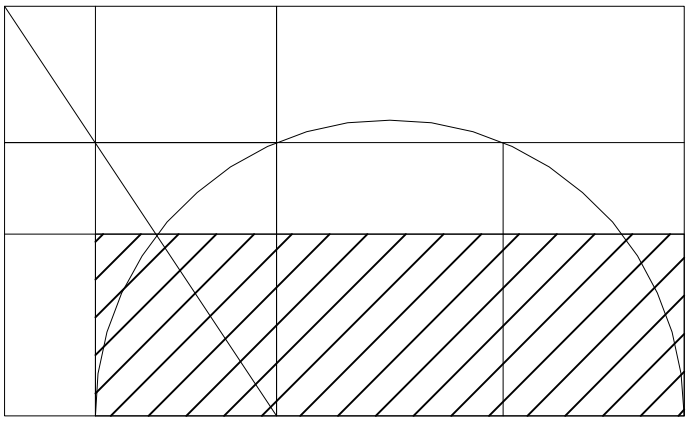
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

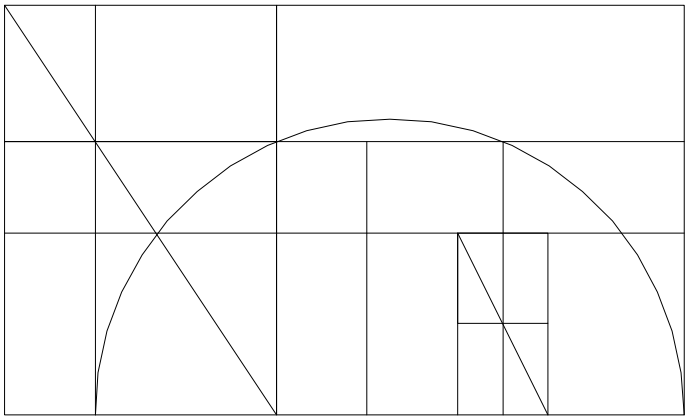
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



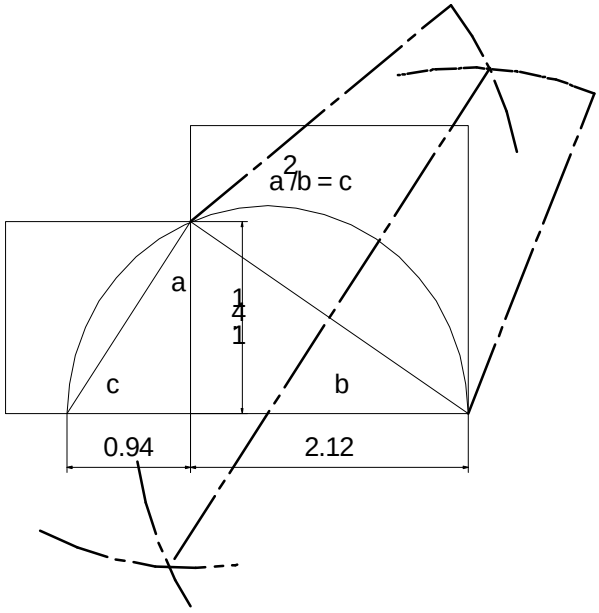
$b^2$ $z^2$



$b^2$ $z^2$

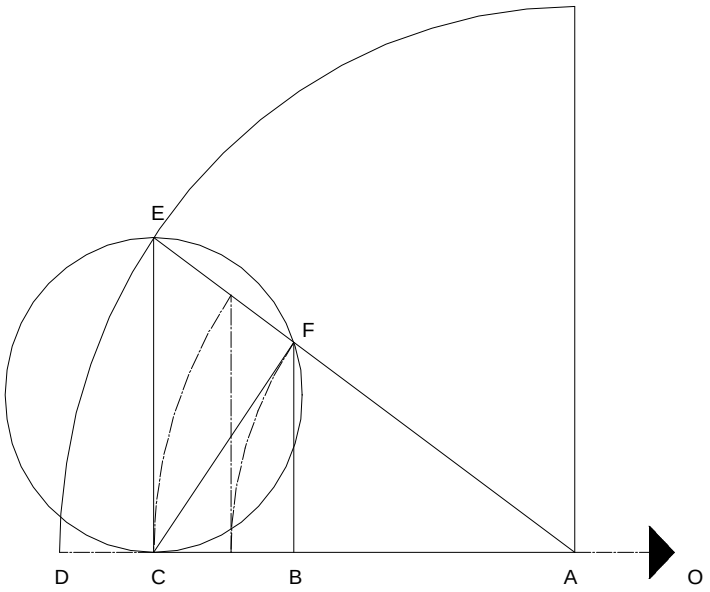


$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

## Just for fun.




$N := 1.3$

$AD := 42 \qquad AC := \frac{AD}{N} \qquad CO := AD + AC$

$CD := AD - AC \qquad CE := \sqrt{CD \cdot CO}$

$AF := \frac{AC^2}{AD} \qquad AB := \frac{AF \cdot AC}{AD}$

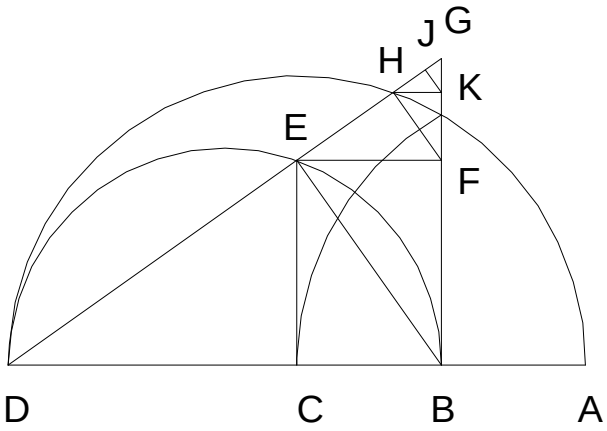
$AC^3 - \left(AD^2 \cdot AB\right) = 0$

$AB = 19.117$

$AC - \left(AD^2 \cdot AB\right)^{\frac{1}{3}} = 0$

The ratio of BC to CE is common to another cube root expression, which?

Pascal's triangle with exponential division.




$AB := 3 \qquad AD := 5$

$BD := AD - AB \qquad AC := \sqrt{AB \cdot AD}$

$CD := AD - AC \qquad BC := BD - CD$

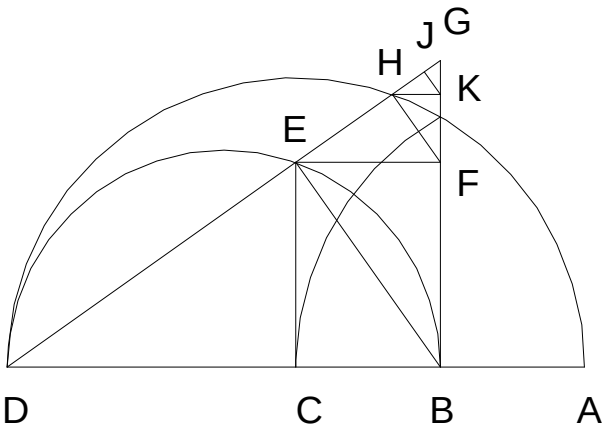
$CD := BD - BC \qquad CE := \sqrt{BC \cdot CD}$

$BE := \sqrt{BC^2 + CE^2} \qquad DE := \sqrt{CD^2 + CE^2}$

$BG := \frac{BD \cdot BE}{DE} \qquad FG := \frac{BC \cdot BG}{BD}$

$EG := \frac{BE \cdot BG}{BD} \qquad DG := DE + EG$

$GH := \frac{BC \cdot EG}{BD} \qquad GJ := \frac{BC \cdot GH}{BD}$




$\frac{BD}{BC} = 2.291 \qquad \frac{DG}{GH} = 5.249 \qquad \frac{DG}{GJ} = 12.025$

$N := 1 .. 3 \qquad \frac{\left(\sqrt{AD} + \sqrt{AB}\right)^N}{\sqrt{AB^N}}$

| |
|---|
| 2.291 |
| 5.249 |
| 12.025 |

$\sqrt{AD} + \sqrt{AB}$

$\frac{A + B}{A}$

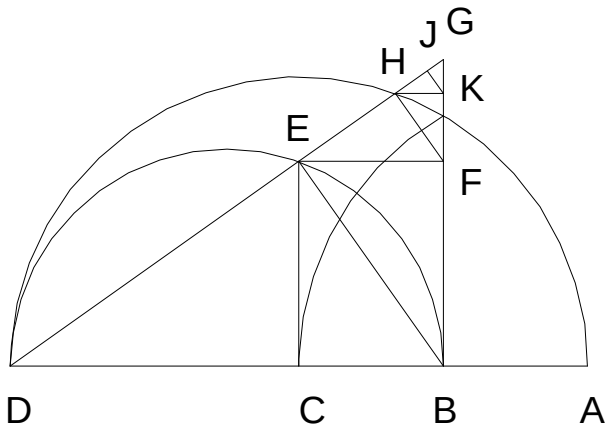
$\left(\sqrt{AD}\right)^2 + 2 \cdot \sqrt{AB} \cdot \sqrt{AD} + \left(\sqrt{AB}\right)^2$

$\frac{A^2 + 2AB + B^2}{A^2}$

$\left(\sqrt{AD}\right)^3 + 3 \cdot AD \cdot \sqrt{AB} + 3 \cdot \sqrt{AD} \cdot AB + \left(\sqrt{AB}\right)^3$

$\frac{A^3 + 3A^2 \cdot B + 3A \cdot B^2 + B^3}{A^3}$

Dividing an exponentiated integer by an exponentiated integer of the same power, straight edge and compass construction. Followed by who knows what!




$CD := 1.5 \qquad BC := .75 \qquad BD := BC + CD$

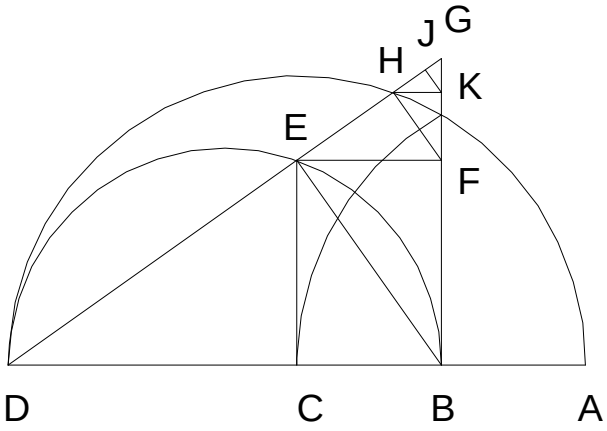
$CE := \sqrt{BC \cdot CD} \qquad BE := \sqrt{BC^2 + CE^2}$

$EF := BC \qquad FG := \frac{BC \cdot EF}{CE} \qquad FG = 0.53$

$DE := \sqrt{CD^2 + CE^2} \qquad EG := \frac{BD \cdot EF}{DE}$

$GH := \frac{BC \cdot EF}{DE} \qquad DG := DE + EG$

$JK := \frac{FG \cdot GH}{EG} \qquad GJ := \frac{BC \cdot GH}{BD} \qquad FH := \frac{CE \cdot EG}{BD} \qquad \frac{BD}{BC} = 3 \qquad \frac{DG}{GH} = 9 \qquad \frac{DG}{GJ} = 27$



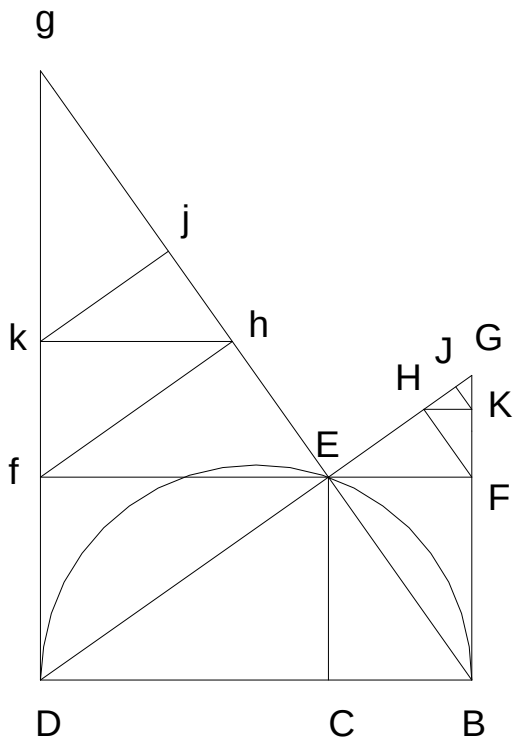

$n := 1..3$

| $\frac{a^n}{b^n}$ | $\frac{BD^n}{BC^n}$ |
|---|---|
| | 3 |
| | 9 |
| | 27 |

$BC + CD = 2.25$ $\sqrt{DG \cdot DE} = 2.25$

$\frac{\sqrt{BC + CD} \cdot BC}{\sqrt{CD}} = 0.919$ $\sqrt{DG \cdot GH} = 0.919$

$\frac{\left(\sqrt{BC}\right)^3}{\sqrt{CD}} = 0.53$ $\sqrt{DG \cdot GJ} = 0.53$




$Ef := CD$ $Eg := \frac{BD \cdot Ef}{BE}$ $fh := \frac{CE \cdot Ef}{BE}$

$hk := \frac{CE \cdot fh}{BE}$ $hj := \frac{BC \cdot hk}{BE}$ $Eg := \frac{BD \cdot Ef}{BE}$

$Bg := BE + Eg$ $hg := \frac{CD \cdot Ef}{BE}$ $jg := \frac{CD \cdot hk}{BE}$

$\frac{BD}{CD} = 1.5$ $\frac{Bg}{hg} = 2.25$ $\frac{Bg}{jg} = 3.375$ $\left(\frac{BD}{CD}\right)^n$

| |
|---|
| 1.5 |
| 2.25 |
| 3.375 |

$\frac{Bg}{EG} = 4.243$ $\frac{Bg}{GH} = 12.728$ $\frac{Bg}{GJ} = 38.184$ $\sqrt{CD} \cdot \frac{(BC + CD)^n}{\left(\sqrt{BC}\right)^{2 \cdot n + 1}}$

| |
|---|
| 4.243 |
| 12.728 |
| 38.184 |

$n := 1 .. 4$

$\sqrt{Bg \cdot GJ} = 0.631$

$\sqrt{Bg \cdot GH} = 1.092$

$\sqrt{Bg \cdot EG} = 1.892$

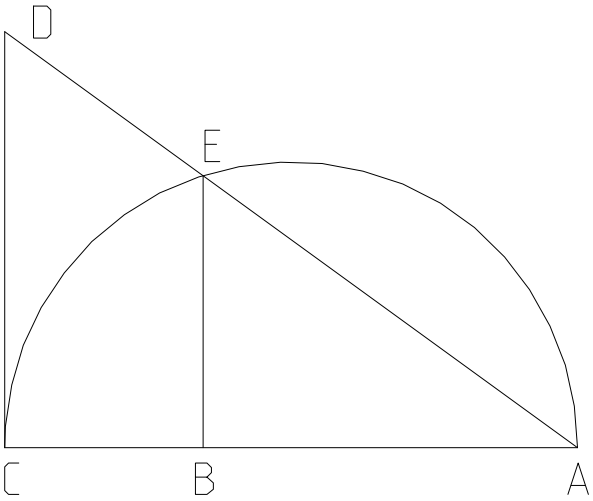
$\sqrt{Bg \cdot DG} = 3.277$

$\frac{BC^{\left(\frac{7}{4}\right)}}{CD^{\left(\frac{1}{4}\right)} \cdot \sqrt{BC + CD}} \cdot \frac{\left(\sqrt{BC + CD}\right)^n}{\left(\sqrt{BC}\right)^n}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |

$\frac{BC^{\left(\frac{7}{4} - \frac{1}{2} \cdot n\right)}}{CD^{\left(\frac{1}{4}\right)}} \cdot (BC + CD)^{\left(\frac{-1}{2} + \frac{1}{2} \cdot n\right)}$

| |
|---|
| 0.631 |
| 1.092 |
| 1.892 |
| 3.277 |




Given AC and CD find BC when it is equal to $\frac{AC \cdot CD^2}{AC^2 + CD^2}$.

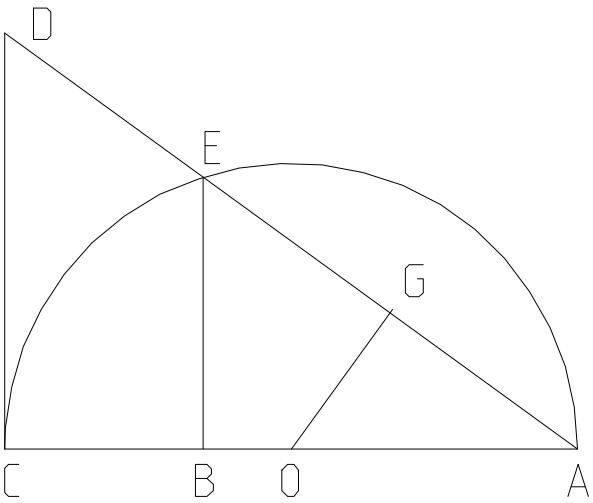
$AC := 15 \quad CD := 5 \quad AO := \frac{AC}{2}$

$AD := \sqrt{AC^2 + CD^2}$

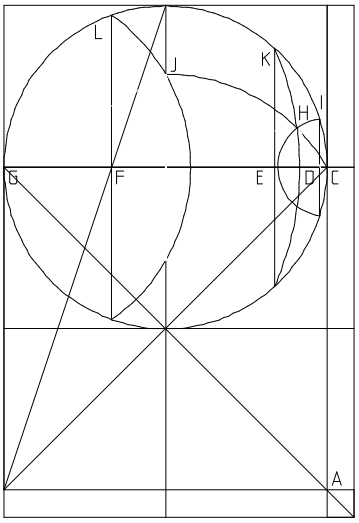
$AG := \frac{AC \cdot AO}{AD} \quad AE := 2 \cdot AG$

$AB := \frac{AC \cdot AE}{AD} \quad BC := AC - AB \quad BC = 1.5$

$BC := \frac{AC \cdot CD^2}{\left(AC^2 + CD^2\right)} \quad BC = 1.5$




A cube divided by the sum of two squares.




# One Circle

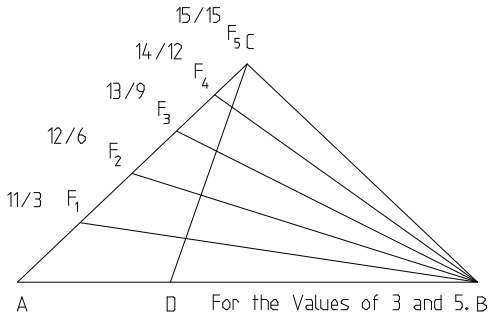
**By John Clark**

# One Square

# One Line

## And the Delian Quest

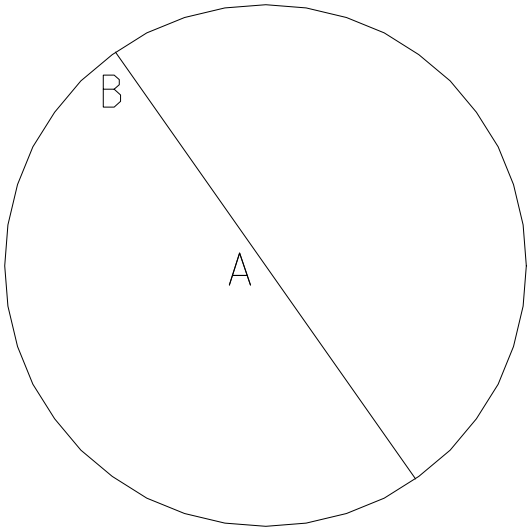
# 1996



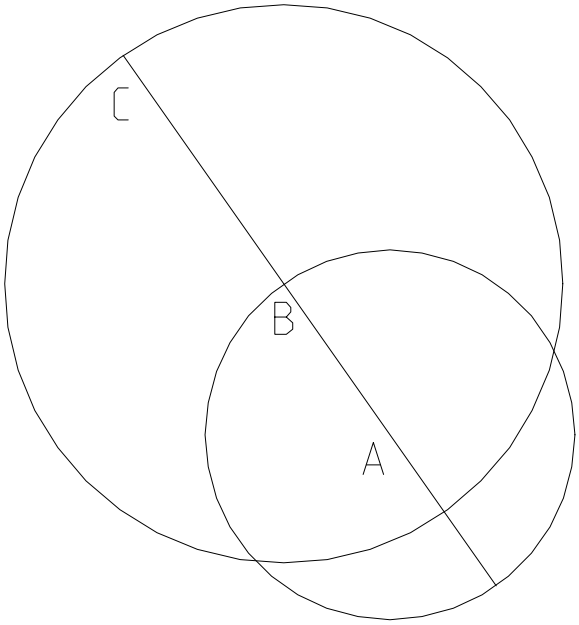
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

## The Archamedian Paper Trisector- Without the Numbers.

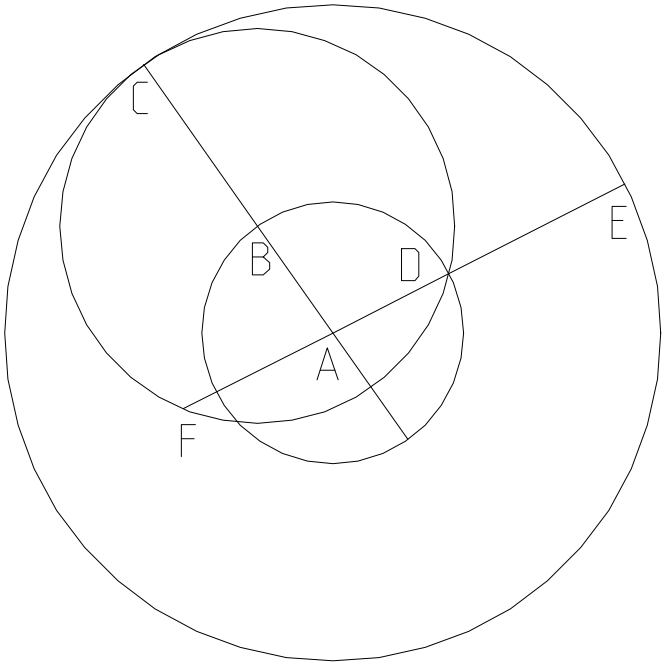
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.




Given any circle AB

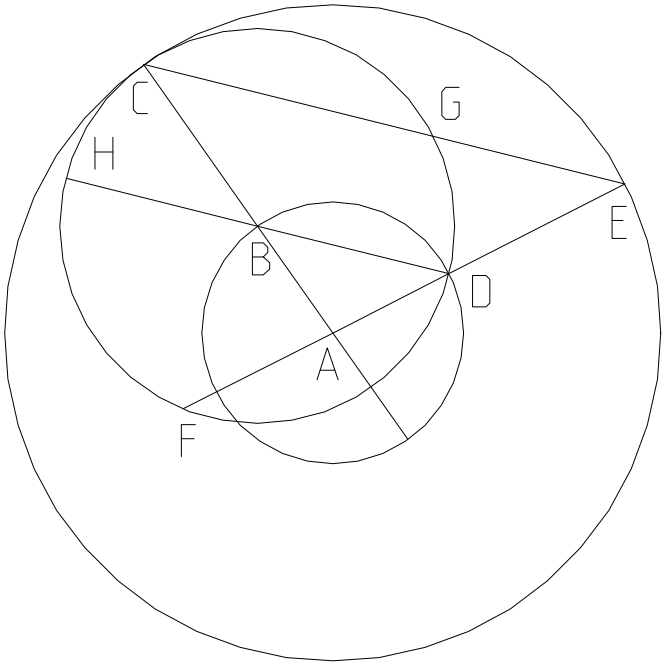



Given any circle BC such that BC≤ 2AB.

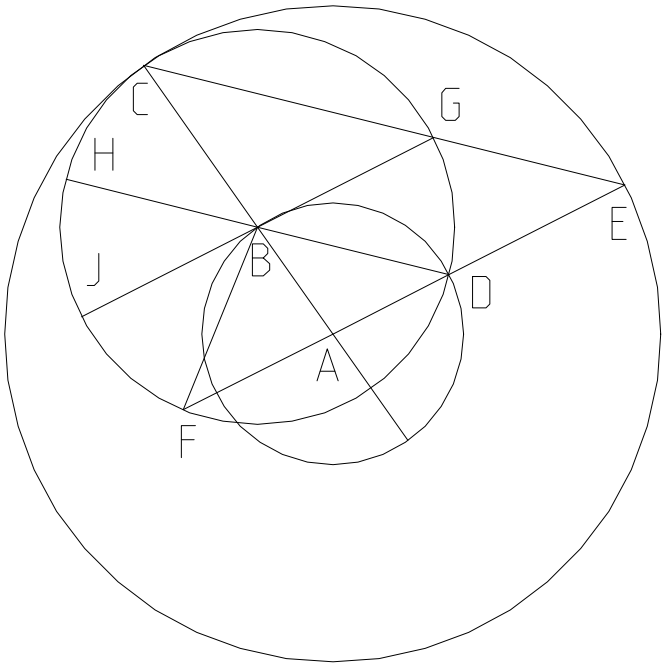



Construct AE such that AE = AC.

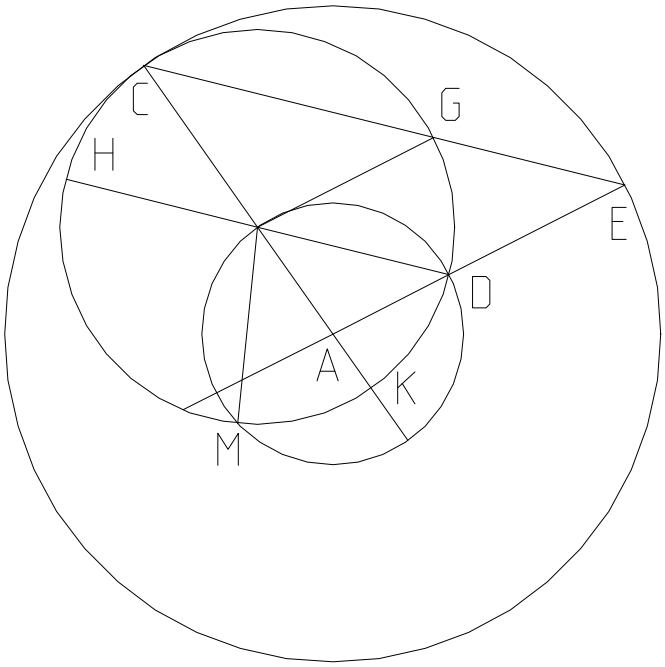
Since AC = AB+ BC and AD = AB, DE = BC.




Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



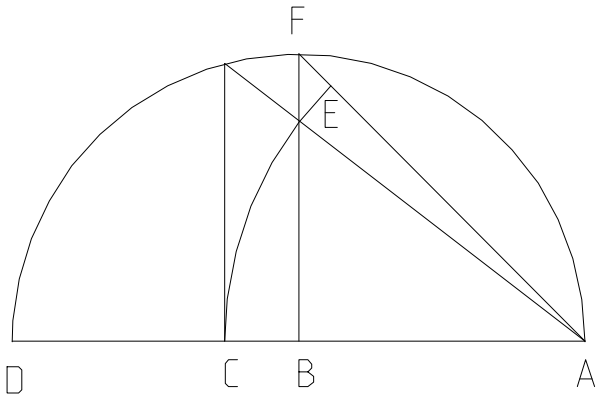

Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.




By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

A rusty Compass construction for the duplication of the cube.

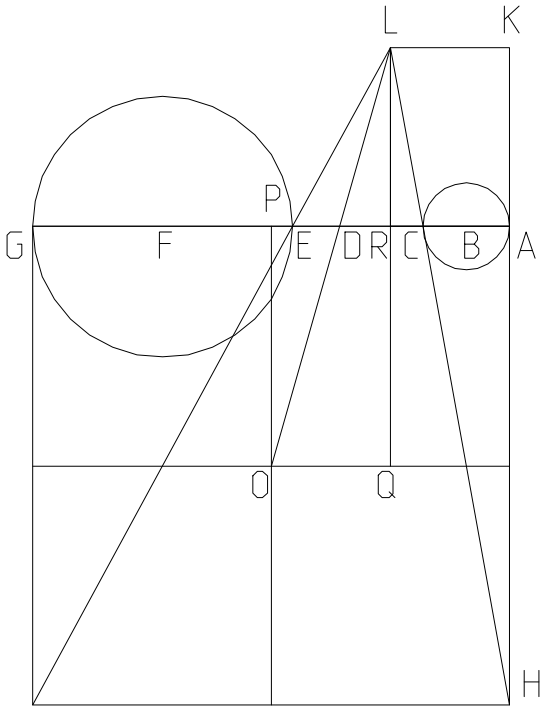



$AD := 2 \quad AB := \frac{AD}{2} \quad AF := \sqrt{2 \cdot AB^2} \quad AE := \frac{AF}{9} \cdot 8$

$AC := AE \quad AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26 \qquad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.




# Alternate Method, Power Line.

Given AB, EF, BF, find the power line intersection on BF. Looking back to 94, it seems I never derived a formula for it either.

$AB := R_1 \quad BF := D \quad EF := R_2$

$BC := AB \quad AC := AB + BC \quad FG := EF$

$EG := EF + FG \quad AG := AB + BF + FG \quad AH := AG$

$GJ := AG \quad AP := \frac{AG}{2} \quad CE := BF - (BC + EF)$

$LR := \frac{AH \cdot CE}{AC + EG} \quad PO := AP \quad AK := LR$

$KL := \frac{AC \cdot (AH + AK)}{AH} \quad AR := KL$

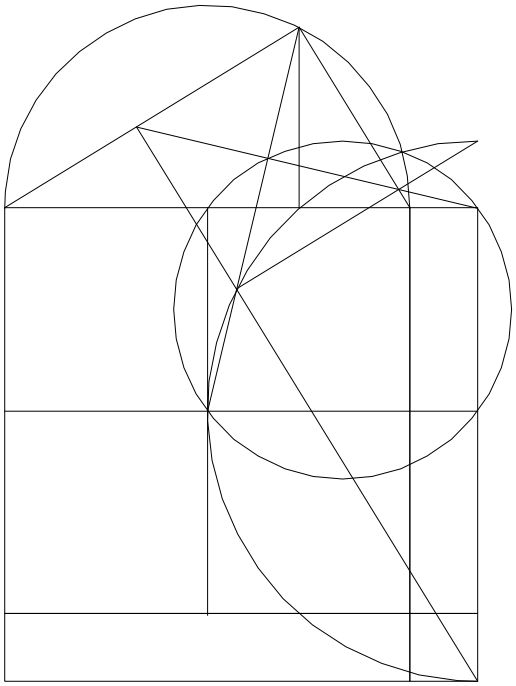
$PR := AP - AR \quad OQ := PR \quad QR := PO$

$LQ := QR + LR \quad DR := \frac{OQ \cdot LR}{LQ} \quad AD := AR + DR$

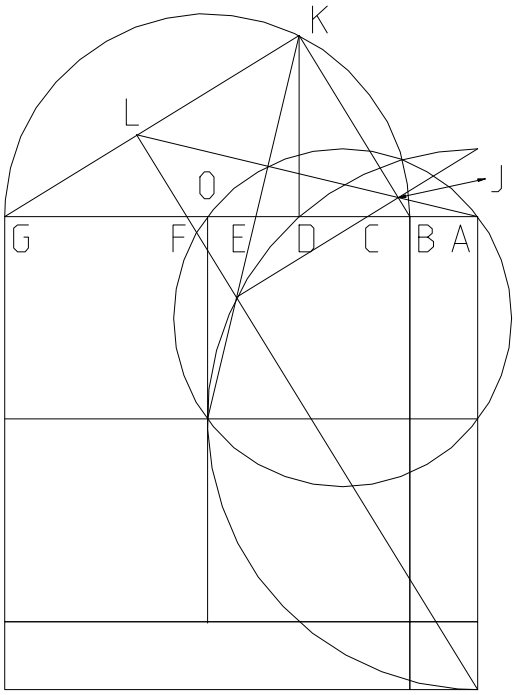
$AD = 25.333$ Plug values in below.

$R_1 \equiv 9 \quad R_2 \equiv 1 \quad D \equiv 30$

$$\frac{(R_1 + D + R_2) \cdot (D - R_2 + R_1)}{2 \cdot D} = 25.333$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.




$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

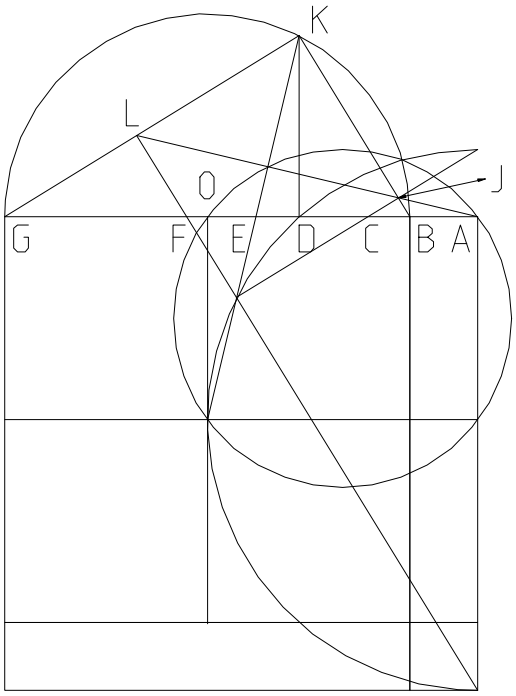
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



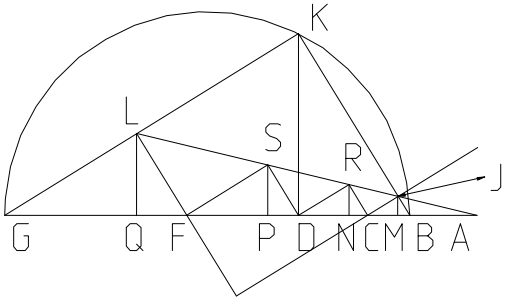

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$




$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$

$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

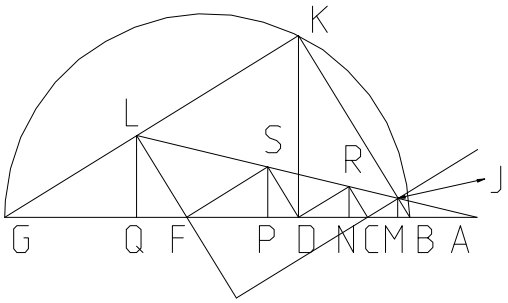
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$

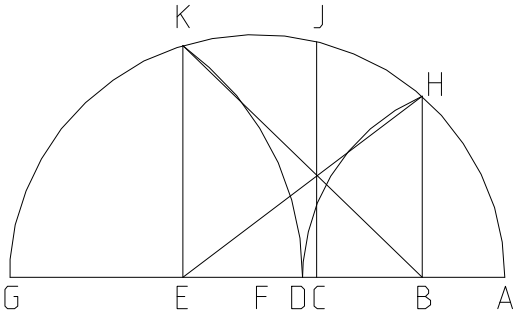



$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

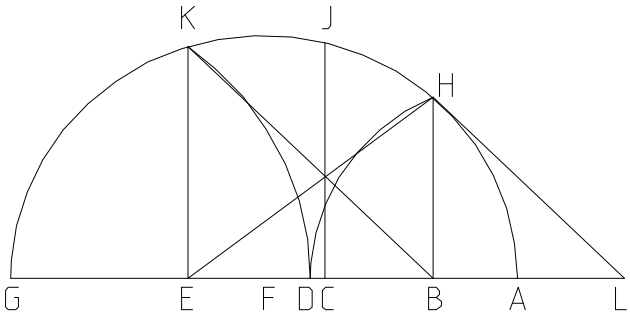
$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$




If the figure was drawn differently, XC would be $\sqrt{XB \cdot XE}$, irregardless of how XB and XE were placed, however that would require part of the figure that is not given here.




$$AG := N \quad AB := \frac{AG}{N}$$

$$BG := AG - AB \quad BH := \sqrt{AB \cdot BG}$$

$$AH := \sqrt{AB^2 + BH^2} \quad AD := AH \quad DG := AG - AD$$

$$GK := DG \quad GE := \frac{GK^2}{AG} \quad AE := AG - GE$$

$$BE := AE - AB$$

$$EG := AG - AE \quad EK := \sqrt{AE \cdot EG} \quad BL := \frac{BE \cdot BH}{EK}$$

$$EL := BE + BL \quad BC := \frac{BL \cdot BE}{EL} \quad AC := AB + BC$$

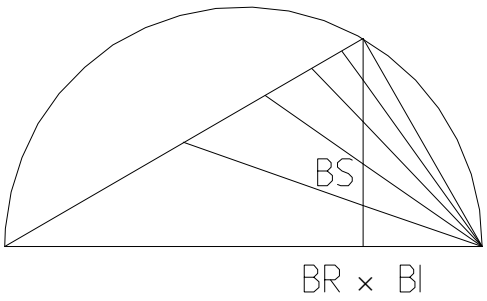
$N \equiv 4$

Make N any number and watch the equations, then make it equal to 1 and see what happens. Now this is strange work, for the formula is an identity with AC, so what happens at 1? This is an example of Binary contradiction.

$AC = 2$

$$\frac{\begin{bmatrix} AB^{\left(\frac{5}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{7}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + 2 \cdot AB \cdot \sqrt{BG} \cdot (AB + BG)^{\left(\frac{3}{2}\right)} \ldots \\ + - AB^{\left(\frac{5}{2}\right)} \cdot \sqrt{BG} - AB^{\left(\frac{3}{2}\right)} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}}{\begin{bmatrix} AB^{\left(\frac{1}{4}\right)} \cdot (AB + BG)^{\left(\frac{5}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} \ldots \\ + - AB^{\left(\frac{3}{4}\right)} \cdot (AB + BG)^{\left(\frac{3}{4}\right)} \cdot \sqrt{2 \cdot AB + 2 \cdot BG - \sqrt{AB} \cdot \sqrt{AB + BG}} + AB^{\left(\frac{3}{2}\right)} \cdot \sqrt{BG} + \sqrt{AB} \cdot BG^{\left(\frac{3}{2}\right)} \end{bmatrix}} = 2$$

# Pyramid of Ratios, Moving the Point



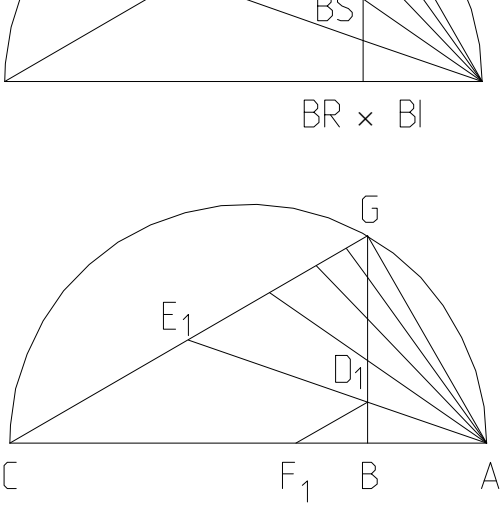

BR = Base Ratio, BS = Bisector Segments, BI = Base Index.

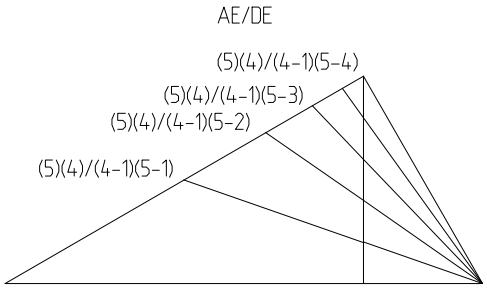
$BR \equiv 4 \quad BS \equiv 5 \quad BI := 2 \quad AC := BR \quad AB := \frac{AC}{BR} \cdot BI$

$BC := AC - AB \quad BG := \sqrt{AB \cdot BC} \quad \delta := 1 .. BS - 1$

$BD_\delta := \frac{BG}{BS} \cdot \delta \quad BF_\delta := \frac{BC \cdot BD_\delta}{BG} \quad AD_\delta := \sqrt{AB^2 + (BD_\delta)^2}$

$AE_\delta := \frac{AD_\delta \cdot AC}{AB + BF_\delta} \quad DE_\delta := AE_\delta - AD_\delta$



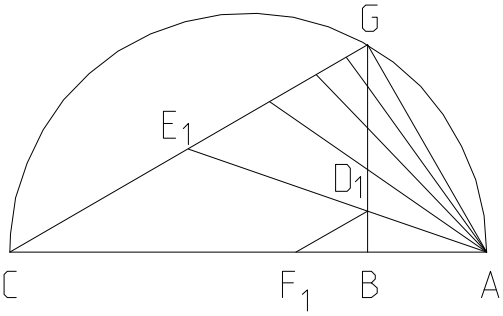




$\frac{AE_\delta}{DE_\delta}$

| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |

$\frac{BS \cdot BR}{(BR - BI) \cdot (BS - \delta)}$

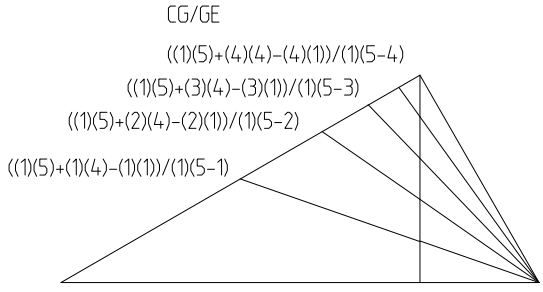
| |
|---|
| 2.5 |
| 3.333 |
| 5 |
| 10 |




$AF_\delta := AB + BF_\delta \quad EJ_\delta := \frac{BD_\delta \cdot AC}{AF_\delta}$

$AJ_\delta := \sqrt{(AE_\delta)^2 - (EJ_\delta)^2} \quad CJ_\delta := AC - AJ_\delta$

$CE_\delta := \sqrt{(CJ_\delta)^2 + (EJ_\delta)^2} \quad CG := \sqrt{BG^2 + BC^2}$




$GE_\delta := CG - CE_\delta$

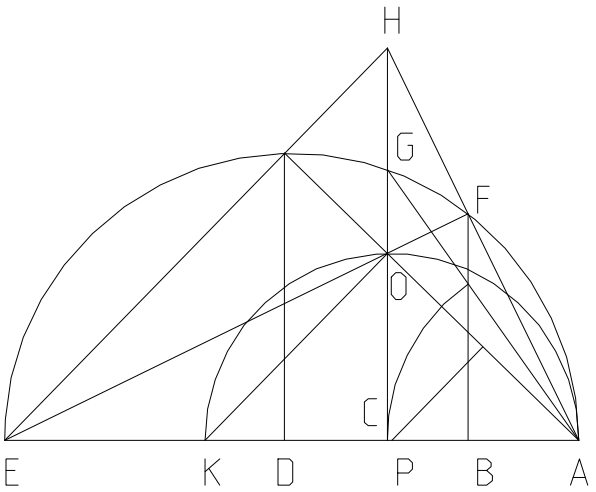
$\frac{CG}{GE_\delta}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

$\frac{(BI \cdot BS + \delta \cdot BR - \delta \cdot BI)}{(BI \cdot (BS - \delta))}$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |

What is AD. What is BD to $\sqrt{AB \cdot DE}$ ?




$AE := 5.5 \quad AB := 1.05 \quad BE := AE - AB$

$AC := \left(AB^2 \cdot AE\right)^{\frac{1}{3}} \quad CE := AE - AC$

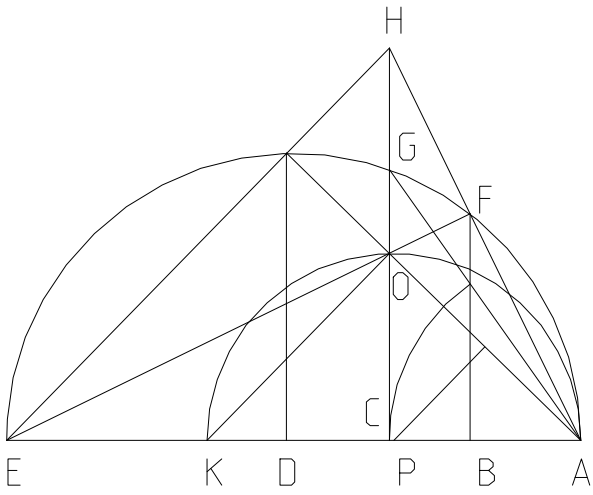
$BF := \sqrt{AB \cdot BE} \quad CO := \frac{BF \cdot CE}{BE} \quad AO := \sqrt{AC^2 + CO^2}$

$AP := \frac{1}{2} \cdot \frac{AO^2}{AC} \quad AK := 2 \cdot AP \quad AD := \frac{AC \cdot AE}{AK}$

$DE := AE - AD \quad BD := AD - AB$

$AD = 2.807$

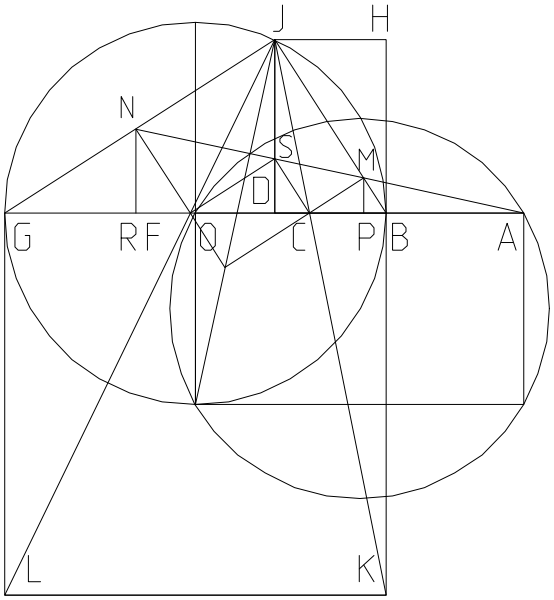
$$\frac{AE^{\frac{3}{3}} \cdot AB^{\frac{1}{3}} + AE^{\frac{2}{3}} \cdot AB^{\frac{2}{3}} + AE^{\frac{1}{3}} \cdot AB^{\frac{3}{3}}}{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}} = 2.807$$




$$\frac{\sqrt{AB \cdot DE}}{BD} = 0.957$$

$$\frac{AB^{\frac{1}{6}} \cdot AE^{\frac{1}{6}} \cdot \sqrt{AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} + AE - AE^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} \cdot \sqrt{2 \cdot AB^{\frac{1}{3}} + AE^{\frac{1}{3}}}}{AE + AE^{\frac{2}{3}} \cdot AB^{\frac{1}{3}} - 2 \cdot AB} = 0.957$$

The figure cuts the base in Cube Roots and provides some interesting ratios.




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

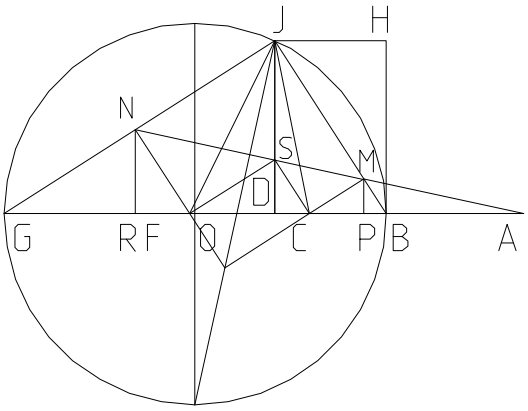
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

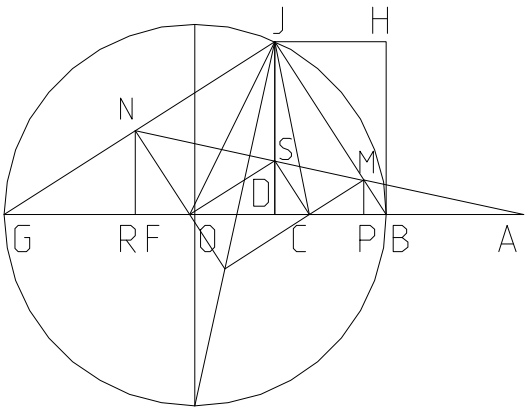
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

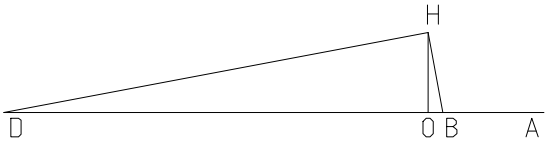
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$




$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

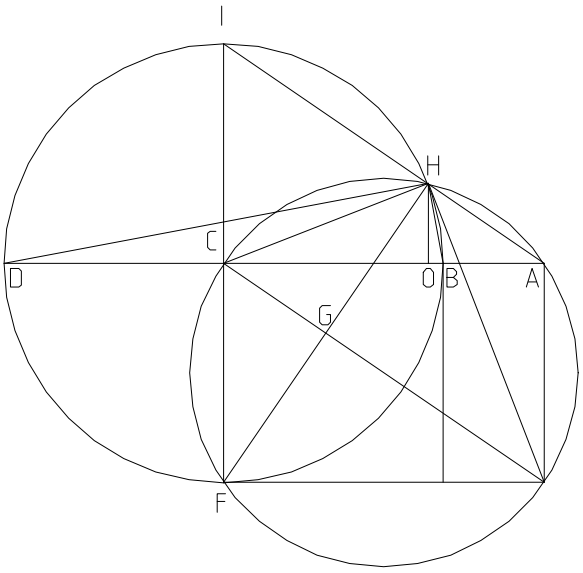
$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$




Given AD and AB on AD, place a right triangle on BD as base such that the opposite sides are in the ratio of AB to AD.

$BD := 8 \quad AB := 2 \quad AD := AB + BD$

$BC := \frac{BD}{2} \quad AC := AB + BC \quad CF := BC$




$CI := BC \quad CH := BC \quad AE := BC$

$CE := \sqrt{AC^2 + AE^2} \quad CG := \frac{CH^2}{CE}$

$GH := \sqrt{CH^2 - CG^2} \quad FH := 2 \cdot GH \quad FI := CF + CI$

$HI := \sqrt{FI^2 - FH^2} \quad AI := \sqrt{AC^2 + CI^2}$

$AH := AI - HI \quad AO := \frac{AC \cdot AH}{AI} \quad HO := \frac{CI \cdot AO}{AC}$

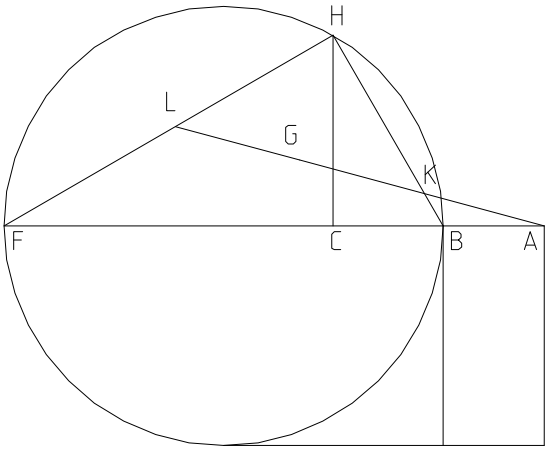
$BO := AO - AB \quad DO := BD - BO$

$DH := \sqrt{DO^2 + HO^2} \quad BH := \sqrt{BO^2 + HO^2}$

$\frac{DH}{BH} = 5 \qquad \frac{AD}{AB} = 5$

Given a straight edge and compass, AB and BD find the sum of six cubes divided by the sum of five squares.

$$\frac{2 \cdot AB^3 + 3 \cdot AB^2 \cdot BD + AB \cdot BD^2}{2 \cdot AB^2 + 2 \cdot AB \cdot BD + BD^2} = 2.308 \qquad AO = 2.308$$




Given AF and AB on AF and a right triangle on BF divide the sides of the triangle such that a section on one side is to the other as AB is to AF.

Now it can be realized that there are stipulations as to possible placements of the given triangle.

$AB := 3 \quad BF := 10 \quad BC := 1$

$AC := AB + BC \quad BE := \frac{BF}{2} \quad AE := AB + BE$

$$DOABLE := \frac{2\cdot AB^3 + 3\cdot AB^2\cdot BF + AB\cdot BF^2}{2\cdot AB^2 + 2\cdot AB\cdot BF + BF^2} \le AC < AE$$

$DOABLE = 1$

$AF := AB + BF$

$CF := BF - BC \quad CH := \sqrt{BC\cdot CF} \quad EJ := BE$

$CE := BE - BC \quad JH := \sqrt{CE^2 + (CH + EJ)^2}$

$DE := \frac{CE\cdot EJ}{EJ + CH} \quad AD := AE - DE$

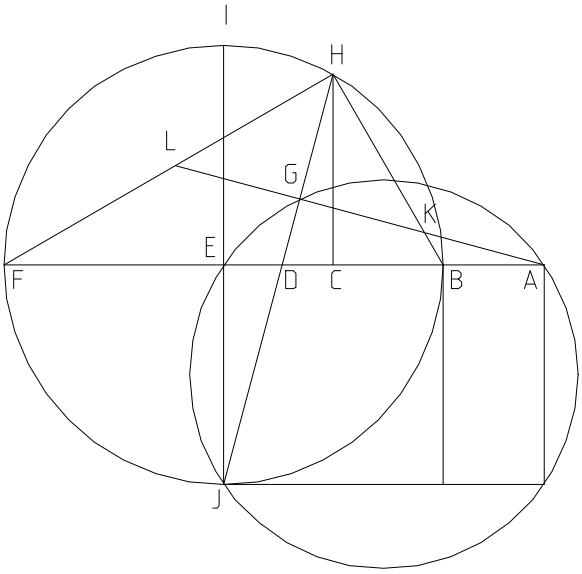
$JD := \sqrt{DE^2 + EJ^2} \quad DG := \frac{DE\cdot AD}{JD}$

$AG := \sqrt{AD^2 - DG^2} \quad GH := JH - (JD + DG)$

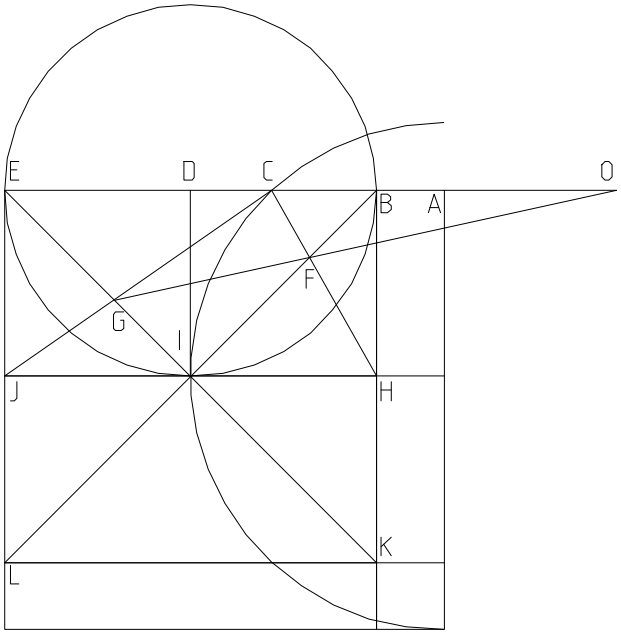
$HK := \sqrt{2\cdot GH^2} \quad BH := \sqrt{BC^2 + CH^2} \quad HL := HK$

$BK := BH - HK \quad FH := \sqrt{CF^2 + CH^2}$

$FL := FH - HL$




$\frac{FL}{BK} = 4.333 \qquad \frac{AF}{AB} = 4.333$




Projecting from KL or HJ is productive, can I find any other productive points?

$AB := 5 \quad BE := 3 \quad BK := BE \quad AE := AB + BE$

$BD := \frac{BE}{2} \quad AC := \sqrt{AB \cdot AE} \quad HI := BD \quad IJ := BD$

$BC := AC - AB \quad CE := BE - BC$

$\Delta := 2 \quad \delta := 1 .. \Delta$

$BH_\delta := \frac{BK}{\Delta} \cdot \delta \quad Ha_\delta := \frac{BH_\delta \cdot HI}{BC} \quad EJ_\delta := BH_\delta$

$Ba_\delta := Ha_\delta + BD \quad Bb_\delta := \frac{(BH_\delta)^2}{Ba_\delta} \quad Jc_\delta := \frac{EJ_\delta \cdot IJ}{CE}$

$Ec_\delta := Jc_\delta + BD \quad Ed_\delta := \frac{(EJ_\delta)^2}{Ec_\delta} \quad Ef_\delta := Bb_\delta$

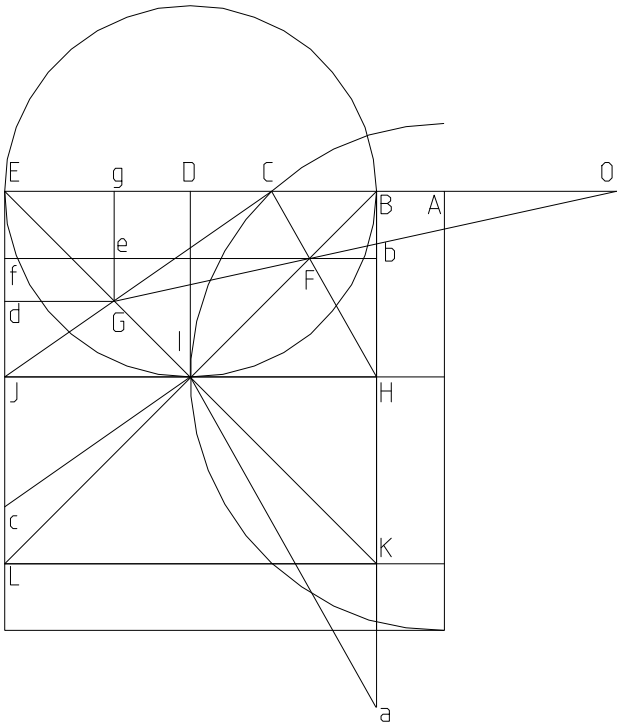
$df_\delta := Ed_\delta - Ef_\delta \quad Ge_\delta := df_\delta \quad Fb_\delta := \frac{HI \cdot BH_\delta}{Ba_\delta}$

$Gd_\delta := \frac{IJ \cdot EJ_\delta}{Ec_\delta} \quad ef_\delta := Gd_\delta \quad Fe_\delta := BE - (ef_\delta + Fb_\delta)$

$Gg_\delta := Ed_\delta \quad Og_\delta := \frac{Fe_\delta \cdot Gg_\delta}{Ge_\delta} \quad Eg_\delta := ef_\delta$

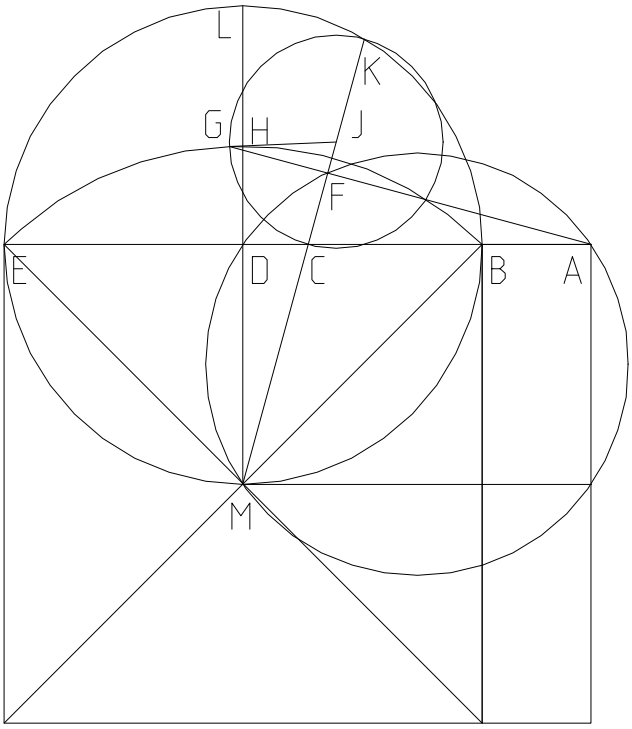
$EO_\delta := Og_\delta + Eg_\delta \quad BO_\delta := EO_\delta - BE$

| $\left(\frac{EO_\delta}{BO_\delta}\right)$ |
|---|
| 1.2649 |
| 1.6 |




$\left(\frac{AE}{AB}\right)^{\frac{1}{2}} = 1.2649 \qquad \frac{AE}{AB} = 1.6$

I have not found any.




The power line for cube root abstraction is developed off from a simple curve.

$$AB := 33 \quad BE := 11 \quad BD := \frac{BE}{2} \quad AE := AB + BE$$

$$R_1 := \frac{\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AB}{2} \quad R_2 := \frac{AE - \left(AB \cdot AE^2\right)^{\frac{1}{3}}}{2}$$

$$D := \left[\left(AB \cdot AE^2\right)^{\frac{1}{3}} - \left(AB^2 \cdot AE\right)^{\frac{1}{3}}\right] + R_1 + R_2$$

$$BC := \frac{\left(R_1 + D + R_2\right) \cdot \left(D - R_2 + R_1\right)}{2 \cdot D}$$

The formula for the power line (BC) was given in 01_08_96.MCD

$$CD := BD - BC \quad AC := BC + AB \quad DM := BD$$

$$DL := BD \quad CM := \sqrt{CD^2 + DM^2} \quad ML := DM + DL$$

$$MK := \frac{DM \cdot ML}{CM} \quad CK := MK - CM \quad CJ := \frac{CK}{2}$$

$$JG := CJ \quad JM := CM + CJ \quad BM := \sqrt{2 \cdot BD^2}$$

$$GM := BM \quad FJ := \frac{JG^2}{JM} \quad FM := JM - FJ$$

$$AD := BD + AB \quad HM := \frac{CM \cdot FM}{DM}$$

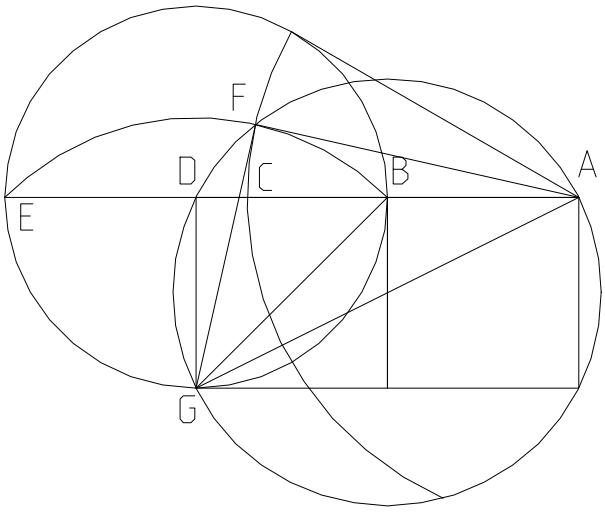
$$DH := HM - DM \quad AH := \sqrt{DH^2 + AD^2}$$

$$CF := FM - CM \quad FH := \frac{CD \cdot HM}{CM}$$

$$AF_1 := AH - FH \quad AF_2 := \sqrt{AC^2 - CF^2}$$

$$AF_1 - AF_2 = 0$$

Trivial Method; Square Root




$N := 9003$

$AE := N \quad AB := \frac{AE}{N} \quad BE := AE - AB \quad BD := \frac{BE}{2}$

$DG := BD \; AD := AB + BD \quad AG := \sqrt{AD^2 + DG^2}$

$BG := \sqrt{BD^2 + DG^2} \quad FG := BG \quad AF := \sqrt{AG^2 - FG^2}$

$AC := \sqrt{AB \cdot AE}$

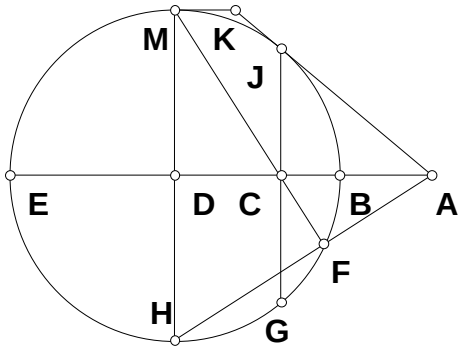
$AC - AF = 0$

$AF = 94.884$

$AC = 94.884$

## 01/24/96 Tangent

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**




$N := 5 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2} \qquad AD := AB + BD$

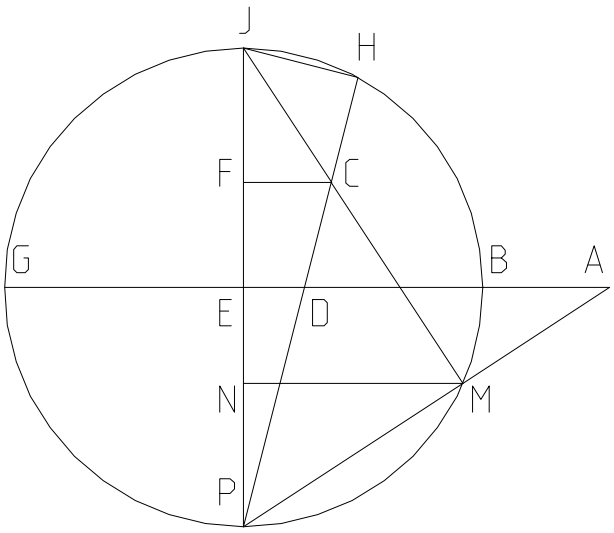
$DM := BD \qquad DH := BD \qquad CD := \frac{DH \cdot DM}{AD}$

$BC := BD - CD \qquad CE := BE - BC$

$CJ := \sqrt{BC \cdot CE} \qquad AJ := \sqrt{(AB + BC)^2 + CJ^2}$

$$\sqrt{(AB + BC)^2 + CJ^2} - \sqrt{AB \cdot AE} = 0$$

Given a point on BG, project to the point of cubic similarity.




$BG := 100 \quad BD := 49 \qquad BE := \frac{BG}{2}$

$EG := BE \quad DE := BE - BD \quad EP := BE$

$EJ := BE \quad DP := \sqrt{DE^2 + EP^2} \quad JP := EP + EJ$

$HP := \frac{EP \cdot JP}{DP} \quad DH := HP - DP \quad CD := \frac{DH}{2}$

$CP := DP + CD \quad CF := \frac{DE \cdot CP}{DP} \quad FP := \frac{EP \cdot CP}{DP}$

$EF := FP - EP \quad FJ := EJ - EF \quad CJ := \sqrt{CF^2 + FJ^2}$

$JM := \frac{FJ \cdot JP}{CJ} \quad JN := \frac{FJ \cdot JM}{CJ} \quad NP := JP - JN$

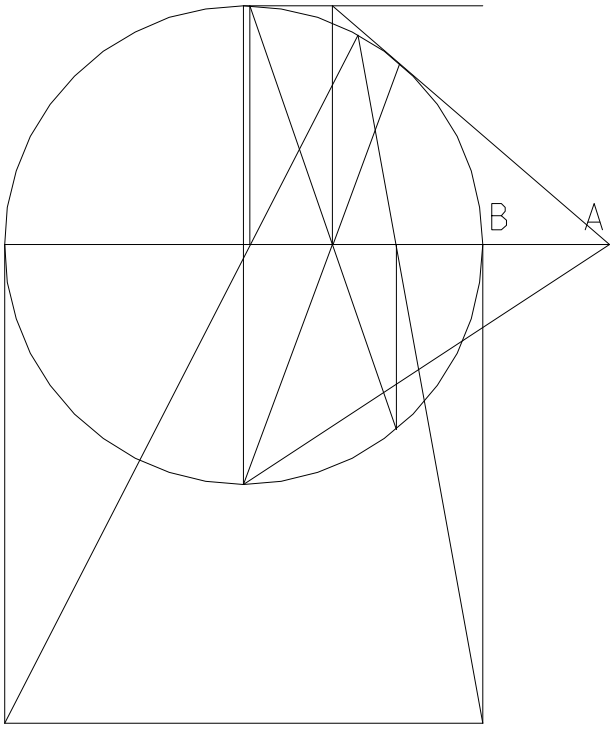
$MP := \frac{CF \cdot JP}{CJ} \quad AP := \frac{MP \cdot EP}{NP} \quad MN := \frac{CF \cdot JM}{CJ}$

$$\frac{BG^4 - 3 \cdot BG^3 \cdot BD + 3 \cdot BG^2 \cdot BD^2 - BD^3 \cdot BG}{BG^3 - 3 \cdot BG^2 \cdot BD + 3 \cdot BG \cdot BD^2 - 2 \cdot BD^3} = 884.222$$

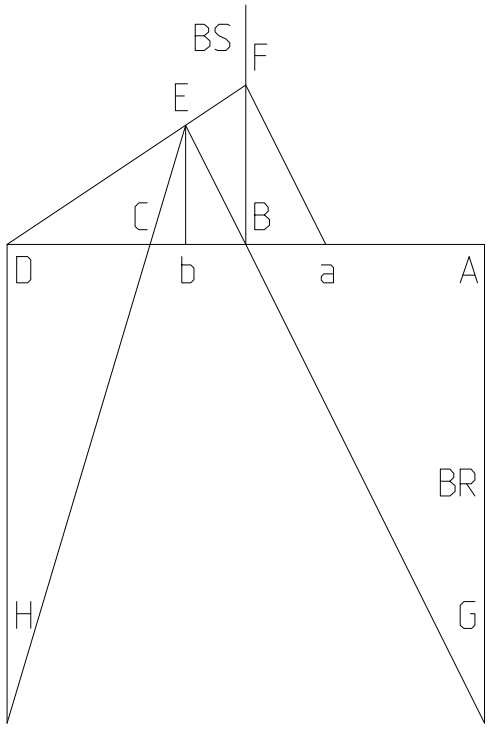
$AE := \frac{MN \cdot AP}{MP} \quad AG := AE + EG$

$$AG = 884.222$$

$$\frac{BG \cdot \left[ (AG - BG)^{\frac{2}{3}} \cdot AG^{\frac{2}{3}} + AG^{\frac{1}{3}} \cdot BG - AG^{\frac{4}{3}} - BG \cdot (AG - BG)^{\frac{1}{3}} + (AG - BG)^{\frac{1}{3}} \cdot AG \right]}{(AG - BG)^{\frac{1}{3}} \cdot (2 \cdot AG - BG)} - BD = 0$$




One may be tempted to state the obvious, $\frac{A^{N+1}}{A^{N}} := A_{\tau}$, but what is not so obvious at first blush is that the processes themselves are assigned dimensional values. This has significance when using mathematics to theorize dimensions beyond three. Dimensions are so generally defined that processes are legitimate dimensional differences, but it is also impossible to defend mathematical theory about dimensions as objective. It becomes a point of Philosophical Mystic contemplation to realize that relationships concerning a single dimensional object and several processes adding dimensionally to the whole, is true of a multidimensional object without those processes!




Linear division $\frac{2\cdot(A + B)}{A}$

$BR := \frac{1}{4}$ $BS := 3$

$AD := \frac{BR}{BR}$ $AG := AD\cdot BR$ $AB := \frac{AD}{2}$

$BF := AB\cdot BS$ $Ba := \frac{AB\cdot BF}{AG}$ $BD := AB$

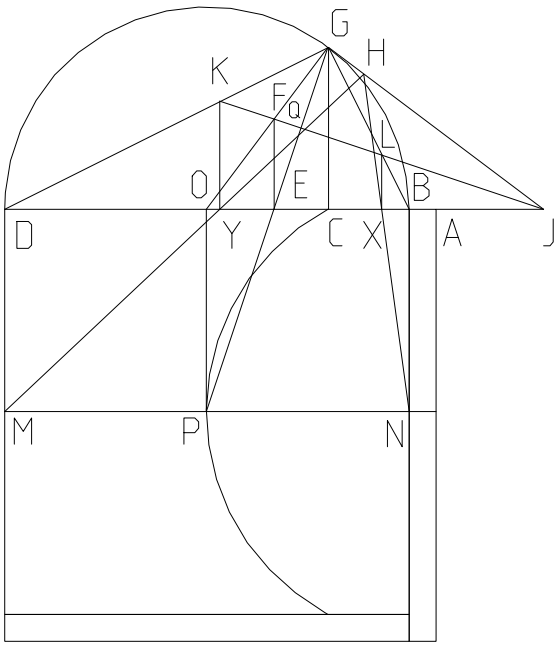
$Da := BD + Ba$ $Bb := \frac{Ba\cdot BD}{Da}$ $Db := BD - Bb$

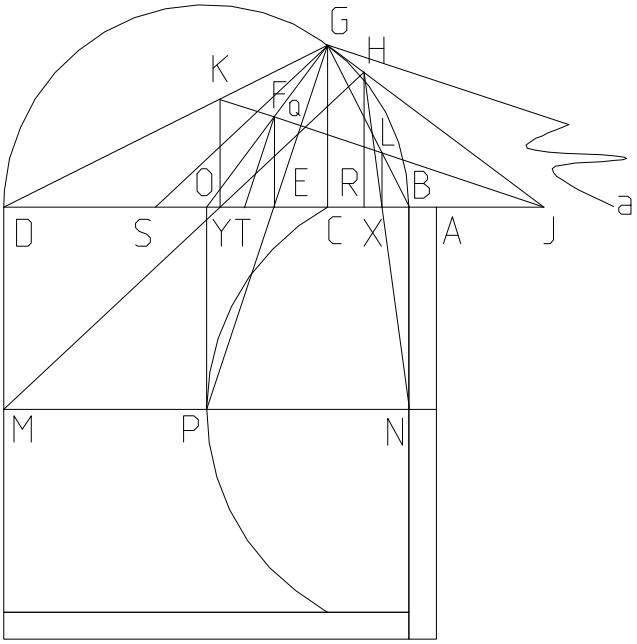
$Eb := \frac{BF\cdot Db}{BD}$ $DH := AG$

$DC := \frac{Db\cdot DH}{DH + Eb}$ $\frac{AD}{DC} = 26$

$\frac{2\cdot(BR + BS)}{BR} = 26$

Hitting JG from any BN while maintaining complimentary roots.




$AB := 2 \quad BD := 5 \quad BO := \frac{BD}{2} \quad AD := AB + BD$

$DO := BO \quad AC := \sqrt{AB \cdot AD} \quad BC := AC - AB$

$CD := BD - BC \quad CG := \sqrt{BC \cdot CD} \quad CO := BO - BC$

$N := 7 \quad OP := BO \cdot N \quad BN := OP \quad DM := OP$

$NP := BO \quad MP := BO \quad EO := \frac{CO \cdot OP}{OP + CG}$

$CE := CO - EO \quad EF := \frac{OP \cdot CE}{CO} \quad GO := BO$

$CJ := \frac{CG^2}{CO} \quad EJ := CJ + CE \quad Ca := \frac{EJ \cdot CG}{EF}$

$DJ := CD + CJ \quad Da := CD + Ca \quad JY := \frac{Ca \cdot DJ}{Da}$

$KY := \frac{EF \cdot JY}{EJ} \quad JK := \sqrt{JY^2 + KY^2} \quad JG := \sqrt{CJ^2 + CG^2}$

$GP := \sqrt{CO^2 + (OP + CG)^2} \quad EP := \sqrt{EO^2 + OP^2}$

$ET := \frac{EO \cdot EF}{OP} \quad JT := EJ + ET \quad FT := \sqrt{ET^2 + EF^2}$

$EG := GP - EP \quad EQ := \frac{FT \cdot EJ}{JT} \quad GQ := EG - EQ$

$KL := 2 \cdot GQ \quad JL := JK - KL$

$DY := DJ - JY \quad CS := \frac{DY \cdot CG}{DM} \quad JS := CS + CJ$

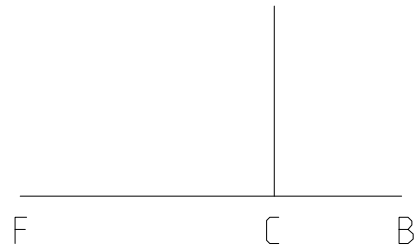
$YR := \frac{CS \cdot JY}{JS} \quad JR := JY - YR \quad HR := \frac{CG \cdot JR}{CJ}$

$AJ := CJ - AC \quad JX := \frac{JY \cdot JL}{JK}$

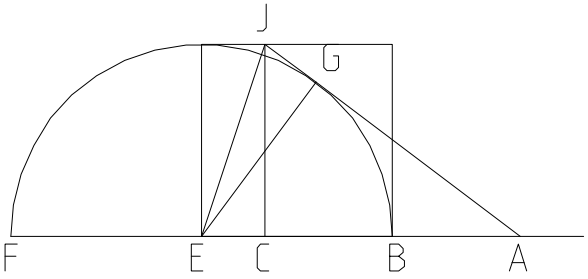
$BR_1 := JR - (AJ + AB) \quad BX := JX - (AJ + AB)$

$BR_2 := \frac{BX \cdot (BN + HR)}{BN}$

$BR_2 - BR_1 = 0$




Given BF and BC such that $\sqrt{(AB + BF)\cdot AB} = AB + BC$, find AB. It is obvious from the construction that answers are obtainable when BC is less than 1/2 of BF.

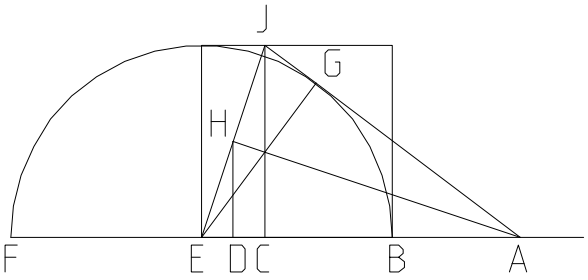



$\Delta := 5 \qquad \delta := 1..\Delta - 1 \quad BE := 5 \qquad BF := BE\cdot 2$

$BC_\delta := \frac{BE}{\Delta}\cdot\delta \qquad CE_\delta := BE - BC_\delta \qquad EG := BE$

$CJ := BE \quad EJ_\delta := \sqrt{\left(CE_\delta\right)^2 + CJ^2} \qquad DH := \frac{CJ}{2}$

$DE_\delta := \frac{CE_\delta}{2} \qquad AD_\delta := \frac{DH^2}{DE_\delta} \qquad AE_\delta := AD_\delta + DE_\delta$




$AC_\delta := AE_\delta - CE_\delta \qquad AJ_\delta := \sqrt{\left(AC_\delta\right)^2 + CJ^2}$

$GJ_\delta := \sqrt{\left(EJ_\delta\right)^2 - EG^2} \qquad AB_\delta := AE_\delta - BE$

$AF_\delta := AB_\delta + BF$

$\left[\frac{1}{2}\cdot\frac{BE}{\Delta}\cdot\delta\cdot\frac{(-2\cdot\Delta + \delta)}{(-\Delta + \delta)}\right] - AC_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$GJ_\delta - CE_\delta$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

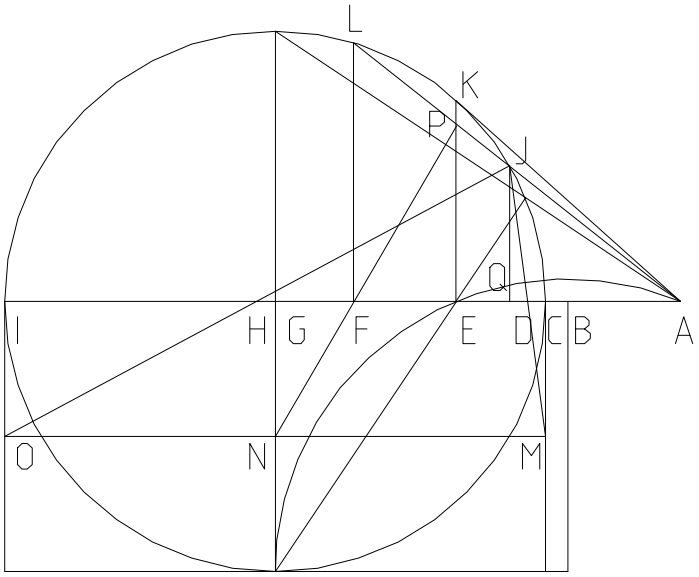
$AB_\delta + BC_\delta - \sqrt{AB_\delta\cdot(AB + BF)_\delta}$

| |
|---|
| 0 |
| 0 |
| 0 |
| 0 |

$A := \frac{BF}{2} \qquad B := BC_2 \qquad \frac{2\cdot A\cdot B - B^2}{2\cdot A - 2\cdot B} - \left(AB_2 + BC_2\right) = 0$

$\sqrt{\left(AB_2 + BF\right)\cdot AB_2} - \left(AB_2 + BC_2\right) = 0$

$\frac{BF\cdot BC_2 - \left(BC_2\right)^2}{BF - 2\cdot BC_2} - \left(AB_2 + BC_2\right) = 0$ A rectangle minus a square divided by a line yeilds a line.




Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

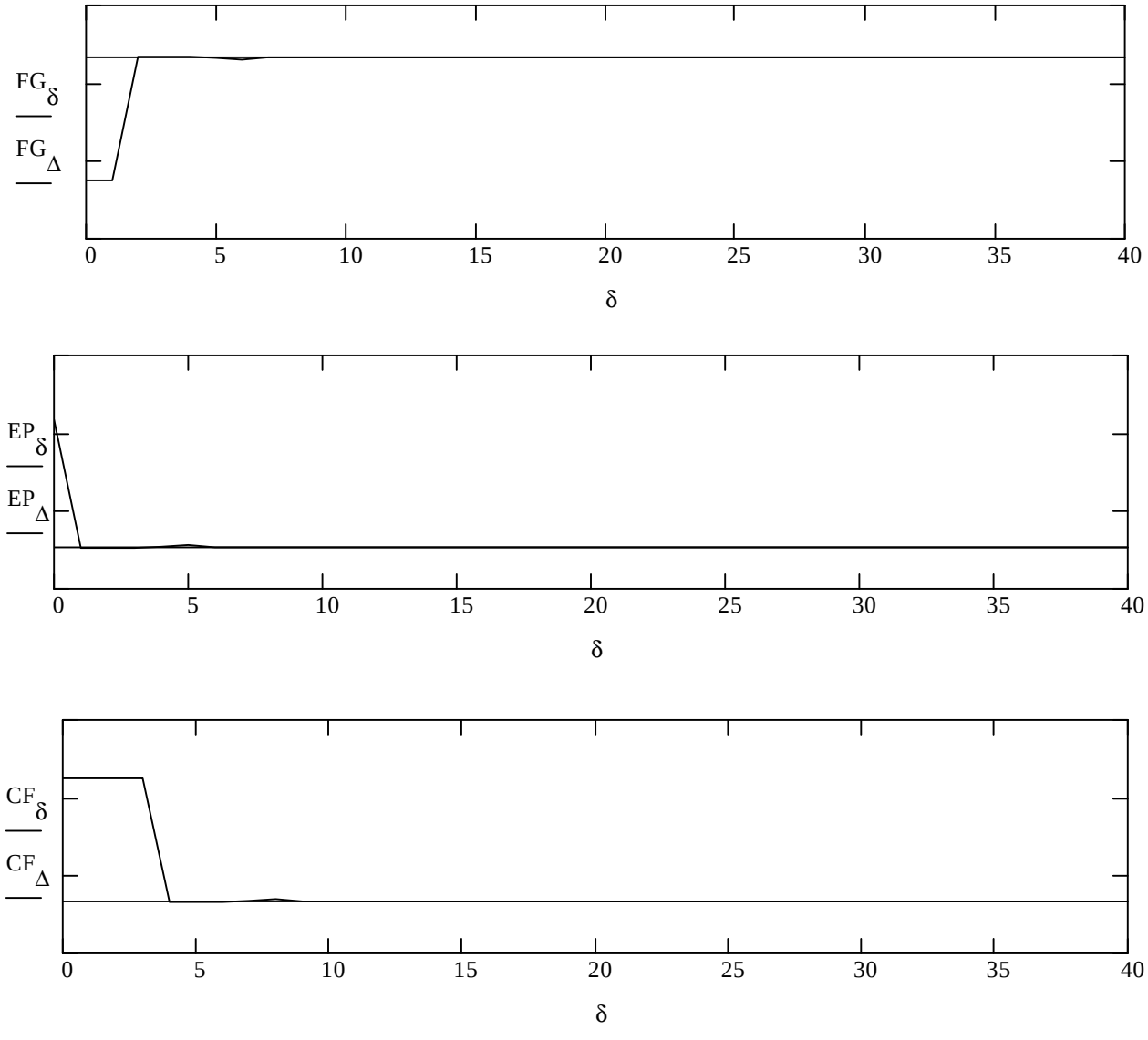
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

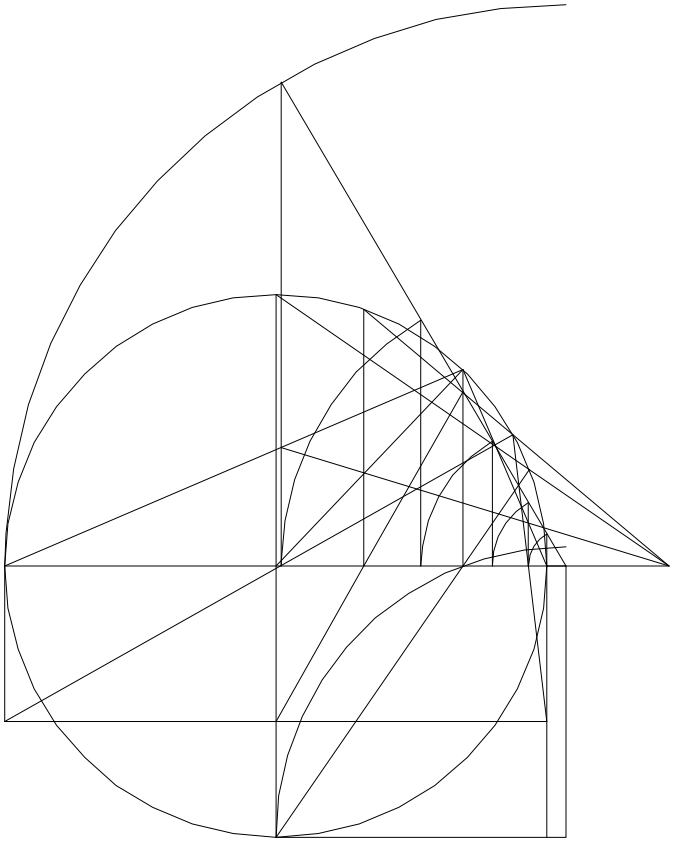
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.



Because of it's long projection, the last vertices is not drawn.

Does every $n^{th}$ root series have at least one square root pair?

$n := 5 \qquad \delta := 0..\frac{n}{2}$

$A := 3 \quad B := 10$

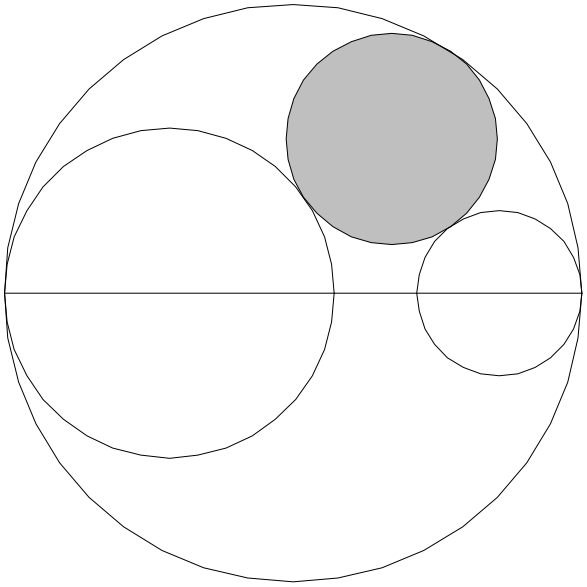
$$\sqrt{\left(A^{n-1}\cdot B^{1}\right)^{\frac{1}{n}}\cdot\left(A^{1}\cdot B^{n-1}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B} = 0$$

$$\sqrt{\left(A^{n-\delta}\cdot B^{\delta}\right)^{\frac{1}{n}}\cdot\left(A^{\delta}\cdot B^{n-\delta}\right)^{\frac{1}{n}}} - \sqrt{A\cdot B}$$

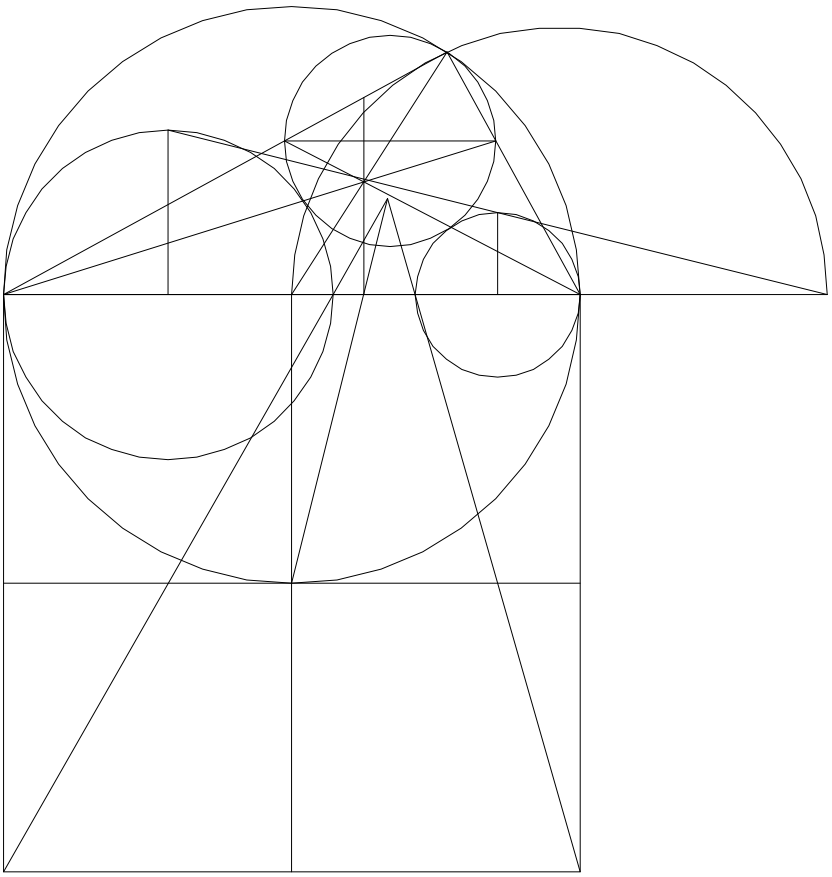
| |
|---|
| 0 |
| 0 |
| 0 |

A root series has as many vertices on a circle as it has square root pairs, and it has the greater whole of n/2 vertices where n is the root series denominator.

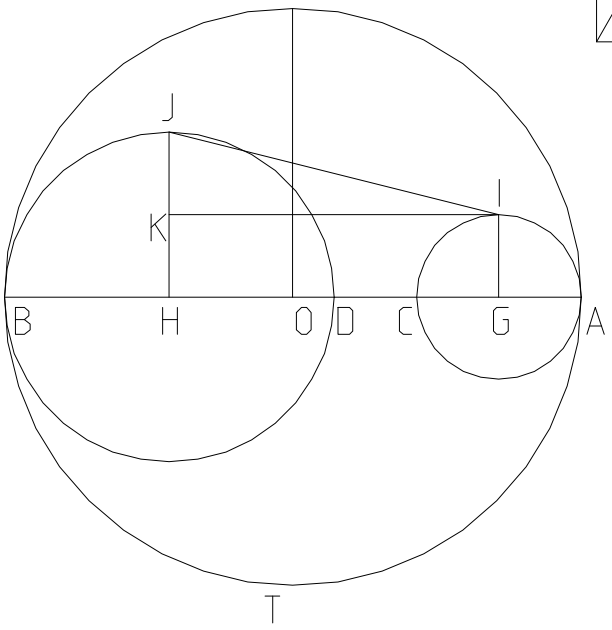
# Method for Unequals



Given three circles in the said configuration, find the fourth.
I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."



Process Summary.

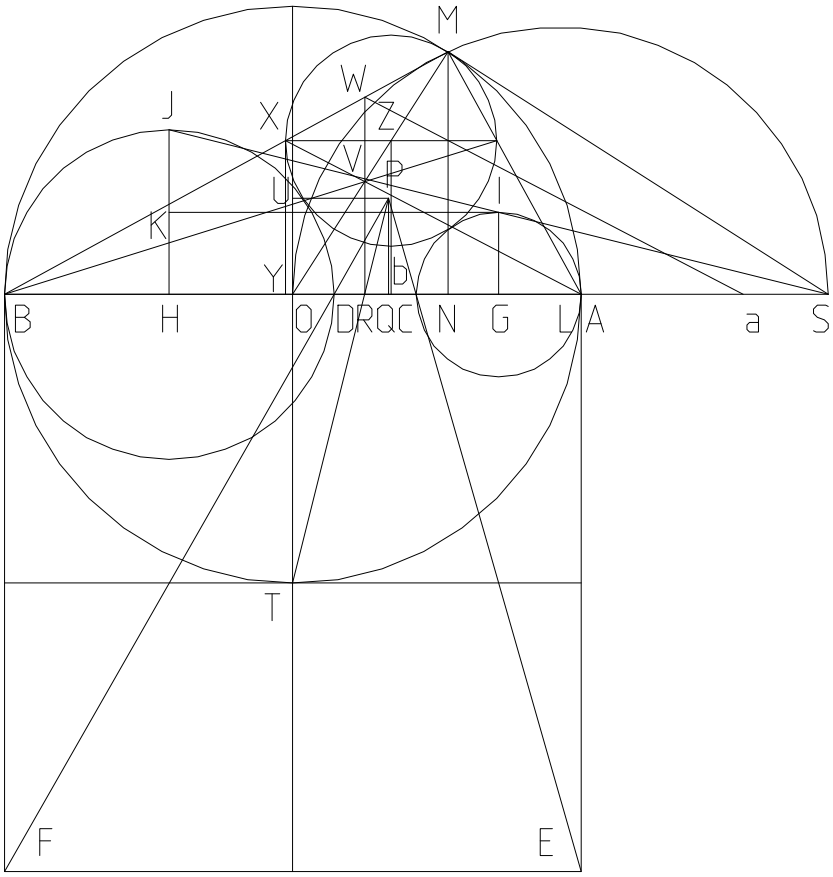


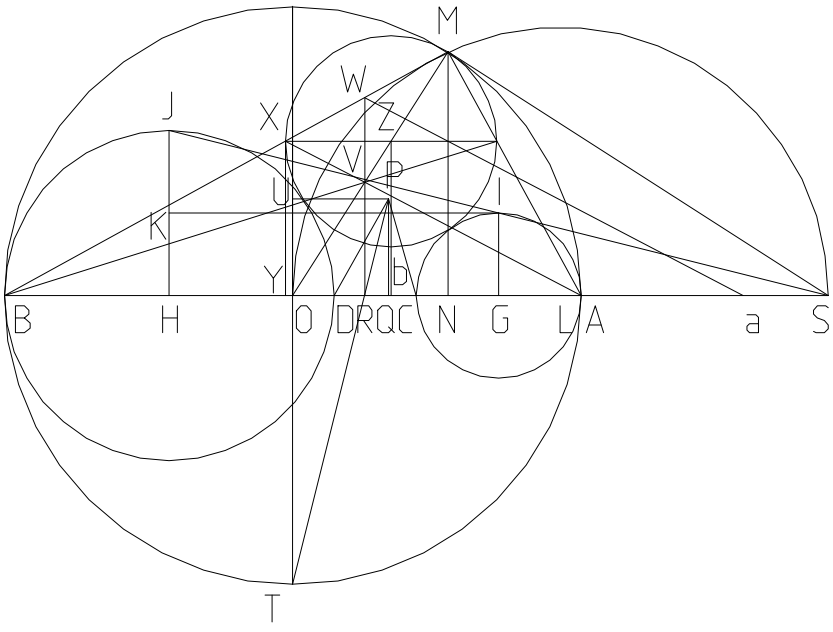

AO := 5 AG := 1 BH := 3 AB := 2·AO

BO := AO CG := AG GI := AG HJ := BH

DH := BH GH := AB – (AG + BH) HK := GI

JK := HJ – HK




$HS := \frac{GH \cdot HJ}{JK}$ $AH := AB - BH$

$AS := HS - AH$ $OS := AO + AS$

$SL := \frac{OS}{2}$ $MO := AO$ $MS := \sqrt{OS^2 - MO^2}$

$MN := \frac{MO \cdot MS}{OS}$ $NS := \frac{MS \cdot MN}{MO}$

$AN := NS - AS$ $ON := AO - AN$

$BN := AB - AN$ $AM := \sqrt{MN^2 + AN^2}$

$AE := AB$ $BF := AB$ $AC := 2 \cdot AG$

$BD := 2 \cdot BH$ $CD := AB - (AC + BD)$

$PQ := \frac{AE \cdot CD}{(AC + BD)}$ $OU := PQ$

$CQ := \frac{AC \cdot PQ}{AE}$ $AQ := AC + CQ$

$OQ := AO - AQ$ $OT := AO$

$TU := OT + OU$ $OR := \frac{OQ \cdot OT}{TU}$

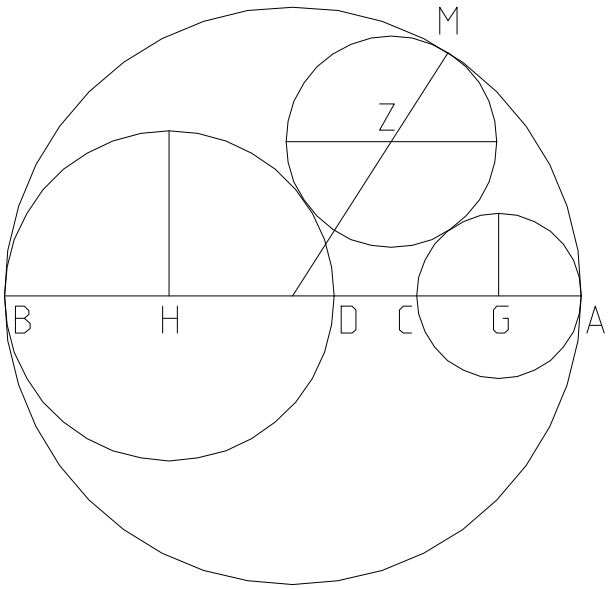
$RV := \frac{MN \cdot OR}{ON}$ $BR := BO + OR$

$RW := \frac{MN \cdot BR}{BN}$ $AR := AO - OR$

$Ra := \frac{AR \cdot RW}{RV}$ $XY := \frac{RW \cdot AB}{BR + Ra}$

$Zb := XY$ $OZ := \frac{MO \cdot Zb}{MN}$

$MZ := MO - OZ$ $MZ = 1.818$

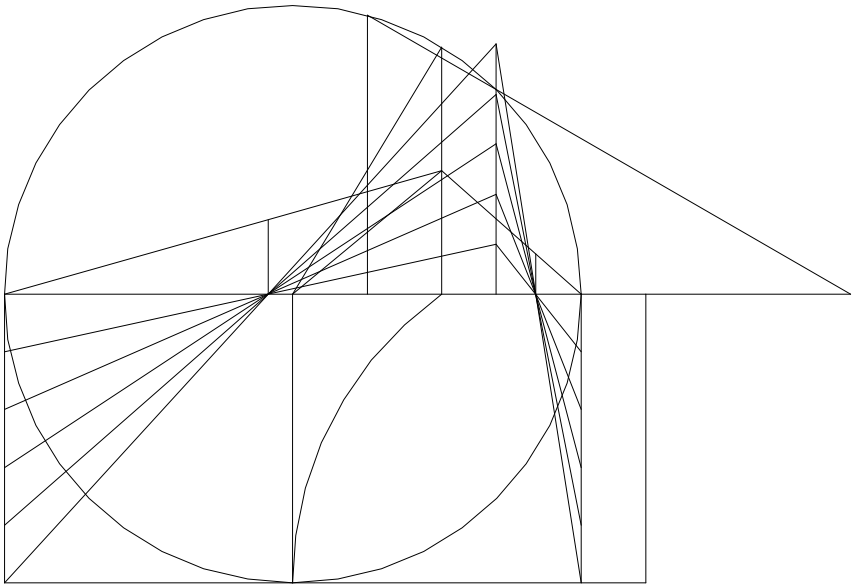


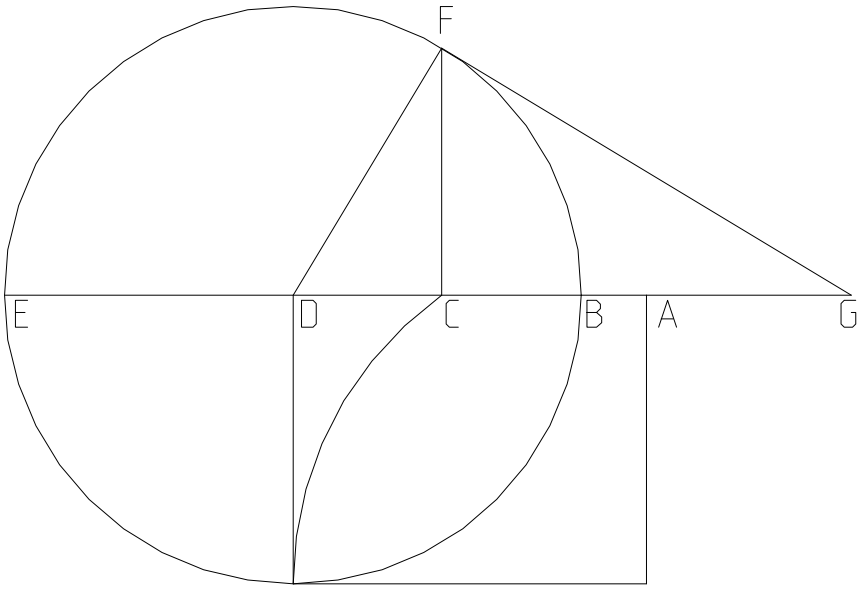

$$MZ = 1.818$$

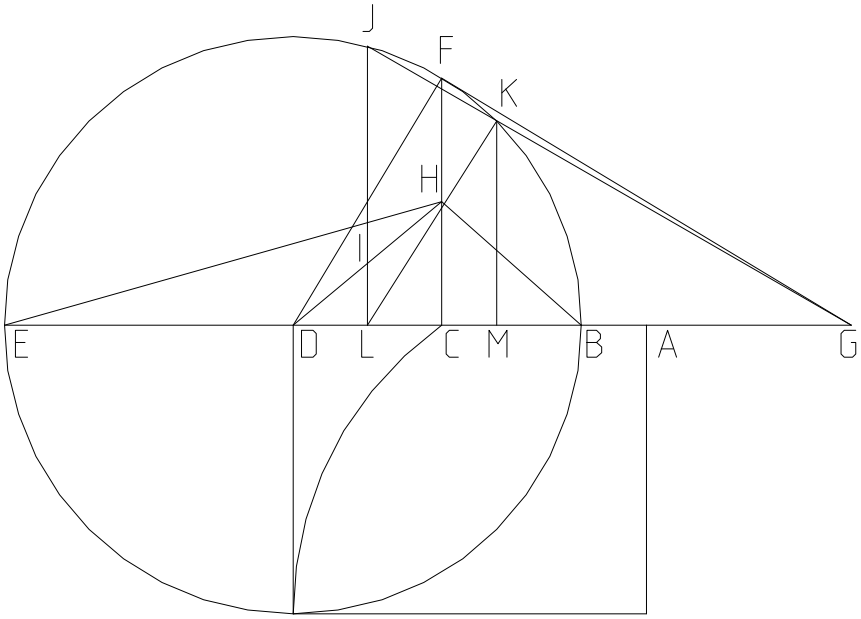
$$\frac{AB^3 - 2\cdot AB^2\cdot AG - 2\cdot AB^2\cdot BH + 4\cdot AB\cdot BH\cdot AG}{2\cdot AB^2 - 8\cdot BH\cdot AG} = 1.818$$

Neither in the process, nor in the Algebraic name. is the order of AG and BH recognized. Neither does it matter if they intersect.

# On Gemini Roots






$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

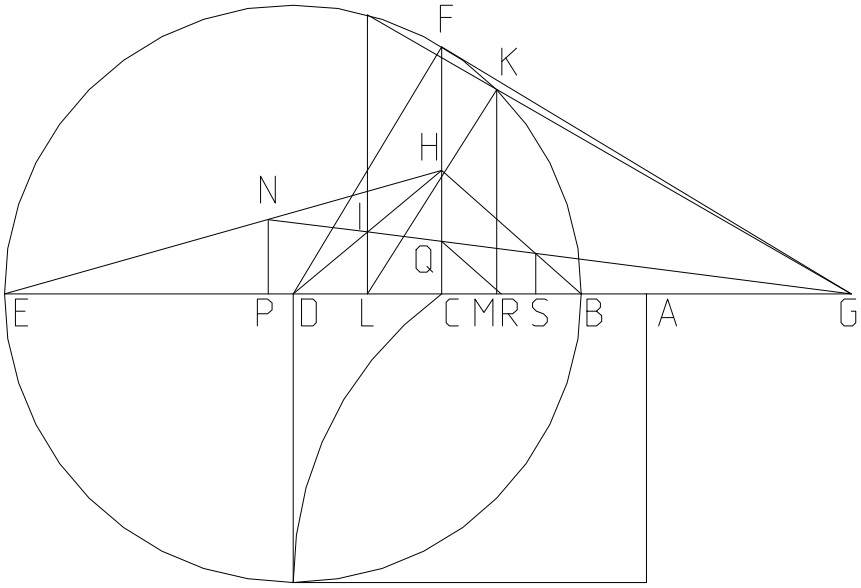
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

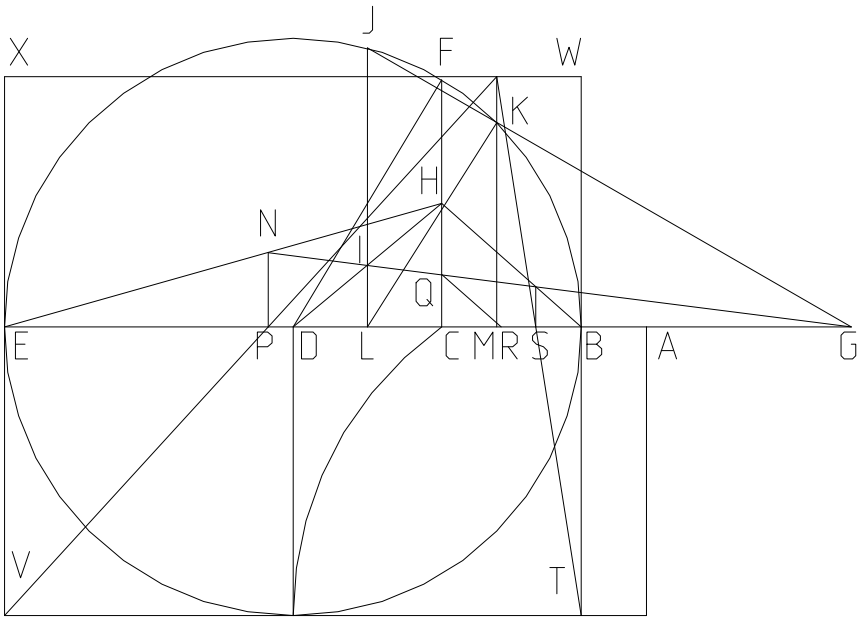



$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

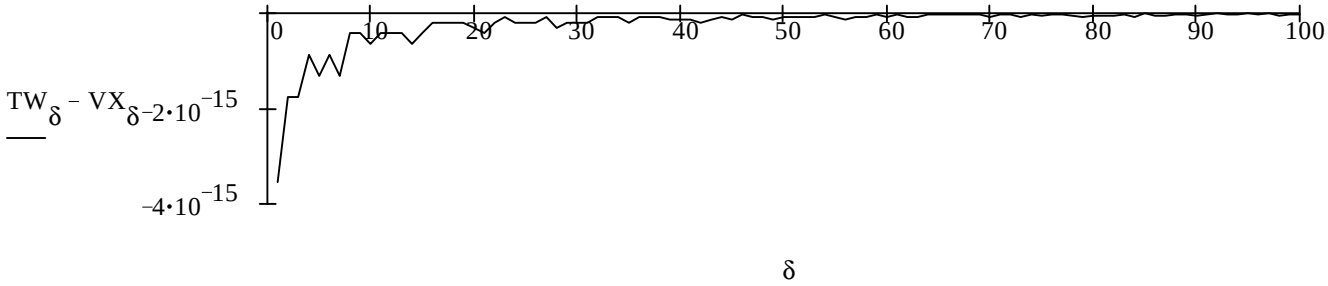
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$

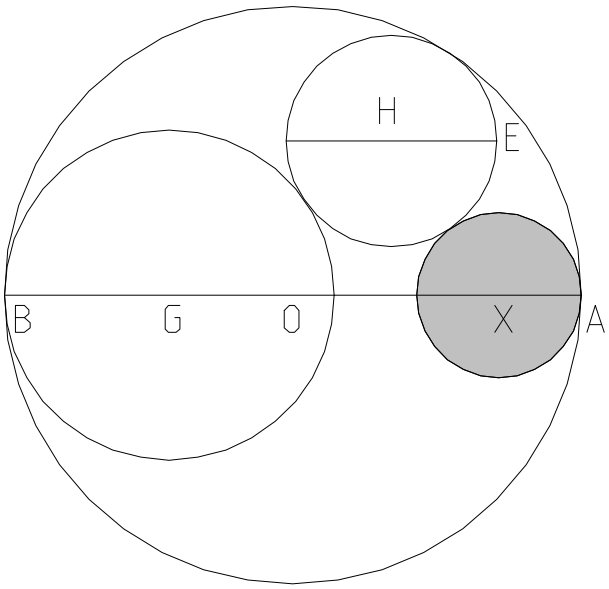



$$\delta := 1..100$$

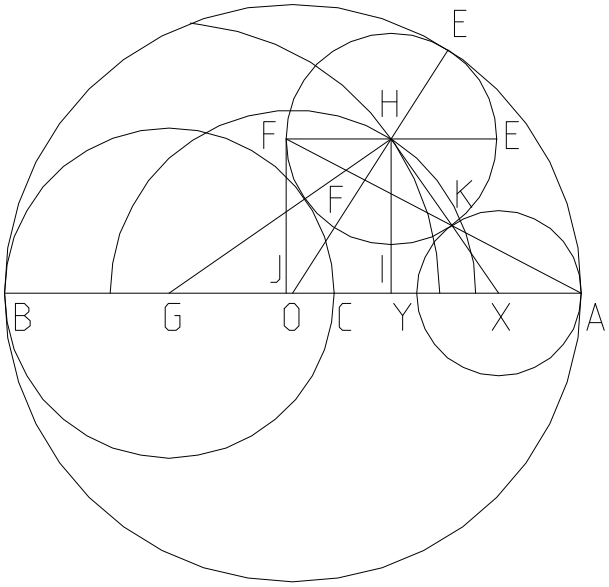
$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$






Given three radii, AO > BG + EH, place them such that two by two they are tangent and find the fourth, AX, such that AX is tangent to EH and AO. This of course means that if the sum of BG and EH is equal to AO, we have no result.




$AO := 5 \quad BG := 2.5 \quad EH := 1.5$

$AB := 2 \cdot AO \quad BC := 2 \cdot BG \quad EF := 2 \cdot EH$

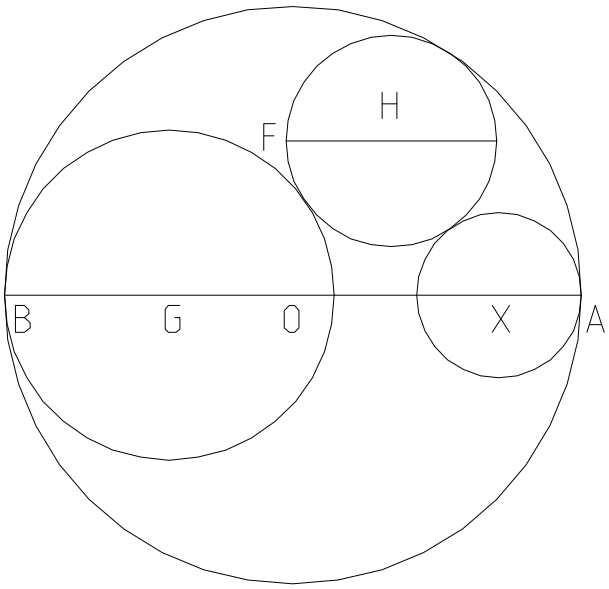
$GH := BG + EH \quad OH := AO - EH \quad GO := AO - BG$

$$GI := \frac{GH^2 + GO^2 - OH^2}{2 \cdot GO} \quad HI := \sqrt{GH^2 - GI^2}$$

$AG := AB - BG \quad AI := AG - GI \quad IJ := EH$

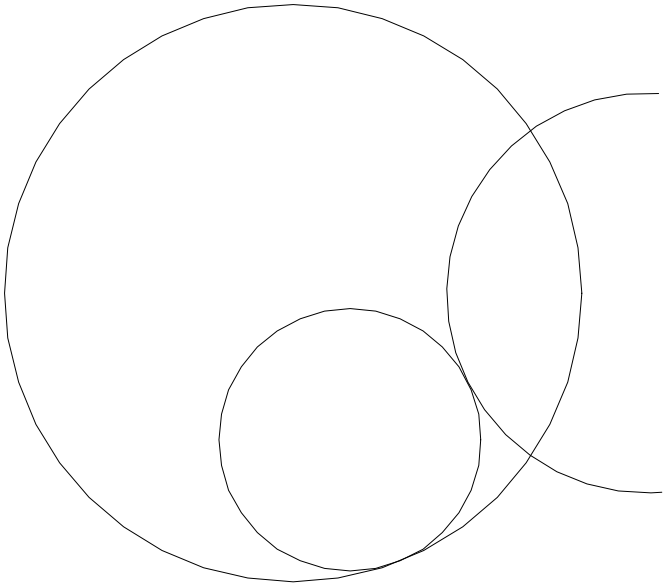
$$AJ := AI + IJ \quad FJ := HI \quad AF := \sqrt{AJ^2 + FJ^2}$$

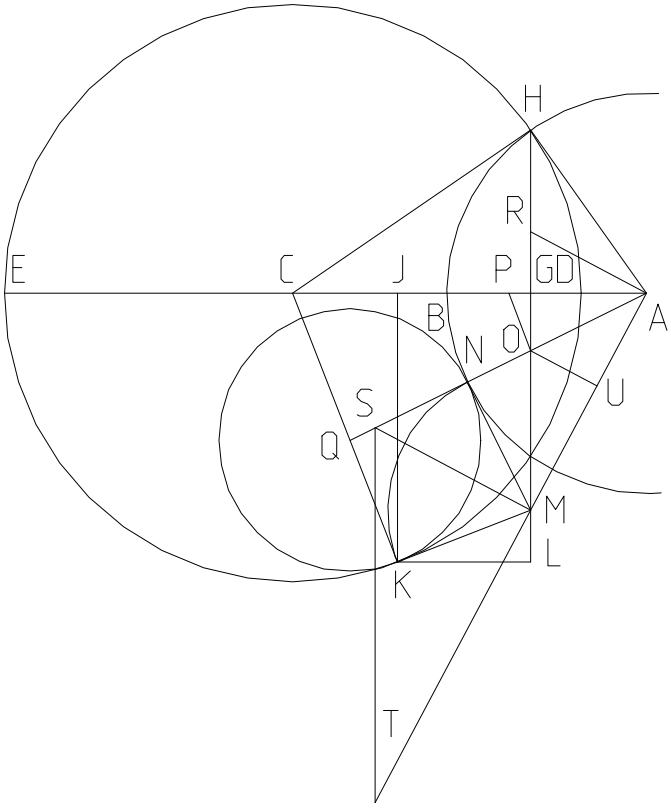
$$FK := \frac{AJ \cdot EF}{AF} \quad AK := AF - FK \quad AY := \frac{AF \cdot AK}{AJ}$$




$$AX := \frac{AY}{2} \qquad AX = 2.857$$

$$\frac{AO^3 - AO^2 \cdot EH - AO^2 \cdot BG}{AO^2 - AO \cdot BG - EH \cdot BG} = 2.857$$






Given a point of tangency, draw a circle in a crescent tangent to the other side. This figure is given for the tangent on the exterior of the crescent, the other will become obvious.

$AB := Concave\_Radius$

$CD := Convex\_Radius \quad AC := Center\_Difference$

$DE := 2 \cdot CD \quad BF := 2 \cdot AB \quad CE := CD$

$AE := AC + CE \quad BE := AE - AB$

$$CG := \frac{CD^2 + AC^2 - AB^2}{2 \cdot AC} \quad AG := AC - CG$$

$GE := AE - AG$

$GJ := Power\_Line\_Tangent \cdot GE$

$EJ := GE - GJ$

$DJ := DE - EJ \quad JK := \sqrt{DJ \cdot EJ}$

$CJ := CG - GJ \quad CK := CD \quad KL := GJ$

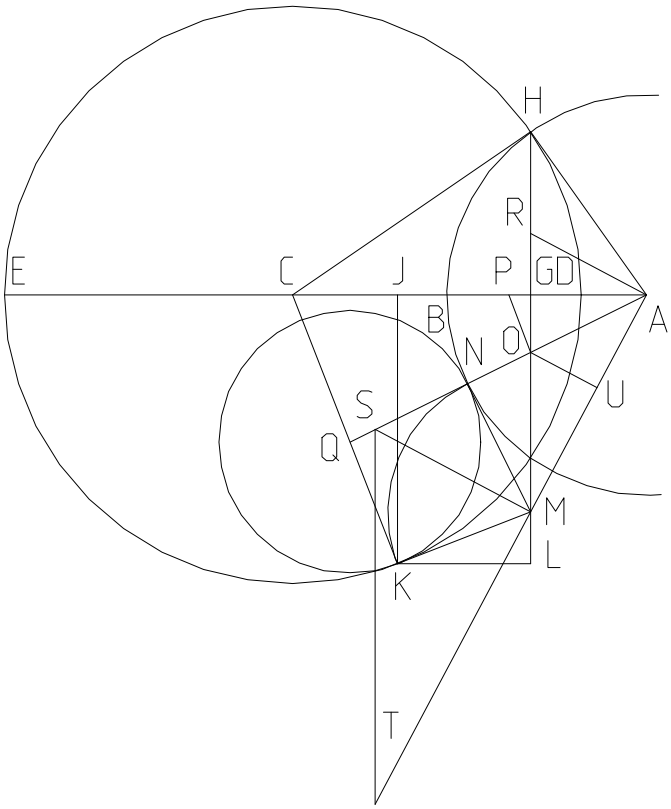
$$LM := \frac{CJ \cdot KL}{JK} \quad GL := JK \quad GM := GL - LM$$

$$AM := \sqrt{AG^2 + GM^2} \quad AN := AB$$

$$AS := \frac{AM^2}{AN} \quad MR := \frac{AM^2}{GM}$$

$$MS := \sqrt{AS^2 - AM^2} \quad AR := \sqrt{MR^2 - AM^2}$$

$$ST := \frac{MR \cdot MS}{AR} \quad MT := \frac{AM \cdot MS}{AR}$$




$$MO := \frac{ST \cdot AM}{AM + MT}$$

$$GO := GM - MO \quad GP := \frac{CJ \cdot GO}{JK}$$

$$AP := AG + GP \quad OP := \frac{CK \cdot GO}{JK}$$

$$CQ := \frac{OP \cdot AC}{AP} \quad QK := CK - CQ$$

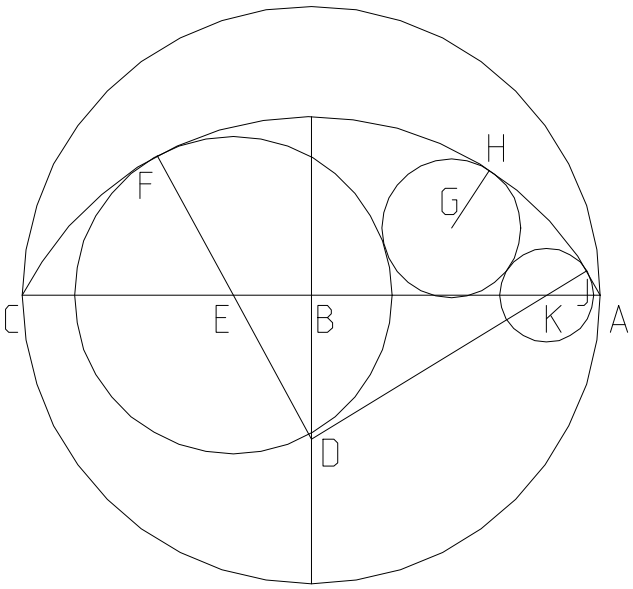
$$QK = 0.206$$

$$Concave\_Radius \equiv 2.37$$

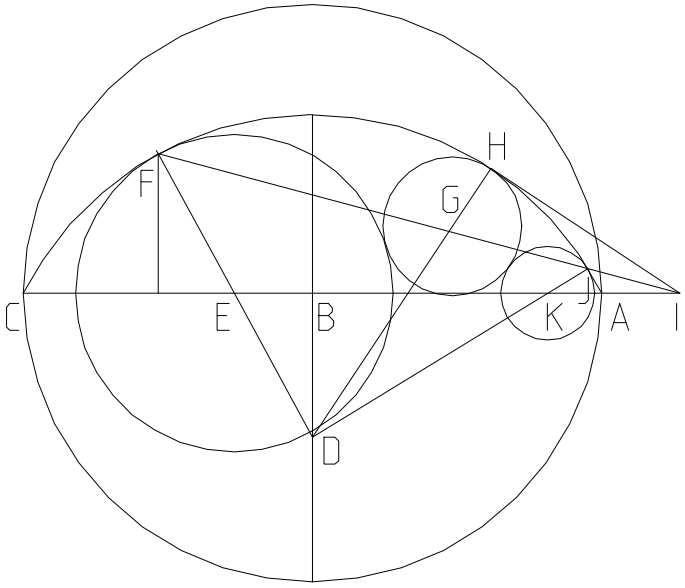
$$Convex\_Radius \equiv 1.5$$

$$Center\_Difference \equiv 1.84$$

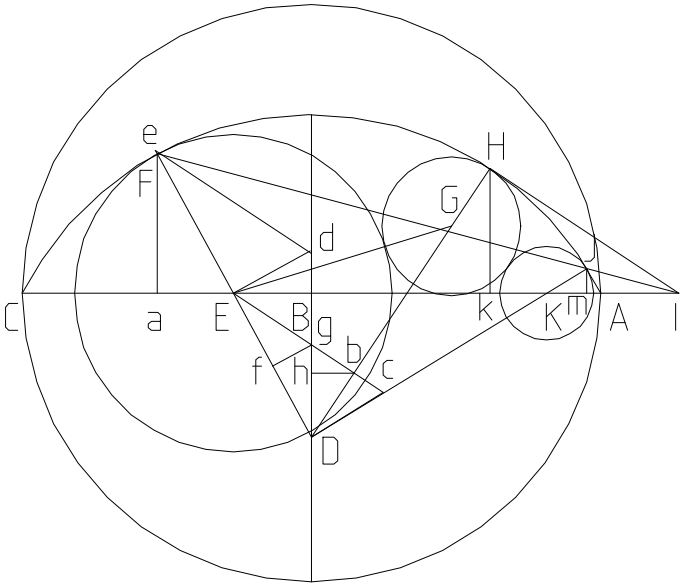
$$Power\_Line\_Tangent \equiv \frac{1}{3} \quad \text{Given as Fraction} < 1.$$




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

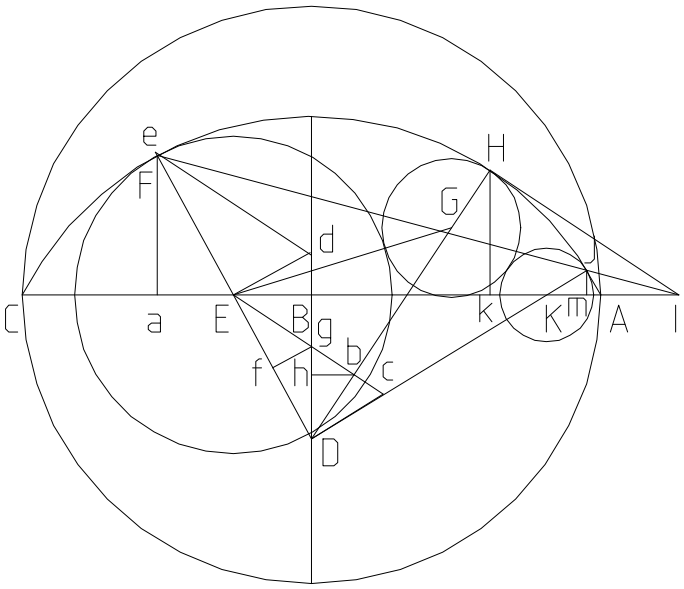
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

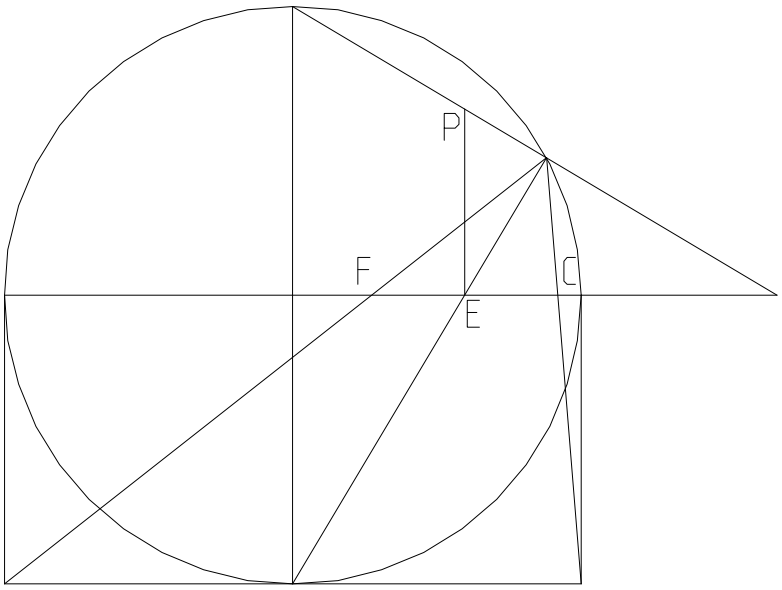
$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

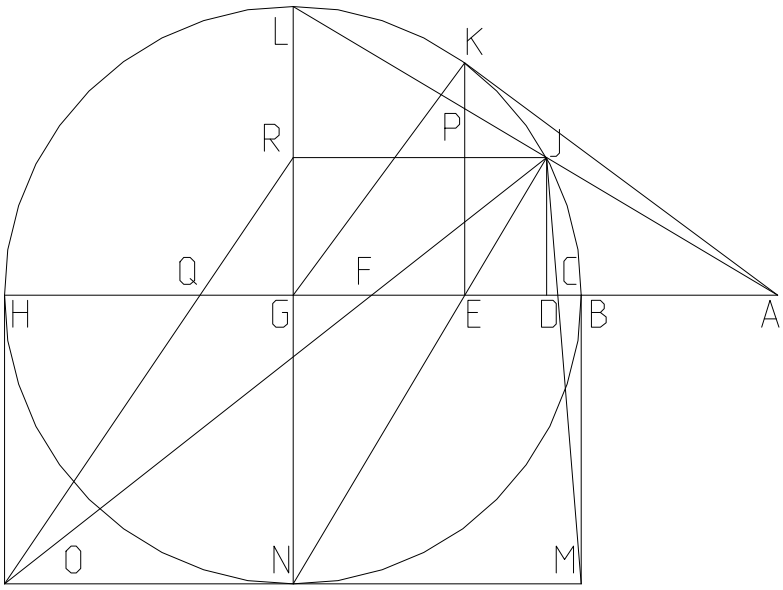
$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.




I found this little sketch in my notebook and have no idea of when I did it or why.

Does CF always equal EP?




$\Delta := 100 \quad \delta := 1..\Delta - 1 \quad BG := 10 \quad GH := BG$

$GN := BG \quad BD_\delta := \frac{BG}{\Delta}\cdot\delta \quad DG_\delta := BG - BD_\delta$

$DH_\delta := GH + DG_\delta \quad DJ_\delta := \sqrt{BD_\delta \cdot DH_\delta}$

$EG_\delta := \frac{DG_\delta \cdot GN}{GN + DJ_\delta} \quad BE_\delta := BG - EG_\delta$

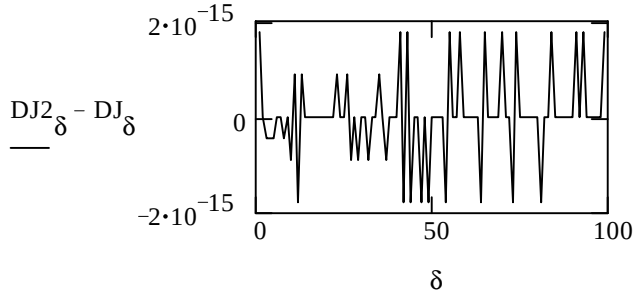
$EH_\delta := EG_\delta + GH \quad EK_\delta := \sqrt{BE_\delta \cdot EH_\delta}$

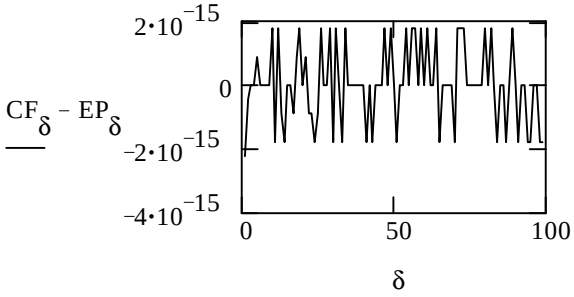
$AE_\delta := \frac{(EK_\delta)^2}{EG_\delta} \quad AB_\delta := AE_\delta - BE_\delta$

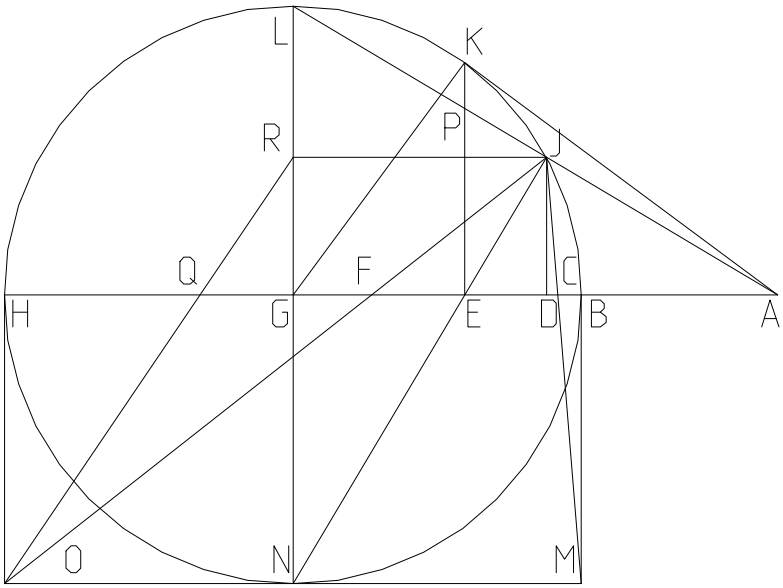
$GL := BG \qquad AD_\delta := AB_\delta + BD_\delta$

$AG_\delta := AB_\delta + BG \qquad DJ2_\delta := \frac{GL \cdot AD_\delta}{AG_\delta}$

$EP_\delta := \frac{GL \cdot AE_\delta}{AG_\delta} \quad GR_\delta := DJ_\delta \quad NO := BG \quad NR_\delta := GN + GR_\delta \quad GQ_\delta := \frac{NO \cdot GR_\delta}{NR_\delta} \quad CF_\delta := 2 \cdot GQ_\delta$






Reducing both by the symbolic processor leaves a little.

$$BG := 10 \qquad BD := \frac{BG}{13}$$

$$CF := 2\cdot BG\cdot\sqrt{BD}\cdot\frac{\sqrt{2\cdot BG - BD}}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)}$$

$$EP := -2\cdot BG\cdot\frac{\left(-BG\cdot\sqrt{BD}\cdot\sqrt{2\cdot BG - BD} - 2\cdot BG\cdot BD + BD^2\right)}{\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)^2}$$

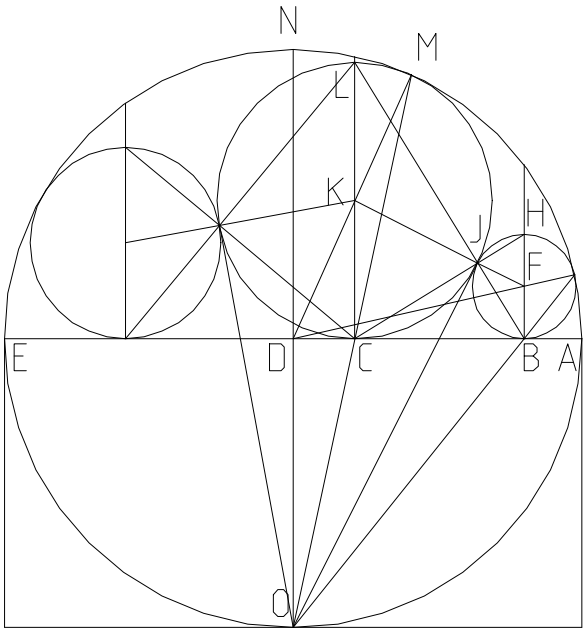
$EP = 5.556 \qquad EP - CF = 0$

And if I divide one by the other and reduce; $\dfrac{\left[BG\cdot\sqrt{2\cdot BG - BD} + 2\cdot BG\cdot\sqrt{BD} - BD^{\left(\frac{3}{2}\right)}\right]}{\left[\left(BG + \sqrt{BD}\cdot\sqrt{2\cdot BG - BD}\right)\cdot\sqrt{2\cdot BG - BD}\right]} = 1$

This is another figure that I had sketched out last year but never got around to writing down.

Given a Circle, place the next on the diameter.

I tried to reduce this series with the symbolic processor, but it is having trouble, at some point it switches AC for EC and I get the other circle.



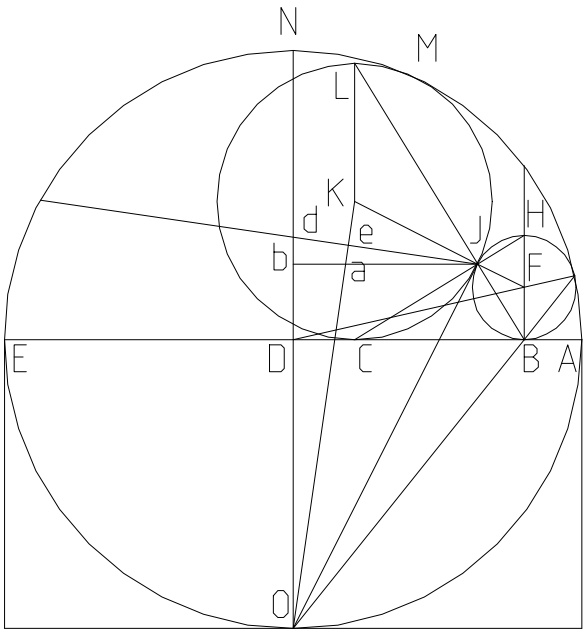

$AD := Radius \quad AE := 2 \cdot AD \quad AC := \frac{AE}{N}$

$CD := AD - AC \quad DO := AD \quad CO := \sqrt{DO^2 + CD^2}$

$NO := AE \quad MO := \frac{DO \cdot NO}{CO} \quad CM := MO - CO$

$CK := \frac{DO \cdot CM}{MO} \quad KO := \sqrt{CD^2 + (DO + CK)^2}$

$JK := CK$

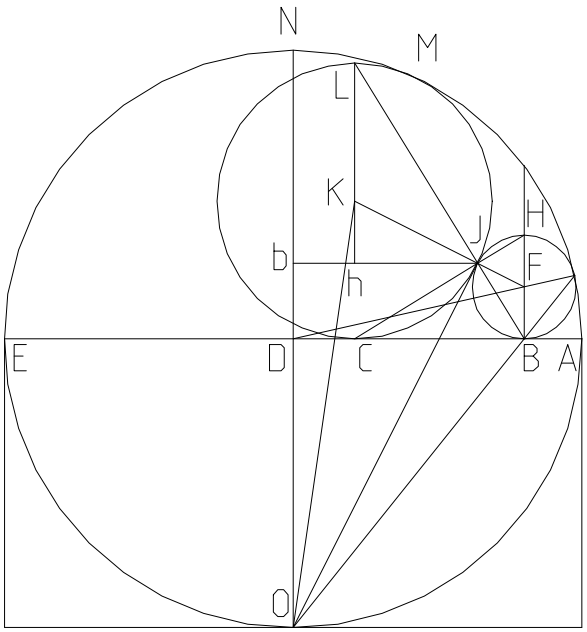



$Ke := \frac{JK^2}{KO} \quad Oe := KO - Ke \quad de := \frac{CD \cdot Oe}{DO + CK}$

$Je := \sqrt{JK^2 - Ke^2}$

$Jd := de + Je \quad bd := \frac{CD \cdot Jd}{KO}$

$Od := \frac{KO \cdot de}{CD} \quad Db := Od - DO - bd$




$Kh := CK - Db$

$Lh := CK + Kh$ $\quad FJ := \frac{JK \cdot Db}{Lh}$

$N \equiv \frac{10}{1}$ $\quad FJ = 3.965$

$Radius \equiv 108$ $\quad CK = 19.44$

And from the other side;

$N \equiv \frac{10}{9}$ $\quad FJ = 51.53$

$Radius \equiv 108$ $\quad CK = 19.44$

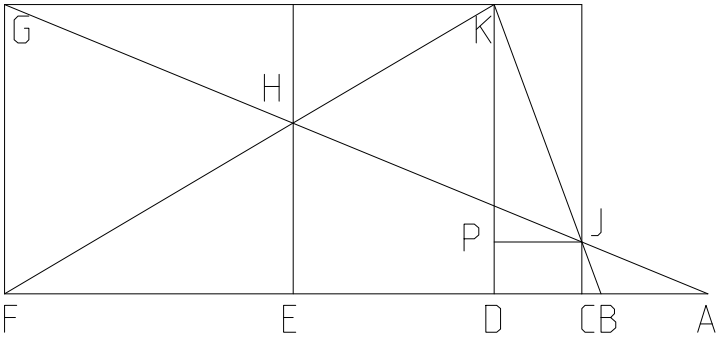
This figure might be recognized as the similarity point for Gemini root projection.

Given AC, CF, and that

$$AC = -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

find BC.

N can be any value whatever, except 0.




$CF := 216 \quad AC := 47.29 \quad N := 100000$

$CE := \frac{CF}{2} \quad AF := AC + CF \quad AE := AC + CE$

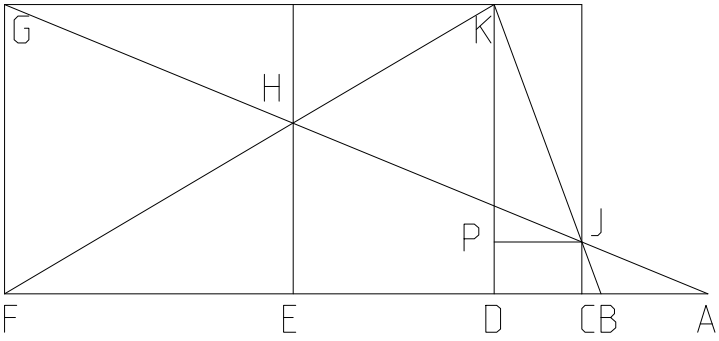
$FG := N \quad EH := \frac{FG\cdot AE}{AF} \quad EF := CE$

$DF := \frac{EF\cdot FG}{EH} \quad CJ := \frac{FG\cdot AC}{AF} \quad DP := CJ$

$CD := CF - DF \quad DK := FG \quad KP := DK - DP$

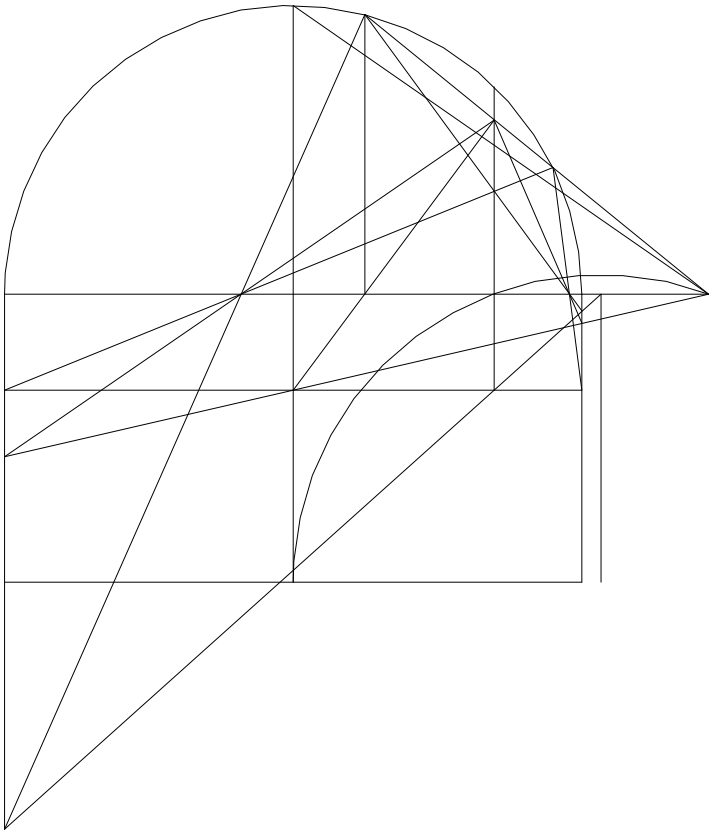
$BD := \frac{CD\cdot DK}{KP} \quad BD = 40.089$

$BC := BD - CD \quad BC = 7.201$



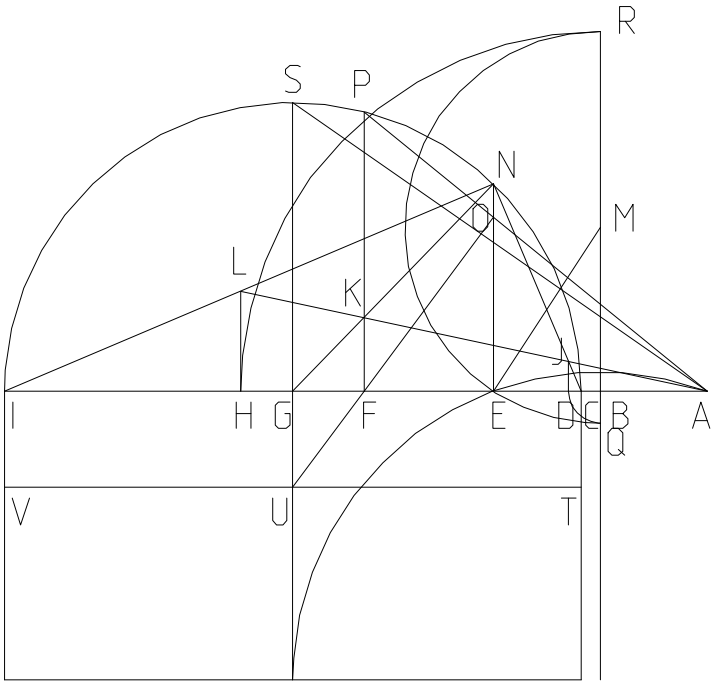

$$AC_f := -CF\cdot\frac{\left(\sqrt{BC}\cdot\sqrt{BC + CF} - BC\right)}{\left(-CF + 2\cdot\sqrt{BC}\cdot\sqrt{BC + CF} - 2\cdot BC\right)}$$

$$\frac{AC_f}{AC} = 1$$



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

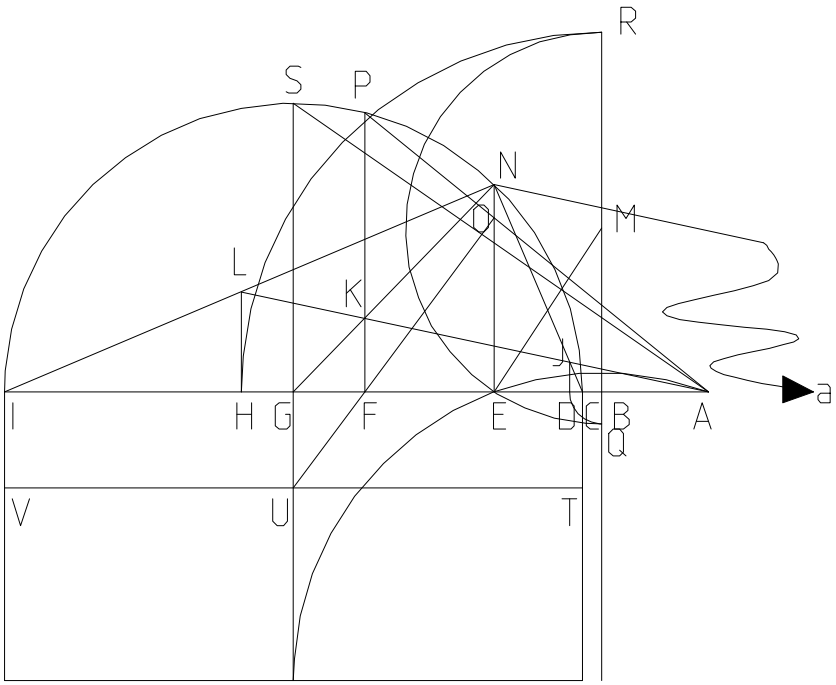
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



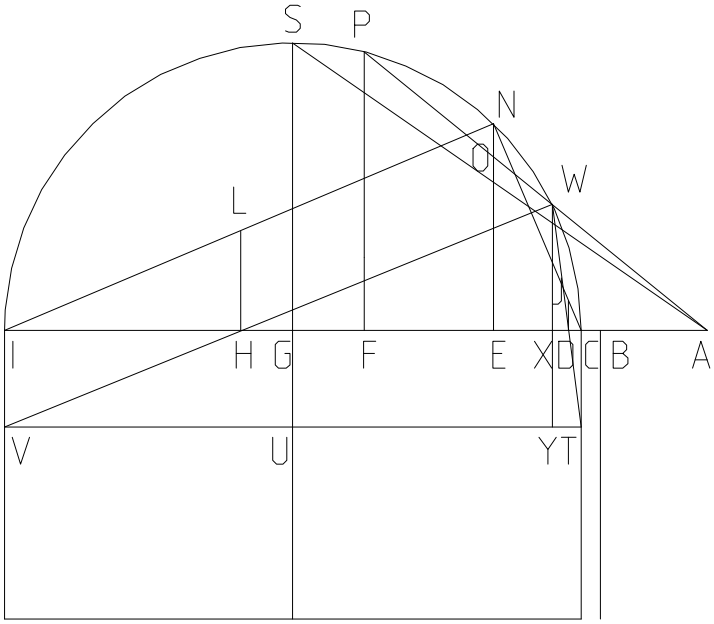

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

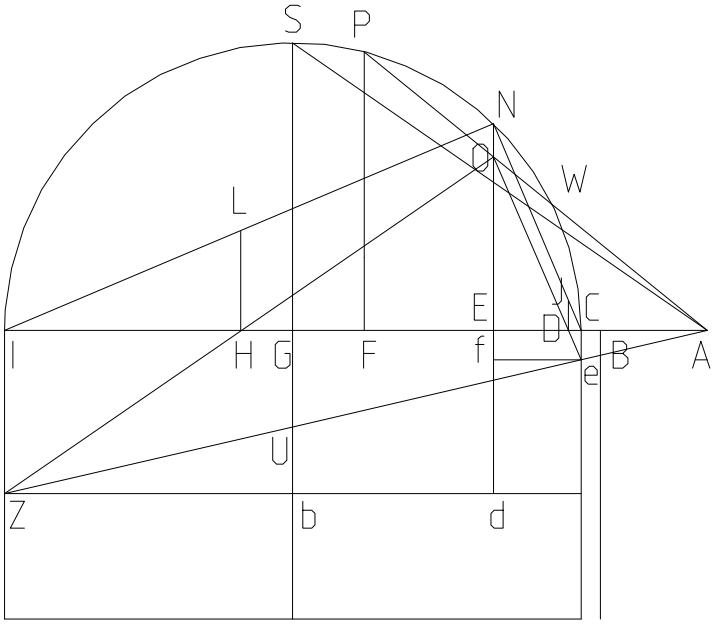
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

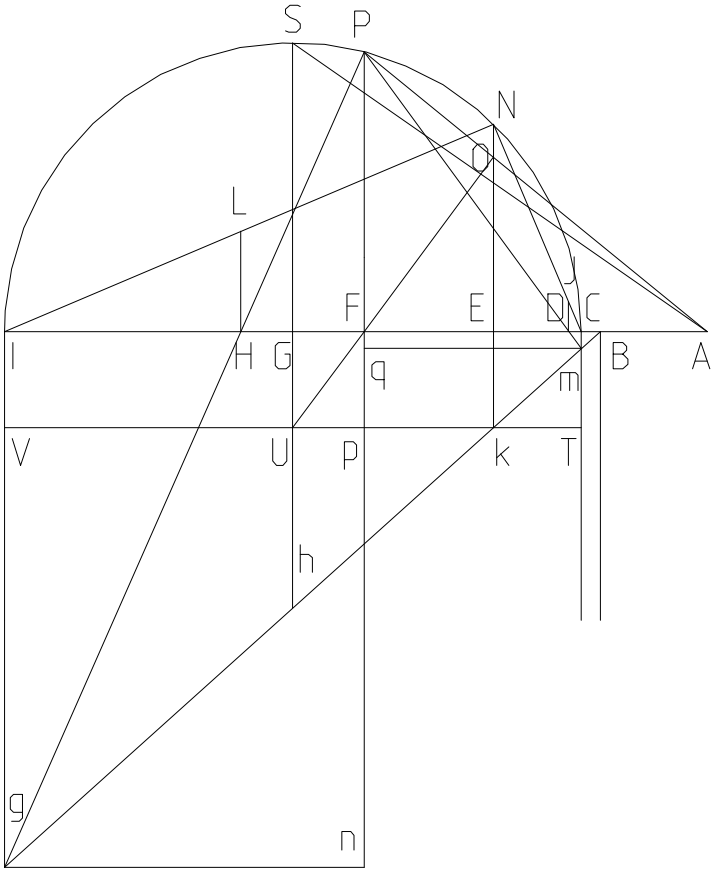
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$



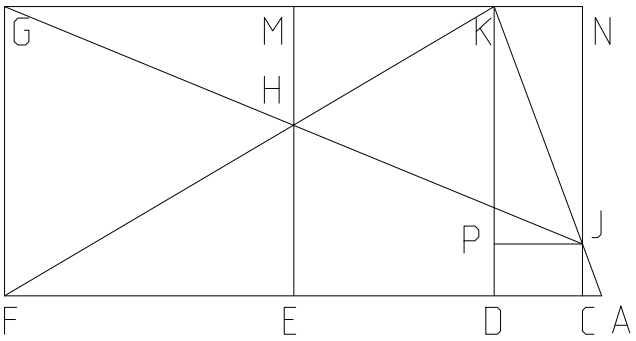

$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

Given CF and CD such that CD = $\sqrt{AF \cdot AC} - AC$ find AC.




$CF := 216 \quad CD := 32.89 \quad CE := \frac{CF}{2} \quad EF := CE$

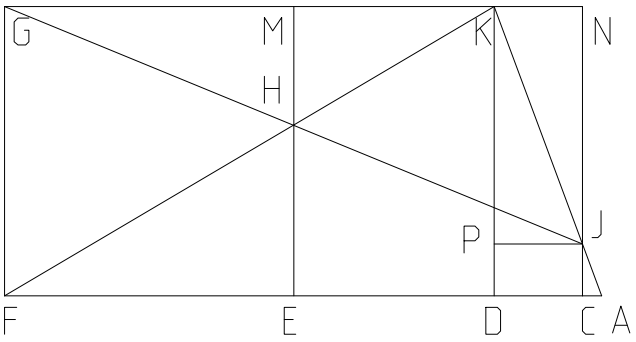
Except for 0, N can be any value.

$N := 108 \quad FG := N \quad DK := N \quad DF := CF - CD$

$EH := \frac{DK \cdot EF}{DF} \quad EM := FG \quad HM := EM - EH$

$CN := FG \quad HM := EM - EH \quad GN := CF$

$GM := EF \quad JN := \frac{HM \cdot GN}{GM} \quad JC := CN - JN$




$KP := JN \quad JP := CD \quad AD := \frac{JP \cdot DK}{KP}$

$AC := AD - CD \quad AC = 7.201$

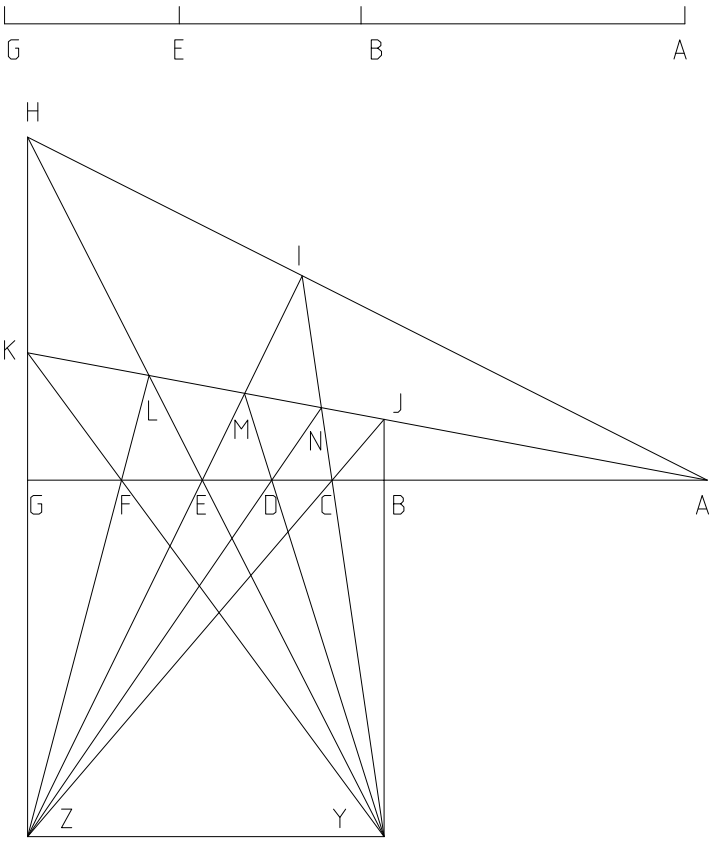
$AF := AC + CF$

$\frac{\sqrt{AF \cdot AC} - AC}{CD} = 1 \qquad \frac{\sqrt{AF \cdot AC}}{AD} = 1$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use $5^{th}$ root series for example.




$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

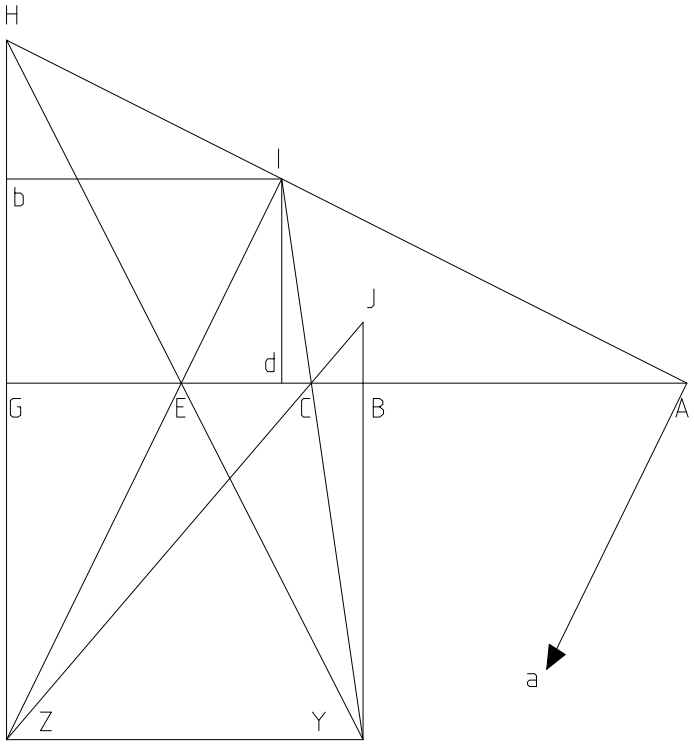
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

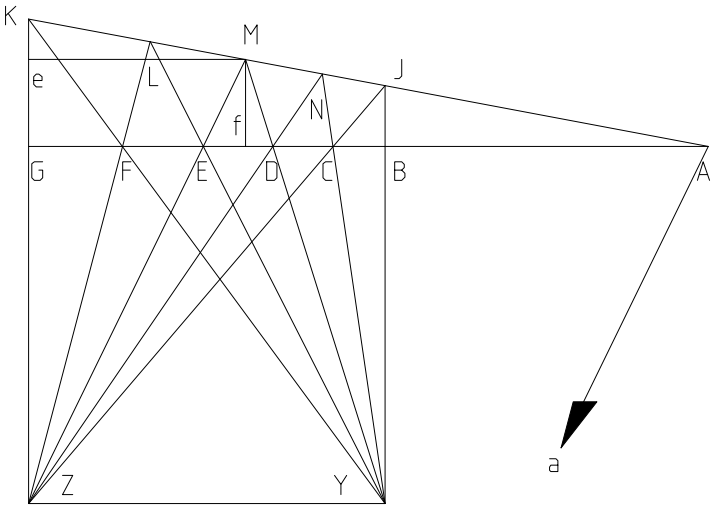
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



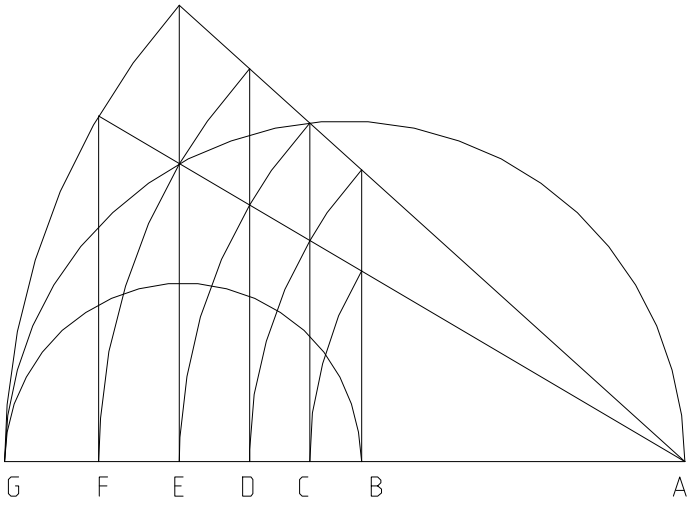



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



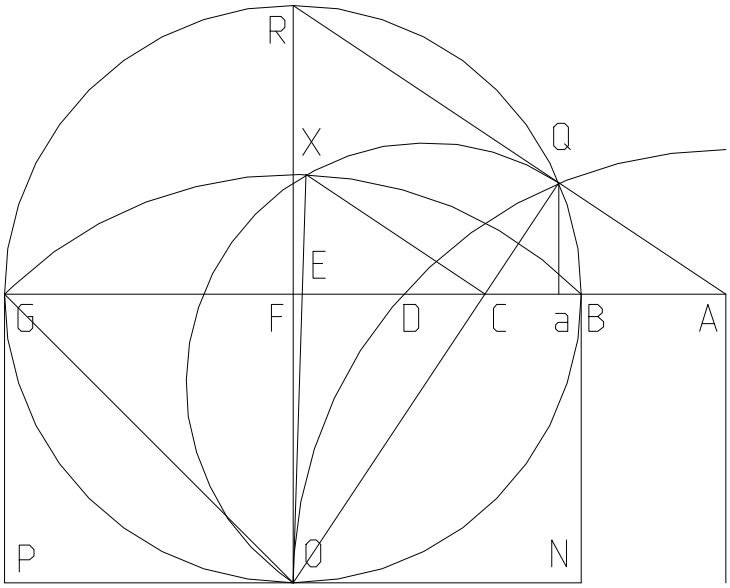

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

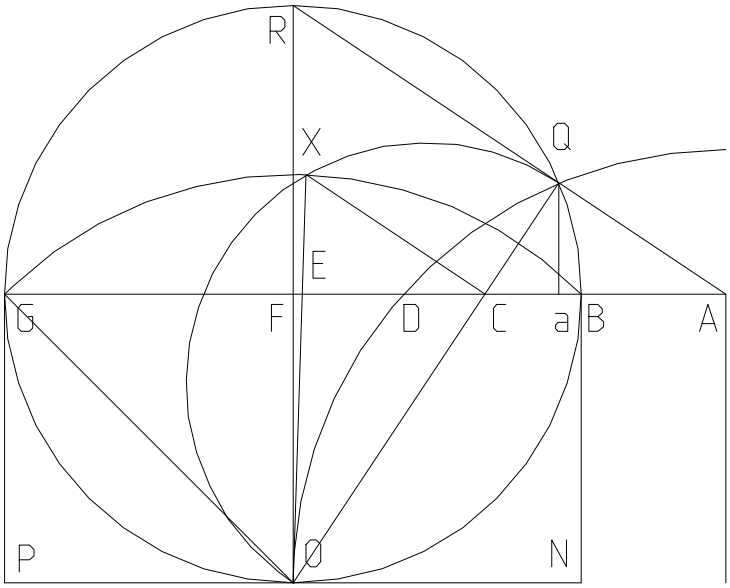
$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.




Is CX a constant?

I have had so much back work to catch up on I post dated a couple.

$AB := 54 \quad AG := 270$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FO := BF$

$AF := AB + BF \quad FR := BF$

$AR := \sqrt{AF^2 + FR^2} \quad AQ := \frac{AB \cdot AG}{AR}$

$Aa := \frac{AF \cdot AQ}{AR} \quad Qa := \frac{FR \cdot AQ}{AR}$

$Fa := AF - Aa \quad OQ := \sqrt{Fa^2 + (FO + Qa)^2}$

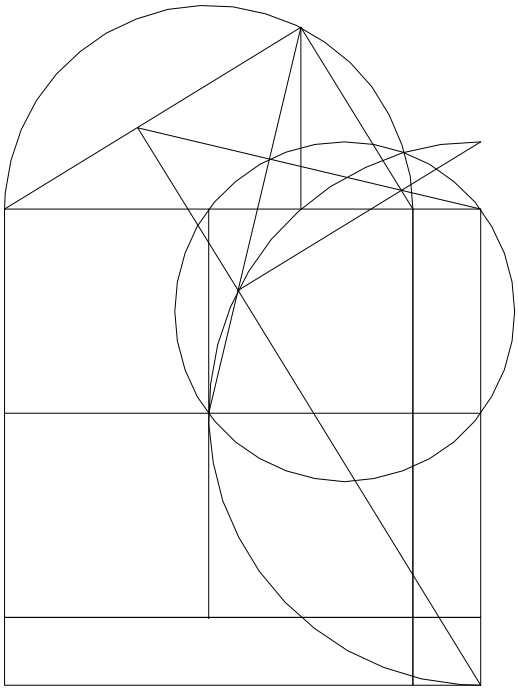
$CQ := \frac{OQ \cdot Qa}{FO + Qa} \quad OX := \sqrt{BF^2 + FO^2}$

$CO := OQ - CQ$

$\frac{\frac{OX^2}{OQ}}{CO} = 1$

Both expressions reduce to,

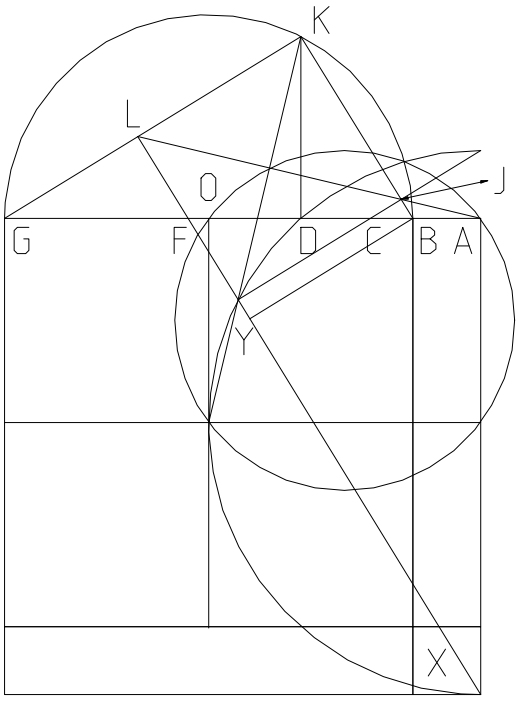
$CQ = 49.923 \qquad OQ - \frac{OX^2}{OQ} = 49.923 \qquad \frac{AG - AB}{AG + AB} \cdot \frac{\sqrt{2 \cdot (AG \cdot AB)}}{\sqrt{AB^2 + AG^2}} = 49.923$



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.




$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

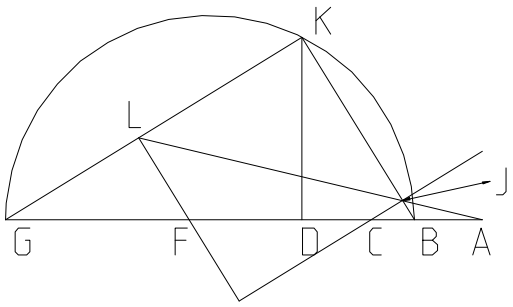
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



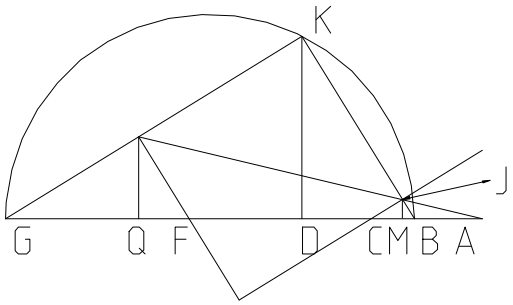

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

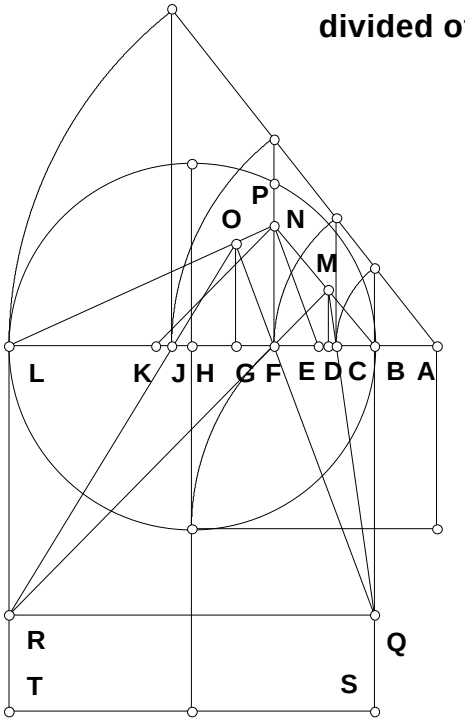
On the left is the first and last of the series, on the right is the entire series.

| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$

# 12/20/96 Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off as shown in the figure.**




$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \qquad BS := BL \qquad LT := BL$

$BH := \frac{BL}{2} \qquad HL := BH \qquad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \qquad FL := AL - AF \qquad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \qquad FN := \frac{BQ \cdot FP}{BS} \qquad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \qquad FG := \frac{EF \cdot FL}{EL} \qquad GO := \frac{FN \cdot FG}{EF}$

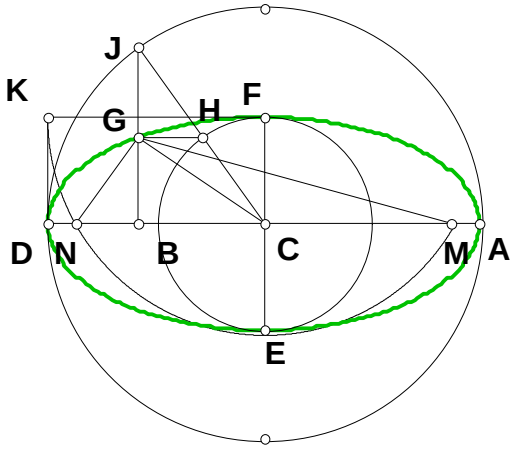
$GL := FL - FG \qquad LR := BQ \qquad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$

## 09/11/97 The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**




$N_1 := 5 \qquad N_2 := 1 \qquad N_3 := \frac{1}{5}$

$AD := N_1 \qquad EF := N_2 \qquad AB := AD \cdot N_3$

$BD := AD - AB \qquad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \qquad BC := AC - AB \qquad CH := \frac{EF}{2}$

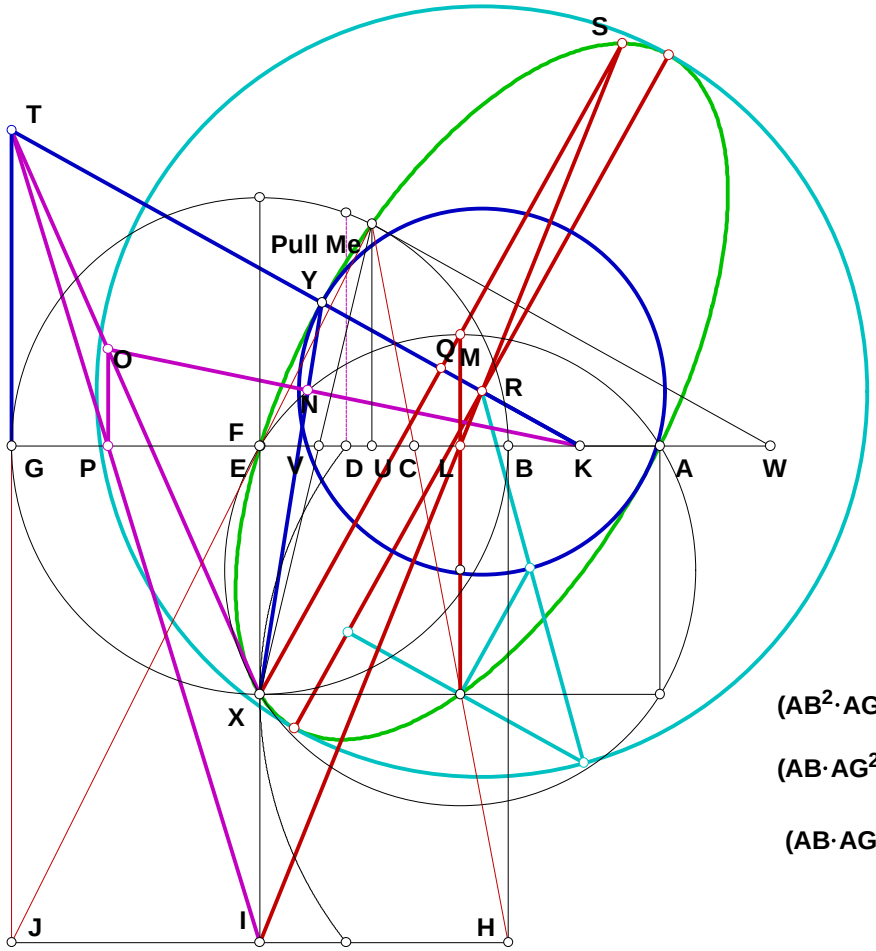
$CJ := AC \qquad BG := \frac{BJ \cdot CH}{CJ}$

$CG := \sqrt{BG^2 + BC^2} \qquad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$CG := \frac{1}{2} \cdot \sqrt{4 \cdot N_3 \cdot N_2^2 - 4 \cdot N_3^2 \cdot N_2^2 + N_1^2 - 4 \cdot N_1^2 \cdot N_3 + 4 \cdot N_1^2 \cdot N_3^2}$$

$MN := \sqrt{(N_1 - N_2) \cdot (N_1 + N_2)} \qquad BG3 := \sqrt{N_3 - N_3^2} \cdot N_2$

For the delian quest
John Clark 08/11/99
jclark8659@wwnet.net




··· Clear
··· Ellipse
··· Step One Center of Ellipse
··· Step Two
··· Step Three
··· Step Four Minimum
··· Step Five Maximum

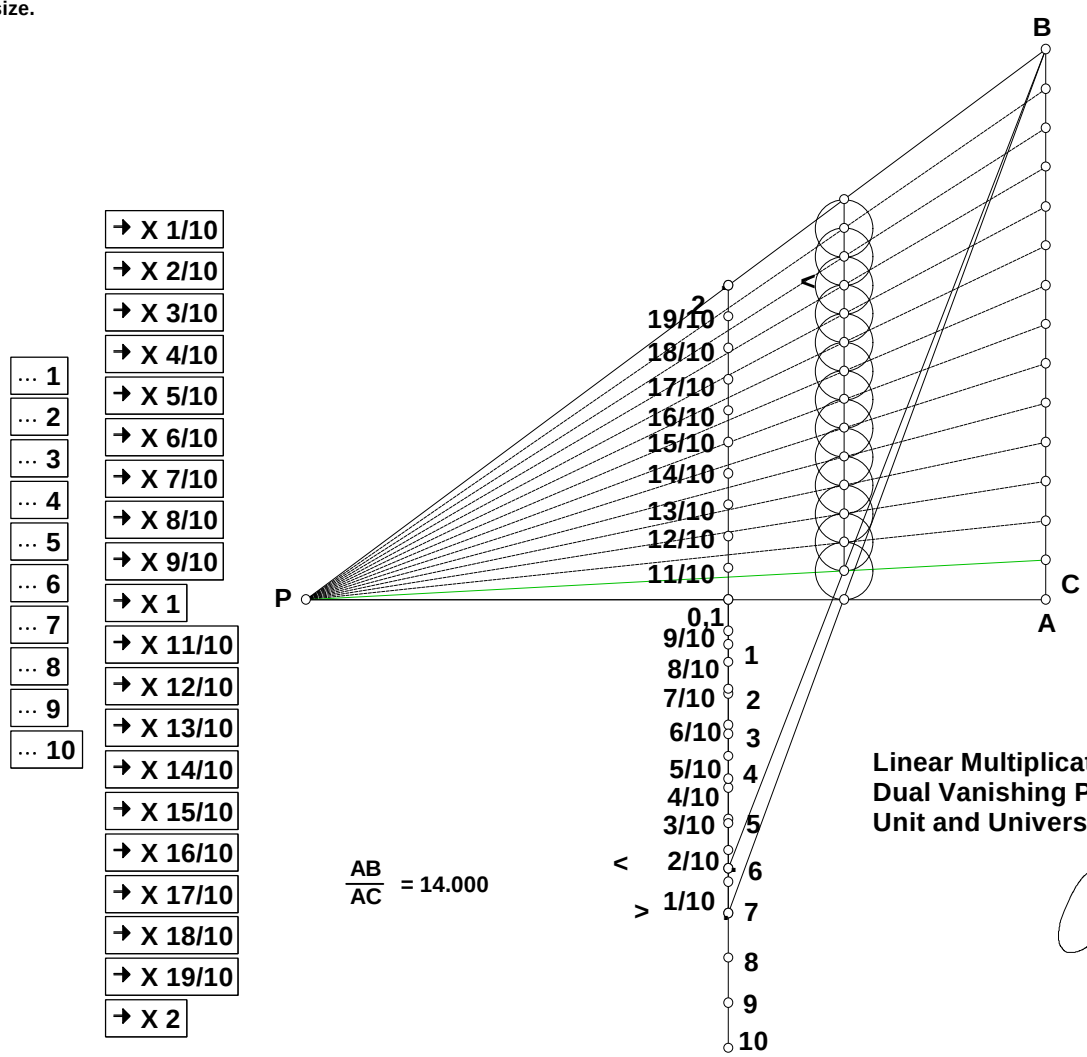
$(AB^2 \cdot AG)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AG^2)^{\left(\frac{1}{3}\right)} - AE = 0.000$

$(AB \cdot AG)^{\left(\frac{1}{2}\right)} - AD = 0.000$

# a two Dimensional Solution to the Delian Problem.

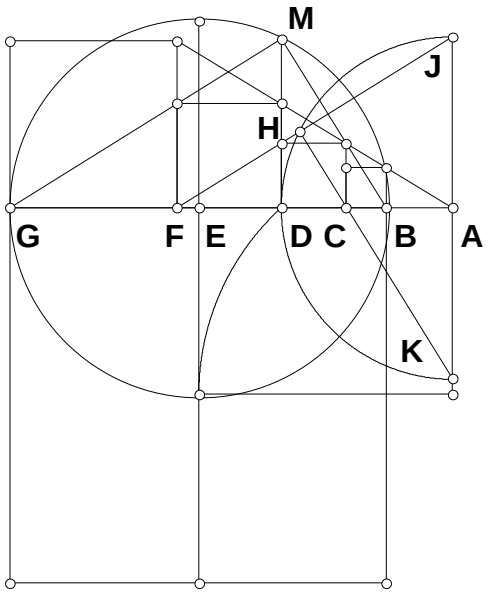
**Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.**




$\frac{AB}{AC}$ = 14.000

**Linear Multiplication and Division,**
**Dual Vanishing Point Method**
**Unit and Universe of Discourse.**

# 07/09/00 Alternate Method Quad Roots




$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

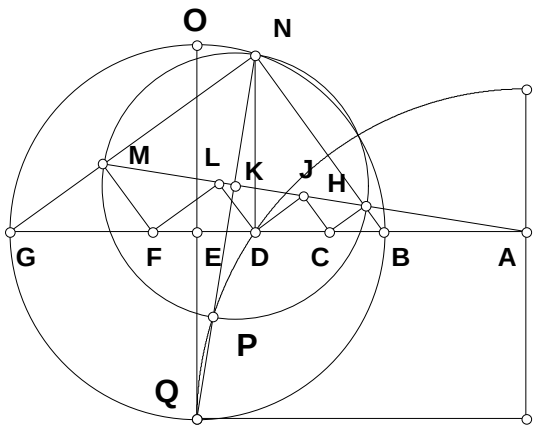
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

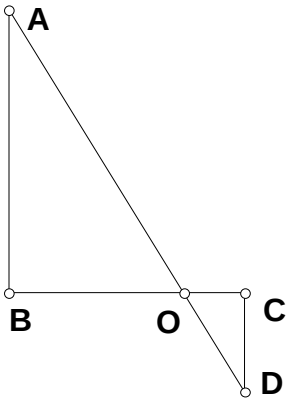
$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

# 08/01/00 Alternate Method Quad Roots



## 08/07/00 Proportion Series II

**Two unknowns have the same proportion as two givens and the sum of the unknowns are known. Find the two unknowns.**




$AB := 9 \qquad CD := 3 \qquad BC := 5$

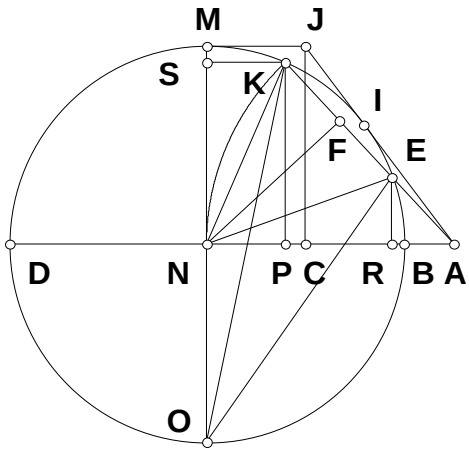
$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

$\frac{AB}{CD} - \frac{BO}{CO} = 0$

# 08/23/00 Trisection In A Square Root Figure




$N := 5 \qquad AB := 4 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN \qquad CJ := BN$

$AJ := \sqrt{AC^2 + CJ^2} \qquad AK := AJ$

$AN := AB + BN \qquad AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$

$AF := \frac{AP \cdot AN}{AK} \qquad FK := AK - AF$

$EF := FK \qquad AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK}$

$$AB \cdot (AB + BD) \cdot \frac{\left(8 \cdot AB^2 + 8 \cdot AB \cdot BD + BD^2\right)}{(2 \cdot AB + BD)^3} - AR = 0$$

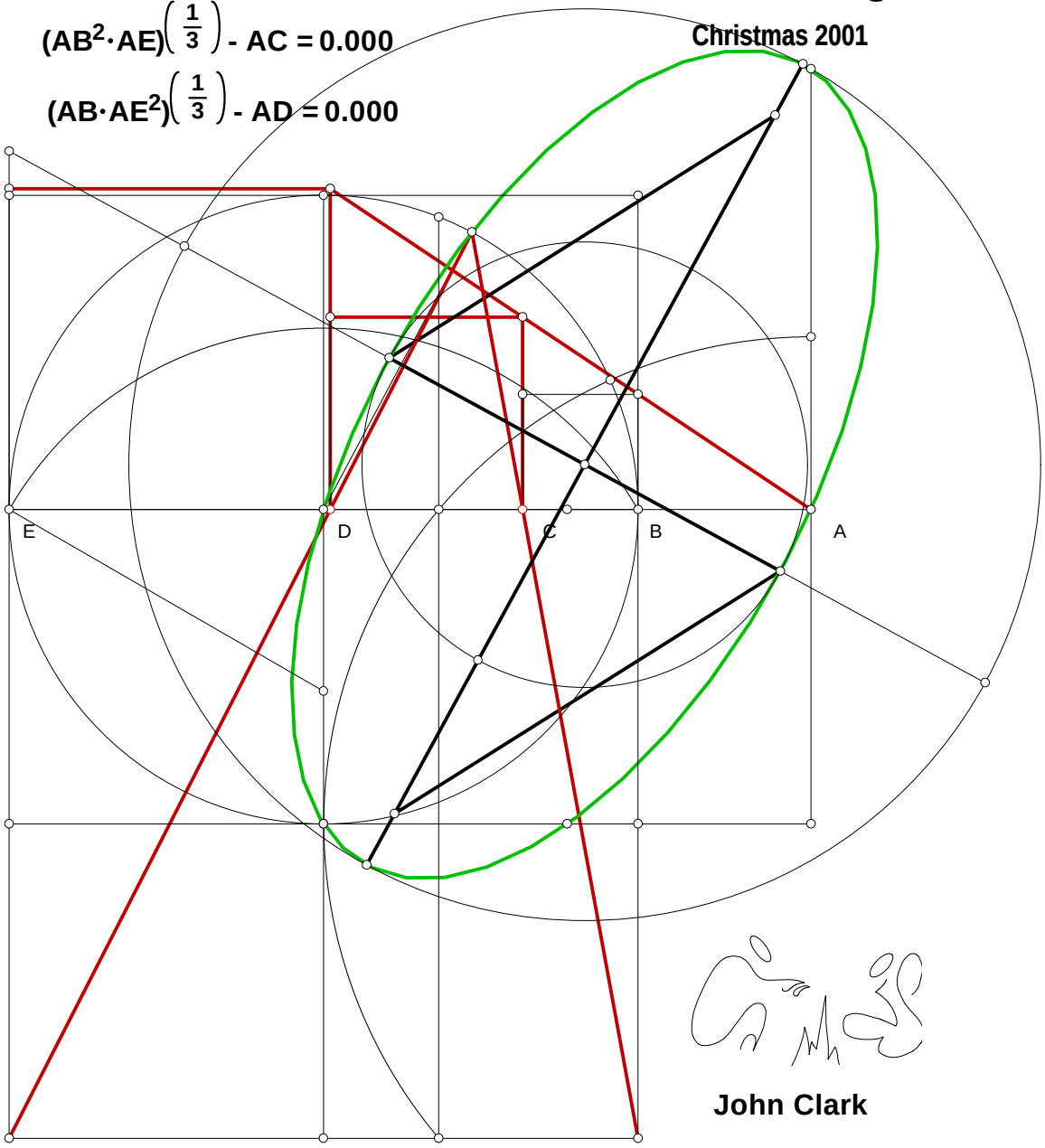
$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$

# The Delian Quest

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$

Christmas 2001






John Clark

## Foreword

One can deny a convention and thus deny a concurrence afforded by that convention. For example, if one wanted to prove that a house could not be built from a set of plans, simply deny the convention of units from which measure is effected. Does this denial afford a proof of the non-existence of the ability to construct that house from a certain set of plans?

If one denies a convention, one cannot solve for the Name Of The Beast. If one denies a convention, one cannot solve The Delian Problem. Solutions are then only for those who have a certain respect for The Law Of Reciprocity–those who aspire to have but one measure in their purse, those who seek to perceive by a single eye–those that that seek the attainment to a civil human state.

There have been hard drive failures and viruses that have claimed some work–sometimes many months worth. A few plates are still in folders, waiting for attention, they have not made it in this text, perhaps they will in a future revision. A sequel, ***Three Pieces Of Paper*** is underway and is on angle trisection. There is so much I want to do, so much I am doing, that revision is inevitable and completions rare.

# The Delian Quest

> "Socrates: And I, Meno, like what I am saying. Some things I have said of which I am not altogether confident. But that we shall be better and braver and less helpless if we think that we ought to inquire, than we should have been if we indulged in the idle fancy that there was no knowing and no use in seeking to know what we do not know;—that is a theme upon which I am ready to fight, in word and deed, to the utmost of my power." ***Meno***, by Plato

## Introduction

The Delian Quest is a novel written primarily in two languages, Geometry and Algebra. These two languages are commensurate with the definition of a thing.

**Definition**: A thing is any difference what so ever circumscribed by any form what so ever.

Commensurate with difference in the definition is the ***Geometric Grammar***. Commensurate with the form in the definition is the ***Algebraic Grammar***. The smattering of English is of no great consequence. I take exception to any teaching that does not recognize the geometric figure as ***the Grammar of Geometry***. Much of what is called geometry is apparently stated by those who have no clue. The ***Geometric Grammar*** system is a ***relatiologic*** and ***Algebraic Grammar*** is a ***tautologic***–together they make a proposition–and together and only together can a ***thing*** called ***knowledge*** be constructed–by definition.

By way of definition, so called non-Euclidean Geometries which cannot construct the figures is not knowledge and those that use Euclidean figures but not standard definitions and practices are also not knowledge. Since the ***Geometric Figure*** is the ***Geometric Grammar***, there is no such thing as a Grammar without Grammar. And by the principle of well-defined terms, not knowing how to define a thing, or the inability to be consistent in what has been defined, does not constitute literacy.

## The Delian Problem

From what I remember, the Delian Problem acquired its name from the circumstances of the problem's inception. In order to stop a plague in ancient Greece an oracle at Delos suggested–supposedly by direction of a *God*–that the altar of Apollo be doubled. The altar of Apollo was a cube. What it boiled down to was the abstraction of the cube root of 2–the cube root of twice its current volume. Now if, in geometry, one can precisely abstract the square root of 2, it seemed reasonable that, in geometry, one might abstract the cube root of 2 as well. What this means is that an act of will may be performed for which no Arithmetic name is possible in the case of the square maybe it can be done for the cube as well. The conventions of Arithmetic Grammar cannot provide an Arithmetic name for the square root of 2. Algebraic grammar overcomes this problem of irrationality in Arithmetic Grammar by incorporating operands as part of the Algebraic name. The problem in Geometric Grammar was, no one knew how to abstract cube roots. No one knew how to assert that act of will. In time, the general response to the Delian Problem was that it is an impossible problem to solve. No name could be provided in Arithmetic Grammar, and none in the Grammar of Geometry, while in Algebraic grammar, a name

could be simply had, but no clue as to how to render the fact.

## A Class Idea

Those things that are grouped in accordance by a common characteristic all have the same name by that measure. Some people may recognize this notion as Set Theory, some may see it as the founding principle of all Grammars because it is the foundation upon which the concept of words are based. A fact of the craft of measurement itself is derived from $A = A$ and $A - A = 0$ which is derived from knowing and not knowing, some may realize that it is the ***Sensor Model*** of reality. This has a very important application. If there is no difference between two things, then there is no knowledge of 'two-ness' under the class definition–therefore both items are regarded as the same–treated the same, respected the same. By definition, it is not possible to act differently in any case or any instance toward members of the same set. Action is a reaction from cause, but one has stated that all members are the same, no difference or cause can then exist between them–under that class definition. The definition of a set provides Universality circumscribing all members of that set.

In regard to the Delian problem one might apply the idea of definition in this manner; take the straightedge and compass and abstract a common characteristic that would group them into one class (to my knowledge such an application of the foundation of logic has never been proposed to solve the Delian Problem). If one finds that common characteristic then one can use it as a measure to determine that other tools, which have that same characteristic, are included in that class. Perhaps these tools are what is needed to solve the Delian Problem. Perhaps the real intention of the problem was to learn to think in terms of definition–a founding principle of judgment.

The tool one would be looking for would have the class characteristic that would define a Two Dimensional Geometer's Writing Instrument defined by a common characteristic. This is the true custom upon which all language is based, the same notion that determines the meaning of any word in any language that one speaks. Logically, reasonably, factually, one cannot state that Geometry is to be written in only two tools of a class of tools if that class is well defined, for there is no way to distinguish these tools under the class concept, the definition of class membership. Providing of course one can abstract the class concept that the tools reside in. Aristotle quite rightly determined that a definition renders both ***substance*** and ***form***–every bit of knowledge is known that way, ***substance*** and ***form***. Even in our mind, to think of ***substance*** we supply a generic ***form*** to it, and to think of ***form*** we assert a generic ***substance*** for it–otherwisde we could not conceive of either. What is the ***substance*** and the ***form*** of the Geometric Writing Instruments we are familiar with?

A straight edge provides one with one and only one difference (***substance***) between two points (***form***). A compass provides us with one and only one difference (***substance***) between two points (***form***). One may comprehend the straightedge however as providing the Unit of discourse, while the compass the Universe. Has anyone ever comprehended the ellipse as a figure that is produced as one and only one difference (***substance***)–a sum–between two points (***form***)? Is it possible that the *Gods* gave a problem, just like the problem of understanding the ***Name Of The Beast*** in the ***Bible***, whereby the only solution is to think by one measure–class concepts–the very foundation of language

itself? Perhaps the only possible path to a solution is when men start to reason by respecting what, as recorded in the Bible, is a Holy understanding–the ***Law of Reciprocity***? ***A*** = ***A***. I don't know. I do know that the tool that constructs an ellipse is in the same class as the straightedge and compass in regard to the same definition of ***substance*** and ***form***, and also that the ellipse can solve the Delian Problem. I would strongly suspect that the problems given by the *Gods* all have a common solution which involves a growth in men's understanding. I did not suspect such an understanding when I started my search. I started this search because either no *God* gave the Delian Problem or ,if given, it would seem reasonable that a *God* would only ask for the possible and, if it were possible, then it was possible for me to find it.

## Proof And Prove

Proofing and proving are two entirely different concepts. Proofing is the compliance of conventions within a grammar. Proving is the verification of existence–which means attaining to an experience.

The proof of a proposition in a tautologic is then relies on a tautology, ***name*** = ***name***. One might have a name, but no meaning of that name, but still attain to proof. Proof in a relatiologic is constructions tendered through a given set of tools used in a prescribed manner. One might have a drawing with no meaning, but still it is proofed.

It is typical even for scholars, whole communities and histories of them, to confuse and make a mess out of the two ideas. To prove a thing, one must supply both logic systems, the tautological and the relatiological. To proof a thing, one needs only demonstrate compliance with the conventions of the grammar. Proving implies proofing, but proofing does not imply proofing–Aristotle said very nearly the same thing.

To prove a thing, is to tender both the experience and the names in compliance with a given set of conventions–however, one cannot say that one has attained to conventionality by listing those conventions in the Particular, avoiding or not knowing the Universal upon which they stand–matter and form must be known and knowable.

## Proposition

Even the process of group construction (propositional construction) follows the definition of proposition itself. One method is particular (the ***substance***), each member is enumerated, the other method is universal (the ***form***), each member has the same definition. Each method renders a judgment as to inclusion and exclusion of a member into a class. As Plato pointed out, only by comprehending all the details of both methods can one be said to have understanding. The methods themselves follow propositional form in another respect–the particular method is not effected through understanding but the method of forms (definition) is–this Plato pointed out too and is why he championed that method. Using class definition to define Geometric Writing Instruments is an idea that has not been promoted or conceived, and if enacted many mistakes will be made, but nevertheless, we may not be altogether certain as to what we are doing, but we can be certain that life is in that quest for knowing. Perception determines conception, conception determines will.

## Reading

Since perception determines conception and conception determines will, learning to 'say' what we 'see' can be improved by practice. One must read the figure and the equations together in this work. j.c.

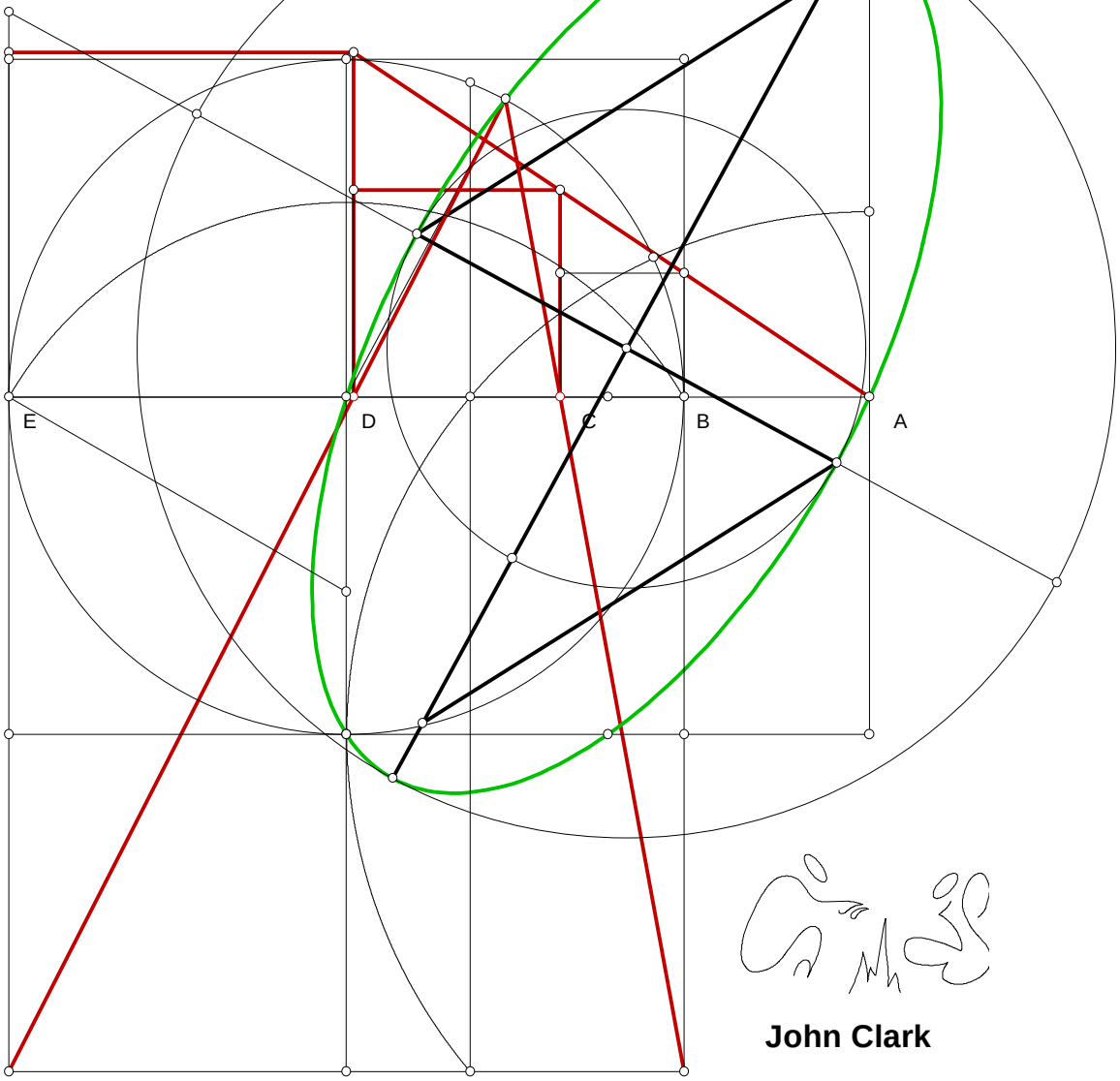
> "For the person who deliberates seems to investigate and analyze in the way described as though he were analyzing a geometrical construction" ***Nicomachean Ethics*** by Aristotle

# The Delian Quest

## 1989

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$






John Clark

This was originally type written and hand drawn figures. I ran copies off to send out into a non receptive world. I did not blush at my own ignorance.

# THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefor this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

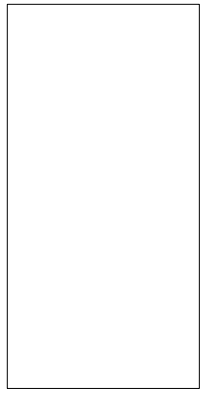


Plate 1

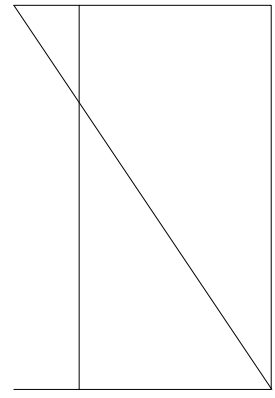


Plate 2

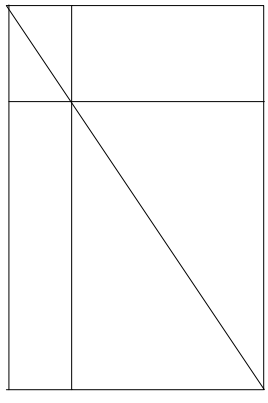


Plate 3

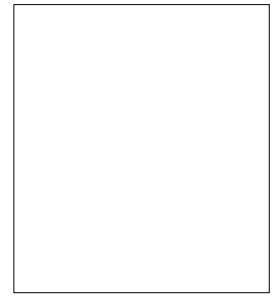


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

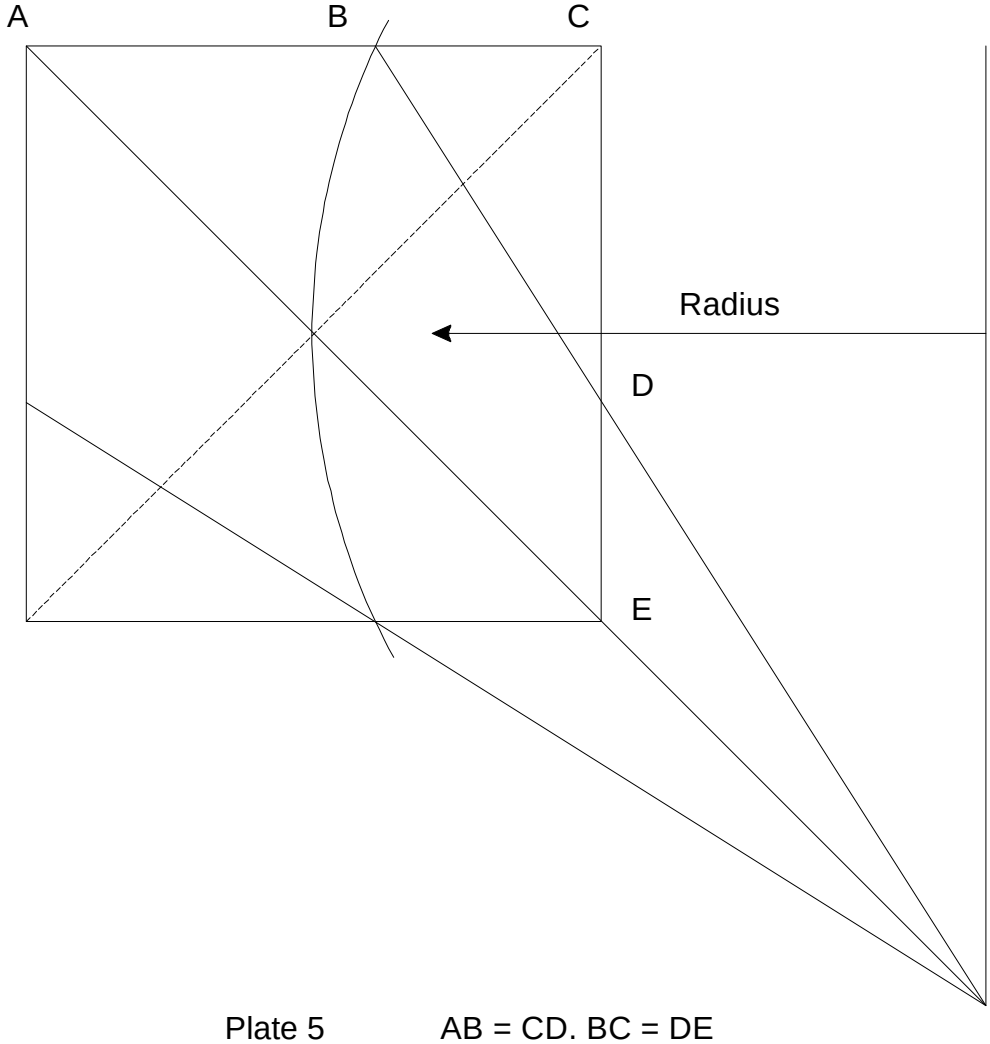
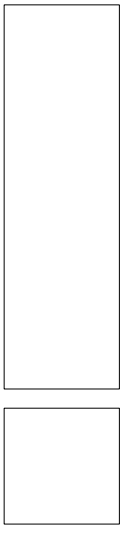
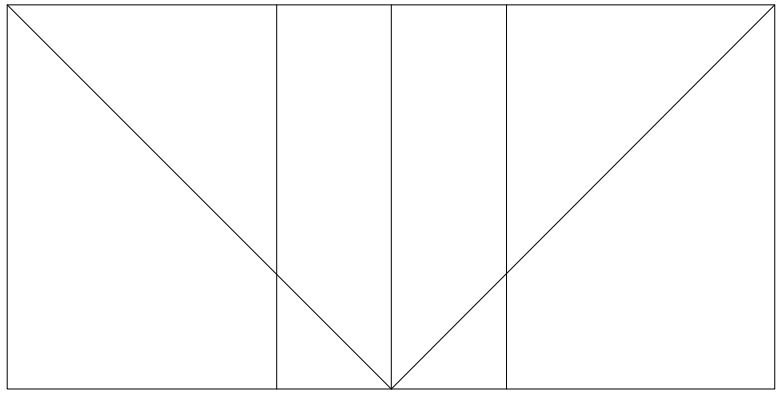



Plate 5 AB = CD. BC = DE

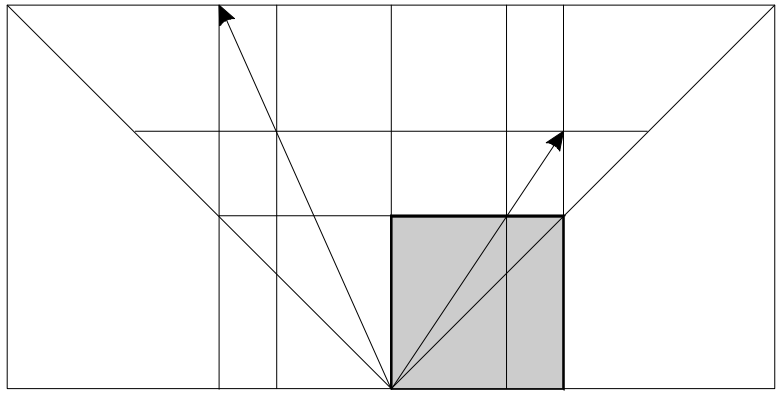
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



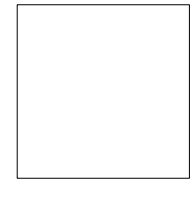
P.6



P.7

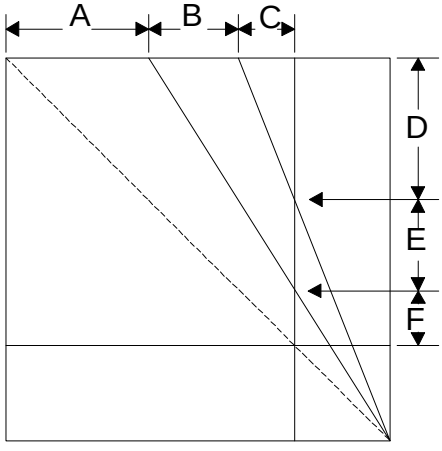


P.8

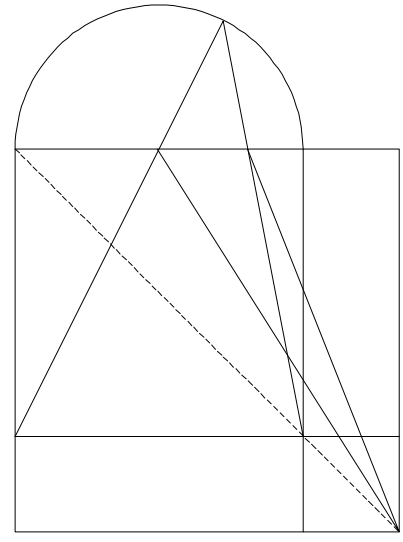


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

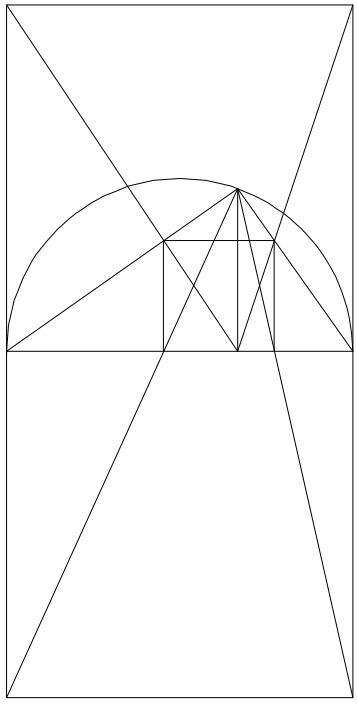



P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

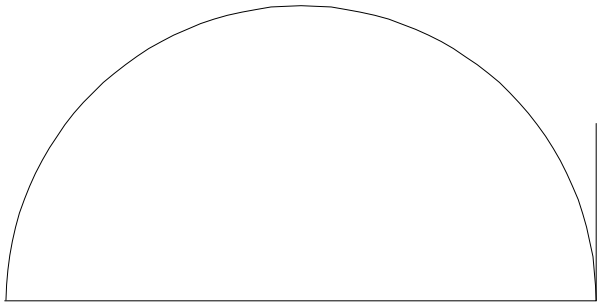


Plate 13

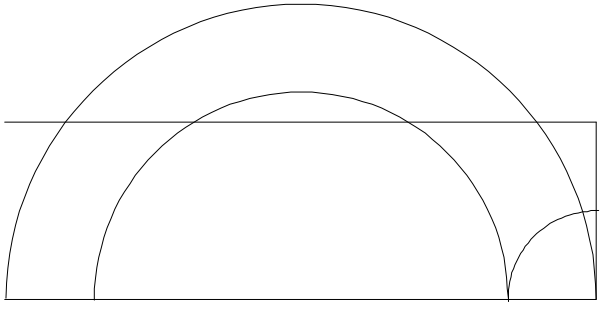


Plate 14

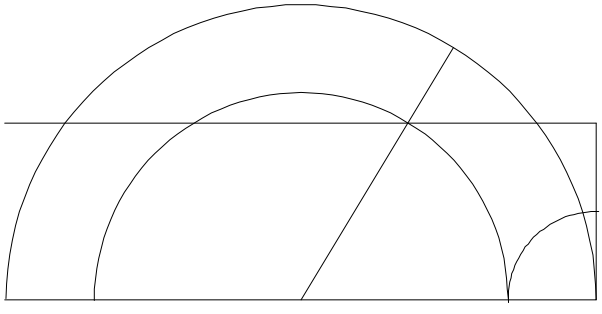


Plate 15

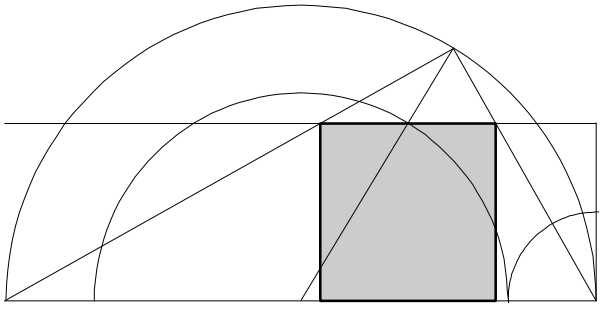
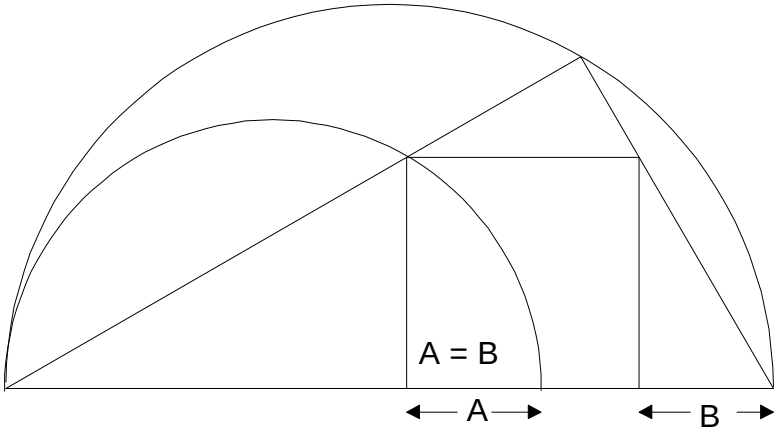
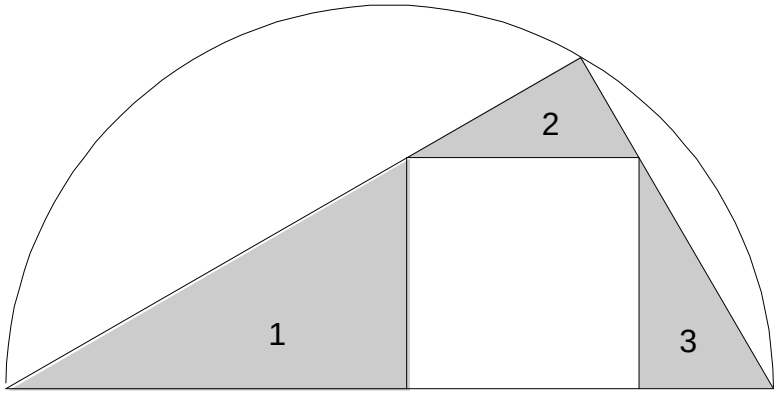


Plate 16

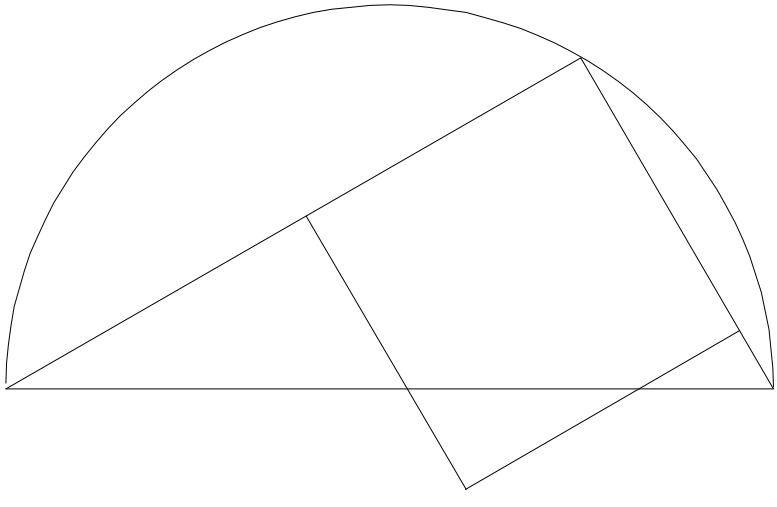
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.




In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"




P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

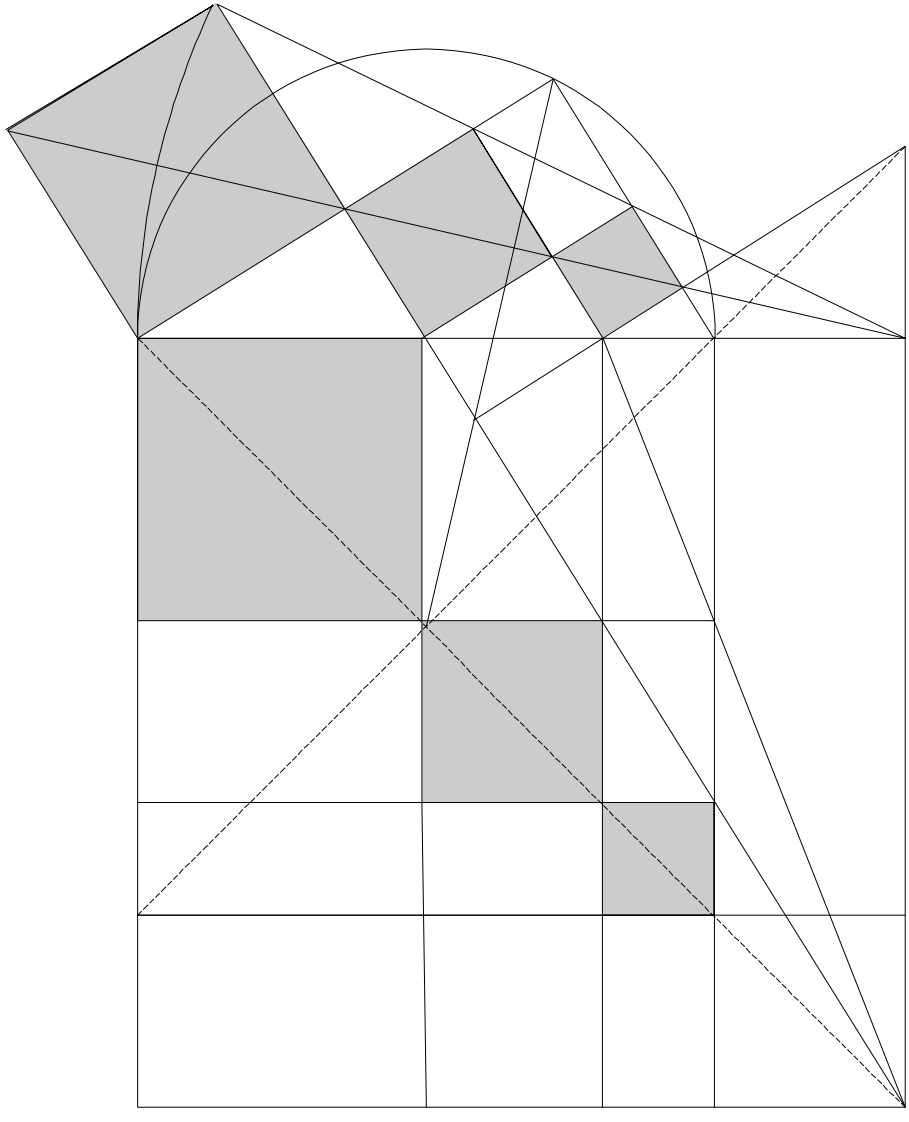
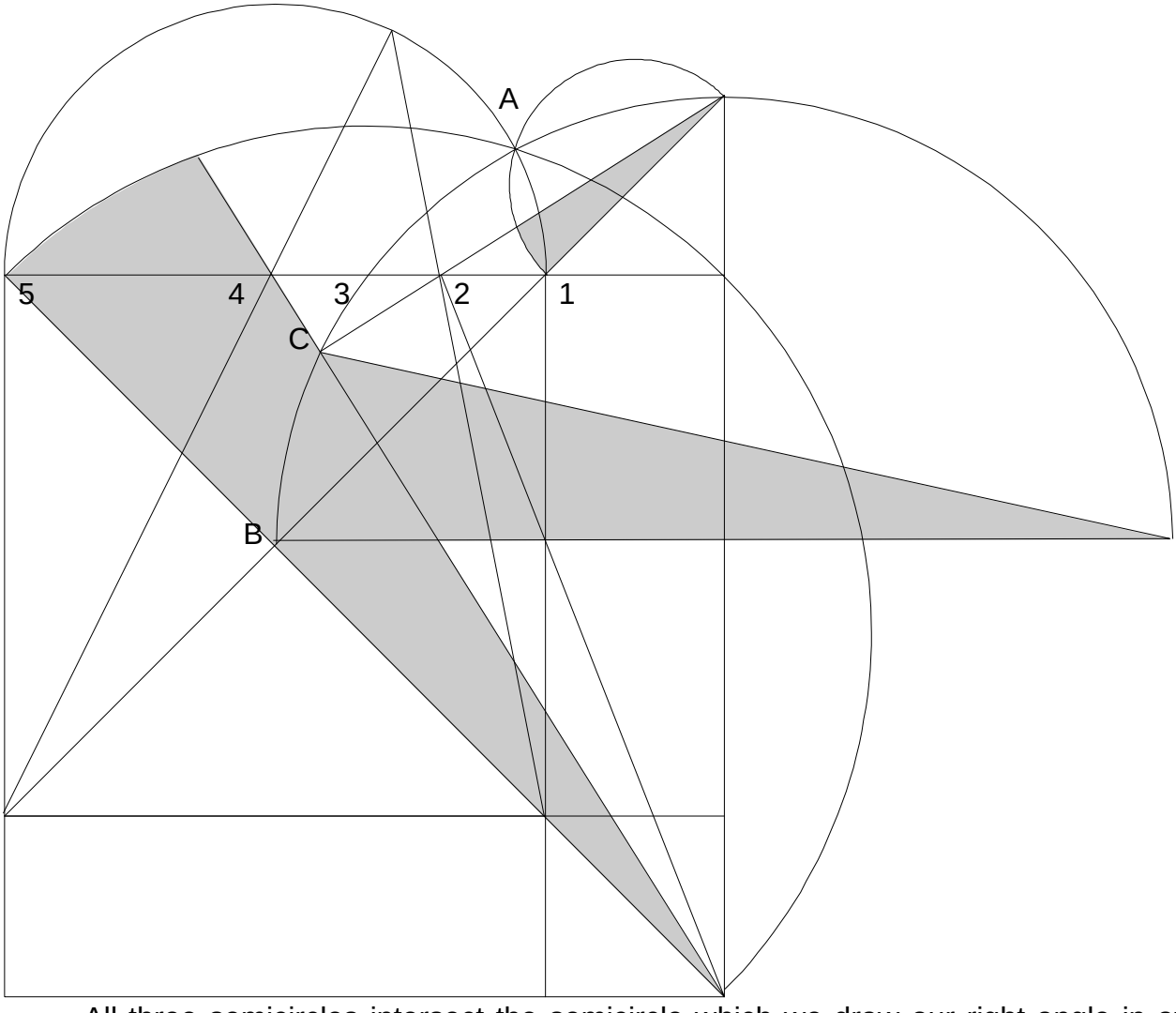


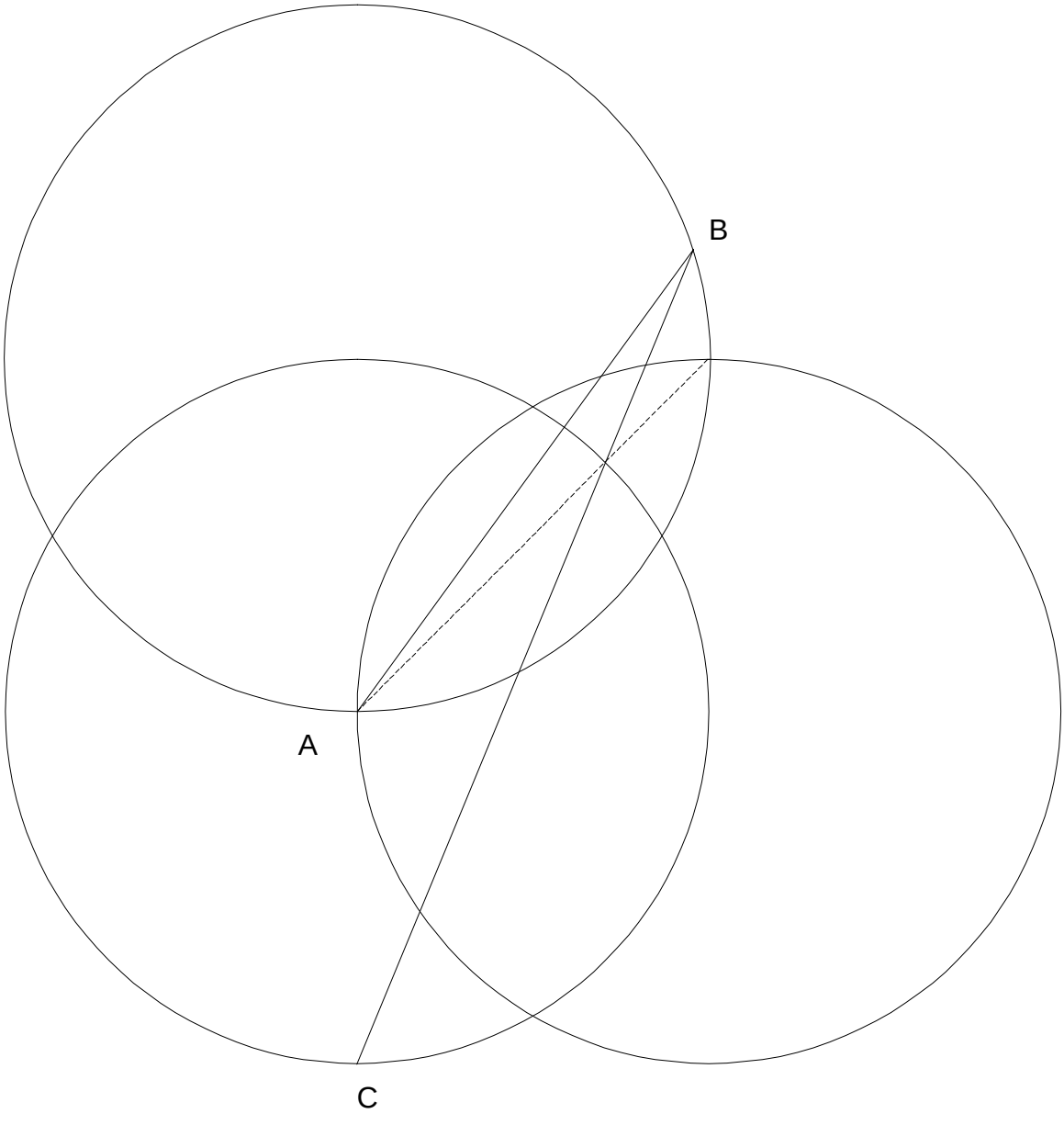
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.




All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.




How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

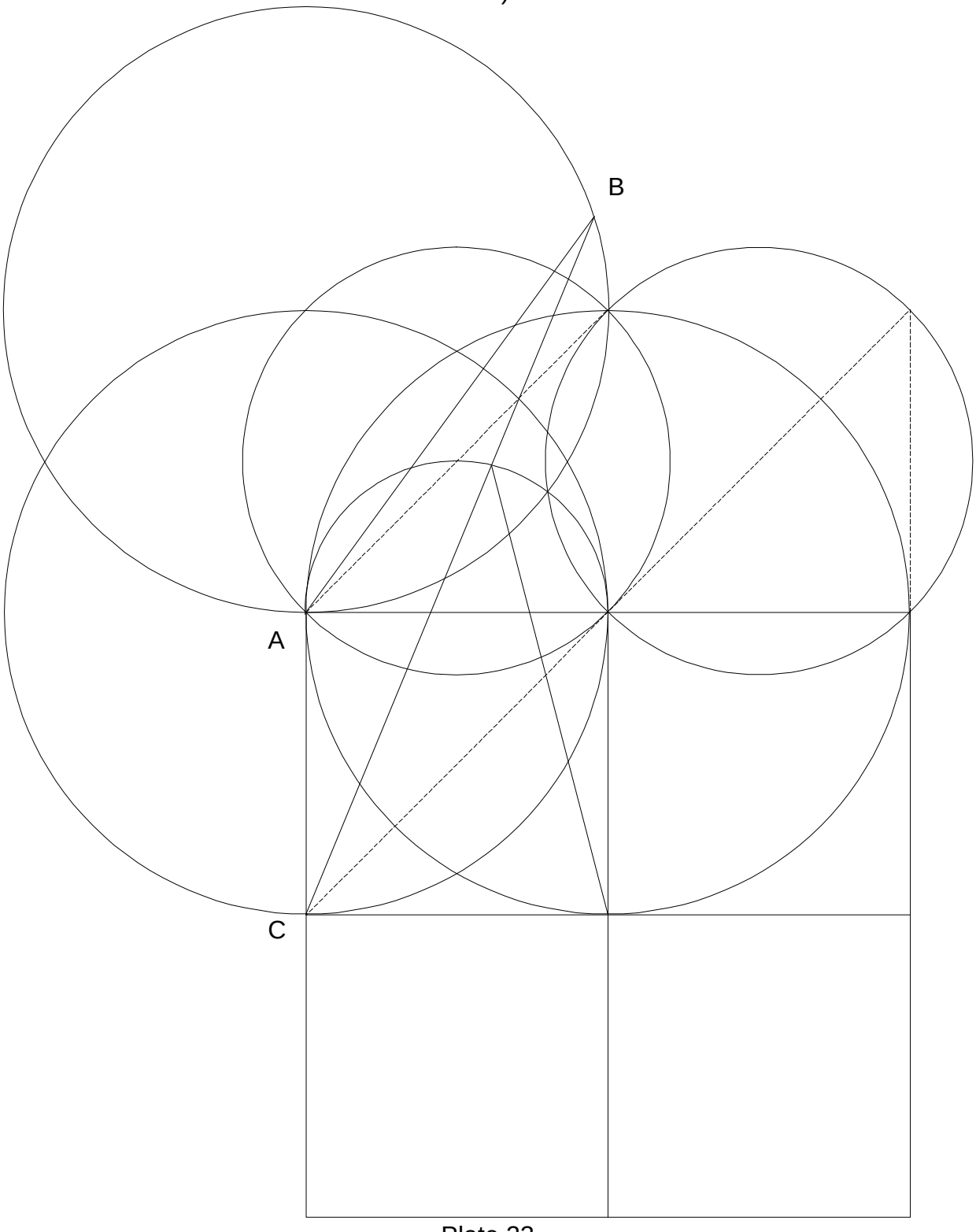
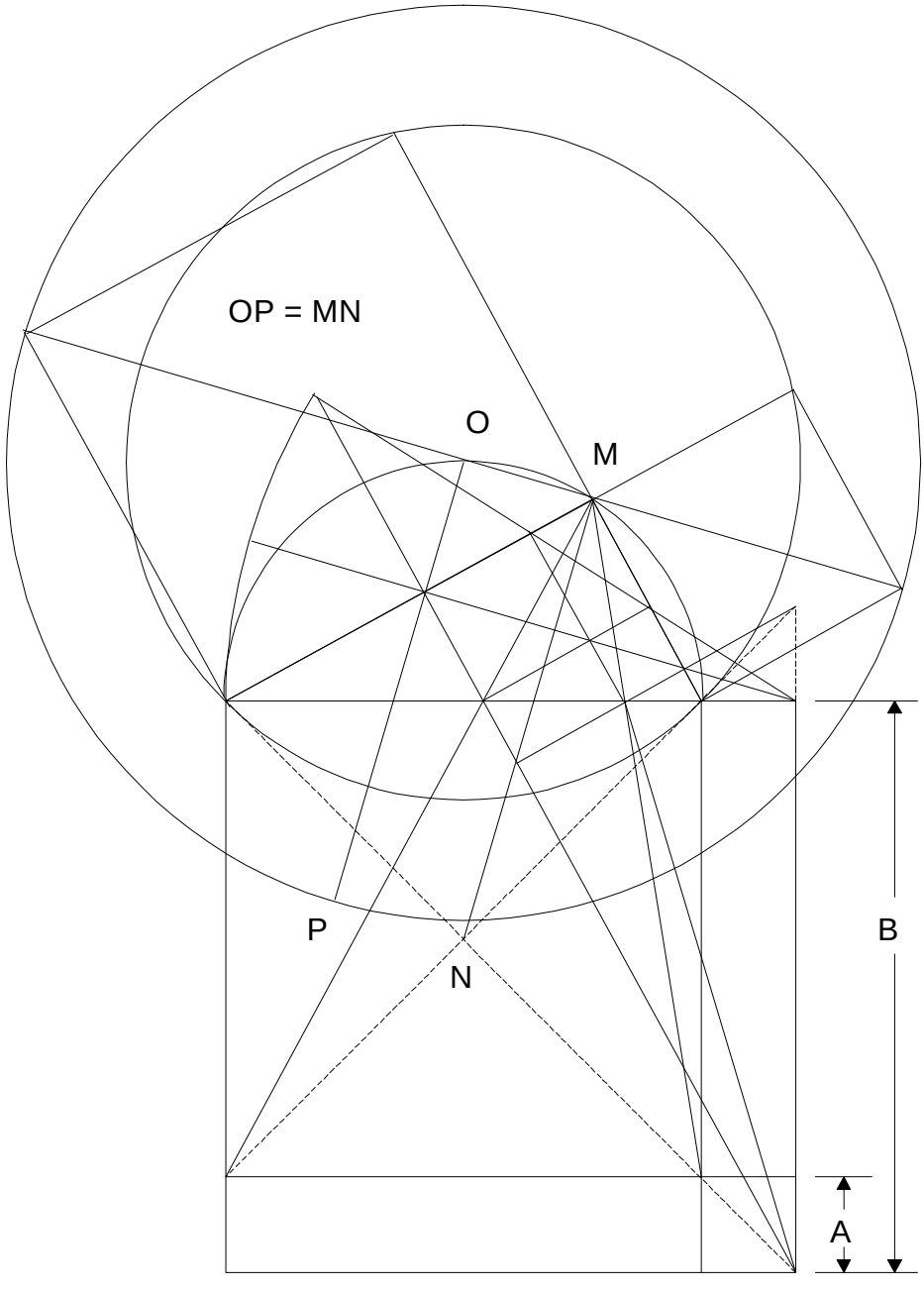



Plate 23

On Plate 24 the radius for the circle OP is given by MN.




One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of $B^2A$ (if you have missed it, the figure gives both roots, $A^2B$ and $B^2A$) there is a series of intersects, (three of them). When these intersects form a line

parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

I was so happy with myself that I found all this out on my own that I sent it out to see if anyone would publish it. The returns indicate that it was stillborn, however I continue my explorations. Good books on Geometrical constructions are not readily available and I am quite ignorant of what has been done in the way of plane Geometry. I strike out more or less on my own on the Delian Quest. I take only One Cirlce, One Square, and One Line as my travel companions, not to mention Elementary Algebra as a consultant.

# GEOMETRIAE DEDICATA

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3508 TA Utrecht
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Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

# American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA  Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

**Location:**
201 Charles **Street**
Providence, **RI**
02904


December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Mathematics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

*Serving the mathematical community for over 100 years*

# American Mathematical Society

Department of Mathematics
**Roger E. Howe**
Bulletin
Editorial Committee

Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

# JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches lnstitut
Technische Universitiit Munchen
Arcisstr. 21

D-8000 Munchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".
Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H. -J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:
You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.
All together the article in the given version is not understandable.

JOURNAL OF GEOMETRY
Editor's Office

München, 1 June 1990

Dear Professor Clark,
Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.
We are very sorry that we could not be of any help to you.

Sincerely yours,
H. -J. Kroll

(This one is a form letter.)

**société mathématique defrance**

paris, le
BULLETIN

n. réf.
v. réf.

a l'attention de

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Delian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

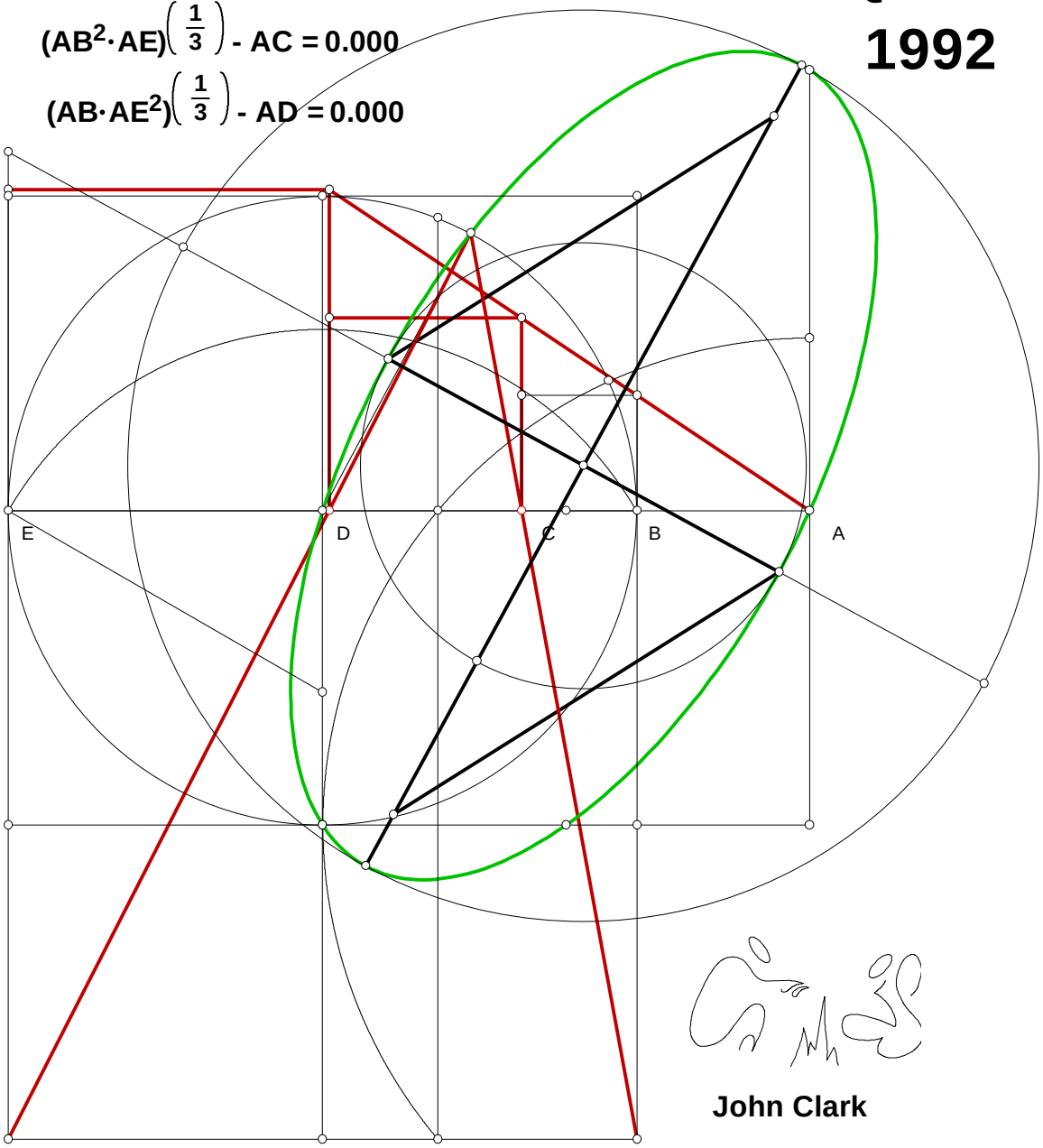
P.J. : Manuscrit

# The Delian Quest

## 1992

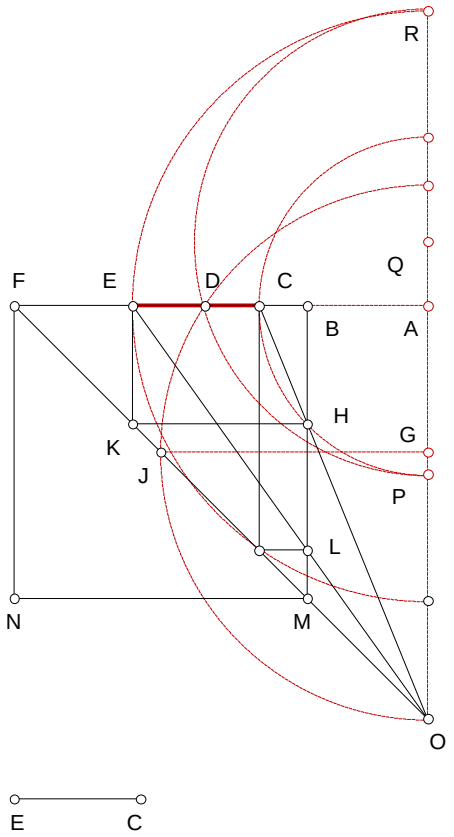
$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$






John Clark







# A Triplicate Ratio 06/20/92

**Given some point O place CE on BF such that O is the point of similarity.**

$N := 1.52 \quad N_2 := .89 \quad N_3 := .66$

$BF := N \quad MO := N_2 \quad CE := N_3$

$FM := \sqrt{2 \cdot BF^2} \quad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \quad AD := \sqrt{AB \cdot AF} \quad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \quad QR := DQ \quad QP := DQ$

$AP := QP - AQ \quad AR := QR + AQ \quad AC := AP$

$AE := AR \quad AO := AF \quad BC := AC - AB$

$EF := AF - AE \quad EK := EF \quad BH := EK$

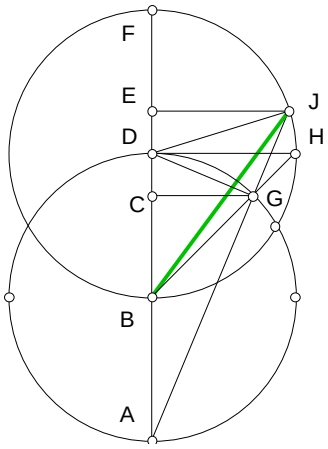
$\frac{BC}{BH} - \frac{AC}{AO} = 0$

**The last ratio can be tediously proved by reducing each term to the givens.**

**Edit 062800**

# 08/12/92 Rusty Cube of a Sphere

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**




$N := 9$ $\quad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

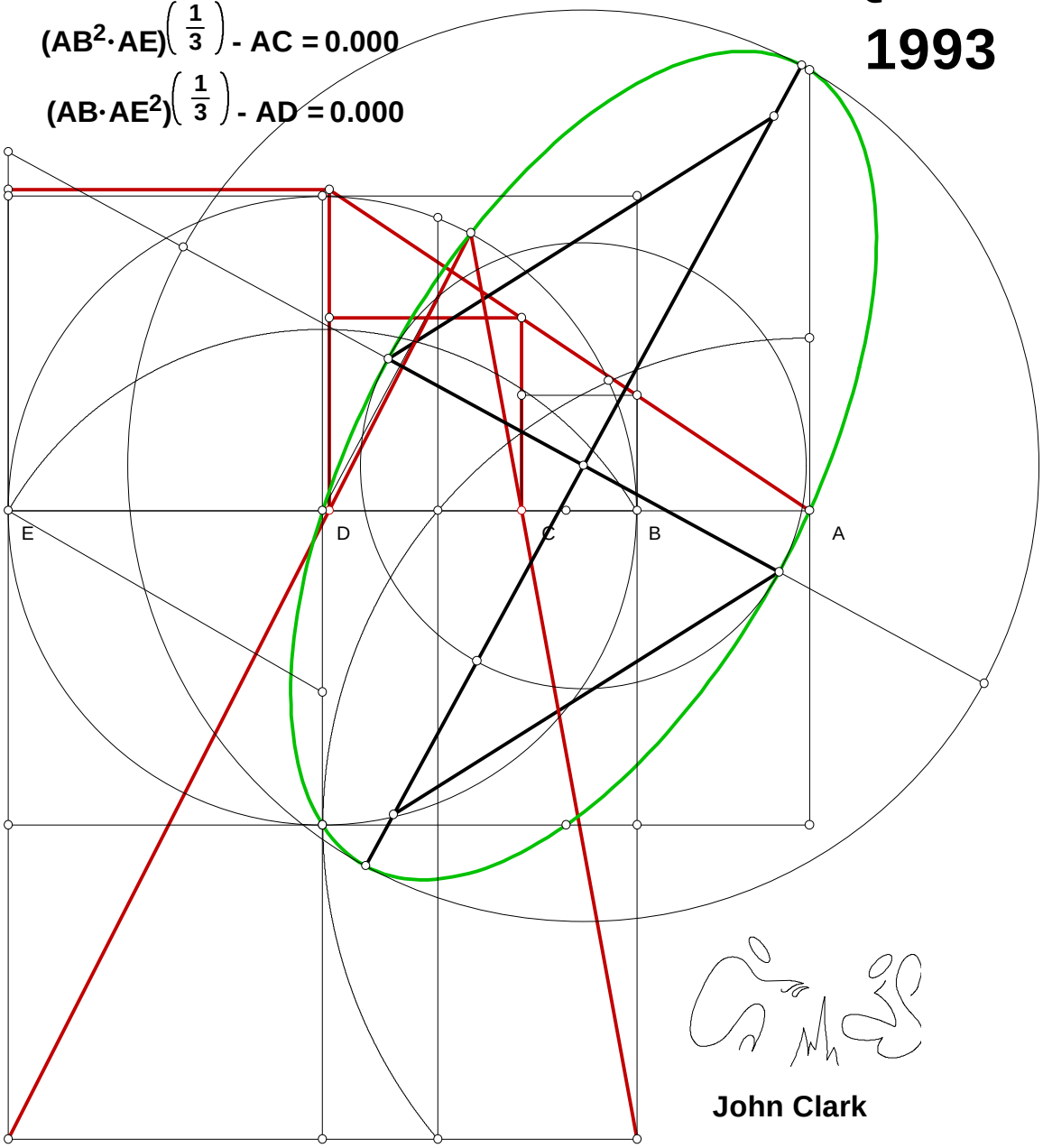
$\frac{CUBE\_ROOT}{BJ} = 0.99906$

# The Delian Quest

## 1993

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

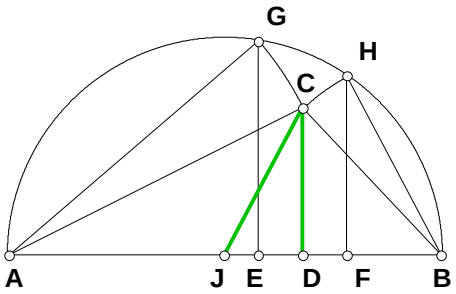
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$






John Clark

## 01/08/93 Pythagoras Revisited




**Given AB, BC, AC, what is CD, AD, BD and CJ?**

$S_1 := 2 \quad S_2 := 3 \quad S_3 := 4$

$AB := S_1 \quad BC := S_2 \quad AC := S_3$

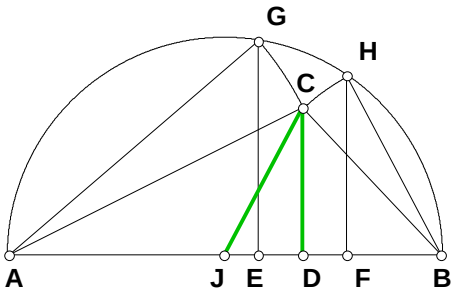
$AG := AC \quad BH := BC \quad AE := \frac{AG^2}{AB} \quad BF := \frac{BH^2}{AB}$

$EF := AB - (AE + BF) \quad DE := \frac{EF}{2} \quad DF := DE$

$AD := AE + DE \qquad AD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$BD := BF + DF \qquad BD := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CD := \sqrt{AC^2 - AD^2} \qquad CD := \frac{\sqrt{(-S_1 + S_2 - S_3) \cdot (S_1 + S_2 + S_3) \cdot (S_1 - S_2 - S_3) \cdot (S_1 + S_2 - S_3)}}{2 \cdot S_1}$



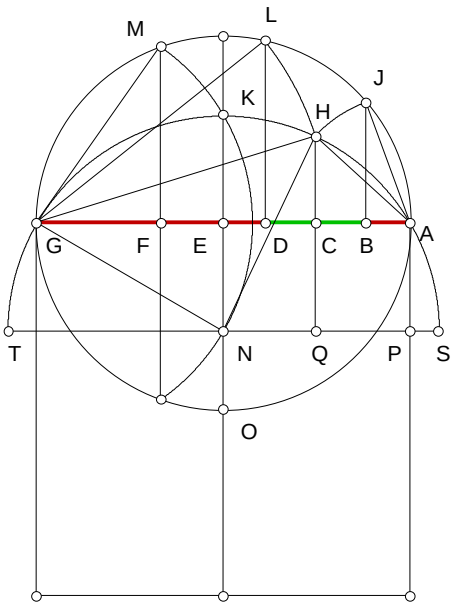

$AJ := \frac{AB}{2} \quad JD := AD - AJ \quad CJ := \sqrt{JD^2 + CD^2}$

$CJ := \frac{1}{2} \cdot \sqrt{2 \cdot S_3^2 - S_1^2 + 2 \cdot S_2^2}$

$\left(AC^2 + BC^2\right) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

**The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).**

# 06/03/93.MCD Exploring The Curve CJ




**Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?**

$N := 2 \qquad N_2 := 4$

$AG := N \qquad AC := \frac{AG}{N_2} \qquad GF := \frac{AG}{3} \qquad AE := \frac{AG}{2}$

$EG := AE \qquad AF := AG - GF \qquad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \qquad GN := GM \qquad EN := \sqrt{GN^2 - EG^2}$

$NH := GM \qquad NS := GM \qquad PN := AE \qquad PS := NS - PN$

$ST := 2 \cdot GM \qquad SQ := AC + PS \qquad QT := ST - SQ$

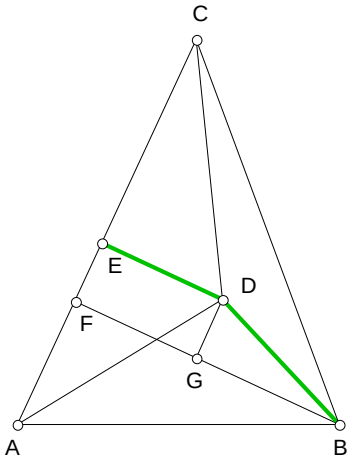
$QH := \sqrt{SQ \cdot QT} \qquad CQ := EN \qquad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \qquad CG := AG - AC \qquad GH := \sqrt{CG^2 + CH^2} \qquad AJ := AH \qquad AB := \frac{AJ^2}{AG}$

$GL := GH \qquad DG := \frac{GL^2}{AG} \qquad BD := AG - (AB + DG) \qquad \sqrt{AB \cdot DG} - BD = 0$

$$\left( \frac{-N}{3} + \frac{N}{3} \cdot \frac{\sqrt{N_2^{\,2} + 12 \cdot N_2 - 12}}{N_2} \right) - BD = 0$$

# 06/07/93 For All Triangles Find BD




## Given AB, BC, AC, CD, AD, find BD

## To simplify use line names found in 010893

$S_1 := 5.55$ $S_2 := 4.61$ $S_3 := 1.5$

$S_5 := 4.5$ $S_6 := 5.48$

$AB := S_5$ $BC := S_6$ $AC := S_1$ $CD := S_3$ $AD := S_2$

$$CE := \frac{CD^2 + AC^2 - AD^2}{2 \cdot AC} \qquad CF := \frac{BC^2 + AC^2 - AB^2}{2 \cdot AC} \qquad EF := CF - CE$$

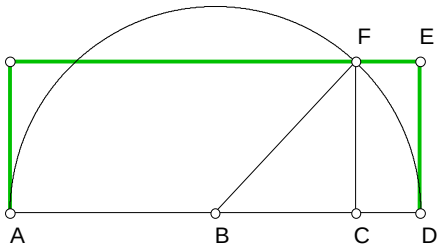
$$DE := \sqrt{CD^2 - CE^2} \qquad BF := \sqrt{BC^2 - CF^2} \qquad GF := DE$$

$$BG := BF - GF \qquad DG := EF \qquad BD := \sqrt{DG^2 + BG^2} \qquad BD = 3.983$$

**OR**

$$BG_2 := BF + GF \qquad DG_2 := EF \qquad BD_2 := \sqrt{DG^2 + BG_2^2} \qquad BD_2 = 5.757$$

# 06/09/93 Rectangular Roots




**Given AD and DE divide AD into the rectangular roots of DE.**

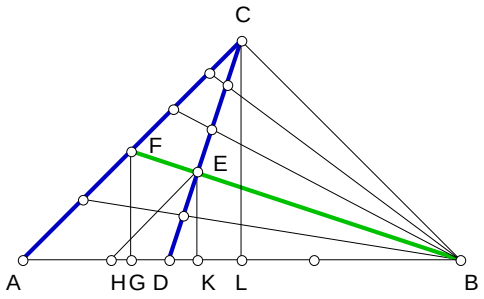
$N := 5 \qquad N_2 := 2$

$AD := N \quad DE := N_2 \quad CF := DE \quad BF := \frac{AD}{2}$

$AB := BF \quad BC := \sqrt{BF^2 - CF^2} \quad AC := AB + BC \quad CD := AD - AC \quad \sqrt{CD \cdot AC} - DE = 0$

$AC := \frac{1}{2} \cdot N + \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2} \qquad CD := \frac{1}{2} \cdot N - \frac{1}{2} \cdot \sqrt{-4 \cdot N_2^2 + N^2}$

# 06/21/93 Pyramid of Ratios I




**Divide AB by $N_1$ then divide CD by $N_2$, what are BF/EF and AC/AF?**

$N_1 := 3 \qquad N_2 := 5 \qquad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AL := \frac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

$\delta$

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

$DE_\delta := \frac{CD \cdot \delta}{N_2} \quad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\text{if}\left(N_2 - \delta, \frac{N_1 \cdot N_2}{N_2 - \delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\frac{AC}{AF_\delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

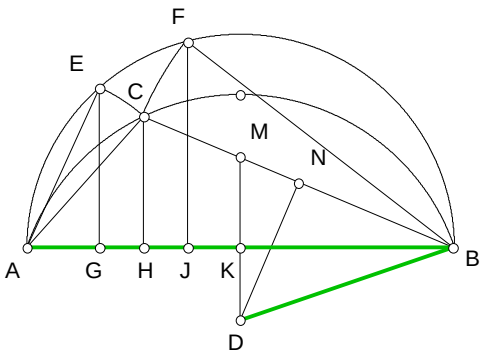
$\frac{(N_1 - 1) \cdot N_2 + \delta}{N_1 \cdot \delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

# 06/27/93 Describe A Circle About a Triangle

## Given the difference between three non-collinear points, find the radius of the circle that circumscribes them

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $\quad NOT(X) := X = 0$ $\quad \delta := 0 .. 2$




$BK := \frac{AB}{2}$ $\quad AE := AC$ $\quad BF := BC$

$AG := \frac{AE^2}{AB}$ $\quad BJ := \frac{BF^2}{AB}$ $\quad GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $\quad BH := BJ + HJ$ $\quad CH := \sqrt{BC^2 - BH^2}$

$BN := \frac{BC}{2}$ $\quad BM := \frac{BC \cdot BK}{BH}$ $\quad MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $\quad BD := \sqrt{BN^2 + DN^2}$

$radius := if(\Delta, BD, 0)$ $\quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary\_radius = 0$

$\Delta = 1$

$S_1 := \begin{bmatrix} AB \\ AC \\ BC \end{bmatrix}$ $\quad S_2 := \begin{bmatrix} AC \\ BC \\ AB \end{bmatrix}$ $\quad S_3 := \begin{bmatrix} BC \\ AB \\ AC \end{bmatrix}$

$AB \equiv 3$ $\quad AC \equiv 4$ $\quad BC \equiv 6$

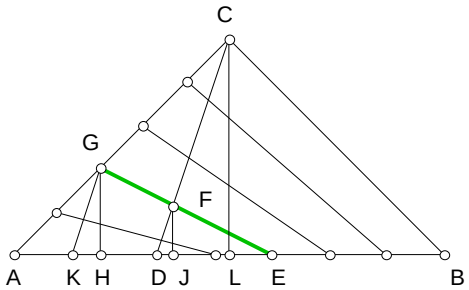
## The name of the Radius as a proportion of the given names

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$R^T = [\, 3.375 \quad 3.375 \quad 3.375 \,]$

# 07/15/93 Pyramid of Ratios II

**AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?**




$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

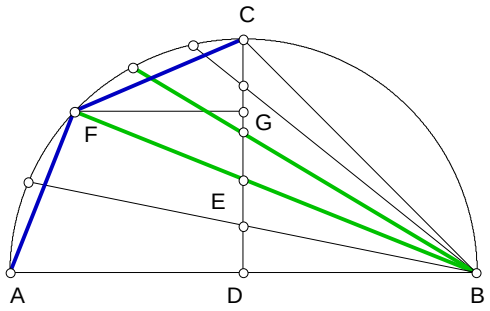
| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

# 07/25/93 Pyramid of Ratios III

## Dividing DC by an number provides wht in terms of BE/BF and AF/CF?




$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

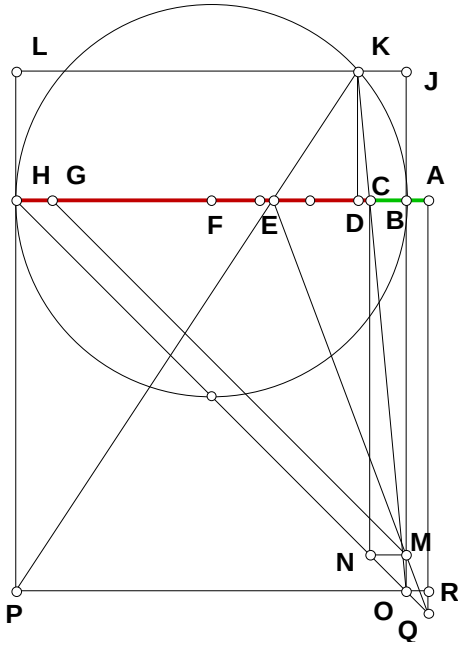
$DG := DE + EG \qquad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$ | $\frac{BE_\delta}{BF_\delta}$ | $\frac{BF_\delta}{BE_\delta}$ | $\frac{2}{\left(\frac{\delta}{N}\right)^2 + 1}$ |
|---|---|---|---|---|
| 0.514 | 0.514 | 0.514 | 1.946 | 1.946 |
| 0.556 | 0.556 | 0.556 | 1.8 | 1.8 |
| 0.625 | 0.625 | 0.625 | 1.6 | 1.6 |
| 0.722 | 0.722 | 0.722 | 1.385 | 1.385 |
| 0.847 | 0.847 | 0.847 | 1.18 | 1.18 |
| 1 | 1 | 1 | 1 | 1 |

| $\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$ | $\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$ |
|---|---|
| 0.283 | 0.283 |
| 0.707 | 0.707 |
| 1.414 | 1.414 |
| 2.828 | 2.828 |
| 7.071 | 7.071 |
| 0 | 0 |

# 11/06/93 Gruntwork I on the Delian Solution




**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?**

$N := 4$ $BH := 1$

$BF := \frac{BH}{2}$ $BD := \frac{BF}{N}$ $DH := BH - BD$

$DK := \sqrt{BD \cdot DH}$ $BJ := DK$ $BO := BH$

$JO := BJ + BO$ $JK := BD$ $CD := \frac{JK \cdot DK}{JO}$

$KL := DH$ $LP := JO$ $DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD$ $CE := CD + DE$ $MN := BC$

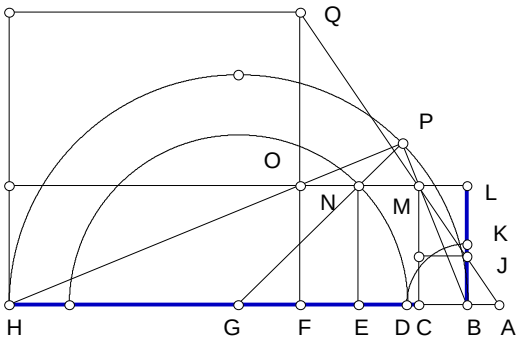
$GH := MN$ $CH := BH - BC$ $HN := \sqrt{2 \cdot CH^2}$ $GM := HN$ $EH := CH - CE$ $EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG}$ $HO := \sqrt{2 \cdot BH^2}$ $OQ := HQ - HO$ $OR := \sqrt{\frac{OQ^2}{2}}$ $AB := OR$ $AC := AB + BC$

$AE := AB + BD + DE$ $AH := AB + BH$ $(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0$ $(AB \cdot AH^2)^{\frac{1}{3}} - AE = 0$

## 11/09/93 Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem.**
**BH is the difference between the segments AH and AB.**
**CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**



$N := 2 \qquad BH := N - 1$

$BG := \frac{BH}{2} \qquad CF := N^{\frac{2}{3}} - N^{\frac{1}{3}}$

$BL := CF \qquad GP := BG \qquad BK := \frac{BL}{2}$

$BD := BK \qquad NP := BD \qquad GN := GP - NP$

$EN := BL \qquad GE := \sqrt{GN^2 - EN^2}$

$CE := BD \qquad BC := BG - (GE + CE)$

$GH := BG \qquad EF := BD$

$FH := GH + GE - EF \qquad FQ := FH \qquad FO := BL \qquad OQ := FQ - FO \qquad MO := CF \qquad AF := \frac{MO \cdot FQ}{OQ}$

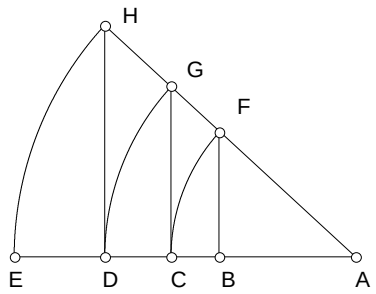
$AC := AF - CF \qquad AH := AF + FH \qquad AB := AH - BH$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 2$$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

# 11/10/93 Gruntwork II on the Delian Solution

**Given any acute angle in the isosceles, divide the base leg as shown. Do the resultant segments show any particular relationship to one another?**




$N := 4.846$ $AE := 1$

$DE := \frac{AE}{N}$ $AD := AE - DE$ $AH := AE$

$AG := AD$ $AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2$ $\frac{AD}{AB} = 1.588$ $\frac{AC}{AB} = 1.26$

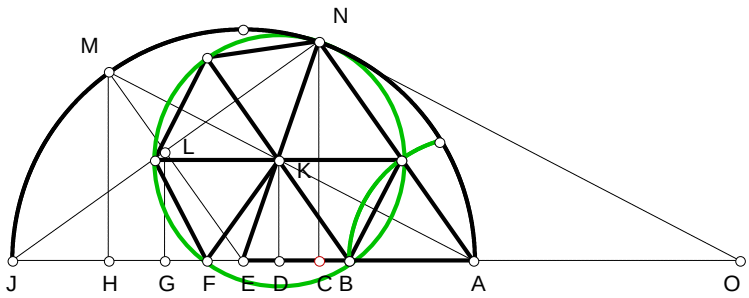
## Albebraic Names:

$DE := \frac{1}{N}$ $AD := 1 - \frac{1}{N}$ $AC := \frac{(N-1)^2}{N^2}$ $AB := \frac{(N-1)^3}{N^3}$

$\left[\left[\frac{(N-1)^3}{N^3}\right]^2\right]^{\frac{1}{3}} - \frac{(N-1)^2}{N^2} = 0$ $\left[\frac{(N-1)^3}{N^3}\right]^{\frac{1}{3}} - \left(1 - \frac{1}{N}\right) = 0$

## 11/11/93 The Archamedian Paper Trisector

**If one accepts the facts of the original figure, one only need prove that BK = AB.**
**If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?**




$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

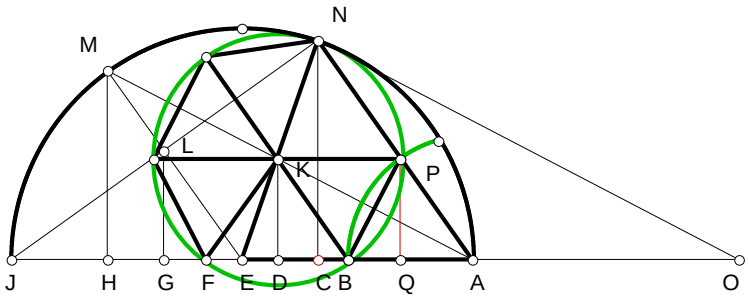
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

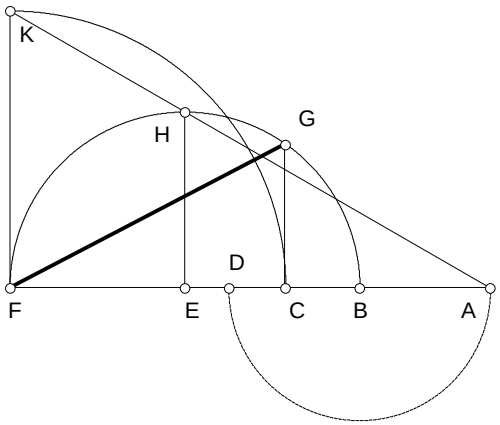
$AB - BK = 0$ $AB = 0.25$

**If PK is parallel to AJ, then . . . .**




$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

**11/12/93 To Square A Circle Off The Base Of A Right Triangle.**




**Sometime in 1992, I remembered reading that some guy spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost it again, so I set out to find it and did. It took a couple hours so I wonder what he did with the rest of his time?**

**Using the approximation, $\pi$ = 22/7, square the circle off the base of a right triangle.**

$$BF := 1 \qquad BE := \frac{BF}{2} \qquad EF := BE \qquad EH := BE \qquad BD := \frac{3}{4}\cdot BE \qquad AB := BD$$

$$AE := AB + BE \qquad AF := AB + BF \qquad FK := \frac{EH \cdot AF}{AE} \qquad CF := FK \qquad BC := BF - CF$$

$$CG := \sqrt{BC \cdot CF} \qquad FG := \sqrt{CF^2 + CG^2} \qquad \pi\_A := \frac{FG^2}{BE^2}$$

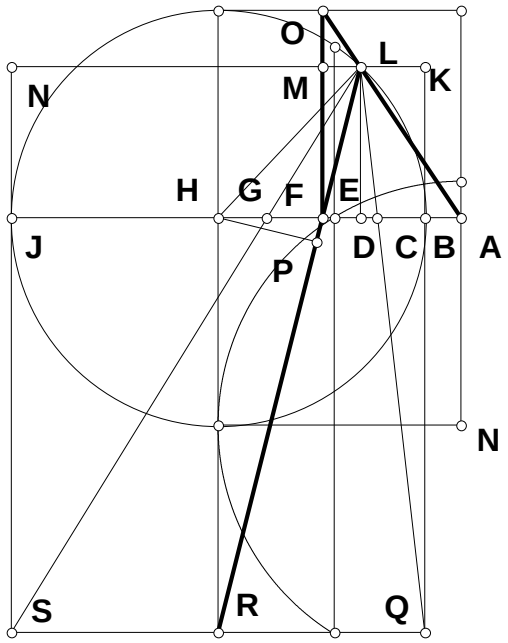
$$\pi = 3.14159265359$$

$$\pi\_A = 3.142857142857$$

$$\frac{\pi}{\pi\_A} = 0.999597662505843$$

# 11/18/93 Exploring Cube Roots Plate A

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.**




$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM$

$LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

$BQ := BJ \quad BK := DL \quad BD := BF - DF$

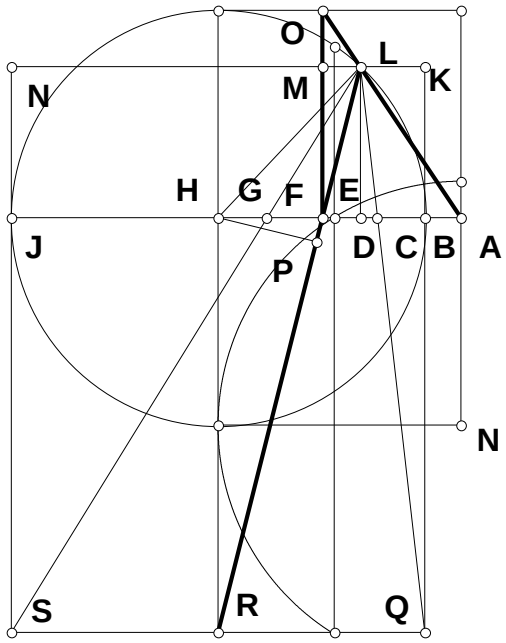
$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad HJ := BH \quad DJ := HJ + FH + DF \quad LN := DJ$

$JS := BJ \quad JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC$

$AG := AB + BG \quad AJ := AB + BJ \quad \left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

# 11/18/93 Exploring Cube Roots Plate A

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.**




$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM$

$LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

$BQ := BJ \quad BK := DL \quad BD := BF - DF$

$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad DJ := BJ - BD \quad LN := DJ \quad JS := BJ$

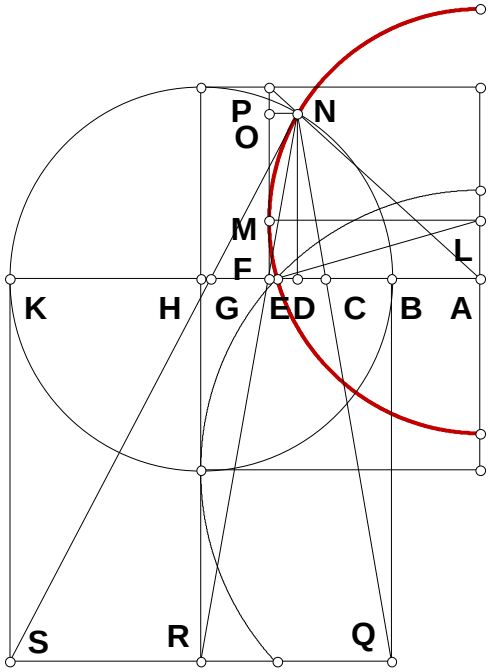
$JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC \quad AG := AB + BG$

$AJ := AB + BJ$

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

# 11/18/93 Exploring Cube Roots Plate B

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**




$N := 1.2$ $BK := 1$

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

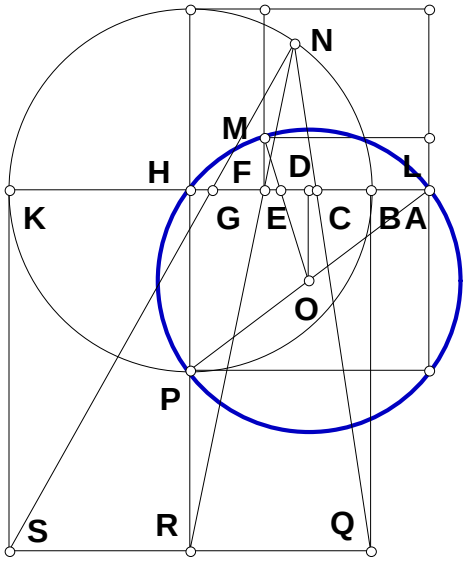
$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$ $AB = 1.523$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

# 11/18/93 Exploring Cube Roots Plate C

## The circle AO passes through point M.




$N := 2$

$AB := 1 \quad AK := AB \cdot N \quad BK := AK - AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

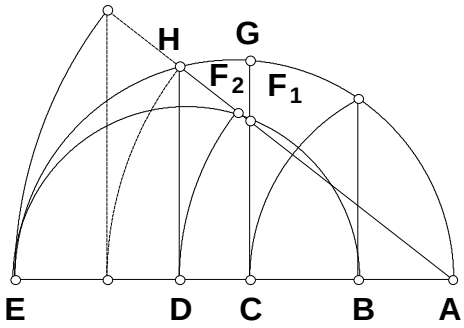
$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$




## 11/22/93 Cube by Iteration

**When $F_1$ and $F_2$ are the same point on C, then a sixth root series has been constructed. Use iteration to place $F_2$ on $F_1$.**

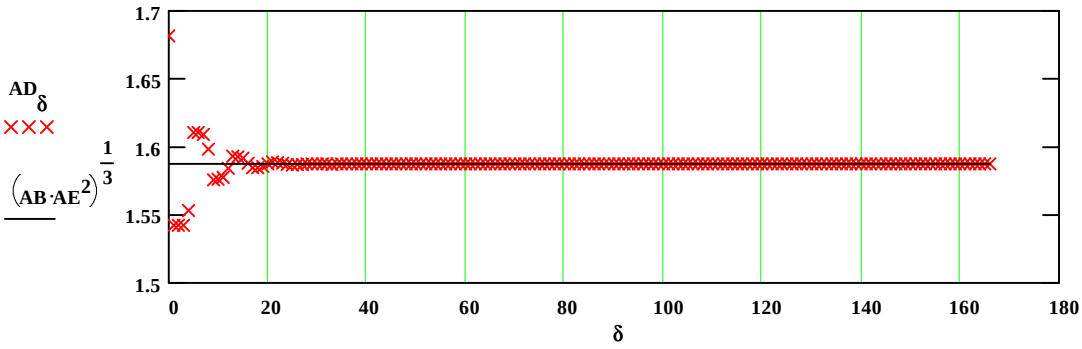
$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG) \cdot AG} \\ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \\ \sqrt{AC^2 + \left[ \dfrac{\sqrt{(AE - AG) \cdot AG} \cdot AC}{AG} \right]^2} \end{bmatrix}$$

$$\begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \dfrac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + \left(CF_\delta\right)^2} \end{bmatrix}$$

$\Delta \equiv 166$

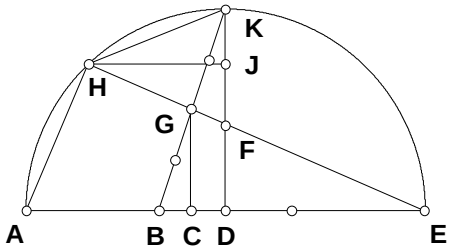



$\left(AB^2 \cdot AE^4\right)^{\frac{1}{6}} - AD_\Delta = 0$

$\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD_\Delta = 0$

# 11/24/93 POR Series IV

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**




$N_1 := 3 \qquad N_2 := 5 \qquad AE := 1$

$\alpha := 1 .. N_1 - 1 \qquad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

$\frac{AH}{HK} = 0.265 \qquad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \qquad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

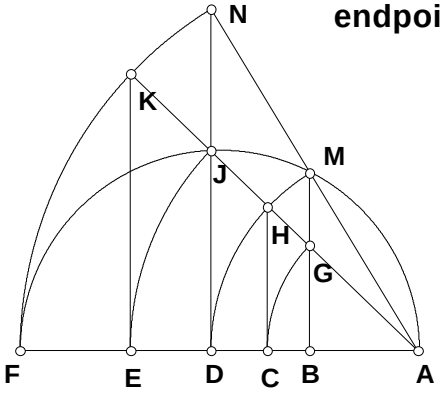
$\frac{EH}{GH} = 2.85 \qquad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$

## 12/04/93 Exponential Series M^(1/2^N)

**Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.**




$M := 8$

$AF := M \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

$AN := AF \quad AD := AM \qquad DF := AF - AD$

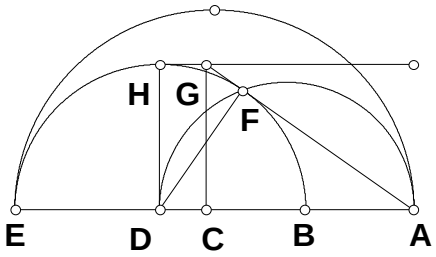
$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

$AK := AF \quad AE := AJ \qquad AH := AD \qquad AC := \frac{AD \cdot AH}{AJ}$

$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$M^{\frac{1}{4}} - AC = 0 \qquad M^{\frac{2}{4}} - AD = 0 \qquad M^{\frac{3}{4}} - AE = 0$

# 12/06/93 Alternate Method: Square Root
# Common Segment Common Endpoint




$N := 6 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2}$

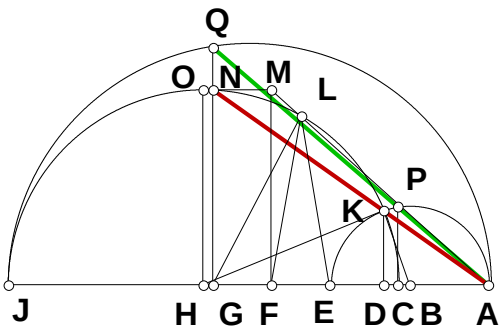
$DF := BD \qquad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$

# 12/06/93B Gruntwork IV on the Delian Solution

## Are A,P and Q collinear? Are A, K and N collinear?




$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$AE := \left(AC^2 \cdot AJ\right)^{\left(\frac{1}{3}\right)} \qquad AG := \left(AC \cdot AJ^2\right)^{\left(\frac{1}{3}\right)}$

$CG := AG - AC \qquad CJ := AJ - AC$

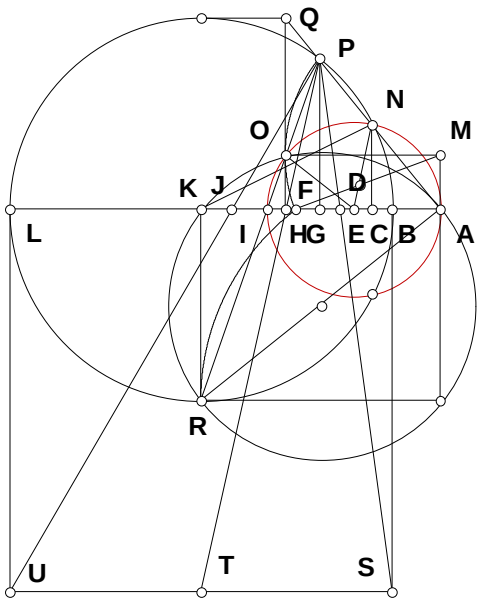
$GJ := CJ - CG \qquad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \qquad CE := AE - AC \qquad CH := \frac{CJ}{2}$

$BK := AB \qquad HK := CH \qquad HJ := CH \qquad AH := AJ - HJ \qquad BH := AH - AB \qquad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$AD := AB + BD \qquad DE := AE - AD \qquad DK := \sqrt{AD \cdot DE} \qquad GQ := \sqrt{AG \cdot GJ} \qquad CP := \sqrt{AC \cdot CE}$

$$\frac{AG}{GN} - \frac{AD}{DK} = 0 \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$$




## 12/11/93

**The structure in red appears to be a constant.**

$N := 6$ $AB := 1$ $AL := AB \cdot N$

$BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$

$AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$ $BE := AE - AB$ $BJ := AJ - AB$

$JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$

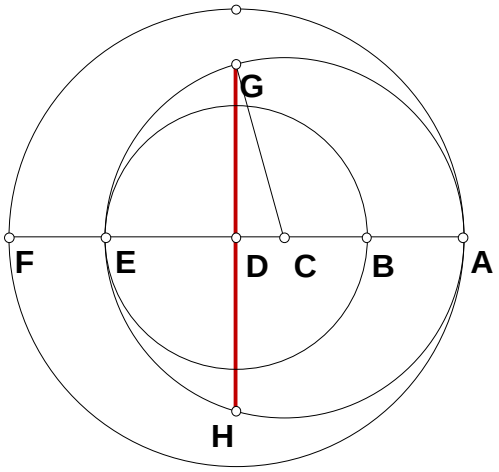
$KR := BK$ $KL := BK$ $FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$

$AD := \frac{AI}{2}$ $KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

# 12/12/93 The Square Root




**Square root by common segment common midpoint. Given AFand BE is GH their root?**

$N := 5$ $BE := 1$ $AF := BE \cdot N$

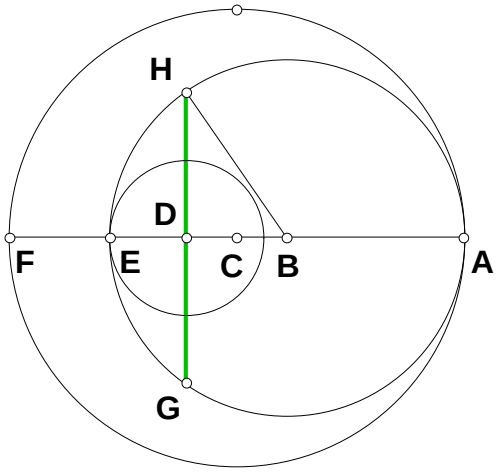
$AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$ $AB := AD - BD$

$AE := BE + AB$ $AC := \frac{AE}{2}$ $CG := AC$

$CD := AD - AC$ $GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$

# 12/12/93 Generalize The Previous Square Root Figure




$N_1 := 1 \qquad N_2 := 3 \qquad N_3 := 2$

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

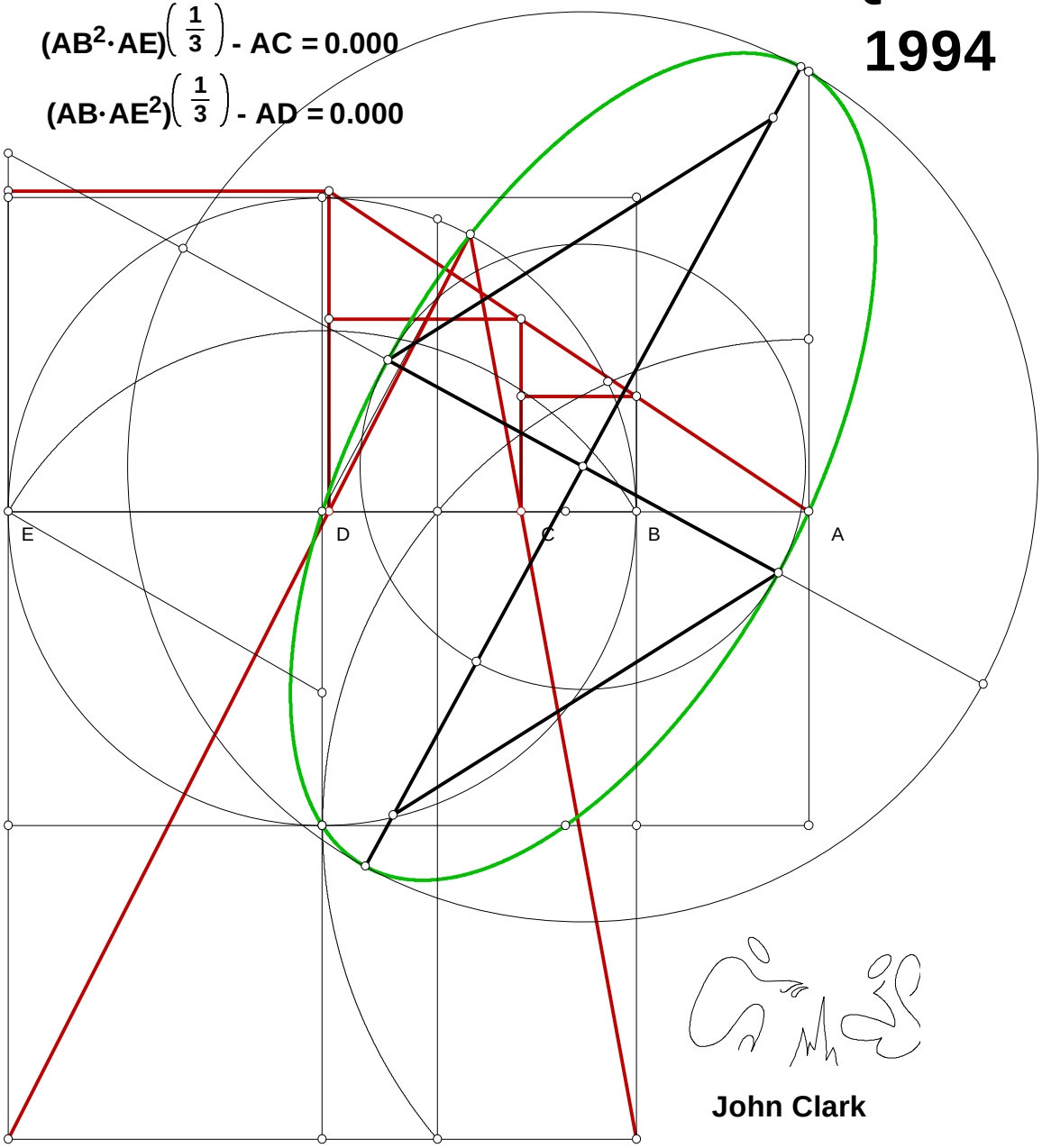
$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

# The Delian Quest

## 1994

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

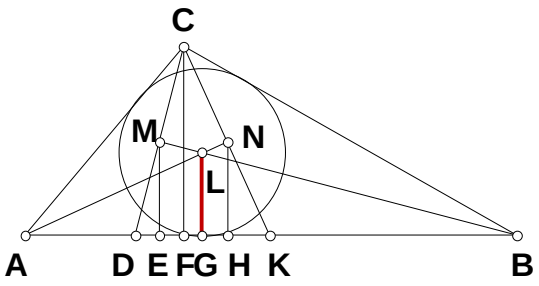





John Clark

# 04/06/94 Inscribing A Circle In A Given Triangle

**Given three sides of a triangle, what is the length of the inscribed radius?**




$AB := 3 \qquad BC := 4 \qquad AC := 5$

$AK := AC \quad BD := BC \quad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$

$FK := AK - AF \quad CF := \sqrt{AC^2 - AF^2}$

$CK := \sqrt{FK^2 + CF^2} \qquad AN := \frac{CF \cdot AK}{CK}$

$AH := \frac{CF \cdot AN}{CK} \quad HN := \frac{FK \cdot AN}{CK} \quad BF := AB - AF$

$DF := BD - BF \quad CD := \sqrt{CF^2 + DF^2} \quad BM := \frac{CF \cdot BD}{CD} \quad BE := \frac{CF \cdot BM}{CD} \quad GL := \frac{HN \cdot AB}{AH + BE}$

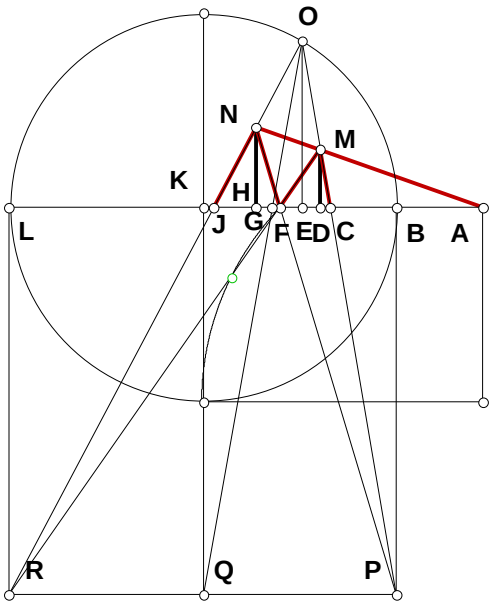
$GL = 1$

$S_1 := \begin{bmatrix} AB \\ BC \\ AC \end{bmatrix} \qquad S_2 := \begin{bmatrix} BC \\ AC \\ AB \end{bmatrix} \qquad S_3 := \begin{bmatrix} AC \\ AB \\ BC \end{bmatrix} \qquad \delta := 0 .. 2$

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

# 04/21/94 The Cradle

## Are A,M,N colinear?




$N := 5 \qquad AB := 1 \qquad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \qquad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

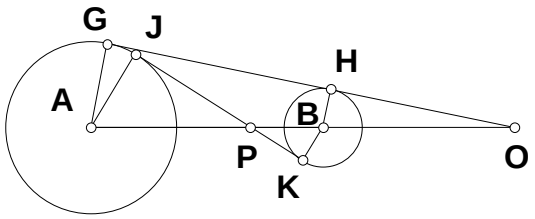
$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

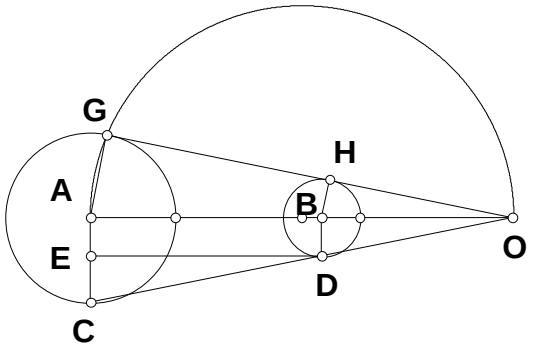
$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$

# 04/26/94 Tangents and Similarity Points




**What is the Algebraic names of the similarity points O and P in relation to the radius of each circle and the difference between their centers?**

**I will work with point O first.**

**Given $R_L$ = large radius**
**$R_S$ = small radius**
**D = difference between origins.**

$R_L := 4 \qquad R_S := 1 \qquad D := 8$

$AC := R_L \qquad BD := R_S \qquad AB := D$

$DE := AB \qquad AE := BD \qquad CE := AC - AE$

$$AO := \frac{DE \cdot AC}{CE} \qquad AO = 10.667$$

**$EOR_L$ "External similarity point Origin to center of Radius Large"**

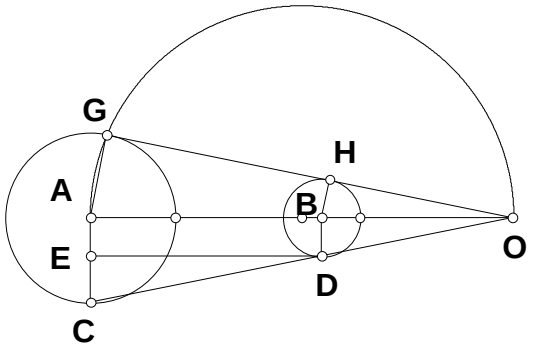
$$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right) \qquad EOR_L = 10.667$$

**What is the length of line (OG) tangent to both circles?**

$AG := AC \qquad GO := \sqrt{AO^2 - AG^2} \qquad GO = 9.888$

**And what is the formula?**
**$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"**

$$EOT_{LR} := R_L \cdot \frac{\sqrt{\left(R_L - R_S + D\right) \cdot \left(-R_L + R_S + D\right)}}{R_L - R_S} \qquad EOT_{LR} = 9.888$$




**What is the length of the line tangent to the least circle (HO)?**

$BH := BD$ $BO := AO - AB$ $BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

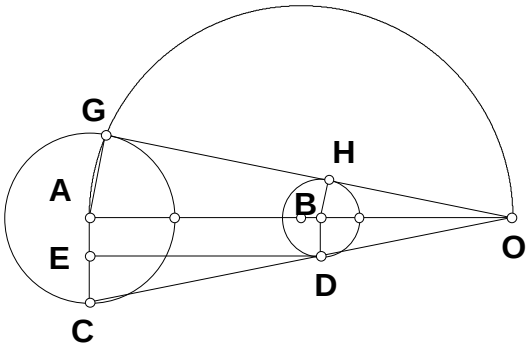
$HO = 2.472$

**And what is the formula?**

**$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"**

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}}{R_L - R_S} \qquad EOT_{SR} = 2.472$$

**Lastly what is the length of line from tangent to tangent of these circles?**



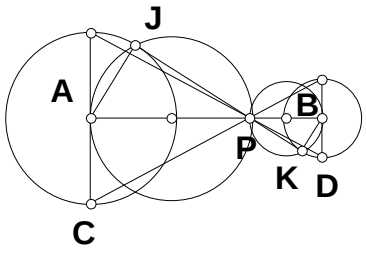

$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

**And what is the formula?**
**ETT "Tangent to Tangent"**

$$ETT := \sqrt{-(R_L - R_S + D)\cdot(R_L - R_S - D)}$$

$ETT = 7.416$




**I will now turn my attention to the point P, the internal similarity point.**

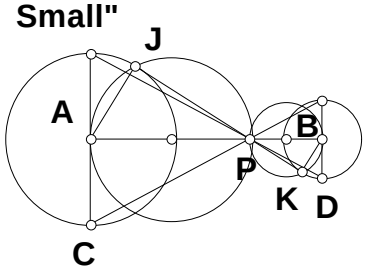
$$AP := \frac{AB \cdot AC}{AC + BD} \qquad AP = 6.4$$

**$IOR_L$ "Internal similarity point to center of Radius Large"**

$$IOR_L := D \cdot \frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

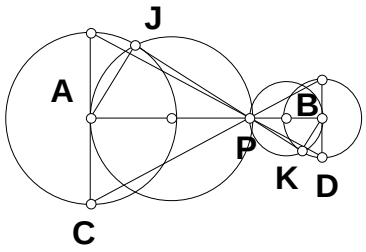
**$IOR_S$ "Internal similarity point to center of Radius Small"**




$$IOR_S := D \cdot \frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$

$AJ := AC \qquad BK := BD \qquad JP := \sqrt{AP^2 - AJ^2} \qquad JP = 4.996$

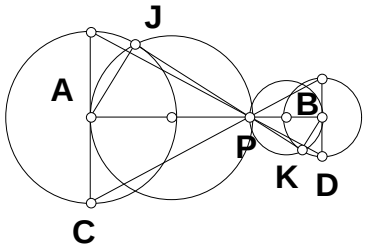
**$IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"**




$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996 \qquad KP := \sqrt{BP^2 - BK^2} \qquad KP = 1.249$

**$IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"**



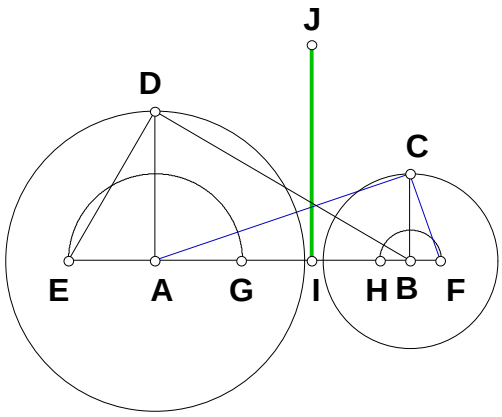

$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249 \qquad JK := JP + KP \qquad JK = 6.245$

**ITT "Internal similarity point Tangent to Tangent"**

$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)} \qquad ITT = 6.245$$

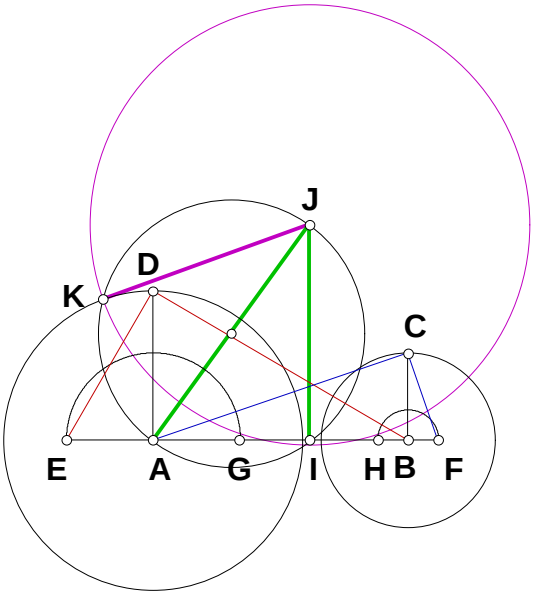
# 04/27/94 The Chordal or Power Line of two Circles




**The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**
**Given two circles find their chordal or power line given just their radius and difference between their centers.**

$AB := 1.323 \quad AD := .771 \quad BC := .448 \quad AE := \frac{AD^2}{AB} \quad BF := \frac{BC^2}{AB} \quad GH := AB - (AE + BF)$

$GI := \frac{GH}{2} \quad AI := AE + GI \quad BI := BF + GI$




$D := AB \quad R_1 := AD \quad R_2 := BC$

$$AI - \frac{\left(R_1^2 + D^2 - R_2^2\right)}{2 \cdot D} = 0$$

$$BI - \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D} = 0$$

**If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.**

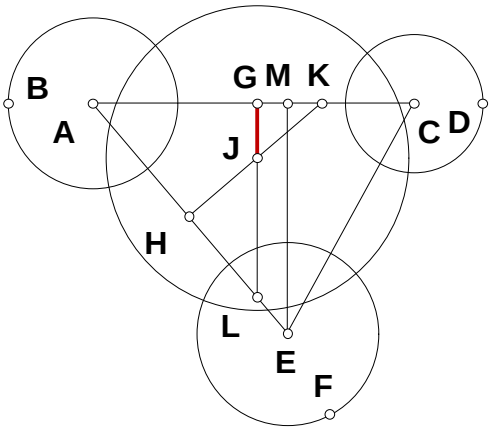
$IJ := 1.112 \quad AJ := \sqrt{AI^2 + IJ^2} \quad AK := AD$

$JK := \sqrt{AJ^2 - AK^2} \quad P := IJ$

$$JK - \frac{\sqrt{R_1^4 - 2 \cdot R_1^2 \cdot D^2 - 2 \cdot R_1^2 \cdot R_2^2 + D^4 - 2 \cdot R_2^2 \cdot D^2 + R_2^4 + 4 \cdot P^2 \cdot D^2}}{2 \cdot D} = 0$$

# 04/28/94 Power Point

**Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate the Algebraic name for the power point and the length of the resultant tangent.**




$AB := .438 \quad CD := .354 \quad EF := .471$

$AC := 1.667 \quad AE := 1.559 \quad CE := 1.357$

$R_1 := AB \quad R_2 := CD \quad R_3 := EF$

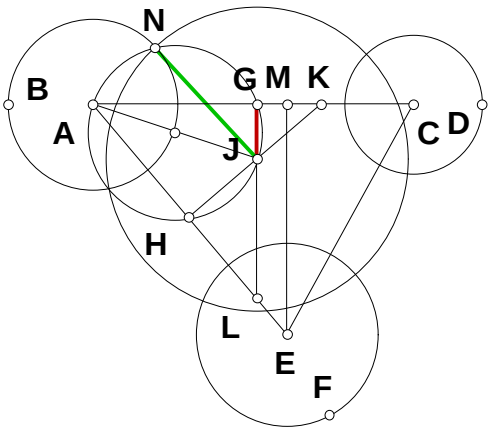
$D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{R_1^2 + D_1^2 - R_2^2}{2 \cdot D_1}$$

$$AH := \frac{R_1^2 + D_2^2 - R_3^2}{2 \cdot D_2}$$

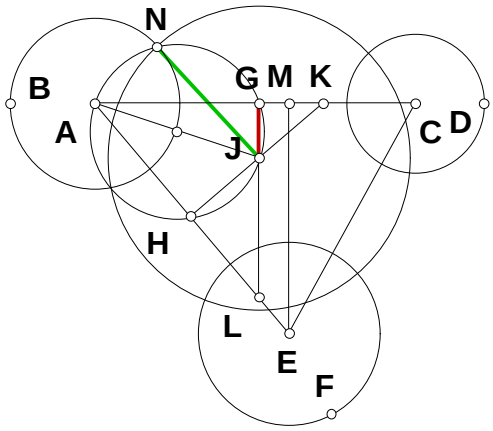
$$AM := \frac{D_2^2 + D_1^2 - D_3^2}{2 \cdot D_1} \qquad EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{1}{2} \cdot \frac{\left[\begin{array}{l}\left(D_2^2 \cdot D_1^2 - D_2^2 \cdot R_1^2 + D_2^2 \cdot R_2^2 + D_3^2 \cdot R_1^2 - 2 \cdot D_1^2 \cdot R_3^2\right) \ldots \\ + R_2^2 \cdot D_1^2 - D_3^2 \cdot R_2^2 - D_1^4 + D_3^2 \cdot D_1^2 + D_1^2 \cdot R_1^2\end{array}\right]}{D_1 \cdot \sqrt{-\left(D_2 + D_1 - D_3\right) \cdot \left(D_2 + D_1 + D_3\right) \cdot \left(D_2 - D_1 - D_3\right) \cdot \left(D_2 - D_1 + D_3\right)}} = 0$$



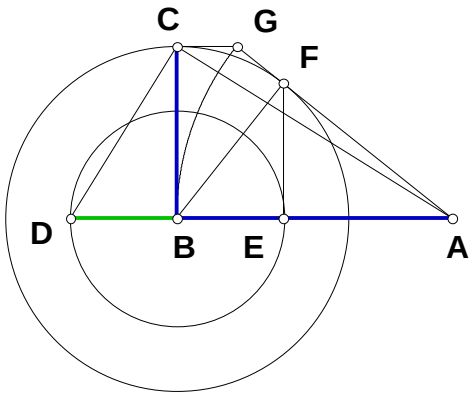

$$AJ := \sqrt{AG^2 + GJ^2} \qquad AN := AB \qquad JN := \sqrt{AJ^2 - AN^2}$$

$JN = 0.786$




$$JN - \sqrt{\frac{-\left[\begin{array}{l}\left(-D_3^2 \cdot R_1^2 \cdot D_1^2 + R_1^2 \cdot D_3^4 + R_2^4 \cdot D_2^2 + D_1^2 \cdot D_3^2 \cdot D_2^2\right) \ldots \\ + \left(R_1^4 \cdot D_3^2 - R_2^2 \cdot D_1^2 \cdot D_2^2 + R_2^2 \cdot D_2^4 + R_1^2 \cdot R_2^2 \cdot D_1^2\right) \ldots \\ + \left(-D_2^2 \cdot D_1^2 \cdot R_3^2 - D_1^2 \cdot R_3^2 \cdot R_1^2 - D_1^2 \cdot R_3^2 \cdot D_3^2 - D_1^2 \cdot R_3^2 \cdot R_2^2\right) \ldots \\ + \left(-D_2^2 \cdot R_1^2 \cdot D_3^2 + R_3^2 \cdot D_3^2 \cdot R_2^2 - R_1^2 \cdot R_2^2 \cdot D_2^2 + D_1^4 \cdot R_3^2\right) \ldots \\ + \left(-R_1^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot D_3^2 - D_2^2 \cdot R_2^2 \cdot R_3^2 - D_3^2 \cdot R_1^2 \cdot R_3^2\right) \ldots \\ + D_2^2 \cdot R_1^2 \cdot R_3^2 + D_1^2 \cdot R_3^4\end{array}\right]}{\left(D_2 - D_3 + D_1\right) \cdot \left(D_2 + D_3 + D_1\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(-D_2 - D_3 + D_1\right)}} = 0$$

## 04/30/94 Division №



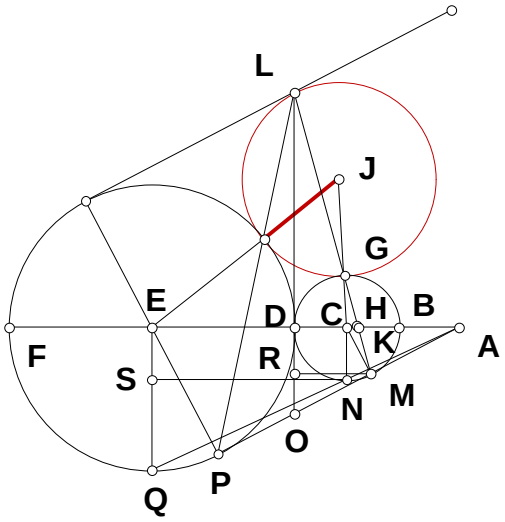

$N_1 := 5$ $N_2 := 2$

$AB := N_1$ $BC := N_2$ $AC := \sqrt{AB^2 + BC^2}$

$CD := \frac{BC \cdot AC}{AB}$ $BD := \sqrt{CD^2 - BC^2}$

$\frac{N_2^2}{N_1} - BD = 0$

## 05/01/94 Two Circles And A Parallel




**Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.**

$R_1 := 3 \quad R_2 := 2$

$DE := R_1 \quad BC := R_2 \quad CN := BC$

$EQ := DE \quad CD := BC \quad CE := CD + DE$

$ES := CN \quad NS := CE \quad SQ := EQ - ES$

$AE := \frac{NS \cdot EQ}{SQ} \quad AD := AE - DE \quad EP := DE$

$AP := \sqrt{AE^2 - EP^2} \quad DO := \frac{EP \cdot AD}{AP}$

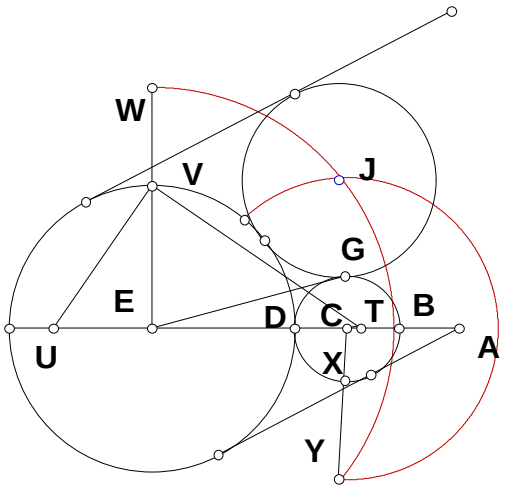
$DL := \frac{DO \cdot DE}{CD} \quad AC := AD - CD \quad AM := \frac{AP \cdot AC}{AE} \quad AO := \frac{AE \cdot AD}{AP} \quad MO := AO - AM$

$MR := \frac{AD \cdot MO}{AO} \quad RO := \frac{DO \cdot MR}{AD} \quad DR := DO - RO \quad LR := DR + DL \quad ML := \sqrt{MR^2 + LR^2}$

$DK := \frac{MR \cdot DL}{LR} \quad CK := DK - CD \quad CH := \frac{LR \cdot CK}{ML} \quad CM := BC \quad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \quad GL := ML - MG \quad GJ := \frac{CM \cdot GL}{MG}$

**The Algebraic name for GJ suggests a simpler method of construction.**




$R_3 := \frac{R_1^2}{4 \cdot R_2} \quad GJ - R_3 = 0$

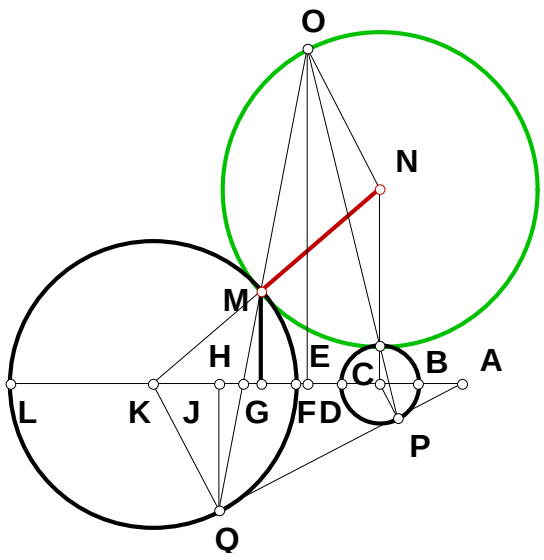
$ET := 4 \cdot BC \quad EV := DE \quad EU := \frac{DE^2}{ET}$

$VW := EU \quad XY := EU \quad EV := DE \quad CX := BC$

$EW := EV + VW \quad CY := CX + XY$

$GJ_2 := EU \quad GJ - GJ_2 = 0$

# 05/04/94 Two Circles And A Tangent




**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**

$R_1 := 1 \quad R_2 := .5 \quad D := 2 \quad N := 2$

$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK$

$$AK := \frac{D \cdot R_1}{R_1 - R_2} \qquad EK := \frac{R_1^2 + D^2 - R_2^2}{2 \cdot D}$$

$$AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$$

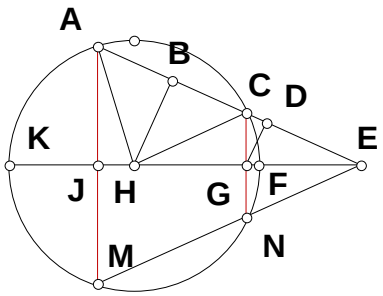
$$FG := \frac{FL}{N} \quad GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK} \quad AF := AK - FK \quad FJ := AJ - AF$$

$$JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG \quad QM := \sqrt{(JQ + GM)^2 + GJ^2} \quad GH := \frac{GJ \cdot GM}{JQ + GM}$$

$$HM := \frac{QM \cdot GM}{JQ + GM} \quad EF := EK - FK \quad EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \quad MO := HO - HM$$

$$KM := FK \quad MN := \frac{KM \cdot MO}{QM} \qquad MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2\,N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

# 05/06/94 A Ratio In Trisection




## What is AJ to CG?

$N := 2.423$ $FH := 1$ $CE := FH$

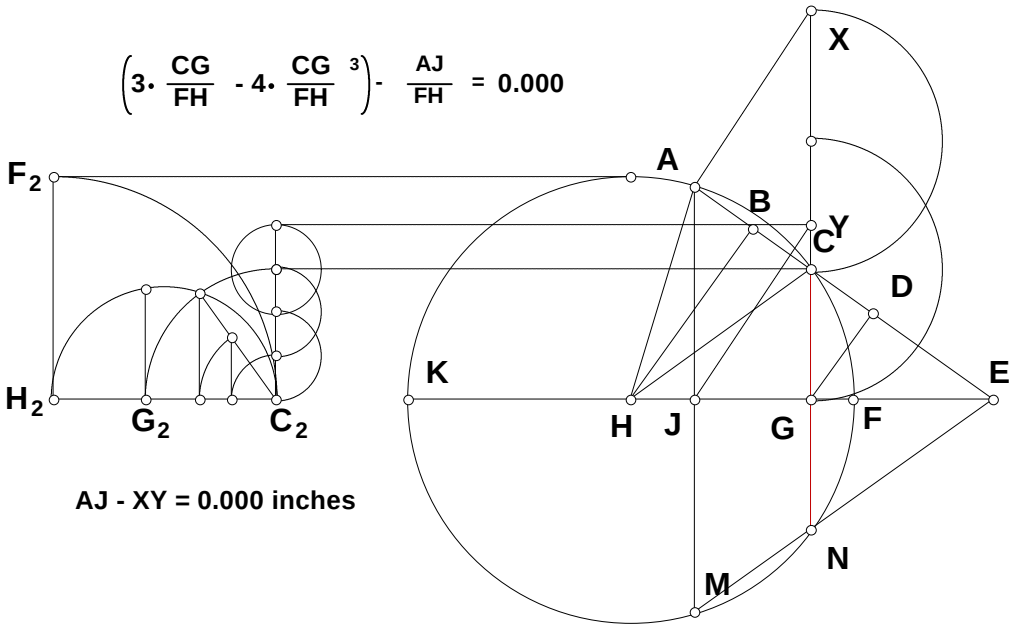
$CG := \frac{FH}{N}$ $EG := \sqrt{CE^2 - CG^2}$ $CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2}$ $EH := 2 \cdot EG$ $BH := \frac{DG \cdot EH}{EG}$ $CH := FH$

$BC := \sqrt{CH^2 - BH^2}$ $AC := 2 \cdot BC$ $AE := AC + CE$ $AJ := \frac{CG \cdot AE}{CE}$ $3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0$

$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$ $AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$ $AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

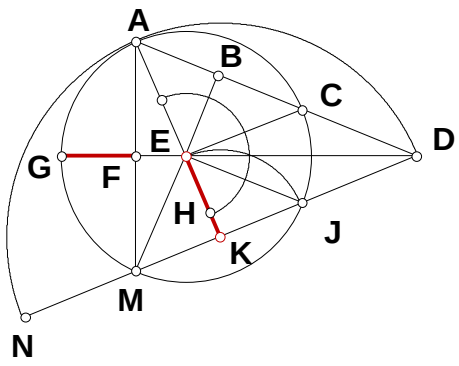
$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG}{FH}^3\right) - \frac{AJ}{FH} = 0.000$




AJ - XY = 0.000 inches

# 05/07/94 A Trisection Ratio

## In trisection, what is the ratio of FG/EK?




$N := 2$

$AE := 1 \quad EH := \frac{AE}{2} \quad HK := AE \cdot N \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

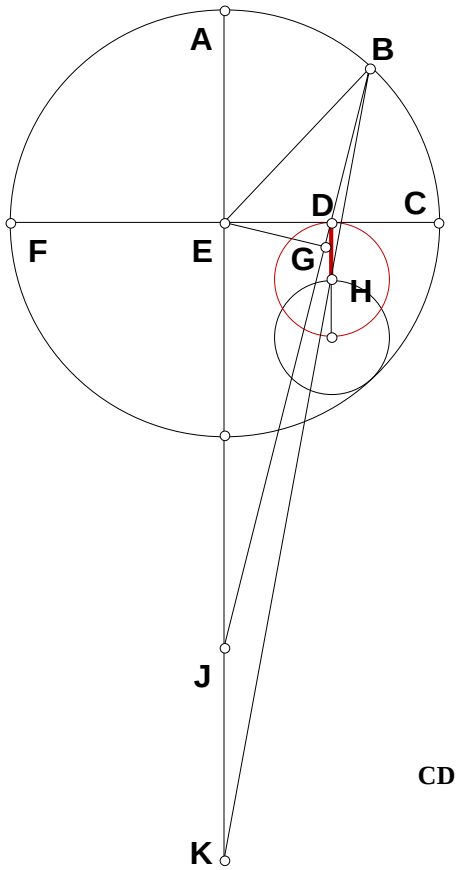
### Algebraic Names,

$$FG - \left[1 + \frac{(N+1)\cdot(2\cdot N-1)}{(2N+1)\cdot\sqrt{(N+1)\cdot(2N+1)}}\right] = 0 \qquad EK - \frac{1}{2} - N = 0$$

$$\frac{FG}{EK} - 2\cdot\frac{\left[2\cdot N\cdot\sqrt{(N+1)\cdot(2\cdot N+1)} + \sqrt{(N+1)\cdot(2\cdot N+1)} + (N+1)\cdot(2\cdot N-1)\right]}{\left[(2\cdot N+1)^2\cdot\sqrt{(N+1)\cdot(2\cdot N+1)}\right]} = 0$$

## 05/16/94A Tangent Diameter and Circles

**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.**




$CF := 1 \qquad CE := \frac{CF}{2} \qquad N_1 := 2 \qquad N_2 := 3$

$CD := \frac{CE}{N_1} \qquad DE := CE - CD \qquad EJ := CE \cdot N_2$

$DJ := \sqrt{DE^2 + EJ^2} \qquad JG := \frac{EJ^2}{DJ} \qquad BE := CE$

$EG := \sqrt{EJ^2 - JG^2} \qquad BG := \sqrt{BE^2 - EG^2} \qquad BJ := BG + JG$

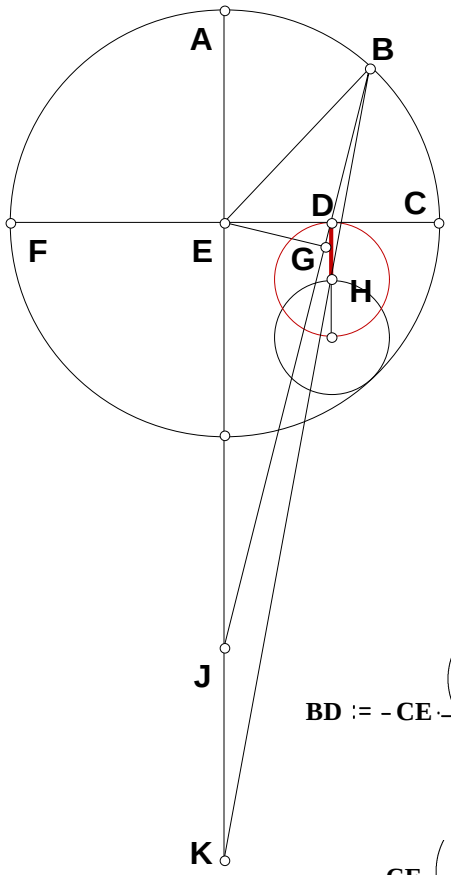
$JK := CE \qquad BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$

### Algebraic Names:

$$CD := \frac{CE}{N_1} \qquad DE := CE \cdot \frac{(N_1 - 1)}{N_1} \qquad EJ := CE \cdot N_2$$

$$DJ := CE \cdot \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{N_1}$$

$$JG := CE \cdot \frac{N_1 \cdot N_2^2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} \qquad EG := CE \cdot \frac{(N_1 - 1) \cdot N_2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$



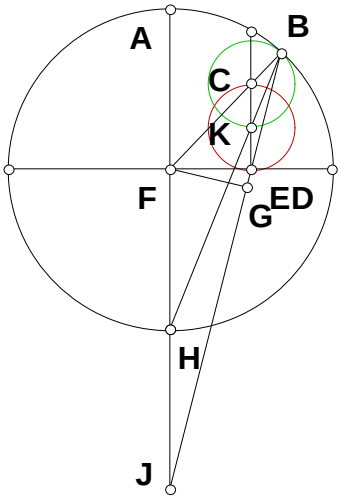

$$BG := CE \cdot \sqrt{\frac{\left(N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2\right)}{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}}$$

$$BJ := CE \cdot \frac{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

$$BD := -CE \cdot \frac{\left(-N_1 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_1^2 - 2 \cdot N_1 + 1\right)}{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1\right)}$$

$$DH := \frac{CE \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1\right)}{N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}$$

# 05/16/94B Tangent Diameter and Circles




**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.**

$N_1 := 2 \quad N_2 := 2 \quad AF := 1$

$DF := AF \quad DE := \frac{DF}{N_1} \quad AJ := AF \cdot N_2$

$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$

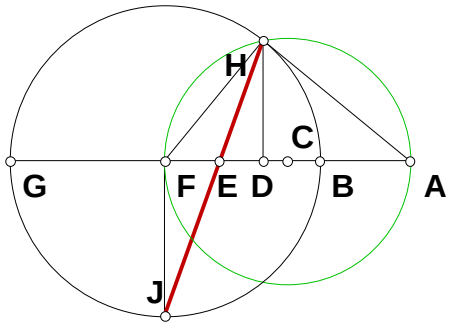
$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad KE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot KE\right]^2} \quad BC - KE = 0 \quad KE = 0.375$

$$1 - \frac{2 \cdot N_1^2 - 2 \cdot N_1 + 1 + N_1^2 \cdot N_2^2 - 2 \cdot N_1^2 \cdot N_2}{N_1 \cdot \left(\sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 4 \cdot N_1 \cdot N_2 - N_2^2 + 2 \cdot N_2} + N_1 + N_1 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2\right)} - BC = 0$$

# 10/27/94 Trivial Method Square Root

## AE is the square root of AB x AG.




$N := 5$ $AB := 1$

$AG := AB \cdot N$ $BG := AG - AB$ $BF := \frac{BG}{2}$

$AF := AB + BF$ $FH := BF$ $DF := \frac{FH^2}{AF}$ $AD := AF - DF$

$BD := AD - AB$ $DG := BG - BD$ $FJ := BF$

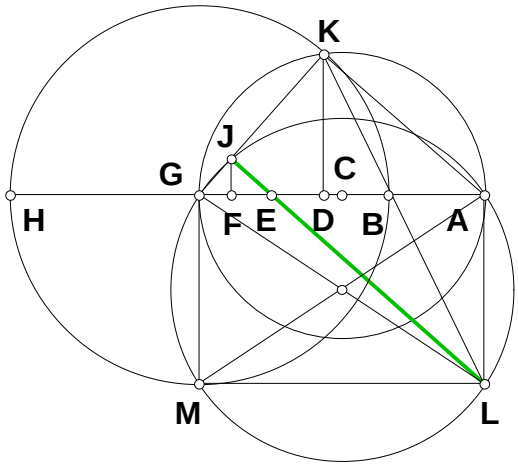
$DH := \sqrt{BD \cdot DG}$ $DE := \frac{DF \cdot DH}{DH + FJ}$ $AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$

# 10/28/94 Trivial Method Square Root

## AE is the square root of AB x AH.

$N := 5 \quad AB := 1 \quad AH := AB \cdot N \quad BH := AH - AB$




$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

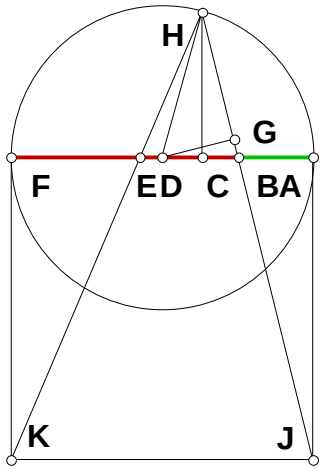
$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad JL := \sqrt{GL^2 - GJ^2}$

$FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$

# 10/31/94 Square Root of a Segment

**Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.**




$N := 11 \qquad AB := 1$

$AF := AB \cdot N \qquad BF := AF - AB \qquad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \qquad BC := \frac{AB \cdot (BG + GH)}{BJ} \qquad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \qquad CE := \frac{CF \cdot CH}{(CH + FK)}$

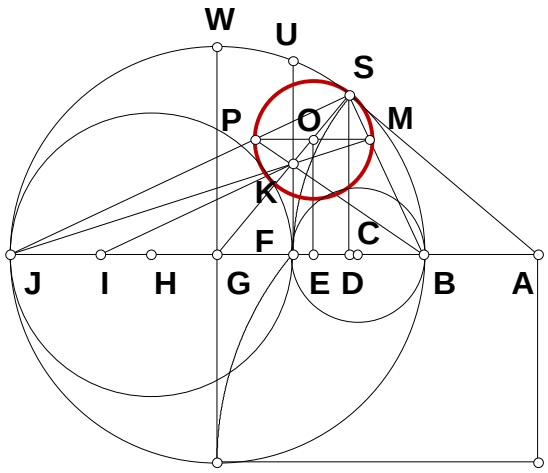
$EF := CF - CE \qquad BE := BC + CE \qquad DF := BF - BE \qquad CD := AD - AC \qquad BE^2 - EF = 0$

$$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$$

$$EF - \left( N - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} - 1 \right) = 0$$

# 12/24/94 Power Line At Square Root

**In this square root figure, what is the Algebraic name of the tangent circle OS?**




$N := 5 \qquad AB := 1 \qquad AJ := AB \cdot N$

$AF := \sqrt{AB \cdot AJ} \qquad BJ := AJ - AB \qquad BG := \frac{BJ}{2}$

$AG := AB + BG \qquad GS := BG \qquad DG := \frac{GS^2}{AG}$

$FG := AG - AF \qquad BD := BG - DG \qquad DJ := BJ - BD$

$DS := \sqrt{BD \cdot DJ} \qquad FK := \frac{DS \cdot FG}{DG} \qquad BF := AF - AB$

$BK := \sqrt{BF^2 + FK^2} \qquad FI := \frac{DJ \cdot FK}{DS} \qquad BI := FI + BF$

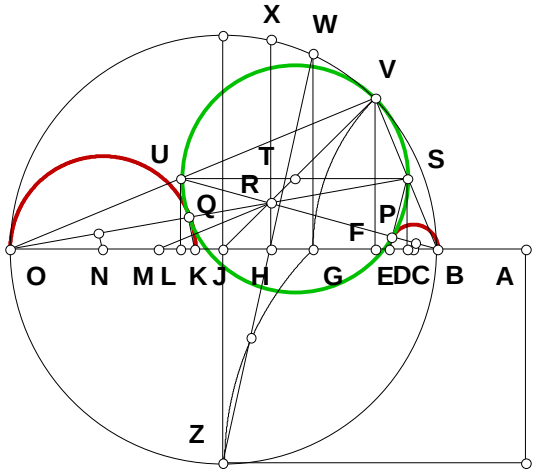
$BP := \frac{BK \cdot BJ}{BI} \qquad KP := BP - BK \qquad MP := \frac{BJ \cdot KP}{BK} \qquad OS := \frac{MP}{2}$

## Algebraic Names

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0$$

$$OS - \frac{-2 \cdot N^2 + 2 \cdot N - \sqrt{N} + N^{\left(\frac{5}{2}\right)}}{2 \cdot \left[N^2 - \sqrt{N} - N^{\left(\frac{3}{2}\right)} + 1\right]} = 0$$

# 12/25/95 Two Prime Exponential Series Developed Through The Powerline Progression




$\Delta := 2$ $\quad \delta := 1 .. \Delta$

$N := 5$ $\quad AB := 1$

$AO := AB \cdot N$ $\quad AG := \sqrt{AB \cdot AO}$ $\quad BO := AO - AB$

$BJ := \frac{BO}{2}$ $\quad JZ := BJ$ $\quad JV := BJ$ $\quad JO := BJ$

$BG_1 := AG - AB$ $\quad GO_1 := BO - BG_1$ $\quad GW_1 := \sqrt{BG_1 \cdot GO_1}$

$GJ_1 := BJ - BG_1$ $\quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{bmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{bmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - \left(BG_\delta + GH_\delta\right) \\ \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]} \\ BJ - \left(BG_\delta + GH_\delta\right) \\ \frac{\left[BJ - \left(BG_\delta + GH_\delta\right)\right] \cdot \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}}{JZ + \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}} \end{bmatrix}$$

$HJ := BJ - BG_\Delta$

$FJ := \frac{(N-1)^2}{2 \cdot (N+1)}$

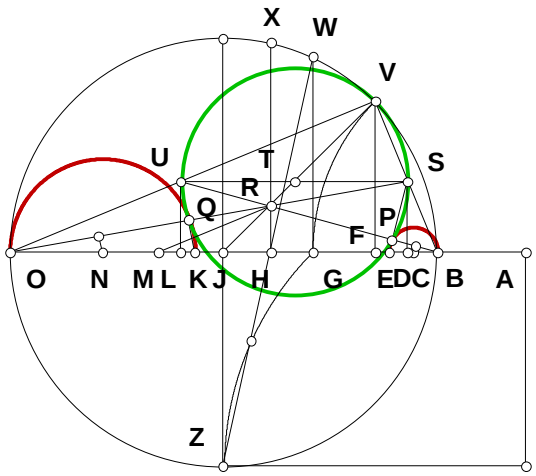
$BF := BJ - FJ$ $\quad FO := FJ + JO$ $\quad FV := \sqrt{BF \cdot FO}$

$HR := \frac{FV \cdot HJ}{FJ}$ $\quad BH := BJ - HJ$ $\quad BR := \sqrt{HR^2 + BH^2}$

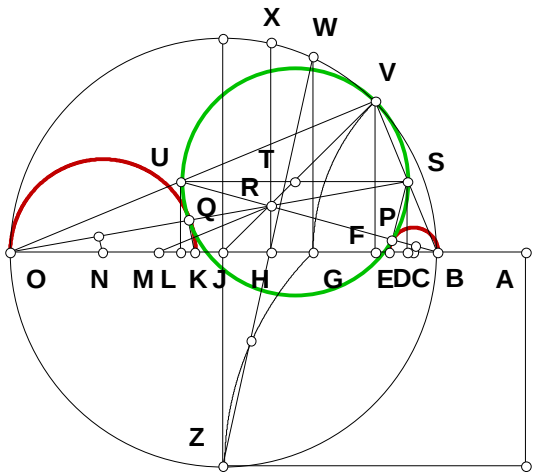
$HM := \frac{FO \cdot HR}{FV}$ $\quad BU := \frac{BR \cdot BO}{BH + HM}$ $\quad RU := BU - BR$

$SU := \frac{BO \cdot RU}{BR}$ $\quad TV := \frac{SU}{2}$ $\quad PU := \frac{BH \cdot SU}{BR}$

$BP := BU - PU$ $\quad BE := \frac{BR \cdot BP}{BH}$ $\quad AE := AB + BE$




$$N^{2^{\frac{1}{\Delta}}} - AE = 0$$




$$LU := \frac{HR \cdot BU}{BR} \qquad BL := \frac{BH \cdot BU}{BR} \qquad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \qquad DS := LU \qquad OS := \frac{OR \cdot DS}{HR}$$

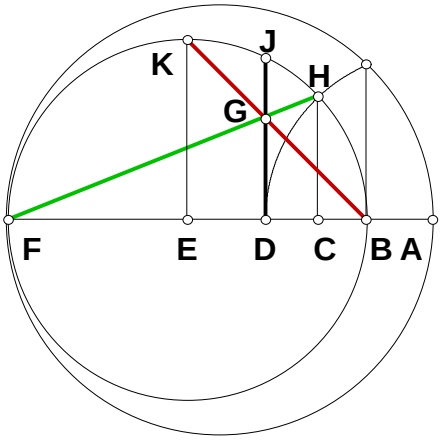
$$DO := \frac{HO \cdot DS}{HR} \qquad QS := \frac{DO \cdot SU}{OS} \qquad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \qquad AK := AO - KO$$

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

# 12/26/94 Is Point G on DJ?

## Is G, the intersection of FH and BK, on DJ?




$N := 5$ $AB := 1$ $AF := AB \cdot N$ $BF := AF - AB$

$BE := \frac{BF}{2}$ $EF := BE$ $EK := BE$

$AD := \sqrt{AB \cdot AF}$ $CE := \frac{(N-1)^2}{2 \cdot (N+1)}$ $CF := CE + EF$

$DF := AF - AD$ $BC := BF - CF$ $CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF}$

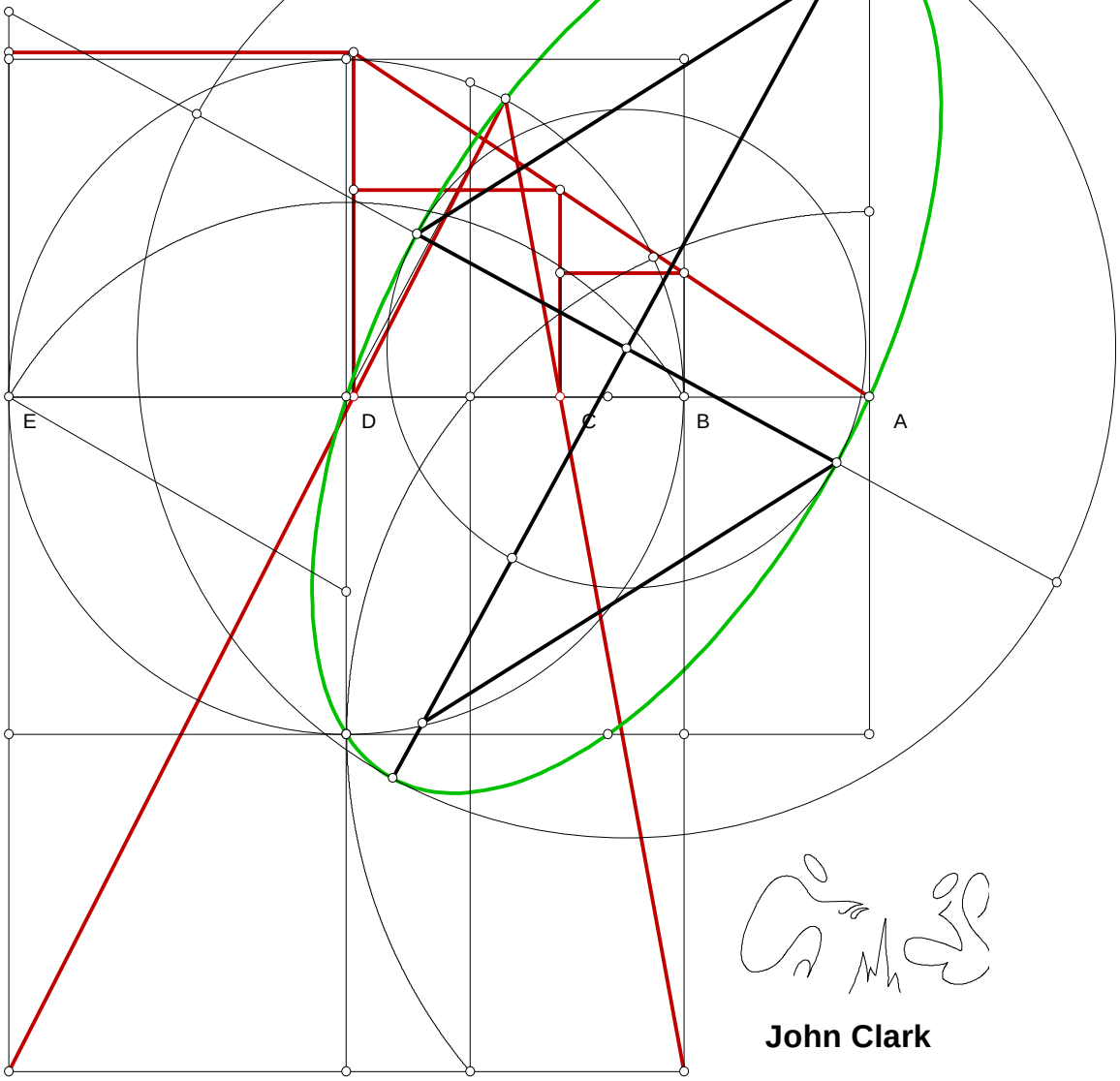
$BD := AD - AB$ $DG_2 := \frac{EK \cdot BD}{BE}$ $DG_1 - DG_2 = 0$

# The Delian Quest

## 1995

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

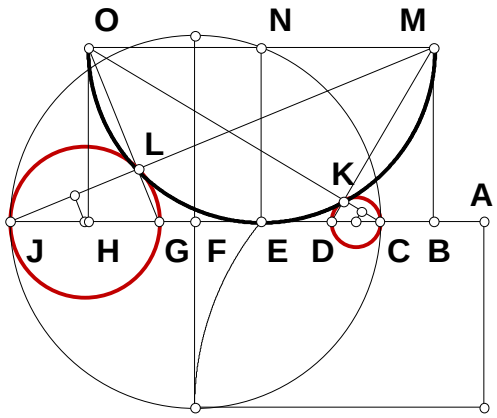
$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$






John Clark

# 01/06/95 Alternate Method Quad Roots




$N := 5$ $AC := 1$ $AJ := AC \cdot N$

$CJ := AJ - AC$ $AE := \sqrt{AC \cdot AJ}$ $CE := AE - AC$

$EJ := CJ - CE$ $EN := \sqrt{CE \cdot EJ}$ $BM := EN$ $HO := EN$

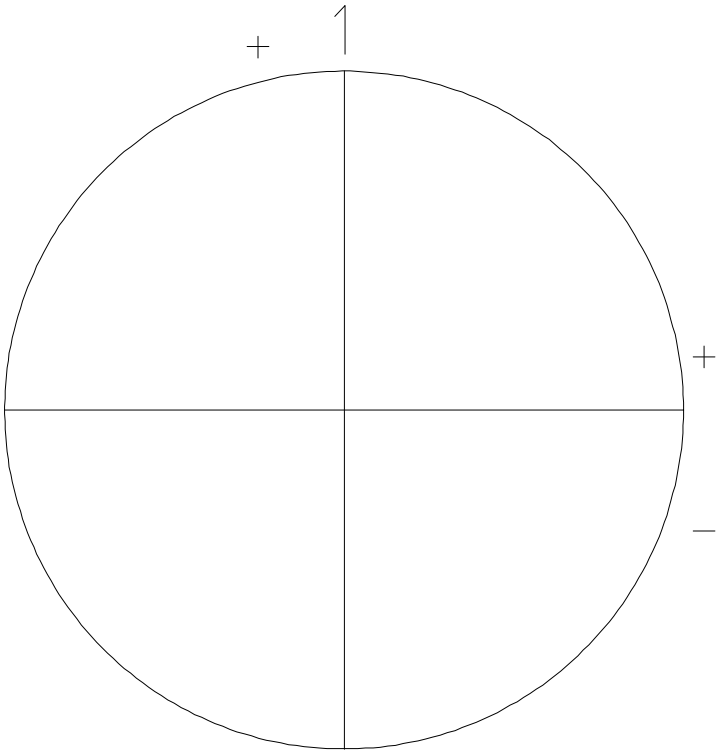
$MN := EN$ $NO := EN$ $BE := EN$ $EH := EN$

$BJ := BE + EJ$ $MJ := \sqrt{BJ^2 + BM^2}$ $MO := MN + NO$

$ML := \frac{BJ \cdot MO}{MJ}$ $JL := MJ - ML$ $GJ := \frac{MJ \cdot JL}{BJ}$ $AG := AJ - GJ$ $\left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH$ $CO := \sqrt{CH^2 + HO^2}$ $KO := \frac{CH \cdot MO}{CO}$ $CK := CO - KO$ $CD := \frac{CO \cdot CK}{CH}$ $AD := AC + CD$

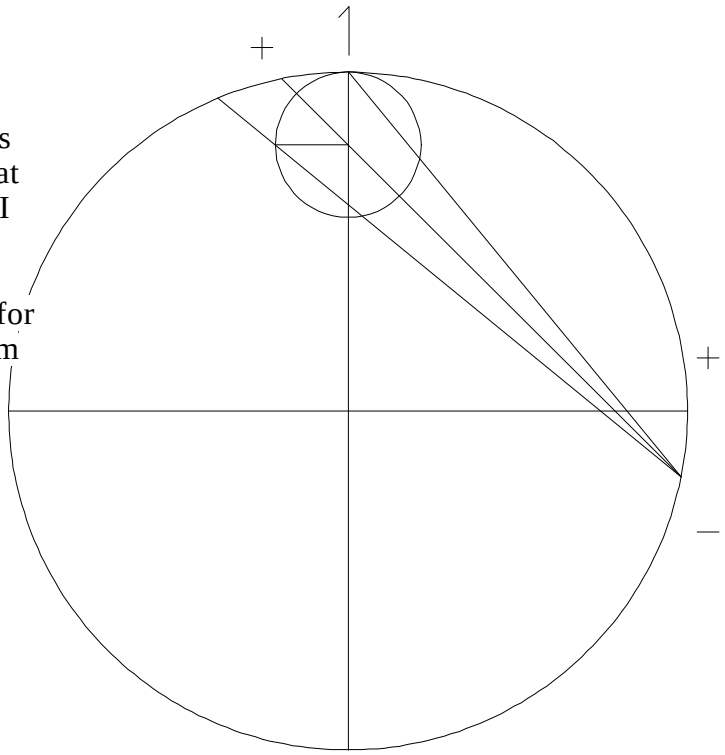
$\left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0$ $N^{\frac{3}{4}} - AG = 0$ $N^{\frac{1}{4}} - AD = 0$




Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

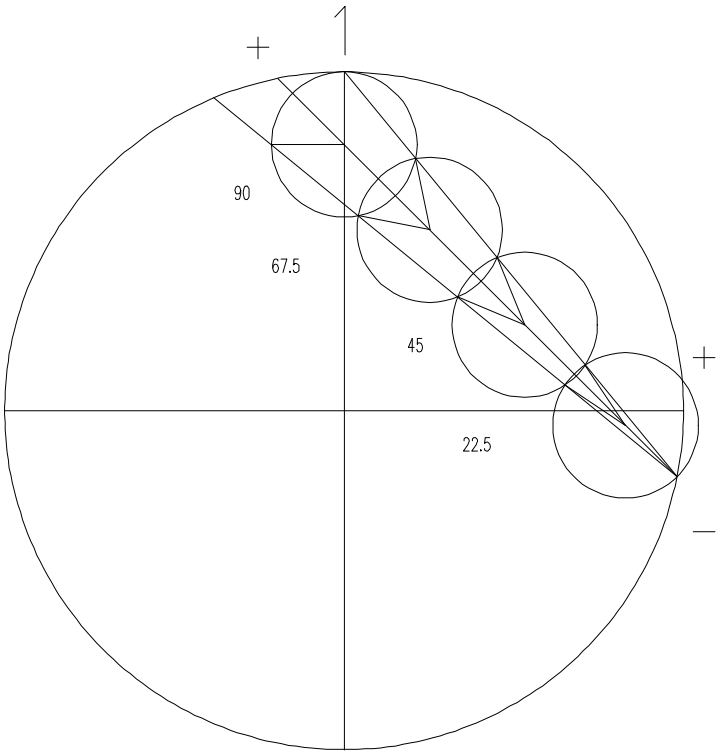
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right)\cdot 90 = 90$$




$B := 1 + \frac{1}{8} - \frac{1}{8}$ $B = 1$

$\frac{B \cdot 4}{4} \cdot 90 = 90$ $\frac{B \cdot 3}{4} \cdot 90 = 67.5$

$\frac{B \cdot 2}{4} \cdot 90 = 45$ $\frac{B}{4} \cdot 90 = 22.5$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$\mathrm{mod}(8 + 1 - 1, 2) = 0$

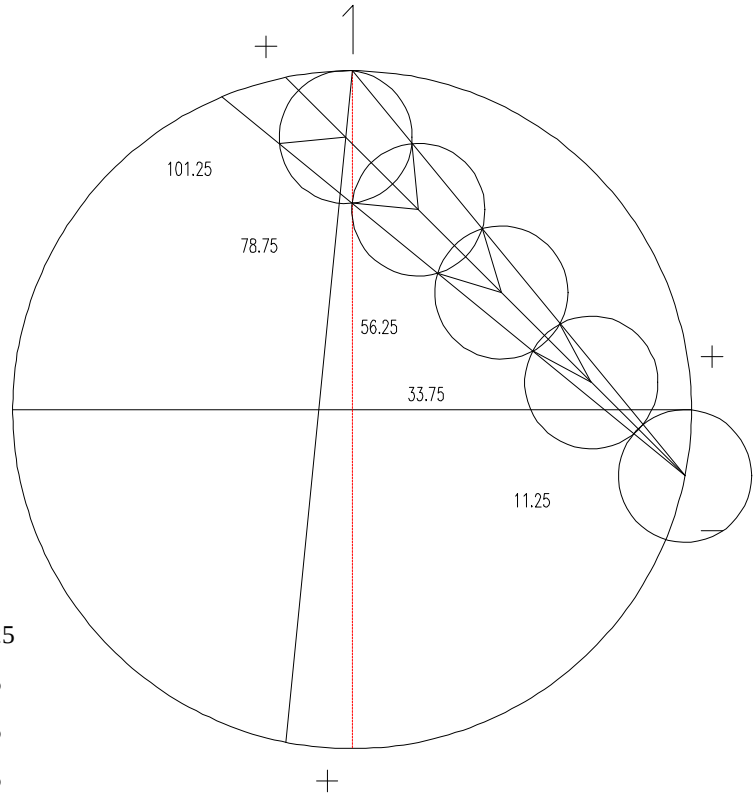
I have added another plus to a quadrant at the bottom of the figure.

$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

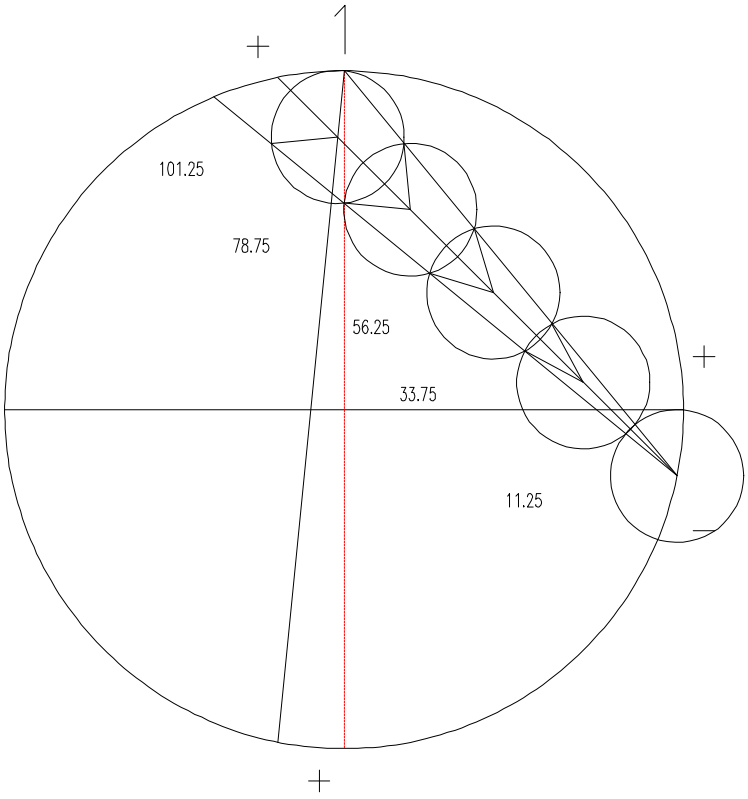
$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$




| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

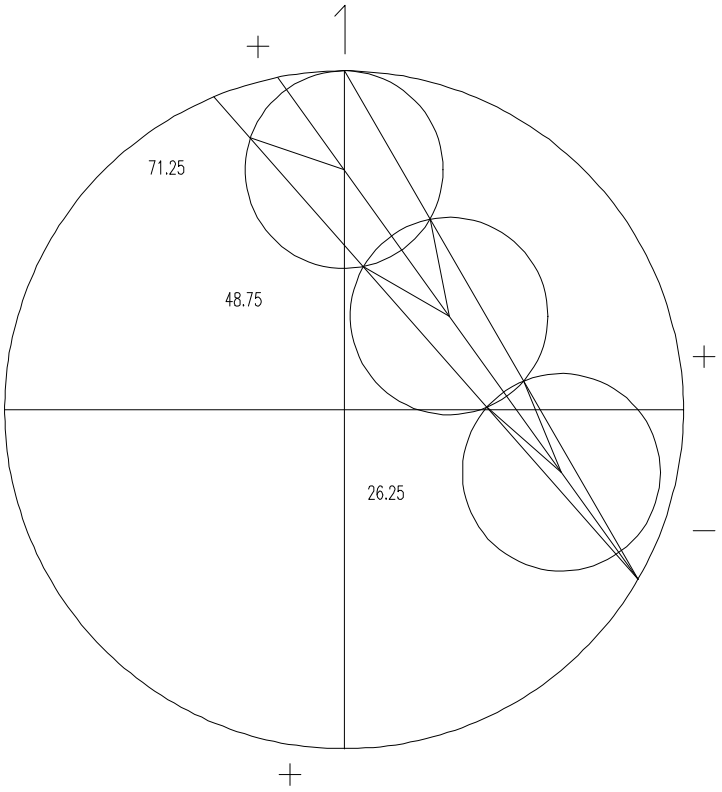
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

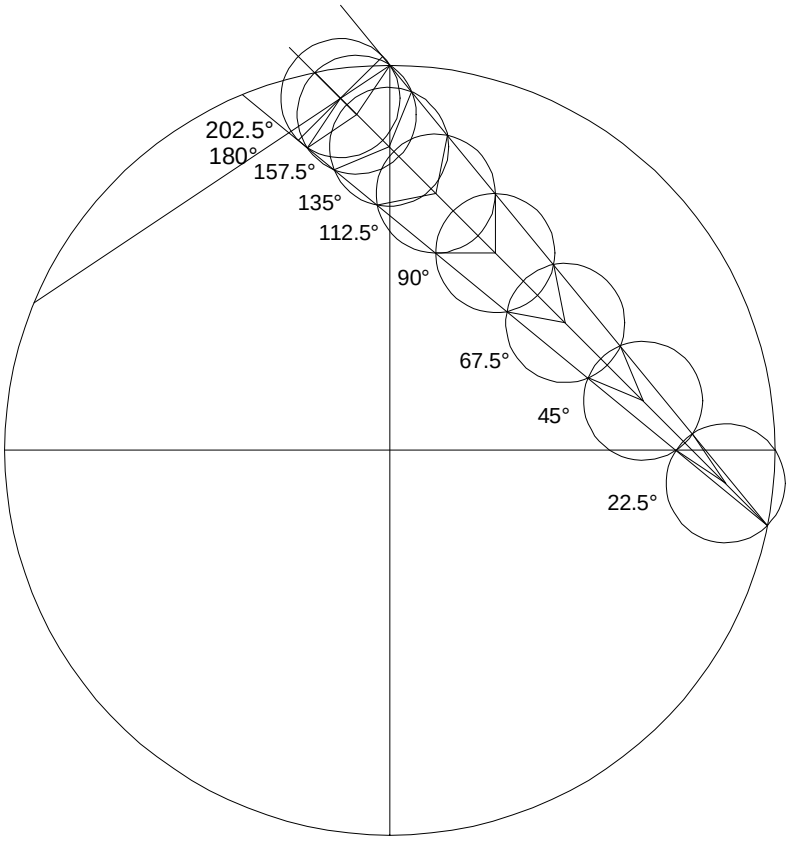
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

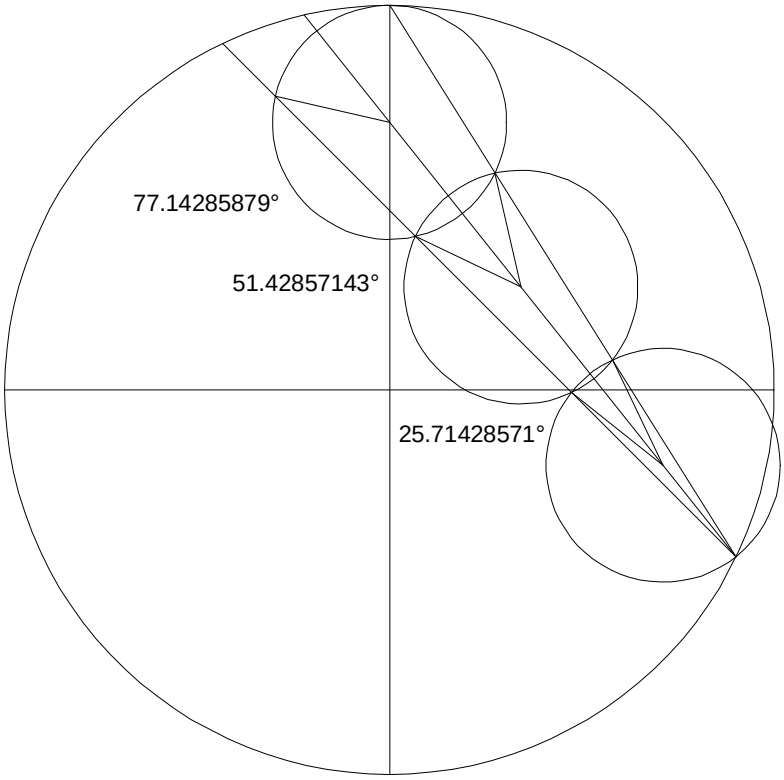
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

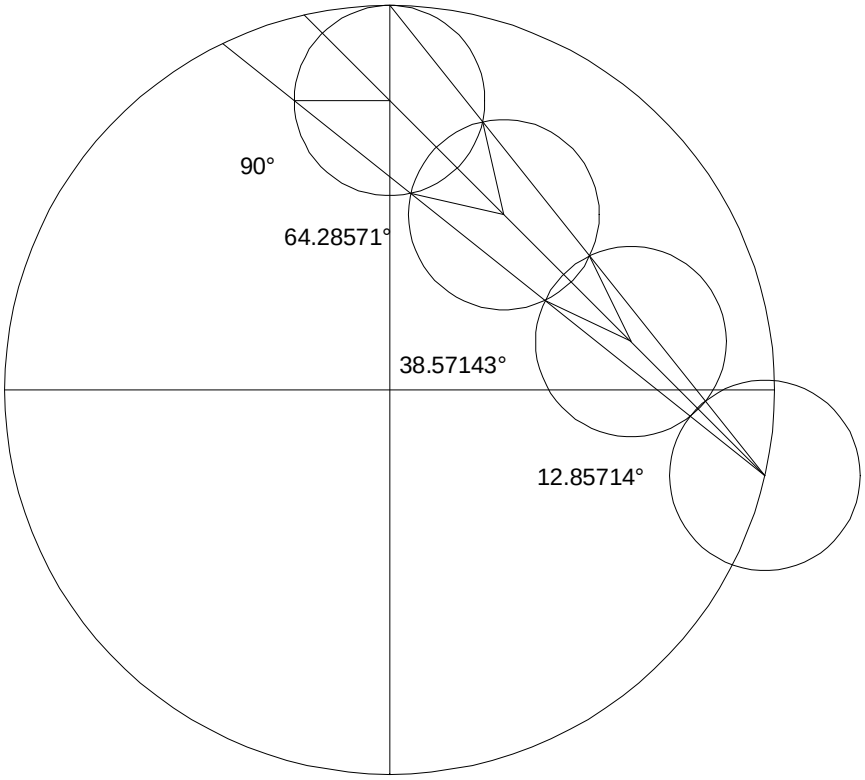
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

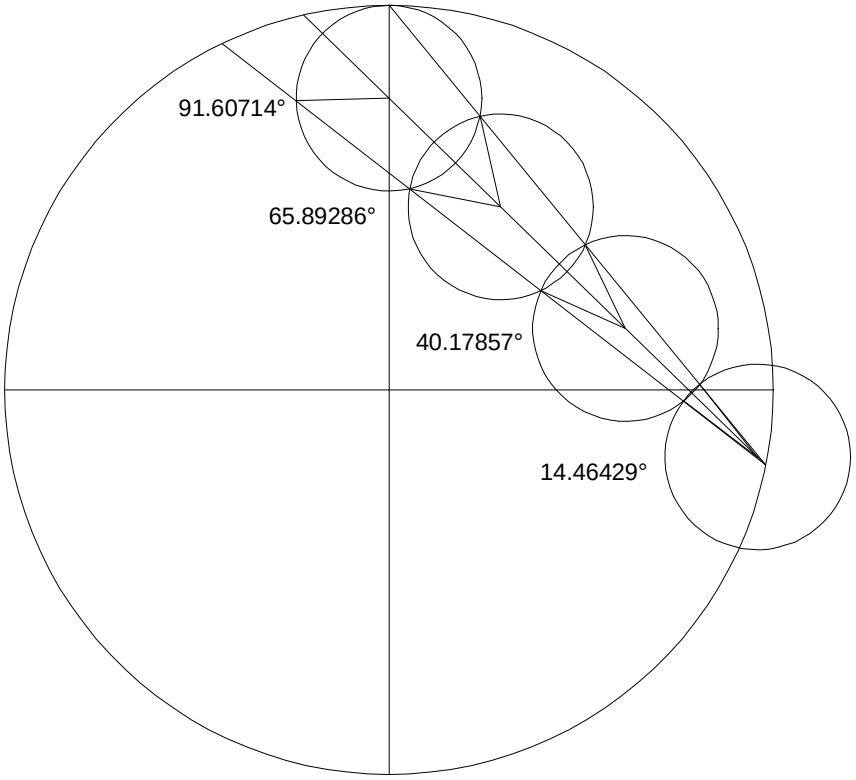
$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

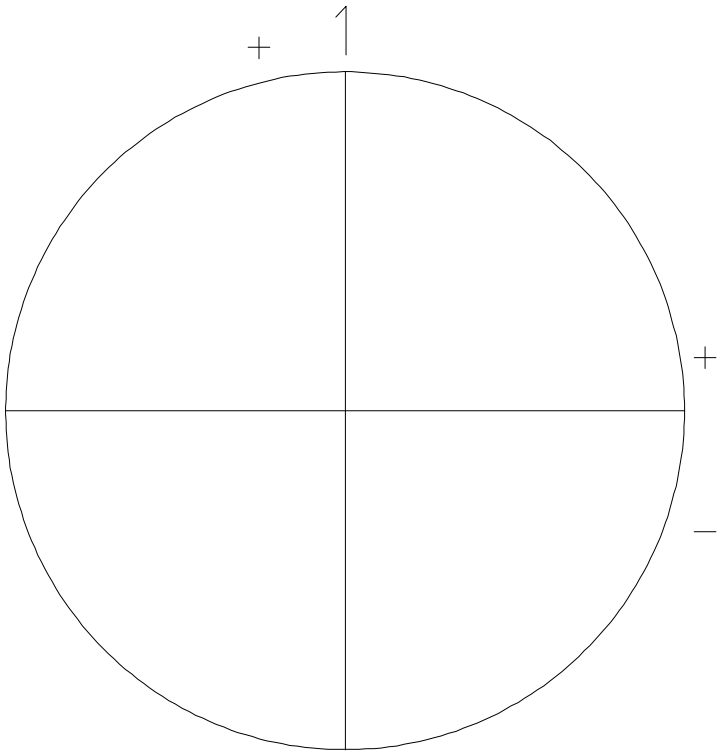
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

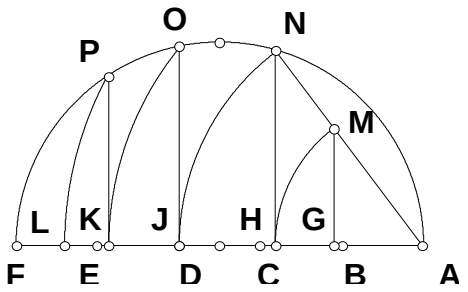
$\mathrm{mod}(7 + 1 - 1, 2) = 1$




Work in progress.

# 04/01/95 Exponential Series-Roots and Powers

**I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.**




$N_1 := 5$ $\quad AB := 1$ $\quad AF := AB \cdot N_1$

$\delta := 0 .. 3$

$N_2 := 3$ $\quad AJ_0 := AB \cdot N_2$ $\quad AN := AJ_0$

$$AJ_1 := \frac{AN^2}{AF} \qquad AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF}$$

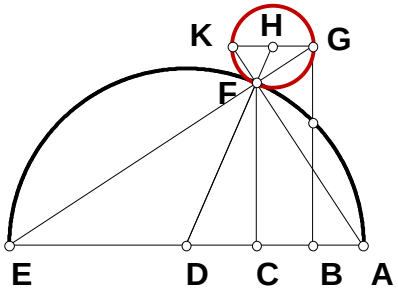
$AD_0 := AJ_0$ $\quad DF_0 := AF - AD_0$ $\quad DO_0 := \sqrt{AD_0 \cdot DF_0}$ $\quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{bmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{bmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

$$\sum_\delta \left[ \frac{AF}{AJ_\delta} - \left( \frac{N_1}{N_2} \right)^{\delta+1} \right] = 0$$

$$\sum_\delta \left[ \frac{AF}{AD_\delta} - \left( \frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$

## 09/13/95 A Study In Placement




### Given AE, AB, AC what is GH?

$N_1 := 1 \quad N_2 := 4 \quad N_3 := 1 \quad N_4 := 5 \quad AE := 1$

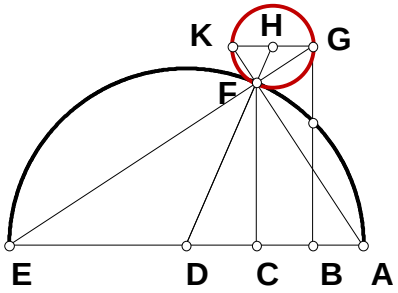
$AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad AC := AE \cdot \frac{N_3}{N_4} \quad DF := AD$

$DE := AD \quad CD := AD - AC \quad CF := \sqrt{DF^2 - CD^2}$

$CE := AE - AC \quad BD := AD - AB \quad BE := AE - AB \quad EF := \sqrt{CE^2 + CF^2} \quad EG := \frac{EF \cdot BE}{CE}$

$FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right| \quad GH - \left| \frac{N_3 \cdot N_2 - N_1 \cdot N_4}{2 \cdot N_2 \cdot (N_4 - N_3)} \right| = 0$

### Given AE, AB, EF what is GH?



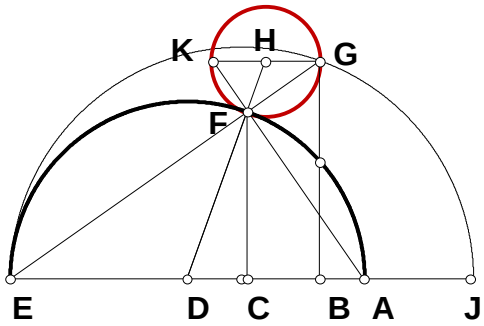

$AE := 1 \quad AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad DE := AD$

$EF := AE \cdot \frac{N_3}{N_4} \quad CE := \frac{EF^2}{AE} \quad BD := AD - AB$

$BE := BD + DE \quad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF$

$GH := \frac{AD \cdot FG}{EF} \quad GH - \frac{N_2 \cdot N_4^2 - N_1 \cdot N_4^2 - N_3^2 \cdot N_2}{2 \cdot N_2 \cdot N_3^2} = 0$

### Given AE, AB, BG what is GH?




$AE := 1 \quad AD := \frac{AE}{2} \quad AB := AE \cdot \frac{N_1}{N_2} \quad BG := AE \cdot \frac{N_3}{N_4}$

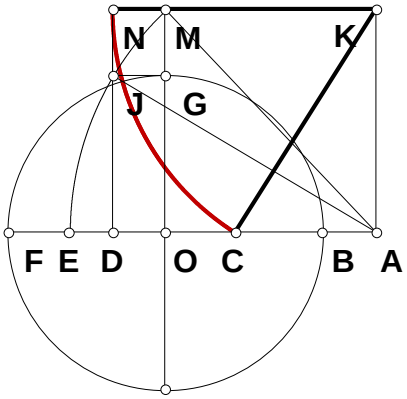
$BE := AE - AB \quad EG := \sqrt{BE^2 + BG^2} \quad EJ := \frac{EG^2}{BE}$

$EF := \frac{EG \cdot AE}{EJ} \quad FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right|$

$GH - \left| \frac{N_1 \cdot N_4^2 \cdot N_2 - N_1^2 \cdot N_4^2 - N_3^2 \cdot N_2^2}{2 \cdot N_2 \cdot \left[ N_4^2 \cdot (N_1 - N_2) \right]} \right| = 0$

# 10/14/95 Alternate Method Square Root

## For any AK is AC the root of AB x AF?




$N_1 := 5 \quad N_2 := 4$

$AB := 1 \quad AF := AB \cdot N_1 \quad AK := AB \cdot N_2 \quad BF := AF - AB$

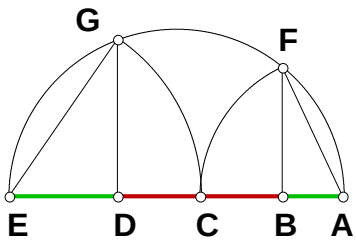
$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$GO := BO \quad DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$KN := AD \quad CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$

# 10/20/95 Four Times The Square




$N_1 := 2$ $AE := 1$ $AB := \frac{AE}{N_1}$

$BE := AE - AB$ $BF := \sqrt{AB \cdot BE}$ $AF := \sqrt{AB^2 + BF^2}$

$AC := AF$ $CE := AE - AC$ $EG := CE$ $DE := \frac{EG^2}{AE}$

$BD := AE - (AB + DE)$ $\frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$

$AD := AE - DE$ $DG := \sqrt{AD \cdot DE}$

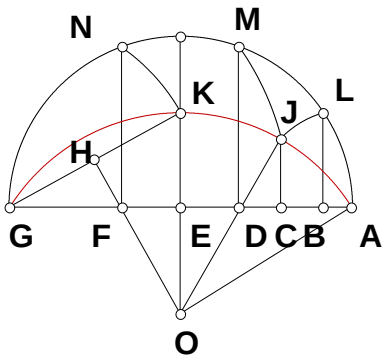
## Algebraic Names:

$\frac{1}{N_1} - AB = 0$ $1 - \frac{1}{N_1} - BE = 0$ $\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0$ $\sqrt{\frac{N_1}{N_1^2}} - AF = 0$

$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0$ $1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0$ $2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$

$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0$ $\frac{\sqrt{2 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1^2 - 5 \cdot N_1 + 4 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1 - 1}}{N_1} - DG = 0$

## 11/01/95 A Modification Of The Square Root Figure, Gemini Roots




**On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.**

$N_1 := 9 \qquad N_2 := 2 \qquad AG := 1$

$AE := \frac{AG}{2} \quad EG := AE \quad EF := \frac{AG}{2 \cdot N_1} \quad AF := AE + EF$

$FG := EG - EIFN := \sqrt{AF \cdot FG} \quad GN := \sqrt{FN^2 + FG^2} \quad GK := GN$

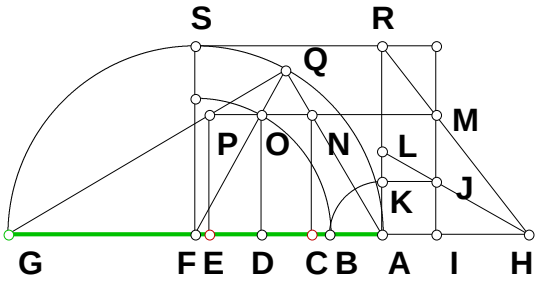
$EK := \sqrt{GK^2 - EG^2} \quad EO := \frac{EG \cdot EF}{EK} \quad OK := EO + EK \quad DE := \frac{AE}{N_2} \qquad DO := \sqrt{DE^2 + EO^2}$

$DJ := OK - DO \quad CD := \frac{DE \cdot DJ}{DO} \quad CE := CD + DE \; CJ := \frac{EO \cdot DJ}{DO} \quad AC := AE - CE \quad AJ := \sqrt{AC^2 + CJ^2}$

$AL := AJ \quad AB := \frac{AL^2}{AG} \quad CG := AG - AC \quad GJ := \sqrt{CG^2 + CJ^2} \quad GM := GJ \quad DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG) \qquad \frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$

# 11/05/95 Alternate Method Gemini Roots




$N_1 := 3 \quad N_2 := 5$

$AG := 1 \quad AF := \frac{AG}{2} \quad AR := AF \quad FQ := AF \quad FG := AF$

$AL := \frac{AR}{N_1} \qquad IM := \frac{AR}{N_2} \qquad AK := \frac{AL \cdot IM}{AR}$

$DO := IM \quad AB := AK \quad BF := AF - AB \quad FO := BF$

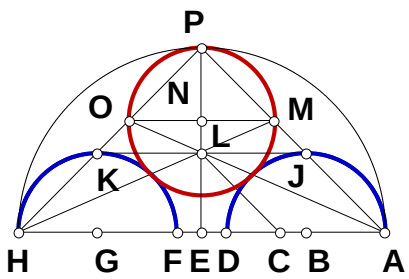
$OQ := FQ - FO \quad NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO} \quad NP - 2 \cdot AK = 0 \quad CD := AK \quad DE := AK$

$DF := \sqrt{FO^2 - DO^2} \quad AD := AF - DF \quad AC := AD - CD \quad EG := FG + DF - DE$

$CE := NP \qquad \frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$

## 12/01/95 Method For Equals

### Given AB find NP.




$N_1 := 3$

$AH := 1 \quad AE := \frac{AH}{2} \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{AE}{N_1} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$

$$\frac{1}{2} \cdot \frac{\left(N_1 - 1\right)}{\left(N_1 + 1\right)} - NP = 0$$

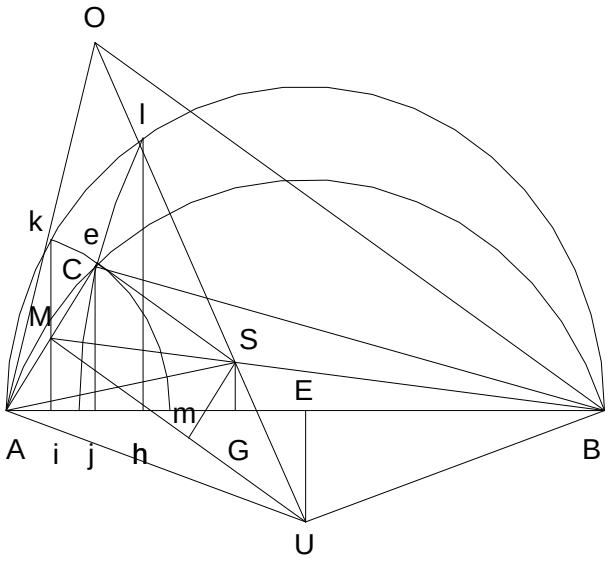
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

12_07_95.MCD

$\delta := 0..2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

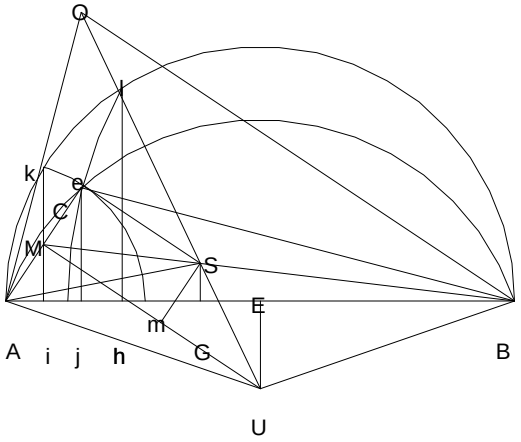
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

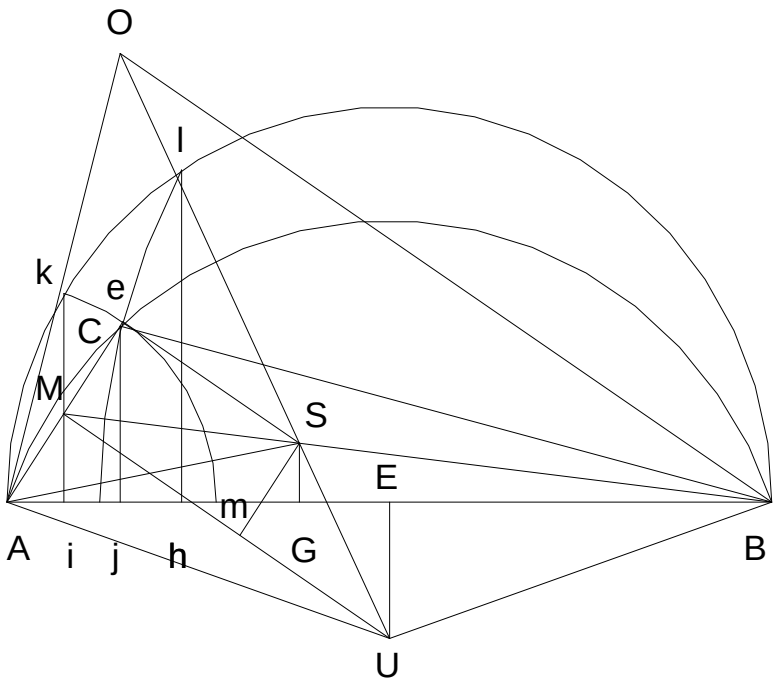
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta Sm_\delta := eM_\delta \qquad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \qquad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75

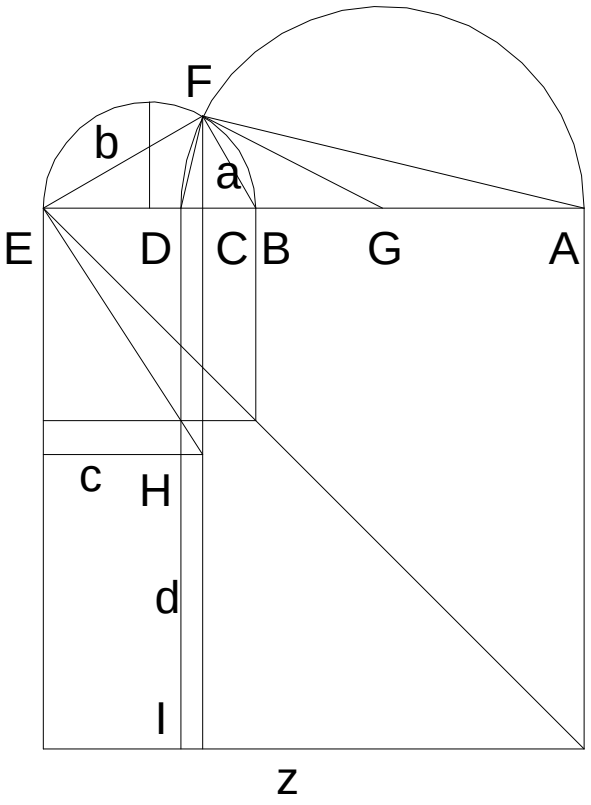



$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.




12_16_95.MCD

Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \frac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \frac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.622$

$c := DE$ $\quad c = 0.622$

$d := HI$ $\quad d = 6.186$

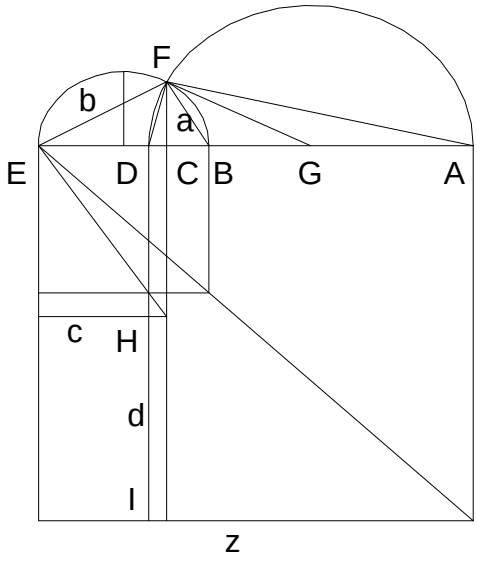
$$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

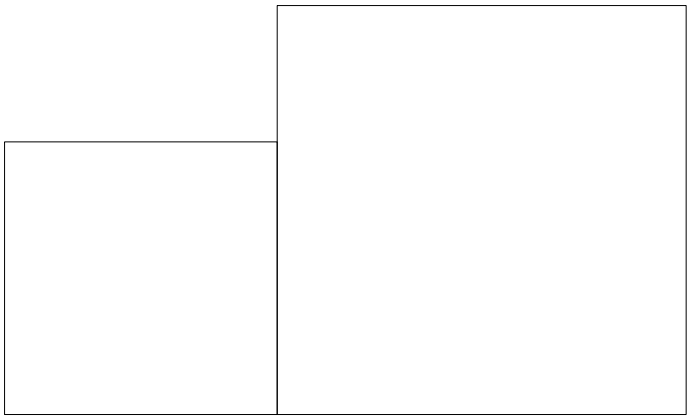
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

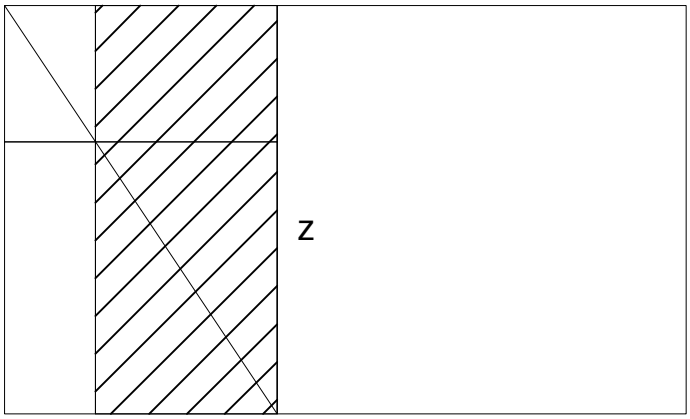
$$(c\cdot d)-p=0$$

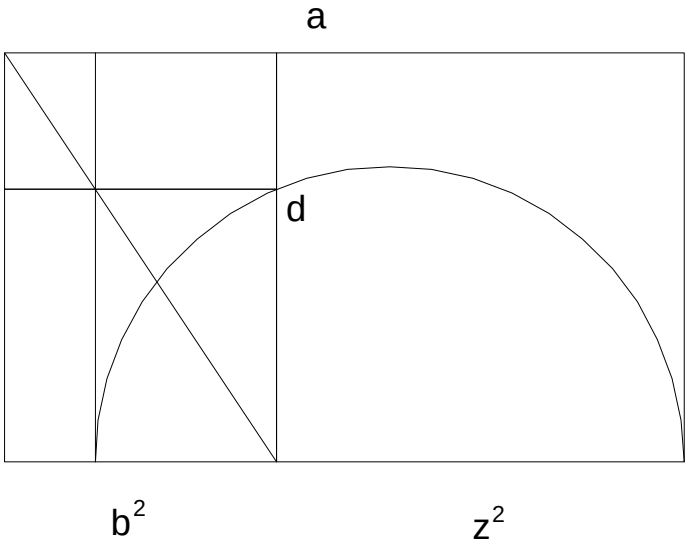
$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for z below.

$$\begin{bmatrix} \frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}} \end{bmatrix}$$




12_18_95.MCD

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.

$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

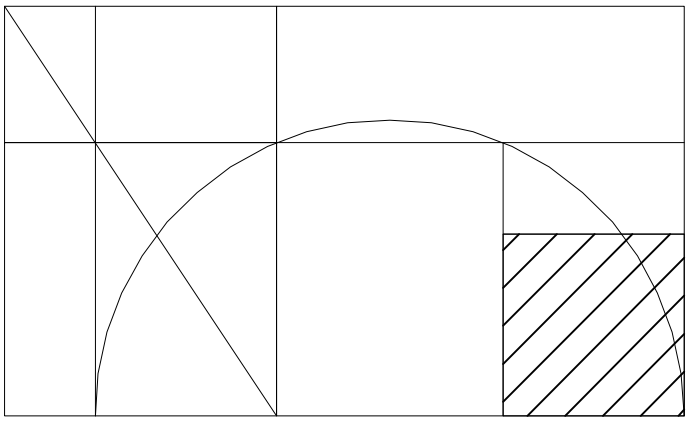
$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

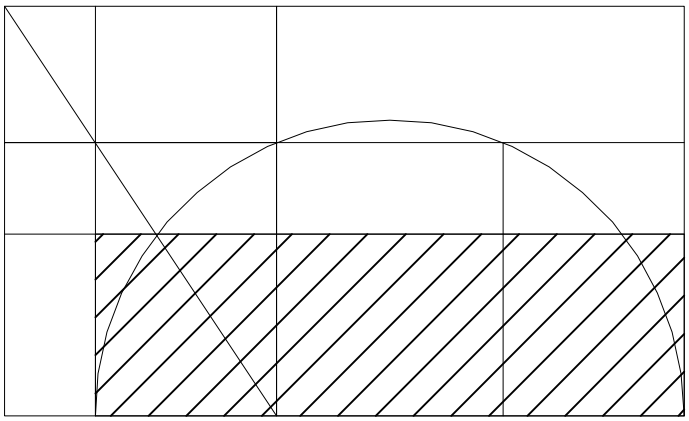
$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

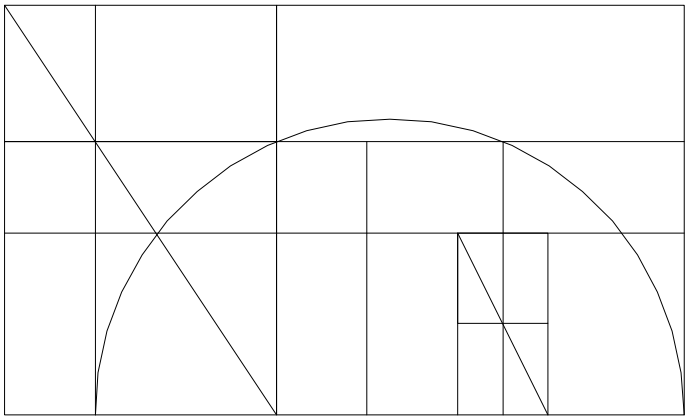
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



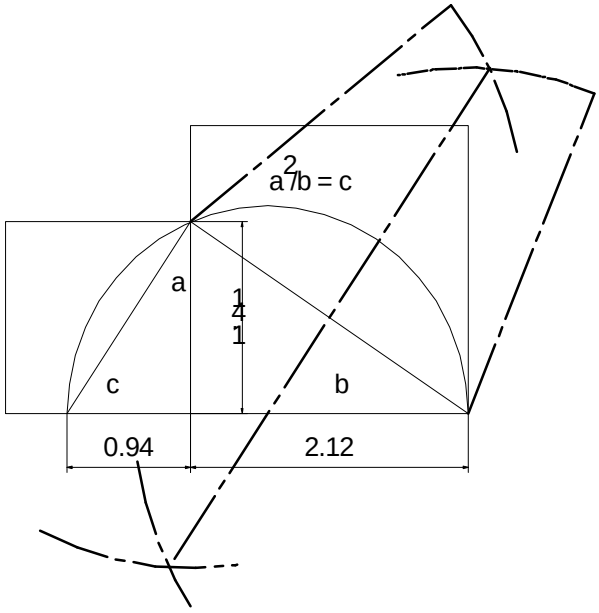
$b^2$ $z^2$



$b^2$ $z^2$

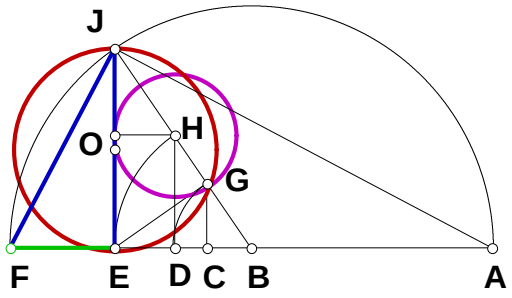


$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.

# 12/20/95 Just For Fun




$N := 2$

$EF := 1 \quad EJ := EF \cdot N \quad AE := \frac{EJ^2}{EF} \quad AF := AE + EF$

$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF \quad BH := BE$

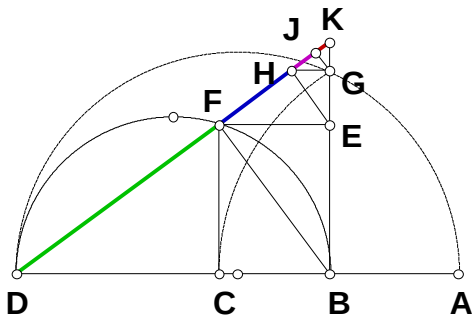
$BJ := \sqrt{EJ^2 + BE^2} \quad BD := \frac{BE \cdot BH}{BJ} \quad BG := BD$

$BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD \quad HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \quad \frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1 \quad \left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

$$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2+1\right)^2} = 0$$

# 12/21/95 Pascal's Triangle With Exponential Division




$N := 7$

$AB := 1 \qquad AD := AB \cdot N \qquad AC := \sqrt{AB \cdot AD}$

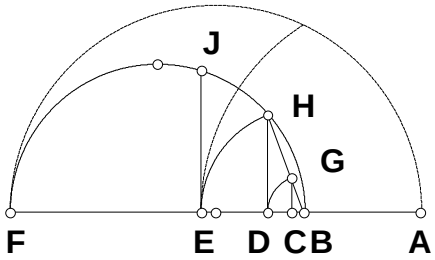
$BD := AD - AB \qquad CD := AD - AC \qquad BC := BD - CD$

$CF := \sqrt{BC \cdot CD} \qquad CF := \sqrt{BC \cdot CD} \qquad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \qquad FK := \frac{DK \cdot BC}{BD} \qquad HK := \frac{FK \cdot FK}{DK} \qquad JK := \frac{HK \cdot HK}{FK}$

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{DK}{JK} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

## A more civil figure.

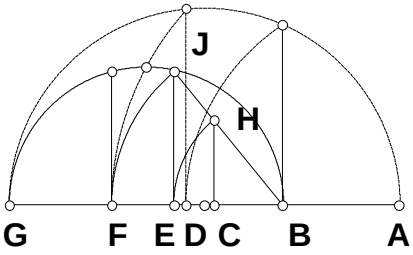



$AB := 1 \qquad AF := N \qquad BF := AF - AB \qquad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \qquad BH := BE \qquad BD := \frac{BE \cdot BH}{BF} \qquad BG := BD$

$BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$




$AB := 1 \qquad AG := N \qquad BG := AG - AB \qquad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}}$

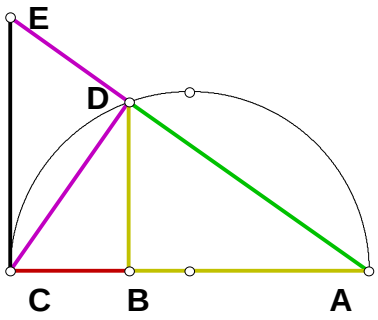
$BF := AF - AB \qquad BJ := BF \qquad BE := \frac{BJ \cdot BF}{BG} \qquad BH := BE$

$BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^2 - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0 \qquad \frac{BG}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$

## 12/29/95

**Given AC and CE find BC when it is equal to** $\frac{AC \cdot CE^2}{AC^2 + CE^2}$.




$N_1 := 1.854 \quad N_2 := 1.313$

$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$

$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$

$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \qquad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \qquad AB - \left( N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} \right) = 0$

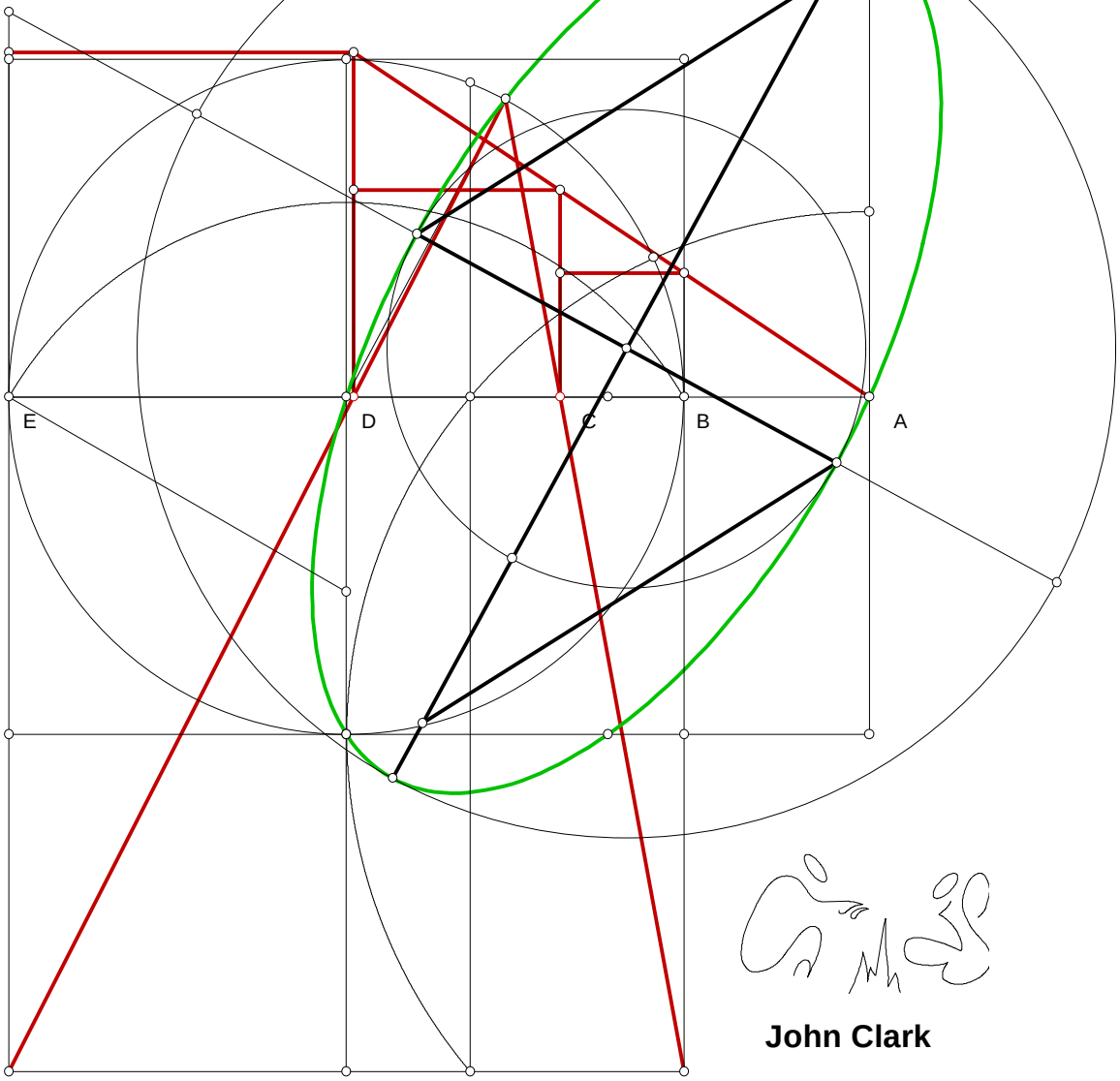
$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \qquad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

# The Delian Quest

## 1996

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$



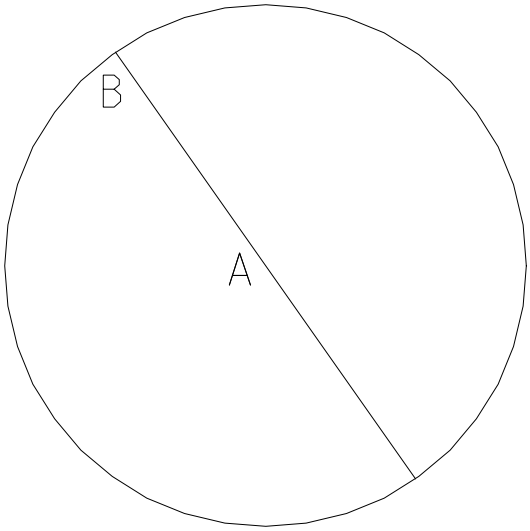



John Clark

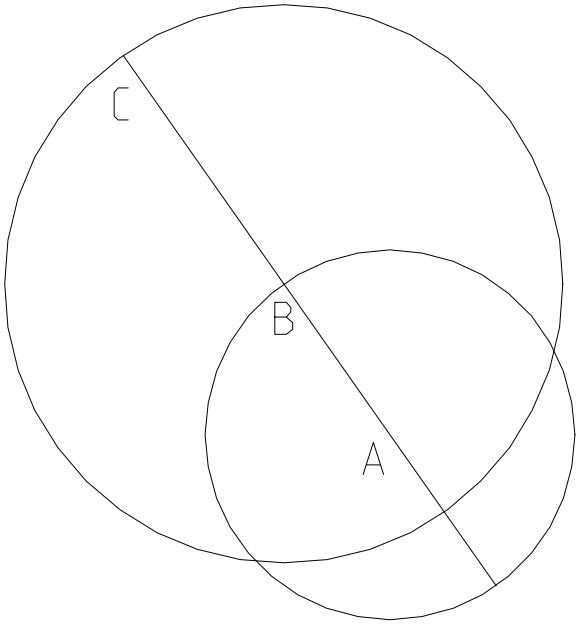
The Euclidean proof of 11_11_93.MCD may be reminiscent of trimming hedges with a jack knife, but the method is for exercise of those methodical parts which comprise it. I can never get too much of those practices. There is however a golden approach to proofing the figure which has almost no regard for the practices of basic moves- a eunuch in regards to teaching, but whose simplicity implants the concepts of the figure with a clarity unrivaled by more energetic methods.

## The Archamedian Paper Trisector- Without the Numbers.

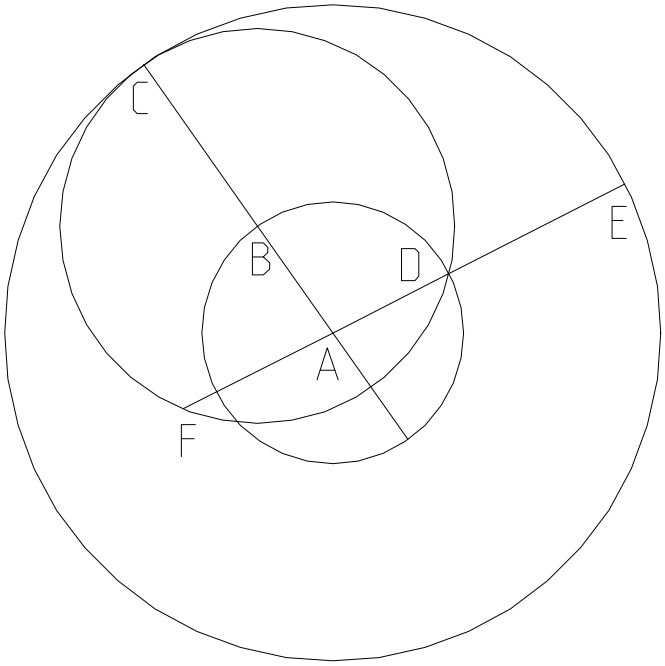
One of the distinctions that this and the paper of 11_11_93.MCD bring to the subject is that the construction of the figure is not assumed, but done.




Given any circle AB

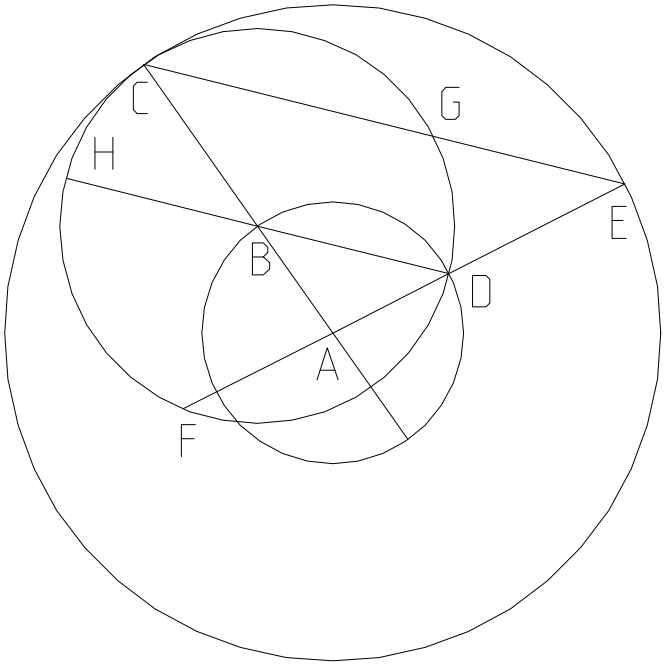



Given any circle BC such that BC≤ 2AB.

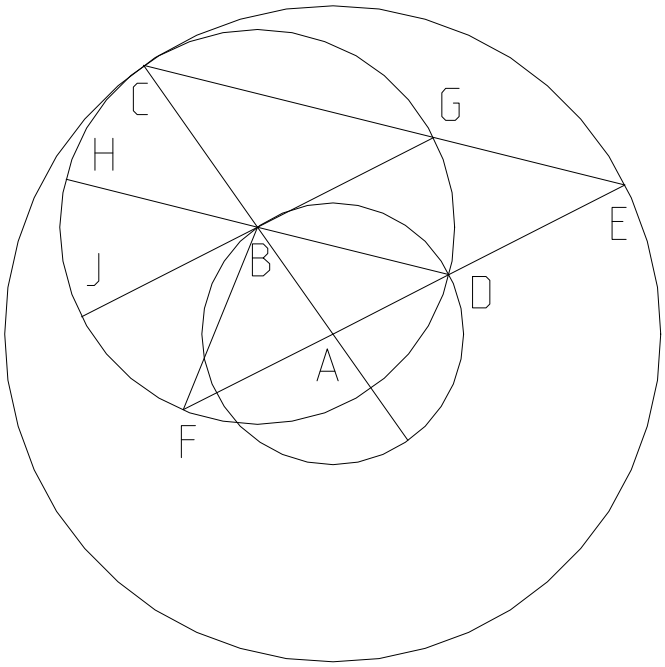



Construct AE such that AE = AC.

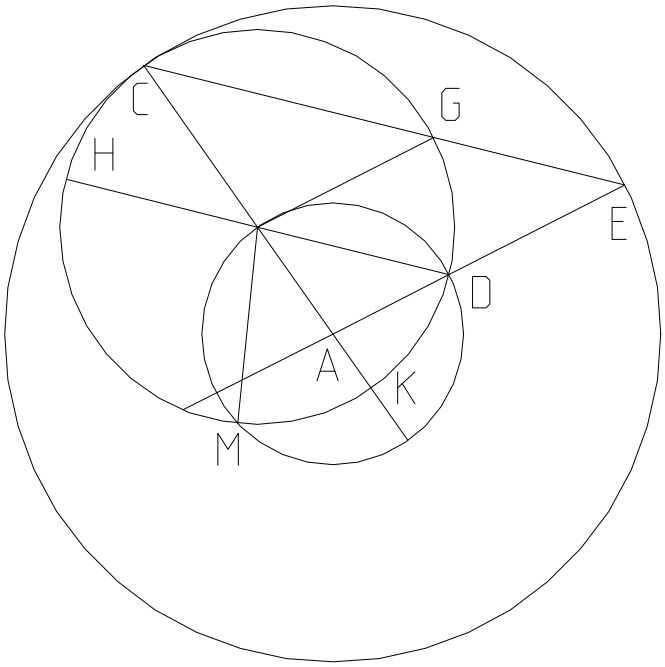
Since AC = AB+ BC and AD = AB, DE = BC.




Construct DH parallel to BD. Construct CE. Since AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD. From here one can take two paths.



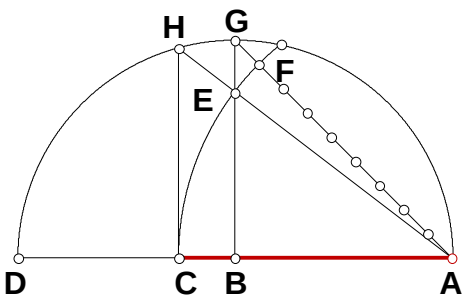

Construct GJ parallel to EF. Now Since CE is parallel to DH, DG = CH. Since GJ is parallel to EF, DG = FJ. Since $\angle$ HBJ is opposite and equal to $\angle$ GBD, DG = HJ, therefore DG is $\frac{1}{3}$ CF.
Since CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.




By construction DK = KM. Since DH is parallel to CE, CH = DG. Since DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.
But like I said at the start, there is no real work in this figure.

I have given two constructions for the figure, I cannot understand why sliding paper is still used to demonstrate it. The figure adds a few moves to Euclid's figure for demonstrating that the angle from the circumference is half the angle from the center of the circle.

## 01/07/96 Rusty Cubes




**A rusty Compass construction for the duplication of the cube.**

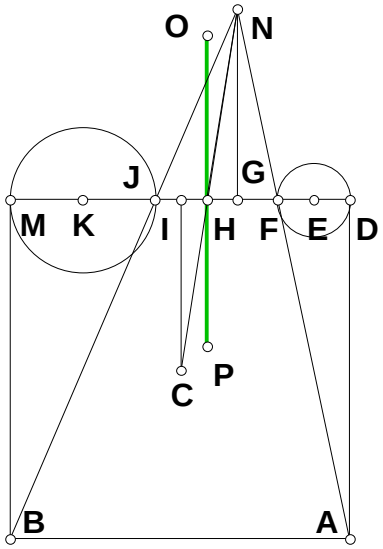
$AD := 2$ $AB := \frac{AD}{2}$ $AG := \sqrt{2 \cdot AB^2}$ $AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26$ $\frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

**I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.**

# 01/08/96 Alternate Method Power Line




$R_1 := 2 \qquad R_2 := 5 \qquad D := 4$

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + JK) \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \qquad GJ := FJ - FG \qquad DI := \frac{DM}{2}$

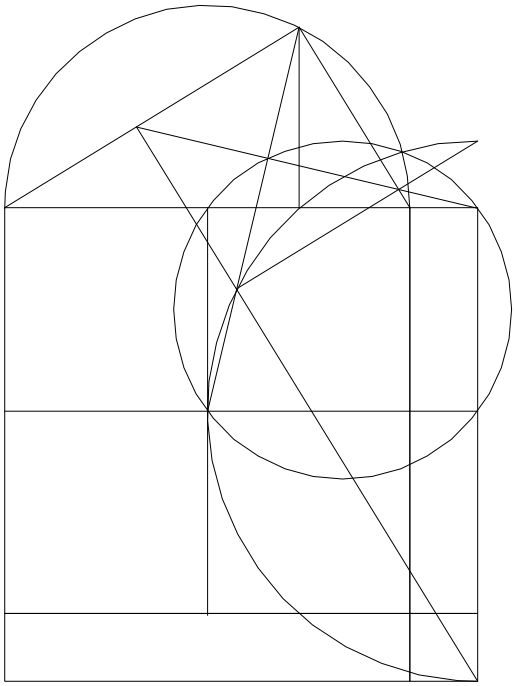
$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

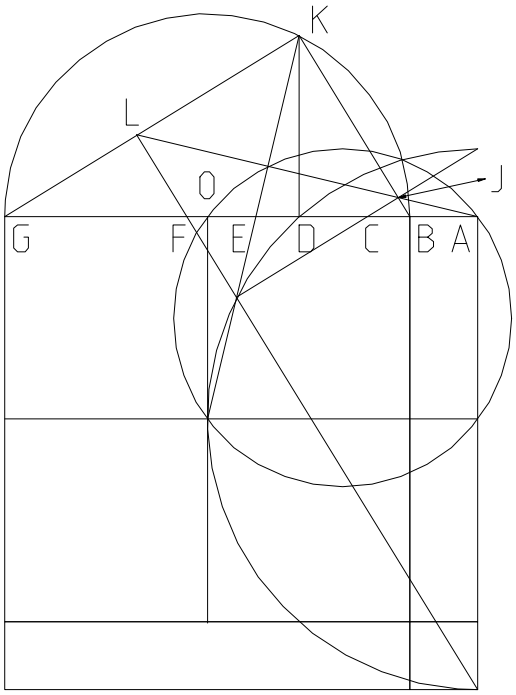
$HM := DM - DH$

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$



The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.




$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$

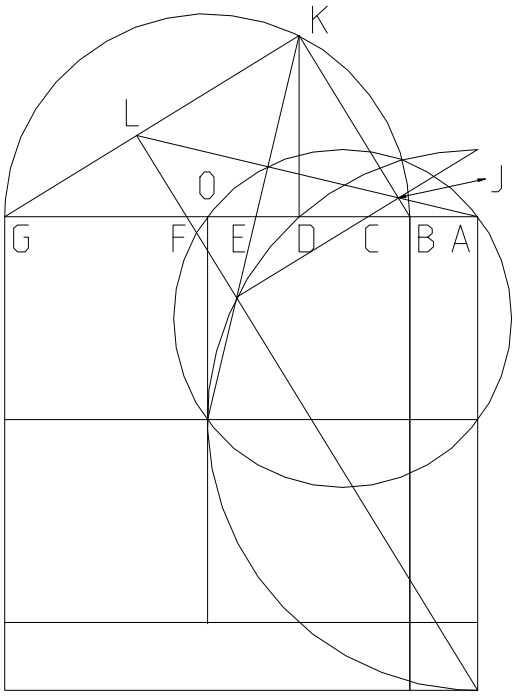
$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



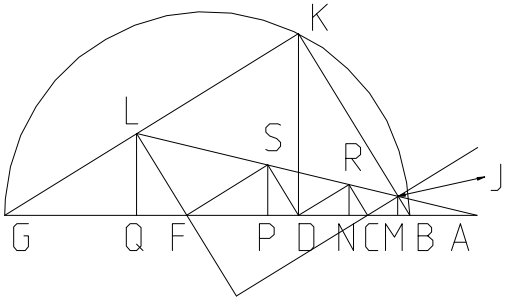

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

Plug in AG here. AB will become "1".

$N \equiv 5$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} = 2.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 8.075 \qquad \frac{BK}{BJ} = 8.075 \qquad N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 8.075$$




$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG}$

$CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$$\left(\frac{AG}{AB}\right)^{\frac{5}{4}} + \frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} = 26.132 \qquad \frac{BG}{BM} = 26.132$$

$$N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} = 26.132$$

$$\frac{AG}{AB} + \left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} = 17.475 \qquad \frac{BG}{CN} = 17.475$$

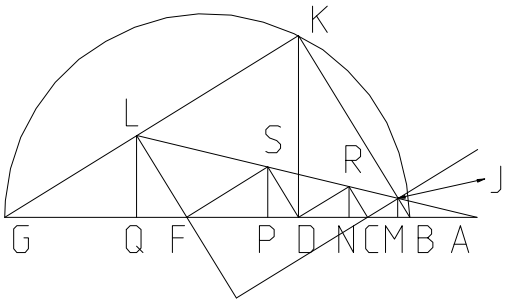
$$N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} = 17.475$$

$$\left(\frac{AG}{AB}\right)^{\frac{3}{4}} + \left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} = 11.686 \quad \frac{BG}{DP} = 11.686$$

$$N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} = 11.686$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{4}} + \left(\frac{AG}{AB}\right)^{\frac{1}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AG}{AB}\right)^{\frac{0}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{1}{4}} + \left(\frac{AB}{AG}\right)^{\frac{2}{4}} + \left(\frac{AB}{AG}\right)^{\frac{3}{4}} = 7.815 \quad \frac{BG}{FQ} = 7.815$$

$$N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{1}{4}}} + \frac{1}{N^{\frac{2}{4}}} + \frac{1}{N^{\frac{3}{4}}} = 7.815$$



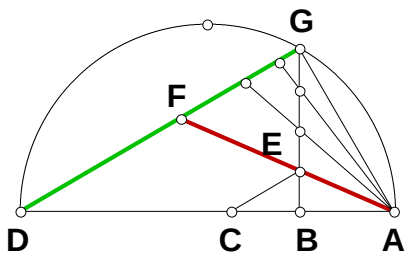

$$\frac{AG^{\frac{6}{4}} + AG^{\frac{4}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{5}{4}} - AB^{\frac{6}{4}}} = 32.665 \qquad \frac{AG}{BM} = 32.665 \qquad \frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 32.665$$

$$\frac{AG^{\frac{5}{4}} + AG^{\frac{3}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{4}{4}} - AB^{\frac{5}{4}}} = 21.844 \qquad \frac{AG}{CN} = 21.844 \qquad \frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 21.844$$

$$\frac{AG^{\frac{4}{4}} + AG^{\frac{2}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{3}{4}} - AB^{\frac{4}{4}}} = 14.608 \qquad \frac{AG}{DP} = 14.608 \qquad \frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 14.608$$

$$\frac{AG^{\frac{3}{4}} + AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}} \cdot AB^{\frac{2}{4}} - AB^{\frac{3}{4}}} = 9.769 \qquad \frac{AG}{FQ} = 9.769 \qquad \frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} = 9.769$$

# 01/13/96 Pyramid of Ratios VI, Moving the Point




$N_1 := 5 \quad N_2 := 7 \quad N_1 := N_1 \quad N_2 := N_2$

$AD := 1 \quad AB := \frac{AD}{N_1} \quad BD := AD - AB$

$BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG}{N_2} \quad BC := \frac{BD \cdot BE}{BG}$

$AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \quad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0$

$\Delta := 2 .. N_1 \quad \delta := 2 .. N_2$

$SeriesAF_{\Delta,\delta} := \frac{\Delta \cdot \delta}{(\Delta - 1) \cdot (\delta - 1)}$

$SeriesAF = \begin{bmatrix} 4 & 3 & 2.667 & 2.5 & 2.4 & 2.333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.667 & 2 & 1.778 & 1.667 & 1.6 & 1.556 \\ 2.5 & 1.875 & 1.667 & 1.563 & 1.5 & 1.458 \end{bmatrix}$

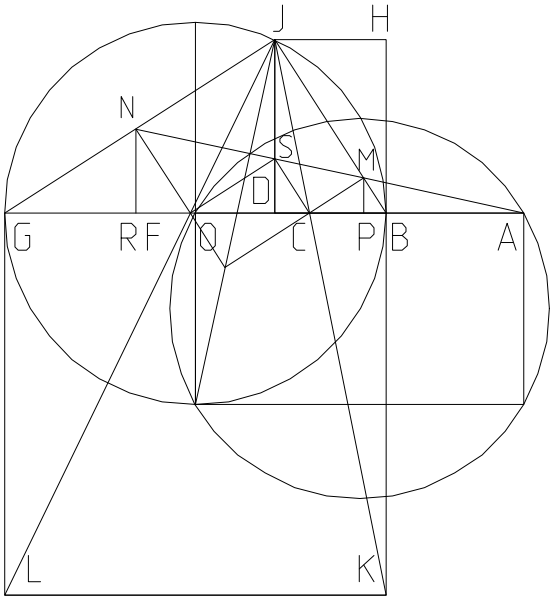
$DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2} \quad DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF$

$\frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$

$SeriesDG_{\Delta,\delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)}$

$SeriesDG = \begin{bmatrix} 3 & 2 & 1.667 & 1.5 & 1.4 & 1.333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333 & 2 & 1.8 & 1.667 \\ 6 & 3.5 & 2.667 & 2.25 & 2 & 1.833 \end{bmatrix}$

The figure cuts the base in Cube Roots and provides some interesting ratios.




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

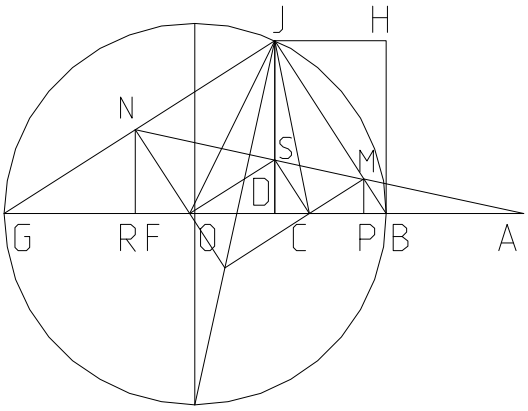
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \qquad BP := \frac{BD \cdot BC}{BG} \qquad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

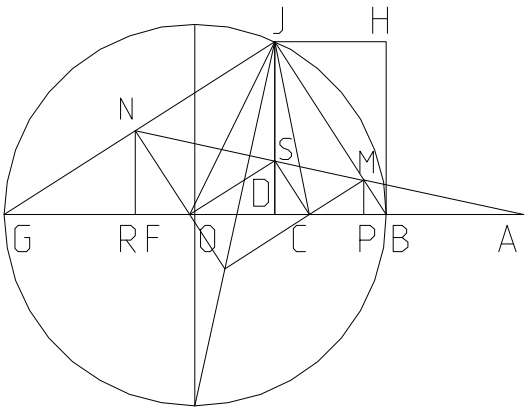
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$

$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476$$



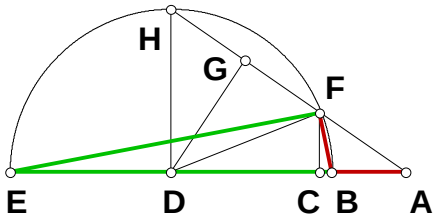

$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$

# 01/17/96 A Right Triangle In A Given Ratio

**Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.**




$N := 3$

$AB := 1 \quad AE := AB \cdot N \quad BE := AE - AB$

$BD := \frac{BE}{2} \quad DF := BD \quad DE := BD \quad AD := AB + BD$

$DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$S_1 := AD \quad S_2 := AF \quad S_3 := DF$

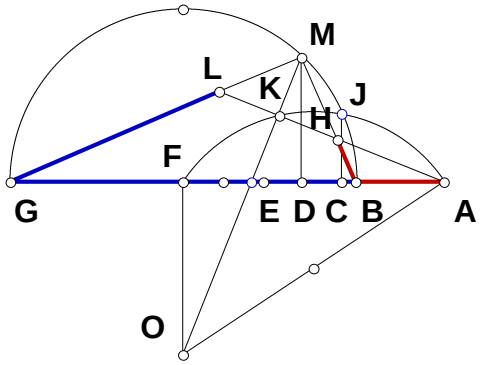
$CD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad BC := BD - CD$

$CE := CD + DE \quad CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2} \quad AC := AD - CD$

$\frac{AE}{AB} - \frac{EF}{BF} = 0 \qquad AC - \frac{N^2 + N}{N^2 + 1} = 0 \qquad BF - \frac{N - 1}{\sqrt{N^2 + 1}} = 0 \qquad EF - \frac{N^2 - N}{\sqrt{N^2 + 1}} = 0$

# 01/17/96B Divide The Sides Of A Triangle In A Given Ratio

**Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.**




$N_1 := 5 \qquad N_2 := 9$

$AB := 1 \qquad AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \frac{BG}{2}$

$AC := \frac{N_1^2 + N_1}{N_1^2 + 1} \qquad AF := AB + BF \quad CF := AF - AC$

$BC := AC - AB \quad BD := \frac{CF}{N_2} + BC \qquad DF := BF - BD$

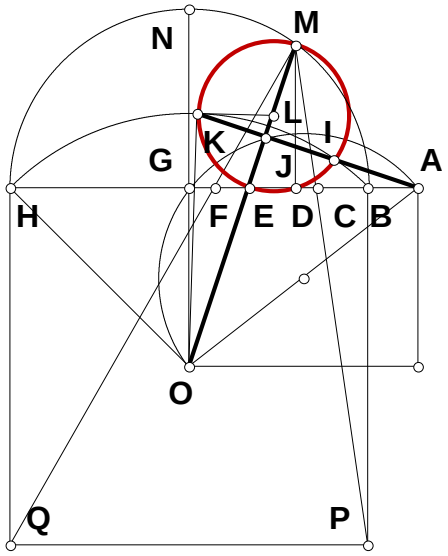
$DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF \quad EF := \frac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \frac{EO \cdot (DM + FO)}{FO} \quad EK := \frac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO) HK := KM \quad HM := \sqrt{2 \cdot KM^2}$

$BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM \; GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM$

$\frac{AG}{AB} - \frac{GL}{BH} = 0$

# 01/21/96 More On Cube Roots




$N := 5$ $BH := 1$

$BP := BH$ $HQ := BH$

$BG := \frac{BH}{2}$ $GO := BG$ $GN := BG$ $NO := BH$ $GH := BG$

$BE := \frac{BG}{N}$ $EG := BG - BE$ $EO := \sqrt{EG^2 + GO^2}$

$MO := \frac{GO \cdot NO}{EO}$ $EM := MO - EO$ $EL := \frac{EM}{2}$ $LK := EL$

$LO := EO + EL$ $LJ := \frac{LK^2}{LO}$ $EJ := EL - LJ$

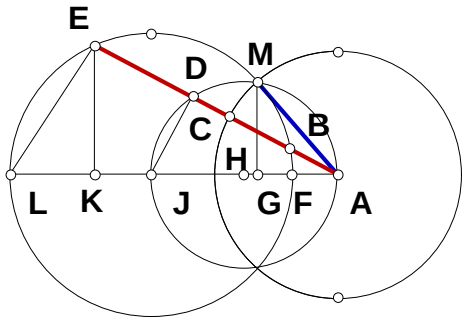
$AE := \frac{EO \cdot EJ}{EG}$ $AH := AE + EG + GH$ $AB := AH - BH$

$DE := \frac{EG \cdot EM}{EO}$ $DM := \frac{GO \cdot EM}{EO}$ $BD := BG - (EG + DE)$

$BC := \frac{BD \cdot BP}{BP + DM}$ $DH := BH - BD$ $DF := \frac{DH \cdot DM}{DM + HQ}$ $AC := AB + BC$ $AF := AB + BD + DF$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$

# 01/22/96 Trivial Method Square Root




**For any E between M and L, AM is the square root of AB x AE.**

$N_1 := 6 \qquad N_2 := 3$

$AF := 1 \quad AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

$GL := AL - AG \quad GK := \frac{GL}{N_2} \qquad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

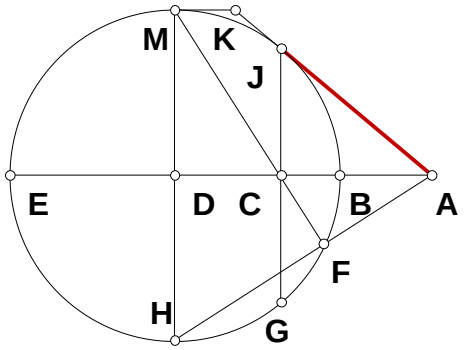
$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

# 01/24/96 Tangent

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**




$N := 5$ $AB := 1$ $AE := AB \cdot N$

$BE := AE - AB$ $BD := \frac{BE}{2}$ $AD := AB + BD$

$DM := BD$ $DH := BD$ $CD := \frac{DH \cdot DM}{AD}$

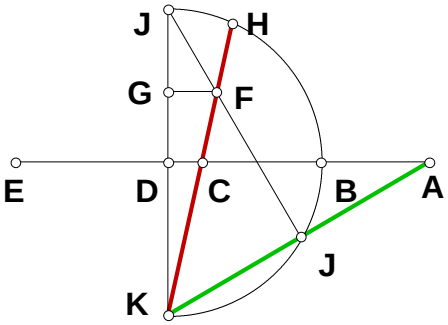
$BC := BD - CD$ $CE := BE - BC$

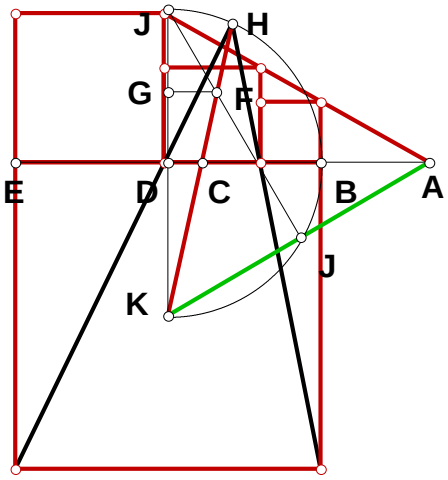
$CJ := \sqrt{BC \cdot CE}$ $AJ := \sqrt{(AB + BC)^2 + CJ^2}$

$AJ - \sqrt{N} = 0$

$AC := AB + BC$ $2 \cdot AB \cdot \frac{N}{(1 + N)} - AC = 0$

## 01/25/96 On Cubes




**Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.**

$N := 9 \qquad BE := 1$

$BD := \frac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \frac{BD}{N} \qquad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$

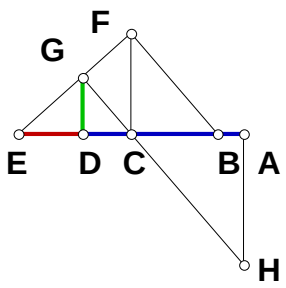
$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

$AB := AE - BE$

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

## 01/29/96 Linear division $\frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)}$




$N_1 := 3 \qquad N_2 := 9 \qquad AE := 1$

$AH := AE \cdot N_1 \quad AC := \frac{AE}{2} \quad CF := AE \cdot N_2 \quad BC := \frac{AC \cdot CF}{AH}$

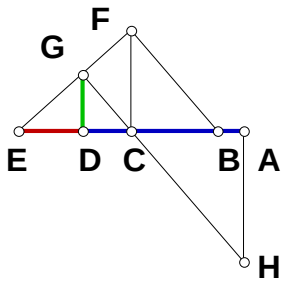
$CE := AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{AH \cdot CD}{AC}$

$DE - \frac{N_1}{2 \cdot (N_1 + N_2)} = 0 \qquad AD - \frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)} = 0 \qquad CD - \frac{N_2}{2 \cdot (N_1 + N_2)} = 0$

$DG - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

## Linear division $\frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)}$




$N_1 := 4 \qquad N_2 := 4 \qquad N_3 := 2 \qquad AE := 1$

$AH := AE \cdot N_1 \quad AC := \frac{AE}{N_2} \quad CF := AE \cdot N_3 \quad BC := \frac{AC \cdot CF}{AH}$

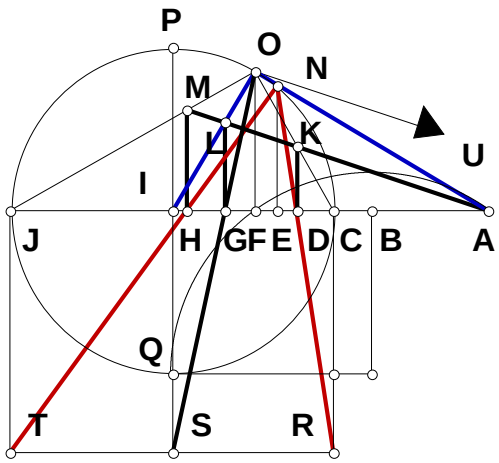
$CE := AE - AC \; BE := CE + BC \; CD := \frac{BC \cdot CE}{BE} \; DE := CE - CD$

$AD := AC + CD \quad DG := \frac{CF \cdot CE}{BE}$

$DE - AE \cdot \frac{N_1 \cdot (N_2 - 1)^2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0 \qquad AD - AE \cdot \frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

$DG - AE \cdot \frac{N_1 \cdot N_3 \cdot (N_2 - 1)}{N_1 \cdot N_2 - N_1 + N_3} = 0 \qquad CD - AE \cdot \frac{N_3 \cdot (N_2 - 1)}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

# 01/31/96 On Gemini Roots




## Hitting AO from any RT while maintaining Gemini Roots.

$N_1 := 5 \qquad N_2 := 3 \qquad BC := 1$

$BJ := BC \cdot N_1 \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF$

$FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2 \quad HS := CR$

$FI := FJ - IJ \quad FG := \frac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \frac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \frac{HS \cdot GO}{OS} \quad FU := \frac{AG \cdot FO}{GL}$

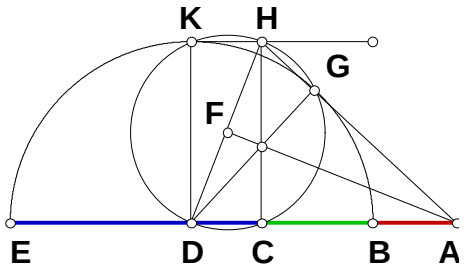
$AH := \frac{FU \cdot AJ}{FU + FJ} \quad DK := \frac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \frac{AG \cdot DK}{GL} \quad AC := AF - CF \quad CD := AD - AC$

$CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \frac{CR \cdot DH}{CD + HJ} \quad CE := \frac{CD \cdot (CR + EN)}{CR}$

$AE := AC + CE \quad \frac{AF}{FO} - \frac{AE}{EN} = 0 \quad BD := BC + CD \quad BH := BC + CH \quad \sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$

## 02/02/96 Find A Segment

**Given BE and BC such that $\sqrt{(AB + BE)\cdot AB}$ = AB + BC, find AB.**

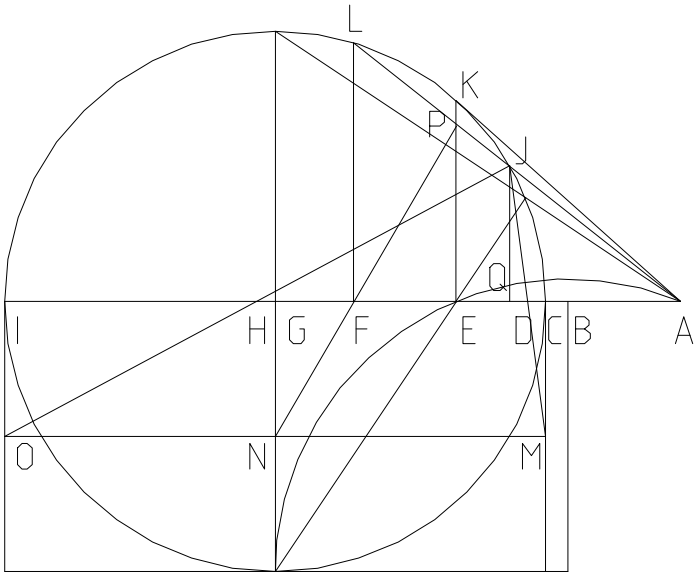



$N_1 := 5$ $N_1 > 2 = 1$

$BC := 1$ $BE := BC \cdot N_1$ $BD := \frac{BE}{2}$ $CD := BD - BC$

$CH := BD$ $DH := \sqrt{CD^2 + CH^2}$ $DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD}$ $AB := AD - BD$ $AB - \frac{1}{(N_1 - 2)} = 0$ $\sqrt{(AB + BE)\cdot AB} - (AB + BC) = 0$




Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$

$\Delta := 40 \quad \delta := 0 .. \Delta$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$

$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$

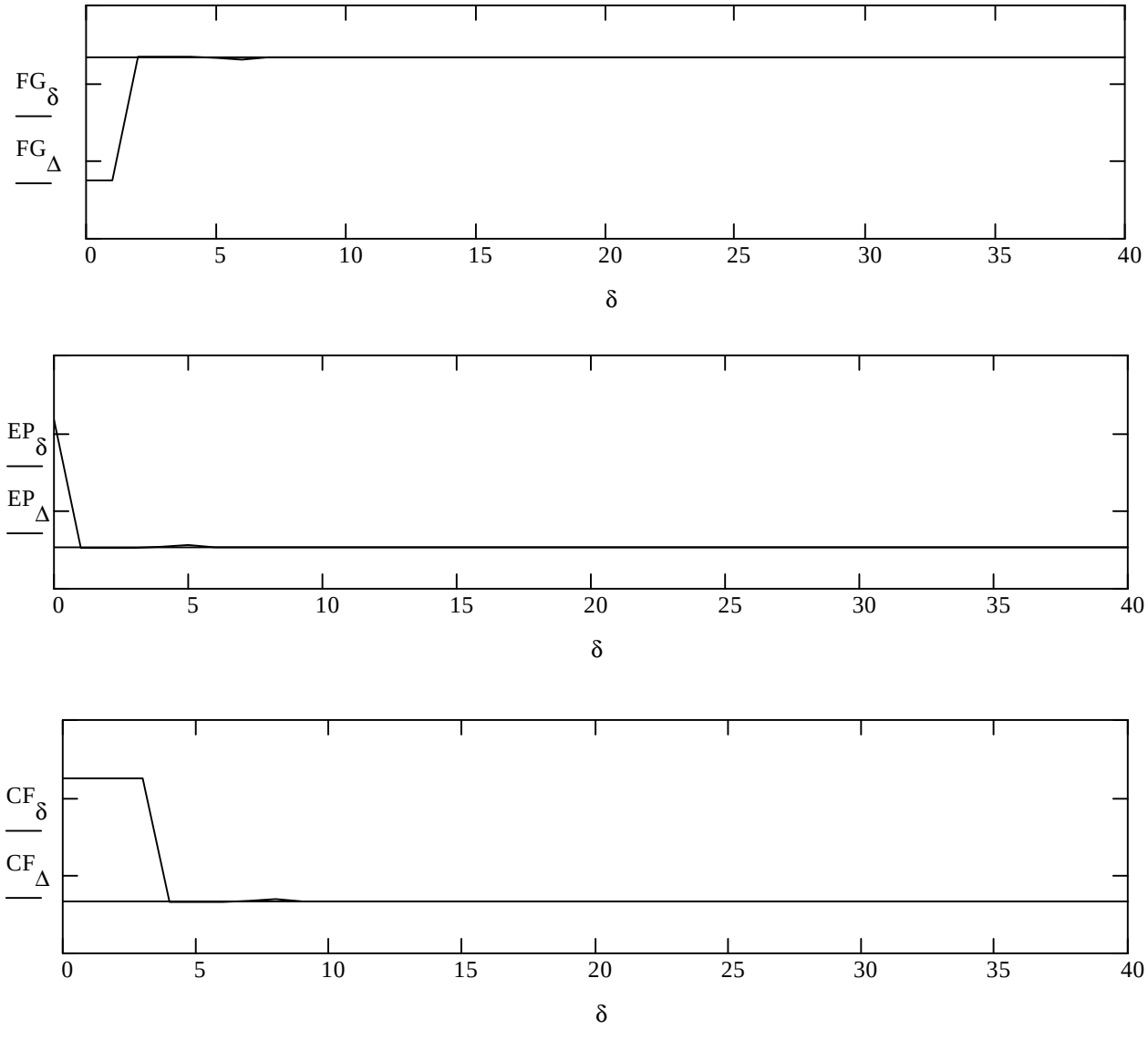
$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$

The next two equations are for the Delian Problem only.

$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

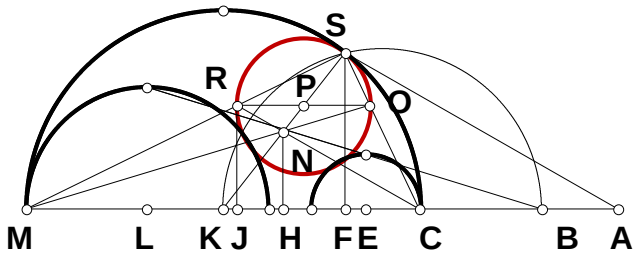
$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.

## 04/14/96 Method for Unequals

**Given three circles in the said configuration, find the fourth. I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."**




$N_1 := 7$ $N_2 := 3$

$CM := 1$ $CK := \frac{CM}{2}$ $CE := \frac{CM}{N_1}$

$LM := \frac{CM}{N_2}$ $EL := CM - (CE + LM)$

$BL := \frac{EL \cdot LM}{LM - CE}$ $BM := BL + LM$ $BC := BM - CM$ $BK := \frac{CM}{2} + BC$

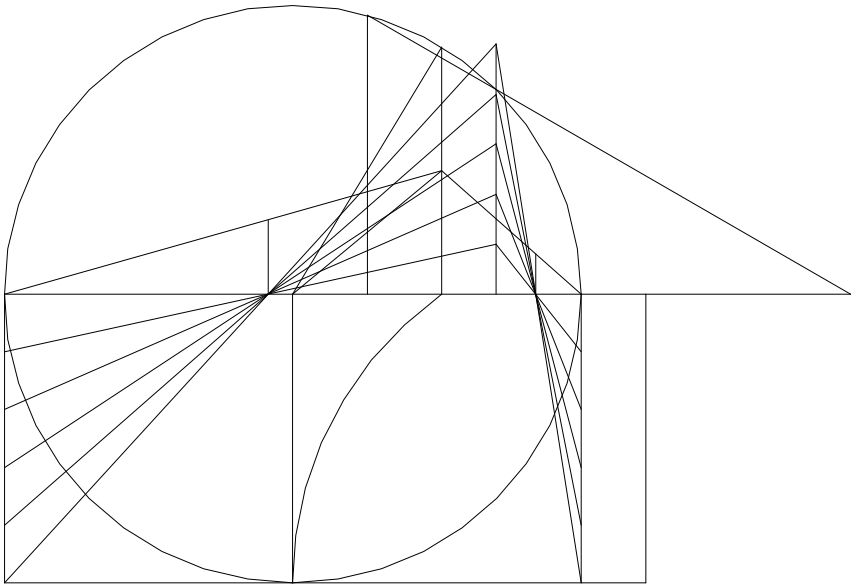
$R_1 := LM$ $R_2 := CE$ $D := EL$ $KS := CK$ $EH := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$

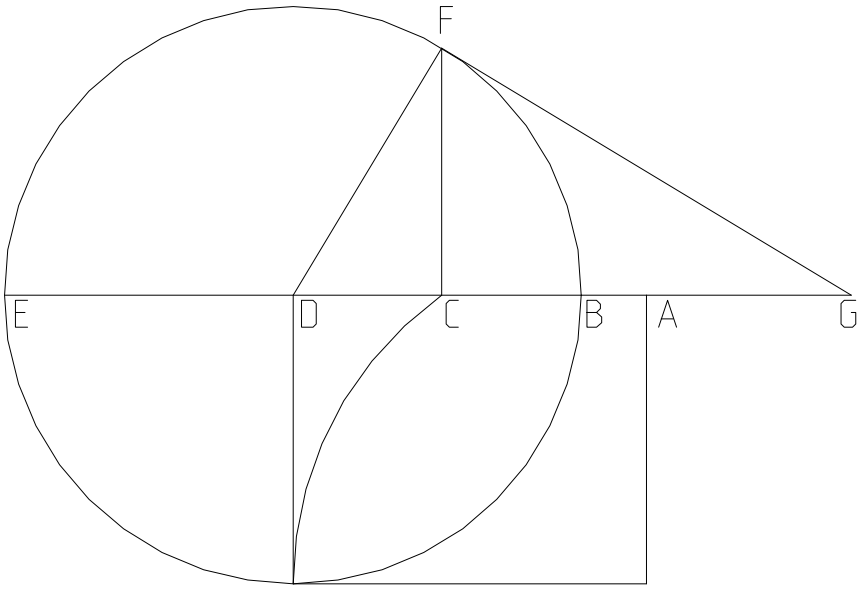
$FK := \frac{KS^2}{BK}$ $CF := CK - FK$ $FM := CM - CF$ $FS := \sqrt{CF \cdot FM}$

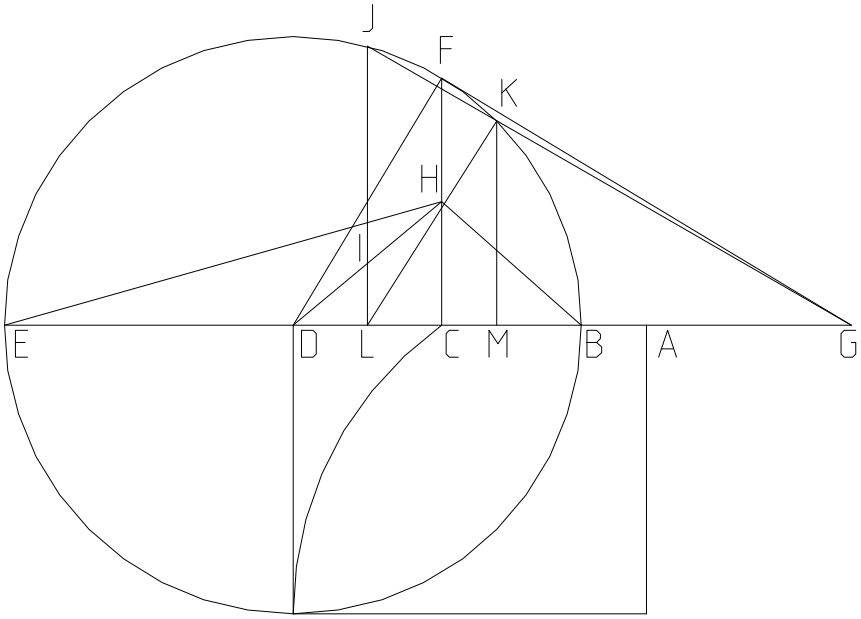
$HK := CK - (CE + EH)$ $CH := CK - HK$ $HN := \frac{FS \cdot HK}{FK}$ $AF := \frac{CH \cdot FS}{HN}$ $JR := \frac{FS \cdot CM}{AF + FM}$

$RO := \frac{CM \cdot (FS - JR)}{FS}$ $PS := \frac{RO}{2}$ $PS - \frac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$

# On Gemini Roots






$AB := 1 \quad BE := 10 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

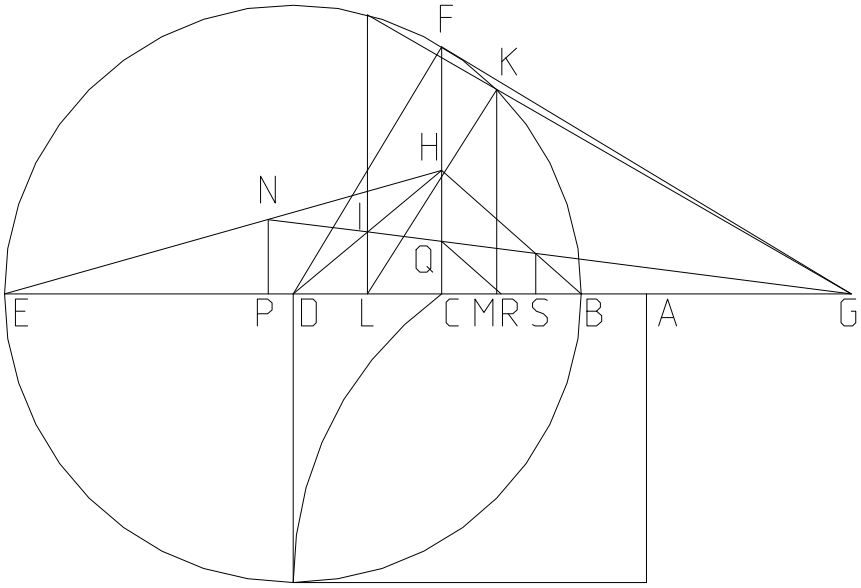
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \quad BM := GM - BG$

$EM := BE - BM$

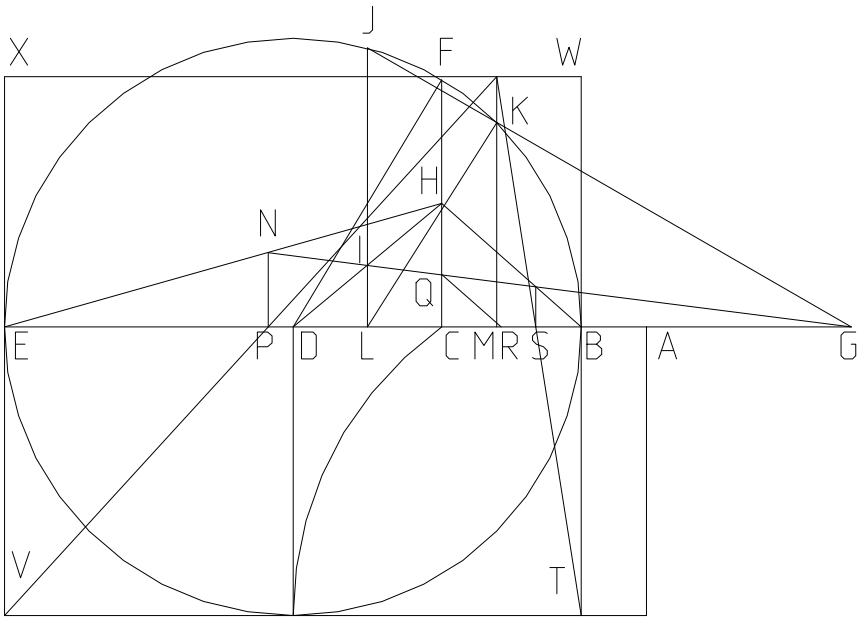



$$IL := \sqrt{DI^2 - DL^2} \quad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \quad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \quad CR := \frac{BC \cdot CQ}{CH}$$

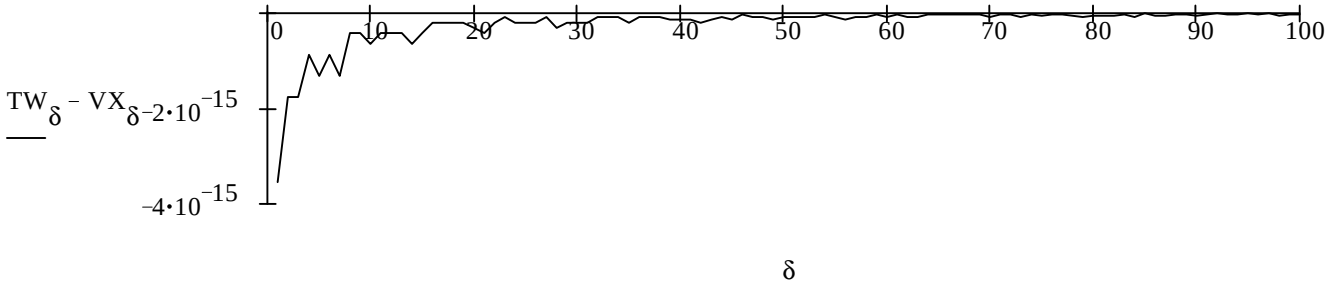
$$GR := CG - CR \quad BS := \frac{CR \cdot BG}{GR}$$



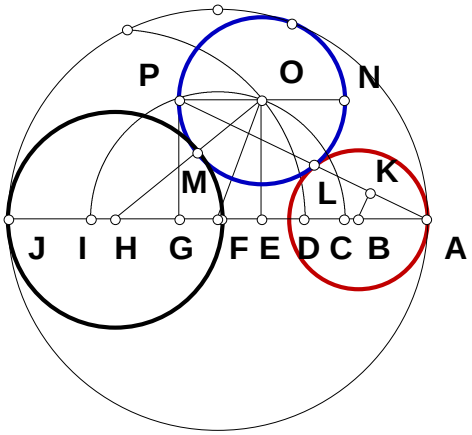

$$\delta := 1..100$$

$$E_\delta := \frac{BE}{\delta} \quad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \quad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$



## 04/16/96 Given Three Radii




**Given AJ, HJ and NO find AB such that AB is collinear with AJ and HJ.**

$N_1 := 4 \qquad N_2 := 16$

$AJ := 1 \qquad AF := \frac{AJ}{2} \qquad HJ := \frac{AJ}{N_1} \qquad NO := \frac{AJ}{N_2}$

$HM := HJ \quad MO := NO \quad HO := HM + MO$

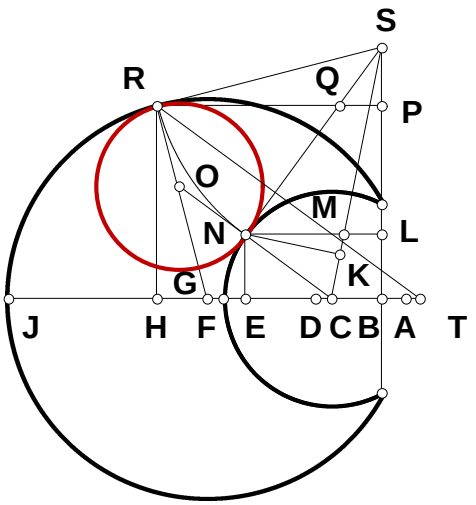
$FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$S_1 := FH \quad S_2 := FO \quad S_3 := HO \quad EH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO \quad AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP}$

$AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG} \qquad AB - \frac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot (N_2 \cdot N_1 - 2 \cdot N_2 - 4)} = 0$

# 04/17/96 A Circle In A Crescent




$N_1 := 2$ $N_2 := 2$ $N_3 := 3$

$AJ := 1$ $AG := \frac{AJ}{2}$ $FR := AG$ $AF := \frac{AJ}{N_1}$

$AD := \frac{AF}{2}$ $AC := \frac{AD}{N_2}$ $CF := AF - AC$

$CG := AG - AC$ $R_1 := AG$ $R_2 := CF$ $D := CG$

$BC := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$ $AB := AC + BC$

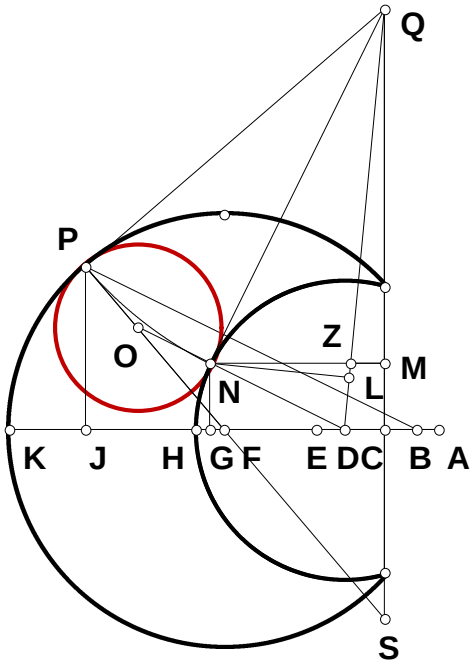
$BJ := AJ - AB$ $BH := \frac{BJ}{N_3}$ $AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{FR^2 - GH^2}$

$AP := HR$ $PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := AP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$ $KN := \sqrt{CN^2 - CK^2}$ $KM := \frac{BC \cdot KN}{BS}$

$SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$ $BL := BS - SL$ $EN := BL$ $CE := \sqrt{CN^2 - EN^2}$

$HT := \frac{CE \cdot HR}{EN}$ $GT := HT - GH$ $GO := \frac{FR \cdot CG}{GT}$ $OR := FR - GO$ $OR = 0.125$




# 04/17/96b A Circle In A Crescent

$N_1 := 1.7 \quad N_2 := 2 \quad N_3 := 1.3$

$AK := 1 \quad AF := \frac{AK}{2} \quad FP := AF \quad FK := AF$

$AH := \frac{AK}{N_1} \quad AE := \frac{AH}{2} \quad EH := AE \quad AD := \frac{AE}{N_2}$

$DE := AE - AD \quad DH := EH + DE \quad DN := DH$

$DF := AF - AD \quad R_1 := AF \quad R_2 := DH \quad D := DF$

$CD := \frac{R_2^2 + D^2 - R_1^2}{2 \cdot D} \quad AC := AD + CD$

$CK := AK - AC \quad CJ := \frac{CK}{N_3} \quad CF := CK - FK$

$FJ := CJ - CF \quad JP := \sqrt{FP^2 - FJ^2} \quad FS := \frac{FP \cdot CF}{FJ} \quad PS := FS + FP \quad QS := \frac{FP \cdot PS}{JP} \quad PQ := \frac{FJ \cdot QS}{FP}$

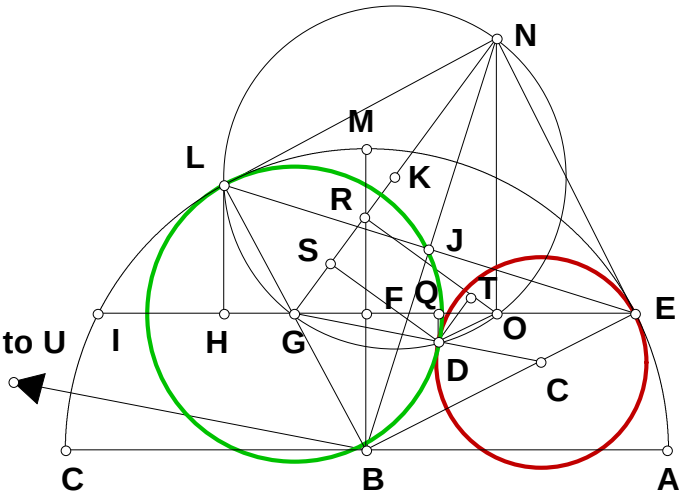
$CS := \frac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad NQ := PQ \quad DQ := \sqrt{NQ^2 + DN^2} \quad DL := \frac{DN^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2}$

$LZ := \frac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ \quad MQ := \frac{CQ \cdot QZ}{DQ} \quad CM := CQ - MQ \quad GN := CM$

$CG := \sqrt{DN^2 - GN^2} \quad BJ := \frac{CG \cdot JP}{GN} \quad BF := BJ - FJ \quad FO := \frac{FP \cdot DF}{BF} \quad OP := FP - FO \quad OP = 0.175$

**04/22/96**

## Given BF as a ratio to BM and EG as a ratio to EI, what is CE?




$N_1 := 2.5$ $N_2 := 1.5$ $AC := 1$

$AB := \frac{AC}{2}$ $BM := AB$ $BE := AB$

$BL := AB$ $BF := \frac{BM}{N_1}$

$EF := \sqrt{BE^2 - BF^2}$ $EI := 2 \cdot EF$

$EG := \frac{EI}{N_2}$ $FG := EG - EF$

$BG := \sqrt{BF^2 + FG^2}$ $GL := BL - BG$ $DG := GL$ $GH := \frac{FG \cdot GL}{BG}$ $HL := \sqrt{GL^2 - GH^2}$ $EH := EG + GH$

$EL := \sqrt{EH^2 + HI}$ $JL := \frac{EL}{2}$ $BJ := \sqrt{BL^2 - JL^2}$ $LN := \frac{BL \cdot JL}{BJ}$ $GN := \sqrt{LN^2 + GL^2}$ $JN := \sqrt{LN^2 - JL^2}$

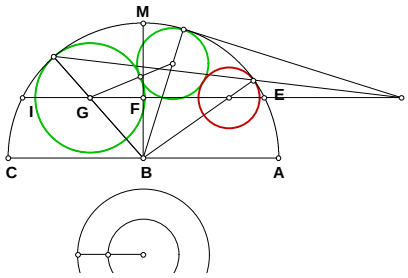
$EJ := JL$ $EN := \sqrt{JN^2 + EJ^2}$ $S_1 := EG$ $S_2 := EN$ $S_3 := GN$ $GO := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$NO := \sqrt{GN^2 - GO^2}$ $NR := \frac{NO^2}{GN}$ $GS := \frac{DG^2}{GN}$ $RS := GN - (NR + GS)$ $DT := RS$ $DS := \sqrt{DG^2 - GS^2}$

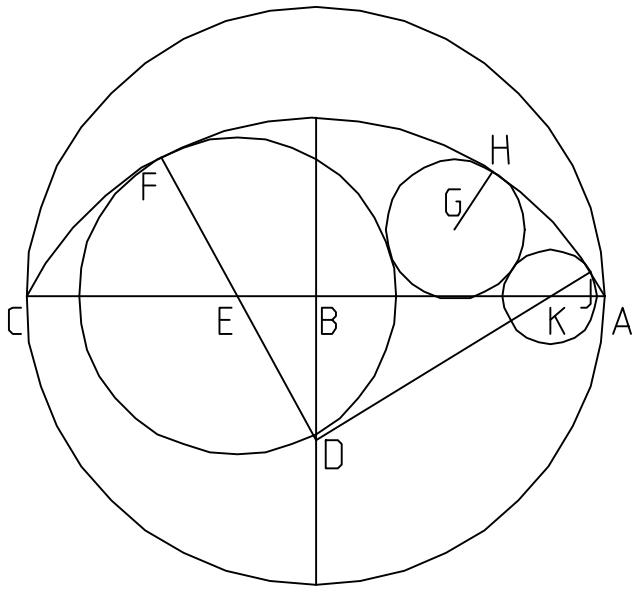
$RT := DS$ $OR := \sqrt{NO^2 - NR^2}$ $OT := OR - RT$ $DO := \sqrt{DT^2 + OT^2}$

$S_1 := GO$ $S_2 := DG$ $S_3 := DO$ $OQ := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$ $GQ := GO - OQ$ $DQ := \sqrt{DO^2 - OQ^2}$

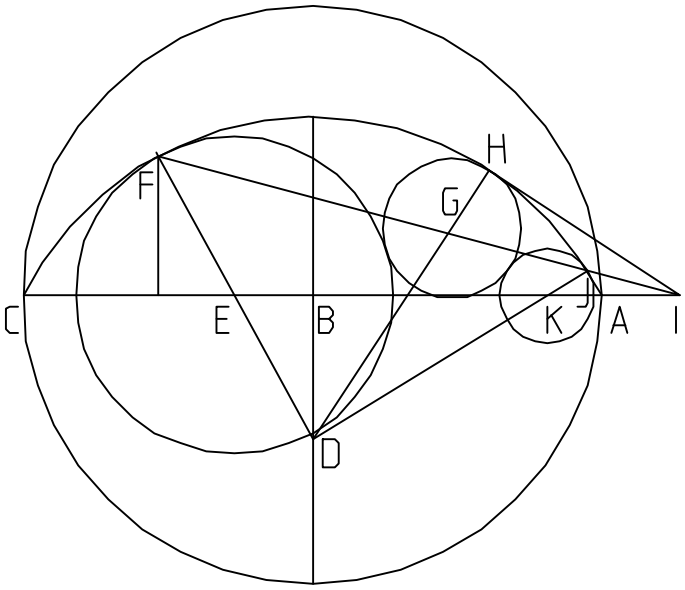
$FU := \frac{GQ \cdot BF}{DQ}$ $CE := \frac{BE \cdot EG}{FU + EF}$ $CE = 0.193$



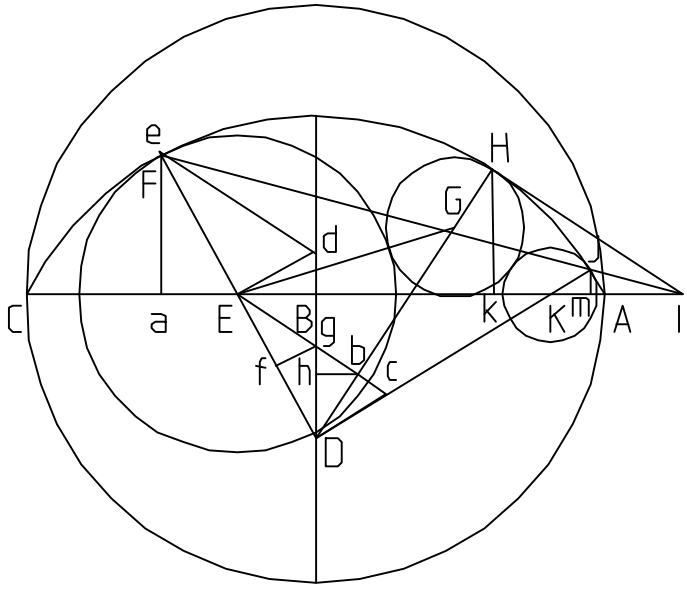

**Edit In progress for three circles.**




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

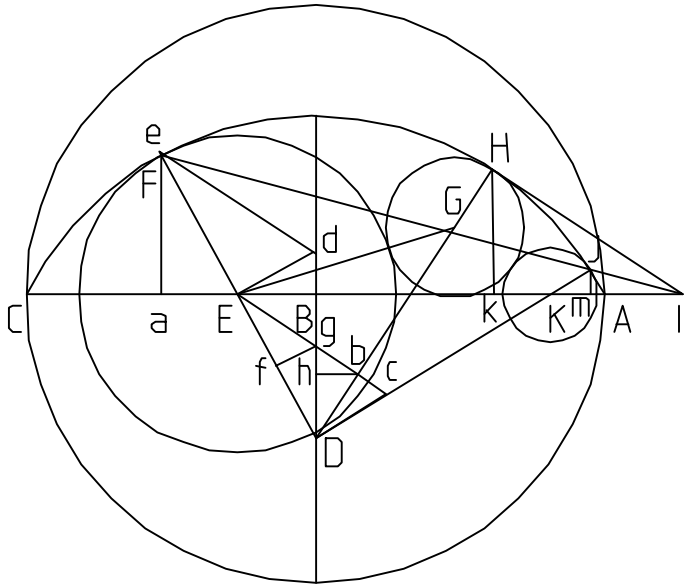
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

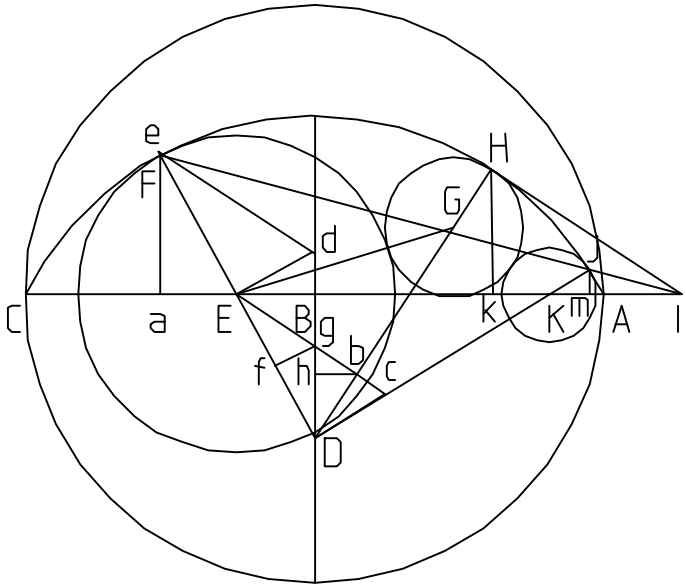
$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

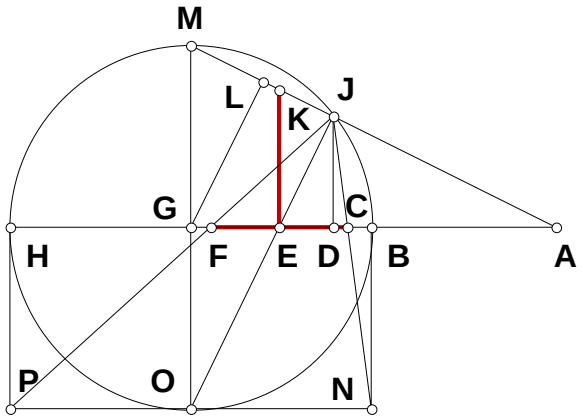
$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

$JK = 17.571$

When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.




## Is CF always equal to EK?

$N_1 := 3$ $AB := 1$

$AH := AB \cdot N_1$ $BH := AH - AB$

$BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

$GM := BG$ $GH := BG$ $AG := AH - GH$

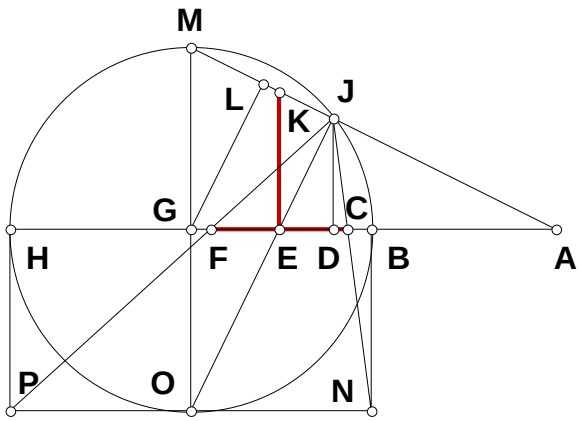
$AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$ $BD := AD - AB$

$DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$ $DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$

$CE := \frac{CF}{2}$ $BE := BC + CE$ $AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$

## 04/23/96 Plate II - Definition And Proof




**Proof of equality in a tautologic is simply the demonstration that two names are synonyms. Is CF always equal to EK?**

$N := 3$ $AB := 1$ $AH := N$

$BH := N - 1$ $BG := \frac{N - 1}{2}$

$AG := \frac{1}{2} \cdot N + \frac{1}{2}$ $AM := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$AL := \frac{1}{2} \cdot \frac{(N + 1)^2}{\sqrt{2 \cdot N^2 + 2}}$ $LM := \frac{1}{2} \cdot \frac{(N - 1)^2}{\sqrt{2 \cdot N^2 + 2}}$ $AJ := \frac{2}{\sqrt{2 \cdot N^2 + 2}} \cdot N$ $AD := (N + 1) \cdot \frac{N}{(N^2 + 1)}$

$BD := \frac{(N - 1)}{(N^2 + 1)}$ $DH := N^2 \cdot \frac{(N - 1)}{(N^2 + 1)}$ $DJ := \frac{(N - 1)}{(N^2 + 1)} \cdot N$ $BC := \frac{(N - 1)}{(N + 1)^2}$

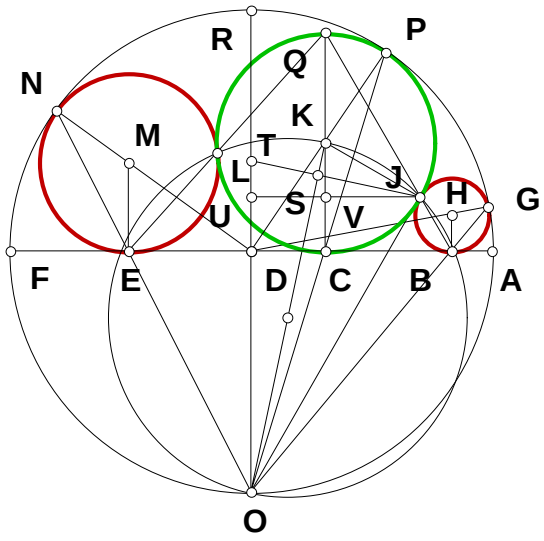
$DF := 2 \cdot N^3 \cdot \frac{(N - 1)}{[(N + 1)^2 \cdot (N^2 + 1)]}$ $CD := 2 \cdot (N - 1) \cdot \frac{N}{[(N^2 + 1) \cdot (N + 1)^2]}$

$CF := 2 \cdot N \cdot \frac{(N - 1)}{(N + 1)^2}$ $CE := N \cdot \frac{(N - 1)}{(N + 1)^2}$ $BE := \frac{(N - 1)}{(N + 1)}$ $AE := 2 \cdot \frac{N}{(N + 1)}$

$EK := 2 \cdot (N - 1) \cdot \frac{N}{(N + 1)^2}$ $EK - CF = 0$ $EK = 0.75$

**Meditation : Do these equations satisfy the requirement that a definition must contain both form and matter?**

# Three Circles 04/24/96




## Given AC, find CK and BH.

$N := 1.5$ $AF := 1$ $AD := \frac{AF}{2}$

$AC := \frac{AF}{N}$ $DO := AD$ $OR := AF$

$CD := AD - AC$ $CO := \sqrt{CD^2 + DO^2}$

$PO := \frac{DO \cdot OR}{CO}$ $CP := PO - CO$ $CK := \frac{DO \cdot CP}{PO}$

$JK := CK$ $KO := \sqrt{CD^2 + (DO + CK)^2}$

$JO := \sqrt{KO^2 - JK^2}$ $KS := \frac{JK^2}{KO}$ $SO := KO - KS$ $JS := \frac{JK \cdot SO}{JO}$ $ST := \frac{CD \cdot SO}{DO + CK}$ $JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD}$ $TU := \frac{CD \cdot JT}{KO}$ $DU := TO - (DO + TU)$ $CV := DU$ $CQ := 2 \cdot CK$ $QV := CQ - CV$

$BH := \frac{CK \cdot CV}{QV}$ $BH = 0.19$

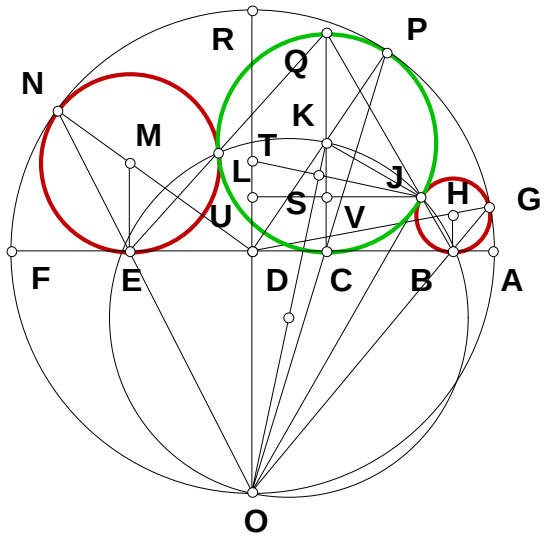
## Some Algebraic Names

$AC := \frac{1}{N}$ $CD := \frac{1}{2} - \frac{1}{N}$ $CO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^2 - 2 \cdot N + 2}}{N}$ $PO := \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{N^2 - 2 \cdot N + 2}} \cdot N$

$CP := \sqrt{2} \cdot \frac{(N - 1)}{\left(\sqrt{N^2 - 2 \cdot N + 2} \cdot N\right)}$ $CK := \frac{(N - 1)}{N^2}$ $KO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}{N^2}$

$JO := \frac{1}{2} \cdot \sqrt{2}$ $KS := \frac{(N - 1)^2}{N^2} \cdot \frac{\sqrt{2}}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $SO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{N^2}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$

$JS := \frac{(N - 1)}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $ST := \frac{1}{2} \cdot (N - 2) \cdot \sqrt{2} \cdot \frac{N^3}{\left[\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2} \cdot \left(N^2 + 2 \cdot N - 2\right)\right]}$

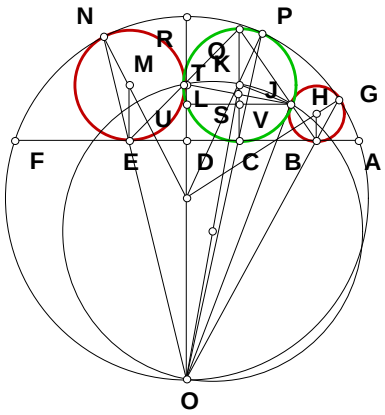



$$JT := \frac{1}{\left(2\cdot\sqrt{N^4 + 2\cdot N^2 - 4\cdot N + 2}\right)}\cdot\frac{\left(2\cdot N^3 + 2\cdot N^2 - 8\cdot N + 4 + \sqrt{2}\cdot N^4 - 2\cdot\sqrt{2}\cdot N^3\right)}{\left(N^2 + 2\cdot N - 2\right)} \qquad TO := \frac{N^2}{\left(N^2 + 2\cdot N - 2\right)}$$

$$TU := \frac{1}{2}\cdot\left(N - \sqrt{2}\right)\cdot(N - 2)\cdot\left(N - 2 + \sqrt{2}\right)\cdot\frac{N}{\left[\left(N^2 + 2\cdot N - 2\right)\cdot\left(N^2 - \sqrt{2}\cdot N + 2 - \sqrt{2}\right)\right]}$$

$$DU := \frac{-1}{2}\cdot\left(\sqrt{2} - 2\right)\cdot\frac{(N - 1)}{\left(N^2 - \sqrt{2}\cdot N + 2 - \sqrt{2}\right)} \qquad CQ := 2\cdot\frac{(N - 1)}{N^2}$$

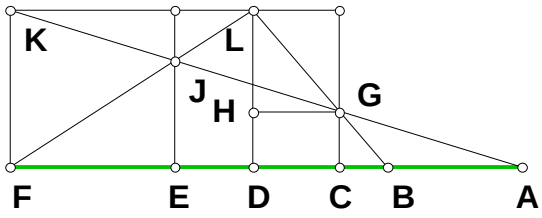
$$QV := \frac{1}{2}\cdot\left(\sqrt{2} + 2\right)\cdot(N - 1)\cdot\frac{\left(N + 2 - 2\cdot\sqrt{2}\right)^2}{\left[\left(N^2 - \sqrt{2}\cdot N + 2 - \sqrt{2}\right)\cdot N^2\right]} \qquad BH := \frac{(N - 1)\cdot\left(3 - 2\cdot\sqrt{2}\right)}{\left[(N + 2) - 2\cdot\sqrt{2}\right]^2}$$




**Working on Moving D.**

## One Over N + One 04/25/96

### Construct 1/(N+1)




$N_1 := 2.817 \quad N_2 := 3 \quad AC := 1$

$AF := AC \cdot N_1 \quad CF := AF - AC \quad CE := \frac{CF}{2}$

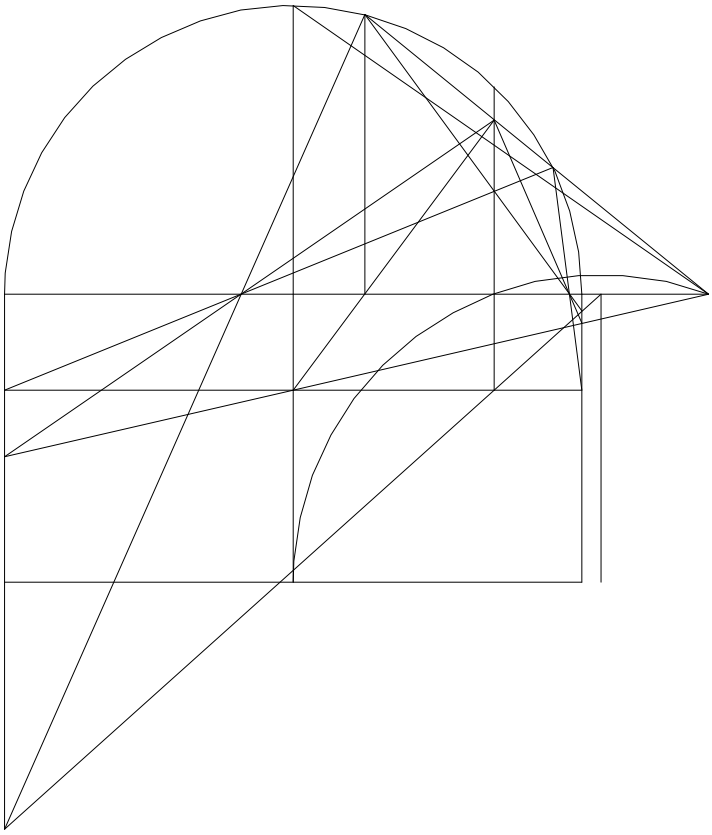
$AE := AC + CE \quad FK := AC \cdot N_2 \quad EJ := \frac{FK \cdot AE}{AF}$

$DL := FK \quad EF := CE \quad DF := \frac{EF \cdot DL}{EJ} \quad CG := \frac{FK \cdot AC}{AF} \quad CD := CF - DF \quad DH := CG \quad HL := DL - DH$

$BC := \frac{CD \cdot CG}{HL}$

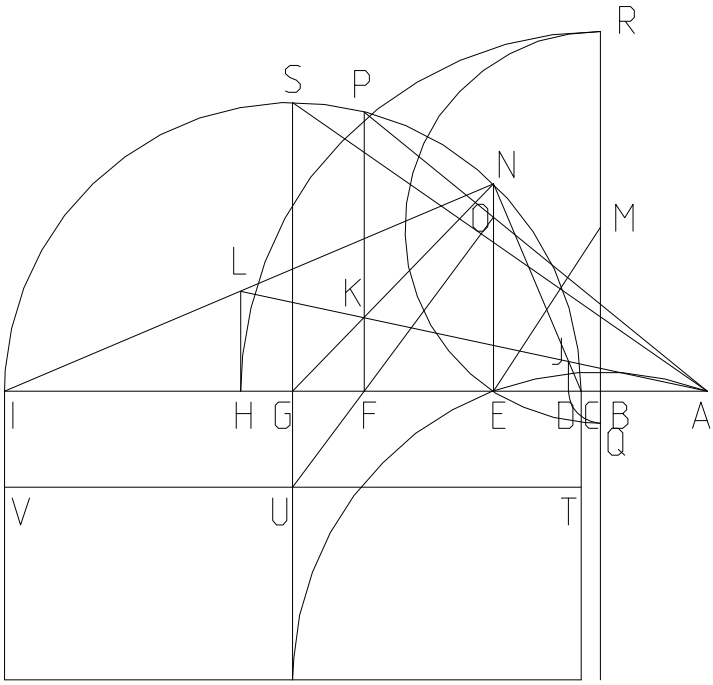
$CF := N_1 - 1 \quad CE := \frac{1}{2} \cdot (N_1 - 1) \quad AE := \frac{1}{2} \cdot (1 + N_1) \quad FK := N_2 \quad EJ := \frac{1}{2} \cdot N_2 \cdot \frac{(1 + N_1)}{N_1}$

$DF := \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 \quad CG := \frac{N_2}{N_1} \quad CD := \frac{(N_1 - 1)}{(1 + N_1)} \quad HL := N_2 \cdot \frac{(N_1 - 1)}{N_1} \quad \frac{1}{N_1 + 1} - BC = 0$



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

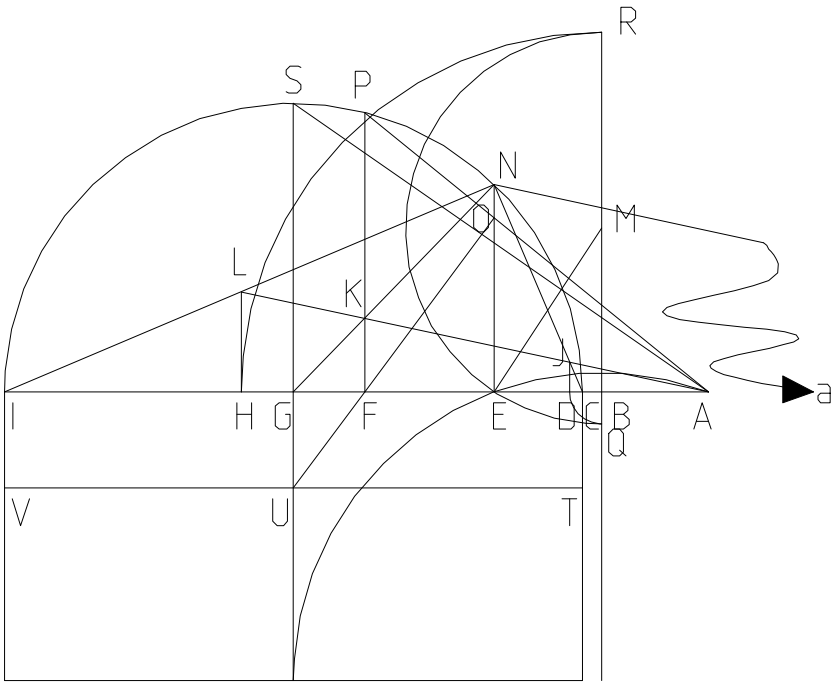
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



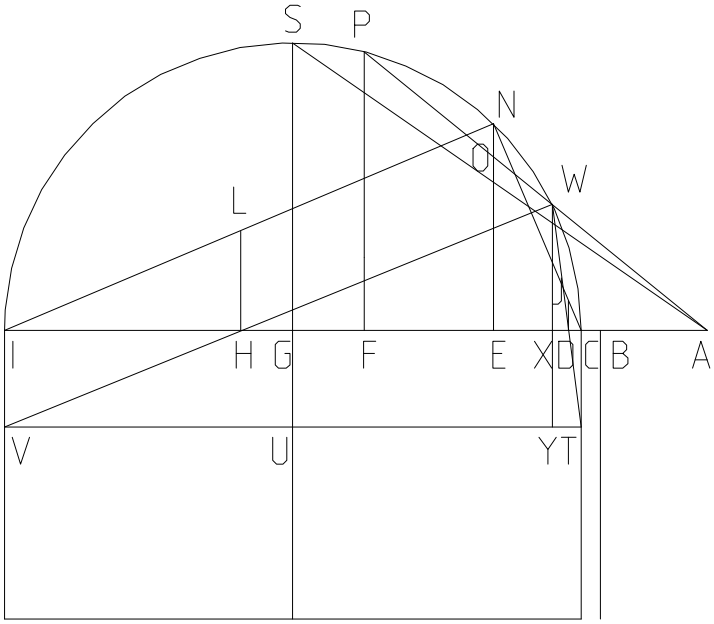

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

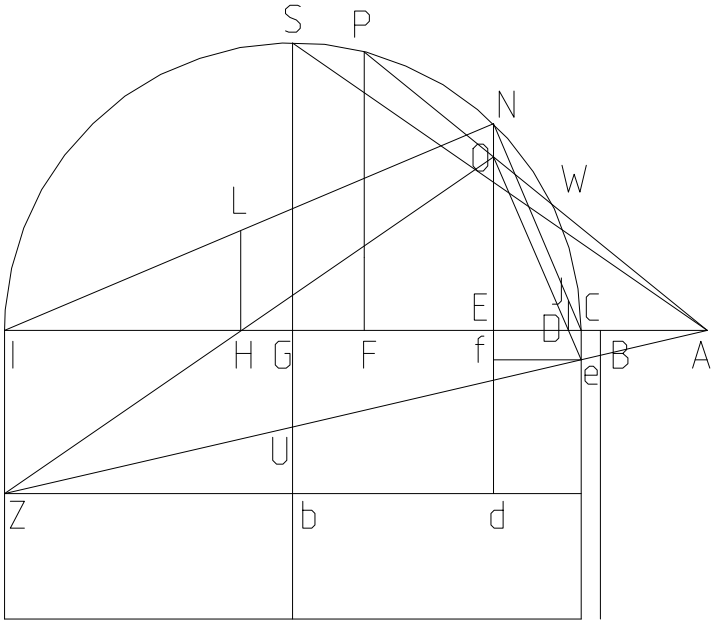
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

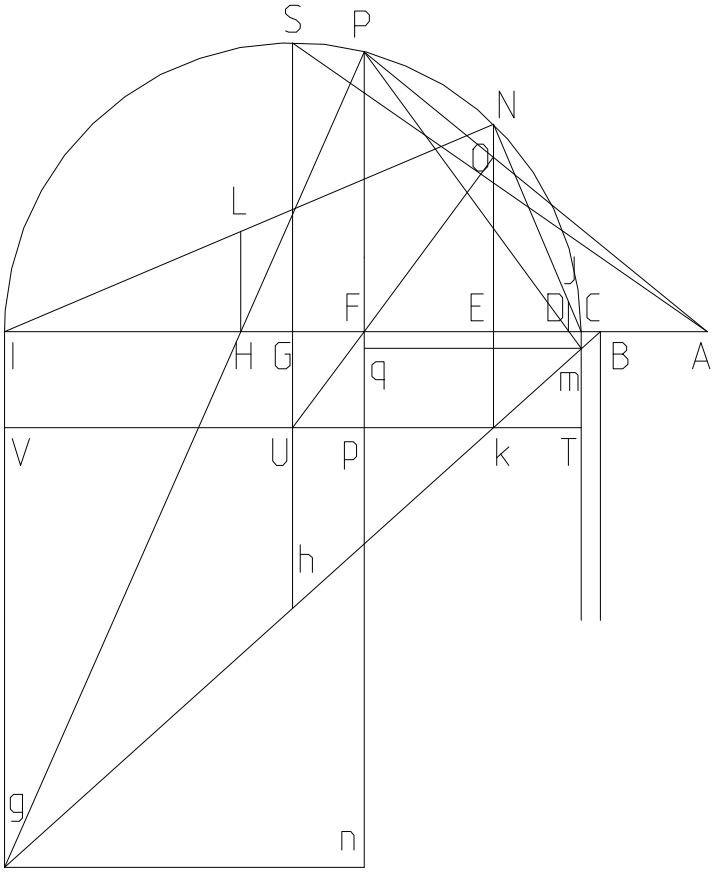
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$




$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

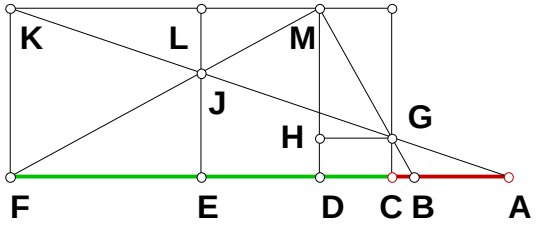
$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

# A Root FIgure 04/27/96

**Given CD, CE, and EF = CE, and that CD is the square root of BCxBF, find BC.**




$N_1 := 5 \qquad N_2 := 3 \qquad CD := 1$

$CE := CD \cdot N_1 \qquad CF := 2 \cdot CE \qquad FK := CD \cdot N_2$

$DM := FK \qquad EL := FK \qquad DF := CF - CD$

$EF := \frac{CF}{2} \qquad EJ := \frac{DM \cdot EF}{DF} \qquad JL := EL - EJ$

$KL := EF \qquad AF := \frac{KL \cdot FK}{JL} \qquad AC := AF - CF \qquad CG := \frac{FK \cdot AC}{AF} \qquad DH := CG \qquad HM := DM - DH \qquad BC := \frac{CD \cdot DH}{HM}$

$BF := BC + CF \qquad BD := BC + CD \qquad \sqrt{BC \cdot BF} - BD = 0$

$CF := 2 \cdot N_1 \qquad FK := N_2 \qquad DF := \left(2 \cdot N_1 - 1\right) \qquad EF := N_1 \qquad EJ := \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1}$

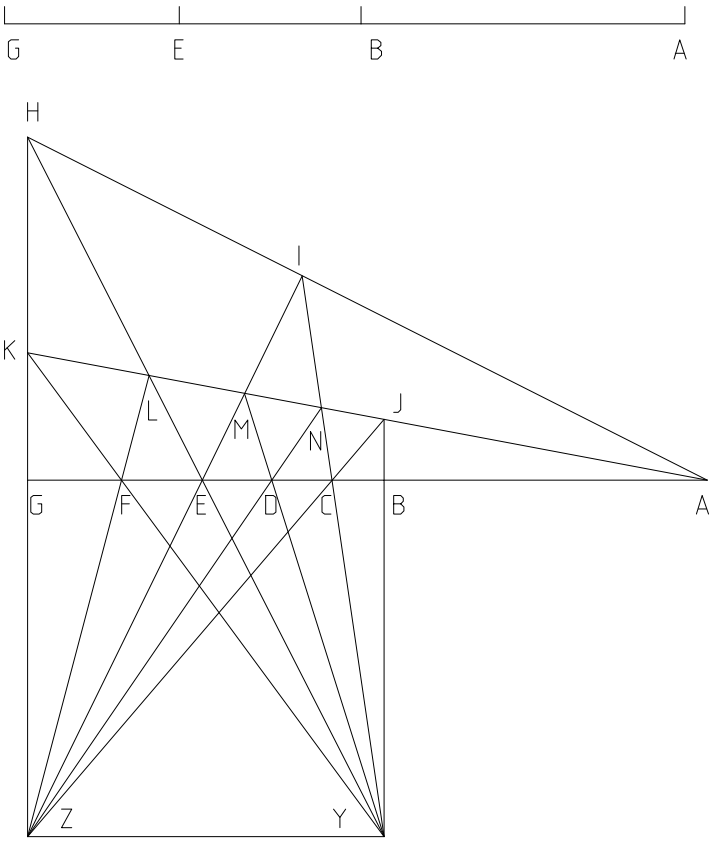
$JL := N_2 \cdot \frac{\left(N_1 - 1\right)}{\left(2 \cdot N_1 - 1\right)} \qquad AF := \frac{N_1 \cdot \left(2 \cdot N_1 - 1\right)}{N_1 - 1} \qquad AC := \frac{N_1}{\left(N_1 - 1\right)} \qquad CG := \frac{N_2}{\left(2 \cdot N_1 - 1\right)}$

$HM := 2 \cdot N_2 \cdot \frac{\left(N_1 - 1\right)}{\left(2 \cdot N_1 - 1\right)} \qquad BC := \frac{1}{2 \cdot \left(N_1 - 1\right)} \qquad BF := \frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 1\right)^2}{\left(N_1 - 1\right)} \qquad BD := \frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 1\right)}{\left(N_1 - 1\right)}$

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use $5^{th}$ root series for example.




$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$GH := \dfrac{BY \cdot EG}{BE}$

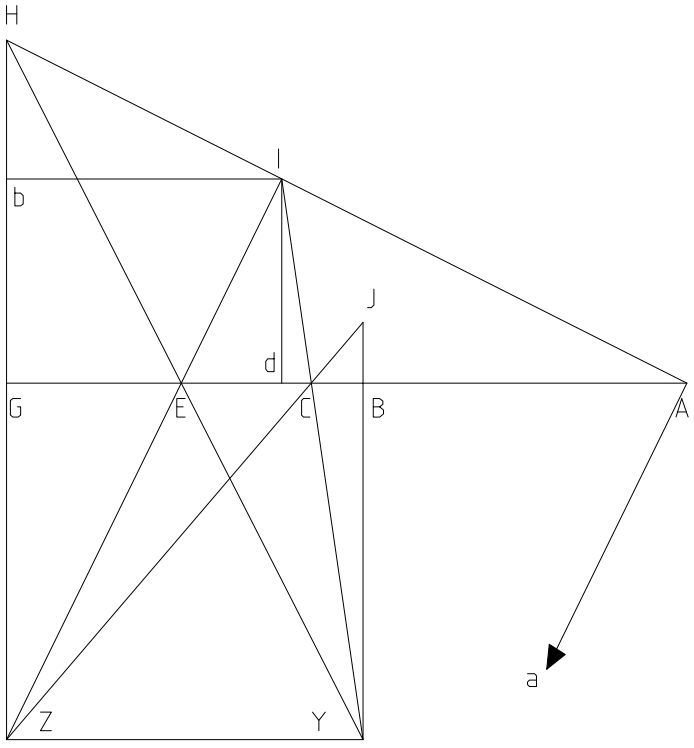
$Ga := \dfrac{GZ \cdot AG}{EG} \qquad Hb := \dfrac{GH \cdot (GH + GZ)}{GH + Ga}$

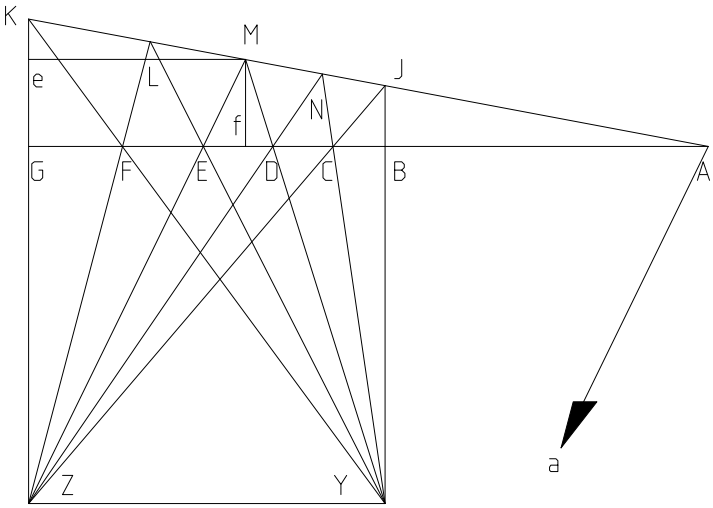
$Gb := GH - Hb \quad Ib := \dfrac{AG \cdot (GH + GZ)}{GH + Ga}$

$Bd := BG - Ib \quad BC := \dfrac{Bd \cdot BY}{BY + Gb}$

$AC := AB + BC$

$CG := BG - BC \quad BJ := \dfrac{GZ \cdot BC}{CG}$



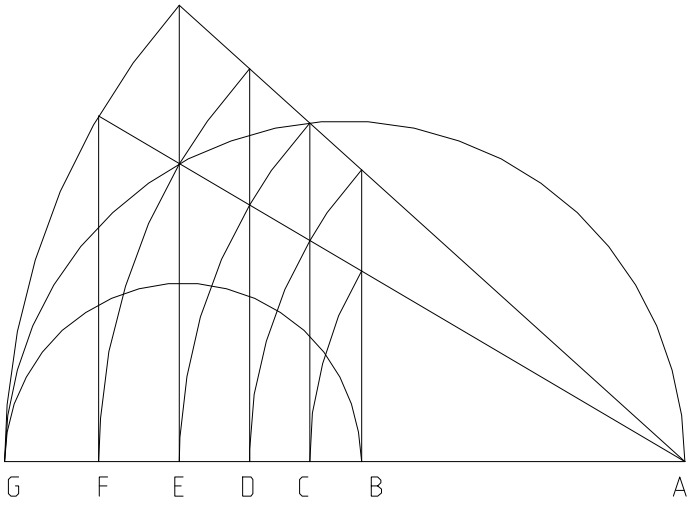



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$




$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

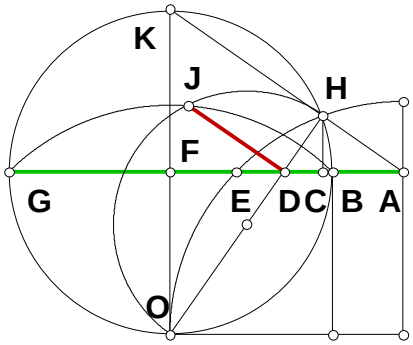
$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.

# What is DJ? 04/29/96

**DJ is the Geometric name, what is its Algebraic name?**




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N \qquad BG := AG - AB$

$BF := \frac{BG}{2} \qquad FK := BF \qquad FO := BF \qquad AF := BF + AB$

$DF := \frac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

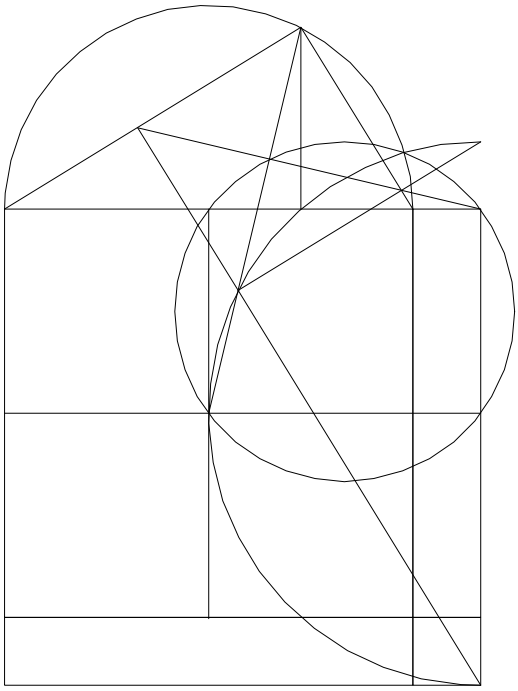
$HO := \frac{AF \cdot KO}{AK} \qquad DO := \frac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

$AG := N \qquad BG := N - 1 \qquad BF := \frac{N-1}{2} \qquad AF := \frac{1}{2} \cdot N + \frac{1}{2} \qquad DF := \frac{1}{2} \cdot \frac{(N-1)^2}{(N+1)} \qquad AK := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$HO := (N+1) \cdot \frac{(N-1)}{\sqrt{2 \cdot N^2 + 2}} \qquad DO := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2} \cdot \frac{(N-1)}{(N+1)} \qquad DH := 2 \cdot N \cdot \frac{(N-1)}{\left[\sqrt{2 \cdot N^2 + 2} \cdot (N+1)\right]}$

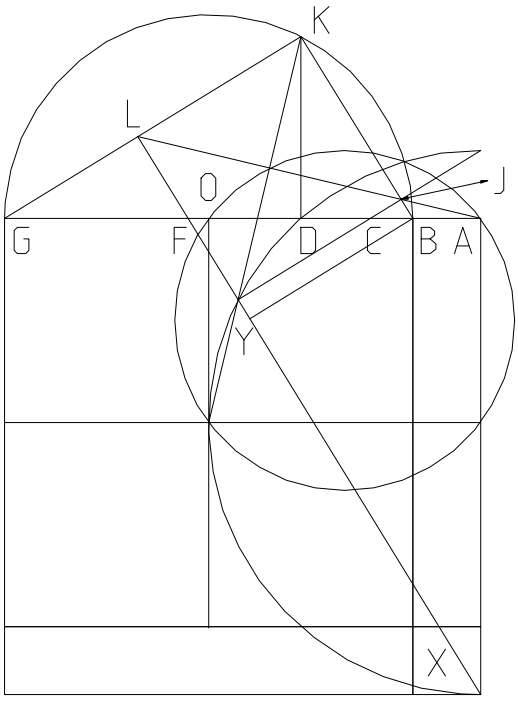
$$DJ := \sqrt{N} \cdot \frac{(N-1)}{(N+1)}$$



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta}^{N^{\frac{Root-\delta}{Root}}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta}^{N^{\frac{Root-\delta}{Root}}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 01_08_96 and 01_16_96 for the sides of the right triangle.




$N = 4 \quad Root = 4 \quad M = 1 \quad BG := N \quad AB := M$

$AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

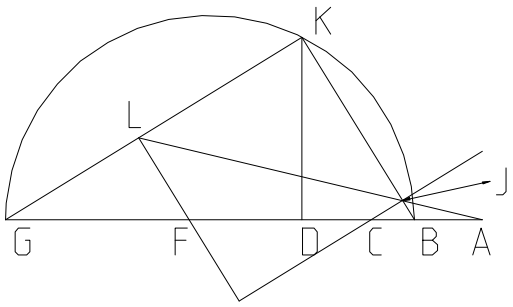
$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$



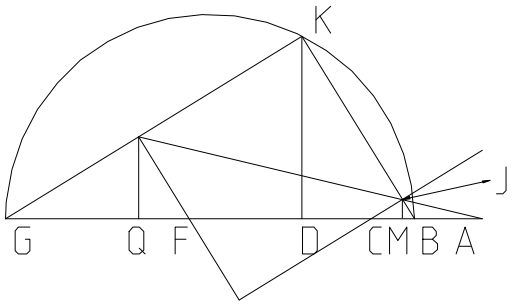

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

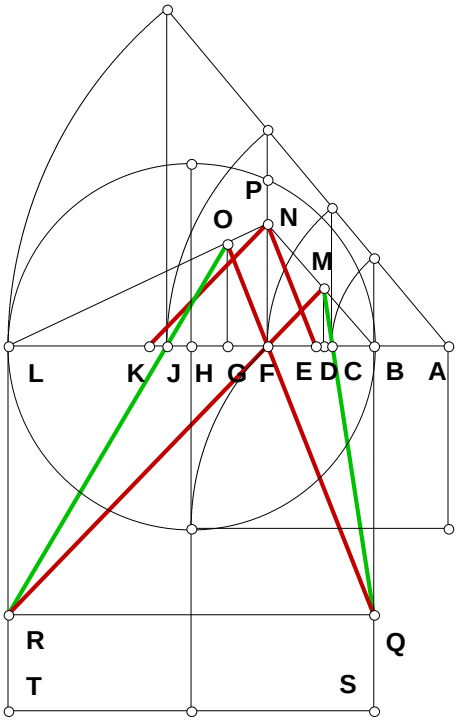
On the left is the first and last of the series, on the right is the entire series.

| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$

# 12/20/96 Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off in the figure.**




$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL$

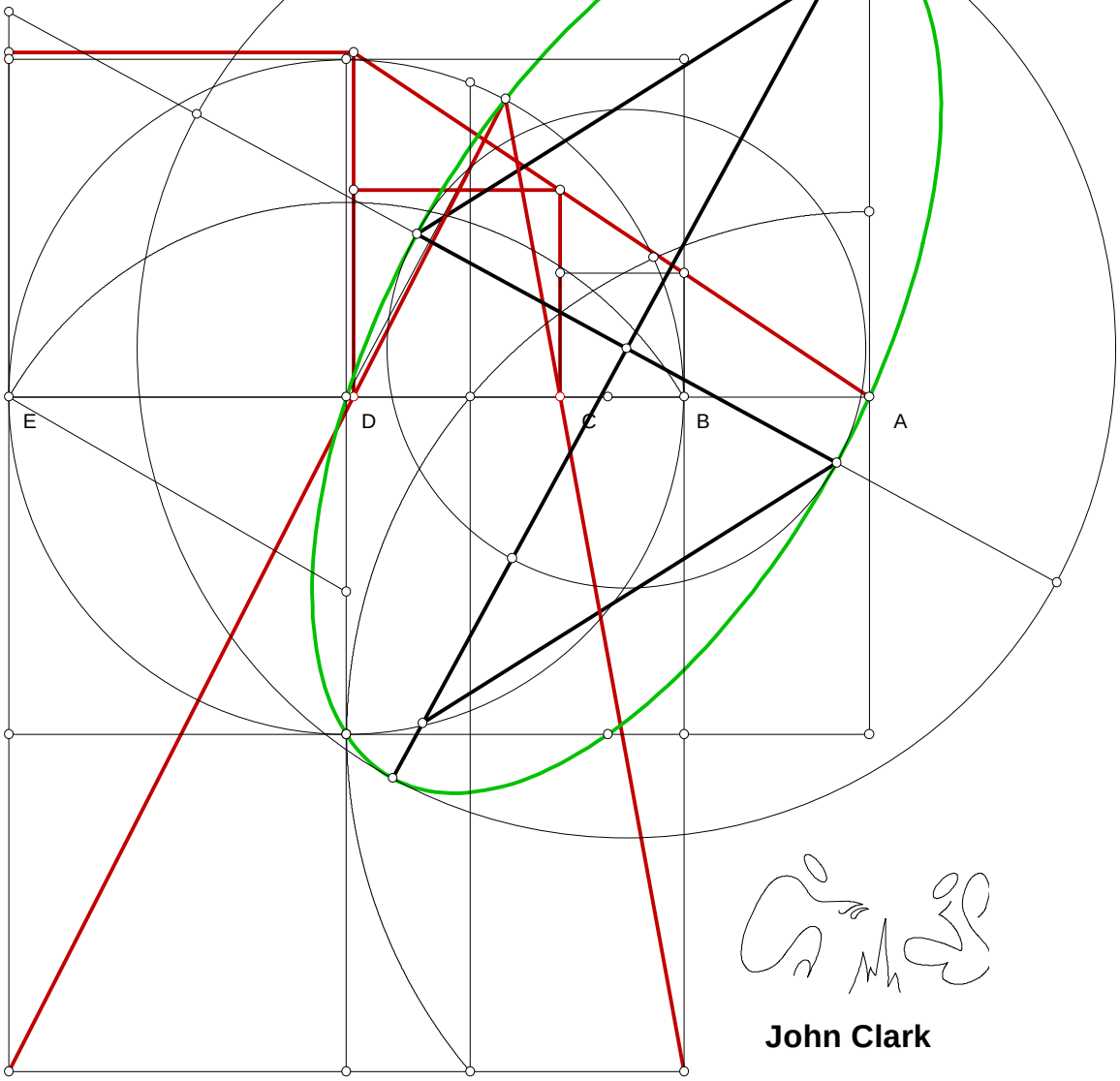
$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$

# The Delian Quest

## 1997

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

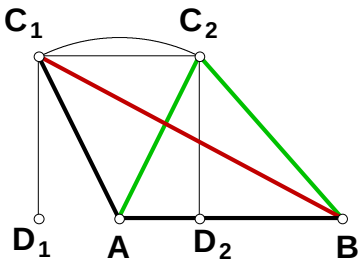





John Clark

# 04/04/97 Triangles

**Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.**




$AB := 5 \qquad AC := 4 \qquad CD := 3$

$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$

$BC_1 := \sqrt{CD^2 + BD_1^2} \qquad BC_2 := \sqrt{CD^2 + BD_2^2}$

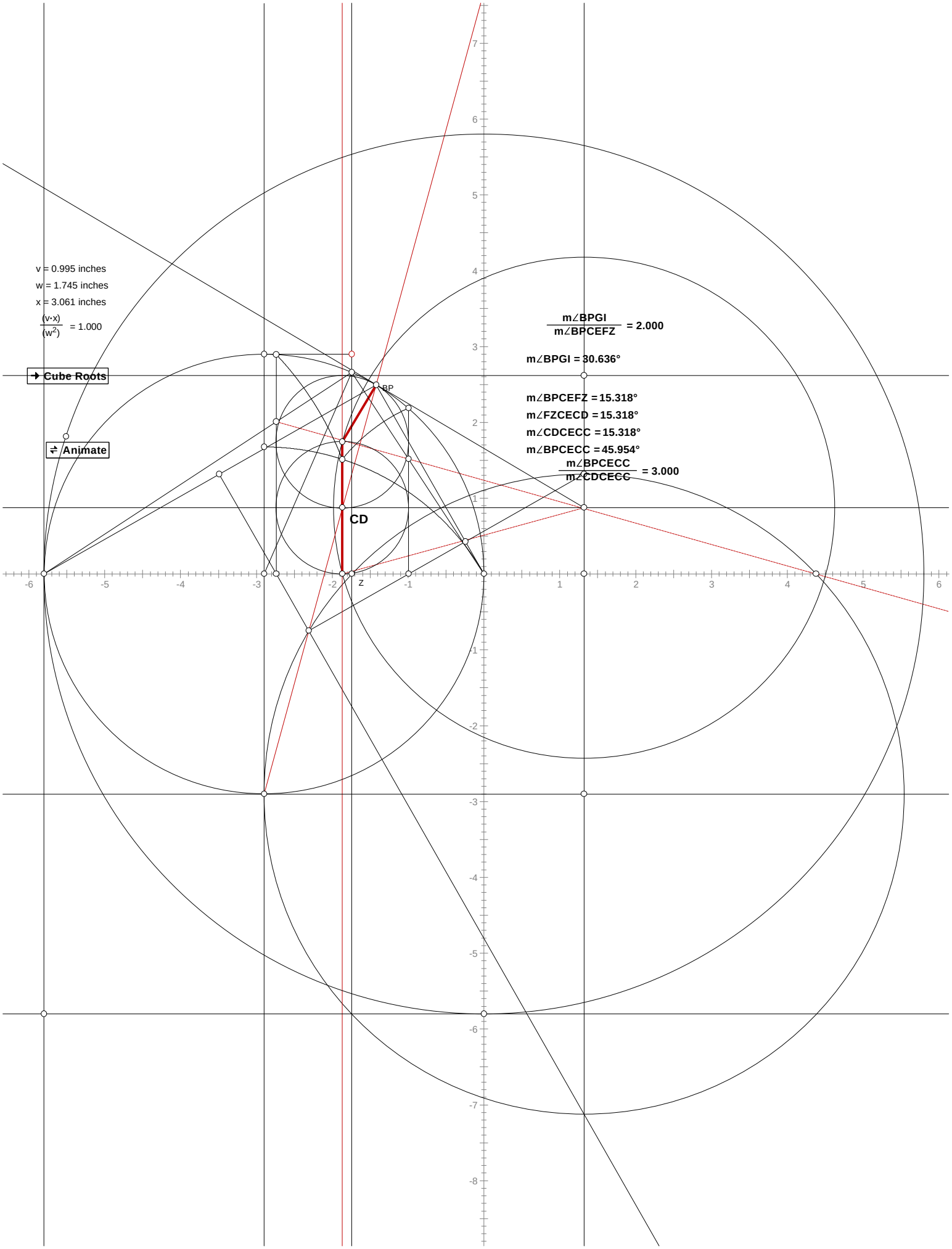
$BC_1 = 8.213 \qquad BC_2 = 3.813$

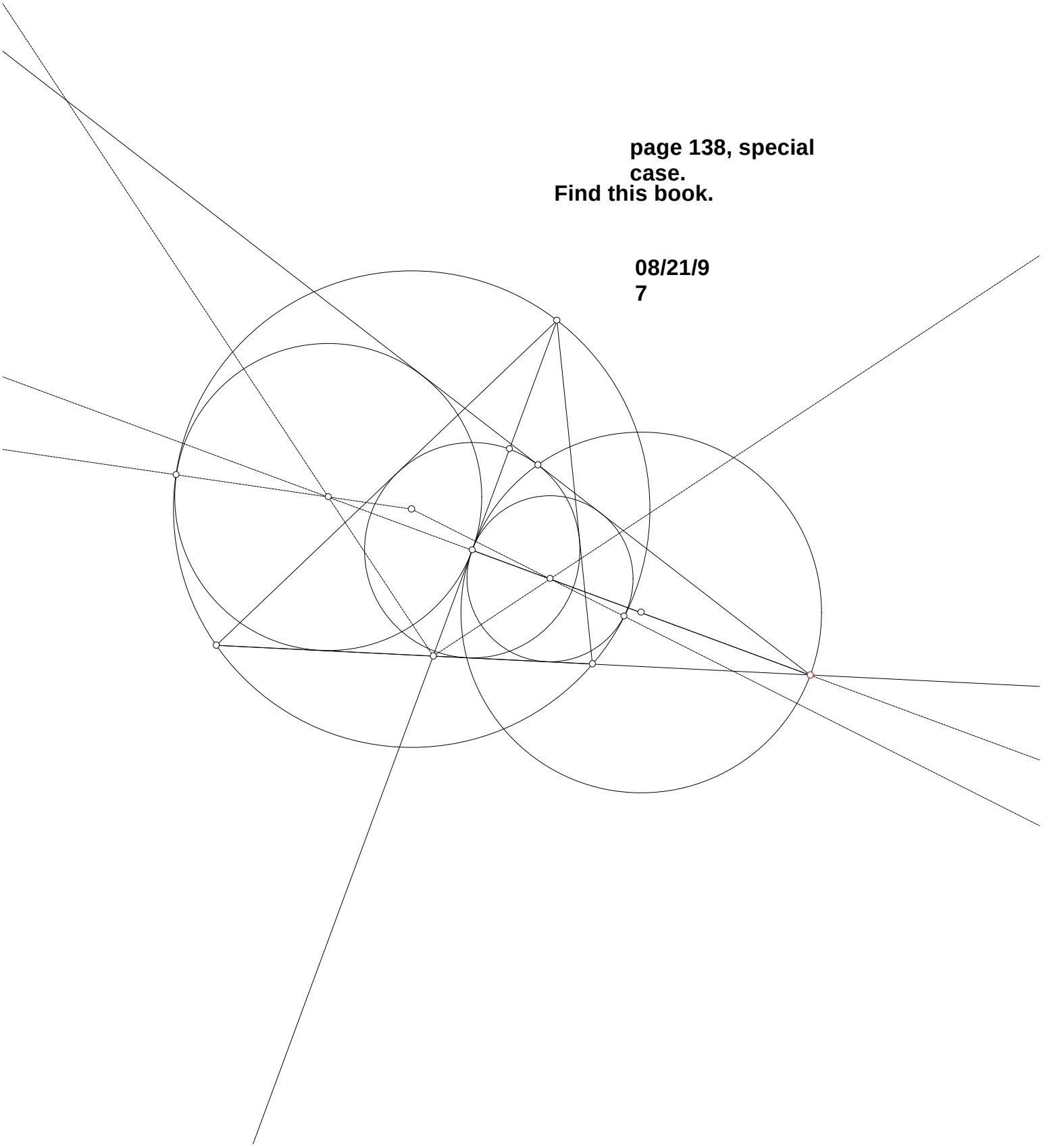
$S_1 := AB \quad S_2 := AC \quad S_3 := BC_1$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

$S_1 := AB \quad S_2 := AC \quad S_3 := BC_2$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$


v = 0.995 inches
w = 1.745 inches
x = 3.061 inches
(v·x)/(w²) = 1.000
Cube Roots
Animate
m∠BPGI / m∠BPCEFZ = 2.000
m∠BPGI = 30.636°
m∠BPCEFZ = 15.318°
m∠FZCECD = 15.318°
m∠CDCECC = 15.318°
m∠BPCECC = 45.954°
m∠BPCECC / m∠CDCECC = 3.000
BP
CD
Z

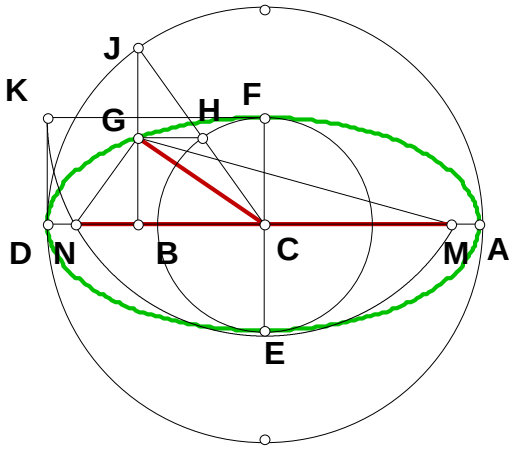


**page 138, special case.**
**Find this book.**

**08/21/97**

# 09/11/97 The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**




$N_1 := 3 \qquad N_2 := 4$

$AD := 1 \qquad EF := \frac{AD}{N_1} \qquad AB := \frac{AD}{N_2}$

$BD := AD - AB \qquad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \qquad BC := AC - AB \qquad CH := \frac{EF}{2}$

$CJ := AC \qquad BG := \frac{BJ \cdot CH}{CJ}$

$CG := \sqrt{BG^2 + BC^2} \qquad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$\frac{\sqrt{4 \cdot N_2 - 4 + N_2^2 \cdot N_1^2 - 4 \cdot N_2 \cdot N_1^2 + 4 \cdot N_1^2}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{\sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0$$

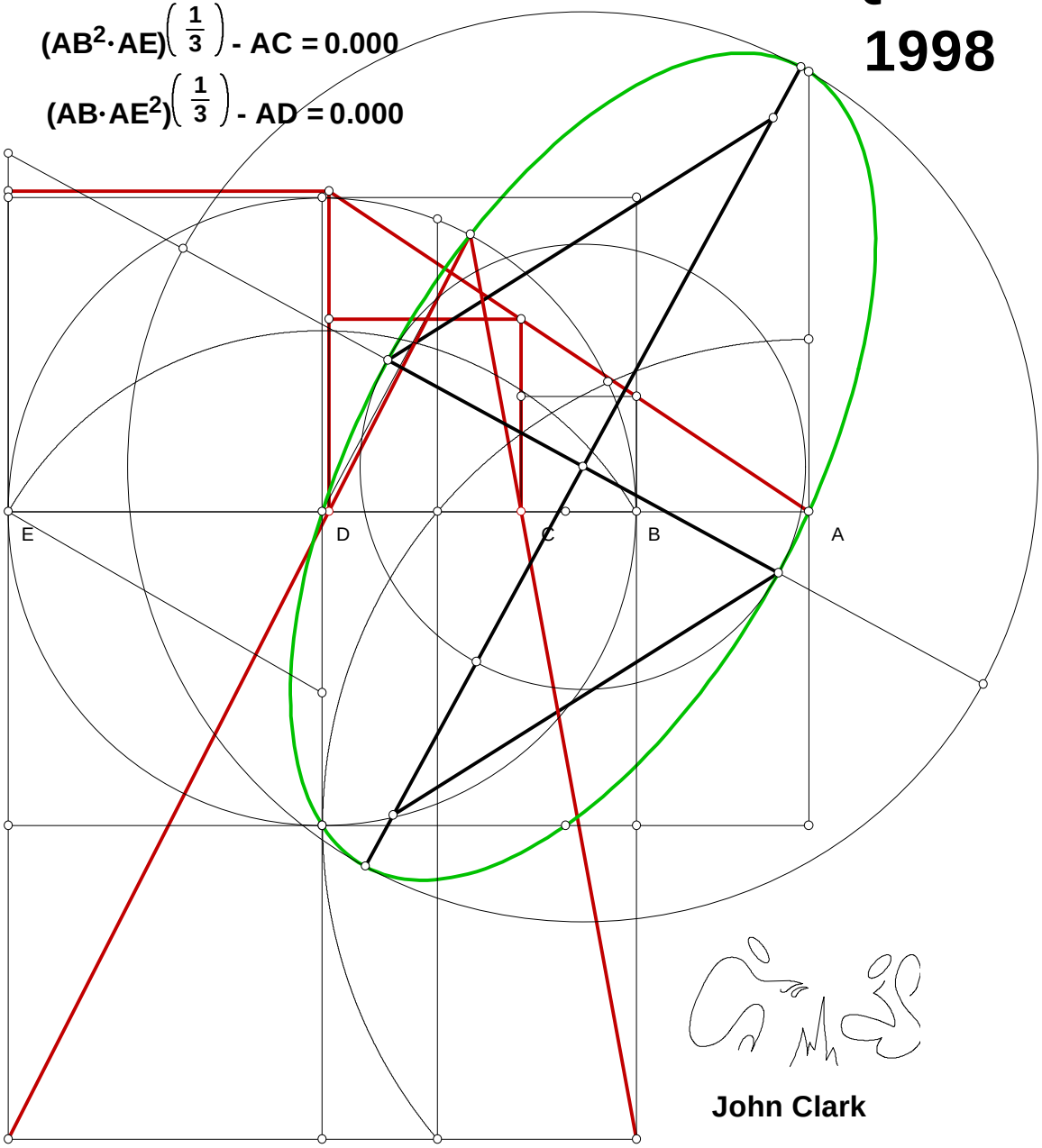
$$\frac{\sqrt{\left(N_1^2 - 1\right)}}{N_1} - MN = 0$$

# The Delian Quest

## 1998

$(AB^2 \cdot AE)\left(\frac{1}{3}\right) - AC = 0.000$

$(AB \cdot AE^2)\left(\frac{1}{3}\right) - AD = 0.000$

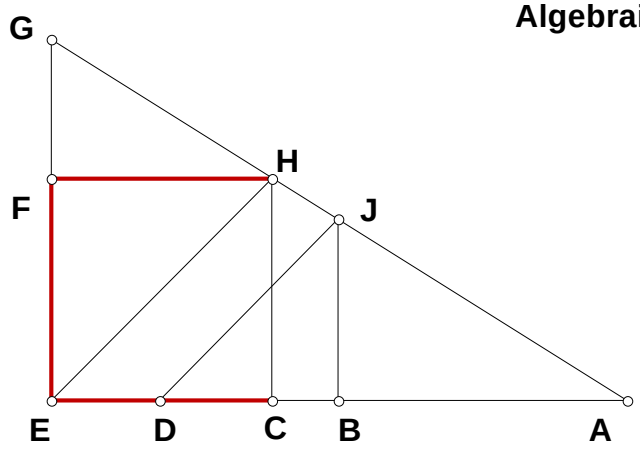





John Clark

# A Square In A Triangle 02/10/98

**What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?**




$N := 4 \quad AE := 1 \quad EG := N$

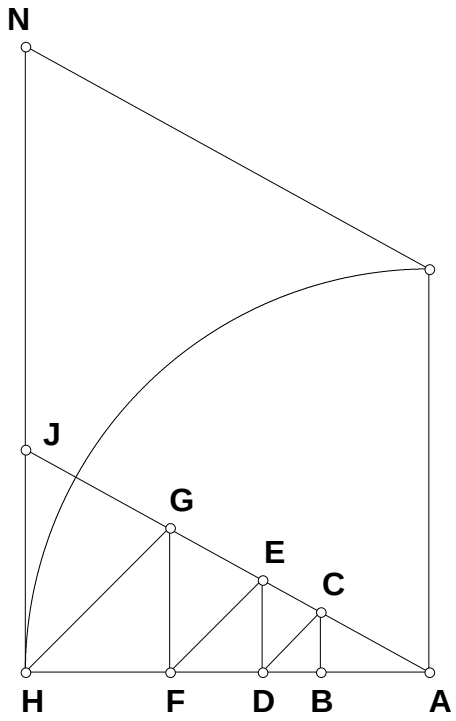
$AB := \frac{AE}{2} \quad BJ := \frac{EG}{2} \quad BD := BJ$

$AD := AB + BD \quad CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE \quad FG := EG - CE$

$\frac{AE}{AC} - (N + 1) = 0 \quad AC - \frac{1}{N + 1} = 0 \quad CE - \frac{N}{N + 1} = 0 \quad FG - \frac{N^2}{N + 1} = 0$

# Alternate Method Root Series 02/25/98




**Given a length and a unit, raise that length to any whole power.**

**Given for the third power.**

$N := 1.3 \quad AH := 1 \quad HN := AH \cdot N$

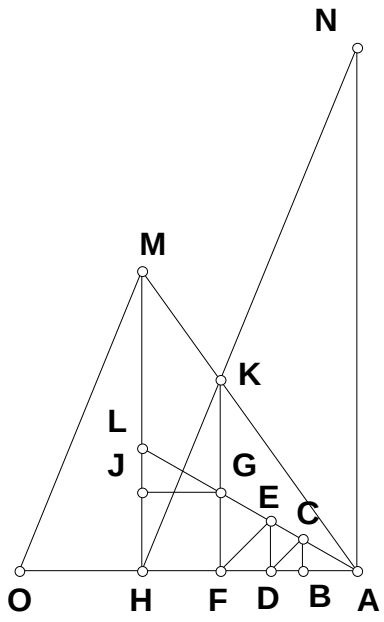
$HJ := HN - AH \quad FH := \frac{AH \cdot HJ}{AH + HJ} \quad AF := AH - FH$

$FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$\frac{AH}{AF} - N^1 = 0 \quad \frac{AH}{AD} - N^2 = 0 \quad \frac{AH}{AB} - N^3 = 0$

# Sum DIvided by One Powered 02/25/98B




$N_1 := 5 \quad N_2 := 6 \quad AH := 1 \quad HM := AH \cdot N_1 \quad AN := AH \cdot N_2$

$HO := \frac{AH \cdot HM}{AN} \quad AO := AH + HO \quad AF := \frac{AH^2}{AO}$

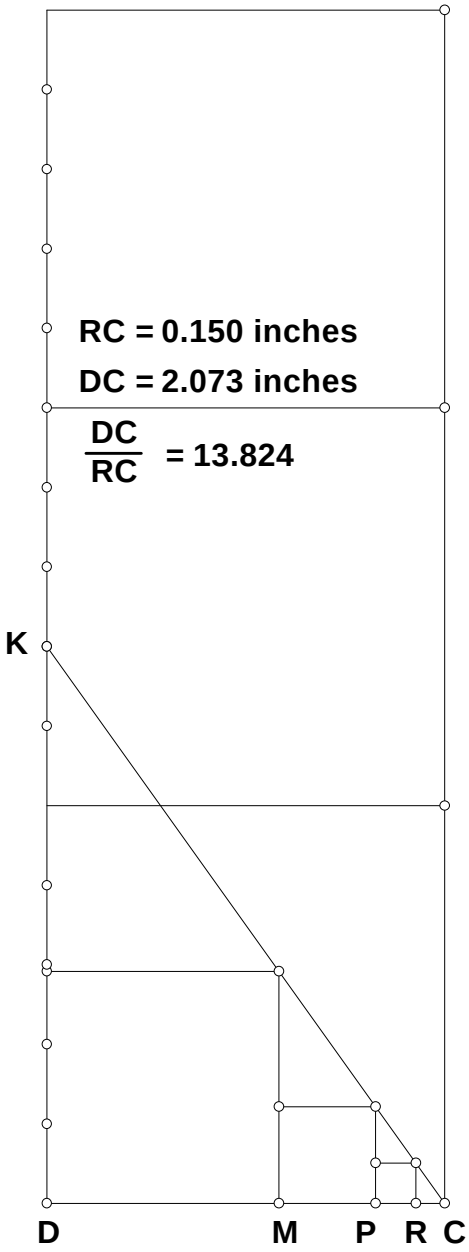
$FH := AH - AF \quad FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0$$

→ 4
→ 3+4/5
→ 3+3/5
→ 3+2/5
→ 3+1/5
→ 3
→ 2+4/5
→ 2+3/5
→ 2+2/5
→ 2+1/5
→ 2
→ 1+4/5
→ 1+3/5
→ 1+2/5
→ 1+1/5
▲ Show
△ Hide

# M^N




$4^3 = 64.000$

$\left(3 + \frac{4}{5}\right)^3 = 54.872$

$\left(3 + \frac{3}{5}\right)^3 = 46.656$

$\left(3 + \frac{2}{5}\right)^3 = 39.304$

$\left(3 + \frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2 + \frac{4}{5}\right)^3 = 21.952$

$\left(2 + \frac{3}{5}\right)^3 = 17.576$

$\left(2 + \frac{2}{5}\right)^3 = 13.824$

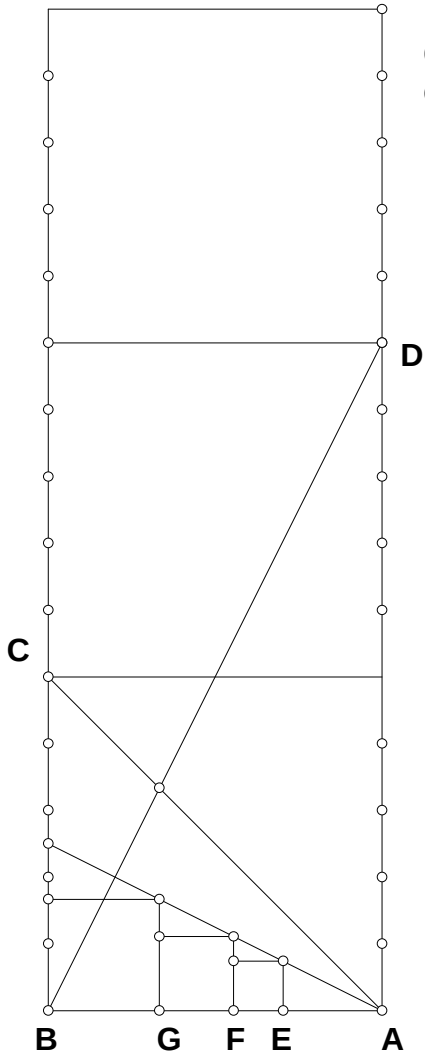
$\left(2 + \frac{1}{5}\right)^3 = 10.648$

$\left(1 + \frac{4}{5}\right)^3 = 5.832$

$\left(1 + \frac{3}{5}\right)^3 = 4.096$

$\left(1 + \frac{2}{5}\right)^3 = 2.744$

$\left(1 + \frac{1}{5}\right)^3 = 1.728$




**Given line AB, divide it by the equation**

$$\left( \frac{(BC + AD)}{AD} \right)^{W}$$

**where W is a whole number.**

- → 2+4/5
- → 2+3/5
- → 2+2/5
- → 2+1/5
- → 2
- → 1+4/5
- → 1+3/5
- → 1+2/5
- → 1+1/5

**BC**

- → 4
- → 3+4/5
- → 3+3/5
- → 3+2/5
- → 3+1/5
- → 3

**AD**

- → 2+4/5
- → 2+3/5
- → 2+2/5
- → 2+1/5
- → 2
- → 1+4/5
- → 1+3/5
- → 1+2/5
- → 1+1/5

- → 4
- → 3+4/5
- → 3+3/5
- → 3+2/5
- → 3+1/5
- → 3

$$\frac{AB}{AG} = 1.500$$

$$\frac{AB}{AF} = 2.250$$

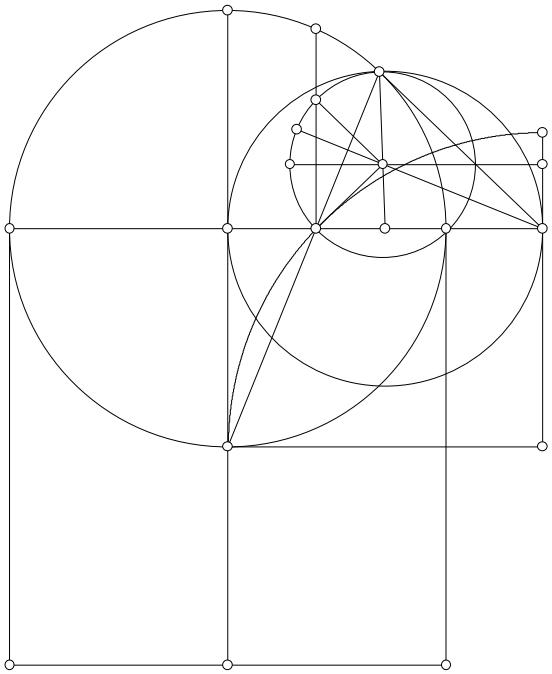
$$\frac{AB}{AE} = 3.375$$

$$\left( \frac{(BC + AD)}{AD} \right)^{1} = 1.500$$

$$\left( \frac{(BC + AD)}{AD} \right)^{2} = 2.250$$

$$\left( \frac{(BC + AD)}{AD} \right)^{3} = 3.375$$

**042398**



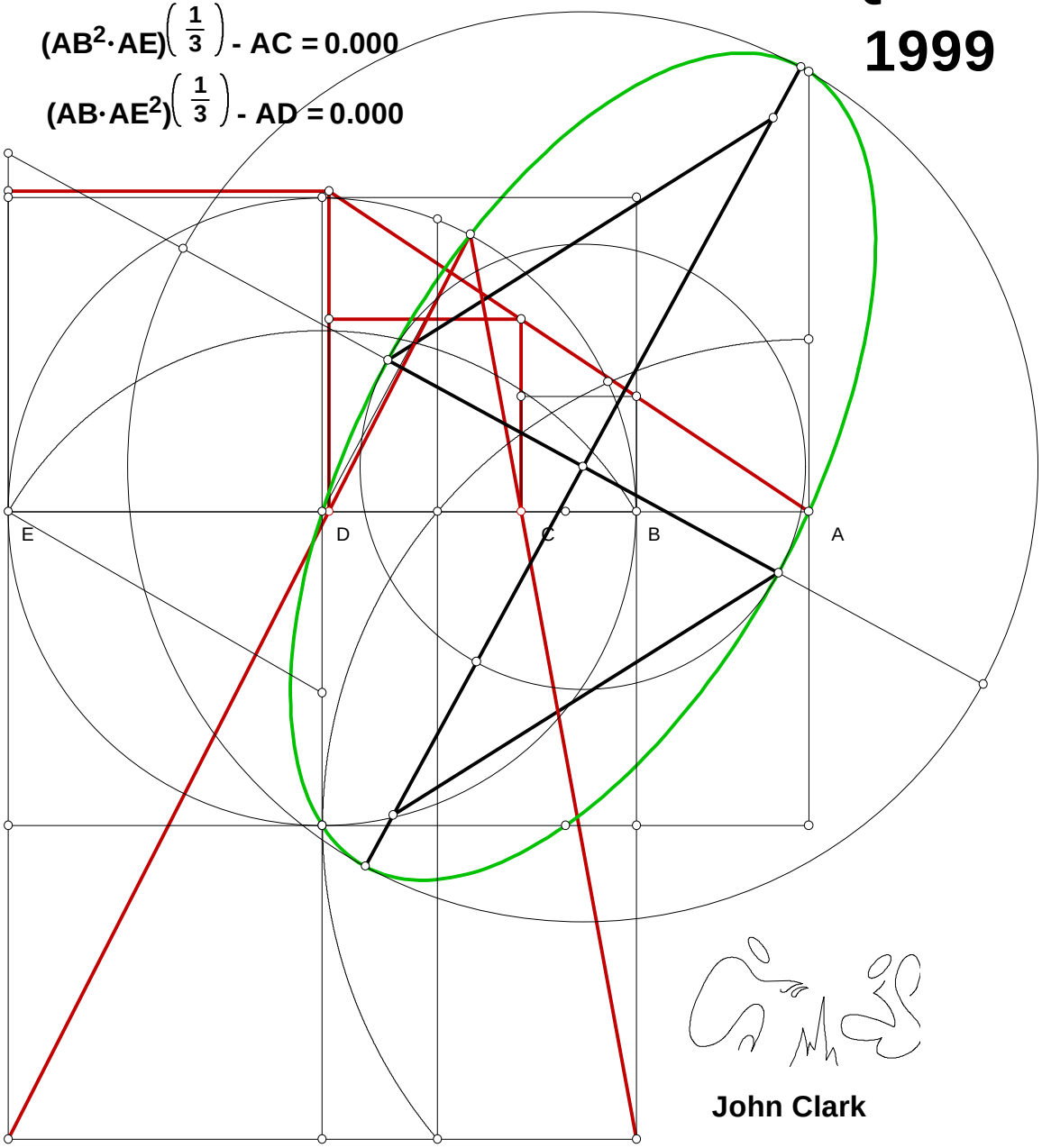
# Trisection and Square Roots

# The Delian Quest

## 1999

$(AB^2 \cdot AE)^{\left(\frac{1}{3}\right)} - AC = 0.000$

$(AB \cdot AE^2)^{\left(\frac{1}{3}\right)} - AD = 0.000$

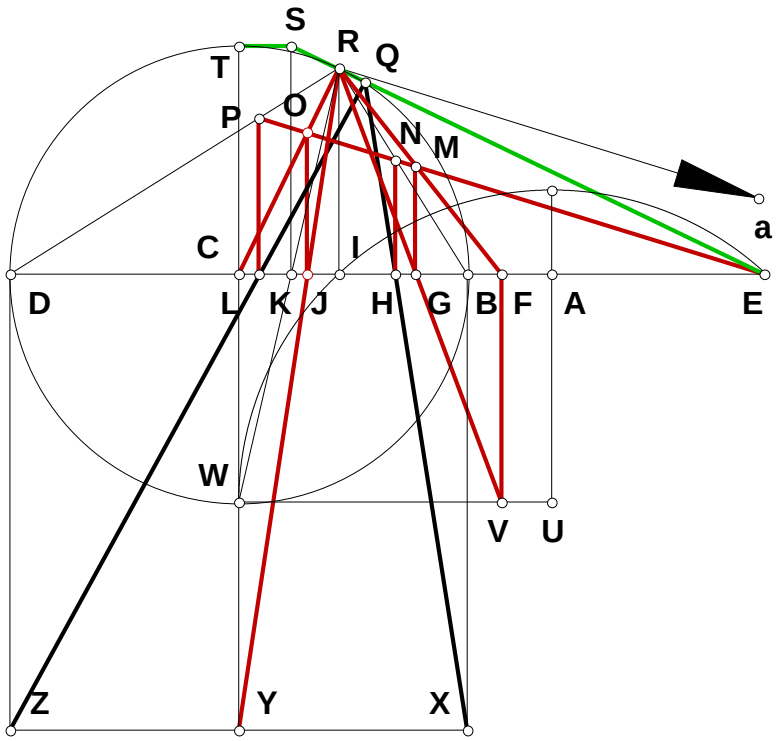





John Clark

# On Gemini Roots 07/24/99

## CE is to EF as CY is to CW




$N_1 := 6.429$ $N_2 := 2$

$AB := .438$ $AD := AB \cdot N_1$

$BD := AD - AB$ $BC := \frac{BD}{2}$

$CW := BC$ $CT := BC$

$FV := BC$ $AI := \sqrt{AB \cdot AD}$

$BI := AI - AB$ $DI := BD - BI$

$IR := \sqrt{BI \cdot DI}$ $AE := AI$

$AC := AB + BC$ $ED := AD + AE$

$CE := AC + AE$ $EF := \frac{CE}{N_2}$

$BE := AE + AB$ $EI := AE + AI$

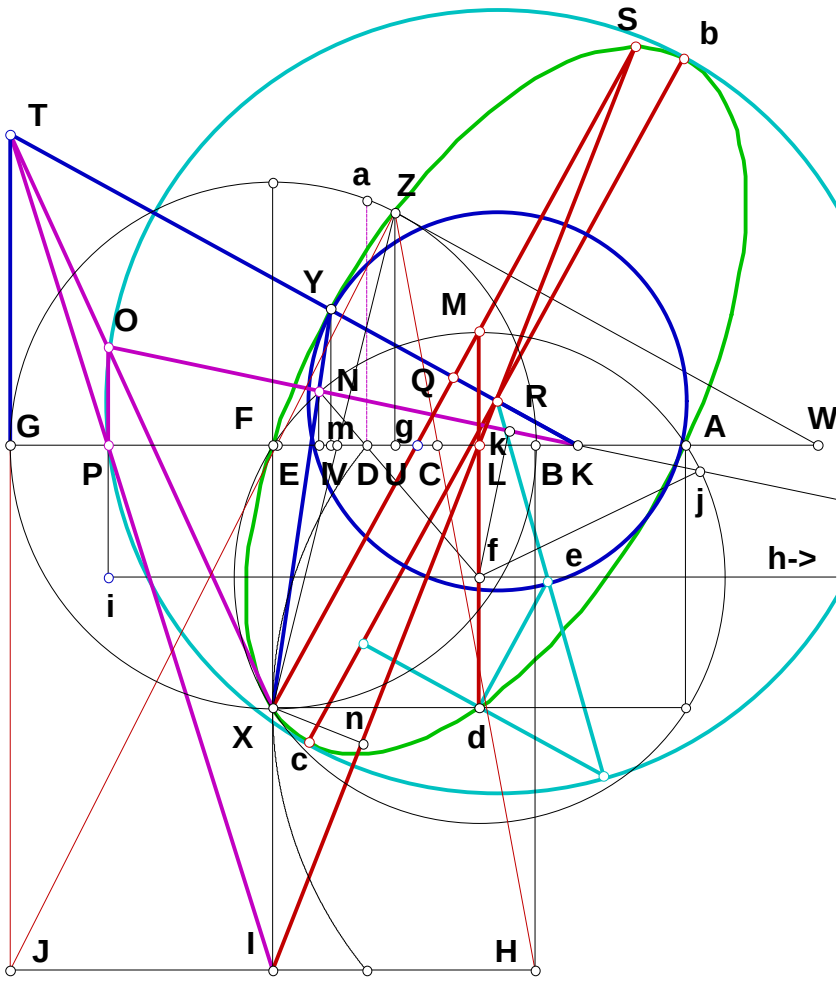
$FI := EI - EF$ $FG := \frac{FI \cdot FV}{FV + IR}$

$EG := EF + FG$ $GI := FI - FG$ $GM := \frac{FV \cdot GI}{FI}$ $Ia := \frac{EG \cdot IR}{GM}$ $EL := \frac{Ia \cdot ED}{Ia + DI}$ $BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI$ $BH := \frac{BI \cdot BE}{Ba}$ $EH := BE + BH$ $CI := AC - AI$ $JO := \frac{IR \cdot CE}{CI + Ia}$ $CJ := \frac{CI \cdot JO}{IR}$

$JI := CI - CJ$ $CY := \frac{IR \cdot CJ}{JI}$ $\frac{CY}{CW} - \frac{CE}{EF} = 0$

# A Delian Solution 08/11/99




**What are the minor and major axis for the ellipse that will give point Z for the cube root?**

$AB := 1 \qquad N := 16 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad AD := \sqrt{AB \cdot AG}$

$BF := \frac{BG}{2} \qquad FG := BF \qquad AF := AB + BF$

$FX := BF \qquad Mf := \frac{\sqrt{AF^2 + FX^2}}{2} \qquad Lf := \frac{FX}{2}$

$ML := Mf - Lf \qquad FL := \frac{AF}{2} \qquad Xd := FL \qquad df := Lf$

$IX := FX \qquad Md := Mf + df \qquad MX := \sqrt{Xd^2 + Md^2}$

$SX := \frac{MX \cdot IX}{IX - ML} \qquad Lg := \frac{FL \cdot ML}{ML + FX} \qquad QX := \frac{SX}{2}$

$Fg := FL - Lg \qquad Xg := \frac{MX \cdot Fg}{Xd} \qquad Qg := QX - Xg$

$Kg := \frac{Xg \cdot Qg}{Fg} \qquad GK := FG + Fg + Kg \qquad GJ := BG$

$GT := \frac{Fg \cdot GK}{FX} \qquad JT := GJ + GT \qquad IJ := BF$

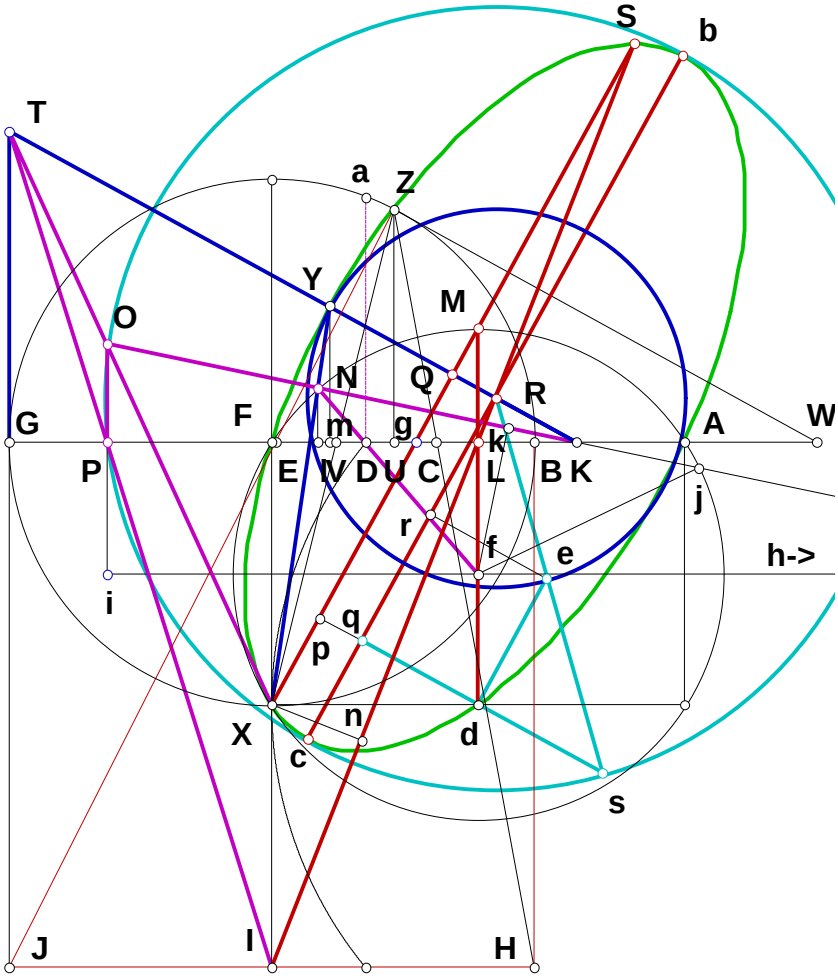
$FP := \frac{IJ \cdot GJ}{JT} \qquad OP := \frac{IX \cdot GT}{JT}$

$KP := Fg + Kg + FP \qquad Pi := Lf \qquad Oi := OP + Pi \qquad hi := \frac{KP \cdot Oi}{OP} \qquad fi := FP + FL \qquad fh := hi - fi \qquad KO := \sqrt{KP^2 + OP^2}$

$hk := \frac{KP \cdot fh}{KO} \qquad fk := \frac{OP \cdot fh}{KO} \qquad Nf := Mf \qquad Nk := \sqrt{Nf^2 - fk^2} \qquad Nh := hk + Nk \qquad Oh := \frac{KO \cdot Oi}{OP} \qquad NO := Oh - Nh \qquad KN := KO - NO$

$$Nl := \frac{OP \cdot KN}{KO} \quad Kl := \frac{KP \cdot KN}{KO} \quad FK := KP - FP \quad Fl := FK - Kl \quad NX := \sqrt{(FX + Nl)^2 + Fl^2} \quad XY := \frac{NX \cdot IX}{IX - Nl} \quad Fm := \frac{Fl \cdot XY}{NX}$$

$$Km := FK - Fm \quad Fo := \frac{Fl \cdot FX}{FX + Nl} \quad Xo := \frac{NX \cdot Fo}{Fl} \quad mo := Fm - Fo \quad Ym := \frac{FX \cdot mo}{Fo} \quad FI := 2 \cdot BF \quad IL := \sqrt{FL^2 + FI^2}$$




$$IS := \frac{IL \cdot IX}{IX - ML} \quad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

$$Xn := \sqrt{IX^2 - In^2} \quad QR := \frac{Xn \cdot QX}{IS - In}$$

$$KG := FK + FG \quad KT := \sqrt{KG^2 + GT^2}$$

$$KY := \frac{KT \cdot Ym}{GT} \quad KQ := \frac{KG \cdot Qg}{GT}$$

$$RY := KY - KQ + QR \quad RY = 4.119$$

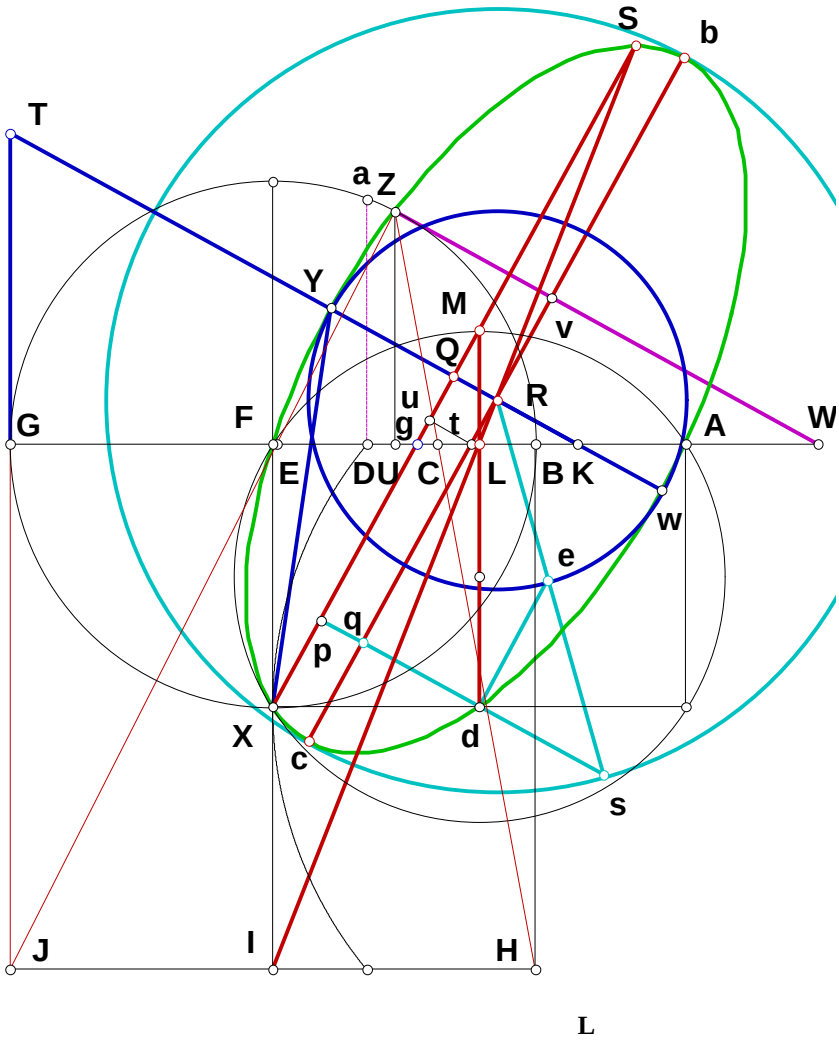
$$Xd := FL \quad dp := \frac{GK \cdot Xd}{KT} \quad pq := QR \quad dq := dp - pq$$

$$Xp := \frac{GT \cdot Xd}{KT} \quad Qp := QX - Xp \quad Rq := Qp$$

$$er := dq \quad Re := RY \quad Rr := \sqrt{Re^2 - er^2}$$

$$Rs := \frac{Re \cdot Rq}{Rr} \quad Rs = 7.171$$

**Is the segment Zv equal to the perpendicular for the ellipse?**




$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \qquad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}}$$

$$BC := AC - AB \qquad BE := AE - AB$$

$$CE := BE - BC \qquad EG := BG - BE$$

$$CU := \frac{BC \cdot CE}{BC + EG} \qquad BU := BC + CU$$

$$GU := BG - BU \qquad UZ := \sqrt{BU \cdot GU}$$

$$UW := \frac{KG \cdot UZ}{GT} \qquad Gg := GK - Kg$$

$$tu := QR \qquad gt := \frac{KT \cdot tu}{GK} \qquad Gt := Gg + gt$$

$$GW := GU + UW \qquad Wt := GW - Gt$$

$$tv := \frac{GT \cdot Wt}{KT} \qquad Kt := GK - Gt \qquad Rt := \frac{GT \cdot Kt}{KT}$$

$$Rv := tv - Rt \qquad bc := 2 \cdot Rs \qquad Rc := Rs$$

$$cv := Rc + Rv \qquad Yw := 2 \cdot RY \qquad WZ := \frac{KT \cdot UZ}{GT}$$

$$Wv := \frac{GK \cdot tv}{GT} \qquad Zv := \left| WZ - Wv \right|$$

$N_1 := Yw \qquad N_2 := bc \qquad N_3 := cv \qquad N_4 := bc \qquad \sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2} - Zv = 0 \qquad \frac{AC}{2 \cdot AB} - 2^{\frac{1}{3}} = 0$

$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

PD = 0.601 inches

PC = 5.299 inches

$\frac{PC}{PD}$ = 8.821

ARAS = 1.323 inches

QR = 1.323 inches

ARAS - QR = 0.000 inches

PD = 0.601 inches

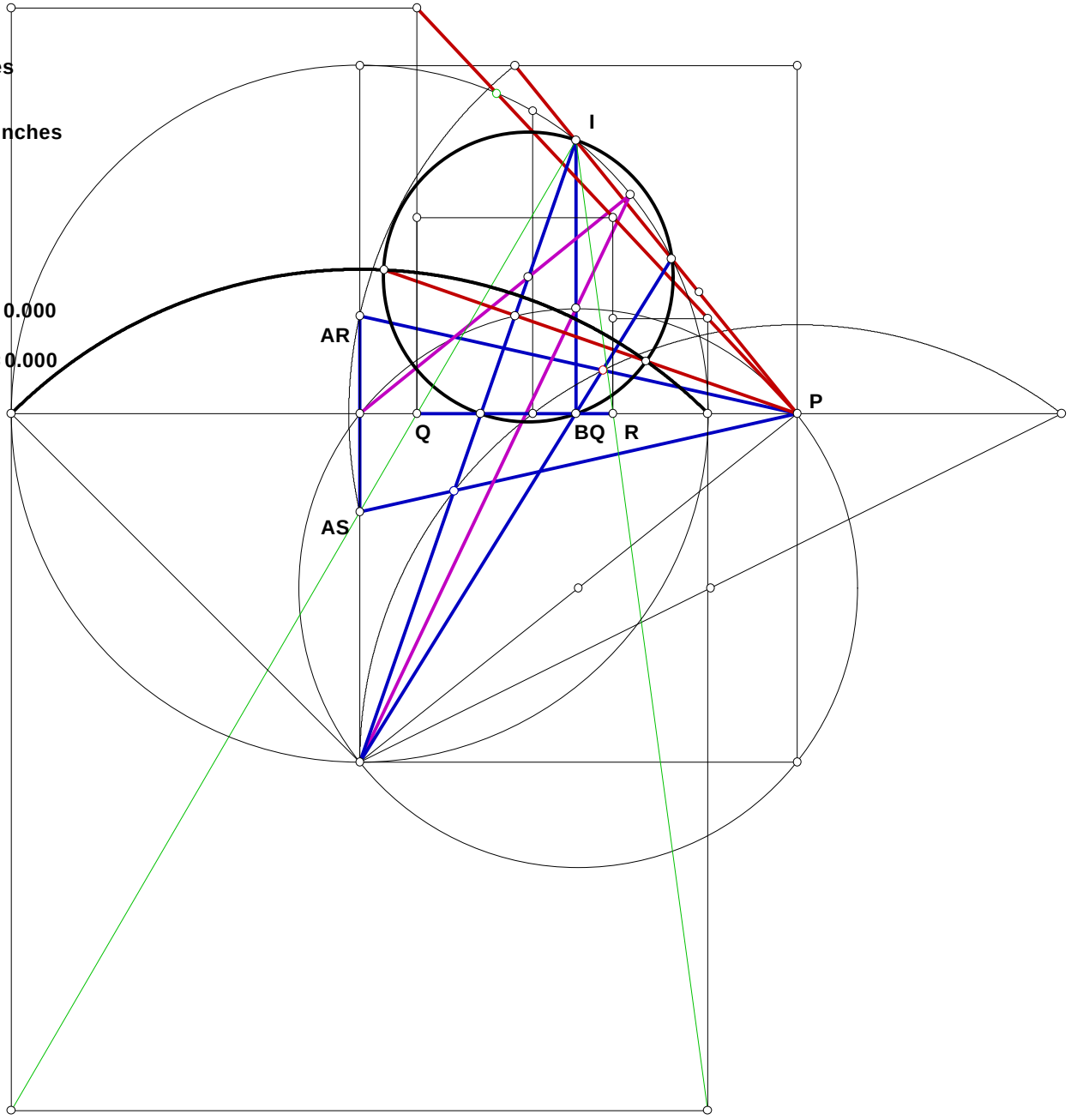
PC = 5.299 inches

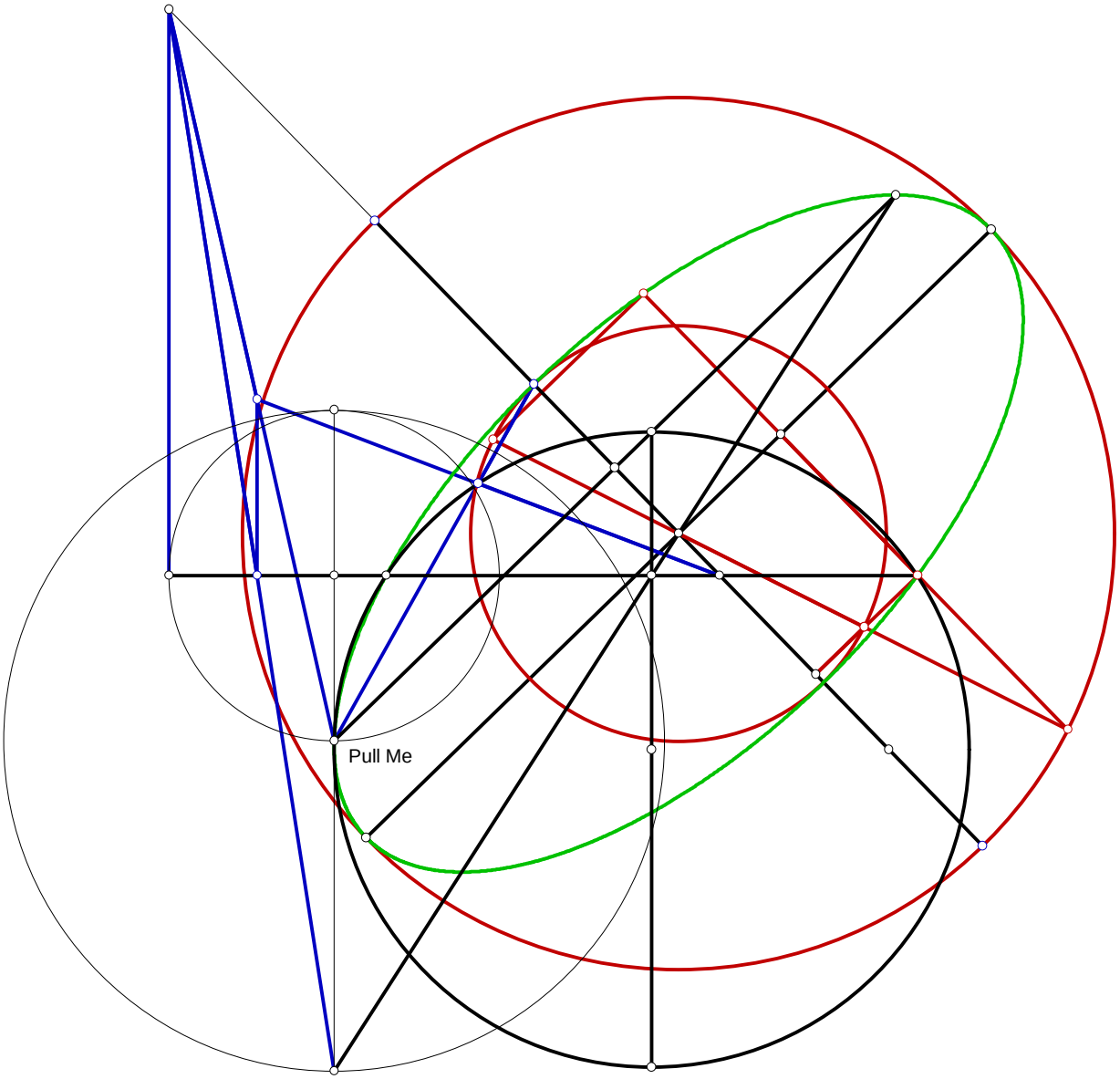
PR = 1.241 inches

PQ = 2.564 inches

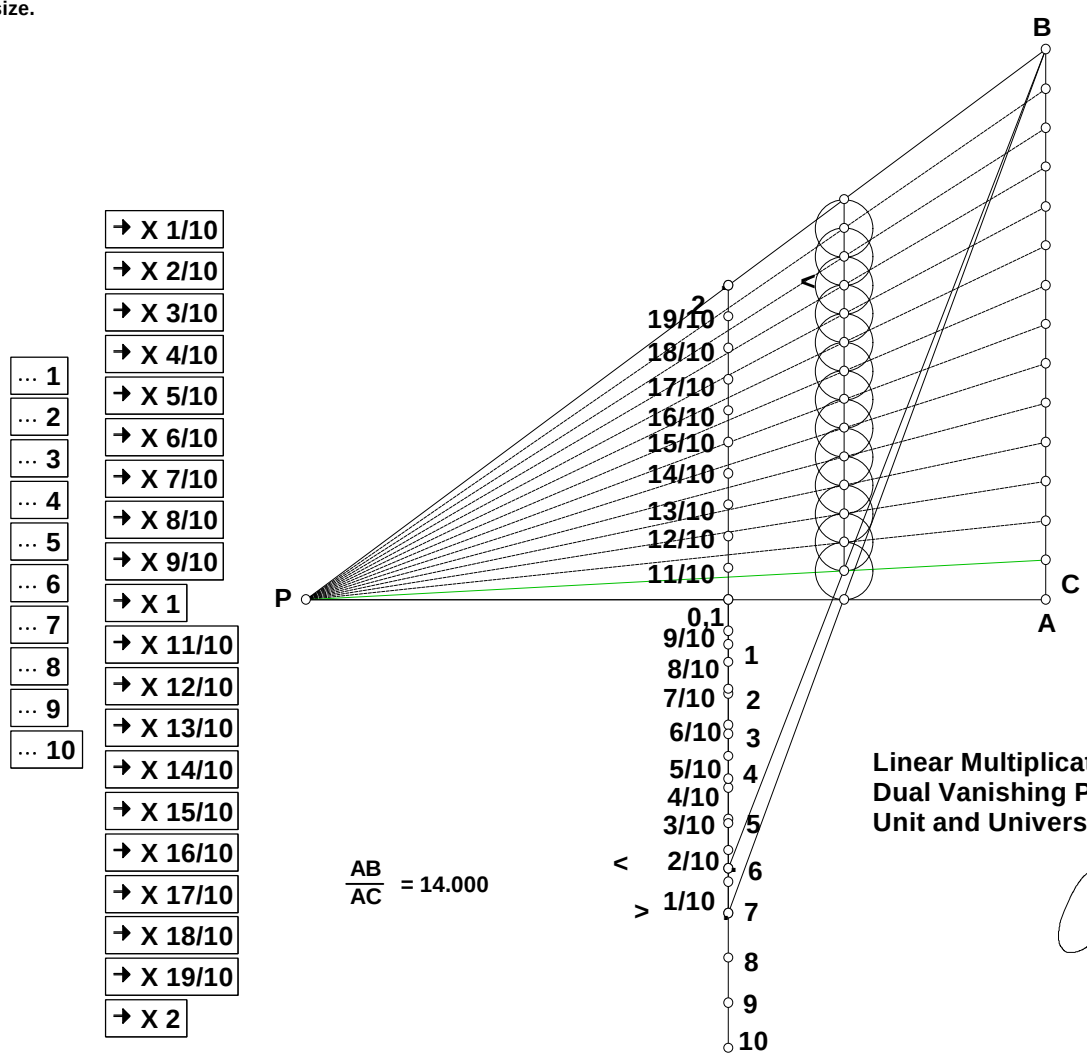
$(PD^2 \cdot PC)\left(\frac{1}{3}\right)$ - PR = 0.000

$(PD \cdot PC^2)\left(\frac{1}{3}\right)$ - PQ = 0.000



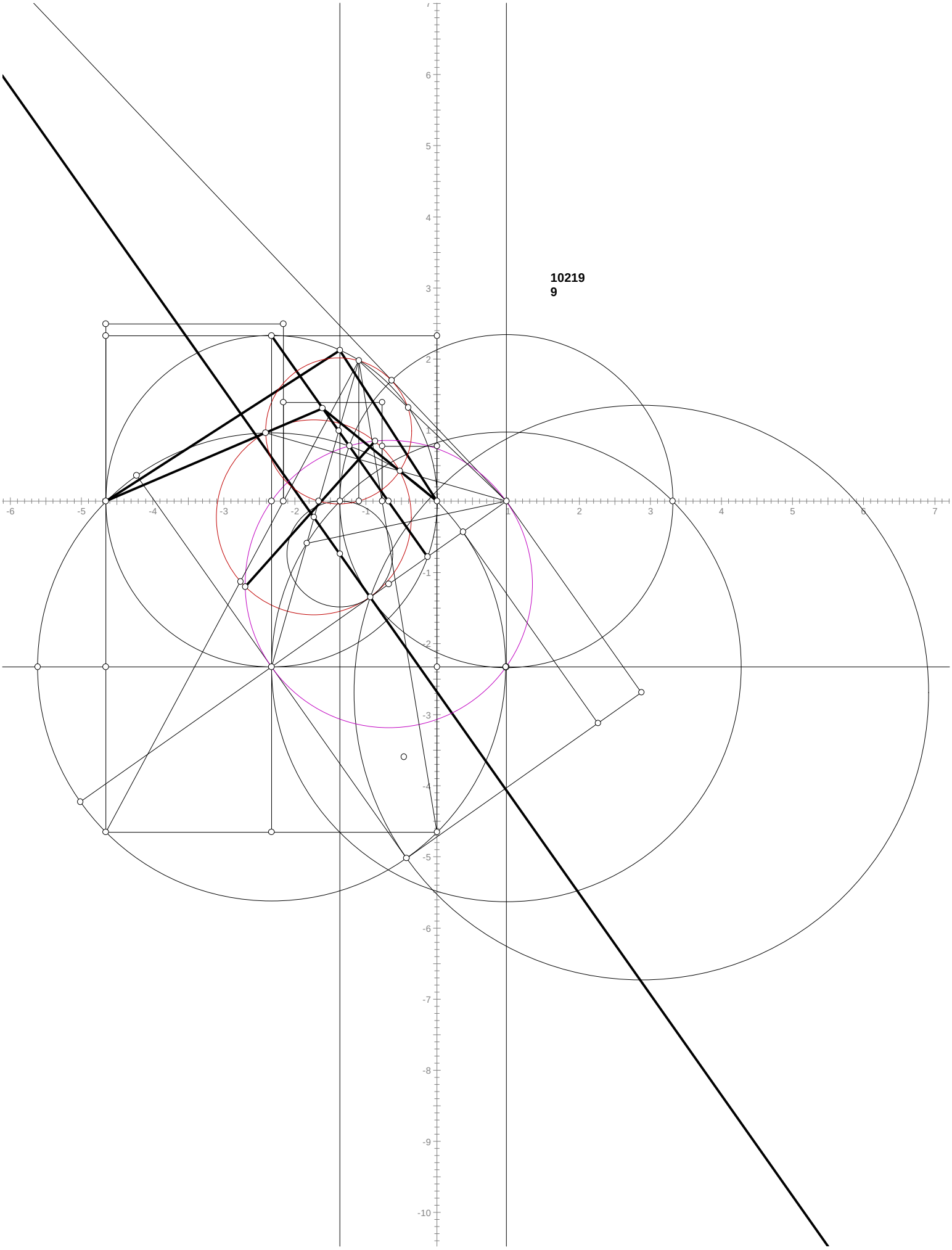

Pull Me

**Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.**



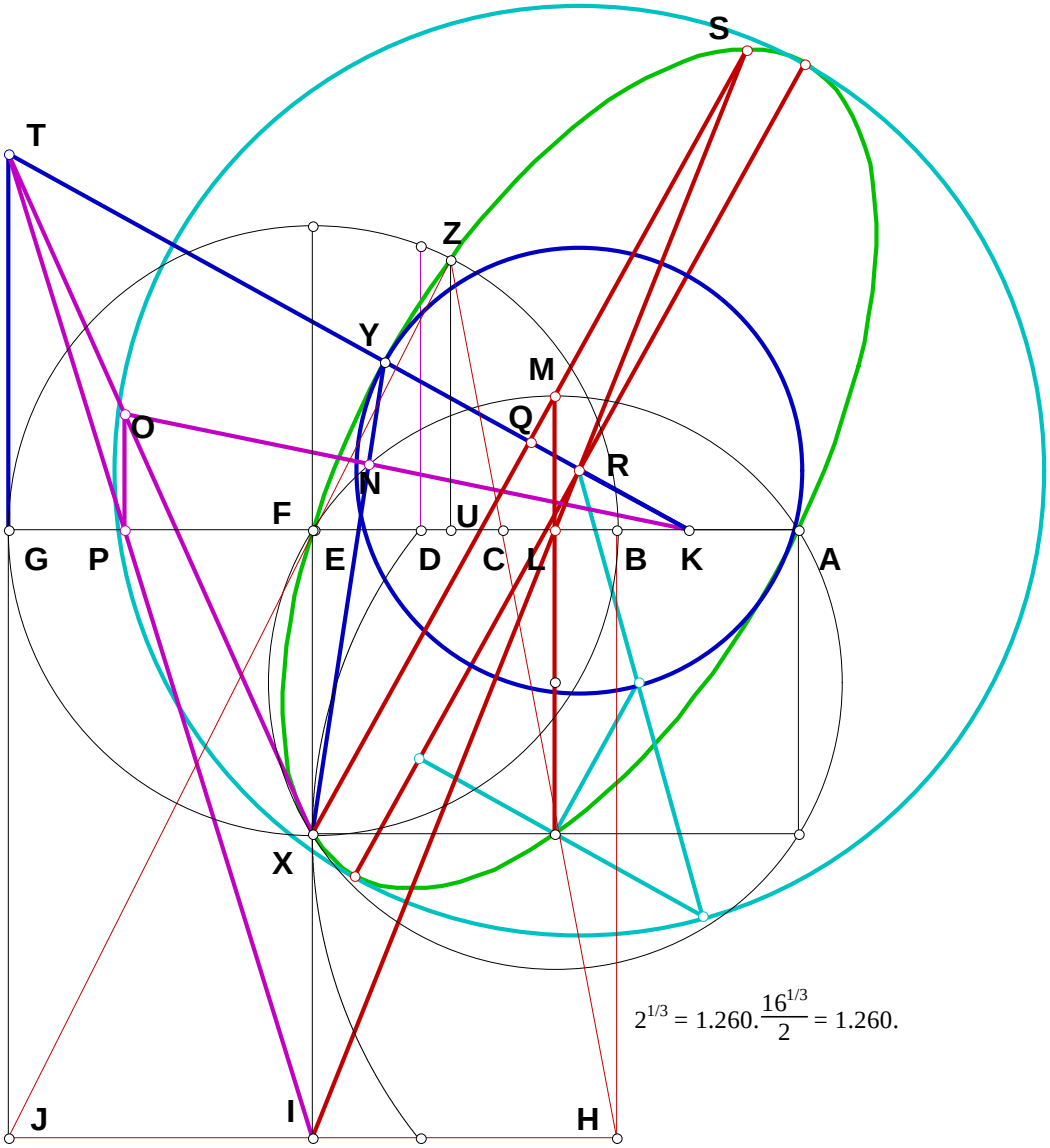

$\frac{AB}{AC}$ = 14.000

**Linear Multiplication and Division,**
**Dual Vanishing Point Method**
**Unit and Universe of Discourse.**


10219
9

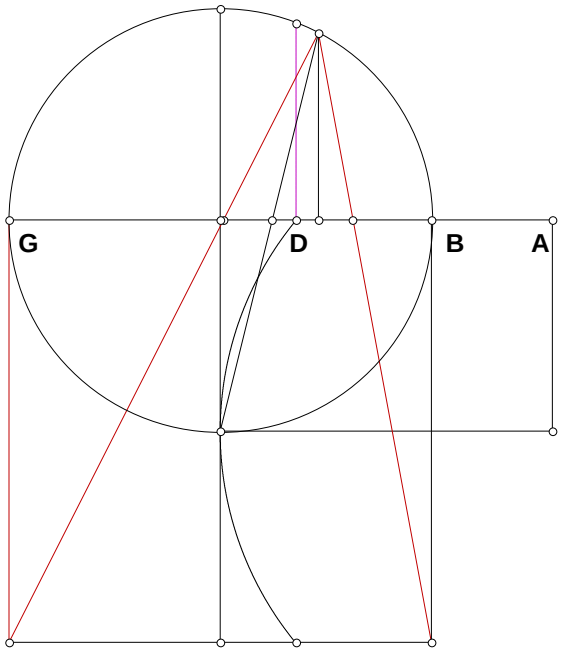
**Appendix A**

Step by Step




**As this method of construction does not quite make it to two, sixteen will do as well.**

**Plate 1**




$AB = 1, N = 16, AG = AB \times N$

$BG = AG - AB, AD = \sqrt{AB \times AG}$

**Plate 2**

$BF = \frac{BG}{2}, FG = BF, AF = AB + BF$

$FX = BF, Mf = \frac{\sqrt{AF^2 + FX^2}}{2}, LF = \frac{FX}{2}$

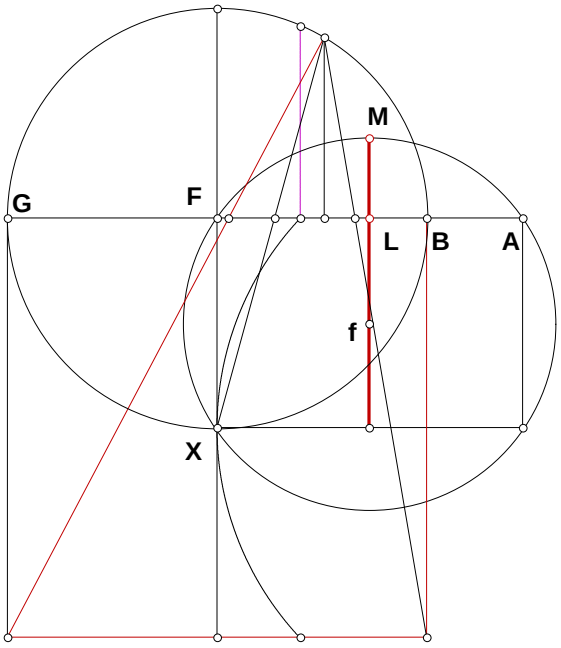
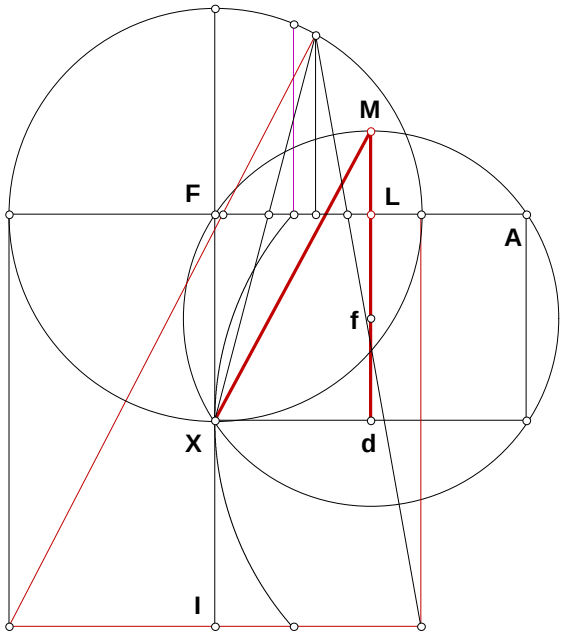


**Plate 3**




$$ML = Mf - Lf,\ FL = \frac{AF}{2},\ Xd = FL,\ df = Lf$$

$$IX = FX,\ Md = Mf + df,\ MX = \sqrt{Xd^2 + Md^2}$$

**Plate 4**

$$SX = \frac{MX \times IX}{IX - ML},\ Lg = \frac{FL \times ML}{ML + FX},\ QX = \frac{SX}{2}$$

$$Fg = FL - Lg,\ Xg = \frac{MX \times Fg}{Xd},\ Qg = QX - Xg$$

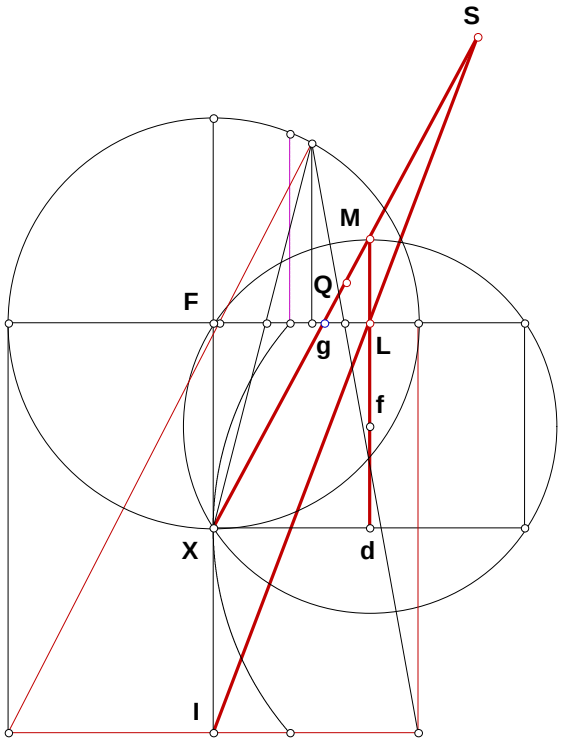
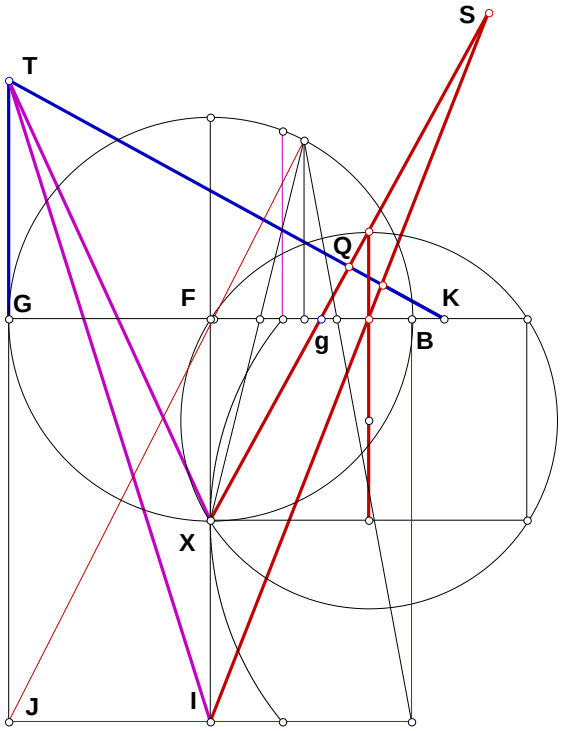


**Plate 5**




$$Kg = \frac{Xg \times Qg}{Fg},\ GK = FG + Fg + Kg,\ GJ = BG$$

$$GT = \frac{Fg \times GK}{FX},\ JT = GJ + GT,\ IJ = BF$$

**Plate 6**

$$FP = \frac{IJ \times GJ}{JT}$$

$$OP = \frac{IX \times GT}{JT}$$

$KP = Fg +$

$Kg + FP$

$Pi = Lf$

$Oi = OP + Pi$

$$hi = \frac{KP \times Oi}{OP}$$

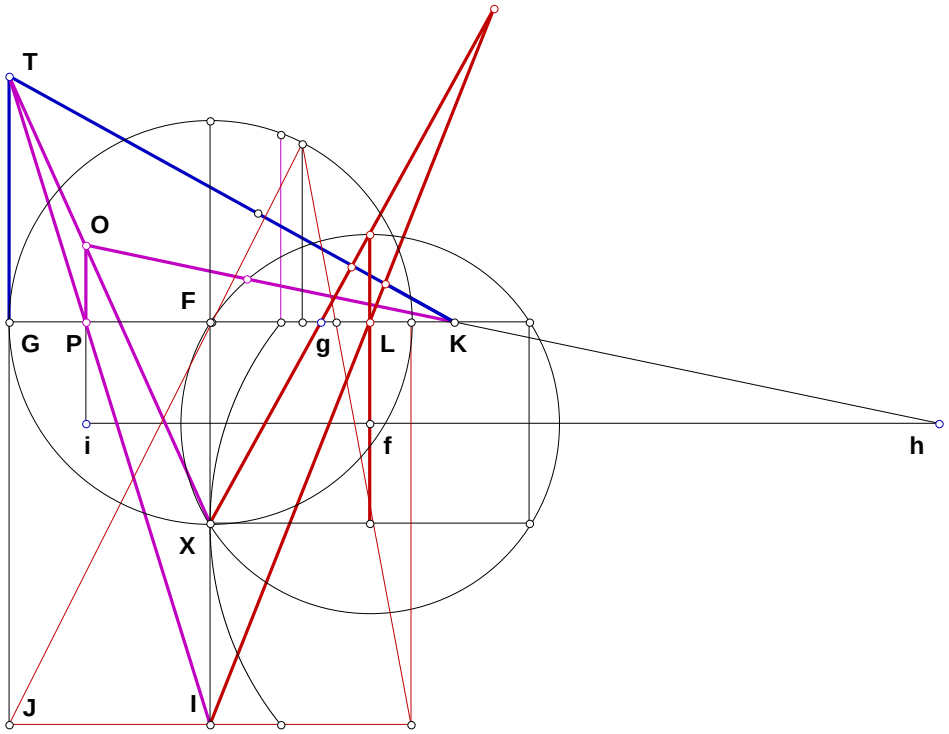
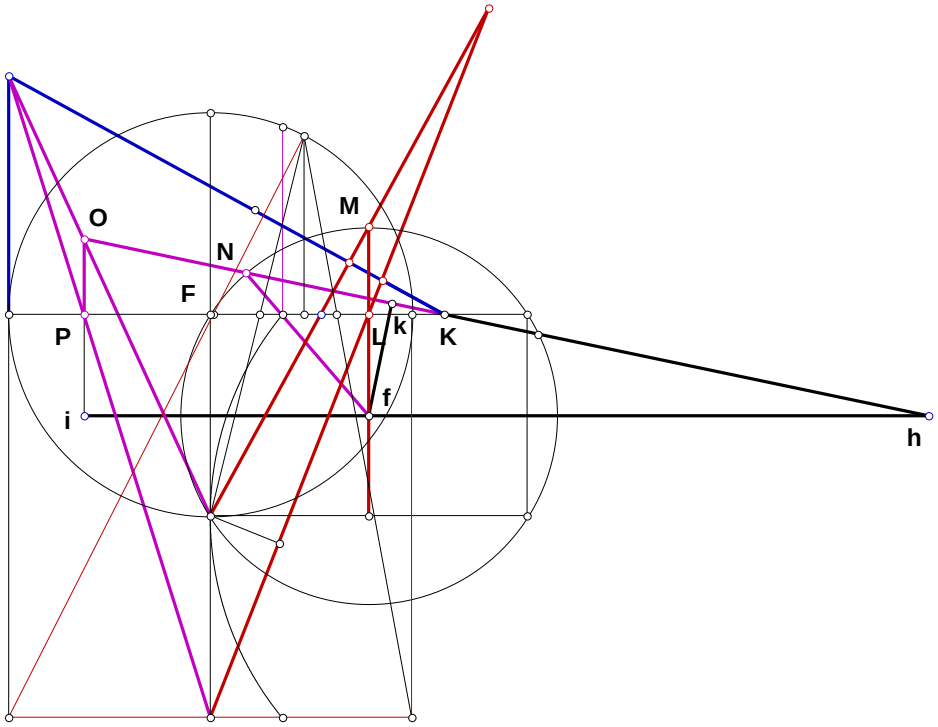


**Plate 7**




$fi = FP + FL$

$fh = hi - fi$

$KO = \sqrt{KP^2 + OP^2}$

$hk = \frac{KP \times fh}{KO}$

$fk = \frac{OP \times fh}{KO}$

$Nf = Mf$

$Nk = \sqrt{Nf^2 - fk^2}$

**Plate 8**

$Nh = hk + Nk$

$Oh = \frac{KO \times Oi}{OP}$

$NO = Oh - Nh$

$KN = KO - NO$

$NI = \frac{OP \times KN}{KO}$

$KI = \frac{KP \times KN}{KO}$

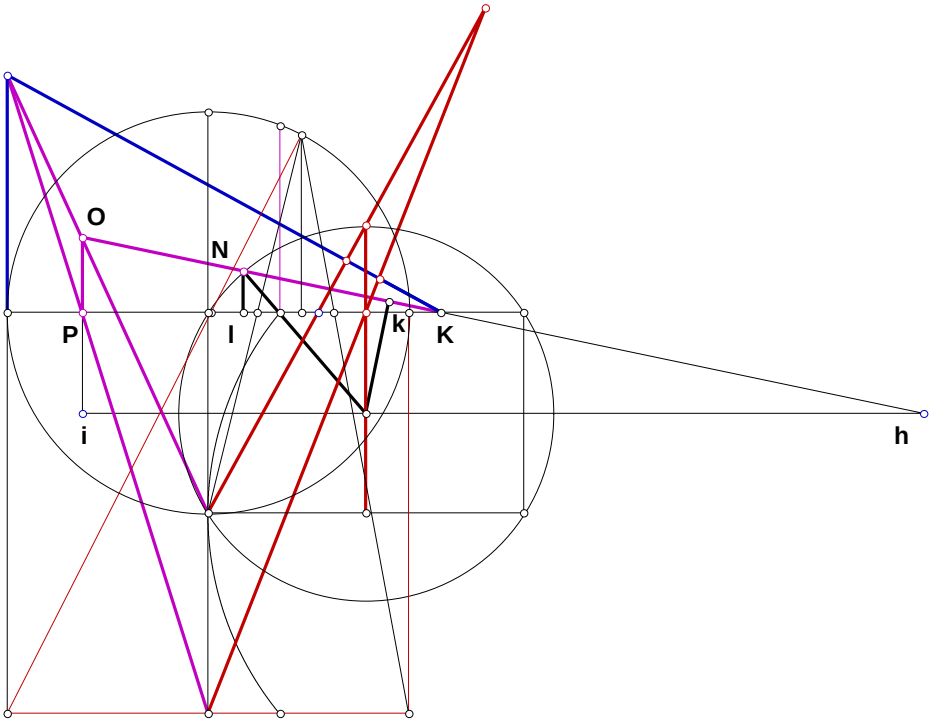
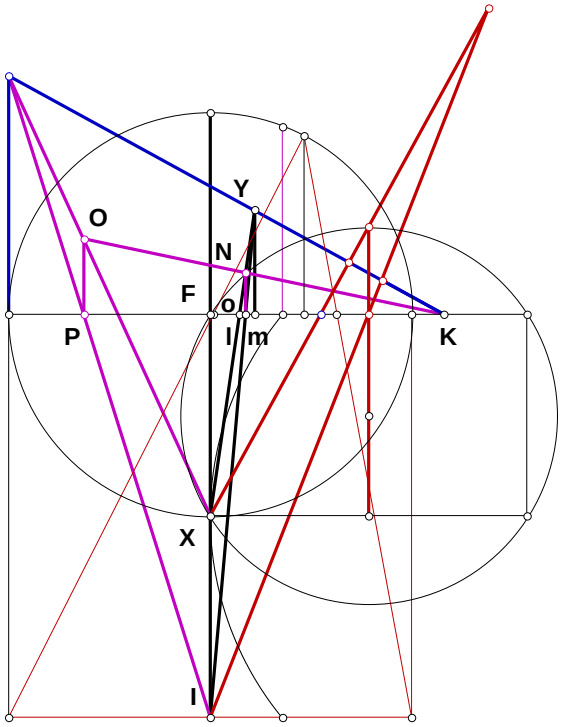


**Plate 9**




$$FK = KP - FP,\ Fl = FK - Kl,\ NX = \sqrt{(FX + Nl)^2 + Fl^2}$$

$$XY = \frac{NX \times IX}{IX - Nl},\ Fm = \frac{Fl \times XY}{NX},\ Km = FK - Fm$$

**Plate 10**

$$Fo = \frac{Fl \times FX}{FX + Nl},\ Xo = \frac{NX \times Fo}{Fl},\ mo = Fm - Fo$$

$$Ym = \frac{FX \times mo}{Fo},\ FI = 2 \times BF,\ IL = \sqrt{FL^2 + FI^2}$$

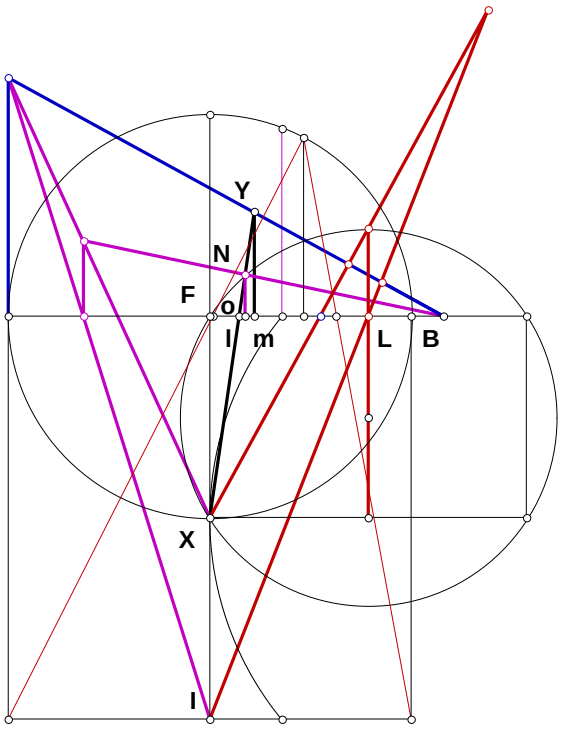
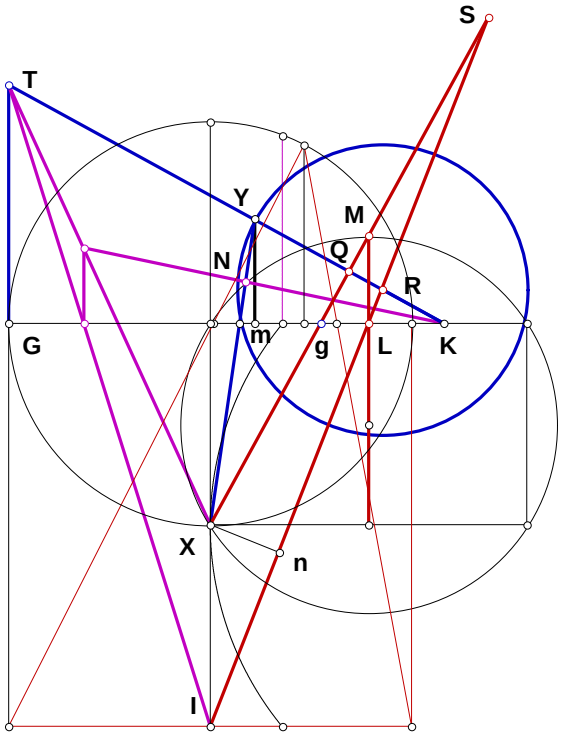


**Plate 11 The Minor Axis**




$$IS = \frac{IL \times IX}{IX - ML},\ In = \frac{IS^2 - SX^2 + IX^2}{2 \times IS}$$

$$Xn = \sqrt{IX^2 - In^2},\ QR = \frac{Xn \times QX}{IS - In}$$

$$KT = \sqrt{GK^2 + GT^2},\ KY = \frac{KT \times Ym}{GT}$$

$$KQ = \frac{GK \times Qg}{GT},\ RY = KY - KQ + QR$$

**Plate 12**

$$Xd = FL, dp = \frac{GK \times Xd}{KT},\ pq = QR$$

$$dq = dp - pq,\ Xp = \frac{GT \times Xd}{KT},\ Op = QX - Xp$$

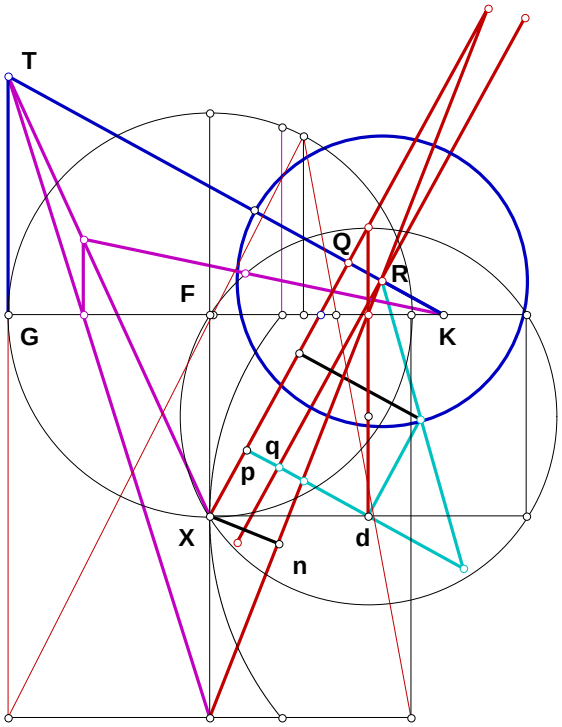
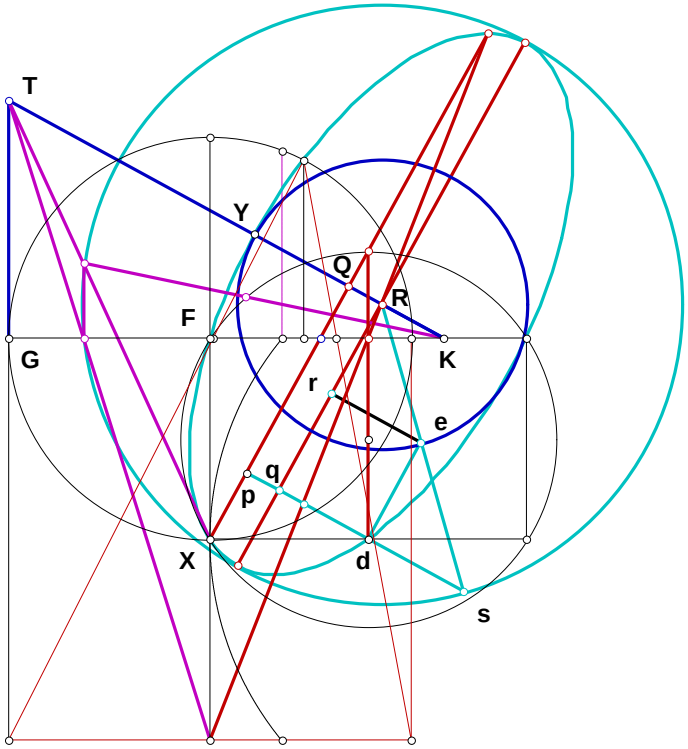


**Plate 13 The Major Axis**




$er = dq,\ Re = RY,\ Rr = \sqrt{Re^2 - er^2},$

$Rq = Qp,\ Rs = \dfrac{Re \times Rq}{Rr}$

**Plate 14**

$AC = (AB^2 \times AG)^{1/3},\ AE = (AB \times AG^2)^{1/3}$

$BC = AC - AB,\ BE = AE - AB,\ CE = BE - BC$

$EG = BG - BE$

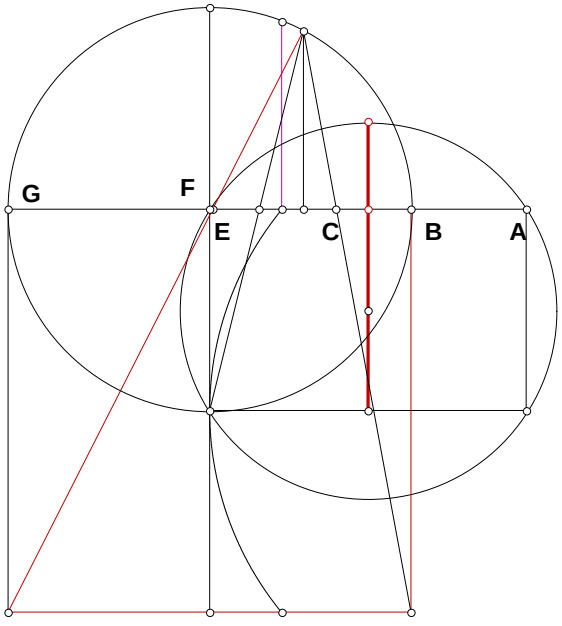
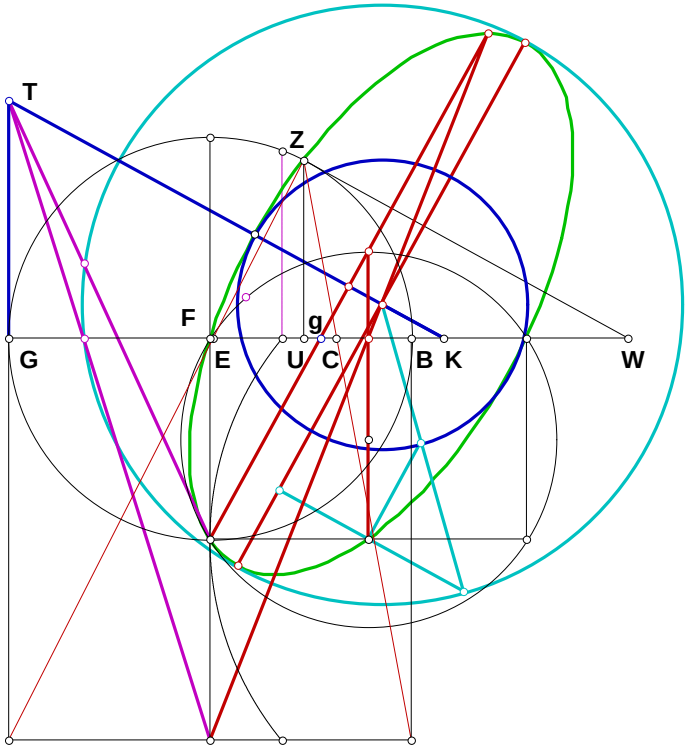


**Plate 15**




$$CU = \frac{BC \times CE}{BC + EG},\ BU = BC + CU$$

$$GU = BG - BU, UZ = \sqrt{BU \times GU}$$

$$UW = \frac{GK \times UZ}{GT},\ Gg = GK - Kg$$

**Plate 16**

$$tu = QR,\ gt = \frac{KT \times tu}{GK},\ Gt = Gg + gt$$

$$GW = GU + UW,\ Wt = GW - Gt$$

$$tv = \frac{GT \times Wt}{KT},\ Kt = GK - Gt$$

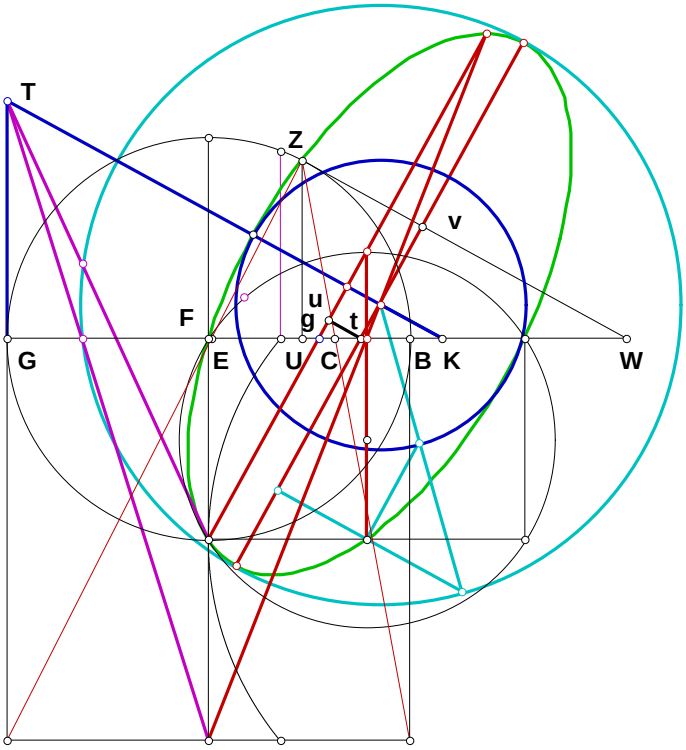
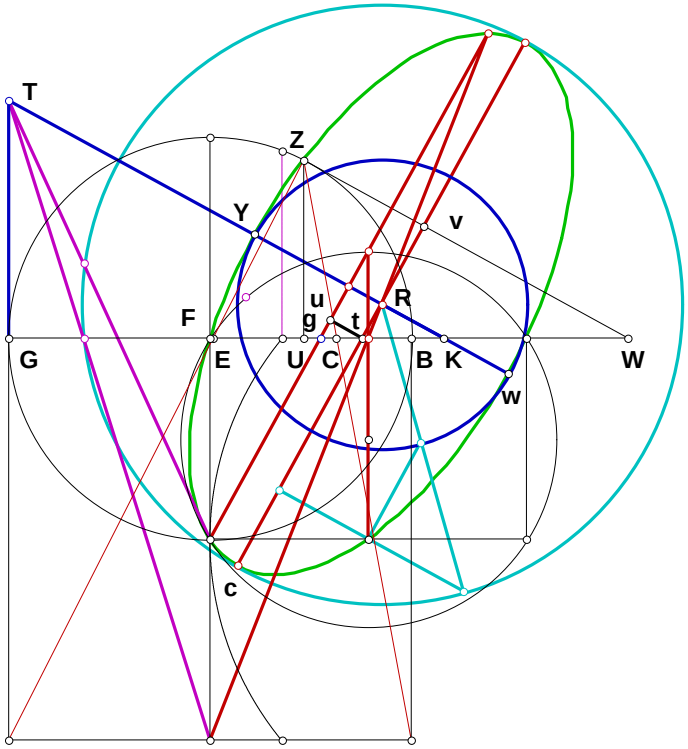


**Plate 17**




$$Rt = \frac{GT \times Kt}{KT},\ Rv = tv - Rt$$

$$bc = 2 \times Rs,\ Rc = Rs,\ cv = Rc + Rv$$

$$Yw = 2 \times RY$$

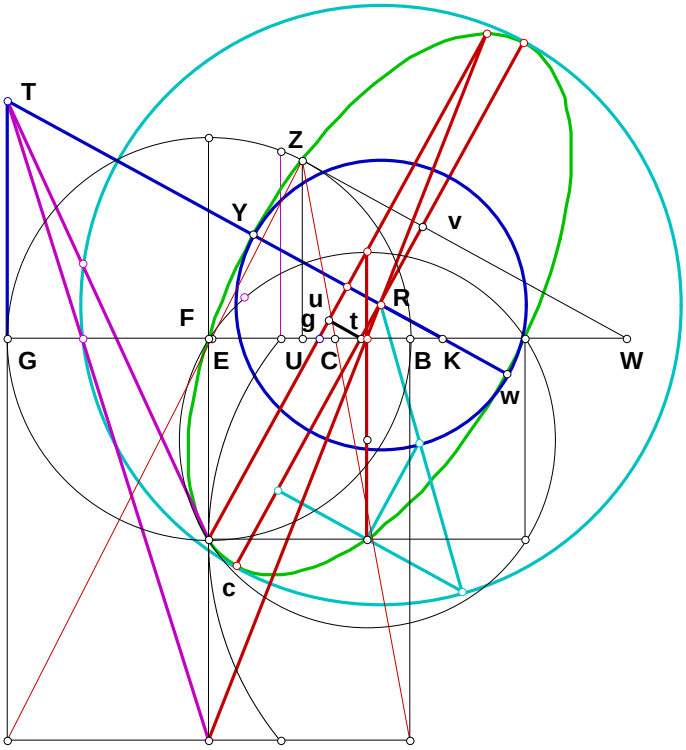
**Plate 18**

$$WZ = \frac{KT \times UZ}{GT},\ Wv = \frac{GK \times tv}{GT}$$

$$Zv = |WZ - Wv|$$

$$N_1 = Yw,\ N_2 = bc,\ N_3 = cv,\ N_4 = bc$$

$$\sqrt{N_3 \times (N_4 - N_3)} \times \frac{N_1}{N_2} - Zv = 0$$



# First Philosophy:

## The Body Prophet
## And
## The Path To Enlightenment
## Through
## The Theory Of Forms

Number of Words: 86629

Number Of Pages: 192

# First Philosophy: The Body Prophet And The Path To Enlightenment Through The Theory Of Forms

as presented by Plato and Aristotle
with attempted explanations by John Clark

Dedicated to those individuals in history who envisioned a humanity embracing ***Judgment Day*** as the day that men have come to learn judgment–to have learned how to reason–for it is reason that is the enlightenment of the soul:

> "Thus that in the soul which is called mind (by mind I mean that whereby the soul thinks and judges) is, before it thinks, not actually any real **thing**." ***On The Soul*** by Aristotle

> How can we ask our fellow man to be reasonable when we ourselves know not what reason is? Author

# Forewarning–Worth!

I set out on a journey to find me a wise man that took me to the highest reaches of the earth. Upon arriving atop the tallest mountain, upon which the wisest of men sat, I discovered, much to my dismay, that he had but half a brain. And so I set out again to find me a wise man that took me to the deepest reaches of the sea. Upon arriving beneath the deepest ravine, within which sat the wisest of men, I discovered, much to my dismay, that she had but half a brain. And so I set off to find me two asses, one to carry the wisest man of the tallest mountain and the wisest man of the deepest watery ravine and carried them both, upon their respective ass, to a marshy plain. There I set them upon the damp ground, between heaven and hell, before each other but each other they did not see. Upon realizing the meaning of all my vain labor, I sat beneath a Dumnudumn tree and began to cry. **Janus** by Devoid Void

## What is Your Value?

Answer these questions.

1). What are you worth or what is your value?

2). What is the value of any thing?

When does a toaster *quâ* toaster have value, when it cannot make toast or when it can?

When does a car *quâ* car have value, when it can not function or when it provides safe and dependable transportation?

When does an ink-pen *quâ* ink-pen have value, when it can no longer write or when it can write?

When does a toaster *quâ* toaster have the higher value, when it makes toast erratically or when it makes consistently fine toast?

When does a car *quâ* car attain to the highest value, when it performs erratically or when it performs dependably?

When does an ink-pen *quâ* ink-pen have the highest value? Etc., etc., etc.

From these examples and many more one may come to believe that the greatest value of a thing is attained in the perfection of the function of that thing–especially if they have but one definition for value–one measure in their purse–see through a single eye. It may follow that in order to know what our own value is, we must know what our function is. In order to attain to our greatest value, our greatest worth as human beings, we will have to find a perfection of that function.

Do you know what your function is? Do you care to know what your function is?

When is a worker ready to work, when he has excused himself from all labor or when he seeks out his job?

Why do you cry? asked the Dumnundumn tree, whereupon, I told the tree the tale of the two wise men. Most unfortunate. quipped the tree, but perhaps you should try beating the dog out of the frequency. Of course I recalled that when two frequencies are added together the resulting frequencies were the sum, the difference and the original two frequencies but I did not understand the meaning of the tree. A Dumnundumn tree can be most annoying with its crypticism. Give me kudos or give me criticism but please spare me the crypticism! A fact of reality, I know, is that crypticism is the spice of the Dumnundumn tree. So, in resolve, I sat the afternoon, after drying my tears, in meditations. How could I add the two wise men together to get the

dog of wisdom? Should I find me a surgeon? No, who could perform such a thing, besides that would not leave the original two. Perhaps they could have children!

The man of the mountain and the woman of the sea had children. It was not easy getting them to marry and have a family–she would cry, *you have no feelings*, and he would yell, *you know not reason*!–but over the years they had four children. One child had no brains at all; one was like its mother destined to live in a sea of tears; one like its father never aware of the warmth of a sun; and one had a whole brain. **Janus** by Devoid Void

## Heart andor Head?

Commonly one will hear people speak of ignoring reason because it cannot be trusted–to follow one's heart or feelings, meaning emotion, while others speak of following reason–to ignore feelings because they cannot be trusted. Who is right? Should we rise above sensory experience or drown in a sea of tears? Perhaps neither or perhaps both.

Is it possible to learn what our function is if we *à priori* demand that our function is not really our function? If the emotional deny reason or the reasonable deny emotion can we attain to any kind of value? Maybe the human brain is divided into two based on something real and necessary, say the definition of ***a thing*** itself? Maybe the human brain is divided the same way our senses are divided, between those that abstract ***matter*** and those that abstract ***form***–and perhaps ***form*** is to ***reason*** as ***matter*** is to ***emotion***? Perhaps it is not possible to attain to the perfection of our function until emotion becomes circumscribed by reason–emotion enrobed by reason.

**Definition of Thing**: A definition provides that name of both ***a thing's form*** and ***a thing's matter***.

Form is absolute and is never difference, matter is relative and is always difference.

By analogy then,

**Definition of Motive**: Motive is emotion (matter) circumscribed by reason (form).

There once was a child with a whole brain who set out on a journey to find a wise man that took him to the highest reaches of the earth. Upon arriving atop the tallest mountain, upon which the wisest of men sat, he discovered, much to his dismay, that the man had but half a brain. And so the child with a whole brain set out again to find a wise man, a journey that took him to the deepest reaches of the sea. Upon arriving beneath the deepest ravine, within which sat the wisest of men, the child with a whole brain discovered, much to his dismay, that the woman had but half a brain. And so he set off to find him two asses, one to carry the wisest man of the tallest mountain and the wisest man of the deepest watery ravine and carried them both, upon their respective ass, to a marshy plain. There he set them upon the damp ground, between heaven and hell, before each other but each other they did not see. The child with a whole brain bid each wise man to speak–in his own tongue, both about the same thing and at the same time. One of the wise men, the woman of the sea, spoke with matter–crafts–and the other wise man, the man of the mountain, spoke with form–words. The devoid (matter) and the void (form) came together to create a thing called knowledge. Thus the child achieved a formal education through his parents and became truly wise. **Janus** by Devoid Void

At this point, the reader may not understand, perhaps Plato and Aristotle can help explain.

# NOTE

There are sections in this book with extra wide margins within which to take notes, wisdom, however, prefers something erasable.

In order to acclimatize the reader into the use of synonyms by Plato and Aristotle, I have compiled a table. The table is not exhaustive. How the reader may use a word to mean something must be set aside and they must make an attempt to understand the use of words as the authors used them.

Every body sense acquires and manipulates things, even the human mind. In order to start one on the way to comprehension, let me render a definition;

**Thing**: ***A thing*** is any ***differences*** what so ever circumscribed by any ***form*** what so ever.

**Context of Synonyms**

| | |
|---|---|
| ***Actuality***: | "the **actuality** or **shape** is smoothness. It is obvious then, from what has been said, what sensible **substance** is and how it exists–one kind of it as **matter**, another as **form** or **actuality**"<br>"for the **essence** certainly attaches to the **form** and the **actuality**.[1]"<br>"in a formula there is always an element of **matter** as well as one of **actuality**; e.g. the circle is 'a plane figure'.[2]"<br>"But, as has been said, the proximate **matter** and the **form** are one and the same ***thing***, the one ***potentially***, and the other ***actually***.[3]"<br>"Obviously, therefore, the **substance** or **form** is **actuality**."<br>"Further, then, these **substances** must be without **matter**; for they must be eternal, if **anything** is eternal. Therefore they must be **actuality**."<br>"Now **matter** is **potentiality**, **form actuality**;"<br>"For, as we said, the word **substance** has three meanings **form**, **matter**, and the complex of both and of these three what is called **matter** is **potentiality**, what is called **form actuality**." |
| ***Archetype***: | "In another sense (2) the **form** or the **archetype**, i.e. the statement of the **essence**, and its **genuses**, are called 'causes' (e.g. of the octave the relation of 2:, and generally number), and the parts in the definition." |
| ***Differentiae***: | "the contraries as **differentiae**, i.e. **Forms**)" |
| ***Essence***: | "but they furnish the **forms** as the **essence** of every other thing"<br>"The **form** or **pattern**, i.e. the definition of the **essence**"<br>"(By **form** I mean the **essence** of each thing and its primary **substance**.)"<br>"when I speak of **substance** without **matter** I mean the **essence**."<br>"'A part' may be a part either of the **form** (i.e. of the **essence**)" |

[1] Notice that after defining *actuality* to mean *form* he uses the two words as if they are now different.

[2] By substitution 'in a formula there is always an element of ***matter*** as well as one of ***form***; e.g. the circle is 'a plane figure'.'

[3] Is this a translator's error? Should be, 'But, as has been said, the proximate ***matter*** and the ***form*** are of one and the same thing, the one ***potentially***, and the other ***actually***.'

| | |
|---|---|
| | "And since the **essence** is **substance**" |
| ***Genus***: | "Moreover, primary **substances** are most properly called **substances** in virtue of the fact that they are the entities which underlie everything else, and that everything else is either **predicated** of them or present in them. Now the same relation which subsists between primary **substance** and everything else subsists also between the species and the **genus**: for the species is to the **genus** as **subject** is to **predicate**, since the **genus** is **predicated** of the species, whereas the species cannot be **predicated** of the **genus**. Thus we have a second ground for asserting that the species is more truly **substance** than the **genus**."<br><br>"the **genus** is the **pattern** of the various **forms**–of–a–**genus**; therefore the same thing will be **pattern** and **copy**."<br><br>"As the **genus** is 'in' the species and generally the part of the specific **form** 'in' the definition of the specific **form**." |
| ***Nature***: | "and not only is the first **matter nature** (and this in two senses, either the first, counting from the thing, or the first in general; e.g. in the case of works in bronze, bronze is first with reference to them, but in general perhaps water is first, if all things that can be melted are water), but also the **form** or **essence**, which is the end of the process of becoming.–(6) By an extension of meaning from this sense of 'nature' every **essence** in general has come to be called a 'nature', because the **nature** of a thing is one kind of **essence**."<br><br>"For when we state the essential **nature** of the sphere or circle we do not include in the formula gold or bronze, because they do not belong to the **essence**, but if we are speaking of the copper or gold sphere we do include them."<br><br>"Whether the **form** or the **substratum** is the essential **nature** of a physical object is not yet clear."<br><br>"Another account is that '**nature'** is the **shape** or **form** which is specified in the definition of the **thing**."<br><br>"Thus in the second sense of '**nature**' it would be the **shape** or **form** (not separable except in statement) of things which have in themselves a source of motion. (The combination of the two, e.g. man, is not '**nature**' but 'by **nature**' or '**natural**'.) [4]"<br><br>"The **form** indeed is '**nature**' rather than the **matter**; for a thing is more properly said to be what it is when it has attained to fulfillment than when it exists potentially."<br><br>"Since '**nature**' has two senses, the **form** and the **matter**, we must investigate its objects as we would the **essence** of snubness. That is, such things are neither independent of **matter** nor can be defined in terms of **matter** only. Here too indeed one might raise a difficulty. Since there are two **natures**, with which is the physicist concerned?"<br><br>"And since '**nature**' means two things, the **matter** and the **form**, of which the latter is the end, and since all the rest is for the sake of the end, the **form** must be the cause in the sense of 'that for the sake of which'." |
| ***Order***: | "By the 'opposite' I mean the 'unmusical', by the 'subject' 'man', and similarly I call the absence of **shape** or **form** or **order** the 'opposite', and the bronze or stone or gold the 'subject'." |
| ***Potentiality***: | Now **matter** is **potentiality**, **form actuality**;<br><br>For, as we said, the word **substance** has three meanings **form**, **matter**, and the complex of both and of these three what is called **matter** is **potentiality**, what is called **form actuality**. |
| ***Predicate***: | it is evident what the underlying **matter** is, of which the **forms** are **predicated** in the case of |

[4] The phrase, ***not separable except in statement*** meaning that the perception is always of things, has a great deal of significance and number of implications.

| | |
|---|---|
| | sensible things<br><br>**a thing** must share in its **form** as in something not **predicated** of a **subject**<br><br>the **predicate** is a **form** and a 'this', the ultimate **subject** is **matter** and **material substance**<br><br>In the case of **predicates** constituting the essential **nature** of a thing, it clearly terminates, seeing that if definition is possible, or in other words, if essential **form** is knowable, and an infinite series cannot be traversed, **predicates** constituting a thing's essential **nature** must be finite in number.<br><br>And the positive **form** is one–the **order**, the acquired art of music, or any similar **predicate**. |
| ***Quality***: | "Alike in the sphere of **quality** and in that of **quantity** there is that which corresponds rather to **form** and that which corresponds to **matter**." |
| ***Subject***: | the **predicate** is a **form** and a 'this', the ultimate **subject** is **matter** and **material substance**.<br><br>I call the absence of **shape** or **form** or **order** the 'opposite', and the bronze or stone or gold the '***subject'***.<br><br>it follows that the soul must be a **ratio** or formulable **essence**, not a **matter** or **subject**. |
| ***Substance***: | "It follows, then, that '**substance'** has two senses, (A) ultimate **substratum**, which is no longer **predicated** of **anything** else, and (B) that which, being a 'this', is also separable and of this **nature** is the **shape** or **form** of each thing."<br><br>"The word '**substance'** is applied, if not in more senses, still at least to four main objects; for both the **essence** and the universal and the **genus**, are thought to be the **substance** of each thing, and fourthly the **substratum**."<br><br>"(By **form** I mean the **essence** of each thing and its primary **substance**.)"<br><br>"(for we should have looked for **forms** in these cases if in any; for these are **substances** if **anything** is so)"<br><br>"It is obvious then, from what has been said, what sensible **substance** is and how it exists–one kind of it as **matter**, another as **form** or **actuality**, while the third kind is that which is composed of these two." |

One might see that Socrates, Plato and Aristotle were trying to abstract the definition of ***thing*** and elucidate the implications of this definition as the paradigm upon which all of reasoning is derived. The senses of the human body start with things and all manipulation follows from ***a thing***. The first task of processing ***a thing*** is the abstraction of either ***form*** or ***matter***.

## Paradigm of A Thing

As one asserts a point (***form***) of linearity (***matter***), or elucidates the linearity (***matter***) between two points (***form***) to create ***a thing*** called a ***line***, so too can one only assert or deny ***form*** or ***matter*** of ***a thing*** in the unit sentence. How these unit sentences are added and subtracted to form more complex sentences is a topic for another time. Suffice it to say, that without understanding the paradigm upon which a sentence is founded, it is not possible to write a grammar, consequently what has come down to us as grammar may have been overly embellished by miscomprehension–much like Bertrand Russell's ***Theory Of Types***. As ***a thing*** has one and only one definition, the founding paradigm for all logics is the same, be that the logic of a Geometric Figure (Grammar) or of English Grammar.

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# Introduction

A man must be gifted with very considerable ability before he can learn that everything has a class and an absolute essence; and still more remarkable will he be who discovers all these things for himself, and having thoroughly investigated them is able to teach them to others. ***Parmenides*** by Plato

## What Is This Text About?

Tsze-lu said, "The ruler of Wei has been waiting for you, in order with you to administer the government. What will you consider the first thing to be done?"

The Master replied, "What is necessary is to rectify names."

"So! indeed!" said Tsze-lu. "You are wide of the mark! Why must there be such rectification?"

The Master said, "How uncultivated you are, Yu! A superior man, in regard to what he does not know, shows a cautious reserve.

"If names be not correct, language is not in accordance with the truth of things. If language be not in accordance with the truth of things, affairs cannot be carried on to success. ***Analects*** by Confucius

The quest for reason seems to be indigenous to a very few individuals in every culture. This paper is about ***The Theory Of Forms*** found in the early Greek works of Plato and Aristotle. The importance of ***The Theory Of Forms*** cannot be overestimated. I am familiar with no work by any author who, in my estimation, understood what the theory is or its importance in the psychological foundation of the human mind [5]. How I became interested in the study this theory and its importance to me is outlined in this introduction–but in a nutshell, ***The Theory Of Forms*** is an examination of the definition of the word ***thing*** and what the definition implies for all of reasoning. The paradigm upon which all reasoning is and must be based is the definition of ***a thing***–the consequences of that definition is ***The Theory Of Forms***. ***A thing*** is any ***difference*** what so ever circumscribed by any ***form*** what so ever. ***A thing–being***–in the application of primary predication–is the foundation of reason and the foundation of the subject matter of ***First Philosophy***.

The definition of ***a thing*** not only divides all of logic into two major categories, one of which has not yet been understood as a logic, but also determines what the simple sentence is, what predication is, and how language itself must be conceived. The definition of ***a thing*** determines even what a ***Formal Presentation*** is. A ***Formal Presentation*** is two logic systems, one a relatiologic and one a tautologic, side by side saying the same thing, an idea that is based on the very definition of the word ***thing***. This idea has been so misunderstood that almost all of what has been called ***Formal Systems*** are no more than waste paper–equivalent to mental masturbation. Plato stated (Seventh Letter) that the foundation of philosophy (***First Philosophy***) was very simple and could not be forgotten once understood (***Elementary Predication***), but he also said that he would never set it out in expository manner–Aristotle's method. Neither method, the pull method of Plato or the push method of Aristotle, seems to have been effective in teaching basic logic–grammar–historically Buridan's ass just wont move toward reason.

## What Are My Qualifications?

**I am a factory worker–a pseudo-intellectual**. After winning a couple of writing awards, I

[5] This does not mean that I am familiar with much that has been written on the subject.

dropped out of college to pursue my own path–there was and is to my mind something horribly wrong with man's ability, my ability, to think. I don't live in an age where the examination, the exercise, of our ability to think is *a manly pursuit*, I live in an age where even the smallest courtesy in examining reasoning can get one thrown in jail [6]. Trying to entice people into an examination of how we think is met with reactions from people who are actually pained at the prospect of self-examination. Public entertainment has about reached the pits of common denomination–personal and social masturbation. How easily amused is the mass man–and how proud he is of such a vulgar achievement. This is a very strange world I live in. And so I dropped out. I set my own studies and do the best I can, alone, to answer some very fundamental questions that I don't believe I have ever seen rightly answered. I work on the Yoga Sutras, a translation is hinted at the likes of which I have never seen–a translation commensurate with the literary master Patanjali, not the Patanjali of mythology. I work in geometry and have developed a figure that leads one to all the information needed to solve the Delian Problem–the abstraction of cube roots. This work is written in Algebraic and Geometric Grammar. All of the things I work on are to improve my understanding. But perhaps I am just another non-entity, just another lost and meaningless soul. All the same **I am just a factory worker–a pseudo-intellectual**.

## Where Am I Going?

What is my purpose in life? Why do I live? What does it mean to be a better or worse person? What is the path to personal self-improvement? I believe that there is a right way to be, therefore I believe that these questions can be answered unequivocally. If one were to want a toaster that was the best that a toaster could be–a toaster that would make reliable breakfast toast every morning, not too light, not too dark, not an uneven baked surface, one that would not take *all day* to make the toast–then one would outline what the function of a toaster was and use these certain parameters as criteria for the accomplishments of that toaster. I must know what a toaster is suppose to do–what its designed function is–before I could even offer a suggestion in regard to perfecting that toaster. The same holds true for mankind–one must know what a man is for, what is his function, because the recognition of function precedes any means of objective determination of improvement by that measure.

Does man have a function? Many claim that he does not by using a misdefined concept of *free will* or even by denying obvious and simple causality. Does man have a purpose? One must know what a *man is* before that question can be answered. There can be no criteria for ethics, law, or any other form of individual or social behavior until someone/something somewhere can give us, mankind, a hint as to what *a man is*! Although Plato and ***The Theory Of Forms*** is the intention of this paper, getting to Plato is going to take a seemingly very strange path. Maybe someone has given us a hint, but that hint cannot be seen until a certain goal is reached–the same as any other marker. Is there a class concept by which a man can be measured?

[6] I was trying to defend myself in court, when the judge ask if anyone had any questions. I had not spoken a word for the entire proceedings, the judge acted as if I were not there. I raised my hand and asked for the states definition of marriage. The judge went into an immediate rage and threatened to have me thrown in jail for daring to ask that question. Today I am an American Slave. I am being forcibly dispossessed of my earnings and have been dispossessed of all my property.

## The Number of His Name

> "there will be no cessation of evils for the sons of men, till either those who are pursuing a right and true philosophy receive sovereign power in the States, or those in power in the States by some dispensation of providence become true philosophers." ***The Seventh Letter*** by Plato.
>
> "The wise man should go through life with the same attitude of mind towards his country. If she should appear to him to be following a policy which is not a good one, he should say so, provided that his words are not likely either to fall on deaf ears or to lead to the loss of his own life. But force against his native land he should not use in order to bring about a change of constitution, when it is not possible for the best constitution to be introduced without driving men into exile or putting them to death; he should keep quiet and offer up prayers for his own welfare and for that of his country." ***The Seventh Letter*** by Plato.

Ever had someone tell you, "It can't be done." but those words sound so absurd that you yearn to prove them wrong? Sometimes that very word *impossible* grates on the very fabric of one's soul. Sometimes the problem seems to be trivial, but once solved is so much more than trivial, for example, ***Number Theory and Its History*** by O. Ore © 1948 contains;

> "The names of the Bible have been a favorite field for gematry [7]. Most famous is the Number of the Beast, given in the Revelation of St. John (13:18) "Here is wisdom. Let him that hath understanding count the number of the beast; for it is the number of a man and his number is six hundred three score and six." In spite of the innumerable researches on this question through the centuries it seems impossible to arrive at any definite solution. Clearly many names will have the same number. In the violent theological feuds of the Reformation it was a vicious stroke to write the opponent's name in such a way that his number became the fatal 666 of the beast."

Why a number problem? What are numbers? I have always been amazed at man's conception as to what arithmetic and numbers were. They are just names in an order naming convention. Here, in the very fact of numbers is a hint as to why the problem was given in numbers–an ***ordered naming convention***. There are unordered naming conventions, and there are ordered naming conventions–but traditionally, historically, currently Arithmetic is not seen simply as another grammar-another logic system and the myths about it are promoted by those *who have been better paper trained than the average dog*. At any rate I had come to wonder if I could figure out this bad boy's name. I went to a Hebrew bookstore and purchased a dictionary set.

First of all, if *how* a number is translated into letters were discretional and not conventional, then there is no answer. Trying to push out an answer discretionally and not conventionally is simply a waste of time, motivated by some lack of reason. I remember being bidden in the text of the scripture to have but one measure in my purse–to see with a single eye–therefore I would suspect that this problem could only be solved by such an ideal. I reject cabalism and resort to the social convention of the Hebrew numbering system itself. Cabalism, by its very methods, violates the very tenets promoted by the Bible–and is thus a methodology based on contradiction. The fact that the answer depends on convention is no trivial statement. Reason is not possible, judgment is not possible, judgment day is not possible, until the human mind recognizes and uses standard conventions. The first stipulation then is that there is one and only one method of translating from number to letter and letter to number, and that is the established convention of the Hebrew number system. The **number** of his name is ***a*** name–not ***some*** name.

[7] Gematria: a cabalistic method of interpretation of the Hebrew scripture based upon the numerical value of the letters of the word.

"or the **number of his name** … for it is the number of a man, and his number is Six Hundred Threescore and Six."

A numerology chart found in ***From One to Zero*** by George Ifrah provided me with ,

| **Letter Name** | **Sound** | **Number value** | | **Letter Name** | **Sound** | **Number value** |
|---|---|---|---|---|---|---|
| Aleph | A | 1 | | Lamed | L | 30 |
| Beth | B | 2 | | Mem | M | 40 |
| Gimel | G | 3 | | Nun | N | 50 |
| Daleth | D | 4 | | Samekh | S | 60 |
| He | H | 5 | | Ayin | ‘ | 70 |
| Vav | V | 6 | | Pe | P | 80 |
| Zayin | Z | 7 | | Tsade | Ts | 90 |
| Heth | H | 8 | | Qoph | Q | 100 |
| Teth | T | 9 | | Resh | R | 200 |
| Yod | Y | 10 | | Shin | Sh | 300 |
| Kaph | K | 20 | | Tav | T | 400 |

666 = 400 + 200 + 60 + 6 by the convention of Hebrew numbering.

| | | |
|---|---|---|
| Tav | T | 400 |
| Resh | R | 200 |
| Samekh | S | 60 |
| Vav | V | 6 |

I do not know Hebrew, in fact the only language that I do know is a portion of my native tongue. Taking the four letters, I searched the dictionary for 400, 200, 60, 6. TRSV. There is no word listed in Hebrew Dictionary. Curious, the starting letter, according to the dictionary, is *a letter that ends words*–this implies that the numbers are backwards.

Why would anyone, construct a meaningless puzzle? Certainly it cannot be meaningless–unless it was written by a moron. Perhaps there are hints in the text itself as to what to do with the four letters.

Re:13;2 And the beast which I saw was like unto a leopard, and his feet were as the feet of a bear, and his mouth as the mouth of a lion: and the dragon gave him his power, and his seat, and great authority.

Who described themselves in the Bible using all three images?

Ho:13;7 Therefore I will be unto them as a lion; as a leopard by the way will I observe them:

Ho:12;8 I will meet them as a bear that is bereaved of he whelps, and will rend the caul of their heart, and there will I devour them like a lion; the wild beast shall tear them.

The image is God but it is not God because,

> Re:12;18 Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six Hundred Threescore and six.

It is a man. What is the relationship of God to man given in the scripture?

> Ge:1;27 So God created man in his own image, in the image of God created he him; male and female created he them.

The image, perception is by image. When I look in the mirror I see an image; an image is backwards.

> Re:13;14 … "that they should make an image to the beast…" (Quote from Moses here) Turn it around, 6,60,200,400, or VSRT.

Now I must bid the reader, in to understand the number of his name, once again, make an image of the beast. Now what Hebrew word does this image make? "to shutter" and the conversive (turning the past into the future and the future into the past.) Found in the dictionary written by R. Alcalay. The shutter that turns the past into the future and the future into the past. I recalled to mind the Law, the answer must be repeated two or three times in the text.

> Re:12;18 Here is wisdom. Let him that has understanding ***count the number of the beast***: for it is the number of a man; and his number is Six Hundred Threescore and six.

Putting the numbers in counting sequence is the same as taking their image–for at least the third time, make an image of the beast. And so there are at least three ways the text tells one to turn the letters around–to take their image. What is the significance of ***the shutter that turns the past into the future and the future into the past***?

## Shutters

> "And these words, which I command thee this day, shall be in thine heart; And thou shalt teach them diligently unto thy children, and shalt talk of them when thou walkest by the way, and when thou liest down, and when thou riset up. ***And thou shalt bind them for a sign upon thine hand***, ***and they shall be as frontlets between thine eyes***. And thou shalt write them upon the posts of thy house, and on thy gates.
>
> De:27;2 "And it shall be on the day when ye shall pass over Jordan unto the land which the Lord thy God giveth thee, that thou shalt set thee up great stones, and plaister them with plaister: And thou shalt write upon them all the words of the law, when thou art passed over, that thou mayest go in unto the land which the Lord thy God giveth thee, a land that floweth with milk and honey; as the Lord God of thy fathers hath promised thee. Therefore it shall be when ye be gone over Jordan, that ye shall set up these stones, which I command you this day, in mount Ebal, and thou shalt plaister them with plaister."

## Seals

> Is:29;11 And the vision of all is become unto you as the words of a book that is sealed.
>
> Re:5;1 And I say in the right hand of him that sat on the throne a book written within and on the backside, sealed with seven seals.
>
> Re:7;3 "…***till we have sealed the servants of our God in their foreheads.***"
>
> Ez:9:4 "***And the Lord said unto him, Go through the midst of the city, through the midst of Jerusalem, and set a mark upon the foreheads of the men that sigh and that cry for all the abominations that be done in the midst thereof***."
>
> 7 ***But in the days of the voice of the seventh angel, when he shall begin to sound, the mystery of God should be finished, as he hath declared to his servants the prophets.***
>
> 6 …having seven horns and seven eyes which are the seven Spirits of God sent forth into all the earth.

Seven horns are seven ways to protect our life and seven eyes are seven ways to perceive–if one can think in terms of one and only one measure, man has seven senses. What is the sense that turns the past into the future and the future into the past? How is prophecy and judgment the same? Is not the goal of reason to infer consequences? Do we not learn by past experience to predict the results of our behavior? Hello World! How simple does a puzzle have to be? And lastly, a solution is effected yet another way. I am not my stomach, nor my eyes, I am mind–a man is named, is addressed through his mind.

> Re:12;18 Here is wisdom. Let him that has understanding ***count the number of the beast***: for it is the number of a man; and his number is Six Hundred Threescore and six.

Apparently only one man in history was suppose to solve for the name of the beast. This man is supposed to have understanding, but I have no understanding. Now this is strange. At any rate one can consider a man as a collection of, seven–as a matter of biological fact–senses. The beast itself is a metaphor for a human body sense of mind responsible for judgment. The beast is man come unto judgment day–a day he has learned to distinguish right from wrong.

What is the defining characteristic that allows one to classify all the body senses into one group such that he becomes the lamp of seven lights or a beast of seven eyes and seven horns or again as a Jew with seven wives hiding his nakedness? What is the *simile in multis* involved?

## Sense

A definition, rightly described by Aristotle, states both the ***form*** and the ***substance*** of ***a thing***. The definition of this ***thing*** called *a sense* is used like any other unit by which we can determine if another thing is a member of a class or not. It is a convention by which judgment is rendered. Definition determines how things are ordered.

**Body Sense:** A body sense system is that which acquires something from the environment, processes that which it has acquired and the product is used to sustain and promote that human life.

1) Gastro-intestinal system is a sense.

   Acquires food.

2) Cardio-vascular system is a sense.

   Acquires oxygen.

3) Visual system is a sense.

   Acquires perspective.

4) The auditory system for gravitational orientation is a sense.

   Acquires balance.

5) Tactile system for manipulation for crafts is a sense.

   Acquires the raw material for crafts.

6) Reproductive system is a sense.

   Acquires D. N. A. for parenting.

7) The human mind is a sense.

Acquires experience and process it so that we can predict the results of human behavior in order to sustain and promote our life.

**The Human Mind:** That body system which acquires experiences, processes those experiences in order to predict the results of human behavior for the purpose of sustaining and promoting human life.

Of course, I don't have it exactly right, what can be expected of a factory worker? But still, the beast is the sense of judgment performed by the human mind. The name of the beast is acquired from the very biological construction of man himself–objective reality as convention.

Perception determines conception and conception determines will is the causal chain reflected by the definition of every human body sense. The definition of body sense demands that life is initiated through reconnaissance–acts of perception–experience and ultimately terminates in expression–what we do. By accomplishing through the function of each and every body sense those things that sustain and promote the life of the individual–through those things that are peculiar to each and every body sense–we attain to life and a life of reason–judgment. It is this individual concern that sums to the concerns of any social enterprise. This does not vary from individual to individual–or from society to society for there is one and only one definition of man. How we, as individuals and as a collective, attain to those ends which correlate to judgment vary as functions of environment and personal intelligence–like any other craft. Thought is a craft that initiates expression–action–enterprise–that is measured in longevity and quality of life. Grammar, logic, reasoning, judgment serve the purpose *to have life and have it more abundantly*.

By definition the human mind is a body sense and it acquires ***something*** from the environment, like all senses, and processes those ***things***. The processing of ***things*** by the body's senses entails the abstraction of either ***form*** from ***thing*** or ***matter*** from ***a thing***. The human body senses are then divided into two groups, those that abstract ***matter*** (breathing, eating, and craft) from ***a thing***, and those which abstract ***form*** from ***a thing*** (sight, sex, reason). This is why those who have stated that we never know the *thing in itself* were entirely right, as a body sense abstracts either ***matter*** or ***form***–otherwise the sense would not *process* ***a thing*** and not be a sense at all. This fact about abstraction leads us to ***The Theory Of Forms*** expounded in the works of Plato and Aristotle.

The human mind processes ***a thing*** called experience. Experience is gathered during time and it is our experiences that feed the mind so that it can make a product that sustains and promotes our life. That product, as was pointed out by the grammarian Patanjali in a proper translation of the ***Yoga Sutras***, is human behavior mitigated by judgment–predication–prediction–reason. We are the ***Body Prophets*** and it is our experiences that lead us to prophesy. Prophecy is the prediction of the results of human behavior or more generally predicting the results of propositional manipulations–propositional calculus, if you will. Or one might say we are to become philosophers by experiences leading us to judgment. The puzzle about the name of the beast is a children's story for minds just learning to reason. Was it not written that by eating (consuming) of the tree of life (experience) one might learn judgment (reason) and come to judgment day–a day when man can reason in truth–truely reason that is? Reason starts by understanding the definition of a word.

## Definitions

As Aristotle pointed out,

> It is also clear that the loss of any one of the senses entails the loss of a corresponding portion of knowledge, and that, since we learn either by induction or by demonstration, this knowledge cannot be acquired. ***Posterior Analytics*** by Aristotle

the deprivation of experience (induction) is equitable to the loss of sense for without the acquisition of experience there is nothing for the mind to process (demonstration)–words are only ***form***, they must be endowed with ***matter***–experience. What kinds of experiences teach us how ***things*** are manipulated? Crafts. Our own psychology demands that we invest in crafts on a personal level. As ***words*** are the ***form***, ***objects*** of sense the ***matter*** so too ***knowledge*** is ***the thing***. It is these experiences that provide our mind with the material whereby we make abstractions needed to construct and understand definitions that are used to fulfill our function guided by right judgment. Learning how words are associated by rote is not the learning of reason.

If things are defined in terms of ***matter*** and ***form***, how are ***matter*** and ***form*** defined?

> But none of these primeval elements can be defined; they can only be named, for they have nothing but a name, and the things which are compounded of them, as they are complex, are expressed by a combination of names, for the combination of names is the essence of a definition. ***Theaetetus*** by Plato

The definition of the phrase ***a thing*** not only divides logic into two branches [8] but also provides one with a standard to divide words into three groups–those that stand for ***a thing***, those that stand for ***forms***, and those that stand for ***matter***. It has been errantly believed that one can define any word in any group. This may have been a matter of contention between Aristotle and Plato. Plato rightly asserted that ***form*** and ***matter*** could only be named and must be learned by direct experience. Aristotle however called descriptions for constructing those experiences definitions also–thus making it appear that ***form*** and ***matter*** could be defined. As explained by Plato–demonstratred in ***Parmenides***, this is not true, only ***things*** are defined–***form*** and ***matter*** must be abstracted, or as Aristotle pointed out, inducted. Definition is the denotation of ***a things form*** and ***matter***. Neither ***form*** nor ***matter*** can be defined because they are not ***things***. As only ***things*** exist, one cannot say that either ***form*** or ***matter*** exists without contradicting the definition of ***a thing***. In ***Parmenides*** Plato will also point out that one can neither deny form and matter existence.

The meaning of words that denote ***things*** can be transmitted though a definition, how is the meaning of ***forms*** and ***matter*** transmitted? By abstraction–the so-called definition of a circle is not a definition at all, but it is a description for producing a circle from which experience may be acquired. Knowledge of ***matter*** is acquired also by direct experience. Again, ***forms*** and ***matter***, as Plato pointed out, are simply named. One can *indicate* an abstraction–but that is not a definition. What is plainly indicated is that since a social structure relies on common elements–individuals–a common set of crafts allied with a common set of texts, situated in a formal system, that is taught through an educational system is the only means by which to perpetuate a common set of social values needed to maintain said structure. The survival of a social structure depends on shared meaning.

[8] Tautological and relatiological.

## Forms Of Enlightenment

Plato outlined a path to enlightenment by describing what the human mind must accomplish to attain it.

> "But take the case of the other, who recognizes the existence of absolute beauty and is able to distinguish the ***idea*** from the ***objects which participate in the idea***, neither putting the ***objects*** in the place of the ***idea*** nor the ***idea*** in the place of the ***objects***–is he a dreamer, or is he awake?" ***The Republic*** by Plato

This statement is one key to understanding the dialog called ***Parmenides***. Plato will run through all the ways, and consequences of confusing ***form***, ***matter*** and ***a thing***. Putting this statement in general terms of ***The Theory Of Forms*** yields,

> But take the case of one who recognizes the existence of ***definitions*** and is able to distinguish the ***form*** from the ***matter*** which is contained by the ***form***, neither putting the ***matter*** in the place of the ***form*** nor the ***form*** in the place of the ***matter***–his mind is enlightened.

Grammar only becomes operative when it is clear in the mind that a name is a ***form*** and that which is designated–contained–by that name is ***matter***. The concept of definition means that the human mind is manipulating ***real abstract things*** comprised of names as ***form*** and what those names stand for as ***matter*** when it becomes grammatically functional. We must, as a result of this realization, first learn simple predication. What are the mistakes in thinking that the mind must learn to overcome? What does he mean by ***form*** and ***matter***? This brings us to ***The Theory Of Forms***.

## In Short

> ***On The Soul***: Since according to common agreement ***there is nothing outside and separate in existence from sensible spatial magnitudes***, ***the objects of thought are in the sensible forms***, viz. ***both the abstract objects and all the states and affections of sensible things***. ***Hence*** (1) ***no one can learn or understand anything in the absence of sense***, and (when the mind is actively aware of **anything** it is necessarily aware of it along with an ***image***; for ***images are like sensuous contents except in that they contain no matter***. Aristotle.

Thought is ***form*** without ***matter***, thought is possible through the abstraction of ***form*** from ***things***. The early Greek writers were on the brink of comprehending man as a group of senses where reason is divided in the same way ***form*** and ***matter*** are abstracted from the very experiences of ***things*** themselves which are the resources of those senses.

Plato, probably initiated by Parmenides, realized that the definition of ***a thing*** provided the paradigm from which all reasoning is developed–it is *the* great paradigm of logic. This means that grammar, all grammars, are derived from that paradigm. The definition of ***a thing*** is really quite simple, any ***difference*** what so ever circumscribed by any ***form*** what so ever, is ***a thing***. If one is not aware of which words stand for ***things***, which for ***forms***, and which for ***matter***, one cannot help but think non-sense and express non-sense–one is in reality mentally dysfunctional. In that regard the following two quotes are equivalent.

> "But take the case of the other, who recognizes the existence of absolute beauty and is able to distinguish the ***idea*** from the ***objects which participate in the idea***, neither putting the ***objects*** in the place of the ***idea*** nor the ***idea*** in the place of the ***objects***–is he a dreamer, or is he awake?" ***The Republic*** by Plato

> "Thus that in the soul which is called mind (by mind I mean that whereby the soul thinks and judges) is, before it thinks, not actually any real **thing**." ***On The Soul*** by Aristotle

This is what is meant in the Judeo-Christian Scripture that we are dead. This is why the

resurrection of the dead is on judgment day. Judgment Day is the day that man learns to reason, learns judgment, he comes to life. Not a mystical event at all–it is a simple biological fact. Certain wise men preached the resurrection of the dead through the only means, by definition, possible–the exercise of reason. These were and are the true preachers of truth in history.

Every body sense processes ***things*** that it gathers from the environment–fundamentally the process abstracts and uses either the ***matter*** or the ***form*** from ***a thing***. Only when the human mind functions by abstraction derived from direct experience can it be said to live. If it cannot perform its function, it is dead–that man's name will not be written in the Book Of Life. The denial of objective epistemology is the death of mankind. For example, digestion abstracts the ***matter*** and discards the ***form*** of ***the things*** we eat–it does not fabricate its own food. Breathing is the same. Tactile manipulation in crafts the same. Sex, hearing, seeing however all abstract ***form*** in a different way–leaving the ***matter***. Reasoning uses ***forms*** abstracted from ***things*** called experience–it is for this reason that the paradigm is called ***The Theory Of Forms***–if our concern was eating, breathing or crafts, we would call it ***The Theory Of Matter*** [9]. Plato demonstrated that if we removed the crafts from the definition of wisdom, then wisdom cannot be found.

Another way of saying ***a thing*** is that it is ***matter*** that participates in a ***form*** and another way of saying a ***form*** contains ***matter*** is *simile in multis*. Plato stressed the *simile in multis*–the similar ***idea*** in the many ***examples***–the one ***form*** (***set***) comprised of many ***examples*** (***members***). This fact is universal in propositional (***a thing***) manipulation and is a fundamental to the naming convention of any grammar \ logic system. For example, in Euclidean Geometry there is a point (***form***), linear difference (***matter***) and a line (***a thing***). ***Form*** and ***matter*** are always inseparable from ***a thing***. In common grammar one asserts a name of experience. Imagine that someone were to say to you, given any of two of the three elements of Geometry, one may be *asserted* of the other or not. We can ***assert*** (***predicate***) a point of linear difference or the linear difference of a line but we cannot ***assert*** (***predicate***) a point of a point. These elemental assertions and denials are the procedures for constructing any Geometric figure–it is a relatiologic–a grammar. Imagine that someone were to say to you, given any two words in English Grammar one word may be ***asserted*** of the other or not. Have you, the reader, ever been taught in school the fundamentals of assertion and denial–a basic to understanding grammar? I, for myself, cannot recall such teachings–but then, I may have slept through it.

Potatoes are food. A = B.

Tomatoes are not weapons! D ≠ E.

Elemental assertions and denials are what is used to construct all sentences–these Aristotle called *premises*. Two premises construct a definition when the assertion is of ***a thing's*** ***form*** and ***a things*** ***matter***.

There is one set of ideas that underlies all grammar systems, one set of ideas that underlies all the manipulations we engage in with the environment–each system may be called a craft or a grammar or a logic if one recognize the *simile in multis* underlying them all. How do we know in any logic system which element may be asserted of which especially in a grammar system, like common grammars, where all words are composed from the same alphabet? Just as there are

[9] ***The Ethics*** by Spinoza was an attempt at ***The Theory Of Substance*** that he had offered as a counter to ***The Theory Of Forms***, however Spinoza was not particularly bright, having no understanding of the abstraction, and botched the whole affair from the start.

causal relationships for constructing a Geometric figure, there are causal relationships for constructing the English figure called a sentence. Because the abstraction is ***The Common Universal Of Sense***, common sense for short, if one cannot comprehend, for example, the principles of Euclidean Geometry, that they be denied, then one has no hope of ever being literate in any grammar. There is one and only one founding paradigm upon which all reasoning is based, be it expressed in common grammar, geometry, arithmetic, algebra–or even frying an egg. One of the goals one must achieve to become enlightened is the real and objective process of abstracting the *simile in multis* and it in all the grammars we speak.

## Methods

We cannot command the eyes to see, nor can sanity be legislated, but we can discover and engage in the exercise of the eyes and mind we do have. Short of a formal system exampled by Euclid, there are two general methods of writing to try and teach a subject–one is covert and one is overt. Both of these methods rely upon the memory of the individual readers. Plato tried covertly to educate men in the ways of reason through entertaining dialogs and was opposed (Seventh Letter) to the overt Aristotelian method of exposition. Examining Plato's prohibition against writing, perhaps acquired from Socrates, the dialogs, modeled after Aesop's fables, appears to be a compromise with his natural abhorrence for the mass man.

# Definition

> Stranger. We ought always to come to an understanding about the thing itself in terms of a definition, and not merely about the name minus the definition. ***The Sophist*** by Plato

How does a word get its meaning? What does *definition* mean? How is a word defined? Where does the definition of a word come from? Does *definition* supply the *meaning* of a word? What is the difference between *definition* and *description*?

## Plato on Definition

***The Seventh Letter***: For everything that exists there are three instruments by which the knowledge of it is necessarily imparted; fourth, there is the knowledge itself, and, as fifth, we must count the thing itself which is known and truly exists. The first is the name, the, second the definition, the third the image, and the fourth the knowledge. If you wish to learn what I mean, take these in the case of one instance, and so understand them in the case of all.

1). A circle is a thing spoken of, and its name is that very word which we have just uttered.

2). The second thing belonging to it is its definition, made up names and verbal forms. For that which has the name "round," "annular," or, "circle," might be defined as that which has the distance from its circumference to its center everywhere equal.

3). Third (image), comes that which is drawn and rubbed out again, or turned on a lathe and broken up–none of which things can happen to the circle itself–to which the other things, mentioned have reference; for it is something of a different order from them.

4). Fourth, comes knowledge, intelligence and right opinion about these things. Under this one head we must group everything which has its existence, not in words nor in bodily shapes, but in souls–from which it is clear that it is something different from the nature of the circle itself and from the three things mentioned before. Of these things intelligence comes closest in kinship and likeness to the fifth, and the others are farther distant [10].

***The Seventh Letter***: Again you must learn the point which comes next. Every circle, of those which are by the act of man drawn or even turned on a lathe, is full of that which is opposite to the fifth thing. For everywhere it has contact with the straight. But the circle itself, we say, has nothing in either smaller or greater, of that which is its opposite. We say also that the name is not a thing of permanence for any of them, and that nothing prevents the things now called round from being called straight, and the straight things round; for those who make changes and call things by opposite names, nothing will be less permanent (than a name). Again with regard to the definition, if it is made up of names and verbal forms, the same remark holds that there is no sufficiently durable permanence in it. And there is no end to the instances of the ambiguity from which each of the four suffers; but the greatest of them is that which we mentioned a little earlier, that, whereas there are two things, that which has real being, and that which is only a quality, when the soul is seeking to know, not the quality, but the essence, each of the four, presenting to the soul [11] by word and in act that which it is not seeking (i.e., the quality), a thing

[10] Notice that Plato's five actually reduce to three–the paradigm of a thing. 1). Names and the ability to manipulate names. (forms) 2). Experience and the ability to manipulate experience. (matter) 3). The thing itself.

[11] In his use of *soul* mind should have been used by the translator.

open to refutation by the senses, being merely the thing presented to the soul in each particular case whether by statement or the act of showing, fills, one may say, every man with puzzlement and perplexity.

Here Plato is well on his way, but he is not there yet.

***The Seventh Letter***: In one word, the man who has no natural kinship with this matter cannot be made akin to it by quickness of learning or memory; for it cannot be engendered at all in natures which are foreign to it. Therefore, if men are not by nature kinship allied to justice and all other things that are honorable, though they may be good at learning and remembering other knowledge of various kinds–or if they have the kinship but are slow learners and have no memory–none of all these will ever learn to the full the truth about virtue and vice. For both must be learnt together; and together also must be learnt, by complete and long continued study, as I said at the beginning, the true and the false about all that has real being. After much effort, as names, definitions, sights, and other data of sense, are brought into contact and friction one with another, in the course of ***scrutiny and kindly testing by men who proceed by question and answer without ill will,*** with a sudden flash there shines forth understanding about every problem, and an intelligence whose efforts reach the furthest limits of human powers. Therefore every man of worth, when dealing with matters of worth, will be far from exposing them to ill feeling and misunderstanding among men by committing them to writing. In one word, then, it may be known from this that, if one sees written treatises composed by anyone, either the laws of a lawgiver, or in any other form whatever, these are not for that man the things of most worth, if he is a man of worth, but that his treasures are laid up in the fairest spot that he possesses. But if these things were worked at by him as things of real worth, and committed to writing, then surely, not gods, but men "have themselves bereft him of his wits [12]."

*scrutiny and kindly testing by men who proceed by question and answer without ill will*, here is the form by which the Platonic dialogs are based. This was a pastime of early Greek Philosophers. In the Seventh Letter one learns what topics Plato will be exampling and by what method he will produce them. There is a dialog, ***Phaedrus***, which also gives a very valuable clue as to what the outline of a piece will be based on–a principle. Where Aesop used moral principles, Plato used principles of reason–when the reader grasps the principle of reason, they will then grasp the outline of a piece.

***Phaedrus***: Socrates. First, the comprehension of scattered particulars in one idea; as in our definition of love, which whether true or false certainly gave clearness and consistency to the discourse, the speaker should define his several notions and so make his meaning clear.

One of the foundations of reasoning.

***The Sophist***: Stranger. We ought always to come to an understanding about the thing itself in terms of a definition, and not merely about the name minus the definition.

Today it is common for a highly educated individual to not even notice the error in a statement such as 'Plato discussed knowledge of the absolute, but failed to discuss absolute knowledge.' which is so much like the mystics who translate *life's future* into *future lives*.

***Theaetetus***: Socrates. Come, you made a good beginning just now; let your own answer about roots be your model, and as you comprehended them all in one class, try and bring the many sorts of knowledge under one definition.

[12] Plato did not write in the expository manner, like Aristotle, instead he wrote in a way meant to stimulate thinking–unfortunately those who are not so quick witted still try to claim that Plato had expository meaning when he often had quite the opposite intention.

A founding principle of the *name* is the simple expression of an identity, *name* = *name*. By a name a class/set of objects is denoted. A definition is a standard of convention by which judgement of class membership, class inclusion or class exclusion, is determined–thus how names are deployed–part of a naming convention. Definition is a naming convention by which association of experiences, provided by the bodies senses, with names for ***a thing***, ***a thing's form*** and ***a thing's matter*** is effected. If ***A*** is ***a thing***, ***B*** its ***form*** and ***C*** its ***matter*** then ***A*** = ***B*** + ***C***, ***B*** = ***A*** – ***C***, and ***C*** = ***A*** – ***B***. The first equation is definitive, the second and third are abstractive.

***Theaetetus***: Socrates. Let me give you, then, a dream in return for a dream:–I thought that I too had a dream, and I heard in my dream that the primeval letters or elements out of which you and I and all other things are compounded, have no reason or explanation; you can only name them, but no predicate can be either affirmed or denied of them, for in the one case existence, in the other non-existence is already implied, neither of which must be added, if you mean to speak of this or that thing by itself alone. It should not be called itself, or that, or each, or alone, or this, or the like; for these go about everywhere and are applied to all things, but are distinct from them; whereas, if the first elements could be described, and had a definition of their own, they would be spoken of apart from all else. But none of these primeval elements can be defined; they can only be named, for they have nothing but a name, and the things which are compounded of them, as they are complex, are expressed by a combination of names, for the combination of names is the essence of a definition. Thus, then, the elements or letters are only objects of perception, and cannot be defined or known; but the syllables or combinations of them are known and expressed, and are apprehended by true opinion. When, therefore, any one forms the true opinion of anything without rational explanation, you may say that his mind is truly exercised, but has no knowledge; for he who cannot give and receive a reason for a thing, has no knowledge of that thing; but when he adds rational explanation, then, he is perfected in knowledge and may be all that I have been denying of him. Was that the form in which the dream appeared to you?

This, that nothing can be predicated of ***form*** or ***matter*** (elements of ***a thing***), is what Plato will demonstrate in ***Parmenides***.

***Theaetetus***: Socrates. Understand why:–the reason is, as I was just now saying, that if you get at the ***difference*** and ***distinguishing characteristic*** of each ***thing***, then, as many persons affirm, you will get at the definition or explanation of it; but while you lay hold only of the common and not of the characteristic notion, you will only have the definition of those things to which this common quality belongs.

***Difference*** is ***matter*** and ***distinguishing characteristic*** is ***form***. Here, Plato states that definition is composed by denoting ***a thing's matter*** and its ***form***.

## Aristotle On Definition

***Posterior Analytics***: But induction is impossible for those who have not sense perception. For it is sense perception alone which is adequate for grasping the particulars: they cannot be objects of scientific knowledge, because neither can universals give us knowledge of them without induction, nor can we get it through induction without sense–perception.~

Induction too will sufficiently convince us of this difference; for never yet by defining anything–essential attribute or accident–did we get knowledge of it [13].

[13] Aristotle along with Plato affirms that knowledge is not imparted through definition but only through direct experience with things.

***Posterior Analytics***: If a thesis assumes one part or the other of an enunciation, i.e. asserts either the existence or the non-existence of a subject, it is a hypothesis; if it does not so assert, it is a definition. Definition is a 'thesis' or a 'laying something down', since the arithmetician lays it down that to be a unit is to be quantitatively indivisible; but it is not a hypothesis, for to define what a unit *is* is not the same as to affirm its existence.

***Posterior Analytics***: Our own doctrine is that not all knowledge is demonstrative: on the contrary, *knowledge of the immediate premises* is independent of demonstration. (The necessity of this is obvious; for since we must know the prior premises from which the demonstration is drawn, and since the regress must end in immediate truths, those truths must be indemonstrable.) Such, then, is our doctrine, and in addition we maintain that besides scientific knowledge *there is its originative source which enables us to recognize the definitions.*

Another affirmation that knowledge starts by direct experience.

***Posterior Analytics***: In the case of predicates constituting the essential nature of a thing, it clearly terminates, seeing that if definition is possible, or in other words, if essential ***form*** is knowable, and an infinite series cannot be traversed, predicates constituting a thing's essential nature must be finite in number.

Reduction to experience.

***Posterior Analytics***: (2) The universal has not a separate being over against groups of singulars.

As the ***form*** of ***a thing*** does not separately exist apart from the ***matter*** of ***the thing*** so too the Universal (the Absolute) does not exist separable from the many particulars–thus the abstract ***thing*** is ***real***, by definition, as Plato asserted.

***Posterior Analytics***: (2) If there is a single identical definition i.e. if the commensurate universal is unequivocal–then the universal will possess being not less but more than some of the particulars, inasmuch as it is universals which comprise the imperishable, particulars that tend to perish.

This was Plato's doctrine–since there is one and only one definition for ***a thing***, a class of objects is ***a thing***. By class definition, the member is not individually distinguishable. Keep this in mind when trying to figure out Plato's ***Parmenides***.

***Posterior Analytics***: The truth perhaps is that if a man grasp truths that cannot be other than they are, in the way in which he grasps the definitions through which demonstrations take place, he will have not opinion but knowledge: if on the other hand he apprehends these attributes as inhering in their subjects, but not in virtue of the subjects' ***substance*** and ***essential nature*** possesses opinion and not genuine knowledge; and his opinion, if obtained through immediate premises, will be both of the fact and of the reasoned fact; if not so obtained, of the fact alone.

Since demonstration takes place through definitions and definitions are a complex of names for ***a thing's*** ***form*** and ***matter*** and since the names of ***form*** and ***matter*** are learned by direct experience, without experience reason is not possible. Therefor, the production of a social class of individuals demonstrating a lack of reasoning ability may be directly related to sensory deprivation imposed by a sterile classroom environment. These confused and pointless individuals may be destined to a confused and pointless life. Perception determines conception and conception determines will is a causal chain originating in direct experience, a fact that makes those of us who would be parents wonder if there is not a standard set of experiences by which standards of civil society may be achieved. One set of experiences needed to come to understanding was located and formalized as ***The Elements*** by Euclid, it is ***Euclidean*** Geometry. Straightedge, compass and paper make it very

portable. Music and Gymnastics were other such sets of experiences.

***Posterior Analytics***: Moreover, the basic premises of demonstrations are definitions, and it has already been shown that these will be found indemonstrable; either the basic premises will be demonstrable and will depend on prior premises, and the regress will be endless; or the primary truths will be indemonstrable definitions.

In other words the ***limit*** of ***knowledge*** starts at the beginning with direct experience. I once asked a group of people, 'How does a word get its meaning?' All different walks of life and educational levels, no one knew and so I said, 'Don't you remember? Mama, what is that? asks a child pointing to something, That is a chair.'

***Posterior Analytics***: Moreover it is clear, if we consider the methods of defining actually in use, that definition does not prove that the thing defined exists: since even if there does actually exist something which is equidistant from a center, yet why should the thing named in the definition exist? Why, in other words, should this be the formula defining circle? One might equally well call it the definition of mountain copper. For definitions do not carry a further guarantee that the thing defined can exist or that it is what they claim to define: one can always ask why.

And yet still today most so-called formalist actually think that they can demand existence through words, "there exists" . . . non-sense.

***Posterior Analytics***: Thus it follows that the degree of our knowledge of a thing's essential nature is determined by the sense in which we are aware that it exists.

The degree of knowledge is determined not only by the sense, but also by which ones and how many of them, also by duration and intimacy of sensual examination of the thing, experience with it. It follows then that a curriculum based on books alone does not produce knowledgeable students, but stupid ones. It is not the wise who challenges conventions nor is it the wise that cherishes a conventionless tradition or culture.

***Posterior Analytics***: Since definition is said to be the statement of a thing's nature, obviously one kind of definition will be a statement of the meaning of the name, or of an equivalent nominal formula. A definition in this sense tells you, e.g. the meaning of the phrase 'triangular character'. When we are aware that triangle exists, we inquire the reason why it exists. But it is difficult thus to learn the definition of things the existence of which we do not genuinely know– the cause of this difficulty being, as we said before, that we only know accidentally whether or not the thing exists. Moreover, a statement may be a unity in either of two ways, by conjunction, like the Iliad, or because it exhibits a single predicate as inhering not accidentally in a single subject.

A premise asserts either ***form*** or ***substance*** of ***a thing***, not both so a definition is not accomplished by a premise. A definition is a sentence constructed from two premises, one denoting ***a thing's substance*** and the other its ***form***.

***Posterior Analytics***: Another kind of definition is a formula exhibiting the cause of a thing's existence. Thus the former signifies without proving, but the latter will clearly be a quasi–demonstration of essential nature, differing from demonstration in the arrangement of its terms. For there is a difference between stating why it thunders, and stating what is the essential nature of thunder; since the first statement will be 'Because fire is quenched in the clouds', while the statement of what the nature of thunder is will be 'The noise of fire being quenched in the clouds'. Thus the same statement takes a different form: in one form it is continuous demonstration, in the other definition. Again, thunder can be defined as noise in the clouds,

which is the conclusion of the demonstration embodying essential nature. On the other hand the definition of immediates is an indemonstrable positing of essential nature.

Immediates, direct experience with ***a thing*** and learning its ***form*** and its ***matter***.

***Posterior Analytics***: We conclude then that definition is (a) an indemonstrable statement of essential nature, or (b) a syllogism of essential nature differing from demonstration in grammatical form, or (c) the conclusion of a demonstration giving essential nature.

***Posterior Analytics***: In establishing a definition by division one should keep three objects in view: (1) the admission only of elements in the definable form, (2) the arrangement of these in the right order, (3) the omission of no such elements. The first is feasible because one can establish genus and differentia through the topic of the genus, just as one can conclude the inherence of an accident through the topic of the accident. The right order will be achieved if the right term is assumed as primary, and this will be ensured if the term selected is predicable of all the others but not all they of it; since there must be one such term. Having assumed this we at once proceed in the same way with the lower terms; for our second term will be the first of the remainder, our third the first of those which follow the second in a 'contiguous' series, since when the higher term is excluded, that term of the remainder which is 'contiguous' to it will be primary, and so on. Our procedure makes it clear that no elements in the definable form have been omitted: we have taken the differentia that comes first in the order of division, pointing out that animal, e.g. is divisible exhaustively into A and B, and that the subject accepts one of the two as its predicate. Next we have taken the differentia of the whole thus reached, and shown that the whole we finally reach is not further divisible–i.e. that as soon as we have taken the last differentia to form the concrete totality, this totality admits of no division into species. For it is clear that there is no superfluous addition, since all these terms we have selected are elements in the definable form; and nothing lacking, since any omission would have to be a genus or a differentia. Now the primary term is a genus, and this term taken in conjunction with its differentiae is a genus: moreover the differentiae are all included, because there is now no further differentia; if there were, the final concrete would admit of division into species, which, we said, is not the case.

For an example of definition by division see Plato's ***Sophist*** and ***Statesman***.

***Posterior Analytics***: We may add that if dialectical disputation must not employ metaphors, clearly metaphors and metaphorical expressions are precluded in definition: otherwise dialectic would involve metaphors.

***Categories***: It is plain from what has been said that both the name and the definition of the predicate must be predicable of the subject. For instance, 'man' is predicted of the individual man. Now in this case the name of the species man' is applied to the individual, for we use the term 'man' in describing the individual; and the definition of 'man' will also be predicated of the individual man, for the individual man is both man and animal. Thus, both the name and the definition of the species are predicable of the individual.

***Categories***: With regard, on the other hand, to those things which are present in a subject, it is generally the case that neither their name nor their definition is predicable of that in which they are present. Though, however, the definition is never predicable, there is nothing in certain cases to prevent the name being used. For instance, 'white' being present in a body is predicated of that in which it is present, for a body is called white: the definition, however, of the color white' is never predicable of the body.

***Categories***: Of species themselves, except in the case of such as are genera, no one is more truly substance than another. We should not give a more appropriate account of the individual man by stating the species to which he belonged, than we should of an individual horse by adopting the same method of definition. In the same way, of primary substances, no one is more truly substance than another; an individual man is not more truly substance than an individual ox.

***Categories***: It is, then, with good reason that of all that remains, when we exclude primary substances, we concede to species and genera alone the name 'secondary substance', for these alone of all the predicates convey a knowledge of primary substance. For it is by stating the species or the genus that we appropriately define any individual man; and we shall make our definition more exact by stating the former than by stating the latter. All other things that we state, such as that he is white, that he runs, and so on, are irrelevant to the definition. Thus it is just that these alone, apart from primary substances, should be called substances.

***Categories***: It is a common characteristic of all substance that it is never present in a subject. For primary substance is neither present in a subject nor predicated of a subject; while, with regard to secondary substances, it is clear from the following arguments (apart from others) that they are not present in a subject. For 'man' is predicated of the individual man, but is not present in any subject: for manhood is not present in the individual man. In the same way, 'animal' is also predicated of the individual man, but is not present in him. Again, when a thing is present in a subject, though the name may quite well be applied to that in which it is present, the definition cannot be applied. Yet of secondary substances, not only the name, but also the definition, applies to the subject: we should use both the definition of the species and that of the genus with reference to the individual man. Thus substance cannot be present in a subject.

> The ***form*** is not in the ***matter***. The ***class*** is not in a ***member***. Aristotle calls ***matter*** the ***subject*** (subjected to form) and ***form*** the ***predicate*** therefore the ***subject*** is in the ***predicate***, not the ***predicate*** in the ***subject***.

***Categories***: Yet this is not peculiar to substance, for it is also the case that differentiae cannot be present in subjects. The characteristics 'terrestrial' and 'two-footed' are predicated of the species 'man', but not present in it. For they are not in man. Moreover, the definition of the differentia may be predicated of that of which the differentia itself is predicated. For instance, if the characteristic 'terrestrial' is predicated of the species 'man', the definition also of that characteristic may be used to form the predicate of the species 'man': for 'man' is terrestrial.

> ***Differentia*** (***form***) cannot be present in ***subjects*** (***matter***).

***On Generation And Corruption***: It is therefore better to suppose that in all instances of coming-to-be the ***matter*** is inseparable, being numerically identical and one with the '***containing***' body, though isolable from it by definition. But the same reasons also forbid us to regard the ***matter***, out of which the body comes-to-be, as ***points*** or lines. The ***matter*** is that of which ***points*** and ***lines*** are ***limits***, and it is something that can never exist without ***quality*** and without ***form***.

> Here is a very plain statement. In one dimension, ***points*** are ***form***, ***linearity*** is ***matter*** and ***line*** is ***a thing***. In two dimensions, ***planarity*** is ***matter***, ***lines form*** and ***plane figure a thing***. etc. The ***boundary*** of n–dimension is n-1 and when the subject is n dimensions, n-1 dimensions can never be considered as ***matter***, only ***form***. In this model, one sees definition by division in its most perfectly plain state. If getting just the definition of ***a thing*** has been so very difficult in history, really coming to understand dimensional progression may be much harder.

***Metaphysics***: Parmenides *seems* to fasten on that which is one in ***definition***, Melissus on that

which is one in ***matter***, for which reason the former says that it is limited, the latter that it is unlimited;

***Metaphysics***: 'Cause' means (1) that from which, as immanent ***material***, a thing comes into being, e.g. the bronze is the cause of the statue and the silver of the saucer, and so are the classes which include these. (2) The ***form*** or ***pattern***, i.e. the ***definition of the essence***, and the ***classes*** which include this (e.g. the ratio 2:1 and number in general are causes of the octave), and the parts included in the definition. (3) That from which the change or the resting from change first begins; e.g. the adviser is a cause of the action, and the father a cause of the child, and in general the maker a cause of the thing made and the change–producing of the changing. (4) The end, i.e. that for the sake of which a thing is; e.g. health is the cause of walking. For 'Why does one walk?' we say; 'that one may be healthy'; and in speaking thus we think we have given the cause. The same is true of all the means that intervene before the end, when something else has put the process in motion, as e.g. thinning or purging or drugs or instruments intervene before health is reached; for all these are for the sake of the end, though they differ from one another in that some are instruments and others are actions.

***Metaphysics***:–(4) The essence, the formula of which is a definition, is also called the substance of each thing.

> Here is where Aristotle is most confusing, I for one would never call a ***form–substance***. Aristotle will use the word ***substance*** to indicate either or both ***form*** and ***matter*** and sometimes he will use it to indicate one or the other contrasting it against the other, as in the following.

***Metaphysics***: Or has 'definition', like 'what a thing is', several meanings? 'What a thing is' in one sense means substance and the 'this', in another one or other of the predicates, quantity, quality, and the like. For as 'is' belongs to all things, not however in the same sense, but to one sort of thing primarily and to others in a secondary way, so too 'what a thing is' belongs in the simple sense to substance, but in a limited sense to the other ***categories***. For even of a quality we might ask what it is, so that quality also is a 'what a thing is',–not in the simple sense, however, but just as, in the case of that which is not, some say, emphasizing the linguistic form, that that is which is not is–not is simply, but is non-existent; so too with quality.

> See Aristotle's ***Categories***.

***Metaphysics***: Since a definition is a formula, and every formula has parts, and as the formula is to the thing, so is the part of the formula to the part of the thing, the question is already being asked whether the formula of the parts must be present in the formula of the whole or not. For in some cases the formulae of the parts are seen to be present, and in some not. The formula of the circle does not include that of the segments, but that of the syllable includes that of the letters; yet the circle is divided into segments as the syllable is into letters.–And further if the parts are prior to the whole, and the acute angle is a part of the right angle and the finger a part of the animal, the acute angle will be prior to the right angle and finger to the man. But the latter are thought to be prior; for in formula the parts are explained by reference to them, and in respect also of the power of existing apart from each other the wholes are prior to the parts.

> Would Aristotle be chasing his own tail if he distinguished between definition and description? A ***definition*** of ball would be, "A ball is a sphere of rubber." A ***deductive description*** of sphere would be, "A sphere is a ball without rubber." and a ***deductive description*** of rubber would be, "Rubber is a ball without its form." A = B + C, B = A – C, C = A – B. If one knew what a sphere was and what rubber was, they could construct a ball. If one had a ball, then one could abstract meanings for both *rubber* and *sphere*.

***Metaphysics***: But when we come to the concrete thing, e.g. this circle, i.e. one of the individual circles, whether perceptible or intelligible (I mean by intelligible circles the mathematical, and by perceptible circles those of bronze and of wood),–of these there is no definition, but they are known by the aid of intuitive thinking or of perception; and when they pass out of this complete realization it is not clear whether they exist or not; but they are always stated and recognized by means of the universal formula. But matter is unknowable in itself. And some matter is perceptible and some intelligible, perceptible matter being for instance bronze and wood and all matter that is changeable, and intelligible matter being that which is present in perceptible things not quâ perceptible, i.e. the objects of mathematics.

> Some ***matter*** is perceptible, like wood or metal, and some intelligible like linearity. We have several senses, many of which abstract ***matter*** by which we abstract the notion that all differences, even apparently the nothingness of space is ***matter***. ***Linearity***, ***planarity***, ***spatiality*** or even ***time***, are not perceptible ***matter***, but they are, at least to some, intelligible ***matter***.

***Metaphysics***: Another question is naturally raised, viz. what sort of parts belong to the ***form*** and what sort not to the ***form***, but to the concrete ***thing***. Yet if this is not plain it is not possible to define any thing; for definition is of the universal and of the ***form***. If then it is not evident what sort of parts are of the nature of ***matter*** and what sort are not, neither will the formula of the ***thing*** be evident. In the case of things which are found to occur in specifically different ***materials***, as a ***circle*** may exist in ***bronze*** or ***stone*** or ***wood***, it seems plain that these, the ***bronze*** or the ***stone***, are no part of the ***essence*** of the circle, since it is found apart from them. Of things which are not seen to exist apart, there is no reason why the same may not be true, just as if all circles that had ever been seen were of bronze; for none the less the bronze would be no part of the ***form***; but it is hard to eliminate it in thought. E.g. the ***form*** of man is always found in ***flesh*** and ***bones*** and parts of this kind; are these then also parts of the ***form*** and the formula? *No, they are* ***matter***; but because man is not found also in other ***matters*** we are unable to perform the abstraction.

> And the same applies to matter, iron makes many things, etc.

***Metaphysics***: We have pointed out, then, that the question of definitions contains some difficulty, and why this is so. And so to reduce all things thus to ***forms*** and to eliminate the ***matter*** is useless labor; for *some* ***things*** *surely are a particular* ***form*** *in a particular* ***matter***, or particular things in a particular state. And the comparison which Socrates the younger used to make in the case of ‘animal’ is not sound; for it leads away from the truth, and makes one suppose that man can possibly exist without his parts, as the circle can without the bronze. But the case is not similar; for an animal is something perceptible, and it is not possible to define it without reference to movement–nor, therefore, without reference to the parts’ being in a certain state. For it is not a hand in any and every state that is a part of man, but only when it can fulfil its work, and therefore only when it is alive; if it is not alive it is not a part.

***Metaphysics***: Obviously, then, the actuality or the formula is different when the matter is different; for in some cases it is the composition, in others the mixing, and in others some other of the attributes we have named. And so, of the people who go in for defining, those who define a house as stones, bricks, and timbers are speaking of the potential house, for these are the matter; but those who propose ‘a receptacle to shelter chattels and living beings’, or something of the sort, speak of the actuality. Those who combine both of these speak of the third kind of substance, which is composed of ***matter*** and ***form*** (for the formula that gives the differentiae seems to be an account of the form or actuality, while that which gives the components is rather

an account of the matter); and the same is true of the kind of definitions which Archytas used to accept; they are accounts of the combined ***form*** and ***matter***. E.g. what is still weather? Absence of motion in a large expanse of air; air is the ***matter***, and absence of motion is the ***actuality*** and ***substance***. What is a calm? Smoothness of sea; the ***material substratum*** is the ***sea***, and the actuality or ***shape*** is ***smoothness***. It is obvious then, from what has been said, what sensible substance is and how it exists–one kind of it as ***matter***, another as ***form*** or ***actuality***, while the third kind is that which is composed of these two.

> The third kind is ***a thing***. Aristotle and Plato were on the verge, the very precipice of understanding that there are two main branches in logic, tautologics, based on form and relatiologics based on matter. In one, the form is given and the matter asserted, in the other the matter is given and the form asserted. Thus English grammar is a tautologic, but Geometric grammar (the figures) is a relatiologic–when given side by side, both saying the same thing, we have a ***Formal Presentation***, the presentation is ***a thing*** called teaching. This fact will distress those theorists who claim that a circle-jerk of words is a ***Formal System***.

***Metaphysics***: Therefore the difficulty which used to be raised by the school of Antisthenes and other such uneducated people has a certain timeliness. They said that the 'what' cannot be defined (for the definition so called is a 'long rigmarole') but of what sort a thing, e.g. silver, is, they thought it possible actually to explain, not saying what it is, but that it is like tin. Therefore one kind of substance can be defined and formulated, i.e. the composite kind, whether it be perceptible or intelligible; but the primary parts of which this consists cannot be defined, since a definitory formula predicates something of something, and one part of the definition must play the part of ***matter*** and the other that of ***form***.

> Then it is to Antisthenes that one would owe the idea of description when definition is not possible?

***Metaphysics***: To return to the difficulty which has been stated with respect both to definitions and to numbers, what is the cause of their unity? In the case of all things which have several parts and in which the totality is not, as it were, a mere heap, but the whole is something beside the parts, there is a cause; for even in bodies contact is the cause of unity in some cases, and in others viscosity or some other such quality. And a definition is a set of words which is one not by being connected together, like the Iliad, but by dealing with one object.–What then, is it that makes man one; why is he one and not many, e.g. animal biped, especially if there are, as some say, an animal–itself and a biped–itself? Why are not those ***forms*** themselves the man, so that men would exist by participation not in man, nor in-one ***form***, but in two, animal and biped, and in general man would be not one but more than one ***thing***, animal and biped?

> There are many think that the concept of addition is just that, putting things in a heap, however, the idea of concatenation is itself a *simile in multis* derived from many different processes of propositional construction. This is one of the points that Plato was driving at in ***Philebus***.
>
> Socrates. Were we not saying that God revealed a finite element of existence, and also an infinite?
>
> Protarchus. Certainly.
>
> Socrates. Let us assume these two principles, and also a third, which is compounded out of them; but I fear that am ridiculously clumsy at these processes of division and enumeration.
>
> Protarchus. What do you mean, my good friend?
>
> Socrates. I say that a fourth class is still wanted.
>
> Protarchus. What will that be?

Socrates. Find the cause of the third or compound, and add this as a fourth class to the three others.

In the text *finite* is referring to ***form***, infinite to ***matter***, the compound to ***a thing***, and causality the processes by which ***form*** is added to ***matter*** to produce ***a thing***.

***Metaphysics***: Clearly, then, if people proceed thus in their usual manner of definition and speech, they cannot explain and solve the difficulty. But if, as we say, one element is ***matter*** and another is ***form***, and one is ***potentially*** and the other ***actually***, the question will no longer be thought a difficulty. For this difficulty is the same as would arise if 'round bronze' were the definition of 'cloak'; for this word would be a sign of the definitory formula, so that the question is, what is the cause of the unity of 'round' and 'bronze'? The difficulty disappears, because the one is ***matter***, the other ***form***. What, then, causes this–that which was ***potentially*** to be ***actually***–except, in the case of ***things*** which are generated, the agent? For there is no other cause of the potential sphere's becoming actually a sphere, but this was the essence of either. Of ***matter*** some is *intelligible*, some *perceptible*, and in a formula there is always an element of ***matter*** as well as one of ***actuality***; e.g. the ***circle*** is 'a ***plane figure***'. But of the ***things*** which have no ***matter***, either intelligible or perceptible, each is by its nature essentially a kind of unity, as it is essentially a kind of being–individual substance, quality, or quantity (*and so neither 'existent' nor 'one' is present in their definitions*), and the essence of each of them is by its very nature a kind of unity as it is a kind of being–and so none of these has any reason outside itself, for being one, nor for being a kind of being; for each is by its nature a kind of being and a kind of unity, not as being in the genus 'being' or 'one' nor in the sense that being and unity can exist apart from particulars.

If ***a thing***, errantly called ***a thing***, is lacking either ***form*** (***a***) or ***matter*** (***thing***), then existence and unity cannot be asserted of it. ***Things*** exist. Here is a key to understanding the errors in ***Parmenides*** that causes the intentional antinomies. Aristotle tries to convey the difficulty in the idea–*a kind of being but not being.*

***Physics***: If 'bounded by a surface' is the definition of body there cannot be an infinite body either intelligible or sensible.

Since ***a thing*** is defined as finite (***formed***), to state that ***a thing*** is infinite (without (***a***) ***form***) is a contradiction in definition and of obvious grammar.

***Physics***: (4) As the genus is 'in' the species and generally the part of the specific ***form*** 'in' the definition of the specific ***form***.

***Physics***: Time, then, also is both made continuous by the 'now' and divided at it. For here too there is a correspondence with the locomotion and the moving body. For the motion or locomotion is made one by the thing which is moved, because it is one-not because it is one in its own nature (for there might be pauses in the movement of such a thing)-but because it is one in definition: for this determines the movement as 'before' and 'after'. Here, too there is a correspondence with the point; for the point also both connects and terminates the length-it is the beginning of one and the end of another. But when you take it in this way, using the one point as two, a pause is necessary, if the same point is to be the beginning and the end. The 'now' on the other hand, since the body carried is moving, is always different.

Aristotle is trying to convey how it is that motion and time are ***things***, for they too are ***matter*** of which ***form*** is asserted. ***Matter*** is always continuous, any ***boundary*** is not ***matter***, it is ***form***.

***On The Soul***: It seems not only useful for the discovery of the causes of the derived properties of substances to be acquainted with the essential nature of those substances (as in mathematics it is useful for the understanding of the property of the equality of the interior

angles of a triangle to two right angles to know the essential nature of the straight and the curved or of the line and the plane) but also conversely, for the knowledge of the essential nature of a substance is largely promoted by an acquaintance with its properties: for, *when we are able to give an account conformable to* ***experience*** of all or most of the properties of a substance, we shall be in the most favorable position to say something worth saying about the essential nature of that subject; in all demonstration a definition of the essence is required as a starting-point, so that definitions which do not enable us to discover the derived properties, or which fail to facilitate even a conjecture about them, must obviously, one and all, be dialectical and futile.

What does this say about an educational system that denies experience and replaces that experience with books?

***On The Soul***: If the circular movement is eternal, there must be something which mind is always thinking-what can this be? *For all practical processes of thinking have limits*-they all go on for the sake of something outside the process, and *all theoretical processes come to a close* in the same way as the phrases in speech which express processes and results of thinking. *Every such linguistic phrase is either definitory or demonstrative. Demonstration has both a starting-point and may be said to end in a conclusion or inferred result; even if the process never reaches final completion, at any rate it never returns upon itself again to its starting-point, it goes on assuming a fresh middle term or a fresh extreme, and moves straight forward, but circular movement returns to its starting-point. Definitions, too, are closed groups of terms.*

One can say that ***a thing*** exhibits ***closure***. Since ***a thing*** is defined as finite, one cannot possible say or think infinite ***things***, without using words empty of meaning.

***On The Soul***: Since what is clear or logically more evident emerges from what in itself is confused but more observable by us, we must reconsider our results from this point of view. For it is not enough for a definitive formula to express as most now do the mere fact; it must include and exhibit the ground also. At present definitions are given in a form analogous to the conclusion of a syllogism; e.g. What is squaring? The construction of an equilateral rectangle equal to a given oblong rectangle. Such a definition is in form equivalent to a conclusion. One that tells us that squaring is the discovery of a line which is a mean proportional between the two unequal sides of the given rectangle discloses the ground of what is defined.

There is a hint here, I think, that a demonstration would be Plato's fourth class, how ***matter*** and ***form*** are combined to produce ***a thing***.

***On The Soul***: Assertion is the saying of something concerning something, e.g. affirmation, and is in every case either true or false: this is not always the case with mind: the thinking of the definition in the sense of the constitutive essence is never in error nor is it the assertion of something concerning something, but, just as while the seeing of the special object of sight can never be in error, the belief that the white object seen is a man may be mistaken, so too in the case of objects which are without ***matter***.

There are those who make erroneous statements denouncing perception because they make an erroneous assumption that perception *quâ* perception leads to error, that the senses deceive us, as if they were capable of deception–the fact of the matter is, it is always the interpretation of the perception that is in error, i.e., the reasoner. Transference of blame even extends to one's own senses. Here Aristotle uses abstraction, B = A – C. ***Form*** is ***object*** without ***matter***.

***Topics***: We must now say what are 'definition', 'property', 'genus', and 'accident'. A 'definition' is a phrase signifying a thing's essence. It is rendered in the form either of a phrase in lieu of a term, or of a phrase in lieu of another phrase; for it is sometimes possible to define

the meaning of a phrase as well. People whose rendering consists of a term only, try it as they may, clearly do not render the definition of the thing in question, because a definition is always a phrase of a certain kind. One may, however, use the word ‘definitory’ also of such a remark as ‘The “becoming” is “beautiful”’, and likewise also of the question, ‘Are sensation and knowledge the same or different?’, for argument about definitions is mostly concerned with questions of sameness and difference. In a word we may call ‘definitory’ everything that falls under the same branch of inquiry as definitions; and that all the above–mentioned examples are of this character is clear on the face of them. For if we are able to argue that two things are the same or are different, we shall be well supplied by the same turn of argument with lines of attack upon their definitions as well: for when we have shown that they are not the same we shall have demolished the definition. Observe, please, that the converse of this last statement does not hold: for to show that they are the same is not enough to establish a definition. To show, however, that they are not the same is enough of itself to overthrow it.

***Topics***: To detect errors of this sort, exchange the word for its definition, e.g. the definition of ‘day’ as the ‘passage of the sun over the earth’. Clearly, whoever has said ‘the passage of the sun over the earth’ has said ‘the sun’, so that in bringing in the ‘day’ he has brought in the sun.

> The substitution of a definition in place of a name works for examining many sentences and often very nicely when examining the validity of adjectival usage. Plato, in ***Parmenides***, will, in demonstrating our inability to reason, use adjectives that are erroneous, for example, the construction *absolute knowledge*. Are there two kinds of knowledge, absolute and relative? And when given any knowledge about a thing, how do I tell if it is absolute or relative knowledge? I mean, that the same knowledge is either permanent (absolute) or that it is changing (relative). I would suspect that adjectival rules are the same between names from the one name naming convention than from the two name naming convention–meaning of course that what can be an adjective in one of them is not an adjective in the other. So much I do not understand.

# The Theory of Forms

## Plato on Form

**Cratylus**: Socrates. And suppose the ***shuttle*** to be broken in making, will he make another, looking to the broken one? or will he look to the ***form*** according to which he made the other?

> Tactile manipulation abstracts ***matter*** and discards ***form*** before imposing a new ***form*** on the ***matter***.

**Cratylus**: Socrates. And whatever ***shuttles*** are wanted, for the manufacture of garments, thin or thick, of flaxen, woolen, or other material, ought all of them to have the true ***form*** of the ***shuttle***; and whatever is the ***shuttle*** best adapted to each kind of work, that ought to be the ***form*** which the maker produces in each case.

> A ***universal form*** and a ***particular form***.

**Cratylus**: Socrates. And the same holds of other instruments: when a man has discovered the instrument which is naturally adapted to each work, he must express this natural ***form,*** and not others which he fancies, in the ***material***, whatever it may be, which he employs; for example, he ought to know how to put into ***iron*** the ***forms*** of ***awls*** adapted by nature to their several uses?

> ***An awl*** is ***matter*** (***iron***) circumscribed by ***form***. A definition states both ***form*** and ***matter*** of ***a thing***.

**Cratylus**: Socrates. And how to put into ***wood forms*** of ***shuttles*** adapted by nature to their uses?

> ***A shuttle*** is ***matter*** (***wood***) circumscribed by ***form***. ***A thing*** is ***matter*** with a ***form***. As one would not say *a wood* neither would one say *a form* (form form).

**Cratylus**: Socrates. For the several ***forms*** of ***shuttles*** naturally answer to the several kinds of webs; and this is true of instruments in general.

**Cratylus**: Socrates. Then, as to names: ought not our legislator also to know how to put the true natural names of each thing into sounds and syllables and to make and give all names with a view to the ideal name, if he is to be a namer in any true sense? And we must remember that different legislators will not use the same syllables. For neither does every smith, although he may be making the same instrument for the same purpose, make them all of the same iron. The ***form*** must be the same, but the ***material*** may vary, and still the ***instrument*** may be equally good of whatever ***iron*** made, whether in Hellas or in a foreign country;–there is no difference .

> Here Plato asserts that the ***material*** of a ***word*** are the ***syllables*** and the resultant ***word*** has a certain ***form***. One will get around to the idea that a name is a ***form*** and all the things that are given that name are the ***matter*** in ***a thing*** called knowledge. In this dialog, Plato is describing grammatical construction principles that remind me of Sanskrit.

**Cratylus**: Socrates. And the legislator, whether he be Hellene or barbarian, is not therefore to be deemed by you a worse legislator, provided he gives the true and proper ***form*** of the name in whatever syllables; this or that country makes no matter.

**Cratylus**: Socrates. But who then is to determine whether the proper ***form*** is given to the ***shuttle***, whatever sort of ***wood*** may be used? the carpenter who makes, or the weaver who is to use them?

**Cratylus**: Socrates. Then, Hermogenes, I should say that this giving of names can be no such light matter as you fancy, or the work of light or chance persons; and Cratylus is right in saying that things have names by nature, and that not every man is an artificer of names, but he only who looks to the name which each thing by nature has, and is able to express the true ***forms*** of ***things*** in letters and syllables.

> Names are of no light matter, for as we see they can be grouped into three categories, those which designate ***things***, those which designate ***forms*** and those which designates ***mater***. It might be found that there is also, like mathematics a fourth group, causal–operands–which of course, denote the ***form*** of operation.

**Meno**: Socrates. And now, as Pindar says, "read my meaning" color is an effluence of ***form*****,** commensurate with sight, and palpable to sense.

> Aristotle will repeat this, that color is a ***form***–those senses that do not abstract ***matter*** abstract ***form***. Logic systems are divided between the tautological and the relatiological.

**Phaedo**: Then now let us return to the previous discussion. Is that ***idea*** or ***essence***, which in the dialectical process we define as ***essence*** of ***true existence***–whether ***essence*** of equality, beauty, or anything else: are these ***essences***, I say, liable at times to some degree of change? or are they each of them always what they are, having the same simple, self-existent and unchanging ***forms***, and not admitting of variation at all, or in any way, or at any time?

> If ***matter*** is ***difference*** of ***a thing*** and ***form*** is ***non-difference*** of ***a thing***–which is ***eternal*** and ***never changing***? ***non-difference*** or ***difference***? By definition, ***form*** is not ***difference*** or again not ***change*** and never ***changing***.

**The Republic**: Well then, shall we begin the inquiry in our usual manner: Whenever a number of individuals have a common name, we assume them to have also a corresponding ***idea*** or ***form***. Do you understand me?

> At the very foundation of spoken grammars, logic, is what has become called ***Set Theory***–a name (word) designates a group of objects each of which has a common definition denoted by that name. All men are equal, by law, not a social law, not a political law, but a law of grammar. One might say that the first social laws are abstractions from the environment and are above, or in spite, of man. The claim that men give up their individual rights, their own sovereign rights to join a community is denied by the very grammar common to all reason.

**The Republic**: But there are only two ideas or ***forms*** of them, one the ***idea*** of a ***bed***–the other of a ***table***.

**Theaetetus**: Socrates. Quite true, Theaetetus, and therefore, according to our present view, a syllable must surely be some indivisible ***form***?

> The divisor (***form***) cannot be divided, this is not a stipulation, but a physical fact–for example, a ***surface*** is not the ***surface*** of a ***surface*** but the ***surface*** of ***some thing***. To say that 'the point is that which has no part.' is the same as saying that the point is a ***form–boundary–limit–divisor–cut***–etc. No one can ***cut*** a ***cut*** or ***divide*** a ***form***. To claim that a line is composed of an infinite number of points is the same as saying that a tomato can be made by waving a knife through the air and ungodly number of times.

**Theaetetus**: Socrates. Then is not the syllable in the same case as the elements or letters, if it has no parts and is one ***form***?

> If *it has no parts*, that is if it cannot be parted it must be ***form***, for there is only ***form*** and ***matter*** that is predicable of ***a thing***,–***form*** parts ***matter*** as the definition of ***a thing***. If a point is that which has no part, then a point is that which is the part (form, cut, boundary, limit, etc.).

## Plato on Participation in the Form (Matter)

To ***participate*** in a ***form*** can also be said as to be a ***member*** of a ***class***, is the same as to be the ***matter*** of ***a thing***.

In this section, I will not place any of the quotations from the work ***Parmenides*** as that work is a demonstration on the impossibility of reason when the names for ***form***, ***matter*** and ***a thing*** are not used in accordance with their meaning.

**Euthydemus**: Socrates. Yes, Crito, there is more speciousness than truth; they cannot be made to understand the nature of intermediates. For all persons or things, which are intermediate between two other things, and **participate** in both of them–if one of these two things is good and the other evil, are better than the one and worse than the other; but if they are in a mean between two good things which do not tend to the same end, they fall short of either of their component elements in the attainment of their ends. Only in the case when the two component elements which do not tend to the same end are evil is the participant better than either. Now, if philosophy and political action are both good, but tend to different ends, and they **participate** in both, and are in a mean between them, then they are talking nonsense, for they are worse than either; or, if the one be good and the other evil, they are better than the one and worse than the other; only on the supposition that they are both evil could there be any truth in what they say. I do not think that they will admit that their two pursuits are either wholly or partly evil; but the truth is, that these philosopher-politicians who aim at both fall short of both in the attainment of their respective ends, and are really third, although they would like to stand first. There is no need, however, to be angry at this ambition of theirs–which may be forgiven; for every man ought to be loved who says and manfully pursues and works out anything which is at all like wisdom: at the same time we shall do well to see them as they really are.

**Meno**: Socrates. Then all men are good in the same way, and by **participation** in the same ***virtues***?

***Virtue*** is a ***form*** of behavior.

**Phaedo**: He proceeded: I know nothing and can understand nothing of any other of those wise causes which are alleged; and if a person says to me that the bloom of ***color***, or ***form***, or anything else of that sort is a source of beauty, I leave all that, which is only confusing to me, and simply and singly, and perhaps foolishly, hold and am assured in my own mind that nothing makes a thing beautiful but the presence and **participation** of beauty in whatever way or manner obtained; for as to the manner I am uncertain, but I stoutly contend that by beauty all beautiful things become beautiful. That appears to me to be the only safe answer that I can give, either to myself or to any other, and to that I cling, in the persuasion that I shall never be overthrown, and that I may safely answer to myself or any other that by beauty beautiful things become beautiful. Do you not agree to that?

Plato reduces all causality of ***being*** to ***form*** and ***matter*** i.e., ***a things*** definition. The cause of anything being what it *is*–is because it is a certain ***matter*** in a certain ***form***.

**Phaedo**: Again, would you not be cautious of affirming that the addition of one to one, or the division of one, is the cause of two? And you would loudly asseverate that you know of no way in which anything comes into existence except by **participation** in its own proper essence, and consequently, as far as you know, the only cause of two is the **participation** in duality; that is the way to make two, and the **participation** in one is the way to make one. You would say: I will let

alone puzzles of division and addition–wiser heads than mine may answer them; inexperienced as I am, and ready to start, as the proverb says, at my own shadow, I cannot afford to give up the sure ground of a principle. And if anyone assails you there, you would not mind him, or answer him until you had seen whether the consequences which follow agree with one another or not, and when you are further required to give an explanation of this principle, you would go on to assume a higher principle, and the best of the higher ones, until you found a resting-place; but you would not refuse the principle and the consequences in your reasoning like the Eristics–at least if you wanted to discover real existence. Not that this confusion signifies to them who never care or think about the matter at all, for they have the wit to be well pleased with themselves, however great may be the turmoil of their ideas. But you, if you are a philosopher, will, I believe, do as I say.

> The principle of causality, ***form*** and ***matter***, is not to be given up for the sake of demonstration, but demonstration must always affirm and rest in the definition. The recipes in a cookbook are demonstrations–they lead one to the thing defined. All directions that lead one to construct ***a thing*** are demonstrations. The Greek demand for constructability was not confined to geometry. Demonstration, if it is demonstration (scientific knowledge) leads to ***the thing*** defined. When so called scholars tossed out the criteria of constructability, all meaning was left behind.

**Phaedo**: Phaedo. After all this was admitted, and they had agreed about the existence of ideas and the **participation** in them of the other things which derive their names from them, Socrates, if I remember rightly, said:–

This is your way of speaking; and yet when you say that Simmias is greater than Socrates and less than Phaedo, do you not predicate of Simmias both greatness and smallness?

**Philebus**: Socrates. Does not the right **participation** in the finite give health–in disease, for instance?

**Protagoras**: And you would call pleasant, I said, the things which **participate** in pleasure or create pleasure?

**The Republic**: But take the case of the other, who recognizes the existence of absolute beauty and is able to distinguish the idea from the objects which **participate** in the idea, neither putting the objects in the place of the idea nor the idea in the place of the objects–is he a dreamer, or is he awake?

> But take the case of those who recognize ***forms*** and is able to distinguish the ***form*** of ***a thing*** from the ***substance*** of ***a thing***, neither putting the ***substance*** in place of the ***form*** nor the ***form*** in place of the ***substance***, his mind is awake.

**The Republic**: Well, and do not all these qualities, which we have been enumerating, go together, and are they not, in a manner, necessary to a soul, which is to have a full and perfect **participation** of being?

**The Republic**: And if there be a pleasure in being filled with that which is according to nature, that which is more really filled with more real being will more really and truly enjoy true pleasure; whereas that which **participates** in less real being will be less truly and surely satisfied, and will **participate** in an illusory and less real pleasure?

**The Sophist**: Stranger. And you would allow that we **participate** in generation, with the body, and through perception, but we **participate** with the soul through thought in true essence; and essence you would affirm to be always the same and immutable, whereas generation or becoming varies?

**The Sophist**: Stranger. Well, fair sirs, we say to them, what is this **participation**, which you assert of both? Do you agree with our recent definition?

**The Sophist**: Stranger. Then you conceive of being as some third and distinct nature, under which rest and motion are alike included; and, observing that they both **participate** in being, you declare that they are.

**The Sophist**: Stranger. Shall we refuse to attribute being to motion and rest, or anything to anything, and assume that they do not mingle, and are incapable of **participating** in one another? Or shall we gather all into one class of things communicable with one another? Or are some things communicable and others not?–Which of these alternatives, Theaetetus, will they prefer?

**The Sophist**: Stranger. Very good, and first let us assume them to say that nothing is capable of **participating** in anything else in any respect; in that case rest and motion cannot **participate** in being at all.

**The Sophist**: Stranger. But would either of them be if not **participating** in being?

**The Sophist**: Stranger. Most ridiculous of all will the men themselves be who want to carry out the argument and yet forbid us to call anything, because **participating** in some affection from another, by the name of that other.

**The Sophist**: Stranger. Let not any one say, then, that while affirming the opposition of not-being to being, we still assert the being of not-being; for as to whether there is an opposite of being, to that inquiry we have long said good-bye–it may or may not be, and may or may not be capable of definition. But as touching our present account of not-being, let a man either convince us of error, or, so long as he cannot, he too must say, as we are saying, that there is a communion of classes, and that being, and difference or other, traverse all things and mutually interpenetrate, so that the other partakes of being, and by reason of this **participation** is, and yet is not that of which it partakes, but other, and being other than being, it is clearly a necessity that not-being should be. And again, being, through partaking of the other, becomes a class other than the remaining classes, and being other than all of them, is not each one of them, and is not all the rest, so that undoubtedly there are thousands upon thousands of cases in which being is not, and all other things, whether regarded individually or collectively, in many respects are, and in many respects are not.

> A member of a class cannot be said to exist independently from the class. The upshot of this is that if 'existence' is said to be a class, all the members that make up existence cannot be said to exist for they are other than existence. The reason being is that neither member nor form of a class is separable from the class. ***A part*** of ***a car*** is ***a thing***.

**The Sophist**: Stranger. And now, not-being has been shown to partake of being, and therefore he will not continue fighting in this direction, but he will probably say that some ideas partake of not-being, and some not, and that language and opinion are of the non-partaking class; and he will still fight to the death against the existence of the image-making and fantastic art, in which we have placed him, because, as he will say, opinion and language do not partake of not-being, and unless this **participation** exists, there can be no such thing as falsehood. And, with the view of meeting this evasion, we must begin by inquiring into the nature of language, opinion, and imagination, in order that when we find them we may find also that they have communion with not-being, and, having made out the connection of them, may thus prove that falsehood exists; and therein we will imprison the Sophist, if he deserves it, or, if not, we will let him go again and look for him in another class.

## Synonym Toast: Aristotle And The Theory of Forms

Some of Aristotle's text indicates, to the point of hostility, that he did not grasp the ***Theory Of Forms***, but a majority of the other text indicates that he understood it reasonably well and knew it was fundamental to reasoning. Since his writings were collected long after his death, it should be assumed that he was no different from any writer or any man, his understanding improved with age.

Aristotle liked his synonyms but I, for one, find it difficult to keep several names for the same thing in my head without confusing them.

Aristotle did not realize that he was working with a two name naming convention, and so his theory of predication suffered–his theory of the syllogism suffered.

> **Synonyms**: **shape**, **form**, **pattern**, **essence**, **substance**, **quality**, **differentiae**, **order**, **ratio**, **nature**, **archetype**, **genus**, **actuality**, **idea**, **predicate**.
>
> **Synonyms**: **matter**, **substance**, **material**, **quantity**, **substratum**, **potentiality**, **participation**, **nature**, **subject**.

**Works Mentioned**: ***On Philosophy***, Aristotle. ***Phaedo***, ***Timaeus***, Lectures ***On Philosophy***, Plato.

***Metaphysics***: After the systems we have named came the philosophy of Plato, which in most respects followed these thinkers, but had peculiarities that distinguished it from the philosophy of the Italians. For, having in his youth first become familiar with Cratylus and with the Heraclitean doctrines (that all sensible **things** are ever in a state of flux and there is no knowledge about them [14]), these views he held even in later years. Socrates, however, was busying himself about ethical matters and neglecting the world of nature as a whole but seeking the universal in these ethical matters, and fixed thought for the first time on definitions; Plato accepted his teaching, but held that the problem applied not to sensible **things** but to entities of another kind–for this reason, that the common definition could not be a definition of any sensible **thing**, as they were always changing. **Things** of this other sort, then, he called **Ideas**, and sensible **things**, he said, were all named after these, and in virtue of a relation to these; for the many existed by **participation** in the **Ideas** that have the same name as they. Only the name '**participation**' was new; for the Pythagoreans say that **things** exist by 'imitation' of numbers, and Plato says they exist by **participation**, changing the name. But what the **participation** or the imitation of the **forms** could be they left an open question.

> Since all knowledge is of ***things***–***matter*** circumscribed by ***form***–it must represent the most primitive paradigm upon which all reasoning is based. It's importance to the metaphysics is inestimable.
>
> Actually there is a very big difference between the Pythagorean conception and the Platonic–that which participates in a ***form*** is the ***matter***–that which imitates is ***form***. One starts to see two psychological foundations in thought–***matter*** based and ***form*** based–lack of difference (***form***, ***equality***, ***same***) and difference (***matter***, ***inequality***). The idea is not only the founding paradigm of reason, but also the foundation of psychology.

***Metaphysics***: Further, besides sensible **things** and **forms** he says there are the objects of

[14] Can one then say **anything** determinate about that which they have no knowledge? If not, then how can they even make this statement? The Liars Paradox.

mathematics, which occupy an intermediate position, differing from sensible **things** in being eternal and unchangeable, from **forms** in that there are many alike, while the **form** itself is in each case unique.

It is strange that numbers are generally not understood to be just a naming convention in a grammar system. Neither the name nor the naming convention changes that which it names. Here Aristotle indicates that Plato fell short on his concepts.

***Metaphysics***: Since the **forms** were the causes of all other **things**, he thought their elements were the elements of all **things**. As **matter**, the great and the small were principles; as essential reality, the One; for from the great and the small, by **participation** in the One, come the Numbers.

Why would the ***form*** of ***a thing*** be a cause any more than the ***matter***?

***Metaphysics***: But he agreed with the Pythagoreans in saying that the One is **substance** and not a **predicate** of something else; and in saying that the Numbers are the causes of the reality of other **things** he agreed with them; but positing a dyad and constructing the infinite out of great and small, instead of treating the infinite as one, is peculiar to him; and so is his view that the Numbers exist apart from sensible **things**, while they say that the **things** themselves are Numbers, and do not place the objects of mathematics between **forms** and sensible **things**. His divergence from the Pythagoreans in making the One and the Numbers separate from **things**, and his introduction of the **forms**, were due to his inquiries in the region of definitions (for the earlier thinkers had no tincture of dialectic), and his making the other entity besides the One a dyad was due to the belief that the numbers, except those which were prime, could be neatly produced out of the dyad as out of some plastic material. Yet what happens is the contrary; the theory is not a reasonable one. For they make many **things** out of the **matter**, and the **form** generates only once, but what we observe is that one **table** is made from one **matter**, while the man who applies the **form**, though he is one, makes many tables. And the relation of the male to the female is similar; for the latter is impregnated by one copulation, but the male impregnates many females; yet these are analogues of those first principles.

Since Arithmetic is a younger grammar than conversational grammars, one will find mythology about numbers that were once held about language in general. Numbers are names in an **ordered** naming convention–so far, I am the only one I know that states this.

What about Aristotle's counter-point–the one ***matter*** and the many ***forms***? Are there not some very simple arguments against it? For example, is the divisor divisible? What is countable, ***forms***, ***substances***, or ***things***?

In the following, Aristotle will expand upon his objection to the one **idea** in the many examples–Plato's *simile in multis*. Look at the word *table*, by what criteria can we call a great number of **things** *table*–there are a great number of individual ***forms*** and a great number of individual ***matters***, but they all have the same name table–the word *table* is a ***form***, all those **things** that can be called a table is the ***matter*** that fills that ***form***. Together the *word* and all the ***matter*** the word circumscribes is ***a thing***–an abstract ***thing***–that is more real, than any particular table–particular as ***matter*** and particular in individual ***form***. It is more real because the individual tables we sense with our body, the ideal table we have learned to sense with our mind, and we, as mind only truly sense as mind. It is a subtle **idea**, but it follows by strict definition of ***a thing***.

A particular table can exist as ***a thing*** but when one understands particular tables as the ***matter*** in the ***form*** provided by the word ***table*** then one has, by definition, a real abstract table which is never really in front of our face or felt with our hands. In this manner one can understand the ***table ideal*** as a real entity–a reality brought about by the very concept and function of reason, of grammar. When this becomes part of one's psychology, then one will see the only true, only real table–the real

meaning of ideal–functioning on the grammatical level of the human mind. This realization is reason–the Ideal Table is the only real table–itis the single measure and we have developed a singularity of perception. This also infers that the ideal table can never be accomplished in mind without objective experience with the matter, physical tables that complete the definition and meaning of the ideal. These are ideas Plato tried to stress because they are true. *******

***Metaphysics***: Plato, then, declared himself thus on the points in question; it is evident from what has been said that he has used only two causes, that of the **essence** and the **material** cause (for the **forms** are the causes of the **essence** of all other **things**, and the One is the cause of the **essence** of the **forms**); and it is evident what the underlying **matter** is, of which the **forms** are **predicated** in the case of sensible **things**, and the One in the case of **forms**, viz. that this is a dyad, the great and the small. Further, he has assigned the cause of good and that of evil to the elements, one to each of the two, as we say some of his predecessors sought to do, e.g. Empedocles and Anaxagoras.

***Essence*** is a synonym for ***form***, and ***material*** cause or caused by ***matter***. In other words, Plato only recognized ***form*** and ***matter*** as cause–compare this with Geometry, point and linearity–from these two *causes* all else follows. Since every bit of knowledge and knowing involves only these two, Plato was correct.

Notice that here Aristotle is stating the idea of *what* is predicated of *what* in the case of sensible things, ***form*** is predicated of ***matter***, point is asserted of linearity. Thus to assert a point to a line, one can say that one is predicating a point of linearity.

***Metaphysics***: The **essence**, i.e. the substantial reality, no one has expressed distinctly. It is hinted at chiefly by those who believe in the **forms**; for they do not suppose either that the **forms** are the **matter** of sensible **things**, and the One the **matter** of the **forms**, or that they are the source of movement (for they say these are causes rather of immobility and of being at rest), but they furnish the **forms** as the **essence** of every other **thing**, and the One as the **essence** of the **forms**.

***Form*** is not ***matter*** and ***matter*** is not ***form***–in the words of Euclid 'The point (**form**) is that which has no part (**matter**) i.e. we do not assume that a line is composed of an infinite number of points because that would be a contradiction.

***Metaphysics***: Let us leave the Pythagoreans for the present; for it is enough to have touched on them as much as we have done. But as for those who posit the **Ideas** as causes, firstly, in seeking to grasp the causes of the **things** around us, they introduced others equal in number to these, as if a man who wanted to count **things** thought he would not be able to do it while they were few, but tried to count them when he had added to their number. For the **forms** are practically equal to–or not fewer than–the **things**, in trying to explain which these thinkers proceeded from them to the **forms**. For to each **thing** there answers an entity which has the same name and exists apart from the **substances**, and so also in the case of all other groups there is a one over many, whether the many are in this world or are eternal.

If ***form*** is not separable from ***a thing***, just like ***matter*** is not separable from ***a thing***, can one number ***forms*** as something distinct from ***a thing***? or ***matter*** for that matter? This is one of the paradoxes examined in ***Parmenides***–can one use the same name for ***form***, ***a thing***, and ***matter*** without contradicting themselves?

***Metaphysics***: Further, of the ways in which we prove that the **forms** exist, none is convincing; for from some no inference necessarily follows, and from some arise **forms** even of **things** of which we think there are no **forms**. For according to the arguments from the existence of the sciences there will be **forms** of all **things** of which there are sciences and according to the 'one

over many' argument there will be **forms** even of negations, and according to the argument that there is an object for thought even when the **thing** has perished, there will be **forms** of perishable **things**; for we have an image of these. Further, of the more accurate arguments, some lead to **Ideas** of relations, of which we say there is no independent class, and others introduce the 'third man'.

> If ***a thing*** defines existence, then neither ***form*** nor ***matter***, independent of the other can. The same is true for the idea of definition. A definition indicates both ***form*** and ***matter***, therefore an indication of either ***form*** or ***matter*** independent of the other is not a definition. If ***A*** stands for a thing, ***b*** for form and ***c*** for matter then definition may be expressed as ***A*** = ***b*** + ***c***. A ***description*** might be either ***b*** = ***A*** – ***c*** or ***c*** = ***A*** – ***b***.

***Metaphysics***: And in general the arguments for the **forms** destroy the **things** for whose existence we are more zealous than for the existence of the **Ideas**; for it follows that not the dyad but number is first, i.e. that the relative is prior to the absolute,–besides all the other points on which certain people by following out the opinions held about the **Ideas** have come into conflict with the principles of the theory.

> Aristotle recognizes that ***matter*** (***relative***) comes before a ***boundary*** (***absolute***) to that ***matter***.

***Metaphysics***: Further, according to the assumption on which our belief in the **Ideas** rests, there will be **forms** not only of **substances** but also of many other **things** (for the concept is single not only in the case of **substances** but also in the other cases, and there are sciences not only of **substance** but also of other **things**, and a thousand other such difficulties confront them). But according to the necessities of the case and the opinions held about the **forms**, if **forms** can be shared in there must be **Ideas** of **substances** only. For they are not shared in incidentally, but a **thing** must share in its **form** as in something not **predicated** of a **subject** (by 'being shared in incidentally' I mean that e.g. if a **thing** shares in 'double itself', it shares also in 'eternal', but incidentally; for 'eternal' happens to be predicable of the 'double'). Therefore the **forms** will be **substance**; but the same terms indicate **substance** in this and in the ideal world (or what will be the meaning of saying that there is something apart from the particulars–the one over many?). And if the **Ideas** and the particulars that share in them have the same **form**, there will be something common to these; for why should '2' be one and the same in the perishable 2's or in those which are many but eternal, and not the same in the '2' itself' as in the particular 2? But if they have not the same **form**, they must have only the name in common, and it is as if one were to call both Callias and a wooden image a 'man', without observing any community between them.

> Therefore in starting to understand the **idea** of '***form'*** one must observe a community between **things**, i.e. the *simile in multis* that Plato had understood.

***Metaphysics***: Above all one might discuss the question what on earth the **forms** contribute to sensible **things**, either to those that are eternal or to those that come into being and cease to be. For they cause neither movement nor any change in them. But again they help in no wise either towards the knowledge of the other **things** (for they are not even the **substance** of these, else they would have been in them), or towards their being, if they are not in the particulars which share in them; though if they were, they might be thought to be causes, as white causes whiteness in a white object by entering into its composition. But this argument, which first Anaxagoras and later Eudoxus and certain others used, is very easily upset; for it is not difficult to collect many insuperable objections to such a view.

***Metaphysics***: But, further, all other **things** cannot come from the **forms** in any of the usual

senses of ‘from’. And to say that they are **patterns** and the other **things** share in them is to use empty words and poetical metaphors. For what is it that works, looking to the **Ideas**? And **anything** can either be, or become, like another without being copied from it, so that whether Socrates or not a man Socrates like might *come to be*; and evidently this might be so even if Socrates were eternal. And there will be several **patterns** of the same **thing**, and therefore several **forms**; e.g. ‘animal’ and ‘two-footed’ and also ‘man himself’ will be **forms** of man. Again, the **forms** are **patterns** not only sensible **things**, but of **forms** themselves also; i.e. the **genus**, as **genus** of various species, will be so; therefore the same **thing** will be **pattern** and copy.

> In these lines Aristotle is demonstrating that he understood one of Plato’s points in ***Parmenides***. A ***form*** of a form is self-referential–they cannot be ***different*** (***matter***) yet the ***same*** (***form***). In the abstraction of ***form*** from ***thing***, one cannot say that the abstracted ***form*** is ***different*** than the ***form*** abstracted. Our grammar can quickly trip us up.

***Metaphysics***: Again, it would seem impossible that the **substance** and that of which it is the **substance** should exist apart; how, therefore, could the **Ideas**, being the **substances** of **things**, exist apart? In the ***Phaedo*** the case is stated in this way–that the **forms** are causes both of being and of becoming; yet when the **forms** exist, still the **things** that share in them do not come into being, unless there is something to originate movement; and many other **things** come into being (e.g. a house or a ring) of which we say there are no **forms**. Clearly, therefore, even the other **things** can both be and come into being owing to such causes as produce the **things** just mentioned.

> If one does not accept that the ***individuals*** are the ***matter*** that are circumscribed by the ***idea*** as ***form***, then one violates the definition and fact that ***matter*** and ***form*** cannot exist apart. Or ***matter*** for the ***form*** provided by the word might be called the matter gray. Aristotle seems to be grasping for this. At any rate, the ***experience***, as ***matter*** must accompany the ***word*** as ***form***.

***Metaphysics***: Again, if the **forms** are numbers, how can they be causes? Is it because existing **things** are other numbers, e.g. one number is man, another is Socrates, another Callias? Why then are the one set of numbers causes of the other set? It will not make any difference even if the former are eternal and the latter are not. But if it is because **things** in this sensible world (e.g. harmony) are **ratios** of numbers, evidently the **things** between which they are **ratios** are some one class of **things**. If, then, this–the **matter**–is some definite **thing**, evidently the numbers themselves too will be **ratios** of something to something else. E.g. if Callias is a numerical **ratio** between fire and earth and water and air, his **Idea** also will be a number of certain other underlying **things**; and man himself, whether it is a number in a sense or not, will still be a numerical **ratio** of certain **things** and not a number proper, nor will it be a of number merely because it is a numerical **ratio**.

> Numbers are Arithmetic names and names are ***forms***. In “the **matter**–is some definite **thing**” Aristotle is on the verge that ***things*** (as experience) are the ***matter*** and the name the ***form***.

***Metaphysics***: Again, from many numbers one number is produced, but how can one **form** come from many **forms**? And if the number comes not from the many numbers themselves but from the units in them, e.g. in 10,000, how is it with the units? If they are specifically alike, numerous absurdities will follow, and also if they are not alike (neither the units in one number being themselves like one another nor those in other numbers being all like to all); for in what will they differ, as they are without **quality**? This is not a plausible view, nor is it consistent with our thought on the matter.

In his own way, Aristotle is reflecting on some of the paradox's exampled in ***Parmenides***.

***Metaphysics***: Nor have the **forms** any connection with what we see to be the cause in the case of the arts, that for whose sake both all mind and the whole of **nature** are operative,–with this cause which we assert to be one of the first principles; but mathematics has *come to be* identical with philosophy for modern thinkers, though they say that it should be studied for the sake of other **things**. Further, one might suppose that the **substance** which according to them underlies as **matter** is too mathematical, and is a **predicate** and **differentia** of the **substance**, i.e. of the **matter**, rather than **matter** itself; i.e. the great and the small are like the rare and the dense which the physical philosophers speak of, calling these the primary **differentiae** of the **substratum**; for these are a kind of excess and defect. And regarding movement, if the great and the small are to be movement, evidently the **forms** will be moved; but if they are not to be movement, whence did movement come? The whole study of **nature** has been annihilated.

Can one say that ***form*** *quâ* ***form*** or ***matter*** *quâ* ***matter*** can move or be moved?

***Metaphysics***: Nor can it be explained either how the lines and planes and solids that come after the numbers exist or can exist, or what significance they have; for these can neither be **forms** (for they are not numbers), nor the intermediates (for those are the objects of mathematics), nor the perishable **things**. This is evidently a distinct fourth class.

***Metaphysics***: (4) Further, must we say that sensible **substances** alone exist, or that there are others besides these? And are **substances** of one kind or are there in fact several kinds of **substances**, as those say who assert the existence both of the **forms** and of the intermediates, with which they say the mathematical sciences deal?–The sense in which we say the **forms** are both causes and self-dependent **substances** has been explained in our first remarks about them; while the theory presents difficulties in many ways, the most paradoxical **thing** of all is the statement that there are certain **things** besides those in the **material** universe, and that these are the same as sensible **things** except that they are eternal while the latter are perishable. For they say there is a man-himself and a horse-itself and health-itself, with no further qualification,–a procedure like that of the people who said there are gods, but in human **form**. For they were positing **nothing** but eternal men, nor are the Platonists making the **forms** **anything** other than eternal sensible **things**.

***Metaphysics***: Further, if we are to posit besides the **forms** and the sensibles the intermediates between them, we shall have many difficulties. For clearly on the same principle there will be lines besides the lines-themselves and the sensible lines, and so with each of the other classes of **things**; so that since astronomy is one of these mathematical sciences there will also be a heaven besides the sensible heaven, and a sun and a moon (and so with the other heavenly bodies) besides the sensible. Yet how are we to believe in these **things**? It is not reasonable even to suppose such a body immovable, but to suppose it moving is quite impossible.–And similarly with the **things** of which optics and mathematical harmonics treat; for these also cannot exist apart from the sensible **things**, for the same reasons. For if there are sensible **things** and sensations intermediate between **form** and individual, evidently there will also be animals intermediate between animals-themselves and the perishable animals.–We might also raise the question, with reference to which kind of existing **things** we must look for these sciences of intermediates. If geometry is to differ from mensuration only in this, that the latter deals with **things** that we perceive, and the former with **things** that are not perceptible, evidently there will also be a science other than medicine, intermediate between medical–science-itself and this individual medical science, and so with each of the other sciences. Yet how is this possible?

There would have to be also healthy **things** besides the perceptible healthy **things** and the healthy-itself.–And at the same time not even this is true, that mensuration deals with perceptible and perishable magnitudes; for then it would have perished when they perished.

Aristotle poses questions that should be asked, 'What are the paradoxes involved when one posits existence of something other than ***things***? this does not mean he has no answer, or that he believes some hasty answer he gives to the question.

***Metaphysics***: But on the other hand astronomy cannot be dealing with perceptible magnitudes nor with this heaven above us. For neither are perceptible lines such lines as the geometer speaks of (for no perceptible **thing** is straight or round in the way in which he defines 'straight' and 'round'; for a hoop touches a straight edge not at a point, but as Protagoras used to say it did, in his refutation of the geometers), nor are the movements and spiral orbits in the heavens like those of which astronomy treats, nor have geometrical points the same **nature** as the actual stars.–Now there are some who say that these so-called intermediates between the **forms** and the perceptible **things** exist, not apart from the perceptible **things**, however, but in these; the impossible results of this view would take too long to enumerate, but it is enough to consider even such points as the following:–It is not reasonable that this should be so only in the case of these intermediates, but clearly the **forms** also might be in the perceptible **things**; for both statements are parts of the same theory. Further, it follows from this theory that there are two solids in the same place, and that the intermediates are not immovable, since they are in the moving perceptible **things**. And in general to what purpose would one suppose them to exist indeed, but to exist in perceptible **things**? For the same paradoxical results will follow which we have already mentioned; there will be a heaven besides the heaven, only it will be not apart but in the same place; which is still more impossible.

If one were to assert existence of either **form** *quâ* **form** or **matter** *quâ* **matter**, paradoxes result.

***Metaphysics***: Further, if we admit in the fullest sense that something exists apart from the concrete **thing**, whenever something is **predicated** of the **matter**, must there, if there is something apart, be something apart from each set of individuals, or from some and not from others, or from none? (A) If there is **nothing** apart from individuals, there will be no object of thought, but all **things** will be objects of sense, and there will not be knowledge of **anything**, unless we say that sensation is knowledge. Further, **nothing** will be eternal or unmovable; for all perceptible **things** perish and are in movement. But if there is **nothing** eternal, neither can there be a process of *coming to be*; for there must be something that *comes to be*, i.e. from which *something comes to be*, and the ultimate term in this series cannot have *come to be*, since the series has a limit and since **nothing** can *come to be* out of that which is not. Further, if generation and movement exist there must also be a limit; for no movement is infinite, but every movement has an end, and that which is incapable of completing its *coming to be* cannot be in process of *coming to be*; and that which has completed its *coming to be* must be as soon as it has *come to be*. Further, since the **matter** ***exists***, because it is ***ungenerated***, it is *a fortiori* reasonable that the **substance** or **essence**, that ***which the matter is at any time coming to be, should exist***; for if neither **essence** nor **matter** is to be, **nothing** will be at all, and since this is impossible there must be something besides the concrete **thing**, viz. the **shape** or **form**.

This amounts to Aristotle's proof of the existence of **forms**. One will notice, he really has not made a direct abstraction–he reasoned his way to it. He knows that to assert the existence of **forms** produces paradoxes, but if they do not exist then thought–as he defined it–would be impossible. Instead of the paradoxes turning him back to his definition of thought, he accepts the paradoxes as undefeatable–whereupon his mind cannot see the simple tense errors in his statements. Sometimes

our habit of thought is overpowering and we cannot learn. Never actually making the abstraction, he will be clumsy and contradictory in his expressions of ***The Theory Of Forms***.

Without ***form*** (**essence**) nor ***matter***, **nothing** could exist–as everything is defined in terms of *difference* (a Universal term) circumscribed by ***form***. . . . stated sometimes as a proposition (***thing***) is a relation (***matter***) between terms (***form***). If he could have realized that his body is a number of sense systems, each sense either abstracting ***form*** or ***matter*** from ***things***, he may have then seen that his mind is constrained to reason as branches from this physical fact of abstraction.

***Metaphysics***: In general one might raise the question why after all, besides perceptible **things** and the intermediates, we have to look for another class of **things**, i.e. the **forms** which we posit. If it is for this reason, because the objects of mathematics, while they differ from the **things** in this world in some other respect, differ not at all in that there are many of the same kind, so that their first principles cannot be limited in number (just as the elements of all the language in this sensible world are not limited in number, but in kind, unless one takes the elements of this individual syllable or of this individual articulate sound–whose elements will be limited even in number; so is it also in the case of the intermediates; for there also the members of the same kind are infinite in number), so that if there are not–besides perceptible and mathematical objects–others such as some maintain the **forms** to be, there will be no **substance** which is one in number, but only in kind, nor will the first principles of **things** be determinate in number, but only in kind:–if then this must be so, the **forms** also must therefore be held to exist. Even if those who support this view do not express it articulately, still this is what they mean, and they must be maintaining the **forms** just because each of the **forms** is a **substance** and none is by accident.

***Metaphysics***: But if we are to suppose both that the **forms** exist and that the principles are one in number, not in kind, we have mentioned the impossible results that necessarily follow.

***Metaphysics***: But we shall say in answer to this argument also that while there is some justification for their thinking that the changing, when it is changing, does not exist, yet it is after all disputable; for that which is losing a **quality** has something of that which is being lost, and of that which is *coming to be*, something must already be. And in general if a **thing** is perishing, will be present something that exists; and if a **thing** is *coming to be*, there must be something from which it *comes to be* and something by which it is generated, and this process cannot go on ad infinitum.–But, leaving these arguments, let us insist on this, that it is not the same **thing** to change in **quantity** and in **quality**. Grant that in **quantity** a **thing** is not constant; still it is in respect of its **form** that we know each **thing**.–And again, it would be fair to criticize those who hold this view for asserting about the whole **material** universe what they saw only in a minority even of sensible **things**. For only that region of the sensible world which immediately surrounds us is always in process of destruction and generation; but this is–so to speak–not even a fraction of the whole, so that it would have been juster to acquit this part of the world because of the other part, than to condemn the other because of this.–And again, obviously we shall make to them also the same reply that we made long ago; we must show them and persuade them that there is something whose **nature** is changeless. Indeed, those who say that **things** at the same time are and are not, should in consequence say that all **things** are at rest rather than that they are in movement; for there is **nothing** into which they can change, since all attributes belong already to all **subjects**.

***Metaphysics***: ‘Cause’ means (1) that from which, as immanent **material**, a **thing** comes into being, e.g. the bronze is the cause of the statue and the silver of the saucer, and so are the classes which include these. (2) The **form** or **pattern**, i.e. the definition of the **essence**, and the classes

which include this (e.g. the **ratio** 2: and number in general are causes of the octave), and the parts included in the definition. (3) That from which the change or the resting from change first begins; e.g. the adviser is a cause of the action, and the father a cause of the child, and in general the maker a cause of the **thing** made and the change–producing of the changing. (4) The end, i.e. that for the sake of which a **thing** is; e.g. health is the cause of walking. For 'Why does one walk?' we say; 'that one may be healthy'; and in speaking thus we think we have given the cause. The same is true of all the means that intervene before the end, when something else has put the process in motion, as e.g. thinning or purging or drugs or instruments intervene before health is reached; for all these are for the sake of the end, though they differ from one another in that some are instruments and others are actions.

***Metaphysics***: Hence as regards the **things** that are or *come to be* by **nature**, though that from which they naturally *come to be* or are is already present, we say they have not their **nature** yet, unless they have their **form** or **shape**. That which comprises both of these exists by **nature**, e.g. the animals and their parts; and not only is the first **matter nature** (and this in two senses, either the first, counting from the **thing**, or the first in general; e.g. in the case of works in bronze, bronze is first with reference to them, but in general perhaps water is first, if all **things** that can be melted are water), but also the **form** or **essence**, which is the end of the process of becoming.– (6) By an extension of meaning from this sense of 'nature' every **essence** in general has *come to be* called a 'nature', because the **nature** of a **thing** is one kind of **essence**.

***Metaphysics***: (c) Two **things** are called one, when the definition which states the **essence** of one is indivisible from another definition which shows us the other (though in itself every definition is divisible). Thus even that which has increased or is diminishing is one, because its definition is one, as, in the case of plane figures, is the definition of their **form**. In general those **things** the thought of whose **essence** is indivisible, and cannot separate them either in time or in place or in definition, are most of all one, and of these especially those which are **substances**. For in general those **things** that do not admit of division are called one in so far as they do not admit of it; e.g. if two **things** are indistinguishable *quâ* man, they are one kind of man; if *quâ* animal, one kind of animal; if *quâ* magnitude, one kind of magnitude.–Now most **things** are called one because they either do or have or suffer or are related to something else that is one, but the **things** that are primarily called one are those whose **substance** is one,–and one either in continuity or in **form** or in definition; for we count as more than one either **things** that are not continuous, or those whose **form** is not one, or those whose definition is not one.

***Metaphysics***: It follows, then, that 'substance' has two senses, (A) ultimate **substratum**, which is no longer **predicated** of **anything** else, and (B) that which, being a 'this', is also separable and of this **nature** is the **shape** or **form** of each **thing**.

***Metaphysics***: 'Limit' means (1) the last point of each **thing**, i.e. the first point beyond which it is not possible to find any part, and the first point within which every part is; (2) the **form**, whatever it may be, of a spatial magnitude or of a **thing** that has magnitude; (3) the end of each **thing** (and of this **nature** is that towards which the movement and the action are, not that from which they are–though sometimes it is both, that from which and that to which the movement is, i.e. the final cause); (4) the **substance** of each **thing**, and the **essence** of each; for this is the limit of knowledge; and if of knowledge, of the object also. Evidently, therefore, 'limit' has as many senses as 'beginning', and yet more; for the beginning is a limit, but not every limit is a beginning.

Aristotle often has a difficult time with things because he cannot complete an abstraction.

***Metaphysics***: 'That in virtue of which' has several meanings:–(1) the **form** or **substance** of each **thing**, e.g. that in virtue of which a man is good is the good itself, (2) the proximate **subject** in which it is the **nature** of an attribute to be found, e.g. color in a surface. 'That in virtue of which', then, in the primary sense is the **form**, and in a secondary sense the **matter** of each **thing** and the proximate **substratum** of each.–In general 'that in virtue of which' will found in the same number of senses as 'cause'; for we say indifferently (3) in virtue of what has he come?' or 'for what end has he come?'; and (4) in virtue of what has he inferred wrongly, or inferred?' or 'what is the cause of the inference, or of the wrong inference?'–Further (5) Kath' d is used in reference to position, e.g. 'at which he stands' or 'along which he walks; for all such phrases indicate place and position.

***Metaphysics***: 'To come from something' means (1) to come from something as from **matter**, and this in two senses, either in respect of the highest **genus** or in respect of the lowest species; e.g. in a sense all **things** that can be melted come from water, but in a sense the statue comes from bronze.–(2) As from the first moving principle; e.g. 'what did the fight come from?' from abusive language, because this was the origin of the fight.–(3) from the compound of **matter** and **shape**, as the parts come from the whole, and the verse from the Iliad, and the stones from the house; (in every such case the whole is a compound of **matter** and **shape**,) for the **shape** is the end, and only that which attains an end is complete.–(4) As the **form** from its part, e.g. man from 'two-footed' and syllable from 'letter'; for this is a different sense from that in which the statue comes from bronze; for the composite **substance** comes from the sensible **matter**, but the **form** also comes from the **matter** of the **form**.–Some **things**, then, are said to come from something else in these senses; but (5) others are so described if one of these senses is applicable to a part of that other **thing**; e.g. the child comes from its father and mother, and plants come from the earth, because they come from a part of those **things**.–(6) It means coming after a **thing** in time, e.g. night comes from day and storm from fine weather, because the one comes after the other. Of these **things** some are so described because they admit of change into one another, as in the cases now mentioned; some merely because they are successive in time, e.g. the voyage took place 'from' the equinox, because it took place after the equinox, and the festival of the Thargelia comes 'from' the Dionysia, because after the Dionysia.

***Metaphysics***: 'Part' means (1) (a) that into which a quantum can in any way be divided; for that which is taken from a quantum *quâ* quantum is always called a part of it, e.g. two is called in a sense a part of three. It means (b), of the parts in the first sense, only those which measure the whole; this is why two, though in one sense it is, in another is not, called a part of three.–(2) The elements into which a kind might be divided apart from the **quantity** are also called parts of it; for which reason we say the species are parts of the **genus**.–(3) The elements into which a whole is divided, or of which it consists–the 'whole' meaning either the **form** or that which has the **form**; e.g. of the bronze sphere or of the bronze cube both the bronze–i.e. the **matter** in which the **form** is–and the characteristic angle are parts.–(4) The elements in the definition which explains a **thing** are also parts of the whole; this is why the **genus** is called a part of the species, though in another sense the species is part of the **genus**.

***Metaphysics***: Further, some do not think there is **anything** substantial besides sensible **things**, but others think there are eternal **substances** which are more in number and more real; e.g. Plato posited two kinds of **substance**–the **forms** and objects of mathematics–as well as a third kind, viz. the **substance** of sensible bodies. And Speusippus made still more kinds of **substance**,

beginning with the One, and assuming principles for each kind of **substance**, one for numbers, another for spatial magnitudes, and then another for the soul; and by going on in this way he multiplies the kinds of **substance**. And some say **forms** and numbers have the same **nature**, and the other **things** come after them–lines and planes–until we come to the **substance** of the **material** universe and to sensible bodies.

***Metaphysics***: The word 'substance' is applied, if not in more senses, still at least to four main objects; for both the **essence** and the universal and the **genus**, are thought to be the **substance** of each **thing**, and fourthly the **substratum**. Now the **substratum** is that of which everything else is **predicated**, while it is itself not **predicated** of **anything** else. And so we must first determine the **nature** of this; for that which underlies a **thing** primarily is thought to be in the truest sense its **substance**. And in one sense **matter** is said to be of the **nature** of **substratum**, in another, **shape**, and in a third, the compound of these. (By the **matter** I mean, for instance, the bronze, by the **shape** the **pattern** of its **form**, and by the compound of these the statue, the concrete whole.) Therefore if the **form** is prior to the **matter** and more real, it will be prior also to the compound of both, for the same reason.

***Metaphysics***: If we adopt this point of view, then, it follows that **matter** is **substance**. But this is impossible; for both separability and 'this-ness' are thought to belong chiefly to **substance**. And so **form** and the compound of **form** and **matter** would be thought to be **substance**, rather than **matter**. The **substance** compounded of both, i.e. of **matter** and **shape**, may be dismissed; for it is posterior and its **nature** is obvious. And **matter** also is in a sense manifest. But we must inquire into the third kind of **substance**; for this is the most perplexing.

***Metaphysics***: Thus, then, are natural products produced; all other productions are called 'makings'. And all makings proceed either from art or from a faculty or from thought. Some of them happen also spontaneously or by luck just as natural products sometimes do; for there also the same **things** sometimes are produced without seed as well as from seed. Concerning these cases, then, we must inquire later, but from art proceed the **things** of which the **form** is in the soul of the artist. (By **form** I mean the **essence** of each **thing** and its primary **substance**.) for even contraries have in a sense the same **form**; for the **substance** of a privation is the opposite **substance**, e.g. health is the **substance** of disease (for disease is the absence of health); and health is the formula in the soul or the knowledge of it. The healthy **subject** is produced as the result of the following train of thought:–since this is health, if the **subject** is to be healthy this must first be present, e.g. a uniform state of body, and if this is to be present, there must be heat; and the physician goes on thinking thus until he reduces the **matter** to a final something which he himself can produce. Then the process from this point onward, i.e. the process towards health, is called a 'making'. Therefore it follows that in a sense health comes from health and house from house, that with **matter** from that without **matter**; for the medical art and the building art are the **form** of health and of the house, and when I speak of **substance** without **matter** I mean the **essence**.

***Metaphysics***: Since **anything** which is produced is produced by something (and this I call the starting-point of the production), and from something (and let this be taken to be not the privation but the **matter**; for the meaning we attach to this has already been explained), and since something is produced (and this is either a sphere or a circle or whatever else it may chance to be), just as we do not make the **substratum** (the brass), so we do not make the sphere, except incidentally, because the brazen sphere is a sphere and we make the **form**. For to make a 'this' is to make a 'this' out of the **substratum** in the full sense of the word. (I mean that to make the

brass round is not to make the round or the sphere, but something else, i.e. to produce this **form** in something different from itself. For if we make the **form**, we must make it out of something else; for this was assumed. E.g. we make a brazen sphere; and that in the sense that out of this, which is brass, we make this other, which is a sphere.) If, then, we also make the **substratum** itself, clearly we shall make it in the same way, and the processes of making will regress to infinity. Obviously then the **form** also, or whatever we ought to call the **shape** present in the sensible **thing**, is not produced, nor is there any production of it, nor is the **essence** produced; for this is that which is made to be in something else either by art or by **nature** or by some faculty. But that there is a brazen sphere, this we make. For we make it out of brass and the sphere; we bring the **form** into this particular **matter**, and the result is a brazen sphere. But if the **essence** of sphere in general is to be produced, something must be produced out of something. For the product will always have to be divisible, and one part must be this and another that; I mean the one must be **matter** and the other **form**. If, then, a sphere is 'the figure whose circumference is at all points equidistant from the center', part of this will be the medium in which the **thing** made will be, and part will be in that medium, and the whole will be the **thing** produced, which corresponds to the brazen sphere. It is obvious, then, from what has been said, that that which is spoken of as **form** or **substance** is not produced, but the concrete **thing** which gets its name from this is produced, and that in everything which is generated **matter** is present, and one part of the **thing** is **matter** and the other **form**.

***Metaphysics***: Is there, then, a sphere apart from the individual spheres or a house apart from the bricks? Rather we may say that no 'this' would ever have been *coming to be*, if this had been so, but that the 'form' means the 'such', and is not a 'this'–a definite **thing**; but the artist makes, or the father begets, a 'such' out of a 'this'; and when it has been begotten, it is a 'this such'. And the whole 'this', Callias or Socrates, is analogous to 'this brazen sphere', but man and animal to 'brazen sphere' in general. Obviously, then, the cause which consists of the **forms** (taken in the sense in which some maintain the existence of the **forms**, i.e. if they are something apart from the individuals) is useless, at least with regard to comings-to-be and to **substances**; and the **forms** need not, for this reason at least, be self-subsistent **substances**. In some cases indeed it is even obvious that the begetter is of the same kind as the begotten (not, however, the same nor one in number, but in **form**), i.e. in the case of natural products (for man begets man), unless something happens contrary to **nature**, e.g. the production of a mule by a horse. (And even these cases are similar; for that which would be found to be common to horse and ass, the **genus** next above them, has not received a name, but it would doubtless be both in fact something like a mule.) Obviously, therefore, it is quite unnecessary to set up a **form** as a **pattern** (for we should have looked for **forms** in these cases if in any; for these are **substances** if **anything** is so); the begetter is adequate to the making of the product and to the causing of the **form** in the **matter**. And when we have the whole, such and such a **form** in this flesh and in these bones, this is Callias or Socrates; and they are different in virtue of their **matter** (for that is different), but the same in **form**; for their **form** is indivisible.

***Metaphysics***: And it is clear also from what has been said that in a sense every product of art is produced from a **thing** which shares its name (as natural products are produced), or from a part of itself which shares its name (e.g. the house is produced from a house, *quâ* produced by reason; for the art of building is the **form** of the house), or from something which contains an art of it,–if we exclude **things** produced by accident; for the cause of the **thing**'s producing the product directly *per se* is a part of the product. The heat in the movement caused heat in the body, and this is either health, or a part of health, or is followed by a part of health or by health itself. And

so it is said to cause health, because it causes that to which health attaches as a consequence.

***Metaphysics***: But not only regarding **substance** does our argument prove that its **form** does not *come to be*, but the argument applies to all the primary classes alike, i.e. **quantity**, **quality**, and the other categories. For as the brazen sphere *comes to be*, but not the sphere nor the brass, and so too in the case of brass itself, if it *comes to be*, it is its concrete unity that *comes to be* (for the **matter** and the **form** must always exist before), so is it both in the case of **substance** and in that of **quality** and **quantity** and the other categories likewise; for the **quality** does not *come to be*, but the wood of that **quality**, and the **quantity** does not *come to be*, but the wood or the animal of that size. But we may learn from these instances a peculiarity of **substance**, that there must exist beforehand in complete reality another **substance** which produces it, e.g. an animal if an animal is produced; but it is not necessary that a **quality** or **quantity** should pre-exist otherwise than potentially.

***Metaphysics***: Perhaps we should rather say that 'part' is used in several senses. One of these is 'that which measures another **thing** in respect of quantity'. But let this sense be set aside; let us inquire about the parts of which **substance** consists. If then **matter** is one **thing**, **form** another, the compound of these a third, and both the **matter** and the **form** and the compound are **substance** even the **matter** is in a sense called part of a **thing**, while in a sense it is not, but only the elements of which the formula of the **form** consists. E.g. of concavity flesh (for this is the **matter** in which it is produced) is not a part, but of snubness it is a part; and the bronze is a part of the concrete statue, but not of the statue when this is spoken of in the sense of the **form**. (for the **form**, or the **thing** as having **form**, should be said to be the **thing**, but the **material** element by itself must never be said to be so.) And so the formula of the circle does not include that of the segments, but the formula of the syllable includes that of the letters; for the letters are parts of the formula of the **form**, and not **matter**, but the segments are parts in the sense of **matter** on which the **form** supervenes; yet they are nearer the **form** than the bronze is when roundness is produced in bronze. But in a sense not even every kind of letter will be present in the formula of the syllable, e.g. particular waxen letters or the letters as movements in the air; for in these also we have already something that is part of the syllable only in the sense that it is its perceptible **matter**. For even if the line when divided passes away into its halves, or the man into bones and muscles and flesh, it does not follow that they are composed of these as parts of their **essence**, but rather as **matter**; and these are parts of the concrete **thing**, but not also of the **form**, i.e. of that to which the formula refers; wherefore also they are not present in the formulae. In one kind of formula, then, the formula of such parts will be present, but in another it must not be present, where the formula does not refer to the concrete object. For it is for this reason that some **things** have as their constituent principles parts into which they pass away, while some have not. Those **things** which are the **form** and the **matter** taken together, e.g. the snub, or the bronze circle, pass away into these **materials**, and the **matter** is a part of them; but those **things** which do not involve **matter** but are without **matter**, and whose formulae are formulae of the **form** only, do not pass away,–either not at all or at any rate not in this way. Therefore these **materials** are principles and parts of the concrete **things**, while of the **form** they are neither parts nor principles. And therefore the clay statue is resolved into clay and the ball into bronze and Callias into flesh and bones, and again the circle into its segments; for there is a sense of 'circle' in which involves **matter**. For 'circle' is used ambiguously, meaning both the circle, unqualified, and the individual circle, because there is no name peculiar to the individuals.

***Metaphysics***: The truth has indeed now been stated, but still let us state it yet more clearly,

taking up the question again. The parts of the formula, into which the formula is divided, are prior to it, either all or some of them. The formula of the right angle, however, does not include the formula of the acute, but the formula of the acute includes that of the right angle; for he who defines the acute uses the right angle; for the acute is 'less than a right angle'. The circle and the semicircle also are in a like relation; for the semicircle is defined by the circle; and so is the finger by the whole body, for a finger is 'such and such a part of a man'. Therefore the parts which are of the **nature** of **matter**, and into which as its **matter** a **thing** is divided, are posterior; but those which are of the **nature** of parts of the formula, and of the **substance** according to its formula, are prior, either all or some of them. And since the soul of animals (for this is the **substance** of a living being) is their **substance** according to the formula, i.e. the **form** and the **essence** of a body of a certain kind (at least we shall define each part, if we define it well, not without reference to its function, and this cannot belong to it without perception), so that the parts of soul are prior, either all or some of them, to the concrete 'animal', and so too with each individual animal; and the body and parts are posterior to this, the essential **substance**, and it is not the **substance** but the concrete **thing** that is divided into these parts as its **matter**:–this being so, to the concrete **thing** these are in a sense prior, but in a sense they are not. For they cannot even exist if severed from the whole; for it is not a finger in any and every state that is the finger of a living **thing**, but a dead finger is a finger only in name. Some parts are neither prior nor posterior to the whole, i.e. those which are dominant and in which the formula, i.e. the essential **substance**, is immediately present, e.g. perhaps the heart or the brain; for it does not **matter** in the least which of the two has this **quality**. But man and horse and terms which are thus applied to individuals, but universally, are not **substance** but something composed of this particular formula and this particular **matter** treated as universal; and as regards the individual, Socrates already includes in him ultimate individual **matter**; and similarly in all other cases. 'A part' may be a part either of the **form** (i.e. of the **essence**), or of the compound of the **form** and the **matter**, or of the **matter** itself. But only the parts of the **form** are parts of the formula, and the formula is of the universal; for 'being a circle' is the same as the circle, and 'being a soul' the same as the soul. But when we come to the concrete **thing**, e.g. this circle, i.e. one of the individual circles, whether perceptible or intelligible (I mean by intelligible circles the mathematical, and by perceptible circles those of bronze and of wood),–of these there is no definition, but they are known by the aid of intuitive thinking or of perception; and when they pass out of this complete realization it is not clear whether they exist or not; but they are always stated and recognized by means of the universal formula. But **matter** is unknowable in itself. And some **matter** is perceptible and some intelligible, perceptible **matter** being for instance bronze and wood and all **matter** that is changeable, and intelligible **matter** being that which is present in perceptible **things** not *quâ* perceptible, i.e. the objects of mathematics.

***Metaphysics***: Another question is naturally raised, viz. what sort of parts belong to the **form** and what sort not to the **form**, but to the concrete **thing**. Yet if this is not plain it is not possible to define any **thing**; for definition is of the universal and of the **form**. If then it is not evident what sort of parts are of the **nature** of **matter** and what sort are not, neither will the formula of the **thing** be evident. In the case of **things** which are found to occur in specifically different **materials**, as a circle may exist in bronze or stone or wood, it seems plain that these, the bronze or the stone, are no part of the **essence** of the circle, since it is found apart from them. Of **things** which are not seen to exist apart, there is no reason why the same may not be true, just as if all circles that had ever been seen were of bronze; for none the less the bronze would be no part of the **form**; but it is hard to eliminate it in thought. E.g. the **form** of man is always found in flesh

and bones and parts of this kind; are these then also parts of the **form** and the formula? No, they are **matter**; but because man is not found also in other **matters** we are unable to perform the abstraction.

***Metaphysics***: Since this is thought to be possible, but it is not clear when it is the case, some people already raise the question even in the case of the circle and the triangle, thinking that it is not right to define these by reference to lines and to the continuous, but that all these are to the circle or the triangle as flesh and bones are to man, and bronze or stone to the statue; and they reduce all **things** to numbers, and they say the formula of 'line' is that of 'two'. And of those who assert the **Ideas** some make 'two' the line-itself, and others make it the **form** of the line; for in some cases they say the **form** and that of which it is the **form** are the same, e.g. 'two' and the **form** of two; but in the case of 'line' they say this is no longer so.

***Metaphysics***: It follows then that there is one **form** for many **things** whose **form** is evidently different (a conclusion which confronted the Pythagoreans also); and it is possible to make one **thing** the **form**-itself of all, and to hold that the others are not **forms**; but thus all **things** will be one.

***Metaphysics***: We have pointed out, then, that the question of definitions contains some difficulty, and why this is so. And so to reduce all **things** thus to **forms** and to eliminate the **matter** is useless labor; for some **things** surely are a particular **form** in a particular **matter**, or particular **things** in a particular state. And the comparison which Socrates the younger used to make in the case of 'animal' is not sound; for it leads away from the truth, and makes one suppose that man can possibly exist without his parts, as the circle can without the bronze. But the case is not similar; for an animal is something perceptible, and it is not possible to define it without reference to movement–nor, therefore, without reference to the parts' being in a certain state. For it is not a hand in any and every state that is a part of man, but only when it can fulfil its work, and therefore only when it is alive; if it is not alive it is not a part.

***Metaphysics***: Regarding the objects of mathematics, why are the formulae of the parts not parts of the formulae of the wholes; e.g. why are not the semicircles included in the formula of the circle? It cannot be said, 'because these parts are perceptible **things**'; for they are not. But perhaps this makes no difference; for even some **things** which are not perceptible must have **matter**; indeed there is some **matter** in everything which is not an **essence** and a bare **form** but a 'this'. The semicircles, then, will not be parts of the universal circle, but will be parts of the individual circles, as has been said before; for while one kind of **matter** is perceptible, there is another which is intelligible.

***Metaphysics***: If then a **differentia** of a **differentia** be taken at each step, one **differentia**–the last–will be the **form** and the **substance**; but if we divide according to accidental **qualities**, e.g. if we were to divide that which is endowed with feet into the white and the black, there will be as many **differentiae** as there are cuts. Therefore it is plain that the definition is the formula which contains the **differentiae**, or, according to the right method, the last of these. This would be evident, if we were to change the **order** of such definitions, e.g. of that of man, saying 'animal which is two-footed and endowed with feet'; for 'endowed with feet' is superfluous when 'two-footed' has been said. But there is no **order** in the **substance**; for how are we to think the one element posterior and the other prior? Regarding the definitions, then, which are reached by the method of divisions, let this suffice as our first attempt at stating their **nature**.

***Metaphysics***: It is clear also from these very facts what consequence confronts those who say

the **Ideas** are **substances** capable of separate existence, and at the same time make the **form** consist of the **genus** and the **differentiae**. For if the **forms** exist and ‘animal’ is present in ‘man’ and ‘horse’, it is either one and the same in number, or different. (In formula it is clearly one; for he who states the formula will go through the formula in either case.) If then there is a ‘man-in-himself’ who is a ‘this’ and exists apart, the parts also of which he consists, e.g. ‘animal’ and ‘two-footed’, must indicate ‘this’es’, and be capable of separate existence, and **substances**; therefore ‘animal’, as well as ‘man’, must be of this sort.

***Metaphysics***: Further, (3) in the case of sensible **things** both these consequences and others still more absurd follow. If, then, these consequences are impossible, clearly there are not **forms** of sensible **things** in the sense in which some maintain their existence.

***Metaphysics***: But those who say the **forms** exist, in one respect are right, in giving the **forms** separate existence, if they are **substances**; but in another respect they are not right, because they say the one over many is a **form**. The reason for their doing this is that they cannot declare what are the **substances** of this sort, the imperishable **substances** which exist apart from the individual and sensible **substances**. They make them, then, the same in kind as the perishable **things** (for this kind of **substance** we know)–’man-himself’ and ‘horse-itself’, adding to the sensible **things** the word ‘itself’. Yet even if we had not seen the stars, none the less, I suppose, would they have been eternal **substances** apart from those which we knew; so that now also if we do not know what non-sensible **substances** there are, yet it is doubtless necessary that there should be some.–Clearly, then, no universal term is the name of a **substance**, and no **substance** is composed of **substances**.

***Metaphysics***: The object of the inquiry is most easily overlooked where one term is not expressly **predicated** of another (e.g. when we inquire ‘what man is’), because we do not distinguish and do not say definitely that certain elements make up a certain whole. But we must articulate our meaning before we begin to inquire; if not, the inquiry is on the border-line between being a search for something and a search for **nothing**. Since we must have the existence of the **thing** as something given, clearly the question is why the **matter** is some definite **thing**; e.g. why are these **materials** a house? Because that which was the **essence** of a house is present. And why is this individual **thing**, or this body having this **form**, a man? Therefore what we seek is the cause, i.e. the **form**, by reason of which the **matter** is some definite **thing**; and this is the **substance** of the **thing**. Evidently, then, in the case of simple terms no inquiry nor teaching is possible; our attitude towards such **things** is other than that of inquiry.

***Metaphysics***: WE must reckon up the results arising from what has been said, and compute the sum of them, and put the finishing touch to our inquiry. We have said that the causes, principles, and elements of **substances** are the object of our search. And some **substances** are recognized by every one, but some have been advocated by particular schools. Those generally recognized are the natural **substances**, i.e. Fire, earth, water, air, &c., the simple bodies; second plants and their parts, and animals and the parts of animals; and finally the physical universe and its parts; while some particular schools say that **forms** and the objects of mathematics are **substances**. But there are arguments which lead to the conclusion that there are other **substances**, the **essence** and the **substratum**. Again, in another way the **genus** seems more substantial than the various species, and the universal than the particulars. And with the universal and the **genus** the **Ideas** are connected; it is in virtue of the same argument that they are thought to be **substances**. And since the **essence** is **substance**, and the definition is a formula of the **essence**, for this reason we have discussed definition and essential predication. Since the

definition is a formula, and a formula has parts, we had to consider also with respect to the notion of 'part', what are parts of the **substance** and what are not, and whether the parts of the **substance** are also parts of the definition. Further, too, neither the universal nor the **genus** is a **substance**; we must inquire later into the **Ideas** and the objects of mathematics; for some say these are **substances** as well as the sensible **substances**.

***Metaphysics***: Obviously, then, the **actuality** or the formula is different when the **matter** is different; for in some cases it is the composition, in others the mixing, and in others some other of the attributes we have named. And so, of the people who go in for defining, those who define a house as stones, bricks, and timbers are speaking of the potential house, for these are the **matter**; but those who propose 'a receptacle to shelter chattels and living beings', or something of the sort, speak of the **actuality**. Those who combine both of these speak of the third kind of **substance**, which is composed of **matter** and **form** (for the formula that gives the **differentiae** seems to be an account of the **form** or **actuality**, while that which gives the components is rather an account of the **matter**); and the same is true of the kind of definitions which Archytas used to accept; they are accounts of the combined **form** and **matter**. E.g. what is still weather? Absence of motion in a large expanse of air; air is the **matter**, and absence of motion is the **actuality** and **substance**. What is a calm? Smoothness of sea; the **material substratum** is the sea, and the **actuality** or **shape** is smoothness. It is obvious then, from what has been said, what sensible **substance** is and how it exists–one kind of it as **matter**, another as **form** or **actuality**, while the third kind is that which is composed of these two.

***Metaphysics***: We must not fail to notice that sometimes it is not clear whether a name means the composite **substance**, or the **actuality** or **form**, e.g. whether 'house' is a sign for the composite **thing**, 'a covering consisting of bricks and stones laid thus and thus', or for the **actuality** or **form**, 'a covering', and whether a line is 'two-ness in length' or 'two-ness', and whether an animal is soul in a body' or 'a soul'; for soul is the **substance** or **actuality** of some body. 'Animal' might even be applied to both, not as something definable by one formula, but as related to a single **thing**. But this question, while important for another purpose, is of no importance for the inquiry into sensible **substance**; for the **essence** certainly attaches to the **form** and the **actuality**. For 'soul' and 'to be soul' are the same, but 'to be man' and 'man' are not the same, unless even the bare soul is to be called man; and thus on one interpretation the **thing** is the same as its **essence**, and on another it is not.

***Metaphysics***: (This, then, must either be eternal or it must be destructible without being ever in course of being destroyed, and must have *come to be* without ever being in course of *coming to be*. But it has been proved and explained elsewhere that no one makes or begets the **form**, but it is the individual that is made, i.e. the complex of **form** and **matter** that is generated. Whether the **substances** of destructible **things** can exist apart, is not yet at all clear; except that obviously this is impossible in some cases–in the case of **things** which cannot exist apart from the individual instances, e.g. house or utensil. Perhaps, indeed, neither these **things** themselves, nor any of the other **things** which are not **formed** by **nature**, are **substances** at all; for one might say that the **nature** in natural objects is the only **substance** to be found in destructible **things**.)

***Metaphysics***: Therefore the difficulty which used to be raised by the school of Antisthenes and other such uneducated people has a certain timeliness. They said that the 'what' cannot be defined (for the definition so called is a 'long rigmarole') but of what sort a **thing**, e.g. silver, is, they thought it possible actually to explain, not saying what it is, but that it is like tin. Therefore one kind of **substance** can be defined and formulated, i.e. the composite kind, whether it be

perceptible or intelligible; but the primary parts of which this consists cannot be defined, since a definitory formula **predicates** something of something, and one part of the definition must play the part of **matter** and the other that of **form**.

***Metaphysics***: It is also obvious that, if **substances** are in a sense numbers, they are so in this sense and not, as some say, as numbers of units. For a definition is a sort of number; for (1) it is divisible, and into indivisible parts (for definitory formulae are not infinite), and number also is of this **nature**. And (2) as, when one of the parts of which a number consists has been taken from or added to the number, it is no longer the same number, but a different one, even if it is the very smallest part that has been taken away or added, so the definition and the **essence** will no longer remain when **anything** has been taken away or added. And (3) the number must be something in virtue of which it is one, and this these thinkers cannot state, what makes it one, if it is one (for either it is not one but a sort of heap, or if it is, we ought to say what it is that makes one out of many); and the definition is one, but similarly they cannot say what makes it one. And this is a natural result; for the same reason is applicable, and **substance** is one in the sense which we have explained, and not, as some say, by being a sort of unit or point; each is a complete reality and a definite **nature**. And (4) as number does not admit of the more and the less, neither does **substance**, in the sense of **form**, but if any **substance** does, it is only the **substance** which involves **matter**. Let this, then, suffice for an account of the generation and destruction of so-called **substances** in what sense it is possible and in what sense impossible–and of the reduction of **things** to number.

***Metaphysics***: Since some **things** are and are not, without *coming to be* and ceasing to be, e.g. points, if they can be said to be, and in general **forms** (for it is not 'white' *comes to be*, but the wood *comes to be* white, if everything that *comes to be* comes from something and *comes to be* something), not all contraries can come from one another, but it is in different senses that a pale man comes from a dark man, and pale comes from dark. Nor has everything **matter**, but only those **things** which *come to be* and change into one another. Those **things** which, without ever being in course of changing, are or are not, have no **matter**.

***Metaphysics***: There is difficulty in the question how the **matter** of each **thing** is related to its contrary states. E.g. if the body is potentially healthy, and disease is contrary to health, is it potentially both healthy and diseased? And is water potentially wine and vinegar? We answer that it is the **matter** of one in virtue of its positive state and its **form**, and of the other in virtue of the privation of its positive state and the corruption of it contrary to its **nature**. It is also hard to say why wine is not said to be the **matter** of vinegar nor potentially vinegar (though vinegar is produced from it), and why a living man is not said to be potentially dead. In fact they are not, but the corruptions in question are accidental, and it is the **matter** of the animal that is itself in virtue of its corruption the potency and **matter** of a corpse, and it is water that is the **matter** of vinegar. For the corpse comes from the animal, and vinegar from wine, as night from day. And all the **things** which change thus into one another must go back to their **matter**; e.g. if from a corpse is produced an animal, the corpse first goes back to its **matter**, and only then becomes an animal; and vinegar first goes back to water, and only then becomes wine.

***Metaphysics***: To return to the difficulty which has been stated with respect both to definitions and to numbers, what is the cause of their unity? In the case of all **things** which have several parts and in which the totality is not, as it were, a mere heap, but the whole is something beside the parts, there is a cause; for even in bodies contact is the cause of unity in some cases, and in others viscosity or some other such **quality**. And a definition is a set of words which is one not

by being connected together, like the Iliad, but by dealing with one object.–What then, is it that makes man one; why is he one and not many, e.g. animal biped, especially if there are, as some say, an animal-itself and a biped-itself? Why are not those **forms** themselves the man, so that men would exist by **participation** not in man, nor in one **form**, but in two, animal and biped, and in general man would be not one but more than one **thing**, animal and biped?

***Metaphysics***: Clearly, then, if people proceed thus in their usual manner of definition and speech, they cannot explain and solve the difficulty. But if, as we say, one element is **matter** and another is **form**, and one is potentially and the other actually, the question will no longer be thought a difficulty. For this difficulty is the same as would arise if 'round bronze' were the definition of 'cloak'; for this word would be a sign of the definitory formula, so that the question is, what is the cause of the unity of 'round' and 'bronze'? The difficulty disappears, because the one is **matter**, the other **form**. What, then, causes this–that which was potentially to be actually–except, in the case of **things** which are generated, the agent? For there is no other cause of the potential sphere's becoming actually a sphere, but this was the **essence** of either. Of **matter** some is intelligible, some perceptible, and in a formula there is always an element of **matter** as well as one of **actuality**; e.g. the circle is 'a plane figure'. But of the **things** which have no **matter**, either intelligible or perceptible, each is by its **nature** essentially a kind of unity, as it is essentially a kind of being–individual **substance**, **quality**, or **quantity** (and so neither 'existent' nor 'one' is present in their definitions), and the **essence** of each of them is by its very **nature** a kind of unity as it is a kind of being–and so none of these has any reason outside itself, for being one, nor for being a kind of being; for each is by its **nature** a kind of being and a kind of unity, not as being in the **genus** 'being' or 'one' nor in the sense that being and unity can exist apart from particulars.

> i.e. One cannot say that '***forms'*** or '***matter'*** 'exist' as '**things**' 'exist'. Existence cannot be divided against itself.

***Metaphysics***: Owing to the difficulty about unity some speak of 'participation', and raise the question, what is the cause of **participation** and what is it to participate; and others speak of 'communion', as Lycophron says knowledge is a communion of knowing with the soul; and others say life is a 'composition' or 'connexion' of soul with body. Yet the same account applies to all cases; for being healthy, too, will on this showing be either a 'communion' or a 'connexion' or a 'composition' of soul and health, and the fact that the bronze is a triangle will be a 'composition' of bronze and triangle, and the fact that a **thing** is white will be a 'composition' of surface and whiteness. The reason is that people look for a unifying formula, and a difference, between potency and complete reality. But, as has been said, the proximate **matter** and the **form** are one and the same **thing**, the one potentially, and the other actually. Therefore it is like asking what in general is the cause of unity and of a **thing**'s being one; for each **thing** is a unity, and the potential and the actual are somehow one. Therefore there is no other cause here unless there is something which caused the movement from potency into **actuality**. And all **things** which have no **matter** are without qualification essentially unities.

***Metaphysics***: It seems that when we call a **thing** not something else but 'that-en'–e.g. a casket is not 'wood' but 'wooden', and wood is not 'earth' but 'earthen', and again earth will illustrate our point if it is similarly not something else but 'that-en'–that other **thing** is always potentially (in the full sense of that word) the **thing** which comes after it in this series. E.g. a casket is not 'earthen' nor 'earth', but 'wooden'; for this is potentially a casket and this is the **matter** of a casket, wood in general of a casket in general, and this particular wood of this particular casket.

And if there is a first **thing**, which is no longer, in reference to something else, called ‘that-en’, this is prime **matter**; e.g. if earth is ‘airy’ and air is not ‘fire’ but ‘fiery’, fire is prime **matter**, which is not a ‘this’. For the **subject** or **substratum** is differentiated by being a ‘this’ or not being one; i.e. the **substratum** of modifications is, e.g. a man, i.e. a body and a soul, while the modification is ‘musical’ or ‘pale’. (The **subject** is called, when music *comes to be* present in it, not ‘music’ but ‘musical’, and the man is not ‘paleness’ but ‘pale’, and not ‘ambulation’ or ‘movement’ but ‘walking’ or ‘moving’,–which is akin to the ‘that-en’.) Wherever this is so, then, the ultimate **subject** is a **substance**; but when this is not so but the **predicate** is a **form** and a ‘this’, the ultimate **subject** is **matter** and **material substance**. And it is only right that ‘that-en’ should be used with reference both to the **matter** and to the accidents; for both are indeterminates.

***Metaphysics***: But (3) it is also prior in substantiality; firstly, (a) because the **things** that are posterior in becoming are prior in **form** and in substantiality (e.g. man is prior to boy and human being to seed; for the one already has its **form**, and the other has not), and because everything that *comes to be* moves towards a principle, i.e. an end (for that for the sake of which a **thing** is, is its principle, and the becoming is for the sake of the end), and the **actuality** is the end, and it is for the sake of this that the potency is acquired. For animals do not see in **order** that they may have sight, but they have sight that they may see. And similarly men have the art of building that they may build, and theoretical science that they may theorize; but they do not theorize that they may have theoretical science, except those who are learning by practice; and these do not theorize except in a limited sense, or because they have no need to theorize. Further, **matter** exists in a potential state, just because it may come to its **form**; and when it exists actually, then it is in its **form**. And the same holds good in all cases, even those in which the end is a movement. And so, as teachers think they have achieved their end when they have exhibited the pupil at work, **nature** does likewise. For if this is not the case, we shall have Pauson’s Hermes over again, since it will be hard to say about the knowledge, as about the figure in the picture, whether it is within or without. For the action is the end, and the **actuality** is the action. And so even the word ‘actuality’ is derived from ‘action’, and points to the complete reality.

***Metaphysics***: Obviously, therefore, the **substance** or **form** is **actuality**. According to this argument, then, it is obvious that **actuality** is prior in substantial being to potency; and as we have said, one **actuality** always precedes another in time right back to the **actuality** of the eternal prime mover.

***Metaphysics***: To the one belong, as we indicated graphically in our distinction of the contraries, the same and the like and the equal, and to plurality belong the other and the unlike and the unequal. ‘The same’ has several meanings; (1) we sometimes mean ‘the same numerically’; again, (2) we call a **thing** the same if it is one both in definition and in number, e.g. you are one with yourself both in **form** and in **matter**; and again, (3) if the definition of its primary **essence** is one; e.g. equal straight lines are the same, and so are equal and equal-angled quadrilaterals; there are many such, but in these equality constitutes unity.

***Metaphysics***: **Things** are like if, not being absolutely the same, nor without difference in respect of their concrete **substance**, they are the same in **form**; e.g. the larger square is like the smaller, and unequal straight lines are like; they are like, but not absolutely the same. Other **things** are like, if, having the same **form**, and being **things** in which difference of degree is possible, they have no difference of degree. Other **things**, if they have a **quality** that is in **form** one and same–e.g. whiteness–in a greater or less degree, are called like because their **form** is

one. Other **things** are called like if the **qualities** they have in common are more numerous than those in which they differ–either the **qualities** in general or the prominent **qualities**; e.g. tin is like silver, *quâ* white, and gold is like fire, *quâ* yellow and red.

***Metaphysics***: The primary contrariety is that between positive state and privation–not every privation, however (for 'privation' has several meanings), but that which is complete. And the other contraries must be called so with reference to these, some because they possess these, others because they produce or tend to produce them, others because they are acquisitions or losses of these or of other contraries. Now if the kinds of opposition are contradiction and privation and contrariety and relation, and of these the first is contradiction, and contradiction admits of no intermediate, while contraries admit of one, clearly contradiction and contrariety are not the same. But privation is a kind of contradiction; for what suffers privation, either in general or in some determinate way, either that which is quite incapable of having some attribute or that which, being of such a **nature** as to have it, has it not; here we have already a variety of meanings, which have been distinguished elsewhere. Privation, therefore, is a contradiction or incapacity which is determinate or taken along with the receptive **material**. This is the reason why, while contradiction does not admit of an intermediate, privation sometimes does; for everything is equal or not equal, but not everything is equal or unequal, or if it is, it is only within the sphere of that which is receptive of equality. If, then, the comings-to-be which happen to the **matter** start from the contraries, and proceed either from the **form** and the possession of the **form** or from a privation of the **form** or **shape**, clearly all contrariety must be privation, but presumably not all privation is contrariety (the reason being that that has suffered privation may have suffered it in several ways); for it is only the extremes from which changes proceed that are contraries.

***Metaphysics***: Evidently, then, there cannot be **forms** such as some maintain, for then one man would be perishable and another imperishable. Yet the **forms** are said to be the same in **form** with the individuals and not merely to have the same name; but **things** which differ in kind are farther apart than those which differ in **form**.

***Metaphysics***: But again the science we are looking for must not be supposed to deal with the causes which have been mentioned in the ***Physics***. For (A) it does not deal with the final cause (for that is the **nature** of the good, and this is found in the field of action and movement; and it is the first mover–for that is the **nature** of the end–but in the case of **things** unmovable there is **nothing** that moved them first), and (B) in general it is hard to say whether perchance the science we are now looking for deals with perceptible **substances** or not with them, but with certain others. If with others, it must deal either with the **forms** or with the objects of mathematics. Now (a) evidently the **forms** do not exist. (But it is hard to say, even if one suppose them to exist, why in the world the same is not true of the other **things** of which there are **forms**, as of the objects of mathematics. I mean that these thinkers place the objects of mathematics between the **forms** and perceptible **things**, as a kind of third set of **things** apart both from the **forms** and from the **things** in this world; but there is not a third man or horse besides the ideal and the individuals. If on the other hand it is not as they say, with what sort of **things** must the mathematician be supposed to deal? Certainly not with the **things** in this world; for none of these is the sort of **thing** which the mathematical sciences demand.) Nor (b) does the science which we are now seeking treat of the objects of mathematics; for none of them can exist separately. But again it does not deal with perceptible **substances**; for they are perishable.

Like most, Aristotle does not see Arithmetic as simply a grammar, and numbers only names in an

ordered naming convention.

Apparently the ‘third man’ is a claim that between the Universal and the Particular there is some third state. One must see the Universal as the **Form**, Definition, etc., and the Particular as the **matter**, member, etc., of that **form**. This is an abstraction from ‘proposition’ which is **matter** and **form**. If there is **matter**, **form** and something other, one can make a claim for the third man. Aristotle actually supports the ‘third man’ when he says that **matter**, **form**, and the compound are each of them ‘**things**’ when in fact only the compound, **thing**, proposition, exists by definition of existence.]

***Metaphysics***: Further, must we suppose something apart from individual **things**, or is it these that the science we are seeking treats of? But these are infinite in number. Yet the **things** that are apart from the individuals are **genuses** or species; but the science we now seek treats of neither of these. The reason why this is impossible has been stated. Indeed, it is in general hard to say whether one must assume that there is a separable **substance** besides the sensible **substances** (i.e. the **substances** in this world), or that these are the real **things** and Wisdom is concerned with them. For we seem to seek another kind of **substance**, and this is our problem, i.e. to see if there is something which can exist apart by itself and belongs to no sensible **thing**.–further, if there is another **substance** apart from and corresponding to sensible **substances**, which kinds of sensible **substance** must be supposed to have this corresponding to them? Why should one suppose men or horses to have it, more than either the other animals or even all lifeless **things**? On the other hand to set up other and eternal **substances** equal in number to the sensible and perishable **substances** would seem to fall beyond the bounds of probability.–But if the principle we now seek is not separable from corporeal **things**, what has a better claim to the name **matter**? This, however, does not exist in **actuality**, but exists in potency. And it would seem rather that the **form** or **shape** is a more important principle than this; but the **form** is perishable, so that there is no eternal **substance** at all which can exist apart and independent. But this is paradoxical; for such a principle and **substance** seems to exist and is sought by nearly all the most refined thinkers as something that exists; for how is there to be **order** unless there is something eternal and independent and permanent?

***Metaphysics***: Further, is there **anything** apart from the concrete **thing** (by which I mean the **matter** and that which is joined with it), or not? If not, we are met by the objection that all **things** that are in **matter** are perishable. But if there is something, it must be the **form** or **shape**. Now it is hard to determine in which cases this exists apart and in which it does not; for in some cases the **form** is evidently not separable, e.g. in the case of a house.

***Metaphysics***: There are three kinds of **substance**–one that is sensible (of which one subdivision is eternal and another is perishable; the latter is recognized by all men, and includes e.g. plants and animals), of which we must grasp the elements, whether one or many; and another that is immovable, and this certain thinkers assert to be capable of existing apart, some dividing it into two, others identifying the **forms** and the objects of mathematics, and others positing, of these two, only the objects of mathematics. The former two kinds of **substance** are the **subject** of physics (for they imply movement); but the third kind belongs to another science, if there is no principle common to it and to the other kinds.

***Metaphysics***: One might raise the question from what sort of non-being generation proceeds; for ‘non-being’ has three senses. If, then, one form of non-being exists potentially, still it is not by virtue of a **potentiality** for any and every **thing**, but different **things** come from different

**things**; nor is it satisfactory to say that ‘all **things** were together’; for they differ in their **matter**, since otherwise why did an infinity of **things** *come to be*, and not one **thing**? For ‘reason’ is one, so that if **matter** also were one, that must have *come to be* in **actuality** which the **matter** was in potency. The causes and the principles, then, are three, two being the pair of contraries of which one is definition and **form** and the other is privation, and the third being the **matter**.

***Metaphysics***: Note, next, that neither the **matter** nor the **form** *comes to be*–and I mean the last **matter** and **form**. For everything that changes is something and is changed by something and into something. That by which it is changed is the immediate mover; that which is changed, the **matter**; that into which it is changed, the **form**. The process, then, will go on to infinity, if not only the bronze *comes to be* round but also the round or the bronze *comes to be*; therefore there must be a stop.

***Metaphysics***: There are three kinds of **substance**–the **matter**, which is a ‘this’ in appearance (for all **things** that are characterized by contact and not, by organic unity are **matter** and **substratum**, e.g. Fire, flesh, head; for these are all **matter**, and the last **matter** is the **matter** of that which is in the full sense **substance**); the **nature**, which is a ‘this’ or positive state towards which movement takes place; and again, thirdly, the particular **substance** which is composed of these two, e.g. Socrates or Callias. Now in some cases the ‘this’ does not exist apart from the composite **substance**, e.g. the **form** of house does not so exist, unless the art of building exists apart (nor is there generation and destruction of these **forms**, but it is in another way that the house apart from its **matter**, and health, and all ideals of art, exist and do not exist); but if the ‘this’ exists apart from the concrete **thing**, it is only in the case of natural objects. And so Plato was not far wrong when he said that there are as many **forms** as there are kinds of natural object (if there are **forms** distinct from the **things** of this earth). The moving causes exist as **things** preceding the effects, but causes in the sense of definitions are simultaneous with their effects. For when a man is healthy, then health also exists; and the **shape** of a bronze sphere exists at the same time as the bronze sphere. (But we must examine whether any **form** also survives afterwards. For in some cases there is **nothing** to prevent this; e.g. the soul may be of this sort–not all soul but the reason; for presumably it is impossible that all soul should survive.) Evidently then there is no necessity, on this ground at least, for the existence of the **Ideas**. For man is begotten by man, a given man by an individual father; and similarly in the arts; for the medical art is the formal cause of health.

***Metaphysics***: Or, as we are wont to put it, in a sense they have and in a sense they have not; e.g. perhaps the elements of perceptible bodies are, as **form**, the hot, and in another sense the cold, which is the privation; and, as **matter**, that which directly and of itself potentially has these attributes; and **substances** comprise both these and the **things** composed of these, of which these are the principles, or any unity which is produced out of the hot and the cold, e.g. Flesh or bone; for the product must be different from the elements. These **things** then have the same elements and principles (though specifically different **things** have specifically different elements); but all **things** have not the same elements in this sense, but only analogically; i.e. one might say that there are three principles–the **form**, the privation, and the **matter**. But each of these is different for each class; e.g. in color they are white, black, and surface, and in day and night they are light, darkness, and air.

***Metaphysics***: Since not only the elements present in a **thing** are causes, but also something external, i.e. the moving cause, clearly while ‘principle’ and ‘element’ are different both are causes, and ‘principle’ is divided into these two kinds; and that which acts as producing

movement or rest is a principle and a **substance**. Therefore analogically there are three elements, and four causes and principles; but the elements are different in different **things**, and the proximate moving cause is different for different **things**. Health, disease, body; the moving cause is the medical art. **Form**, disorder of a particular kind, bricks; the moving cause is the building art. And since the moving cause in the case of natural **things** is–for man, for instance, man, and in the products of thought the **form** or its contrary, there will be in a sense three causes, while in a sense there are four. For the medical art is in some sense health, and the building art is the **form** of the house, and man begets man; Further, besides these there is that which as first of all **things** moves all **things**.

***Metaphysics***: And in yet another way, analogically identical **things** are principles, i.e. **actuality** and potency; but these also are not only different for different **things** but also apply in different ways to them. For in some cases the same **thing** exists at one time actually and at another potentially, e.g. wine or flesh or man does so. (And these too fall under the above-named causes. For the **form** exists actually, if it can exist apart, and so does the complex of **form** and **matter**, and the privation, e.g. darkness or disease; but the **matter** exists potentially; for this is that which can become qualified either by the **form** or by the privation.) But the distinction of **actuality** and **potentiality** applies in another way to cases where the **matter** of cause and of effect is not the same, in some of which cases the **form** is not the same but different; e.g. the cause of man is (1) the elements in man (viz. Fire and earth as **matter**, and the peculiar **form**), and further (2) something else outside, i.e. the father, and (3) besides these the sun and its oblique course, which are neither **matter** nor **form** nor privation of man nor of the same species with him, but moving causes.

***Metaphysics***: Further, if the causes of **substances** are the causes of all **things**, yet different **things** have different causes and elements, as was said; the causes of **things** that are not in the same class, e.g. of colors and sounds, of **substances** and **quantities**, are different except in an analogical sense; and those of **things** in the same species are different, not in species, but in the sense that the causes of different individuals are different, your **matter** and **form** and moving cause being different from mine, while in their universal definition they are the same. And if we inquire what are the principles or elements of **substances** and relations and **qualities**–whether they are the same or different–clearly when the names of the causes are used in several senses the causes of each are the same, but when the senses are distinguished the causes are not the same but different, except that in the following senses the causes of all are the same. They are (1) the same or analogous in this sense, that **matter**, **form**, privation, and the moving cause are common to all **things**; and (2) the causes of **substances** may be treated as causes of all **things** in this sense, that when **substances** are removed all **things** are removed; Further, (3) that which is first in respect of complete reality is the cause of all **things**. But in another sense there are different first causes, viz. all the contraries which are neither generic nor ambiguous terms; and, further, the **matters** of different **things** are different. We have stated, then, what are the principles of sensible **things** and how many they are, and in what sense they are the same and in what sense different.

***Metaphysics***: But if there is something which is capable of moving **things** or acting on them, but is not actually doing so, there will not necessarily be movement; for that which has a potency need not exercise it. Nothing, then, is gained even if we suppose eternal **substances**, as the believers in the **forms** do, unless there is to be in them some principle which can cause change; nay, even this is not enough, nor is another **substance** besides the **forms** enough; for if it is not

to act, there will be no movement. Further even if it acts, this will not be enough, if its **essence** is potency; for there will not be eternal movement, since that which is potentially may possibly not be. There must, then, be such a principle, whose very **essence** is **actuality**. Further, then, these **substances** must be without **matter**; for they must be eternal, if **anything** is eternal. Therefore they must be **actuality**.

***Metaphysics***: Further, why should there always be becoming, and what is the cause of becoming?–this no one tells us. And those who suppose two principles must suppose another, a superior principle, and so must those who believe in the **forms**; for why did **things** come to participate, or why do they participate, in the **forms**? And all other thinkers are confronted by the necessary consequence that there is something contrary to Wisdom, i.e. to the highest knowledge; but we are not. For there is **nothing** contrary to that which is primary; for all contraries have **matter**, and **things** that have **matter** exist only potentially; and the ignorance which is contrary to any knowledge leads to an object contrary to the object of the knowledge; but what is primary has no contrary.

***Metaphysics***: Again, if besides sensible **things** no others exist, there will be no first principle, no **order**, no becoming, no heavenly bodies, but each principle will have a principle before it, as in the accounts of the theologians and all the natural philosophers. But if the **forms** or the numbers are to exist, they will be causes of **nothing**; or if not that, at least not of movement. Further, how is extension, i.e. a continuum, to be produced out of unextended parts? For number will not, either as mover or as **form**, produce a continuum. But again there cannot be any contrary that is also essentially a productive or moving principle; for it would be possible for it not to be. Or at least its action would be posterior to its potency. The world, then, would not be eternal. But it is; one of these premises, then, must be denied. And we have said how this must be done. Further, in virtue of what the numbers, or the soul and the body, or in general the **form** and the **thing**, are one–of this no one tells us **anything**; nor can any one tell, unless he says, as we do, that the mover makes them one. And those who say mathematical number is first and go on to generate one kind of **substance** after another and give different principles for each, make the **substance** of the universe a mere series of episodes (for one **substance** has no influence on another by its existence or nonexistence), and they give us many governing principles; but the world refuses to be governed badly.

***Metaphysics***: Again, the solid is a sort of **substance**; for it already has in a sense completeness. But how can lines be **substances**? Neither as a **form** or **shape**, as the soul perhaps is, nor as **matter**, like the solid; for we have no experience of **anything** that can be put together out of lines or planes or points, while if these had been a sort of **material substance**, we should have observed **things** which could be put together out of them.

***Metaphysics***: So much then for the objects of mathematics; we have said that they exist and in what sense they exist, and in what sense they are prior and in what sense not prior. Now, regarding the **Ideas**, we must first examine the ***Ideal Theory*** itself, not connecting it in any way with the **nature** of numbers, but treating it in the form in which it was originally understood by those who first maintained the existence of the **Ideas**. The supporters of the ***Ideal Theory*** were led to it because on the question about the truth of **things** they accepted the Heraclitean sayings which describe all sensible **things** as ever passing away, so that if knowledge or thought is to have an object, there must be some other and permanent entities, apart from those which are sensible; for there could be no knowledge of **things** which were in a state of flux. But when Socrates was occupying himself with the excellences of character, and in connexion with them

became the first to raise the problem of universal definition (for of the physicists Democritus only touched on the **subject** to a small extent, and defined, after a fashion, the hot and the cold; while the Pythagoreans had before this treated of a few **things**, whose definitions–e.g. those of opportunity, justice, or marriage–they connected with numbers; but it was natural that Socrates should be seeking the **essence**, for he was seeking to syllogize, and 'what a **thing** is' is the starting-point of syllogisms; for there was as yet none of the dialectical power which enables people even without knowledge of the **essence** to speculate about contraries and inquire whether the same science deals with contraries; for two **things** may be fairly ascribed to Socrates–inductive arguments and universal definition, both of which are concerned with the starting-point of science):–but Socrates did not make the universals or the definitions exist apart: they, however, gave them separate existence, and this was the kind of **thing** they called **Ideas**. Therefore it followed for them, almost by the same argument, that there must be **Ideas** of all **things** that are spoken of universally, and it was almost as if a man wished to count certain **things**, and while they were few thought he would not be able to count them, but made more of them and then counted them; for the **forms** are, one may say, more numerous than the particular sensible **things**, yet it was in seeking the causes of these that they proceeded from them to the **forms**. For to each **thing** there answers an entity which has the same name and exists apart from the **substances**, and so also in the case of all other groups there is a one over many, whether these be of this world or eternal.

***Metaphysics***: Again, of the ways in which it is proved that the **forms** exist, none is convincing; for from some no inference necessarily follows, and from some arise **forms** even of **things** of which they think there are no **forms**. For according to the arguments from the sciences there will be **forms** of all **things** of which there are sciences, and according to the argument of the 'one over many' there will be **forms** even of negations, and according to the argument that thought has an object when the individual object has perished, there will be **forms** of perishable **things**; for we have an image of these. Again, of the most accurate arguments, some lead to **Ideas** of relations, of which they say there is no independent class, and others introduce the 'third man'.

***Metaphysics***: And in general the arguments for the **forms** destroy **things** for whose existence the believers in **forms** are more zealous than for the existence of the **Ideas**; for it follows that not the dyad but number is first, and that prior to number is the relative, and that this is prior to the absolute–besides all the other points on which certain people, by following out the opinions held about the **forms**, came into conflict with the principles of the theory.

***Metaphysics***: Again, according to the assumption on the belief in the **Ideas** rests, there will be **forms** not only of **substances** but also of many other **things**; for the concept is single not only in the case of **substances**, but also in that of non-**substances**, and there are sciences of other **things** than **substance**; and a thousand other such difficulties confront them. But according to the necessities of the case and the opinions about the **forms**, if they can be shared in there must be **Ideas** of **substances** only. For they are not shared in incidentally, but each **form** must be shared in as something not **predicated** of a **subject**. (By 'being shared in incidentally' I mean that if a **thing** shares in 'double itself', it shares also in 'eternal', but incidentally; for 'the double' happens to be eternal.) Therefore the **forms** will be **substance**. But the same names indicate **substance** in this and in the ideal world (or what will be the meaning of saying that there is something apart from the particulars–the one over many?). And if the **Ideas** and the **things** that share in them have the same **form**, there will be something common: For why should '2' be one

and the same in the perishable 2's, or in the 2's which are many but eternal, and not the same in the '2 itself' as in the individual 2? But if they have not the same **form**, they will have only the name in common, and it is as if one were to call both Callias and a piece of wood a 'man', without observing any community between them.

***Metaphysics***: But if we are to suppose that in other respects the common definitions apply to the **forms**, e.g. that 'plane figure' and the other parts of the definition apply to the circle itself, but 'what really is' has to be added, we must inquire whether this is not absolutely meaningless. For to what is this to be added? To 'center' or to 'plane' or to all the parts of the definition? For all the elements in the **essence** are **Ideas**, e.g. 'animal' and 'two-footed'. Further, there must be some Ideal answering to 'plane' above, some **nature** which will be present in all the **forms** as their **genus**.

***Metaphysics***: But, further, all other **things** cannot come from the **forms** in any of the usual senses of 'from'. And to say that they are **patterns** and the other **things** share in them is to use empty words and poetical metaphors. For what is it that works, looking to the **Ideas**? And any **thing** can both be and come into being without being copied from something else, so that, whether Socrates exists or not, a man like Socrates might *come to be*. And evidently this might be so even if Socrates were eternal. And there will be several **patterns** of the same **thing**, and therefore several **forms**; e.g. 'animal' and 'two-footed', and also 'man-himself', will be **forms** of man. Again, the **forms** are **patterns** not only of sensible **things**, but of **forms** themselves also; i.e. the **genus** is the **pattern** of the various **forms**-of-a-**genus**; therefore the same **thing** will be **pattern** and copy.

***Metaphysics***: Again, if number can exist separately, one might ask which is prior–1, or 3 or 2? Inasmuch as the number is composite, 1 is prior, but inasmuch as the universal and the **form** is prior, the number is prior; for each of the units is part of the number as its **matter**, and the number acts as **form**. And in a sense the right angle is prior to the acute, because it is determinate and in virtue of its definition; but in a sense the acute is prior, because it is a part and the right angle is divided into acute angles. As **matter**, then, the acute angle and the element and the unit are prior, but in respect of the **form** and of the **substance** as expressed in the definition, the right angle, and the whole consisting of the **matter** and the **form**, are prior; for the concrete **thing** is nearer to the **form** and to what is expressed in the definition, though in generation it is later. How then is 1 the starting-point? Because it is not divisible, they say; but both the universal, and the particular or the element, are indivisible. But they are starting-points in different ways, one in definition and the other in time. In which way, then, is 1 the starting-point? As has been said, the right angle is thought to be prior to the acute, and the acute to the right, and each is one. Accordingly they make 1 the starting-point in both ways. But this is impossible for the universal is one as **form** or **substance**, while the element is one as a part or as **matter**. For each of the two is in a sense one–in truth each of the two units exists potentially (at least if the number is a unity and not like a heap, i.e. if different numbers consist of differentiated units, as they say), but not in complete reality; and the cause of the error they fell into is that they were conducting their inquiry at the same time from the standpoint of mathematics and from that of universal definitions, so that (1) from the former standpoint they treated unity, their first principle, as a point; for the unit is a point without position. They put **things** together out of the smallest parts, as some others also have done. Therefore the unit becomes the **matter** of numbers and at the same time prior to 2; and again posterior, 2 being treated as a whole, a unity, and a **form**. But (2) because they were seeking the universal they treated the unity which can be

**predicated** of a number, as in this sense also a part of the number. But these characteristics cannot belong at the same time to the same **thing**.

***Metaphysics***: Number, then, whether it be number in general or the number which consists of abstract units, is neither the cause as agent, nor the **matter**, nor the **ratio** and **form** of **things**. Nor, of course, is it the final cause.

***Posterior Analytics***: In the case of **predicates** constituting the essential **nature** of a **thing**, it clearly terminates, seeing that if definition is possible, or in other words, if essential **form** is knowable, and an infinite series cannot be traversed, **predicates** constituting a **thing**'s essential **nature** must be finite in number. But as regards **predicates** generally we have the following prefatory remarks to make. (1) We can affirm without falsehood 'the white (**thing**) is walking', and that big (**thing**) is a log'; or again, 'the log is big', and 'the man walks'. But the affirmation differs in the two cases. When I affirm 'the white is a log', I mean that something which happens to be white is a log–not that white is the **substratum** in which log inheres, for it was not *quâ* white or *quâ* a species of white that the white (**thing**) came to be a log, and the white (**thing**) is consequently not a log except incidentally. On the other hand, when I affirm 'the log is white', I do not mean that something else, which happens also to be a log, is white (as I should if I said 'the musician is white,' which would mean 'the man who happens also to be a musician is white'); on the contrary, log is here the **substratum**–the **substratum** which actually came to be white, and did so *quâ* wood or *quâ* a species of wood and *quâ* **nothing** else.

The order of a sentence does not change which word stands for ***matter***, ***form*** or ***thing***.

***Posterior Analytics***: (2) **Predicates** which signify **substance** signify that the **subject** is identical with the **predicate** or with a species of the **predicate**. **Predicates** not signifying **substance** which are **predicated** of a **subject** not identical with themselves or with a species of themselves are accidental or coincidental; e.g. white is a coincident of man, seeing that man is not identical with white or a species of white, but rather with animal, since man is identical with a species of animal. These **predicates** which do not signify **substance** must be **predicates** of some other **subject**, and **nothing** can be white which is not also other than white. The **forms** we can dispense with, for they are mere sound without sense; and even if there are such **things**, they are not relevant to our discussion, since demonstrations are concerned with **predicates** such as we have defined.

Here is an Aristotle hostile to ***The Theory of Forms***–is this a passage from an early work? Contrast to the following.

***On Generation And Corruption***: It is therefore better to suppose that in all instances of coming-to-be the **matter** is inseparable, being numerically identical and one with the 'containing' body, though isolable from it by definition. But the same reasons also forbid us to regard the **matter**, out of which the body comes-to-be, as points or lines. The **matter** is that of which ***points*** and ***lines*** are ***limits***, and it is something that can never exist without **quality** and without **form**.

By definition there is no such ***thing*** as infinitely large or small.

***On Generation And Corruption***: Two preliminary distinctions will prepare us to grasp the cause of growth. We must note (i) that the organic parts grow by the growth of the tissues (for every organ is composed of these as its constituents); and (ii) that flesh, bone, and every such part–like every other **thing** which has its **form** immersed in **matter**–has a twofold **nature**: For the **form** as well as the **matter** is called 'flesh' or 'bone'.

***On Generation And Corruption***: Now, that any and every part of the tissue *quâ* **form** should grow–and grow by the accession of something–is possible, but not that any and every part of the tissue *quâ* **matter** should do so. For we must think of the tissue after the image of flowing water that is measured by one and the same measure: particle after particle comes-to-be, and each successive particle is different. And it is in this sense that the **matter** of the flesh grows, some flowing out and some flowing in fresh; not in the sense that fresh **matter** accedes to every particle of it. There is, however, an accession to every part of its figure or 'form'.

***On Generation And Corruption***: That growth has taken place proportionally, is more manifest in the organic parts–e.g. in the hand. For there the fact that the **matter** is distinct from the **form** is more manifest than in flesh, i.e. than in the tissues. That is why there is a greater tendency to suppose that a corpse still possesses flesh and bone than that it still has a hand or an arm.

***On Generation And Corruption***: Hence in one sense it is true that any and every part of the flesh has grown; but in another sense it is false. For there has been an accession to every part of the flesh in respect to its **form**, but not in respect to its **matter**. The whole, however, has become larger. And this increase is due (a) on the one hand to the accession of something, which is called 'food' and is said to be 'contrary' to flesh, but (b) on the other hand to the transformation of this food into the same **form** as that of flesh as if, e.g. 'moist' were to accede to 'dry' and, having acceded, were to be transformed and to become 'dry'. For in one sense 'Like grows by Like', but in another sense 'Unlike grows by Unlike'.

***On Generation And Corruption***: The **form** of which we have spoken is a kind of power immersed in **matter**–a duct, as it were. If, then, a **matter** accedes-a **matter**, which is potentially a duct and also potentially possesses determinate **quantity** the ducts to which it accedes will become bigger. But if it is no longer able to act–if it has been weakened by the continued influx of **matter**, just as water, continually mixed in greater and greater **quantity** with wine, in the end makes the wine watery and converts it into water–then it will cause a diminution of the quantum; though still the **form** persists.

***On Generation And Corruption***: Those active powers, then, whose **forms** are not embodied in **matter**, are unaffected: but those whose **forms** are in **matter** are such as to be affected in acting. For we maintain that one and the same '**matter'** is equally, so to say, the basis of either of the two opposed **things**–being as it were a 'kind'; and that that which can be hot must be made hot, provided the heating agent is there, i.e. comes near. Hence (as we have said) some of the active powers are unaffected while others are such as to be affected; and what holds of motion is true also of the active powers. For as in motion 'the first mover' is unmoved, so among the active powers 'the first agent' is unaffected.

***On Generation And Corruption***: But the third 'originative source' must be present as well–the cause vaguely dreamed of by all our predecessors, definitely stated by none of them. On the contrary (a) some amongst them thought the **nature** of 'the forms' was adequate to account for coming-to-be. Thus Socrates in the ***Phaedo*** first blames everybody else for having given no explanation; and then lays it down; that 'some **things** are **forms**, others participants in the forms', and that 'while a **thing** is said to "be" in virtue of the **form**, it is said to "come-to-be" *quâ* sharing in," to "pass-away" *quâ* "losing," the 'form'. Hence he thinks that 'assuming the truth of these theses, the **forms** must be causes both of coming-to-be and of passing-away'. On the other hand (b) there were others who thought 'the matter' was adequate by itself to account

for coming-to-be, since ‘the movement originates from the matter’.

***On Generation And Corruption***: Neither of these theories, however, is sound. For (a) if the **forms** are causes, why is their generating activity intermittent instead of perpetual and continuous–since there always are participants as well as **forms**? Besides, in some instances we see that the cause is other than the **form**. For it is the doctor who implants health and the man of science who implants science, although ‘Health itself’ and ‘Science itself’ are as well as the participants: and the same principle applies to everything else that is produced in accordance with an art. On the other hand (b) to say that ‘**matter** generates owing to its movement’ would be, no doubt, more scientific than to make such statements as are made by the thinkers we have been criticizing. For what ‘alters’ and transfigures plays a greater part in bringing **things** into being; and we are everywhere accustomed, in the products of **nature** and of art alike, to look upon that which can initiate movement as the producing cause. Nevertheless this second theory is not right either.

> Since neither ***form*** *quâ* ***form*** or ***matter*** *quâ* ***matter*** are ***things***–it stands to reason they cannot be causes. Again ***things*** are causes–neither ***forms*** nor ***matter*** exist independent of ***a thing***.

***On The Heavens***: We must show not only that the heaven is one, but also that more than one heaven is and, further, that, as exempt from decay and generation, the heaven is eternal. We may begin by raising a difficulty. From one point of view it might seem impossible that the heaven should be one and unique, since in all formations and products whether of **nature** or of art we can distinguish the **shape** in itself and the **shape** in combination with **matter**. For instance the **form** of the sphere is one **thing** and the gold or bronze sphere another; the **shape** of the circle again is one **thing**, the bronze or wooden circle another. For when we state the essential **nature** of the sphere or circle we do not include in the formula gold or bronze, because they do not belong to the **essence**, but if we are speaking of the copper or gold sphere we do include them. We still make the distinction even if we cannot conceive or apprehend any other example beside the particular **thing**. This may, of course, sometimes be the case: it might be, for instance, that only one circle could be found; yet none the less the difference will remain between the being of circle and of this particular circle, the one being **form**, the other **form** in **matter**, i.e. a particular **thing**. Now since the universe is perceptible it must be regarded as a particular; for everything that is perceptible subsists, as we know, in **matter**. But if it is a particular, there will be a distinction between the being of ‘this universe’ and of ‘universe’ unqualified. There is a difference, then, between ‘this universe’ and simple ‘universe’; the second is **form** and **shape**, the first **form** in combination with **matter**; and any **shape** or **form** has, or may have, more than one particular instance.

***On The Heavens***: On the supposition of **forms** such as some assert, this must be the case, and equally on the view that no such entity has a separate existence. For in every case in which the **essence** is in **matter** it is a fact of observation that the particulars of like **form** are several or infinite in number. Hence there either are, or may be, more heavens than one. On these grounds, then, it might be inferred either that there are or that there might be several heavens. We must, however, return and ask how much of this argument is correct and how much not.

***On The Heavens***: These, then, are the views which have been advanced by others and the terms in which they state them. We may begin our own statement by settling a question which to some has been the main difficulty–the question why some bodies move always and naturally upward and others downward, while others again move both upward and downward. After that we will inquire into light and heavy and of the various phenomena connected with them. The

local movement of each body into its own place must be regarded as similar to what happens in connexion with other forms of generation and change. There are, in fact, three kinds of movement, affecting respectively the size, the **form**, and the place of a **thing**, and in each it is observable that change proceeds from a contrary to a contrary or to something intermediate: it is never the change of any chance **subject** in any chance direction, nor, similarly, is the relation of the mover to its object fortuitous: the **thing** altered is different from the **thing** increased, and precisely the same difference holds between that which produces alteration and that which produces increase. In the same manner it must be thought that produces local motion and that which is so moved are not fortuitously related. Now, that which produces upward and downward movement is that which produces weight and lightness, and that which is moved is that which is potentially heavy or light, and the movement of each body to its own place is motion towards its own **form**. (It is best to interpret in this sense the common statement of the older writers that 'like moves to like'. For the words are not in every sense true to fact. If one were to remove the earth to where the moon now is, the various fragments of earth would each move not towards it but to the place in which it now is. In general, when a number of similar and undifferentiated bodies are moved with the same motion this result is necessarily produced, viz. that the place which is the natural goal of the movement of each single part is also that of the whole. But since the place of a **thing** is the boundary of that which contains it, and the continent of all **things** that move upward or downward is the extremity and the center, and this boundary *comes to be*, in a sense, the **form** of that which is contained, it is to its like that a body moves when it moves to its own place. For the successive members of the series are like one another: water, I mean, is like air and air like fire, and between intermediates the relation may be converted, though not between them and the extremes; thus air is like water, but water is like earth: For the relation of each outer body to that which is next within it is that of **form** to **matter**.) Thus to ask why fire moves upward and earth downward is the same as to ask why the healable, when moved and changed *quâ* healable, attains health and not whiteness; and similar questions might be asked concerning any other **subject** of alteration. Of course the **subject** of increase, when changed *quâ* increasable, attains not health but a superior size. The same applies in the other cases. One **thing** changes in **quality**, another in **quantity**: and so in place, a light **thing** goes upward, a heavy **thing** downward. The only difference is that in the last case, viz. that of the heavy and the light, the bodies are thought to have a spring of change within themselves, while the **subjects** of healing and increase are thought to be moved purely from without. Sometimes, however, even they change of themselves, i.e., in response to a slight external movement reach health or increase, as the case may be. And since the same **thing** which is healable is also receptive of disease, it depends on whether it is moved *quâ* healable or *quâ* liable to disease whether the motion is towards health or towards disease. But the reason why the heavy and the light appear more than these **things** to contain within themselves the source of their movements is that their **matter** is nearest to being. This is indicated by the fact that locomotion belongs to bodies only when isolated from other bodies, and is generated last of the several kinds of movement; in **order** of being then it will be first. Now whenever air comes into being out of water, light out of heavy, it goes to the upper place. It is forthwith light: becoming is at an end, and in that place it has being. Obviously, then, it is a **potentiality**, which, in its passage to **actuality**, comes into that place and **quantity** and **quality** which belong to its **actuality**. And the same fact explains why what is already actually fire or earth moves, when **nothing** obstructs it, towards its own place. For motion is equally immediate in the case of nutriment, when **nothing** hinders, and in the case of the **thing** healed, when **nothing** stays the healing. But the movement is also due to the original

creative force and to that which removes the hindrance or off which the moving **thing** rebounded, as was explained in our opening discussions, where we tried to show how none of these **things** moves itself. The reason of the various motions of the various bodies, and the meaning of the motion of a body to its own place, have now been explained.

***On The Heavens***: The following account will make it plain that there is an absolutely light and an absolutely heavy body. And by absolutely light I mean one which of its own **nature** always moves upward, by absolutely heavy one which of its own **nature** always moves downward, if no obstacle is in the way. There are, I say, these two kinds of body, and it is not the case, as some maintain, that all bodies have weight. Different views are in fact agreed that there is a heavy body, which moves uniformly towards the center. But is also similarly a light body. For we see with our eyes, as we said before, that earthy **things** sink to the bottom of all **things** and move towards the center. But the center is a fixed point. If therefore there is some body which rises to the surface of all **things**-and we observe fire to move upward even in air itself, while the air remains at rest-clearly this body is moving towards the extremity. It cannot then have any weight. If it had, there would be another body in which it sank: and if that had weight, there would be yet another which moved to the extremity and thus rose to the surface of all moving **things**. In fact, however, we have no evidence of such a body. Fire, then, has no weight. Neither has earth any lightness, since it sinks to the bottom of all **things**, and that which sinks moves to the center. That there is a center towards which the motion of heavy **things**, and away from which that of light **things** is directed, is manifest in many ways. First, because no movement can continue to infinity. For what cannot be can no more come-to-be than be, and movement is a coming to-be in one place from another. Secondly, like the upward movement of fire, the downward movement of earth and all heavy **things** makes equal angles on every side with the earth's surface: it must therefore be directed towards the center. Whether it is really the center of the earth and not rather that of the whole to which it moves, may be left to another inquiry, since these are coincident. But since that which sinks to the bottom of all **things** moves to the center, necessarily that which rises to the surface moves to the extremity of the region in which the movement of these bodies takes place. For the center is opposed as contrary to the extremity, as that which sinks is opposed to that which rises to the surface. This also gives a reasonable ground for the duality of heavy and light in the spatial duality center and extremity. Now there is also the intermediate region to which each name is given in opposition to the other extreme. For that which is intermediate between the two is in a sense both extremity and center. For this reason there is another heavy and light; namely, water and air. But in our view the continent pertains to **form** and the contained to **matter**: and this distinction is present in every **genus**. Alike in the sphere of **quality** and in that of **quantity** there is that which corresponds rather to **form** and that which corresponds to **matter**. In the same way, among spatial distinctions, the above belongs to the determinate, the below to **matter**. The same holds, consequently, also of the **matter** itself of that which is heavy and light: as potentially possessing the one character, it is **matter** for the heavy, and as potentially possessing the other, for the light. It is the same **matter**, but its being is different, as that which is receptive of disease is the same as that which is receptive of health, though in being different from it, and therefore diseased-ness is different from healthiness.

***Physics***: The principles in question must be either (a) one or (b) more than one. If (a) one, it must be either (i) motionless, as Parmenides and Melissus assert, or (ii) in motion, as the physicists hold, some declaring air to be the first principle, others water. If (b) more than one, then either (i) a finite or (ii) an infinite plurality. If (i) finite (but more than one), then either two

or three or four or some other number. If (ii) infinite, then either as Democritus believed one in kind, but differing in **shape** or **form**; or different in kind and even contrary.

***Physics***: If, then, we approach the thesis in this way it seems impossible for all **things** to be one. Further, the arguments they use to prove their position are not difficult to expose. For both of them reason contentiously-I mean both Melissus and Parmenides. [Their premises are false and their conclusions do not follow. Or rather the argument of Melissus is gross and palpable and offers no difficulty at all: admit one ridiculous proposition and the rest follows–a simple enough proceeding.] The fallacy of Melissus is obvious. For he supposes that the assumption 'what has come into being always has a beginning' justifies the assumption 'what has not come into being has no beginning'. Then this also is absurd, that in every case there should be a beginning of the **thing**–not of the time and not only in the case of *coming to be* in the full sense but also in the case of *coming to have a* ***quality***–as if change never took place suddenly. Again, does it follow that *being*, if one, is motionless? Why should it not move, the whole of it within itself, as parts of it do which are unities, e.g. this water? Again, why is qualitative change impossible? But, further, *being* cannot be one in **form**, though it may be in what it is made of. (Even some of the physicists hold it to be one in the latter way, though not in the former.) Man obviously differs from horse in **form**, and contraries from each other.

***Physics***: The first set make the underlying body one either one of the three or something else which is denser than fire and rarer than air then generate everything else from this, and obtain multiplicity by condensation and rarefaction. Now these are contraries, which may be generalized into 'excess and defect'. (Compare Plato's 'Great and Small'–except that he make these his **matter**, the one his **form**, while the others treat the one which underlies as **matter** and the contraries as **differentiae**, i.e. **Forms**).

> Some of a ***thing*** (***matter***) none of a ***thing*** (***form***) are the Great (***matter***) and the Small (***form***).

***Physics***: Thus, clearly, from what has been said, whatever *comes to be* is always complex. There is, on the one hand, (a) something which comes into existence, and again (b) something which becomes that–the latter (b) in two senses, either the **subject** or the opposite. By the 'opposite' I mean the 'unmusical', by the 'subject' 'man', and similarly I call the absence of **shape** or **form** or **order** the 'opposite', and the bronze or stone or gold the 'subject'.

> Now here is something that I don't think Plato would have made a mistake about but which Aristotle seems to, the 'opposite' refers not to an object but to predication–an assertion in lieu of a denial or a denial in lieu of an assertion, the opposite is not in ***the thing***, but in the predication–***the thing*** in of itself has no opposite.

***Physics***: Plainly then, if there are conditions and principles which constitute natural objects and from which they primarily *are* or have *come to be*–have *come to be*, I mean, what each is said *to be* in its essential **nature**, not what each is in respect of a concomitant attribute–plainly, I say, everything *comes to be* from both **subject** and **form**. For 'musical man' is composed (in a way) of 'man' and 'musical': you can analyze it into the definitions of its elements. It is clear then that what *comes to be* will *come to be* from these elements.

> ***Everything*** comes ***to be*** from both ***matter*** and ***form*** . . .

***Physics***: Now the **subject** is one numerically, though it is two in **form**. (For it is the man, the gold–the '**matter**' generally–that is counted, for it is more of the **nature** of a 'this', and what *comes to be* does not come from it in virtue of a concomitant attribute; the privation, on the other hand, and the contrary are incidental in the process.) And the positive **form** is one–the **order**, the

acquired art of music, or any similar **predicate**.

***Physics***: The underlying **nature** is an object of scientific knowledge, by an analogy. For as the bronze is to the statue, the wood to the bed, or the **matter** and the formless before receiving **form** to any **thing** which has **form**, so is the underlying **nature** to **substance**, i.e. the 'this' or existent.

***Physics***: This then is one principle (though not one or existent in the same sense as the 'this'), and the definition was one as we agreed; then further there is its contrary, the privation. In what sense these are two, and in what sense more, has been stated above. Briefly, we explained first that only the contraries were principles, and later that a **substratum** was indispensable, and that the principles were three; our last statement has elucidated the difference between the contraries, the mutual relation of the principles, and the **nature** of the **substratum**. Whether the **form** or the **substratum** is the essential **nature** of a physical object is not yet clear. But that the principles are three, and in what sense, and the way in which each is a principle, is clear.

***Physics***: Now we distinguish **matter** and privation, and hold that one of these, namely the **matter**, is *not-being* only in virtue of an attribute which it has, while the privation in its own **nature** is *not-being*; and that the **matter** is nearly, in a sense is, **substance**, while the privation in no sense is. They, on the other hand, identify their Great and Small alike with *not-being*, and that whether they are taken together as one or separately. Their triad is therefore of quite a different kind from ours. For they got so far as to see that there must be some underlying **nature**, but they make it one–for even if one philosopher makes a dyad of it, which he calls Great and Small, the effect is the same, for he overlooked the other **nature**. For the one which persists is a joint cause, with the **form**, of what *comes to be*–a mother, as it were. But the negative part of the contrariety may often seem, if you concentrate your attention on it as an evil agent, not to exist at all.

***Physics***: For admitting with them that there is something divine, good, and desirable, we hold that there are two other principles, the one contrary to it, the other such as of its own **nature** to desire and yearn for it. But the consequence of their view is that the contrary desires its extinction. Yet the **form** cannot desire itself, for it is not defective; nor can the contrary desire it, for contraries are mutually destructive. The truth is that what desires the **form** is **matter**, as the female desires the male and the ugly the beautiful–only the ugly or the female not *per se* but *per accidens*.

***Physics***: The accurate determination of the first principle in respect of **form**, whether it is one or many and what it is or what they are, is the province of the primary type of science; so these questions may stand over till then. But of the natural, i.e. perishable, **forms** we shall speak in the expositions which follow.

***Physics***: Another account is that '**nature'** is the **shape** or **form** which is specified in the definition of the **thing**.

***Physics***: For the word '**nature'** is applied to what is according to **nature** and the natural in the same way as 'art' is applied to what is artistic or a work of art. We should not say in the latter case that there is **anything** artistic about a **thing**, if it is a bed only potentially, not yet having the **form** of a bed; nor should we call it a work of art. The same is true of natural compounds. What is potentially flesh or bone has not yet its own '**nature'**, and does not exist until it receives the **form** specified in the definition, which we name in defining what flesh or bone is. Thus in the second sense of '**nature'** it would be the **shape** or **form** (not separable except in statement) of

**things** which have in themselves a source of motion. (The combination of the two, e.g. man, is not '**nature'** but 'by nature' or 'natural'.)

***Physics***: The **form** indeed is '**nature'** rather than the **matter**; for a **thing** is more properly said to be what it is when it has attained to fulfillment than when it exists potentially. Again man is born from man, but not bed from bed. That is why people say that the figure is not the **nature** of a bed, but the wood is–if the bed sprouted not a bed but wood would come up. But even if the figure is art, then on the same principle the **shape** of man is his **nature**. For man is born from man.

***Physics***: '***Shape'*** and '**nature'**, it should be added, are in two senses. For the privation too is in a way **form**. But whether in unqualified *coming to be* there is privation, i.e. a contrary to what *comes to be*, we must consider later.

> One might go a step further and declare that neither existence nor non-existence can be **predicated** of either ***form*** *quâ* ***form*** or ***matter*** *quâ* ***matter***–as existence is either asserted of or denied of ***things***. This would illuminate that the paradox of denying the existence of ***form*** or ***matter*** as being only apparent–it is simply not predicable.

***Physics***: Since '**nature'** has two senses, the **form** and the **matter**, we must investigate its objects as we would the **essence** of snubness. That is, such **things** are neither independent of **matter** nor can be defined in terms of **matter** only. Here too indeed one might raise a difficulty. Since there are two **natures**, with which is the physicist concerned? Or should he investigate the combination of the two? But if the combination of the two, then also each severally. Does it belong then to the same or to different sciences to know each severally?

***Physics***: If we look at the ancients, physics would to be concerned with the **matter**. (It was only very slightly that Empedocles and Democritus touched on the **forms** and the **essence**.)

***Physics***: But if on the other hand art imitates **nature**, and it is the part of the same discipline to know the **form** and the **matter** up to a point (e.g. the doctor has a knowledge of health and also of bile and phlegm, in which health is realized, and the builder both of the **form** of the house and of the **matter**, namely that it is bricks and beams, and so forth): if this is so, it would be the part of physics also to know **nature** in both its senses.

***Physics***: For the arts make their **material** (some simply 'make' it, others make it serviceable), and we use everything as if it was there for our sake. (We also are in a sense an end. 'That for the sake of which' has two senses: the distinction is made in our work ***On Philosophy***.) The arts, therefore, which govern the **matter** and have knowledge are two, namely the art which uses the product and the art which directs the production of it. That is why the using art also is in a sense directive; but it differs in that it knows the **form**, whereas the art which is directive as being concerned with production knows the **matter**. For the helmsman knows and prescribes what sort of **form** a helm should have, the other from what wood it should be made and by means of what operations. In the products of art, however, we make the **material** with a view to the function, whereas in the products of **nature** the **matter** is there all along.

***Physics***: Again, **matter** is a relative term: to each **form** there corresponds a special **matter**. How far then must the physicist know the **form** or **essence**? Up to a point, perhaps, as the doctor must know sinew or the smith bronze (i.e. until he understands the purpose of each): and the physicist is concerned only with **things** whose **forms** are separable indeed, but do not exist apart from **matter**. Man is begotten by man and by the sun as well. The mode of existence and **essence** of the separable it is the business of the primary type of philosophy to define.

***Physics***: In another sense (2) the **form** or the **archetype**, i.e. the statement of the **essence**, and its **genuses**, are called ‘causes’ (e.g. of the octave the relation of 2:, and generally number), and the parts in the definition.

***Physics***: All the causes now mentioned fall into four familiar divisions. The letters are the causes of syllables, the **material** of artificial products, fire, &c., of bodies, the parts of the whole, and the premises of the conclusion, in the sense of ‘that from which’. Of these pairs the one set are causes in the sense of **substratum**, e.g. the parts, the other set in the sense of **essence**–the whole and the combination and the **form**. But the seed and the doctor and the adviser, and generally the maker, are all sources whence the change or stationariness originates, while the others are causes in the sense of the end or the good of the rest; for ‘that for the sake of which’ means what is best and the end of the **things** that lead up to it. (Whether we say the ‘good itself or the ‘apparent good’ makes no difference.)

***Physics***: Now, the causes being four, it is the business of the physicist to know about them all, and if he refers his problems back to all of them, he will assign the ‘why’ in the way proper to his science–the **matter**, the **form**, the mover, ‘that for the sake of which’. The last three often coincide; for the ‘what’ and ‘that for the sake of which’ are one, while the primary source of motion is the same in species as these (for man generates man), and so too, in general, are all **things** which cause movement by being themselves moved; and such as are not of this kind are no longer inside the province of physics, for they cause motion not by possessing motion or a source of motion in themselves, but being themselves incapable of motion. Hence there are three branches of study, one of **things** which are incapable of motion, the second of **things** in motion, but indestructible, the third of destructible **things**.

***Physics***: The question ‘why’, then, is answered by reference to the **matter**, to the **form**, and to the primary moving cause. For in respect of *coming to be* it is mostly in this last way that causes are investigated–’what *comes to be* after what? what was the primary agent or patient?’ and so at each step of the series.

***Physics***: Now the principles which cause motion in a physical way are two, of which one is not physical, as it has no principle of motion in itself. Of this kind is whatever causes movement, not being itself moved, such as (1) that which is completely unchangeable, the primary reality, and (2) the **essence** of that which is *coming to be*, i.e. the **form**; for this is the end or ‘that for the sake of which’. Hence since **nature** is for the sake of something, we must know this cause also. We must explain the ‘why’ in all the senses of the term, namely, (1) that from this that will necessarily result (‘from this’ either without qualification or in most cases); (2) that ‘this must be so if that is to be so’ (as the conclusion presupposes the premises); (3) that this was the **essence** of the **thing**; and (4) because it is better thus (not without qualification, but with reference to the essential **nature** in each case).

***Physics***: Further, where a series has a completion, all the preceding steps are for the sake of that. Now surely as in intelligent action, so in **nature**; and as in **nature**, so it is in each action, if nothing interferes. Now intelligent action is for the sake of an end; therefore the **nature** of **things** also is so. Thus if a house, e.g. had been a **thing** made by **nature**, it would have been made in the same way as it is now by art; and if **things** made by **nature** were made also by art, they would *come to be* in the same way as by **nature**. Each step then in the series is for the sake of the next; and generally art partly completes what **nature** cannot bring to a finish, and partly imitates her. If, therefore, artificial products are for the sake of an end, so clearly also are natural products.

The relation of the later to the earlier terms of the series is the same in both. This is most obvious in the animals other than man: they make **things** neither by art nor after inquiry or deliberation. Wherefore people discuss whether it is by intelligence or by some other faculty that these creatures work, spiders, ants, and the like. By gradual advance in this direction we come to see clearly that in plants too that is produced which is conducive to the end–leaves, e.g. grow to provide shade for the fruit. If then it is both by **nature** and for an end that the swallow makes its nest and the spider its web, and plants grow leaves for the sake of the fruit and send their roots down (not up) for the sake of nourishment, it is plain that this kind of cause is operative in **things** which *come to be* and are by **nature**. And since '**nature'** means two **things**, the **matter** and the **form**, of which the latter is the end, and since all the rest is for the sake of the end, the **form** must be the cause in the sense of 'that for the sake of which'.

***Physics***: Now each of these belongs to all its **subjects** in either of two ways: namely (1) **substance**–the one is positive **form**, the other privation; (2) in **quality**, white and black; (3) in **quantity**, complete and incomplete; (4) in respect of locomotion, upwards and downwards or light and heavy. Hence there are as many types of motion or change as there are meanings of the word 'is'.

***Physics***: The mover too is moved, as has been said–every mover, that is, which is capable of motion, and whose immobility is rest–when a **thing** is subject to motion its immobility is rest. For to act on the movable as such is just to move it. But this it does by contact, so that at the same time it is also acted on. Hence we can define motion as the fulfillment of the movable *quâ* movable, the cause of the attribute being contact with what can move so that the mover is also acted on. The mover or agent will always be the vehicle of a **form**, either a 'this' or 'such', which, when it acts, will be the source and cause of the change, e.g. the full-**formed** man begets man from what is potentially man.

***Physics***: (1) Some, as the Pythagoreans and Plato, make the infinite a principle in the sense of a self-subsistent **substance**, and not as a mere attribute of some other **thing**. Only the Pythagoreans place the infinite among the objects of sense (they do not regard number as separable from these), and assert that what is outside the heaven is infinite. Plato, on the other hand, holds that there is no body outside (the **forms** are not outside because they are nowhere), yet that the infinite is present not only in the objects of sense but in the **forms** also.

> Actually, since ***a thing*** is defined, neither ***form*** nor ***matter*** *quâ* ***form*** or ***matter*** is defined. ***Matter*** without ***form*** is infinitely large. ***Form*** without ***matter*** is infinitely small. Neither can exist, by definition.

***Physics***: Hence Parmenides must be thought to have spoken better than Melissus. The latter says that the whole is infinite, but the former describes it as limited, 'equally balanced from the middle'. For to connect the infinite with the all and the whole is not like joining two pieces of string; for it is from this they get the dignity they ascribe to the infinite–its containing all **things** and holding the all in itself–from its having a certain similarity to the whole. It is in fact the **matter** of the completeness which belongs to size, and what is potentially a whole, though not in the full sense. It is divisible both in the direction of reduction and of the inverse addition. It is a whole and limited; not, however, in virtue of its own **nature**, but in virtue of what is other than it. It does not contain, but, in so far as it is infinite, is contained. Consequently, also, it is unknowable, *quâ* infinite; for the **matter** has no **form**. (Hence it is plain that the infinite stands in the relation of part rather than of whole. For the **matter** is part of the whole, as the bronze is of the bronze statue.) If it contains in the case of sensible **things**, in the case of intelligible **things** the great and the small ought to contain them. But it is absurd and impossible to suppose that the

unknowable and indeterminate should contain and determine.

***Physics***: It is reasonable that there should not be held to be an infinite in respect of addition such as to surpass every magnitude, but that there should be thought to be such an infinite in the direction of division. For the **matter** and the infinite are contained inside what contains them, while it is the **form** which contains. It is natural too to suppose that in number there is a limit in the direction of the minimum, and that in the other direction every assigned number is surpassed. In magnitude, on the contrary, every assigned magnitude is surpassed in the direction of smallness, while in the other direction there is no infinite magnitude. The reason is that what is one is indivisible whatever it may be, e.g. a man is one man, not many. Number on the other hand is a plurality of 'ones' and a certain **quantity** of them. Hence number must stop at the indivisible: For 'two' and 'three' are merely derivative terms, and so with each of the other numbers. But in the direction of largeness it is always possible to think of a larger number: For the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, never actual: the number of parts that can be taken always surpasses any assigned number. But this number is not separable from the process of bisection, and its infinity is not a permanent **actuality** but consists in a process of *coming to be*, like time and the number of time.

***Physics***: (4) Also we may ask: of what in **things** is space the cause? None of the four modes of causation can be ascribed to it. It is neither in the sense of the **matter** of existents (for **nothing** is composed of it), nor as the **form** and definition of **things**, nor as end, nor does it move existents.

***Physics***: Now if place is what primarily contains each body, it would be a limit, so that the place would be the **form** or **shape** of each body by which the magnitude or the **matter** of the magnitude is defined: For this is the limit of each body.

***Physics***: If, then, we look at the question in this way the place of a **thing** is its **form**. But, if we regard the place as the extension of the magnitude, it is the **matter**. For this is different from the magnitude: it is what is contained and defined by the **form**, as by a bounding plane. **Matter** or the indeterminate is of this **nature**; when the boundary and attributes of a sphere are taken away, **nothing** but the **matter** is left.

> In **order** to regard place as an extension of magnitude one would have to think in terms of place to place, i.e., as a difference between places–but not, however, place *quâ* place.

***Physics***: In view of these facts we should naturally expect to find difficulty in determining what place is, if indeed it is one of these two **things**, **matter** or **form**. They demand a very close scrutiny, especially as it is not easy to recognize them apart.

> Why, when Aristotle knows that ***thing*** is defined in terms of ***matter*** and ***form***, does he continue to call ***matter*** and ***form*** *quâ* ***matter*** and ***form*** ***things***?

***Physics***: But it is at any rate not difficult to see that place cannot be either of them. The **form** and the **matter** are not separate from the **thing**, whereas the place can be separated. As we pointed out, where air was, water in turn *comes to be*, the one replacing the other; and similarly with other bodies. Hence the place of a **thing** is neither a part nor a state of it, but is separable from it. For place is supposed to be something like a vessel–the vessel being a transportable place. But the vessel is no part of the **thing**.

***Physics***: In so far then as it is separable from the **thing**, it is not the **form**: *quâ* containing, it is different from the **matter**.

***Physics***: Also it is held that what is anywhere is both itself something and that there is a different **thing** outside it. (Plato of course, if we may digress, ought to tell us why the **form** and the numbers are not in place, if 'what participates' is place–whether what participates is the Great and the Small or the **matter**, as he called it in writing in the ***Timaeus***.)

***Physics***: Further, how could a body be carried to its own place, if place was the **matter** or the **form**? It is impossible that what has no reference to motion or the distinction of up and down can be place. So place must be looked for among **things** which have these characteristics.

***Physics***: If the place is in the **thing** (it must be if it is either **shape** or **matter**) place will have a place: For both the **form** and the indeterminate undergo change and motion along with the **thing**, and are not always in the same place, but are where the **thing** is. Hence the place will have a place.

> An odd way of saying that *place* is neither here nor there.

***Physics***: (4) As the **genus** is 'in' the species and generally the part of the specific **form** 'in' the definition of the specific **form**.

***Physics***: (5) As health is 'in' the hot and the cold and generally the **form** 'in' the **matter**.

> This will elucidate Aristotle's notion of Set Theory–opposite of actual Venn. Instead of conceiving the **matter** in the **form**, he conceives the **form** in the **matter**!

***Physics***: Another **thing** is plain: since the vessel is no part of what is in it (what contains in the strict sense is different from what is contained), place could not be either the **matter** or the **form** of the **thing** contained, but must be different–for the latter, both the **matter** and the **shape**, are parts of what is contained.

> Apparently Aristotle never understood Plato's ***Parmenides*** perhaps he never even read it.

***Physics***: (1) The **shape** is supposed to be place because it surrounds, for the extremities of what contains and of what is contained are coincident. Both the **shape** and the place, it is true, are boundaries. But not of the same **thing**: the **form** is the boundary of the **thing**, the place is the boundary of the body which contains it.

***Physics***: Well, then, if place is none of the three–neither the **form** nor the **matter** nor an extension which is always there, different from, and over and above, the extension of the **thing** which is displaced–place necessarily is the one of the four which is left, namely, the boundary of the containing body at which it is in contact with the contained body. (By the contained body is meant what can be moved by way of locomotion.)

> Aristotle's inability to come to a complete abstraction gives him two ***forms*** and two ***matters***. Two differences, (**matter** and extension) and two **forms** (**form** and place).

***Physics***: Nor (7) is it without reason that each should remain naturally in its proper place. For this part has the same relation to its place, as a separable part to its whole, as when one moves a part of water or air: so, too, air is related to water, for the one is like **matter**, the other **form**–water is the **matter** of air, air as it were the **actuality** of water, for water is potentially air, while air is potentially water, though in another way.

***On The Soul***: In the same way Plato in the ***Timaeus*** fashions soul out of his elements; for like, he holds, is known by like, and **things** are **formed** out of the principles or elements, so that soul must be so too. Similarly also in his lectures '***On Philosophy***' it was set forth that the Animal-itself is compounded of the **Idea** itself of the One together with the primary length,

breadth, and depth, everything else, the objects of its perception, being similarly constituted. Again he puts his view in yet other terms: Mind is the monad, science or knowledge the dyad (because it goes undeviatingly from one point to another), opinion the number of the plane, sensation the number of the solid; the numbers are by him expressly identified with the **forms** themselves or principles, and are **formed** out of the elements; now **things** are apprehended either by mind or science or opinion or sensation, and these same numbers are the **forms** of **things**.

> Plato got part of logic, the perception by the apprehension of ***form***, but not the perception by the apprehension of ***matter***. Eating, breathing, crafts (like the figures in geometry) are perceptions by the apprehension of ***matter***.

***On The Soul***: We are in the habit of recognizing, as one determinate kind of what is, **substance**, and that in several senses, (a) in the sense of **matter** or that which in itself is not 'a this', and (b) in the sense of **form** or **essence**, which is that precisely in virtue of which a **thing** is called 'a this', and thirdly (c) in the sense of that which is compounded of both (a) and (b). Now **matter** is **potentiality**, **form actuality**; of the latter there are two grades related to one another as e.g. knowledge to the exercise of knowledge.

***On The Soul***: Since the expression 'that whereby we live and perceive' has two meanings, just like the expression 'that whereby we know'–that may mean either (a) knowledge or (b) the soul, for we can speak of knowing by or with either, and similarly that whereby we are in health may be either (a) health or (b) the body or some part of the body; and since of the two terms thus contrasted knowledge or health is the name of a **form**, **essence**, or **ratio**, or if we so express it an **actuality** of a recipient **matter**–knowledge of what is capable of knowing, health of what is capable of being made healthy (for the operation of that which is capable of originating change terminates and has its seat in what is changed or altered); Further, since it is the soul by or with which primarily we live, perceive, and think:–it follows that the soul must be a **ratio** or formulable **essence**, not a **matter** or **subject**. For, as we said, the word **substance** has three meanings **form**, **matter**, and the complex of both and of these three what is called **matter** is **potentiality**, what is called **form actuality**. Since then the complex here is the living **thing**, the body cannot be the **actuality** of the soul; it is the soul which is the **actuality** of a certain kind of body. Hence the rightness of the view that the soul cannot be without a body, while it cannot be a body; it is not a body but something relative to a body. That is why it is in a body, and a body of a definite kind. It was a mistake, therefore, to do as former thinkers did, merely to fit it into a body without adding a definite specification of the kind or character of that body. Reflection confirms the observed fact; the **actuality** of any given **thing** can only be realized in what is already potentially that **thing**, i.e. in a **matter** of its own appropriate to it. From all this it follows that soul is an **actuality** or formulable **essence** of something that possesses a **potentiality** of being besouled.

***On The Soul***: (A) By a 'sense' is meant what has the power of receiving into itself the sensible **forms** of **things** without the **matter**. This must be conceived of as taking place in the way in which a piece of wax takes on the impress of a signet-ring without the iron or gold; we say that what produces the impression is a signet of bronze or gold, but its particular metallic constitution makes no difference: in a similar way the sense is affected by what is colored or flavored or sounding, but it is indifferent what in each case the **substance** is; what alone **matters** is what **quality** it has, i.e. in what **ratio** its constituents are combined.

> I on the other hand define sense quite differently; That human body system which acquires something from the environment, processes it, and the product is used to sustain and promote

human life. This gives us seven senses. By this definition, half of logic goes unrecognized as logic for thousands of years.

***On The Soul***: The sense and its organ are the same in fact, but their **essence** is not the same. What perceives is, of course, a spatial magnitude, but we must not admit that either the having the power to perceive or the sense itself is a magnitude; what they are is a certain **ratio** or power in a magnitude. This enables us to explain why objects of sense which possess one of two opposite sensible **qualities** in a degree largely in excess of the other opposite destroy the organs of sense; if the movement set up by an object is too strong for the organ, the equipoise of contrary **qualities** in the organ, which just is its sensory power, is disturbed; it is precisely as concord and tone are destroyed by too violently twanging the strings of a lyre. This explains also why plants cannot perceive in spite of their having a portion of soul in them and obviously being affected by tangible objects themselves; for undoubtedly their temperature can be lowered or raised. The explanation is that they have no mean of contrary **qualities**, and so no principle in them capable of taking on the **forms** of sensible objects without their **matter**; in the case of plants the affection is an affection by **form**-and-**matter** together. The problem might be raised: Can what cannot smell be said to be affected by smells or what cannot see by colors, and so on? It might be said that a smell is just what can be smelt, and if it produces any effect it can only be so as to make something smell it, and it might be argued that what cannot smell cannot be affected by smells and further that what can smell can be affected by it only in so far as it has in it the power to smell (similarly with the proper objects of all the other senses). Indeed that this is so is made quite evident as follows. Light or darkness, sounds and smells leave bodies quite unaffected; what does affect bodies is not these but the bodies which are their vehicles, e.g. what splits the trunk of a tree is not the sound of the thunder but the air which accompanies thunder. Yes, but, it may be objected, bodies are affected by what is tangible and by flavors. If not, by what are **things** that are without soul affected, i.e. altered in **quality**? Must we not, then, admit that the objects of the other senses also may affect them? Is not the true account this, that all bodies are capable of being affected by smells and sounds, but that some on being acted upon, having no boundaries of their own, disintegrate, as in the instance of air, which does become odorous, showing that some effect is produced on it by what is odorous? But smelling is more than such an affection by what is odorous–what more? Is not the answer that, while the air owing to the momentary duration of the action upon it of what is odorous does itself become perceptible to the sense of smell, smelling is an observing of the result produced?

***On The Soul***: But, it may be objected, it is impossible that what is self-identical should be moved at me and the same time with contrary movements in so far as it is undivided, and in an undivided moment of time. For if what is sweet be the **quality** perceived, it moves the sense or thought in this determinate way, while what is bitter moves it in a contrary way, and what is white in a different way. Is it the case then that what discriminates, though both numerically one and indivisible, is at the same time divided in its being? In one sense, it is what is divided that perceives two separate objects at once, but in another sense it does so *quâ* undivided; for it is divisible in its being but spatially and numerically undivided. Is not this impossible? For while it is true that what is self-identical and undivided may be both contraries at once potentially, it cannot be self-identical in its being–it must lose its unity by being put into activity. It is not possible to be at once white and black, and therefore it must also be impossible for a **thing** to be affected at one and the same moment by the **forms** of both, assuming it to be the case that sensation and thinking are properly so described.

***On The Soul***: If thinking is like perceiving, it must be either a process in which the soul is acted upon by what is capable of being thought, or a process different from but analogous to that. The thinking part of the soul must therefore be, while impassible, capable of receiving the **form** of an object; that is, must be potentially identical in character with its object without being the object. Mind must be related to what is thinkable, as sense is to what is sensible.

> By his definition, thinking is also a sense, how is it that the mind has the same objects of sense as the other senses? He does not see the contradiction. Each sense has a domain of stimulus it responds to, his definition does not make it apparent.

***On The Soul***: Therefore, since everything is a possible object of thought, mind in order, as Anaxagoras says, to dominate, that is, to know, must be pure from all admixture; for the co-presence of what is alien to its **nature** is a hindrance and a block: it follows that it too, like the sensitive part, can have no **nature** of its own, other than that of having a certain capacity. Thus that in the soul which is called mind (by mind I mean that whereby the soul thinks and judges) is, before it thinks, not actually any real **thing**. For this reason it cannot reasonably be regarded as blended with the body: if so, it would acquire some **quality**, e.g. warmth or cold, or even have an organ like the sensitive faculty: as it is, it has none. It was a good **idea** to call the soul 'the place of **forms**', though (1) this description holds only of the intellective soul, and (2) even this is the **forms** only potentially, not actually.

> An example of schizophrenia induced by definition. If the man thinks and a man has a soul and the soul thinks, and if the thoughts of a man guide the body, and the thoughts of the soul guide the soul–so who is following who? Does the soul follow the body or does the man follow the soul?

***On The Soul***: Since we can distinguish between a spatial magnitude and what it is to be such, and between water and what it is to be water, and so in many other cases (though not in all; for in certain cases the **thing** and its **form** are identical), flesh and what it is to be flesh are discriminated either by different faculties, or by the same faculty in two different states: For flesh necessarily involves **matter** and is like what is snub-nosed, a this in a this. Now it is by means of the sensitive faculty that we discriminate the hot and the cold, i.e. the factors which combined in a certain **ratio** constitute flesh: the essential character of flesh is apprehended by something different either wholly separate from the sensitive faculty or related to it as a bent line to the same line when it has been straightened out.

***On The Soul***: The faculty of thinking then thinks the **forms** in the images, and as in the former case what is to be pursued or avoided is marked out for it, so where there is no sensation and it is engaged upon the images it is moved to pursuit or avoidance. E.g.. perceiving by sense that the beacon is fire, it recognizes in virtue of the general faculty of sense that it signifies an enemy, because it sees it moving; but sometimes by means of the images or thoughts which are within the soul, just as if it were seeing, it calculates and deliberates what is to come by reference to what is present; and when it makes a pronouncement, as in the case of sensation it pronounces the object to be pleasant or painful, in this case it avoids or pursues and so generally in cases of action.

***On The Soul***: Knowledge and sensation are divided to correspond with the realities, potential knowledge and sensation answering to **potentialities**, actual knowledge and sensation to **actualities**. Within the soul the faculties of knowledge and sensation are potentially these objects, the one what is knowable, the other what is sensible. They must be either the **things** themselves or their **forms**. The former alternative is of course impossible: it is not the stone which is present in the soul but its **form**.

Not having one definition for sense, Aristotle missed the fact that some senses abstract ***matter*** and some abstract ***form***.

***On The Soul***: It follows that the soul is analogous to the hand; for as the hand is a tool of tools, so the mind is the **form** of **forms** and sense the **form** of sensible **things**.

***On The Soul***: Since according to common agreement there is **no thing** outside and separate in existence from sensible spatial magnitudes, the objects of thought are in the sensible **forms**, viz. both the abstract objects and all the states and affections of sensible **things**. Hence (1) no one can learn or understand **anything** in the absence of sense, and (when the mind is actively aware of **any thing** it is necessarily aware of it along with an image; for images are like sensuous contents except in that they contain no **matter**.

No experience, no knowledge. Words do not provide experience.

***On The Soul***: But sensation need not be found in all **things** that live. For it is impossible for touch to belong either (1) to those whose body is uncompounded or (2) to those which are incapable of taking in the **forms** without their **matter**.

***Topics***: Or again, look and see if anything has been said about something, of such a kind that if it be true, contrary **predicates** must necessarily belong to the **thing**: e.g. if he has said that the '***Ideas'*** ***exist*** in us. For then the result will be that they are both in motion and at rest, and moreover that they are ***objects*** both of sensation and of thought. For according to the views of those who posit the existence of **Ideas**, those **Ideas** are at rest and are objects of thought; while if they ***exist*** in us, it is impossible that they should be unmoved: For when we move, it follows necessarily that all that is in us moves with us as well. Clearly also they are objects of sensation, if they exist in us: For it is through the sensation of sight that we recognize the **form** present in each individual.

# Categories by Aristotle

translated by E. M. Edghill

This section of this book is set up so that the reader can write many notes in the margins. Aristotle's writings do not reflect a clear clean abstraction of concepts and thus he leaves a great deal to critique. I have perhaps not left a wide enough margin.

How are things named? In the simple sentence, from which all sentences are composed, one name is asserted or denied of another. What are the possible categories of names? In order to demonstrate his point that reason was not possible without well-defined terms, Plato will go through what was believed to be the ***Categories of Predication***.

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Things are said to be named '***equivocally'*** when, though they ***have a common name, the definition corresponding with the name differs for each.*** Thus, a real man and a figure in a picture can both lay claim to the name 'animal'; yet these are equivocally so named, for, though they have a common name, the definition corresponding with the name differs for each. For should any one define in what sense each is an animal, his definition in the one case will be appropriate to that case only.

On the other hand, things are said to be named '***univocally'*** which ***have both the name and the definition answering to the name in common.*** A man and an ox are both 'animal', and these are univocally so named, inasmuch as not only the name, but also the definition, is the same in both cases: for if a man should state in what sense each is an animal, the statement in the one case would be identical with that in the other.

Things are said to be named '***derivatively'***, which ***derive their name from some other name, but differ from it in termination.*** Thus the grammarian derives his name from the word 'grammar', and the courageous man from the word 'courage'.

Forms of speech are either simple or composite. Examples of the latter are such expressions as ‘the man runs’, ‘the man wins’; of the former ‘man’, ‘ox’, ‘runs’, ‘wins’.

Of things themselves some are predicable of a subject, and are never present in a subject. Thus ‘man’ is predicable of the individual man, and is never present in a subject.

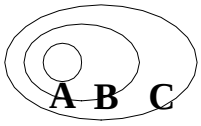
By being ‘present in a subject’ I do not mean present as parts are present in a whole, but being incapable of existence apart from the said subject.

Some things, again, are present in a subject, but are never predicable of a subject. For instance, a certain point of grammatical knowledge is present in the mind, but is not predicable of any subject; or again, a certain whiteness may be present in the body (for color requires a material basis), yet it is never predicable of anything.

Other things, again, are both predicable of a subject and present in a subject. Thus while knowledge is present in the human mind, it is predicable of grammar.

There is, lastly, a class of things which are neither present in a subject nor predicable of a subject, such as the individual man or the individual horse. But, to speak more generally, that which is individual and has the character of a unit is never predicable of a subject. Yet in some cases there is nothing to prevent such being present in a subject. Thus a certain point of grammatical knowledge is present in a subject.

When one thing is predicated of another, all that which is predicable of the predicate will be predicable also of the subject. Thus, ‘man’ is predicated of the individual man; but ‘animal’ is predicated of ‘man’; it will, therefore, be predicable of the individual man also: for the individual man is both ‘man’ and ‘animal’.

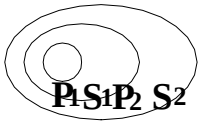



A = Individual, B = Man, C = Animal

Socrates (A) is a man (B).

All Men (B) are animals (C).

Therefore, Socrates (A) is an animal. (C)




If genera are different and co-ordinate, their differentiae are themselves different in kind. Take as an instance the genus ‘animal’ and the genus ‘knowledge’. ‘With feet’, ‘two-footed’, ‘winged’, ‘aquatic’, are differentiae of ‘animal’; the species of knowledge are not distinguished by the same differentiae. One species of knowledge does not differ from another in being ‘two-footed’.

But where one genus is subordinate to another, there is nothing to prevent their having the same differentiae: for the greater class is predicated of the lesser, so that all the differentiae of the predicate will be differentiae also of the subject.

Expressions which are in no way composite signify ***substance, quantity, quality, relation, place, time, position, state, action, or affection*** [15]. To sketch my meaning roughly, examples of

1). substance are ‘man’ or ‘the horse’,

2). of quantity, such terms as ‘two cubits long’ or ‘three cubits long’,

3). of quality, such attributes as ‘white’, ‘grammatical’.

4). ‘Double’, ‘half’, ‘greater’, fall under the category of relation;

5). ‘in the market place’, ‘in the Lyceum’, under that of place;

6). ‘yesterday’, ‘last year’, under that of time.

7). ‘Lying’, ‘sitting’, are terms indicating position,

8). ‘shod’, ‘armed’, state;

9). ‘to lance’, ‘to cauterize’, action;

[15] ***The Categories***.

10). ‘to be lanced’, ‘to be cauterized’, affection.

No one of these terms, in and by itself, involves an affirmation; it is by the combination of such terms that positive or negative statements arise. For every assertion must, as is admitted, be either true or false, whereas expressions which are not in any way composite such as ‘man’, ‘white’, ‘runs’, ‘wins’, cannot be either true or false.

## Substance

Substance, in the truest and primary and most definite sense of the word, is that which is neither predicable of a subject nor present in a subject; for instance, the individual man or horse. But in a secondary sense those things are called substances within which, as species, the primary substances are included; also those which, as genera, include the species. For instance, the individual man is included in the species ‘man’, and the genus to which the species belongs is ‘animal’; these, therefore–that is to say, the species ‘man’ and the genus ‘animal’–are termed secondary substances.

It is plain from what has been said that both the name and the definition of the predicate must be predicable of the subject. For instance, ‘man’ is predicted of the individual man. Now in this case the name of the species ‘man’ is applied to the individual, for we use the term ‘man’ in describing the individual; and the definition of ‘man’ will also be predicated of the individual man, for the individual man is both man and animal. Thus, both the name and the definition of the species are predicable of the individual.

With regard, on the other hand, to those things which are present in a subject, it is generally the case that neither their name nor their definition is predicable of that in which they are present. Though, however, the definition is never predicable, there is nothing in certain cases to prevent the name being used. For instance, ‘white’ being present in a body is predicated of that in which it is present, for a body is called white: the definition, however, of the color ‘white’ is never predicable of the body.

Everything except primary substances is either

predicable of a primary substance or present in a primary substance. This becomes evident by reference to particular instances which occur. ‘Animal’ is predicated of the species ‘man’, therefore of the individual man, for if there were no individual man of whom it could be predicated, it could not be predicated of the species ‘man’ at all. Again, color is present in body, therefore in individual bodies, for if there were no individual body in which it was present, it could not be present in body at all. Thus everything except primary substances is either predicated of primary substances, or is present in them, and if these last did not exist, it would be impossible for anything else to exist.

Of secondary substances, the species is more truly substance than the genus, being more nearly related to primary substance. For if any one should render an account of what a primary substance is, he would render a more instructive account, and one more proper to the subject, by stating the species than by stating the genus. Thus, he would give a more instructive account of an individual man by stating that he was man than by stating that he was animal, for the former description is peculiar to the individual in a greater degree, while the latter is too general. Again, the man who gives an account of the nature of an individual tree will give a more instructive account by mentioning the species ‘tree’ than by mentioning the genus ‘plant’.

Moreover, primary substances are most properly called substances in virtue of the fact that they are the entities which underlie everything else, and that everything else is either predicated of them or present in them. Now the same relation which subsists between primary substance and everything else subsists also between the species and the genus: for the species is to the genus as subject is to predicate, since the genus is predicated of the species, whereas the species cannot be predicated of the genus. Thus we have a second ground for asserting that the species is more truly substance than the genus.

Of species themselves, except in the case of such as are genera, no one is more truly substance

than another. We should not give a more appropriate account of the individual man by stating the species to which he belonged, than we should of an individual horse by adopting the same method of definition. In the same way, of primary substances, no one is more truly substance than another; an individual man is not more truly substance than an individual ox.

It is, then, with good reason that of all that remains, when we exclude primary substances, we concede to species and genera alone the name ‘secondary substance’, for these alone of all the predicates convey a knowledge of primary substance. For it is by stating the species or the genus that we appropriately define any individual man; and we shall make our definition more exact by stating the former than by stating the latter. All other things that we state, such as that he is white, that he runs, and so on, are irrelevant to the definition. Thus it is just that these alone, apart from primary substances, should be called substances.

Further, primary substances are most properly so called, because they underlie and are the subjects of everything else. Now the same relation that subsists between primary substance and everything else subsists also between the species and the genus to which the primary substance belongs, on the one hand, and every attribute which is not included within these, on the other. For these are the subjects of all such. If we call an individual man ‘skilled in grammar’, the predicate is applicable also to the species and to the genus to which he belongs. This law holds good in all cases.

It is a common characteristic of all substance that it is never present in a subject. For primary substance is neither present in a subject nor predicated of a subject; while, with regard to secondary substances, it is clear from the following arguments (apart from others) that they are not present in a subject. For ‘man’ is predicated of the individual man, but is not present in any subject: for manhood is not present in the individual man. In the same way, ‘animal’ is also predicated of the individual man, but is not present in him. Again, when a thing is present in a subject, though the

name may quite well be applied to that in which it is present, the definition cannot be applied. Yet of secondary substances, not only the name, but also the definition, applies to the subject: we should use both the definition of the species and that of the genus with reference to the individual man. Thus substance cannot be present in a subject.

Yet this is not peculiar to substance, for it is also the case that differentiae cannot be present in subjects. The characteristics ‘terrestrial’ and ‘two-footed’ are predicated of the species ‘man’, but not present in it. For they are not in man. Moreover, the definition of the differentia may be predicated of that of which the differentia itself is predicated. For instance, if the characteristic ‘terrestrial’ is predicated of the species ‘man’, the definition also of that characteristic may be used to form the predicate of the species ‘man’: for ‘man’ is terrestrial.

The fact that the parts of substances appear to be present in the whole, as in a subject, should not make us apprehensive lest we should have to admit that such parts are not substances: for in explaining the phrase ‘being present in a subject’, we stated that we meant ‘otherwise than as parts in a whole’.

It is the mark of substances and of differentiae that, in all propositions of which they form the predicate, they are predicated univocally. For all such propositions have for their subject either the individual or the species. It is true that, inasmuch as primary substance is not predicable of anything, it can never form the predicate of any proposition. But of secondary substances, the species is predicated of the individual, the genus both of the species and of the individual. Similarly the differentiae are predicated of the species and of the individuals. Moreover, the definition of the species and that of the genus are applicable to the primary substance, and that of the genus to the species. For all that is predicated of the predicate will be predicated also of the subject. Similarly, the definition of the differentiae will be applicable to the species and to the individuals. But it was stated above that the word ‘univocal’ was applied to those things which had both name and definition in common. It is,

therefore, established that in every proposition, of which either substance or a differentia forms the predicate, these are predicated univocally.

All substance appears to signify that which is individual. In the case of primary substance this is indisputably true, for the thing is a unit. In the case of secondary substances, when we speak, for instance, of ‘man’ or ‘animal’, our form of speech gives the impression that we are here also indicating that which is individual, but the impression is not strictly true; for a secondary substance is not an individual, but a class with a certain qualification; for it is not one and single as a primary substance is; the words ‘man’, ‘animal’, are predicable of more than one subject.

Yet species and genus do not merely indicate quality, like the term ‘white’; ‘white’ indicates quality and nothing further, but species and genus determine the quality with reference to a substance: they signify substance qualitatively differentiated. The determinate qualification covers a larger field in the case of the genus that in that of the species: he who uses the word ‘animal’ is herein using a word of wider extension than he who uses the word ‘man’.

Another mark of substance is that it has no contrary. What could be the contrary of any primary substance, such as the individual man or animal? It has none. Nor can the species or the genus have a contrary. Yet this characteristic is not peculiar to substance, but is true of many other things, such as quantity. There is nothing that forms the contrary of ‘two cubits long’ or of ‘three cubits long’, or of ‘ten’, or of any such term. A man may contend that ‘much’ is the contrary of ‘little’, or ‘great’ of ‘small’, but of definite quantitative terms no contrary exists.

Substance, again, does not appear to admit of variation of degree. I do not mean by this that one substance cannot be more or less truly substance than another, for it has already been stated that this is the case; but that no single substance admits of varying degrees within itself. For instance, one particular substance, ‘man’, cannot be more or less man either than himself at some other time or than

some other man. One man cannot be more man than another, as that which is white may be more or less white than some other white object, or as that which is beautiful may be more or less beautiful than some other beautiful object. The same quality, moreover, is said to subsist in a thing in varying degrees at different times. A body, being white, is said to be whiter at one time than it was before, or, being warm, is said to be warmer or less warm than at some other time. But substance is not said to be more or less that which it is: a man is not more truly a man at one time than he was before, nor is anything, if it is substance, more or less what it is. Substance, then, does not admit of variation of degree.

The most distinctive mark of substance appears to be that, while remaining numerically one and the same, it is capable of admitting contrary qualities. From among things other than substance, we should find ourselves unable to bring forward any which possessed this mark. Thus, one and the same color cannot be white and black. Nor can the same one action be good and bad: this law holds good with everything that is not substance. But one and the selfsame substance, while retaining its identity, is yet capable of admitting contrary qualities. The same individual person is at one time white, at another black, at one time warm, at another cold, at one time good, at another bad. This capacity is found nowhere else, though it might be maintained that a statement or opinion was an exception to the rule. The same statement, it is agreed, can be both true and false. For if the statement ‘he is sitting’ is true, yet, when the person in question has risen, the same statement will be false. The same applies to opinions. For if any one thinks truly that a person is sitting, yet, when that person has risen, this same opinion, if still held, will be false. Yet although this exception may be allowed, there is, nevertheless, a difference in the manner in which the thing takes place. It is by themselves changing that substances admit contrary qualities. It is thus that that which was hot becomes cold, for it has entered into a different state. Similarly that which was white becomes black, and that which was bad good, by a process of change; and in the same way in all other

cases it is by changing that substances are capable of admitting contrary qualities. But statements and opinions themselves remain unaltered in all respects: it is by the alteration in the facts of the case that the contrary quality comes to be theirs. The statement ‘he is sitting’ remains unaltered, but it is at one time true, at another false, according to circumstances. What has been said of statements applies also to opinions. Thus, in respect of the manner in which the thing takes place, it is the peculiar mark of substance that it should be capable of admitting contrary qualities; for it is by itself changing that it does so.

If, then, a man should make this exception and contend that statements and opinions are capable of admitting contrary qualities, his contention is unsound. For statements and opinions are said to have this capacity, not because they themselves undergo modification, but because this modification occurs in the case of something else. The truth or falsity of a statement depends on facts, and not on any power on the part of the statement itself of admitting contrary qualities. In short, there is nothing which can alter the nature of statements and opinions. As, then, no change takes place in themselves, these cannot be said to be capable of admitting contrary qualities.

But it is by reason of the modification which takes place within the substance itself that a substance is said to be capable of admitting contrary qualities; for a substance admits within itself either disease or health, whiteness or blackness. It is in this sense that it is said to be capable of admitting contrary qualities.

To sum up, it is a distinctive mark of substance, that, while remaining numerically one and the same, it is capable of admitting contrary qualities, the modification taking place through a change in the substance itself.

Let these remarks suffice on the subject of substance.

## Quantity

Quantity is either discrete or continuous.

Moreover, some quantities are such that each part of the whole has a relative position to the other parts: others have within them no such relation of part to part.

Instances of discrete quantities are number and speech; of continuous, lines, surfaces, solids, and, besides these, time and place.

In the case of the parts of a number, there is no common boundary at which they join. For example: two fives make ten, but the two fives have no common boundary, but are separate; the parts three and seven also do not join at any boundary. Nor, to generalize, would it ever be possible in the case of number that there should be a common boundary among the parts; they are always separate. Number, therefore, is a discrete quantity.

The same is true of speech. That speech is a quantity is evident: for it is measured in long and short syllables. I mean here that speech which is vocal. Moreover, it is a discrete quantity for its parts have no common boundary. There is no common boundary at which the syllables join, but each is separate and distinct from the rest.

A line, on the other hand, is a continuous quantity, for it is possible to find a common boundary at which its parts join. In the case of the line, this common boundary is the point; in the case of the plane, it is the line: for the parts of the plane have also a common boundary. Similarly you can find a common boundary in the case of the parts of a solid, namely either a line or a plane.

Space and time also belong to this class of quantities. Time, past, present, and future, forms a continuous whole. Space, likewise, is a continuous quantity; for the parts of a solid occupy a certain space, and these have a common boundary; it follows that the parts of space also, which are occupied by the parts of the solid, have the same common boundary as the parts of the solid. Thus, not only time, but space also, is a continuous quantity, for its parts have a common boundary.

Quantities consist either of parts which bear a relative position each to each, or of parts which do not. The parts of a line bear a relative position to

each other, for each lies somewhere, and it would be possible to distinguish each, and to state the position of each on the plane and to explain to what sort of part among the rest each was contiguous. Similarly the parts of a plane have position, for it could similarly be stated what was the position of each and what sort of parts were contiguous. The same is true with regard to the solid and to space. But it would be impossible to show that the arts of a number had a relative position each to each, or a particular position, or to state what parts were contiguous. Nor could this be done in the case of time, for none of the parts of time has an abiding existence, and that which does not abide can hardly have position. It would be better to say that such parts had a relative order, in virtue of one being prior to another. Similarly with number: in counting, 'one' is prior to 'two', and 'two' to 'three', and thus the parts of number may be said to possess a relative order, though it would be impossible to discover any distinct position for each. This holds good also in the case of speech. None of its parts has an abiding existence: when once a syllable is pronounced, it is not possible to retain it, so that, naturally, as the parts do not abide, they cannot have position. Thus, some quantities consist of parts which have position, and some of those which have not.

Strictly speaking, only the things which I have mentioned belong to the category of quantity: everything else that is called quantitative is a quantity in a secondary sense. It is because we have in mind some one of these quantities, properly so called, that we apply quantitative terms to other things. We speak of what is white as large, because the surface over which the white extends is large; we speak of an action or a process as lengthy, because the time covered is long; these things cannot in their own right claim the quantitative epithet. For instance, should any one explain how long an action was, his statement would be made in terms of the time taken, to the effect that it lasted a year, or something of that sort. In the same way, he would explain the size of a white object in terms of surface, for he would state the area which it covered. Thus the things already mentioned, and

these alone, are in their intrinsic nature quantities; nothing else can claim the name in its own right, but, if at all, only in a secondary sense.

Quantities have no contraries. In the case of definite quantities this is obvious; thus, there is nothing that is the contrary of ‘two cubits long’ or of ‘three cubits long’, or of a surface, or of any such quantities. A man might, indeed, argue that ‘much’ was the contrary of ‘little’, and ‘great’ of ‘small’. But these are not quantitative, but relative; things are not great or small absolutely, they are so called rather as the result of an act of comparison. For instance, a mountain is called small, a grain large, in virtue of the fact that the latter is greater than others of its kind, the former less. Thus there is a reference here to an external standard, for if the terms ‘great’ and ‘small’ were used absolutely, a mountain would never be called small or a grain large. Again, we say that there are many people in a village, and few in Athens, although those in the city are many times as numerous as those in the village: or we say that a house has many in it, and a theatre few, though those in the theatre far outnumber those in the house. The terms ‘two cubits long,’ ‘three cubits long,’ and so on indicate quantity, the terms ‘great’ and ‘small’ indicate relation, for they have reference to an external standard. It is, therefore, plain that these are to be classed as relative.

Again, whether we define them as quantitative or not, they have no contraries: for how can there be a contrary of an attribute which is not to be apprehended in or by itself, but only by reference to something external? Again, if ‘great’ and ‘small’ are contraries, it will come about that the same subject can admit contrary qualities at one and the same time, and that things will themselves be contrary to themselves. For it happens at times that the same thing is both small and great. For the same thing may be small in comparison with one thing, and great in comparison with another, so that the same thing comes to be both small and great at one and the same time, and is of such a nature as to admit contrary qualities at one and the same moment. Yet it was agreed, when substance was being discussed, that nothing admits contrary qualities at one and the same moment. For though

substance is capable of admitting contrary qualities, yet no one is at the same time both sick and healthy, nothing is at the same time both white and black. Nor is there anything which is qualified in contrary ways at one and the same time.

Moreover, if these were contraries, they would themselves be contrary to themselves. For if ‘great’ is the contrary of ‘small’, and the same thing is both great and small at the same time, then ‘small’ or ‘great’ is the contrary of itself. But this is impossible. The term ‘great’, therefore, is not the contrary of the term ‘small’, nor ‘much’ of ‘little’. And even though a man should call these terms not relative but quantitative, they would not have contraries.

It is in the case of space that quantity most plausibly appears to admit of a contrary. For men define the term ‘above’ as the contrary of ‘below’, when it is the region at the center they mean by ‘below’; and this is so, because nothing is farther from the extremities of the universe than the region at the center. Indeed, it seems that in defining contraries of every kind men have recourse to a spatial metaphor, for they say that those things are contraries which, within the same class, are separated by the greatest possible distance.

Quantity does not, it appears, admit of variation of degree. One thing cannot be two cubits long in a greater degree than another. Similarly with regard to number: what is ‘three’ is not more truly three than what is ‘five’ is five; nor is one set of three more truly three than another set. Again, one period of time is not said to be more truly time than another. Nor is there any other kind of quantity, of all that have been mentioned, with regard to which variation of degree can be predicated. The category of quantity, therefore, does not admit of variation of degree.

The most distinctive mark of quantity is that equality and inequality are predicated of it. Each of the aforesaid quantities is said to be equal or unequal. For instance, one solid is said to be equal or unequal to another; number, too, and time can have these terms applied to them, indeed can all those kinds of quantity that have been mentioned.

That which is not a quantity can by no means, it would seem, be termed equal or unequal to anything else. One particular disposition or one particular quality, such as whiteness, is by no means compared with another in terms of equality and inequality but rather in terms of similarity. Thus it is the distinctive mark of quantity that it can be called equal and unequal.

Those things are called relative, which, being either said to be of something else or related to something else, are explained by reference to that other thing. For instance, the word ‘superior’ is explained by reference to something else, for it is superiority over something else that is meant. Similarly, the expression ‘double’ has this external reference, for it is the double of something else that is meant. So it is with everything else of this kind. There are, moreover, other relatives, e.g. habit, disposition, perception, knowledge, and attitude. The significance of all these is explained by a reference to something else and in no other way. Thus, a habit is a habit of something, knowledge is knowledge of something, attitude is the attitude of something. So it is with all other relatives that have been mentioned. Those terms, then, are called relative, the nature of which is explained by reference to something else, the preposition ‘of’ or some other preposition being used to indicate the relation. Thus, one mountain is called great in comparison with another; for the mountain claims this attribute by comparison with something. Again, that which is called similar must be similar to something else, and all other such attributes have this external reference. It is to be noted that lying and standing and sitting are particular attitudes, but attitude is itself a relative term. To lie, to stand, to be seated, are not themselves attitudes, but take their name from the aforesaid attitudes.

It is possible for relatives to have contraries. Thus virtue has a contrary, vice, these both being relatives; knowledge, too, has a contrary, ignorance. But this is not the mark of all relatives; ‘double’ and ‘triple’ have no contrary, nor indeed has any such term.

It also appears that relatives can admit of

variation of degree. For ‘like’ and ‘unlike’, ‘equal’ and ‘unequal’, have the modifications ‘more’ and ‘less’ applied to them, and each of these is relative in character: for the terms ‘like’ and ‘unequal’ bear a reference to something external. Yet, again, it is not every relative term that admits of variation of degree. No term such as ‘double’ admits of this modification. All relatives have correlatives: by the term ‘slave’ we mean the slave of a master, by the term ‘master’, the master of a slave; by ‘double’, the double of its half; by ‘half’, the half of its double; by ‘greater’, greater than that which is less; by ‘less,’ less than that which is greater.

So it is with every other relative term; but the case we use to express the correlation differs in some instances. Thus, by knowledge we mean knowledge of the knowable; by the knowable, that which is to be apprehended by knowledge; by perception, perception of the perceptible; by the perceptible, that which is apprehended by perception.

Sometimes, however, reciprocity of correlation does not appear to exist. This comes about when a blunder is made, and that to which the relative is related is not accurately stated. If a man states that a wing is necessarily relative to a bird, the connexion between these two will not be reciprocal, for it will not be possible to say that a bird is a bird by reason of its wings. The reason is that the original statement was inaccurate, for the wing is not said to be relative to the bird *quâ* bird, since many creatures besides birds have wings, but *quâ* winged creature. If, then, the statement is made accurate, the connexion will be reciprocal, for we can speak of a wing, having reference necessarily to a winged creature, and of a winged creature as being such because of its wings.

Occasionally, perhaps, it is necessary to coin words, if no word exists by which a correlation can adequately be explained. If we define a rudder as necessarily having reference to a boat, our definition will not be appropriate, for the rudder does not have this reference to a boat *quâ* boat, as there are boats which have no rudders. Thus we cannot use the terms reciprocally, for the word

‘boat’ cannot be said to find its explanation in the word ‘rudder’. As there is no existing word, our definition would perhaps be more accurate if we coined some word like ‘ruddered’ as the correlative of ‘rudder’. If we express ourselves thus accurately, at any rate the terms are reciprocally connected, for the ‘ruddered’ thing is ‘ruddered’ in virtue of its rudder. So it is in all other cases. A head will be more accurately defined as the correlative of that which is ‘headed’, than as that of an animal, for the animal does not have a head *quâ* animal, since many animals have no head.

Thus we may perhaps most easily comprehend that to which a thing is related, when a name does not exist, if, from that which has a name, we derive a new name, and apply it to that with which the first is reciprocally connected, as in the aforesaid instances, when we derived the word ‘winged’ from ‘wing’ and from ‘rudder’.

All relatives, then, if properly defined, have a correlative. I add this condition because, if that to which they are related is stated as haphazard and not accurately, the two are not found to be interdependent. Let me state what I mean more clearly. Even in the case of acknowledged correlatives, and where names exist for each, there will be no interdependence if one of the two is denoted, not by that name which expresses the correlative notion, but by one of irrelevant significance. The term ‘slave,’ if defined as related, not to a master, but to a man, or a biped, or anything of that sort, is not reciprocally connected with that in relation to which it is defined, for the statement is not exact. Further, if one thing is said t$^{o}$ be correlative with another, and the terminology used is correct, then, though all irrelevant attributes should be removed, and only that one attribute left in virtue of which it was correctly stated to be correlative with that other, the stated correlation will still exist. If the correlative of ‘the slave’ is said to be ‘the master’, then, though all irrelevant attributes of the said ‘master’, such as ‘biped’, ‘receptive of knowledge’, ‘human’, should be removed, and the attribute ‘master’ alone left, the stated correlation existing between him and the slave will remain the same, for it is of a master that

a slave is said to be the slave. On the other hand, if, of two correlatives, one is not correctly termed, then, when all other attributes are removed and that alone is left in virtue of which it was stated to be correlative, the stated correlation will be found to have disappeared.

For suppose the correlative of ‘the slave’ should be said to be ‘the man’, or the correlative of ‘the wing’ ‘the bird’; if the attribute ‘master’ be withdrawn from ‘the man’, the correlation between ‘the man’ and ‘the slave’ will cease to exist, for if the man is not a master, the slave is not a slave. Similarly, if the attribute ‘winged’ be withdrawn from ‘the bird’, ‘the wing’ will no longer be relative; for if the so-called correlative is not winged, it follows that ‘the wing’ has no correlative.

Thus it is essential that the correlated terms should be exactly designated; if there is a name existing, the statement will be easy; if not, it is doubtless our duty to construct names. When the terminology is thus correct, it is evident that all correlatives are interdependent.

Correlatives are thought to come into existence simultaneously. This is for the most part true, as in the case of the double and the half. The existence of the half necessitates the existence of that of which it is a half. Similarly the existence of a master necessitates the existence of a slave, and that of a slave implies that of a master; these are merely instances of a general rule. Moreover, they cancel one another; for if there is no double it follows that there is no half, and vice versa; this rule also applies to all such correlatives. Yet it does not appear to be true in all cases that correlatives come into existence simultaneously. The object of knowledge would appear to exist before knowledge itself, for it is usually the case that we acquire knowledge of objects already existing; it would be difficult, if not impossible, to find a branch of knowledge the beginning of the existence of which was contemporaneous with that of its object.

Again, while the object of knowledge, if it ceases to exist, cancels at the same time the knowledge which was its correlative, the converse of this is not true. It is true that if the object of

knowledge does not exist there can be no knowledge: for there will no longer be anything to know. Yet it is equally true that, if knowledge of a certain object does not exist, the object may nevertheless quite well exist. Thus, in the case of the squaring of the circle, if indeed that process is an object of knowledge, though it itself exists as an object of knowledge, yet the knowledge of it has not yet come into existence. Again, if all animals ceased to exist, there would be no knowledge, but there might yet be many objects of knowledge.

This is likewise the case with regard to perception: for the object of perception is, it appears, prior to the act of perception. If the perceptible is annihilated, perception also will cease to exist; but the annihilation of perception does not cancel the existence of the perceptible. For perception implies a body perceived and a body in which perception takes place. Now if that which is perceptible is annihilated, it follows that the body is annihilated, for the body is a perceptible thing; and if the body does not exist, it follows that perception also ceases to exist. Thus the annihilation of the perceptible involves that of perception.

But the annihilation of perception does not involve that of the perceptible. For if the animal is annihilated, it follows that perception also is annihilated, but perceptibles such as body, heat, sweetness, bitterness, and so on, will remain.

Again, perception is generated at the same time as the perceiving subject, for it comes into existence at the same time as the animal. But the perceptible surely exists before perception; for fire and water and such elements, out of which the animal is itself composed, exist before the animal is an animal at all, and before perception. Thus it would seem that the perceptible exists before perception.

It may be questioned whether it is true that no substance is relative, as seems to be the case, or whether exception is to be made in the case of certain secondary substances. With regard to primary substances, it is quite true that there is no such possibility, for neither wholes nor parts of primary substances are relative. The individual man or ox is not defined with reference to something

external. Similarly with the parts: a particular hand or head is not defined as a particular hand or head of a particular person, but as the hand or head of a particular person. It is true also, for the most part at least, in the case of secondary substances; the species ‘man’ and the species ‘ox’ are not defined with reference to anything outside themselves. Wood, again, is only relative in so far as it is some one’s property, not in so far as it is wood. It is plain, then, that in the cases mentioned substance is not relative. But with regard to some secondary substances there is a difference of opinion; thus, such terms as ‘head’ and ‘hand’ are defined with reference to that of which the things indicated are a part, and so it comes about that these appear to have a relative character. Indeed, if our definition of that which is relative was complete, it is very difficult, if not impossible, to prove that no substance is relative. If, however, our definition was not complete, if those things only are properly called relative in the case of which relation to an external object is a necessary condition of existence, perhaps some explanation of the dilemma may be found.

The former definition does indeed apply to all relatives, but the fact that a thing is explained with reference to something else does not make it essentially relative.

From this it is plain that, if a man definitely apprehends a relative thing, he will also definitely apprehend that to which it is relative. Indeed this is self-evident: for if a man knows that some particular thing is relative, assuming that we call that a relative in the case of which relation to something is a necessary condition of existence, he knows that also to which it is related. For if he does not know at all that to which it is related, he will not know whether or not it is relative. This is clear, moreover, in particular instances. If a man knows definitely that such and such a thing is ‘double’, he will also forthwith know definitely that of which it is the double. For if there is nothing definite of which he knows it to be the double, he does not know at all that it is double. Again, if he knows that a thing is more beautiful, it follows necessarily that he will forthwith definitely know that also than which it is more beautiful. He will not merely know

indefinitely that it is more beautiful than something which is less beautiful, for this would be supposition, not knowledge. For if he does not know definitely that than which it is more beautiful, he can no longer claim to know definitely that it is more beautiful than something else which is less beautiful: for it might be that nothing was less beautiful. It is, therefore, evident that if a man apprehends some relative thing definitely, he necessarily knows that also definitely to which it is related.

Now the head, the hand, and such things are substances, and it is possible to know their essential character definitely, but it does not necessarily follow that we should know that to which they are related. It is not possible to know forthwith whose head or hand is meant. Thus these are not relatives, and, this being the case, it would be true to say that no substance is relative in character. It is perhaps a difficult matter, in such cases, to make a positive statement without more exhaustive examination, but to have raised questions with regard to details is not without advantage.

## Quality

By ‘quality’ I mean that in virtue of which people are said to be such and such.

Quality is a term that is used in many senses. One sort of quality let us call ‘habit’ or ‘disposition’. Habit differs from disposition in being more lasting and more firmly established. The various kinds of knowledge and of virtue are habits, for knowledge, even when acquired only in a moderate degree, is, it is agreed, abiding in its character and difficult to displace, unless some great mental upheaval takes place, through disease or any such cause. The virtues, also, such as justice, self-restraint, and so on, are not easily dislodged or dismissed, so as to give place to vice.

By a disposition, on the other hand, we mean a condition that is easily changed and quickly gives place to its opposite. Thus, heat, cold, disease, health, and so on are dispositions. For a man is disposed in one way or another with reference to

these, but quickly changes, becoming cold instead of warm, ill instead of well. So it is with all other dispositions also, unless through lapse of time a disposition has itself become inveterate and almost impossible to dislodge: in which case we should perhaps go so far as to call it a habit.

It is evident that men incline to call those conditions habits which are of a more or less permanent type and difficult to displace; for those who are not retentive of knowledge, but volatile, are not said to have such and such a 'habit' as regards knowledge, yet they are disposed, we may say, either better or worse, towards knowledge. Thus habit differs from disposition in this, that while the latter in ephemeral, the former is permanent and difficult to alter.

Habits are at the same time dispositions, but dispositions are not necessarily habits. For those who have some specific habit may be said also, in virtue of that habit, to be thus or thus disposed; but those who are disposed in some specific way have not in all cases the corresponding habit.

Another sort of quality is that in virtue of which, for example, we call men good boxers or runners, or healthy or sickly: in fact it includes all those terms which refer to inborn capacity or incapacity. Such things are not predicated of a person in virtue of his disposition, but in virtue of his inborn capacity or incapacity to do something with ease or to avoid defeat of any kind. Persons are called good boxers or good runners, not in virtue of such and such a disposition, but in virtue of an inborn capacity to accomplish something with ease. Men are called healthy in virtue of the inborn capacity of easy resistance to those unhealthy influences that may ordinarily arise; unhealthy, in virtue of the lack of this capacity. Similarly with regard to softness and hardness. Hardness is predicated of a thing because it has that capacity of resistance which enables it to withstand disintegration; softness, again, is predicated of a thing by reason of the lack of that capacity.

A third class within this category is that of affective qualities and affections. Sweetness, bitterness, sourness, are examples of this sort of

quality, together with all that is akin to these; heat, moreover, and cold, whiteness, and blackness are affective qualities. It is evident that these are qualities, for those things that possess them are themselves said to be such and such by reason of their presence. Honey is called sweet because it contains sweetness; the body is called white because it contains whiteness; and so in all other cases.

The term ‘affective quality’ is not used as indicating that those things which admit these qualities are affected in any way. Honey is not called sweet because it is affected in a specific way, nor is this what is meant in any other instance. Similarly heat and cold are called affective qualities, not because those things which admit them are affected. What is meant is that these said qualities are capable of producing an ‘affection’ in the way of perception. For sweetness has the power of affecting the sense of taste; heat, that of touch; and so it is with the rest of these qualities.

Whiteness and blackness, however, and the other colors, are not said to be affective qualities in this sense, but because they themselves are the results of an affection. It is plain that many changes of color take place because of affections. When a man is ashamed, he blushes; when he is afraid, he becomes pale, and so on. So true is this, that when a man is by nature liable to such affections, arising from some concomitance of elements in his constitution, it is a probable inference that he has the corresponding complexion of skin. For the same disposition of bodily elements, which in the former instance was momentarily present in the case of an excess of shame, might be a result of a man’s natural temperament, so as to produce the corresponding coloring also as a natural characteristic. All conditions, therefore, of this kind, if caused by certain permanent and lasting affections, are called affective qualities. For pallor and duskiness of complexion are called qualities, inasmuch as we are said to be such and such in virtue of them, not only if they originate in natural constitution, but also if they come about through long disease or sunburn, and are difficult to remove, or indeed remain throughout life. For in the same

way we are said to be such and such because of these.

Those conditions, however, which arise from causes which may easily be rendered ineffective or speedily removed, are called, not qualities, but affections: for we are not said to be such in virtue of them. The man who blushes through shame is not said to be a constitutional blusher, nor is the man who becomes pale through fear said to be constitutionally pale. He is said rather to have been affected.

Thus such conditions are called affections, not qualities.

In like manner there are affective qualities and affections of the soul. That temper with which a man is born and which has its origin in certain deep-seated affections is called a quality. I mean such conditions as insanity, irascibility, and so on: for people are said to be mad or irascible in virtue of these. Similarly those abnormal psychic states which are not inborn, but arise from the concomitance of certain other elements, and are difficult to remove, or altogether permanent, are called qualities, for in virtue of them men are said to be such and such.

Those, however, which arise from causes easily rendered ineffective are called affections, not qualities. Suppose that a man is irritable when vexed: he is not even spoken of as a bad-tempered man, when in such circumstances he loses his temper somewhat, but rather is said to be affected. Such conditions are therefore termed, not qualities, but affections.

The fourth sort of quality is figure and the shape that belongs to a thing; and besides this, straightness and curvedness and any other qualities of this type; each of these defines a thing as being such and such. Because it is triangular or quadrangular a thing is said to have a specific character, or again because it is straight or curved; in fact a thing's shape in every case gives rise to a qualification of it.

Rarity and density, roughness and smoothness, seem to be terms indicating quality: yet these, it would appear, really belong to a class different from

that of quality. For it is rather a certain relative position of the parts composing the thing thus qualified which, it appears, is indicated by each of these terms. A thing is dense, owing to the fact that its parts are closely combined with one another; rare, because there are interstices between the parts; smooth, because its parts lie, so to speak, evenly; rough, because some parts project beyond others.

There may be other sorts of quality, but those that are most properly so called have, we may safely say, been enumerated.

These, then, are qualities, and the things that take their name from them as derivatives, or are in some other way dependent on them, are said to be qualified in some specific way. In most, indeed in almost all cases, the name of that which is qualified is derived from that of the quality. Thus the terms ‘whiteness’, ‘grammar’, ‘justice’, give us the adjectives ‘white’, ‘grammatical’, ‘just’, and so on.

There are some cases, however, in which, as the quality under consideration has no name, it is impossible that those possessed of it should have a name that is derivative. For instance, the name given to the runner or boxer, who is so called in virtue of an inborn capacity, is not derived from that of any quality; for those capacities have no name assigned to them. In this, the inborn capacity is distinct from the science, with reference to which men are called, e.g. boxers or wrestlers. Such a science is classed as a disposition; it has a name, and is called ‘boxing’ or ‘wrestling’ as the case may be, and the name given to those disposed in this way is derived from that of the science. Sometimes, even though a name exists for the quality, that which takes its character from the quality has a name that is not a derivative. For instance, the upright man takes his character from the possession of the quality of integrity, but the name given him is not derived from the word ‘integrity’. Yet this does not occur often.

We may therefore state that those things are said to be possessed of some specific quality which have a name derived from that of the aforesaid quality, or which are in some other way dependent on it.

One quality may be the contrary of another; thus justice is the contrary of injustice, whiteness of blackness, and so on. The things, also, which are said to be such and such in virtue of these qualities, may be contrary the one to the other; for that which is unjust is contrary to that which is just, that which is white to that which is black. This, however, is not always the case. Red, yellow, and such colors, though qualities, have no contraries.

If one of two contraries is a quality, the other will also be a quality. This will be evident from particular instances, if we apply the names used to denote the other categories; for instance, granted that justice is the contrary of injustice and justice is a quality, injustice will also be a quality: neither quantity, nor relation, nor place, nor indeed any other category but that of quality, will be applicable properly to injustice. So it is with all other contraries falling under the category of quality.

Qualities admit of variation of degree. Whiteness is predicated of one thing in a greater or less degree than of another. This is also the case with reference to justice. Moreover, one and the same thing may exhibit a quality in a greater degree than it did before: if a thing is white, it may become whiter.

Though this is generally the case, there are exceptions. For if we should say that justice admitted of variation of degree, difficulties might ensue, and this is true with regard to all those qualities which are dispositions. There are some, indeed, who dispute the possibility of variation here. They maintain that justice and health cannot very well admit of variation of degree themselves, but that people vary in the degree in which they possess these qualities, and that this is the case with grammatical learning and all those qualities which are classed as dispositions. However that may be, it is an incontrovertible fact that the things which in virtue of these qualities are said to be what they are vary in the degree in which they possess them; for one man is said to be better versed in grammar, or more healthy or just, than another, and so on.

The qualities expressed by the terms ‘triangular’ and ‘quadrangular’ do not appear to admit of

variation of degree, nor indeed do any that have to do with figure. For those things to which the definition of the triangle or circle is applicable are all equally triangular or circular. Those, on the other hand, to which the same definition is not applicable, cannot be said to differ from one another in degree; the square is no more a circle than the rectangle, for to neither is the definition of the circle appropriate. In short, if the definition of the term proposed is not applicable to both objects, they cannot be compared. Thus it is not all qualities which admit of variation of degree.

Whereas none of the characteristics I have mentioned are peculiar to quality, the fact that likeness and unlikeness can be predicated with reference to quality only, gives to that category its distinctive feature. One thing is like another only with reference to that in virtue of which it is such and such; thus this forms the peculiar mark of quality.

## Relative, Action, Affection

We must not be disturbed because it may be argued that, though proposing to discuss the category of quality, we have included in it many relative terms. We did say that habits and dispositions were relative. In practically all such cases the genus is relative, the individual not. Thus knowledge, as a genus, is explained by reference to something else, for we mean a knowledge of something. But particular branches of knowledge are not thus explained. The knowledge of grammar is not relative to anything external, nor is the knowledge of music, but these, if relative at all, are relative only in virtue of their genera; thus grammar is said be the knowledge of something, not the grammar of something; similarly music is the knowledge of something, not the music of something.

Thus individual branches of knowledge are not relative. And it is because we possess these individual branches of knowledge that we are said to be such and such. It is these that we actually possess: we are called experts because we possess knowledge in some particular branch. Those

particular branches, therefore, of knowledge, in virtue of which we are sometimes said to be such and such, are themselves qualities, and are not relative. Further, if anything should happen to fall within both the category of quality and that of relation, there would be nothing extraordinary in classing it under both these heads.

Action and affection both admit of contraries and also of variation of degree. Heating is the contrary of cooling, being heated of being cooled, being glad of being vexed. Thus they admit of contraries. They also admit of variation of degree: for it is possible to heat in a greater or less degree; also to be heated in a greater or less degree. Thus action and affection also admit of variation of degree. So much, then, is stated with regard to these categories.

## Time, Place, State

We spoke, moreover, of the category of position when we were dealing with that of relation, and stated that such terms derived their names from those of the corresponding attitudes.

As for the rest, time, place, state, since they are easily intelligible, I say no more about them than was said at the beginning, that in the category of state are included such states as ‘shod’, ‘armed’, in that of place ‘in the Lyceum’ and so on, as was explained before.

The proposed categories have, then, been adequately dealt with.

## Opposition

We must next explain the various senses in which the term ‘opposite’ is used. Things are said to be opposed in four senses: (i) as correlatives to one another, (ii) as contraries to one another, (iii) as privatives to positives, (iv) as affirmatives to negatives.

Let me sketch my meaning in outline. An instance of the use of the word ‘opposite’ with reference to correlatives is afforded by the expressions ‘double’ and ‘half’; with reference to contraries by ‘bad’ and ‘good’. Opposites in the

sense of ‘privatives’ and ‘positives’ are’ blindness’ and ‘sight’; in the sense of affirmatives and negatives, the propositions ‘he sits’, ‘he does not sit’.

### Correlatives

(i) Pairs of opposites which fall under the category of relation are explained by a reference of the one to the other, the reference being indicated by the preposition ‘of’ or by some other preposition. Thus, double is a relative term, for that which is double is explained as the double *of* something. Knowledge, again, is the opposite *of* the thing known, in the same sense; and the thing known also is explained by its relation to its opposite, knowledge. For the thing known is explained as that which is known *by* something, that is, *by* knowledge. Such things, then, as are opposite the one to the other in the sense of being correlatives are explained by a reference of the one to the other.

### Contraries

(ii) Pairs of opposites which are contraries are not in any way interdependent, but are contrary the one to the other. The good is not spoken of as the good of the bad, but as the contrary of the bad, nor is white spoken of as the white of the black, but as the contrary of the black. These two types of opposition are therefore distinct. Those contraries which are such that the subjects in which they are naturally present, or of which they are predicated, must necessarily contain either the one or the other of them, have no intermediate, but those in the case of which no such necessity obtains, always have an intermediate. Thus disease and health are naturally present in the body of an animal, and it is necessary that either the one or the other should be present in the body of an animal. Odd and even, again, are predicated of number, and it is necessary that the one or the other should be present in numbers. Now there is no intermediate between the terms of either of these two pairs. On the other hand, in those contraries with regard to which no such necessity obtains, we find an intermediate. Blackness and whiteness are naturally present in the body, but it is

not necessary that either the one or the other should be present in the body, inasmuch as it is not true to say that everybody must be white or black. Badness and goodness, again, are predicated of man, and of many other things, but it is not necessary that either the one quality or the other should be present in that of which they are predicated: it is not true to say that everything that may be good or bad must be either good or bad. These pairs of contraries have intermediates: the intermediates between white and black are gray, sallow, and all the other colors that come between; the intermediate between good and bad is that which is neither the one nor the other.

Some intermediate qualities have names, such as gray and sallow and all the other colors that come between white and black; in other cases, however, it is not easy to name the intermediate, but we must define it as that which is not either extreme, as in the case of that which is neither good nor bad, neither just nor unjust.

## Privatives To Positives

(iii) 'privatives' and 'positives' have reference to the same subject. Thus, sight and blindness have reference to the eye. It is a universal rule that each of a pair of opposites of this type has reference to that to which the particular 'positive' is natural. We say that that is capable of some particular faculty or possession has suffered privation when the faculty or possession in question is in no way present in that in which, and at the time at which, it should naturally be present. We do not call that toothless which has not teeth, or that blind which has not sight, but rather that which has not teeth or sight at the time when by nature it should. For there are some creatures which from birth are without sight, or without teeth, but these are not called toothless or blind.

To be without some faculty or to possess it is not the same as the corresponding 'privative' or 'positive'. 'Sight' is a 'positive', 'blindness' a 'privative', but 'to possess sight' is not equivalent to 'sight', 'to be blind' is not equivalent to 'blindness'. Blindness is a 'privative', to be blind is to be in a state of privation, but is not a 'privative'.

Moreover, if ‘blindness’ were equivalent to ‘being blind’, both would be predicated of the same subject; but though a man is said to be blind, he is by no means said to be blindness.

To be in a state of ‘possession’ is, it appears, the opposite of being in a state of ‘privation’, just as ‘positives’ and ‘privatives’ themselves are opposite. There is the same type of antithesis in both cases; for just as blindness is opposed to sight, so is being blind opposed to having sight.

That which is affirmed or denied is not itself affirmation or denial. By ‘affirmation’ we mean an affirmative proposition, by ‘denial’ a negative. Now, those facts which form the matter of the affirmation or denial are not propositions; yet these two are said to be opposed in the same sense as the affirmation and denial, for in this case also the type of antithesis is the same. For as the affirmation is opposed to the denial, as in the two propositions ‘he sits’, ‘he does not sit’, so also the fact which constitutes the matter of the proposition in one case is opposed to that in the other, his sitting, that is to say, to his not sitting.

It is evident that ‘positives’ and ‘privatives’ are not opposed each to each in the same sense as relatives. The one is not explained by reference to the other; sight is not sight of blindness, nor is any other preposition used to indicate the relation. Similarly blindness is not said to be blindness of sight, but rather, privation of sight. Relatives, moreover, reciprocate; if blindness, therefore, were a relative, there would be a reciprocity of relation between it and that with which it was correlative. But this is not the case. Sight is not called the sight of blindness.

That those terms which fall under the heads of ‘positives’ and ‘privatives’ are not opposed each to each as contraries, either, is plain from the following facts: Of a pair of contraries such that they have no intermediate, one or the other must needs be present in the subject in which they naturally subsist, or of which they are predicated; for it is those, as we proved, in the case of which this necessity obtains, that have no intermediate. Moreover, we cited health and disease, odd and

even, as instances. But those contraries which have an intermediate are not subject to any such necessity. It is not necessary that every substance, receptive of such qualities, should be either black or white, cold or hot, for something intermediate between these contraries may very well be present in the subject. We proved, moreover, that those contraries have an intermediate in the case of which the said necessity does not obtain. Yet when one of the two contraries is a constitutive property of the subject, as it is a constitutive property of fire to be hot, of snow to be white, it is necessary determinately that one of the two contraries, not one or the other, should be present in the subject; for fire cannot be cold, or snow black. Thus, it is not the case here that one of the two must needs be present in every subject receptive of these qualities, but only in that subject of which the one forms a constitutive property. Moreover, in such cases it is one member of the pair determinately, and not either the one or the other, which must be present.

In the case of ‘positives’ and ‘privatives’, on the other hand, neither of the aforesaid statements holds good. For it is not necessary that a subject receptive of the qualities should always have either the one or the other; that which has not yet advanced to the state when sight is natural is not said either to be blind or to see. Thus ‘positives’ and ‘privatives’ do not belong to that class of contraries which consists of those which have no intermediate. On the other hand, they do not belong either to that class which consists of contraries which have an intermediate. For under certain conditions it is necessary that either the one or the other should form part of the constitution of every appropriate subject. For when a thing has reached the stage when it is by nature capable of sight, it will be said either to see or to be blind, and that in an indeterminate sense, signifying that the capacity may be either present or absent; for it is not necessary either that it should see or that it should be blind, but that it should be either in the one state or in the other. Yet in the case of those contraries which have an intermediate we found that it was never necessary that either the one or the other should be present in every appropriate subject, but only that in certain subjects one of the pair

should be present, and that in a determinate sense. It is, therefore, plain that ‘positives’ and ‘privatives’ are not opposed each to each in either of the senses in which contraries are opposed.

Again, in the case of contraries, it is possible that there should be changes from either into the other, while the subject retains its identity, unless indeed one of the contraries is a constitutive property of that subject, as heat is of fire. For it is possible that that which is healthy should become diseased, that which is white, black, that which is cold, hot, that which is good, bad, that which is bad, good. The bad man, if he is being brought into a better way of life and thought, may make some advance, however slight, and if he should once improve, even ever so little, it is plain that he might change completely, or at any rate make very great progress; for a man becomes more and more easily moved to virtue, however small the improvement was at first. It is, therefore, natural to suppose that he will make yet greater progress than he has made in the past; and as this process goes on, it will change him completely and establish him in the contrary state, provided he is not hindered by lack of time. In the case of ‘positives’ and ‘privatives’, however, change in both directions is impossible. There may be a change from possession to privation, but not from privation to possession. The man who has become blind does not regain his sight; the man who has become bald does not regain his hair; the man who has lost his teeth does not grow his grow a new set.

### Affirmatives To Negatives

(iv) Statements opposed as affirmation and negation belong manifestly to a class which is distinct, for in this case, and in this case only, it is necessary for the one opposite to be true and the other false.

Neither in the case of contraries, nor in the case of correlatives, nor in the case of ‘positives’ and ‘privatives’, is it necessary for one to be true and the other false. Health and disease are contraries: neither of them is true or false. ‘Double’ and ‘half’ are opposed to each other as correlatives: neither of

them is true or false. The case is the same, of course, with regard to ‘positives’ and ‘privatives’ such as ‘sight’ and ‘blindness’. In short, where there is no sort of combination of words, truth and falsity have no place, and all the opposites we have mentioned so far consist of simple words.

At the same time, when the words which enter into opposed statements are contraries, these, more than any other set of opposites, would seem to claim this characteristic. ‘Socrates is ill’ is the contrary of ‘Socrates is well’, but not even of such composite expressions is it true to say that one of the pair must always be true and the other false. For if Socrates exists, one will be true and the other false, but if he does not exist, both will be false; for neither ‘Socrates is ill’ nor ‘Socrates is well’ is true, if Socrates does not exist at all.

In the case of ‘positives’ and ‘privatives’, if the subject does not exist at all, neither proposition is true, but even if the subject exists, it is not always the fact that one is true and the other false. For ‘Socrates has sight’ is the opposite of ‘Socrates is blind’ in the sense of the word ‘opposite’ which applies to possession and privation. Now if Socrates exists, it is not necessary that one should be true and the other false, for when he is not yet able to acquire the power of vision, both are false, as also if Socrates is altogether non-existent.

But in the case of affirmation and negation, whether the subject exists or not, one is always false and the other true. For manifestly, if Socrates exists, one of the two propositions ‘Socrates is ill’, ‘Socrates is not ill’, is true, and the other false. This is likewise the case if he does not exist; for if he does not exist, to say that he is ill is false, to say that he is not ill is true. Thus it is in the case of those opposites only, which are opposite in the sense in which the term is used with reference to affirmation and negation, that the rule holds good, that one of the pair must be true and the other false.

That the contrary of a good is an evil is shown by induction: the contrary of health is disease, of courage, cowardice, and so on. But the contrary of an evil is sometimes a good, sometimes an evil. For defect, which is an evil, has excess for its contrary,

this also being an evil, and the mean. which is a good, is equally the contrary of the one and of the other. It is only in a few cases, however, that we see instances of this: in most, the contrary of an evil is a good.

In the case of contraries, it is not always necessary that if one exists the other should also exist: for if all become healthy there will be health and no disease, and again, if everything turns white, there will be white, but no black. Again, since the fact that Socrates is ill is the contrary of the fact that Socrates is well, and two contrary conditions cannot both obtain in one and the same individual at the same time, both these contraries could not exist at once: for if that Socrates was well was a fact, then that Socrates was ill could not possibly be one.

It is plain that contrary attributes must needs be present in subjects which belong to the same species or genus. Disease and health require as their subject the body of an animal; white and black require a body, without further qualification; justice and injustice require as their subject the human soul.

Moreover, it is necessary that pairs of contraries should in all cases either belong to the same genus or belong to contrary genera or be themselves genera. White and black belong to the same genus, color; justice and injustice, to contrary genera, virtue and vice; while good and evil do not belong to genera, but are themselves actual genera, with terms under them.

## Prior

There are four senses in which one thing can be said to be ‘prior’ to another. Primarily and most properly the term has reference to time: in this sense the word is used to indicate that one thing is older or more ancient than another, for the expressions ‘older’ and ‘more ancient’ imply greater length of time.

Secondly, one thing is said to be ‘prior’ to another when the sequence of their being cannot be reversed. In this sense ‘one’ is ‘prior’ to ‘two’. For if ‘two’ exists, it follows directly that ‘one’ must exist, but if ‘one’ exists, it does not follow

necessarily that ‘two’ exists: thus the sequence subsisting cannot be reversed. It is agreed, then, that when the sequence of two things cannot be reversed, then that one on which the other depends is called ‘prior’ to that other.

In the third place, the term ‘prior’ is used with reference to any order, as in the case of science and of oratory. For in sciences which use demonstration there is that which is prior and that which is posterior in order; in geometry, the elements are prior to the propositions; in reading and writing, the letters of the alphabet are prior to the syllables. Similarly, in the case of speeches, the exordium is prior in order to the narrative.

Besides these senses of the word, there is a fourth. That which is better and more honorable is said to have a natural priority. In common parlance men speak of those whom they honor and love as ‘coming first’ with them. This sense of the word is perhaps the most far-fetched.

Such, then, are the different senses in which the term ‘prior’ is used.

Yet it would seem that besides those mentioned there is yet another. For in those things, the being of each of which implies that of the other, that which is in any way the cause may reasonably be said to be by nature ‘prior’ to the effect. It is plain that there are instances of this. The fact of the being of a man carries with it the truth of the proposition that he is, and the implication is reciprocal: for if a man is, the proposition wherein we allege that he is true, and conversely, if the proposition wherein we allege that he is true, then he is. The true proposition, however, is in no way the cause of the being of the man, but the fact of the man’s being does seem somehow to be the cause of the truth of the proposition, for the truth or falsity of the proposition depends on the fact of the man’s being or not being.

Thus the word ‘prior’ may be used in five senses.

## Simultaneity

The term ‘simultaneous’ is primarily and most appropriately applied to those things the genesis of

the one of which is simultaneous with that of the other; for in such cases neither is prior or posterior to the other. Such things are said to be simultaneous in point of time. Those things, again, are ‘simultaneous’ in point of nature, the being of each of which involves that of the other, while at the same time neither is the cause of the other’s being. This is the case with regard to the double and the half, for these are reciprocally dependent, since, if there is a double, there is also a half, and if there is a half, there is also a double, while at the same time neither is the cause of the being of the other.

Again, those species which are distinguished one from another and opposed one to another within the same genus are said to be ‘simultaneous’ in nature. I mean those species which are distinguished each from each by one and the same method of division. Thus the ‘winged’ species is simultaneous with the ‘terrestrial’ and the ‘water’ species. These are distinguished within the same genus, and are opposed each to each, for the genus ‘animal’ has the ‘winged’, the ‘terrestrial’, and the ‘water’ species, and no one of these is prior or posterior to another; on the contrary, all such things appear to be ‘simultaneous’ in nature. Each of these also, the terrestrial, the winged, and the water species, can be divided again into subspecies. Those species, then, also will be ‘simultaneous’ point of nature, which, belonging to the same genus, are distinguished each from each by one and the same method of differentiation.

But genera are prior to species, for the sequence of their being cannot be reversed. If there is the species ‘water-animal’, there will be the genus ‘animal’, but granted the being of the genus ‘animal’, it does not follow necessarily that there will be the species ‘water-animal’.

Those things, therefore, are said to be ‘simultaneous’ in nature, the being of each of which involves that of the other, while at the same time neither is in any way the cause of the other’s being; those species, also, which are distinguished each from each and opposed within the same genus. Those things, moreover, are ‘simultaneous’ in the unqualified sense of the word which come into

being at the same time.

## Motion

There are six sorts of movement: generation, destruction, increase, diminution, alteration, and change of place.

It is evident in all but one case that all these sorts of movement are distinct each from each. Generation is distinct from destruction, increase and change of place from diminution, and so on. But in the case of alteration it may be argued that the process necessarily implies one or other of the other five sorts of motion. This is not true, for we may say that all affections, or nearly all, produce in us an alteration which is distinct from all other sorts of motion, for that which is affected need not suffer either increase or diminution or any of the other sorts of motion. Thus alteration is a distinct sort of motion; for, if it were not, the thing altered would not only be altered, but would forthwith necessarily suffer increase or diminution or some one of the other sorts of motion in addition; which as a matter of fact is not the case. Similarly that which was undergoing the process of increase or was subject to some other sort of motion would, if alteration were not a distinct form of motion, necessarily be subject to alteration also. But there are some things which undergo increase but yet not alteration. The square, for instance, if a gnomon is applied to it, undergoes increase but not alteration, and so it is with all other figures of this sort. Alteration and increase, therefore, are distinct.

Speaking generally, rest is the contrary of motion. But the different forms of motion have their own contraries in other forms; thus destruction is the contrary of generation, diminution of increase, rest in a place of change of place. As for this last, change in the reverse direction would seem to be most truly its contrary; thus motion upwards is the contrary of motion downwards and vice versa.

In the case of that sort of motion which yet remains, of those that have been enumerated, it is not easy to state what is its contrary. It appears to have no contrary, unless one should define the

contrary here also either as ‘rest in its quality’ or as ‘change in the direction of the contrary quality’, just as we defined the contrary of change of place either as rest in a place or as change in the reverse direction. For a thing is altered when change of quality takes place; therefore either rest in its quality or change in the direction of the contrary may be called the contrary of this qualitative form of motion. In this way becoming white is the contrary of becoming black; there is alteration in the contrary direction, since a change of a qualitative nature takes place.

## Possessives

The term ‘to have’ is used in various senses. In the first place it is used with reference to habit or disposition or any other quality, for we are said to ‘have’ a piece of knowledge or a virtue. Then, again, it has reference to quantity, as, for instance, in the case of a man’s height; for he is said to ‘have’ a height of three or four cubits. It is used, moreover, with regard to apparel, a man being said to ‘have’ a coat or tunic; or in respect of something which we have on a part of ourselves, as a ring on the hand: or in respect of something which is a part of us, as hand or foot. The term refers also to content, as in the case of a vessel and wheat, or of a jar and wine; a jar is said to ‘have’ wine, and a corn-measure wheat. The expression in such cases has reference to content. Or it refers to that which has been acquired; we are said to ‘have’ a house or a field. A man is also said to ‘have’ a wife, and a wife a husband, and this appears to be the most remote meaning of the term, for by the use of it we mean simply that the husband lives with the wife.

Other senses of the word might perhaps be found, but the most ordinary ones have all been enumerated.

-THE END-

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## Conclusion of Categories

What principle does Aristotle use to construct the Categories? I would be hard pressed to find one.

He does not use ***form*** and ***matter***, for in many categories we find ***forms*** and ***matter*** side by side.

I would think that there would be some advantage to making a primary division of the bodies senses between those that abstract ***matter*** and those that abstract ***form***. Then locate a primitive vocabulary specific to each sense. etc.

Be this as it may, one will find a striking similarity in Plato's construction of Parmenides, which uses these categories to see if anything can be predicated of the basic elements–***matter*** and ***form***.

## Leaving the Cave; Human Enlightenment

Once we learn that every thing has a class and things are divided by definition, how far can we divide or does division end at some place beyond which no further predication can take place?

> Let me give you, then, a dream in return for a dream:–I thought that I too had a dream, and I heard in my dream that the primeval letters or ***elements out of which you and I and all other things are compounded, have no reason or explanation; you can only name them***, but ***no predicate can be either affirmed or denied of them***, for in the one case existence, in the other non-existence is already implied, neither of which must be added, if you mean to speak of this or that thing by itself alone. ~
>
> ~ But none of these primeval elements can be defined; they can only be named, for they have nothing but a name, and the things which are compounded of them, as they are complex, are expressed by a combination of names, for the combination of names is the essence of a definition. ***Theaetetus*** by Plato

Nothing can be predicated of either ***matter*** or ***form***–this is what Plato will demonstrate in the dialog called ***Parmenides***. ***Parmenides*** amounts to Plato's proof of the fundamentality of a definition. Neither ***form*** nor ***matter*** can be defined, as they are not ***things***. Predication starts and stops at perception–direct experience.

If something can be defined, it can take a predicate. If something cannot be defined, it cannot take a predicate. If we remember some of Aristotle, we will find ***form***, ***matter*** and their combination–the sequence was learned from Plato. Here Plato will go through the ***Categories*** using assertion and denial, singly and compounded of ***form*** and ***matter*** and their combination making the demonstration divided into six sections. Plato will be quite methodical, choosing his words and phrases carefully.

1) Assertion of ***form*** (one will mean ***form***).

2) Assertion of ***matter*** (one will mean ***matter***).

3) Assertion of both ***form*** and ***matter***.

4) Denial of ***form***.

5) Denial of ***matter***.

6) Denial of ***form*** and ***matter*** (a very short section).

Words that denote ***form*** or ***matter*** may be described, but not defined–the meaning of these names must be learned by experience. Plato, in this demonstration, proves that reasoning begins at direct experience–perception. His dialogs demonstrating that if one removes all the crafts as the sole source of wisdom would leave one without any source of wisdom were more than a hint. A society whose individuals have forgotten crafts degenerate into meaningless unreasonable mass of human flesh.

# Plato on Definition; Parmenides

by Plato 370 BC translated by Benjamin Jowett

This section of this book is set up so that the reader can write many notes in the margins.

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Persons Of The Dialogue: Cephalus; Adeimantus; Glaucon; Antiphon; Pythodorus; Socrates; Zeno; Parmenides; Aristoteles.

Cephalus rehearses a dialogue which is supposed to have been narrated in his presence by Antiphon, the half–brother of Adeimantus and Glaucon, to certain Clazomenians.

We had come from our home at Clazomenae to Athens, and met Adeimantus and Glaucon in the Agora. Welcome, Cephalus, said Adeimantus, taking me by the hand; is there anything which we can do for you in Athens?

Yes; that is why I am here; I wish to ask a favor of you.

What may that be? he said.

I want you to tell me the name of your half brother, which I have forgotten; he was a mere child when I last came hither from Clazomenae, but that was a long time ago; his father's name, if I remember rightly, was Pyrilampes?

Yes, he said, and the name of our brother, Antiphon; but why do you ask?

Let me introduce some countrymen of mine, I said; they are lovers of philosophy, and have heard that Antiphon was intimate with a certain Pythodorus, a friend of Zeno, and remembers a conversation which took place between Socrates, Zeno, and Parmenides many years ago, Pythodorus having often recited it to him.

Quite true.

And could we hear it? I asked.

Nothing easier, he replied; when he was a youth he made a careful study of the piece; at present his

thoughts run in another direction; like his grandfather Antiphon he is devoted to horses. But, if that is what you want, let us go and look for him; he dwells at Melita, which is quite near, and he has only just left us to go home.

Accordingly we went to look for him; he was at home, and in the act of giving a bridle to a smith to be fitted. When he had done with the smith, his brothers told him the purpose of our visit; and he saluted me as an acquaintance whom he remembered from my former visit, and we asked him to repeat the dialogue. At first he was not very willing, and complained of the trouble, but at length he consented. He told us that Pythodorus had described to him the appearance of Parmenides and Zeno; they came to Athens, as he said, at the great Panathenaea; the former was, at the time of his visit, about 65 years old, very white with age, but well favored. Zeno was nearly 40 years of age, tall and fair to look upon; in the days of his youth he was reported to have been beloved by Parmenides. He said that they lodged with Pythodorus in the Ceramicus, outside the wall, whither Socrates, then a very young man, came to see them, and many others with him; they wanted to hear the writings of Zeno, which had been brought to Athens for the first time on the occasion of their visit. These Zeno himself read to them in the absence of Parmenides, and had very nearly finished when Pythodorus entered, and with him Parmenides and Aristoteles who was afterwards one of the Thirty, and heard the little that remained of the dialogue. Pythodorus had heard Zeno repeat them before.

When the recitation was completed, Socrates requested that the first thesis of the first argument might be read over again, and this having been done, he said: What is your meaning, Zeno? Do you maintain that if being is many, it must be both like and unlike, and that this is impossible, for neither can the like be unlike, nor the unlike like–is that your position?

Just so, said Zeno.

And if the unlike cannot be like, or the like unlike, then according to you, being could not be many; for this would involve an impossibility. In all

that you say have you any other purpose except to disprove the being of the many? and is not each division of your treatise intended to furnish a separate proof of this, there being in all as many proofs of the not-being of the many as you have composed arguments? Is that your meaning, or have I misunderstood you?

No, said Zeno; you have correctly understood my general purpose.

I see, Parmenides, said Socrates, that Zeno would like to be not only one with you in friendship but your second self in his writings too; he puts what you say in another way, and would fain make believe that he is telling us something which is new. For you, in your poems, say The All is one, and of this you adduce excellent proofs; and he on the other hand says there is not many; and on behalf of this he offers overwhelming evidence. You affirm unity, he denies plurality. And so you deceive the world into believing that you are saying different things when really you are saying much the same. This is a strain of art beyond the reach of most of us.

Yes, Socrates, said Zeno. But although you are as keen as a Spartan hound in pursuing the track, you do not fully apprehend the true motive of the composition, which is not really such an artificial work as you imagine; for what you speak of was an accident; there was no pretence of a great purpose; nor any serious intention of deceiving the world. The truth is, that these writings of mine were meant to protect the arguments of Parmenides against those who make fun of him and seek to show the many ridiculous and contradictory results which they suppose to follow from the affirmation of the one. My answer is addressed to the partisans of the many, whose attack I return with interest by retorting upon them that their hypothesis of the being of many, if carried out, appears to be still more ridiculous than the hypothesis of the being of one. Zeal for my master led me to write the book in the days of my youth, but some one stole the copy; and therefore I had no choice whether it should be published or not; the motive, however, of writing, was not the ambition of an elder man, but the

pugnacity of a young one. This you do not seem to see, Socrates; though in other respects, as I was saying, your notion is a very just one.

I understand, said Socrates, and quite accept your account. But tell me, Zeno, do you not further think that there is an idea of likeness in itself, and another idea of unlikeness, which is the opposite of likeness, and that in these two, you and I and all other things to which we apply the term many, participate–things which participate in likeness become in that degree and manner like; and so far as they participate in unlikeness become in that degree unlike, or both like and unlike in the degree in which they participate in both? And may not all things partake of both opposites, and be both like and unlike, by reason of this participation?–Where is the wonder? Now if a person could prove the absolute like to become unlike, or the absolute unlike to become like, that, in my opinion, would indeed be a wonder; but there is nothing extraordinary, Zeno, in showing that the things which only partake of likeness and unlikeness experience both. Nor, again, if a person were to show that all is one by partaking of one, and at the same time many by partaking of many, would that be very astonishing. But if he were to show me that the absolute one was many, or the absolute many one, I should be truly amazed. And so of all the rest: I should be surprised to hear that the natures or ideas themselves had these opposite qualities; but not if a person wanted to prove of me that I was many and also one. When he wanted to show that I was many he would say that I have a right and a left side, and a front and a back, and an upper and a lower half, for I cannot deny that I partake of multitude; when, on the other hand, he wants to prove that I am one, he will say, that we who are here assembled are seven, and that I am one and partake of the one. In both instances he proves his case. So again, if a person shows that such things as wood, stones, and the like, being many are also one, we admit that he shows the coexistence of the one and many, but he does not show that the many are one or the one many; he is uttering not a paradox but a truism. If however, as I just now suggested, some one were to abstract simple notions of like,

unlike, one, many, rest, motion, and similar ideas, and then to show that these admit of admixture and separation in themselves, I should be very much astonished. This part of the argument appears to be treated by you, Zeno, in a very spirited manner; but, as I was saying, I should be far more amazed if any one found in the ideas themselves which are apprehended by reason, the same puzzle and entanglement which you have shown to exist in visible objects.

While Socrates was speaking, Pythodorus thought that Parmenides and Zeno were not altogether pleased at the successive steps of the argument; but still they gave the closest attention and often looked at one another, and smiled as if in admiration of him. When he had finished, Parmenides expressed their feelings in the following words:–

Socrates, he said, I admire the bent of your mind towards philosophy; tell me now, was this your own distinction between ideas in themselves and the things which partake of them? and do you think that there is an idea of likeness apart from the likeness which we possess, and of the one and many, and of the other things which Zeno mentioned?

I think that there are such ideas, said Socrates.

Parmenides proceeded: And would you also make absolute ideas of the just and the beautiful and the good, and of all that class?

Yes, he said, I should.

And would you make an idea of man apart from us and from all other human creatures, or of fire and water?

I am often undecided, Parmenides, as to whether I ought to include them or not.

And would you feel equally undecided, Socrates, about things of which the mention may provoke a smile?–I mean such things as hair, mud, dirt, or anything else which is vile and paltry; would you suppose that each of these has an idea distinct from the actual objects with which we come into contact, or not?

Certainly not, said Socrates; visible things like

these are such as they appear to us, and I am afraid that there would be an absurdity in assuming any idea of them, although I sometimes get disturbed, and begin to think that there is nothing without an idea; but then again, when I have taken up this position, I run away, because I am afraid that I may fall into a bottomless pit of nonsense, and perish; and so I return to the ideas of which I was just now speaking, and occupy myself with them.

Yes, Socrates, said Parmenides; that is because you are still young; the time will come, if I am not mistaken, when philosophy will have a firmer grasp of you, and then you will not despise even the meanest things; at your age, you are too much disposed to regard opinions of men. But I should like to know whether you mean that there are certain ideas of which all other things partake, and from which they derive their names; that similars, for example, become similar, because they partake of similarity; and great things become great, because they partake of greatness; and that just and beautiful things become just and beautiful, because they partake of justice and beauty.

Yes, certainly, said Socrates that is my meaning.

Then each individual partakes either of the whole of the idea or else of a part of the idea? Can there be any other mode of participation?

There cannot be, he said.

Then do you think that the whole idea is one, and yet, being one, is in each one of the many?

Why not, Parmenides? said Socrates.

Because one and the same thing will exist as a whole at the same time in many separate individuals, and will therefore be in a state of separation from itself.

Nay, but the idea may be like the day which is one and the same in many places at once, and yet continuous with itself; in this way each idea may be one; and the same in all at the same time.

I like your way, Socrates, of making one in many places at once. You mean to say, that if I were to spread out a sail and cover a number of men, there would be one whole including many–is not that

your meaning?

I think so.

And would you say that the whole sail includes each man, or a part of it only, and different parts different men?

The latter.

Then, Socrates, the ideas themselves will be divisible, and things which participate in them will have a part of them only and not the whole idea existing in each of them?

That seems to follow.

Then would you like to say, Socrates, that the one idea is really divisible and yet remains one?

Certainly not, he said.

Suppose that you divide absolute greatness, and that of the many great things, each one is great in virtue of a portion of greatness less than absolute greatness–is that conceivable?

No.

Or will each equal thing, if possessing some small portion of equality less than absolute equality, be equal to some other thing by virtue of that portion only?

Impossible.

Or suppose one of us to have a portion of smallness; this is but a part of the small, and therefore the absolutely small is greater; if the absolutely small be greater, that to which the part of the small is added will be smaller and not greater than before.

How absurd!

Then in what way, Socrates, will all things participate in the ideas, if they are unable to participate in them either as parts or wholes?

Indeed, he said, you have asked a question which is not easily answered.

Well, said Parmenides, and what do you say of another question?

What question?

I imagine that the way in which you are led to assume one idea of each kind is as follows:–You see a number of great objects, and when you look at them there seems to you to be one and the same idea (or nature) in them all; hence you conceive of greatness as one.

Very true, said Socrates.

And if you go on and allow your mind in like manner to embrace in one view the idea of greatness and of great things which are not the idea, and to compare them, will not another greatness arise, which will appear to be the source of all these?

It would seem so.

Then another idea of greatness now comes into view over and above absolute greatness, and the individuals which partake of it; and then another, over and above all these, by virtue of which they will all be great, and so each idea instead of being one will be infinitely multiplied.

But may not the ideas, asked Socrates, be thoughts only, and have no proper existence except in our minds, Parmenides? For in that case each idea may still be one, and not experience this infinite multiplication.

And can there be individual thoughts which are thoughts of nothing?

Impossible, he said.

The thought must be of something?

Yes.

Of something which is or which is not?

Of something which is.

Must it not be of a single something, which the thought recognizes as attaching to all, being a single form or nature?

Yes.

And will not the something, which is apprehended as one and the same in all, be an idea?

From that, again, there is no escape.

Then, said Parmenides, if you say that

everything else participates in the ideas, must you not say either that everything is made up of thoughts, and that all things think; or that they are thoughts but have no thought?

The latter view, Parmenides, is no more rational than the previous one. In my opinion, the ideas are, as it were, patterns fixed in nature, and other things are like them, and resemblances of them–what is meant by the participation of other things in the ideas, is really assimilation to them.

But if, said he, the individual is like the idea, must not the idea also be like the individual, in so far as the individual is a resemblance of the idea? That which is like, cannot be conceived of as other than the like of like.

Impossible.

And when two things are alike, must they not partake of the same idea?

They must.

And will not that of which the two partake, and which makes them alike, be the idea itself?

Certainly.

Then the idea cannot be like the individual, or the individual like the idea; for if they are alike, some further idea of likeness will always be coming to light, and if that be like anything else, another; and new ideas will be always arising, if the idea resembles that which partakes of it?

Quite true.

The theory, then that other things participate in the ideas by resemblance, has to be given up, and some other mode of participation devised?

It would seem so.

Do you see then, Socrates, how great is the difficulty of affirming the ideas to be absolute?

Yes, indeed.

And, further, let me say that as yet you only understand a small part of the difficulty which is involved if you make of each thing a single idea, parting it off from other things.

What difficulty? he said.

There are many, but the greatest of all is this:–If an opponent argues that these ideas, being such as we say they ought to be, must remain unknown, no one can prove to him that he is wrong, unless he who denies their existence be a man of great ability and knowledge, and is willing to follow a long and laborious demonstration; he will remain unconvinced, and still insist that they cannot be known.

What do you mean, Parmenides? said Socrates.

In the first place, I think, Socrates, that you, or any one who maintains the existence of absolute essences, will admit that they cannot exist in us.

No, said Socrates; for then they would be no longer absolute.

True, he said; and therefore when ideas are what they are in relation to one another, their essence is determined by a relation among themselves, and has nothing to do with the resemblances, or whatever they are to be termed, which are in our sphere, and from which we receive this or that name when we partake of them. And the things which are within our sphere and have the same names with them, are likewise only relative to one another, and not to the ideas which have the same names with them, but belong to themselves and not to them.

What do you mean? said Socrates.

I may illustrate my meaning in this way, said Parmenides:–A master has a slave; now there is nothing absolute in the relation between them, which is simply a relation of one man to another. But there is also an idea of mastership in the abstract, which is relative to the idea of slavery in the abstract. These natures have nothing to do with us, nor we with them; they are concerned with themselves only, and we with ourselves. Do you see my meaning?

Yes, said Socrates, I quite see your meaning.

And will not knowledge–I mean absolute knowledge–answer to absolute truth?

Certainly.

And each kind of absolute knowledge will answer to each kind of absolute being?

Yes.

But the knowledge which we have, will answer to the truth which we have; and again, each kind of knowledge which we have, will be a knowledge of each kind of being which we have?

Certainly.

But the ideas themselves, as you admit, we have not, and cannot have?

No, we cannot.

And the absolute natures or kinds are known severally by the absolute idea of knowledge?

Yes.

And we have not got the idea of knowledge?

No.

Then none of the ideas are known to us, because we have no share in absolute knowledge?

I suppose not.

Then the nature of the beautiful in itself, and of the good in itself, and all other ideas which we suppose to exist absolutely, are unknown to us?

It would seem so.

I think that there is a stranger consequence still.

What is it?

Would you, or would you not say, that absolute knowledge, if there is such a thing, must be a far more exact knowledge than our knowledge; and the same of beauty and of the rest?

Yes.

And if there be such a thing as participation in absolute knowledge, no one is more likely than God to have this most exact knowledge?

Certainly.

But then, will God, having absolute knowledge, have a knowledge of human things?

Why not?

Because, Socrates, said Parmenides, we have admitted that the ideas are not valid in relation to human things; nor human things in relation to them; the relations of either are limited to their respective spheres.

Yes, that has been admitted.

And if God has this perfect authority, and perfect knowledge, his authority cannot rule us, nor his knowledge know us, or any human thing; just as our authority does not extend to the gods, nor our knowledge know anything which is divine, so by parity of reason they, being gods, are not our masters, neither do they know the things of men.

Yet, surely, said Socrates, to deprive God of knowledge is monstrous.

These, Socrates, said Parmenides, are a few, and only a few of the difficulties in which we are involved if ideas really are and we determine each one of them to be an absolute unity. He who hears what may be said against them will deny the very existence of them–and even if they do exist, he will say that they must of necessity be unknown to man; and he will seem to have reason on his side, and as we were remarking just now, will be very difficult to convince; a man must be gifted with very considerable ability before he can learn that everything has a class and an absolute essence; and still more remarkable will he be who discovers all these things for himself, and having thoroughly investigated them is able to teach them to others.

I agree with you, Parmenides, said Socrates; and what you say is very much to my mind.

And yet, Socrates, said Parmenides, if a man, fixing his attention on these and the like difficulties, does away with ideas of things and will not admit that every individual thing has its own determinate idea which is always one and the same, he will have nothing on which his mind can rest; and so he will utterly destroy the power of reasoning, as you seem to me to have particularly noted.

Very true, he said.

But, then, what is to become of philosophy? Whither shall we turn, if the ideas are unknown?

I certainly do not see my way at present.

Yes, said Parmenides; and I think that this arises, Socrates, out of your attempting to define the beautiful, the just, the good, and the ideas generally, without sufficient previous training. I noticed your deficiency, when I heard you talking here with your friend Aristoteles, the day before yesterday. The impulse that carries you towards philosophy is assuredly noble and divine; but there is an art which is called by the vulgar idle talking, and which is of imagined to be useless; in that you must train and exercise yourself, now that you are young, or truth will elude your grasp.

And what is the nature of this exercise, Parmenides, which you would recommend?

That which you heard Zeno practicing; at the same time, I give you credit for saying to him that you did not care to examine the perplexity in reference to visible things, or to consider the question that way; but only in reference to objects of thought, and to what may be called ideas.

Why, yes, he said, there appears to me to be no difficulty in showing by this method that visible things are like and unlike and may experience anything.

Quite true, said Parmenides; but I think that you should go a step further, and consider not only the consequences which flow from a given hypothesis, but also the consequences which flow from denying the hypothesis; and that will be still better training for you.

What do you mean? he said.

I mean, for example, that in the case of this very hypothesis of Zeno's about the many, you should inquire not only what will be the consequences to the many in relation to themselves and to the one, and to the one in relation to itself and the many, on the hypothesis of the being of the many, but also what will be the consequences to the one and the many in their relation to themselves and to each other, on the opposite hypothesis. Or, again, if likeness is or is not, what will be the consequences in either of these cases to the subjects of the

hypothesis, and to other things, in relation both to themselves and to one another, and so of unlikeness; and the same holds good of motion and rest, of generation and destruction, and even of being and not-being. In a word, when you suppose anything to be or not to be, or to be in any way affected, you must look at the consequences in relation to the thing itself, and to any other things which you choose–to each of them singly, to more than one, and to all; and so of other things, you must look at them in relation to themselves and to anything else which you suppose either to be or not to be, if you would train yourself perfectly and see the real truth.

That, Parmenides, is a tremendous business of which you speak, and I do not quite understand you; will you take some hypothesis and go through the steps?–then I shall apprehend you better.

That, Socrates, is a serious task to impose on a man of my years.

Then will you, Zeno? said Socrates.

Zeno answered with a smile:–Let us make our petition to Parmenides himself, who is quite right in saying that you are hardly aware of the extent of the task which you are imposing on him; and if there were more of us I should not ask him, for these are not subjects which any one, especially at his age, can well speak of before a large audience; most people are not aware that this round-about progress through all things is the only way in which the mind can attain truth and wisdom. And therefore, Parmenides, I join in the request of Socrates, that I may hear the process again which I have not heard for a long time.

When Zeno had thus spoken, Pythodorus, according to Antiphon's report of him, said, that he himself and Aristoteles and the whole company entreated Parmenides to give an example of the process. I cannot refuse, said Parmenides; and yet I feel rather like Ibycus, who, when in his old age, against his will, he fell in love, compared himself to an old racehorse, who was about to run in a chariot race, shaking with fear at the course he knew so well–this was his simile of himself. And I also

experience a trembling when I remember through what an ocean of words I have to wade at my time of life. But I must indulge you, as Zeno says that I ought, and we are alone. Where shall I begin? And what shall be our first hypothesis, if I am to attempt this laborious pastime? Shall I begin with myself, and take my own hypothesis the one? and consider the consequences which follow on the supposition either of the being or of the not being of one?

By all means, said Zeno.

And who will answer me? he said. Shall I propose the youngest? He will not make difficulties and will be the most likely to say what he thinks; and his answers will give me time to breathe.

I am the one whom you mean, Parmenides, said Aristoteles; for I am the youngest and at your service. Ask, and I will answer [16].

Parmenides proceeded: If one is, he said, the one cannot be many?

Impossible.

Then the one cannot have parts, and cannot be a whole?

Why not?

Because every part is part of a whole; is it not?

Yes.

And what is a whole? would not that of which no part is wanting be a whole?

Certainly.

Then, in either case, the one would be made up of parts; both as being a whole, and also as having parts?

To be sure.

And in either case, the one would be many, and not one?

True.

But, surely, it ought to be one and not many?

It ought.

[16] One is form. (1)

Then, if the one is to remain one, it will not be a whole, and will not have parts?

No.

But if it has no parts, it will have neither beginning, middle, nor end; for these would of course be parts of it.

Right.

But then, again, a beginning and an end are the limits of everything?

Certainly.

Then the one, having neither beginning nor end, is unlimited?

Yes, unlimited.

And therefore formless; for it cannot partake either of round or straight.

But why?

Why, because the round is that of which all the extreme points are equidistant from the center?

Yes.

And the straight is that of which the center intercepts the view of the extremes?

True.

Then the one would have parts and would be many, if it partook either of a straight or of a circular form?

Assuredly.

But having no parts, it will be neither straight nor round?

Right.

And, being of such a nature, it cannot be in any place, for it cannot be either in another or in itself.

How so?

Because if it were in another, it would be encircled by that in which it was, and would touch it at many places and with many parts; but that which is one and indivisible, and does not partake of a circular nature, cannot be touched all round in many places.

Certainly not.

But if, on the other hand, one were in itself, it would also be contained by nothing else but itself; that is to say, if it were really in itself; for nothing can be in anything which does not contain it.

Impossible.

But then, that which contains must be other than that which is contained? for the same whole cannot do and suffer both at once; and if so, one will be no longer one, but two?

True.

Then one cannot be anywhere, either in itself or in another?

No.

Further consider, whether that which is of such a nature can have either rest or motion.

Why not?

Why, because the one, if it were moved, would be either moved in place or changed in nature; for these are the only kinds of motion.

Yes.

And the one, when it changes and ceases to be itself, cannot be any longer one.

It cannot.

It cannot therefore experience the sort of motion which is change of nature?

Clearly not.

Then can the motion of the one be in place?

Perhaps.

But if the one moved in place, must it not either move round and round in the same place, or from one place to another?

It must.

And that which moves in a circle must rest upon a center; and that which goes round upon a center must have parts which are different from the center; but that which has no center and no parts cannot possibly be carried round upon a center?

Impossible.

But perhaps the motion of the one consists in change of place?

Perhaps so, if it moves at all.

And have we not already shown that it cannot be in anything?

Yes.

Then its coming into being in anything is still more impossible; is it not?

I do not see why.

Why, because anything which comes into being in anything, can neither as yet be in that other thing while still coming into being, nor be altogether out of it, if already coming into being in it.

Certainly not.

And therefore whatever comes into being in another must have parts, and then one part may be in, and another part out of that other; but that which has no parts can never be at one and the same time neither wholly within nor wholly without anything.

True.

And is there not a still greater impossibility in that which has no parts, and is not a whole, coming into being anywhere, since it cannot come into being either as a part or as a whole?

Clearly.

Then it does not change place by revolving in the same spot, not by going somewhere and coming into being in something; nor again, by change in itself?

Very true.

Then in respect of any kind of motion the one is immoveable?

Immoveable.

But neither can the one be in anything, as we affirm.

Yes, we said so.

Then it is never in the same?

Why not?

Because if it were in the same it would be in something.

Certainly.

And we said that it could not be in itself, and could not be in other?

True.

Then one is never in the same place?

It would seem not.

But that which is never in the same place is never quiet or at rest?

Never.

One then, as would seem, is neither rest nor in motion?

It certainly appears so.

Neither will it be the same with itself or other; nor again, other than itself or other.

How is that?

If other than itself it would be other than one, and would not be one.

True.

And if the same with other, it would be that other, and not itself; so that upon this supposition too, it would not have the nature of one, but would be other than one?

It would.

Then it will not be the same with other, or other than itself?

It will not.

Neither will it be other than other, while it remains one; for not one, but only other, can be other than other, and nothing else.

True.

Then not by virtue of being one will it be other?

Certainly not.

But if not by virtue of being one, not by virtue of

itself; and if not by virtue of itself, not itself, and itself not being other at all, will not be other than anything?

Right.

Neither will one be the same with itself.

How not?

Surely the nature of the one is not the nature of the same.

Why not?

It is not when anything becomes the same with anything that it becomes one.

What of that?

Anything which becomes the same with the many, necessarily becomes many and not one.

True.

But, if there were no difference between the one and the same, when a thing became the same, it would always become one; and when it became one, the same?

Certainly.

And, therefore, if one be the same with itself, it is not one with itself, and will therefore be one and also not one.

Surely that is impossible.

And therefore the one can neither be other than other, nor the same with itself.

Impossible.

And thus the one can neither be the same, nor other, either in relation to itself or other?

No.

Neither will the one be like anything or unlike itself or other.

Why not?

Because likeness is sameness of affections.

Yes.

And sameness has been shown to be of a nature distinct from oneness?

That has been shown.

But if the one had any other affection than that of being one, it would be affected in such a way as to be more than one; which is impossible.

True.

Then the one can never be so affected as to be the same either with another or with itself?

Clearly not.

Then it cannot be like another, or like itself?

No.

Nor can it be affected so as to be other, for then it would be affected in such a way as to be more than one.

It would.

That which is affected otherwise than itself or another, will be unlike itself or another, for sameness of affections is likeness.

True.

But the one, as appears, never being affected otherwise, is never unlike itself or other?

Never.

Then the one will never be either like or unlike itself or other?

Plainly not.

Again, being of this nature, it can neither be equal nor unequal either to itself or to other.

How is that?

Why, because the one if equal must be of the same measures as that to which it is equal.

True.

And if greater or less than things which are commensurable with it, the one will have more measures than that which is less, and fewer than that which is greater?

Yes.

And so of things which are not commensurate with it, the one will have greater measures than that

which is less and smaller than that which is greater.

Certainly.

But how can that which does not partake of sameness, have either the same measures or have anything else the same?

Impossible.

And not having the same measures, the one cannot be equal either with itself or with another?

It appears so.

But again, whether it have fewer or more measures, it will have as many parts as it has measures; and thus again the one will be no longer one but will have as many parts as measures.

Right.

And if it were of one measure, it would be equal to that measure; yet it has been shown to be incapable of equality.

It has.

Then it will neither partake of one measure, nor of many, nor of few, nor of the same at all, nor be equal to itself or another; nor be greater or less than itself, or other?

Certainly.

Well, and do we suppose that one can be older, or younger than anything, or of the same age with it?

Why not?

Why, because that which is of the same age with itself or other, must partake of equality or likeness of time; and we said that the one did not partake either of equality or of likeness?

We did say so.

And we also said, that it did not partake of inequality or unlikeness.

Very true.

How then can one, being of this nature, be either older or younger than anything, or have the same age with it?

In no way.

Then one cannot be older or younger, or of the same age, either with itself or with another?

Clearly not.

Then the one, being of this nature, cannot be in time at all; for must not that which is in time, be always growing older than itself?

Certainly.

And that which is older, must always be older than something which is younger?

True.

Then, that which becomes older than itself, also becomes at the same time younger than itself, if it is to have something to become older than.

What do you mean?

I mean this:–A thing does not need to become different from another thing which is already different; it is different, and if its different has become, it has become different; if its difference will be, it will be different; but of that which is becoming different, there cannot have been, or be about to be, or yet be, a difference–the only difference possible is one which is becoming.

That is inevitable.

But, surely, the elder is a difference relative to the younger, and to nothing else.

True.

Then that which becomes older than itself must also, at the same time, become younger than itself?

Yes.

But again, it is true that it cannot become for a longer or for a shorter time than itself, but it must become, and be, and have become, and be about to be, for the same time with itself?

That again is inevitable.

Then things which are in time, and partake of time, must in every case, I suppose, be of the same age with themselves; and must also become at once older and younger than themselves?

Yes.

But the one did not partake of those affections?

Not at all.

Then it does not partake of time, and is not in any time?

So the argument shows.

Well, but do not the expressions “was,” and “has become,” and “was becoming,” signify a participation of past time?

Certainly.

And do not “will be,” “will become,” “will have become,” signify a participation of future time?

Yes.

And “is,” or “becomes,” signifies a participation of present time?

Certainly.

And if the one is absolutely without participation in time, it never had become, or was becoming, or was at any time, or is now become or is becoming, or is, or will become, or will have become, or will be, hereafter.

Most true.

But are there any modes of partaking of being other than these?

There are none.

Then the one cannot possibly partake of being?

That is the inference.

Then the one is not at all?

Clearly not.

Then the one does not exist in such way as to be one; for if it were and partook of being, it would already be; but if the argument is to be trusted, the one neither is nor is one?

True.

But that which is not admits of no attribute or relation?

Of course not.

Then there is no name, nor expression, nor perception, nor opinion, nor knowledge of it?

Clearly not.

Then it is neither named, nor expressed, nor opined, nor known, nor does anything that is perceive it.

So we must infer.

But can all this be true about the one?

I think not.

Suppose, now, that we return once more to the original hypothesis; let us see whether, on a further review, any new aspect of the question appears.

I shall be very happy to do so.

We say that we have to work out together all the consequences, whatever they may be, which follow, if the one is [17]?

Yes.

Then we will begin at the beginning:–If one is, can one be, and not partake of being?

Impossible.

Then the one will have being, but its being will not be the same with the one; for if the same, it would not be the being of the one; nor would the one have participated in being, for the proposition that one is would have been identical with the proposition that one is one; but our hypothesis is not if one is one, what will follow, but if one is:–am I not right?

Quite right.

We mean to say, that being has not the same significance as one?

Of course.

And when we put them together shortly, and say "One is," that is equivalent to saying, "partakes of being"?

Quite true.

[17] One is matter. (2)

Once more then let us ask, if one is what will follow. Does not this hypothesis necessarily imply that one is of such a nature as to have parts?

How so?

In this way:–If being is predicated of the one, if the one is, and one of being, if being is one; and if being and one are not the same; and since the one, which we have assumed, is, must not the whole, if it is one, itself be, and have for its parts, one and being?

Certainly.

And is each of these parts–one and being to be simply called a part, or must the word "part" be relative to the word "whole"?

The latter.

Then that which is one is both a whole and has a part?

Certainly.

Again, of the parts of the one, if it is–I mean being and one–does either fail to imply the other? is the one wanting to being, or being to the one?

Impossible.

Thus, each of the parts also has in turn both one and being, and is at the least made up of two parts; and the same principle goes on for ever, and every part whatever has always these two parts; for being always involves one, and one being; so that one is always disappearing, and becoming two.

Certainly.

And so the one, if it is, must be infinite in multiplicity?

Clearly.

Let us take another direction.

What direction?

We say that the one partakes of being and therefore it is?

Yes.

And in this way, the one, if it has being, has turned out to be many?

True.

But now, let us abstract the one which, as we say, partakes of being, and try to imagine it apart from that of which, as we say, it partakes–will this abstract one be one only or many?

One, I think.

Let us see:–Must not the being of one be other than one? for the one is not being, but, considered as one, only partook of being?

Certainly.

If being and the one be two different things, it is not because the one is one that it is other than being; nor because being is being that it is other than the one; but they differ from one another in virtue of other-ness and difference.

Certainly.

So that the other is not the same either with the one or with being?

Certainly not.

And therefore whether we take being and the other, or being and the one, or the one and the other, in every such case we take two things, which may be rightly called both.

How so.

In this way–you may speak of being?

Yes.

And also of one?

Yes.

Then now we have spoken of either of them?

Yes.

Well, and when I speak of being and one, I speak of them both?

Certainly.

And if I speak of being and the other, or of the one and the other–in any such case do I not speak of both?

Yes.

And must not that which is correctly called both, be also two?

Undoubtedly.

And of two things how can either by any possibility not be one?

It cannot.

Then, if the individuals of the pair are together two, they must be severally one?

Clearly.

And if each of them is one, then by the addition of any one to any pair, the whole becomes three?

Yes.

And three are odd, and two are even?

Of course.

And if there are two there must also be twice, and if there are three there must be thrice; that is, if twice one makes two, and thrice one three?

Certainly.

There are two, and twice, and therefore there must be twice two; and there are three, and there is thrice, and therefore there must be thrice three?

Of course.

If there are three and twice, there is twice three; and if there are two and thrice, there is thrice two?

Undoubtedly.

Here, then, we have even taken even times, and odd taken odd times, and even taken odd times, and odd taken even times.

True.

And if this is so, does any number remain which has no necessity to be?

None whatever.

Then if one is, number must also be?

It must.

But if there is number, there must also be many, and infinite multiplicity of being; for number is infinite in multiplicity, and partakes also of being:

am I not right?

Certainly.

And if all number participates in being, every part of number will also participate?

Yes.

Then being is distributed over the whole multitude of things, and nothing that is, however small or however great, is devoid of it? And, indeed, the very supposition of this is absurd, for how can that which is, be devoid of being?

In no way.

And it is divided into the greatest and into the smallest, and into being of all sizes, and is broken up more than all things; the divisions of it have no limit.

True.

Then it has the greatest number of parts?

Yes, the greatest number.

Is there any of these which is a part of being, and yet no part?

Impossible.

But if it is at all and so long as it is, it must be one, and cannot be none?

Certainly.

Then the one attaches to every single part of being, and does not fail in any part, whether great or small, or whatever may be the size of it?

True.

But reflect:–Can one in its entirety, be in many places at the same time?

No; I see the impossibility of that.

And if not in its entirety, then it is divided; for it cannot be present with all the parts of being, unless divided.

True.

And that which has parts will be as many as the parts are?

Certainly.

Then we were wrong in saying just now, that being was distributed into the greatest number of parts. For it is not distributed into parts more than the one, into parts equal to the one; the one is never wanting to being, or being to the one, but being two they are co-equal and coextensive.

Certainly that is true.

The one itself, then, having been broken up into parts by being, is many and infinite?

True.

Then not only the one which has being is many, but the one itself distributed by being, must also be many?

Certainly.

Further, inasmuch as the parts are parts of a whole, the one, as a whole, will be limited; for are not the parts contained the whole?

Certainly.

And that which contains, is a limit?

Of course.

Then the one if it has being is one and many, whole and parts, having limits and yet unlimited in number?

Clearly.

And because having limits, also having extremes?

Certainly.

And if a whole, having beginning and middle and end. For can anything be a whole without these three? And if any one of them is wanting to anything, will that any longer be a whole?

No.

Then the one, as appears, will have beginning, middle, and end.

It will.

But, again, the middle will be equidistant from the extremes; or it would not be in the middle?

Yes.

Then the one will partake of figure, either rectilinear or round, or a union of the two?

True.

And if this is the case, it will be both in itself and in another too.

How?

Every part is in the whole, and none is outside the whole.

True.

And all the parts are contained by the whole?

Yes.

And the one is all its parts, and neither more nor less than all?

No.

And the one is the whole?

Of course.

But if all the parts are in the whole, and the one is all of them and the whole, and they are all contained by the whole, the one will be contained by the one; and thus the one will be in itself.

That is true.

But then, again, the whole is not in the parts– neither in all the parts, nor in some one of them. For if it is in all, it must be in one; for if there were any one in which it was not, it could not be in all the parts; for the part in which it is wanting is one of all, and if the whole is not in this, how can it be in them all?

It cannot.

Nor can the whole be in some of the parts; for if the whole were in some of the parts, the greater would be in the less, which is impossible.

Yes, impossible.

But if the whole is neither in one, nor in more than one, nor in all of the parts, it must be in something else, or cease to be anywhere at all?

Certainly.

If it were nowhere, it would be nothing; but being a whole, and not being in itself, it must be in another.

Very true.

The one then, regarded as a whole, is in another, but regarded as being all its parts, is in itself; and therefore the one must be itself in itself and also in another.

Certainly.

The one then, being of this nature, is of necessity both at rest and in motion?

How?

The one is at rest since it is in itself, for being in one, and not passing out of this, it is in the same, which is itself.

True.

And that which is ever in the same, must be ever at rest?

Certainly.

Well, and must not that, on the contrary, which is ever in other, never be in the same; and if never in the same, never at rest, and if not at rest, in motion?

True.

Then the one being always itself in itself and other, must always be both at rest and in motion?

Clearly.

And must be the same with itself, and other than itself; and also the same with the others, and other than the others; this follows from its previous affections.

How so?

Every thing in relation to every other thing, is either the same or other; or if neither the same nor other, then in the relation of a part to a whole, or of a whole to a part.

Clearly.

And is the one a part of itself?

Certainly not.

Since it is not a part in relation to itself it cannot be related to itself as whole to part?

It cannot.

But is the one other than one?

No.

And therefore not other than itself?

Certainly not.

If then it be neither other, nor a whole, nor a part in relation to itself, must it not be the same with itself?

Certainly.

But then, again, a thing which is in another place from “itself,” if this “itself” remains in the same place with itself, must be other than “itself,” for it will be in another place?

True.

Then the one has been shown to be at once in itself and in another?

Yes.

Thus, then, as appears, the one will be other than itself?

True.

Well, then, if anything be other than anything, will it not be other than that which is other?

Certainly.

And will not all things that are not one, be other than the one, and the one other than the not-one?

Of course.

Then the one will be other than the others?

True.

But, consider:–Are not the absolute same, and the absolute other, opposites to one another?

Of course.

Then will the same ever be in the other, or the other in the same?

They will not.

If then the other is never in the same, there is nothing in which the other is during any space of time; for during that space of time, however small, the other would be in the same. Is not that true?

Yes.

And since the other is never in the same, it can never be in anything that is.

True.

Then the other will never be either in the not-one, or in the one?

Certainly not.

Then not by reason of other-ness is the one other than the not-one, or the not-one other than the one.

No.

Nor by reason of themselves will they be other than one another, if not partaking of the other.

How can they be?

But if they are not other, either by reason of themselves or of the other, will they not altogether escape being other than one another?

They will.

Again, the not-one cannot partake of the one; otherwise it would not have been not-one, but would have been in some way one.

True.

Nor can the not-one be number; for having number, it would not have been not-one at all.

It would not.

Again, is the not-one part of the one; or rather, would it not in that case partake of the one?

It would.

If then, in every point of view, the one and the not-one are distinct, then neither is the one part or whole of the not-one, nor is the not-one part or whole of the one?

No.

But we said that things which are neither parts nor wholes of one another, nor other than one

another, will be the same with one another:–so we said?

Yes.

Then shall we say that the one, being in this relation to the not-one, is the same with it?

Let us say so.

Then it is the same with itself and the others, and also other than itself and the others.

That appears to be the inference. And it will also be like and unlike itself and the others?

Perhaps.

Since the one was shown to be other than the others, the others will also be other than the one.

Yes.

And the one is other than the others in the same degree that the others are other than it, and neither more nor less?

True.

And if neither more nor less, then in a like degree?

Yes.

In virtue of the affection by which the one is other than others and others in like manner other than it, the one will be affected like the others and the others like the one.

How do you mean?

I may take as an illustration the case of names: You give a name to a thing?

Yes.

And you may say the name once or oftener?

Yes.

And when you say it once, you mention that of which it is the name? and when more than once, is it something else which you mention? or must it always be the same thing of which you speak, whether you utter the name once or more than once?

Of course it is the same.

And is not “other” a name given to a thing?

Certainly.

Whenever, then, you use the word “other,” whether once or oftener, you name that of which it is the name, and to no other do you give the name?

True.

Then when we say that the others are other than the one, and the one other than the others, in repeating the word “other” we speak of that nature to which the name is applied, and of no other?

Quite true.

Then the one which is other than others, and the other which is other than the one, in that the word “other” is applied to both, will be in the same condition; and that which is in the same condition is like?

Yes.

Then in virtue of the affection by which the one is other than the others, every thing will be like every thing, for every thing is other than every thing.

True.

Again, the like is opposed to the unlike?

Yes.

And the other to the same?

True again.

And the one was also shown to be the same with the others?

Yes.

And to be, the same with the others is the opposite of being other than the others?

Certainly.

And in that it was other it was shown to be like?

Yes.

But in that it was the same it will be unlike by virtue of the opposite affection to that which made it and this was the affection of other-ness.

Yes.

The same then will make it unlike; otherwise it will not be the opposite of the other.

True.

Then the one will be both like and unlike the others; like in so far as it is other, and unlike in so far as it is the same.

Yes, that argument may be used.

And there is another argument.

What?

In so far as it is affected in the same way it is not affected otherwise, and not being affected otherwise is not unlike, and not being unlike, is like; but in so far as it is affected by other it is otherwise, and being otherwise affected is unlike.

True.

Then because the one is the same with the others and other than the others, on either of these two grounds, or on both of them, it will be both like and unlike the others?

Certainly.

And in the same way as being other than itself, and the same with itself on either of these two grounds and on both of them, it will be like and unlike itself.

Of course.

Again, how far can the one touch or not touch itself and others?–Consider.

I am considering.

The one was shown to be in itself which was a whole?

True.

And also in other things?

Yes.

In so far as it is in other things it would touch other things, but in so far as it is in itself it would be debarred from touching them, and would touch itself only.

Clearly.

Then the inference is that it would touch both?

It would.

But what do you say to a new point of view? Must not that which is to touch another be next to that which it is to touch, and occupy the place nearest to that in which what it touches is situated?

True.

Then the one, if it is to touch itself, ought to be situated next to itself, and occupy the place next to that in which itself is?

It ought.

And that would require that the one should be two, and be in two places at once, and this, while it is one, will never happen.

No.

Then the one cannot touch itself any more than it can be two?

It cannot.

Neither can it touch others.

Why not?

The reason is, that whatever is to touch another must be in separation from, and next to, that which it is to touch, and no third thing can be between them.

True.

Two things, then, at the least are necessary to make contact possible?

They are.

And if to the two a third be added in due order, the number of terms will be three, and the contacts two?

Yes.

And every additional term makes one additional contact, whence it follows that the contacts are one less in number than the terms; the first two terms exceeded the number of contacts by one, and the whole number of terms exceeds the whole number

of contacts by one in like manner; and for every one which is afterwards added to the number of terms, one contact is added to the contacts.

True.

Whatever is the whole number of things, the contacts will be always one less.

True.

But if there be only one, and not two, there will be no contact?

How can there be?

And do we not say that the others being other than the one are not one and have no part in the one?

True.

Then they have no number, if they have no one in them?

Of course not.

Then the others are neither one nor two, nor are they called by the name of any number?

No.

One, then, alone is one, and two do not exist?

Clearly not.

And if there are not two, there is no contact?

There is not.

Then neither does the one touch the others, nor the others the one, if there is no contact?

Certainly not.

For all which reasons the one touches and does not touch itself and the others?

True.

Further–is the one equal and unequal to itself and others?

How do you mean?

If the one were greater or less than the others, or the others greater or less than the one, they would not be greater or less than each other in virtue of their being the one and the others; but, if in addition

to their being what they are they had equality, they would be equal to one another, or if the one had smallness and the others greatness, or the one had greatness and the others smallness–whichever kind had greatness would be greater, and whichever had smallness would be smaller?

Certainly.

Then there are two such ideas as greatness and smallness; for if they were not they could not be opposed to each other and be present in that which is.

How could they?

If, then, smallness is present in the one it will be present either in the whole or in a part of the whole?

Certainly.

Suppose the first; it will be either co-equal and co-extensive with the whole one, or will contain the one?

Clearly.

If it be co-extensive with the one it will be co-equal with the one, or if containing the one it will be greater than the one?

Of course.

But can smallness be equal to anything or greater than anything, and have the functions of greatness and equality and not its own functions?

Impossible.

Then smallness cannot be in the whole of one, but, if at all, in a part only?

Yes.

And surely not in all of a part, for then the difficulty of the whole will recur; it will be equal to or greater than any part in which it is.

Certainly.

Then smallness will not be in anything, whether in a whole or in a part; nor will there be anything small but actual smallness.

True.

Neither will greatness be in the one, for if

greatness be in anything there will be something greater other and besides greatness itself, namely, that in which greatness is; and this too when the small itself is not there, which the one, if it is great, must exceed; this, however, is impossible, seeing that smallness is wholly absent.

True.

But absolute greatness is only greater than absolute smallness, and smallness is only smaller than absolute greatness.

Very true.

Then other things not greater or less than the one, if they have neither greatness nor smallness; nor have greatness or smallness any power of exceeding or being exceeded in relation to the one, but only in relation to one another; nor will the one be greater or less than them or others, if it has neither greatness nor smallness.

Clearly not.

Then if the one is neither greater nor less than the others, it cannot either exceed or be exceeded by them?

Certainly not.

And that which neither exceeds nor is exceeded, must be on an equality; and being on an equality, must be equal.

Of course.

And this will be true also of the relation of the one to itself; having neither greatness nor smallness in itself, it will neither exceed nor be exceeded by itself, but will be on an equality with and equal to itself.

Certainly.

Then the one will be equal to both itself and the others?

Clearly so.

And yet the one, being itself in itself, will also surround and be without itself; and, as containing itself, will be greater than itself; and, as contained in itself, will be less; and will thus be greater and less

than itself.

It will.

Now there cannot possibly be anything which is not included in the one and the others?

Of course not.

But, surely, that which is must always be somewhere?

Yes.

But that which is in anything will be less, and that in which it is will be greater; in no other way can one thing be in another.

True.

And since there is nothing other or besides the one and the others, and they must be in something, must they not be in one another, the one in the others and the others in the one, if they are to be anywhere?

That is clear.

But inasmuch as the one is in the others, the others will be greater than the one, because they contain the one, which will be less than the others, because it is contained in them; and inasmuch as the others are in the one, the one on the same principle will be greater than the others, and the others less than the one.

True.

The one, then, will be equal to and greater and less than itself and the others?

Clearly.

And if it be greater and less and equal, it will be of equal and more and less measures or divisions than itself and the others, and if of measures, also of parts?

Of course.

And if of equal and more and less measures or divisions, it will be in number more or less than itself and the others, and likewise equal in number to itself and to the others?

How is that?

It will be of more measures than those things which it exceeds, and of as many parts as measures; and so with that to which it is equal, and that than which it is less.

True.

And being greater and less than itself, and equal to itself, it will be of equal measures with itself and of more and fewer measures than itself; and if of measures then also of parts?

It will.

And being of equal parts with itself, it will be numerically equal to itself; and being of more parts, more, and being of less, less than itself?

Certainly.

And the same will hold of its relation to other things; inasmuch as it is greater than them, it will be more in number than them; and inasmuch as it is smaller, it will be less in number; and inasmuch as it is equal in size to other things, it will be equal to them in number.

Certainly.

Once more then, as would appear, the one will be in number both equal to and more and less than both itself and all other things.

It will.

Does the one also partake of time? And is it and does it become older and younger than itself and others, and again, neither younger nor older than itself and others, by virtue of participation in time?

How do you mean?

If one is, being must be predicated of it?

Yes.

But to be (ειναι) is only participation of being in present time, and to have been is the participation of being at a past time, and to be about to be is the participation of being at a future time?

Very true.

Then the one, since it partakes of being, partakes of time?

Certainly.

And is not time always moving forward?

Yes.

Then the one is always becoming older than itself, since it moves forward in time?

Certainly.

And do you remember that the older becomes older than that which becomes younger?

I remember.

Then since the one becomes older than itself, it becomes younger at the same time?

Certainly.

Thus, then, the one becomes older as well as younger than itself?

Yes.

And it is older (is it not?) when in becoming, it gets to the point of time between “was” and “will be,” which is “now”: for surely in going from the past to the future, it cannot skip the present?

No.

And when it arrives at the present it stops from becoming older, and no longer becomes, but is older, for if it went on it would never be reached by the present, for it is the nature of that which goes on, to touch both the present and the future, letting go the present and seizing the future, while in process of becoming between them.

True.

But that which is becoming cannot skip the present; when it reaches the present it ceases to become, and is then whatever it may happen to be becoming.

Clearly.

And so the one, when in becoming older it reaches the present, ceases to become, and is then older.

Certainly.

And it is older than that than which it was

becoming older, and it was becoming older than itself.

Yes.

And that which is older is older than that which is younger?

True.

Then the one is younger than itself, when in becoming older it reaches the present?

Certainly.

But the present is always present with the one during all its being; for whenever it is it is always now.

Certainly.

Then the one always both is and becomes older and younger than itself?

Truly.

And is it or does it become a longer time than itself or an equal time with itself?

An equal time.

But if it becomes or is for an equal time with itself, it is of the same age with itself?

Of course.

And that which is of the same age, is neither older nor younger?

No.

The one, then, becoming and being the same time with itself, neither is nor becomes older or younger than itself?

I should say not.

And what are its relations to other things? Is it or does it become older or younger than they?

I cannot tell you.

You can at least tell me that others than the one are more than the one–other would have been one, but the others have multitude, and are more than one?

They will have multitude.

And a multitude implies a number larger than one?

Of course.

And shall we say that the lesser or the greater is the first to come or to have come into existence?

The lesser.

Then the least is the first? And that is the one?

Yes.

Then the one of all things that have number is the first to come into being; but all other things have also number, being plural and not singular.

They have.

And since it came into being first it must be supposed to have come into being prior to the others, and the others later; and the things which came into being later, are younger than that which preceded them? And so the other things will be younger than the one, and the one older than other things?

True.

What would you say of another question? Can the one have come into being contrary to its own nature, or is that impossible?

Impossible.

And yet, surely, the one was shown to have parts; and if parts, then a beginning, middle and end?

Yes.

And a beginning, both of the one itself and of all other things, comes into being first of all; and after the beginning, the others follow, until you reach the end?

Certainly.

And all these others we shall affirm to be parts of the whole and of the one, which, as soon as the end is reached, has become whole and one?

Yes; that is what we shall say.

But the end comes last, and the one is of such a nature as to come into being with the last; and, since

the one cannot come into being except in accordance with its own nature, its nature will require that it should come into being after the others, simultaneously with the end.

Clearly.

Then the one is younger than the others and the others older than the one.

That also is clear in my judgment.

Well, and must not a beginning or any other part of the one or of anything, if it be *a part* and not parts, being *a part*, be also of necessity one?

Certainly.

And will not the one come into being together with each part–together with the first part when that comes into being, and together with the second part and with all the rest, and will not be wanting to any part, which is added to any other part until it has reached the last and become one whole; it will be wanting neither to the middle, nor to the first, nor to the last, nor to any of them, while the process of becoming is going on?

True.

Then the one is of the same age with all the others, so that if the one itself does not contradict its own nature, it will be neither prior nor posterior to the others, but simultaneous; and according to this argument the one will be neither older nor younger than the others, nor the others than the one, but according to the previous argument the one will be older and younger than the others and the others than the one.

Certainly.

After this manner then the one is and has become. But as to its becoming older and younger than the others, and the others than the one, and neither older nor younger, what shall we say? Shall we say as of being so also of becoming, or otherwise?

I cannot answer.

But I can venture to say, that even if one thing were older or younger than another, it could not

become older or younger in a greater degree than it was at first; for equals added to unequals, whether to periods of time or to anything else, leave the difference between them the same as at first.

Of course. Then that which is, cannot become older or younger than that which is, since the difference of age is always the same; the one is and has become older and the other younger; but they are no longer becoming so.

True.

And the one which is does not therefore become either older or younger than the others which are?

No.

But consider whether they may not become older and younger in another way.

In what way?

Just as the one was proven to be older than the others and the others than the one.

And what of that?

If the one is older than the others, has come into being a longer time than the others.

Yes.

But consider again; if we add equal time to a greater and a less time, will the greater differ from the less time by an equal or by a smaller portion than before?

By a smaller portion.

Then the difference between the age of the one and the age of the others will not be afterwards so great as at first, but if an equal time be added to both of them they will differ less and less in age?

Yes.

And that which differs in age from some other less than formerly, from being older will become younger in relation to that other than which it was older?

Yes, younger.

And if the one becomes younger the others aforesaid will become older than they were before,

in relation to the one.

Certainly.

Then that which had become younger becomes older relatively to that which previously had become and was older; it never really is older, but is always becoming, for the one is always growing on the side of youth and the other on the side of age. And in like manner the older is always in process of becoming younger than the younger; for as they are always going in opposite directions they become in ways the opposite to one another, the younger older than the older and the older younger than the younger. They cannot, however have become; for if they had already become they would be and not merely become. But that is impossible; for they are always becoming both older and younger than one another: the one becomes younger than the others because it was seen to be older and prior, and the others become older than the one because they came into being later; and in the same way the others are in the same relation tothe o ne, because they were seen to be older, and prior to the one.

That is clear.

Inasmuch then, one thing does not become older or younger than another, in that they always differ from each other by an equal number, the one cannot become older or younger than the others, nor the other than the one; but inasmuch as that which came into being earlier and that which came into being later must continually differ from each other by a different portion–in this point of view the others must become older and younger than the one, and the one than the others.

Certainly.

For all these reasons, then, the one is and becomes older and younger than itself and the others, and neither is nor becomes older or younger than itself or the others.

Certainly.

But since the one partakes of time, and partakes of becoming older and younger, must it not also partake of the past, the present, and the future?

Of course it must.

Then the one was and is and will be, and was becoming and is becoming and will become?

Certainly.

And there is and was and will be something which is in relation to it and belongs to it?

True.

And since we have at this moment opinion and knowledge and perception of the one, there is opinion and knowledge and perception of it?

Quite right.

Then there is name and expression for it, and it is named and expressed, and everything of this kind which appertains to other things appertains to the one.

Certainly, that is true.

Yet once more and for the third time, let us consider: If the one is both one and many, as we have described, and is, neither one nor many, and participates in time, must it not, in as far as it is one, at times partake of being, and in as far as it is not one, at times not partake of being [18]?

Certainly.

But can it partake of being when not partaking of being, or not partake of being when partaking of being?

Impossible.

Then the one partakes and does not partake of being at different times, for that is the only way in which it can partake and not partake of the same.

True.

And is there not also a time at which it assumes being and relinquishes being–for how can it have and not have the same thing unless it receives and also gives it up at some time?

Impossible.

And the assuming of being is what you would call becoming?

[18] If one is both form and matter. (3)

I should.

And the relinquishing of being you would call destruction?

I should.

The one then, as would appear, becomes and is destroyed by taking and giving up being.

Certainly.

And being one and many and in process of becoming and being destroyed, when it becomes one it ceases to be many, and when many, it ceases to be one?

Certainly.

And as it becomes one and many, must it not inevitably experience separation and aggregation?

Inevitably.

And whenever it becomes like and unlike it must be assimilated and dissimilated?

Yes.

And when it becomes greater or less or equal it must grow or diminish or be equalized?

True.

And when being in motion it rests, and when being at rest it changes to motion, it can surely be in no time at all?

How can it?

But that a thing which is previously at rest should be afterwards in motion, or previously in motion and afterwards at rest, without experiencing change, is impossible.

Impossible.

And surely there cannot be a time in which a thing can be at once neither in motion nor at rest?

There cannot.

But neither can it change without changing.

True.

When then does it change; for it cannot change either when at rest, or when in motion, or when in

time?

It cannot.

And does this strange thing in which it is at the time of changing really exist?

What thing?

The moment. For the moment seems to imply a something out of which change takes place into either of two states; for the change is not from the state of rest as such, nor, from the state of motion as such; but there is this curious nature, which we call the moment lying between rest and motion, not being in any time; and into this and out of this what is in motion changes into rest, and what is at rest into motion.

So it appears.

And the one then, since it is at rest and also in motion, will change to either, for only in this way can it be in both. And in changing it changes in a moment, and when it is changing it will be in no time, and will not then be either in motion or at rest.

It will not.

And it will be in the same case in relation to the other changes, when it passes from being into cessation of being, or from not-being into becoming–then it passes between certain states of motion and rest, and, neither is nor is not, nor becomes nor is destroyed.

Very true.

And on the same principle, in the passage from one to many and from many to one, the one is neither one nor many, neither separated nor aggregated; and in the passage from like to unlike, and from unlike to like, it is neither like nor unlike, neither in a state of assimilation nor of dissimilation; and in the passage from small to great and equal and back again, it will be neither small nor great, nor equal, nor in a state of increase, or diminution, or equalization.

True.

All these, then, are the affections of the one, if the one has being.

Of course.

But if one is, what will happen to the others–is not that also to be considered?

Yes.

Let us show then, if one is, what will be the affections of the others than the one.

Let us do so.

Inasmuch as there are things other than the one, the others are not the one; for if they were they could not be other than the one.

Very true.

Nor are the others altogether without the one, but in a certain way they participate in the one.

In what way?

Because the others are other than the one inasmuch as they have parts; for if they had no parts they would be simply one.

Right.

And parts, as we affirm, have relation to a whole?

So we say.

And a whole must necessarily be one made up of many; and the parts will be parts of the one, for each of the parts is not a part of many, but of a whole.

How do you mean?

If anything were a part of many, being itself one of them, it will surely be a part of itself, which is impossible, and it will be a part of each one of the other parts, if of all; for if not a part of some one, it will be a part of all the others but this one, and thus will not be a part of each one; and if not a part of each, one it will not be a part of anyone of the many; and not being a part of any one, it cannot be a part or anything else of all those things of none of which it is anything.

Clearly not.

Then the part is not a part of the many, nor of all, but is of a certain single form, which we call a

whole, being one perfect unity framed out of all–of this the part will be a part.

Certainly.

If, then, the others have parts, they will participate in the whole and in the one.

True.

Then the others than the one must be one perfect whole, having parts.

Certainly.

And the same argument holds of each part, for the part must participate in the one; for if each of the parts is a part, this means, I suppose, that it is one separate from the rest and self-related; otherwise it is not each.

True.

But when we speak of the part **participating** in the one, it must clearly be other than one; for if not, it would merely have participated, but would have been one; whereas only the itself can be one.

Very true.

Both the whole and the part must participate in the one; for the whole will be one whole, of which the parts will be parts; and each part will be one part of the whole which is the whole of the part.

True.

And will not the things which participate in the one, be other than it?

Of course.

And the things which are other than the one will be many; for if the things which are other than the one were neither one nor more than one, they would be nothing.

True.

But, seeing that the things which participate in the one as a part, and in the one as a whole, are more than one, must not those very things which participate in the one be infinite in number?

How so?

Let us look at the matter thus:–Is it not a fact that

in partaking of the one they are not one, and do not partake of the one at the very time when they are partaking of it?

Clearly.

They do so then as multitudes in which the one is not present?

Very true.

And if we were to abstract from them in idea the very smallest fraction, must not that least fraction, if it does not partake of the one, be a multitude and not one?

It must.

And if we continue to look at the other side of their nature, regarded simply, and in itself, will not they, as far as we see them, be unlimited in number?

Certainly.

And yet, when each several part becomes a part, then the parts have a limit in relation to the whole and to each other, and the whole in relation to the parts.

Just so.

The result to the others than the one is that of themselves and the one appears to create a new element in them which gives to them limitation in relation to one another; whereas in their own nature they have no limit.

That is clear.

Then the others than the one, both as whole and parts, are infinite, and also partake of limit.

Certainly.

Then they are both like and unlike one another and themselves.

How is that?

Inasmuch as they are unlimited in their own nature, they are all affected in the same way.

True.

And inasmuch as they all partake of limit, they are all affected in the same way.

Of course.

But inasmuch as their state is both limited and unlimited, they are affected in opposite ways.

Yes.

And opposites are the most unlike of things.

Certainly.

Considered, then, in regard to either one of their affections, they will be like themselves and one another; considered in reference to both of them together, most opposed and most unlike.

That appears to be true.

Then the others are both like and unlike themselves and one another?

True.

And they are the same and also different from one another, and in motion and at rest, and experience every sort of opposite affection, as may be proved without difficulty of them, since they have been shown to have experienced the affections aforesaid?

True.

Suppose, now, that we leave the further discussion of these matters as evident, and consider again upon the hypothesis that the one is, whether opposite of all this is or is not equally true of the others.

By all means.

Then let us begin again, and ask, If one is, what must be the affections of the others?

Let us ask that question.

Must not the one be distinct from the others, and the others from the one?

Why so?

Why, because there is nothing else beside them which is distinct from both of them; for the expression “one and the others” includes all things.

Yes, all things.

Then we cannot suppose that there is anything

different from them in which both the one and the others might exist?

There is nothing.

Then the one and the others are never in the same?

True.

Then they are separated from each other?

Yes.

And we surely cannot say that what is truly one has parts?

Impossible.

Then the one will not be in the others as a whole, nor as part, if it be separated from the others, and has no parts?

Impossible.

Then there is no way in which the others can partake of the one, if they do not partake either in whole or in part?

It would seem not.

Then there is no way in which the others are one, or have in themselves any unity?

There is not.

Nor are the others many; for if they were many, each part of them would be a part of the whole; but now the others, not partaking in any way of the one, are neither one nor many, nor whole, nor part.

True.

Then the others neither are nor contain two or three, if entirely deprived of the one?

True.

Then the others are neither like nor unlike the one, nor is likeness and unlikeness in them; for if they were like and unlike, or had in them likeness and unlikeness, they would have two natures in them opposite to one another.

That is clear.

But for that which partakes of nothing to partake of two things was held by us to be impossible?

Impossible.

Then the others are neither like nor unlike nor both, for if they were like or unlike they would partake of one of those two natures, which would be one thing, and if they were both they would partake of opposites which would be two things, and this has been shown to be impossible.

True.

Therefore they are neither the same, nor other, nor in motion, nor at rest, nor in a state of becoming, nor of being destroyed, nor greater, nor less, nor equal, nor have they experienced anything else of the sort; for, if they are capable of experiencing any such affection, they will participate in one and two and three, and odd and even, and in these, as has been proved, they do not participate, seeing that they are altogether and in every way devoid of the one.

Very true.

Therefore if one is, the one is all things, and also nothing, both in relation to itself and to other things.

Certainly.

Well, and ought we not to consider next what will be the consequence if the one is not?

Yes; we ought.

What is the meaning of the hypothesis–If the one is not; is there any difference between this and the hypothesis–If the not one is not?

There is a difference, certainly.

Is there a difference only, or rather are not the two expressions–if the one is not, and if the not one is not, entirely opposed?

They are entirely opposed.

And suppose a person to say:–If greatness is not, if smallness is not, or anything of that sort, does he not mean, whenever he uses such an expression, that “what is not” is other than other things?

To be sure.

And so when he says “If one is not” he clearly means, that what “is not” is other than all others; we

know what he means–do we not?

Yes, we do.

When he says “one,” he says something which is known; and secondly something which is other than all other things; it makes no difference whether he predicate of one being or not being, for that which is said “not to be” is known to be something all the same, and is distinguished from other things.

Certainly.

Then I will begin again, and ask: If one is not, what are the consequences? In the first place, as would appear, there is a knowledge of it, or the very meaning of the words, “if one is not,” would not be known [19].

True.

Secondly, the others differ from it, or it could not be described as different from the others?

Certainly.

Difference, then, belongs to it as well as knowledge; for in speaking of the one as different from the others, we do not speak of a difference in the others, but in the one.

Clearly so.

Moreover, the one that is not is something and partakes of relation to “that,” and “this,” and “these,” and the like, and is an attribute of “this”; for the one, or the others than the one, could not have been spoken of, nor could any attribute or relative of the one that is not have been or been spoken of, nor could it have been said to be anything, if it did not partake of “some,” or of the other relations just now mentioned.

True.

Being, then, cannot be ascribed to the one, since it is not; but the one that is not may or rather must participate in many things, if it and nothing else is not; if, however, neither the one nor the one that is not is supposed not to be, and we are speaking of something of a different nature, we can predicate

[19] If one is not form. (4)

nothing of it. But supposing that the one that is not and nothing else is not, then it must participate in the predicate “that,” and in many others.

Certainly.

And it will have unlikeness in relation to the others, for the others being different from the one will be of a different kind.

Certainly.

And are not things of a different kind also other in kind?

Of course.

And are not things other in kind unlike?

They are unlike.

And if they are unlike the one, that which they are unlike will clearly be unlike them?

Clearly so.

Then the one will have unlikeness in respect of which the others are unlike it?

That would seem to be true.

And if unlikeness to other things is attributed to it, it must have likeness to itself.

How so?

If the one have unlikeness to one, something else must be meant; nor will the hypothesis relate to one; but it will relate to something other than one?

Quite so.

But that cannot be.

No.

Then the one must have likeness to itself?

It must.

Again, it is not equal to the others; for if it were equal, then it would at once be and be like them in virtue of the equality; but if one has no being, then it can neither be nor be like?

It cannot.

But since it is not equal to the others, neither can the others be equal to it?

Certainly not.

And things that are not equal are unequal?

True.

And they are unequal to an unequal?

Of course.

Then the one partakes of inequality, and in respect of this the others are unequal to it?

Very true.

And inequality implies greatness and smallness?

Yes.

Then the one, if of such a nature, has greatness and smallness?

That appears to be true.

And greatness and smallness always stand apart?

True.

Then there is always something between them?

There is.

And can you think of anything else which is between them other than equality?

No, it is equality which lies between them.

Then that which has greatness and smallness also has equality, which lies between them?

That is clear.

Then the one, which is not, partakes, as would appear, of greatness and smallness and equality?

Clearly.

Further, it must surely in a sort partake of being?

How so?

It must be so, for if not, then we should not speak the truth in saying that the one is not. But if we speak the truth, clearly we must say what is. Am I not right?

Yes.

And since we affirm that we speak truly, we must also affirm that we say what is?

Certainly.

Then, as would appear, the one, when it is not, is; for if it were not to be when it is not, but were to relinquish something of being, so as to become not-being, it would at once be.

Quite true.

Then the one which is not, if it is to maintain itself, must have the being of not-being as the bond of not-being, just as being must have as a bond the not-being of not-being in order to perfect its own being; for the truest assertion of the being of being and of the not-being of not being is when being partakes of the being of being, and not of the being of not-being–that is, the perfection of being; and when not-being does not partake of the not-being of not-being but of the being of not-being–that is the perfection of not-being.

Most true.

Since then what is partakes of not-being, and what is not of being, must not the one also partake of being in order not to be?

Certainly.

Then the one, if it is not, clearly has being?

Clearly.

And has not-being also, if it is not?

Of course.

But can anything which is in a certain state not be in that state without changing?

Impossible.

Then everything which is and is not in a certain state, implies change?

Certainly.

And change is motion–we may say that?

Yes, motion.

And the one has been proved both to be and not to be?

Yes.

And therefore is and is not in the same state?

Yes.

Thus the one that is not has been shown to have motion also, because it changes from being to not-being?

That appears to be true.

But surely if it is nowhere among what is, as is the fact, since it is not, it cannot change from one place to another?

Impossible.

Then it cannot move by changing place?

No.

Nor can it turn on the same spot, for it nowhere touches the same, for the same is, and that which is not cannot be reckoned among things that are?

It cannot.

Then the one, if it is not, cannot turn in that in which it is not?

No.

Neither can the one, whether it is or is not, be altered into other than itself, for if it altered and became different from itself, then we could not be still speaking of the one, but of something else?

True.

But if the one neither suffers alteration, nor turns round in the same place, nor changes place, can it still be capable of motion?

Impossible.

Now that which is unmoved must surely be at rest, and that which is at rest must stand still?

Certainly.

Then the one that is not, stands still, and is also in motion?

That seems to be true.

But if it be in motion it must necessarily undergo alteration, for anything which is moved, in so far as it is moved, is no longer in the same state, but in another?

Yes.

Then the one, being moved, is altered?

Yes.

And, further, if not moved in any way, it will not be altered in any way?

No.

Then, in so far as the one that is not is moved, it is altered, but in so far as it is not moved, it is not altered?

Right.

Then the one that is not is altered and is not altered?

That is clear.

And must not that which is altered become other than it previously was, and lose its former state and be destroyed; but that which is not altered can neither come into being nor be destroyed?

Very true.

And the one that is not, being altered, becomes and is destroyed; and not being altered, neither becomes nor is destroyed; and so the one that is not becomes and is destroyed, and neither becomes nor is destroyed?

True.

And now, let us go back once more to the beginning, and see whether these or some other consequences will follow.

Let us do as you say.

If one is not, we ask what will happen in respect of one? That is the question.

Yes.

Do not the words “is not” signify absence of being in that to which we apply them?

Just so.

And when we say that a thing is not, do we mean that it is not in one way but is in another? or do we mean, absolutely, that what is not has in no sort or way or kind participation of being?

Quite absolutely.

Then, that which is not cannot be, or in any way participate in being?

It cannot.

And did we not mean by becoming, and being destroyed, the assumption of being and the loss of being?

Nothing else.

And can that which has no participation in being, either assume or lose being?

Impossible.

The one then, since it in no way is, cannot have or lose or assume being in any way?

True.

Then the one that is not, since it in no way partakes of being, neither nor becomes?

No.

Then it is not altered at all; for if it were it would become and be destroyed?

True.

But if it be not altered it cannot be moved?

Certainly not.

Nor can we say that it stands, if it is nowhere; for that which stands must always be in one and the same spot?

Of course.

Then we must say that the one which is not never stands still and never moves?

Neither.

Nor is there any existing thing which can be attributed to it; for if there had been, it would partake of being?

That is clear.

And therefore neither smallness, nor greatness, nor equality, can be attributed to it?

No.

Nor yet likeness nor difference, either in relation to itself or to others?

Clearly not.

Well, and if nothing should be attributed to it, can other things be attributed to it?

Certainly not.

And therefore other things can neither be like or unlike, the same, or different in relation to it?

They cannot.

Nor can what is not, be anything, or be this thing, or be related to or the attribute of this or that or other, or be past, present, or future. Nor can knowledge, or opinion, or perception, or expression, or name, or any other thing that is, have any concern with it?

No.

Then the one that is not has no condition of any kind?

Such appears to be the conclusion.

Yet once more; if one is not, what becomes of the others? Let us determine that [20].

Yes; let us determine that.

The others must surely be; for if they, like the one, were not, we could not be now speaking of them.

True.

But to speak of the others implies difference–the terms “other” and “different” are synonymous?

True.

Other means other than other, and different, different from the different?

Yes.

Then, if there are to be others, there is something than which they will be other?

Certainly.

And what can that be?–for if the one is not, they will not be other than the one.

[20] If one is not matter (5)

They will not.

Then they will be other than each other; for the only remaining alternative is that they are other than nothing.

True.

And they are each other than one another, as being plural and not singular; for if one is not, they cannot be singular but every particle of them is infinite in number; and even if a person takes that which appears to be the smallest fraction, this, which seemed one, in a moment evanesces into many, as in a dream, and from being the smallest becomes very great, in comparison with the fractions into which it is split up?

Very true.

And in such particles the others will be other than one another, if others are, and the one is not?

Exactly.

And will there not be many particles, each appearing to be one, but not being one, if one is not?

True.

And it would seem that number can be predicated of them if each of them appears to be one, though it is really many?

It can.

And there will seem to be odd and even among them, which will also have no reality, if one is not?

Yes.

And there will appear to be a least among them; and even this will seem large and manifold in comparison with the many small fractions which are contained in it?

Certainly.

And each particle will be imagined to be equal to the many and little; for it could not have appeared to pass from the greater to the less without having appeared to arrive at the middle; and thus would arise the appearance of equality.

Yes.

And having neither beginning, middle, nor end, each separate particle yet appears to have a limit in relation to itself and other.

How so?

Because, when a person conceives of any one of these as such, prior to the beginning another beginning appears, and there is another end, remaining after the end, and in the middle truer middles within but smaller, because no unity can be conceived of any of them, since the one is not.

Very true.

And so all being, whatever we think of, must be broken up into fractions, for a particle will have to be conceived of without unity?

Certainly.

And such being when seen indistinctly and at a distance, appears to be one; but when seen near and with keen intellect, every single thing appears to be infinite, since it is deprived of the one, which is not?

Nothing more certain.

Then each of the others must appear to be infinite and finite, and one and many, if others than the one exist and not the one.

They must.

Then will they not appear to be like and unlike?

In what way?

Just as in a picture things appear to be all one to a person standing at a distance, and to be in the same state and alike?

True.

But when you approach them, they appear to be many and different; and because of the appearance of the difference, different in kind from, and unlike, themselves?

True.

And so must the particles appear to be like and unlike themselves and each other.

Certainly.

And must they not be the same and yet different

from one another, and in contact with themselves, although they are separated, and having every sort of motion, and every sort of rest, and becoming and being destroyed, and in neither state, and the like, all which things may be easily enumerated, if the one is not and the many are?

Most true.

Once more, let us go back to the beginning, and ask if the one is not, and the others of the one are, what will follow.

Let us ask that question.

In the first place, the others will not be one?

Impossible.

Nor will they be many; for if they were many one would be contained in them. But if no one of them is one, all of them are naught, and therefore they will not be many.

True.

If there be no one in the others, the others are neither many nor one.

They are not.

Nor do they appear either as one or many.

Why not?

Because the others have no sort or manner or way of communion with any sort of not-being, nor can anything which is not, be connected with any of the others; for that which is not has no parts.

True.

Nor is there an opinion or any appearance of not-being in connection with the others, nor is not-being ever in any way attributed to the others.

No.

Then if one is not, the others neither are, nor any of the others either as one or many; for you cannot conceive the many without the one [21].

You cannot.

[21] If one is neither. (6)

Then if one is not, there is no conception of can be conceived to be either one or many?

It would seem not.

Nor as like or unlike?

No.

Nor as the same or different, nor in contact or separation, nor in any of those states which we enumerated as appearing to be;–the others neither are nor appear to be any of these, if one is not?

True.

Then may we not sum up the argument in a word and say truly: If one is not, then nothing is?

Certainly.

Let thus much be said; and further let us affirm what seems to be the truth, that, whether one is or is not, one and the others in relation to themselves and one another, all of them, in every way, are and are not, and appear to be and appear not to be.

Most true.

–THE END–

## Conclusion

What is the significance of knowing about definitions? Underlying the work of the early Greeks, and a very few others is a principle that is not often, if ever, been well voiced. How the names of any particular grammar \ logic are manipulated is determined exclusively on the fundamental level of that grammar \ logic by the naming convention. The naming convention is a model of the paradigm of logical manipulation for that logic. A naming convention is developed from an abstraction denoting some causal relation. For example, both the definition of unit and Place Value Notation are the ordered naming convention in the grammar of Arithmetic. Every operation in Arithmetic, must maintain this naming convention or that method of manipulating the Arithmetic Names is not valid–it will not yield true results. Although common grammars are not ordered, they still have a convention of naming that they must adhere too in order for valid manipulation of those names to take place. The discovery of the significance of this order was attributed by Aristotle to Socrates. That order is established by the definition of a name. This means that there are two naming conventions in common grammars–one based on ***thing***, one on ***form*** and ***matter***. A grammar always starts with a naming convention and the significance of that convention–all methods of manipulation of those names must respect and maintain that convention. This is another way of saying, as Socrates and Plato did, that reason must follow from the definition of a word.

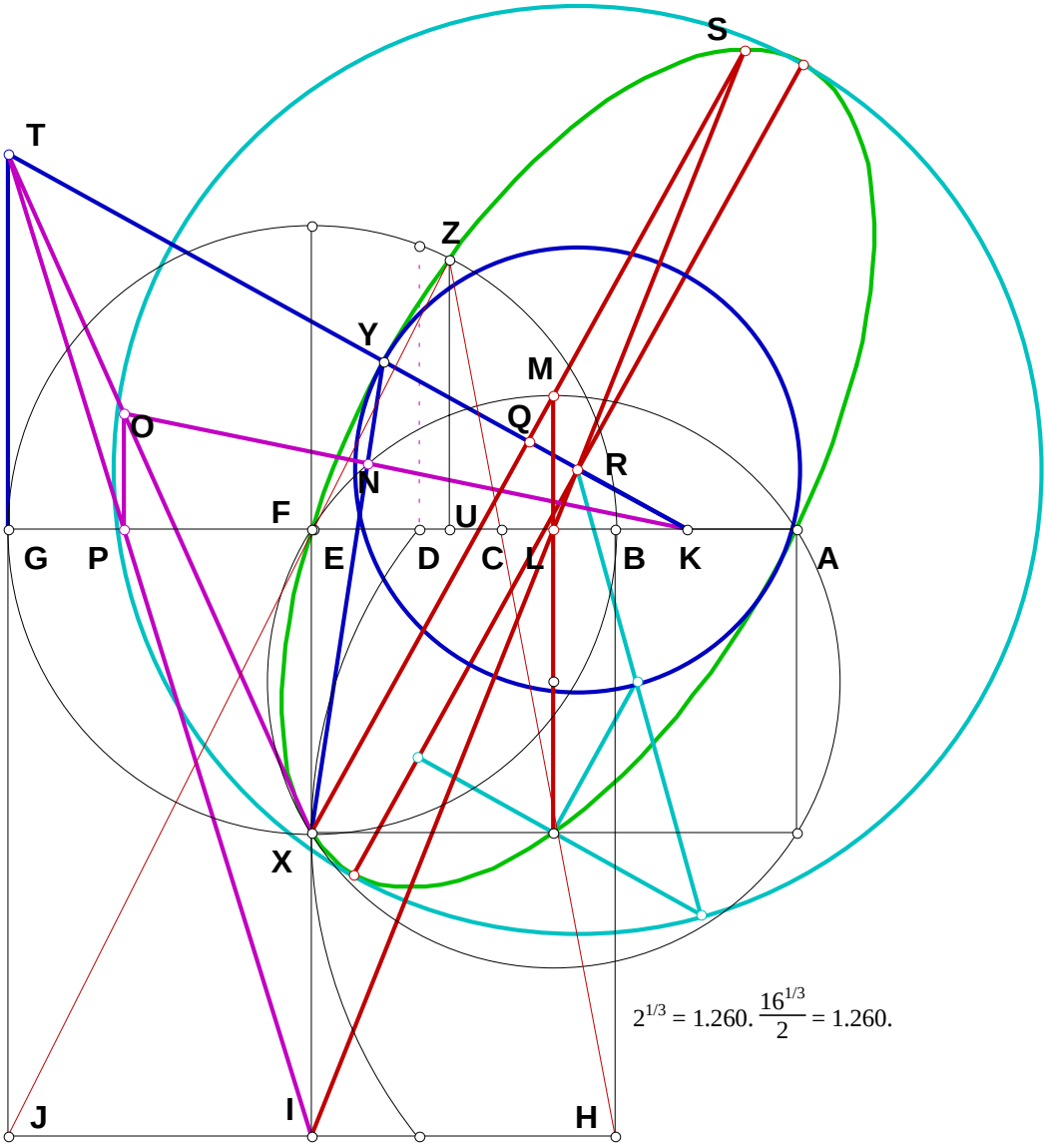
One would not think of calling someone a mathematician who did not have some notion of what a unit was or the significance of Place Value Notation, yet when it comes to common grammars, what a definition is and its significance to reasoning more often than not escapes a Ph.D. in any field–most cannot even tell you where the meaning of a word comes from. Definition is not an infinite circle-jerk. It is still true today what Socrates tried to warn men about, we think we know what we do not know. Man is still thoughtlessly celebrating the death of Socrates. We are still wandering about in a hemlock induced mental stupor.

Only after one knows the fundamentals of predication can one then develop the grammatical manipulations for constructing larger sentences, just like Arithmetic. For example, The car is red. becomes, The red car is fast. The fast red car is mine. etc. The addition of names to a simple premise in order to construct larger and larger sentences is determined by the definitions of the various words used in that sentence and rules of placement and parsing established by some social convention. In other words, despite the technology of our times, we are still fundamentally preliterate–the horror of which cannot be imagined by the common man until he finds himself in the grips of a judicial system that violates every tenet of human rights–simply because those judges are sociopathically expressing their illiteracy. This I know first hand.

At some point I have to stop editing this, if I am to get it out for Christmas presents, and this seems to be a favorable time. Perhaps at some future date I may be enticed to revise this. 11/26/01

# The Idiot Crafticus

A workbook for the Delian Quest.




$2^{1/3} = 1.260. \frac{16^{1/3}}{2} = 1.260.$

## 06/20/92 A Triplicate Ratio

Given BF = $N_1$, MO = $N_2$ =

CE = $N_3$ =

AQ = CE/2

BM =

FM =

AB =

AF =

AD =

DQ =

QR =

QP =

AP =

AR =

AC =

AE =

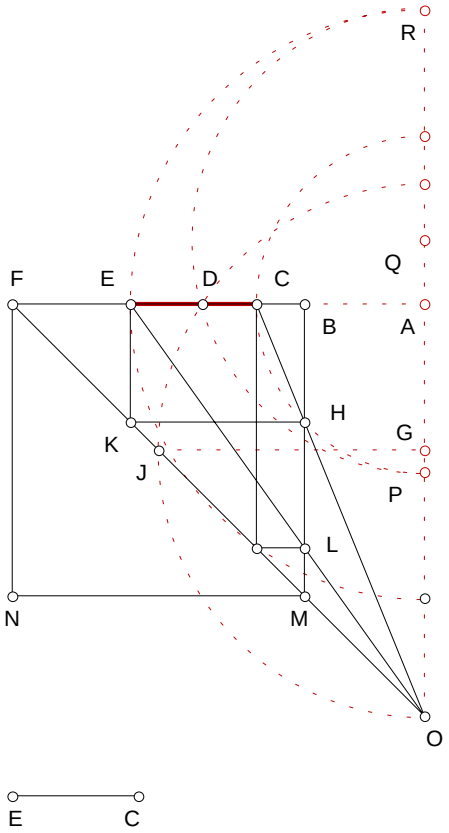
AO =

BC =

EF =

EK =

BH =







$$\frac{BC}{BH} - \frac{AC}{AO} =$$

## 08/12/92 Rusty Cube of a Sphere

Given AB = N.

BD =

BG =

DJ =

CG =

BC =

CD =

DG =

GJ =

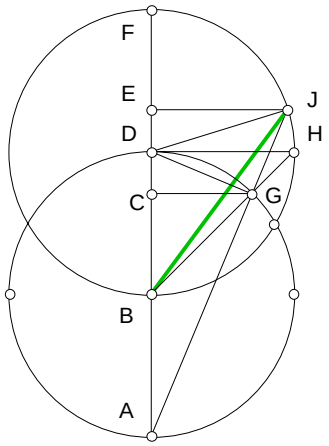
AC =

AG =

AJ =

AE =

EJ =




BE =

BJ =

$$\frac{(4/3 \times \pi \times N^3)^{1/3}}{BJ} =$$

## 01/08/93 Pythagoras Revisited

Given AB = $S_1$, BC = $S_2$, AC = $S_3$

AG =

BH =

AE =

BF =

EF =

DE =

DF =

AD =

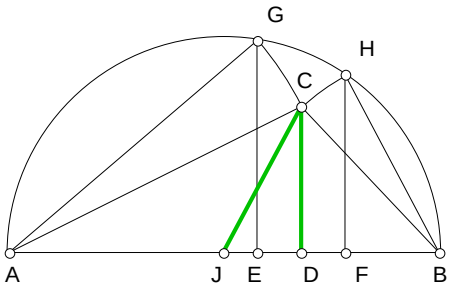
BD =

CD =

AJ =

JD =

CJ =



## 06/03/93 Exploring The Properties Of The Curve AH

Given AG = $N_1$, AC = AG/$N_2$

GF = AG/3

AE =

EG =

AF =

FM =

GM =

GN =

EN =

NH =

NS =

PN =

PS =

ST =

PQ =

SQ =

QT =

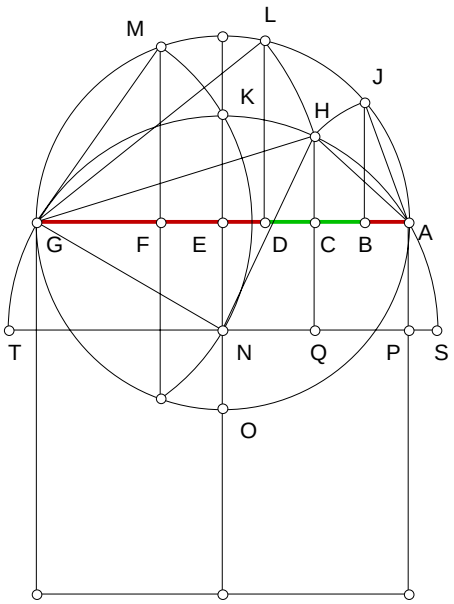
QH =

CQ =

CH =

AH =

CG =




GH =

AJ =

AB =

GL =

DG =

BD =

$\sqrt{AB \times DG} - BD =$

## 06/07/93 For All Triangles Find BD

Given AB, BC, AC, CD, AD.

CE =

CF =

EF =

DE =

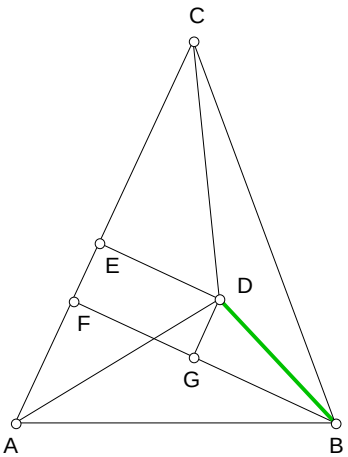
BF =

GF =

BG =

DG =

BD =



## 06/09/93 Rectangular Roots

Given AD = $N_1$, DE = $N_2$

CF =

BF =

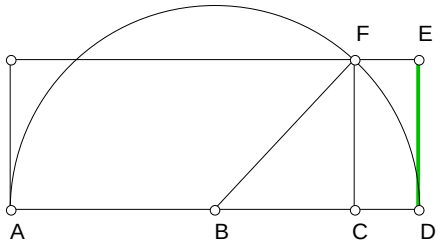
AB =

BC =

AC =

CD =

$\sqrt{CD \times AC} - DE =$




## 06/21/93 A Pyramid of Ratios I

Given AB = 1, AD = $\frac{AB}{N_1}$, AL = $\frac{AB}{2}$

$\delta = 1 \ldots N_2$, DE = $\frac{CD \times \delta}{N_2}$

DL =

AC =

CL =

CD =

DK =

AK =

BK =

EK =

BE =

HK =

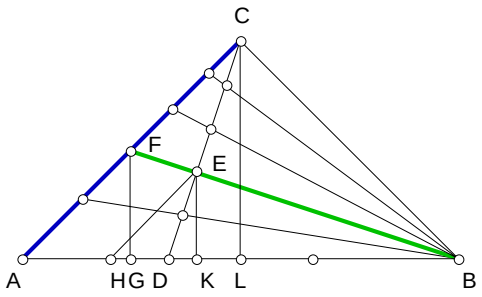
BH =

EH =

AF =

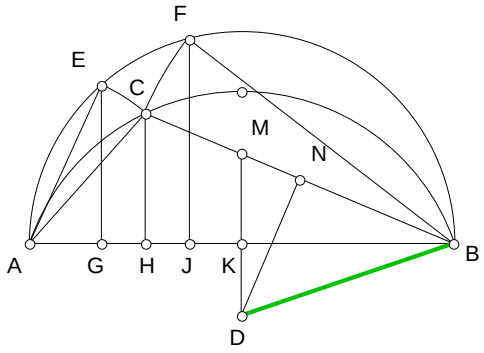
BF =

EF =



## 06/27/93 Describe a Circle about a Triangle

Given AB, BC, AC, find BD.




BK =

AE =

BF =

AG =

BJ =

GJ =

HJ =

BH =

CH =

BN =

BM =

MN =

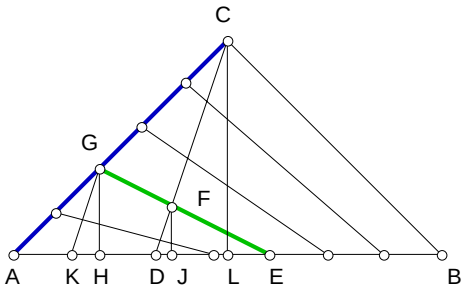
DN =

BD =

## 07/15/93 Pyramid of Ratios II

Given AB = 1, AD = $\frac{AB}{N_1}$, DE = $\frac{BD}{N_2}$

AG = $\frac{AC}{N_2}$.




AC =

BD =

AE =

AH =

GH =

EH =

EG =

AL =

DL =

CL =

HK =

EK =

DJ =

FJ =

DF =

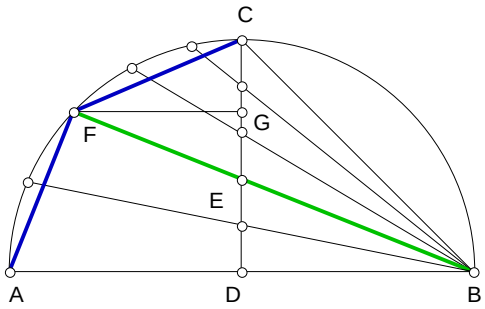
EF =

FG =

CD =

## 07/25/93 Pyramid of Ratios III

Given AB = 1, AD = $\frac{AB}{2}$ , DE = $\frac{CD}{N}$.




BD =

CD =

BC =

BE =

BF =

AF =

EF =

EG =

FG =

DG =

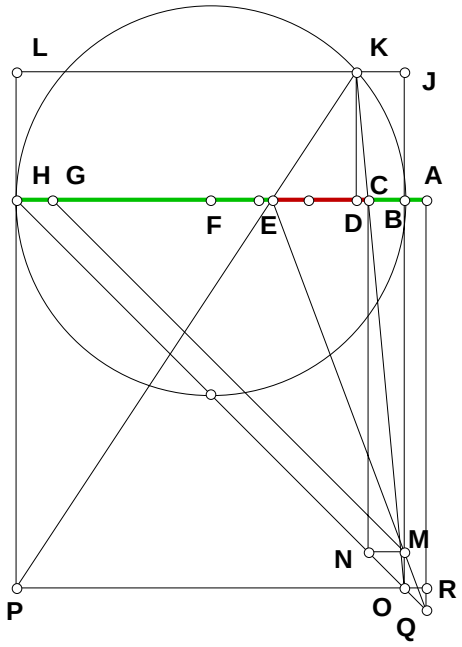
GC =

CF =

## 11/06/93 Gruntwork I on the Delian Solution

Given N, BH = 1, BF = $\frac{BH}{2}$

BD = $\frac{BF}{N}$.




DH =

DK =

BJ =

BO =

JO =

JK =

CD =

KL =

LP =

DE =

BC =

CE =

MN =

GH =

CH =

HN =

GM =

EH =

EG =

HQ =

HO =

OQ =

OR =

AB =

AC =

AE =

AH =

AD – $(AB \times AC^2)^{1/3}$ =

AF – $(AB^2 \times AC)^{1/3}$ =

## 11/09/93 Solve for Cube Placement

Given N, BH = N – 1

BG = $\frac{BH}{2}$, CF = $N^{2/3} - N^{1/3}$

BL = CF.

GP =

BK =

BD =

NP =

GN =

EN =

GE =

CE =

BC =

GH =

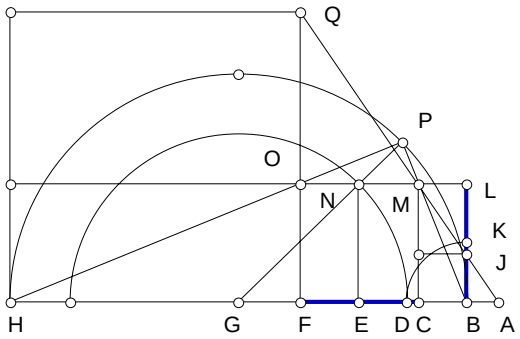
EF =

FH =

FQ =

FO =

OQ =



MO =

AF =

AC =

AH =

AB =

$(AB^2 \times AH)^{1/3} - AC$ =

$(AB \times AH2)^{1/3} - AF$ =

## 11/10/93 Gruntwork II on the Delian Solution

Given N, DE = $\frac{AE}{N}$.

AD =

AH =

AG =

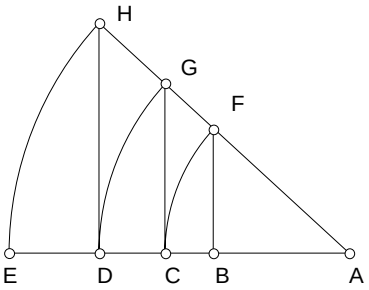
AC =

AF =

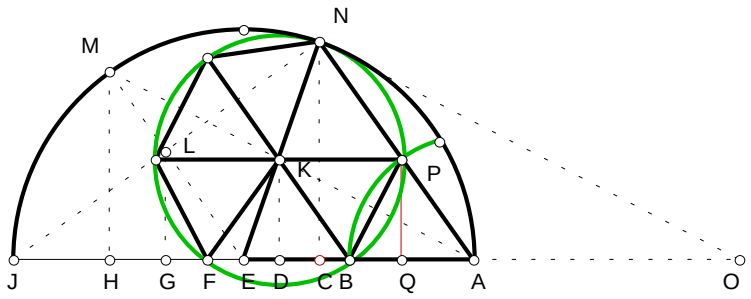
AB =

$(AB^2 \times AE)^{1/3} - AC$ =

$(AB \times AE^2)^{1/3} - AD$ =



## 11/11/93 The Archamedian Paper Trisector.




Given N, AJ = 1, AE = $\frac{AJ}{2}$ and AC = $\frac{AJ}{N}$.

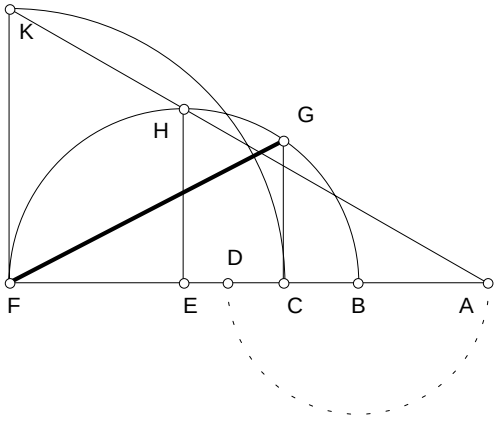
EJ =
EN =
EM =
CJ =
CN =
JN =
JL =
GL =
GJ =
EG =
EL =
EH =
HM =
AH =
CO =

CE =
EO =
EK =
DE =
DK =
AD =
KN =
BK =
BD =
AB =
AB – BK =
AN =
AP =
PQ =
PQ – DK =

## 11/12/93 To Square a Circle

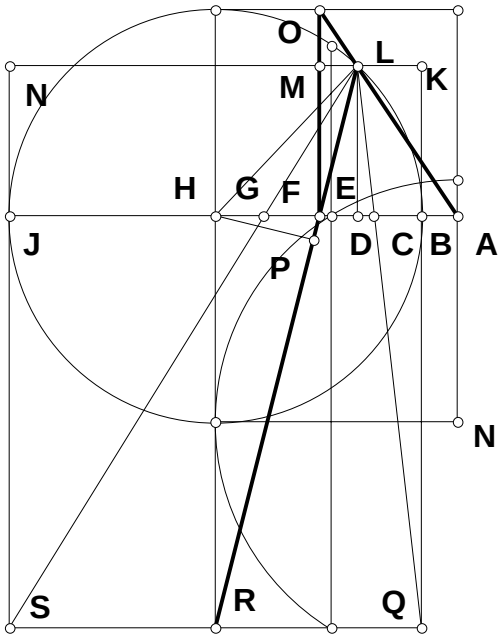
Given BF = 1, BD = ¾ × BE.

BE =
EF =
EH =
AB =
AE =
AF =
FK =
CF =
BC =
CG =
FG =



## 11/18/93 Exploring Cube Roots Plate A

Given N, BJ, BH = $\frac{BH}{2}$ , BF = $\frac{BH}{N}$.



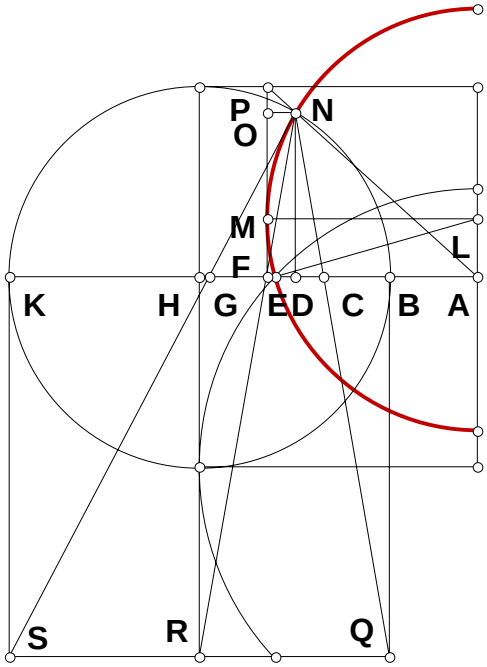

HL =
FH =
HR =
FR =
FP =
PH =
LP =
FL =
DF =
DL =
FO =
MO =
LM =
AF =
AB =
BQ =
BK =
BD =
KQ =
KL =
BC =
DJ =
LN =

JS =
JN =
NS =
GJ =
BG =
AC =
AG =
AJ =
$(AB^2 \times AJ)^{1/3} - AC =$
$(AB \times AJ^2)^{1/3} - AG =$

## 11/18/93 Exploring Cube Roots Plate B

Given N, BK = 1, BH = $\frac{BK}{2}$, BD = $\frac{BH}{N}$.

DK =
DN =
BQ =
KS =
HR =
BC =
GK =
BG =
DH =
FH =
BF =
CF =
AL =
DF =
NO =
FP =
PO =
AD =

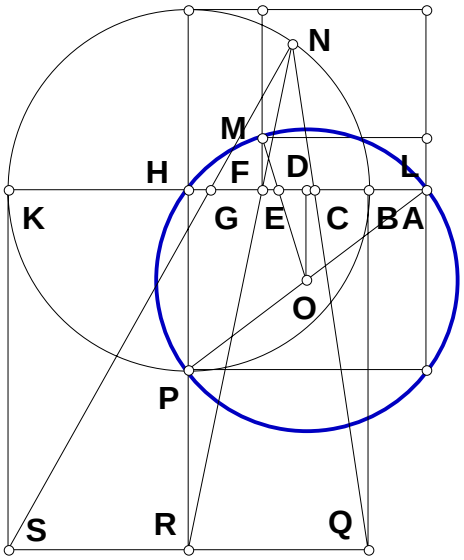



AB =
AF =
LM =
EL =
AK =
$\sqrt{EL^2 - AL^2} - \sqrt{AB \times AK}$ =

## 11/18/93 Exploring Cube Roots Plate C

Given N, AB = 1, AK = AB × N,

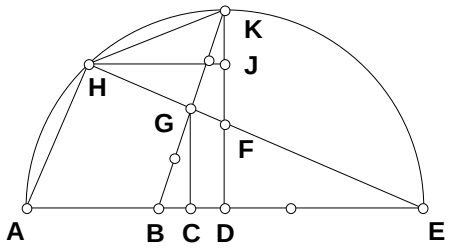
BK =
AC =
AG =
CG =
CF =
BH =
AH =
HP =
AP =
AO =
DO =
AF =
AD =
DF =
MO =




FM =
$MO^2 - [DF^2 + (DO + FM)^2]$ =

## 11/24/93 Pyramid of Ratios IV

Given $N_1$, $N_2$, AE = 1, AB = $\frac{AE}{N_1}$




BG = $\frac{BK}{N_2}$

AD =

DK =

DE =

BD =

BK =

BC =

CG =

AC =

CE =

DF =

EG =

EF =

AH =

EH =

GH =

FH =

FJ =

HJ =

DJ =

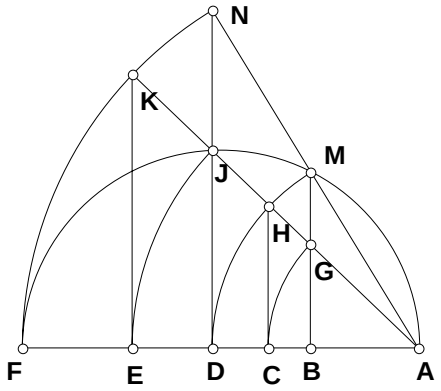
JK =

HK =

$\frac{AH}{HK} - \frac{\sqrt{2} \times N_1}{2 \times (N_1 - 1)\,(N_2 - 1)} =$

## 12/04/93 $M^{1/2^N}$ Exponential Series

Given M, AF = M, AB = 1.




BF =

BM =

AM =

AN =

AD =

DF =

DJ =

AJ =

AK =

AE =

AH =

AC =

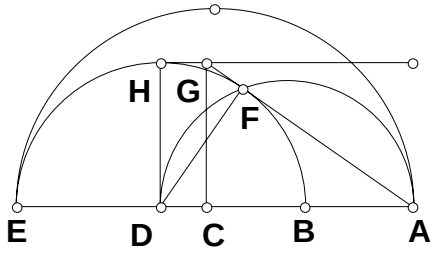
$M^{1/4}$ – AC =

$M^{2/4}$ – AD =

$M^{3/4}$ – AE =

## 12/06/93 Methods: Square Roots

Given N, AB = 1, AE = AB × N.




BE =

BD =

DF =

AD =

AF =

AC =

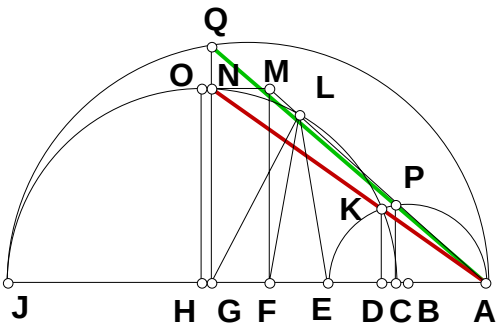
$\sqrt{AB \times AE} - AC =$

## 12/06/93b Gruntwork on the Delian Solution IV

Given N, AC = 1, AJ = AC × N,

$AE = (AC^2 \times AJ)^{1/3}$

$AG = (AC \times AJ^2)^{1/3}$




CG =

CJ =

GJ =

GN =

AB =

CE =

CH =

BK =

HK =

HJ =

AH =

BH =

BD =

DE =

AD =

DK =

GQ =

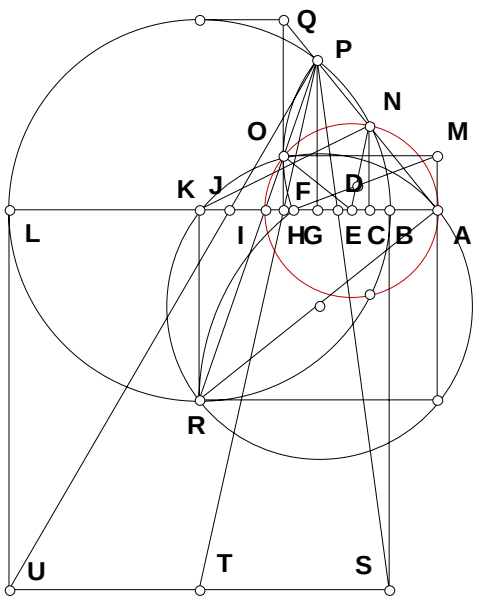
CP =

$\frac{AG}{GN} - \frac{AD}{DK} =$

$\frac{AG}{GQ} - \frac{AC}{CP} =$

## 12/11/93 The Structure in Red

Given N, AB = 1, AL = AB × N.




BL =
BK =
AE =
AJ =
BE =
BJ =
JL =
EJ =
FJ =
FL =
BF =
FP =
KR =
KL =
FK =
IK =
AK =
AI =
AD =
KT =
FH =
AF =
AH =
HI =
HO =

DN =
KN =
DK =
CK =
AC =
CI =
CN =

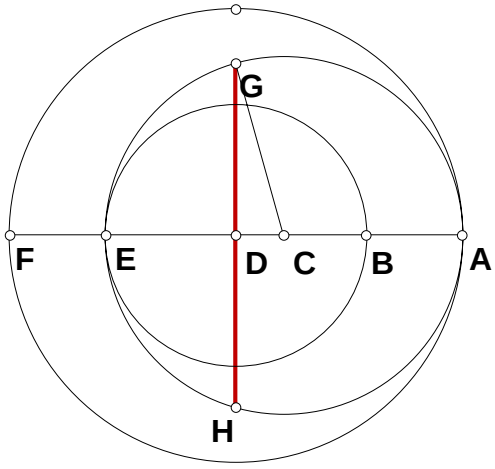
$$\frac{KR}{IK} - \frac{HO}{HI} =$$

$$\frac{AF}{FP} - \frac{AC}{CN} =$$

## 12/12/93 The Square Root

Given N, BE = 1, AF = BE × N,

$AD = \frac{AF}{2}$, $BD = \frac{BE}{2}$.




AB =
AE =
AC =
CG =
CD =
GH =

$GH - \sqrt{AF \times BE} =$

## 12/12/93B Generalize 12/12/93

Given $N_1$, $N_2$, $N_3$, AF = $N_1$

DF = $\frac{AF}{N_2}$, DE = $\frac{DF}{N_3}$

AD =

AE =

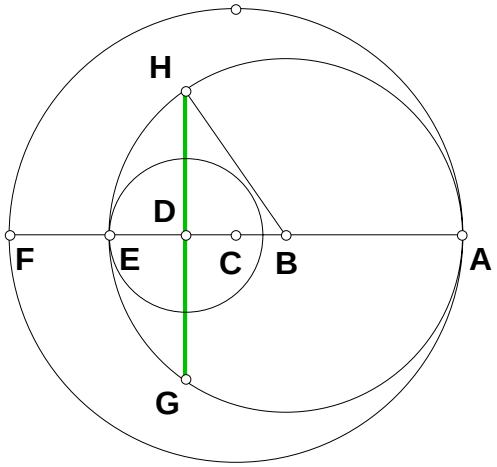
AB =

BD =

BH =

GH =

GH – $2 \times \frac{N_1 \times \sqrt{N_2 - 1}}{N_2 \times \sqrt{N_3}}$ =




## 04/06/94 Inscribe a Circle in a Triangle

Given AB, BC, AC.

AK =

BD =

AF =

FK =

CF =

CK =

AN =

AH =

HN =

BF =

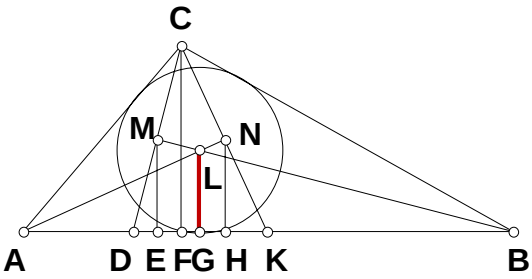
DF =

CD =

BM =

BE =

GL =



## 04/21/94 The Cradle

Given N, AB = 1, AL = AB × N.

AF = `

AC =

AJ =

BL =

BP =

LR =

FL =

BC =

BJ =

JL =

BF =

FJ =

CF =

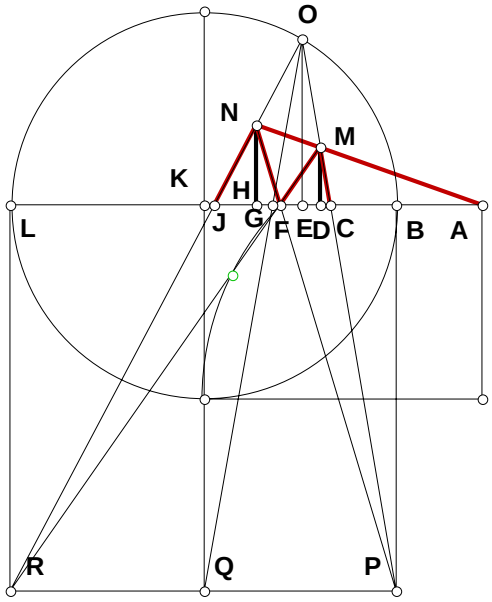
FG =

GN =

CD =

DM =

AD =




AG =

$$\frac{AG}{GN} - \frac{AD}{DM} =$$

## 04/26/94 Tangents and Similarity Points

Given $R_L$, $R_S$, D, AC = $R_L$,

BD = $R_S$, AB = D.

DE =

AE =

CE =

AO =

AG =

GO =

BH =

BO =

HO =

GH =

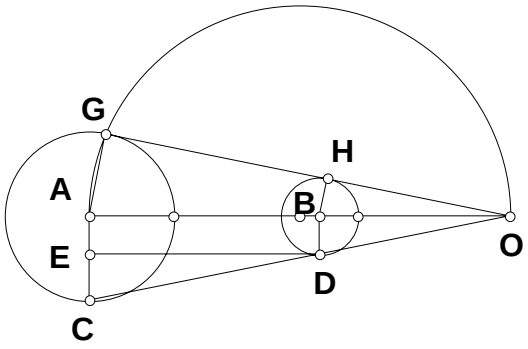
AP =

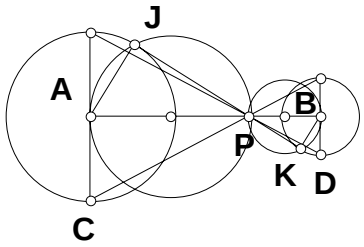
BP =

AJ =

BK =

JP =




KP =

JK =

## 04/27/94 The Chordal or Power Line of two Circles

Given AB, AD, BC, IJ.

AE =

BF =

GH =

GI =

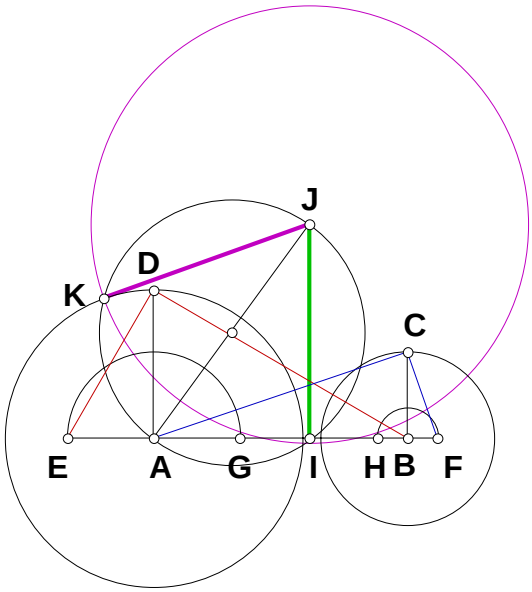
AI =

BI =

AJ =

AK =

JK =




## 04/28/94 The Power Point

Given AB, CD, EF, AC, AE, CE.

AG =

AH =

AM =

EM =

AK =

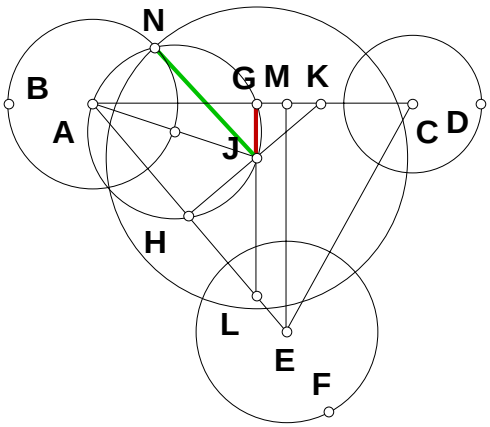
GK =

GJ =

AJ =

AN =

JN =




## 04/30/94 Division $N^2$

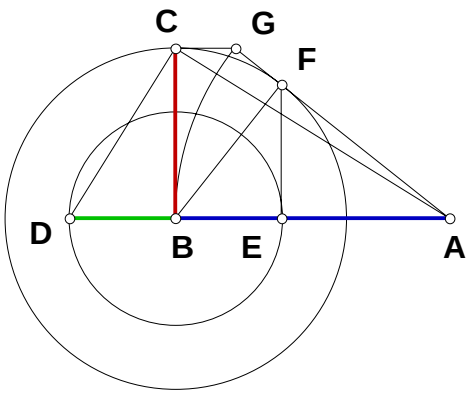
Given $N_1$, $N_2$, AB = $N_1$, BC = $N_2$

AC =

CD =

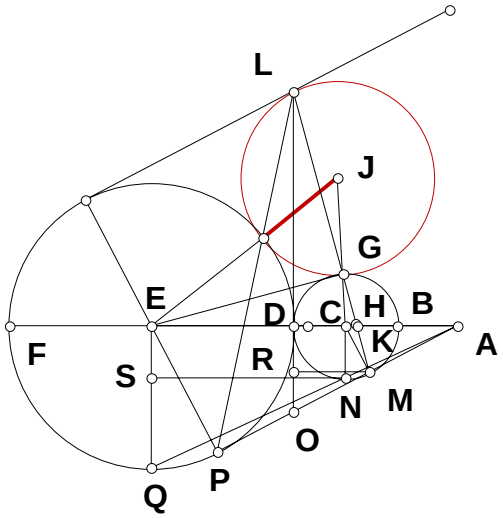
BD =

$\frac{N_2^2}{N_1}$ – BD =



## 05/01/94 Two Circles and a Parallel

Given $R_1$, $R_2$, DE = $R_1$, BC = $R_2$.




CN =
EQ =
CD =
CE =
ES =
NS =
SQ =
AE =
AD =
EP =
AP =
DO =
DL =
AC =
AM =
AO =
MO =
MR =
RO =

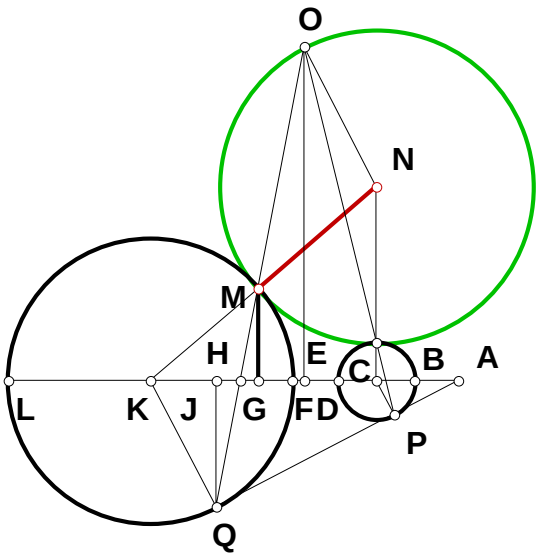
DR =
ML =
CK =
CM =
MG =
GJ =

LR =
DK =
CH =
MH =
GL =

## 05/04/94 Two Circles and a Tangent

Given $R_1$, $R_2$, D, N, FK = R1

BC = $R_2$, CH = D, FG = $\frac{FL}{N}$.




FL =
AK =
EK =
AQ =
GL =
GM =
AJ =
AF =
FJ =
JL =
JQ =
GJ =
QM =
GH =
HM =

EF =
HO =
KM =

EH =
MO =
MH =

## 05/16/94 Tangent to Diameter and Circles

Given $N_1$, $N_2$, CF = 1, CD = $\frac{CE}{N_1}$

EJ = CE × $N_2$.

CE =

DE =

DJ =

JG =

BE =

EG =

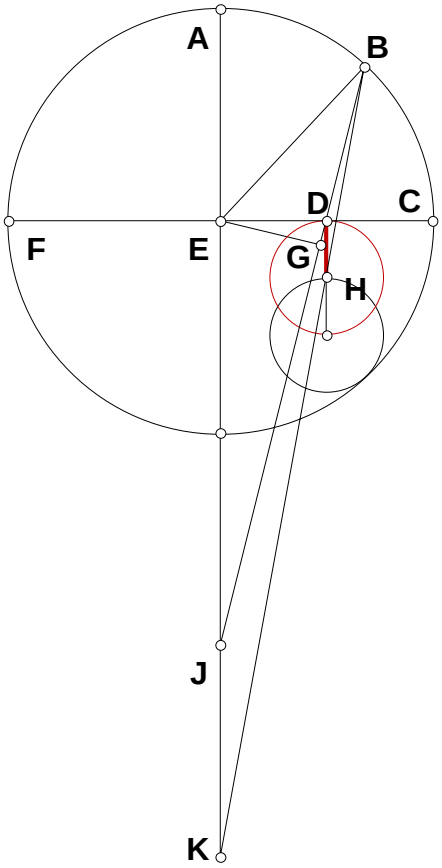
BG =

BJ =

JK =

BD =

BH =




## 05/16/94b Tangent to Diameter and Circles

Given $N_1$, $N_2$, AF = 1, DE = $\frac{DF}{N_1}$

AJ = AF × $N_2$.

DF =

HJ =

EF =

FJ =

EJ =

EG =

BF =

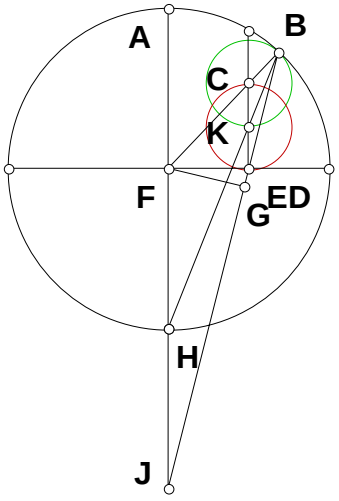
FG =

BG =

BE =

BJ =

KE =




BC =

BC – KE =

## 10/27/94 Method: Square Root

Given N, AB = 1, AG = AB × N.

BG =

BF =

AF =

FH =

DF =

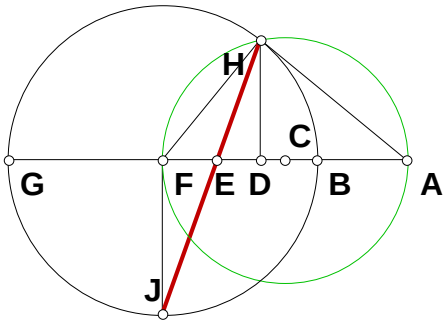
AD =

BD =

DG =

FJ =

DH =




DE =

AE =

$\sqrt{AB \times AG} - AE =$

## 10/28/94 Method: Square Root

Given N, AB = 1, AH = AB × N.

BH =

BG =

GK =

AG =

DG =

AD =

AL =

GL =

BD =

DH =

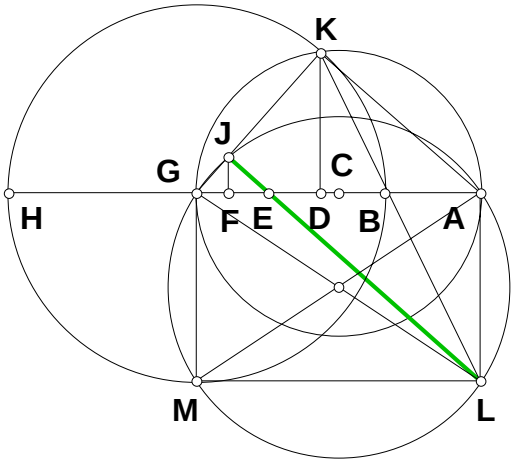
DK =

KL =

GJ =

JL =

FG =




AF =

FJ =

EF =

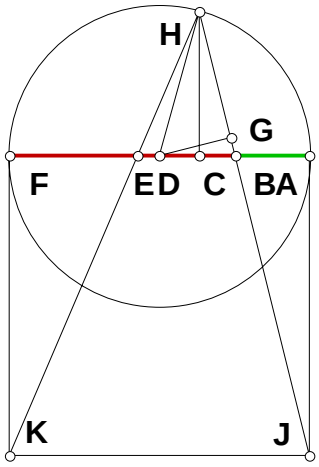
AE =

$\sqrt{AB \times AH} - AE =$

## 10/31/94 Divide a Segment Such

Given N, AB = 1, AF = AB × N.

BF =
AD =
AJ =
FK =
BD =
BJ =
BG =
DH =
DG =
GH =
HJ =
BC =
AC =
CF =
CH =
CE =

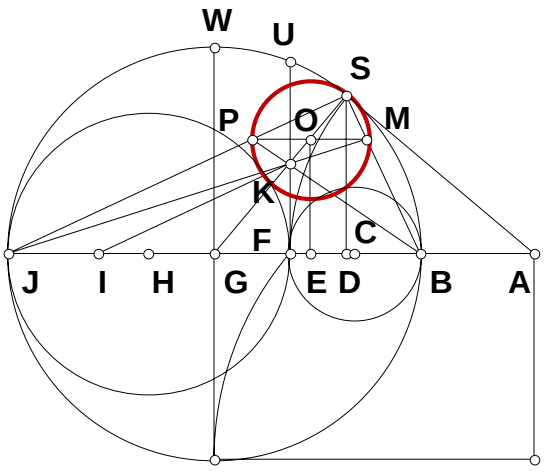



EF =
BE =
DF =
CD =
$BE^2 - EF$ =

## 12/24/94 Power Line at Square Root

Given N, AB = 1, AJ = AB × N.

AF =
BJ =
BG =
AG =
GS =
DG =
FG =
BD =
DJ =
DS =
FK =
BF =
BK =
FI =
BI =




BP =
KP =
MP =
OS =

## 12/26/94 Point G

Given N, AB = 1, AF = AB × N.

BF =

BE =

EF =

EK =

AD =

CE =

CF =

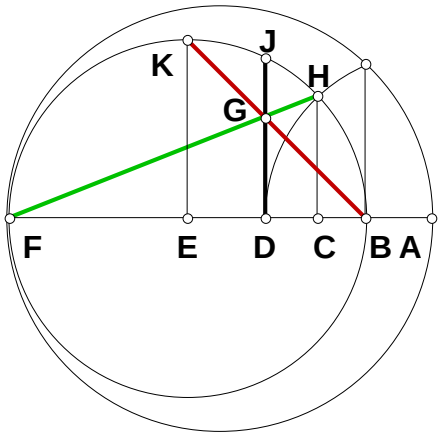
DF =

BC =

CH =

$DG_1$ =

BD =




$DG_2$ =

$DG_1 - DG_2$ =

## 01/06/95 Alternate Method Quad Roots

Given N, AC = 1, AJ = AC × N.

CJ =

AE =

CE =

EJ =

EN =

BM =

HO =

MN =

NO =

BE =

EH =

BJ =

MJ =

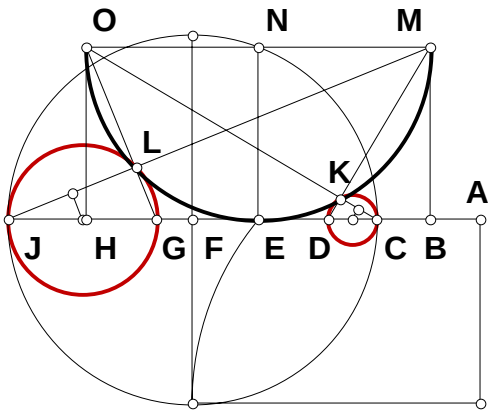
MO =

ML =

JL =

GJ =

AG =




$(AC \times AJ^3)^{1/4} - AG$ =

CH =

CO =

KO =

CK =

CD =

AD =

$(AC^3 \times AJ)^{1/4} - AD$ =

## 09/13/95 A Study in Placement

Given $N_1$, $N_2$, $N_3$, $N_4$.

Given AE = 1, $AB = AE \times \frac{N_1}{N_2}$,

$AC = \frac{N_3}{N_4}$ what is GH?

AD =

DF =

DE =

CD =

CF =

CE =

BD =

BE =

EF =

EG =

FG =

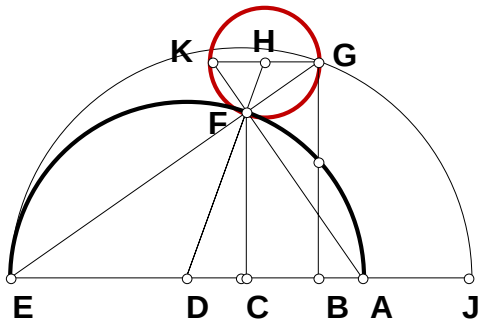
GJ =

Given AE = 1, $AB = AE \times \frac{N_1}{N_2}$

$EF = AE \times \frac{N_3}{N_4}$.

DE =

BD =




BE =

EG =

FG =

GH =

Given AE = 1, $AB = AE \times \frac{N_1}{N_2}$

$BG = AE \times \frac{N_3}{N_4}$.

AD =

BE =

EG =

EJ =

EF =

FG =

GH =

## 10/14/95 Alternate Method: Square Root

Given $N_1$, $N_2$, AB = 1

$AF = AB \times N_1$, $AK = AB \times N_2$.

BF =

BO =

AO =

KM =

AM =

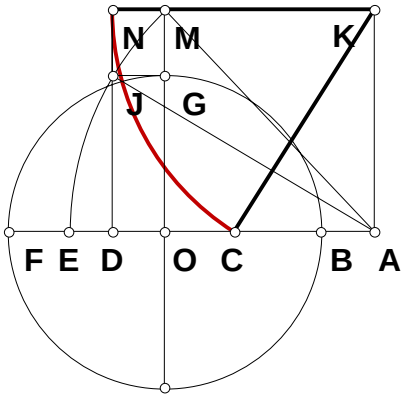
GO =

DJ =

AJ =

AD =

KN =



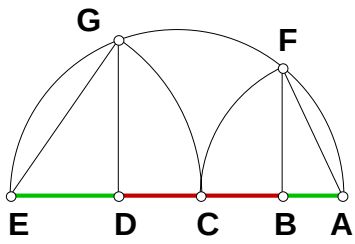

CK =

AC =

$\sqrt{AB \times AF} - AC =$

## 10/20/95 Four Times The Square

Given N, AE = 1, AB = $\frac{AE}{N}$.



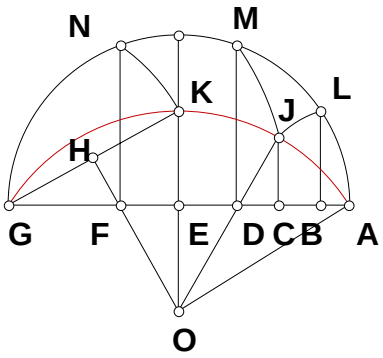

BE =
BF =
AF =
AC =
CE =
EG =
DE =
BD =

$$\frac{BD^2}{4 \times (AB \times DE)} =$$

## 11/01/95 Gemini Roots

Given $N_1$, $N_2$, $N_3$, AG = 1

EF = $\frac{AG}{2 \times N_1}$, DE = AE × $\frac{N_2}{N_3}$.




AE =
EG =
AF =
FG =
FN =
GN =
GK =
EK =
EO =
OK =
DE =
DO =
DJ =
CD =
CE =
CJ =

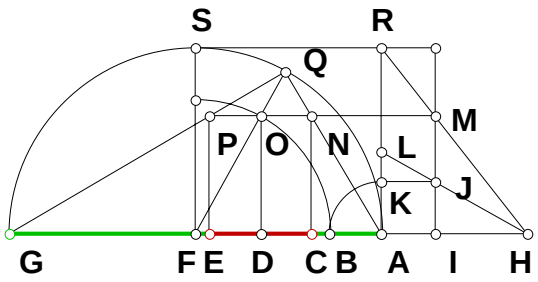
AC =
AJ =
AL =
AB =
CG =
GJ =
GM =
DG =
BD =

$$\frac{N_1 - 1}{2} - \frac{AB \times DG}{BD} =$$

## 11/05/95 Gemini Roots Again

Given N1, N2, AG = 1

AL = $\frac{AR}{N_1}$, IM = $\frac{AR}{N_2}$.




AF =

AR =

FQ =

FG =

AK =

DO =

AB =

BF =

FO =

OQ =

NP =

CD =

DE =

DF =

AD =

AC =

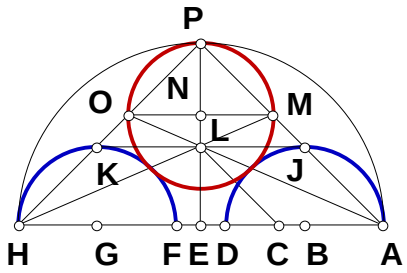
EG =

CE =

$\frac{N_1}{2} - \frac{\sqrt{AC \times EG}}{CE} =$

## 12/01/95 Method for Equals

Given N, AH = 1, AB = $\frac{AE}{N}$.




AE =

EH =

EP =

AP =

CE =

CH =

CL =

AM =

MP =

NP =

$NP - \frac{N - 1}{2 \times (N + 1)} =$

## 12/20/95 Just for Fun

Given N, EF = 1, EJ = EF × N.

AE =

AF =

AB =

BF =

BE =

BH =

BJ =

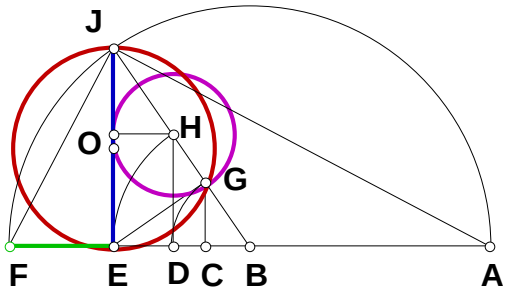
BD =

BG =

BC =

GH =

DE =




HO =

$EG_1$ =

GJ =

$EG_2$ =

$\frac{EG_1}{EG_2}$ =

## 12/29/95

Given $N_1$, $N_2$, AC = $N_1$, CE = $N_2$.

AE =

AD =

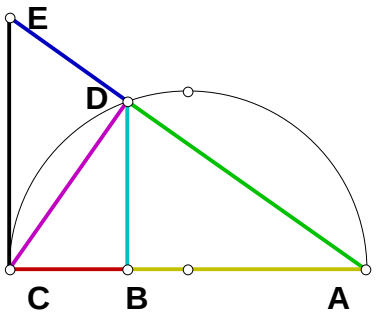
AB =

BC =

BD =

CD =

DE =



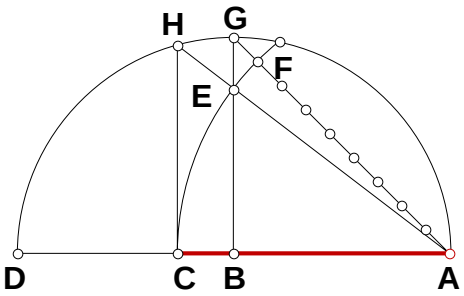

## 01/07/96 Two Cube Rustic

Given AD = 2, AF = $\frac{8 \times AG}{9}$.

AB =

AG =

AC =



## 01/08/96 Alternate Method: Power Line

Given $R_1$, $R_2$, D, DE = $R_1$

KM = $R_2$, EK = D.

DM =

EF =

JK =

FJ =

AD =

BM =

DF =

JM =

FG =

GJ =

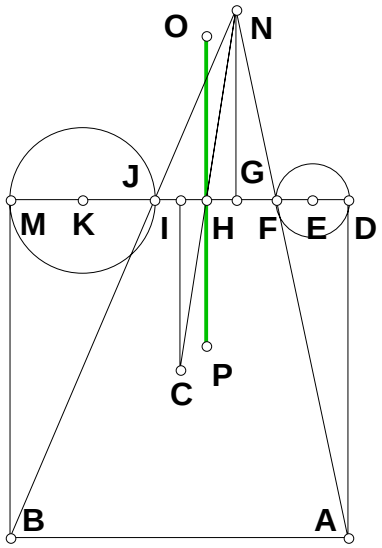
DI =

DG =

GI =

CI =

GN =




GH =

DH =

HM =

## 01/13/96 Pyramid of Ratios VI: Moving the Point

Given $N_1$, $N_2$, AD = 1

AB = $\frac{AD}{N_1}$, BE = $\frac{BG}{N_2}$.

BD =

BG =

BC =

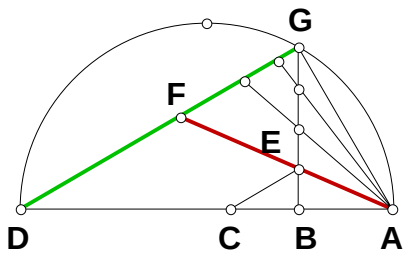
AE =

AC =

AF =

EF =

$\frac{AF}{EF} - \frac{N_2 N_1}{(N_2 - 1)(N_1 - N_1)} =$




DG =

CE =

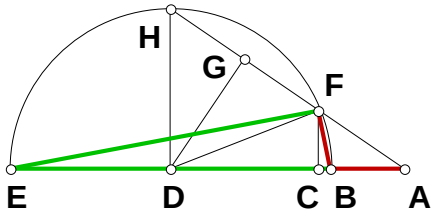
DF =

GF =

$\frac{DG}{GF} - \frac{N_2 + N_1 - 1}{N_2 - 1} =$

## 01/17/96 A Right Triangle In A Given Ratio

Given N, AB = 1, AE = AB × N.



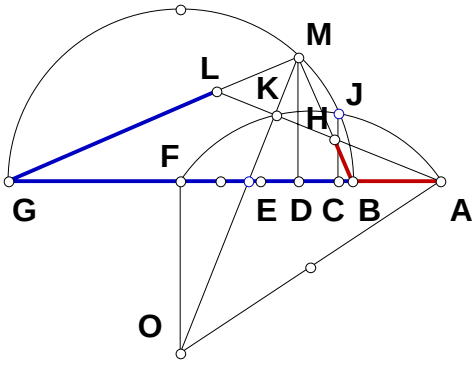

BE =
BD =
DF =
DE =
AD =
DH =
AH =
AG =
GH =
FG =
AF =
CD =

BC =
CE =
CF =
BF =
EF =
AC =
$\frac{AE}{AB} - \frac{EF}{BF} =$

## 01/17/96b Divide The Sides Of A Triangle In A Given Ratio

Given N1, N2, AB = 1

$AG = AB \times N_1$, $BD = \frac{CF}{N_2} + BC$

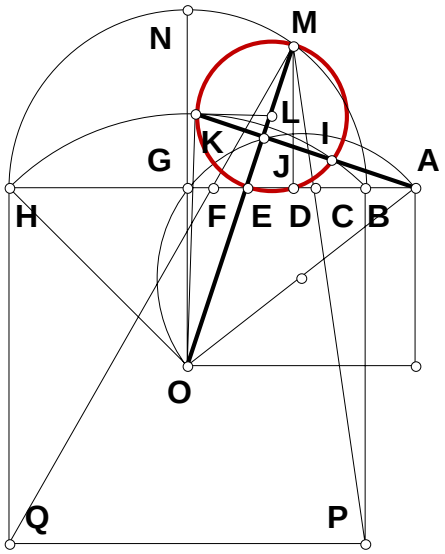



BG =
BF =
AC =
AF =
CF =
BC =
DF =
DG =
DM =
FO =
EF =
AE =
EO =
MO =
EK =

KM =
HK =
HM =
BM =
BH =
GM =
LM =
GL =
$\frac{AG}{AB} - \frac{GL}{BH} =$

## 01/21/96 The Power Line for Cube Roots

Given N, BH = 1, BE = $\frac{BG}{N}$.



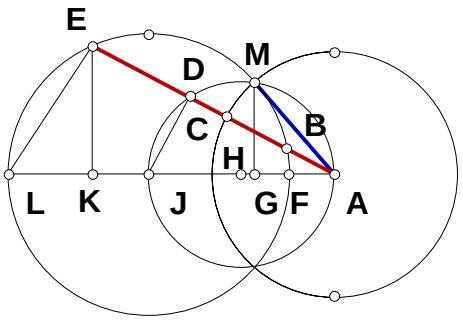

BP =
HQ =
BG =
GO =
GN =
NO =
GH =
EG =
EO =
MO =
EM =
EL =
LK =
LO =
LJ =
EJ =
AE =
AH =
AB =
DE =

DM =
BD =
BC =
DH =
DF =
AC =
AF =
$(AB^2 \times AH)^{1/3} - AC =$
$(AB \times AH^2)^{1/3} - AF =$

## 01/22/96 Alternate Method: Square Root

Given $N_1$, $N_2$, AF = 1

AL = AF × $N_1$, GK = $\frac{GL}{N_2}$.




FL =
FJ =
AM =
AJ =
AG =
GL =
FG =
FK =
KL =
EK =
AK =

AE =
AD =
DE =
BD =
AB =
$\sqrt{AB \times AE} - AM =$

## 01/24/96 Tangent

Given N, AB = 1, AE = AB × N.

BE =

BD =

AD =

DM =

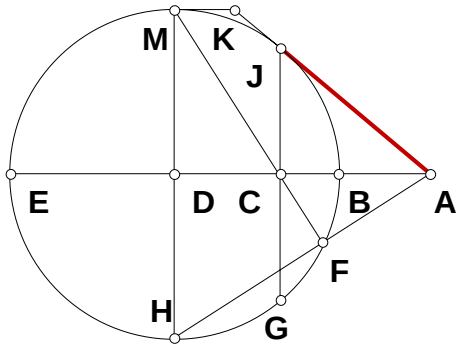
DH =

CD =

BC =

CE =

CJ =




AJ =

AJ – $\sqrt{N}$ =

## 01/25/96 Projecting to the Cubic Similarity Point

Given N, BE = 1, BC = $\frac{BD}{N}$.

BD =

DK =

DJ =

JK =

DE =

CD =

CK =

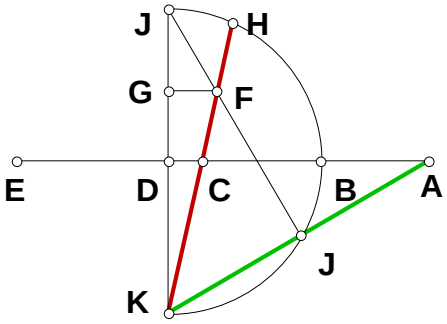
HK =

CH =

CF =

FK =

GK =




FG =

GJ =

AD =

AE =

AB =

## 01/29/96 Linear Division

Given $N_1$, $N_2$, AE = 1

$AH = AE \times N_1$, $AC = \frac{AE}{2}$

$CF = AE \times N_2$.

BC =

CE =

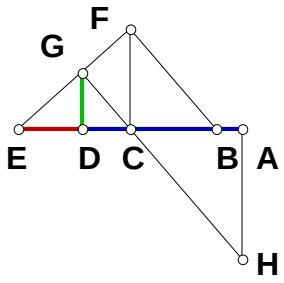
BE =

CD =

DE =

AD =

DG =




Given $N_1$, $N_2$, $N_3$, AE = 1

$AH = AE \times N_1$, $AC = \frac{AE}{N_2}$

$CF = AE \times N_3$.

BC =

CE =

BE =

CD =

DE =

AD =

DG =

## 01/31/96 Gemini Roots

Given $N_1$, $N_2$, BC = 1

$BJ = BC \times N_1$, $CR = CJ \times N_2$.

CJ =

CI =

IJ =

BF =

AB =

AF =

CF =

FJ =

FO =

HS =

FI =

FG =

AG =

OS =

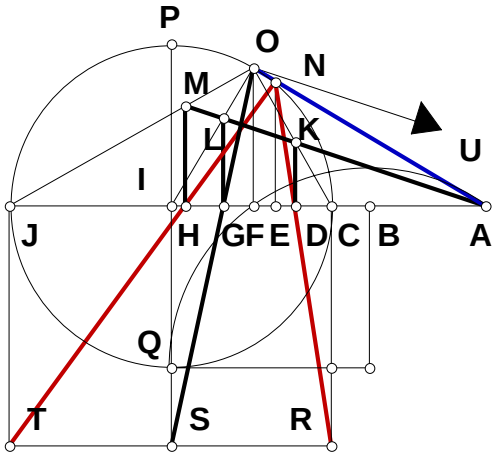
GO =

AJ =

GL =

FU =

AH =




DK =

AC =

CH =

HJ =

CE =

$\frac{AF}{FO} - \frac{AE}{EN} =$

BD =

$\sqrt{BC \times BJ} - \sqrt{BD \times BH} =$

AD =

CD =

DH =

EN =

AE =

BH =

## 02/02/96 Find A Segment

Given N, N > 2 = 1, BC = 1

BE = BC × N.

BD =

CD =

CH =

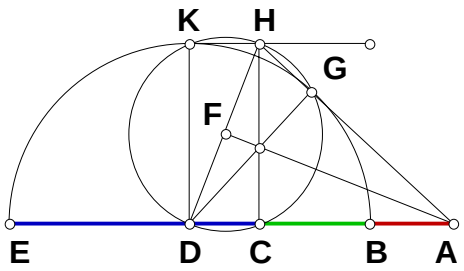
DH =

DF =

AD =

AB =

$AB - \frac{1}{N-2}$




## 04/14/96 Method for Unequals

Given $N_1$, $N_2$, CM = 1

$CE = \frac{CM}{N_1}$, $LM = \frac{CM}{N_2}$.

CK =

EL =

BL =

BM =

BC =

BK =

KS =

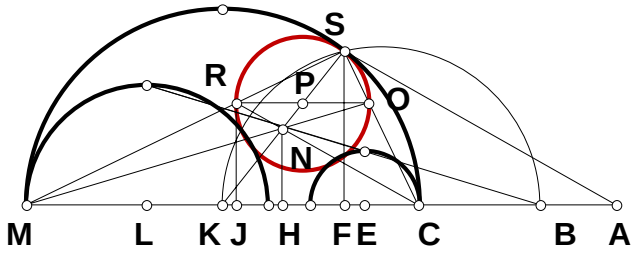
EH =

FK =

CF =

FM =

FS =




HK =

CH =

HN =

AF =

JR =

RO =

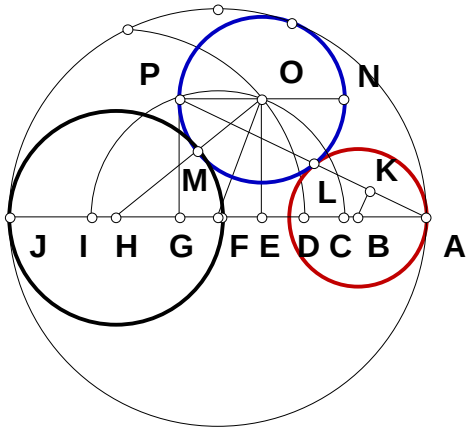
PS =

$PS - \frac{(R_2-2)(N_1-2)}{2\,(N_1N_2-4)} =$

## 04/16/96 Given Three Radii

Given $N_1$, $N_2$, AJ = 1,

$HJ = \frac{AJ}{N_1}$, $NO = \frac{AJ}{N_2}$.




AF =
HM =
MO =
HO =
FO =
AH =
FH =
EH =
EO =
OP =

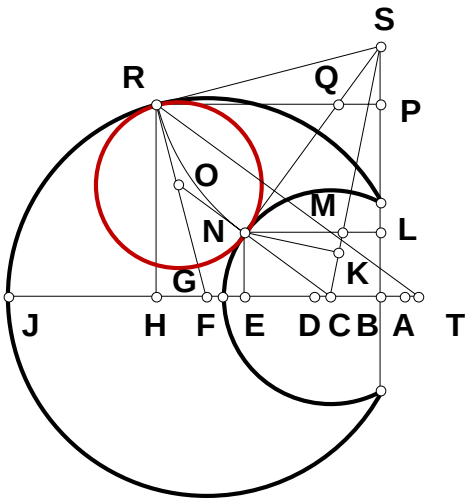
EG =
AG =
AP =
AL =

AE =
GP =
PL =
AB =

## 04/17/96 A Circle In A Crescent

Given $N_1$, $N_2$, $N_3$, AJ = 1

$AF = \frac{AJ}{N_1}$, $AC = \frac{AD}{N_2}$, $BH = \frac{BJ}{N_3}$.




AG =
FR =
AD =
CF =
CG =
BC =
AB =
BJ =
AH =
GH =
HR =
AP =
PR =
PS =
BS =
RS =
NS =
CN =

CS =
SK =
KM =
SL =
EN =
HT =
GO =

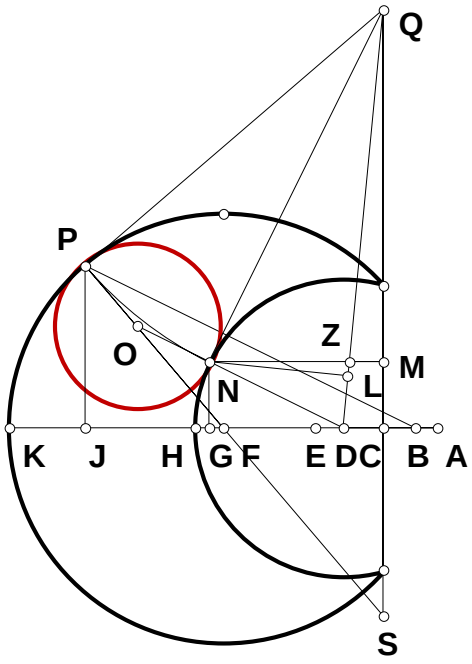
CK =
KN =
SM =
BL =
CE =
GT =
OR =

## 04/17/96b A Circle In A Crescent

Given $N_1$, $N_2$, $N_3$, AK = 1

$AH = \frac{AK}{N_1}$, $AD = \frac{AE}{N_2}$, $CJ = \frac{CK}{N_3}$.

AF =
FP =
FK =
AE =
EH =
DE =
DH =
DN =
DF =
CD =
AC =
CF =
FJ =
JP =
FS =
PS =
QS =
PQ =
CS =
CQ =
NQ =
DQ =
DL =



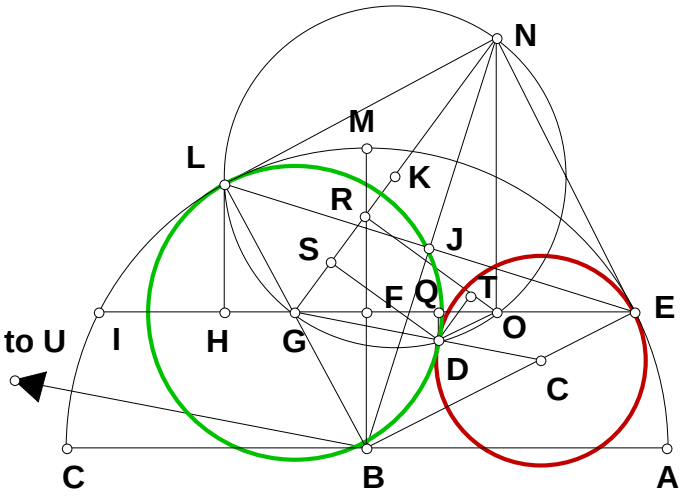

LN =
LZ =
QZ =
MQ =
CM =
GN =
CG =
BJ =
BF =
FO =
OP =

**04/22/96**

Given $N_1$, $N_2$, AC

BF = $\frac{BM}{N_1}$, EG = $\frac{EI}{N_2}$.

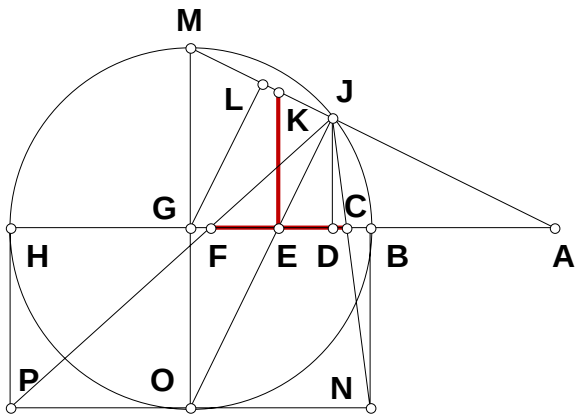
AB =
BM =
BE =
BL =
EF =
EI =
FG =
BG =
GL =
DG =
GH =
HL =
EH =
EL =
JL =
BJ =
LN =
GN =
JN =
EJ =
EN =
GO =
NO =




NR =
GS =
RS =
DT =
DS =
RT =
OR =
OT =
DO =
OQ =
GQ =
DQ =
FU =
CE =

## 04/23/96 Found a Sketch

Given N, AB = 1, AH = AB × N.

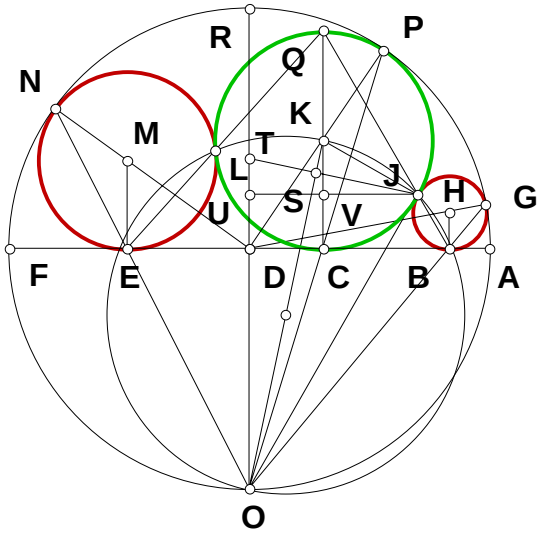



BH =
BG =
BN =
GO =
HP =
GM =
GH =
AG =
AM =
AL =
LM =
JL =
AJ =
AD =
BD =
DH =
DJ =
BC =

DF =
CD =
CF =
CE =
BE =
AE =
EK =
EK – CF =

## 04/24/96 Three Circles Tangent to a Chord

Given N, AF = 1, AC = $\frac{AF}{N}$.




AD =
DO =
OR =
CD =
CO =
PO =
CP =
CK =
JK =
KO =
JO =
KS =
SO =
JS =
ST =
JT =
TO =

TU =
DU =
CV =
CQ =
QV =
BH =

## 04/25/96 One Over N + One

Given $N_1$, $N_2$, AC = 1

$AF = AC \times N_1$, $FK = AC \times N_2$.

CF =

CE =

AE =

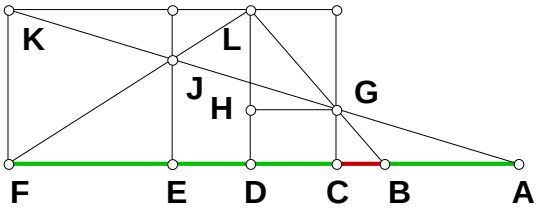
EJ =

DL =

EF =

DF =

CG =




CD =

DH =

HL =

BC =

$BC - \frac{1}{N_1 + 1} =$

## 04/27/96 A Root Figure

Given $N_1$, $N_2$, CD = 1

$CE = CD \times N_1$, $FK = CD \times N_2$.

CF =

DM =

EL =

DF =

EF =

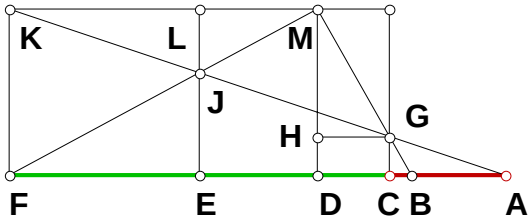
EJ =

JL =

KL =

AF =

AC =




CG =

DH =

HM =

BC =

BF =

BD =

$\sqrt{BD \times BF} - BD =$

## 04/29/96 What is DJ?

Given N, AB = 1, $AG = AB \times N$.

BG =

BF =

FK =

FO =

AF =

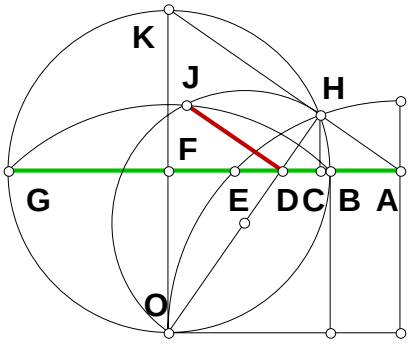
DF =

AK =

KO =

HO =

DO =




DH =

DJ =

$DJ - \frac{\sqrt{N}\,(N - 1)}{N + 1} =$

## 12/20/96 Alternate Method Quad Roots

Given $N_1$, $N_2$, AB = 1

AL = AB × $N_1$, BQ = BS × $N_2$.

BL =

BS =

LT =

BH =

HL =

AF =

FL =

BF =

FP =

FN =

EF =

EL =

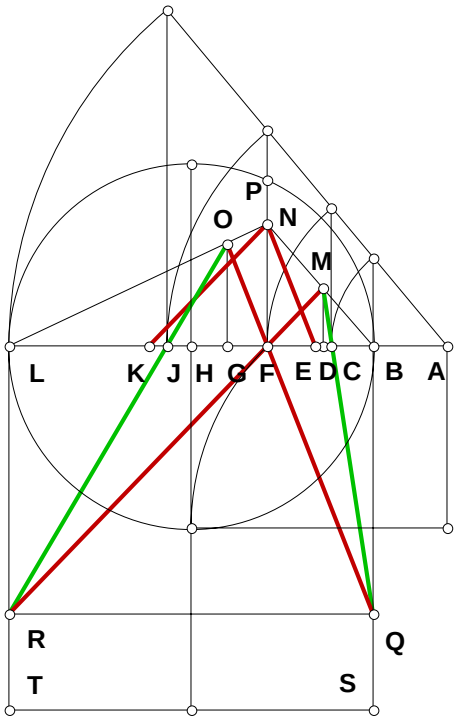
FG =

GO =

GL =

LR =

JL =




AJ =

$(AB \times AL^3)^{1/4} - AJ$ =

## 04/04/97 Triangles

Given AB, AC, CD.

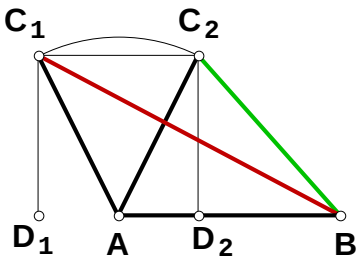
AD =

$BD_1$ =

$BD_2$ =

$BC_1$ =

$BC_2$ =



## 09/11/97 The Ellipse

Given $N_1$, $N_2$, AD = 1

$EF = \frac{AD}{N_1}$, $AB = \frac{AD}{N_2}$.

BD =

BJ =

AC =

BC =

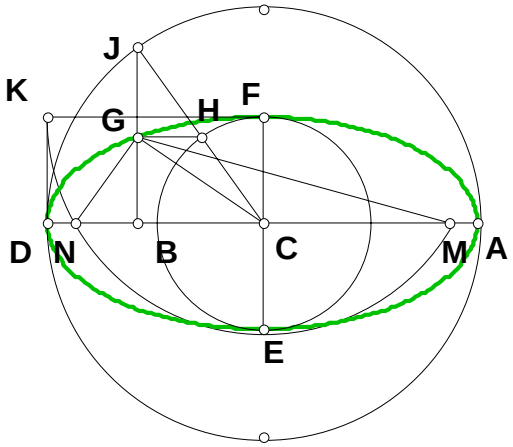
CH =

CJ =

BG =

CG =

MN =




## 02/10/98 A Square In A Triangle

Given N, AE = 1, EG = N.

AB =

BJ =

BD =

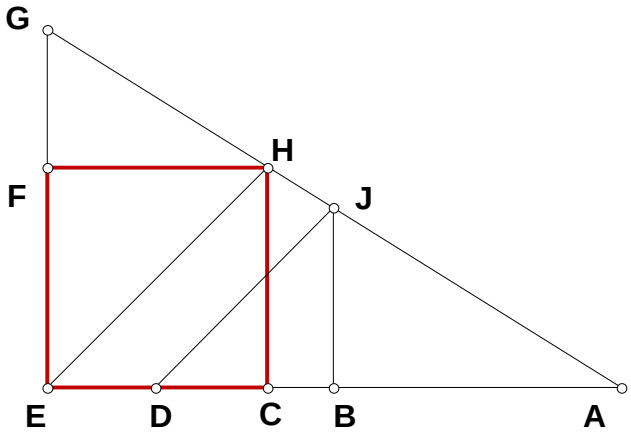
AD =

CE =

AC =

FG =

$\frac{AE}{AC} - (N + 1) =$



## 02/25/98 Alternate Method Root Series

Given N, AH = 1, HN = AH × N.

HJ =

FH =

AF =

FG =

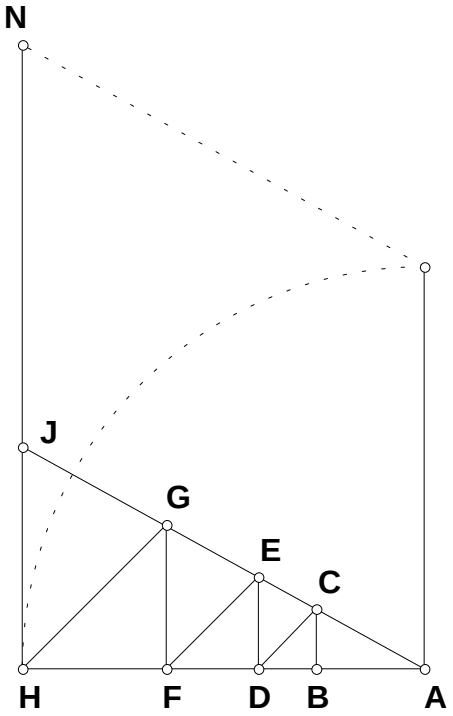
DF =

AD =

DE =

BD =

AB =




## 02/25/98B Sum Divided by One Powered

Given $N_1$, $N_2$, AH = 1

HM = AH × $N_1$, AN = AH × $N_2$.

HO =

AO =

AF =

FH =

FG =

DF =

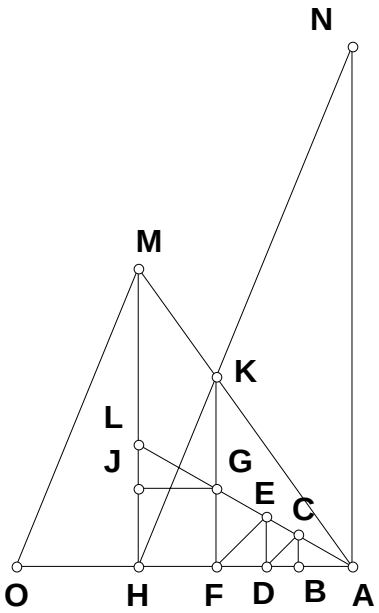
AD =

DE =

BD =

AB =

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3$$

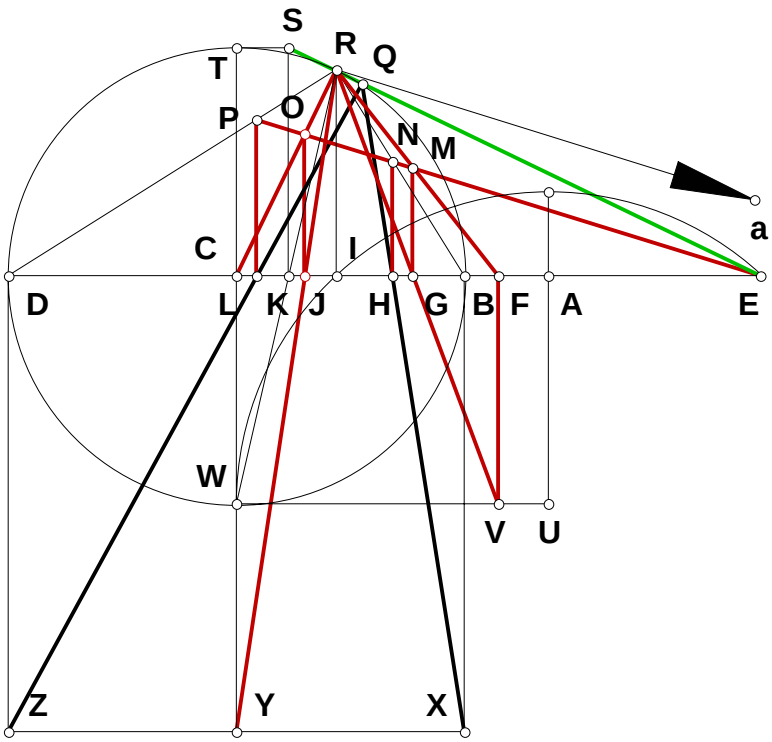


## 07/24/99 On Gemini Roots

Given $N_1$, $N_2$, AB = 1

$AD = AB \times N_1$, $EF = \frac{CE}{N_2}$.

BD =
BC =
CW =
CT =
FV =
AI =
BI =
DI =
IR =
AE =
AC =
ED =
CE =
BE =
EI =
FI =
FG =
EG =
GI =
GM =
Ia =
EL =
BR =
Ba =




BH =
EH =
CI =
JO =
CJ =
JI =
CY =
$\frac{CY}{CW} - \frac{CE}{EF} =$

## 08/11/99 A Delian Solution

**Plate 1**

Given N, AB = 1, AG = AB × N.

BG =

AD =

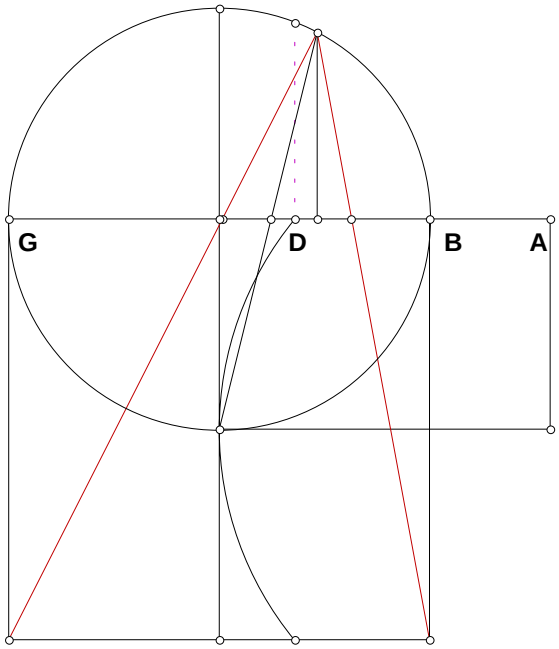



**Plate 2**

BF =

FG =

AF =

FX =

Mf =

Lf =

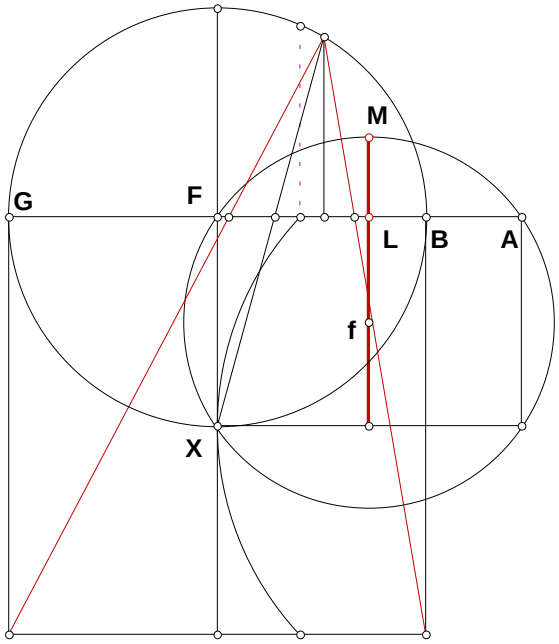


**Plate 3**

ML =
FL =
Xd =
df =
IX =
Md =
MX =

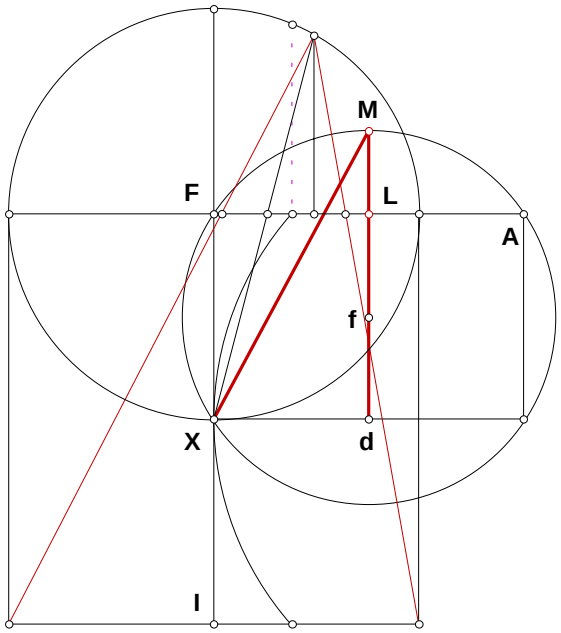



**Plate 4**

SX =
Lg =
QX =
Fg =
Xg =
Qg =

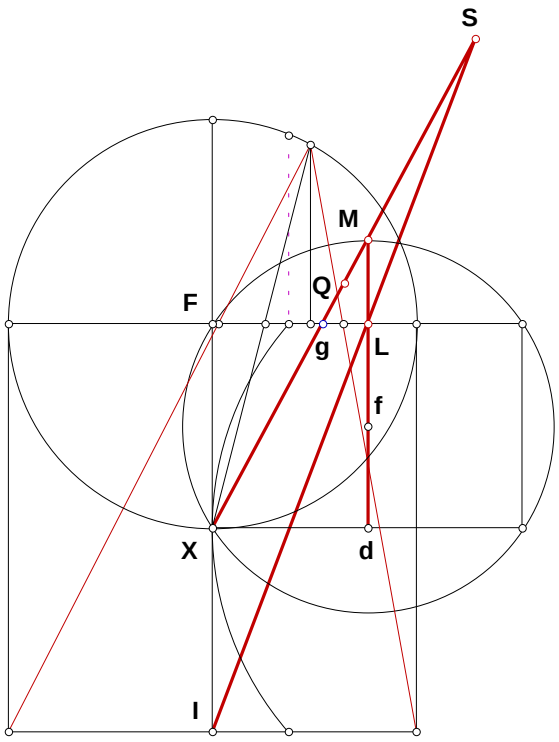


**Plate 5**

Kg =
GK =
GJ =
GT =
JT =
IJ =

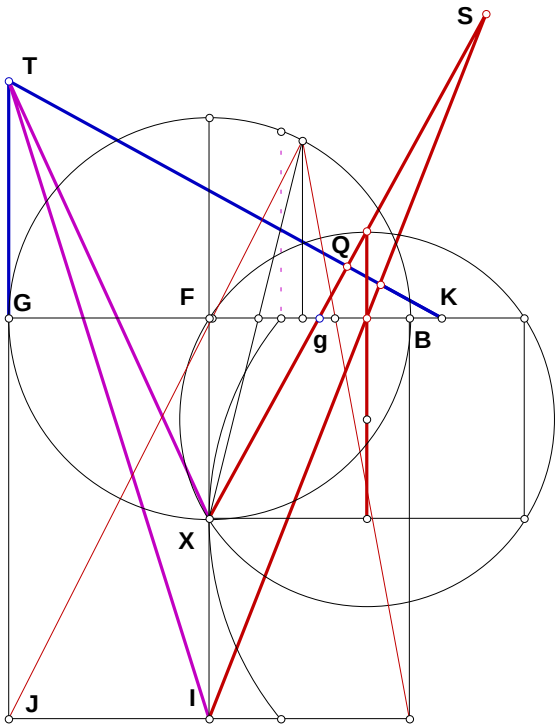



**Plate 6**

FP =
OP =
KP =
Pi =
Oi =
hi =

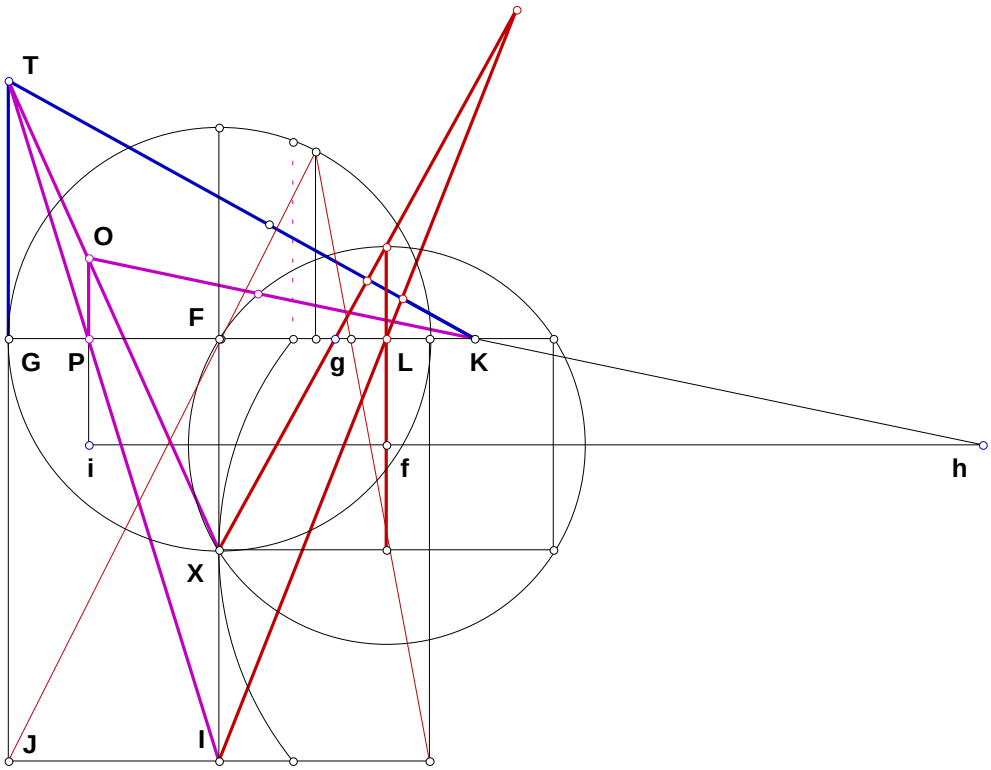


**Plate 7**

fi =

fh =

KO =

hk =

fk =

Nf =

Nk =

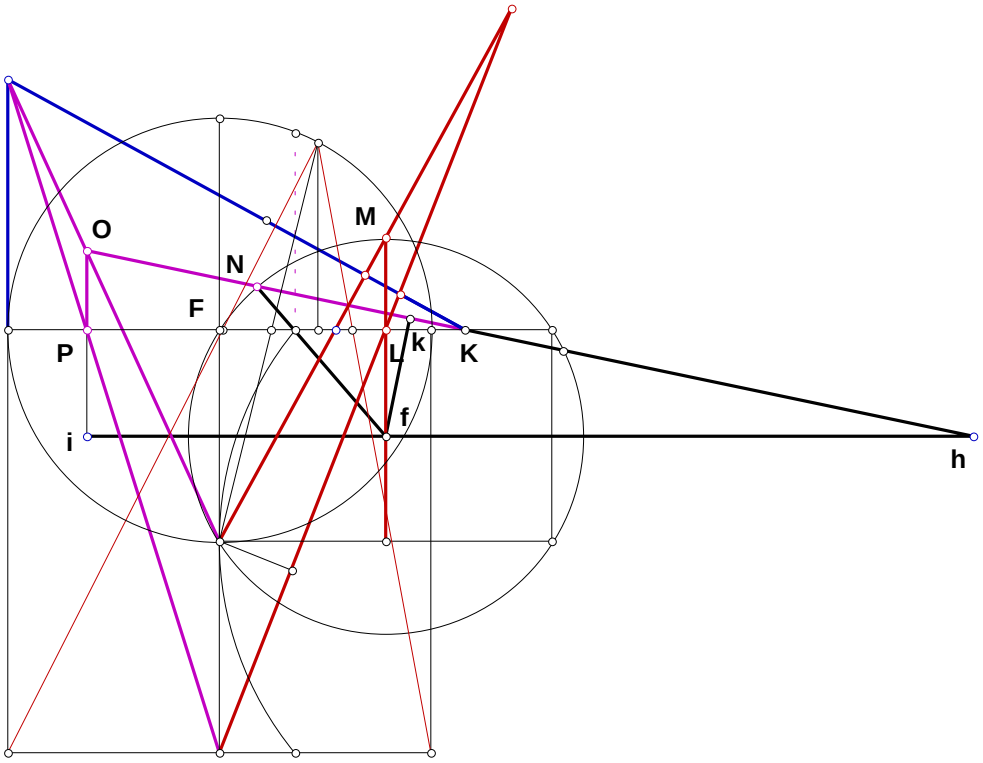



**Plate 8**

Nh =

Oh =

NO =

KN =

NI =

KI =

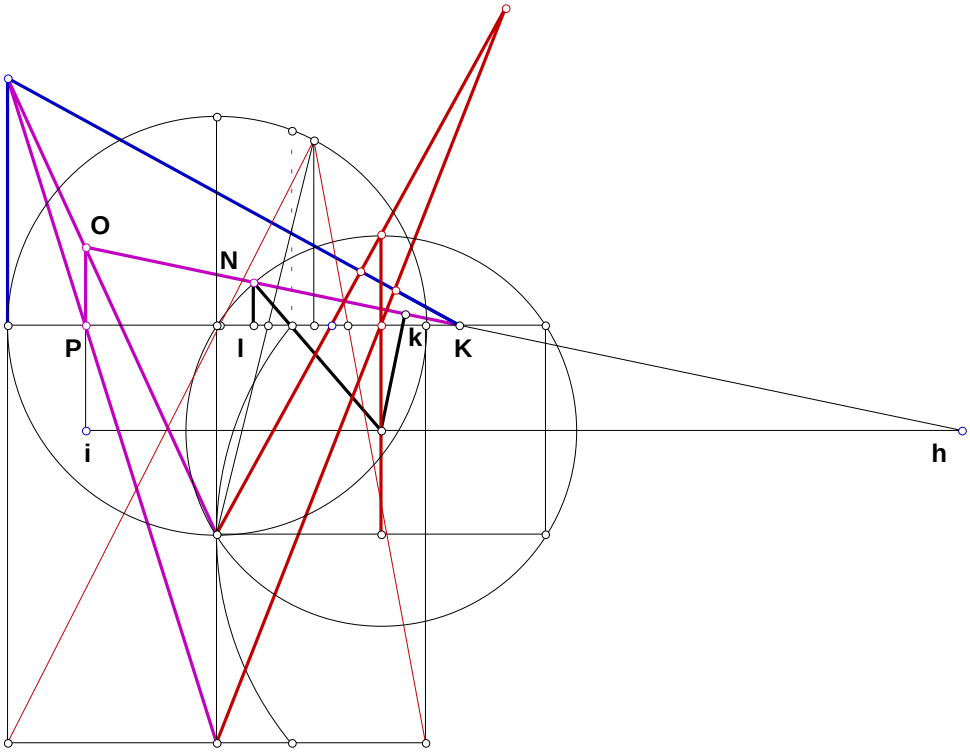


**Plate 9**

FK =

Fl =

NX =

XY =

Fm =

Km =

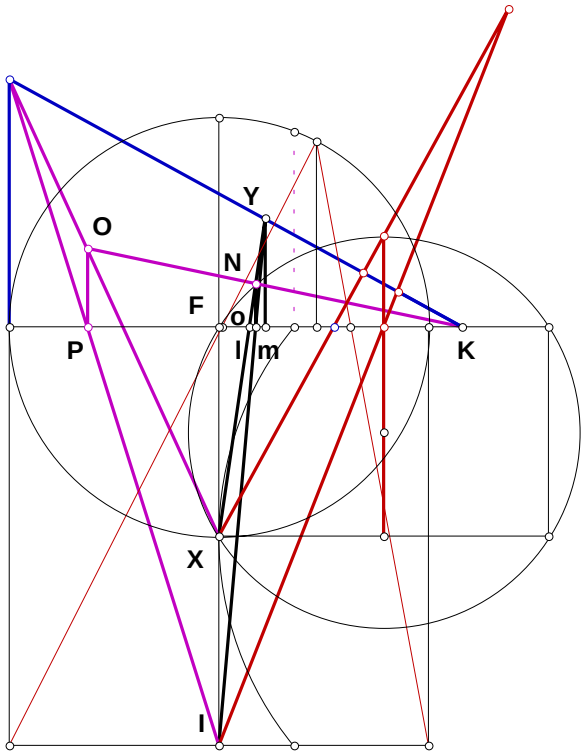



**Plate 10**

Fo =

Xo =

mo =

Ym =

FI =

IL =

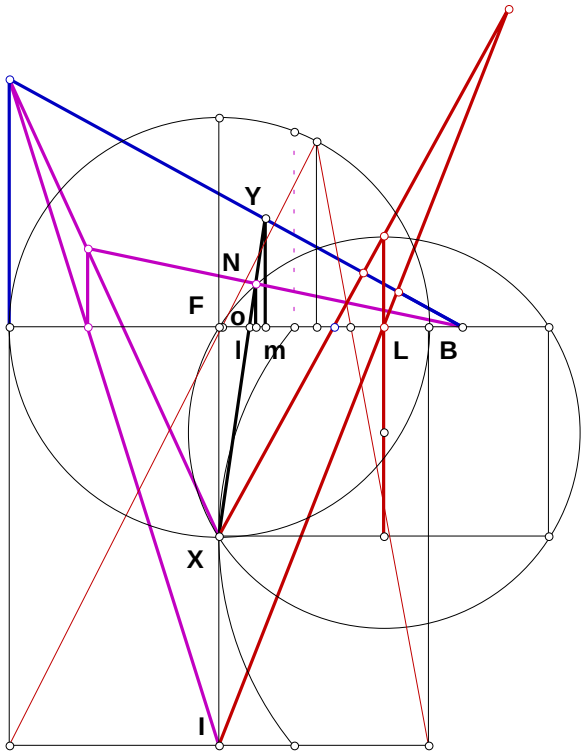


**Plate 11**

IS =

In=

Xn =

QR =

KT =

KY =

KQ =

RY =

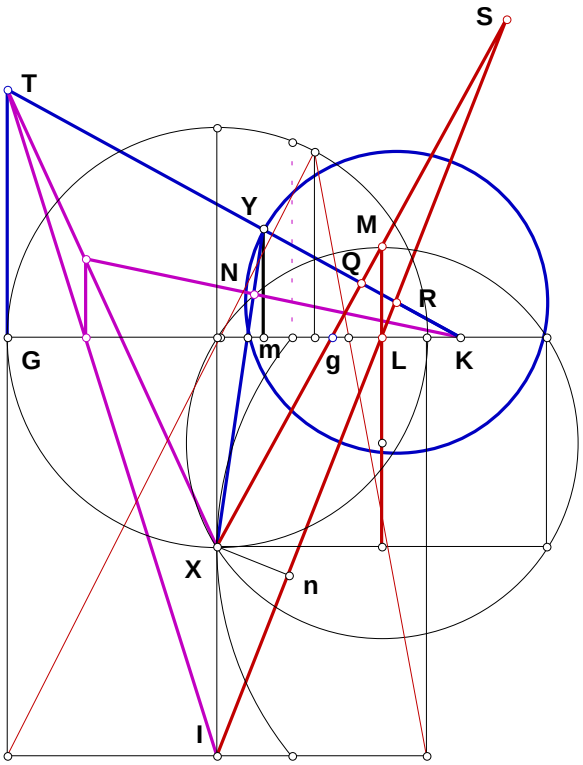



**Plate 12**

Xd =

dp =

pq =

dq =

Xp =

Op =

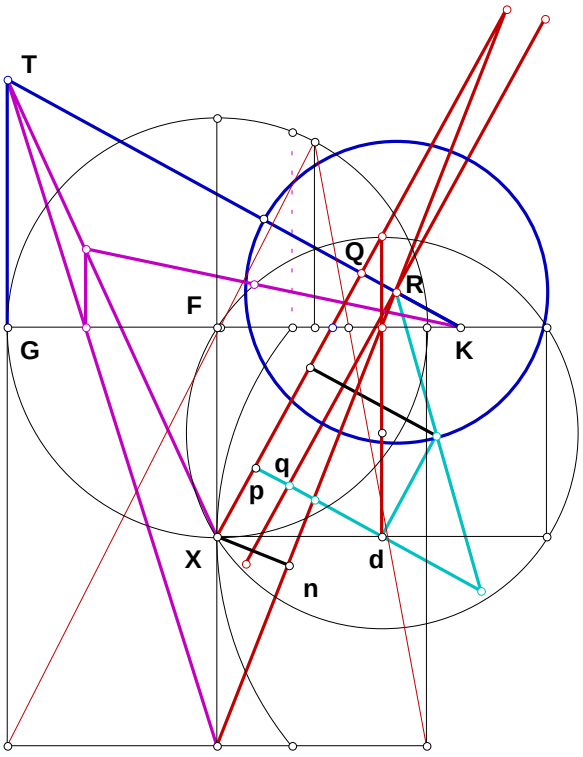


**Plate 13**

er =

Re =

Rr =

Rq =

Rs =

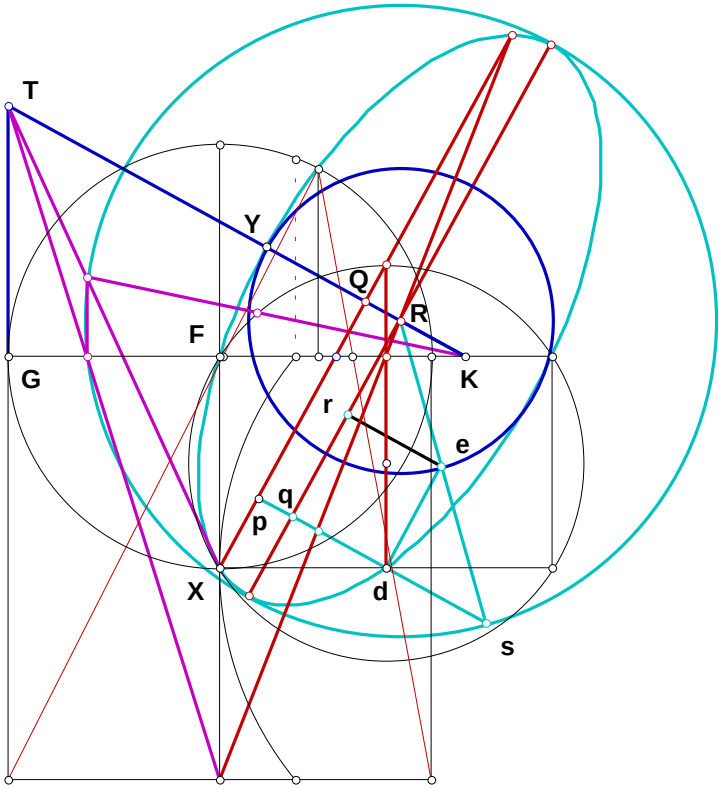



**Plate 14**

AC =

AE =

BC =

BE =

CE =

EG =

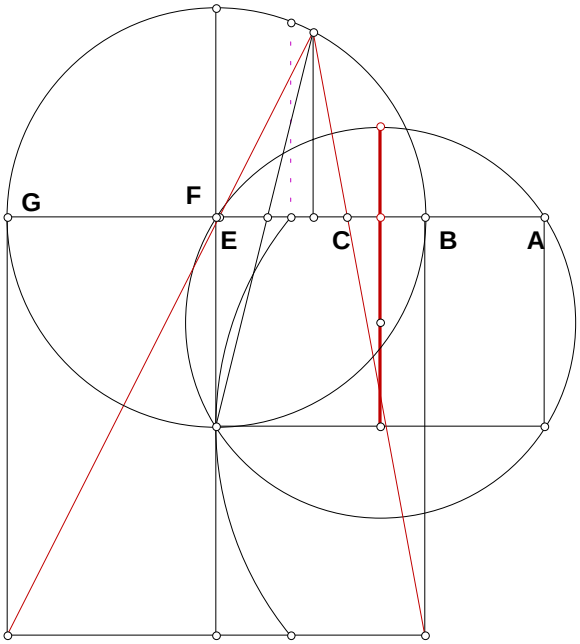


**Plate 15**

CU =

BU =

GU =

UZ =

UW =

Gg=

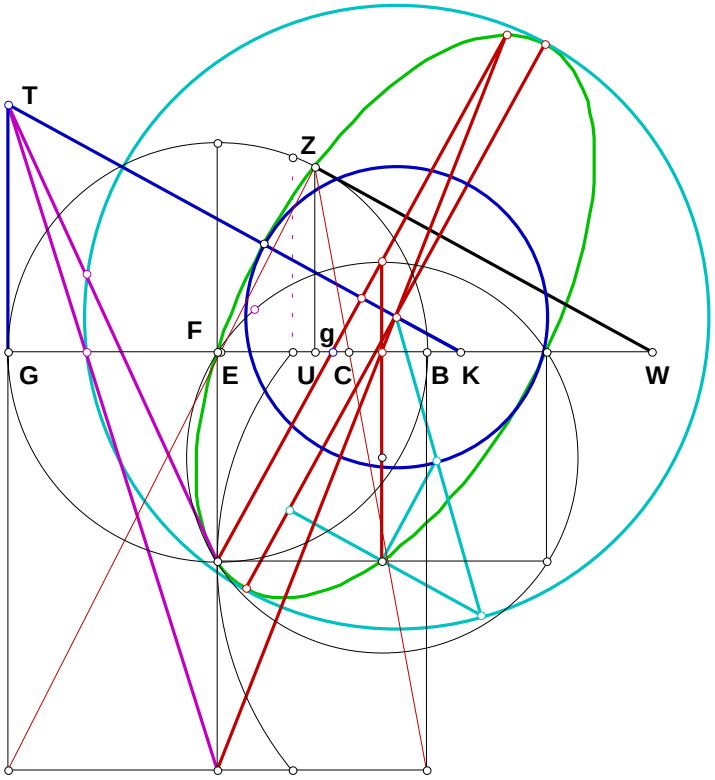



**Plate 16**

tu =

gt =

Gt =

GW =

Wt =-

tv =

Kt =

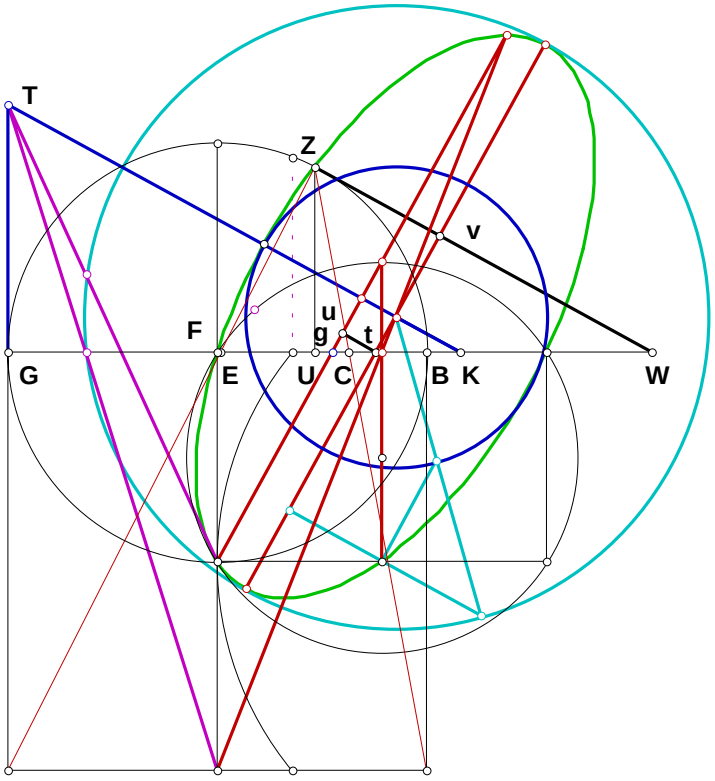


**Plate 17**

Rt =

Rv =

bc =

Rc =

cv =

Yw =

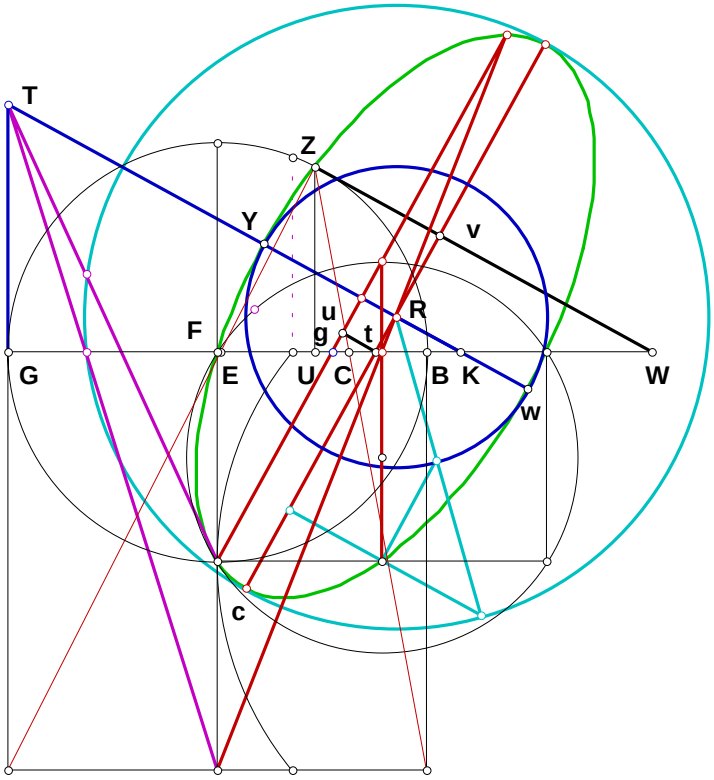
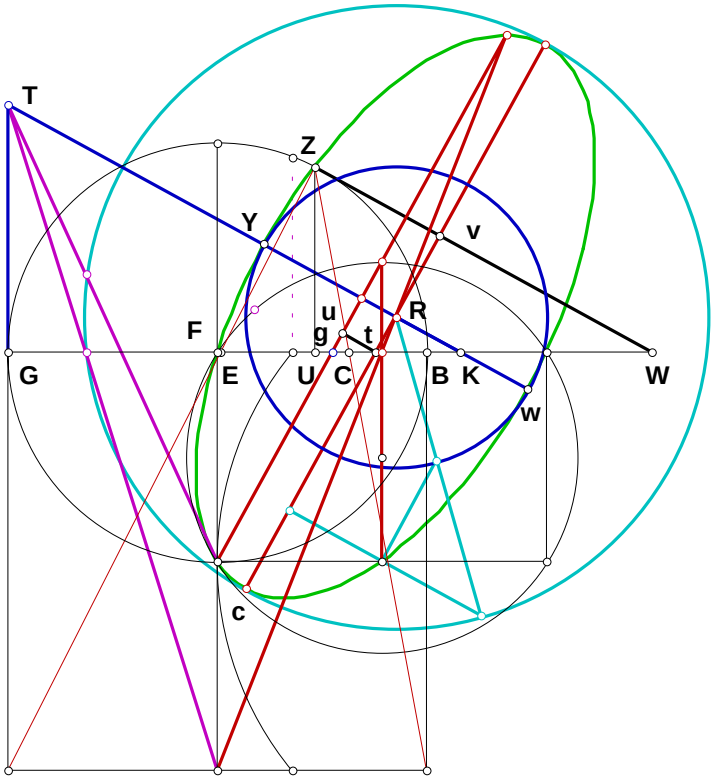



**Plate 18**

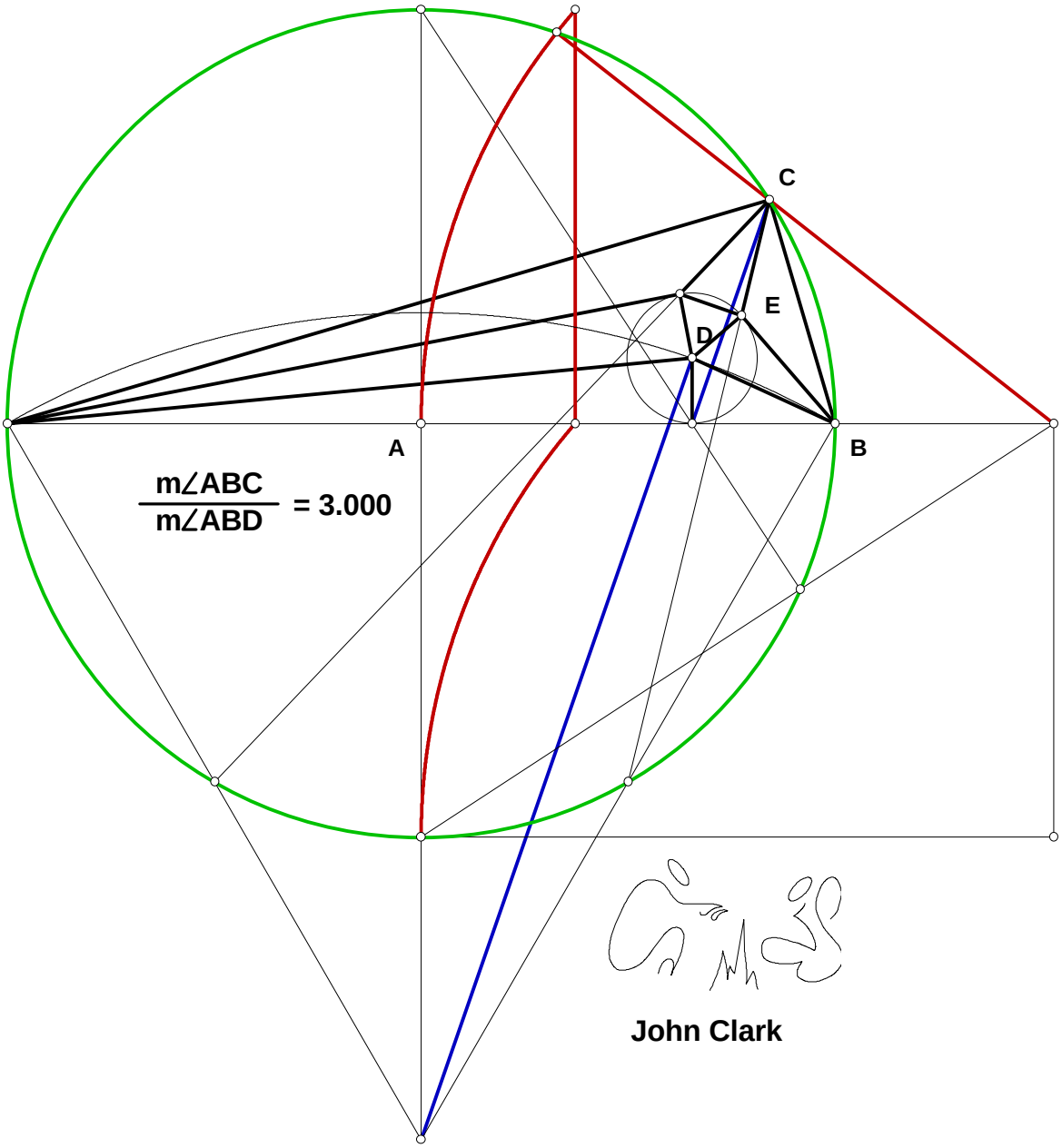
WZ =

Wv =

Zv =



# Three Pieces Of Paper




John Clark

## Three Pieces Of Paper

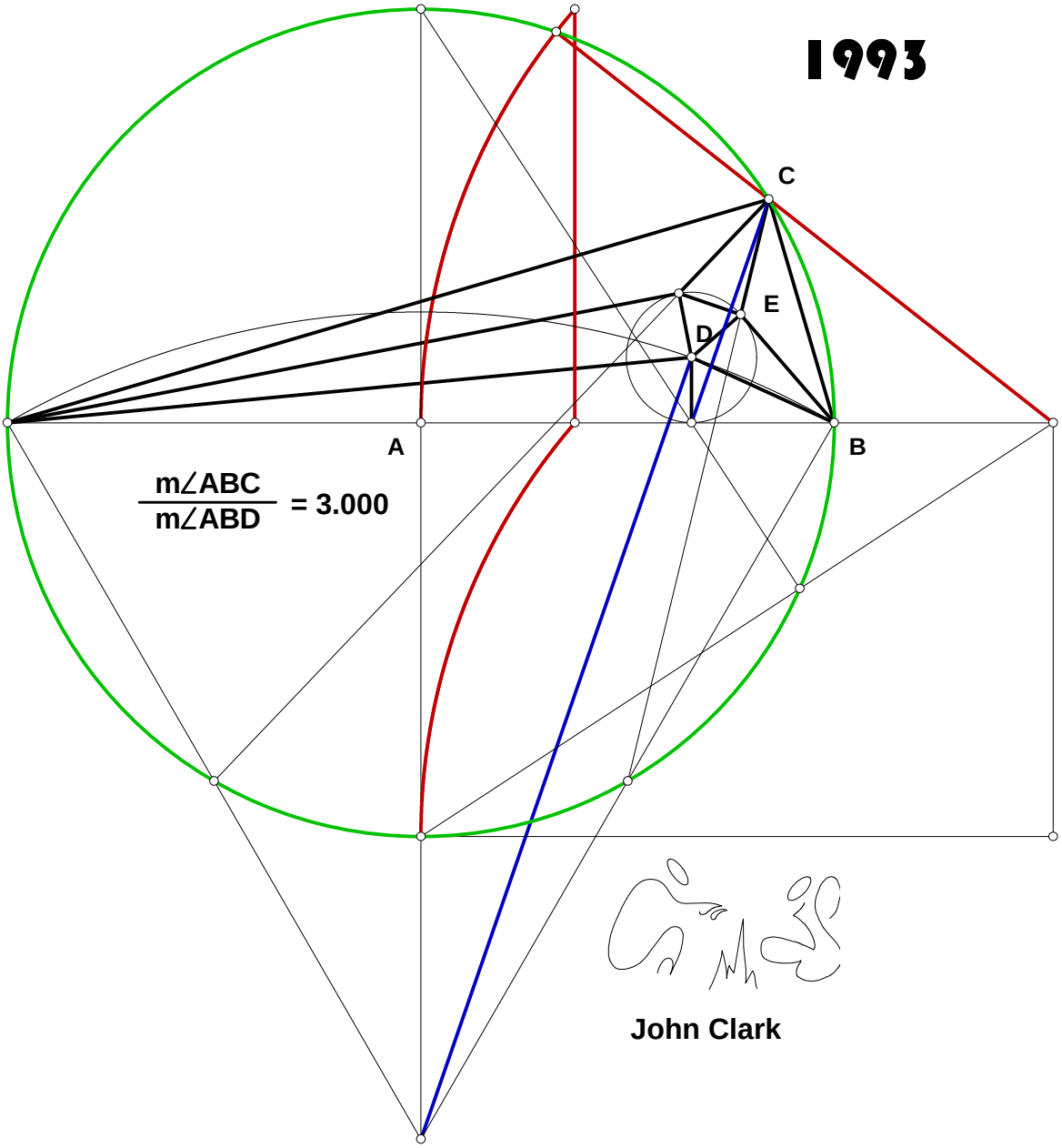
### Introduction

Is a novel about the search for a method to divide angles into at least three equal parts. The novel is written in the relatiologic of Geometry and the tautologic of basic Algebra.

I suspect that someday I might have something of significance to write here. J. C.

# Three Pieces Of Paper

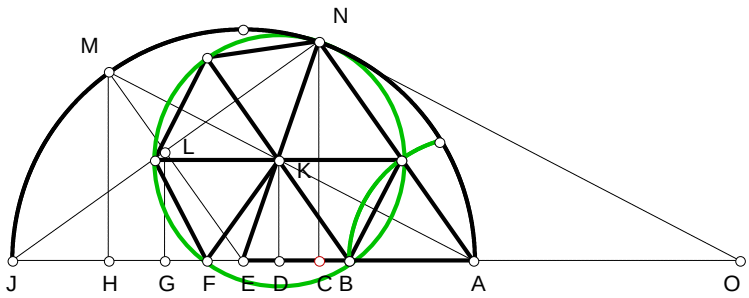
1993




John Clark

## 11/11/93 The Archamedian Paper Trisector

**If one accepts the facts of the original figure, one only need prove that BK = AB.**
**If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?**




$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

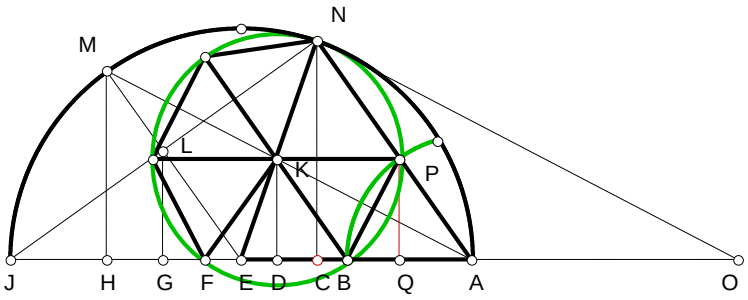
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

$AB - BK = 0$ $AB = 0.25$

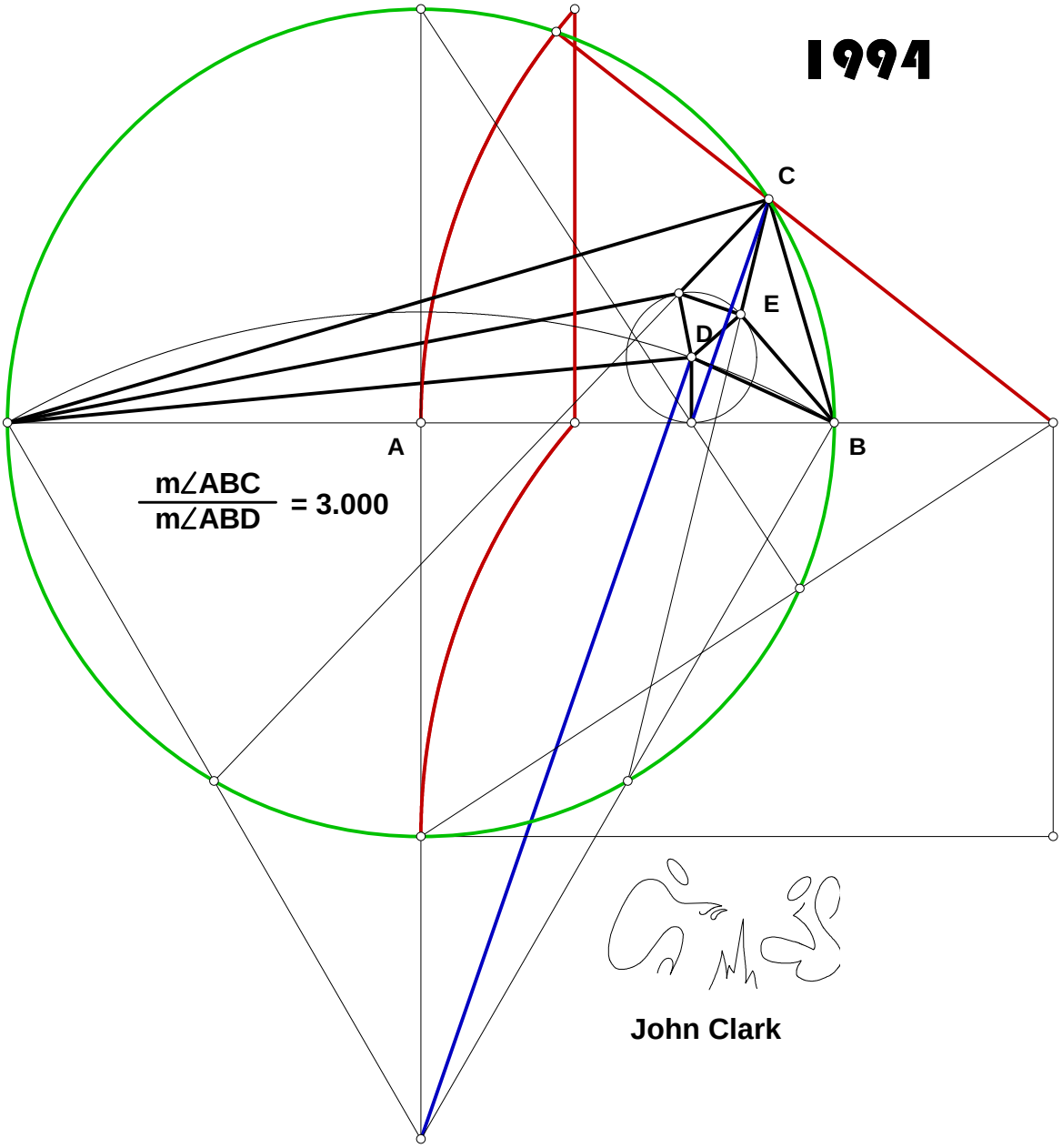
**If PK is parallel to AJ, then . . . .**




$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

# Three Pieces Of Paper

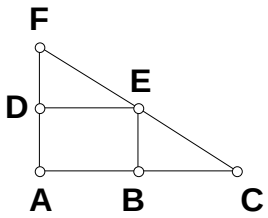
1994




John Clark

# 042994

## Given AB, AF, BE, what is EF?



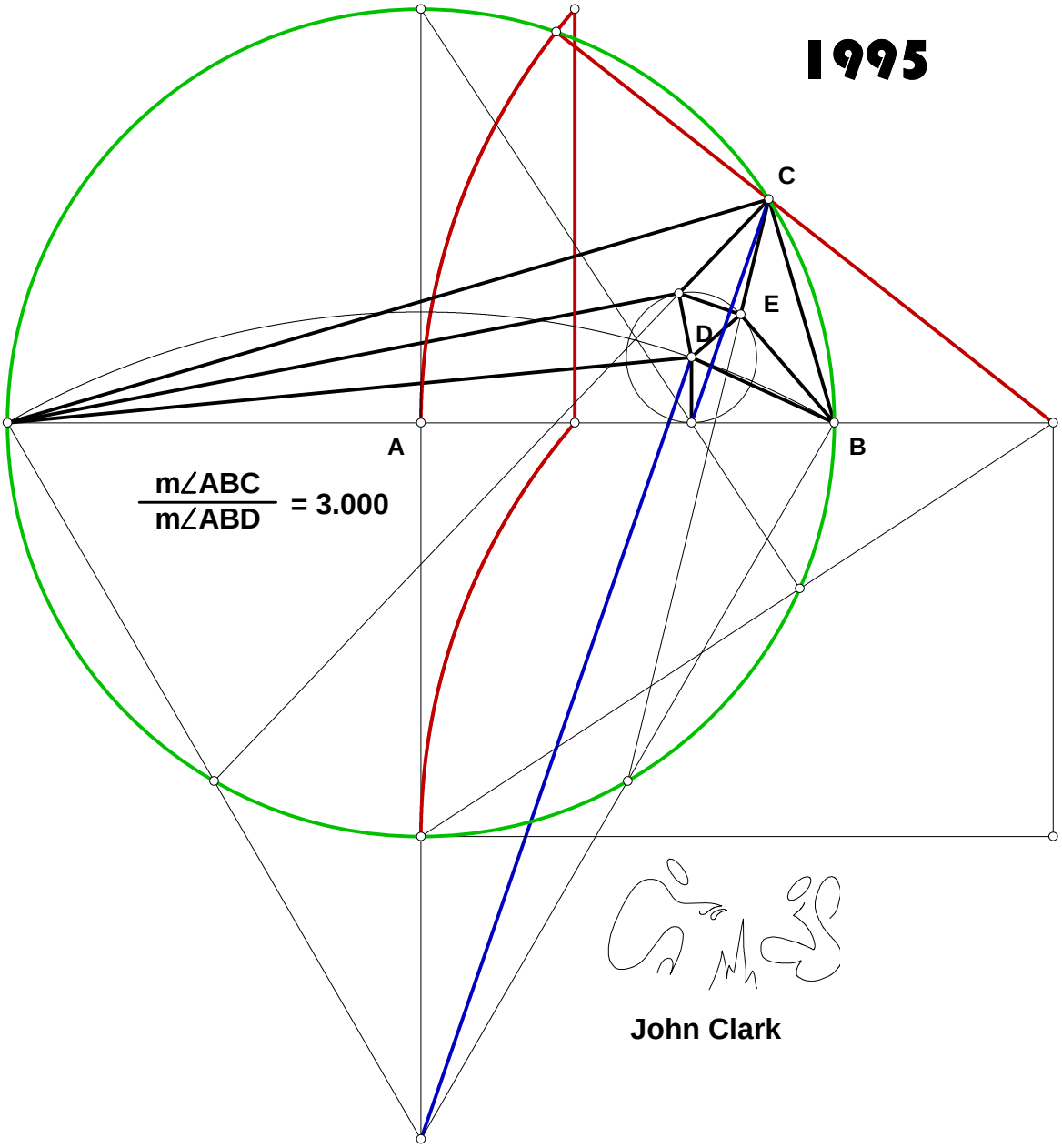

$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$

# Three Pieces Of Paper

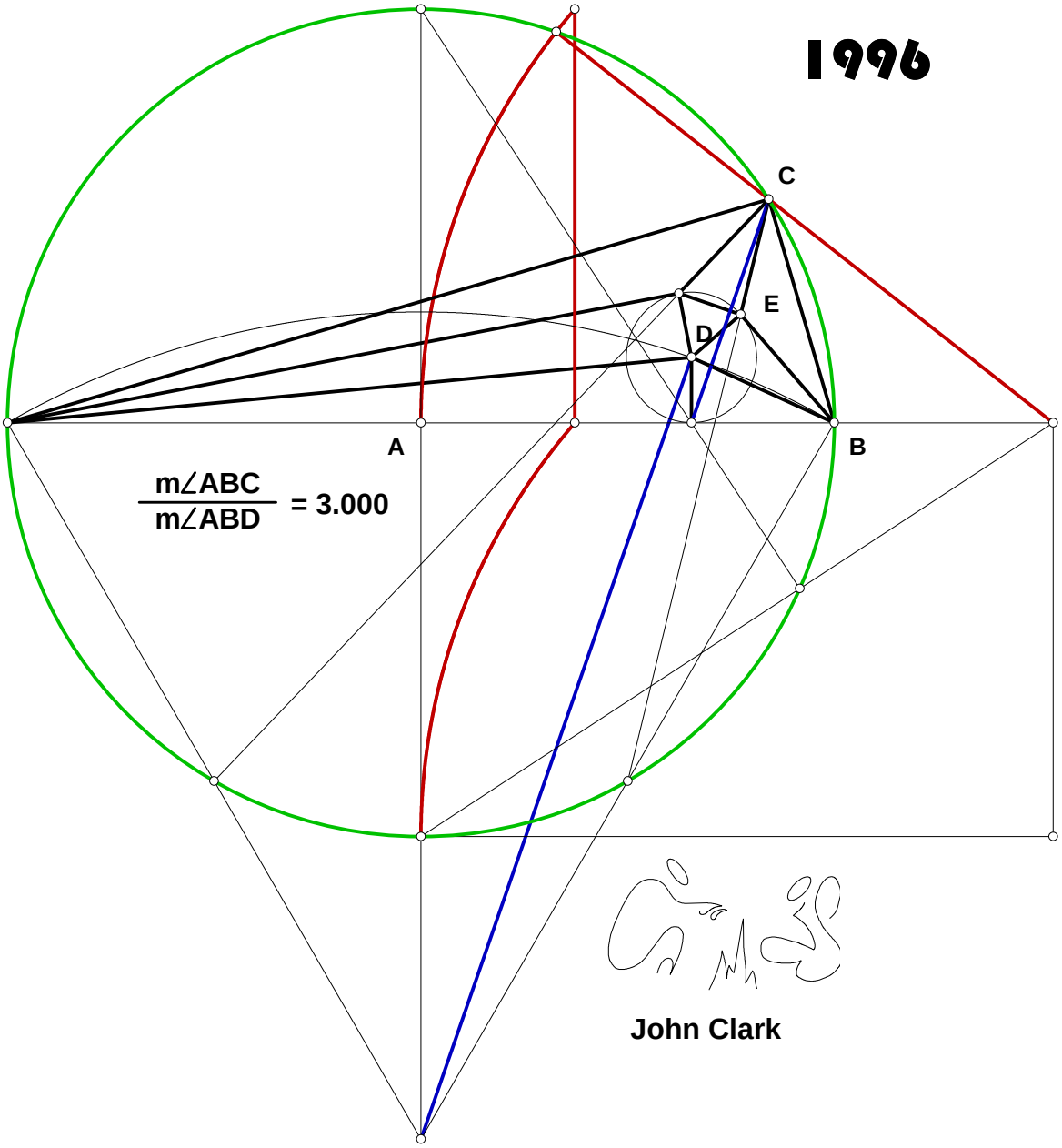
1995




John Clark

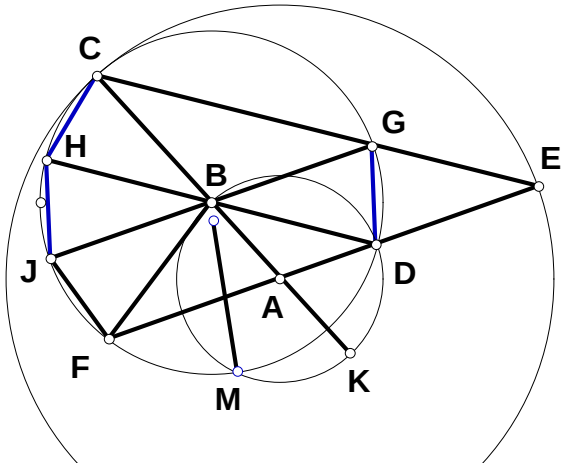
# Three Pieces Of Paper

1996



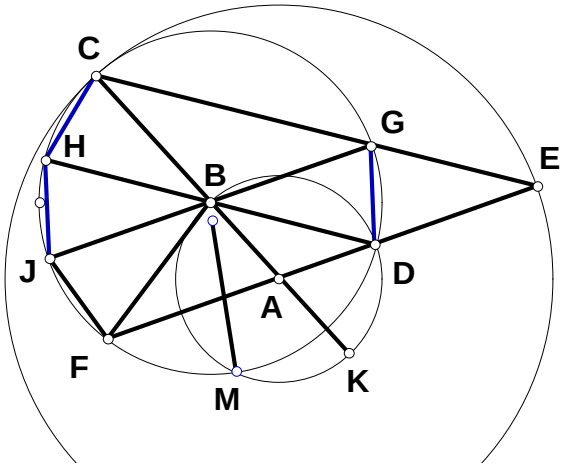

John Clark

## 010496 The Archamedian Paper Trisector- Without the Numbers.



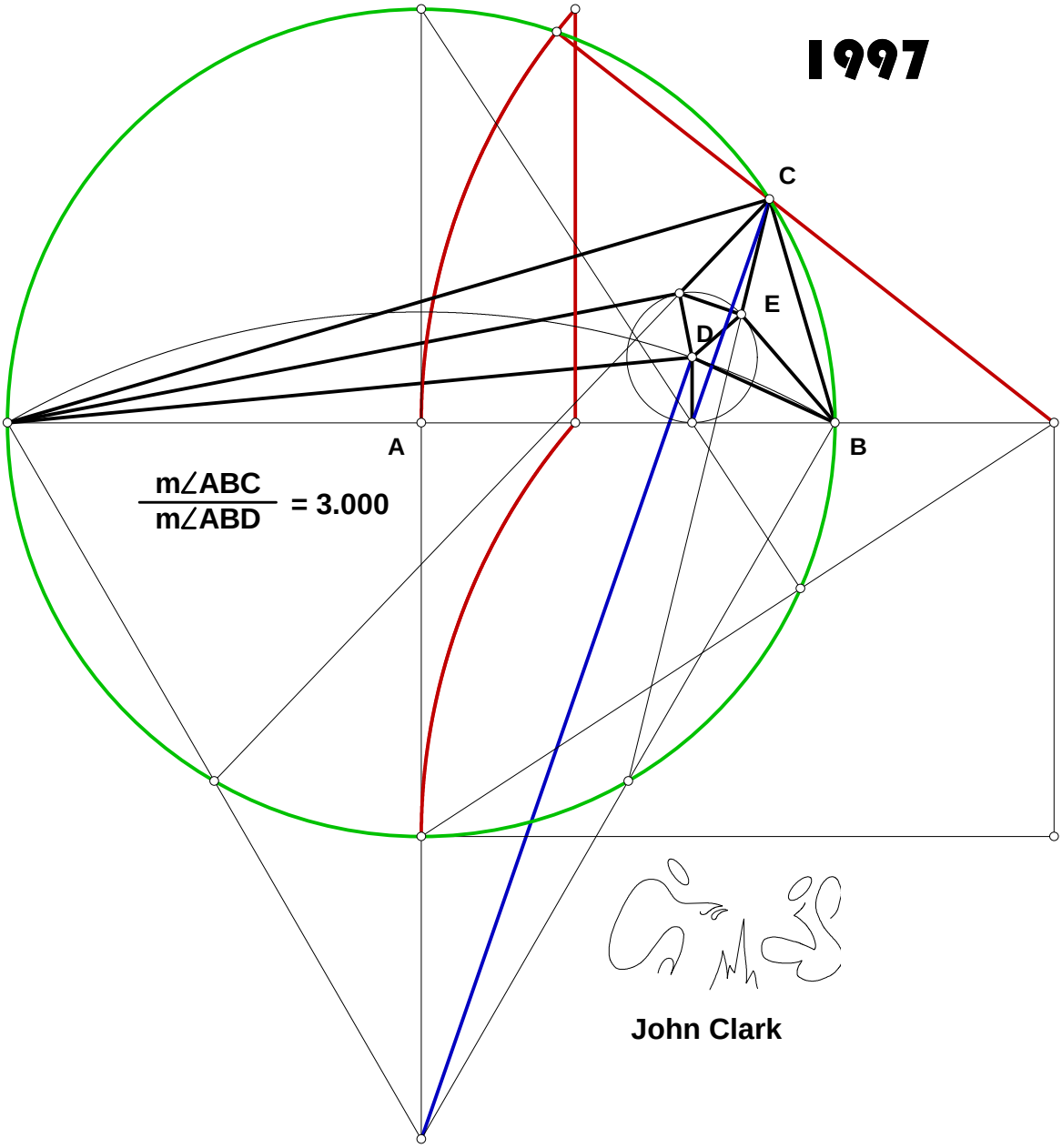

**Given any circle AB.**
**Given any circle BC such that BC ≤ 2AB.**
**Construct AE such that AE = AC.**
**As AC = AB + BC**
**and AD = AB so too DE = BC.**
**Construct DH parallel to BD.**
**Construct CE.**
**As AB = AD and AC = AE,**
**Δ ABD is proportional to Δ ACE,**
**therefore CE is parallel to BD.**
**From here one can take two paths.**
**Construct GJ parallel to EF.**
**As CE is parallel to DH,**
**DG = CH.**
**As GJ is parallel to EF,**
**DG = FJ.**

**As ∠ HBJ is opposite**
**and equal to ∠ GBD,**
**DG = HJ,**
**therefore ∠ DG is $\frac{1}{3}$ CF.**
**As CE is parallel to DH**
**and EF is parallel to GJ,**
**EGBD is equilateral.**
**By construction DK = KM.**
**As DH is parallel to CE,**
**CH = DG.**
**As DK is equal and opposite CH,**
**MK + DK + DG is $\frac{1}{3}$ DG.**



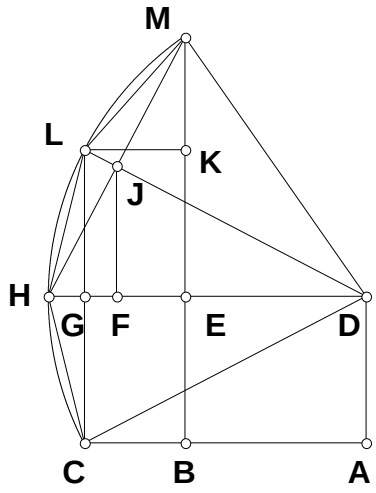
# Three Pieces Of Paper

1997




John Clark

# 042897




$N := 1.458 \quad AD := 1 \quad AC := AD \cdot N \quad N := N$

$CD := \sqrt{AD^2 + AC^2} \quad DH := CD$

$CG := AD \quad DG := AC \quad GH := DH - DG \quad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \quad DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \quad EF := FH \quad DE := DH - (EF + FH)$

$AB := DE \quad EG := DG - DE \quad LM := CH \quad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \quad BE := AD \quad BK := 2 \cdot BE \quad BM := BK + KM$

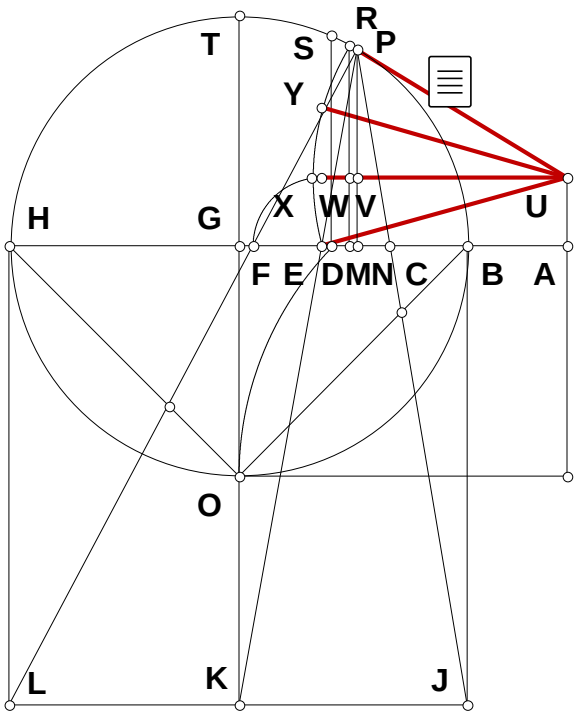
## Some Algebraic Names:

$\sqrt{N^2 + 1} - CD = 0 \quad N - AC = 0 \quad \sqrt{N^2 + 1} - N - GH = 0 \quad \sqrt{2 \cdot N^2 - 2 \cdot \sqrt{N^2 + 1} \cdot N + 2} - CH = 0$

$\frac{1}{\sqrt{N^2 + 1}} - FH = 0 \quad \frac{(N - 1) \cdot (N + 1)}{\sqrt{N^2 + 1}} - AB = 0 \quad 2 + \sqrt{\frac{5 \cdot N^2}{N^2 + 1} - \frac{4 \cdot N}{\sqrt{N^2 + 1}} + \frac{1}{N^2 + 1}} - BM = 0$

# Trisection and the Cube Roots 042997

**If trisection can be placed at RUE, then PV is proportional to RW.**




$N := 5 \qquad AB := 1 \qquad AH := AB \cdot N$

$BH := AH - AB \quad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2}$

$AE := AC + CE$

$AU := CE \quad NV := AU \quad MW := AU$

**(For the next two equations see 042897.)**

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

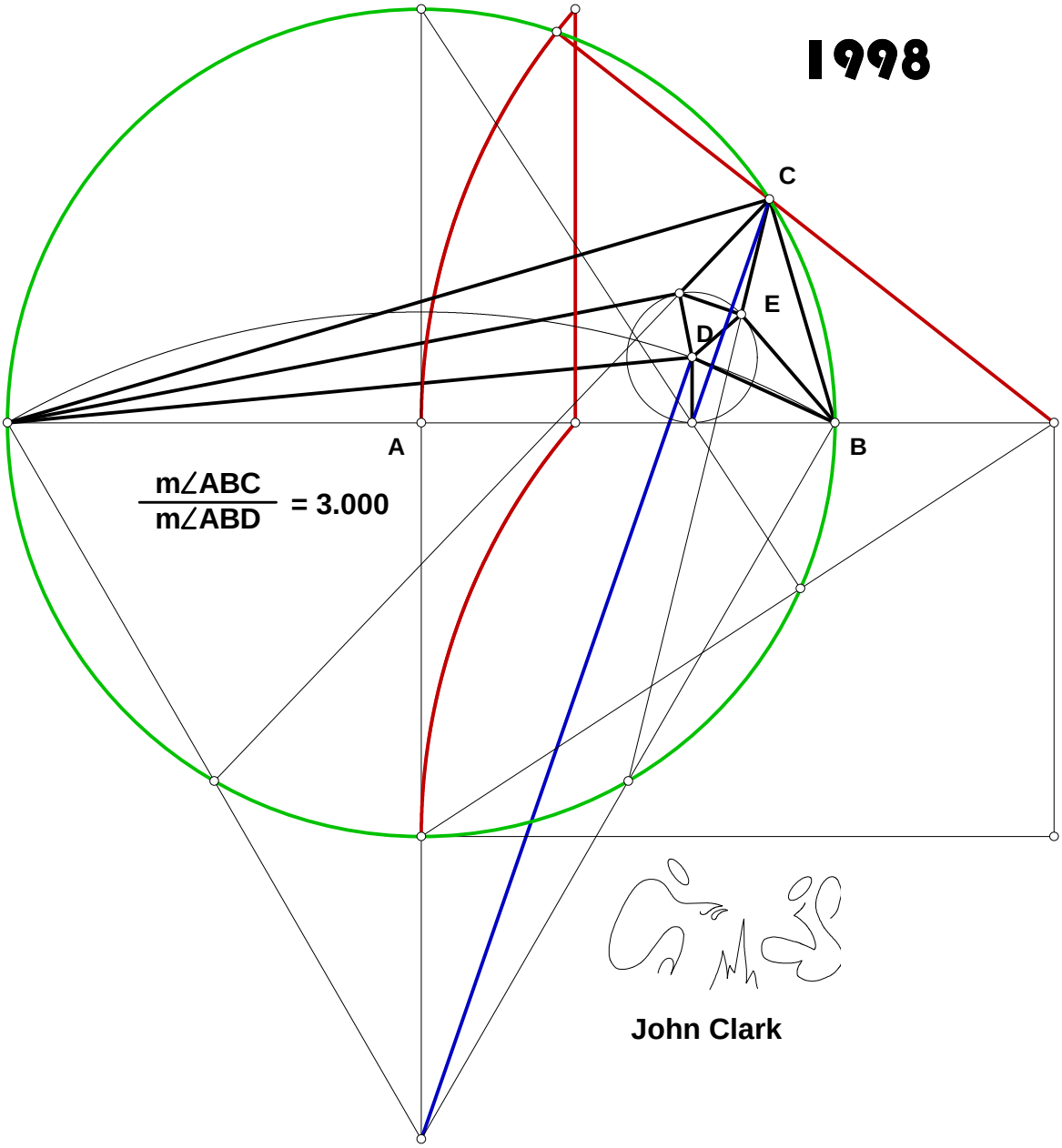
$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad RW := MR - MW$$

$BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH} \qquad NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV$

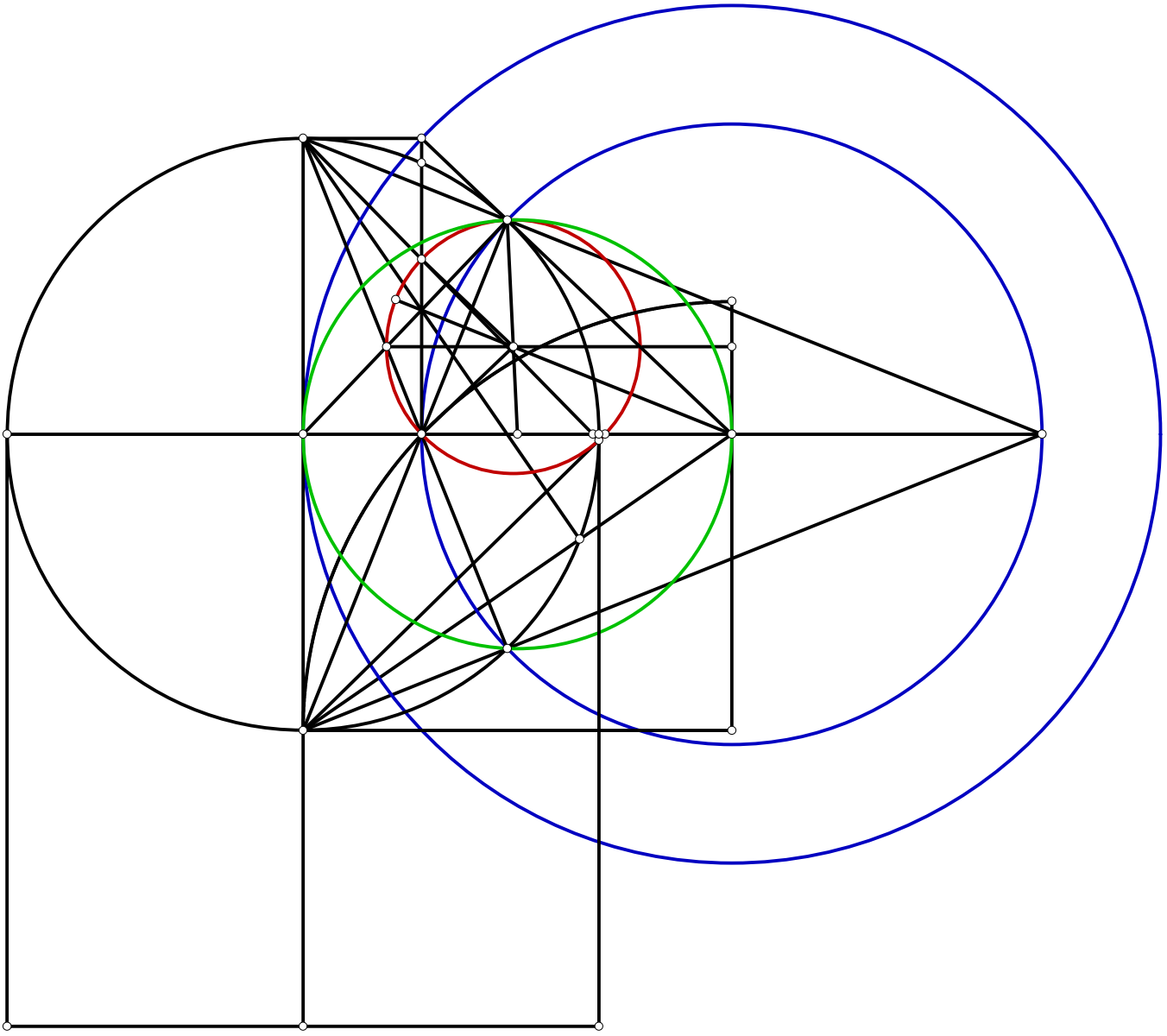
$AN := AC + CN \qquad UV := AN \quad UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

# Three Pieces Of Paper

1998

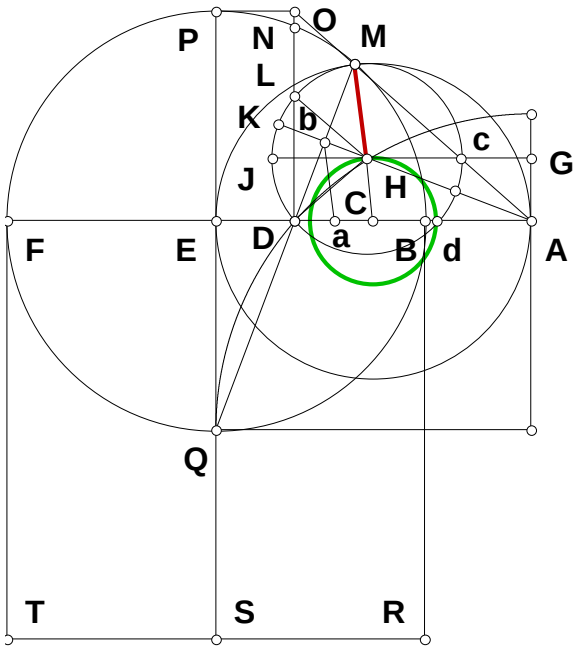



John Clark



# A Square Root Figure And Triseciton 042398

**In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?**




$N := 5 \quad AB := 1 \quad AF := AB \cdot N \quad BF := AF - AB$

$AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2} \quad BD := AD - AB$

$DE := BE - BD \quad EQ := BE \quad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}}$

$PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ} \quad DM := QM - DQ$

$AE := AB + BE \quad AC := \frac{AE}{2} \quad Db := \frac{DM}{2}$

$CM := AC \quad ab := \frac{CM \cdot Db}{DM} \quad CD := AD - AC$

$Ca := \frac{CD}{2} \quad Aa := AC + Ca \quad CH := \frac{ab \cdot AC}{Aa}$

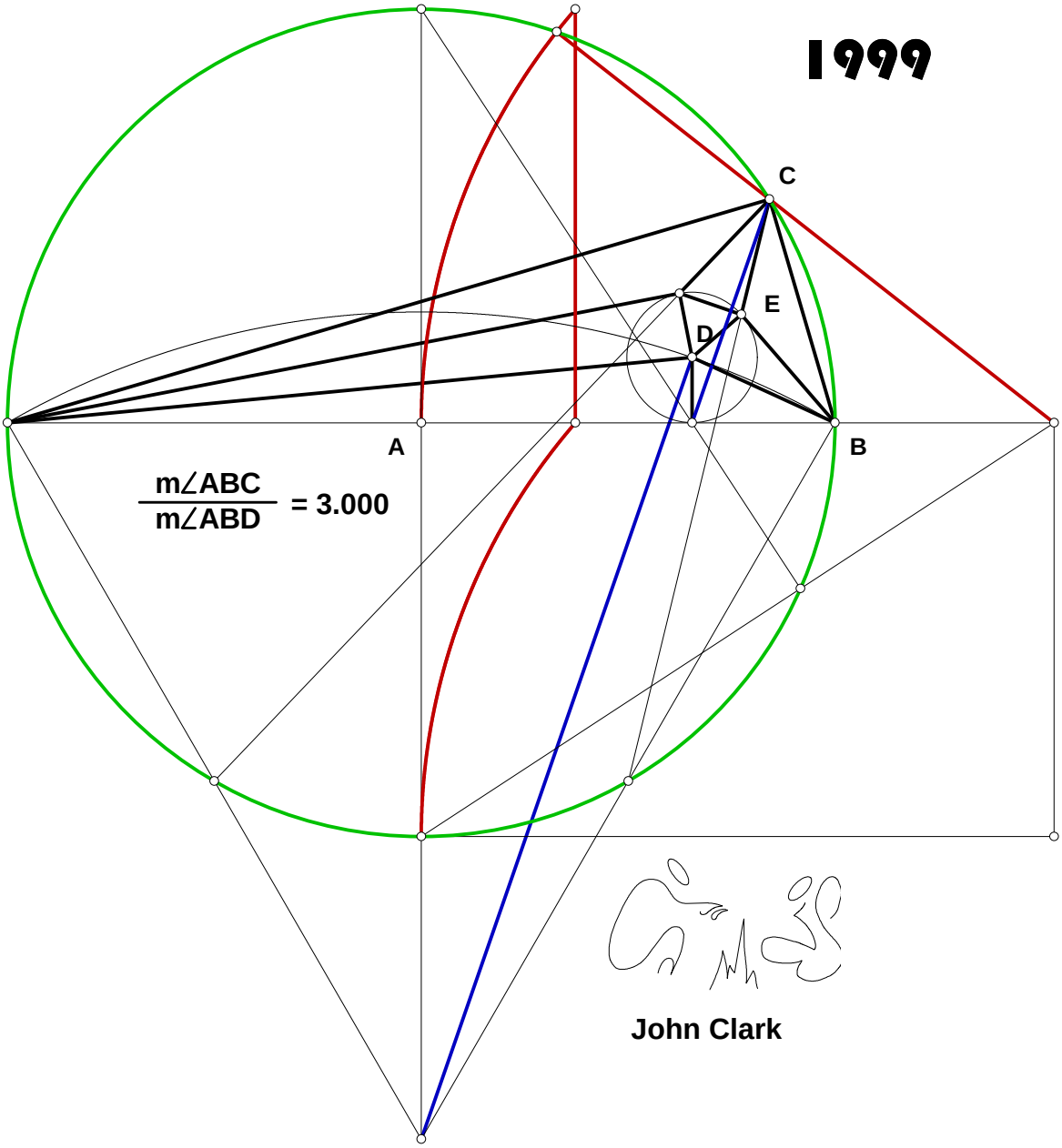
$AM := AD \quad Ac := \frac{AM \cdot CH}{CM} \quad HM := CM - CH$

$HM - Ac = 0$

$$HM - AB \cdot \frac{\sqrt{N} \cdot (N+1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \qquad CH - AB \cdot \frac{(N+1)^2}{4 \cdot N + 16 \cdot \sqrt{N} + 4} = 0$$

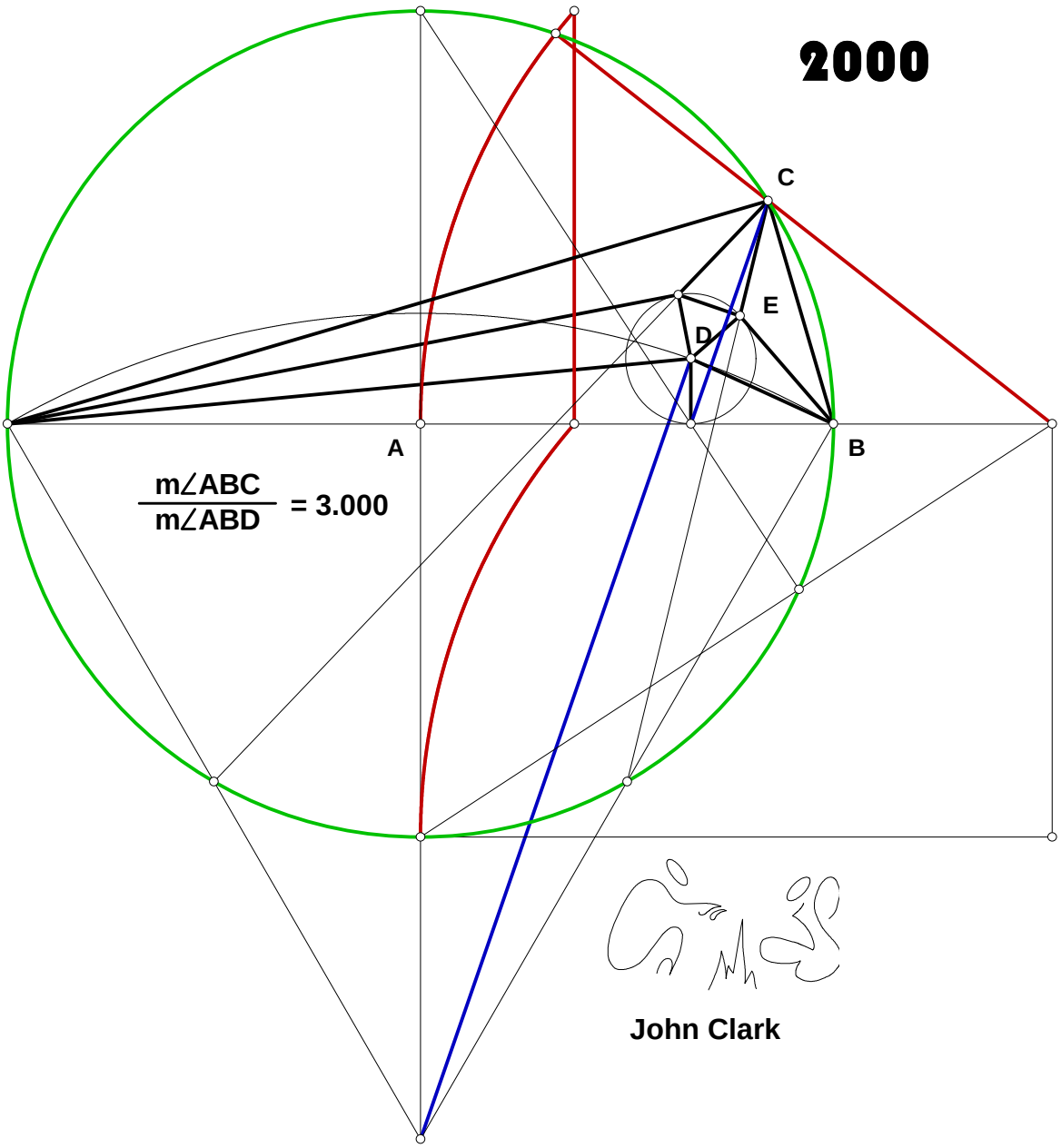
# Three Pieces Of Paper

1999



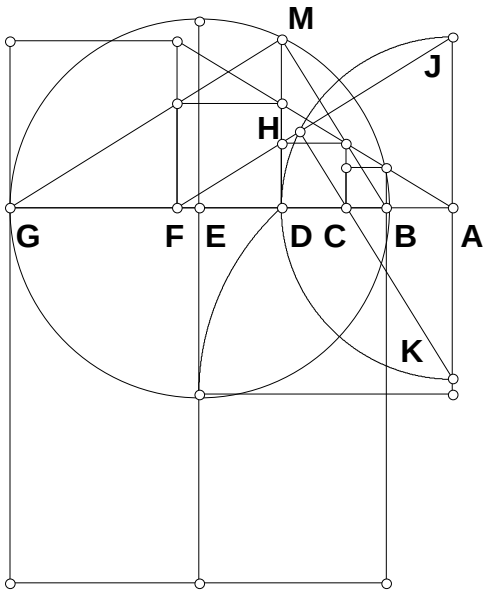

John Clark

# Three Pieces Of Paper




John Clark

# 07/09/00 Alternate Method Quad Roots




$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

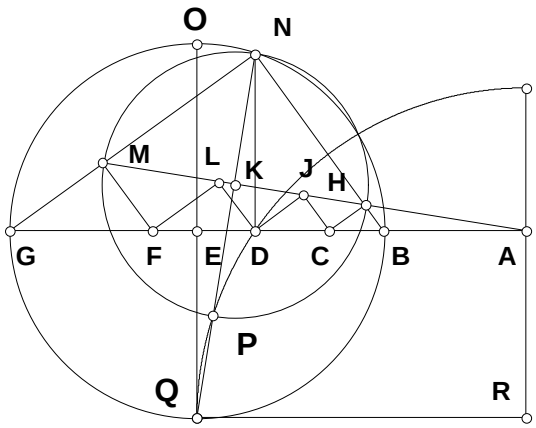
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

# 08/01/00 Alternate Method Quad Roots




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

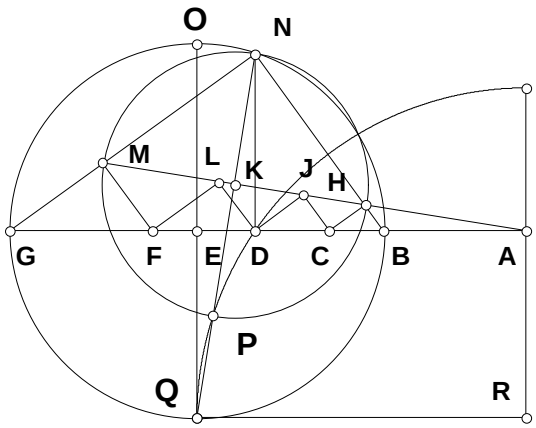
$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}} \qquad BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

# 08/01/00B Alternate Method Quad Roots




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

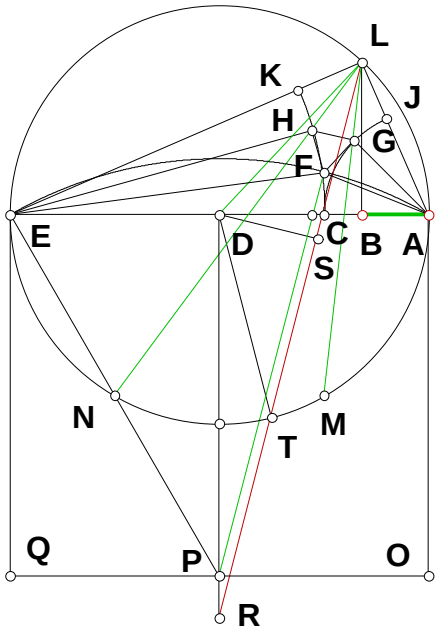
$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2} \qquad AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2} \qquad GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN}$

$AF := AG - GF \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

# In Trisection What Is AB? 08/02/00




**In the trisection figure given and given AC as the Unit what is AB?**

$AC := 1 \quad N := 5 \quad AE := AC \cdot N \quad AD := \frac{AE}{2}$

$EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2} \quad FP := EP$

$CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF \quad DR := DP + PR \quad CR := \sqrt{CD^2 + DR^2}$

$CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD$

$LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS \quad LR := RS + LS$

$BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$

**In trisection the length RT to the similarity point is equal to the radius of the circle.**

$RT - \frac{N}{2} = 0 \quad AE := N \quad AD := \frac{N}{2}$

$DP := \frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{N^2} \quad CE := N - 1 \quad CD := \frac{1}{2} \cdot N - 1 \quad CF := \frac{1}{2} \cdot \sqrt{(3 \cdot N - 2) \cdot (N + 2)} - \frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{N^2}$

$DR := \frac{1}{2} \cdot \sqrt{(3 \cdot N - 2) \cdot (N + 2)} \quad N - CR = 0 \quad CS := \frac{1}{4} \cdot \frac{(N - 2)^2}{N} \quad LS := \frac{1}{4} \cdot \frac{\left(N^2 + 4 \cdot N - 4\right)}{N}$

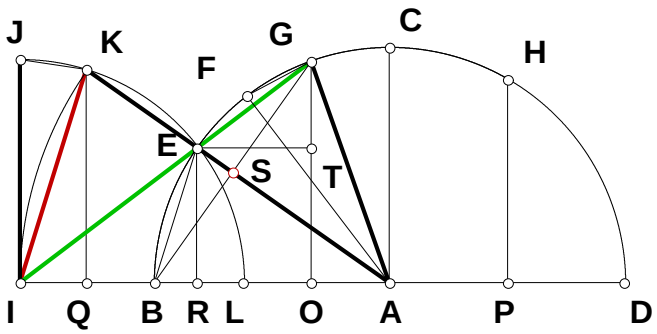
$DS := \frac{1}{4} \cdot \frac{\sqrt{(3 \cdot N - 2) \cdot (N - 2)^2 \cdot (N + 2)}}{N} \quad RS := \frac{1}{4} \cdot (3 \cdot N - 2) \cdot \frac{(N + 2)}{N} \quad LR := \frac{1}{N} \cdot \left(N^2 + 2 \cdot N - 2\right)$

$BD := \frac{1}{2} \cdot (N - 2) \cdot \frac{\left(N^2 + 2 \cdot N - 2\right)}{N^2} \quad \frac{(3 \cdot N - 2)}{N^2} - AB = 0 \quad AB \cdot N^2 - 3 \cdot N + 2 = 0$

**If the ratio of AB to N is given, such as N/AB = X, then the Unit AC can be found from a cubic equation.**

$X := 4 \quad \frac{\left[\left[-X + \sqrt{-X^2 \cdot (X - 1)}\right]^{\left(\frac{2}{3}\right)} + X\right]}{\left[-X + \sqrt{-X^2 \cdot (X - 1)}\right]^{\left(\frac{1}{3}\right)}} = 3.064$

# 080300 Trisection




**If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yeild a trisected figure JKL.**

$N := 3 \qquad BD := 1$

$AB := \frac{BD}{2} \qquad AD := AB \qquad AP := \frac{AD}{2}$

$BP := AB + AP \qquad BO := \frac{BP}{N} \qquad AE := AB$

$DO := BD - BO \qquad GO := \sqrt{BO \cdot DO}$

$BG := \sqrt{GO^2 + BO^2} \qquad BS := \frac{BG}{2} \qquad ER := BS \qquad TO := ER \qquad GT := GO - TO \qquad AS := \sqrt{AB^2 - BS^2}$

$ES := AE - AS \qquad BR := ES \qquad OR := BO - BR \qquad ET := OR \qquad IO := \frac{ET \cdot GO}{GT} \qquad BI := IO - BO$

$AI := BI + AB \qquad BE := \sqrt{ER^2 + BR^2} \qquad GE := BE \qquad GI := \frac{GE \cdot GO}{GT} \qquad EI := GI - GE \qquad AK := AI \qquad IK := EI$

$IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI} \qquad EK := AK - AE \qquad \frac{EK}{IQ} = 2$

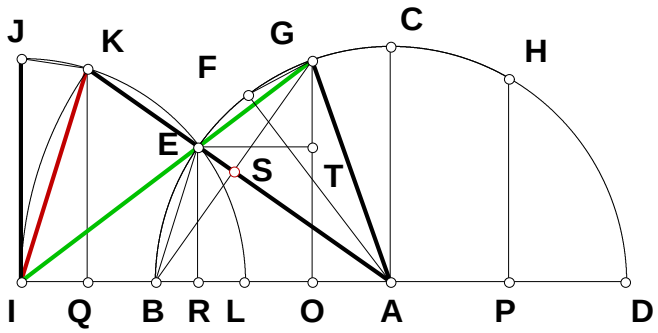
## Some Algebraic Names:

$\frac{3}{4 \cdot N} - BO = 0 \qquad 1 - \frac{3}{4 \cdot N} - DO = 0 \qquad \frac{1}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0$

$\frac{1}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0 \qquad \frac{1}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0 \qquad \frac{1}{4} \cdot \frac{\sqrt{3}}{N} \cdot \sqrt{4 \cdot N - 3} - \frac{1}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - GT = 0$

$\frac{1}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0 \qquad \frac{1}{2} - \frac{1}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - ES = 0 \qquad \frac{3}{(4 \cdot N)} - \frac{1}{2} + \frac{1}{4} \cdot \sqrt{4 - \frac{3}{N}} - OR = 0$

$\frac{-1}{4} \cdot \frac{\left[3 \cdot \sqrt{4 \cdot N - 3} \cdot \sqrt{N} - 2 \cdot \sqrt{4 \cdot N - 3} \cdot N^{\left(\frac{3}{2}\right)} + 4 \cdot N^2 - 3 \cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)} \cdot \left(-\sqrt{4 \cdot N - 3} + \sqrt{N}\right)\right]} - IO = 0 \qquad \frac{-1}{2} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - 2 \cdot \sqrt{N}\right)}{\left(\sqrt{4 \cdot N - 3} - \sqrt{N}\right)} - BI = 0$




$$\frac{-1}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

$$\frac{1}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

$$\frac{1}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

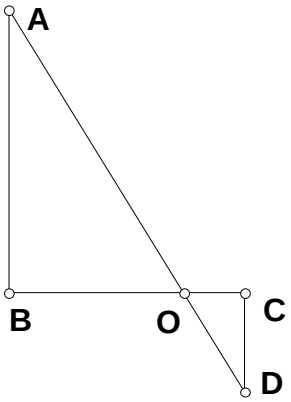
$$\frac{-1}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

$$\frac{1}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0$$

$$\frac{1}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$

## 08/07/00 Proportion Series II

**Two unknowns have the same proportion as two givens and the sum of the unknowns are known. Find the two unknowns.**




$AB := 9 \qquad CD := 3 \qquad BC := 5$

$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

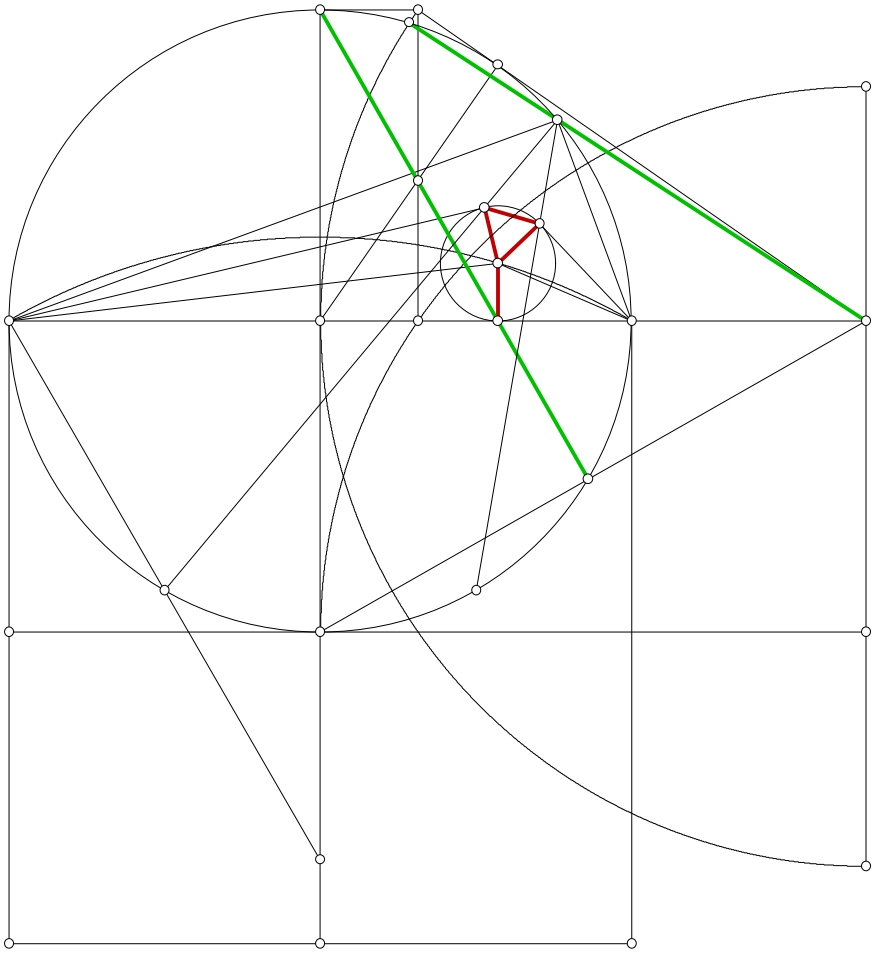
$\frac{AB}{CD} - \frac{BO}{CO} = 0$

m∠UDG = 69.867°

m∠UDCI = 23.289°

$\frac{m\angle UDG}{m\angle UDCI} = 3.000$

# Trisection and the square root figure.

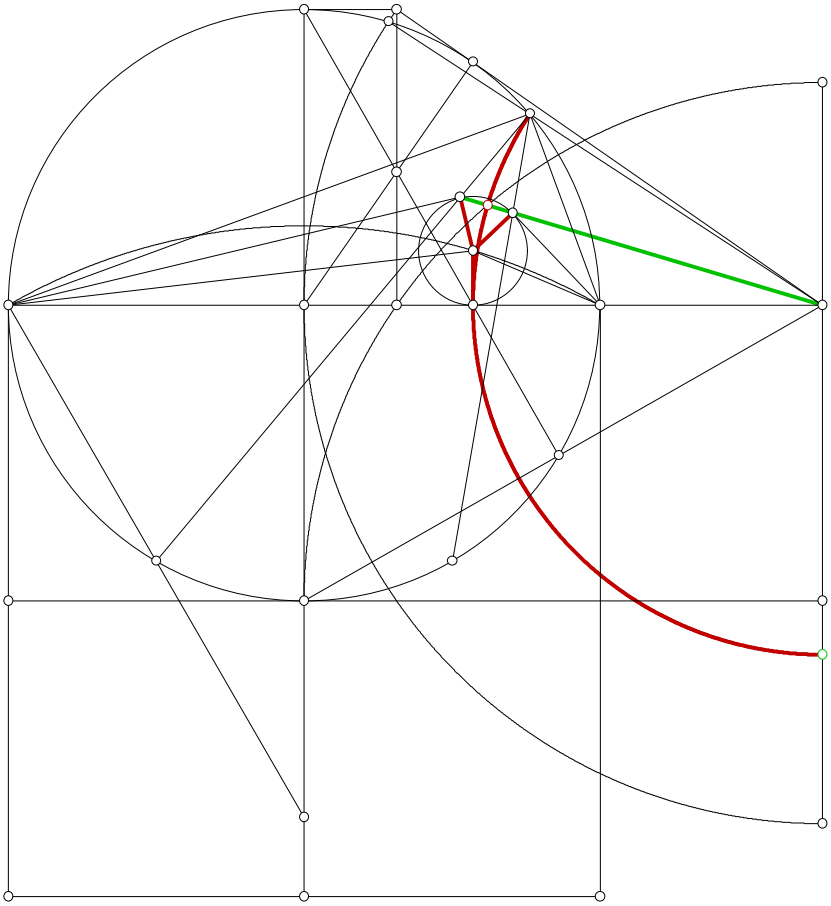


m∠UDG = 69.867°

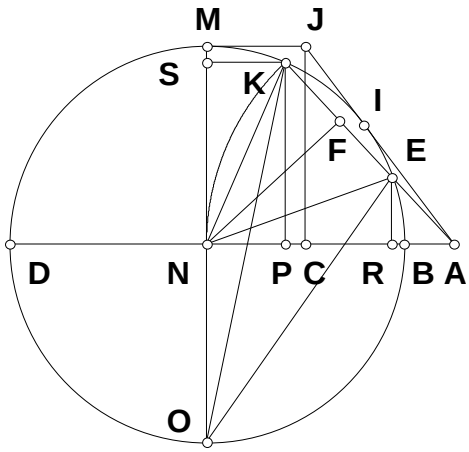
m∠UDCI = 23.289°

$$\frac{m\angle UDG}{m\angle UDCI} = 3.000$$

# Trisection and the square root figure.



# 08/23/00 Trisection In A Square Root Figure




**Given the square root figure drawn for trisection, what is AR given AB and AD?**

$N := 3 \qquad AB := 1 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN \qquad CJ := BN$

$AJ := \sqrt{AC^2 + CJ^2} \qquad AK := AJ$

$AN := AB + BN \qquad AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$

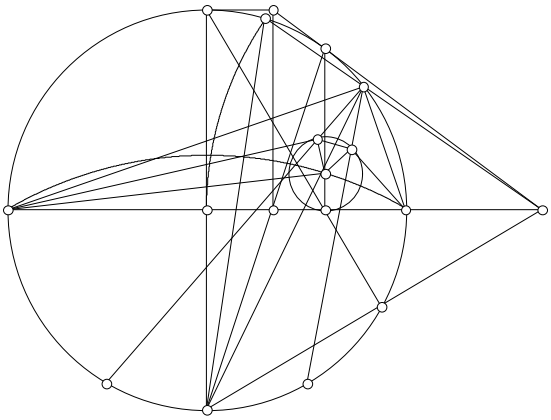
$AF := \frac{AP \cdot AN}{AK} \qquad FK := AK - AF \qquad EF := FK$

$AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK} \qquad N \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \qquad BR := AR - AB$

$(3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} - BR = 0$

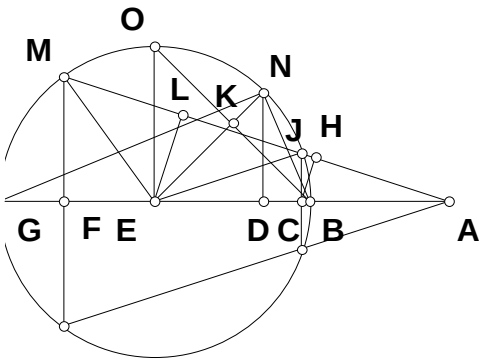
## Does KS = FK?

$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$



## 09/03/00 Ratios In Trisection

**How does BF vary with BC? How does DF vary with BC?**




$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2}$

$EN := BE \quad EK := \frac{BE \cdot BE}{BO} \quad KN := EN - EK \quad BK := \frac{BO}{2}$

$BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE$

$AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC \quad AE := AB + BE$

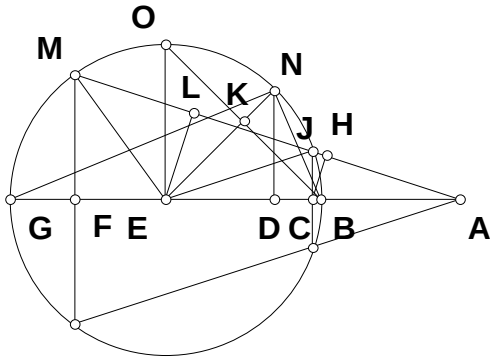
$JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB$

$$\frac{-1}{4} \cdot \left(-2 + \sqrt{2}\right) \cdot \frac{N_1}{N_2} - BC = 0$$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2 \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right)\right]}{\left(N_2^2 \cdot N_1\right)} - \frac{BF}{BC} = 0$$

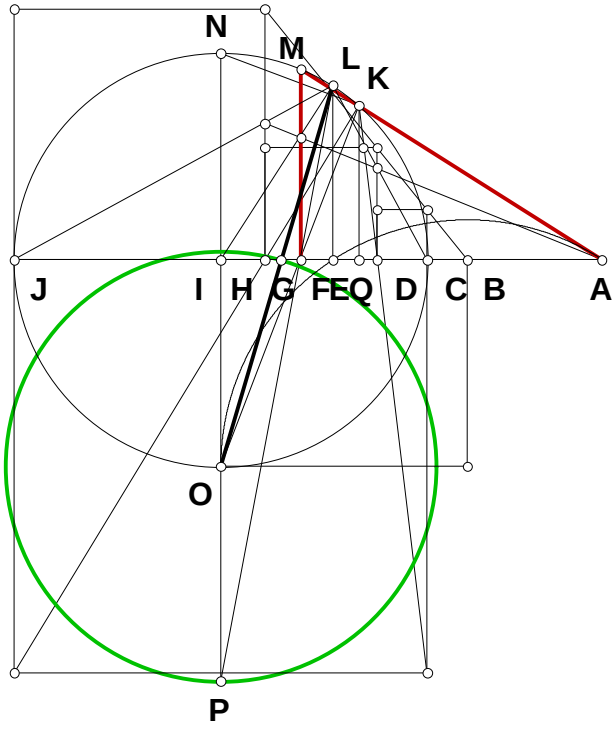



$$DF := BF - BD$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(7\cdot\sqrt{2} - 10\right)\cdot\left(12\cdot\sqrt{2}\cdot N_2^{\,2} + 17\cdot N_2^{\,2} + 2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2\right)\right]}{4\,N_2^{\,3}} - DF = 0$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(2\cdot\sqrt{2} - 3\right)\cdot\left(2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2 + 17\cdot N_2^{\,2} + 12\cdot\sqrt{2}\cdot N_2^{\,2}\right)\right]}{N_1\cdot N_2^{\,2}} - \frac{DF}{BC} = 0$$

# Goshdarn Good Pencil 09/16/00




$N := 2$

$BC := 1 \qquad BJ := BC \cdot N \qquad BE := \sqrt{BC \cdot BJ}$

$CJ := BJ - BC \qquad CI := \frac{CJ}{2} \qquad IO := CI$

$NO := CJ \qquad CR := CJ$

$CE := BE - BC \qquad EI := CI - CE \qquad EJ := CJ - CE$

$EL := \sqrt{CE \cdot EJ} \qquad EG := \frac{EI \cdot EL}{EL + IO} \qquad GI := EI - EG$

$GO := \sqrt{GI^2 + IO^2} \qquad OP := GO$

$IP := IO + OP \qquad EF := \frac{EI \cdot EL}{EL + IP} \qquad FI := EI - EF$

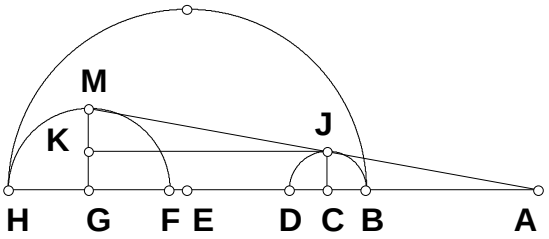
$FO := \sqrt{FI^2 + IO^2} \qquad OK := \frac{IO \cdot NO}{FO}$

$FK := OK - FO \qquad FQ := \frac{FI \cdot FK}{FO} \qquad QI := FQ + FI \qquad CQ := CI - QI \qquad QJ := CJ - CQ \qquad QK := \sqrt{CQ \cdot QJ}$

$CD := \frac{CQ \cdot CR}{CR + QK} \qquad BD := CD + BC \qquad \left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 0.000004486957912$

## Midpoints and Similarity Points 09/18/00

**What is AE given the radius of the two circles and the difference between their centers? (External Unit).**




$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8$

$BC := N_1 \quad GH := N_2 \quad CG := N_3$

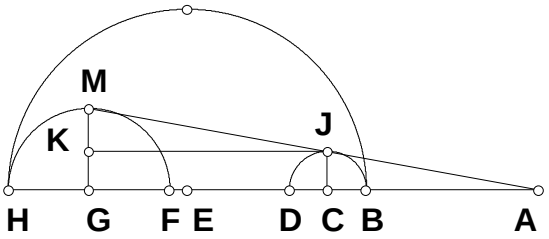
$CJ := BC \quad GM := GH \quad GK := CJ$

$BH := BC + CG + GH \quad BE := \frac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1^2 + N_3 \cdot N_1 + N_2^2}{2 \cdot (N_2 - N_1)} = 0$$

**What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).**




$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2}$

$GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$

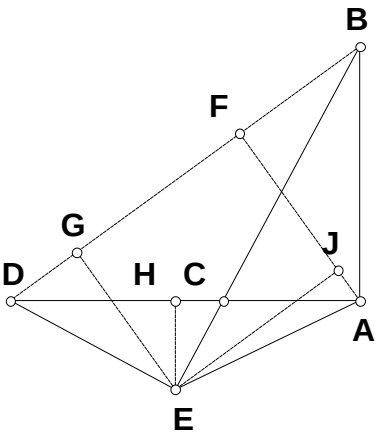
$CJ := BC \quad GM := GH \quad GK := CJ$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$$

**11/13/00**

## Given AB, DE, AD find BE, AC, CD, EC, BC.




$N_1 := 2 \qquad N_2 := 4 \qquad N_3 := 1$

$AB := N_1 \qquad AD := N_2 \qquad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \qquad BF := \frac{AB^2}{BD} \qquad DG := \frac{DE^2}{BD} \qquad BE := \sqrt{AD^2 - DE^2}$

$AF := \sqrt{AB^2 - BF^2} \qquad EG := \sqrt{DE^2 - DG^2}$

$FG := BD - (BF + DG) \qquad EJ := FG \qquad FJ := EG \qquad AJ := AF - FJ \qquad AE := \sqrt{EJ^2 + AJ^2} \qquad S_1 := AD \qquad S_2 := DE \qquad S_3 := AE$

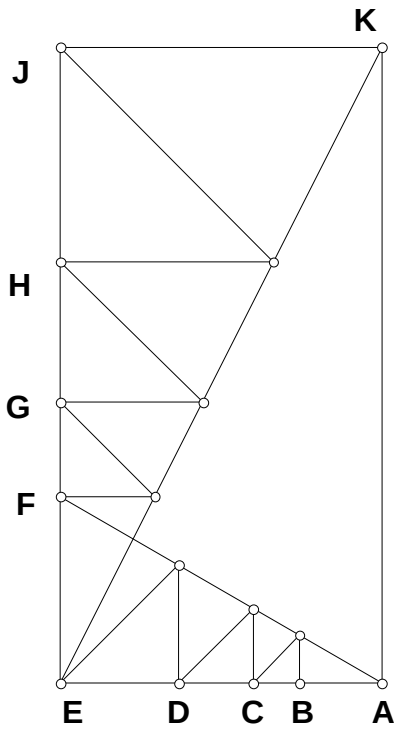
$AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad EH := \sqrt{AE^2 - AH^2} \qquad CH := \frac{EH \cdot AH}{AB + EH} \qquad AC := AH - CH \qquad CE := \frac{AC \cdot DE}{AB} \qquad CD := AD - AC$

$$N_1 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_1^2 \cdot N_2 - N_1 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - AC = 0$$

$$N_3 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_2 \cdot N_1^2 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - CE = 0$$

# Means On Means 11/28/00

**Modify 02/28/98 for Mean proportionals between E and J.**




$AE := 1 \qquad N := 3 \qquad EJ := AE \cdot N \qquad JK := AE$

$HJ := \frac{JK \cdot EJ}{JK + EJ} \qquad EH := EJ - HJ \qquad GH := \frac{EH \cdot HJ}{EH + HJ}$

$EG := EH - GH \qquad FG := \frac{EG \cdot GH}{EG + GH} \qquad EF := EG - FG$

$DE := \frac{EF \cdot AE}{EF + AE} \qquad AD := AE - DE \qquad CD := \frac{AD \cdot DE}{AD + DE}$

$AC := AD - CD \qquad BC := \frac{AC \cdot CD}{AC + CD} \qquad AB := AC - BC$

$M := 0 .. 3 \qquad P := 0 .. 3 \qquad AEAB_{M,P} := \left[ \frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

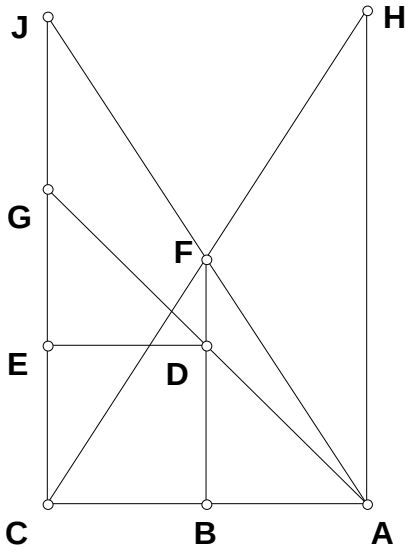
$$AEAB = \begin{bmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.563 & 34.328 \\ 1 & 2.688 & 7.223 & 19.411 \\ 1 & 2.266 & 5.133 & 11.63 \end{bmatrix}$$

$AEAB_{3,3} - \frac{AE}{AB} = 0 \qquad AEAB_{3,2} - \frac{AE}{AC} = 0 \qquad AEAB_{3,1} - \frac{AE}{AD} = 0 \qquad AEAB_{3,0} - \frac{AE}{AE} = 0$

$AEAB_{3,3} - \frac{AE}{AB} = 0$

# Multiplication and Division-Line By A Line 11/29/00

**Given some unit, and two differences, multiply or divide the one difference by the other.**
**For Division:**




$AC := 1 \qquad N_1 := 3 \qquad N_2 := 12 \qquad AH := N_1$

$CJ := N_2 \qquad AB := \frac{AH}{(CJ + AH)} \cdot AC$

$BC := AC - AB \quad BD := BC \quad CG := \frac{BD \cdot AC}{AB}$

$CG - \frac{N_2}{N_1} = 0 \qquad CG = 4$

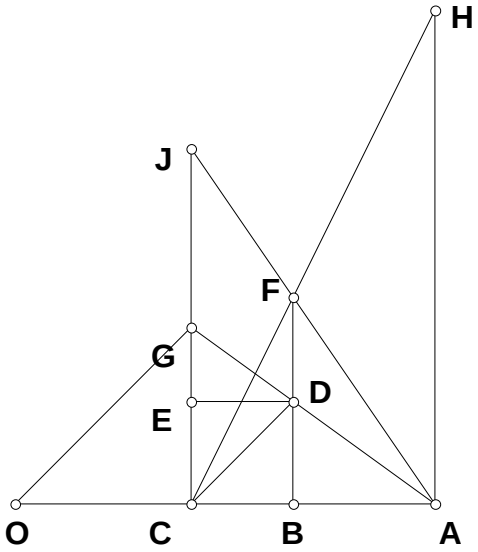
**For Multiplication:**

$AC := 1 \qquad N_1 := 5 \qquad N_2 := 7 \qquad AH := N_1$

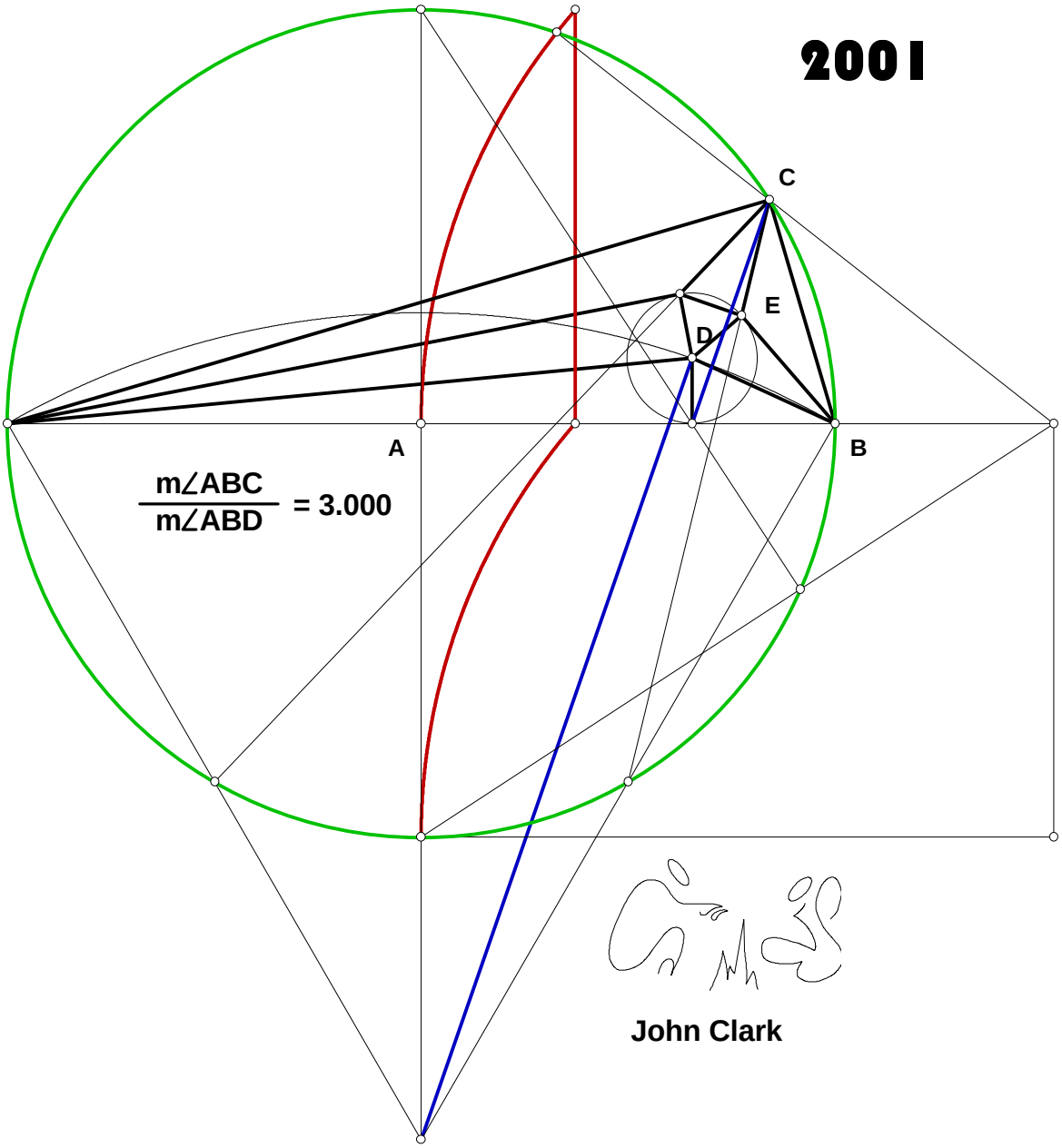
$CG := N_2 \qquad CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

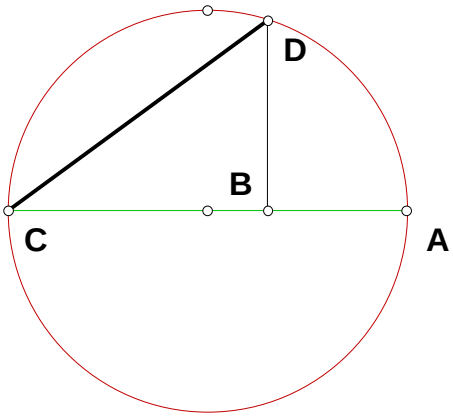
$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$



# Three Pieces Of Paper




John Clark




**Alternate method for common segment common endpoint square root.**

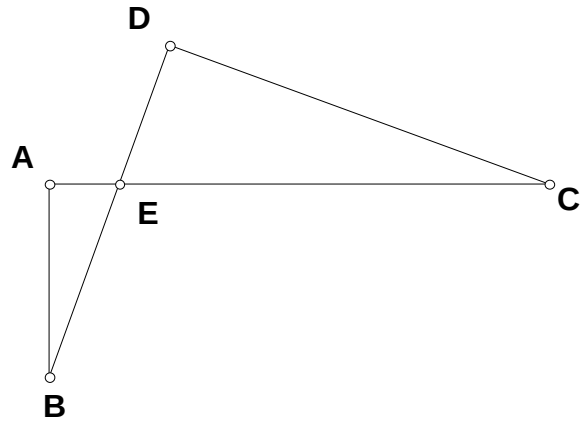
$N_1 := 5$ $N_2 := 3$ $AC := N_1$ $BC := N_2$

$AB := AC - BC$ $BD := \sqrt{AB \cdot BC}$ $CD := \sqrt{BD^2 + BC^2}$

$\sqrt{N_2 \cdot N_1} - CD = 0$

## Three Given Five Taken 042101

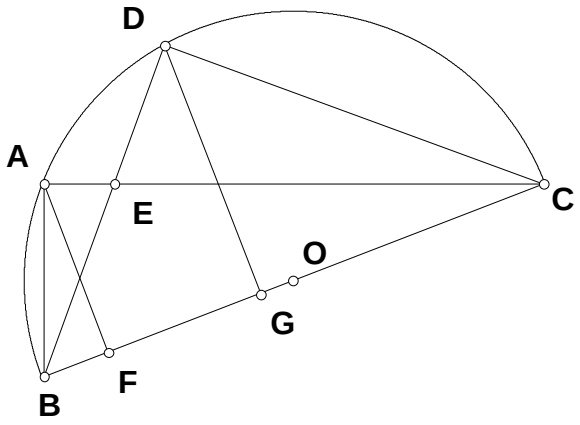
**Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?**




$N_1 := 2 \qquad N_2 := 3 \qquad N_3 := 4$

$AB := N_1 \quad CD := N_2 \quad AC := N_3$

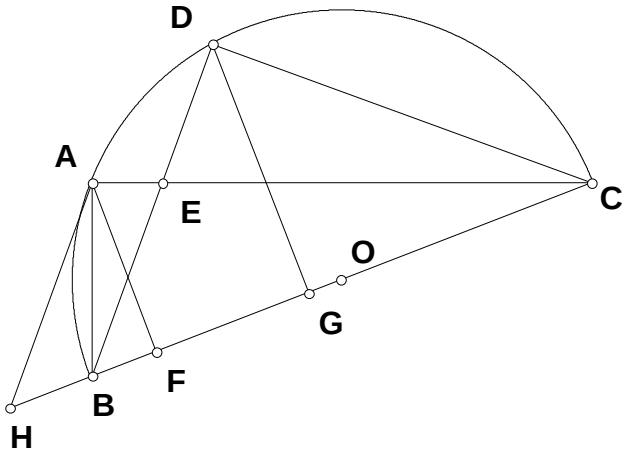
$BC := \sqrt{AB^2 + AC^2} \qquad CG := \frac{CD^2}{BC} \qquad BF := \frac{AB^2}{BC}$




$BG := BC - CG \qquad CF := BC - BF$

$AF := \sqrt{AB^2 - BF^2} \qquad DG := \sqrt{CD^2 - CG^2}$

$FH := \frac{BG \cdot AF}{DG} \qquad CH := CF + FH$



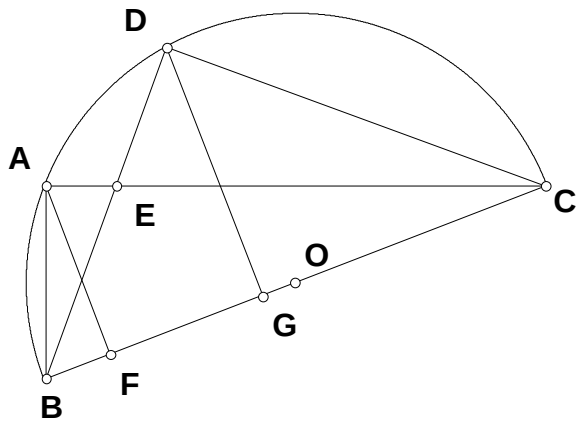

$BD := \sqrt{BG^2 + DG^2}$

$AH := \frac{BD \cdot FH}{BG} \qquad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \qquad CE := \frac{AC \cdot BC}{CH}$

$AE := AC - CE$

**Idea about process progression: Given two differences one renders a set number of processes upon them, then one progresses to three?**




**Some Algebraic Names:**

$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

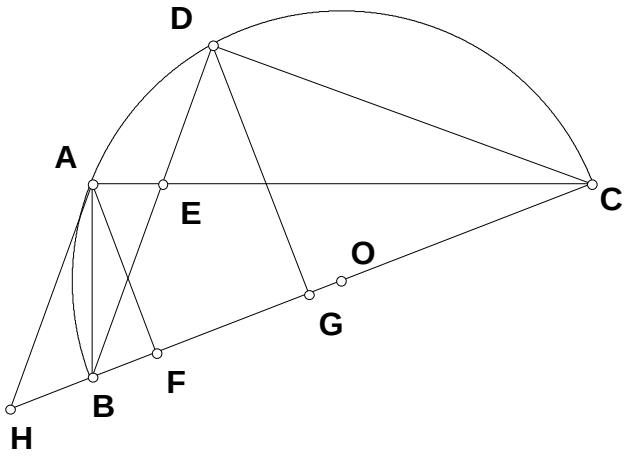
$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0$$

$$\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0$$

$$\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0$$

$$\frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

$$N_2 \cdot \sqrt{\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\left(N_1^2 + N_3^2\right)}} - DG = 0$$

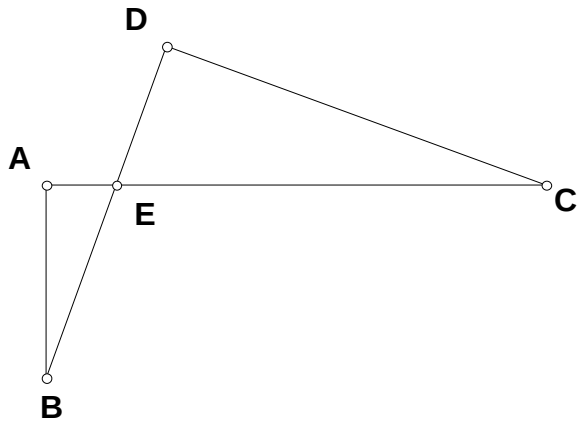



$$\frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} + \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}}\right] - CH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

**The Five Sought:**

$$\sqrt{N_1^2 + N_3^2 - N_2^2} - BD = 0$$



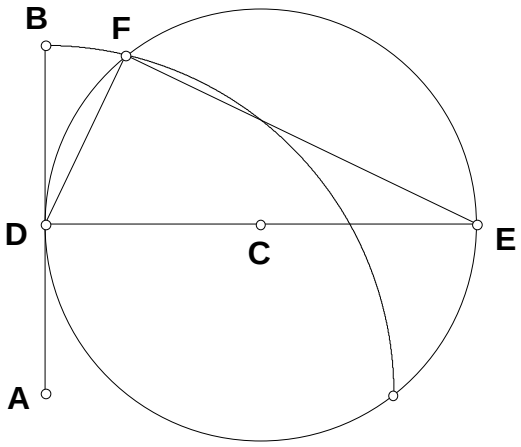

$$N_2 \cdot \frac{\left(\sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - DE = 0$$

$$N_1 \cdot \frac{\left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - BE = 0$$

$$\frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1} - CE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - AE = 0$$

**Given AB as unit, AD and DC, what is EF and DF?**




$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2\ CD$

$AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$

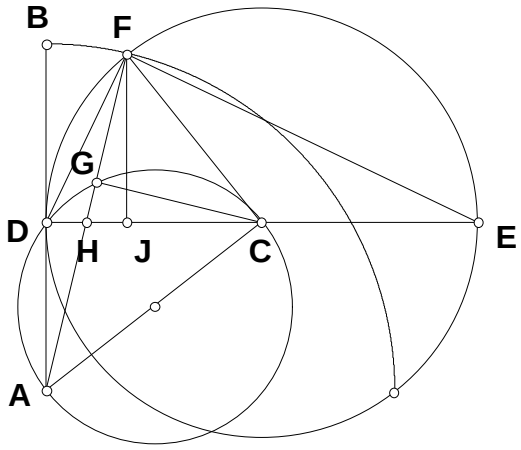
$AG := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

$L_1 := AD \quad L_2 := CG \quad L_3 := CD$

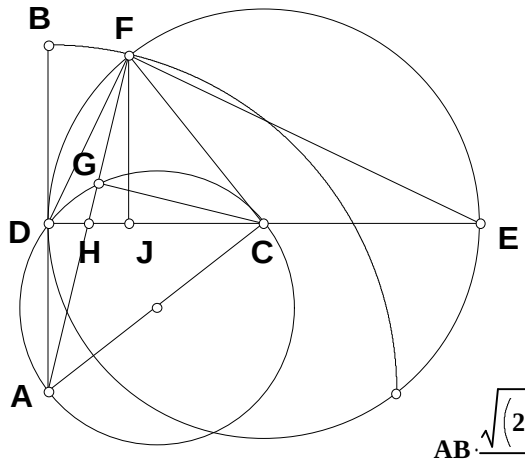
$DH := L_3 - \frac{L_2 \cdot \left(L_1^2 + L_3^2\right)}{L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1}$

$AH := L_1 \cdot \frac{\left(L_1^2 + L_3^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1\right)} \quad FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$

$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$



**Some Algebraic Names:**




$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2\,AB) \cdot N_2 - DE = 0 \qquad AB \cdot \sqrt{\frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{N_1^{\,2}}} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0$$

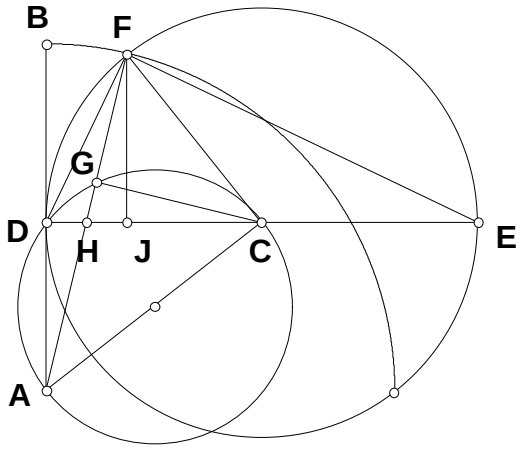
$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2} \cdot AB \cdot \frac{\begin{bmatrix} -N_1^{\,4} \cdot N_2^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + 2 \cdot N_1^{\,3} \cdot N_2 - 2 \cdot N_1^{\,5} \cdot N_2 \ldots \\ + 6 \cdot N_1^{\,5} \cdot N_2^{\,3} - 3 \cdot N_2^{\,2} \cdot N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + 2 \cdot N_2^{\,3} \cdot N_1^{\,3} - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ldots \\ + N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \end{bmatrix}}{\left[N_1 \cdot \left[\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right) \cdot \left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)\right]\right]} - HJ = 0$$




$$\frac{1}{2}\cdot AB\cdot\frac{\left(N_1{}^3\cdot N_2+N_2\cdot N_1-\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N_2\cdot N_1+4\cdot N_2{}^3\cdot N_1{}^3-N_1{}^3\cdot N_2+\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}-EJ=0$$

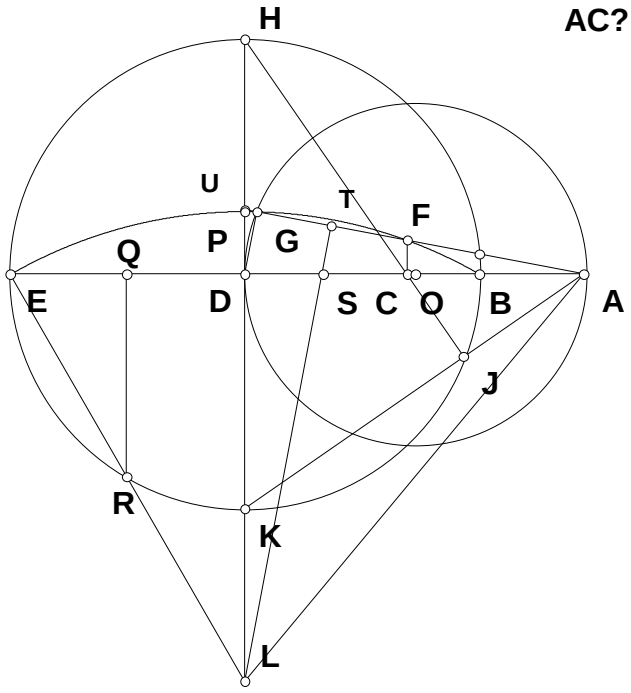
$$\frac{-1}{2}\cdot AB\,\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}+1-N_1{}^2+2\cdot N_2{}^2\cdot N_1{}^2\right)}{\left[N_1\cdot\left(1+N_2{}^2\cdot N_1{}^2\right)\right]}-FJ=0$$

$$\sqrt{AB^2\cdot N_2\cdot\frac{\left(3\cdot N_1\cdot N_2+4\cdot N_2{}^3\cdot N_1{}^3-N_1{}^3\cdot N_2+\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)}{\left[\left(1+N_2{}^2\cdot N_1{}^2\right)\cdot N_1\right]}}-EF=0$$

$$\sqrt{\left(N_1{}^3\cdot N_2+N_1\cdot N_2-\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}\right)\cdot AB^2\cdot\frac{N_2}{\left[N_1\cdot\left(1+N_2{}^2\cdot N_1{}^2\right)\right]}}-DF=0$$

# Counterpoint 042301

**CF is the radius of a circle expressed in trisection. Is AF the counterpart of AC?**




$N := 5 \qquad AB := 1$

$AE := AB \cdot N \qquad BE := AE - AB$

$BD := \frac{BE}{2} \qquad AD := AB + BD$

$AO := \frac{AD}{2} \qquad DE := BD \qquad DQ := \frac{DE}{2}$

$EQ := DQ \qquad BQ := BD + DQ$

$QR := \sqrt{BQ \cdot EQ} \qquad ER := \sqrt{QR^2 + EQ^2}$

$EL := 2 \cdot ER \qquad LP := EL \qquad DL := \frac{QR \cdot EL}{ER}$

$L_1 := \frac{LP}{DL} \qquad L_2 := \frac{AO}{LP}$

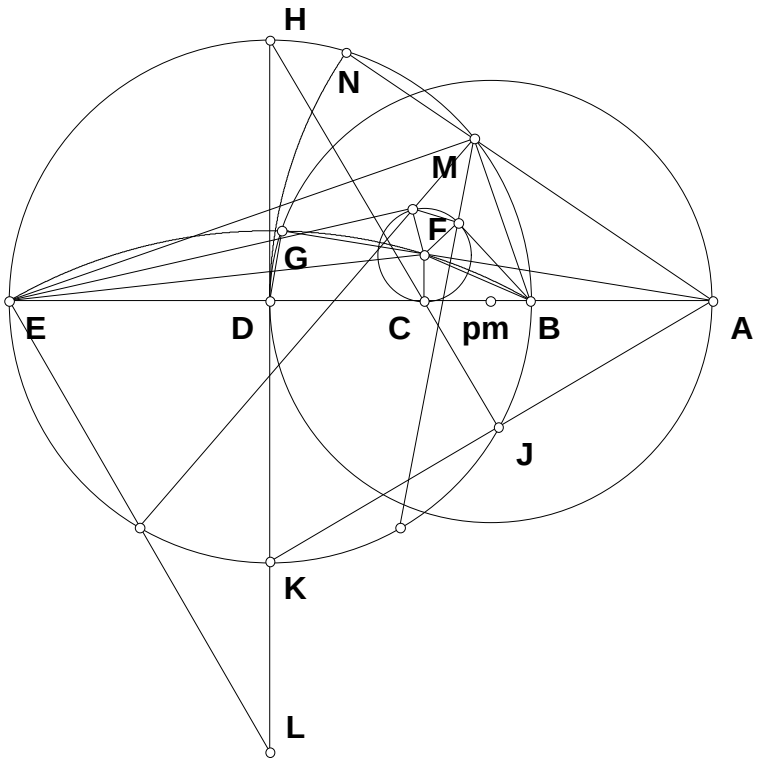
$$AG := \sqrt{LP^2 \cdot L_2 \cdot \frac{\left(3 \cdot L_1 \cdot L_2 + 4 \cdot L_2^3 \cdot L_1^3 - L_1^3 \cdot L_2 + \sqrt{2 \cdot L_1^2 + 4 \cdot L_1^4 \cdot L_2^2 - 1 - L_1^4}\right)}{\left[\left(1 + L_2^2 \cdot L_1^2\right) \cdot L_1\right]}}$$

$DG := \sqrt{AD^2 - AG^2} \qquad AU := \frac{AD \cdot AD}{AG} \qquad DU := \frac{DG \cdot AD}{AG} \qquad LU := DL + DU \qquad TU := \frac{DG \cdot LU}{AD} \qquad GU := AU - AG$

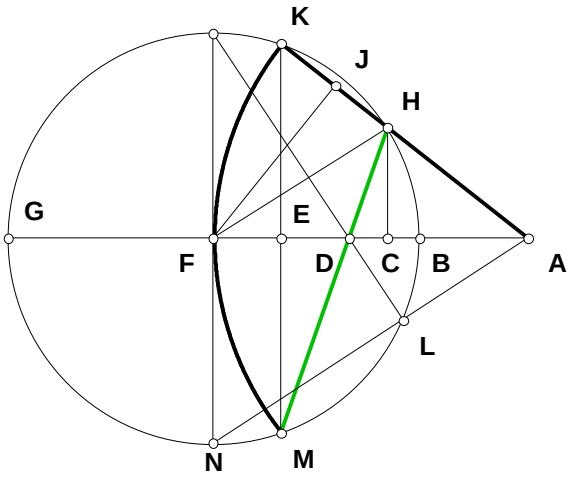
$TG := TU - GU \qquad FT := TG \qquad AF := AG - (FT + TG) \qquad AC := \frac{AD \cdot AF}{AU}$

**From 012496 for AC**

$$2 \cdot AB \cdot \frac{N}{(1+N)} \qquad AC - 2 \cdot AB \cdot \frac{N}{(1+N)} = 0$$



**042401**




**Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?**

$N := 5$ $AB := 1$ $AG := AB \cdot N$

$BG := AG - AB$ $BF := \frac{BG}{2}$ $AF := AB + BF$

$AK := AF$ $FK := BF$ $AE := \frac{2\,AK^2 - FK^2}{2\,AF}$

$AJ := AE$ $JK := AK - AJ$ $HJ := JK$

$AH := AK - (JK + HJ)$ $AC := \frac{AE \cdot AH}{AK}$ $CE := AE - AC$ $BE := AE - AB$ $EG := BG - BE$

$EK := \sqrt{BE \cdot EG}$ $BC := AC - AB$ $CG := BG - BC$ $CH := \sqrt{BC \cdot CG}$ $DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE$ $HM := \sqrt{CE^2 + (EK + CH)^2}$

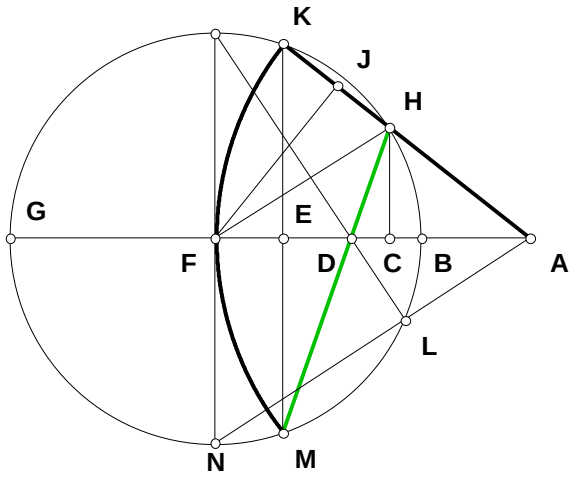
**Some Algebraic Names:**

$AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$

$\frac{1}{4} \cdot AB \cdot \frac{(N^2 + 6 \cdot N + 1)}{(1 + N)} - AE = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(1 - 2 \cdot N + N^2)}{(1 + N)} - JK = 0$ $2 \cdot AB \cdot \frac{N}{(1 + N)} - AH = 0$

$AB \cdot \frac{(1 + 6 \cdot N + N^2)}{(1 + N)^3} \cdot N - AC = 0$ $\frac{1}{4} \cdot AB \cdot (N^2 + 6 \cdot N + 1) \cdot \frac{(N - 1)^2}{(1 + N)^3} - CE = 0$

$\frac{1}{4} \cdot AB \cdot (N + 3) \cdot \frac{(N - 1)}{(1 + N)} - BE = 0$ $\frac{1}{4} \cdot AB \cdot (3 \cdot N + 1) \cdot \frac{(N - 1)}{(1 + N)} - EG = 0$




$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-EK=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BC=0$$

$$AB\cdot N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3}-CG=0$$

$$AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3}-CH=0$$

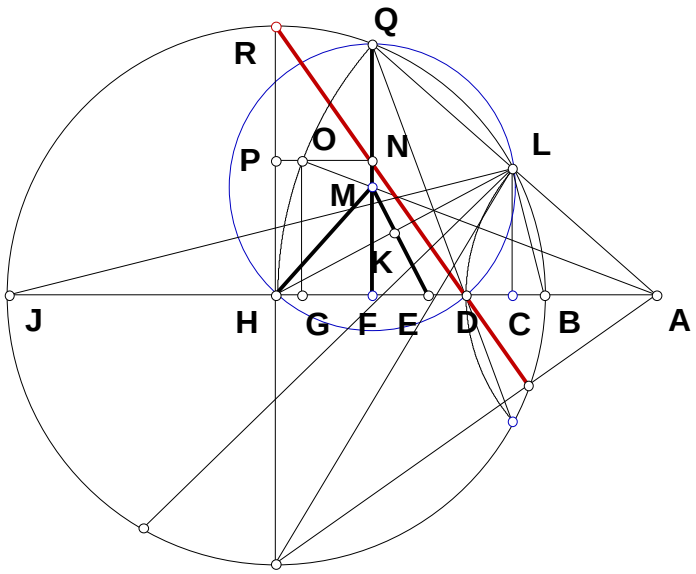
$$\frac{1}{4}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DE=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(N+1)}-DF=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)^2}-HM=0$$

**042501**

**What is the Algebraic name of the circle HM? Does point N divide DR in half?**




$N := 5.768$ $AB := .583$ $AJ := AB \cdot N$

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$ $HM - MQ = 0$

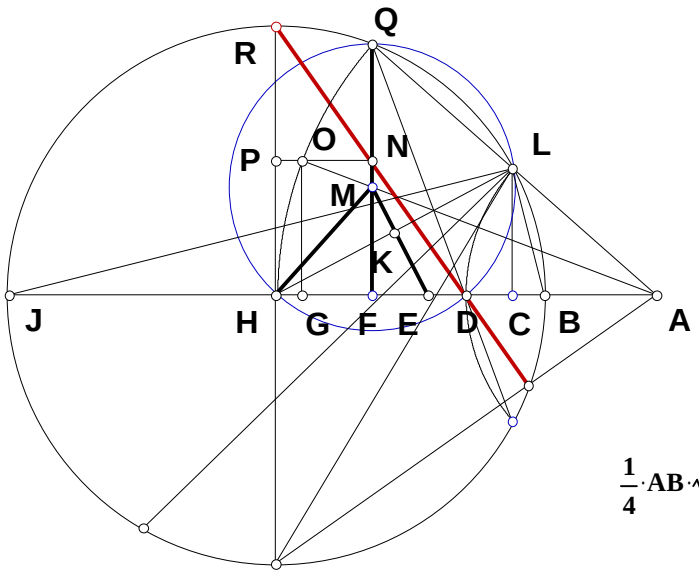
$DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

**Some Algebraic Names:**

$AB \cdot N - AB - BJ = 0$ $\frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0$ $\frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0$ $\frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0$ $\frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$




$$\frac{1}{4}\cdot AB\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)}-FJ=0$$

$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)}-BF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-FQ=0$$

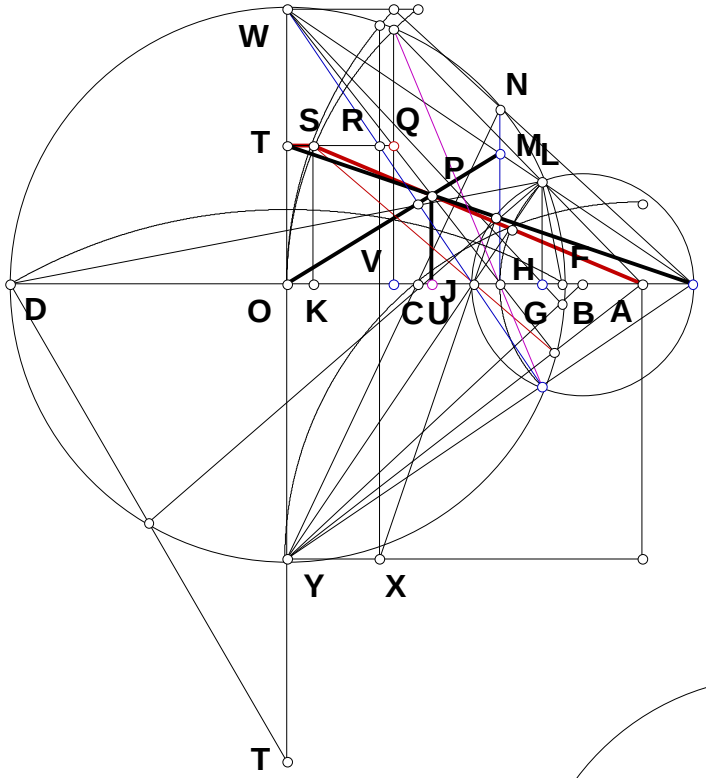
$$\frac{1}{2}\cdot(1+N)\cdot AB\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-HM=0$$

$$HM-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DH=0 \qquad \frac{DH}{2}-FH=0$$

# Four Lines To A Point 042901

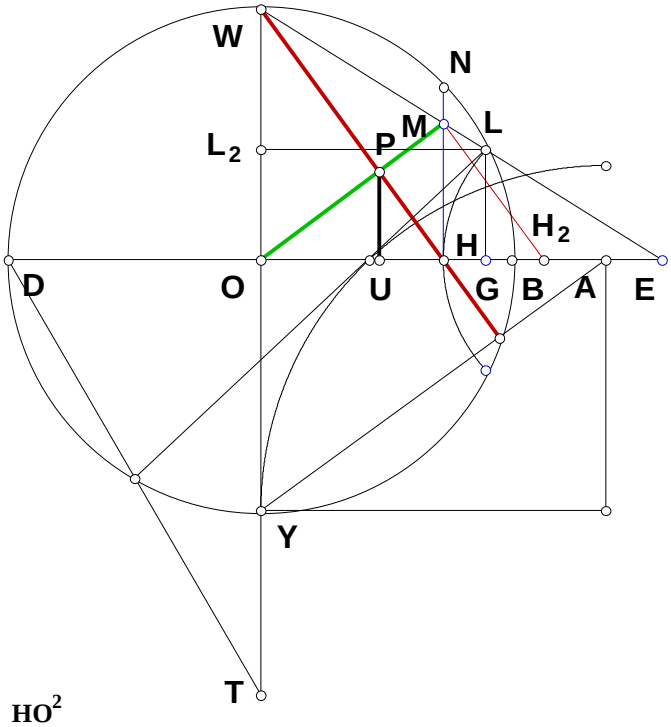



**Does the difference OU and PU each have but one Algebraic name?**

$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \frac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$




$HO := \frac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \frac{OL^2 + AO^2 - AL^2}{2\,AO}$

$BG := BO - GO \qquad DG := GO + DO$

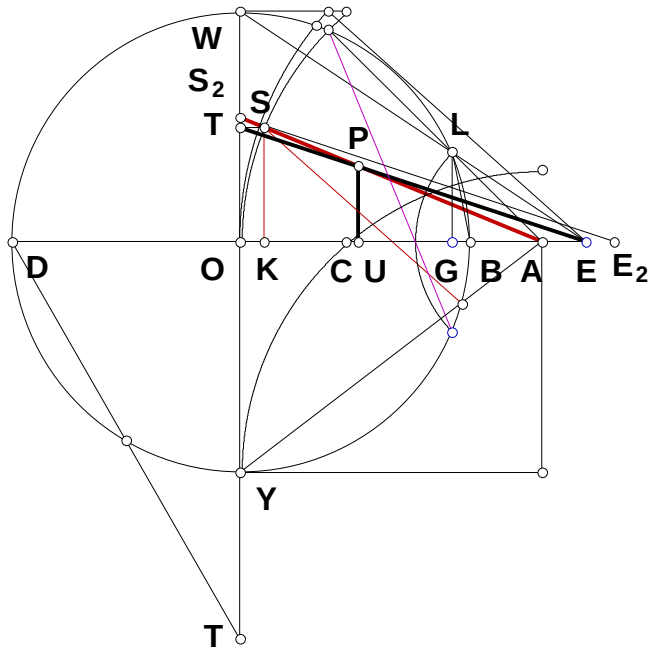
$GL := \sqrt{BG \cdot DG} \qquad EO := \frac{GO \cdot OW}{OW - GL}$

$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \frac{GL \cdot EH}{EG} \qquad HH_2 := \frac{HO \cdot HM}{OW} \qquad OU := \frac{HO^2}{HH_2 + HO}$

$UP := \frac{HM \cdot OU}{HO}$




$$OT := \frac{OW}{2} \quad KS := OT \quad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \quad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \quad AS_2 := \sqrt{AO^2 + OS_2{}^2}$$

$$AE_2 := OE_2 - AO \quad AE := EO - AO$$

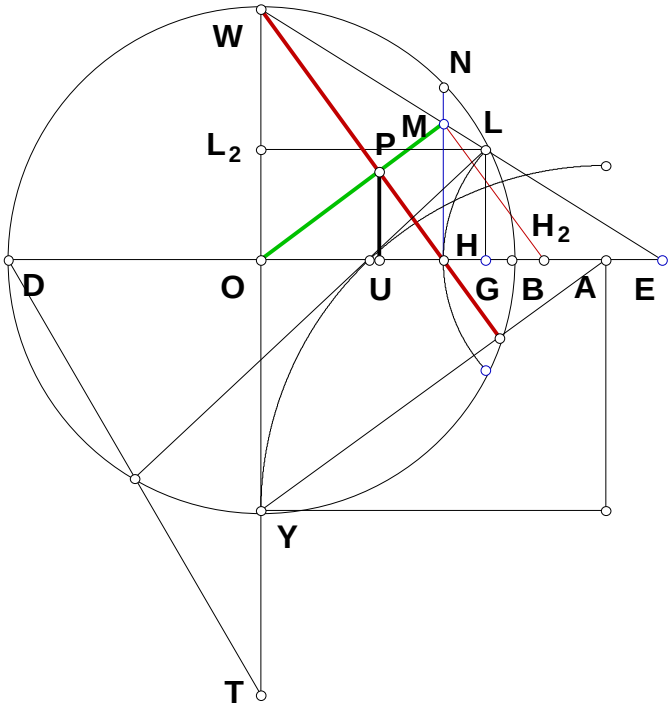
$$AP := \frac{AS_2 \cdot AE}{AE_2} \quad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \quad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$

## Some Algebraic Names:




$$AB\cdot(N-1)-BD=0$$

$$\frac{AB\cdot(N-1)}{2}-BO=0$$

$$\frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0$$

$$2\cdot AB\cdot\frac{N}{(1+N)}-AH=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0$$

$$AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0$$

$$AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

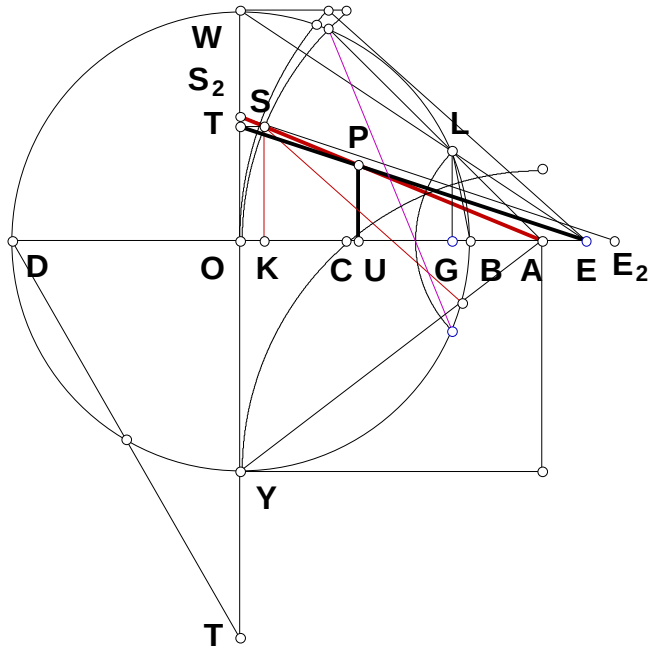
$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0$$

$$N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$




$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0$$

$$\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

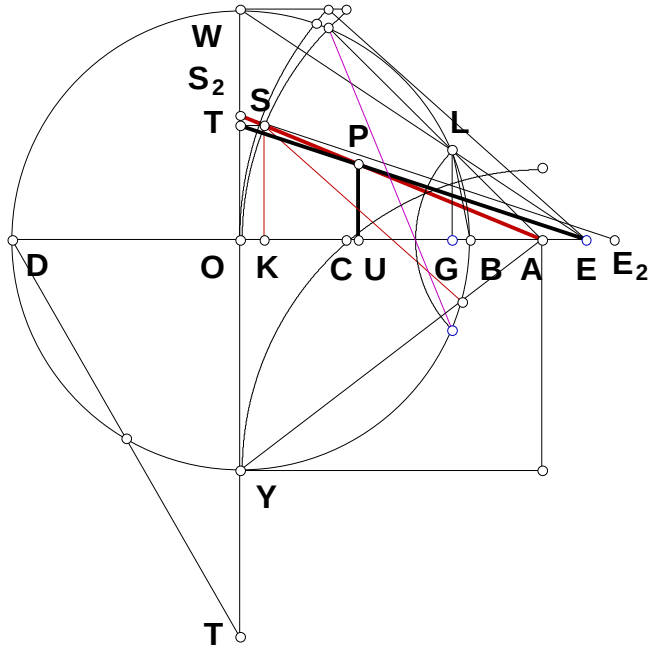
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AP=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$




$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AU=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

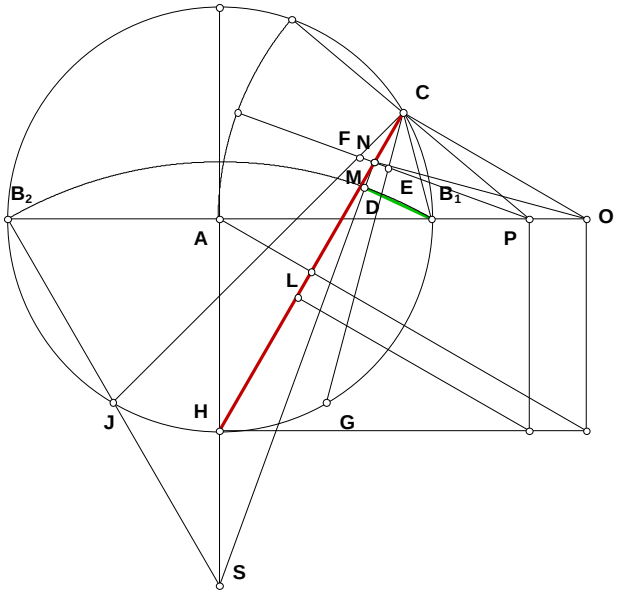
$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

# Compass Construction 043001

$m\angle AB_1C = 75.000°$ $m\angle AB_2C = 15.000°$

$m\angle AB_1D = 25.000°$ $m\angle AB_2D = 5.000°$

$\frac{m\angle AB_1C}{m\angle AB_1D} = 3.000$ $\frac{m\angle AB_2C}{m\angle AB_2D} = 3.000$



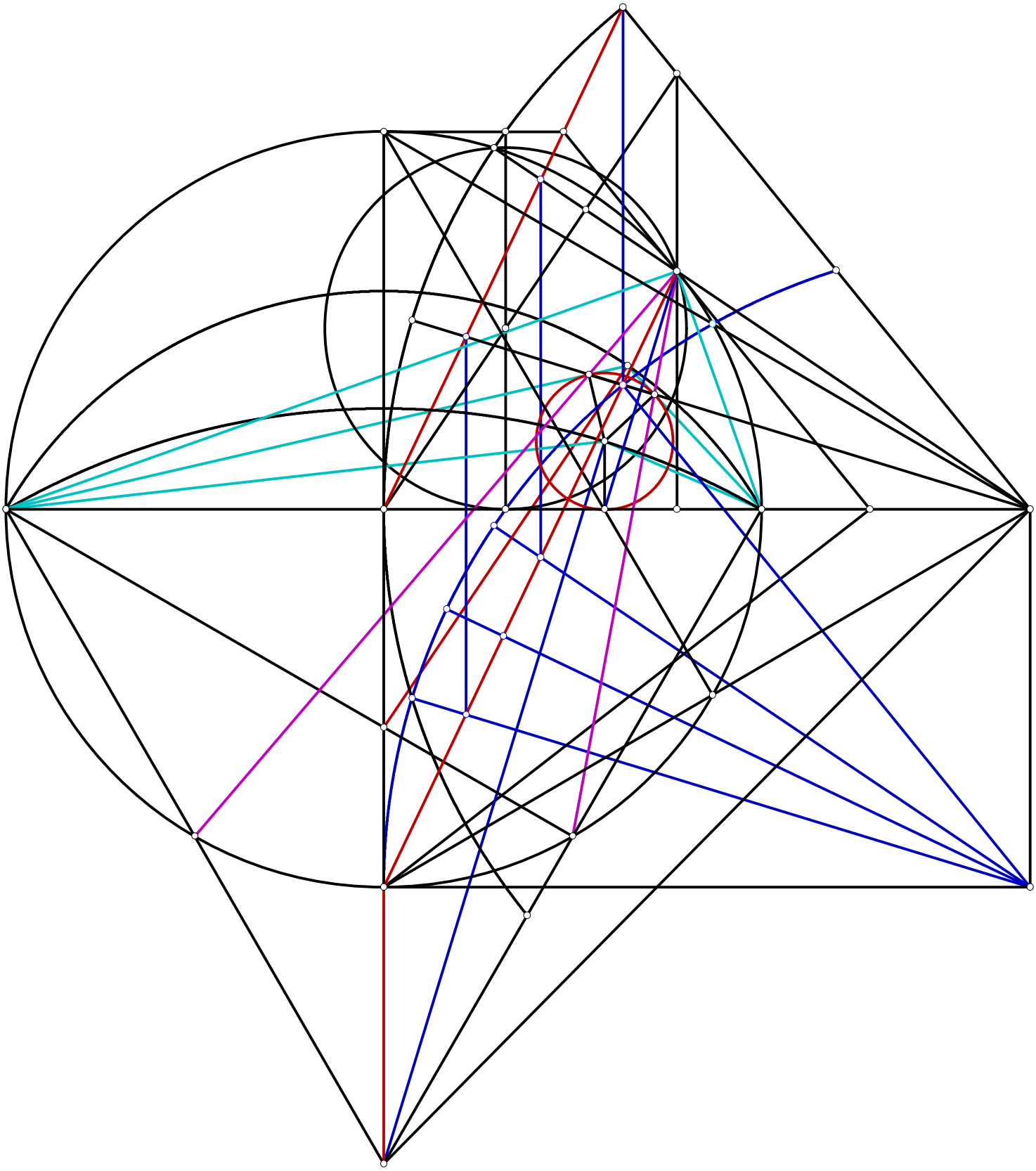

$m\angle ABC - 3 \cdot m\angle ABD = 0.000°$

$m\angle AB_2C - 3 \cdot m\angle AB_2D = -0.000°$

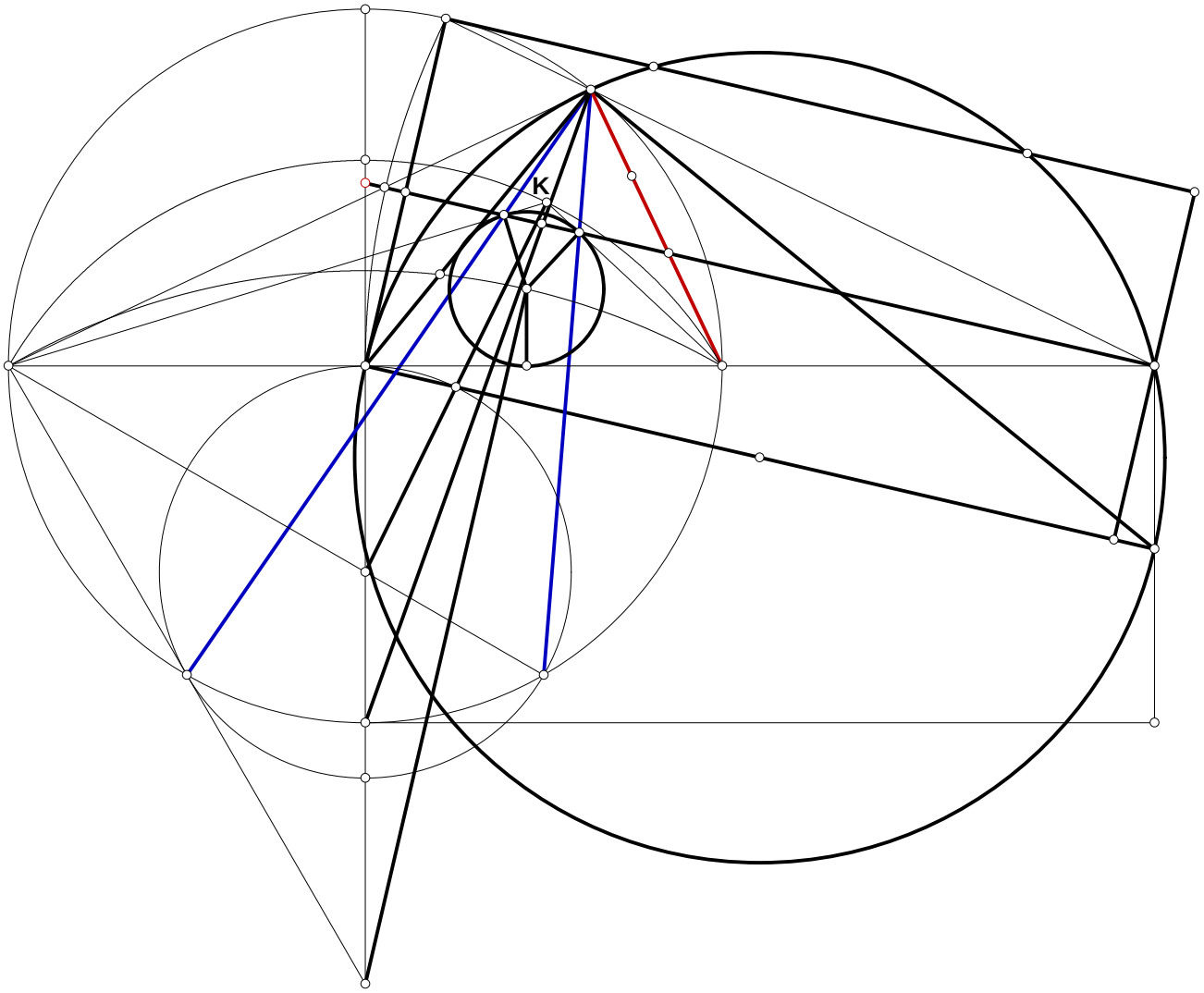
Move C->60 Move C->75

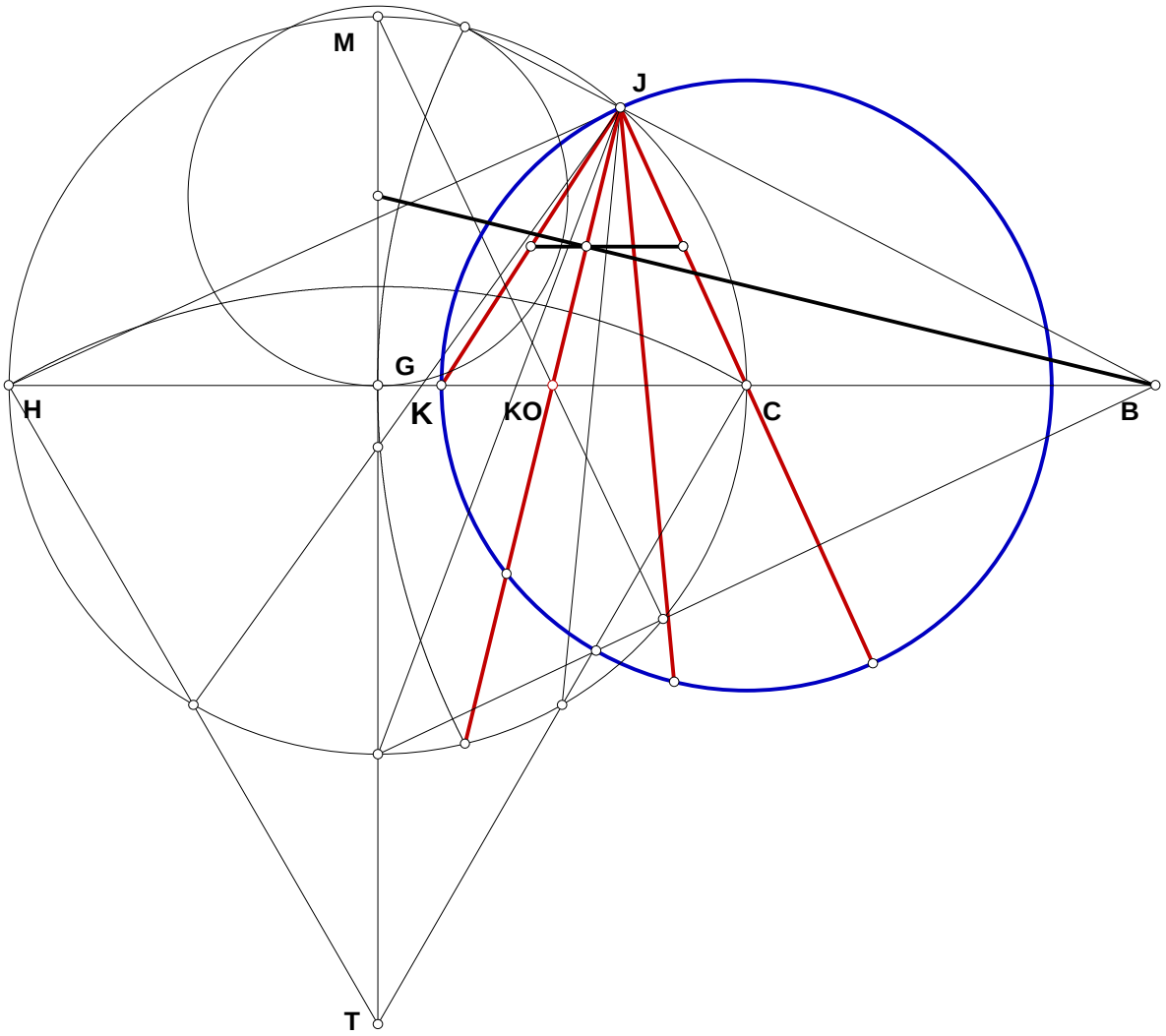
**The process is simple, construct point O and then bisect AOC. From point N construct P from the perpendicular L-which is the midpoint of NS.**
**Bisect APC constructing E and F which are bisected at point M. Draw S to M to construct point D.**

**When there is no difference between two things, those things are said to be equal. This is true even of tolerance. If a figure is more accurate than the tools used to construct the figure, then that figure is equal, in that tool, to a solution that is tool independant.**



**Pull K.**




M
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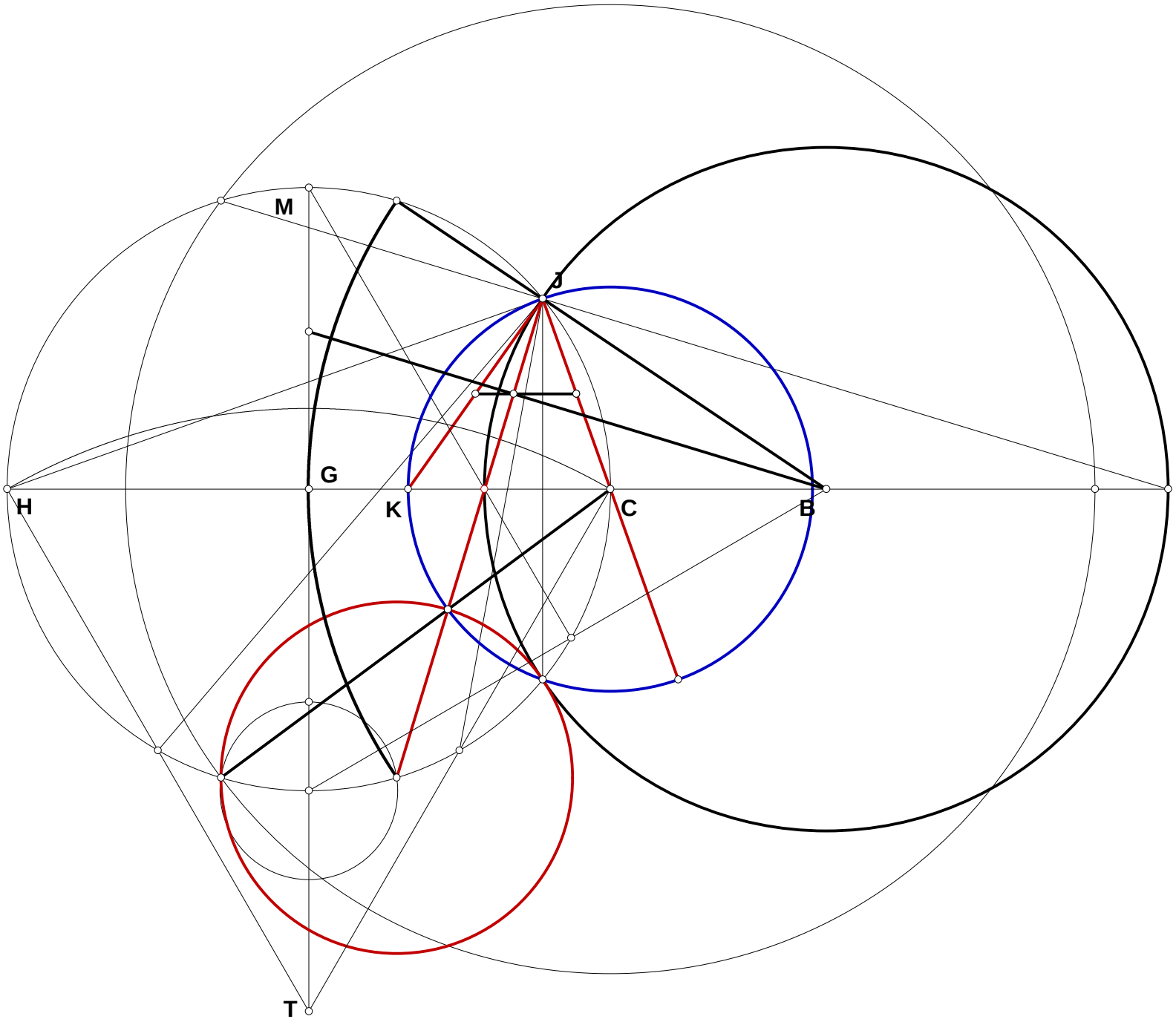
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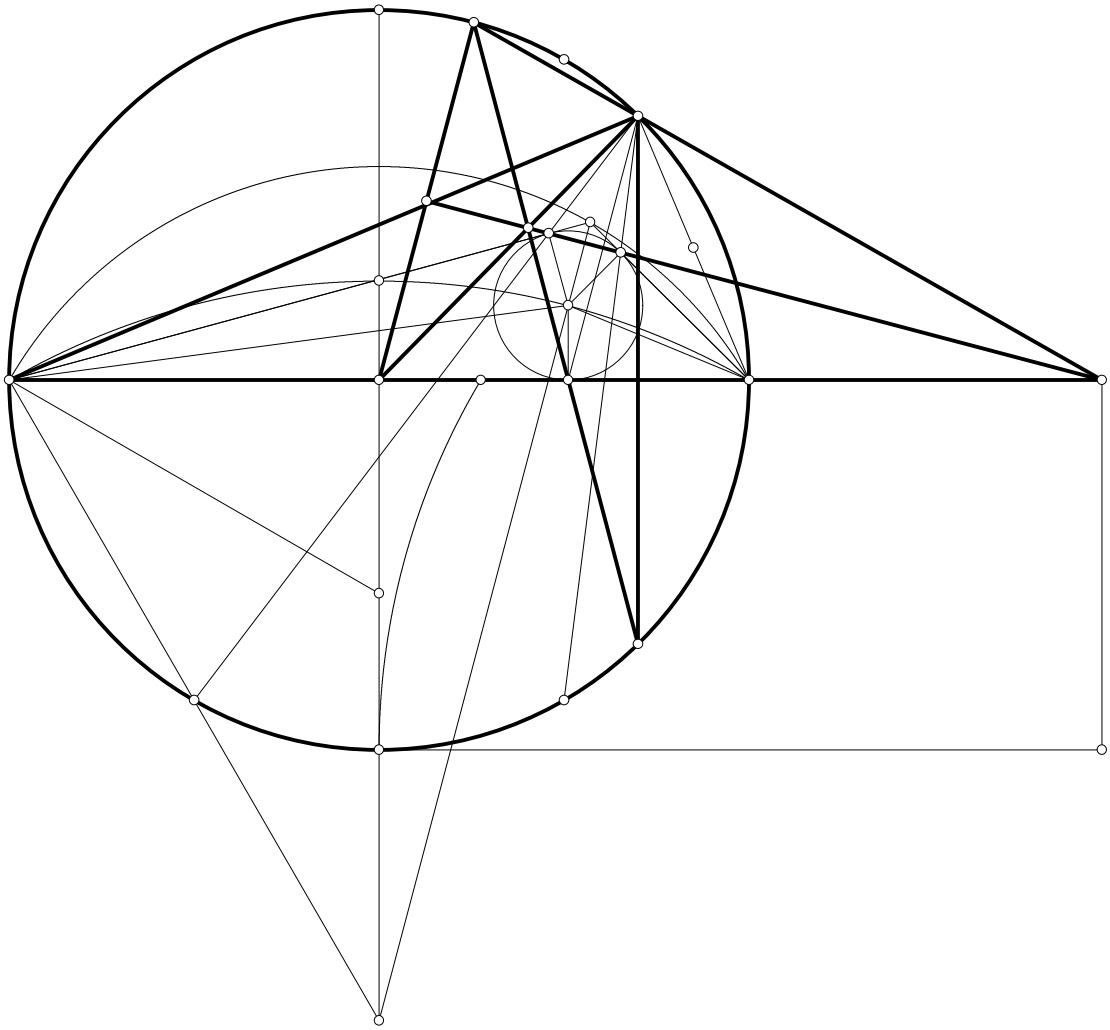

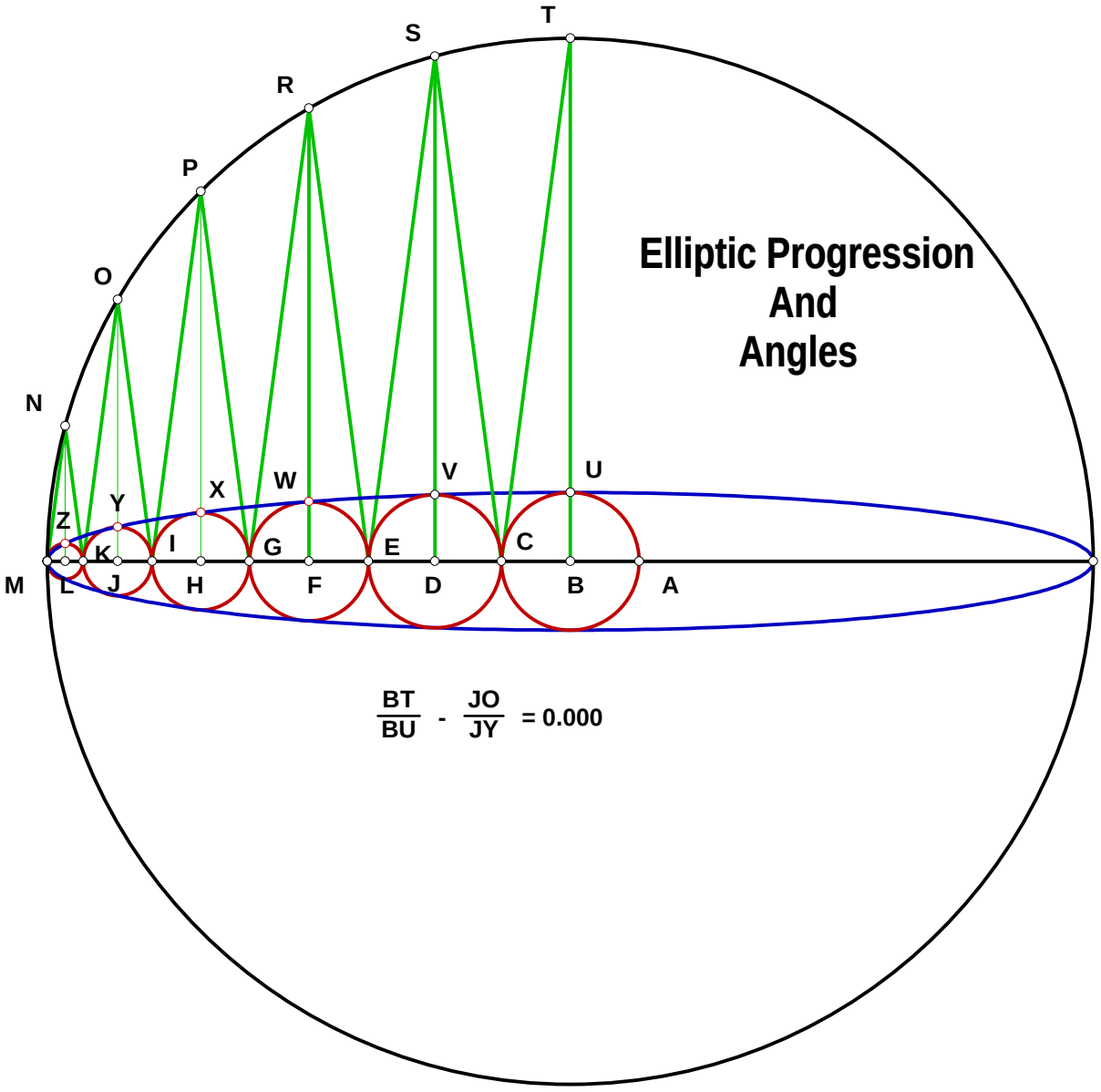
T
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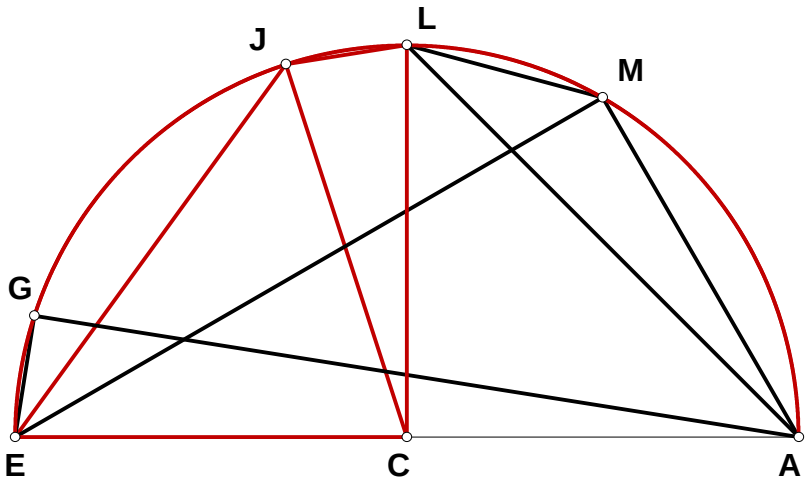
X
W
V
U
Elliptic Progression
And
Angles
BT/BU - JO/JY = 0.000




$EM := \frac{1}{2}\cdot\sqrt{3}$

$AL := \frac{1}{2}\cdot\sqrt{2}$

$JL := \frac{1}{4}\cdot\sqrt{8 - 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

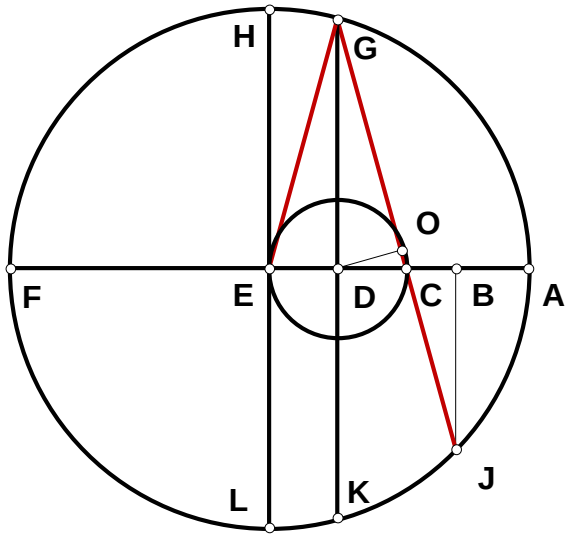
$LM := \frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}$

$AM := \frac{1}{2}$

$EJ := \frac{1}{4}\cdot\sqrt{10 - 2\cdot\sqrt{5}}$

$AG := \frac{1}{4}\cdot\sqrt{8 + 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

## Elliptic Progression Outtake One 0507011




**A method of trisection Algebraically.**

$N := M$

$N \geq 4 = 1 \qquad AF := 6 \qquad AE := \frac{AF}{2}$

$DE := \frac{AF}{N} \qquad AD := AE - DE \qquad DF := AF - AD$

$DG := \sqrt{AD \cdot DF} \qquad CD := DE \qquad EG := AE$

$CO := \frac{CD^2}{EG} \qquad CG := EG \qquad CJ := CG - 4 \cdot CO$

$BC := \frac{CD \cdot CJ}{CG} \qquad AB := AE - (2 \cdot DE + BC)$

$BJ := \frac{DG \cdot BC}{CD} \qquad BD := BC + CD$

$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2} \qquad \frac{JK}{2 \cdot DE} = 1$

**Some Algebraic Names,**

**Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.**

$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0 \qquad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0 \qquad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$

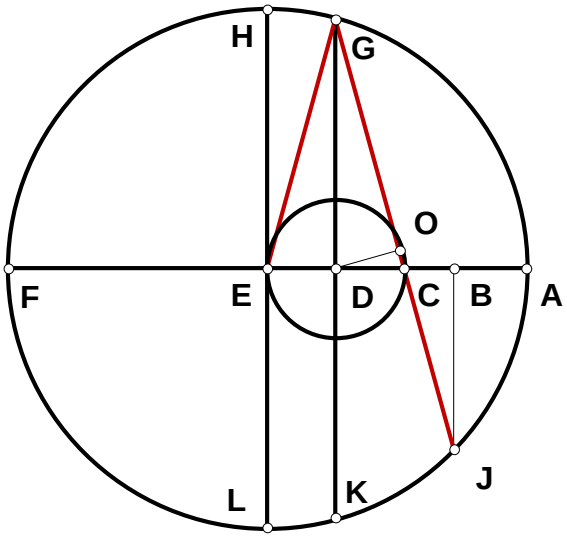
$\frac{2\ AF}{N^2} - CO = 0 \qquad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0 \qquad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$

$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$

**One of the meanings of trisection is solving for the following equation when given AF and AB.**

$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0$

$AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0 \qquad AF \cdot \frac{2 \cdot (N^2 - 8)}{N^3} - BD = 0 \qquad \frac{2 \cdot AF}{N} - JK = 0$




$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - \frac{AF}{AB} = 0 \qquad \frac{AF}{AB} = 6$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - 6 = 0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6$$

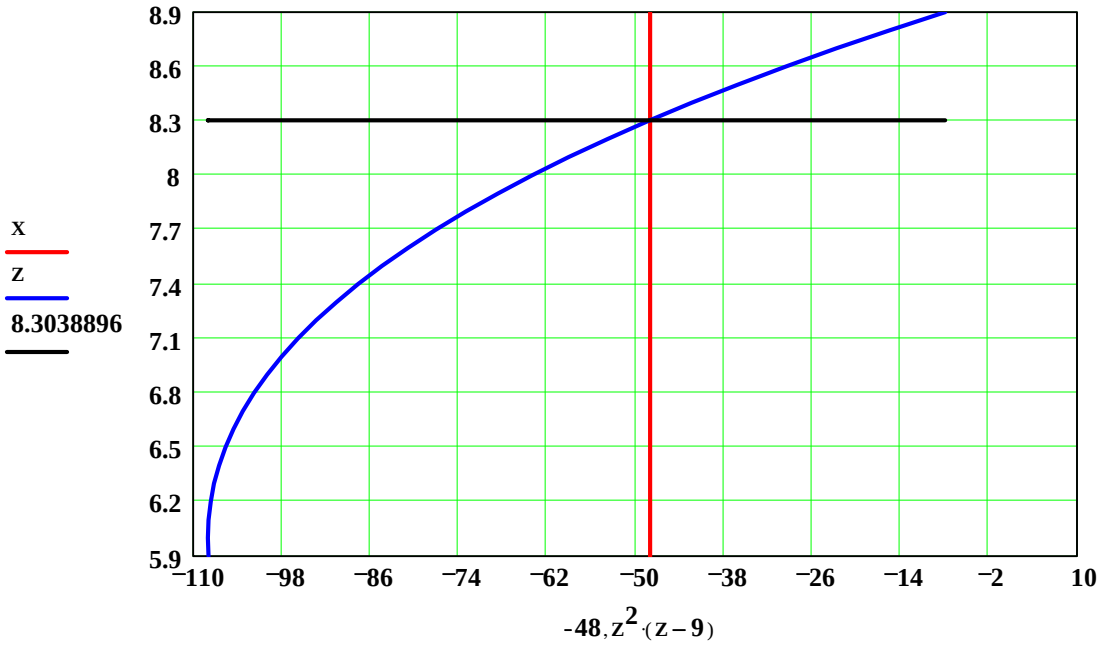
$$2\cdot M^3 - 6\cdot\left[(M+2)\cdot(M-4)^2\right] = 0$$

$$2\cdot M^3 - \left(6\cdot M^3 - 36\cdot M^2 + 192\right) = 0$$

$$4\cdot M^3 - 36\cdot M^2 + 192 = 0$$

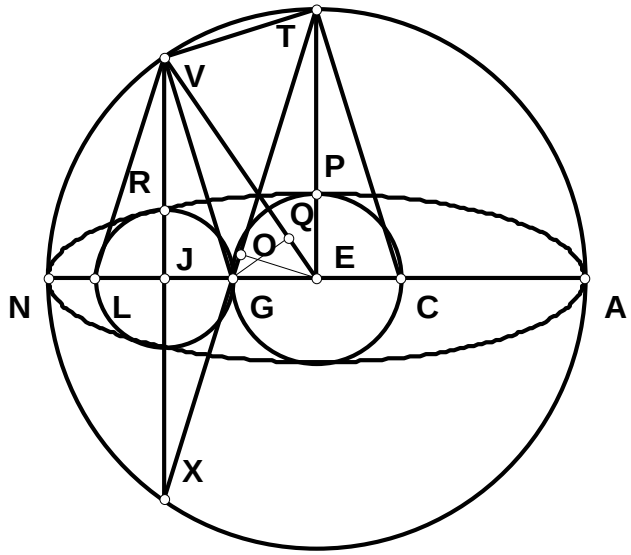
$$M^3 - 9\cdot M^2 + 48 = 0 \qquad M^2\cdot(M-9) + 48 = 0 \qquad M \equiv 8.303889634816388$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6 \qquad DE - \frac{AF}{M} = 0 \qquad Z := 5.9, 6 .. 8.9$$



# Elliptic Progression Outtake Two 0507012

## Angles TEV and EVJ equals CTG.




$N := 6.381 \quad AN := 2.792$

$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$

$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$

$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$

$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$

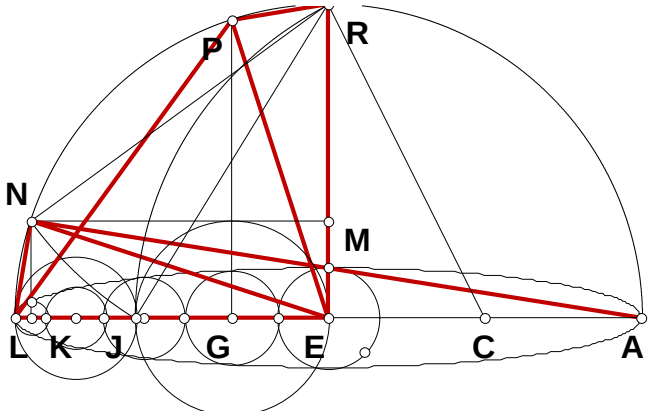
$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \qquad JV := JX$

$EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$

$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \qquad EQ := \frac{EJ \cdot EG}{EV} \quad OQ := \frac{JV \cdot EQ}{EJ} \quad OQ - GJ = 0$

## Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013




$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

***Irrational*** **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

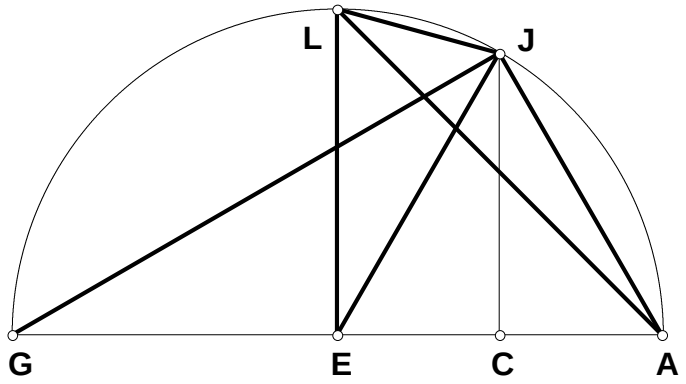
$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

## Outtake Four: Some Names 0507014




$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$

$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$

$EL := AE \quad CE := AC$

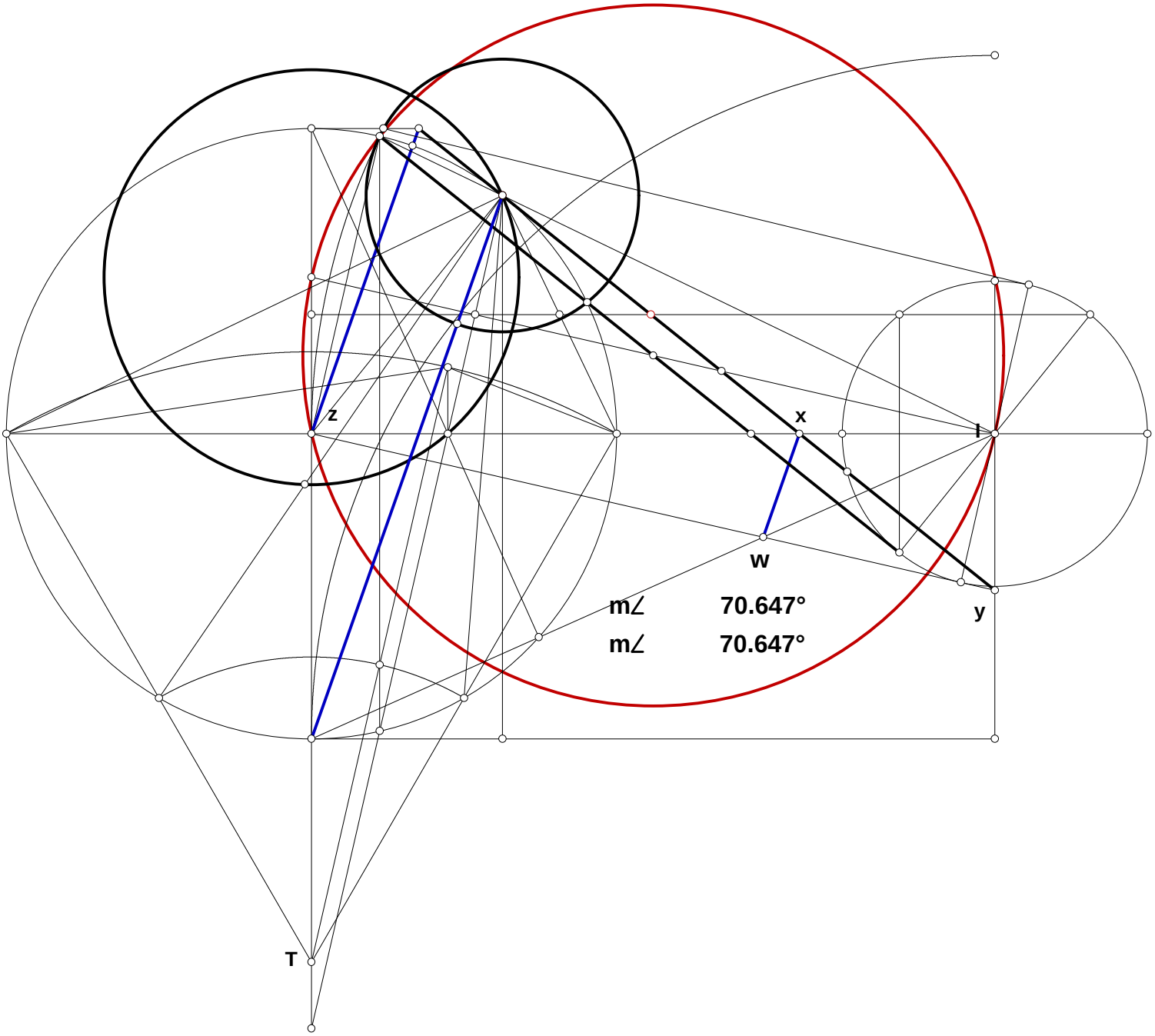
$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

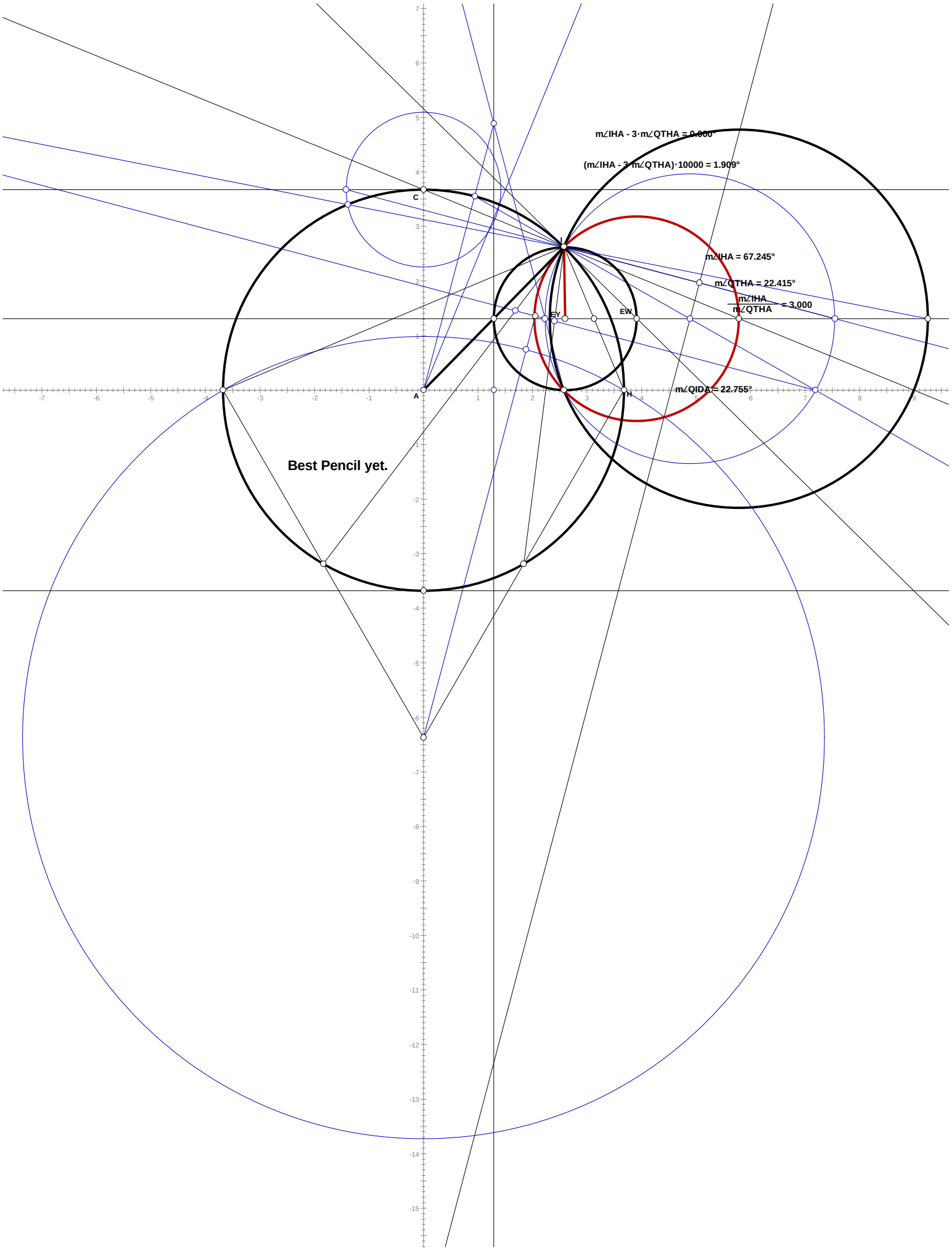
$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$

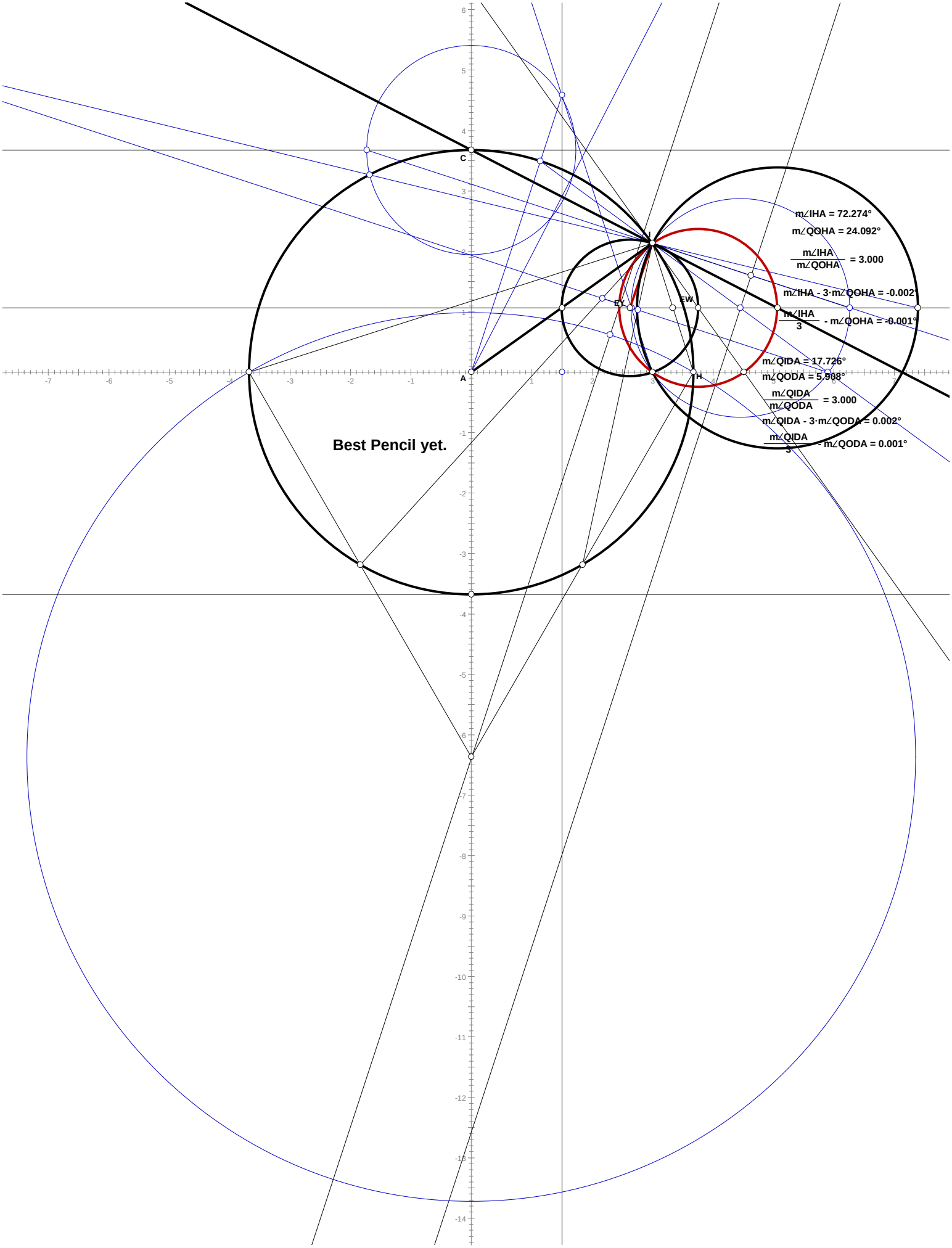
$AL := \sqrt{AE^2 + EL^2}$

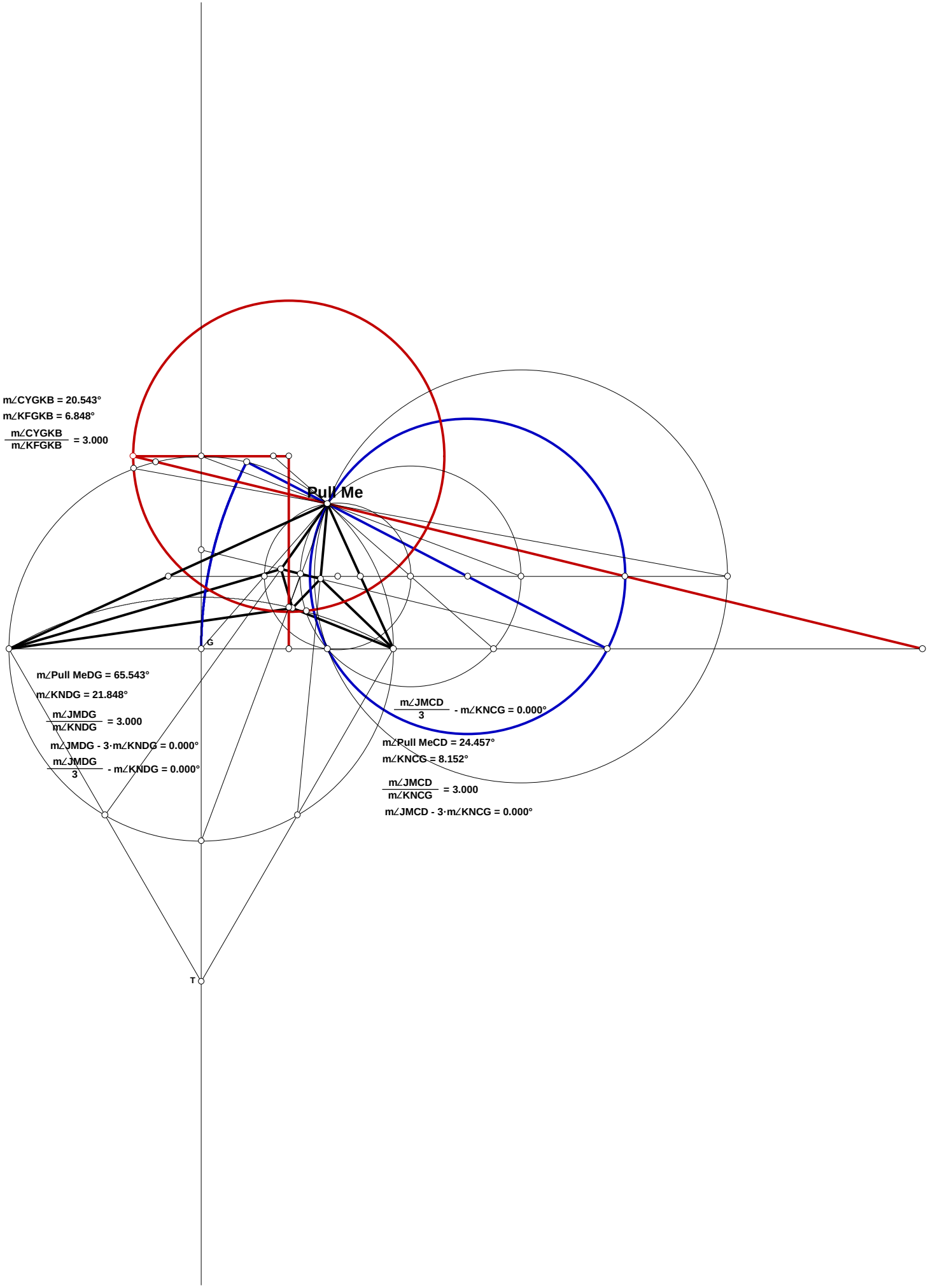
$AE := \frac{1}{2} \quad AC := \frac{1}{4} \quad CG := 1 - \frac{1}{4} \quad CJ := \frac{1}{4} \cdot \sqrt{3} \quad JL := \frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}$

$AJ := \frac{1}{2} \quad GJ := \frac{1}{2} \cdot \sqrt{3}_{*} \quad AL := \frac{1}{2} \cdot \sqrt{2}$


z
x
l
w
y
T
m∠ 70.647°
m∠ 70.647°

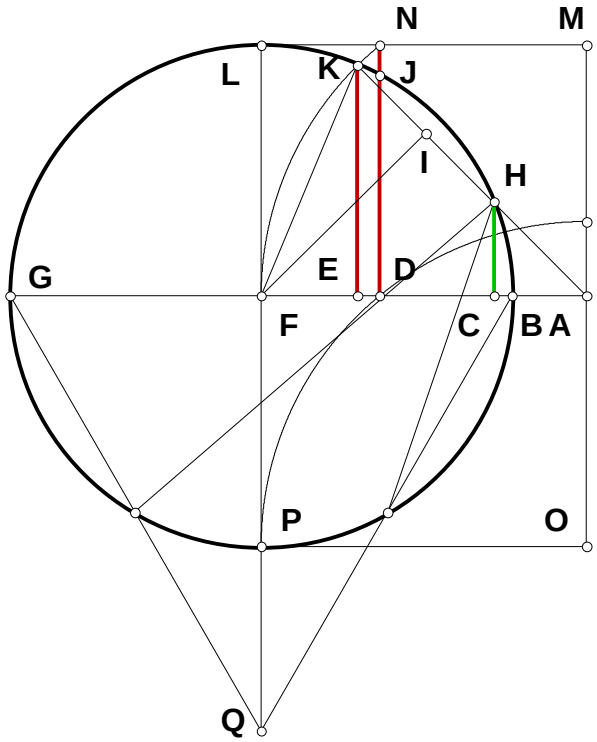

m∠IHA - 3·m∠QTHA = 0.000°
(m∠IHA - 3·m∠QTHA)·10000 = 1.909°
C
I
m∠IHA = 67.245°
m∠QTHA = 22.415°
m∠IHA / m∠QTHA = 3.000
EY
EW
A
H
m∠QIDA = 22.755°
Best Pencil yet.


C
I
EW
H
A
m∠IHA = 72.274°
m∠QOHA = 24.092°
m∠IHA / m∠QOHA = 3.000
m∠IHA - 3·m∠QOHA = -0.002°
m∠IHA / 3 - m∠QOHA = -0.001°
m∠QIDA = 17.726°
m∠QODA = 5.908°
m∠QIDA / m∠QODA = 3.000
m∠QIDA - 3·m∠QODA = 0.002°
m∠QIDA / 3 - m∠QODA = 0.001°
Best Pencil yet.


m∠CYGKB = 20.543°
m∠KFGKB = 6.848°
m∠CYGKB / m∠KFGKB = 3.000
Pull Me
G
T
m∠Pull MeDG = 65.543°
m∠KNDG = 21.848°
m∠JMDG / m∠KNDG = 3.000
m∠JMDG - 3·m∠KNDG = 0.000°
m∠JMDG / 3 - m∠KNDG = 0.000°
m∠JMCD / 3 - m∠KNCG = 0.000°
m∠Pull MeCD = 24.457°
m∠KNCG = 8.152°
m∠JMCD / m∠KNCG = 3.000
m∠JMCD - 3·m∠KNCG = 0.000°

# On Trisection 051301

## For any given trisection what is the Algebraic names of BC and BE taking BG as unit?




$N := 9 \qquad BG := 1 \qquad AG := BG \cdot N$

$AB := AG - BG \qquad BF := \frac{BG}{2}$

$AF := AB + BF \qquad AN := AF \qquad AK := AN$

$FK := BF \qquad S_1 := AF \qquad S_2 := AK \qquad S_3 := FK$

$AE := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad AI := AE$

$IK := AK - AI \qquad HI := IK \qquad AH := AK - (HI + IK)$

$AC := \frac{AE \cdot AH}{AK} \qquad BC := AC - AB \qquad BE := AE - AB$

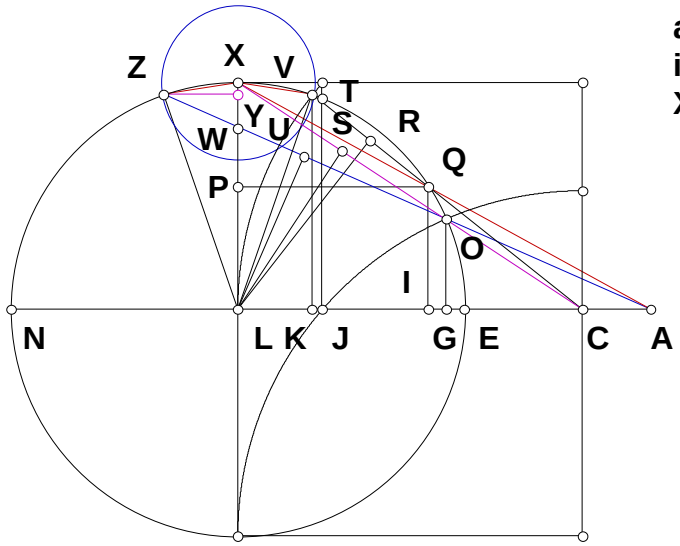
## Some Algebraic Names:

$N - 1 - AB = 0 \qquad \frac{1}{2} - BF = 0 \qquad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \qquad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$

$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \qquad 2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \qquad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \qquad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

## On Trisection 051401.MCD




**For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?**

$N := 5 \quad CE := 1 \quad CN := CE \cdot N$

$EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$CL := CE + EL$

$CV := CL \quad S_1 := CL \quad S_2 := CV$

$S_3 := LV \quad CK := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CR := CK \quad RV := CV - CR \quad QR := RV \quad CQ := CV - (QR + RV) \quad CI := \frac{CK \cdot CQ}{CV} \quad EI := CI - CE$

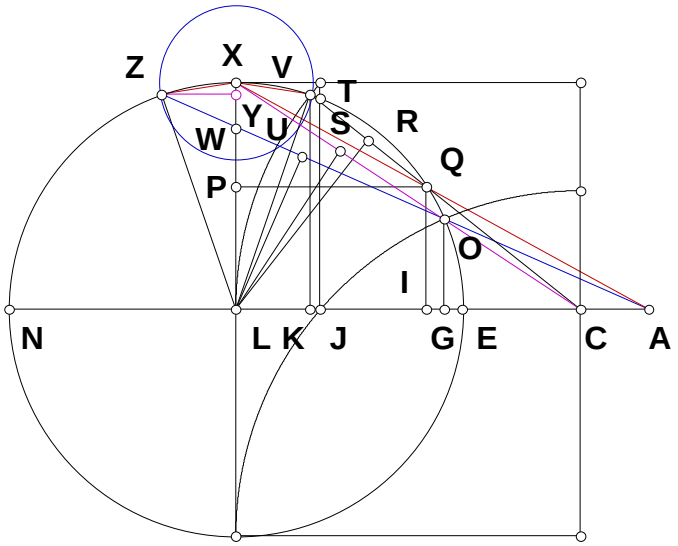
$IN := EN - EI \quad IQ := \sqrt{EI \cdot IN} \quad LP := IQ \quad LX := EL \quad IL := EL - EI \quad PX := LX - LP \quad AL := \frac{IL \cdot LX}{PX}$

$AE := AL - EL \quad AC := AE - CE \quad AI := AC + CI \quad AX := \sqrt{AL^2 + LX^2} \quad AQ := \frac{AX \cdot AI}{AL} \quad QX := AX - AQ$

$CX := \sqrt{CL^2 + LX^2} \quad CS := \frac{CL^2}{CX} \quad SX := CX - CS \quad OS := SX \quad CO := CX - (SX + OS)$

$GO := \frac{LX \cdot CO}{CX} \quad CG := \frac{CL \cdot CO}{CX} \quad EG := CG - CE \quad AG := AE + EG \quad AO := \sqrt{AG^2 + GO^2}$

$AU := \frac{AG \cdot AL}{AO} \quad OU := AU - AO \quad UZ := OU \quad AZ := AO + (OU + UZ)$




$$LW := \frac{GO \cdot AL}{AG} \qquad AW := \frac{AO \cdot AL}{AG}$$

$$WZ := AZ - AW \qquad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \qquad YX := LX - (LW + WY)$$

$$XZ := \sqrt{YX^2 + YZ^2}$$

**Some Algebraic Names:**

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0$$

$$2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0 \qquad CE \cdot \frac{N^3 + 6\,N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$

$$\frac{1}{2}\cdot CE\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AL=0$$

$$CE\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AE=0$$

$$-CE\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AC=0$$

$$\frac{CE\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-AI=0$$

$$\frac{1}{2}\cdot CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-AX=0$$

$$CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-AQ=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}}\cdot CE\cdot(N-1)\cdot\frac{\left[\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}{2\cdot(1+N)^3}\ \ldots=0$$

$$\left.+3\cdot N+1\right]$$

$$\left.+3\cdot N+1\right]$$

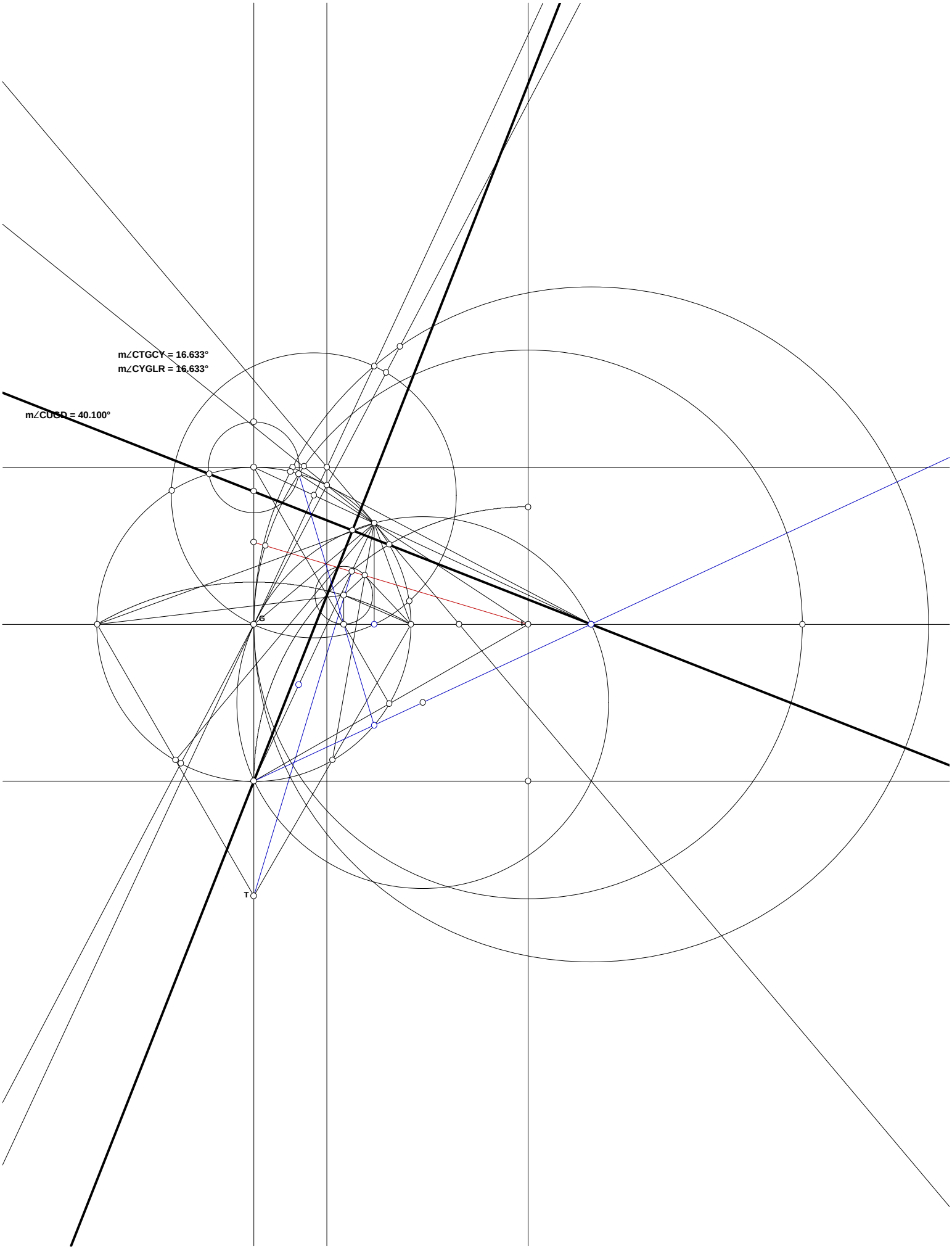
$$+-QX$$

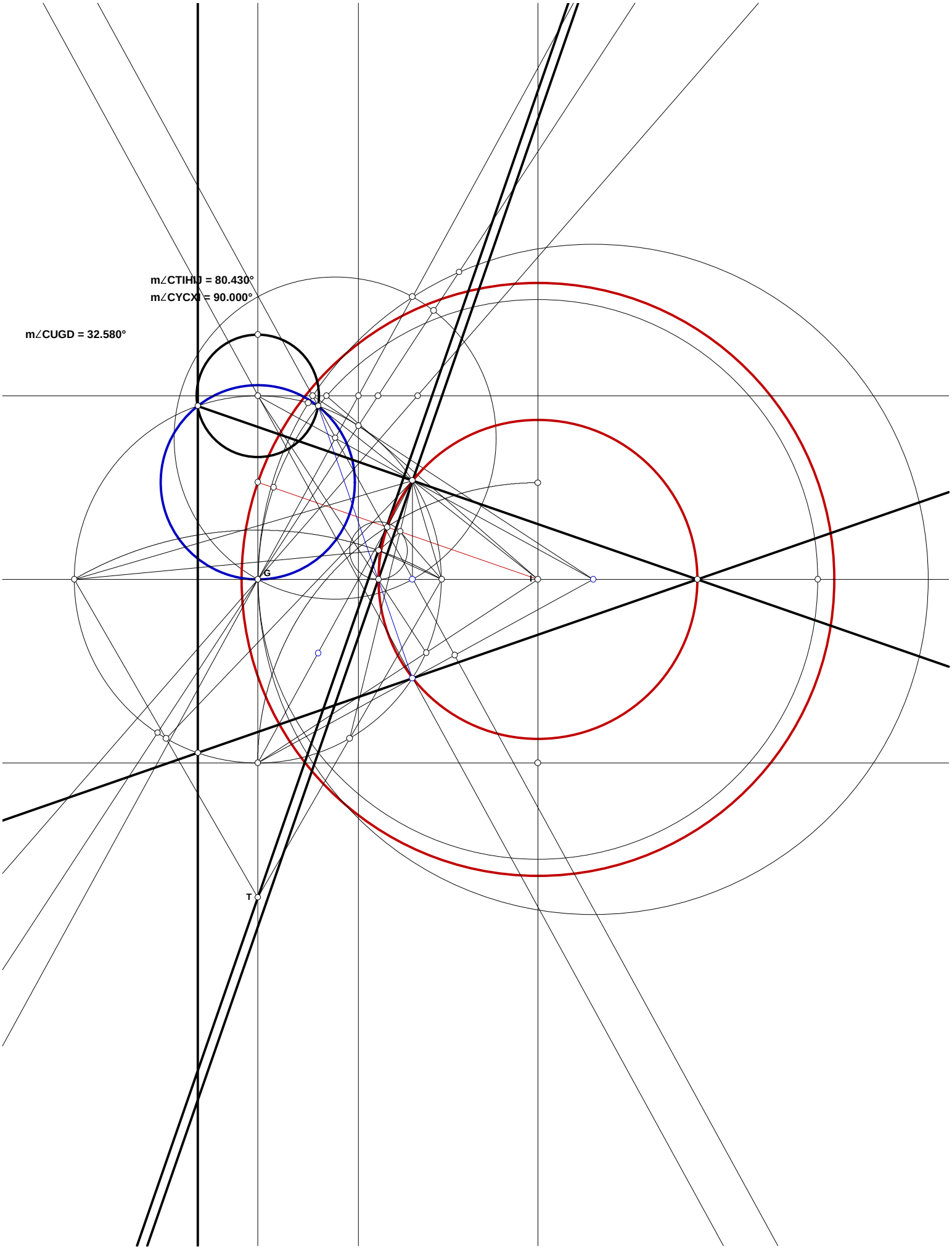
$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)}-CX=0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}}-CS=0 \qquad \frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}}-SX=0$$

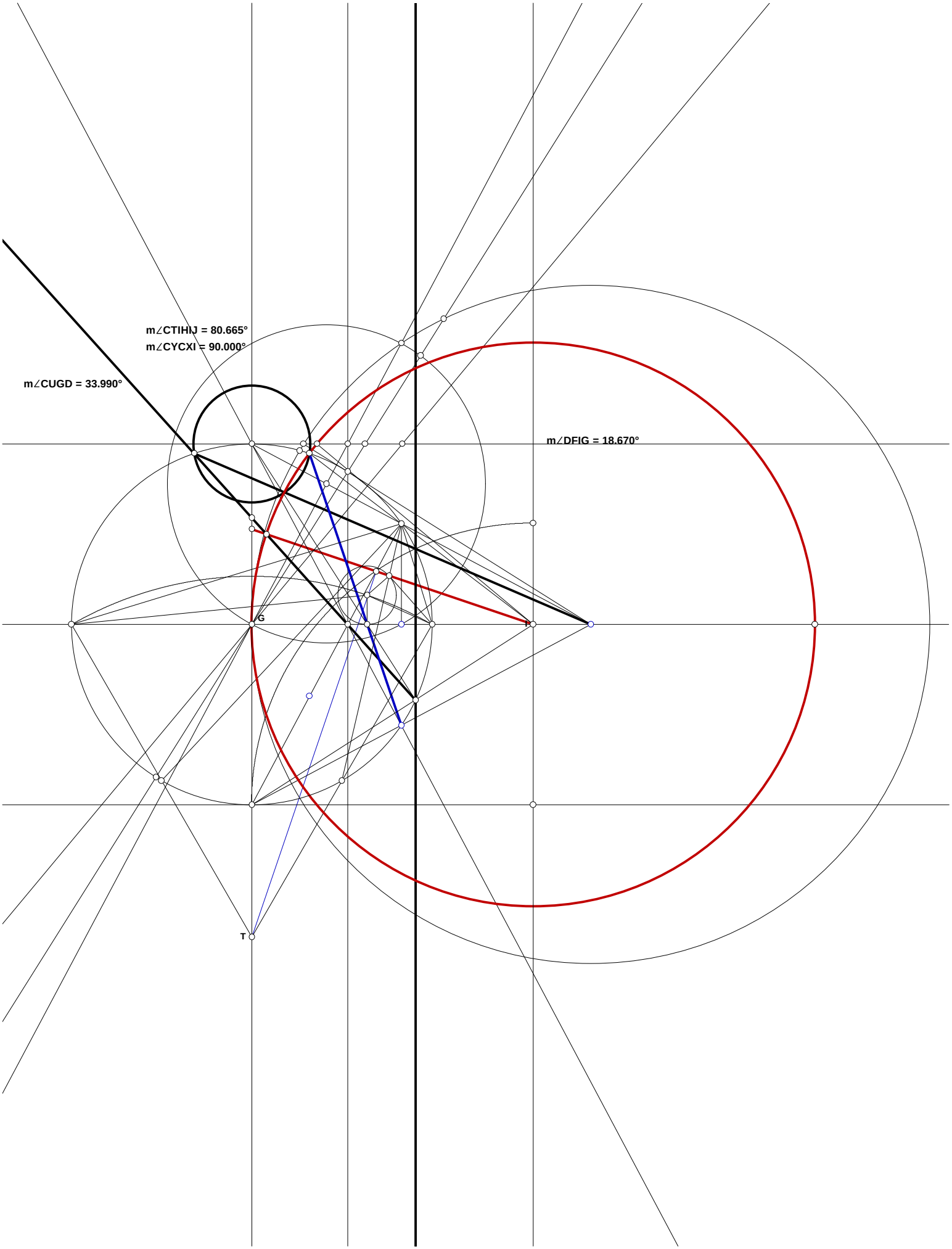
$$\frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N-CO=0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)}-GO=0 \qquad CE\cdot\frac{N^2+N}{\left(1+N^2\right)}-CG=0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)}-EG=0$$

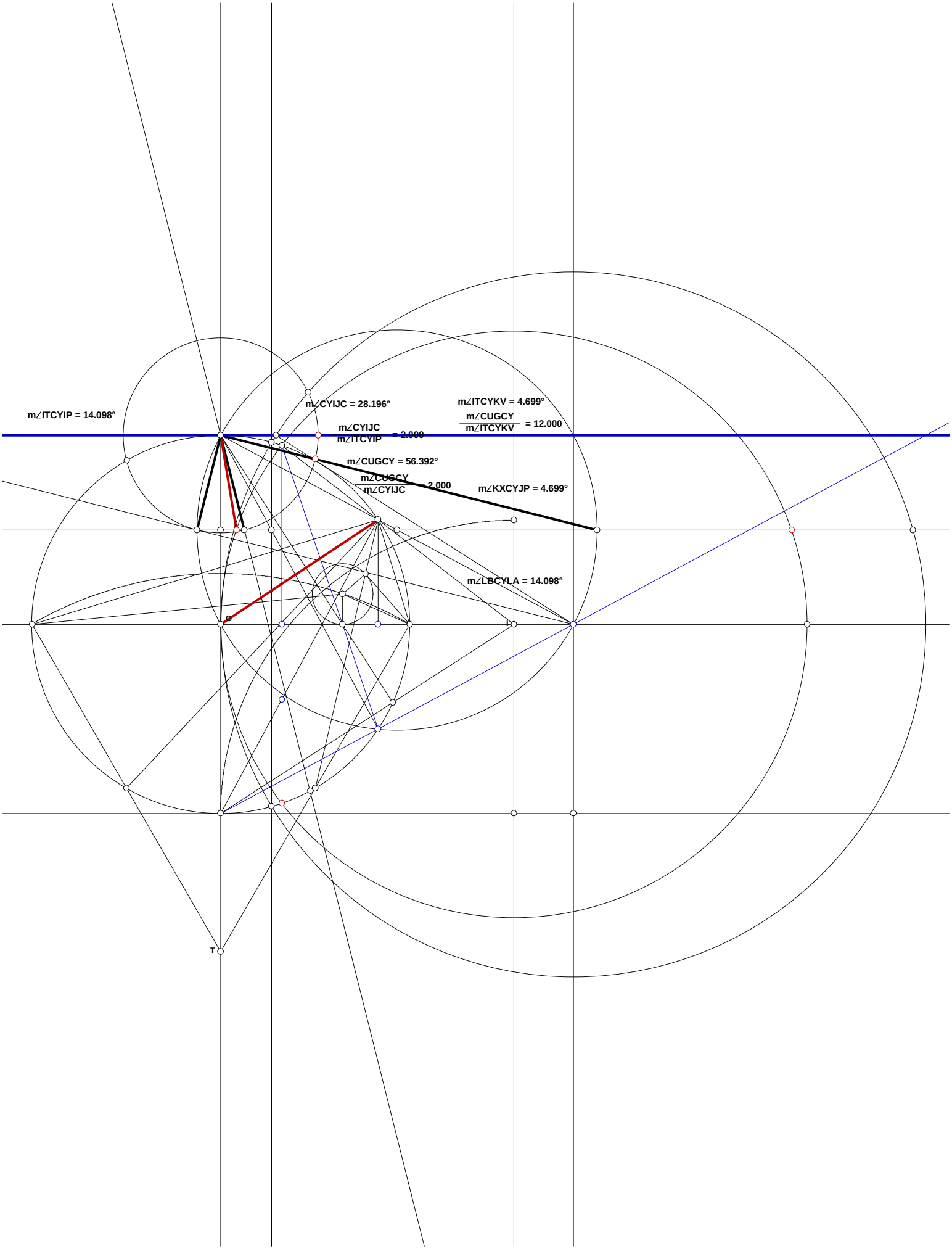
$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-2\cdot N+\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]}-AG=0$$

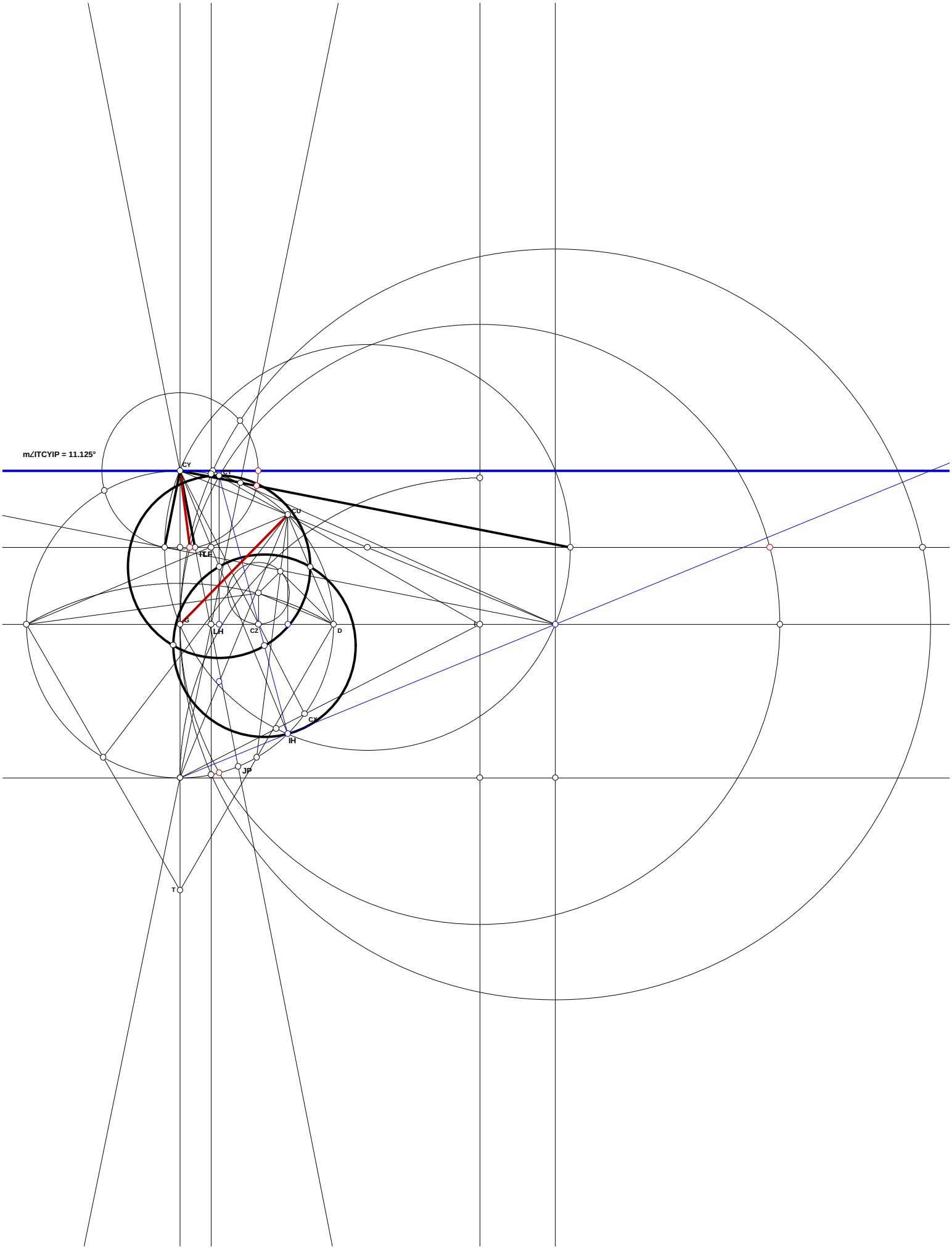
$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\begin{array}{l}\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\,-7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\end{array}\right]}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}}-XZ=0$$

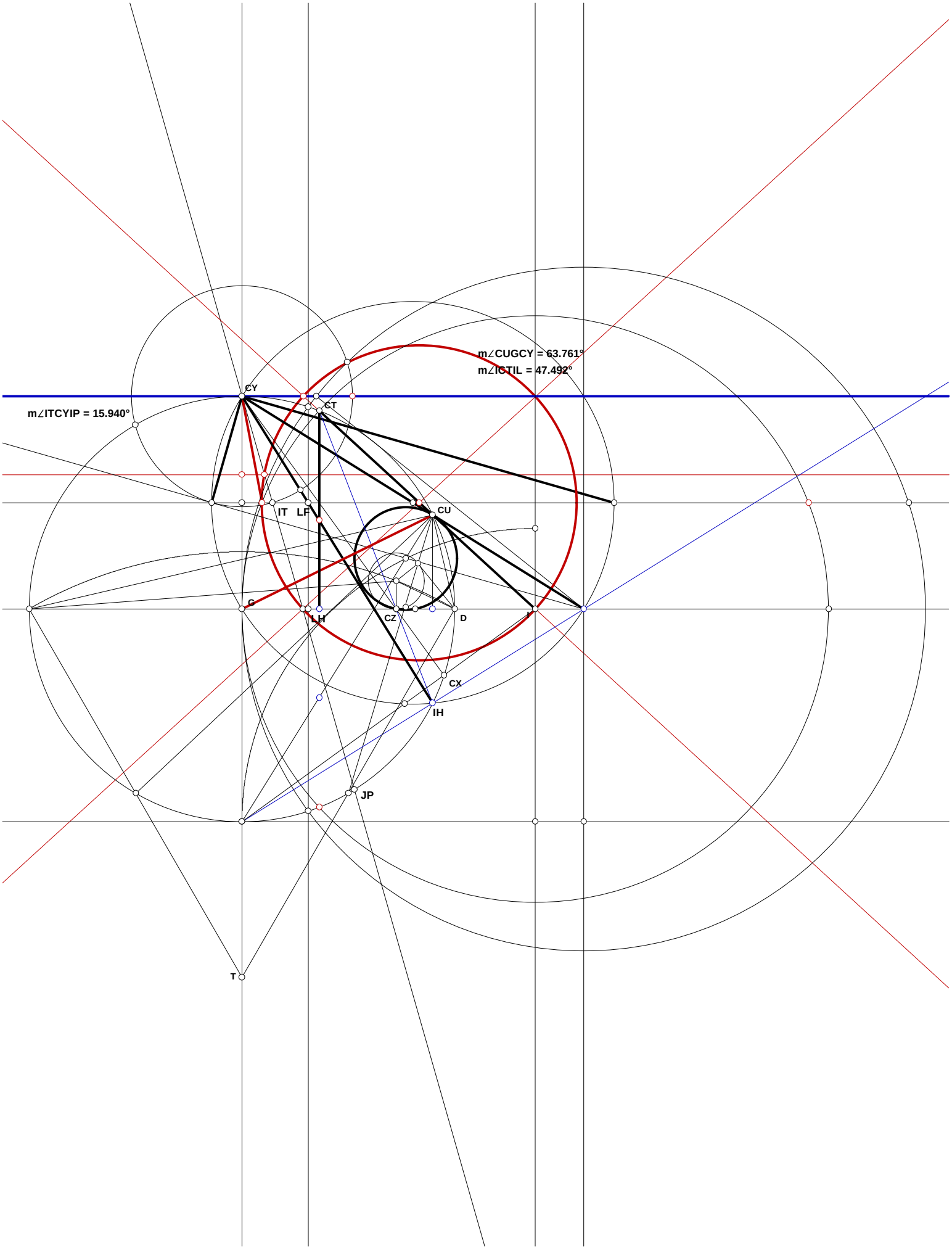

m∠CTGCY = 16.633°
m∠CYGLR = 16.633°
m∠CUGD = 40.100°
G
T

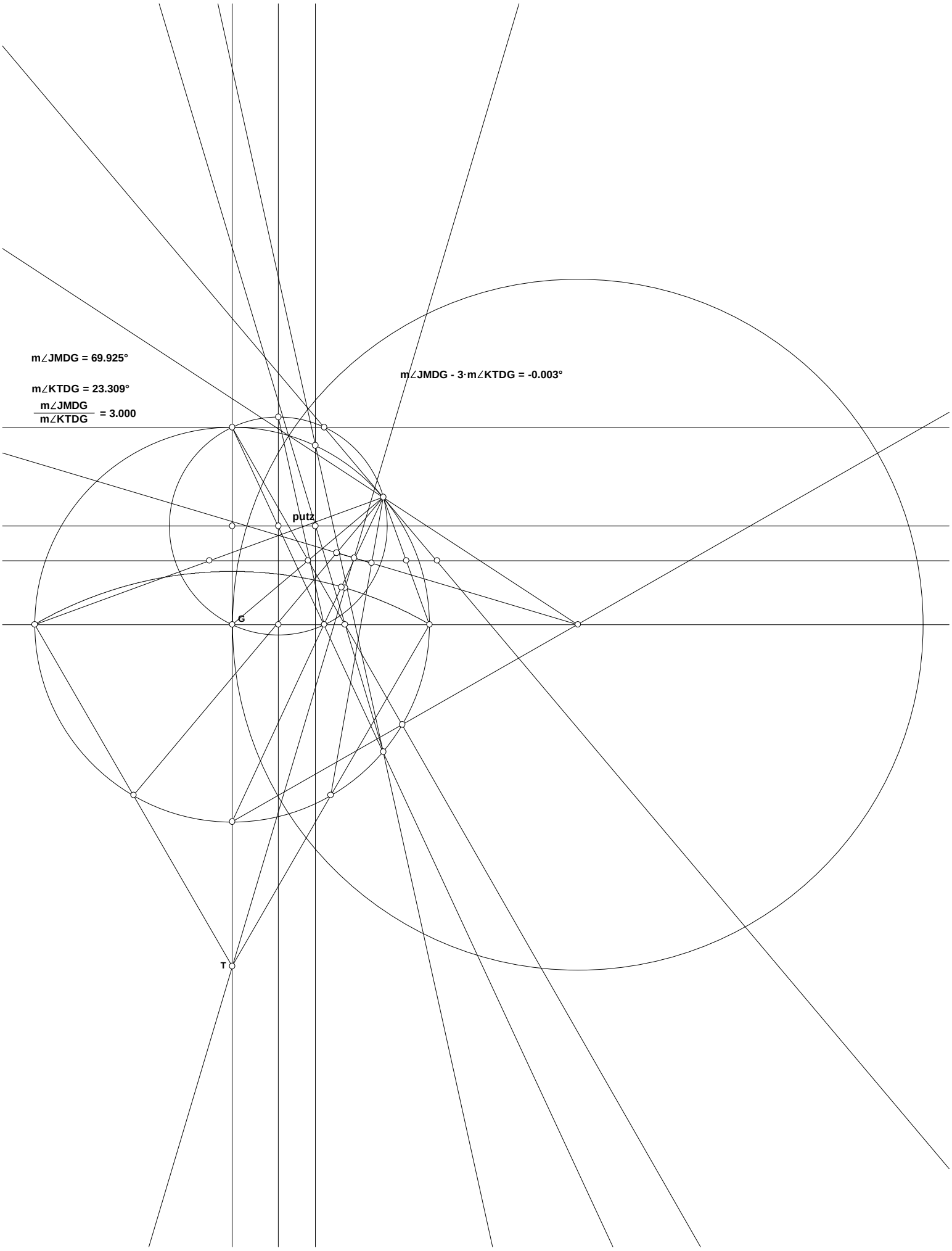

m∠CTIHIJ = 80.430°
m∠CYCX = 90.000°
m∠CUGD = 32.580°
G
I
T

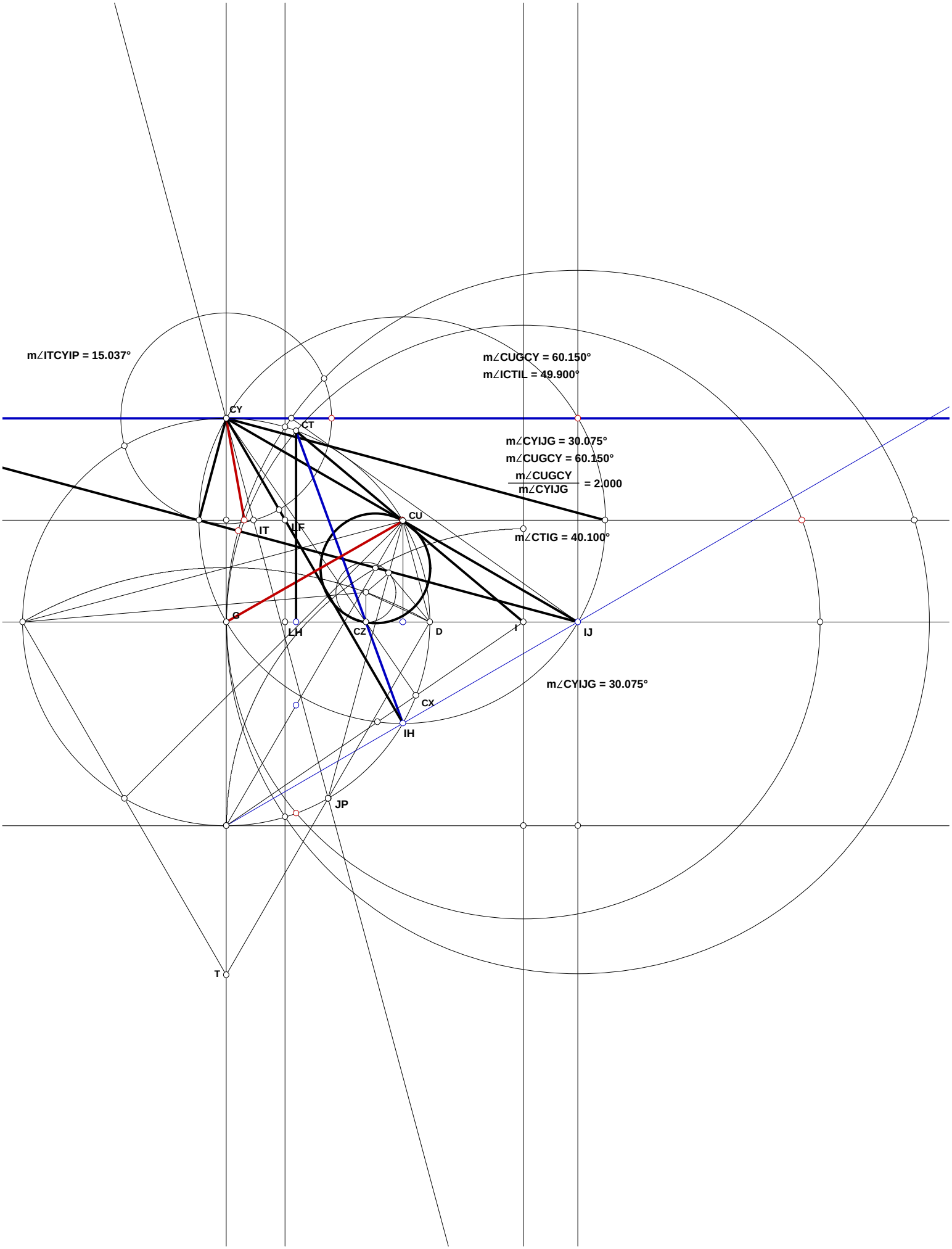

m∠CTIHIJ = 80.665°
m∠CYCXI = 90.000°
m∠CUGD = 33.990°
m∠DFIG = 18.670°
G
T

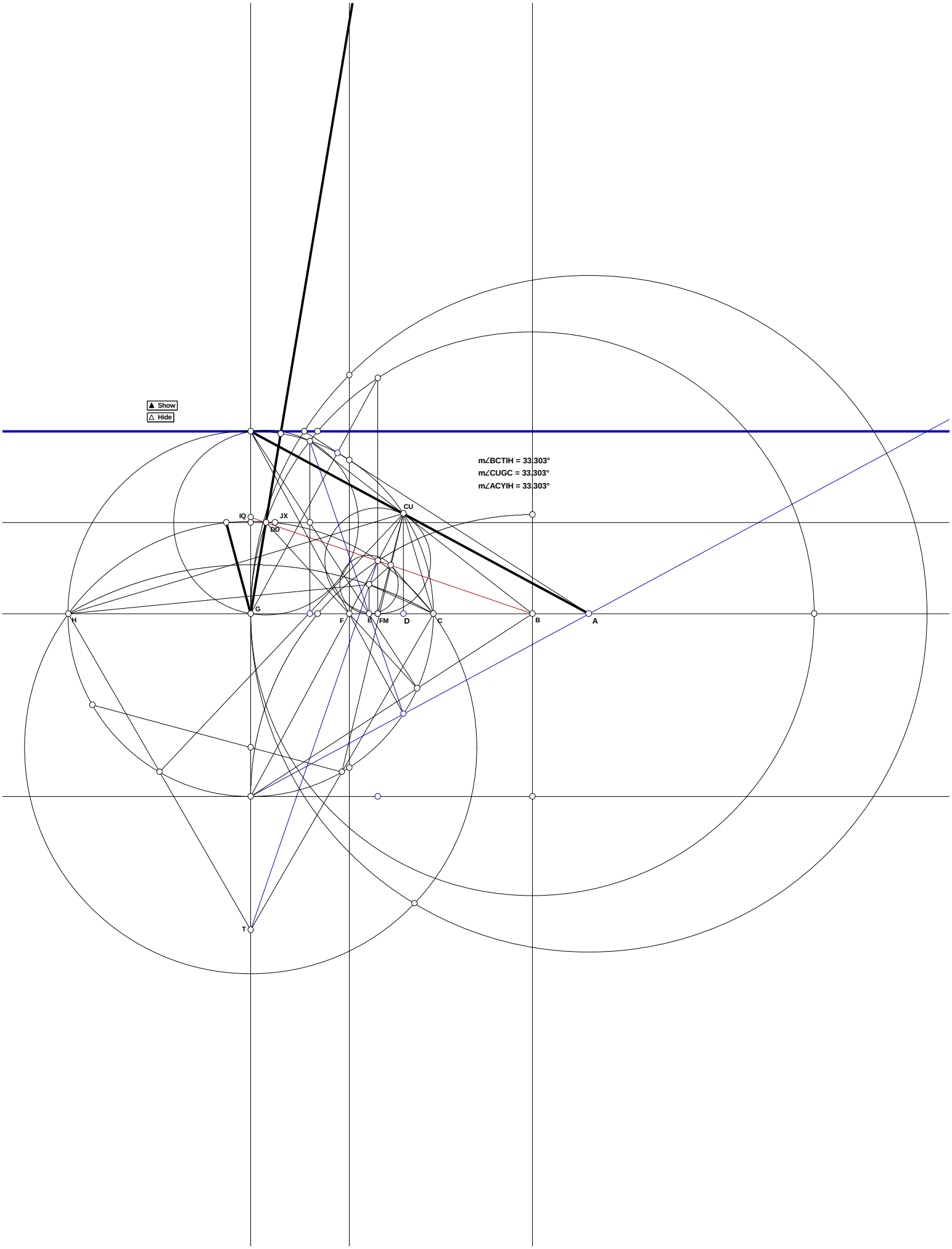

m∠ITCYIP = 14.098°
m∠CYIJC = 28.196°
m∠ITCYKV = 4.699°
m∠CUGCY / m∠ITCYKV = 12.000
m∠CYIJC / m∠ITCYIP = 2.000
m∠CUGCY = 56.392°
m∠CUGCY / m∠CYIJC = 2.000
m∠KXCYJP = 4.699°
m∠LBCYLA = 14.098°
G
I
T

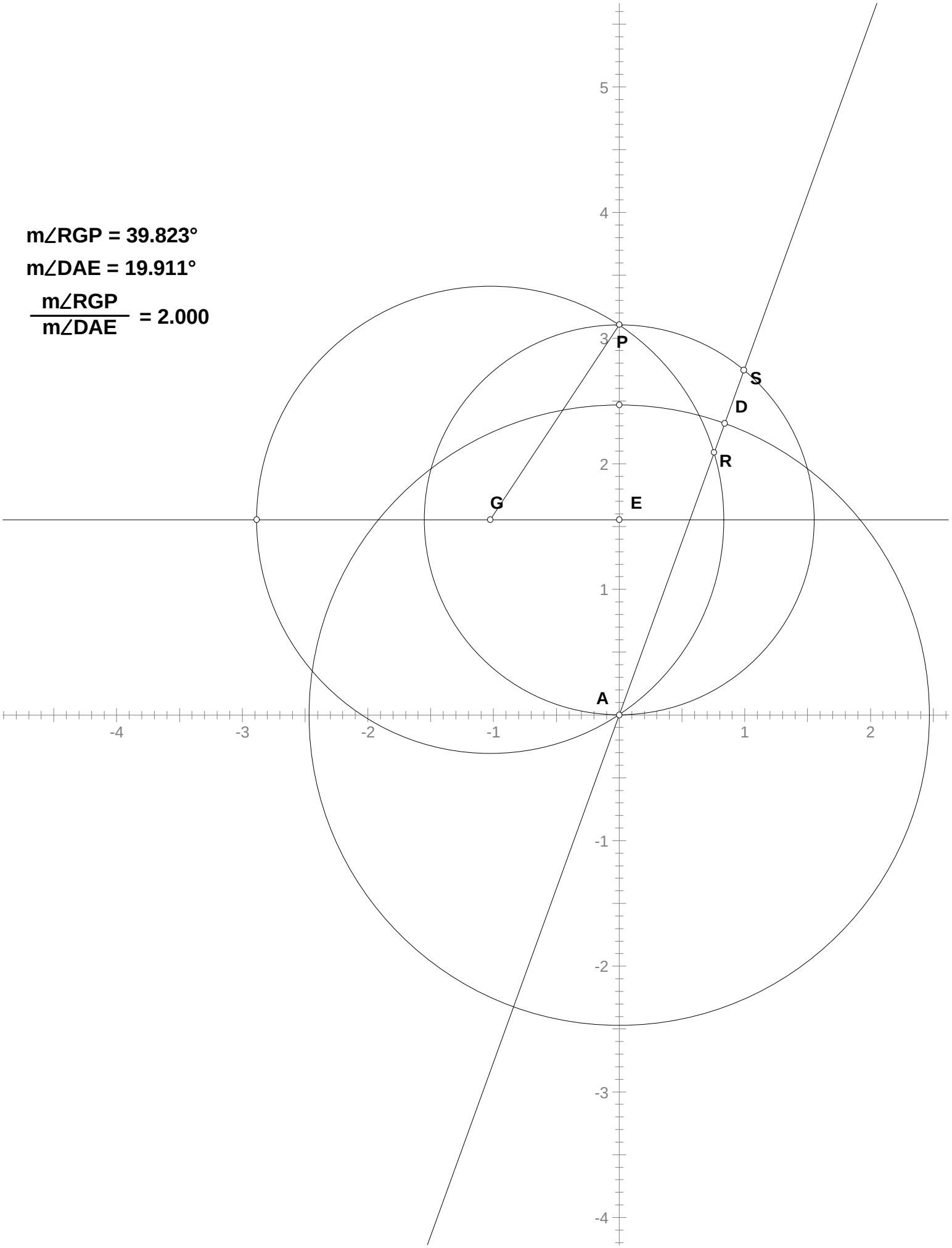

m∠ITCYIP = 11.125°
CY
CT
CO
ITLE
C
LH
CZ
D
CX
IH
JP
T

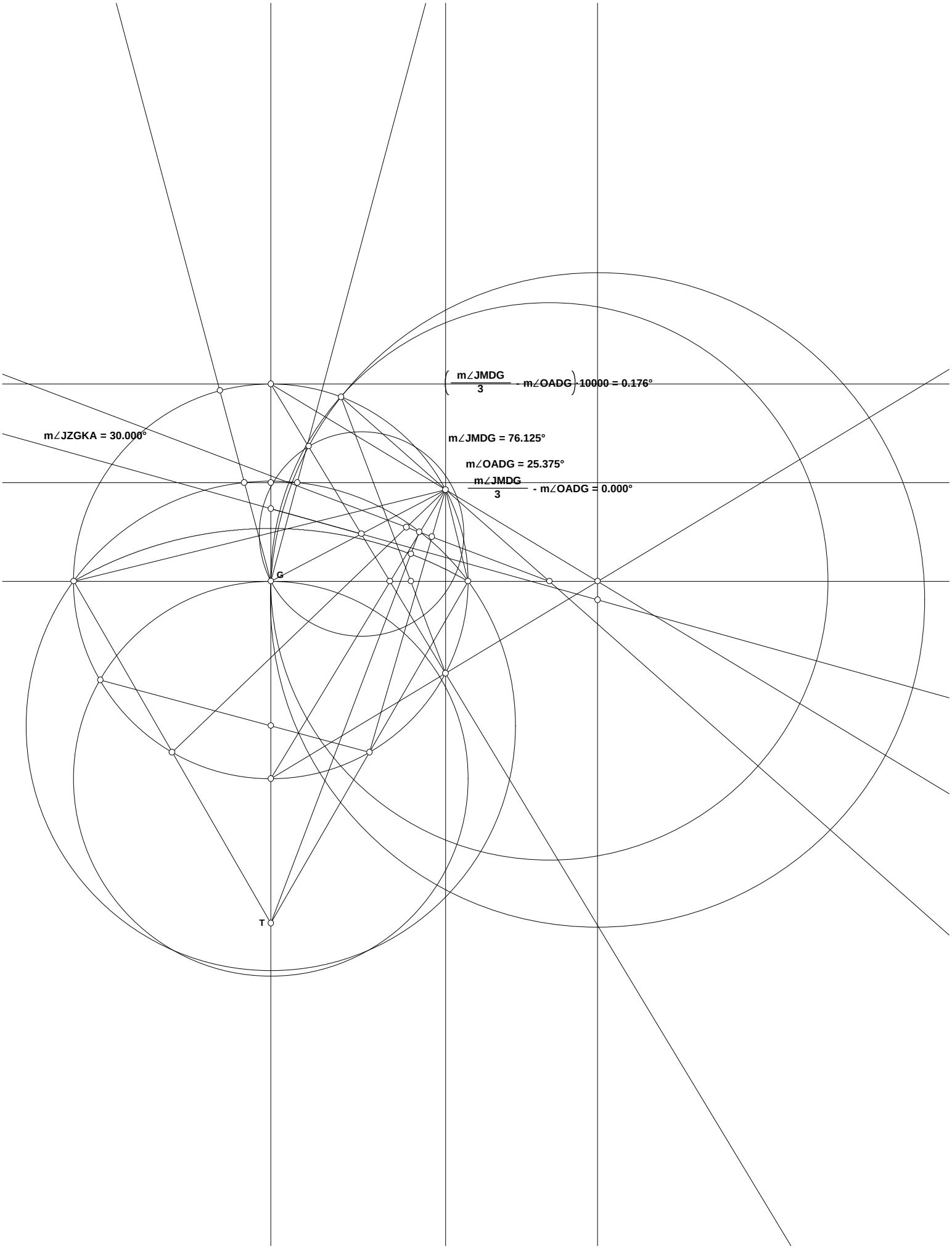

m∠CUGCY = 63.761°
m∠ICTIL = 47.492°
m∠ITCYIP = 15.940°
CY
CT
IT
LF
CU
G
LH
CZ
D
I
CX
IH
JP
T

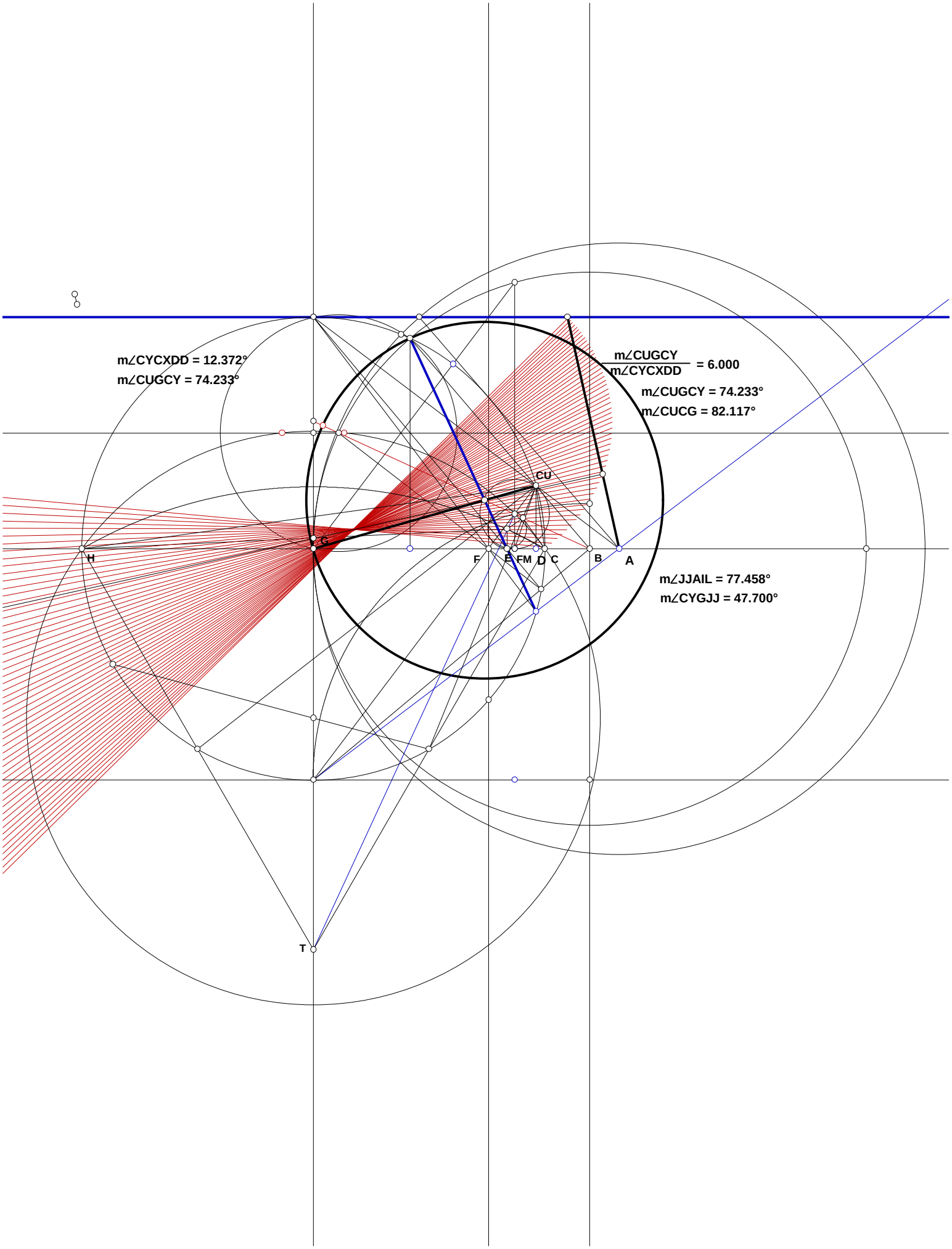

m∠JMDG = 69.925°
m∠KTDG = 23.309°
m∠JMDG / m∠KTDG = 3.000
m∠JMDG - 3·m∠KTDG = -0.003°
putz
G
T

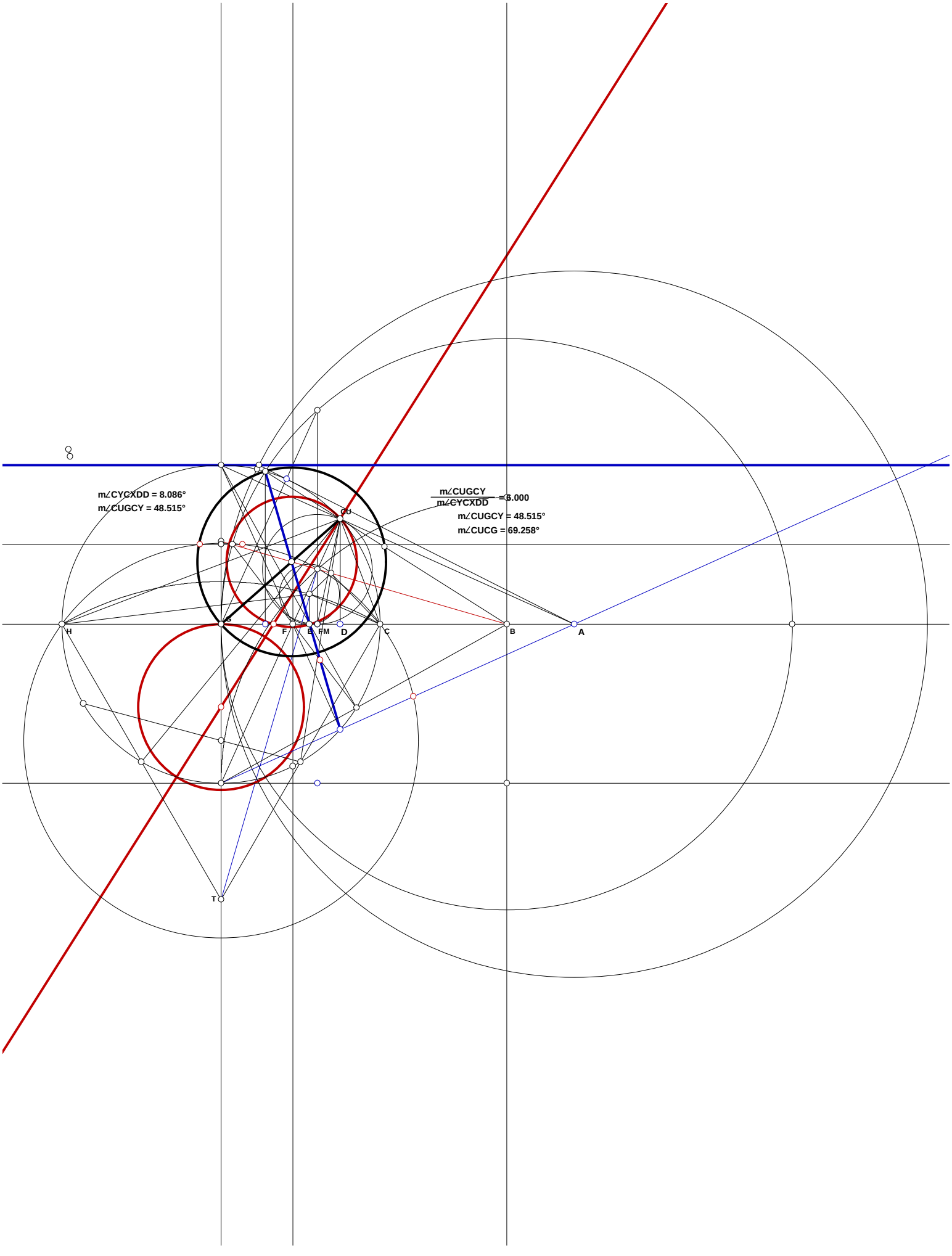

m∠ITCYIP = 15.037°
m∠CUGCY = 60.150°
m∠ICTIL = 49.900°
CY
CT
m∠CYIJG = 30.075°
m∠CUGCY = 60.150°
m∠CUGCY / m∠CYIJG = 2.000
CU
IT
LF
m∠CTIG = 40.100°
G
LH
CZ
D
I
IJ
m∠CYIJG = 30.075°
CX
IH
JP
T

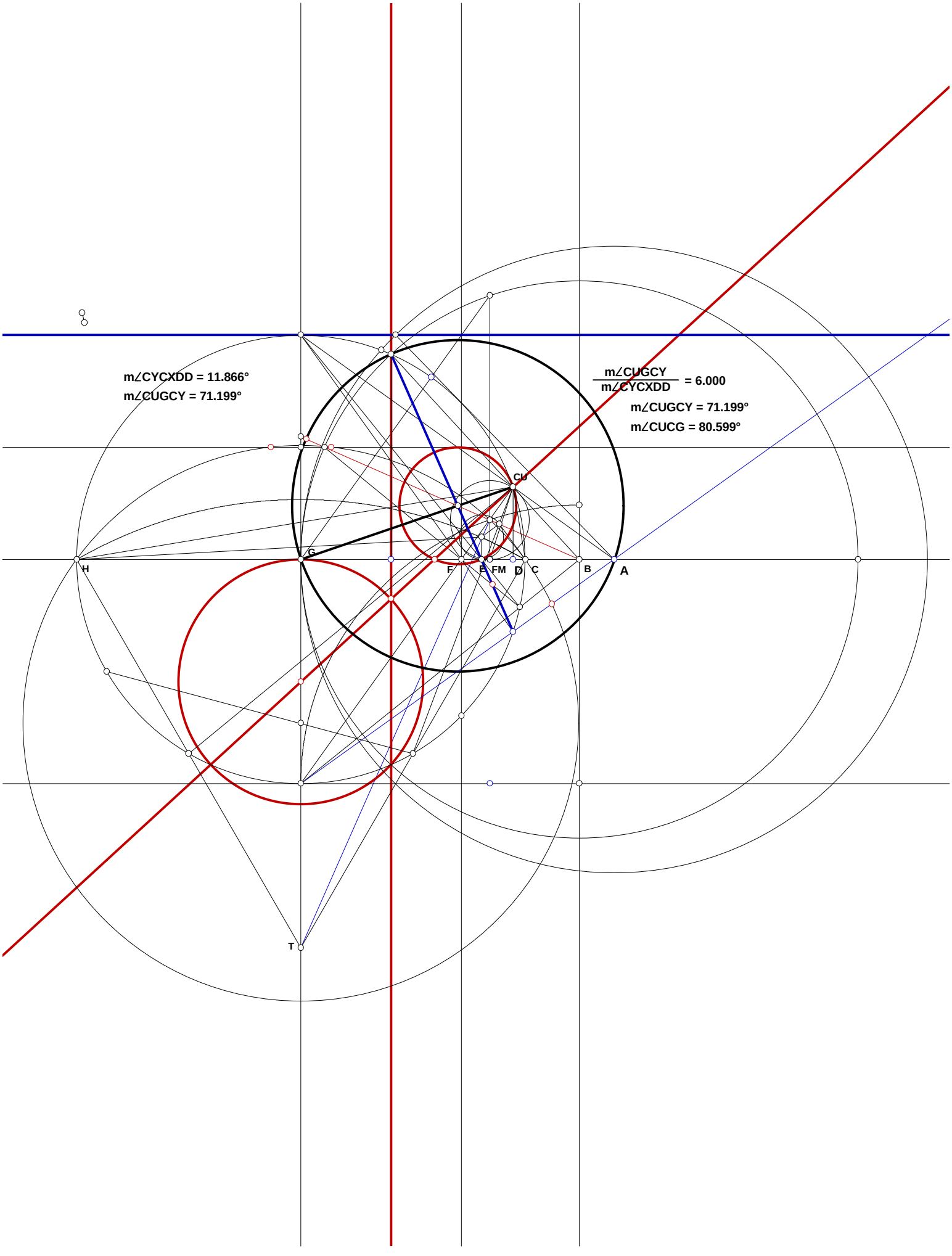

Show
Hide
m∠BCTIH = 33.303°
m∠CUGC = 33.303°
m∠ACYIH = 33.303°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

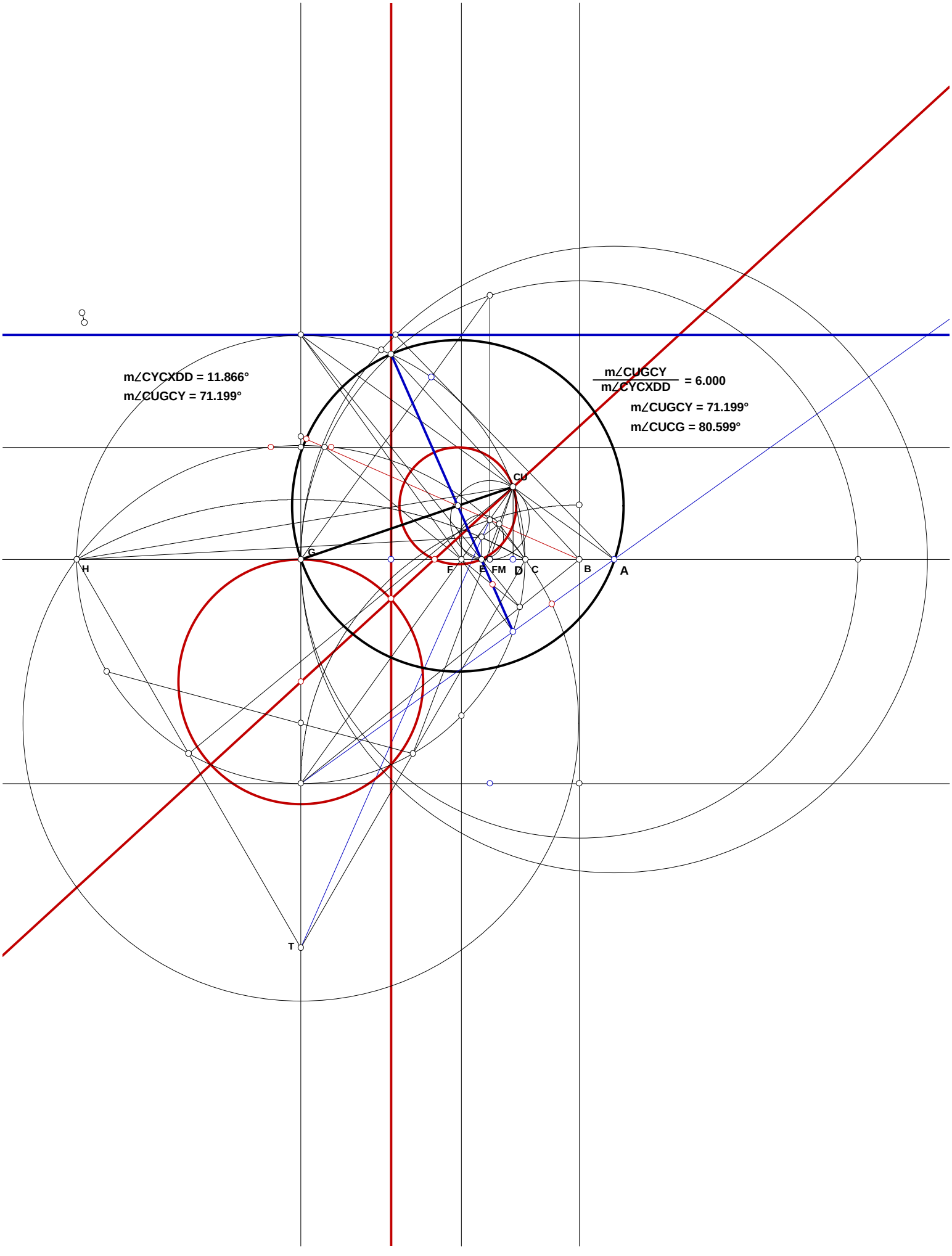

m∠RGP = 39.823°
m∠DAE = 19.911°
m∠RGP / m∠DAE = 2.000
P
S
D
R
G
E
A
5
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2

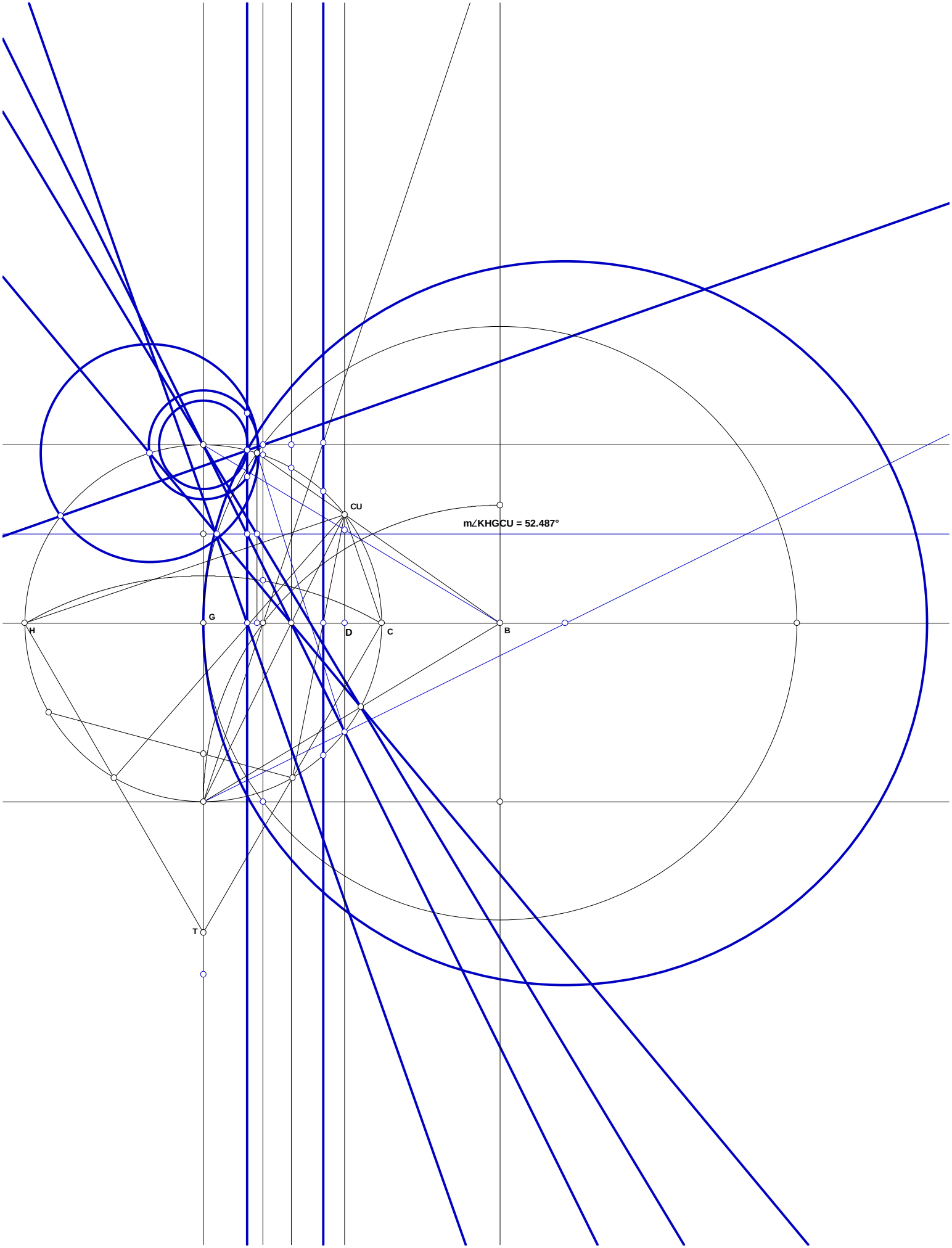

$\left(\frac{m\angle JMDG}{3} - m\angle OADG\right) 10000 = 0.176°$
$m\angle JZGKA = 30.000°$
$m\angle JMDG = 76.125°$
$m\angle OADG = 25.375°$
$\frac{m\angle JMDG}{3} - m\angle OADG = 0.000°$
G
T


m∠CYCXDD = 12.372°
m∠CUGCY = 74.233°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 74.233°
m∠CUCG = 82.117°
m∠JJAIL = 77.458°
m∠CYGJJ = 47.700°
CU
G
H
F
E
FM
D
C
B
A
T

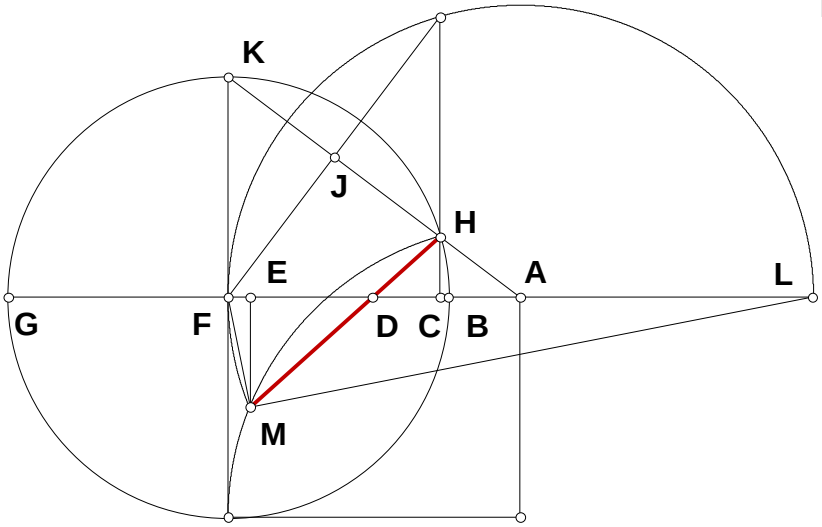

m∠CYCXDD = 8.086°
m∠CUGCY = 48.515°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 48.515°
m∠CUCG = 69.258°
U
H
G
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


CU
m∠KHGCU = 52.487°
G
H
D
C
B
T

# Segment DF And HM 052201




**Given AB and AG, what is HM and DF?**

$N := 7.111$ $AB := .375$

$AG := AB \cdot N$ $BG := AG - AB$

$BF := \frac{BG}{2}$ $FK := BF$

$AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2}$

$AJ := \frac{AF^2}{AK}$ $JK := AK - AJ$

$HJ := JK$ $AH := AK - (JK + HJ)$ $AC := \frac{AF \cdot AH}{AK}$ $EM := \frac{BF}{2}$ $FL := 2 \cdot AF$ $EF := \frac{FL}{2} - \frac{\sqrt{-4\ EM^2 + FL^2}}{2}$

$AE := AF - EF$ $CE := AE - AC$ $CH := \frac{FK \cdot AH}{AK}$ $HM := \sqrt{(EM + CH)^2 + CE^2}$ $DE := \frac{CE \cdot EM}{EM + CH}$

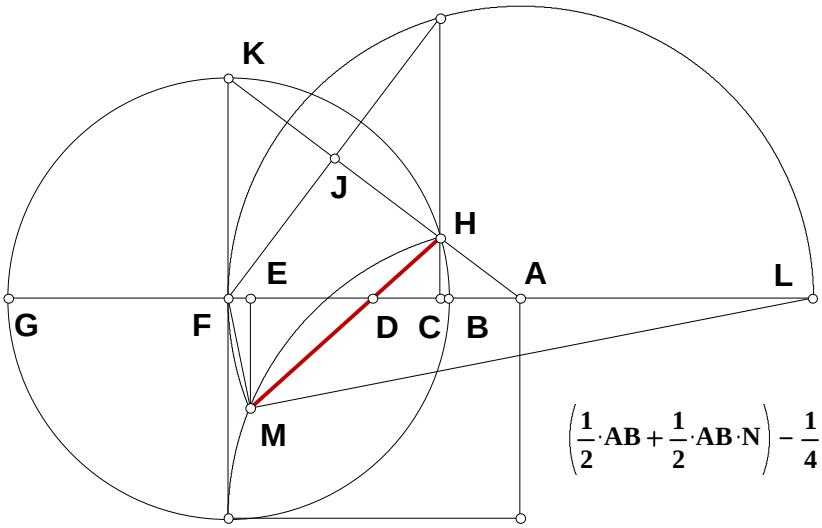
$DF := DE + EF$

## Some Algebraic Names:

$AB \cdot N - AG = 0$ $AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$

$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$ $\frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0$ $\frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$

$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{(1 + N^2 - 2 \cdot N)}{\sqrt{1 + N^2}} - JK = 0$ $AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0$ $AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$




$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0$$

$$AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

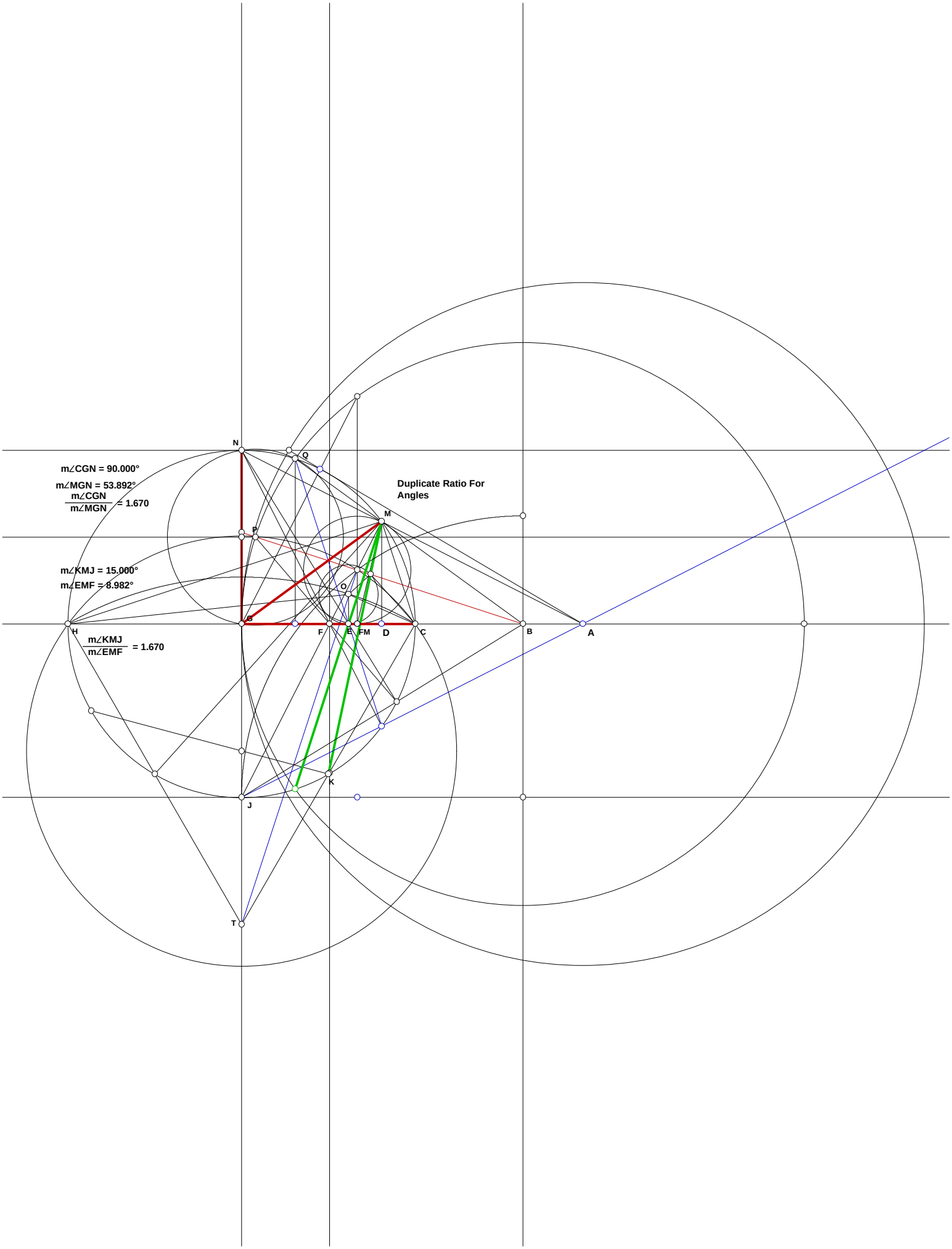
$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

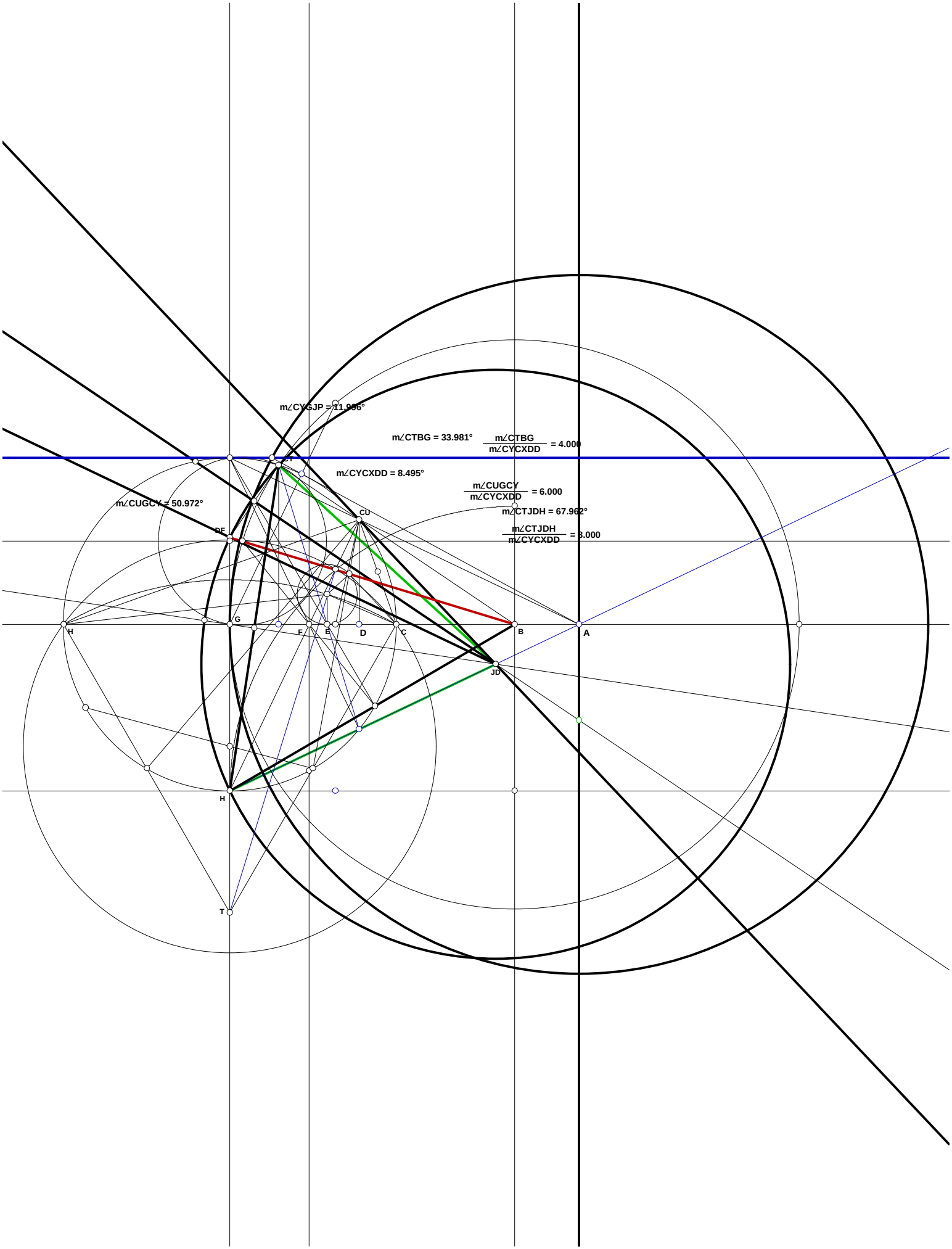
$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

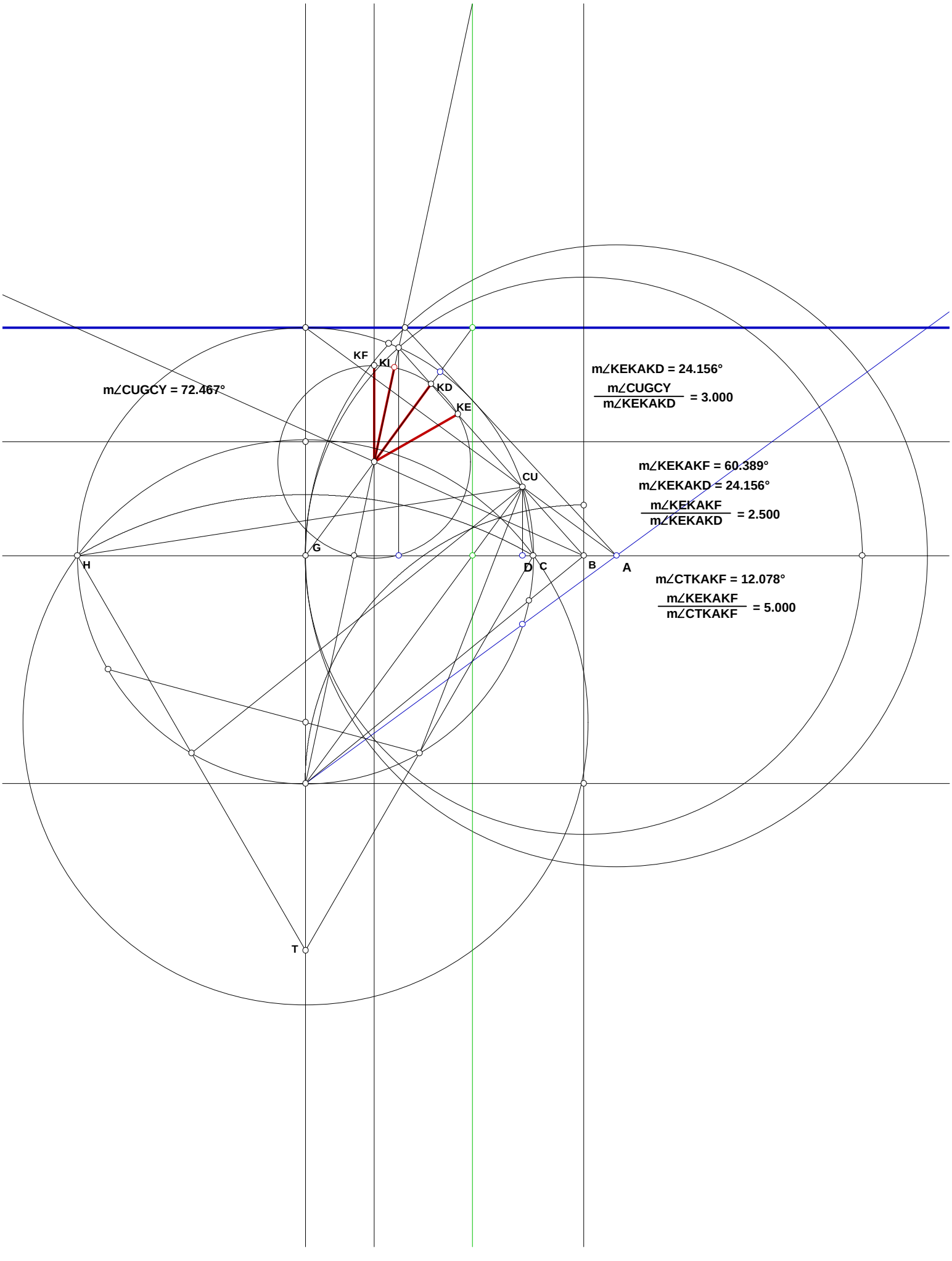
$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

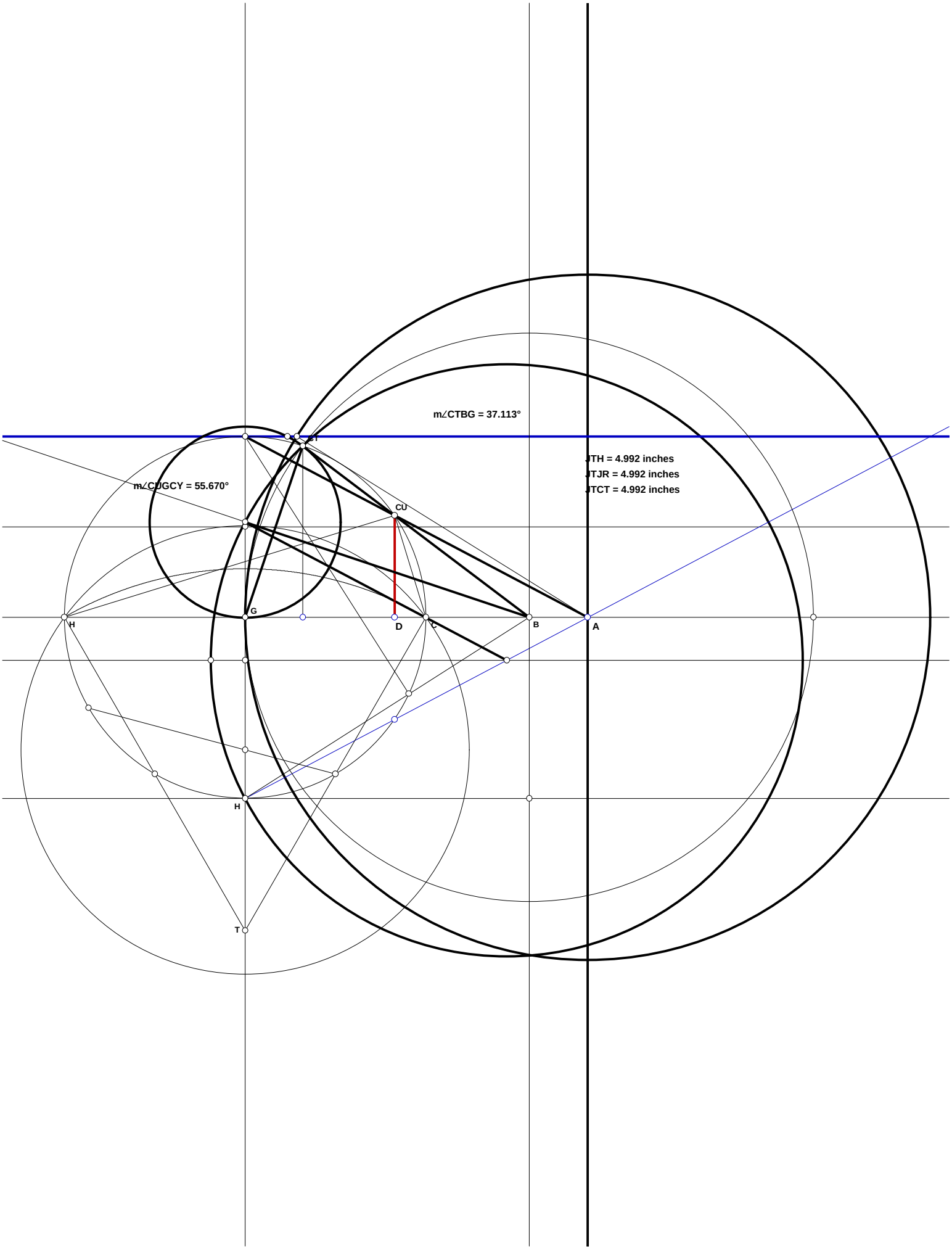
$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

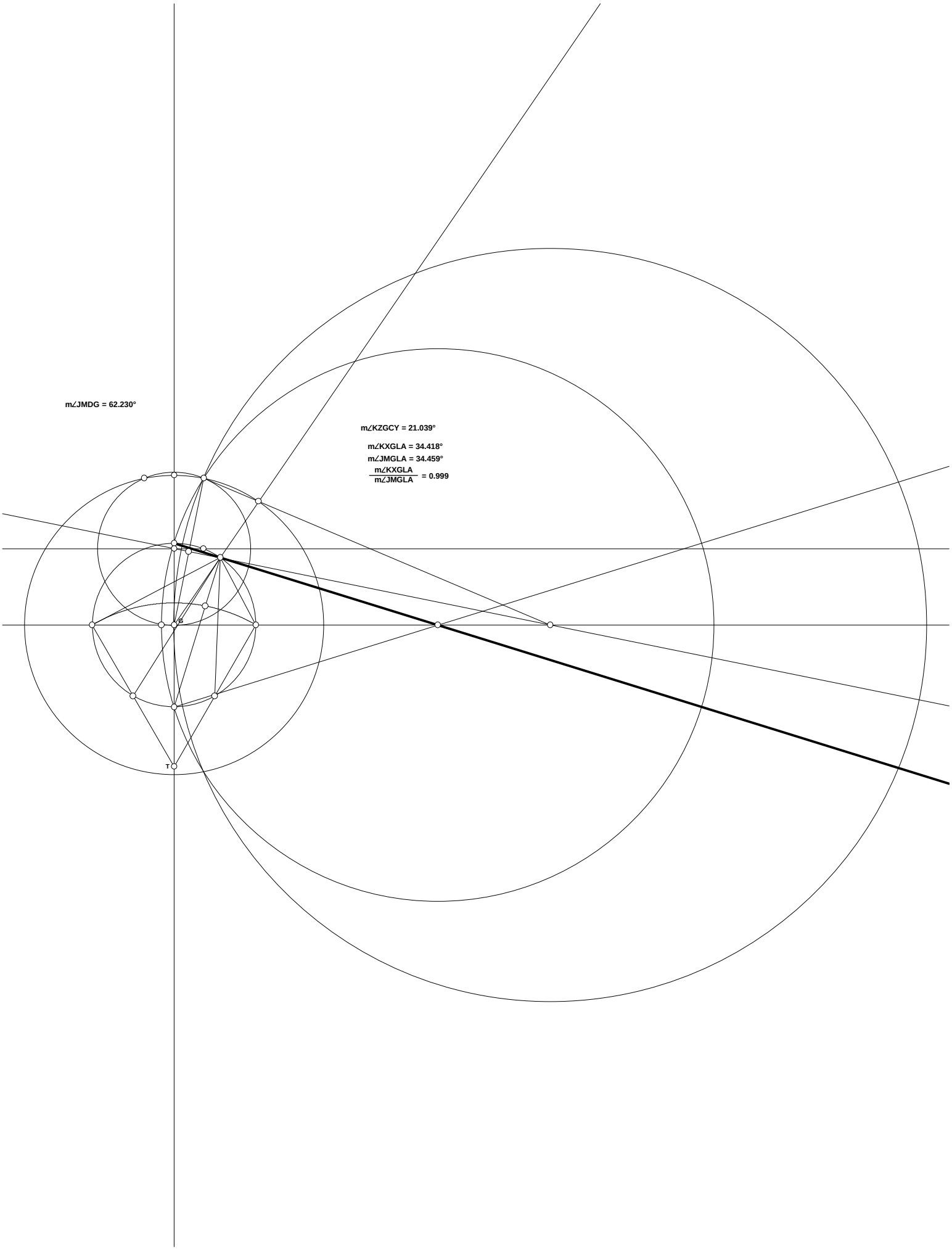
$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$

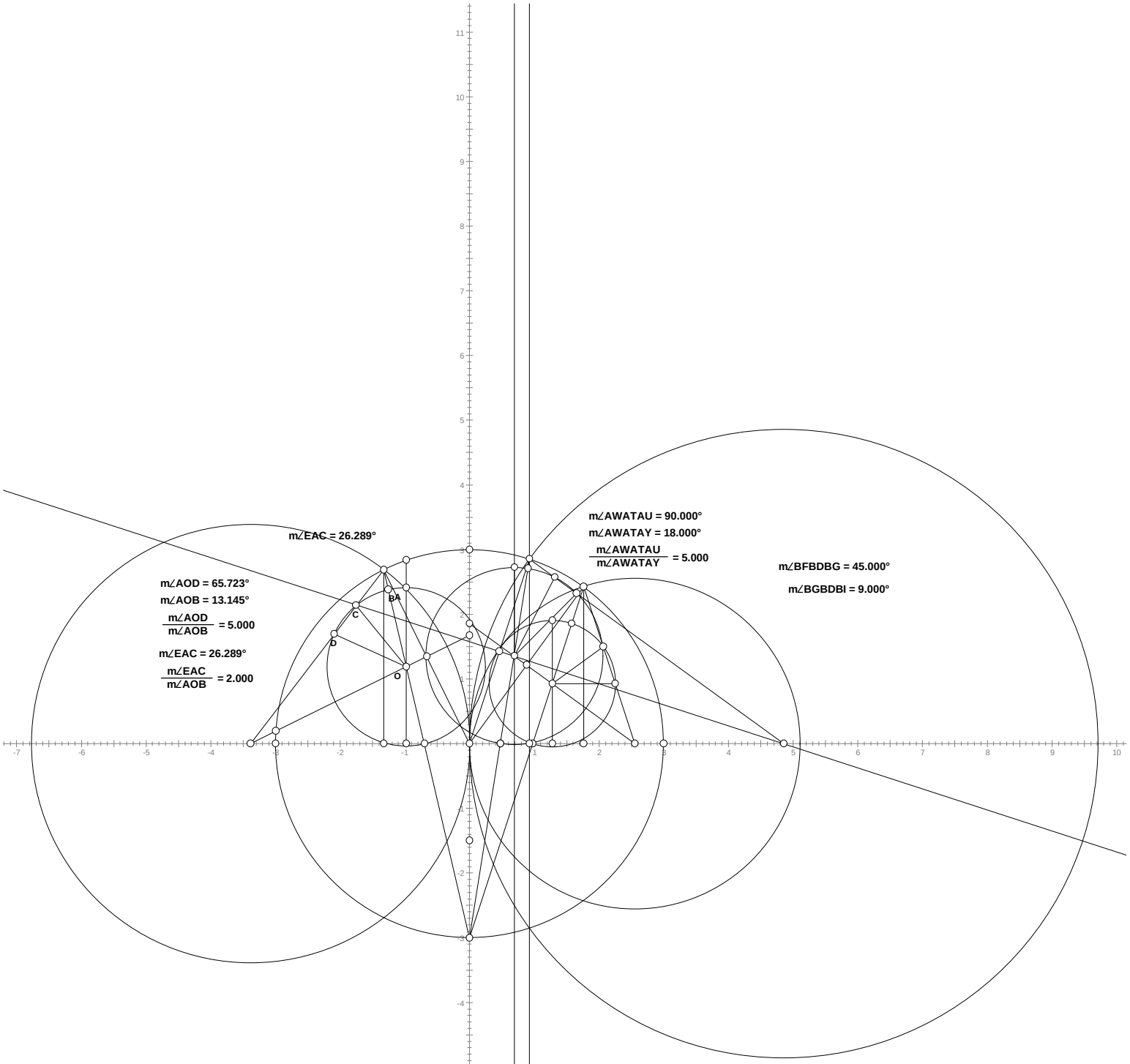

m∠CGN = 90.000°
m∠MGN = 53.892°
m∠CGN / m∠MGN = 1.670
Duplicate Ratio For Angles
m∠KMJ = 15.000°
m∠EMF = 8.982°
m∠KMJ / m∠EMF = 1.670
N
Q
M
P
O
G
H
F
E
FM
D
C
B
A
K
J
T

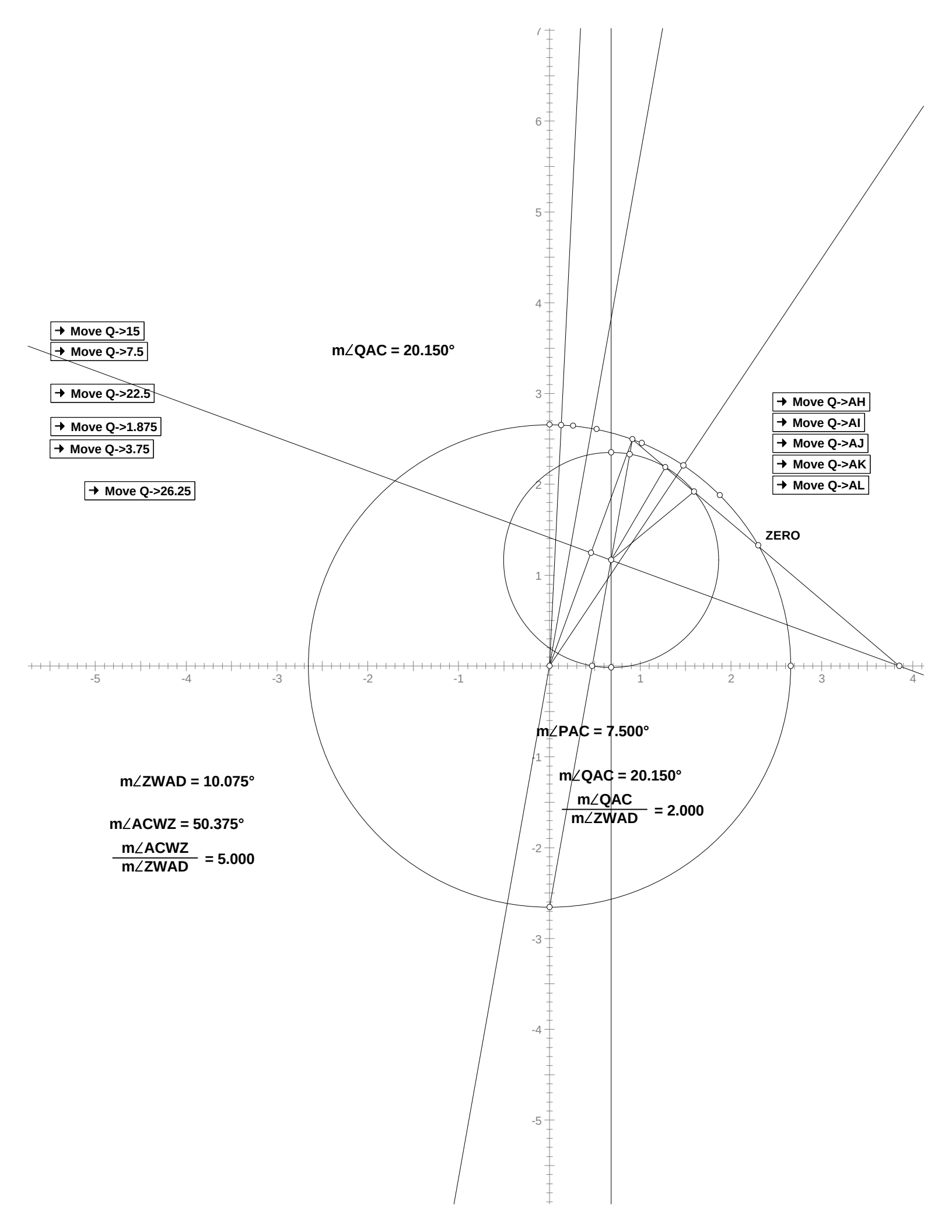

m∠CYGJP = 11.996°
m∠CTBG = 33.981°
m∠CTBG / m∠CYCXDD = 4.000
m∠CYCXDD = 8.495°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 50.972°
m∠CTJDH = 67.962°
m∠CTJDH / m∠CYCXDD = 8.000
CU
DE
G
H
F
E
D
C
B
A
JD
H
T

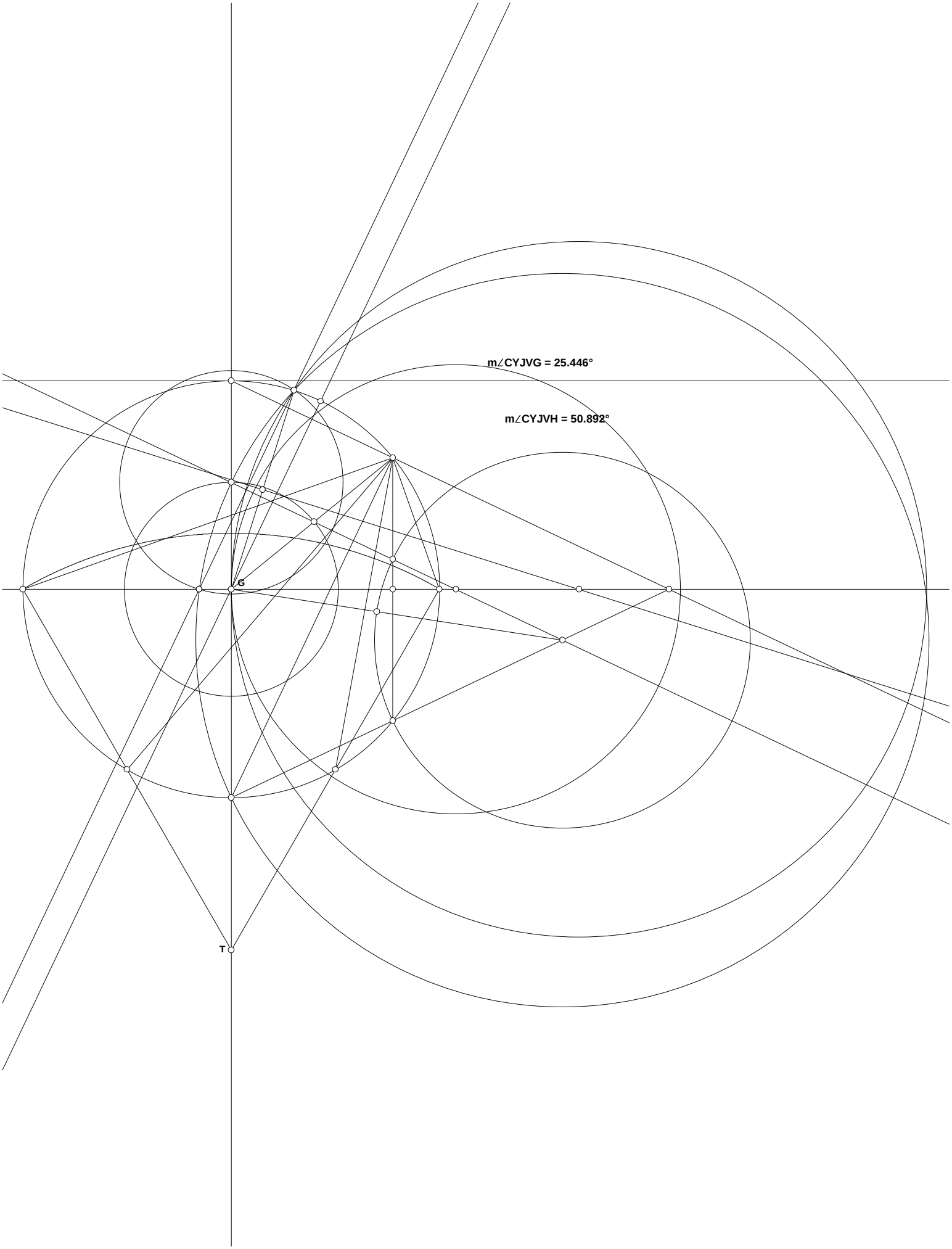

KF
KI
KD
KE
CU
G
H
D
C
B
A
T
m∠CUGCY = 72.467°
m∠KEKAKD = 24.156°
m∠CUGCY / m∠KEKAKD = 3.000
m∠KEKAKF = 60.389°
m∠KEKAKD = 24.156°
m∠KEKAKF / m∠KEKAKD = 2.500
m∠CTKAKF = 12.078°
m∠KEKAKF / m∠CTKAKF = 5.000

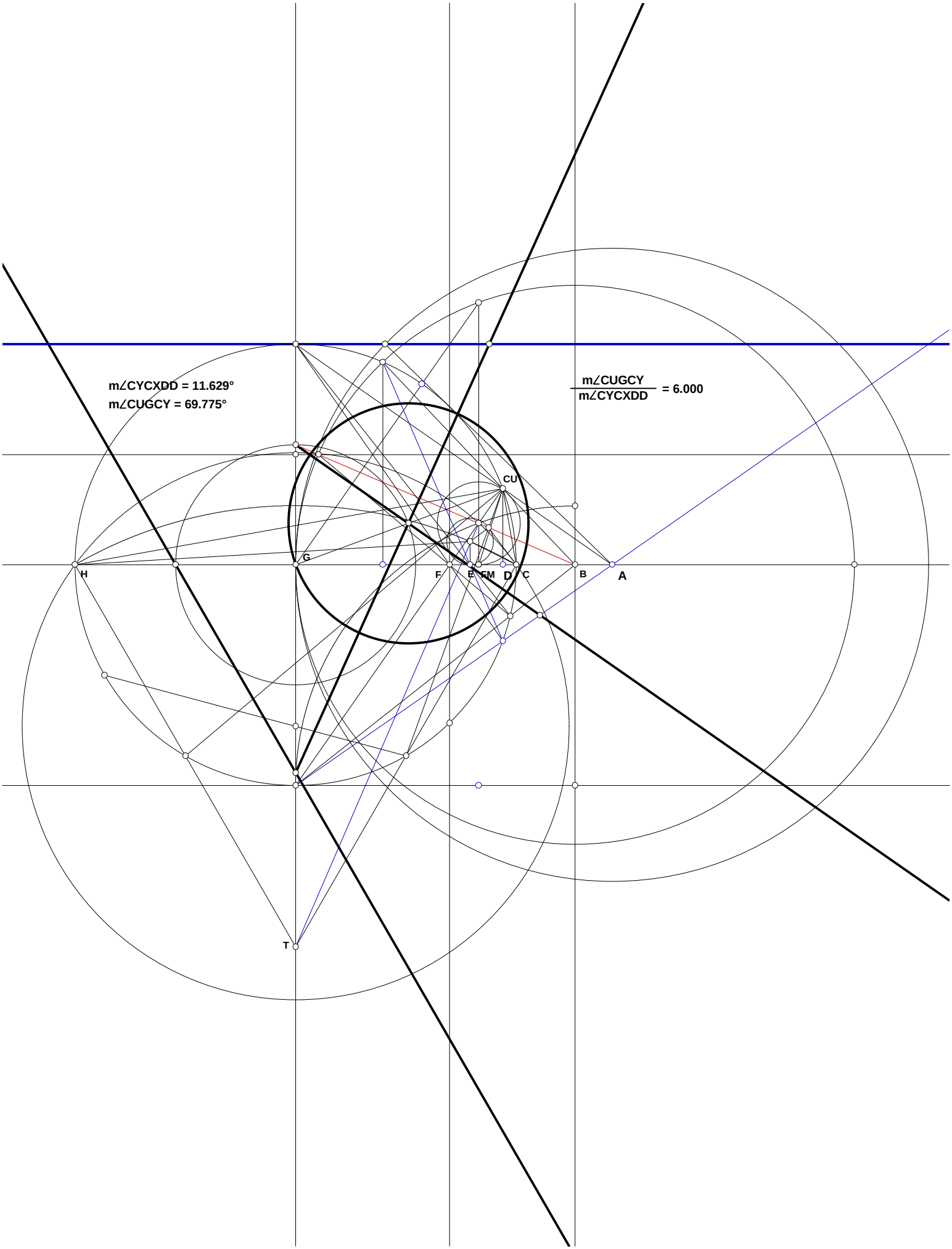

m∠CTBG = 37.113°
JTH = 4.992 inches
JTJR = 4.992 inches
JTCT = 4.992 inches
m∠CUGCY = 55.670°
CU
G
H
D
C
B
A
H
T

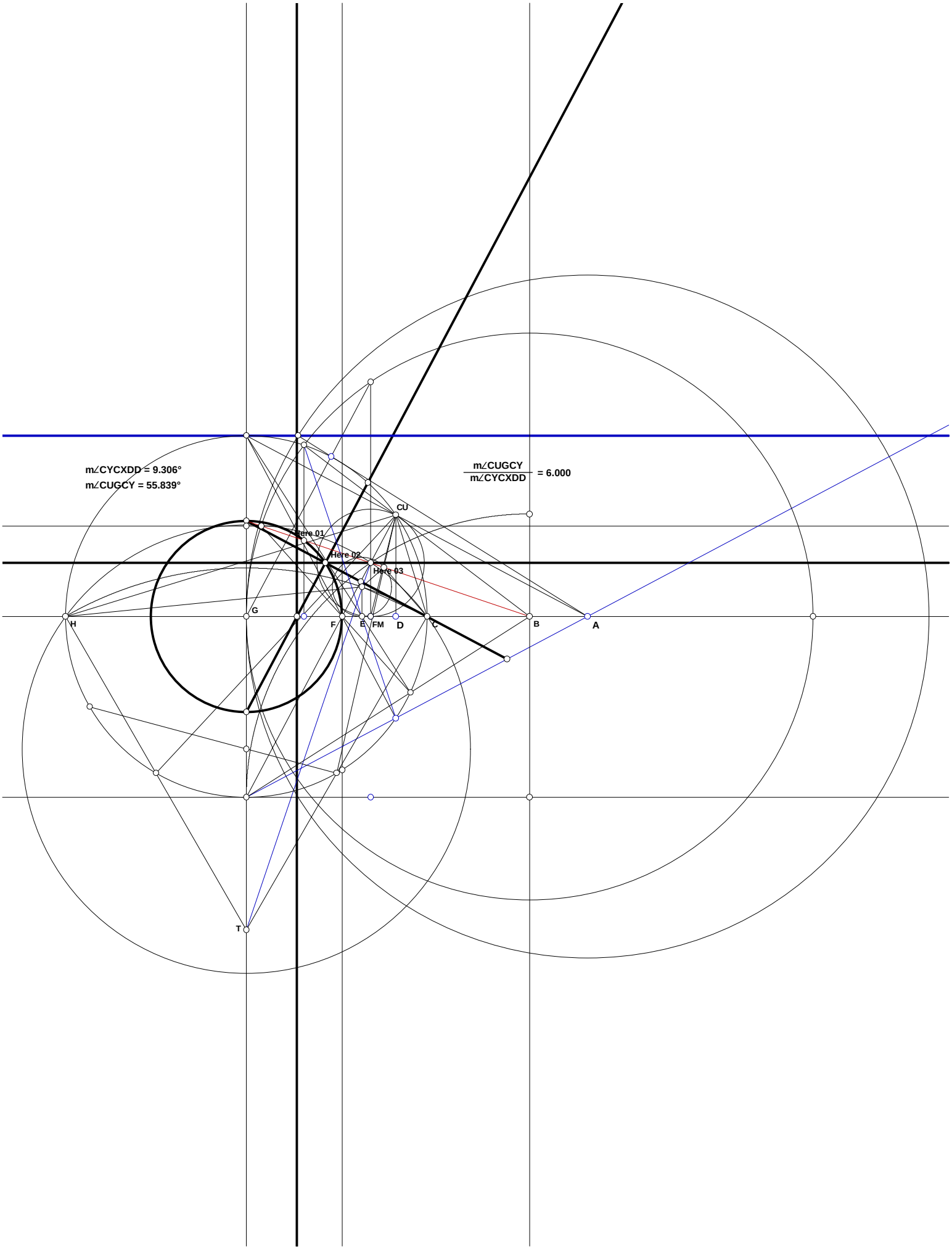

m∠JMDG = 62.230°
m∠KZGCY = 21.039°
m∠KXGLA = 34.418°
m∠JMGLA = 34.459°
m∠KXGLA / m∠JMGLA = 0.999
T


m∠EAC = 26.289°
m∠AOD = 65.723°
m∠AOB = 13.145°
m∠AOD / m∠AOB = 5.000
m∠EAC = 26.289°
m∠EAC / m∠AOB = 2.000
m∠AWATAU = 90.000°
m∠AWATAY = 18.000°
m∠AWATAU / m∠AWATAY = 5.000
m∠BFBDBG = 45.000°
m∠BGBDBI = 9.000°
A
B
C
D
O

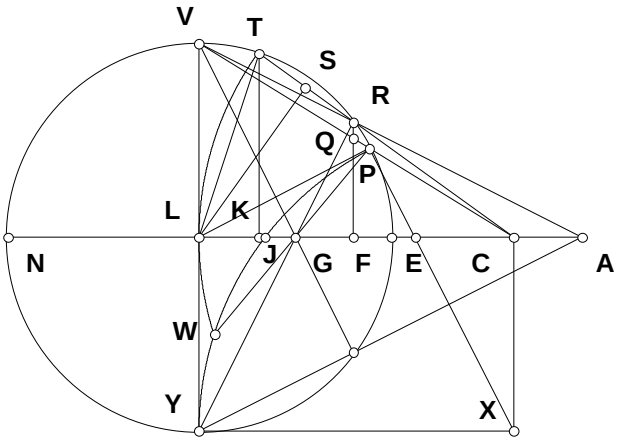

Move Q->15
Move Q->7.5
Move Q->22.5
Move Q->1.875
Move Q->3.75
Move Q->26.25
m∠QAC = 20.150°
Move Q->AH
Move Q->AI
Move Q->AJ
Move Q->AK
Move Q->AL
ZERO
m∠PAC = 7.500°
m∠QAC = 20.150°
m∠QAC / m∠ZWAD = 2.000
m∠ZWAD = 10.075°
m∠ACWZ = 50.375°
m∠ACWZ / m∠ZWAD = 5.000


m∠CYJVG = 25.446°
m∠CYJVH = 50.892°
G
T


m∠CYCXDD = 11.629°
m∠CUGCY = 69.775°
m∠CUGCY / m∠CYCXDD = 6.000
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 9.306°
m∠CUGCY = 55.839°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Here 01
Here 02
Here 03
G
H
F
E
FM
D
C
B
A
T

# Point of Intersection 052701




## Do RY and PW intersect at G?

$N := 5 \qquad CE := 1 \qquad CN := CE \cdot N$

$EN := CN - CE \qquad EL := \frac{EN}{2} \qquad LV := EL$

$LT := EL \qquad LY := EL \qquad CL := CE + EL \qquad CT := CL$

$S_1 := CL \qquad S_2 := CT \qquad S_3 := LT$

$KL := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad CK := CL - KL \qquad CS := CK \qquad ST := CT - CS \qquad RS := ST$

$CR := CT - (ST + RS) \qquad CF := \frac{CK \cdot CR}{CT} \qquad FR := \sqrt{CR^2 - CF^2} \qquad FQ := \frac{LV \cdot CF}{CL} \qquad FL := CL - CF$

$GL := \frac{FL \cdot LY}{(LY + FR)}$

## Some Algebraic Names:

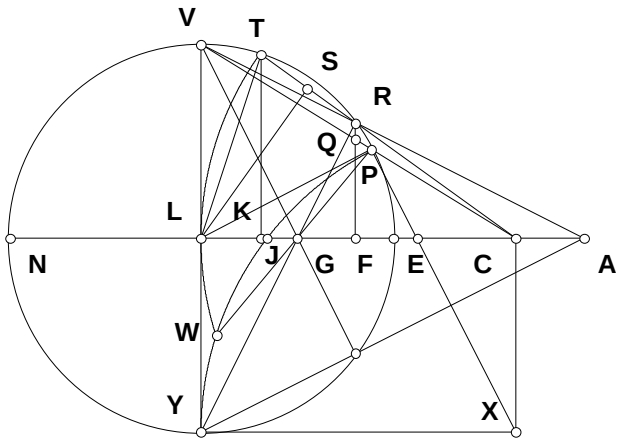
$CE \cdot N - CN = 0 \qquad CE \cdot N - CE - EN = 0 \qquad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0$

$2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \qquad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0$

$\frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$




$$GL := \frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}$$

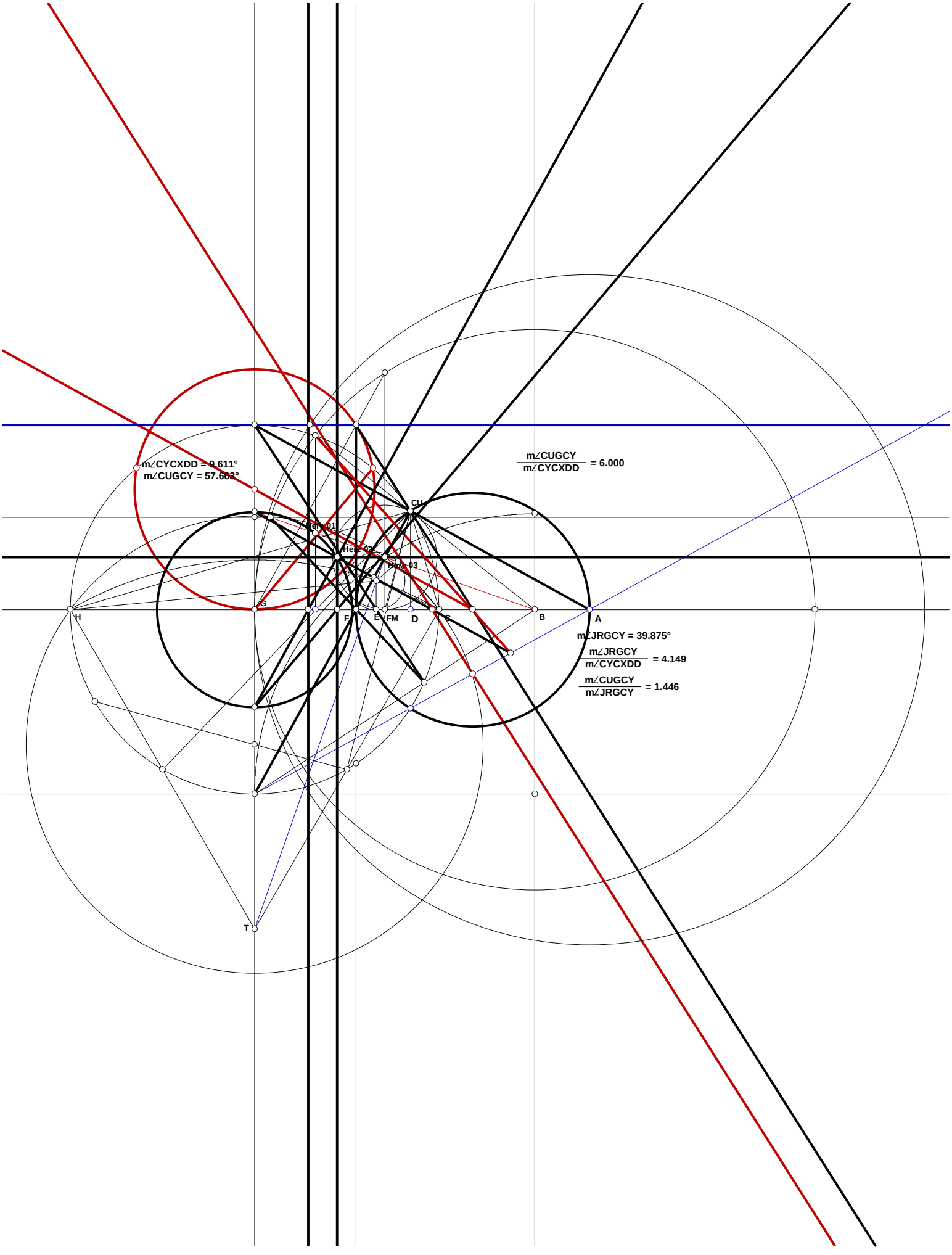
## From Segment DF And HM 052201:

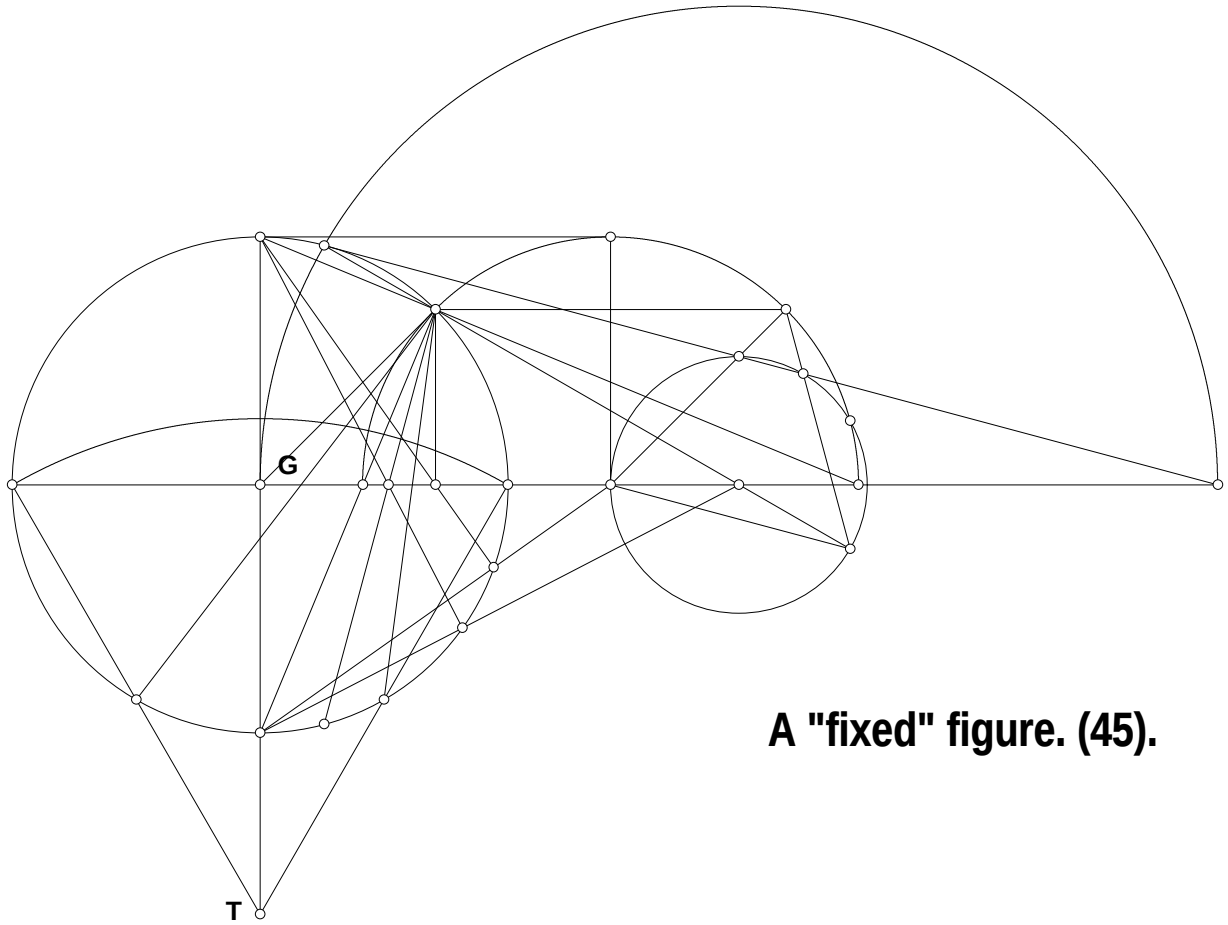
$$\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)} - GL = 0$$

$$\frac{\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)}}{\frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}} = 1$$

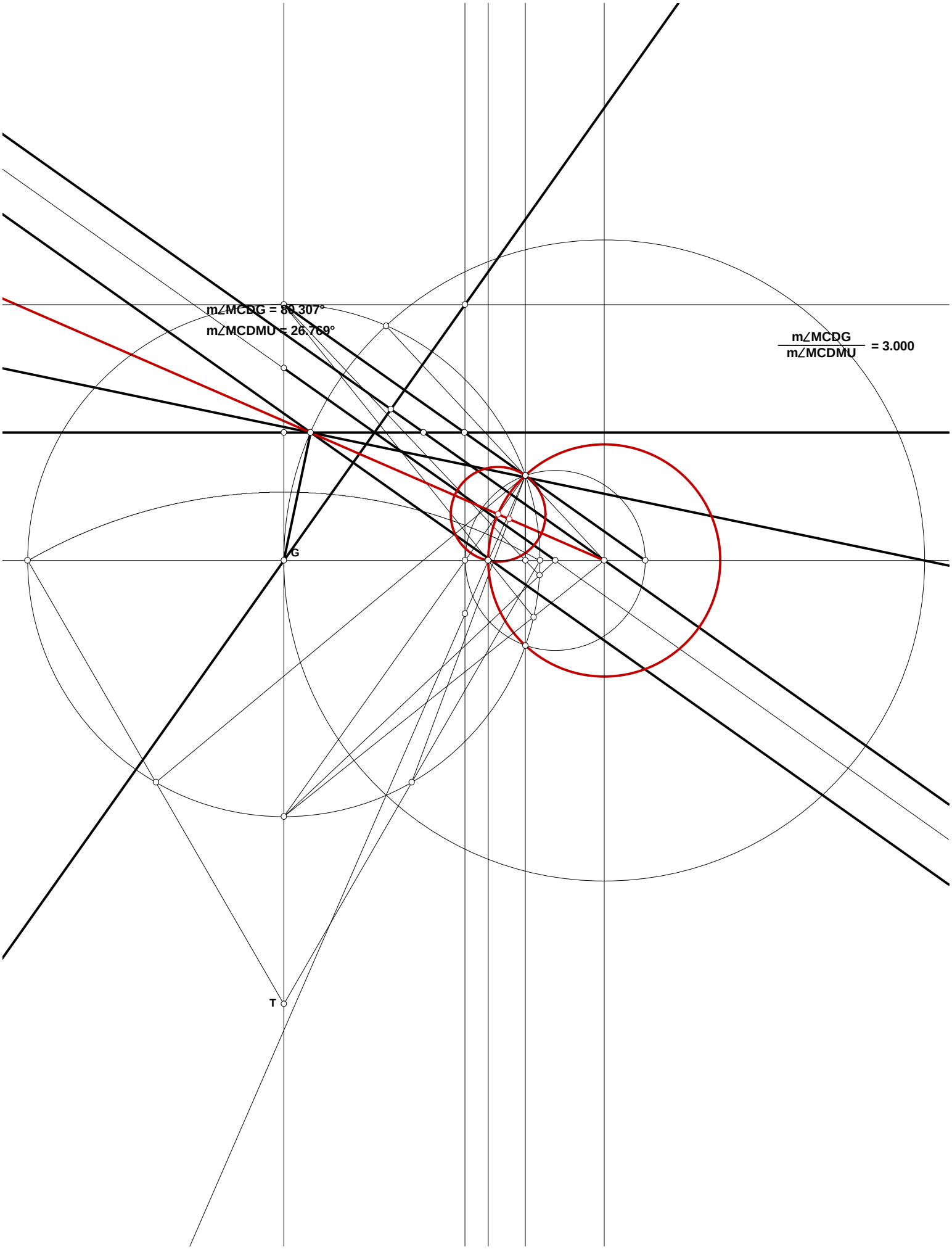
**Which does reduce to,**

$$1 = 1$$


m∠CYCXDD = 9.611°
m∠CUGCY = 57.668°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Hero 01
Hero 02
Hero 03
G
H
F
E
FM
D
C
B
A
T
m∠JRGCY = 39.875°
m∠JRGCY / m∠CYCXDD = 4.149
m∠CUGCY / m∠JRGCY = 1.446

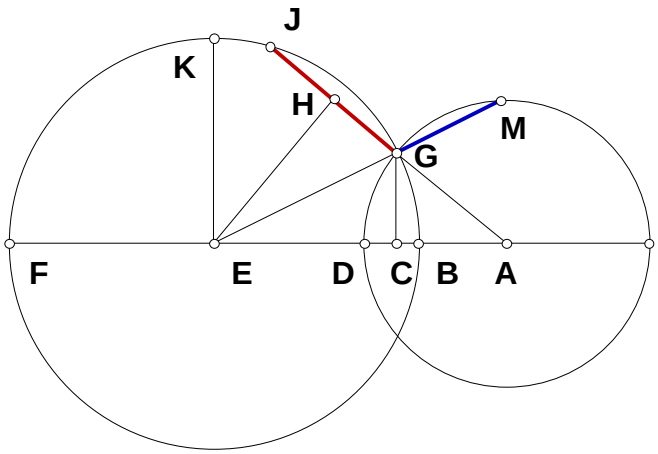



**A "fixed" figure. (45).**


m∠MCDG = 80.307°
m∠MCDMU = 26.769°
m∠MCDG / m∠MCDMU = 3.000
G
T

# A Small Extrapolation 060101

**Given AE, AG, and EG, what is the Algebraic name of the segment GJ?**




$S_1 := 3 \quad S_2 := 2 \quad S_3 := 1$

$AE := S_1 \quad AG := S_2 \quad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2\,AE}$$

$$AH := \frac{AC \cdot AE}{AG} \quad GH := AH - AG$$

$HJ := GH \quad GJ := GH + HJ$

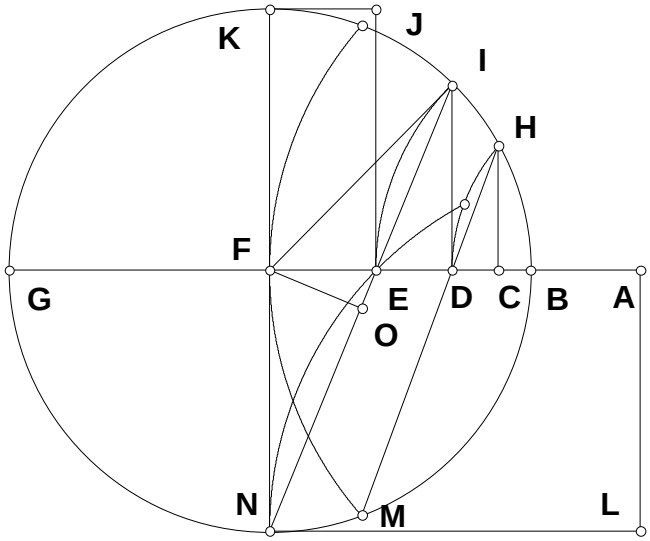
**Some Algebraic Names:**

$$\frac{S_2^2 + S_1^2 - S_3^2}{2\,S_1} - AC = 0 \qquad \frac{S_1^2 + S_2^2 - S_3^2}{2\,S_2} - AH = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{2\,S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{S_3} - GM = \ ?$$

## Units From Both Sides 060201

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




$N := 5.727 \qquad AB := .573 \qquad AG := AB \cdot N$

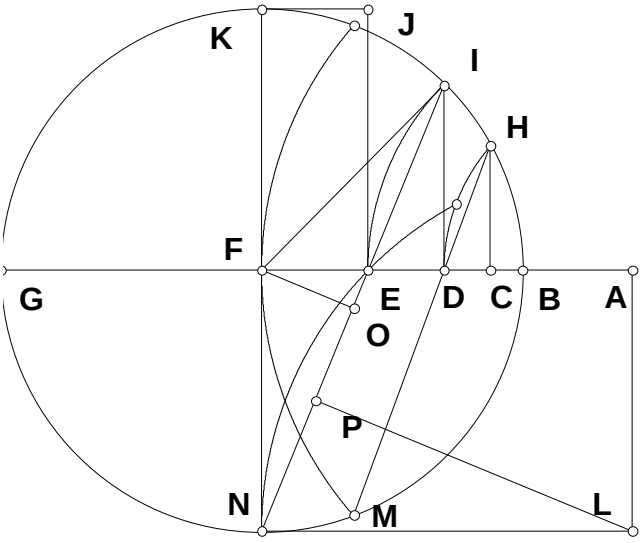
$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

$AB \cdot (N - 1) - BG = 0$

$\frac{1}{2} \cdot AB \cdot (N - 1) - BF = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0 \qquad AB\sqrt{N} - AE = 0 \qquad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$



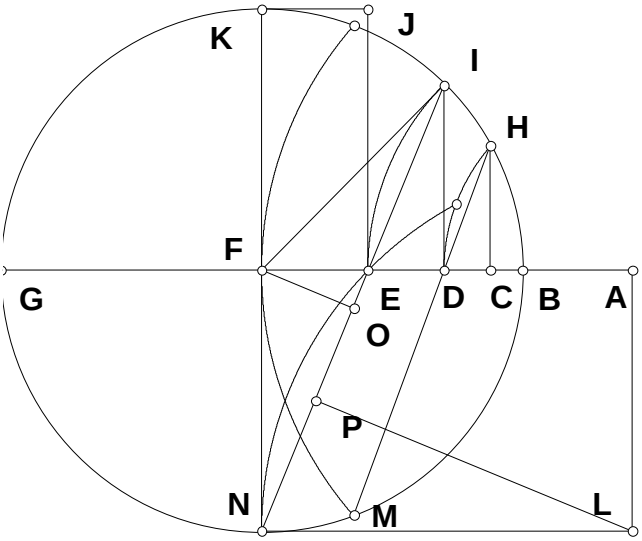

$N_2 := 2 \qquad BG_2 := 1 \qquad BF_2 := \frac{BG_2}{2}$

$BE_2 := \frac{BF_2}{N_2} \qquad EF_2 := BF_2 - BE_2$

$FN_2 := BF_2 \qquad EN_2 := \sqrt{EF_2^2 + FN_2^2}$

$NP_2 := \frac{EN_2}{2} \qquad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$

$AF_2 := LN_2 \qquad AB_2 := AF_2 - BF_2$




$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0$$

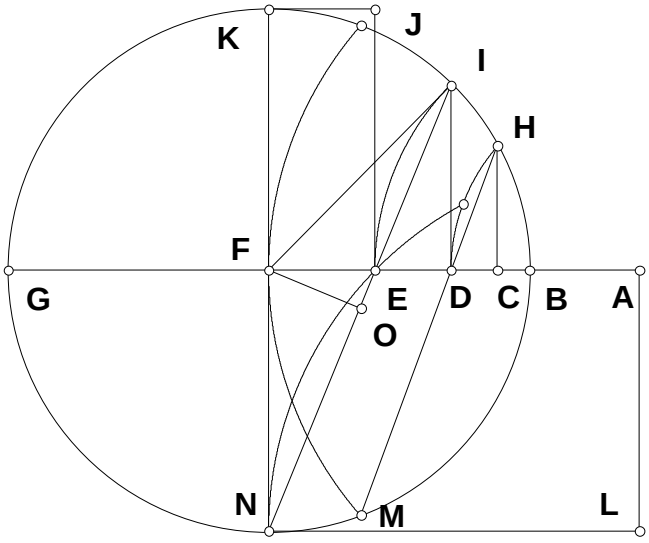
$$\frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{(2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{[N_2 \cdot (N_2 - 1)]} - LN_2 = 0$$

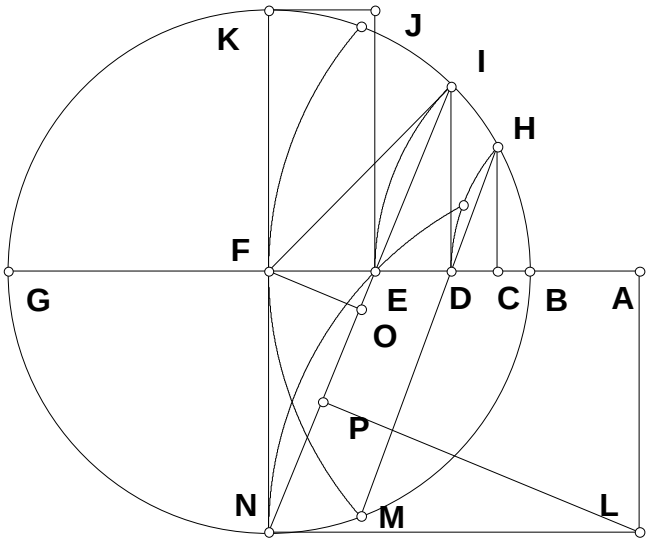
$$\frac{BG_2}{4 \cdot N_2 \cdot (N_2 - 1)} - AB_2 = 0$$




$$AI := AE \quad FI := BF \quad AD := \frac{AI^2 + AF^2 - FI^2}{2\,AF}$$

$$BD := AD - AB$$

$$AD := 2 \cdot AB \cdot \frac{N}{(N+1)} \qquad BD := AB \cdot \frac{(N-1)}{(1+N)}$$




$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2}$

$BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$

$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3$

$FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$

$EN_3 := \sqrt{EF_3^2 + FN_3^2} \quad NP_3 := \frac{EN_3}{2}$

$LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$

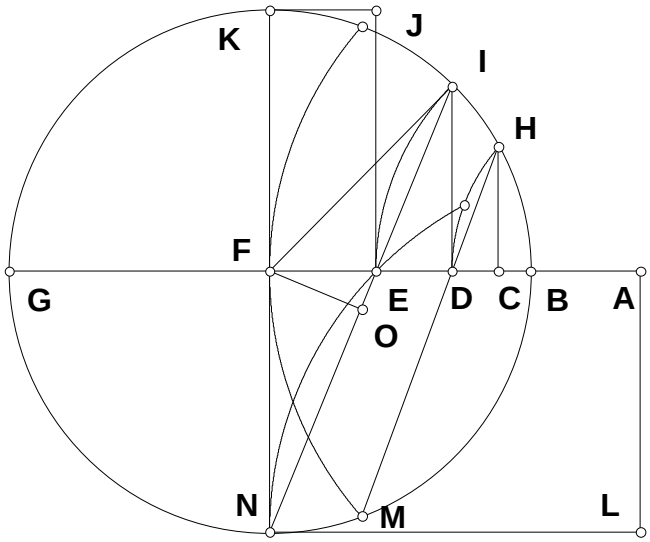
$AB_3 := AF_3 - BF_3 \quad AB_3 = 0.573$

$BD_3 := \frac{1}{2} \cdot \frac{BG_3}{N_3} \quad DG_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} \quad DI_3 := \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1}$

$DF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} \quad EF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{(N_3 + \sqrt{2 \cdot N_3 - 1})}$

$EN_3 := \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} \quad NP_3 := \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}}$

$LN_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} \quad AB_3 := \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)}$




$AH := AD$

$FH := BF \quad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$

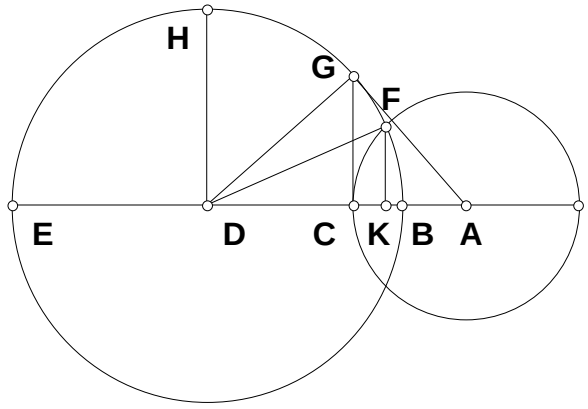
$BC := AC - AB$

$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$

$AC := AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N+1)^3} \qquad BC := AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3}$

# Isolating A Problem 060301

**If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?**




$N := 4 \qquad BE := 1$

$BD := \frac{BE}{2} \qquad BC := \frac{BE}{N} \qquad CE := BE - BC$

$CG := \sqrt{BC \cdot CE} \qquad CD := BD - BC \qquad AC := \frac{CG^2}{CD}$

$AF := AC \qquad AD := AC + CD \qquad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2\ AD} \qquad BK := BD - DK$

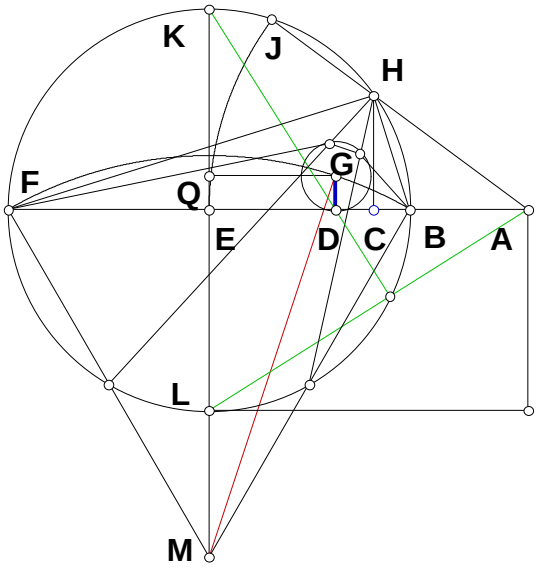
$CK := BC - BK$

$BE \cdot \frac{(N-1)}{N} - CE = 0 \qquad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \qquad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$

$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \qquad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \qquad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \qquad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \qquad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

# For Any AB 061001




## For any AB, AF what is DG?

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ$

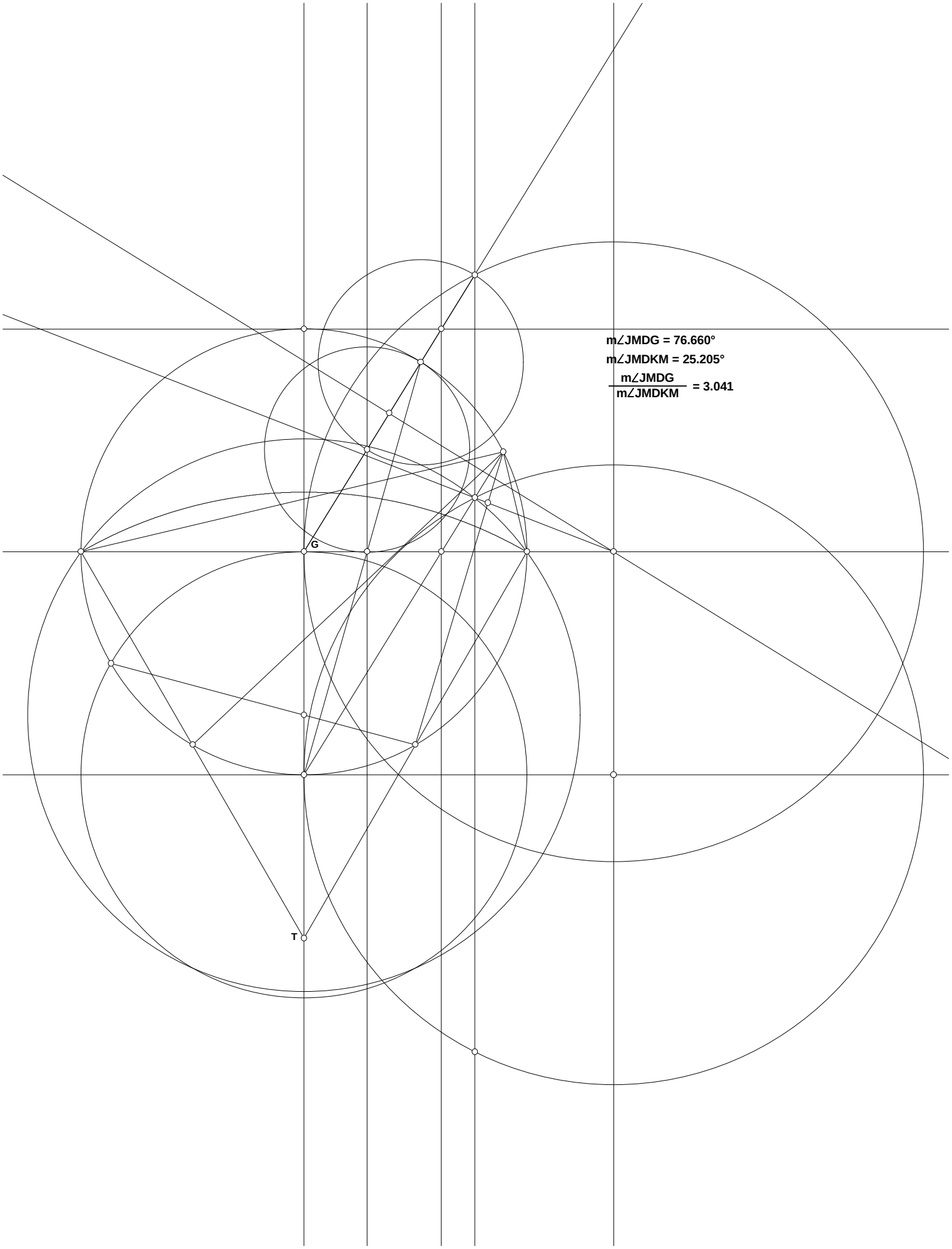
## Some Algebraic Names:

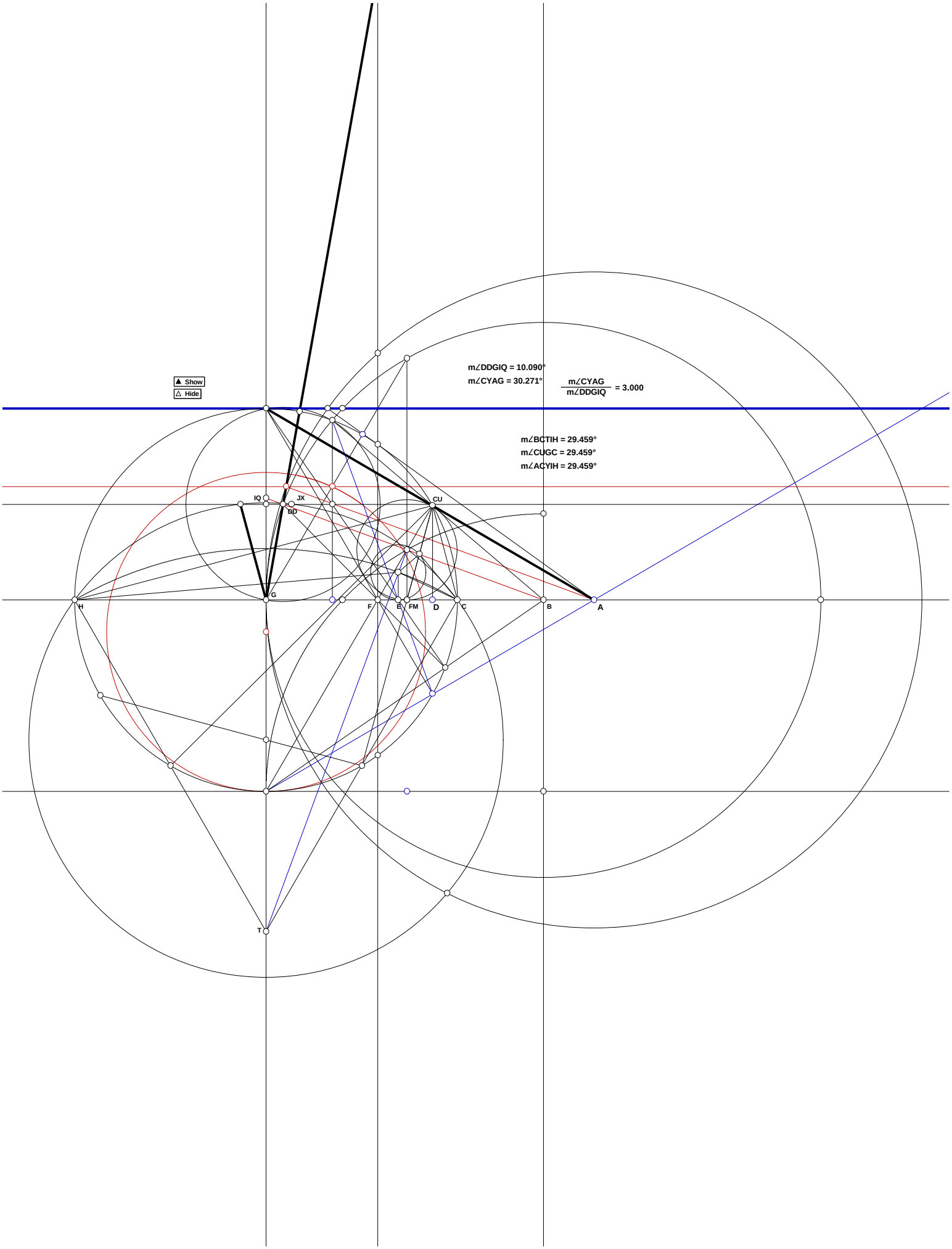
$AB \cdot (N - 1) - BF = 0 \qquad \frac{AB \cdot (N - 1)}{2} - BE = 0 \qquad \frac{1}{2} \cdot AB \cdot (N + 1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N - 1)^2}{(N + 1)} - DE = 0 \qquad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N - 1) - EM = 0$

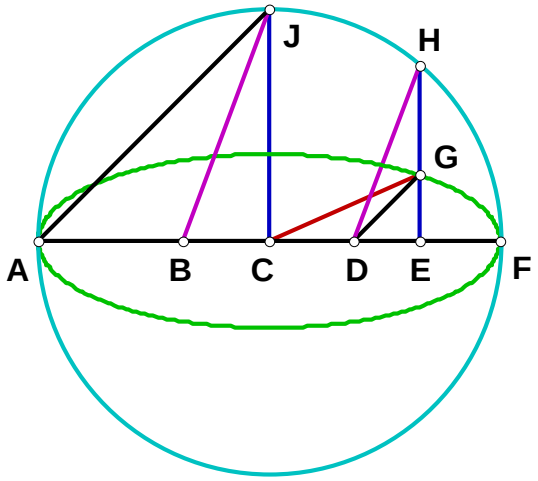
$$\frac{AB \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)\right]}{2 \cdot (N + 1)} - MQ = 0$$

$$\frac{AB \cdot (N - 1) \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N + 1)} - DG = 0$$


m∠JMDG = 76.660°
m∠JMDKM = 25.205°
m∠JMDG / m∠JMDKM = 3.041
G
T


Show
Hide
m∠DDGIQ = 10.090°
m∠CYAG = 30.271°
m∠CYAG / m∠DDGIQ = 3.000
m∠BCTIH = 29.459°
m∠CUGC = 29.459°
m∠ACYIH = 29.459°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

# Elipse By Parallels 082601




$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \frac{AF}{2} \quad CJ := AC$

$BC := \frac{AC}{N_1} \qquad AE := \frac{AF}{N_2}$

$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \frac{BC \cdot EH}{CJ}$

$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

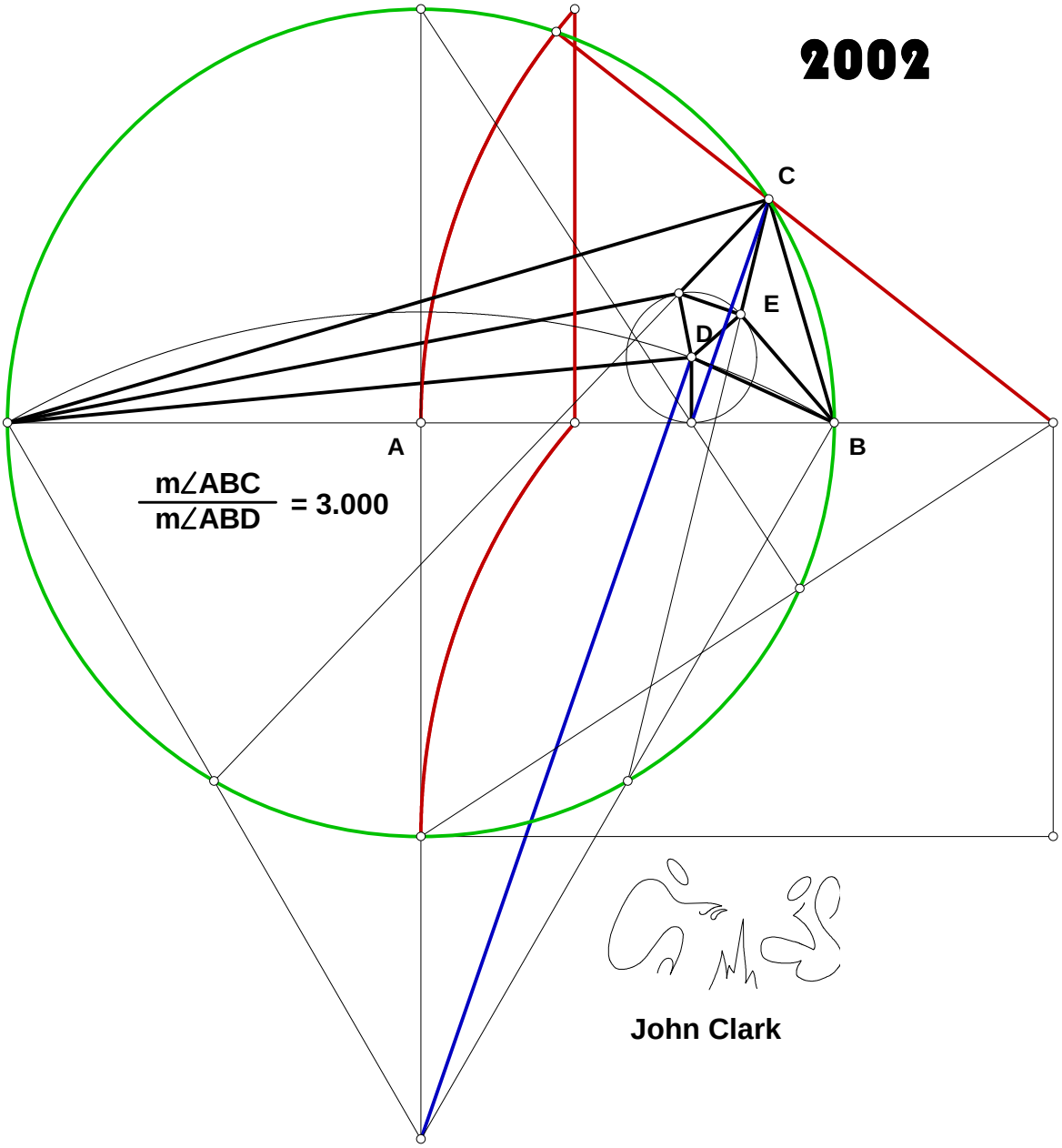
## Some Algebraic Names:

$BC := \frac{1}{(2 \cdot N_1)} \quad AE := \frac{1}{N_2} \quad EF := 1 - \frac{1}{N_2} \quad EH := \frac{\sqrt{N_2 - 1}}{N_2} \quad EG := \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2}$

$CE := \frac{1}{N_2} - \frac{1}{2} \quad CG := \frac{1}{2} \cdot \frac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)}$

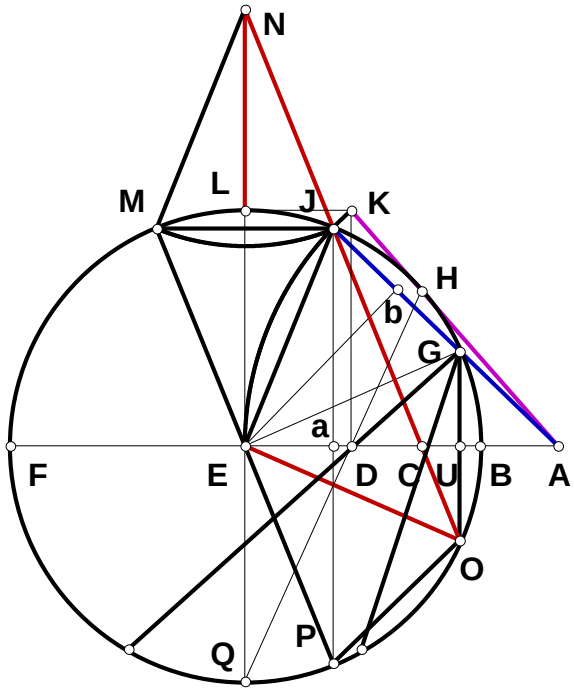
# Three Pieces Of Paper

2002




John Clark

# Just Another Proof Of Paper 010202




$N := 7 \quad AB := 1 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$

$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$

$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$

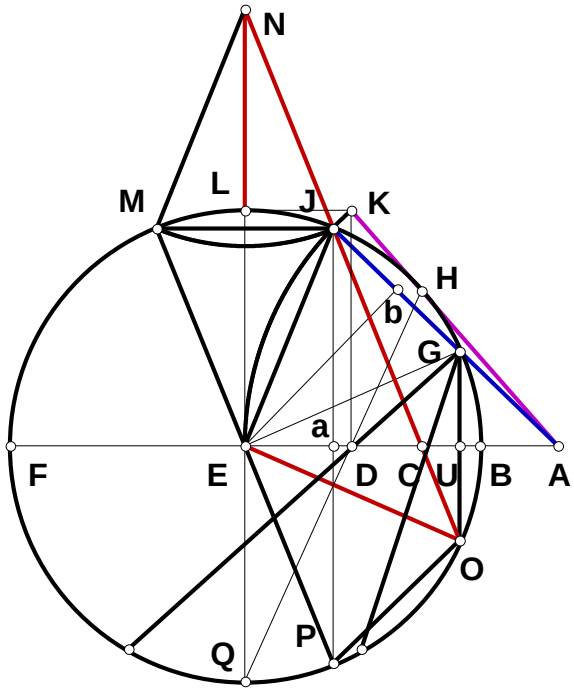
$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$

**From 4/29/94** $OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$

**Some Algebraic Names;**




$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0$$

$$\frac{N+1}{2} - AE = 0 \qquad \frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0$$

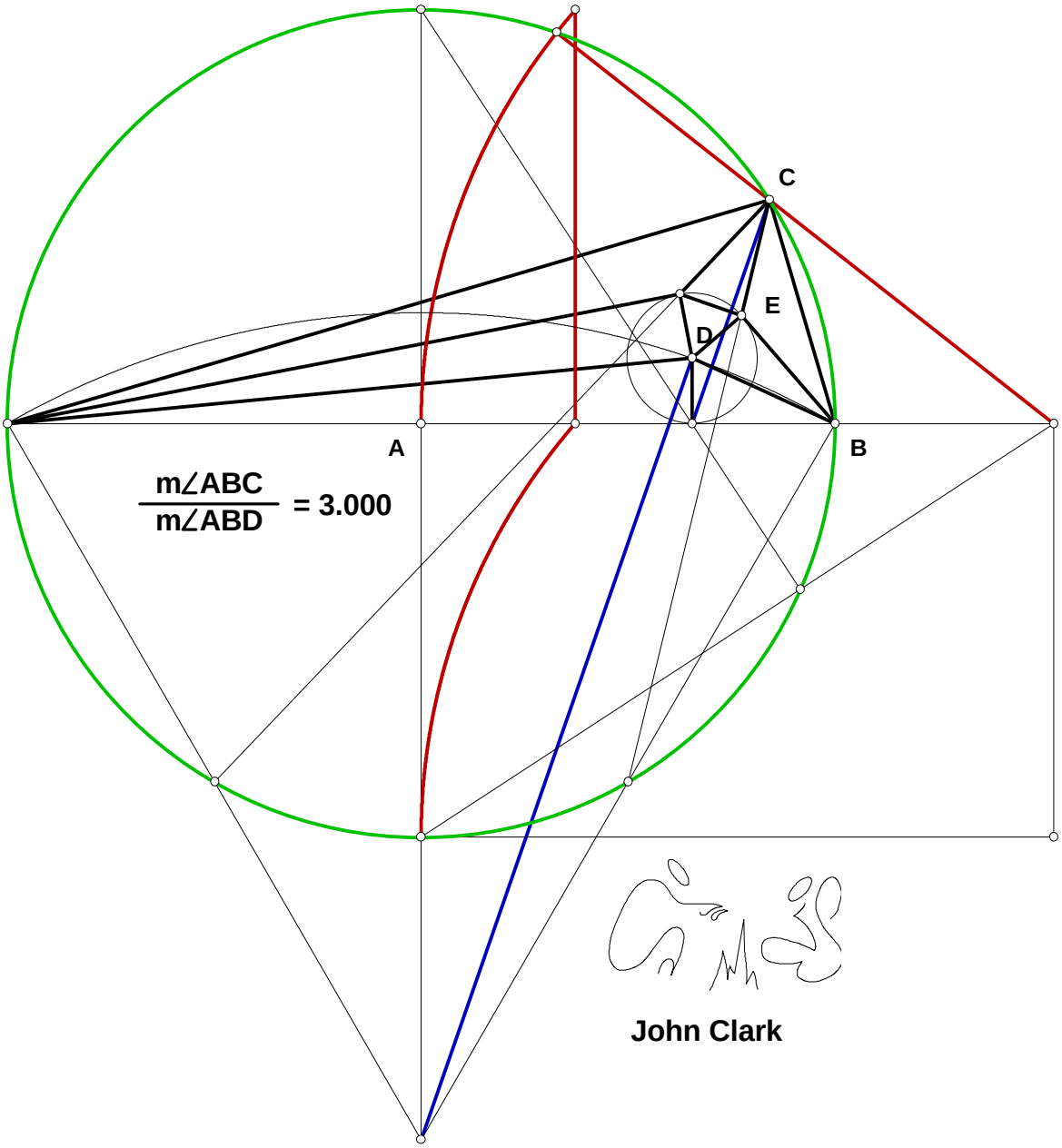
$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0 \qquad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - OP = 0 \qquad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$

# Three Pieces Of Paper




John Clark

## Three Pieces Of Paper

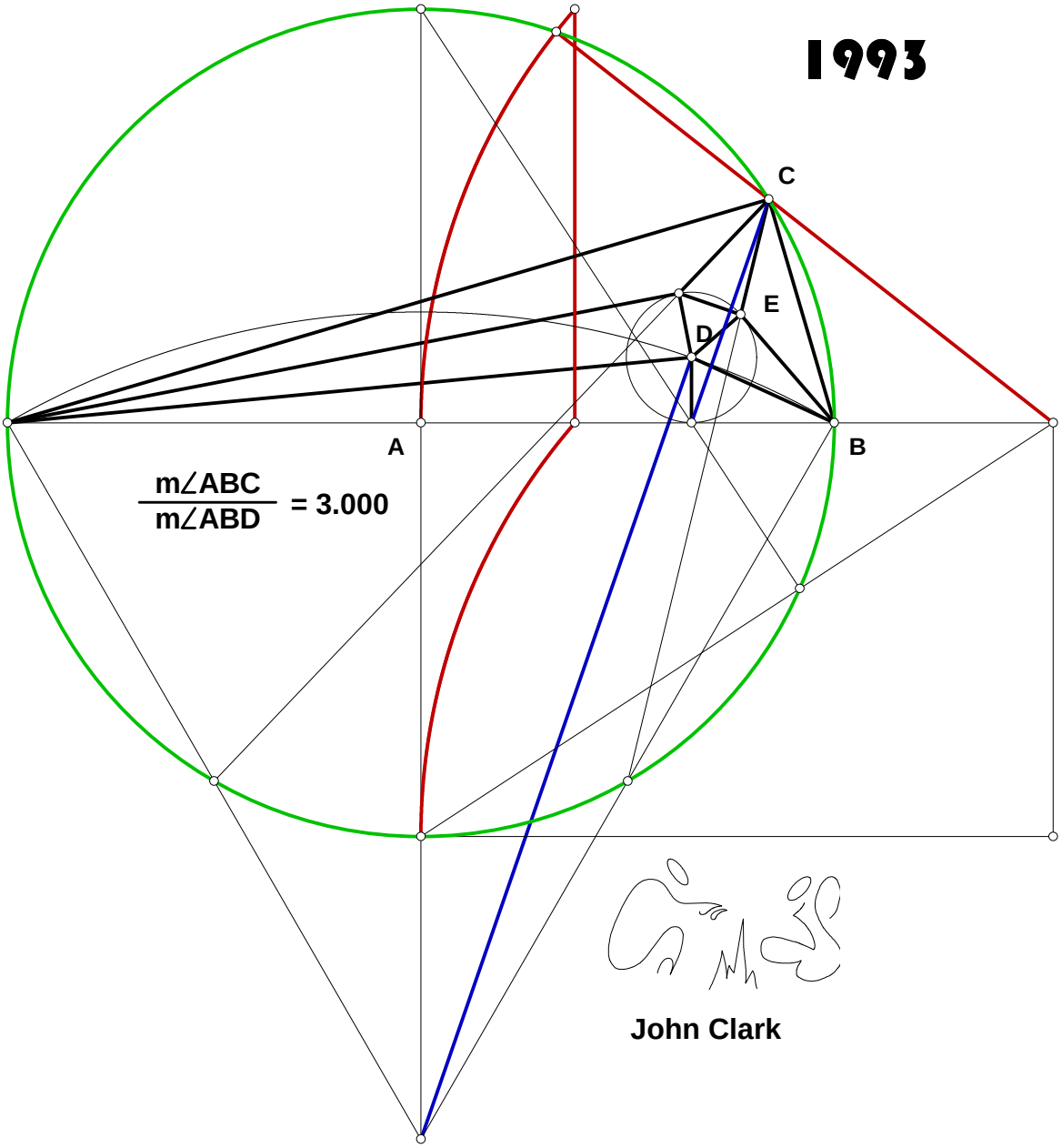
### Introduction

Is a novel about the search for a method to divide angles into at least three equal parts. The novel is written in the relatiologic of Geometry and the tautologic of basic Algebra.

I suspect that someday I might have something of significance to write here. J. C.

# Three Pieces Of Paper

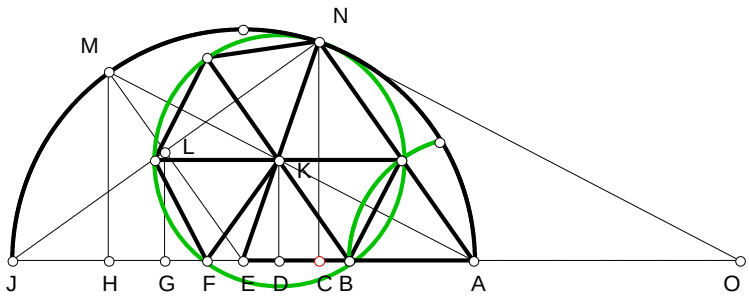
1993




John Clark

## 11/11/93 The Archamedian Paper Trisector

**If one accepts the facts of the original figure, one only need prove that BK = AB.**
**If one does not accept the facts, examination of the construction should make it apparent. Does FK = BK = AB?**




$N := 4$ $AJ := 1$ $AE := \frac{AJ}{2}$ $EJ := AE$ $EN := AE$ $EM := AE$ $AC := \frac{AJ}{N}$

$CJ := AJ - AC$ $CN := \sqrt{AC \cdot CJ}$ $JN := \sqrt{CN^2 + CJ^2}$ $JL := \frac{JN}{2}$ $GL := \frac{CN}{2}$ $GJ := \frac{CJ}{2}$

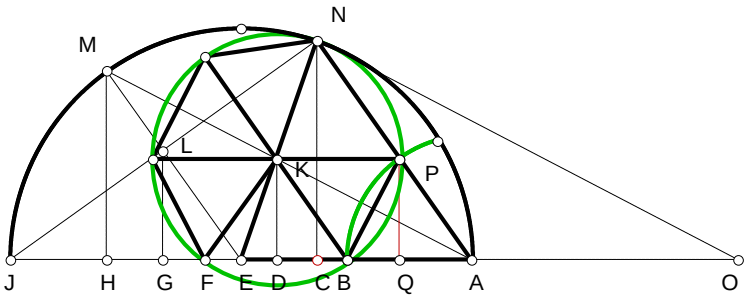
$EG := EJ - GJ$ $EL := \sqrt{EG^2 + GL^2}$ $EH := \frac{EG \cdot EM}{EL}$ $HM := \frac{GL \cdot EM}{EL}$ $AH := AE + EH$

$CO := \frac{AH \cdot CN}{HM}$ $CE := AE - AC$ $EO := CO + CE$ $EK := \frac{EN \cdot AE}{EO}$ $DE := \frac{CE \cdot AE}{EO}$ $DK := \frac{CN \cdot EK}{EN}$

$AD := AE - DE$ $KN := EN - EK$ $BK := KN$ $BD := \sqrt{BK^2 - DK^2}$ $AB := AD - BD$

$AB - BK = 0$ $AB = 0.25$

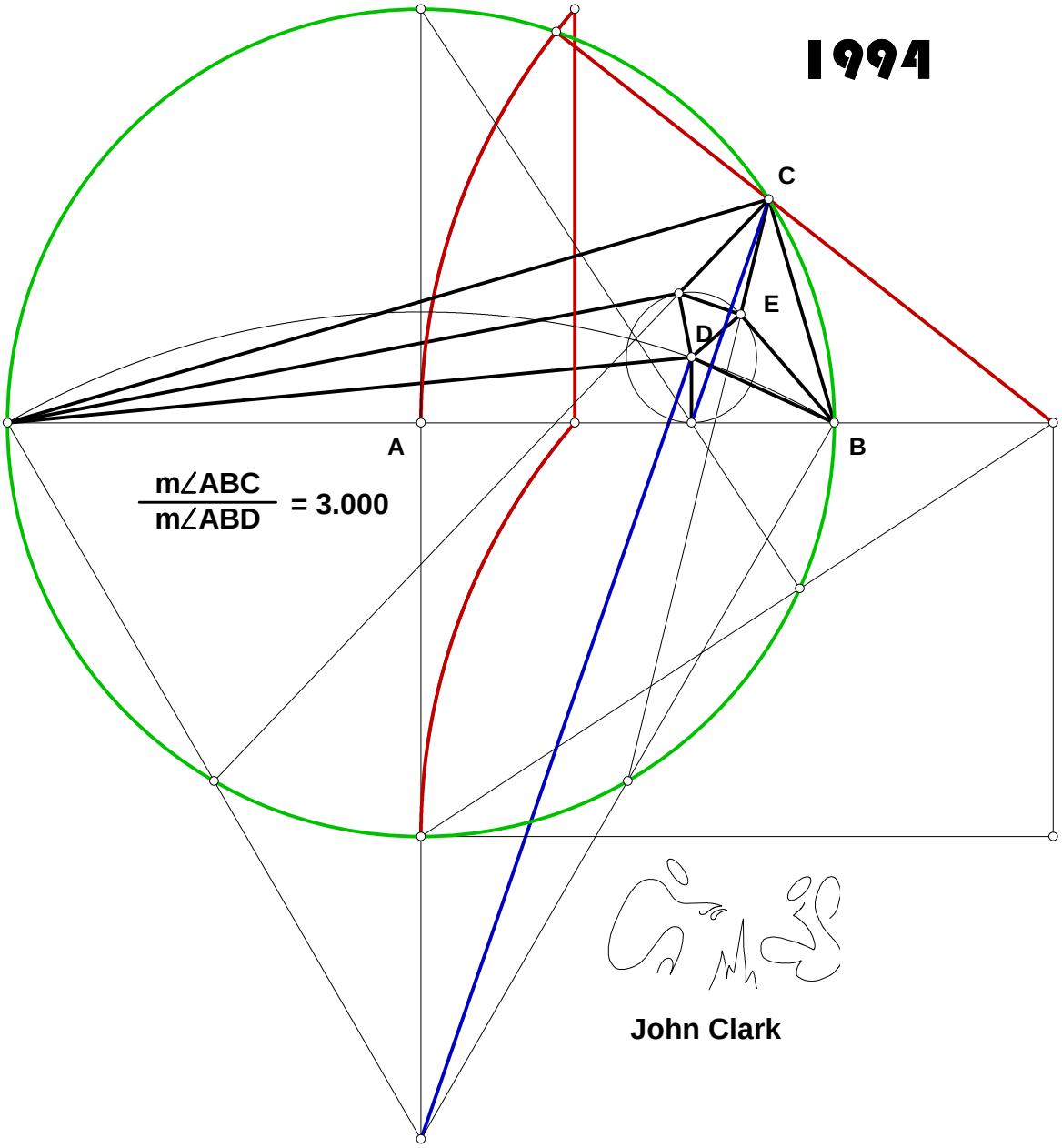
**If PK is parallel to AJ, then . . . .**




$AN := 2 \cdot EL$ $AP := AB$ $PQ := \frac{CN \cdot AP}{AN}$ $PQ - DK = 0$

# Three Pieces Of Paper

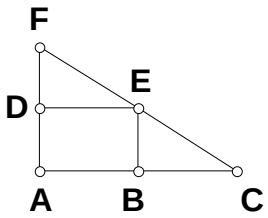
1994




John Clark

# 042994

## Given AB, AF, BE, what is EF?



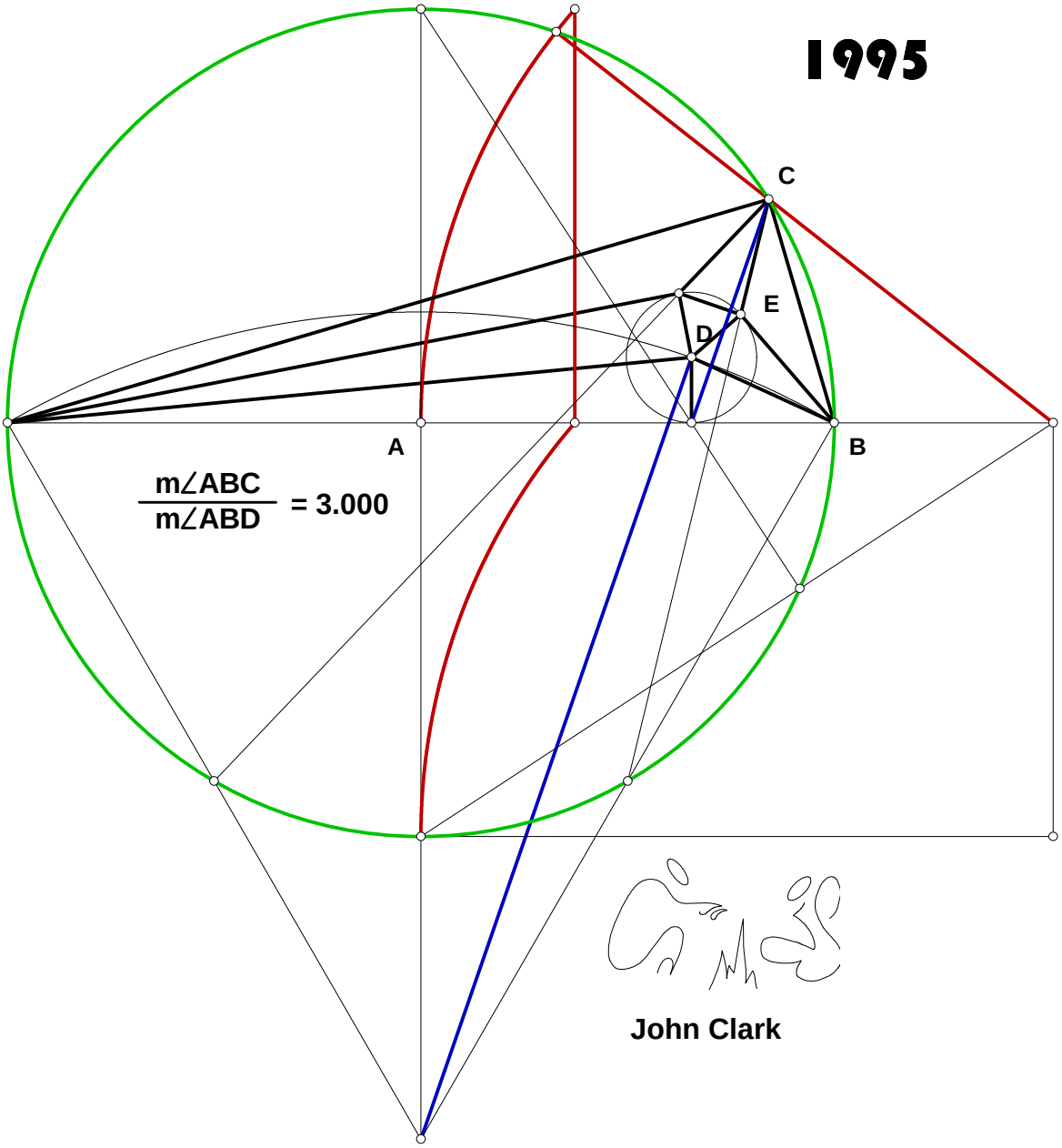

$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

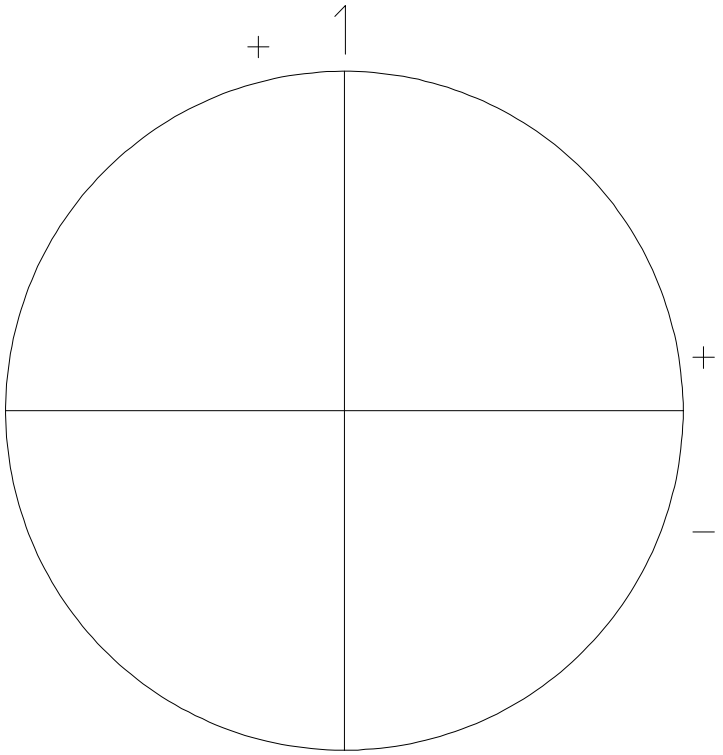
$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$

# Three Pieces Of Paper

1995




John Clark

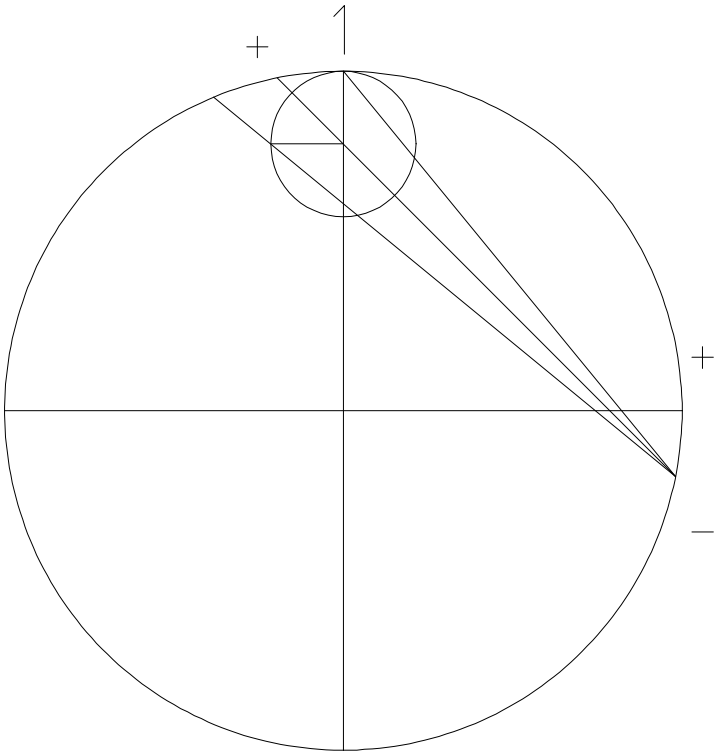



**01_12_95.MCD**

**Archamedian Trisection Revisited.**

**I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.**

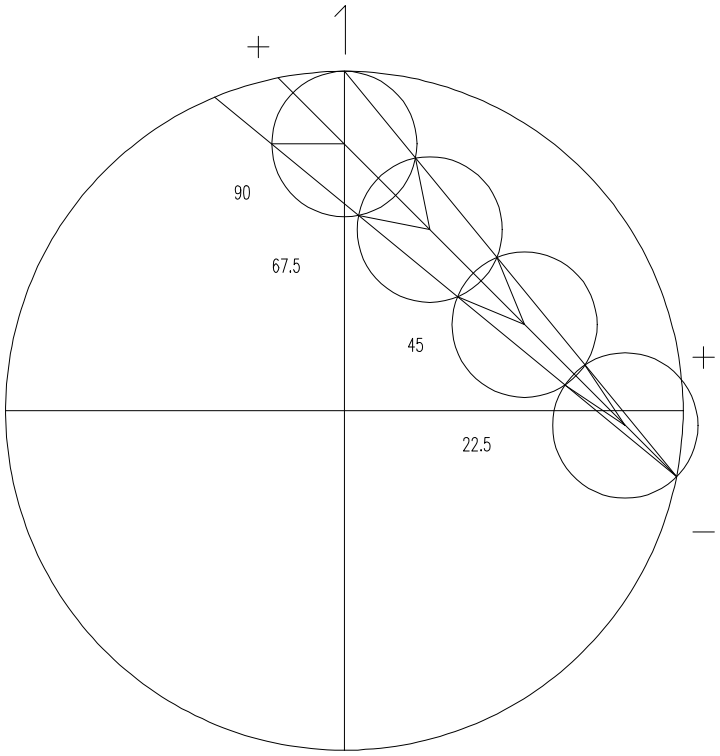
**I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.**




**The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.**

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$




$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

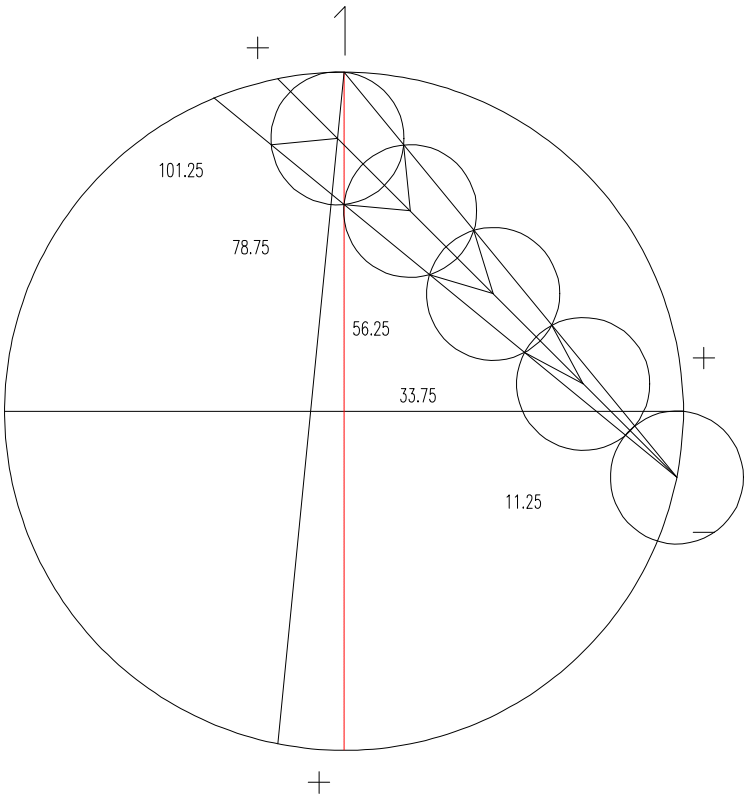
$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$$\mathrm{mod}(8 + 1 - 1, 2) = 0$$

**I have added another plus to a quadrant at the bottom of the figure.**




$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

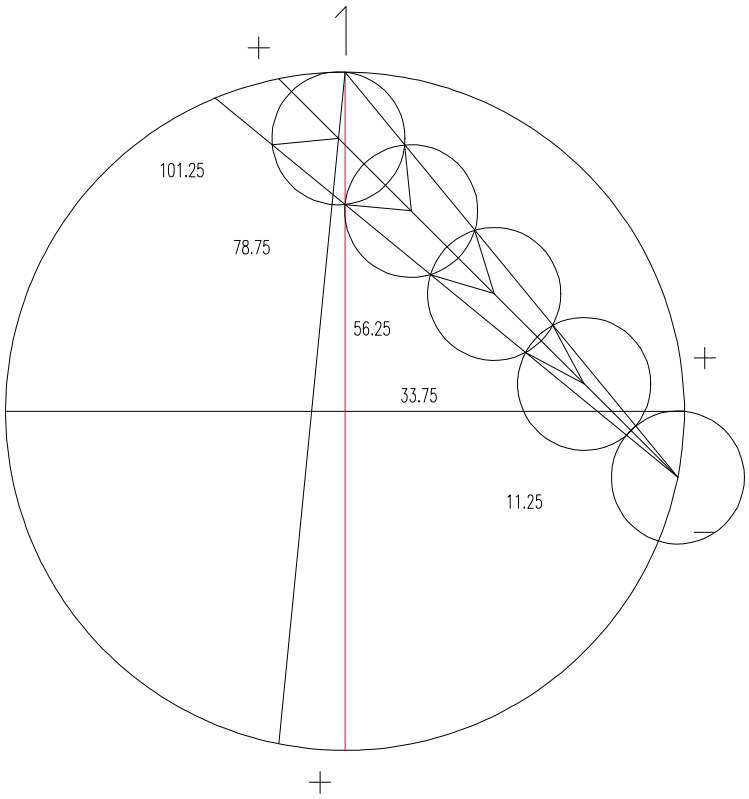
$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$$




$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

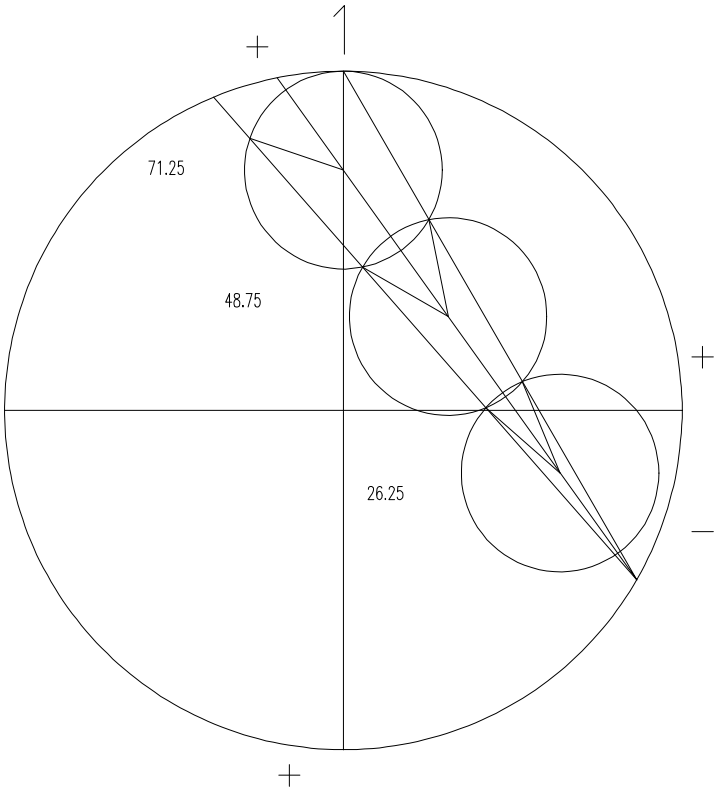
$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$$\mathbf{mod}(8 + 1, 2) = 1$$




$$B := 1 + \frac{3}{24} - \frac{8}{24} \qquad B = 0.7917 \qquad \frac{19}{24} = 0.7917$$

$$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25 \qquad \frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$$

$$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499 \qquad \frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$$

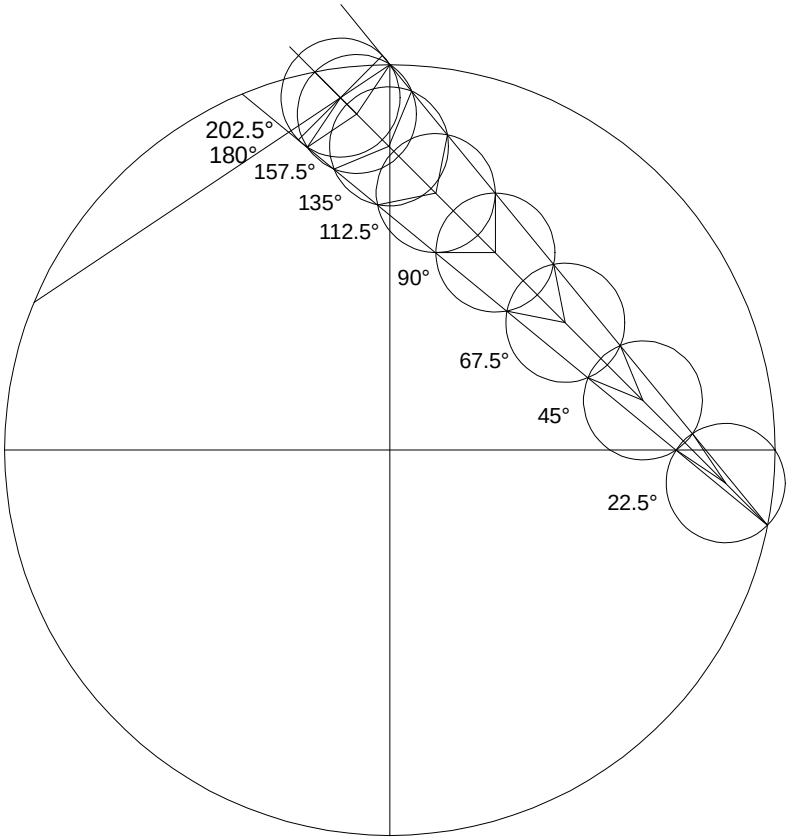
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$$\mathbf{mod}(24 + 3 - 8, 2) = 1$$

$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8} \qquad B = 2.25 \qquad \frac{18}{8} = 2.25$$

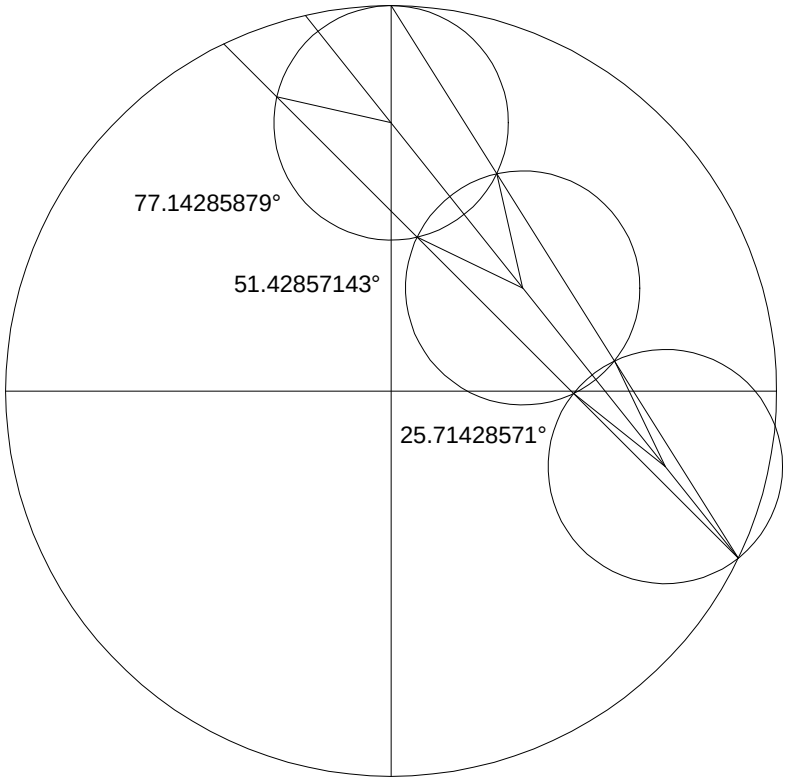
$$\frac{B \cdot 9}{9} \cdot 90 = 202.5 \qquad \frac{B \cdot 8}{9} \cdot 90 = 180$$

$$\frac{B \cdot 7}{9} \cdot 90 = 157.5 \qquad \frac{B \cdot .6}{9} \cdot 90 = 13.5$$




| | |
|---|---|
| $8 + 1 - 1 + 10 = 18$ | $18 \cdot 11.25 = 202.5$ |
| $8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ | $16 \cdot 11.25 = 180$ |
| $8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ | $14 \cdot 11.25 = 157.5$ |
| $8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ | $12 \cdot 11.25 = 135$ |
| $8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ | $10 \cdot 11.25 = 112.5$ |
| $8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ | $2 \cdot 11.25 = 22.5$ |

$$\mathbf{mod}((8 + 1 - 1) + 10, 2) = 0$$




$$B := 1 + \frac{1}{7} - \frac{2}{7} \qquad B = 0.8571 \qquad \frac{6}{7} = 0.8571$$

$$\frac{B \cdot 6}{6} \cdot 90 = 77.1429 \qquad \frac{B \cdot 4}{6} \cdot 90 = 51.4286$$

$$\frac{B \cdot 2}{6} \cdot 90 = 25.7143 \qquad c := \frac{90}{7}$$

| | |
|---|---|
| $7 + 1 - (1 \cdot 2) = 6$ | $6 \cdot c = 77.1429$ |
| $7 + 1 - (2 \cdot 2) = 4$ | $4 \cdot c = 51.4286$ |
| $7 + 1 - (3 \cdot 2) = 2$ | $2 \cdot c = 25.7143$ |

$$\mathbf{mod}(7 + 1 - 2, 2) = 0$$

$$B := 1 + \frac{1}{7} - \frac{1}{7} \qquad B = 1 \qquad \frac{7}{7} = 1$$

$$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.2857$$

$$\frac{B \cdot 3}{7} \cdot 90 = 38.5714 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.8571$$

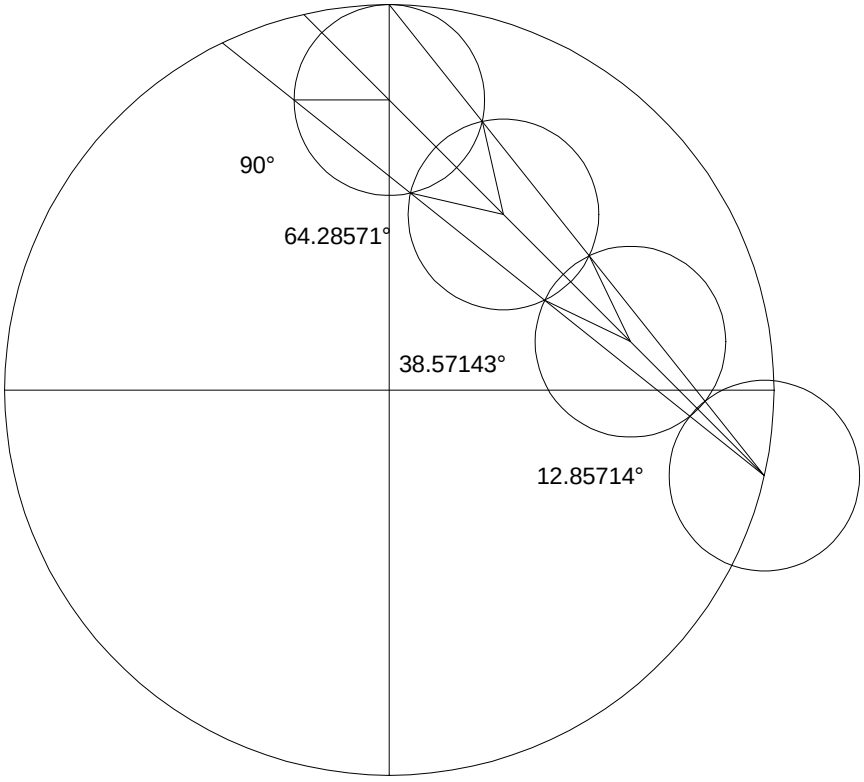
$7 + 1 - 1 = 7$ $\quad 7 \cdot c = 90$

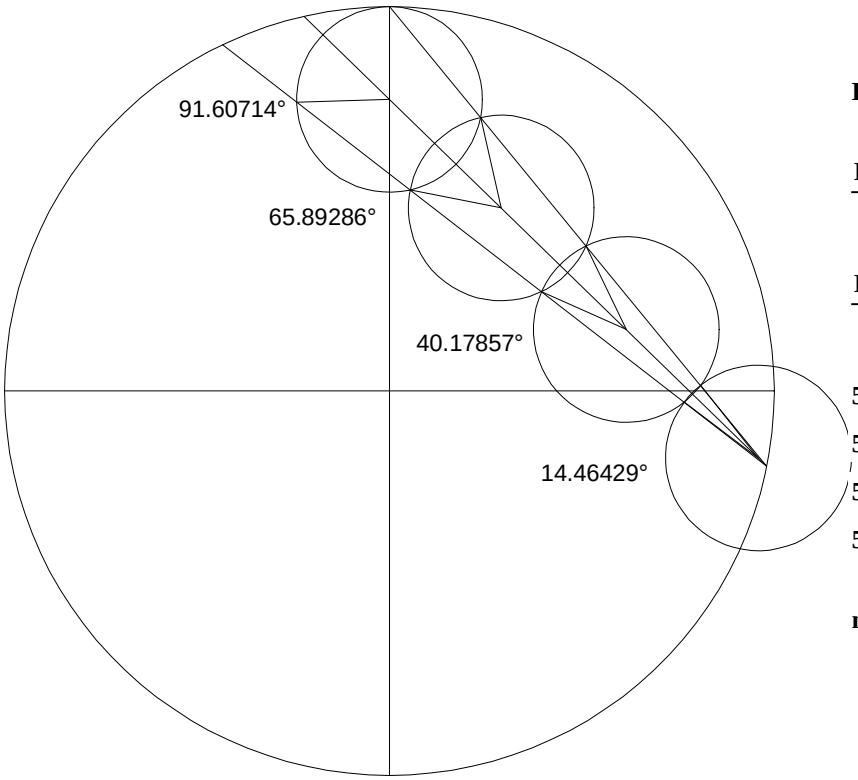
$7 + 1 - 1 - (1 \cdot 2) = 5$ $\quad 5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $\quad 3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $\quad 1 \cdot c = 12.8571$

$\mathbf{mod}(7 + 1 - 1, 2) = 1$




$$B := 1 + \frac{8}{56} - \frac{7}{56} \qquad B = 1.0179$$

$$\frac{B \cdot 57}{57} \cdot 90 = 91.6071 \qquad \frac{B \cdot 41}{57} \cdot 90 = 65.8929$$

$$\frac{B \cdot 25}{57} \cdot 90 = 40.1786 \qquad c := \frac{90}{56}$$

$56 + 8 - 7 = 57$ $\quad 57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $\quad 41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $\quad 25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $\quad 9 \cdot c = 14.4643$

$\mathbf{mod}(56 + 8 - 7, 16) = 9$

$$B := 1 + \frac{1}{7} - \frac{1}{7} \qquad B = 1$$

$$\frac{B \cdot 7}{7} \cdot 90 = 90 \qquad \frac{B \cdot 5}{7} \cdot 90 = 64.2857$$

$$\frac{B \cdot 3}{7} \cdot 90 = 38.5714 \qquad \frac{B \cdot 1}{7} \cdot 90 = 12.8571$$

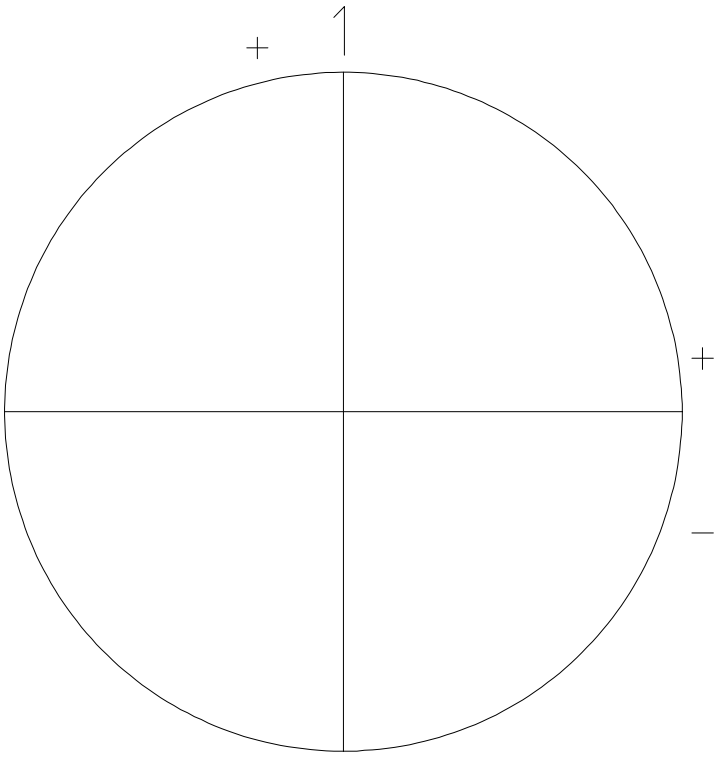
$$7 + 1 - 1 = 7$$

$$7 + 1 - 1 - (1 \cdot 2) = 5$$

$$7 + 1 - 1 - (2 \cdot 2) = 3$$

$$7 + 1 - 1 - 2 - 2 - 2 = 1$$

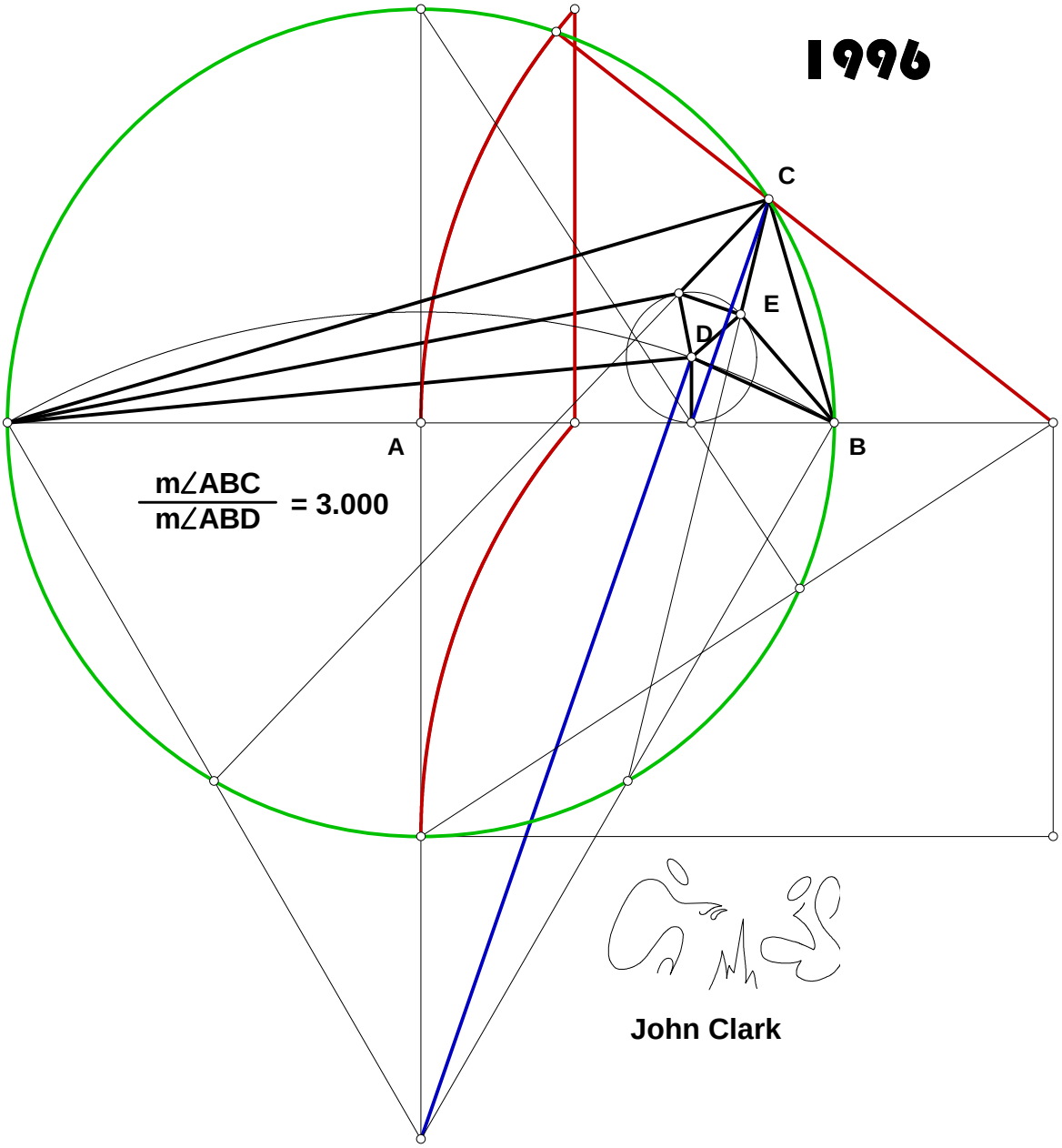
$$\mathbf{mod}(7 + 1 - 1, 2) = 1$$




**Work in progress.**

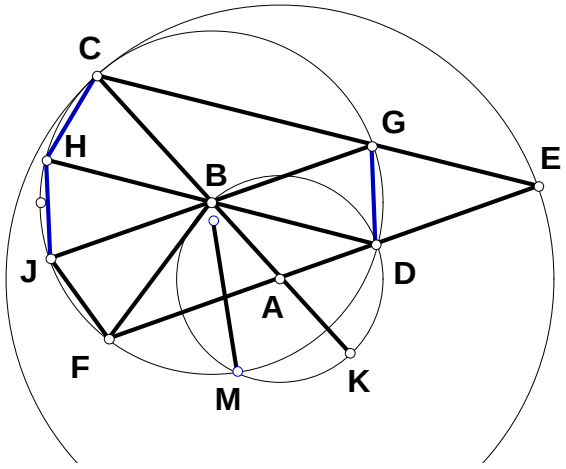
# Three Pieces Of Paper

1996



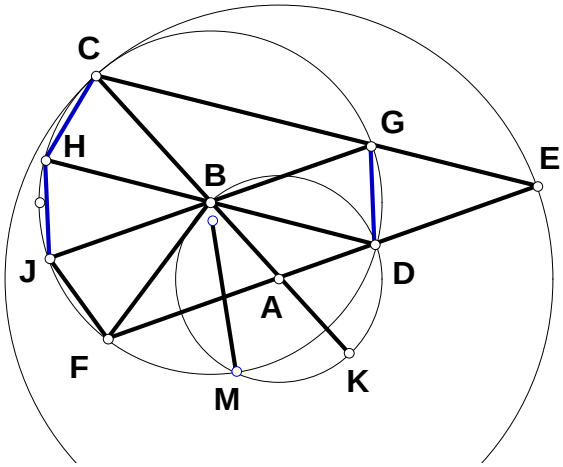

John Clark

## 010496 The Archamedian Paper Trisector- Without the Numbers.



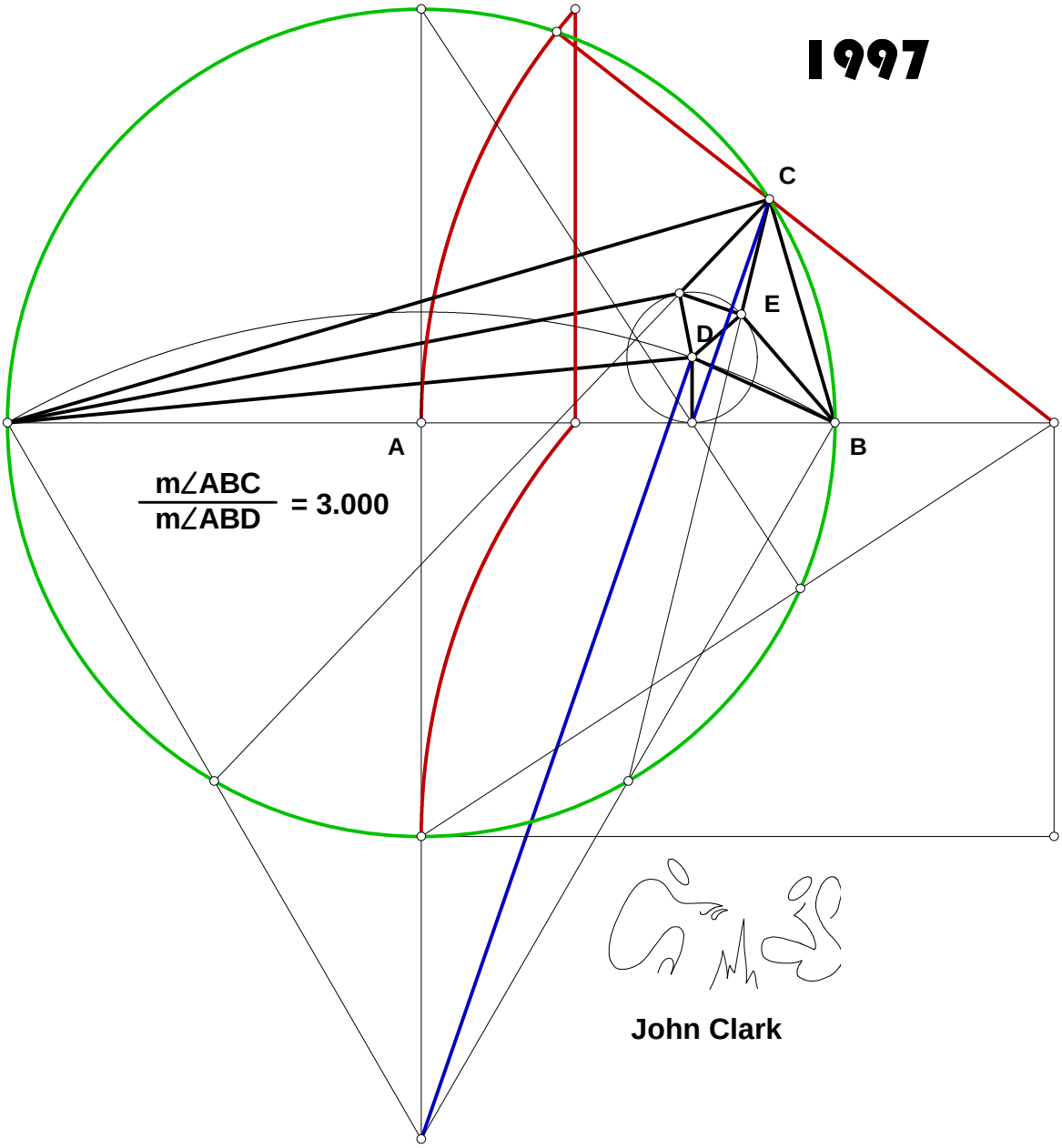

**Given any circle AB.**
**Given any circle BC such that BC ≤ 2AB.**
**Construct AE such that AE = AC.**
**As AC = AB + BC**
**and AD = AB so too DE = BC.**
**Construct DH parallel to BD.**
**Construct CE.**
**As AB = AD and AC = AE,**
**Δ ABD is proportional to Δ ACE,**
**therefore CE is parallel to BD.**
**From here one can take two paths.**
**Construct GJ parallel to EF.**
**As CE is parallel to DH,**
**DG = CH.**
**As GJ is parallel to EF,**
**DG = FJ.**

**As ∠ HBJ is opposite**
**and equal to ∠ GBD,**
**DG = HJ,**
**therefore ∠ DG is $\frac{1}{3}$ CF.**
**As CE is parallel to DH**
**and EF is parallel to GJ,**
**EGBD is equilateral.**
**By construction DK = KM.**
**As DH is parallel to CE,**
**CH = DG.**
**As DK is equal and opposite CH,**
**MK + DK + DG is $\frac{1}{3}$ DG.**



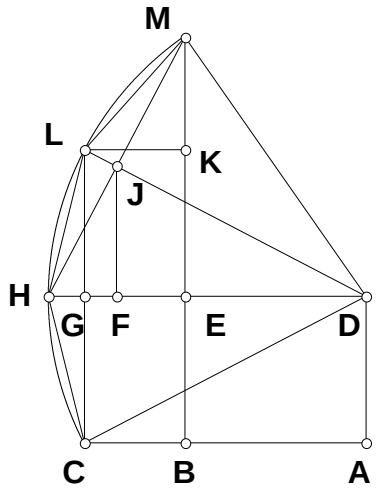
# Three Pieces Of Paper

1997




John Clark

# 042897




$N := 1.458 \quad AD := 1 \quad AC := AD \cdot N \quad N := N$

$CD := \sqrt{AD^2 + AC^2} \quad DH := CD$

$CG := AD \quad DG := AC \quad GH := DH - DG \quad CH := \sqrt{GH^2 + CG^2}$

$HJ := CG \quad DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH} \quad EF := FH \quad DE := DH - (EF + FH)$

$AB := DE \quad EG := DG - DE \quad LM := CH \quad LK := EG$

$KM := \sqrt{LM^2 - LK^2} \quad BE := AD \quad BK := 2 \cdot BE \quad BM := BK + KM$

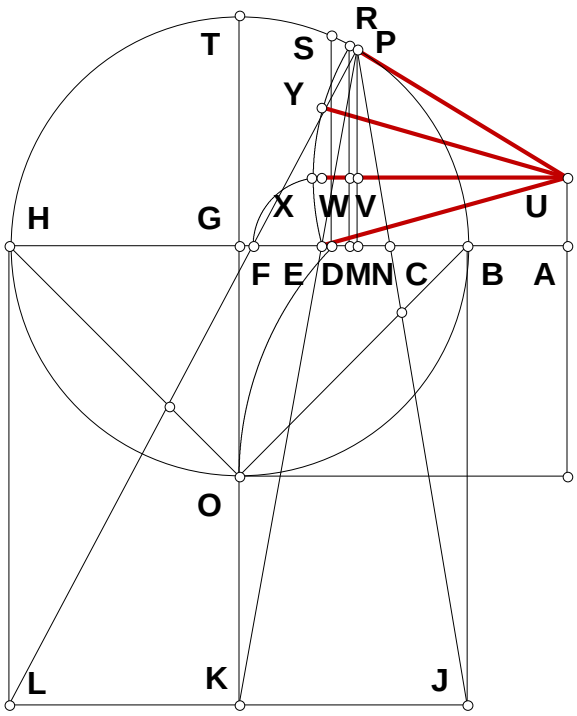
## Some Algebraic Names:

$\sqrt{N^2 + 1} - CD = 0 \quad N - AC = 0 \quad \sqrt{N^2 + 1} - N - GH = 0 \quad \sqrt{2 \cdot N^2 - 2 \cdot \sqrt{N^2 + 1} \cdot N + 2} - CH = 0$

$\frac{1}{\sqrt{N^2 + 1}} - FH = 0 \quad \frac{(N - 1) \cdot (N + 1)}{\sqrt{N^2 + 1}} - AB = 0 \quad 2 + \sqrt{\frac{5 \cdot N^2}{N^2 + 1} - \frac{4 \cdot N}{\sqrt{N^2 + 1}} + \frac{1}{N^2 + 1}} - BM = 0$

# Trisection and the Cube Roots 042997

**If trisection can be placed at RUE, then PV is proportional to RW.**




$N := 5 \qquad AB := 1 \qquad AH := AB \cdot N$

$BH := AH - AB \quad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2}$

$AE := AC + CE$

$AU := CE \quad NV := AU \quad MW := AU$

**(For the next two equations see 042897.)**

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

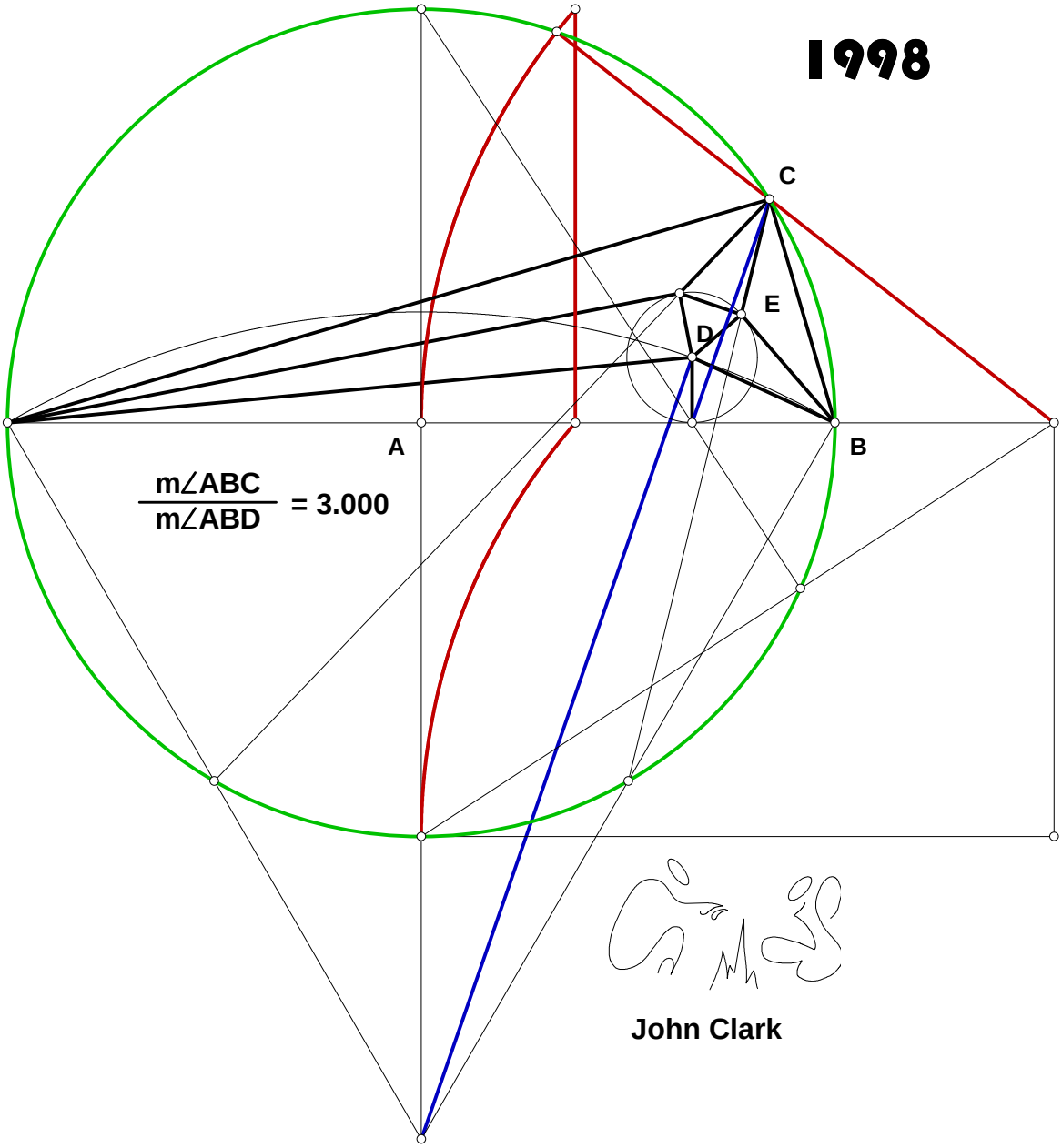
$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad RW := MR - MW$$

$BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH} \qquad NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV$

$AN := AC + CN \qquad UV := AN \quad UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

# Three Pieces Of Paper

1998




John Clark

m∠OKL = 30.872°

m∠HAQ = 20.581°

m∠OTK = 15.436°

m∠IJV = 10.291°

m∠IBJ = 5.145°

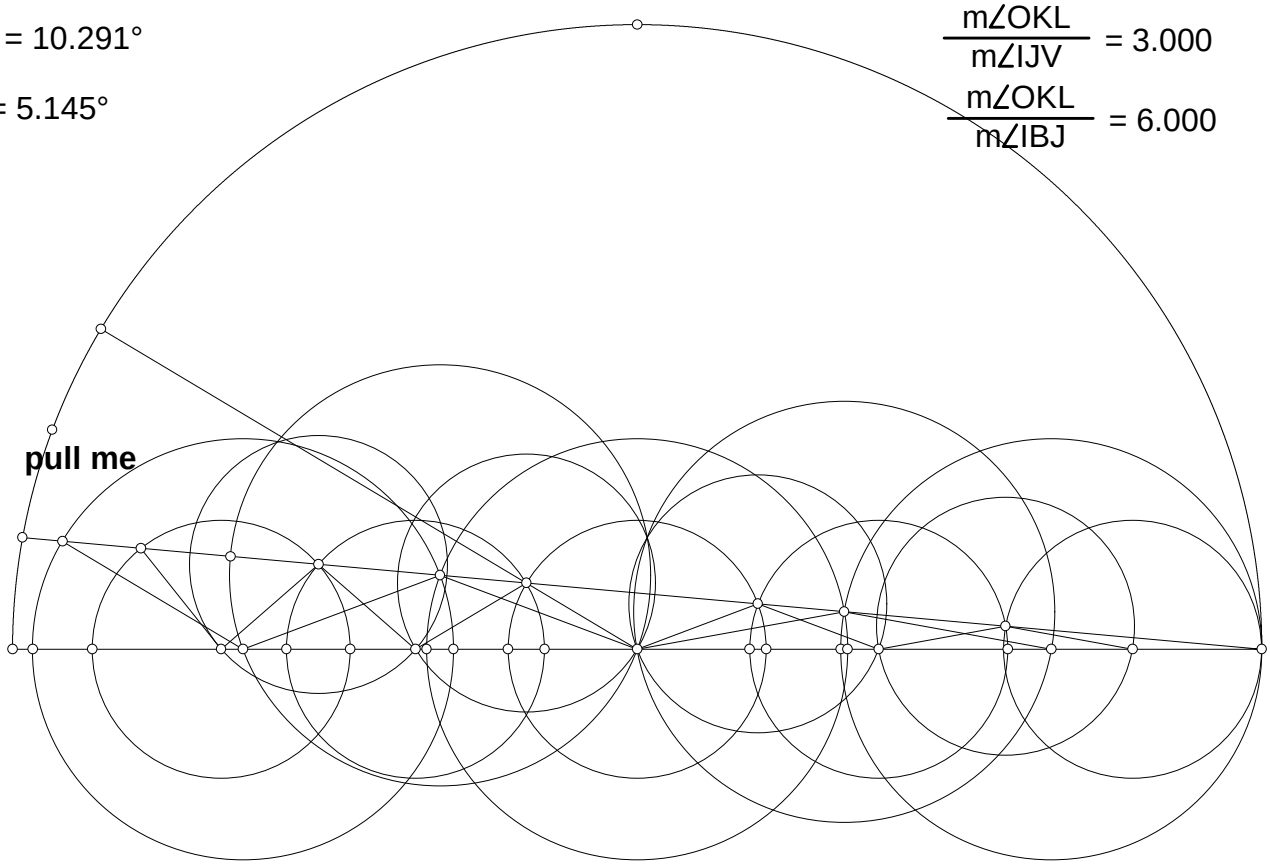
▲ **Show** △ **Hide**

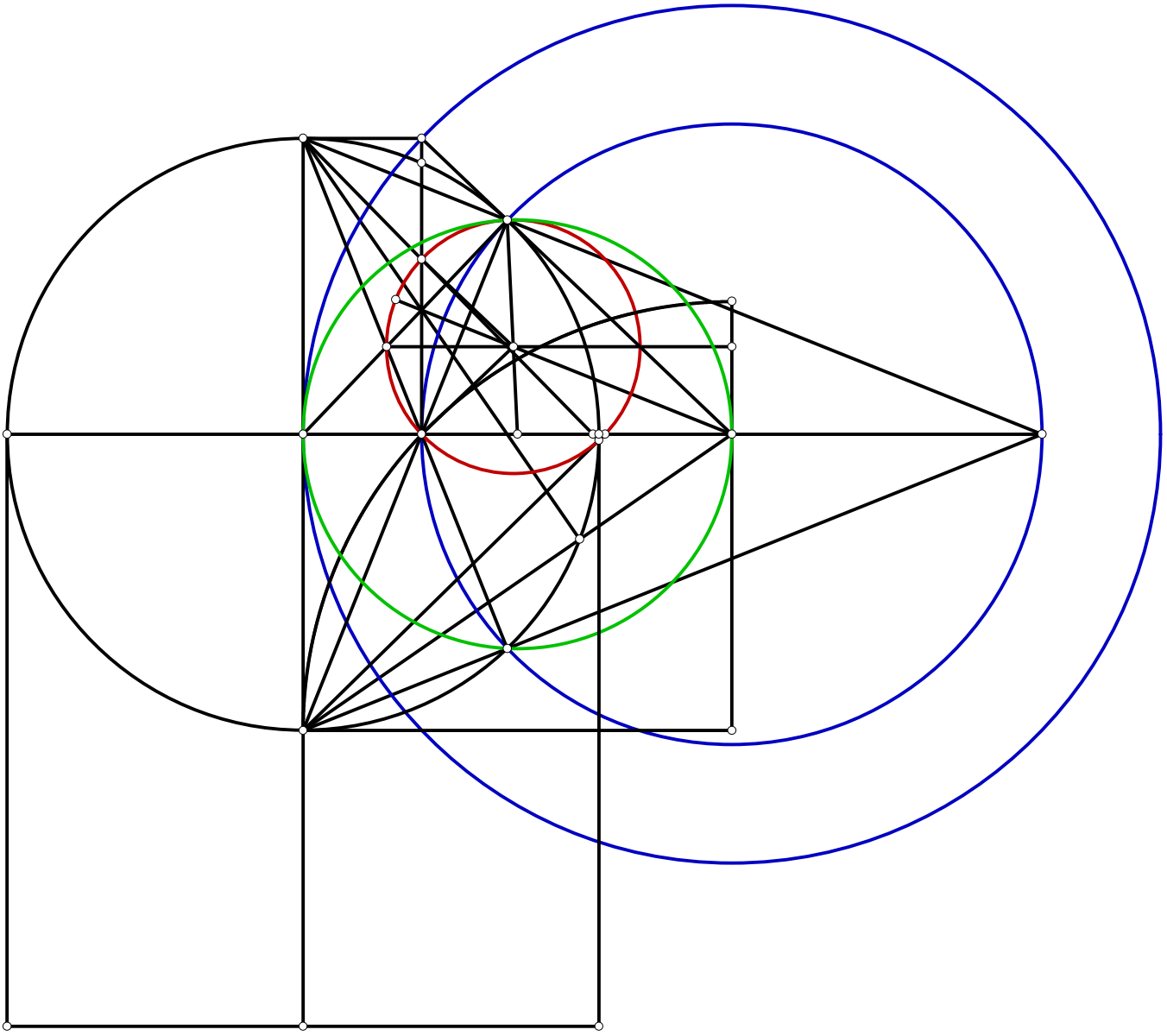
$\frac{m\angle OKL}{m\angle HAQ} = 1.500$

$\frac{m\angle OKL}{m\angle OTK} = 2.000$

$\frac{m\angle OKL}{m\angle IJV} = 3.000$

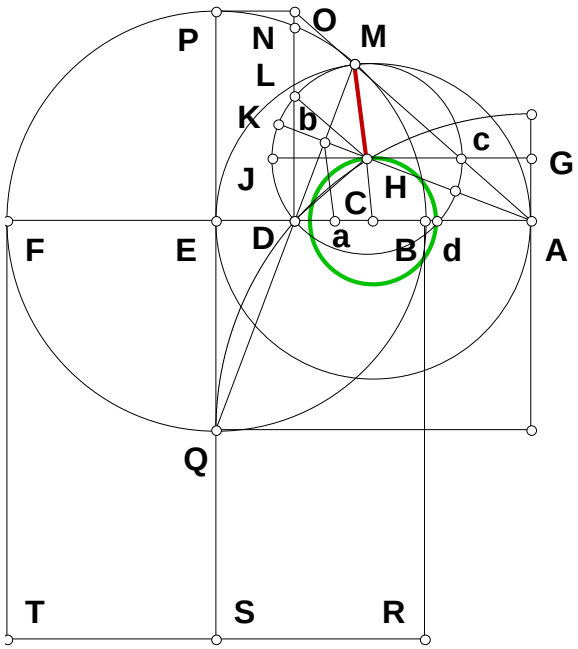
$\frac{m\angle OKL}{m\angle IBJ} = 6.000$





# A Square Root Figure And Triseciton 042398

**In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?**




$N := 5 \quad AB := 1 \quad AF := AB \cdot N \quad BF := AF - AB$

$AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2} \quad BD := AD - AB$

$DE := BE - BD \quad EQ := BE \quad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}}$

$PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ} \quad DM := QM - DQ$

$AE := AB + BE \quad AC := \frac{AE}{2} \quad Db := \frac{DM}{2}$

$CM := AC \quad ab := \frac{CM \cdot Db}{DM} \quad CD := AD - AC$

$Ca := \frac{CD}{2} \quad Aa := AC + Ca \quad CH := \frac{ab \cdot AC}{Aa}$

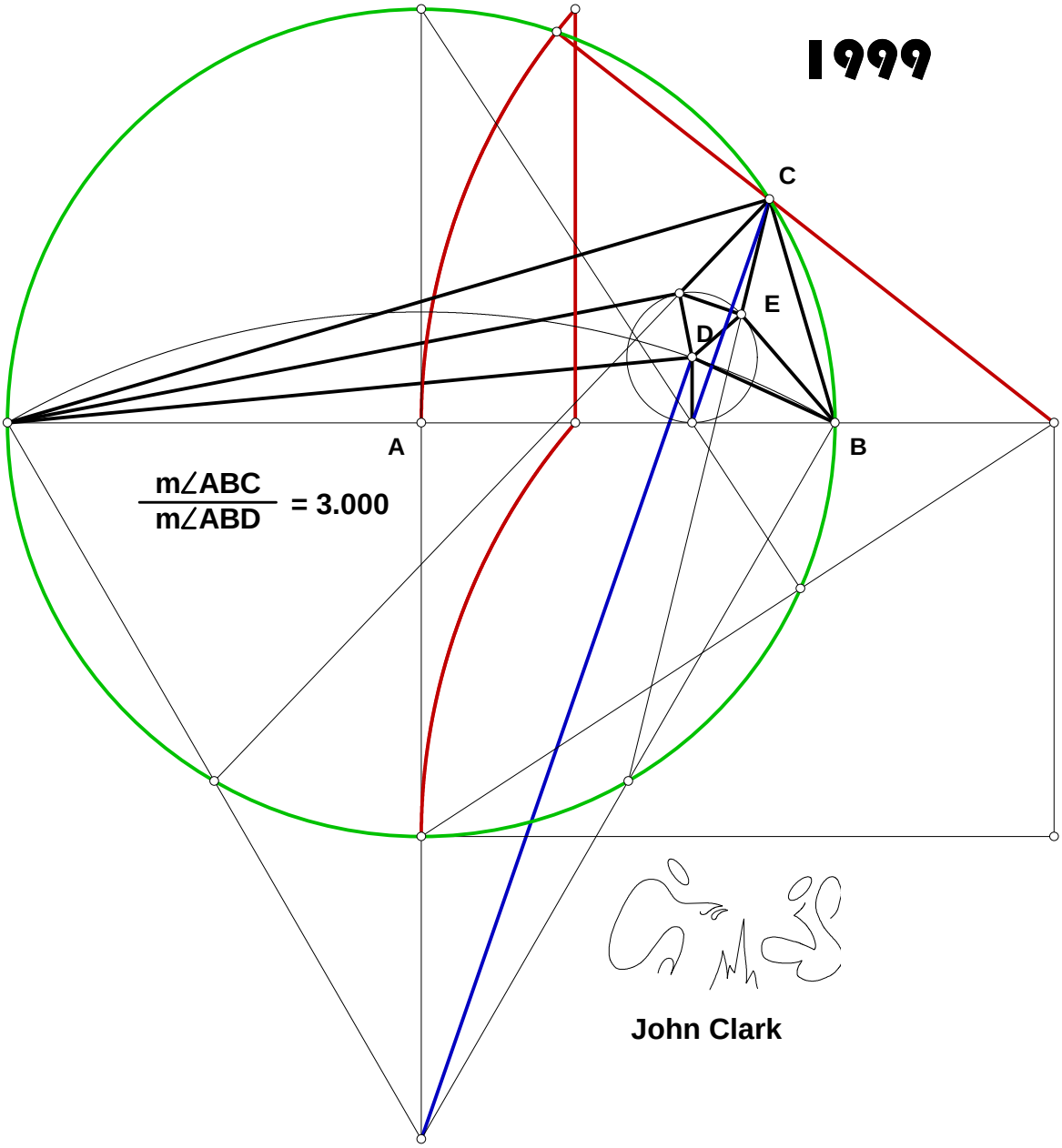
$AM := AD \quad Ac := \frac{AM \cdot CH}{CM} \quad HM := CM - CH$

$HM - Ac = 0$

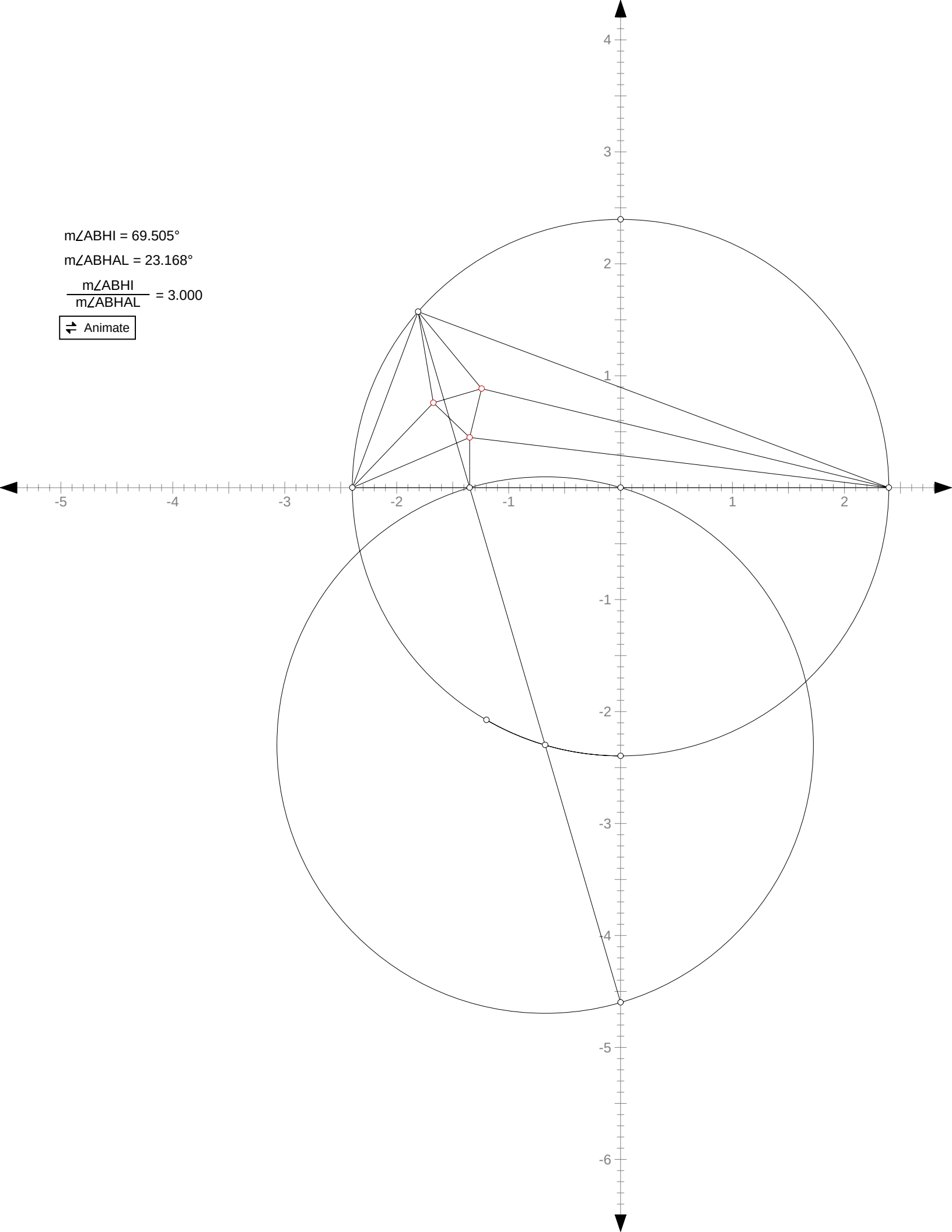
$$HM - AB \cdot \frac{\sqrt{N} \cdot (N+1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \qquad CH - AB \cdot \frac{(N+1)^2}{4 \cdot N + 16 \cdot \sqrt{N} + 4} = 0$$

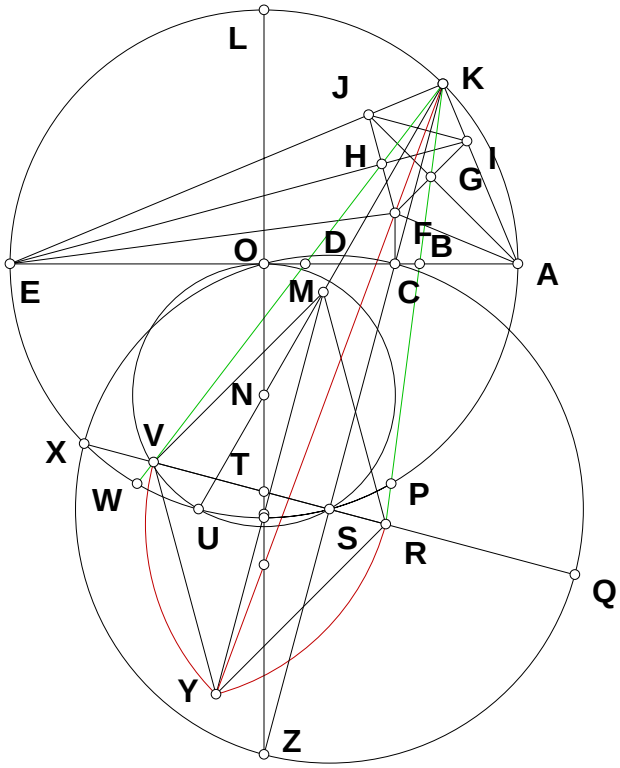
# Three Pieces Of Paper

1999




John Clark


m∠ABHI = 69.505°
m∠ABHAL = 23.168°
m∠ABHI / m∠ABHAL = 3.000
Animate
4
3
2
1
-1
-2
-3
-4
-5
-6
-5
-4
-3
-2
-1
1
2


L
K
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H
I
G
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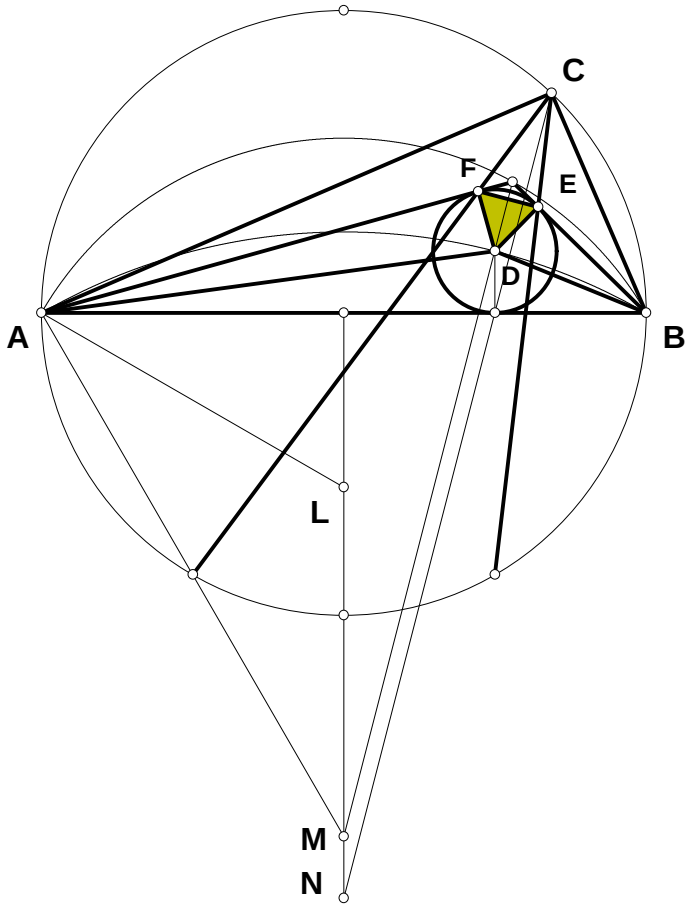
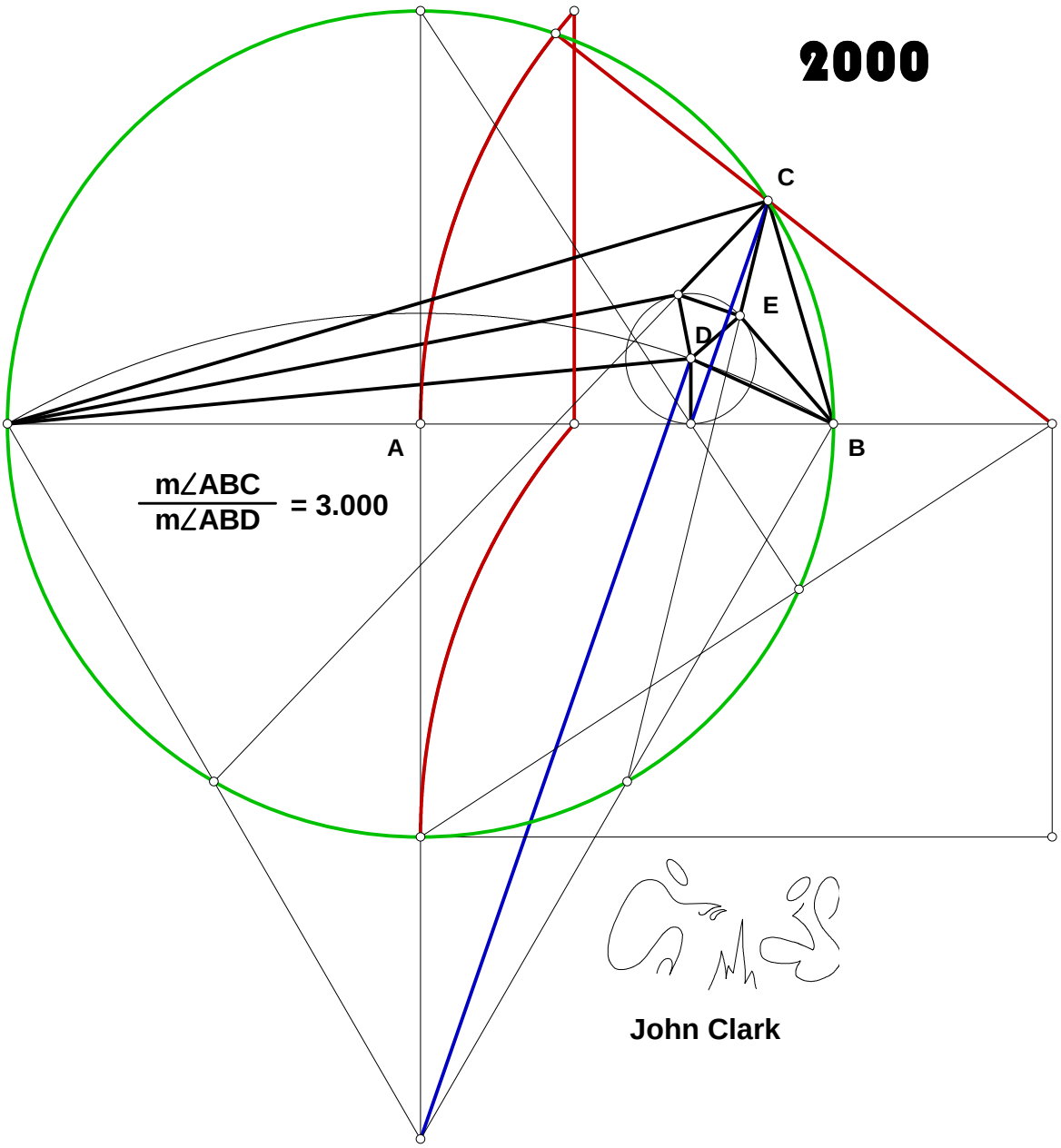
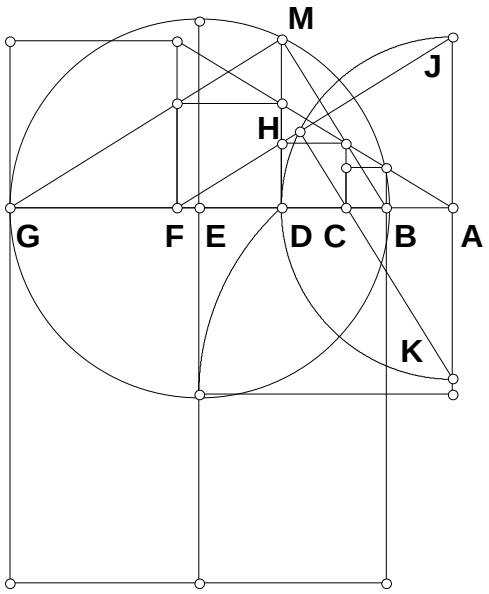

C
F
E
D
A
B
L
M
N

# Three Pieces Of Paper




John Clark

# 07/09/00 Alternate Method Quad Roots




$N := 3 \quad AB := 1 \quad AG := AB \cdot N \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

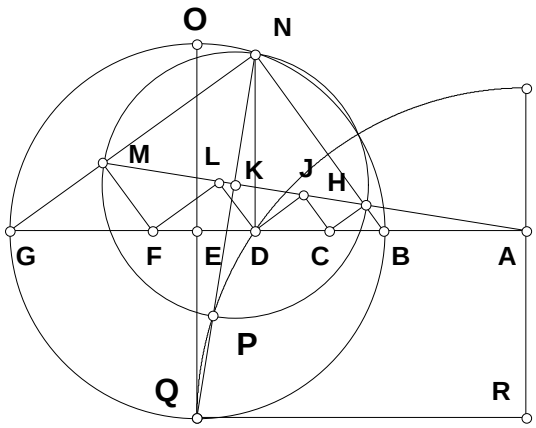
$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \quad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

# 08/01/00 Alternate Method Quad Roots




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

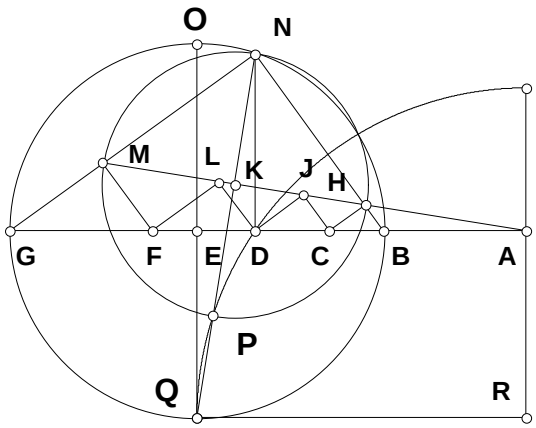
$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}} \qquad BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

# 08/01/00B Alternate Method Quad Roots




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

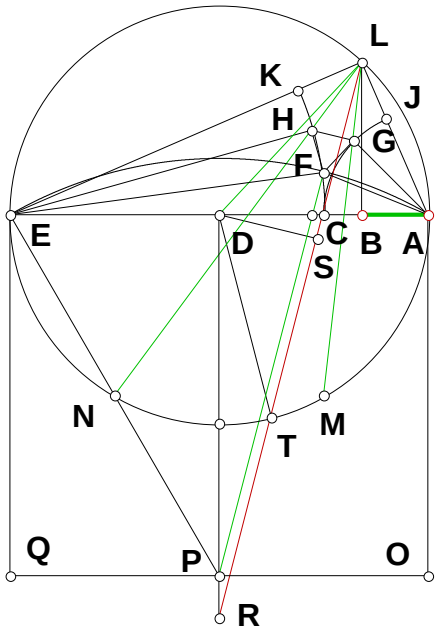
$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2} \qquad AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2} \qquad GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN}$

$AF := AG - GF \qquad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

# In Trisection What Is AB? 08/02/00




**In the trisection figure given and given AC as the Unit what is AB?**

$AC := .884 \quad AE := 3.521 \quad AD := \frac{AE}{2}$

$EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2} \quad FP := EP$

$CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF \quad DR := DP + PR \quad CR := \sqrt{CD^2 + DR^2}$

$CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD$

$LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS \quad LR := RS + LS$

$BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$

**In trisection the length RT to the similarity point is equal to the radius of the circle.**

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

$$AD - \frac{AE}{2} = 0 \qquad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \qquad CF - \left[\frac{1}{2} \cdot \sqrt{-(AE + 2 \cdot AC) \cdot (-3 \cdot AE + 2 \cdot AC)} - \frac{1}{2} \cdot AE \cdot \sqrt{3}\right] = 0$$

$$CE - (AE - AC) = 0 \qquad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0 \qquad DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0$$

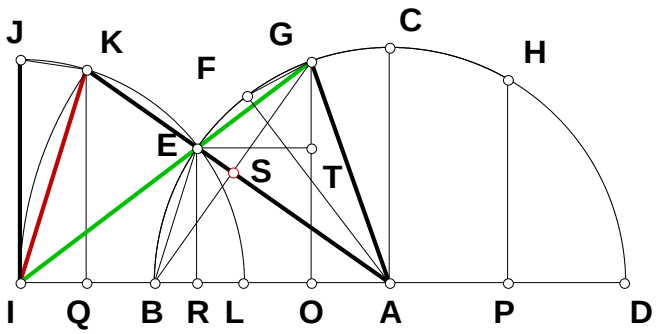
$$CR - AE = 0 \qquad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \qquad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0$$

$$RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0 \qquad BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0$$

$$AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \qquad AB \cdot AE^2 - AC^2 (3 \cdot AE - 2 \cdot AC) = 0$$

# 080300 Trisection




**If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yeild a trisected figure JKL.**

$N := 3 \qquad BD := 2$

$AB := \frac{BD}{2} \quad AD := AB \quad AP := \frac{AD}{2}$

$BP := AB + AP \quad BO := \frac{BP}{N} \quad AE := AB$

$DO := BD - BO \quad GO := \sqrt{BO \cdot DO}$

$BG := \sqrt{GO^2 + BO^2} \quad BS := \frac{BG}{2} \quad ER := BS \quad TO := ER \quad GT := GO - TO \quad AS := \sqrt{AB^2 - BS^2}$

$ES := AE - AS \quad BR := ES \quad OR := BO - BR \quad ET := OR \quad IO := \frac{ET \cdot GO}{GT} \quad BI := IO - BO$

$AI := BI + AB \quad BE := \sqrt{ER^2 + BR^2} \quad GE := BE \quad GI := \frac{GE \cdot GO}{GT} \quad EI := GI - GE \quad AK := AI \quad IK := EI$

$IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI} \quad EK := AK - AE \quad \frac{EK}{IQ} = 2$

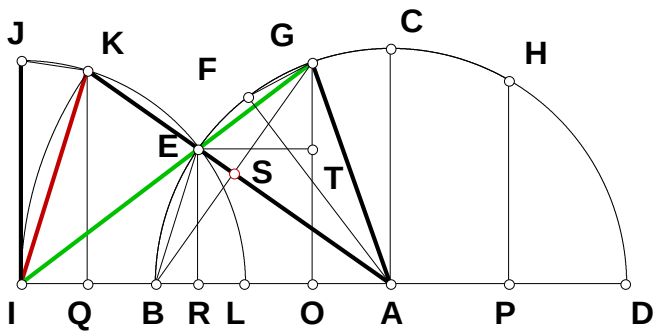
## Some Algebraic Names:

$$\frac{3 \cdot BD}{4 \cdot N} - BO = 0 \qquad \frac{BD}{4} \cdot \frac{(4 \cdot N - 3)}{N} - DO = 0 \qquad \frac{BD}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0 \qquad \frac{BD}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0$$

$$\frac{BD}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0 \qquad \frac{BD}{4} \cdot \sqrt{3} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - \sqrt{\frac{1}{N}} \cdot N\right)}{N} - GT = 0 \qquad \frac{BD}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0$$

$$\frac{BD}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N - 3)}{N}}\right] - ES = 0 \qquad \frac{BD}{4} \cdot \frac{\left[3 - 2 \cdot N + \sqrt{\frac{(4 \cdot N - 3)}{N}} \cdot N\right]}{N} - OR = 0$$

$$\frac{-BD}{4} \cdot \frac{\left[3 \cdot \sqrt{4 \cdot N - 3} \cdot \sqrt{N} - 2 \cdot \sqrt{4 \cdot N - 3} \cdot N^{\left(\frac{3}{2}\right)} + 4 \cdot N^2 - 3 \cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)} \cdot \left(-\sqrt{4 \cdot N - 3} + \sqrt{N}\right)\right]} - IO = 0 \qquad \frac{-BD}{2} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - 2 \cdot \sqrt{N}\right)}{\left(\sqrt{4 \cdot N - 3} - \sqrt{N}\right)} - BI = 0$$



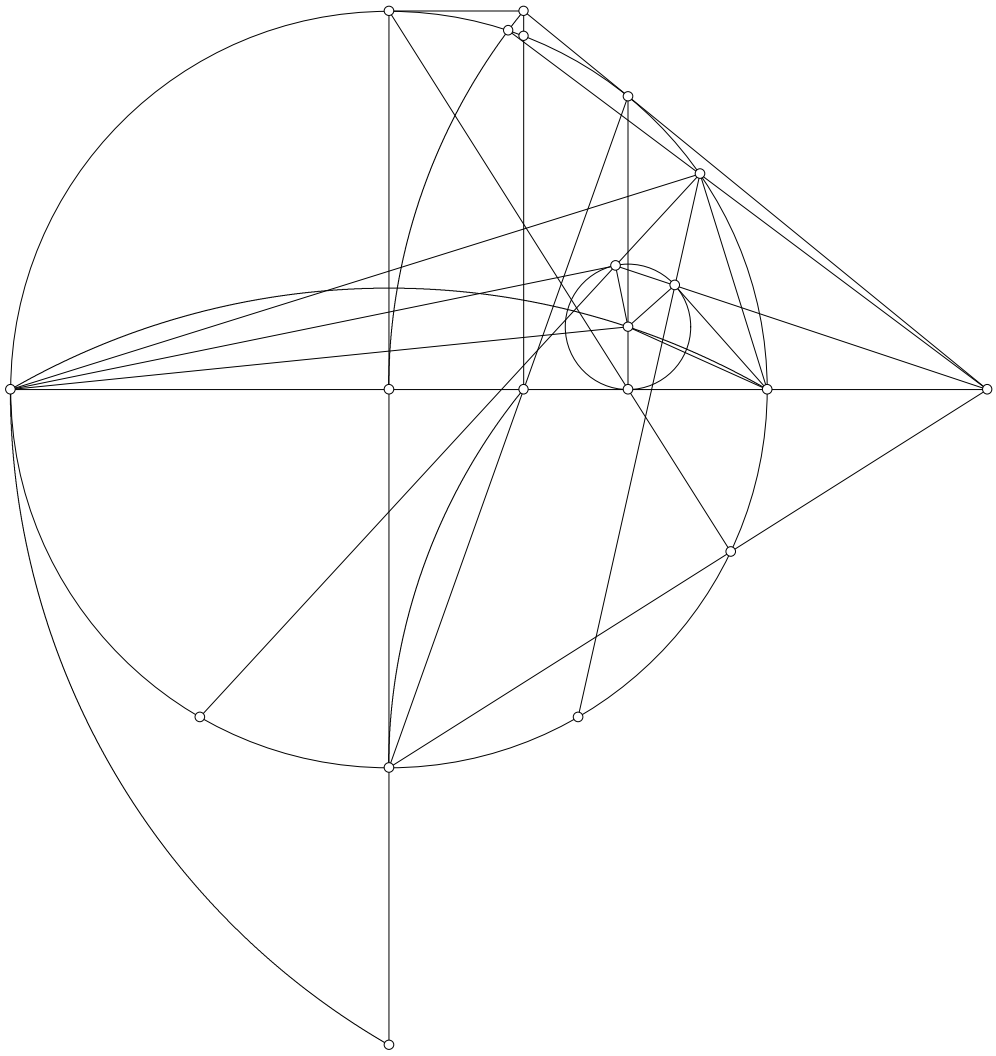

$$\frac{-BD}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

$$\frac{BD}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

$$\frac{BD}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

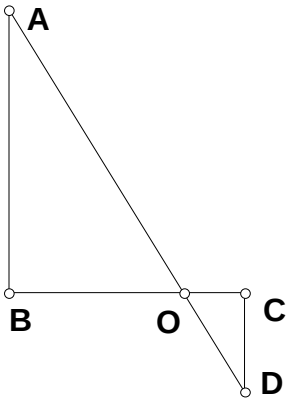
$$\frac{-BD}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

$$\frac{BD}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0\qquad\frac{BD}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$



## 08/07/00 Proportion Series II

**Two unknowns have the same proportion as two givens and the sum of the unknowns are known. Find the two unknowns.**




$AB := 9 \qquad CD := 3 \qquad BC := 5$

$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$

$BO = 3.75 \qquad CO = 1.25$

$BO + CO - BC = 0$

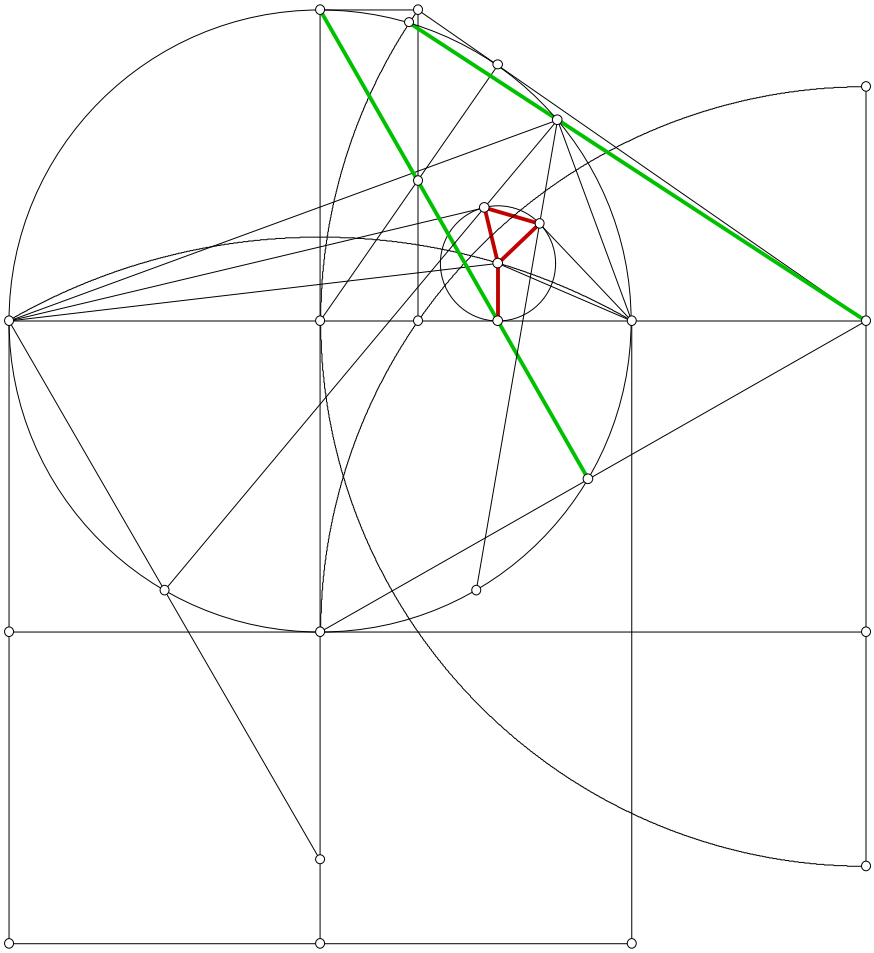
$\frac{AB}{CD} - \frac{BO}{CO} = 0$

m∠UDG = 69.867°

m∠UDCI = 23.289°

$\frac{m\angle UDG}{m\angle UDCI} = 3.000$

# Trisection and the square root figure.

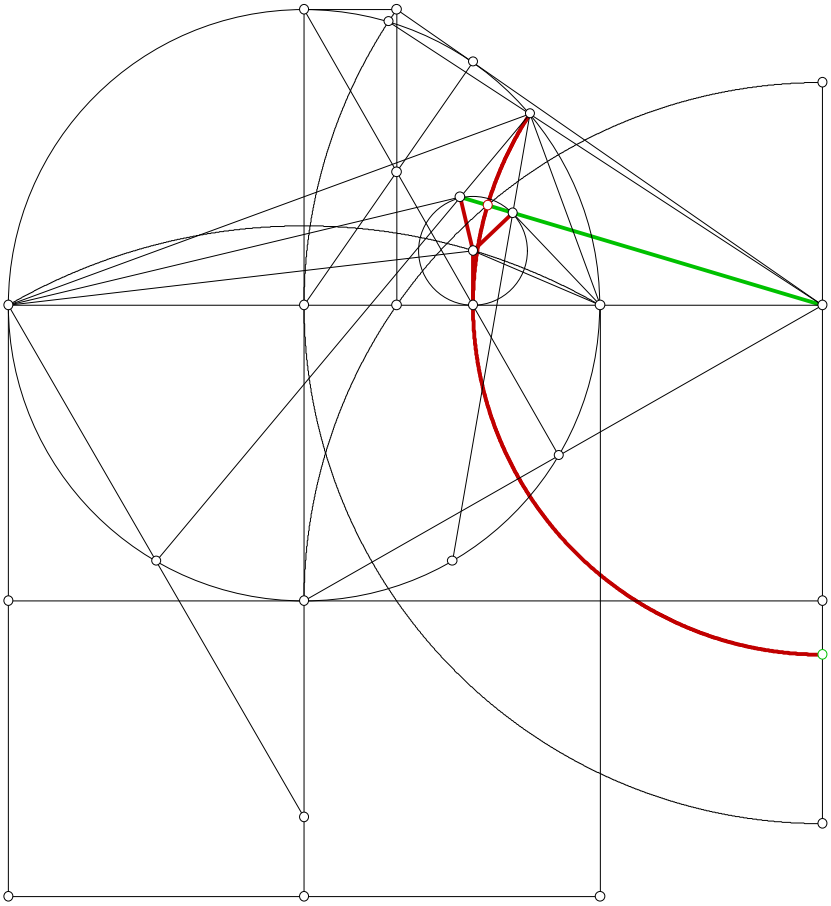


m∠UDG = 69.867°

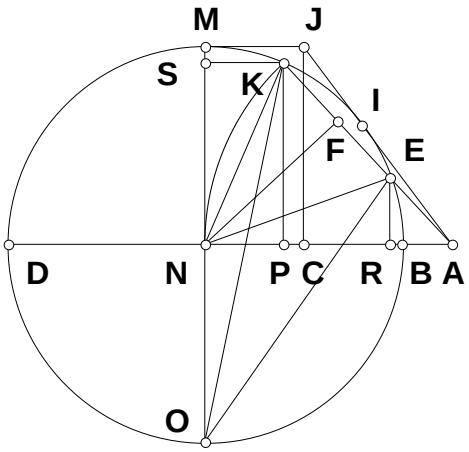
m∠UDCI = 23.289°

$$\frac{m\angle UDG}{m\angle UDCI} = 3.000$$

# Trisection and the square root figure.



# 08/23/00 Trisection In A Square Root Figure




**Given the square root figure drawn for trisection, what is AR given AB and AD?**

$N := 4 \qquad AB := 2 \qquad AD := AB \cdot N$

$AC := \sqrt{AB \cdot AD} \qquad BD := AD - AB$

$BN := \frac{BD}{2} \qquad KN := BN$

$AN := AB + BN \qquad AK := AN$

$AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN} \qquad AF := \frac{AP \cdot AN}{AK}$

$FK := AK - AF \qquad EF := FK$

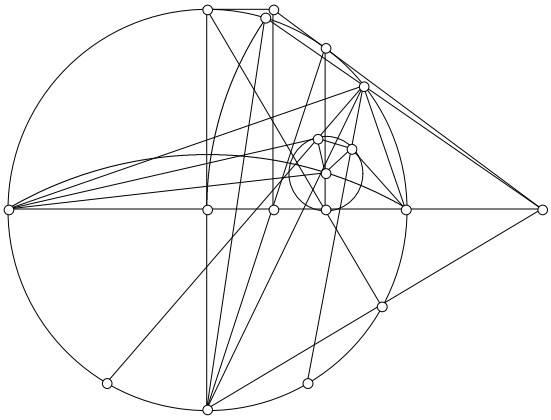
$AE := AK - 2 \cdot (EF) \qquad AR := \frac{AP \cdot AE}{AK} \qquad AB \cdot N \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \qquad BR := AR - AB$

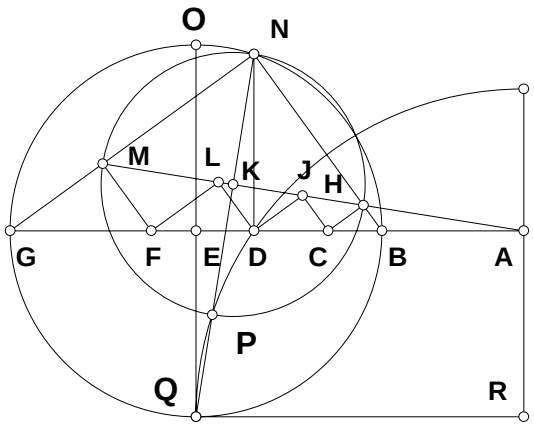
$AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} - BR = 0$

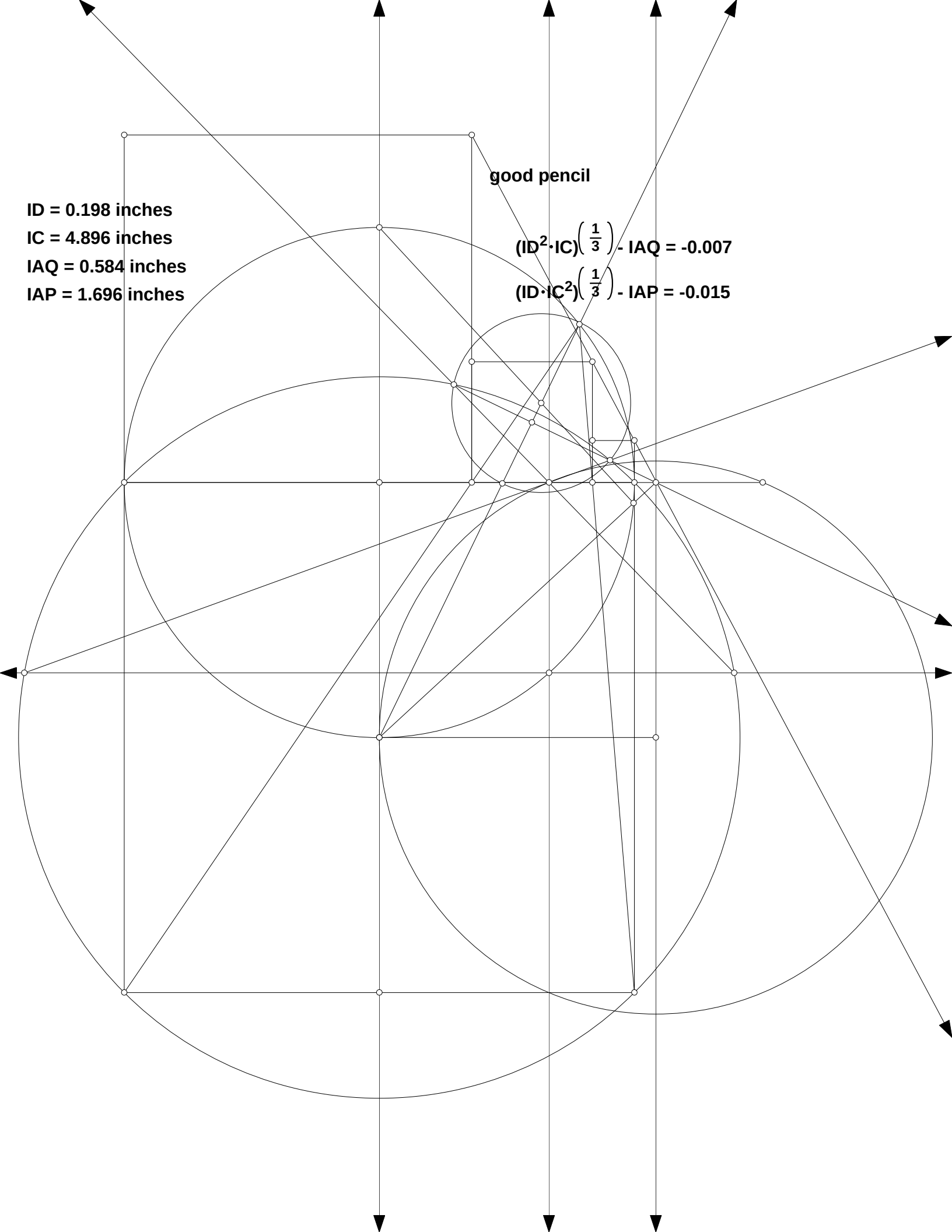
**Does KS = FK?**

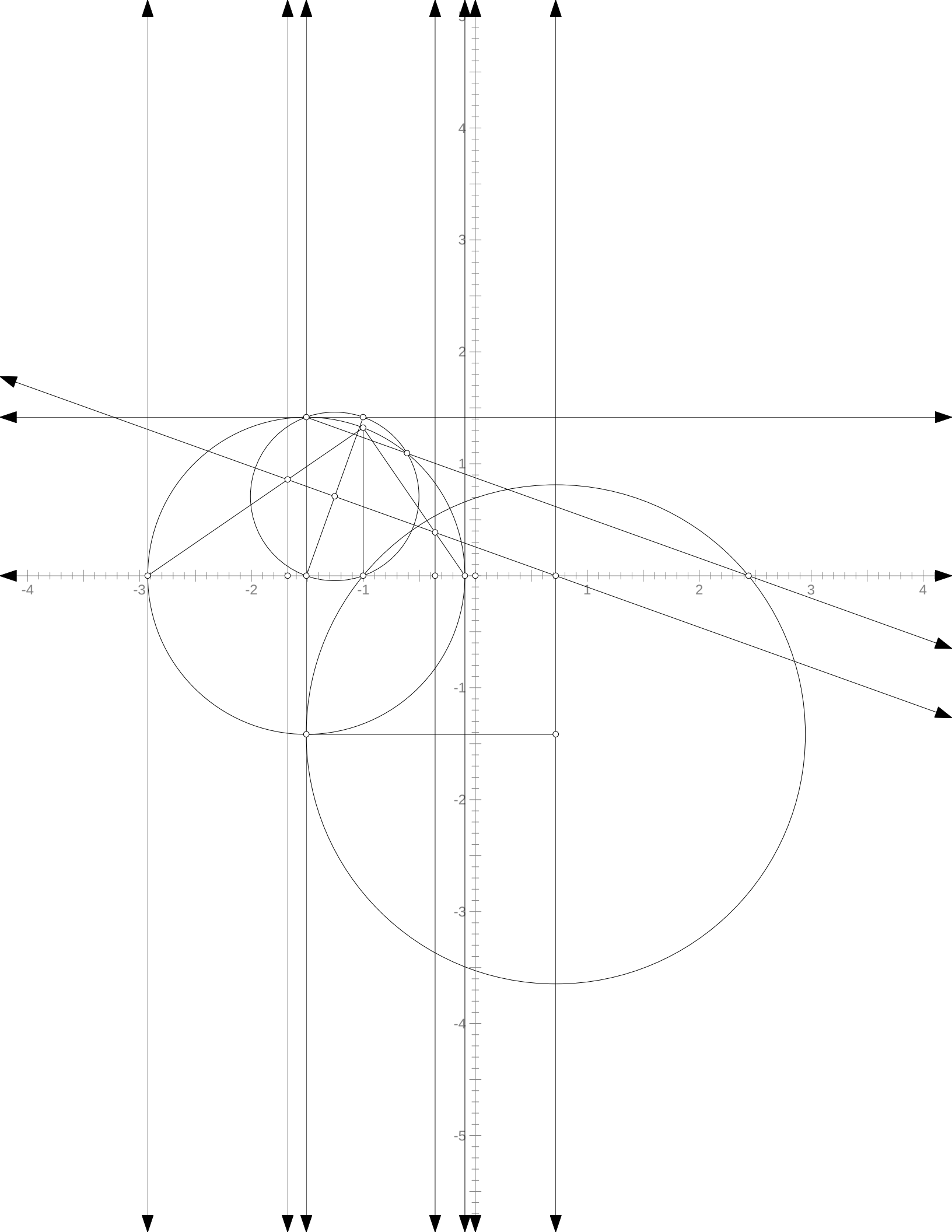
$BP := AP - AB \qquad DP := BD - BP \qquad NP := BN - BP \qquad KS := NP \qquad KS - FK = 0$

$\frac{AB \cdot (N-1)^2}{4 \cdot (N+1)} - KS = 0$



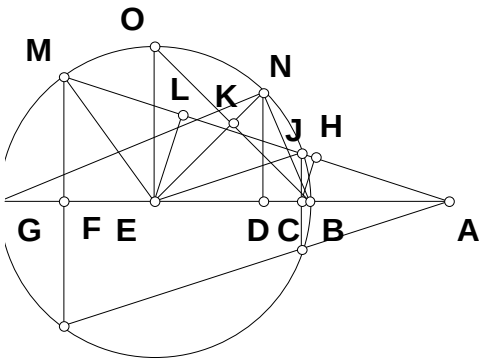

O
N
M
L
K
J
H
G
F
E
D
C
B
A
P
Q
R


good pencil
ID = 0.198 inches
IC = 4.896 inches
IAQ = 0.584 inches
IAP = 1.696 inches
$(ID^2 \cdot IC)^{\left(\frac{1}{3}\right)}$ - IAQ = -0.007
$(ID \cdot IC^2)^{\left(\frac{1}{3}\right)}$ - IAP = -0.015


-4
-3
-2
-1
1
2
3
4
-1
-2
-3
-4
-5

## 09/03/00 Ratios In Trisection

**How does BF vary with BC? How does DF vary with BC?**




$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2}$

$EN := BE \quad EK := \frac{BE \cdot BE}{BO} \quad KN := EN - EK \quad BK := \frac{BO}{2}$

$BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE$

$AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC \quad AE := AB + BE$

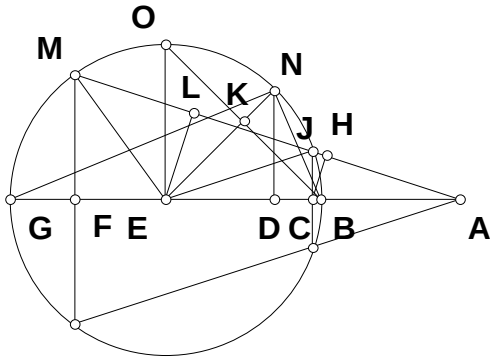
$JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB$

$$\frac{-1}{4} \cdot \left(-2 + \sqrt{2}\right) \cdot \frac{N_1}{N_2} - BC = 0$$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{1}{2} \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2 \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right)\right]}{\left(N_2^2 \cdot N_1\right)} - \frac{BF}{BC} = 0$$

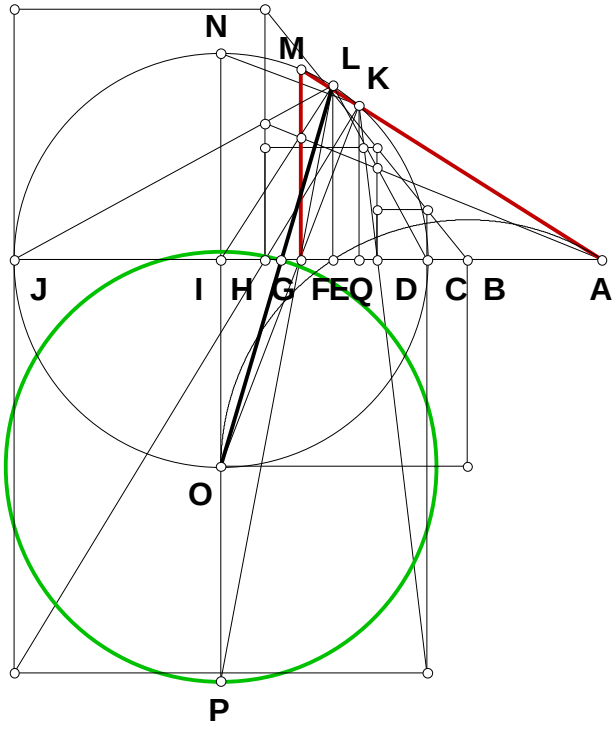



$$DF := BF - BD$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(7\cdot\sqrt{2} - 10\right)\cdot\left(12\cdot\sqrt{2}\cdot N_2^{\,2} + 17\cdot N_2^{\,2} + 2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2\right)\right]}{4\,N_2^{\,3}} - DF = 0$$

$$\frac{\left(N_1 - N_2\right)\cdot\left[\left(2\cdot\sqrt{2} - 3\right)\cdot\left(2\cdot N_1^{\,2} - 10\cdot N_1\cdot N_2 - 6\cdot N_1\cdot\sqrt{2}\cdot N_2 + 17\cdot N_2^{\,2} + 12\cdot\sqrt{2}\cdot N_2^{\,2}\right)\right]}{N_1\cdot N_2^{\,2}} - \frac{DF}{BC} = 0$$

# Goshdarn Good Pencil 09/16/00




$N := 2$

$BC := 1 \qquad BJ := BC \cdot N \qquad BE := \sqrt{BC \cdot BJ}$

$CJ := BJ - BC \qquad CI := \frac{CJ}{2} \qquad IO := CI$

$NO := CJ \qquad CR := CJ$

$CE := BE - BC \qquad EI := CI - CE \qquad EJ := CJ - CE$

$EL := \sqrt{CE \cdot EJ} \qquad EG := \frac{EI \cdot EL}{EL + IO} \qquad GI := EI - EG$

$GO := \sqrt{GI^2 + IO^2} \qquad OP := GO$

$IP := IO + OP \qquad EF := \frac{EI \cdot EL}{EL + IP} \qquad FI := EI - EF$

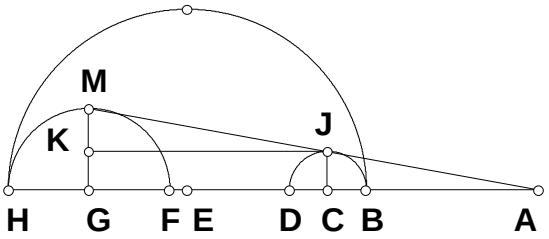
$FO := \sqrt{FI^2 + IO^2} \qquad OK := \frac{IO \cdot NO}{FO}$

$FK := OK - FO \qquad FQ := \frac{FI \cdot FK}{FO} \qquad QI := FQ + FI \qquad CQ := CI - QI \qquad QJ := CJ - CQ \qquad QK := \sqrt{CQ \cdot QJ}$

$CD := \frac{CQ \cdot CR}{CR + QK} \qquad BD := CD + BC \qquad \left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 0.000004486957912$

## Midpoints and Similarity Points 09/18/00

**What is AE given the radius of the two circles and the difference between their centers? (External Unit).**




$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8$

$BC := N_1 \quad GH := N_2 \quad CG := N_3$

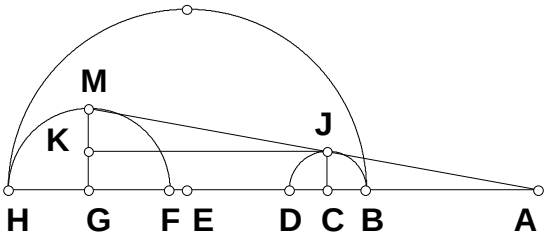
$CJ := BC \quad GM := GH \quad GK := CJ$

$BH := BC + CG + GH \quad BE := \frac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1^2 + N_3 \cdot N_1 + N_2^2}{2 \cdot (N_2 - N_1)} = 0$$

**What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).**




$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2}$

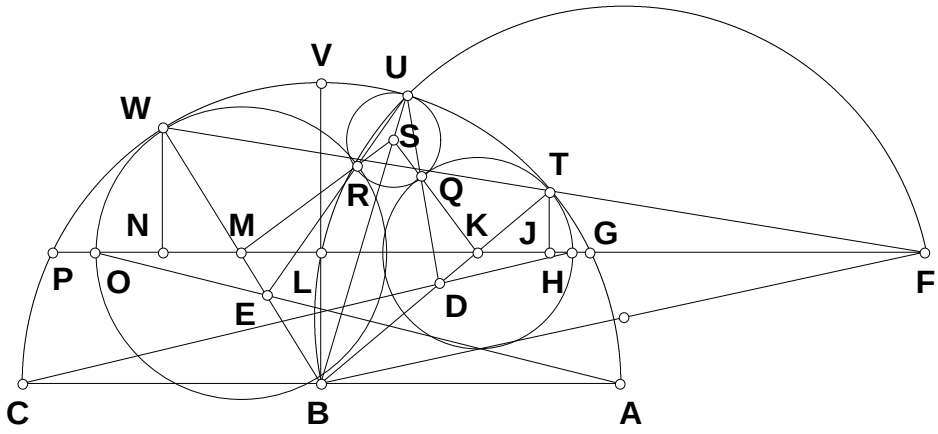
$GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$

$CJ := BC \quad GM := GH \quad GK := CJ$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

$$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$$

**100600**




$N_1 := 2.286$

$N_2 := 4.805$

$N_3 := 1.5364$

$AC := 3.104$

$AB := \frac{AC}{2}$ $BW := AB$

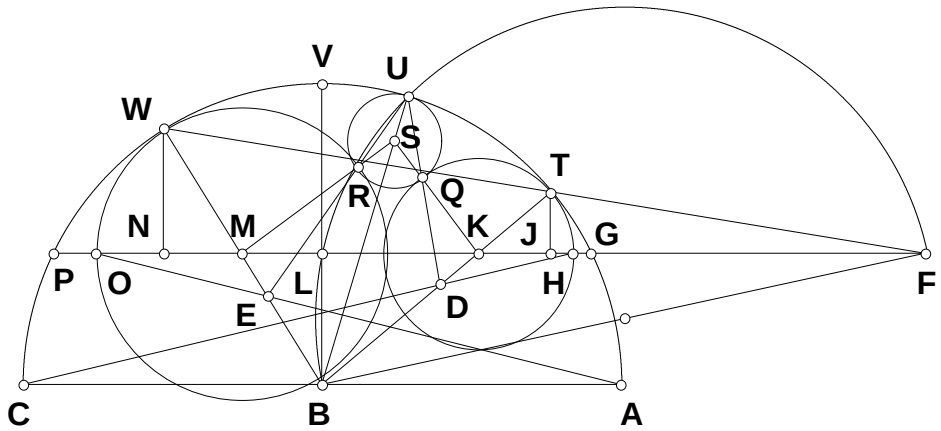
$BT := AB$ $BV := AB$

$BL := \frac{BV}{N_1}$ $LV := BV - BL$ $LP := (LV \cdot (BL + BV))^{\frac{1}{2}}$ $GL := LP$ $GP := GL + LP$ $GK := \frac{GP}{N_2}$

$GM := \frac{GP}{N_3}$ $KL := GL - GK$ $LM := GM - GL$ $BK := \left(BL^2 + KL^2\right)^{\frac{1}{2}}$ $BM := \left(BL^2 + LM^2\right)^{\frac{1}{2}}$

$KT := BT - BK$ $MW := BW - BM$ $KM := GM - GK$ $D := KM$ $R_L := MW$ $R_S := KT$

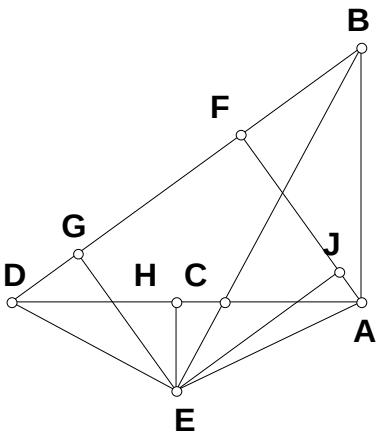
$FM := \frac{D \cdot R_L}{R_L - R_S}$ $FL := FM - LM$ $BF := \left(BL^2 + FL^2\right)^{\frac{1}{2}}$



**11/13/00**

**Given AB, DE, AD find BE, AC, CD, CE, BC.**

**BAD and BED are right.**




$N_1 := 3 \qquad N_2 := 5 \qquad N_3 := 1$

$AB := N_1 \quad AD := N_2 \quad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \quad BF := \frac{AB^2}{BD} \quad DG := \frac{DE^2}{BD} \qquad BE := \sqrt{BD^2 - DE^2}$

$AF := \sqrt{AB^2 - BF^2} \quad EG := \sqrt{DE^2 - DG^2} \qquad FG := BD - (BF + DG)$

$EJ := FG \quad FJ := EG \quad AJ := AF - FJ \qquad AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD \quad S_2 := DE \quad S_3 := AE \qquad AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

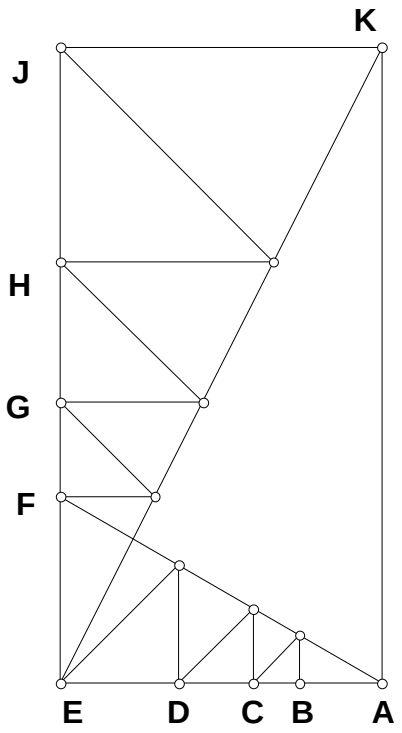
$EH := \sqrt{AE^2 - AH^2} \quad CH := \frac{EH \cdot AH}{AB + EH} \quad AC := AH - CH \quad CE := \frac{AC \cdot DE}{AB} \quad CD := AD - AC \quad BC := BE - CE$

$$N_1 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_1^2 \cdot N_2 - N_1 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - AC = 0$$

$$N_3 \cdot \frac{\left(-N_3^2 \cdot N_2 + N_2^3 + N_2 \cdot N_1^2 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{\left(N_1^3 + N_1 \cdot N_2^2 + N_3 \cdot \sqrt{-N_3^2 \cdot N_2^2 + N_2^4 + N_1^2 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2} \cdot N_3 + N_3^2 \cdot N_1^2}\right)} - CE = 0$$

# Means On Means 11/28/00

**Modify 02/28/98 for Mean proportionals between E and J.**




$AE := 1 \qquad N := 3 \qquad EJ := AE \cdot N \qquad JK := AE$

$HJ := \frac{JK \cdot EJ}{JK + EJ} \qquad EH := EJ - HJ \qquad GH := \frac{EH \cdot HJ}{EH + HJ}$

$EG := EH - GH \qquad FG := \frac{EG \cdot GH}{EG + GH} \qquad EF := EG - FG$

$DE := \frac{EF \cdot AE}{EF + AE} \qquad AD := AE - DE \qquad CD := \frac{AD \cdot DE}{AD + DE}$

$AC := AD - CD \qquad BC := \frac{AC \cdot CD}{AC + CD} \qquad AB := AC - BC$

$M := 0 .. 3 \qquad P := 0 .. 3 \qquad AEAB_{M,P} := \left[ \frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

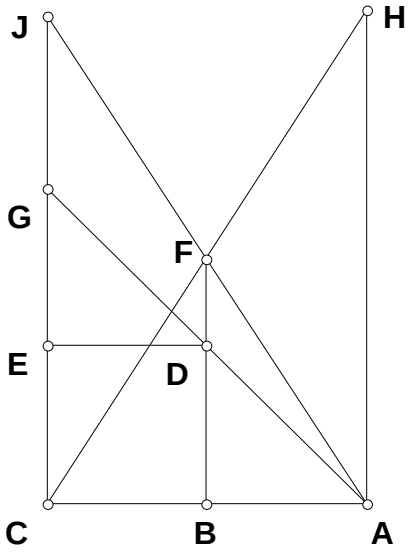
$$AEAB = \begin{bmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.563 & 34.328 \\ 1 & 2.688 & 7.223 & 19.411 \\ 1 & 2.266 & 5.133 & 11.63 \end{bmatrix}$$

$AEAB_{3,3} - \frac{AE}{AB} = 0 \qquad AEAB_{3,2} - \frac{AE}{AC} = 0 \qquad AEAB_{3,1} - \frac{AE}{AD} = 0 \qquad AEAB_{3,0} - \frac{AE}{AE} = 0$

$AEAB_{3,3} - \frac{AE}{AB} = 0$

# Multiplication and Division-Line By A Line 11/29/00

**Given some unit, and two differences, multiply or divide the one difference by the other.**
**For Division:**




$AC := 1 \qquad N_1 := 3 \qquad N_2 := 12 \qquad AH := N_1$

$CJ := N_2 \qquad AB := \frac{AH}{(CJ + AH)} \cdot AC$

$BC := AC - AB \quad BD := BC \quad CG := \frac{BD \cdot AC}{AB}$

$CG - \frac{N_2}{N_1} = 0 \qquad CG = 4$

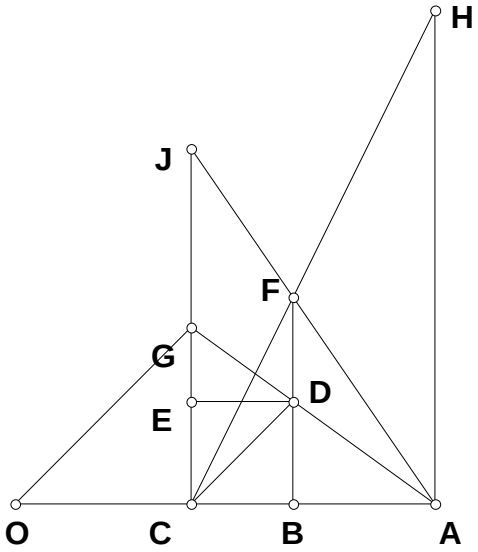
**For Multiplication:**

$AC := 1 \qquad N_1 := 5 \qquad N_2 := 7 \qquad AH := N_1$

$CG := N_2 \qquad CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

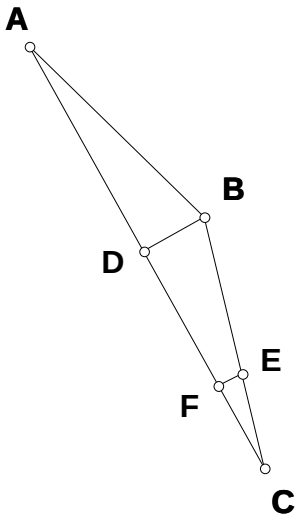
$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$



**120500**

**From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.**




$N_1 := 5 \qquad N_2 := 25 \qquad N_3 := 1 \qquad N_4 := .5$

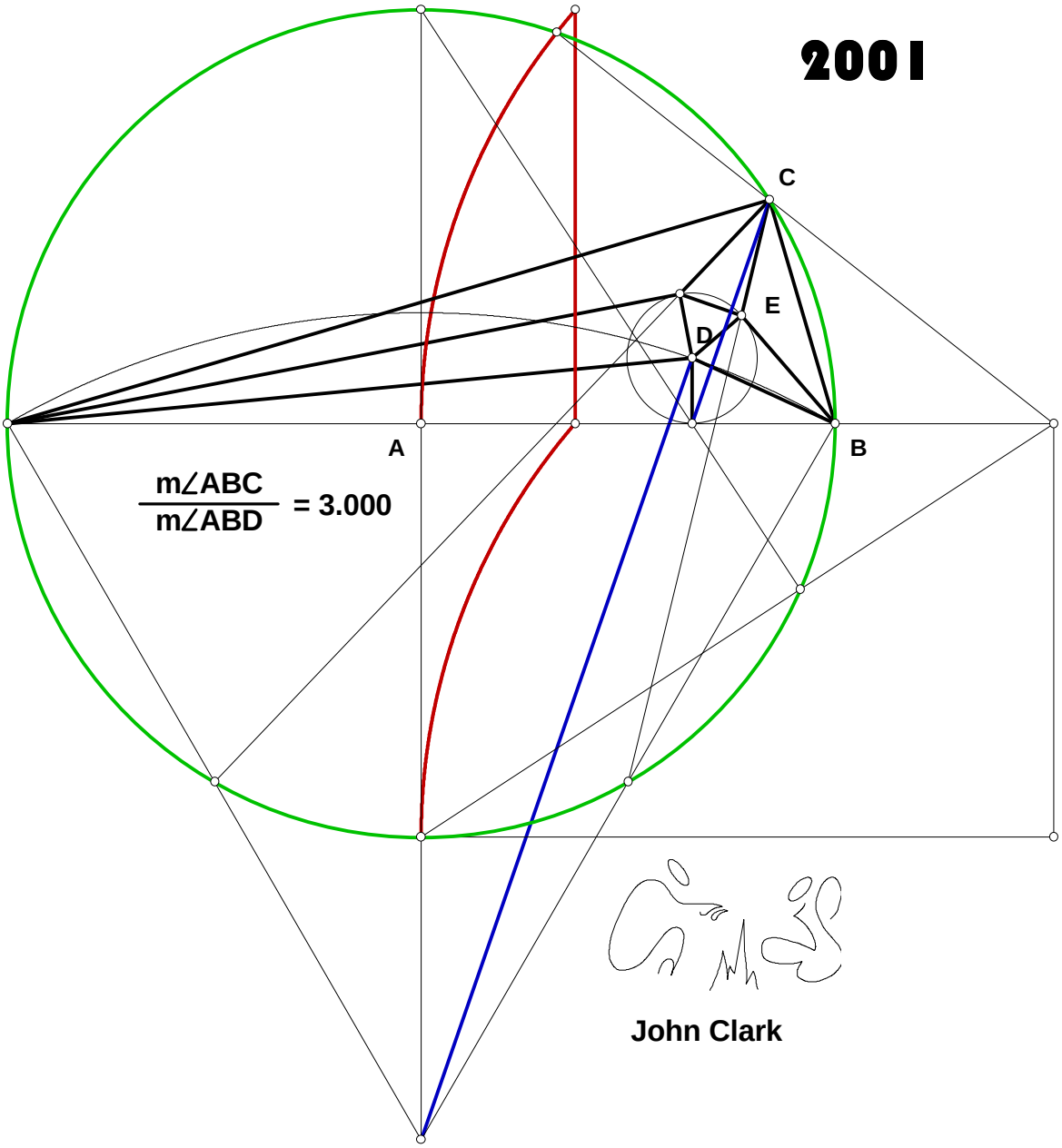
$BC := N_1 \qquad AC := N_2 \qquad CE := N_3 \qquad EF := N_4$

$BD := \frac{EF \cdot BC}{CE} \qquad CF := \sqrt{CE^2 - EF^2} \qquad CD := \frac{CF \cdot BC}{CE}$

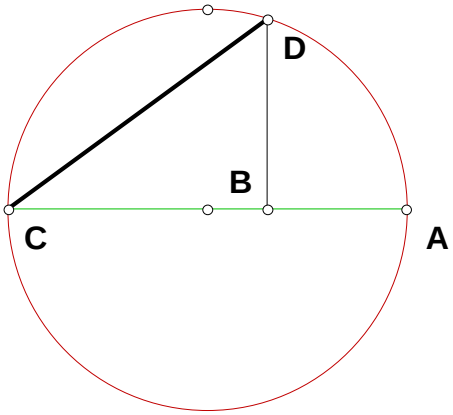
$AD := AC - CD \qquad AB := \sqrt{BD^2 + AD^2}$

$$AB - \sqrt{\frac{1}{N_3} \cdot \left( N_2^2 \cdot N_3 - 2 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2} \cdot N_1 + N_1^2 \cdot N_3 \right)} = 0$$

# Three Pieces Of Paper




John Clark




**Alternate method for common segment common endpoint square root.** $\sqrt{AC \cdot BC} = CD$

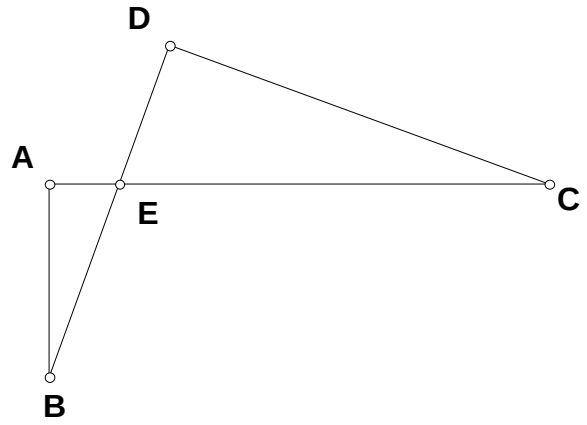
$N_1 := 5 \quad N_2 := 3 \quad AC := N_1 \quad BC := N_2$

$AB := AC - BC \quad BD := \sqrt{AB \cdot BC} \quad CD := \sqrt{BD^2 + BC^2}$

$\sqrt{N_2 \cdot N_1} - CD = 0$

## Three Given Five Taken 042101

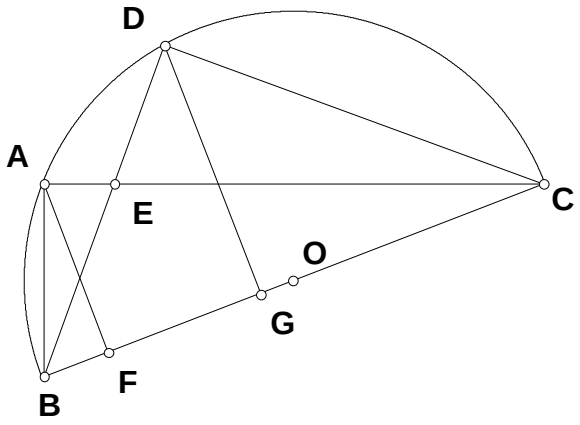
**Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?**




$N_1 := 2 \qquad N_2 := 3 \qquad N_3 := 4$

$AB := N_1 \quad CD := N_2 \quad AC := N_3$

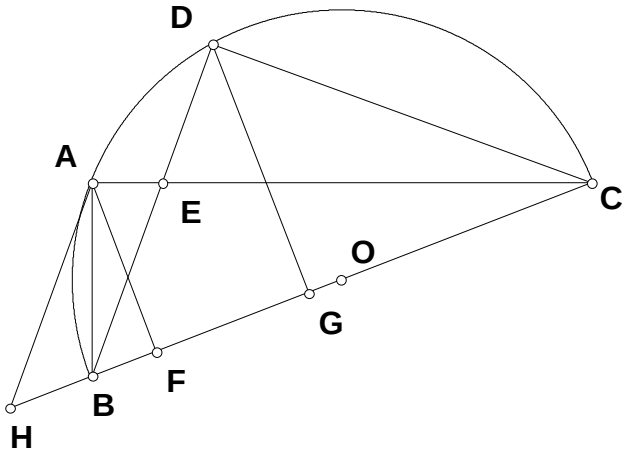
$BC := \sqrt{AB^2 + AC^2} \qquad CG := \frac{CD^2}{BC} \qquad BF := \frac{AB^2}{BC}$




$BG := BC - CG \qquad CF := BC - BF$

$AF := \sqrt{AB^2 - BF^2} \qquad DG := \sqrt{CD^2 - CG^2}$

$FH := \frac{BG \cdot AF}{DG} \qquad CH := CF + FH$



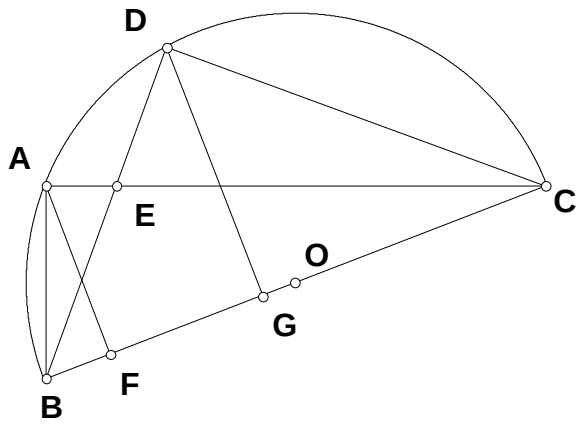

$BD := \sqrt{BG^2 + DG^2}$

$AH := \frac{BD \cdot FH}{BG} \qquad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \qquad CE := \frac{AC \cdot BC}{CH}$

$AE := AC - CE$

**Idea about process progression: Given two differences one renders a set number of processes upon them, then one progresses to three?**




**Some Algebraic Names:**

$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

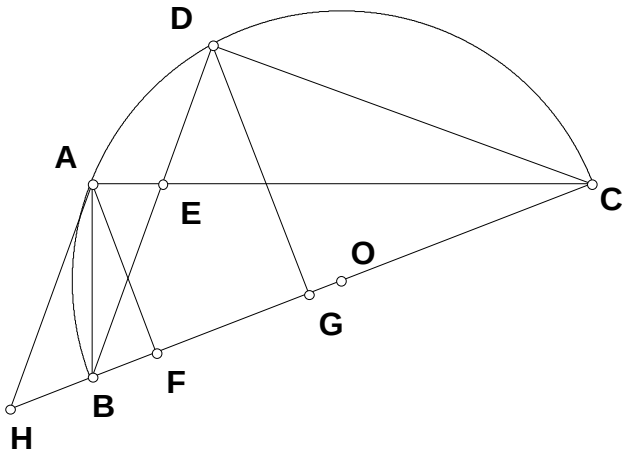
$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0$$

$$\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0$$

$$\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0$$

$$\frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

$$N_2 \cdot \sqrt{\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\left(N_1^2 + N_3^2\right)}} - DG = 0$$

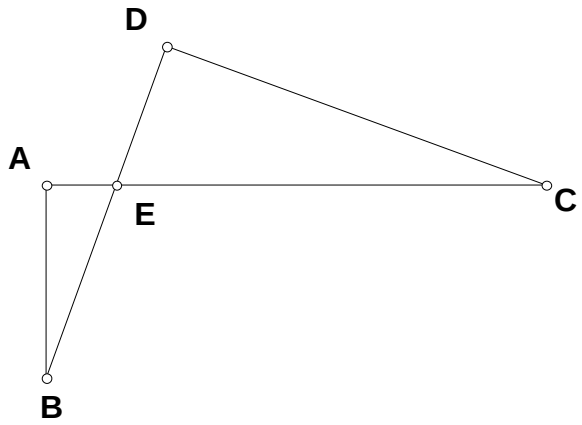



$$\frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} + \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}}\right] - CH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

**The Five Sought:**

$$\sqrt{N_1^2 + N_3^2 - N_2^2} - BD = 0$$




$$N_2 \cdot \frac{\left(\sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - DE = 0$$

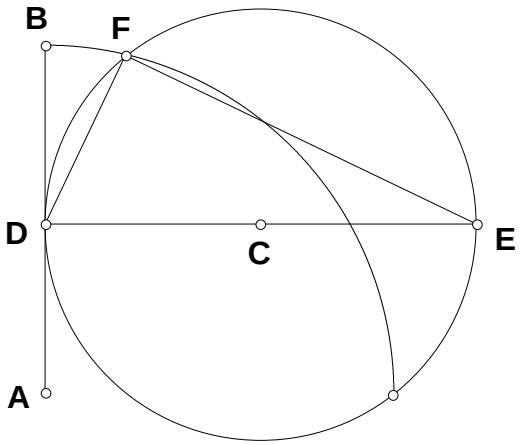
$$N_1 \cdot \frac{\left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - BE = 0$$

$$\frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1} - CE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^2 + N_3^2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^2 + N_3^2 - N_2^2} \cdot N_1\right)} - AE = 0$$

**042201**

**Given AB as unit, AD and DC, what is EF and DF?**

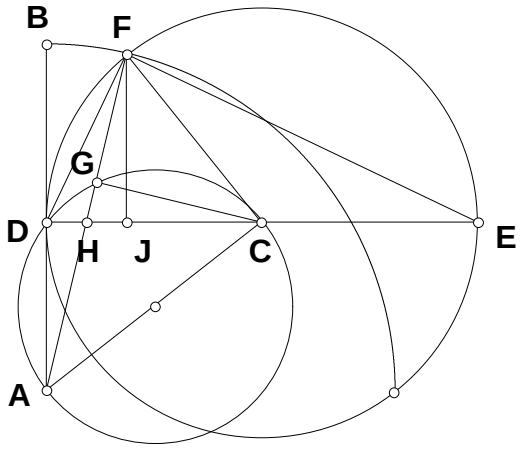



$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2\ CD$

$AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$




$AG := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

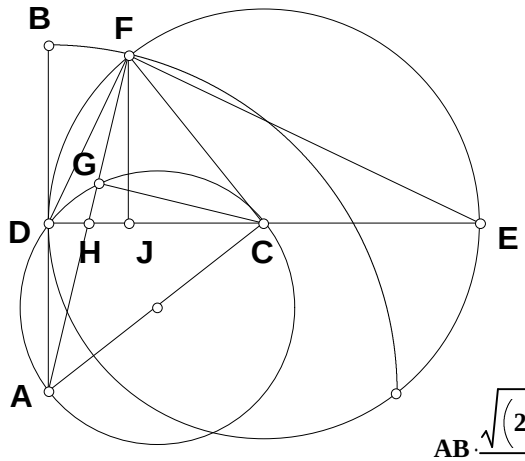
$L_1 := AD \quad L_2 := CG \quad L_3 := CD$

$DH := L_3 - \frac{L_2 \cdot \left(L_1^2 + L_3^2\right)}{L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1}$

$AH := L_1 \cdot \frac{\left(L_1^2 + L_3^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1\right)} \quad FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$

$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$

**Some Algebraic Names:**




$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2\ AB) \cdot N_2 - DE = 0 \qquad AB \cdot \frac{\sqrt{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}}{N_1} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0$$

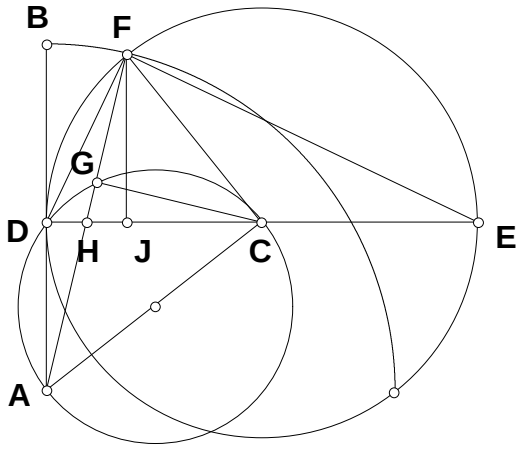
$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2} \cdot AB \cdot \frac{\begin{bmatrix} -N_1^{\,4} \cdot N_2^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + 2 \cdot N_1^{\,3} \cdot N_2 - 2 \cdot N_1^{\,5} \cdot N_2 \ \ldots \\ + 6 \cdot N_1^{\,5} \cdot N_2^{\,3} - 3 \cdot N_2^{\,2} \cdot N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + 2 \cdot N_2^{\,3} \cdot N_1^{\,3} - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \ \ldots \\ + N_1^{\,2} \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} \end{bmatrix}}{\left[N_1 \cdot \left[\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right) \cdot \left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)\right]\right]} - HJ = 0$$




$$\frac{1}{2}\cdot AB\cdot\frac{\left(N_1^{\,3}\cdot N_2+N_2\cdot N_1-\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N_2\cdot N_1+4\cdot N_2^{\,3}\cdot N_1^{\,3}-N_1^{\,3}\cdot N_2+\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}-EJ=0$$

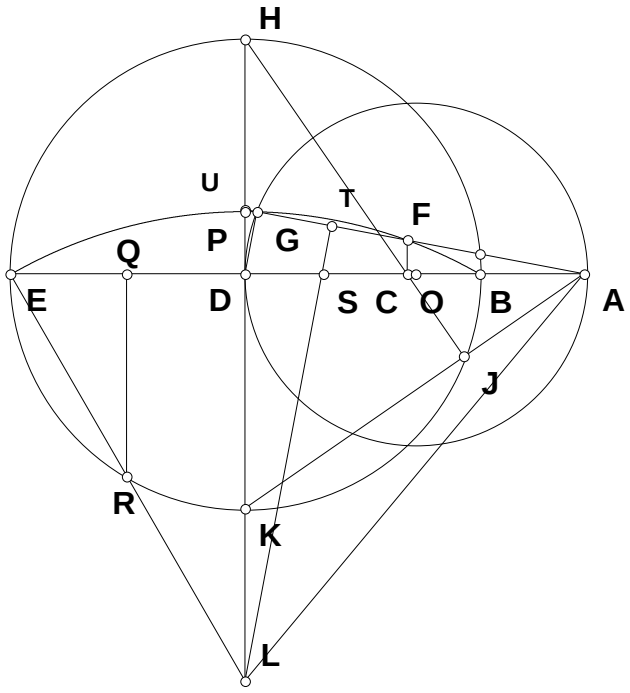
$$\frac{-1}{2}\cdot AB\,\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\sqrt{N_2\cdot\frac{\left(3\cdot N_1\cdot N_2+4\cdot N_2^{\,3}\cdot N_1^{\,3}-N_1^{\,3}\cdot N_2+\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)}{\left[\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\cdot N_1\right]}}-EF=0$$

$$AB\cdot\sqrt{\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}\right)\cdot\frac{N_2}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}}-DF=0$$

# Counterpoint 042301

**CF is the radius of a circle expressed in trisection. Is AF the counterpart of AC?**




$N := 5.48643 \qquad AB := .54331$

$AE := AB \cdot N \qquad BE := AE - AB$

$BD := \frac{BE}{2} \qquad AD := AB + BD$

$AO := \frac{AD}{2} \qquad DE := BD \qquad DQ := \frac{DE}{2}$

$EQ := DQ \qquad BQ := BD + DQ$

$QR := \sqrt{BQ \cdot EQ} \qquad ER := \sqrt{QR^2 + EQ^2}$

$EL := 2 \cdot ER \qquad LP := EL \qquad DL := \frac{QR \cdot EL}{ER}$

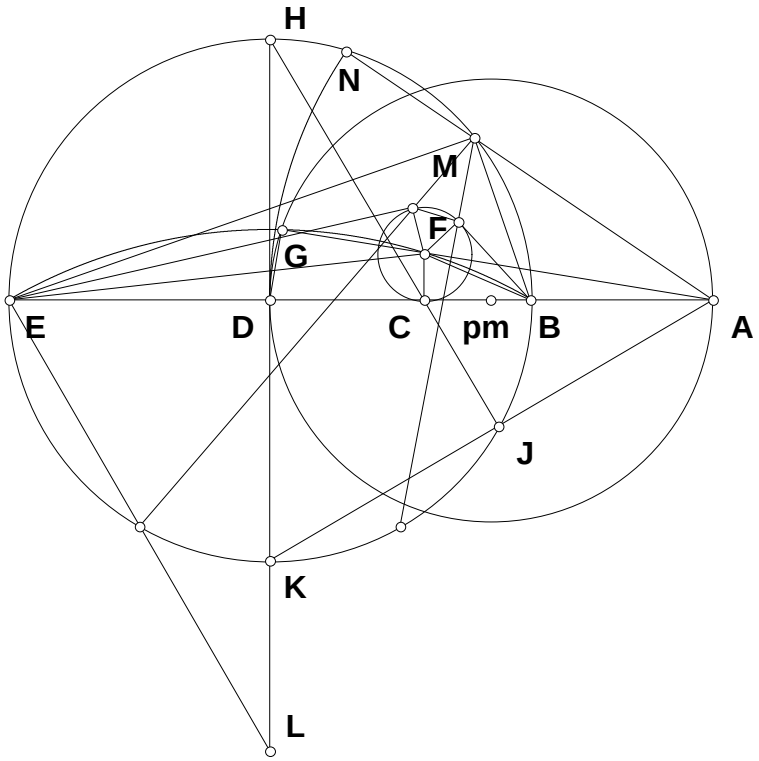
**Try to figure out how I do this;**

$$AG := \frac{AB \cdot (N-1)}{4} \cdot \sqrt{2} \cdot \sqrt{(1+N) \cdot \sqrt{3}} \cdot \sqrt{\frac{\left[\frac{5}{18} \cdot \sqrt{3} \cdot \frac{(1+N)}{(N-1)} + \frac{1}{18} \cdot \frac{(1+N)^3}{(N-1)^3} \cdot \sqrt{3} + \frac{1}{3} \cdot \frac{\sqrt{(N+3) \cdot (3 \cdot N+1)}}{(N-1)}\right]}{\left[\frac{1}{12} \cdot \frac{\left(13 \cdot N^2 - 22 \cdot N + 13\right)}{(N-1)}\right]}}$$

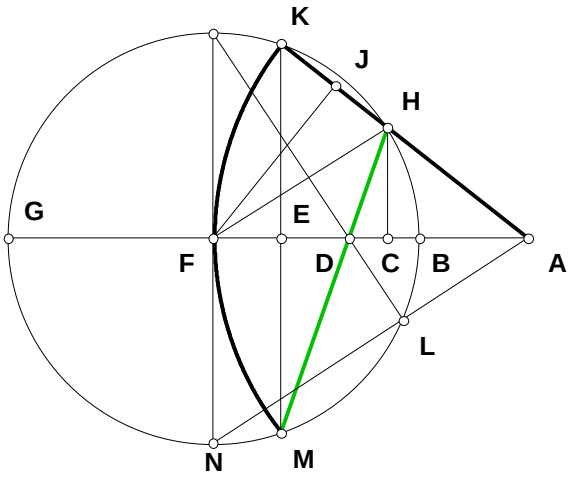
$DG := \sqrt{AD^2 - AG^2} \qquad AU := \frac{AD \cdot AD}{AG} \qquad DU := \frac{DG \cdot AD}{AG} \qquad LU := DL + DU \qquad TU := \frac{DG \cdot LU}{AD} \qquad GU := AU - AG$

$TG := TU - GU \qquad FT := TG \qquad AF := AG - (FT + TG) \qquad AC := \frac{AD \cdot AF}{AU}$

**From 012496 for AC** $2 \cdot AB \cdot \frac{N}{(1+N)}$ $AC - 2 \cdot AB \cdot \frac{N}{(1+N)} = 0$



**042401**




**Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?**

$N := 5$ $AB := 1$ $AG := AB \cdot N$

$BG := AG - AB$ $BF := \frac{BG}{2}$ $AF := AB + BF$

$AK := AF$ $FK := BF$ $AE := \frac{2\,AK^2 - FK^2}{2\,AF}$

$AJ := AE$ $JK := AK - AJ$ $HJ := JK$

$AH := AK - (JK + HJ)$ $AC := \frac{AE \cdot AH}{AK}$ $CE := AE - AC$ $BE := AE - AB$ $EG := BG - BE$

$EK := \sqrt{BE \cdot EG}$ $BC := AC - AB$ $CG := BG - BC$ $CH := \sqrt{BC \cdot CG}$ $DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE$ $HM := \sqrt{CE^2 + (EK + CH)^2}$

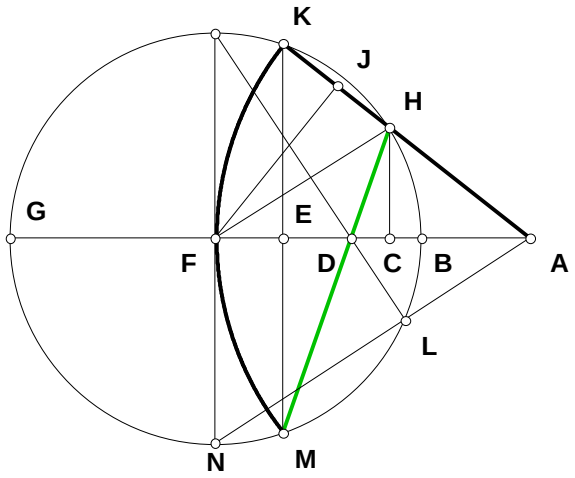
**Some Algebraic Names:**

$AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$

$\frac{1}{4} \cdot AB \cdot \frac{(N^2 + 6 \cdot N + 1)}{(1 + N)} - AE = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(1 - 2 \cdot N + N^2)}{(1 + N)} - JK = 0$ $2 \cdot AB \cdot \frac{N}{(1 + N)} - AH = 0$

$AB \cdot \frac{(1 + 6 \cdot N + N^2)}{(1 + N)^3} \cdot N - AC = 0$ $\frac{1}{4} \cdot AB \cdot (N^2 + 6 \cdot N + 1) \cdot \frac{(N - 1)^2}{(1 + N)^3} - CE = 0$

$\frac{1}{4} \cdot AB \cdot (N + 3) \cdot \frac{(N - 1)}{(1 + N)} - BE = 0$ $\frac{1}{4} \cdot AB \cdot (3 \cdot N + 1) \cdot \frac{(N - 1)}{(1 + N)} - EG = 0$




$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-EK=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BC=0$$

$$AB\cdot N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3}-CG=0$$

$$AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3}-CH=0$$

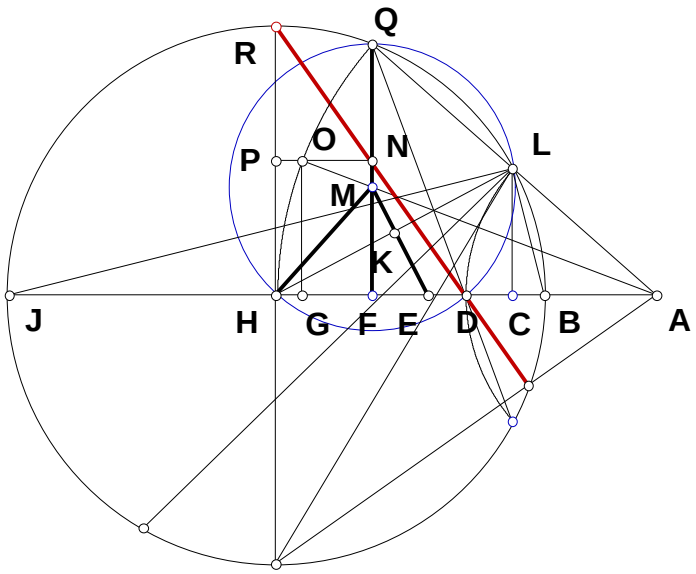
$$\frac{1}{4}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DE=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(N+1)}-DF=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)^2}-HM=0$$

**042501**

**What is the Algebraic name of the circle HM? Does point N divide DR in half?**




$N := 5.768$ $AB := .583$ $AJ := AB \cdot N$

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$ $HM - MQ = 0$

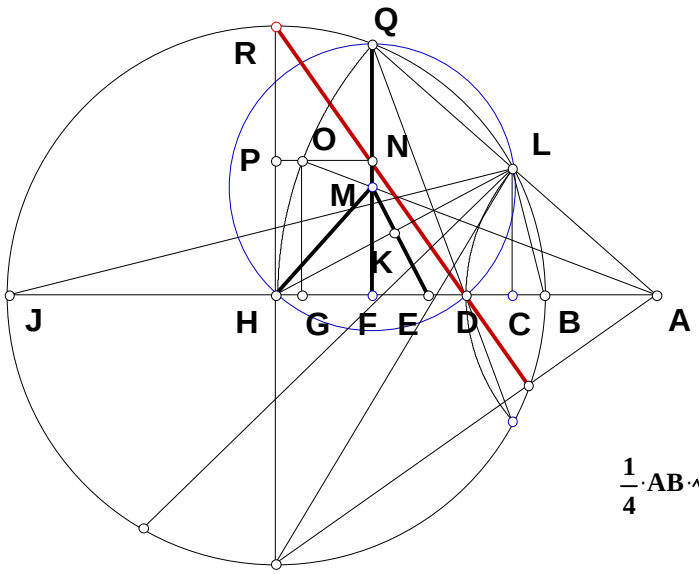
$DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

**Some Algebraic Names:**

$AB \cdot N - AB - BJ = 0$ $\frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0$ $\frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0$ $\frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0$ $\frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0$ $\frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$




$$\frac{1}{4}\cdot AB\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)}-FJ=0$$

$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)}-BF=0$$

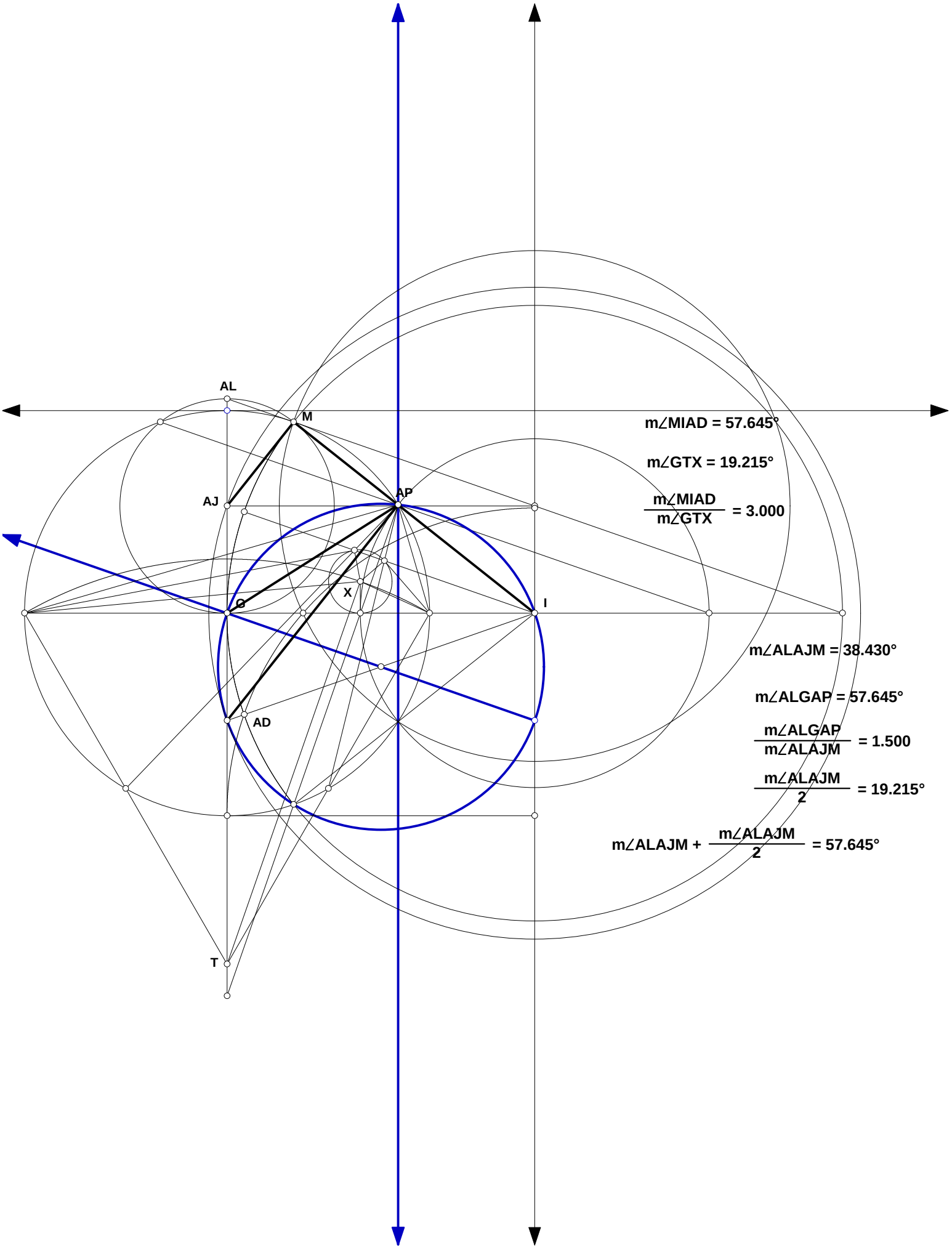
$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-FQ=0$$

$$\frac{1}{2}\cdot(1+N)\cdot AB\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-MQ=0$$

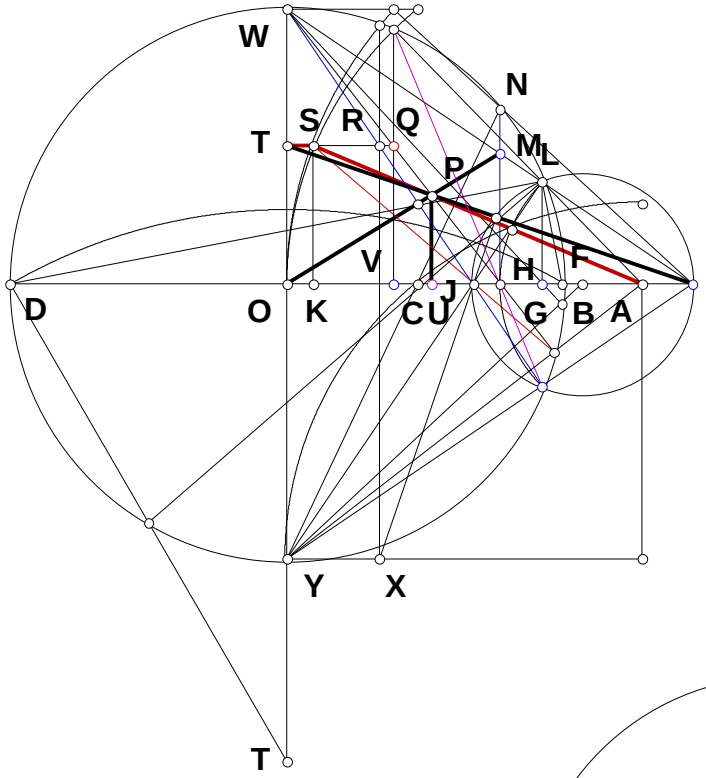
$$\frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-HM=0$$

$$HM-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DH=0 \qquad \frac{DH}{2}-FH=0$$


AL
M
AJ
AP
X
G
I
AD
T
m∠MIAD = 57.645°
m∠GTX = 19.215°
m∠MIAD / m∠GTX = 3.000
m∠ALAJM = 38.430°
m∠ALGAP = 57.645°
m∠ALGAP / m∠ALAJM = 1.500
m∠ALAJM / 2 = 19.215°
m∠ALAJM + m∠ALAJM / 2 = 57.645°

# Four Lines To A Point 042901

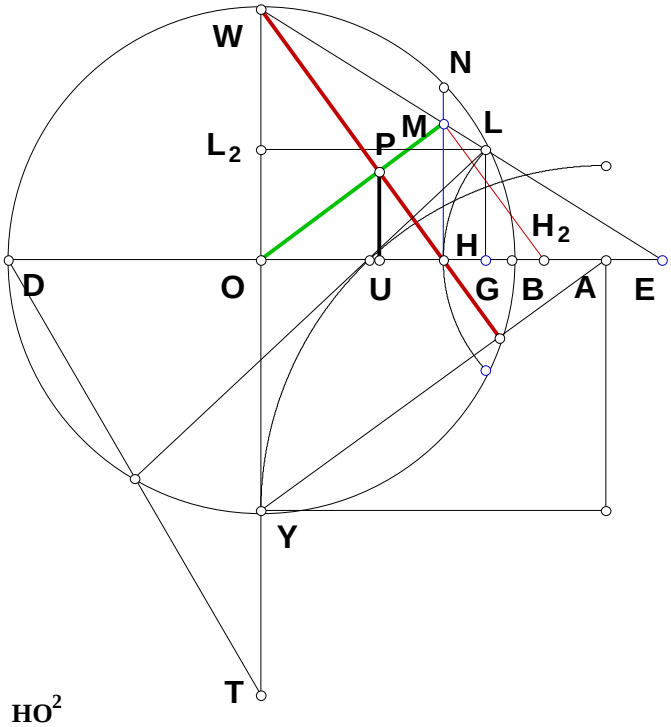



**Does the difference OU and PU each have but one Algebraic name?**

$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \frac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$




$HO := \frac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \frac{OL^2 + AO^2 - AL^2}{2\,AO}$

$BG := BO - GO \qquad DG := GO + DO$

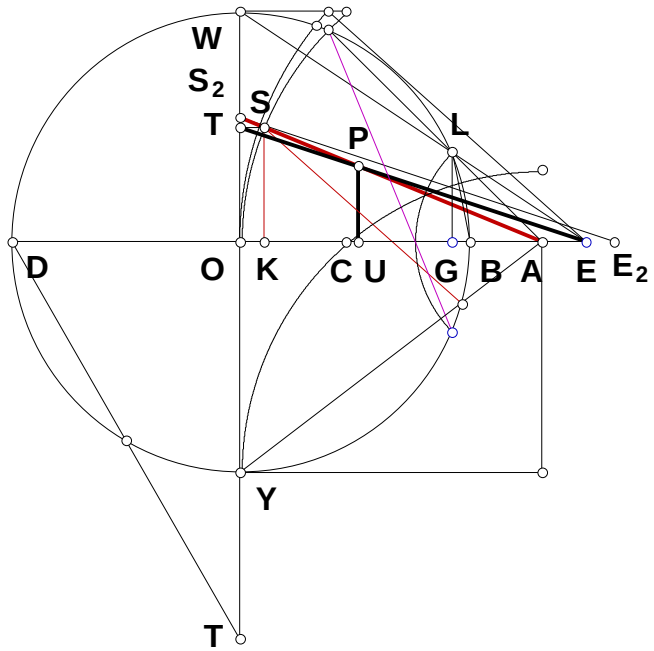
$GL := \sqrt{BG \cdot DG} \qquad EO := \frac{GO \cdot OW}{OW - GL}$

$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \frac{GL \cdot EH}{EG} \qquad HH_2 := \frac{HO \cdot HM}{OW} \qquad OU := \frac{HO^2}{HH_2 + HO}$

$UP := \frac{HM \cdot OU}{HO}$




$$OT := \frac{OW}{2} \quad KS := OT \quad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \quad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \quad AS_2 := \sqrt{AO^2 + OS_2{}^2}$$

$$AE_2 := OE_2 - AO \quad AE := EO - AO$$

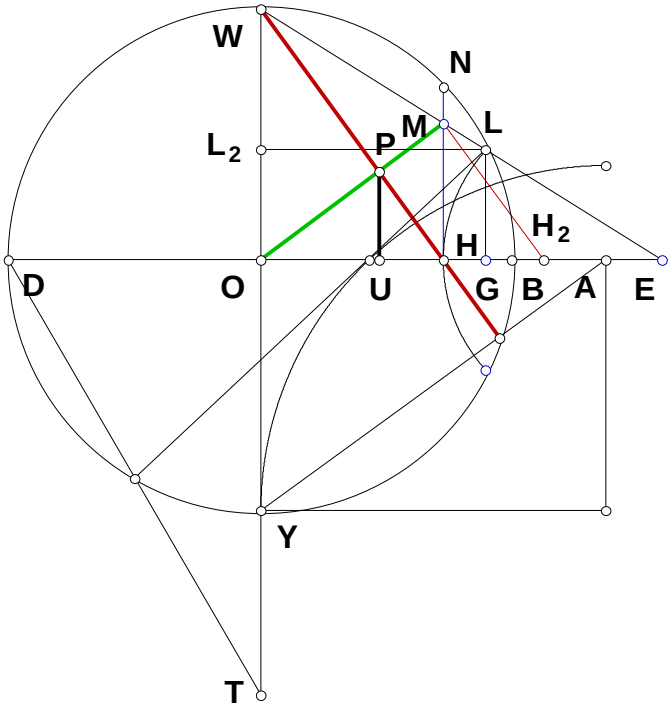
$$AP := \frac{AS_2 \cdot AE}{AE_2} \quad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \quad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$

## Some Algebraic Names:




$$AB\cdot(N-1)-BD=0$$

$$\frac{AB\cdot(N-1)}{2}-BO=0$$

$$\frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0$$

$$2\cdot AB\cdot\frac{N}{(1+N)}-AH=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0$$

$$AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0$$

$$AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

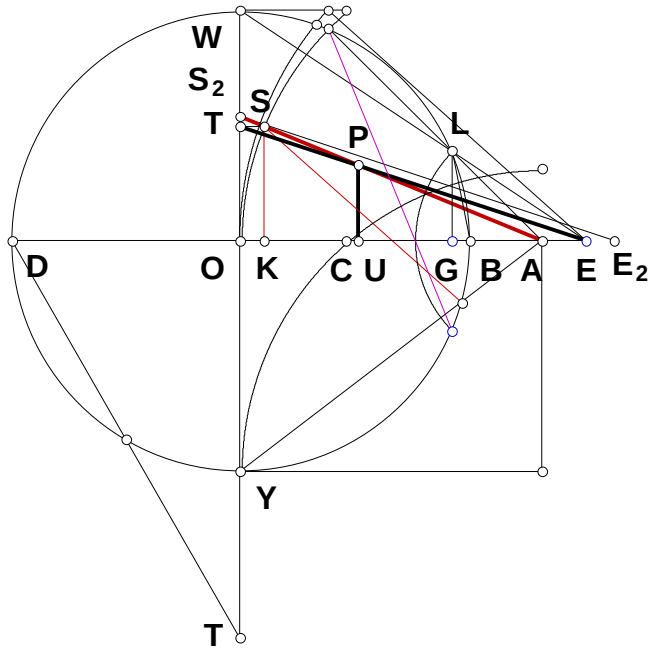
$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0$$

$$N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$




$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0$$

$$\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

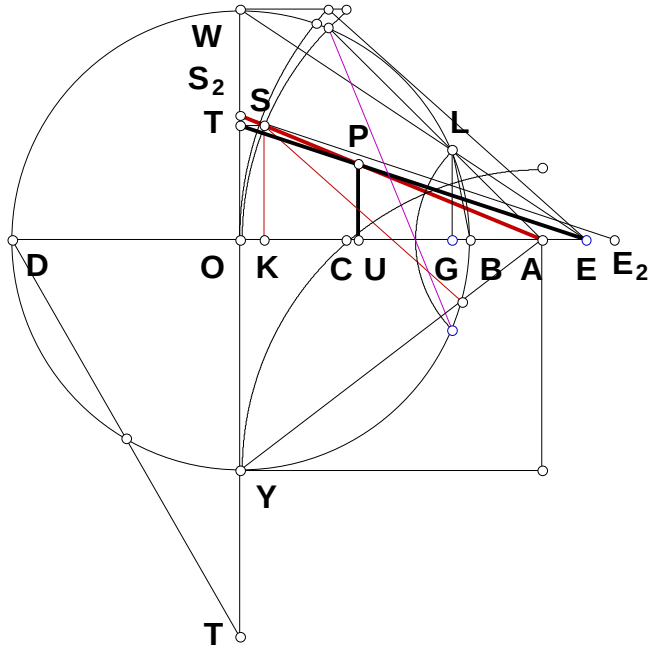
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AP=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$




$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AU=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

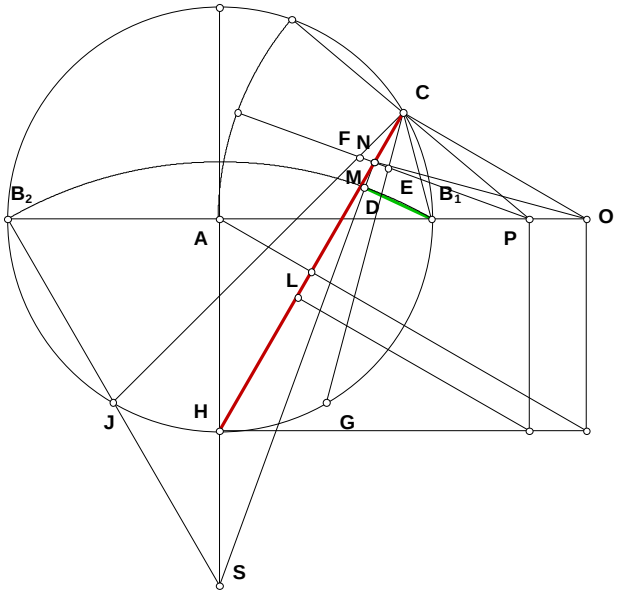
$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

# Compass Construction 043001

$m\angle AB_1C = 75.000°$ $m\angle AB_2C = 15.000°$

$m\angle AB_1D = 25.000°$ $m\angle AB_2D = 5.000°$

$\frac{m\angle AB_1C}{m\angle AB_1D} = 3.000$ $\frac{m\angle AB_2C}{m\angle AB_2D} = 3.000$



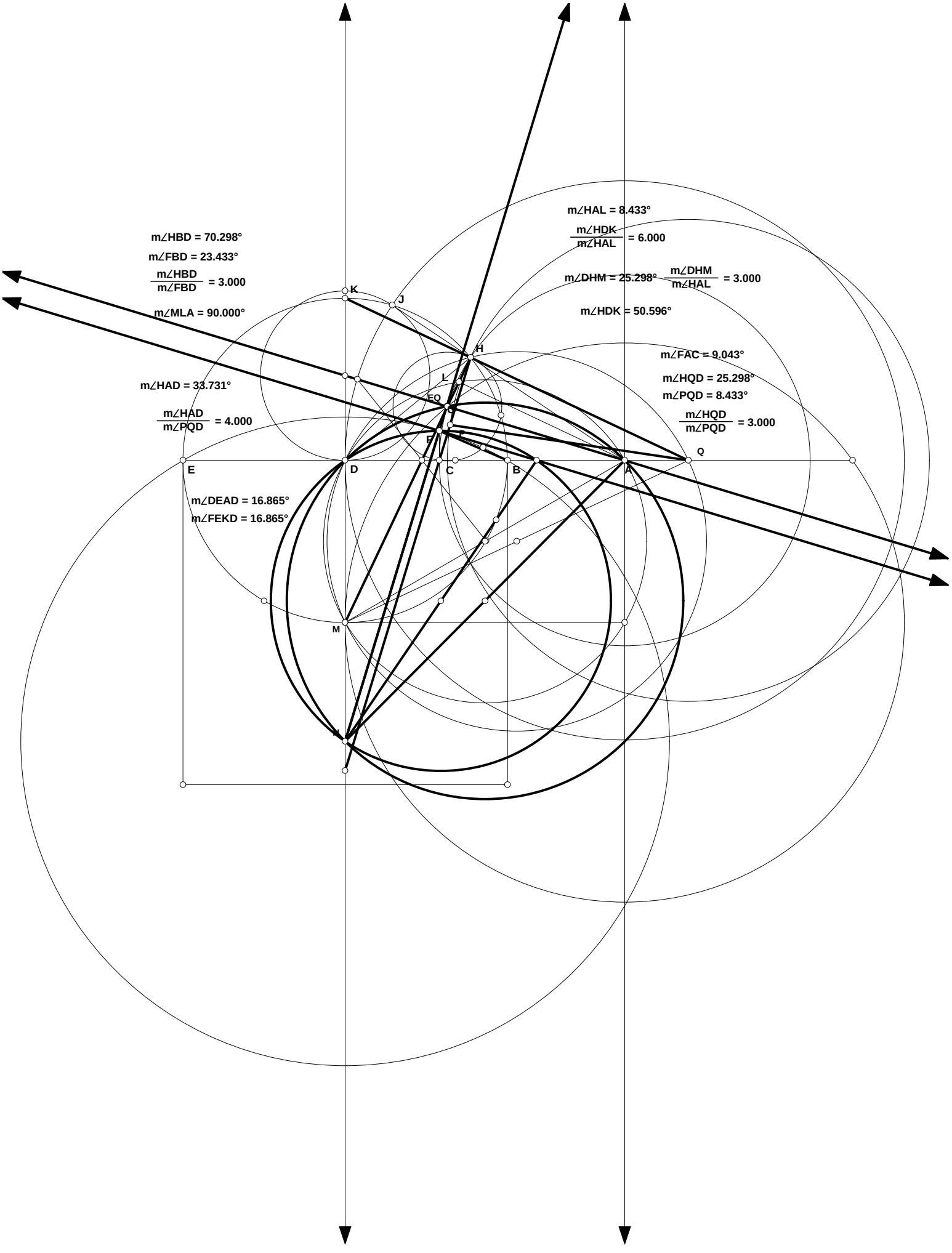

$m\angle ABC - 3 \cdot m\angle ABD = 0.000°$

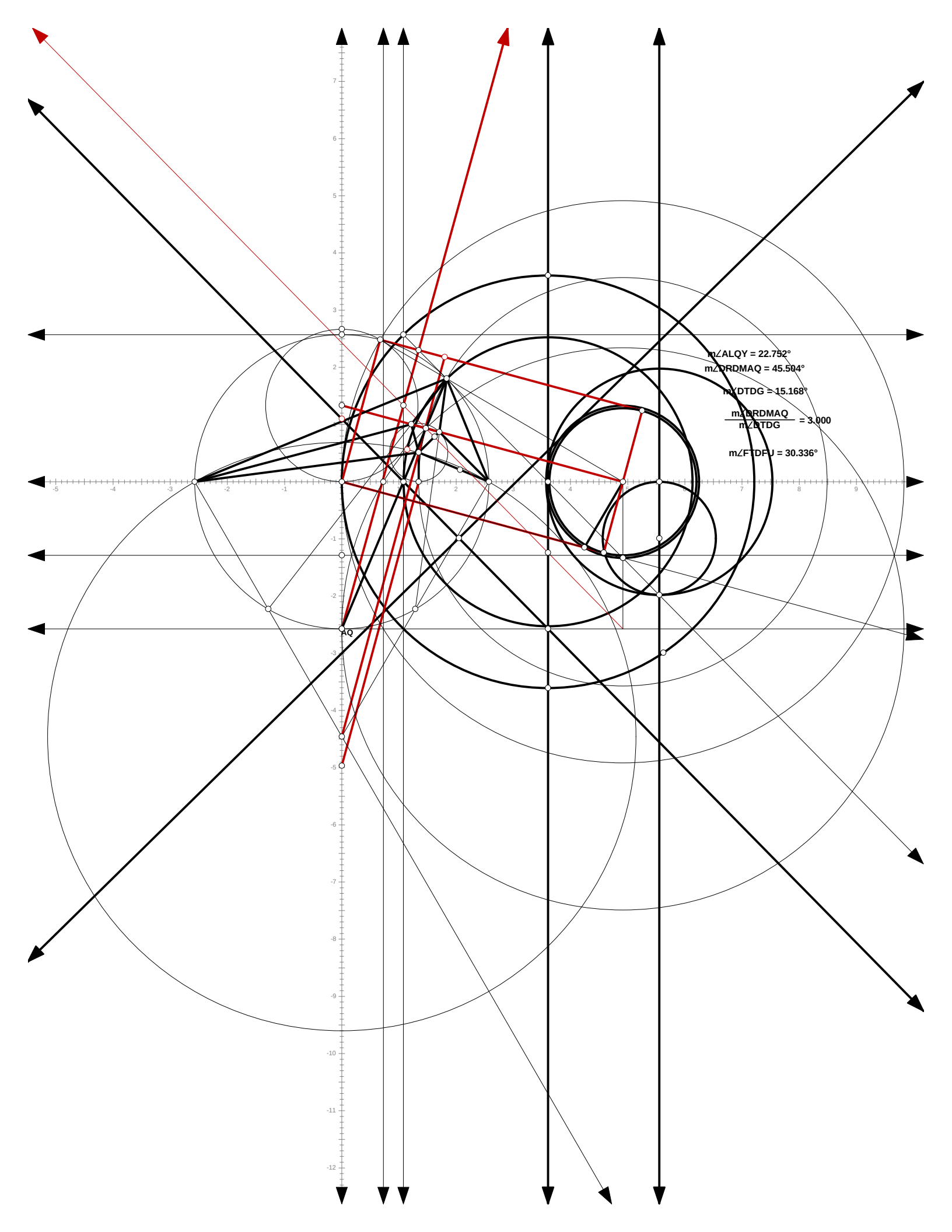
$m\angle AB_2C - 3 \cdot m\angle AB_2D = -0.000°$

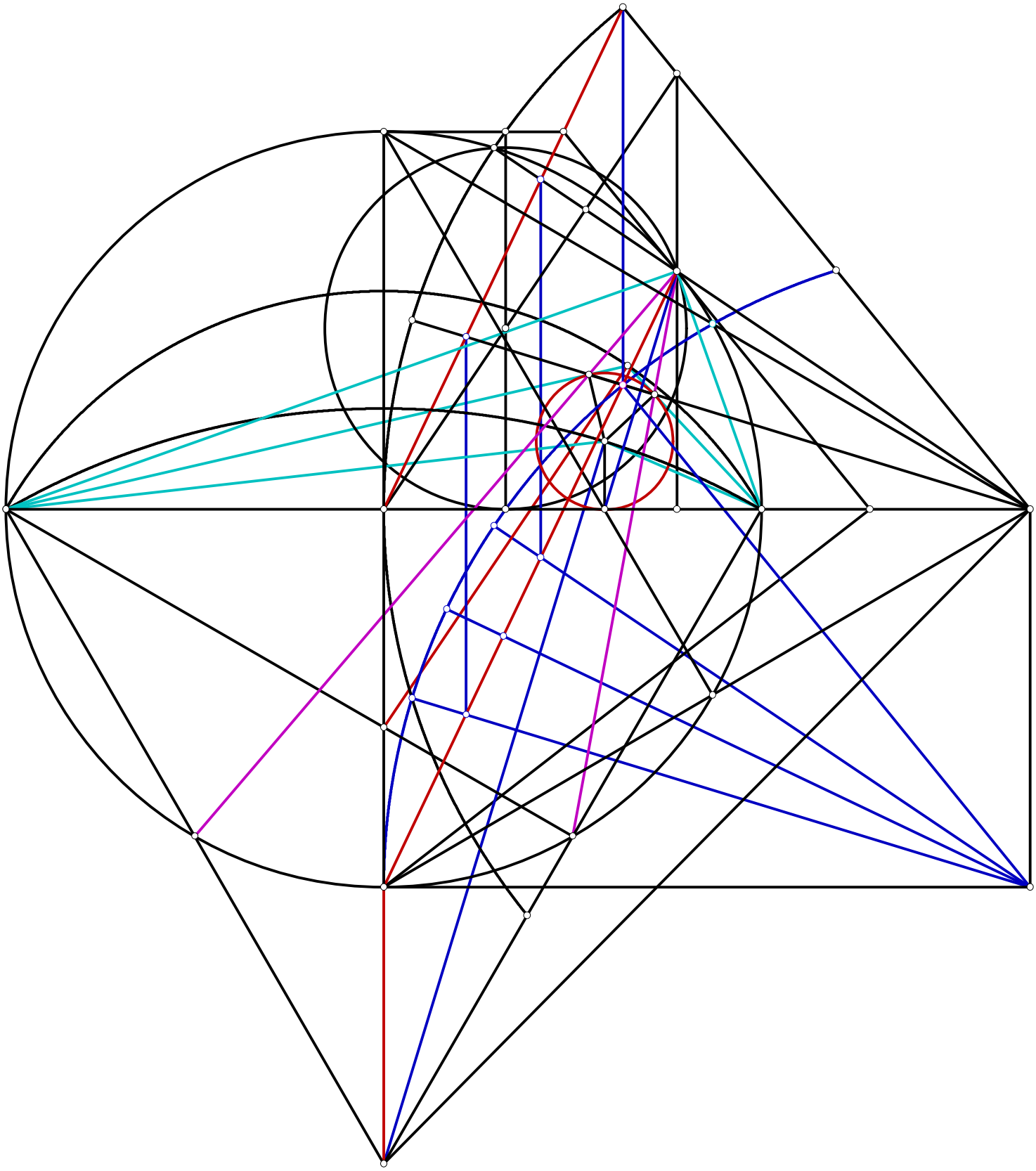
Move C->60 Move C->75

**The process is simple, construct point O and then bisect AOC. From point N construct P from the perpendicular L-which is the midpoint of NS.**
**Bisect APC constructing E and F which are bisected at point M. Draw S to M to construct point D.**

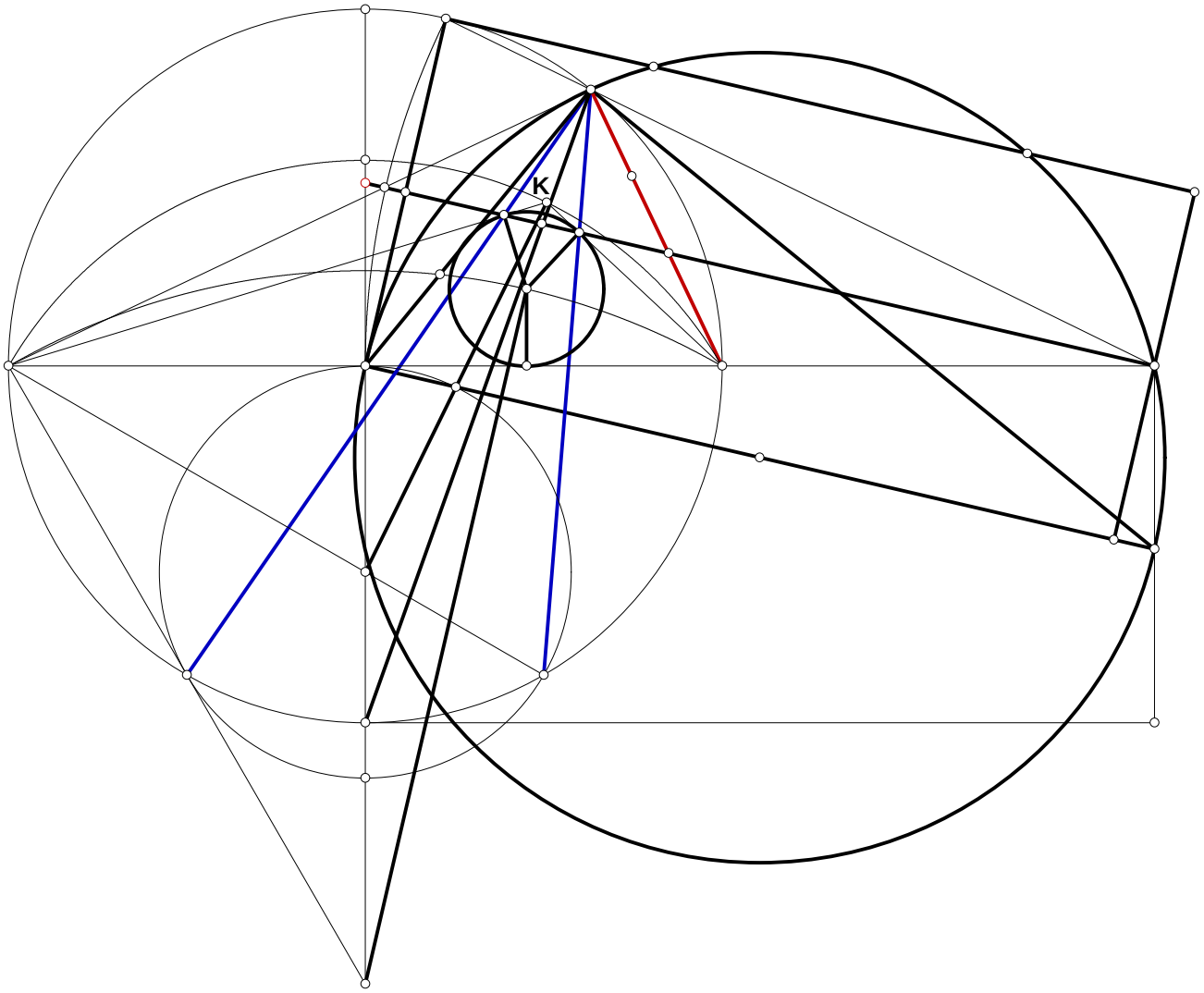
**When there is no difference between two things, those things are said to be equal. This is true even of tolerance. If a figure is more accurate than the tools used to construct the figure, then that figure is equal, in that tool, to a solution that is tool independant.**

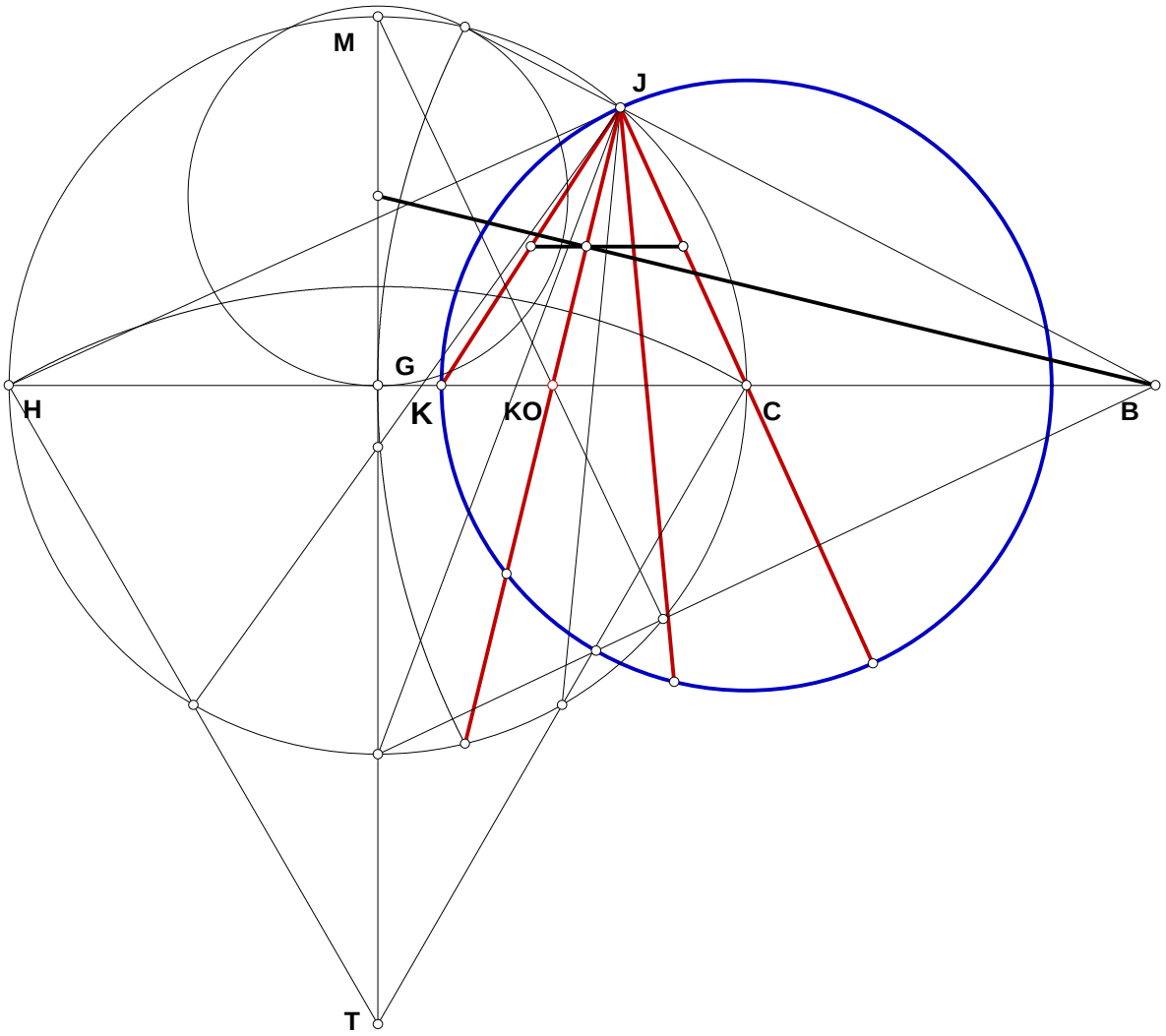

m∠HAL = 8.433°
m∠HDK / m∠HAL = 6.000
m∠HBD = 70.298°
m∠FBD = 23.433°
m∠HBD / m∠FBD = 3.000
m∠DHM = 25.298°
m∠DHM / m∠HAL = 3.000
m∠MLA = 90.000°
m∠HDK = 50.596°
m∠FAC = 9.043°
m∠HAD = 33.731°
m∠HQD = 25.298°
m∠PQD = 8.433°
m∠HAD / m∠PQD = 4.000
m∠HQD / m∠PQD = 3.000
m∠DEAD = 16.865°
m∠FEKD = 16.865°
K
J
H
L
EQ
G
F
I
Q
E
D
C
B
A
M

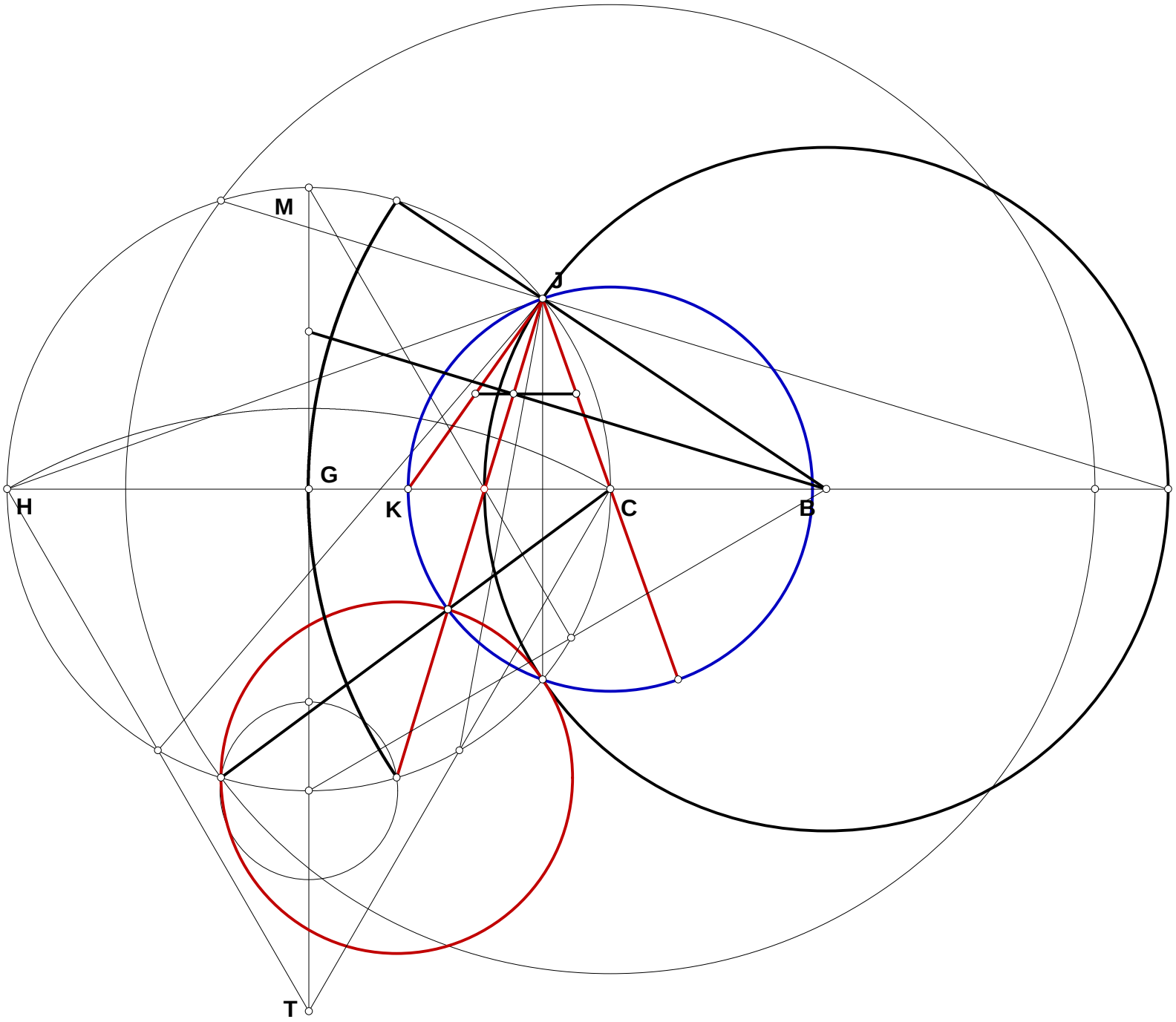

m∠ALQY = 22.752°
m∠DRDMAQ = 45.504°
m∠DTDG = 15.168°
m∠DRDMAQ / m∠DTDG = 3.000
m∠FTDFU = 30.336°
AQ



**Pull K.**




M
J
G
H
K
KO
C
B
T


M
J
G
H
K
C
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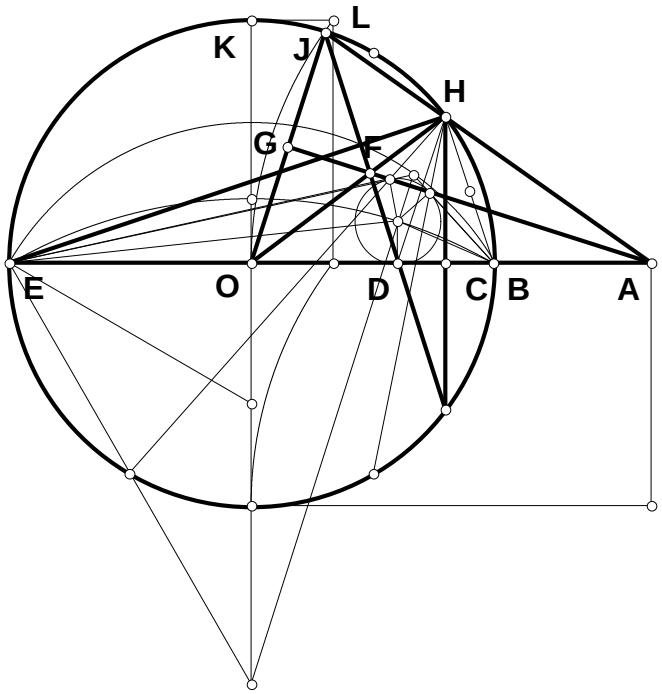
T

# 050601.MCD

## Just some Algebraic Names



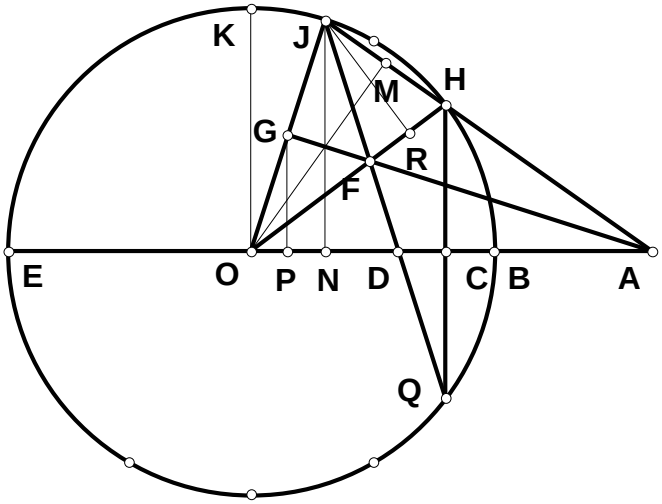

$AB := .818 \qquad N := 4.082$

$AE := AB \cdot N \qquad BE := AE - AB$

$BO := \frac{BE}{2} \qquad AO := AB + BO$

$AJ := AO \qquad JO := BO \qquad GO := \frac{JO}{2}$

$AG := \sqrt{AO^2 - GO^2}$




$AP := \frac{AG^2}{AO} \qquad OP := AO - AP$

$NO := 2 \cdot OP \qquad AN := AO - NO$

$JM := NO \qquad HO := BO$

$HJ := 2 \cdot JM \qquad AH := AJ - HJ$

$AC := \frac{AN \cdot AH}{AJ} \qquad CH := \sqrt{AH^2 - AC^2}$

$HQ := 2 \cdot CH \qquad CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH} \qquad CQ := CH$

$JQ := \sqrt{(CQ + JN)^2 + CN^2} \qquad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO} \qquad JR := \sqrt{JO^2 - OR^2}$

$FO := \frac{JO \cdot GO}{OR} \qquad FJ := FO \qquad DQ := \frac{JQ \cdot CQ}{CQ + JN} \qquad DF := JQ - (DQ + FJ) \qquad FH := HO - FO$

$FG := \frac{JR \cdot GO}{OR} \qquad AF := AG - FG$

$$AE - AB \cdot N = 0 \quad BE - (AB \cdot N - AB) = 0 \quad BO - \frac{1}{2} \cdot AB \cdot (N - 1) = 0$$

$$AO - \frac{1}{2} \cdot AB \cdot (1 + N) = 0 \qquad AG - \frac{AB}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} = 0$$

$$AP - \frac{AB}{8} \cdot (N + 3) \cdot \frac{(3 \cdot N + 1)}{(1 + N)} = 0 \qquad OP - \frac{AB}{8} \cdot \frac{(N - 1)^2}{(1 + N)} = 0 \qquad NO - \frac{AB}{4} \cdot \frac{(N - 1)^2}{(1 + N)} = 0$$

$$AN - \frac{AB}{4} \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} = 0 \qquad HJ - \frac{AB}{2} \cdot \frac{(N - 1)^2}{(1 + N)} = 0 \qquad AH - 2 \cdot AB \cdot \frac{N}{(1 + N)} = 0$$

$$AC - AB \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1 + N)^3} = 0 \qquad CH - AB \cdot N \cdot \sqrt{N + 3} \cdot \sqrt{3 \cdot N + 1} \cdot \frac{(N - 1)}{(1 + N)^3} = 0$$

$$HQ - 2 \cdot AB \cdot N \cdot \sqrt{N + 3} \cdot \sqrt{3 \cdot N + 1} \cdot \frac{(N - 1)}{(1 + N)^3} = 0 \quad CN - \frac{AB}{4} \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)^2}{(1 + N)^3} = 0$$

$$JN - \frac{AB}{4} \cdot (N - 1) \cdot \sqrt{3 \cdot N + 1} \cdot \frac{\sqrt{N + 3}}{(1 + N)} = 0 \qquad JQ - \frac{AB}{2} \cdot (N - 1) \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(1 + N)^2} = 0$$

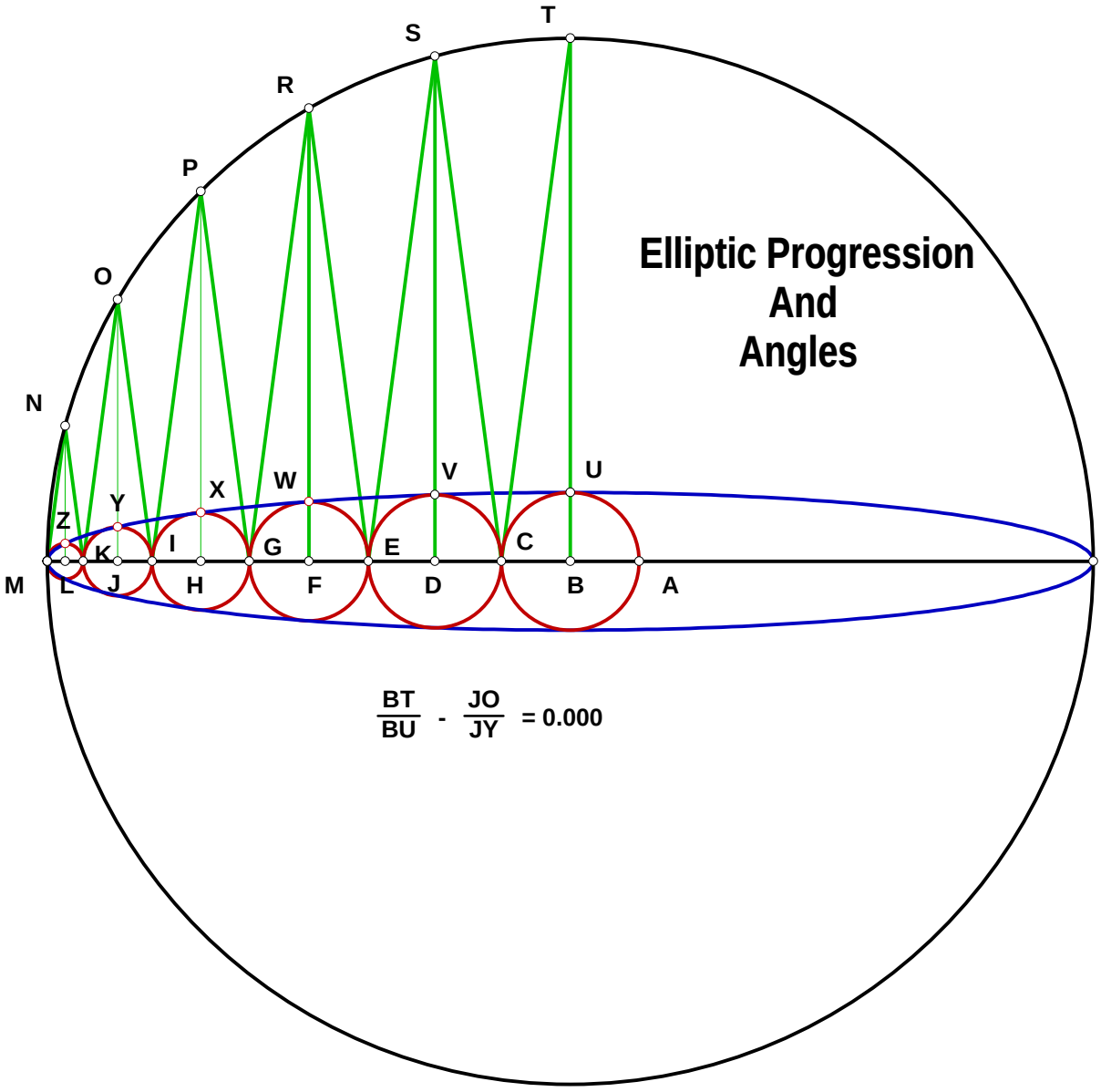
$$OR - \frac{AB}{4} \cdot \left(N^2 + 6 \cdot N + 1\right) \cdot \frac{(N - 1)}{(1 + N)^2} = 0 \qquad JR - \frac{AB}{4} \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{(1 + N)^2} = 0$$

$$FO - \frac{AB}{2} \cdot (1 + N)^2 \cdot \frac{(N - 1)}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad DQ - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{(1 + N)^2} = 0$$

$$DF - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad FH - 2 \cdot N \cdot (N - 1) \cdot \frac{AB}{\left(N^2 + 6 \cdot N + 1\right)} = 0$$

$$FG - \frac{AB}{4} \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{\left(N^2 + 6 \cdot N + 1\right)} = 0 \qquad GO - \frac{AB}{4} \cdot (N - 1) = 0$$

$$AF - \left[ \frac{AB}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \frac{1}{4} \cdot AB \cdot (N - 1)^2 \cdot \sqrt{N + 3} \cdot \frac{\sqrt{3 \cdot N + 1}}{\left(N^2 + 6 \cdot N + 1\right)} \right] = 0$$


T
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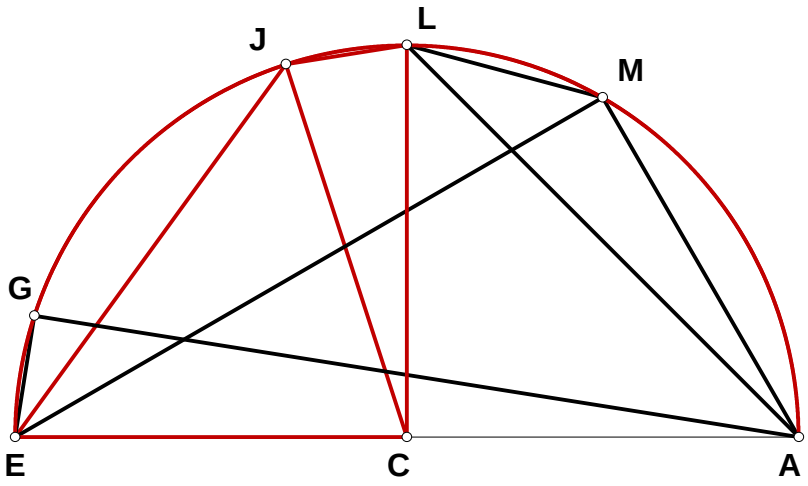
Y
X
W
V
U
Elliptic Progression
And
Angles
BT/BU - JO/JY = 0.000




$EM := \frac{1}{2}\cdot\sqrt{3}$

$AL := \frac{1}{2}\cdot\sqrt{2}$

$JL := \frac{1}{4}\cdot\sqrt{8 - 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

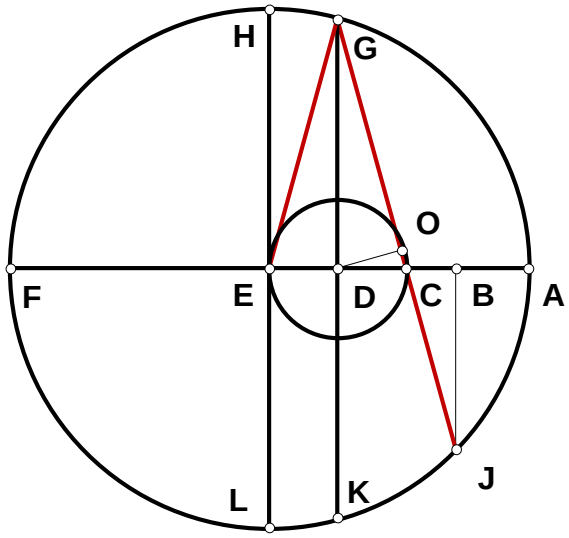
$LM := \frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}$

$AM := \frac{1}{2}$

$EJ := \frac{1}{4}\cdot\sqrt{10 - 2\cdot\sqrt{5}}$

$AG := \frac{1}{4}\cdot\sqrt{8 + 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

## Elliptic Progression Outtake One 0507011




**A method of trisection Algebraically.**

$N := M$

$N \geq 4 = 1 \qquad AF := 6 \qquad AE := \frac{AF}{2}$

$DE := \frac{AF}{N} \qquad AD := AE - DE \qquad DF := AF - AD$

$DG := \sqrt{AD \cdot DF} \qquad CD := DE \qquad EG := AE$

$CO := \frac{CD^2}{EG} \qquad CG := EG \qquad CJ := CG - 4 \cdot CO$

$BC := \frac{CD \cdot CJ}{CG} \qquad AB := AE - (2 \cdot DE + BC)$

$BJ := \frac{DG \cdot BC}{CD} \qquad BD := BC + CD$

$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2} \qquad \frac{JK}{2 \cdot DE} = 1$

**Some Algebraic Names,**

**Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.**

$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0 \qquad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0 \qquad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$

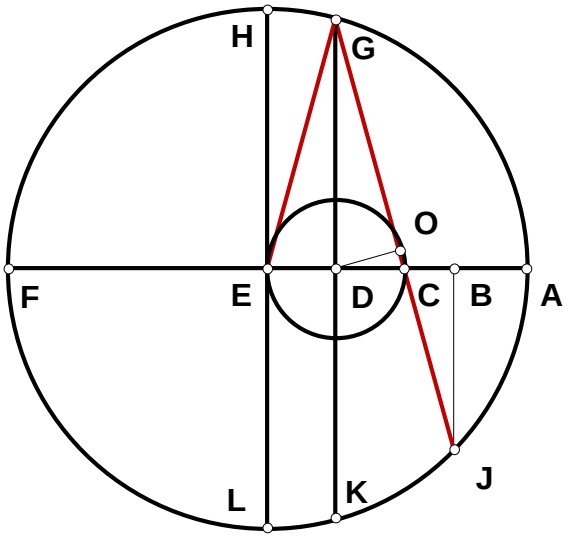
$\frac{2\ AF}{N^2} - CO = 0 \qquad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0 \qquad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$

$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$

**One of the meanings of trisection is solving for the following equation when given AF and AB.**

$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0$

$AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0 \qquad AF \cdot \frac{2 \cdot (N^2 - 8)}{N^3} - BD = 0 \qquad \frac{2 \cdot AF}{N} - JK = 0$




$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - \frac{AF}{AB} = 0 \qquad \frac{AF}{AB} = 6$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - 6 = 0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6$$

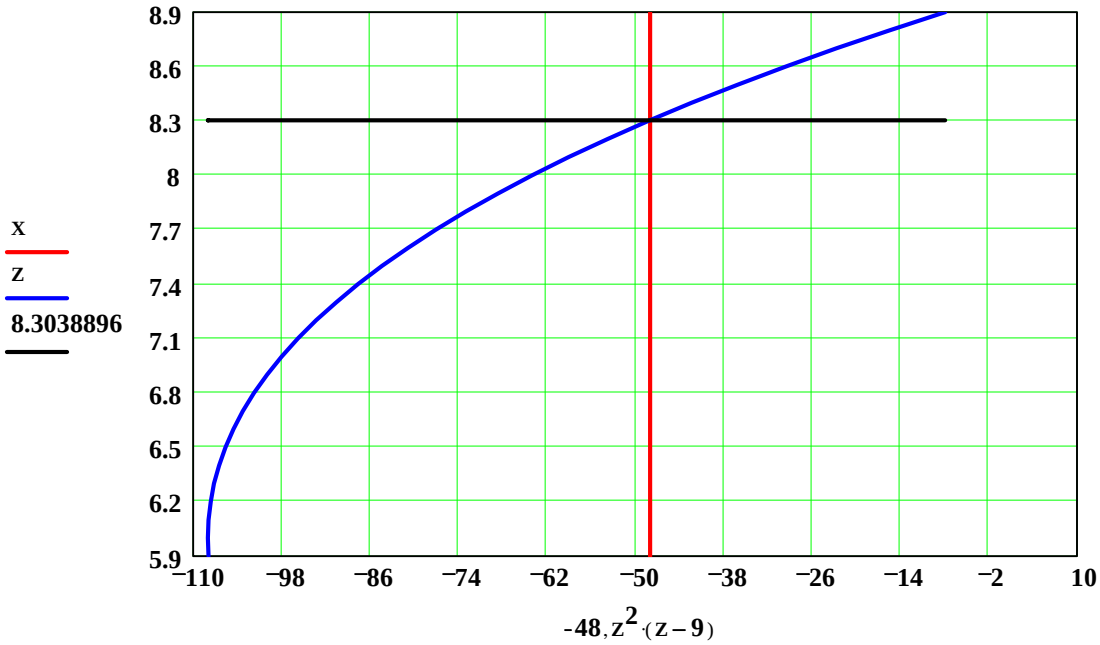
$$2\cdot M^3 - 6\cdot\left[(M+2)\cdot(M-4)^2\right] = 0$$

$$2\cdot M^3 - \left(6\cdot M^3 - 36\cdot M^2 + 192\right) = 0$$

$$4\cdot M^3 - 36\cdot M^2 + 192 = 0$$

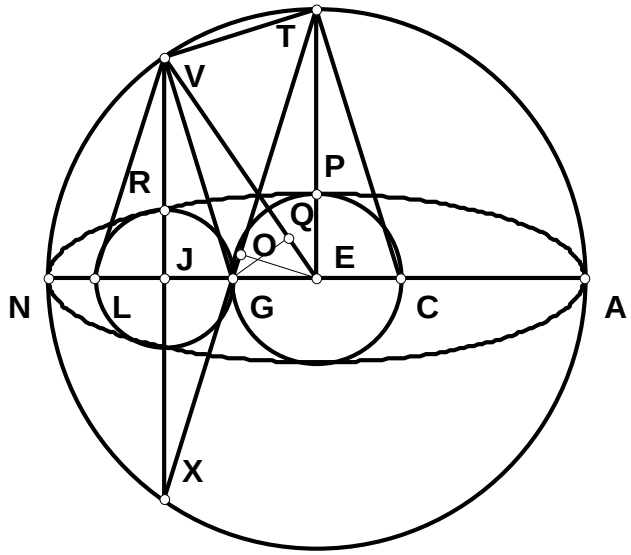
$$M^3 - 9\cdot M^2 + 48 = 0 \qquad M^2\cdot(M-9) + 48 = 0 \qquad M \equiv 8.303889634816388$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6 \qquad DE - \frac{AF}{M} = 0 \qquad Z := 5.9, 6 .. 8.9$$



# Elliptic Progression Outtake Two 0507012

## Angles TEV and EVJ equals CTG.




$N := 6.381 \quad AN := 2.792$

$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$

$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$

$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$

$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$

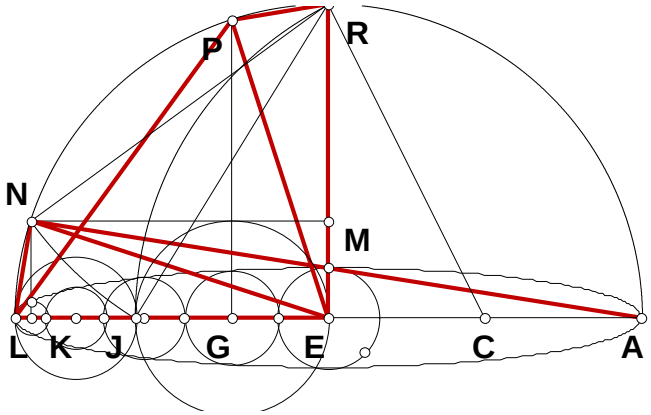
$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \qquad JV := JX$

$EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$

$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \qquad EQ := \frac{EJ \cdot EG}{EV} \quad OQ := \frac{JV \cdot EQ}{EJ} \quad OQ - GJ = 0$

## Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013




$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

***Irrational*** **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

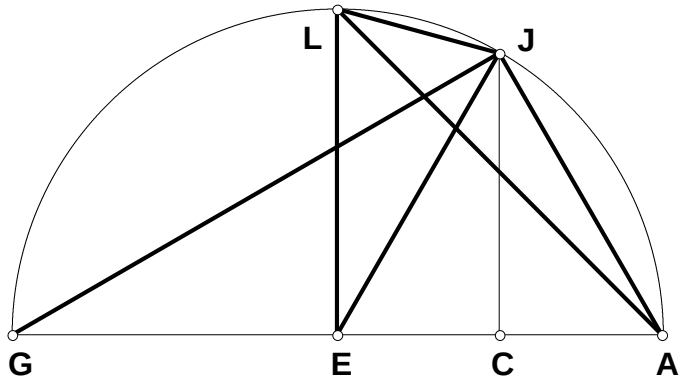
$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

## Outtake Four: Some Names 0507014




$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$

$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$

$EL := AE \quad CE := AC$

$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$

$AL := \sqrt{AE^2 + EL^2}$

$AE := \frac{1}{2} \quad AC := \frac{1}{4} \quad CG := 1 - \frac{1}{4} \quad CJ := \frac{1}{4} \cdot \sqrt{3} \quad JL := \frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}$

$AJ := \frac{1}{2} \quad GJ := \frac{1}{2} \cdot \sqrt{3}_{*} \quad AL := \frac{1}{2} \cdot \sqrt{2}$

# Angles and the Ellipse  050801.mcd

$AC := 1 \qquad CP := \sqrt{2\ AC^2}$

$AP := AC \qquad EP := CP$

$CE := \sqrt{AC^2 + (AP + EP)^2}$

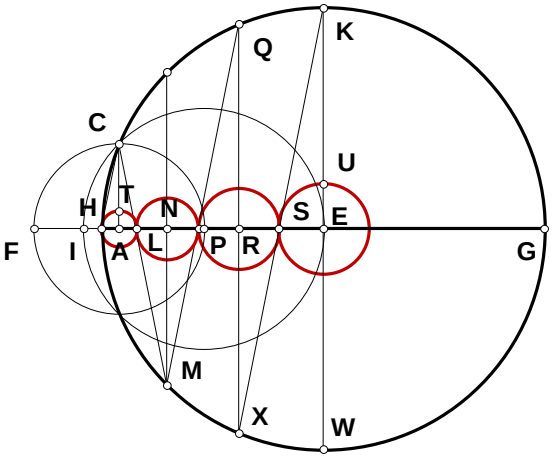
$EK := CE \quad EH := CE$

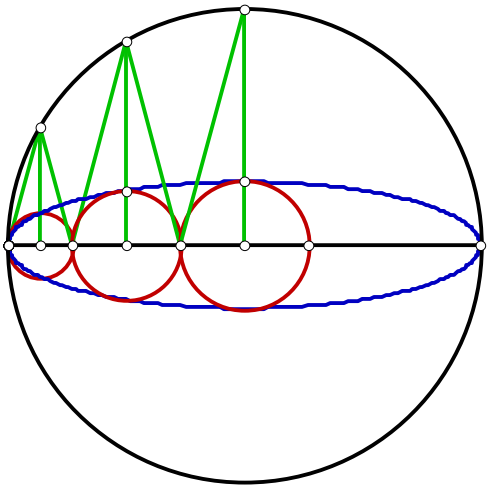
$AE := \sqrt{CE^2 - AC^2}$

$AH := EH - AE$

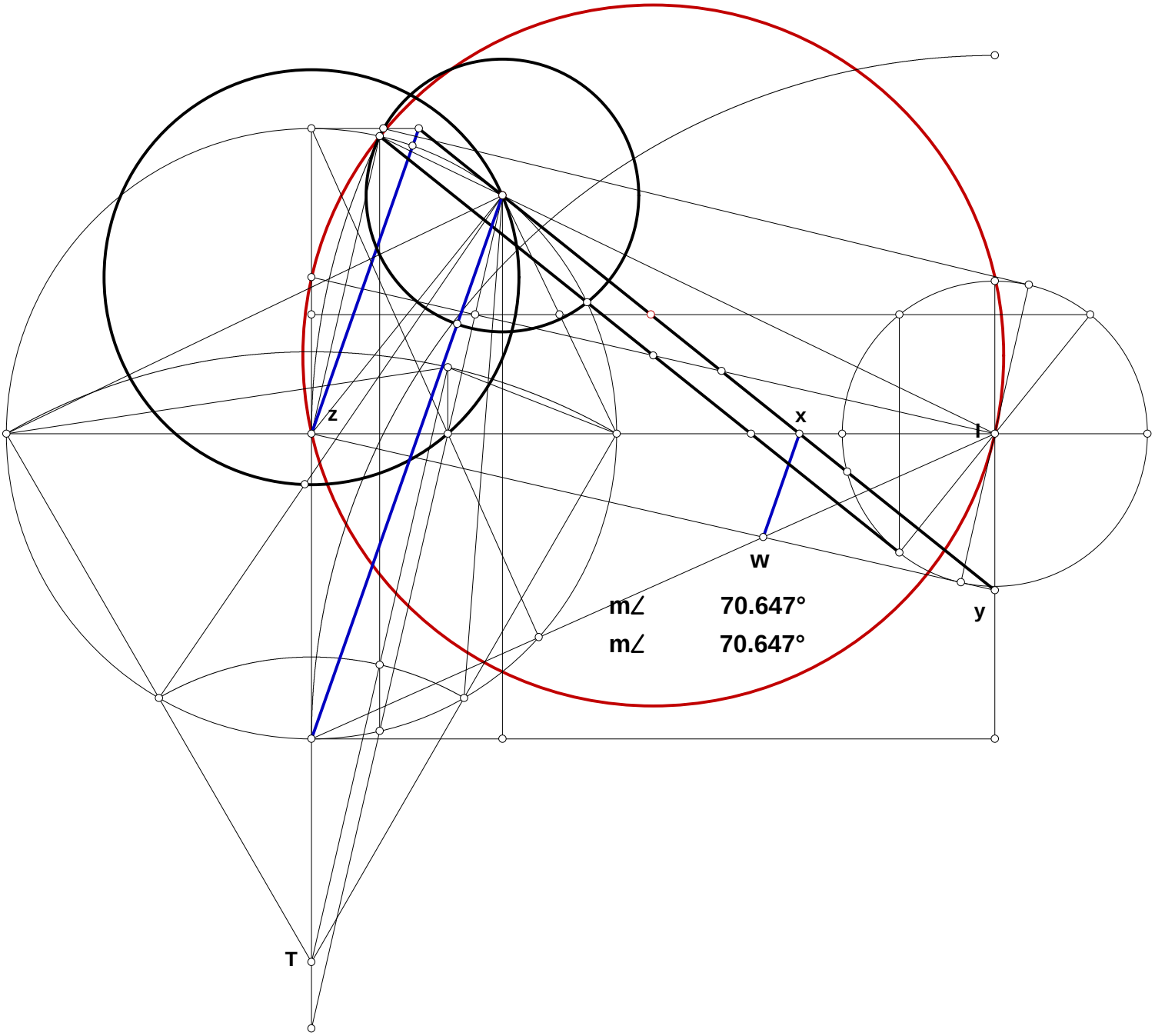
$AT := AH \qquad EU := \frac{AT \cdot EK}{AC}$

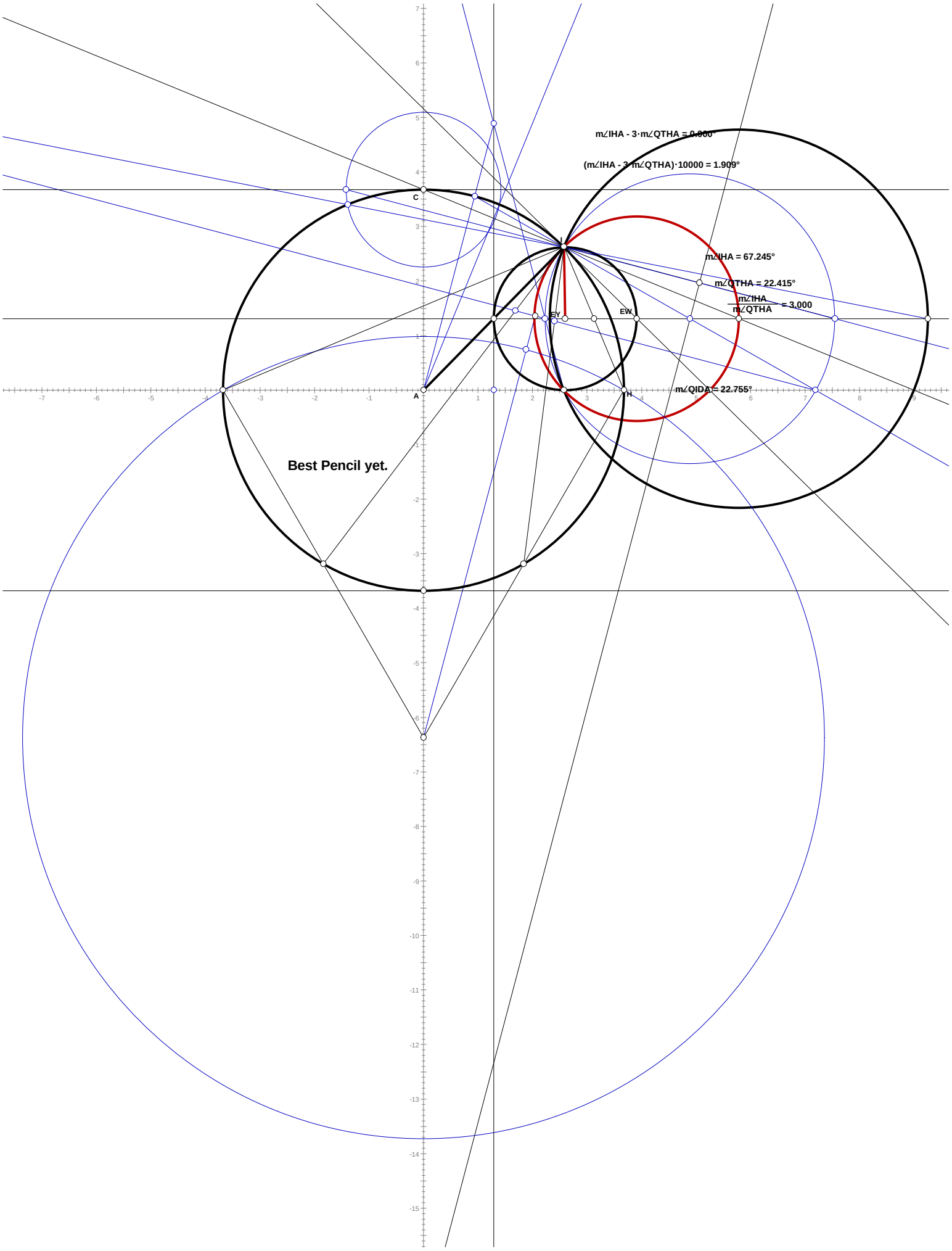
$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$

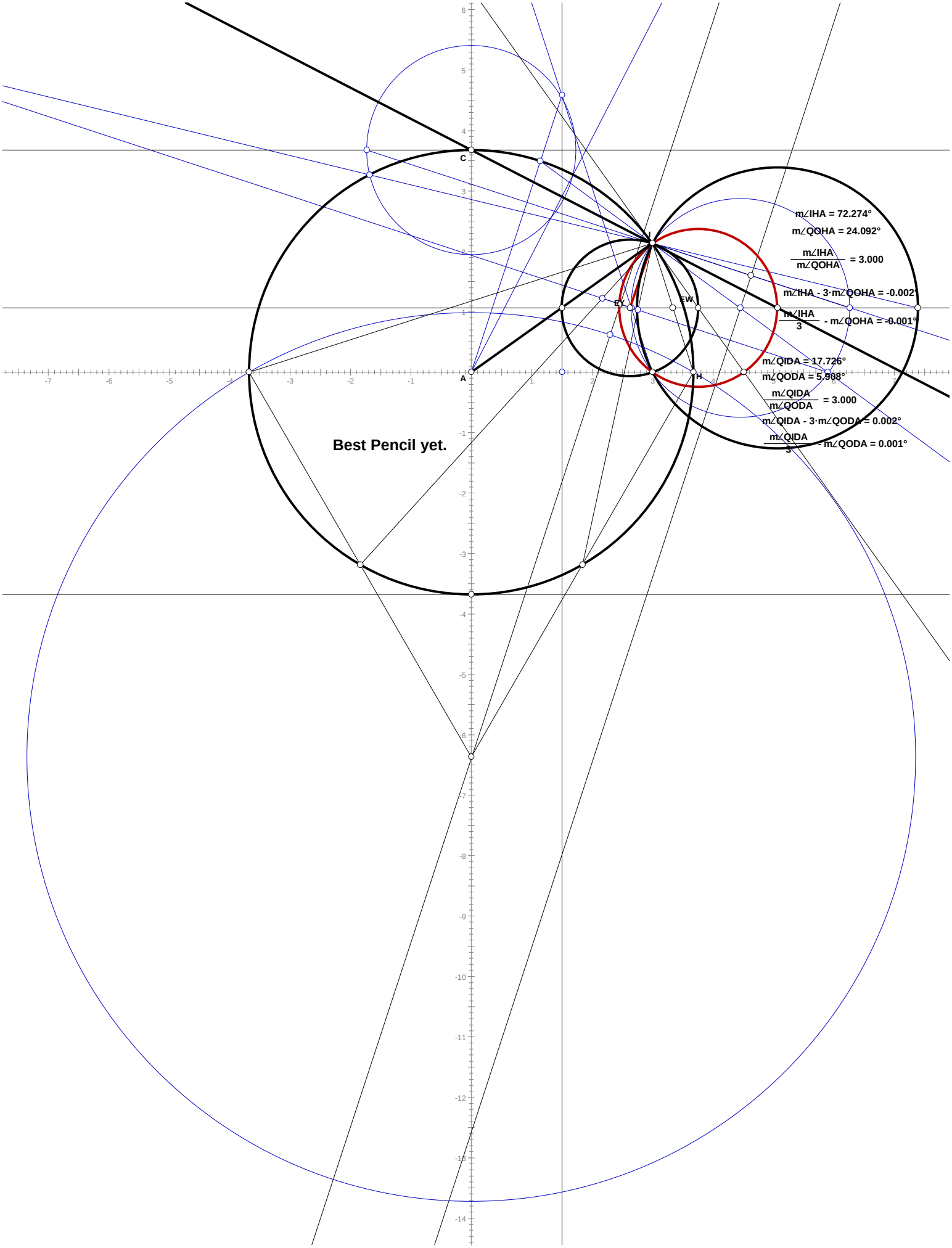


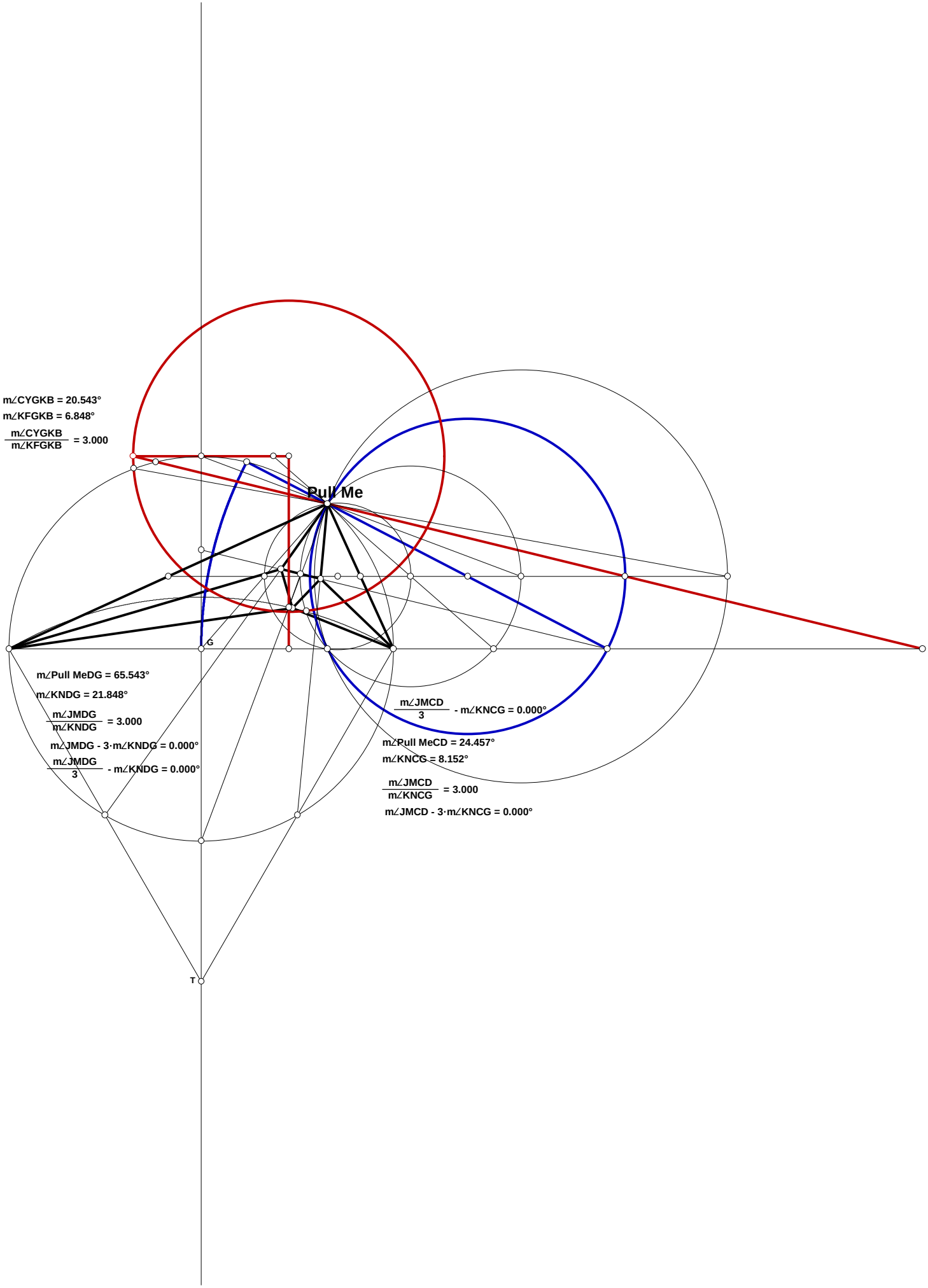



**An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle and from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles. This means that the division of a circle into equal angles may be expressed as an elliptic function.**


z
x
l
w
y
T
m∠ 70.647°
m∠ 70.647°

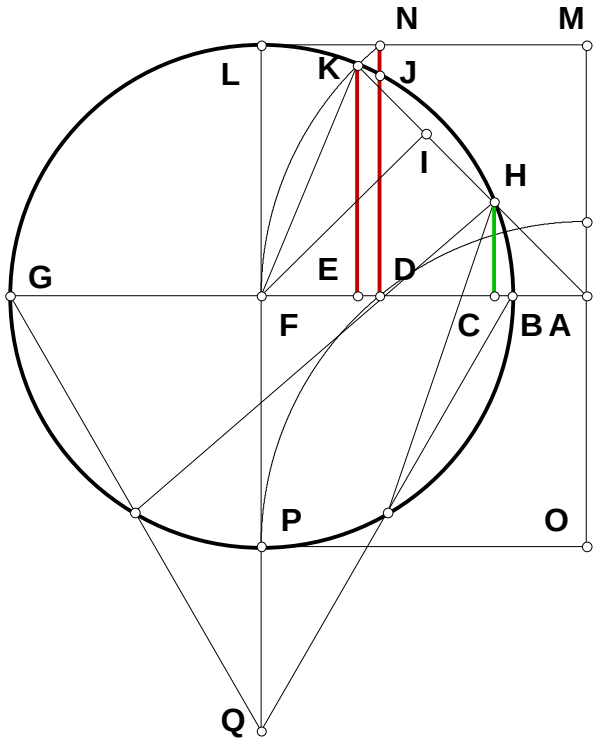

m∠IHA - 3·m∠QTHA = 0.000°
(m∠IHA - 3·m∠QTHA)·10000 = 1.909°
C
I
m∠IHA = 67.245°
m∠QTHA = 22.415°
m∠IHA / m∠QTHA = 3.000
EY
EW
A
H
m∠QIDA = 22.755°
Best Pencil yet.


C
I
EW
H
A
m∠IHA = 72.274°
m∠QOHA = 24.092°
m∠IHA / m∠QOHA = 3.000
m∠IHA - 3·m∠QOHA = -0.002°
m∠IHA / 3 - m∠QOHA = -0.001°
m∠QIDA = 17.726°
m∠QODA = 5.908°
m∠QIDA / m∠QODA = 3.000
m∠QIDA - 3·m∠QODA = 0.002°
m∠QIDA / 3 - m∠QODA = 0.001°
Best Pencil yet.


m∠CYGKB = 20.543°
m∠KFGKB = 6.848°
m∠CYGKB / m∠KFGKB = 3.000
Pull Me
G
T
m∠Pull MeDG = 65.543°
m∠KNDG = 21.848°
m∠JMDG / m∠KNDG = 3.000
m∠JMDG - 3·m∠KNDG = 0.000°
m∠JMDG / 3 - m∠KNDG = 0.000°
m∠JMCD / 3 - m∠KNCG = 0.000°
m∠Pull MeCD = 24.457°
m∠KNCG = 8.152°
m∠JMCD / m∠KNCG = 3.000
m∠JMCD - 3·m∠KNCG = 0.000°

# On Trisection 051301

## For any given trisection what is the Algebraic names of BC and BE taking BG as unit?




$N := 9 \qquad BG := 1 \qquad AG := BG \cdot N$

$AB := AG - BG \qquad BF := \frac{BG}{2}$

$AF := AB + BF \qquad AN := AF \qquad AK := AN$

$FK := BF \qquad S_1 := AF \qquad S_2 := AK \qquad S_3 := FK$

$AE := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad AI := AE$

$IK := AK - AI \qquad HI := IK \qquad AH := AK - (HI + IK)$

$AC := \frac{AE \cdot AH}{AK} \qquad BC := AC - AB \qquad BE := AE - AB$

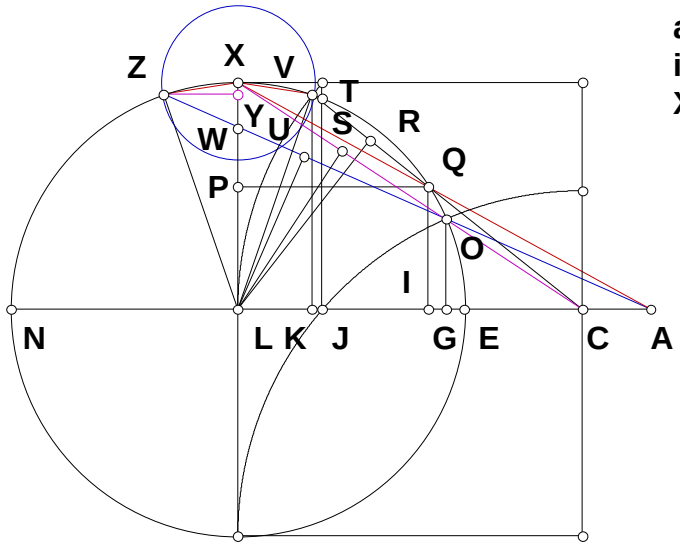
## Some Algebraic Names:

$N - 1 - AB = 0 \qquad \frac{1}{2} - BF = 0 \qquad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \qquad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$

$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \qquad 2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \qquad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \qquad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

## On Trisection 051401.MCD




**For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?**

$N := 5 \quad CE := 1 \quad CN := CE \cdot N$

$EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$CL := CE + EL$

$CV := CL \quad S_1 := CL \quad S_2 := CV$

$S_3 := LV \quad CK := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1}$

$CR := CK \quad RV := CV - CR \quad QR := RV \quad CQ := CV - (QR + RV) \quad CI := \frac{CK \cdot CQ}{CV} \quad EI := CI - CE$

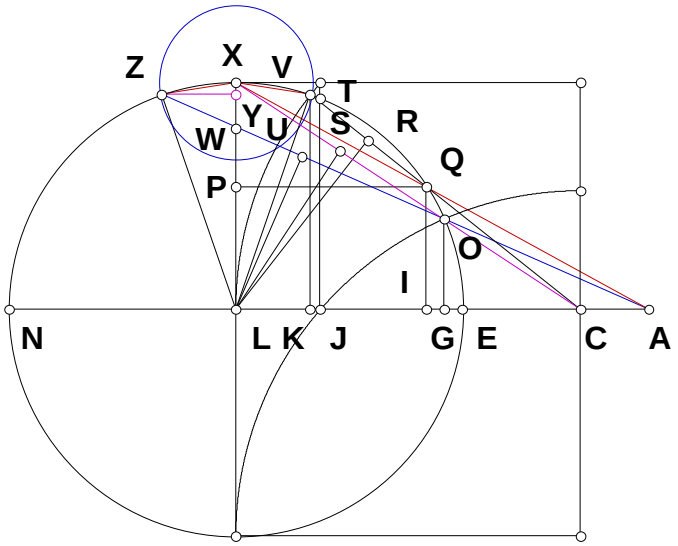
$IN := EN - EI \quad IQ := \sqrt{EI \cdot IN} \quad LP := IQ \quad LX := EL \quad IL := EL - EI \quad PX := LX - LP \quad AL := \frac{IL \cdot LX}{PX}$

$AE := AL - EL \quad AC := AE - CE \quad AI := AC + CI \quad AX := \sqrt{AL^2 + LX^2} \quad AQ := \frac{AX \cdot AI}{AL} \quad QX := AX - AQ$

$CX := \sqrt{CL^2 + LX^2} \quad CS := \frac{CL^2}{CX} \quad SX := CX - CS \quad OS := SX \quad CO := CX - (SX + OS)$

$GO := \frac{LX \cdot CO}{CX} \quad CG := \frac{CL \cdot CO}{CX} \quad EG := CG - CE \quad AG := AE + EG \quad AO := \sqrt{AG^2 + GO^2}$

$AU := \frac{AG \cdot AL}{AO} \quad OU := AU - AO \quad UZ := OU \quad AZ := AO + (OU + UZ)$




$$LW := \frac{GO \cdot AL}{AG} \qquad AW := \frac{AO \cdot AL}{AG}$$

$$WZ := AZ - AW \qquad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \qquad YX := LX - (LW + WY)$$

$$XZ := \sqrt{YX^2 + YZ^2}$$

**Some Algebraic Names:**

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0$$

$$2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0 \qquad CE \cdot \frac{N^3 + 6\,N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$

$$\frac{1}{2}\cdot CE\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AL=0$$

$$CE\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AE=0$$

$$-CE\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AC=0$$

$$\frac{CE\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-AI=0$$

$$\frac{1}{2}\cdot CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-AX=0$$

$$CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-AQ=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}}\cdot CE\cdot(N-1)\cdot\frac{\left[\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right.}{2\cdot(1+N)^3}\ \ldots=0$$

$$\left.+3\cdot N+1\right]$$

$$\left.+3\cdot N+1\right]$$

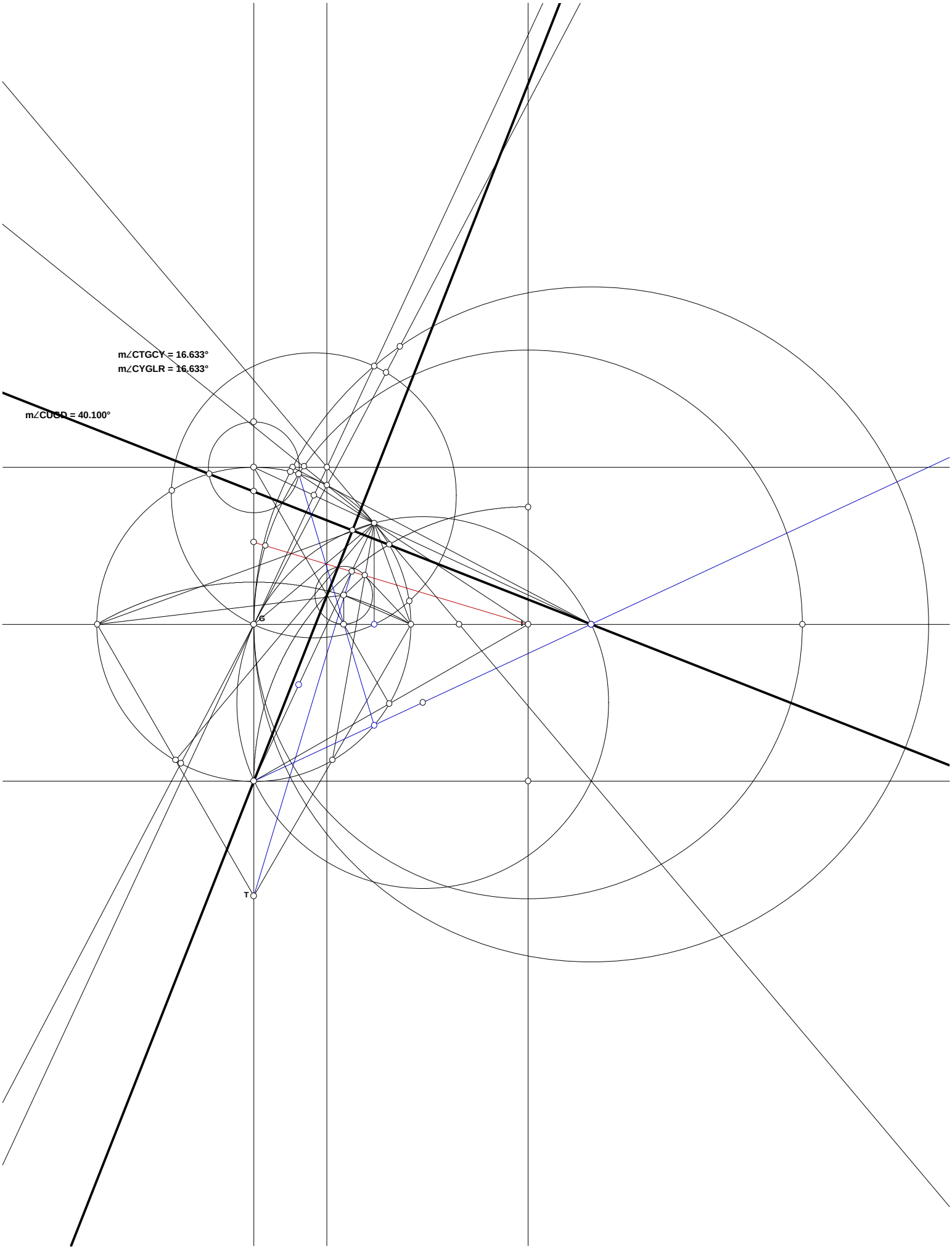
$$+-QX$$

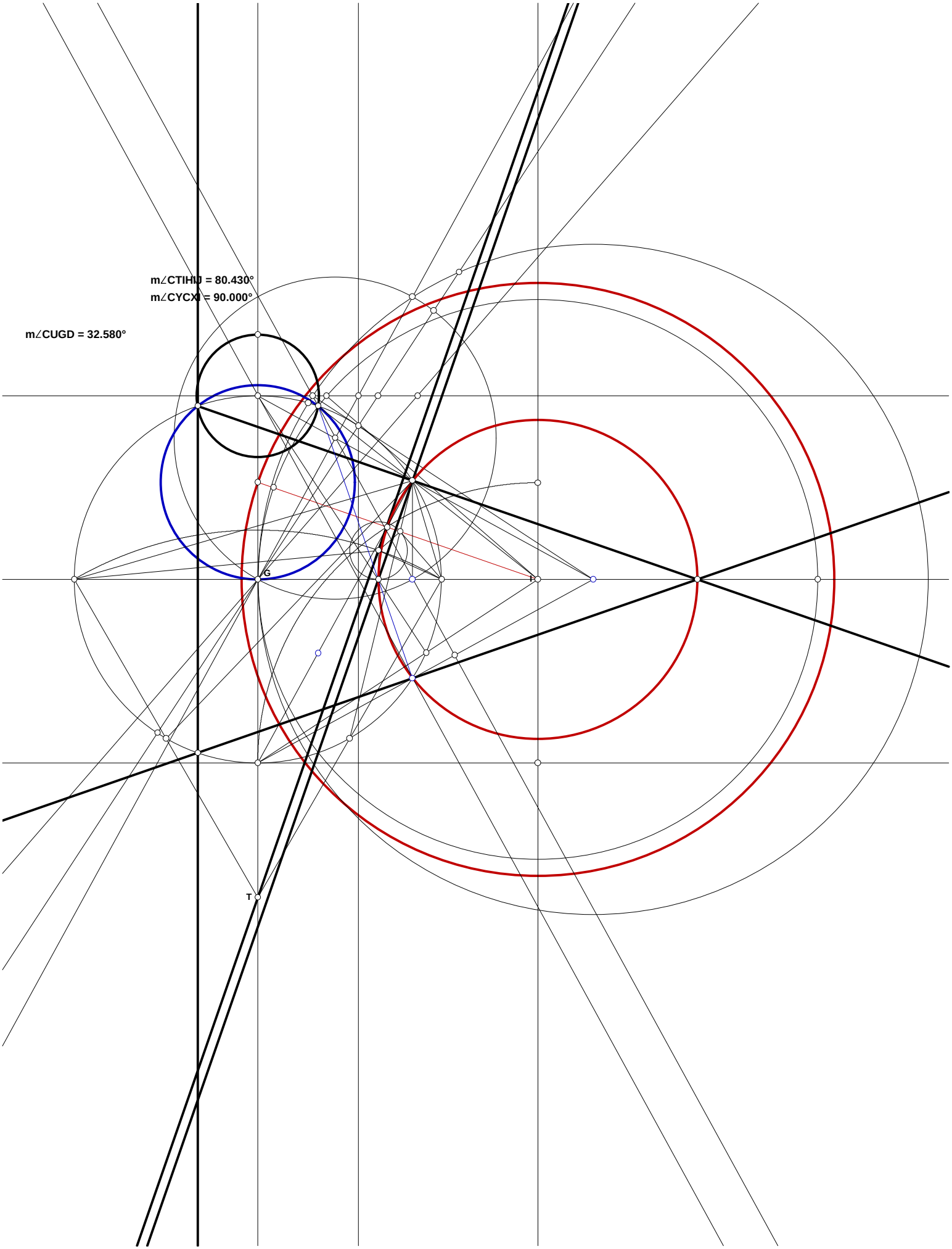
$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)}-CX=0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}}-CS=0 \qquad \frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}}-SX=0$$

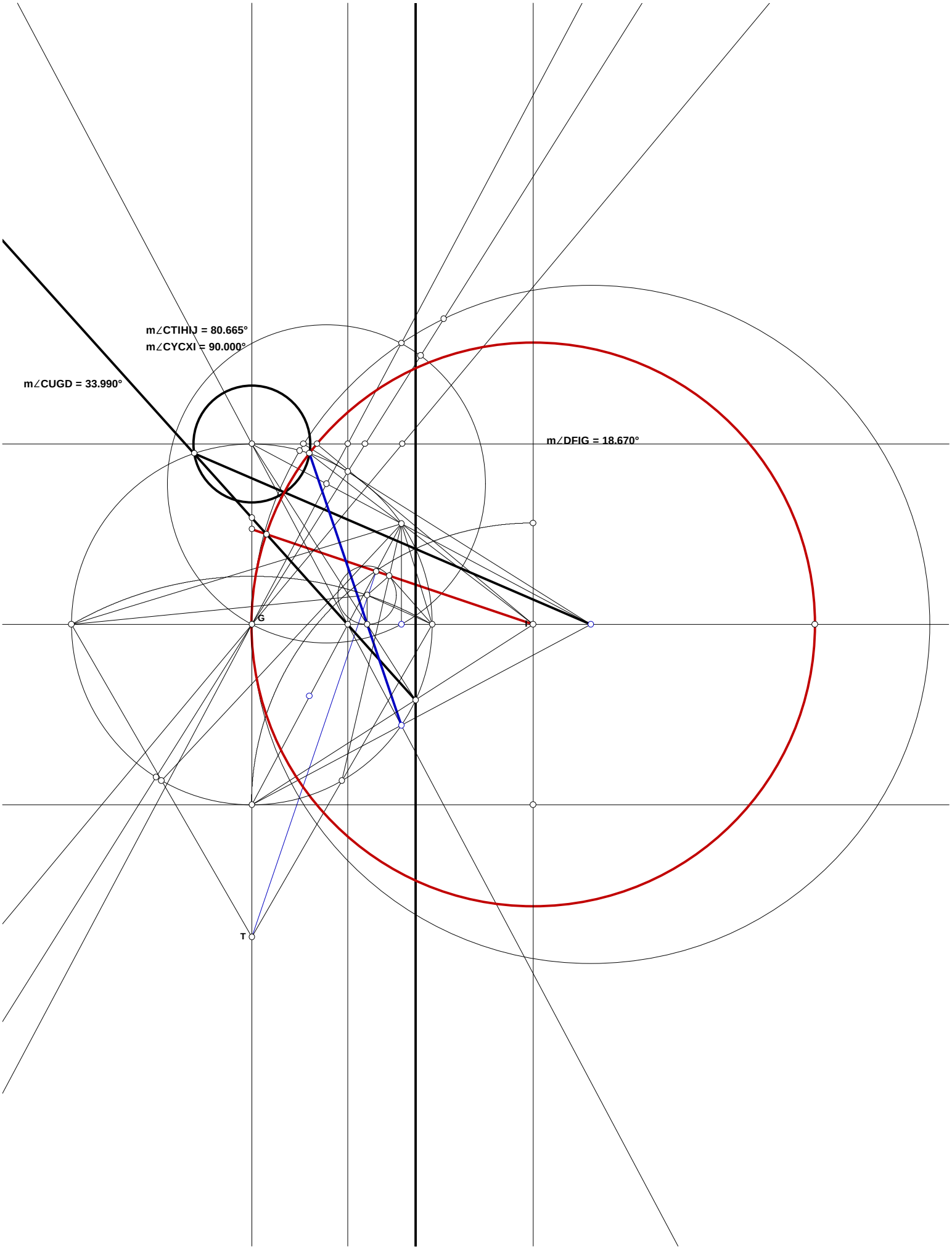
$$\frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N-CO=0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)}-GO=0 \qquad CE\cdot\frac{N^2+N}{\left(1+N^2\right)}-CG=0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)}-EG=0$$

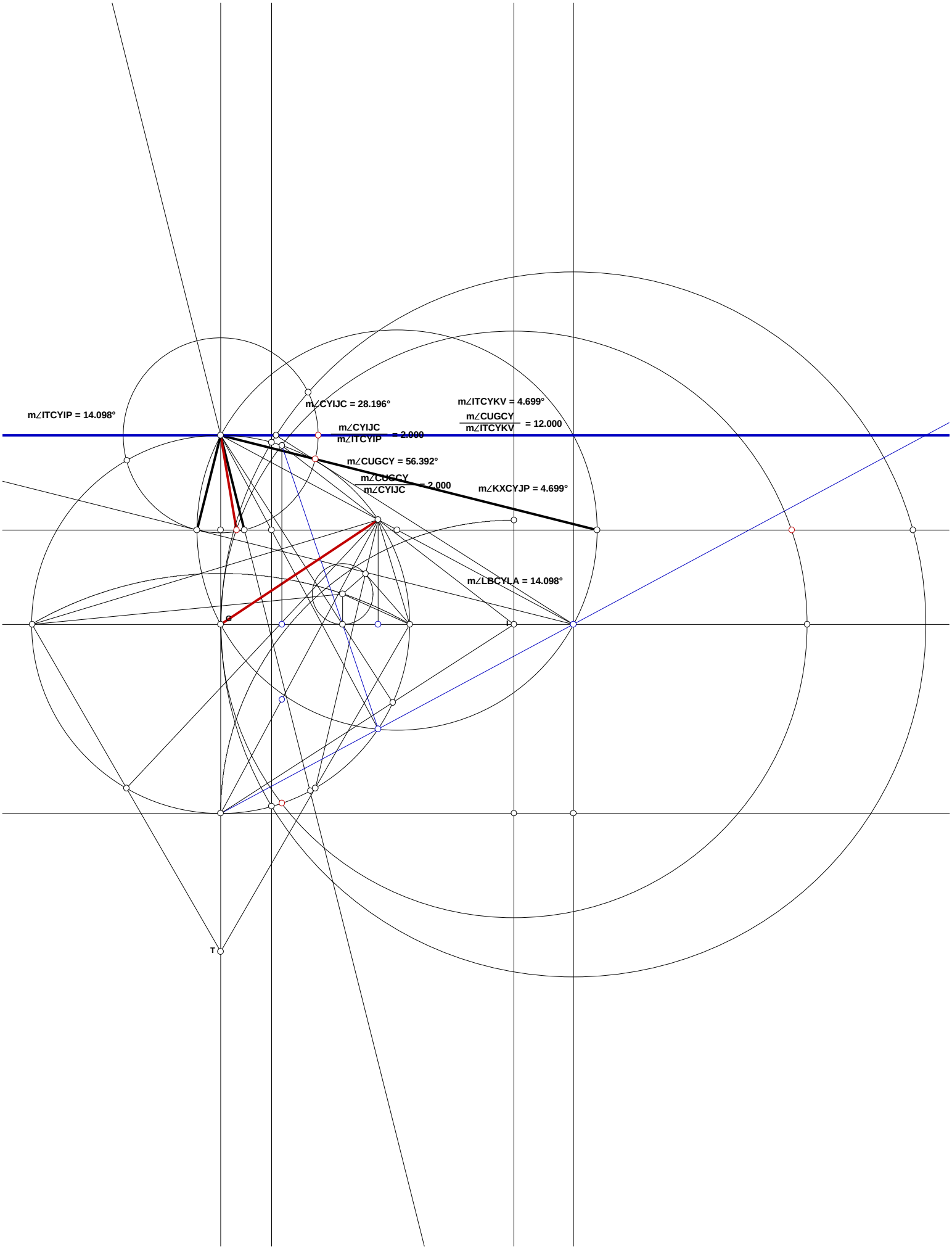
$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-2\cdot N+\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]}-AG=0$$

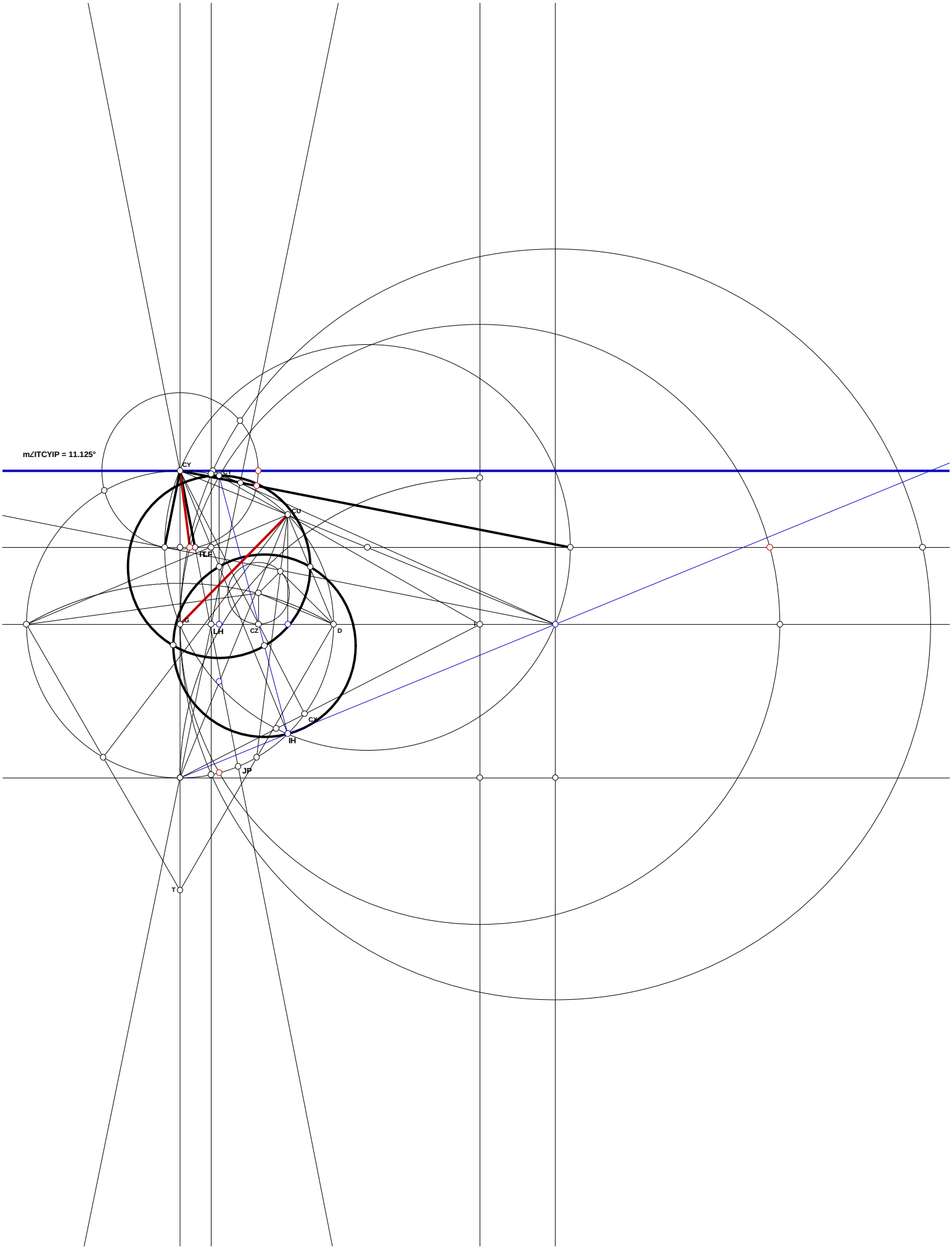
$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\begin{array}{l}\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\ +\,-7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\end{array}\right]}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}}-XZ=0$$

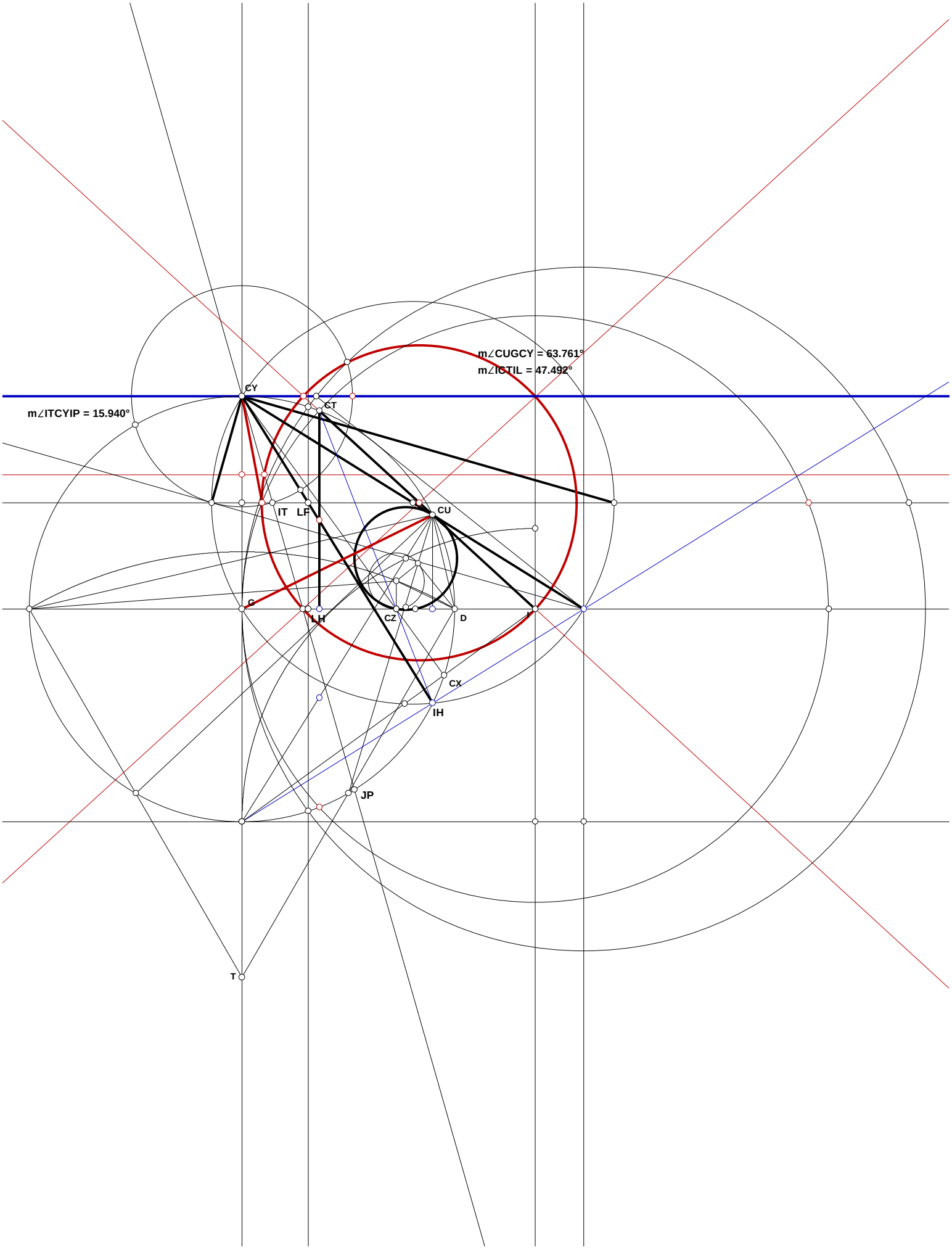

m∠CTGCY = 16.633°
m∠CYGLR = 16.633°
m∠CUGD = 40.100°
G
T

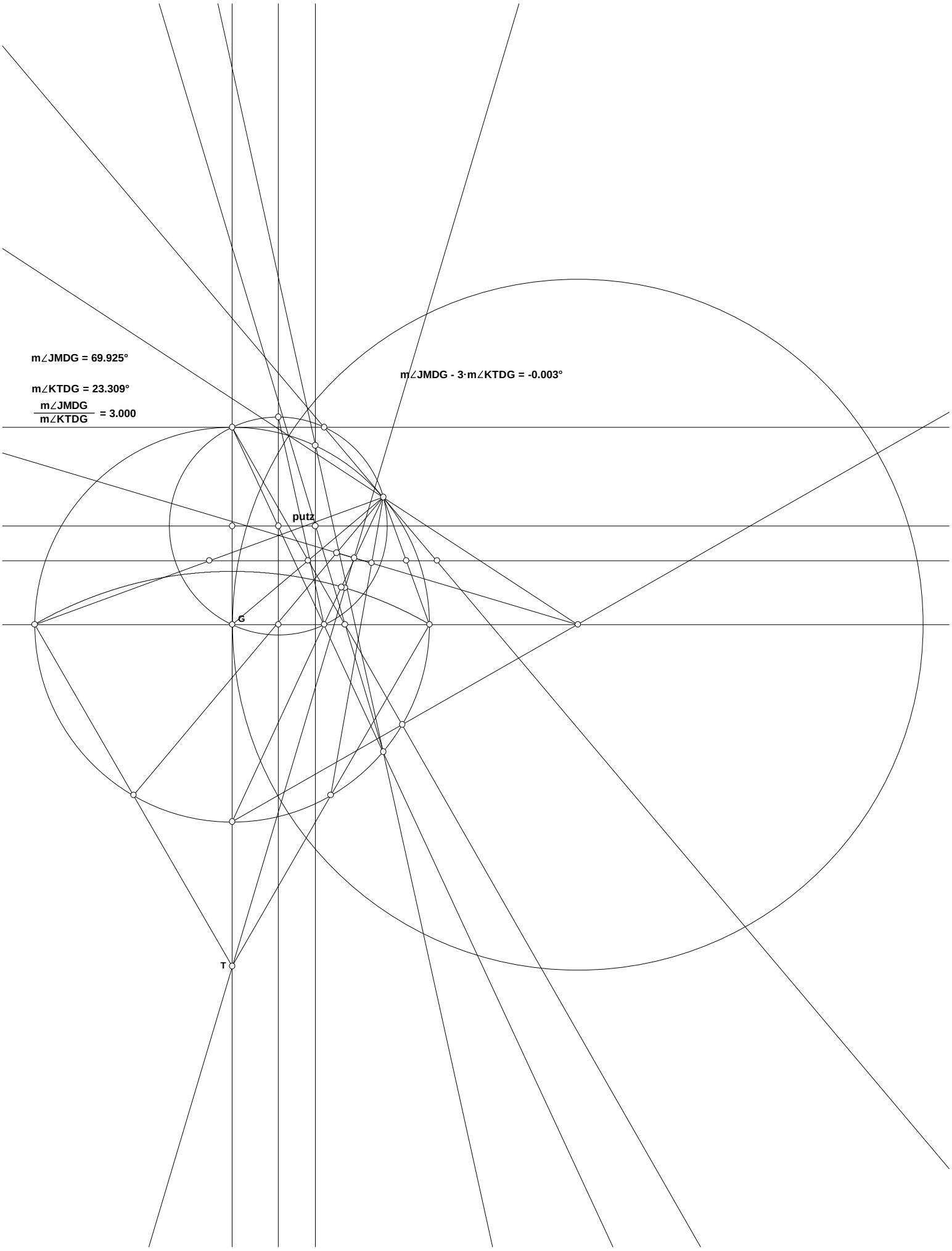

m∠CTIHIJ = 80.430°
m∠CYCX = 90.000°
m∠CUGD = 32.580°
G
I
T

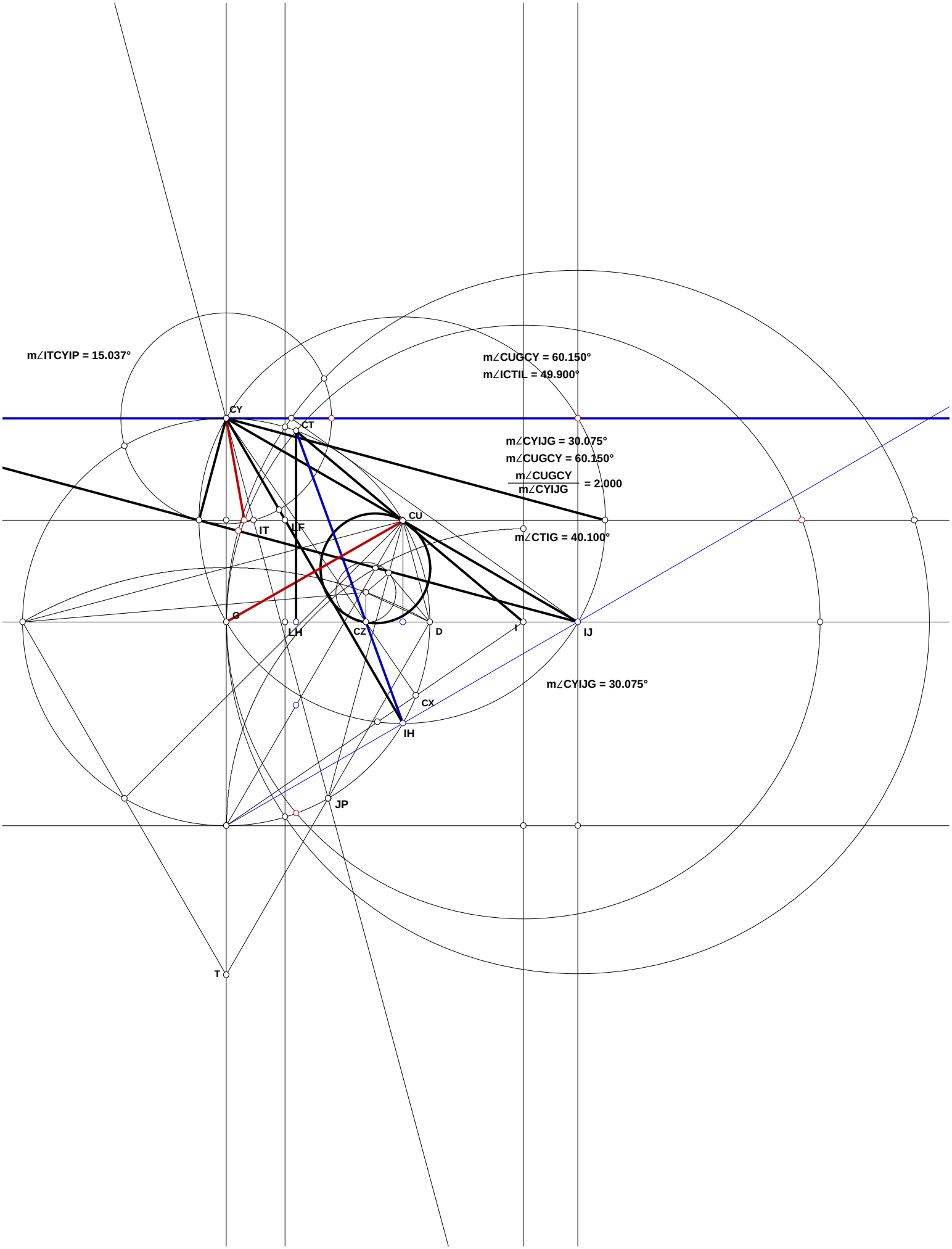

m∠CTIHIJ = 80.665°
m∠CYCXI = 90.000°
m∠CUGD = 33.990°
m∠DFIG = 18.670°
G
T

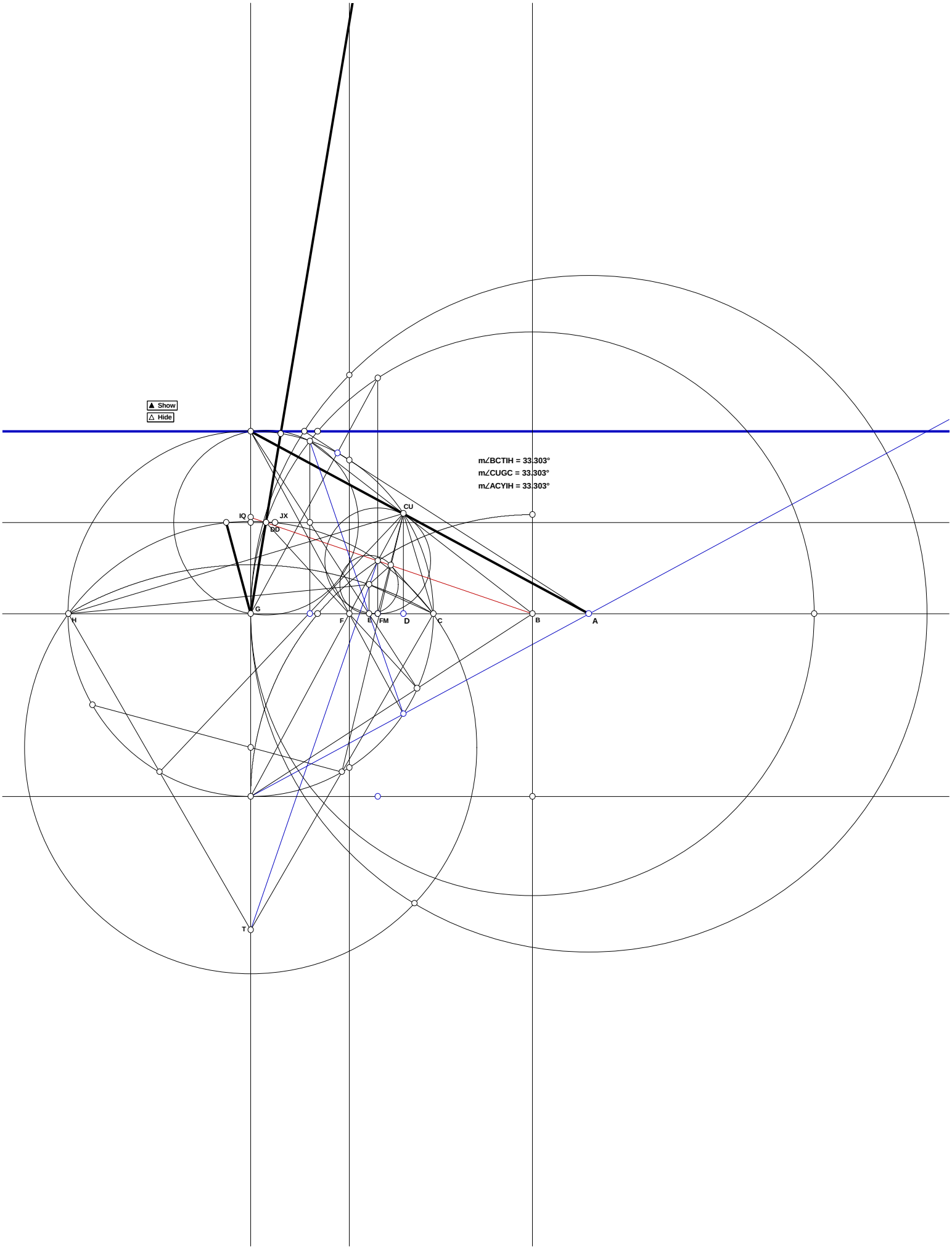

m∠ITCYIP = 14.098°
m∠CYIJC = 28.196°
m∠ITCYKV = 4.699°
m∠CUGCY / m∠ITCYKV = 12.000
m∠CYIJC / m∠ITCYIP = 2.000
m∠CUGCY = 56.392°
m∠CUGCY / m∠CYIJC = 2.000
m∠KXCYJP = 4.699°
m∠LBCYLA = 14.098°
G
I
T

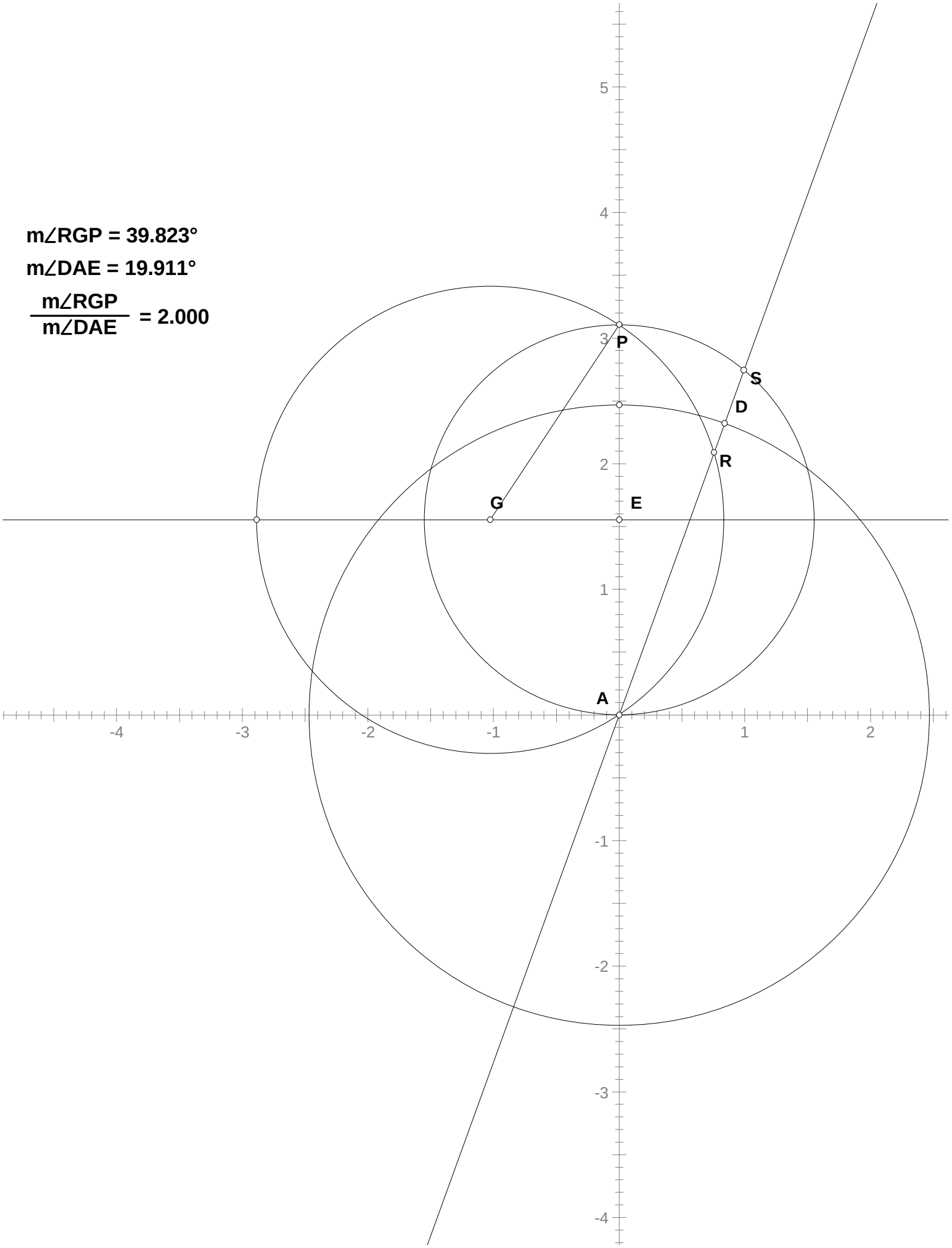

m∠ITCYIP = 11.125°
CY
CT
CO
ITLE
C
LH
CZ
D
CX
IH
JP
T

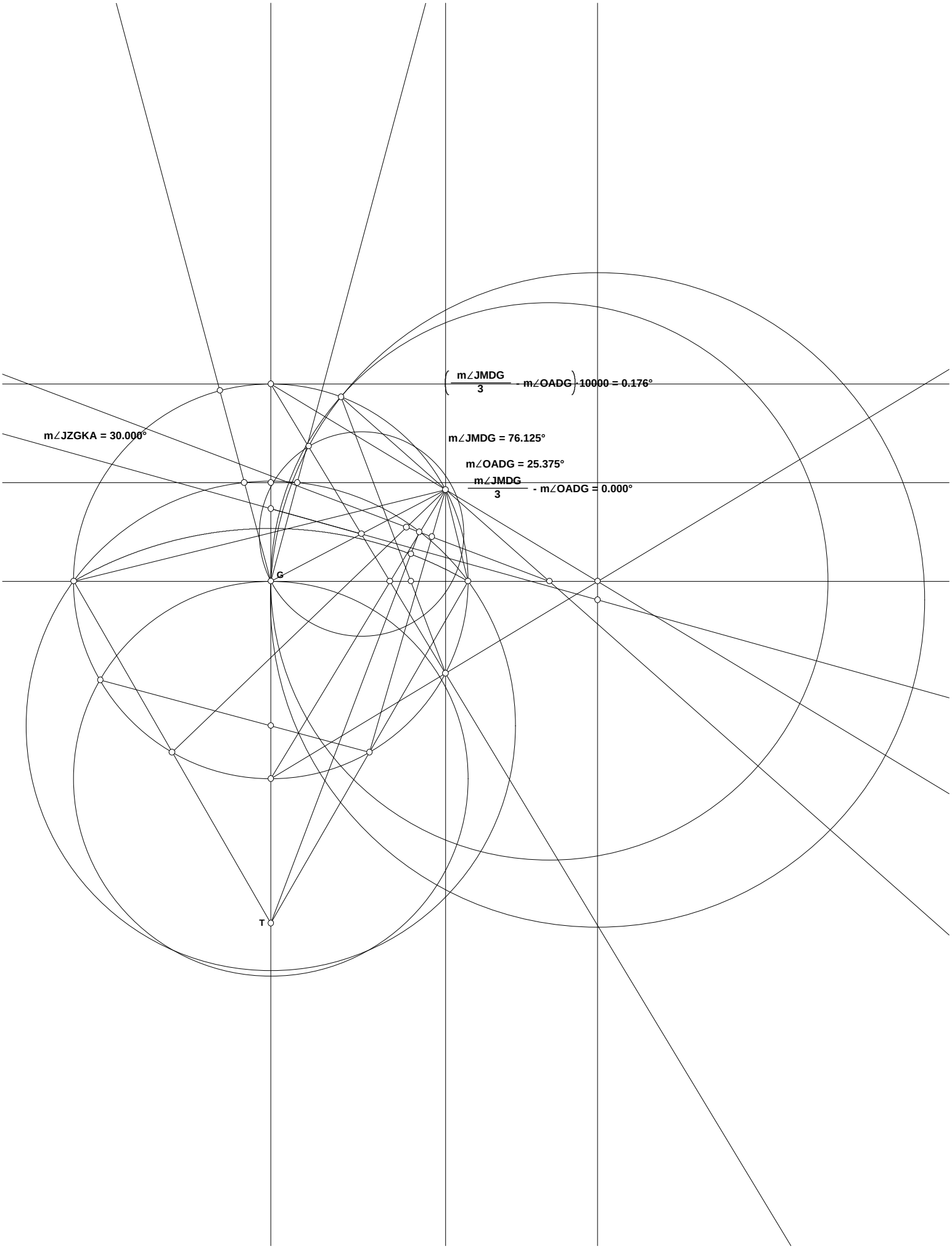

m∠CUGCY = 63.761°
m∠ICTIL = 47.492°
m∠ITCYIP = 15.940°
CY
CT
IT
LF
CU
G
LH
CZ
D
I
CX
IH
JP
T

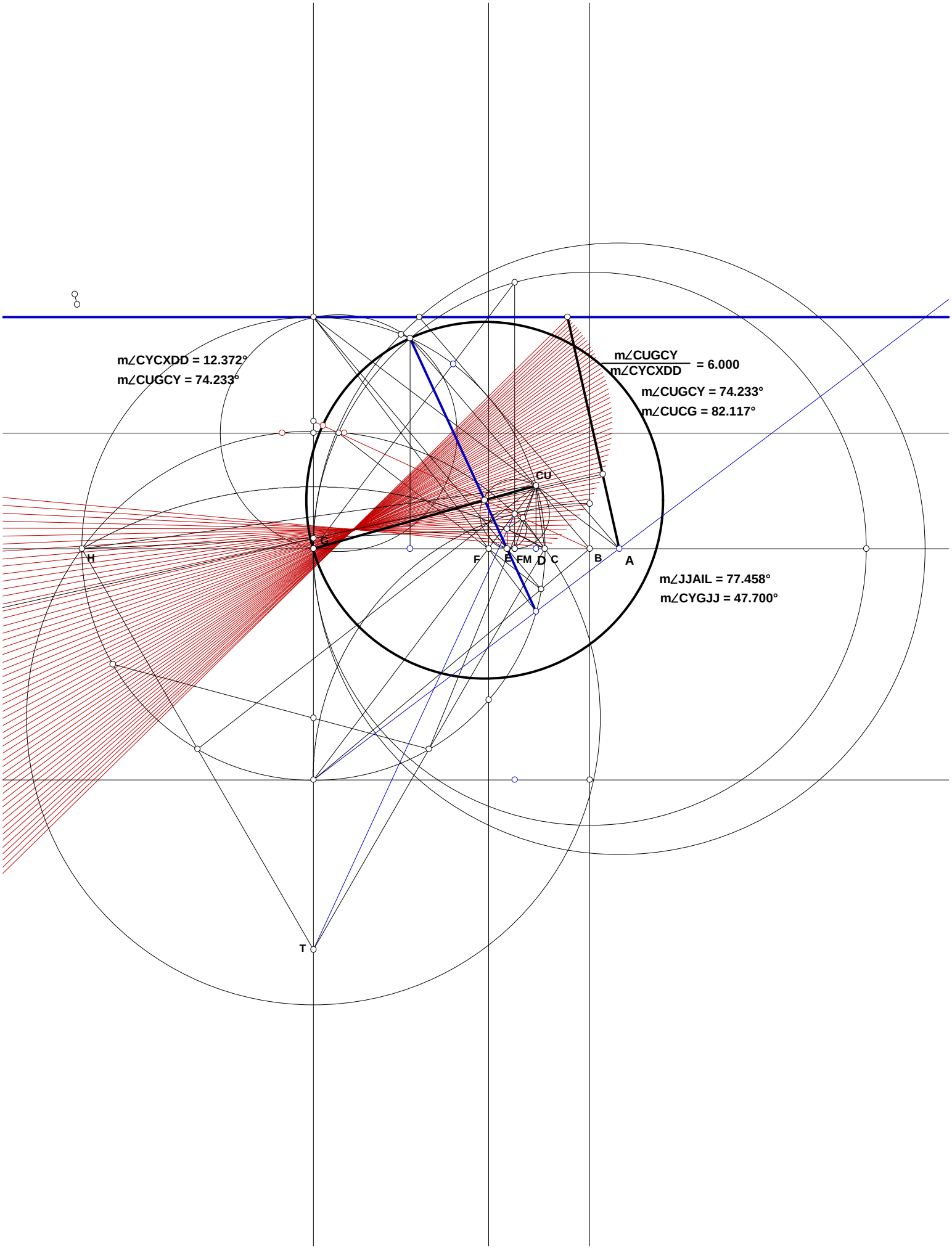

m∠JMDG = 69.925°
m∠KTDG = 23.309°
m∠JMDG / m∠KTDG = 3.000
m∠JMDG - 3·m∠KTDG = -0.003°
putz
G
T

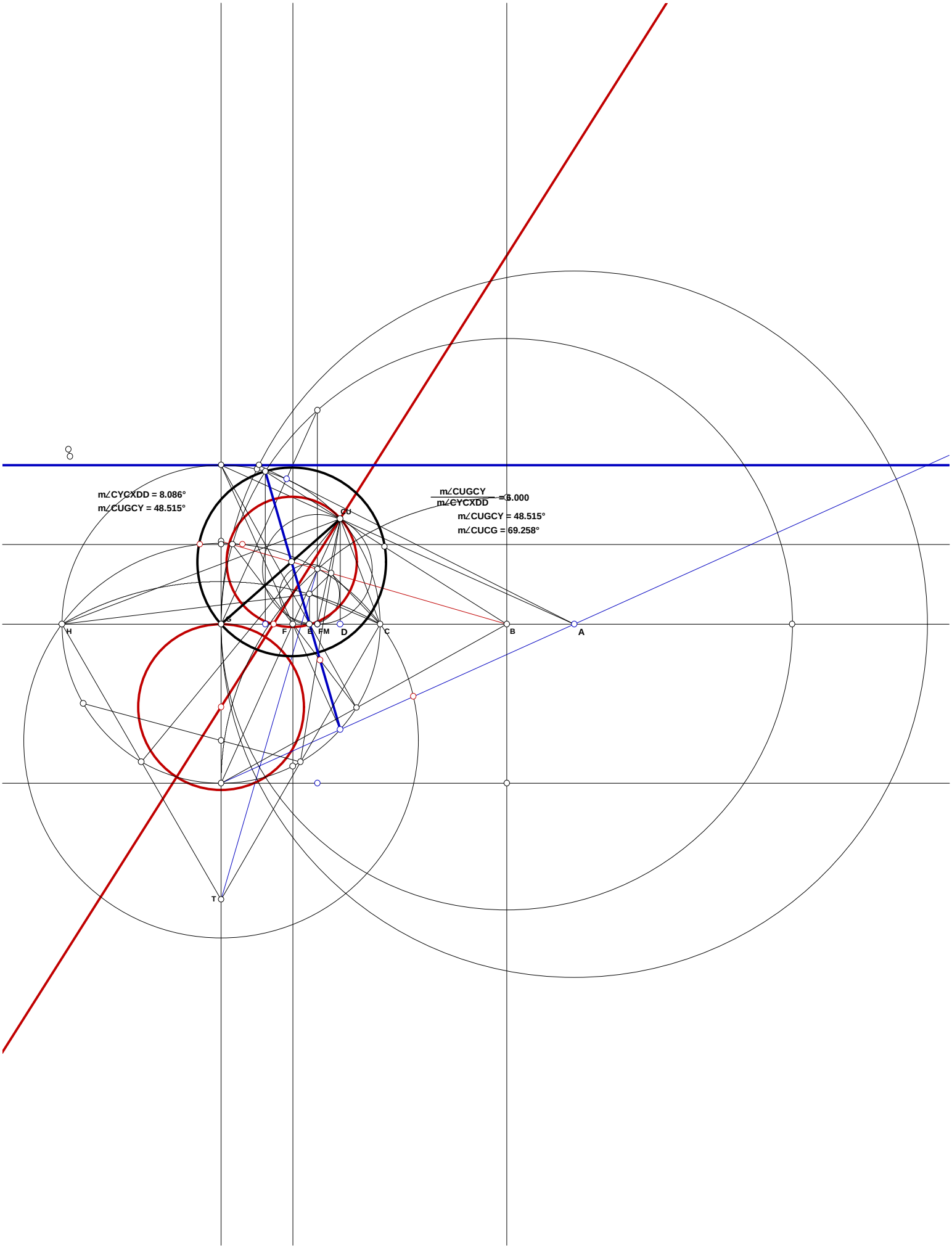

m∠ITCYIP = 15.037°
m∠CUGCY = 60.150°
m∠ICTIL = 49.900°
CY
CT
m∠CYIJG = 30.075°
m∠CUGCY = 60.150°
m∠CUGCY / m∠CYIJG = 2.000
CU
IT
LF
m∠CTIG = 40.100°
G
LH
CZ
D
I
IJ
m∠CYIJG = 30.075°
CX
IH
JP
T

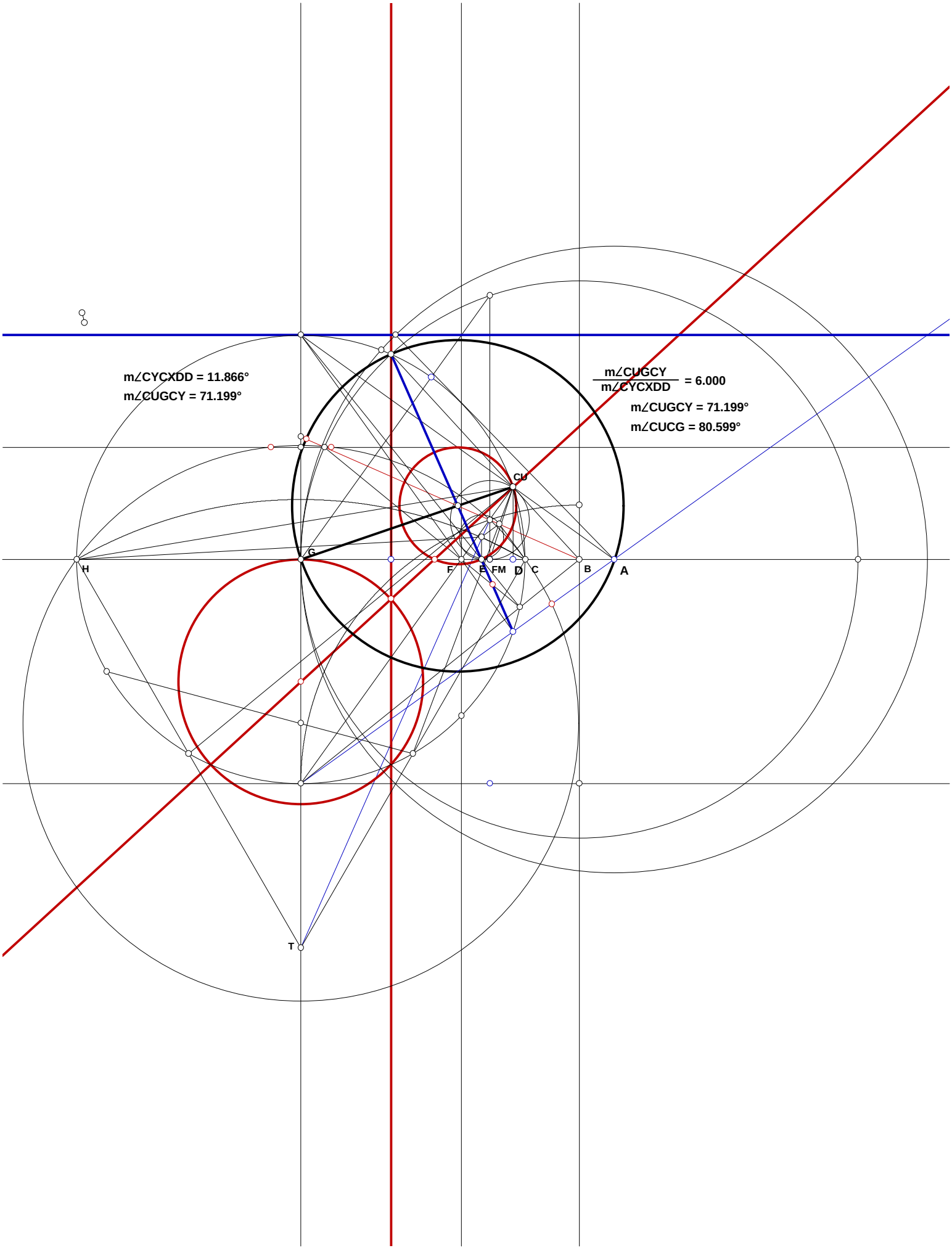

Show
Hide
m∠BCTIH = 33.303°
m∠CUGC = 33.303°
m∠ACYIH = 33.303°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

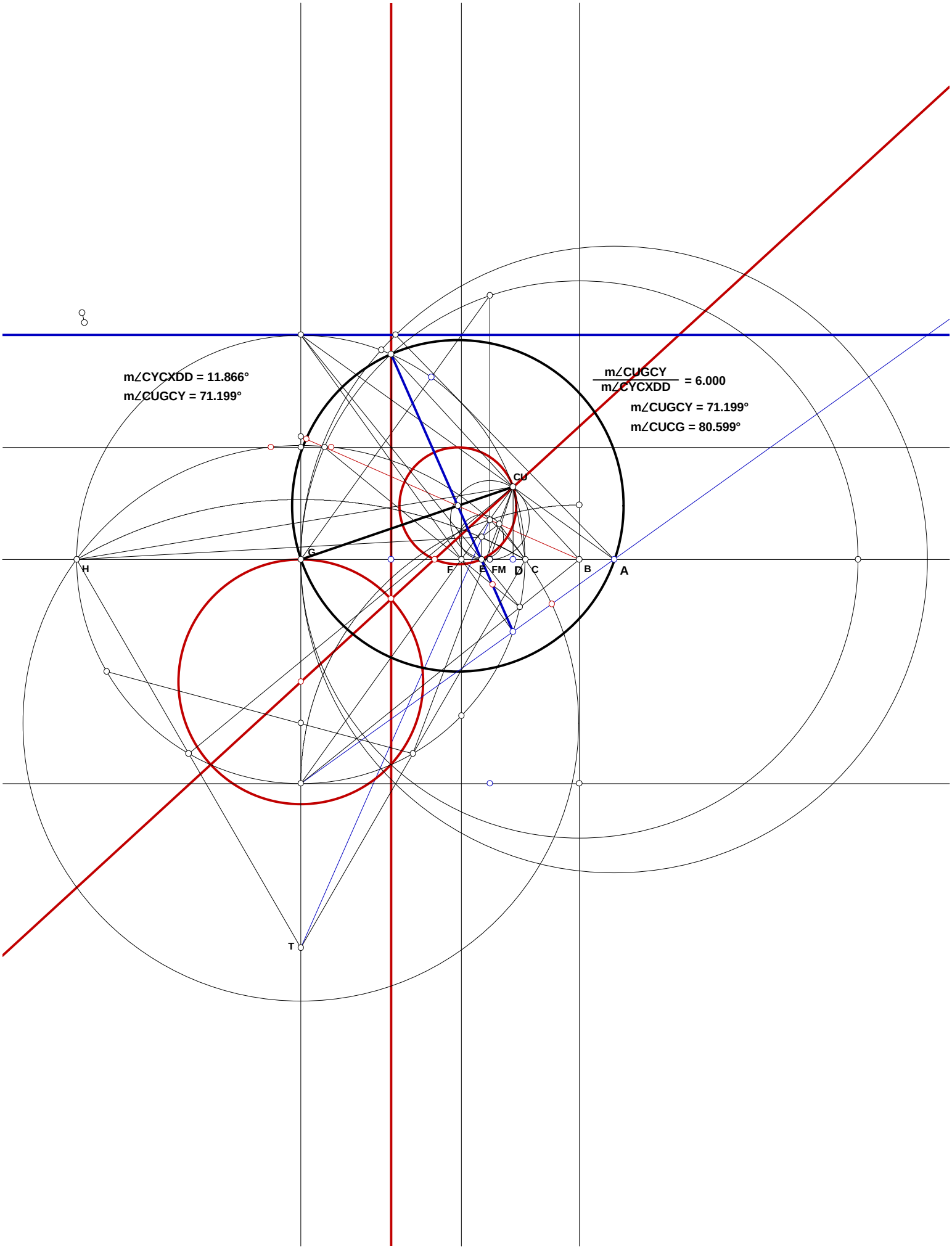

m∠RGP = 39.823°
m∠DAE = 19.911°
m∠RGP / m∠DAE = 2.000
P
S
D
R
G
E
A
5
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2

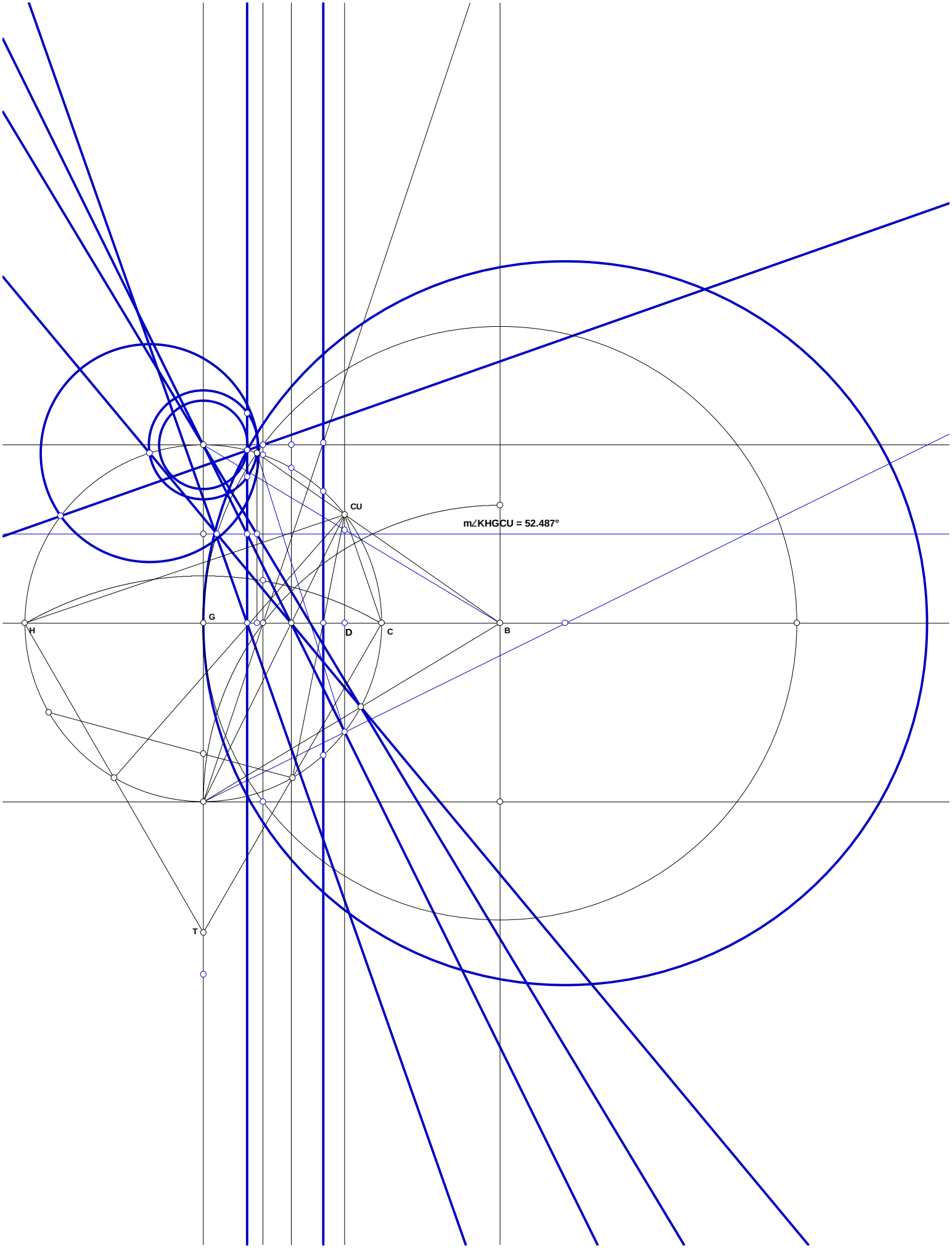

$\left(\frac{m\angle JMDG}{3} - m\angle OADG\right) 10000 = 0.176°$
$m\angle JZGKA = 30.000°$
$m\angle JMDG = 76.125°$
$m\angle OADG = 25.375°$
$\frac{m\angle JMDG}{3} - m\angle OADG = 0.000°$
G
T


m∠CYCXDD = 12.372°
m∠CUGCY = 74.233°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 74.233°
m∠CUCG = 82.117°
m∠JJAIL = 77.458°
m∠CYGJJ = 47.700°
CU
G
H
F
E
FM
D
C
B
A
T

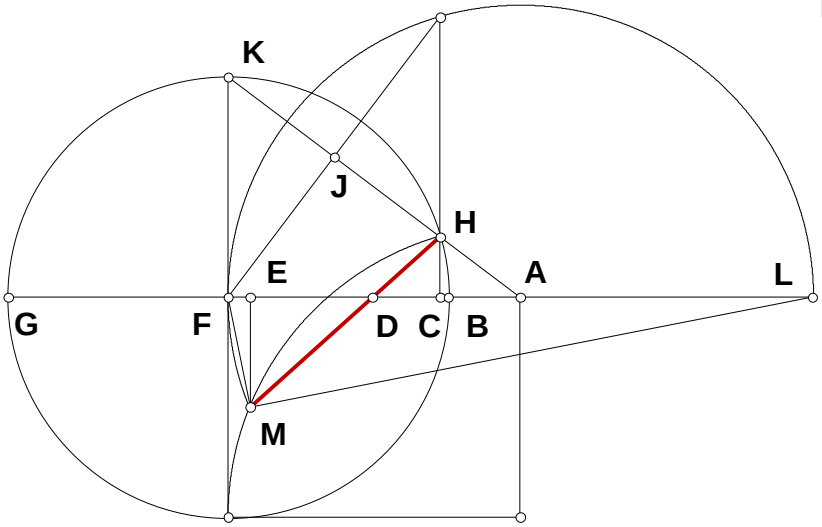

m∠CYCXDD = 8.086°
m∠CUGCY = 48.515°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 48.515°
m∠CUCG = 69.258°
U
H
G
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 11.866°
m∠CUGCY = 71.199°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 71.199°
m∠CUCG = 80.599°
CU
G
H
F
E
FM
D
C
B
A
T


CU
m∠KHGCU = 52.487°
G
H
D
C
B
T

# Segment DF And HM 052201




**Given AB and AG, what is HM and DF?**

$N := 7.111$ $AB := .375$

$AG := AB \cdot N$ $BG := AG - AB$

$BF := \frac{BG}{2}$ $FK := BF$

$AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2}$

$AJ := \frac{AF^2}{AK}$ $JK := AK - AJ$

$HJ := JK$ $AH := AK - (JK + HJ)$ $AC := \frac{AF \cdot AH}{AK}$ $EM := \frac{BF}{2}$ $FL := 2 \cdot AF$ $EF := \frac{FL}{2} - \frac{\sqrt{-4\ EM^2 + FL^2}}{2}$

$AE := AF - EF$ $CE := AE - AC$ $CH := \frac{FK \cdot AH}{AK}$ $HM := \sqrt{(EM + CH)^2 + CE^2}$ $DE := \frac{CE \cdot EM}{EM + CH}$

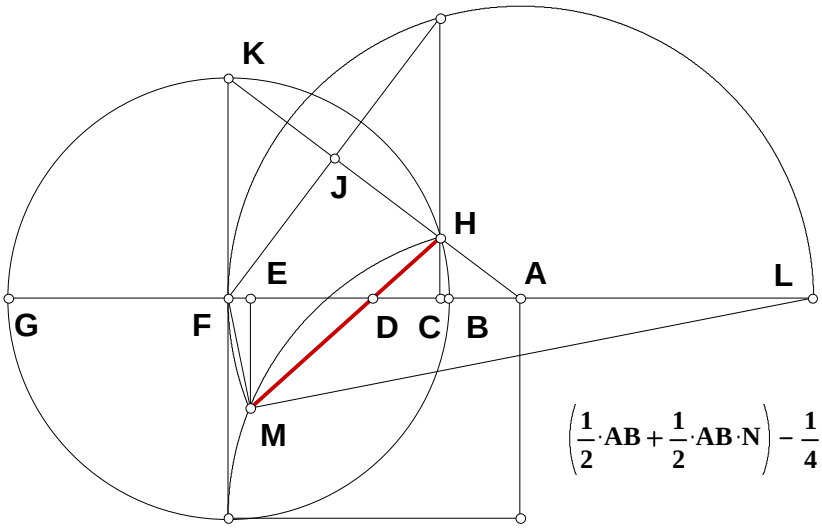
$DF := DE + EF$

## Some Algebraic Names:

$AB \cdot N - AG = 0$ $AB \cdot N - AB - BG = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0$ $\frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$

$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0$ $\frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0$ $\frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$

$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{(1 + N^2 - 2 \cdot N)}{\sqrt{1 + N^2}} - JK = 0$ $AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0$ $AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$




$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0$$

$$AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

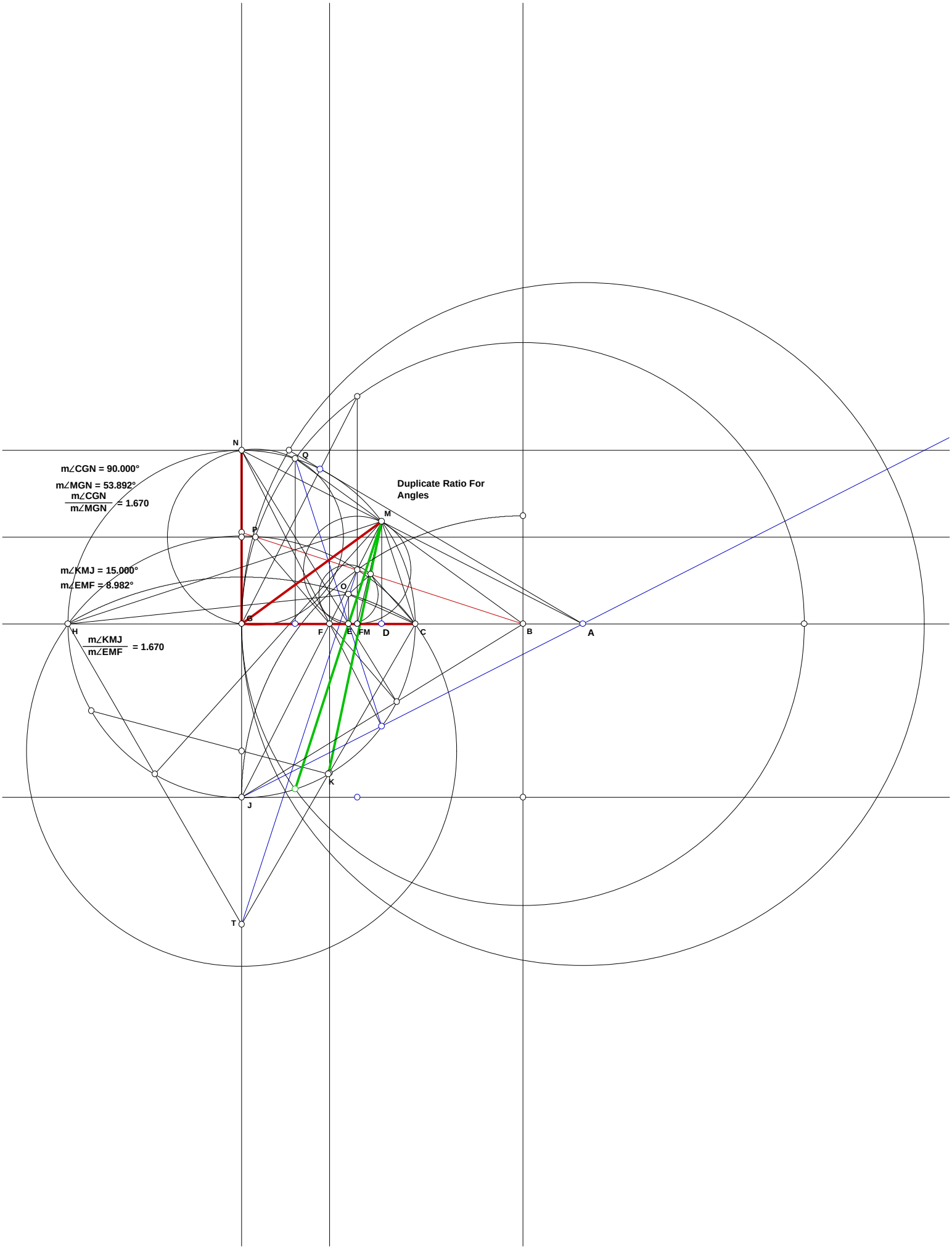
$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

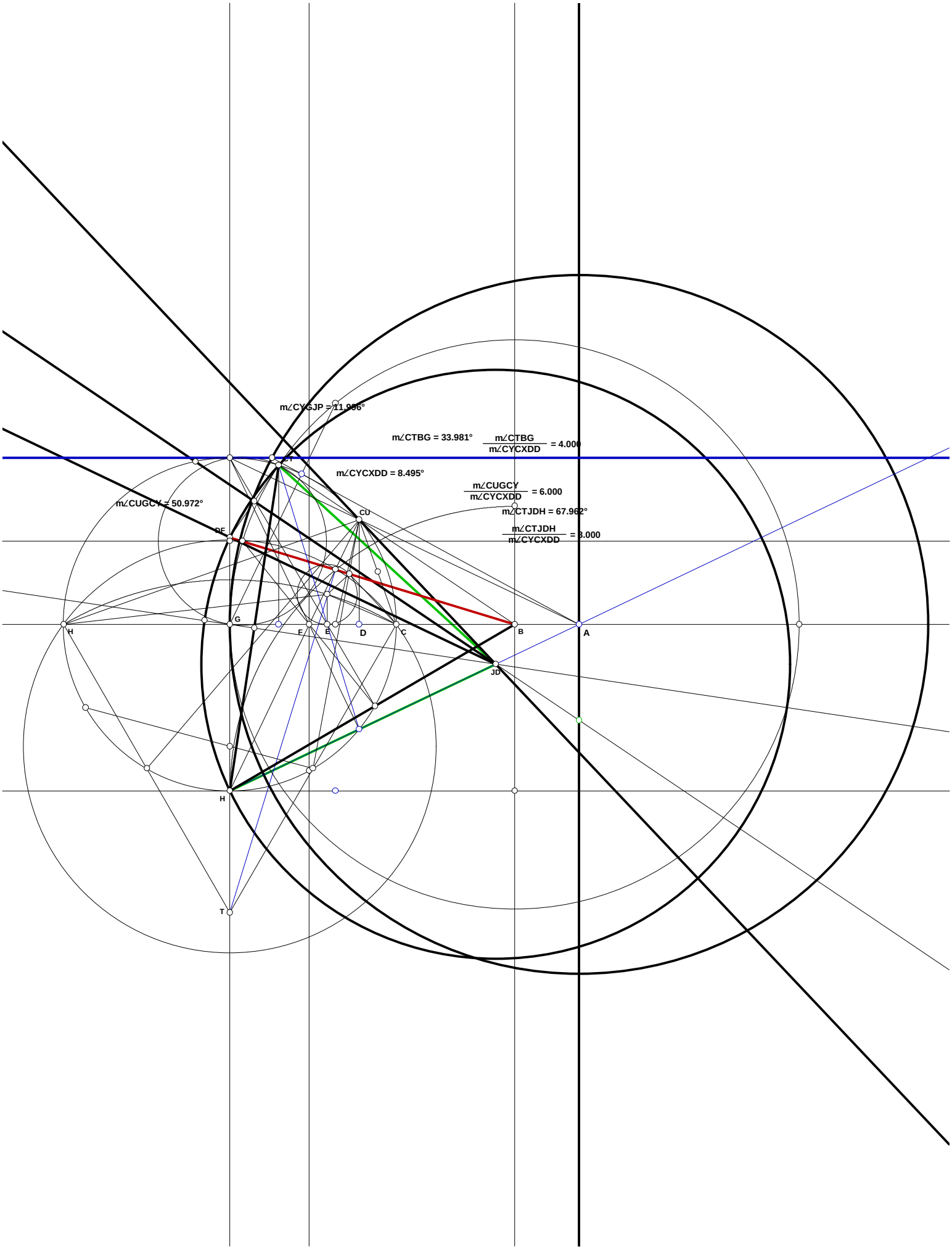
$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

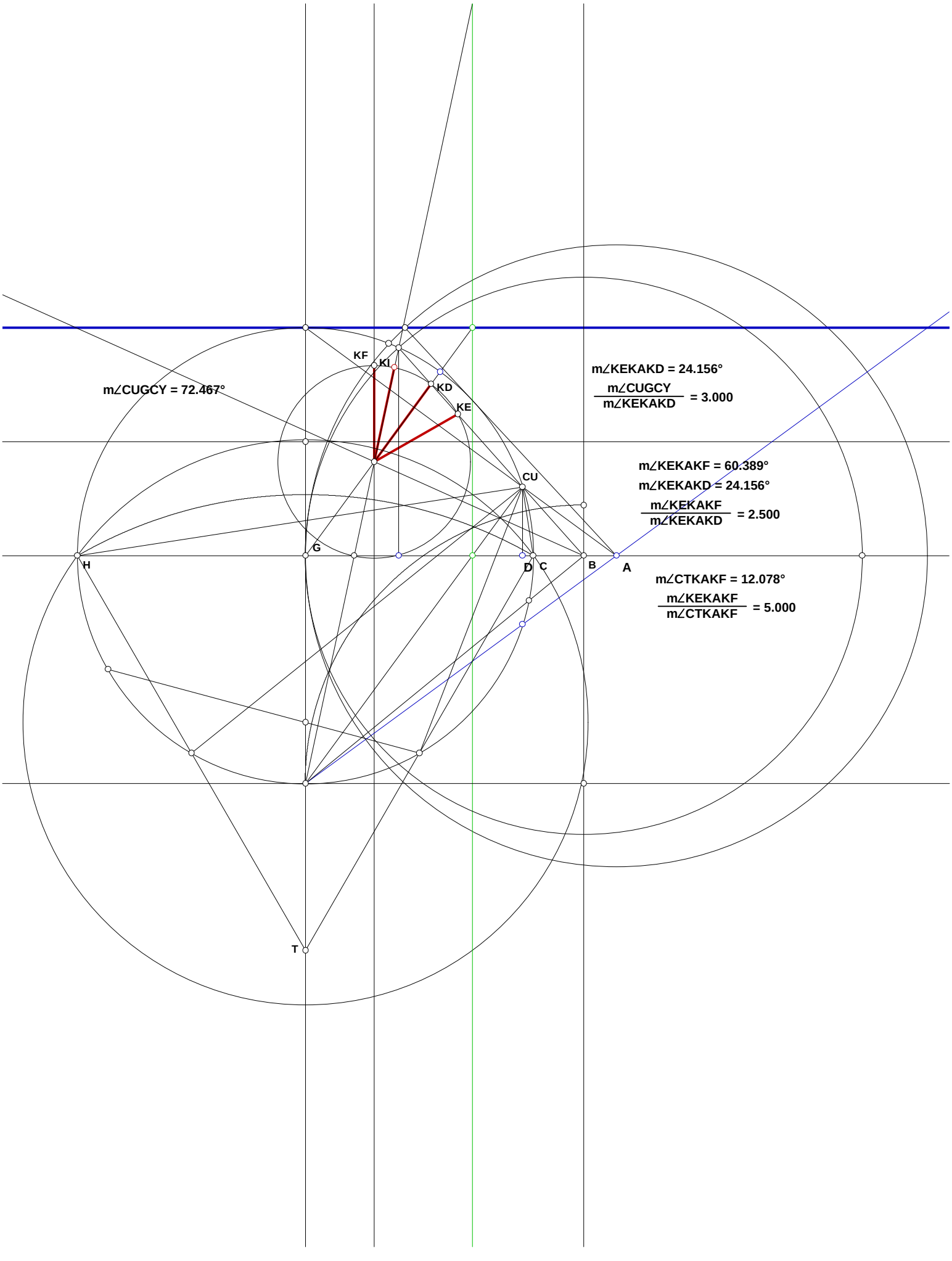
$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

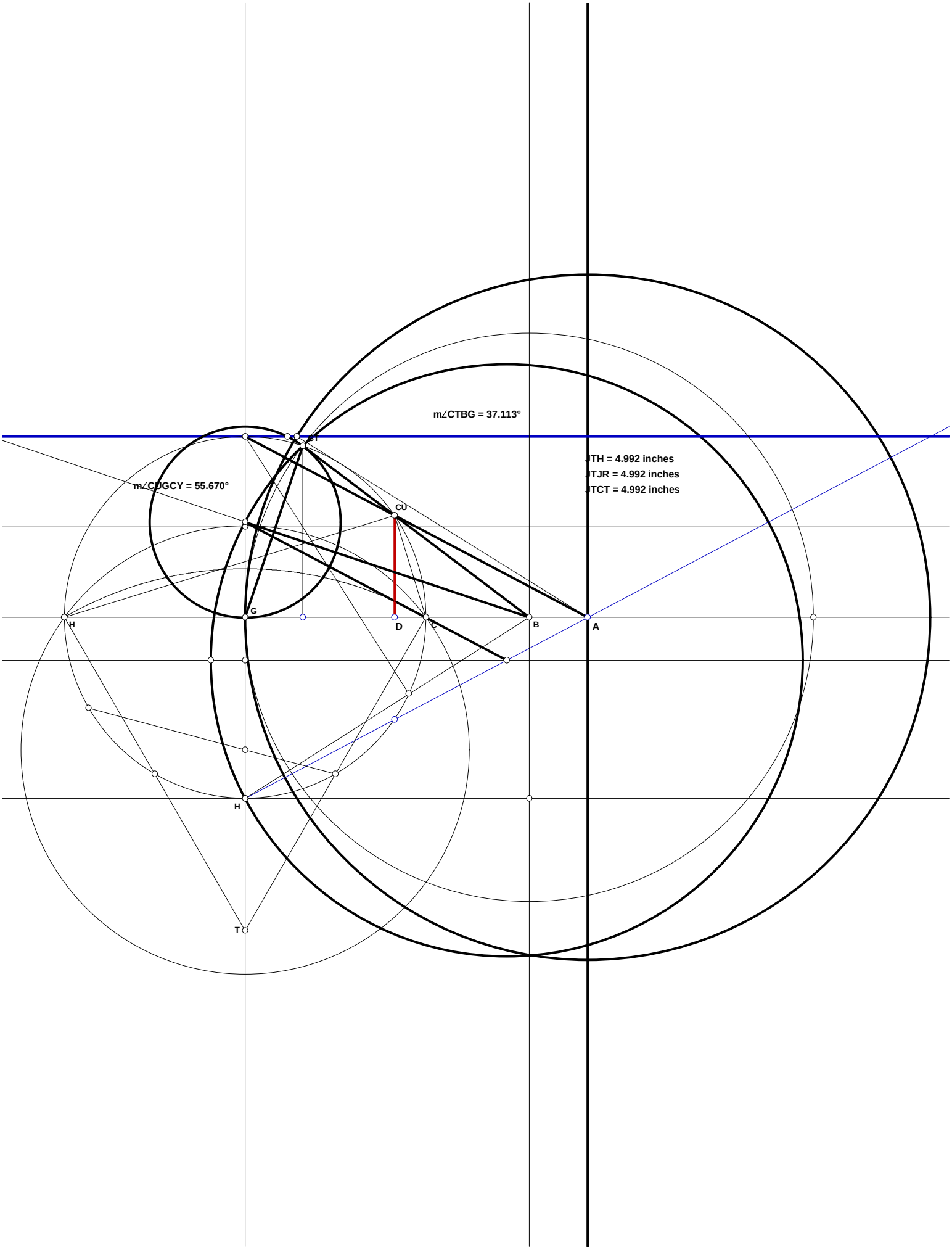
$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

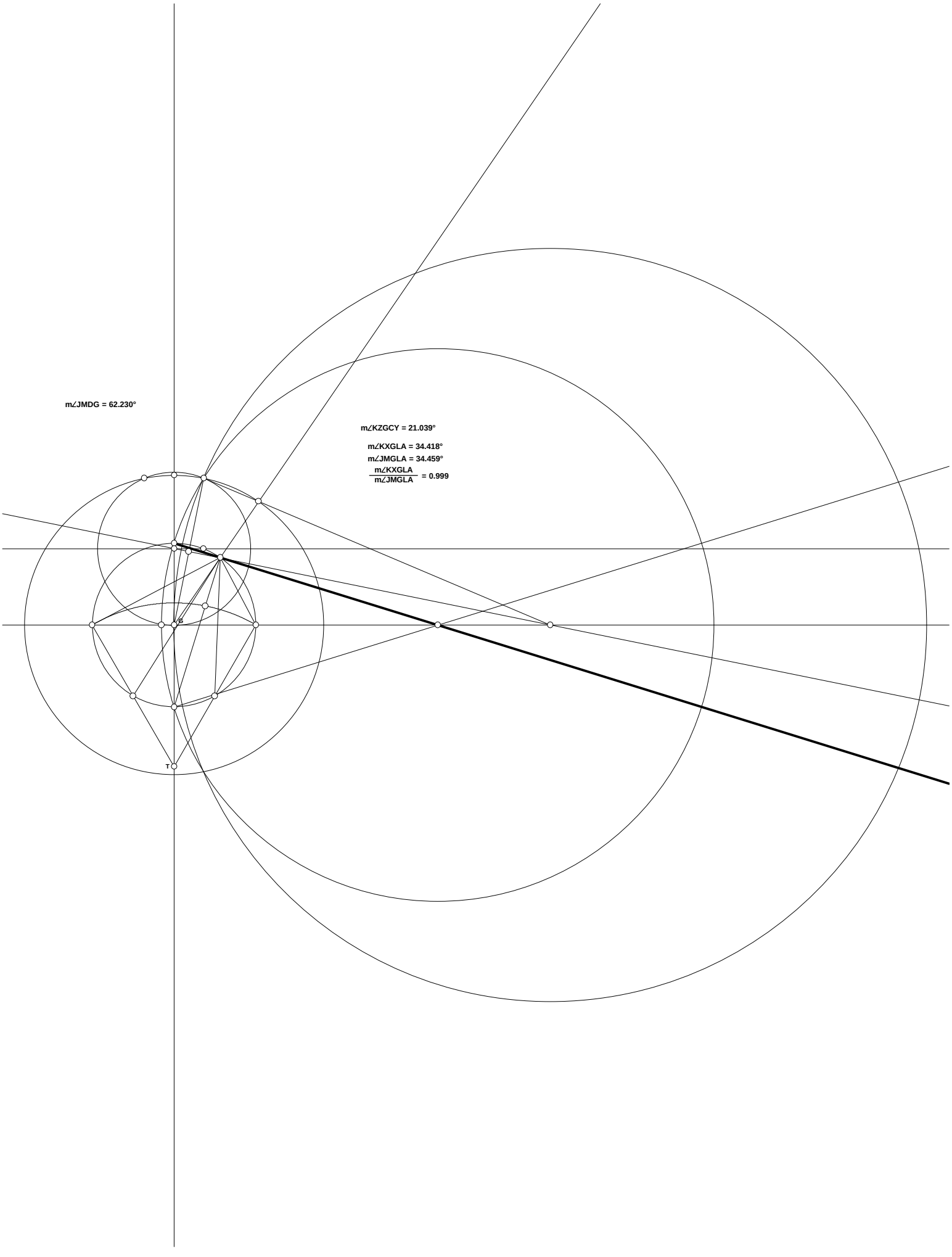
$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$

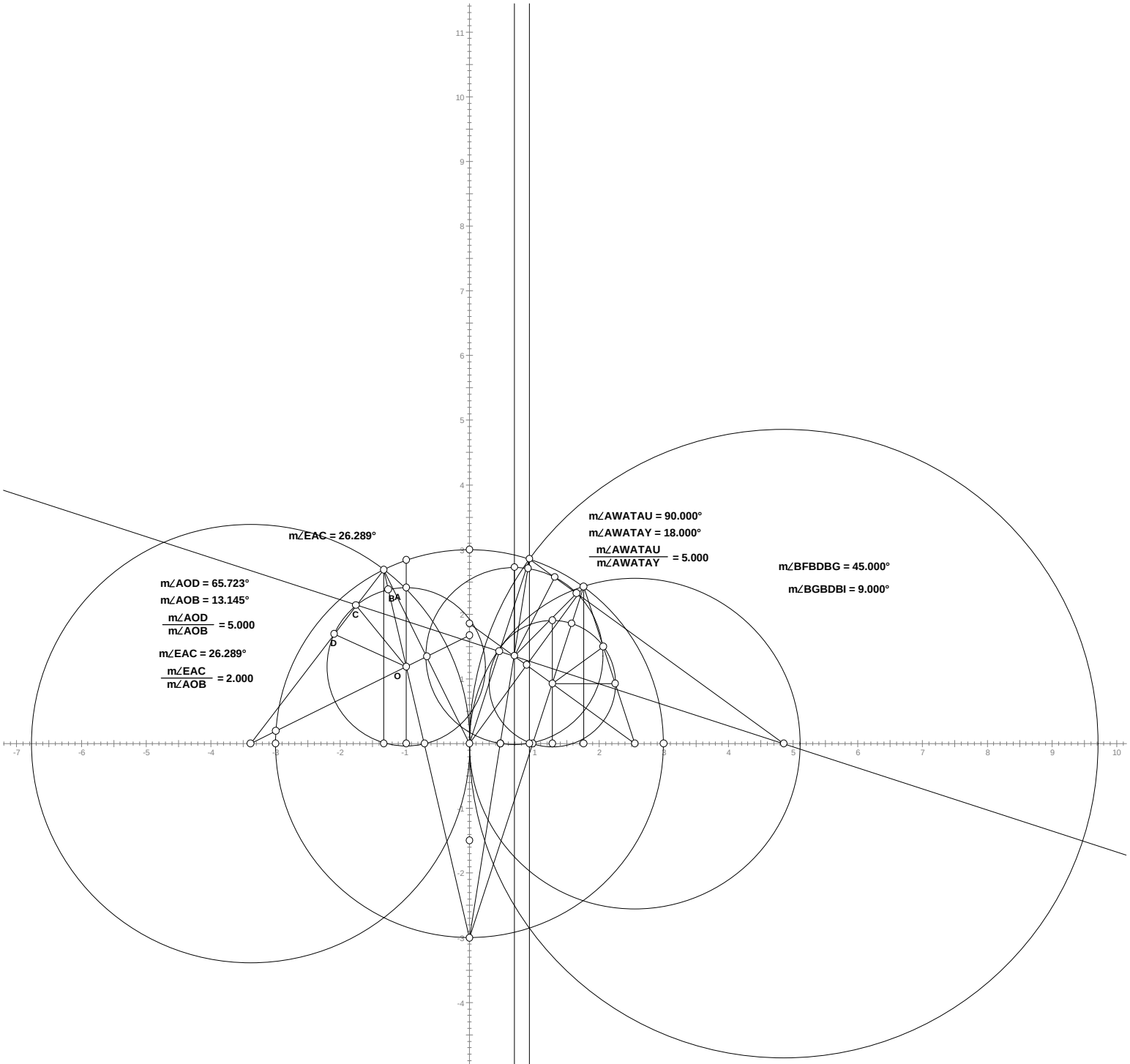

m∠CGN = 90.000°
m∠MGN = 53.892°
m∠CGN / m∠MGN = 1.670
Duplicate Ratio For Angles
m∠KMJ = 15.000°
m∠EMF = 8.982°
m∠KMJ / m∠EMF = 1.670
N
Q
M
P
O
G
H
F
E
FM
D
C
B
A
K
J
T

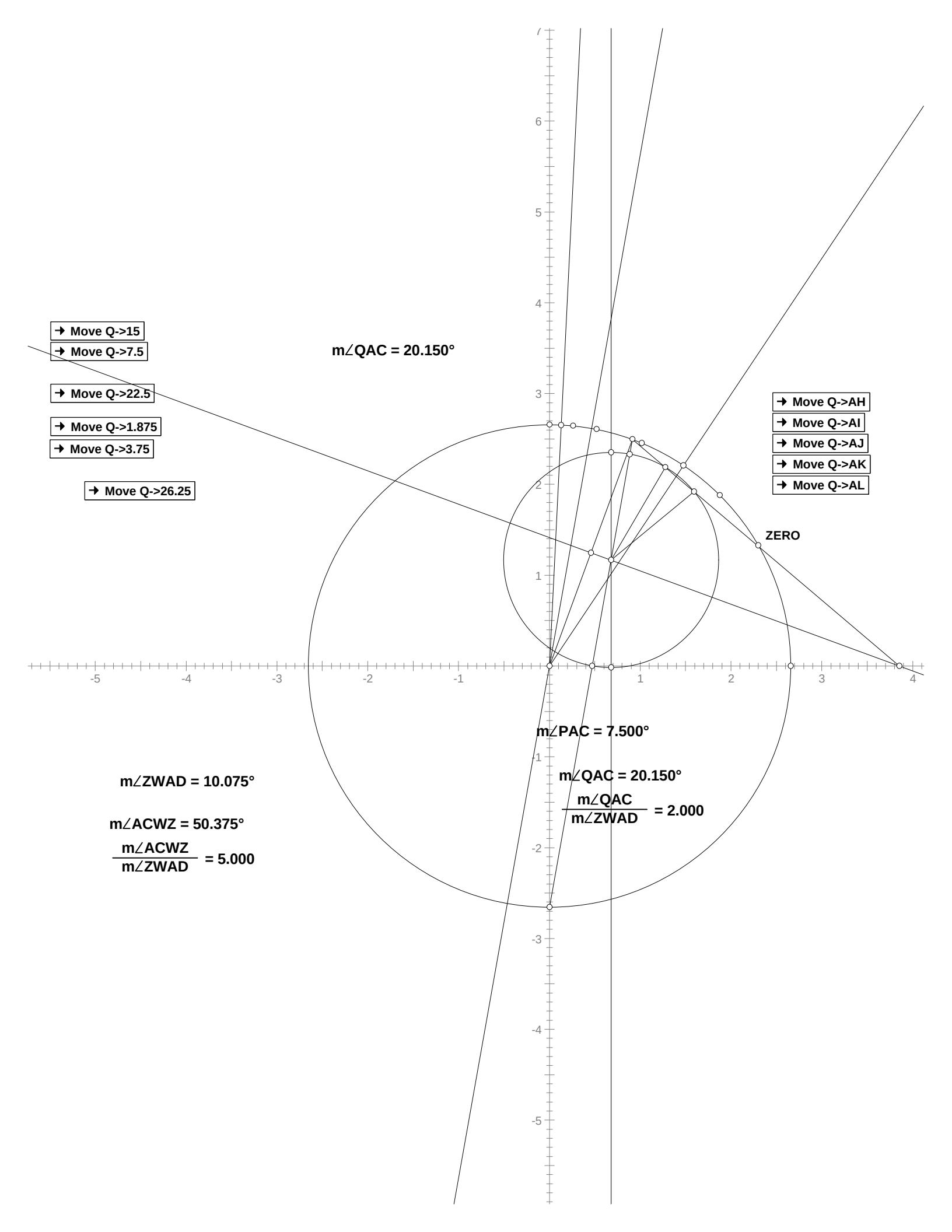

m∠CYGJP = 11.996°
m∠CTBG = 33.981°
m∠CTBG / m∠CYCXDD = 4.000
m∠CYCXDD = 8.495°
m∠CUGCY / m∠CYCXDD = 6.000
m∠CUGCY = 50.972°
m∠CTJDH = 67.962°
m∠CTJDH / m∠CYCXDD = 8.000
CU
DE
G
H
F
E
D
C
B
A
JD
H
T

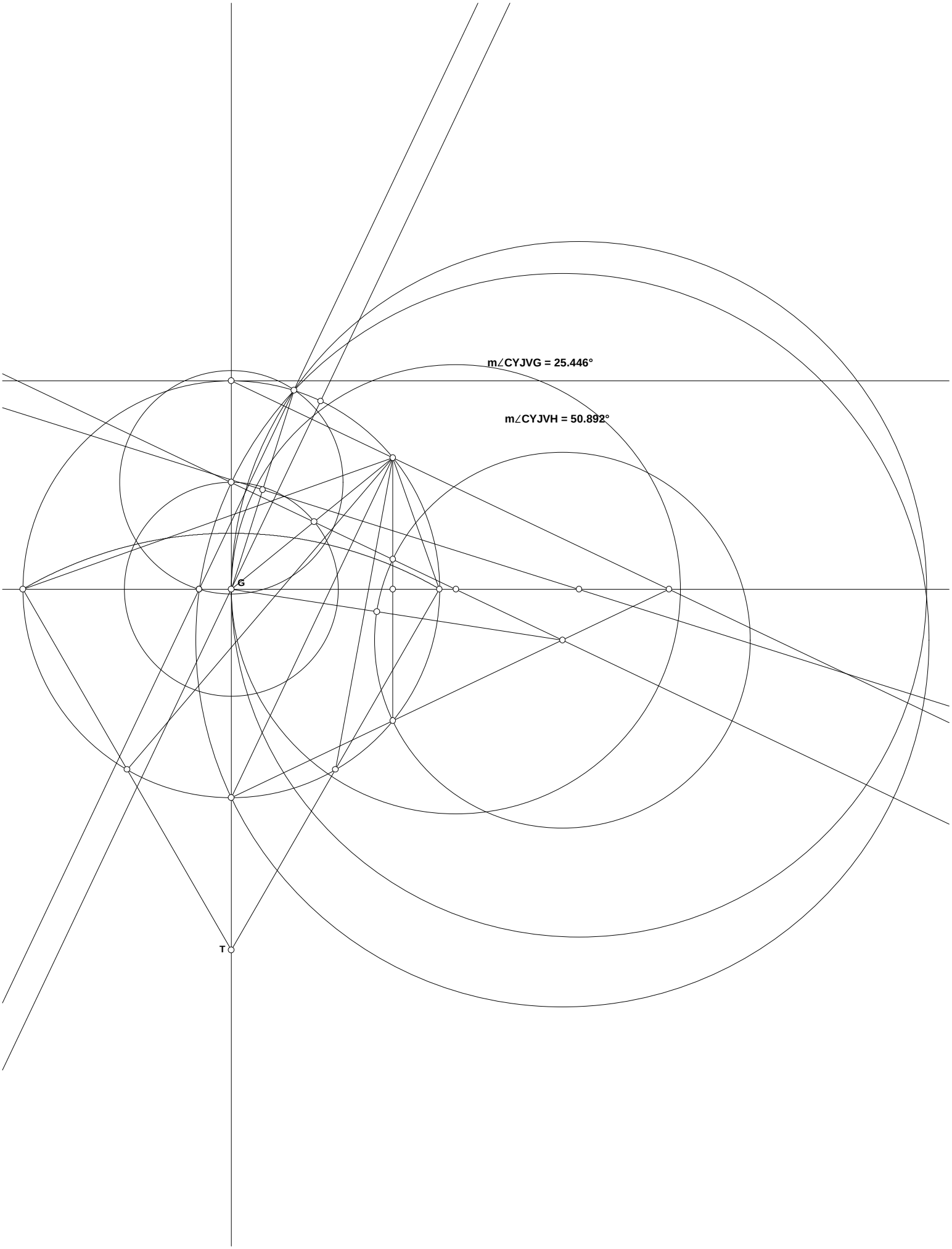

KF
KI
KD
KE
CU
G
H
D
C
B
A
T
m∠CUGCY = 72.467°
m∠KEKAKD = 24.156°
m∠CUGCY / m∠KEKAKD = 3.000
m∠KEKAKF = 60.389°
m∠KEKAKD = 24.156°
m∠KEKAKF / m∠KEKAKD = 2.500
m∠CTKAKF = 12.078°
m∠KEKAKF / m∠CTKAKF = 5.000

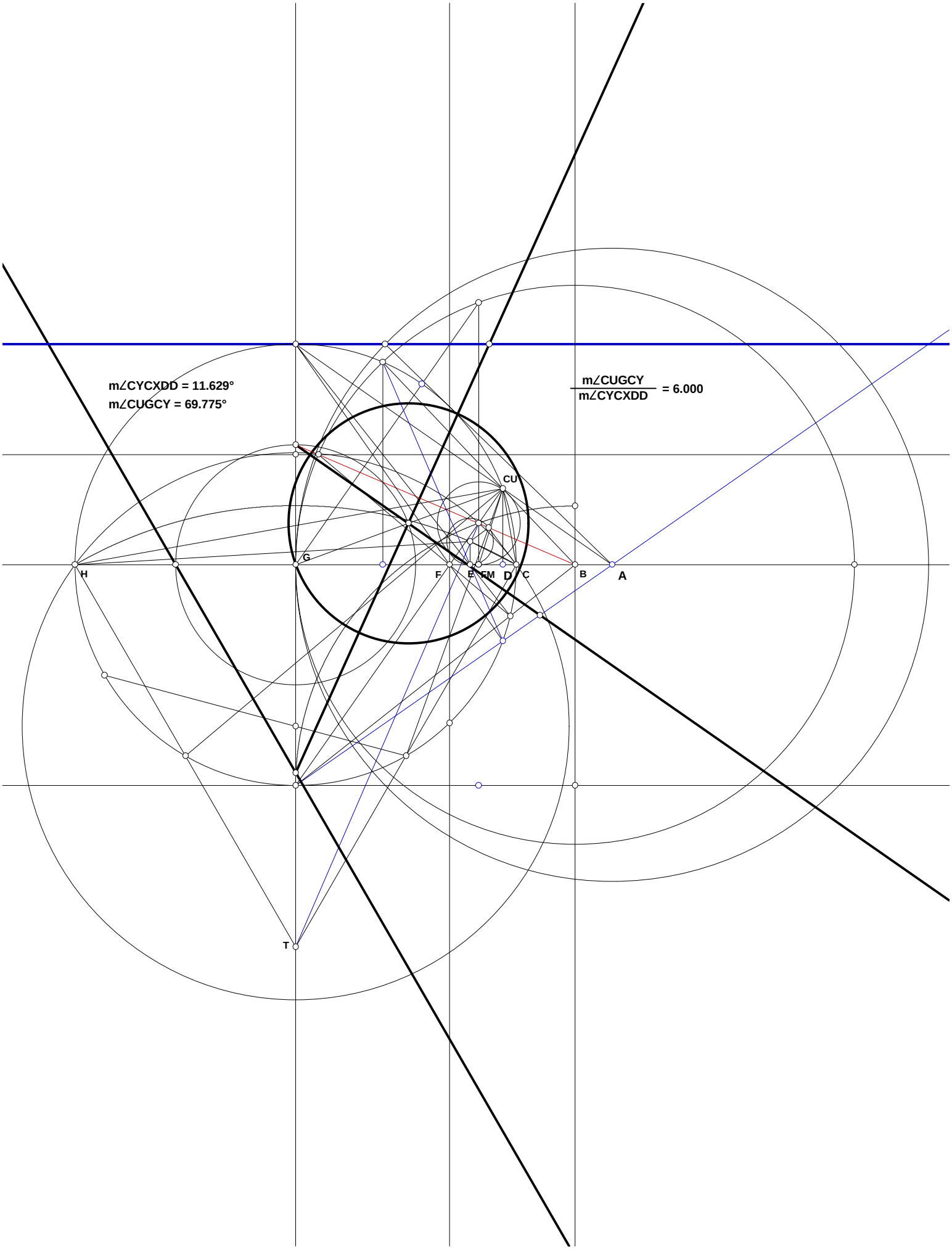

m∠CTBG = 37.113°
JTH = 4.992 inches
JTJR = 4.992 inches
JTCT = 4.992 inches
m∠CUGCY = 55.670°
CU
G
H
D
C
B
A
H
T

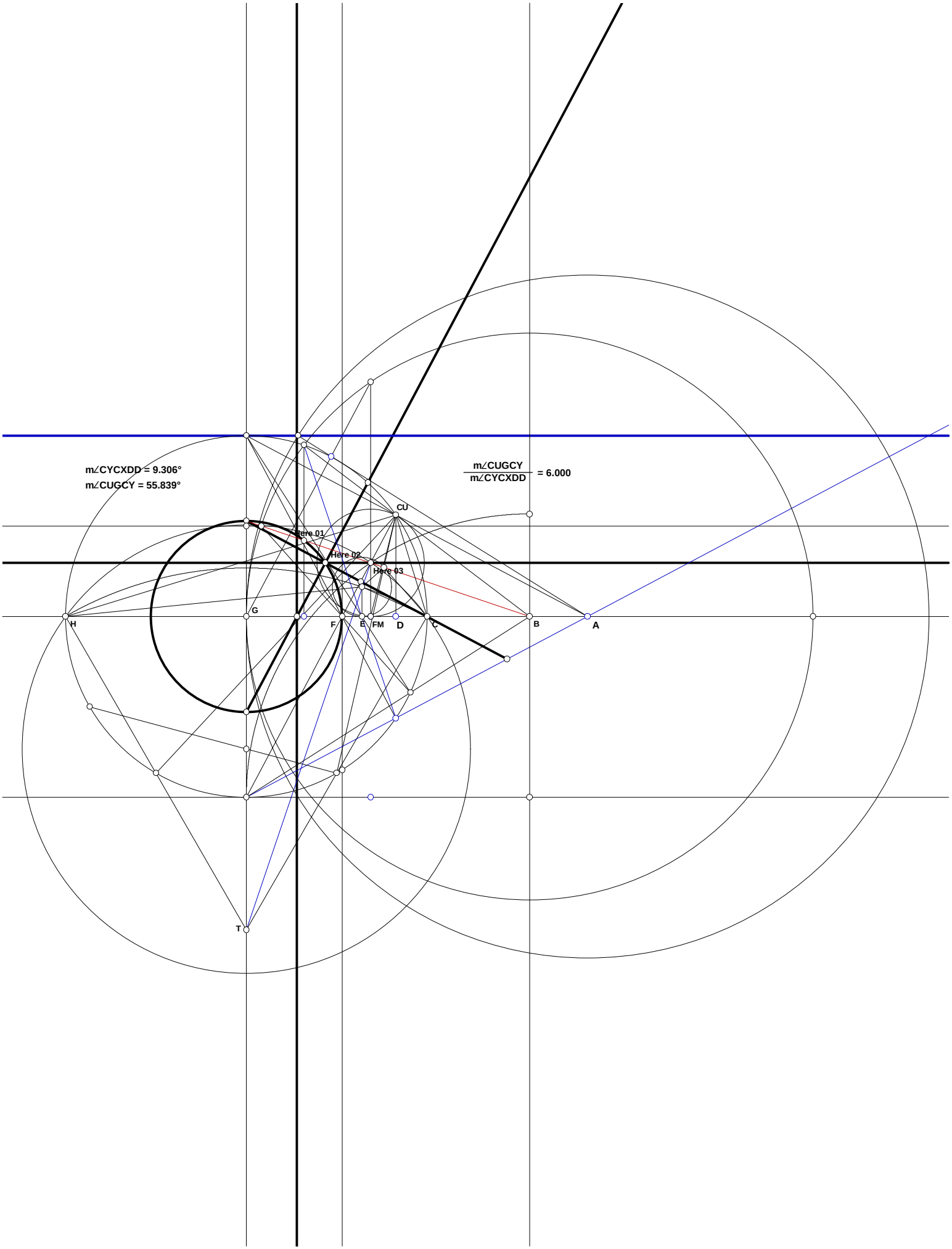

m∠JMDG = 62.230°
m∠KZGCY = 21.039°
m∠KXGLA = 34.418°
m∠JMGLA = 34.459°
m∠KXGLA / m∠JMGLA = 0.999
T


m∠EAC = 26.289°
m∠AOD = 65.723°
m∠AOB = 13.145°
m∠AOD / m∠AOB = 5.000
m∠EAC = 26.289°
m∠EAC / m∠AOB = 2.000
m∠AWATAU = 90.000°
m∠AWATAY = 18.000°
m∠AWATAU / m∠AWATAY = 5.000
m∠BFBDBG = 45.000°
m∠BGBDBI = 9.000°
A
B
C
D
O

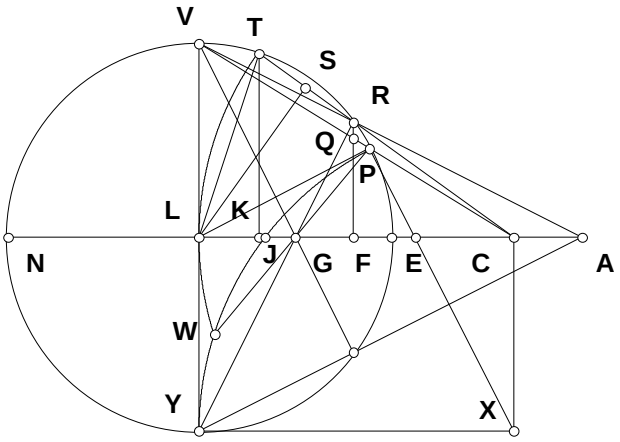

Move Q->15
Move Q->7.5
Move Q->22.5
Move Q->1.875
Move Q->3.75
Move Q->26.25
m∠QAC = 20.150°
Move Q->AH
Move Q->AI
Move Q->AJ
Move Q->AK
Move Q->AL
ZERO
m∠PAC = 7.500°
m∠QAC = 20.150°
m∠QAC / m∠ZWAD = 2.000
m∠ZWAD = 10.075°
m∠ACWZ = 50.375°
m∠ACWZ / m∠ZWAD = 5.000


m∠CYJVG = 25.446°
m∠CYJVH = 50.892°
G
T


m∠CYCXDD = 11.629°
m∠CUGCY = 69.775°
m∠CUGCY / m∠CYCXDD = 6.000
CU
G
H
F
E
FM
D
C
B
A
T


m∠CYCXDD = 9.306°
m∠CUGCY = 55.839°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Here 01
Here 02
Here 03
G
H
F
E
FM
D
C
B
A
T

# Point of Intersection 052701




## Do RY and PW intersect at G?

$N := 5 \qquad CE := 1 \qquad CN := CE \cdot N$

$EN := CN - CE \qquad EL := \frac{EN}{2} \qquad LV := EL$

$LT := EL \qquad LY := EL \qquad CL := CE + EL \qquad CT := CL$

$S_1 := CL \qquad S_2 := CT \qquad S_3 := LT$

$KL := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \qquad CK := CL - KL \qquad CS := CK \qquad ST := CT - CS \qquad RS := ST$

$CR := CT - (ST + RS) \qquad CF := \frac{CK \cdot CR}{CT} \qquad FR := \sqrt{CR^2 - CF^2} \qquad FQ := \frac{LV \cdot CF}{CL} \qquad FL := CL - CF$

$GL := \frac{FL \cdot LY}{(LY + FR)}$

## Some Algebraic Names:

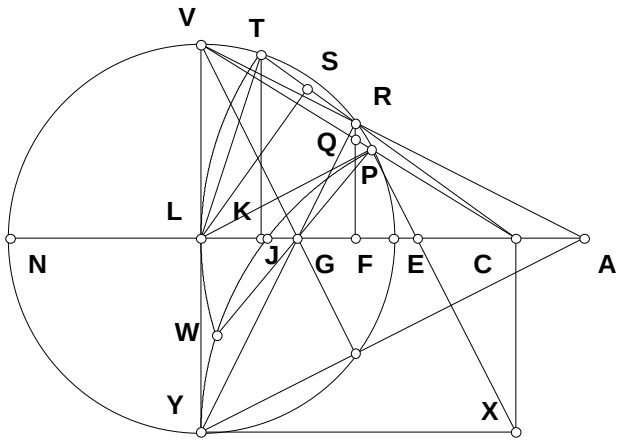
$CE \cdot N - CN = 0 \qquad CE \cdot N - CE - EN = 0 \qquad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0$

$2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \qquad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0$

$\frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$




$$GL := \frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}$$

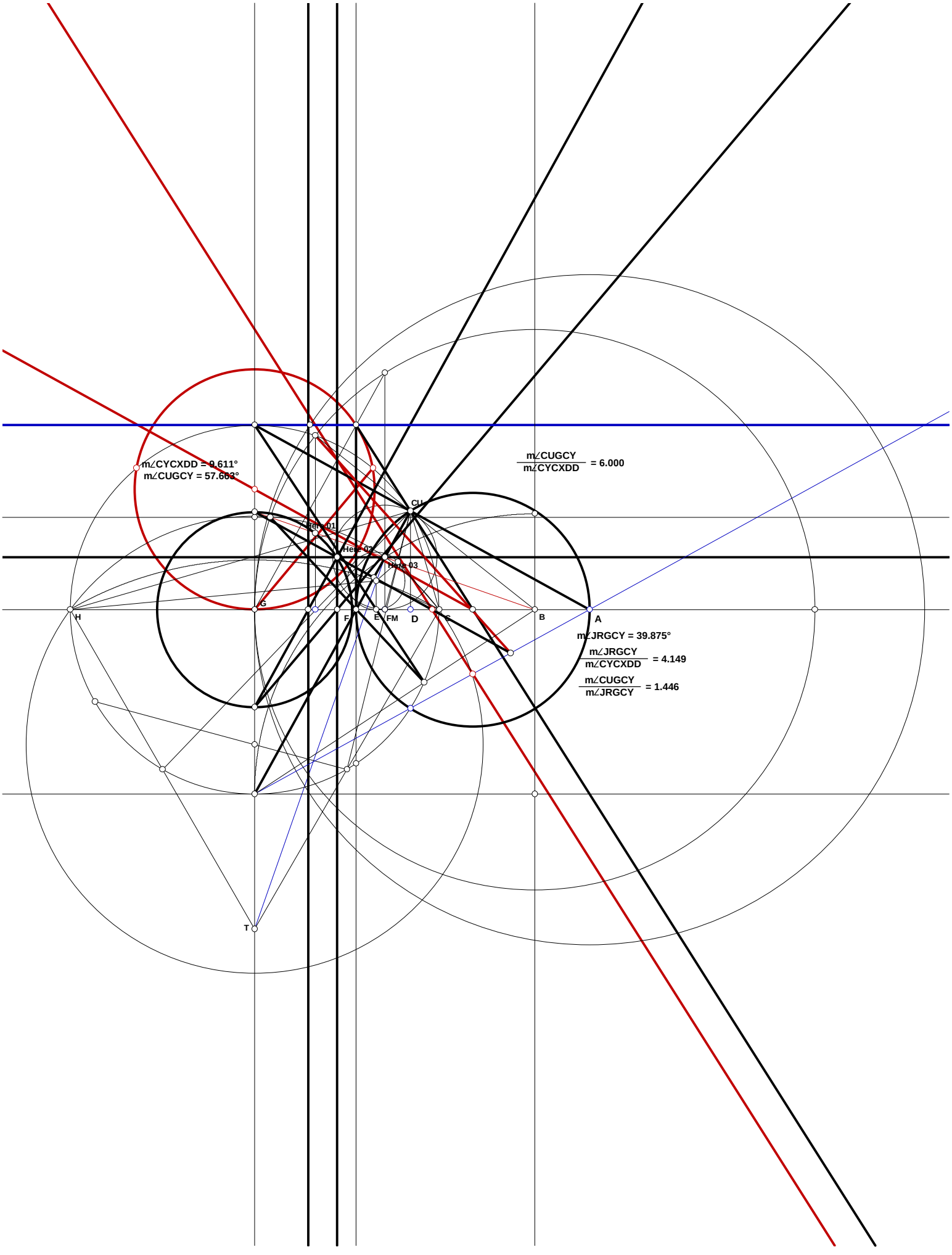
## From Segment DF And HM 052201:

$$\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)} - GL = 0$$

$$\frac{\frac{1}{2} \cdot \frac{CE \cdot \left(N^3 + 3 \cdot N^2 + 3 \cdot N - 2\ N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 1\right)}{\left(N^2 + 4 \cdot N + 1\right)}}{\frac{1}{2} \cdot \frac{CE \cdot \left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{\left[N^3 + 3 \cdot N^2 + 2\ N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}} = 1$$

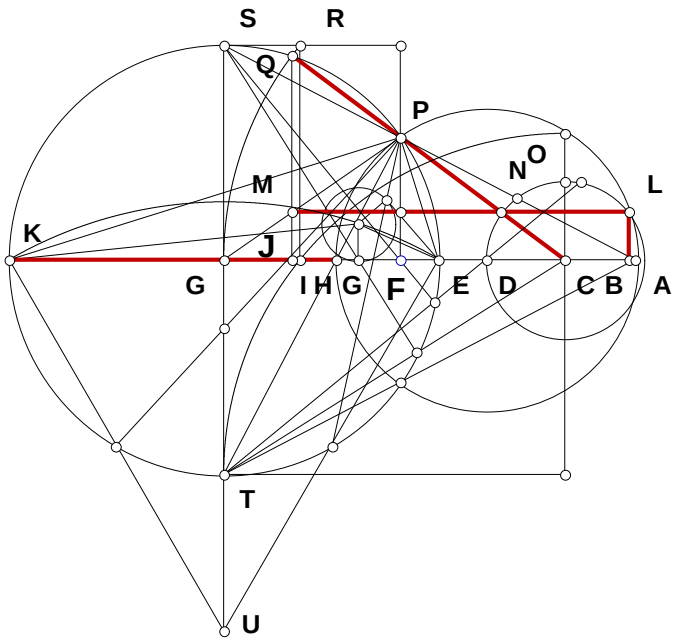
**Which does reduce to,**

$$1 = 1$$


m∠CYCXDD = 9.611°
m∠CUGCY = 57.668°
m∠CUGCY / m∠CYCXDD = 6.000
CU
Hero 01
Hero 02
Hero 03
G
H
F
E
FM
D
C
B
A
T
m∠JRGCY = 39.875°
m∠JRGCY / m∠CYCXDD = 4.149
m∠CUGCY / m∠JRGCY = 1.446

**What is CD and BL?**

**0530011**




$N_1 := 1 \qquad N_2 := 5$

$CE := N_1 \qquad EK := N_2$

$EG := \frac{EK}{2} \qquad CG := CE + EG$

$CQ := CG \qquad PQ := \frac{EG^2}{CQ}$

$CP := CQ - PQ \qquad GT := EG$

$CF := \frac{CP^2 + CG^2 - EG^2}{2 \cdot CG}$

$PF := \sqrt{CP^2 - CF^2} \qquad GS := EG$

$FG := CG - CF \qquad FS := \sqrt{FG^2 + GS^2}$

$DG := \frac{GS \cdot GT}{FG} \qquad CD := CG - DG$

$BL := \frac{PF \cdot CD}{CP}$

**Some Algebraic Names;**

$$CE := N_1 \qquad EK := N_2 \qquad EG - \frac{N_2}{2} = 0 \qquad CG - \left(N_1 + \frac{N_2}{2}\right) = 0$$

$$PQ - \frac{1}{2} \cdot \frac{{N_2}^2}{\left(2 \cdot N_1 + N_2\right)} = 0 \qquad CP - 2 \cdot N_1 \cdot \frac{\left(N_1 + N_2\right)}{\left(2 \cdot N_1 + N_2\right)} = 0$$

$$CF := N_1 \cdot \left(N_1 + N_2\right) \cdot \frac{\left(8 \cdot {N_1}^2 + 8 \cdot N_1 \cdot N_2 + {N_2}^2\right)}{\left(2 \cdot N_1 + N_2\right)^3}$$

$$PF - \left(N_1 + N_2\right) \cdot N_1 \cdot \sqrt{\left(N_2 + 4 \cdot N_1\right) \cdot \left(3 \cdot N_2 + 4 \cdot N_1\right)} \cdot \frac{N_2}{\left(2 \cdot N_1 + N_2\right)^3} = 0$$

$$FG - \frac{1}{2} \cdot {N_2}^2 \cdot \frac{\left(6 \cdot {N_1}^2 + 6 \cdot N_1 \cdot N_2 + {N_2}^2\right)}{\left(2 \cdot N_1 + N_2\right)^3} = 0$$

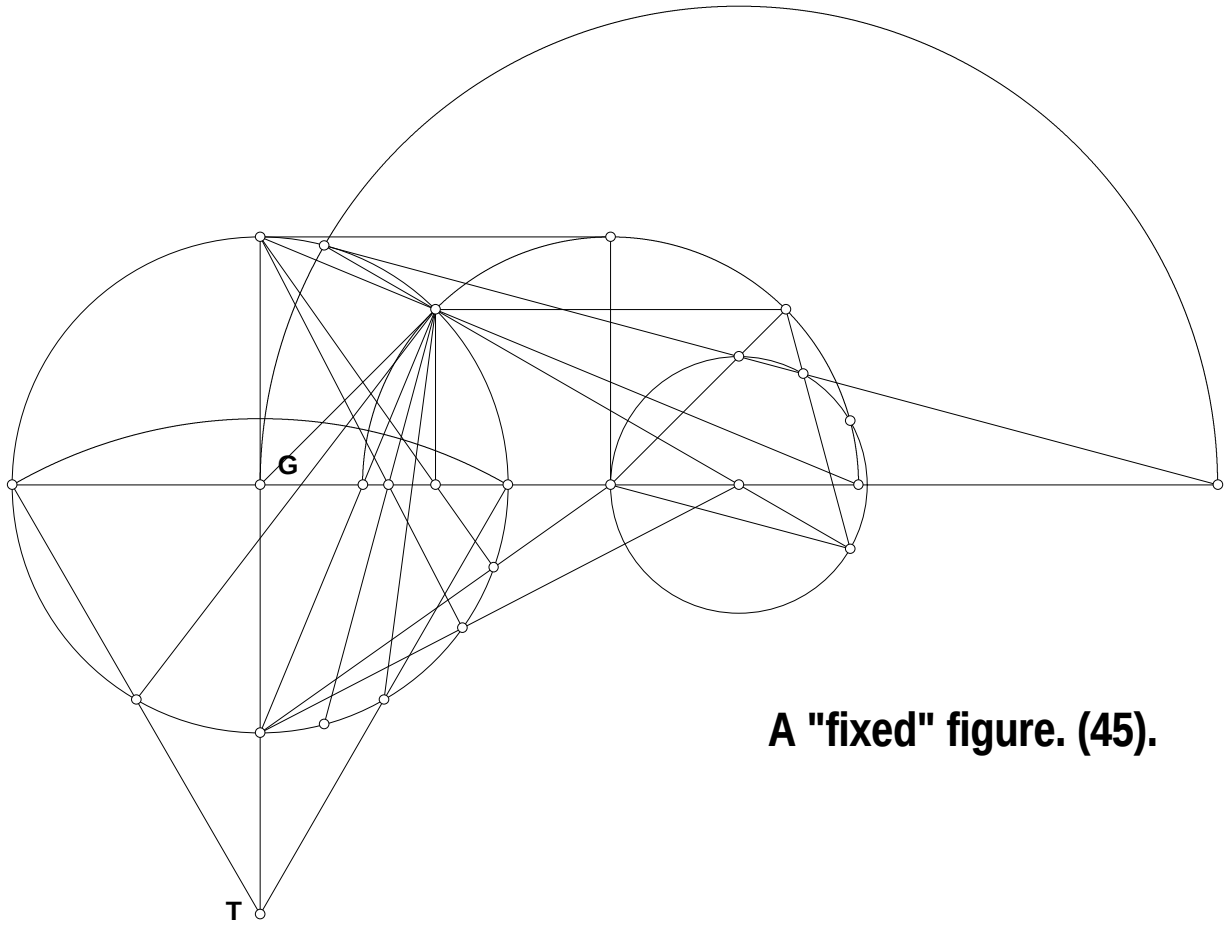
$$FS - \sqrt{\left[\frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}\right]^2+\left(\frac{N_2}{2}\right)^2}=0 \qquad \text{OR}$$

$$FS - \left(\frac{1}{2}\right)\cdot N_2\cdot\sqrt{2}\cdot\frac{\sqrt{\left(138\cdot N_2{}^2\cdot N_1{}^4+116\cdot N_2{}^3\cdot N_1{}^3+54\cdot N_2{}^4\cdot N_1{}^2+12\cdot N_2{}^5\cdot N_1\right)\ldots}}{\left(2\cdot N_1+N_2\right)^3}=0$$
$$+\ N_2{}^6+32\cdot N_1{}^6+96\cdot N_1{}^5\cdot N_2$$

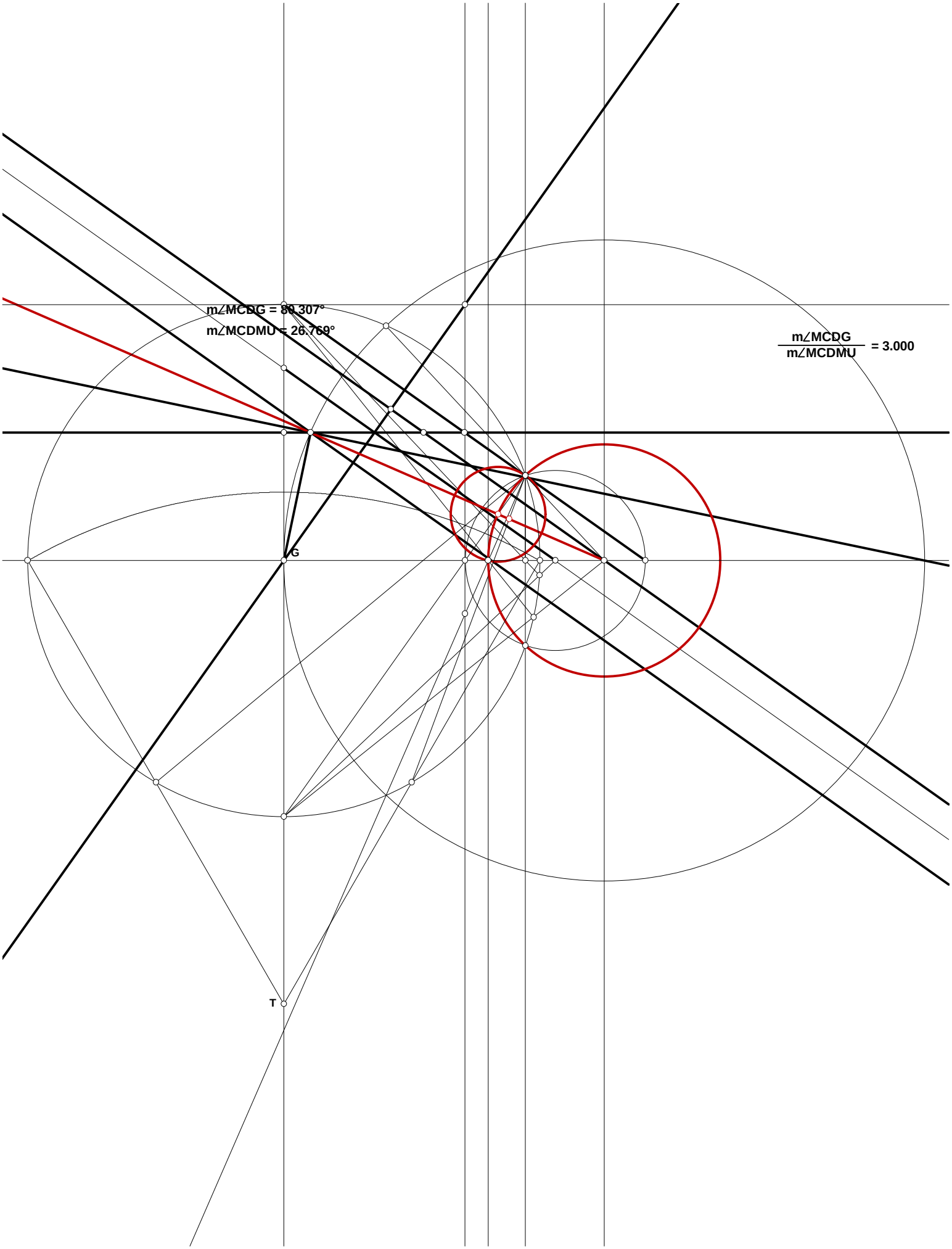
$$DG - \frac{\left(2\cdot N_1+N_2\right)^3}{\left[2\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)\right]}=0$$

$$CD - N_1\cdot\left(N_1+N_2\right)\cdot\frac{\left(2\cdot N_1+N_2\right)}{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$

$$BL - \frac{\left(N_1+N_2\right)\cdot N_1\cdot\sqrt{\left(N_2+4\cdot N_1\right)\cdot\left(3\cdot N_2+4\cdot N_1\right)}\cdot N_2}{2\cdot\left(2\cdot N_1+N_2\right)\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$

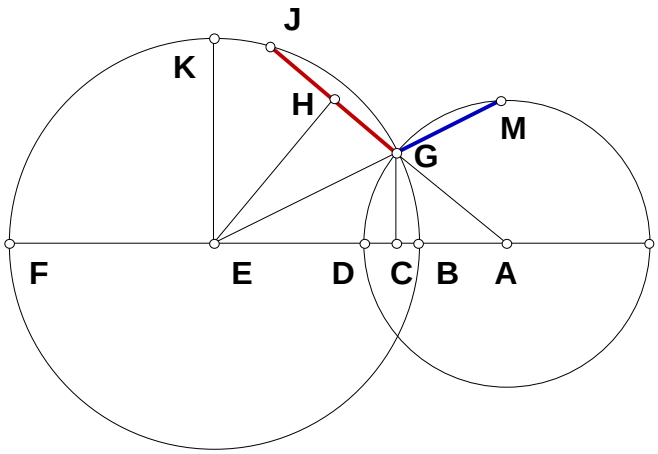



**A "fixed" figure. (45).**


m∠MCDG = 80.307°
m∠MCDMU = 26.769°
m∠MCDG / m∠MCDMU = 3.000
G
T

# A Small Extrapolation 060101

**Given AE, AG, and EG, what is the Algebraic name of the segment GJ?**




$S_1 := 3 \quad S_2 := 2 \quad S_3 := 1$

$AE := S_1 \quad AG := S_2 \quad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2\,AE}$$

$$AH := \frac{AC \cdot AE}{AG} \quad GH := AH - AG$$

$HJ := GH \quad GJ := GH + HJ$

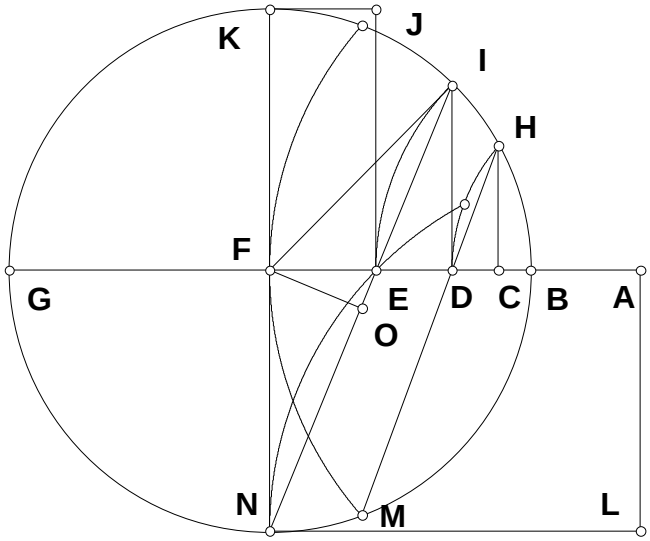
**Some Algebraic Names:**

$$\frac{S_2^2 + S_1^2 - S_3^2}{2\,S_1} - AC = 0 \qquad \frac{S_1^2 + S_2^2 - S_3^2}{2\,S_2} - AH = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{2\,S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{S_3} - GM = \ ?$$

## Units From Both Sides 060201

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




$N := 5.727 \qquad AB := .573 \qquad AG := AB \cdot N$

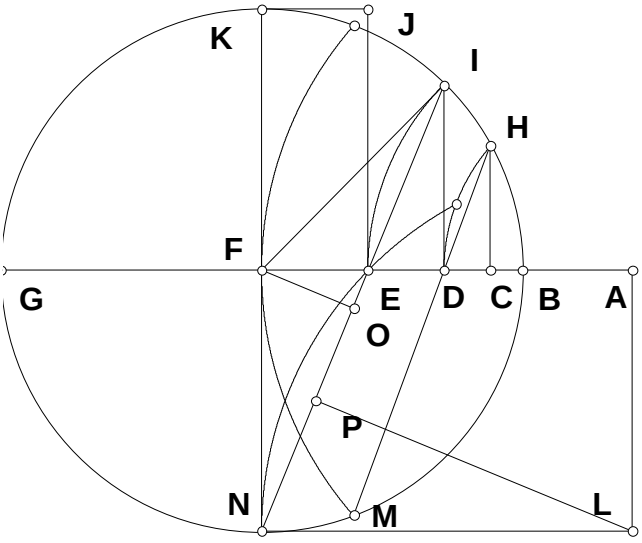
$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

$AB \cdot (N - 1) - BG = 0$

$\frac{1}{2} \cdot AB \cdot (N - 1) - BF = 0$

$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0 \qquad AB\sqrt{N} - AE = 0 \qquad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$



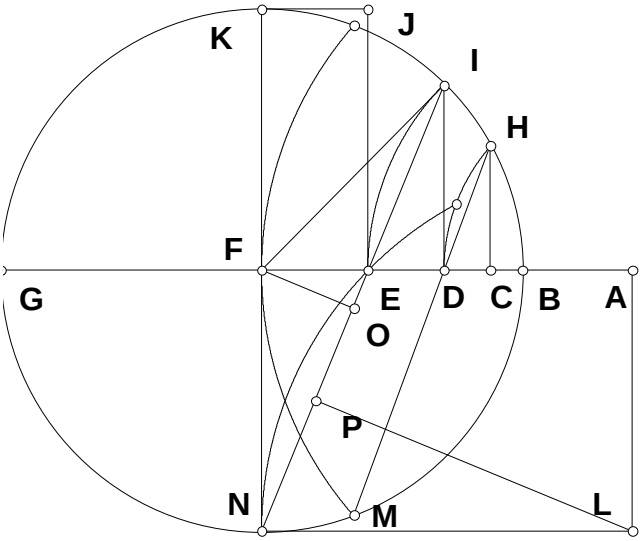

$N_2 := 2 \qquad BG_2 := 1 \qquad BF_2 := \frac{BG_2}{2}$

$BE_2 := \frac{BF_2}{N_2} \qquad EF_2 := BF_2 - BE_2$

$FN_2 := BF_2 \qquad EN_2 := \sqrt{EF_2^2 + FN_2^2}$

$NP_2 := \frac{EN_2}{2} \qquad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$

$AF_2 := LN_2 \qquad AB_2 := AF_2 - BF_2$




$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0$$

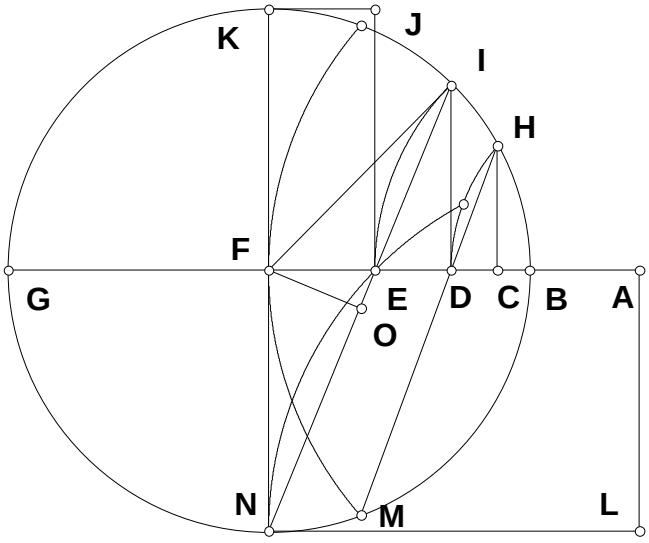
$$\frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{(2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{[N_2 \cdot (N_2 - 1)]} - LN_2 = 0$$

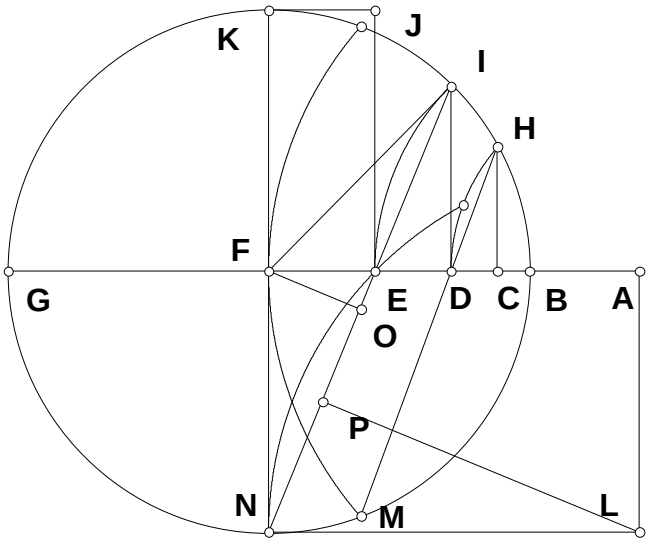
$$\frac{BG_2}{4 \cdot N_2 \cdot (N_2 - 1)} - AB_2 = 0$$




$$AI := AE \quad FI := BF \quad AD := \frac{AI^2 + AF^2 - FI^2}{2\,AF}$$

$$BD := AD - AB$$

$$AD := 2 \cdot AB \cdot \frac{N}{(N+1)} \qquad BD := AB \cdot \frac{(N-1)}{(1+N)}$$




$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2}$

$BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$

$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3$

$FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$

$EN_3 := \sqrt{EF_3^2 + FN_3^2} \quad NP_3 := \frac{EN_3}{2}$

$LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$

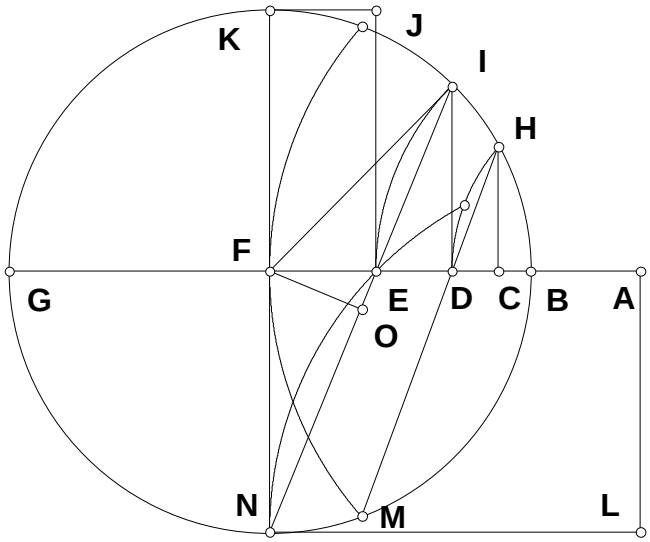
$AB_3 := AF_3 - BF_3 \quad AB_3 = 0.573$

$BD_3 := \frac{1}{2} \cdot \frac{BG_3}{N_3} \quad DG_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} \quad DI_3 := \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1}$

$DF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} \quad EF_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{(N_3 + \sqrt{2 \cdot N_3 - 1})}$

$EN_3 := \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} \quad NP_3 := \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}}$

$LN_3 := \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} \quad AB_3 := \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)}$




$AH := AD$

$FH := BF \quad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$

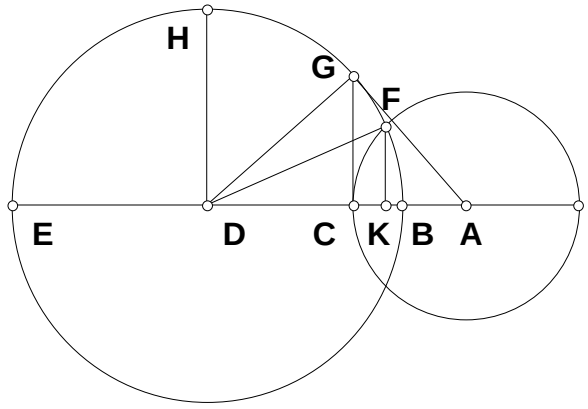
$BC := AC - AB$

$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$

$AC := AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N+1)^3} \qquad BC := AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3}$

# Isolating A Problem 060301

**If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?**




$N := 4 \qquad BE := 1$

$BD := \frac{BE}{2} \qquad BC := \frac{BE}{N} \qquad CE := BE - BC$

$CG := \sqrt{BC \cdot CE} \qquad CD := BD - BC \qquad AC := \frac{CG^2}{CD}$

$AF := AC \qquad AD := AC + CD \qquad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2\ AD} \qquad BK := BD - DK$

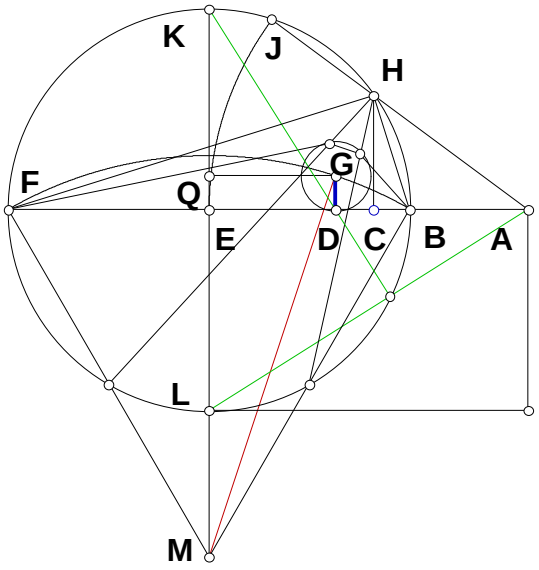
$CK := BC - BK$

$BE \cdot \frac{(N-1)}{N} - CE = 0 \qquad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \qquad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$

$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \qquad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \qquad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \qquad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \qquad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

# For Any AB 061001




## For any AB, AF what is DG?

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM$

$DG := EQ$

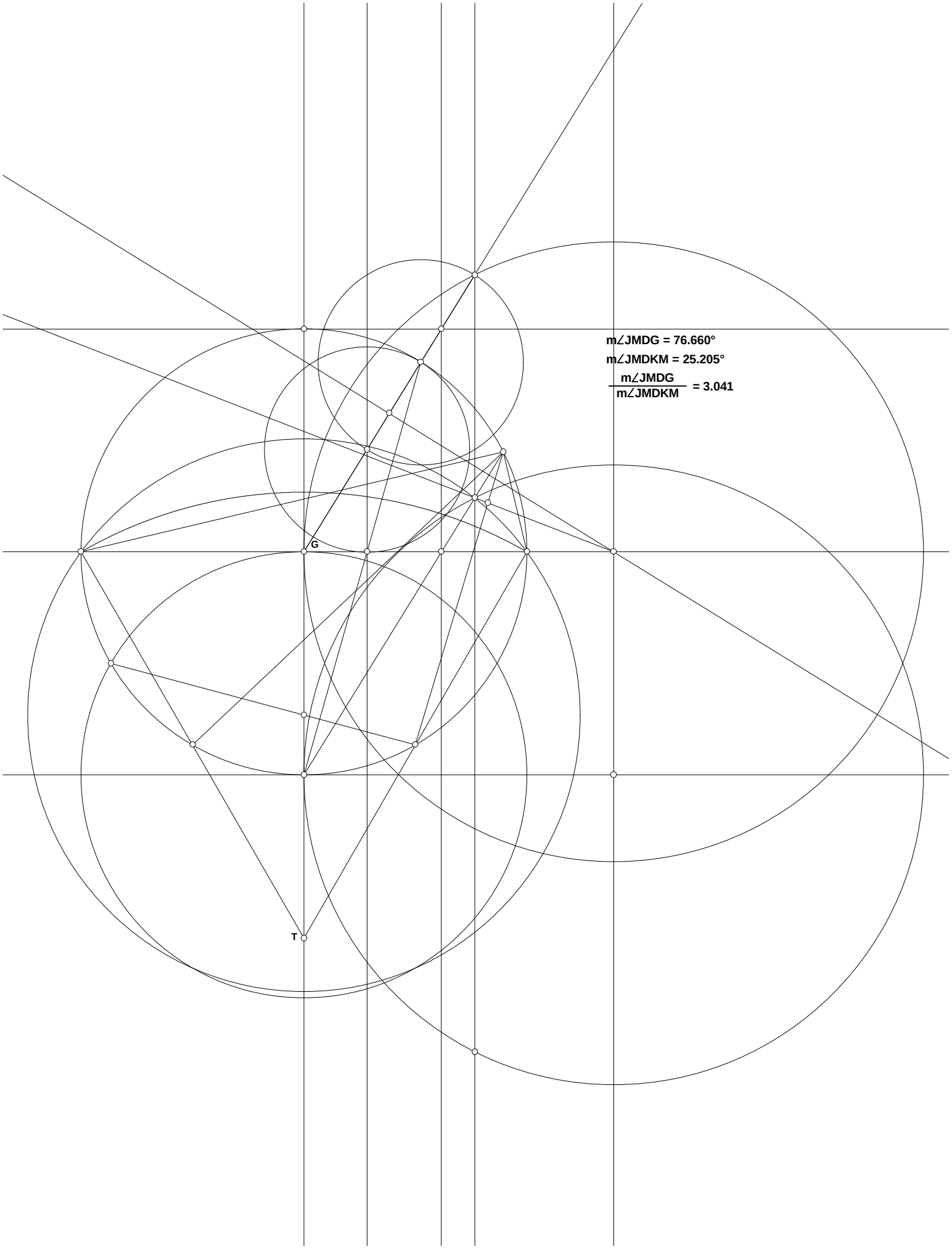
## Some Algebraic Names:

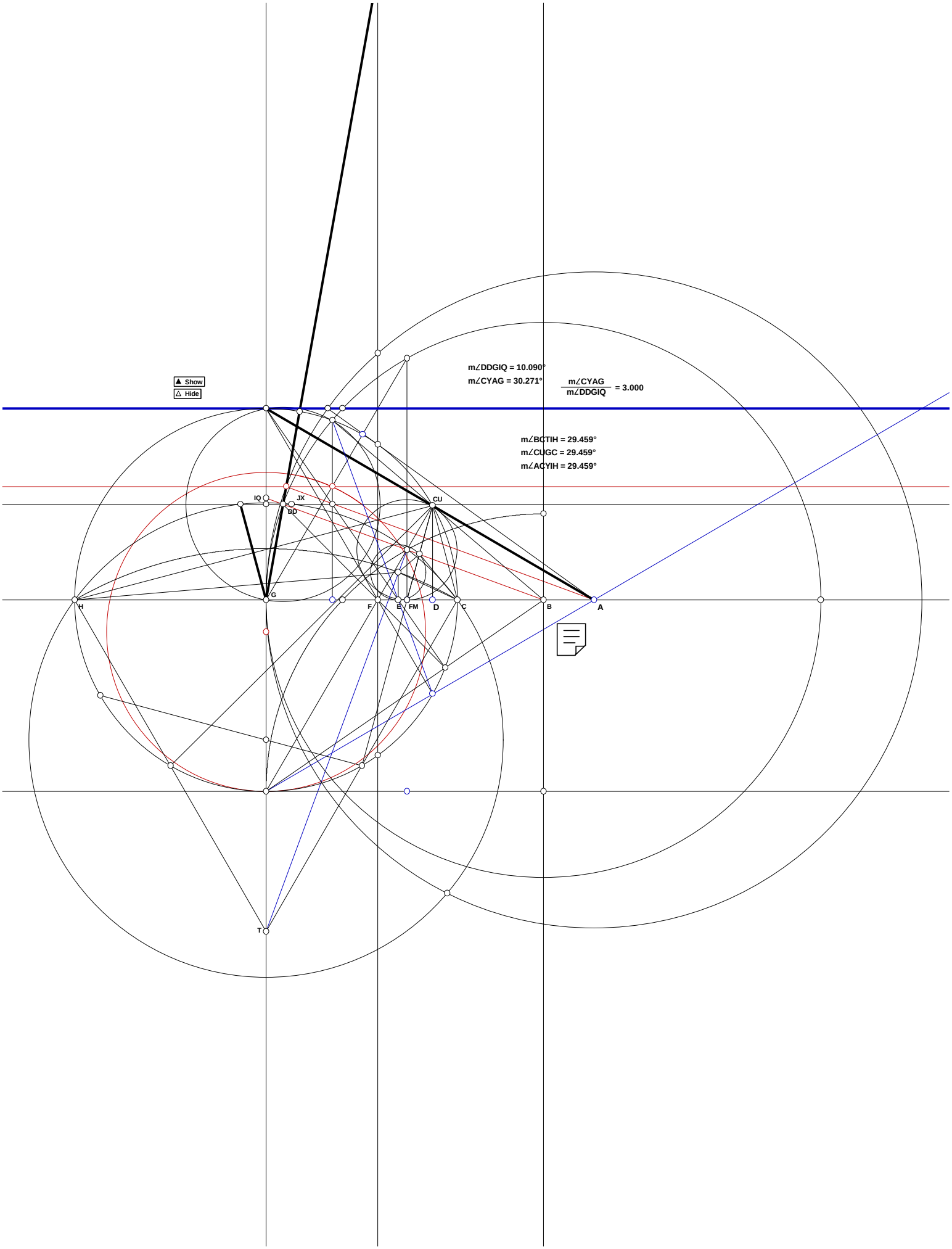
$AB \cdot (N - 1) - BF = 0 \qquad \frac{AB \cdot (N - 1)}{2} - BE = 0 \qquad \frac{1}{2} \cdot AB \cdot (N + 1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N - 1)^2}{(N + 1)} - DE = 0 \qquad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N - 1) - EM = 0$

$$\frac{AB \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)\right]}{2 \cdot (N + 1)} - MQ = 0$$

$$\frac{AB \cdot (N - 1) \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N + 1)} - DG = 0$$

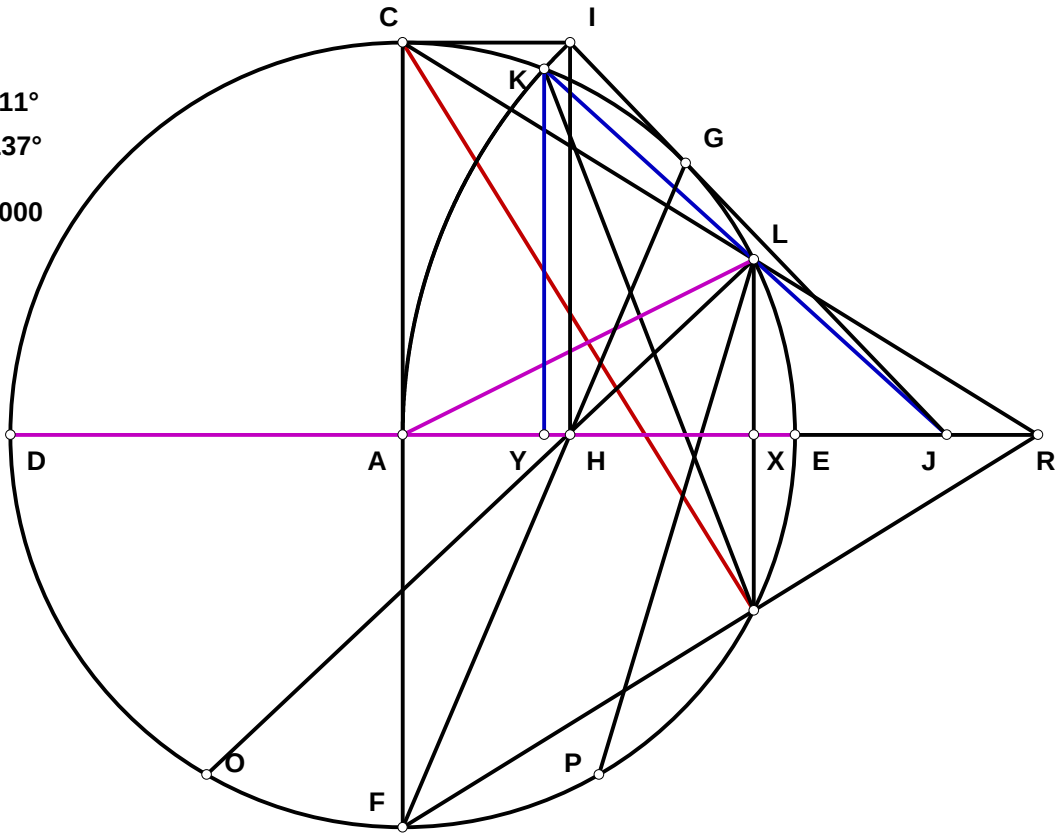

m∠JMDG = 76.660°
m∠JMDKM = 25.205°
m∠JMDG / m∠JMDKM = 3.041
G
T


Show
Hide
m∠DDGIQ = 10.090°
m∠CYAG = 30.271°
m∠CYAG / m∠DDGIQ = 3.000
m∠BCTIH = 29.459°
m∠CUGC = 29.459°
m∠ACYIH = 29.459°
IQ
JX
DD
CU
G
H
F
E
FM
D
C
B
A
T

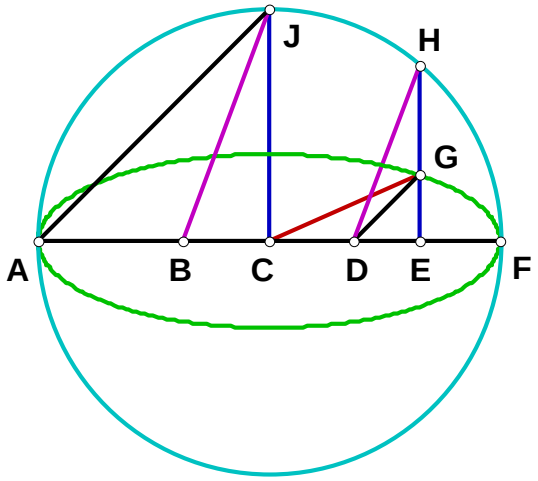
$m\angle LAC = 63.411°$

$m\angle KAC = 21.137°$

$\frac{m\angle LAC}{m\angle KAC} = 3.000$



# Elipse By Parallels 082601




$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \frac{AF}{2} \quad CJ := AC$

$BC := \frac{AC}{N_1} \qquad AE := \frac{AF}{N_2}$

$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \frac{BC \cdot EH}{CJ}$

$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

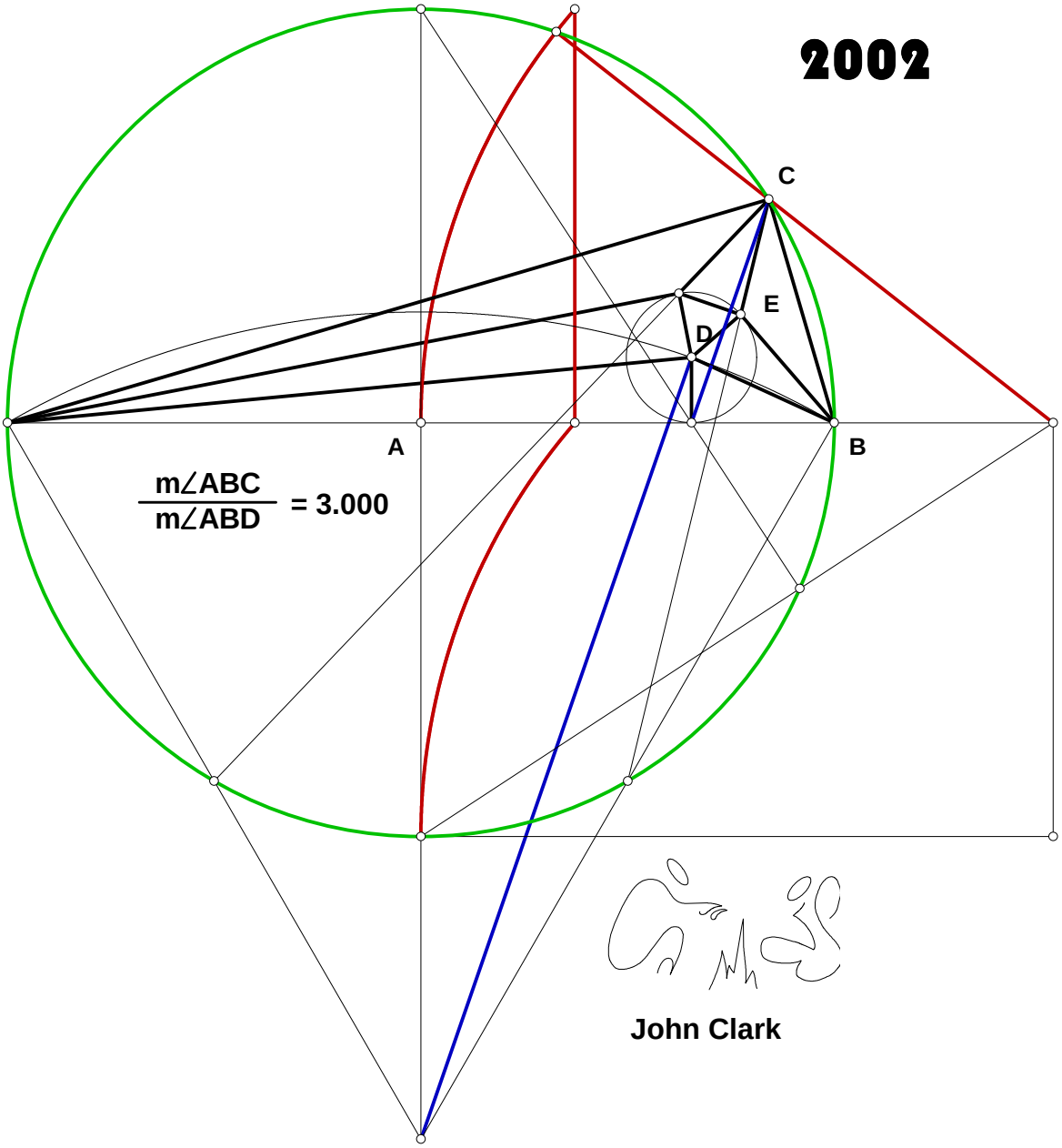
## Some Algebraic Names:

$BC := \frac{1}{(2 \cdot N_1)} \quad AE := \frac{1}{N_2} \quad EF := 1 - \frac{1}{N_2} \quad EH := \frac{\sqrt{N_2 - 1}}{N_2} \quad EG := \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2}$

$CE := \frac{1}{N_2} - \frac{1}{2} \quad CG := \frac{1}{2} \cdot \frac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)}$

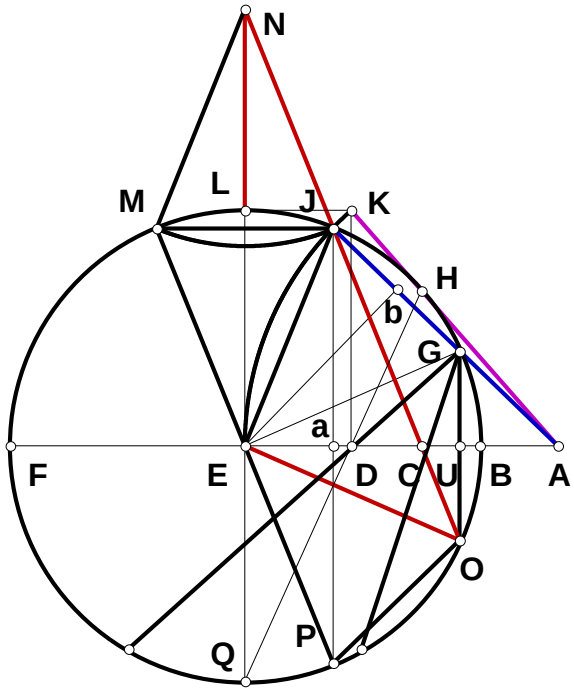
# Three Pieces Of Paper

2002




John Clark

# Just Another Proof Of Paper 010202




$N := 7 \quad AB := 1 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$

$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$

$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$

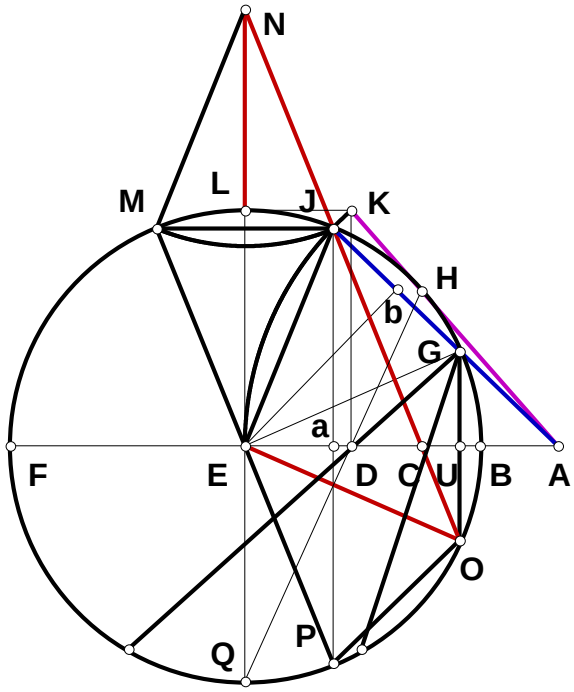
$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$

**From 4/29/94** $OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$

**Some Algebraic Names;**




$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0$$

$$\frac{N+1}{2} - AE = 0 \qquad \frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

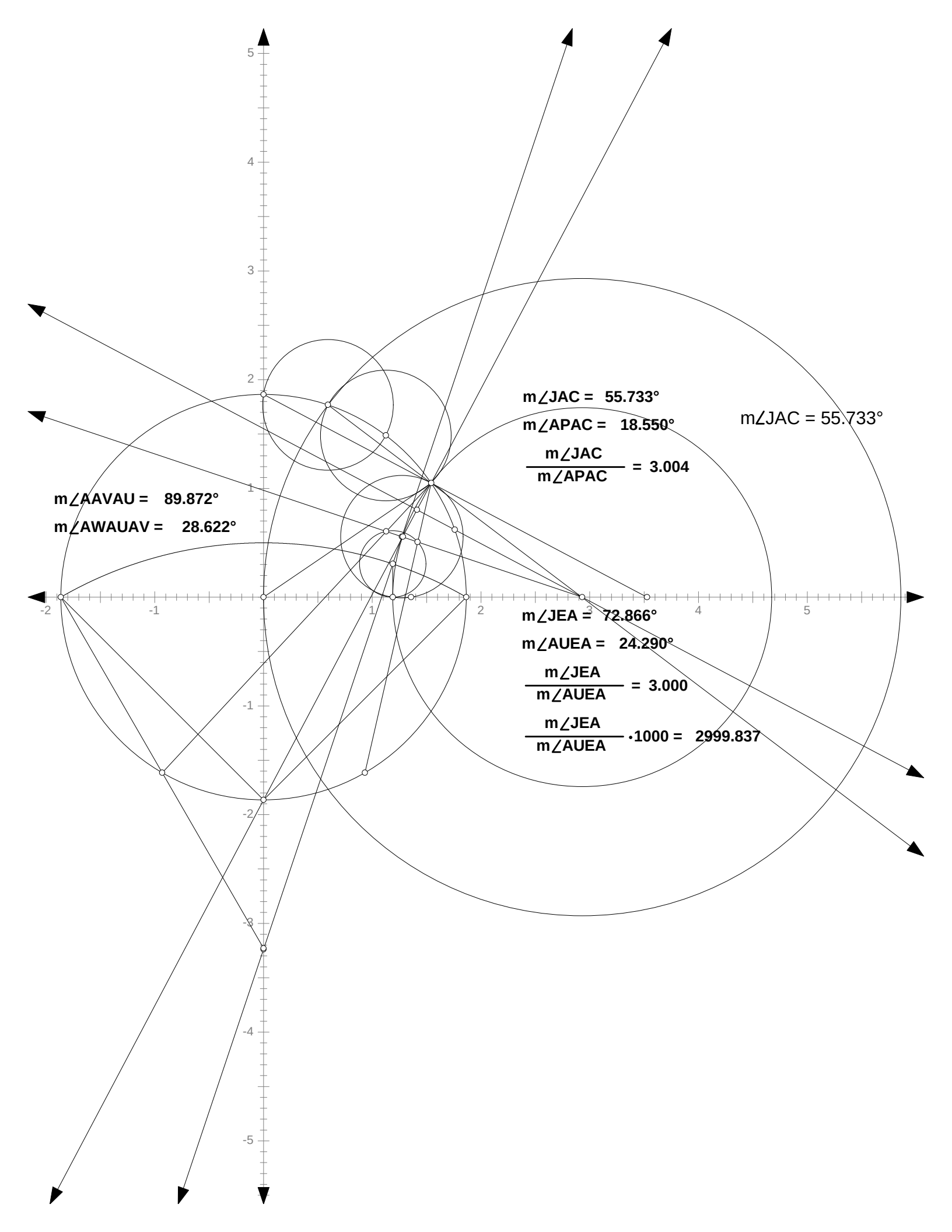
$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0$$

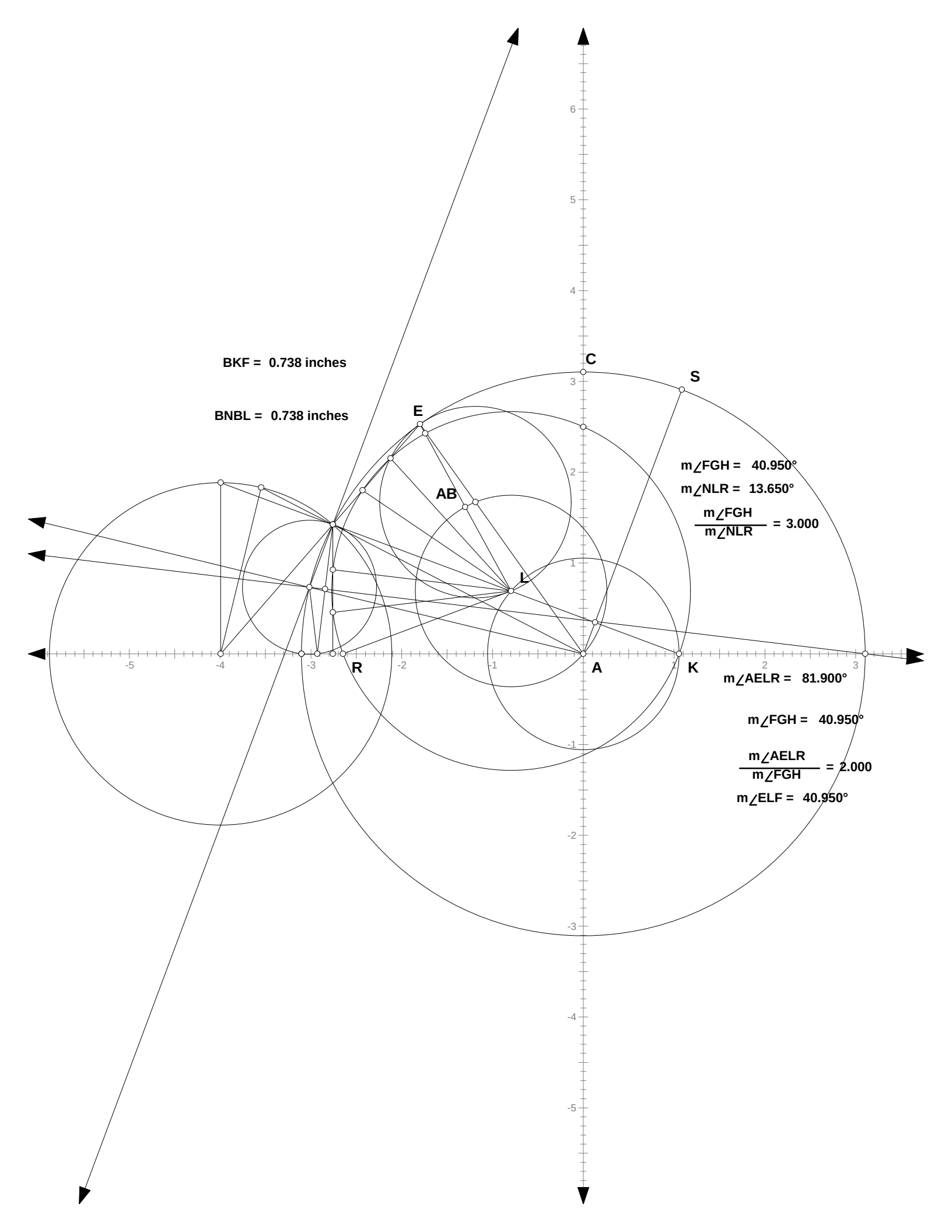
$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0 \qquad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - OP = 0 \qquad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

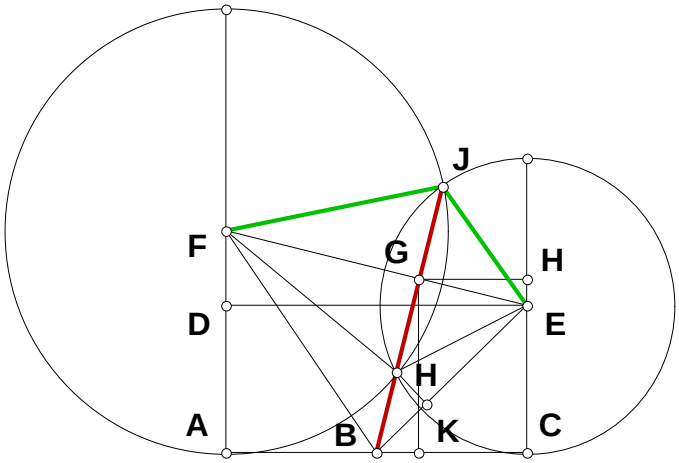
$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$


m∠JAC = 55.733°
m∠JAC = 55.733°
m∠APAC = 18.550°
m∠JAC / m∠APAC = 3.004
m∠AAVAU = 89.872°
m∠AWAUAV = 28.622°
m∠JEA = 72.866°
m∠AUEA = 24.290°
m∠JEA / m∠AUEA = 3.000
m∠JEA / m∠AUEA ·1000 = 2999.837


BKF = 0.738 inches
BNBL = 0.738 inches
C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

## Easy Power-Line 06_20_02

**For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.**




$AC := 5 \qquad AF := 4$

$CE := 3 \qquad FH := AF \quad EH := CE$

$AD := CE \quad DF := AF - AD$

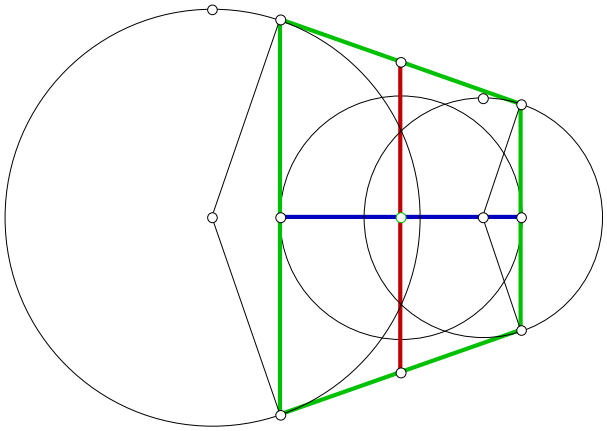
$EJ := CE \quad FJ := AF$

$DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

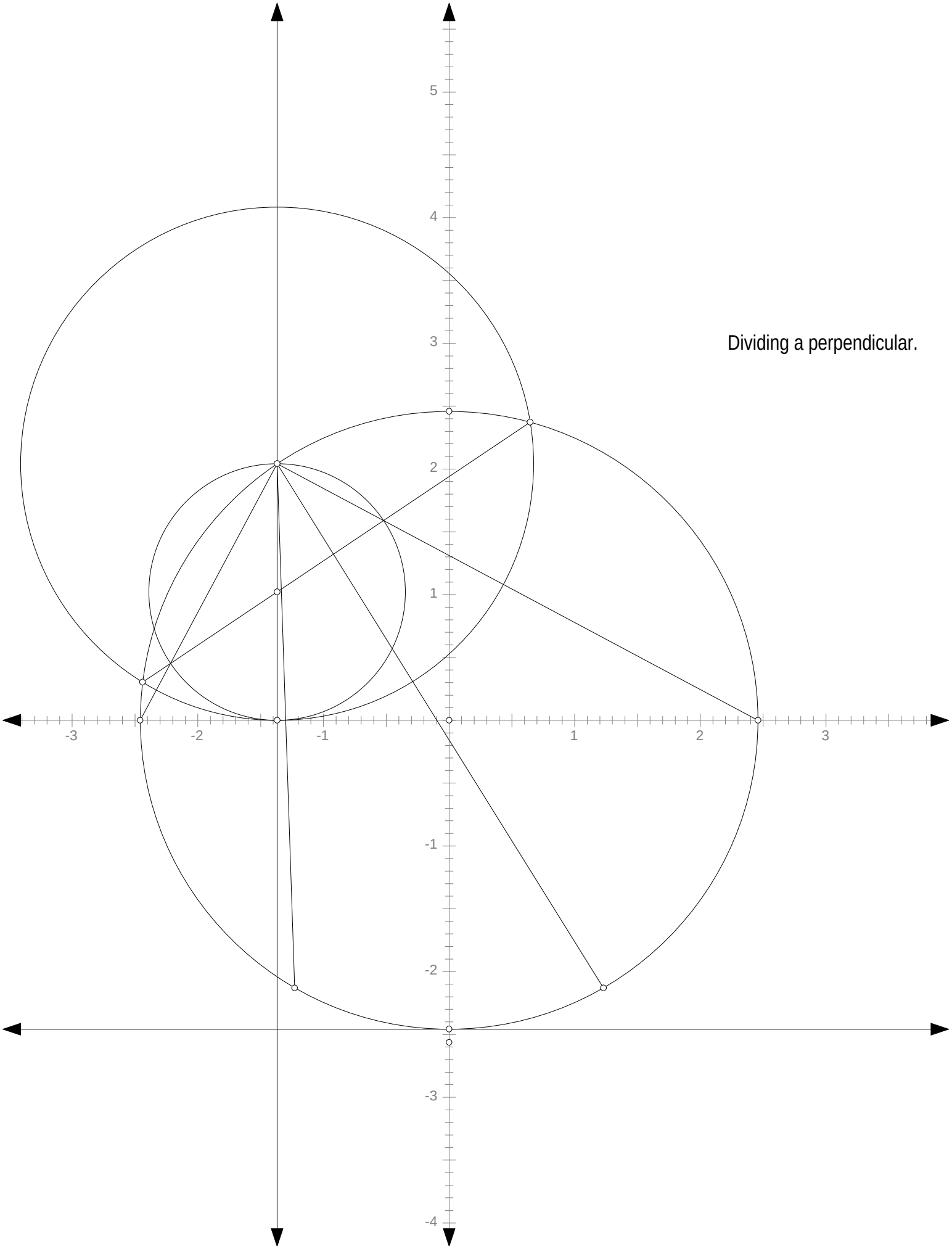
$EG := \dfrac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF}$

$EH := \dfrac{DF \cdot EG}{EF} \quad CH := CE + EH \quad GH := \dfrac{DE \cdot EG}{EF} \quad CK := GH \quad KG := CH \quad BK := \dfrac{DF \cdot KG}{DE} \quad BC := CK + BK$

$BC - \dfrac{AC}{2} = 0$



**The power line is the line upon which lays a segment between the two midpoints on the two tangents of two given circles or between a point and a circle.**
**The intersection of two circles lays halfway between their two tangents.**
**A power plane lays between two spheres or a sphere and a point.**

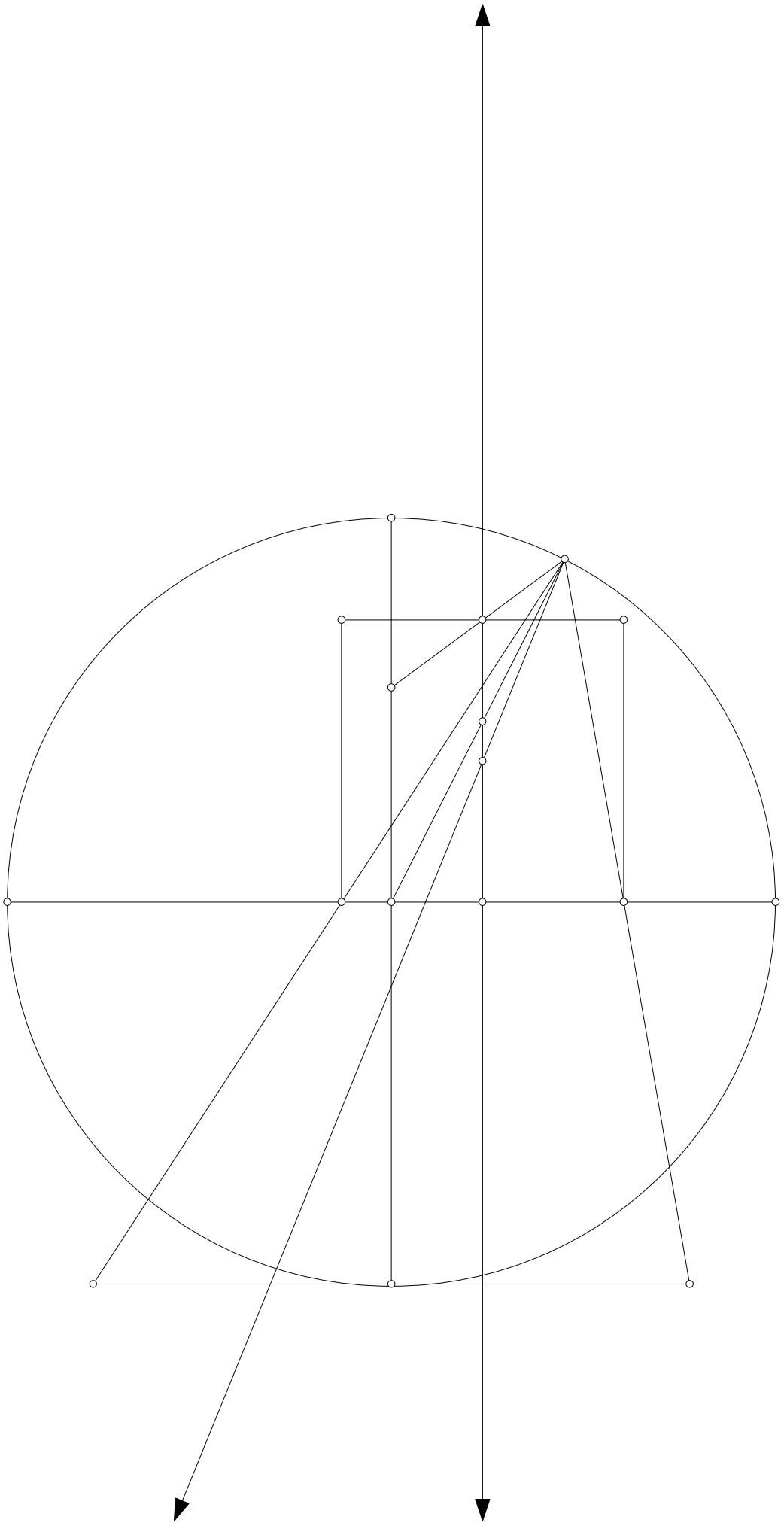


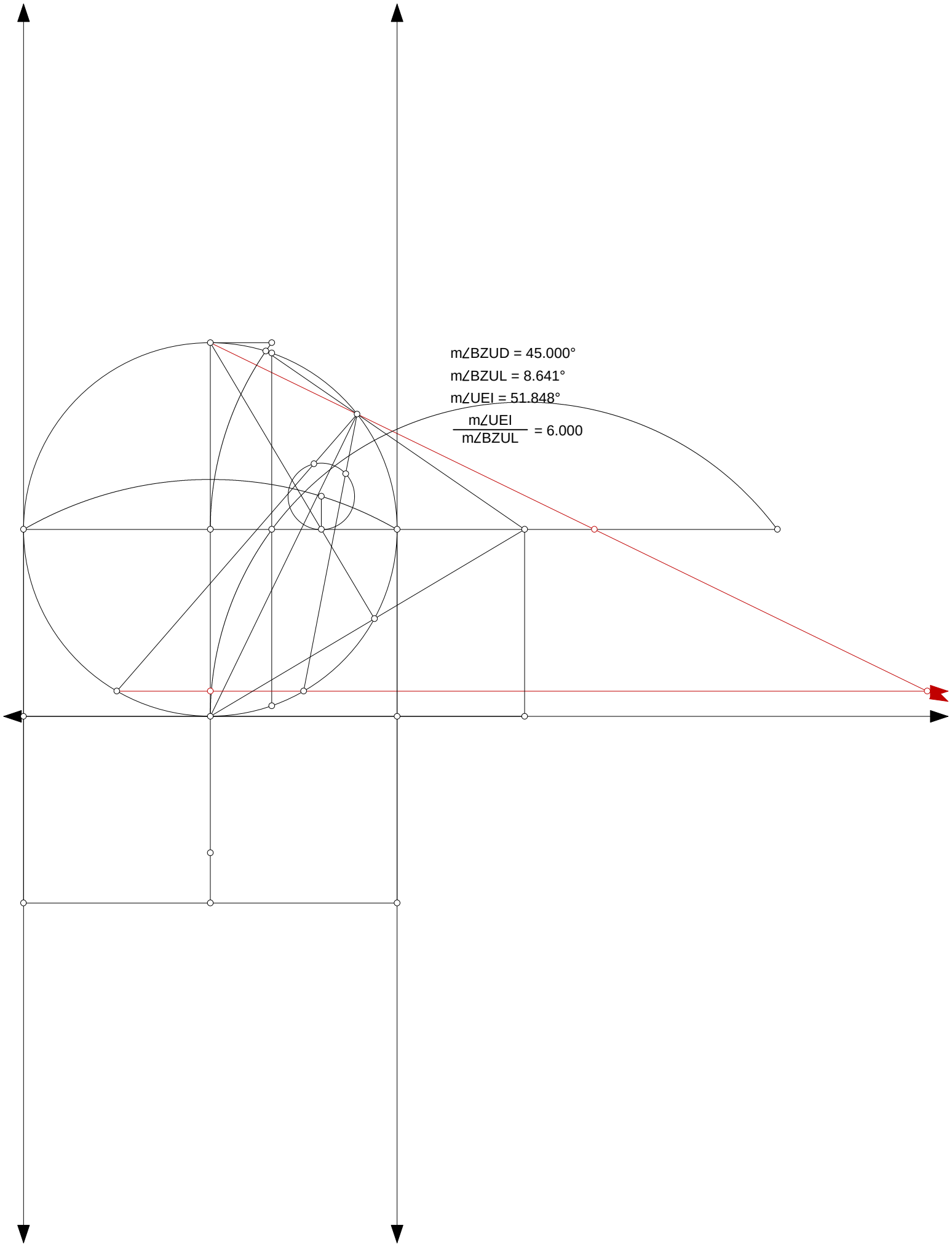

Dividing a perpendicular.

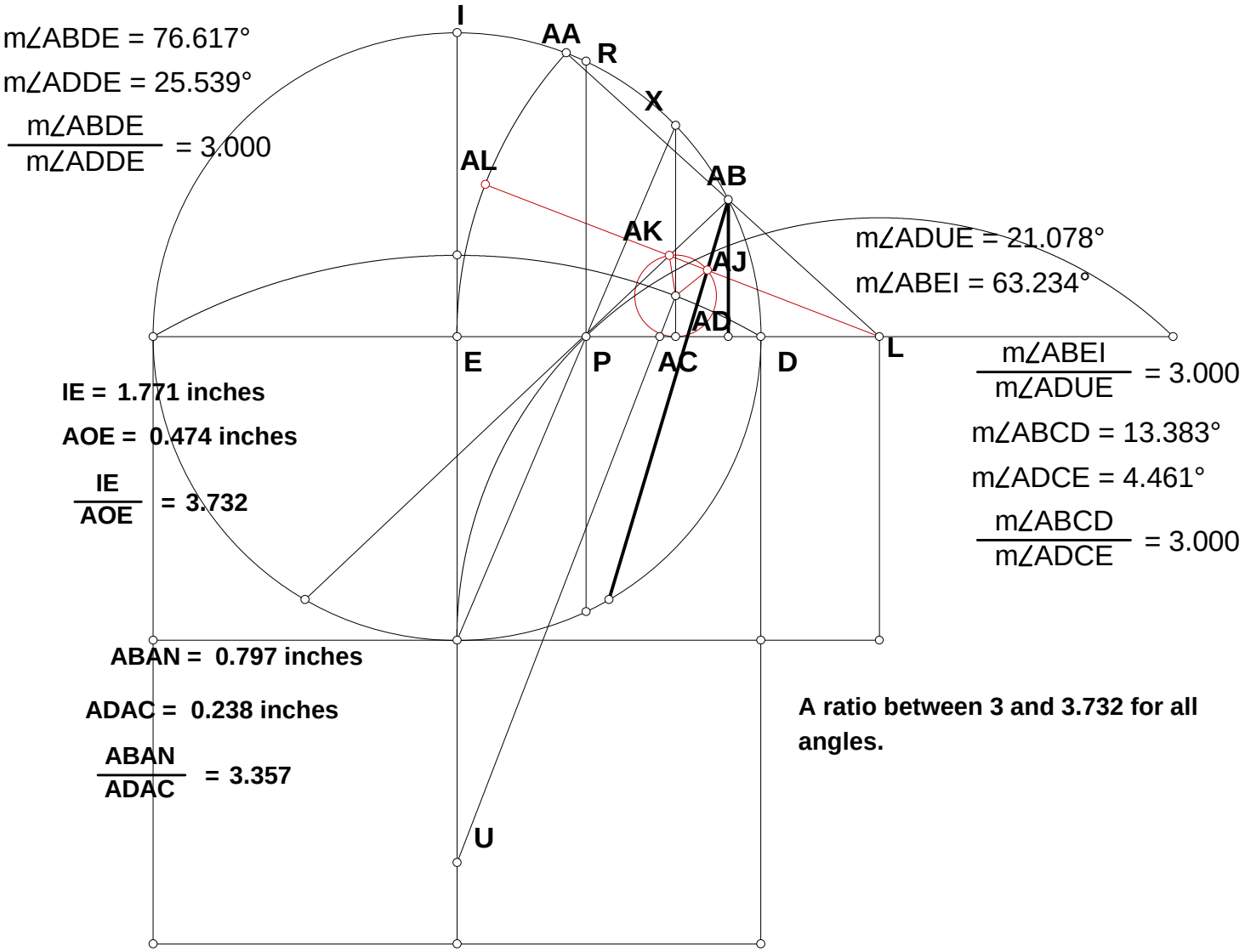
**ZX = 1.951 inches**

**ABX = 0.702 inches**

$$\frac{ZX}{ABX} = 2.780$$




m∠BZUD = 45.000°
m∠BZUL = 8.641°
m∠UEI = 51.848°
m∠UEI / m∠BZUL = 6.000




m∠ABDE = 76.617°

m∠ADDE = 25.539°

$\frac{m\angle ABDE}{m\angle ADDE} = 3.000$

m∠ADUE = 21.078°

m∠ABEI = 63.234°

$\frac{m\angle ABEI}{m\angle ADUE} = 3.000$

m∠ABCD = 13.383°

m∠ADCE = 4.461°

$\frac{m\angle ABCD}{m\angle ADCE} = 3.000$

**IE = 1.771 inches**

**AOE = 0.474 inches**

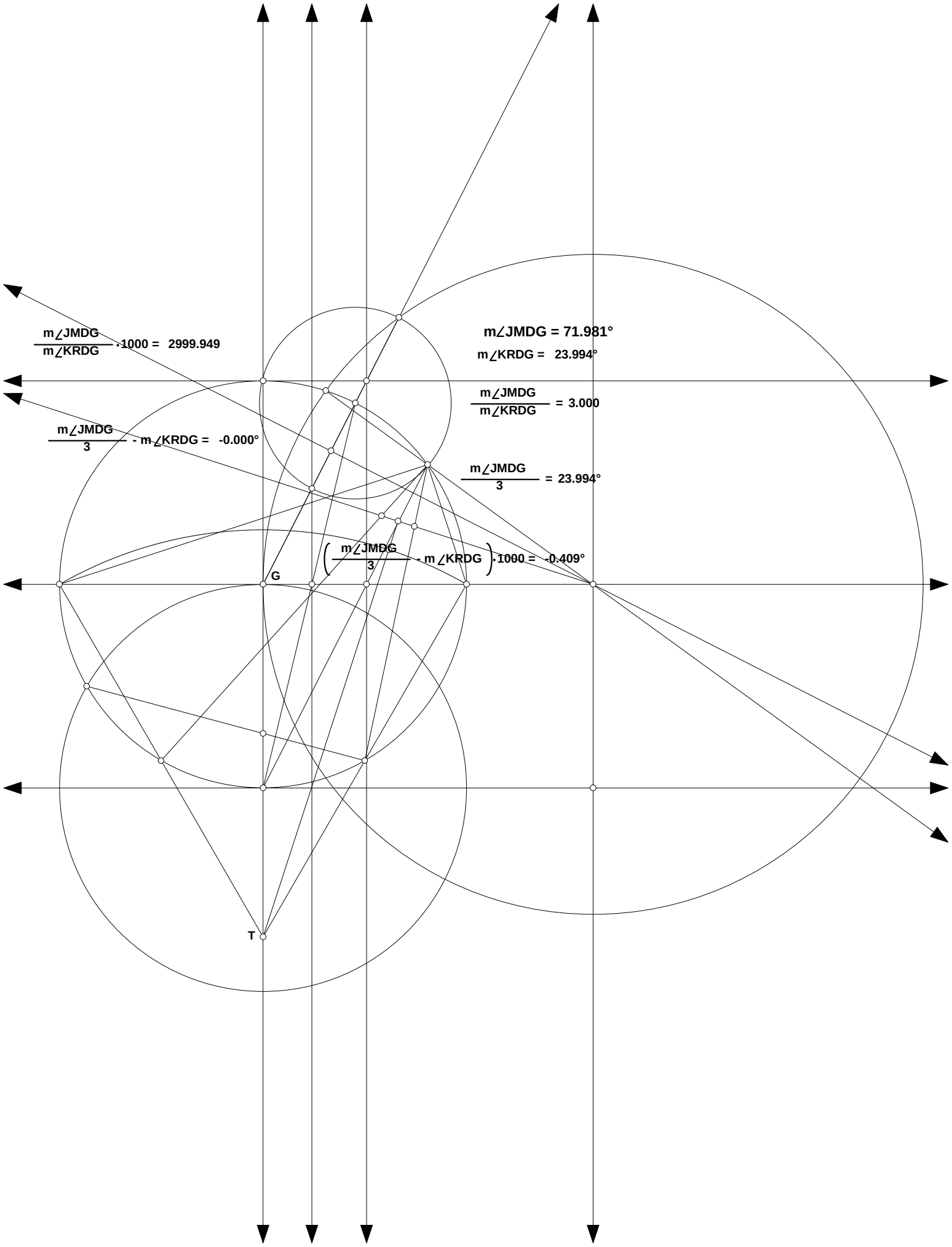
$\frac{IE}{AOE} = 3.732$

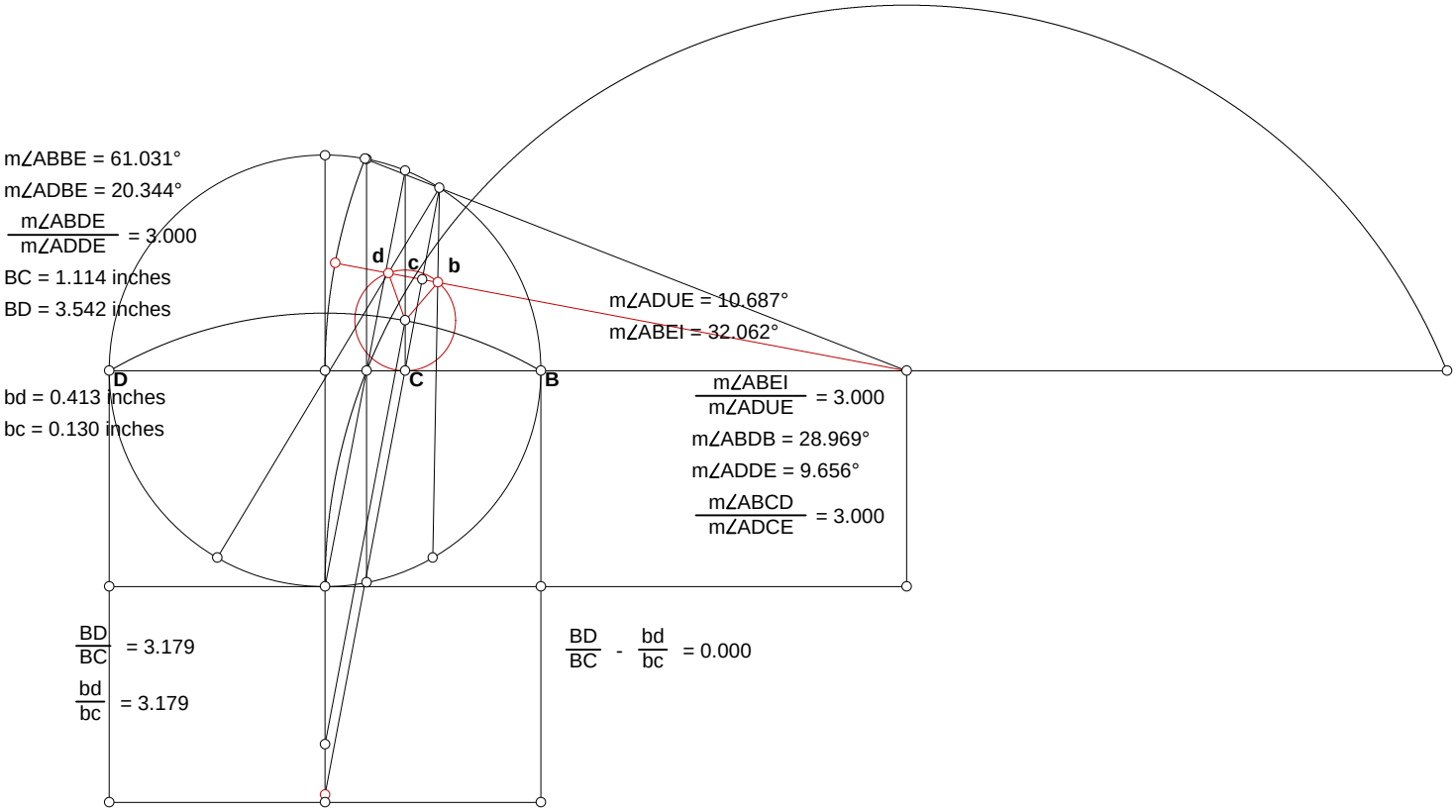
**ABAN = 0.797 inches**

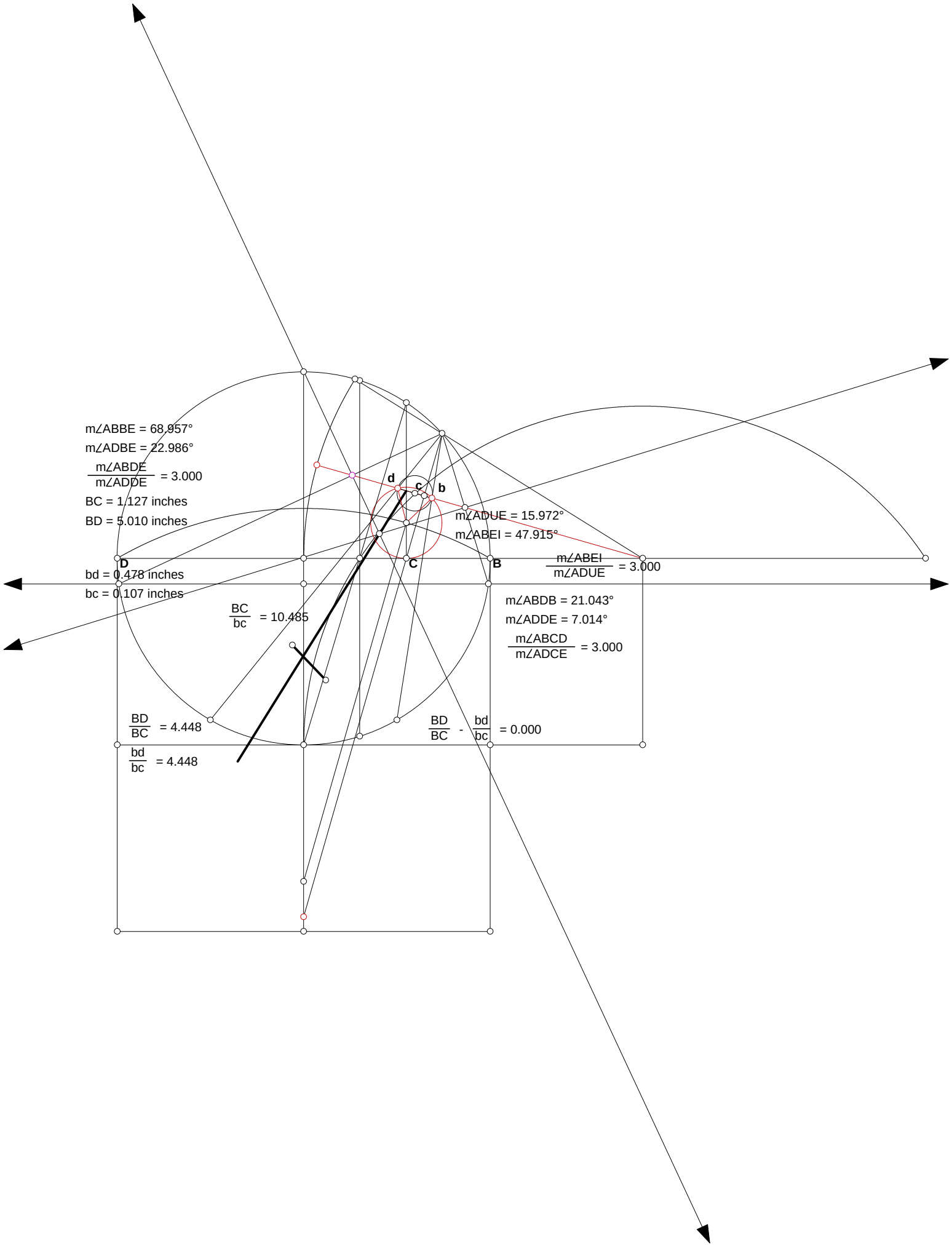
**ADAC = 0.238 inches**

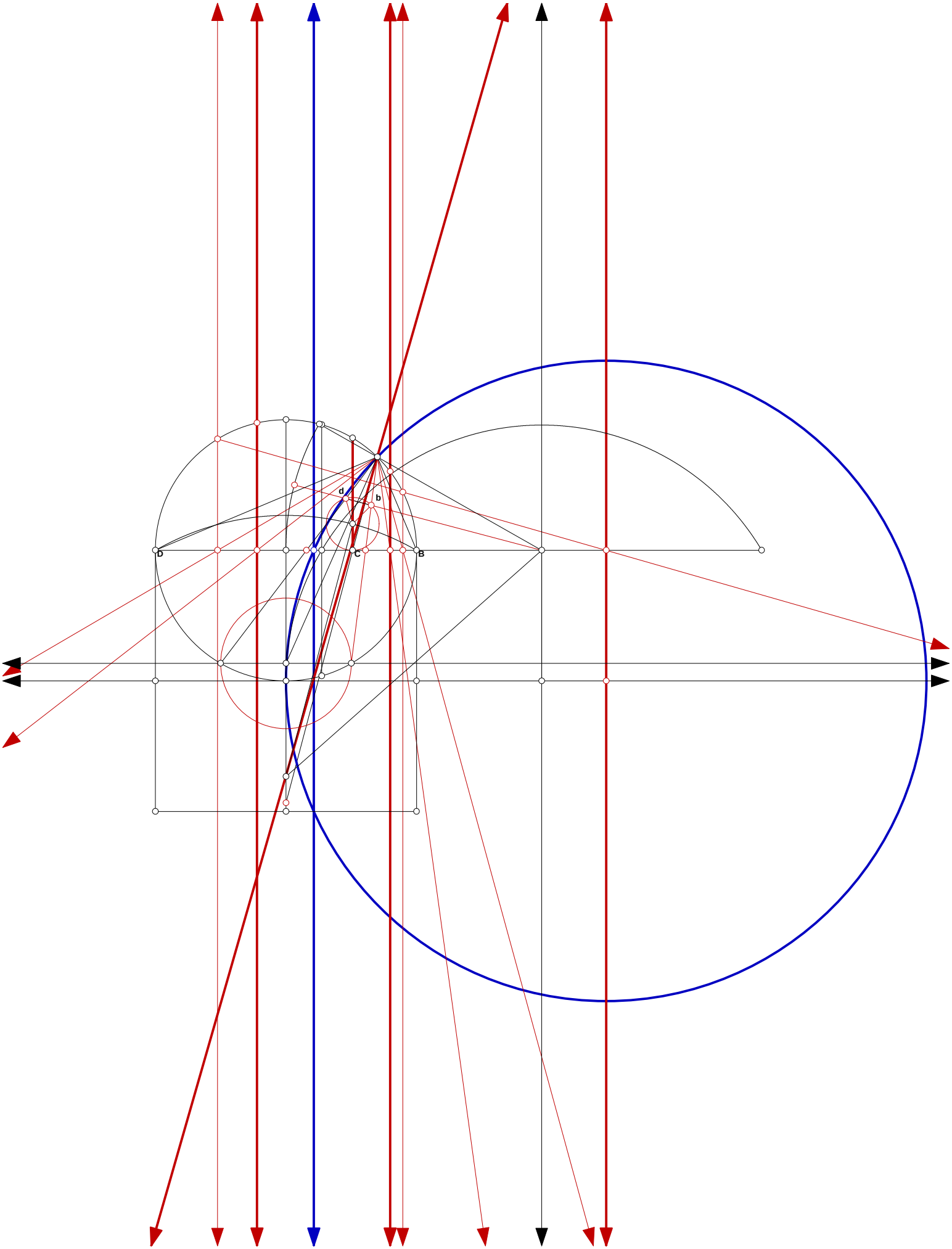
$\frac{ABAN}{ADAC} = 3.357$

**A ratio between 3 and 3.732 for all angles.**

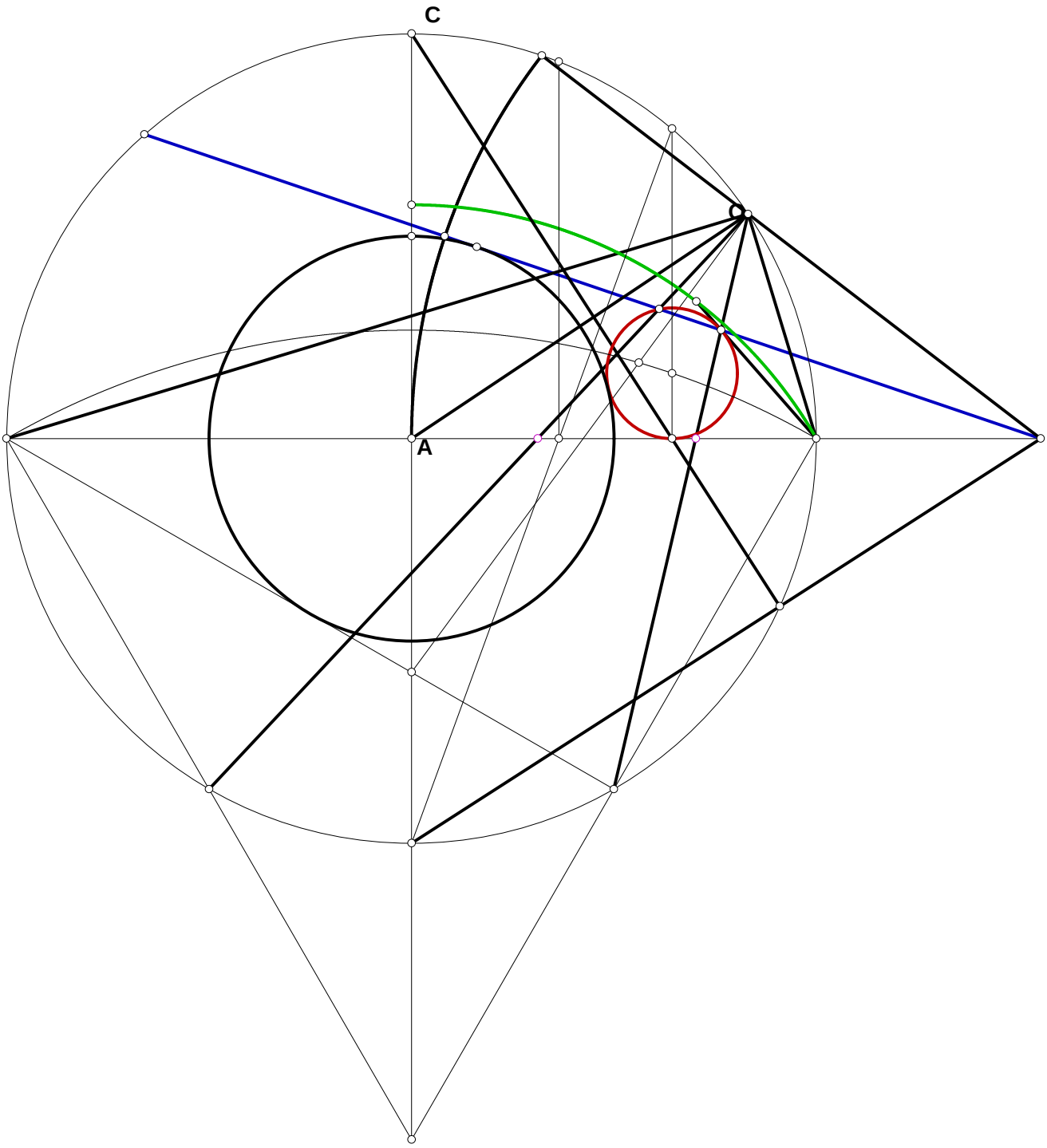

$\frac{m\angle JMDG}{m\angle KRDG} \cdot 1000 = 2999.949$
$m\angle JMDG = 71.981°$
$m\angle KRDG = 23.994°$
$\frac{m\angle JMDG}{m\angle KRDG} = 3.000$
$\frac{m\angle JMDG}{3} - m\angle KRDG = -0.000°$
$\frac{m\angle JMDG}{3} = 23.994°$
$\left(\frac{m\angle JMDG}{3} - m\angle KRDG\right) \cdot 1000 = -0.409°$
G
T


m∠ABBE = 61.031°
m∠ADBE = 20.344°
m∠ABDE / m∠ADDE = 3.000
BC = 1.114 inches
BD = 3.542 inches
bd = 0.413 inches
bc = 0.130 inches
d
c
b
D
C
B
m∠ADUE = 10.687°
m∠ABEI = 32.062°
m∠ABEI / m∠ADUE = 3.000
m∠ABDB = 28.969°
m∠ADDE = 9.656°
m∠ABCD / m∠ADCE = 3.000
BD / BC = 3.179
bd / bc = 3.179
BD / BC - bd / bc = 0.000


m∠ABBE = 68.957°
m∠ADBE = 22.986°
m∠ABDE / m∠ADDE = 3.000
BC = 1.127 inches
BD = 5.010 inches
bd = 0.478 inches
bc = 0.107 inches
BC / bc = 10.485
BD / BC = 4.448
bd / bc = 4.448
BD / BC - bd / bc = 0.000
m∠ADUE = 15.972°
m∠ABEI = 47.915°
m∠ABEI / m∠ADUE = 3.000
m∠ABDB = 21.043°
m∠ADDE = 7.014°
m∠ABCD / m∠ADCE = 3.000
D
C
B
d
c
b


d
b
D
C
B

m∠OKJ = 18.765°

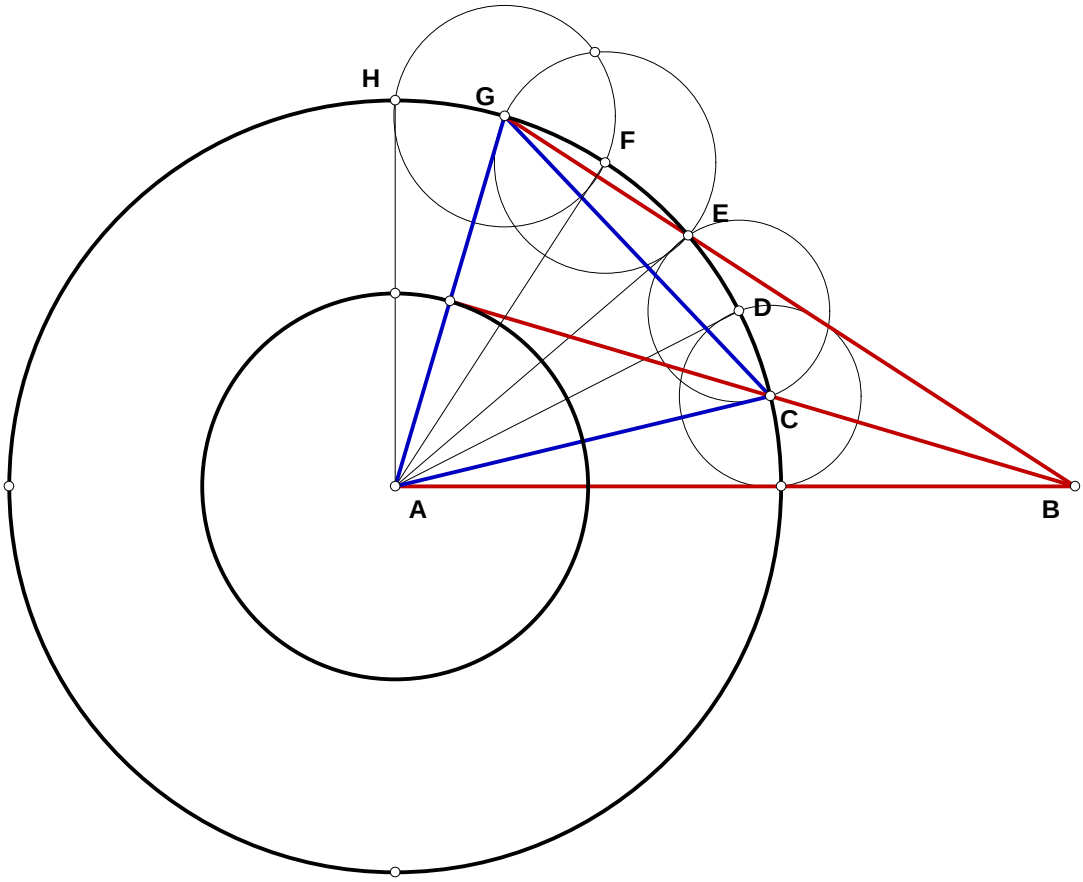


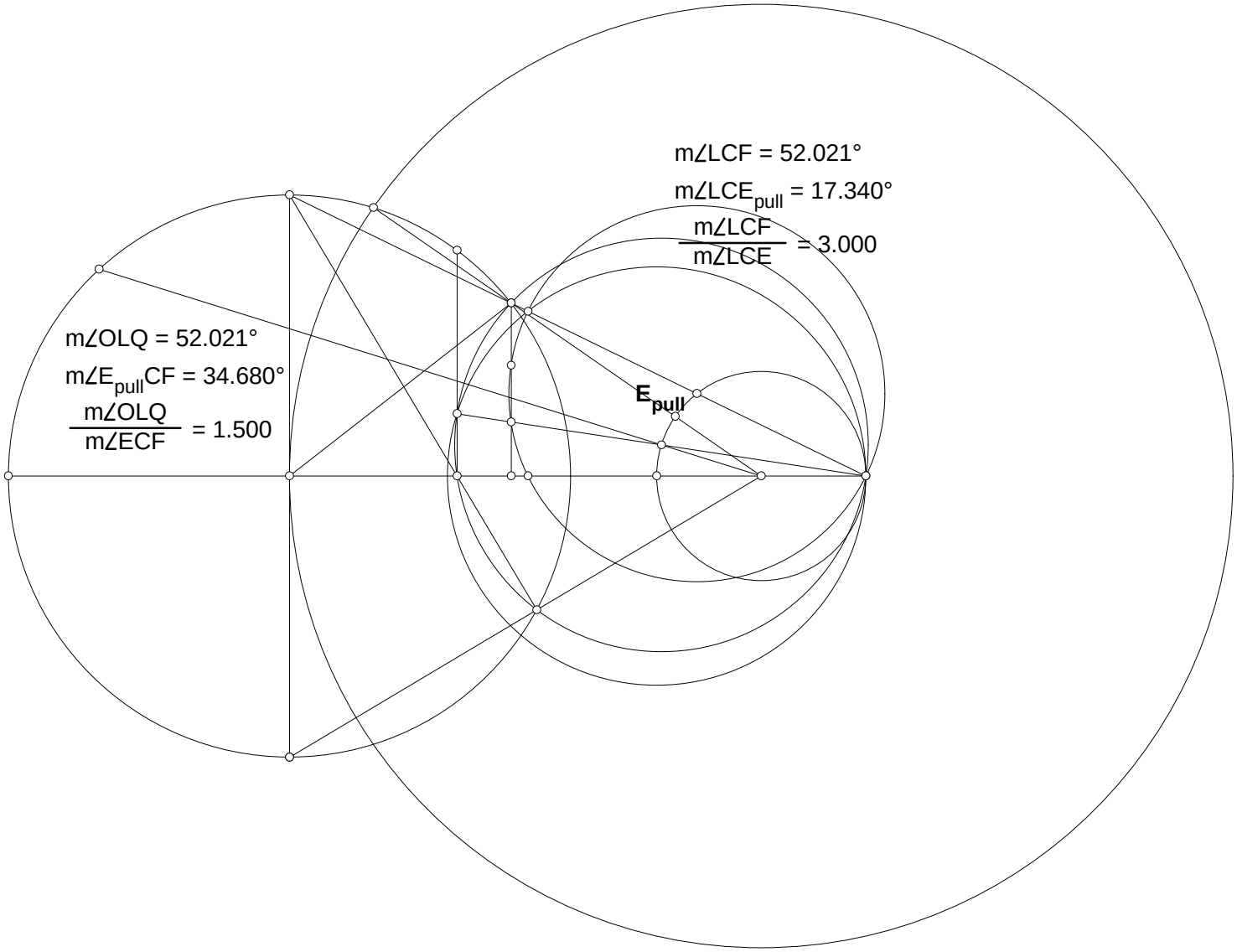
m∠EAH = 49.456°

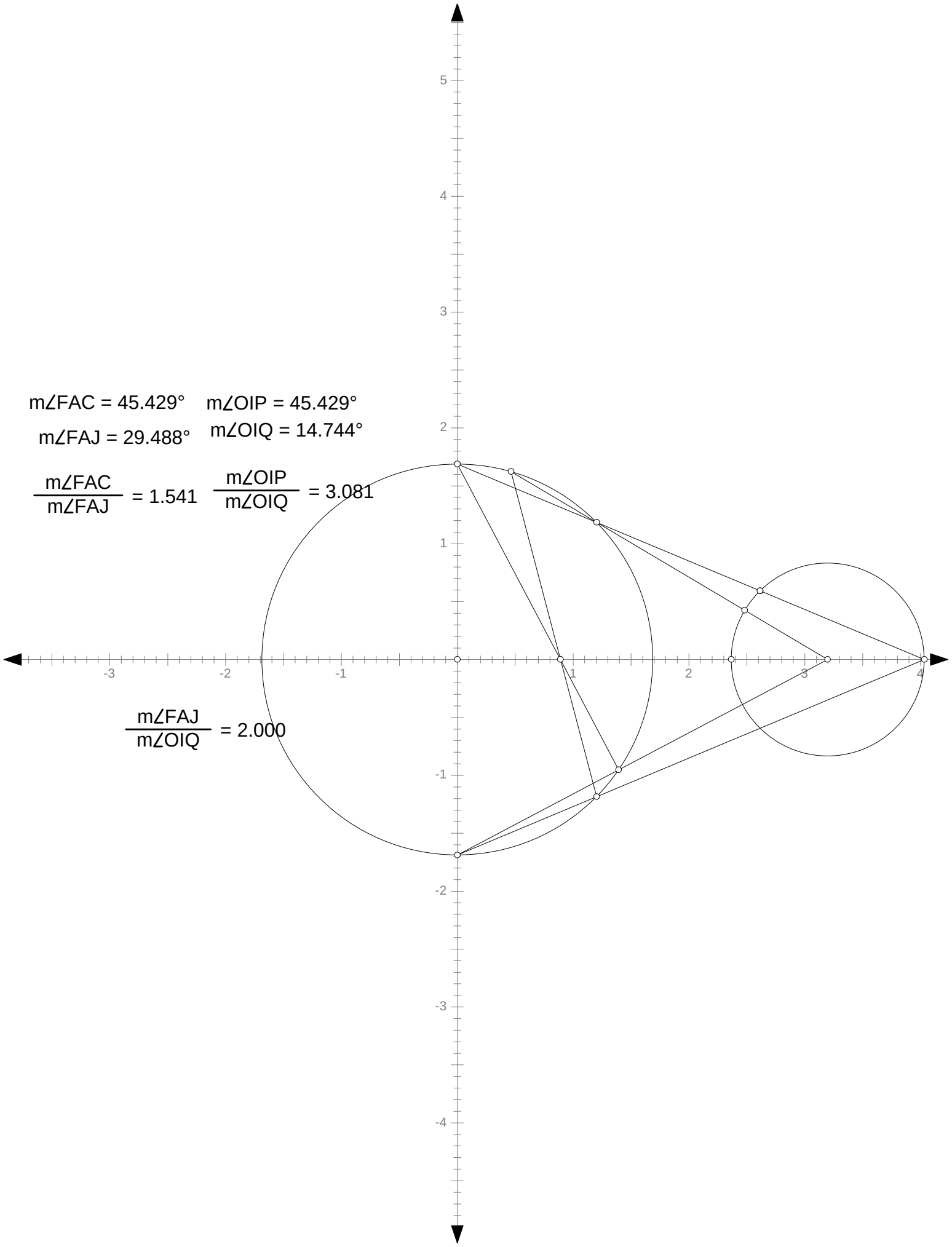
m∠EAH = 49.456°

m∠EAH = 40.544°

m∠EAH + m∠EAH = 90.000°




m∠LCF = 52.021°
m∠LCE$_{pull}$ = 17.340°
$\frac{m\angle LCF}{m\angle LCE}$ = 3.000
m∠OLQ = 52.021°
m∠E$_{pull}$CF = 34.680°
$\frac{m\angle OLQ}{m\angle ECF}$ = 1.500
**E$_{pull}$**

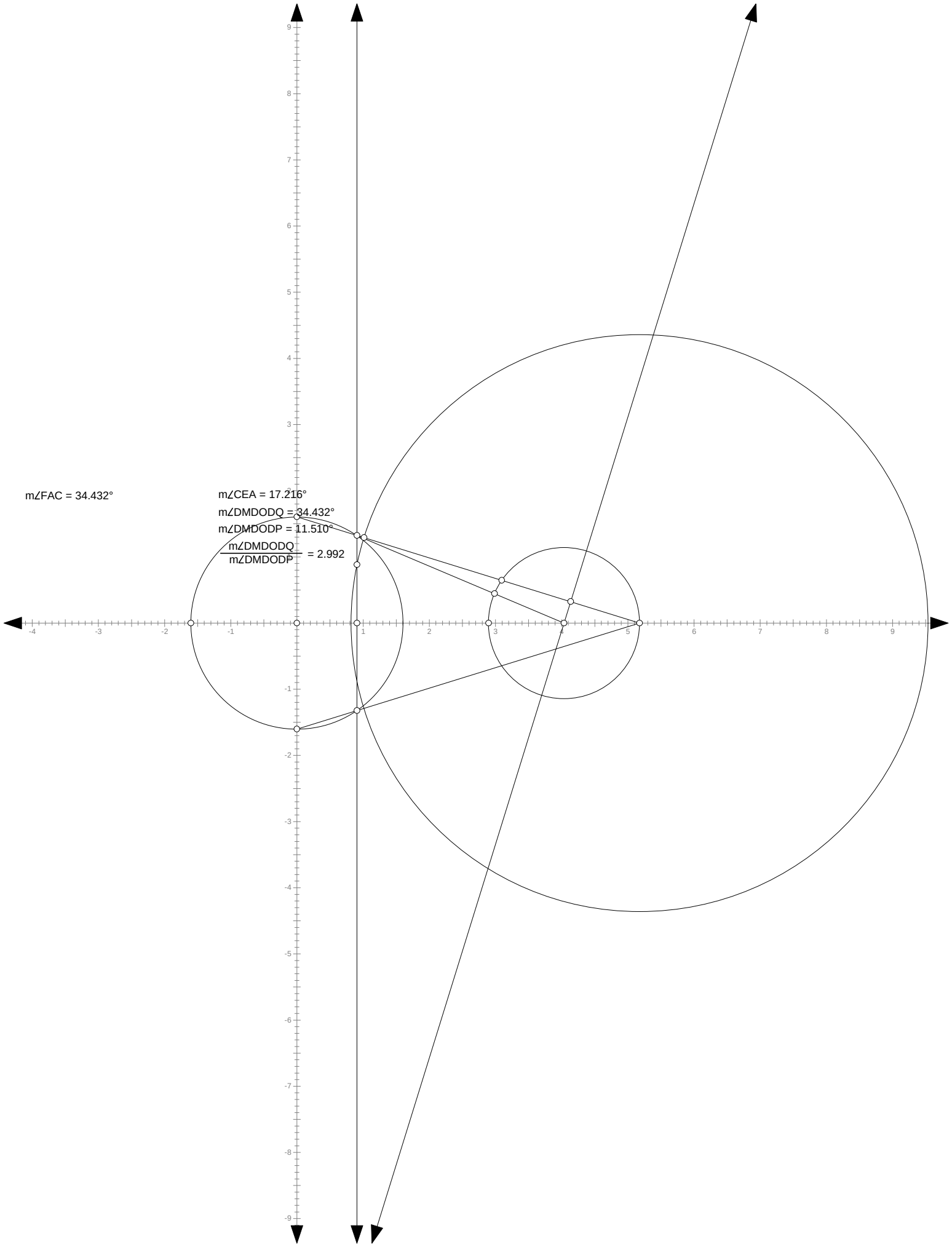


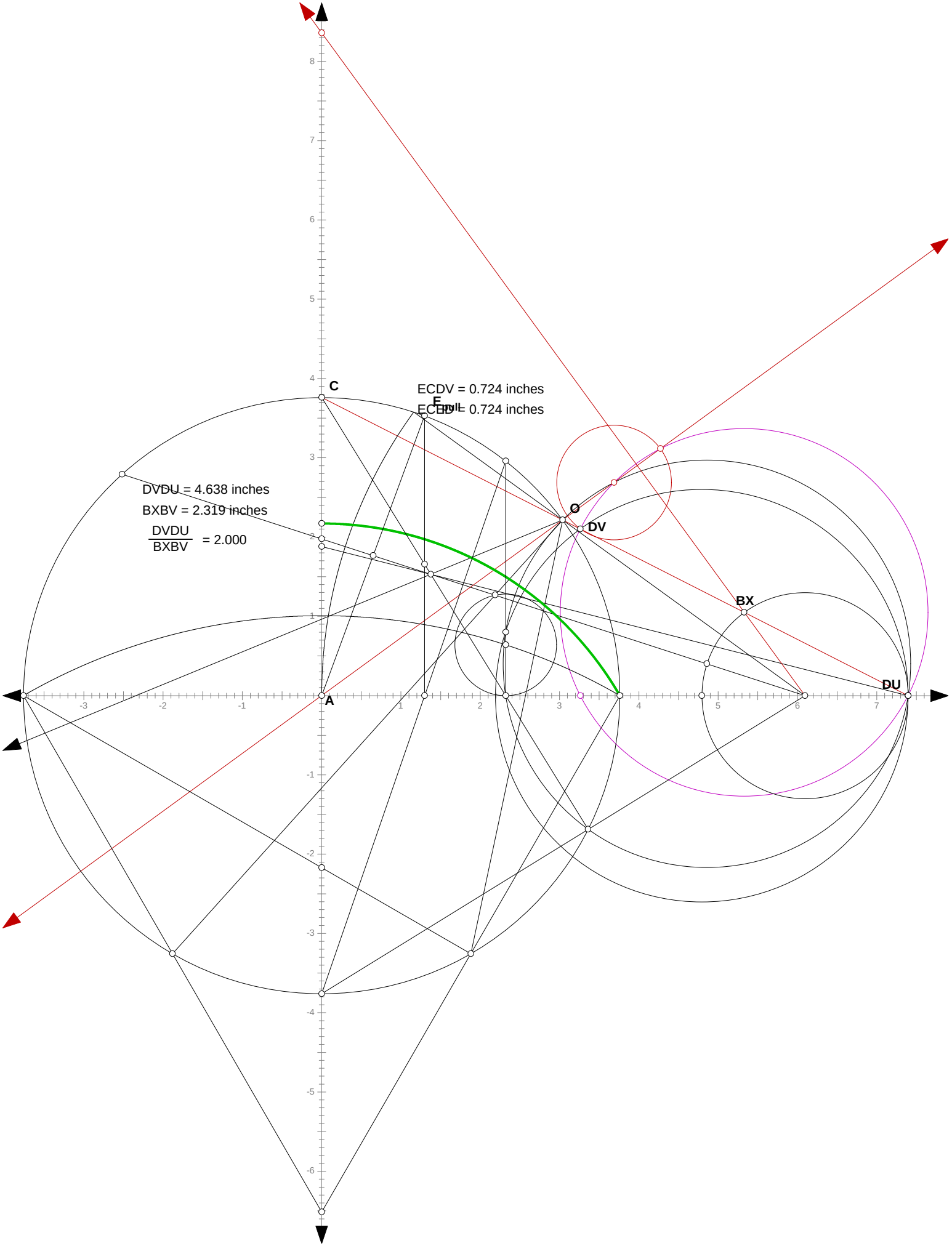

$m\angle FAC = 45.429°$ $m\angle OIP = 45.429°$

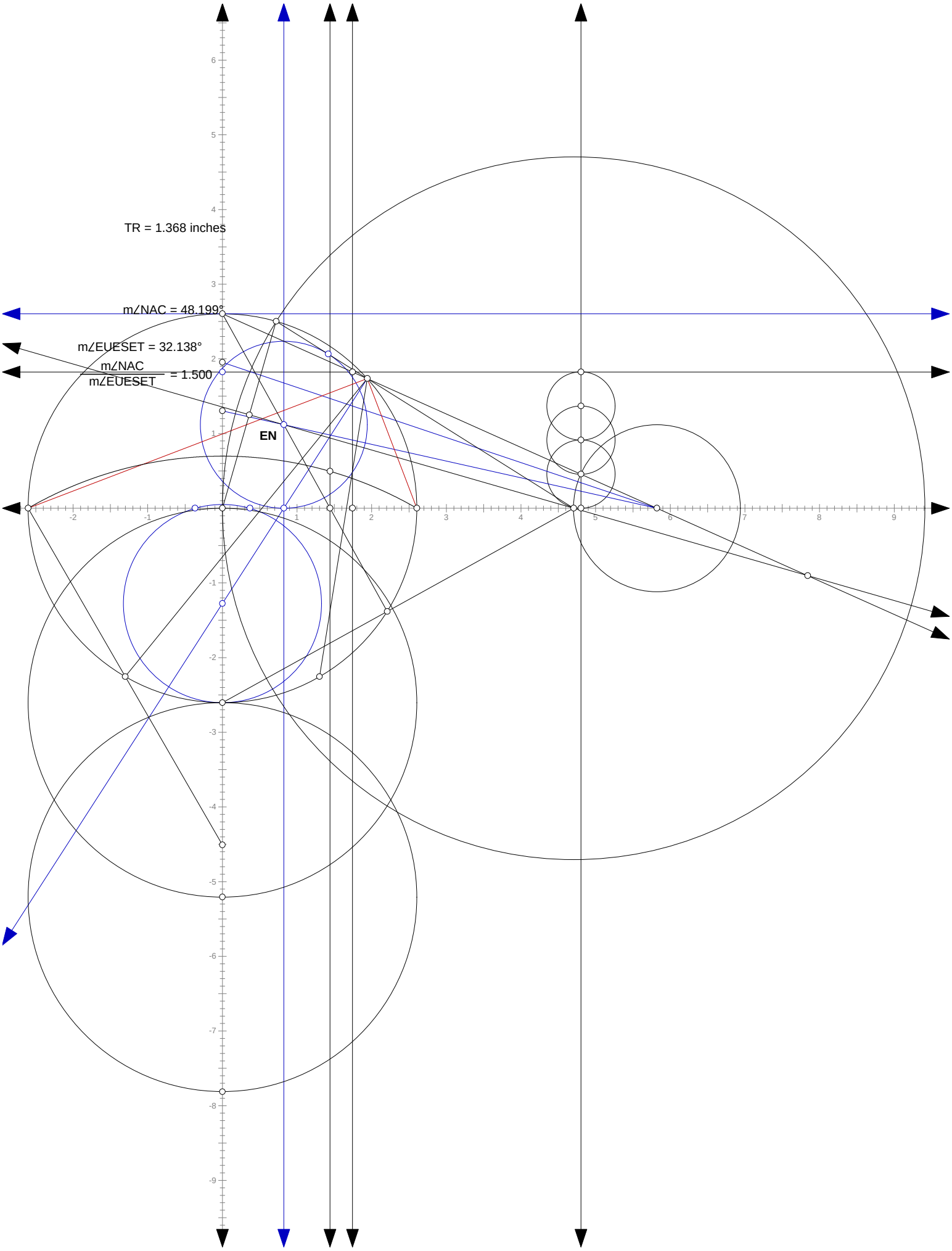
$m\angle FAJ = 29.488°$ $m\angle OIQ = 14.744°$

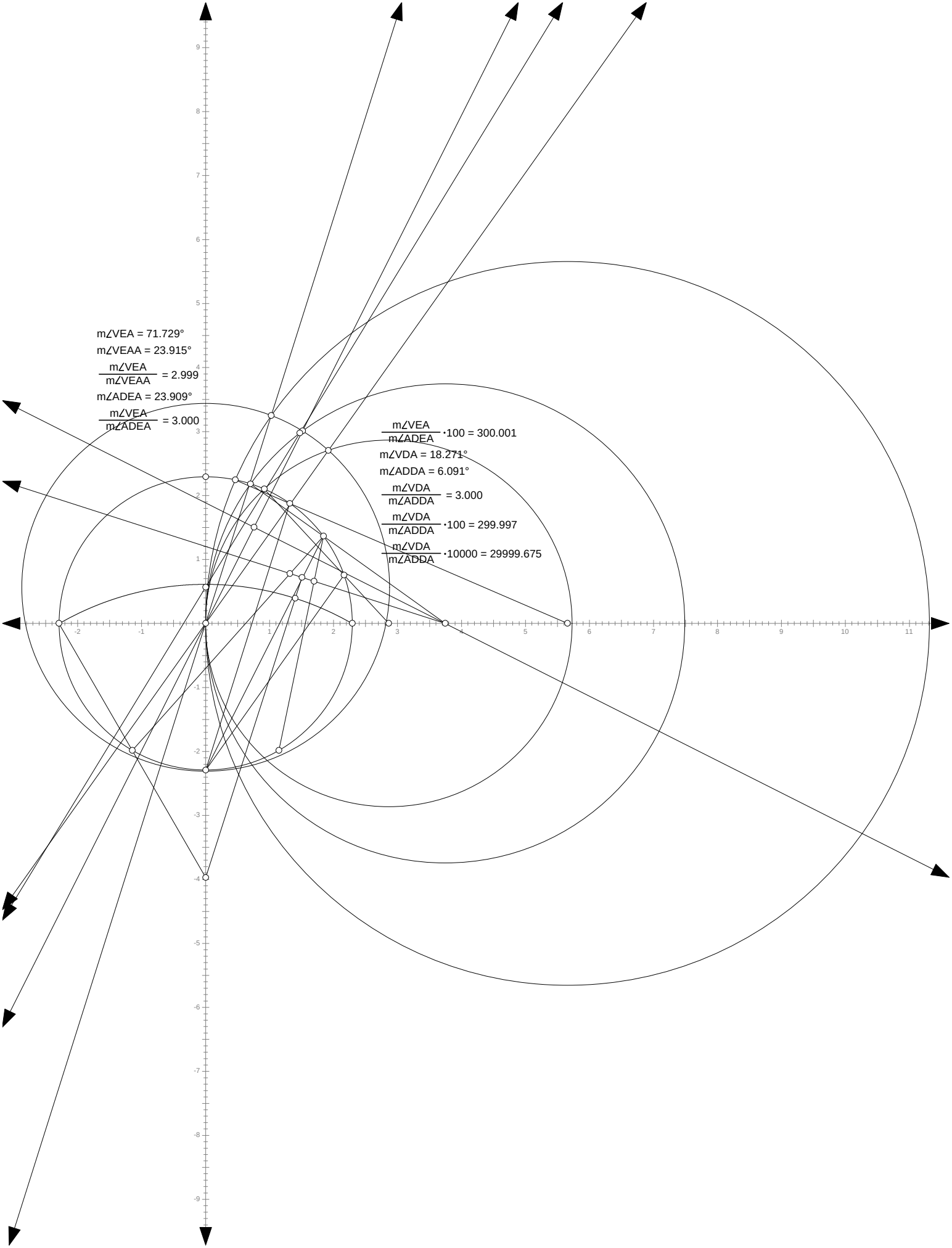
$\frac{m\angle FAC}{m\angle FAJ} = 1.541$ $\frac{m\angle OIP}{m\angle OIQ} = 3.081$

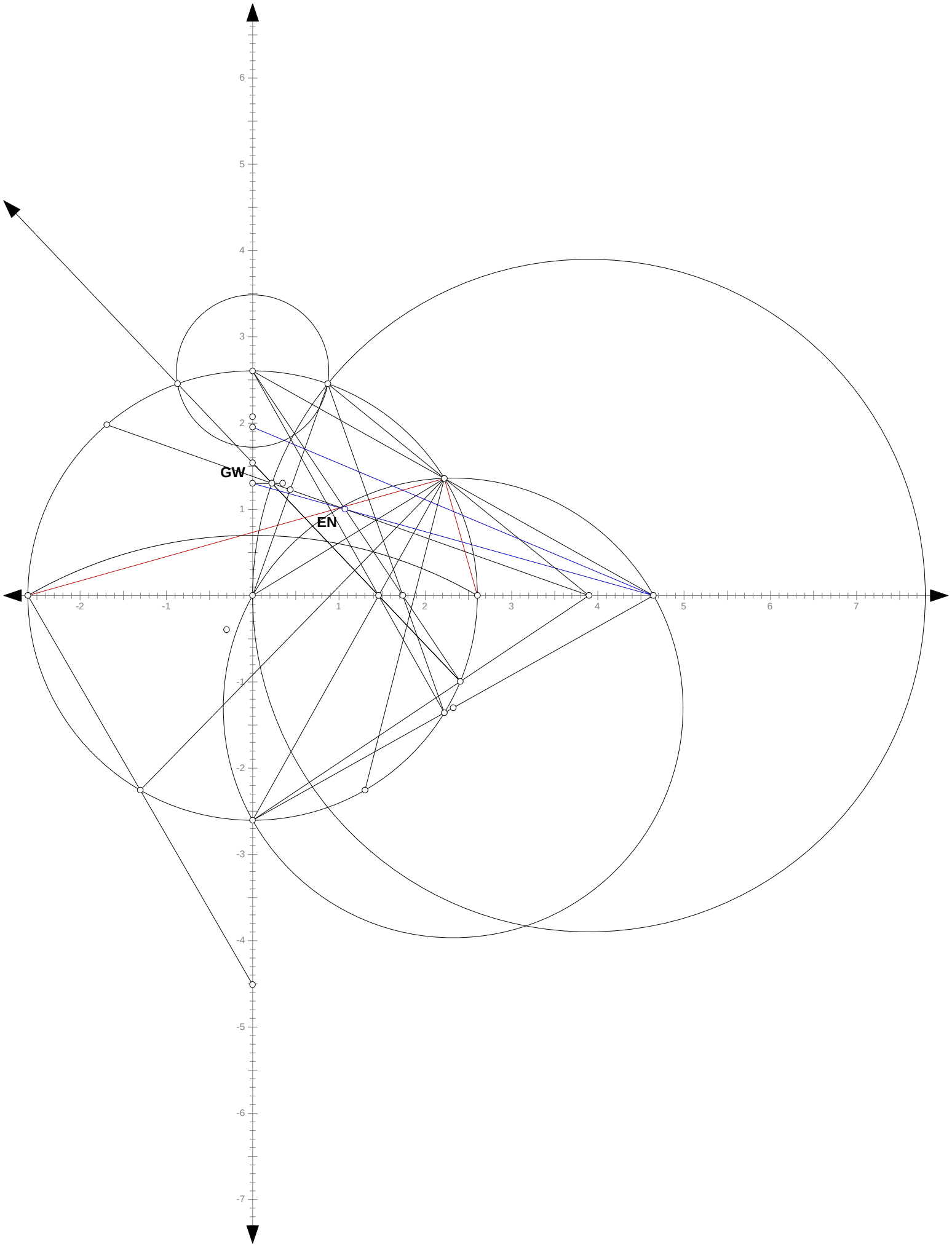
$\frac{m\angle FAJ}{m\angle OIQ} = 2.000$

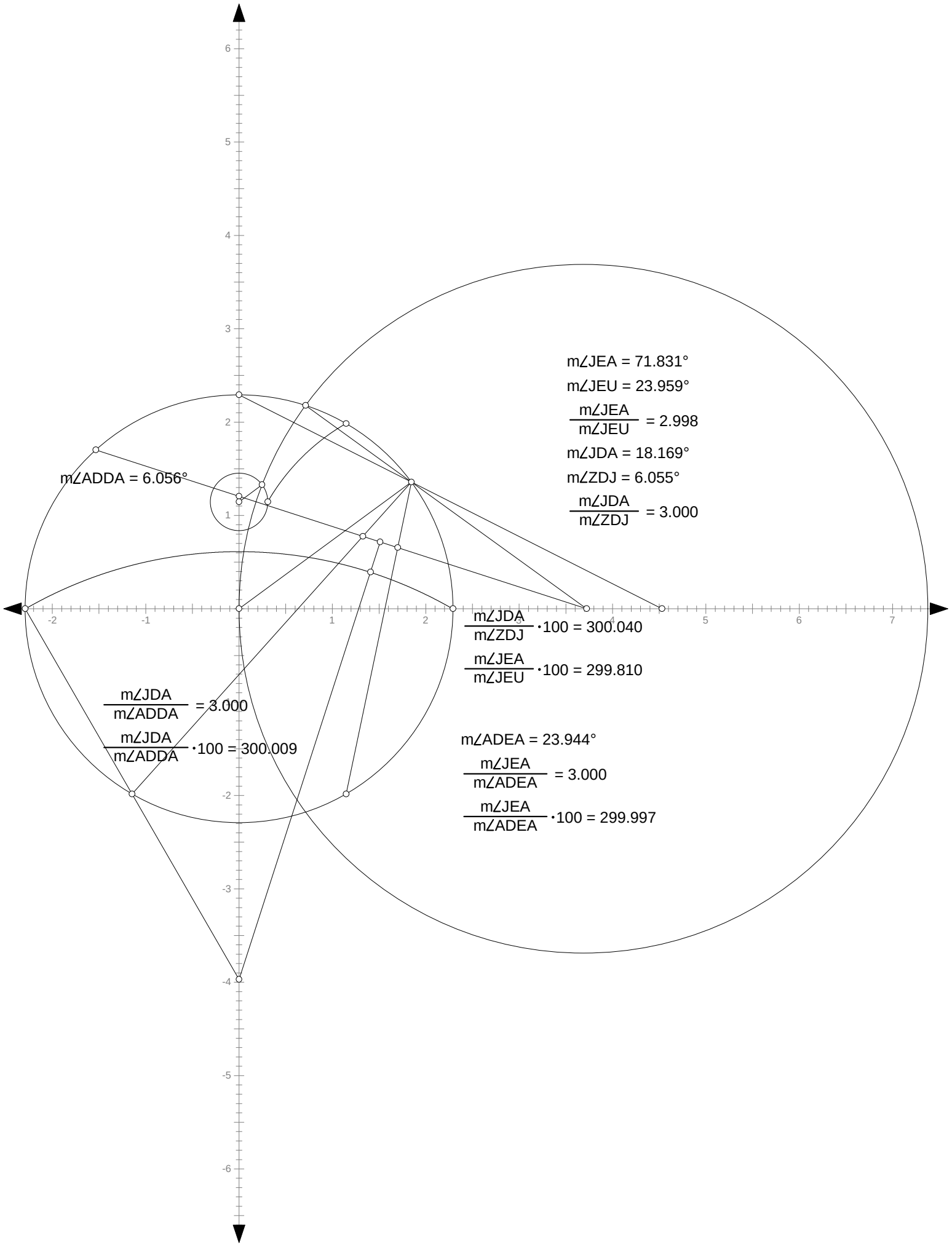

m∠FAC = 34.432°
m∠CEA = 17.216°
m∠DMDODQ = 34.432°
m∠DMDODP = 11.510°
m∠DMDODQ / m∠DMDODP = 2.992

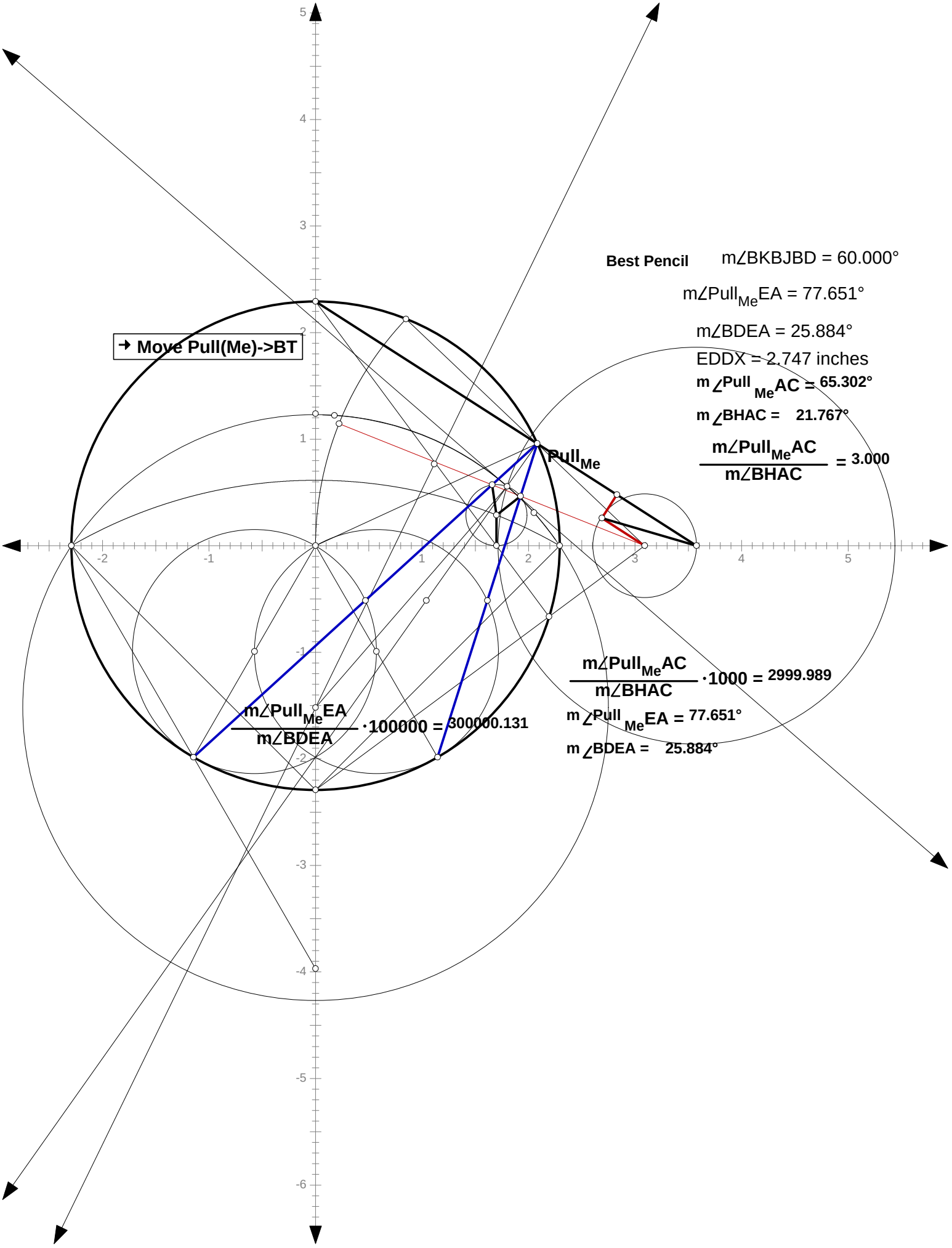

C
ECDV = 0.724 inches
ECBD = 0.724 inches
DVDU = 4.638 inches
BXBV = 2.319 inches
DVDU / BXBV = 2.000
O
DV
BX
DU
A

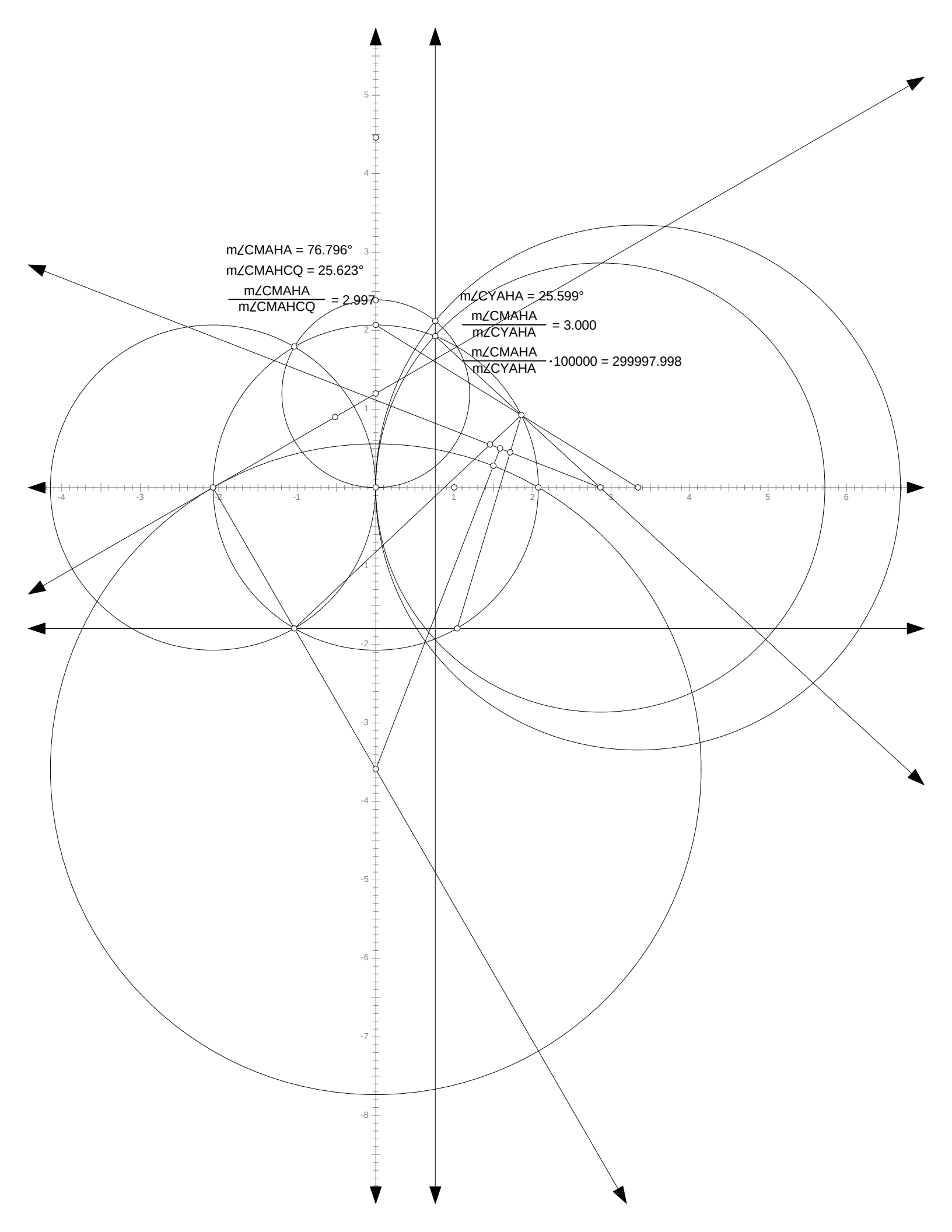

TR = 1.368 inches
m∠NAC = 48.199°
m∠EUESET = 32.138°
m∠NAC / m∠EUESET = 1.500
EN

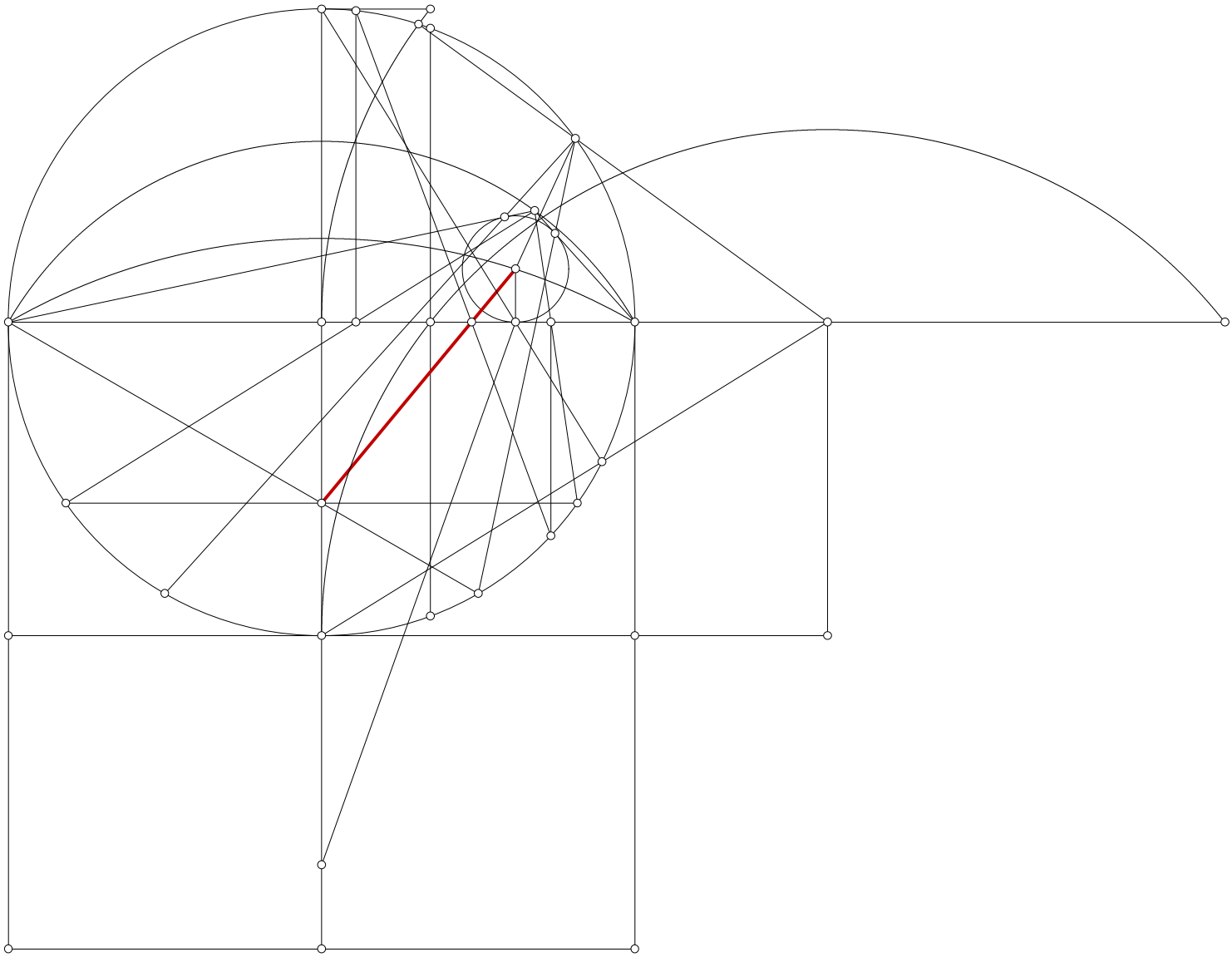

m∠VEA = 71.729°
m∠VEAA = 23.915°
m∠VEA / m∠VEAA = 2.999
m∠ADEA = 23.909°
m∠VEA / m∠ADEA = 3.000
m∠VEA / m∠ADEA ·100 = 300.001
m∠VDA = 18.271°
m∠ADDA = 6.091°
m∠VDA / m∠ADDA = 3.000
m∠VDA / m∠ADDA ·100 = 299.997
m∠VDA / m∠ADDA ·10000 = 29999.675


GW
EN
6
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-7
-2
-1
1
2
3
4
5
6
7


m∠JEA = 71.831°
m∠JEU = 23.959°
m∠JEA / m∠JEU = 2.998
m∠JDA = 18.169°
m∠ZDJ = 6.055°
m∠JDA / m∠ZDJ = 3.000
m∠ADDA = 6.056°
m∠JDA / m∠ZDJ ·100 = 300.040
m∠JEA / m∠JEU ·100 = 299.810
m∠JDA / m∠ADDA = 3.000
m∠JDA / m∠ADDA ·100 = 300.009
m∠ADEA = 23.944°
m∠JEA / m∠ADEA = 3.000
m∠JEA / m∠ADEA ·100 = 299.997


Best Pencil
m∠BKBJBD = 60.000°
m∠Pull$_{Me}$EA = 77.651°
m∠BDEA = 25.884°
EDDX = 2.747 inches
m∠Pull$_{Me}$AC = 65.302°
m∠BHAC = 21.767°
$\frac{m\angle Pull_{Me}AC}{m\angle BHAC}$ = 3.000
Move Pull(Me)->BT
Pull$_{Me}$
$\frac{m\angle Pull_{Me}EA}{m\angle BDEA}\cdot 100000$ = 300000.131
$\frac{m\angle Pull_{Me}AC}{m\angle BHAC}\cdot 1000$ = 2999.989
m∠Pull$_{Me}$EA = 77.651°
m∠BDEA = 25.884°


m∠CMAHA = 76.796°
m∠CMAHCQ = 25.623°
m∠CMAHA / m∠CMAHCQ = 2.997
m∠CYAHA = 25.599°
m∠CMAHA / m∠CYAHA = 3.000
m∠CMAHA / m∠CYAHA · 100000 = 299997.998



m∠FVFUBV = 18.895°

m∠OAC = 56.685°

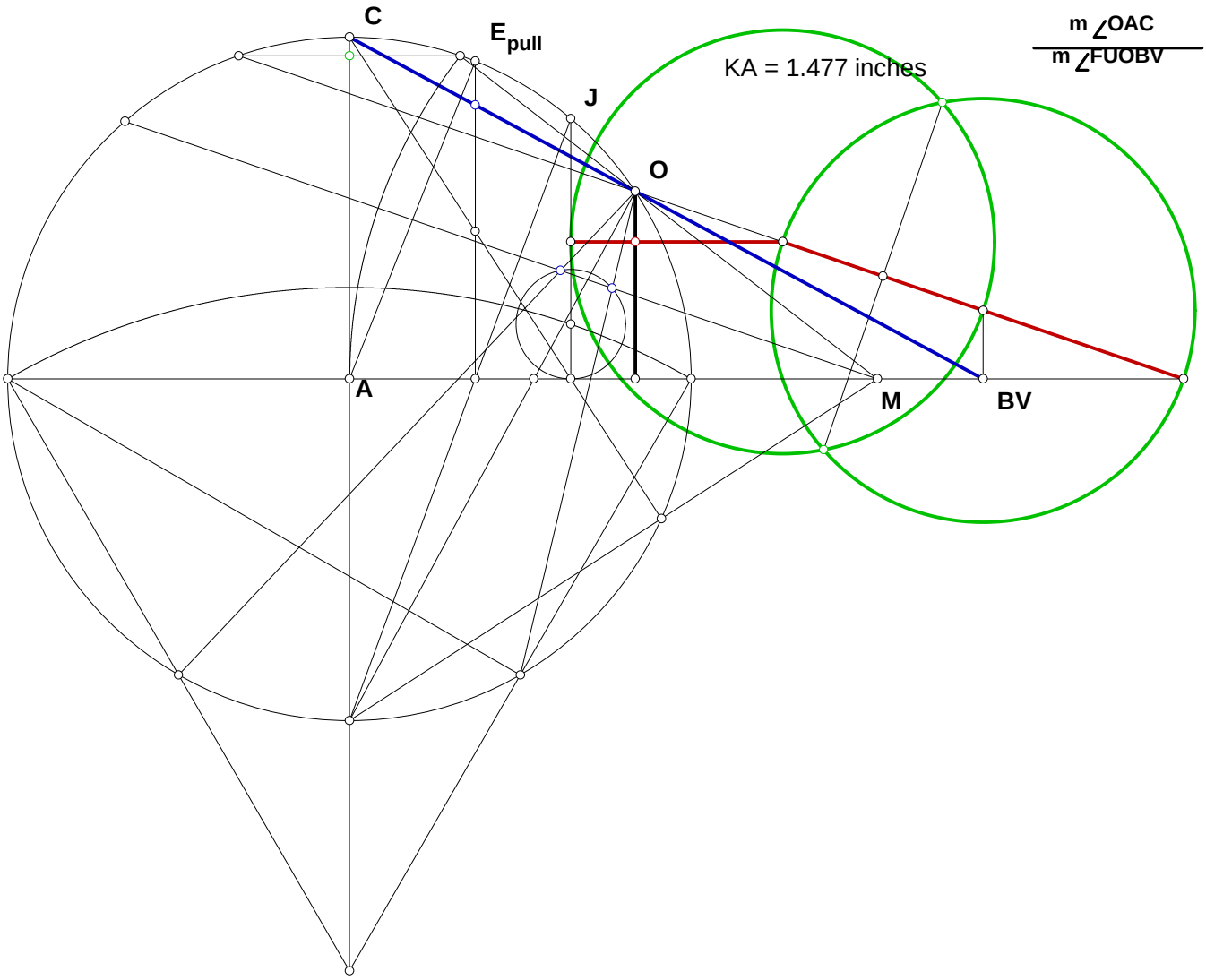
$\frac{m\angle OAC}{m\angle FVFUBV} = 3.000$

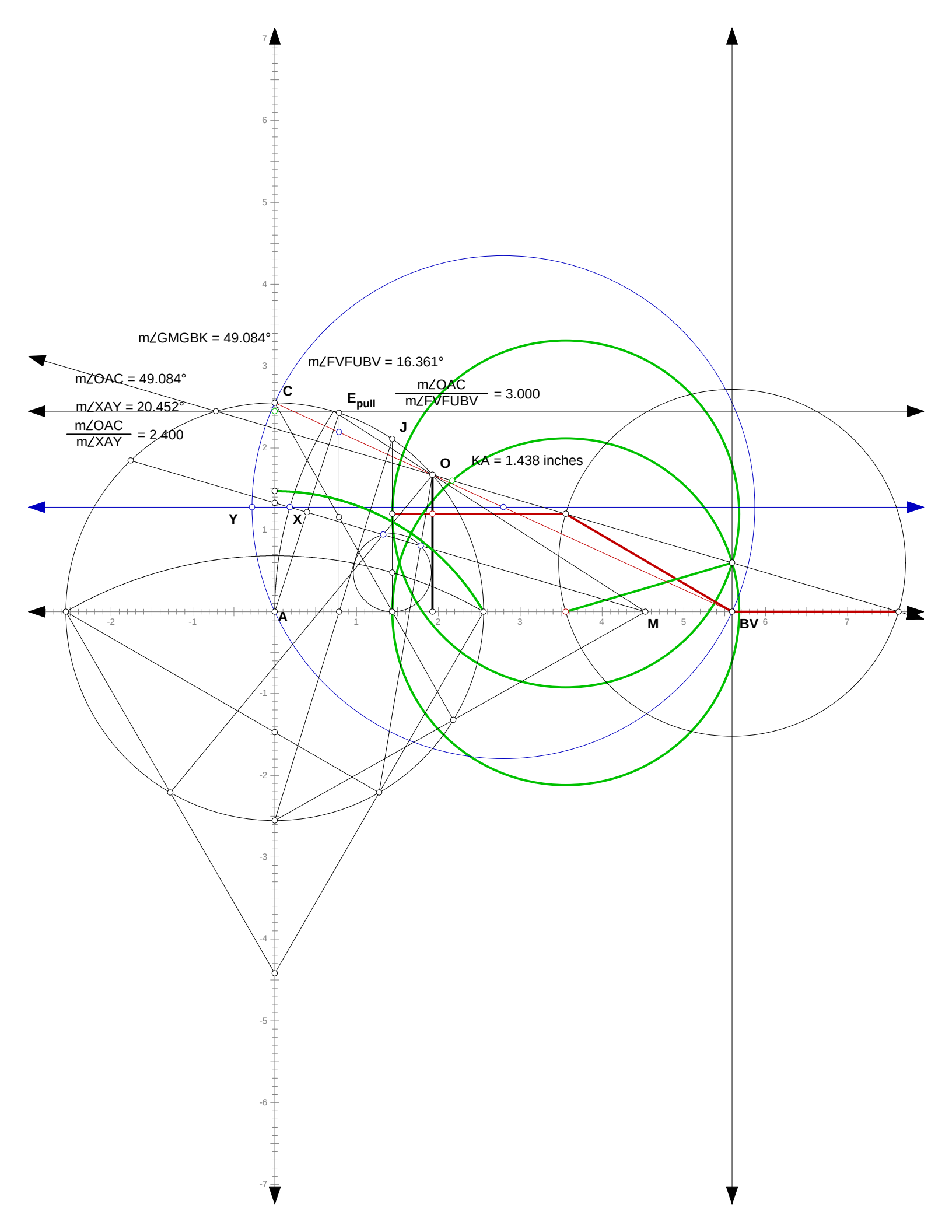
**m ∠FUOBV = 9.447°**

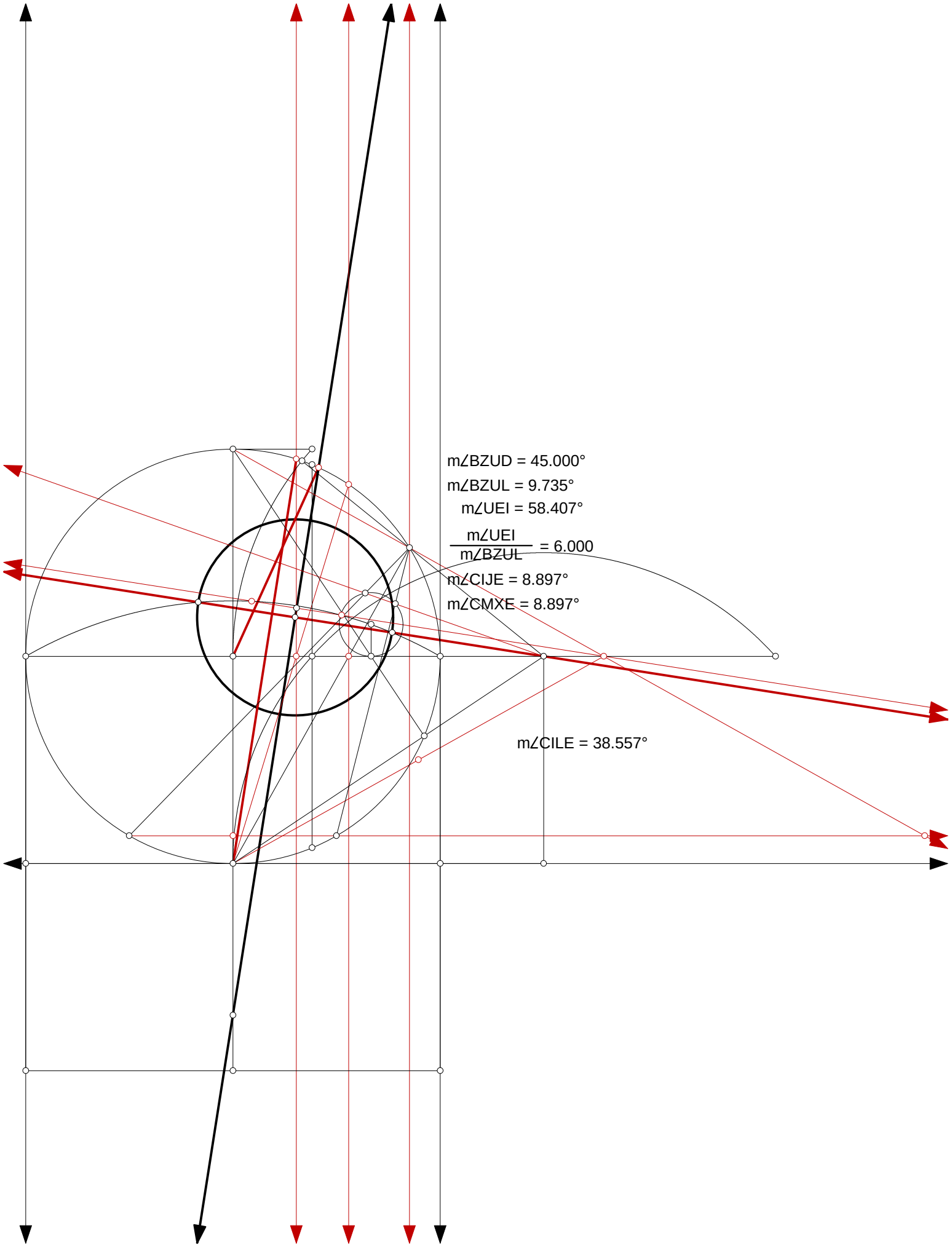
**m ∠OAC = 56.685°**

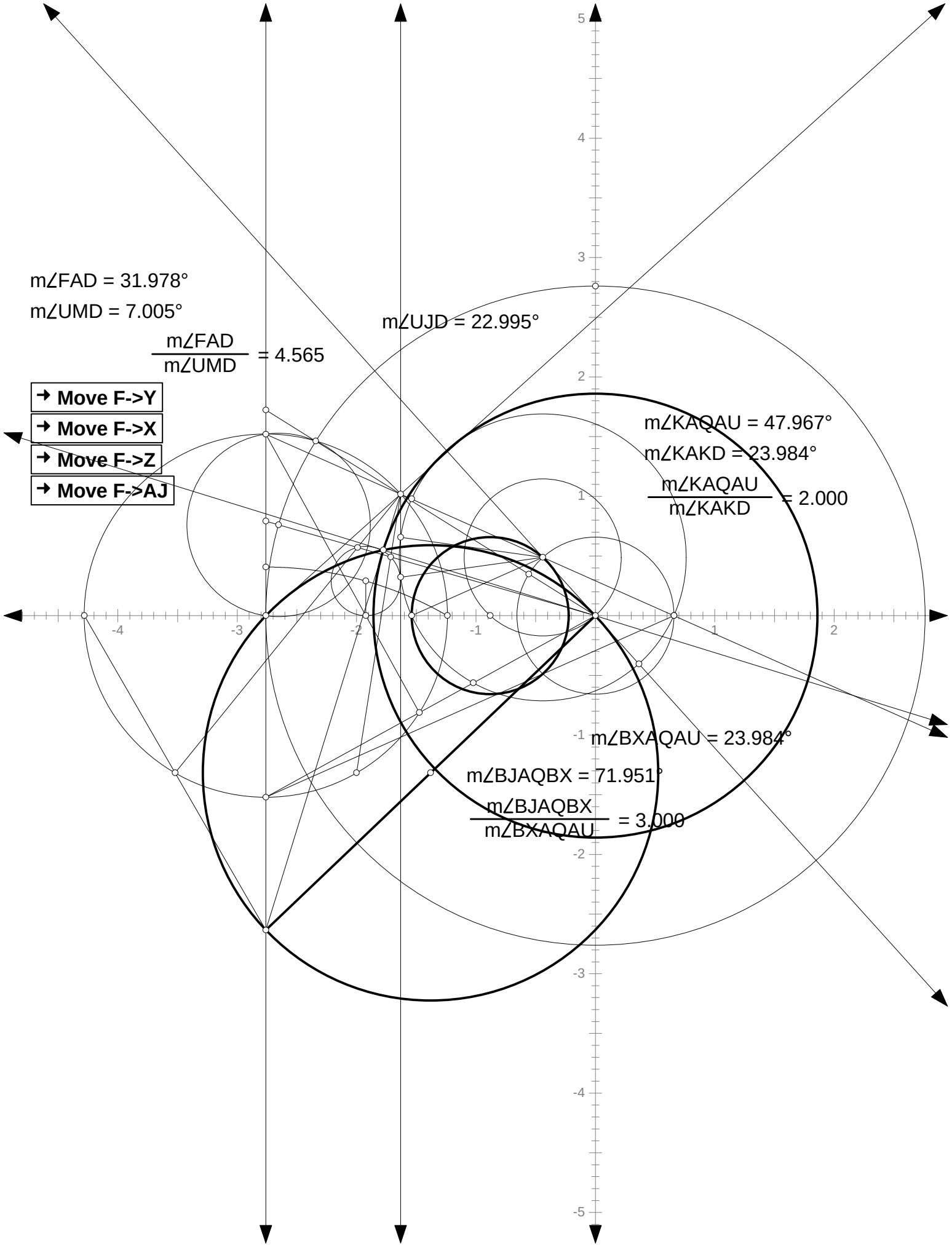
$\frac{\mathbf{m\angle OAC}}{\mathbf{m\angle FUOBV}} = \mathbf{6.000}$

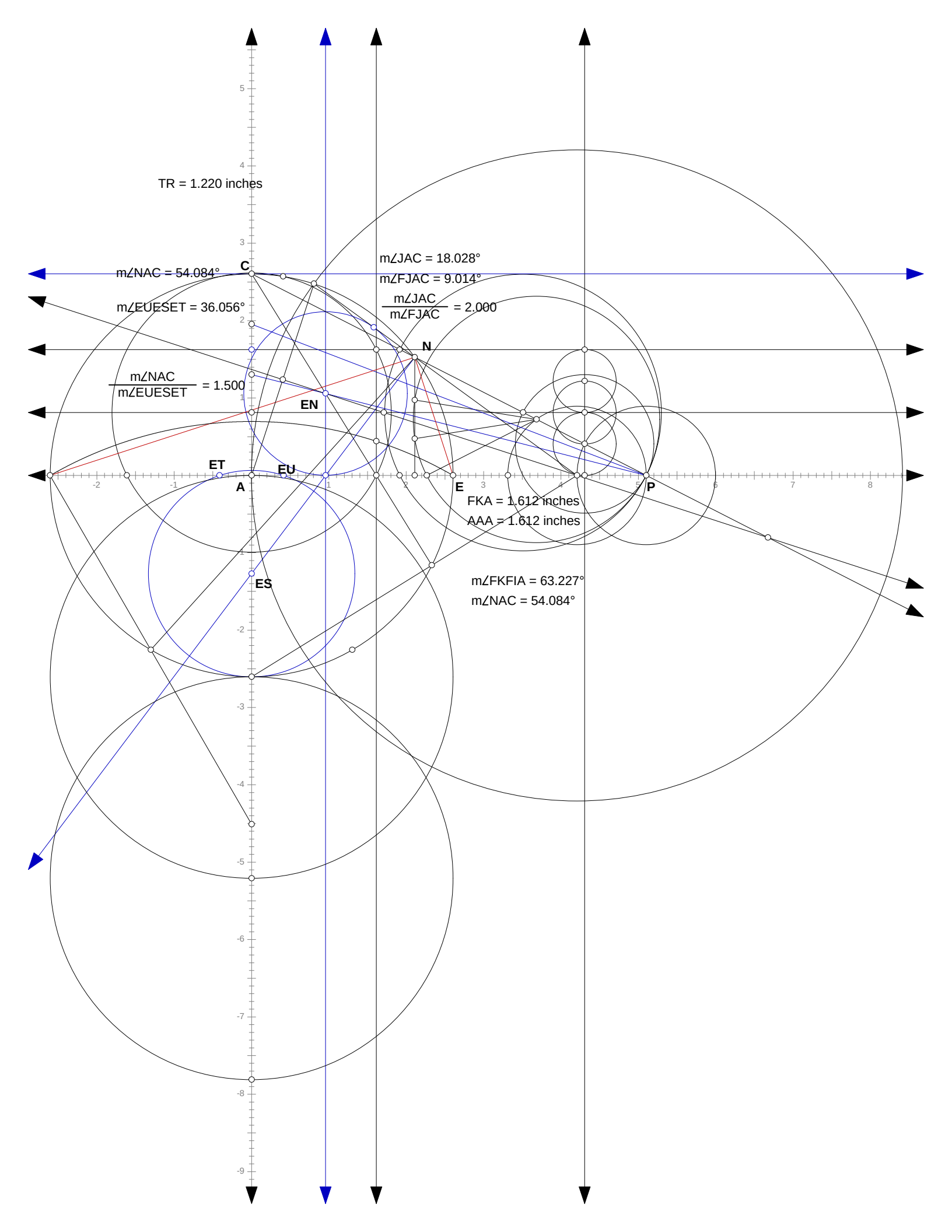
KA = 1.477 inches

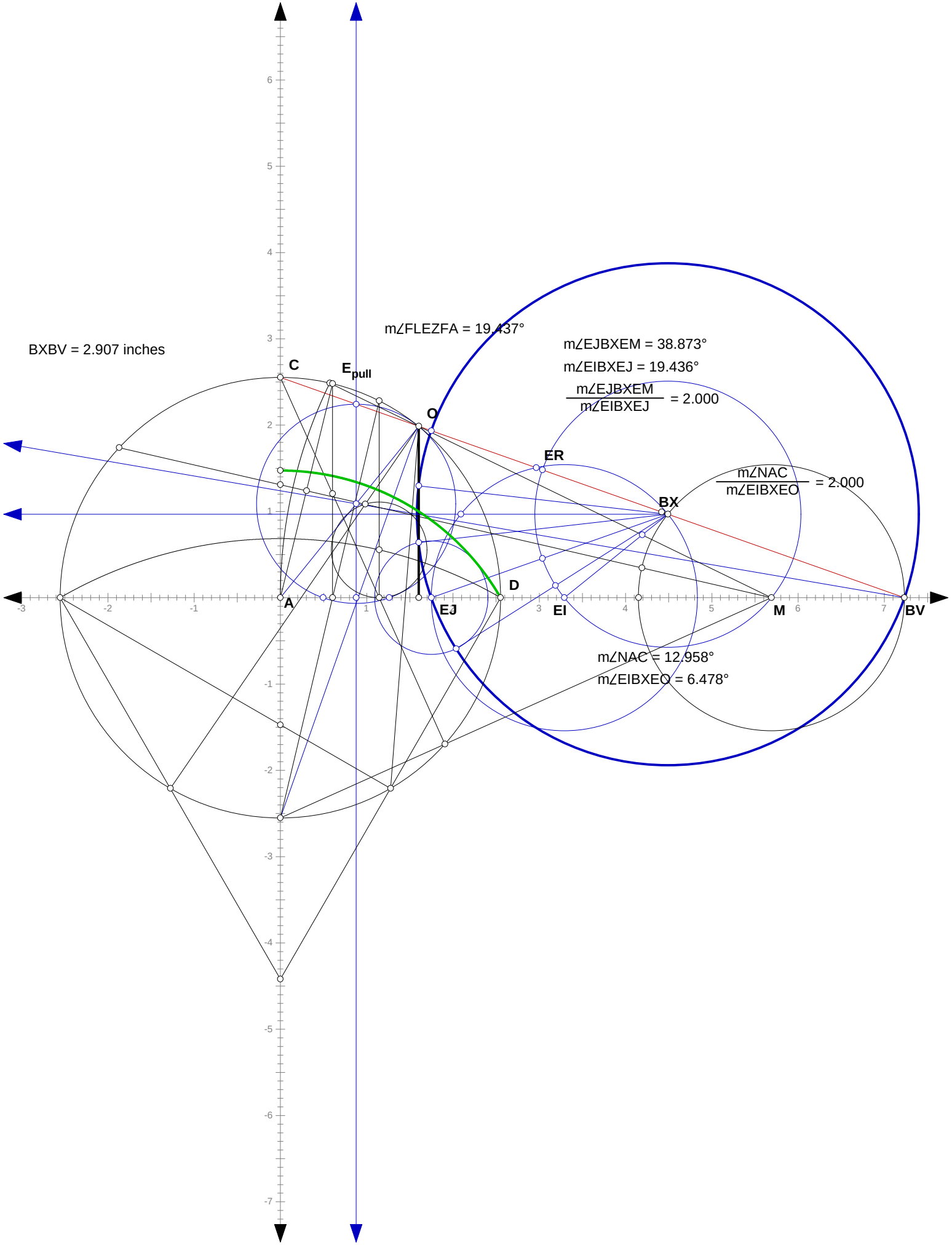


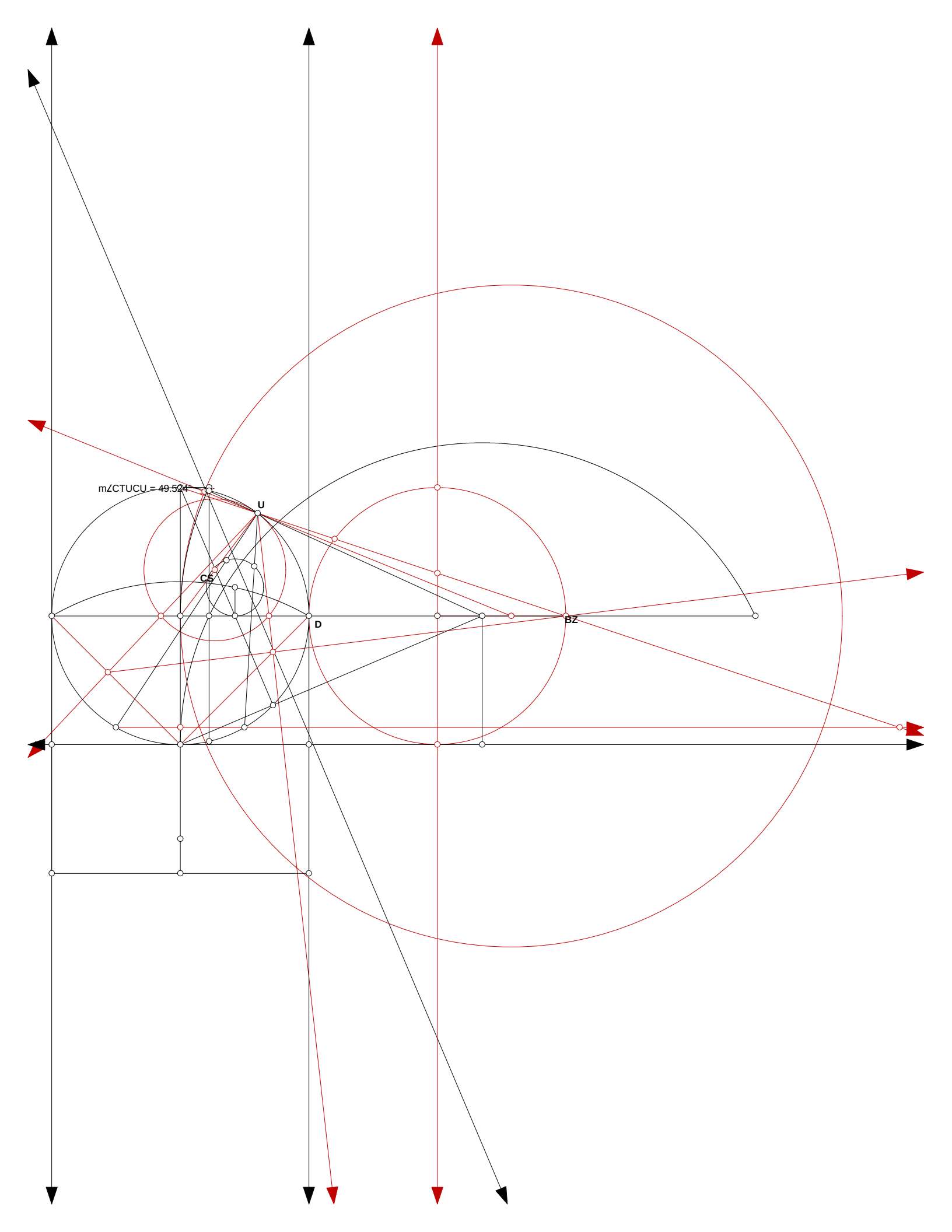

m∠GMGBK = 49.084°
m∠FVFUBV = 16.361°
m∠OAC = 49.084°
m∠XAY = 20.452°
m∠OAC / m∠XAY = 2.400
m∠OAC / m∠FVFUBV = 3.000
KA = 1.438 inches
C
E pull
J
O
Y
X
A
M
BV

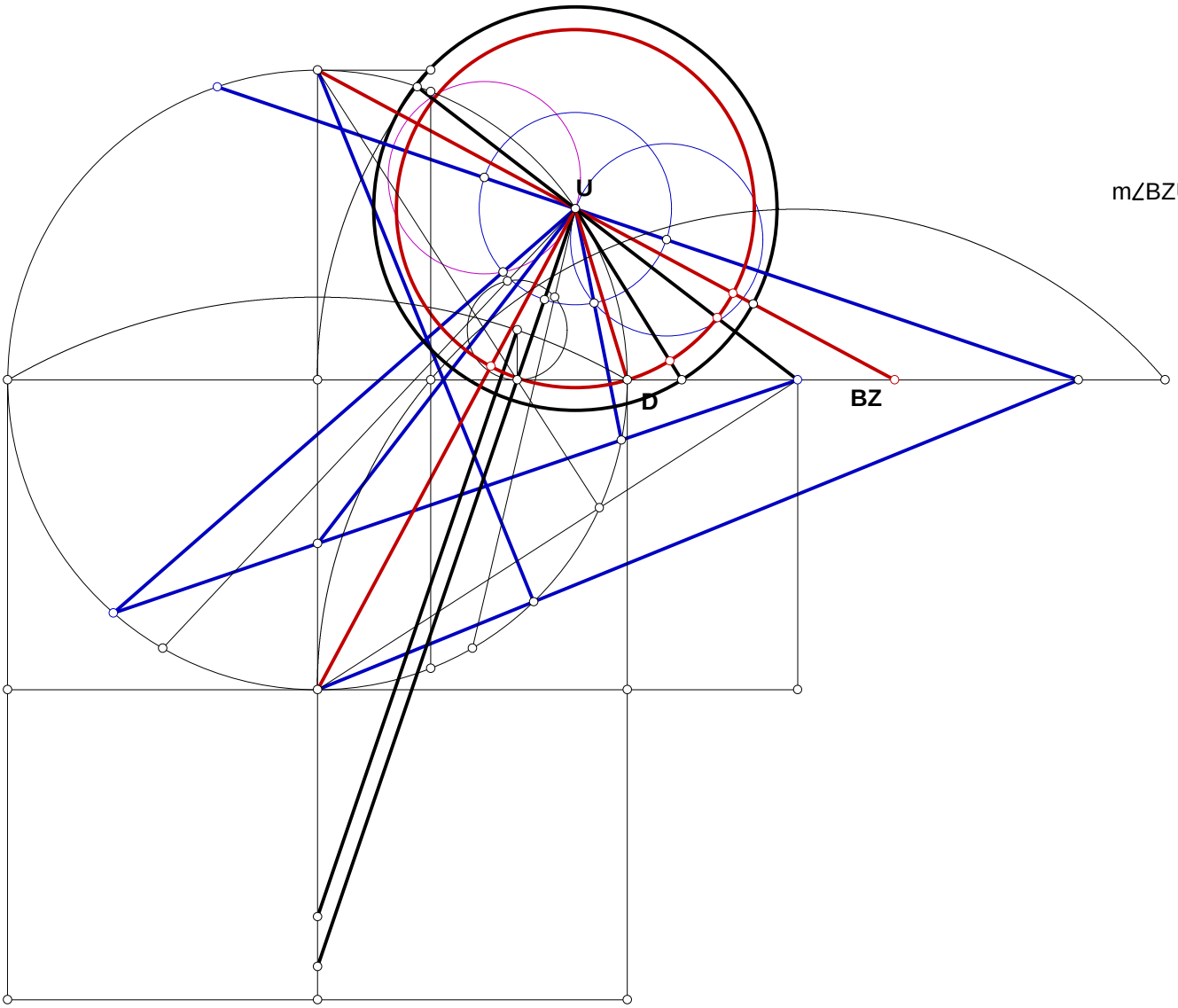

m∠BZUD = 45.000°
m∠BZUL = 9.735°
m∠UEI = 58.407°
m∠UEI / m∠BZUL = 6.000
m∠CIJE = 8.897°
m∠CMXE = 8.897°
m∠CILE = 38.557°


m∠FAD = 31.978°
m∠UMD = 7.005°
m∠UJD = 22.995°
m∠FAD / m∠UMD = 4.565
Move F->Y
Move F->X
Move F->Z
Move F->AJ
m∠KAQAU = 47.967°
m∠KAKD = 23.984°
m∠KAQAU / m∠KAKD = 2.000
m∠BXAQAU = 23.984°
m∠BJAQBX = 71.951°
m∠BJAQBX / m∠BXAQAU = 3.000

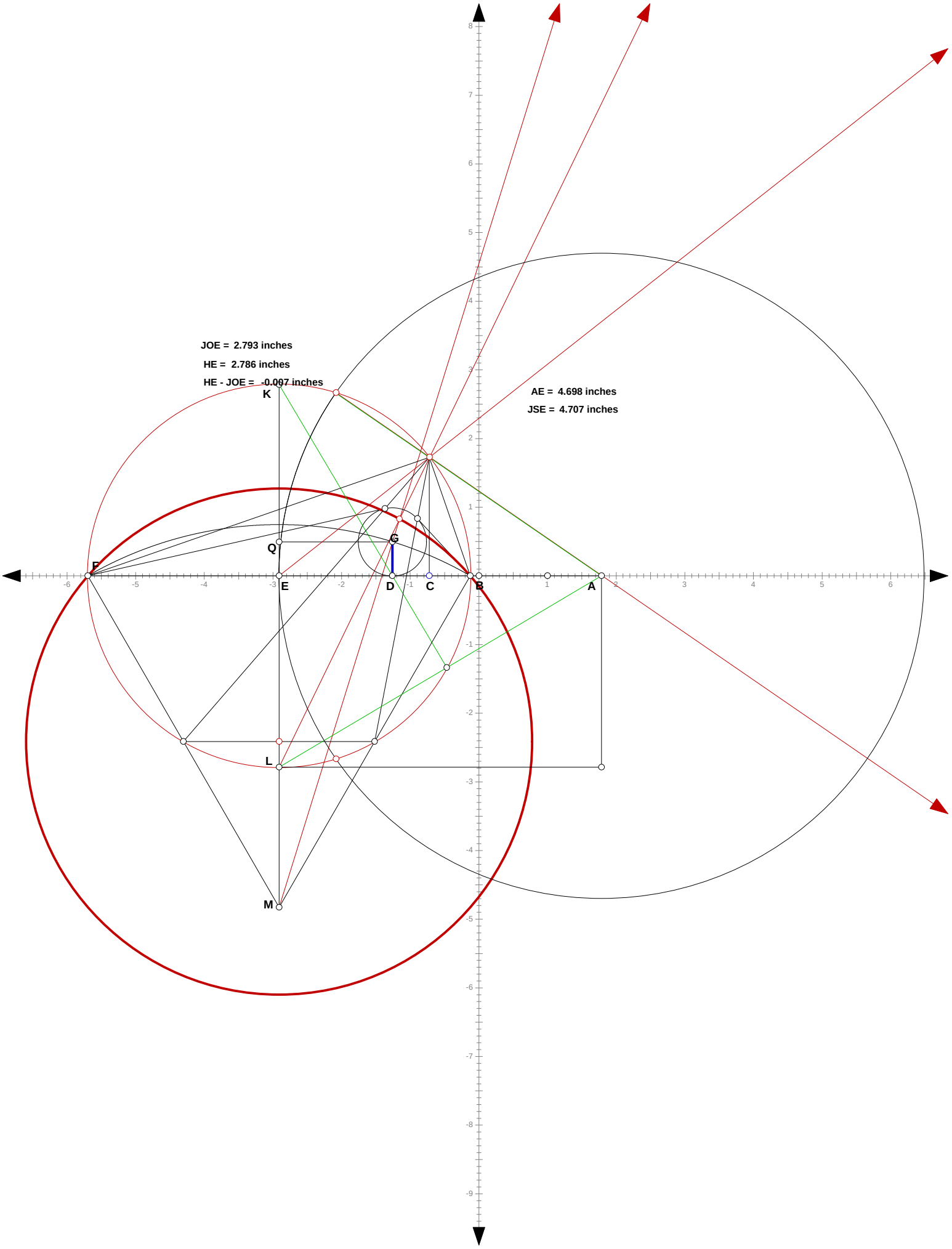

TR = 1.220 inches
C
m∠NAC = 54.084°
m∠EUESET = 36.056°
m∠NAC / m∠EUESET = 1.500
m∠JAC = 18.028°
m∠FJAC = 9.014°
m∠JAC / m∠FJAC = 2.000
N
EN
ET
EU
A
E
P
ES
FKA = 1.612 inches
AAA = 1.612 inches
m∠FKFIA = 63.227°
m∠NAC = 54.084°

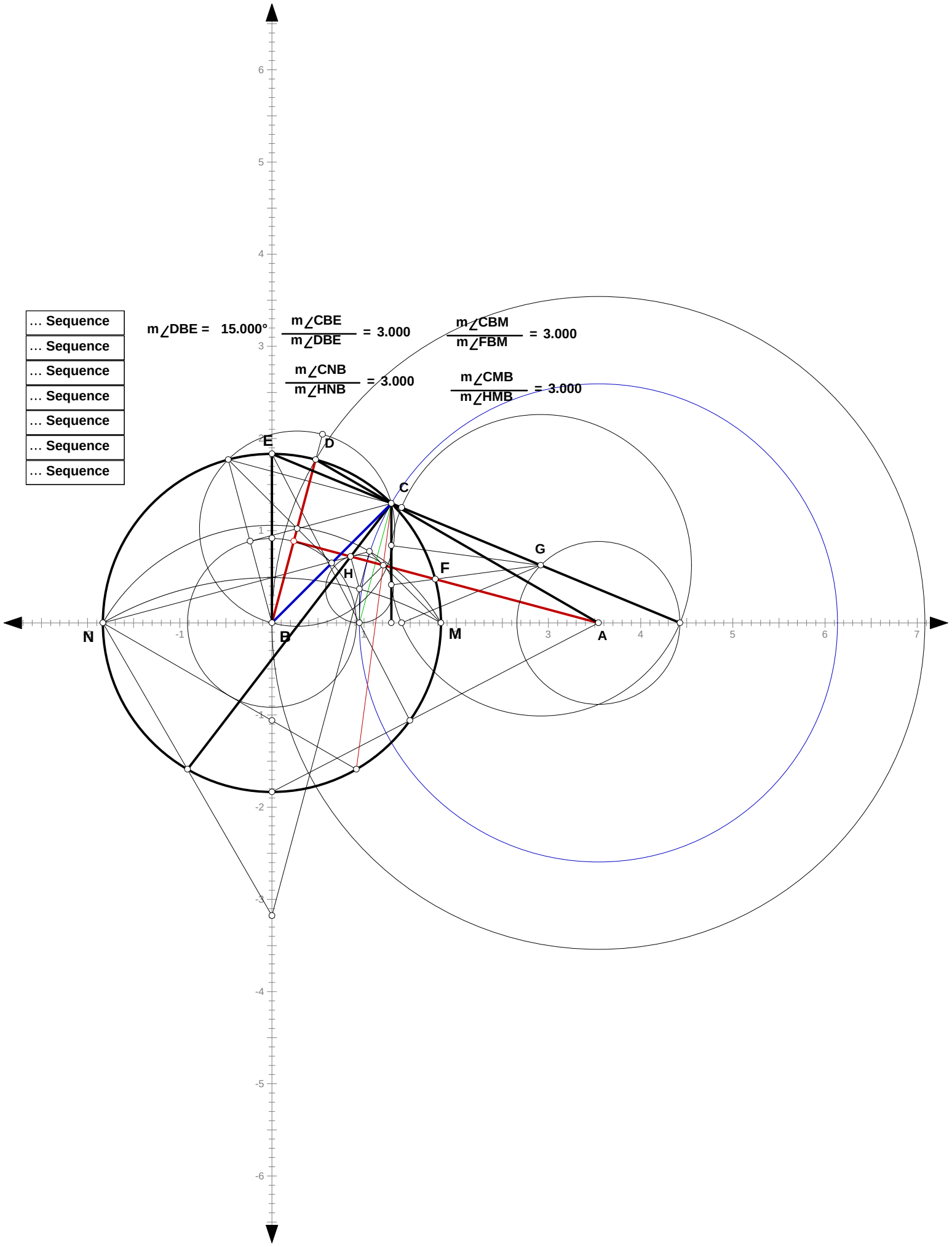

BXBV = 2.907 inches
m∠FLEZFA = 19.437°
m∠EJBXEM = 38.873°
m∠EIBXEJ = 19.436°
m∠EJBXEM / m∠EIBXEJ = 2.000
m∠NAC / m∠EIBXEO = 2.000
m∠NAC = 12.958°
m∠EIBXEO = 6.478°
C
E pull
O
ER
BX
D
A
EJ
EI
M
BV

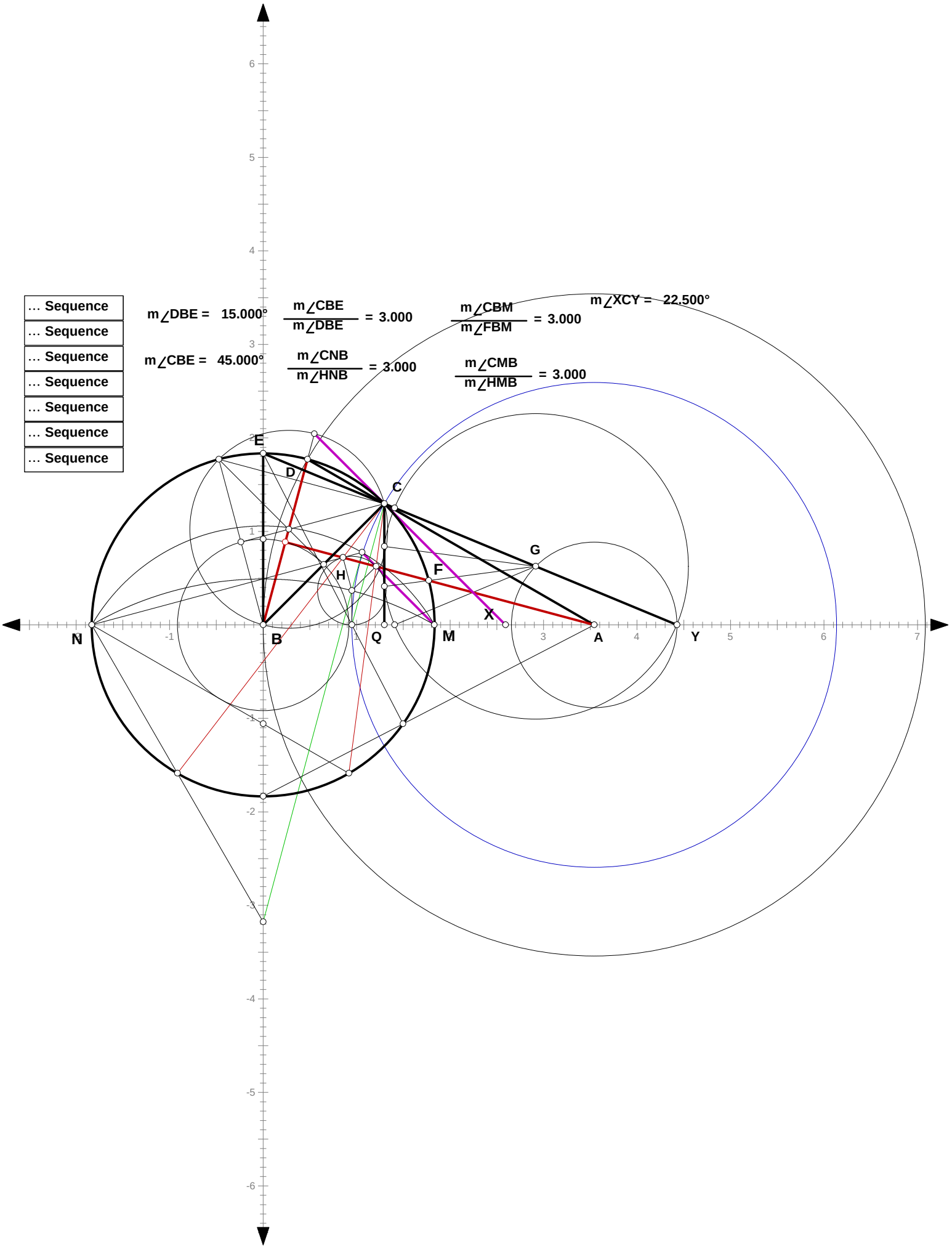

m∠CTUCU = 49.524°
U
CS
D
BZ

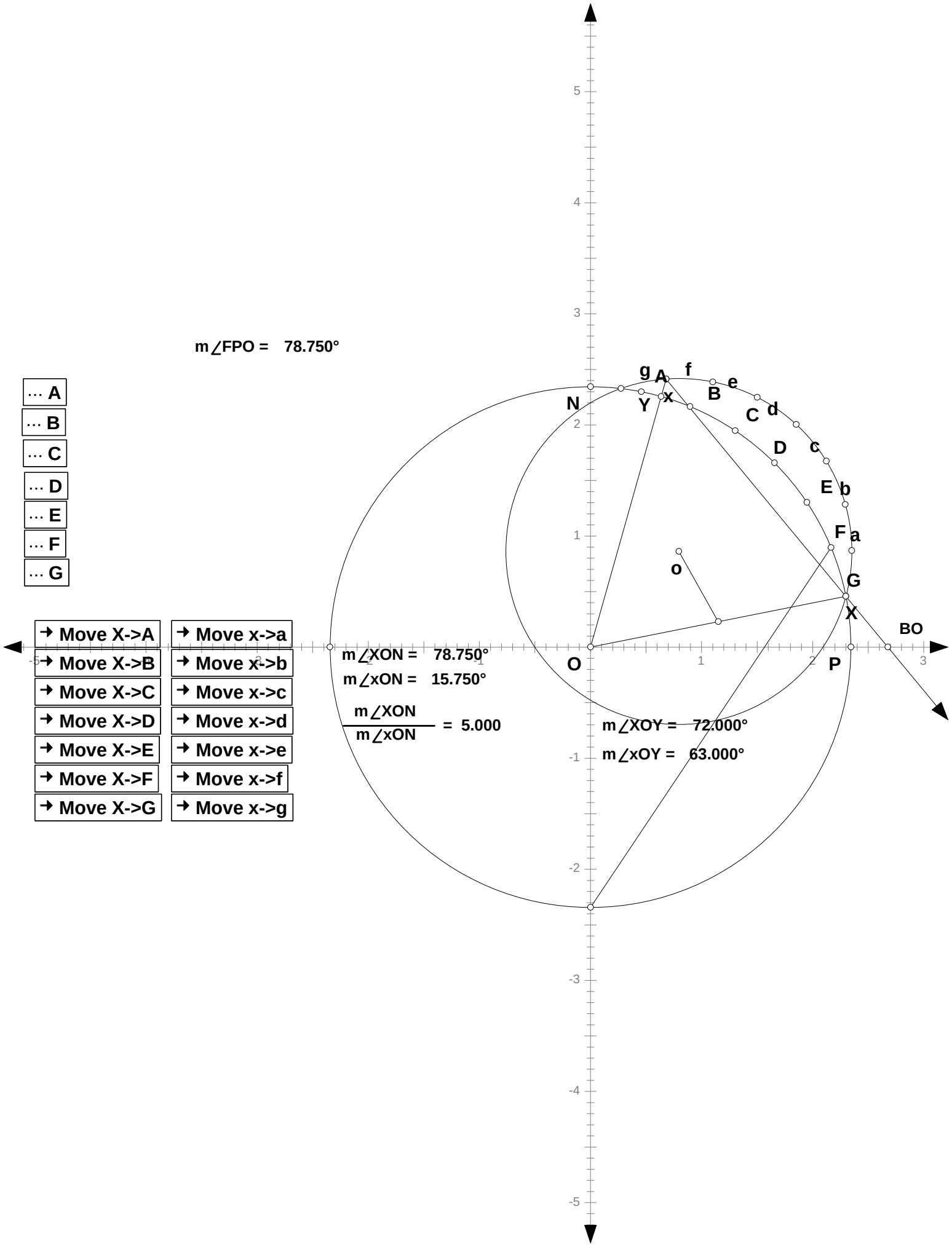


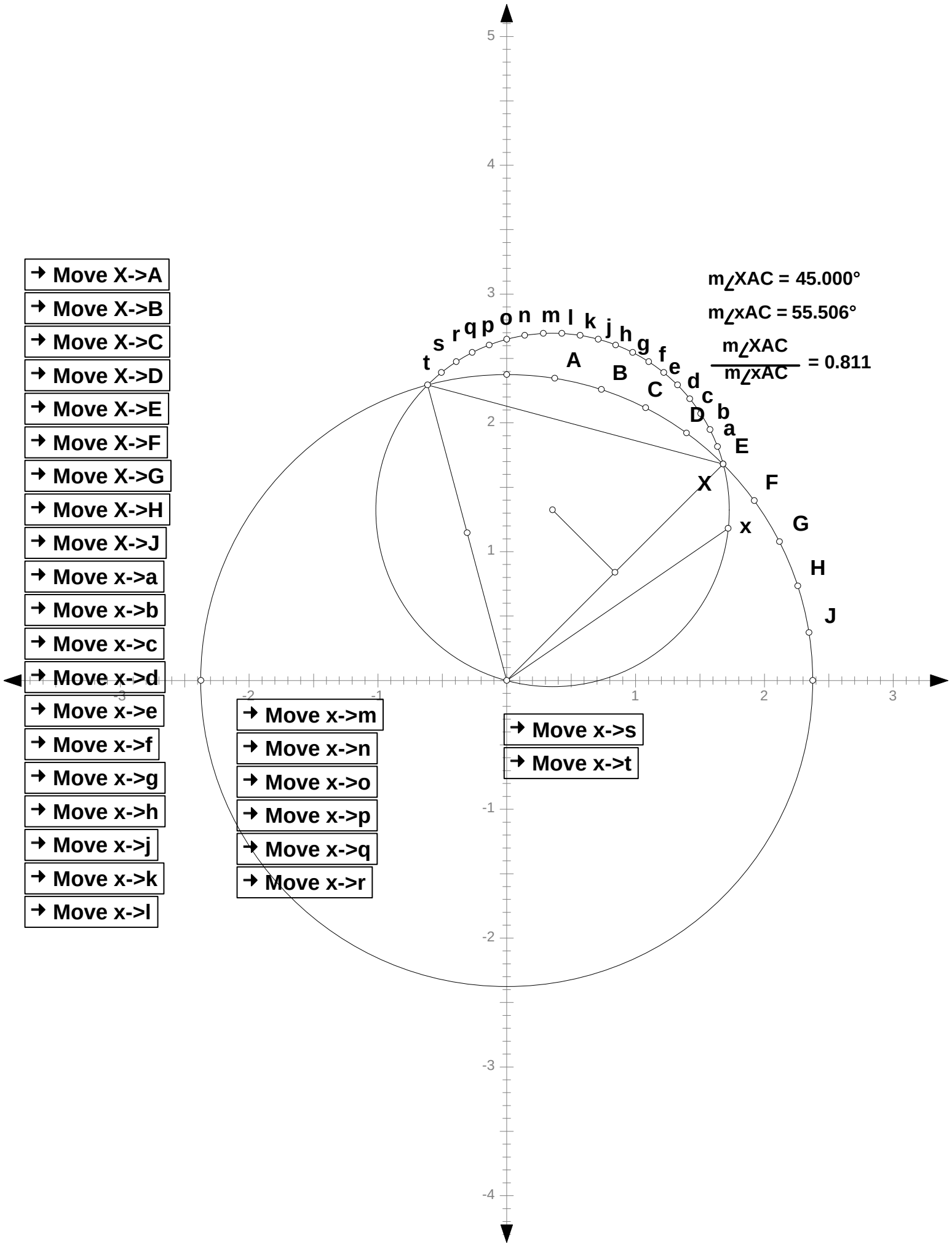

m∠BZUL = 9.404°

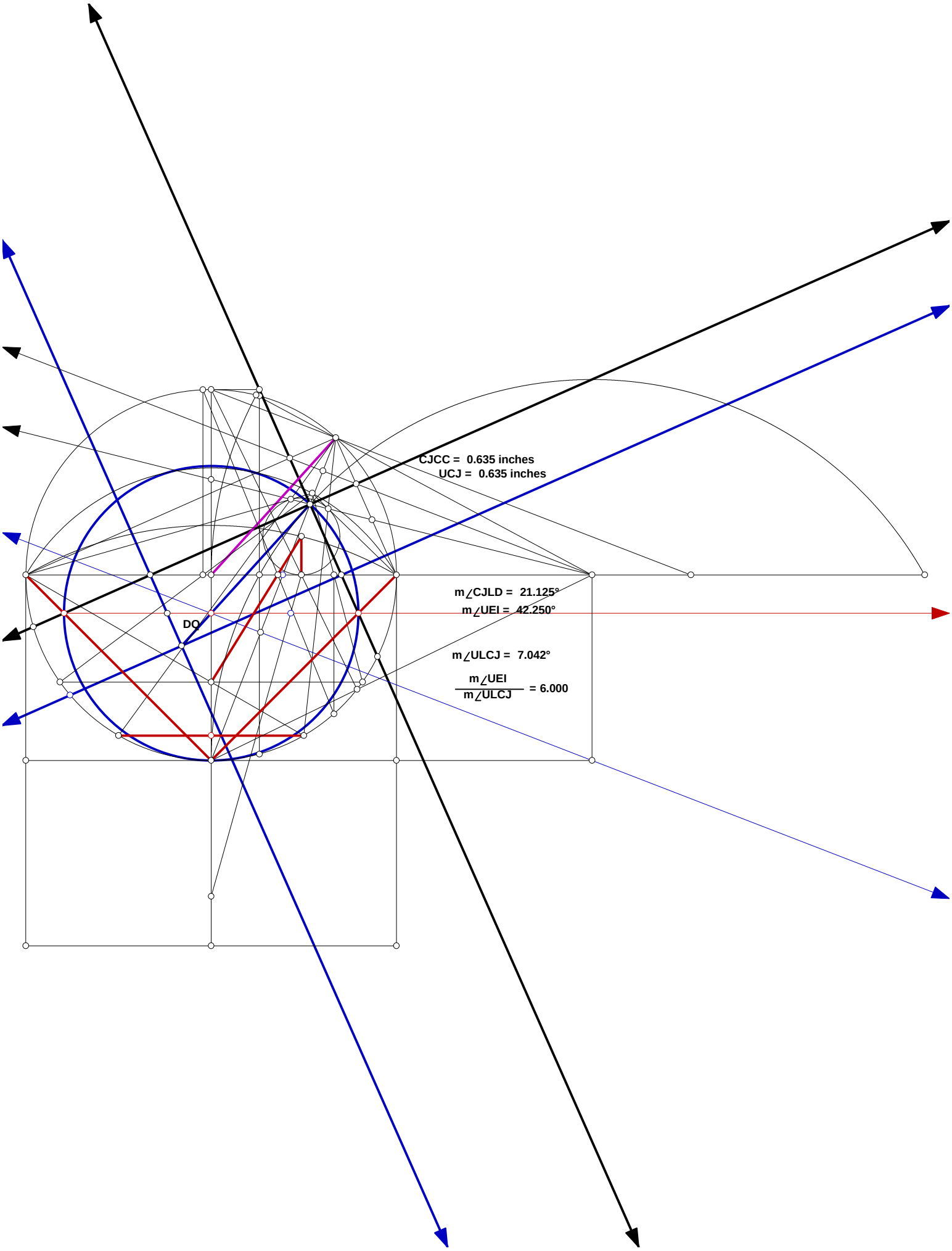

JOE = 2.793 inches
HE = 2.786 inches
HE - JOE = -0.007 inches
AE = 4.698 inches
JSE = 4.707 inches
K
Q
G
F
E
D
C
B
A
L
M

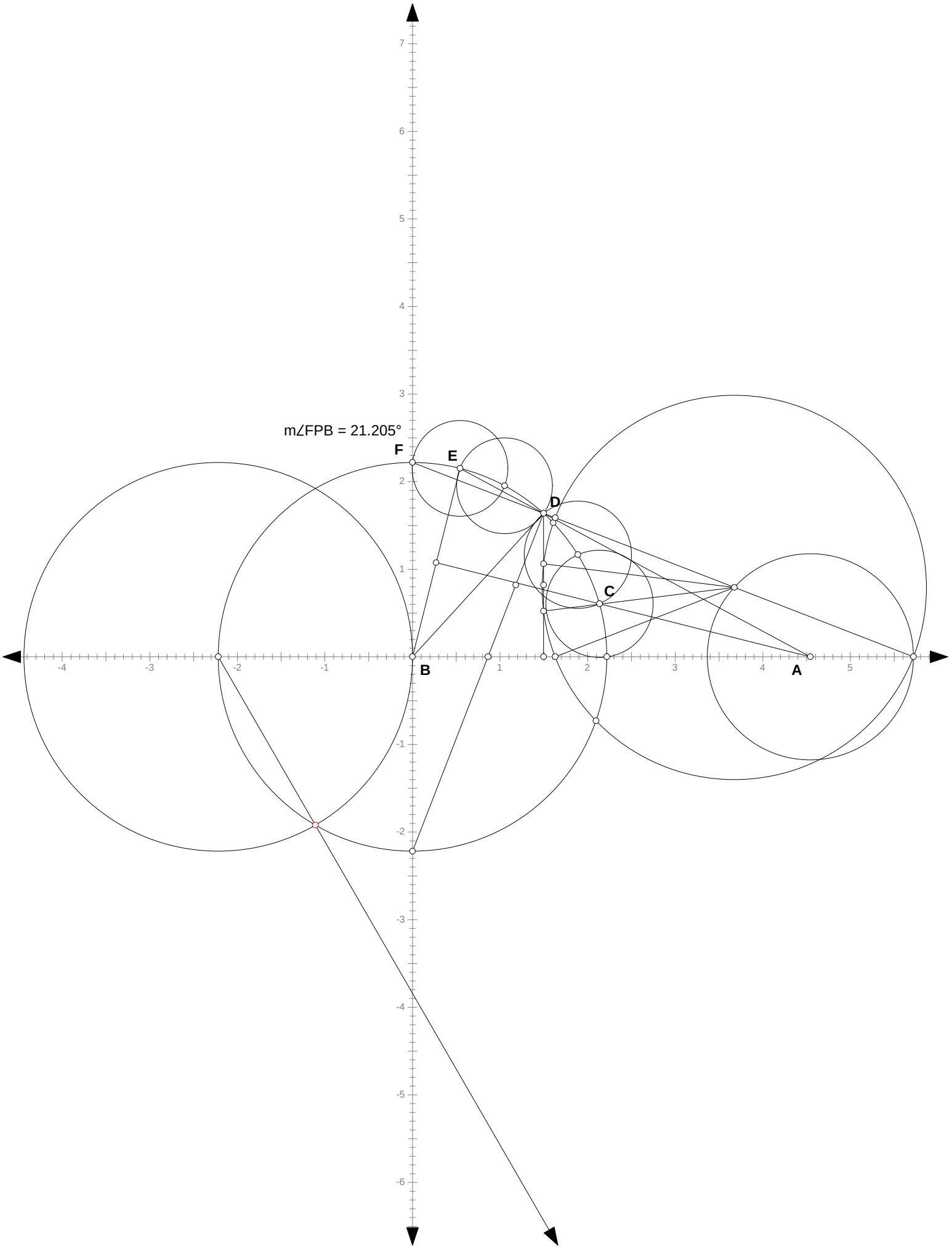

… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
m∠DBE = 15.000°
m∠CBE / m∠DBE = 3.000
m∠CBM / m∠FBM = 3.000
m∠CNB / m∠HNB = 3.000
m∠CMB / m∠HMB = 3.000
E
D
C
G
F
H
N
B
M
A

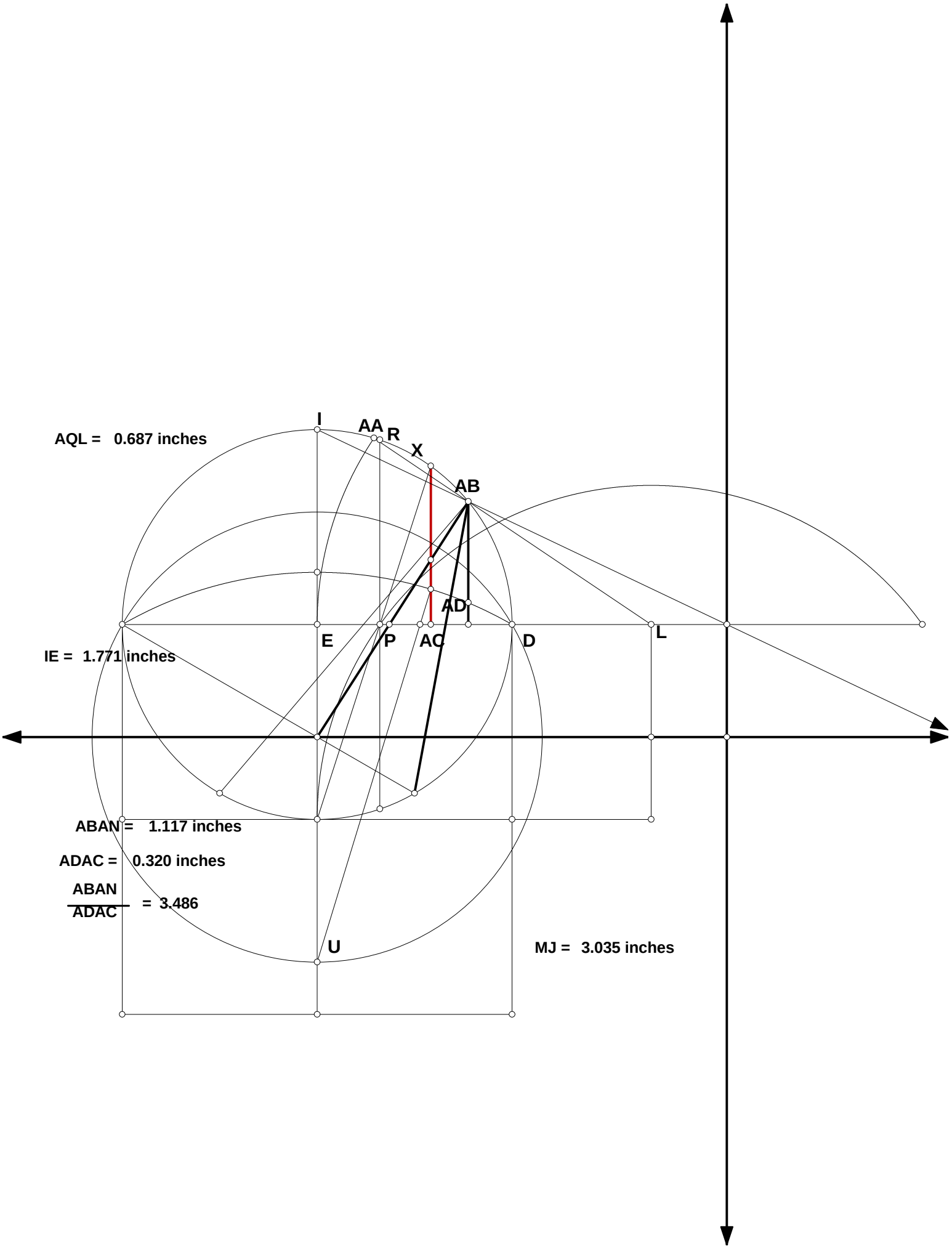

… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
… Sequence
m∠DBE = 15.000°
m∠CBE / m∠DBE = 3.000
m∠CBM / m∠FBM = 3.000
m∠XCY = 22.500°
m∠CBE = 45.000°
m∠CNB / m∠HNB = 3.000
m∠CMB / m∠HMB = 3.000
E
D
C
G
H
F
X
N
B
Q
M
A
Y

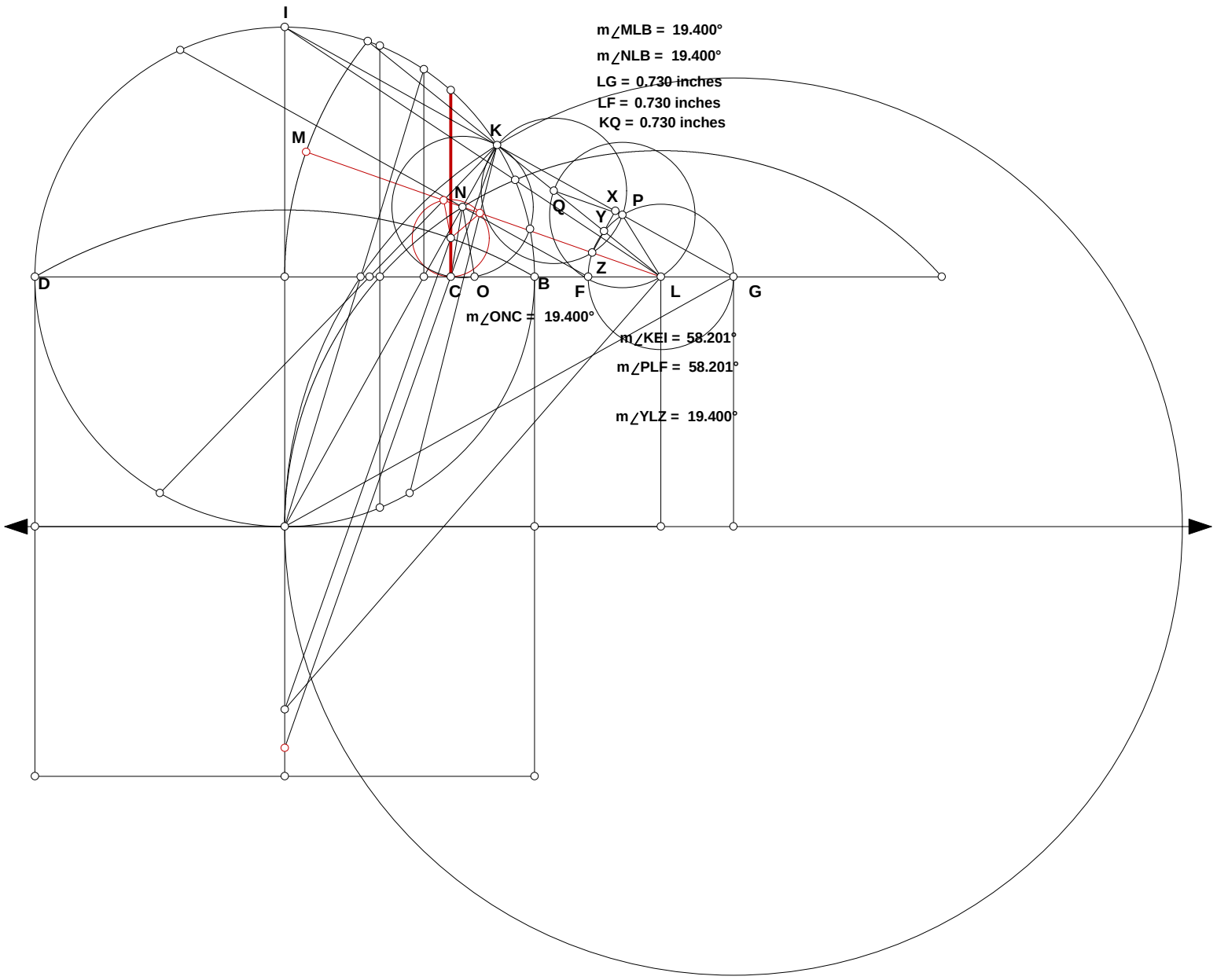

m∠FPO = 78.750°
… A
… B
… C
… D
… E
… F
… G
→ Move X->A
→ Move X->B
→ Move X->C
→ Move X->D
→ Move X->E
→ Move X->F
→ Move X->G
→ Move x->a
→ Move x->b
→ Move x->c
→ Move x->d
→ Move x->e
→ Move x->f
→ Move x->g
m∠XON = 78.750°
m∠xON = 15.750°
m∠XON / m∠xON = 5.000
m∠XOY = 72.000°
m∠xOY = 63.000°
N
Y
x
A
g
f
e
B
C
d
D
c
E
b
F
a
G
X
o
O
P
BO

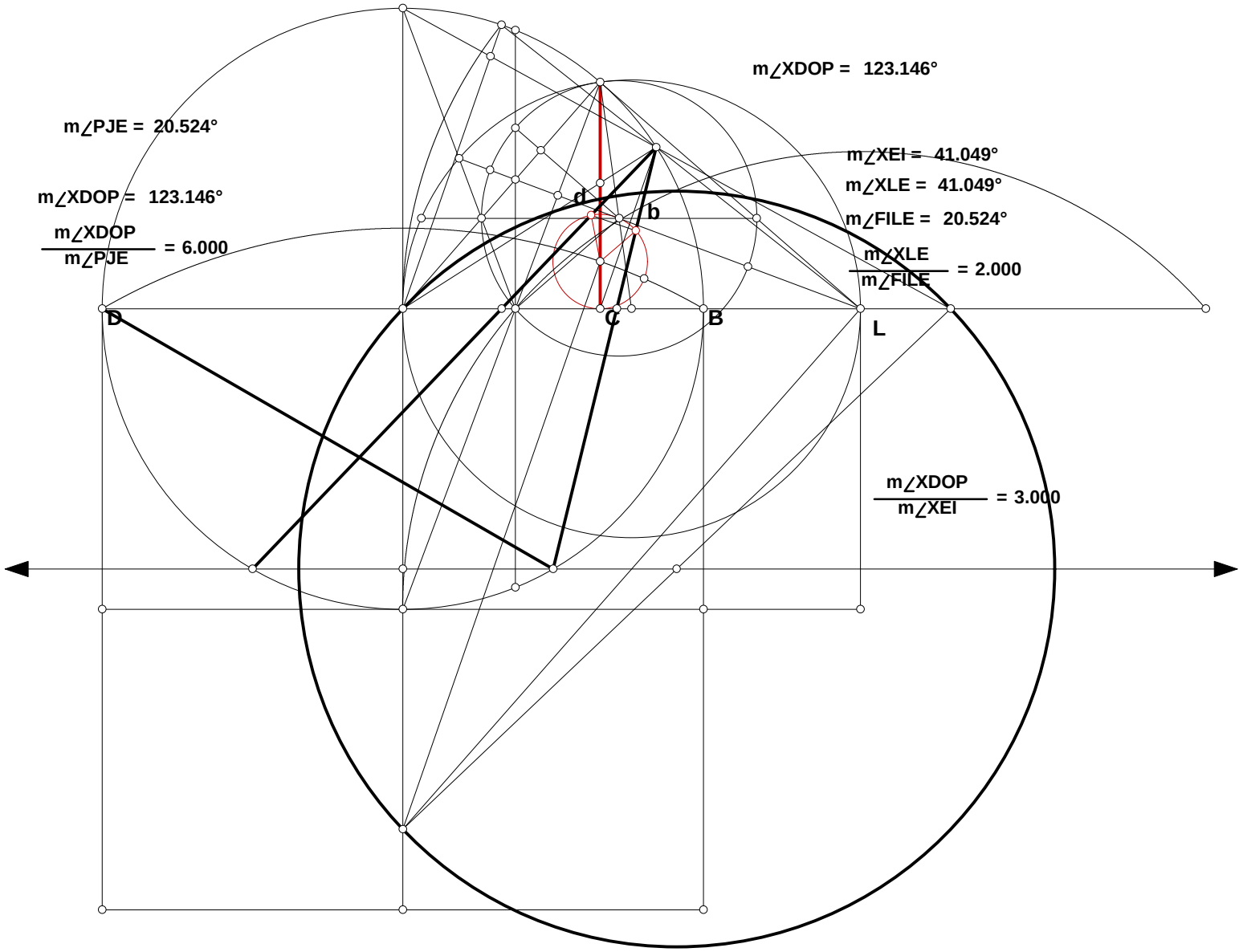

→ Move X->A
→ Move X->B
→ Move X->C
→ Move X->D
→ Move X->E
→ Move X->F
→ Move X->G
→ Move X->H
→ Move X->J
→ Move x->a
→ Move x->b
→ Move x->c
→ Move x->d
→ Move x->e
→ Move x->f
→ Move x->g
→ Move x->h
→ Move x->j
→ Move x->k
→ Move x->l
→ Move x->m
→ Move x->n
→ Move x->o
→ Move x->p
→ Move x->q
→ Move x->r
→ Move x->s
→ Move x->t
m∠XAC = 45.000°
m∠xAC = 55.506°
m∠XAC / m∠xAC = 0.811
t s r q p o n m l k j h g f e d c b a
A B C D E F G H J
X x
5 4 3 2 1 -1 -2 -3 -4
-3 -2 -1 1 2 3

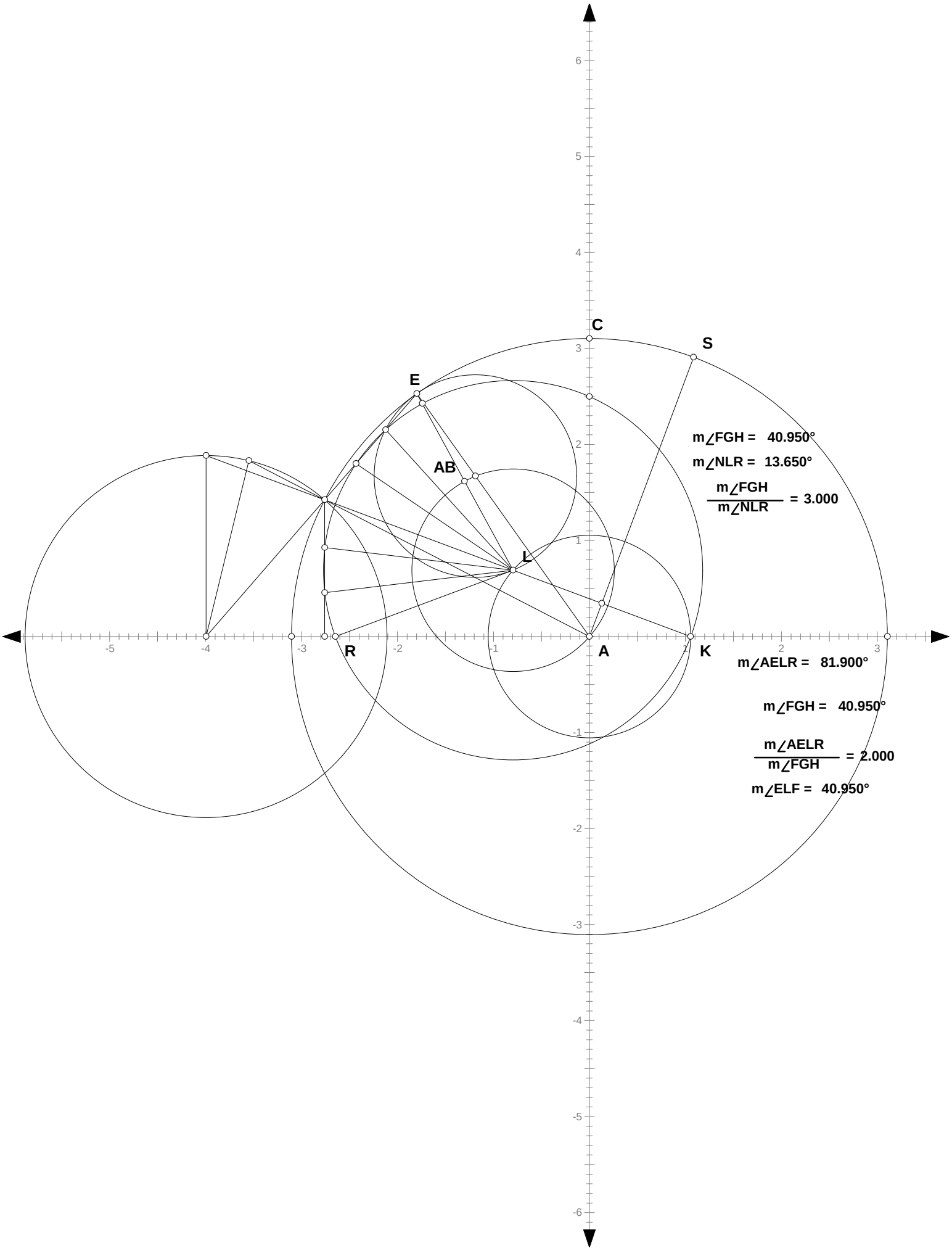

CJCC = 0.635 inches
UCJ = 0.635 inches
m∠CJLD = 21.125°
m∠UEI = 42.250°
DO
m∠ULCJ = 7.042°
m∠UEI / m∠ULCJ = 6.000

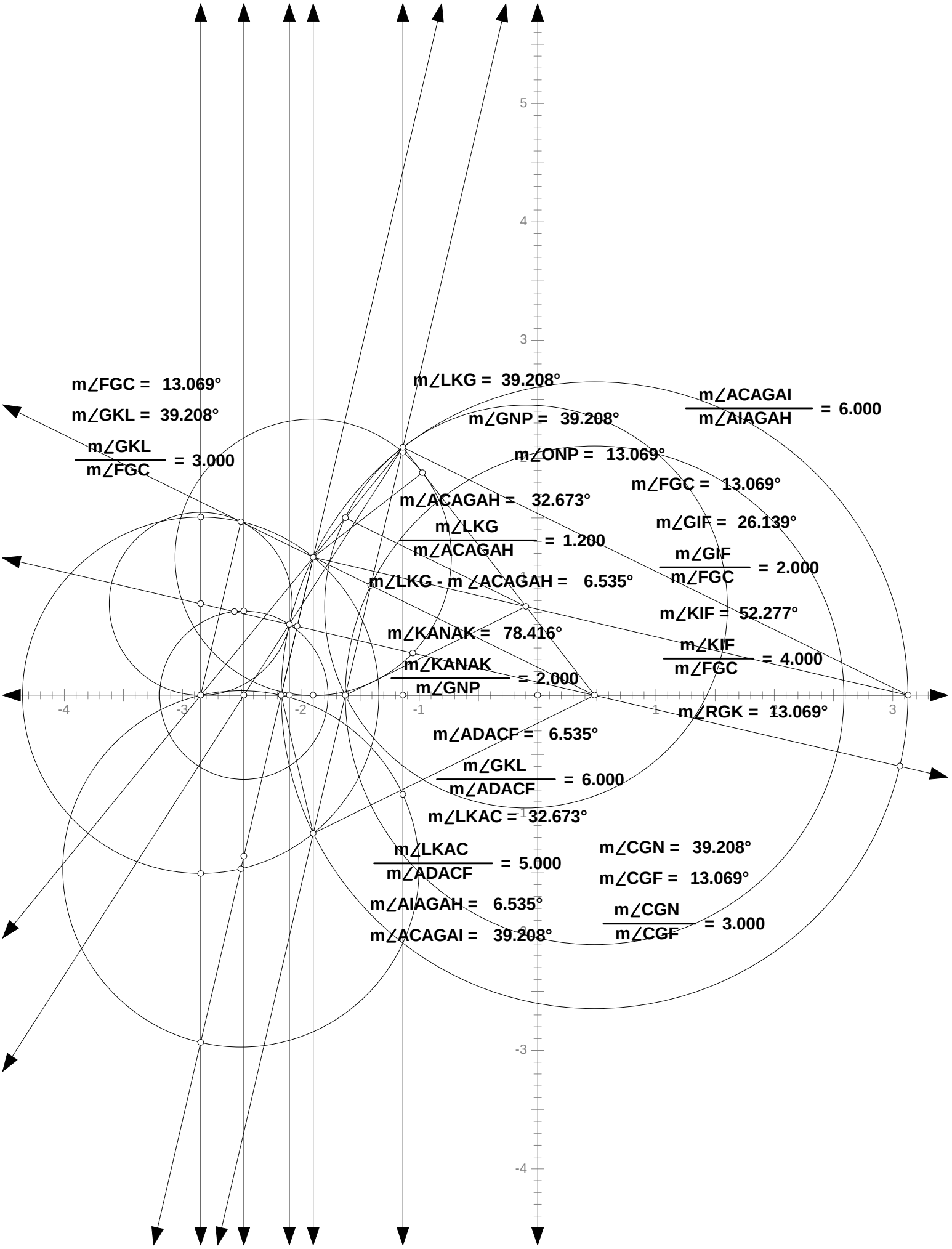

m∠FPB = 21.205°
F
E
D
C
B
A

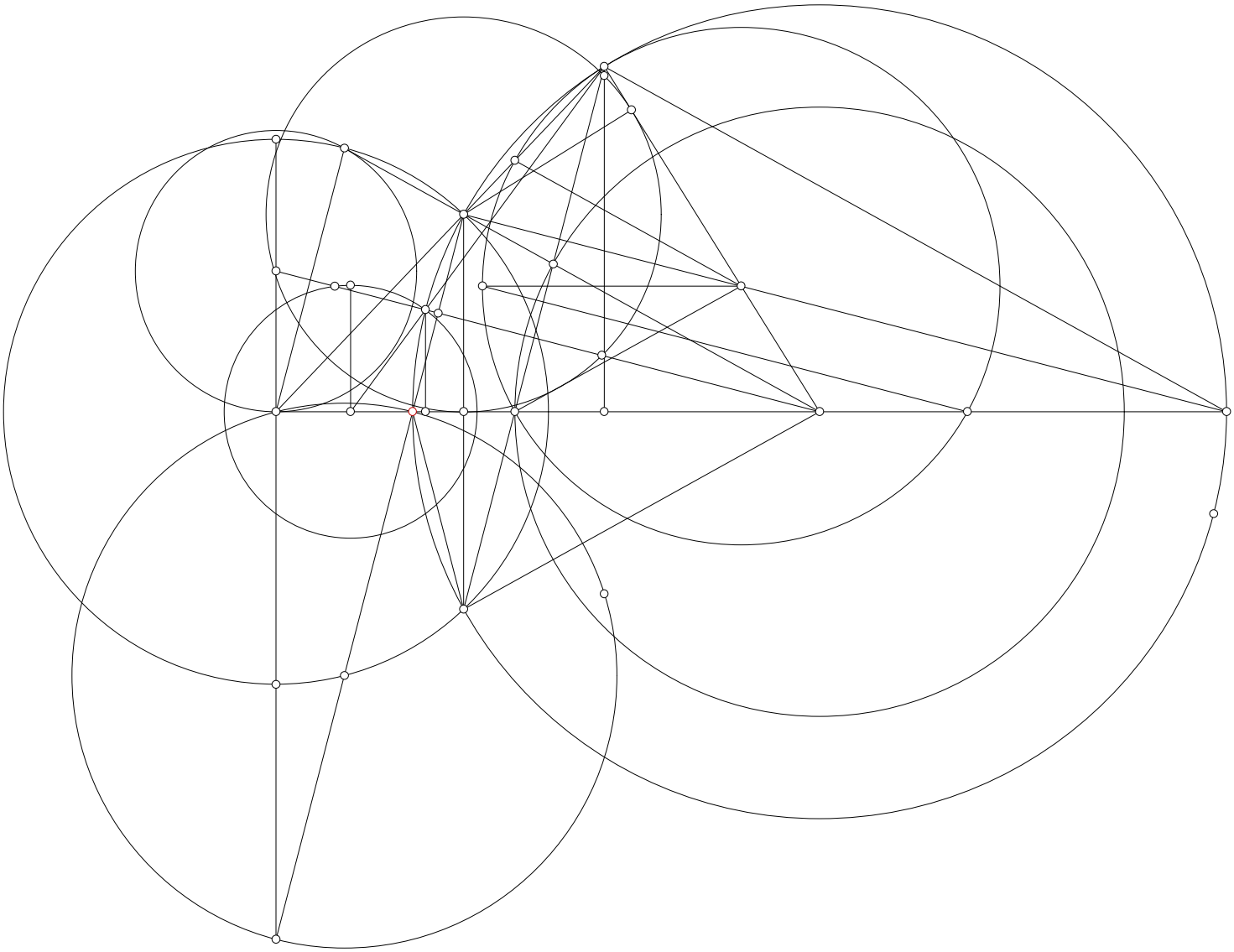

I
AA
R
X
AB
AD
E
P
AC
D
L
U
AQL = 0.687 inches
IE = 1.771 inches
ABAN = 1.117 inches
ADAC = 0.320 inches
ABAN / ADAC = 3.486
MJ = 3.035 inches

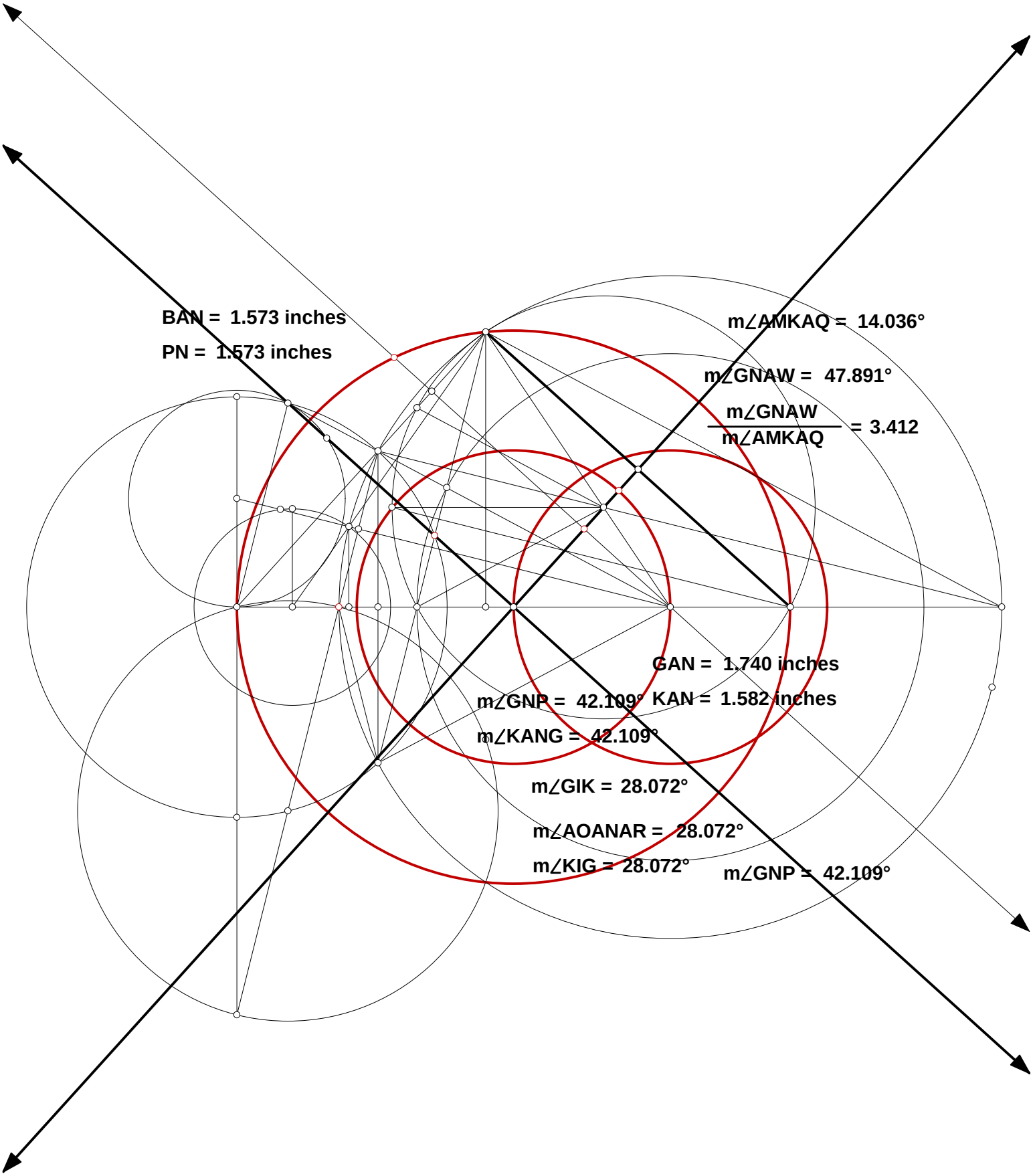

I
m∠MLB = 19.400°
m∠NLB = 19.400°
LG = 0.730 inches
LF = 0.730 inches
KQ = 0.730 inches
M
K
N
Q
X
P
Y
Z
D
C
O
B
F
L
G
m∠ONC = 19.400°
m∠KEI = 58.201°
m∠PLF = 58.201°
m∠YLZ = 19.400°

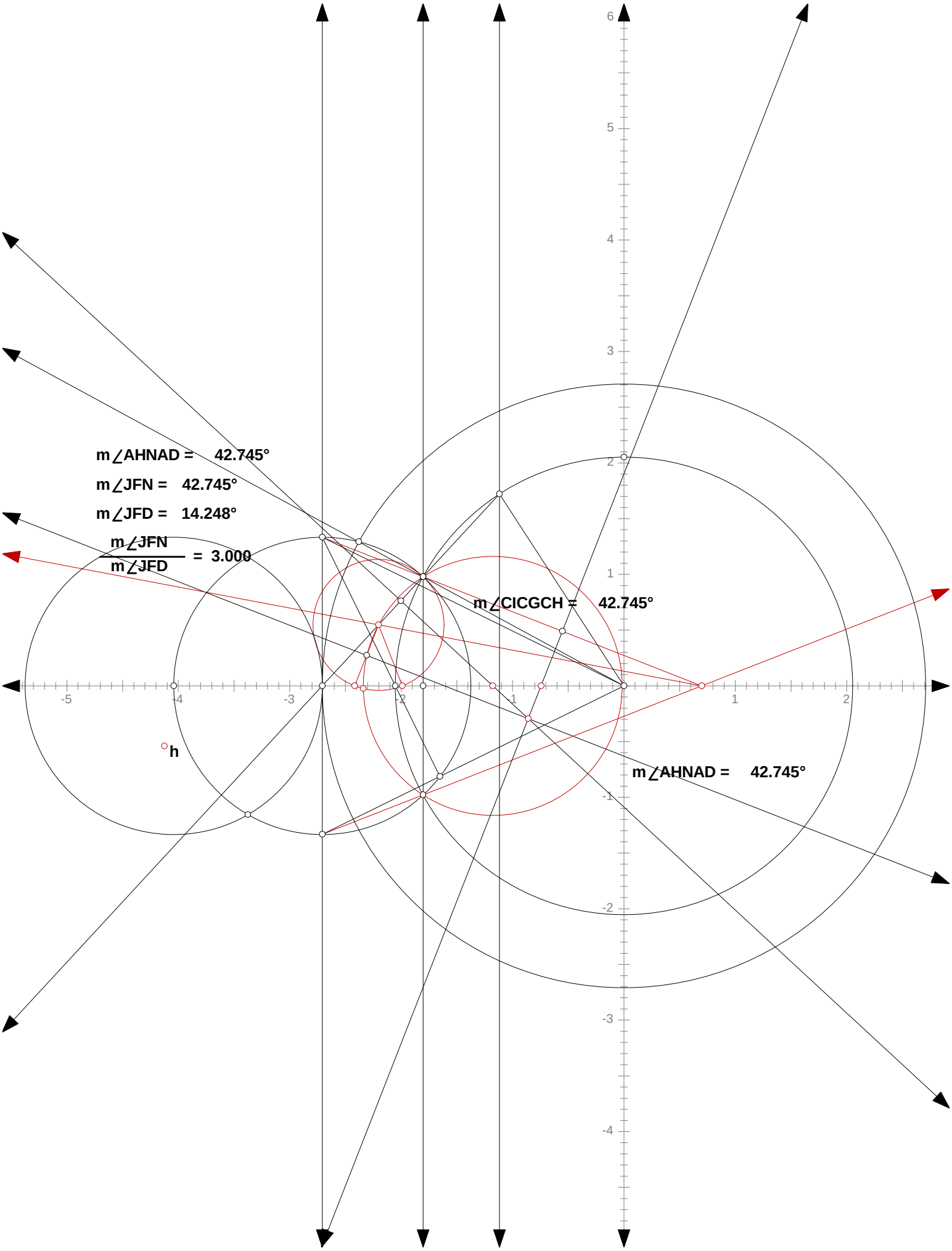

m∠XDOP = 123.146°
m∠PJE = 20.524°
m∠XEI = 41.049°
m∠XLE = 41.049°
m∠XDOP = 123.146°
m∠FILE = 20.524°
m∠XDOP / m∠PJE = 6.000
m∠XLE / m∠FILE = 2.000
d
b
D
C
B
L
m∠XDOP / m∠XEI = 3.000

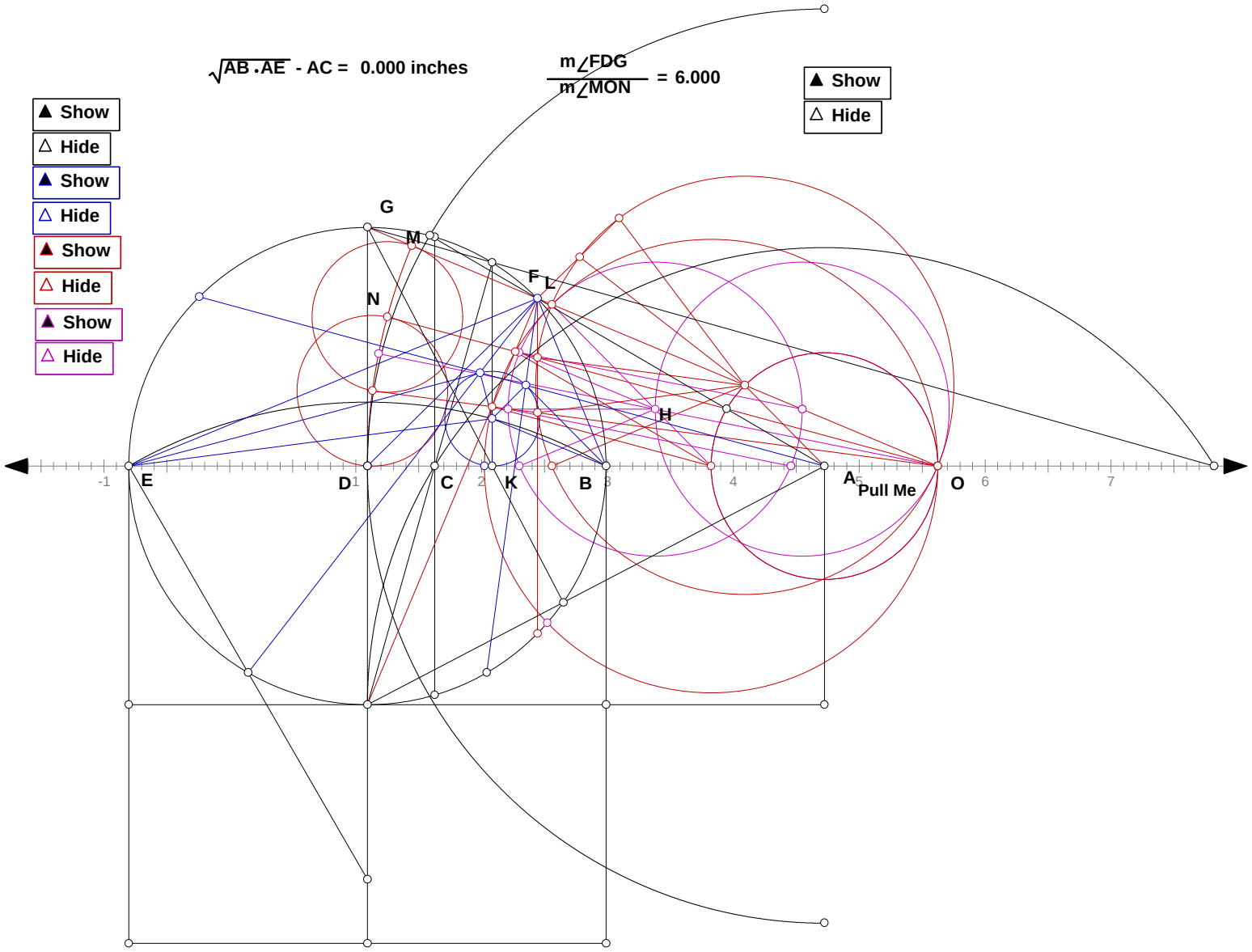

C
S
E
AB
L
R
A
K
m∠FGH = 40.950°
m∠NLR = 13.650°
m∠FGH / m∠NLR = 3.000
m∠AELR = 81.900°
m∠FGH = 40.950°
m∠AELR / m∠FGH = 2.000
m∠ELF = 40.950°

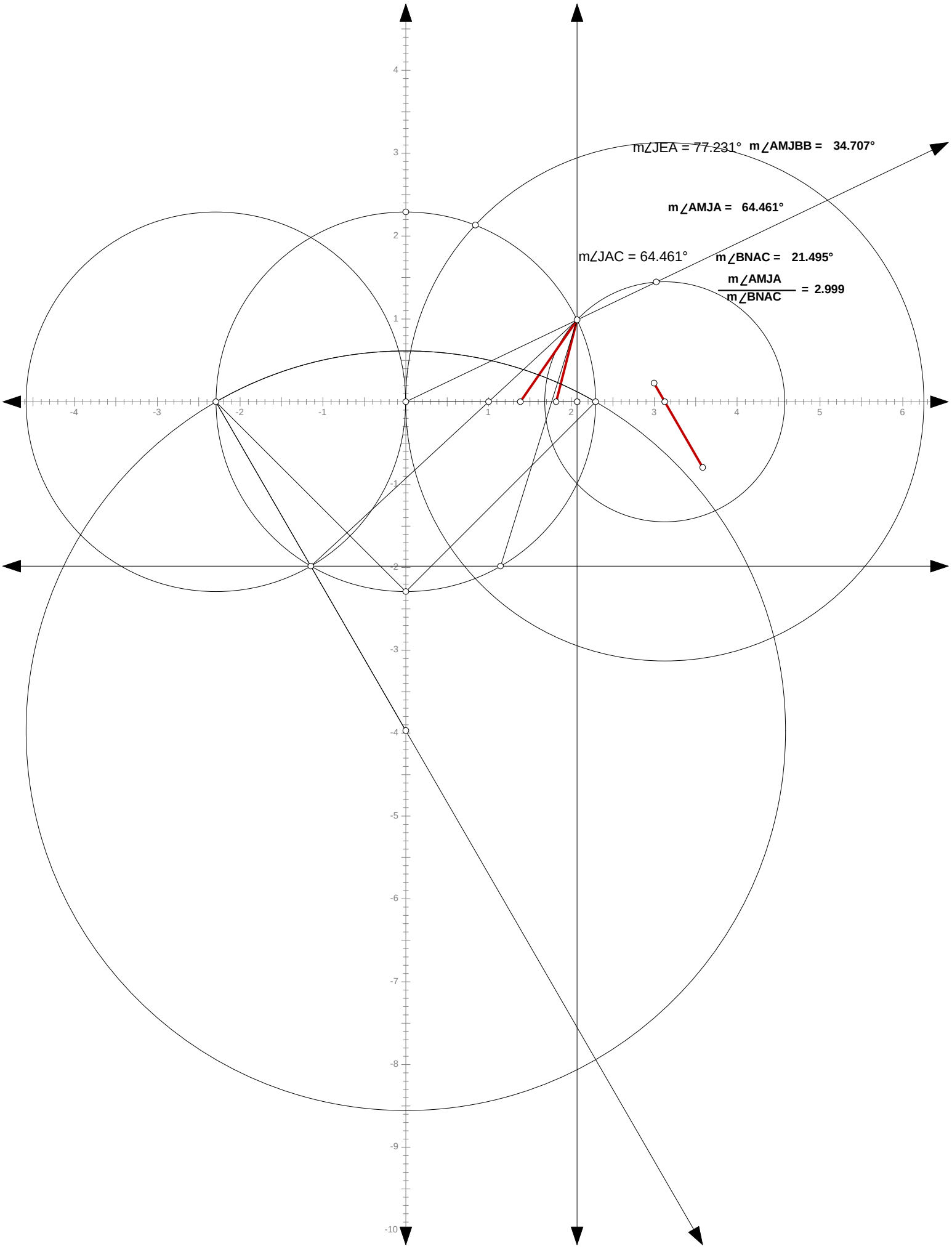

m∠FGC = 13.069°
m∠GKL = 39.208°
m∠GKL / m∠FGC = 3.000
m∠LKG = 39.208°
m∠GNP = 39.208°
m∠ONP = 13.069°
m∠FGC = 13.069°
m∠ACAGAH = 32.673°
m∠LKG / m∠ACAGAH = 1.200
m∠LKG - m ∠ACAGAH = 6.535°
m∠KANAK = 78.416°
m∠KANAK / m∠GNP = 2.000
m∠ADACF = 6.535°
m∠GKL / m∠ADACF = 6.000
m∠LKAC = 32.673°
m∠LKAC / m∠ADACF = 5.000
m∠AIAGAH = 6.535°
m∠ACAGAI = 39.208°
m∠ACAGAI / m∠AIAGAH = 6.000
m∠GIF = 26.139°
m∠GIF / m∠FGC = 2.000
m∠KIF = 52.277°
m∠KIF / m∠FGC = 4.000
m∠RGK = 13.069°
m∠CGN = 39.208°
m∠CGF = 13.069°
m∠CGN / m∠CGF = 3.000

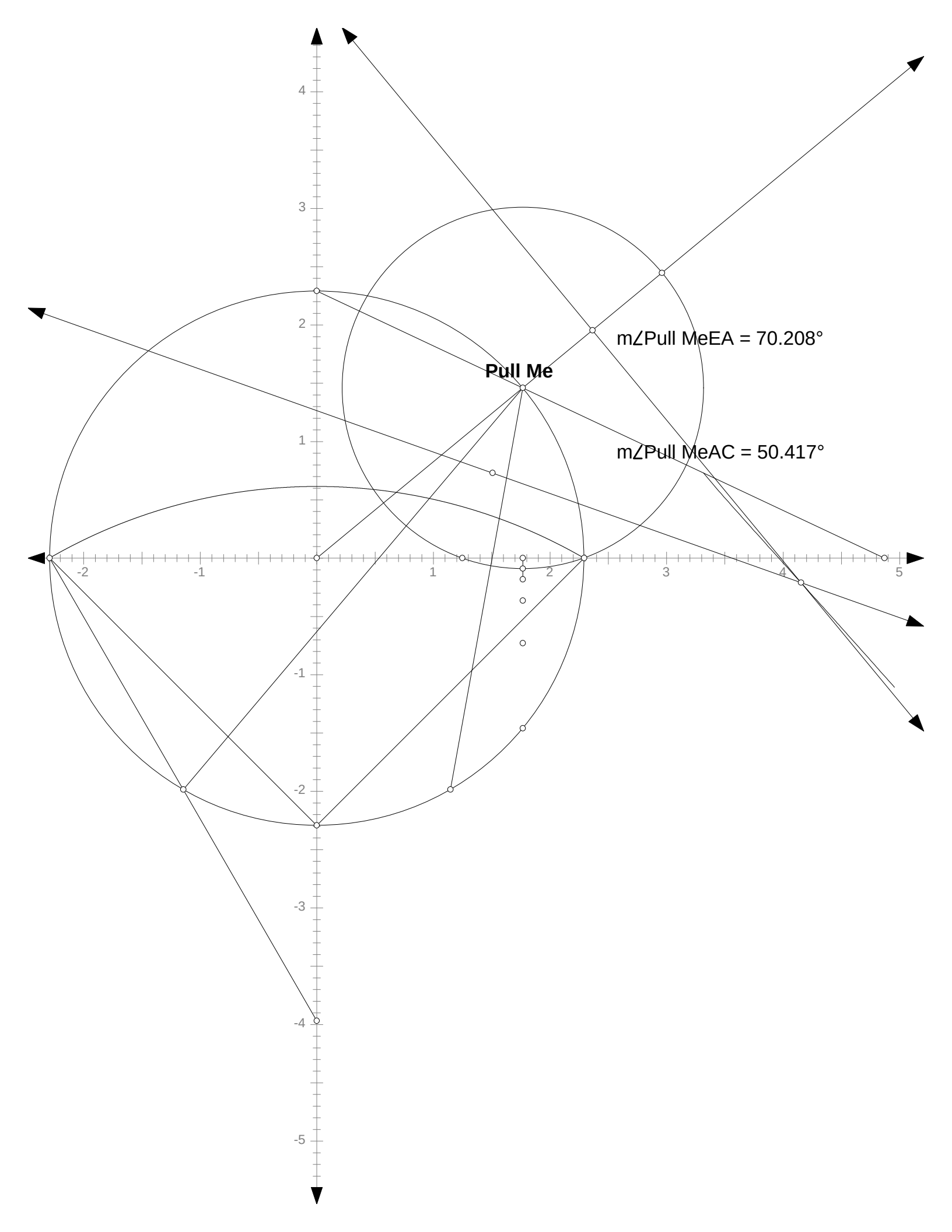


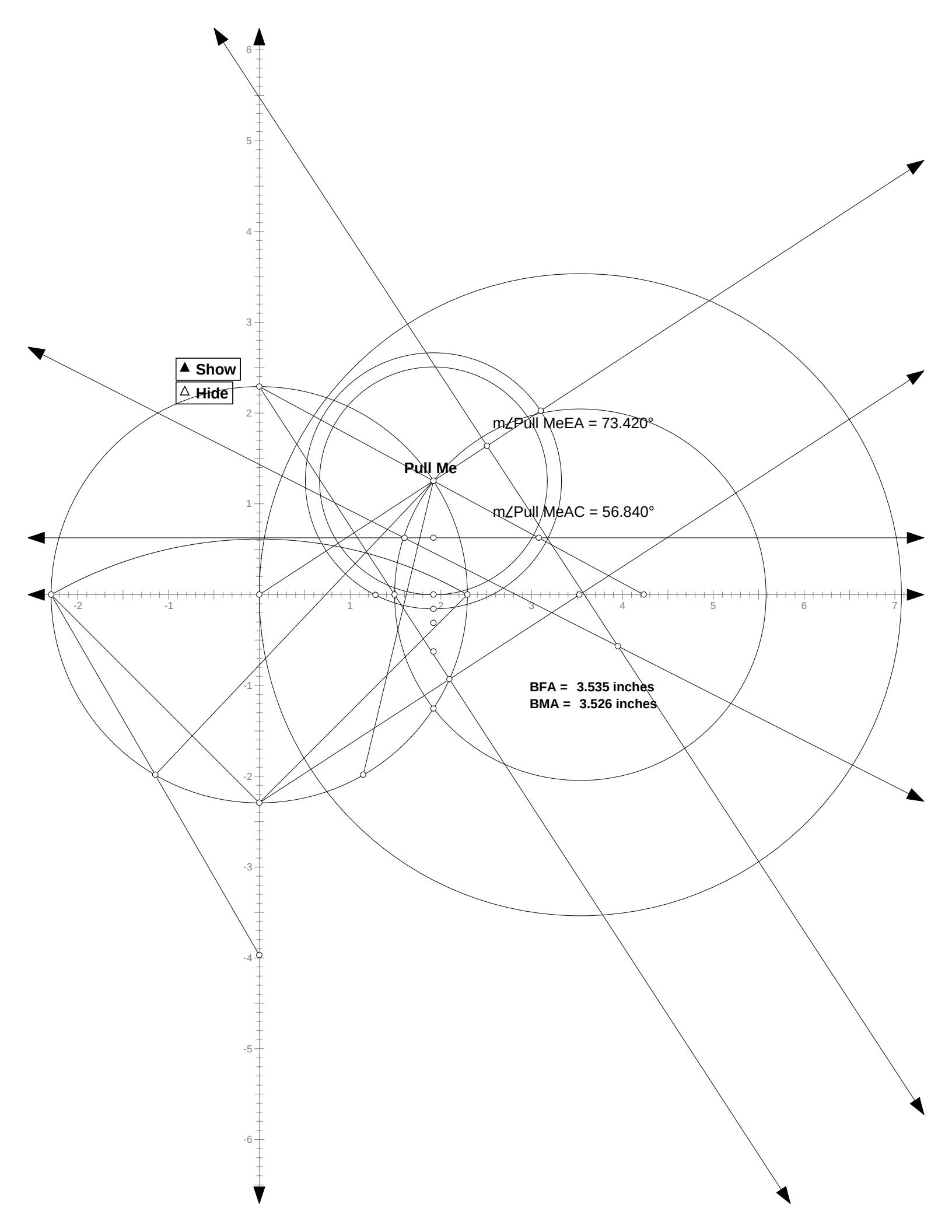

BAN = 1.573 inches
PN = 1.573 inches
m∠AMKAQ = 14.036°
m∠GNAW = 47.891°
m∠GNAW / m∠AMKAQ = 3.412
GAN = 1.740 inches
m∠GNP = 42.109°
KAN = 1.582 inches
m∠KANG = 42.109°
m∠GIK = 28.072°
m∠AOANAR = 28.072°
m∠KIG = 28.072°
m∠GNP = 42.109°

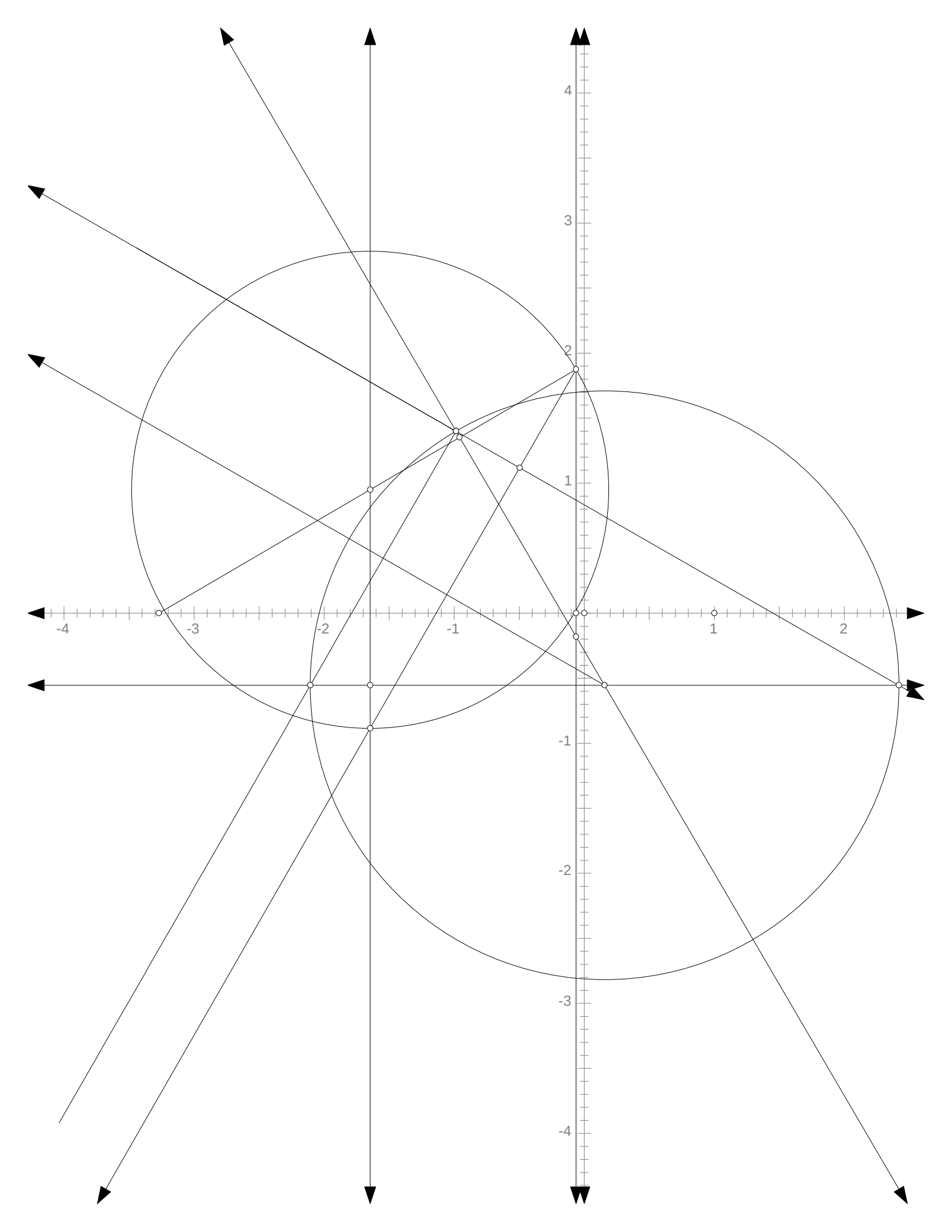

m∠AHNAD = 42.745°
m∠JFN = 42.745°
m∠JFD = 14.248°
m∠JFN / m∠JFD = 3.000
m∠CICGCH = 42.745°
h
m∠AHNAD = 42.745°

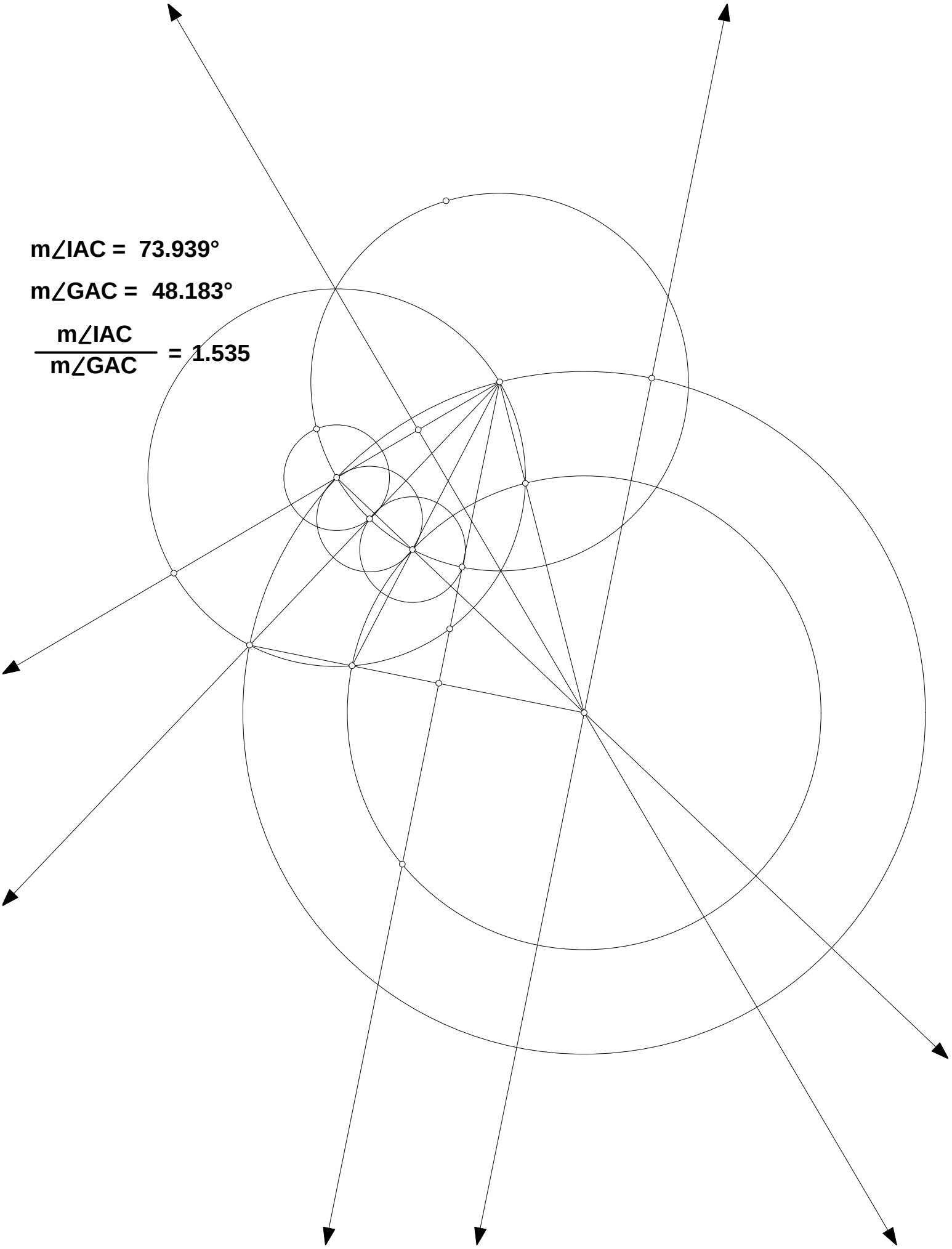

√AB • AE - AC = 0.000 inches
m∠FDG / m∠MON = 6.000
Show
Hide
Show
Hide
Show
Hide
Show
Hide
Show
Hide
G
M
N
F
L
H
E
D
C
K
B
A
O
Pull Me
-1
1
2
3
4
5
6
7

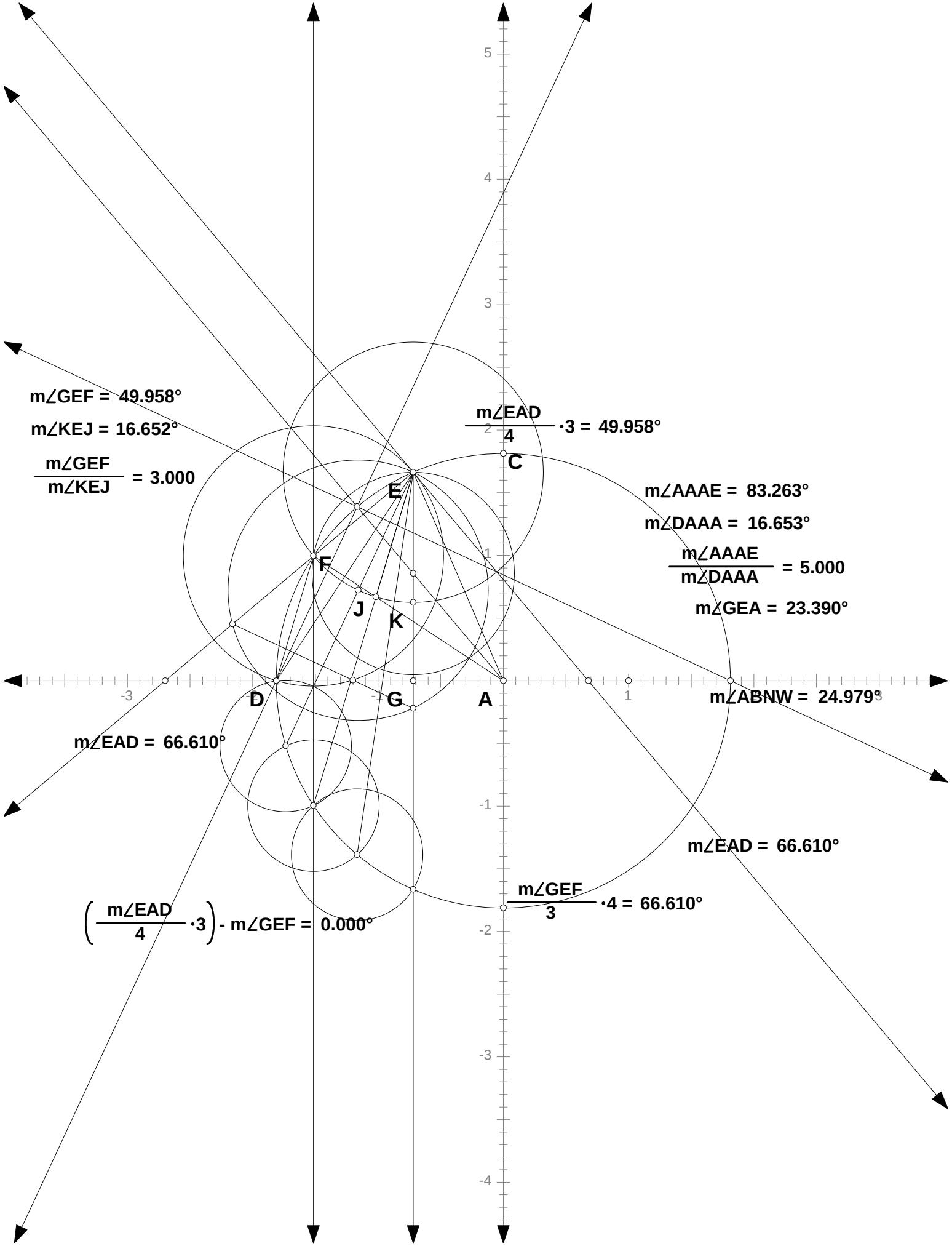

m∠JEA = 77.231°
m∠AMJBB = 34.707°
m∠AMJA = 64.461°
m∠JAC = 64.461°
m∠BNAC = 21.495°
m∠AMJA / m∠BNAC = 2.999

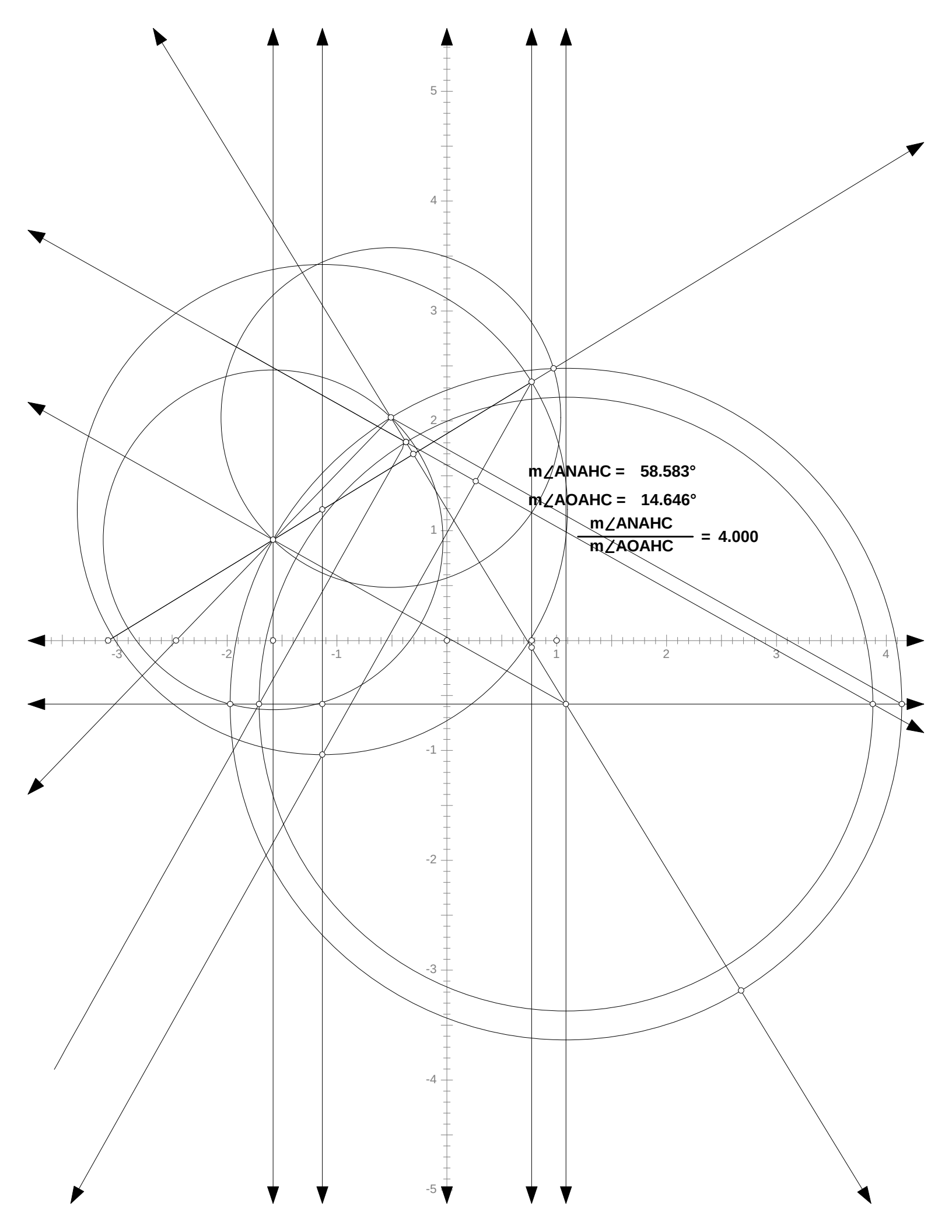

m∠Pull MeEA = 70.208°
Pull Me
m∠Pull MeAC = 50.417°
4
3
2
1
-1
-2
-3
-4
-5
-2
-1
1
2
3
4
5

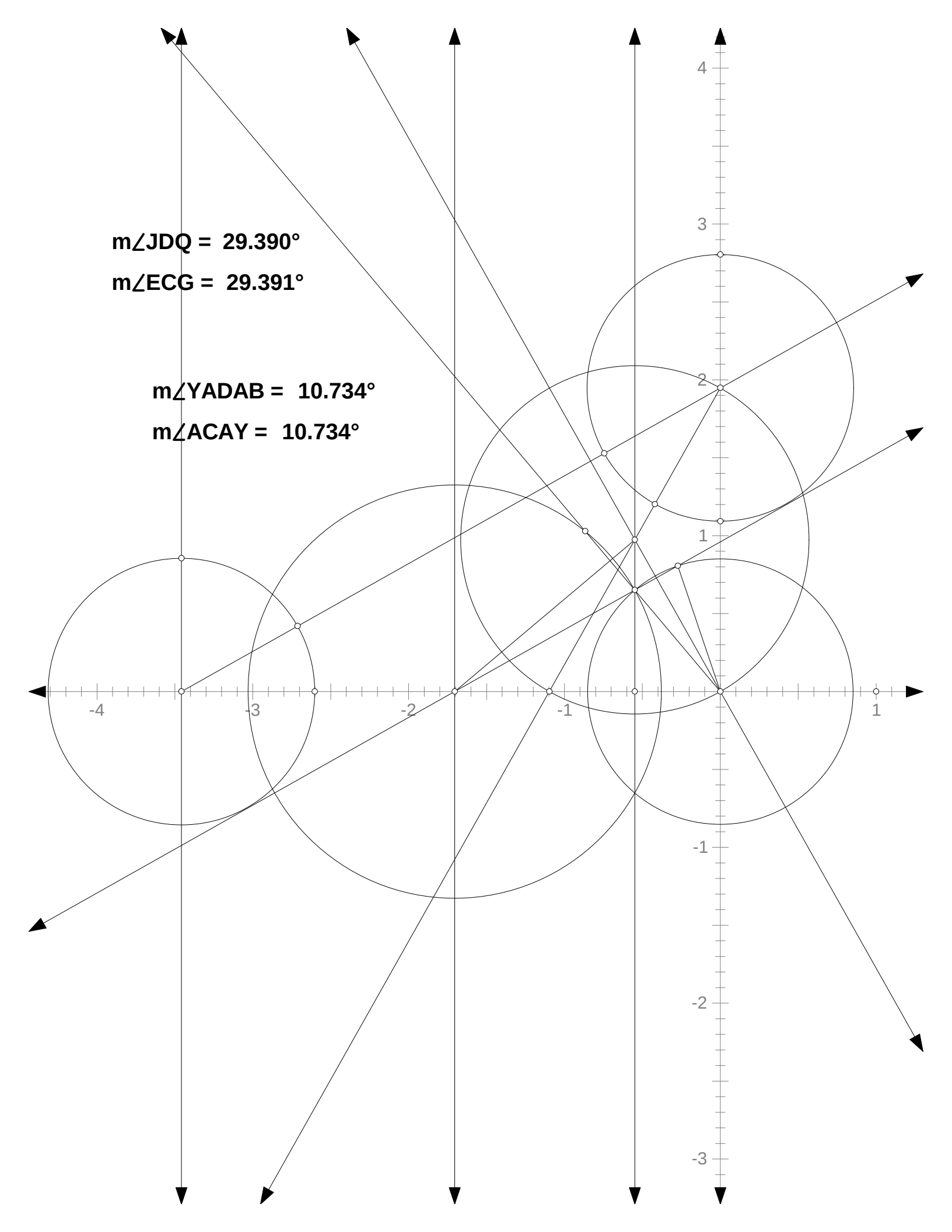

▲ Show
△ Hide
m∠Pull MeEA = 73.420°
Pull Me
m∠Pull MeAC = 56.840°
BFA = 3.535 inches
BMA = 3.526 inches

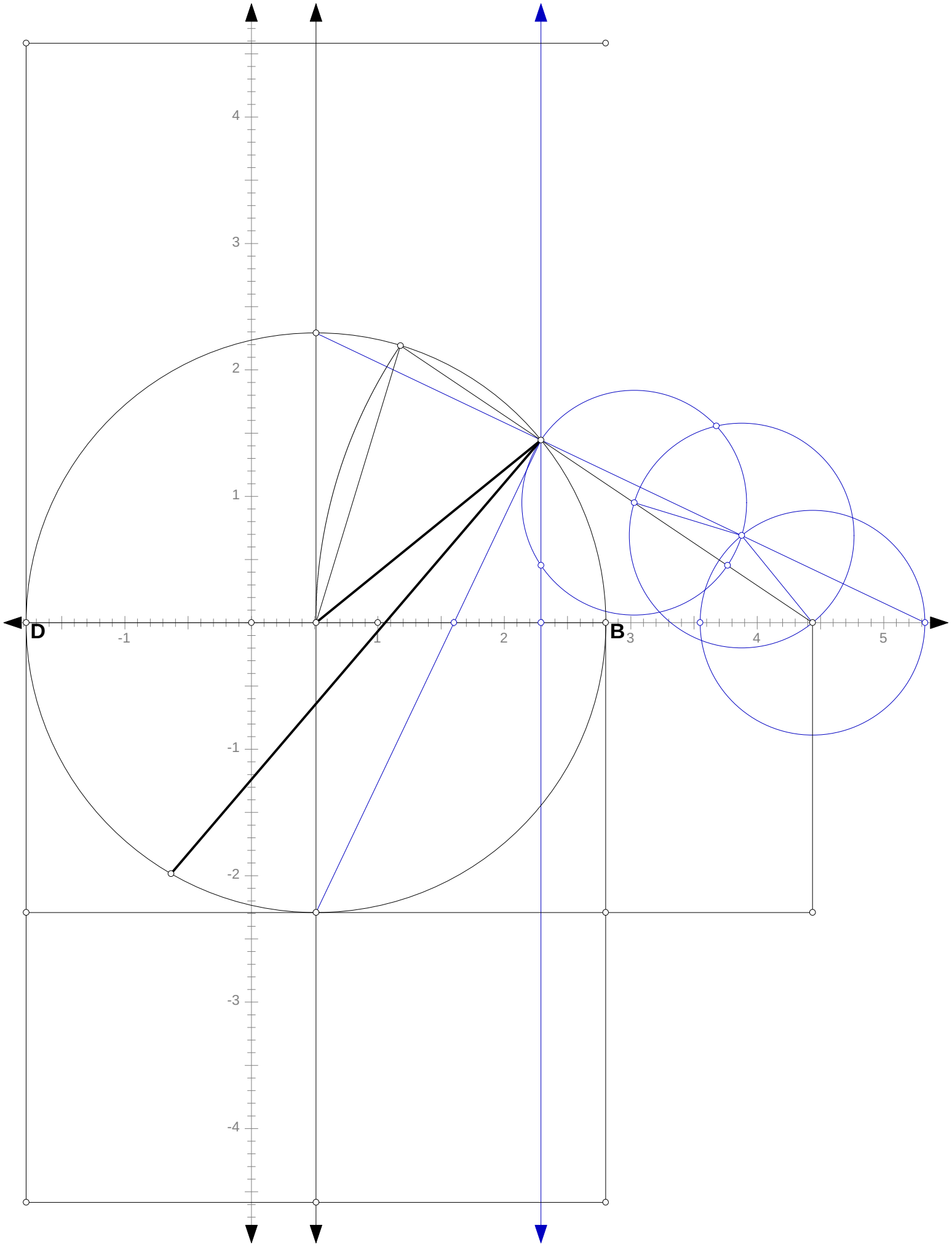

4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2

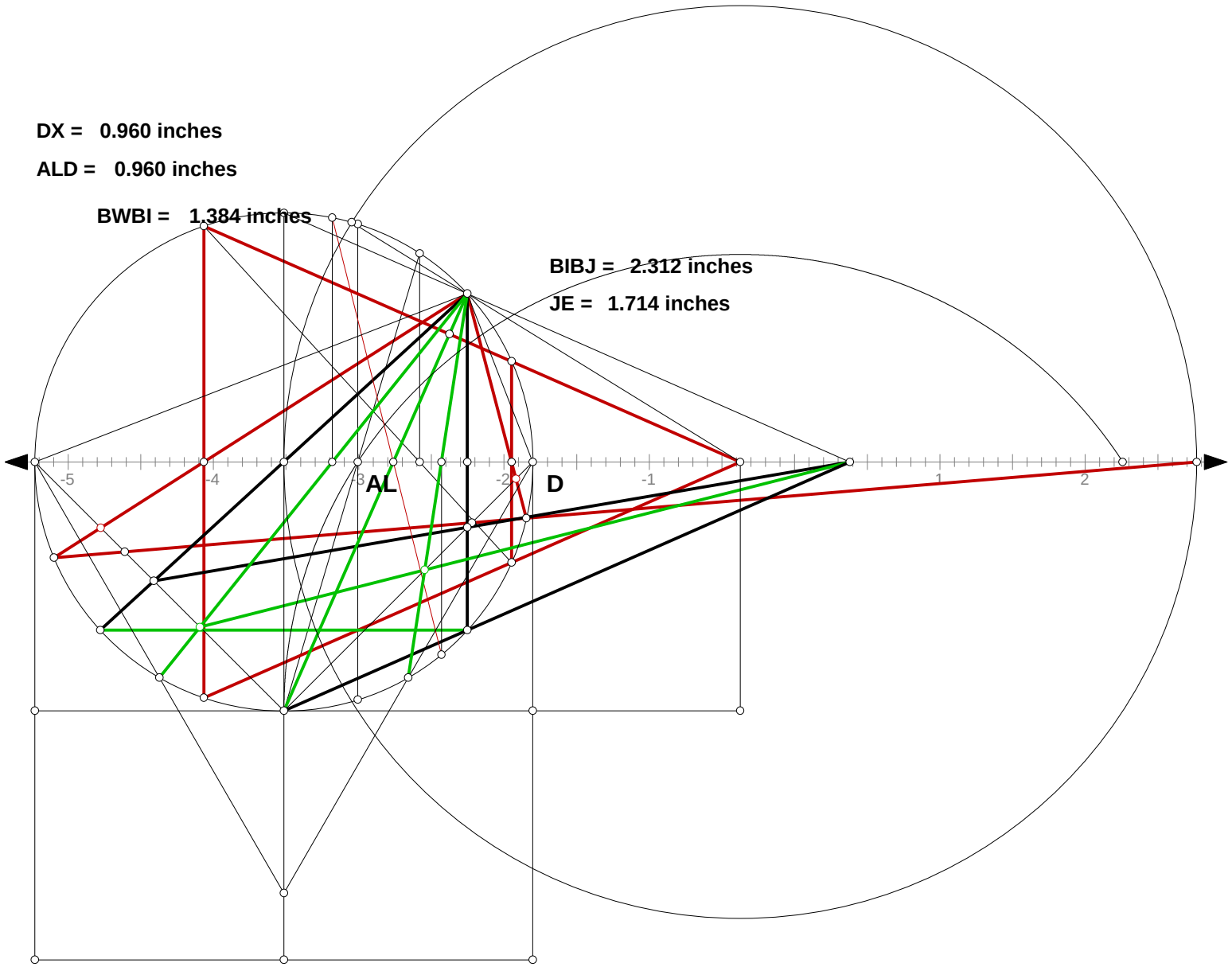

m∠IAC = 73.939°
m∠GAC = 48.183°
m∠IAC / m∠GAC = 1.535


m∠GEF = 49.958°
m∠KEJ = 16.652°
m∠GEF / m∠KEJ = 3.000
m∠EAD / 4 ·3 = 49.958°
m∠AAAE = 83.263°
m∠DAAA = 16.653°
m∠AAAE / m∠DAAA = 5.000
m∠GEA = 23.390°
m∠ABNW = 24.979°
m∠EAD = 66.610°
m∠EAD = 66.610°
m∠GEF / 3 ·4 = 66.610°
(m∠EAD / 4 ·3) - m∠GEF = 0.000°
C
E
F
J
K
D
G
A


m∠ANAHC = 58.583°
m∠AOAHC = 14.646°
m∠ANAHC / m∠AOAHC = 4.000

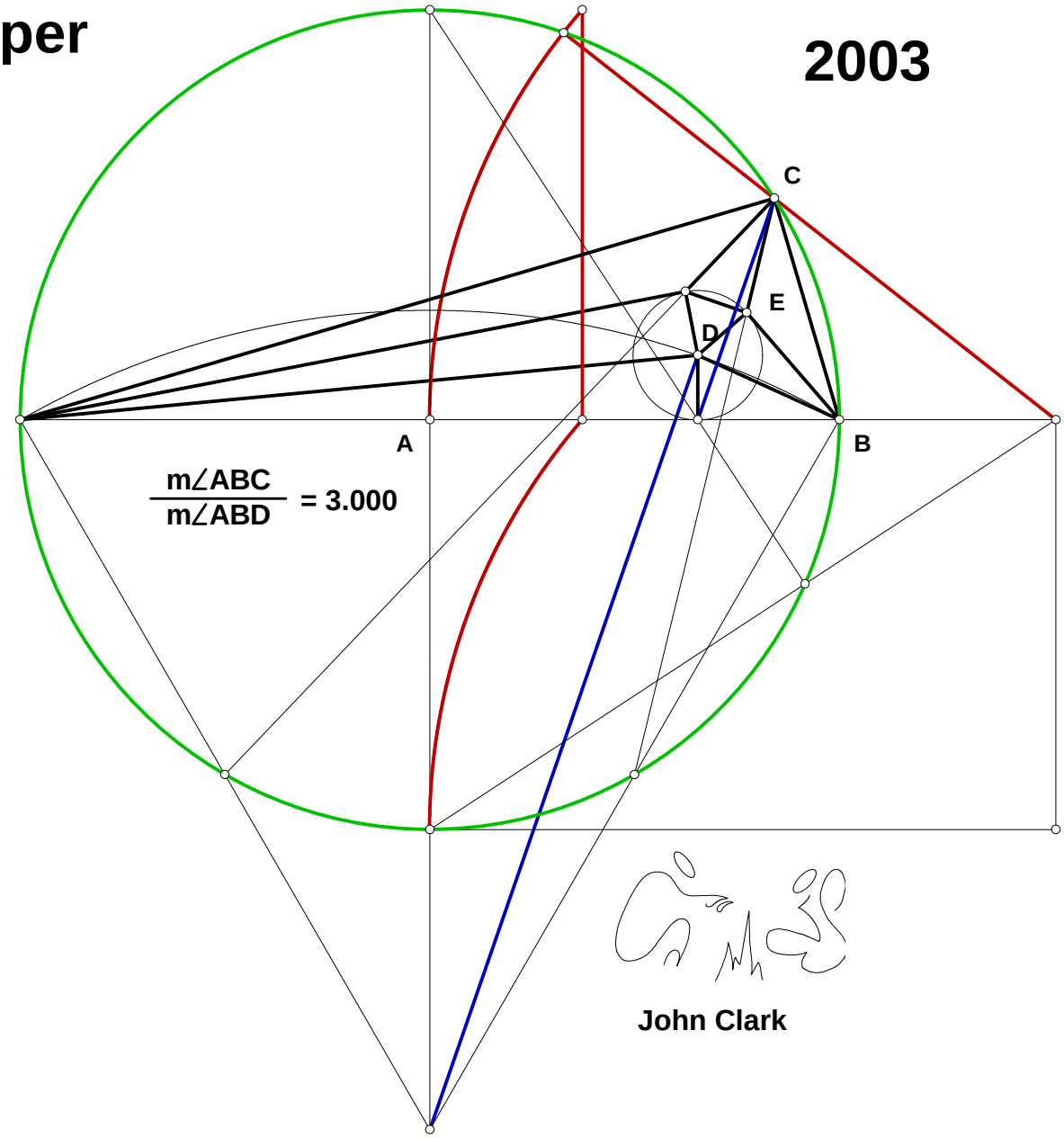

m∠JDQ = 29.390°
m∠ECG = 29.391°
m∠YADAB = 10.734°
m∠ACAY = 10.734°
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
1


4
3
2
1
-1
-2
-3
-4
D
-1
1
2
B
3
4
5


DX = 0.960 inches
ALD = 0.960 inches
BWBI = 1.384 inches
BIBJ = 2.312 inches
JE = 1.714 inches
-5
-4
-3
AL
-2
D
-1
1
2

# Three Pieces Of Paper

**2003**

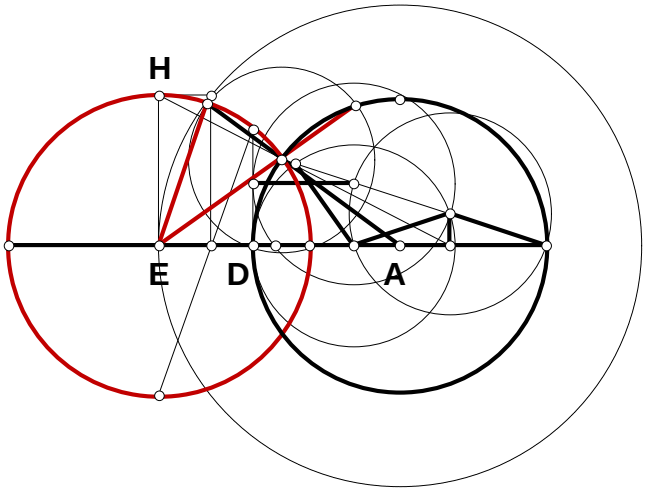



$$\frac{m\angle ABC}{m\angle ABD} = 3.000$$

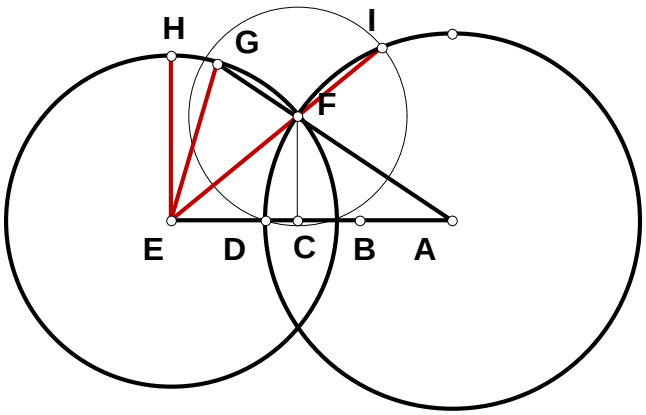


John Clark

# 021603




**From AD project a trisection to EH.**




$AD := .969 \quad AB := \frac{AD}{2} \quad N := 1.51$

$BD := AD - AB \quad BC := \frac{BD}{N}$

$CD := BD - BC \quad AC := AB + BC$

$AF := AD \quad CF := \sqrt{AF^2 - AC^2}$

$FI := \sqrt{CD^2 + CF^2}$

$DI := 2 \cdot CF \quad AI := AD$

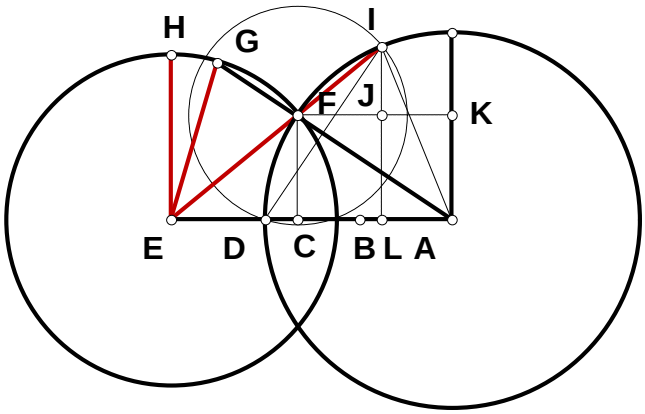
$AL := \frac{\left(2\ AD^2 - DI^2\right)}{2 \cdot AD}$

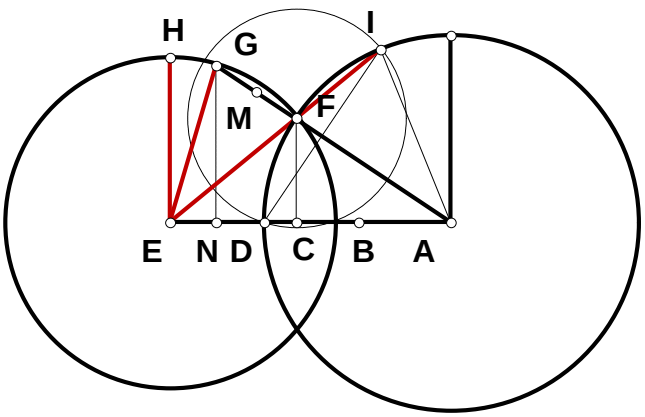
$FJ := AC - AL \quad IL := \sqrt{AI^2 - AL^2}$

$IJ := IL - CF \quad EL := \frac{FJ \cdot IL}{IJ}$

$AE := AL + EL \quad EI := \frac{FI \cdot IL}{IJ}$

$EF := EI - FI$






$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF}$$

$$AG := AF + FG$$

$$AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \quad FM := \frac{FG}{2}$$

$$FM - EN = 0$$

**Some Algebraic Names;**

$$BC - \frac{AD - AB}{N} = 0 \qquad CD - (N-1)\cdot\frac{(AD - AB)}{N} = 0 \qquad AC - \frac{(AB \cdot N + AD - AB)}{N} = 0$$

$$CF - \frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0$$

$$FI - \sqrt{2}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot\frac{AD}{N}} = 0$$

$$DI - 2\cdot\frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0$$

$$AL - \frac{-\left(AD^2 \cdot N^2 - 4 \cdot AD \cdot N \cdot AB + 4 \cdot AB^2 \cdot N - 2 \cdot N^2 \cdot AB^2 - 2 \cdot AD^2 + 4 \cdot AD \cdot AB - 2 \cdot AB^2\right)}{\left(AD \cdot N^2\right)} = 0$$

$$FJ - (N-1)\cdot(AD - AB)\cdot\frac{(AD \cdot N + 2 \cdot AD - 2 \cdot AB + 2 \cdot AB \cdot N)}{\left(AD \cdot N^2\right)} = 0$$

$$IL - \frac{(AB \cdot N + AD - AB)}{\left(AD \cdot N^2\right)}\cdot\sqrt{4}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)} = 0$$

$$IJ - -\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}\cdot\frac{(-2 \cdot AB \cdot N - 2 \cdot AD + 2 \cdot AB + AD \cdot N)}{\left(AD \cdot N^2\right)} = 0$$

$$EL - \frac{-2\cdot(N-1)\cdot(AD-AB)\cdot(AD\cdot N+2\cdot AD-2\cdot AB+2\cdot AB\cdot N)\cdot(AB\cdot N+AD-AB)}{AD\cdot\left[N^2\cdot(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)\right]} = 0$$

$$AE - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$EI - 2\cdot\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot\frac{(AB\cdot N+AD-AB)}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

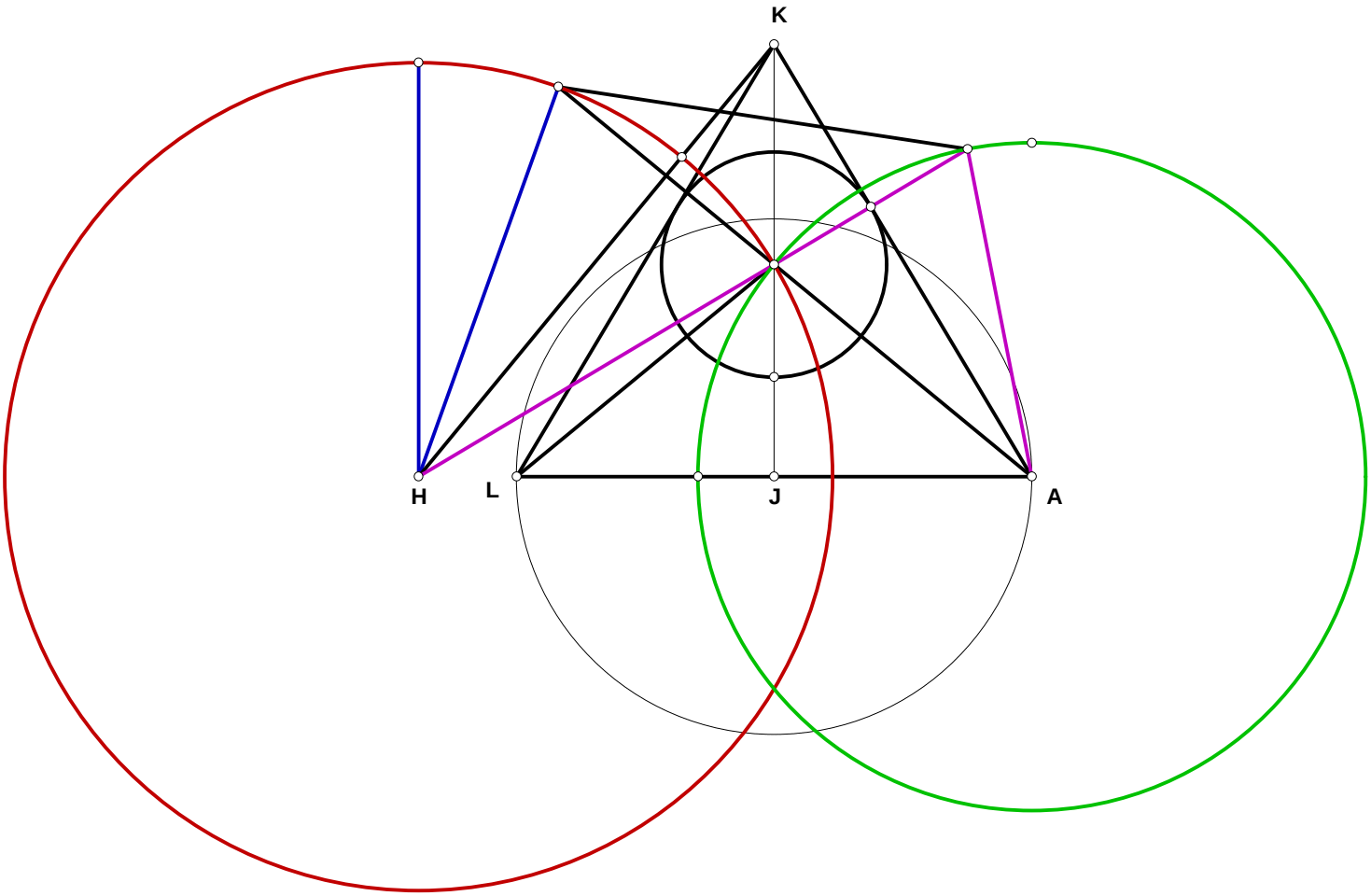
$$EF - -\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot AD\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

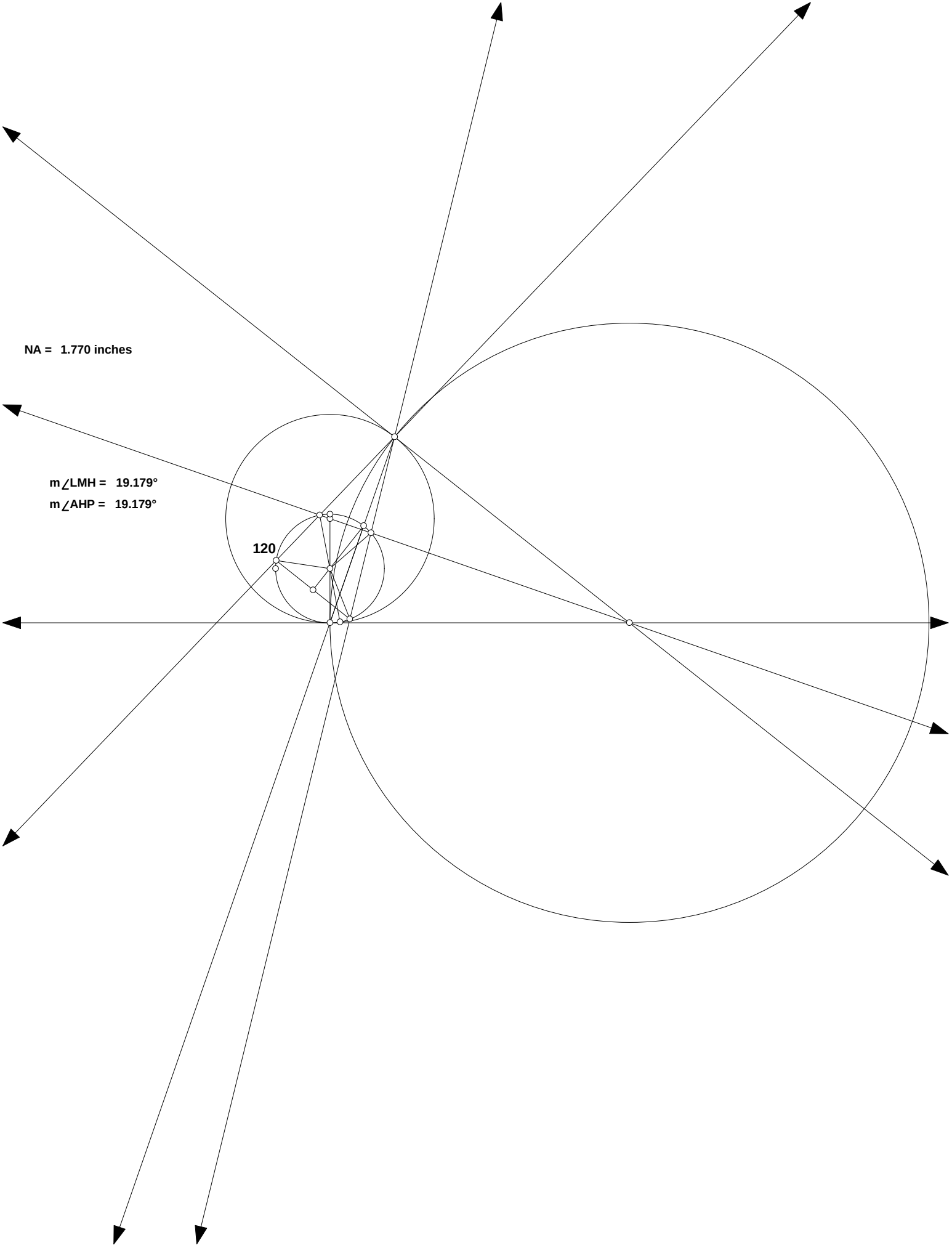
$$FG - -2\cdot AD\cdot(AD-AB)\cdot\frac{(N-1)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

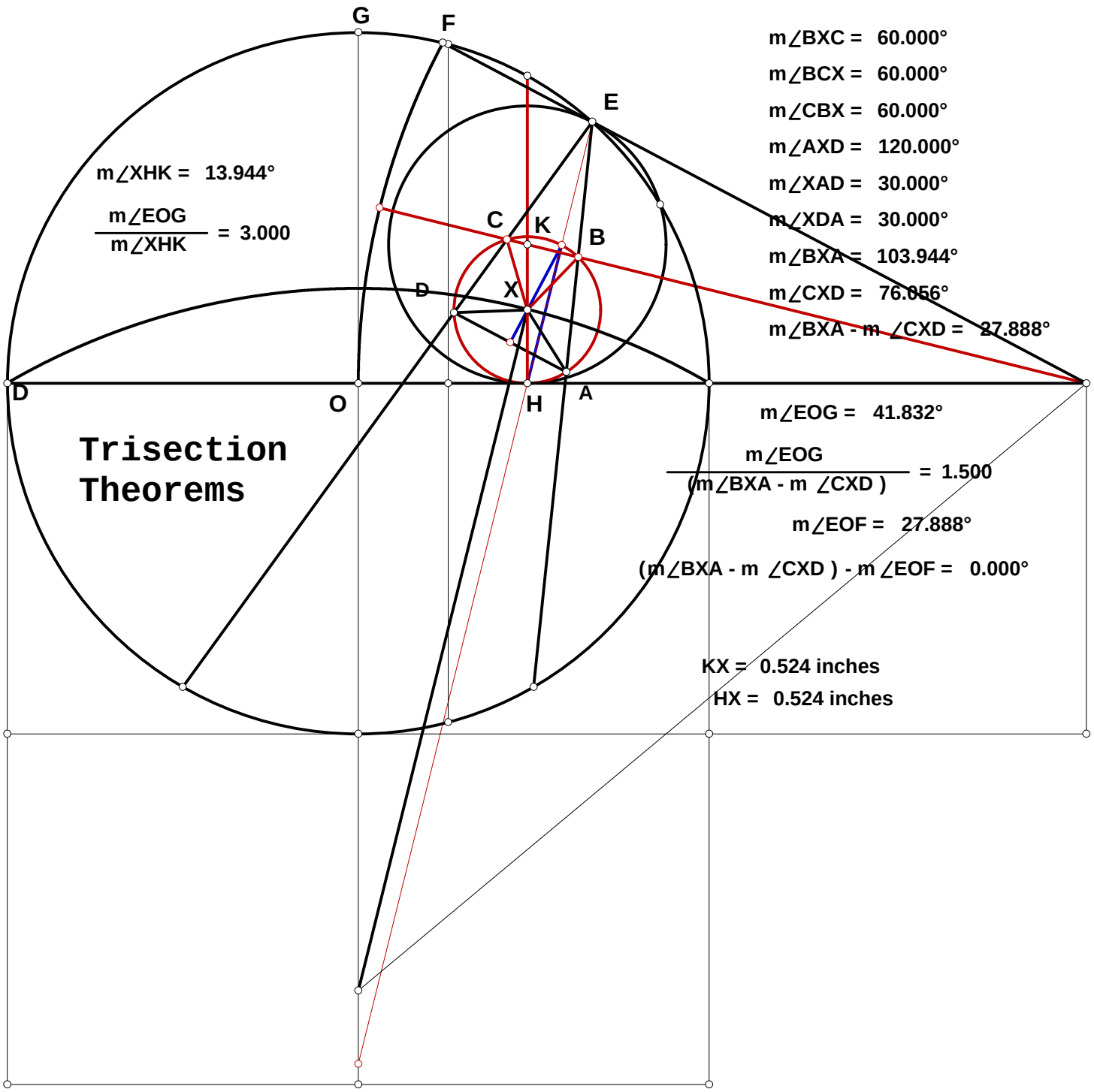
$$AG - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0$$

$$AN - (AB\cdot N+AD-AB)\cdot\frac{AD}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)} = 0$$

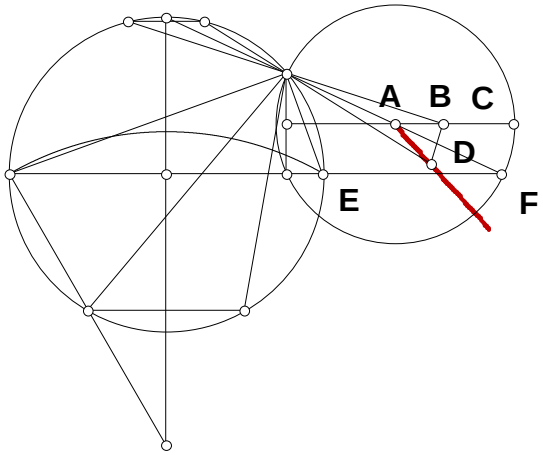
$$EN - -AD\cdot(N-1)\cdot\frac{(AD-AB)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)} = 0 \qquad FM - EN = 0$$


K
H
L
J
A

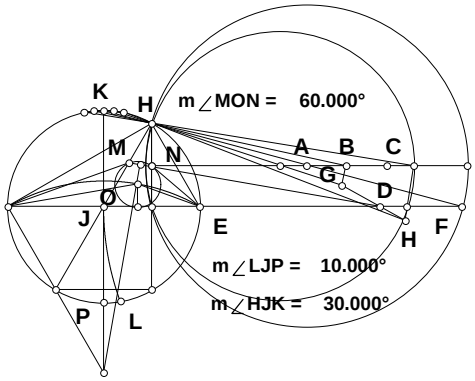

NA = 1.770 inches
m∠LMH = 19.179°
m∠AHP = 19.179°
120


G
F
E
C
K
B
D
X
D
O
H
A
Trisection
Theorems
m∠BXC = 60.000°
m∠BCX = 60.000°
m∠CBX = 60.000°
m∠AXD = 120.000°
m∠XAD = 30.000°
m∠XDA = 30.000°
m∠BXA = 103.944°
m∠CXD = 76.056°
m∠BXA - m ∠CXD = 27.888°
m∠XHK = 13.944°
m∠EOG / m∠XHK = 3.000
m∠EOG = 41.832°
m∠EOG / (m∠BXA - m ∠CXD ) = 1.500
m∠EOF = 27.888°
(m∠BXA - m ∠CXD ) - m∠EOF = 0.000°
KX = 0.524 inches
HX = 0.524 inches

# 022803

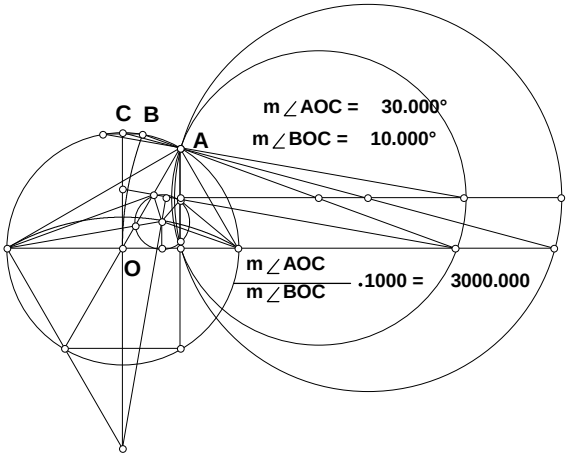



**If D were between EF, then it would be the point sought for angle trisection. When is moved between A and C the locus AD is formed. This locus is not straight, but it is fairly straight.**

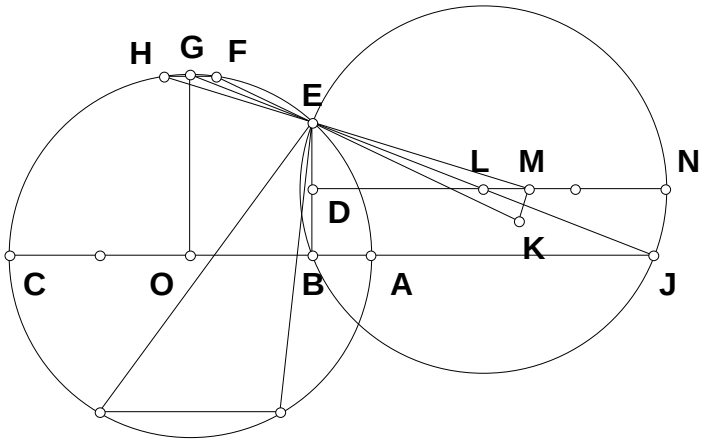



**Even if a large segment is used from two points on AD, say GH for an intercept, this drawing program claims that I have attained a trisection for any angle.**

**If a very small segment is taken, as below, tolerance is beyond the drawing program. What does it look like using Algebra? Trisection to within millionths, in some circumstances, may be tolerable.**




**As the different points of intersection is no actually viewable on the finer figure, the rougher figure will be used for drawings, but the equations will refer to the finer.**




$N_1 := 1 \qquad N_2 := 4$

$N_3 := 2$

$AC := N_1 \quad AO := \frac{AC}{2}$

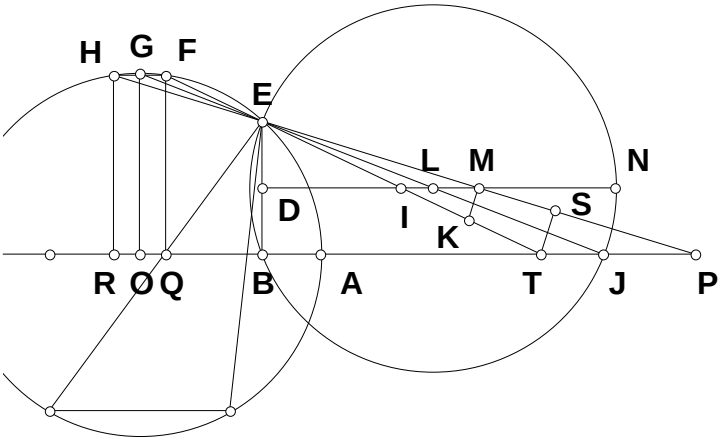
$AB := \frac{AC}{N_2} \quad BC := AC - AB$

$BE := \sqrt{AB \cdot BC} \quad DE := \frac{BE}{2}$

$BO := AO - AB \quad GO := AO$

$DL := \frac{BO \cdot DE}{GO - BE} \quad EL := \sqrt{DL^2 + DE^2}$

$LN := EL \quad LM := \frac{LN}{N_3} \quad DM := DL + LM \quad EM := \sqrt{DE^2 + DM^2} \quad EP := \frac{EM \cdot BE}{DE}$




$BP := \frac{DM \cdot BE}{DE} \quad PO := BP + BO$

$EH := \frac{AO^2 + EP^2 - PO^2}{-EP}$

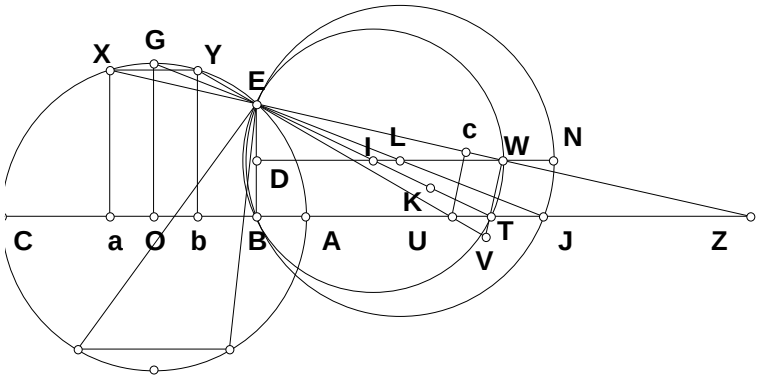
$HP := EP + EH \quad PR := \frac{BP \cdot HP}{EP}$

$OR := PR - PO \quad BQ := BO - OR$

$HR := \frac{BE \cdot HP}{EP} \quad BT := \frac{BQ \cdot BE}{HR - BE}$

$ET := \sqrt{BE^2 + BT^2} \quad PT := BP - BT \quad PS := \frac{PT^2 + EP^2 - ET^2}{2 \cdot EP} \quad ES := EP - PS \quad ST := \sqrt{ET^2 - ES^2}$

$KM := \frac{ST \cdot EM}{ES} \quad EI := \frac{ET}{2} \quad EK := \frac{ET \cdot EM}{ES} \quad OT := BT + BO$




$IW := EI \quad DI := \sqrt{EI^2 - DE^2}$

$DW := DI + IW$

$EW := \sqrt{DE^2 + DW^2}$

$EZ := \frac{EW \cdot BE}{DE} \quad BZ := \frac{DW \cdot BE}{DE}$

$OZ := BZ + BO$

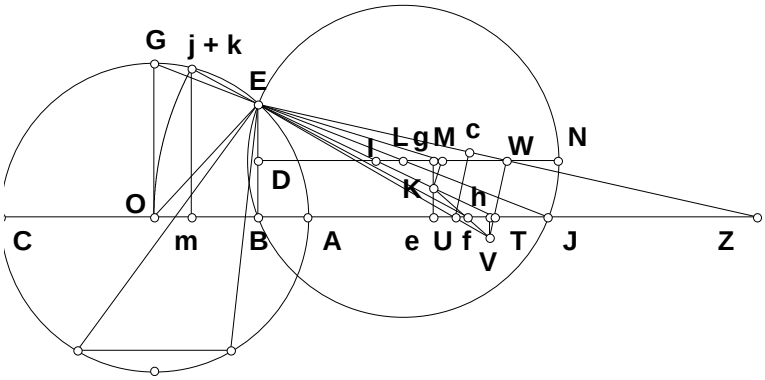
$TW := \sqrt{ET^2 - EW^2}$

$EX := \frac{AO^2 + EZ^2 - OZ^2}{-EZ} \quad XZ := EZ + EX \quad Za := \frac{BZ \cdot XZ}{EZ} \quad Oa := Za - OZ \quad Bb := BO - Oa$

$Xa := \frac{BE \cdot XZ}{EZ} \quad BU := \frac{Bb \cdot BE}{Xa - BE} \quad EU := \sqrt{BE^2 + BU^2} \quad UZ := BZ - BU \quad Zc := \frac{UZ^2 + EZ^2 - EU^2}{2 \cdot EZ}$

$Uc := \sqrt{UZ^2 - Zc^2} \quad VW := \frac{Uc \cdot EW}{EZ - Zc} \quad OU := BU + BO$

$IM := DM - DI \quad IK := EK - EI$




$Ig := \frac{IK^2 + IM^2 - KM^2}{2 \cdot IM}$

$EV := \frac{EU \cdot EW}{EZ - Zc}$

$TU := OT - OU$

$UV := EV - EU$

$TV := VW - TW$

$Uh := \frac{UV^2 + TU^2 - TV^2}{2 \cdot TU}$

$Kg := \sqrt{IK^2 - Ig^2} \quad Ke := DE - Kg \quad Vh := \sqrt{UV^2 - Uh^2} \quad Dg := DI + Ig \quad Bh := BU + Uh$

$eh := Bh - Dg \quad Ke := DE - Kg \quad ef := \frac{eh \cdot Ke}{(Ke + Vh)} \quad Bf := Dg + ef \quad Of := Bf + BO$

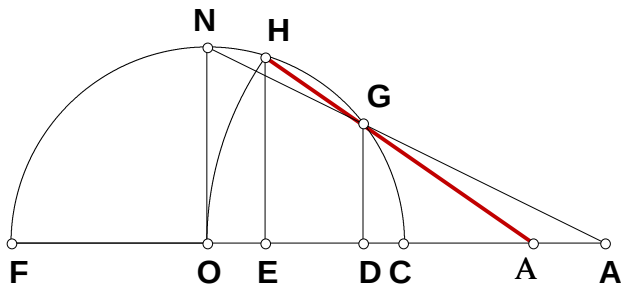
$Om := \frac{AO^2}{2 \cdot Of} \quad Ej := \frac{AO^2}{Of} \quad Om - \frac{Ej}{2} = 0 \quad Ef := \sqrt{BE^2 + Bf^2} \quad Ek := \frac{AO^2 + Ef^2 - Of^2}{-Ef}$

$Om - \frac{Ek}{2} = -0.000000000002639 \quad \frac{Ek}{Ej} = 1.000000000030396$

**And so the Trisection is accurate to within a few decimal places. 30 degree angle shown.**

# The Gravitating Answer. 030503

**How many itterations to go beyond 15 decimal place precision in trisection? The itteration is from AO where GH determines a new A.**




$\Delta := 22 \qquad \delta := 0 .. \Delta \qquad N := 2$

$CF := 1 \qquad CO := \frac{CF}{2}$

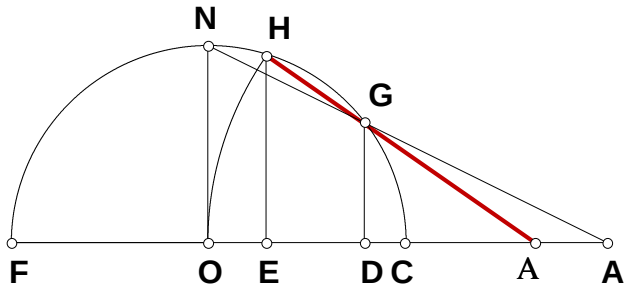
$CD := \frac{CO}{N} \qquad DO := CO - CD$

$DF := DO + CO \qquad DG := \sqrt{CD \cdot DF}$

$AD_0 := \frac{DO \cdot DG}{CO - DG} \qquad AO_0 := AD_0 + DO \qquad EO_0 := \frac{CO^2}{2 \cdot (AD_0 + DO)} \qquad DE_0 := DO - EO_0$

$EH_0 := \sqrt{(CO + EO_0) \cdot (CO - EO_0)}$

$$\begin{bmatrix} AD_{\delta+1} \\ AO_{\delta+1} \\ EO_{\delta+1} \\ DE_{\delta+1} \\ EF_{\delta+1} \\ EH_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{DE_\delta \cdot DG}{EH_\delta - DG} \\ \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \\ \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ DO - \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ CO + \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ \sqrt{\left[ CO - \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right] \cdot \left[ CO + \frac{CO^2}{2 \cdot \left( \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right]} \end{bmatrix}$$




$$AG := \sqrt{DG^2 + \left(AD_{\Delta}\right)^2}$$

$$GH := \frac{CO^2 + AG^2 - \left(AO_{\Delta}\right)^2}{-AG}$$

$$EO_{\Delta} - \frac{GH}{2} = 0$$

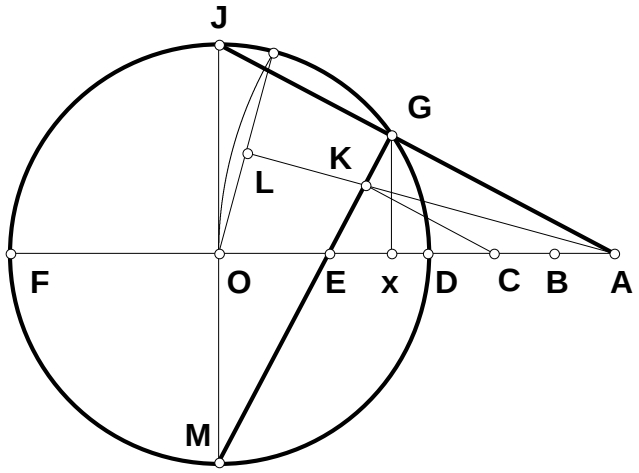
**The displayed precision is for 15 decimal places. Trisection is beyond that. Since the physical world is quantitized, physical trisection is possible.**

| $EO_{\delta} - \frac{GH}{2}$ | $\delta$ |
|---|---|
| -0.019836790725685 | 0 |
| -0.004498504834066 | 1 |
| -0.001019750164038 | 2 |
| -0.000231158851096 | 3 |
| -0.000052399460482 | 4 |
| -0.000011877993414 | 5 |
| -0.000002692522516 | 6 |
| -0.000000610345304 | 7 |
| -0.000000138354048 | 8 |
| -0.000000031362317 | 9 |
| -0.000000007109260 | 10 |
| -0.000000001611539 | 11 |
| -0.000000000365306 | 12 |
| -0.000000000082808 | 13 |
| -0.000000000018771 | 14 |
| -0.000000000004255 | 15 |
| -0.000000000000965 | 16 |
| -0.000000000000219 | 17 |
| -0.000000000000050 | 18 |
| -0.000000000000011 | 19 |
| $-2.789435349370706 \cdot 10^{-15}$ | 20 |
| 0 | 21 |
| 0 | 22 |

**One can see how rapidly each recursion increases precision. And so, for any required precision, one can trisect an angle grearter than that, relatively rapidly--especially if one combine yesterdays plate with today's.**

# 030803

**Is anything saved by starting from a much more precise point for itteration demonstrated in 0305?**




$DF := 1 \qquad DO := \frac{DF}{2}$

$N := 2 \qquad Dx := \frac{DO}{N}$

$Ox := DO - Dx \qquad Fx := DF - Dx$

$Gx := \sqrt{Dx \cdot Fx} \qquad AO := \frac{Ox \cdot DO}{DO - Gx}$

$HO := DO \qquad LO := \frac{HO}{2}$

$Ex := \frac{Ox \cdot Gx}{Gx + DO} \qquad Ob := \frac{LO^2}{AO}$

$Lb := \sqrt{LO^2 - Ob^2} \qquad ab := \frac{Ex \cdot Lb}{Gx}$

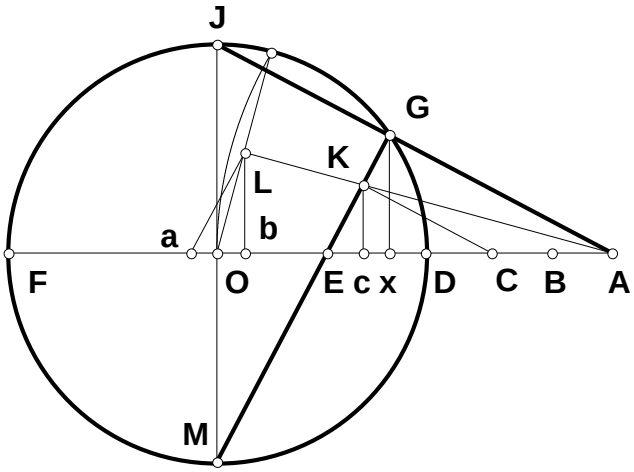
$Aa := AO + ab - Ob$

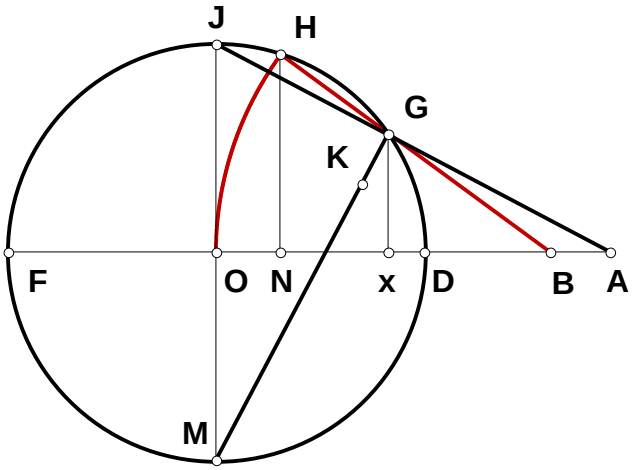
$Ax := AO - Ox \qquad AE := Ax + Ex$

$Kc := \frac{Lb \cdot AE}{Aa} \qquad Cc := \frac{AO \cdot Kc}{DO}$

$Ec := \frac{ab \cdot AE}{Aa} \qquad CE := Cc + Ec \qquad AC := AE - CE$

$AB := \frac{AC}{2}$






$$\Delta := 22 \qquad \delta := 0 .. \Delta$$

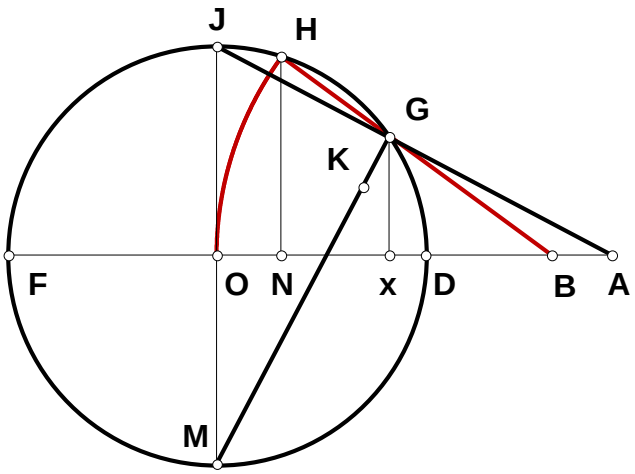
$$BO_0 := AO - AB$$

$$NO_0 := \frac{DO^2}{2\ BO_0}$$

$$Nx_0 := Ox - NO_0$$

$$HN_0 := \sqrt{\left(DO + NO_0\right) \cdot \left(DO - NO_0\right)}$$

$$\begin{bmatrix} BO_{\delta+1} \\ NO_{\delta+1} \\ Nx_{\delta+1} \\ HN_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta \\ \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ Ox - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ \sqrt{\left[DO + \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right] \cdot \left[DO - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]} \end{bmatrix}$$




$$Bx_{\Delta} := BO_{\Delta} - Ox$$

$$BG := \sqrt{Gx^2 + \left(Bx_{\Delta}\right)^2}$$

$$GH := \frac{DO^2 + BG^2 - \left(BO_{\Delta}\right)^2}{-BG}$$

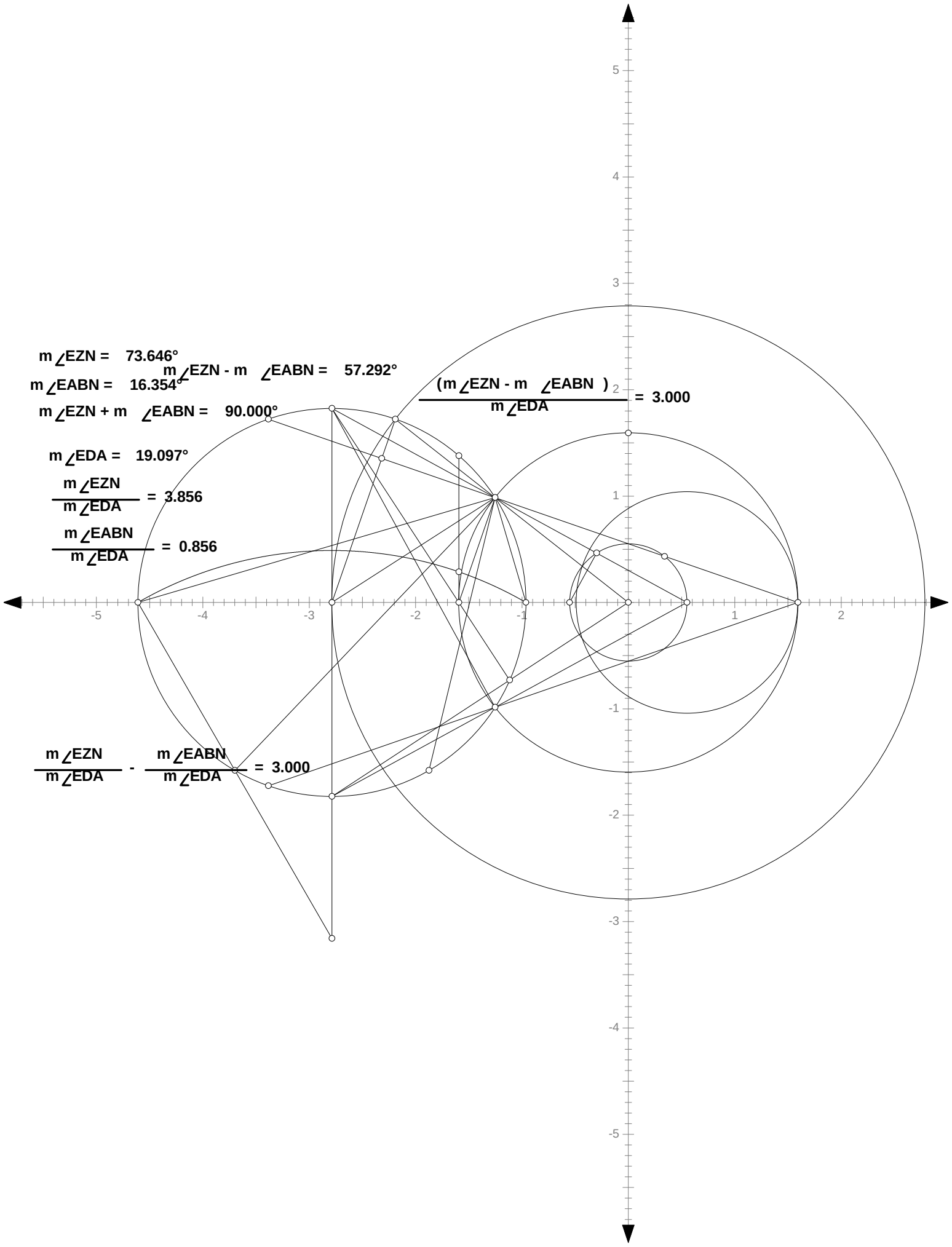
$$NO_{\Delta} - \frac{GH}{2} = 0$$

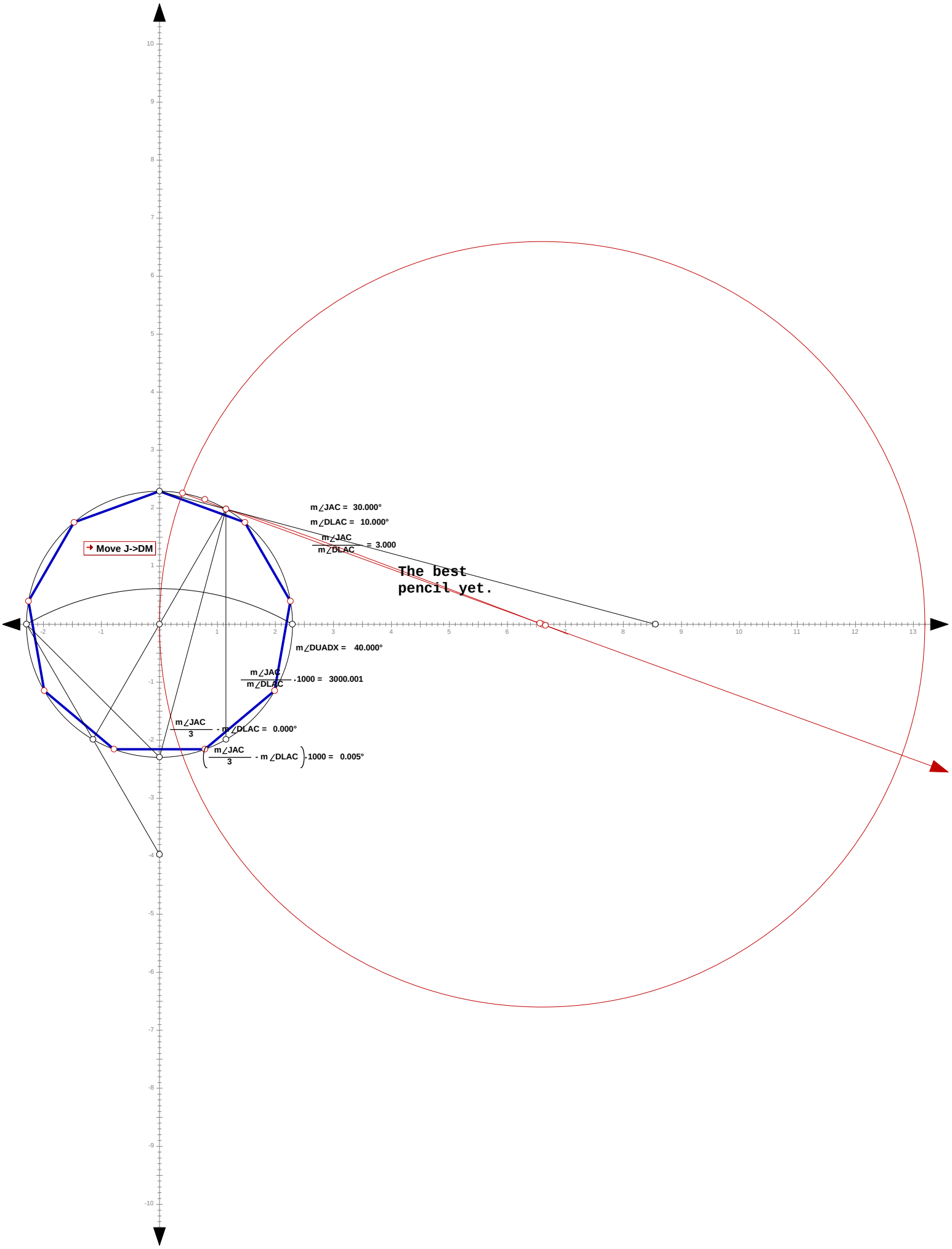
$NO_{\delta} - \frac{GH}{2}$

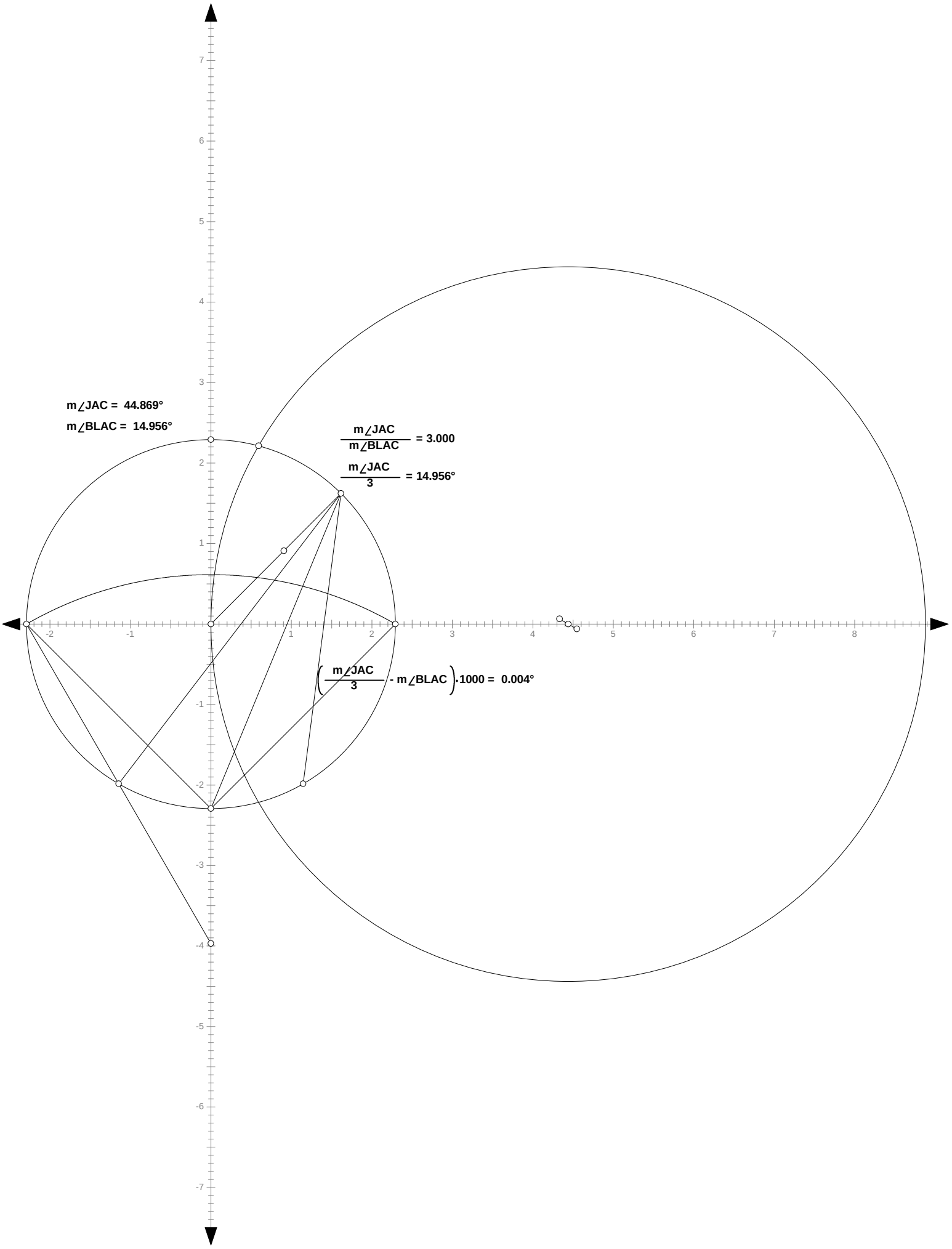
| $\delta$ |
|---|
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |
| 15 |
| 16 |
| 17 |
| 18 |
| 19 |
| 20 |
| 21 |
| 22 |

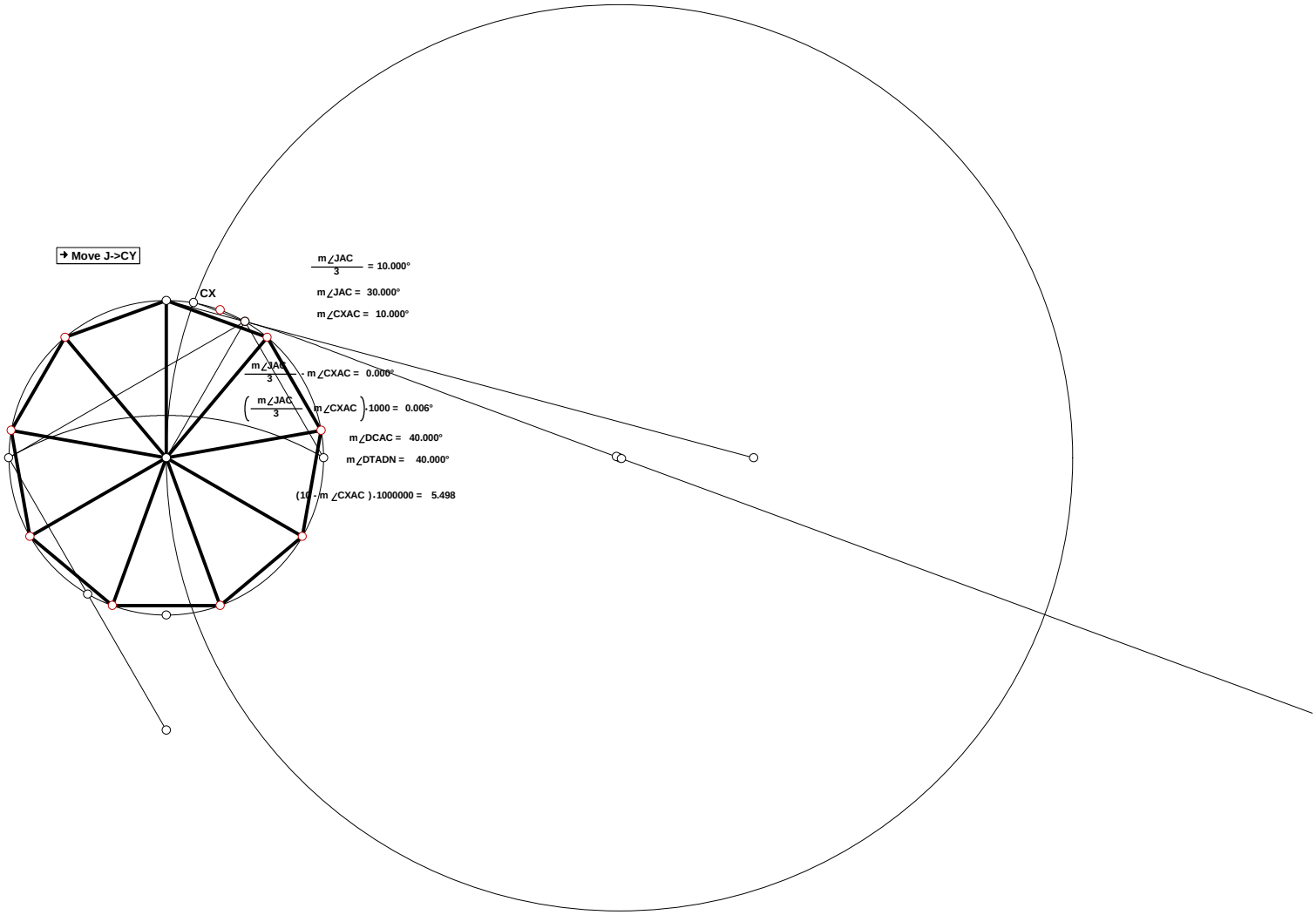
| $NO_{\delta} - \frac{GH}{2}$ |
|---|
| $-0.000733072475044$ |
| $-0.000166174134206$ |
| $-0.000037668619212$ |
| $-0.000008538782769$ |
| $-0.000001935584914$ |
| $-0.000000438761479$ |
| $-0.000000099459153$ |
| $-0.000000022545559$ |
| $-0.000000005110663$ |
| $-0.000000001158493$ |
| $-0.000000000262609$ |
| $-0.000000000059529$ |
| $-0.000000000013494$ |
| $-0.000000000003059$ |
| $-0.000000000000693$ |
| $-0.000000000000157$ |
| $-0.000000000000035$ |
| $-7.965850201685498 \cdot 10^{-15}$ |
| $-1.706967900361178 \cdot 10^{-15}$ |
| 0 |
| 0 |
| 0 |
| 0 |

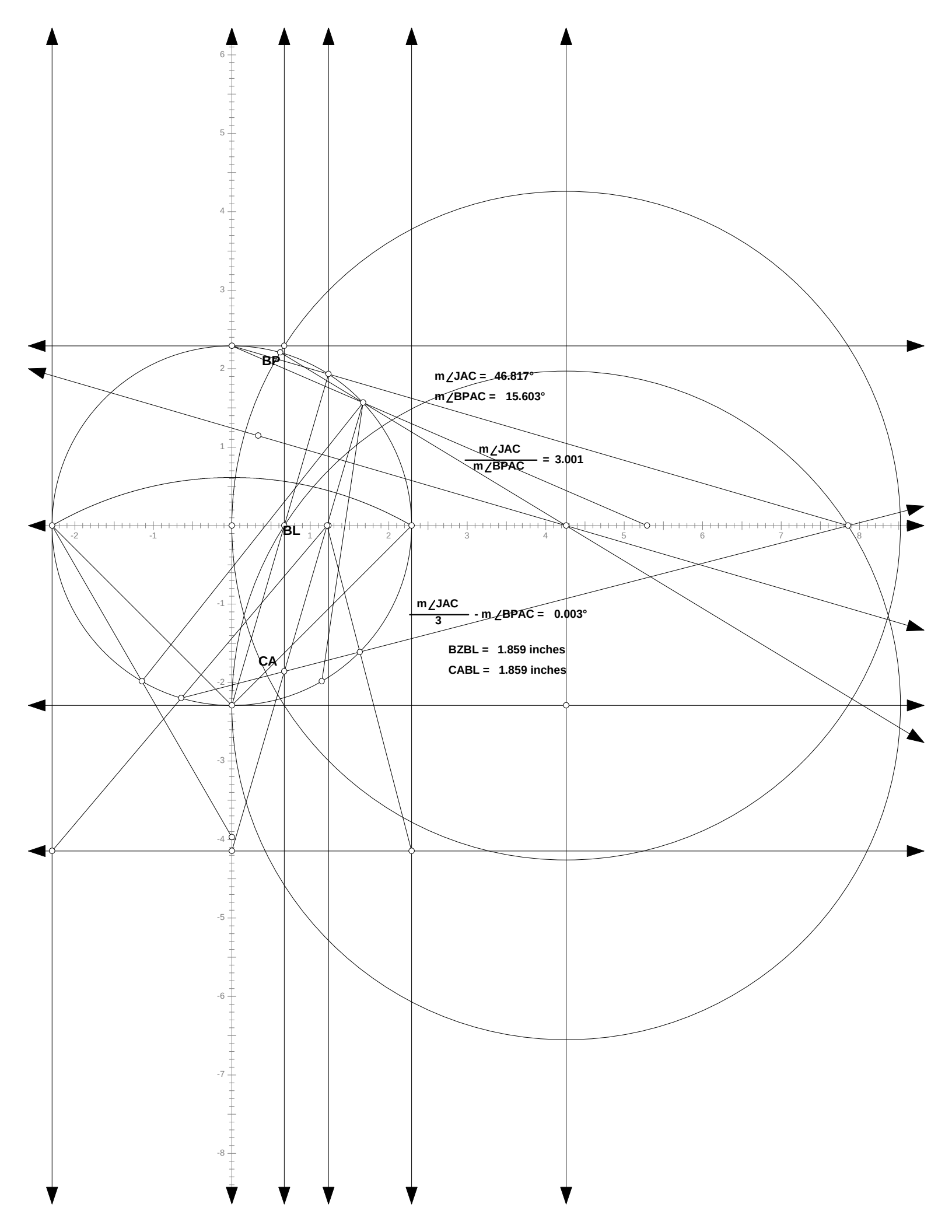
**Although one starts off in a more precise spot, not much in the way of steps for 15 decimal place precision is saved. The steps are a waste of time.**

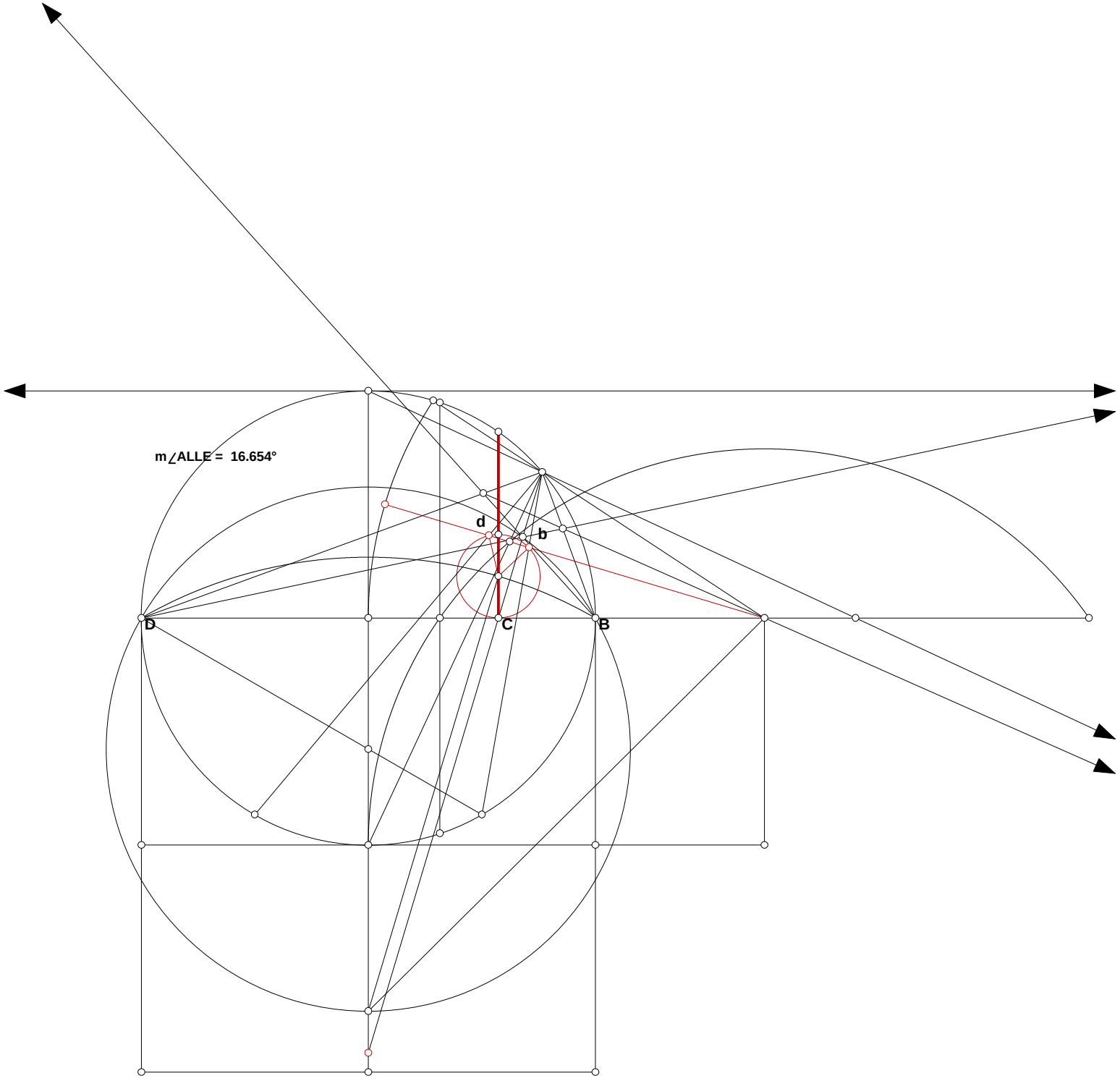

m∠EZN = 73.646°
m∠EABN = 16.354°
m∠EZN + m ∠EABN = 90.000°
m∠EZN - m ∠EABN = 57.292°
(m∠EZN - m ∠EABN ) / m∠EDA = 3.000
m∠EDA = 19.097°
m∠EZN / m∠EDA = 3.856
m∠EABN / m∠EDA = 0.856
m∠EZN / m∠EDA - m∠EABN / m∠EDA = 3.000
5
4
3
2
1
-1
-2
-3
-4
-5
-5
-4
-3
-2
-1
1
2

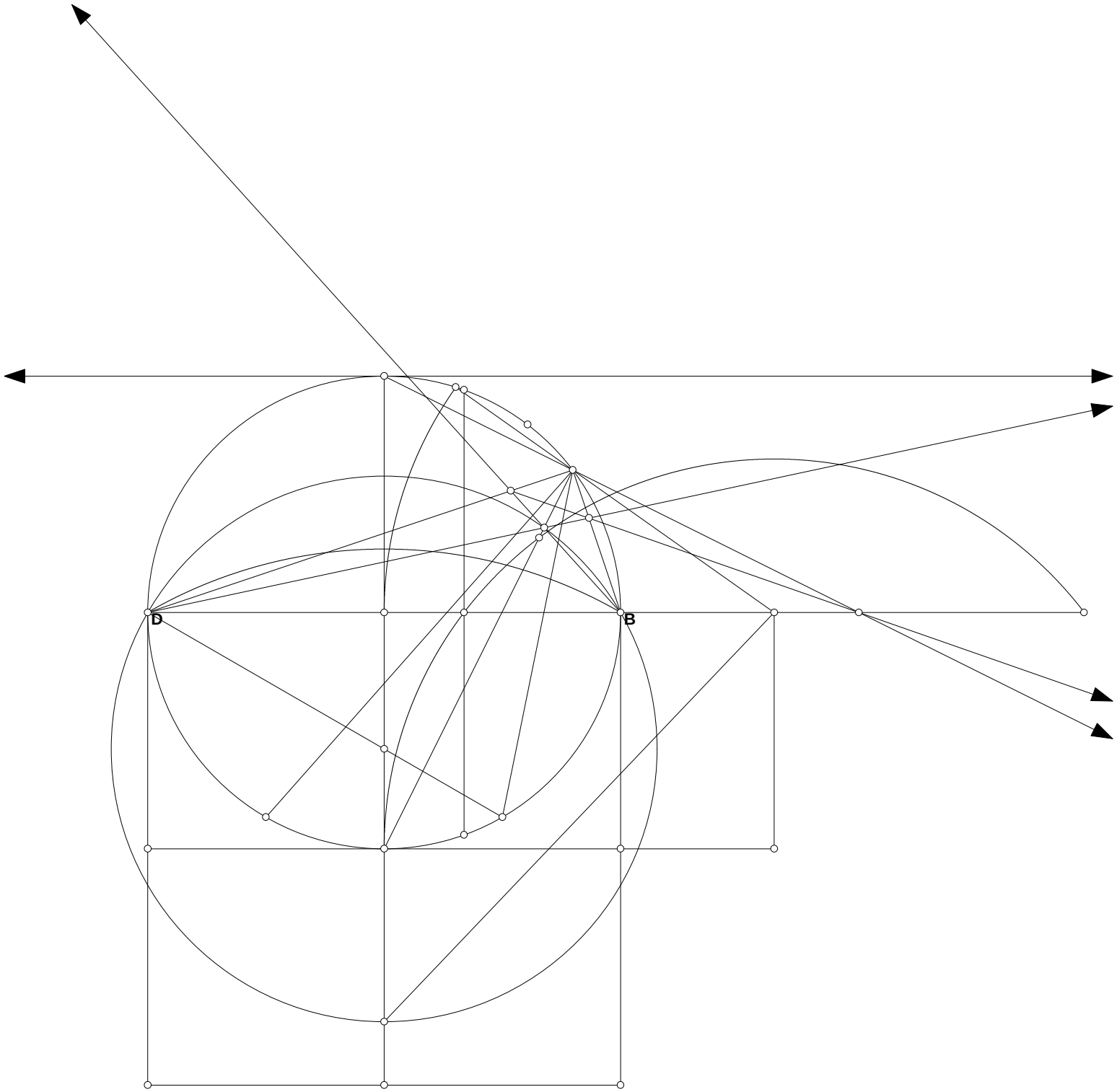

Move J->DM
m∠JAC = 30.000°
m∠DLAC = 10.000°
m∠JAC / m∠DLAC = 3.000
The best pencil yet.
m∠DUADX = 40.000°
m∠JAC / m∠DLAC ·1000 = 3000.001
m∠JAC / 3 - m∠DLAC = 0.000°
( m∠JAC / 3 - m∠DLAC ) ·1000 = 0.005°

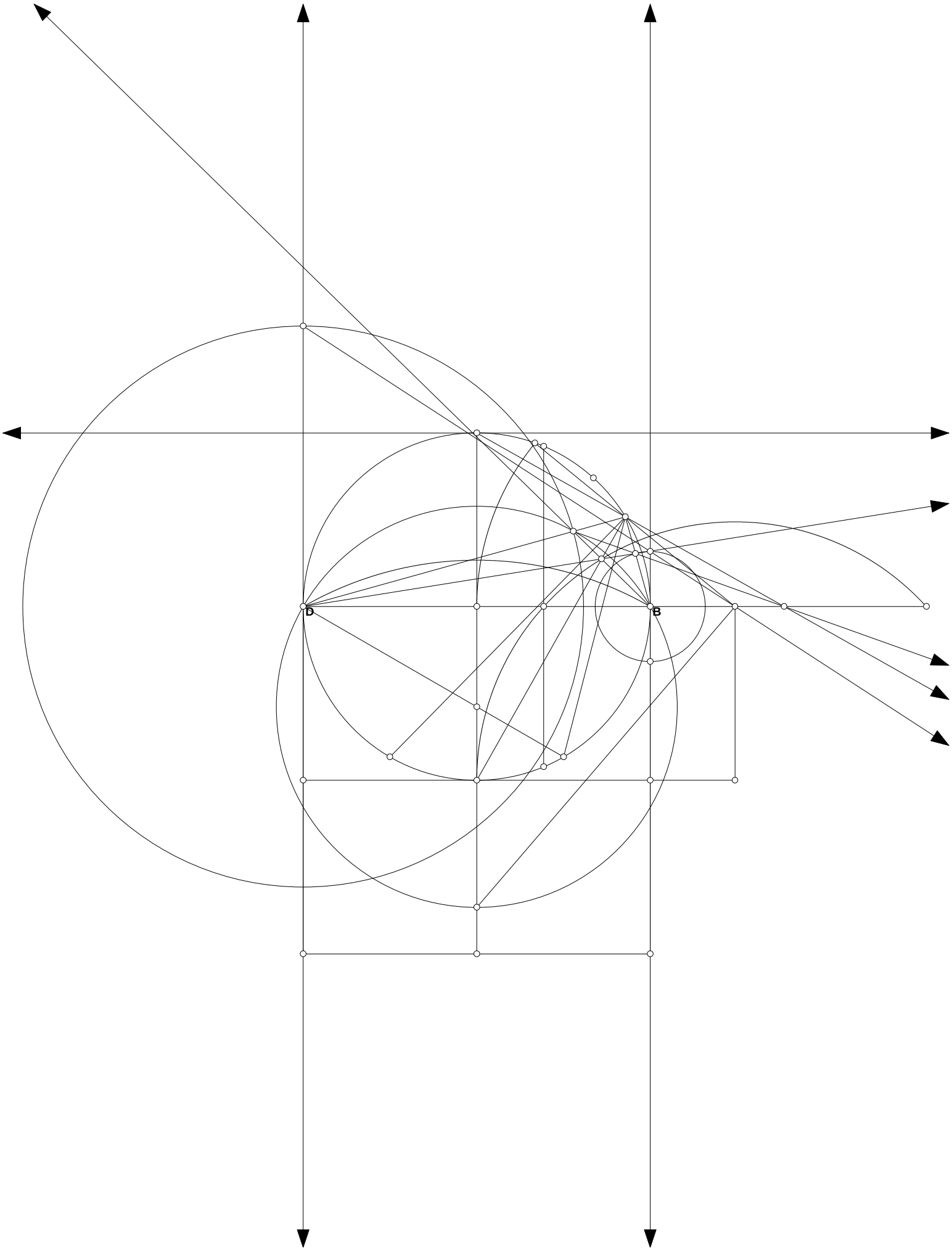

m∠JAC = 44.869°
m∠BLAC = 14.956°
m∠JAC / m∠BLAC = 3.000
m∠JAC / 3 = 14.956°
(m∠JAC / 3 - m∠BLAC)·1000 = 0.004°

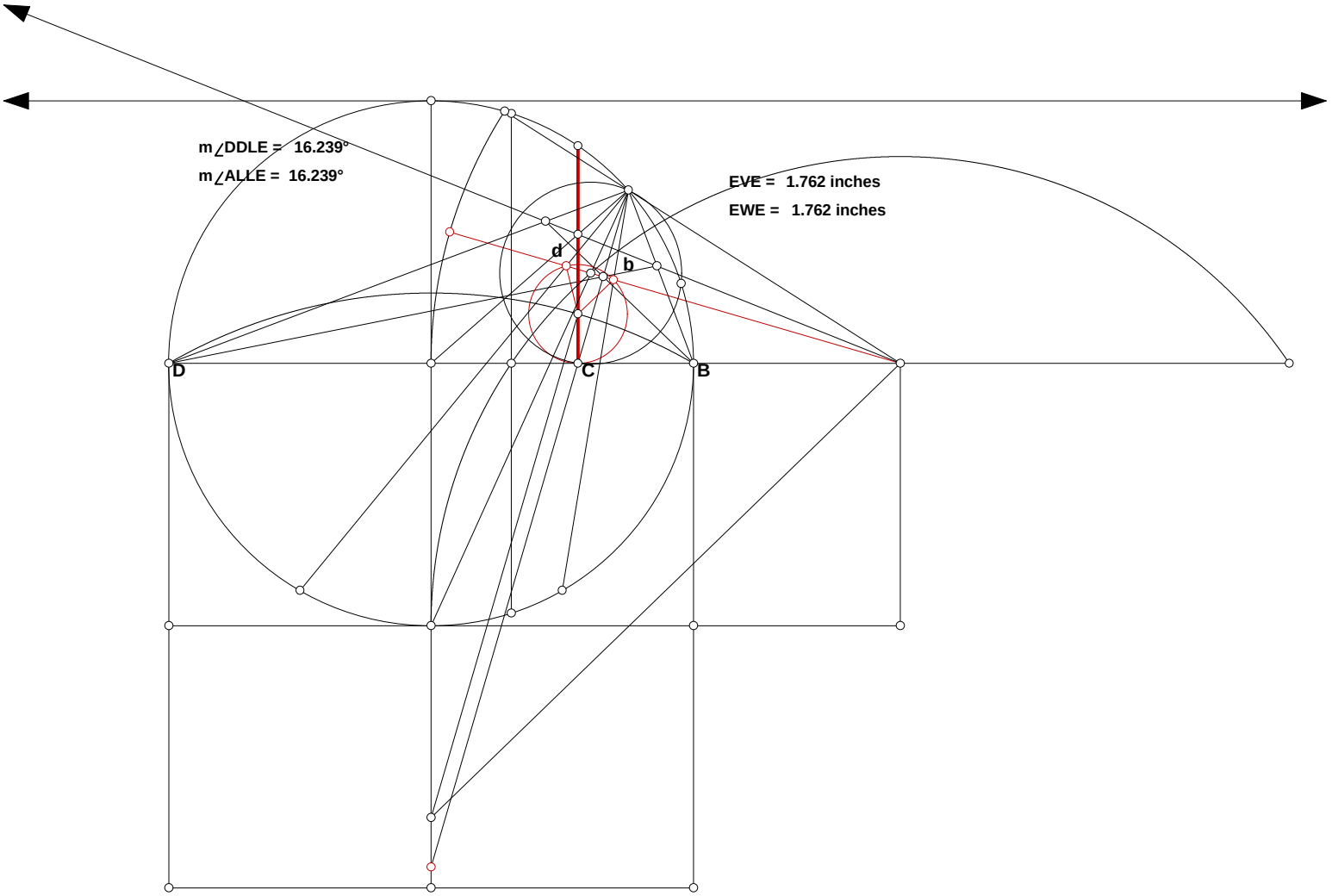

Move J->CY
CX
$\frac{m\angle JAC}{3} = 10.000°$
$m\angle JAC = 30.000°$
$m\angle CXAC = 10.000°$
$\frac{m\angle JAC}{3} - m\angle CXAC = 0.000°$
$\left(\frac{m\angle JAC}{3} - m\angle CXAC\right) \cdot 1000 = 0.006°$
$m\angle DCAC = 40.000°$
$m\angle DTADN = 40.000°$
$(10 - m\angle CXAC) \cdot 1000000 = 5.498$

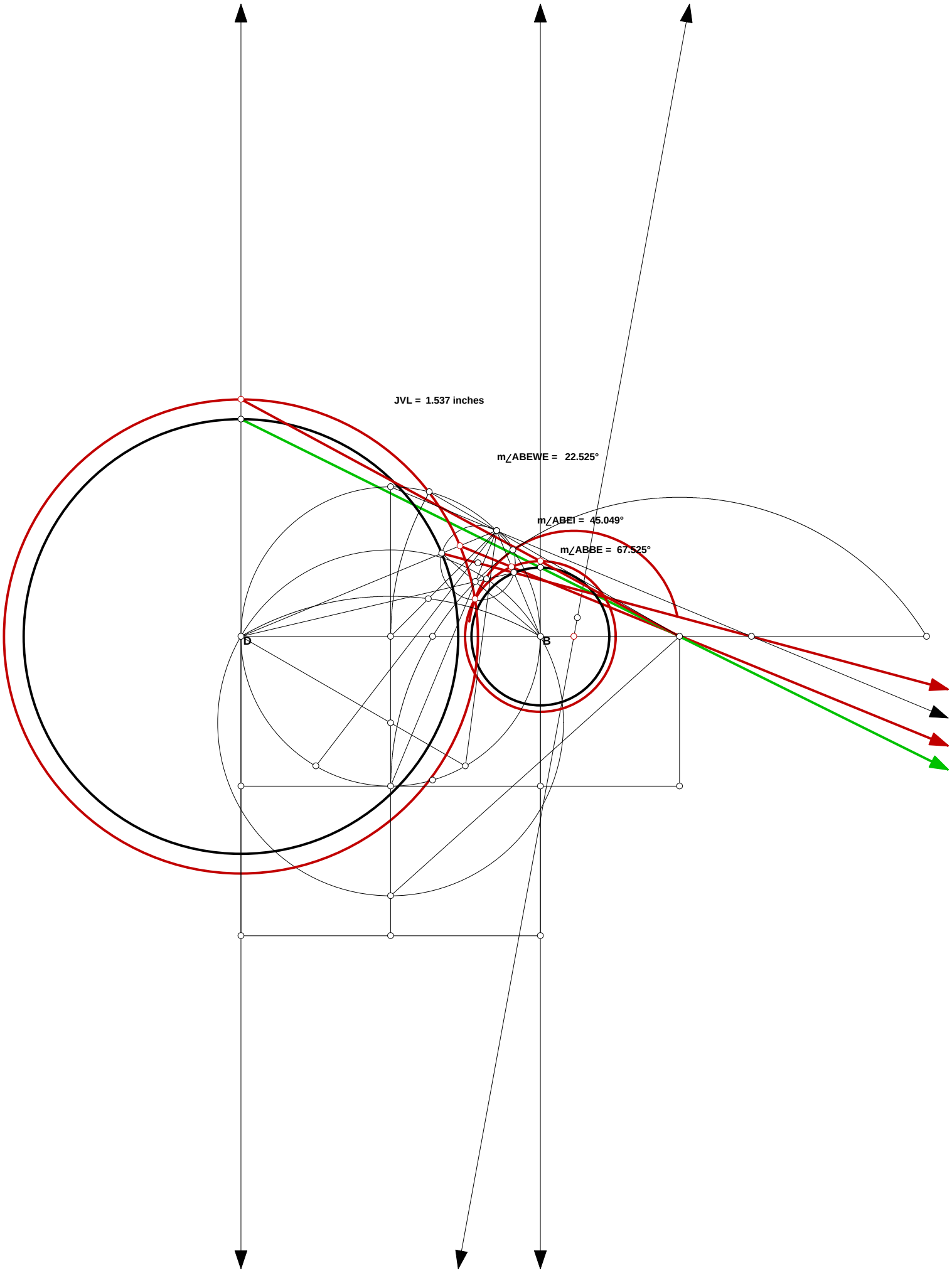

BP
BL
CA
m∠JAC = 46.817°
m∠BPAC = 15.603°
m∠JAC / m∠BPAC = 3.001
m∠JAC / 3 - m∠BPAC = 0.003°
BZBL = 1.859 inches
CABL = 1.859 inches

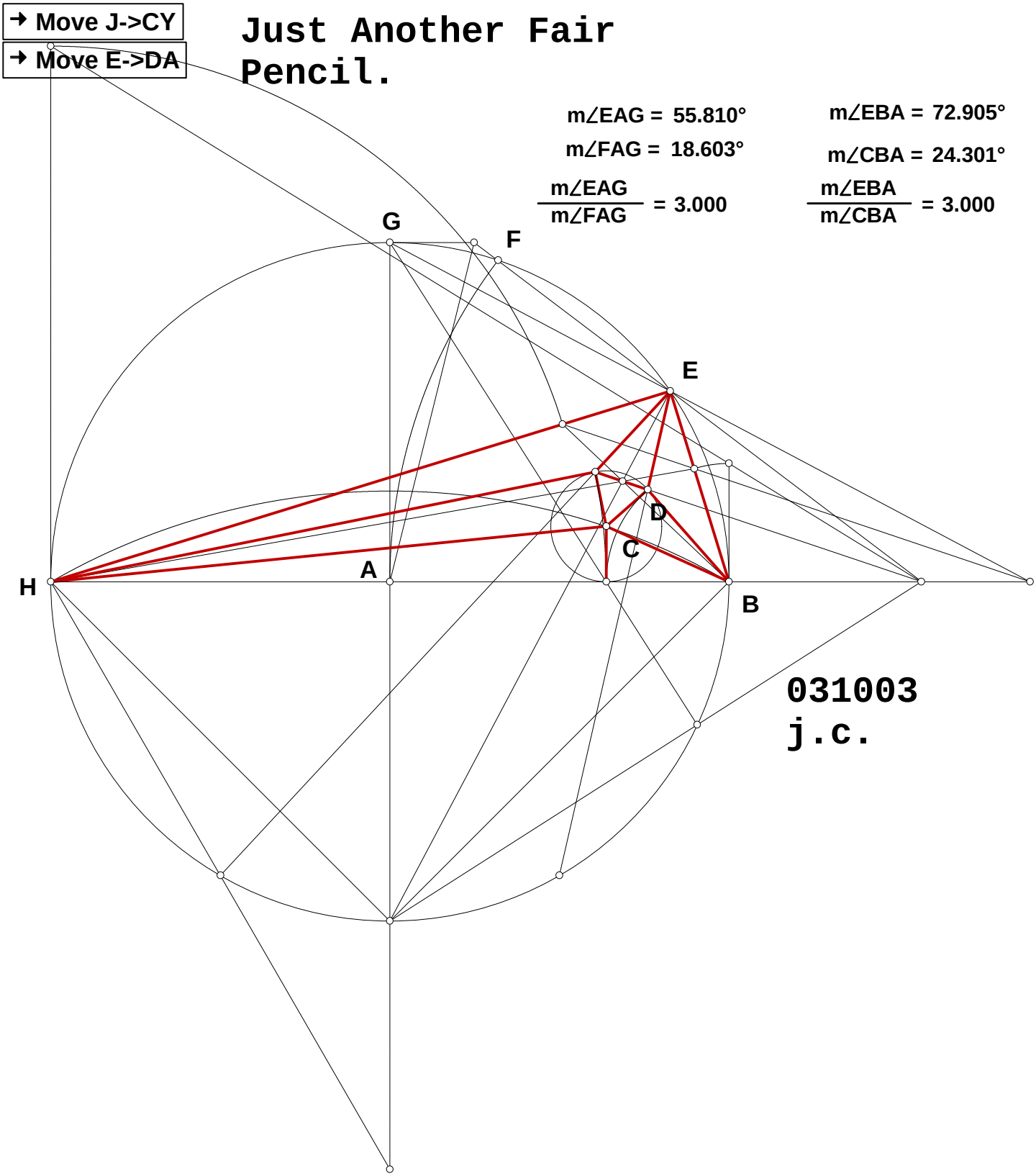

m∠ALLE = 16.654°
d
b
D
C
B

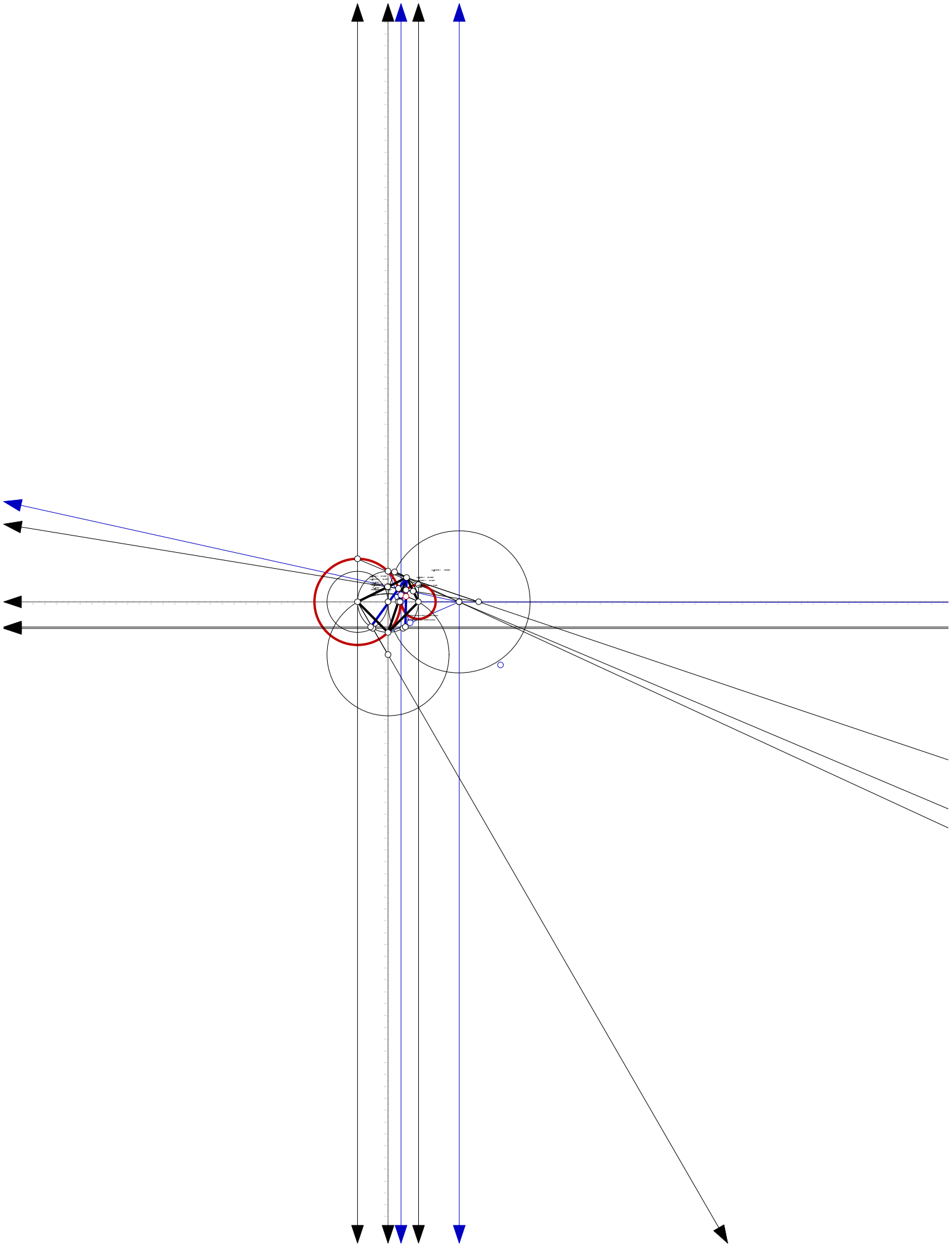

D
B

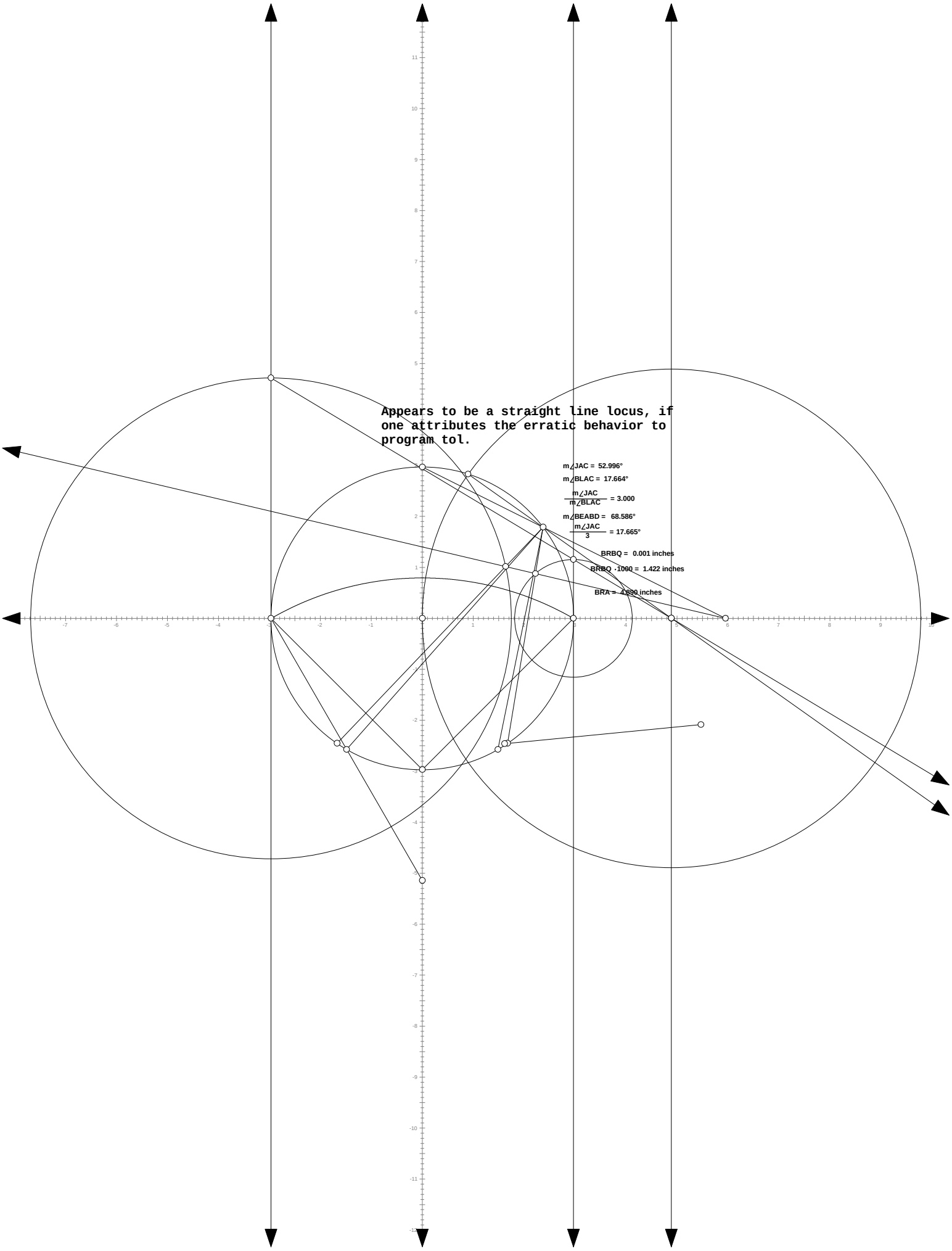

D
B

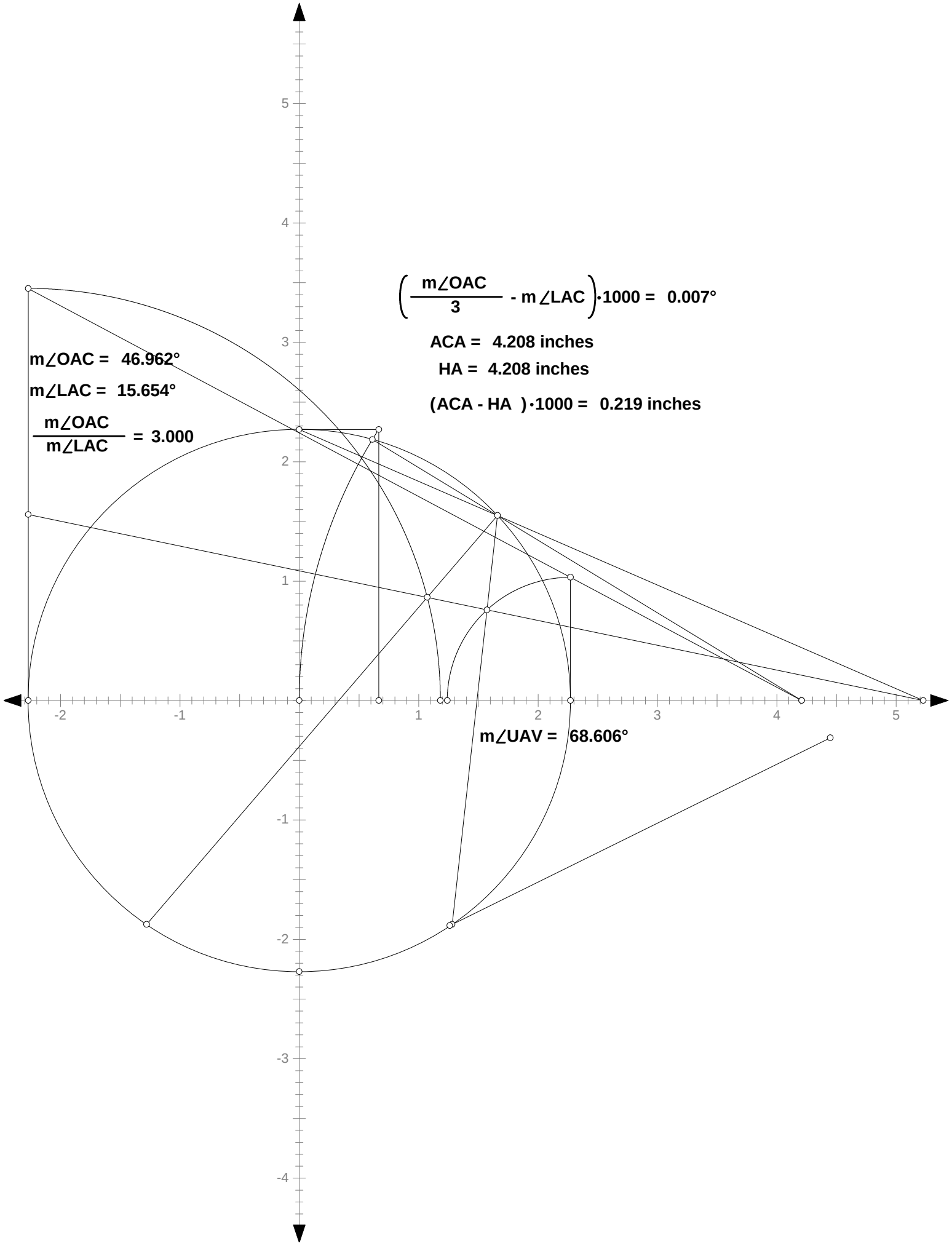

m∠DDLE = 16.239°
m∠ALLE = 16.239°
EVE = 1.762 inches
EWE = 1.762 inches
d
b
D
C
B

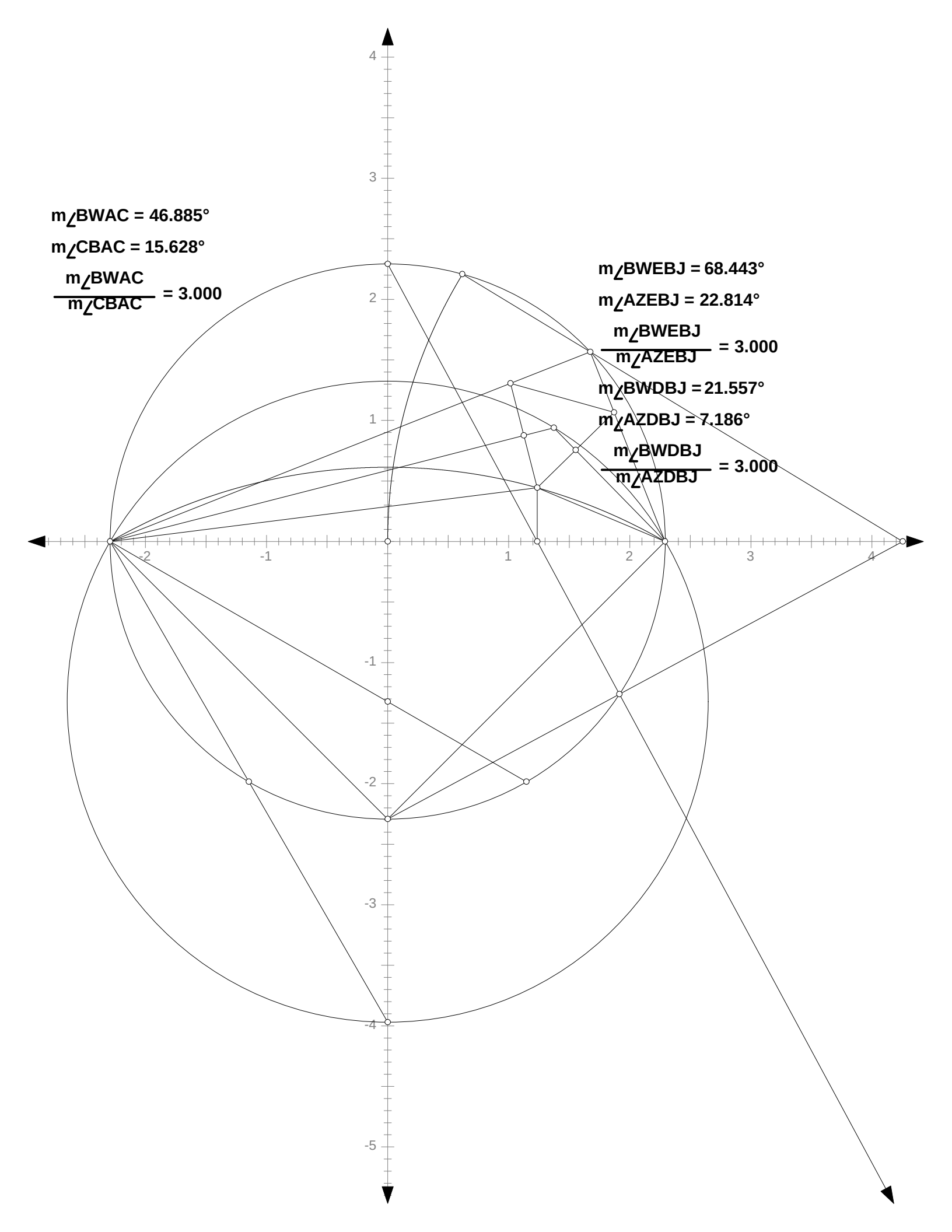

JVL = 1.537 inches
m∠ABEWE = 22.525°
m∠ABEI = 45.049°
m∠ABBE = 67.525°
D
B

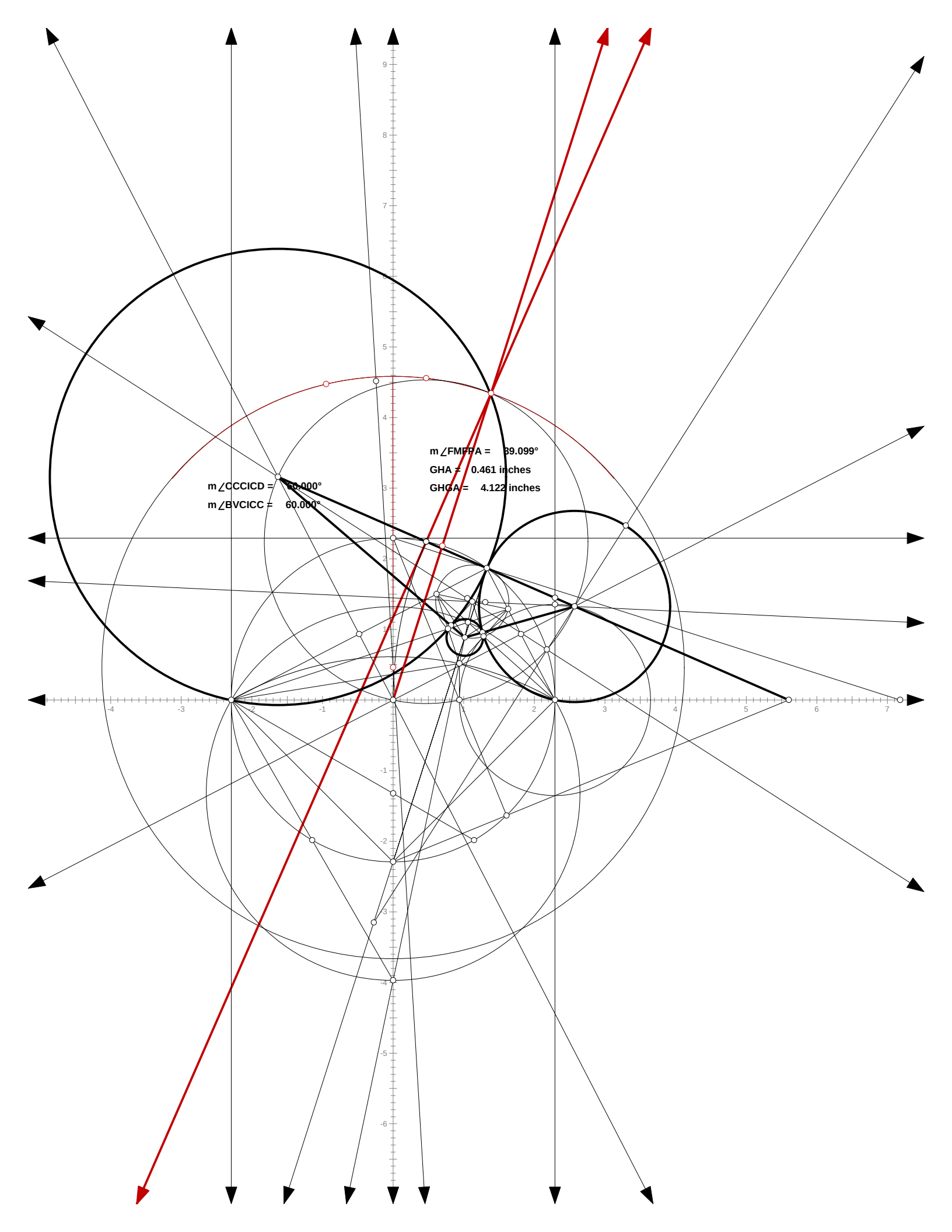

Move J->CY
Move E->DA
Just Another Fair Pencil.
m∠EAG = 55.810°
m∠FAG = 18.603°
m∠EAG / m∠FAG = 3.000
m∠EBA = 72.905°
m∠CBA = 24.301°
m∠EBA / m∠CBA = 3.000
G
F
E
D
C
A
H
B
031003
j.c.

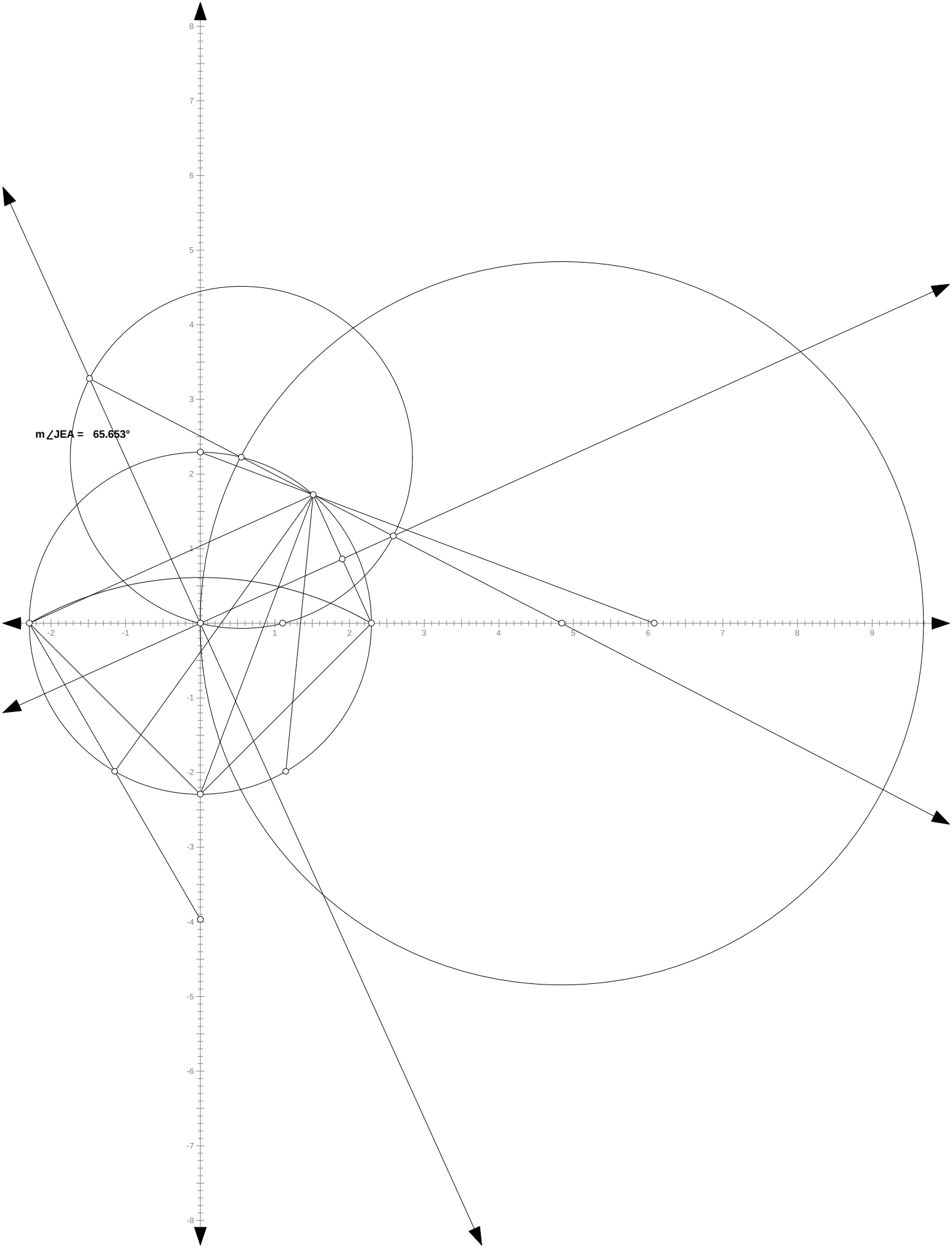


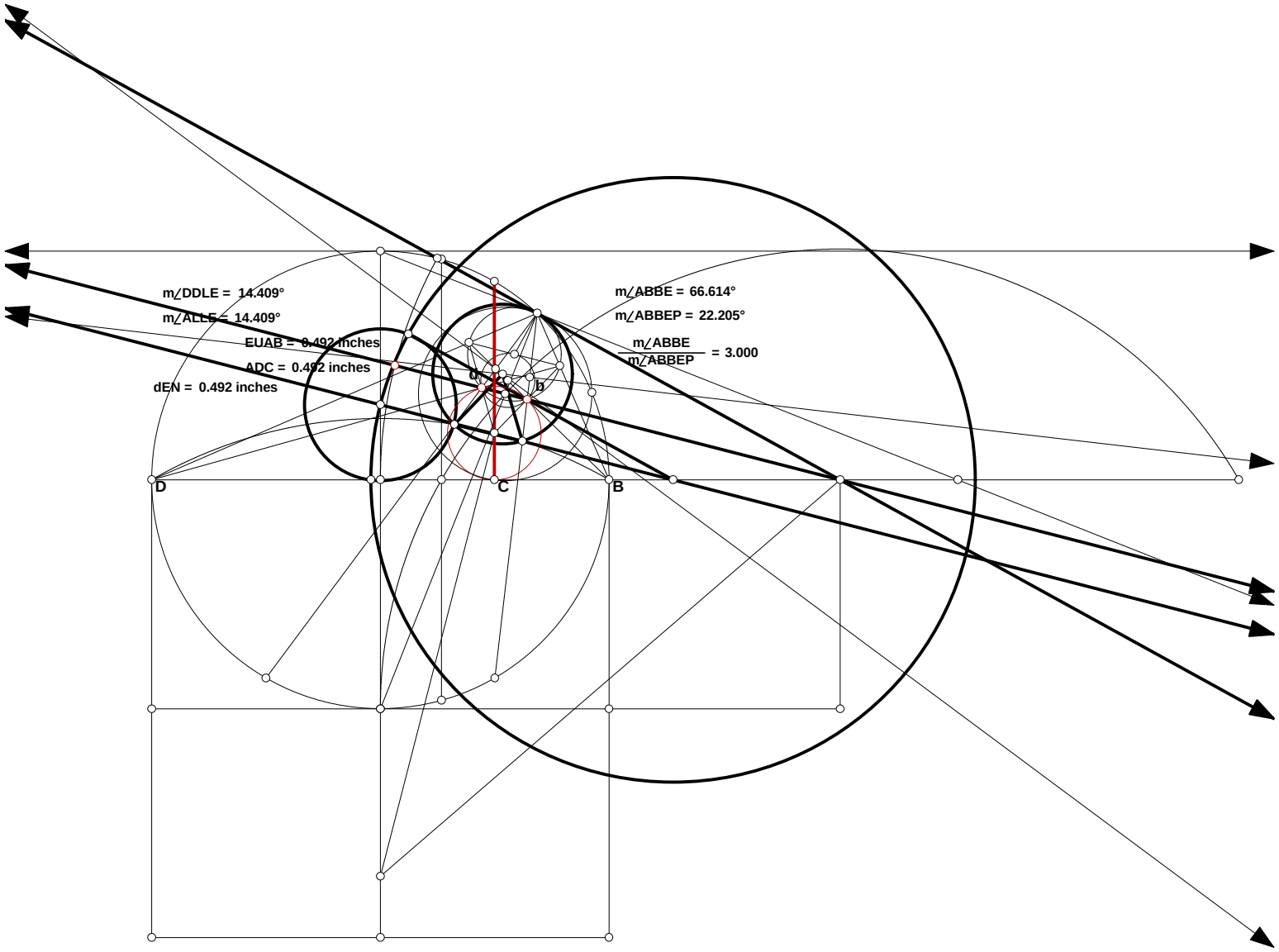

Appears to be a straight line locus, if one attributes the erratic behavior to program tol.
m∠JAC = 52.996°
m∠BLAC = 17.664°
m∠JAC / m∠BLAC = 3.000
m∠BEABD = 68.586°
m∠JAC / 3 = 17.665°
BRBQ = 0.001 inches
BRBQ ·1000 = 1.422 inches
BRA = 4.890 inches

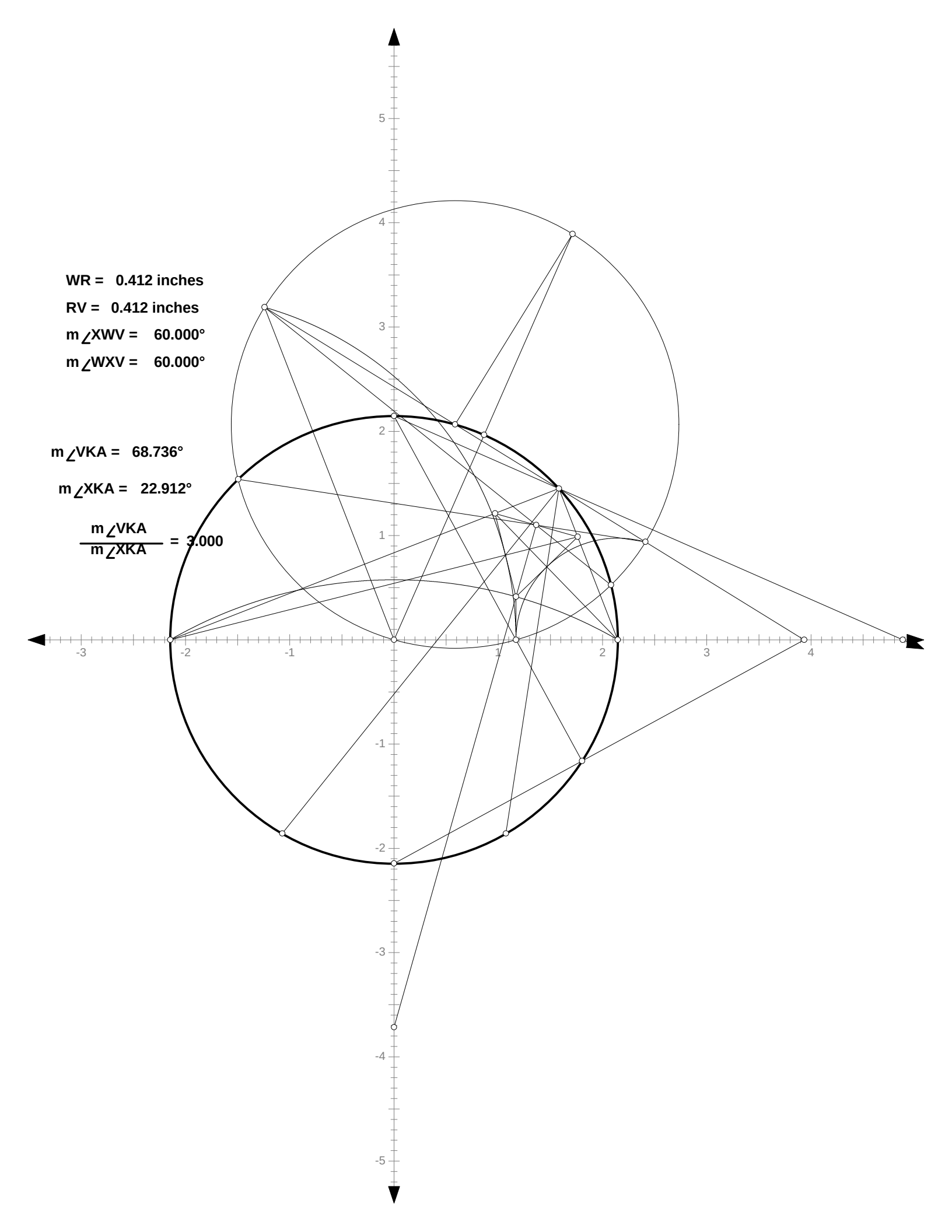

$\left(\frac{m\angle OAC}{3} - m\angle LAC\right)\cdot 1000 = 0.007°$
ACA = 4.208 inches
HA = 4.208 inches
(ACA - HA )·1000 = 0.219 inches
m∠OAC = 46.962°
m∠LAC = 15.654°
$\frac{m\angle OAC}{m\angle LAC} = 3.000$
m∠UAV = 68.606°

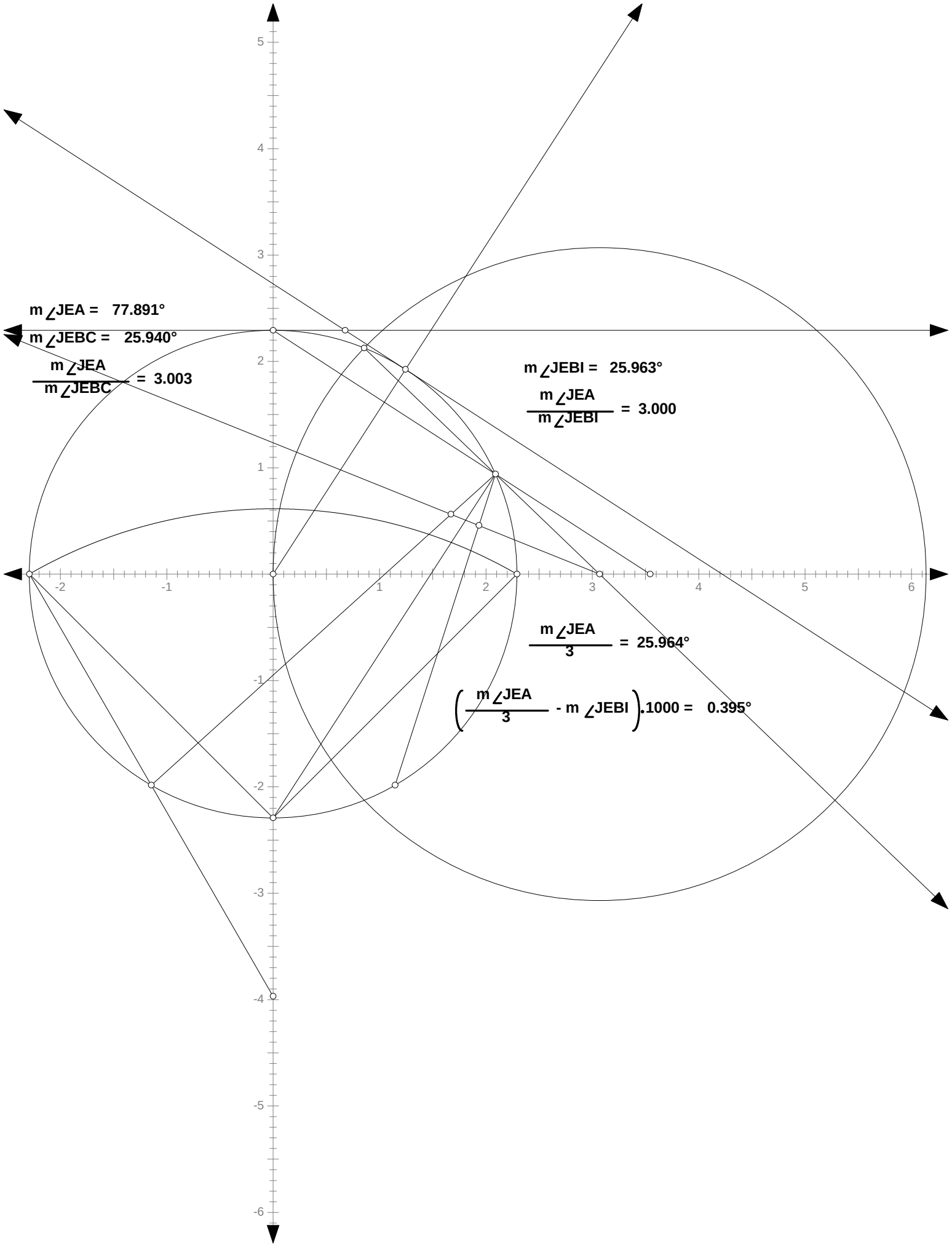

m∠BWAC = 46.885°
m∠CBAC = 15.628°
m∠BWAC / m∠CBAC = 3.000
m∠BWEBJ = 68.443°
m∠AZEBJ = 22.814°
m∠BWEBJ / m∠AZEBJ = 3.000
m∠BWDBJ = 21.557°
m∠AZDBJ = 7.186°
m∠BWDBJ / m∠AZDBJ = 3.000
4
3
2
1
-1
-2
-3
-4
-5
-2
-1
1
2
3
4

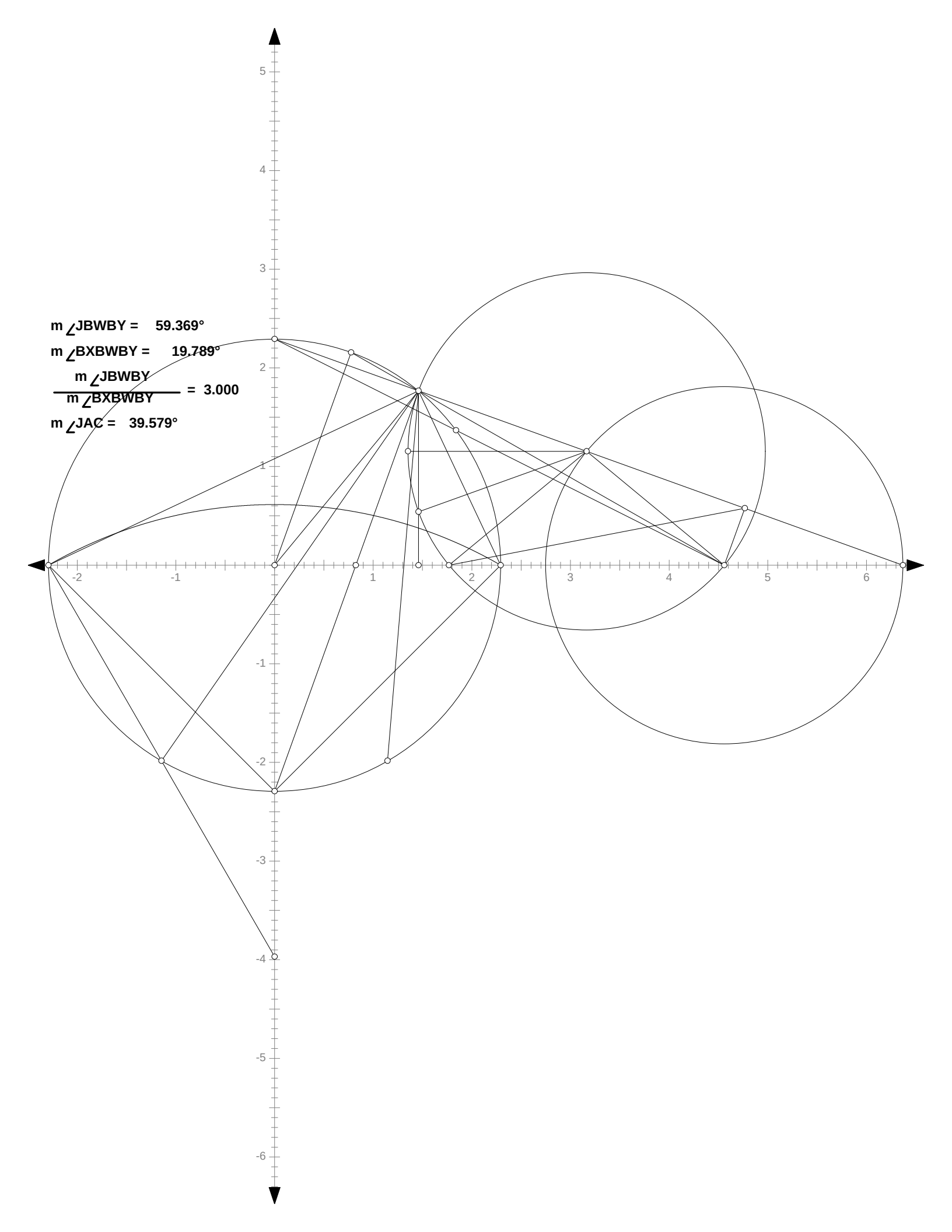

m∠FMPPA = 39.099°
GHA = 0.461 inches
GHGA = 4.122 inches
m∠CCCICD = 60.000°
m∠BVCICC = 60.000°

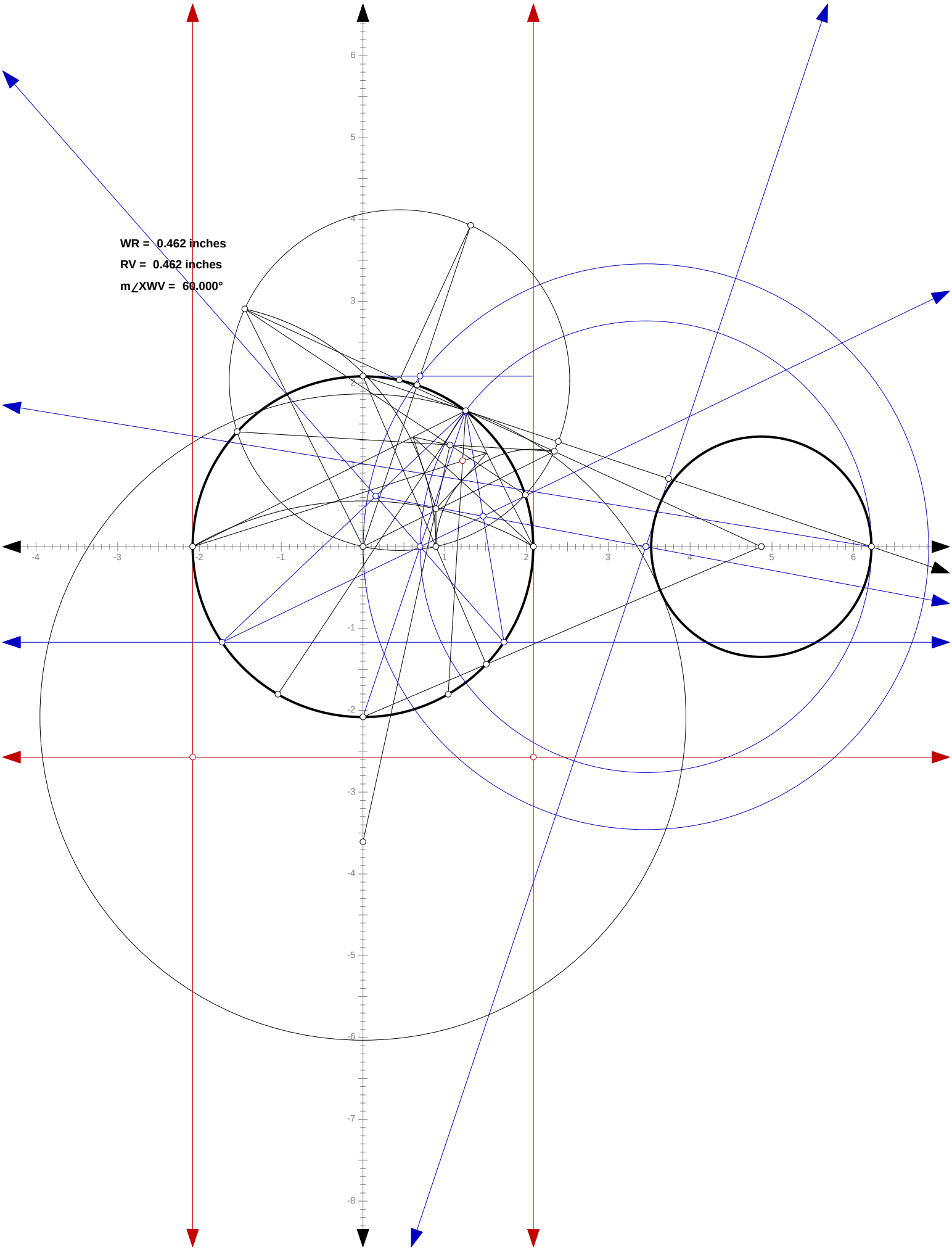

m∠JEA = 65.653°

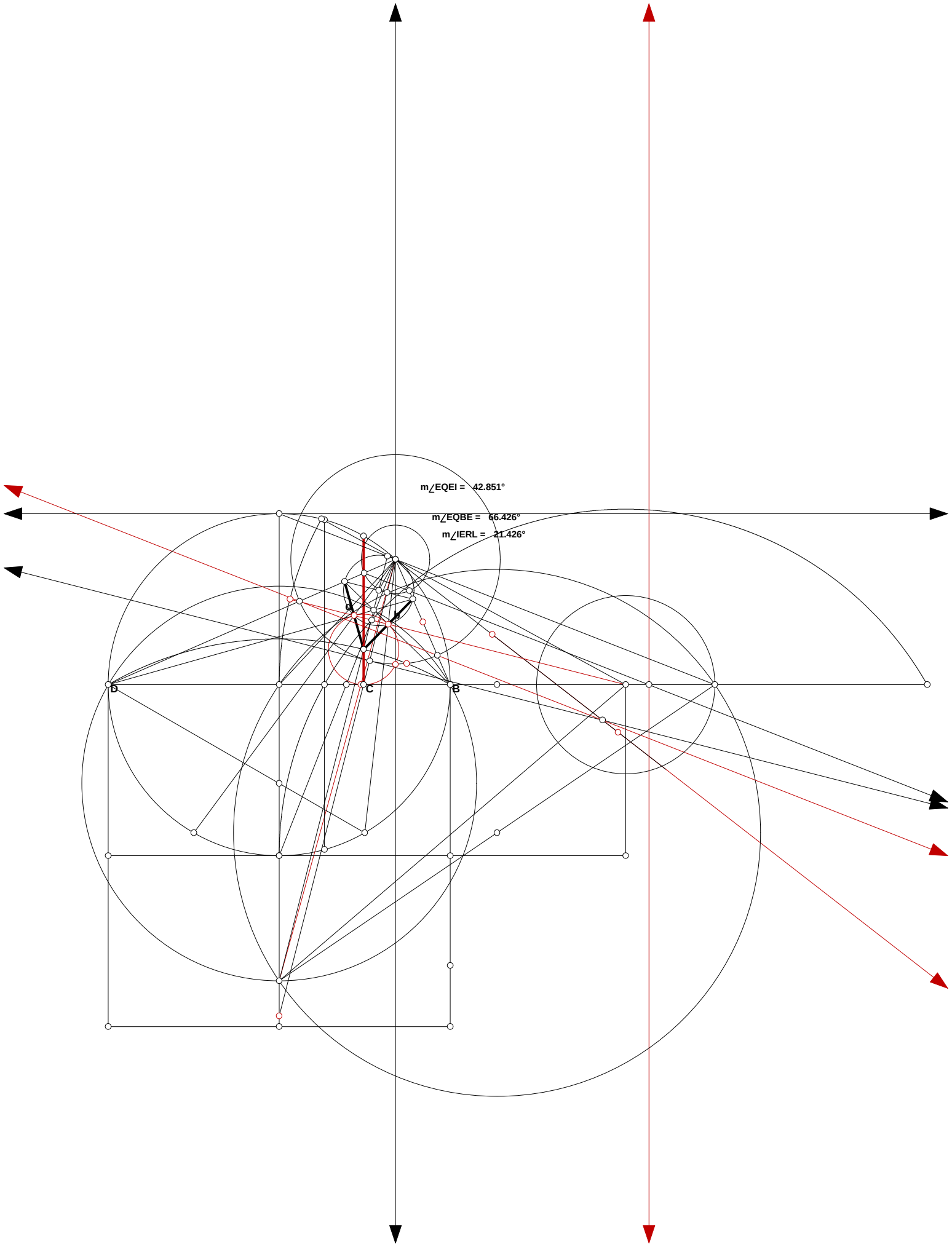

m∠DDLE = 14.409°
m∠ALLE = 14.409°
EUAB = 0.492 inches
ADC = 0.492 inches
dEN = 0.492 inches
m∠ABBE = 66.614°
m∠ABBEP = 22.205°
m∠ABBE / m∠ABBEP = 3.000
D
C
B
a
b

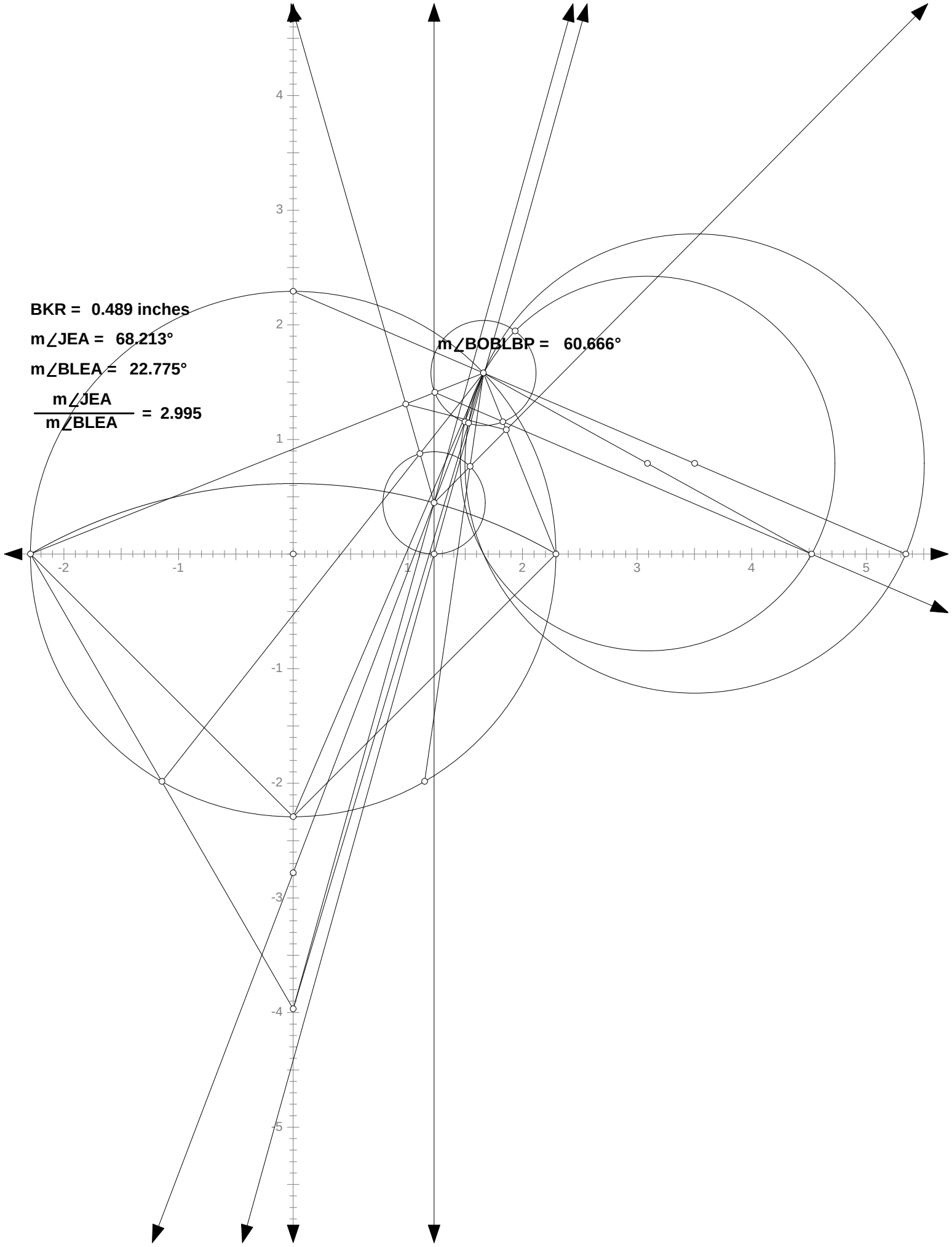

WR = 0.412 inches
RV = 0.412 inches
m∠XWV = 60.000°
m∠WXV = 60.000°
m∠VKA = 68.736°
m∠XKA = 22.912°
m∠VKA / m∠XKA = 3.000
5
4
3
2
1
-1
-2
-3
-4
-5
-3
-2
-1
1
2
3
4

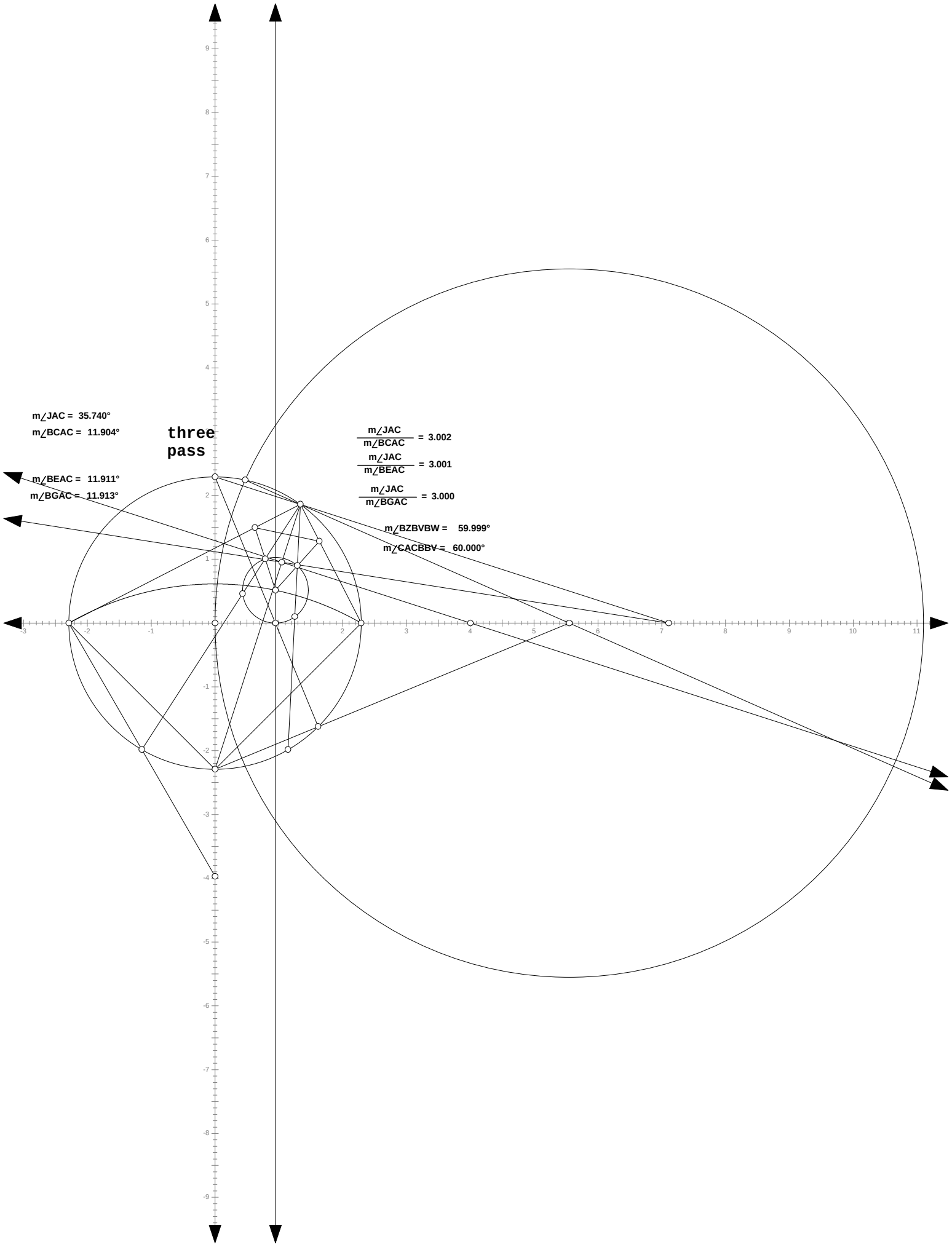

m∠JEA = 77.891°
m∠JEBC = 25.940°
m∠JEA / m∠JEBC = 3.003
m∠JEBI = 25.963°
m∠JEA / m∠JEBI = 3.000
m∠JEA / 3 = 25.964°
(m∠JEA / 3 - m∠JEBI).1000 = 0.395°

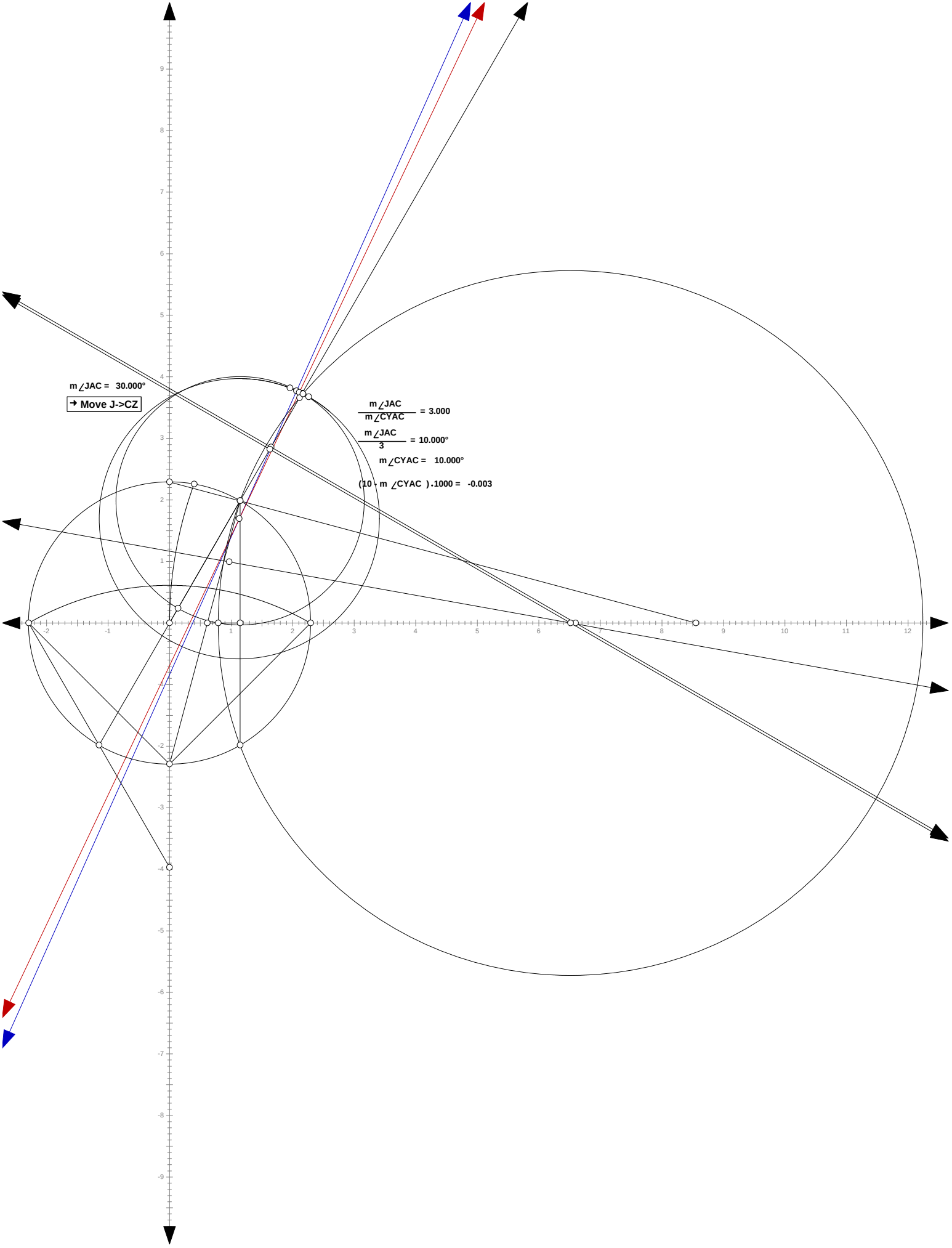

m ∠JBWBY = 59.369°
m ∠BXBWBY = 19.789°
m ∠JBWBY / m ∠BXBWBY = 3.000
m ∠JAC = 39.579°

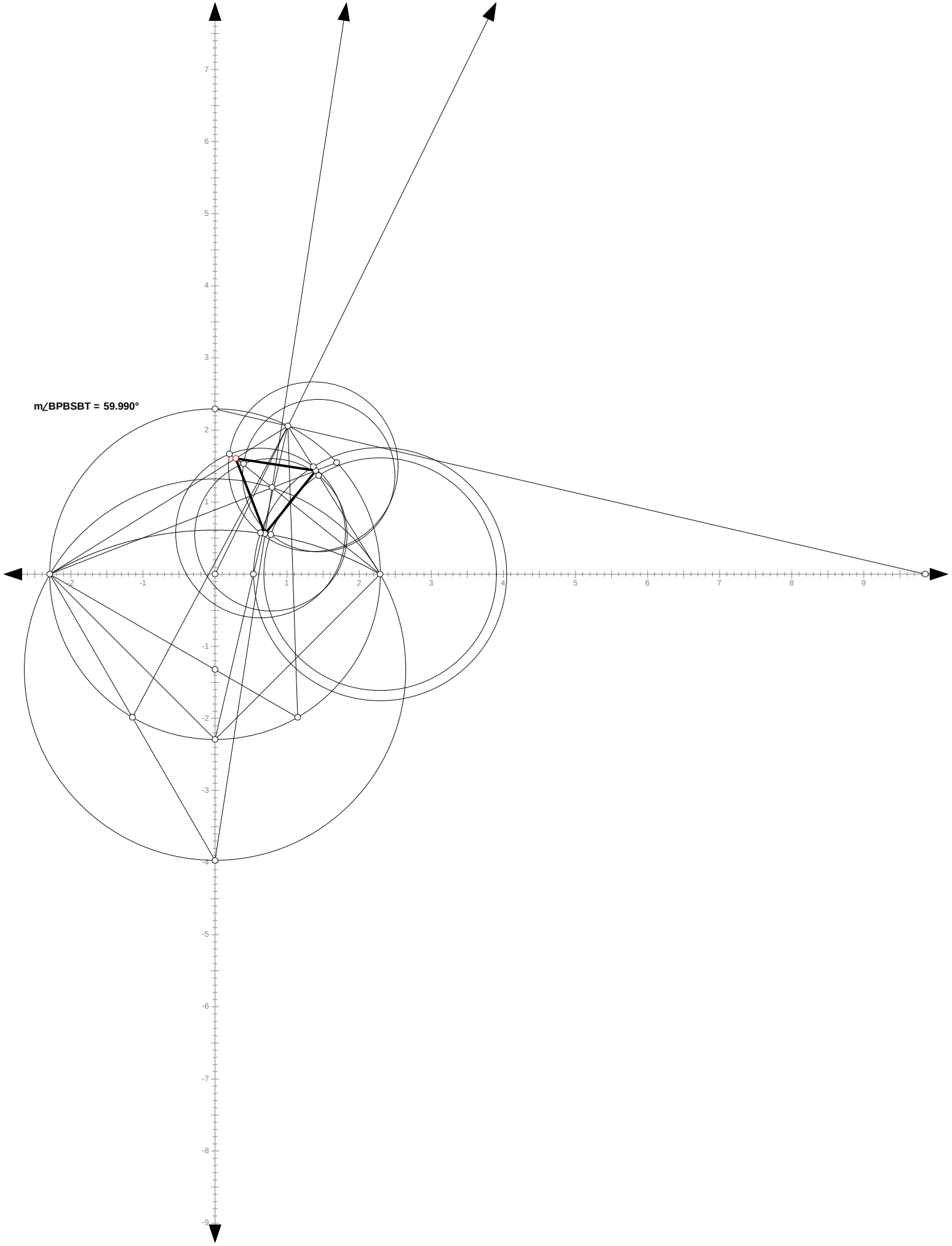

WR = 0.462 inches
RV = 0.462 inches
m∠XWV = 60.000°

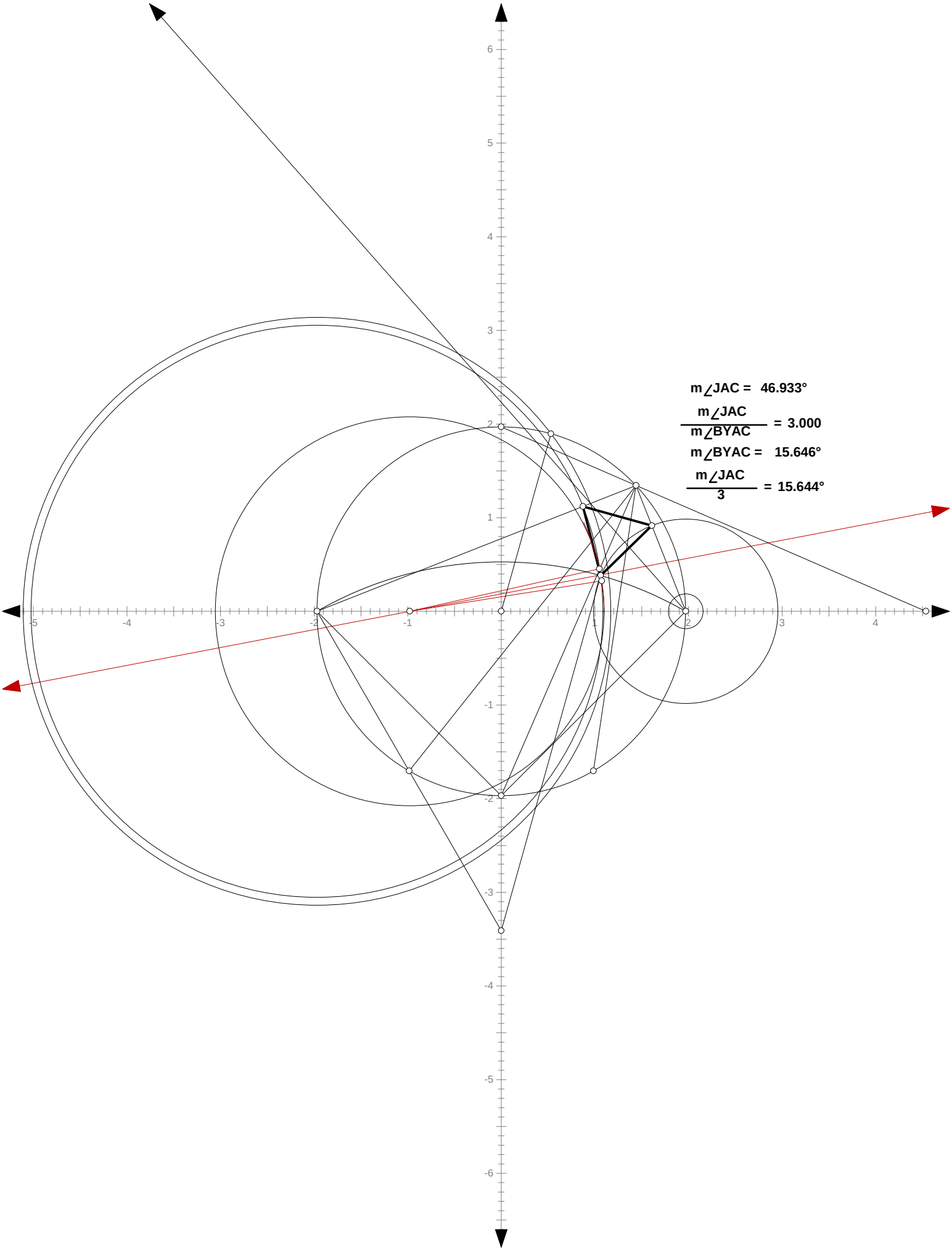

m∠EQEI = 42.851°
m∠EQBE = 66.426°
m∠IERL = 21.426°
D
C
B

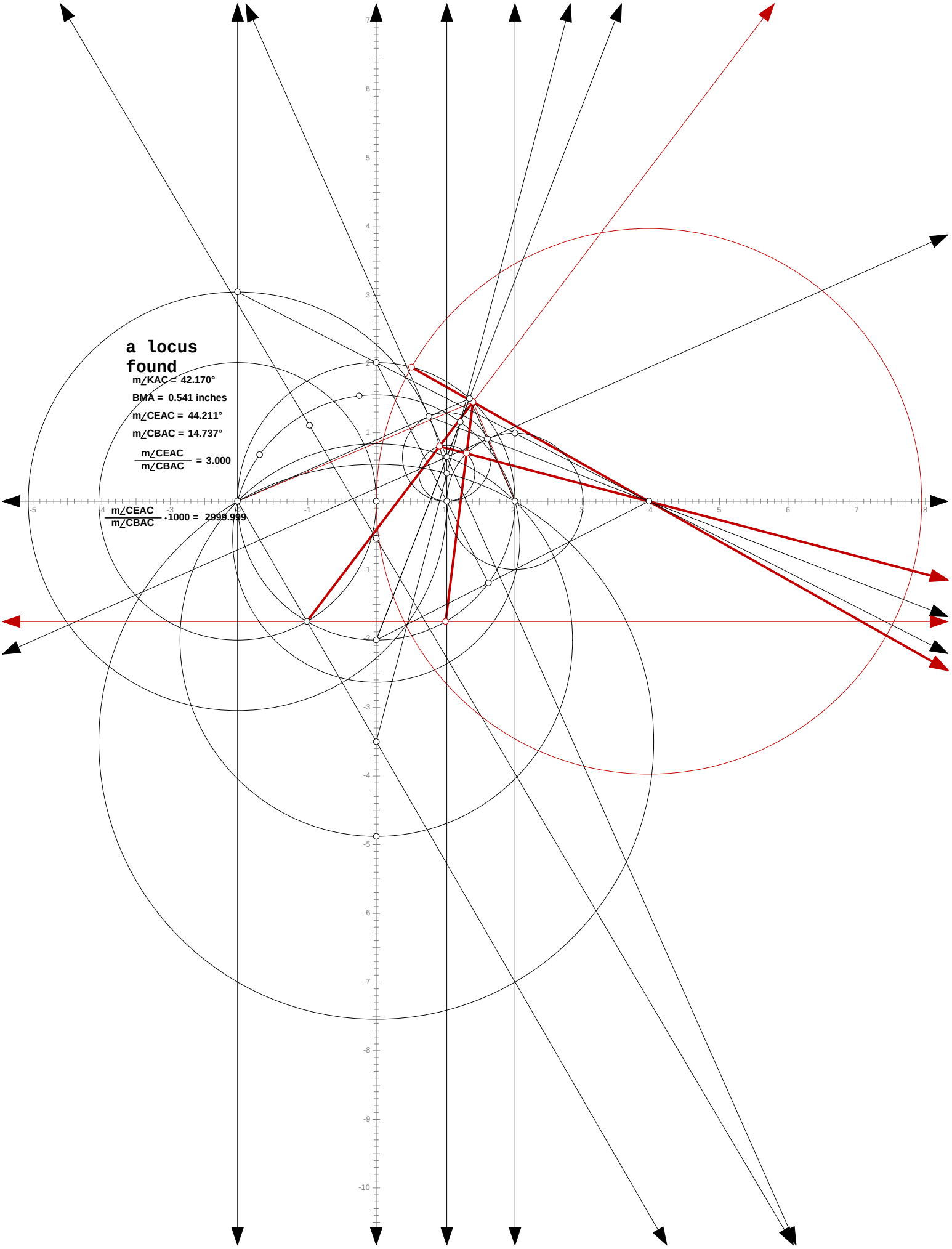

BKR = 0.489 inches
m∠JEA = 68.213°
m∠BLEA = 22.775°
m∠JEA / m∠BLEA = 2.995
m∠BOBLBP = 60.666°

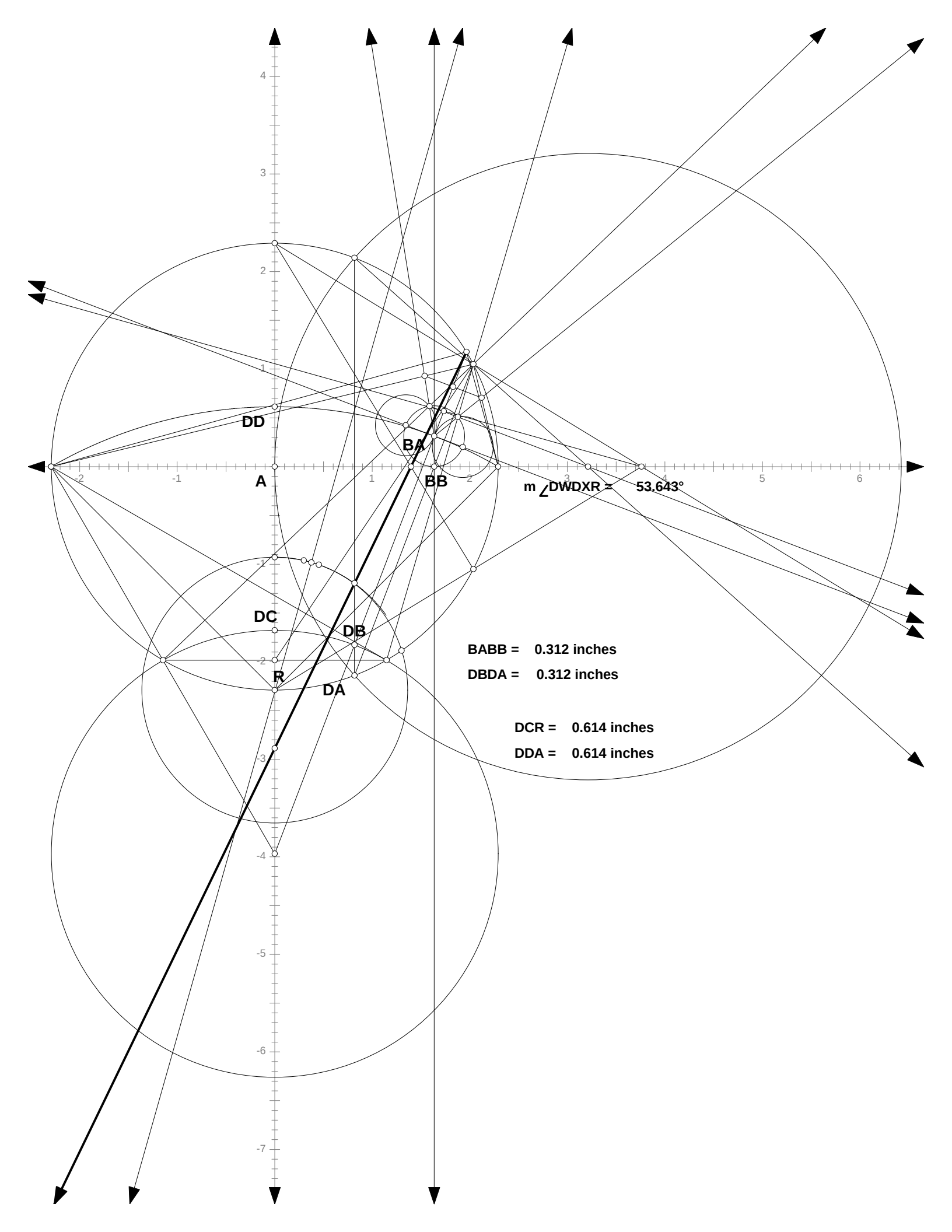

m∠JAC = 35.740°
m∠BCAC = 11.904°
three
pass
m∠BEAC = 11.911°
m∠BGAC = 11.913°
m∠JAC / m∠BCAC = 3.002
m∠JAC / m∠BEAC = 3.001
m∠JAC / m∠BGAC = 3.000
m∠BZBVBW = 59.999°
m∠CACBBV = 60.000°

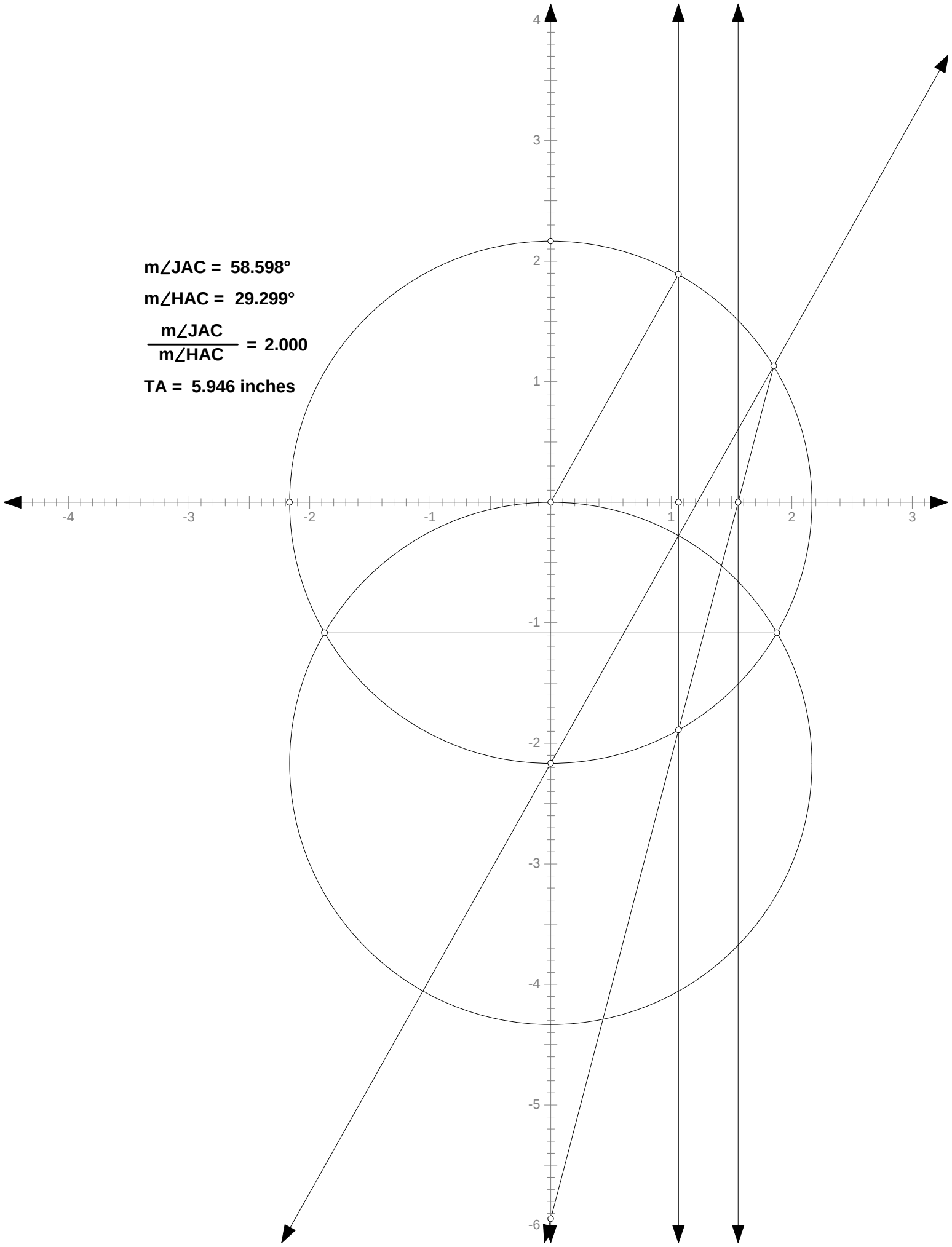

m∠JAC = 30.000°
Move J->CZ
m∠JAC / m∠CYAC = 3.000
m∠JAC / 3 = 10.000°
m∠CYAC = 10.000°
(10 - m∠CYAC ).1000 = -0.003

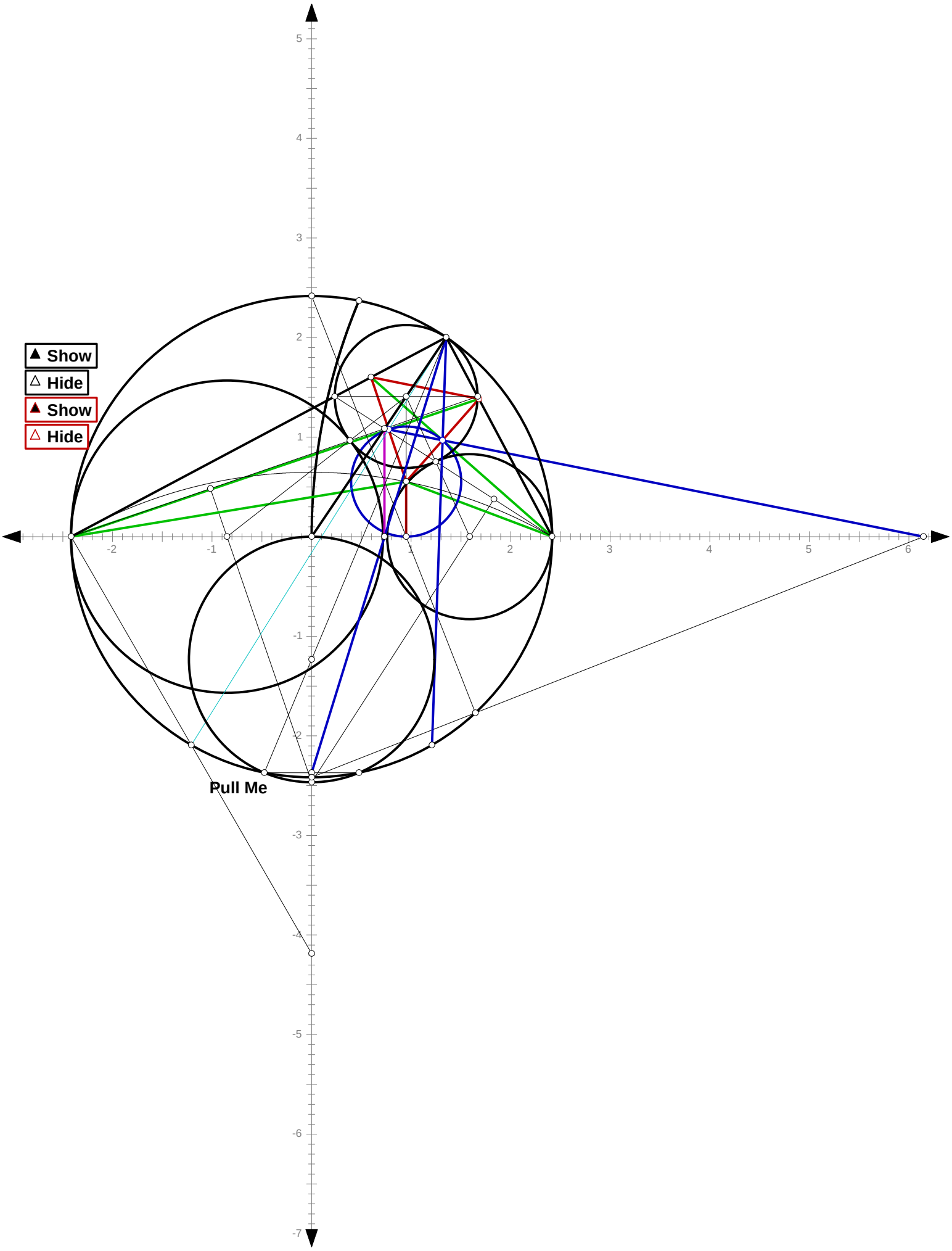

m∠BPBSBT = 59.990°

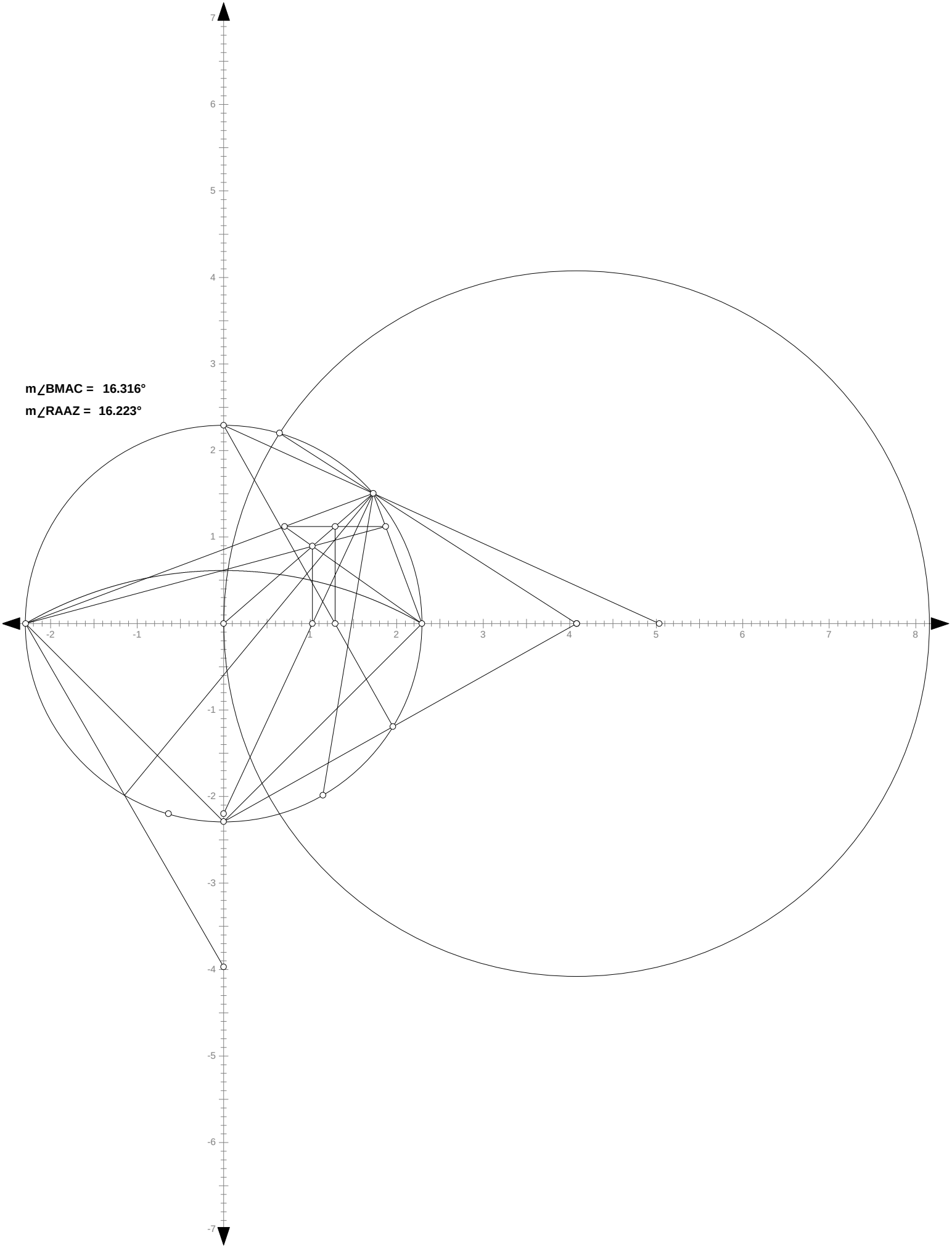

m∠JAC = 46.933°
m∠JAC / m∠BYAC = 3.000
m∠BYAC = 15.646°
m∠JAC / 3 = 15.644°

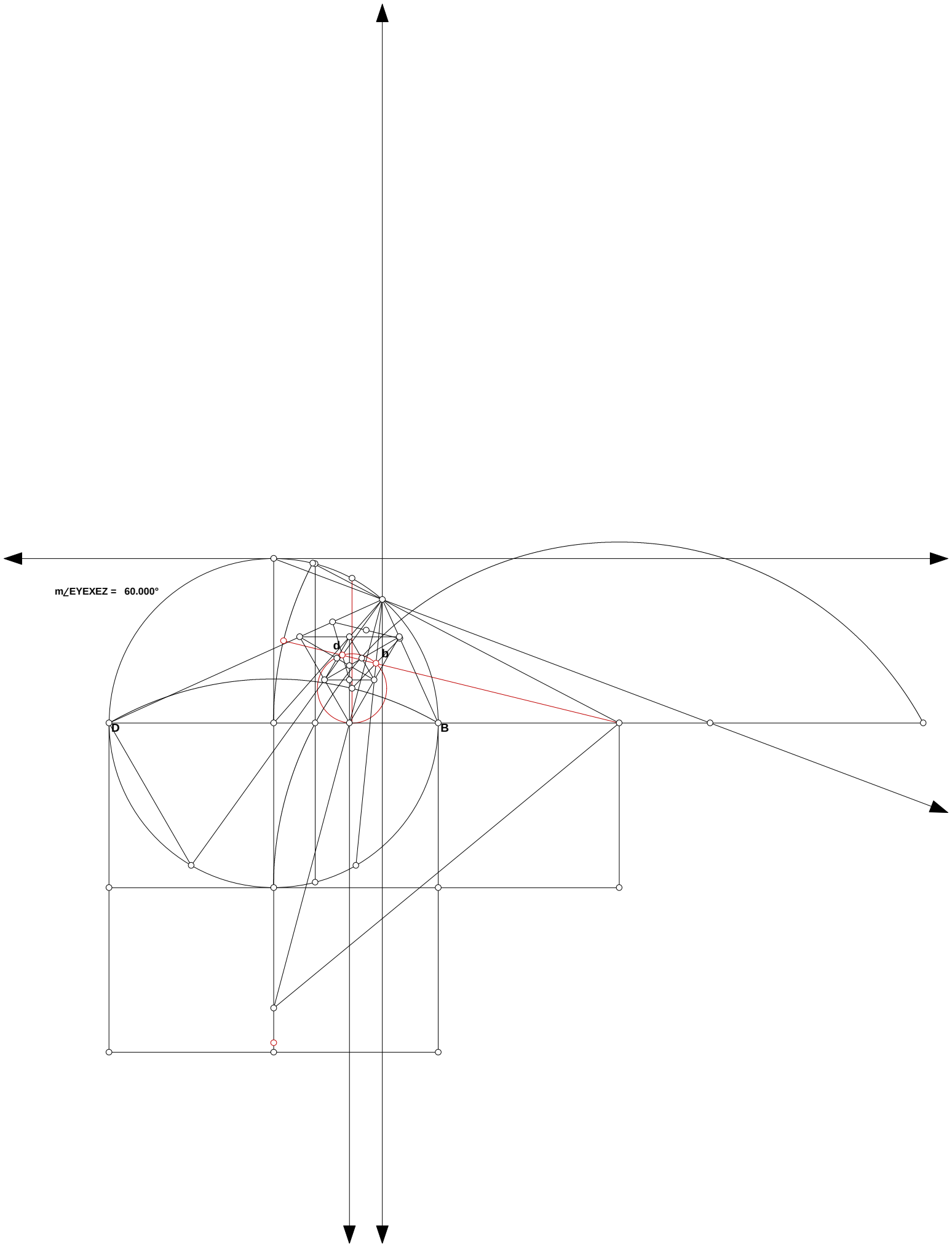

a locus found
m∠KAC = 42.170°
BMA = 0.541 inches
m∠CEAC = 44.211°
m∠CBAC = 14.737°
m∠CEAC / m∠CBAC = 3.000
m∠CEAC / m∠CBAC ·1000 = 2999.999

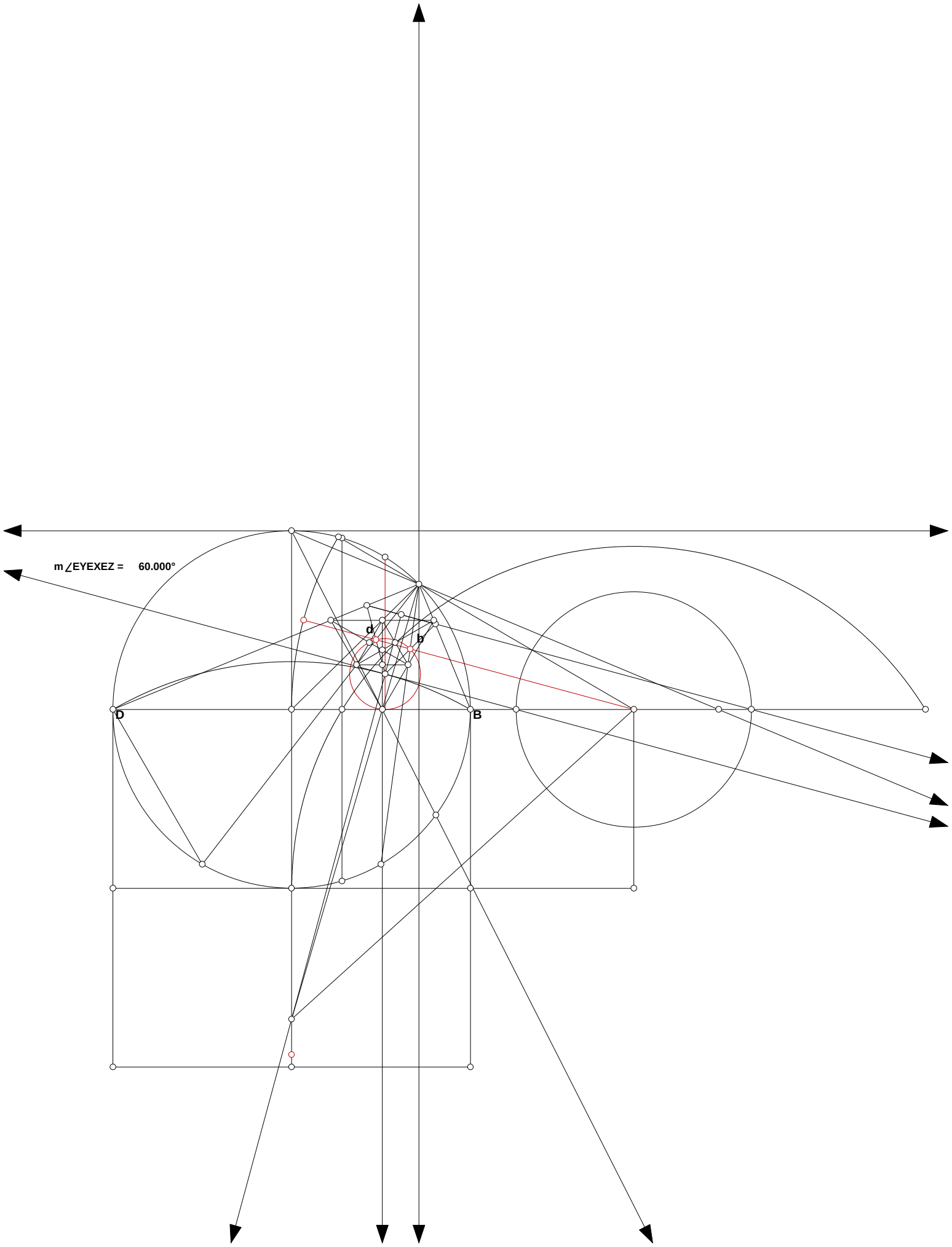

DD
BA
A
BB
m ∠DWDXR = 53.643°
DC
DB
R
DA
BABB = 0.312 inches
DBDA = 0.312 inches
DCR = 0.614 inches
DDA = 0.614 inches

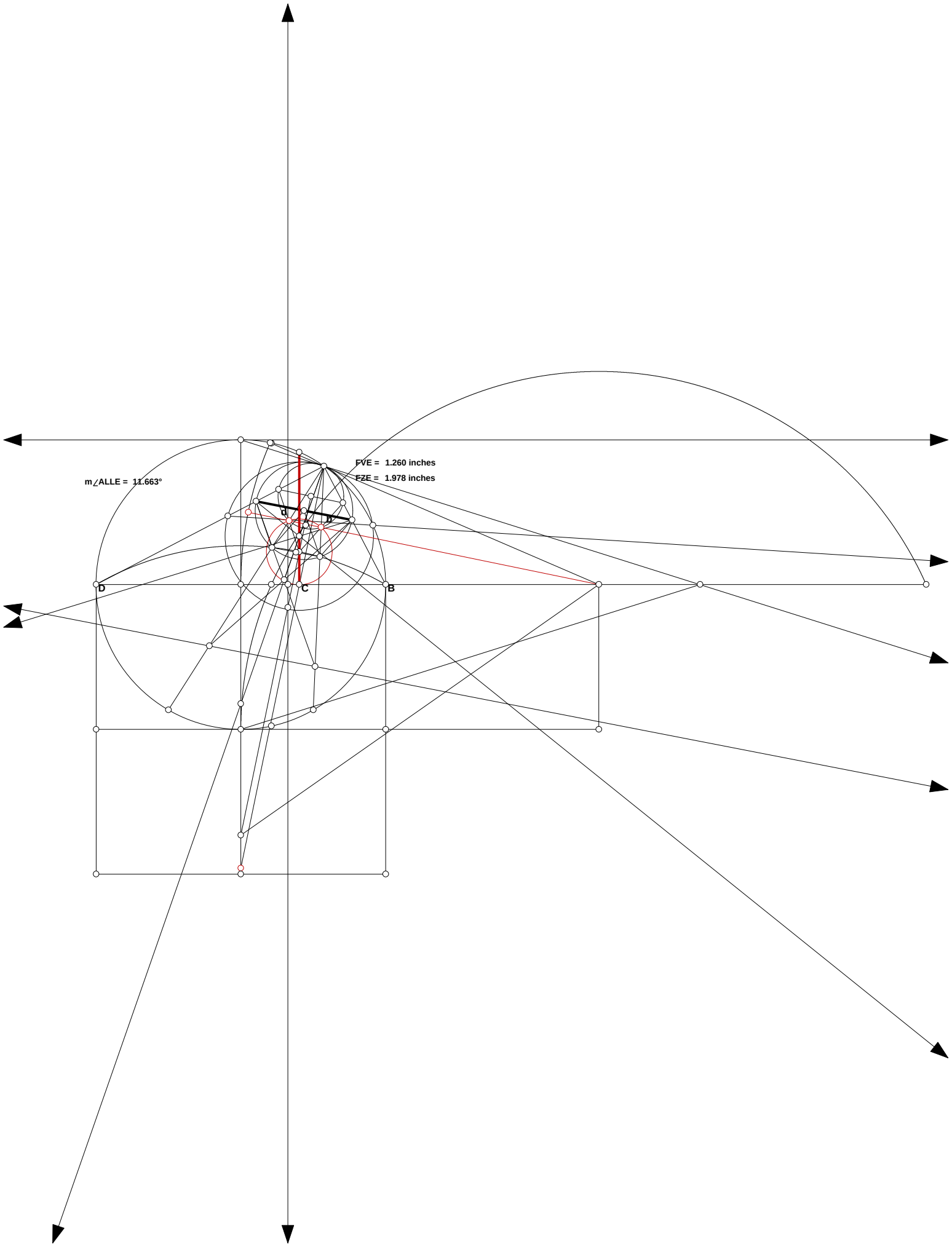

m∠JAC = 58.598°
m∠HAC = 29.299°
m∠JAC / m∠HAC = 2.000
TA = 5.946 inches
4
3
2
1
-1
-2
-3
-4
-5
-6
-4
-3
-2
-1
1
2
3

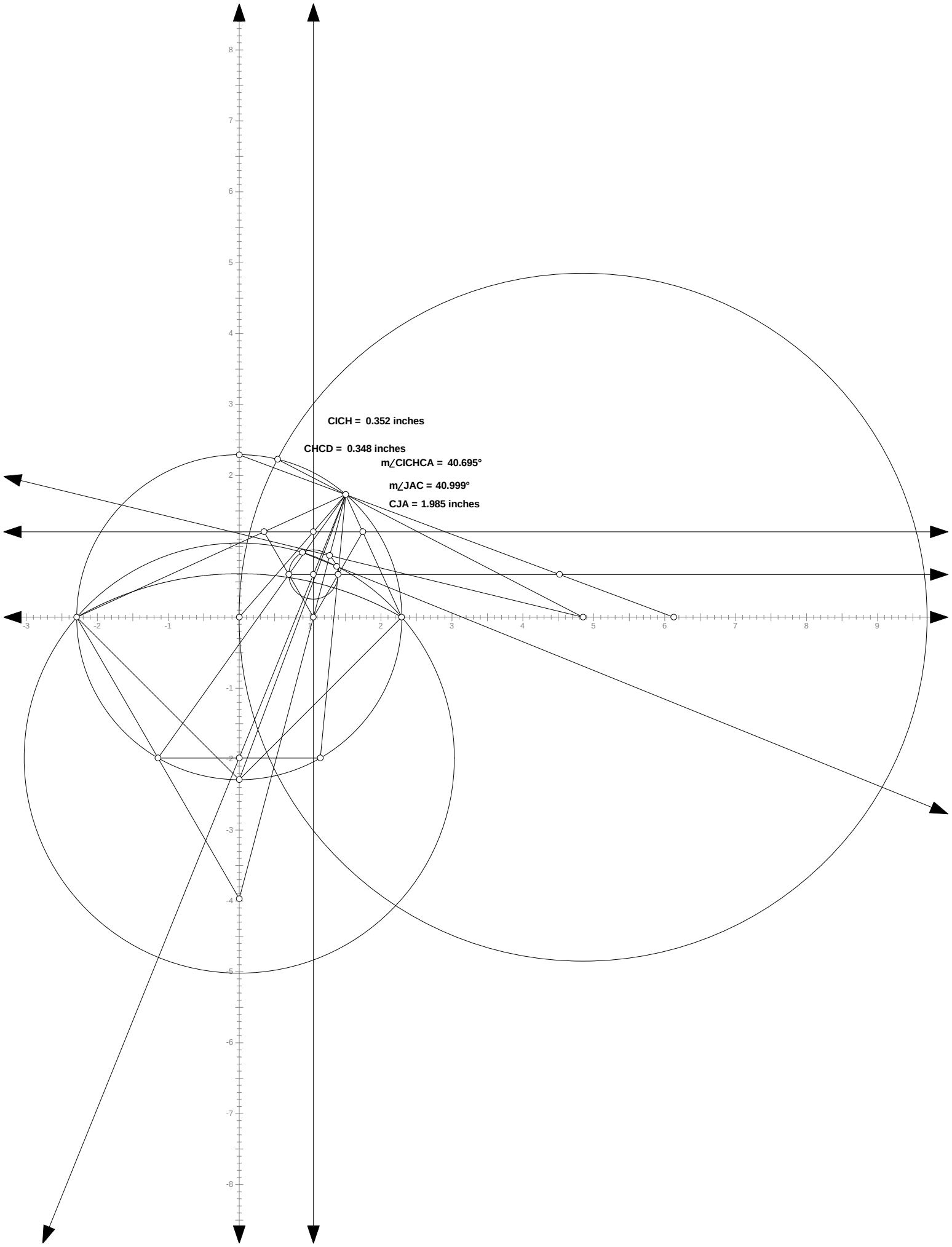

Show
Hide
Show
Hide
Pull Me

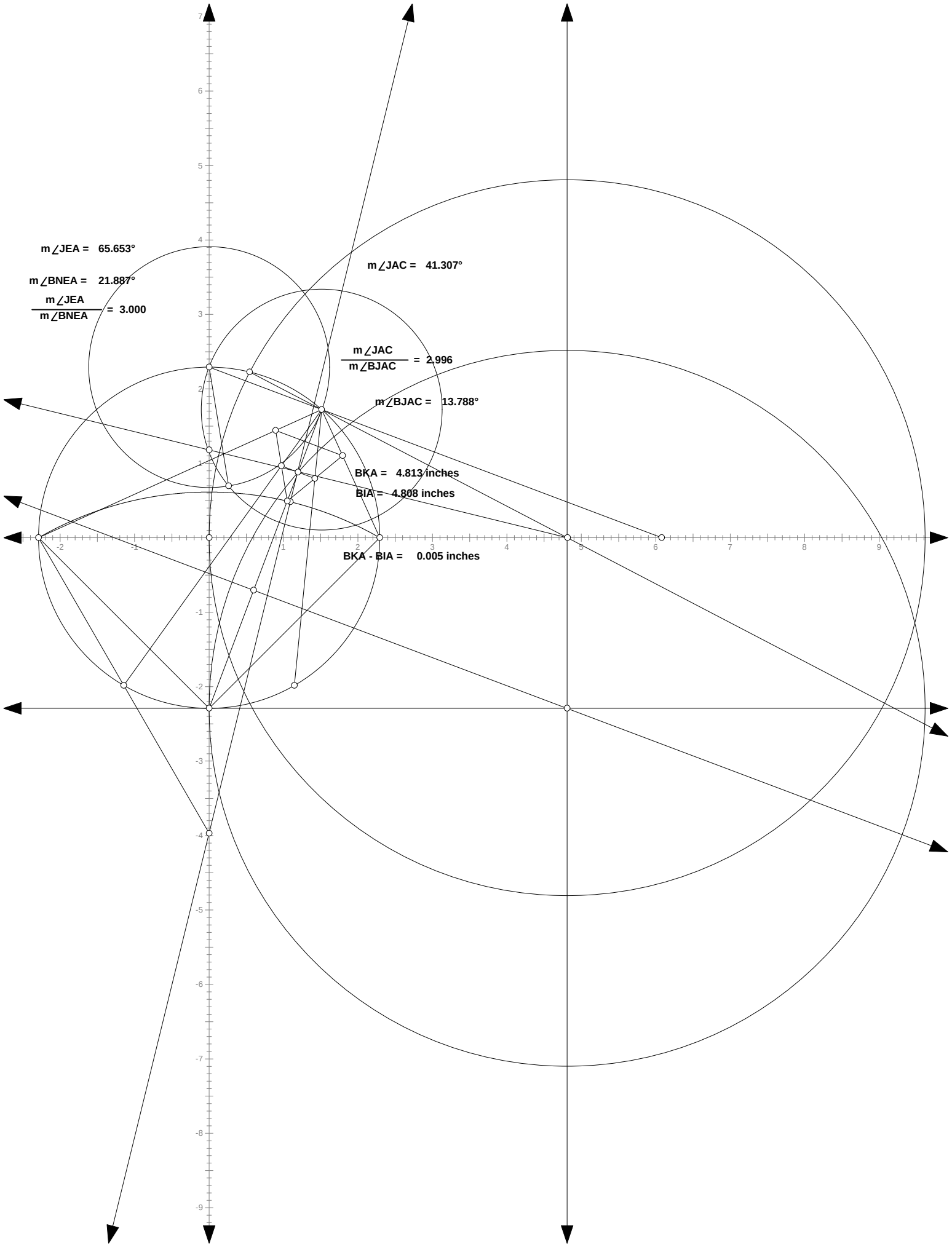

m∠BMAC = 16.316°
m∠RAAZ = 16.223°

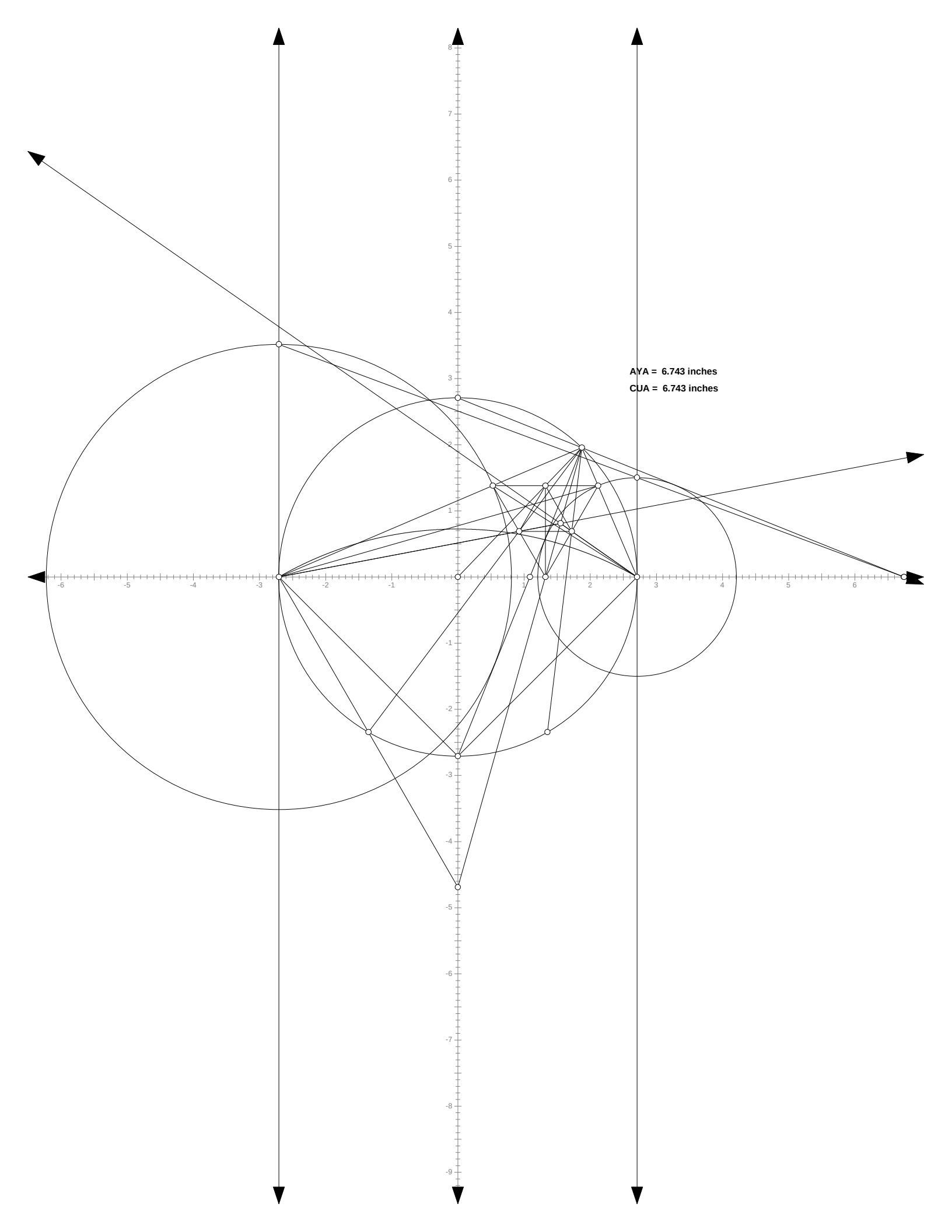

m∠EYEXEZ = 60.000°
d
b
D
B

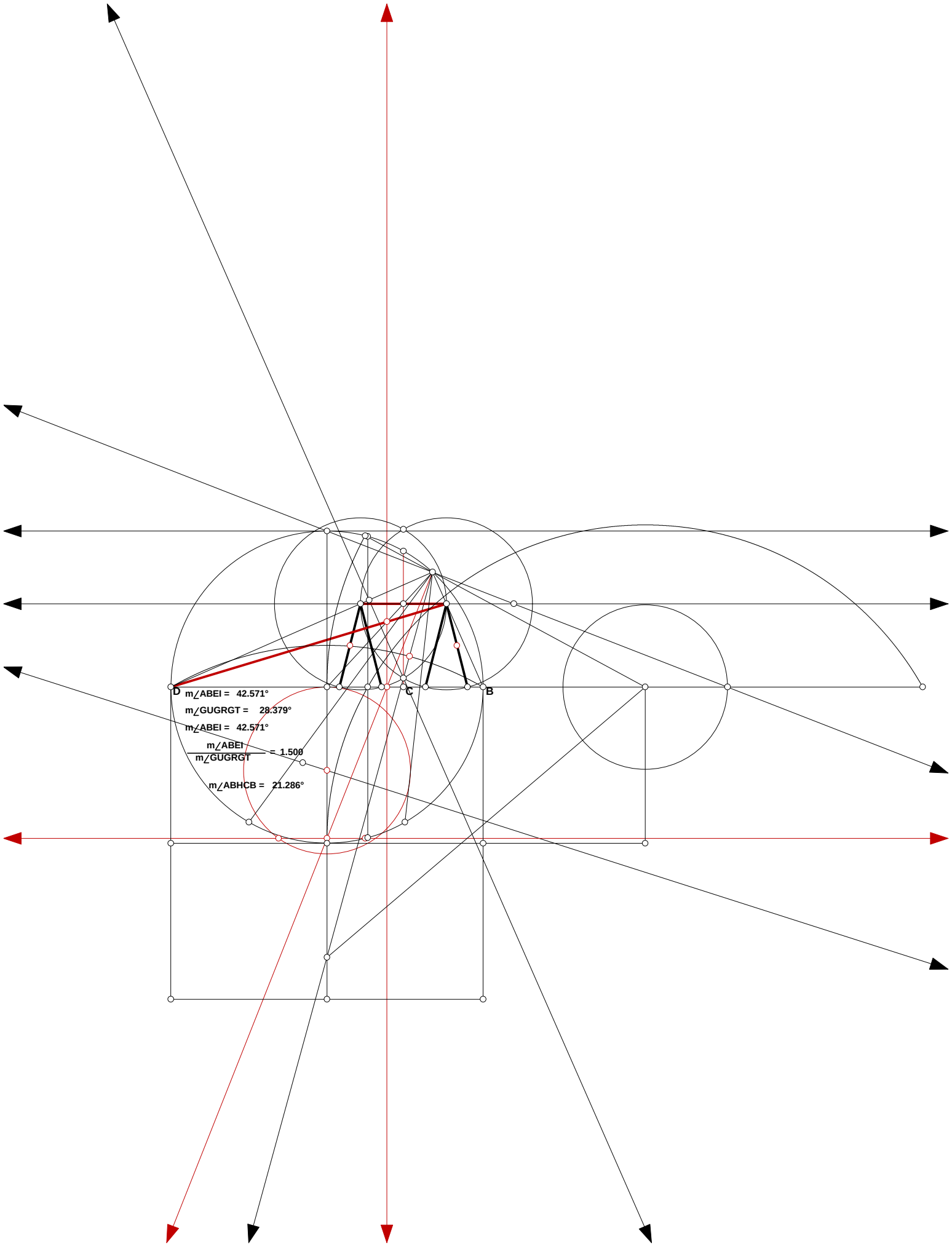

m ∠EYEXEZ = 60.000°
d
b
D
B

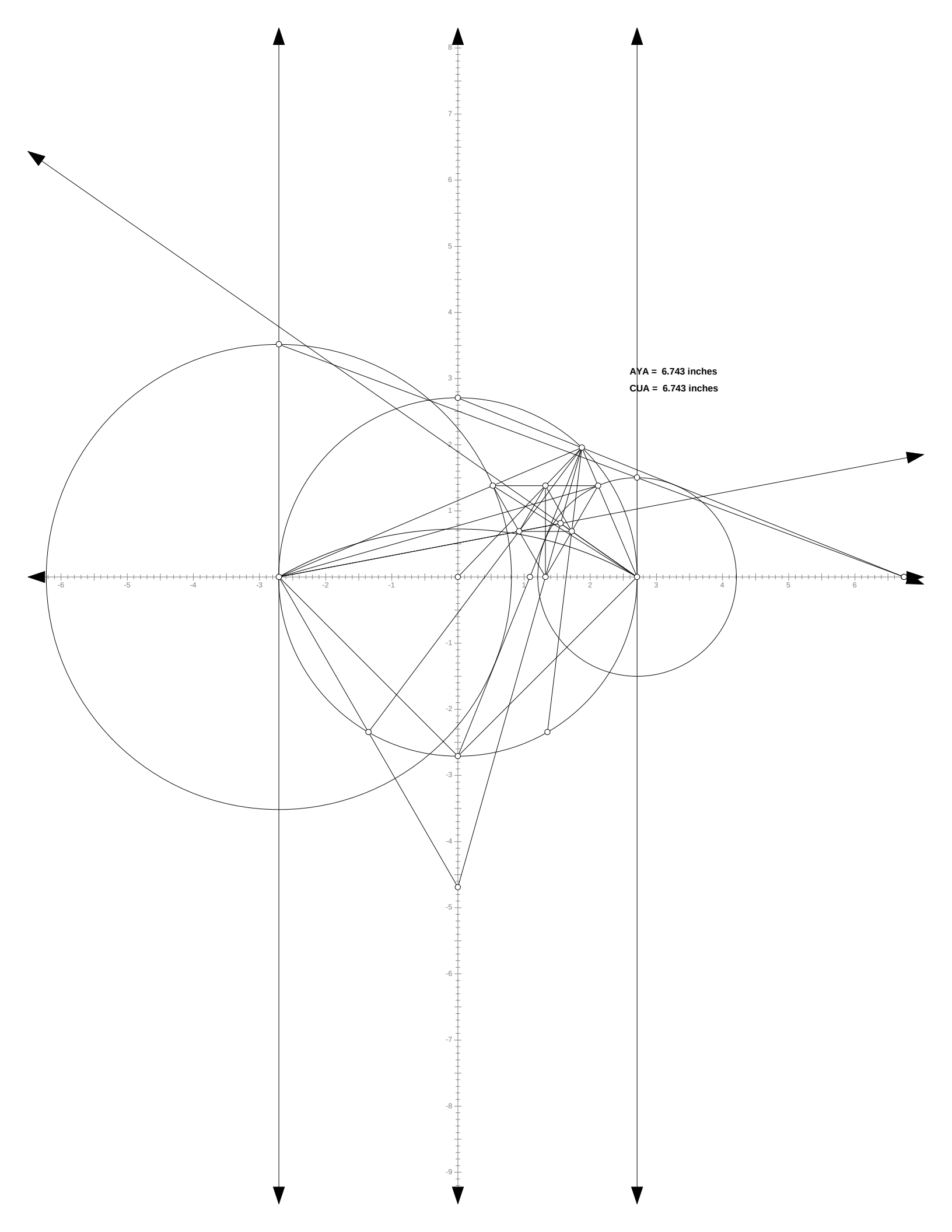

FVE = 1.260 inches
FZE = 1.978 inches
m∠ALLE = 11.663°
D
C
B

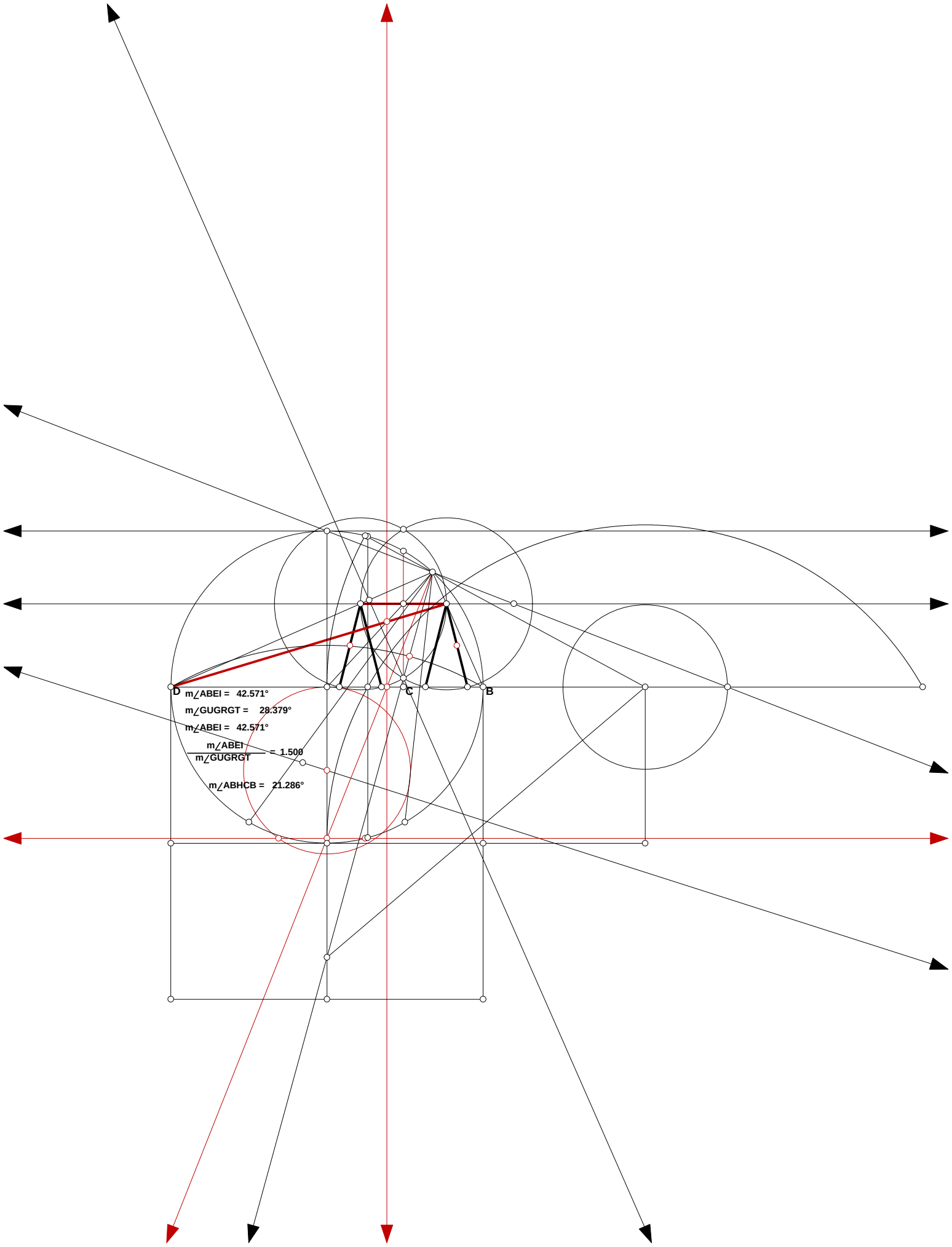

CICH = 0.352 inches
CHCD = 0.348 inches
m∠CICHCA = 40.695°
m∠JAC = 40.999°
CJA = 1.985 inches

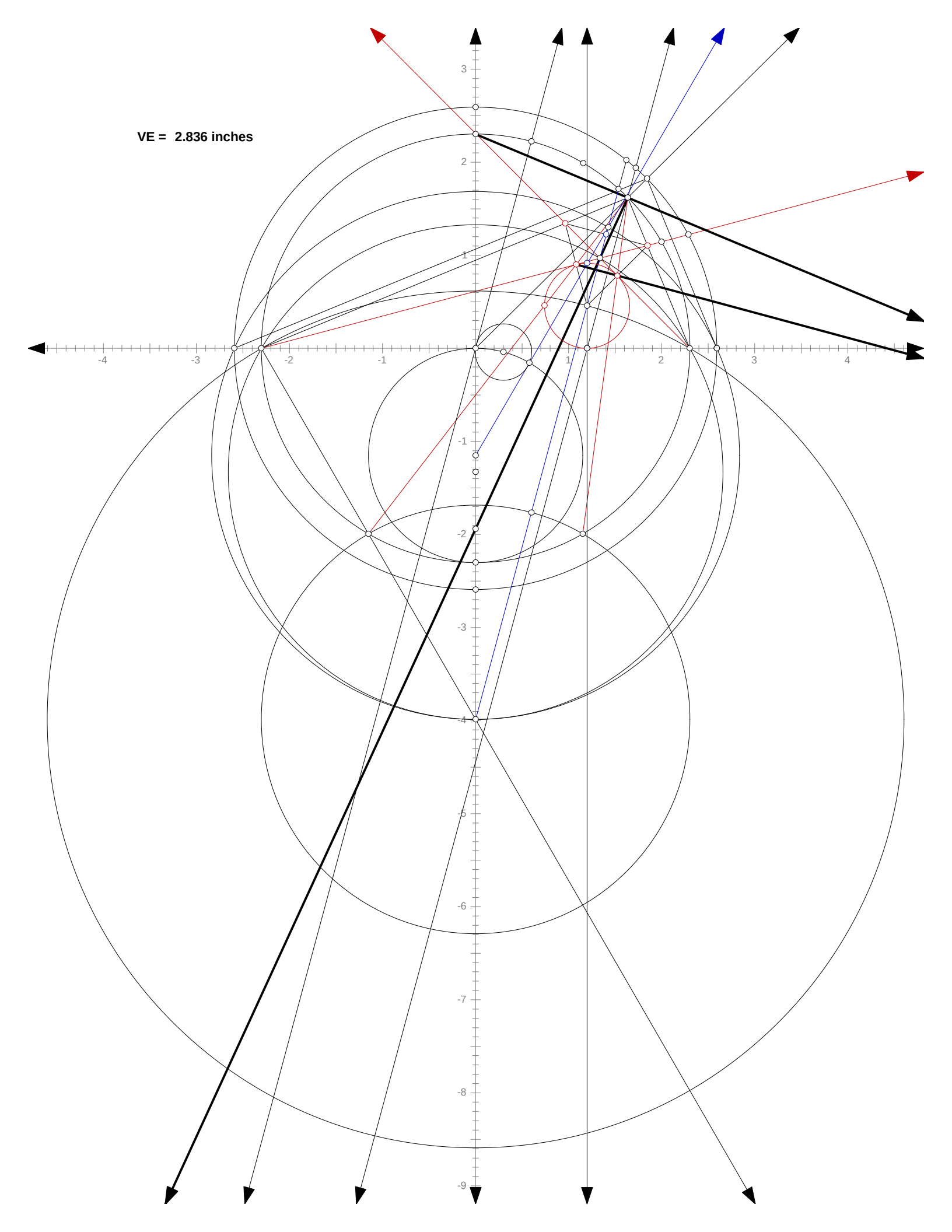

m∠JEA = 65.653°
m∠BNEA = 21.887°
m∠JEA / m∠BNEA = 3.000
m∠JAC = 41.307°
m∠JAC / m∠BJAC = 2.996
m∠BJAC = 13.788°
BKA = 4.813 inches
BIA = 4.808 inches
BKA - BIA = 0.005 inches

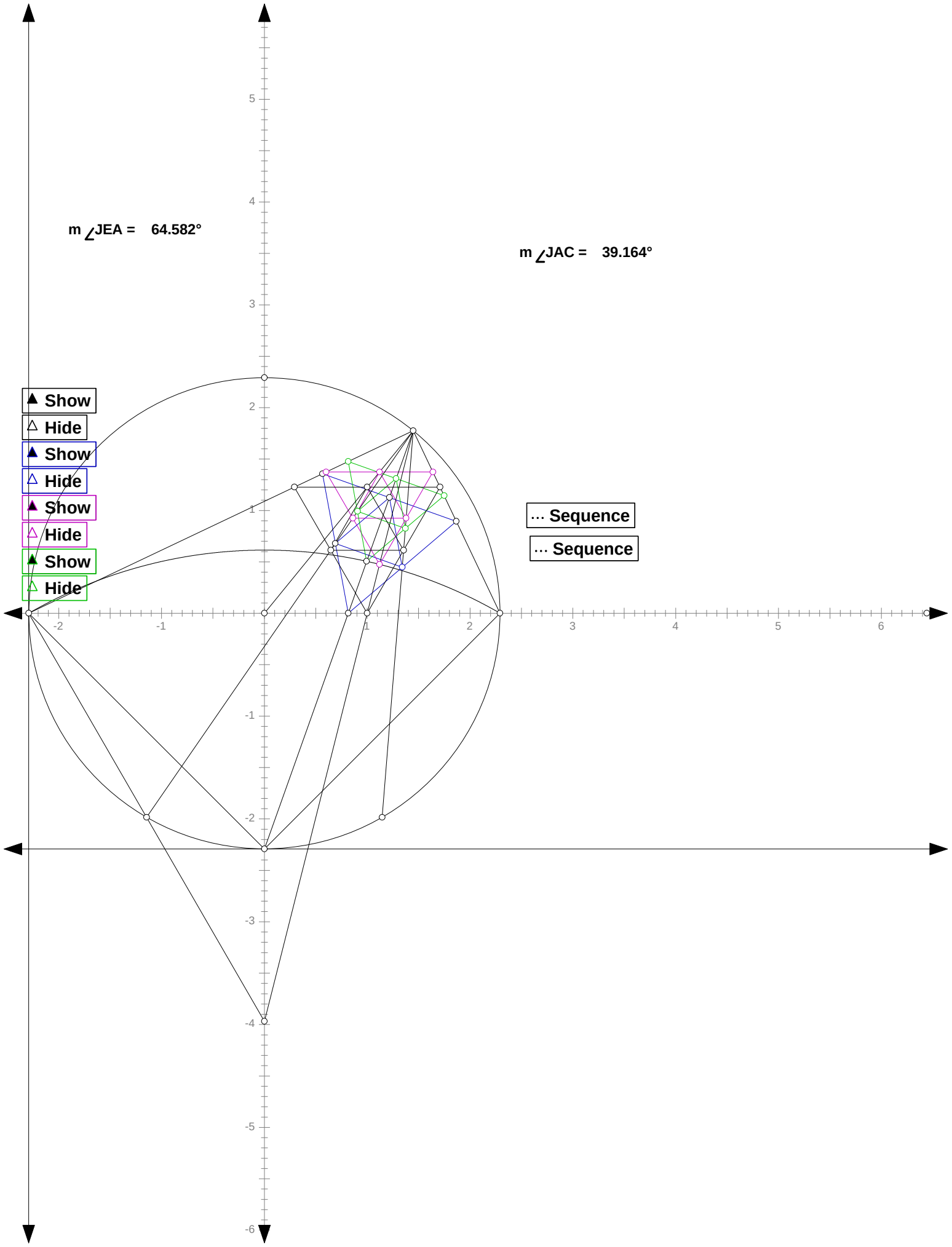

AYA = 6.743 inches
CUA = 6.743 inches

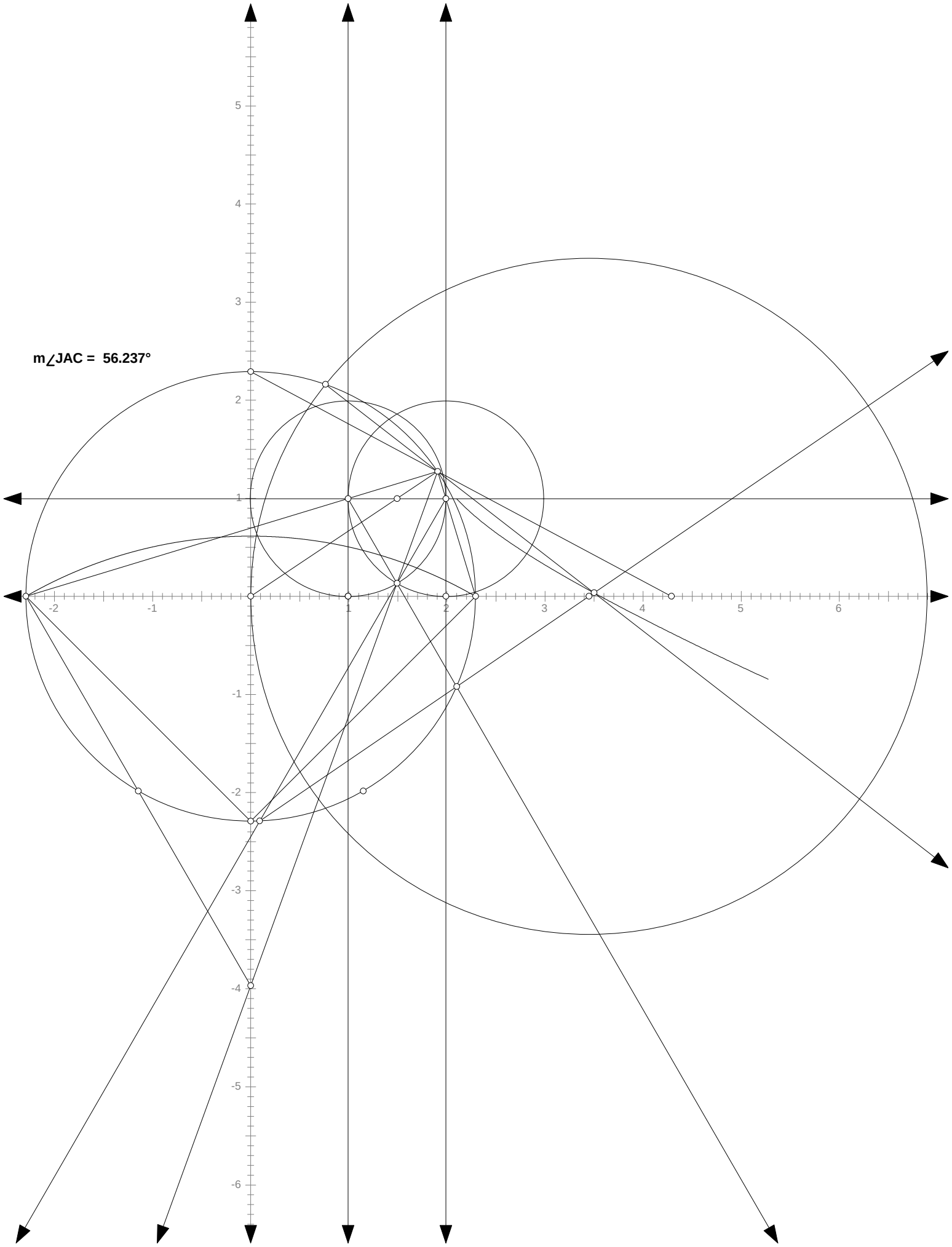

D
C
B
m∠ABEI = 42.571°
m∠GUGRGT = 28.379°
m∠ABEI = 42.571°
m∠ABEI / m∠GUGRGT = 1.500
m∠ABHCB = 21.286°

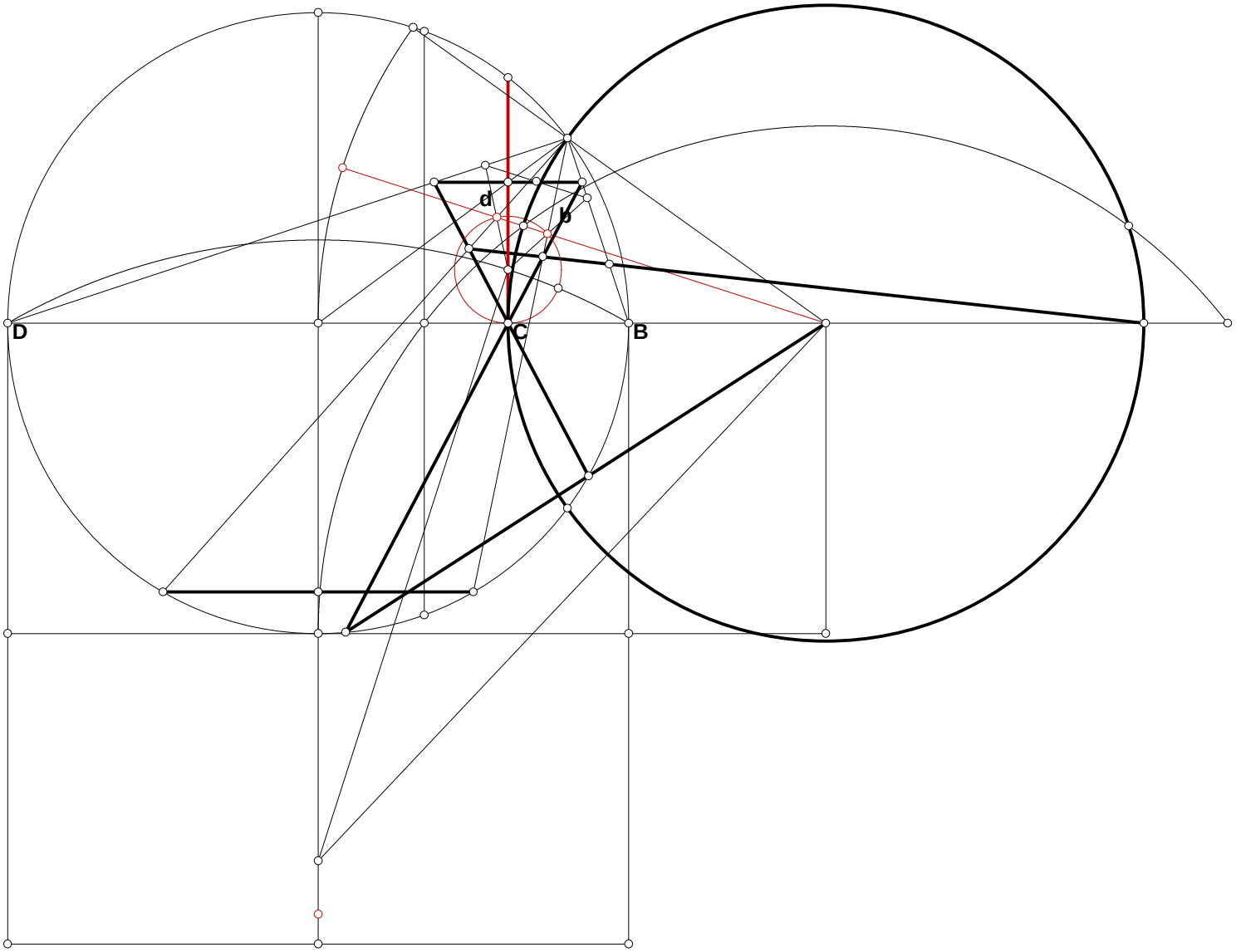

AYA = 6.743 inches
CUA = 6.743 inches

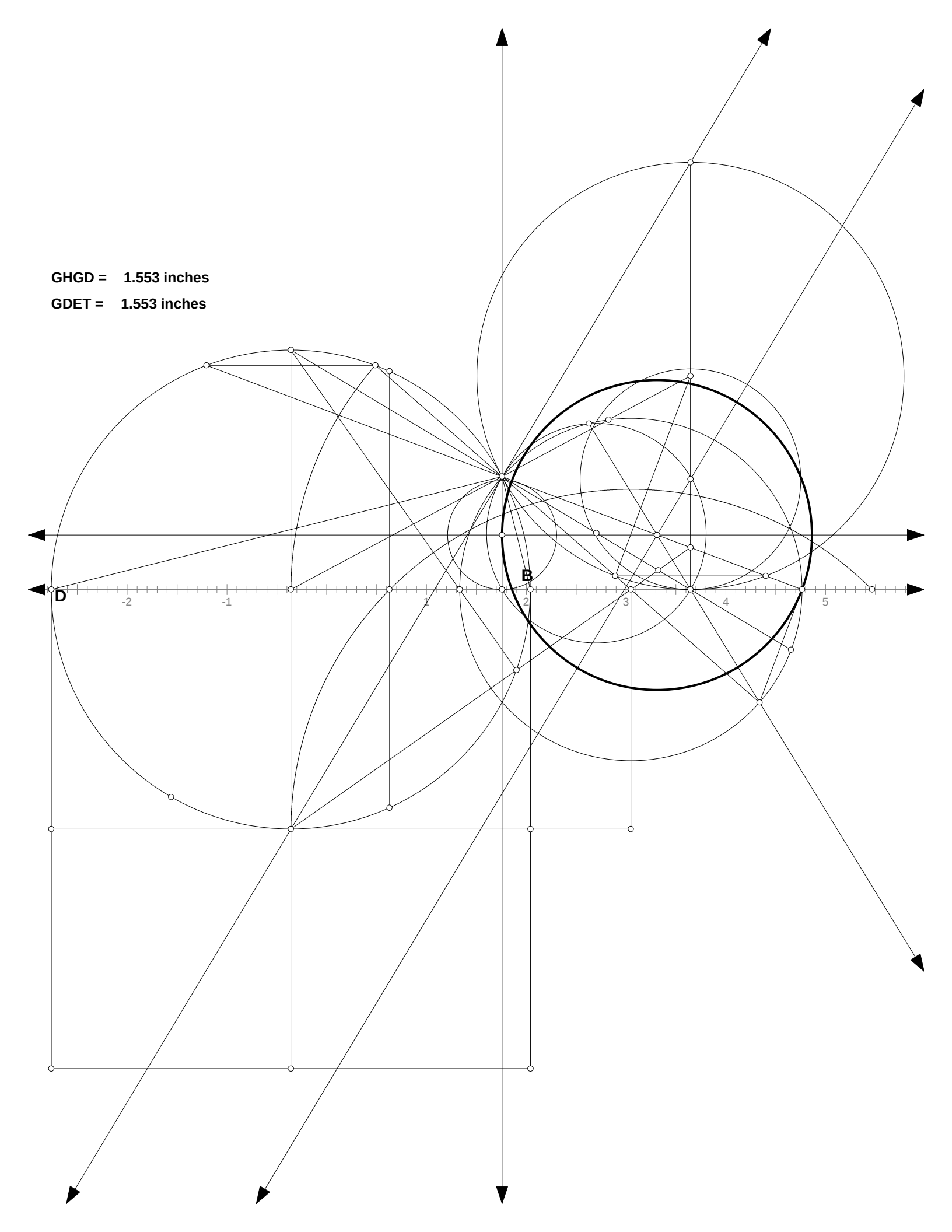

D
C
B
m∠ABEI = 42.571°
m∠GUGRGT = 28.379°
m∠ABEI = 42.571°
m∠ABEI / m∠GUGRGT = 1.500
m∠ABHCB = 21.286°

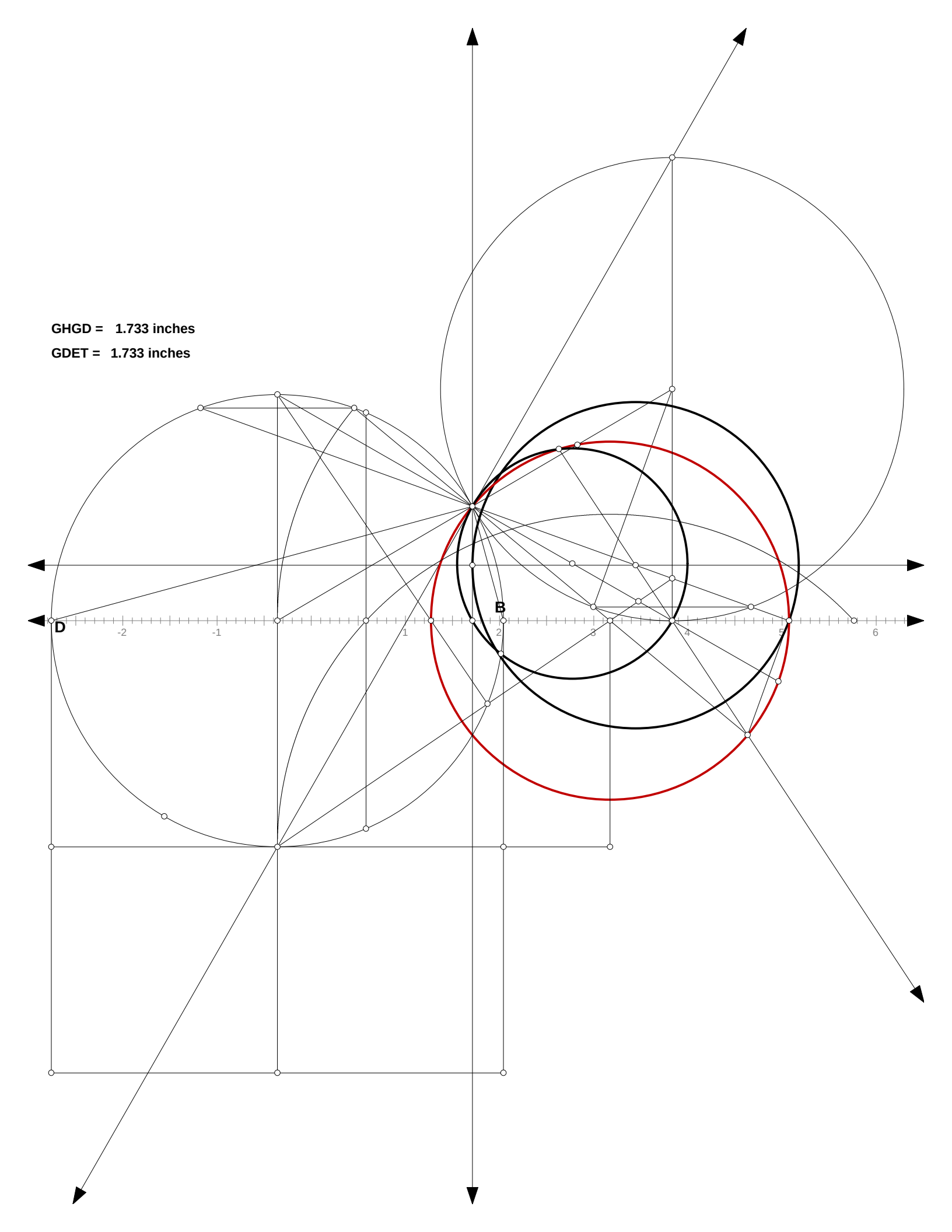

VE = 2.836 inches
3
2
1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-4
-3
-2
-1
1
2
3
4

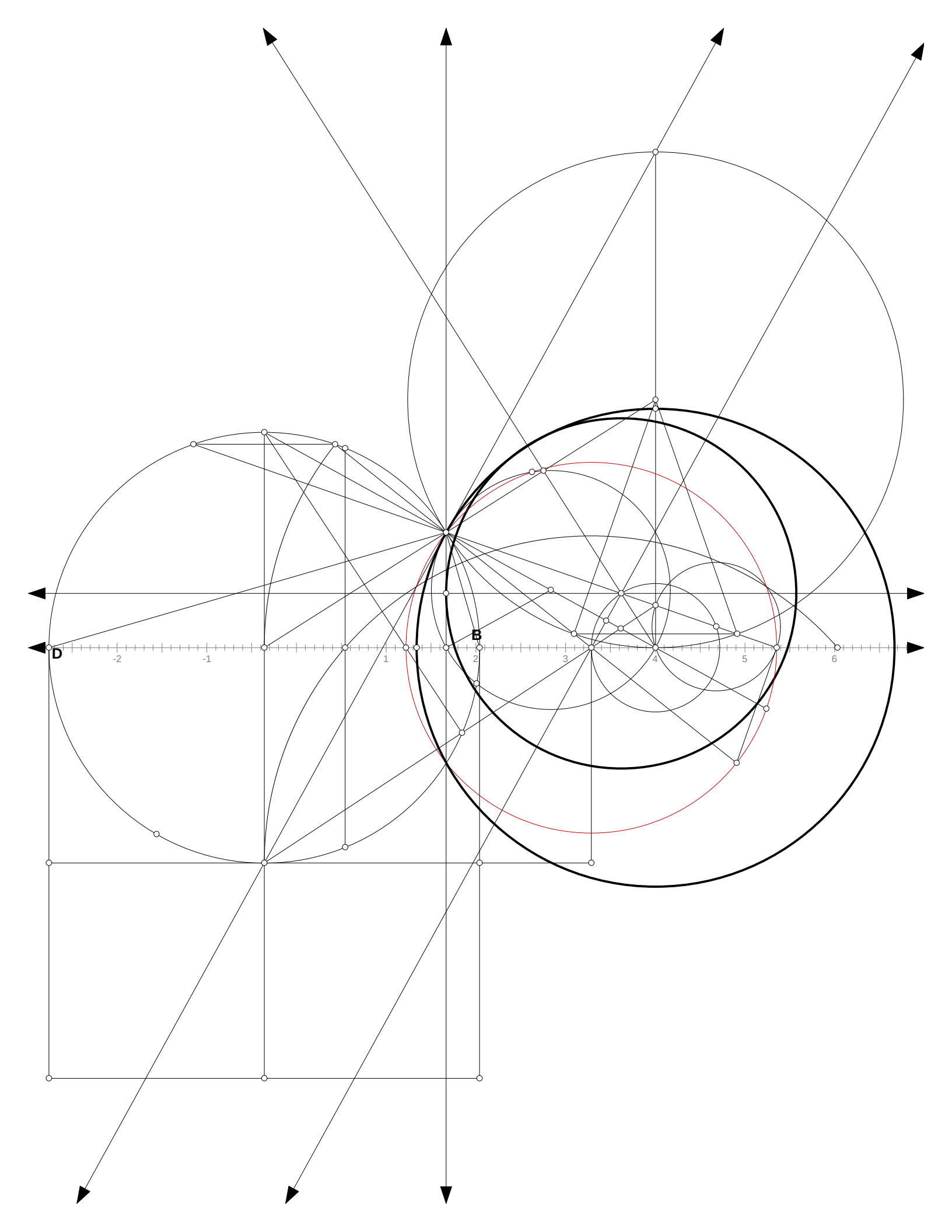

m ∠JEA = 64.582°
m ∠JAC = 39.164°
Show
Hide
Show
Hide
Show
Hide
Show
Hide
··· Sequence
··· Sequence

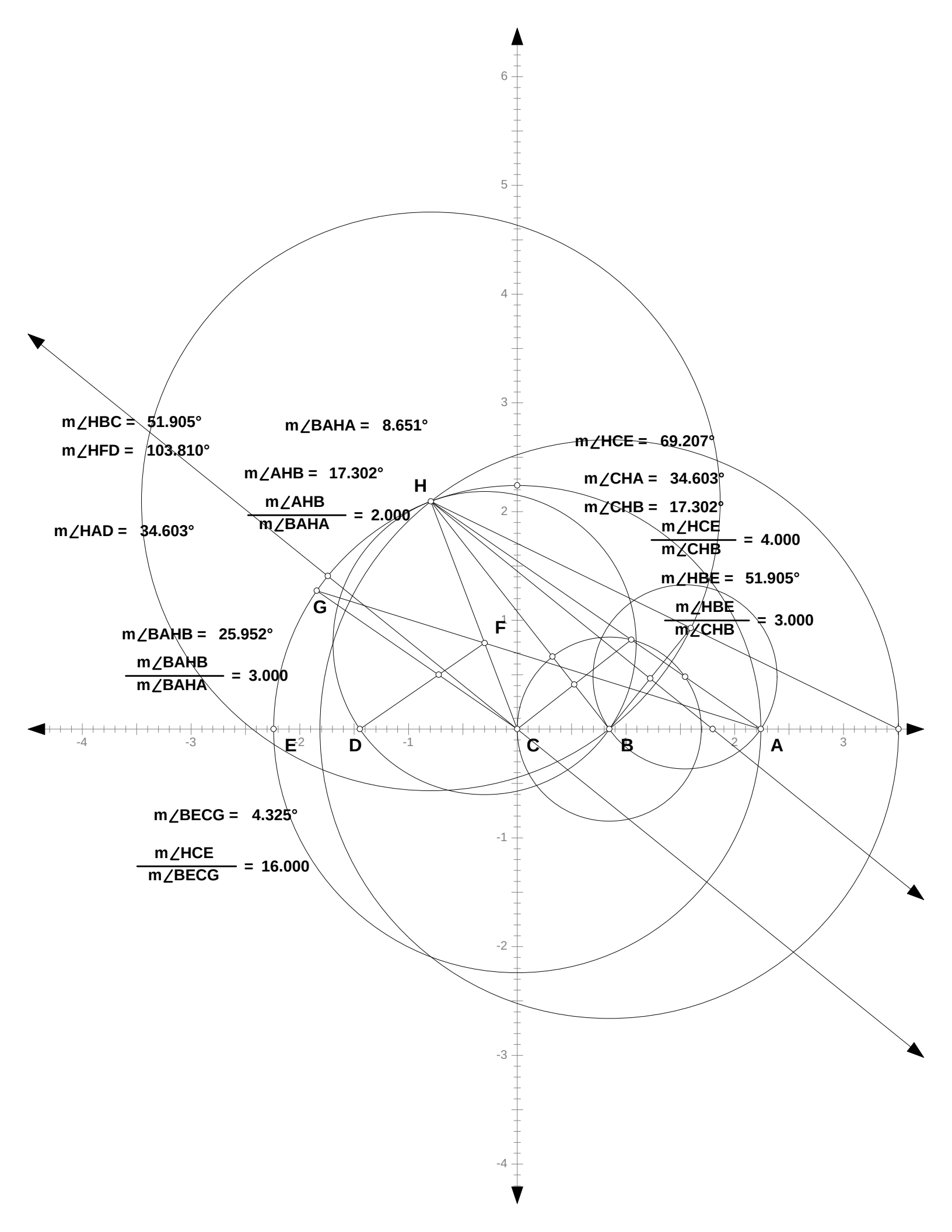

m∠JAC = 56.237°
5
4
3
2
1
-1
-2
-3
-4
-5
-6
-2
-1
1
2
3
4
5
6

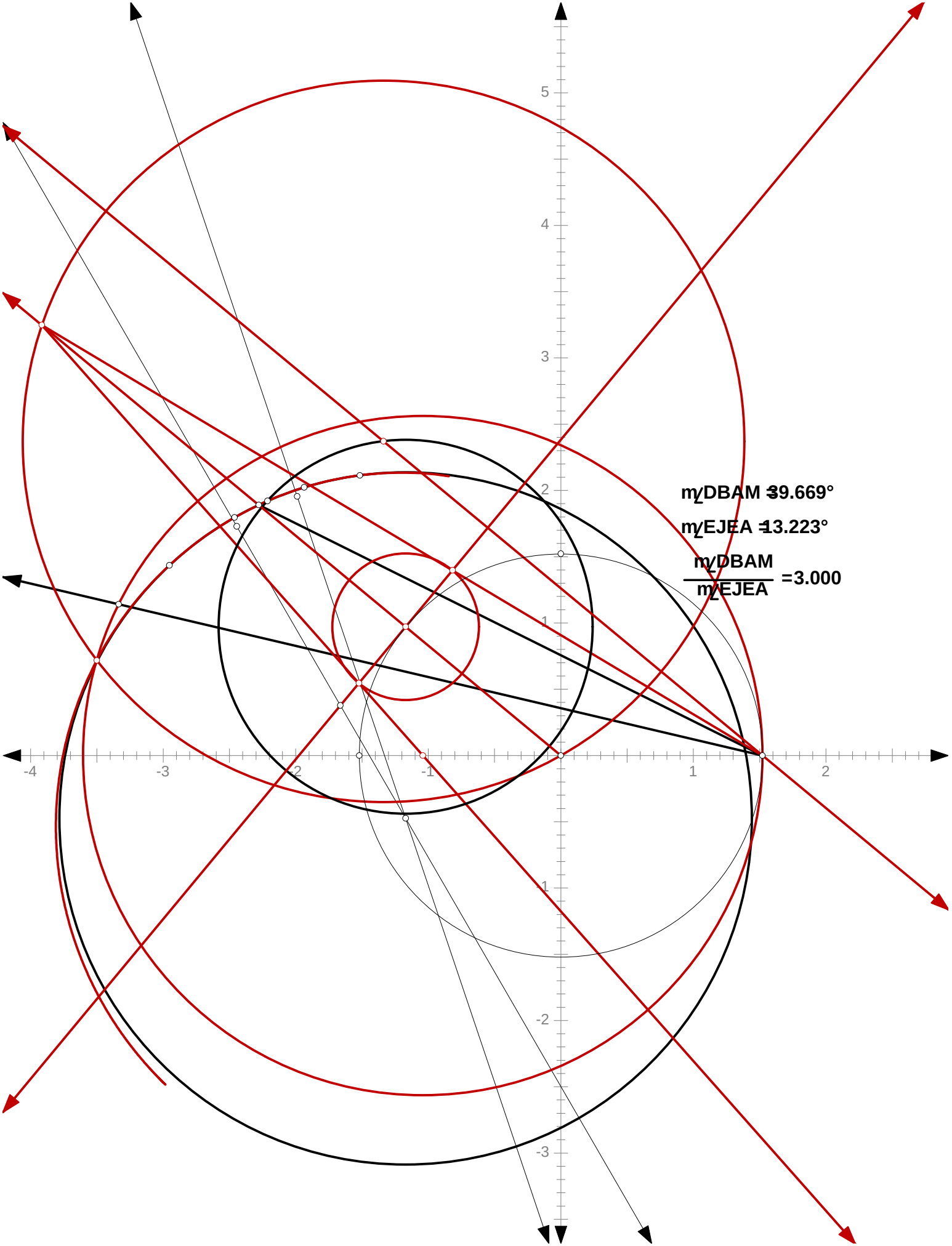

d
b
D
C
B

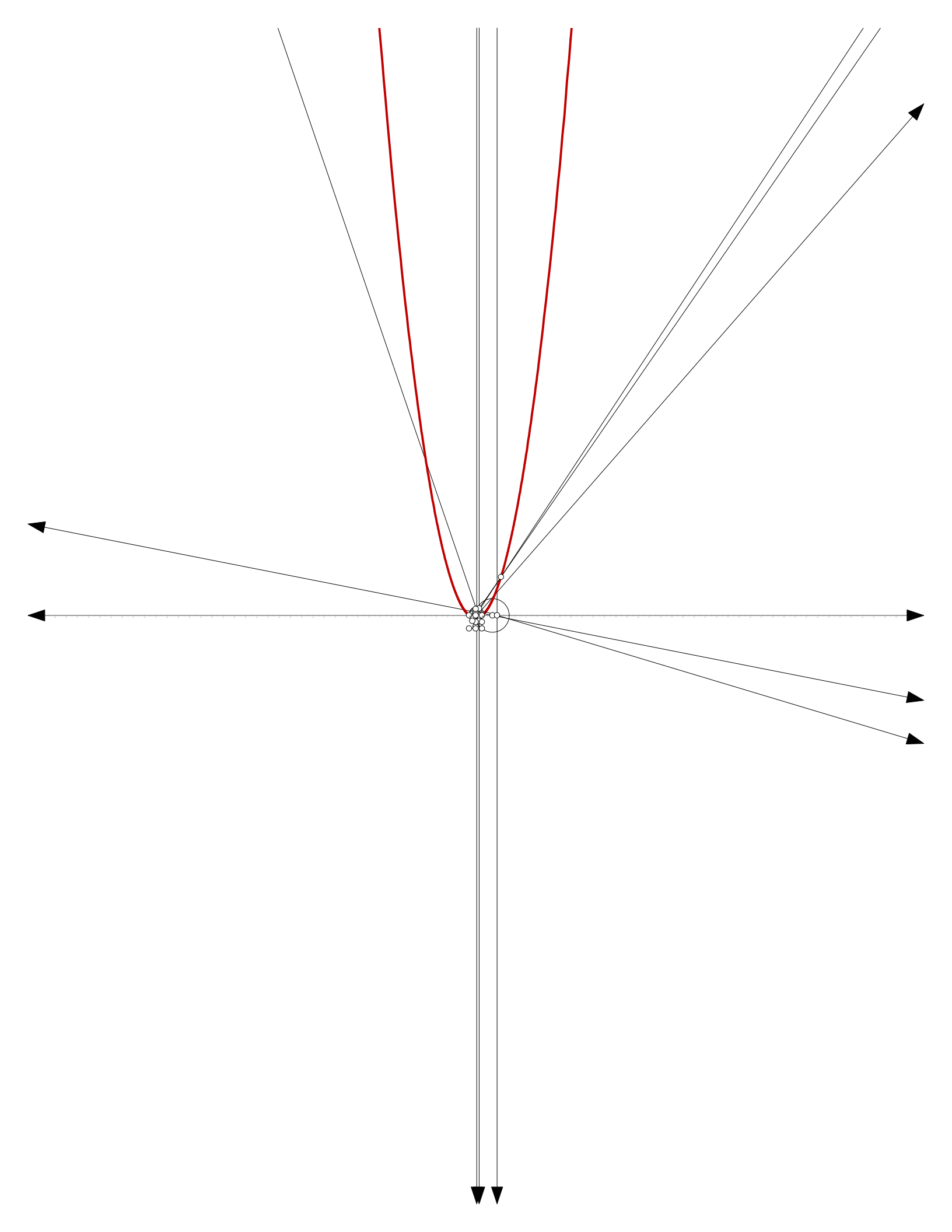

GHGD = 1.553 inches
GDET = 1.553 inches
D
B
-2
-1
1
2
3
4
5

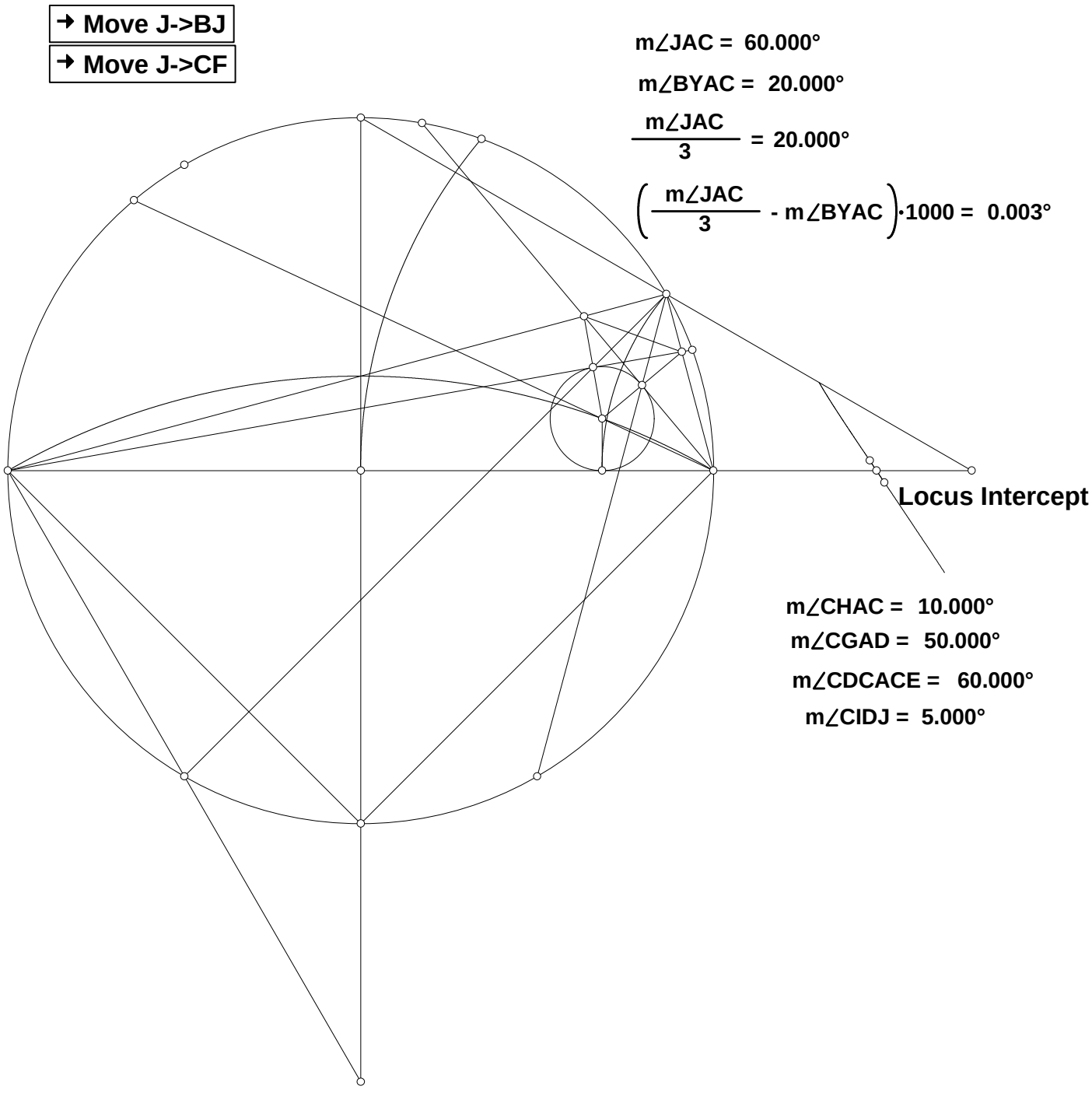

GHGD = 1.733 inches
GDET = 1.733 inches
D
B
-2
-1
1
2
3
4
5
6

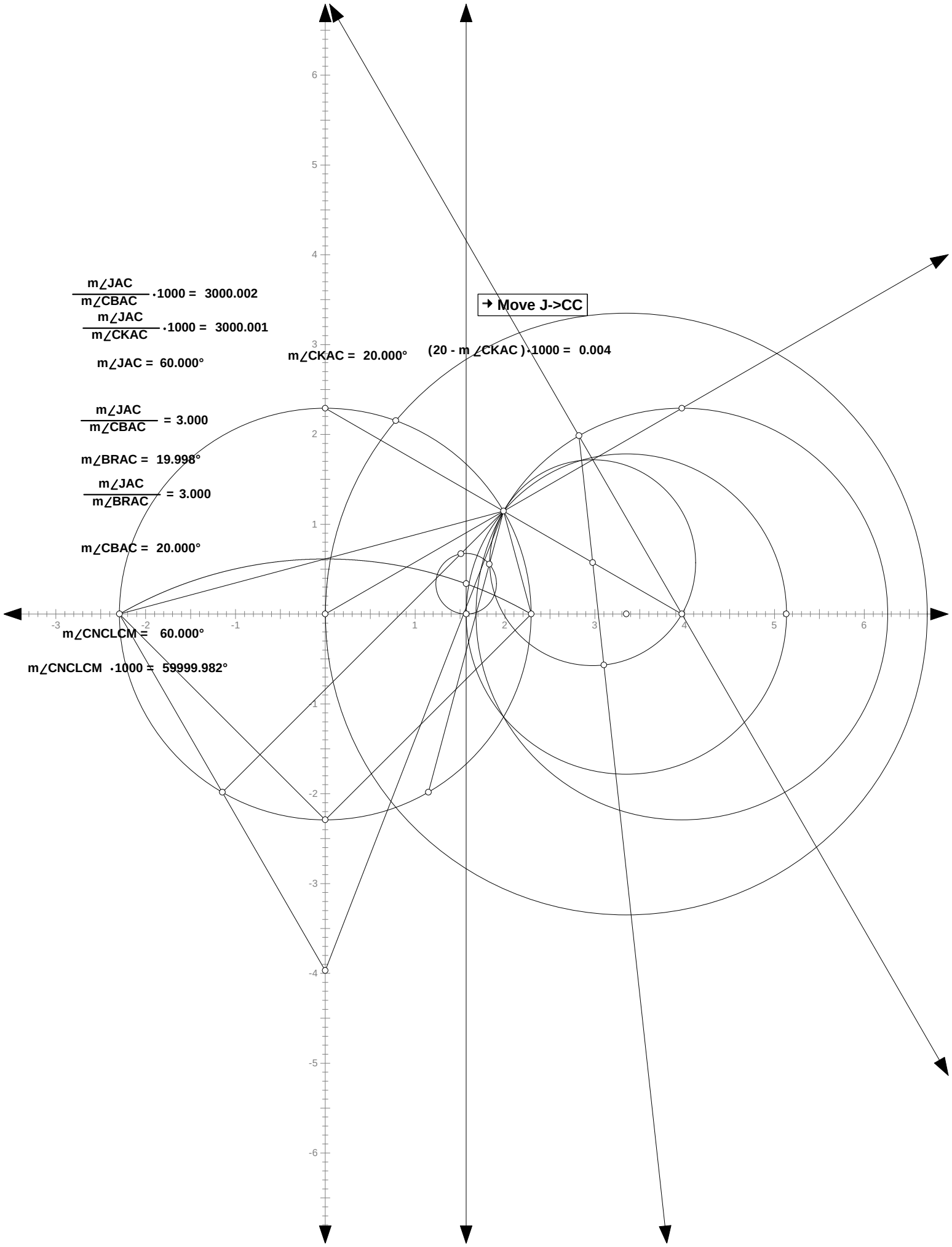

D
B
-2
-1
1
2
3
4
5
6

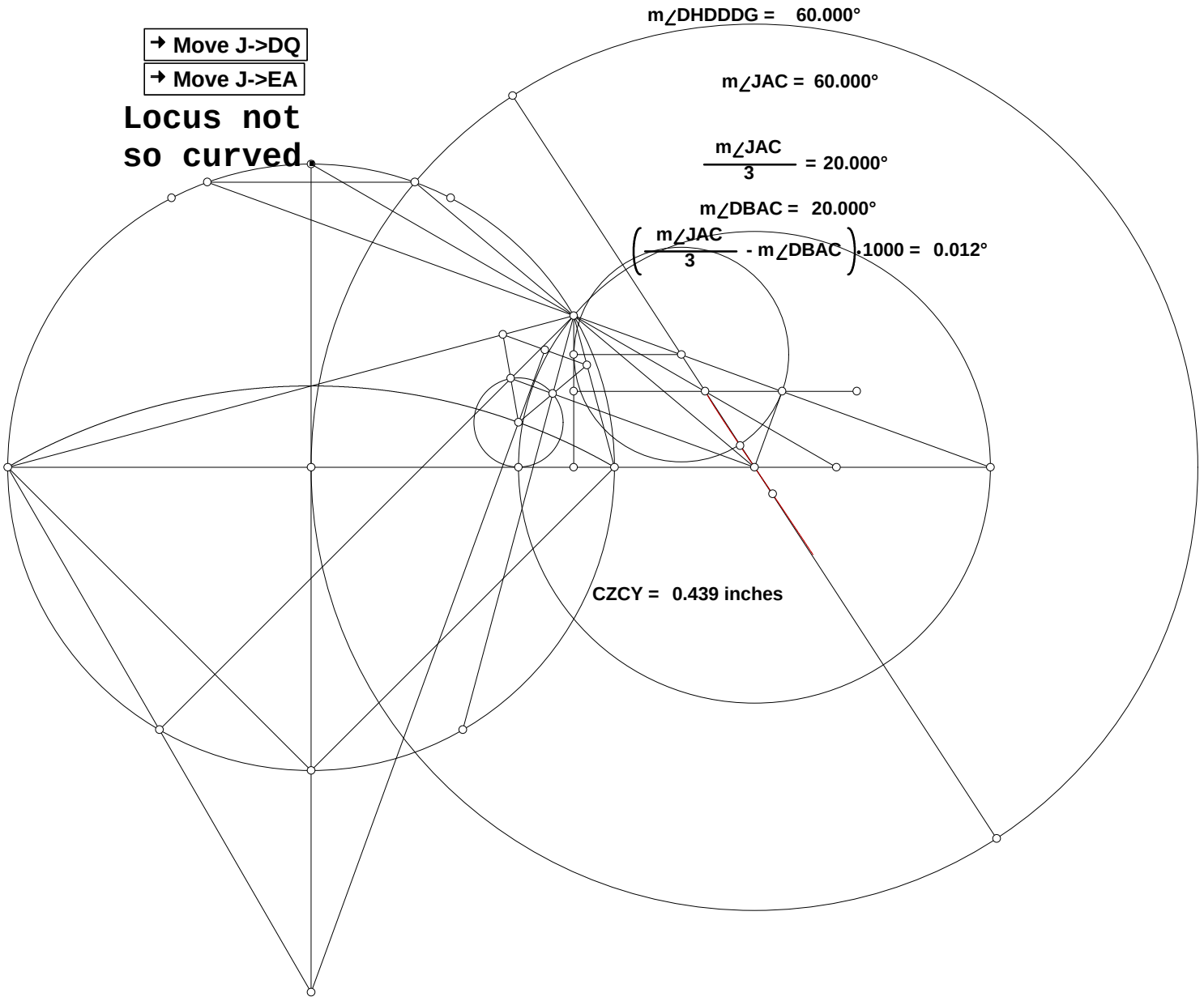

m∠HBC = 51.905°
m∠HFD = 103.810°
m∠BAHA = 8.651°
m∠AHB = 17.302°
m∠AHB / m∠BAHA = 2.000
m∠HAD = 34.603°
m∠HCE = 69.207°
m∠CHA = 34.603°
m∠CHB = 17.302°
m∠HCE / m∠CHB = 4.000
m∠HBE = 51.905°
m∠HBE / m∠CHB = 3.000
m∠BAHB = 25.952°
m∠BAHB / m∠BAHA = 3.000
m∠BECG = 4.325°
m∠HCE / m∠BECG = 16.000
H
G
F
E
D
C
B
A

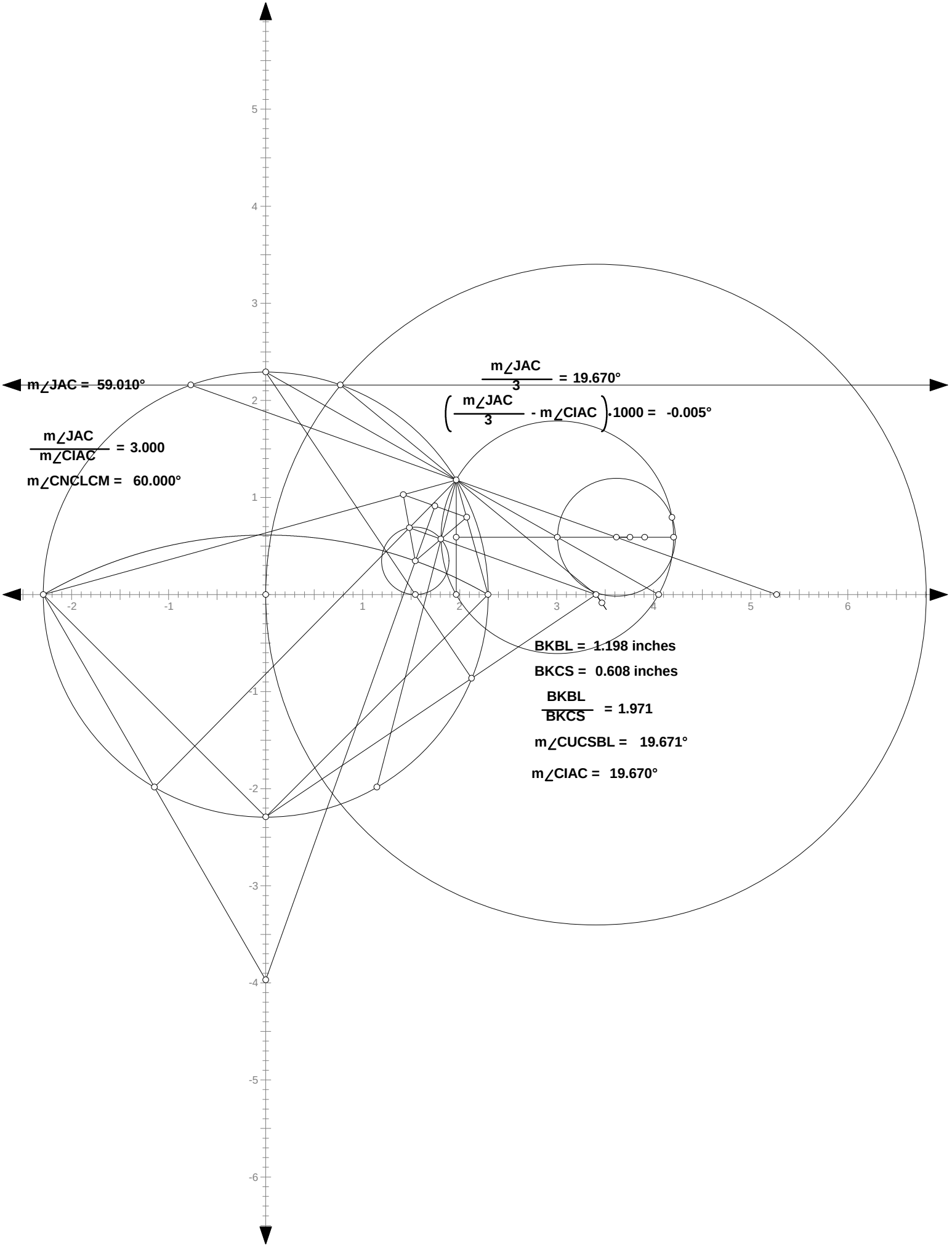

m∠DBAM =39.669°
m∠EJEA =13.223°
m∠DBAM / m∠EJEA =3.000

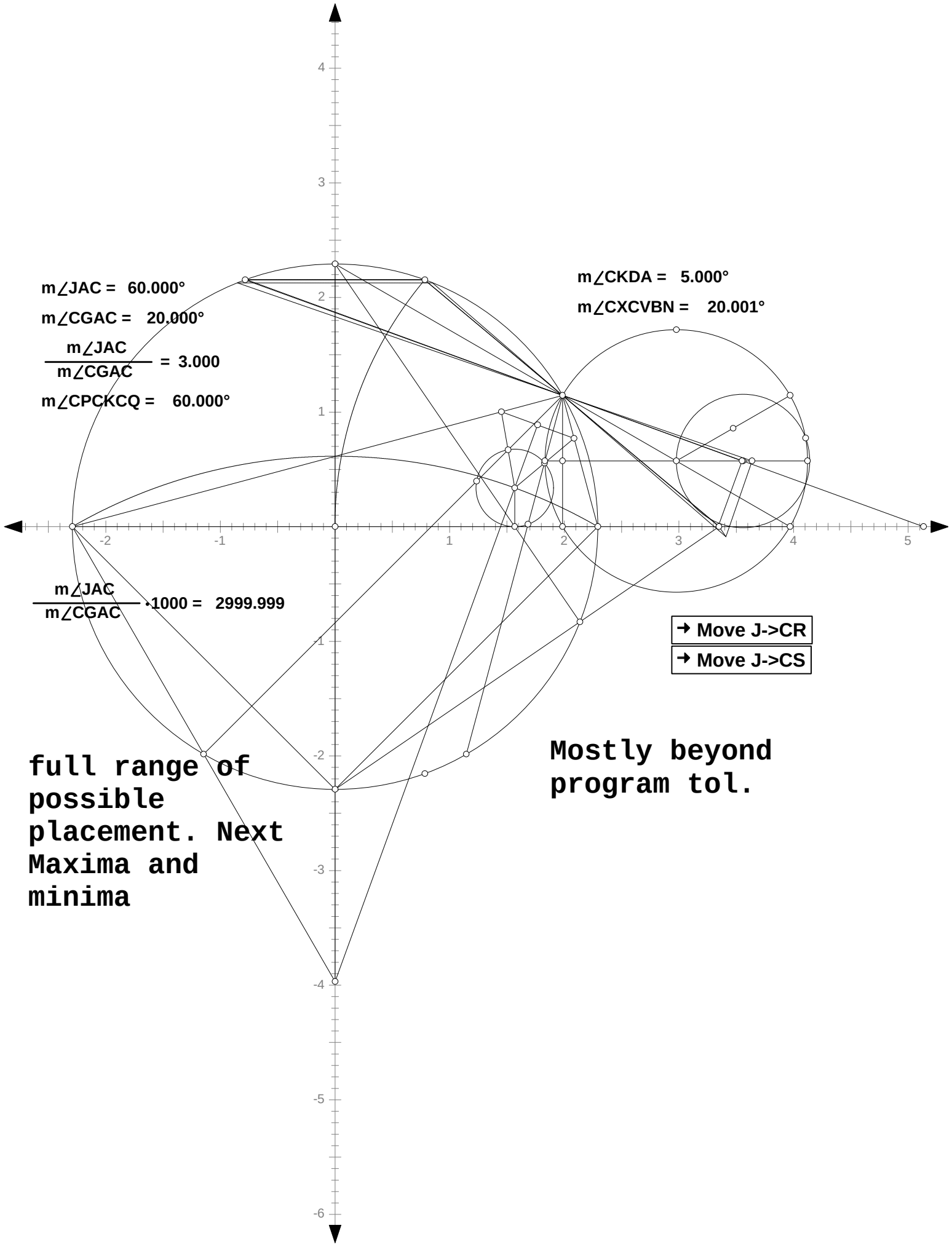


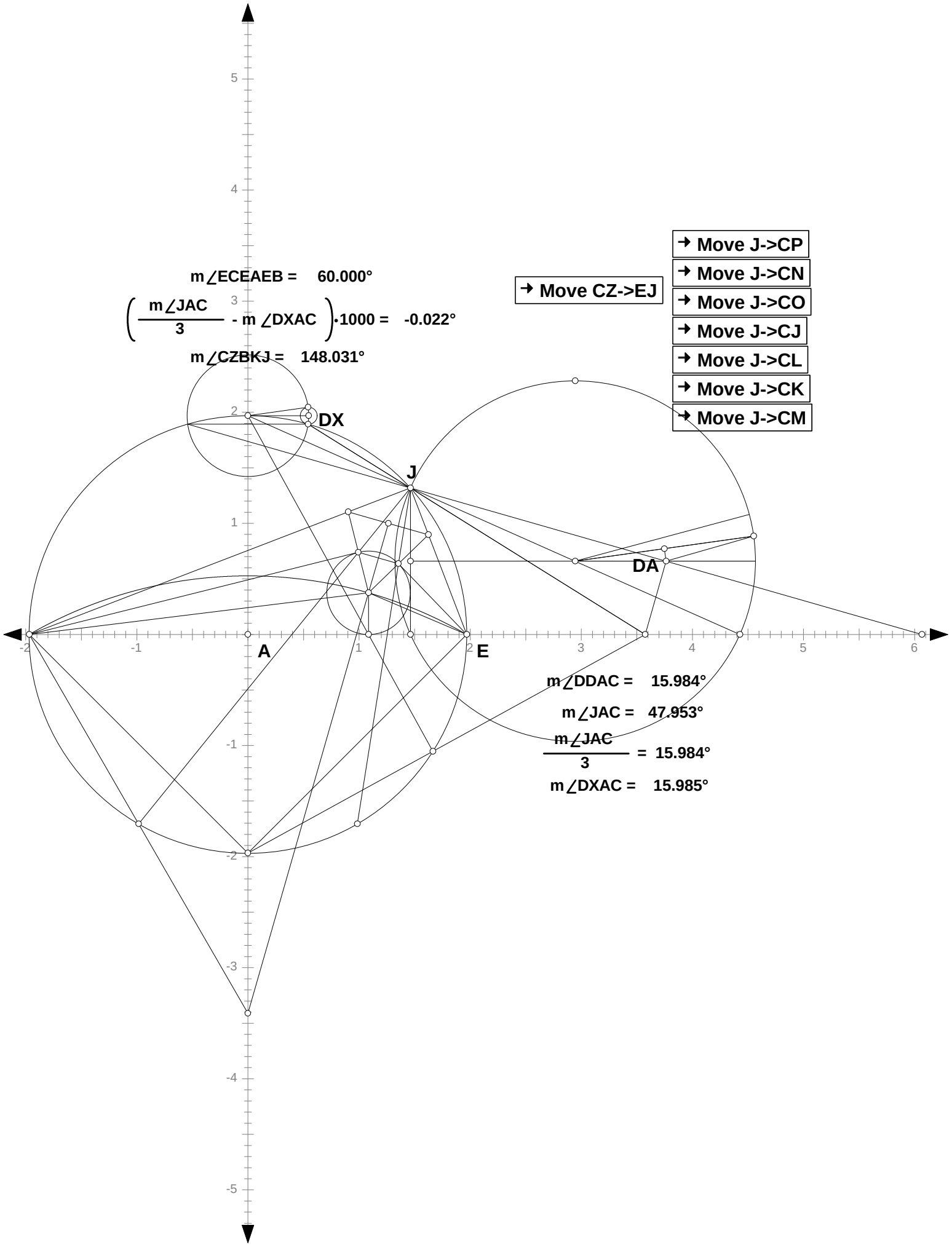

Move J->BJ
Move J->CF
m∠JAC = 60.000°
m∠BYAC = 20.000°
m∠JAC / 3 = 20.000°
( m∠JAC / 3 - m∠BYAC )·1000 = 0.003°
Locus Intercept
m∠CHAC = 10.000°
m∠CGAD = 50.000°
m∠CDCACE = 60.000°
m∠CIDJ = 5.000°

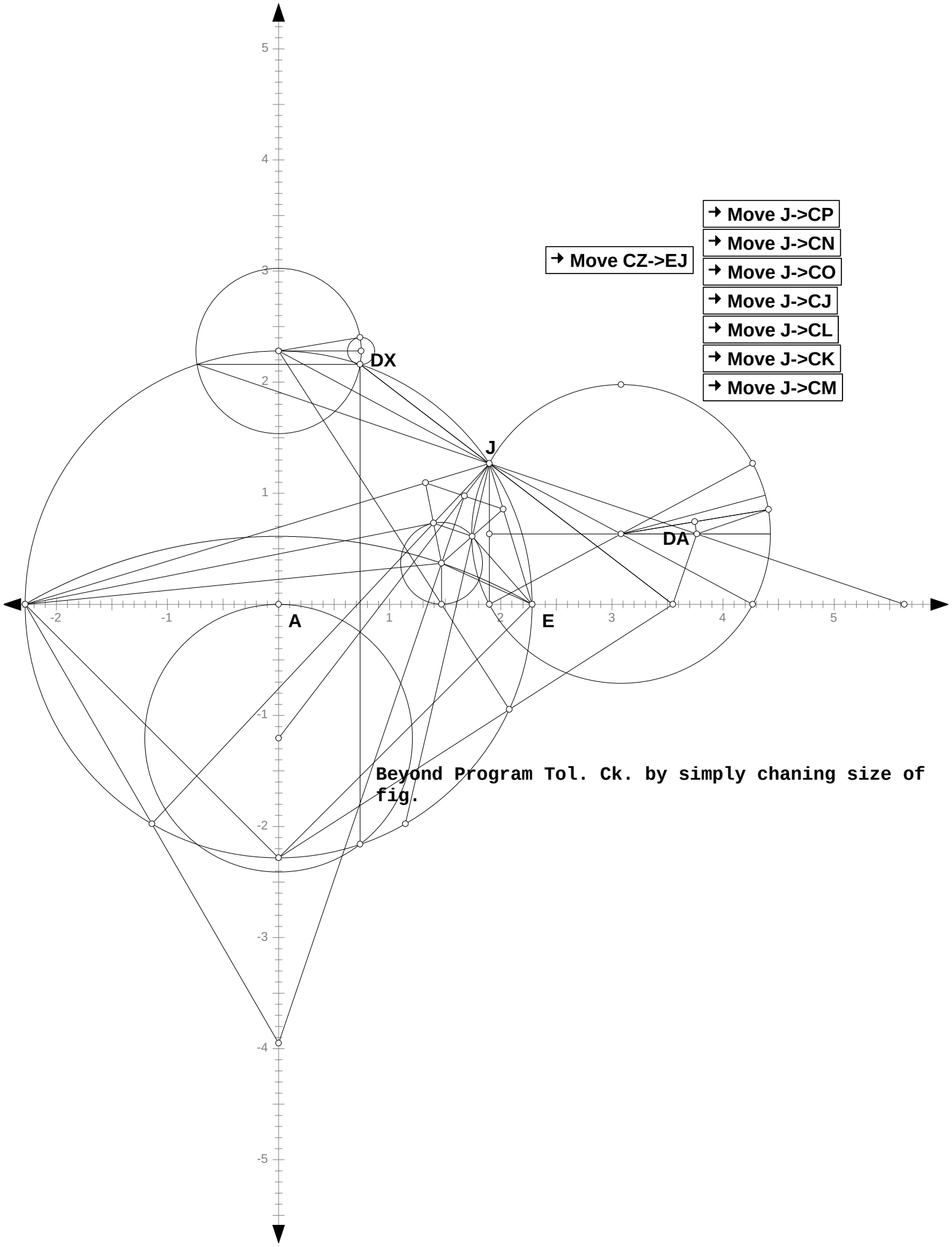

$\frac{m\angle JAC}{m\angle CBAC} \cdot 1000 = 3000.002$
$\frac{m\angle JAC}{m\angle CKAC} \cdot 1000 = 3000.001$
$m\angle JAC = 60.000°$
$\frac{m\angle JAC}{m\angle CBAC} = 3.000$
$m\angle BRAC = 19.998°$
$\frac{m\angle JAC}{m\angle BRAC} = 3.000$
$m\angle CBAC = 20.000°$
$m\angle CKAC = 20.000°$
$(20 - m\angle CKAC) \cdot 1000 = 0.004$
Move J->CC
$m\angle CNCLCM = 60.000°$
$m\angle CNCLCM \cdot 1000 = 59999.982°$

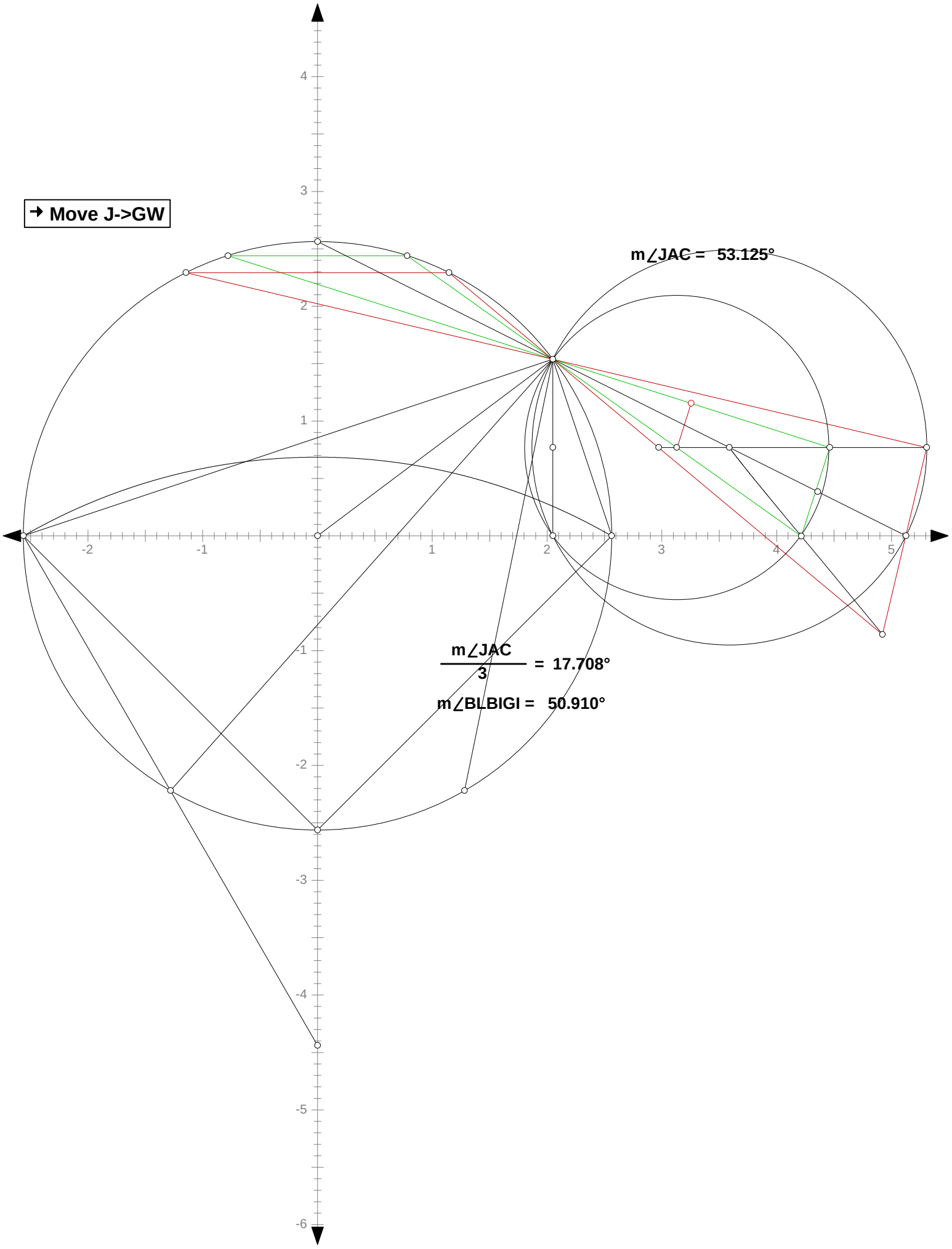

→ Move J->DQ
→ Move J->EA
Locus not so curved
m∠DHDDDG = 60.000°
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠DBAC = 20.000°
( m∠JAC / 3 - m∠DBAC ) ·1000 = 0.012°
CZCY = 0.439 inches

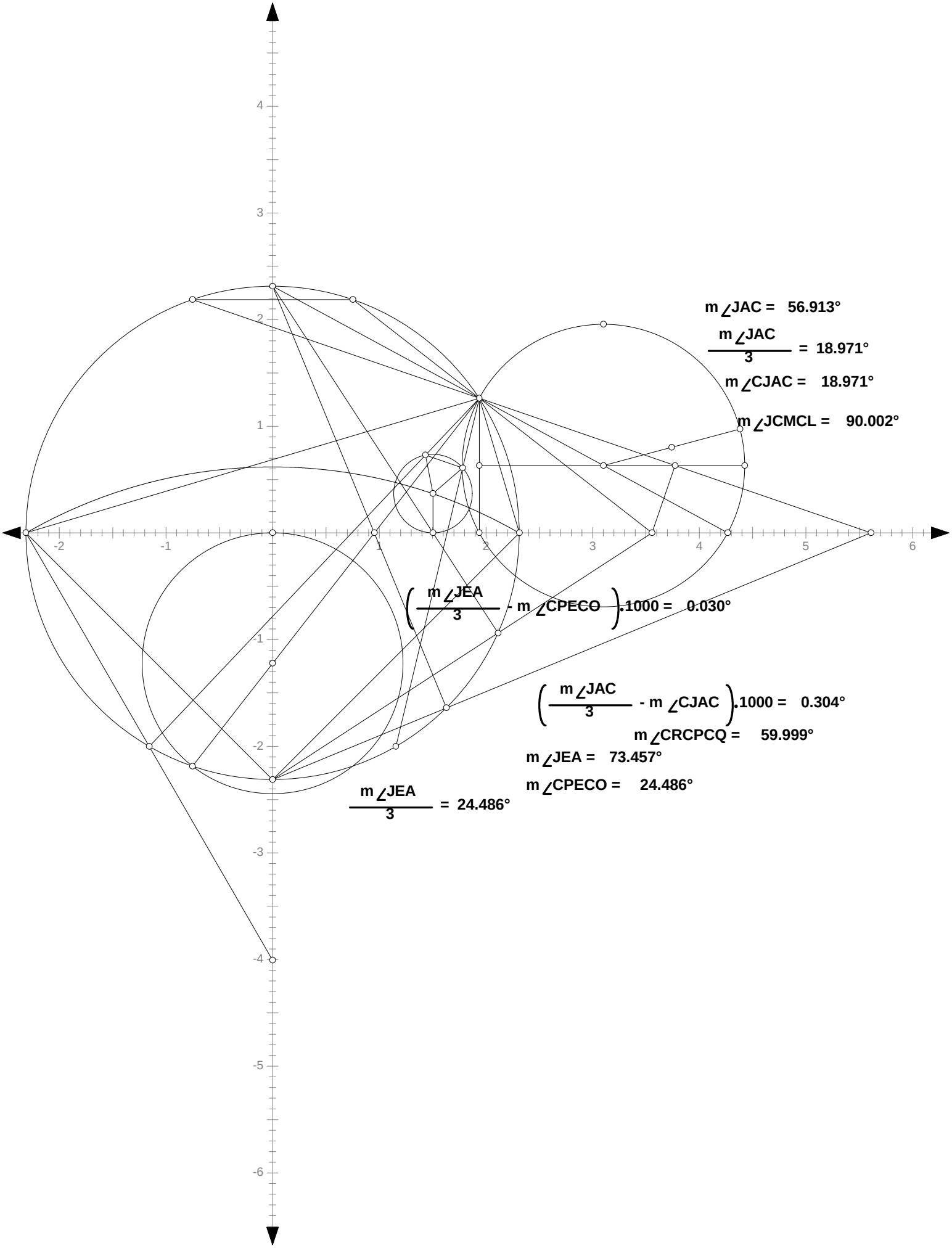

m∠JAC = 59.010°
m∠JAC / 3 = 19.670°
(m∠JAC / 3 - m∠CIAC)·1000 = -0.005°
m∠JAC / m∠CIAC = 3.000
m∠CNCLCM = 60.000°
BKBL = 1.198 inches
BKCS = 0.608 inches
BKBL / BKCS = 1.971
m∠CUCSBL = 19.671°
m∠CIAC = 19.670°

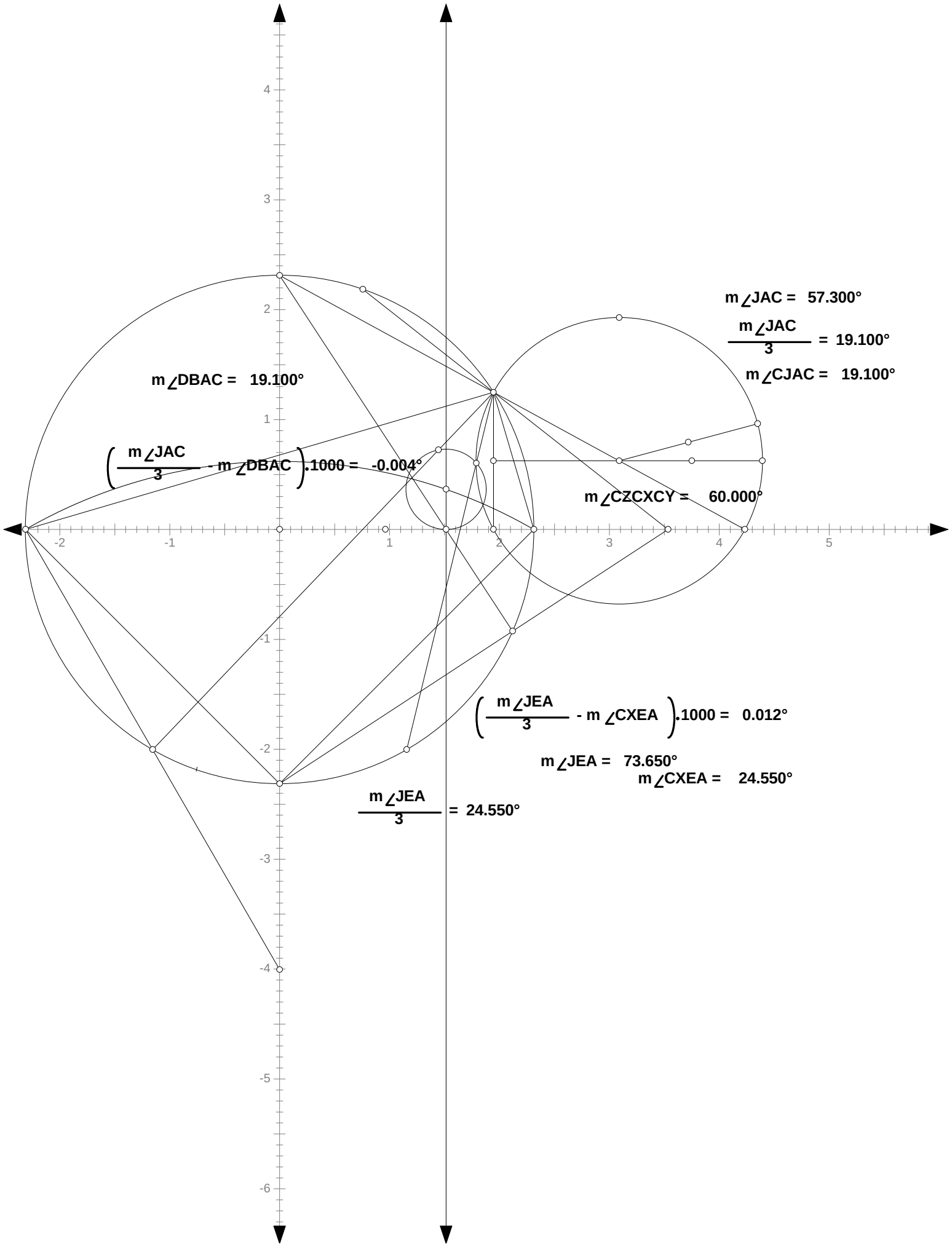

m∠JAC = 60.000°
m∠CGAC = 20.000°
m∠JAC / m∠CGAC = 3.000
m∠CPCKCQ = 60.000°
m∠CKDA = 5.000°
m∠CXCVBN = 20.001°
m∠JAC / m∠CGAC · 1000 = 2999.999
Move J->CR
Move J->CS
full range of possible placement. Next Maxima and minima
Mostly beyond program tol.

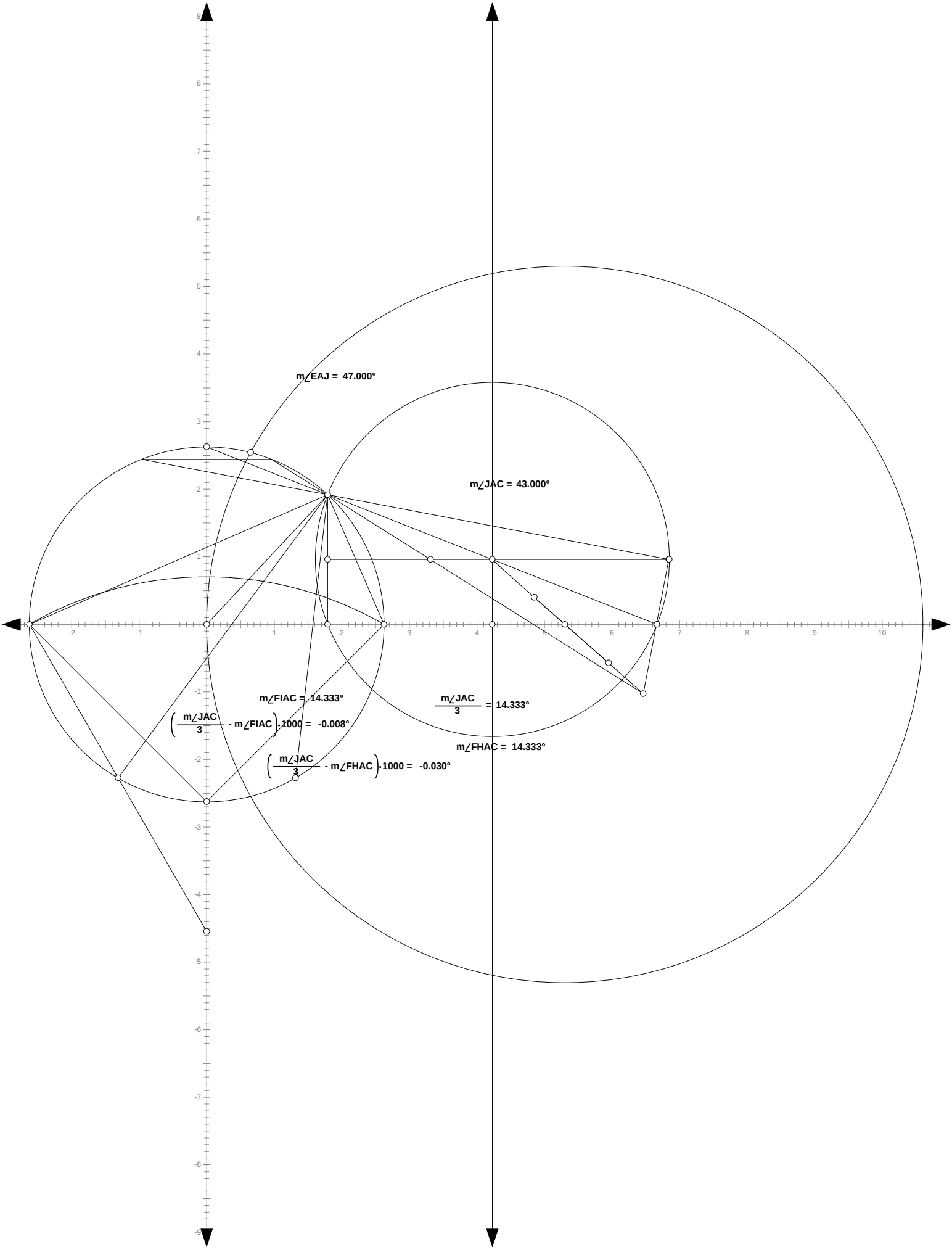

m∠ECEAEB = 60.000°
$\left(\frac{m\angle JAC}{3} - m\angle DXAC\right) \cdot 1000 = -0.022°$
m∠CZBKJ = 148.031°
Move CZ->EJ
Move J->CP
Move J->CN
Move J->CO
Move J->CJ
Move J->CL
Move J->CK
Move J->CM
DX
J
DA
A
E
m∠DDAC = 15.984°
m∠JAC = 47.953°
$\frac{m\angle JAC}{3} = 15.984°$
m∠DXAC = 15.985°

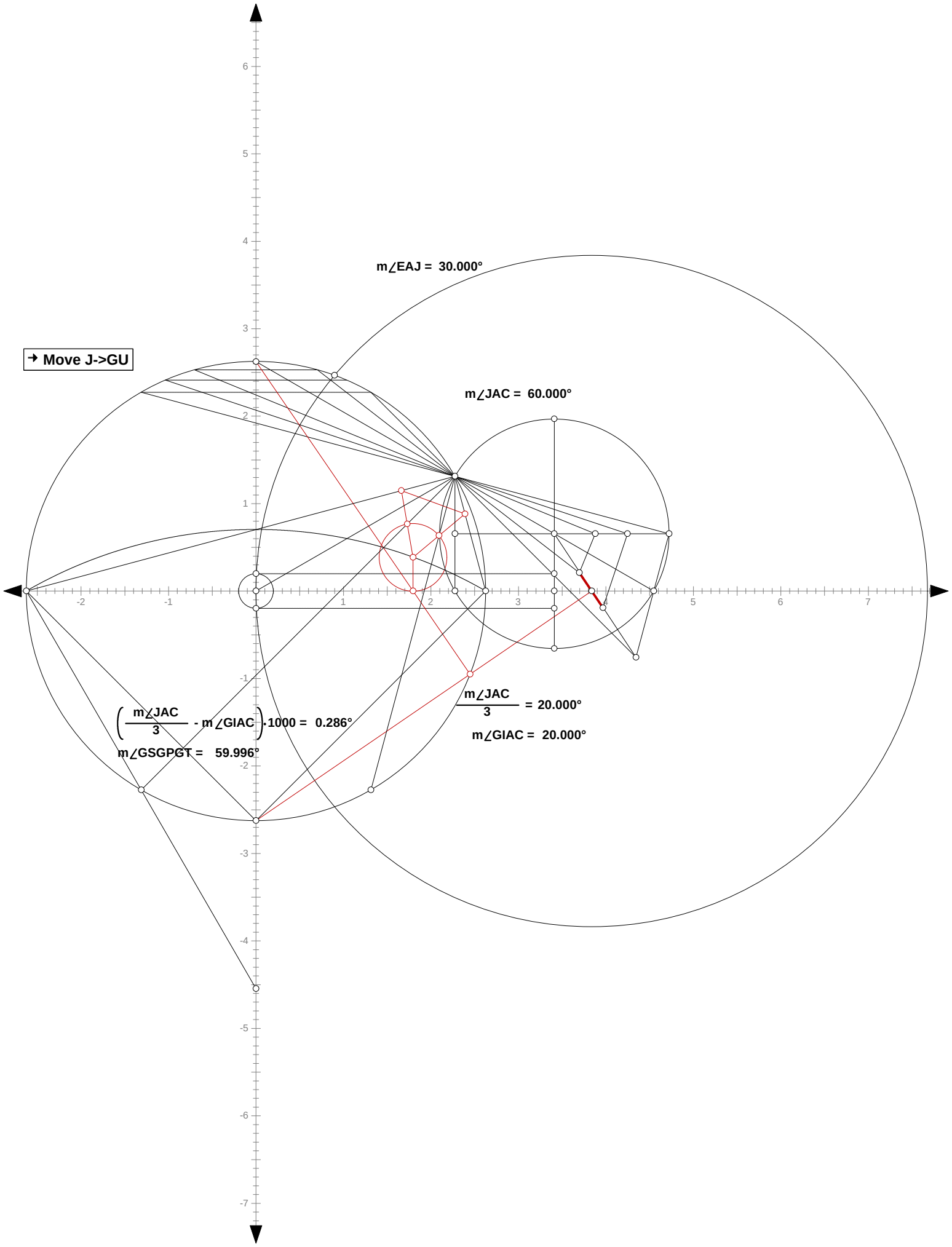

Move J->CP
Move J->CN
Move J->CO
Move J->CJ
Move J->CL
Move J->CK
Move J->CM
Move CZ->EJ
DX
J
DA
A
E
Beyond Program Tol. Ck. by simply chaning size of fig.

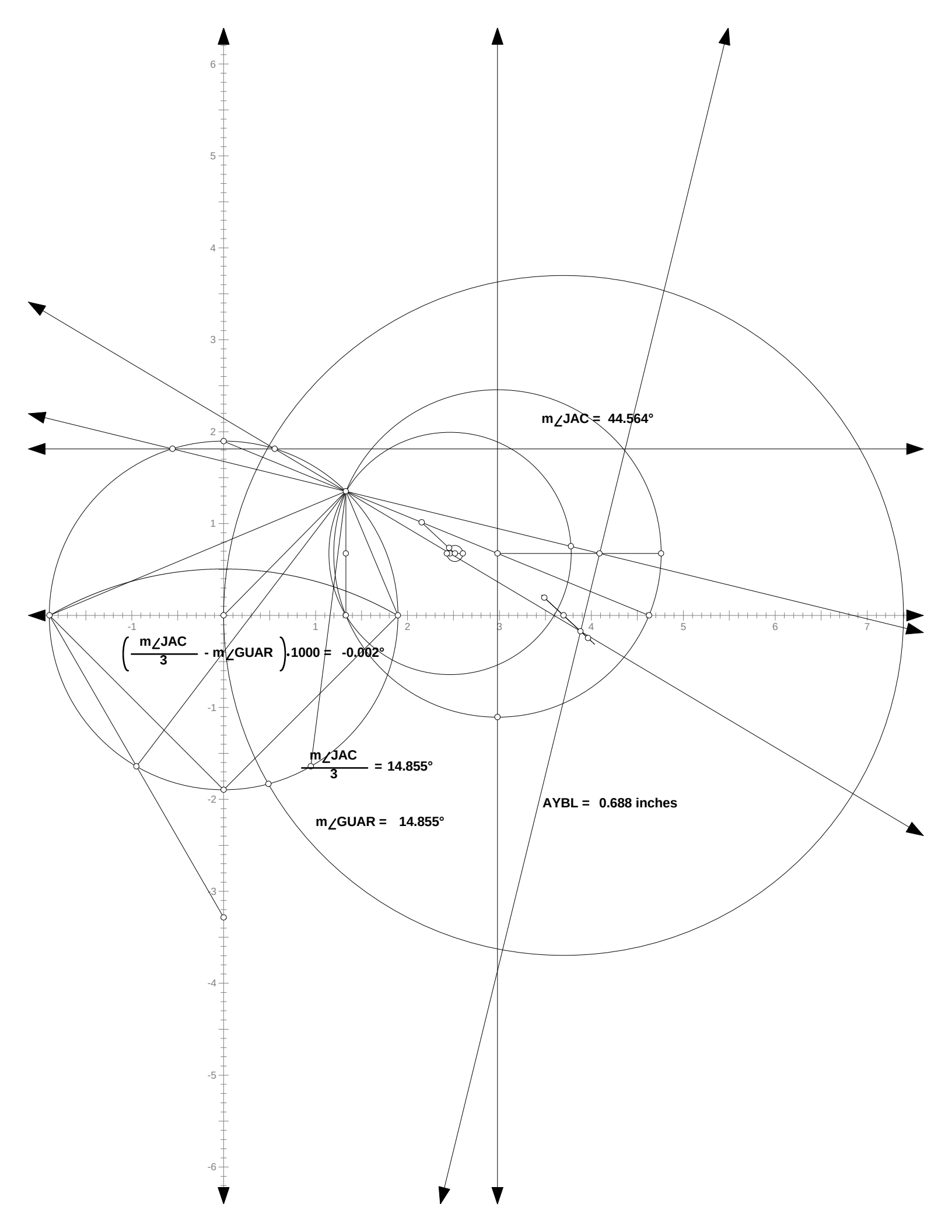

Move J->GW
m∠JAC = 53.125°
m∠JAC / 3 = 17.708°
m∠BLBIGI = 50.910°

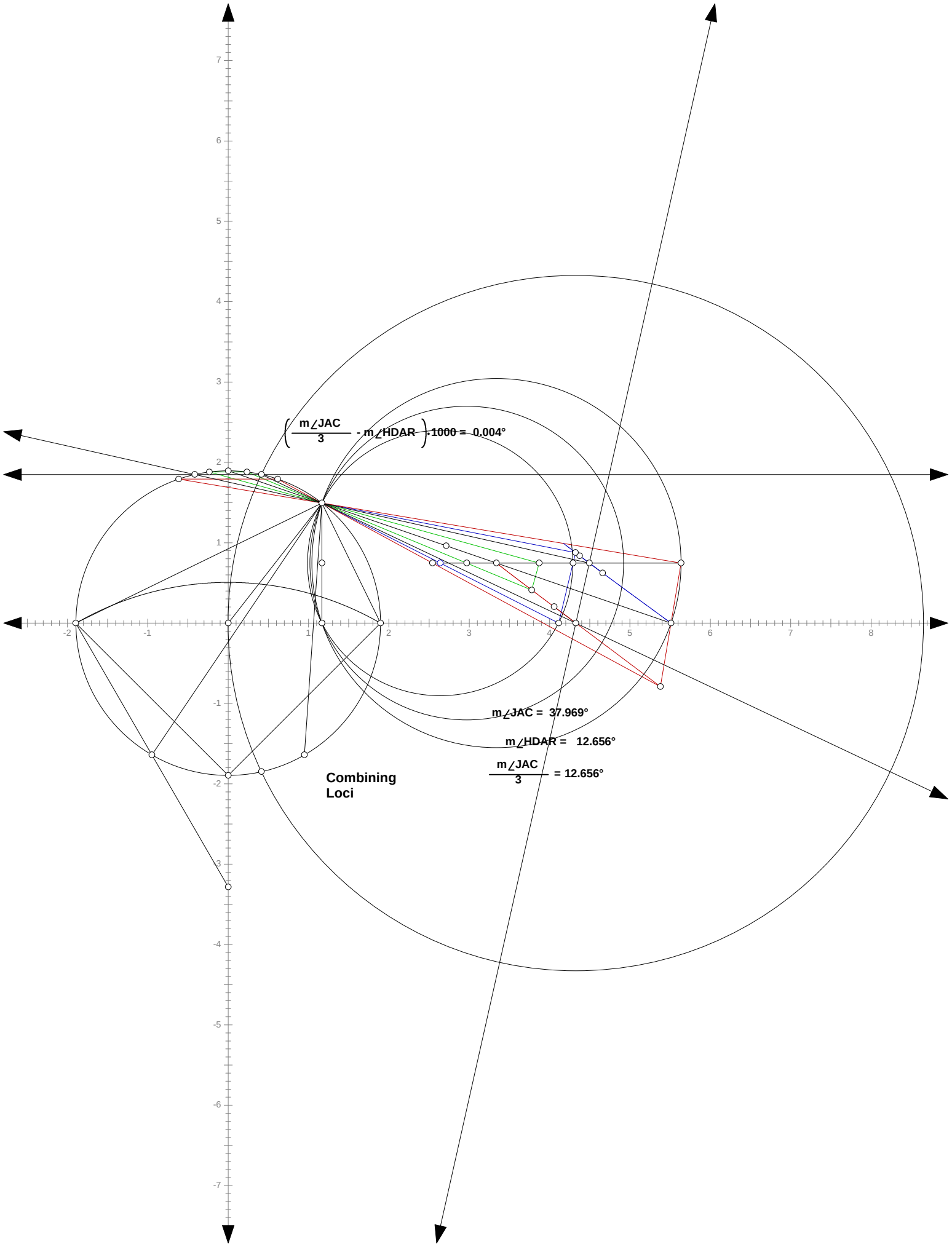

m∠JAC = 56.913°
m∠JAC / 3 = 18.971°
m∠CJAC = 18.971°
m∠JCMCL = 90.002°
(m∠JEA / 3 - m∠CPECO).1000 = 0.030°
(m∠JAC / 3 - m∠CJAC).1000 = 0.304°
m∠CRCPCQ = 59.999°
m∠JEA = 73.457°
m∠CPECO = 24.486°
m∠JEA / 3 = 24.486°


m∠JAC = 57.300°
m∠JAC / 3 = 19.100°
m∠CJAC = 19.100°
m∠DBAC = 19.100°
(m∠JAC / 3 - m∠DBAC).1000 = -0.004°
m∠CZCXCY = 60.000°
(m∠JEA / 3 - m∠CXEA).1000 = 0.012°
m∠JEA = 73.650°
m∠CXEA = 24.550°
m∠JEA / 3 = 24.550°


m∠EAJ = 47.000°
m∠JAC = 43.000°
m∠FIAC = 14.333°
m∠JAC / 3 = 14.333°
(m∠JAC / 3 - m∠FIAC)·1000 = -0.008°
m∠FHAC = 14.333°
(m∠JAC / 3 - m∠FHAC)·1000 = -0.030°


m∠EAJ = 30.000°
Move J->GU
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠GIAC = 20.000°
(m∠JAC / 3 - m∠GIAC) · 1000 = 0.286°
m∠GSGPGT = 59.996°


m∠JAC = 44.564°
$\left(\frac{m\angle JAC}{3} - m\angle GUAR\right) \cdot 1000 = -0.002°$
$\frac{m\angle JAC}{3} = 14.855°$
m∠GUAR = 14.855°
AYBL = 0.688 inches


$\left(\frac{m\angle JAC}{3} - m\angle HDAR\right) \cdot 1000 = 0.004°$
$m\angle JAC = 37.969°$
$m\angle HDAR = 12.656°$
$\frac{m\angle JAC}{3} = 12.656°$
Combining Loci

**Since the trisection is perfect, one can examine the program limitations.**

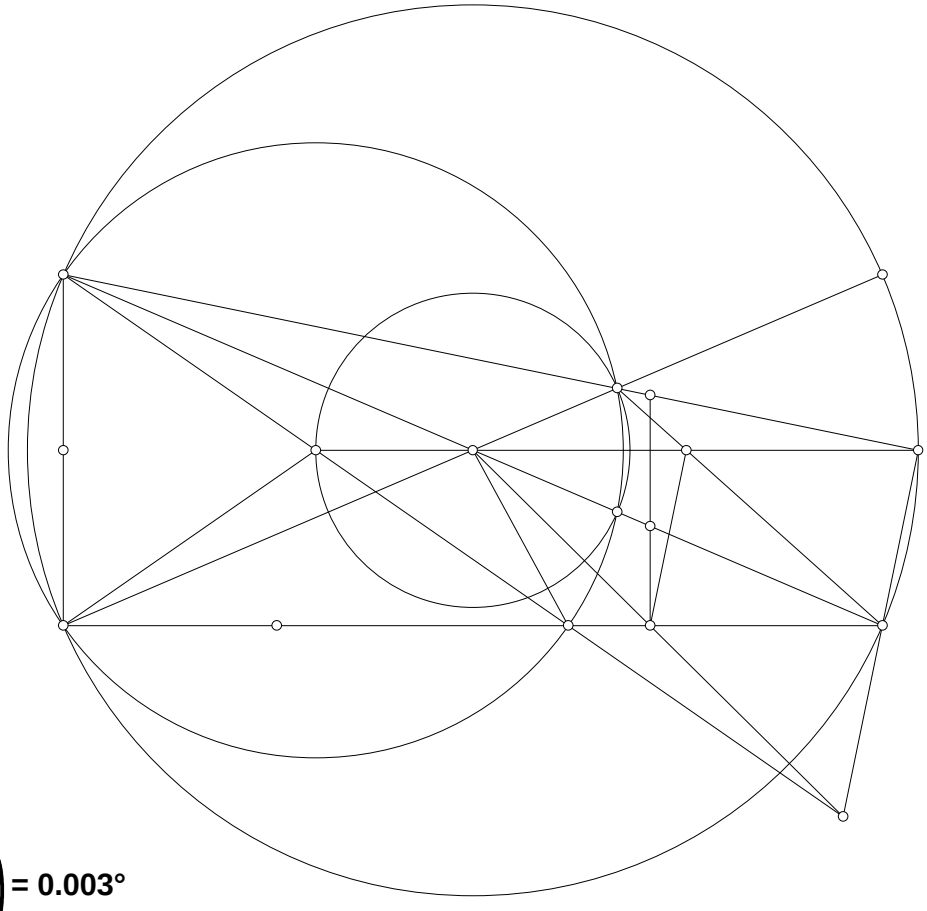
$m\angle UCE = 11.601°$

$\frac{m\angle DUC}{3} = 11.601°$

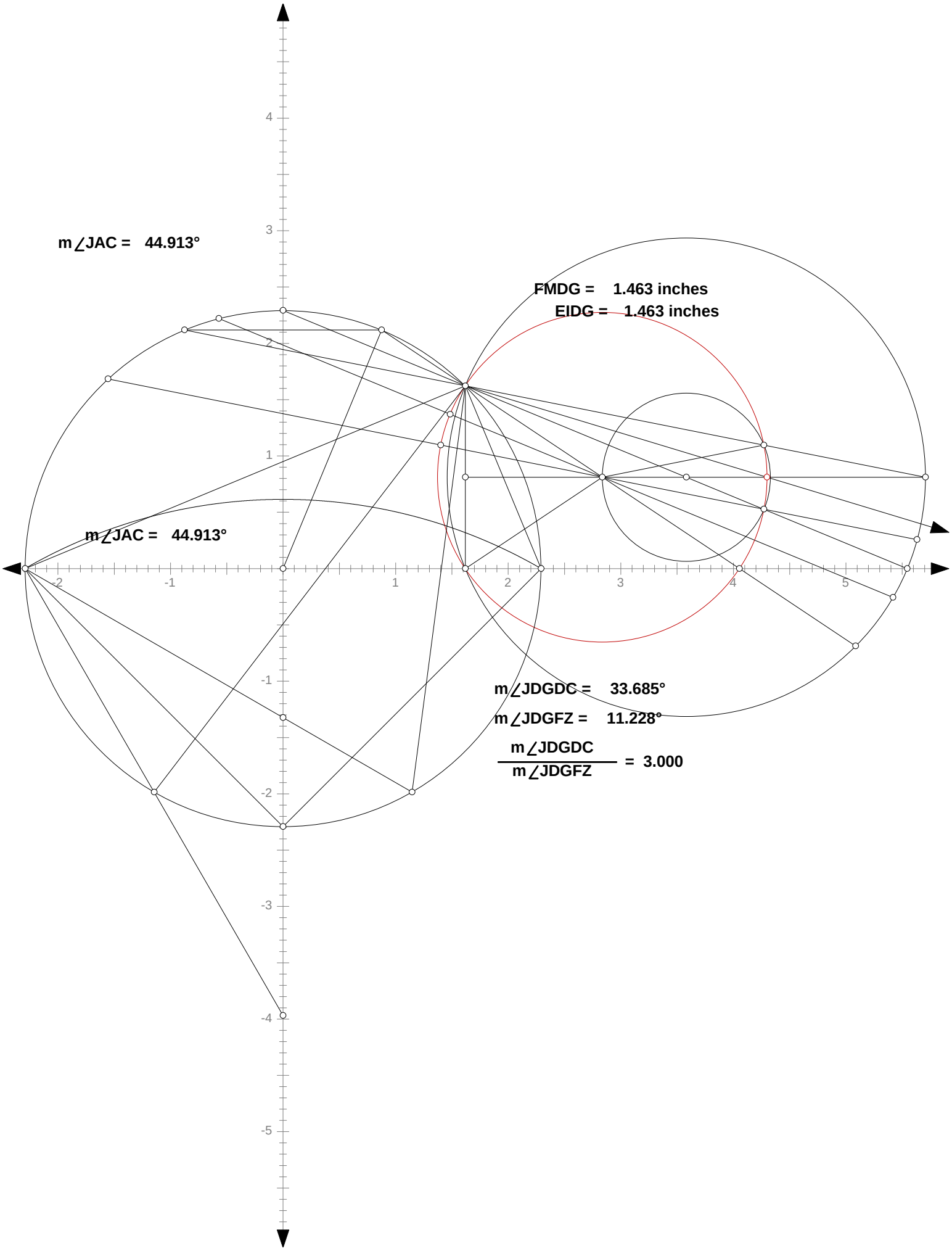
$m\angle ATAMAS = 11.601°$

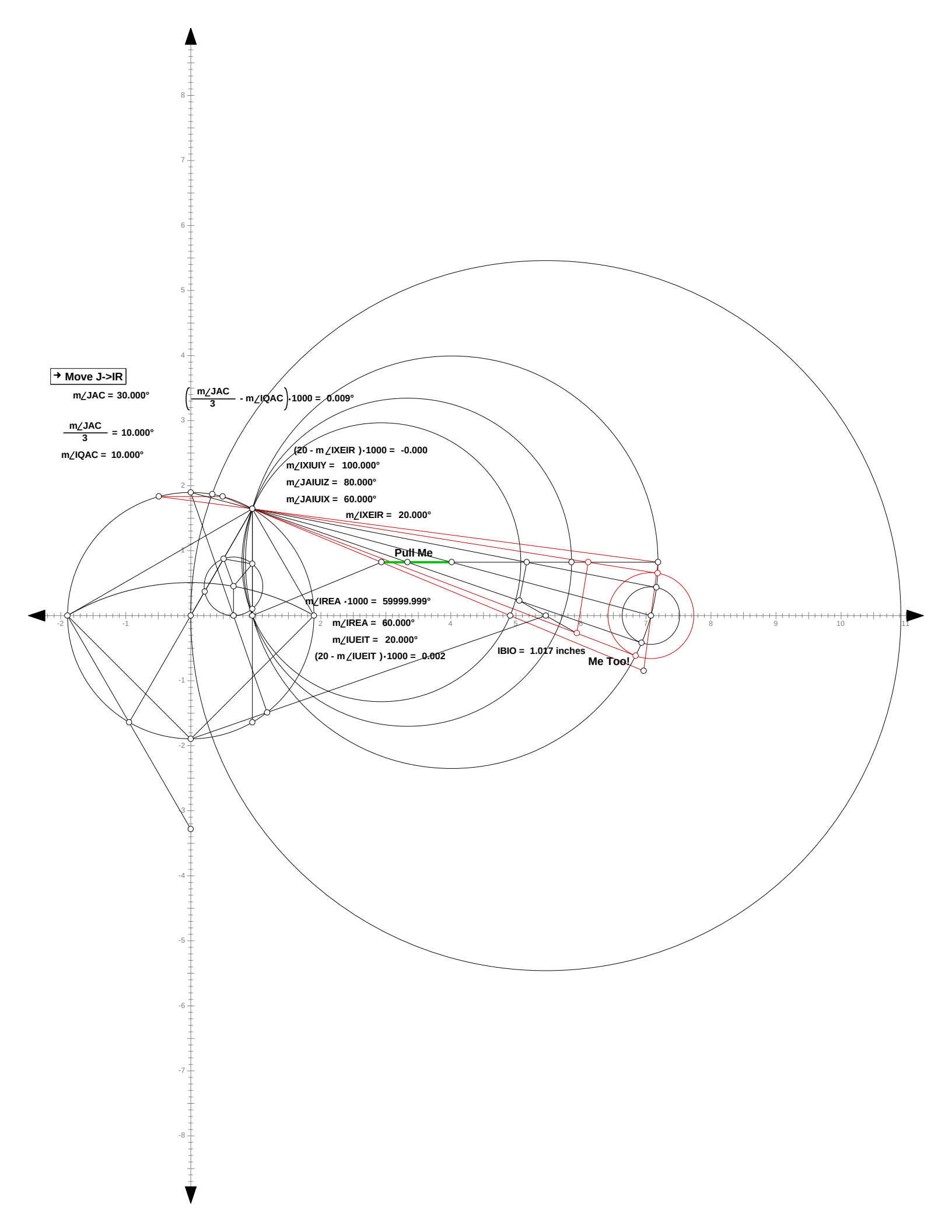
$m\angle DUC = 34.802°$

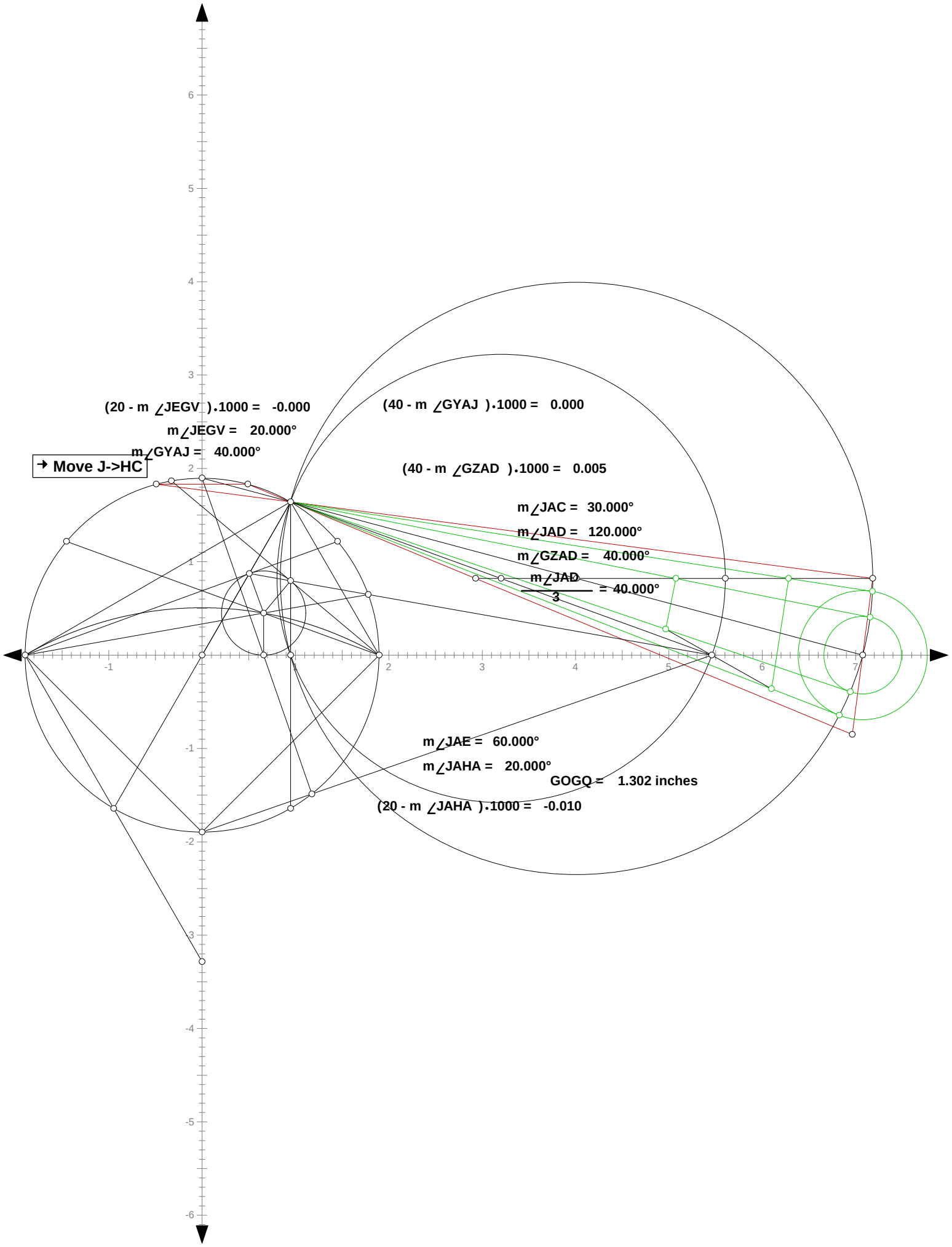
$\frac{m\angle DUC}{m\angle ATAMAS} = 3.000$

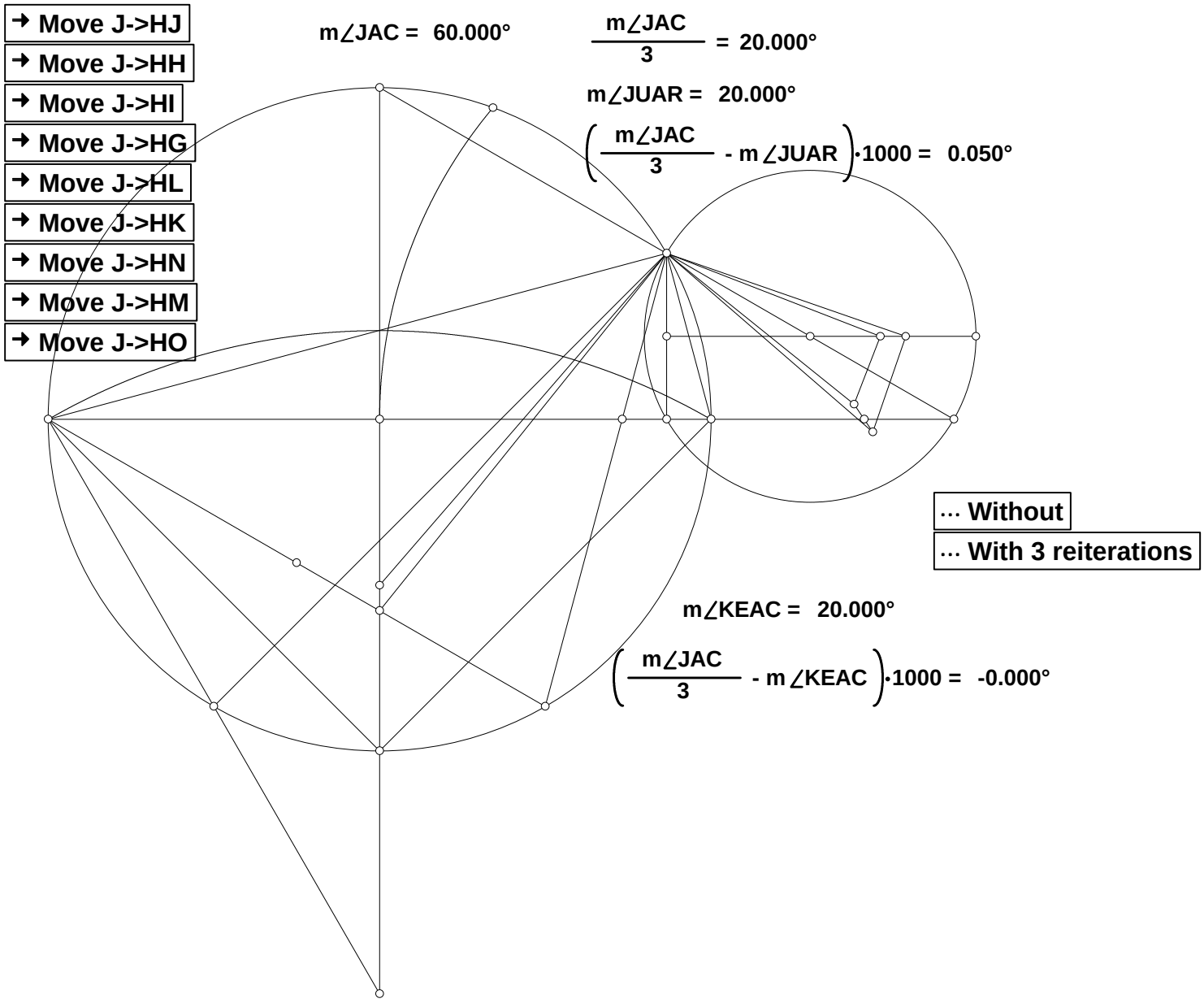


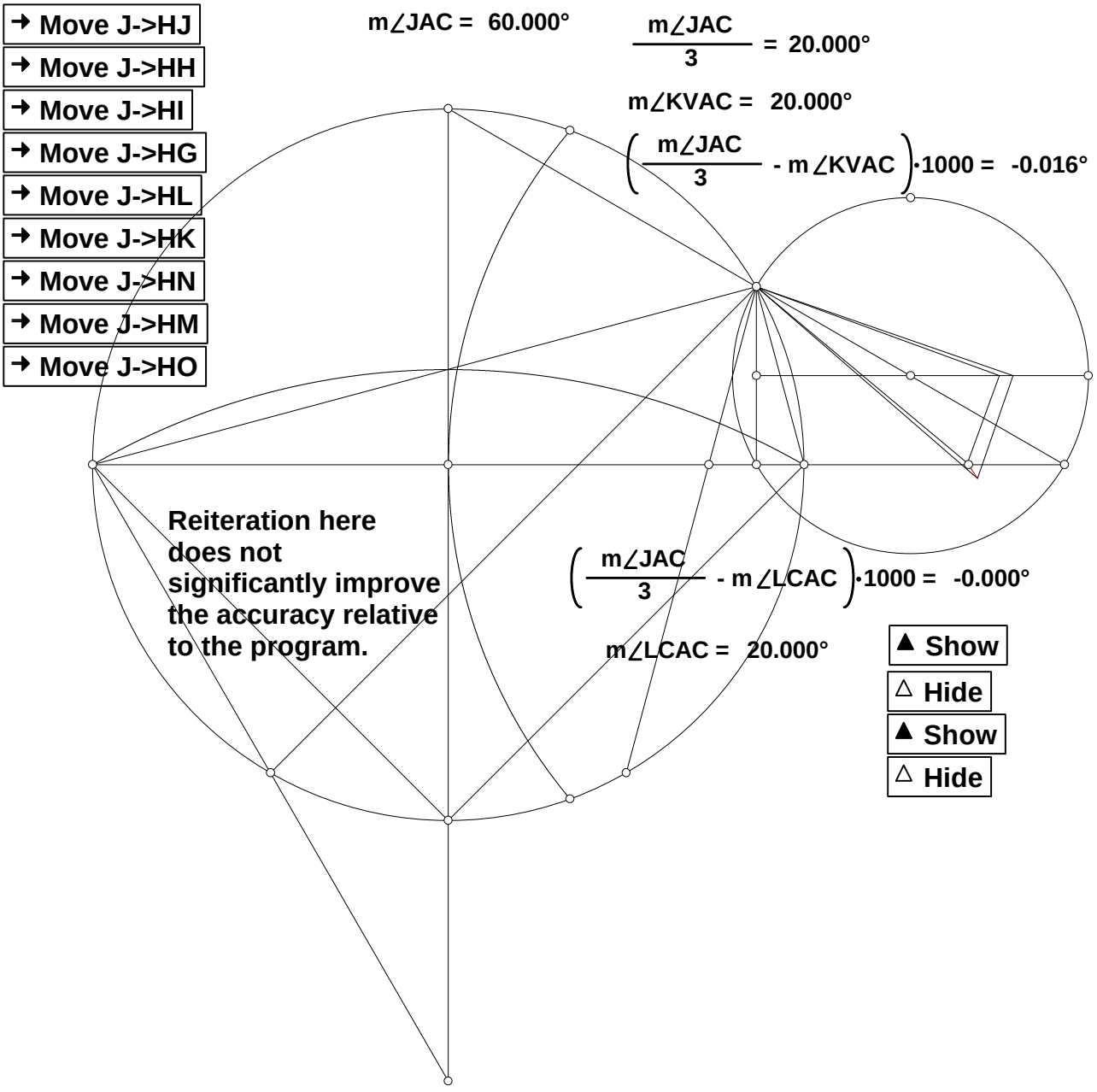
$\left(\left(\frac{m\angle DUC}{3} - m\angle ATAMAS\right).1000\right) = 0.003°$


m∠JAC = 44.913°
FMDG = 1.463 inches
EIDG = 1.463 inches
m∠JAC = 44.913°
m∠JDGDC = 33.685°
m∠JDGFZ = 11.228°
m∠JDGDC / m∠JDGFZ = 3.000


Move J->IR
m∠JAC = 30.000°
m∠JAC / 3 = 10.000°
m∠IQAC = 10.000°
(m∠JAC / 3 - m∠IQAC)·1000 = 0.009°
(20 - m∠IXEIR )·1000 = -0.000
m∠IXIUIY = 100.000°
m∠JAIUIZ = 80.000°
m∠JAIUIX = 60.000°
m∠IXEIR = 20.000°
Pull Me
m∠IREA ·1000 = 59999.999°
m∠IREA = 60.000°
m∠IUEIT = 20.000°
(20 - m∠IUEIT )·1000 = 0.002
IBIO = 1.017 inches
Me Too!


(20 - m ∠JEGV ).1000 = -0.000
m∠JEGV = 20.000°
m∠GYAJ = 40.000°
Move J->HC
(40 - m ∠GYAJ ).1000 = 0.000
(40 - m ∠GZAD ).1000 = 0.005
m∠JAC = 30.000°
m∠JAD = 120.000°
m∠GZAD = 40.000°
m∠JAD / 3 = 40.000°
m∠JAE = 60.000°
m∠JAHA = 20.000°
GOGQ = 1.302 inches
(20 - m ∠JAHA ).1000 = -0.010


→ Move J->HJ
→ Move J->HH
→ Move J->HI
→ Move J->HG
→ Move J->HL
→ Move J->HK
→ Move J->HN
→ Move J->HM
→ Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JUAR = 20.000°
( m∠JAC / 3 - m∠JUAR )·1000 = 0.050°
··· Without
··· With 3 reiterations
m∠KEAC = 20.000°
( m∠JAC / 3 - m∠KEAC )·1000 = -0.000°

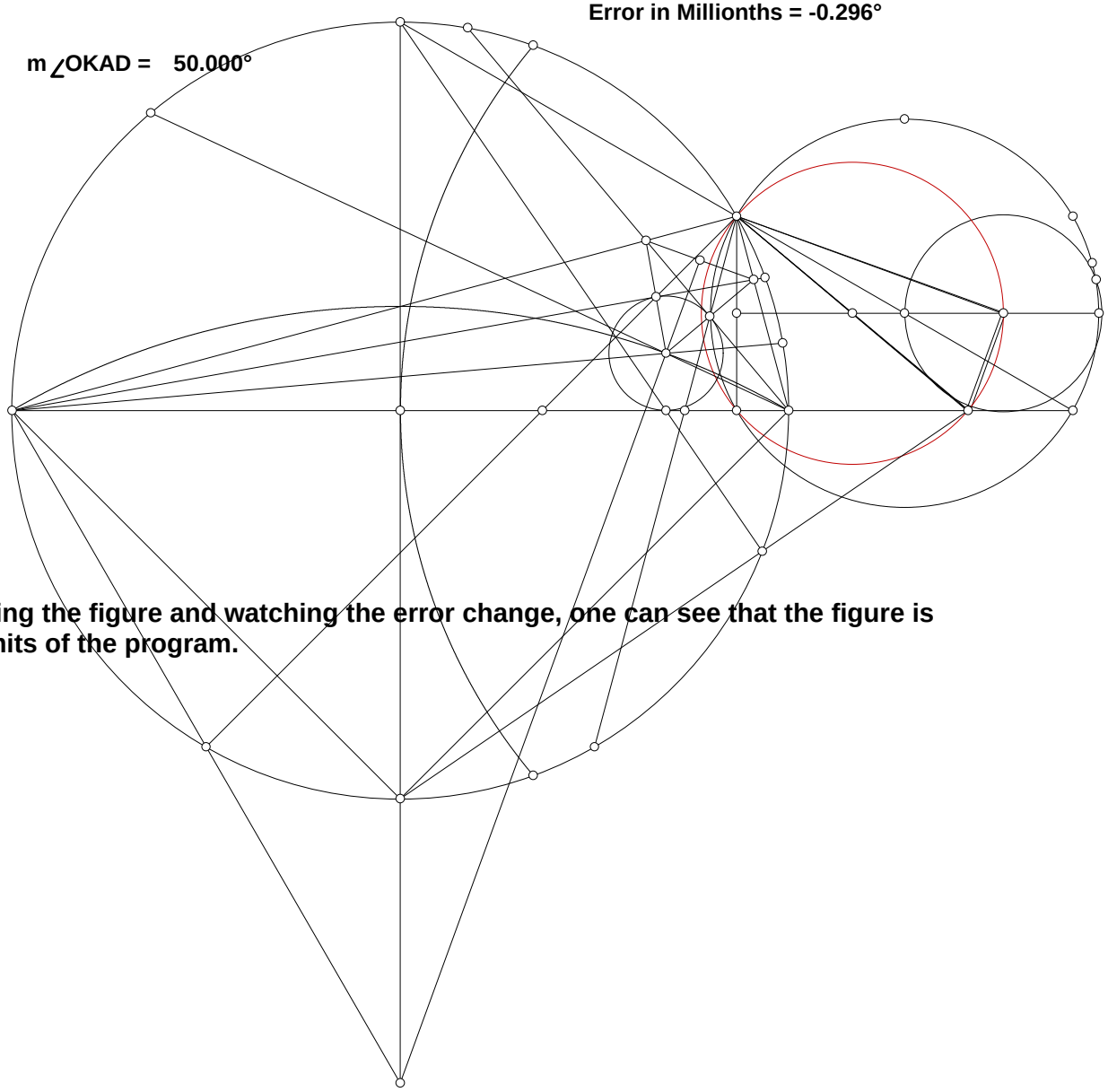

➜ Move J->HJ
➜ Move J->HH
➜ Move J->HI
➜ Move J->HG
➜ Move J->HL
➜ Move J->HK
➜ Move J->HN
➜ Move J->HM
➜ Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠KVAC = 20.000°
( m∠JAC / 3 - m∠KVAC )·1000 = -0.016°
Reiteration here does not significantly improve the accuracy relative to the program.
( m∠JAC / 3 - m∠LCAC )·1000 = -0.000°
m∠LCAC = 20.000°
▲ Show
△ Hide
▲ Show
△ Hide

- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO

$m\angle JAC = 60.000°$ $\quad \frac{m\angle JAC}{3} = 20.000°$ $\quad m\angle NYAC = 20.000°$

Error in Millionths = -0.296°

$m\angle OKAD = 50.000°$



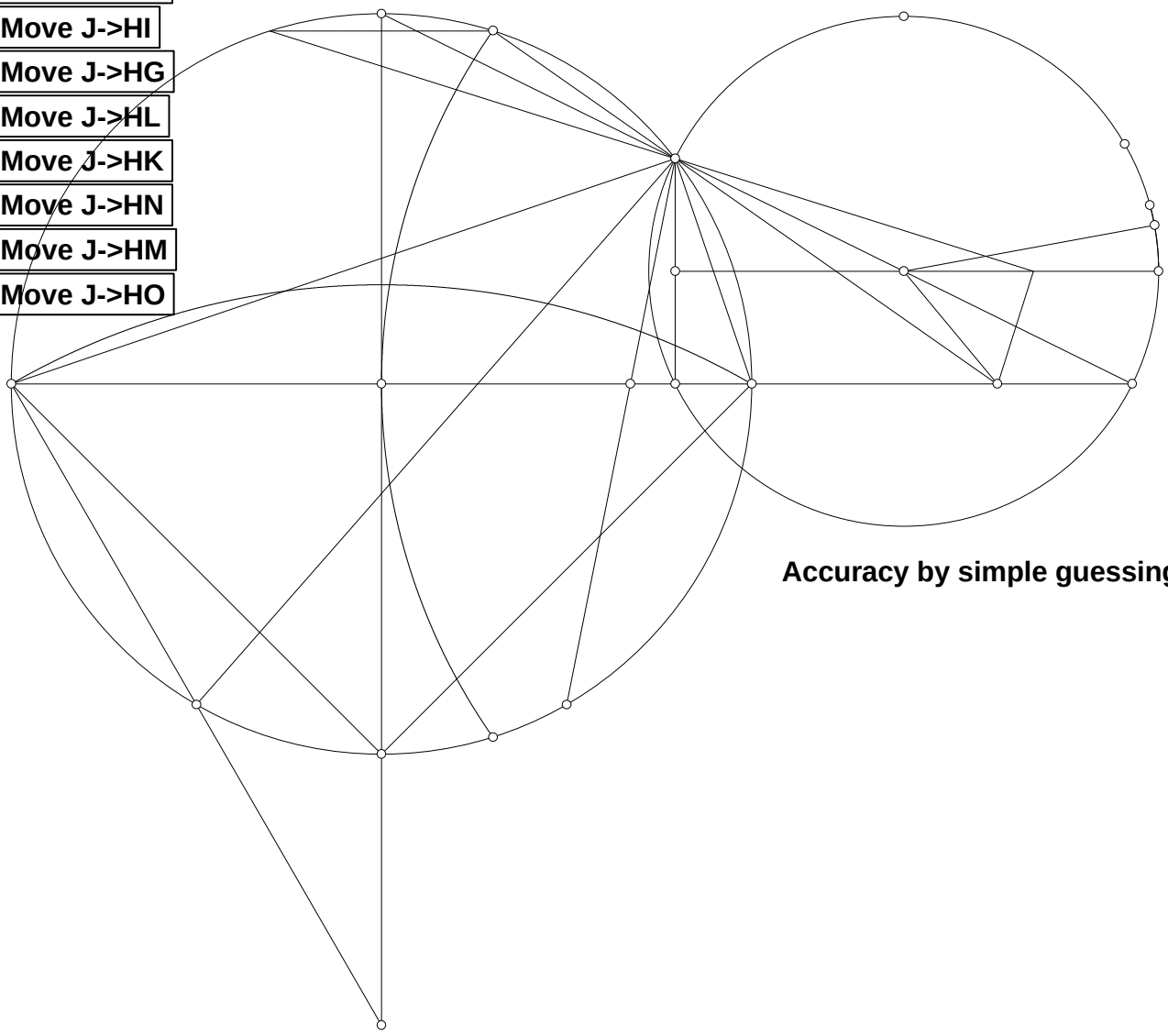
**By proportioning the figure and watching the error change, one can see that the figure is beyond the limits of the program.**

$m\angle JGAC = 17.500°$

$m\angle JAC = 52.500°$

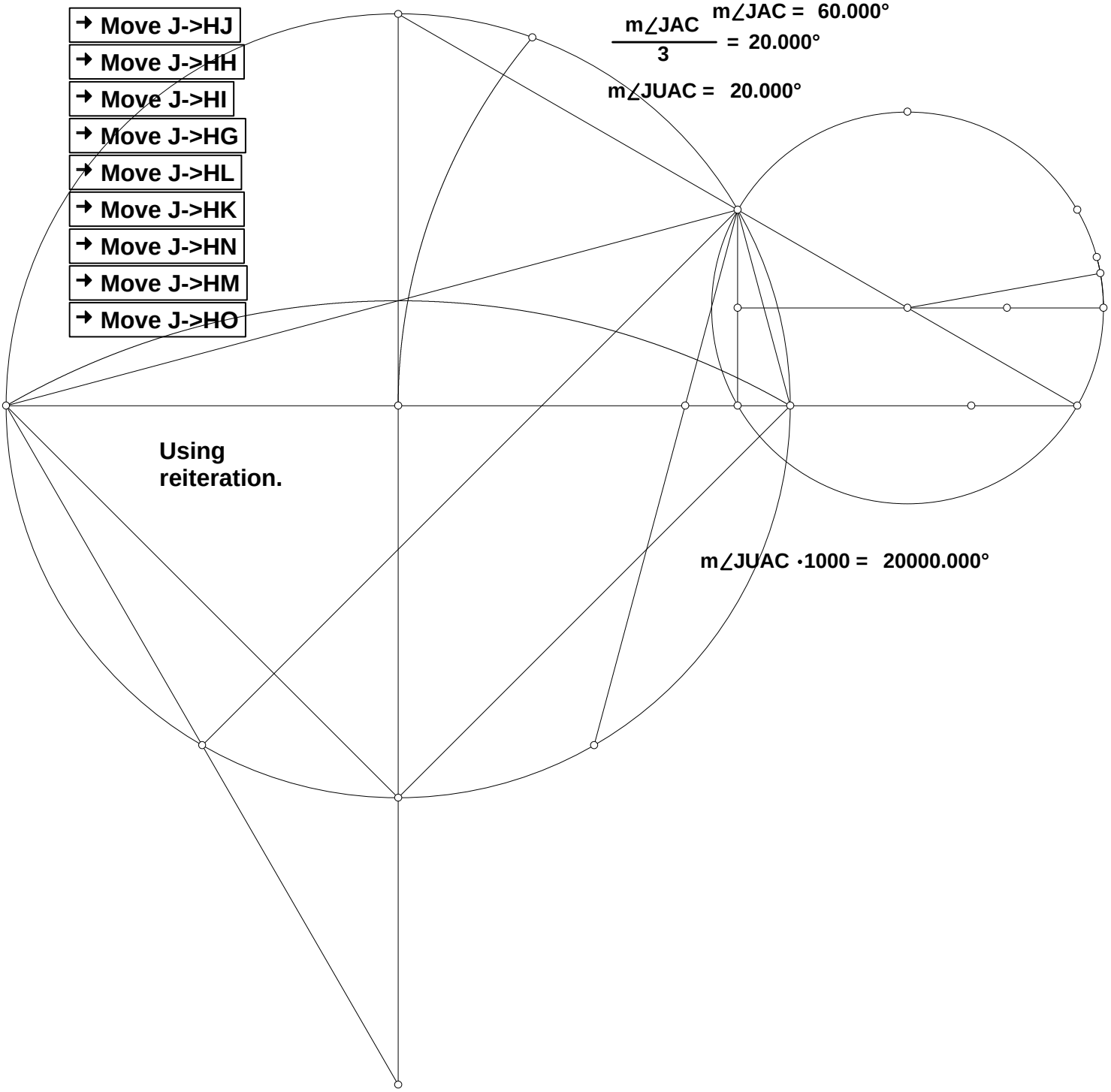
$\frac{m\angle JAC}{3} = 17.500°$

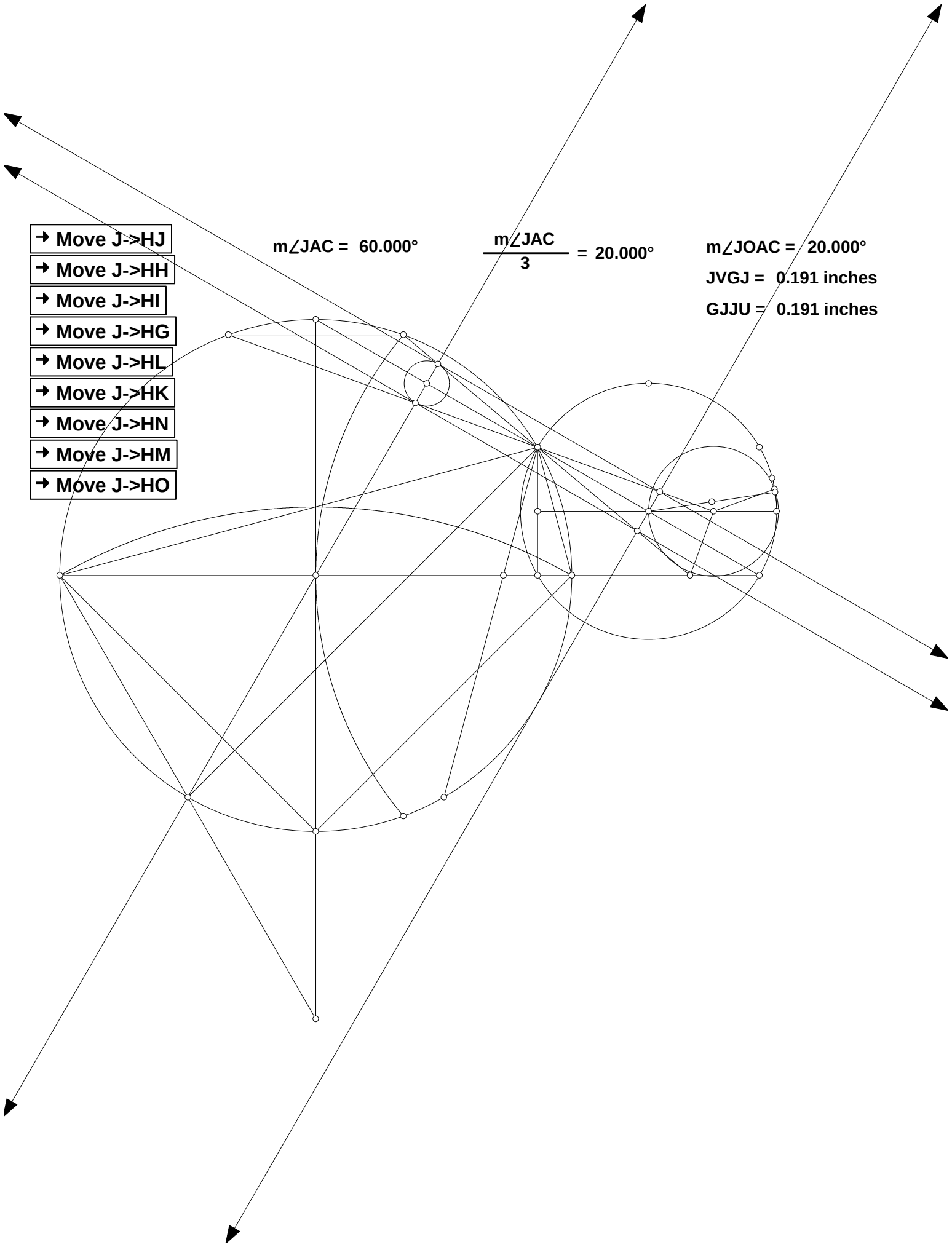
- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO



Guess Me

Accuracy by simple guessing.


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JUAC = 20.000°
Using reiteration.
m∠JUAC ·1000 = 20000.000°

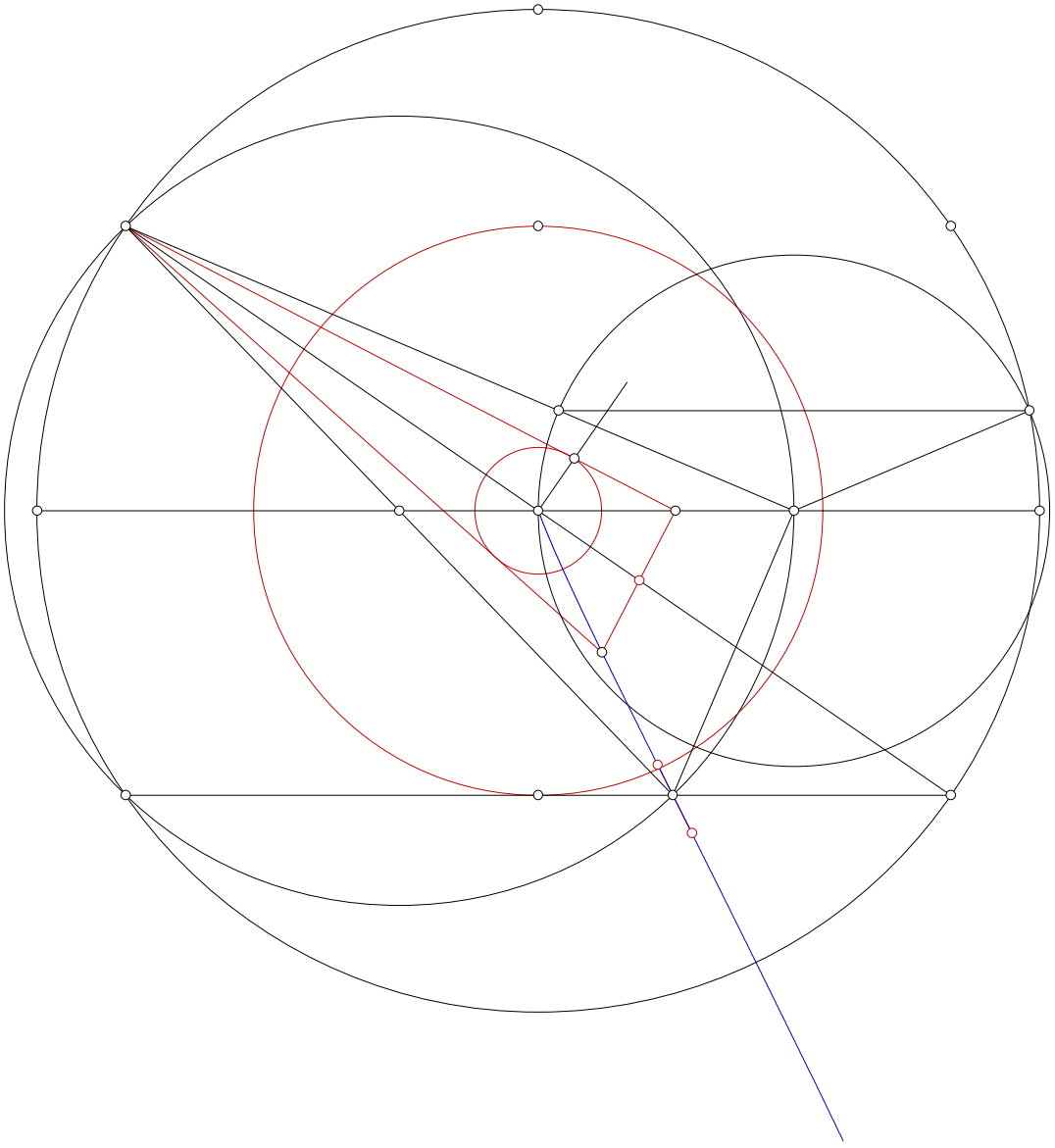

➜ Move J->HJ
➜ Move J->HH
➜ Move J->HI
➜ Move J->HG
➜ Move J->HL
➜ Move J->HK
➜ Move J->HN
➜ Move J->HM
➜ Move J->HO
m∠JAC = 60.000°
m∠JAC / 3 = 20.000°
m∠JOAC = 20.000°
JVGJ = 0.191 inches
GJJU = 0.191 inches

$$\frac{m\angle HAG}{m\angle AGHAC} = 3.000$$

$m\angle HAG = 69.181°$

$m\angle HAGAC = 90.000°$

$m\angle AGHAC = 23.060°$

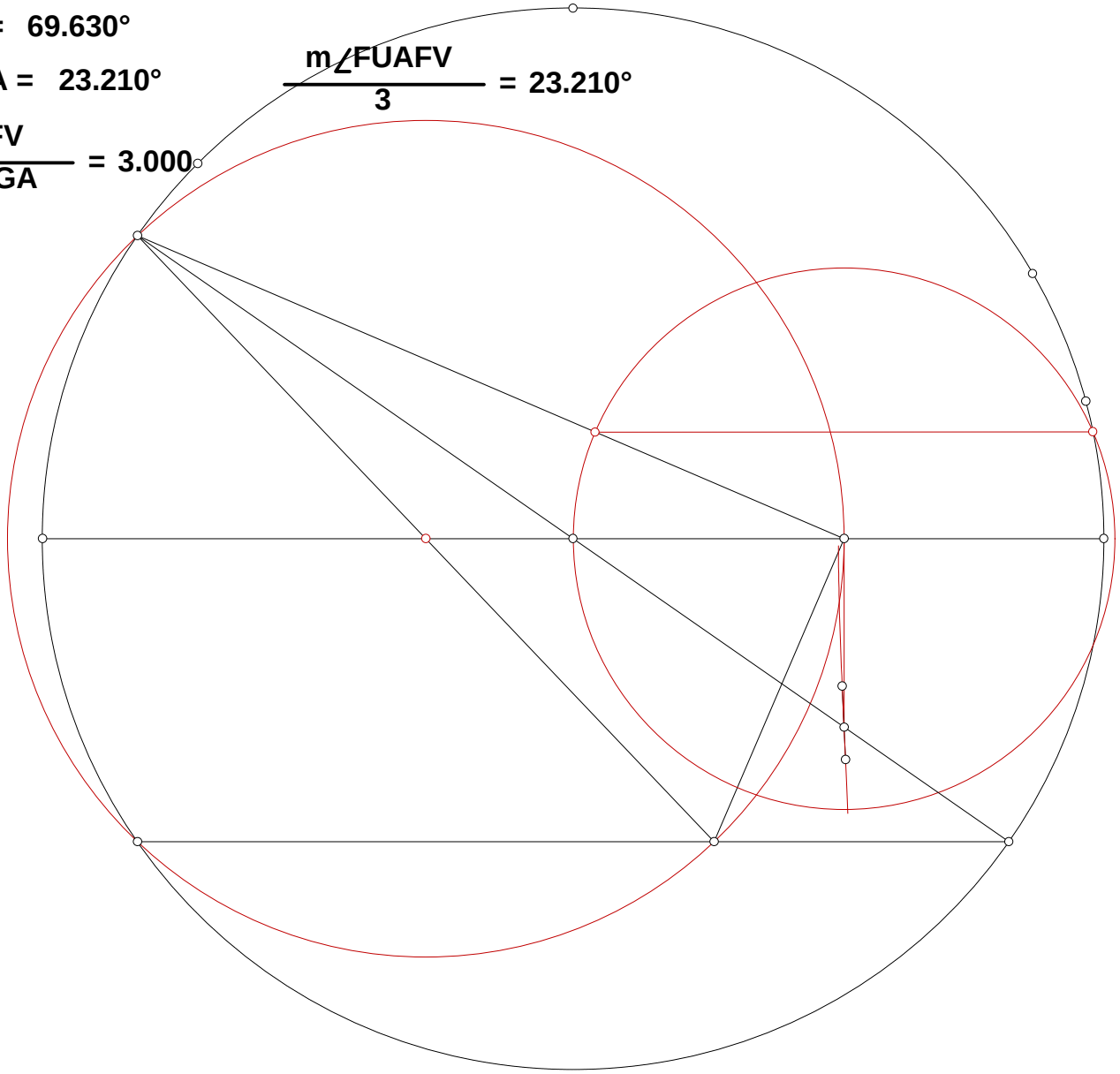


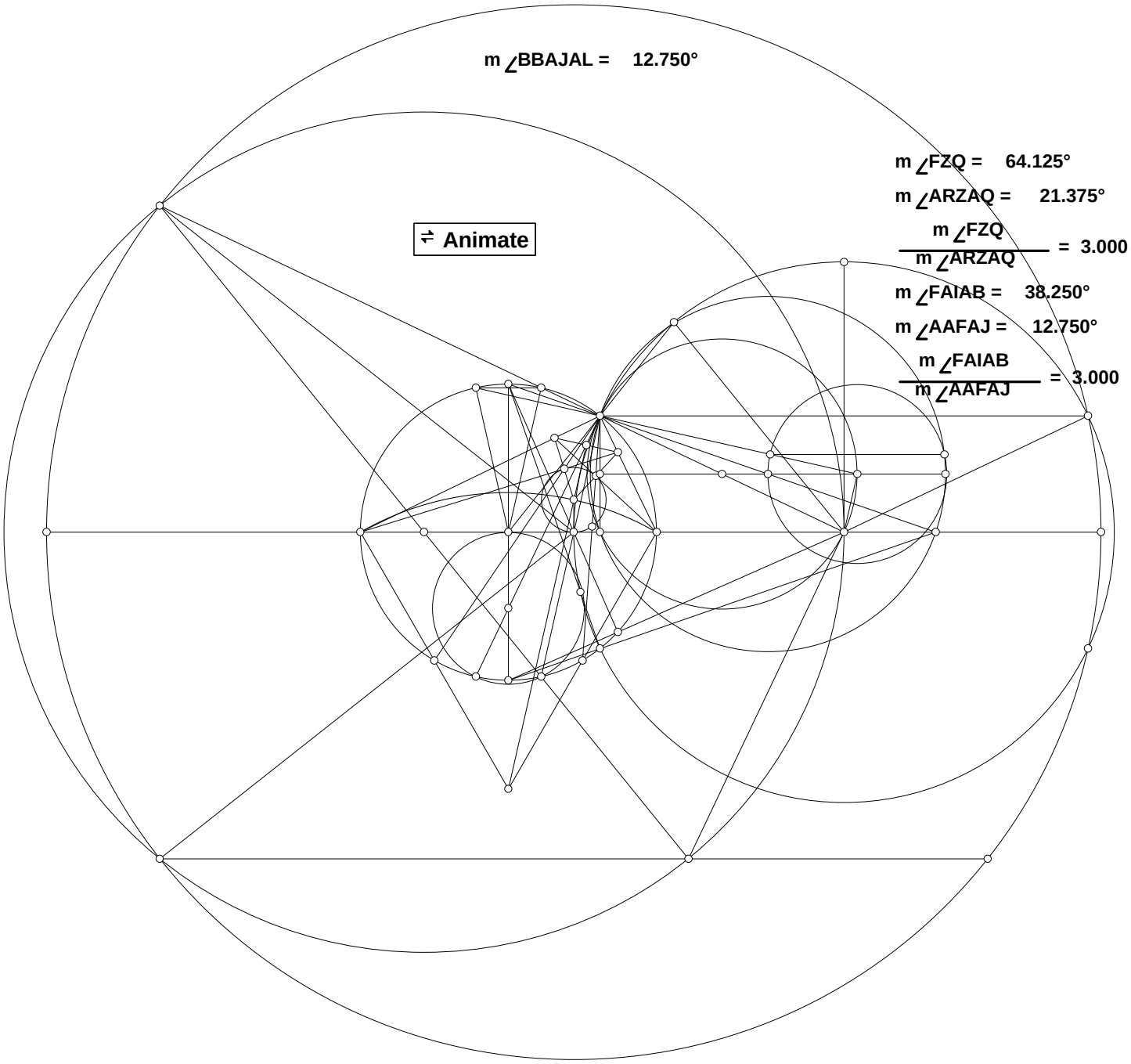
$m\angle FUAFV = 69.630°$

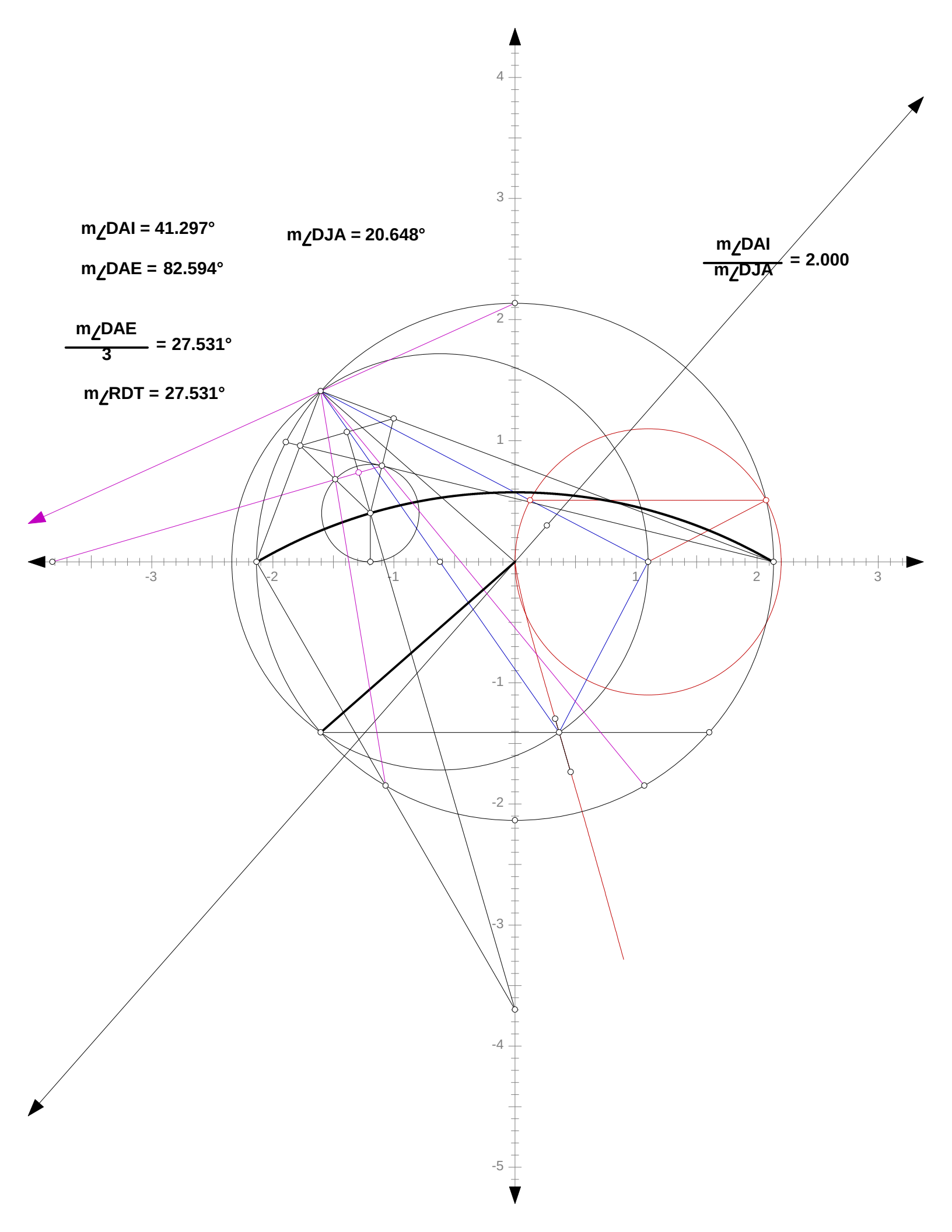
$m\angle GBFUGA = 23.210°$

$\frac{m\angle FUAFV}{m\angle GBFUGA} = 3.000$

$\frac{m\angle FUAFV}{3} = 23.210°$




m ∠BBAJAL = 12.750°
Animate
m ∠FZQ = 64.125°
m ∠ARZAQ = 21.375°
m ∠FZQ / m ∠ARZAQ = 3.000
m ∠FAIAB = 38.250°
m ∠AAFAJ = 12.750°
m ∠FAIAB / m ∠AAFAJ = 3.000

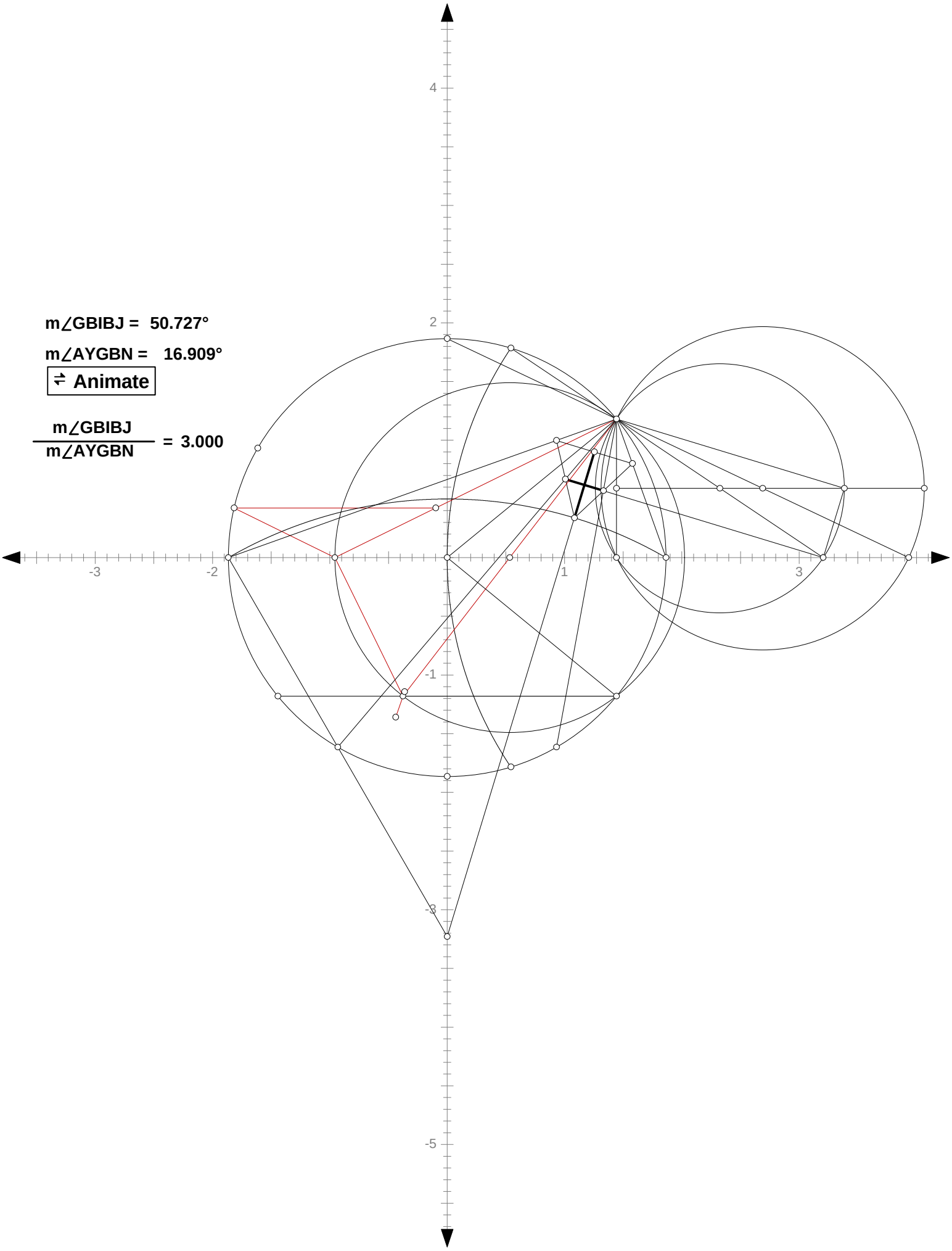

m∠DAI = 41.297°
m∠DJA = 20.648°
m∠DAE = 82.594°
m∠DAE / 3 = 27.531°
m∠RDT = 27.531°
m∠DAI / m∠DJA = 2.000
4
3
2
1
-1
-2
-3
-4
-5
-3
-2
-1
1
2
3

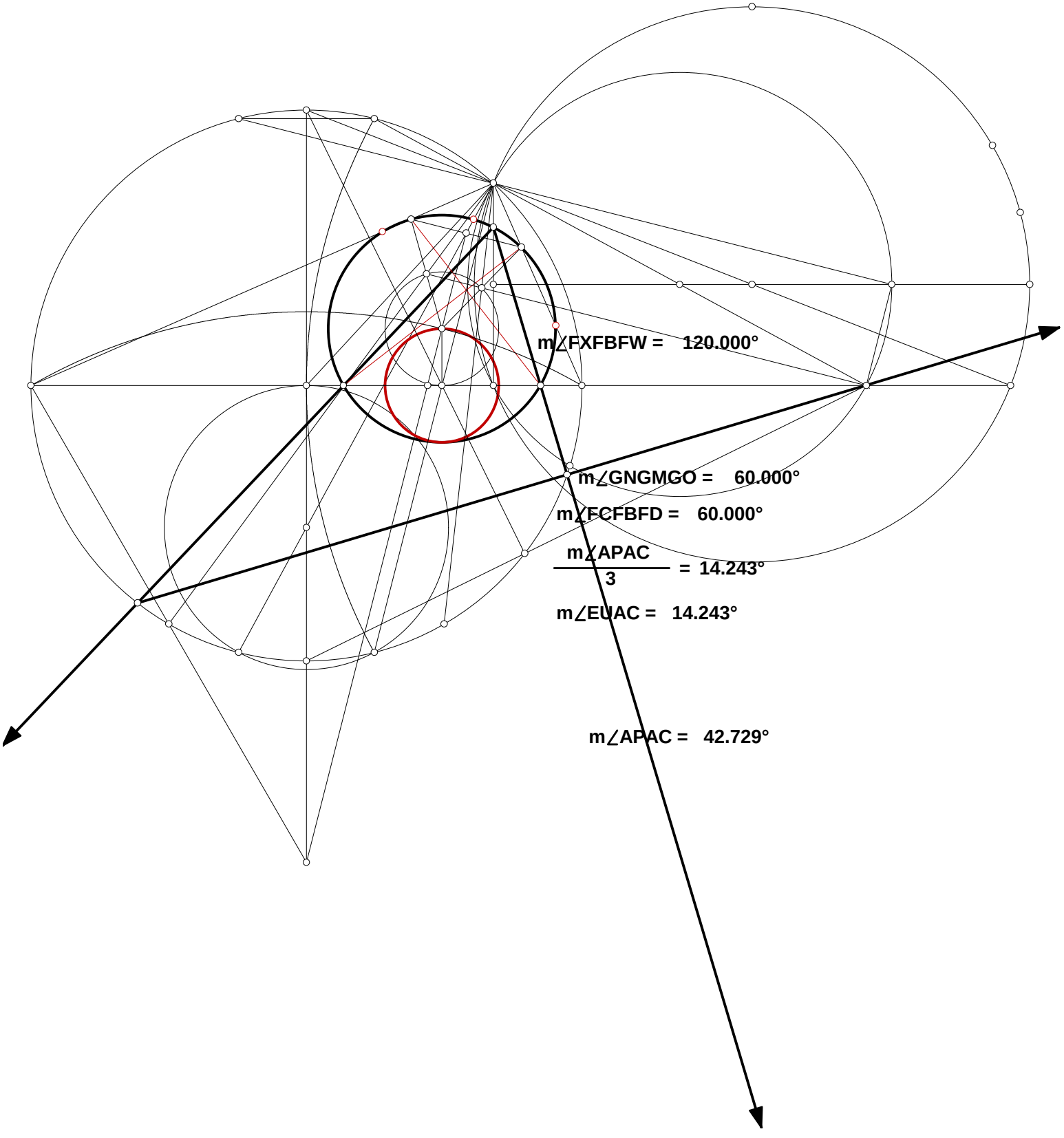
m∠GBIBJ = 50.727°

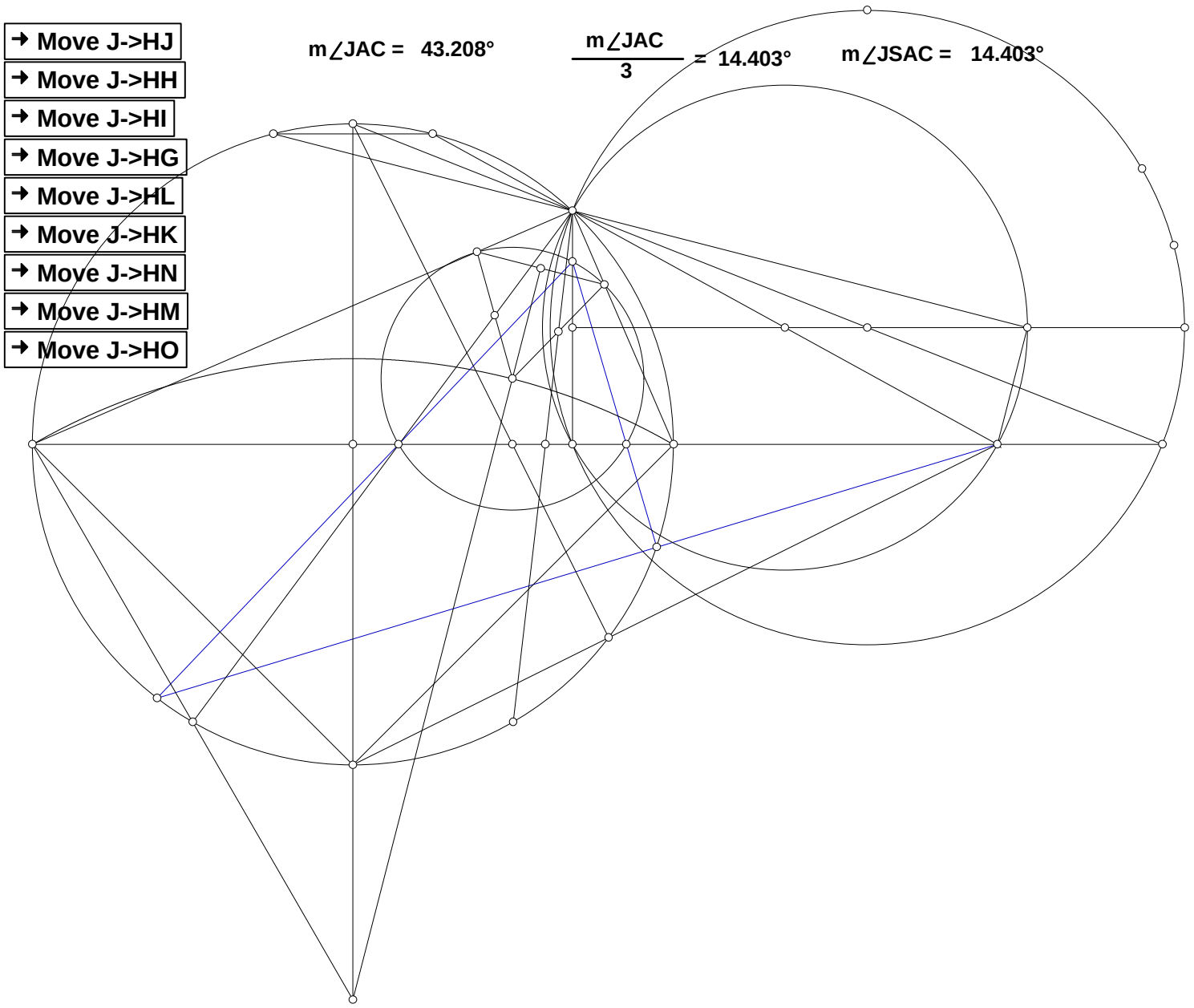
m∠AYGBN = 16.909°

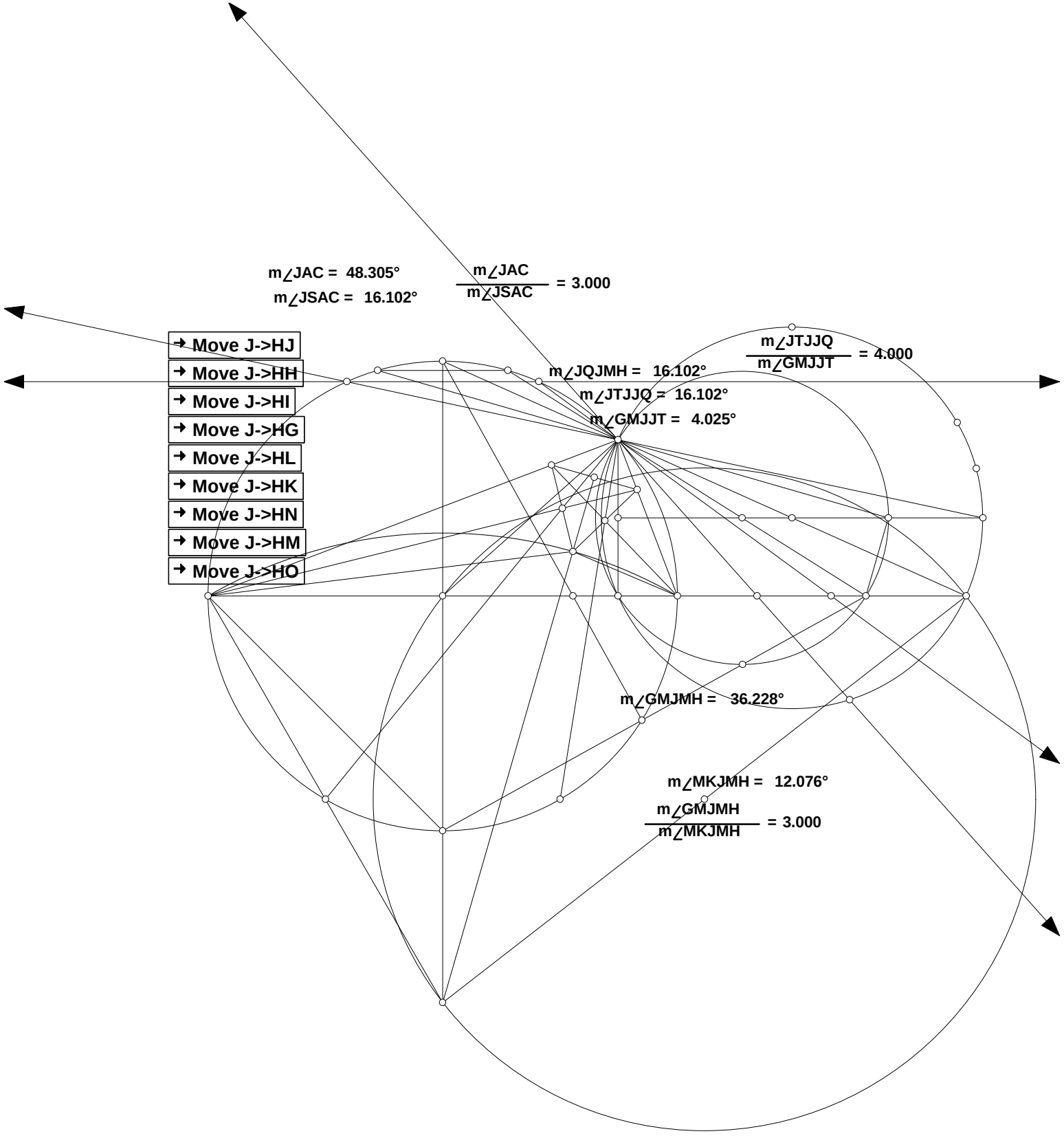
⇄ Animate

$$\frac{m\angle GBIBJ}{m\angle AYGBN} = 3.000$$




m∠FXFBFW = 120.000°
m∠GNGMGO = 60.000°
m∠FCFBFD = 60.000°
m∠APAC / 3 = 14.243°
m∠EUAC = 14.243°
m∠APAC = 42.729°


→ Move J->HJ
→ Move J->HH
→ Move J->HI
→ Move J->HG
→ Move J->HL
→ Move J->HK
→ Move J->HN
→ Move J->HM
→ Move J->HO
m∠JAC = 43.208°
m∠JAC / 3 = 14.403°
m∠JSAC = 14.403°


m∠JAC = 48.305°
m∠JSAC = 16.102°
m∠JAC / m∠JSAC = 3.000
m∠JTJJQ / m∠GMJJT = 4.000
m∠JQJMH = 16.102°
m∠JTJJQ = 16.102°
m∠GMJJT = 4.025°
m∠GMJMH = 36.228°
m∠MKJMH = 12.076°
m∠GMJMH / m∠MKJMH = 3.000
Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO

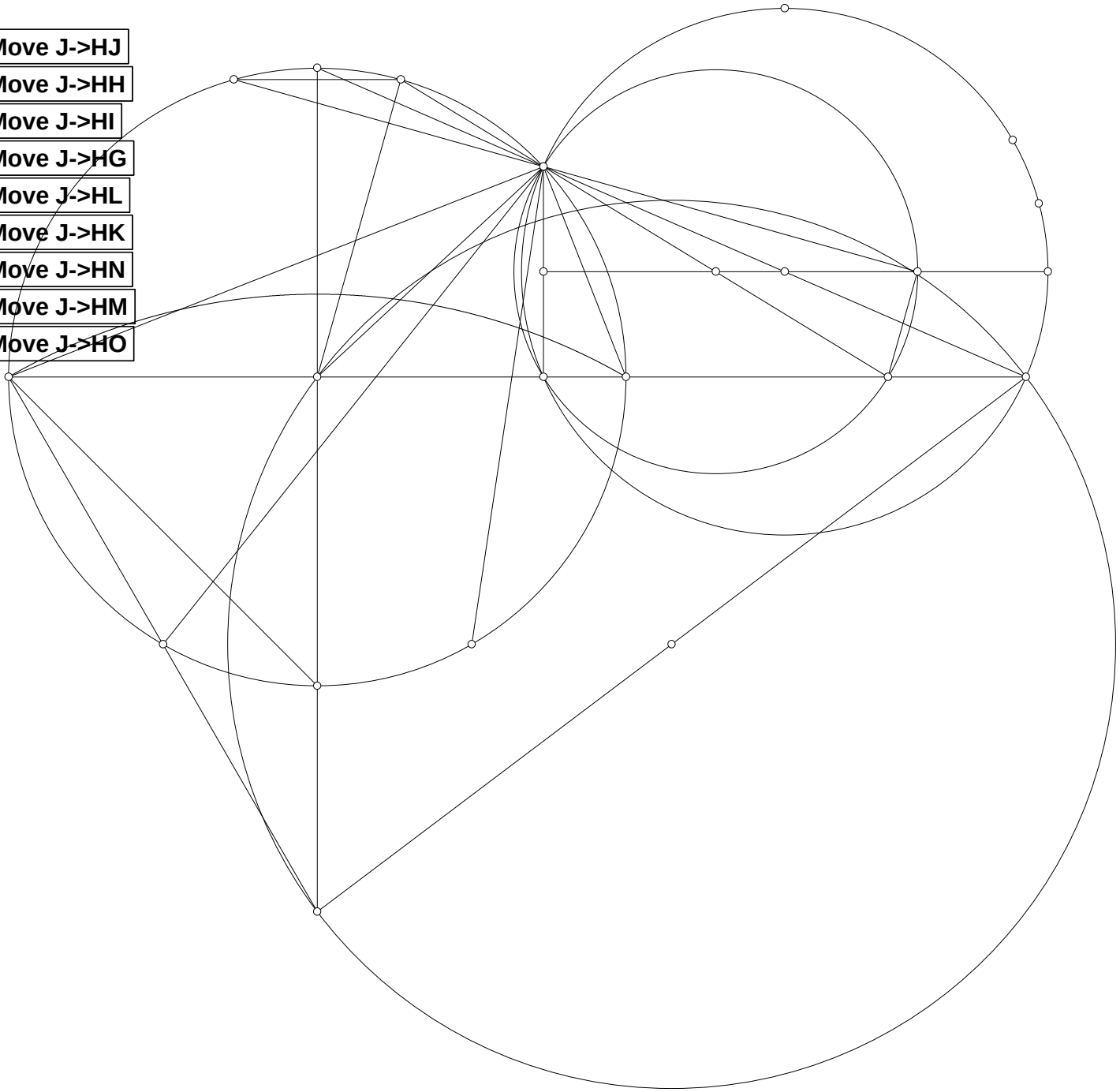
m∠JAC = 47.080°

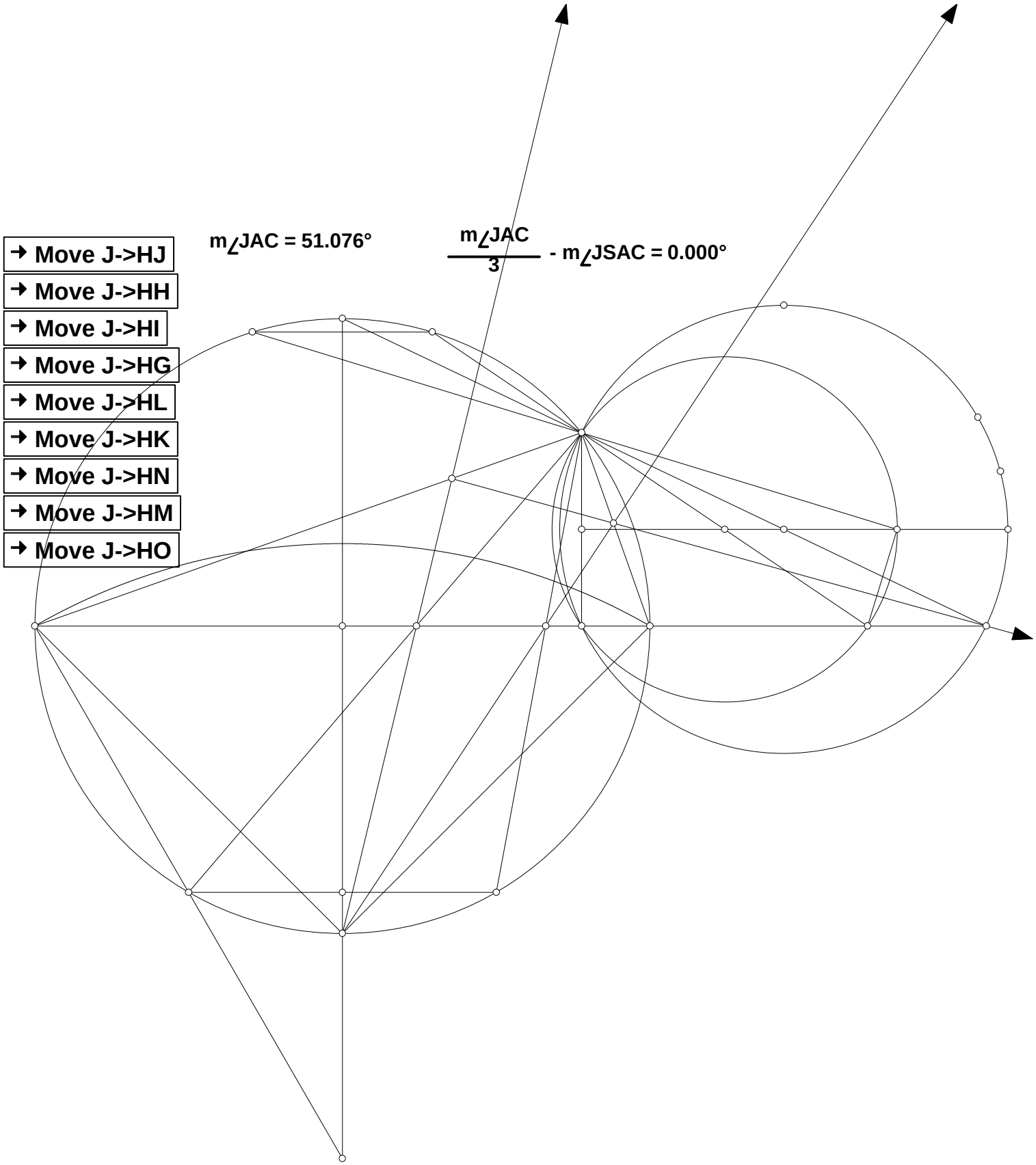
m∠JSAC = 15.693°

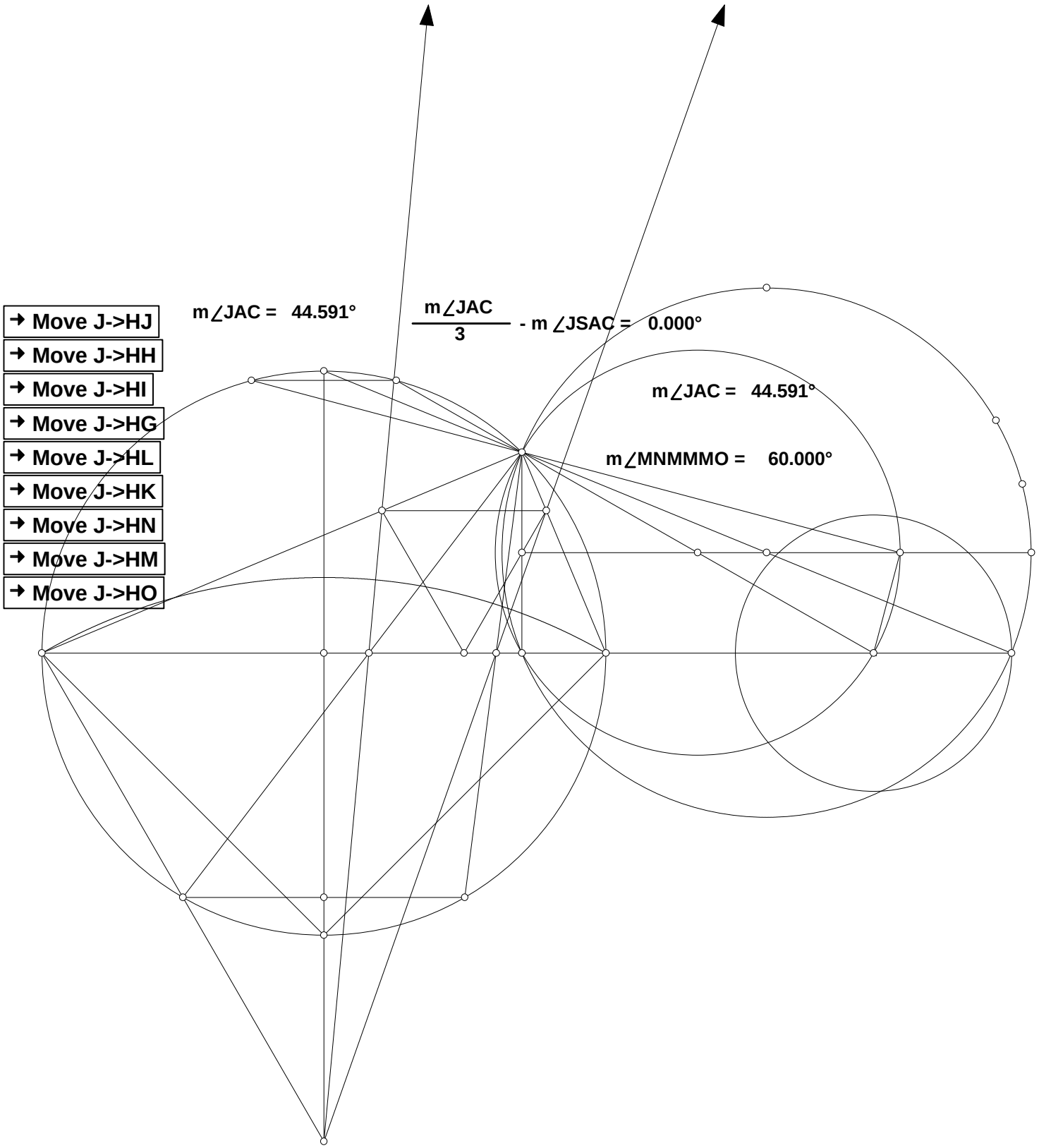
$$\frac{m\angle JAC}{m\angle JSAC} = 3.000$$

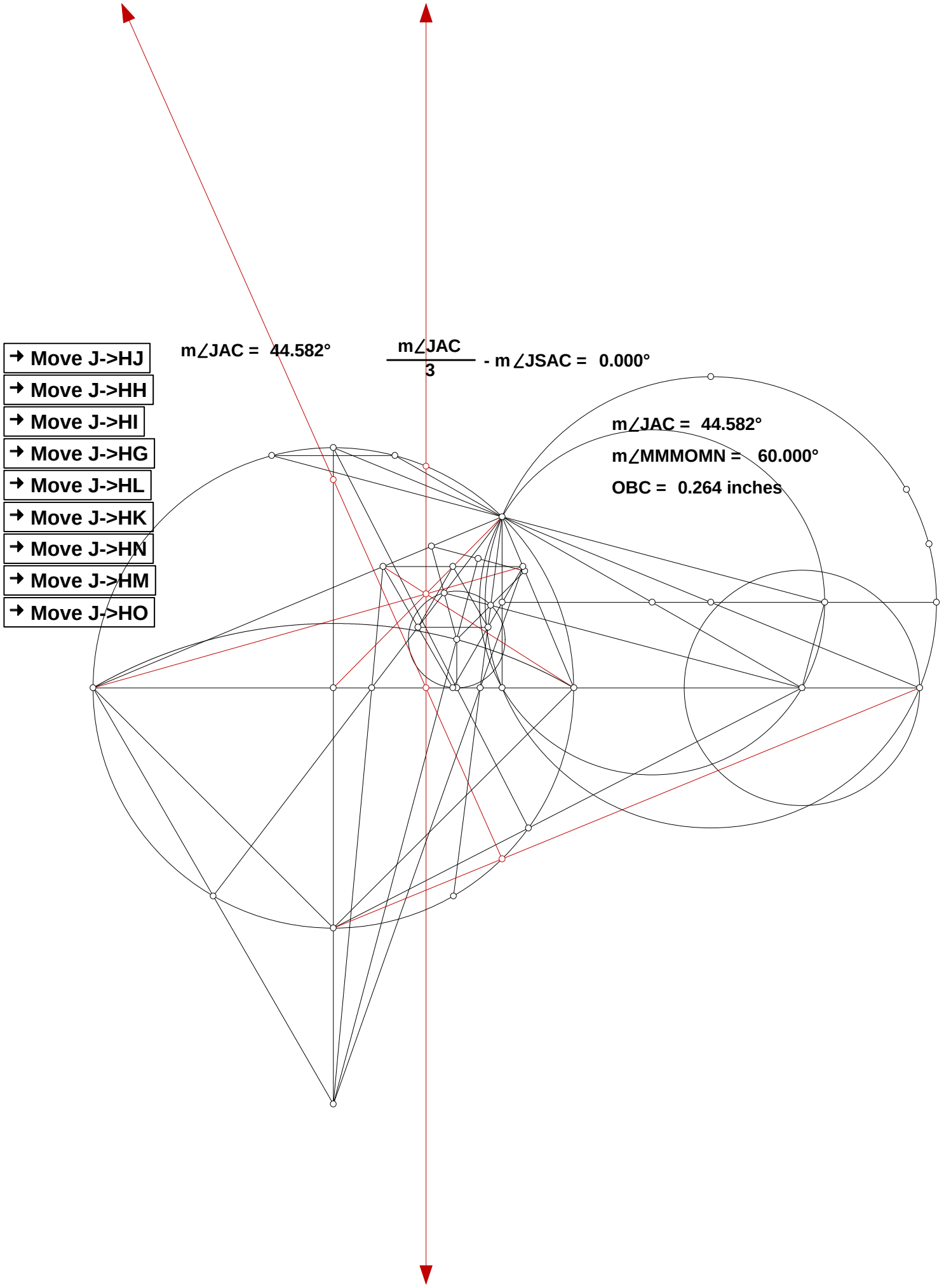
m∠JAC = 47.080°

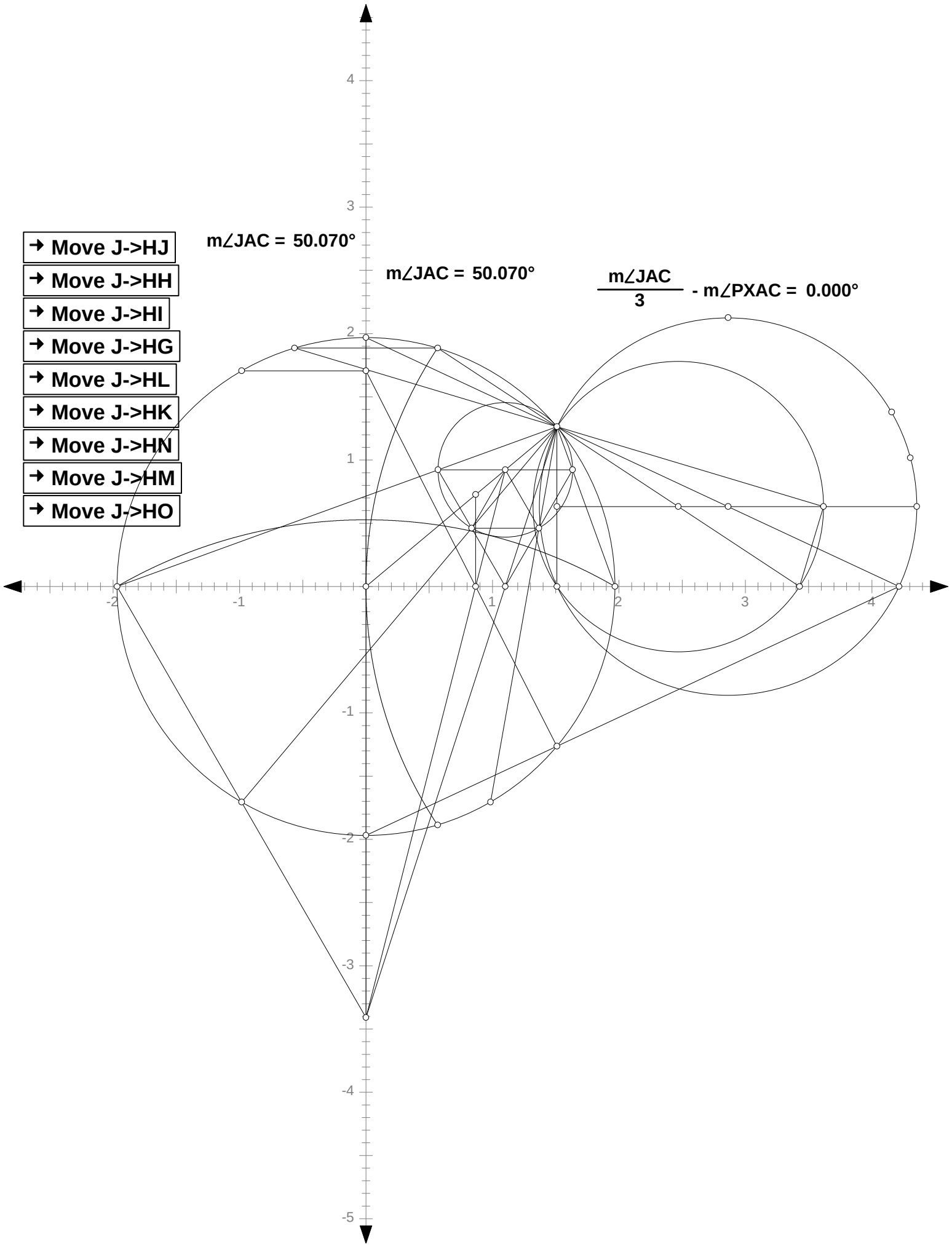
- Move J->HJ
- Move J->HH
- Move J->HI
- Move J->HG
- Move J->HL
- Move J->HK
- Move J->HN
- Move J->HM
- Move J->HO

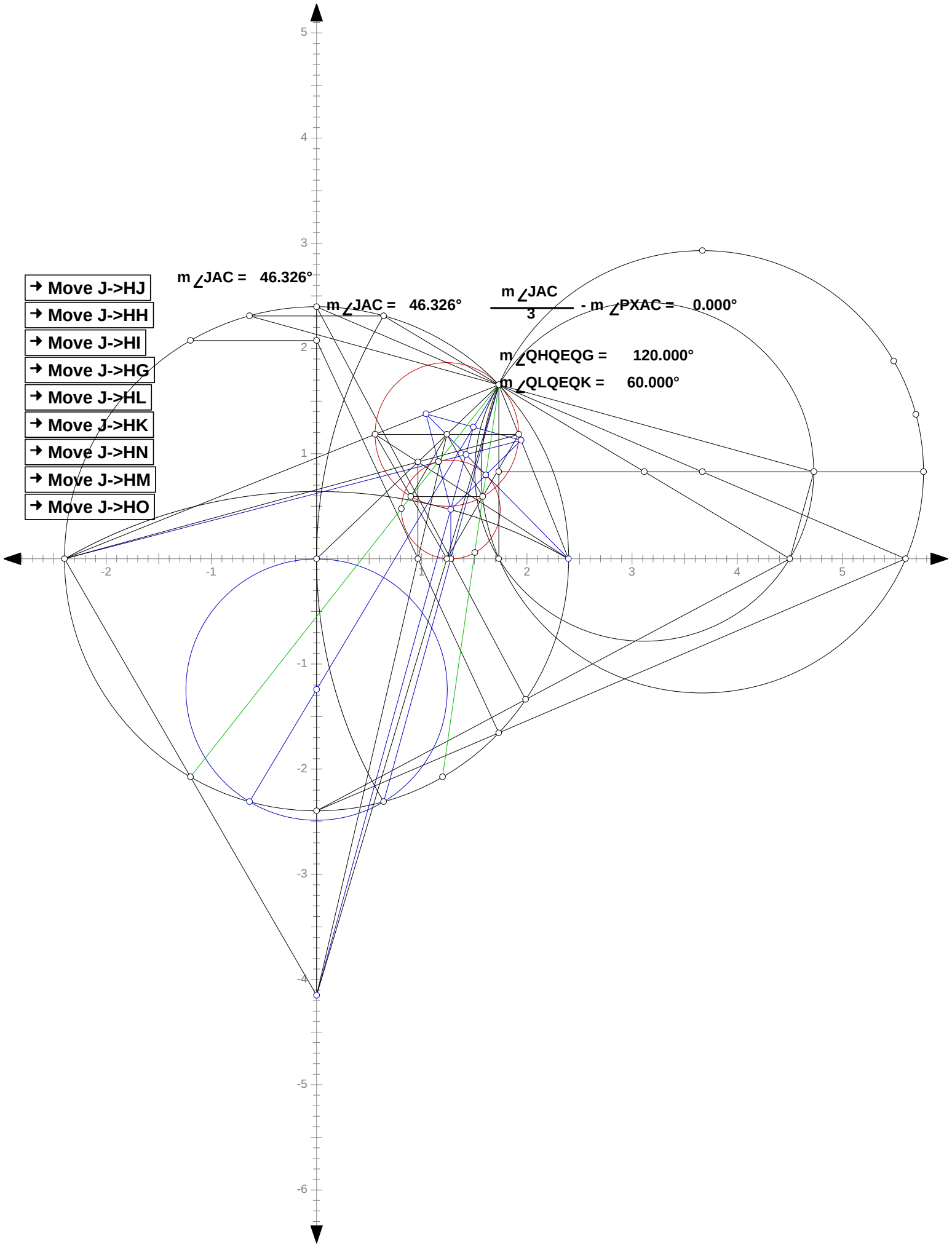


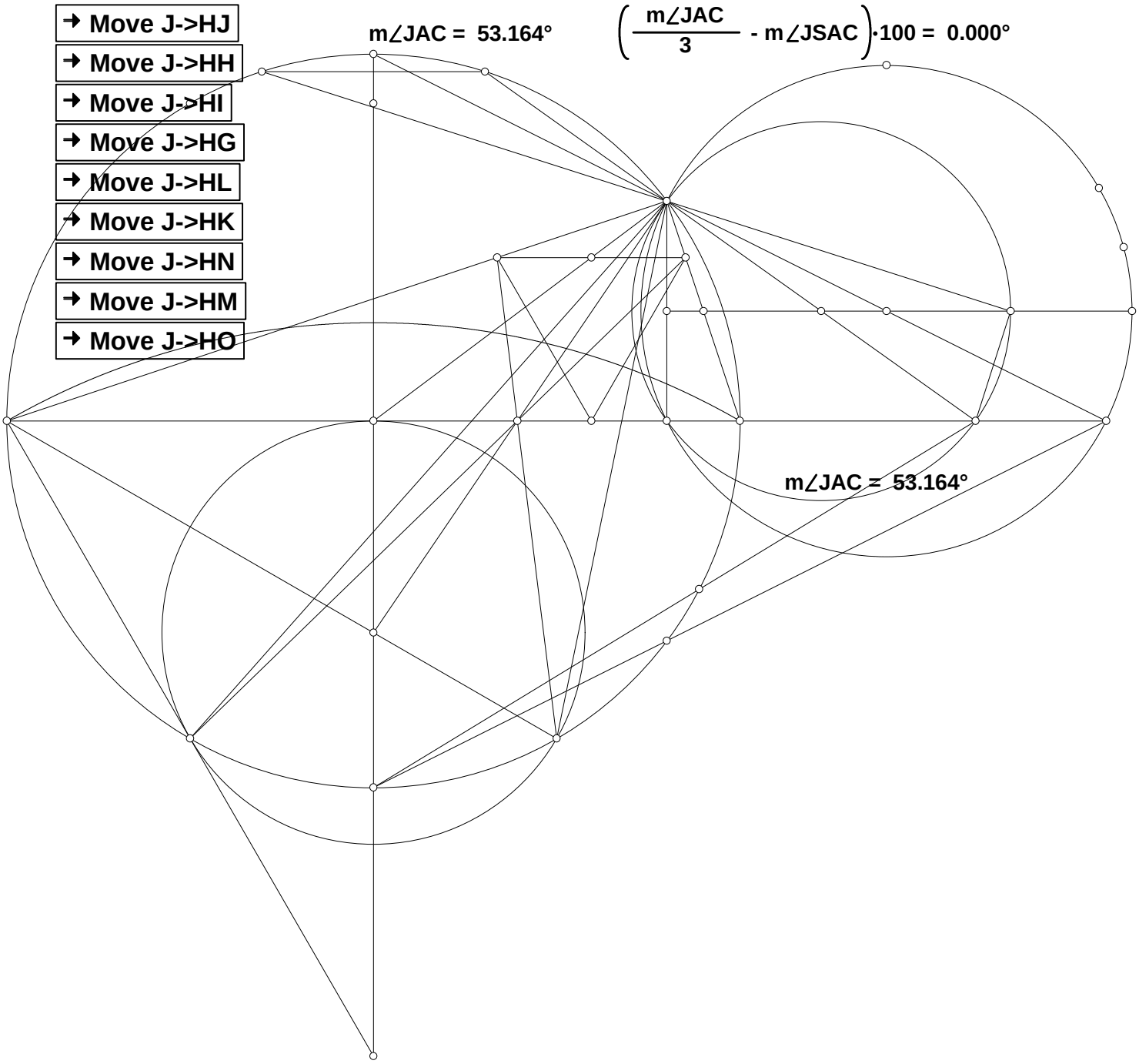

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 51.076°
m∠JAC / 3 - m∠JSAC = 0.000°

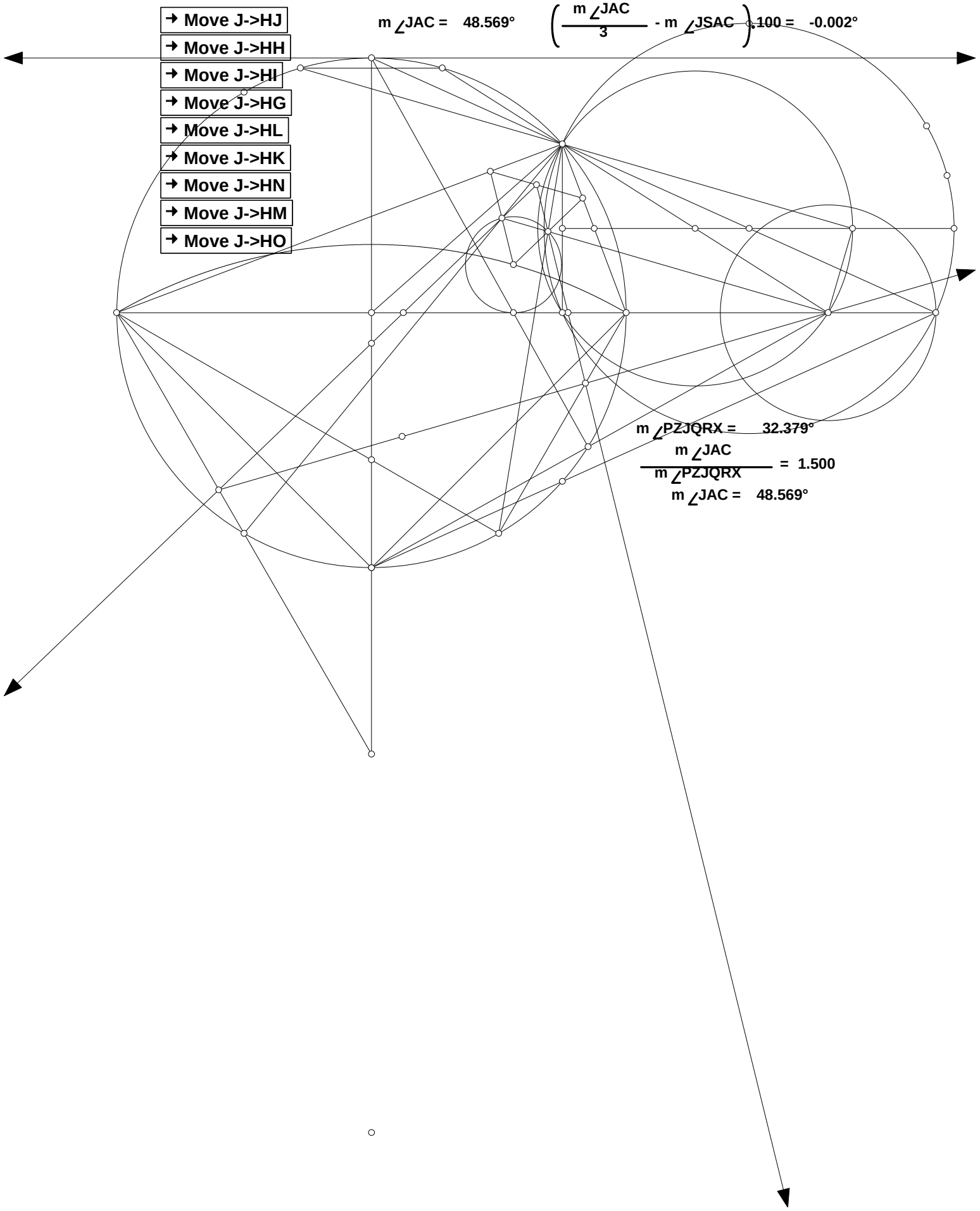

➜ Move J->HJ
➜ Move J->HH
➜ Move J->HI
➜ Move J->HG
➜ Move J->HL
➜ Move J->HK
➜ Move J->HN
➜ Move J->HM
➜ Move J->HO
m∠JAC = 44.591°
m∠JAC / 3 - m∠JSAC = 0.000°
m∠JAC = 44.591°
m∠MNMMMO = 60.000°

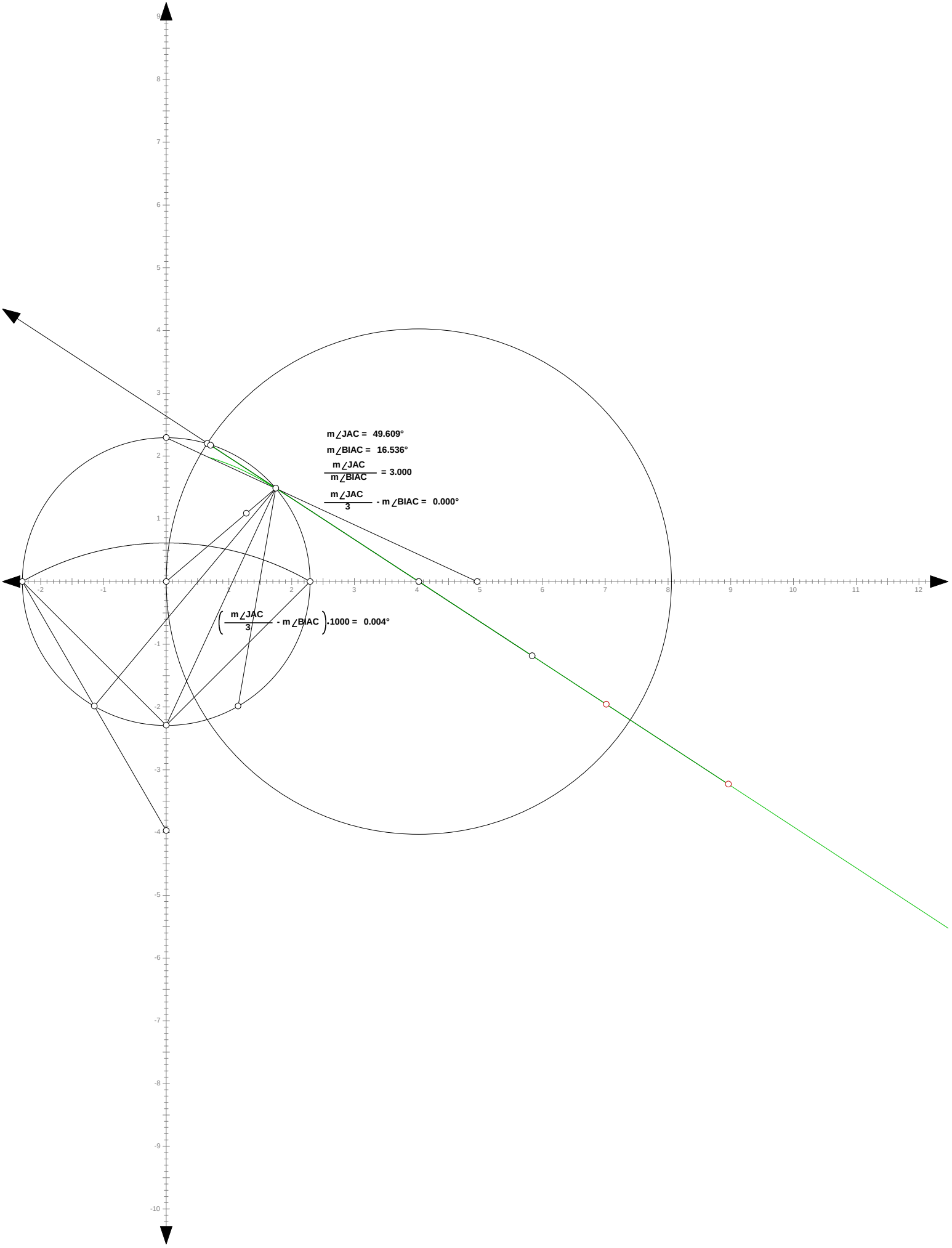

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 44.582°
m∠JAC / 3 - m∠JSAC = 0.000°
m∠JAC = 44.582°
m∠MMMOMN = 60.000°
OBC = 0.264 inches


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 50.070°
m∠JAC = 50.070°
m∠JAC / 3 - m∠PXAC = 0.000°


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 46.326°
m∠JAC = 46.326°
m∠JAC / 3 - m∠PXAC = 0.000°
m∠QHQEQG = 120.000°
m∠QLQEQK = 60.000°

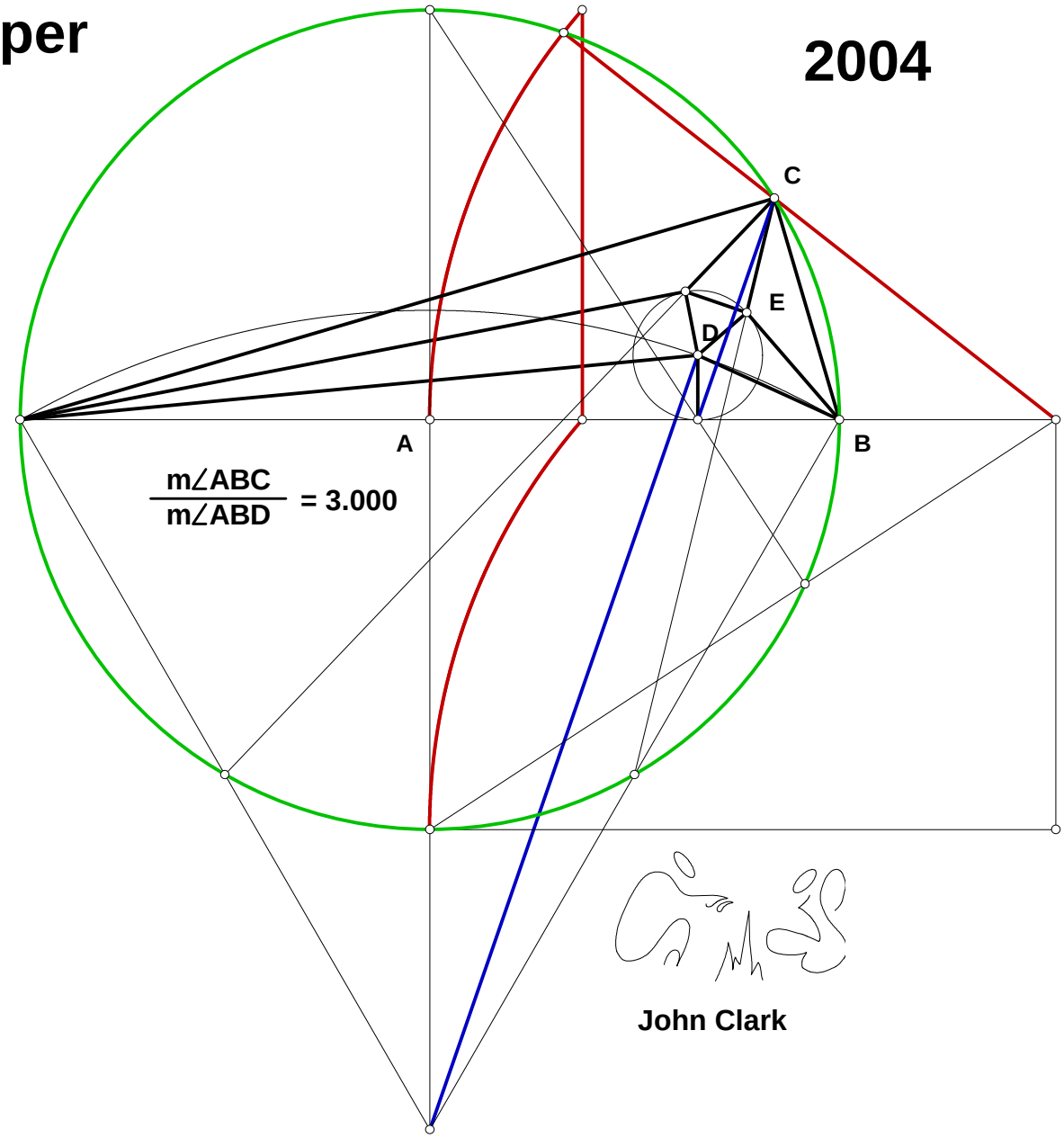

Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 53.164°
$\left(\frac{m\angle JAC}{3} - m\angle JSAC\right)\cdot 100 = 0.000°$
m∠JAC = 53.164°


Move J->HJ
Move J->HH
Move J->HI
Move J->HG
Move J->HL
Move J->HK
Move J->HN
Move J->HM
Move J->HO
m∠JAC = 48.569°
(m∠JAC / 3 - m∠JSAC) ∘100 = -0.002°
m∠PZJQRX = 32.379°
m∠JAC / m∠PZJQRX = 1.500
m∠JAC = 48.569°


m∠JAC = 49.609°
m∠BIAC = 16.536°
m∠JAC / m∠BIAC = 3.000
m∠JAC / 3 - m∠BIAC = 0.000°
( m∠JAC / 3 - m∠BIAC ) • 1000 = 0.004°

# Three Pieces Of Paper

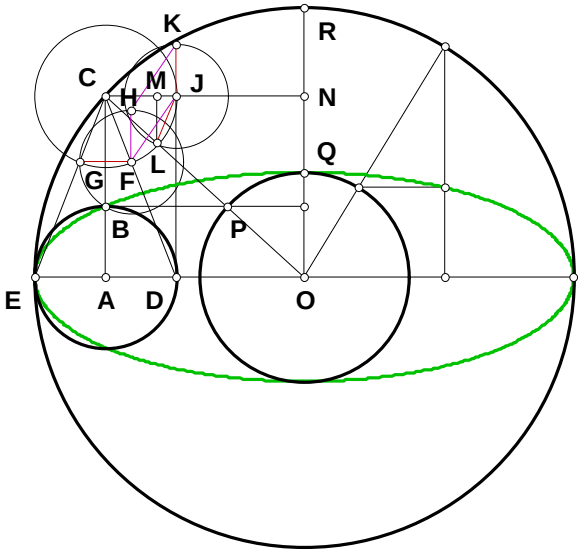
2004




John Clark

## Given AE, AG, AC find the ellipse

**031704.mcd**




**Or given any AE, AC find the diameter of the Circle.**

$N := 3$ $AE := 1$

$AC := AE \cdot N$ $AG := AE$

$DE := AE \cdot 2$ $CJ := AE$

$CE := \sqrt{AE^2 + AC^2}$

$CG := AE$ $FG := \frac{DE \cdot CG}{CE}$

$JL := FG$ $JM := \frac{JL^2}{2 \cdot CJ}$

$CM := CJ - JM$ $LM := \sqrt{JL^2 - JM^2}$ $NO := AC$ $CN := \frac{CM \cdot NO}{LM}$ $AO := CN$ $EO := AO + AE$

$OP := \frac{AE^2}{LM}$ $Major := EO$ $Minor := OP$

$Major = 5$

$Minor = 1.66667$

$Major - \frac{N^2 + 1}{2} = 0$ $Minor - \frac{N^2 + 1}{2 \cdot N} = 0$

$\frac{Major}{Minor} - N = 0$

**Given AC, AB and either pont of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?** **032304**

$N_1 := 1.708$ $N_2 := 1.649$

$N_3 := 1.24$

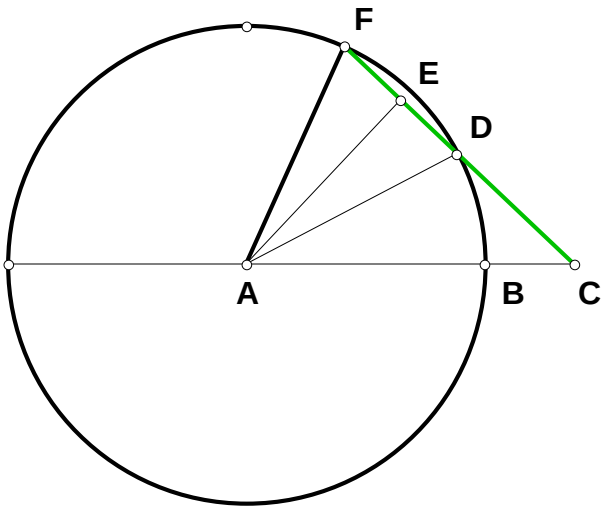
$AC := N_1$ $CF := N_2$

$AF := N_3$

$$DF_1 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

$CD := CF - DF_1$ $N_2 := CD$

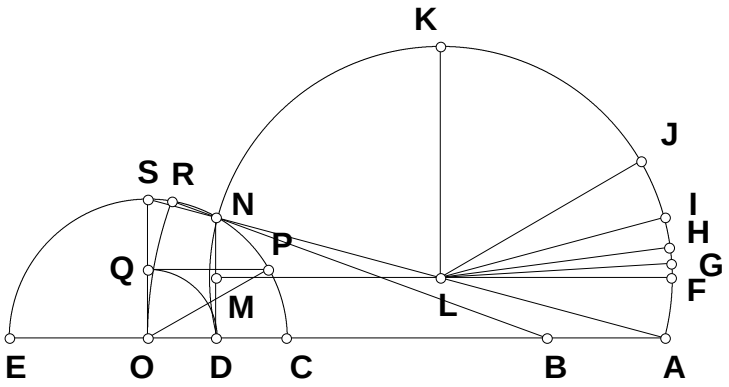
$$DF_2 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$




$DF_1 = 0.812$ $DF_2 = -0.812$ $DF_1 + DF_2 = 0$

**0405043**

**All this extra work and I have lost accuracy from 022803!**



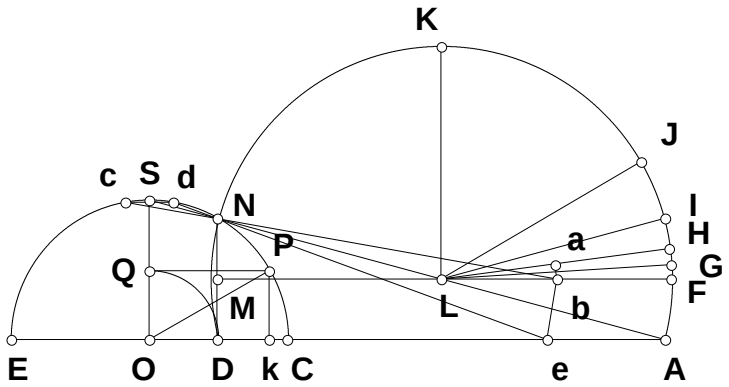

$N := 4 \qquad CE := 1$

$CO := \frac{CE}{2} \qquad CD := \frac{CE}{N}$

$DO := CO - CD$

$DN := \sqrt{(CO + DO)\cdot(CO - DO)}$

$AD := \frac{DO \cdot DN}{CO - DN} \qquad DM := \frac{DN}{2}$




$AN := \sqrt{AD^2 + DN^2} \qquad HL := \frac{AN}{2} \qquad LA := \frac{HL}{2} \qquad Ok := \sqrt{CO^2 - DO^2} \qquad Ck := CO - Ok$

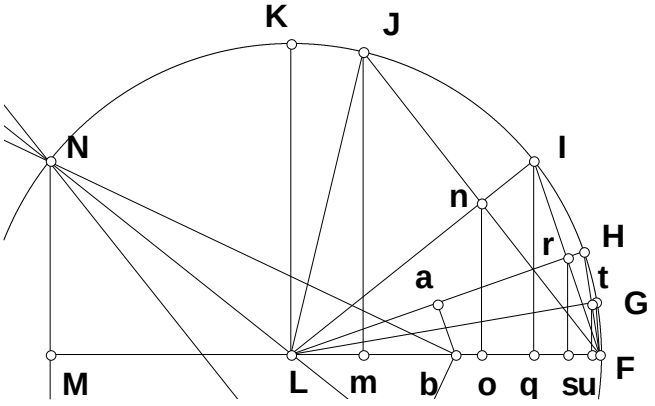
$Fm := \frac{Ck \cdot HL}{CO} \qquad Jm := \frac{DO \cdot HL}{CO}$

$JF := \sqrt{Fm^2 + Jm^2} \qquad Fn := \frac{JF}{2}$

$Fo := \frac{Fm \cdot Fn}{JF} \qquad Lo := HL - Fo$

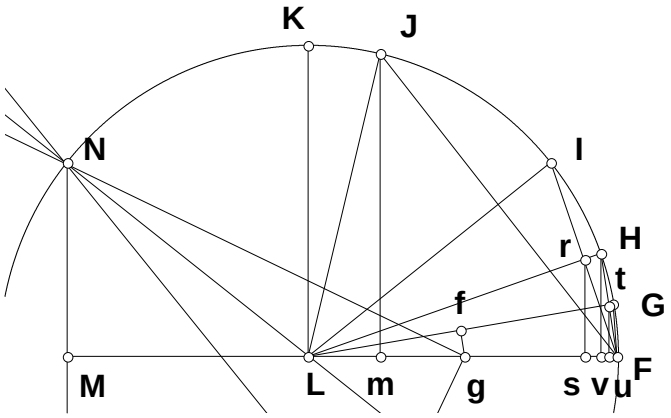
$no := \frac{Jm}{2} \qquad Ln := \sqrt{no^2 + Lo^2}$

$Iq := \frac{no \cdot HL}{Ln} \qquad Lq := \frac{Lo \cdot HL}{Ln}$




$Fq := HL - Lq \qquad FI := \sqrt{Iq^2 + Fq^2} \qquad Fr := \frac{FI}{2} \qquad Fs := \frac{Fq \cdot Fr}{FI} \qquad Ls := HL - Fs \qquad Lr := \sqrt{HL^2 - Fr^2}$

$Lb := \frac{HL^2}{2 \cdot Lr} \qquad rs := \frac{Iq}{2}$




$$Lv := \frac{Ls \cdot HL}{Lr} \quad Hv := \frac{rs \cdot HL}{Lr}$$

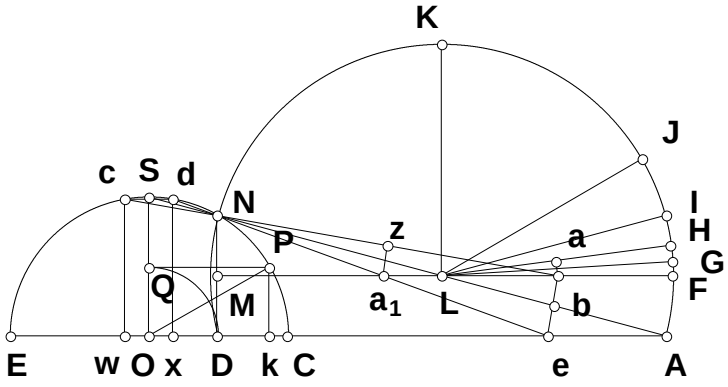
$$Lv := \frac{Ls \cdot HL}{Lr} \quad Fv := HL - Lv$$

$$FH := \sqrt{Hv^2 + Fv^2} \quad Ft := \frac{FH}{2}$$

$$Lf := \frac{HL}{2} \quad Fu := \frac{Fv \cdot Ft}{FH} \quad tu := \frac{Hv}{2}$$

$$Lu := HL - Fu \quad Lt := \sqrt{Lu^2 + tu^2}$$

$$Lg := \frac{Lt \cdot Lf}{Lu}$$




$$MN := \frac{DN}{2} \quad NL := \frac{AN}{2}$$

$$ML := \sqrt{NL^2 - MN^2}$$

$$Mb := ML + Lb$$

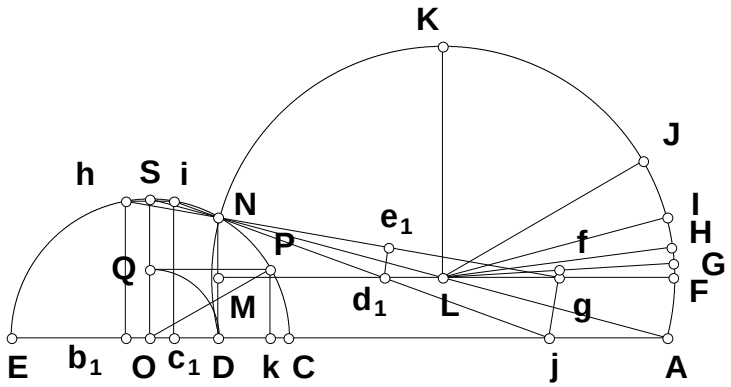
$$Nb := \sqrt{MN^2 + Mb^2}$$

$$Nc := \frac{CO^2 + (2 \cdot Nb)^2 - (2 \cdot Mb + DO)^2}{-2 \cdot Nb} \quad cw := \frac{MN \cdot (Nb + Nc)}{Nb} + MN$$

$$Dw := \frac{2 \cdot Mb \cdot (2 \cdot Nb + Nc)}{2 \cdot Nb} - 2 \cdot Mb \quad Ow := Dw - DO \quad Dx := Dw - 2 \cdot Ow \quad Ma_1 := \frac{Dx \cdot DN}{2 \cdot (cw - DN)}$$

$$Na_1 := \sqrt{MN^2 + (Ma_1)^2} \quad ba_1 := Mb - Ma \quad Nz := \frac{(Na_1)^2 + Nb^2 - (ba_1)^2}{2 \cdot Nb} \quad za_1 := \sqrt{(Na_1)^2 - Nz^2}$$

$$be := \frac{za_1 \cdot Nb}{Nz}$$




$$Mg := ML + Lg$$

$$Ng := \sqrt{MN^2 + Mg^2}$$

$$Nh := \frac{CO^2 + (2 \cdot Ng)^2 - (2 \cdot Mg + DO)^2}{-2 \cdot Ng} \qquad hb_1 := \frac{MN \cdot (Ng + Nh)}{Ng} + MN$$

$$Db_1 := \frac{2 \cdot Mg \cdot (2 \cdot Ng + Nh)}{2 \cdot Ng} - 2 \cdot Mg \qquad Ob_1 := Db_1 - DO \qquad Dc_1 := Db_1 - 2 \cdot Ob_1$$

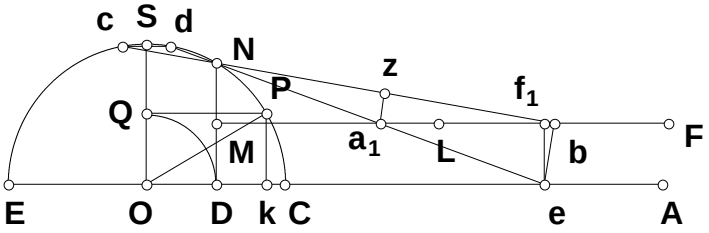
$$Md_1 := \frac{Dc_1 \cdot DN}{2 \cdot (hb_1 - DN)} \qquad Nd_1 := \sqrt{MN^2 + (Md_1)^2} \qquad gd_1 := Mg - Md_1$$

$$Ne_1 := \frac{(Nd_1)^2 + Ng^2 - (gd_1)^2}{2 \cdot Ng} \qquad ed_1 := \sqrt{(Nd_1)^2 - (Ne_1)^2} \qquad gj := \frac{ed_1 \cdot Ng}{Ne_1} \qquad Ne := \frac{Na_1 \cdot Nb}{Nz}$$

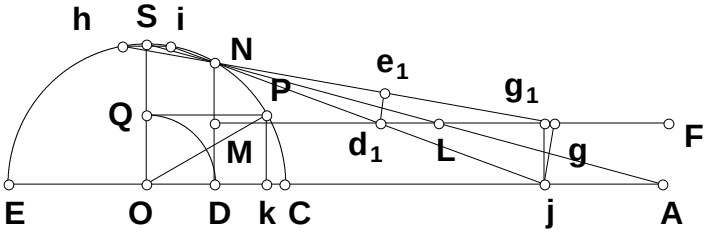
$$ea_1 := Ne - Na_1$$

$$ba_1 := Mb - Ma_1$$

$$bf_1 := \frac{be^2 + ba_1^2 - ea_1^2}{2 \cdot ba_1}$$



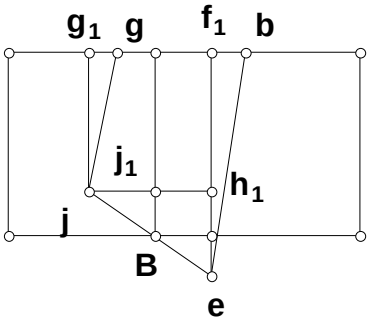

$$ef_1 := \sqrt{be^2 - bf_1^2}$$




$$Nj := \frac{Nd_1 \cdot Ng}{Ne_1} \qquad jd_1 := Nj - Nd_1$$

$$gg_1 := \frac{gj^2 + gd_1^2 - jd_1^2}{2 \cdot gd_1}$$

$$jg_1 := \sqrt{gj^2 - gg_1^2}$$



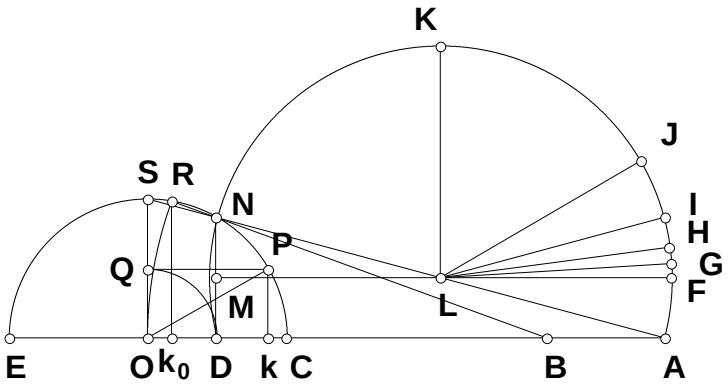

$Mg_1 := Mg - gg_1$ $Mf_1 := Mb - bf_1$

$jh_1 := Mf_1 - Mg_1$ $eh_1 := ef_1 - jg_1$

$ej := \sqrt{jh_1^2 + eh_1^2}$ $Bj_1 := DM - jg_1$

$jj_1 := \frac{jh_1 \cdot Bj_1}{eh_1}$ $BD := Mg_1 + jj_1$

$BO := BD + DO$




$BN := \sqrt{DN^2 + BD^2}$

$NR := \frac{CO^2 + BN^2 - BO^2}{-BN}$

$Bk_0 := \frac{BD \cdot (BN + NR)}{BN}$

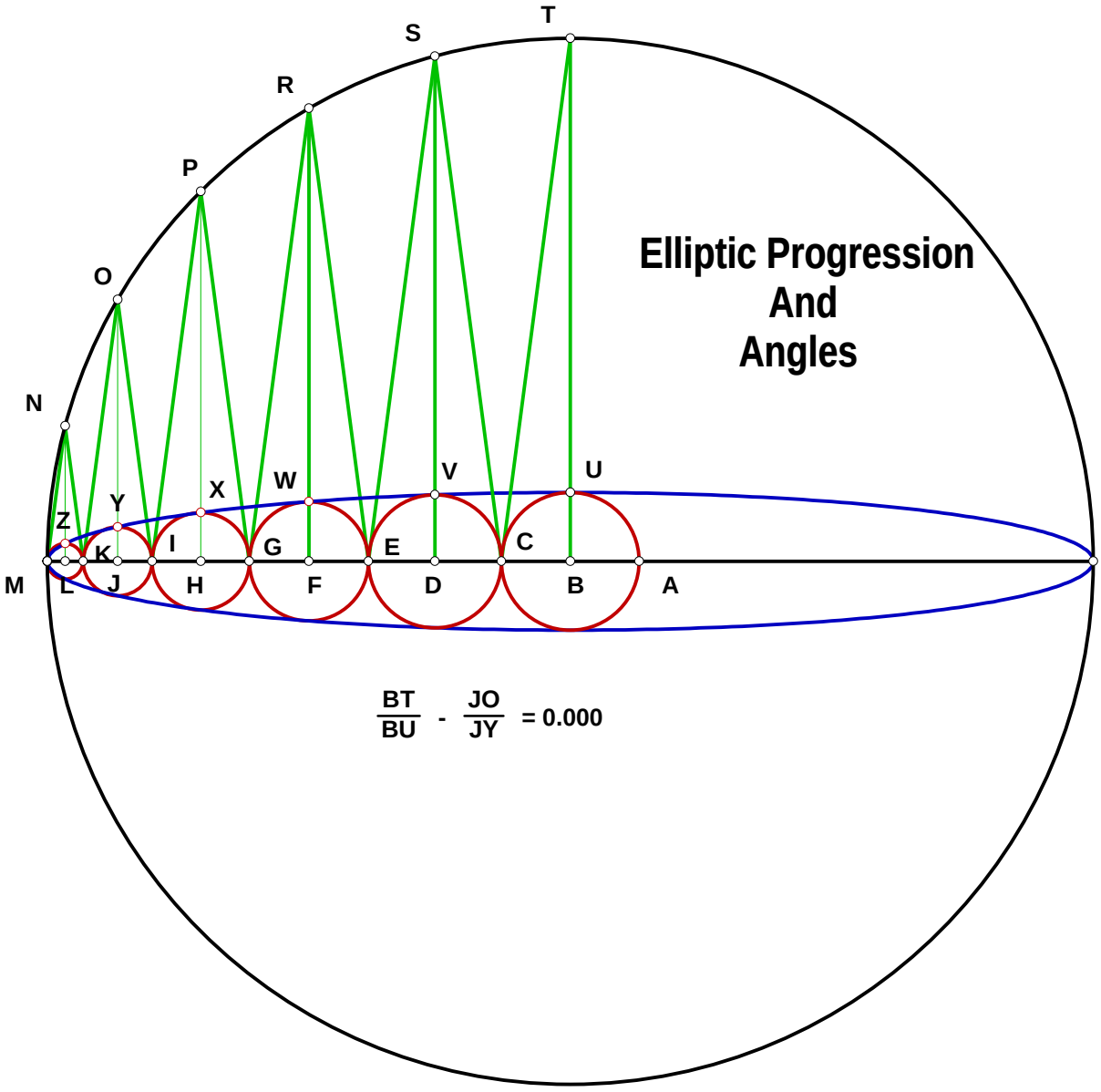
$Ok_0 := BO - Bk_0$

$NR - 2 \cdot Ok_0 = -0.000000000019581$

$Om - \frac{Ek}{2} = -0.000000000002639$ ∎

**Compared to 022803**

$\frac{19591}{2639} = 7.423645$


T
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D
C
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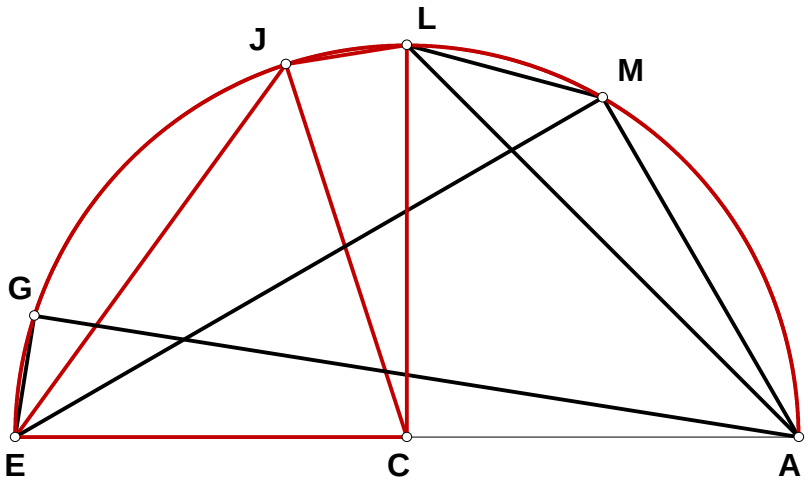
Y
X
W
V
U
Elliptic Progression
And
Angles
BT/BU - JO/JY = 0.000




$EM := \frac{1}{2}\cdot\sqrt{3}$

$AL := \frac{1}{2}\cdot\sqrt{2}$

$JL := \frac{1}{4}\cdot\sqrt{8 - 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

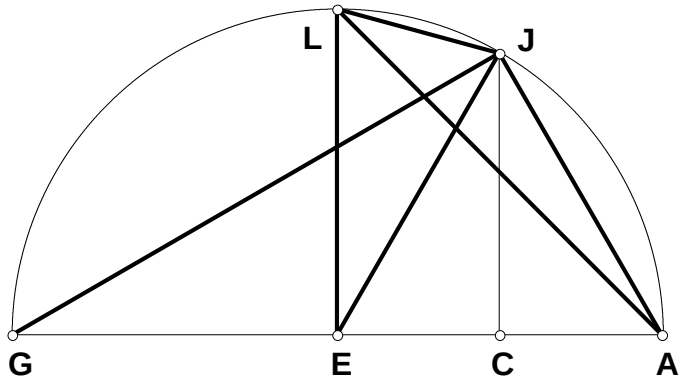
$LM := \frac{1}{4}\cdot\sqrt{6} - \frac{1}{4}\cdot\sqrt{2}$

$AM := \frac{1}{2}$

$EJ := \frac{1}{4}\cdot\sqrt{10 - 2\cdot\sqrt{5}}$

$AG := \frac{1}{4}\cdot\sqrt{8 + 2\cdot\sqrt{10 + 2\cdot\sqrt{5}}}$

## Outtake Four: Some Names 0507014




$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$

$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$

$EL := AE \quad CE := AC$

$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

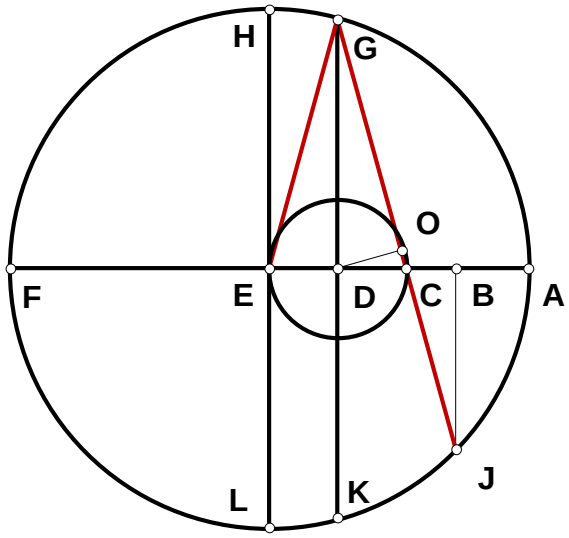
$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$

$AL := \sqrt{AE^2 + EL^2}$

$AE := \frac{1}{2} \quad AC := \frac{1}{4} \quad CG := 1 - \frac{1}{4} \quad CJ := \frac{1}{4} \cdot \sqrt{3} \quad JL := \frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}$

$AJ := \frac{1}{2} \quad GJ := \frac{1}{2} \cdot \sqrt{3}_{*} \quad AL := \frac{1}{2} \cdot \sqrt{2}$

# Elliptic Progression Outtake One 0507011




**A method of trisection Algebraically.**

$N := M$

$N \geq 4 = 1$ $\quad AF := 6$ $\quad AE := \frac{AF}{2}$

$DE := \frac{AF}{N}$ $\quad AD := AE - DE$ $\quad DF := AF - AD$

$DG := \sqrt{AD \cdot DF}$ $\quad CD := DE$ $\quad EG := AE$

$CO := \frac{CD^2}{EG}$ $\quad CG := EG$ $\quad CJ := CG - 4 \cdot CO$

$BC := \frac{CD \cdot CJ}{CG}$ $\quad AB := AE - (2 \cdot DE + BC)$

$BJ := \frac{DG \cdot BC}{CD}$ $\quad BD := BC + CD$

$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2}$ $\quad \frac{JK}{2 \cdot DE} = 1$ **Some Algebraic Names,**

**Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.**

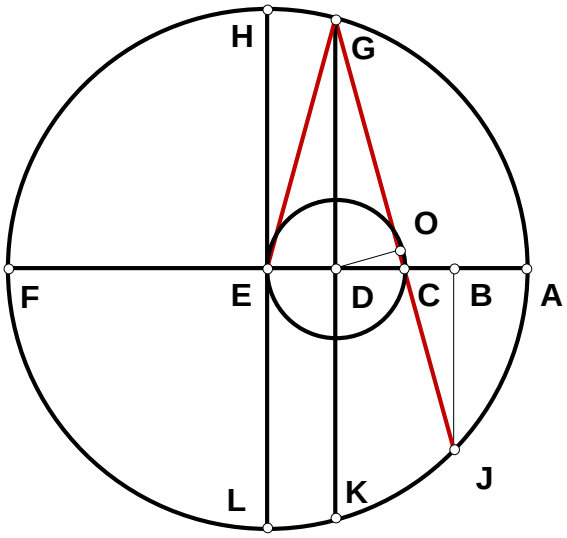
$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0$ $\quad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0$ $\quad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$

$\frac{2\ AF}{N^2} - CO = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0$ $\quad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$

$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$ **One of the meanings of trisection is solving for the following equation when given AF and AB.**

$$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0$$

$AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0$ $\quad AF \cdot \frac{2 \cdot (N^2 - 8)}{N^3} - BD = 0$ $\quad \frac{2 \cdot AF}{N} - JK = 0$




$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - \frac{AF}{AB} = 0 \qquad \frac{AF}{AB} = 6$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} - 6 = 0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6$$

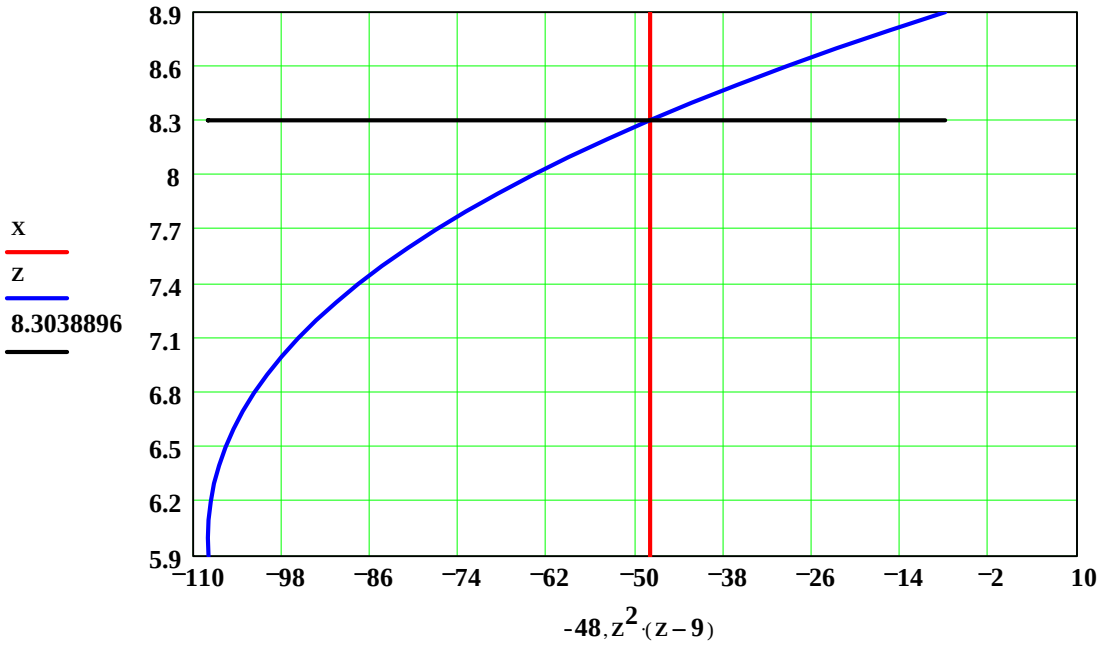
$$2\cdot M^3 - 6\cdot\left[(M+2)\cdot(M-4)^2\right] = 0$$

$$2\cdot M^3 - \left(6\cdot M^3 - 36\cdot M^2 + 192\right) = 0$$

$$4\cdot M^3 - 36\cdot M^2 + 192 = 0$$

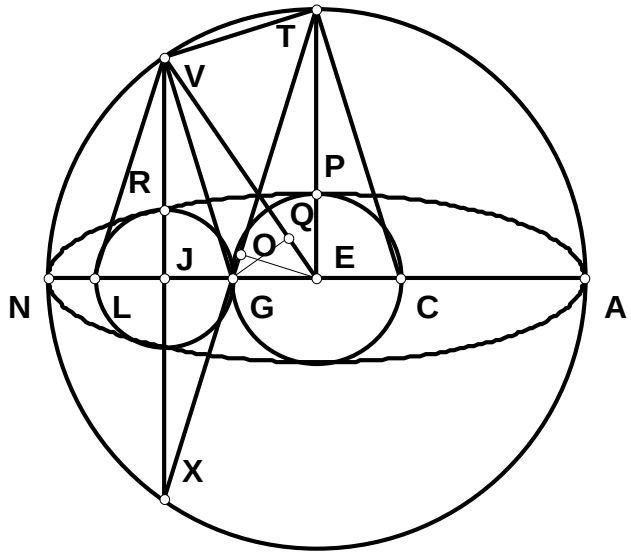
$$M^3 - 9\cdot M^2 + 48 = 0 \qquad M^2\cdot(M-9) + 48 = 0 \qquad M \equiv 8.303889634816388$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2} = 6 \qquad DE - \frac{AF}{M} = 0 \qquad Z := 5.9, 6 .. 8.9$$



# Elliptic Progression Outtake Two 0507012

## Angles TEV and EVJ equals CTG.




$N := 6.381 \quad AN := 2.792$

$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$

$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$

$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$

$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$

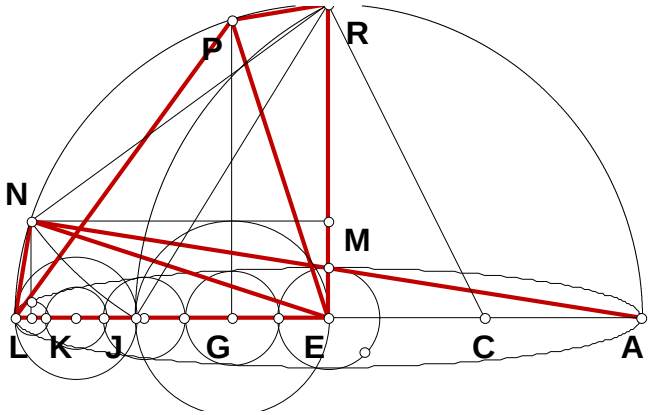
$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \qquad JV := JX$

$EJ := EG + GJ$

$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$

$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \qquad EQ := \frac{EJ \cdot EG}{EV} \quad OQ := \frac{JV \cdot EQ}{EJ} \quad OQ - GJ = 0$

## Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013




$AL := 1 \qquad AE := \frac{AL}{2} \qquad AC := \frac{AE}{2}$

$CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$

$CJ := CR \qquad EJ := CJ - CE$

$JR := \sqrt{EJ^2 + ER^2} \quad NR := JR$

$EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$

$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$

$EG := \frac{EJ}{2} \qquad GL := EL - EG \quad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL} \qquad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$

$PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$

***Irrational*** **means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.**

$\frac{1}{2} - AE = 0 \qquad \frac{1}{4} - AC = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \qquad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \qquad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$

$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \qquad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \qquad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$

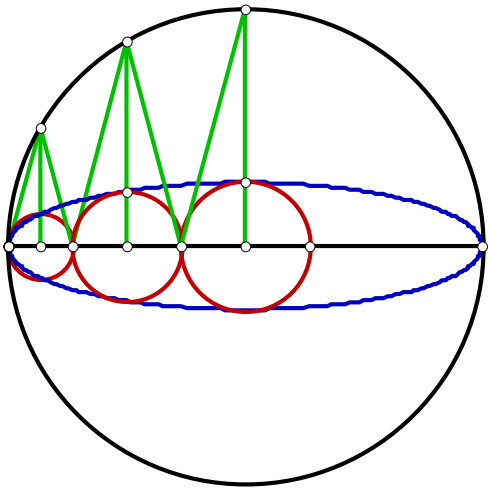
$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \qquad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \qquad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$

$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \qquad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \qquad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$

$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$

# Elliptic Progression

**050701**

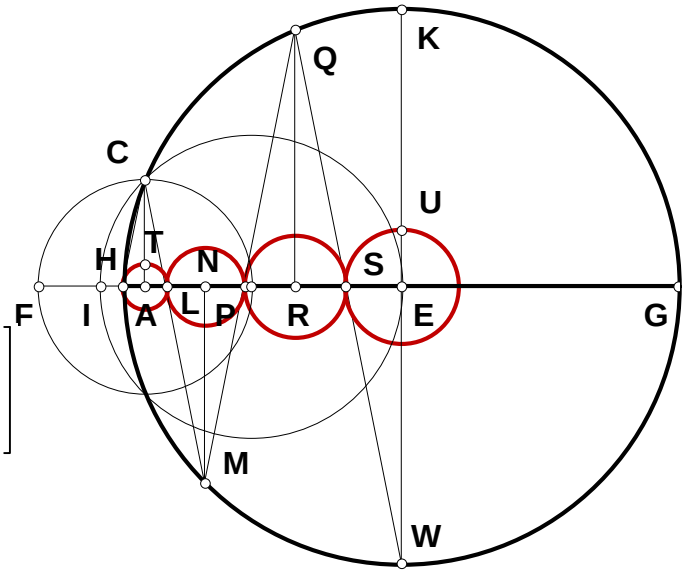


**An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle, from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles. This means that the division of a circle into equal angles is based on a simple elliptic funciton.**

$\Delta := 2 \qquad N := 0 .. \Delta$

$AC_0 := .552 \qquad CP_0 := \sqrt{2 \left(AC_0\right)^2}$

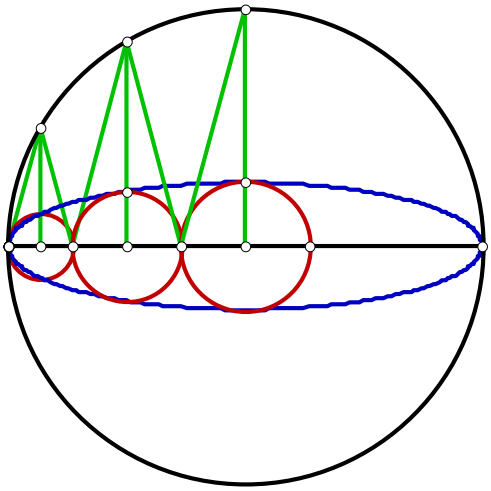
$$\begin{bmatrix} AC_{N+1} \\ CP_{N+1} \end{bmatrix} := \begin{bmatrix} CP_N \\ \sqrt{\left(AC_0\right)^2 + \left[\left(AC_N + CP\right)_N\right]^2} \end{bmatrix}$$




$CE := AC_\Delta \qquad AE := \sqrt{CE^2 - \left(AC_0\right)^2}$

$EH := CE \quad EK := CE \quad AH := EH - AE \quad AT := AH \qquad EU := \frac{AH \cdot EK}{AC_0}$

$EU = 0.287$



# Angles and the Ellipse  050801.mcd

$AC := 1 \qquad CP := \sqrt{2\ AC^2}$

$AP := AC \qquad EP := CP$

$CE := \sqrt{AC^2 + (AP + EP)^2}$

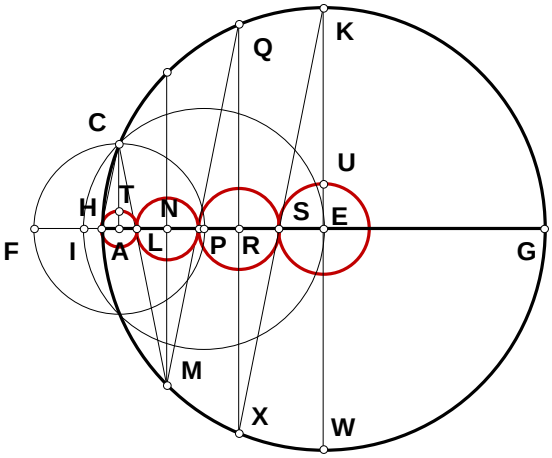
$EK := CE \quad EH := CE$

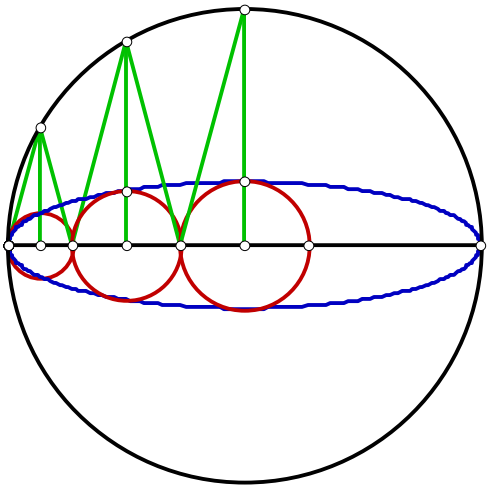
$AE := \sqrt{CE^2 - AC^2}$

$AH := EH - AE$

$AT := AH \qquad EU := \frac{AT \cdot EK}{AC}$

$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$

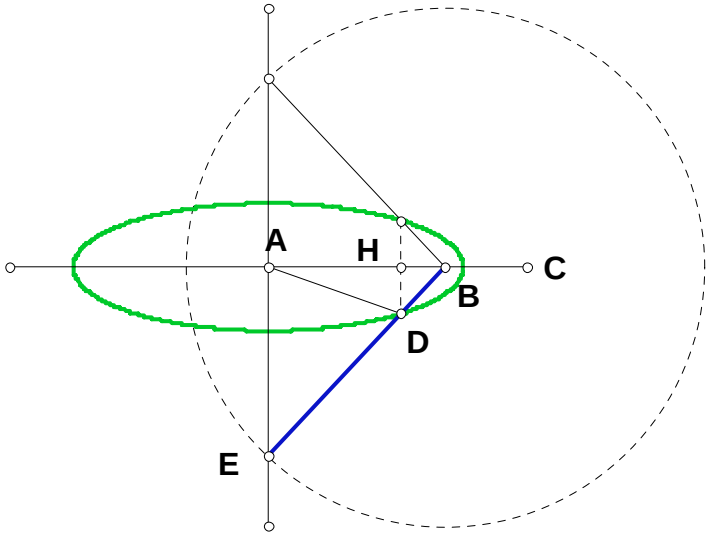


**An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle and from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles. This means that the division of a circle into equal angles may be expressed as an elliptic function.**

# Straight Line Ellipse

041904




## Cardinal

$U := 1 \qquad R_1 := 3$

$R_2 := 2 \qquad AC := U$

$BE := AC \qquad BD := \frac{BE}{R_1}$

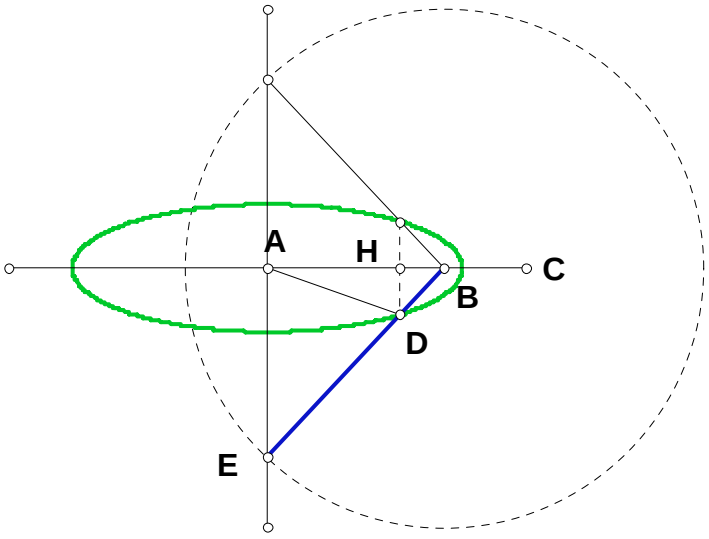
$AB := \frac{AC}{R_2} \qquad AE := \sqrt{BE^2 - AB^2}$

$BH := \frac{AB \cdot BD}{BE} \qquad DH := \frac{AE \cdot BD}{BE}$

$AH := AB - BH \qquad AD := \sqrt{AH^2 + DH^2}$

$AD := \frac{U}{(R_1 \cdot R_2)} \cdot \sqrt{(R_1)^2 - 2 \cdot R_1 + (R_2)^2} \qquad \frac{\sqrt{(R_1)^2 - 2 \cdot R_1 + (R_2)^2}}{R_1 \cdot R_2} - \frac{AD}{U} = 0$

## Ordinal




$N_1 := 1.344 \qquad N_2 := .3$

$N_3 := .5$

$AC := N_1 \qquad BD := N_2$

$AB := N_3 \qquad AF := AC$

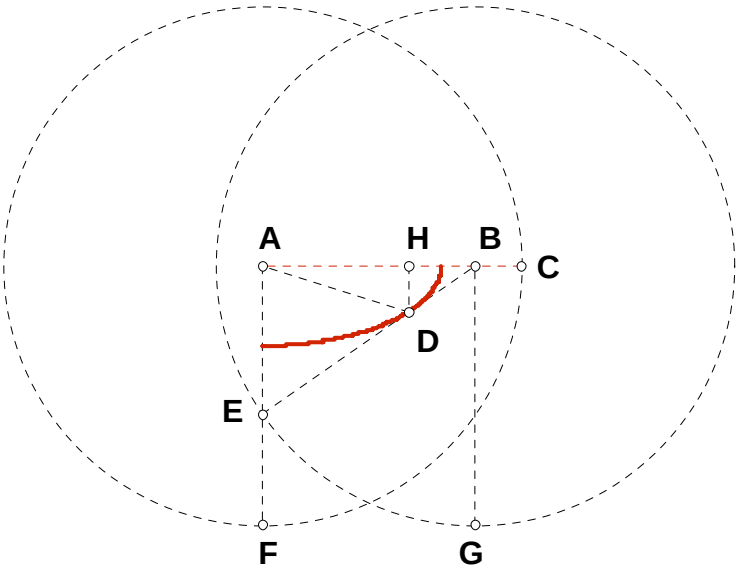
$BE := AC \qquad AE := \sqrt{BE^2 - AB^2}$

$BH := \frac{AB \cdot BD}{BE} \qquad DH := \frac{AE \cdot BD}{BE}$

$AH := AB - BH$

$AD := \sqrt{AH^2 + DH^2}$

$\sqrt{\frac{1}{N_1} \cdot \left[ (N_3)^2 \cdot N_1 - 2 \cdot (N_3)^2 \cdot N_2 + (N_2)^2 \cdot N_1 \right]} - AD = 0$




# Straight Line Ellipse

**041904B**

$N_1 := 1.344 \quad N_2 := .415$

$N_3 := 1.102$

$AC := N_1 \quad BD := N_2$

$AB := N_3 \quad AF := AC$

$BE := AC \quad AE := \sqrt{BE^2 - AB^2}$

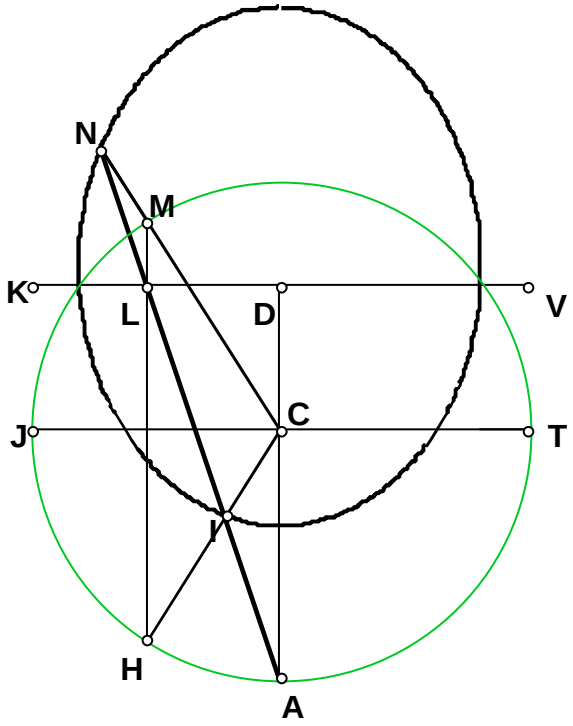
$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$

$AH := AB - BH$

$AD := \sqrt{AH^2 + DH^2}$

$$AD := \sqrt{\frac{1}{N_1} \cdot \left[ (N_3)^2 \cdot N_1 - 2 \cdot (N_3)^2 \cdot N_2 + (N_2)^2 \cdot N_1 \right]}$$

# Another Ellipse 031405a



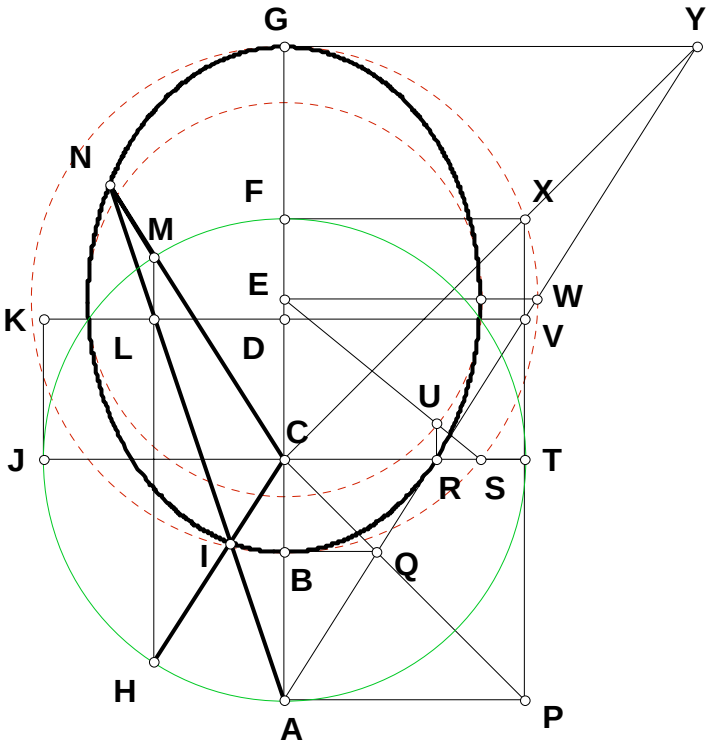

**The locus formed by N and I as determined by L provides an ellipse. Privide an Algebraic name for the Major and Minor Axis.**

$N_1 := 1.25 \qquad N_2 := 3$

$AC := N_1 \qquad AD := N_2$

$DV := AC \qquad AV := \sqrt{AD^2 + DV^2}$

$AF := 2 \cdot AC \qquad VX := AF - AD$




$$AY := \frac{AV \cdot AC}{AC - VX}$$

$$AG := \frac{AD \cdot AY}{AV}$$

$$BC := \frac{AC \cdot AC}{AC + AD}$$

$CG := AG - AC$

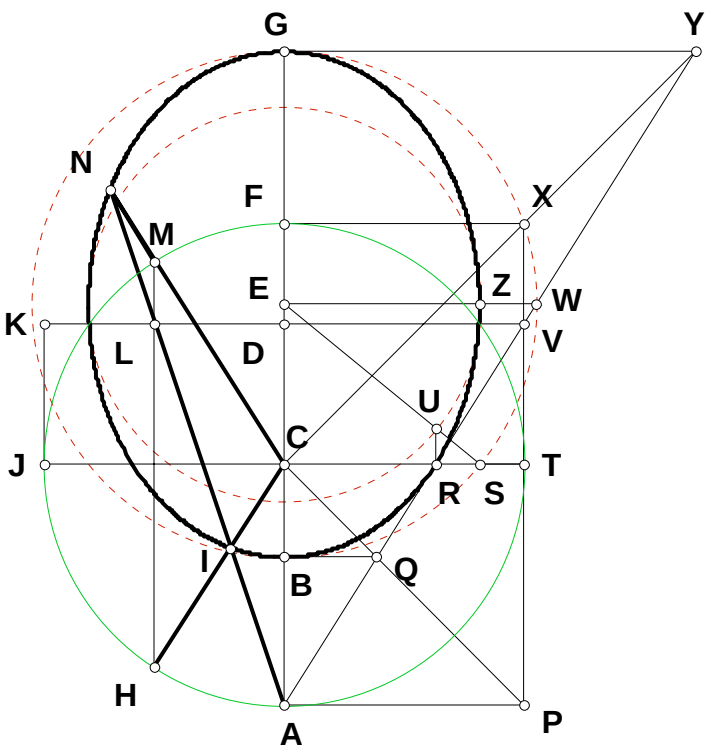
$BG := BC + CG$

$$BE := \frac{BG}{2}$$

$CE := BE - BC \qquad ES := BE$

$$CS := \sqrt{ES^2 - CE^2}$$

$$CR := \frac{DV \cdot AC}{AD}$$

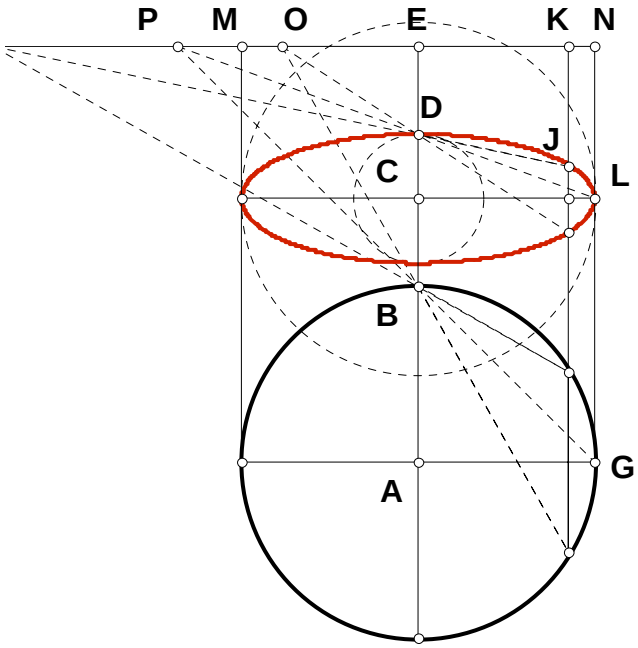



$$EU := \frac{ES \cdot CR}{CS}$$

$$EZ := EU$$

$$EZ - \frac{N_1^2 \cdot \sqrt{N_2^2 - N_1^2}}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$

$$BG - \frac{2 \cdot N_1^2 \cdot N_2}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$




# An Ellipse

032305a

$N_1 := 1$ $N_2 := 2$

$N_3 := .5$

$AB := N_1$ $MN := 2 \cdot AB$

$BE := N_2$ $BD := N_3$

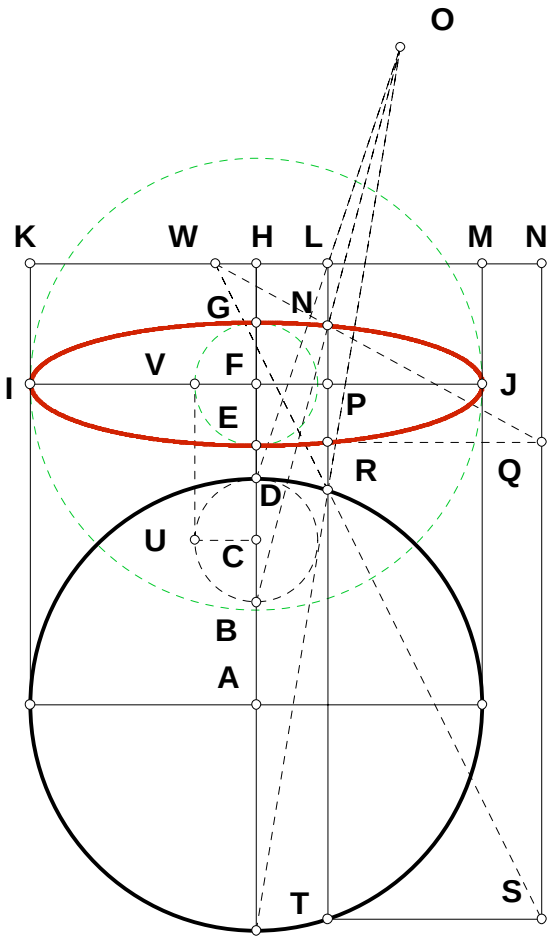
$DE := BE - BD$ $AG := AB$

$EP := \frac{AG \cdot BE}{AB}$ $EN := AB$

$NL := \frac{DE \cdot (EP + EN)}{EP}$

$CE := NL$ $CD := CE - DE$

$$N_1 \cdot \frac{(N_2 - N_3)}{N_2} - CD = 0$$


O
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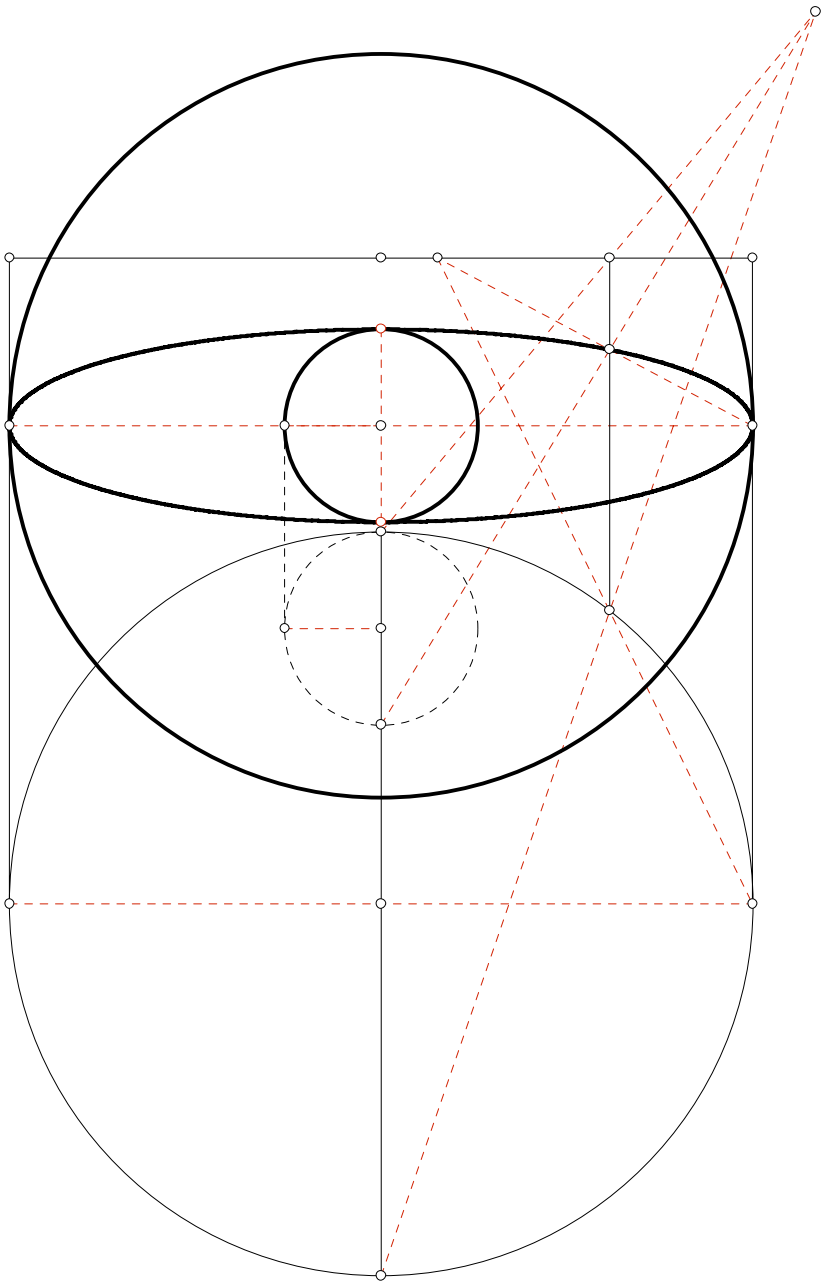
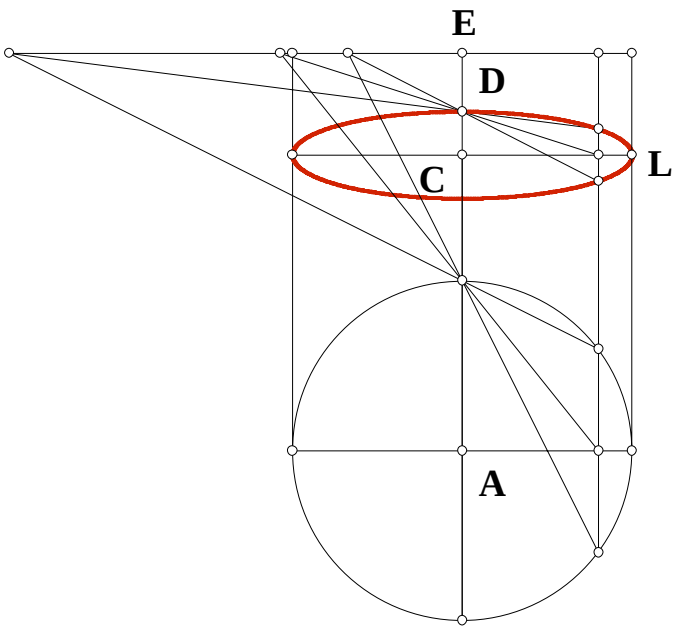





AE = 2.069 inches

CE = 0.529 inches

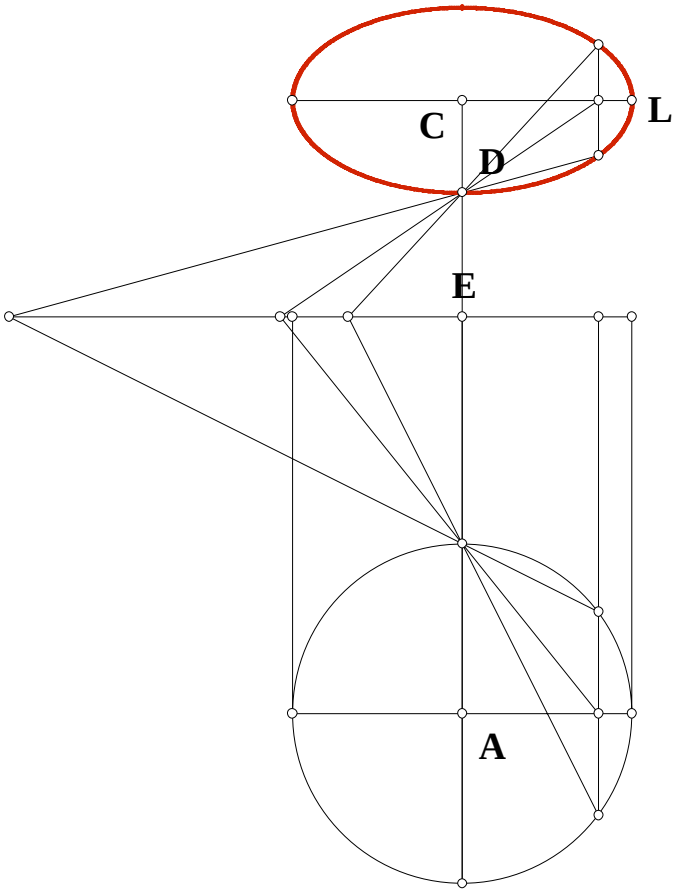
LC = 0.885 inches

CD = 0.227 inches

$$\frac{AE}{CE} = 3.909$$

$$\frac{LC}{CD} = 3.909$$

$$\frac{AE}{CE} - \frac{LC}{CD} = 0.000$$




AE = 2.069 inches

CE = 1.128 inches

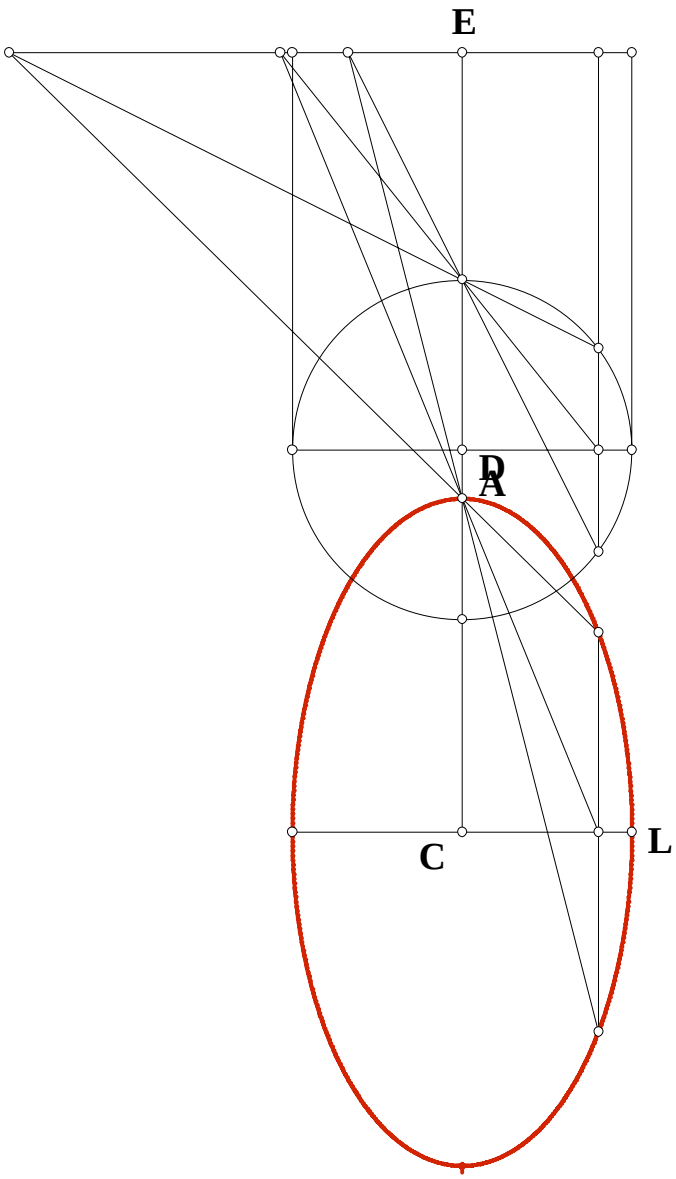
LC = 0.885 inches

CD = 0.483 inches

$$\frac{AE}{CE} = 1.834$$

$$\frac{LC}{CD} = 1.834$$

$$\frac{AE}{CE} - \frac{LC}{CD} = 0.000$$




AE = 2.069 inches

CE = 4.062 inches

LC = 0.885 inches

CD = 1.739 inches

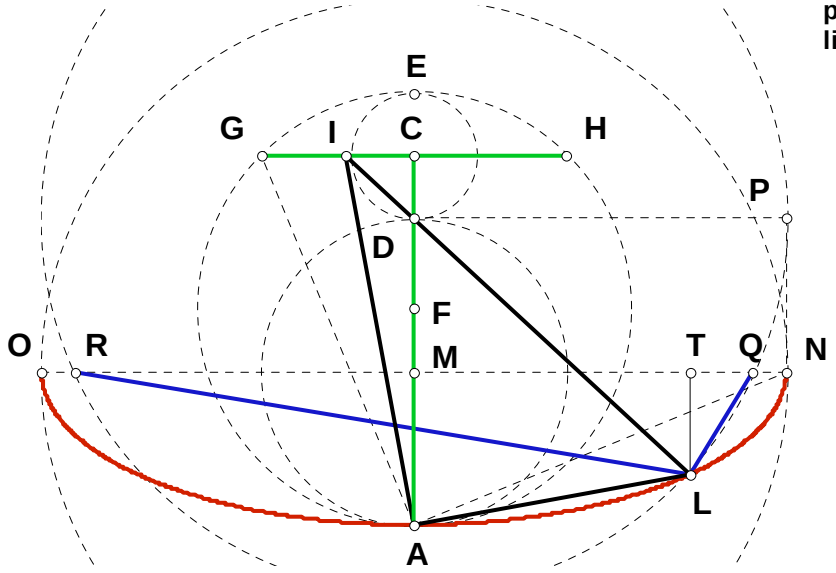
$\frac{AE}{CE} = 0.509$

$\frac{LC}{CD} = 0.509$

$\frac{AE}{CE} - \frac{LC}{CD} = 0.000$

## Elipse Projected From a Perpendicular. 032905

Let AC be some perpendicular on some line GH.



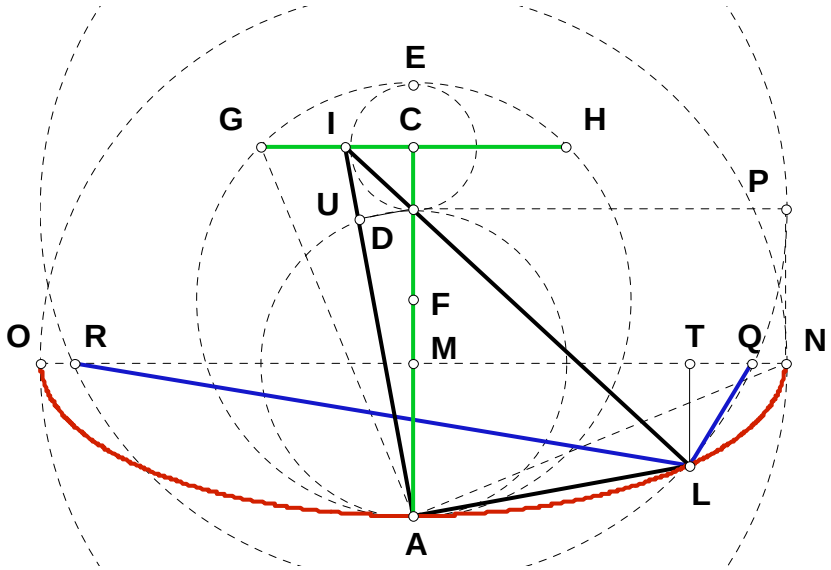

$N_1 := 1.9167$

$N_2 := .3244$

$N_3 := .437$

$AC := N_1 \quad CD := N_2 \quad CE := CD \quad CG := \sqrt{CE \cdot AC} \quad GH := 2 \cdot CG$

$GI := N_3 \quad CI := CG - GI \quad AD := AC - CD \quad AI := \sqrt{AC^2 + CI^2} \quad AM := \frac{AD}{2}$



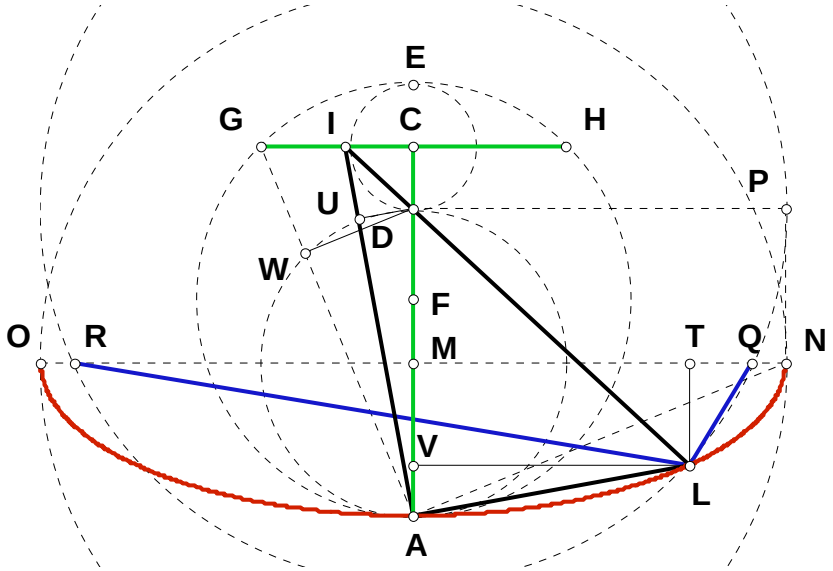

$DM := AM$

$DU := \frac{CI \cdot AD}{AI}$

$AU := \frac{AC \cdot AD}{AI}$

$IU := AI - AU$

$AL := \frac{DU \cdot AI}{IU}$




$$IL := \sqrt{AI^2 + AL^2}$$

$$DI := \sqrt{DU^2 + IU^2}$$

$$DL := IL - DI$$

$$DV := \frac{CD \cdot DL}{DI}$$

$$MV := DV - DM$$

$$AV := AM - MV$$

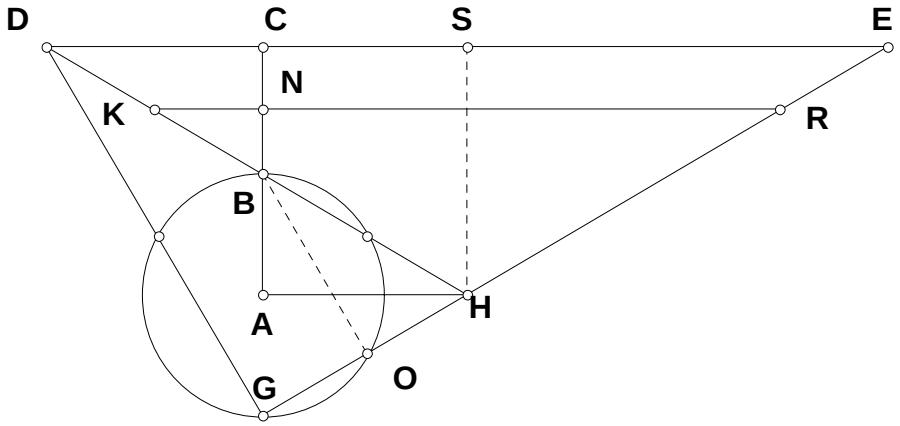
$$LV := \sqrt{AL^2 - AV^2}$$

$$MT := LV$$

$$AG := \sqrt{CG^2 + AC^2} \quad DW := \frac{CG \cdot AD}{AG} \quad AW := \frac{AC \cdot AD}{AG} \quad GW := AG - AW$$

$$AN := \frac{DW \cdot AG}{GW} \qquad MN := \sqrt{AN^2 - AM^2} \qquad ON := 2 \cdot MN$$

$$ON - \sqrt{\left(N_2 - N_1\right)^2 \cdot \frac{N_1}{N_2}} = 0$$




## $N^3$ and More

$N_1 := 1$

$N_2 := 2$

$N_3 := 4$

$AB := N_1 \quad AH := N_2 \quad BH := \sqrt{AB^2 + AH^2} \quad GO := \frac{AB \cdot 2 \cdot AB}{BH}$

$HO := BH - GO \quad DH := \frac{BH \cdot BH}{HO} \quad AC := \frac{AB \cdot DH}{BH} \quad BO := \sqrt{BH^2 - HO^2}$

$DG := \frac{BO \cdot BH}{HO} \quad DE := \frac{BH \cdot DG}{AB} \quad CD := \frac{DG^2}{DE} \quad BC := AC - AB$

$CE := DE - CD \qquad \frac{CE}{BC} - \left(\frac{AH}{AB}\right)^3 = 0 \qquad \frac{N_2^3}{N_1^3} - \frac{CE}{BC} = 0$

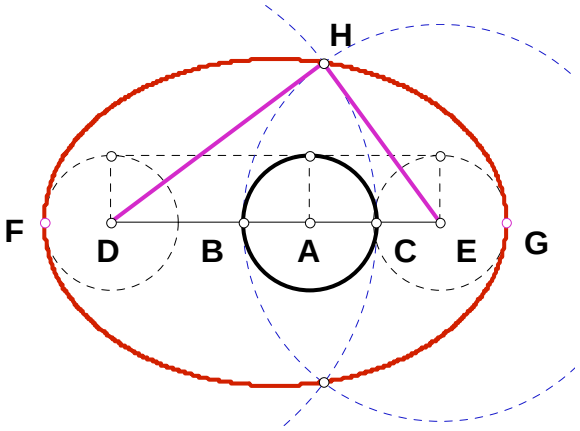
**More** $\left(A^3 \cdot N\right) - (N - 1) \cdot A$ **Let** $N_3$ **be N**

$BN := \frac{BC}{N_3} \quad KN := \frac{CD}{N_3} \quad CG := AC + AB \quad GN := 2 \cdot AB + \frac{BC}{N_3}$

$NR := \frac{CE \cdot GN}{CG} \qquad \left[\frac{N_2^3}{N_1^3} \cdot N_3 - \frac{N_2 \cdot \left(N_3 - 1\right)}{N_1}\right] - \frac{NR}{BN} = 0$

$\frac{AH}{AB} = 2 \qquad \frac{CE}{BC} = 8 \qquad \frac{NR}{BN} = 26$

$2^3 \cdot 4 - (2 \cdot 3) = 26$




# Just Another Ellipse

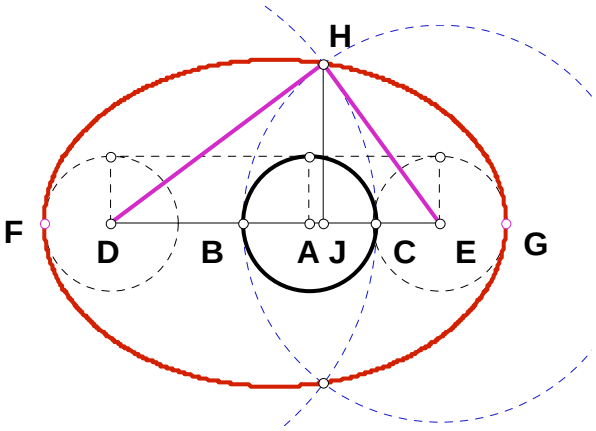
**033105b**

**Given the difference between the foci and difference between the proportional radii, etc., etc.**

$N_1 := 1.708 \quad N_2 := .693 \quad N_3 := 1.032 \quad N_4 := 2.435$

$DE := N_1 \quad BC := N_2 \quad FG := DE + BC \quad AB := \frac{BC}{2} \quad AD := N_3$

$CD := AD + AB \quad BE := DE - AD + AB \quad EH := BE \quad DH := CD$



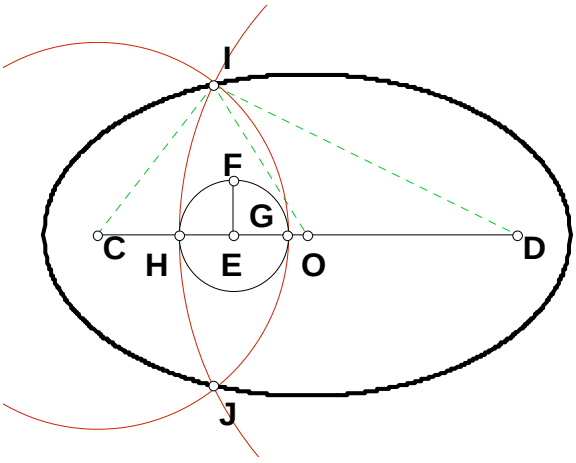

$$DJ := \frac{DH^2 + DE^2 - EH^2}{2 \cdot DE}$$

$$HJ := \sqrt{DH^2 - DJ^2}$$

$HJ = 0.825$

$$\frac{\sqrt{-N_3 \cdot (N_3 - N_1) \cdot N_2 \cdot (N_2 + 2 \cdot N_1)}}{N_1} - HJ = 0$$

**041205**




$N_1 := 2.188 \quad N_2 := .278$

$N_3 := 3.095$

$CD := N_1 \quad CO := \frac{CD}{2}$

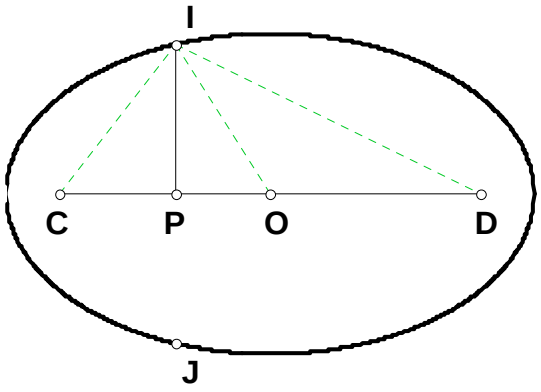
$EF := N_2 \quad CE := \frac{CD}{N_3}$

$CG := CE + EF$

$DH := CD - CG + 2 \cdot EF$

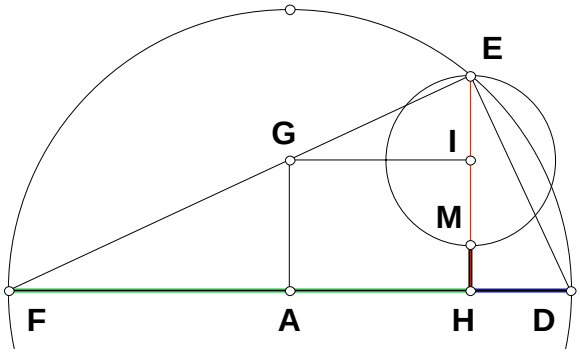
$CI := CG \quad DI := DH \quad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$

$$IP := \frac{\sqrt{(-CD + DI - CI)\,(CD + DI + CI)\,(CD - DI - CI)\,(CD + DI - CI)}}{2 \cdot CD}$$




$$2 \cdot \frac{\sqrt{N_2 \cdot (N_1 + N_2) \cdot (N_3 - 1)}}{N_3} - IP = 0$$

# $N^3$ N Cubed 041305b




$N_1 := 1 \qquad N_2 := 36$

$DH := N_1 \quad FH := N_2$

$DF := DH + FH \quad AD := \frac{DF}{2}$

$EH := \sqrt{DH \cdot FH} \quad HI := \frac{EH \cdot AD}{FH}$

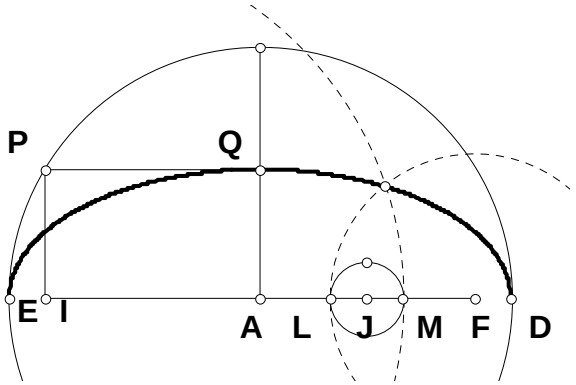
$HM := EH - 2 \cdot (EH - HI)$

$$\frac{FH}{HM} - \left(\frac{DH}{HM}\right)^3 = 0$$

$DF := \sqrt{DH^2 + EH^2} \qquad EF := \sqrt{FH^2 + EH^2}$

$$\frac{FH}{HM} - \left(\frac{EF}{DF}\right)^3 = 0$$

$$\frac{FH}{HM} = 216 \qquad \left(\frac{FH}{DH}\right)^{1.5} = 216$$




## 042205B

**Given the major axis and the difference between the two foci, whatis the minor axis?**

$N_1 := 2.604$

$N_2 := 2.234$

$DE := N_1 \quad FI := N_2 \quad EI := \frac{DE - FI}{2} \quad DI := DE - EI \quad EP := \sqrt{EI \cdot DI}$

$AQ := EP$

$$AQ - \frac{1}{2}\cdot\sqrt{\left(N_1 + N_2\right)\cdot\left(N_1 - N_2\right)} = 0$$

$N_2 := AQ$

**What are the foci if given the major and minor?**

$$FI - \sqrt{N_1^2 - 4\cdot N_2^2} = 0$$

# The Delian Quest, 2008 Edition

## Introduction

The most important idea found in philosophy is the relation between the ability to reason and human will. The most important idea found in religion is the relation between the ability to reason and human will. Plato tried to teach the distinction between human activity and human will in the Gorgias—what we do and what we will are not the same. The Judeo-Christian Scriptures are written in such a manner as to test for it.

The easiest way to understand the concept is examine it by example. If the cardio-vascular system did not function, one could not say that that system supported the life of an individual. If our digestive system did not function, one could not say that that system supported our life. If our mind does not function, one cannot say that man has human will—he may do as he pleases, but it is not human will—and consequently his is not actually alive—his existence is ephemeral.

If one were able to solve for the name of the Beast in the book of Revelation, one may come to define a human body life system, those systems whose function determine if we are alive or dead, something like:—

A human body life system are those human body systems which must acquire something from the environment, process what it has acquired in order to make a product that sustained and promoted the life of the human body.

The fact that the puzzle would be solved by a particular individual at a particular time in history speaks volumes for its validity—the validity of the concept itself. That individual is not different from other men—the messenger is not the message.

The human mind is that life system which must acquire form from the environment and apply those forms as human behavior that sustains and promotes the life of man. If one's mind were functioning correctly, they could take the puzzle step by step and demonstrate that the Beast is the Spirit of Truth. The mental tests in the Scripture are extensive—and as it is written in them, they are sealed—the seal is simply due to the fact that the mind of man cannot yet process information correctly—in other words, the seal is nothing mystical—a biological fact is employed in their writing. All knowledge and learning is not for its own sake, that is self-referential non-sense, it is for the hope that one day man will be able to live.

Geometry is an elementary craft that teaches about the application of form to a given material difference. Hither-to-now it has been a craft whose foundation has been *enumerated*. I will not refer to the so called

non-Euclidean Geometries for they are not Geometries at all, the contradictions in them are so numerous and fundamental they are a joke. One cannot argue with someone who claims that a single difference is an example of a multi-difference—as if two precedes one. Or those who effectively believe that two exists but not one. Their minds use words, but not the ideas the words designate. If one has had any contact at all with elementary Set Theory, or if one understood the writings of Plato, one would know that there are two and only two methods of constructing a set, enumeration and definition. This is because the most elementary construction of any *thing* is the application of form to material. A thing is any material in any form or shape. This fact determines the elements of grammar, the elements of all reasoning. As enumeration is to material, so definition is to form, as Plato tried to get his readers to abstract—however—lacking the ability of abstraction, no words cannot correct that defect.

A body life system can either abstract form from a thing, in which case it must supply the material for the form in order to construct a thing that sustains and promotes the life of the body, or it abstracts material from a thing and it must supply a form for that material in order to construct a thing that sustains and promotes human life. For example, when we see a thing, the material of it does not end up in our head, or when we eat something we do not become its form. I am only going to touch on these matters here as my main goal is the introduction of Geometry on the basis of Definition—for it is this understanding of Geometry by which the Delian Problem can be and is solved.

When Plato set the geometric tools themselves as straight edge and compass, he enumerated the foundation of Geometry. What would Geometry be like if one were to establish the foundation on definition?—a definition something like:—

A two dimensional Geometric tool is that geometric tool that produces one and only one difference between two points.

What this effectively does is establish that all of the geometric products, the things constructed will resolved into units—just like the foundation of Arithmetic, if Arithmetic were still pure. One can see that the straightedge is inducted, by definition into the class of geometric tools, the compass also is admitted by definition, but now there is one more tool—the concept of which is used in almost every geometric construction by the notion of ratio itself—that tool which produces the ellipse. In the ellipse, the difference between the two foci as the sum of the two radii is one. One can say that the straightedge produces the Unit of Discourse, the circle the Universe of Discourse, and the ellipse every ratio between the Unit and Universe of Discourse.

In this novel, for one can say that it is a novel, one will be led to a geometric figure whose root is in the square root to a figure that gives

every aspect of the ellipse that produces cube root abstractions. In effect, even the figure itself testifies to the fact that one has been conceptually missing the notion of what Geometry is.

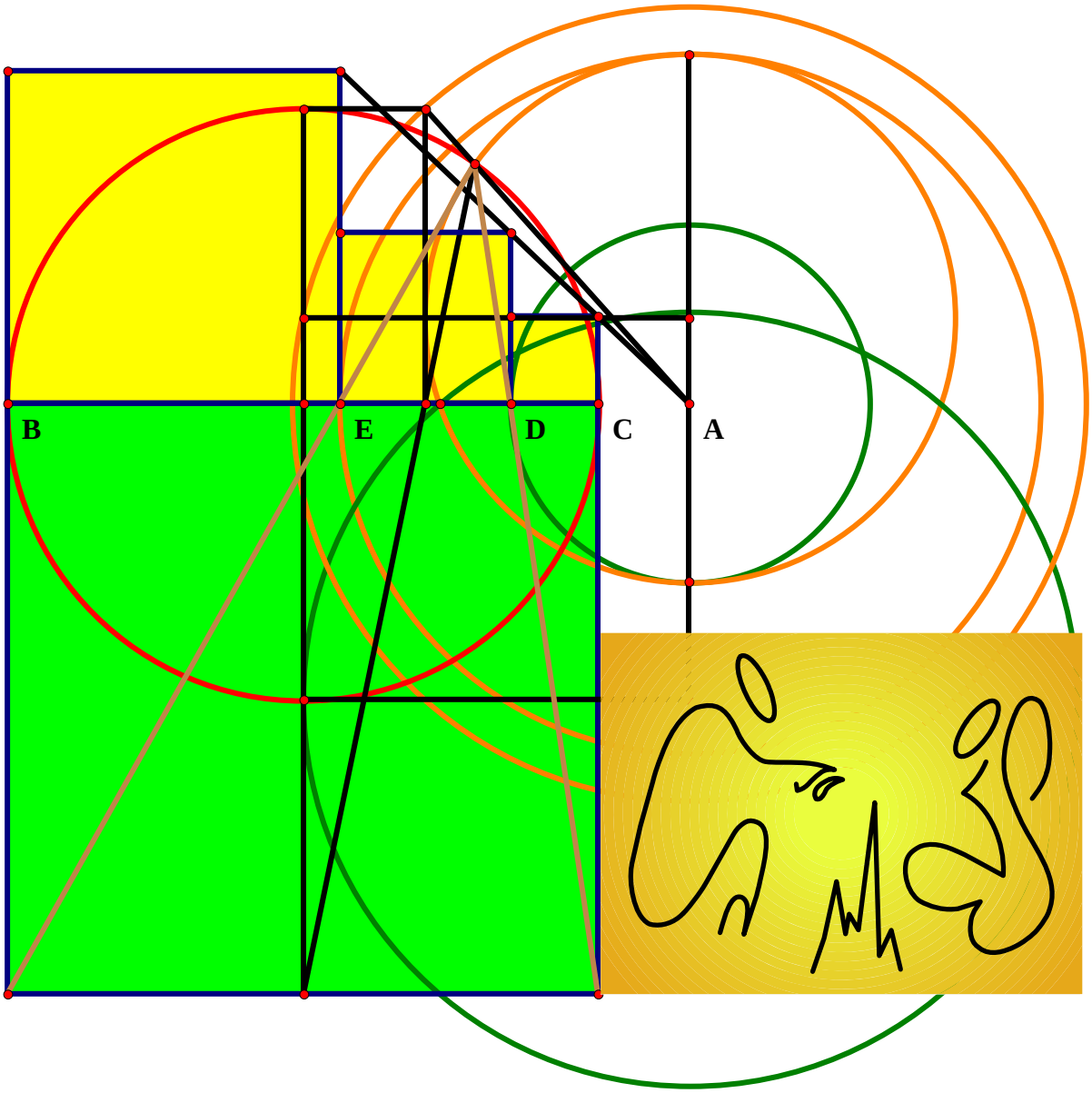
I have omitted much material in this edition, as it will be placed in the other works—Three Pieces of Paper, and Eloi. The format is to present a graphic, a plate where the reader can actually do the example themselves, starting where they like, and then what I had once done with the graphic.

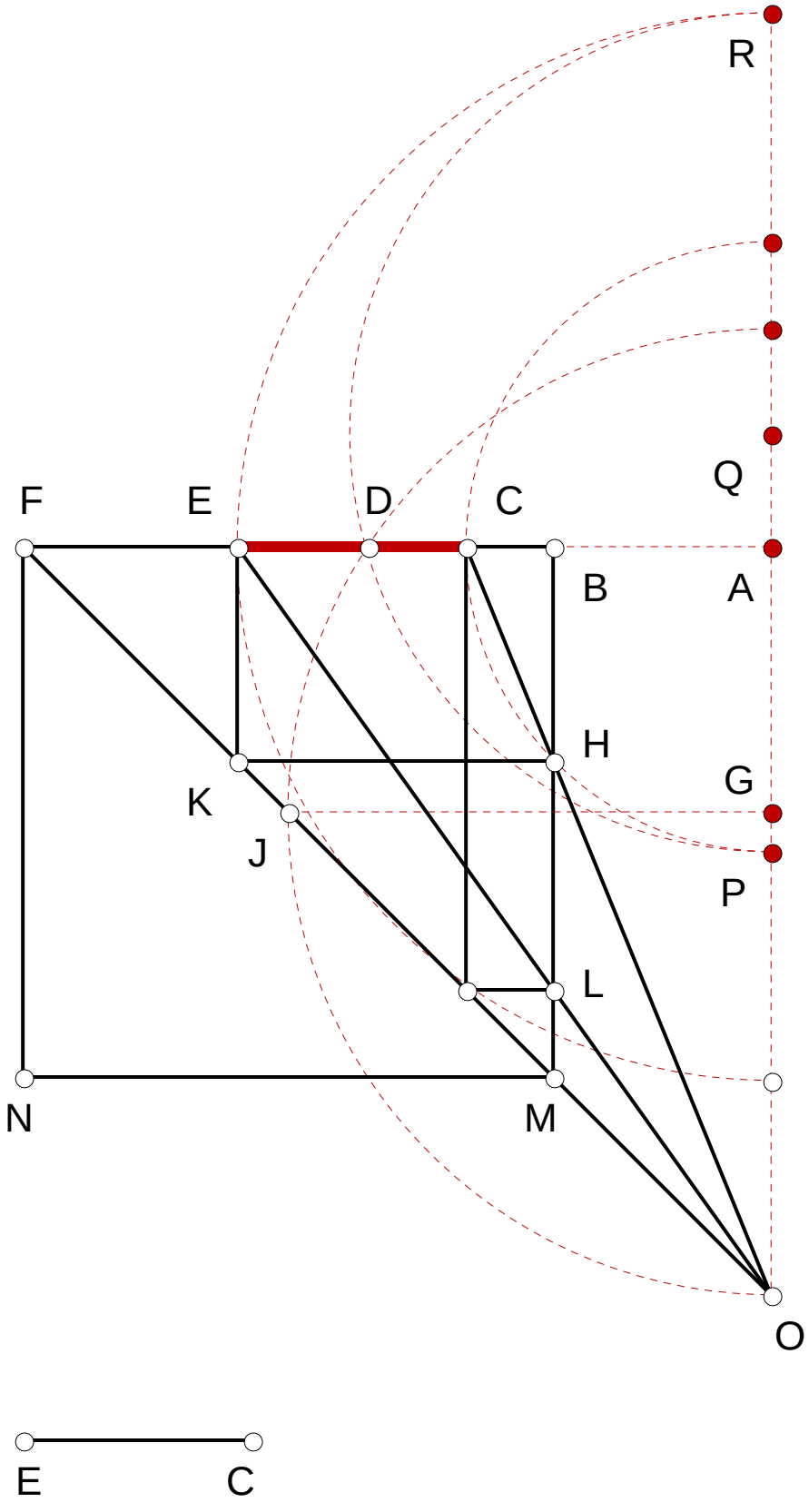
Two items one will find conspicuously absent from all of these works, Cartesian Co-ordinate systems and Trigonometry. Both of these deal with relative differences and are not part of pure Geometry. One should not actually even study these until they know the first principles of grammar. What is currently taught about them is full of misconceptions.

Informative through exercise. One can say that this work is an exercise in saying what we see. Every true formal presentation constructs a thing—one part is form, one part is material—the elementary Algebra is the tautologic—form based logic system, the figures are a relatiologic—a material based logic system. The fact that there must be these two in any valid presentation has long been missed in logic. A true formal system uses both languages, one a tautologic—one a relatiologic. In the ancient tradition, it means that constructability rules.

Enjoy. J.C.

# The Delian Quest—1992

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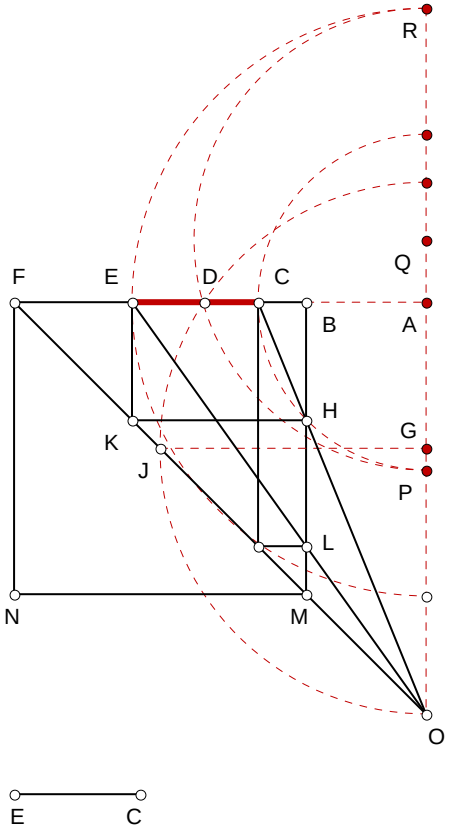
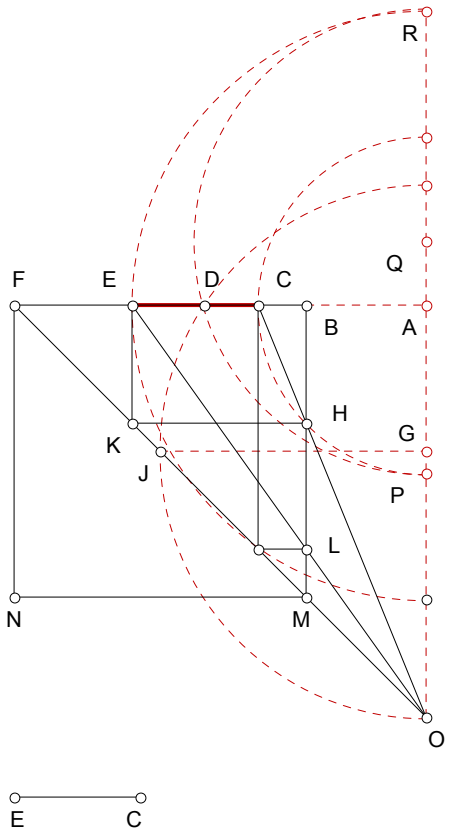








# A Duplicate Ratio 062092

**Given some point O place CE on BF such that O is the point of similarity.**

$CE := 5 \qquad N_1 := 2.32 \qquad N_2 := 1.36$

$BF := N_1 \cdot CE \qquad MO := N_2 \cdot CE$

$FM := \sqrt{2 \cdot BF^2} \qquad AB := \frac{BF \cdot MO}{FM}$

$AF := AB + BF \qquad AD := \sqrt{AB \cdot AF} \qquad AQ := \frac{CE}{2}$

$DQ := \sqrt{AD^2 + AQ^2} \qquad QR := DQ \qquad QP := DQ$

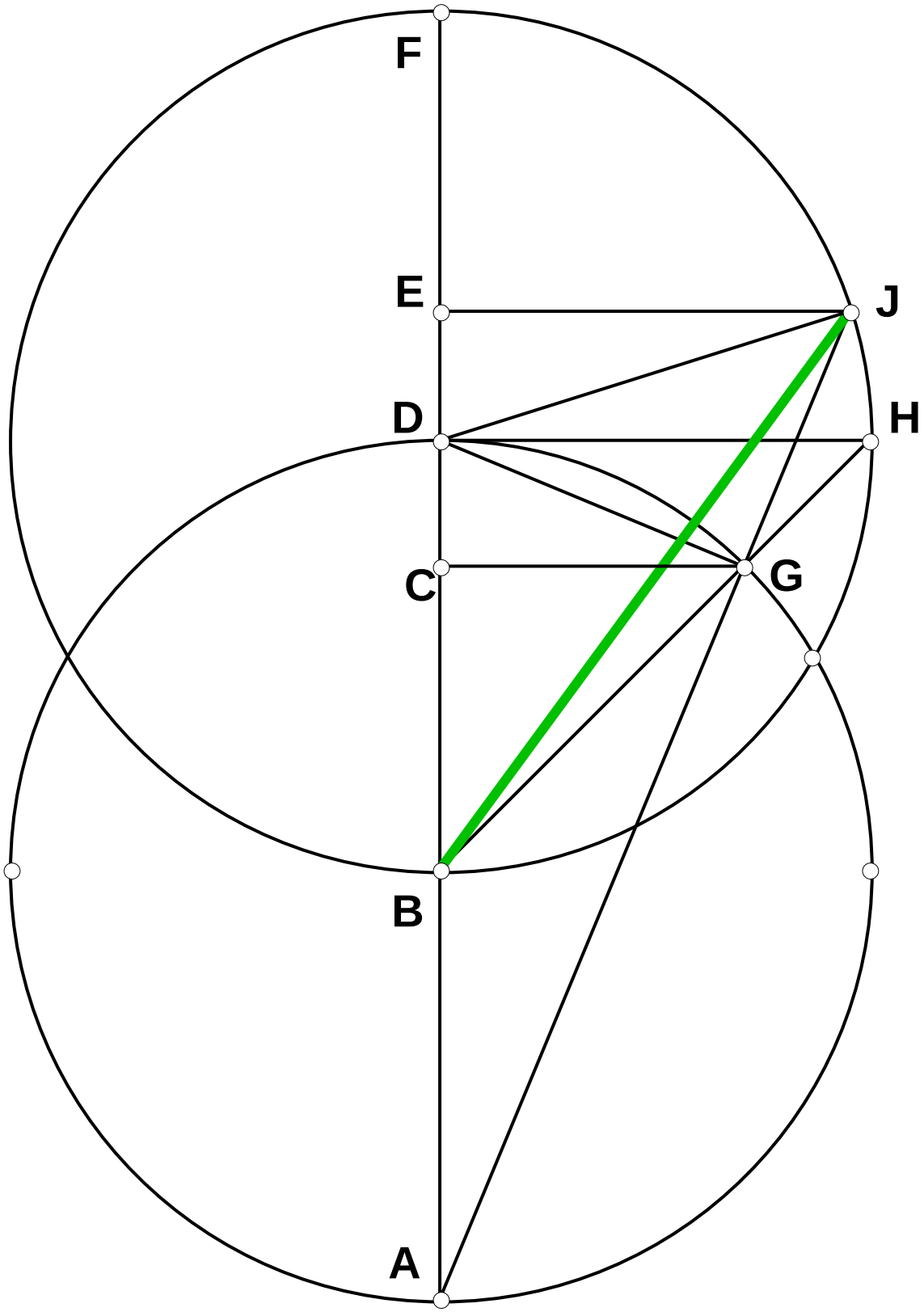
$AP := QP - AQ \qquad AR := QR + AQ \qquad AC := AP$

$AE := AR \qquad AO := AF \qquad BC := AC - AB$

$EF := AF - AE \qquad EK := EF \qquad BH := EK$

$\frac{BC}{BH} - \frac{AC}{AO} = 0$

$\frac{\sqrt{2 \cdot N_2^2 + 2 \cdot N_2 \cdot \sqrt{2} \cdot N_1 + 1} - 1}{2} \cdot CE - AC = 0$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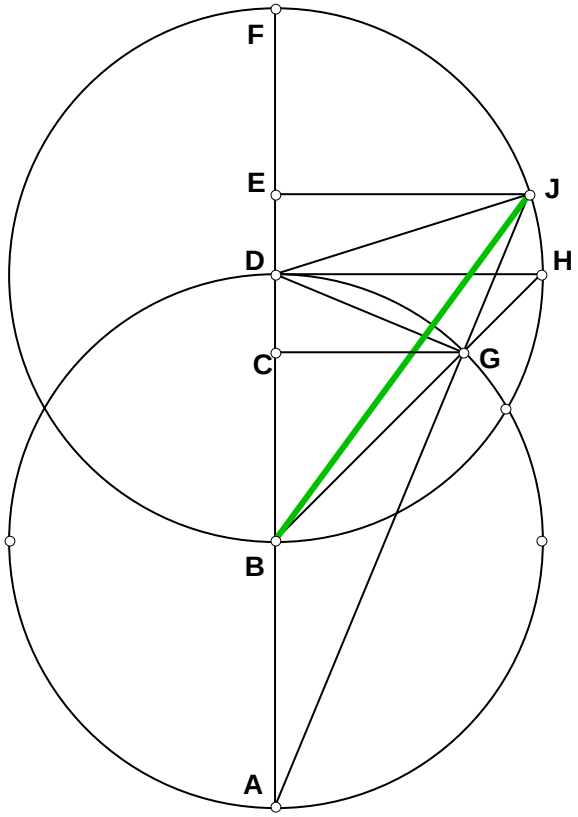
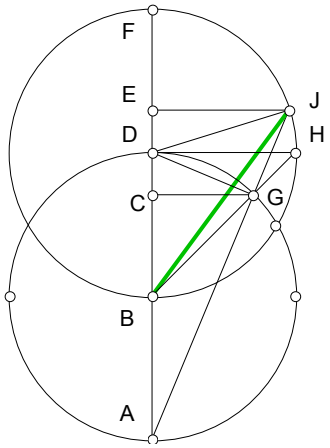



Given:

# 081292 Rusty Cube of a Sphere

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**




$N := 9$ $\quad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N^3\right)^{\frac{1}{3}}$

$AB := N$ $\quad BD := AB$ $\quad BG := AB$

$DJ := AB$ $\quad CG := \sqrt{\frac{BG^2}{2}}$ $\quad BC := CG$

$CD := BD - CG$ $\quad DG := \sqrt{CD^2 + CG^2}$

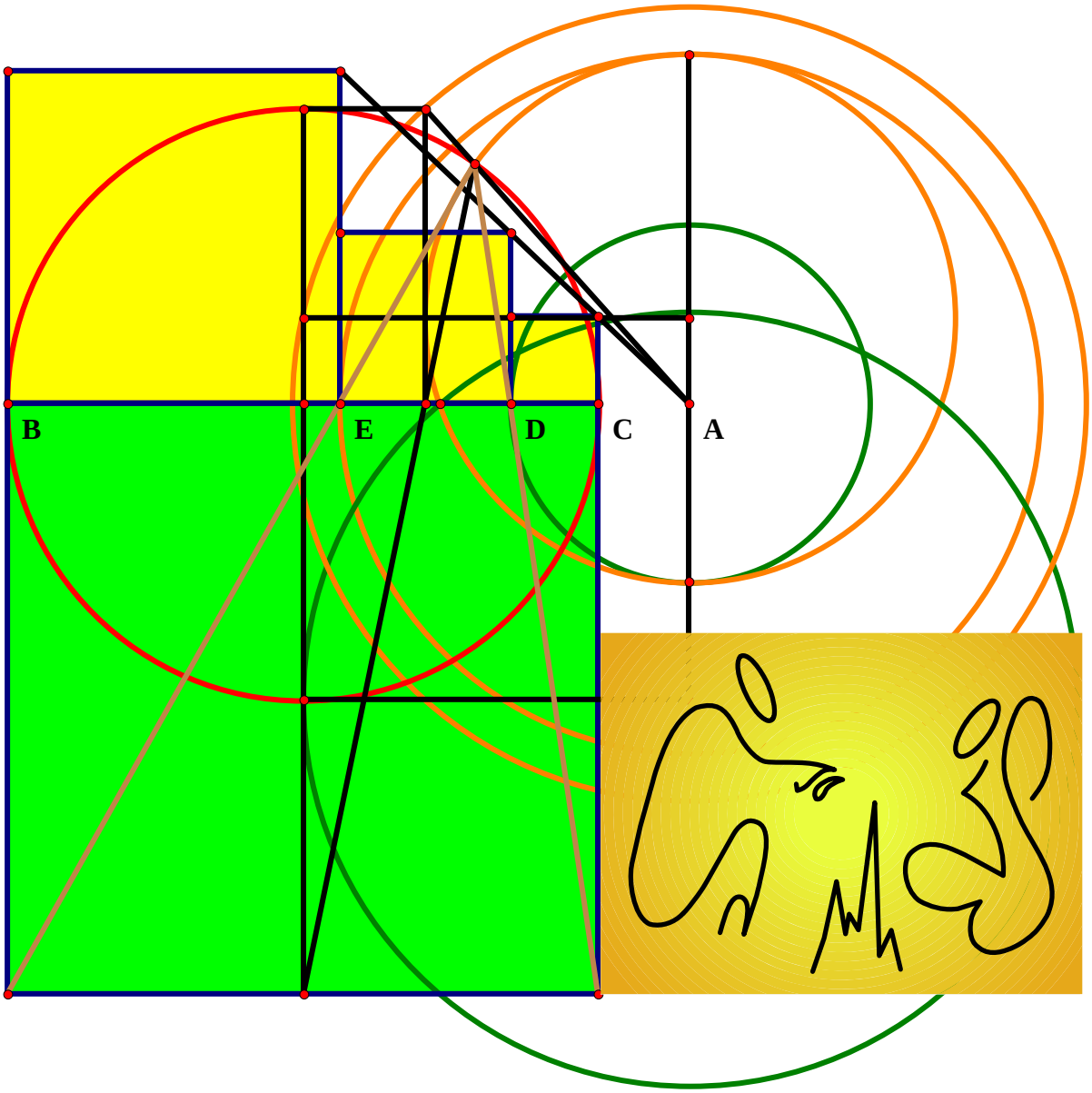
$GJ := \sqrt{DJ^2 - DG^2}$ $\quad AC := AB + BC$

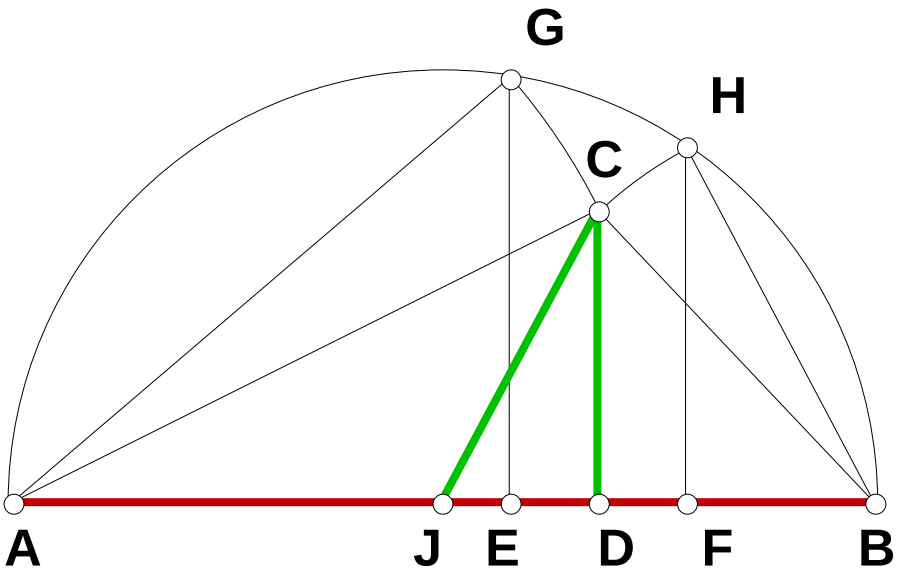
$AG := \sqrt{AC^2 + CG^2}$ $\quad AJ := AG + GJ$

$AE := \frac{AC\cdot AJ}{AG}$ $\quad EJ := \frac{CG\cdot AJ}{AG}$ $\quad BE := AE - AB$ $\quad BJ := \sqrt{EJ^2 + BE^2}$

$$\frac{CUBE\_ROOT}{BJ} = 0.99906$$

# The Delian Quest—1993

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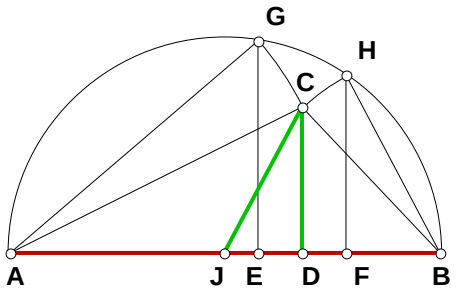
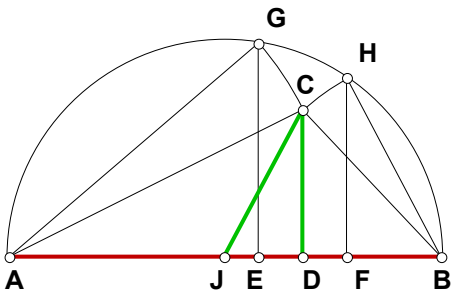



Given:

## 010893 Pythagoras Revisited

**Given AB, BC, AC, what is CD, AD, BD and CJ?**




$AB := 10 \qquad S_1 := .467 \qquad S_2 := .759$

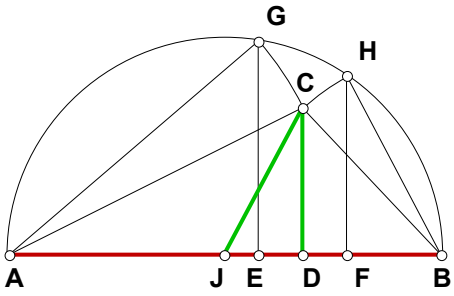
$BC := S_1 \cdot AB \qquad AC := S_2 \cdot AB$

$AG := AC \quad BH := BC \qquad AE := \frac{AG^2}{AB} \qquad BF := \frac{BH^2}{AB}$

$EF := AB - (AE + BF) \quad DE := \frac{EF}{2} \quad DF := DE$

$AD := AE + DE \quad \frac{AB}{2} \cdot \left(S_2^2 + 1 - S_1^2\right) - AD = 0 \quad BD := BF + \frac{AB}{2} \cdot \left(S_1^2 + 1 - S_2^2\right) - BD = 0$

$CD := \sqrt{AC^2 - AD^2} \quad \frac{\sqrt{\left(S_1 + 1 + S_2\right) \cdot \left(-S_1 + 1 + S_2\right) \cdot \left(S_1 - 1 + S_2\right) \cdot \left(S_1 + 1 - S_2\right)}}{2} \cdot AB - CD = 0$

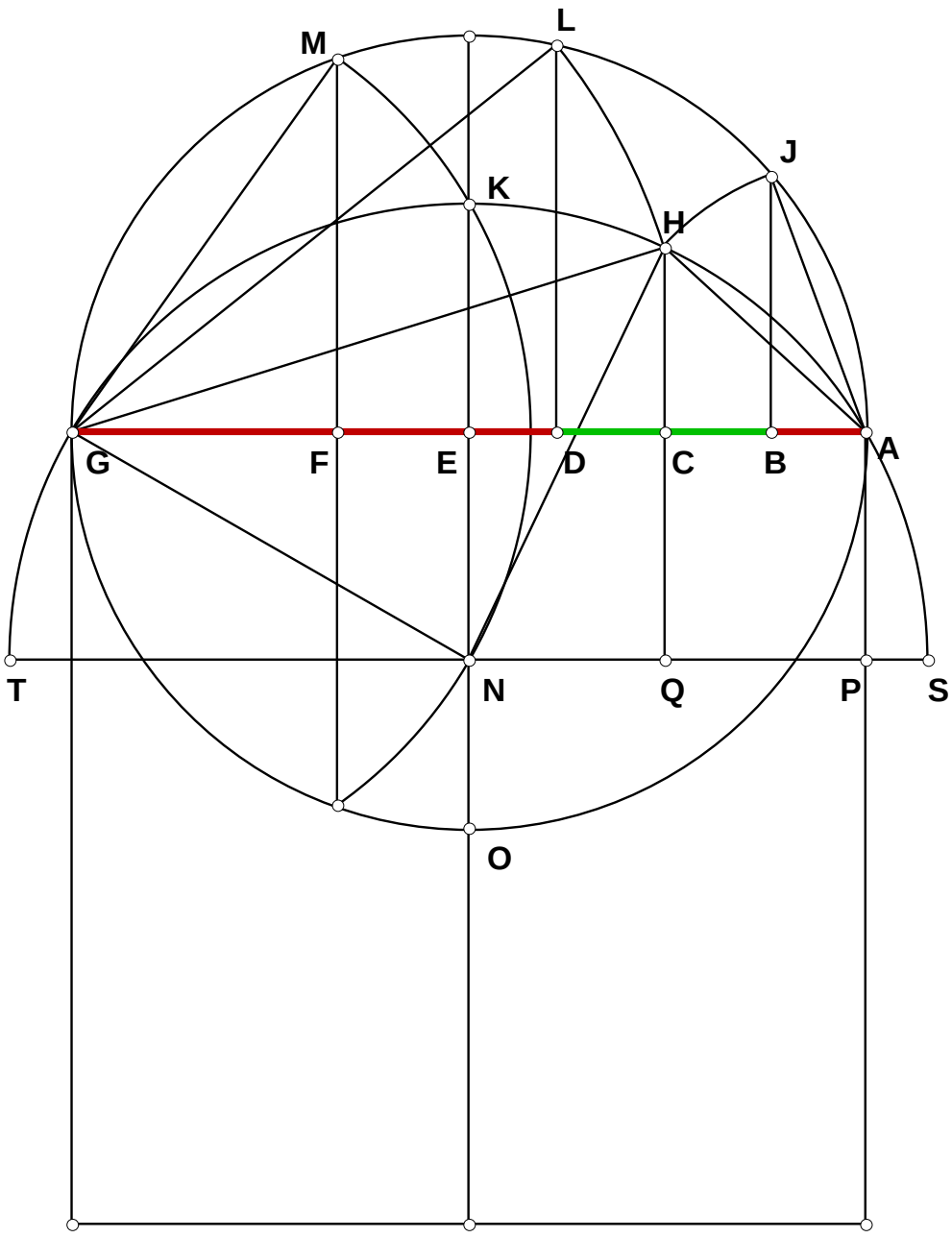



$AJ := \frac{AB}{2} \quad JD := AD - AJ \qquad CJ := \sqrt{JD^2 + CD^2}$

$\frac{AB}{2} \cdot \sqrt{2 \cdot S_2^2 - 1 + 2 \cdot S_1^2} - CJ = 0$

$\left(AC^2 + BC^2\right) - \left(\frac{AB^2}{2} + 2 \cdot CJ^2\right) = 0$

**The sum of the squares on any two sides of any triangle is equal to the sum of half the square on the remaining side plus twice the square on the medial bisector (CJ).**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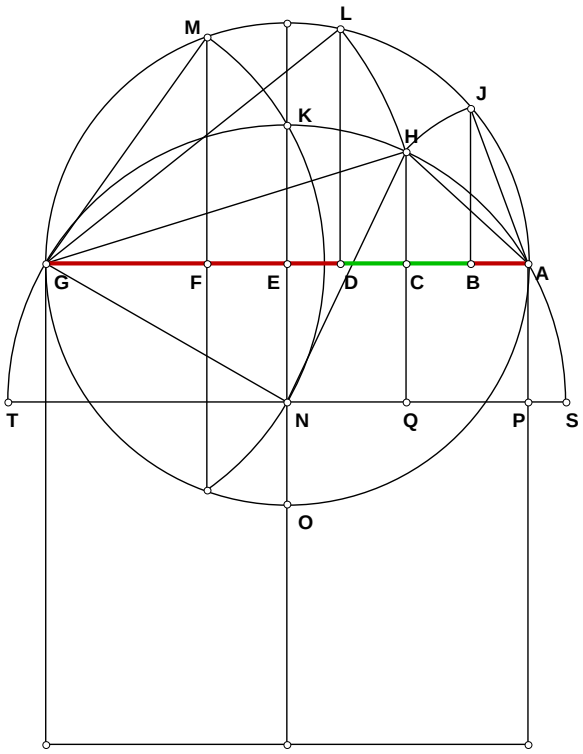
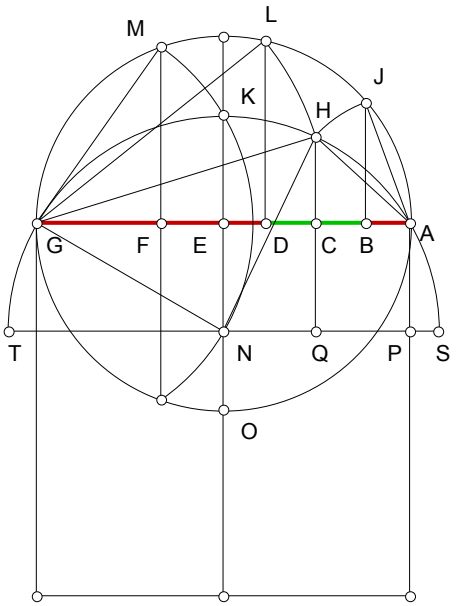



Given:

# 060393 Exploring The Curve CJ




**Given AG and GF = AG/3 and any AC, is BD the square root of AB x DG?**

$AG := 10 \qquad N := .25409$

$AC := N \cdot AG \qquad GF := \frac{AG}{3} \quad AE := \frac{AG}{2}$

$EG := AE \quad AF := AG - GF \quad FM := \sqrt{AF \cdot GF}$

$GM := \sqrt{GF^2 + FM^2} \quad GN := GM \quad EN := \sqrt{GN^2 - EG^2}$

$NS := GM \quad PN := AE \quad PS := NS - PN$

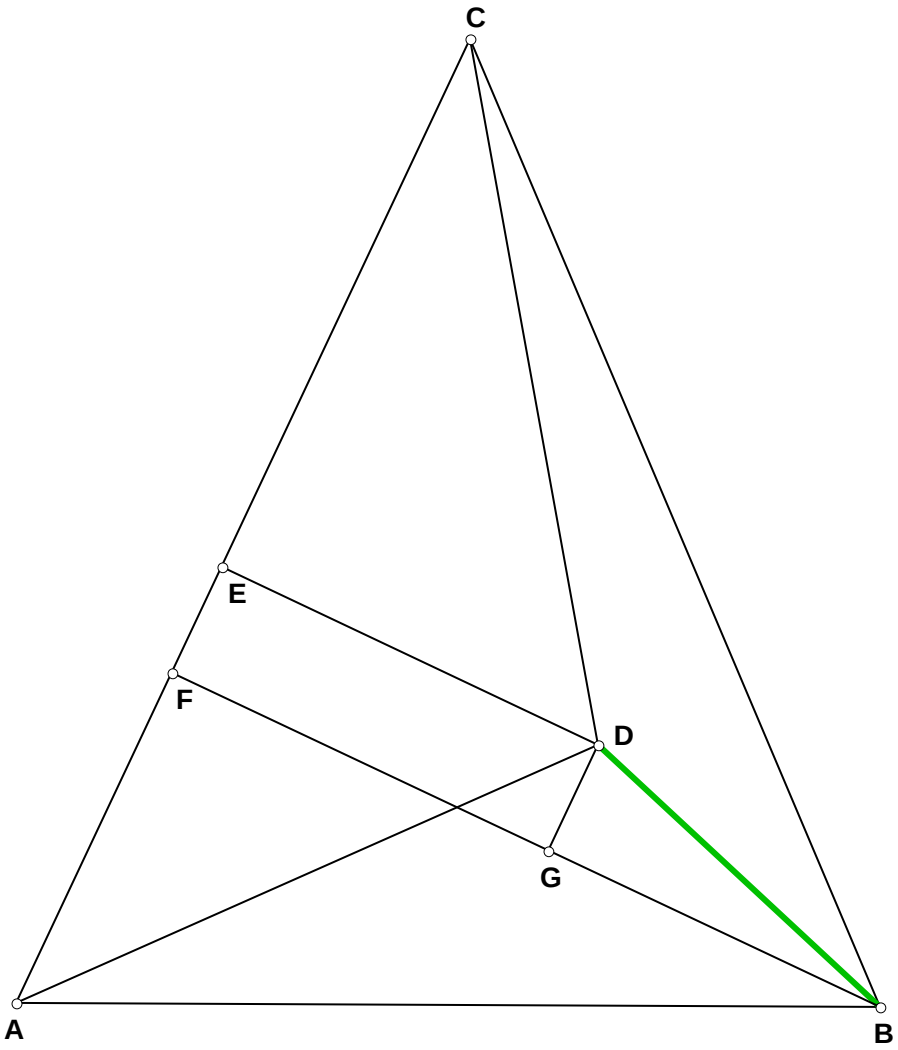
$ST := 2 \cdot GM \quad SQ := AC + PS \quad QT := ST - SQ$

$QH := \sqrt{SQ \cdot QT} \quad CQ := EN \quad CH := QH - CQ$

$AH := \sqrt{AC^2 + CH^2} \quad CG := AG - AC \quad GH := \sqrt{CG^2 + CH^2} \quad AJ := AH \quad AB := \frac{AJ^2}{AG}$

$GL := GH \quad DG := \frac{GL^2}{AG} \quad BD := AG - (AB + DG)\sqrt{AB \cdot DG} - BD = 0$

$\frac{AG}{3} \cdot \left[\sqrt{1 + 12 \cdot \left(N - N^2\right)} - 1\right] - BD = 0$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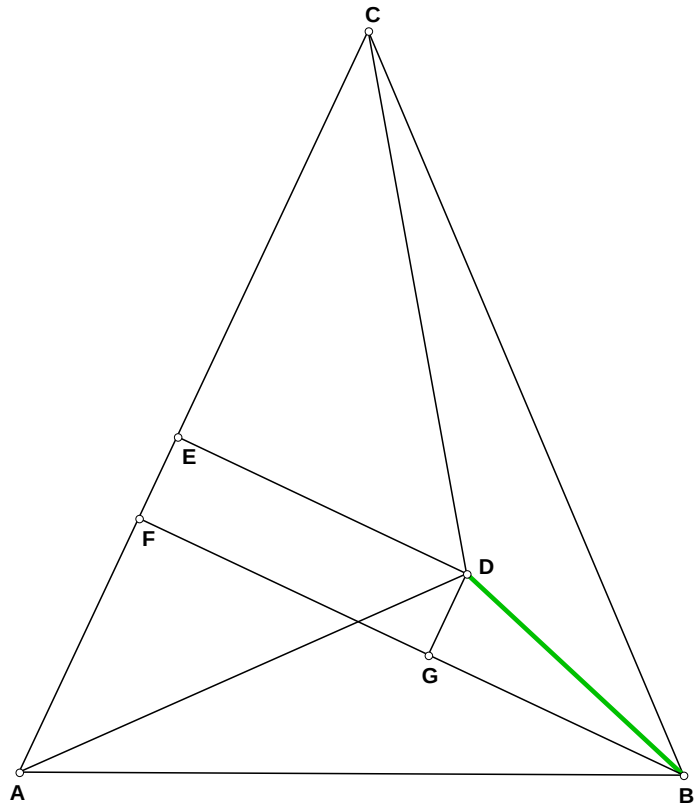
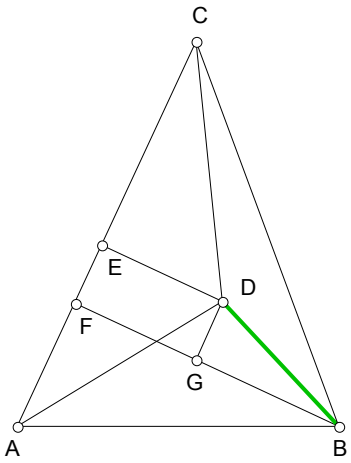



Given:

# 060793 For All Triangles Find BD




**Given AB, BC, AC, CD, AD, find BD**

**To simplify use line names found in 010893**

$S_1 := 5.55 \quad S_2 := 4.61 \quad S_3 := 1.5$

$S_5 := 4.5 \quad S_6 := 5.48$

$AB := S_5 \quad BC := S_6 \quad AC := S_1 \quad CD := S_3 \quad AD := S_2$

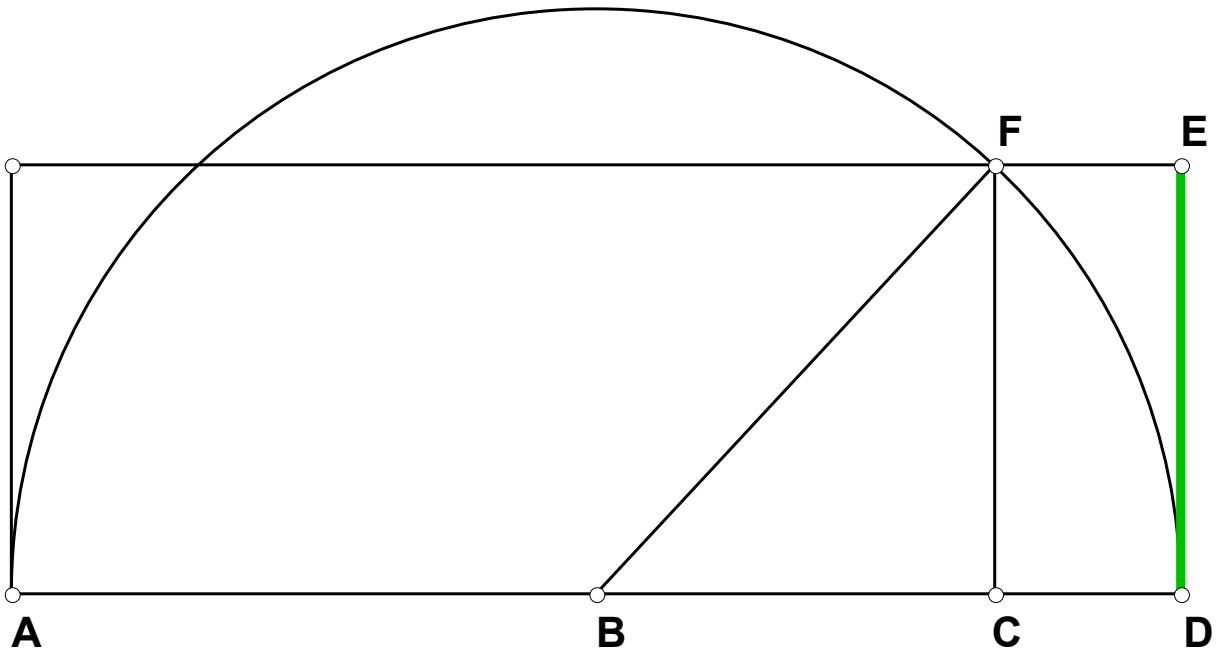
$$CE := \frac{CD^2 + AC^2 - AD^2}{2 \cdot AC} \qquad CF := \frac{BC^2 + AC^2 - AB^2}{2 \cdot AC} \qquad EF := CF - CE$$

$$DE := \sqrt{CD^2 - CE^2} \qquad BF := \sqrt{BC^2 - CF^2} \qquad GF := DE \qquad DG := EF$$

$$BG := BF - GF \qquad BD := \sqrt{DG^2 + BG^2} \qquad BD = 3.983$$

**OR**

$$BG_2 := BF + GF \qquad BD_2 := \sqrt{DG^2 + BG_2^2} \qquad BD_2 = 5.757$$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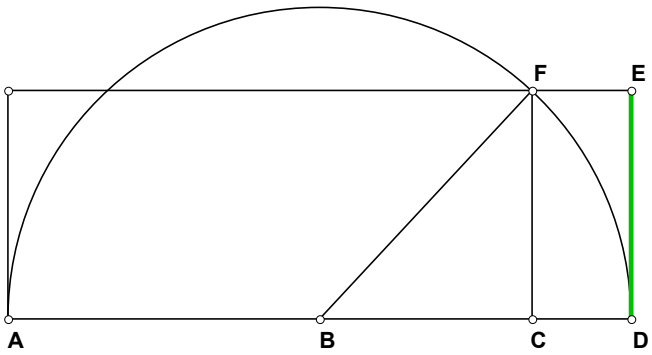
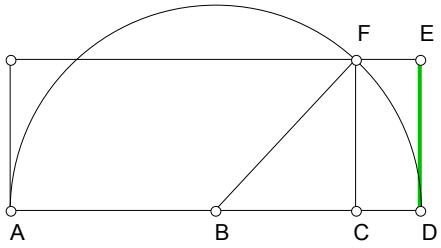



Given:

# 060993 Rectangular Roots




**Given AD and DE divide AD into the rectangular roots of DE.**

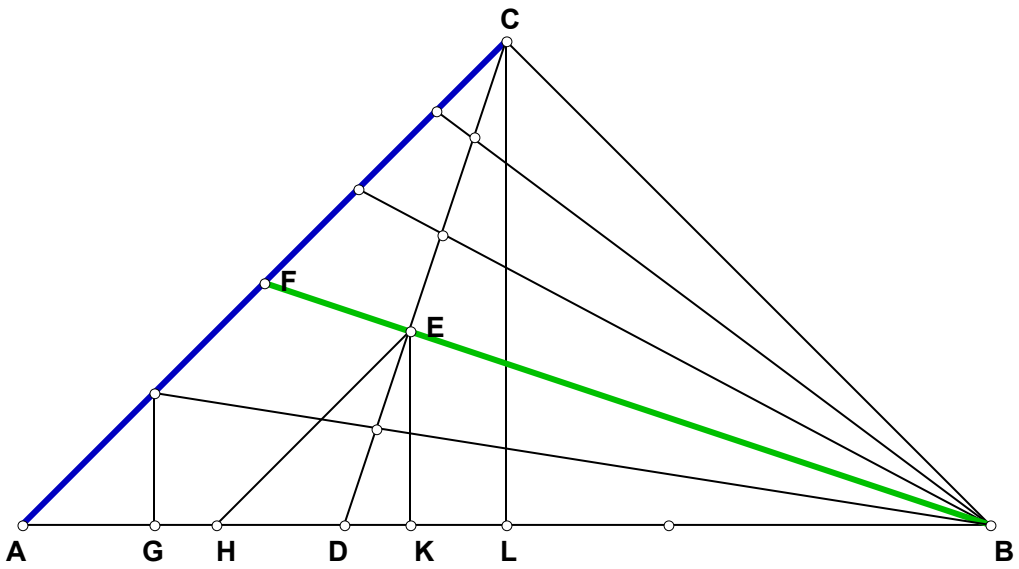
$AD := 10$ $N := .77$

$DE := N \cdot AD$ $CF := DE$ $BF := \frac{AD}{2}$

$AB := BF$ $BC := \sqrt{BF^2 - CF^2}$ $AC := AB + BC$ $CD := AD - AC$ $\sqrt{CD \cdot AC} - DE = 0$

$$\frac{AD}{2} \cdot \left[ 1 + \sqrt{-(2 \cdot N - 1) \cdot (2 \cdot N + 1)} \right] - AC = 0$$

$$\frac{AD}{2} \cdot \left[ 1 - \sqrt{-(2 \cdot N - 1) \cdot (2 \cdot N + 1)} \right] - CD = 0$$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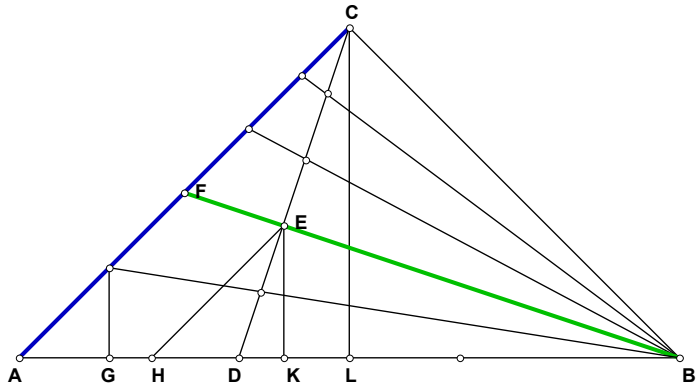
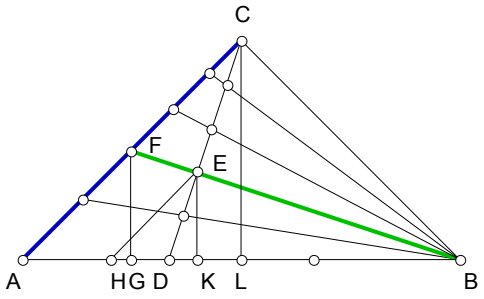



Given:

# 062193 Pyramid of Ratios I




**Divide AB by $N_1$ then divide CD by $N_2$, what are BF/EF and AC/AF?**

$N_1 := 3 \qquad N_2 := 5 \qquad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AL := \frac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

| $\delta$ |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |

$DE_\delta := \frac{CD \cdot \delta}{N_2} \quad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$\text{if}\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\text{if}\left(N_2 - \delta, \frac{N_1 \cdot N_2}{N_2 - \delta}, 0\right)$

| |
|---|
| 3.75 |
| 5 |
| 7.5 |
| 15 |
| 0 |

$\frac{AC}{AF_\delta}$

| |
|---|
| 3.667 |
| 2 |
| 1.444 |
| 1.167 |
| 1 |

$\frac{(N_1 - 1) \cdot N_2 + \delta}{N_1 \cdot \delta}$

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| 3.667 |
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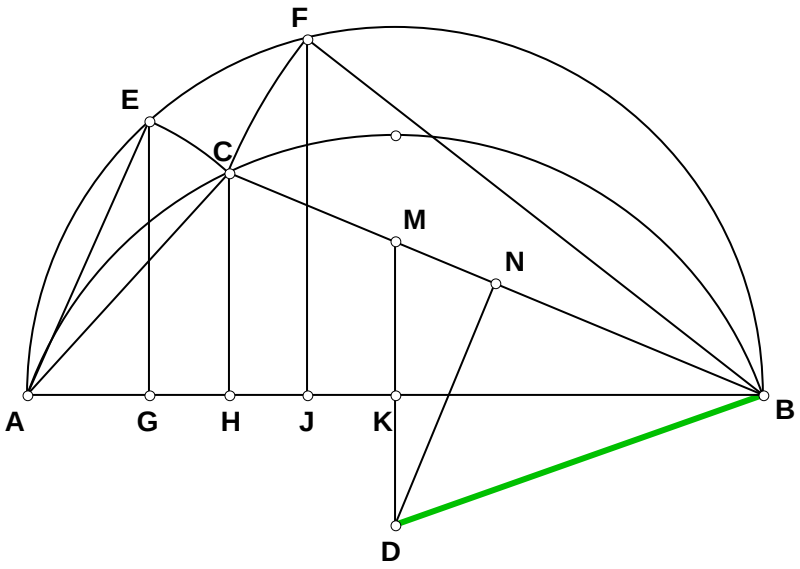
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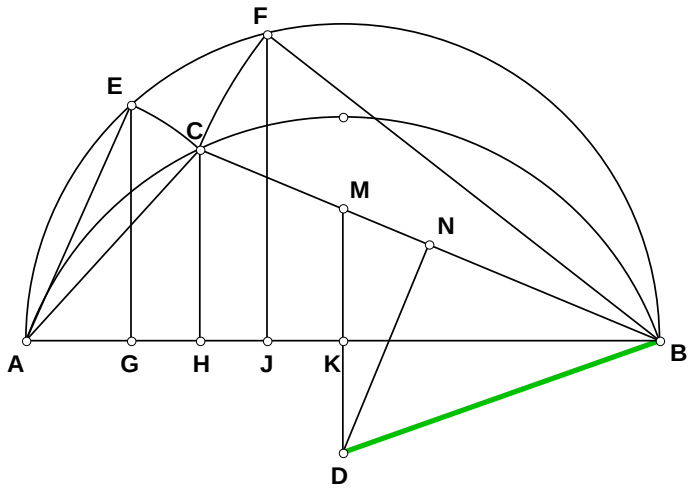
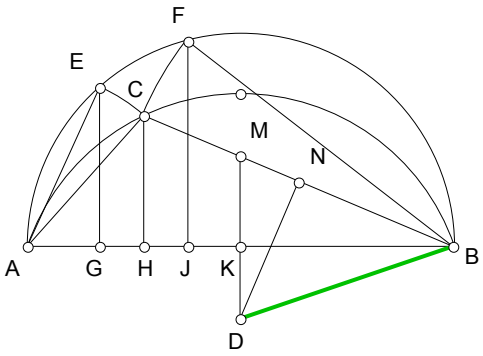



Given:

# 062793 Describe A Circle About a Triangle

**What is the Algebraic name for the Radius circumscribing a Triangle?**
**Let us suppose that the Letters of that name are simply S1, S2 and S3.**




$$AB := 10 \qquad N_1 := .87189$$

$$N_2 := .2855 \qquad BC := AB \cdot N_1 \qquad AC := AB \cdot N_2$$

$$BK := \frac{AB}{2} \qquad AE := AC \qquad BF := BC$$

$$AG := \frac{AE^2}{AB} \qquad BJ := \frac{BF^2}{AB} \qquad GJ := AB - (AG + BJ)$$

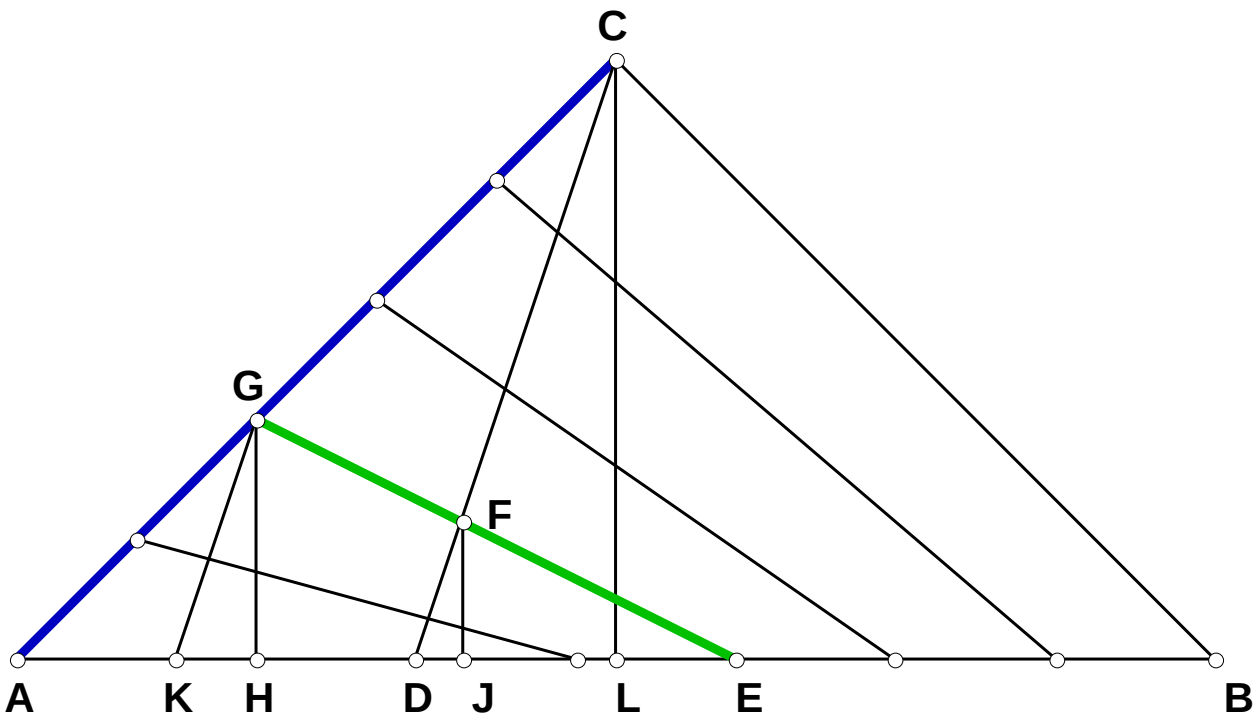
$$HJ := \frac{GJ}{2} \qquad BH := BJ + HJ \qquad CH := \sqrt{BC^2 - BH^2}$$

$$BN := \frac{BC}{2} \qquad BM := \frac{BC \cdot BK}{BH} \qquad MN := BM - BN$$

$$DN := \frac{BH \cdot MN}{CH} \qquad BD := \sqrt{BN^2 + DN^2}$$

**The Algebraic Name of the Radius.**

$$\frac{AB \cdot N_1 \cdot N_2}{\sqrt{(1 + N_1 + N_2)} \cdot \sqrt{(-1 + N_1 + N_2)} \cdot \sqrt{(1 - N_1 + N_2)} \cdot \sqrt{(1 + N_1 - N_2)}} - BD = 0$$


C
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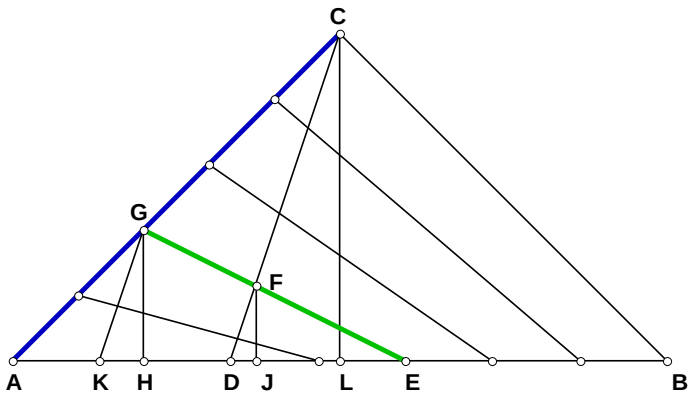
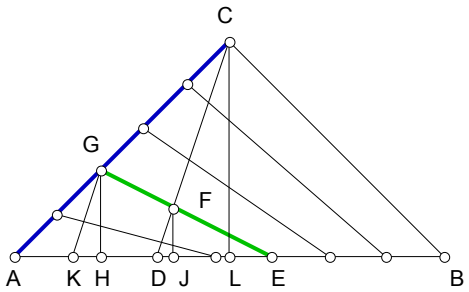
## 930715 EP




Given:

# 071593 Pyramid of Ratios II

**AB is divided by N1 and AC and BD is divided by N2, what are EG/FG and CD/DF?**




$N_1 := 3 \quad N_2 := 5 \quad \delta := 1 .. N_2$

$AB := 1 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD$

$DE_\delta := \frac{BD \cdot \delta}{N_2} \quad AG_\delta := \frac{AC \cdot \delta}{N_2} \quad AE_\delta := AD + DE_\delta$

$AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}} \quad GH_\delta := AH_\delta \quad EH_\delta := AE_\delta - AH_\delta$

$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2} \quad AL := \frac{AB}{2} \quad DL := AL - AD$

$CL := \sqrt{\frac{AC^2}{2}} \quad HK_\delta := \frac{DL \cdot GH_\delta}{CL} \quad EK_\delta := EH_\delta + HK_\delta \quad DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta} \quad FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2} \quad EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta} \quad FG_\delta := EG_\delta - EF_\delta \quad CD := \sqrt{CL^2 + DL^2}$

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right)$

| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot (N_1 - 2)}{N_2 - \delta}, 0\right]$

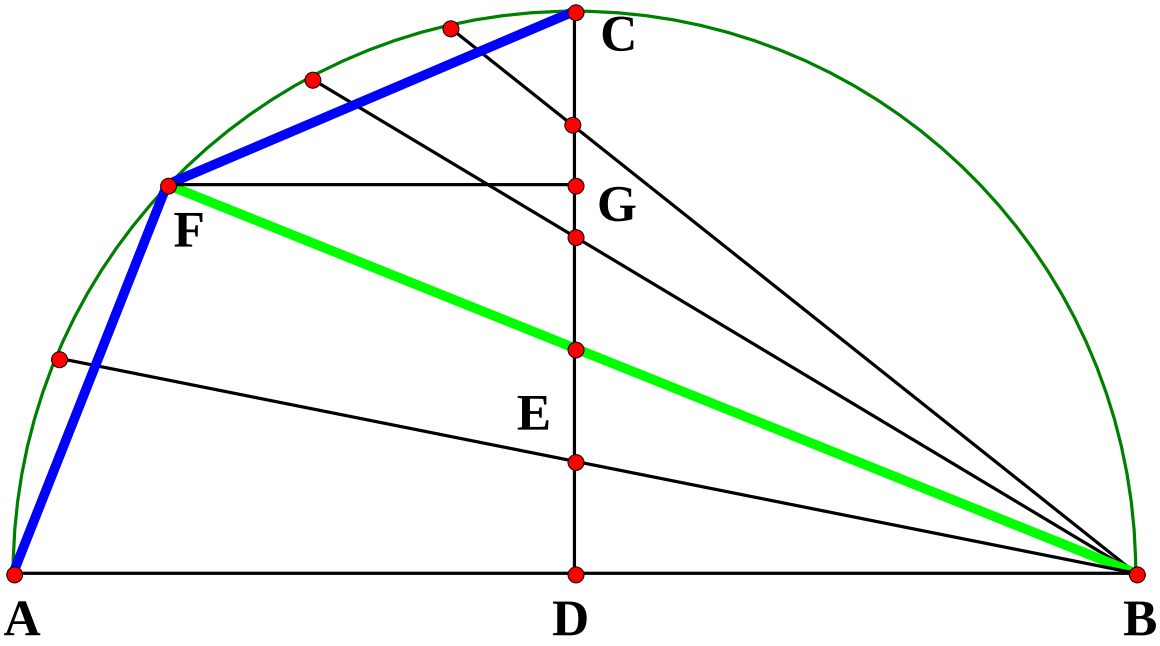
| |
|---|
| 1.5 |
| 2.333 |
| 4 |
| 9 |
| 0 |

$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right)$

| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |

$\text{if}\left[\delta, N_2 \cdot \frac{\left[(N_2 + \delta \cdot N_1) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot (N_1 - 1)\right]}, 0\right]$

| |
|---|
| 15 |
| 4.375 |
| 2.222 |
| 1.406 |
| 1 |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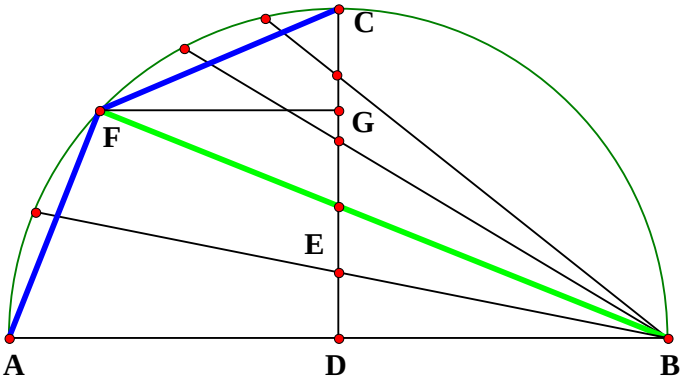
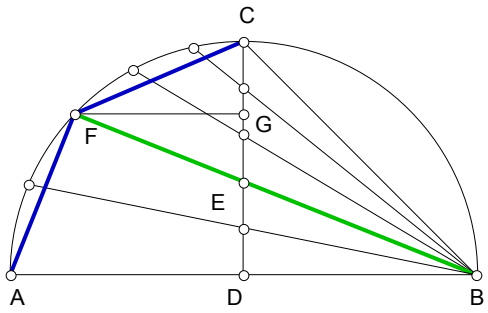
## 930725 EP




Given:

# 072593 Pyramid of Ratios III

## Dividing DC by an number provides wht in terms of BE/BF and AF/CF?




$N := 6 \qquad \delta := 1 .. N$

$AB := 1 \quad AD := \frac{AB}{2} \quad BD := AD \quad CD := BD$

$BC := \sqrt{\frac{BD^2}{2}} \quad DE_\delta := \frac{CD \cdot \delta}{N} \quad BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$

$BF_\delta := \frac{BD \cdot AB}{BE_\delta} \quad AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta} \quad FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$

$DG := DE + EG \quad CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

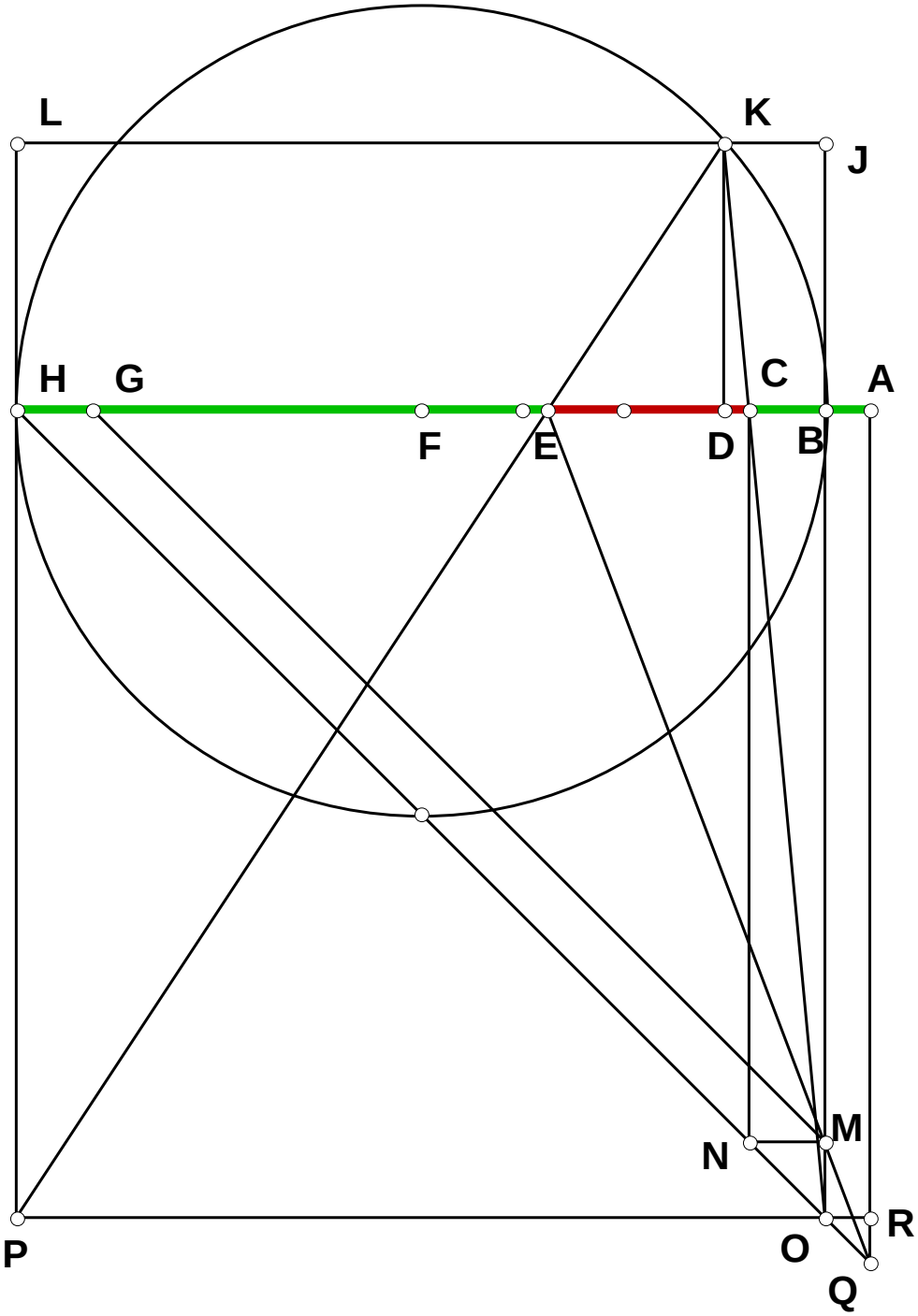
| $\frac{DE_\delta}{DG_\delta}$ | $\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2}$ | $\frac{BE_\delta}{BF_\delta}$ | $\frac{BF_\delta}{BE_\delta}$ | $\frac{2}{\left[\left(\frac{\delta}{N}\right)^2 + 1\right]}$ |
|---|---|---|---|---|
| 0.514 | 0.514 | 0.514 | 1.946 | 1.946 |
| 0.556 | 0.556 | 0.556 | 1.8 | 1.8 |
| 0.625 | 0.625 | 0.625 | 1.6 | 1.6 |
| 0.722 | 0.722 | 0.722 | 1.385 | 1.385 |
| 0.847 | 0.847 | 0.847 | 1.18 | 1.18 |
| 1 | 1 | 1 | 1 | 1 |

$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right)$

| |
|---|
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |

$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right)$

| |
|---|
| 0.283 |
| 0.707 |
| 1.414 |
| 2.828 |
| 7.071 |
| 0 |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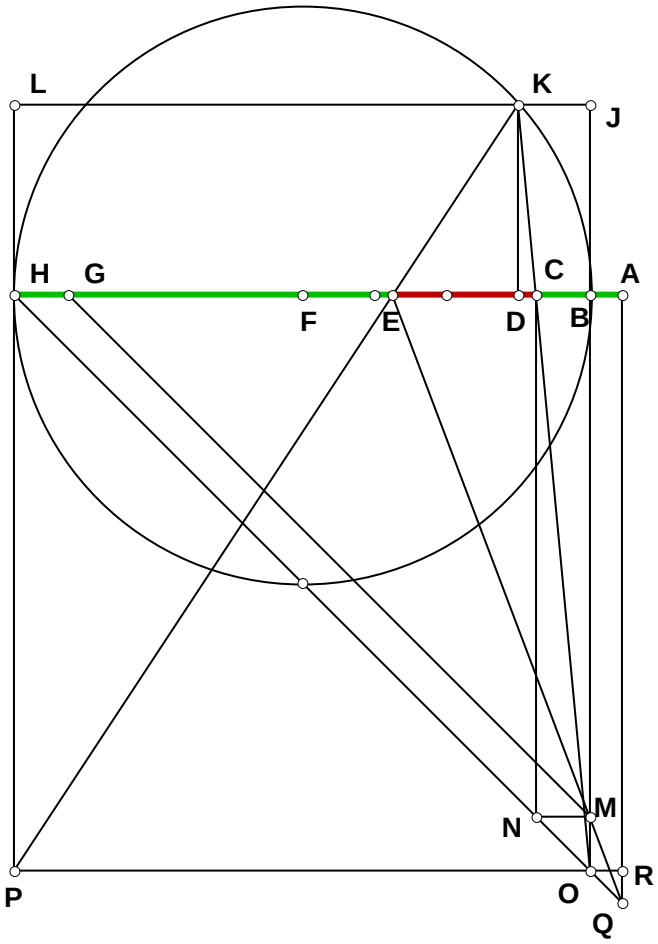
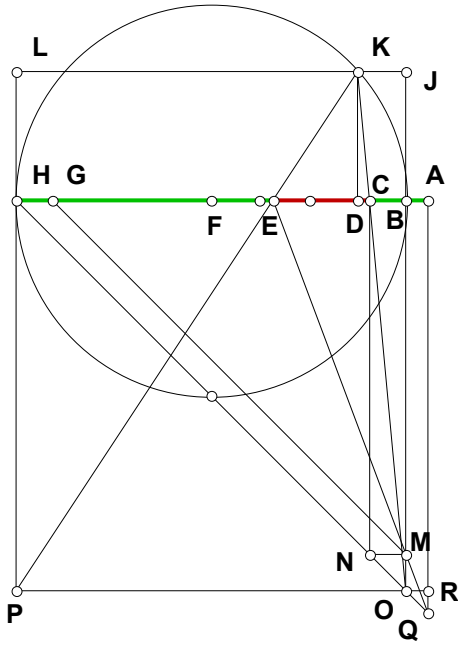
# 931106 EP




Given:

# 110693 Gruntwork I on the Delian Solution




**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/2} = AE$?**

$N := 4 \qquad BH := 1$

$BF := \frac{BH}{2} \quad BD := \frac{BF}{N} \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad BJ := DK \quad BO := BH$

$JO := BJ + BO \quad JK := BD \quad CD := \frac{JK \cdot DK}{JO}$

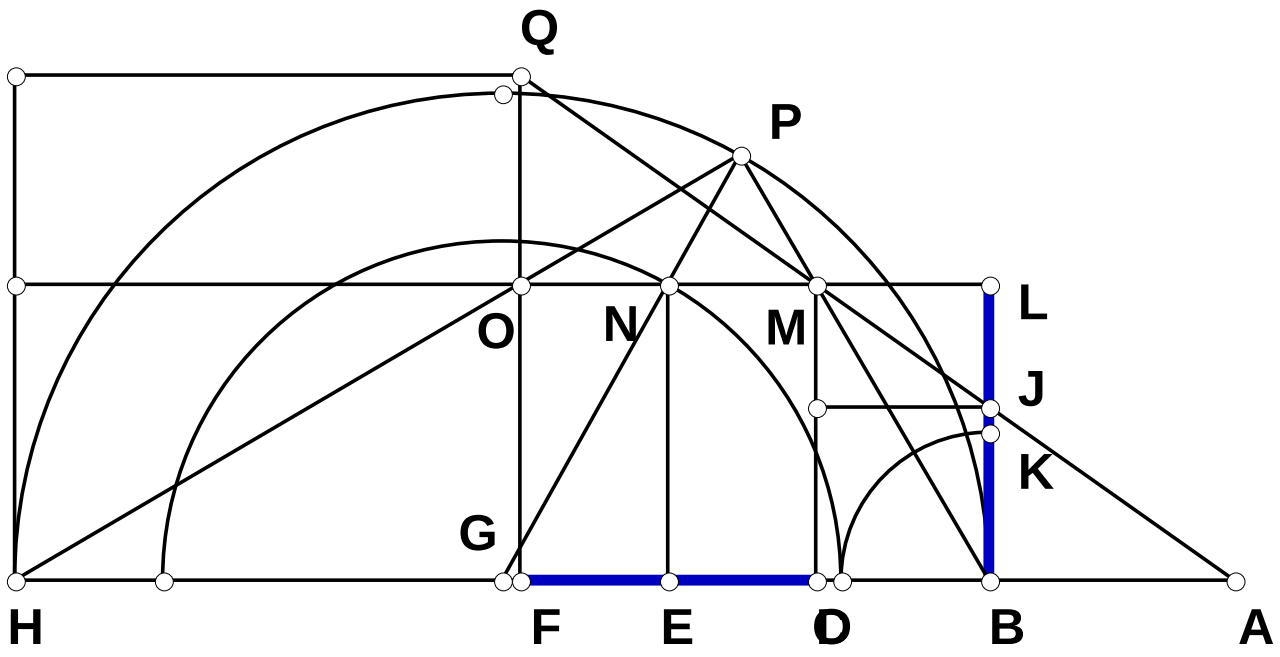
$KL := DH \quad LP := JO \quad DE := \frac{KL \cdot DK}{LP}$

$BC := BD - CD \quad CE := CD + DE \quad MN := BC$

$GH := MN \quad CH := BH - BC \quad HN := \sqrt{2 \cdot CH^2} \quad GM := HN \quad EH := CH - CE \quad EG := EH - GH$

$HQ := \frac{GM \cdot EH}{EG} \quad HO := \sqrt{2 \cdot BH^2} \quad OQ := HQ - HO \quad OR := \sqrt{\frac{OQ^2}{2}} \quad AB := OR \quad AC := AB + BC$

$AE := AB + BD + DE \quad AH := AB + BH \quad \left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$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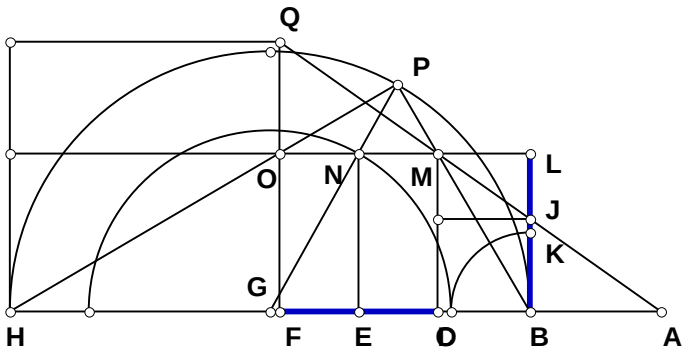
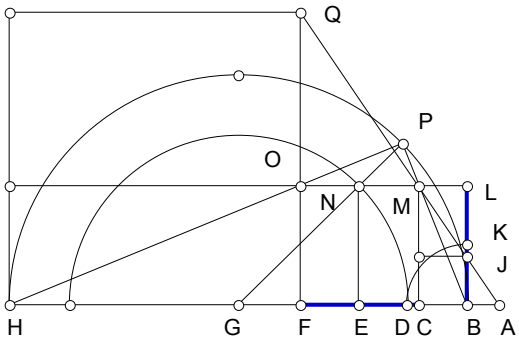
## 931109 EP



Given:

# 110993 Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem.**
**BH is the difference between the segments AH and AB.**
**CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**



$BH := 3.4375 \quad N := 1.3253$

$BG := \frac{BH}{2} \quad CF := \frac{BH}{N \cdot 3}$

$BL := CF \quad GP := BG \quad BK := \frac{BL}{2}$

$BD := BK \quad NP := BD \quad GN := GP - NP$

$EN := BL \quad GE := \sqrt{GN^2 - EN^2}$

$GH := BG \quad EF := BD$

$FH := GH + GE - EF \quad FQ := FH \quad FO := BL \quad OQ := FQ - FO \quad MO := CF \quad AF := \frac{MO \cdot FQ}{OQ}$

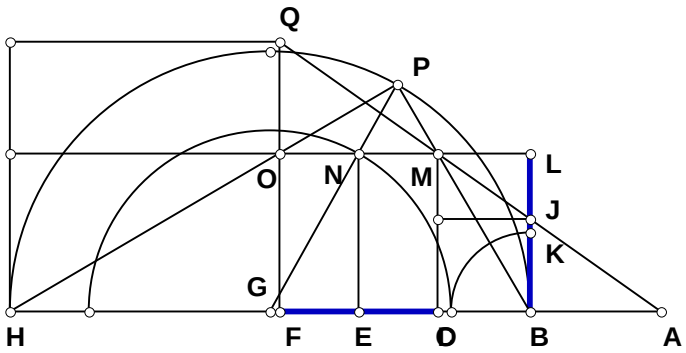
$AC := AF - CF \quad AH := AF + FH \quad AB := AH - BH$

$$\frac{1 \cdot BH}{3} \cdot \frac{\left[3 \cdot N - \sqrt{3} \cdot \sqrt{(3 \cdot N + 1) \cdot (N - 1)} - 1\right]}{\left[N \cdot \left[3 \cdot N + \sqrt{3} \cdot \sqrt{(3 \cdot N + 1) \cdot (N - 1)} - 3\right]\right]} - AB = 0$$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \qquad \frac{AH}{AB} = 17.36924$$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

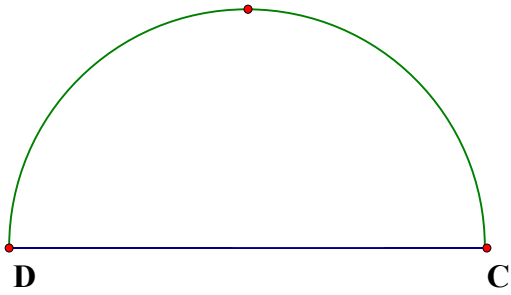
## 931109 EP



Given:

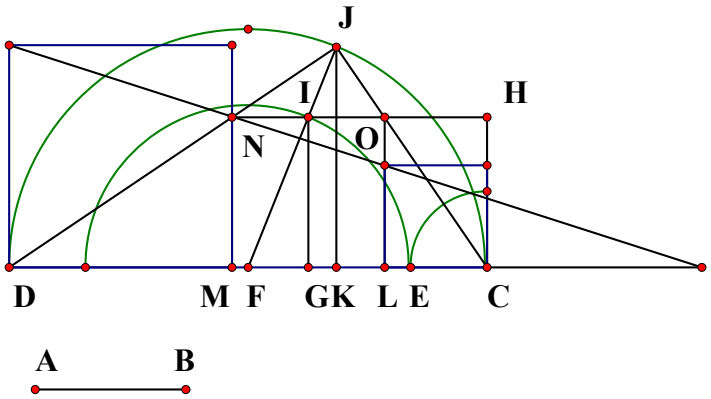
**Demonstrate the duiplicate ratio first.**

**110993 A**




$CD := 10$ $N := 1.2$

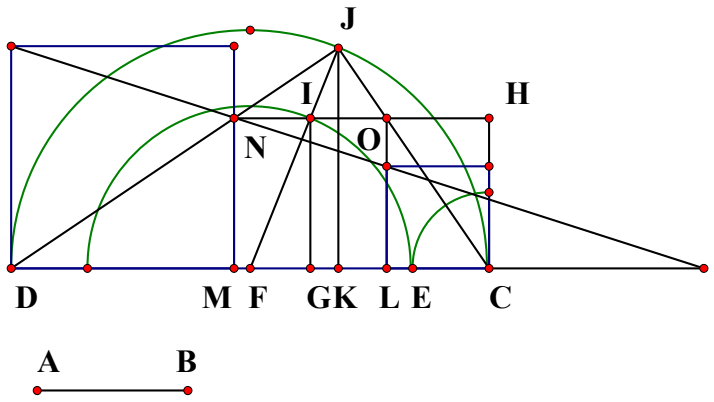
$AB := \frac{CD}{N \cdot 3}$ $AB < \frac{CD}{3} = 1$




$CE := \frac{AB}{2}$ $CF := \frac{CD}{2}$ $GI := AB$ $EF := CF - CE$

$FI := EF$ $FG := \sqrt{FI^2 - GI^2}$ $FJ := CF$ $FK := \frac{FG \cdot FJ}{FI}$ $JK := \frac{GI \cdot FJ}{FI}$

$CK := CF - FK$




$LO := AB \quad DF := CF$

$MN := AB$

$CL := \frac{CK \cdot LO}{JK}$

$DK := DF + FK$

$DM := \frac{DK \cdot MN}{JK}$

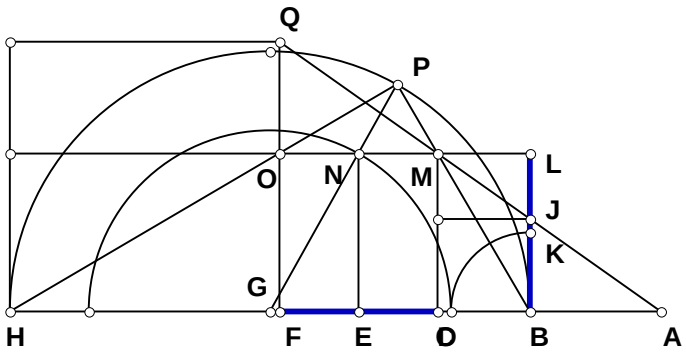
$CD - (DM + CL + AB) = 0 \qquad LM := CD - (DM + CL) \quad LM - AB = 0$

$\frac{DM}{LM} - \frac{LM}{CL} = 0 \qquad \frac{CD}{N \cdot 3} - LM = 0$

$$\left[\frac{CD}{6} \cdot \frac{\left[3 \cdot N - 1 - \sqrt{3} \cdot \sqrt{(3 \cdot N + 1) \cdot (N - 1)}\right]}{N}\right] - CL = 0$$

$$\frac{2}{\left[3 \cdot N - 1 - \sqrt{3} \cdot \sqrt{(3 \cdot N + 1) \cdot (N - 1)}\right]} - \frac{LM}{CL} = 0$$

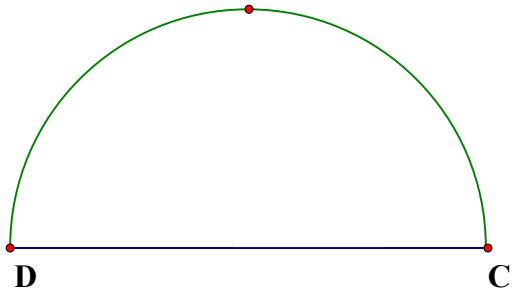
## 931109 EP



Given:

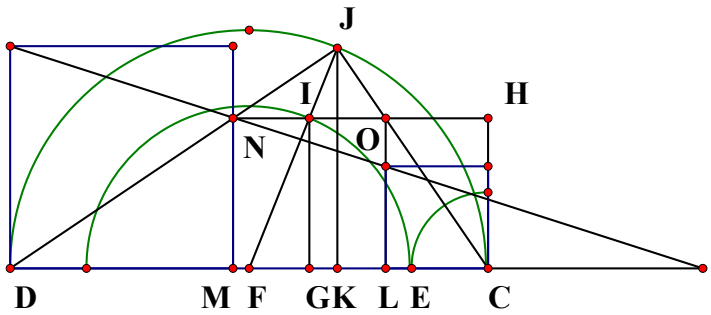
**Demonstrate the duiplicate ratio first.**

**110993 B**




$AB := 10$ $N := .15406$

$CD := 3 \cdot AB + N \cdot AB$

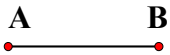



$CE := \frac{AB}{2}$ $CF := \frac{CD}{2}$

$GI := AB$ $EF := CF - CE$

$FI := EF$ $FG := \sqrt{FI^2 - GI^2}$

$FJ := CF$

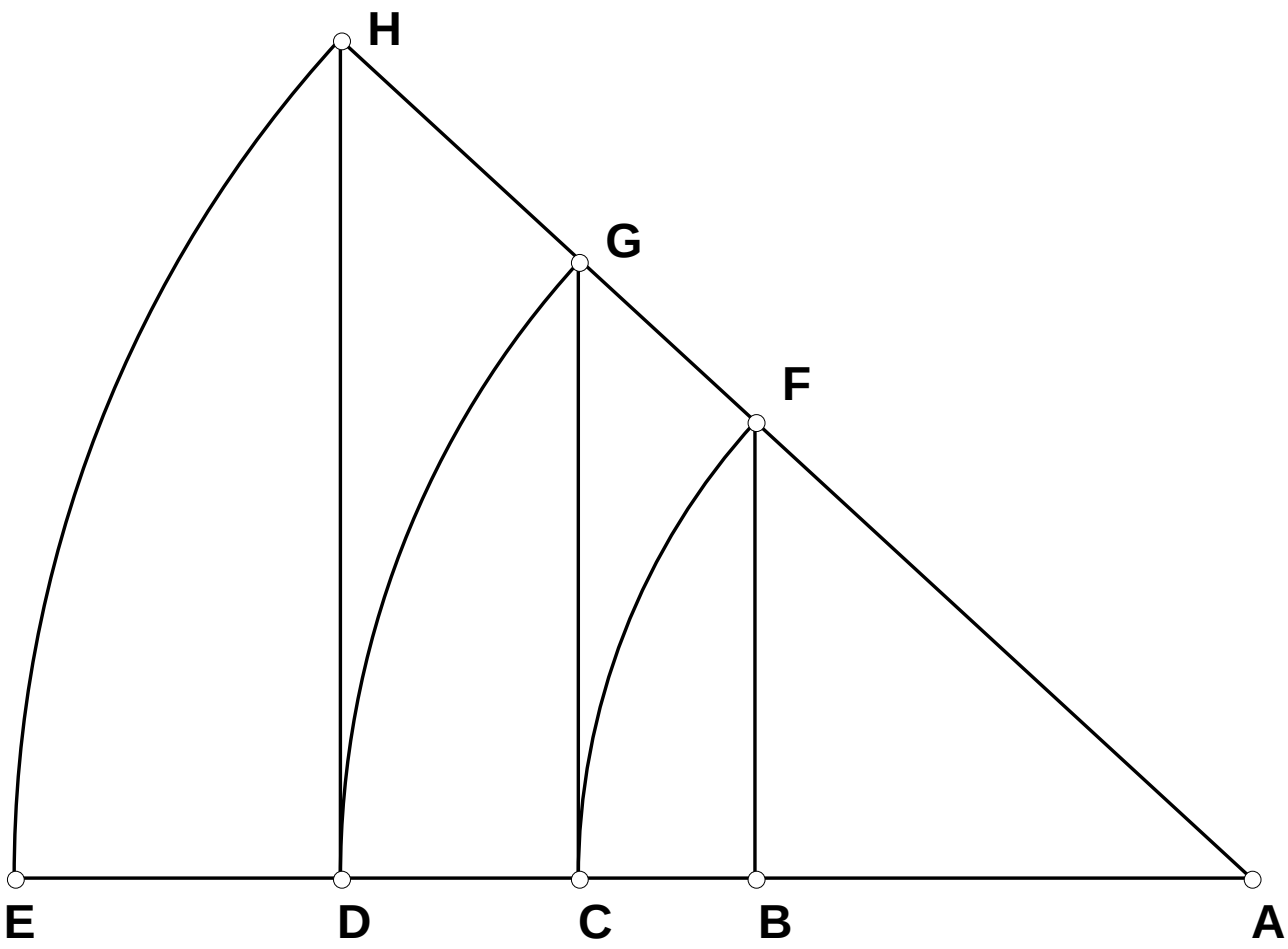



$FK := \frac{FG \cdot FJ}{FI}$ $JK := \frac{GI \cdot FJ}{FI}$ $CK := CF - FK$ $LO := AB$ $DF := CF$

$MN := AB$ $CL := \frac{CK \cdot LO}{JK}$ $DK := DF + FK$ $DM := \frac{DK \cdot MN}{JK}$

$CD - (DM + CL + AB) = 0$ $LM := CD - (DM + CL)$ $LM - AB = 0$

$\frac{DM}{LM} - \frac{LM}{CL} = 0$ $AB \cdot \left(1 + \frac{N - \sqrt{4 \cdot N + N^2}}{2}\right) - CL = 0$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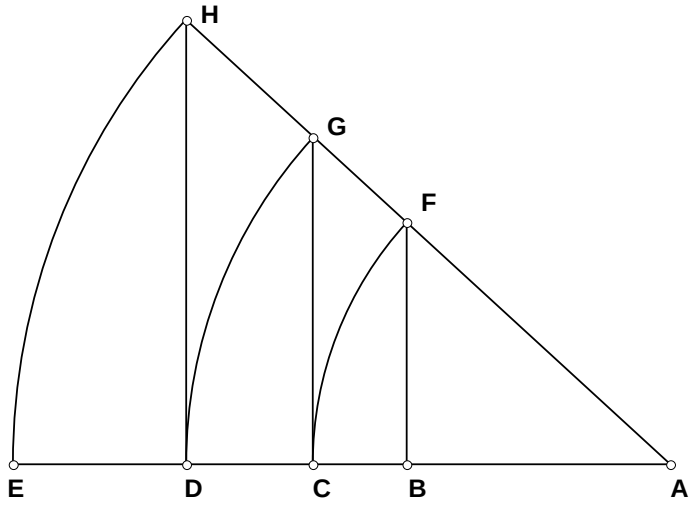
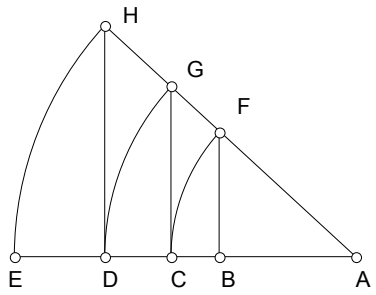
## 931110 EP




Given:

# 111093 Gruntwork II on the Delian Solution




$N := 4.846$ $AE := 1$

$DE := \frac{AE}{N}$ $AD := AE - DE$ $AH := AE$

$AG := AD$ $AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

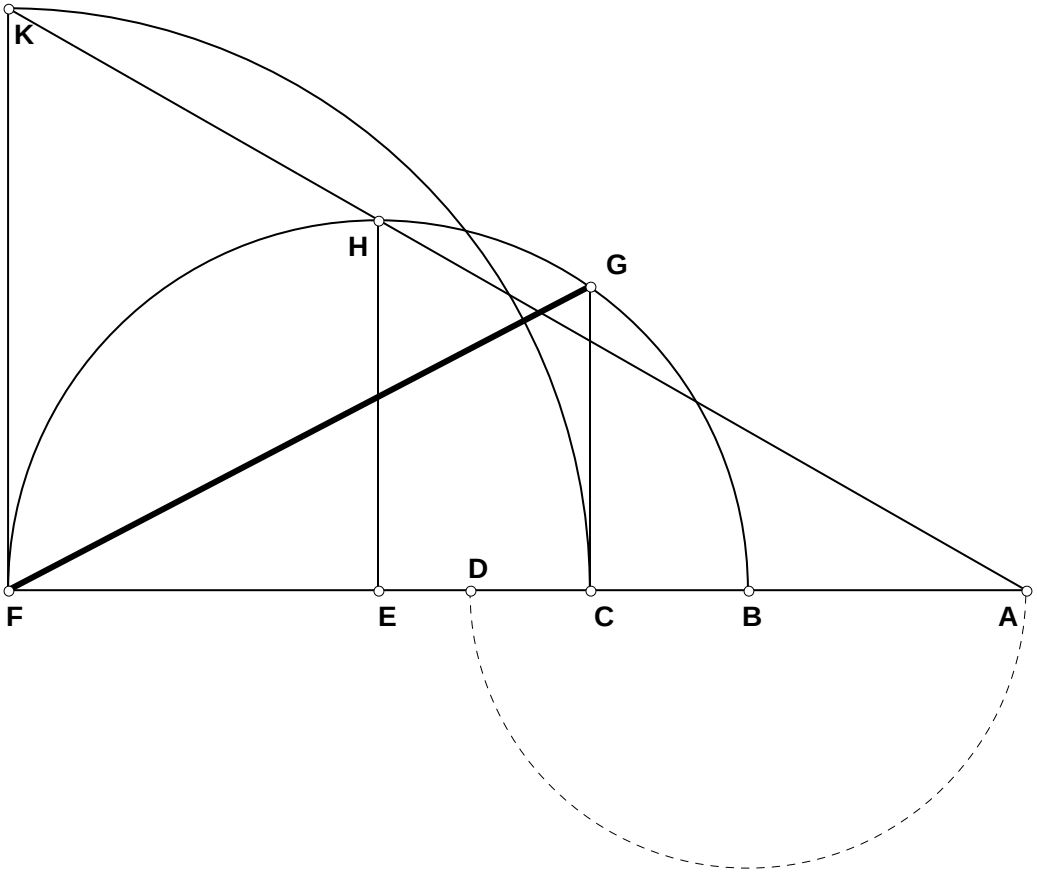
$AB := \frac{AC \cdot AC}{AD}$

$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2$ $\frac{AD}{AB} = 1.588$ $\frac{AC}{AB} = 1.26$

## Albebraic Names:

$\frac{1}{N} - DE = 0$ $1 - \frac{1}{N} - AD = 0$ $\frac{(N-1)^2}{N^2} - AC = 0$ $\frac{(N-1)^3}{N^3} - AB = 0$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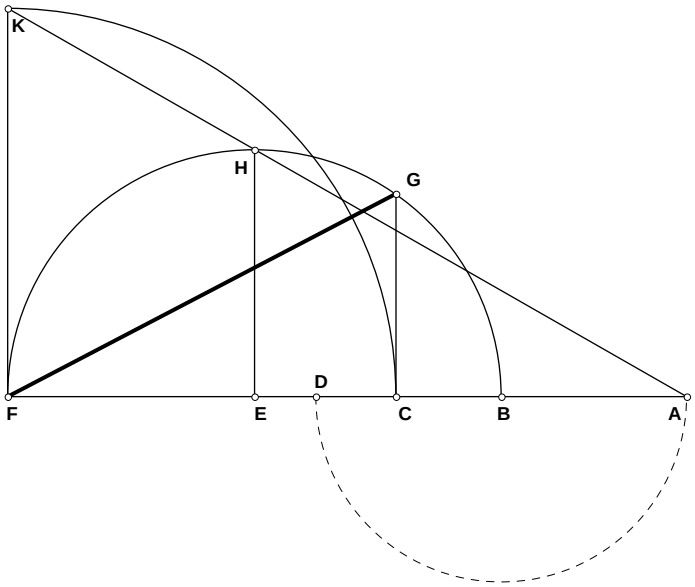
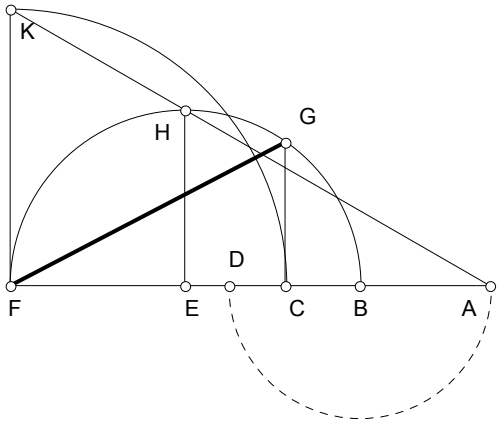



## 931112 EP

Given:

## 111293 To Square A Circle Off The Base Of A Right Triangle.




**Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?**

**Using the approximation, $\pi$ = 22/7, square the circle off the base of a right triangle.**

$$BF := 1 \qquad BE := \frac{BF}{2} \qquad EF := BE \qquad EH := BE \qquad BD := \frac{3}{4}\cdot BE \qquad AB := BD$$

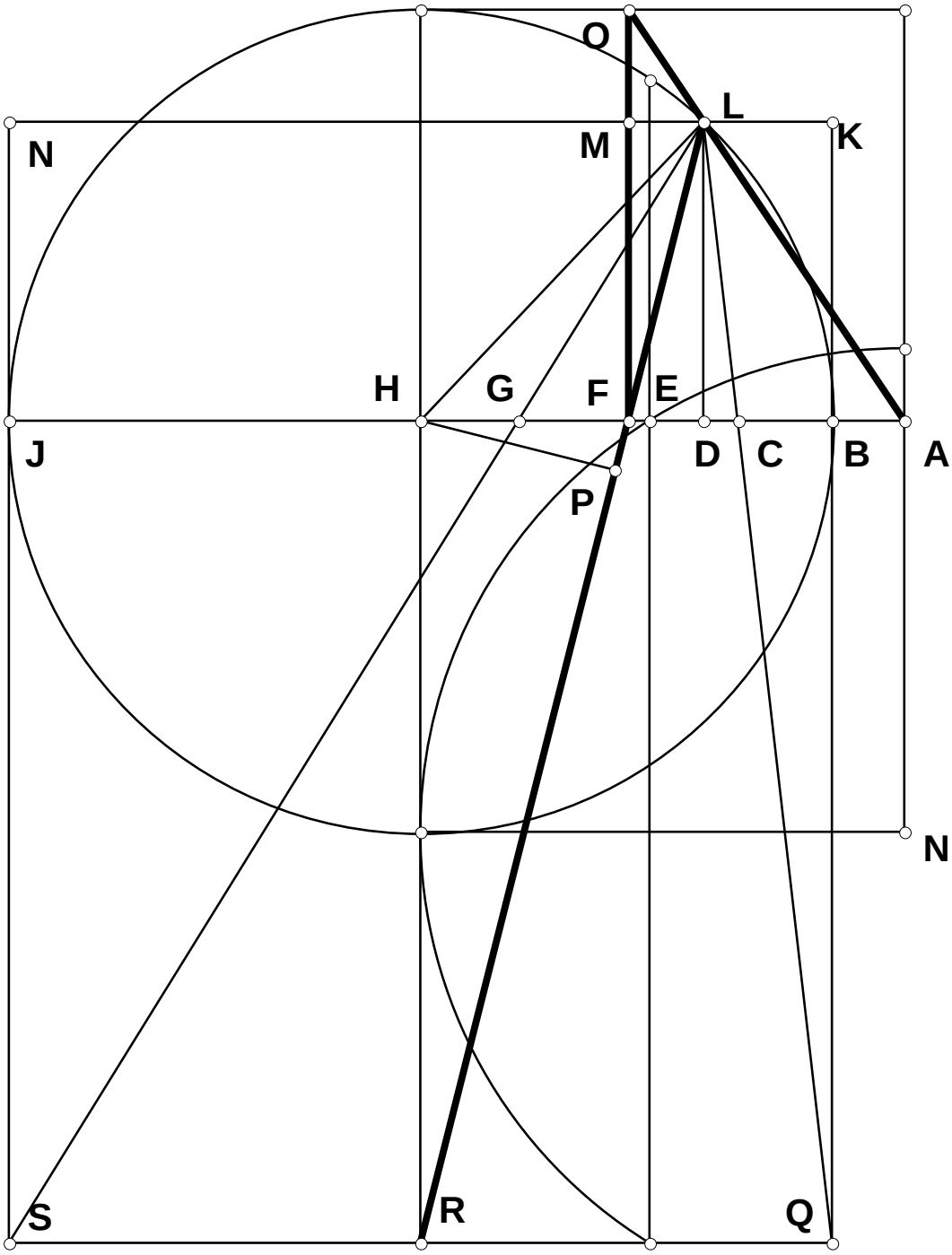
$$AE := AB + BE \qquad AF := AB + BF \qquad FK := \frac{EH \cdot AF}{AE} \qquad CF := FK \qquad BC := BF - CF$$

$$CG := \sqrt{BC \cdot CF} \qquad FG := \sqrt{CF^2 + CG^2} \qquad \pi\_A := \frac{FG^2}{BE^2}$$

$$\pi = 3.14159265359$$

$$\pi\_A = 3.142857142857$$

$$\frac{\pi}{\pi\_A} = 0.999597662505843$$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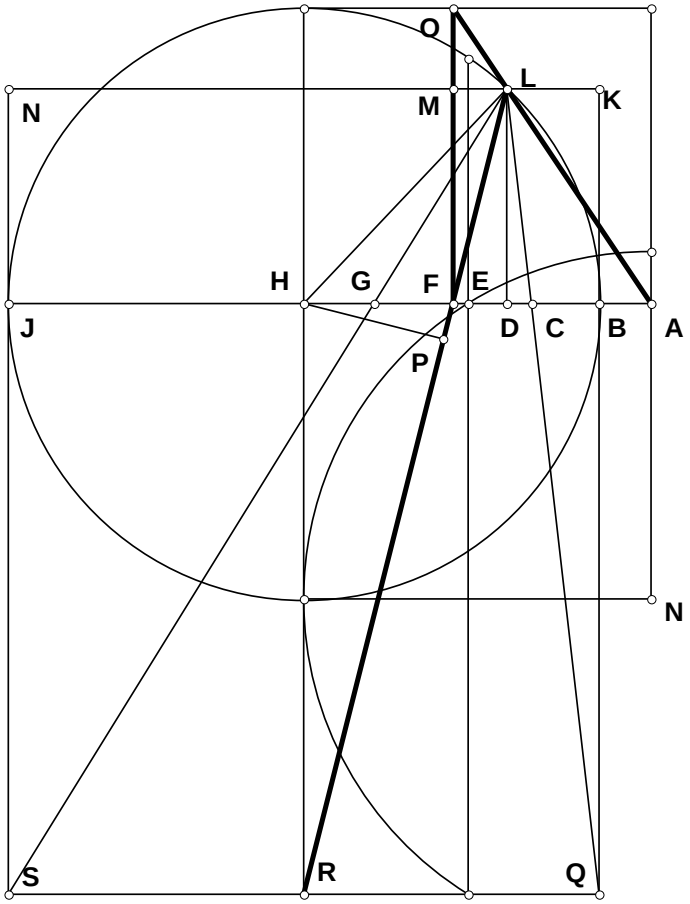
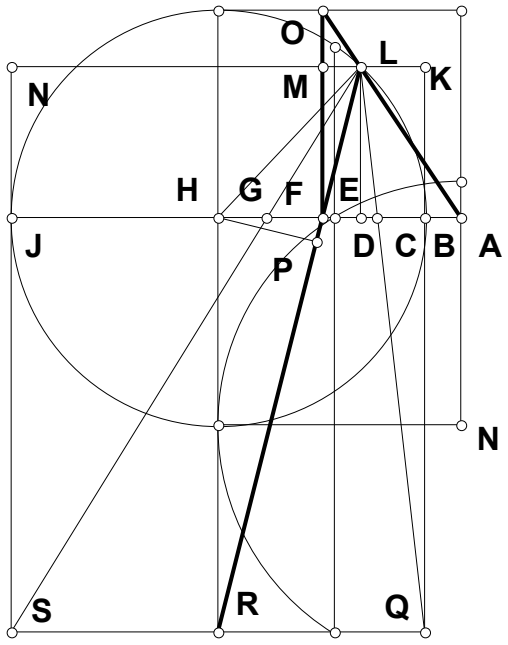



## 931118A EP

Given:

# 111893 Exploring Cube Roots Plate A

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root.**




$N := 2$

$BJ := 1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N}$

$FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2}$

$FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM$

$LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

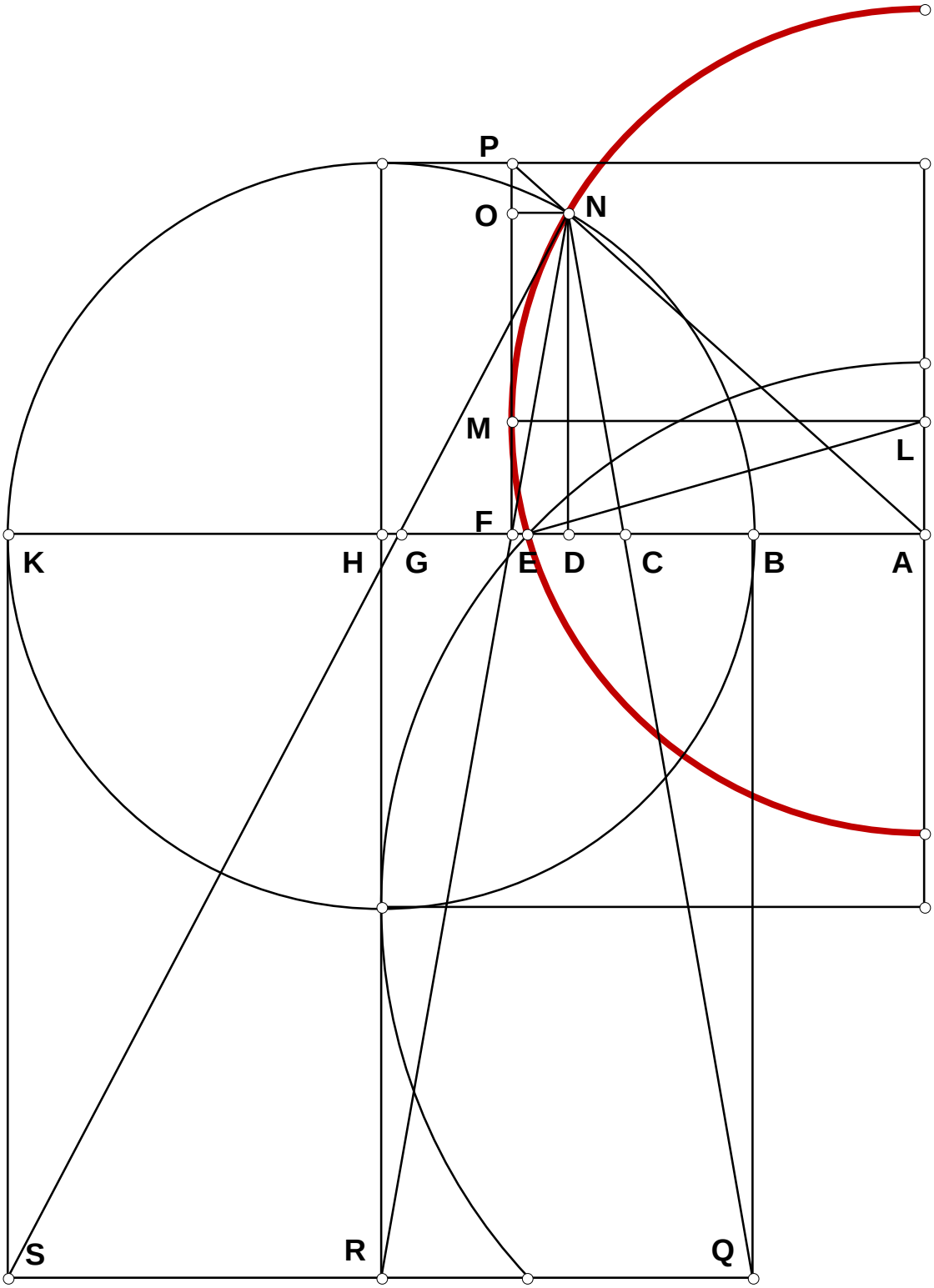
$BQ := BJ \quad BK := DL \quad BD := BF - DF$

$KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad DJ := BJ - BD \quad LN := DJ \quad JS := BJ$

$JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC \quad AG := AB + BG$

$AJ := AB + BJ$

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$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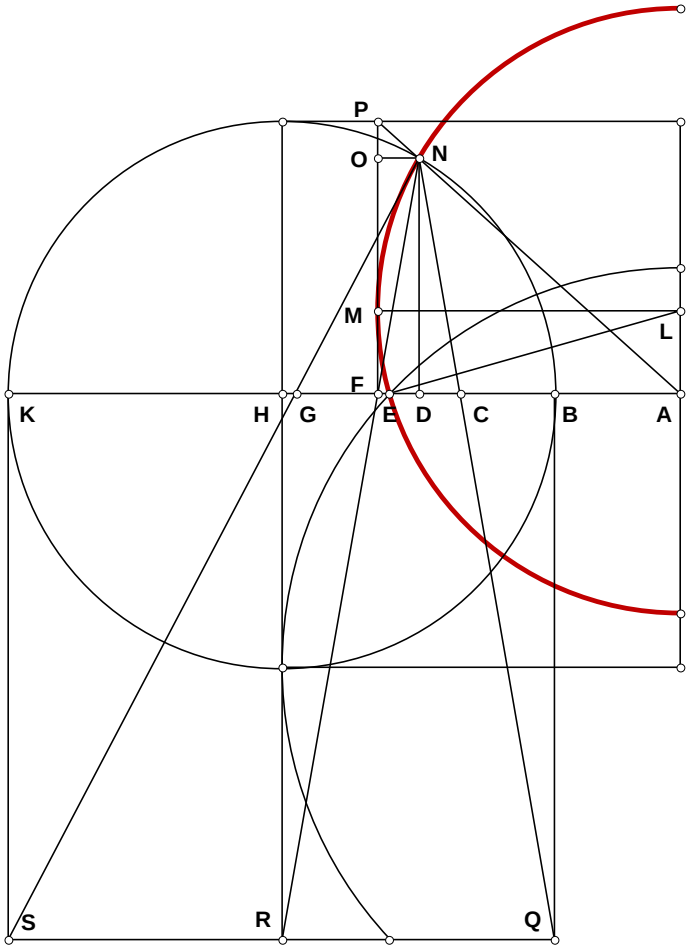
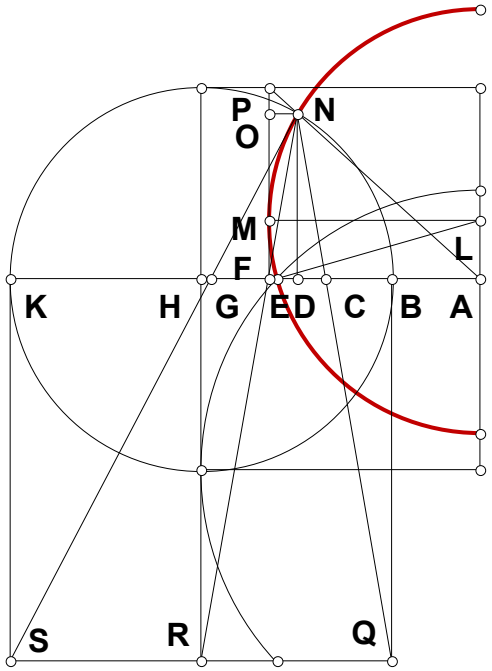
# 931118B EP




Given:

# 111893 Exploring Cube Roots Plate B

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**




$N := 1.2 \qquad BK := 1$

$BH := \frac{BK}{2} \quad BD := \frac{BH}{N} \quad DK := BK - BD$

$DN := \sqrt{BD \cdot DK} \quad BQ := BK \quad KS := BK \quad HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN} \quad GK := \frac{DK \cdot KS}{KS + DN} \quad BG := BK - GK$

$DH := BH - BD \quad FH := \frac{DH \cdot HR}{HR + DN} \quad BF := BH - FH$

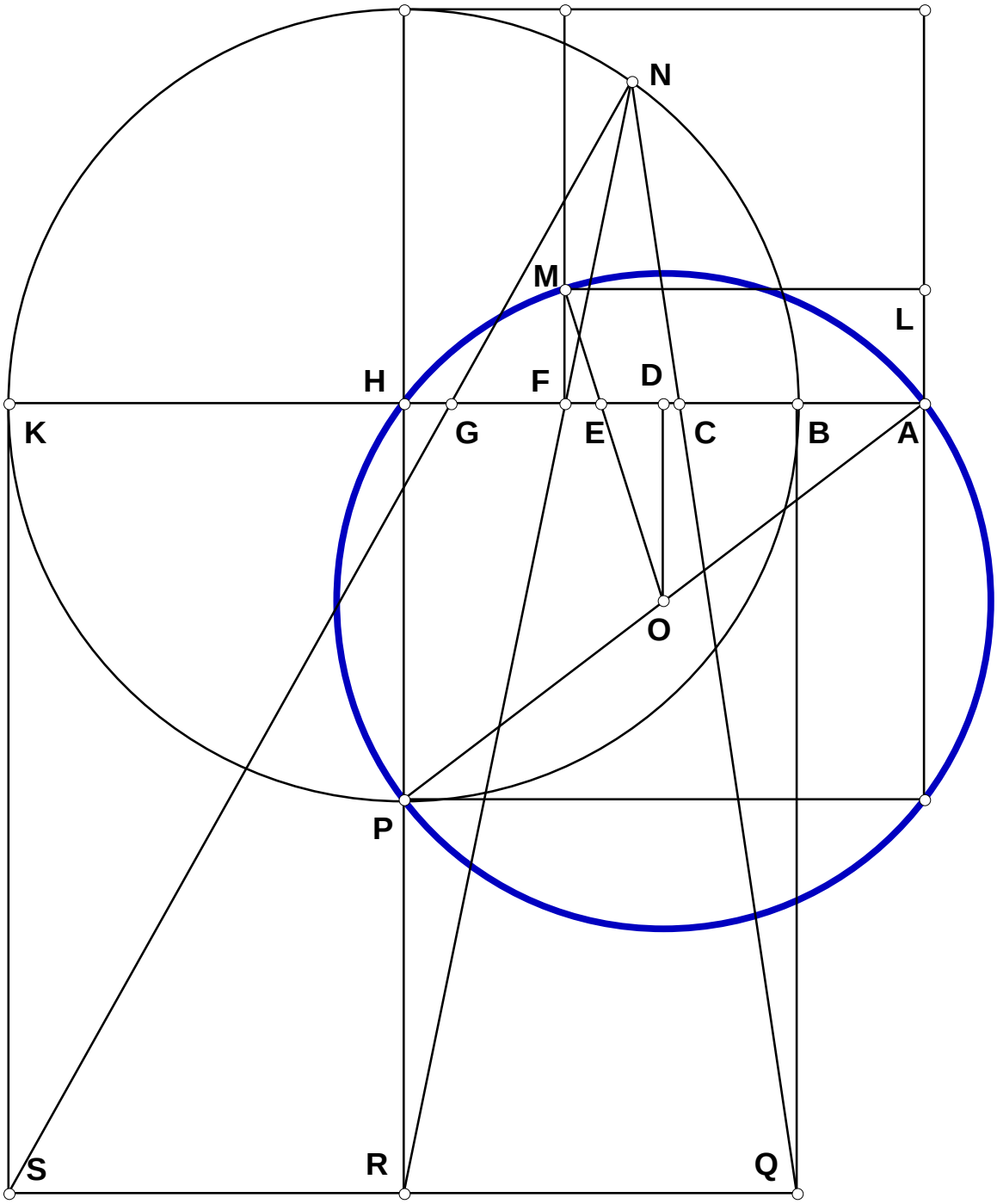
$CF := BF - BC \quad AL := CF \quad DF := BF - BD$

$NO := DF \quad FP := BH \quad PO := FP - DN$

$AD := \frac{NO \cdot DN}{PO} \quad AB := AD - BD \quad AB = 1.523$

$AF := AB + BF \quad LM := AF \quad EL := AF \quad AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2} \qquad AE_2 := \sqrt{AB \cdot AK} \qquad AE_1 - AE_2 = 0$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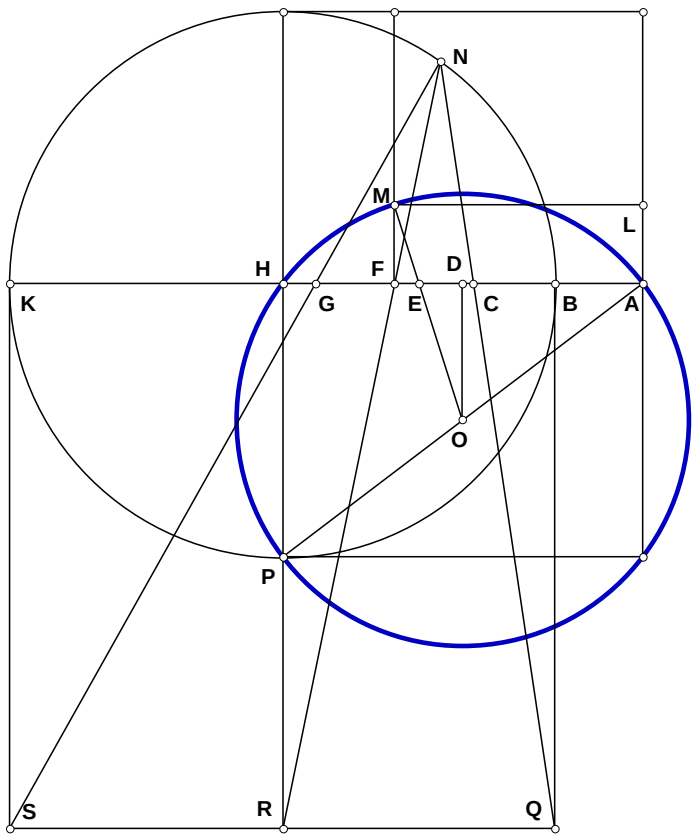
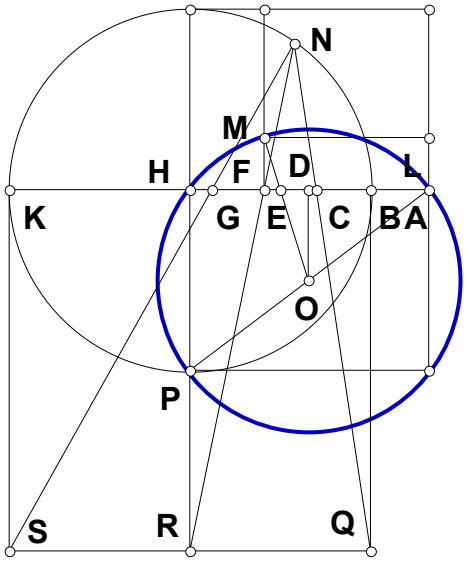
# 931118C EP




Given:

# 111893C Exploring Cube Roots

**The circle AO passes through point M.**




$N := 2$

$AB := N \quad BK := 1 \qquad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

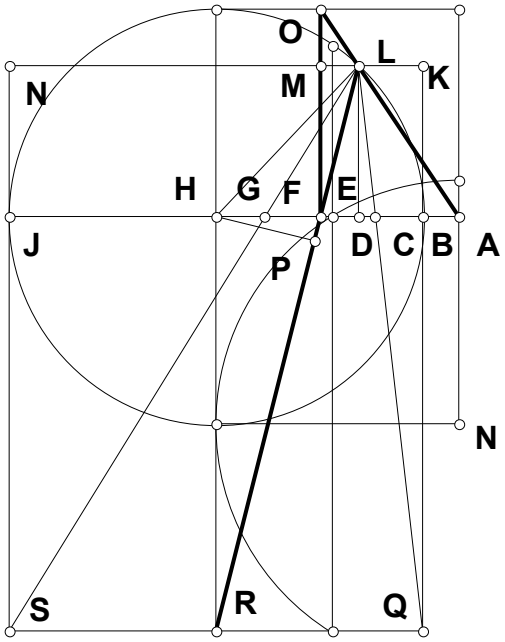
$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$




## 111893N

$N := 1.5$

$BJ := 1 \qquad BH := \frac{BJ}{2} \quad BD := \frac{BH}{N} \quad HJ := BH$

$DH := BH - BD \quad HR := BJ \quad DJ := DH + HJ$

$DL := \sqrt{BD \cdot DJ} \quad DF := \frac{DH \cdot DL}{DL + HR}$

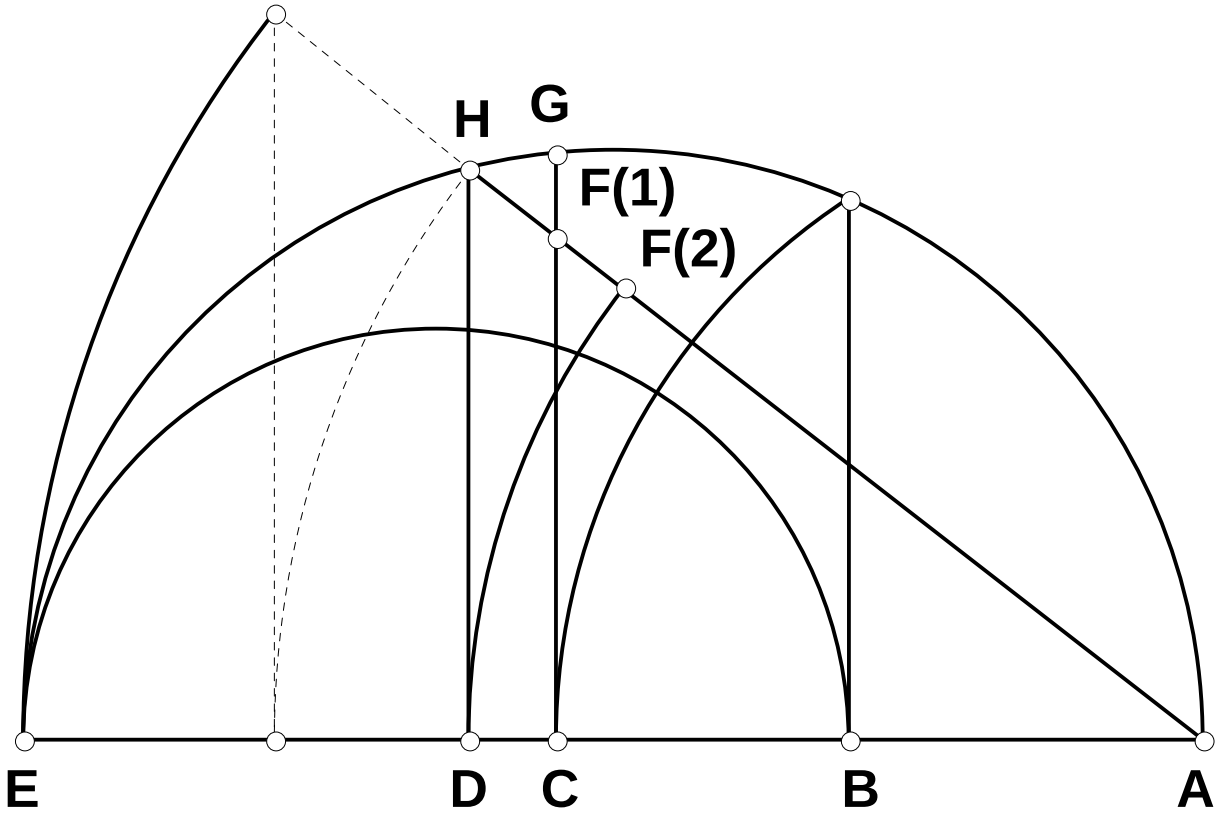
$FO := BH \quad BF := BD + DF$

$MO := FO - DL \quad LM := DF$

$AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$

$AB = 0.547$

$$\frac{\sqrt{2 \cdot N - 1} - 1}{4 \cdot N^2 - 4\,N - 2 \cdot N \cdot \sqrt{2 \cdot N - 1} + 2} - AB = 0$$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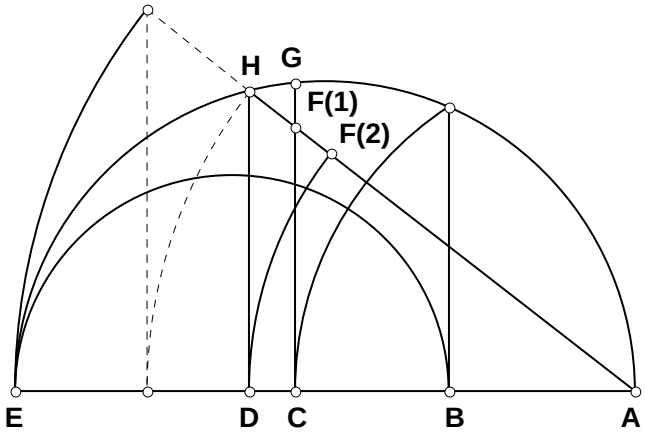
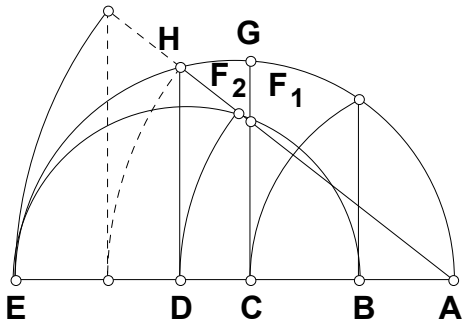
**931122 EP**

Given:





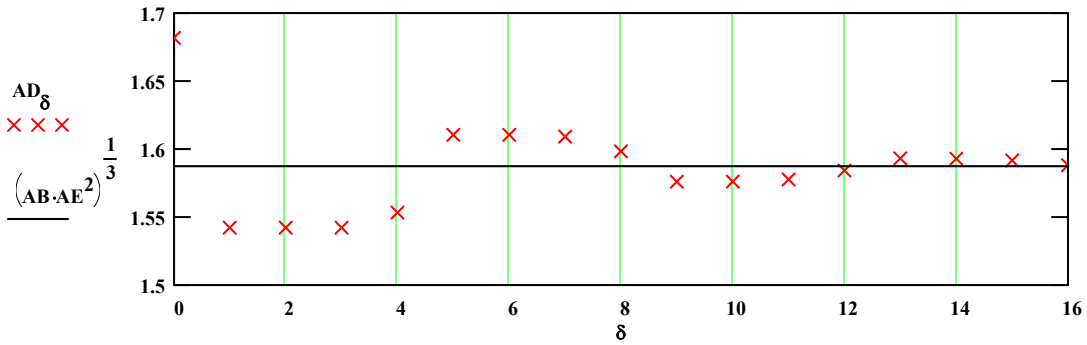

# 112293 Cube by Iteration

**When $F_1$ and $F_2$ are the same point on C, then a sixth root series has been constructed. Use iteration to place $F_2$ on $F_1$.**

$AB := 1 \qquad AE := 2 \qquad \delta := 0 .. \Delta$

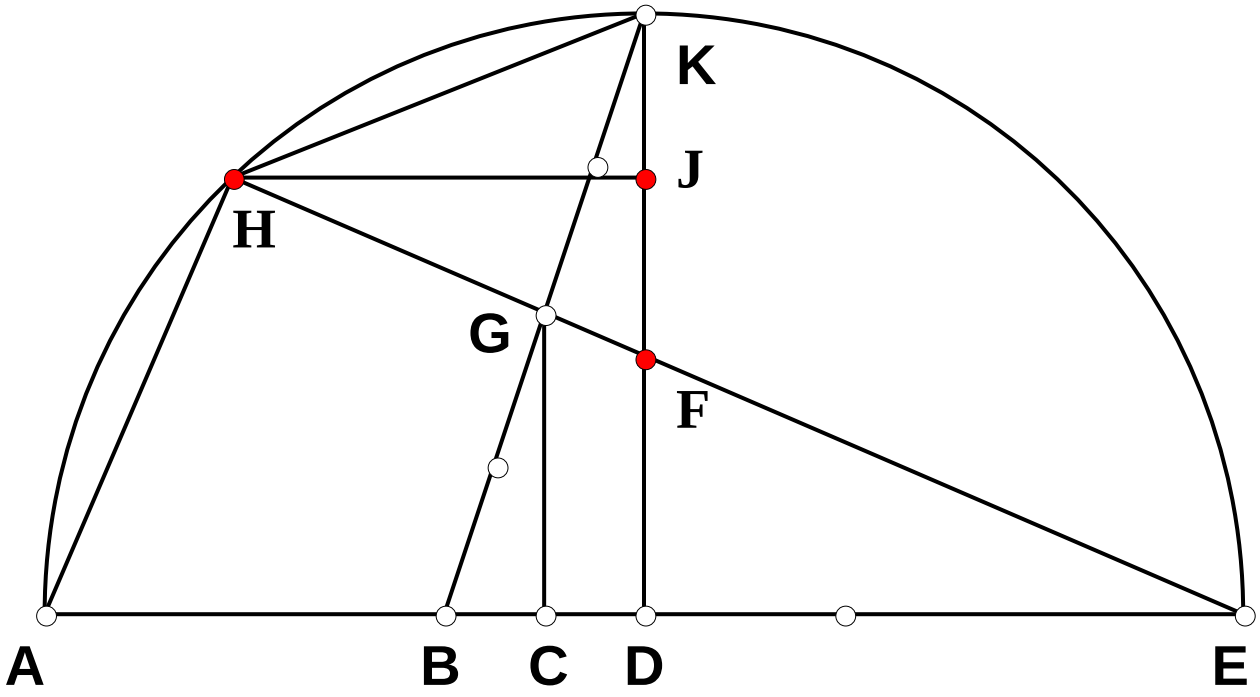
$AC := \sqrt{AB \cdot AE} \quad CE := AE - AC \quad CG := \sqrt{AC \cdot CE} \quad AG := \sqrt{AC^2 + CG^2}$

$$\begin{bmatrix} AD_0 \\ DE_0 \\ DH_0 \\ CF_0 \\ AF_0 \end{bmatrix} := \begin{bmatrix} AG \\ AE - AG \\ \sqrt{(AE - AG)\cdot AG} \\ \frac{\sqrt{(AE - AG)\cdot AG}\cdot AC}{AG} \\ \sqrt{AC^2 + \left[\frac{\sqrt{(AE - AG)\cdot AG}\cdot AC}{AG}\right]^2} \end{bmatrix} \qquad \begin{bmatrix} AD_{\delta+1} \\ DE_{\delta+1} \\ DH_{\delta+1} \\ CF_{\delta+1} \\ AF_{\delta+1} \end{bmatrix} := \begin{bmatrix} AF_\delta \\ AE - AF_\delta \\ \sqrt{AF_\delta \cdot DE_\delta} \\ \frac{DH_\delta \cdot AC}{AD_\delta} \\ \sqrt{AC^2 + (CF_\delta)^2} \end{bmatrix} \qquad \Delta \equiv 16$$




$$(AB^2 \cdot AE^4)^{\frac{1}{6}} - AD_\Delta = -6.761 \cdot 10^{-4} \qquad (AB \cdot AE^2)^{\frac{1}{3}} - AD_\Delta = -6.761 \cdot 10^{-4}$$

**Not a very promising prospect!**


K
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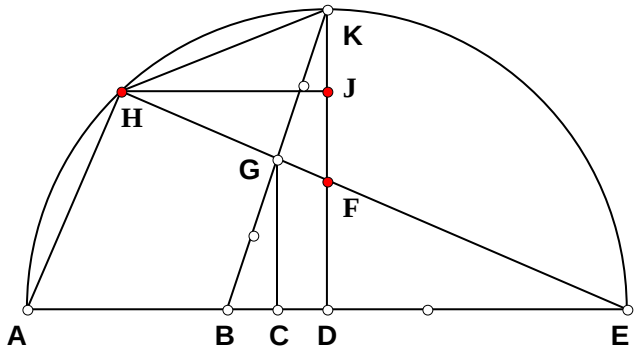
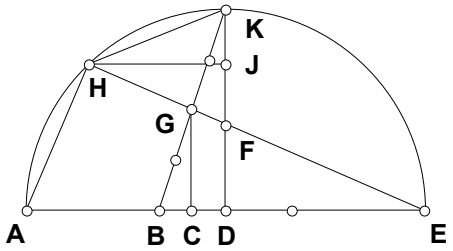



## 931124 EP

Given:

# 112493 POR Series IV

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**




$N_1 := 3 \quad N_2 := 5 \quad AE := 1$

$\alpha := 1 .. N_1 - 1 \quad \beta := 1 .. N_2 - 1$

$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \; DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

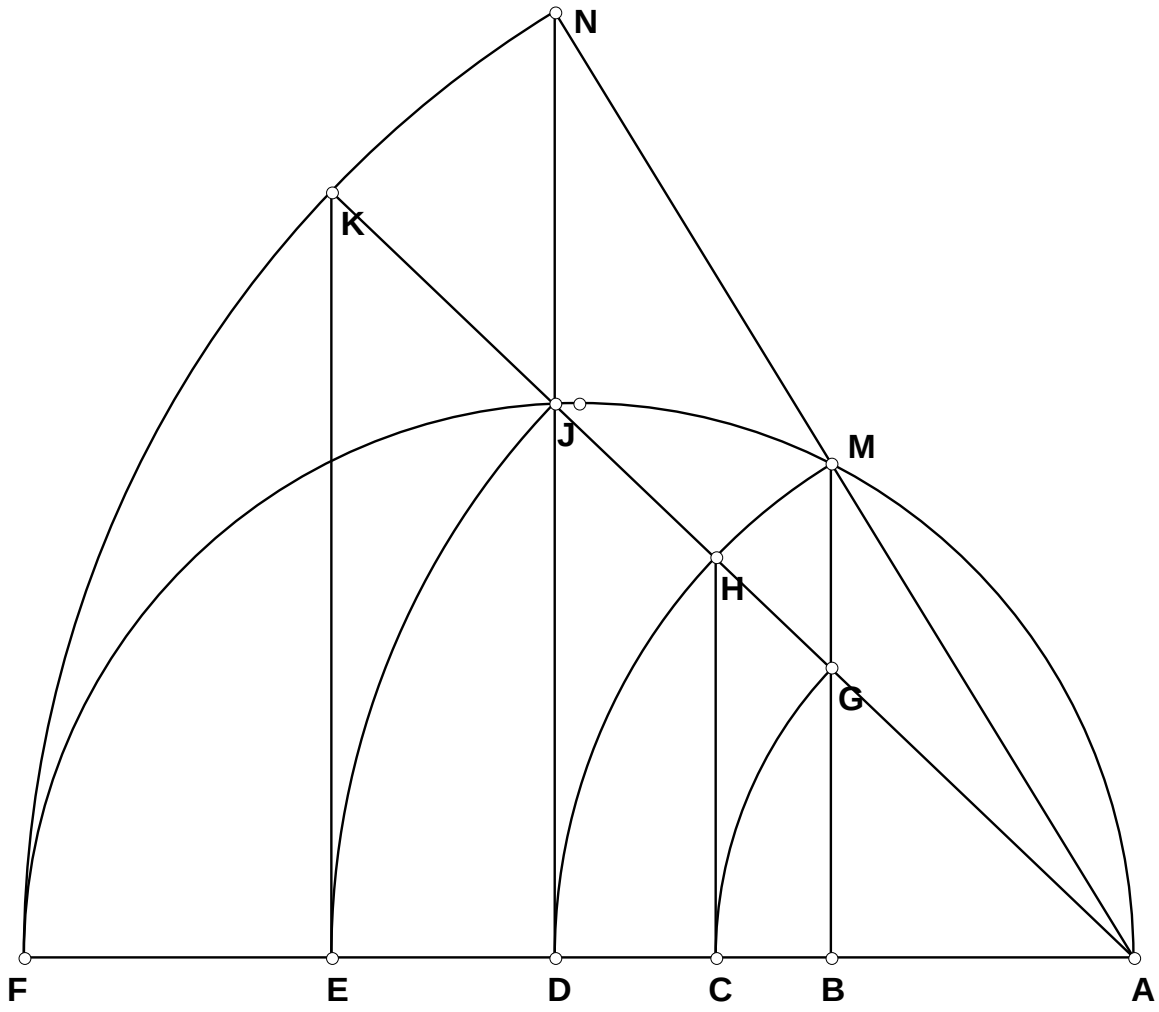
$\frac{AH}{HK} = 0.265 \quad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \quad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$$SeriesAH = \begin{bmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{bmatrix}$$

$\frac{EH}{GH} = 2.85 \quad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$$SeriesEH = \begin{bmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{bmatrix}$$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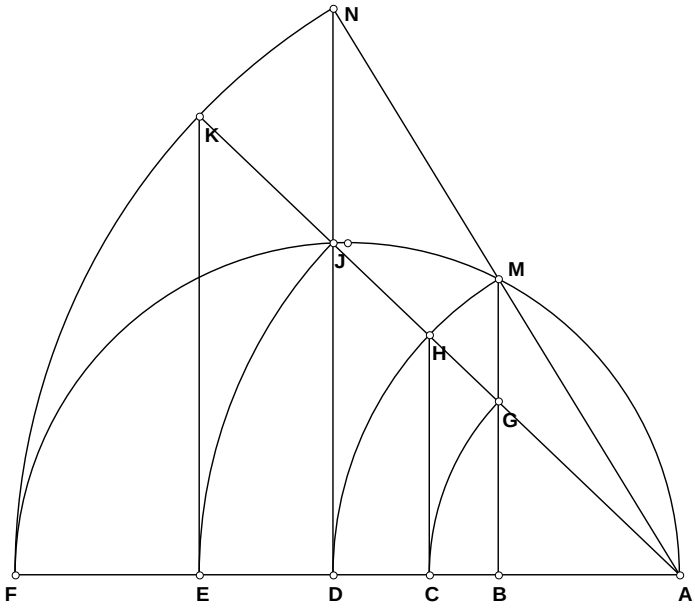
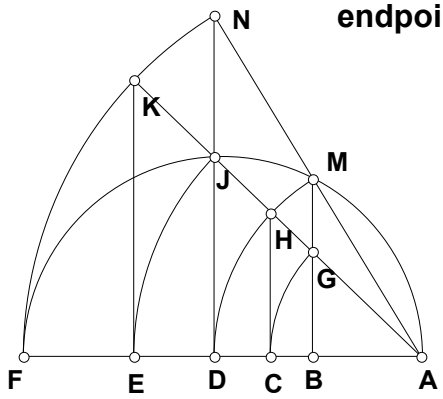
## 931204 EP




Given:

# 120493 Exponential Series M^(1/2^N)

**Given some number, construct a two prime exponential series from it, such as a Quad Root Series, using the common segment, common endpoint and cord method.**




$M := 8$

$AF := M \qquad AB := 1 \qquad BF := AF - AB$

$BM := \sqrt{AB \cdot BF} \qquad AM := \sqrt{AB^2 + BM^2}$

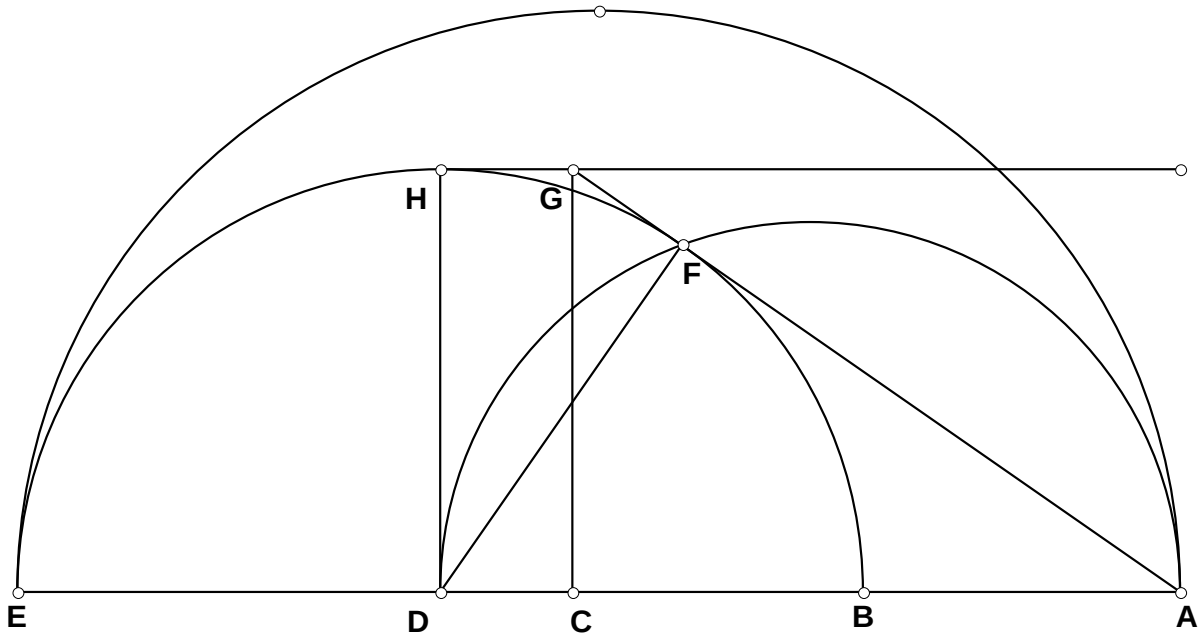
$AN := AF \quad AD := AM \qquad DF := AF - AD$

$DJ := \sqrt{AD \cdot DF} \qquad AJ := \sqrt{AD^2 + DJ^2}$

$AK := AF \quad AE := AJ \qquad AH := AD \qquad AC := \frac{AD \cdot AH}{AJ}$

$\left(AB^3 \cdot AF^1\right)^{\frac{1}{4}} - AC = 0 \qquad \left(AB^2 \cdot AF^2\right)^{\frac{1}{4}} - AD = 0 \qquad \left(AB^1 \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$

$M^{\frac{1}{4}} - AC = 0 \qquad M^{\frac{2}{4}} - AD = 0 \qquad M^{\frac{3}{4}} - AE = 0$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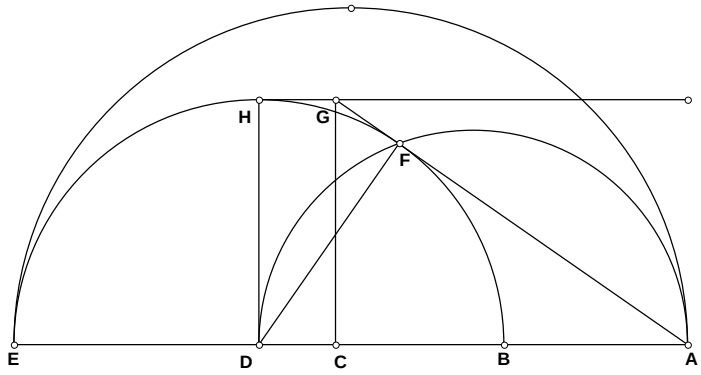
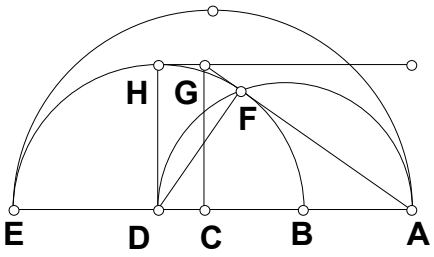
## 931206 EP




Given:

# 120693 Alternate Method: Square Root
# Common Segment Common Endpoint



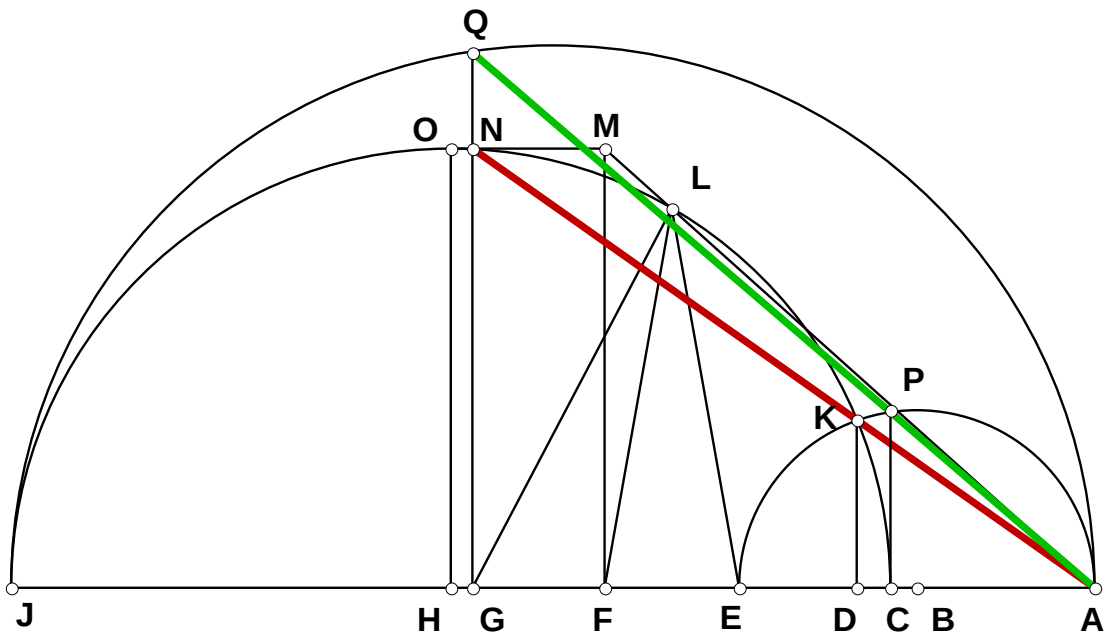

$N := 6 \qquad AB := 1 \quad AE := AB \cdot N$

$BE := AE - AB \quad BD := \frac{BE}{2}$

$DF := BD \quad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$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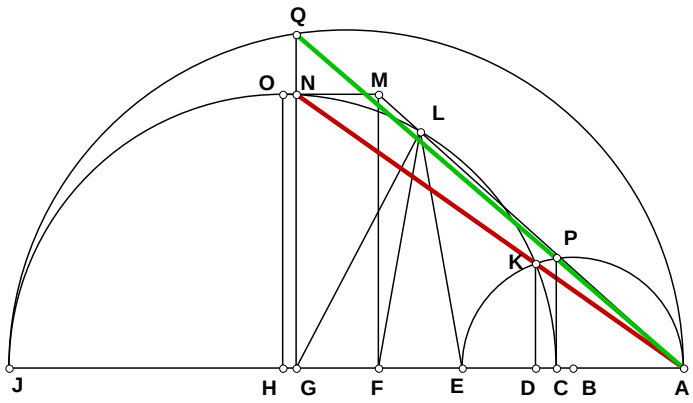
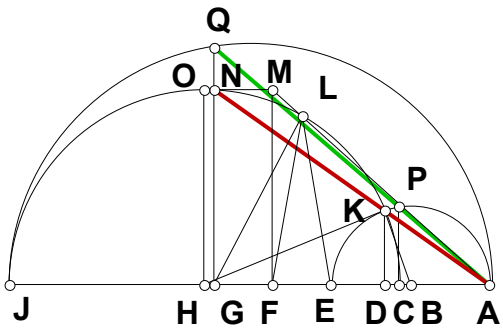
# 120693B EP




Given:

# 120693B Gruntwork IV on the Delian Solution

## Are A, P and Q collinear? Are A, K and N collinear?




$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$AE := \left(AC^2 \cdot AJ\right)^{\left(\frac{1}{3}\right)} \qquad AG := \left(AC \cdot AJ^2\right)^{\left(\frac{1}{3}\right)}$

$CG := AG - AC \qquad CJ := AJ - AC$

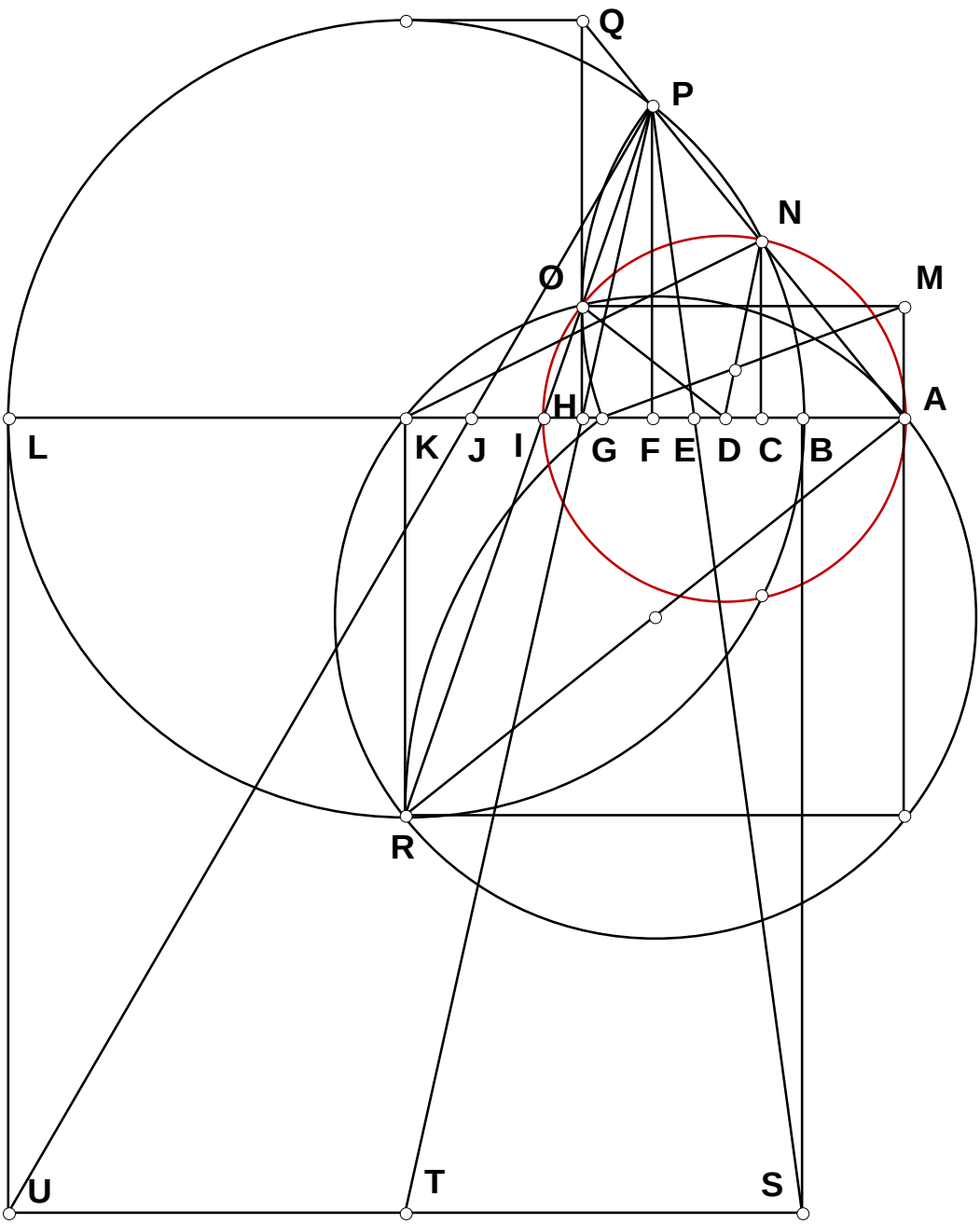
$GJ := CJ - CG \qquad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \qquad CE := AE - AC \qquad CH := \frac{CJ}{2}$

$BK := AB \qquad HK := CH \qquad HJ := CH \qquad AH := AJ - HJ \qquad BH := AH - AB \qquad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

$AD := AB + BD \qquad DE := AE - AD \qquad DK := \sqrt{AD \cdot DE} \qquad GQ := \sqrt{AG \cdot GJ} \qquad CP := \sqrt{AC \cdot CE}$

$\frac{AG}{GN} - \frac{AD}{DK} = 0 \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$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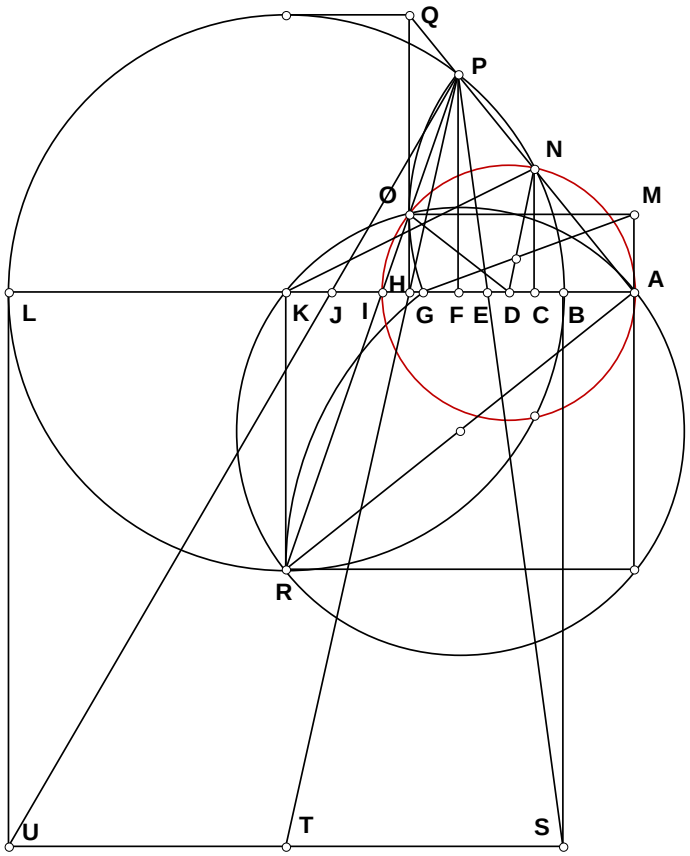
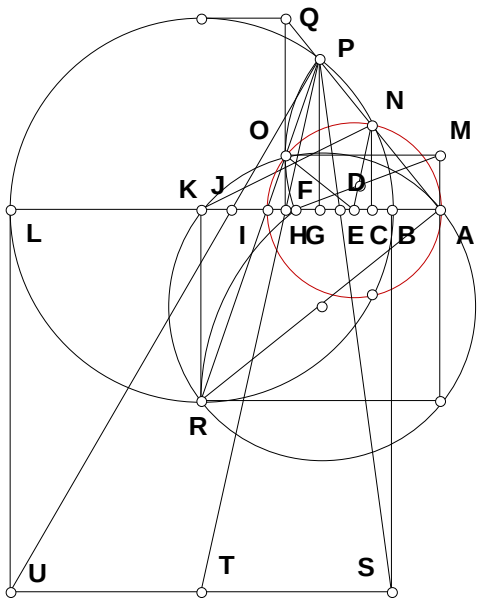



## 931211 EP

Given:




# 121193

**The structure in red appears to be a constant.**

$N := 6 \qquad AB := 1 \qquad AL := AB \cdot N$

$BL := AL - AB \quad BK := \frac{BL}{2} \quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$

$AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}} \quad BE := AE - AB \quad BJ := AJ - AB$

$JL := BL - BJ \quad EJ := AJ - AE \quad FJ := \frac{JL \cdot EJ}{JL + BE}$

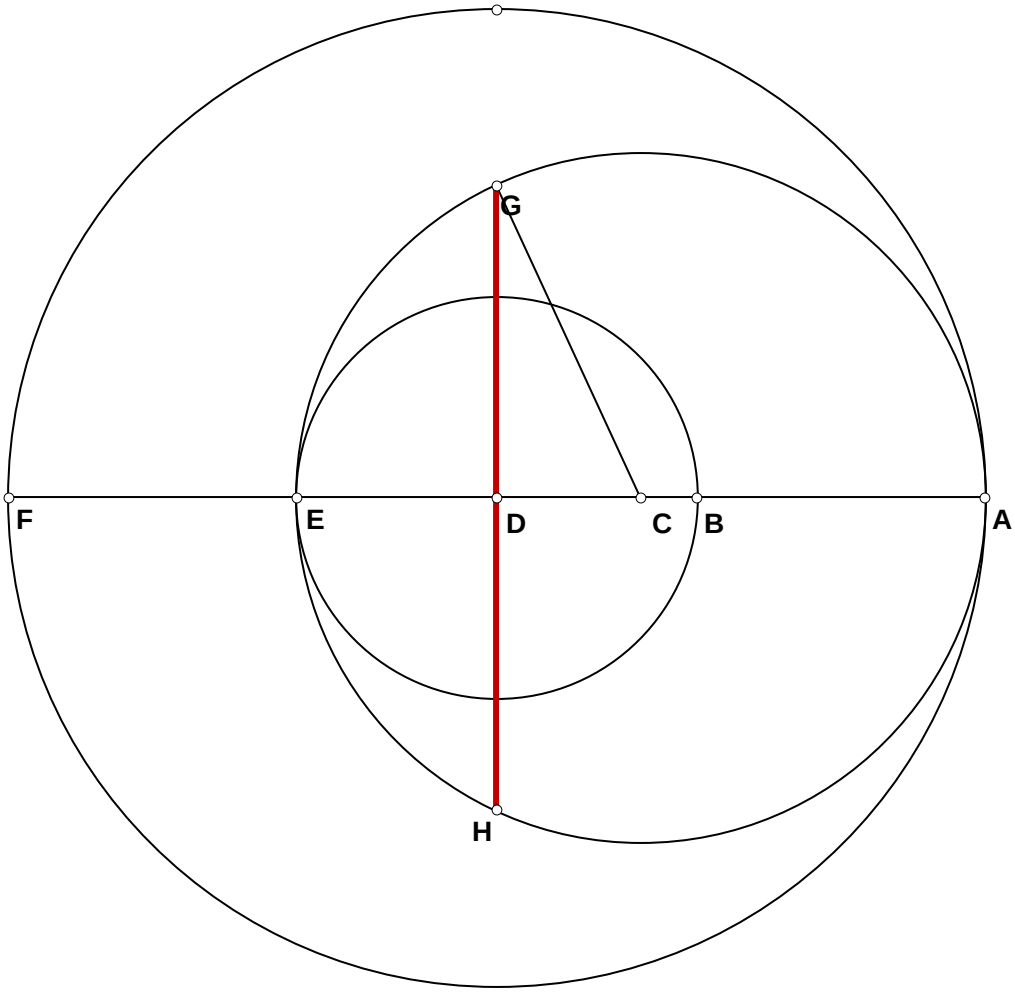
$FL := JL + FJ \quad BF := BL - FL \quad FP := \sqrt{BF \cdot FL}$

$KR := BK \quad KL := BK \quad FK := FL - KL \quad IK := \frac{FK \cdot KR}{KR + FP} \quad AK := BK + AB \quad AI := AK - IK$

$AD := \frac{AI}{2} \quad KT := BL \quad FH := \frac{FK \cdot FP}{KT + FP} \quad AF := BF + AB \quad AH := AF + FH \quad HI := AI - AH$

$HO := \sqrt{AH \cdot HI} \quad DN := AD \quad KN := BK \quad DK := AK - AD \quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK \quad CI := AI - AC \quad CN := \sqrt{AC \cdot CI} \quad \frac{KR}{IK} - \frac{HO}{HI} = 0 \quad \frac{AF}{FP} - \frac{AC}{CN} = 0$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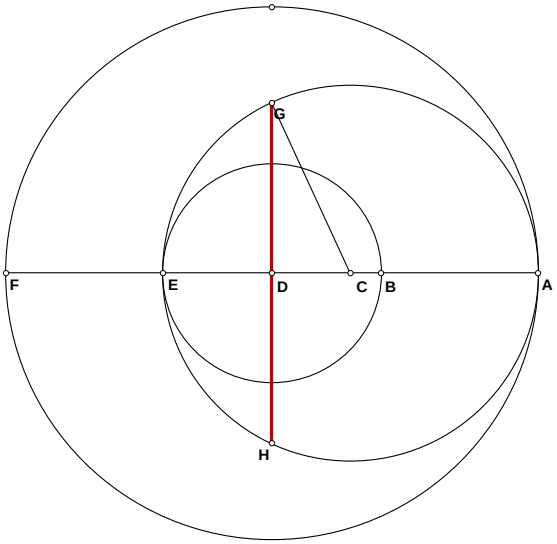
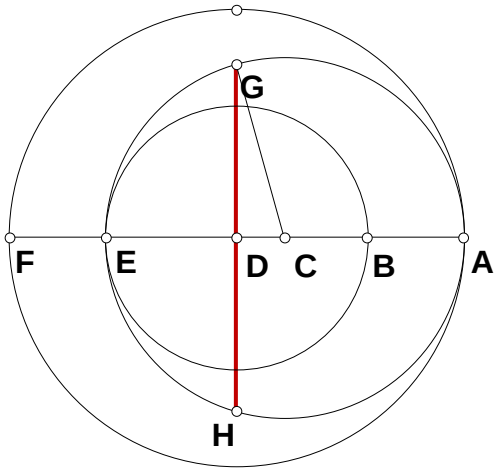
## 931212 EP




Given:

# 121293 The Square Root




**Square root by common segment common midpoint. Given AFand BE is GH their root?**

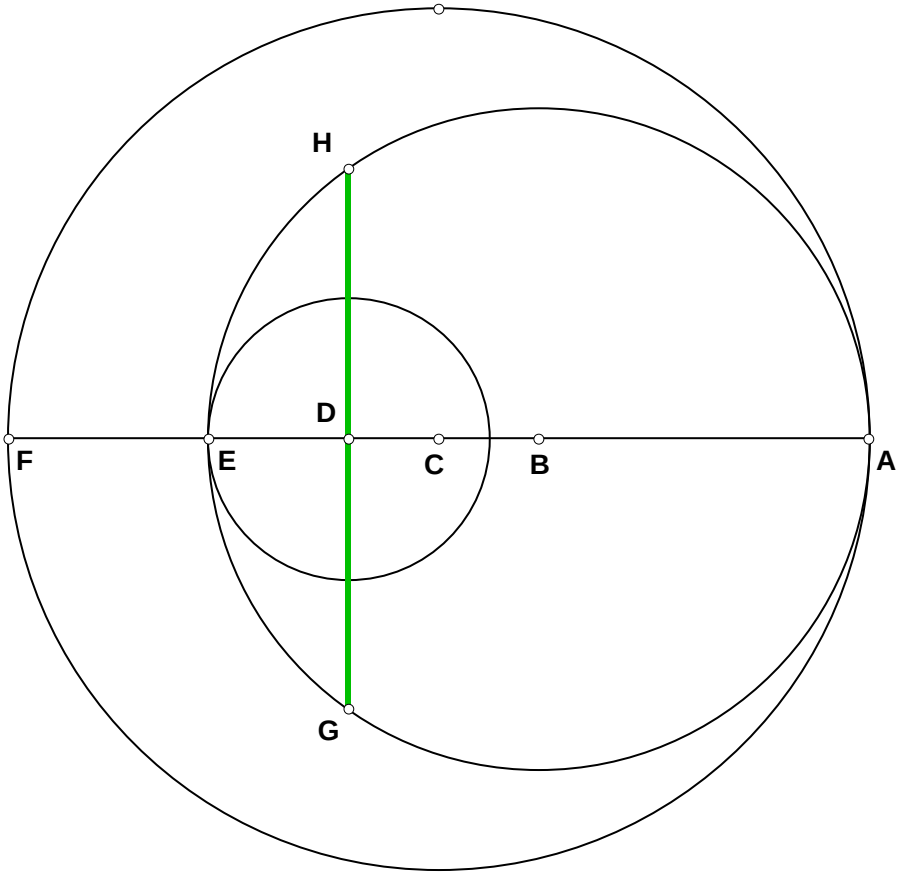
$N := 5 \qquad BE := 1 \qquad AF := BE \cdot N$

$AD := \frac{AF}{2} \quad BD := \frac{BE}{2} \quad AB := AD - BD$

$AE := BE + AB \quad AC := \frac{AE}{2} \qquad CG := AC$

$CD := AD - AC \qquad GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$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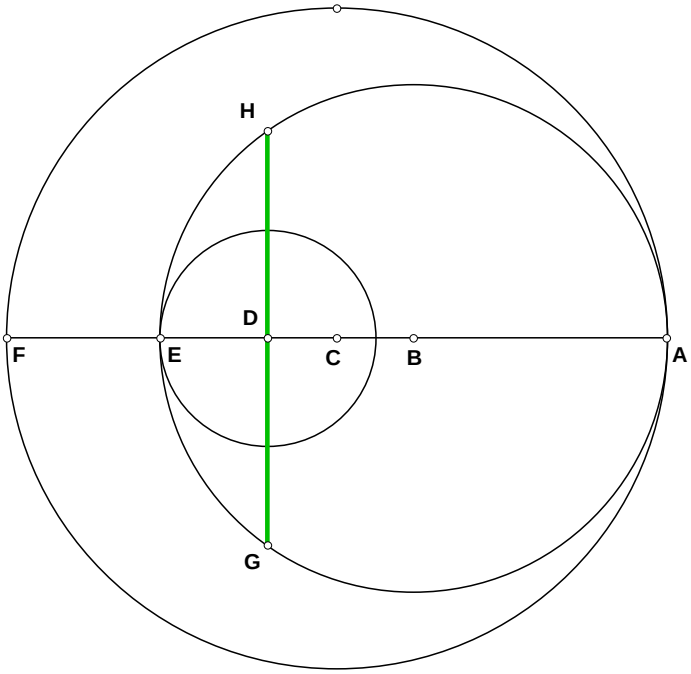
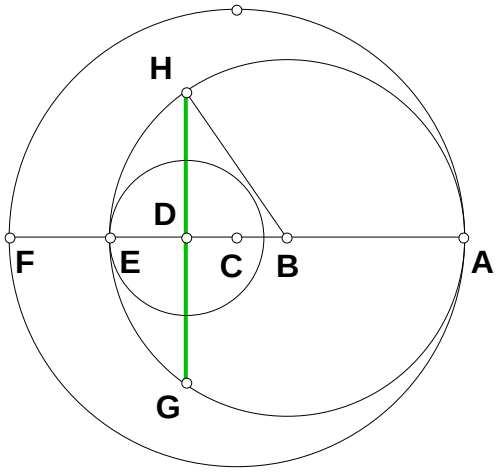
# 931212B EP




Given:

# 121293B Generalize The Previous Square Root Figure




$N_1 := 1 \qquad N_2 := 3 \qquad N_3 := 2$

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

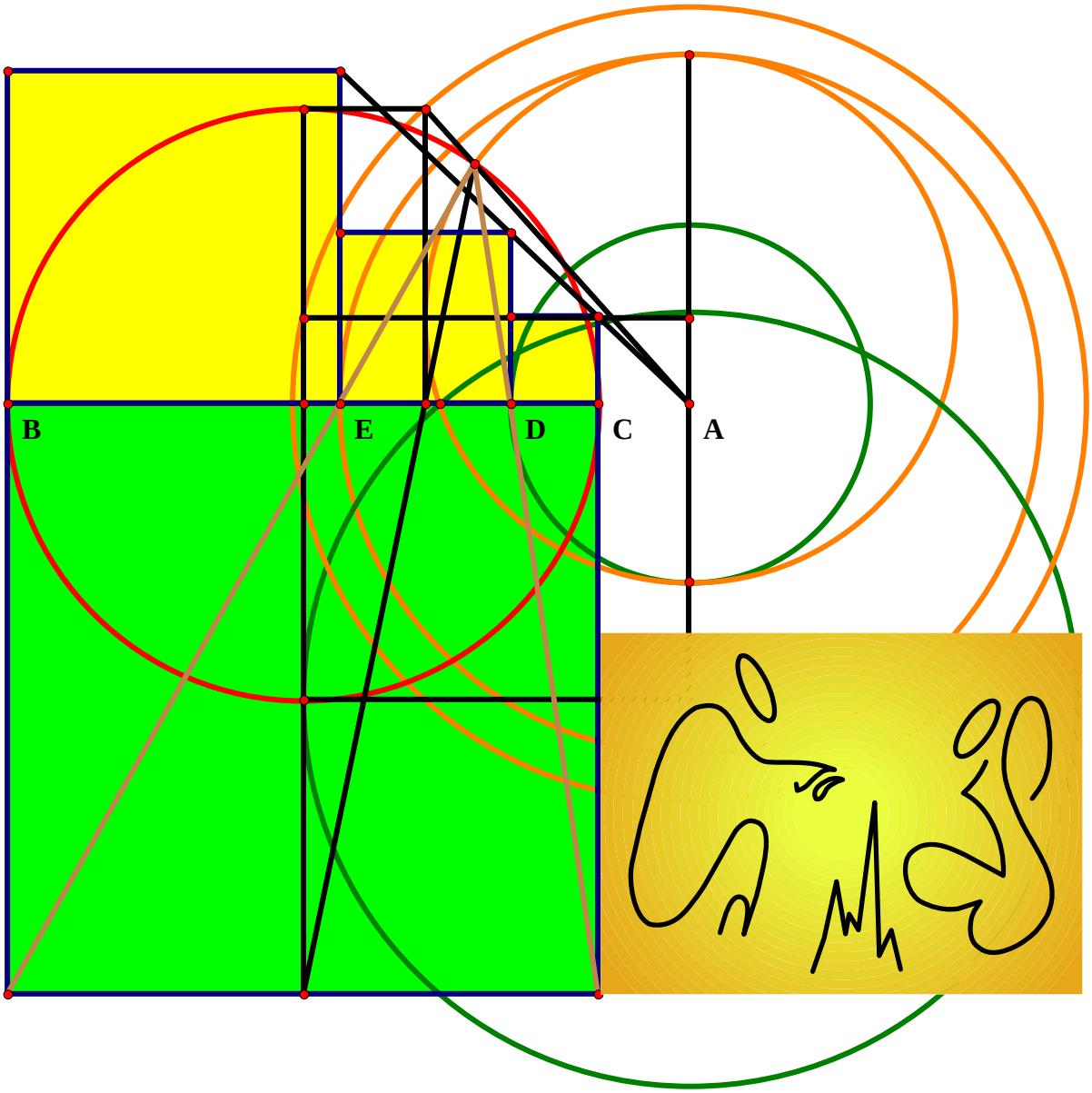
$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

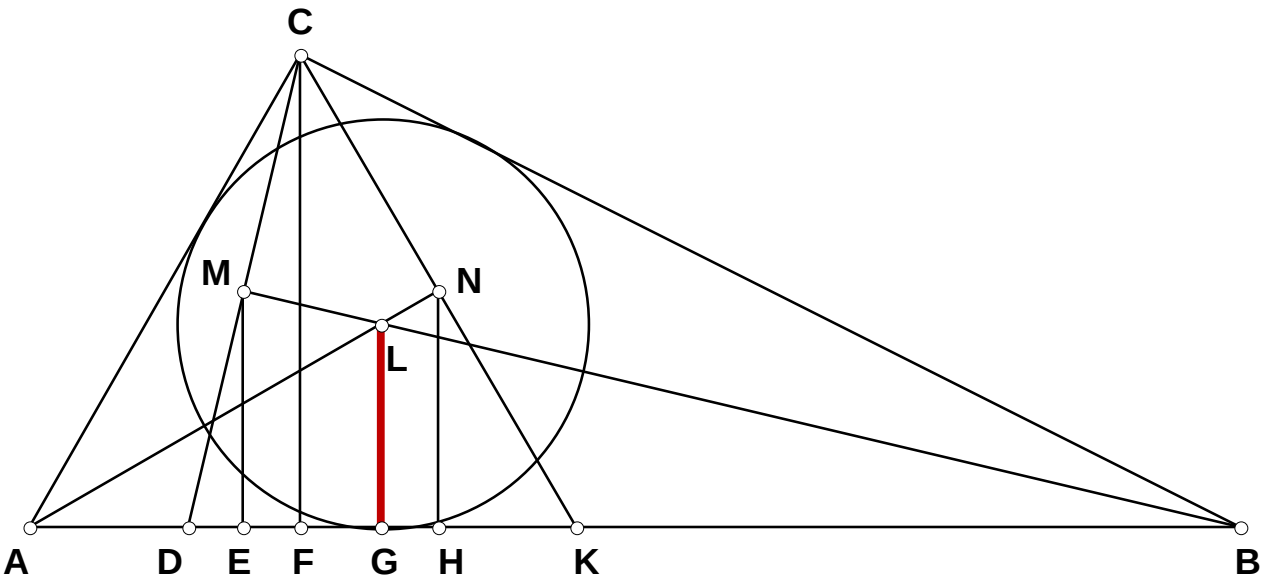
$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

# The Delian Quest—1994

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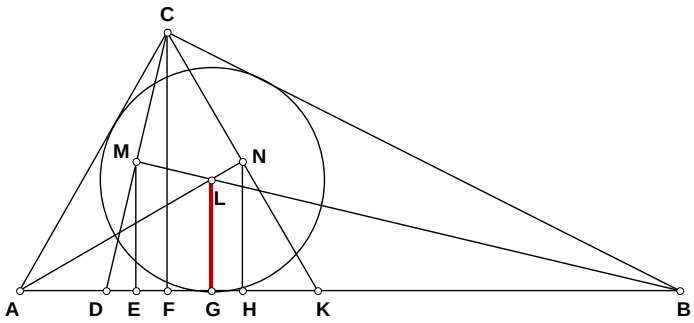
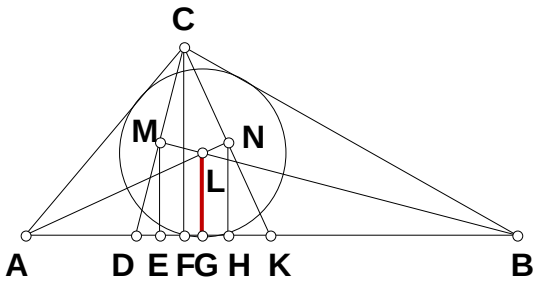



# 940406 EP

Given:

# 040694 Inscribing A Circle In A Given Triangle

**Given three sides of a triangle, what is the length of the inscribed radius?**




$AB := 3 \qquad BC := 4 \qquad AC := 5$

$AK := AC \quad BD := BC \quad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$

$FK := AK - AF \quad CF := \sqrt{AC^2 - AF^2}$

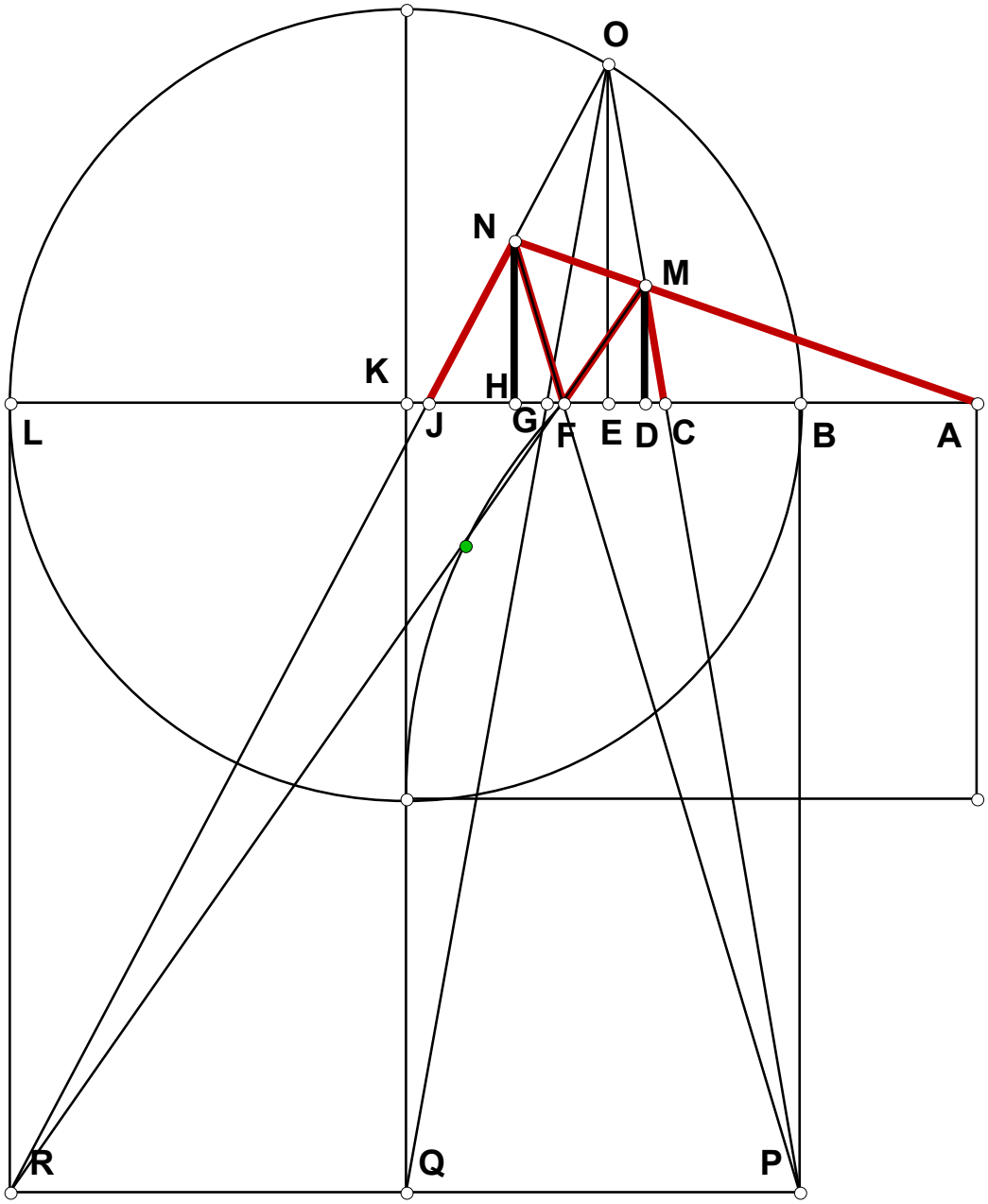
$CK := \sqrt{FK^2 + CF^2} \quad AN := \frac{CF \cdot AK}{CK}$

$AH := \frac{CF \cdot AN}{CK} \quad HN := \frac{FK \cdot AN}{CK} \quad BF := AB - AF$

$DF := BD - BF \quad CD := \sqrt{CF^2 + DF^2} \quad BM := \frac{CF \cdot BD}{CD} \quad BE := \frac{CF \cdot BM}{CD} \quad GL := \frac{HN \cdot AB}{AH + BE} \quad GL = 1$

$S_1 := \begin{bmatrix} AB \\ BC \\ AC \end{bmatrix} \qquad S_2 := \begin{bmatrix} BC \\ AC \\ AB \end{bmatrix} \qquad S_3 := \begin{bmatrix} AC \\ AB \\ BC \end{bmatrix} \qquad \delta := 0 .. 2$

$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$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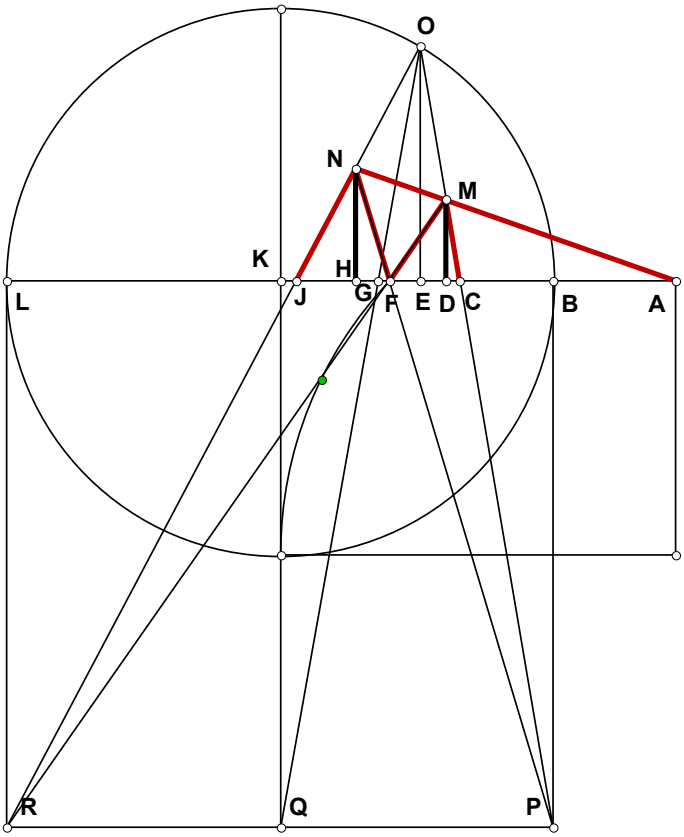
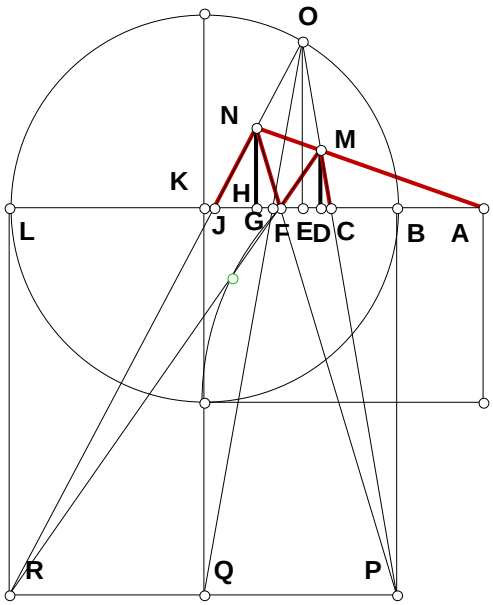
# 940421 EP




Given:

# 042194 The Cradle

## Are A, M, N colinear?




$N := 5 \qquad AB := 1 \qquad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \qquad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

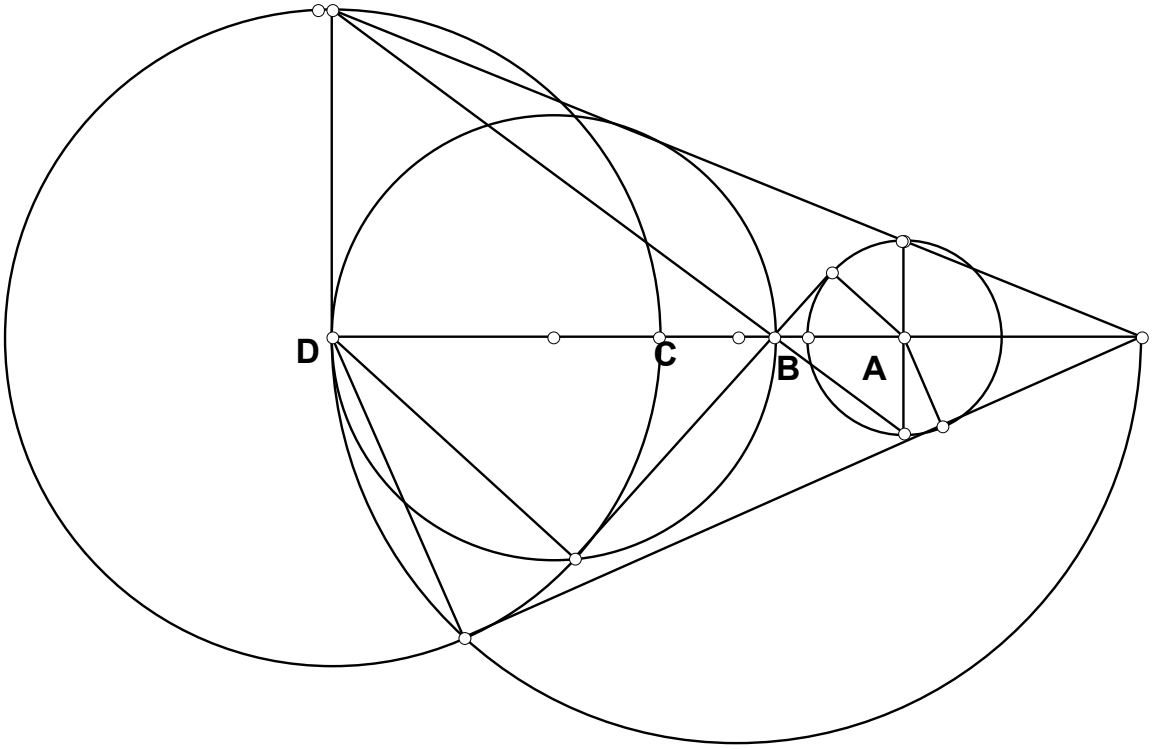
$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$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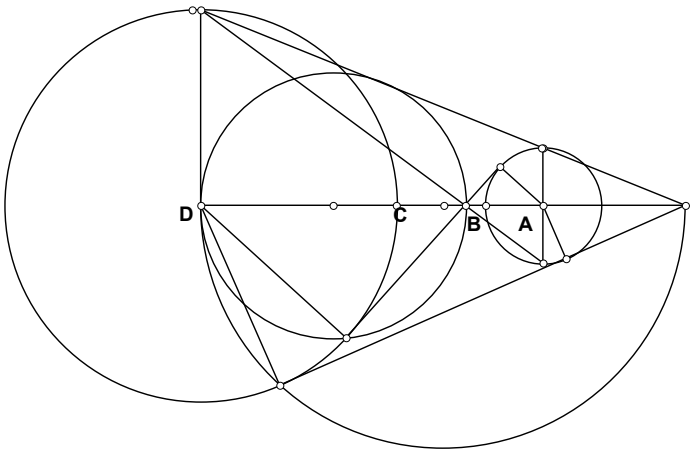
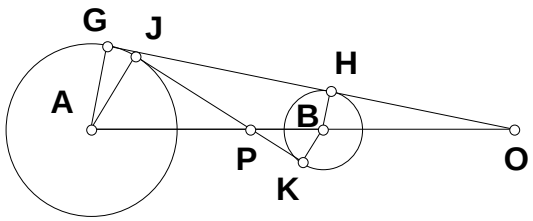
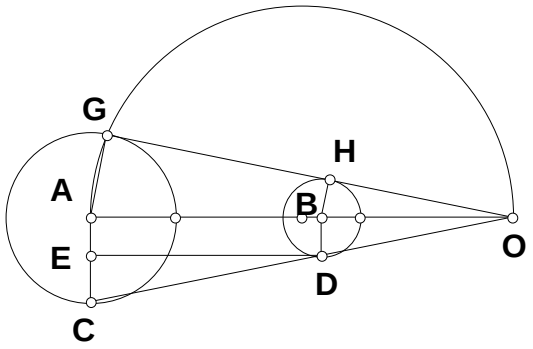



## 940426 EP

Given:

# 042694 Tangents and Similarity Points




**What is the Algebraic names of the similarity points O and P in relation to the radius of each circle and the difference between their centers?**

**I will work with point O first.**

**Given $R_L$ = large radius**
**$R_S$ = small radius**
**D = difference between origins.**

$R_L := 4 \quad R_S := 1 \quad D := 8$

$AC := R_L \quad BD := R_S \quad AB := D$

$DE := AB \quad AE := BD \quad CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE} \quad AO = 10.667$

**$EOR_L$ "External similarity point Origin to center of Radius Large"**

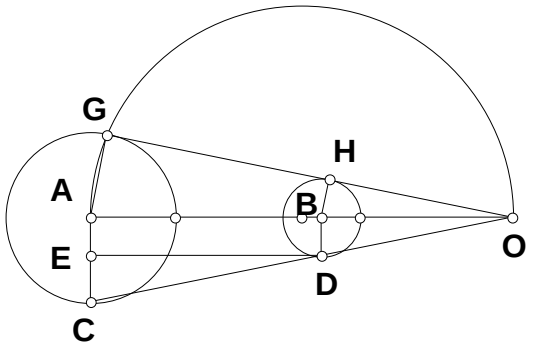
$EOR_L := \text{if}\left(R_L \neq R_S, \text{if}\left(R_S > R_L, 0, \frac{D \cdot R_L}{R_L - R_S}\right), \infty\right) \quad EOR_L = 10.667$

**What is the length of line (OG) tangent to both circles?**

$AG := AC \quad GO := \sqrt{AO^2 - AG^2} \quad GO = 9.888$

**And what is the formula?**
**$EOT_{LR}$ " External similarity point Origin to Tangent (Large Radius)"**

$EOT_{LR} := R_L \cdot \frac{\sqrt{(R_L - R_S + D) \cdot (-R_L + R_S + D)}}{R_L - R_S} \quad EOT_{LR} = 9.888$




**What is the length of the line tangent to the least circle (HO)?**

$BH := BD \qquad BO := AO - AB \qquad BO = 2.667$

$HO := \sqrt{BO^2 - BH^2}$

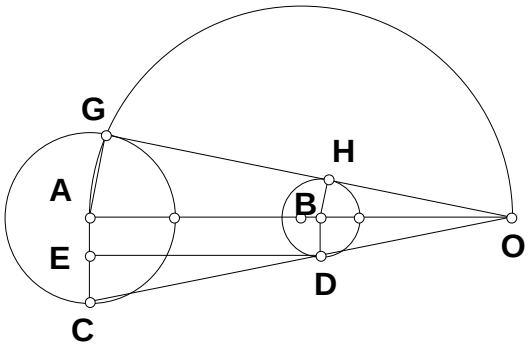
$HO = 2.472$

**And what is the formula?**

**$EOT_{SR}$ " External similarity point Origin to Tangent (Small Radius)"**

$$EOT_{SR} := R_S \cdot \frac{\sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}}{R_L - R_S} \qquad EOT_{SR} = 2.472$$

**Lastly what is the length of line from tangent to tangent of these circles?**



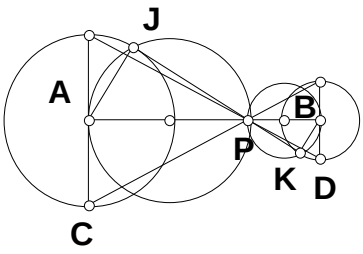

$GH := EOT_{LR} - EOT_{SR}$

$GH = 7.416$

**And what is the formula?**
**ETT "Tangent to Tangent"**

$$ETT := \sqrt{-\left(R_L - R_S + D\right)\cdot\left(R_L - R_S - D\right)}$$

$ETT = 7.416$




**I will now turn my attention to the point P, the internal similarity point.**

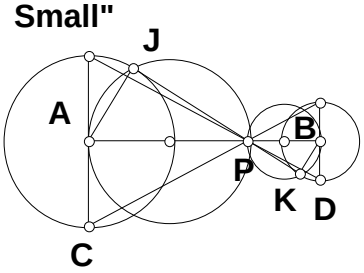
$$AP := \frac{AB \cdot AC}{AC + BD} \qquad AP = 6.4$$

**$IOR_L$ "Internal similarity point to center of Radius Large"**

$$IOR_L := D \cdot \frac{R_L}{R_L + R_S} \qquad IOR_L = 6.4$$

$BP := AB - AP \qquad BP = 1.6$

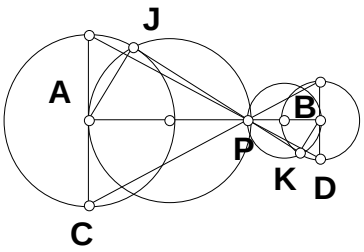
### $IOR_S$ "Internal similarity point to center of Radius Small"




$$IOR_S := D \cdot \frac{R_S}{R_L + R_S} \qquad IOR_S = 1.6$$

$AJ := AC \qquad BK := BD \qquad JP := \sqrt{AP^2 - AJ^2} \qquad JP = 4.996$

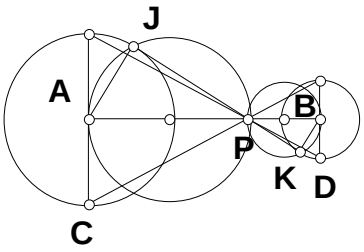
### $IOT_{LR}$ "Internal similarity point Origin to Tangent (Large Radius)"




$$IOT_{LR} := R_L \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{(R_L + R_S)}$$

$IOT_{LR} = 4.996 \qquad KP := \sqrt{BP^2 - BK^2} \qquad KP = 1.249$

### $IOT_{SR}$ "Internal similarity point Origin to Tangent (Small Radius)"

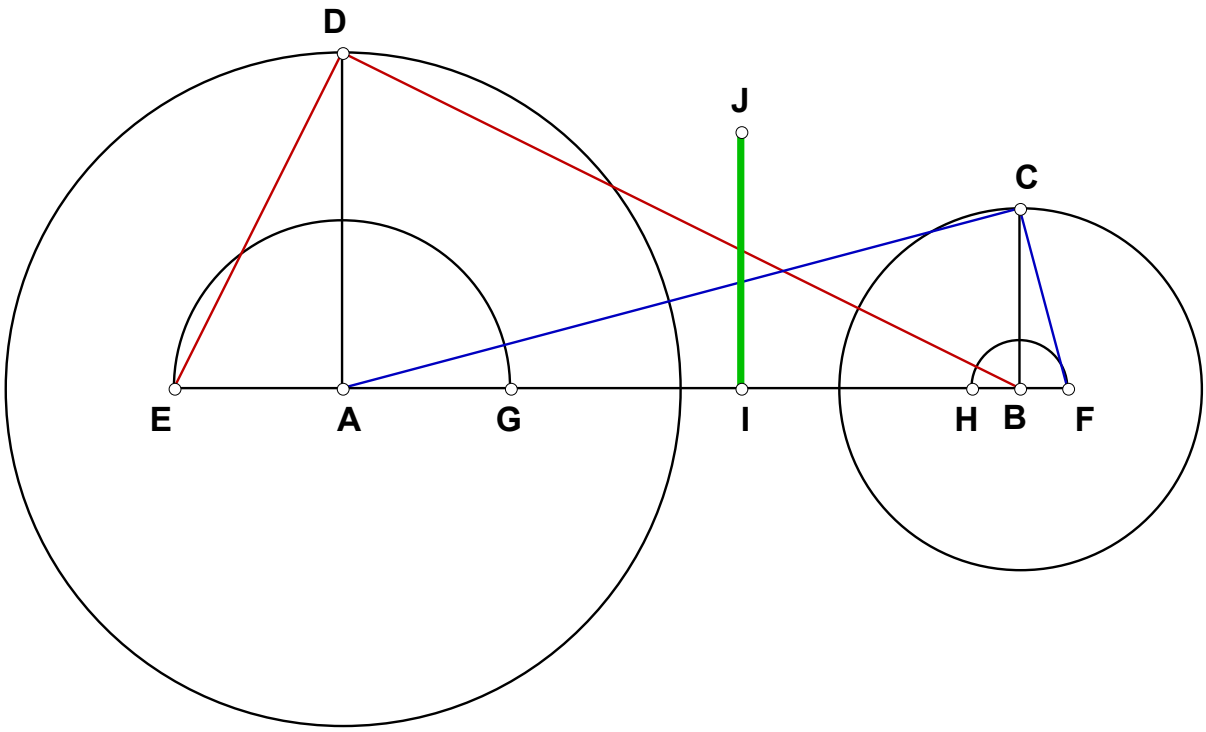



$$IOT_{SR} := R_S \cdot \frac{\sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)}}{R_L + R_S}$$

$IOT_{SR} = 1.249 \qquad JK := JP + KP \qquad JK = 6.245$

### ITT "Internal similarity point Tangent to Tangent"

$$ITT := \sqrt{-(R_L + R_S - D)\cdot(R_L + R_S + D)} \qquad ITT = 6.245$$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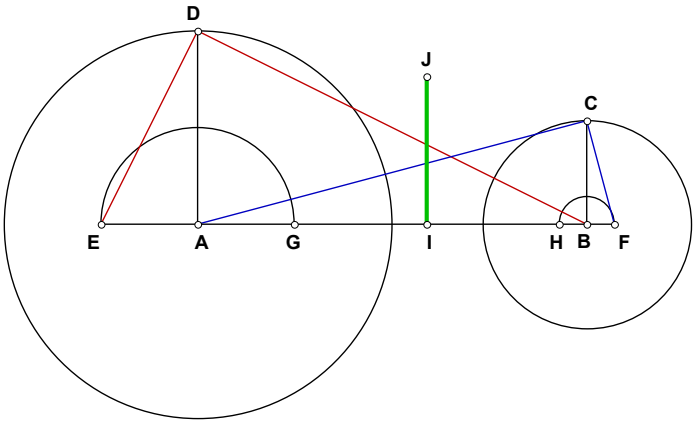
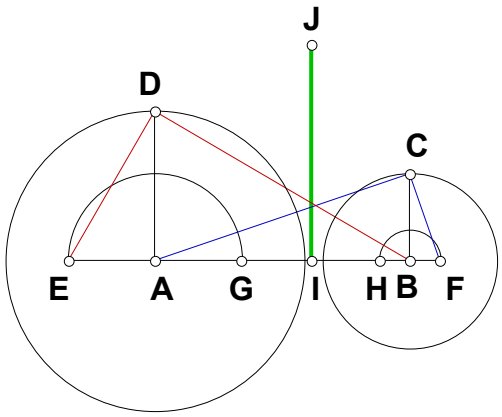
# 940427 EP




Given:

# 042794 The Chordal or Power Line of two Circles




**The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to this kind of process, so I took a couple of minuets (Bach) and developed my own method.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**
**Given two circles find their chordal or power line given just their radius and difference between their centers.**

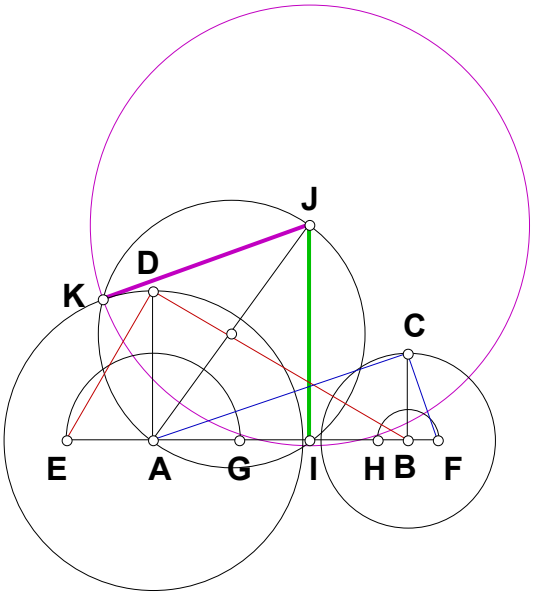
$AB := 1.323 \quad AD := .771 \quad BC := .448 \quad AE := \frac{AD^2}{AB} \quad BF := \frac{BC^2}{AB} \quad GH := AB - (AE + BF)$

$GI := \frac{GH}{2} \quad AI := AE + GI \quad BI := BF + GI$

$D := AB \quad R_1 := AD \quad R_2 := BC$

$$AI - \frac{\left(R_1^2 + D^2 - R_2^2\right)}{2 \cdot D} = 0$$

$$BI - \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D} = 0$$

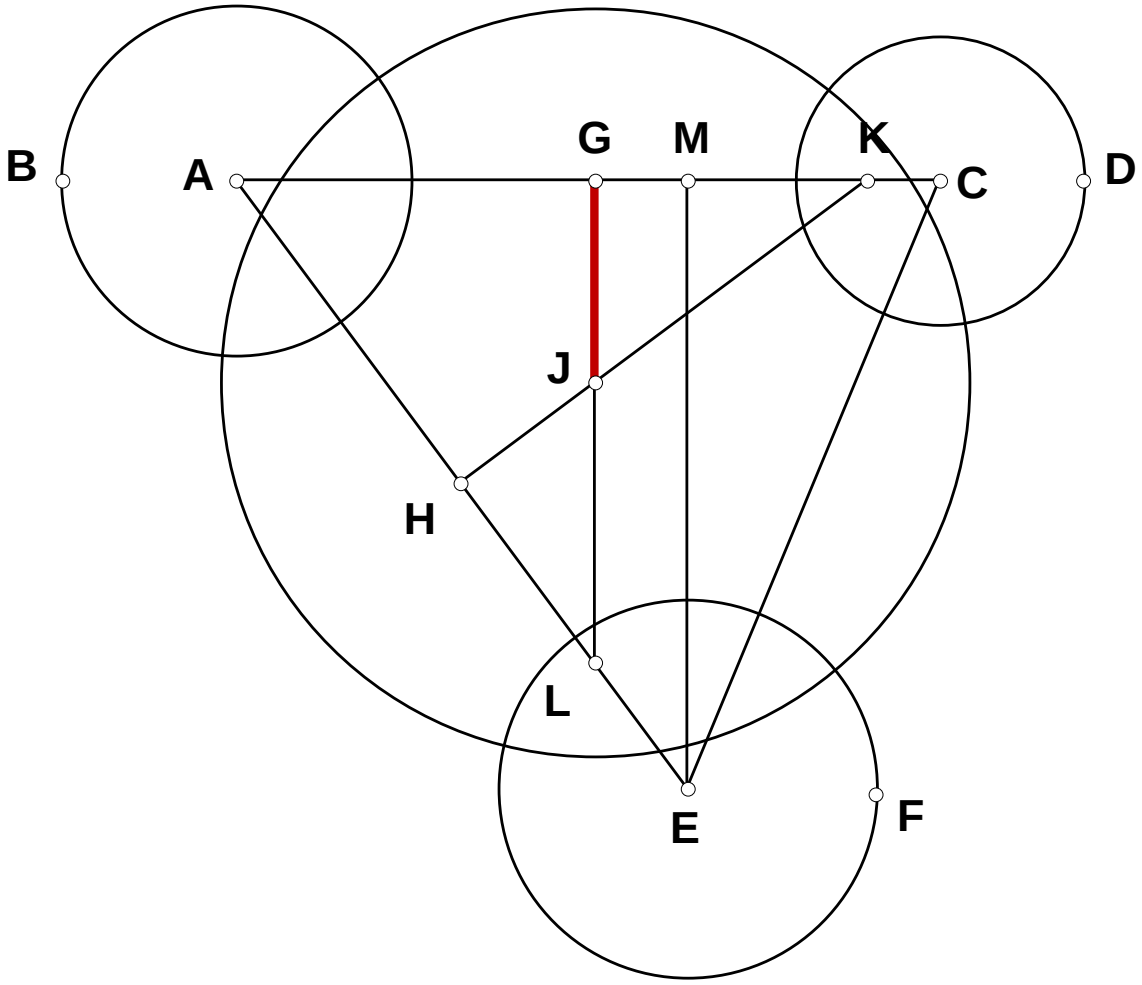



**If these equations look familiar, see 010893 The perpendicular of a Triangle would be on the powerline.**

$IJ := 1.112 \quad AJ := \sqrt{AI^2 + IJ^2} \quad AK := AD$

$JK := \sqrt{AJ^2 - AK^2} \quad P := IJ$

$$JK - \frac{\sqrt{R_1^4 - 2 \cdot R_1^2 \cdot D^2 - 2 \cdot R_1^2 \cdot R_2^2 + D^4 - 2 \cdot R_2^2 \cdot D^2 + R_2^4 + 4 \cdot P^2 \cdot D^2}}{2 \cdot D} = 0$$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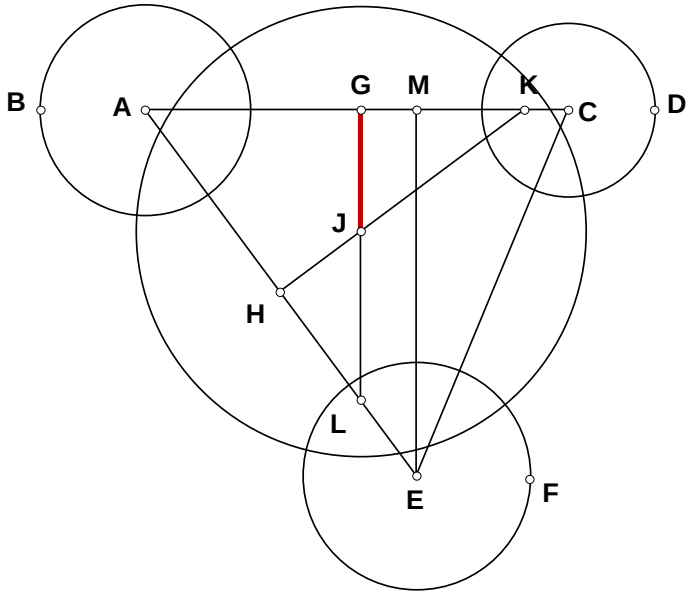
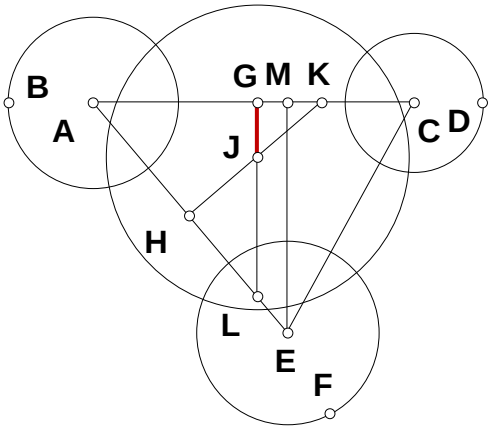



## 940428 EP

Given:

# 042894 Power Point

**Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.**




$AB := .438 \quad CD := .354 \quad EF := .471$

$AC := 1.667 \quad AE := 1.559 \quad CE := 1.357$

$R_1 := AB \quad R_2 := CD \quad R_3 := EF$

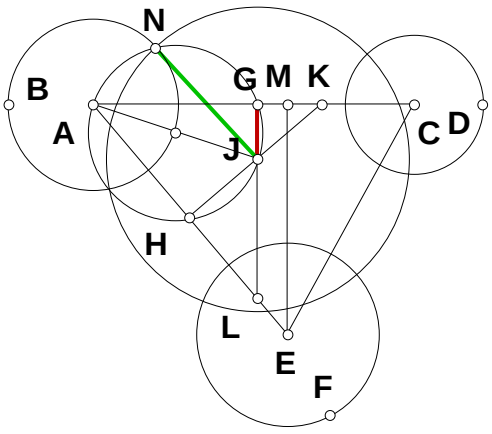
$D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{R_1^2 + D_1^2 - R_2^2}{2 \cdot D_1}$$

$$AH := \frac{R_1^2 + D_2^2 - R_3^2}{2 \cdot D_2}$$

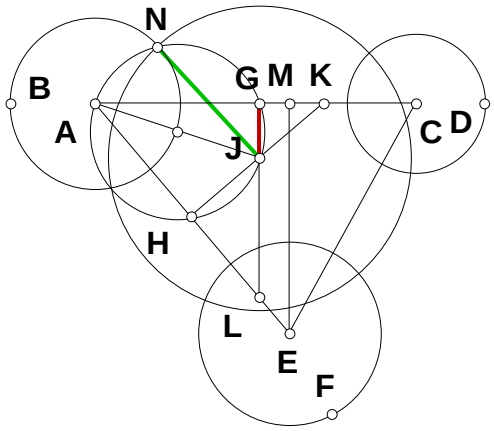
$$AM := \frac{D_2^2 + D_1^2 - D_3^2}{2 \cdot D_1} \qquad EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{1}{2} \cdot \frac{\left[\begin{array}{l}\left(D_2^2 \cdot D_1^2 - D_2^2 \cdot R_1^2 + D_2^2 \cdot R_2^2 + D_3^2 \cdot R_1^2 - 2 \cdot D_1^2 \cdot R_3^2\right) \ldots \\ + R_2^2 \cdot D_1^2 - D_3^2 \cdot R_2^2 - D_1^4 + D_3^2 \cdot D_1^2 + D_1^2 \cdot R_1^2\end{array}\right]}{D_1 \cdot \sqrt{-\left(D_2 + D_1 - D_3\right) \cdot \left(D_2 + D_1 + D_3\right) \cdot \left(D_2 - D_1 - D_3\right) \cdot \left(D_2 - D_1 + D_3\right)}} = 0$$

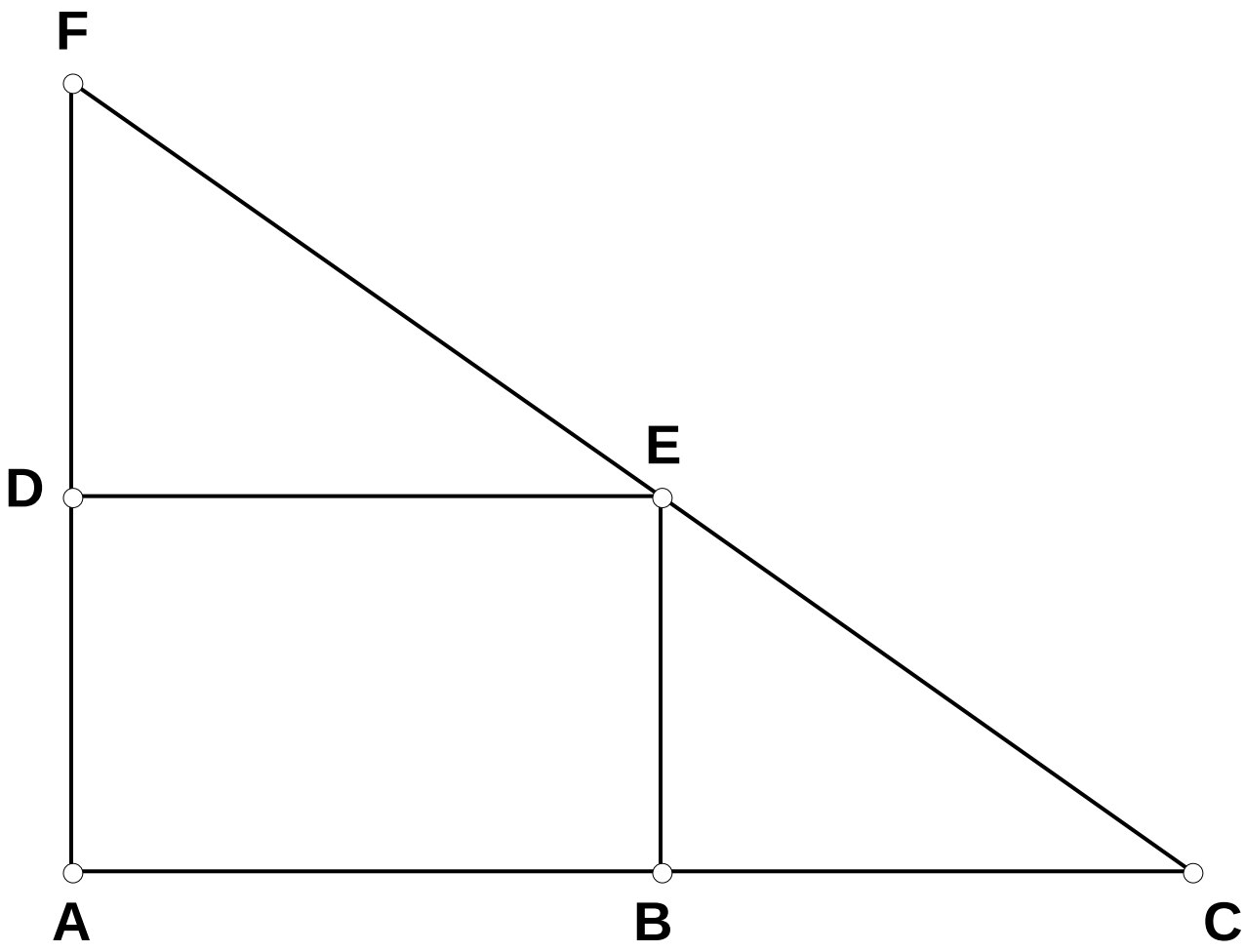



$$AJ := \sqrt{AG^2 + GJ^2} \qquad AN := AB \qquad JN := \sqrt{AJ^2 - AN^2}$$

$JN = 0.786$




$$JN - \sqrt{\frac{-\left[\begin{array}{l}\left(-D_3^{\,2}\cdot R_1^{\,2}\cdot D_1^{\,2}+R_1^{\,2}\cdot D_3^{\,4}+R_2^{\,4}\cdot D_2^{\,2}+D_1^{\,2}\cdot D_3^{\,2}\cdot D_2^{\,2}\right)\ldots\\ +\left(R_1^{\,4}\cdot D_3^{\,2}-R_2^{\,2}\cdot D_1^{\,2}\cdot D_2^{\,2}+R_2^{\,2}\cdot D_2^{\,4}+R_1^{\,2}\cdot R_2^{\,2}\cdot D_1^{\,2}\right)\ldots\\ +\left(-D_2^{\,2}\cdot D_1^{\,2}\cdot R_3^{\,2}-D_1^{\,2}\cdot R_3^{\,2}\cdot R_1^{\,2}-D_1^{\,2}\cdot R_3^{\,2}\cdot D_3^{\,2}-D_1^{\,2}\cdot R_3^{\,2}\cdot R_2^{\,2}\right)\ldots\\ +\left(-D_2^{\,2}\cdot R_1^{\,2}\cdot D_3^{\,2}+R_3^{\,2}\cdot D_3^{\,2}\cdot R_2^{\,2}-R_1^{\,2}\cdot R_2^{\,2}\cdot D_2^{\,2}+D_1^{\,4}\cdot R_3^{\,2}\right)\ldots\\ +\left(-R_1^{\,2}\cdot R_2^{\,2}\cdot D_3^{\,2}-D_2^{\,2}\cdot R_2^{\,2}\cdot D_3^{\,2}-D_2^{\,2}\cdot R_2^{\,2}\cdot R_3^{\,2}-D_3^{\,2}\cdot R_1^{\,2}\cdot R_3^{\,2}\right)\ldots\\ +D_2^{\,2}\cdot R_1^{\,2}\cdot R_3^{\,2}+D_1^{\,2}\cdot R_3^{\,4}\end{array}\right]}{\left(D_2-D_3+D_1\right)\cdot\left(D_2+D_3+D_1\right)\cdot\left(D_1-D_2+D_3\right)\cdot\left(-D_2-D_3+D_1\right)}} = 0$$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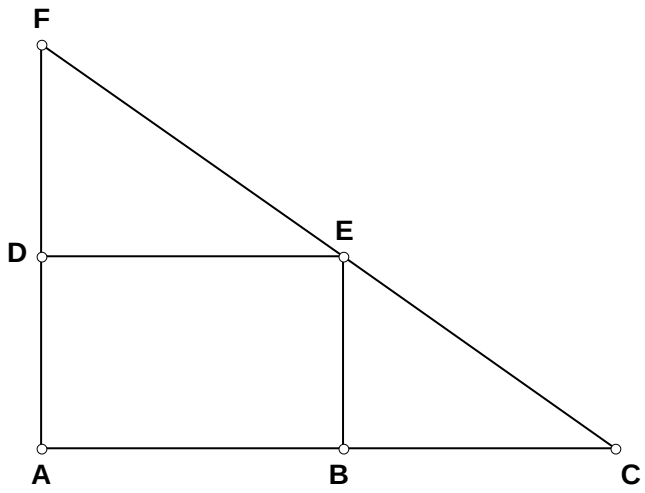
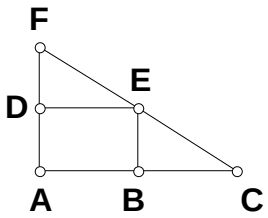



## 940429 EP

Given:

**042994**

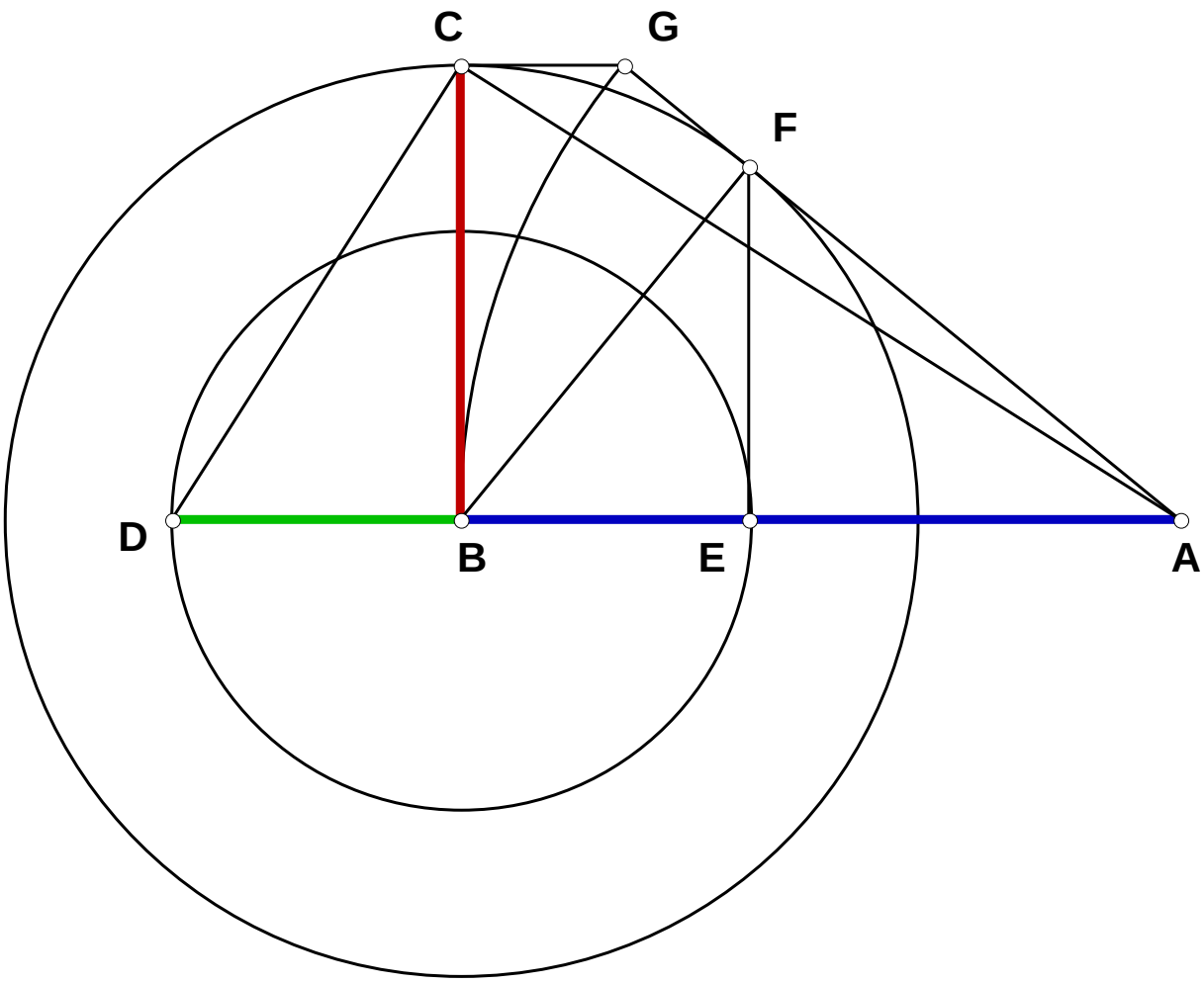
## Given AB, AF, BE, what is EF?




$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$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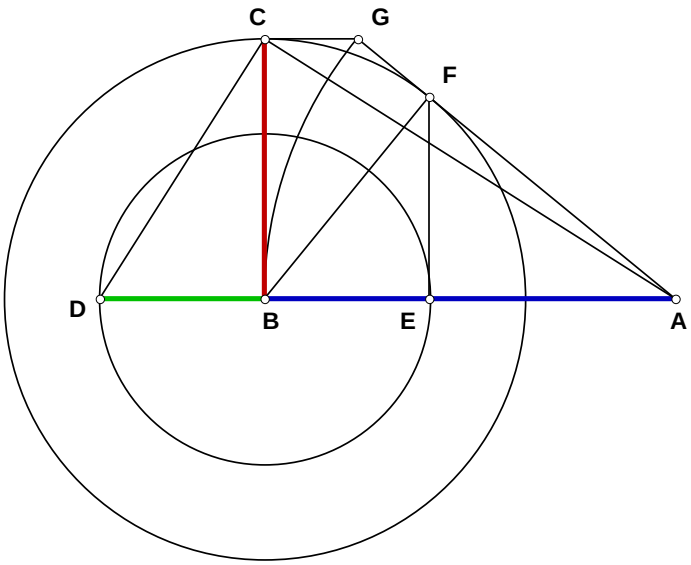
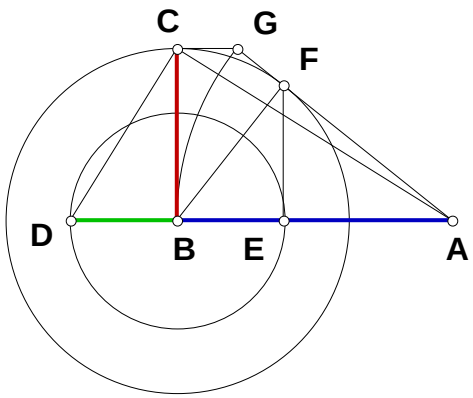
# 940430 EP




Given:

# 043094 Division $N^2$

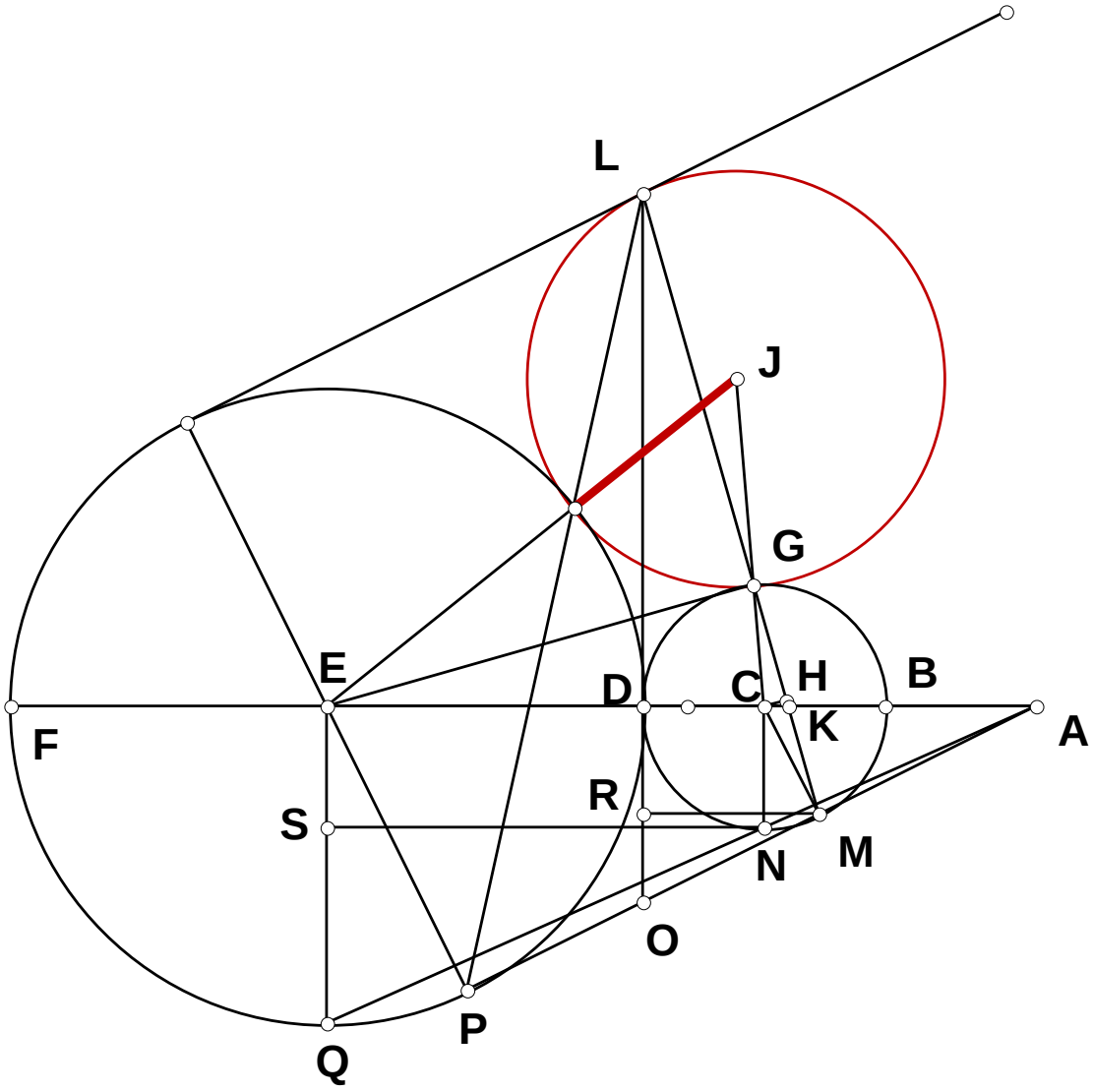



$N_1 := 5$ $N_2 := 2$

$AB := N_1$ $BC := N_2$ $AC := \sqrt{AB^2 + BC^2}$

$CD := \frac{BC \cdot AC}{AB}$ $BD := \sqrt{CD^2 - BC^2}$

$\frac{N_2^2}{N_1} - BD = 0$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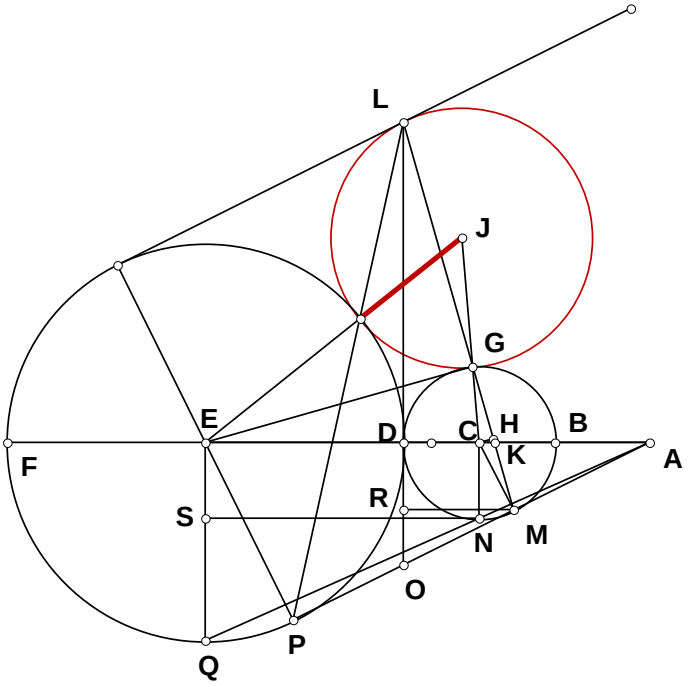
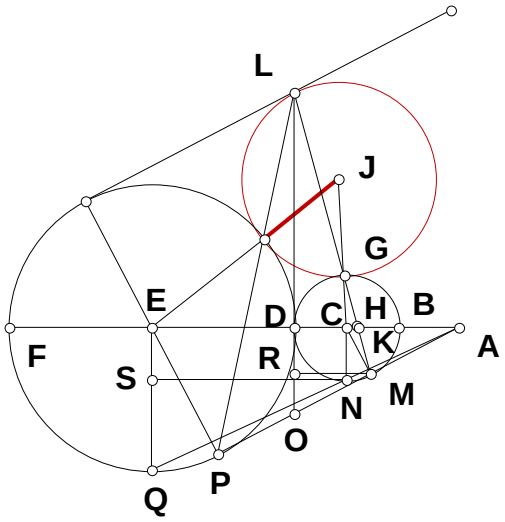
# 940501 EP




Given:

## 050194 Two Circles And A Parallel




**Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.**

$R_1 := 3 \qquad R_2 := 2$

$DE := R_1 \qquad BC := R_2 \qquad CN := BC$

$EQ := DE \quad CD := BC \quad CE := CD + DE$

$ES := CN \quad NS := CE \quad SQ := EQ - ES$

$AE := \frac{NS \cdot EQ}{SQ} \qquad AD := AE - DE \quad EP := DE$

$AP := \sqrt{AE^2 - EP^2} \qquad DO := \frac{EP \cdot AD}{AP}$

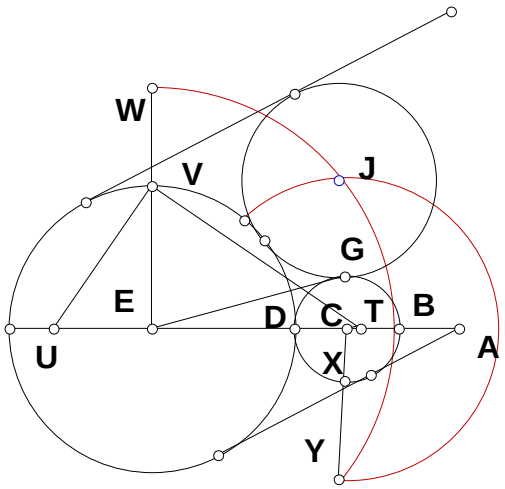
$DL := \frac{DO \cdot DE}{CD} \qquad AC := AD - CD \qquad AM := \frac{AP \cdot AC}{AE} \qquad AO := \frac{AE \cdot AD}{AP} \qquad MO := AO - AM$

$MR := \frac{AD \cdot MO}{AO} \quad RO := \frac{DO \cdot MR}{AD} \qquad DR := DO - RO \quad LR := DR + DL \quad ML := \sqrt{MR^2 + LR^2}$

$DK := \frac{MR \cdot DL}{LR} \qquad CK := DK - CD \quad CH := \frac{LR \cdot CK}{ML} \quad CM := BC \quad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \quad GL := ML - MG \quad GJ := \frac{CM \cdot GL}{MG}$

**The Algebraic name for GJ suggests a simpler method of construction.**



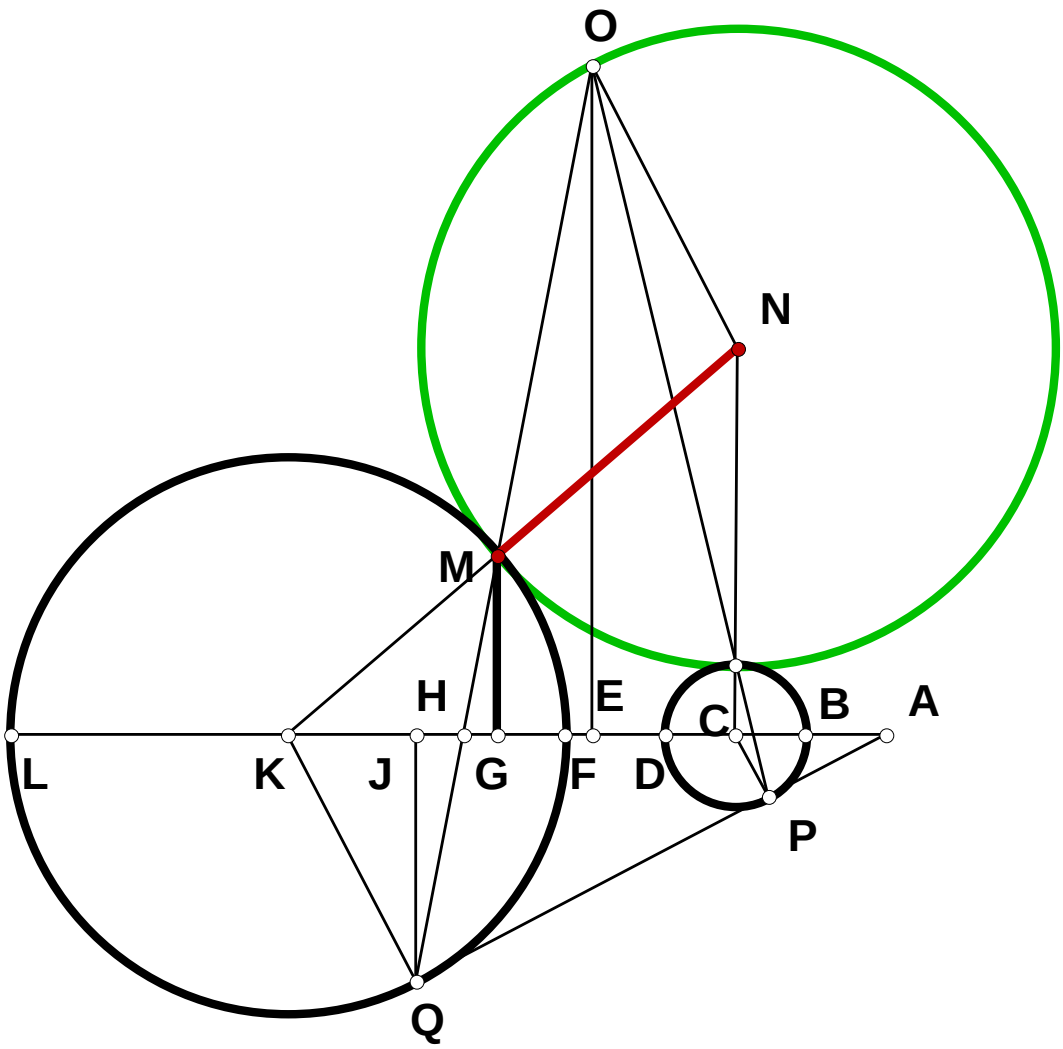

$R_3 := \frac{R_1^2}{4 \cdot R_2} \qquad GJ - R_3 = 0$

$ET := 4 \cdot BC \quad EV := DE \quad EU := \frac{DE^2}{ET}$

$VW := EU \quad XY := EU \quad EV := DE \quad CX := BC$

$EW := EV + VW \quad CY := CX + XY$

$GJ_2 := EU \quad GJ - GJ_2 = 0$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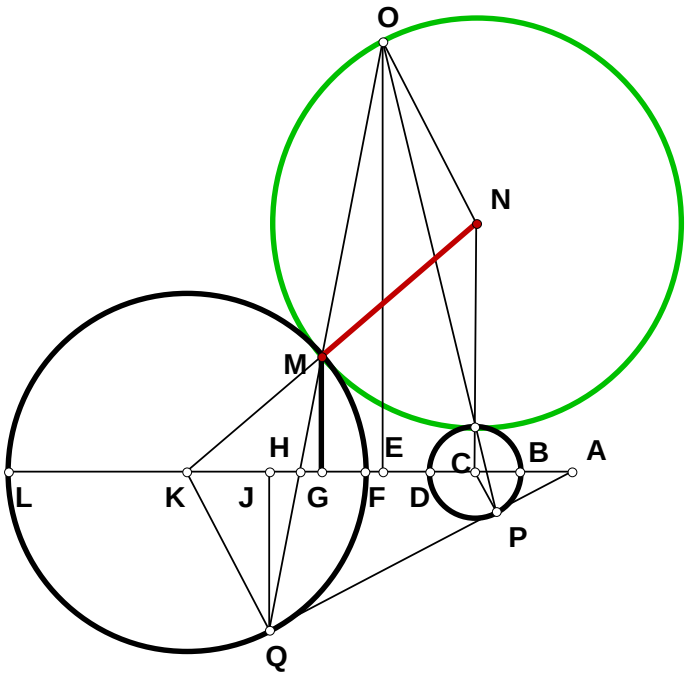
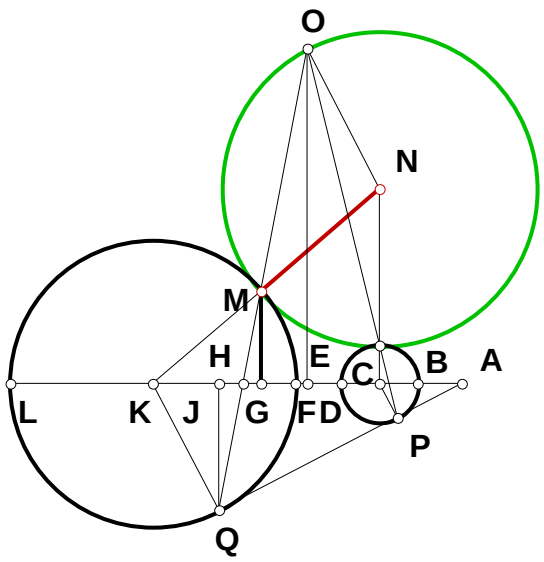
# 940504 EP




Given:

# 050494 Two Circles And A Tangent




**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**

$R_1 := 1 \quad R_2 := .5 \quad D := 2 \quad N := 2$

$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK$

$$AK := \frac{D \cdot R_1}{R_1 - R_2} \qquad EK := \frac{R_1^2 + D^2 - R_2^2}{2 \cdot D}$$

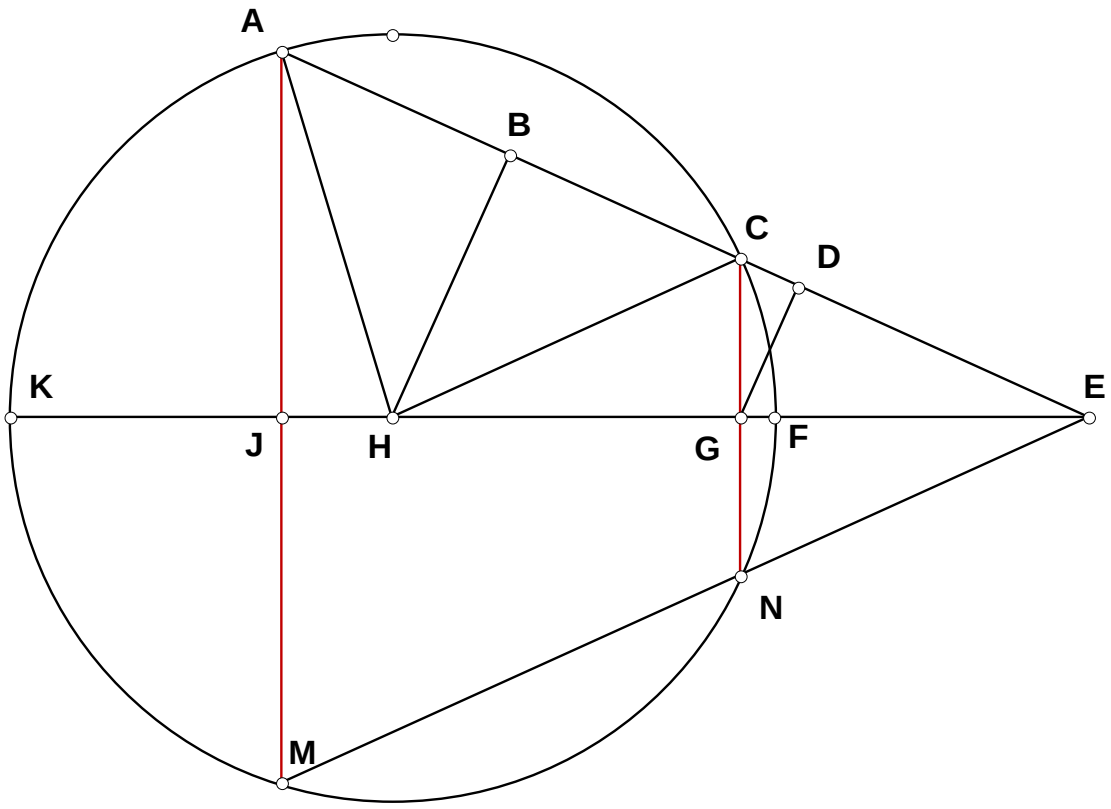
$$AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$$

$$FG := \frac{FL}{N} \quad GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK} \quad AF := AK - FK \quad FJ := AJ - AF$$

$$JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG \quad QM := \sqrt{(JQ + GM)^2 + GJ^2} \quad GH := \frac{GJ \cdot GM}{JQ + GM}$$

$$HM := \frac{QM \cdot GM}{JQ + GM} \quad EF := EK - FK \quad EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \quad MO := HO - HM$$

$$KM := FK \quad MN := \frac{KM \cdot MO}{QM} \qquad MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2\,N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$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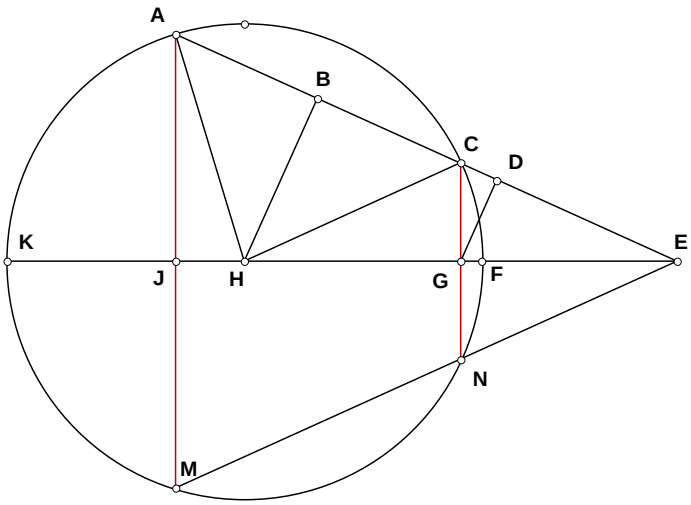
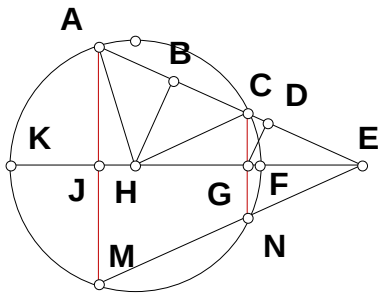
# 940506 EP




Given:

# 050694 A Ratio In Trisection




**What is AJ to CG?**

$N := 2.423$ $FH := 1$ $CE := FH$

$CG := \frac{FH}{N}$ $EG := \sqrt{CE^2 - CG^2}$ $CD := \frac{CG^2}{CE}$

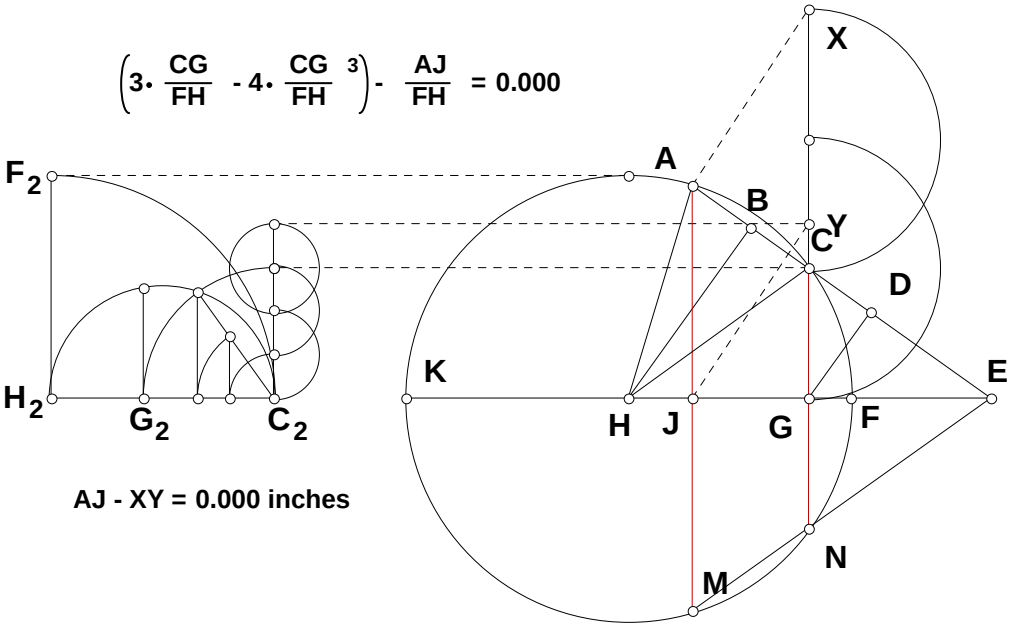
$DG := \sqrt{CG^2 - CD^2}$ $EH := 2 \cdot EG$ $BH := \frac{DG \cdot EH}{EG}$ $CH := FH$

$BC := \sqrt{CH^2 - BH^2}$ $AC := 2 \cdot BC$ $AE := AC + CE$ $AJ := \frac{CG \cdot AE}{CE}$ $3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0$

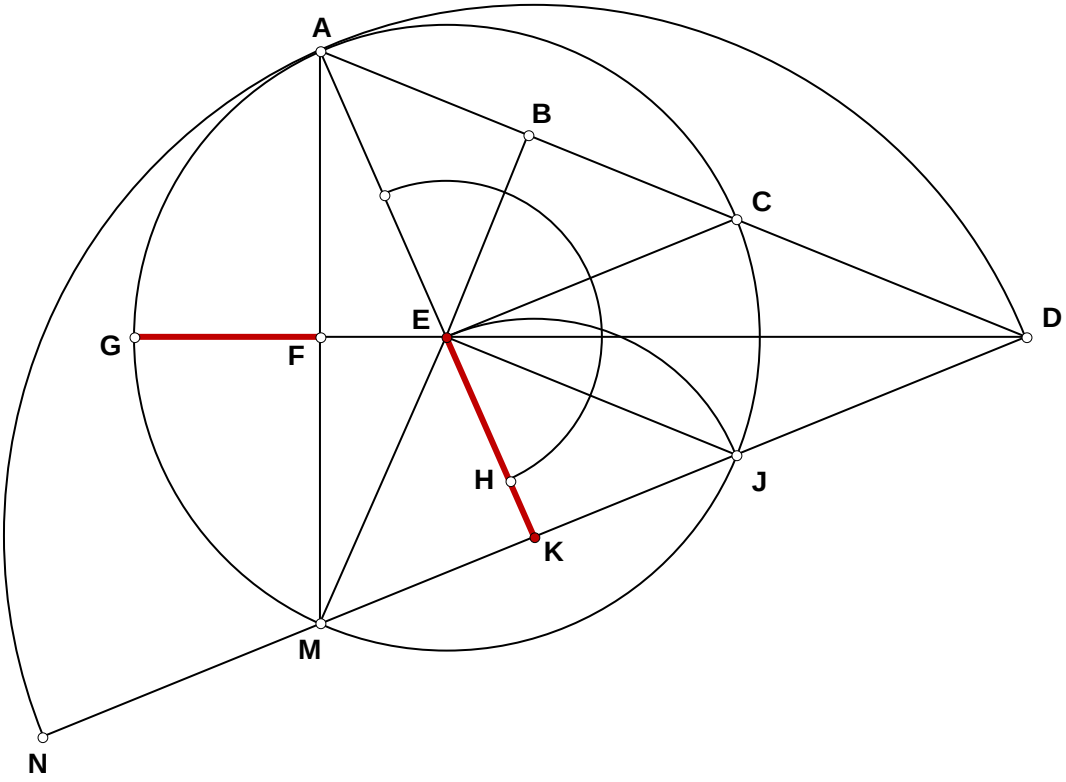
$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$ $AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$ $AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

**The resultant equation suggests this construction.**

$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG}{FH}^3\right) - \frac{AJ}{FH} = 0.000$




AJ - XY = 0.000 inches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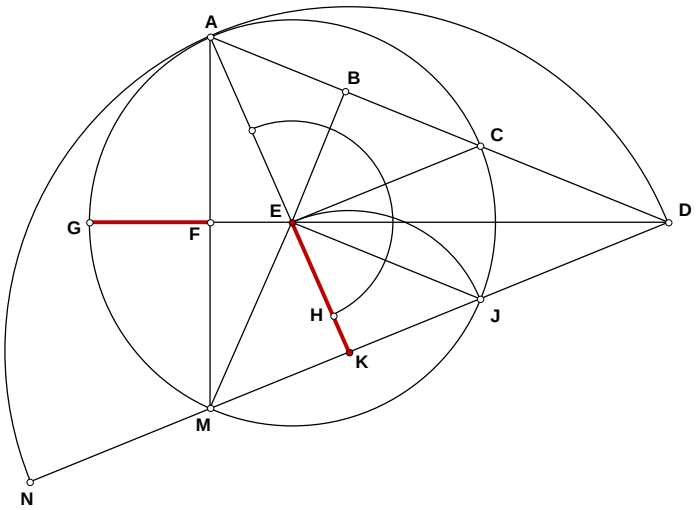
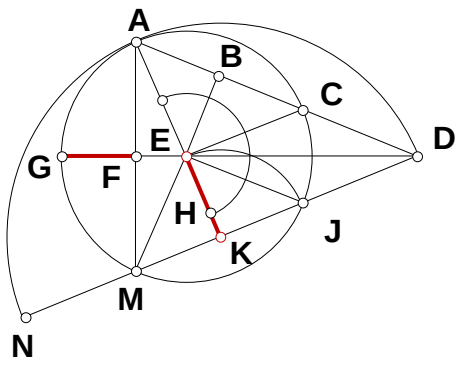
# 940507 EP




Given:

# 050794 A Trisection Ratio

## In trisection, what is the ratio of FG/EK?




$N := 2$

$AE := 1 \quad EH := \frac{AE}{2} \quad HK := AE \cdot N \quad AK := AE + EH + HK$

$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$

$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$

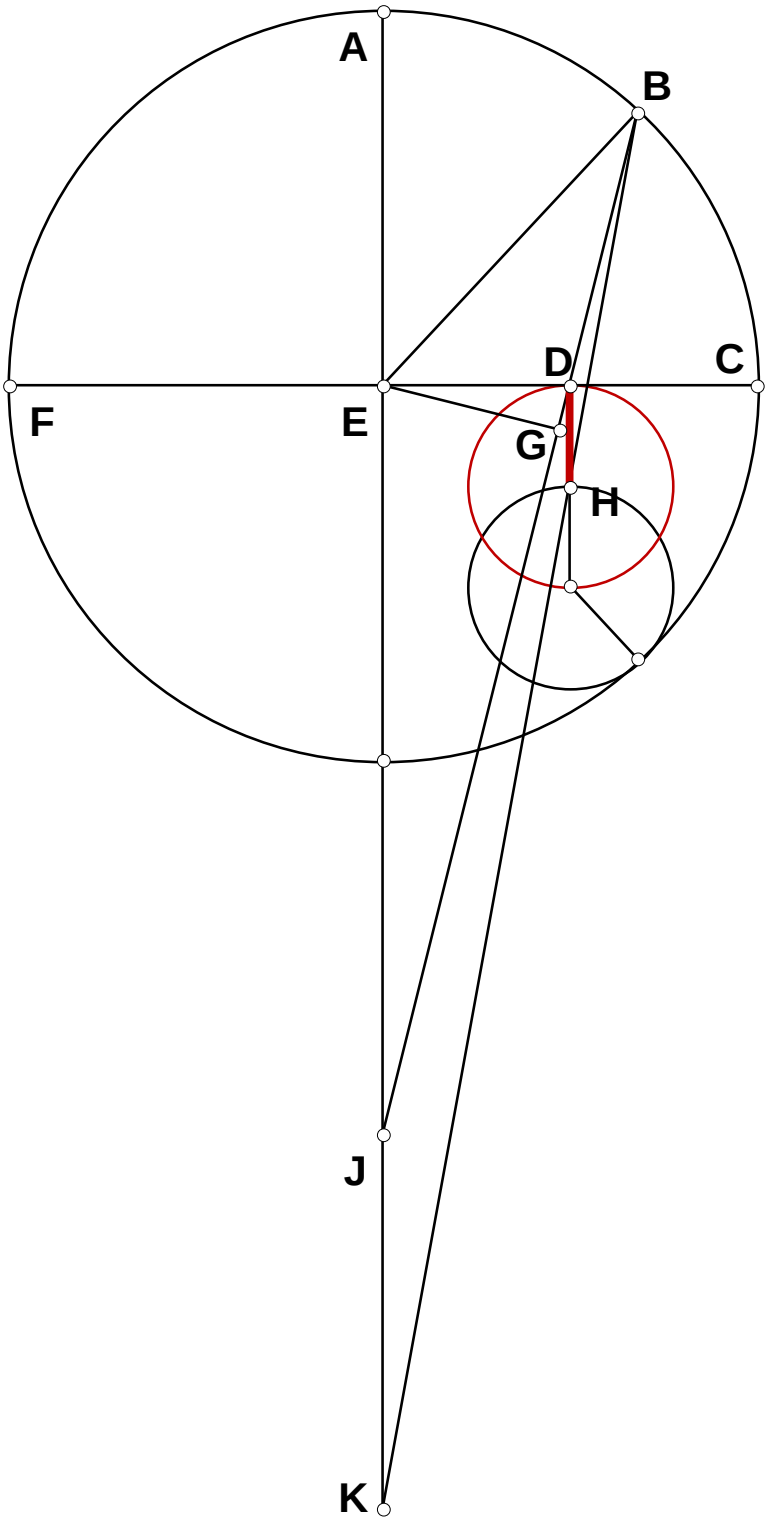
$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$

$EG := AE \quad DG := DE + EG \quad FG := DG - DF$

## Algebraic Names,

$$FG - \left[1 + \frac{(N+1)\cdot(2\cdot N-1)}{(2N+1)\cdot\sqrt{(N+1)\cdot(2N+1)}}\right] = 0 \qquad EK - \frac{1}{2} - N = 0$$

$$\frac{FG}{EK} - 2\cdot\frac{\left[2\cdot N\cdot\sqrt{(N+1)\cdot(2\cdot N+1)} + \sqrt{(N+1)\cdot(2\cdot N+1)} + (N+1)\cdot(2\cdot N-1)\right]}{\left[(2\cdot N+1)^2\cdot\sqrt{(N+1)\cdot(2\cdot N+1)}\right]} = 0$$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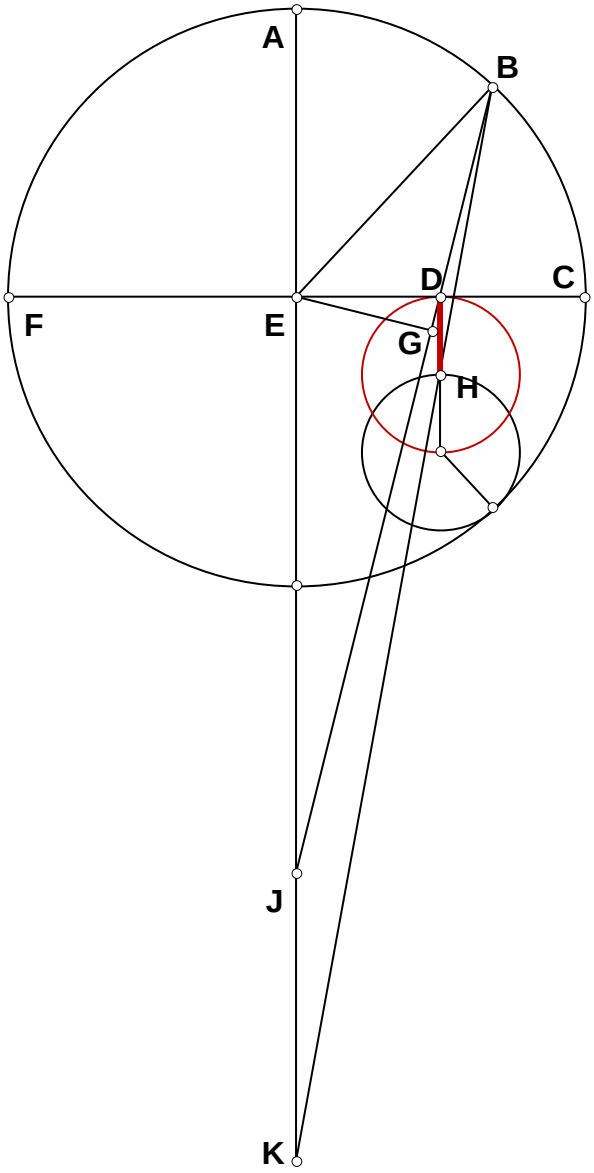
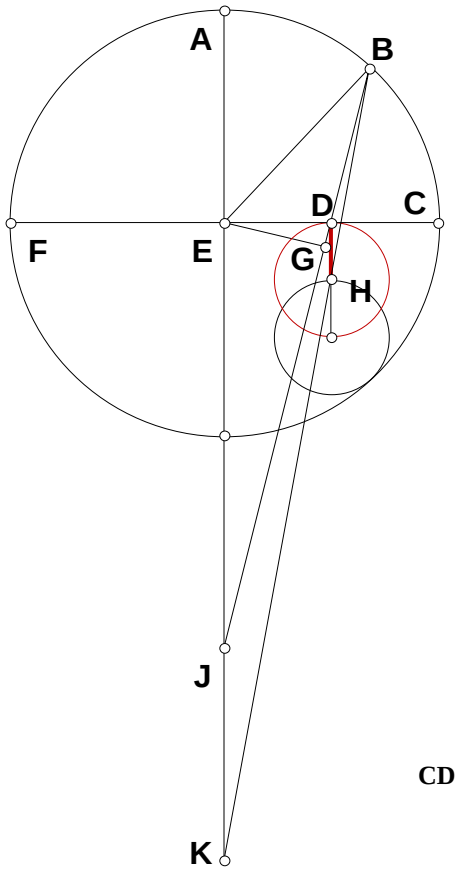



## 940516A EP

Given:

## 051694A Tangent Diameter and Circles

**Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.**




$$CF := 1 \qquad CE := \frac{CF}{2} \qquad N_1 := 2 \qquad N_2 := 3$$

$$CD := \frac{CE}{N_1} \qquad DE := CE - CD \qquad EJ := CE \cdot N_2$$

$$DJ := \sqrt{DE^2 + EJ^2} \qquad JG := \frac{EJ^2}{DJ} \qquad BE := CE$$

$$EG := \sqrt{EJ^2 - JG^2} \qquad BG := \sqrt{BE^2 - EG^2} \qquad BJ := BG + JG$$

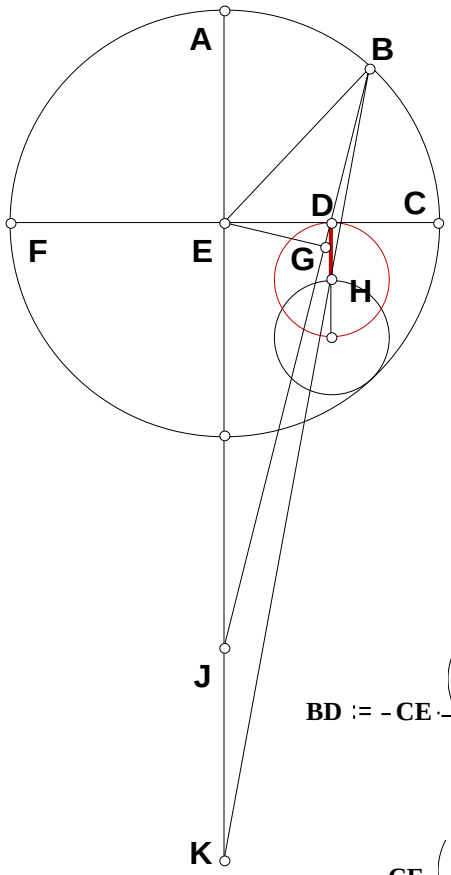
$$JK := CE \qquad BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$$

### Algebraic Names:

$$CD := \frac{CE}{N_1} \qquad DE := CE \cdot \frac{(N_1 - 1)}{N_1} \qquad EJ := CE \cdot N_2$$

$$DJ := CE \cdot \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{N_1}$$

$$JG := CE \cdot \frac{N_1 \cdot N_2^2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} \qquad EG := CE \cdot \frac{(N_1 - 1) \cdot N_2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

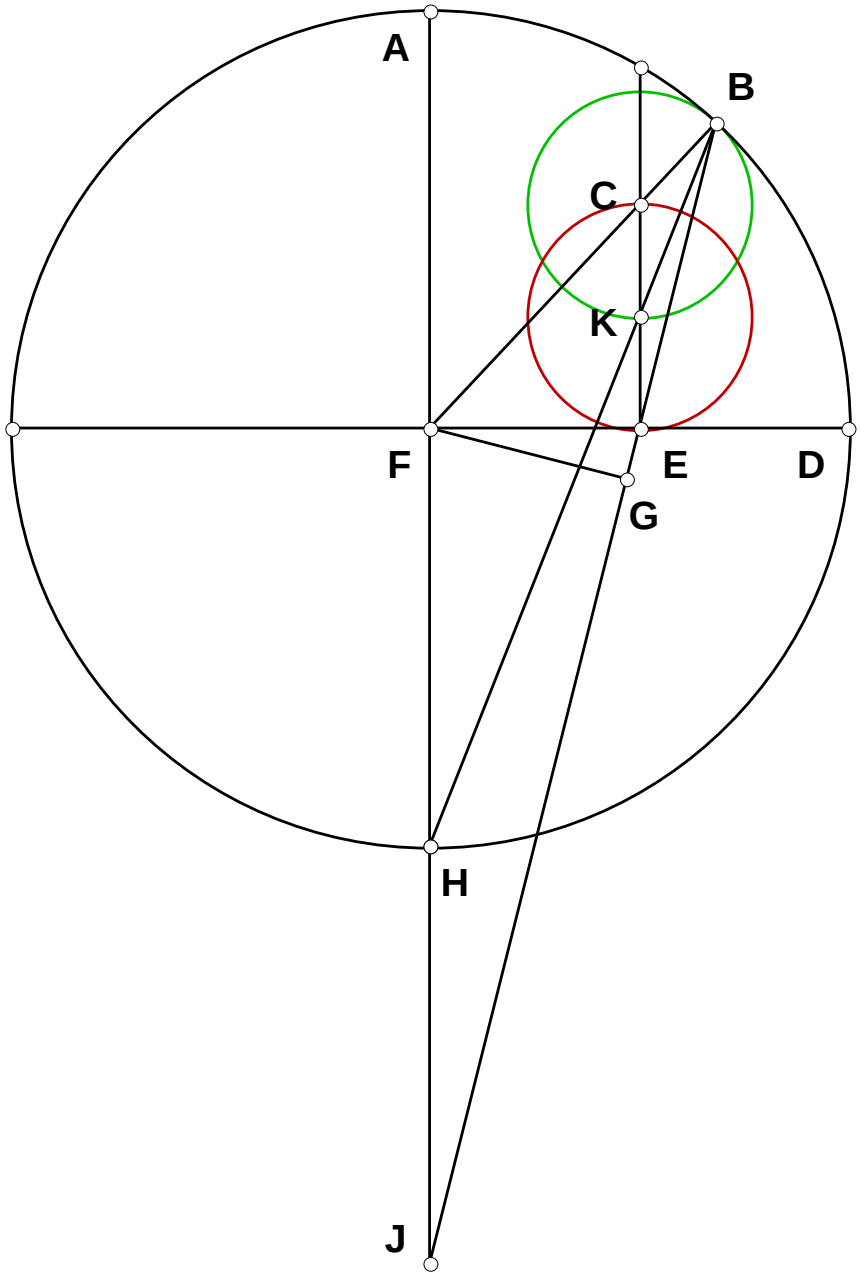



$$BG := CE \cdot \sqrt{\frac{\left(N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2\right)}{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}}$$

$$BJ := CE \cdot \frac{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

$$BD := -CE \cdot \frac{\left(-N_1 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_1^2 - 2 \cdot N_1 + 1\right)}{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1\right)}$$

$$DH := \frac{CE \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1\right)}{N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}$$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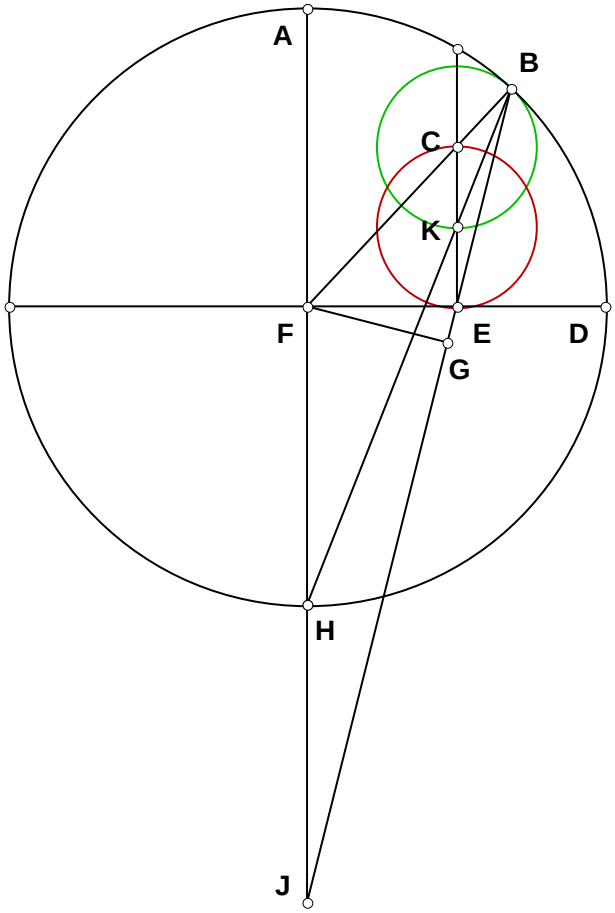
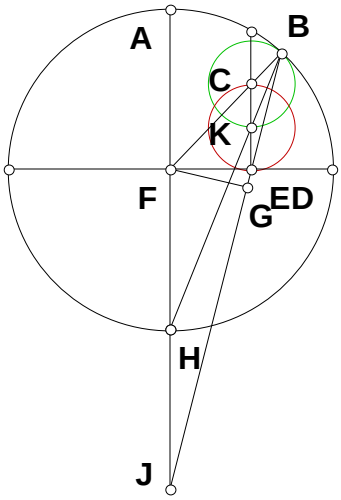
# 940516b EP




Given:

## 05/16/94B Tangent Diameter and Circles




**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.**

$N_1 := 2 \quad N_2 := 2 \quad AF := 1$

$DF := AF \quad DE := \frac{DF}{N_1} \quad AJ := AF \cdot N_2$

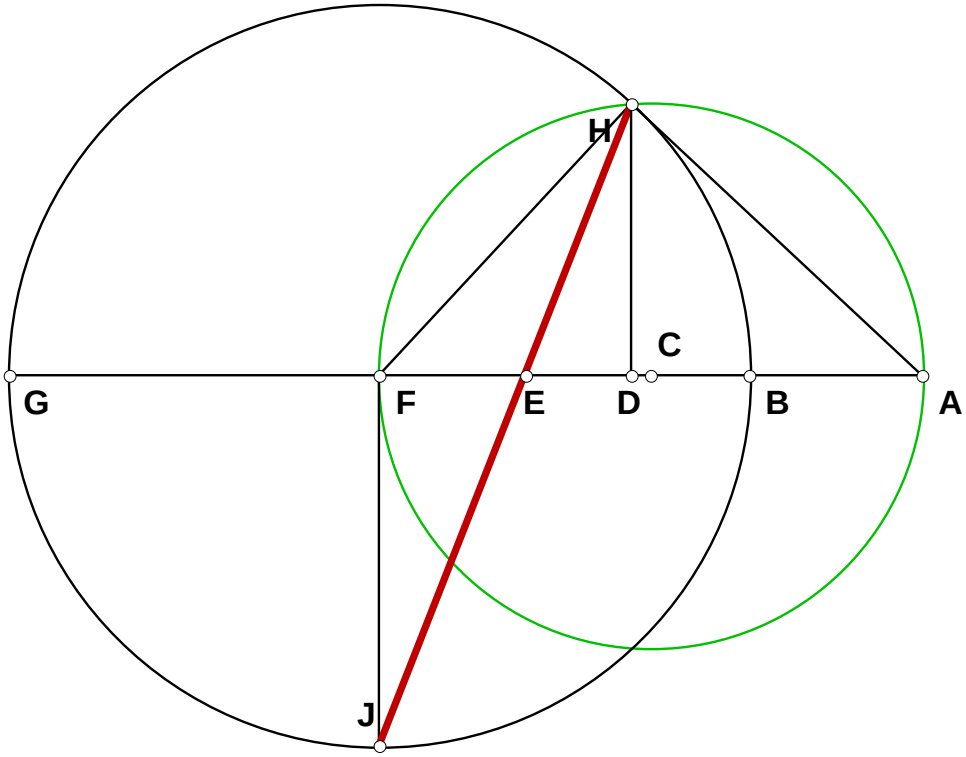
$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$

$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad KE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot KE\right]^2} \quad BC - KE = 0 \quad KE = 0.375$

$$1 - \frac{2 \cdot N_1^2 - 2 \cdot N_1 + 1 + N_1^2 \cdot N_2^2 - 2 \cdot N_1^2 \cdot N_2}{N_1 \cdot \left(\sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 4 \cdot N_1 \cdot N_2 - N_2^2 + 2 \cdot N_2} + N_1 + N_1 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2\right)} - BC = 0$$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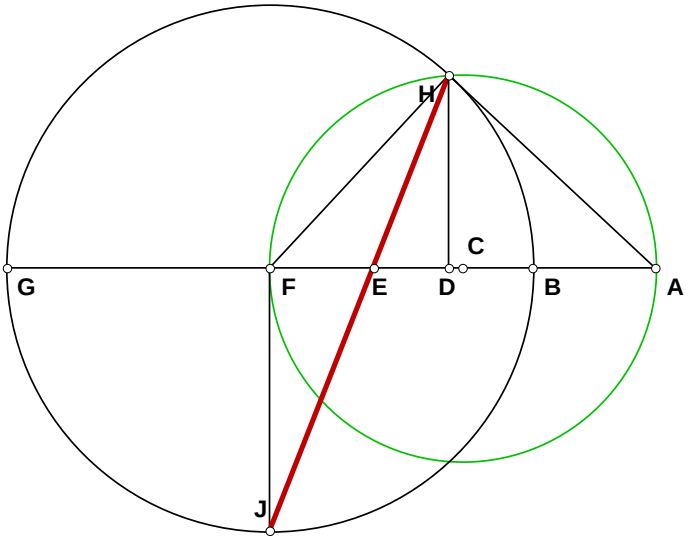
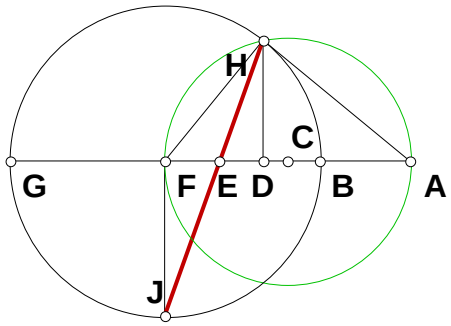
## 941027 EP




Given:

# 102794 Trivial Method Square Root

## AE is the square root of AB x AG.




$N := 5$ $AB := 1$

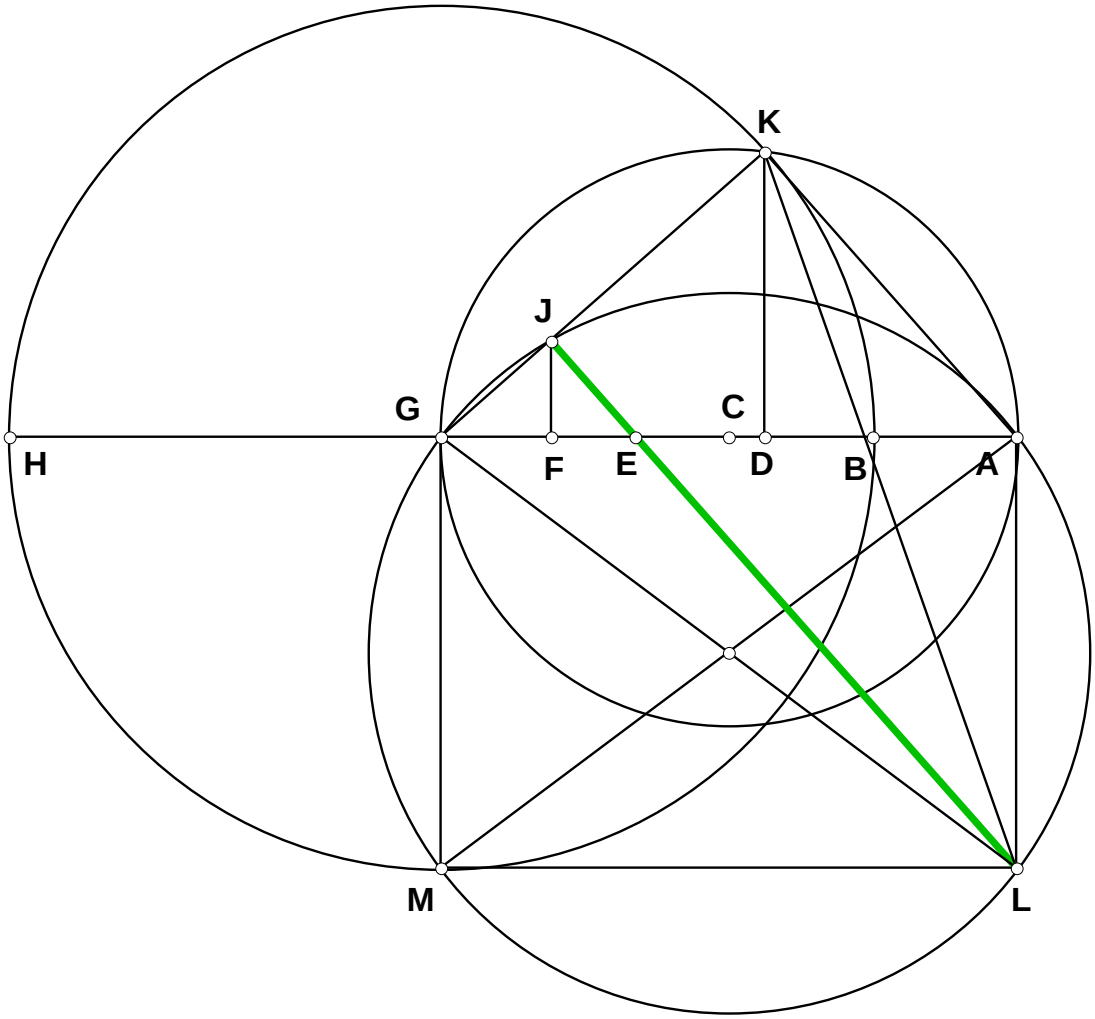
$AG := AB \cdot N$ $BG := AG - AB$ $BF := \frac{BG}{2}$

$AF := AB + BF$ $FH := BF$ $DF := \frac{FH^2}{AF}$ $AD := AF - DF$

$BD := AD - AB$ $DG := BG - BD$ $FJ := BF$

$DH := \sqrt{BD \cdot DG}$ $DE := \frac{DF \cdot DH}{DH + FJ}$ $AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$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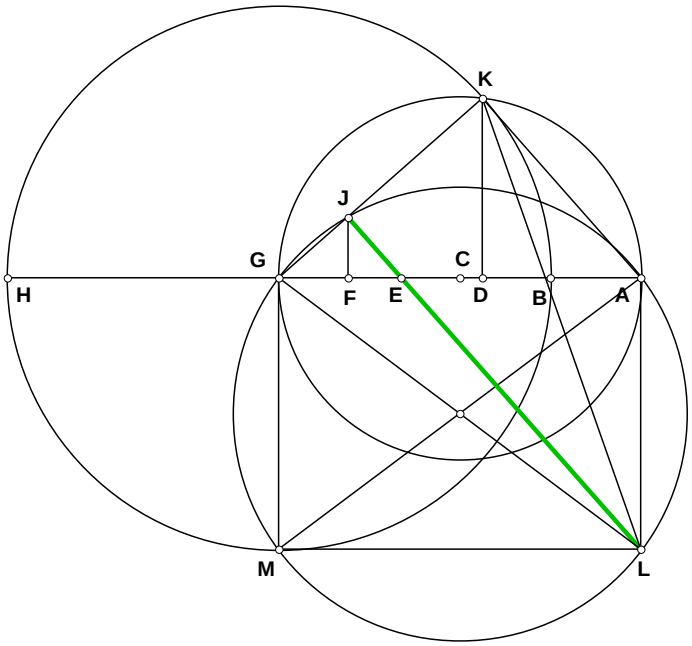
# 941028 EP

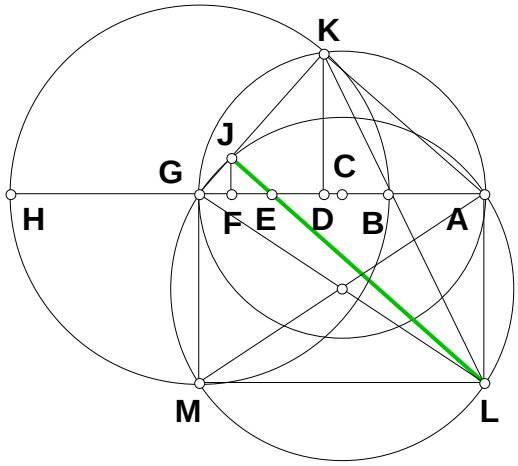



Given:

# 102894 Trivial Method Square Root

## AE is the square root of AB x AH.

$N := 5 \quad AB := 1 \quad AH := AB \cdot N \quad BH := AH - AB$




$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

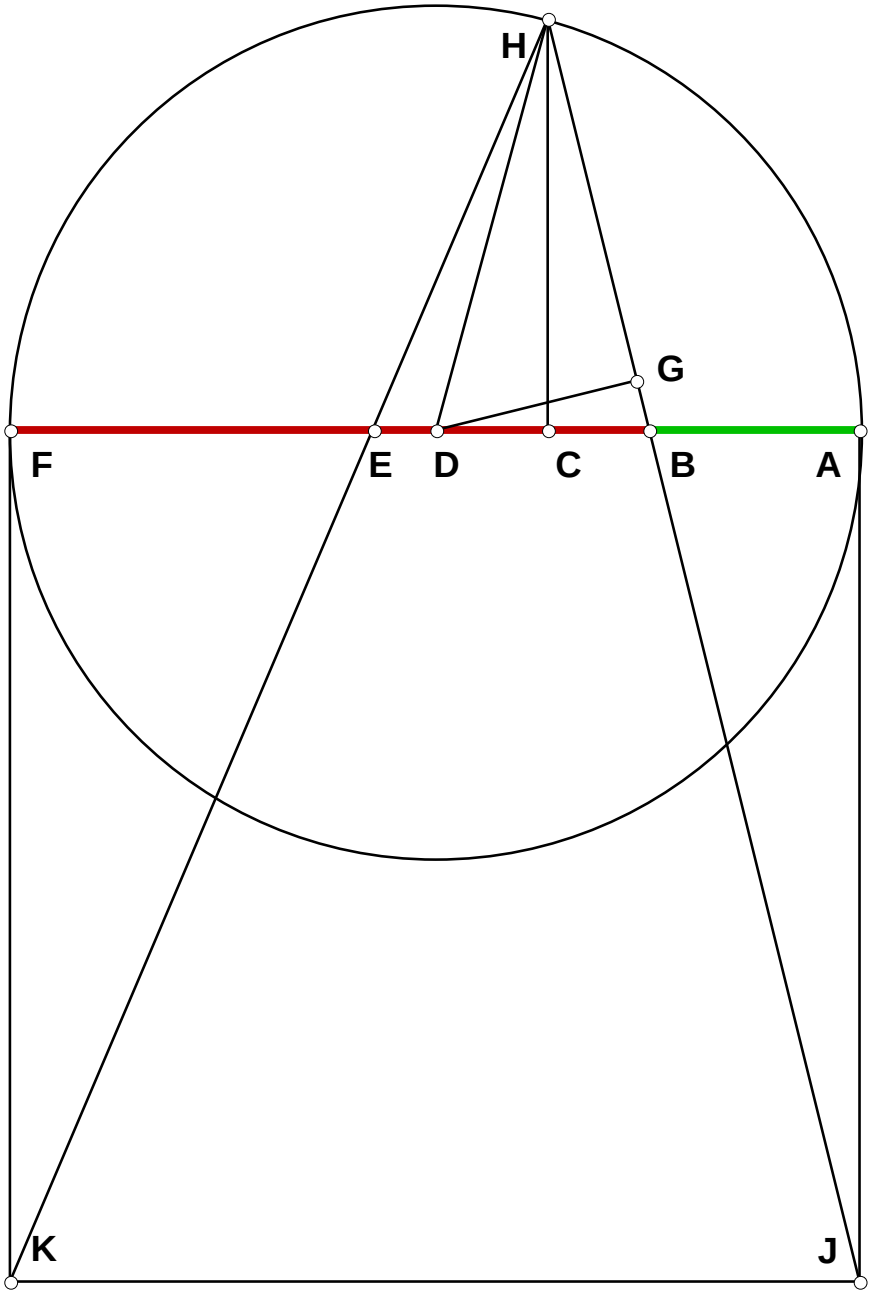
$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \qquad JL := \sqrt{GL^2 - GJ^2}$

$FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$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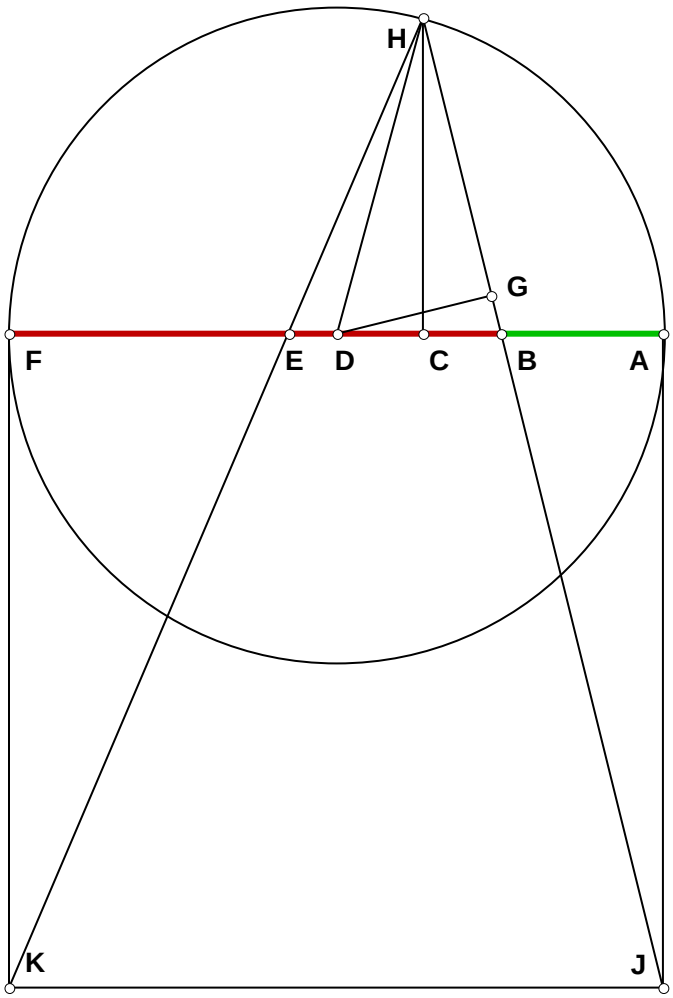
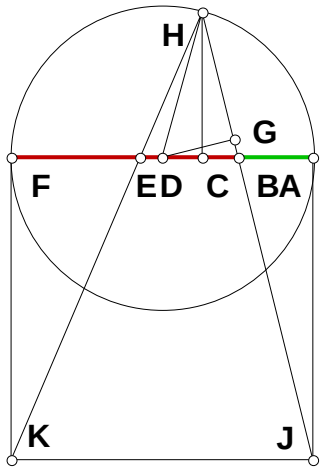



## 941031 EP

Given:

# 103194 Square Root of a Segment

**Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.**




$N := 22 \qquad AB := 1$

$AF := AB \cdot N \qquad BF := AF - AB \qquad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

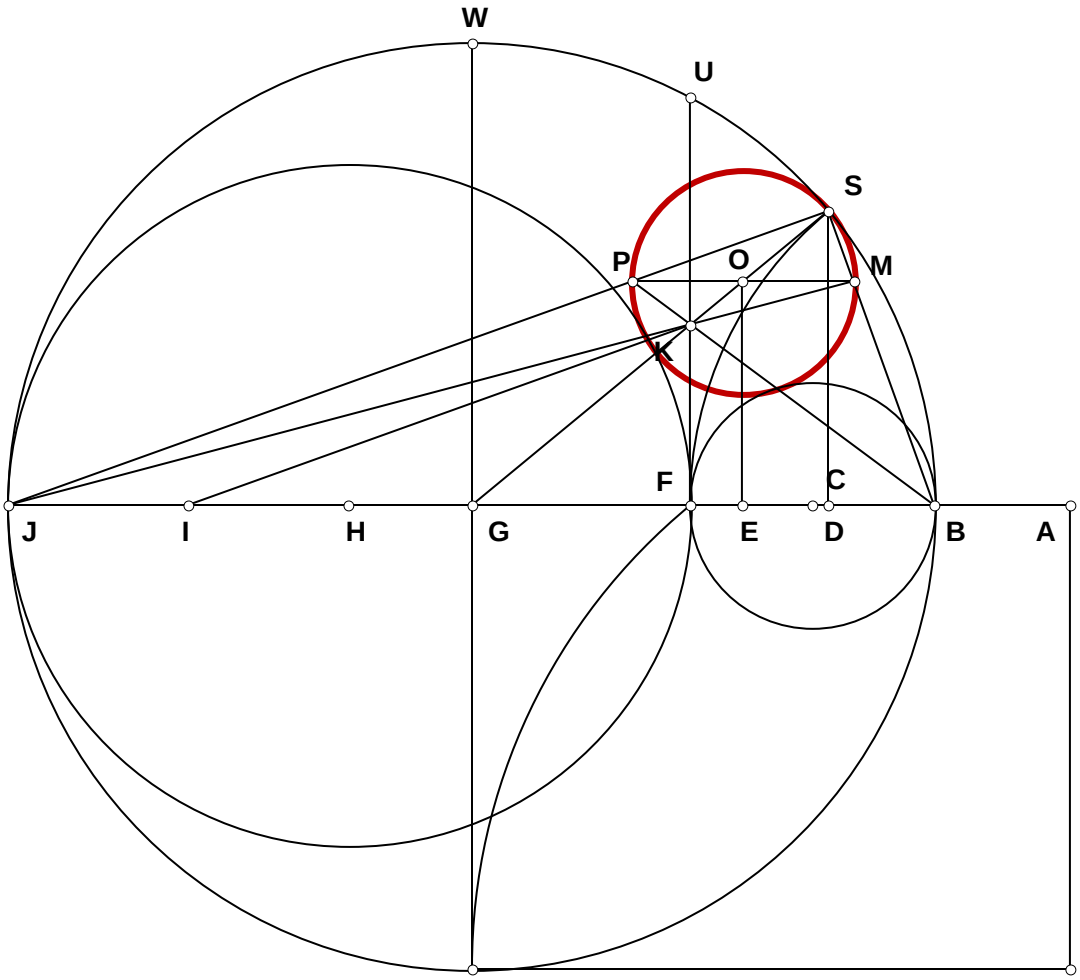
$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \frac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \frac{CF \cdot CH}{(CH + FK)}$

$EF := CF - CE \qquad BE := BC + CE \qquad DF := BF - BE \qquad CD := AD - AC \qquad BE^2 - EF = 0$

$$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0 \qquad EF - \left( N - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} - 1 \right) = 0$$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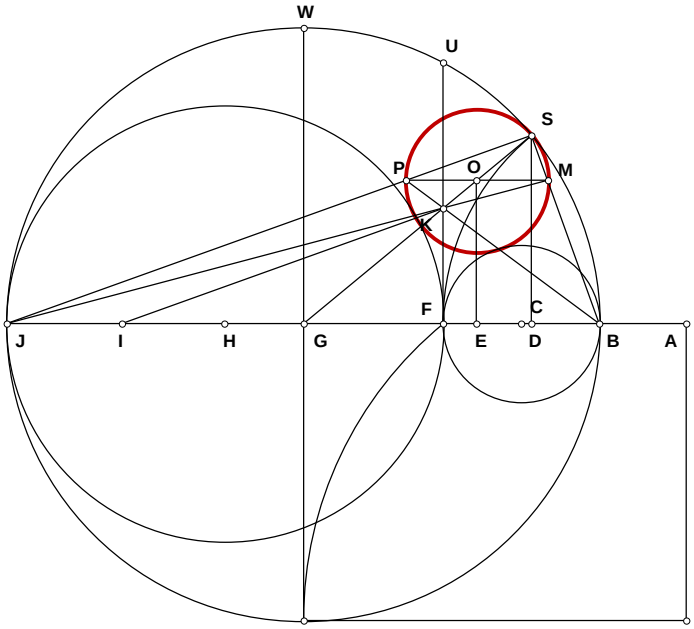
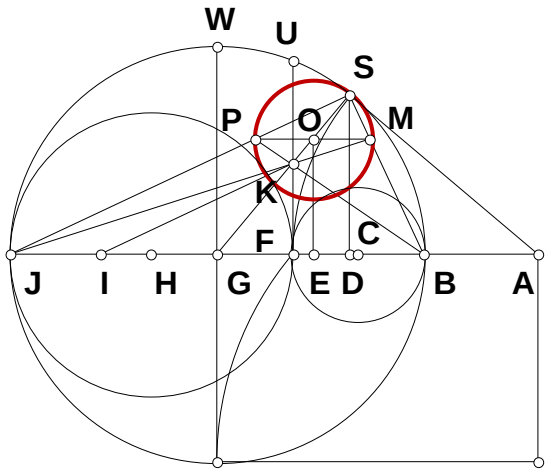
# 941224 EP




Given:

# 122494 Power Line At Square Root

**In this square root figure, what is the Algebraic name of the tangent circle OS?**




$N := 5 \qquad AB := 1 \qquad AJ := AB \cdot N$

$AF := \sqrt{AB \cdot AJ} \quad BJ := AJ - AB \quad BG := \frac{BJ}{2}$

$AG := AB + BG \quad GS := BG \quad DG := \frac{GS^2}{AG}$

$FG := AG - AF \quad BD := BG - DG \quad DJ := BJ - BD$

$DS := \sqrt{BD \cdot DJ} \quad FK := \frac{DS \cdot FG}{DG} \quad BF := AF - AB$

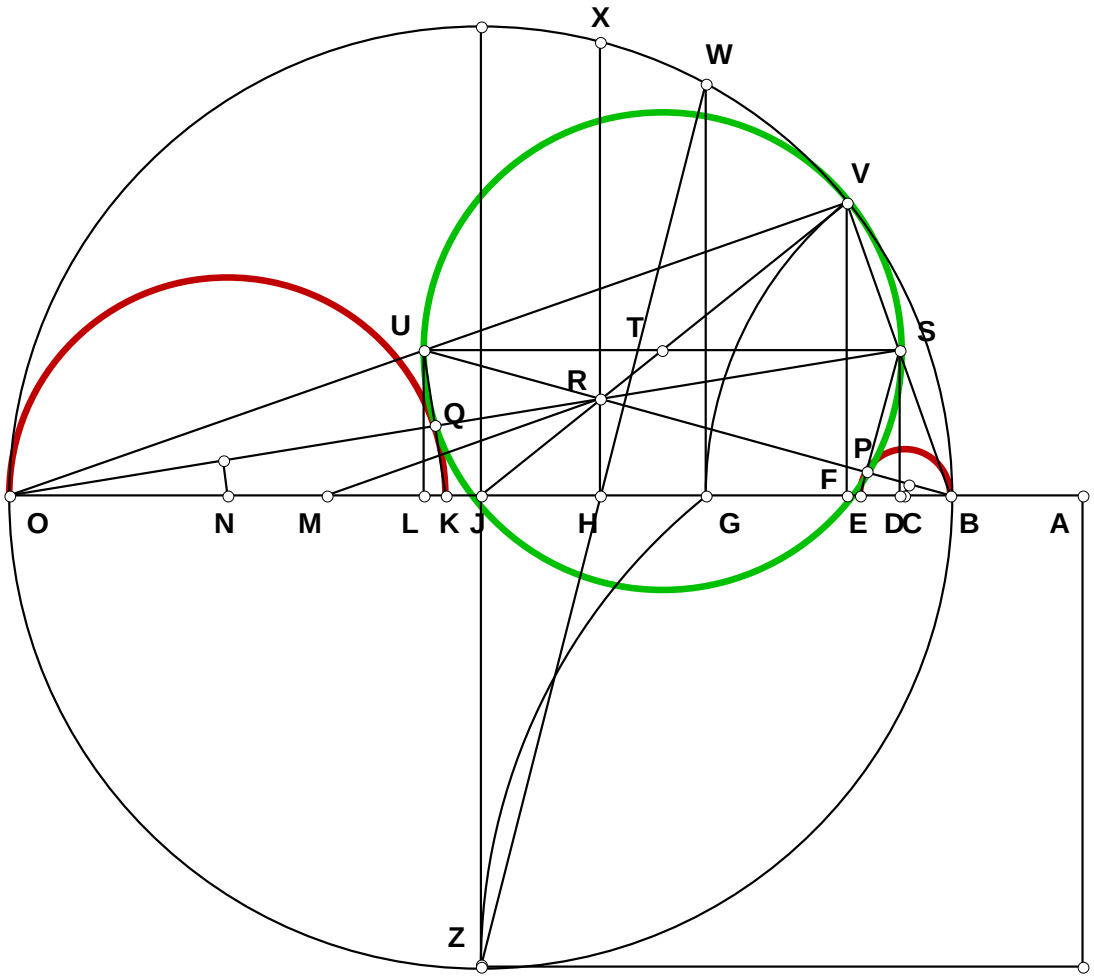
$BK := \sqrt{BF^2 + FK^2} \quad FI := \frac{DJ \cdot FK}{DS} \quad BI := FI + BF$

$BP := \frac{BK \cdot BJ}{BI} \quad KP := BP - BK \quad MP := \frac{BJ \cdot KP}{BK} \quad OS := \frac{MP}{2}$

## Algebraic Names

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0$$

$$OS - \frac{-2 \cdot N^2 + 2 \cdot N - \sqrt{N} + N^{\left(\frac{5}{2}\right)}}{2 \cdot \left[N^2 - \sqrt{N} - N^{\left(\frac{3}{2}\right)} + 1\right]} = 0$$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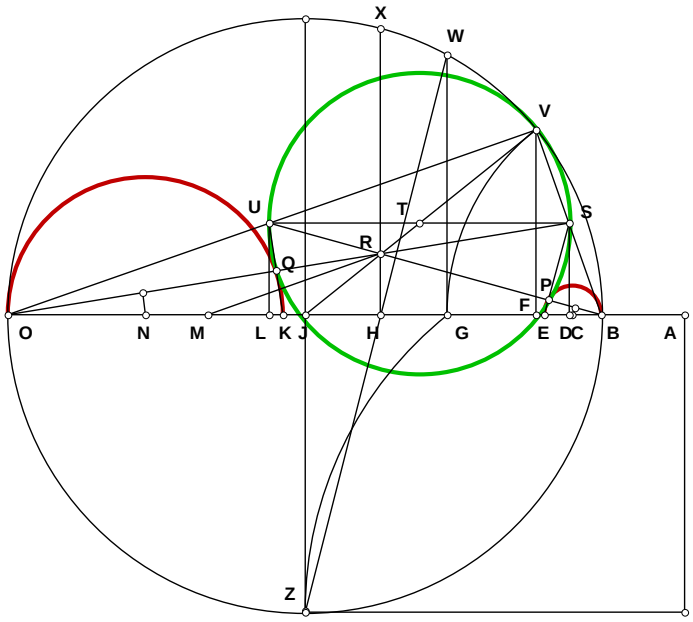
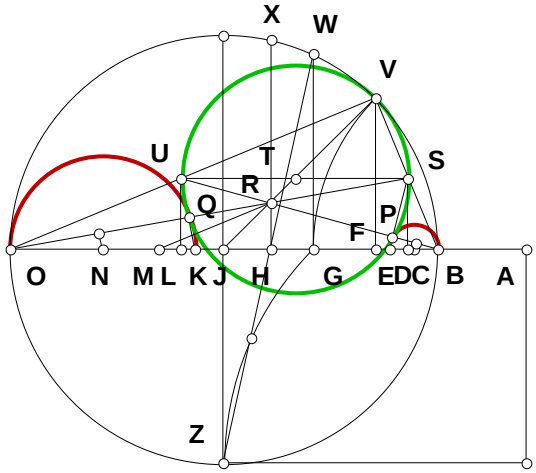
# 941225 EP




Given:

# 122595 Two Prime Exponential Series Developed Through The Powerline Progression




$\Delta := 2 \qquad \delta := 1 .. \Delta$

$N := 5 \qquad AB := 1$

$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB$

$BJ := \frac{BO}{2} \quad JZ := BJ \quad JV := BJ \quad JO := BJ$

$BG_1 := AG - AB \quad GO_1 := BO - BG_1 \quad GW_1 := \sqrt{BG_1 \cdot GO_1}$

$GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{bmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{bmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - \left(BG_\delta + GH_\delta\right) \\ \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]} \\ BJ - \left(BG_\delta + GH_\delta\right) \\ \frac{\left[BJ - \left(BG_\delta + GH_\delta\right)\right] \cdot \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}}{JZ + \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}} \end{bmatrix}$$

$HJ := BJ - BG_\Delta$

$FJ := \frac{(N-1)^2}{2 \cdot (N+1)}$

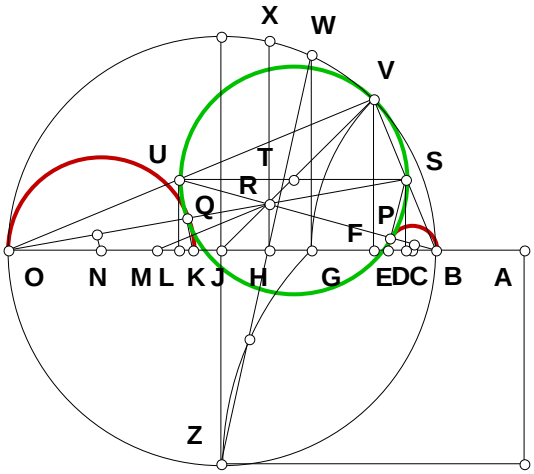
$BF := BJ - FJ \quad FO := FJ + JO \quad FV := \sqrt{BF \cdot FO}$

$HR := \frac{FV \cdot HJ}{FJ} \quad BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2}$

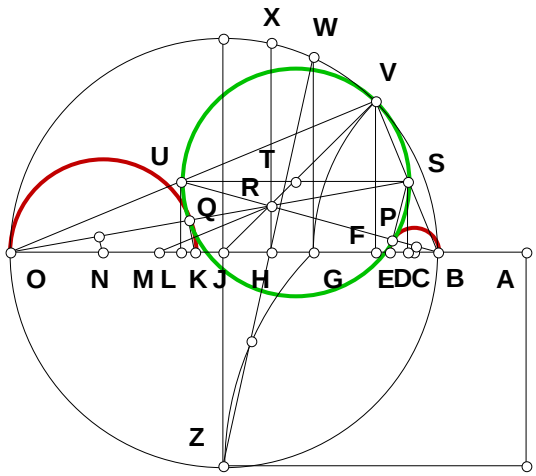
$HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR$

$SU := \frac{BO \cdot RU}{BR} \quad TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR}$

$BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$




$$N^{2^{\frac{1}{\Delta}}} - AE = 0$$



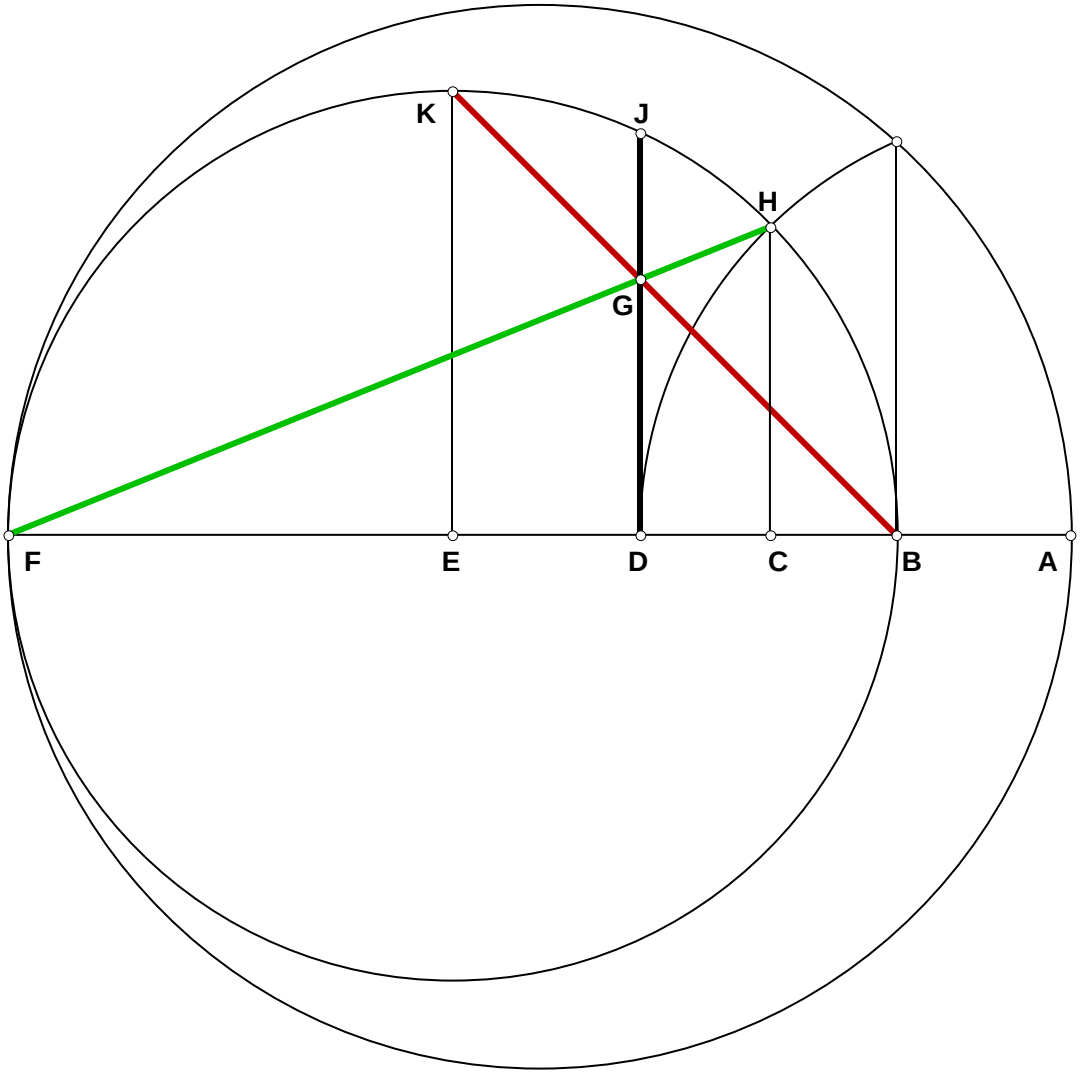

$$LU := \frac{HR \cdot BU}{BR} \qquad BL := \frac{BH \cdot BU}{BR} \qquad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \qquad DS := LU \qquad OS := \frac{OR \cdot DS}{HR}$$

$$DO := \frac{HO \cdot DS}{HR} \qquad QS := \frac{DO \cdot SU}{OS} \qquad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \qquad AK := AO - KO$$

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$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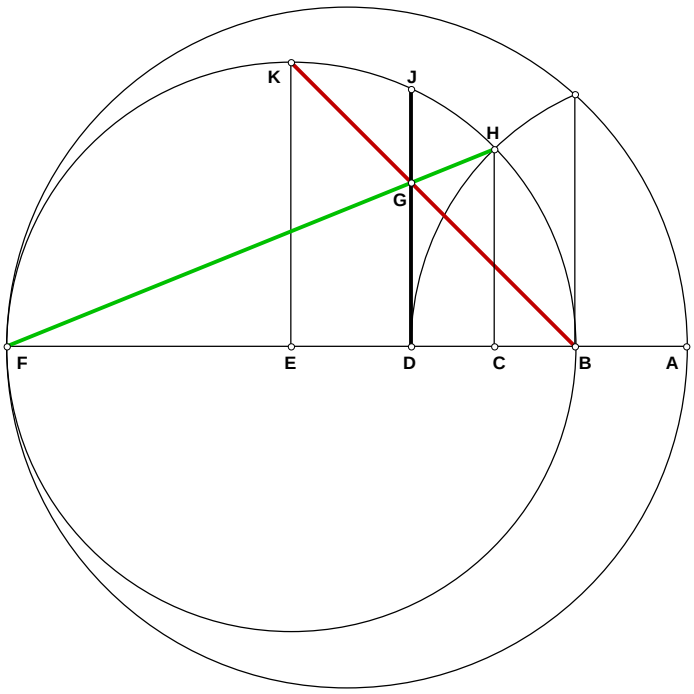
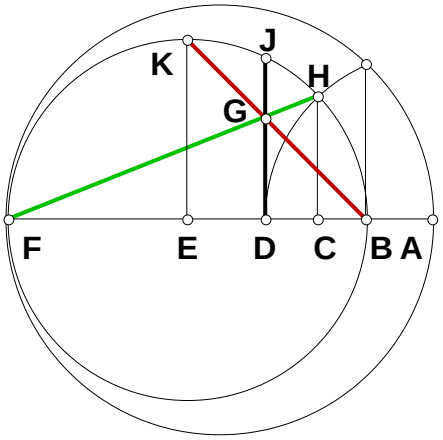
# 941226 EP




Given:

# 122694 Is Point G on DJ?

## Is G, the intersection of FH and BK, on DJ?




$N := 5$ $AB := 1$ $AF := AB \cdot N$ $BF := AF - AB$

$BE := \frac{BF}{2}$ $EF := BE$ $EK := BE$

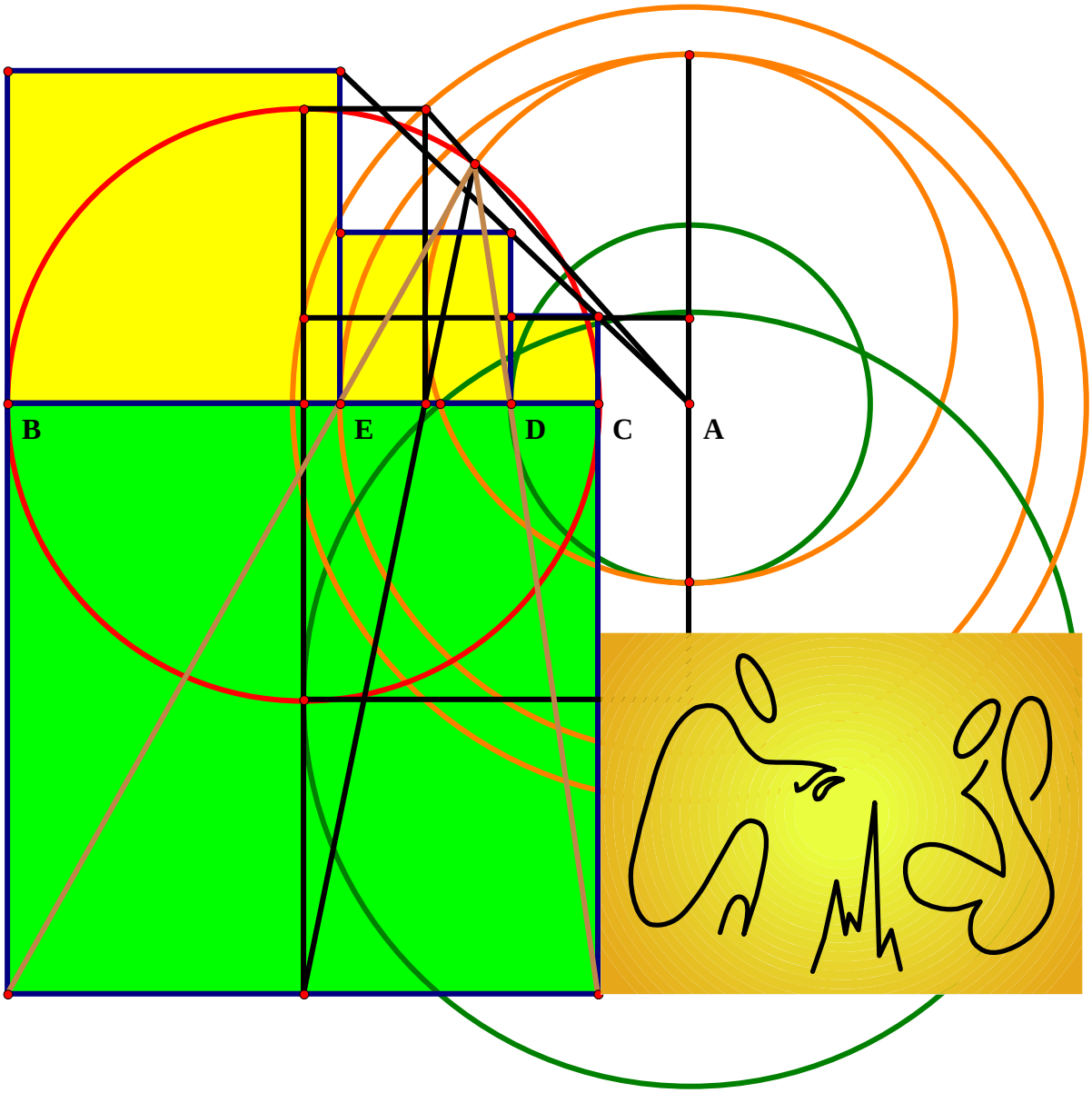
$AD := \sqrt{AB \cdot AF}$ $CE := \frac{(N-1)^2}{2 \cdot (N+1)}$ $CF := CE + EF$

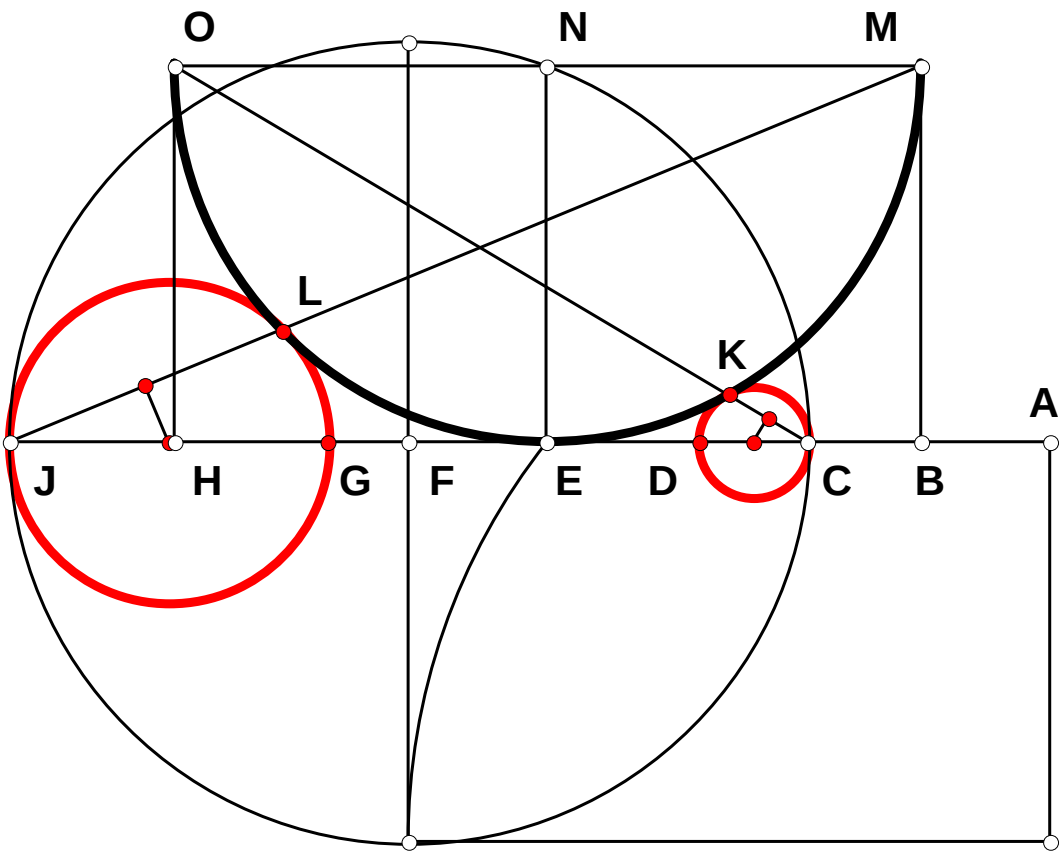
$DF := AF - AD$ $BC := BF - CF$ $CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF}$

$BD := AD - AB$ $DG_2 := \frac{EK \cdot BD}{BE}$ $DG_1 - DG_2 = 0$

# The Delian Quest—1995

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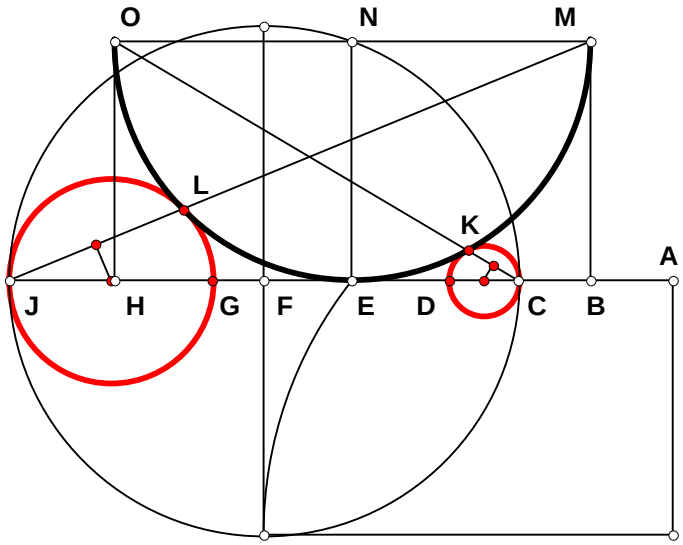
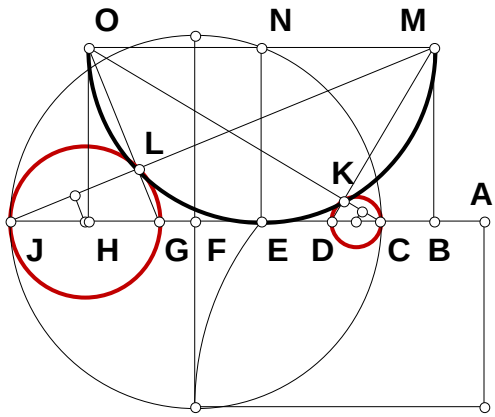
# 950106 EP




Given:

# 010695 Alternate Method Quad Roots




$N := 5 \qquad AC := 1 \qquad AJ := AC \cdot N$

$CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

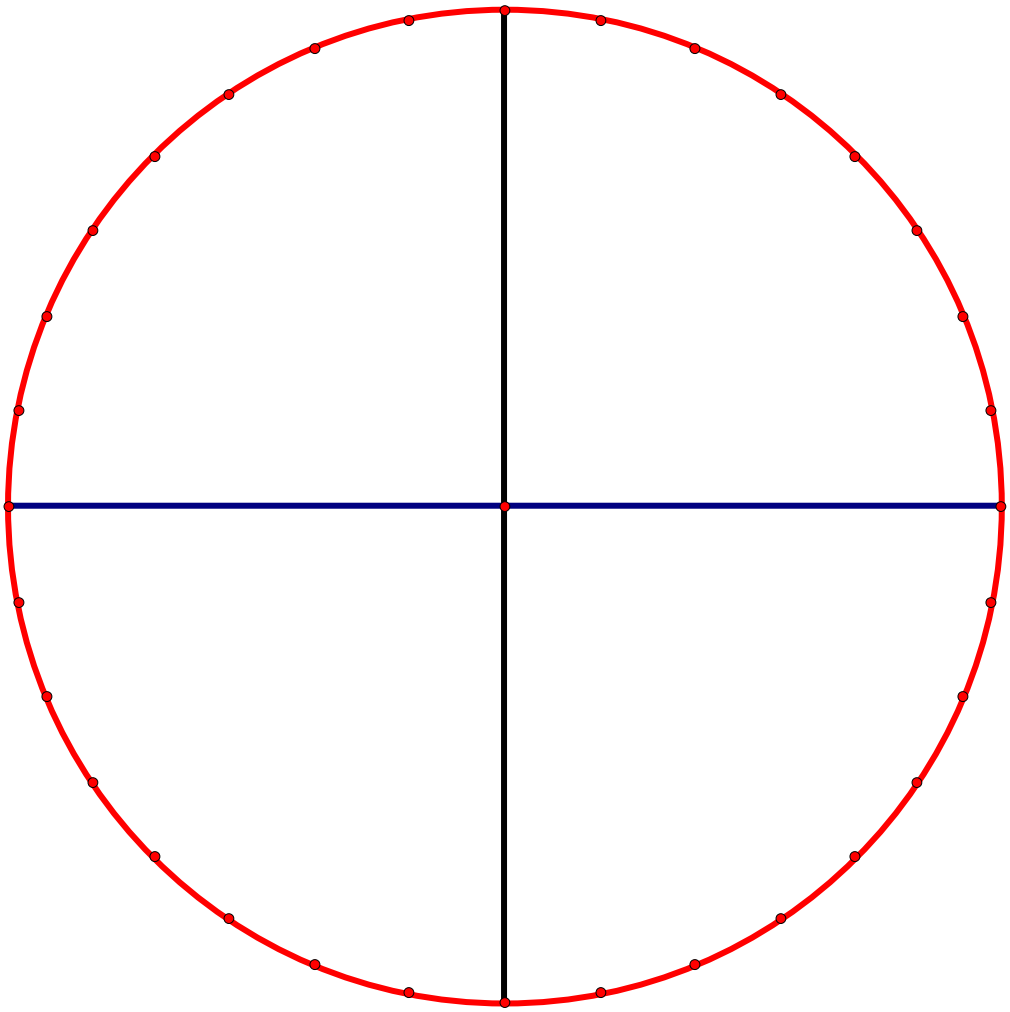
$MN := EN \quad NO := EN \quad BE := EN \quad EH := EN$

$BJ := BE + EJ \quad MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO$

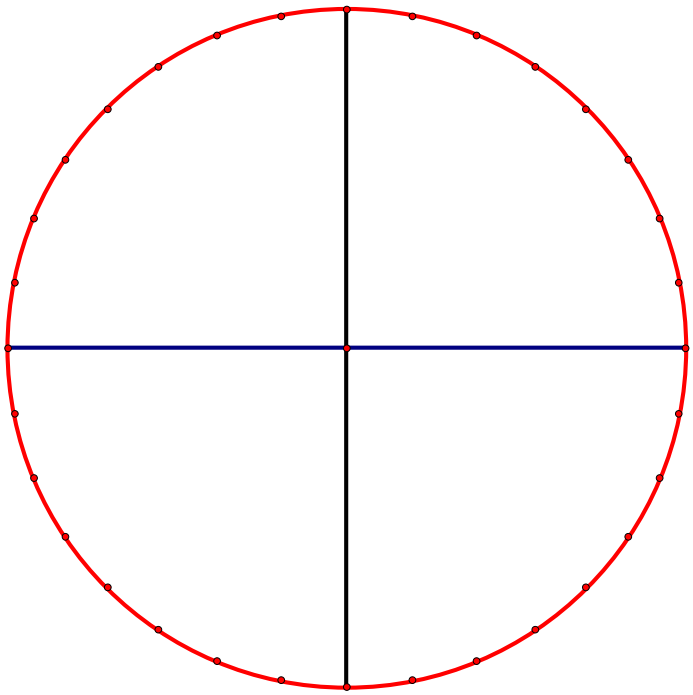
$ML := \frac{BJ \cdot MO}{MJ} \quad JL := MJ - ML \quad GJ := \frac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ \qquad \left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \frac{CH \cdot MO}{CO} \quad CK := CO - KO \quad CD := \frac{CO \cdot CK}{CH} \quad AD := AC + CD$

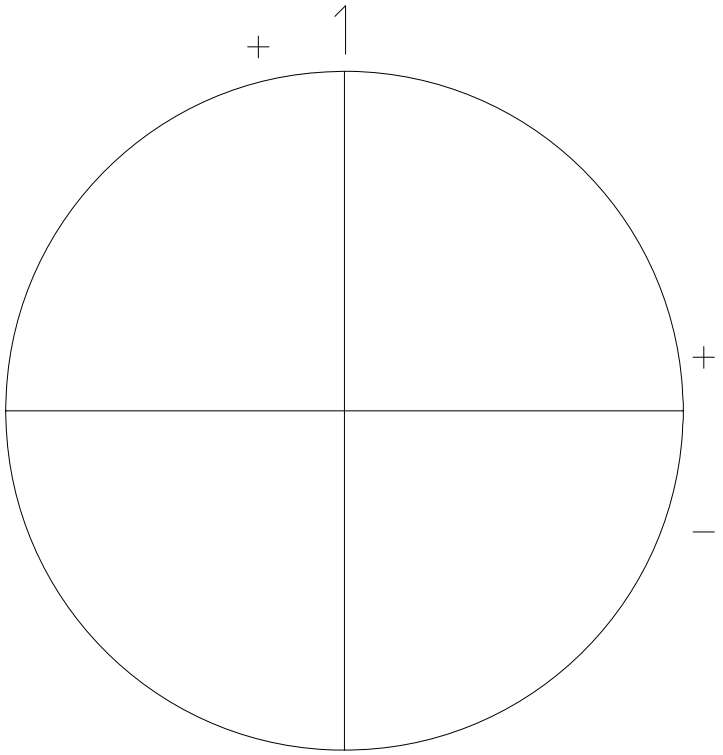
$\left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AG = 0 \qquad N^{\frac{1}{4}} - AD = 0$



**950112 EP**



Given:

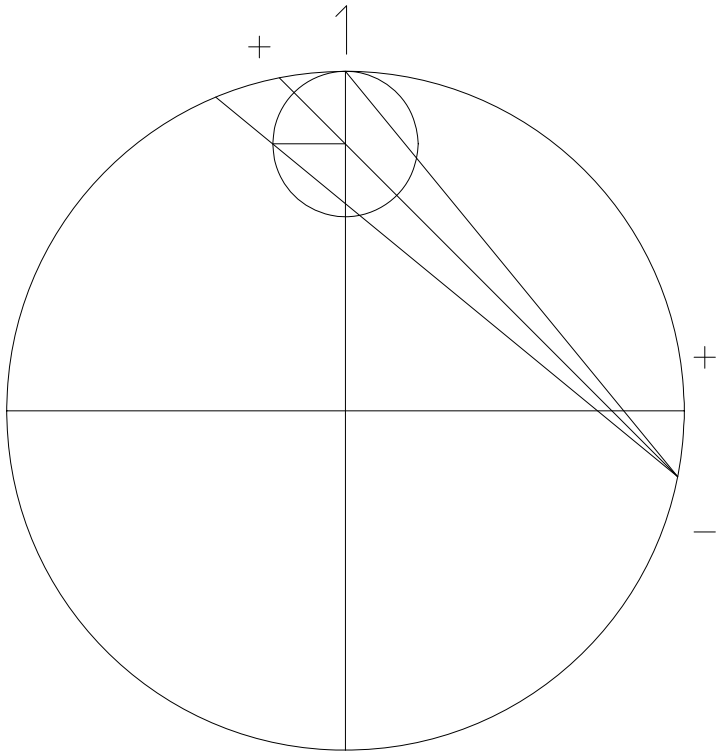



011295.MCD

Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

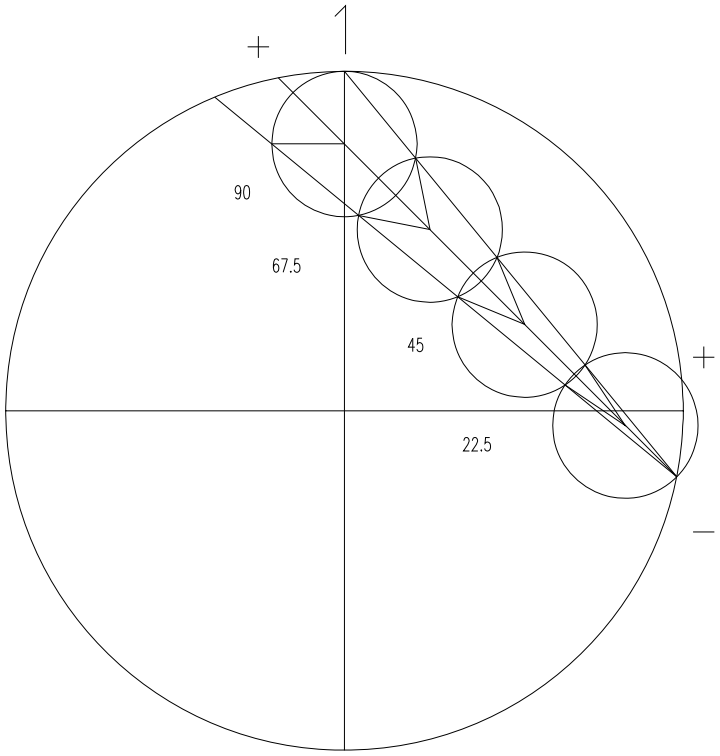
I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.




The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have learned that my starting angle of the smaller circle will be 90 degrees.

$$\frac{90}{8} = 11.25$$

$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$




$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

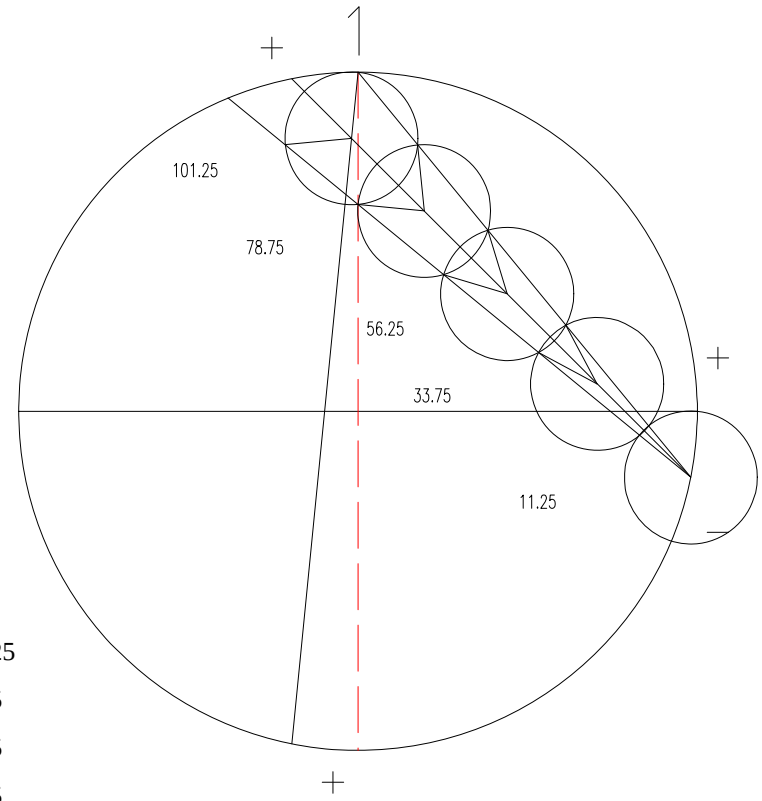
$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

| | |
|---|---|
| $8 + 1 - 1 = 8$ | $8 \cdot 11.25 = 90$ |
| $8 + 1 - 1 - 2 = 6$ | $6 \cdot 11.25 = 67.5$ |
| $8 + 1 - 1 - 2 - 2 = 4$ | $4 \cdot 11.25 = 45$ |
| $8 + 1 - 1 - 2 - 2 - 2 = 2$ | $2 \cdot 11.25 = 22.5$ |
| $8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$ | |

$$\mathrm{mod}(8 + 1 - 1, 2) = 0$$

I have added another plus to a quadrant at the bottom of the figure.




$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

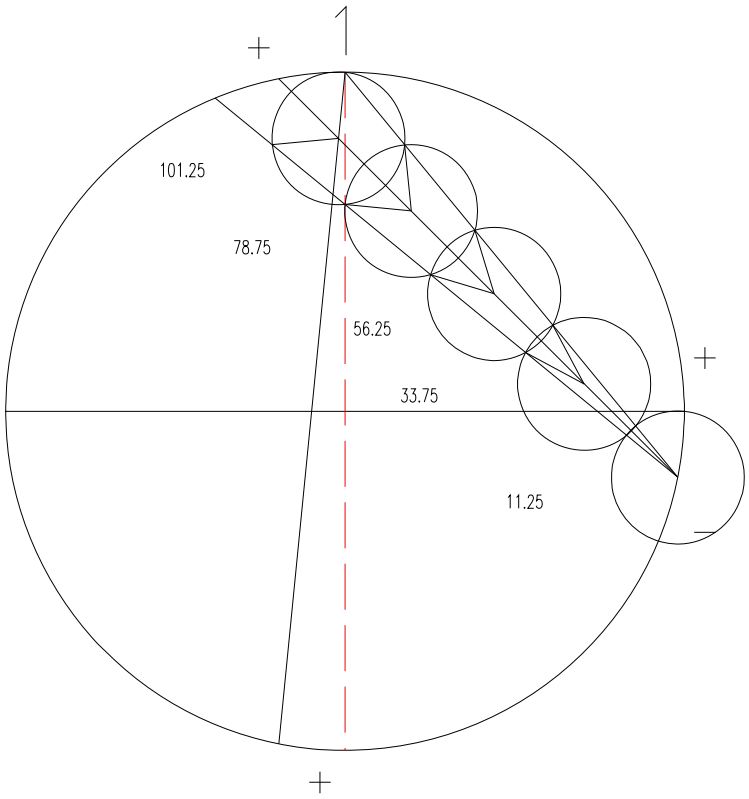
$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

| | |
|---|---|
| $8 + 1 + 1 - 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 + 1 - 1 - 2 = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1$ | $1 \cdot 11.25 = 11.25$ |

$$\mathrm{mod}(8 + 1 + 1 - 1, 2) = 1$$




$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $B = 1.125$ $\frac{9}{8} = 1.125$

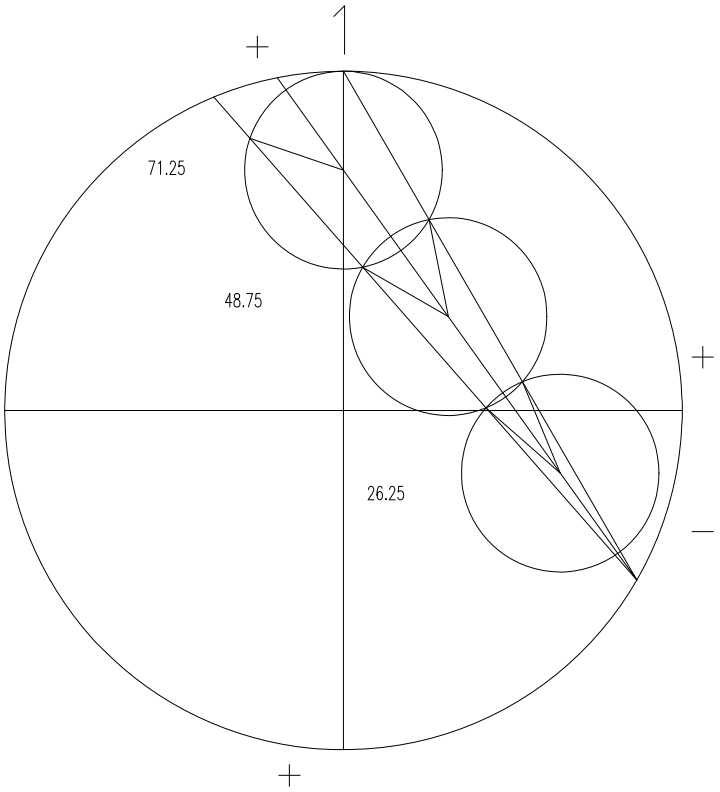
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

| | |
|---|---|
| $8 + 1 = 9$ | $9 \cdot 11.25 = 101.25$ |
| $8 + 1 - (1 \cdot 2) = 7$ | $7 \cdot 11.25 = 78.75$ |
| $8 + 1 - (2 \cdot 2) = 5$ | $5 \cdot 11.25 = 56.25$ |
| $8 + 1 - (3 \cdot 2) = 3$ | $3 \cdot 11.25 = 33.75$ |
| $8 + 1 - (4 \cdot 2) = 1$ | $1 \cdot 11.25 = 11.25$ |

$\mathrm{mod}(8 + 1, 2) = 1$




$B := 1 + \frac{3}{24} - \frac{8}{24}$ $B = 0.7917$ $\frac{19}{24} = 0.7917$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.7495$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.2499$ $\frac{B \cdot .166666}{3.166666} \cdot 90 = 3.75$

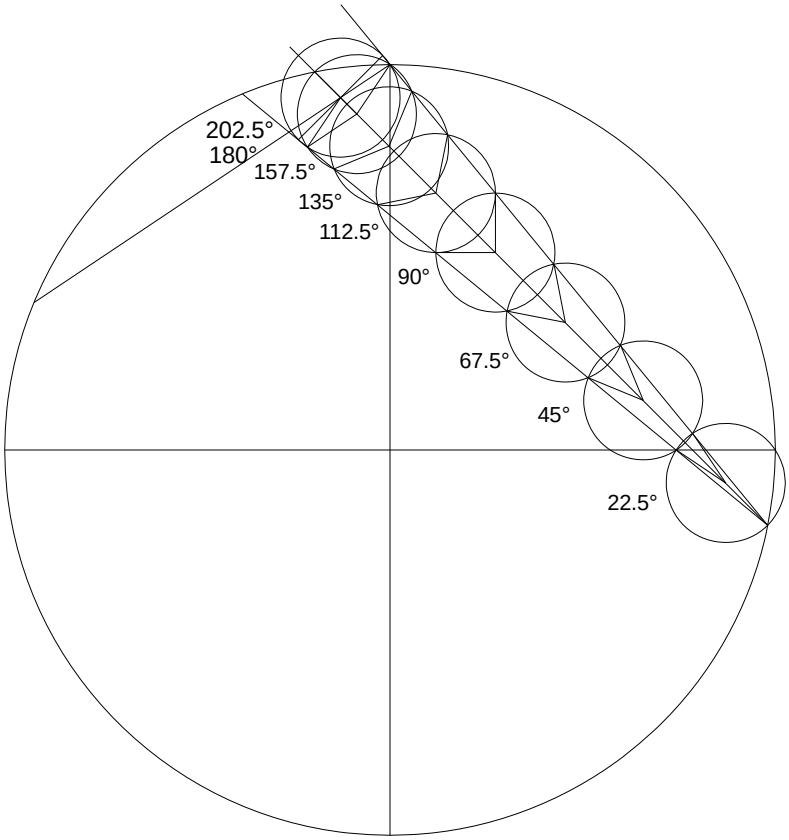
| | |
|---|---|
| $(24 + 3) - 8 = 19$ | $19 \cdot 3.75 = 71.25$ |
| $(24 + 3) - 8 - (1 \cdot 6) = 13$ | $13 \cdot 3.75 = 48.75$ |
| $(24 + 3) - 8 - (2 \cdot 6) = 7$ | $7 \cdot 3.75 = 26.25$ |
| $(24 + 3) - 8 - (3 \cdot 6) = 1$ | $1 \cdot 3.75 = 3.75$ |

$\mathrm{mod}(24 + 3 - 8, 2) = 1$

$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8}$ $B = 2.25$ $\frac{18}{8} = 2.25$

$\frac{B \cdot 9}{9} \cdot 90 = 202.5$ $\frac{B \cdot 8}{9} \cdot 90 = 180$

$\frac{B \cdot 7}{9} \cdot 90 = 157.5$ $\frac{B \cdot .6}{9} \cdot 90 = 13.5$




$8 + 1 - 1 + 10 = 18$ $18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16$ $16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14$ $14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12$ $12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10$ $10 \cdot 11.25 = 112.5$

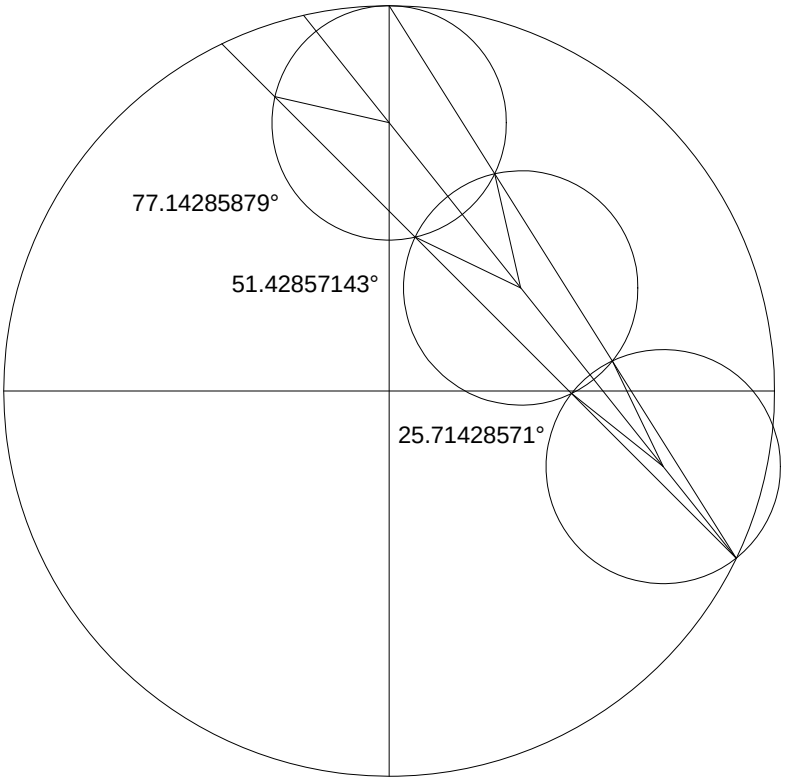
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8$ $8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6$ $6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4$ $4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2$ $2 \cdot 11.25 = 22.5$

$\mathrm{mod}((8 + 1 - 1) + 10, 2) = 0$




$B := 1 + \frac{1}{7} - \frac{2}{7}$ $B = 0.8571$ $\frac{6}{7} = 0.8571$

$\frac{B \cdot 6}{6} \cdot 90 = 77.1429$ $\frac{B \cdot 4}{6} \cdot 90 = 51.4286$

$\frac{B \cdot 2}{6} \cdot 90 = 25.7143$ $c := \frac{90}{7}$

$7 + 1 - (1 \cdot 2) = 6$ $6 \cdot c = 77.1429$

$7 + 1 - (2 \cdot 2) = 4$ $4 \cdot c = 51.4286$

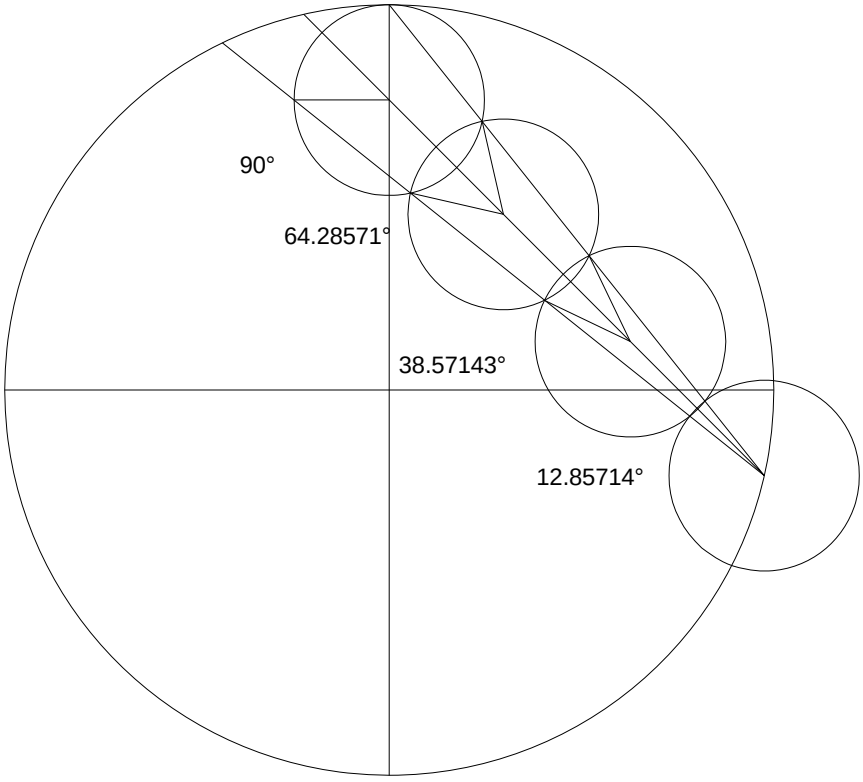
$7 + 1 - (3 \cdot 2) = 2$ $2 \cdot c = 25.7143$

$\mathrm{mod}(7 + 1 - 2, 2) = 0$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$



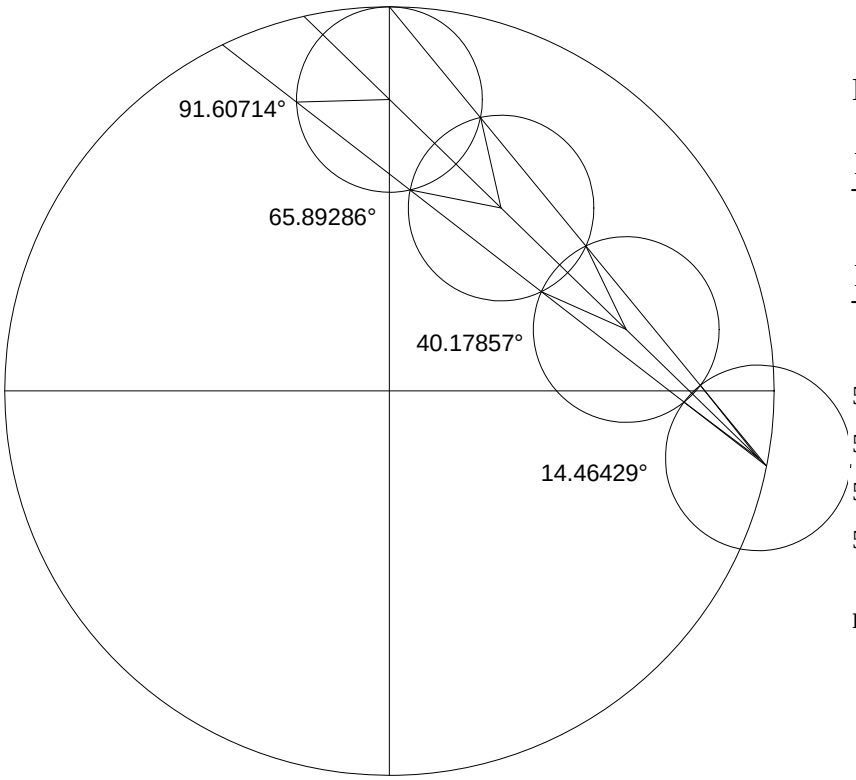

$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.2857$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.5714$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.8571$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.0179$

$\frac{B \cdot 57}{57} \cdot 90 = 91.6071$ $\frac{B \cdot 41}{57} \cdot 90 = 65.8929$

$\frac{B \cdot 25}{57} \cdot 90 = 40.1786$ $c := \frac{90}{56}$

$56 + 8 - 7 = 57$ $57 \cdot c = 91.6071$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.8929$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.1786$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.4643$

$\mathrm{mod}(56 + 8 - 7, 16) = 9$

$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.2857$

$\frac{B \cdot 3}{7} \cdot 90 = 38.5714$ $\frac{B \cdot 1}{7} \cdot 90 = 12.8571$

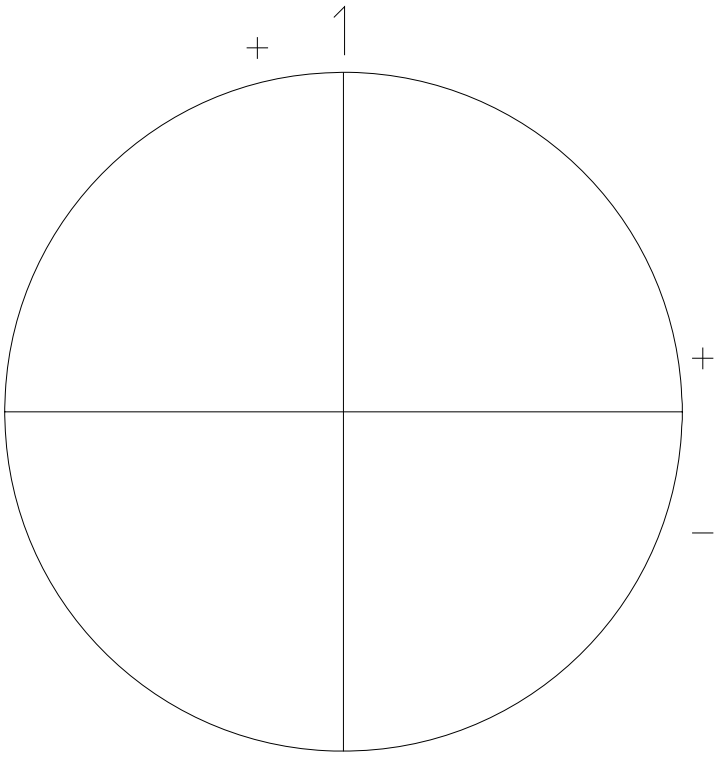
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

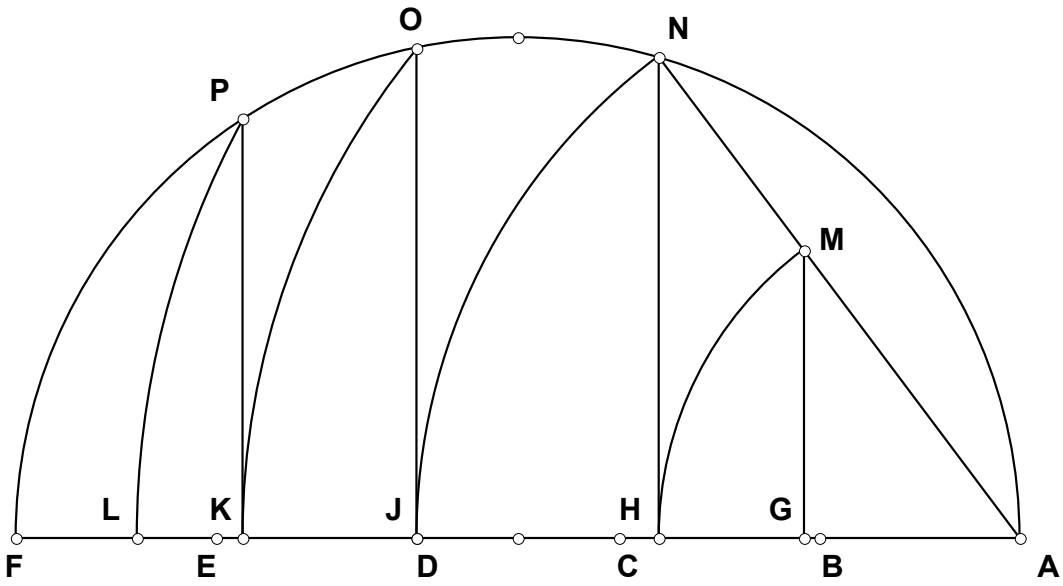
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$




Work in progress.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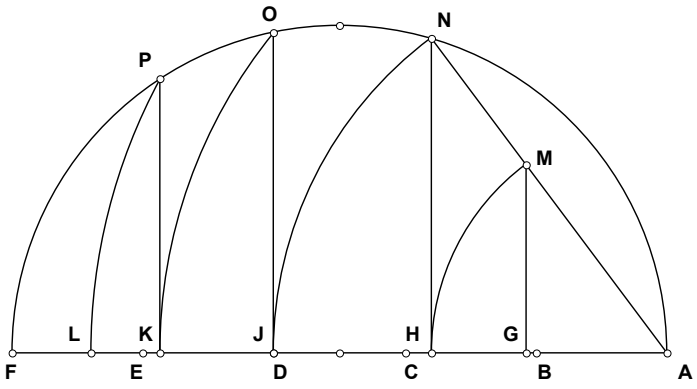
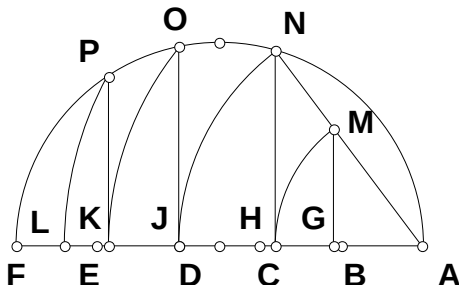
# 95040 EP




Given:

# 040195 Exponential Series-Roots and Powers

**I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.**




$N_1 := 5 \qquad AB := 1 \qquad AF := AB \cdot N_1$

$\delta := 0 .. 3$

$N_2 := 3 \qquad AJ_0 := AB \cdot N_2 \qquad AN := AJ_0$

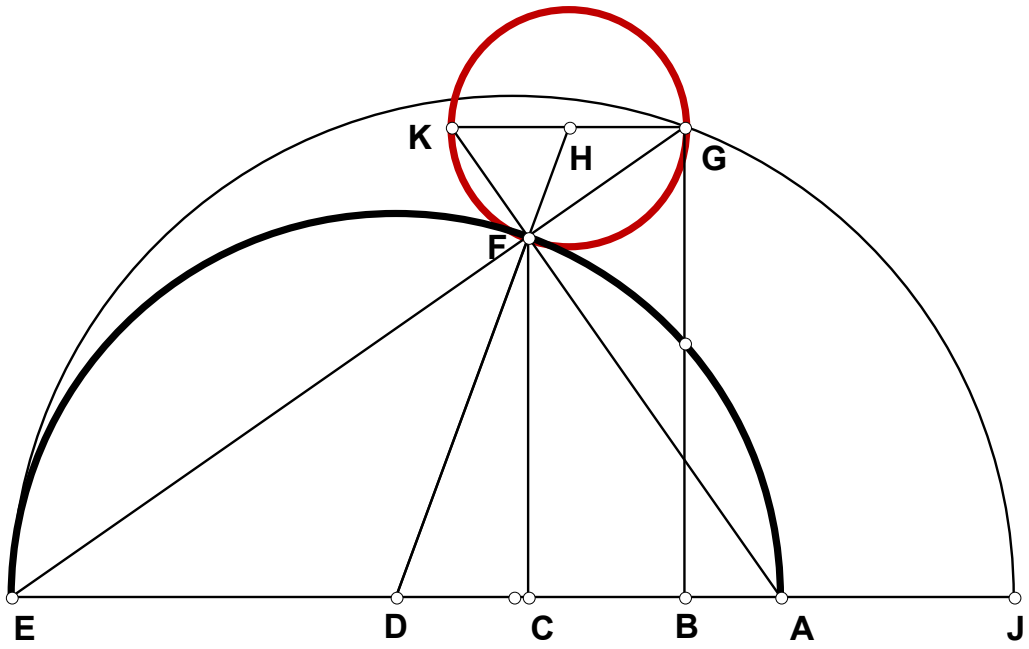
$AJ_1 := \frac{AN^2}{AF} \qquad AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF}$

$AD_0 := AJ_0 \qquad DF_0 := AF - AD_0 \qquad DO_0 := \sqrt{AD_0 \cdot DF_0} \qquad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$

$$\begin{bmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{bmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

$$\sum_\delta \left[ \frac{AF}{AJ_\delta} - \left( \frac{N_1}{N_2} \right)^{\delta+1} \right] = 0$$

$$\sum_\delta \left[ \frac{AF}{AD_\delta} - \left( \frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$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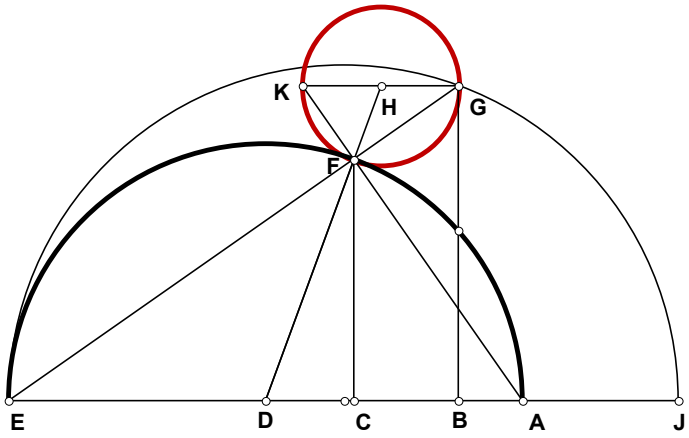
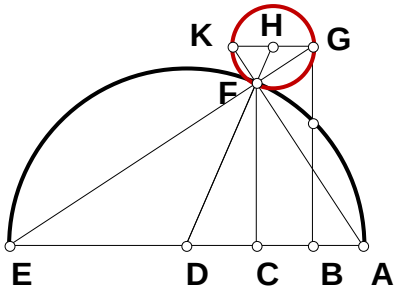
# 950913 EP




Given:

## 091395 A Study In Placement




### Given AE, AB, AC what is GH?

$N_1 := 8.028 \qquad N_2 := 3.044 \qquad AE := 1.833$

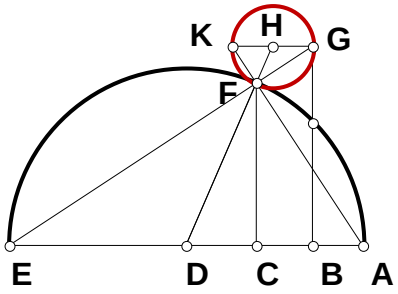
$AD := \frac{AE}{2} \quad AB := \frac{AE}{N_1} \qquad AC := \frac{AE}{N_2} \qquad DF := AD$

$CD := AD - AC \qquad CF := \sqrt{DF^2 - CD^2}$

$CE := AE - AC \quad BE := AE - AB \quad EF := \sqrt{CE^2 + CF^2} \qquad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right|$

$$\frac{AE}{2} \cdot \frac{N_1 - N_2}{N_1 \cdot (N_2 - 1)} - GH = 0$$

### Given AE, AB, EF what is GH?




$N_2 := 1.22$

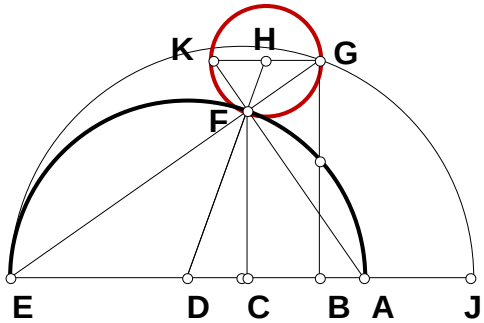
$AE := 1.833 \quad AD := \frac{AE}{2} \quad AB := \frac{AE}{N_1} \qquad DE := AD$

$EF := \frac{AE}{N_2} \qquad CE := \frac{EF^2}{AE} \quad BD := AD - AB$

$BE := BD + DE \quad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF$

$GH := \frac{AD \cdot FG}{EF} \qquad \frac{AE}{2} \cdot \frac{\left(N_2^2 \cdot N_1 - N_2^2 - N_1\right)}{N_1} - GH = 0$

### Given AE, AB, BG what is GH?



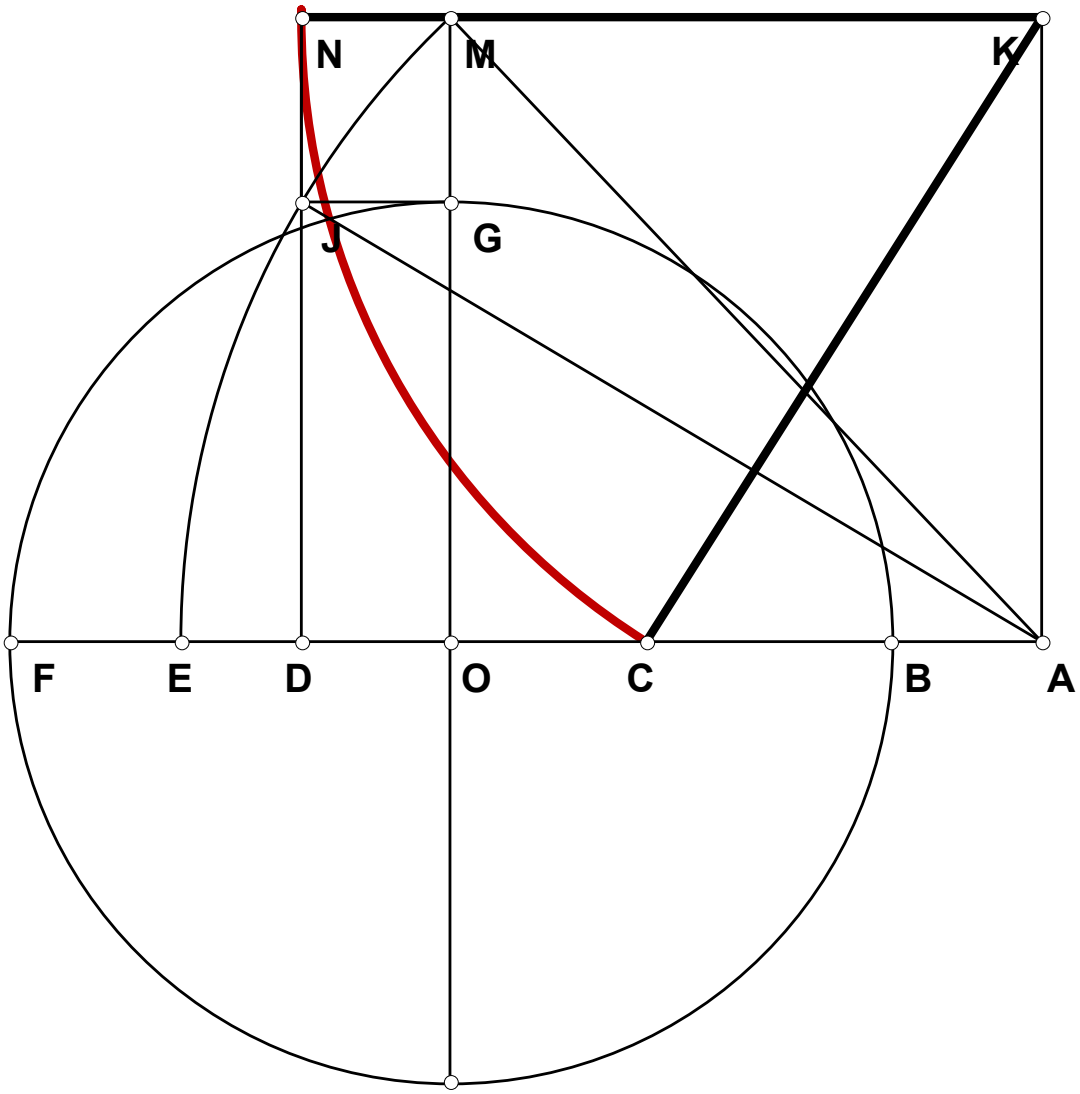

$N_2 := 1.633$

$AE := 1.833 \quad AD := \frac{AE}{2} \quad AB := \frac{AE}{N_1} \qquad BG := \frac{AE}{N_2}$

$BE := AE - AB \quad EG := \sqrt{BE^2 + BG^2} \quad EJ := \frac{EG^2}{BE}$

$EF := \frac{EG \cdot AE}{EJ} \quad FG := EG - EF \quad GH := \left| \frac{AD \cdot FG}{EF} \right|$

$$\frac{AE}{2} \cdot \frac{N_2^2 + N_1^2 - N_2^2 \cdot N_1}{N_2^2 \cdot N_1 \cdot (N_1 - 1)} - GH = 0$$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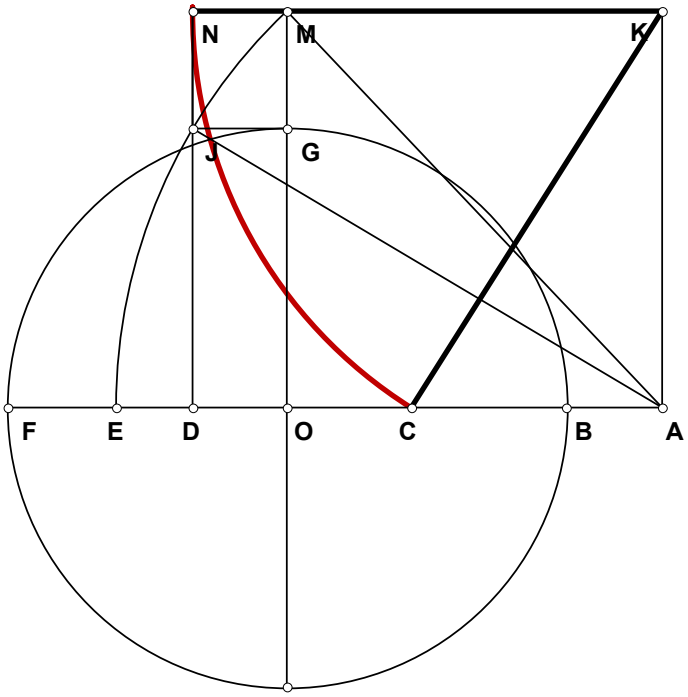
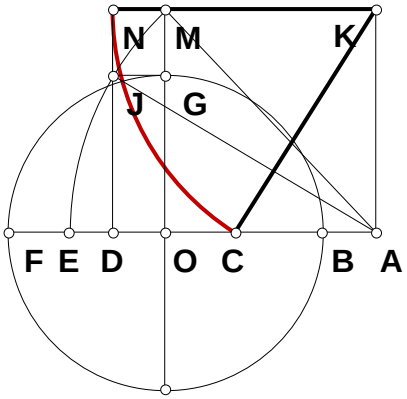
## 951014 EP




Given:

# 101495 Alternate Method Square Root

## For any AK is AC the root of AB x AF?




$N_1 := 5 \quad N_2 := 4$

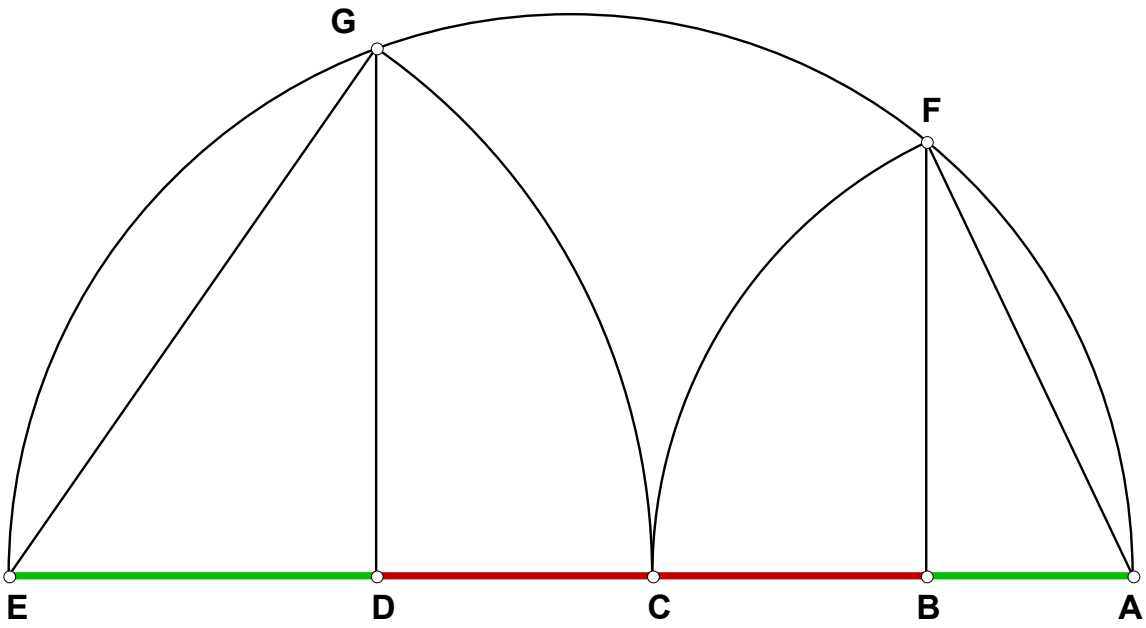
$AB := 1 \quad AF := AB \cdot N_1 \quad AK := AB \cdot N_2 \quad BF := AF - AB$

$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$GO := BO \quad DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$KN := AD \quad CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$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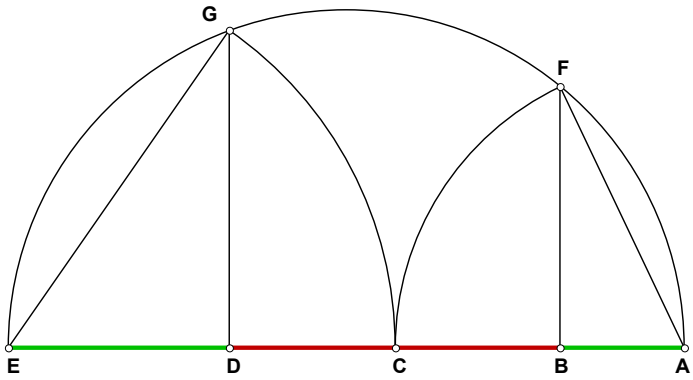
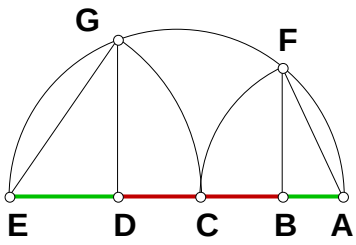
# 951020 EP




Given:

## 102095 Four Times The Square




$N_1 := 2$ $AE := 1$ $AB := \frac{AE}{N_1}$

$BE := AE - AB$ $BF := \sqrt{AB \cdot BE}$ $AF := \sqrt{AB^2 + BF^2}$

$AC := AF$ $CE := AE - AC$ $EG := CE$ $DE := \frac{EG^2}{AE}$

$BD := AE - (AB + DE)$ $\frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$

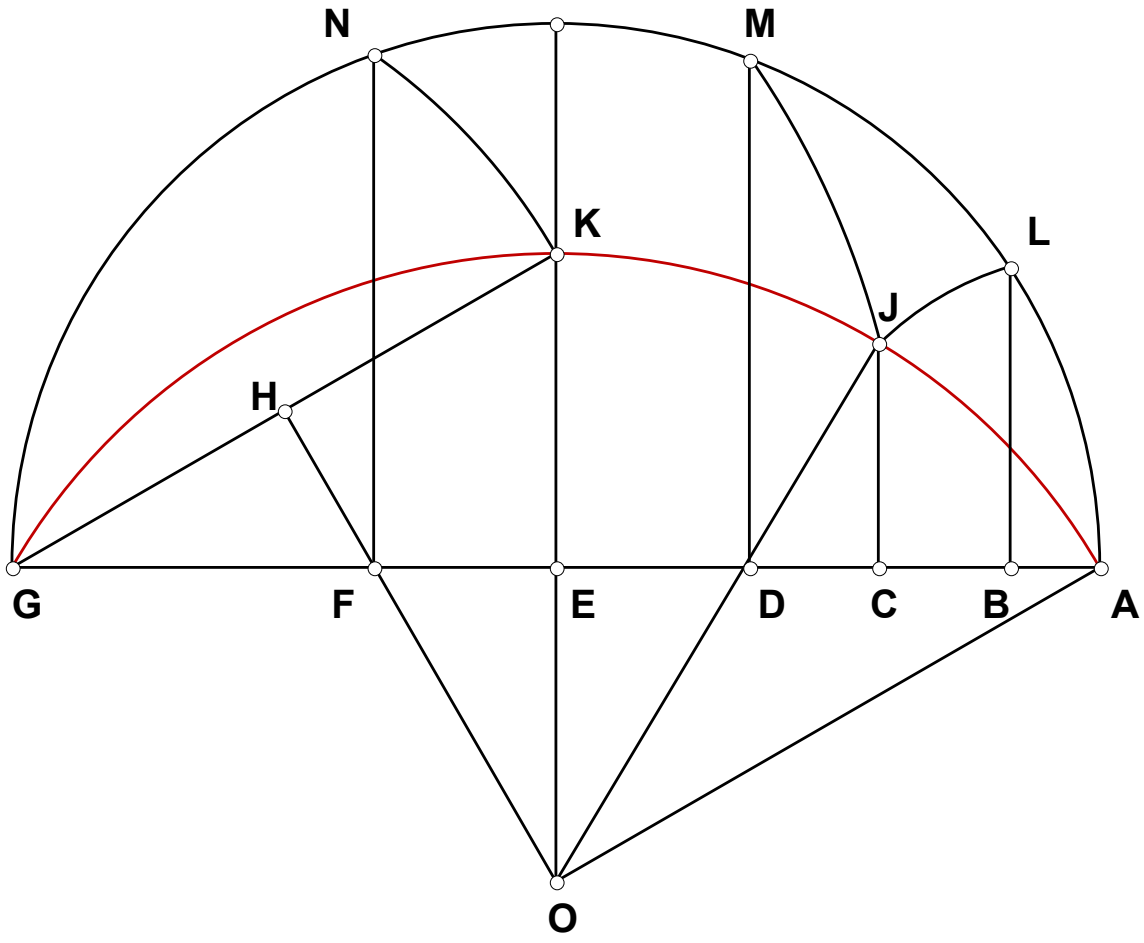
$AD := AE - DE$ $DG := \sqrt{AD \cdot DE}$

### Algebraic Names:

$\frac{1}{N_1} - AB = 0$ $1 - \frac{1}{N_1} - BE = 0$ $\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0$ $\sqrt{\frac{N_1}{N_1^2}} - AF = 0$

$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0$ $1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0$ $2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$

$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0$ $\frac{\sqrt{2 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1^2 - 5 \cdot N_1 + 4 \cdot \sqrt{\frac{1}{N_1}} \cdot N_1 - 1}}{N_1} - DG = 0$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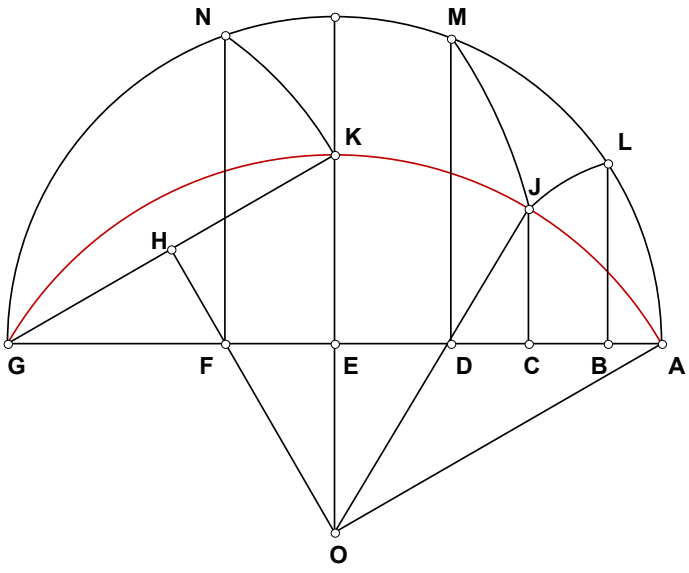
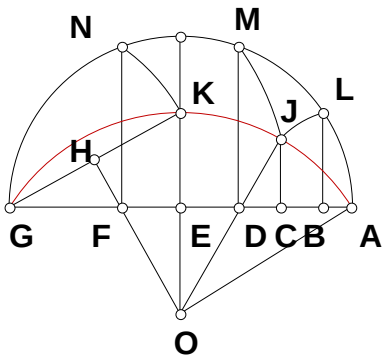
# 951101 EP




Given:

## 110195 A Modification Of The Square Root Figure, Gemini Roots




**On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.**

$N_1 := 5 \qquad N_2 := 2 \qquad AG := 1$

$AE := \frac{AG}{2} \quad EG := AE \quad EF := \frac{AG}{2 \cdot N_1} \quad AF := AE + EF$

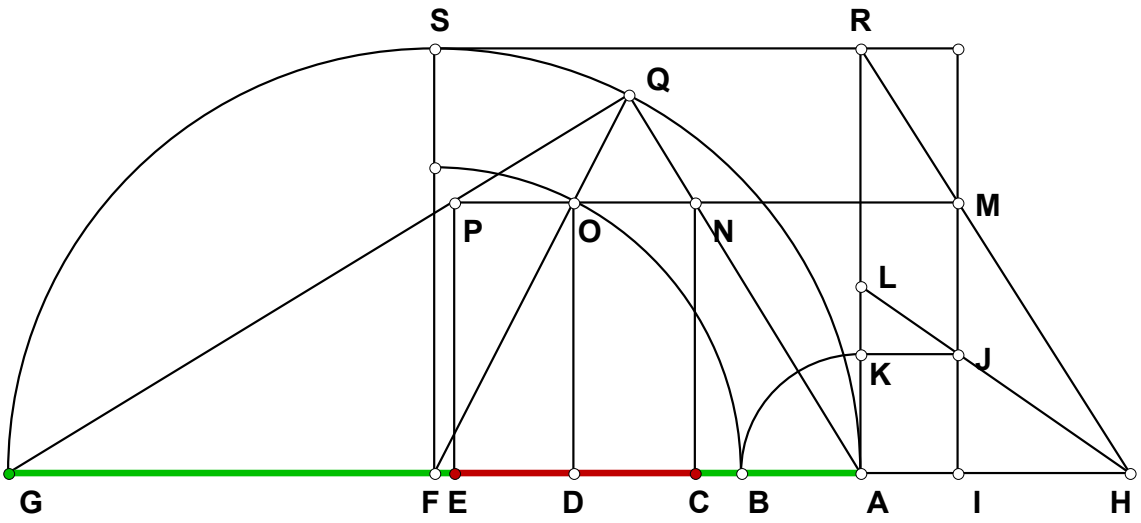
$FG := EG - EI FN := \sqrt{AF \cdot FG} \quad GN := \sqrt{FN^2 + FG^2} \quad GK := GN$

$EK := \sqrt{GK^2 - EG^2} \quad EO := \frac{EG \cdot EF}{EK} \quad OK := EO + EK \quad DE := \frac{AE}{N_2} \quad DO := \sqrt{DE^2 + EO^2}$

$DJ := OK - DO \quad CD := \frac{DE \cdot DJ}{DO} \quad CE := CD + DE \; CJ := \frac{EO \cdot DJ}{DO} \quad AC := AE - CE \quad AJ := \sqrt{AC^2 + CJ^2}$

$AL := AJ \quad AB := \frac{AL^2}{AG} \quad CG := AG - AC \quad GJ := \sqrt{CG^2 + CJ^2} \quad GM := GJ \quad DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG) \qquad \frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$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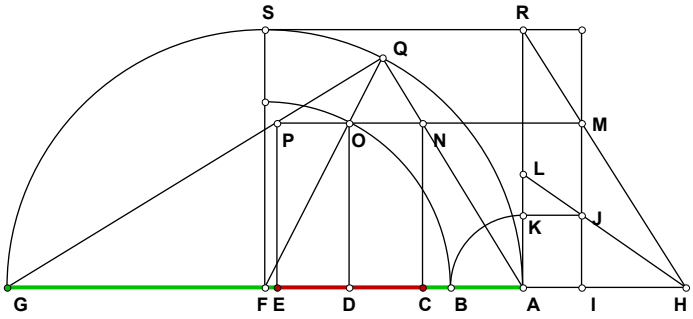
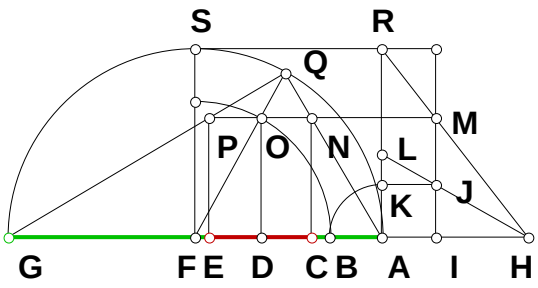
# 951105 EP




Given:

# 110595 Alternate Method Gemini Roots




$N_1 := 3 \quad N_2 := 5$

$AG := 1 \quad AF := \frac{AG}{2} \quad AR := AF \quad FQ := AF \quad FG := AF$

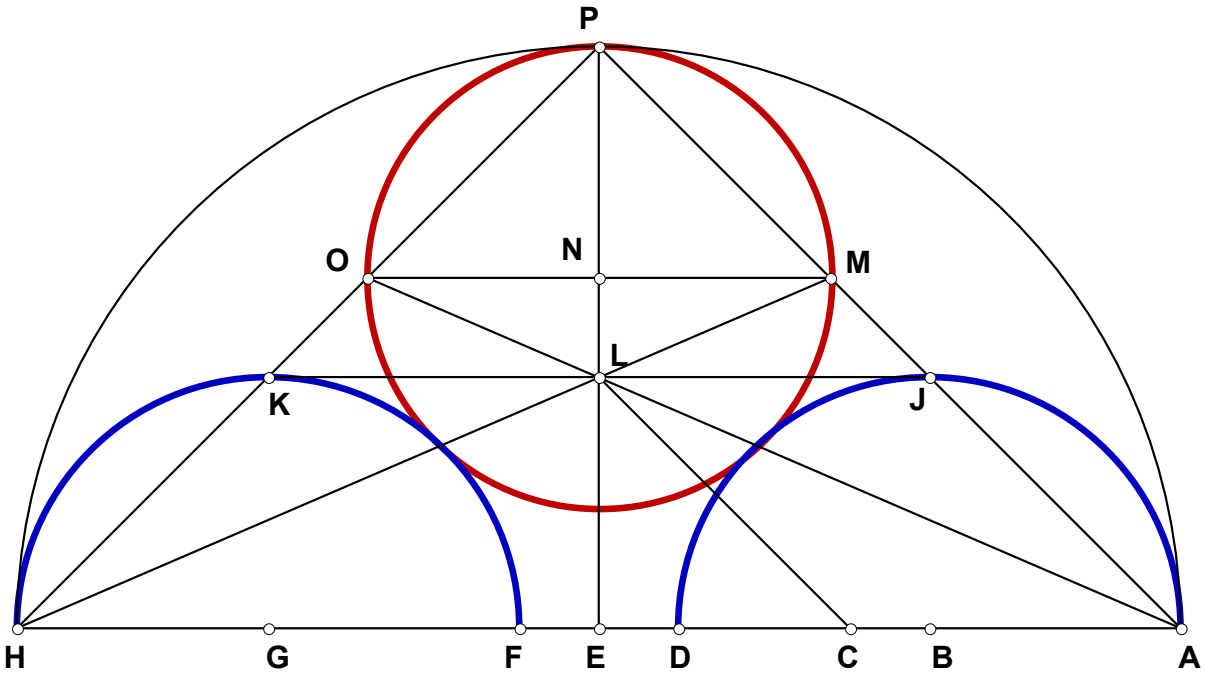
$AL := \frac{AR}{N_1} \qquad IM := \frac{AR}{N_2} \qquad AK := \frac{AL \cdot IM}{AR}$

$DO := IM \quad AB := AK \quad BF := AF - AB \quad FO := BF$

$OQ := FQ - FO \quad NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO} \quad NP - 2 \cdot AK = 0 \quad CD := AK \quad DE := AK$

$DF := \sqrt{FO^2 - DO^2} \quad AD := AF - DF \quad AC := AD - CD \quad EG := FG + DF - DE$

$CE := NP \qquad \frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$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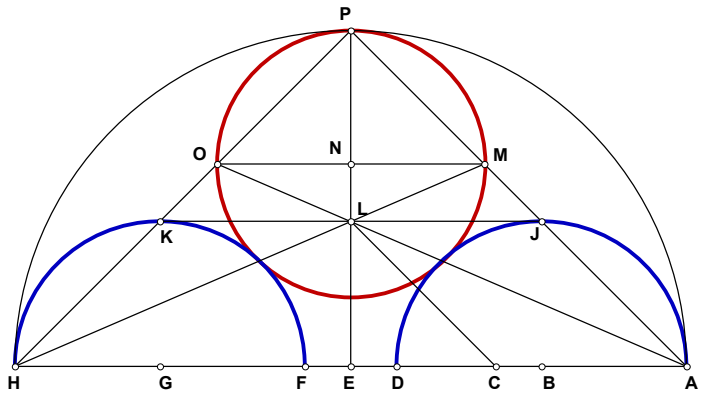
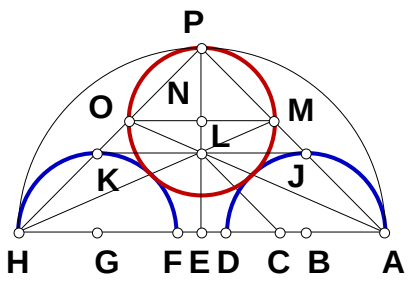
# 951201 EP




Given:

## 120195 Method For Equals

### Given AB find NP.



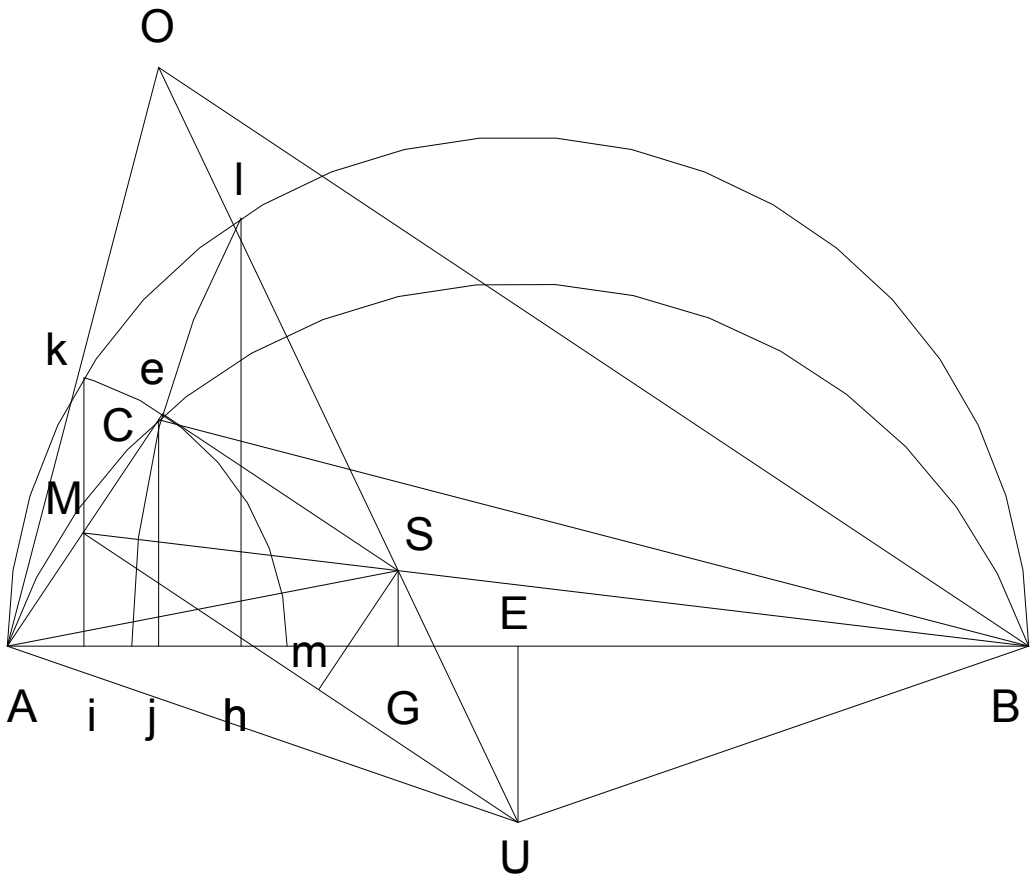

$N_1 := 5$

$AH := 1 \quad AE := \frac{AH}{2} \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$

$AB := \frac{AE}{N_1} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$

$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$

$\frac{1}{2} \cdot \frac{(N_1 - 1)}{(N_1 + 1)} - NP = 0 \qquad \frac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$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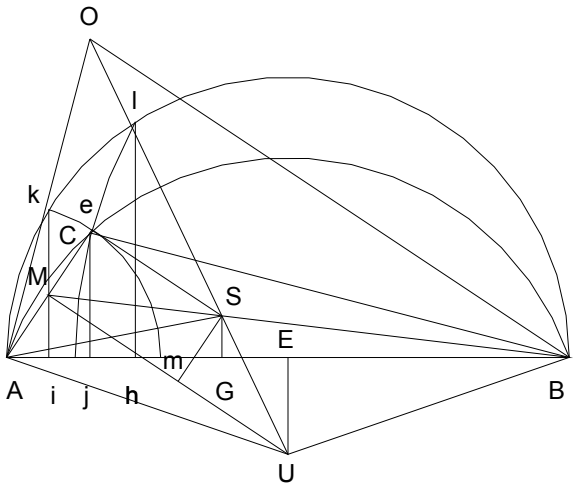
## 951207 EP




Given:

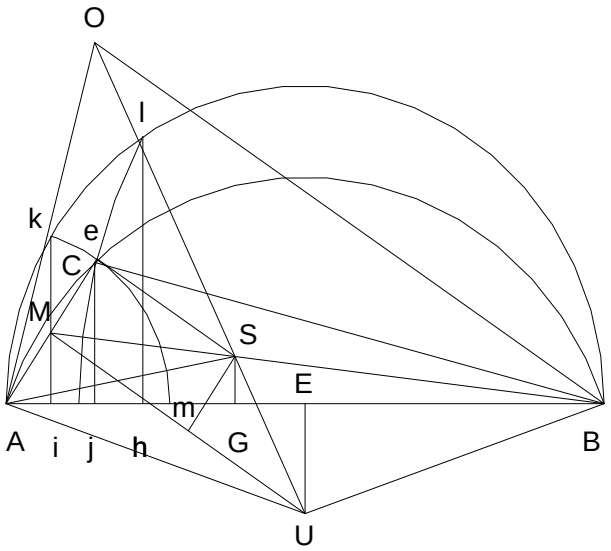
Scholia: *100 Great Problems of Elementary Mathematics* H. Dorrie Problem 27 Euler's Line

**120795.MCD**

$\delta := 0..2 \quad AC := \begin{bmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{bmatrix} \quad BC := \begin{bmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{bmatrix} \quad AB := \begin{bmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{bmatrix}$

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

$TRIANGLE := (Side\_1 + Side\_2 > Side\_3)\cdot(Side\_1 + Side\_3 > Side\_2)\cdot(Side\_2 + Side\_3 > Side\_1)$




$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta$

$Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta} \quad Ah_\delta := AB_\delta - Bh_\delta$

$hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$

$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta$

$Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta}$

$fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$

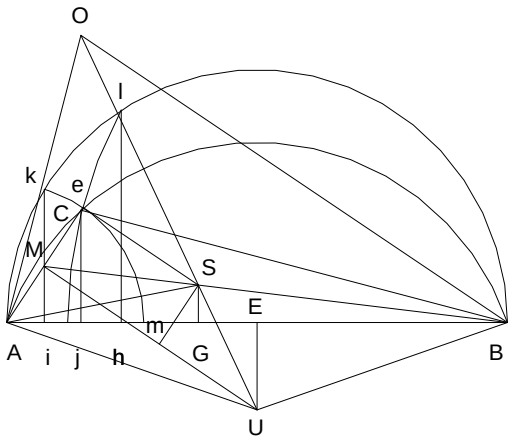
$BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right]$

$AM_\delta := \frac{AC_\delta}{2} \quad AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta$

$GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2} \quad BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2}$

$BS_\delta := \frac{2\cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$

$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2}$




$MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta \quad MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2}\cdot\frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2}\cdot AM_\delta - \frac{1}{2}\cdot\frac{(MS_\delta)^2}{AM_\delta}$

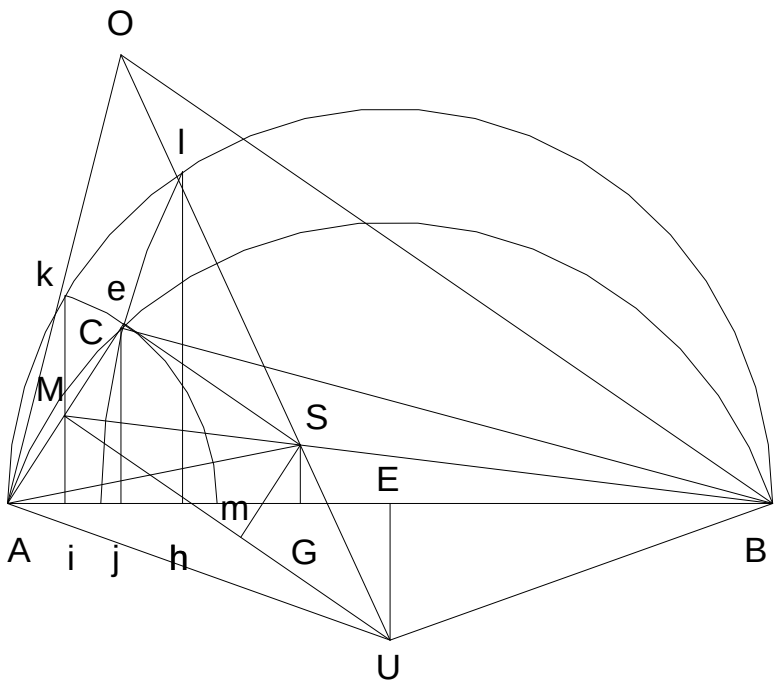
The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta \quad Sm_\delta := eM_\delta \quad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2} \quad Mm_\delta := Se_\delta$

$Um_\delta := \text{if}\left[ AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}, MU_\delta - Mm_\delta, MU_\delta + Mm_\delta \right] \quad SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2} \quad UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

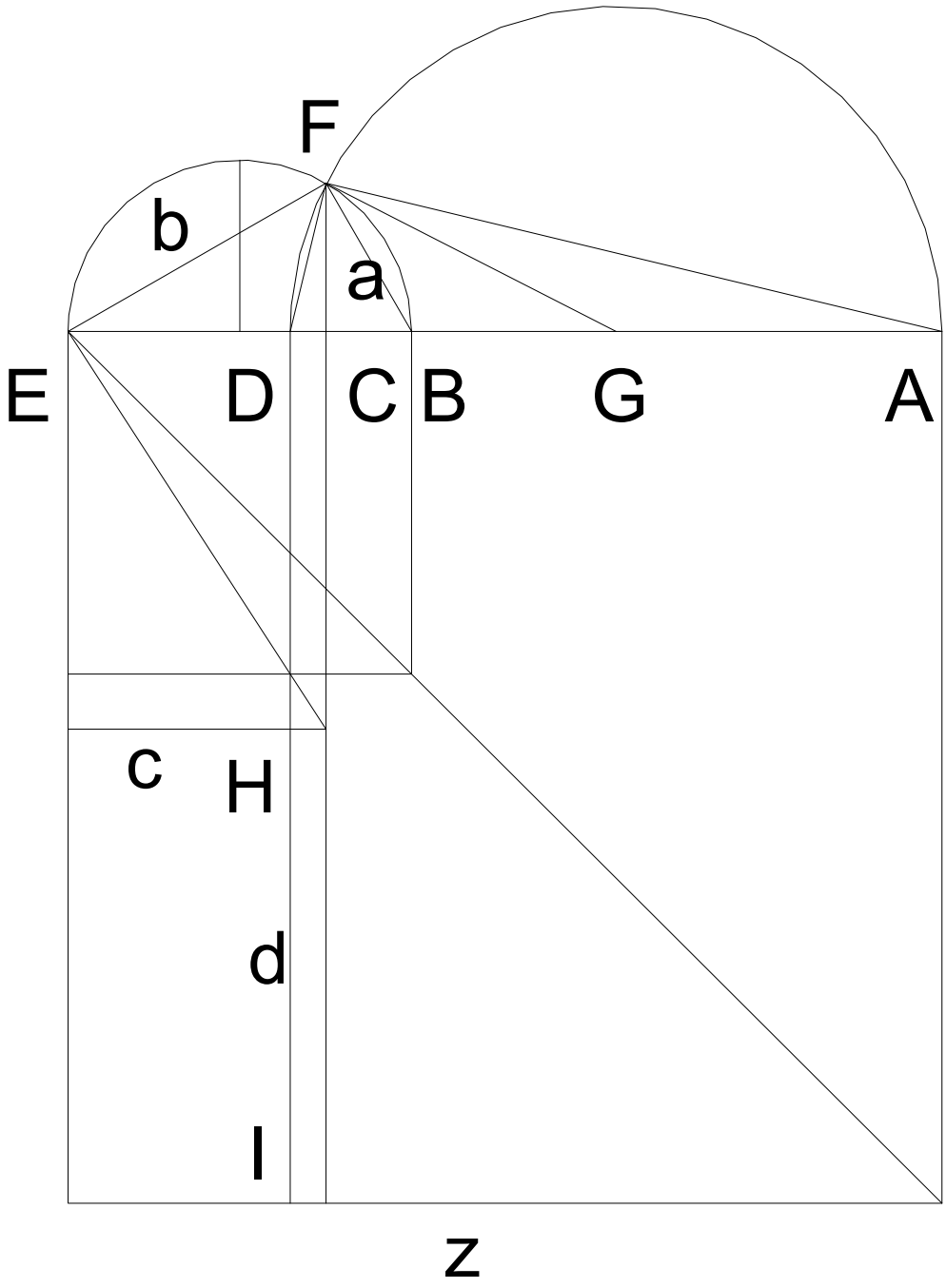
Is this a TRIANGLE = 1 ? Side_1 ≡ 21 Side_2 ≡ 14.4 Side_3 ≡ 7.75




$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$

| |
|---|
| 0 |
| 1 |
| 1 |

| $SU_\delta$ | $UO_\delta$ | $AU_\delta$ |
|---|---|---|
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |
| 14.15083 | 42.45249 | 16.70208 |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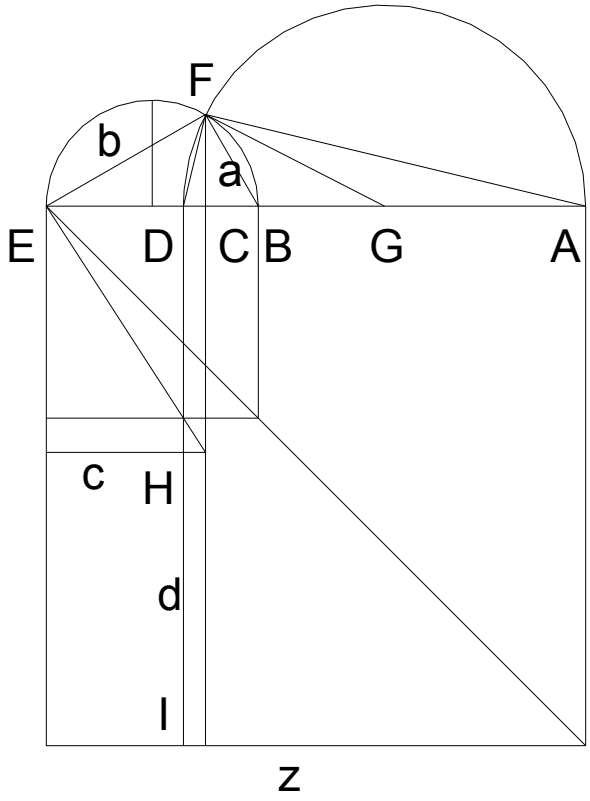
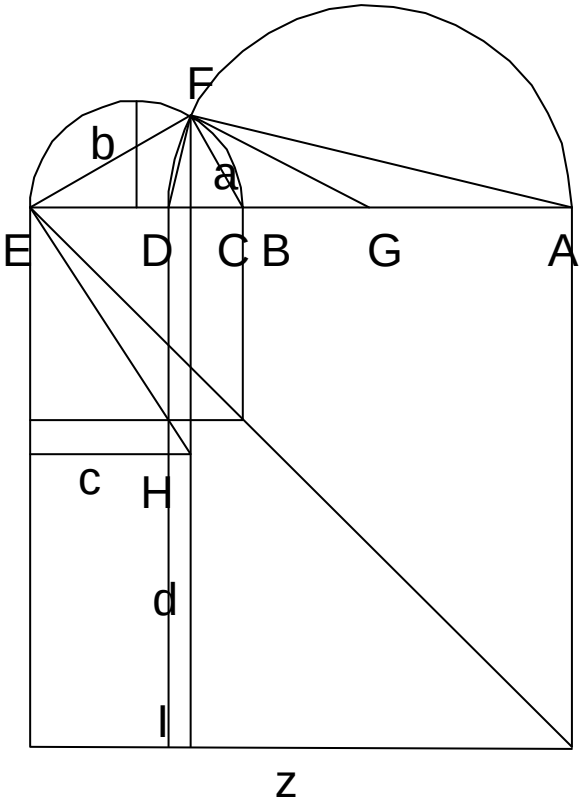



**951216 EP**

Given:

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.

121695.MCD




Given a, n and b for the equation $z^2 = n\,az + b^2 + cd$ find z, c, and d.

$AD := n \cdot a$ $\quad BE := \sqrt{a^2 + b^2}$ $\quad BC := \frac{a^2}{BE}$

$CE := BE - BC$ $\quad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2}$ $\quad CG := \sqrt{FG^2 - CF^2}$

$AG := FG$ $\quad AC := AG + CG$

$BG := CG - BC$ $\quad DG := FG$

$BD := DG - BG$ $\quad AB := AG + BG$

$AE := AB + BE$ $\quad DE := BE - BD$

$DH := \frac{b^2}{DE}$ $\quad DI := AE$ $\quad HI := DI - DH$

$z := AE$ $\quad z = 12.622$

$c := DE$ $\quad c = 0.622$

$d := HI$ $\quad d = 6.186$

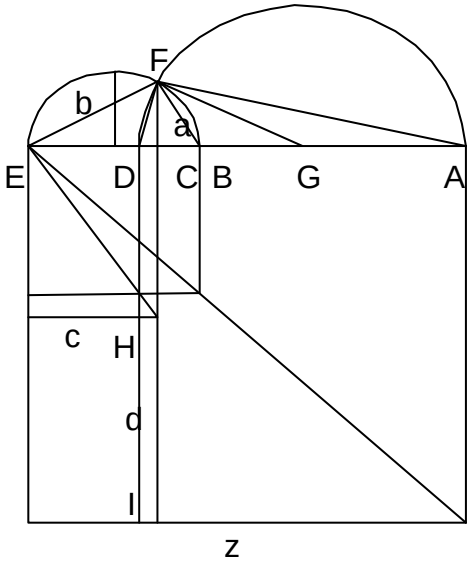
$$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$$

Place values here :

$n \equiv 3$

$a \equiv 4$

$b \equiv 2$



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d := 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}$$

$$c := \frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving z.

$$z := \frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}$$

$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p := -a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

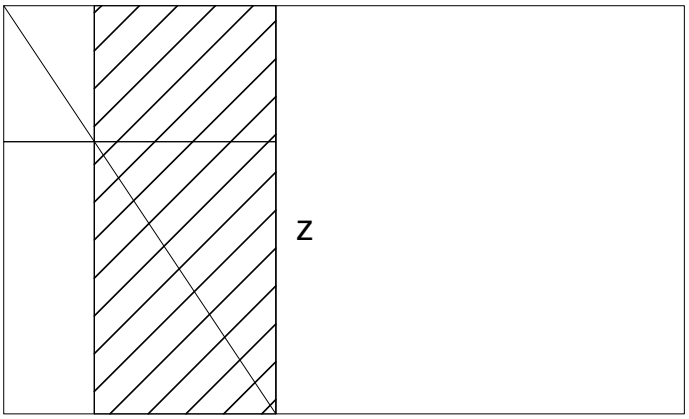
Solve for z below.

$$\begin{bmatrix}\frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\end{bmatrix}$$

Scholia: The Geometry of Rene Descartes
with a facsimile of the first edition.
Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ one can see constants in the figure for solving when only a and b are given.




$b := 2.12 \qquad z := 1.41$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$c := \frac{b^2}{z} \qquad a := z + c$

We find that this c has another relation to z, for i holds a proportion to it in the given equation.

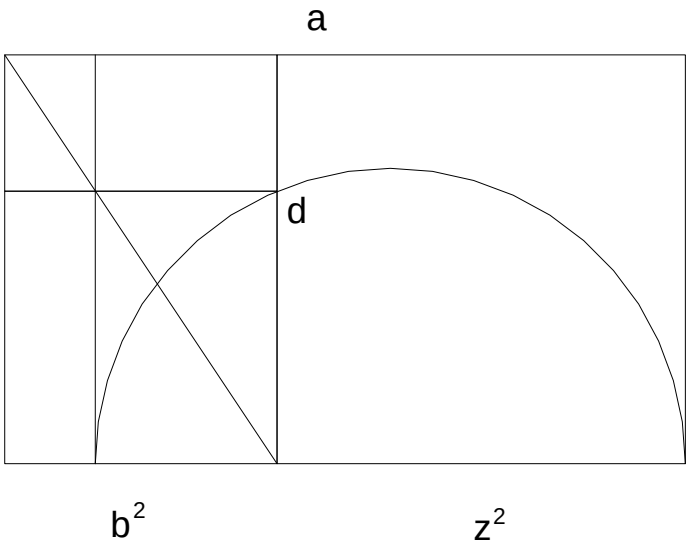
$\left(c^2 + b^2\right) - a \cdot c = 0$

$\left(c^2 + b^2\right) - ((z + c) \cdot c) = 0$

$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

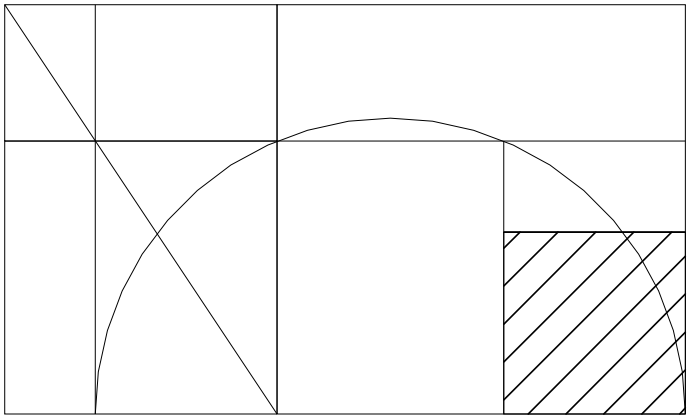
$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

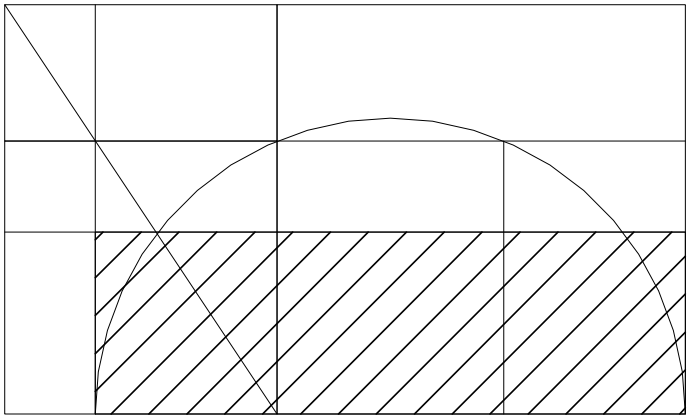



Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

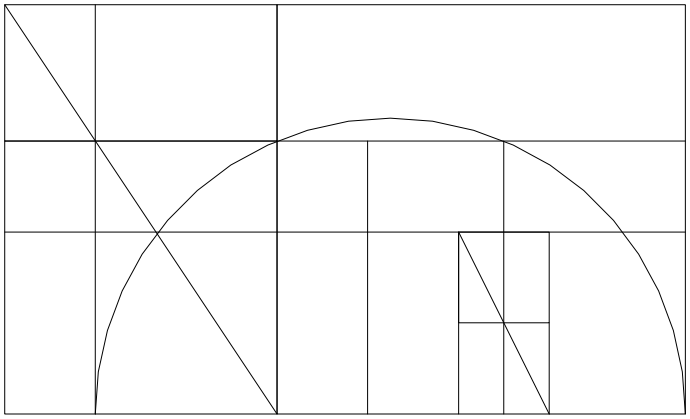
One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



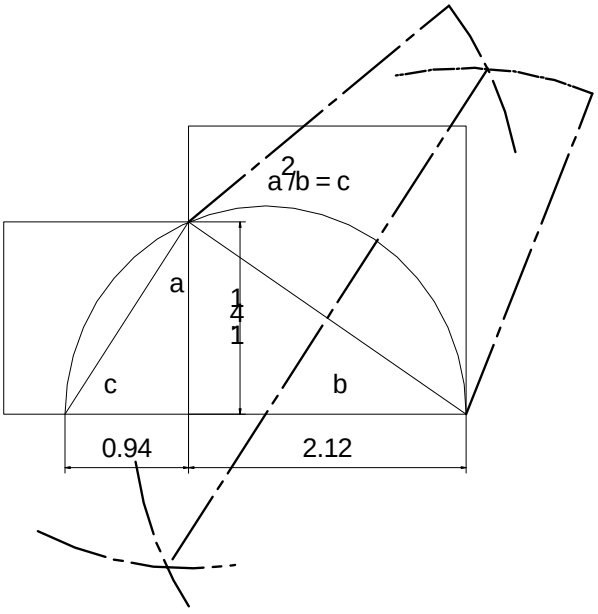
$b^2$ $z^2$



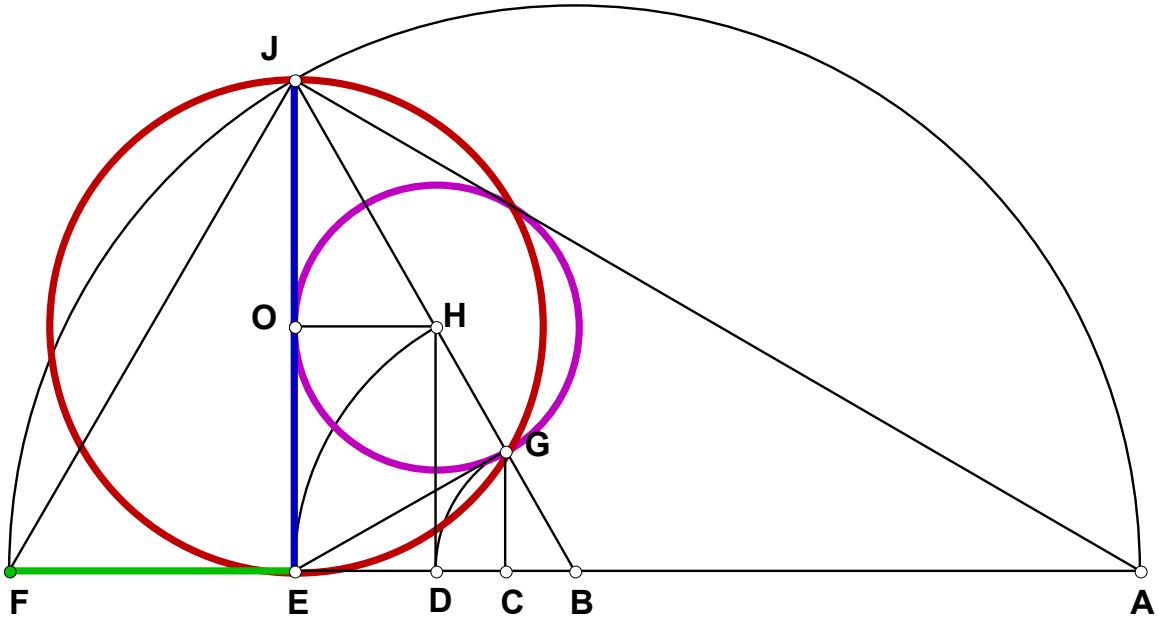
$b^2$ $z^2$



$b^2$ $z^2$




The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.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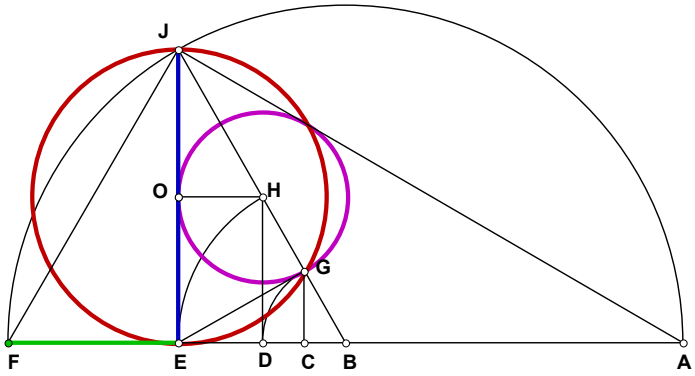
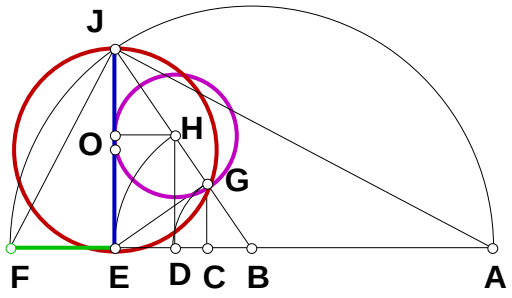
# 951220 EP




Given:

# 122095 Just For Fun




$N := 2$

$EF := 1 \qquad EJ := EF \cdot N \qquad AE := \frac{EJ^2}{EF} \qquad AF := AE + EF$

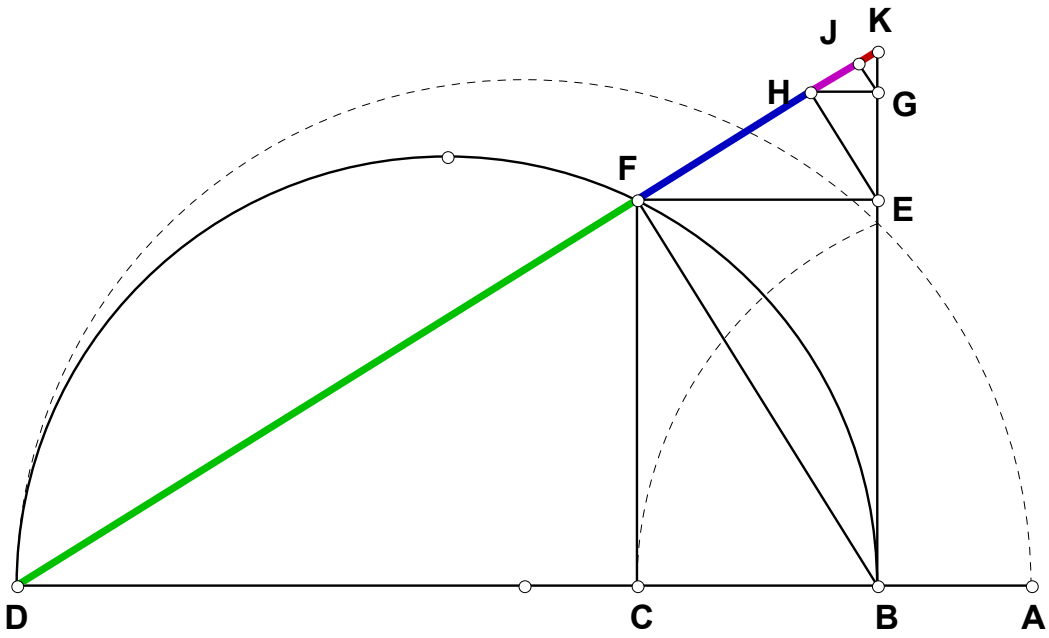
$AB := \frac{AF}{2} \qquad BF := AB \qquad BE := BF - EF \qquad BH := BE$

$BJ := \sqrt{EJ^2 + BE^2} \qquad BD := \frac{BE \cdot BH}{BJ} \qquad BG := BD$

$BC := \frac{BE \cdot BG}{BJ} \qquad GH := BH - BG \qquad DE := BE - BD \qquad HO := DE \qquad EG_1 := \sqrt{BE^2 - BG^2} \qquad GJ := BJ - BG$

$EG_2 := \sqrt{EJ^2 - GJ^2} \qquad \frac{GH}{HO} = 1 \qquad \frac{EG_1}{EG_2} = 1 \qquad \left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \qquad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

$$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2+1\right)^2} = 0$$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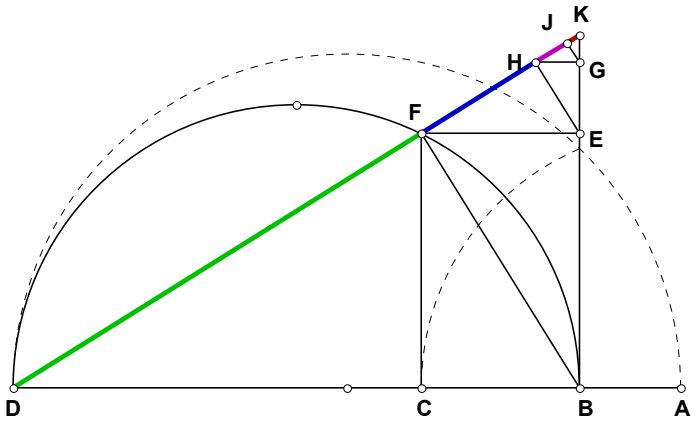
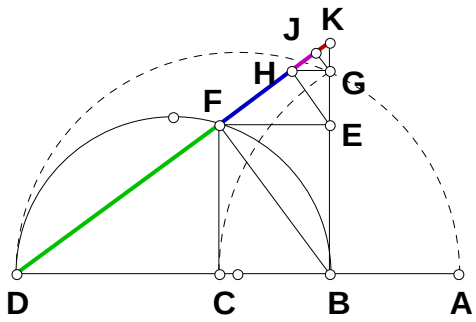
# 951221 EP




Given:

## 122195 Pascal's Triangle With Exponential Division




$N := 7$

$AB := 1 \quad AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD}$

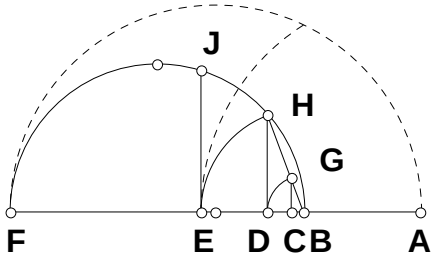
$BD := AD - AB \quad CD := AD - AC \quad BC := BD - CD$

$CF := \sqrt{BC \cdot CD} \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{DK}{JK} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

### A more civil figure.

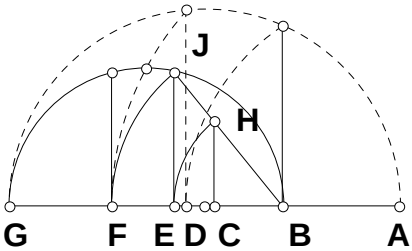



$AB := 1 \quad AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF}$

$BE := AE - AB \quad BH := BE \quad BD := \frac{BE \cdot BH}{BF} \quad BG := BD$

$BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \qquad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \qquad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

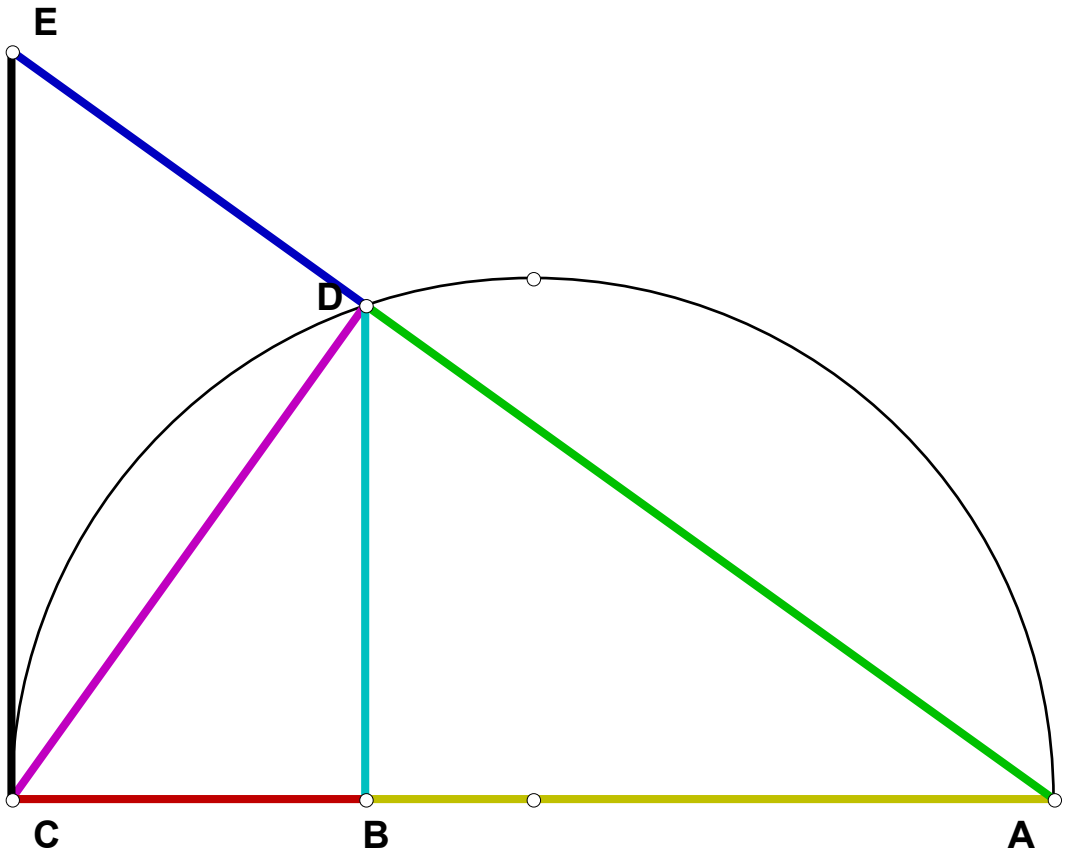



$AB := 1 \quad AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}}$

$BF := AF - AB \quad BJ := BF \quad BE := \frac{BJ \cdot BF}{BG} \quad BH := BE$

$BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \qquad \frac{BG}{BE} - \frac{N^2 - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0 \qquad \frac{BG}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$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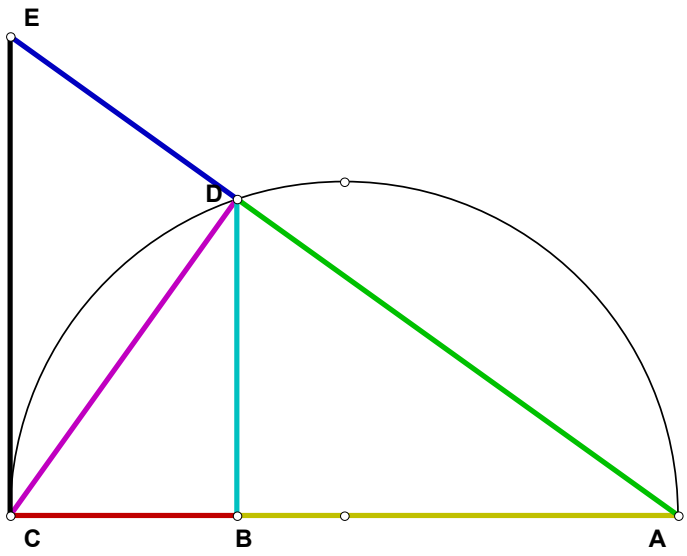
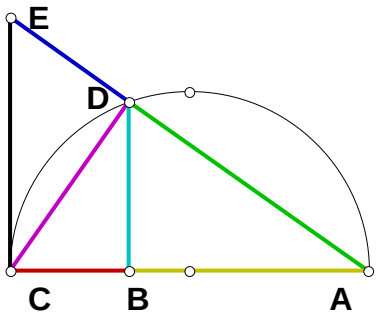
## 951229 EP




Given:

# 122995

## Given AC and CE find BC.




$N_1 := 4 \qquad N_2 := 9$

$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$

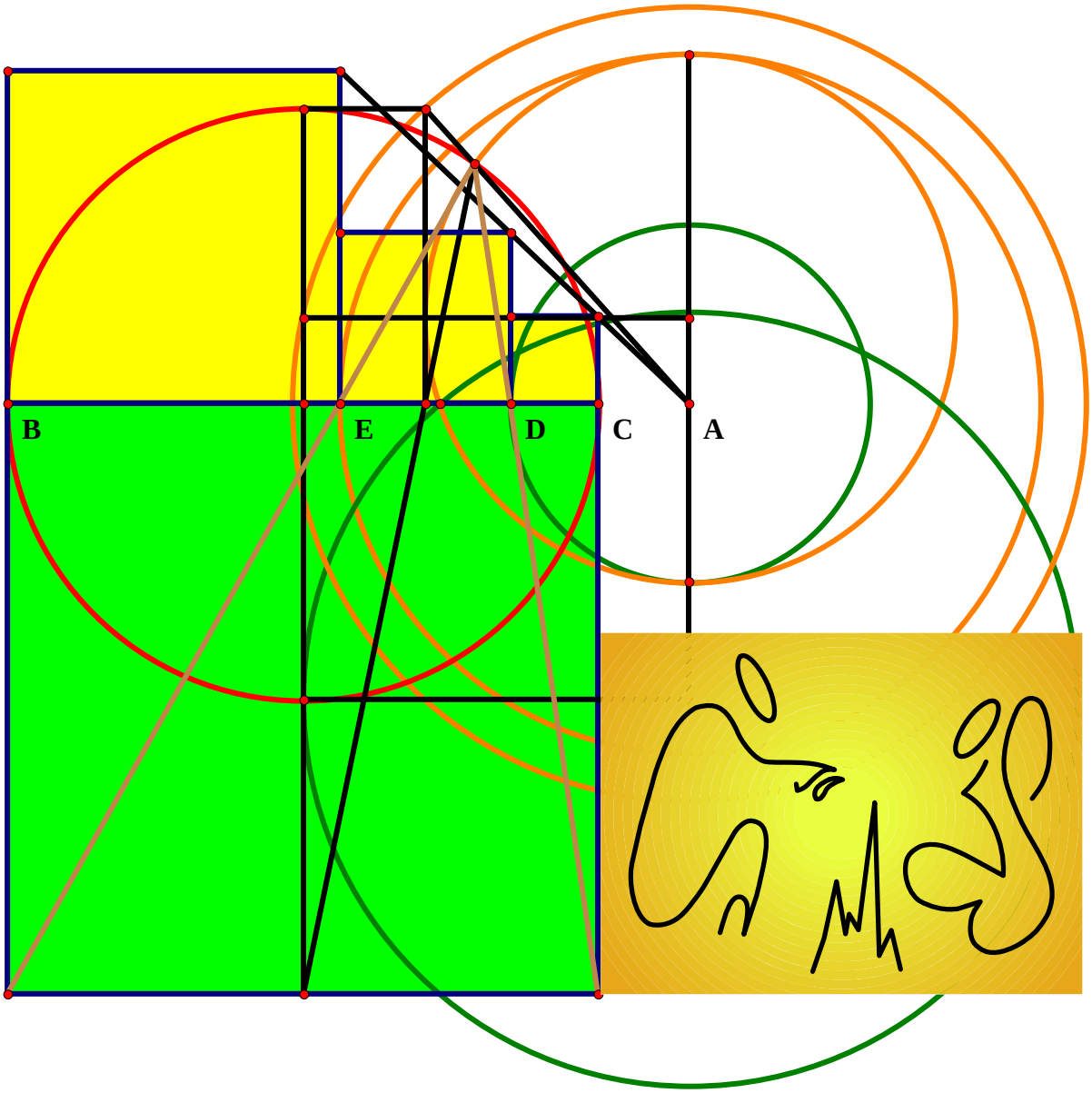
$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$

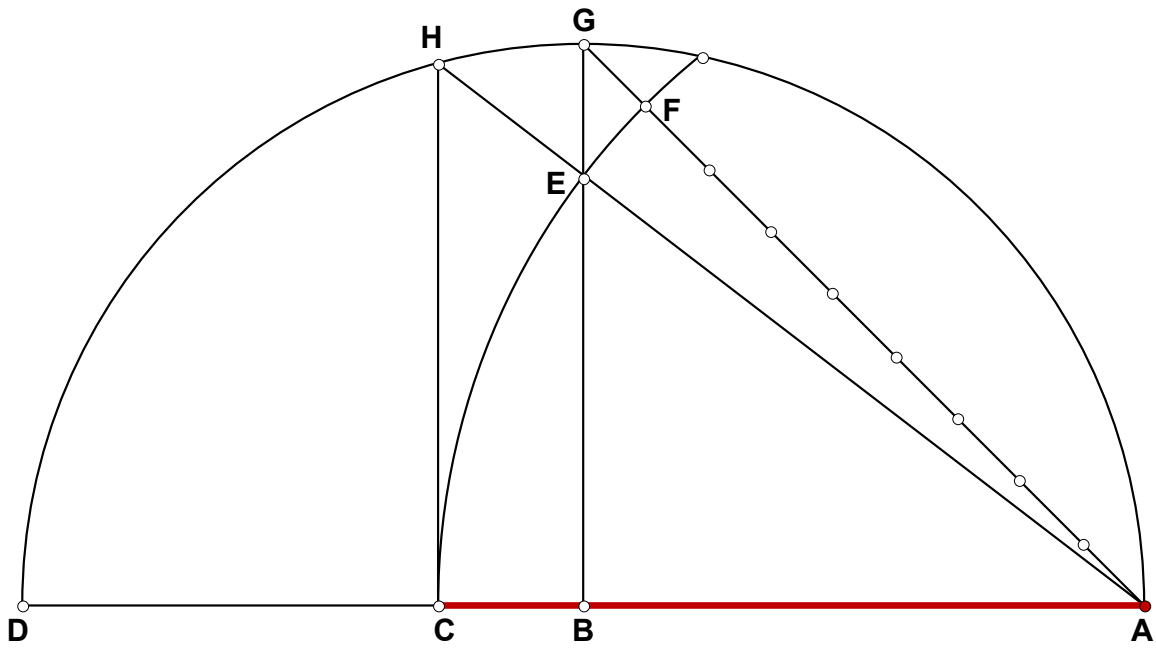
$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$

$$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \qquad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \qquad AB - \left( N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} \right) = 0$$

$$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \qquad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$$

# The Delian Quest—1996

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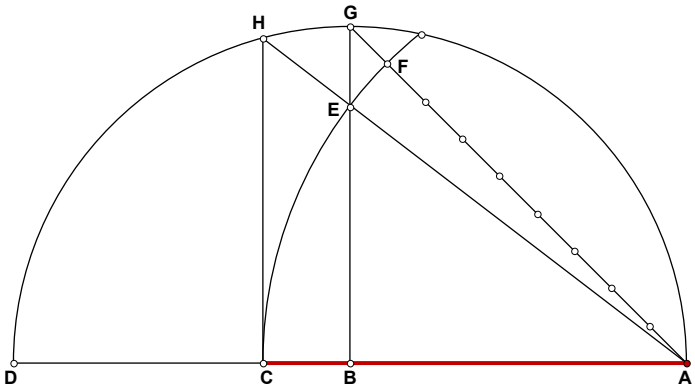
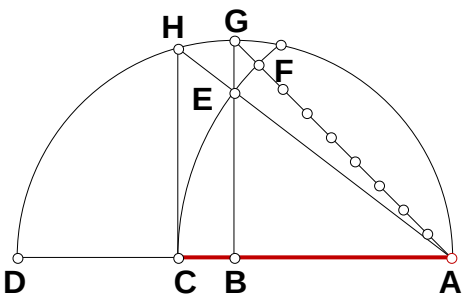
# 960107 EP




Given:

## 010796 Rusty Cubes



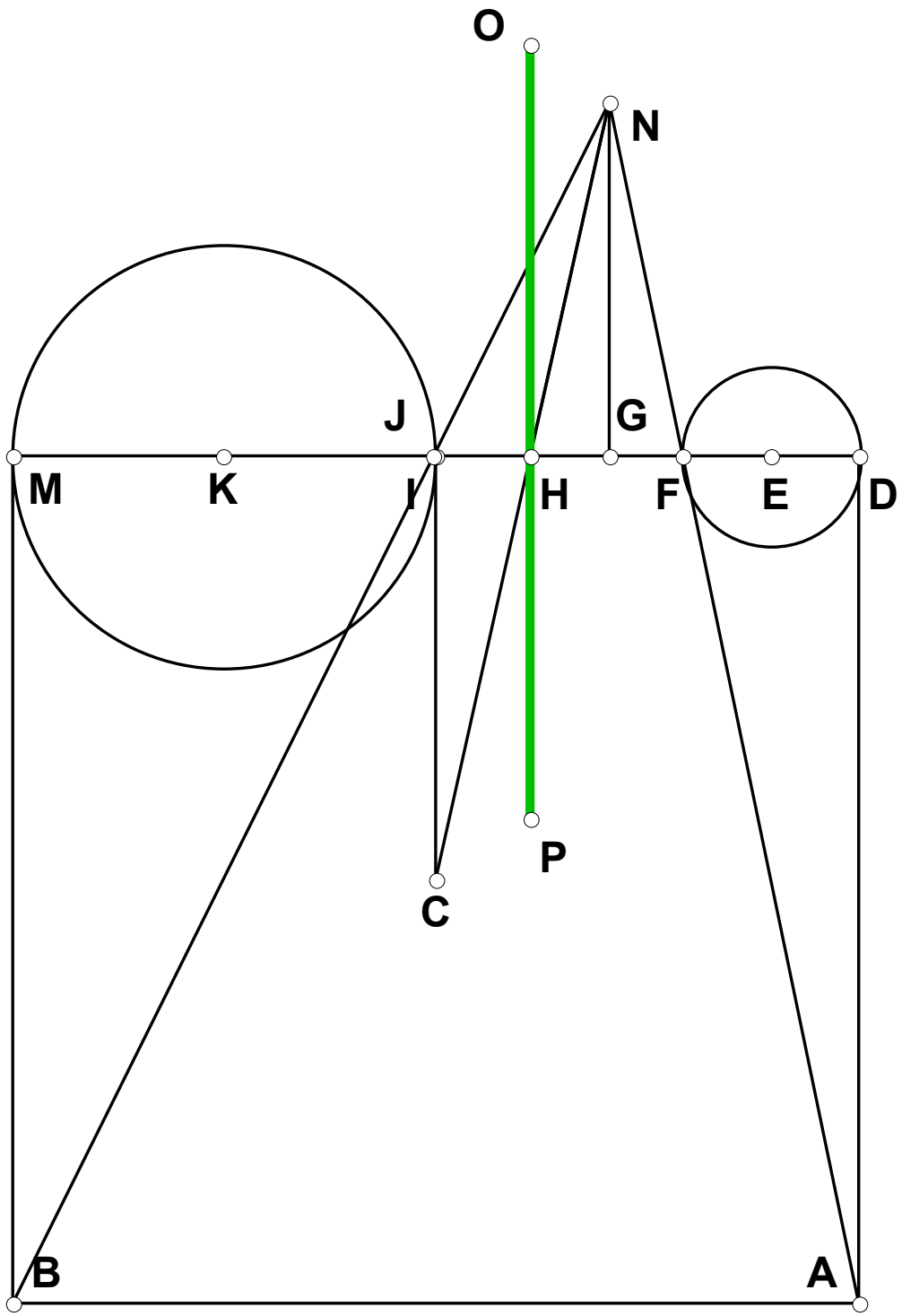

**A rusty Compass construction for the duplication of the cube.**

$AD := 2$ $\quad AB := \frac{AD}{2}$ $\quad AG := \sqrt{2 \cdot AB^2}$ $\quad AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $\quad AC = 1.257$

$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.26$ $\quad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002$

**I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.**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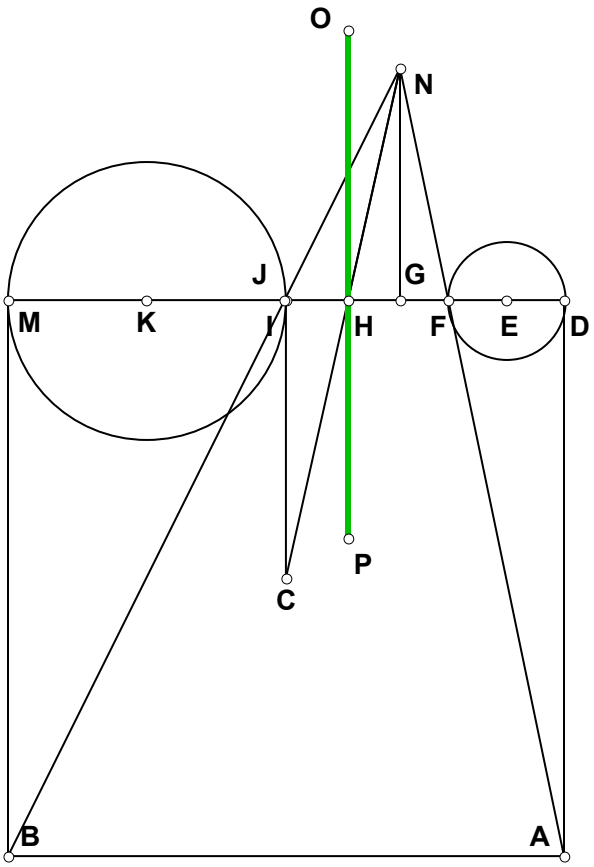
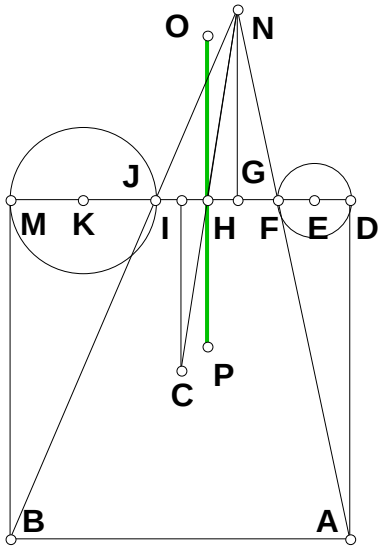
## 960108 EP




Given:

# 010896 Alternate Method Power Line




$R_1 := 2 \qquad R_2 := 5 \qquad D := 4$

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + JK) \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \qquad GJ := FJ - FG \qquad DI := \frac{DM}{2}$

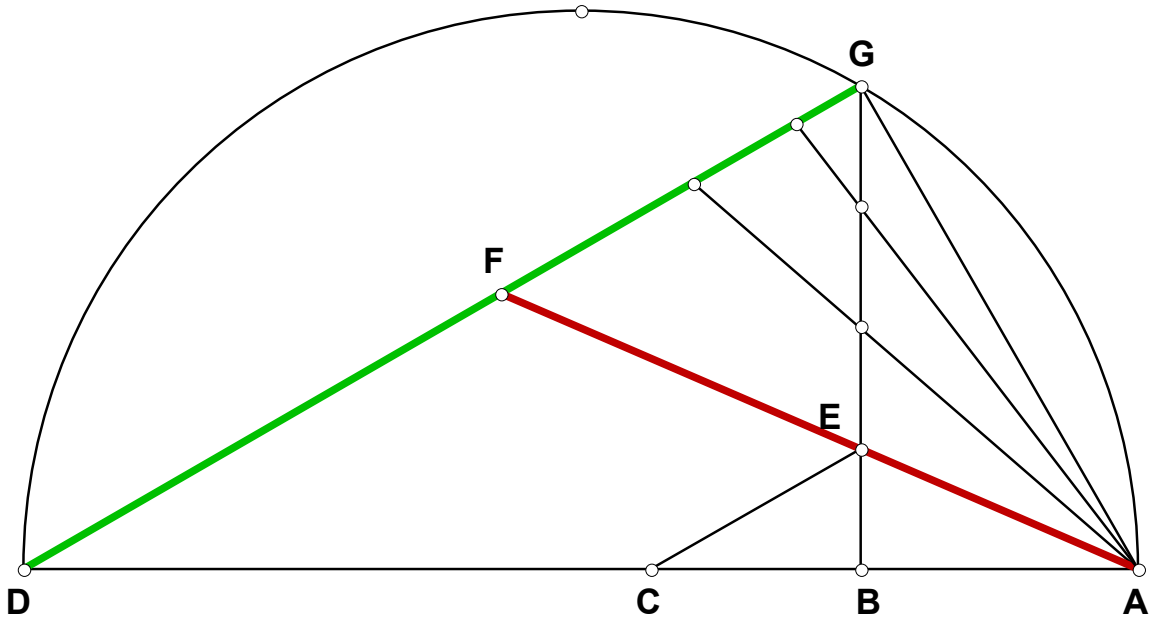
$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

$HM := DM - DH$

$$DH - \frac{\left(R_1 + R_2 + D\right) \cdot \left(R_1 - R_2 + D\right)}{2 \cdot D} = 0$$

$$HM - \frac{\left(R_2 + R_1 + D\right) \cdot \left(R_2 - R_1 + D\right)}{2 \cdot D} = 0$$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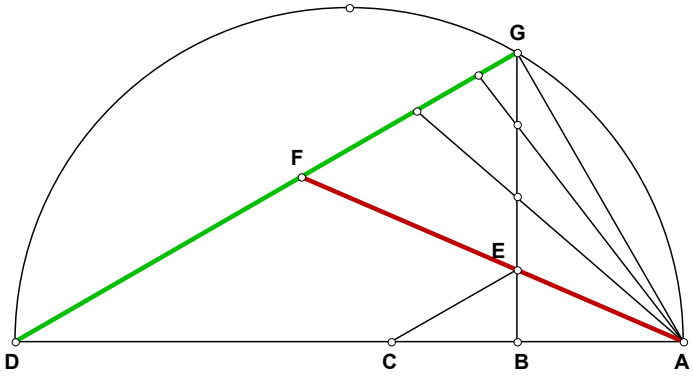
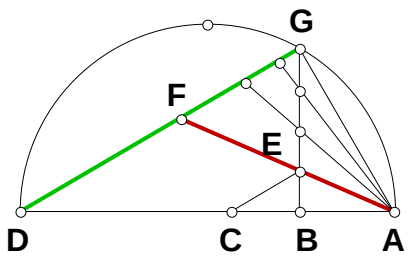
# 960113 EP




Given:

# 011396 Pyramid of Ratios VI, Moving the Point




$N_1 := 5 \quad N_2 := 7 \qquad N_1 := N_1 \quad N_2 := N_2$

$AD := 1 \quad AB := \frac{AD}{N_1} \qquad BD := AD - AB$

$BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG}{N_2} \qquad BC := \frac{BD \cdot BE}{BG}$

$AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \quad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0$

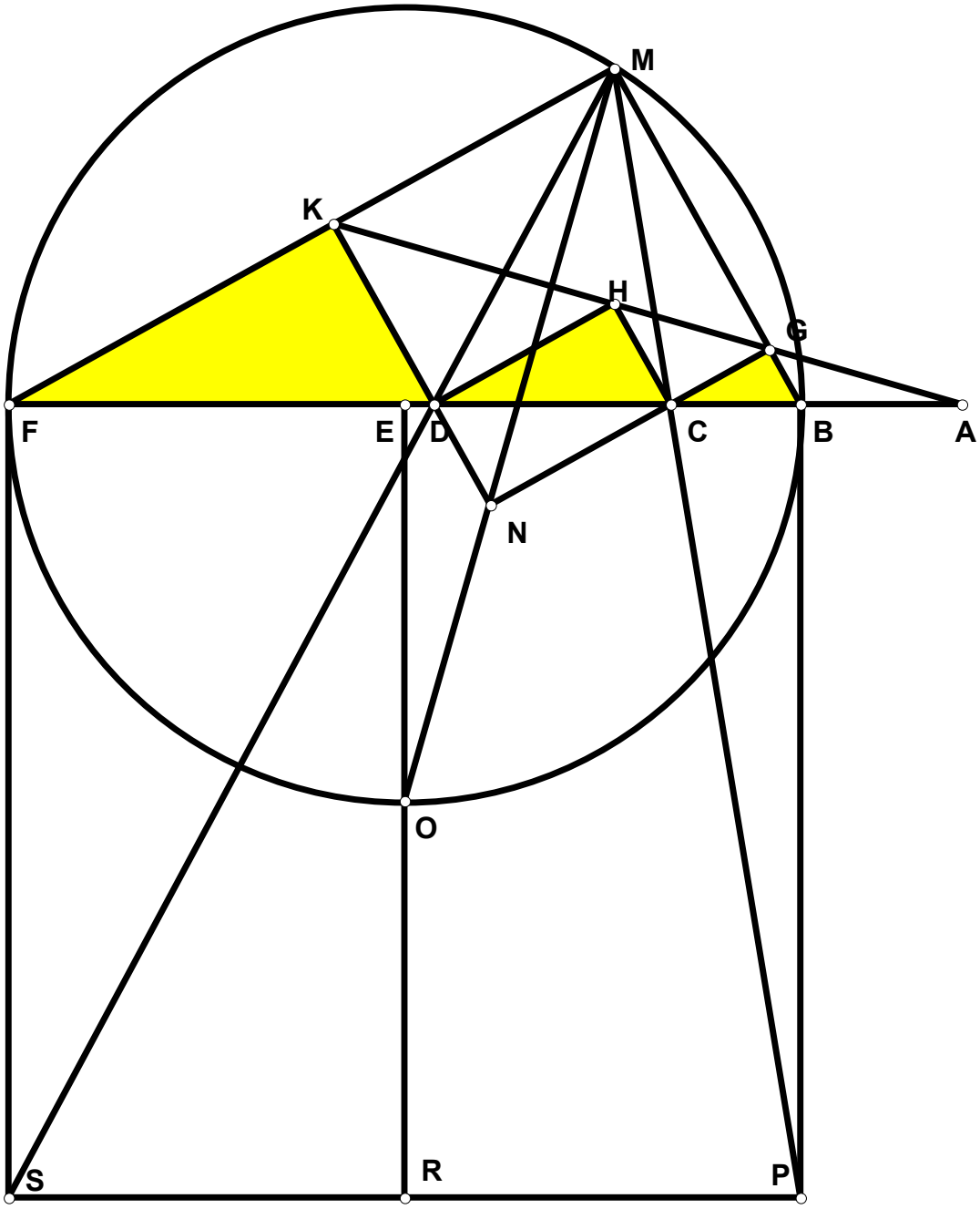
$\Delta := 2 .. N_1 \qquad \delta := 2 .. N_2$

$SeriesAF_{\Delta,\delta} := \frac{\Delta \cdot \delta}{(\Delta - 1) \cdot (\delta - 1)} \qquad SeriesAF = \begin{bmatrix} 4 & 3 & 2.667 & 2.5 & 2.4 & 2.333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.667 & 2 & 1.778 & 1.667 & 1.6 & 1.556 \\ 2.5 & 1.875 & 1.667 & 1.563 & 1.5 & 1.458 \end{bmatrix}$

$DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2} \quad DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF$

$\frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$

$SeriesDG_{\Delta,\delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)} \qquad SeriesDG = \begin{bmatrix} 3 & 2 & 1.667 & 1.5 & 1.4 & 1.333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333 & 2 & 1.8 & 1.667 \\ 6 & 3.5 & 2.667 & 2.25 & 2 & 1.833 \end{bmatrix}$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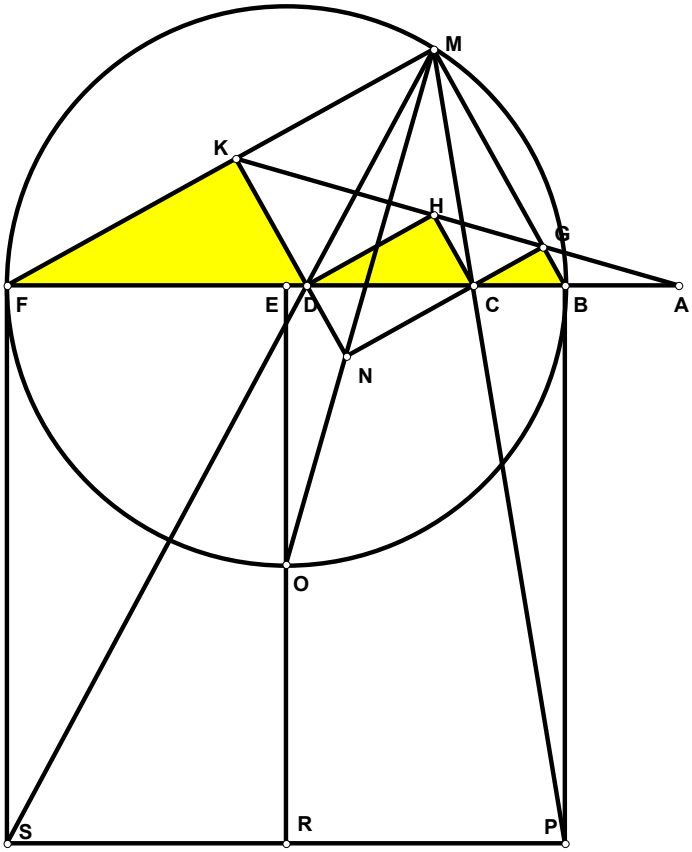
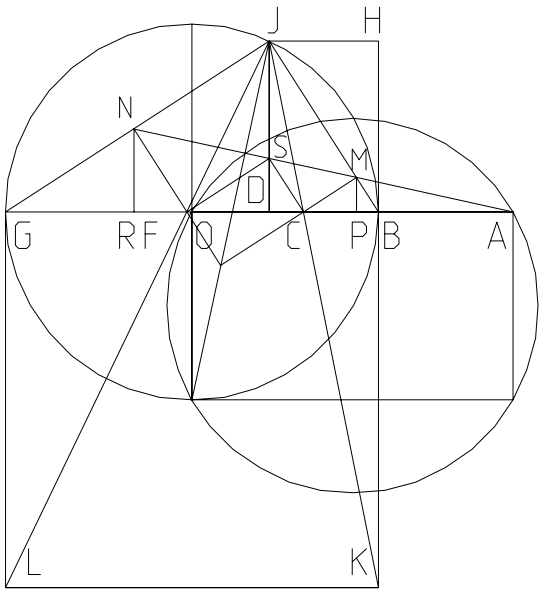
# 960116 EP




Given:

**011696**

**The figure cuts the base in Cube Roots and provides some interesting ratios.**




$N := 10$

$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$

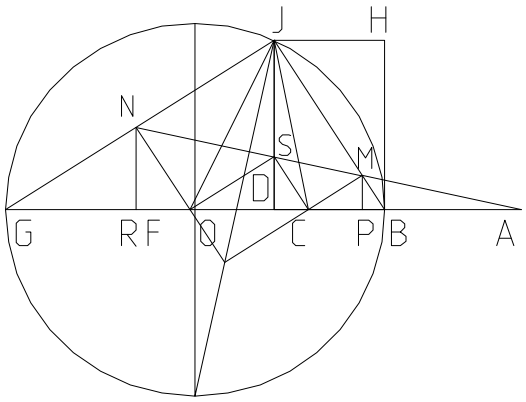
$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$

$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.68 \qquad \frac{GJ}{GN} = 1.68 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.68$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796 \qquad \frac{BJ}{BM} = 7.796 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796$




$$CF := AF - AC \quad BP := \frac{BD \cdot BC}{BG} \quad CD := \frac{BD \cdot CF}{BG}$$

$$FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.982$$

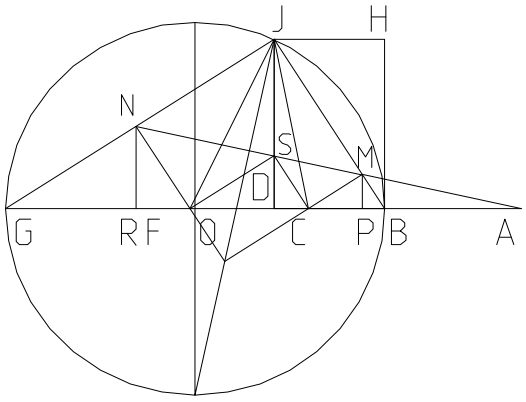
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.982 \qquad \frac{BG}{BP} = 43.982$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.415$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.415 \qquad \frac{BG}{CD} = 20.415$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.476$$

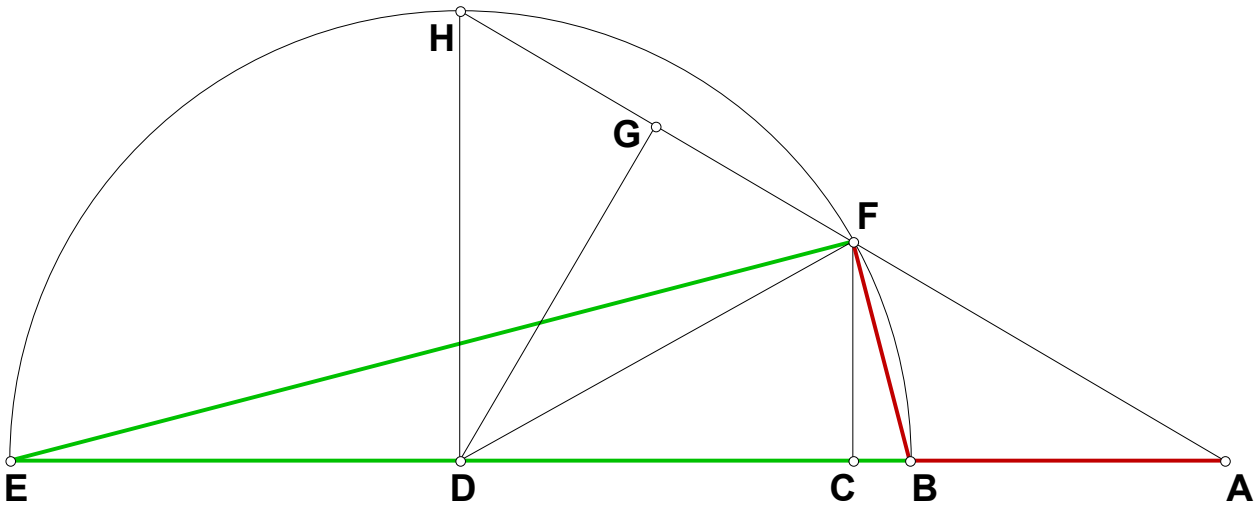
$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.476 \qquad \frac{BG}{FR} = 9.476$$




$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.869 \qquad \frac{AG}{BP} = 48.869 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.869$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.683 \qquad \frac{AG}{CD} = 22.683 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.683$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528 \qquad \frac{AG}{FR} = 10.528 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528$$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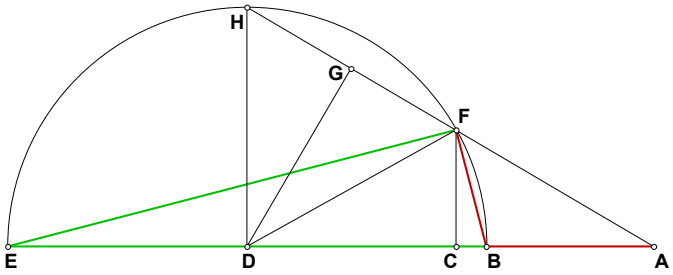
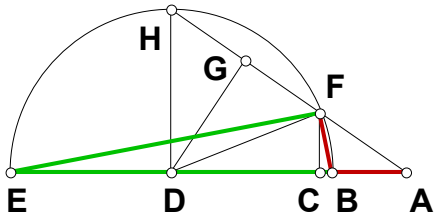
# 960117A EP




Given:

# 011796A Right Triangle In A Given Ratio

**Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.**




$N := 3$

$AB := 1 \quad AE := AB \cdot N \quad BE := AE - AB$

$BD := \frac{BE}{2} \quad DF := BD \quad DE := BD \quad AD := AB + BD$

$DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

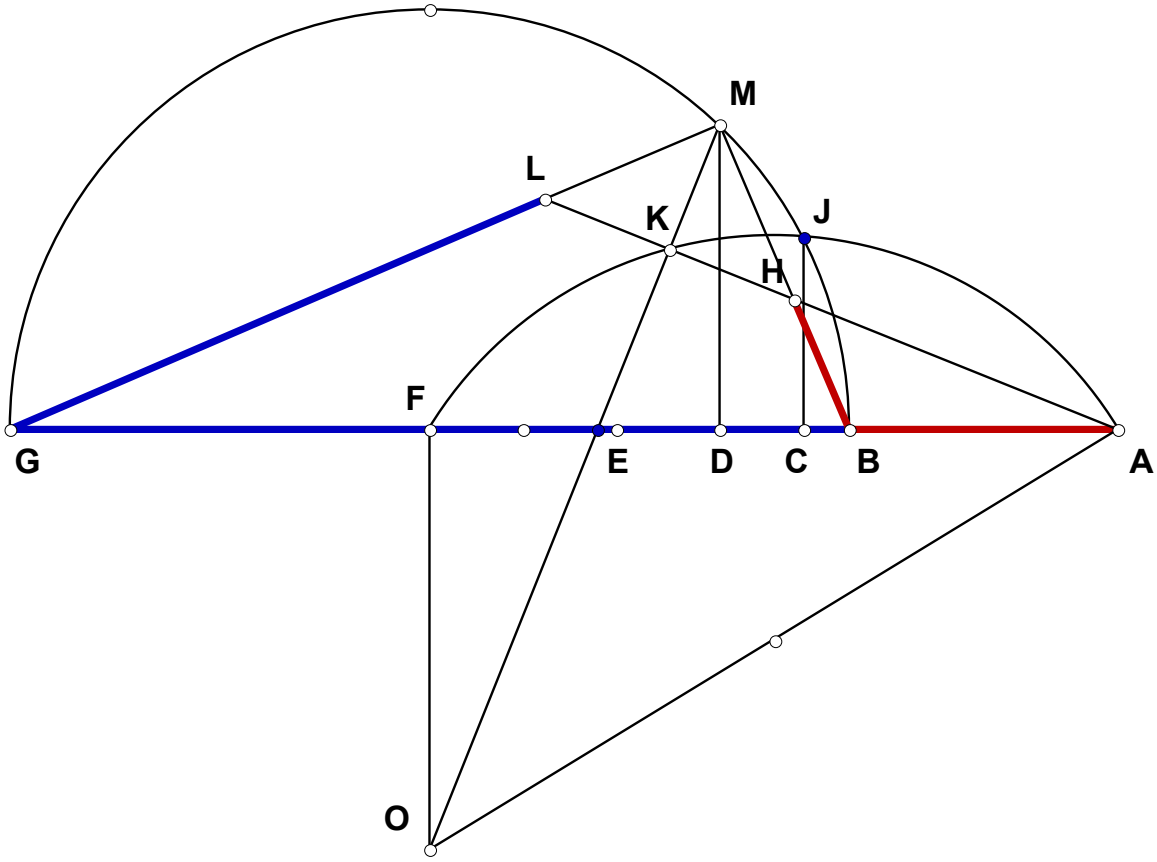
$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$S_1 := AD \quad S_2 := AF \quad S_3 := DF$

$CD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad BC := BD - CD$

$CE := CD + DE \quad CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2} \quad AC := AD - CD$

$\frac{AE}{AB} - \frac{EF}{BF} = 0 \qquad AC - \frac{N^2 + N}{N^2 + 1} = 0 \qquad BF - \frac{N - 1}{\sqrt{N^2 + 1}} = 0 \qquad EF - \frac{N^2 - N}{\sqrt{N^2 + 1}} = 0$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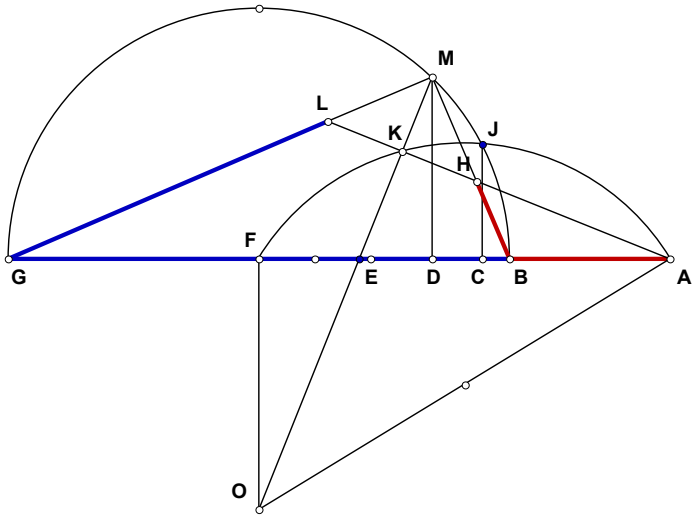
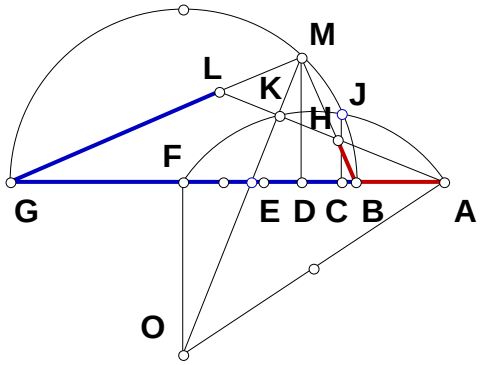



# 960117B EP

Given:

# 011796B Divide The Sides Of A Triangle In A Given Ratio

**Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.**




$N_1 := 5 \qquad N_2 := 9$

$AB := 1 \qquad AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \frac{BG}{2}$

$AC := \frac{N_1^2 + N_1}{N_1^2 + 1} \qquad AF := AB + BF \quad CF := AF - AC$

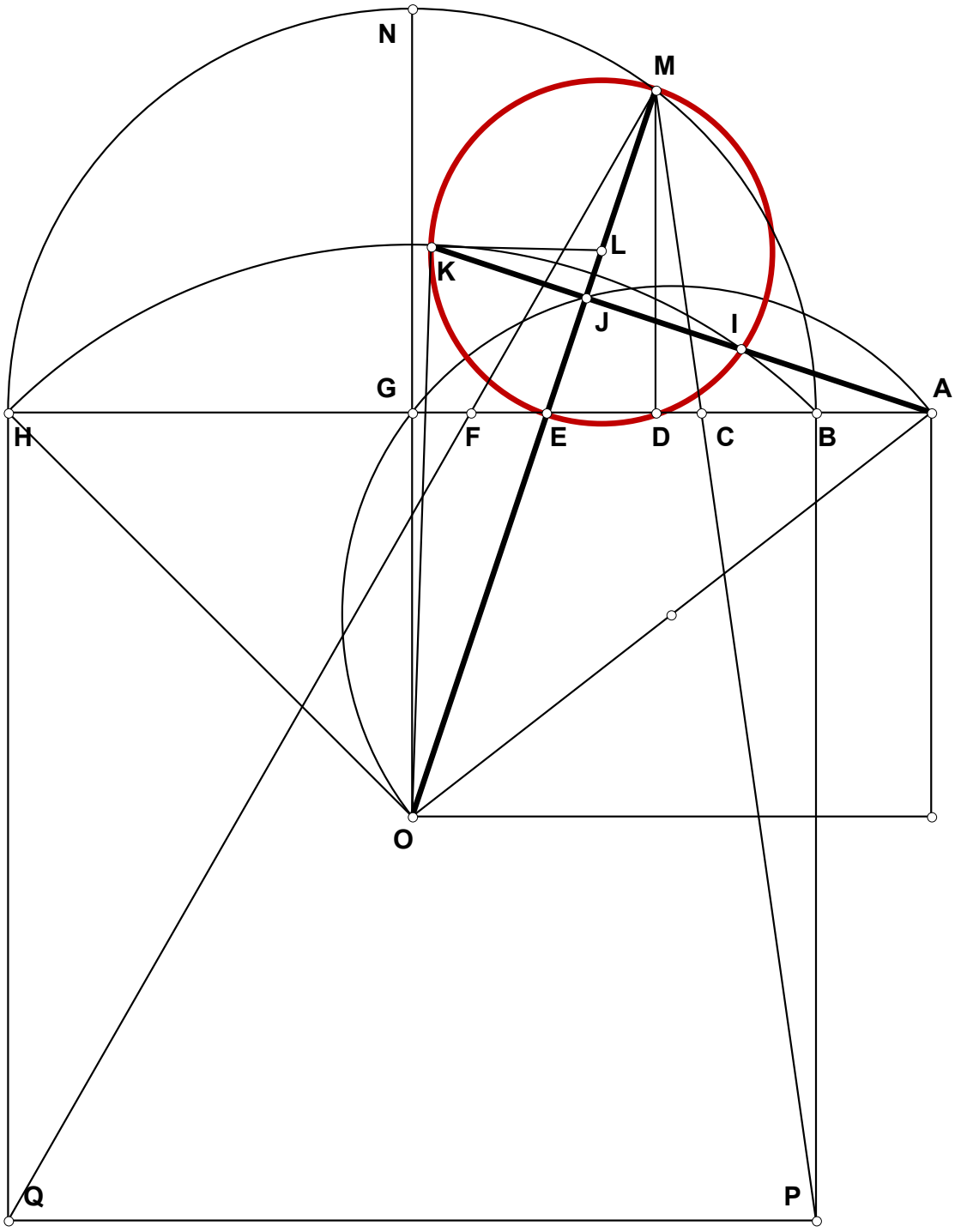
$BC := AC - AB \qquad BD := \frac{CF}{N_2} + BC \qquad DF := BF - BD$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF \quad EF := \frac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \frac{EO \cdot (DM + FO)}{FO} \quad EK := \frac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO) \quad HK := KM \quad HM := \sqrt{2 \cdot KM^2}$

$BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM \quad GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM$

$\frac{AG}{AB} - \frac{GL}{BH} = 0$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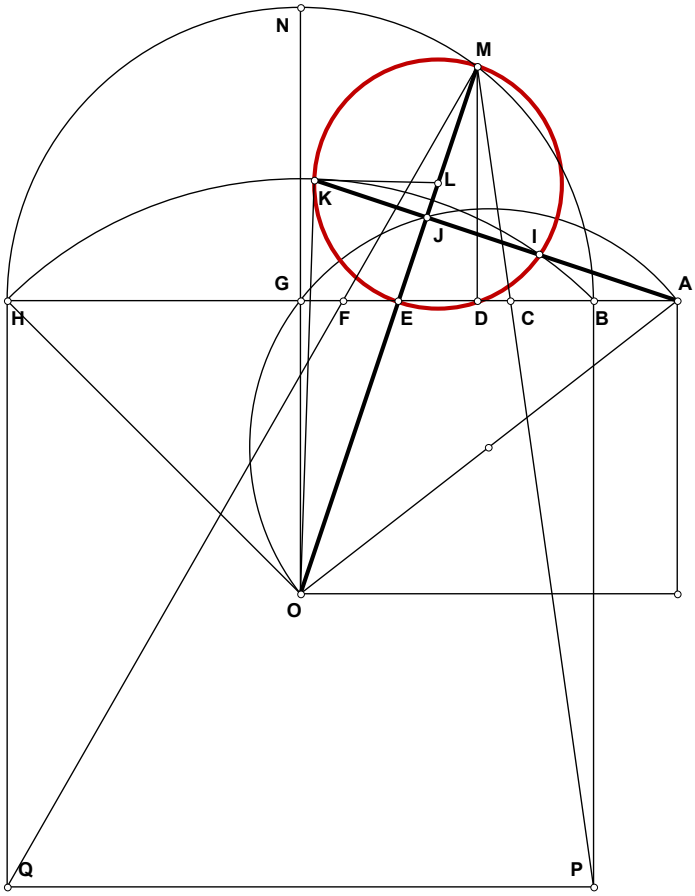
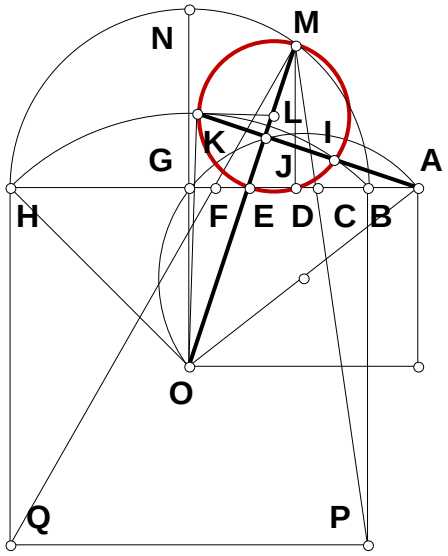



# 960121 EP

Given:

# 012196 More On Cube Roots




$N := 5$ $BH := 1$

$BP := BH$ $HQ := BH$

$BG := \frac{BH}{2}$ $GO := BG$ $GN := BG$ $NO := BH$ $GH := BG$

$BE := \frac{BG}{N}$ $EG := BG - BE$ $EO := \sqrt{EG^2 + GO^2}$

$MO := \frac{GO \cdot NO}{EO}$ $EM := MO - EO$ $EL := \frac{EM}{2}$ $LK := EL$

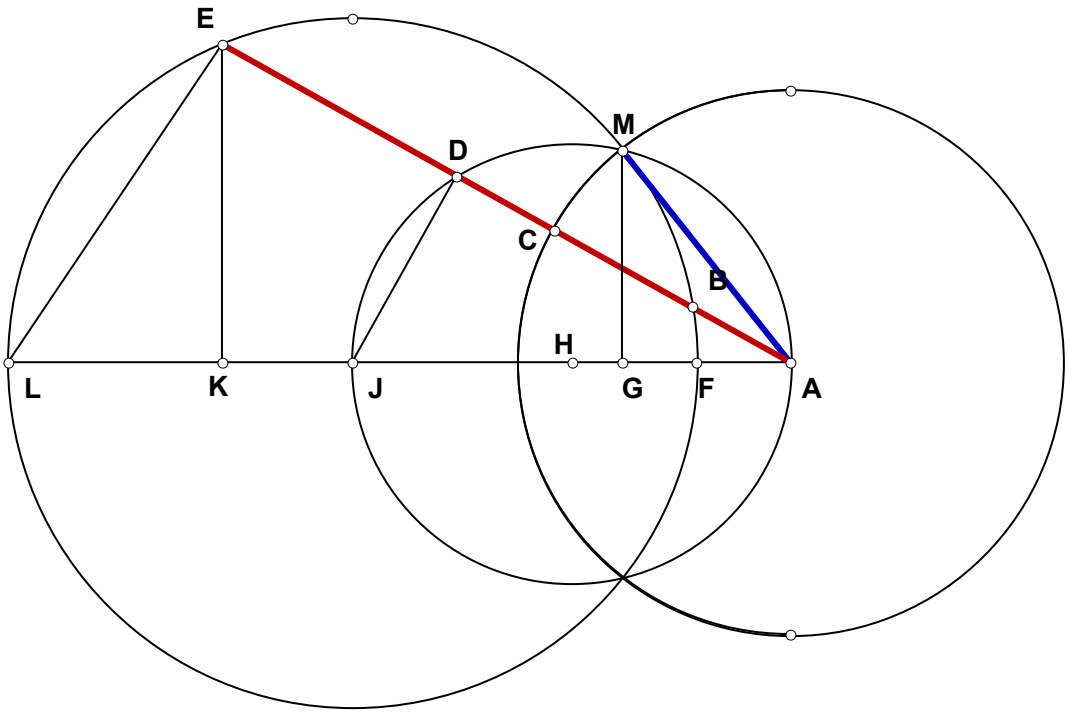
$LO := EO + EL$ $LJ := \frac{LK^2}{LO}$ $EJ := EL - LJ$

$AE := \frac{EO \cdot EJ}{EG}$ $AH := AE + EG + GH$ $AB := AH - BH$

$DE := \frac{EG \cdot EM}{EO}$ $DM := \frac{GO \cdot EM}{EO}$ $BD := BG - (EG + DE)$

$BC := \frac{BD \cdot BP}{BP + DM}$ $DH := BH - BD$ $DF := \frac{DH \cdot DM}{DM + HQ}$ $AC := AB + BC$ $AF := AB + BD + DF$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0$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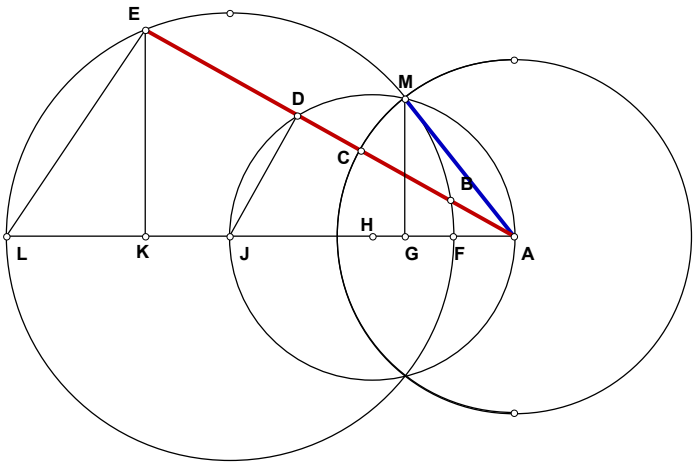
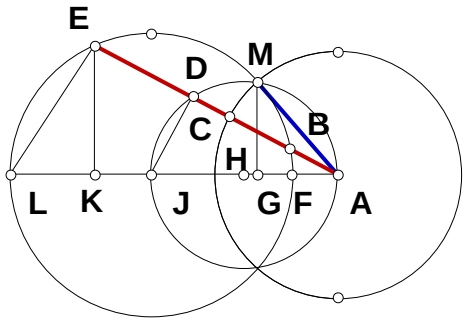



# 960122 EP

Given:

# 012296 Trivial Method Square Root




**For any E between M and L, AM is the square root of AB x AE.**

$N_1 := 6 \qquad N_2 := 3$

$AF := 1 \quad AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

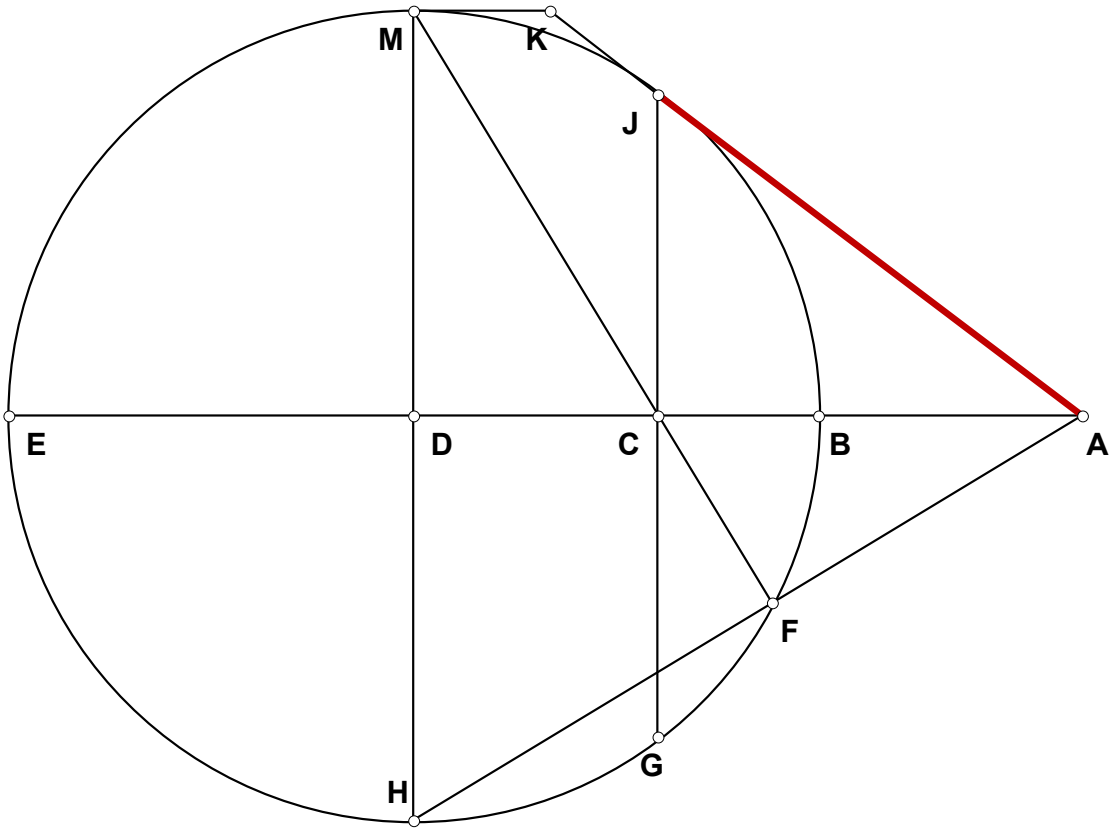
$GL := AL - AG \quad GK := \frac{GL}{N_2} \qquad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$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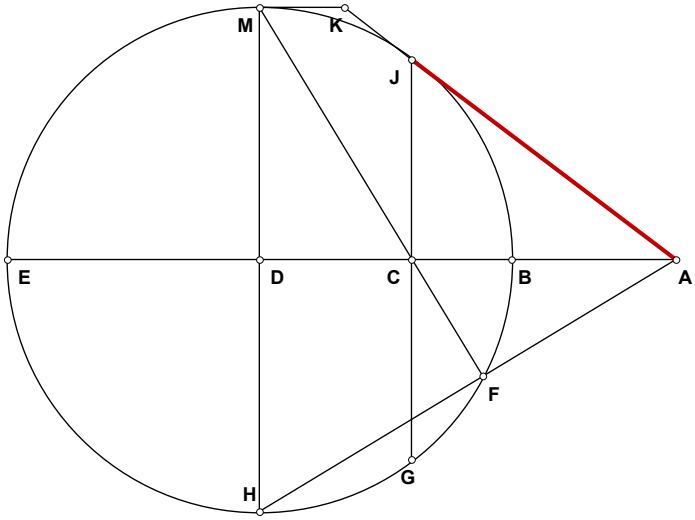
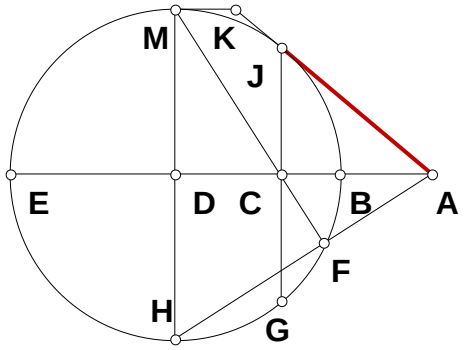



# 960124 EP

Given:

# 012496 Tangent

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**




$N := 5 \qquad AB := 1 \qquad AE := AB \cdot N$

$BE := AE - AB \qquad BD := \frac{BE}{2} \qquad AD := AB + BD$

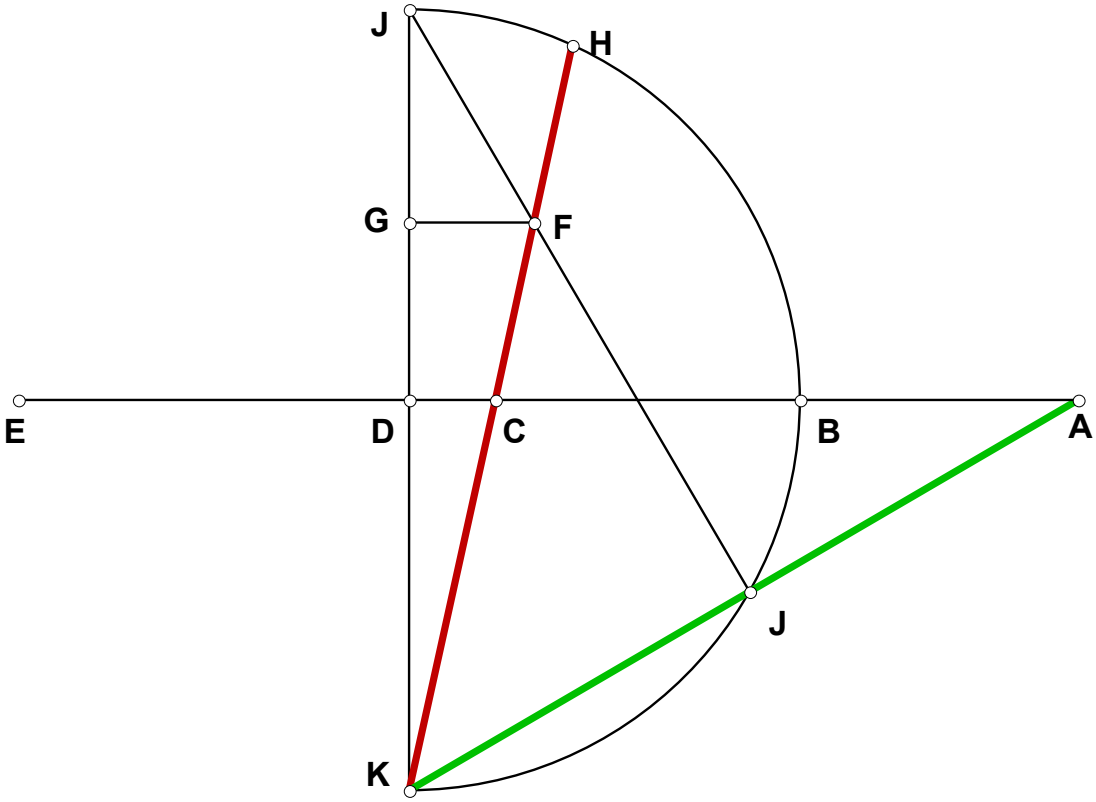
$DM := BD \qquad DH := BD \qquad CD := \frac{DH \cdot DM}{AD}$

$BC := BD - CD \qquad CE := BE - BC$

$CJ := \sqrt{BC \cdot CE} \qquad AJ := \sqrt{(AB + BC)^2 + CJ^2}$

$AJ - \sqrt{N} = 0$

$AC := AB + BC \qquad 2 \cdot AB \cdot \frac{N}{(1 + N)} - AC = 0$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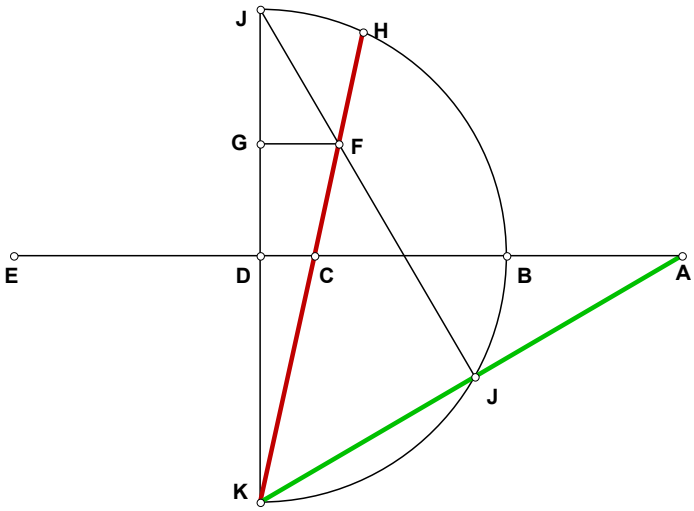
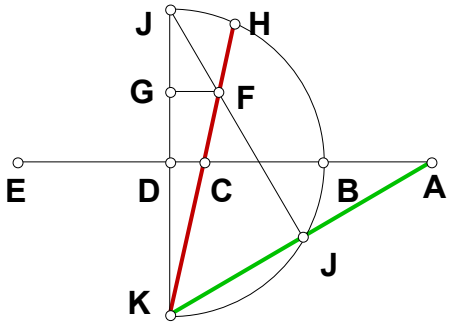
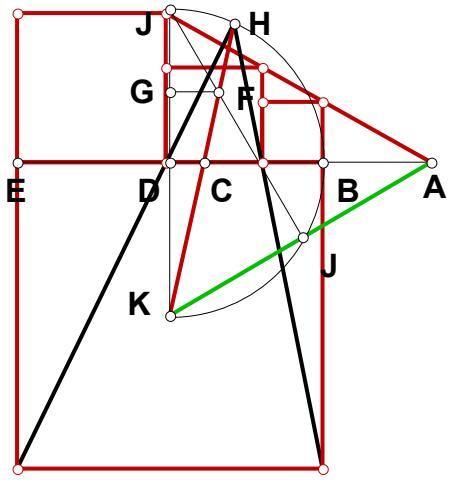



# 960125 EP

Given:

## 012596 On Cubes




**Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.**

$N := 9 \qquad BE := 1$

$BD := \frac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \frac{BD}{N} \qquad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

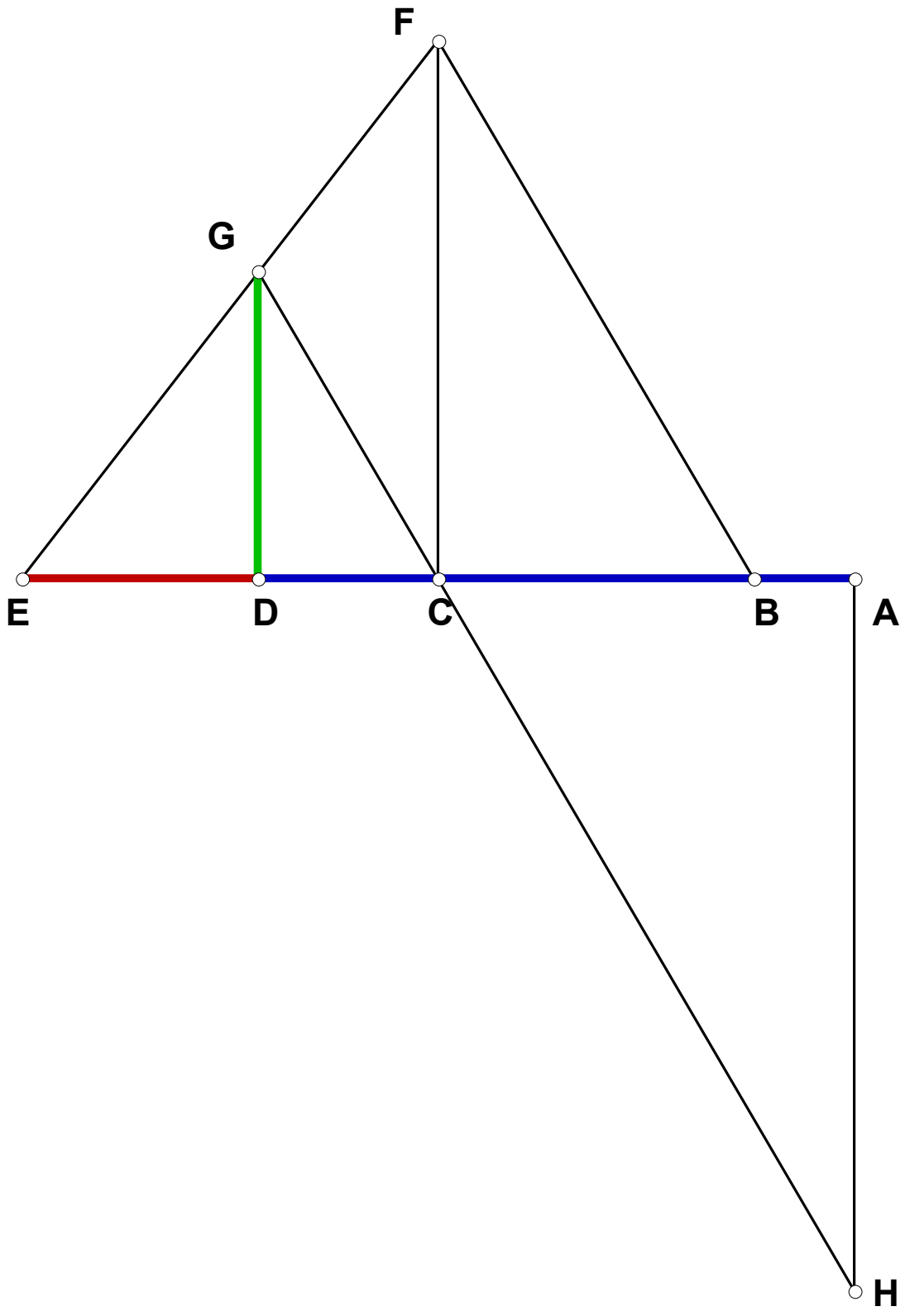
$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$

$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

$AB := AE - BE$

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$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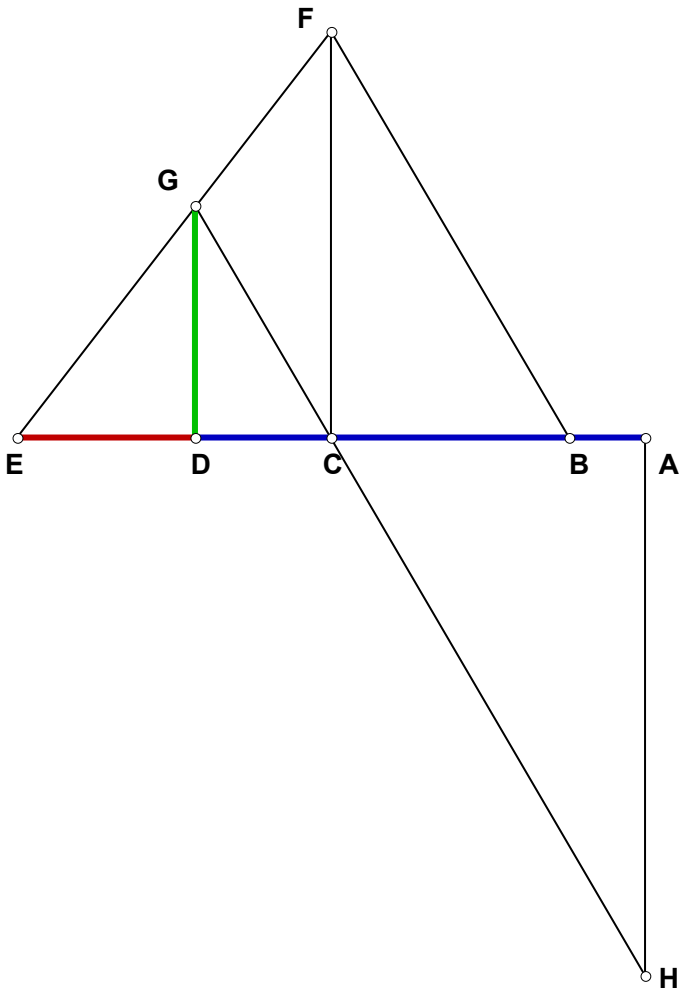
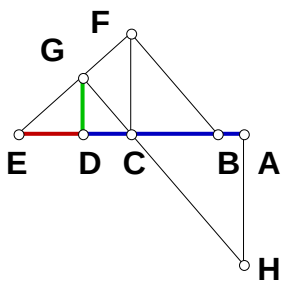



## 960129 EP

Given:

## 012996 Linear division $\frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)}$




$N_1 := 3 \qquad N_2 := 9 \qquad AE := 1$

$AH := AE \cdot N_1 \quad AC := \frac{AE}{2} \quad CF := AE \cdot N_2 \quad BC := \frac{AC \cdot CF}{AH}$

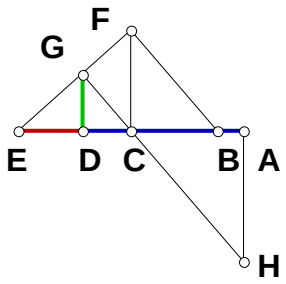
$CE := AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{AH \cdot CD}{AC}$

$DE - \frac{N_1}{2 \cdot (N_1 + N_2)} = 0 \qquad AD - \frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)} = 0 \qquad CD - \frac{N_2}{2 \cdot (N_1 + N_2)} = 0$

$DG - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

## Linear division $\frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)}$




$N_1 := 4 \qquad N_2 := 4 \qquad N_3 := 2 \qquad AE := 1$

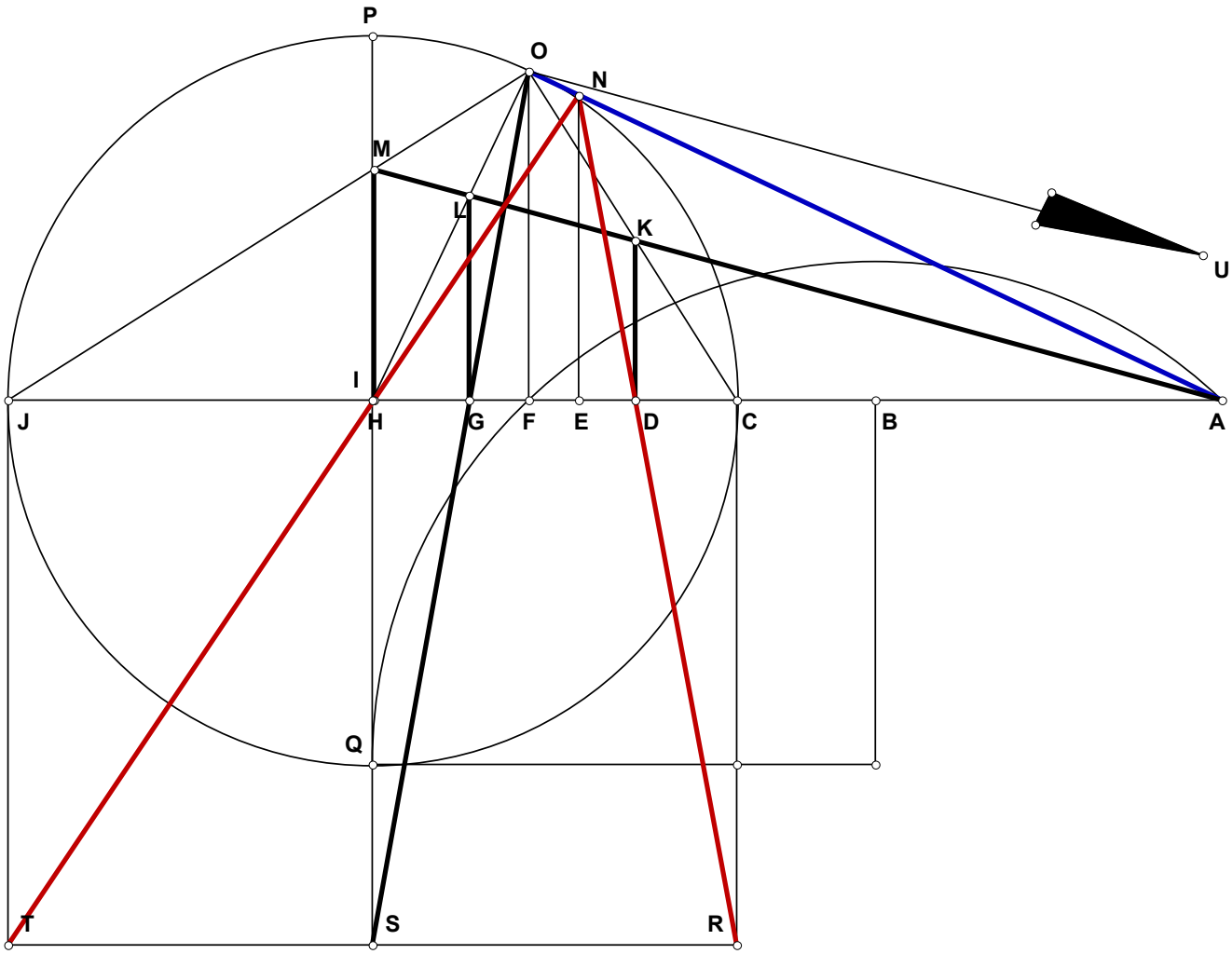
$AH := AE \cdot N_1 \quad AC := \frac{AE}{N_2} \quad CF := AE \cdot N_3 \quad BC := \frac{AC \cdot CF}{AH}$

$CE := AE - AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{CF \cdot CE}{BE}$

$DE - AE \cdot \frac{N_1 \cdot (N_2 - 1)^2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0 \qquad AD - AE \cdot \frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

$DG - AE \cdot \frac{N_1 \cdot N_3 \cdot (N_2 - 1)}{N_1 \cdot N_2 - N_1 + N_3} = 0 \qquad CD - AE \cdot \frac{N_3 \cdot (N_2 - 1)}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$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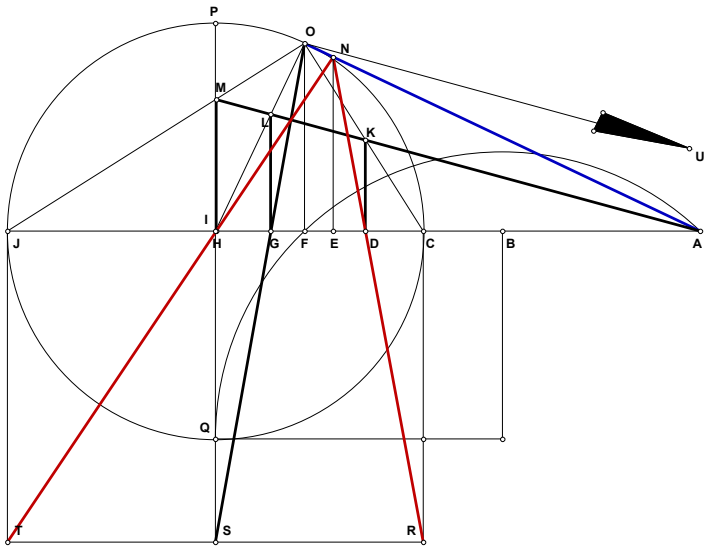
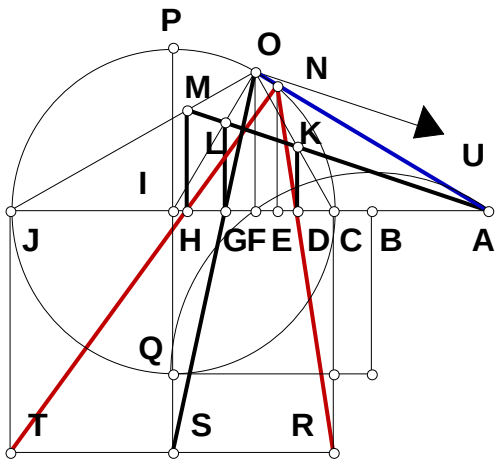



# 960131 EP

Given:

# 013196 On Gemini Roots




## Hitting AO from any RT while maintaining Gemini Roots.

$N_1 := 5 \qquad N_2 := 3 \qquad BC := 1$

$BJ := BC \cdot N_1 \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF$

$FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2 \quad HS := CR$

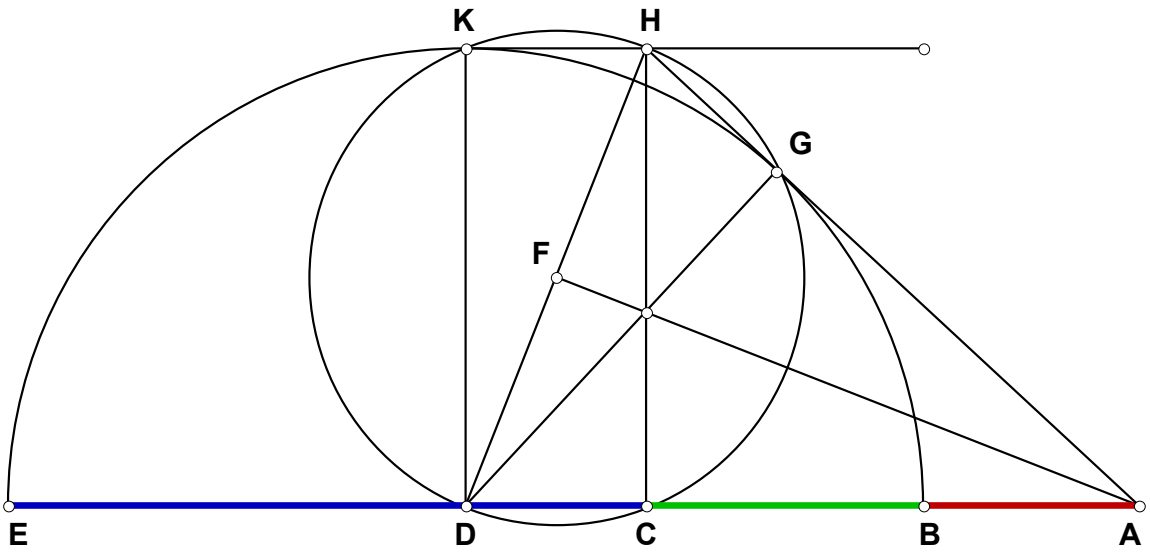
$FI := FJ - IJ \quad FG := \frac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \frac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \frac{HS \cdot GO}{OS} \quad FU := \frac{AG \cdot FO}{GL}$

$AH := \frac{FU \cdot AJ}{FU + FJ} \quad DK := \frac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \frac{AG \cdot DK}{GL} \quad AC := AF - CF \quad CD := AD - AC$

$CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \frac{CR \cdot DH}{CD + HJ} \quad CE := \frac{CD \cdot (CR + EN)}{CR}$

$AE := AC + CE \quad \frac{AF}{FO} - \frac{AE}{EN} = 0 \quad BD := BC + CD \quad BH := BC + CH \quad \sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$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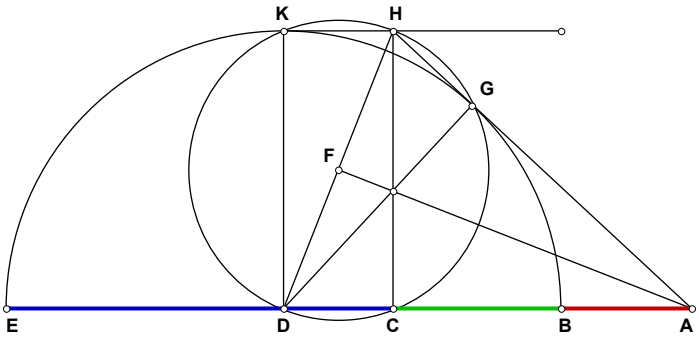
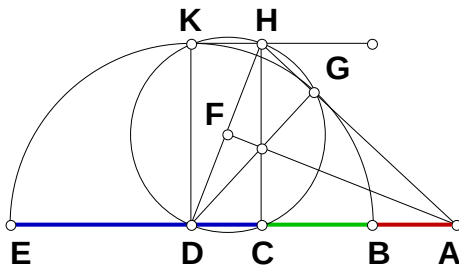



# 960202 EP

Given:

## 020296 Find A Segment

**Given BE and BC such that $\sqrt{(AB + BE)\cdot AB} = AB + BC$, find AB.**

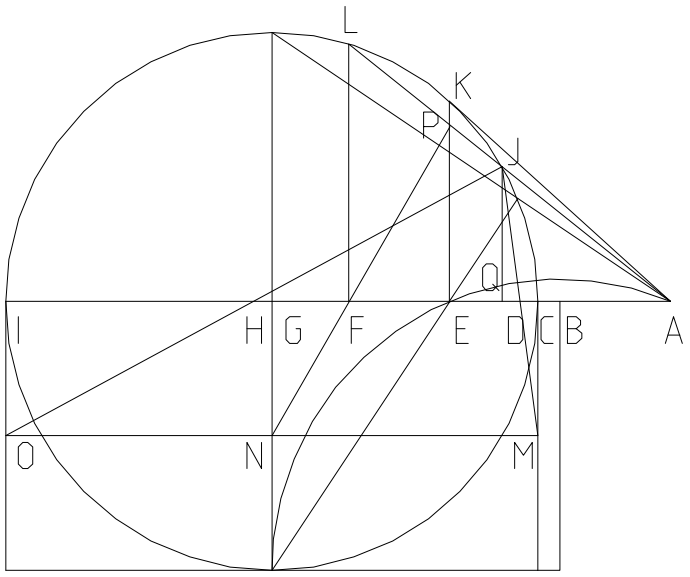



$N_1 := 5 \qquad N_1 > 2 = 1$

$BC := 1 \qquad BE := BC \cdot N_1 \qquad BD := \frac{BE}{2} \qquad CD := BD - BC$

$CH := BD \qquad DH := \sqrt{CD^2 + CH^2} \qquad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \qquad AB := AD - BD \qquad AB - \frac{1}{(N_1 - 2)} = 0 \qquad \sqrt{(AB + BE)\cdot AB} - (AB + BC) = 0$




Use iteration to find any root pair for BE.
Remember that when N is set to 2, we have cube roots.

$$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$$

$$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$$

$$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$$

$$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$$

$$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$$

$$\Delta := 40 \quad \delta := 0 .. \Delta$$

$$\begin{bmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{bmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \\ \sqrt{\left[ \left[ AG - \left( \frac{EG \cdot GN}{GN + EK} \right) \right] - AC \right] \cdot \left[ GI + \left( \frac{EG \cdot GN}{GN + EK} \right) \right]} \end{bmatrix}$$

$$\begin{bmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{bmatrix} := \begin{bmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{bmatrix}$$

$$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$$

$$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$$

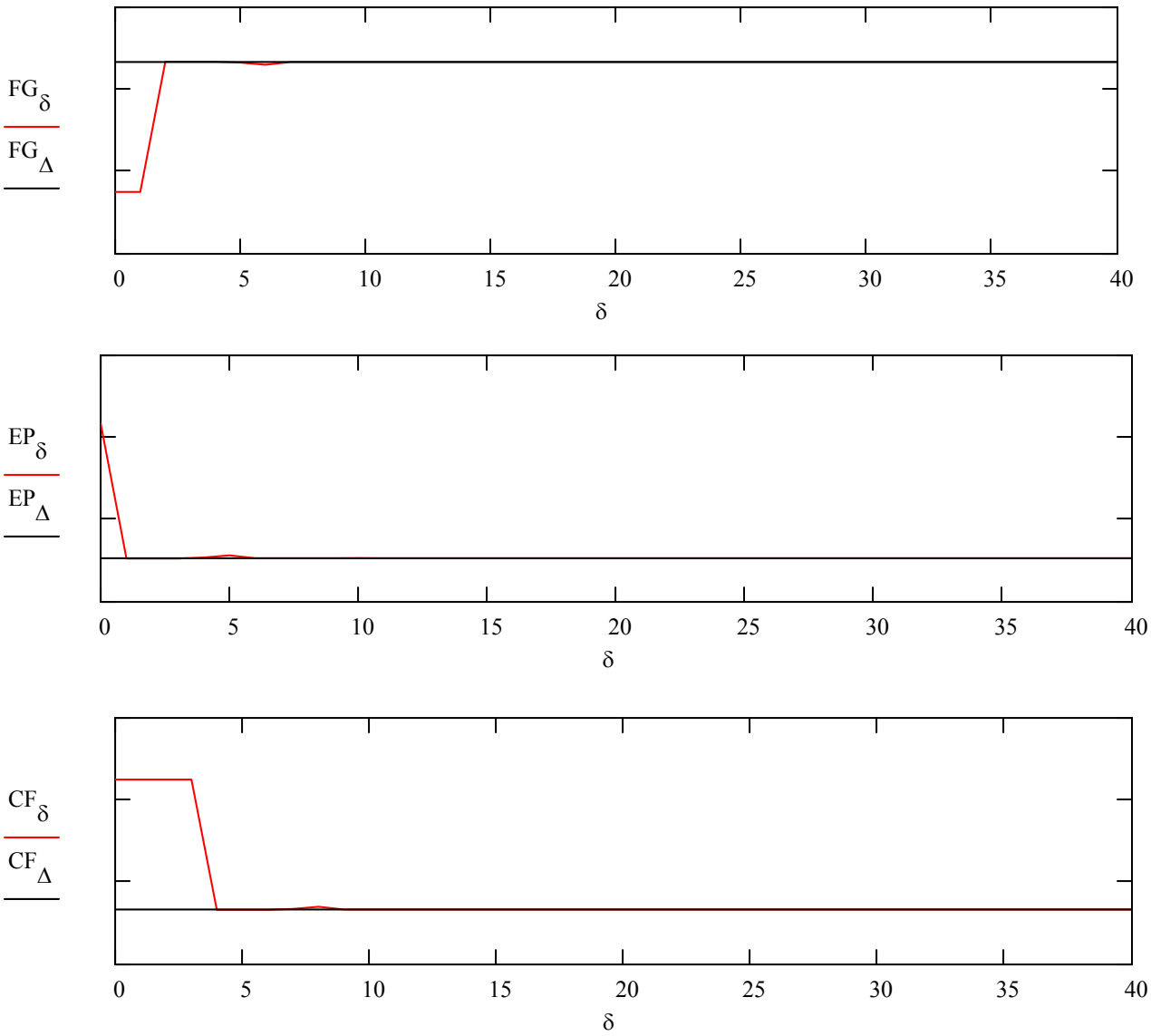
$$BD := BC + CD \quad BH := BC + CD + DH \quad \frac{DH}{\sqrt{CD \cdot HI}} = 1 \quad BE - \sqrt{BD \cdot BH} = 0.000000000000000$$

The next two equations are for the Delian Problem only.

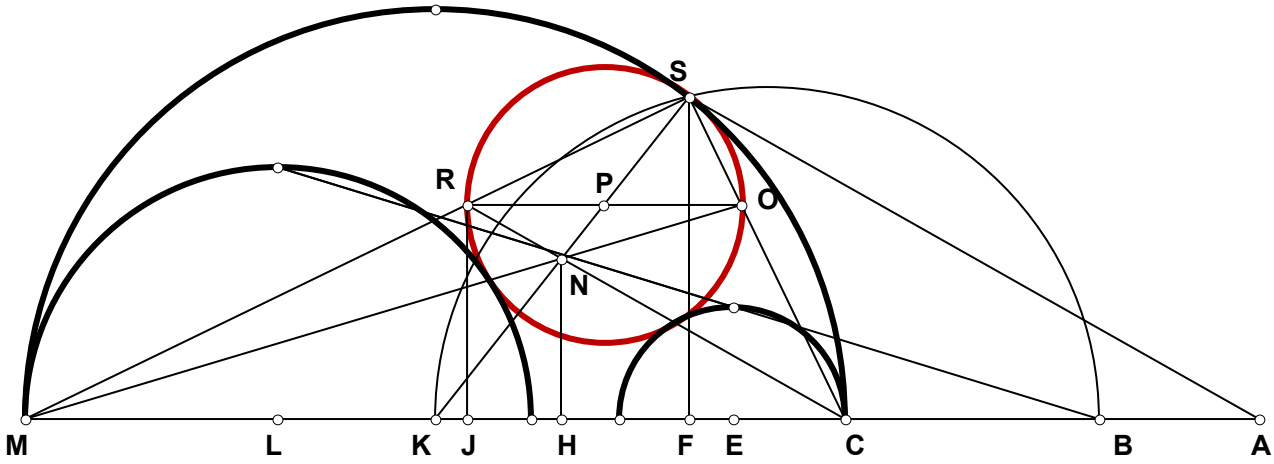
$$\left(BC^2 \cdot BI\right)^{\frac{1}{3}} - BD = 0.000000000000000 \qquad \left(BC \cdot BI^2\right)^{\frac{1}{3}} - BH = 0.000000000000000$$

$$BD = 1.259921049894873 \qquad 2^{\frac{1}{3}} = 1.259921049894873$$

$$BH = 1.587401051968199 \qquad 4^{\frac{1}{3}} = 1.587401051968199$$




The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist.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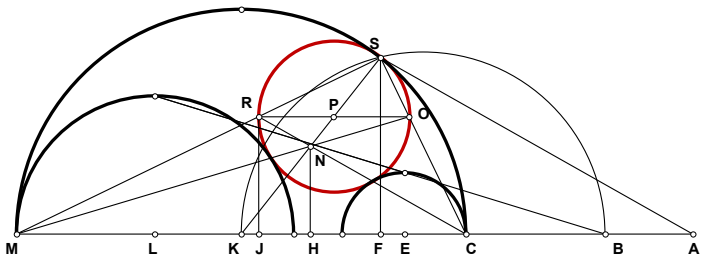
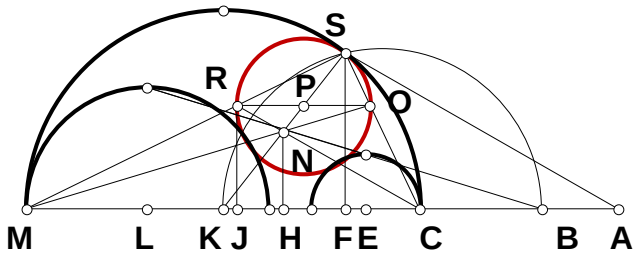
# 960414 EP




Given:

## 041496 Method for Unequals

**Given three circles in the said configuration, find the fourth. I had this sketched out in 95, but if I put it there I would have a lot of document links to redo in "The Quest."**




$N_1 := 7$ $N_2 := 3$

$CM := 1$ $CK := \frac{CM}{2}$ $CE := \frac{CM}{N_1}$

$LM := \frac{CM}{N_2}$ $EL := CM - (CE + LM)$

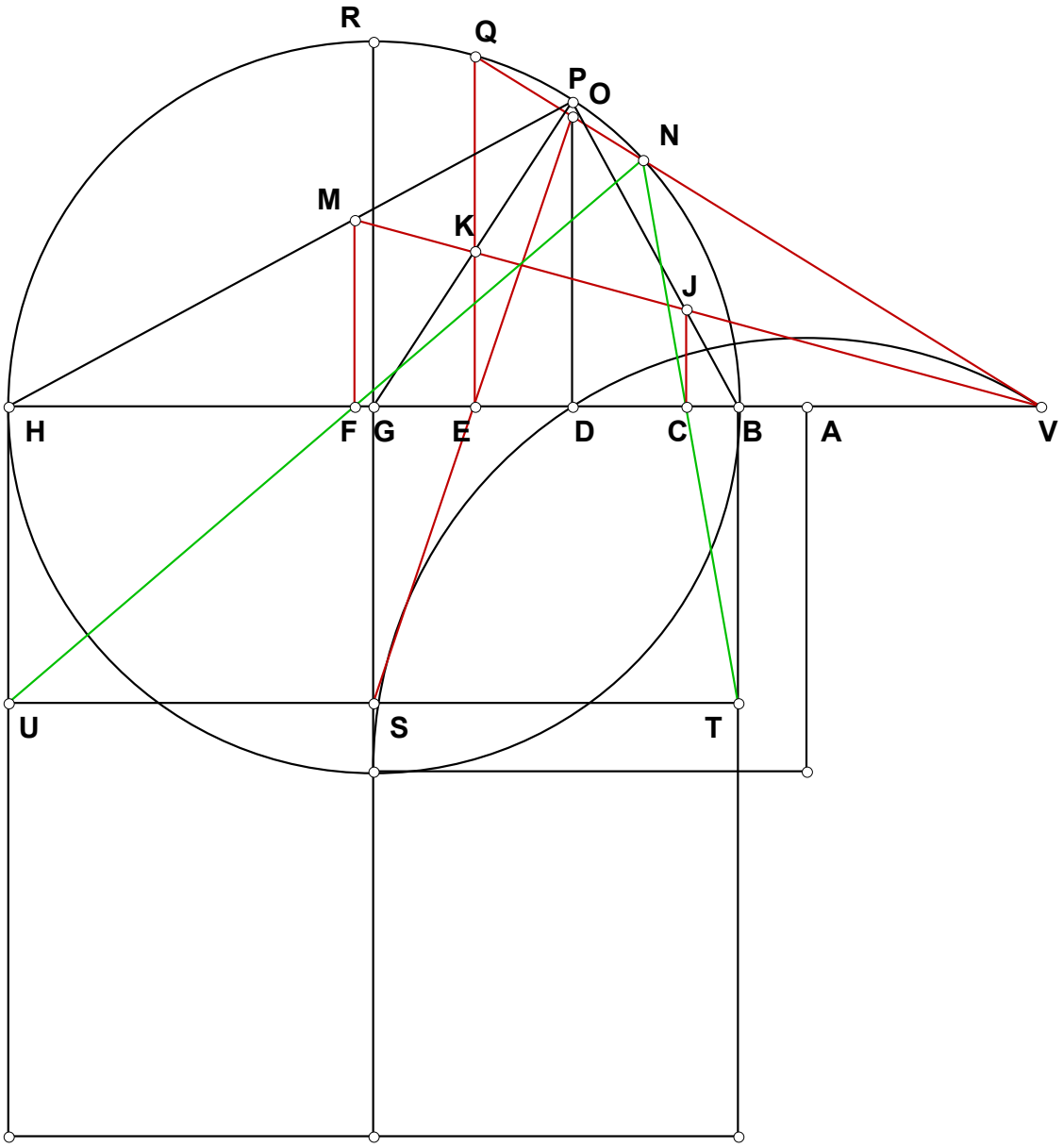
$BL := \frac{EL \cdot LM}{LM - CE}$ $BM := BL + LM$ $BC := BM - CM$ $BK := \frac{CM}{2} + BC$

$R_1 := LM$ $R_2 := CE$ $D := EL$ $KS := CK$ $EH := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$

$FK := \frac{KS^2}{BK}$ $CF := CK - FK$ $FM := CM - CF$ $FS := \sqrt{CF \cdot FM}$

$HK := CK - (CE + EH)$ $CH := CK - HK$ $HN := \frac{FS \cdot HK}{FK}$ $AF := \frac{CH \cdot FS}{HN}$ $JR := \frac{FS \cdot CM}{AF + FM}$

$RO := \frac{CM \cdot (FS - JR)}{FS}$ $PS := \frac{RO}{2}$ $PS - \frac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$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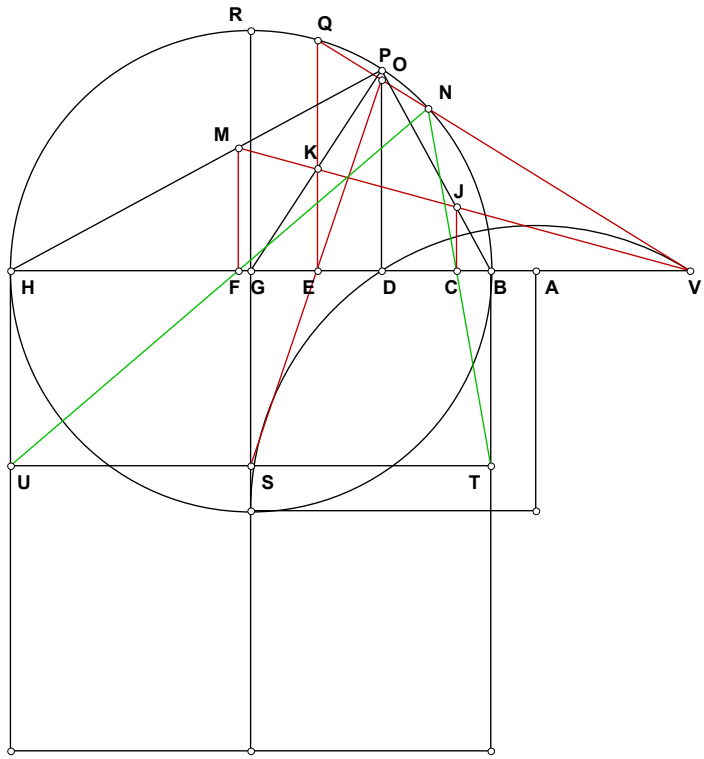
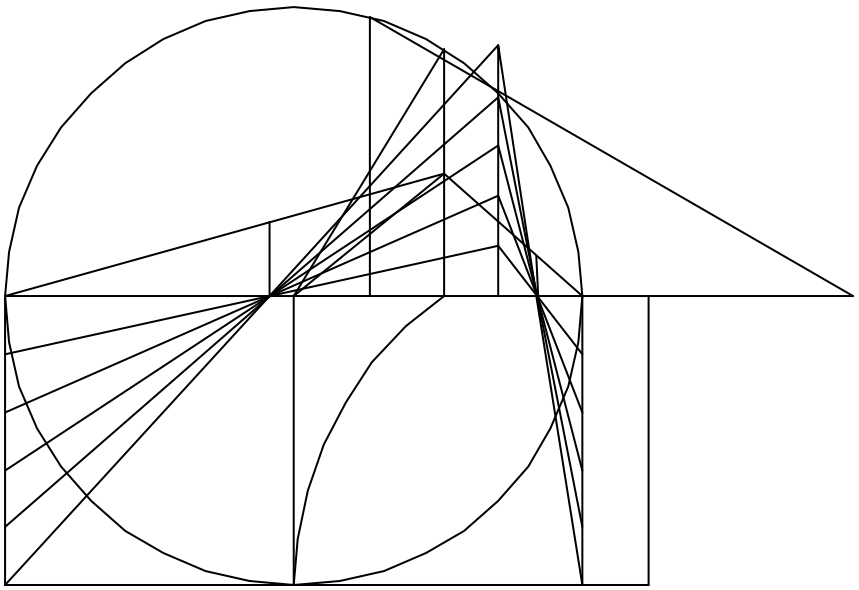
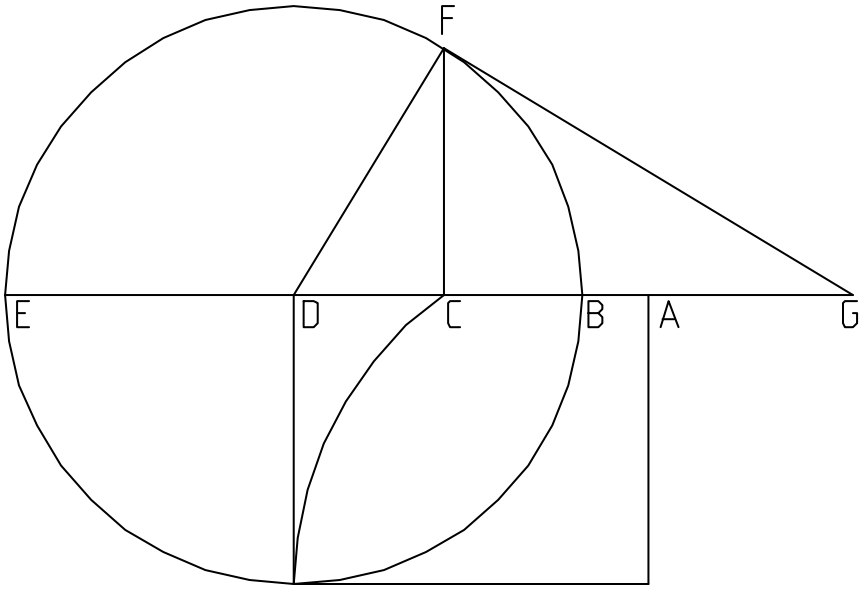


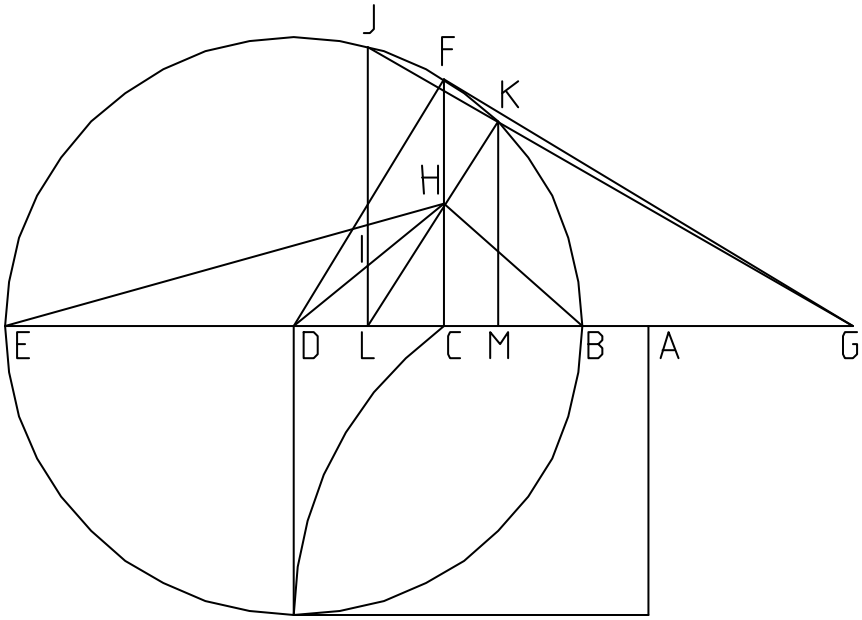

# 960415 EP

Given:



# On Gemini Roots




$N_1 := 10$

$AB := 1 \qquad BE := AB \cdot N_1 \quad BD := \frac{BE}{2}$

$AE := AB + BE \quad AC := \sqrt{AB \cdot AE}$

$BC := AC - AB \quad CE := BE - BC$

$DF := BD \quad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \quad CG := \frac{CF^2}{CD}$

$BG := CG - BC \quad EG := BG + BE$

$CH := \frac{1}{2} \cdot CF \quad DH := \sqrt{CH^2 + CD^2}$

$DI := \frac{1}{2} \cdot DH \quad DL := \frac{CD \cdot DI}{DH}$

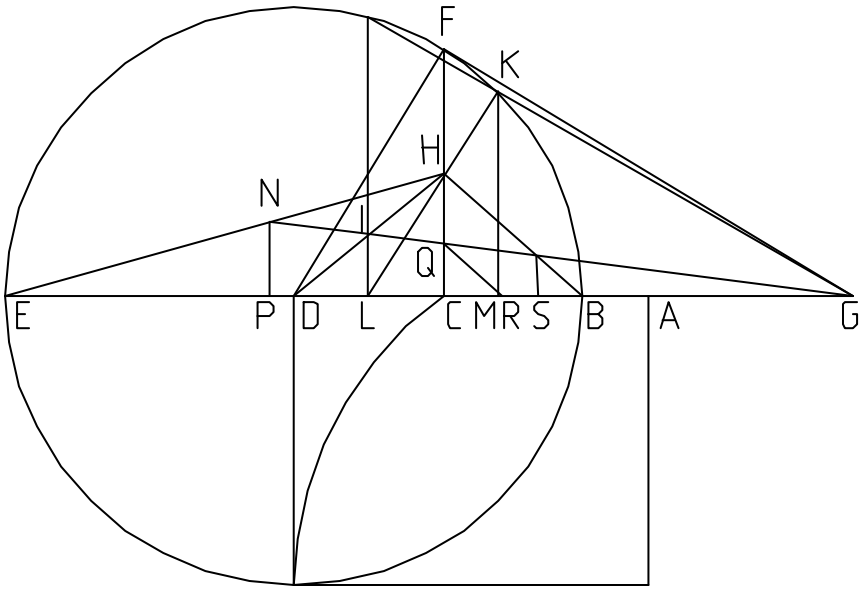
$BL := BD - DL \quad EL := BE - BL$

$JL := \sqrt{BL \cdot EL} \quad GL := BL + BG$

$GJ := \sqrt{JL^2 + GL^2} \quad GK := \frac{BG \cdot EG}{GJ}$

$GM := \frac{GL \cdot GK}{GJ} \qquad BM := GM - BG$

$EM := BE - BM$

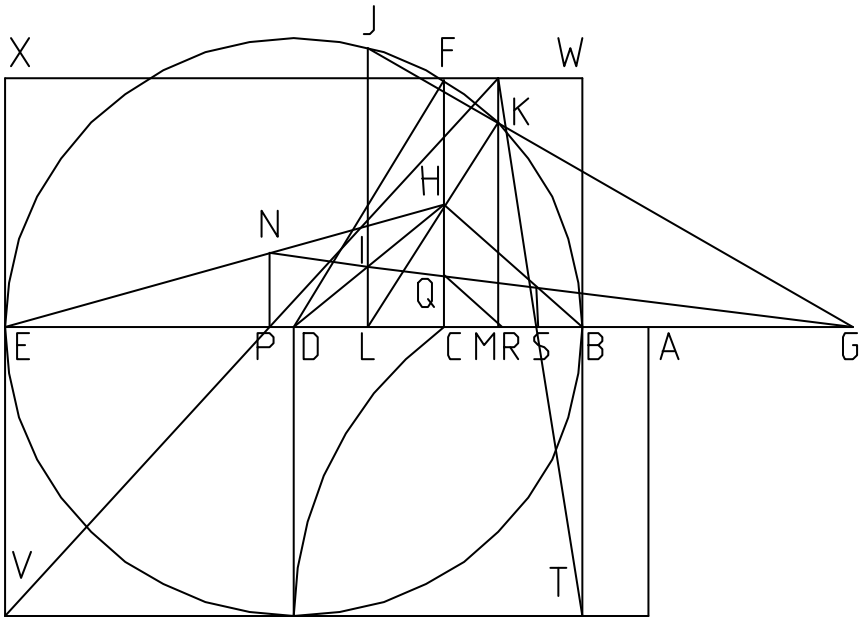



$$IL := \sqrt{DI^2 - DL^2} \qquad CO := \frac{GL \cdot CH}{IL}$$

$$NP := \frac{CH \cdot EG}{(CO + CE)} \qquad EP := \frac{CE \cdot NP}{CH}$$

$$CQ := \frac{IL \cdot CG}{GL} \qquad CR := \frac{BC \cdot CQ}{CH}$$

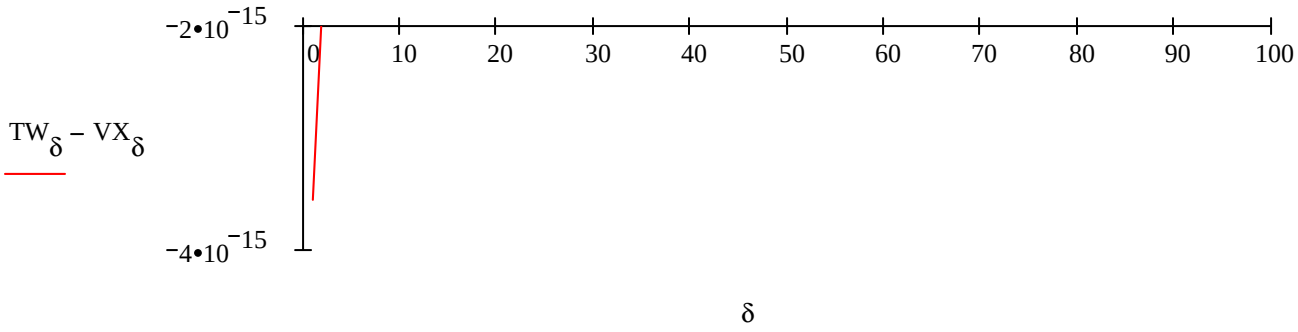
$$GR := CG - CR \qquad BS := \frac{CR \cdot BG}{GR}$$

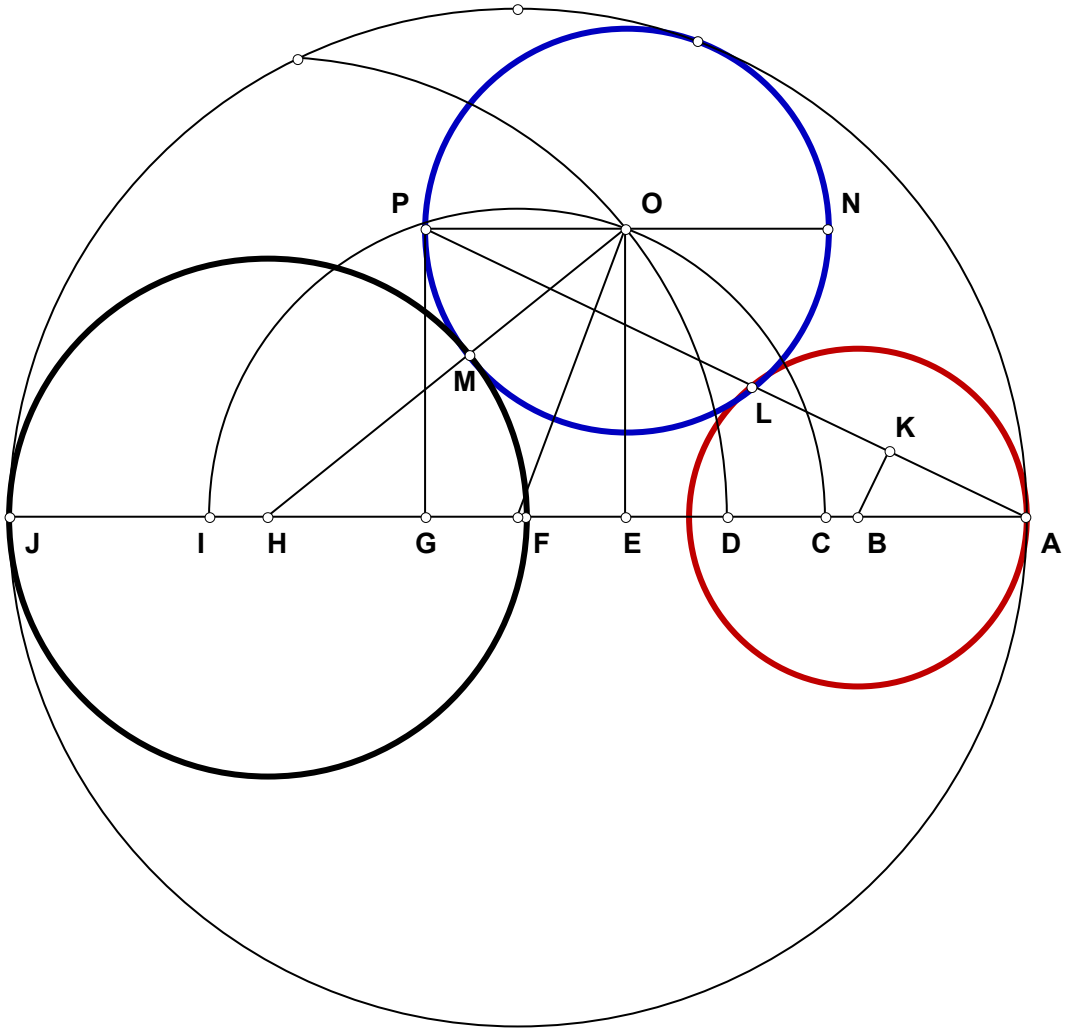



$$\delta := 1 .. 100$$

$$E_\delta := \frac{BE}{\delta} \qquad BT_\delta := E_\delta \quad EV_\delta := E_\delta$$

$$TW_\delta := \frac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$$

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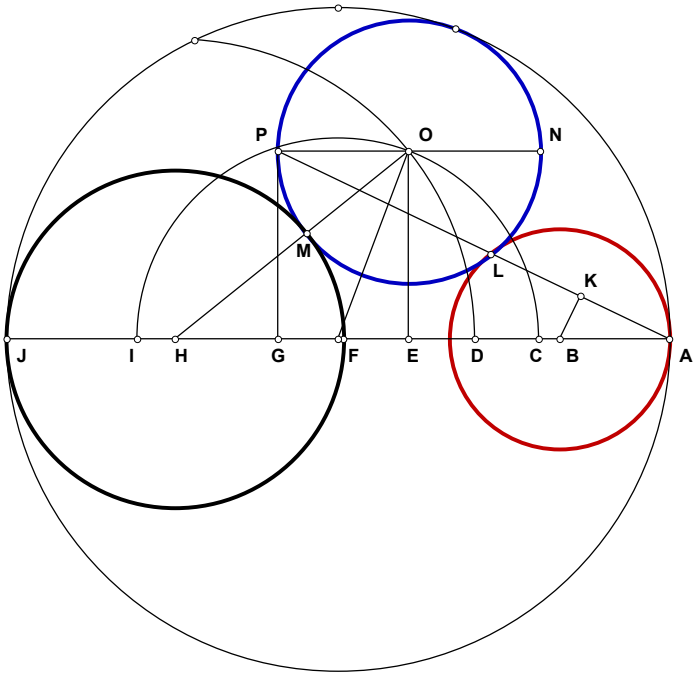
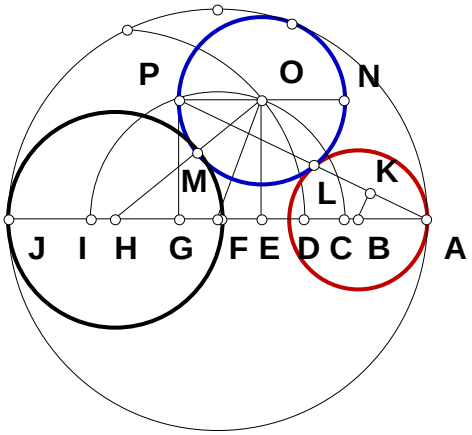



# 960416 EP

Given:

## 041696 Given Three Radii




**Given AJ, HJ and NO find AB such that AB is collinear with AJ and HJ.**

$N_1 := 4 \qquad N_2 := 16$

$AJ := 1 \qquad AF := \frac{AJ}{2} \qquad HJ := \frac{AJ}{N_1} \qquad NO := \frac{AJ}{N_2}$

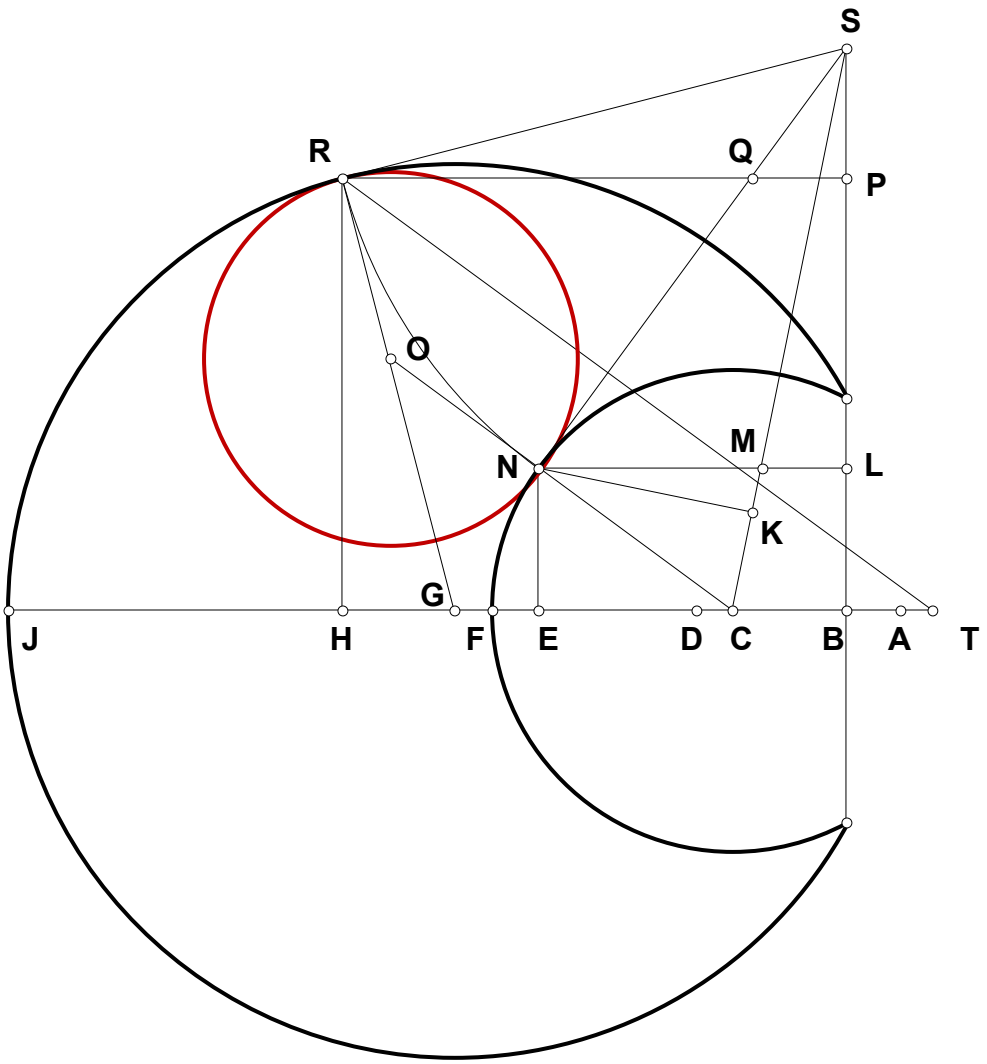
$HM := HJ \quad MO := NO \quad HO := HM + MO$

$FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$S_1 := FH \quad S_2 := FO \quad S_3 := HO \quad EH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO \quad AP := \sqrt{AG^2 + GP^2} \quad PL := \frac{AG \cdot (NO + OP)}{AP}$

$AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG} \qquad AB - \frac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot (N_2 \cdot N_1 - 2 \cdot N_2 - 4)} = 0$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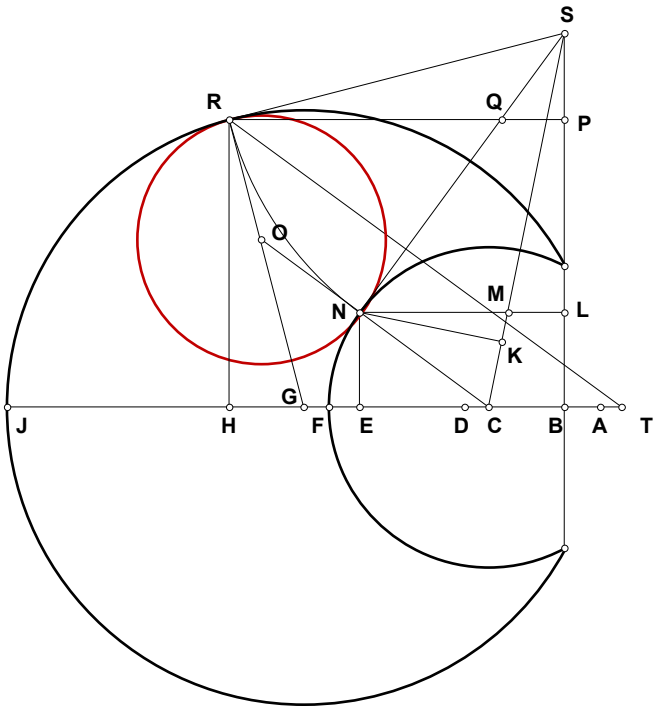
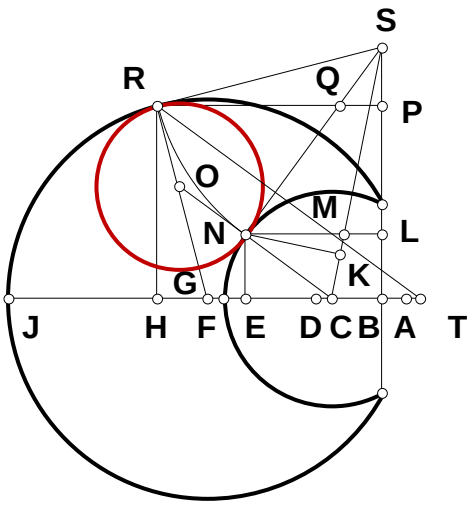
# 960417a EP




Given:

# 041796 A Circle In A Crescent




$N_1 := 2$ $N_2 := 2$ $N_3 := 3$

$AJ := 1$ $AG := \frac{AJ}{2}$ $FR := AG$ $AF := \frac{AJ}{N_1}$

$AD := \frac{AF}{2}$ $AC := \frac{AD}{N_2}$ $CF := AF - AC$

$CG := AG - AC$ $R_1 := AG$ $R_2 := CF$ $D := CG$

$BC := \frac{\left(R_2^2 + D^2 - R_1^2\right)}{2 \cdot D}$ $AB := AC + BC$

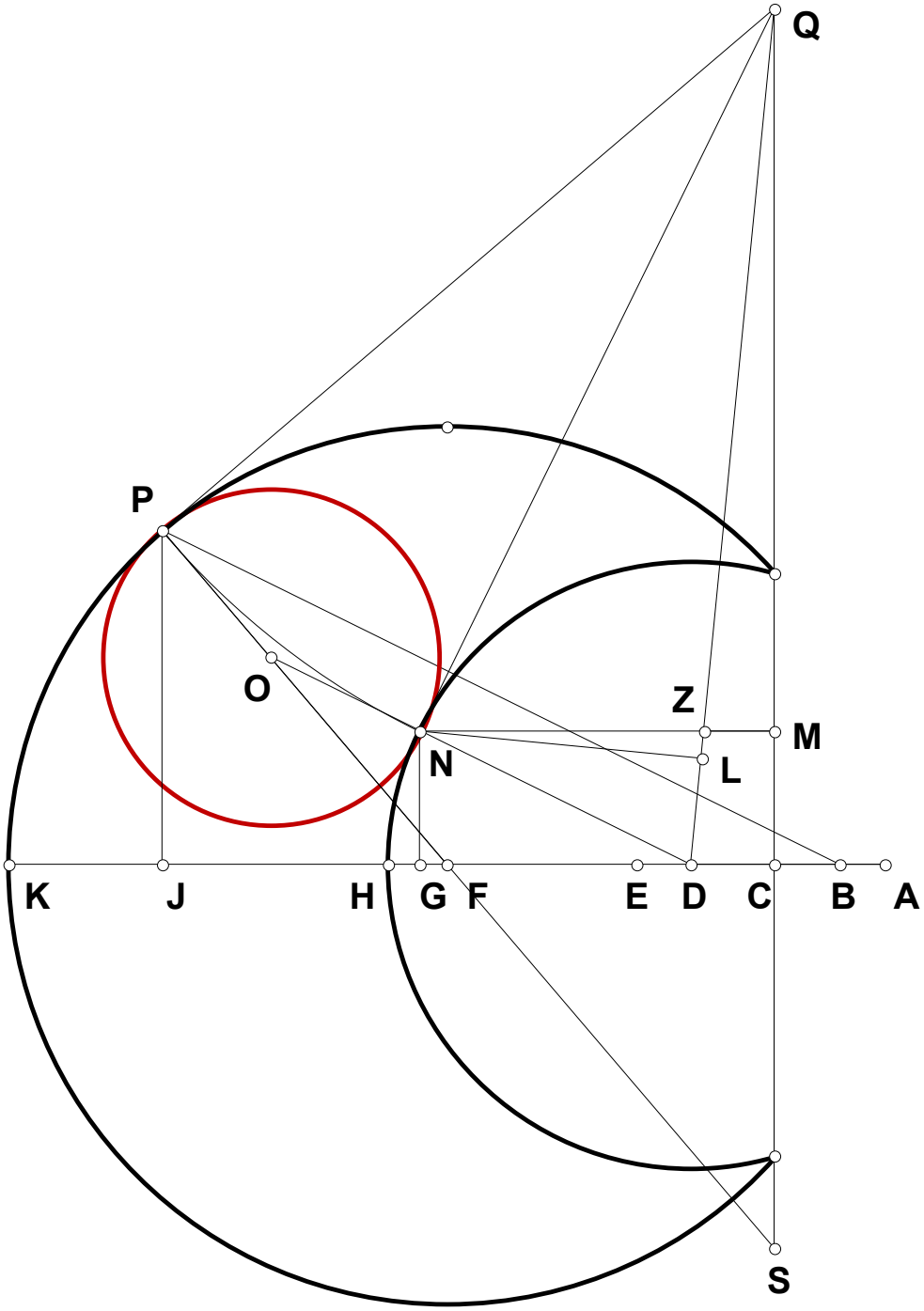
$BJ := AJ - AB$ $BH := \frac{BJ}{N_3}$ $AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{FR^2 - GH^2}$

$AP := HR$ $PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := AP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$ $KN := \sqrt{CN^2 - CK^2}$ $KM := \frac{BC \cdot KN}{BS}$

$SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$ $BL := BS - SL$ $EN := BL$ $CE := \sqrt{CN^2 - EN^2}$

$HT := \frac{CE \cdot HR}{EN}$ $GT := HT - GH$ $GO := \frac{FR \cdot CG}{GT}$ $OR := FR - GO$ $OR = 0.125$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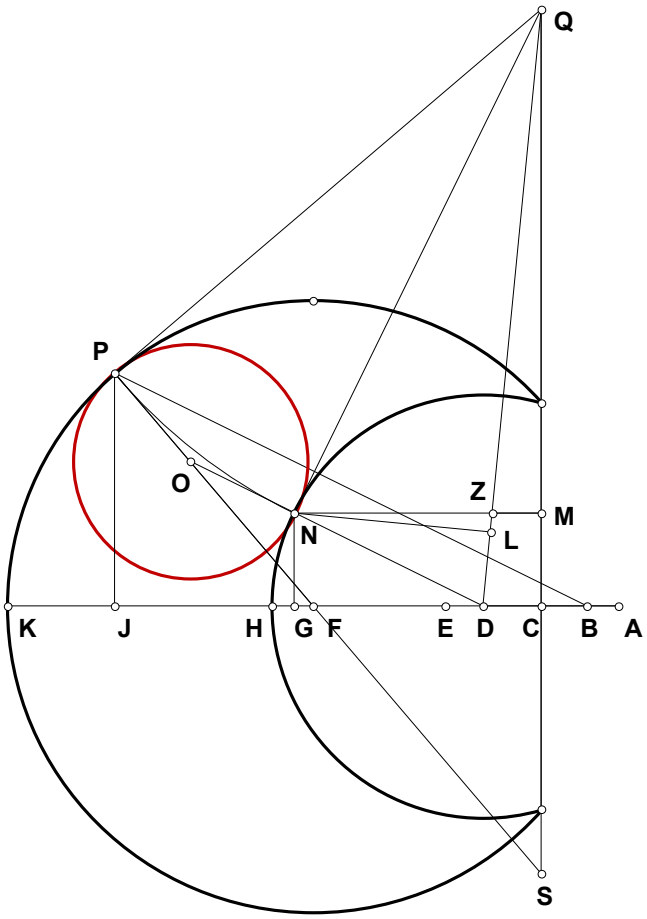
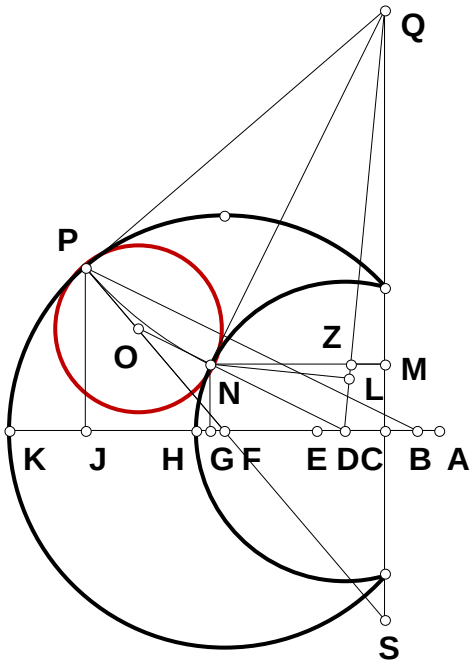



## 960417b EP

Given:




# 041796b A Circle In A Crescent

$N_1 := 1.7 \quad N_2 := 2 \quad N_3 := 1.3$

$AK := 1 \quad AF := \frac{AK}{2} \quad FP := AF \quad FK := AF$

$AH := \frac{AK}{N_1} \quad AE := \frac{AH}{2} \quad EH := AE \quad AD := \frac{AE}{N_2}$

$DE := AE - AD \quad DH := EH + DE \quad DN := DH$

$DF := AF - AD \quad R_1 := AF \quad R_2 := DH \quad D := DF$

$CD := \frac{R_2^2 + D^2 - R_1^2}{2 \cdot D} \quad AC := AD + CD$

$CK := AK - AC \quad CJ := \frac{CK}{N_3} \quad CF := CK - FK$

$FJ := CJ - CF \quad JP := \sqrt{FP^2 - FJ^2} \quad FS := \frac{FP \cdot CF}{FJ} \quad PS := FS + FP \quad QS := \frac{FP \cdot PS}{JP} \quad PQ := \frac{FJ \cdot QS}{FP}$

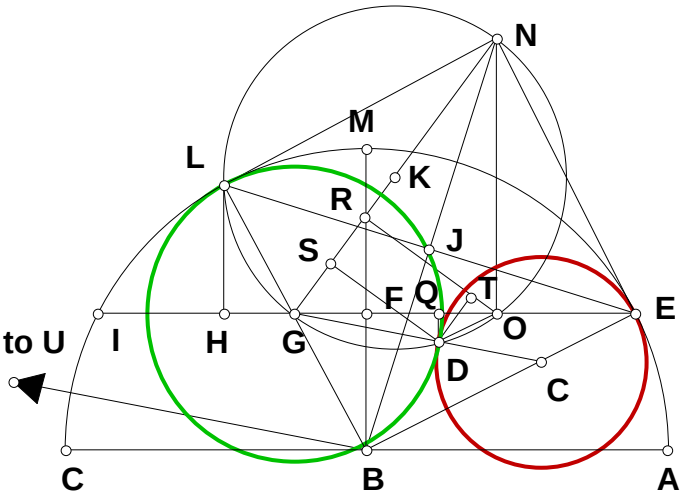
$CS := \frac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad NQ := PQ \quad DQ := \sqrt{NQ^2 + DN^2} \quad DL := \frac{DN^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2}$

$LZ := \frac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ \quad MQ := \frac{CQ \cdot QZ}{DQ} \quad CM := CQ - MQ \quad GN := CM$

$CG := \sqrt{DN^2 - GN^2} \quad BJ := \frac{CG \cdot JP}{GN} \quad BF := BJ - FJ \quad FO := \frac{FP \cdot DF}{BF} \quad OP := FP - FO \quad OP = 0.175$

**042296a**

## Given BF as a ratio to BM and EG as a ratio to EI, what is CE?




$N_1 := 2.5$ $N_2 := 1.5$ $AC := 1$

$AB := \frac{AC}{2}$ $BM := AB$ $BE := AB$

$BL := AB$ $BF := \frac{BM}{N_1}$

$EF := \sqrt{BE^2 - BF^2}$ $EI := 2 \cdot EF$

$EG := \frac{EI}{N_2}$ $FG := EG - EF$

$BG := \sqrt{BF^2 + FG^2}$ $GL := BL - BG$ $DG := GL$ $GH := \frac{FG \cdot GL}{BG}$ $HL := \sqrt{GL^2 - GH^2}$ $EH := EG + GH$

$EL := \sqrt{EH^2 + HI}$ $JL := \frac{EL}{2}$ $BJ := \sqrt{BL^2 - JL^2}$ $LN := \frac{BL \cdot JL}{BJ}$ $GN := \sqrt{LN^2 + GL^2}$ $JN := \sqrt{LN^2 - JL^2}$

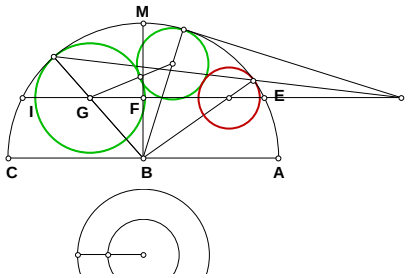
$EJ := JL$ $EN := \sqrt{JN^2 + EJ^2}$ $S_1 := EG$ $S_2 := EN$ $S_3 := GN$ $GO := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$NO := \sqrt{GN^2 - GO^2}$ $NR := \frac{NO^2}{GN}$ $GS := \frac{DG^2}{GN}$ $RS := GN - (NR + GS)$ $DT := RS$ $DS := \sqrt{DG^2 - GS^2}$

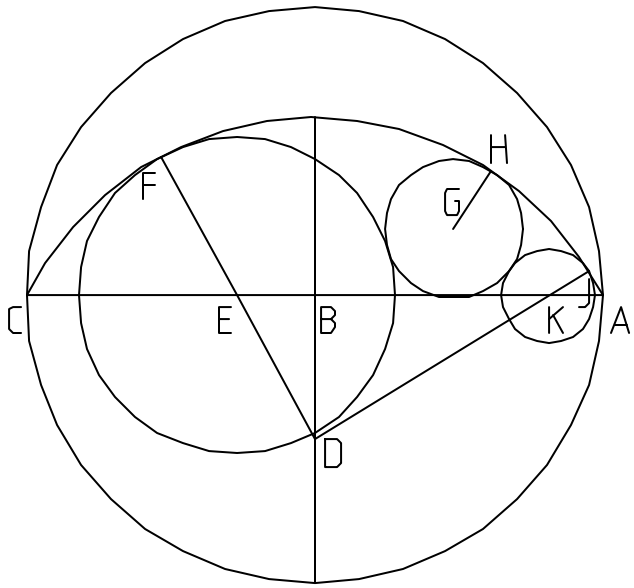
$RT := DS$ $OR := \sqrt{NO^2 - NR^2}$ $OT := OR - RT$ $DO := \sqrt{DT^2 + OT^2}$

$S_1 := GO$ $S_2 := DG$ $S_3 := DO$ $OQ := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$ $GQ := GO - OQ$ $DQ := \sqrt{DO^2 - OQ^2}$

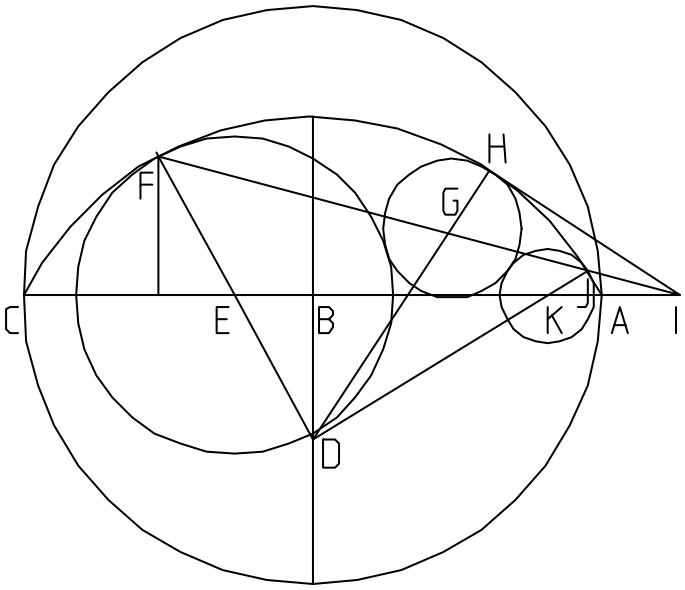
$FU := \frac{GQ \cdot BF}{DQ}$ $CE := \frac{BE \cdot EG}{FU + EF}$ $CE = 0.193$



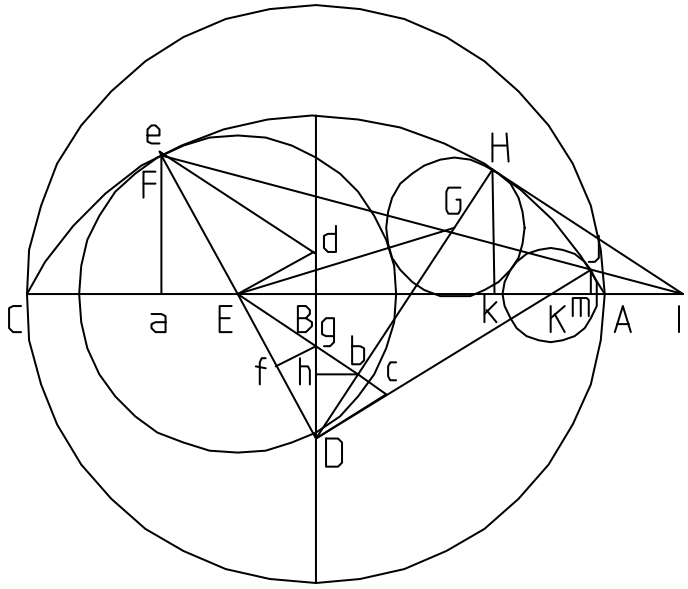

**Edit In progress for three circles.**




Place EF and GH and find JK.




Process summary




$N_1 := \frac{1}{2}$ $N_2 := \frac{9}{8}$ $N_3 := 3$

$AB := 108$ $BC := AB$ $AC := 2 \cdot AB$ $BD := AB \cdot N_1$

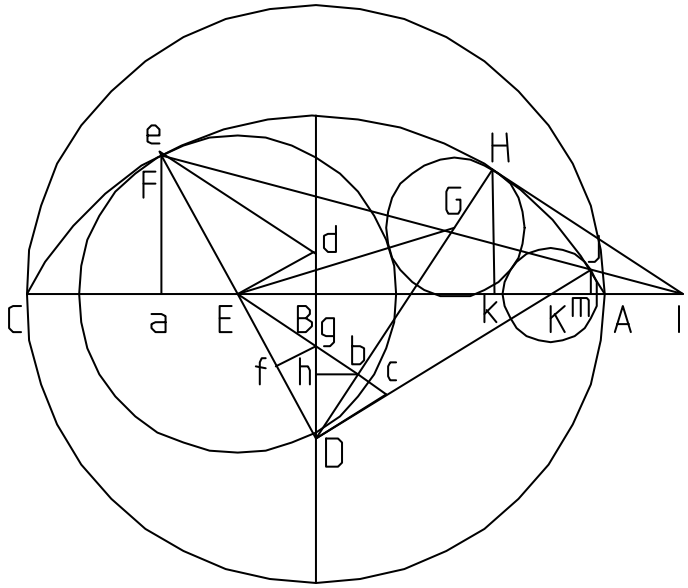
$CD := \sqrt{BC^2 + BD^2}$ $EF := \frac{CD - BD}{N_2}$

$DE := CD - EF$ $BE := \sqrt{DE^2 - BD^2}$

$AE := AB + BE$ $GH := \frac{AE - EF}{N_3}$

$EG := EF + GH$ $DG := CD - GH$ $Ba := \frac{BE \cdot CD}{DE}$

$Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG}$ $Eb := \sqrt{DE^2 - Db^2}$




$$Ec := \frac{DE^2}{Eb} \quad Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD}$$

$$Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$$

$$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db}$$

$$DM := \sqrt{BD^2 + BM^2} \quad Bk := \frac{BM \cdot CD}{DM}$$

$$HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM}$$

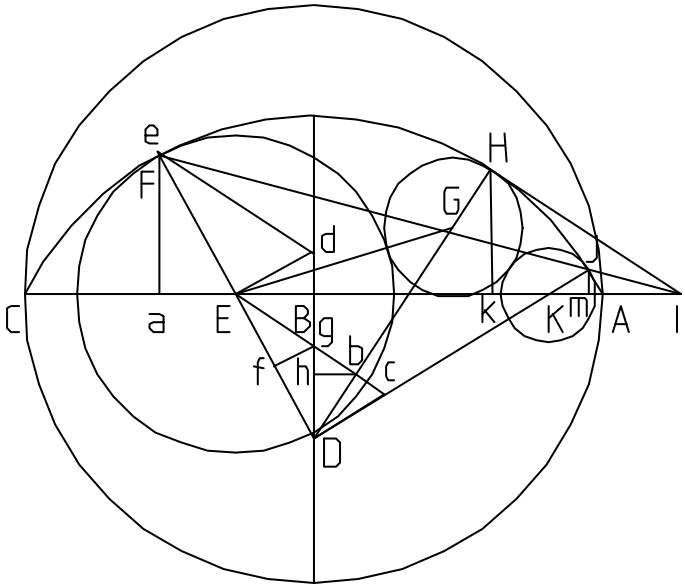
$$Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2}$$

$$Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$$

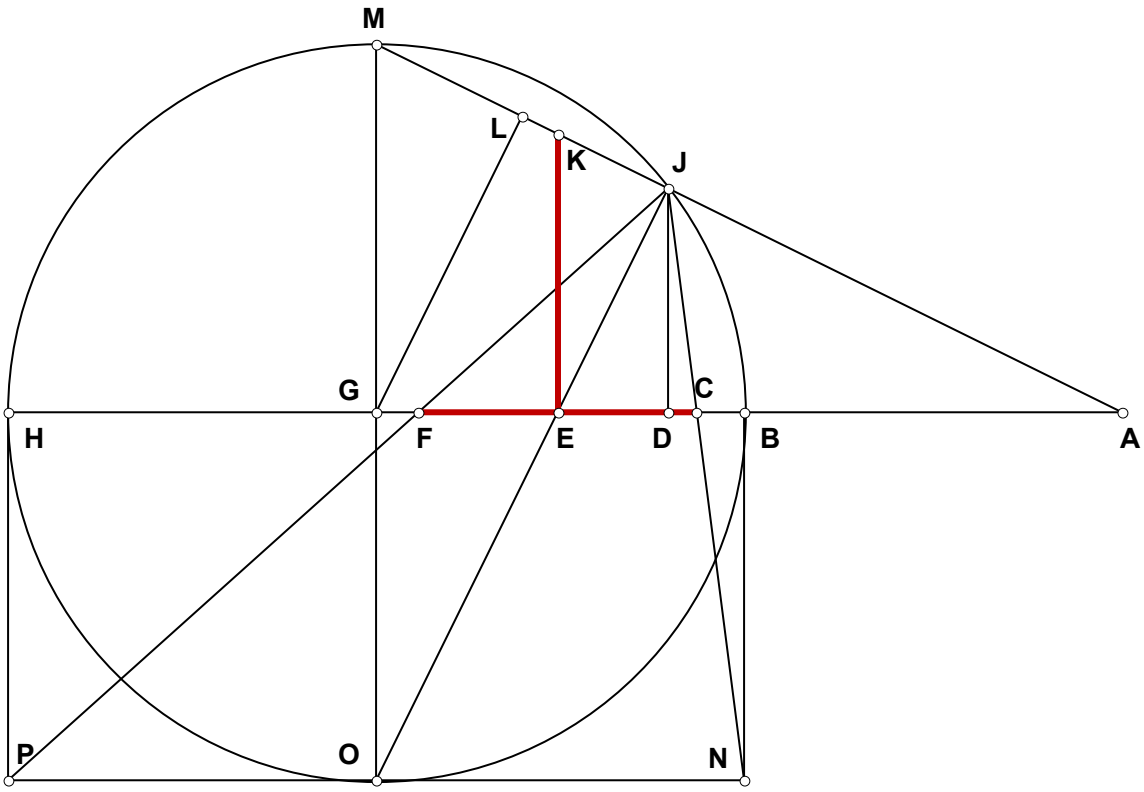
$$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI}$$

$$Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$$

$JK = 17.571$




When GH is small, so that H is on the other side of BD, the similarity point is on the other side of the figure.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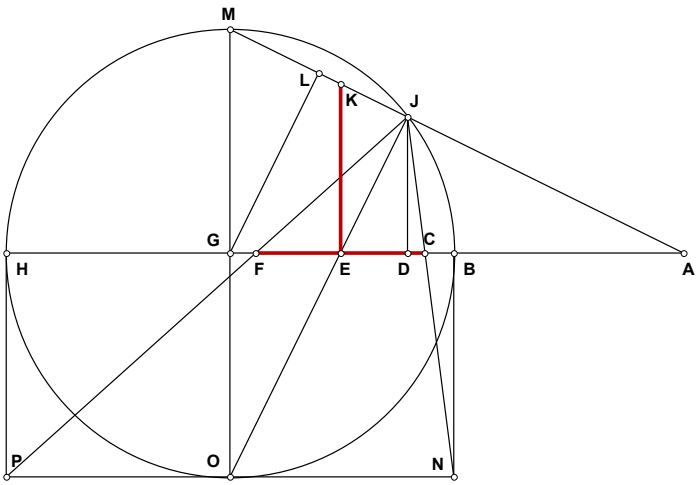
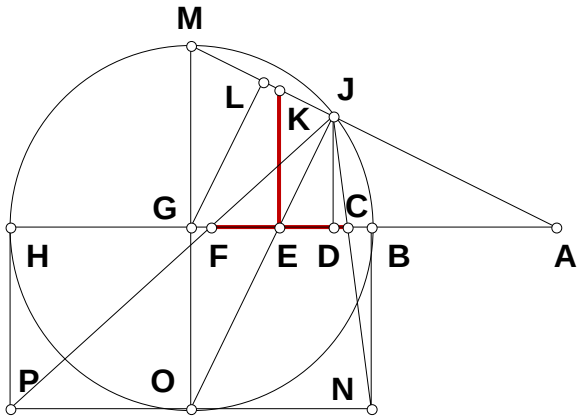



# 960423 EP

Given:

**042396a**




## Is CF always equal to EK?

$N_1 := 3$ $AB := 1$

$AH := AB \cdot N_1$ $BH := AH - AB$

$BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

$GM := BG$ $GH := BG$ $AG := AH - GH$

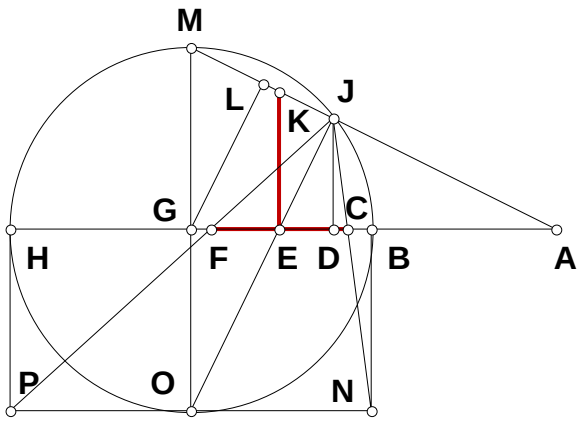
$AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$ $BD := AD - AB$

$DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$ $DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$

$CE := \frac{CF}{2}$ $BE := BC + CE$ $AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$

## 042396b - Definition And Proof




**Proof of equality in a tautologic is simply the demonstration that two names are synonyms. Is CF always equal to EK?**

$N := 3$ $AB := 1$ $AH := N$

$BH := N - 1$ $BG := \frac{N - 1}{2}$

$AG := \frac{1}{2} \cdot N + \frac{1}{2}$ $AM := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$AL := \frac{1}{2} \cdot \frac{(N + 1)^2}{\sqrt{2 \cdot N^2 + 2}}$ $LM := \frac{1}{2} \cdot \frac{(N - 1)^2}{\sqrt{2 \cdot N^2 + 2}}$ $AJ := \frac{2}{\sqrt{2 \cdot N^2 + 2}} \cdot N$ $AD := (N + 1) \cdot \frac{N}{(N^2 + 1)}$

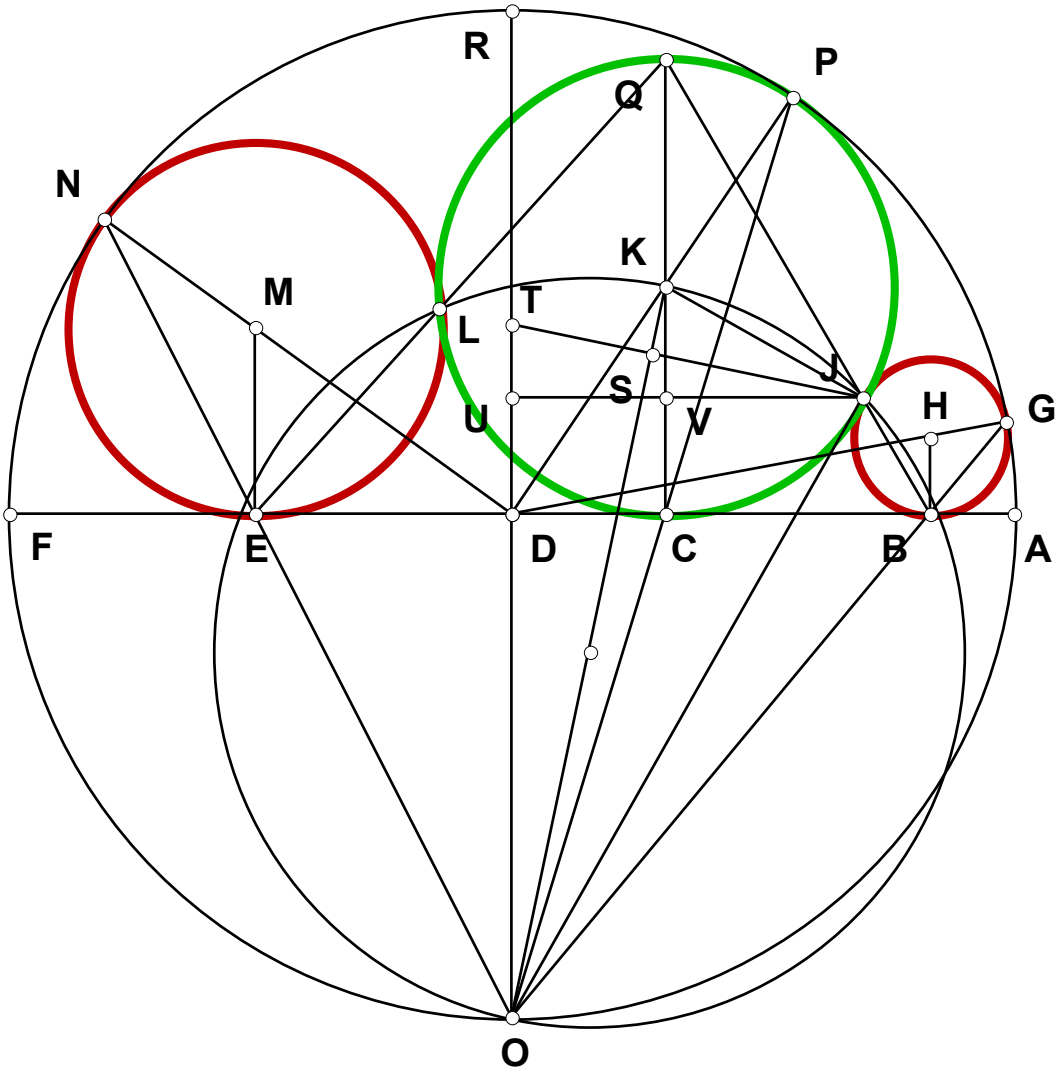
$BD := \frac{(N - 1)}{(N^2 + 1)}$ $DH := N^2 \cdot \frac{(N - 1)}{(N^2 + 1)}$ $DJ := \frac{(N - 1)}{(N^2 + 1)} \cdot N$ $BC := \frac{(N - 1)}{(N + 1)^2}$

$DF := 2 \cdot N^3 \cdot \frac{(N - 1)}{[(N + 1)^2 \cdot (N^2 + 1)]}$ $CD := 2 \cdot (N - 1) \cdot \frac{N}{[(N^2 + 1) \cdot (N + 1)^2]}$

$CF := 2 \cdot N \cdot \frac{(N - 1)}{(N + 1)^2}$ $CE := N \cdot \frac{(N - 1)}{(N + 1)^2}$ $BE := \frac{(N - 1)}{(N + 1)}$ $AE := 2 \cdot \frac{N}{(N + 1)}$

$EK := 2 \cdot (N - 1) \cdot \frac{N}{(N + 1)^2}$ $EK - CF = 0$ $EK = 0.75$

**Meditation : Do these equations satisfy the requirement that a definition must contain both form and matter?**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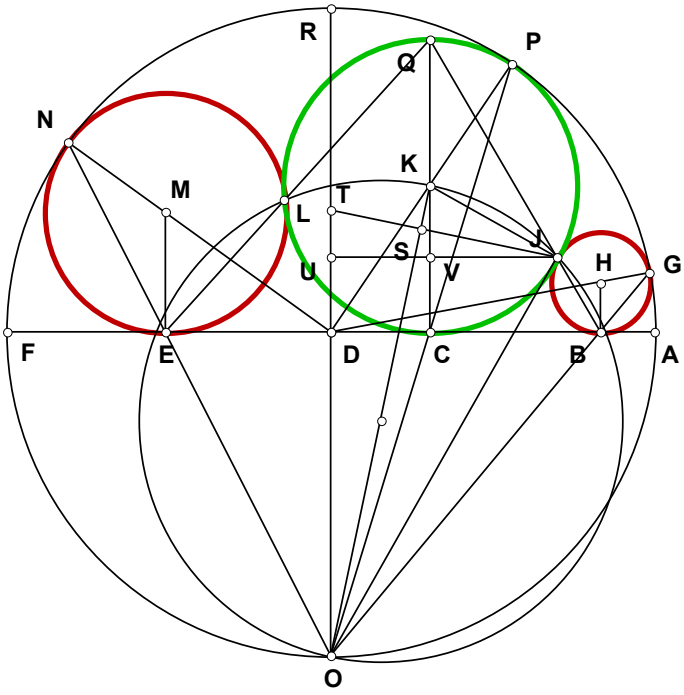
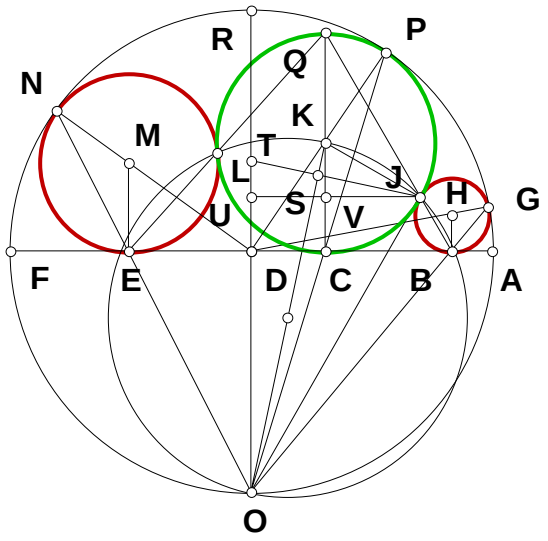



# 960424 EP

Given:

# Three Circles 042496




## Given AC, find CK and BH.

$N := 1.5$ $AF := 1$ $AD := \frac{AF}{2}$

$AC := \frac{AF}{N}$ $DO := AD$ $OR := AF$

$CD := AD - AC$ $CO := \sqrt{CD^2 + DO^2}$

$PO := \frac{DO \cdot OR}{CO}$ $CP := PO - CO$ $CK := \frac{DO \cdot CP}{PO}$

$JK := CK$ $KO := \sqrt{CD^2 + (DO + CK)^2}$

$JO := \sqrt{KO^2 - JK^2}$ $KS := \frac{JK^2}{KO}$ $SO := KO - KS$ $JS := \frac{JK \cdot SO}{JO}$ $ST := \frac{CD \cdot SO}{DO + CK}$ $JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD}$ $TU := \frac{CD \cdot JT}{KO}$ $DU := TO - (DO + TU)$ $CV := DU$ $CQ := 2 \cdot CK$ $QV := CQ - CV$

$BH := \frac{CK \cdot CV}{QV}$ $BH = 0.19$

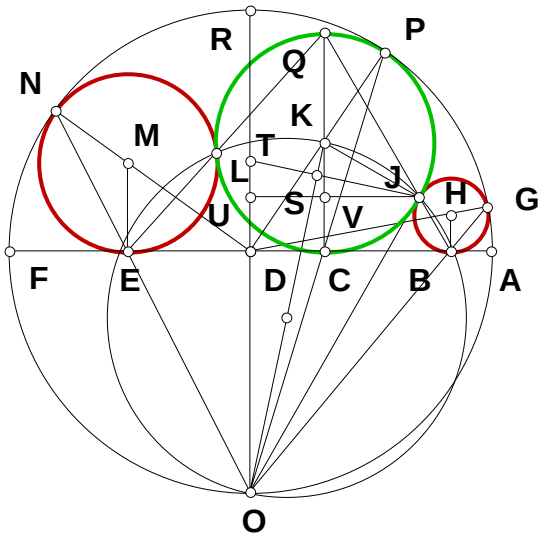
## Some Algebraic Names

$AC := \frac{1}{N}$ $CD := \frac{1}{2} - \frac{1}{N}$ $CO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^2 - 2 \cdot N + 2}}{N}$ $PO := \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{N^2 - 2 \cdot N + 2}} \cdot N$

$CP := \sqrt{2} \cdot \frac{(N - 1)}{\left(\sqrt{N^2 - 2 \cdot N + 2} \cdot N\right)}$ $CK := \frac{(N - 1)}{N^2}$ $KO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}{N^2}$

$JO := \frac{1}{2} \cdot \sqrt{2}$ $KS := \frac{(N - 1)^2}{N^2} \cdot \frac{\sqrt{2}}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $SO := \frac{1}{2} \cdot \sqrt{2} \cdot \frac{N^2}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$

$JS := \frac{(N - 1)}{\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2}}$ $ST := \frac{1}{2} \cdot (N - 2) \cdot \sqrt{2} \cdot \frac{N^3}{\left[\sqrt{N^4 + 2 \cdot N^2 - 4 \cdot N + 2} \cdot \left(N^2 + 2 \cdot N - 2\right)\right]}$

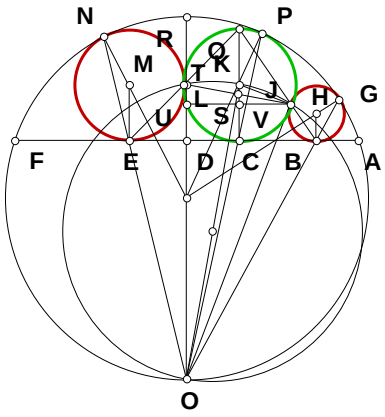



$$JT := \frac{1}{\left(2\cdot\sqrt{N^4+2\cdot N^2-4\cdot N+2}\right)}\cdot\frac{\left(2\cdot N^3+2\cdot N^2-8\cdot N+4+\sqrt{2}\cdot N^4-2\cdot\sqrt{2}\cdot N^3\right)}{\left(N^2+2\cdot N-2\right)} \qquad TO := \frac{N^2}{\left(N^2+2\cdot N-2\right)}$$

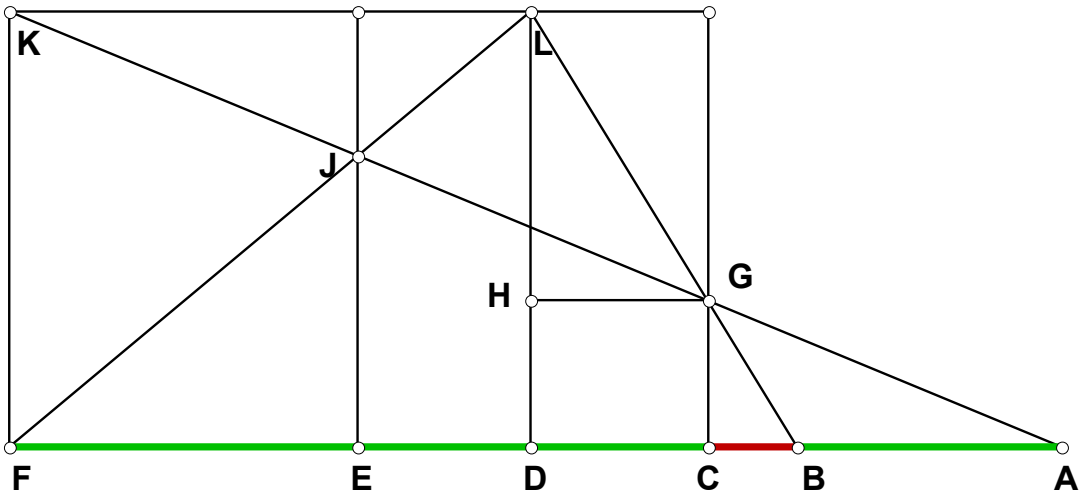
$$TU := \frac{1}{2}\cdot\left(N-\sqrt{2}\right)\cdot(N-2)\cdot\left(N-2+\sqrt{2}\right)\cdot\frac{N}{\left[\left(N^2+2\cdot N-2\right)\cdot\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)\right]}$$

$$DU := \frac{-1}{2}\cdot\left(\sqrt{2}-2\right)\cdot\frac{(N-1)}{\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)} \qquad CQ := 2\cdot\frac{(N-1)}{N^2}$$

$$QV := \frac{1}{2}\cdot\left(\sqrt{2}+2\right)\cdot(N-1)\cdot\frac{\left(N+2-2\cdot\sqrt{2}\right)^2}{\left[\left(N^2-\sqrt{2}\cdot N+2-\sqrt{2}\right)\cdot N^2\right]} \qquad BH := \frac{(N-1)\cdot\left(3-2\cdot\sqrt{2}\right)}{\left[(N+2)-2\cdot\sqrt{2}\right]^2}$$




**Working on Moving D.**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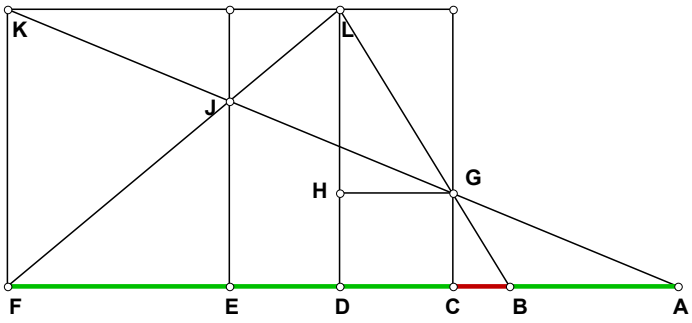
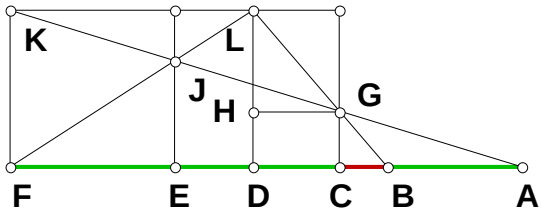



# 960425 EP

Given:

## One Over N + One 042596

### Construct 1/(N+1)




$N_1 := 2.817 \quad N_2 := 3 \quad AC := 1$

$AF := AC \cdot N_1 \quad CF := AF - AC \quad CE := \frac{CF}{2}$

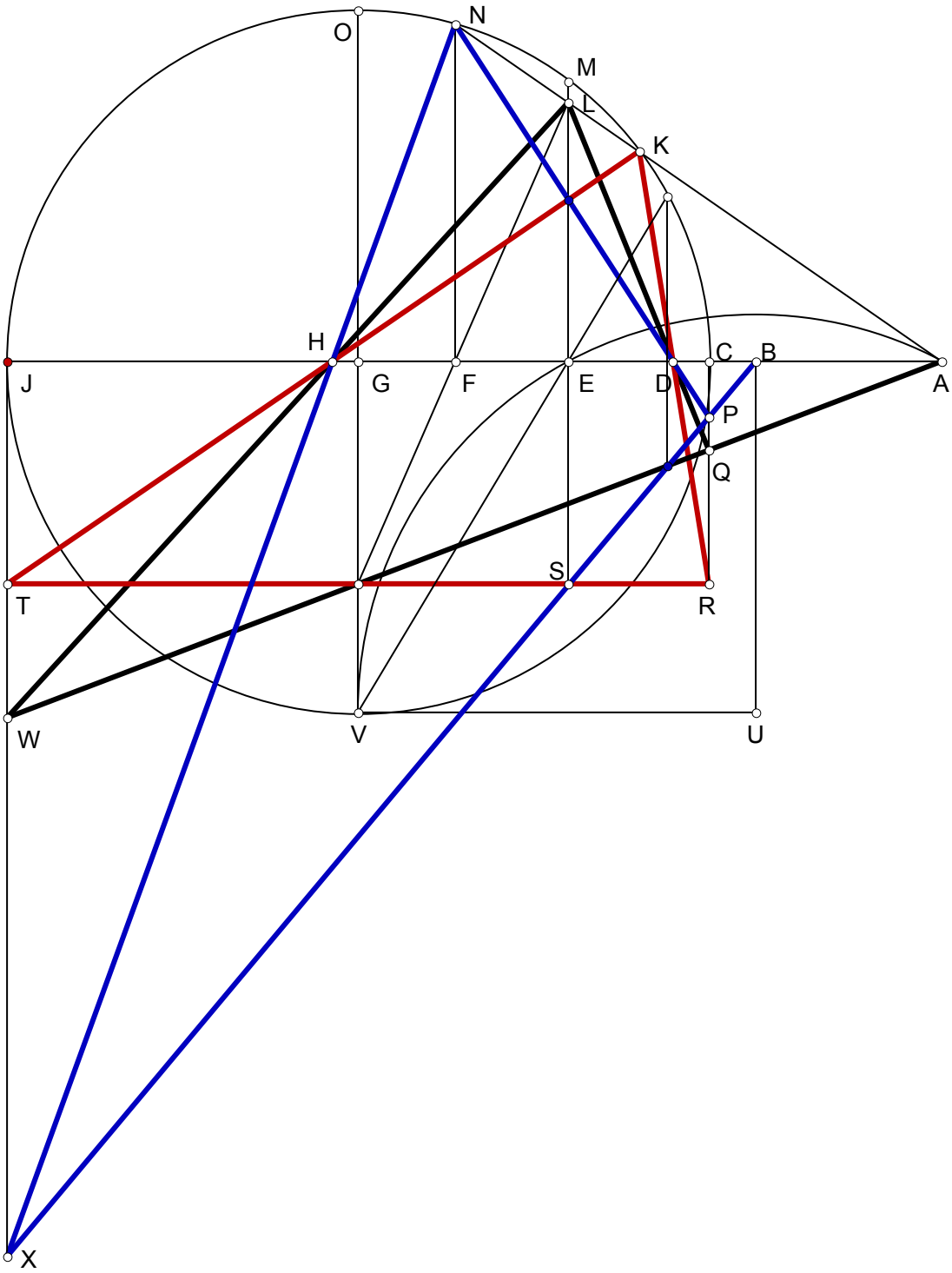
$AE := AC + CE \quad FK := AC \cdot N_2 \quad EJ := \frac{FK \cdot AE}{AF}$

$DL := FK \quad EF := CE \quad DF := \frac{EF \cdot DL}{EJ} \quad CG := \frac{FK \cdot AC}{AF} \quad CD := CF - DF \quad DH := CG \quad HL := DL - DH$

$BC := \frac{CD \cdot CG}{HL}$

$CF := N_1 - 1 \quad CE := \frac{1}{2} \cdot (N_1 - 1) \quad AE := \frac{1}{2} \cdot (1 + N_1) \quad FK := N_2 \quad EJ := \frac{1}{2} \cdot N_2 \cdot \frac{(1 + N_1)}{N_1}$

$DF := \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 \quad CG := \frac{N_2}{N_1} \quad CD := \frac{(N_1 - 1)}{(1 + N_1)} \quad HL := N_2 \cdot \frac{(N_1 - 1)}{N_1} \quad \frac{1}{N_1 + 1} - BC = 0$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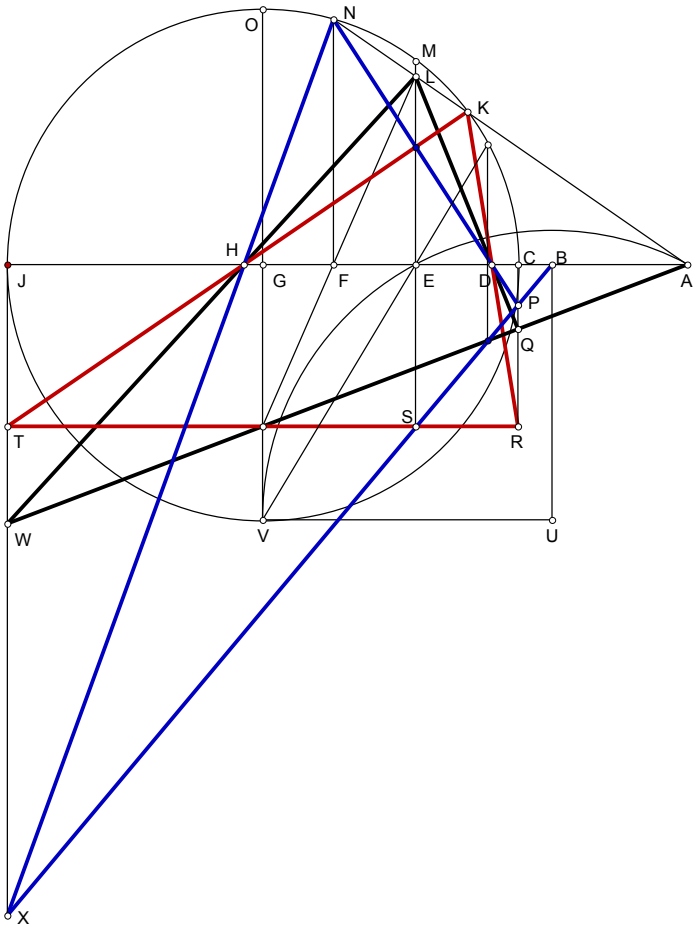
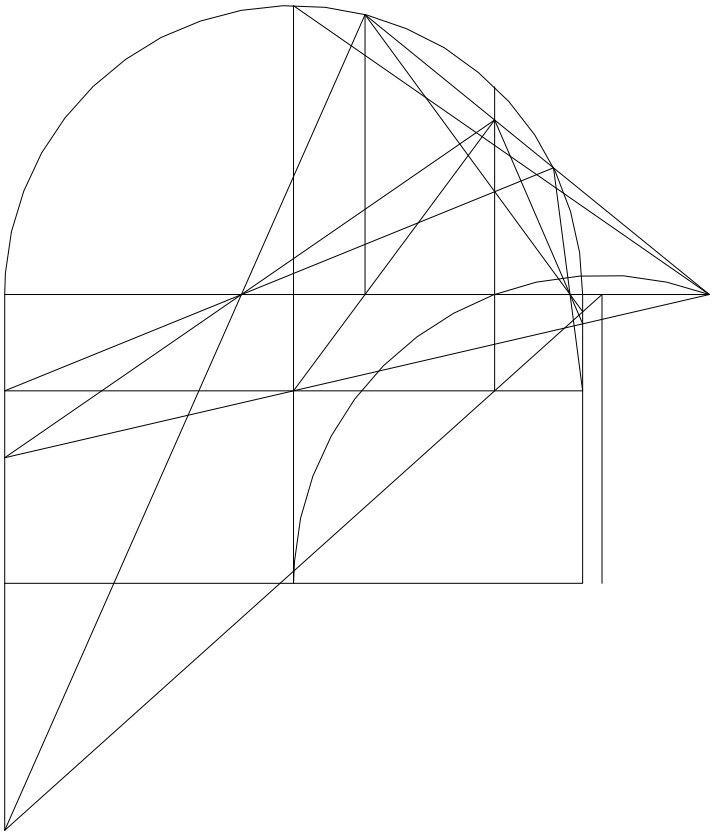


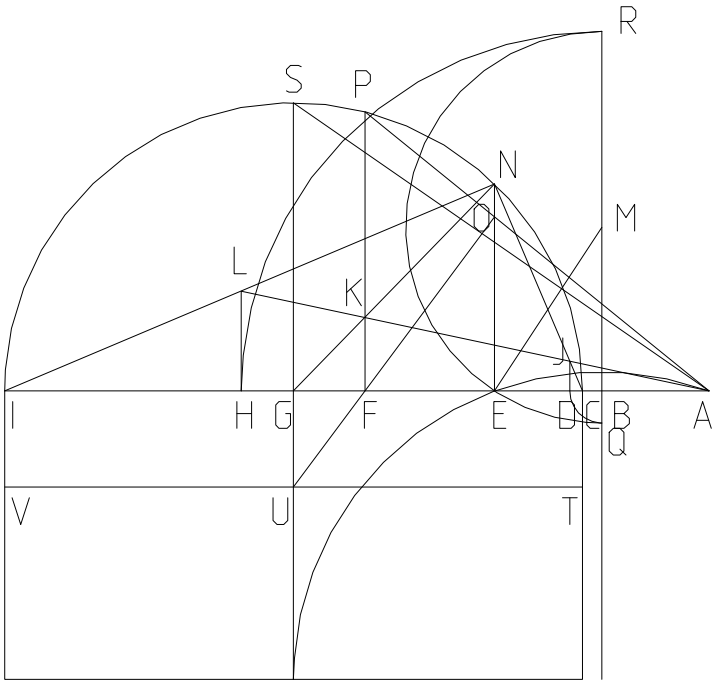

# 960426 EP

Given:



# Three Base Theorem.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.




$BC := 7.2 \quad CI := 216 \quad CG := \frac{CI}{2} \quad BI := BC + CI$

$BE := \sqrt{BC \cdot BI} \quad BM := 61.38$

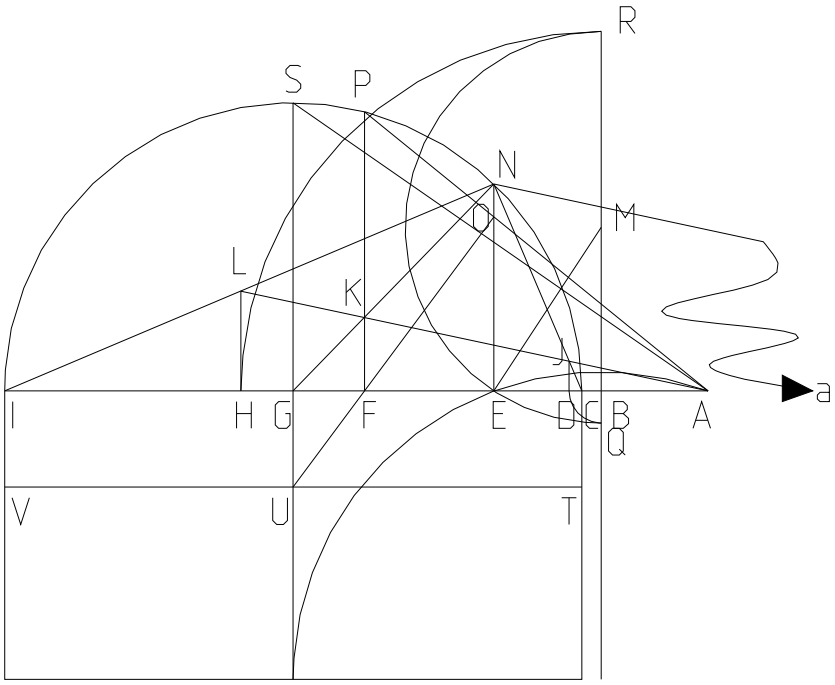
$EM := \sqrt{BM^2 + BE^2} \quad BD := EM - BM$

$BH := BM + EM \quad GN := CG \quad CE := BE - BC$

$EI := CI - CE \quad EN := \sqrt{CE \cdot EI} \quad EH := BH - BE$

$EG := EI - CG \quad AE := \frac{EN^2}{EG} \quad HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI} \quad AG := AE + EG$



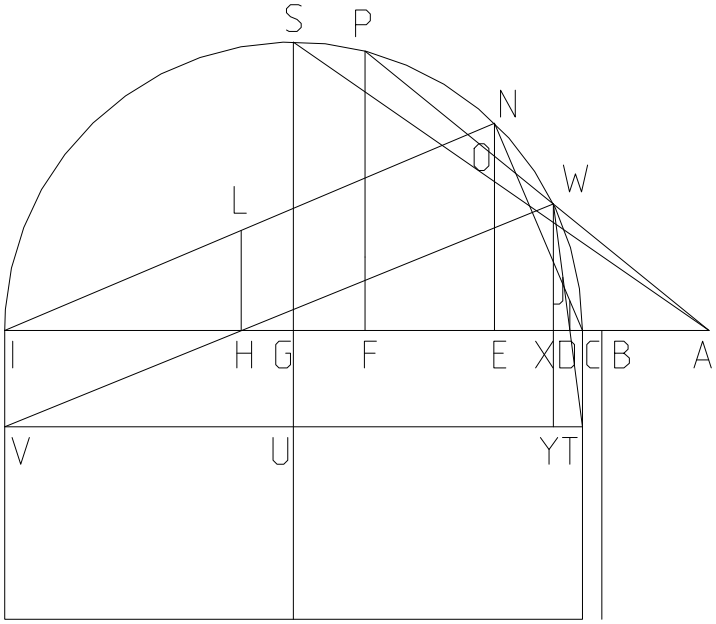

$$AH := AE + EH \quad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \quad CF := CG - FG$$

$$FI := CG + FG \quad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \quad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \quad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \quad AI := AC + CI$$

$$AP := \sqrt{AF^2 + FP^2} \quad AW := \frac{AC \cdot AI}{AP}$$

$$AX := \frac{AF \cdot AW}{AP} \quad CX := AX - AC \quad XI := CI - CX$$

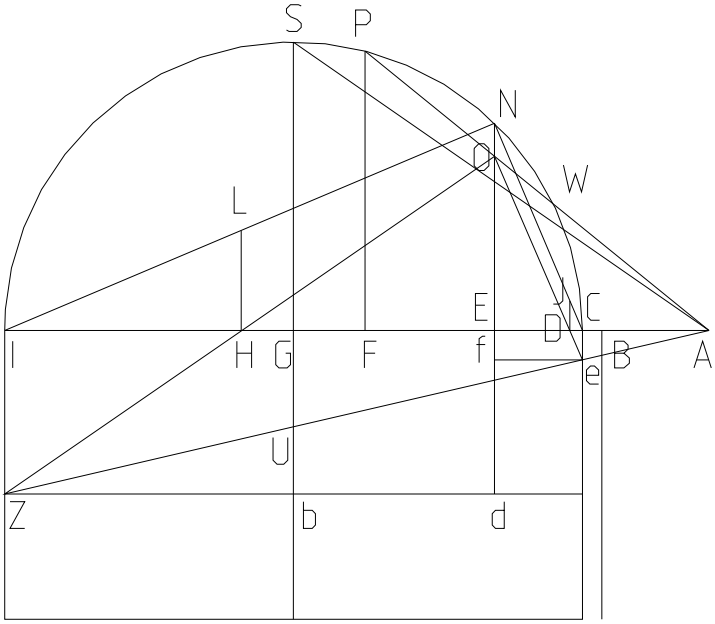
$$WX := \sqrt{CX \cdot XI} \quad XG := CG - CX \quad YU := XG$$

$$UV := CG \quad YV := YU + UV \quad XH := \frac{YV \cdot WX}{WX + GU}$$

$$CH := AH - AC \quad \frac{CH}{XH + CX} = 1$$

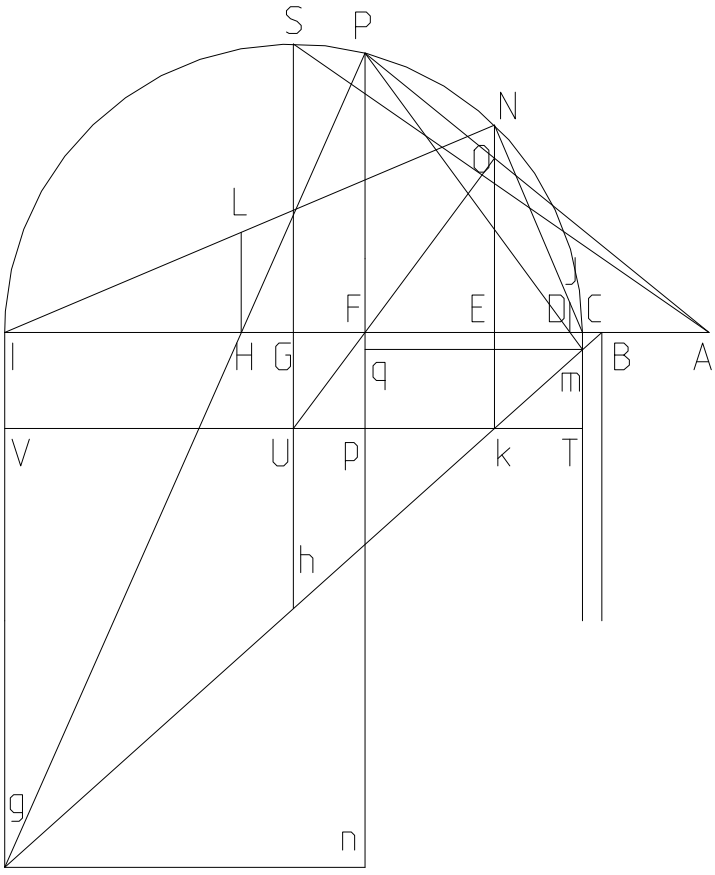
$$CD := BD - BC \quad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$




$$IZ := \frac{GU \cdot AI}{AG} \quad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO + Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \quad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO + Ef}} = 1$$

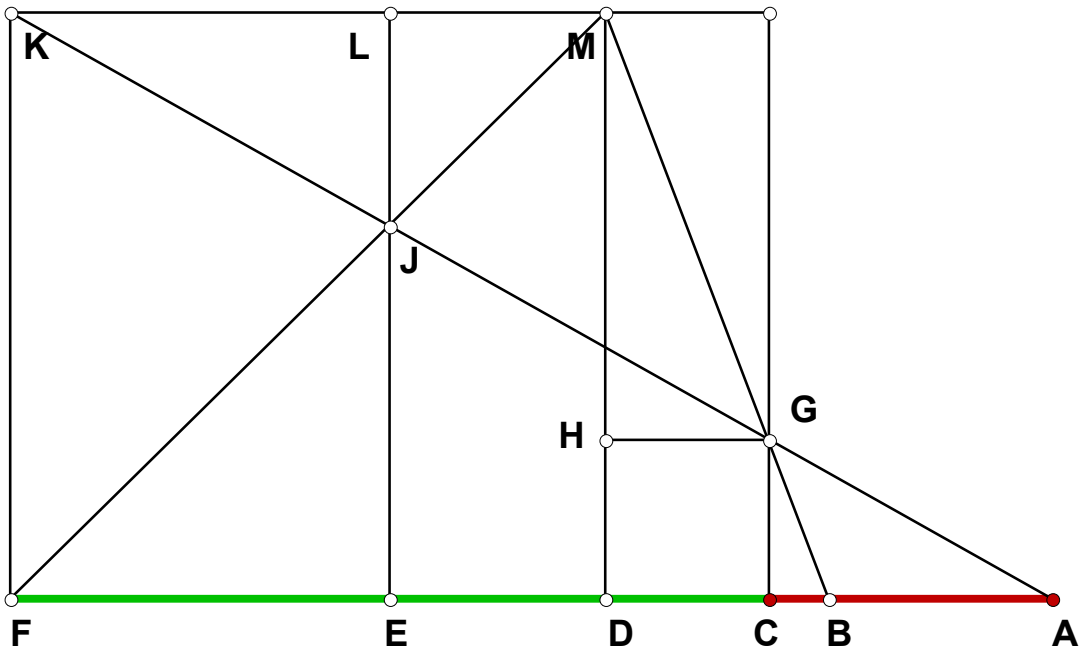



$$Ek := GU \quad Ig := \frac{Ek \cdot BI}{BE} \quad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig \quad gn := FI \quad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \quad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$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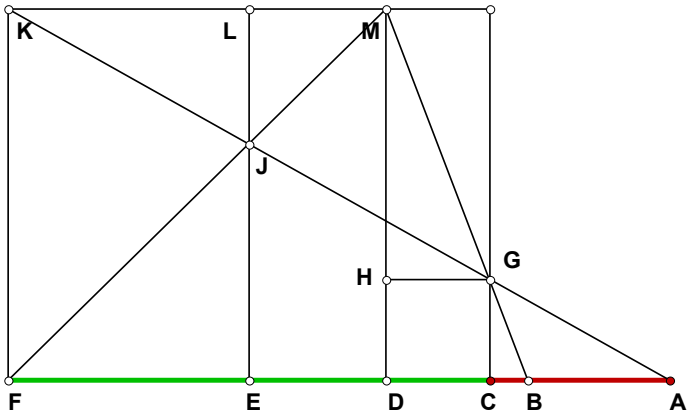
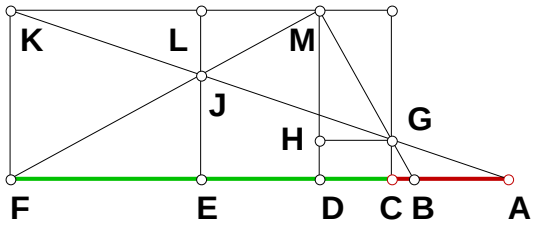



# 960427 EP

Given:

# A Root FIgure 042796

**Given CD, CE, and EF = CE, and that BD is the square root of BCxBF, find BC.**




$N_1 := 5 \quad N_2 := 3 \quad CD := 1$

$CE := CD \cdot N_1 \quad CF := 2 \cdot CE \quad FK := CD \cdot N_2$

$DM := FK \quad EL := FK \quad DF := CF - CD$

$EF := \frac{CF}{2} \quad EJ := \frac{DM \cdot EF}{DF} \quad JL := EL - EJ$

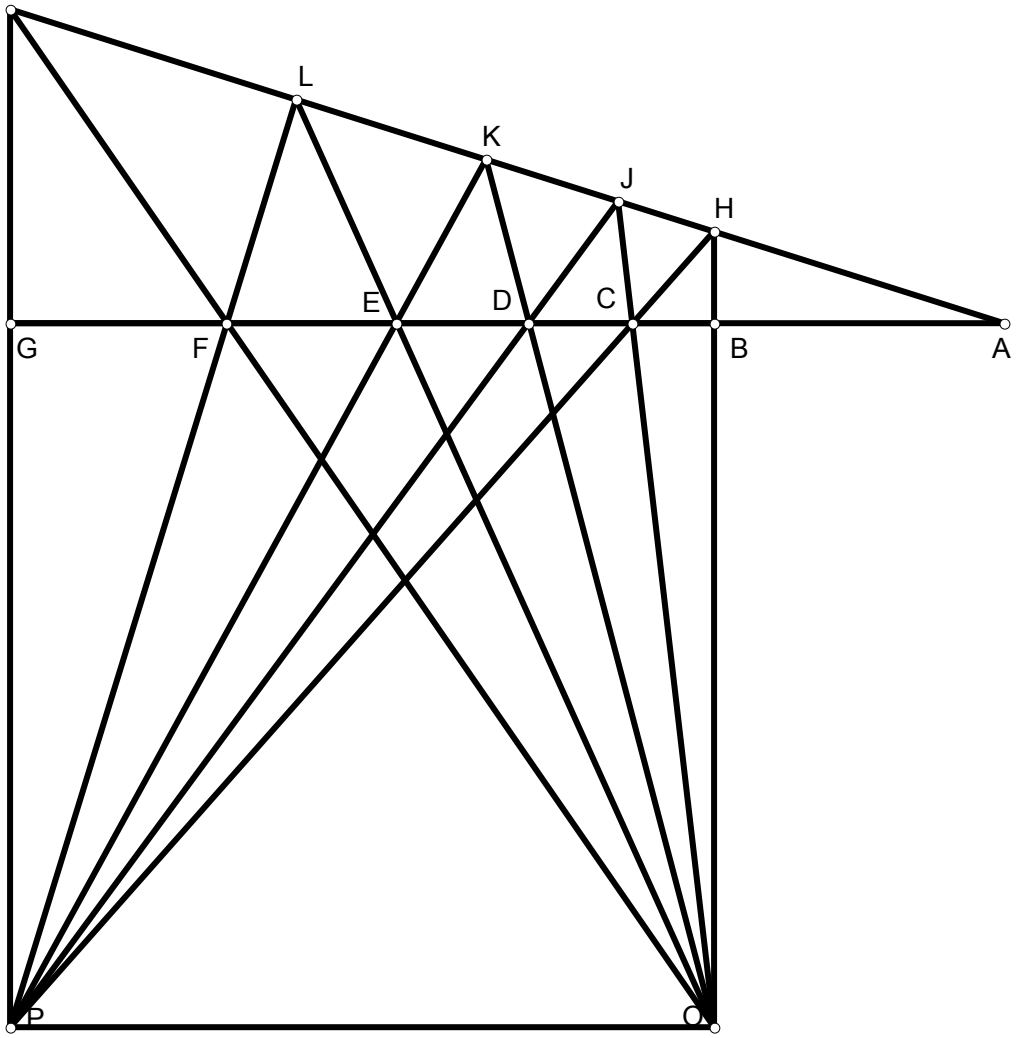
$KL := EF \quad AF := \frac{KL \cdot FK}{JL} \quad AC := AF - CF \quad CG := \frac{FK \cdot AC}{AF} \quad DH := CG \quad HM := DM - DH \quad BC := \frac{CD \cdot DH}{HM}$

$BF := BC + CF \quad BD := BC + CD \quad \sqrt{BC \cdot BF} - BD = 0$

$CF := 2 \cdot N_1 \quad FK := N_2 \quad DF := (2 \cdot N_1 - 1) \quad EF := N_1 \quad EJ := \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1}$

$JL := N_2 \cdot \frac{(N_1 - 1)}{(2 \cdot N_1 - 1)} \quad AF := \frac{N_1 \cdot (2 \cdot N_1 - 1)}{N_1 - 1} \quad AC := \frac{N_1}{(N_1 - 1)} \quad CG := \frac{N_2}{(2 \cdot N_1 - 1)}$

$HM := 2 \cdot N_2 \cdot \frac{(N_1 - 1)}{(2 \cdot N_1 - 1)} \quad BC := \frac{1}{2 \cdot (N_1 - 1)} \quad BF := \frac{1}{2} \cdot \frac{(2 \cdot N_1 - 1)^2}{(N_1 - 1)} \quad BD := \frac{1}{2} \cdot \frac{(2 \cdot N_1 - 1)}{(N_1 - 1)}$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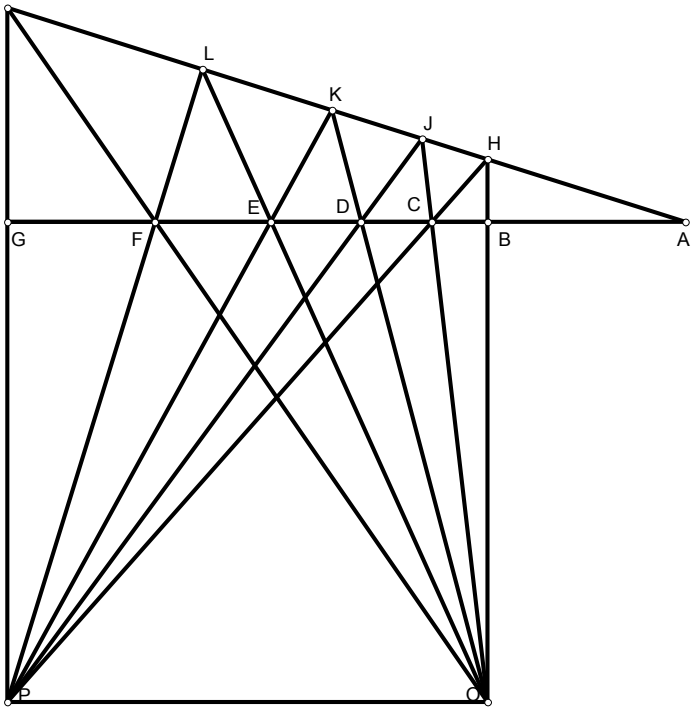



## 960428 EP

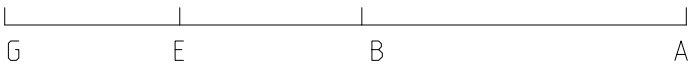
Given:

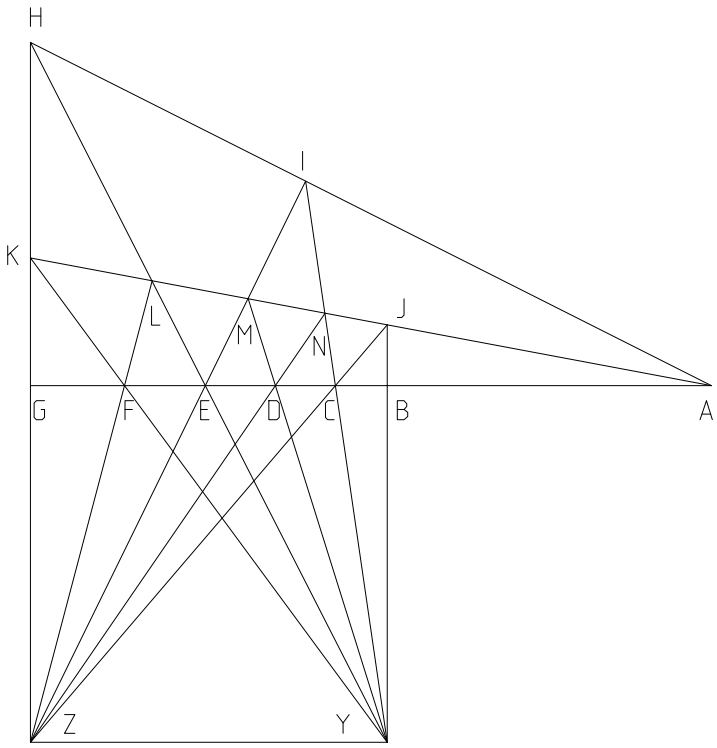
**042896.MCD**

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

Process Summary will use $5^{th}$ root series for example.




$AG := 3^5 \qquad AB := 1 \qquad AE := 3^3$

$BG := AG - AB \quad GZ := BG \quad YZ := BG$

$BY := BG \quad BE := AE - AB \quad EG := BG - BE$

$$GH := \frac{BY \cdot EG}{BE}$$

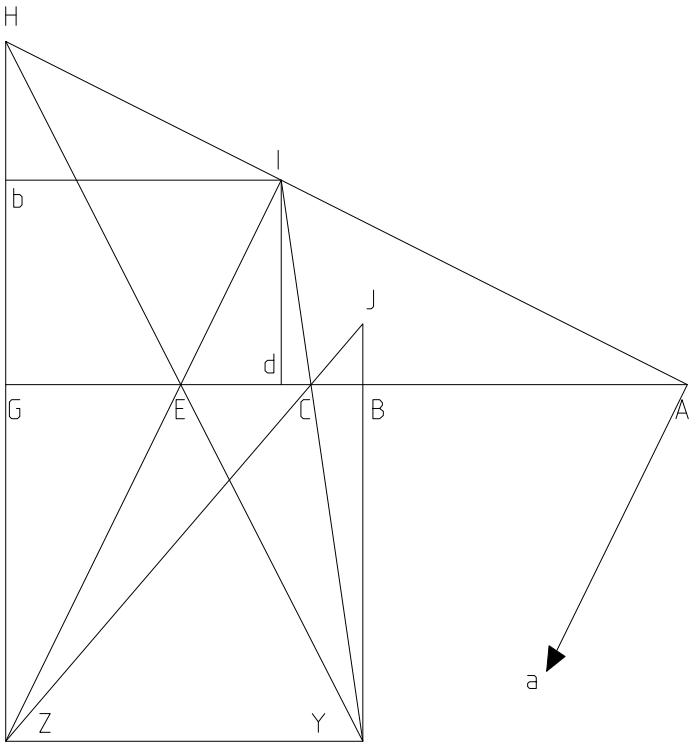
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

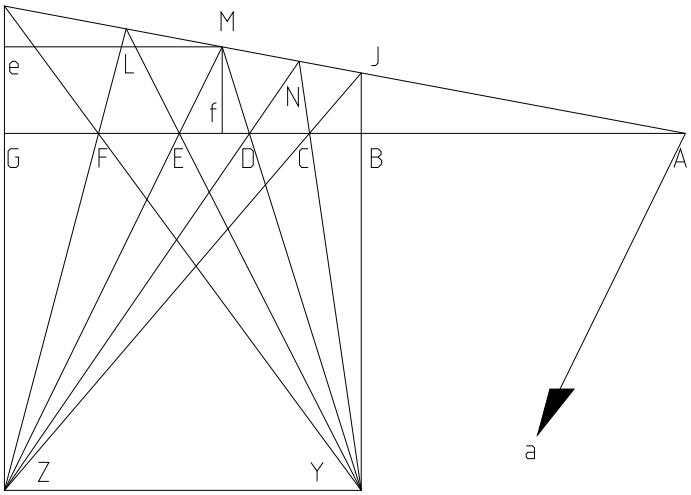
$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$



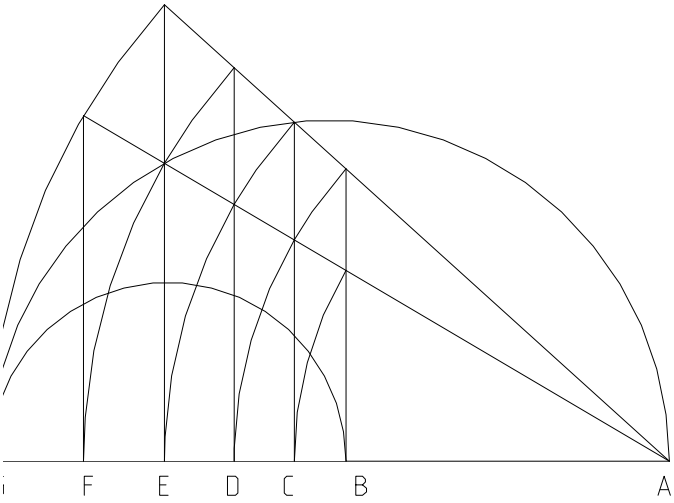



$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



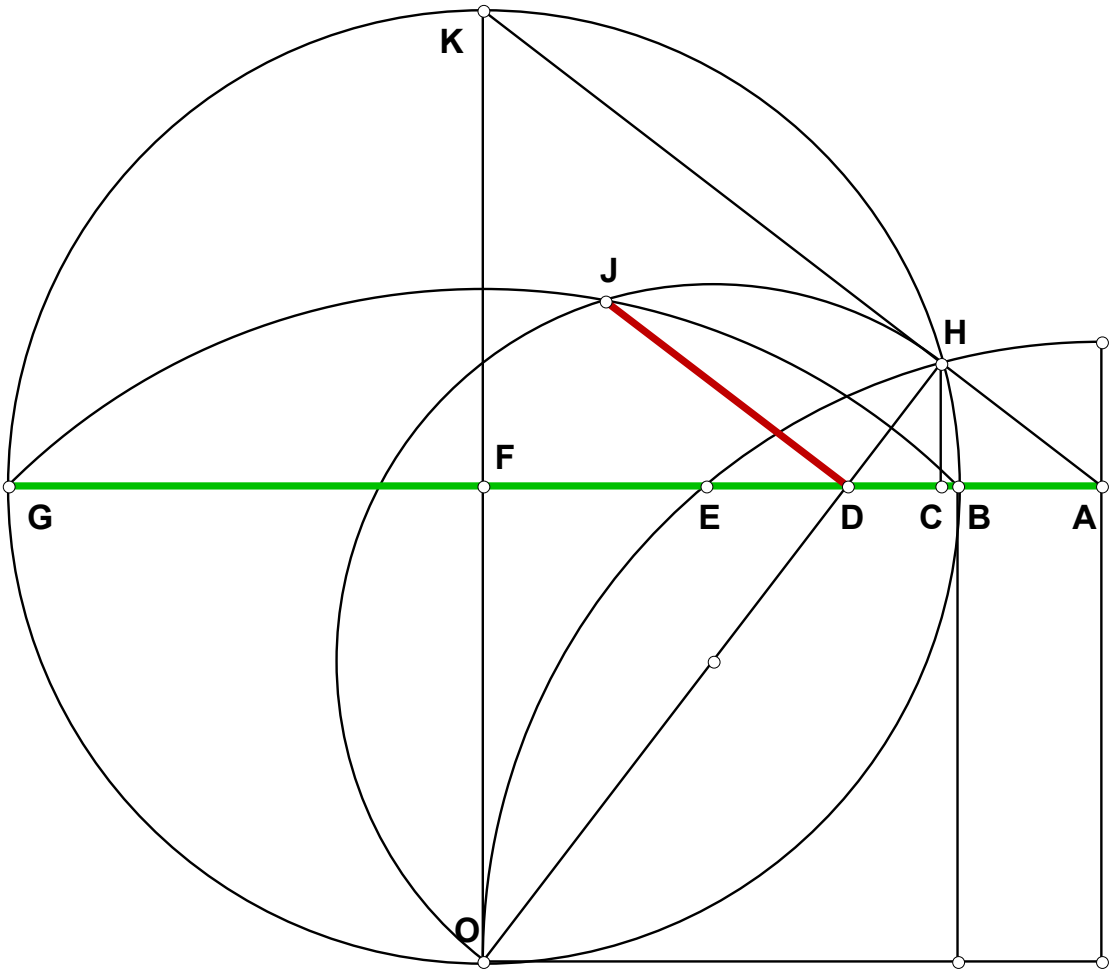

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$

Compass method

If any of a prime root series can be given exactly, every root of the series can be determined exactly.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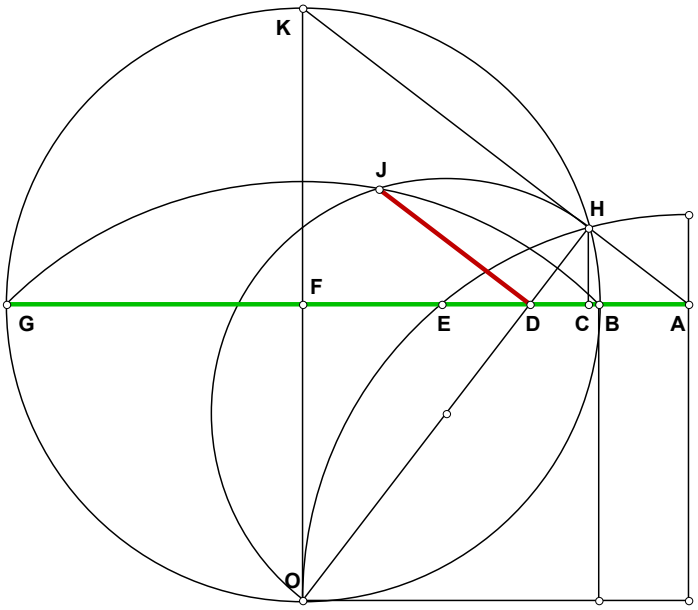
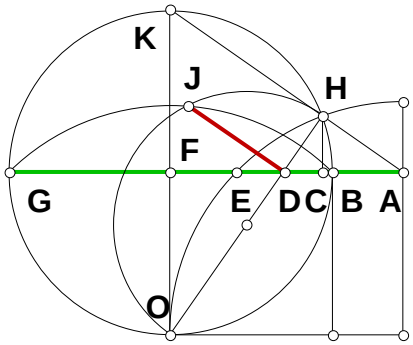



# 960429 EP

Given:

# What is DJ? 042996

**DJ is the Geometric name, what is its Algebraic name?**




$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N \qquad BG := AG - AB$

$BF := \frac{BG}{2} \qquad FK := BF \qquad FO := BF \qquad AF := BF + AB$

$DF := \frac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

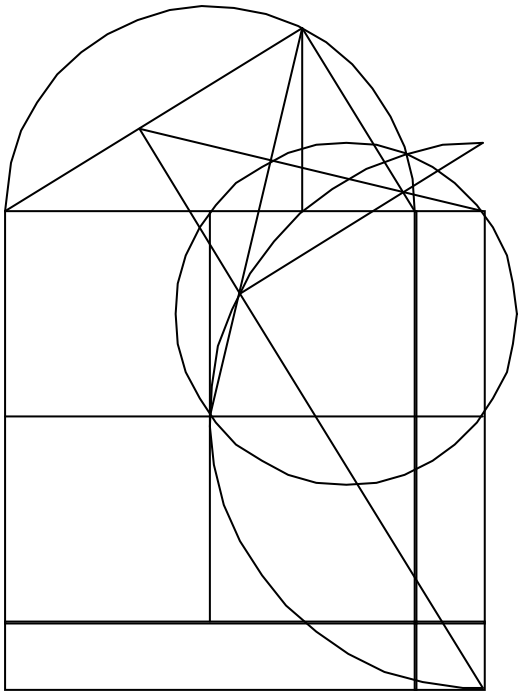
$HO := \frac{AF \cdot KO}{AK} \qquad DO := \frac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

$AG := N \quad BG := N - 1 \quad BF := \frac{N-1}{2} \qquad AF := \frac{1}{2} \cdot N + \frac{1}{2} \qquad DF := \frac{1}{2} \cdot \frac{(N-1)^2}{(N+1)} \qquad AK := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$

$HO := (N+1) \cdot \frac{(N-1)}{\sqrt{2 \cdot N^2 + 2}} \qquad DO := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2} \cdot \frac{(N-1)}{(N+1)} \qquad DH := 2 \cdot N \cdot \frac{(N-1)}{\left[\sqrt{2 \cdot N^2 + 2} \cdot (N+1)\right]}$

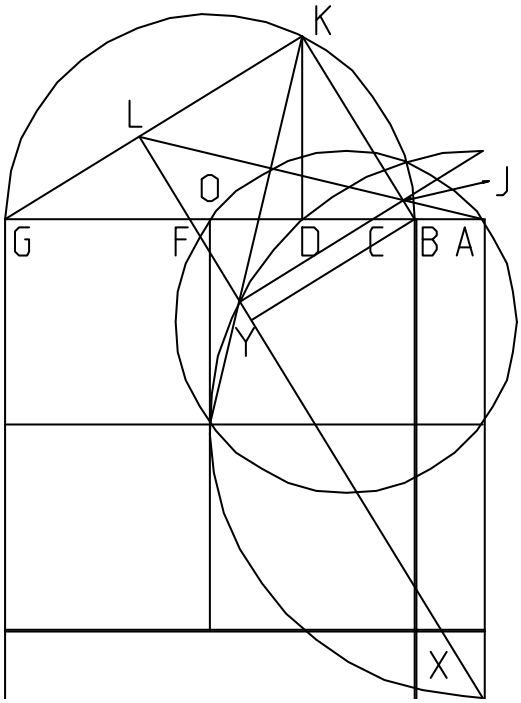
$$DJ := \sqrt{N} \cdot \frac{(N-1)}{(N+1)}$$



# Geometric Exponential Series of the form

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \textbf{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.




$N = 4$ $Root = 4$ $M = 1$ $BG := N$ $AB := M$

$AG := AB + BG$ $BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}}$ $AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

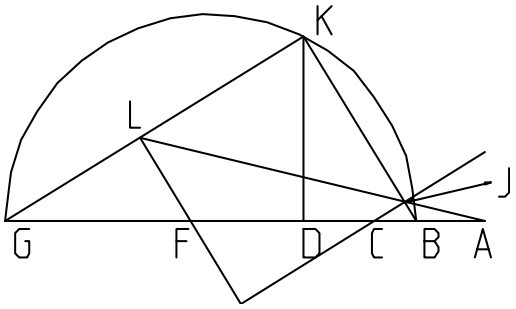
$BC := AC - AB$ $FG := AG - AF$ $FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX}$ $BD := \frac{FY \cdot BG}{FX}$ $AD := BD + AB$

$DG := AG - AD$ $DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2}$ $GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG}$ $GL := \frac{GK \cdot FG}{BG}$



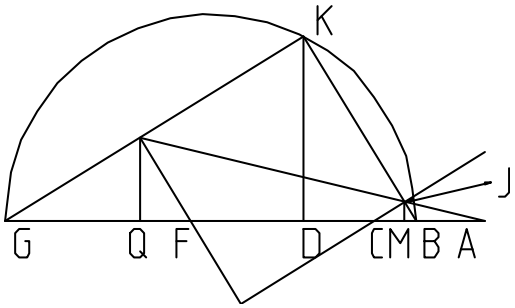

Plug in BG here as N. AB as M. Plug in root series also.

$N \equiv 4$ $\quad Root \equiv 4$ $\quad \delta := 1 .. Root$

$M \equiv 1$

$GL = 1.377$ $\quad BJ = 0.275$ $\quad \frac{GL}{BJ} = 5$ $\quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root - 1}{Root}}} = 2.415 \qquad \frac{GK}{GL} = 2.415 \qquad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root - \delta}{Root}} = 8.075 \qquad \frac{BK}{BJ} = 8.075$$




$$BM := \frac{BD \cdot BC}{BG} \qquad FQ := \frac{BD \cdot FG}{BG}$$

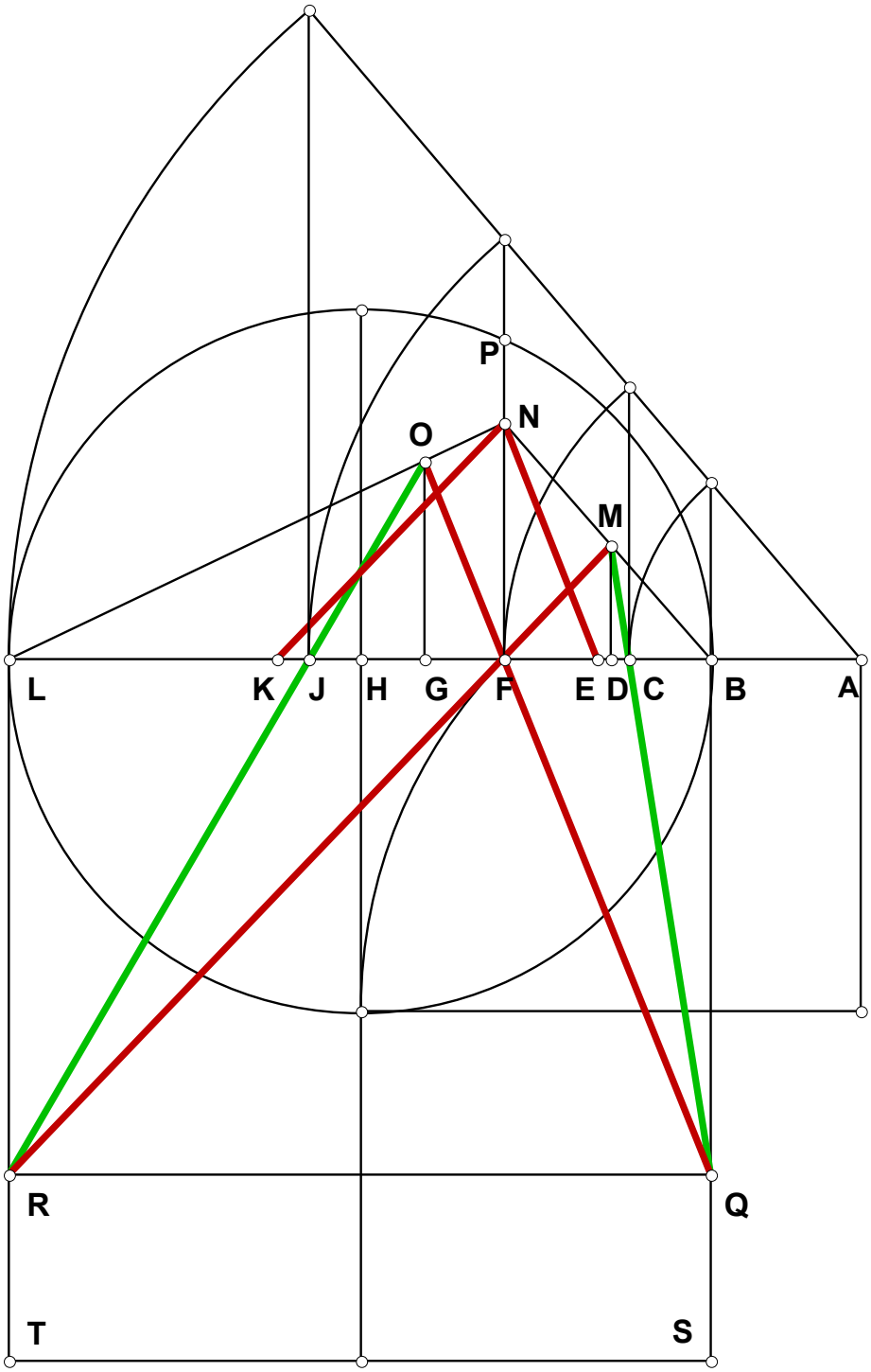
$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta + 2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}}$$

$\frac{AG}{FQ} = 9.769$

On the left is the first and last of the series, on the right is the entire series.

| |
|---|
| 9.769 |
| 14.608 |
| 21.844 |
| 32.665 |

$\frac{AG}{BM} = 32.665$


P
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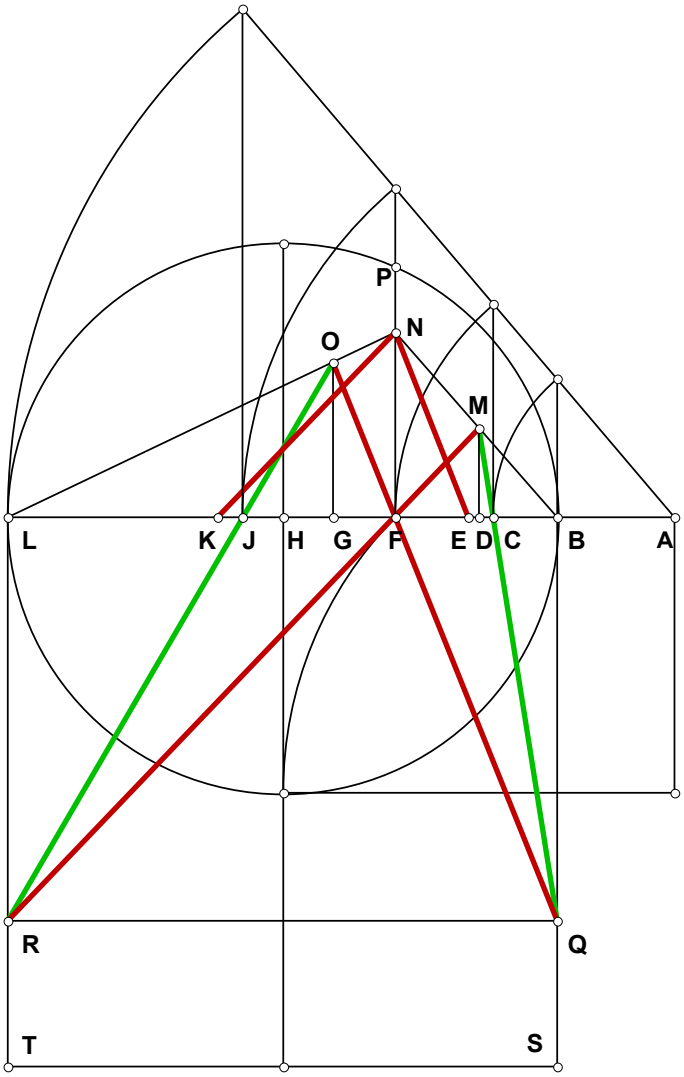
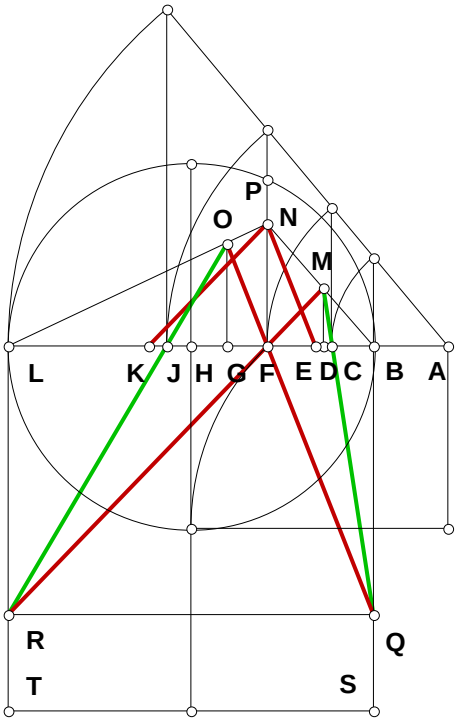
# 961220 EP




Given:

# 122096 Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off in the figure.**




$N_1 := 2 \qquad N_2 := .2$

$AB := 1 \qquad AL := AB \cdot N_1$

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

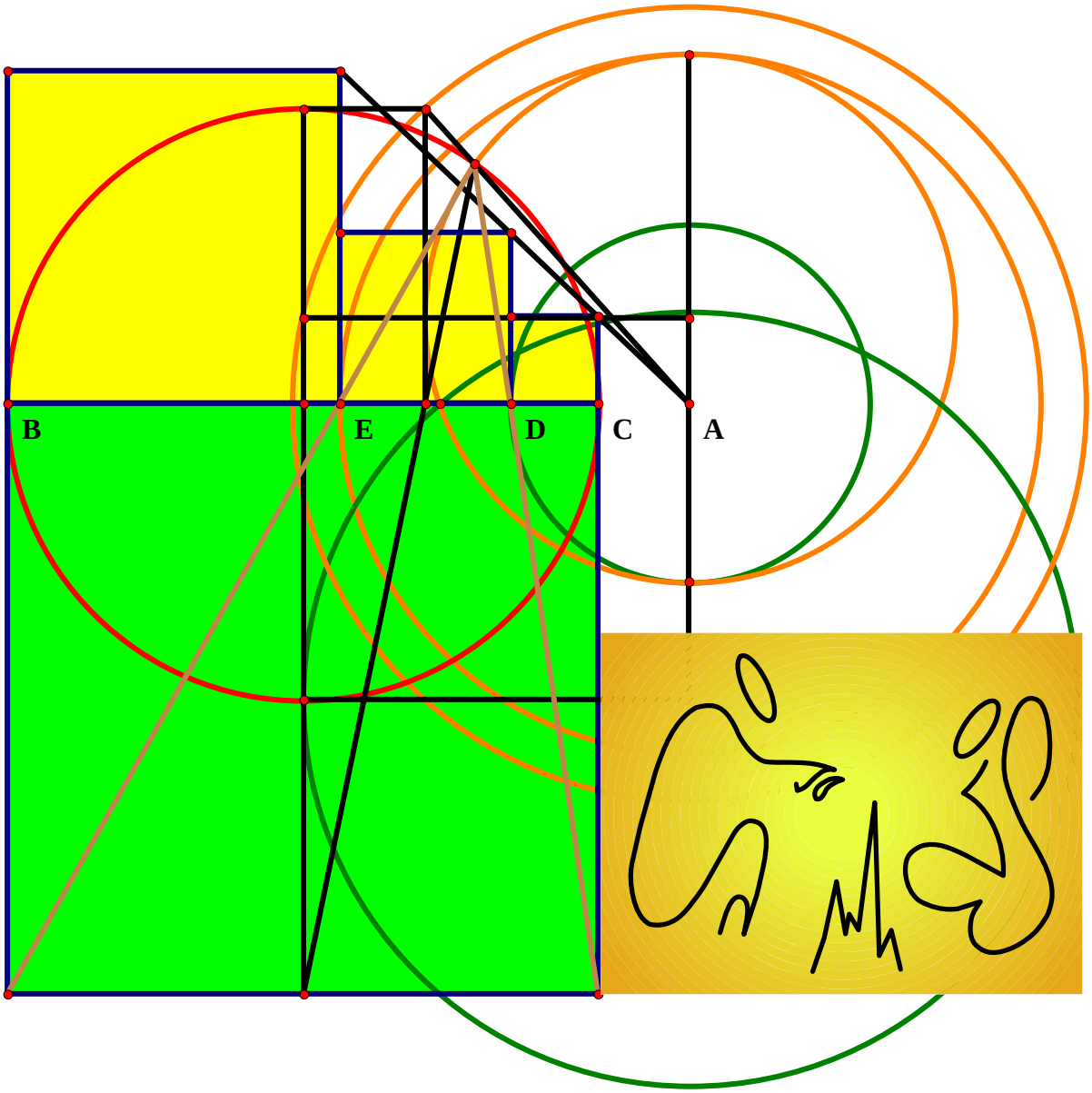
$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

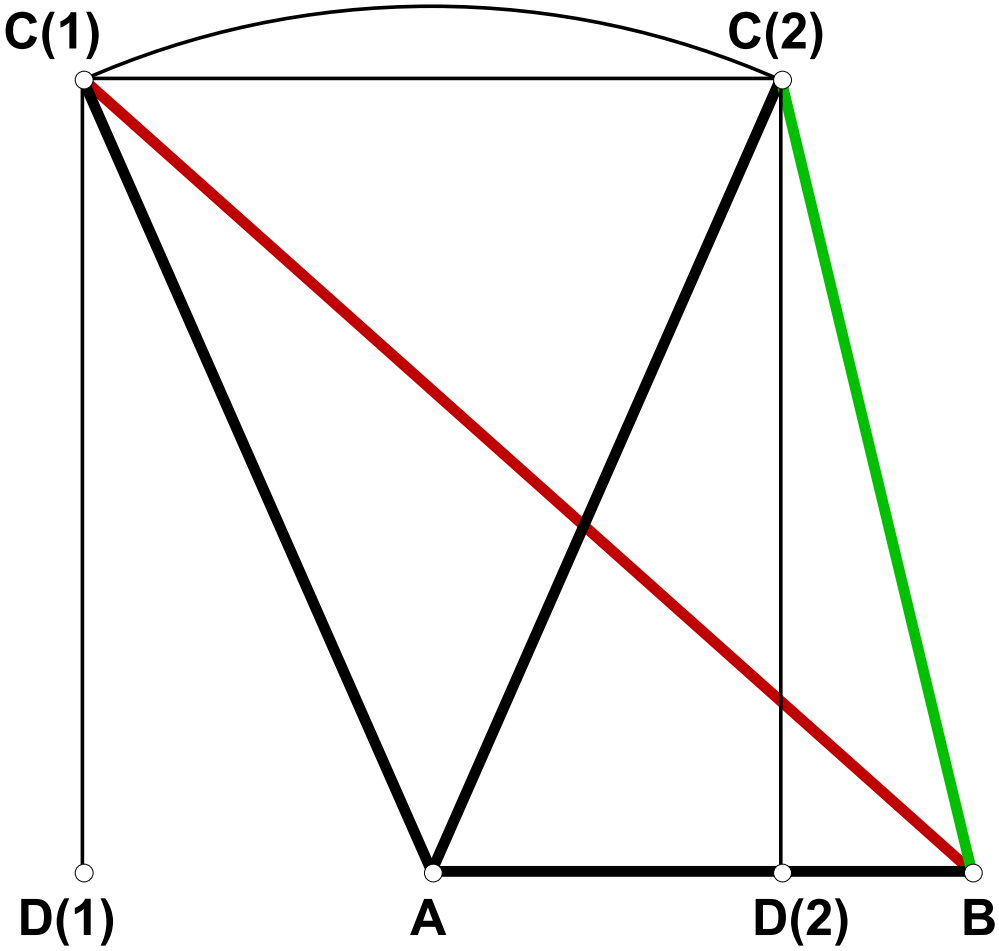
$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

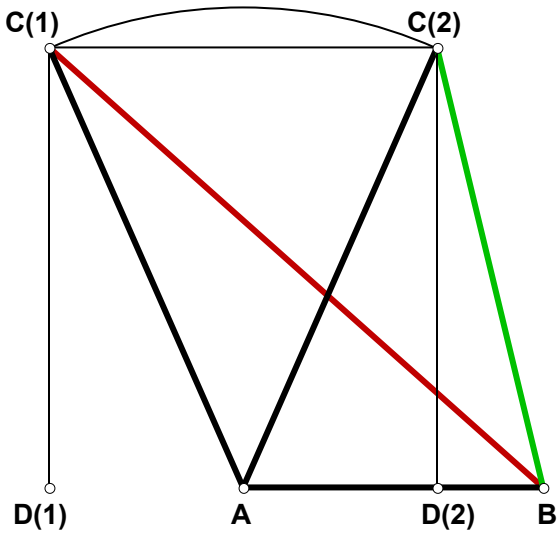
$AJ := AL - JL$

$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$

# The Delian Quest—1997




C(1)
C(2)
D(1)
A
D(2)
B

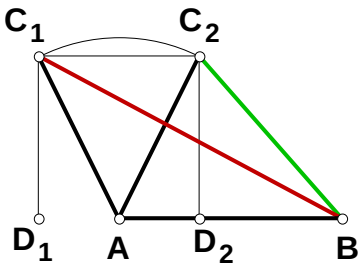



## 970404 EP

Given:

# 040497 Triangles

**Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.**




$AB := 5 \qquad AC := 4 \qquad CD := 3$

$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$

$BC_1 := \sqrt{CD^2 + BD_1^2} \qquad BC_2 := \sqrt{CD^2 + BD_2^2}$

$BC_1 = 8.213 \qquad BC_2 = 3.813$

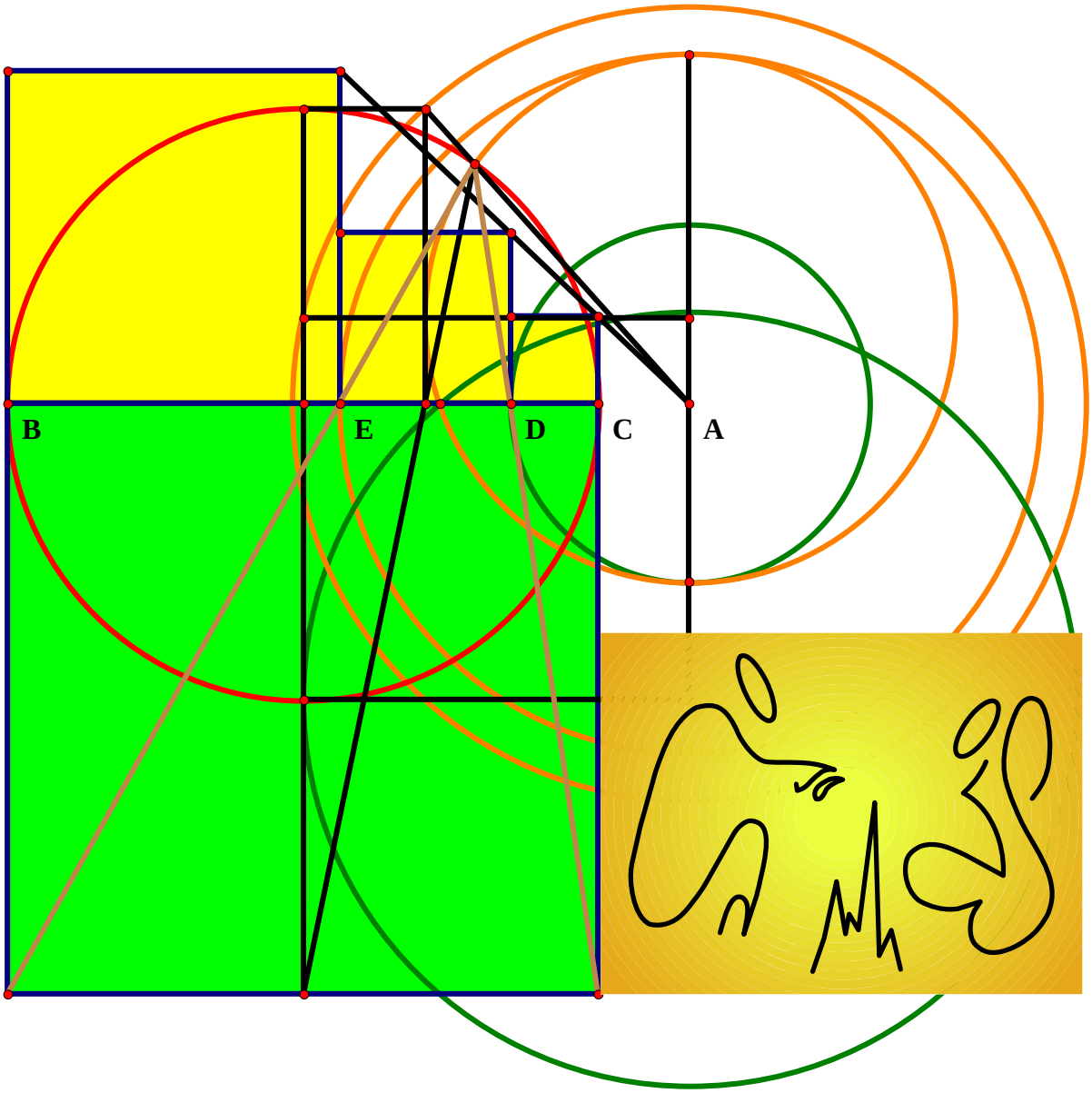
$S_1 := AB \quad S_2 := AC \quad S_3 := BC_1$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

$S_1 := AB \quad S_2 := AC \quad S_3 := BC_2$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

# The Delian Quest—1998



4

3+4/5

3+3/5

3+2/5

3+1/5

3

2+4/5

2+3/5

2+2/5

2+1/5

2

1+4/5

1+3/5

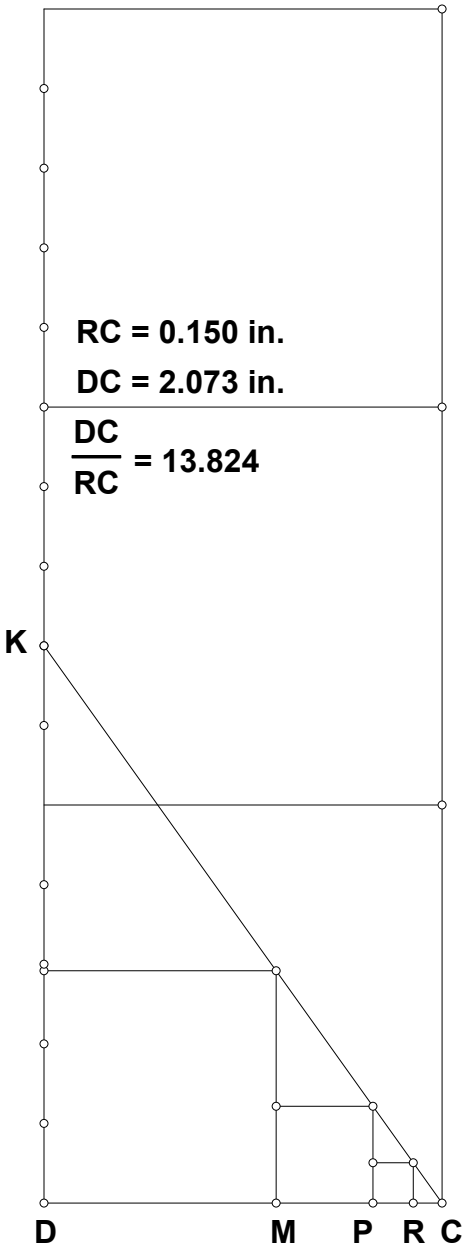
1+2/5

1+1/5

Show

Hide

# M^N




$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

$\left(2+\frac{1}{5}\right)^3 = 10.648$

$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

$\left(1+\frac{1}{5}\right)^3 = 1.728$

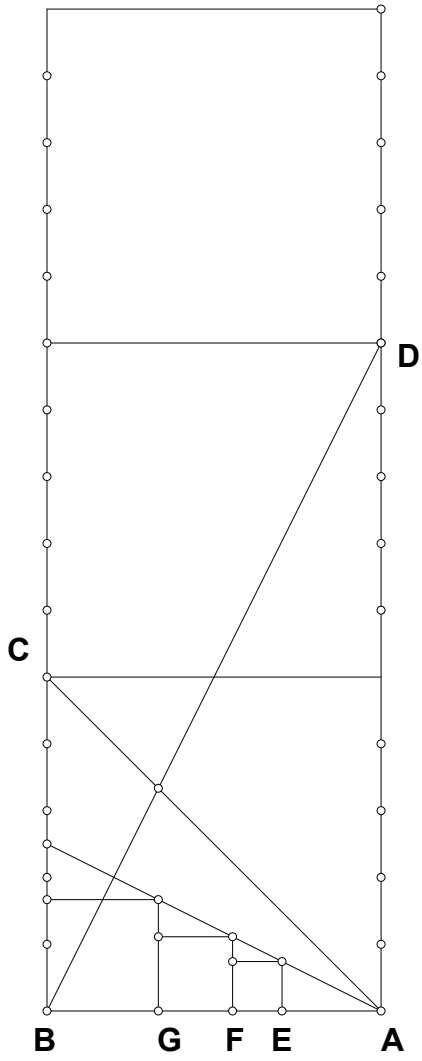
Given line AB, divide it by the equation

| BC |
|---|
| 2+4/5 |
| 2+3/5 |
| 2+2/5 |
| 2+1/5 |
| 2 |
| 1+4/5 |
| 1+3/5 |
| 1+2/5 |
| 1+1/5 |

| AD |
|---|
| 2+4/5 |
| 2+3/5 |
| 2+2/5 |
| 2+1/5 |
| 2 |
| 1+4/5 |
| 1+3/5 |
| 1+2/5 |
| 1+1/5 |

| W |
|---|
| 4 |
| 3+4/5 |
| 3+3/5 |
| 3+2/5 |
| 3+1/5 |
| 3 |

where W is a whole number.




$$\frac{AB}{AG} = 1.500$$

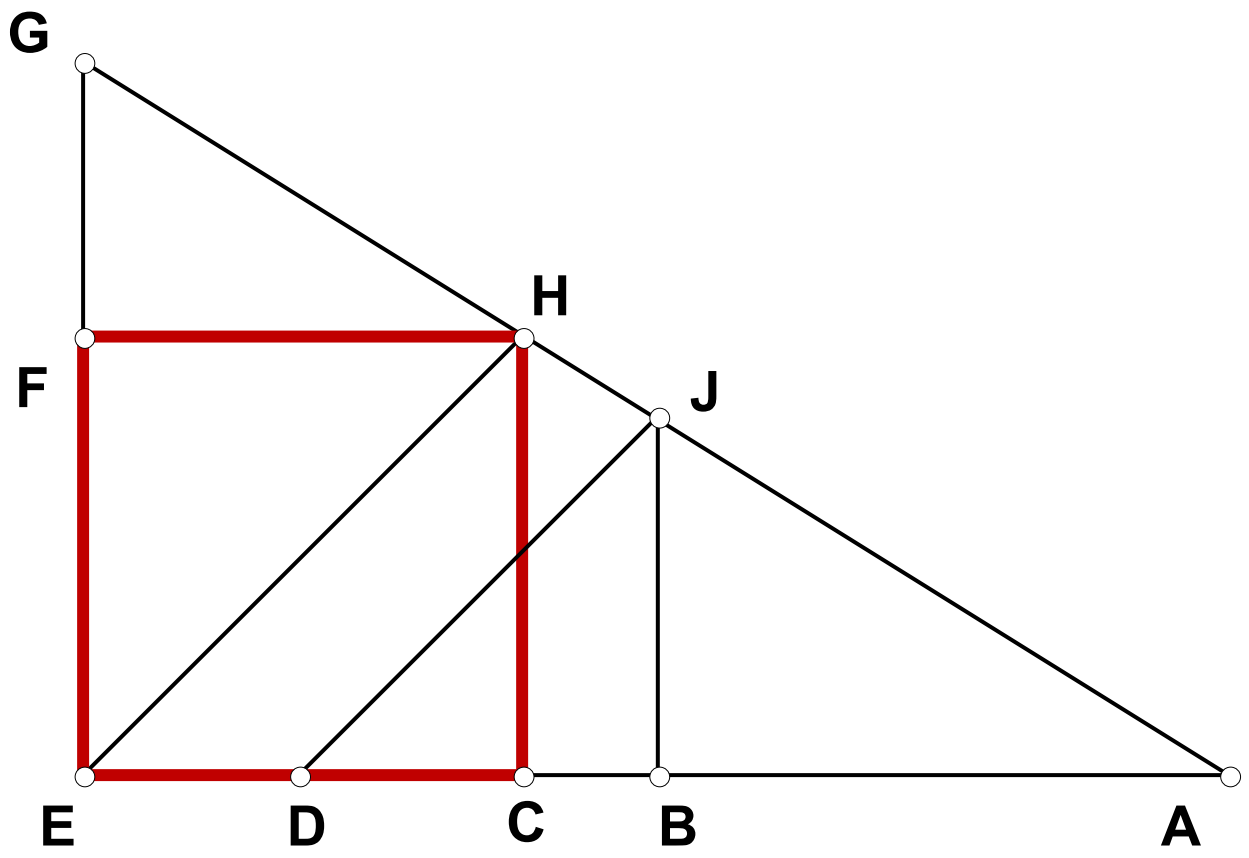
$$\frac{AB}{AF} = 2.250$$

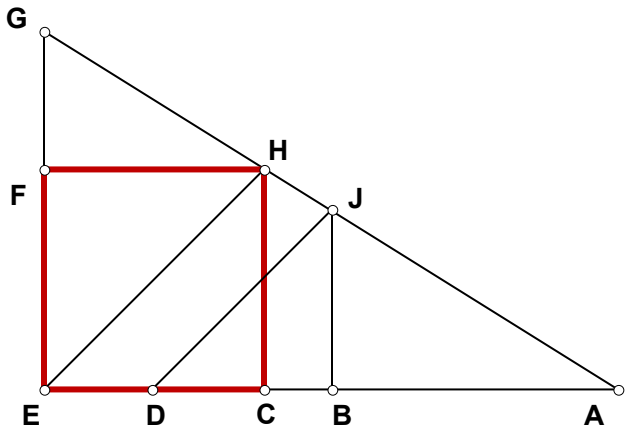
$$\frac{AB}{AE} = 3.375$$

$$\frac{BC+AD\,^{1}}{AD} = 1.500$$

$$\frac{BC+AD\,^{2}}{AD} = 2.250$$

$$\frac{BC+AD\,^{3}}{AD} = 3.375$$


G
H
F
J
E
D
C
B
A

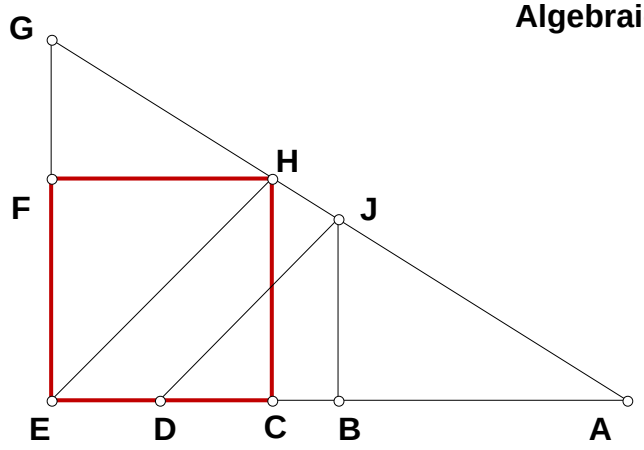



## 980210 EP

Given:

# A Square In A Triangle 021098

**What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?**



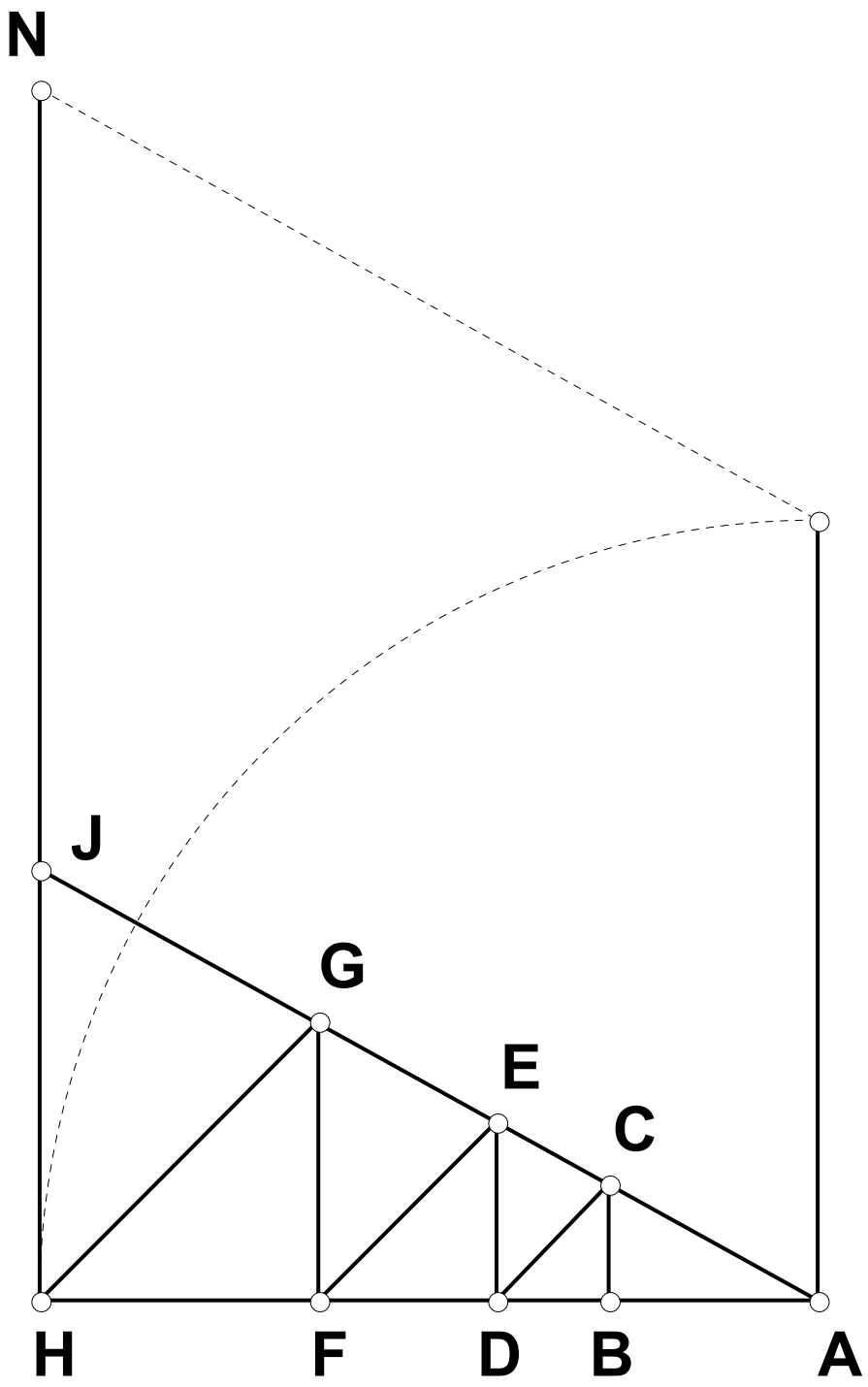

$N := 4 \quad AE := 1 \quad EG := N$

$AB := \frac{AE}{2} \quad BJ := \frac{EG}{2} \quad BD := BJ$

$AD := AB + BD \quad CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE \quad FG := EG - CE$

$\frac{AE}{AC} - (N + 1) = 0 \quad AC - \frac{1}{N + 1} = 0 \quad CE - \frac{N}{N + 1} = 0 \quad FG - \frac{N^2}{N + 1} = 0$


N
J
G
E
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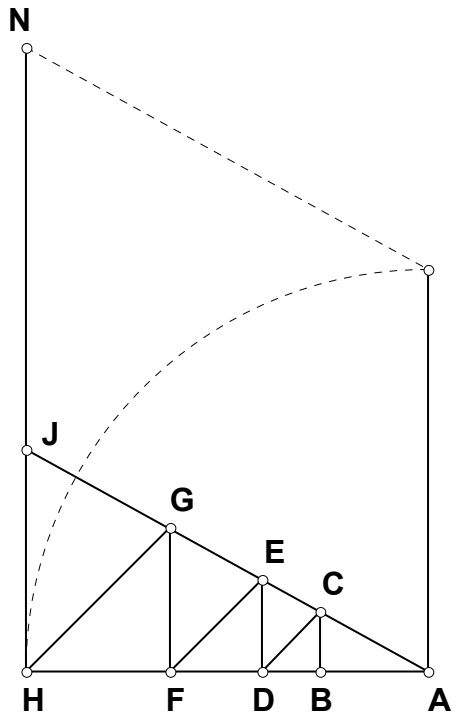
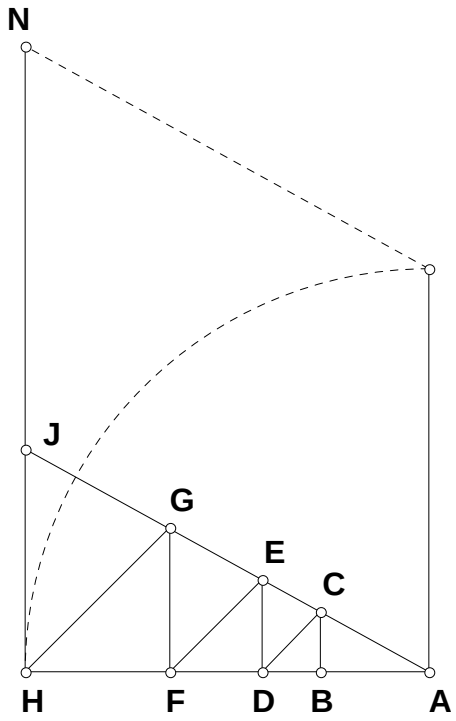



## 980225a EP

Given:

# Alternate Method Root Series 022598




**Given a length and a unit, raise that length to any whole power.**

**Given for the third power.**

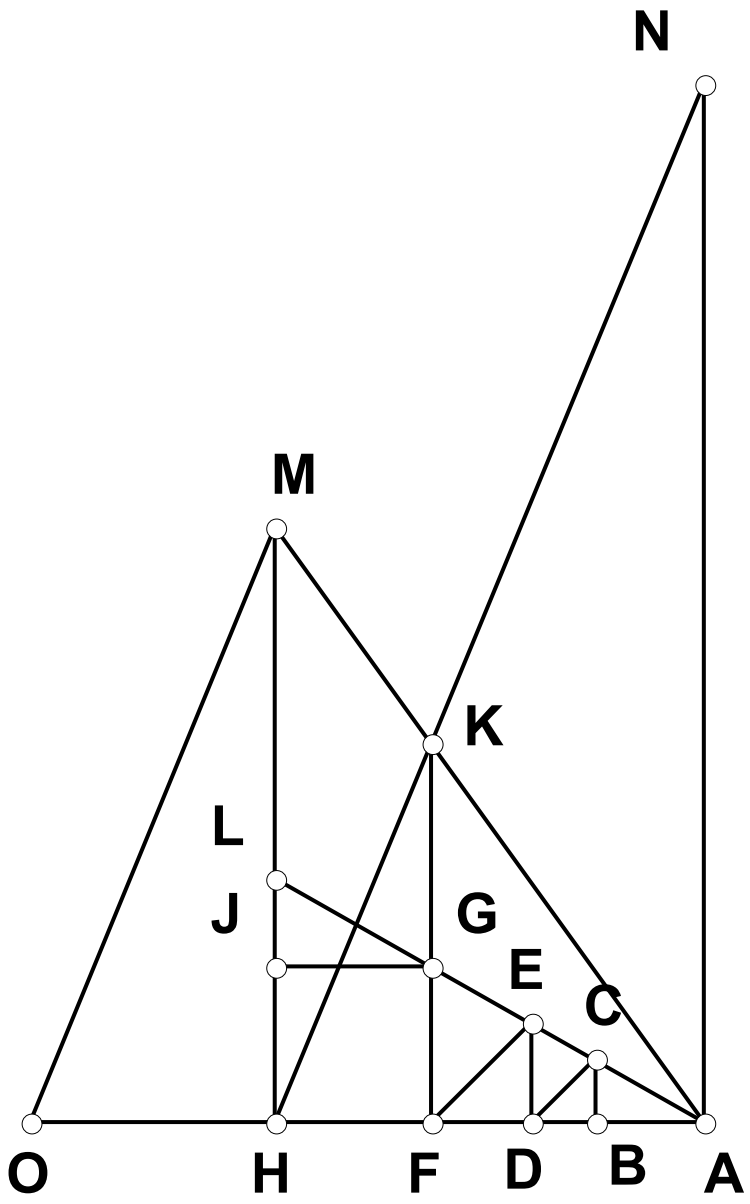
$N := 1.3 \quad AH := 1 \quad HN := AH \cdot N$

$HJ := HN - AH \quad FH := \frac{AH \cdot HJ}{AH + HJ} \quad AF := AH - FH$

$FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$\frac{AH}{AF} - N^1 = 0 \quad \frac{AH}{AD} - N^2 = 0 \quad \frac{AH}{AB} - N^3 = 0$


N
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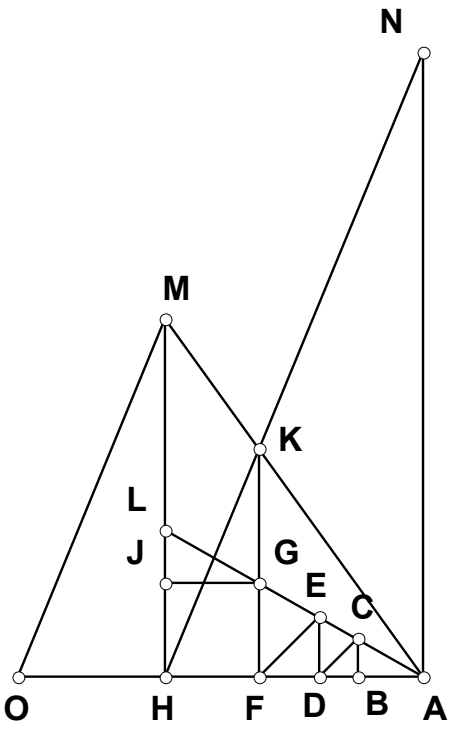
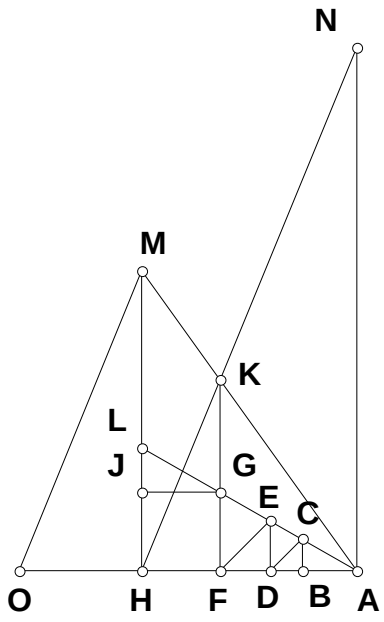



**980225B EP**

Given:

# Sum DIvided by One Powered 022598B




$N_1 := 5 \quad N_2 := 6 \quad AH := 1 \quad HM := AH \cdot N_1 \quad AN := AH \cdot N_2$

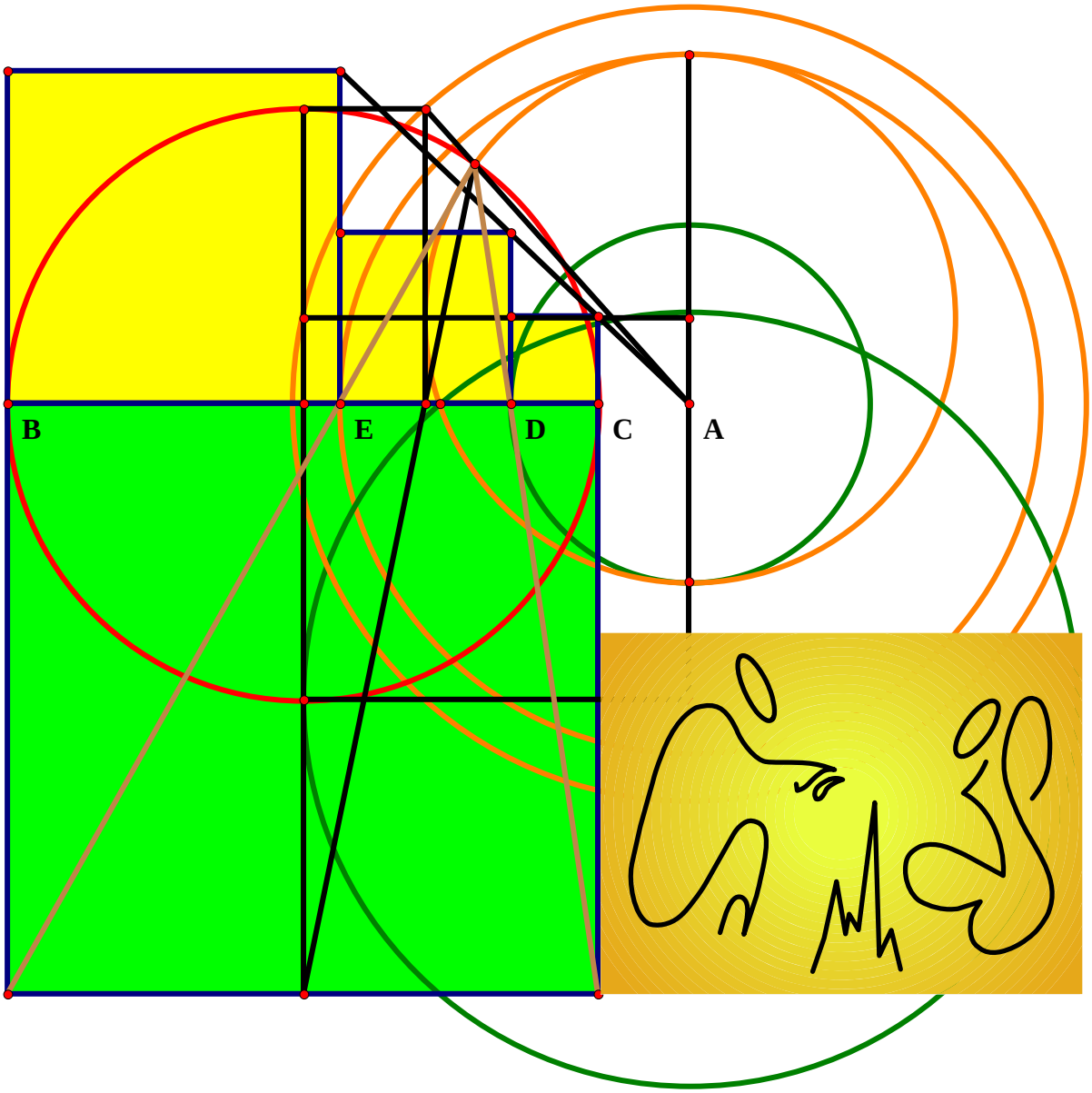
$HO := \frac{AH \cdot HM}{AN} \quad AO := AH + HO \quad AF := \frac{AH^2}{AO}$

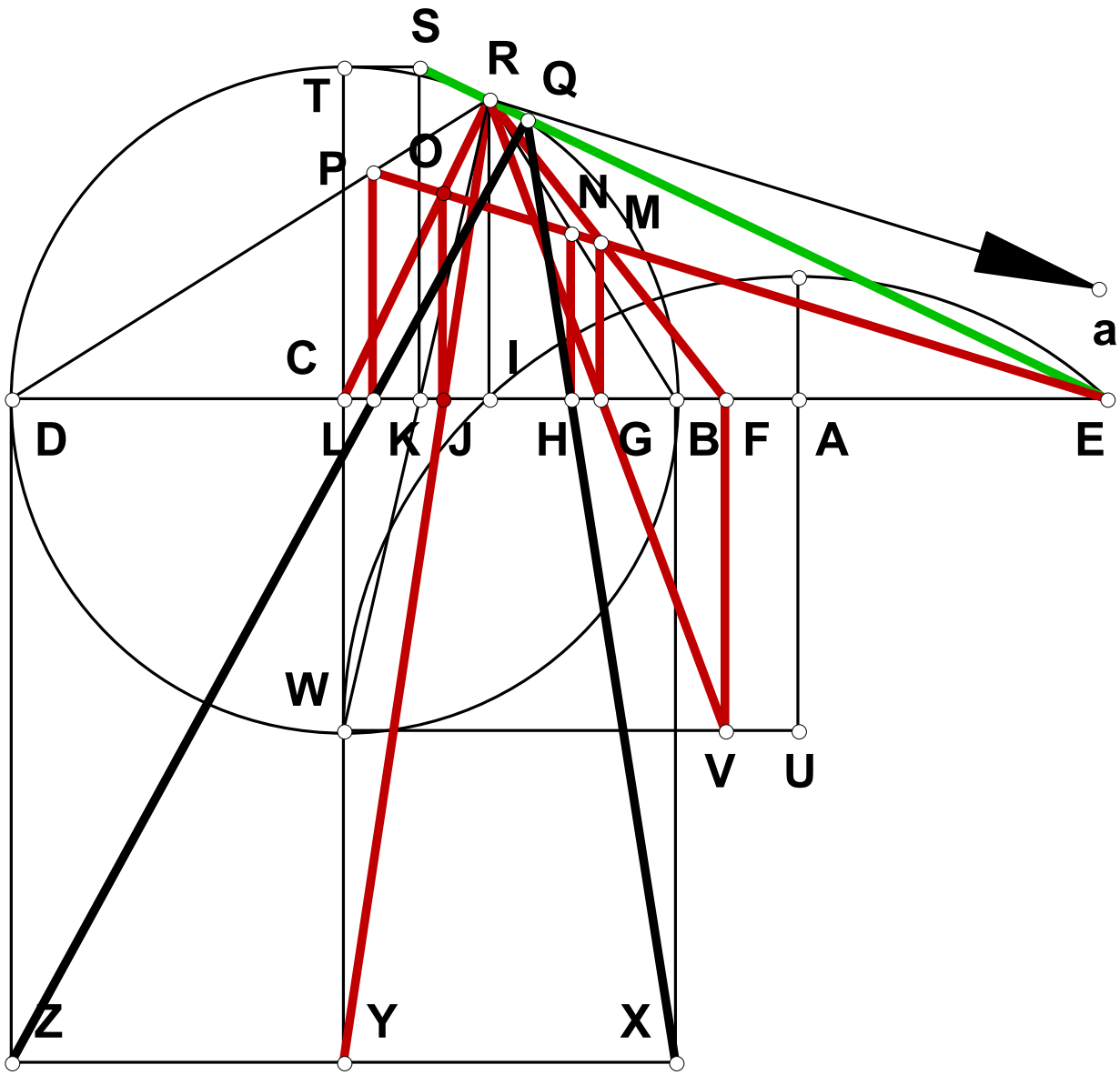
$FH := AH - AF \quad FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0$$

# The Delian Quest—1999




S
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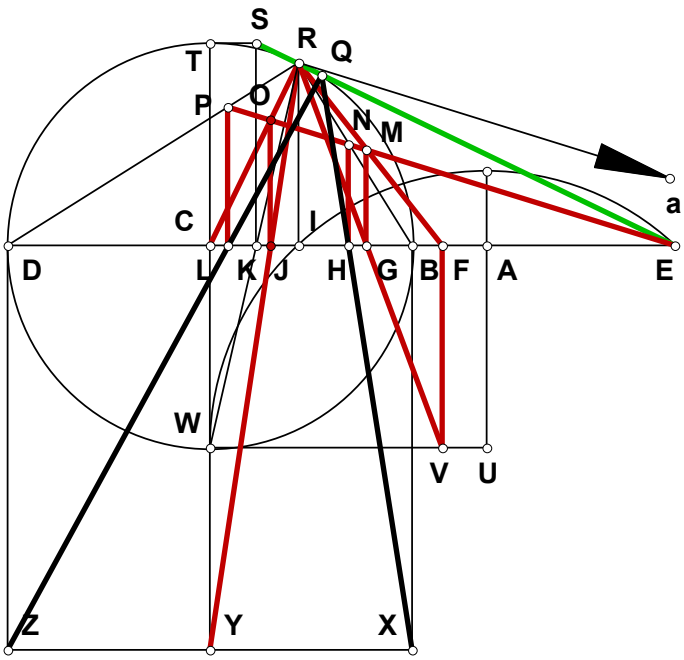
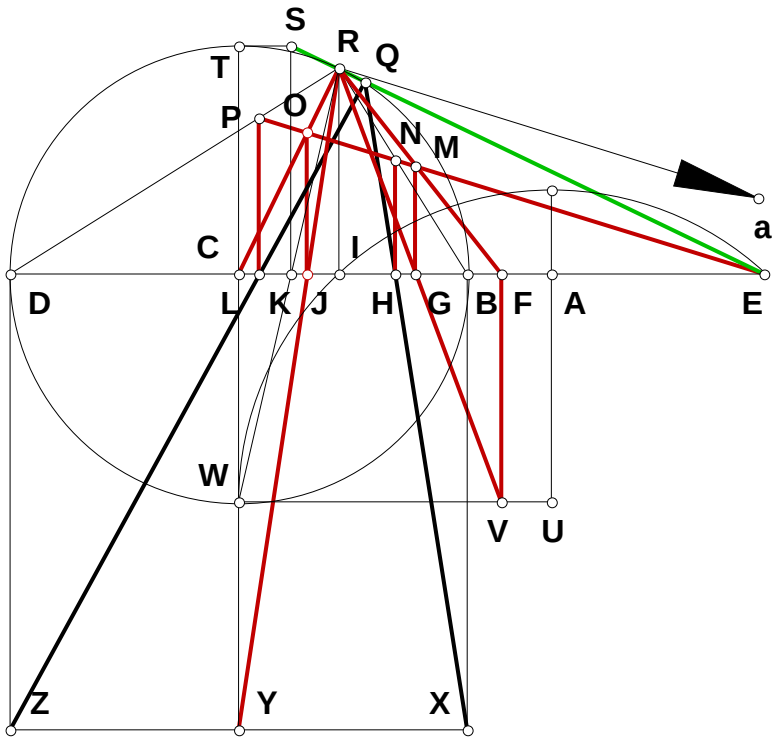



## 990724 EP

Given:

# On Gemini Roots 072499

## CE is to EF as CY is to CW




$N_1 := 6.429$ $N_2 := 2$

$AB := .438$ $AD := AB \cdot N_1$

$BD := AD - AB$ $BC := \frac{BD}{2}$

$CW := BC$ $CT := BC$

$FV := BC$ $AI := \sqrt{AB \cdot AD}$

$BI := AI - AB$ $DI := BD - BI$

$IR := \sqrt{BI \cdot DI}$ $AE := AI$

$AC := AB + BC$ $ED := AD + AE$

$CE := AC + AE$ $EF := \frac{CE}{N_2}$

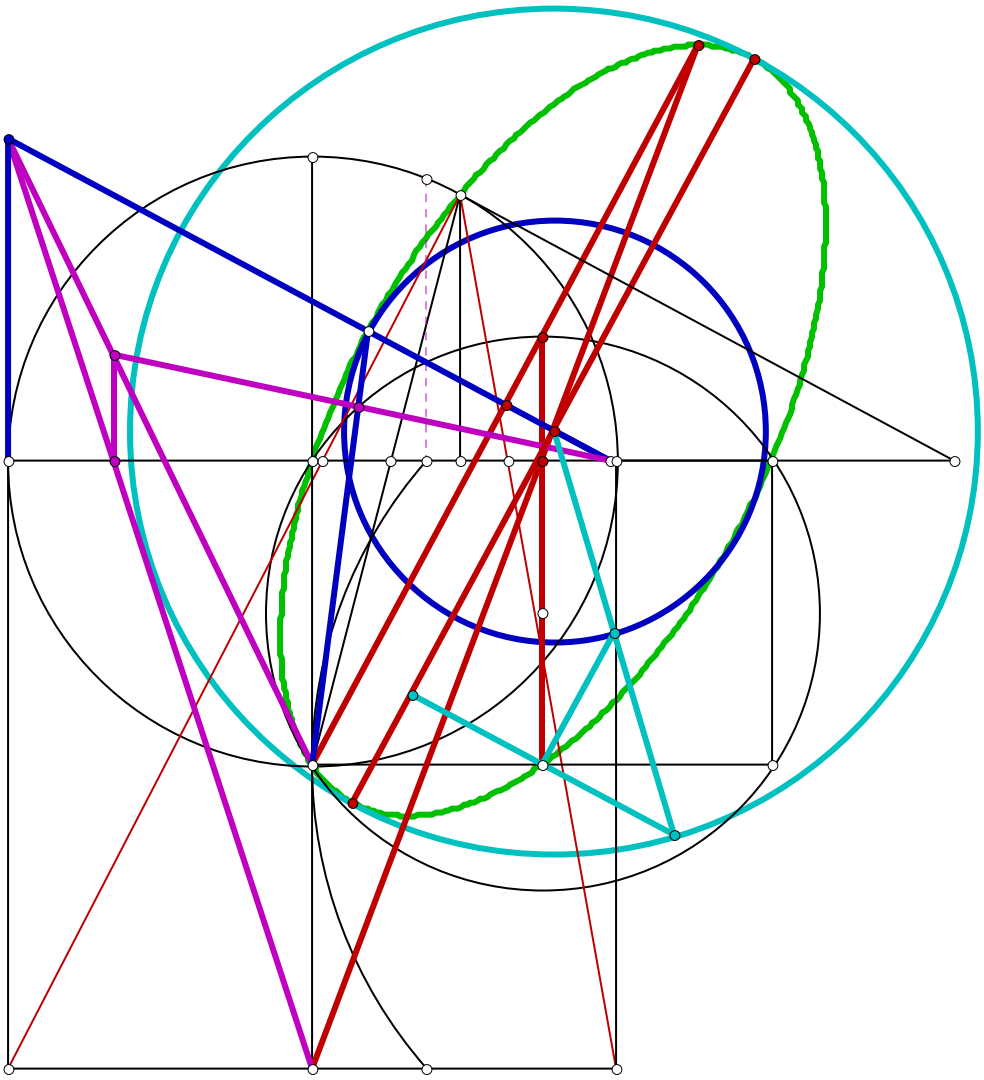
$BE := AE + AB$ $EI := AE + AI$

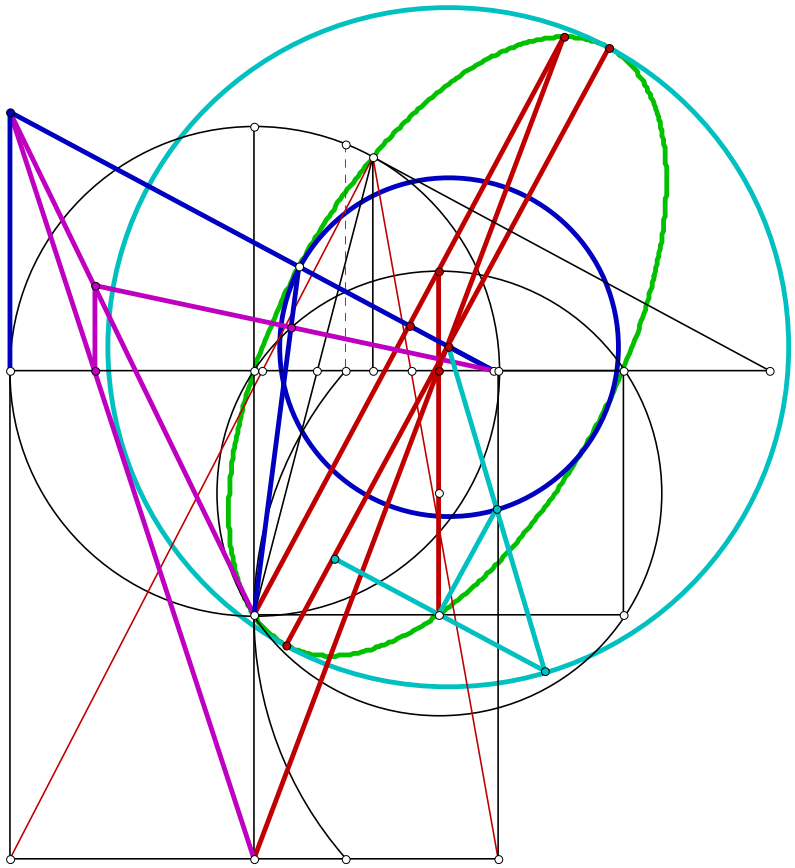
$FI := EI - EF$ $FG := \frac{FI \cdot FV}{FV + IR}$

$EG := EF + FG$ $GI := FI - FG$ $GM := \frac{FV \cdot GI}{FI}$ $Ia := \frac{EG \cdot IR}{GM}$ $EL := \frac{Ia \cdot ED}{Ia + DI}$ $BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI$ $BH := \frac{BI \cdot BE}{Ba}$ $EH := BE + BH$ $CI := AC - AI$ $JO := \frac{IR \cdot CE}{CI + Ia}$ $CJ := \frac{CI \cdot JO}{IR}$

$JI := CI - CJ$ $CY := \frac{IR \cdot CJ}{JI}$ $\frac{CY}{CW} - \frac{CE}{EF} = 0$

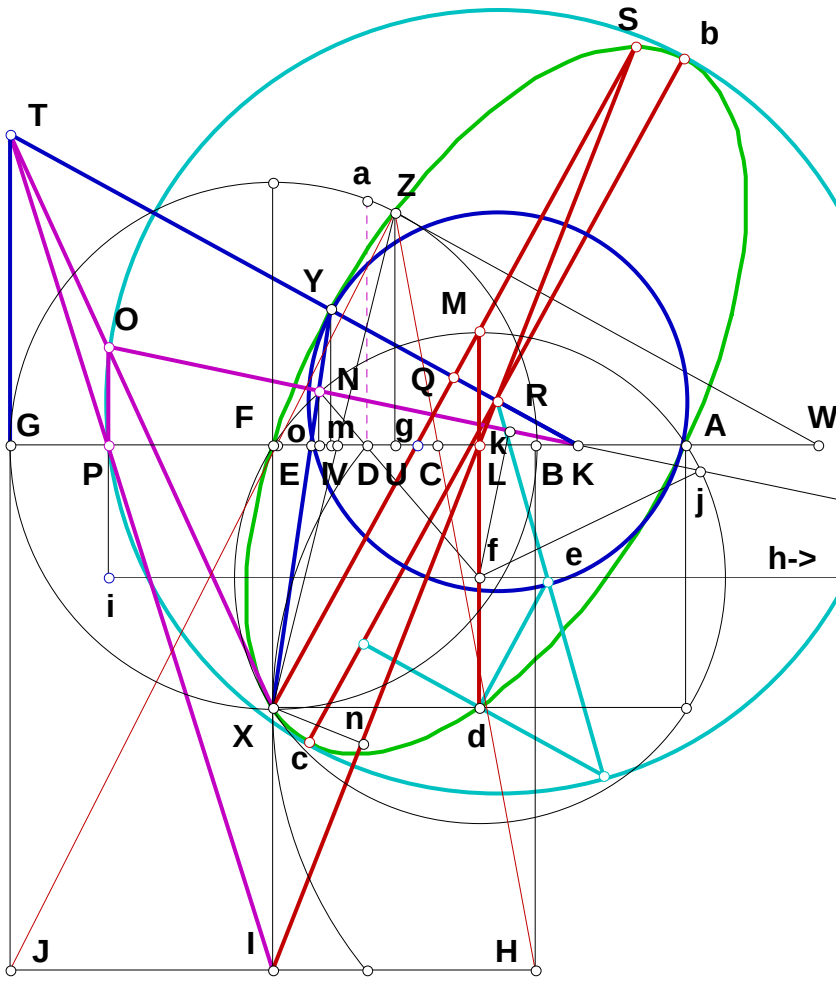




**990811 EP**

Given:

# A Delian Solution 081199




**What are the minor and major axis for the ellipse that will give point Z for the cube root?**

$AB := 1 \qquad N := 16 \qquad AG := AB \cdot N$

$BG := AG - AB$

$BF := \frac{BG}{2} \qquad FG := BF \qquad AF := AB + BF$

$FX := BF \qquad Mf := \frac{\sqrt{AF^2 + FX^2}}{2} \qquad Lf := \frac{FX}{2}$

$ML := Mf - Lf \qquad FL := \frac{AF}{2} \qquad Xd := FL \qquad df := Lf$

$IX := FX \qquad Md := Mf + df \qquad MX := \sqrt{Xd^2 + Md^2}$

$SX := \frac{MX \cdot IX}{IX - ML} \qquad Lg := \frac{FL \cdot ML}{ML + FX} \qquad QX := \frac{SX}{2}$

$Fg := FL - Lg \qquad Xg := \frac{MX \cdot Fg}{Xd} \qquad Qg := QX - Xg$

$Kg := \frac{Xg \cdot Qg}{Fg} \qquad GK := FG + Fg + Kg \qquad GJ := BG$

$GT := \frac{Fg \cdot GK}{FX} \qquad JT := GJ + GT \qquad IJ := BF$

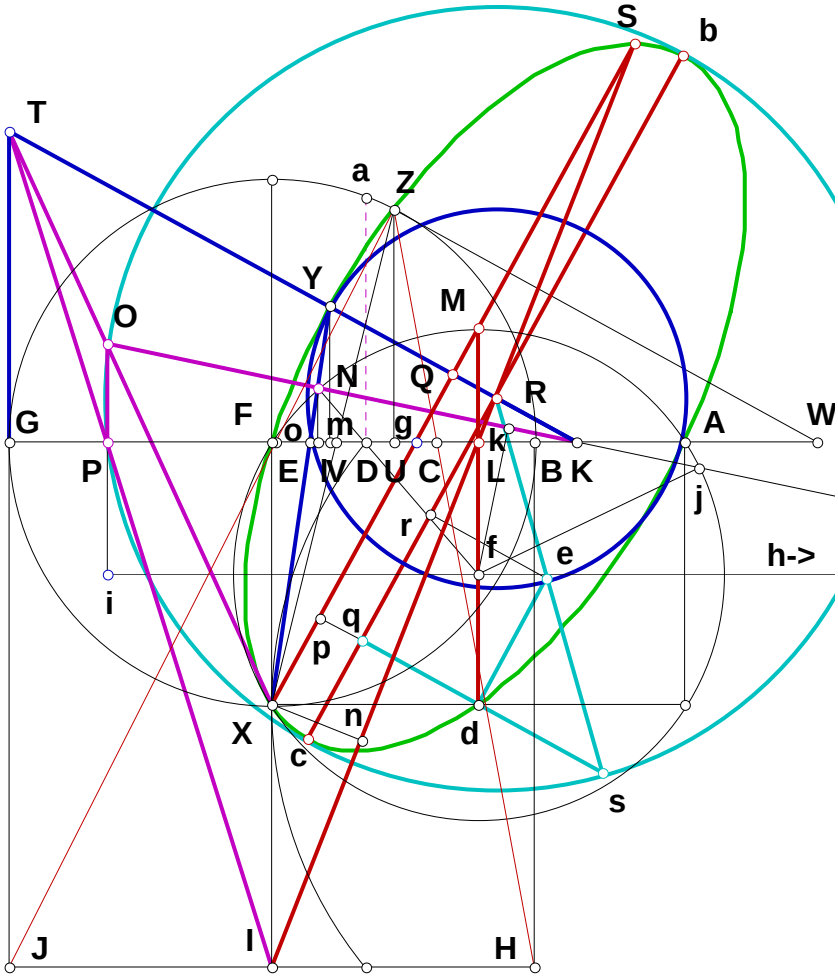
$FP := \frac{IJ \cdot GJ}{JT} \qquad OP := \frac{IX \cdot GT}{JT}$

$KP := Fg + Kg + FP \qquad Pi := Lf \qquad Oi := OP + Pi \qquad hi := \frac{KP \cdot Oi}{OP} \qquad fi := FP + FL \qquad fh := hi - fi \qquad KO := \sqrt{KP^2 + OP^2}$

$hk := \frac{KP \cdot fh}{KO} \qquad fk := \frac{OP \cdot fh}{KO} \qquad Nf := Mf \qquad Nk := \sqrt{Nf^2 - fk^2} \qquad Nh := hk + Nk \qquad Oh := \frac{KO \cdot Oi}{OP} \qquad NO := Oh - Nh \qquad KN := KO - NO$

$$Nl := \frac{OP \cdot KN}{KO} \quad Kl := \frac{KP \cdot KN}{KO} \quad FK := KP - FP \quad Fl := FK - Kl \quad NX := \sqrt{(FX + Nl)^2 + Fl^2} \quad XY := \frac{NX \cdot IX}{IX - Nl} \quad Fm := \frac{Fl \cdot XY}{NX}$$

$$Km := FK - Fm \quad Fo := \frac{Fl \cdot FX}{FX + Nl} \quad Xo := \frac{NX \cdot Fo}{Fl} \quad mo := Fm - Fo \quad Ym := \frac{FX \cdot mo}{Fo} \quad FI := 2 \cdot BF \quad IL := \sqrt{FL^2 + FI^2}$$




$$IS := \frac{IL \cdot IX}{IX - ML} \quad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

$$Xn := \sqrt{IX^2 - In^2} \quad QR := \frac{Xn \cdot QX}{IS - In}$$

$$KT := \sqrt{GK^2 + GT^2} \quad KY := \frac{KT \cdot Ym}{GT}$$

$$KQ := \frac{GK \cdot Qg}{GT} \quad RY := KY - KQ + QR$$

$$RY = 4.119$$

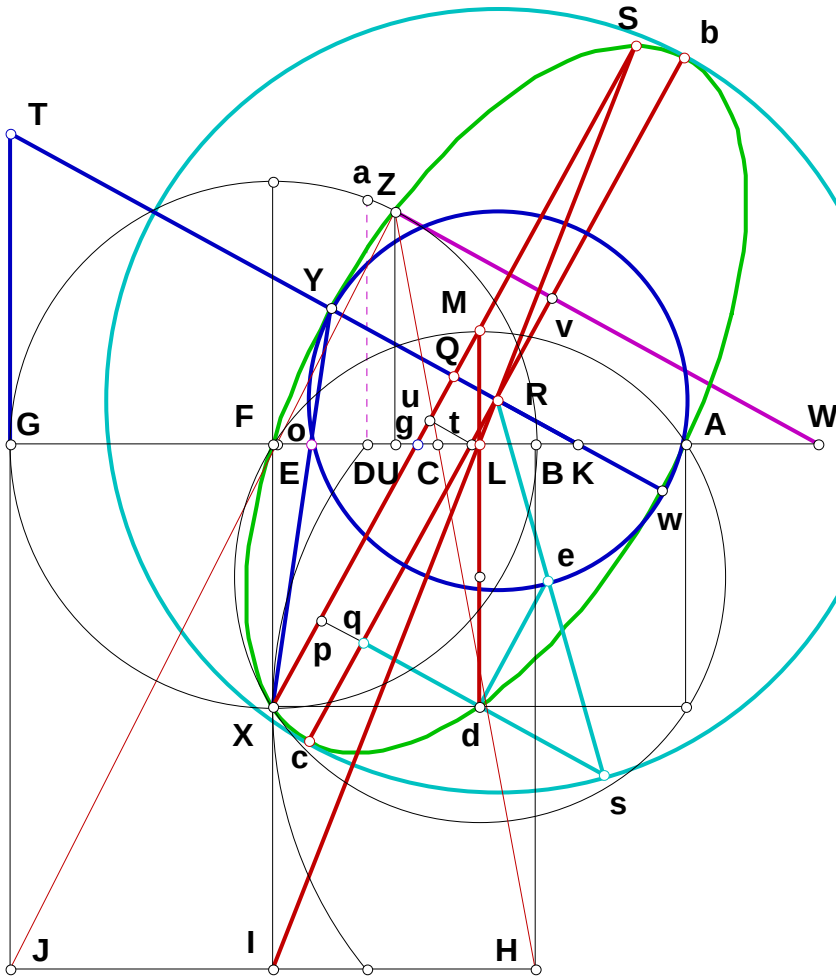
$$Xd := FL \quad dp := \frac{GK \cdot Xd}{KT} \quad pq := QR \quad dq := dp - pq$$

$$Xp := \frac{GT \cdot Xd}{KT} \quad Qp := QX - Xp \quad Rq := Qp$$

$$er := dq \quad Re := RY \quad Rr := \sqrt{Re^2 - er^2}$$

$$Rs := \frac{Re \cdot Rq}{Rr} \quad Rs = 7.171$$

**Is the segment Zv equal to the perpendicular for the ellipse?**




$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}}$$

$$BC := AC - AB \quad BE := AE - AB$$

$$CE := BE - BC \quad EG := BG - BE$$

$$CU := \frac{BC \cdot CE}{BC + EG} \quad BU := BC + CU$$

$$GU := BG - BU \quad UZ := \sqrt{BU \cdot GU}$$

$$UW := \frac{GK \cdot UZ}{GT} \quad Gg := GK - Kg$$

$$tu := QR \quad gt := \frac{KT \cdot tu}{GK} \quad Gt := Gg + gt$$

$$GW := GU + UW \quad Wt := GW - Gt$$

$$tv := \frac{GT \cdot Wt}{KT} \quad Kt := GK - Gt \quad Rt := \frac{GT \cdot Kt}{KT}$$

$$Rv := tv - Rt \quad bc := 2 \cdot Rs \quad Rc := Rs$$

$$cv := Rc + Rv \quad Yw := 2 \cdot RY \quad WZ := \frac{KT \cdot UZ}{GT}$$

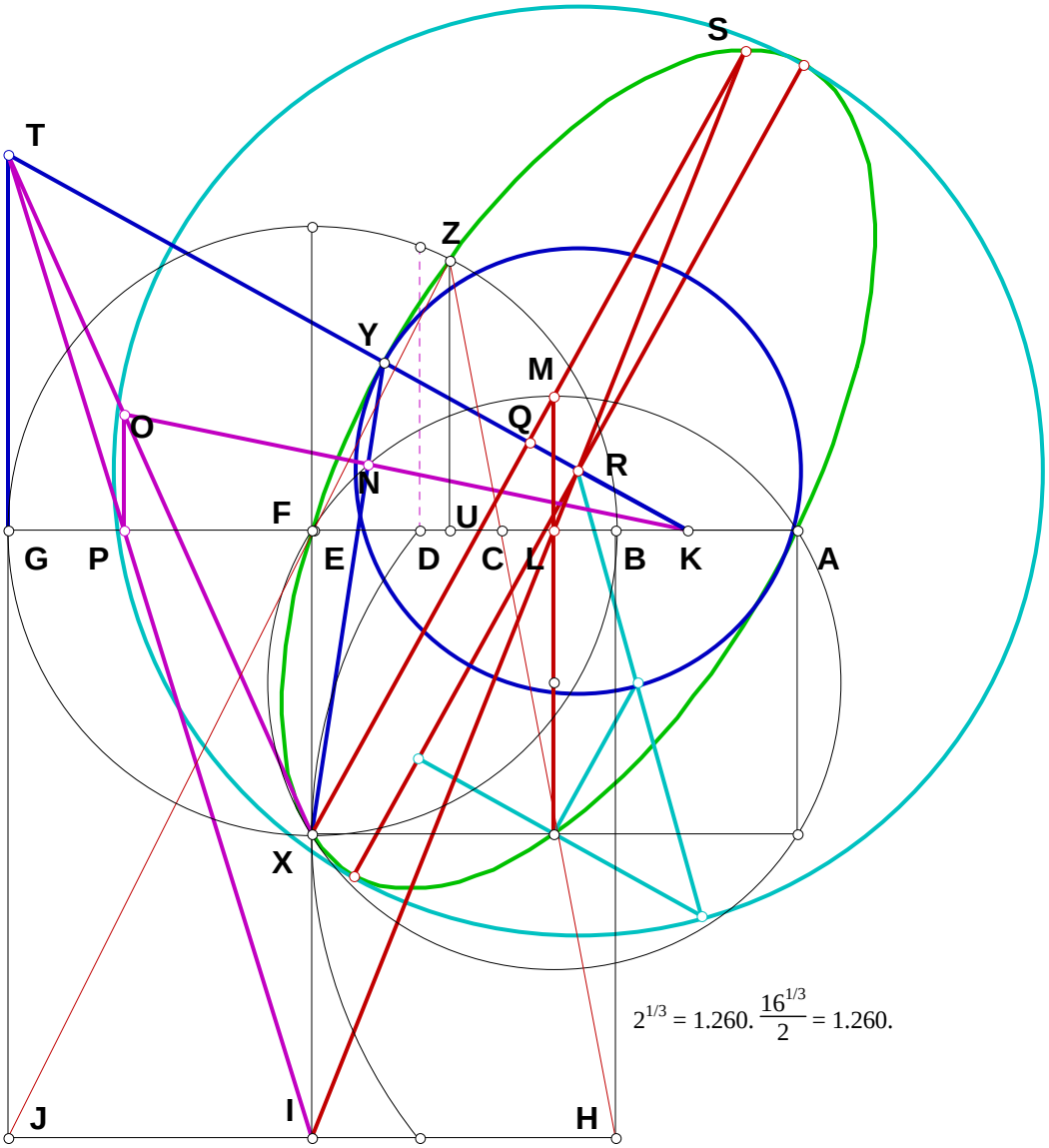
$$Wv := \frac{GK \cdot tv}{GT} \quad Zv := \left| WZ - Wv \right|$$

$N_1 := Yw \quad N_2 := bc \quad N_3 := cv \quad N_4 := bc \quad \sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2} - Zv = 0 \quad \frac{AC}{2 \cdot AB} - 2^{\frac{1}{3}} = 0$

$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

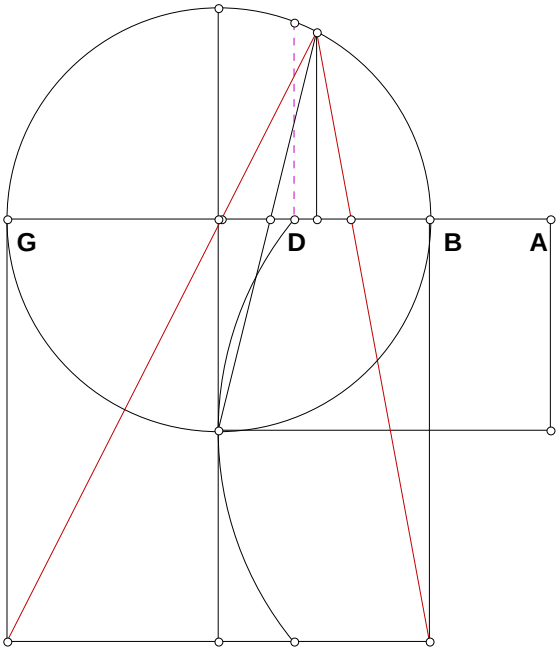
# Appendix A

## Step by Step




As this method of construction does not quite make it to two, sixteen will do as well.

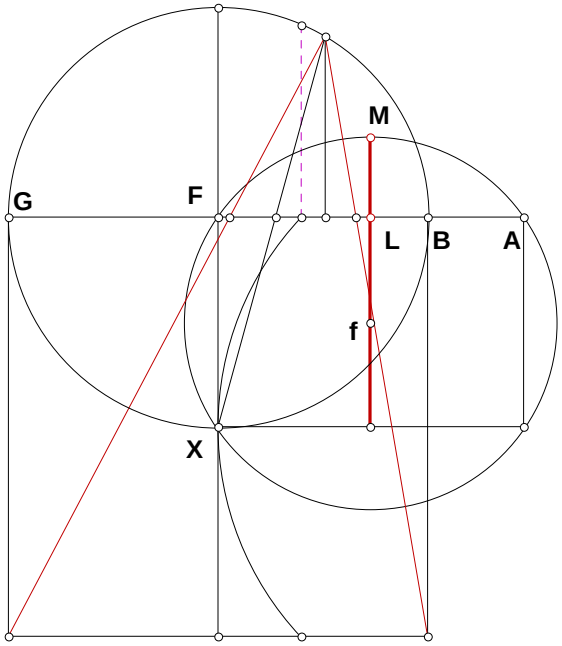
## Plate 1



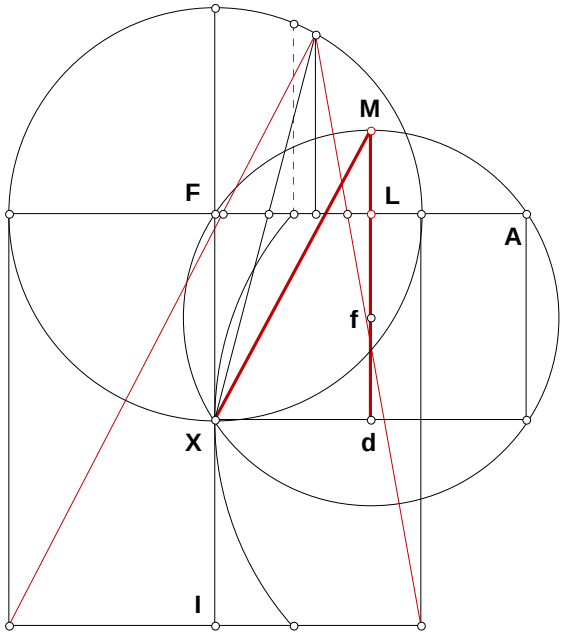

$AB = 1,$
$N = 16,$
$AG = AB \times N$
$BG = AG - AB,$
$AD = \sqrt{AB \times AG}$

## Plate 2

$BF = \frac{BG}{2},$
$FG = BF,$
$AF = AB + BF$
$FX = BF,$
$Mf = \frac{\sqrt{AF^2 + FX^2}}{2},$
$LF = \frac{FX}{2}$



## Plate 3




$ML = Mf - Lf,$

$FL = \frac{AF}{2},$

$Xd = FL,$

$df = Lf$

$IX = FX,$

$Md = Mf + df,$

$MX = \sqrt{Xd^2 + Md^2}$

## Plate 4

$SX = \frac{MX \times IX}{IX - ML},$

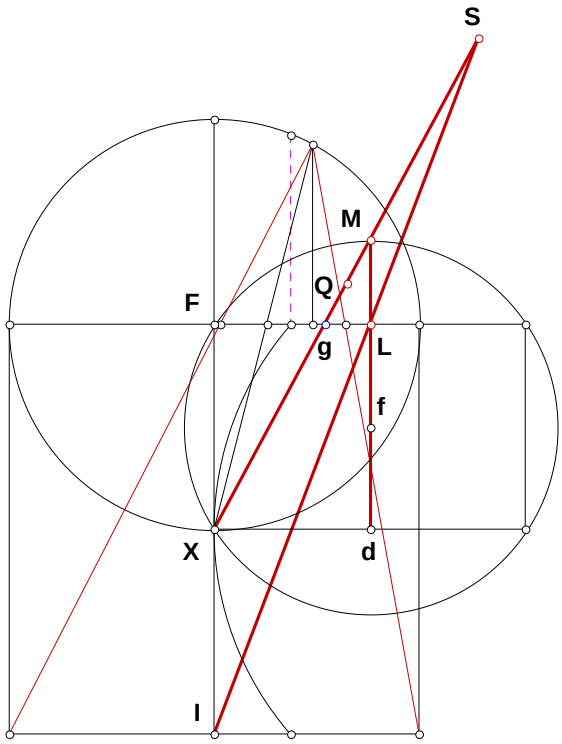
$Lg = \frac{FL \times ML}{ML + FX},$

$QX = \frac{SX}{2}$

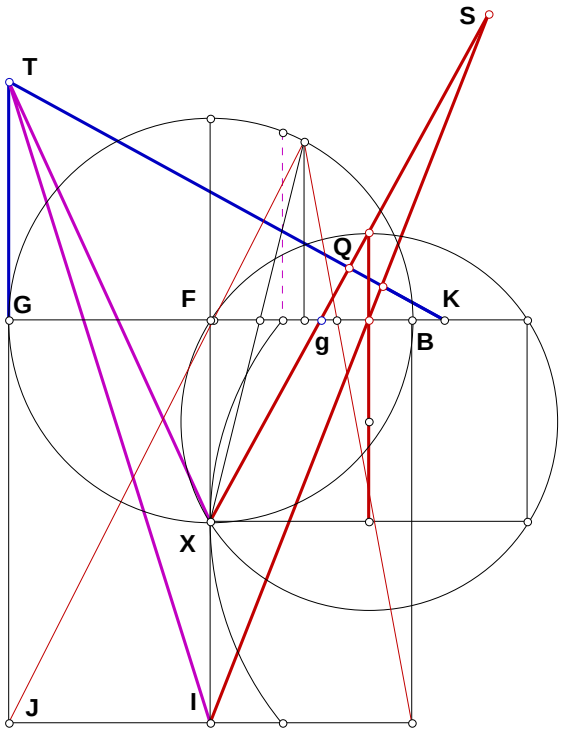
$Fg = FL - Lg,$

$Xg = \frac{MX \times Fg}{Xd},$

$Qg = QX - Xg$



## Plate 5




$$Kg = \frac{Xg \times Qg}{Fg},$$

$GK = FG + Fg + Kg,$

$GJ = BG$

$$GT = \frac{Fg \times GK}{FX},$$

$JT = GJ + GT,$

$IJ = BF$

## Plate 6

$$FP = \frac{IJ \times GJ}{JT},$$

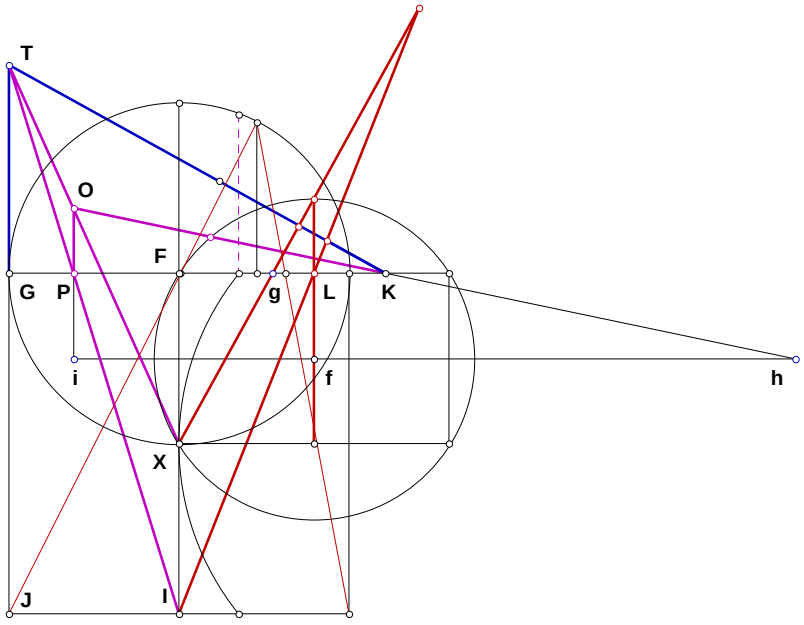
$$OP = \frac{IX \times GT}{JT},$$

$KP = Fg + Kg + FP,$

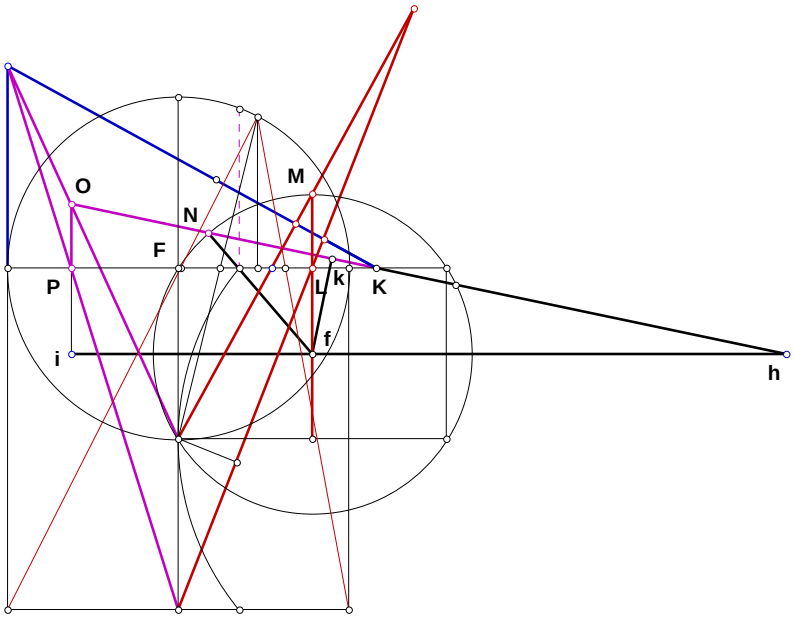
$Pi = Lf,$

$Oi = OP + Pi,$

$$hi = \frac{KP \times Oi}{OP},$$



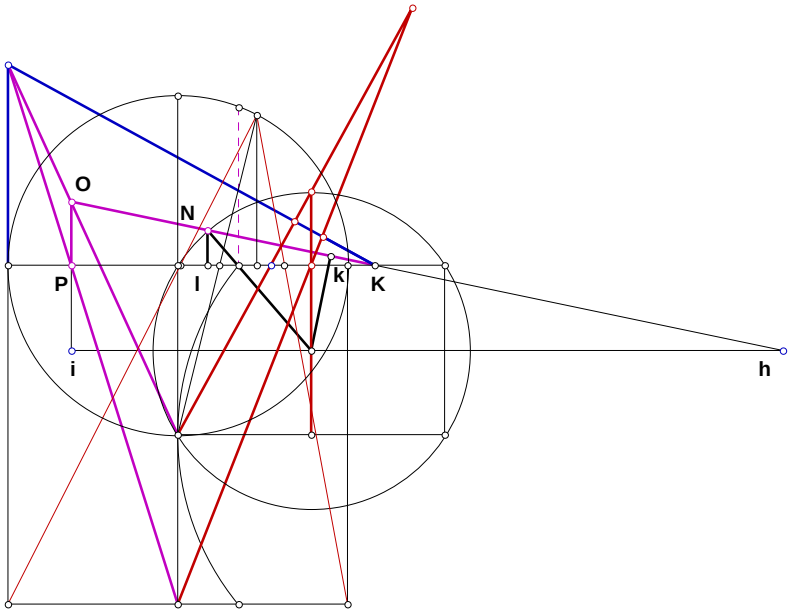
## Plate 7



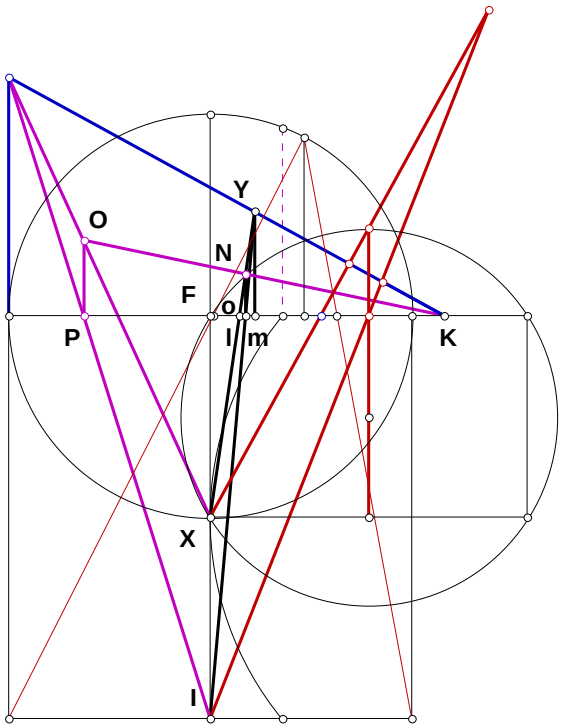

$fi = FP + FL,$
$fh = hi - fi,$
$KO = \sqrt{KP^2 + OP^2},$
$hk = \dfrac{KP \times fh}{KO},$
$fk = \dfrac{OP \times fh}{KO},$
$Nf = Mf,$
$Nk = \sqrt{Nf^2 - fk^2}$

## Plate 8

$Nh = hk + Nk$
$Oh = \dfrac{KO \times Oi}{OP}$
$NO = Oh - Nh$
$KN = KO - NO$
$NI = \dfrac{OP \times KN}{KO}$
$KI = \dfrac{KP \times KN}{KO}$



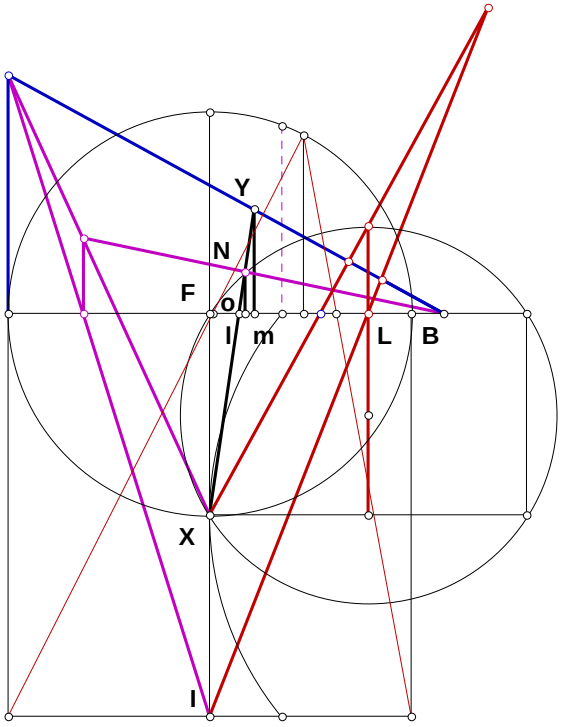
## Plate 9



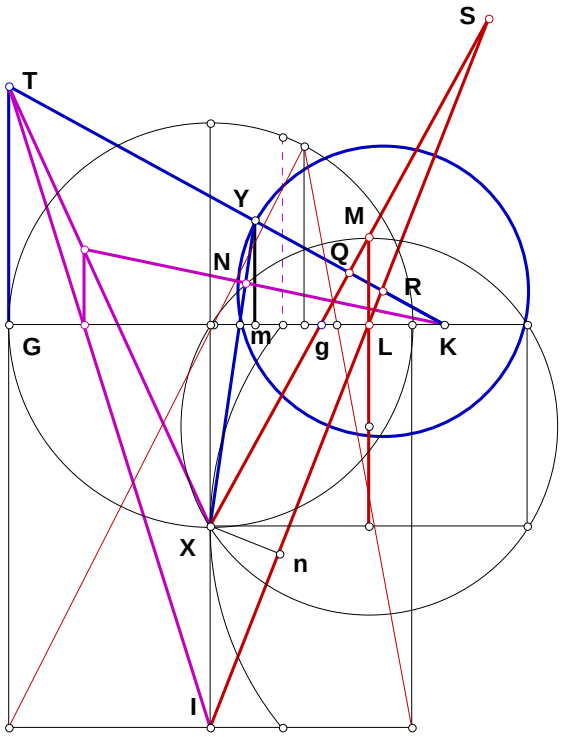

$$FK = KP - FP,$$
$$Fl = FK - Kl,$$
$$NX = \sqrt{(FX + Nl)^2 + Fl^2}$$
$$XY = \frac{NX \times IX}{IX - Nl},$$
$$Fm = \frac{Fl \times XY}{NX},$$
$$Km = FK - Fm$$

## Plate 10

$$Fo = \frac{Fl \times FX}{FX + Nl},$$
$$Xo = \frac{NX \times Fo}{Fl},$$
$$mo = Fm - Fo$$
$$Ym = \frac{FX \times mo}{Fo},$$
$$FI = 2 \times BF,$$
$$IL = \sqrt{FL^2 + FI^2}$$



## Plate 11 The Minor Axis




$$IS = \frac{IL \times IX}{IX - ML},$$

$$In = \frac{IS^2 - SX^2 + IX^2}{2 \times IS}$$

$$Xn = \sqrt{IX^2 - In^2},$$

$$QR = \frac{Xn \times QX}{IS - In}$$

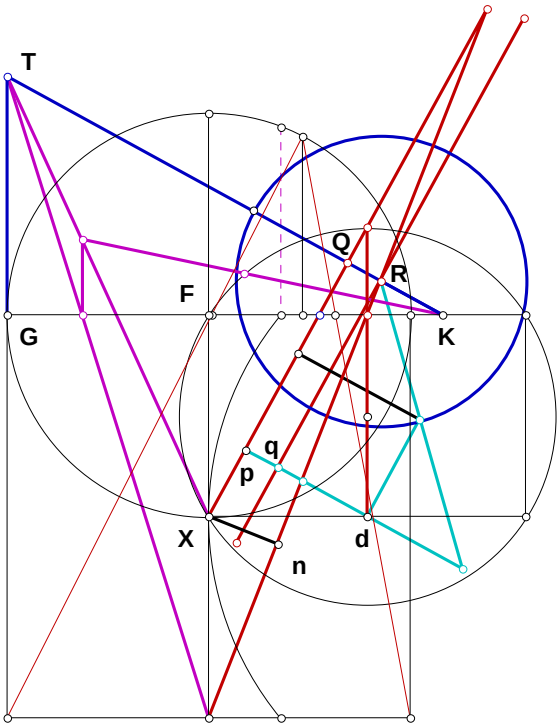
$$KT = \sqrt{GK^2 + GT^2},$$

$$KY = \frac{KT \times Ym}{GT}$$

$$KQ = \frac{GK \times Qg}{GT},$$

$$RY = KY - KQ + QR$$

## Plate 12




$$Xd = FL,$$

$$dp = \frac{GK \times Xd}{KT},$$

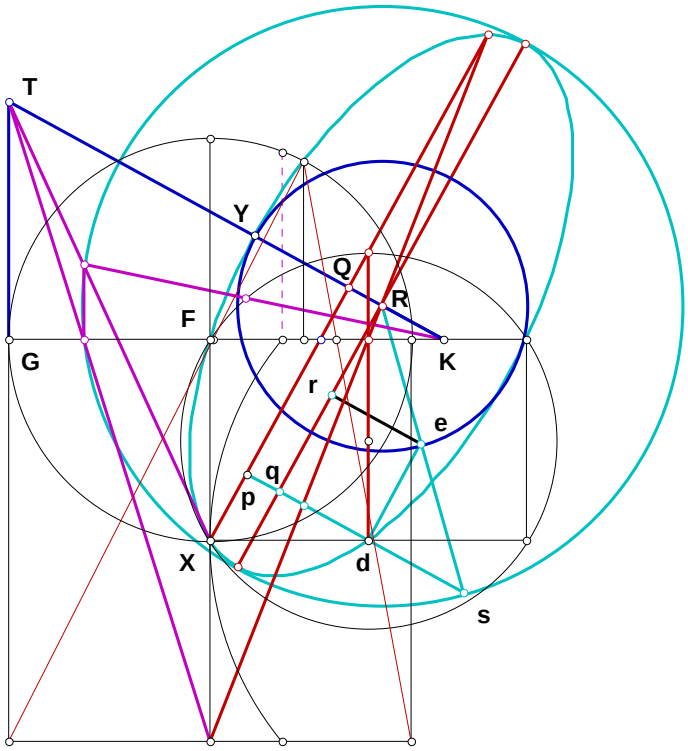
$$pq = QR$$

$$dq = dp - pq,$$

$$Xp = \frac{GT \times Xd}{KT},$$

$$Op = QX - Xp$$

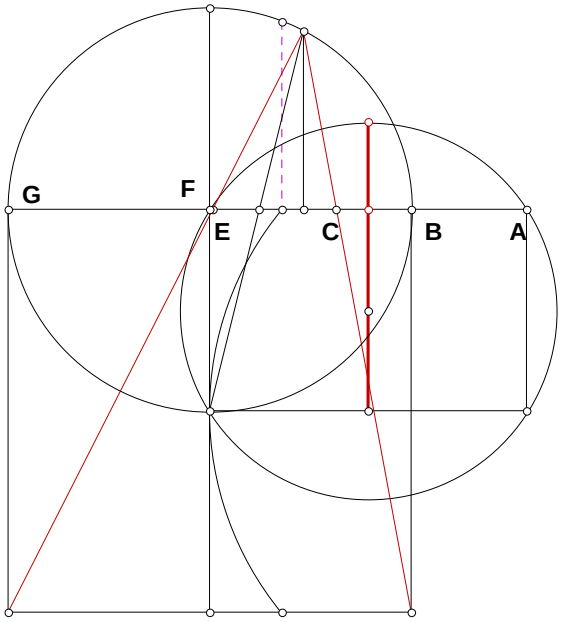
## Plate 13 The Major Axis




$er = dq$, $Re = RY$, $Rr = \sqrt{Re^2 - er^2}$,

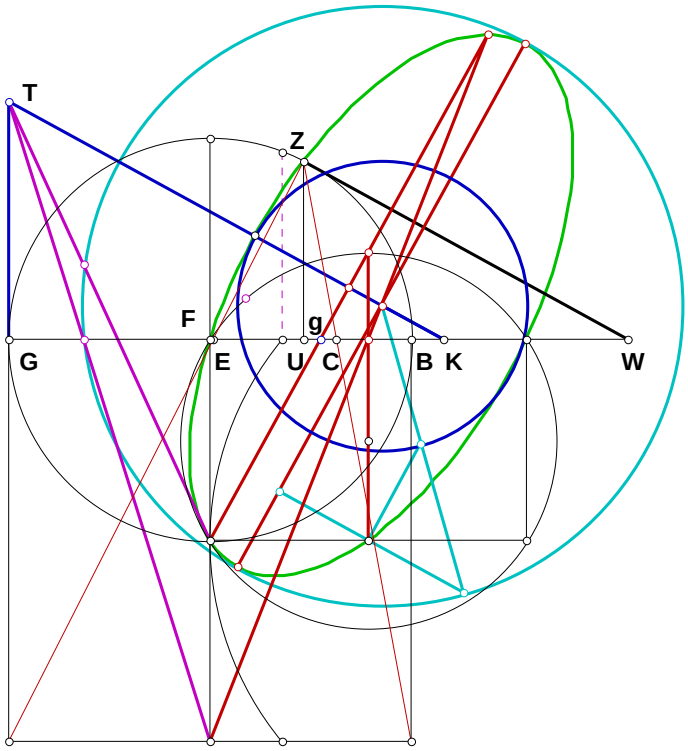
$Rq = Qp$, $Rs = \dfrac{Re \times Rq}{Rr}$

## Plate 14




$AC = (AB^2 \times AG)^{1/3}$,
$AE = (AB \times AG^2)^{1/3}$
$BC = AC - AB$,
$BE = AE - AB$,
$CE = BE - BC$
$EG = BG - BE$

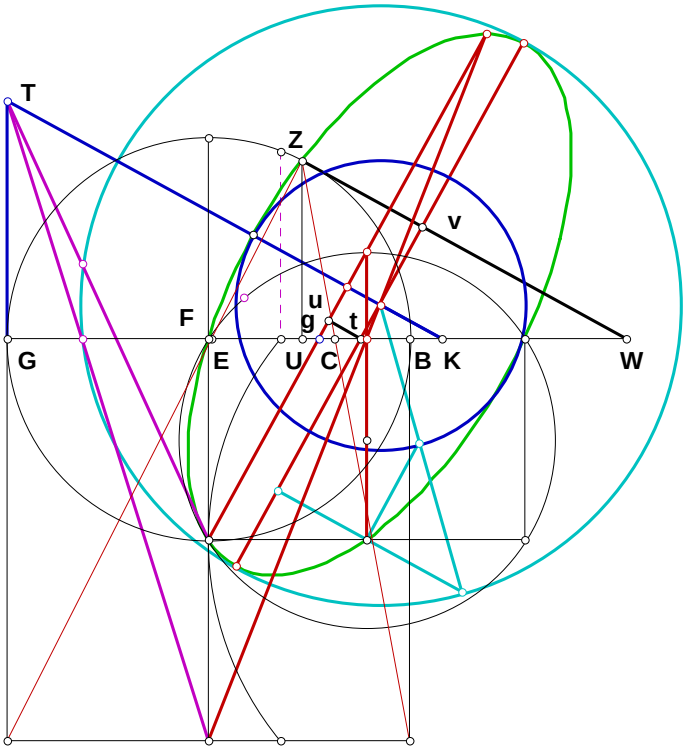
## Plate 15



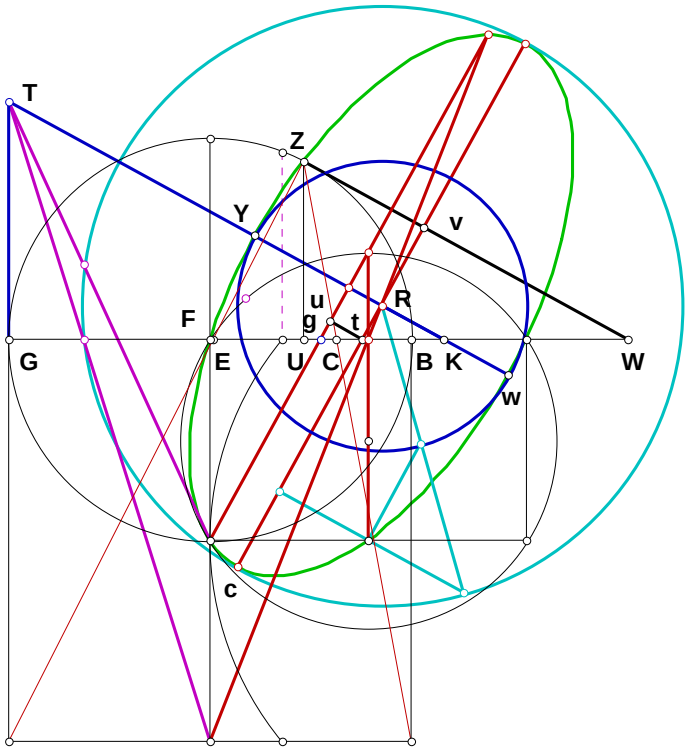

$$CU = \frac{BC \times CE}{BC + EG},$$
$$BU = BC + CU$$
$$GU = BG - BU,$$
$$UZ = \sqrt{BU \times GU}$$
$$UW = \frac{GK \times UZ}{GT},$$
$$Gg = GK - Kg$$

## Plate 16

$$tu = QR,$$
$$gt = \frac{KT \times tu}{GK},$$
$$Gt = Gg + gt$$
$$GW = GU + UW,$$
$$Wt = GW - Gt$$
$$tv = \frac{GT \times Wt}{KT},$$
$$Kt = GK - Gt$$



## Plate 17




$$Rt = \frac{GT \times Kt}{KT},$$

$Rv = tv - Rt$

$bc = 2 \times Rs,$

$Rc = Rs,$

$cv = Rc + Rv$

$Yw = 2 \times RY$

## Plate 18

$$WZ = \frac{KT \times UZ}{GT},$$

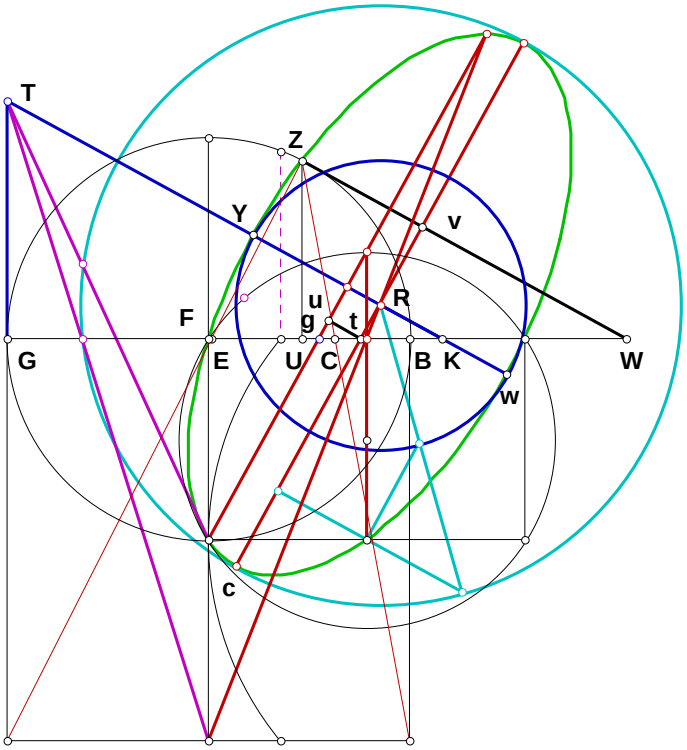
$$Wv = \frac{GK \times tv}{GT}$$

$Zv = | WZ - Wv |$

$N_1 = Yw,\ N_2 = bc,$

$N_3 = cv,\ N_4 = bc$

$$\sqrt{N_3 \times (N_4 - N_3)} \times \frac{N_1}{N_2} - Zv = 0$$



The language in this damn thing is all wrong correct it.

# Basic Arithmetic in Geometry

## Introduction

An outline of basic math moves in Geometry mainly concerning cardinal operations.

A primary mechanism required for language is the ratio. And it is on a biological level first. On a conscious level, one must understand that as things are to each other, so too our mental manipulations of things must be to each other. This identity between reality and mentality is call rationality. It then follows that people who habitually lie, being aware of it or not, are not rational. On a religious level, when one says that God is Truth, they are enunciating a standard in rationality—of judgment.

### Cardinal and Ordinal Operations.

These techniques are primarily focused not on ordinal operations but on cardinal. An example of an ordinal operation is Euclid's Book 1:1. The operations here depend first upon the unit.

## Contents


| | |
|---|---|
| The Unit | Technique 1 |
| Addition Subtraction | Technique 2 |
| Number Construction | Technique 3 |
| Unit Ratio | Technique 4 |
| Fractions. | Technique 5 |
| Ratio Two Numbers. | Technique 6 |
| Multiplication | Technique 7 |
| Division | Technique 8 |

## Basic Techniques

### Technique 1. To construct a unit.

With a given line, assert two points.

$\overline{A0B1}$ is the unit by convention.

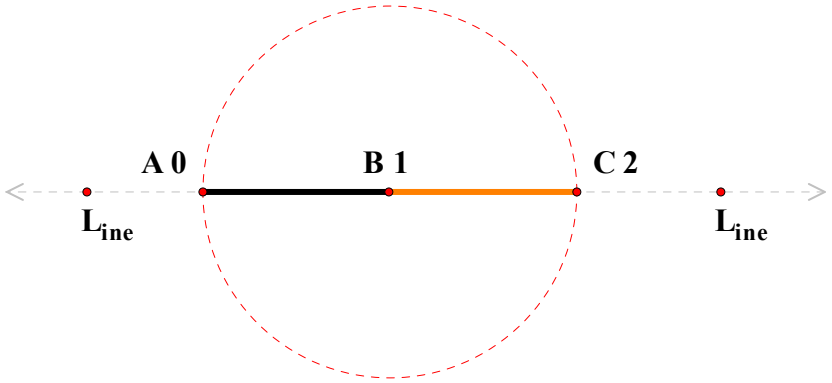
Every formal logic starts the same way—Arithmetic with the definition of the unit, so too in geometry. Craft is all about standards in construction and one starts by constructing our first standard.

Geometry is a relatiologic, which means the material difference is given, and the geometer asserts boundaries. The material difference in geometry is unspecified, of no concern to the geometer. He can neither add to, nor subtract from difference, he can only make things by asserting boundaries to it.

The construction of a unit is understood in this wise:—Between two assertions there is one and only one difference.

Note: Preserve both naming conventions, Geometric and Arithmetic.

### Technique 2. To a given unit add another.



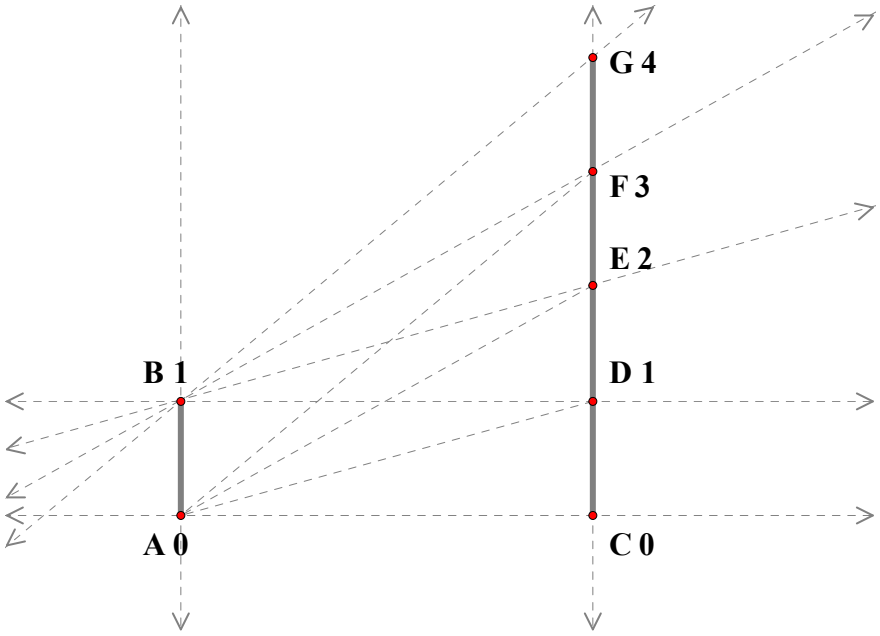

To a given line, Let $\overline{A1B0}$ be the given unit and to it, simply add another. Construct ⊙ B1A0.

$\overline{B1C2}$ is the required addition.

1 + 1 = 2 and 2 – 1 = 1.

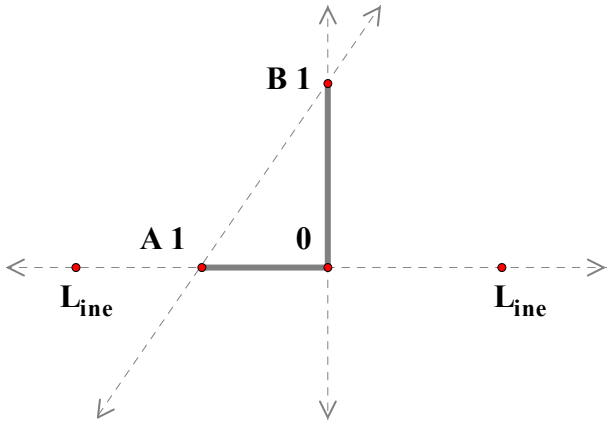
One need not drag this out for subtraction.

## Technique 3: To construct a number.



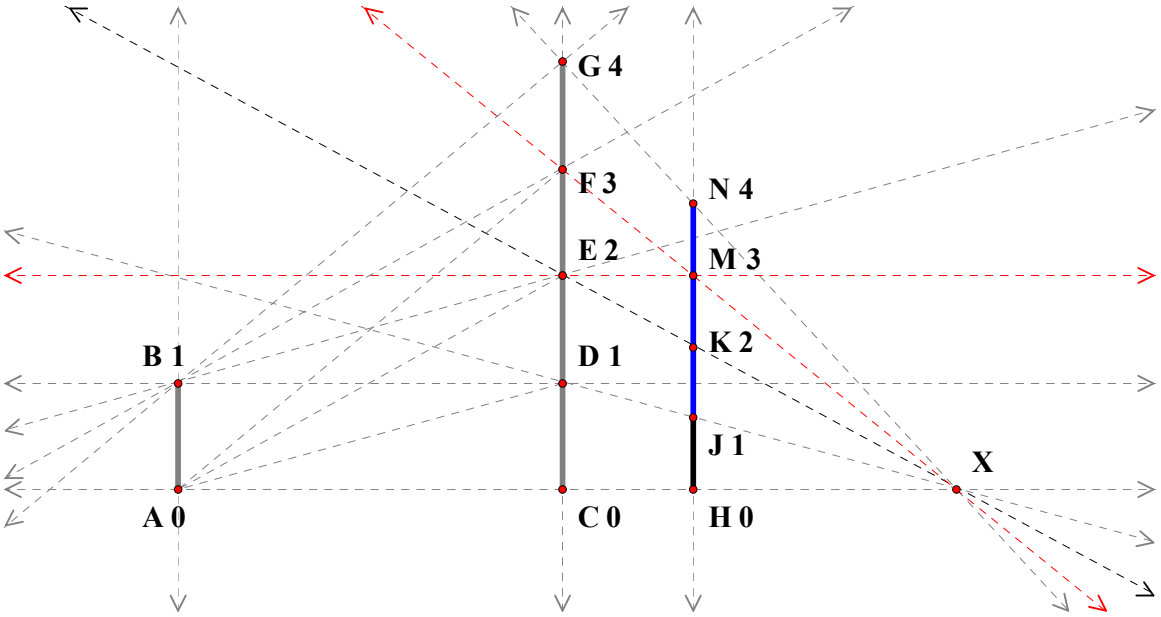

Given $\overline{A0B1}$ as our unit and 4 the number we are to construct, etc.,

## Technique 4. To construct a ratio between linear units.




$\overline{0A1}$ : $\overline{0B1}$ Is what was required .

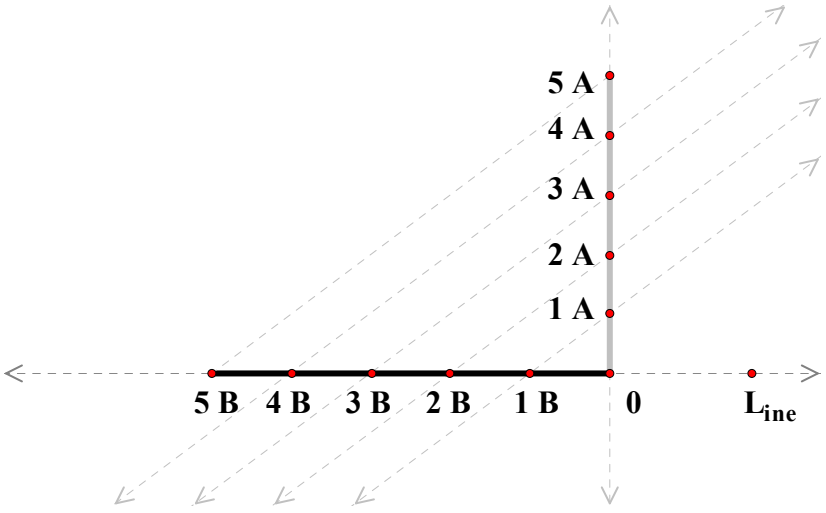
## Technique 5. Construct a fraction.




Let $\overline{A0B1}$ be our given unit, and $\frac{2}{3}$ the fraction which we are to construct.

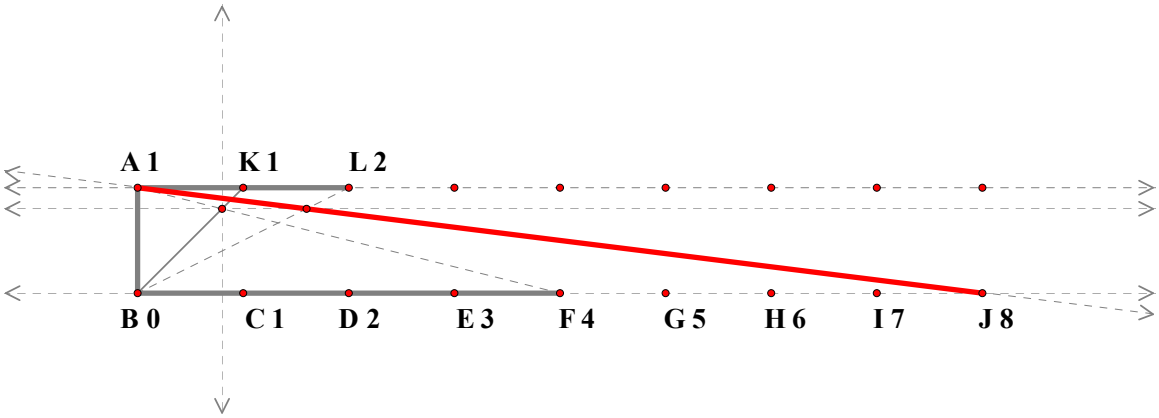
$\overline{H0J1}$ is $\frac{2}{3}$. Furthermore, I say that H0N4 is $\frac{4 \times 2}{3}$ . . .

## Technique 6. Provide a ratio between numbers with two different units.



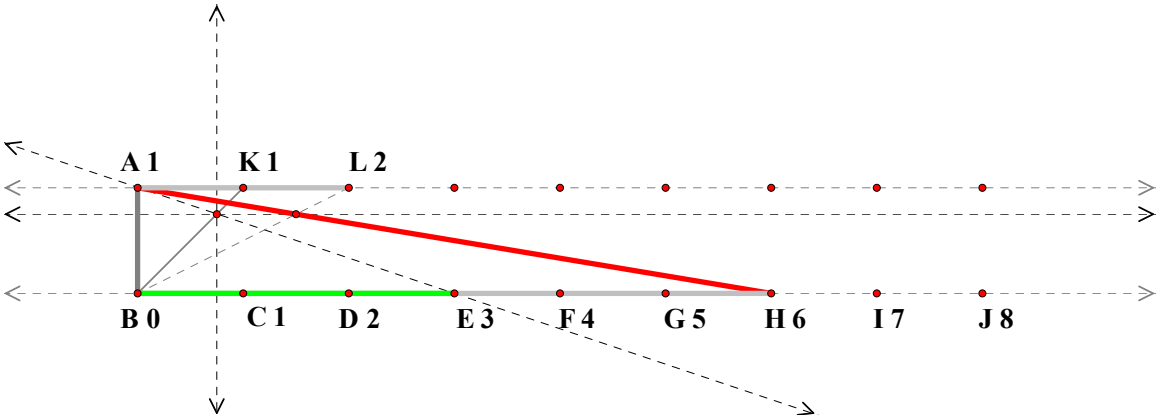

Let the numbers be 5 A and 5 B. Then 1A:1B :: 5A:5B.

## Technique 7. Multiply two numbers.




With the given unit $\overline{A1B0}$ multiply 2 × 4.

## Technique 8. Divide two numbers.




With the given unit $\overline{A1B0}$ divide 6 by 2.

# Multiplication And Division of Lines

**1. An unit is that by virtue of which each of the things that exist is called one.**
**Euclid's Elements**

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**The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.**

**John Clark**

© JClark8659@hotmail.com

Printed via import to MS Word.

# Following the Yellow Brick Road

**Multiplication**

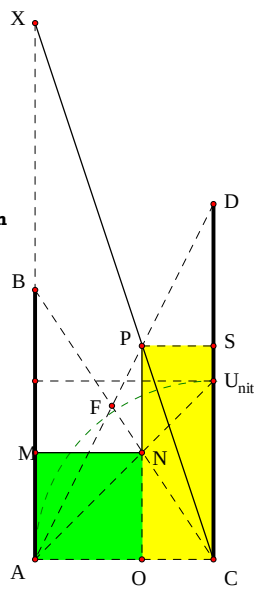
**Unit = 1.00000**

**AB = 1.51095**

**CD = 1.99270**

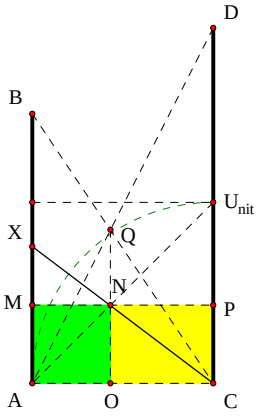
**AX = 3.01087**

**AB·CD-AX = 0.00000**




$$\frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} - \frac{\text{(Area MNOA)}}{\text{(Area OPSC)}} = 0.00000$$

**In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.**




$$\frac{\text{(Area MAON)}}{\text{(Area NOCP)}} - \frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} = 0.00000$$

**Division**

**Unit = 1.00000**

**AB = 1.48921**

**CD = 1.96403**

**AX = 0.75824**

$$\frac{\text{AB}}{\text{CD}} - \text{AX} = 0.00000$$

# Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

# Contents

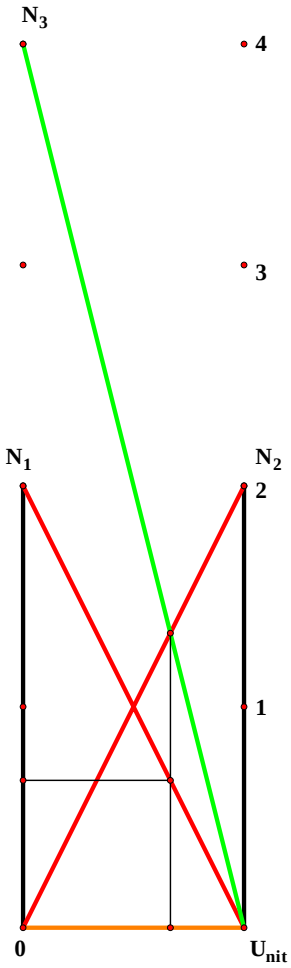
| Function | Page |
|---|---|
| $(N_1 \cdot N_2) - N_3 = 0.00000$ | Link to 1 |
| $\frac{N_1}{N_2} = 1.13725$ $\frac{N_1+N_2}{N_1} = 1.87931$ $\frac{N_1+N_2}{N_2} = 2.13725$ | Link to 2 |
| $\frac{1}{N_1} - N_3 = 0.00000$ | Link to 3 |
| $\frac{N_1}{N_2^2} - N_4 = 0.00000$ $\frac{N_1}{N_2^3} - N_5 = 0.00000$ | Link to 4 |
| $2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$ | Link to 5 |
| $\frac{N_1^2}{(N_2+N_1) \cdot N_2} - N_3 = 0.00000$ | Link to 6 |
| $(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$ | Link to 7 |
| $N_3^2 - \frac{BC}{BD} = 0.00000$ $N_3^3 - \frac{BC}{BE} = 0.00000$ $N_3^4 - \frac{BC}{BF} = 0.00000$ | Link to 8 |
| $\frac{N4_2}{N2_2} = 1.39420$ $\frac{N2_2}{N1_2} = 1.39420$ $\frac{N1_2}{Unit_2} = 1.39420$ $\frac{Unit_2}{N3_2} = 1.39420$ | Link to 9 |
| $\frac{N_1^2}{N_2} - N_5 = 0.00000$ $\frac{N_1^3}{N_2^2} - N_6 = 0.00000$ $\frac{N_2^2}{N_1} - N_7 = 0.00000$ | Link to 10 |

Link to Introdution

| Function | Page |
|---|---|
| $\frac{N_1^2}{N_2^2} - N_5 = 0.00000$ $N_2^2 - N_6 = 0.00000$ | Link to 11 |
| $\sqrt{2} - N_5 = 0.00000$ $\frac{\sqrt{2} \cdot N_1}{N_2} - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{\sqrt{2}} - N_7 = 0.00000$ | Link to 12 |
| $N_5 - 2^{0.75} = 0.00000$ $\left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$ | Link to 13 |
| $N_1^{0.5} - N_2 = 0.00000$ $N_1^{0.25} - N_3 = 0.00000$ $N_1^{0.125} - N_4 = 0.00000$ | Link to 14 |
| $\frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$ $\frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$ $N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$ | Link to 15 |
| $\frac{N_1^2}{(N_1+N_2) \cdot N_2} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+N_2} - M_1 = 0.00000$ | Link to 16 |
| $\frac{N_1^2}{N_2 \cdot (N_1+1)} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+1} - M_1 = 0.00000$ | Link to 17 |
| $\frac{N_1^2}{N_2^4} - N_7 = 0.00000$ $N_2^4 - N_8 = 0.00000$ $N_2^3 - N_{25} = 0.00000$ | Link to 18 |
| $\frac{N_2^4}{N_1} - N_{26} = 0.00000$ $\frac{N_1}{N_2} - N_{27} = 0.00000$ $\frac{N_2^7}{N_1} - N_8 = 0.00000$ | |

# Contents


| Function | Page | Function | Page |
|---|---|---|---|
| $N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$ | Link to 19 | | |
| $N_1 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_5 = 0.00000$ $N_1 \cdot \left(\frac{N_2}{N_1+N_2}\right) - N_6 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_7 = 0.00000$ | Link to 20 | | |
| $N_1^{\frac{1}{2}} - N_2 = 0.00000$ $N_1^{\frac{1}{4}} - N_3 = 0.00000$ $N_1^{\frac{1}{8}} - N_4 = 0.00000$ | Link to 21 | | |
| $N_1^{\frac{8}{8}} - N_1 = 0.00000$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$ | Link to 22 | | |
| $N_1^{\frac{5}{8}} - N_4 = 0.00000$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$ | | | |
| $N_1^{\frac{2}{8}} - N_7 = 0.00000$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$ | | | |




$N_1 = 2.00000$

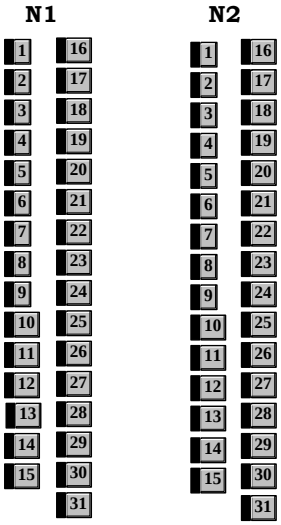
$N_2 = 2.00000$

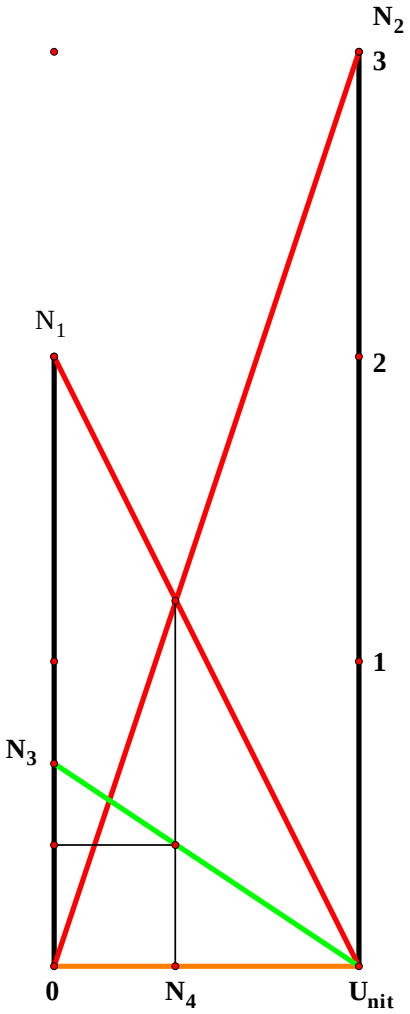
$N_1 \cdot N_2 = 4.00000$

$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

**Multiply N1 by N2**






$N_1 = 2.00000$

$N_2 = 3.00000$

$\frac{N_1}{N_2} = 0.66667$

$N_3 = 0.66667$

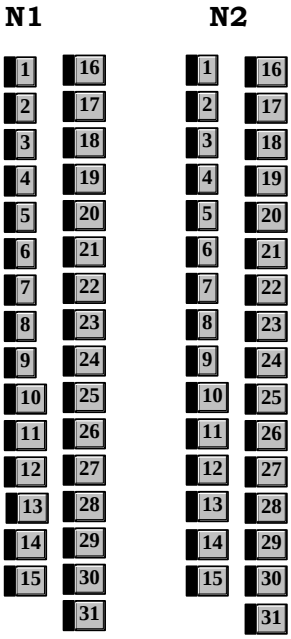
$\frac{U_{nit}}{0N_4} = 2.50000$

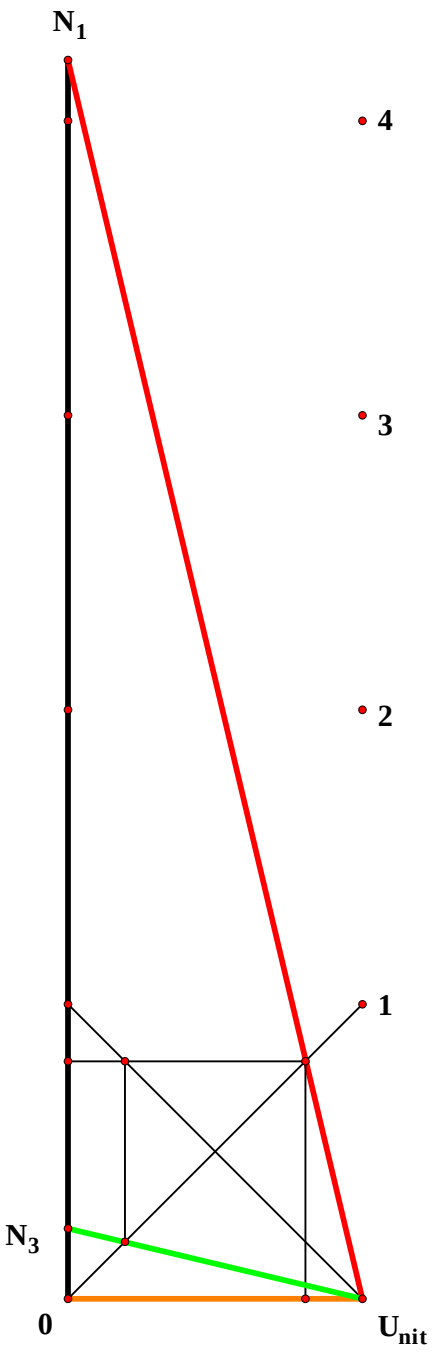
$\frac{N_1+N_2}{N_1} = 2.50000$

$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$

$\frac{N_1+N_2}{N_2} = 1.66667$

**Divide**
**N1 by N2**






**Find the Recprocal N1**

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

**N1**

| | |
|---|---|
| 1 | 16 |
| 2 | 17 |
| 3 | 18 |
| 4 | 19 |
| 5 | 20 |
| 6 | 21 |
| 7 | 22 |
| 8 | 23 |
| 9 | 24 |
| 10 | 25 |
| 11 | 26 |
| 12 | 27 |
| 13 | 28 |
| 14 | 29 |
| 15 | 30 |
| | 31 |

$N_1 = 3.00000$

$N_2 = 2.00000$

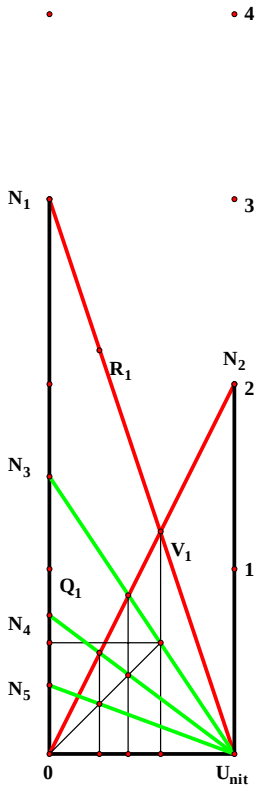
$\frac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

$N_4 = 0.75000$ $\quad \frac{N_1}{N_2^2} - N_4 = 0.00000$

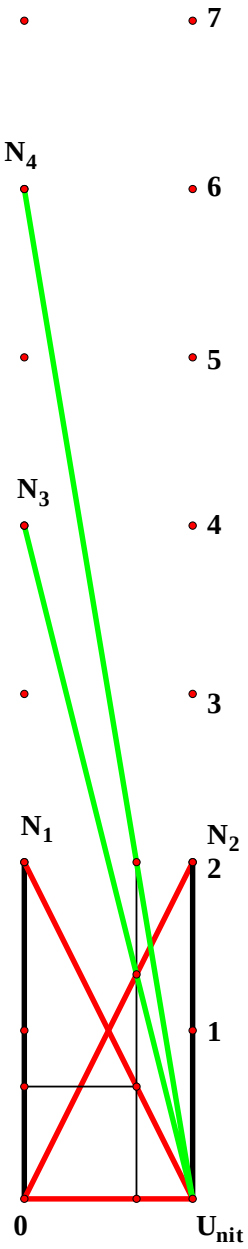
$N_5 = 0.37500$ $\quad \frac{N_1}{N_2^3} - N_5 = 0.00000$

**etc.**




**Divide N1 by a Power of N2**

| | N1 | N2 |
|---|---|---|
| 1 | 16 | 16 |
| 2 | 17 | 17 |
| 3 | 18 | 18 |
| 4 | 19 | 19 |
| 5 | 20 | 20 |
| 6 | 21 | 21 |
| 7 | 22 | 22 |
| 8 | 23 | 23 |
| 9 | 24 | 24 |
| 10 | 25 | 25 |
| 11 | 26 | 26 |
| 12 | 27 | 27 |
| 13 | 28 | 28 |
| 14 | 29 | 29 |
| 15 | 30 | 30 |
| | 31 | 31 |




$N_1 = 2.00000$

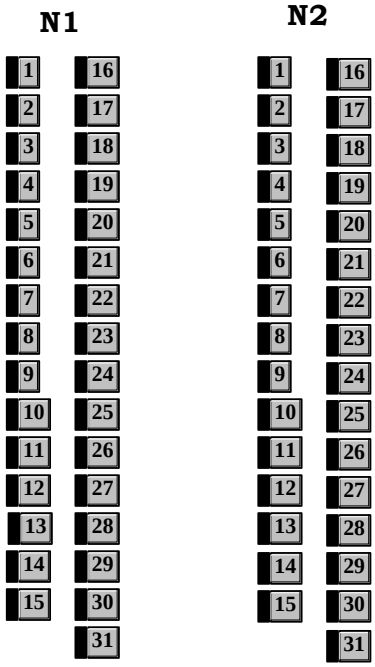
$N_2 = 2.00000$

$N_1 \cdot N_2 = 4.00000$

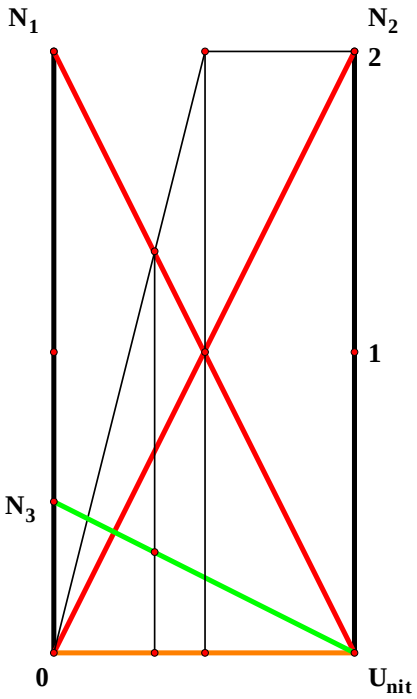
$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



3



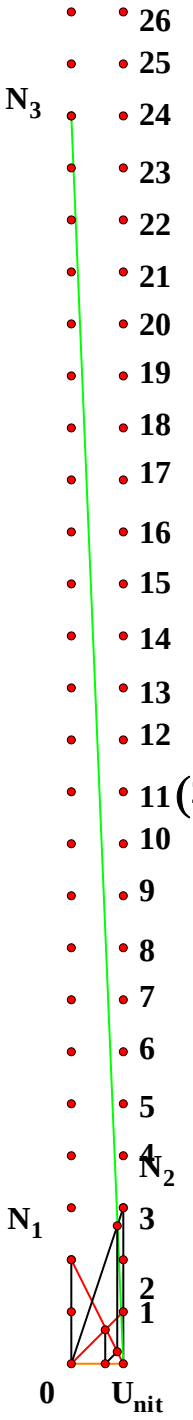

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$

| N1 | | N2 | |
|---|---|---|---|
| 1 | 16 | 1 | 16 |
| 2 | 17 | 2 | 17 |
| 3 | 18 | 3 | 18 |
| 4 | 19 | 4 | 19 |
| 5 | 20 | 5 | 20 |
| 6 | 21 | 6 | 21 |
| 7 | 22 | 7 | 22 |
| 8 | 23 | 8 | 23 |
| 9 | 24 | 9 | 24 |
| 10 | 25 | 10 | 25 |
| 11 | 26 | 11 | 26 |
| 12 | 27 | 12 | 27 |
| 13 | 28 | 13 | 28 |
| 14 | 29 | 14 | 29 |
| 15 | 30 | 15 | 30 |
| | 31 | | 31 |

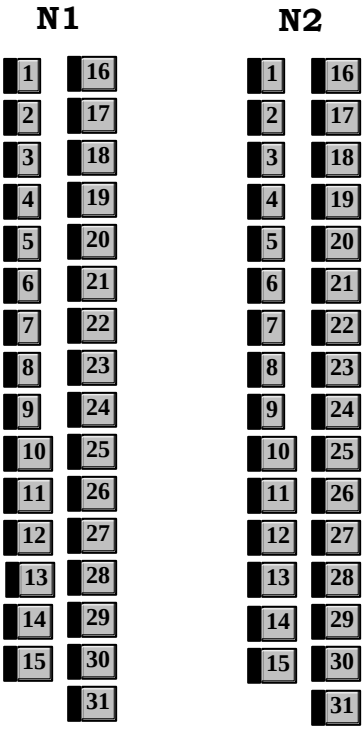



$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$



$\frac{BC}{BV_1} = 0.81650$ $\quad N_3^{-0.5} = 0.81650$

$N_3^{0} = 1.00000$

$\frac{BC}{BU_1} = 1.22474$ $\quad N_3^{0.5} = 1.22474$

$\frac{BC}{BP_1} = 1.50000$

$\frac{BC}{BR_1} = 2.25000$ $\quad N_3^{2} - \frac{BC}{BR_1} = 0.00000$

$\frac{BC}{BS_1} = 3.37500$ $\quad N_3^{3} - \frac{BC}{BS_1} = 0.00000$

$\frac{BC}{BT_1} = 5.06250$ $\quad N_3^{4} - \frac{BC}{BT_1} = 0.00000$

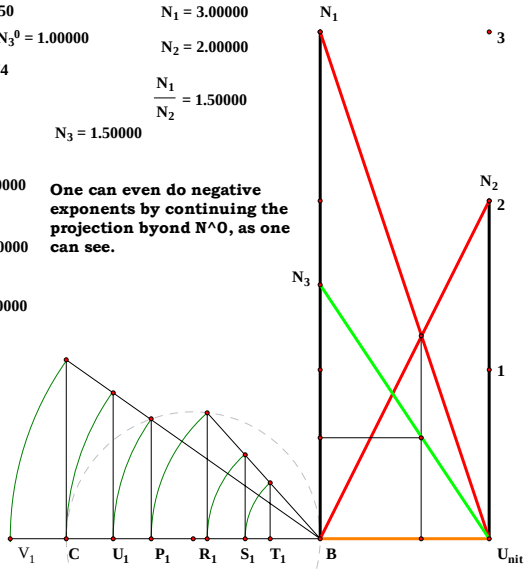
$N_1 = 3.00000$

$N_2 = 2.00000$

$\frac{N_1}{N_2} = 1.50000$

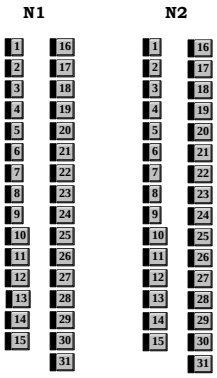
$N_3 = 1.50000$

**One can even do negative exponents by continuing the projection beyond N^0, as one can see.**




**Adding a figure to a figure.**

**I tossed this together, so it is not perfect--it just looks good**



**How to find a Proportional Unit for a given pair of Magnitudes**

$Unit_2 = 1.00000$

$N1_2 = 1.39420$

$N2_2 = 1.94380$

$N3_2 = 0.71726$

$N4_2 = 2.71004$

$\frac{N1_2}{N2_2} - N3_2 = 0.00000$

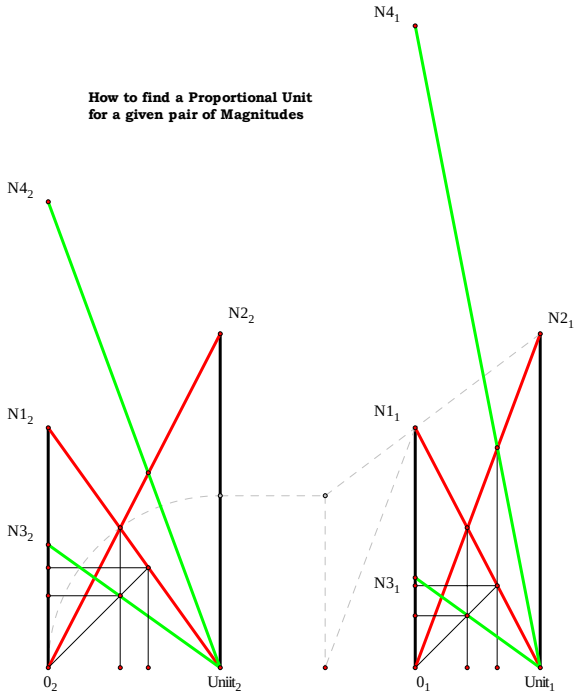
$N1_2 \cdot N2_2 - N4_2 = 0.00000$

$\frac{N4_2}{N2_2} = 1.39420$

$\frac{N2_2}{N1_2} = 1.39420$

$\frac{N1_2}{Unit_2} = 1.39420$

$\frac{Unit_2}{N3_2} = 1.39420$




$Unit_1 = 1.00000$

$N1_1 = 1.91861$

$N2_1 = 2.67492$

$N3_1 = 0.71726$

$N4_1 = 5.13213$

$\frac{N1_1}{N2_1} - N3_1 = 0.00000$

$N1_1 \cdot N2_1 - N4_1 = 0.00000$

$\frac{N4_1}{N2_1} = 1.91861$

$\frac{N2_1}{N1_1} = 1.39420$

$\frac{N1_1}{Unit_1} = 1.91861$

$\frac{Unit_1}{N3_1} = 1.39420$

**Unit = 1.00000**

$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 0.66667$

$N_4 = 6.00000$

$N_5 = 1.33333$

$N_6 = 0.88889$

$N_7 = 4.50000$

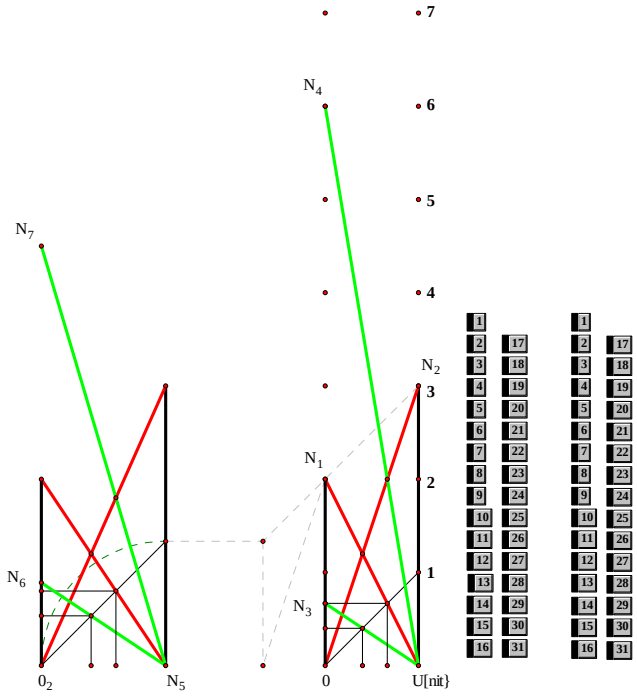
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1^2}{N_2} - N_5 = 0.00000$$

$$\frac{N_1^3}{N_2^2} - N_6 = 0.00000$$

$$\frac{N_2^2}{N_1} - N_7 = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.30952**

**$N_3$ = 0.86598**

**$N_4$ = 4.61905**

**$N_5$ = 0.74992**

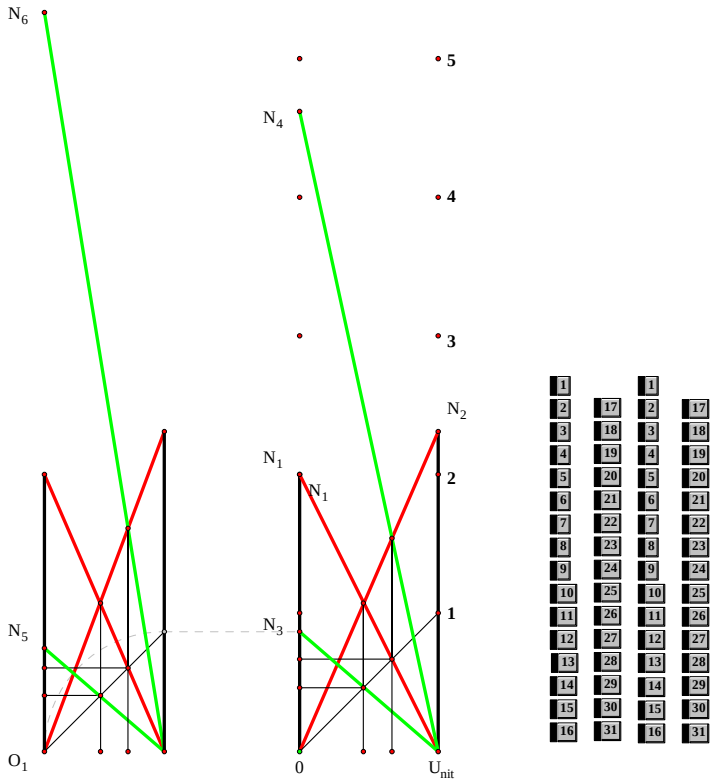
**$N_6$ = 5.33390**

$\frac{N_1}{N_2} - N_3 = 0.00000$

$N_1 \cdot N_2 - N_4 = 0.00000$

$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_2^2 - N_6 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.43939**

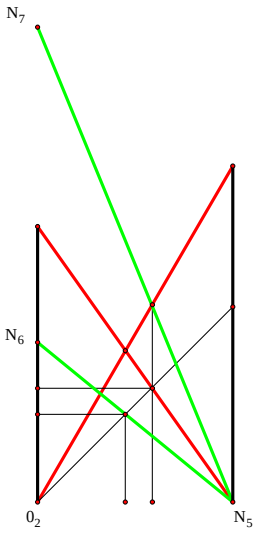
**$N_3$ = 0.81988** $\frac{N_1}{N_2}-N_3 = 0.00000$

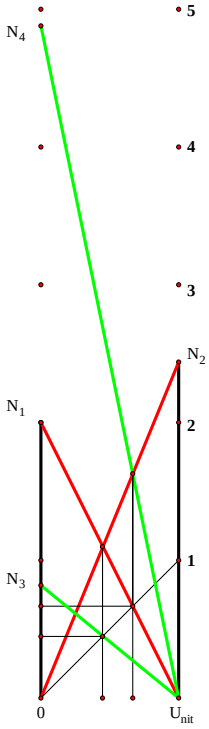
**$N_4$ = 4.87879** $N_1 \cdot N_2 - N_4 = 0.00000$

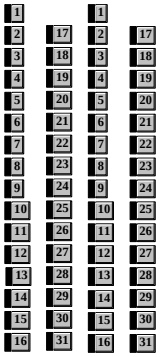
**$N_5$ = 1.41421** $\sqrt{2}-N_5 = 0.00000$

**$N_6$ = 1.15948** $\frac{\sqrt{2}\cdot N_1}{N_2}-N_6 = 0.00000$

**$N_7$ = 3.44982** $\frac{N_1 \cdot N_2}{\sqrt{2}}-N_7 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.00000** $\quad \frac{N_1}{N_2} - N_3 = 0.00000$

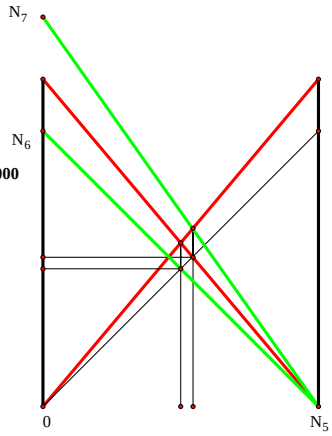
**$N_3$ = 1.00000**

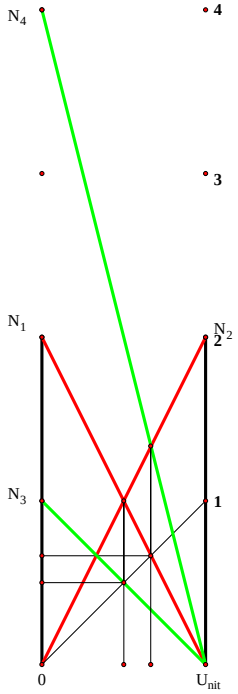
**$N_4$ = 4.00000** $\quad N_1 \cdot N_2 - N_4 = 0.00000$

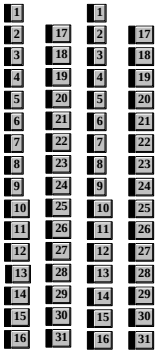
**$N_5$ = 1.68179** $\quad N_5 - 2^{0.75} = 0.00000$

**$N_6$ = 1.68179** $\quad \left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

**$N_7$ = 2.37841** $\quad \frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$



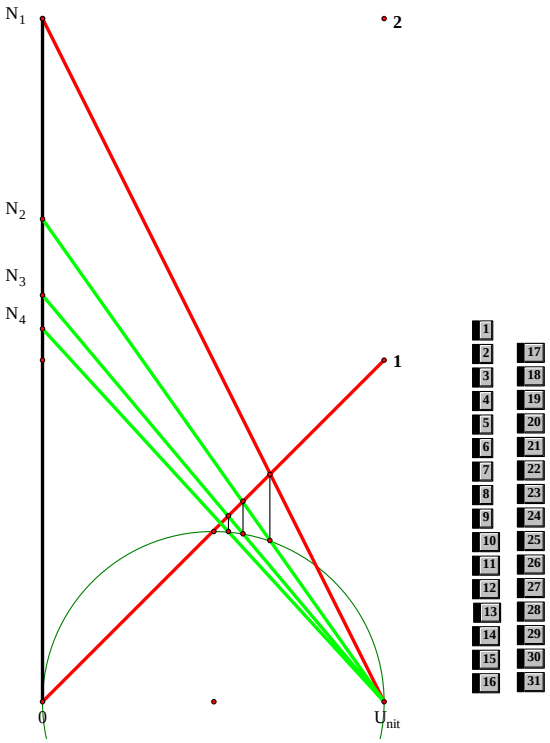
**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 1.41421** **$N_1^{0.5}$-$N_2$ = 0.00000**

**$N_3$ = 1.18921** **$N_1^{0.25}$-$N_3$ = 0.00000**

**$N_4$ = 1.09051** **$N_1^{0.125}$-$N_4$ = 0.00000**



**Unit = 1.00000**

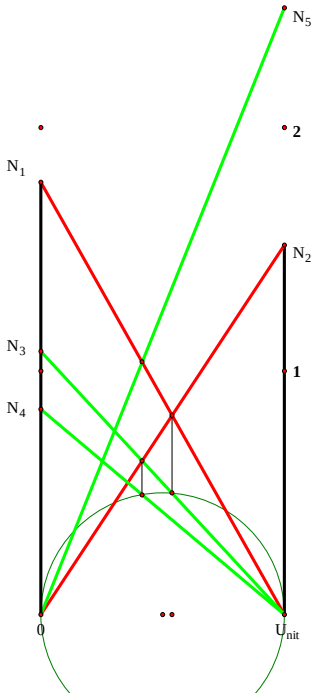
**$N_1$ = 1.77542**

**$N_2$ = 1.51695**

**$N_3$ = 1.08185** $\quad \frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

**$N_4$ = 0.84450** $\quad \frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

**$N_5$ = 2.48947** $\quad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$

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| 3 | 19 | 3 | 19 |
| 4 | 20 | 4 | 20 |
| 5 | 21 | 5 | 21 |
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| 7 | 23 | 7 | 23 |
| 8 | 24 | 8 | 24 |
| 9 | 25 | 9 | 25 |
| 10 | 26 | 10 | 26 |
| 11 | 27 | 11 | 27 |
| 12 | 28 | 12 | 28 |
| 13 | 29 | 13 | 29 |
| 14 | 30 | 14 | 30 |
| 15 | 31 | 15 | 31 |
| 16 | | 16 | |

**Unit = 1.00000**

**$N_1$ = 1.00000**

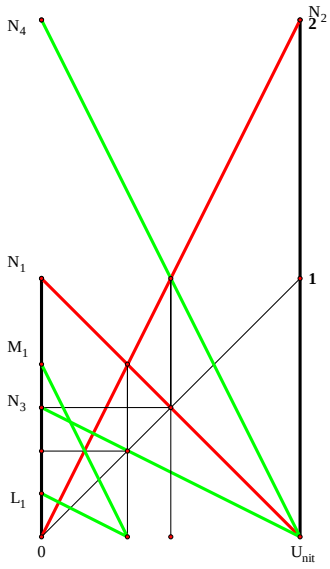
**$N_2$ = 2.00000** $\frac{N_1}{N_2}-N_3 = 0.00000$

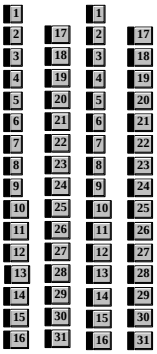
**$N_3$ = 0.50000**

**$N_4$ = 2.00000** $N_1 \cdot N_2 - N_4 = 0.00000$

**$L_1$ = 0.16667** $\frac{N_1^2}{(N_1+N_2)\cdot N_2}-L_1 = 0.00000$

**$M_1$ = 0.66667** $\frac{N_1^2 \cdot N_2}{N_1+N_2}-M_1 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

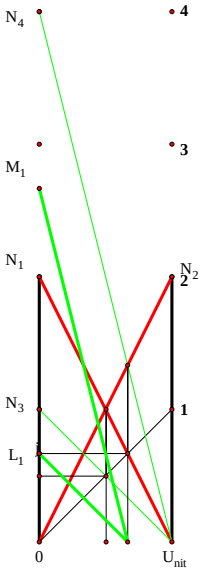
**$N_2$ = 2.00000** $\frac{N_1}{N_2}$**-$N_3$ = 0.00000**

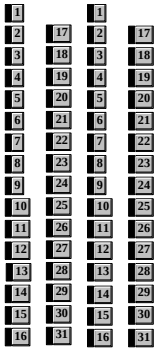
**$N_3$ = 1.00000**

**$N_4$ = 4.00000** **$N_1 \cdot N_2 - N_4 = 0.00000$**

**$L_1$ = 0.66667** $\frac{N_1^2}{N_2 \cdot (N_1+1)}$**-$L_1$ = 0.00000**

**$M_1$ = 2.66667** $\frac{N_1^2 \cdot N_2}{N_1+1}$**-$M_1$ = 0.00000**



$U_1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 1.00000$ $\quad \frac{N_1}{N_2} = 1.00000$

$N_4 = 4.00000$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$U[1] / U_2 = 0.25000$

$N_5 = 2.00000$

$N_6 = 2.00000$

$N_7 = 0.25000$ $\quad \frac{N_1^2}{N_2^4} - N_7 = 0.00000$

$N_8 = 16.00000$ $\quad N_2^4 - N_8 = 0.00000$

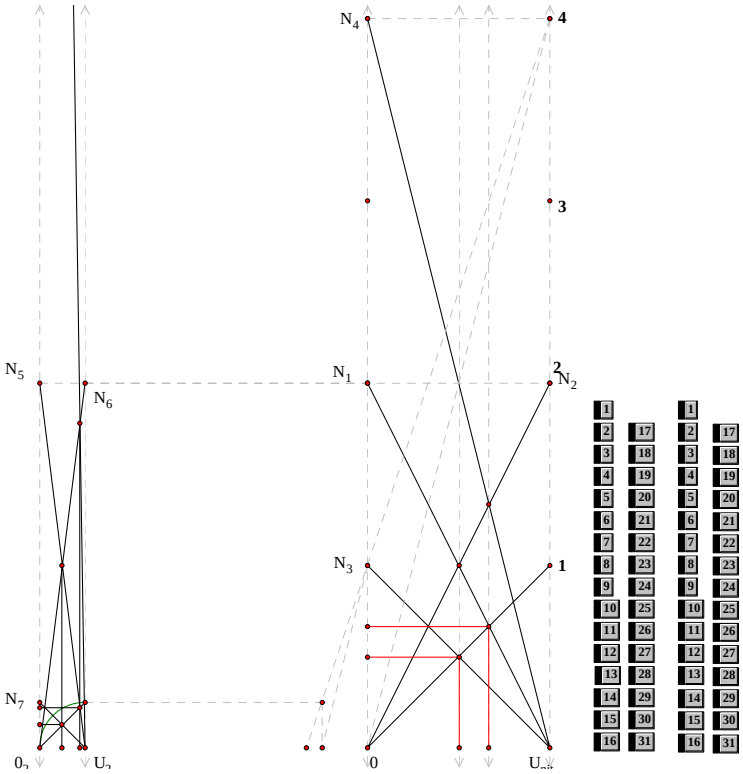
$U_2 = 1.00000$

$N_{25} = 8.00000$ $\quad N_2^3 - N_{25} = 0.00000$

$N_{26} = 8.00000$ $\quad \frac{N_2^4}{N_1} - N_{26} = 0.00000$

$N_{27} = 1.00000$ $\quad \frac{N_1}{N_2} - N_{27} = 0.00000$

$N_8 = 64.00000$ $\quad \frac{N_2^7}{N_1} - N_8 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 1.72159**

**$N_2$ = 2.12500**

**$N_3$ = 0.81016**

**$N_4$ = 3.65838**

**$N_5$ = 0.57386**

**$N_6$ = 1.14773**

**$N_7$ = 0.70833**

**$N_8$ = 1.41667**

**$N_9$ = 1.21946**

**$N_{10}$ = 2.43892**

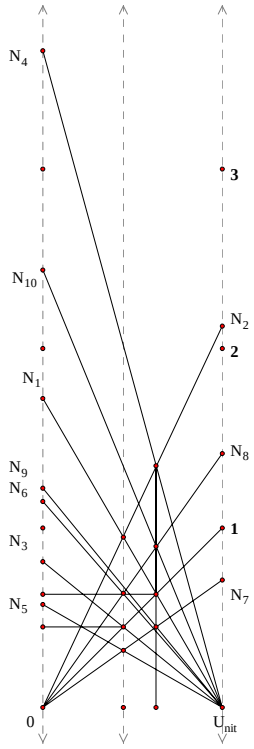
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

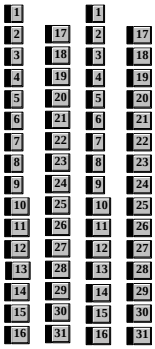
$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1}{N_2} = 0.81016$$

$$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$$N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 1.37056**

**$N_2$ = 1.73096**

**$N_3$ = 0.79179** $\frac{N_1}{N_2} = 0.79179$

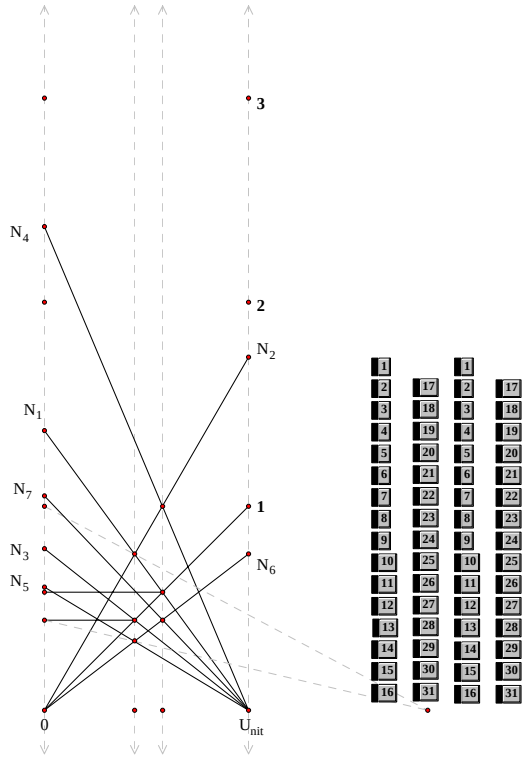
**$N_4$ = 2.37239** $N_1 \cdot N_2 = 2.37239$

$\frac{N_1}{N_2} = 0.79179$

**$N_5$ = 0.60565** $N_1 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000$

**$N_6$ = 0.76491** $N_1 \cdot \left( \frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000$

**$N_7$ = 1.04835** $N_1 \cdot N_2 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000$



**Unit = 1.00000**

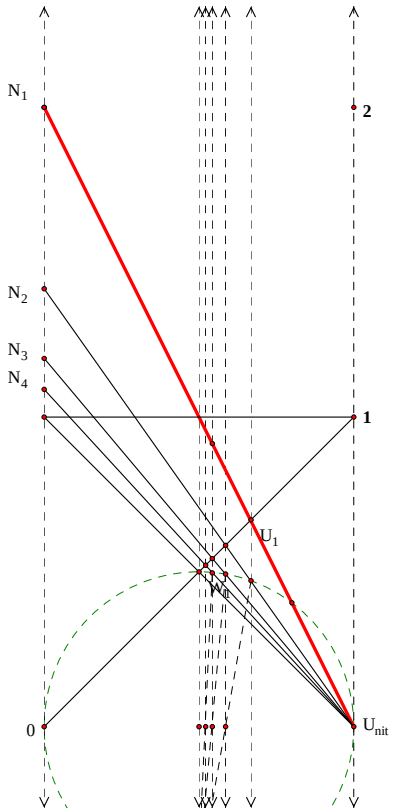
**$N_1$ = 2.00000**

**$N_2$ = 1.41421** $\quad N_1^{\frac{1}{2}} - N_2 = 0.00000$

**$N_3$ = 1.18921** $\quad N_1^{\frac{1}{4}} - N_3 = 0.00000$

**$N_4$ = 1.09051** $\quad N_1^{\frac{1}{8}} - N_4 = 0.00000$

**etc.**




1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

**Unit = 1.00000**

**Eight root series.**

**One can project as long as they like.**

$N_1 = 2.00000$ $N_1^{\frac{8}{8}} - N_1 = 0.00000$

$N_2 = 1.83401$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$

$N_3 = 1.68179$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$

$N_4 = 1.54221$ $N_1^{\frac{5}{8}} - N_4 = 0.00000$

$N_5 = 1.41421$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$

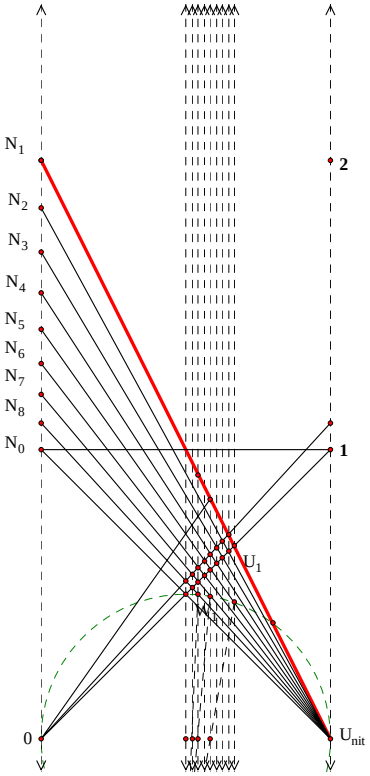
$N_6 = 1.29684$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$

$N_7 = 1.18921$ $N_1^{\frac{2}{8}} - N_7 = 0.00000$

$N_8 = 1.09051$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$

$N_0 = 1.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$

**etc.**





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The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

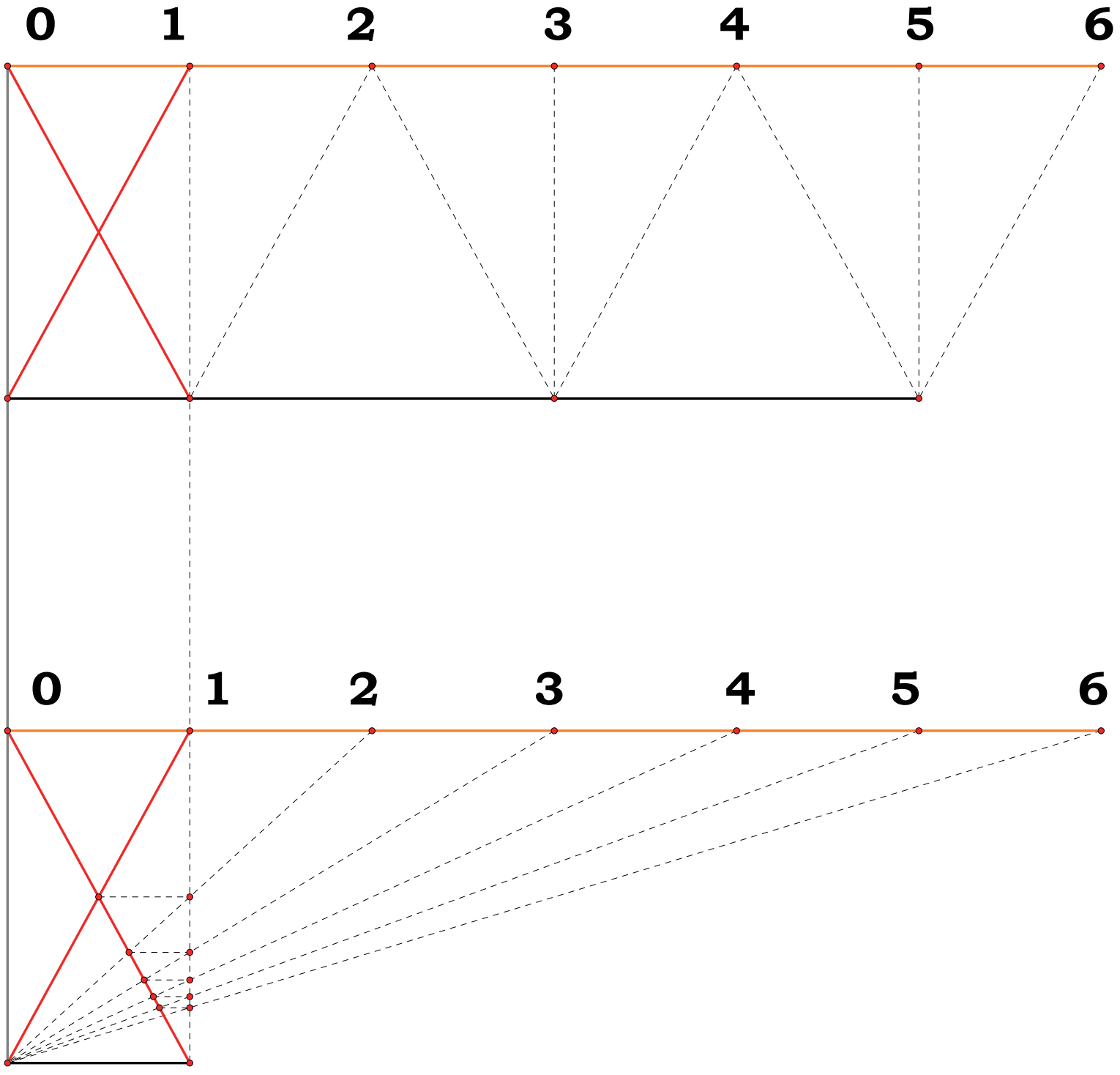
This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.

# Complete Induction.

## or counting.



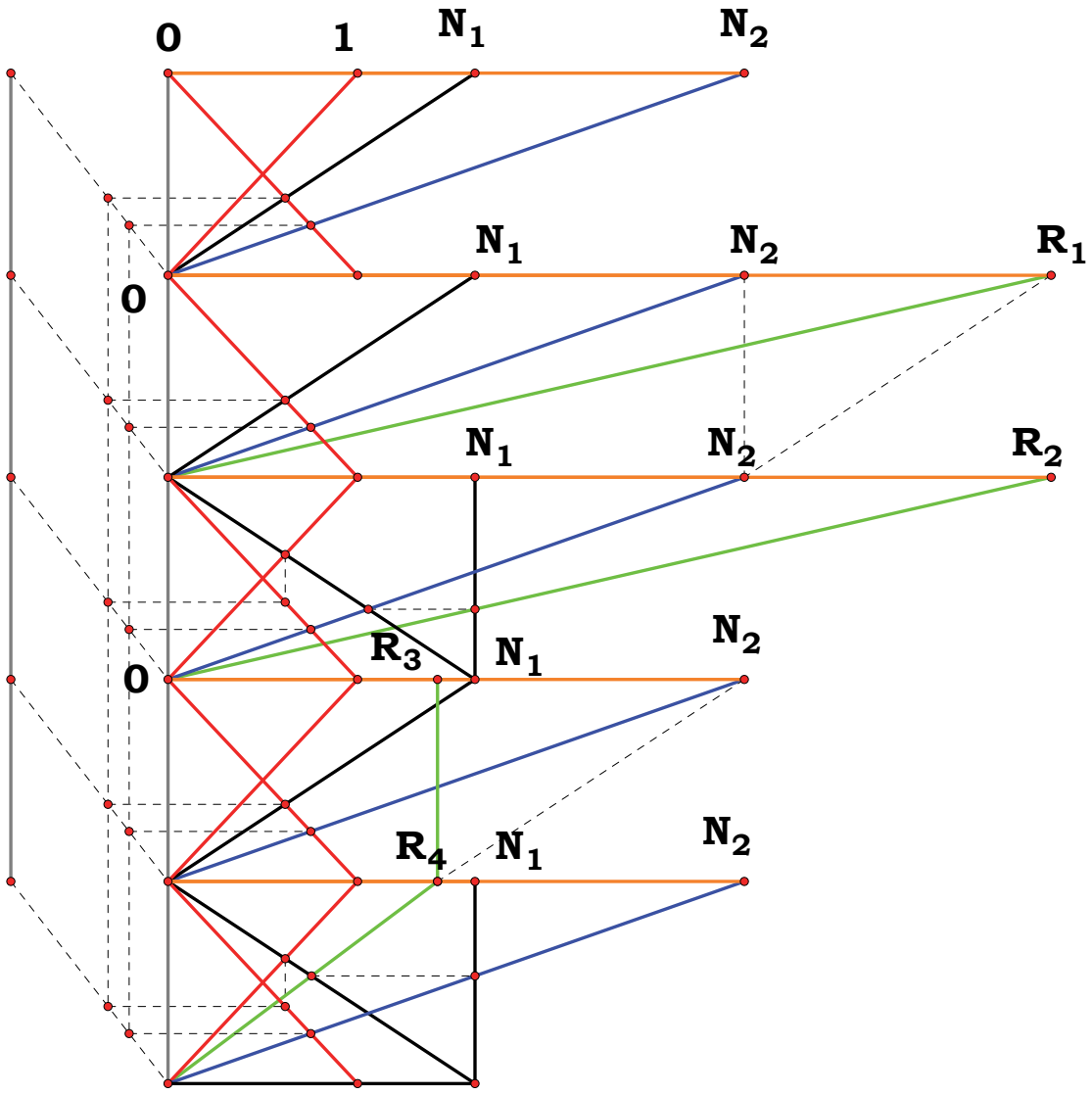
# Addition and Subtraction.

$N_1 = 1.61842$

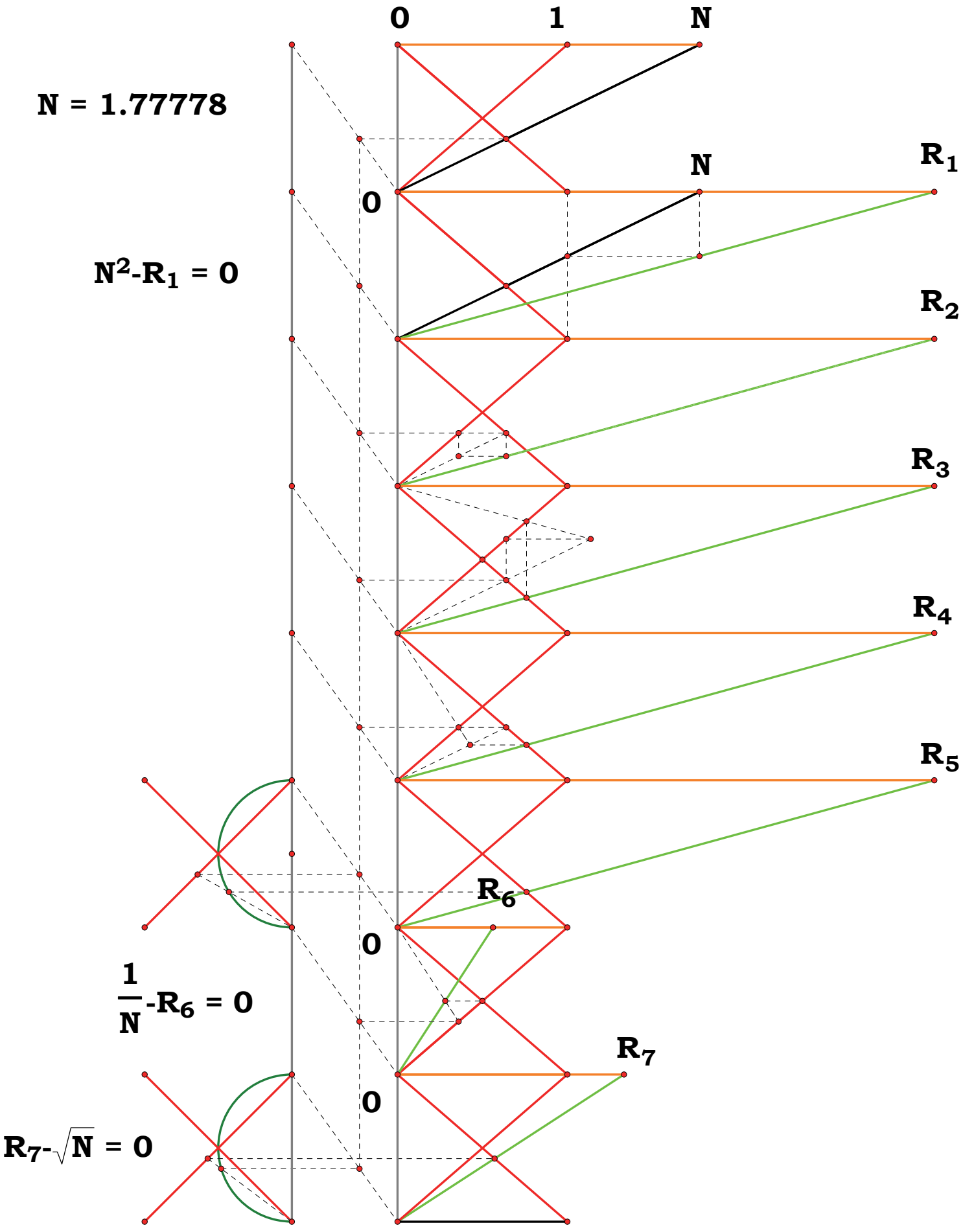
$N_2 = 3.03947$

$(N_1+N_2)-R_1 = 0$

$N_2-N_1-R_3 = 0$



# The Square, Reciprocal and Root.



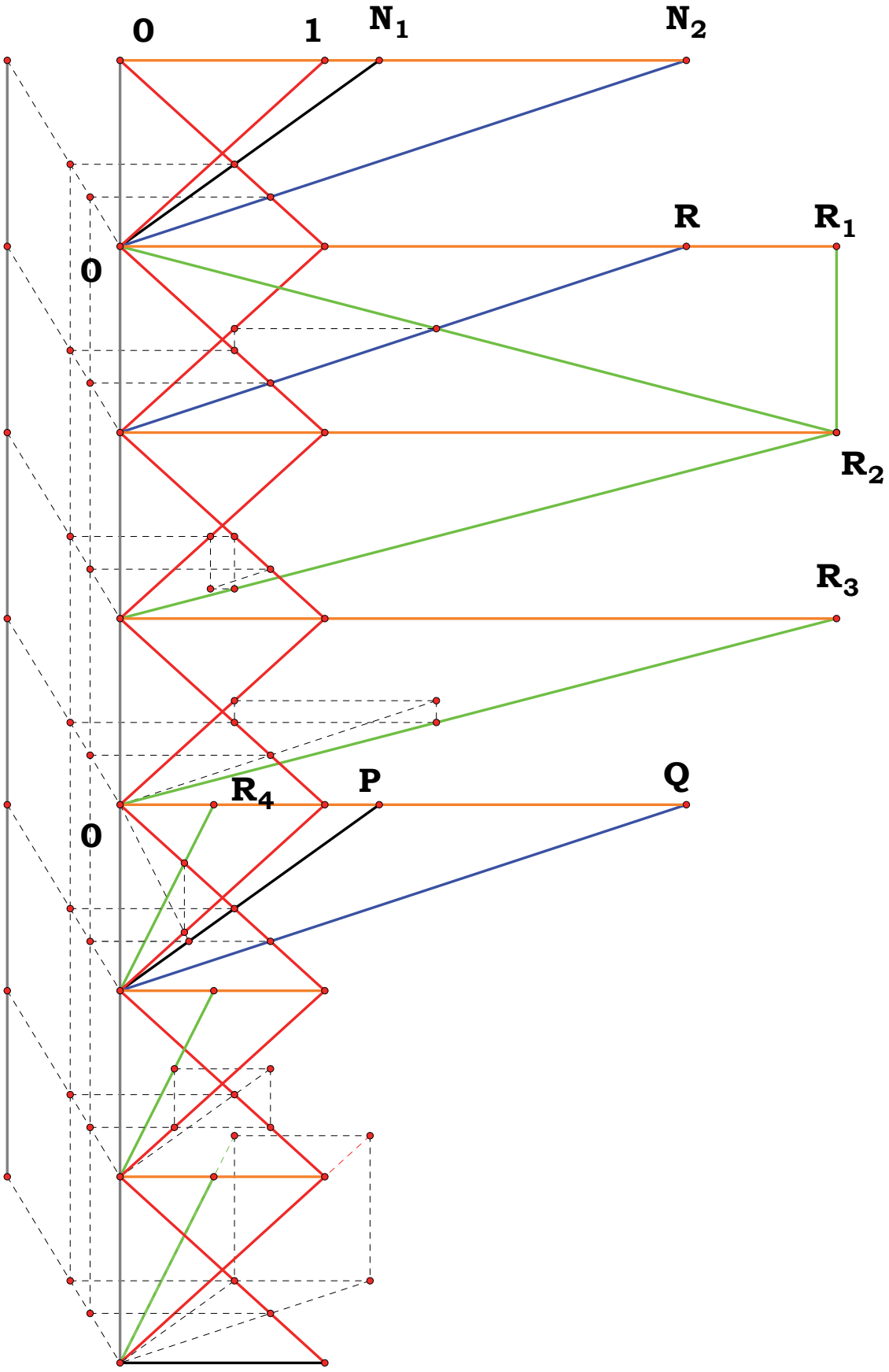
# Multiplication and Division.

$N_1 = 1.26531$

$N_2 = 2.76531$

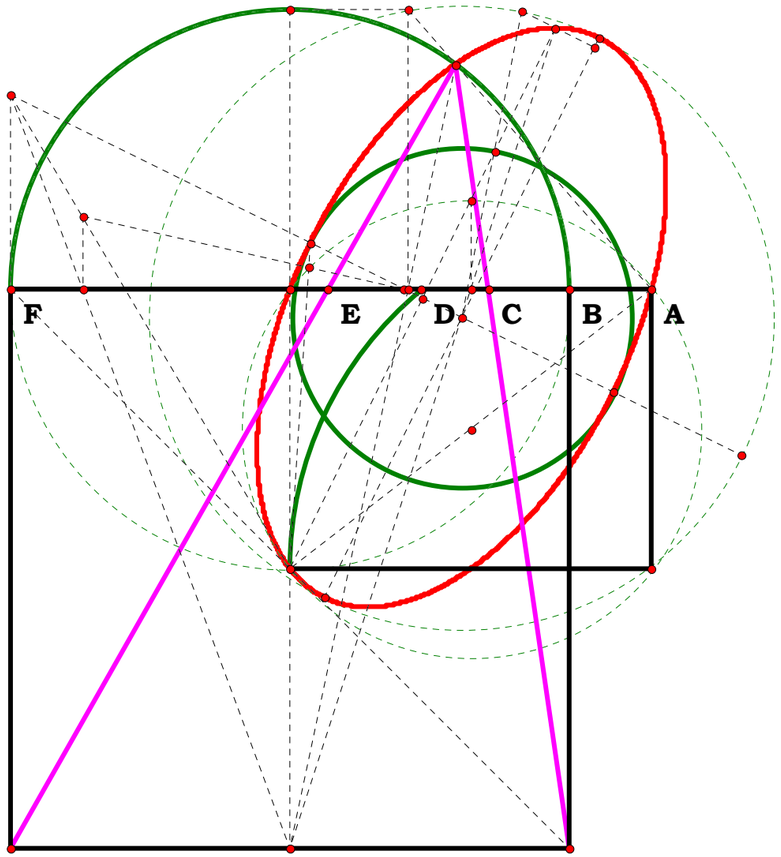
$N_1 \cdot N_2 - R_1 = 0$

$\frac{N_1}{N_2} - R_4 = 0$



# *The Delian Quest.*

**2015 rev.**




**AB = 1.38526 cm**
**AC = 2.74270 cm**
**AD = 3.85923 cm**
**AE = 5.43030 cm**
**AF = 10.75151 cm**

$$(AB^2 \cdot AF)^{\frac{1}{3}} - AC = 0.00000$$

$$(AB \cdot AF^2)^{\frac{1}{3}} - AE = 0.00000$$

$$(AB \cdot AF)^{\frac{1}{2}} - AD = 0.00000$$

# On the Principles of Dialectic.

I am not going to make a long and tedious demonstration here, I will be presenting a more or less concise outline.

Dialectic is used for the maintenance and promotion of the functionality of the human mind in order for it to do its own work. The fact that the mind has a biologically determine job to perform is not an assumption. Nor is it an assumption that other life forms in the universe have guided the development of the mind of man so that one day the mind can perform this biologically determined job. One of the reasons one teaches someone to perform a task is because the teacher is not going to do that task for someone. The concept taught through religions, that a teacher is going to do man's job for him, is the product of a dysfunctional mind and it is a self-referential fallacy which defeats the purpose and efficacy of education. Another fallacy is that philosophy is the product of an individual mind.

The mind is responsible for the behavior of the body within which that mind resides. When that behavior is the product of language, language principles not currently recognized nor taught, it is called *will*. Will is, therefore, a linguistic product of a functional mind. Thus behavior is divided, itself, into two categories which can simply be indexed as "good" and "evil". Will is obtained as a learned process. No one is born free, nor is freedom something bestowed on an individual by another. Freedom is the unobstructed ability to do one's own work. One of the greatest obstructions to human freedom is the lack of education required to learn to perform one's biologically determined job; another is the personal interference from dysfunctional behavior of other individuals in one's environment; and a third is the physical inability of the mind, itself, to attain to that performance. Mankind is currently proto-linguistic but this does not negate the fact that these obstructions must be dealt with commensurately with the specific disability. This small essay and demonstration is educational.

Dialectic is functional as a craft. Like any craft, there are two functional parts. These parts are the material one works with and the processes upon that material in order to render a specific product. This craft can be recorded, retained in memory, as a *Two-Element Metaphysics*. Due to the principles of language, this *Two-Element Metaphysics* can be and have been put into a large number of metaphors such as the *Two-Witnesses of God*, or the *Two-Stone Tablets of Law* and even *The Theory of Forms*. As the mind is wholly linguistic by function, one would be committing a linguistic error and thus behavioral errors to conclude that these terms are either "*religious*" or "*philosophical*"; the grouping function of the mind is factual. Metaphor is used in the exercise of linguistic functionality.

As the mind is wholly linguistic by function, both elements of the crafting processes of the mind must be comprehended in terms of language. Thus language is factually divided into *logic* and *analogic*. analogic being the material and logic the form applied to that material. Again, one can view this pair of functions in terms of metaphor as the *Two Witnesses of God*, or as *The Two Tablets of Law*, primitively this conceptual pair are derived from the definition of a thing and this definition is exampled in living biology. As this pair maintain an image of a thing, language can be effected starting with either of them.

**Definition:** *A thing* is any *material* in some *form*.

Here we see a natural division in things themselves which establishes a crafting paradigm which is universal. The name of a thing is equal to the names of that things elements. One of those elements, form, is a container for the other, material. Even the word dialectic is constructed as a recognition of this division in language which is commensurate with biology.

It is living biology which is the aim of that product called human will, and learning human will entails the comprehension of all human behavior along with the ability to distinguish *good* and *evil* behavior. In short, good and evil is in respect to the product of the mind, human behavior. In this regard I will lay down a concise out line of human behaviors with which the mind functions to regulate in order to maintain and promote life.

In short, the mind is responsible for a product of human behavior that maintains and promotes the life of the body. It performs its job wholly through the artifice of language. Language itself is produced as standards in human behavior to maintain and promote life. Therefore, language is standards of human behavior that maintains and promotes life and is called Law. Law therefore is not the product of any individual or group of individuals, organizations, or so called governing bodies, Law is a derivative of the Law of Identity for the specific purpose of maintaining a particular image which can be put in a concise metaphor as I AM THAT I AM. It means that our life is obtained through the images provided by perception, or in other words by the functions of the seven life support systems of our own body.

The crafting systems of a biological organism can be comprehended as life support systems. Life support systems can be comprehended in terms of environmental acquisition systems of a living organism.

## Environmental Acquisition Systems.

Life support systems which are obvious and which abstract from the environment.

**Definition:** An environmental acquisition system of a living organism is that system of an organism which must acquire from the environment an *element* from some *thing* and process that element which it has acquired for a product that maintains and promotes the life of that organism.

**Those Systems that Acquire Material.**

1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

**Those Systems that Acquire Form.**

4) The Ocular-System.
5) The Vestibular-System.
6) The Procreative-System.
7) The Judgmental-System.

That system, which can be called our *self*, is that system which is responsible for the behavior of the remaining systems through a function called judgment. Judgment determines behavior. Judgment is effected wholly through the artifice of language. Thus, one can list our responsibilities for the rendition of judgment according to the division exhibited in the definition of a thing.

**The Self.**

1) The Judgmental-System.

The human mind is wholly linguistic by function. Its product is behavior. The first order of behavior is language itself. Language is effected as standards of behavior. There are two branches of language:

a) **Analogic**. The application of forms to standard given material.
Examples are provided by:
1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

b) **Logic**. The application of materials to standard given forms.
Examples are provided by:
1) The Ocular-System.
2) The Vestibular-System.
3) The Procreative-System.

From the paradigm expressed in the definition of a thing and exampled in living biology, we learn the fundamentals of both logic and analogic to govern the behaviors specified by the function of our environmental acquisition

systems. We learn to do our own work by example. Refusing these examples only indicates the degree of mental dysfunction.

## The Law of Identity.

The Law, or principle, of Identity is an expression of perception. Its specific expressions such as, *we learn by experience*, and *seeing is believing* are among them as well as *relation to self is inadmissible* and;

*A equals A.*

It also means that we, a biological life form, are not different from ourselves. Language is a biological function. Every functional part of a living organism functions in order to maintain and promote the life of that organism.

> "Everything which has a function exists for its function." On the Heavens, by Aristotle, W. D. Ross.

Therefore, the first place that one starts with, in the study of any language, is the desire to maintain and promote the relationship between language and survival. This desire to do our own work is our most fundamental ally to becoming functional.

## The Paradigm.

Every environmental acquisition system of a living organism, functions by what it can acquire from the environment. It acquires abstractions from things and these abstractions are called a things *elements*. As the mind is responsible for judgment in relation to these things in the environment, the paradigm of judgment starts with the definition of a thing itself. This definition is a product of abstraction by a functional mind and establishes the *unit*, the *foundation*, the *first principle* of language and judgment. If one is not capable of making this abstraction, one cannot do their own work.

**Definition**: A thing is any material in any form.

In regard to the definition of a thing, neither *form*, nor *material* are, in of themselves, things. The part is not the whole.

From the definition of a thing, we acquire, commensurate with our own biology, two, and only two, primitive branches of language; logic commensurate with form and analogic commensurate with material. Also, by the definition of a thing, one of these languages must play the part of form while the other plays

the part of material difference. Also, by the definition of a thing, both branches of reasoning are paired which can only say one and the same thing. Also by definition, neither branch of reasoning is a branch of reasoning if the complementary language is not present—i.e., functionally resident in the mind.

Logics play the part of form. As such logics are seen as containers for the memory of experiences. One can also say that logics are indexing systems, or grouping systems, or again scripting systems for information retrieval and manipulation. The material indexed or contained is the material difference of the memory of perceptions. Memory can be said to be retained fragments of experience or perceptions. These experiences can be either perceptible or intelligible.

Analogics play the part of the material difference. Analogics are best learned through well ordered behaviors which include crafts. One such craft is called geometry.

Logics have the form as a given and the material for those forms must be supplied, while analogics have the material as a given and the form to those materials must be applied.

> 'A part' may be a part either of the form (i.e. of the essence), or of the compound of the form and the matter, or of the matter itself. Metaphysics by Aristotle.

Repairing this translation for accuracy of statement:

'A part' is ether *form*, or the *material* in that *form* of a compound or composite called *a thing*.

> "Therefore one kind of substance can be defined and formulated, i.e. the composite kind, whether it be perceptible or intelligible; but the primary parts of which this consists cannot be defined, since a definitory formula predicates some thing of some thing, and therefore, one part of the definition must play the part of matter and the other that of form." Metaphysics by Aristotle. W. D. Ross.

Things are thus defined in terms of that things *parts*, or *elements*, but neither of these parts can be defined as they are not things; one can only name them.

The method that Plato used to make his reader aware of Aristotle's second statement was as a demonstration in psychotherapy by dialog.

> "SOCRATES: Let me give you, then, a dream in return for a dream:—Methought that I too had a dream, and I heard in my dream that the primeval letters or elements out of which you and I and all other things are compounded, have no reason or explanation; you can only name them, but no predicate can be either affirmed or denied of them, for in the one case existence, in the other non-existence is already implied, neither of which must be added, if you mean to speak of this or

> that thing by itself alone. It should not be called itself, or that, or each, or alone, or this, or the like; for these go about everywhere and are applied to all things, but are distinct from them; whereas, if the first elements could be described, and had a definition of their own, they would be spoken of apart from all else. But none of these primeval elements can be defined; they can only be named, for they have nothing but a name, and the things which are compounded of them, as they are complex, are expressed by a combination of names, for the combination of names is the essence of a definition. Thus, then, the elements or letters are only objects of perception, and cannot be defined or known; but the syllables or combinations of them are known and expressed, and are apprehended by true opinion. When, therefore, any one forms the true opinion of anything without rational explanation, you may say that his mind is truly exercised, but has no knowledge; for he who cannot give and receive a reason for a thing, has no knowledge of that thing; but when he adds rational explanation, then, he is perfected in knowledge and may be all that I have been denying of him. Was that the form in which the dream appeared to you?" *Theætetus*, by Plato, Jowett.

One may be confused as to why Socrates stated that you and I and all things are composed of letters—Plato was very aware that analog information was indeed a language. As the mind is wholly linguistic by function, it can only conceive all information via language itself. What can be determined is that many presentations to teach mankind this concept historically failed. That this would be so is recorded in the Judeo-Christian Scripture. There is no amount of teaching which can change the physical development of the mind itself.

By recursion of the paradigm for a *container* or *form*, *definition* then demonstrates the equality between the name of a thing which contains as a compound of the elements of form and matter which construct that thing by listing names of those elements. Thus we get the *Theory of Forms*, and why Plato was fixated on *definition*. Truth, at the foundation of language is achieved by maintaining the biological image as a convention of names, this is the whole of the correct process of both virtual construction and deconstruction.

Definition is a linguistic convention preserving the identity between the name of a thing and the names of the elements which comprise that thing. It is effected by maintaining then the equality between the name of a thing and the names of that things forms and the names of the materials contained in those forms.

Definition is a standard of individual and social behavior that is critical to the mind's ability to do its own work. The importance of definition, as an image, cannot be overstressed. A mind, or a collection of minds, incapable of this standard of behavior cannot be said to be linguistic, at best it can only be said to be proto-linguistic and fundamentally savage.

This is why the *Theory of Forms* is not a theory at all, but a factual observation concerning the foundation of language.

The fact that all of language resolves to this convention of names also determines what a proof is, and what it functions to do, to resolve a group of words back to the original naming convention which is based on perception. It also determines why we parse statements and parse them to isolate the individual assertions and denials contained in a sentence. Parsing is aimed at examining a statement for the compliance with the original naming convention also. Parsing is therefore part of the proofing process. This convention of names also determines that truth is the compliance with this convention. Naming a thing does not change a thing, but naming is part of the virtualization of experience.

As every environmental acquisition system of a living organism crafts from things to make other things, the paradigm of a thing itself will be used recursively, as a unit of conceptualization in the development of our understanding of language. In the short of it, the psychology of a linguistic species is founded upon a *Two-Element Metaphysics*, or in terms of the product of human behavior, *Two Tablets of Law*, or in other words, *definition* which is functional, by biological fact, only through perception. We learn, like anything else in reality, by experience.

Some important facts to take away from the paradigm. The form is not, nor ever can be the material difference, nor can the material difference ever be the form. Whether or not we use the paradigm as the basis for a name or for an entire system of reasoning, this one fact remains, it cannot exist as a thing in of itself. It takes both, the container and the contained, the form and the material to make any thing and everything—even in the realm of language. One may see this as a simple concept, however a proto-linguistic mind cannot comprehend it enough to effect behavior in accordance with it. A proto-linguistic mind cannot think in accordance with definition.

Within a proto-linguistic species there will always be individuals doing their best to establish another animal, real or imagined, as the standard for human behavior as a means of asserting what they themselves simply desire. They really have no other option than to rely on someone else to set their standards of behavior. This means that in terms of psychology, the psychological foundation of a proto-linguistic species rests upon the self-referential fallacy, or again, ignorance. This is the source of all savage behavior. It also indicates that all governing bodies today are either potentially or actually savage by nature.

## Logic.

There is a metaphor for the fact that language, i.e. Law is based on two branches. The most widely recognized is the *Two Stone Tablets of Law*, which would one day be revealed to man again. Another is in *Set Theory*, that a set can be constructed either through definition which is commensurate with form

or by enumeration which is commensurate with material. Or again as the container and the contained.

Logic is based on form; this is why Plato's so called *Theory of Forms* can also be called the *Theory of Definition*; this is why, in some dialogs, the question was ask if the topic were a thing or not. There is no amount of discourse which can define an element of a thing. The elements of things can only be known by direct perception, perceptible or intelligible, to which an agreed upon name is given to that abstraction. All we can do is name an abstraction, and all language functions to maintain that correspondence.

**Definition**: A thing is any material in any form.

**Categories of Names.**

From the definition of a thing, we find that we have three, and only three primitive categories of names. We can name a thing directly and we can name a thing, as Plato pointed out, by a combination of the names for a thing's elements, those elements being a thing's material and the forms within which those materials reside.

**Parts of Speech.**

These three categories define the parts of speech. Just like an algebraic equation, they are added together as building blocks for more complex statements. *Parts of Speech* begins with the recognition of two fundamental naming conventions. The parts of speech not only include the naming convention for things, but the convention of names for the operations upon those names. For example, in assertion and denial, the words "is" and "is not" are operands. Assertion and denial are operations performed with names. Thus one does not fault the names for an incorrect operation, one faults the operator. Operands are directed towards the operator, or that system which is manipulating the conventions used in naming.

**Naming Conventions.**

From the definition of a thing, we find that we have two distinct naming conventions for logic. We can name a thing directly, which is called the *Subject Naming Convention*, and we can name a thing as a combination of the names of a thing's materials and the names of the forms which contain those materials. This second convention is called the *Predicate Naming Convention*.

The convention of naming things, itself has two categories commensurate with form and material. Some names are standards while others are named *in situ*. Groups of names have been established for assignment *in situ*. Many of those are based on the physical act of pointing to an object from which the

abstraction is to be made. These pointers also can point to the perceptible and the intelligible.

One of the mistakes made in logic is using pointers which cannot, do not, point to a shared abstractable, and again perceptible or intelligible.

**Assertion and Denial.**

Going back to the naming convention itself, assertion and denial turn out to be operands, the first operand in regard to the naming convention, establishing that convention itself. When used in context it becomes a proposition. One can proof this proposition by referring back to the original convention of names, or again the definition.

Assertion is commensurate with the paradigm of form, meaning no difference. Denial is commensurate with the paradigm of material, meaning some difference. The explicit coordinate system of reference, i.e. environmental acquisition system, for comparison for either of these is not in the words.

When we assign names to abstractions, we are establishing a standard behavioral system. Our commitment to doing our own work can be measured by the effort we expend in maintaining that convention as a one-to-one correspondence, or again truth. What is trying to be achieved is a one-to-one correspondence between an abstractable and a name. When we maintain that correspondence is what is meant by truth.

1) Truth is the state of being true between two or more things.
2) True is a lack of difference between two or more things.

Language is functional an intelligible equality. To a simple mind, it is imagined to be mystical.

That state of functionality for a mind called truth is then measured in terms of the maintenance and preservation of the correspondence between a name and the abstraction named, or again by the maintenance and preservation of the naming convention; or again, definition. This is why it was written that we shall know the truth and the truth shall set us free. It is a simple biological fact.

When we name we are not naming our abstractions. We are naming the source of abstraction. As we are not the standard for a name, nor is it possible to standardize the abstracting systems, standardization is afforded by things in the environment itself. These are sometimes established through bureaus of standards. When man becomes functional, this bureau of standards will imply the entire ecosystem of man, including his own behavior, man is destined to have dominion over the earth, a dominion aimed at maintaining and promoting a life supporting ecosystem.

One of the most common fallacies in trying to negate the efficacy of words is by claiming that the name of a thing is referencing a particular persons abstraction, or again, “man is the measure of all things.” It is a simple minded argument which has apparently mastered many so called intellectuals and it is this fallacy which denotes a savage proto-linguistic species.

When we make an assertion we are saying that two names by their respective conventions of names are equal by some means of comparison. When we make a denial, we are saying that two names are not equal and again by some means of comparison. The means of comparison is often implied, i.e. fundamental to an equation is often an ellipsis for the system of comparison itself. It is often taken for granted that as a multi-sensing organism that one can locate the system used by the names given itself to complete the meaning of an assertion or denial. This aids in economy, but detracts from those without what is called “common sense.” Sometimes those without “common sense” can be corrected by affording them the opportunity to participate in the naming convention; this means pointing them to something commonly sensed from which they may make an abstraction; one can afford someone the opportunity, but not the ability. Higher level of processing involves metaphor. As fundamental to language is the ability to abstract the similar concept from many examples, these functions are tested through metaphor. Metaphor tests the range of one’s ability to depend upon definition.

Locating the senses systems used to make either an assertion or denial is a requirement of the indexing system of the mind.

**Unit Sentences.**

Combining the naming conventions with assertion and denial, provide us with unit sentences.

Subject is or is not Subject.

Subject is or is not Predicates.

Predicate equals Predicate.

Thus not every sentence has both subject and predicates. Notice that in these statements the use of both singular and plural for the *Predicate Naming Convention*.

**Parsing.**

Parsing a statement means to break that statement down to all of its expressed assertions and denials.

As every thing is crafted by bringing together material and form, we craft knowledge of things through combinations of a things predicates. The content of predicates come from direct perception; in regard to metaphor, one must not forget that the mind is a environmental acquisition system in its own right and therefore its predicates are, more often than not, intelligibles. We parse a statement in order to proof it, at least to the limit of our ability. One should never expect that the analog domain is equivalent in individuals concerning any statement.

**Proofing.**

Proofing involves the products of parsing for determining both the authors compliance with the original naming convention and testing one's facility in using the indexing system. An assertion or denial is true if it complies with the original naming convention, false if it is not. Proofing does not exclude indexing based on the principles of the naming convention at all, which means that proofing also tests one's ability to following the indexing system through what is called metaphor.

It is not language when the original naming convention is not complied with. It is not comprehensible to a reader when they cannot follow the indexing systems in play. In other words, every proof actually proves the author of the words for that authors linguistic integrity while simultaneously proves the linguistic skill of the reader.

Currently what is called popular education leaves the student product with a false sense security by believing that they understand even common grammar.

Often one does not have the analog references to effect a proof, in this case, the parsing products indicate which analog information one has to acquire. This limitation is why proofing is part and parcel of formal presentation which provide both branches of reasoning in parallel.

As all we can do is assign names to the elements via abstraction, all that proof can do is check that the naming convention has, indeed, been complied with. In order to parse correctly, one has to have a level of ability in using the indexing system itself along with sufficient analog memory required by the naming convention to perform that task.

**Maintenance and Preservation of the Naming Convention.**

Again, commensurate with the paradigm of a thing, there are two methods of maintaining and preserving the naming convention.

**Form: Definition:**

Definition is equating the name of a thing to the names of that things materials and the names of the forms which contain those materials. As the mind is aimed at standards of behavior, a standard definition is then the preservation of the naming convention through preserving the assertions that equate the *Subject Naming Convention* to the *Predicate Naming Convention* in regard to the names of any perceptible and of any intelligible thing.

As mankind is currently proto-linguistic, this process is left to writers of dictionaries who still do not know the foundation of language, nor the difference between a definition and a description. Writers of dictionaries also think it is an improvement in logic that names "grow" by changing their indexing references. Anyone in a right frame of mind does not think it is a growth of language to scramble and obfuscate the indexes.

**Material: Description:**

Descriptions are directions for locating a thing from which an abstraction may be made. Locating a thing can even entail directions for constructing that thing. Descriptions also involve direct experience being pointed out by someone else. By definition the acquisition of analog memory is required for linguistic functionality.

Throughout history many have confused a description with a definition, and thought they were arguing over definition, when in fact they were arguing over descriptions. The only thing provable by this process is that these persons could not even attain to the first principle of language itself, a convention of names.

**The Perceptible and the Intelligible.**

The perceptible are things directly observable within the environment. The intelligible are things constructed within the mind on the same paradigm of any thing. Intelligibles are constructed by a complex combination of perceptibles. Given the time and effort, every intelligible can be parsed back to primary perceptions; thus, even intelligibles have for their origin, perceptible reality. These intelligibles can be proofed by a linguistic process demonstrated, for example, in the works of Euclid such as the *Element*s.

What I have outlined here are basic elements for logic in general. This outline also exhibits a fact in regard to mankind's current place in the process of becoming functional as there has yet to be written a correct book of elementary grammar.

Parsing demonstrates, in logic, that there is only one error possible, that of assertion and denial in reference to the original naming convention upon which a logic resides.

All of logic resolves to yea and nay, in reference only to the original naming convention itself. There never has been anything in logic which would suggest that as we start a logic by naming abstractions, that the results could be anything more than maintaining that original process. In other words, maintaining a simple human behavior. If one cannot do the simple, one certainly cannot do the complex. As names neither adds to, nor subtracts from, that which is named, proofing only insures that we, the user of the indexing system have kept our word. If we cannot give and keep our word, how is it that one can claim that we are linguistic?

**Analogic.**

Analogics are based on material difference. What is added to make something is simply the application of form. Thus, human industry, human will, is the product of analogic.

It is on this wise that the complex customs developed by Confucius was aimed at linguistic functionality. The tie between language and human behavior has always been the goal of a true philosopher, a true prophet, and even true philosophy and true religion—all of them true craftsmen.

> "Tsze-lu said, "The ruler of Wei has been waiting for you, in order with you to administer the government. What will you consider the first thing to be done?"
>
> The Master replied, "What is necessary is to rectify names."
>
> "So! indeed!" said Tsze-lu. "You are wide of the mark! Why must there be such a rectification?"
>
> The Master said, "How uncultivated you are, Yu! A superior man, in regard to what he does not know, shows a cautious reserve.
>
> "If names be not correct, language is not in accordance with the truth of things. If language be not in accordance with the truth of things, affairs cannot be carried on to success.
>
> "When affairs cannot be carried on to success, proprieties and music do not flourish. When proprieties and music do not flourish, punishments will not be properly awarded. When punishments are not properly awarded, the people do not know how to move hand or foot.
>
> "Therefore, a superior man considers it necessary that the names he uses may be spoken appropriately, and also that what he speaks may be carried out appropriately. What the superior man requires is just that, in his words there may be nothing incorrect." *Analects* by Confucius

In regard to analogic as a written language, we have what is called Geometry. Along with the logic of Algebra, one has a formal pairing used in elementary teaching of dialectic. As all that one can do is apply form to material in an analog language, the word geometry can be used metaphorically

for standards of all behaviors of man. The difference in analog language is that concepts are presented as part of the language itself. As the mind of most cannot comply with even the principles of logic, analog presents a greater challenge. One can write an equation analogically that simply will not register at all to a simple mind, they cannot abstract the concepts of what they are perceiving. This is why mankind has not even suspected that lucid dreaming is an analog language.

It is a very naive and simple mind that imagines that one can have many contradicting geometries as a written language, as if form, which is not a difference, is a difference. This only denotes how dysfunctional the mind of man is.

Some of the greatest mistakes in the analogic of geometry is confusing the process of writing the language with the language itself and confusing the writing materials with the grammar itself. The foundation for all languages is simply form and material, and, for linguistic purposes, standards in producing them in the language. Those who formulate non-Euclidean Geometries have been, and are too simple to even notice that at every turn, they violated the principles of language itself. The known parallel between the logic of mathematics and the analog figures in geometry was insufficient for them to realize that if one imagines the corruption of geometry, then because of the parallel, they eradicated any recourse to mathematics. One can, however, congratulate them on their consistency in not seeing contradiction after contradiction. A proto-linguistic mind cannot function in accordance with the principles of language.

## A Duplicate Ratio 062092

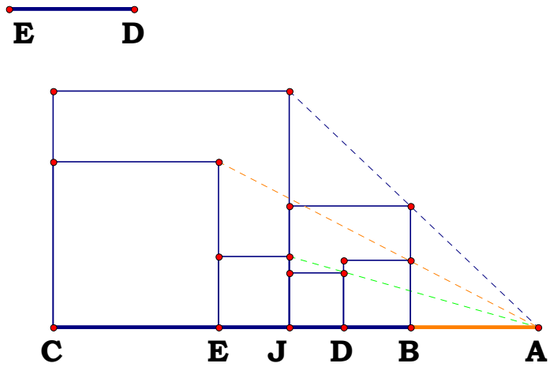
Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

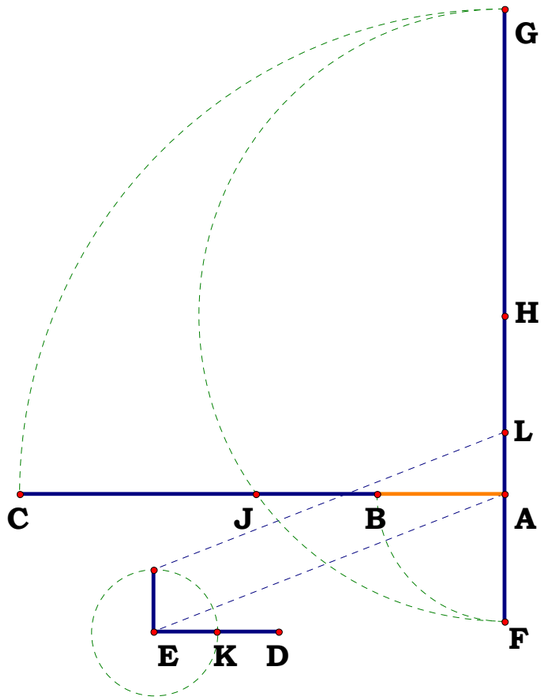
$AB := 1$

$N_1 := 5.99017$

$N_2 := 2.09550$

$BC := N_1 \quad DE := N_2$

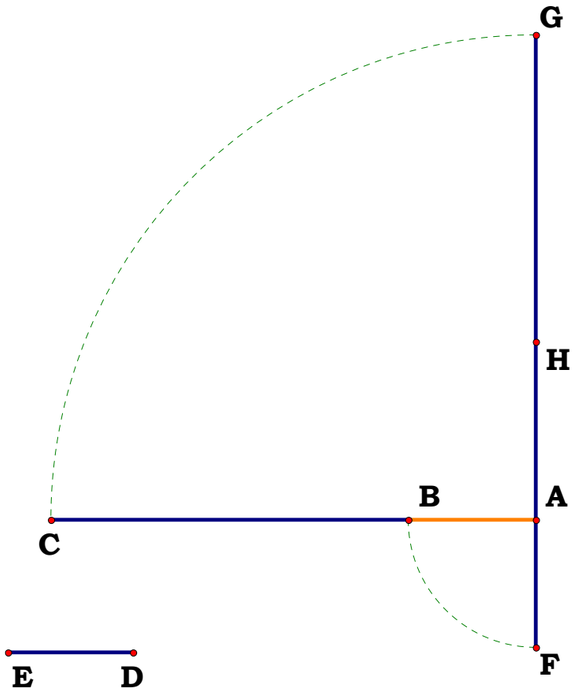



$AC := AB + BC$

$AF := AB \quad AG := AC$

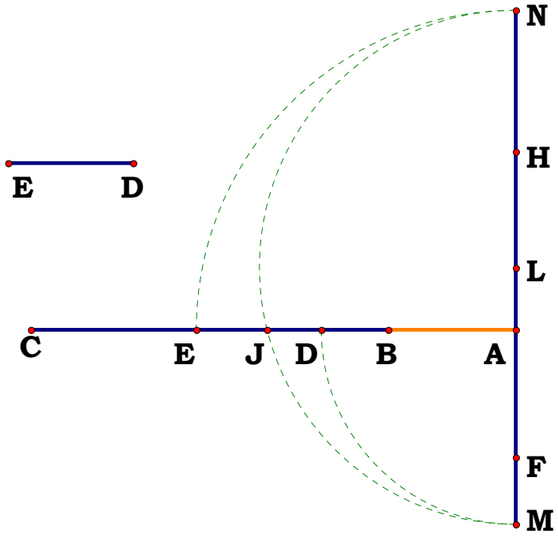
$AJ := \sqrt{AF \cdot AG}$

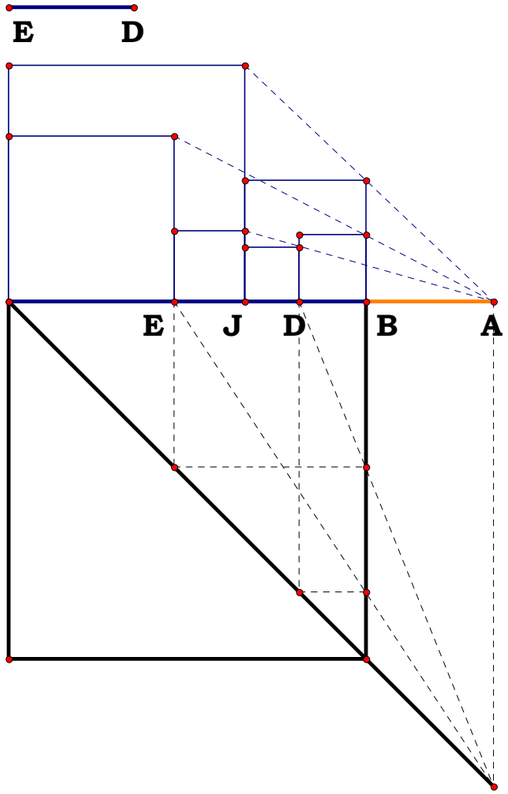



$AL := \frac{DE}{2} \qquad FL := AF + AL \qquad JL := \sqrt{AJ^2 + AL^2}$

$AD := JL - AL \qquad AE := JL + AL$

$\frac{AB}{AD} - \frac{AE}{AC} = 0 \qquad \frac{AD}{AJ} - \frac{AJ}{AE} = 0 \qquad \frac{AB}{AJ} - \frac{AJ}{AC} = 0$






**Definitions**

$$AC - \left(N_1 + 1\right) = 0$$

$$AF - 1 = 0 \qquad AG - \left(N_1 + 1\right) = 0 \qquad AJ - \sqrt{\left(N_1 + 1\right)} = 0$$

$$AL - \frac{N_2}{2} = 0 \qquad FL - \frac{N_2 + 2}{2} = 0 \qquad JL - \sqrt{\frac{N_2^{\,2}}{4} + N_1 + 1} = 0$$

$$AD - \frac{\sqrt{N_2^{\,2} + 4 \cdot N_1 + 4} - N_2}{2} = 0 \qquad AE - \frac{\sqrt{N_2^{\,2} + 4 \cdot N_1 + 4} + N_2}{2} = 0$$

$$-\frac{2}{N_2 - \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}} - \frac{N_2 + \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}}{2 \cdot \left(N_1 + 1\right)} = 0 \qquad \frac{AB}{AD} - \frac{AE}{AC} = 0$$

$$-\frac{N_2 - \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}}{2 \cdot \sqrt{N_1 + 1}} - \frac{2 \cdot \sqrt{N_1 + 1}}{N_2 + \sqrt{N_2^{\,2} + 4 \cdot N_1 + 4}} = 0 \qquad \frac{AD}{AJ} - \frac{AJ}{AE} = 0$$

$$\frac{1}{\sqrt{\left(N_1 + 1\right)}} - \left(N_1 + 1\right)^{\frac{-1}{2}} = 0 \qquad \frac{AB}{AJ} - \frac{AJ}{AC} = 0$$

$$\sqrt{N_1 + 1} - AJ = 0$$

## 081292 Rusty Cube of a Sphere

**Given AB, how close is BJ to the cube root of AB taken as a sphere?**

$$N_1 := 4 \qquad CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}$$

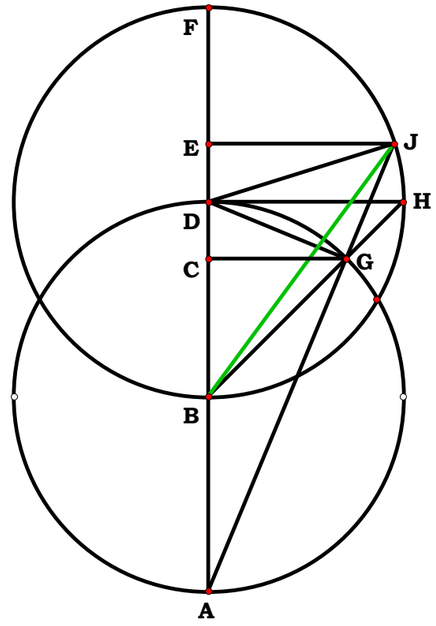
$$AB := N_1 \qquad BH := \sqrt{2\cdot AB^2} \qquad CG := \frac{AB^2}{BH}$$

$$AG := \sqrt{CG^2 + (AB + CG)^2} \qquad DG := CG\cdot\frac{2AB}{AG}$$

$$GJ := \sqrt{AB^2 - DG^2} \qquad AE := \frac{(AB + CG)\cdot(AG + GJ)}{AG}$$

$$EJ := \frac{CG\cdot AE}{AB + CG} \qquad BJ := \sqrt{EJ^2 + (AE - AB)^2}$$

$$\frac{BJ}{\left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}} = 1.000943 \qquad BJ - \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}} = 0.00608$$

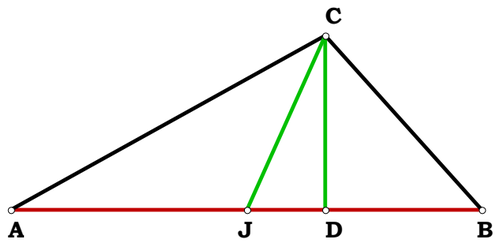



### Definition

$$BJ - N_1\cdot\sqrt{\sqrt{2} + 2^{\frac{1}{4}}} = 0$$

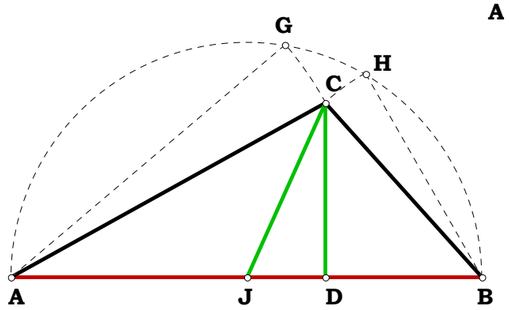
**Pythagoras Revisited** 010893

**Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.**

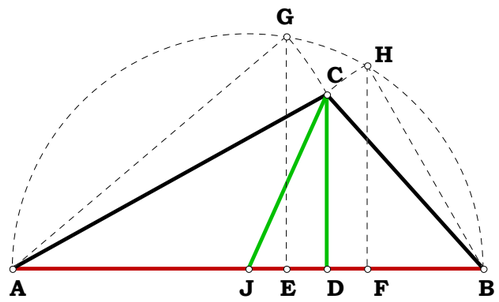



$AB := 7.89517$ $AC := 6.02581$ $BC := 3.92697$

$S_1 := AB$ $S_2 := AC$ $S_3 := BC$



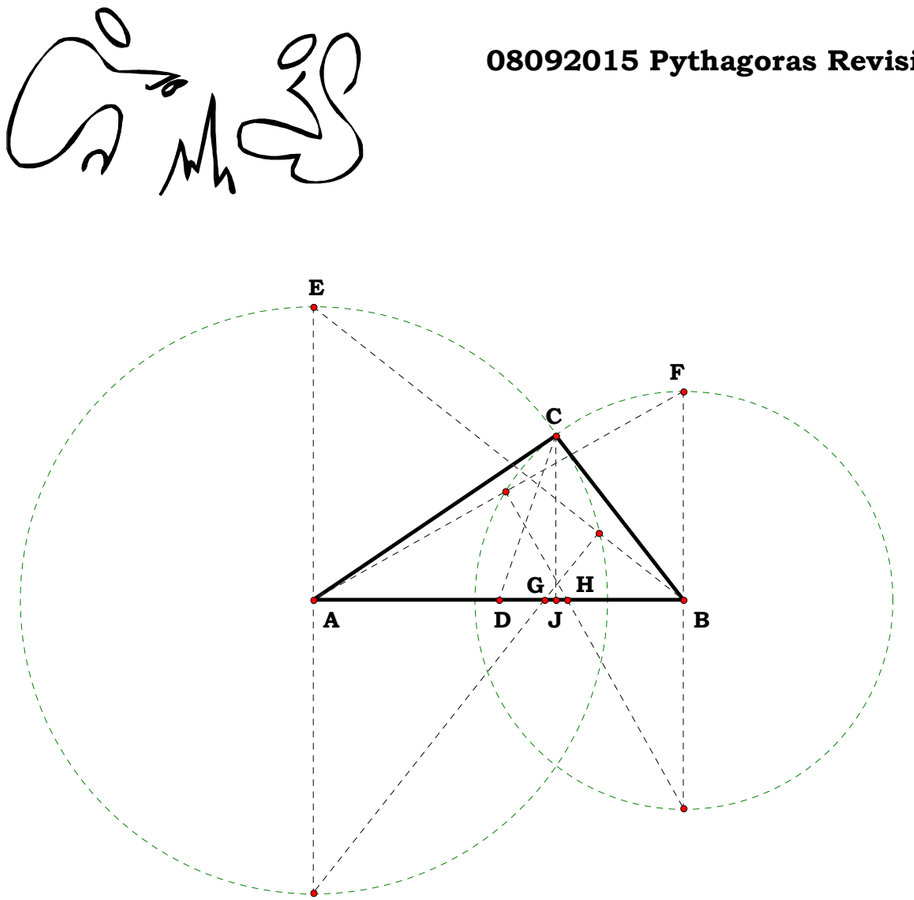

$$AE := \frac{S_2^2}{S_1} \qquad BF := \frac{S_3^2}{S_1} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2} \qquad AD := AE + DE$$




$$BD := AB - AD \qquad CD := \sqrt{S_2^2 - AD^2} \qquad AJ := \frac{S_1}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$$

$$EF - \frac{S_1^2 - S_2^2 - S_3^2}{S_1} = 0 \qquad DE - \frac{S_1^2 - S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad AD - \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^2 - S_2^2 + S_3^2}{2 \cdot S_1} = 0$$

$$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0 \qquad DJ - \frac{\sqrt{\left(S_2^2 - S_3^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^2 - S_1^2 + 2 \cdot S_3^2}}{2} = 0$$



**08092015 Pythagoras Revisited Again!**

**One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.**




$AB := 6.20183 \quad AC := 4.89358 \quad BC := 9.20468$

$$GA := \frac{AC^2}{AB} \qquad HB := \frac{BC^2}{AB} \qquad GH := AB - (GA + HB) \qquad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \qquad CJ := \sqrt{AC^2 - JA^2} \qquad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$

$$JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

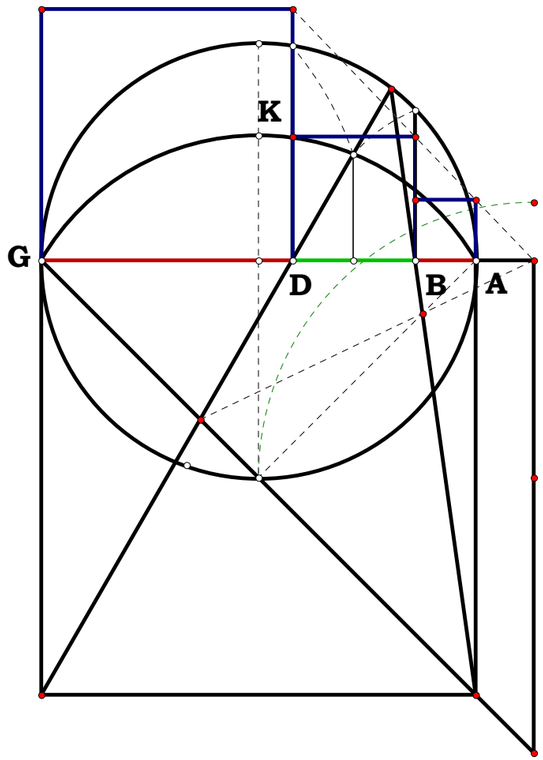
**Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.**

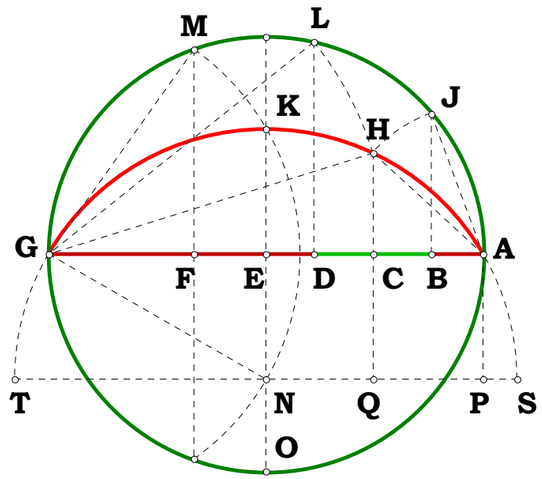
$$AJ := \sqrt{AC^2 - CJ^2} \qquad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \qquad BJ := \sqrt{BC^2 - CJ^2} \qquad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

## 060393 Exploring The Curve AK

The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?
Divide a segment twice such that the mean segment is the root of the extreems.




Givens: $N_1 := 3$ $N_2 := 4$ $AG := N_1$ $AC := \frac{AG}{N_2}$

$GF := \frac{AG}{3}$ $FM := \sqrt{GF \cdot (AG - GF)}$

$GM := \sqrt{GF^2 + FM^2}$ $ST := 2 \cdot GM$ $EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$

$PS := \frac{ST - AG}{2}$ $HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN$ $AH := \sqrt{AC^2 + CH^2}$ $GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \frac{AH^2}{AG}$ $DG := \frac{GH^2}{AG}$ $BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0$ $AB = 0.349$ $BD = 0.803$ $DG = 1.849$

Definitions:

$AC - \frac{N_1}{N_2} = 0$ $GF - \frac{N_1}{3} = 0$ $FM - \frac{\sqrt{2} \cdot N_1}{3} = 0$ $GM - \frac{\sqrt{3} \cdot N_1}{3} = 0$ $ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0$ $EN - \frac{N_1}{\sqrt{12}} = 0$

$PS - \frac{N_1 \cdot \left(2 \cdot \sqrt{3} - 3\right)}{6} = 0$ $HQ - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12}}{N_2 \cdot \sqrt{12}} = 0$ $CH - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0$ $AH - \frac{N_1 \cdot \sqrt{N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6}}{\sqrt{6 \cdot N_2}} = 0$

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0$ $AB - \frac{N_1 \cdot \left(N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6\right)}{6 \cdot N_2} = 0$ $DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{6 \cdot N_2} = 0$

$BD - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0$ $BD^2 - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$ $AB \cdot DG - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$

AG = 2.86667 in.
AC = 0.74179 in.
GF = 0.95556 in.
FM = 1.35136 in.
GM = 1.65507 in.
ST = 3.31014 in.
EN = 0.82754 in.
PS = 0.22174 in.
HQ = 1.50367 in.
CH = 0.67614 in.
AH = 1.00370 in.
GH = 2.22986 in.
AB = 0.35142 in.
BD = 0.78074 in.
DG = 1.73451 in.

$\frac{AG}{AC} = 3.86452$

$N_1 = 2.86667$ in.

$N_2 = 3.86452$

$\frac{AG}{N_2} = 0.74179$ in.

$AC - \frac{N_1}{N_2} = 0.00000$ in.

$GF - \frac{N_1}{3} = 0.00000$ in.

$FM - \frac{\sqrt{2} \cdot N_1}{3} = 0.00000$ in.

$GM - \frac{\sqrt{3} \cdot N_1}{3} = 0.00000$ in.

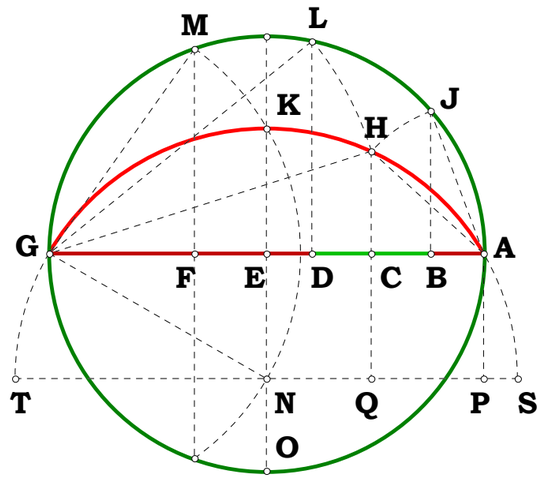
$ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0.00000$ in.

$EN - \frac{N_1}{\sqrt{12}} = 0.00000$ in.

$PS - \frac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0.00000$ in.

$HQ - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12}}{N_2 \cdot \sqrt{12}} = 0.00000$ in.

$CH - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0.00000$ in.




$AH - \frac{N_1 \cdot \sqrt{\left(N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12}\right) + 6}}{\sqrt{6 \cdot N_2}} = 0.00000$ in.

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0.00000$ in.

$AB - \frac{N_1 \cdot \left(\left(N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12}\right) + 6\right)}{6 \cdot N_2} = 0.00000$ in.

$DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - 6\right)}{6 \cdot N_2} = 0.00000$ in.

$BD - \frac{N_1 \cdot \sqrt{(N_2^2 + 12 \cdot N_2) - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0.00000$ in.

## 060793 Two Triangles with a Common Side.

Given two triangles with a common side, find the difference between their free vertices when the triangles do not intersect and when they do intersect.

Let the two triangles ABD and ACD be given.

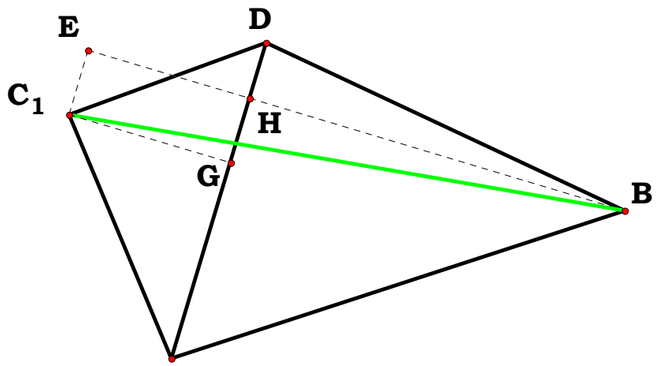
$AD := 2.17506 \quad AB := 3.14654 \quad AC := 1.74732$

$BD := 2.61333 \quad CD := 1.38168$

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD + AB + BD)\cdot(-AD + AB + BD)\cdot(AD - AB + BD)(AD + AB - BD)}}{2\cdot AD}$$

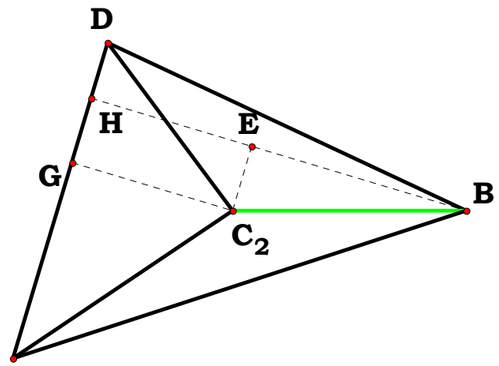
Non-intersecting:




$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Intersecting:




$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + AB^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}\ \ldots}{\sqrt{+\ -AD^4 - AB^2\cdot AC^2 + AB^2\cdot AD^2 + AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2 + AB^2\cdot AD^2 - AD^4 - AB^2\cdot AC^2\ \ldots}}{\sqrt{+\ -\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}}}{2\cdot AD} = 0$$

## 060993 Rectangular Roots.

Given any value $N_1$, any other value, $N_2$, greater than twice the square root of $N_1$ can be divided such that the resulting pair of values equals $N_1$.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

$AB := 11$

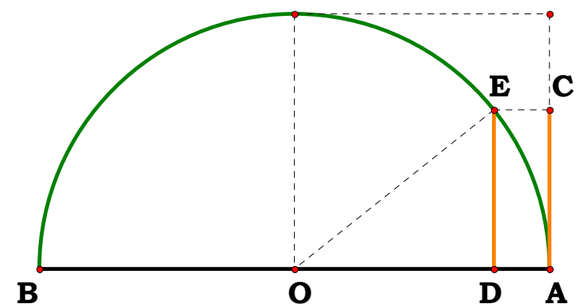
$AC := 5$

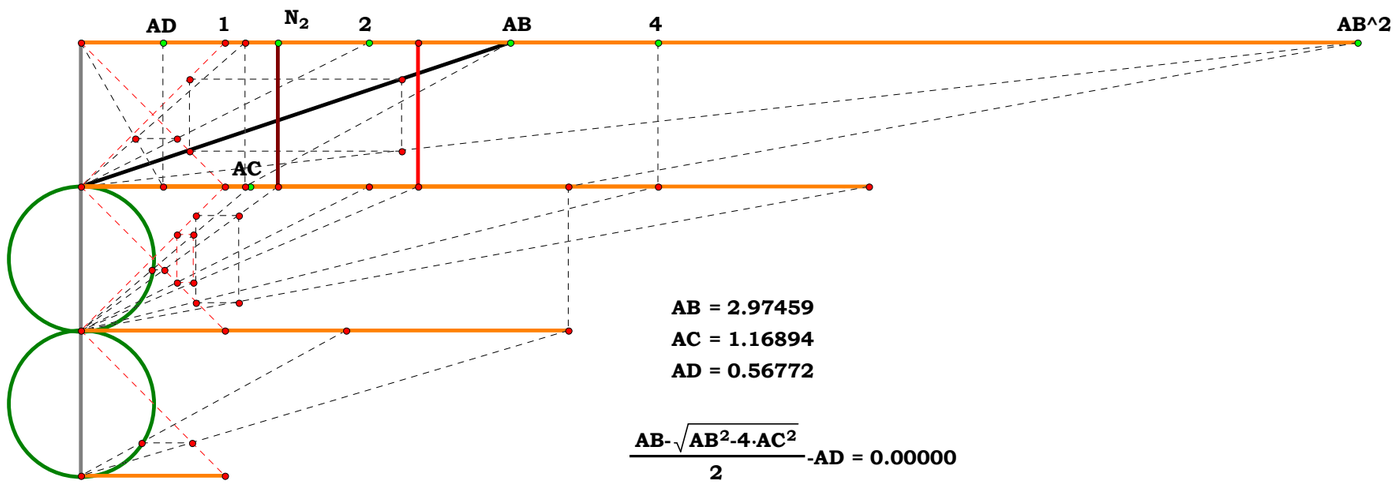
$DE := AC \quad EO := \frac{AB}{2} \quad DO := \sqrt{EO^2 - DE^2} \quad BD := EO + DO$

$AD := AB - BD \quad AD \cdot BD - AC^2 = 0$

$AD - \frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \quad AD = 3.209$

$BD - \frac{AB + \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \quad BD = 7.791 \quad AD + BD = 11$




$AB = 2.97459$

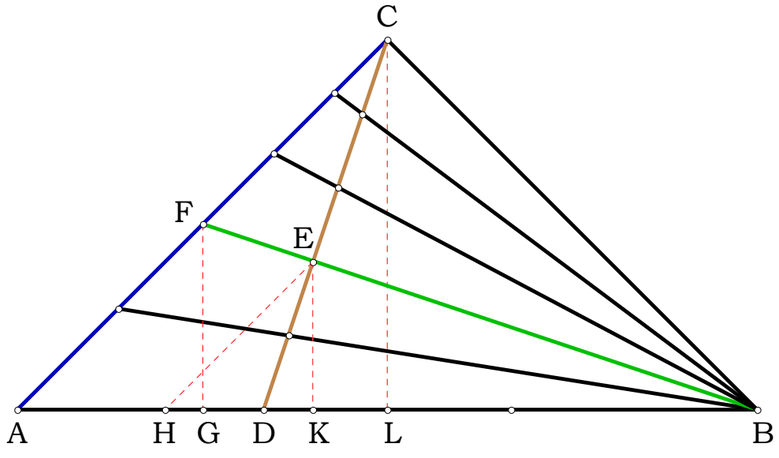
$AC = 1.16894$

$AD = 0.56772$

$\frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} - AD = 0.00000$

## 930621 Pyramid of Ratios I

Divide AB by $N_1$ then divide CD by $N_2$, what are BF/EF and AC/AF?




$$AB := 4.47022 \quad N_1 := 3.82131 \quad N_2 := 2.62629 \quad AD := \frac{AB}{N_1} \quad AL := \frac{AB}{2}$$

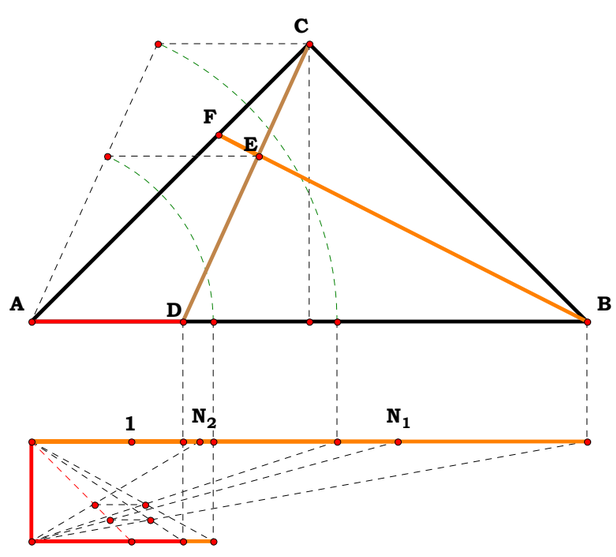
$$AC := \sqrt{2 \cdot AL^2} \quad CL := AL \quad DL := AL - AD \quad CD := \sqrt{DL^2 + CL^2}$$

$$DE := \frac{CD}{N_2} \quad EK := \frac{CL \cdot DE}{CD} \quad DK := \frac{DL \cdot EK}{CL} \quad AK := AD + DK$$

$$BK := AB - AK \quad BE := \sqrt{BK^2 + EK^2} \quad HK := \frac{AL \cdot EK}{CL} \quad BH := BK + HK$$

$$EH := \frac{AC \cdot EK}{CL} \quad AF := \frac{EH \cdot AB}{BH} \quad BF := \frac{BE \cdot AB}{BH} \quad EF := BF - BE$$

$$\frac{BF}{EF} = 6.171 \quad \frac{AC}{AF} = 2.201 \quad \frac{AC}{AF} - \frac{N_1 \cdot N_2 - N_2 + 1}{N_1} = 0 \quad \frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0$$




$$\frac{BF}{EF} = 9.08799 \quad \frac{N_1 \cdot N_2}{N_2 - 1} = 9.08799$$

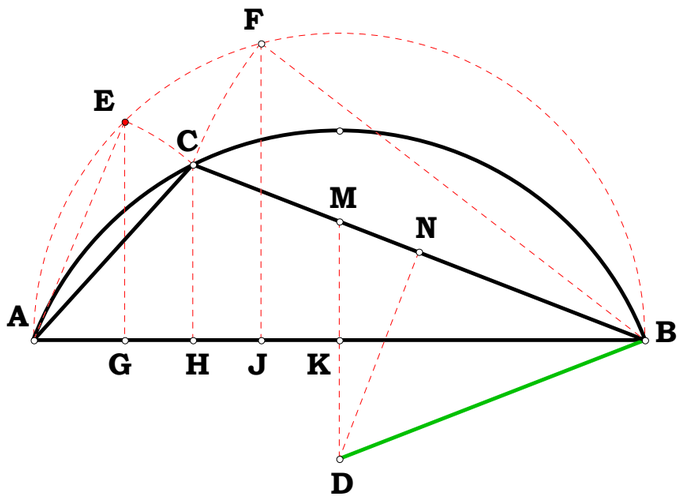
$$\frac{AC}{AF} = 1.49057 \quad \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.49057$$

$$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0.00000 \quad \frac{AC}{AF} - \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 0.00000$$

## 062793 Describe A Circle About a Triangle

**Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.**

$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB)$ $NOT(X) := X =$ $\delta := 0 .. 2$ $AB \equiv 3$ $AC \equiv 4$ $BC \equiv 6$




$BK := \frac{AB}{2}$ $AE := AC$ $BF := BC$ $AG := \frac{AE^2}{AB}$ $BJ := \frac{BF^2}{AB}$ $GJ := AB - (AG + BJ)$

$HJ := \frac{GJ}{2}$ $BH := BJ + HJ$ $CH := \sqrt{BC^2 - BH^2}$ $BN := \frac{BC}{2}$ $BM := \frac{BC \cdot BK}{BH}$ $MN := BM - BN$

$DN := \frac{BH \cdot MN}{CH}$ $BD := \sqrt{BN^2 + DN^2}$

$$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix} \quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix}$$

$$R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$$

$radius := if(\Delta, BD, 0)$ $imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375$

$imaginary\_radius = 0$

$\Delta = 1$

**The construction is independent of the side one starts with.**

**The name of the Radius in terms of the givens.**

$R^T = (3.375 \quad 3.375 \quad 3.375)$

The equation is a statement in regard to the relationship between each side of a triangle.

$\frac{XO}{01} = 1.35962$ $\frac{WO}{01} = 1.35962$ $\frac{ZO}{01} = 1.35962$

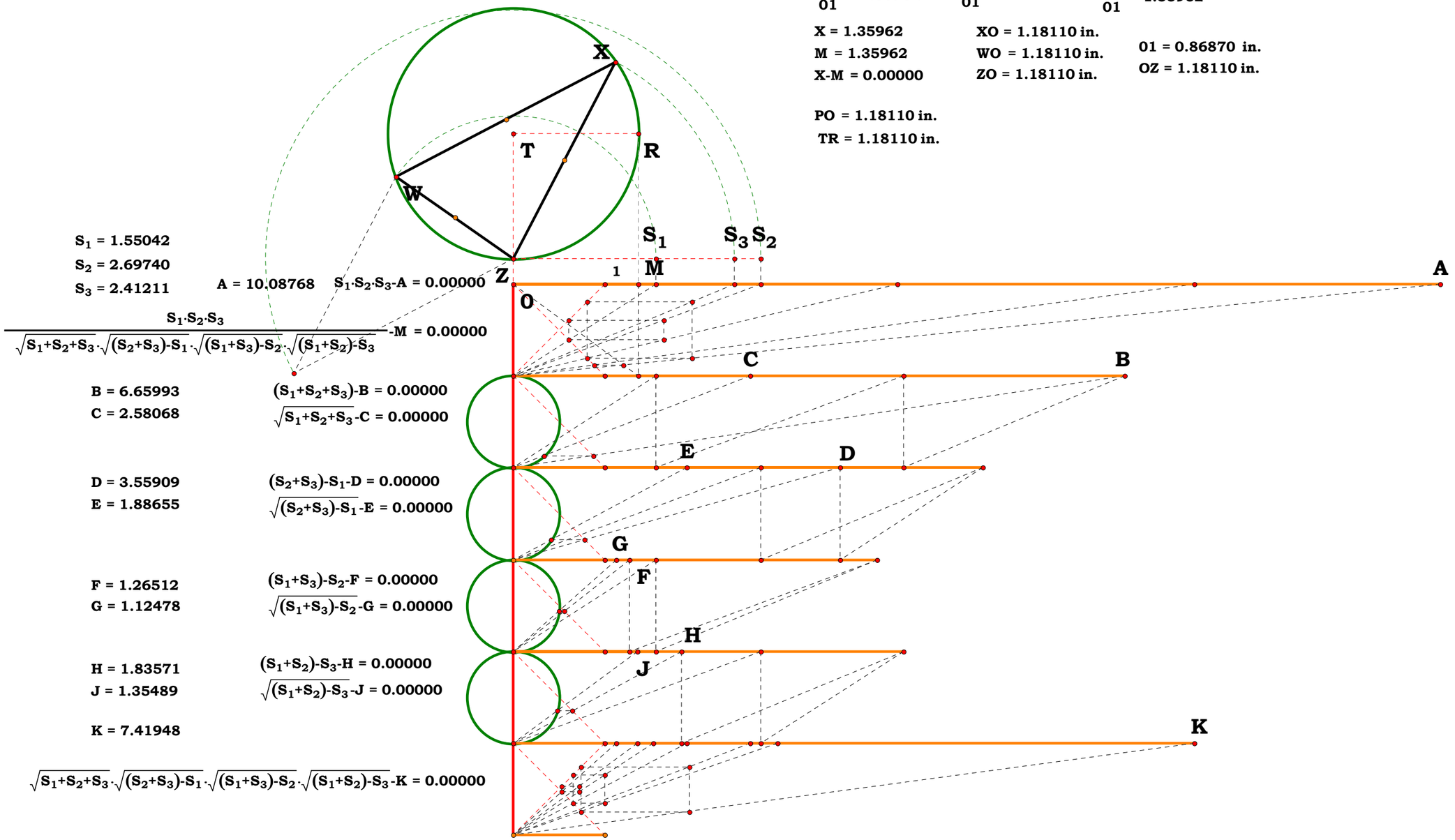
X = 1.35962 XO = 1.18110 in. 01 = 0.86870 in.

M = 1.35962 WO = 1.18110 in. OZ = 1.18110 in.

X-M = 0.00000 ZO = 1.18110 in.

PO = 1.18110 in.

TR = 1.18110 in.




$S_1 = 1.55042$

$S_2 = 2.69740$

$S_3 = 2.41211$

A = 10.08768 $S_1 \cdot S_2 \cdot S_3 - A = 0.00000$

$\frac{S_1 \cdot S_2 \cdot S_3}{\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3}} - M = 0.00000$

B = 6.65993 $(S_1+S_2+S_3) - B = 0.00000$

C = 2.58068 $\sqrt{S_1+S_2+S_3} - C = 0.00000$

D = 3.55909 $(S_2+S_3) - S_1 - D = 0.00000$

E = 1.88655 $\sqrt{(S_2+S_3)-S_1} - E = 0.00000$

F = 1.26512 $(S_1+S_3) - S_2 - F = 0.00000$

G = 1.12478 $\sqrt{(S_1+S_3)-S_2} - G = 0.00000$

H = 1.83571 $(S_1+S_2) - S_3 - H = 0.00000$

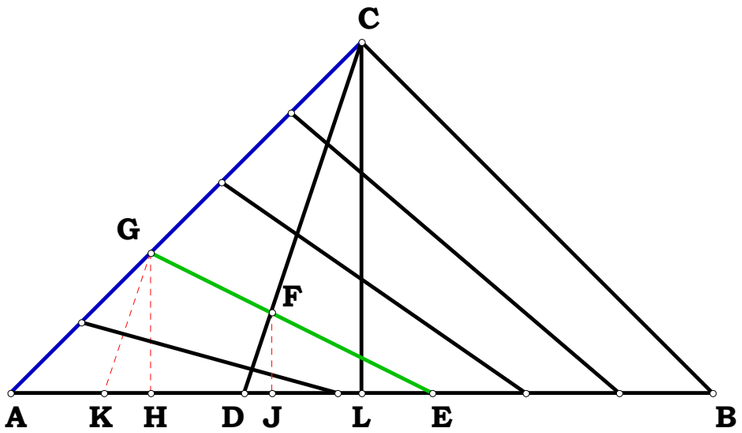
J = 1.35489 $\sqrt{(S_1+S_2)-S_3} - J = 0.00000$

K = 7.41948

$\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3} - K = 0.00000$

## 071593 Pyramid of Ratios II

**AB is divided by $N_1$ and AC and BD is divided by $N_2$, what are EG/FG and CD/DF?**




$$AB := 1 \quad N_1 := 3 \quad N_2 := 5 \quad AD := \frac{AB}{N_1} \quad AC := \sqrt{\frac{AB^2}{2}} \quad BD := AB - AD \quad AL := \frac{AB}{2}$$

$$DE := \frac{BD}{N_2} \quad AG := \frac{AC}{N_2} \quad AE := AD + DE \quad AH := \sqrt{\frac{AG^2}{2}} \quad GH := AH \quad EH := AE - AH$$

$$EG := \sqrt{EH^2 + GH^2} \quad DL := AL - AD \quad CL := AL \quad HK := \frac{DL \cdot AH}{AL} \quad EK := EH + HK$$

$$DJ := \frac{HK \cdot DE}{EK} \quad EF := \frac{EG \cdot DE}{EK} \quad FG := EG - EF \quad FJ := \frac{GH \cdot DE}{EK}$$

$$CD := \sqrt{CL^2 + DL^2} \quad DF := \sqrt{DJ^2 + FJ^2}$$

$$\frac{EG}{FG} = 1.5 \qquad \frac{CD}{DF} = 15 \qquad \frac{N_2 + N_1 - 2}{N_2 - 1} = 1.5 \qquad \frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} = 15$$

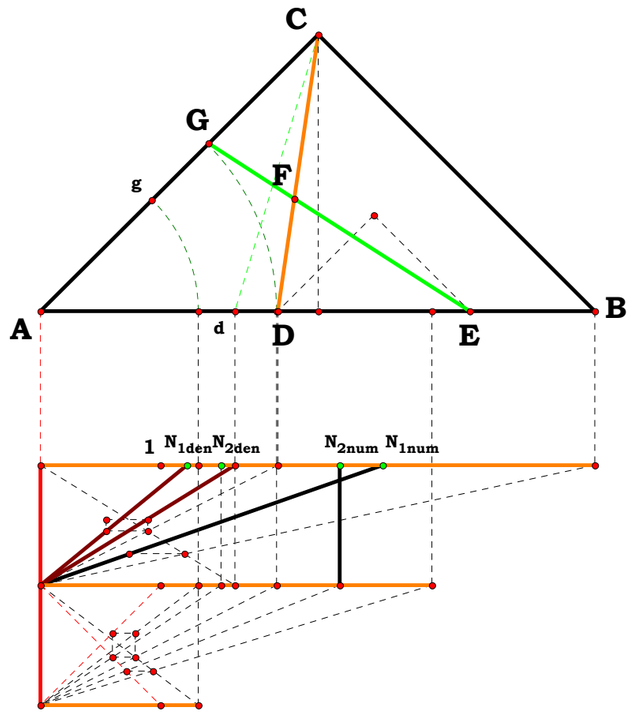
$$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0 \qquad \frac{CD}{DF} - \frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}$$

$N_1 = 2.32900$

$N_2 = 1.65606$

$\frac{N_{1\ num}}{N_{1\ den}} = 2.32900$

$\frac{N_{2\ num}}{N_{2\ dem}} = 1.65606$




$\frac{EG}{FG} = 3.02573$ $\quad \frac{(N_2+N_1)-2}{N_2-1} = 3.02573$ $\quad \frac{EG}{FG}-\frac{(N_2+N_1)-2}{N_2-1} = 0.00000$

$\frac{CD}{DF} = 2.47357$ $\quad \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 2.47357$ $\quad \frac{CD}{DF}-\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 0.00000$

## 072593 Pyramid of Ratios III

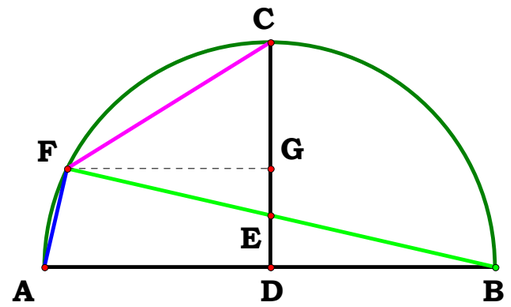
**Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?**

$AB := 1 \qquad N := 5 \qquad CD := \frac{AB}{2} \qquad DE := \frac{CD}{N}$

$BE := \sqrt{CD^2 + DE^2} \qquad BF := \frac{CD \cdot AB}{BE} \qquad AF := \frac{DE \cdot BF}{CD}$

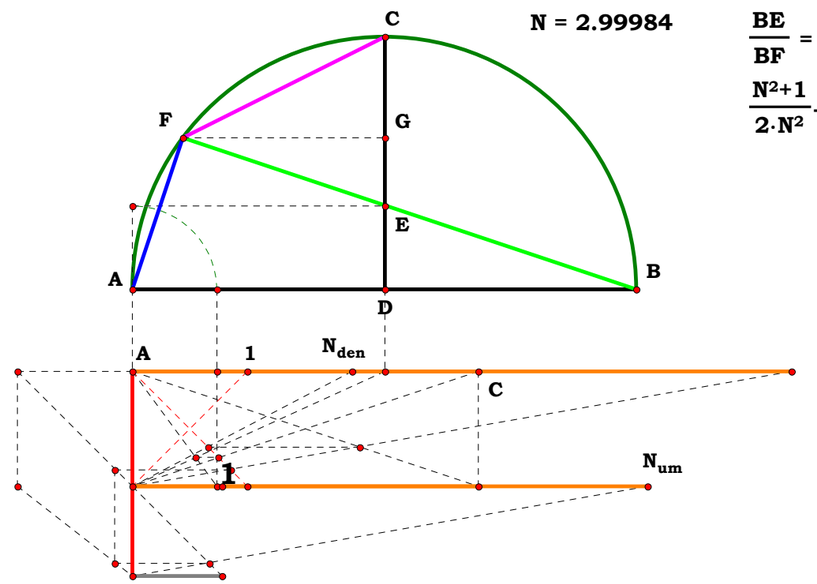
$DG := \frac{DE \cdot BF}{BE} \qquad CG := CD - DG \qquad FG := \frac{CD \cdot (DG - DE)}{DE}$

$CF := \sqrt{FG^2 + CG^2}$




$\frac{BE}{BF} = 0.52 \qquad \frac{N^2 + 1}{2 \cdot N^2} = 0.52 \qquad \frac{AF}{CF} = 0.354 \qquad \frac{\sqrt{2}}{N - 1} = 0.354$

$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \qquad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$




$\frac{N_{um}}{N_{den}} = 2.99984$

$N = 2.99984$

$BE = 1.75445$ in.

$BF = 3.15797$ in.

$\frac{BE}{BF} = 0.55556$

$\frac{N^2+1}{2 \cdot N^2} - \frac{BE}{BF} = 0.00000$

$AF = 1.05271$ in.

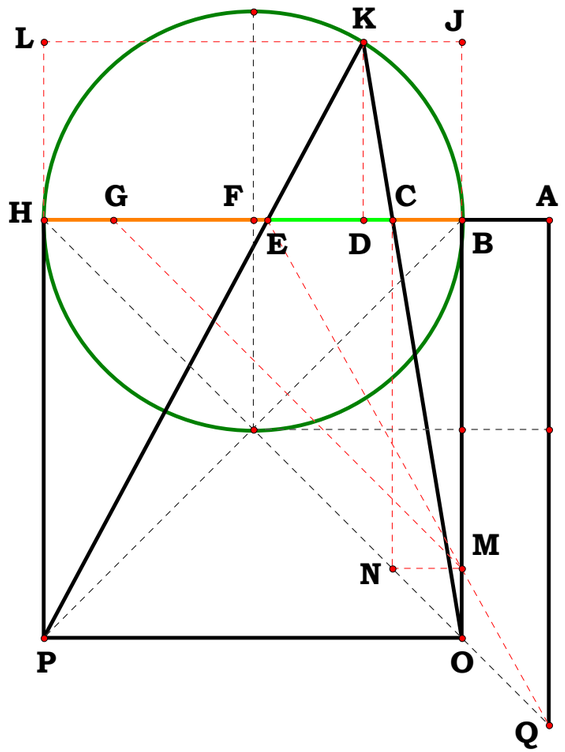
$CF = 1.48864$ in.

$\frac{AF}{CF} = 0.70716$

$\frac{\sqrt{2}}{N-1} - \frac{AF}{CF} = 0.00000$

## 110693 Gruntwork I on the Delian Solution

**Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/3} = AE$?**




$$N := 5 \quad BH := 3 \quad BF := \frac{BH}{2} \quad BD := \frac{BF}{N} \quad DH := BH - BD$$

$$DK := \sqrt{BD \cdot DH} \quad JO := BH + DK \quad BC := \frac{BD \cdot BH}{JO}$$

$$BG := BH - BC \quad EH := \frac{DH \cdot BH}{JO} \quad BE := BH - EH \quad EG := EH - BC$$

$$GM := \sqrt{2 \cdot BG^2} \quad HO := \sqrt{2 \cdot BH^2} \quad HQ := \frac{GM \cdot EH}{EG} \quad OQ := HQ - HO$$

$$AB := \frac{OQ}{\sqrt{2}} \quad AC := AB + BC \quad AE := AB + BE \quad AH := AB + BH$$

$$1 \qquad\qquad 1$$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$$

$$BF - \frac{1}{2} = 1 \quad BD - \frac{BH}{2 \cdot N} = 0 \quad DH - \frac{BH \cdot (2 \cdot N - 1)}{2 \cdot N} = 0 \quad DK - \frac{BH \cdot \sqrt{2 \cdot N - 1}}{(2 \cdot N)} = 0$$

$$JO - \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)}{2 \cdot N} = 0 \quad BC - \frac{BH}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad BG - \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$EH - \frac{BH \cdot (2 \cdot N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad BE - \frac{BH \cdot \left(\sqrt{2 \cdot N - 1} + 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \quad EG - \frac{2 \cdot BH \cdot (N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$GM - \sqrt{2} \cdot \frac{BH \cdot \left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)} = 0 \quad HO - \sqrt{2} \cdot BH = 0$$

$$HQ - \frac{\sqrt{2}\cdot BH\cdot(2\cdot N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

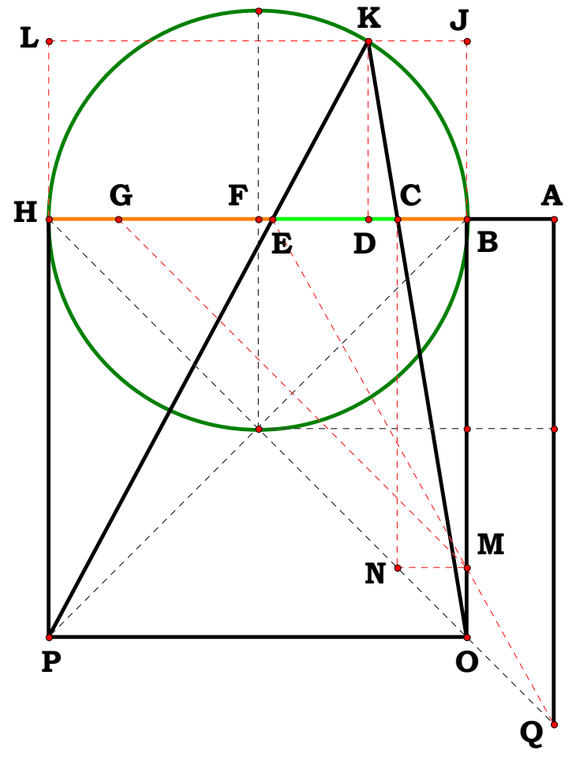
$$OQ - \frac{\sqrt{2}\cdot BH\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{\left(2\cdot N+\sqrt{2\cdot N-1}\right)\cdot(2\cdot N-2)} = 0$$

$$AB - \frac{BH\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AC - \frac{BH\cdot\left[2\cdot N+2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AE - \frac{BH\cdot\left[2\cdot N+(2\cdot N-1)^{\frac{3}{2}}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AH - \frac{BH\cdot\left[(2\cdot N-1)^{\frac{3}{2}}-4\cdot N+4\cdot N^{2}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$




BF = 1.37917 in.

BD = 0.65310 in.

$\frac{BF}{BD}$ = 2.11173

N = 2.11173

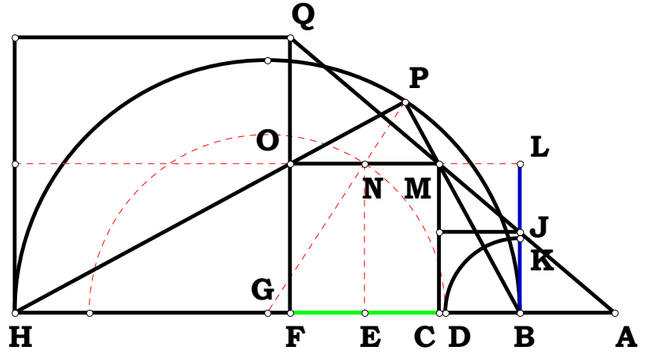
BH = 2.75833 in.

AE = 1.85725 in.

$\frac{BH\cdot\left(\left(2\cdot N+(2\cdot N-1)^{\frac{3}{2}}\right)-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)}$ -AE = 0.00000 in.

## 110993 Solve For Cube Root Placement

**With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.**




$N_1 := 2 \quad N_2 := 4$

$BH := N_1 \quad BG := \frac{BH}{2} \quad CF := \frac{BH}{N_2} \quad BL := CGP := BG$

$BK := \frac{BL}{2} \quad BD := BK \quad NP := BD \quad GN := GP - NP \quad EN := BL$

$GE := \sqrt{GN^2 - EN^2} \quad CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD \quad FH := GH + GE - EF \quad FQ := FH$

$FO := BL \quad OQ := FQ - MO := CF \quad AF := \frac{MO \cdot FQ}{OQ} \quad AC := AF - CF \quad AH := AF + FH \quad AB := AH - BH$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \quad \frac{AH}{AB} = 17.944271909999$

**The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.**

**Algebraic Names**

$$BH - N_1 = 0 \quad BG - \frac{N_1}{2} = 0 \quad CF - \frac{N_1}{N_2} = 0 \quad BK - \frac{N_1}{2 \cdot N_2} = 0 \quad GN - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$$

$$GE - \frac{N_1 \cdot \sqrt{(N_2 + 1) \cdot (N_2 - 3)}}{2 \cdot N_2} = 0 \quad BC - \frac{N_1 \cdot \left[N_2 - \left[\sqrt{(N_2 + 1) \cdot (N_2 - 3)}\right] - 1\right]}{2 \cdot N_2} = 0$$

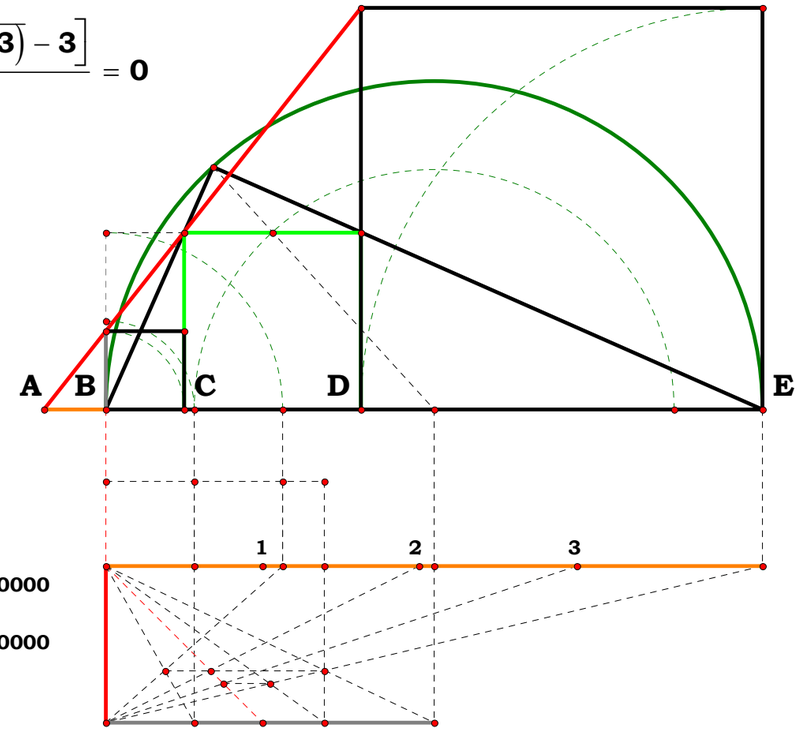
$$FH - \frac{N_1 \cdot \left[N_2 + \left[\sqrt{(N_2 + 1) \cdot (N_2 - 3)}\right] - 1\right]}{2 \cdot N_2} = 0 \quad OQ - \frac{N_1 \cdot \left[N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 3\right]}{2 \cdot N_2} = 0$$

$$AF - \frac{N_1}{N_2} \cdot \frac{N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 1}{N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 3} = 0$$

$$AC - \frac{2N_1}{N_2 \cdot \left[N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 3\right]} = 0$$

$$AH - \frac{N_1 \cdot \left[N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 1\right]^2}{2 \cdot N_2 \cdot \left[N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 3\right]} = 0$$

$$AB - \frac{N_1 \cdot \left[N_2 - \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 1\right]}{N_2 \cdot \left[N_2 + \sqrt{(N_2 + 1) \cdot (N_2 - 3)} - 3\right]} = 0$$




AB = 0.40540 in.
AE = 4.74189 in.
AC = 0.92025 in.
AD = 2.08896 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 11.69681$

# 111093 Gruntwork II on the Delian Solution

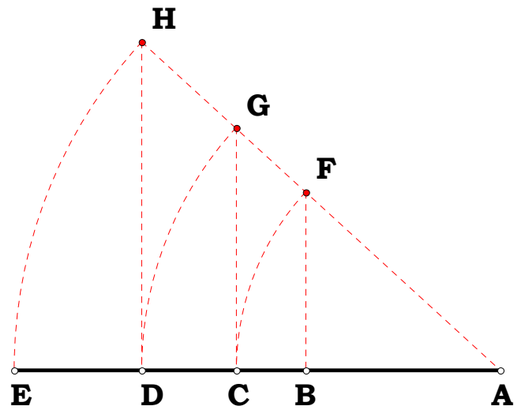
$N := 4 \qquad AE := 1$

$DE := \frac{AE}{N} \qquad AD := AE - DE$

$AH := AE \qquad AG := AD$

$AC := \frac{AD \cdot AD}{AE} \qquad AF := AC$

$AB := \frac{AC \cdot AC}{AD}$




$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2.37 \qquad \frac{AD}{AB} = 1.778 \qquad \frac{AC}{AB} = 1.333$

**Albebraic Names:**

$\frac{1}{N} - DE = 0 \qquad 1 - \frac{1}{N} - AD = 0 \qquad \frac{(N-1)^2}{N^2} - AC = 0$

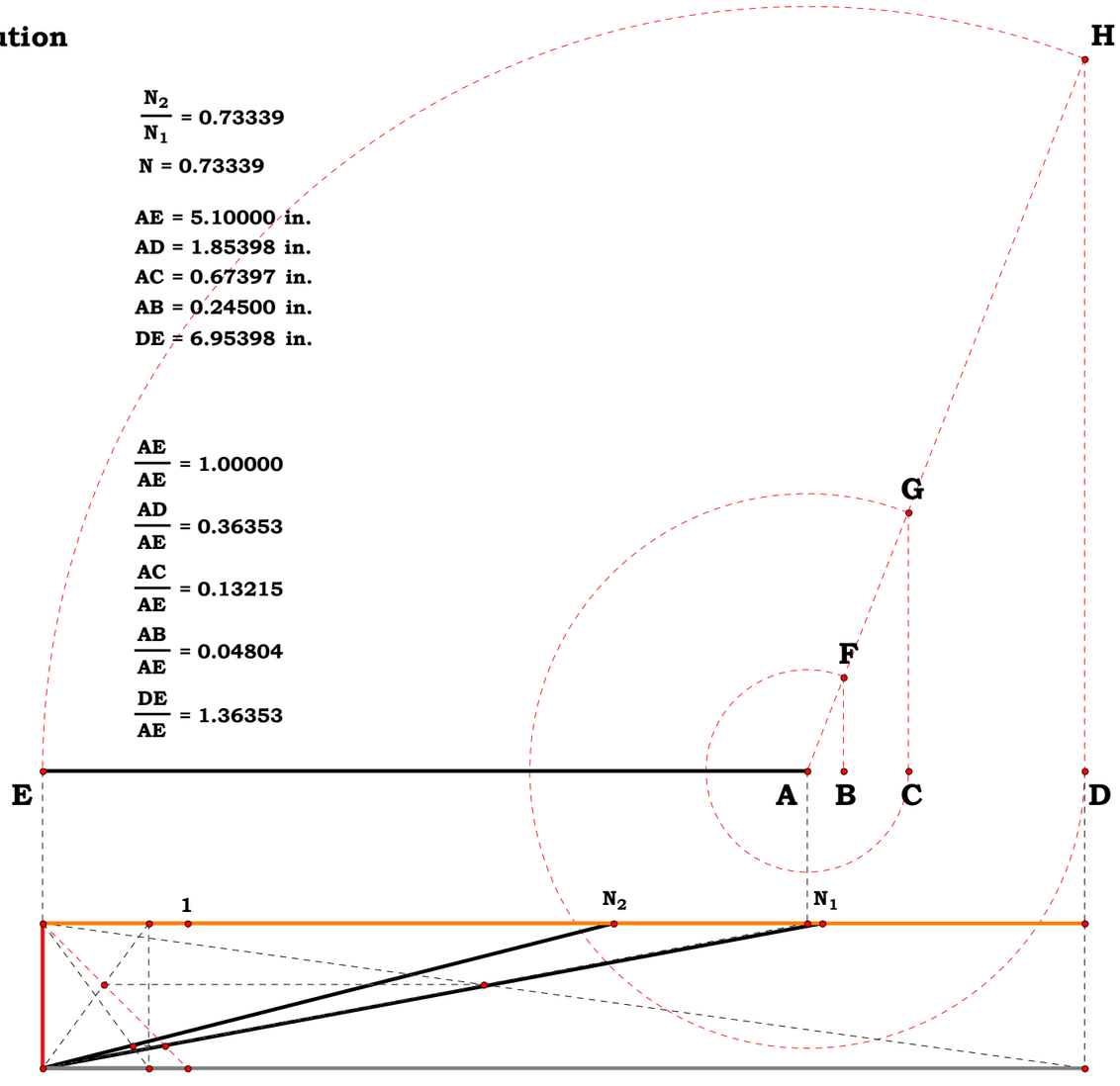
$\frac{(N-1)^3}{N^3} - AB = 0$

$\frac{N_2}{N_1} = 0.73339$

$N = 0.73339$

$AE = 5.10000\ in.$
$AD = 1.85398\ in.$
$AC = 0.67397\ in.$
$AB = 0.24500\ in.$
$DE = 6.95398\ in.$

$\frac{AE}{AE} = 1.00000$
$\frac{AD}{AE} = 0.36353$
$\frac{AC}{AE} = 0.13215$
$\frac{AB}{AE} = 0.04804$
$\frac{DE}{AE} = 1.36353$

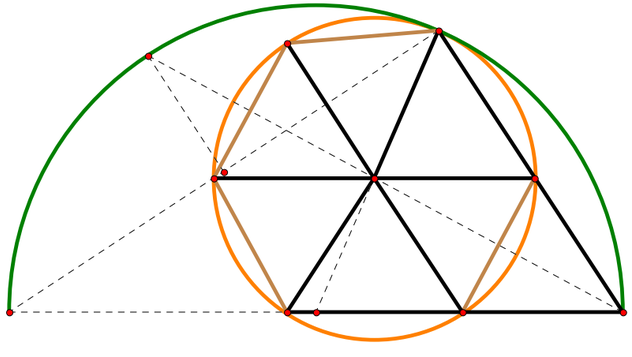



$\sqrt{\left(\frac{1}{N}\right)^2} = 1.36353 \qquad \sqrt{\left(1-\frac{1}{N}\right)^2} = 0.36353 \qquad \frac{(N-1)^2}{N^2} = 0.13215 \qquad \sqrt{\frac{(N-1)^3}{N^3}^2} = 0.04804$

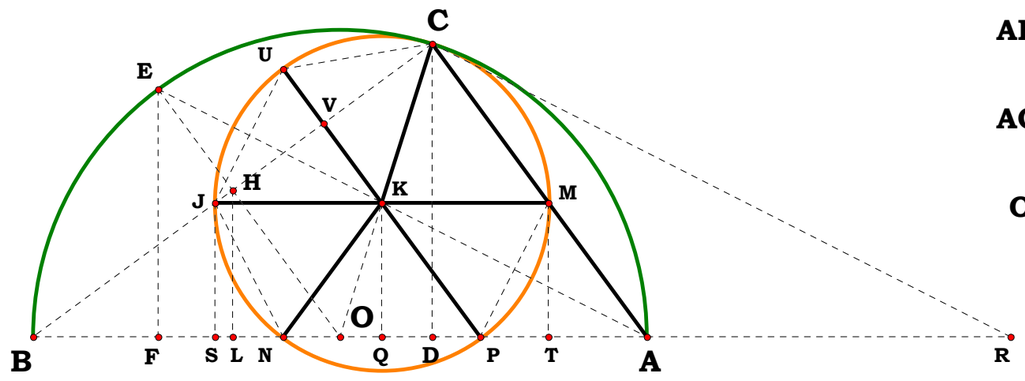
$\sqrt{\left(\frac{1}{N}\right)^2} - \frac{DE}{AE} = 0.00000 \qquad \sqrt{\left(1-\frac{1}{N}\right)^2} - \frac{AD}{AE} = 0.00000 \qquad \frac{(N-1)^2}{N^2} - \frac{AC}{AE} = 0.00000 \qquad \sqrt{\frac{(N-1)^3}{N^3}^2} - \frac{AB}{AE} = 0.00000$

## 111193 The Archimedean Paper Trisector

**When I looked up the Archimedean Paper Trisector, which is all I found. I did not find where anyone had bothered to complete the figure, for it was obvious to me that the figure was simply not complete. The first task then in writing up the figure is to simply complete the figure.**



**Once one understands that the angle on the center is twice the angle from the circumference one can then start to work filling in the figure to include the APT. One can see, not only here, but in other figures that trisection is involved with the right triangle and square roots.**




**First Principles:** $AB := 4.05 \quad N := 3$

**Descriptions:**

$$AD := \frac{AB}{N} \quad BD := AB - AD \quad CD := \sqrt{AD \cdot BD} \quad BC := \sqrt{CD^2 + BD^2}$$

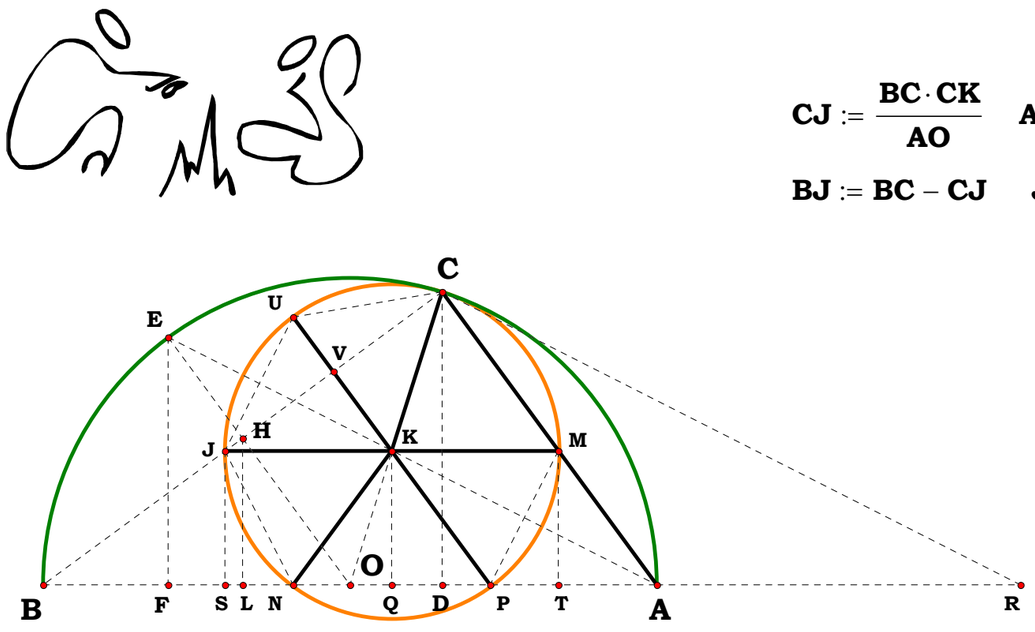
$$AC := \sqrt{AD^2 + CD^2} \quad BH := \frac{BC}{2} \quad AO := \frac{AB}{2} \quad HO := \frac{AC}{2} \quad HL := \frac{CD}{2} \quad LO := \frac{AD}{2}$$

$$OF := \frac{LO \cdot AO}{HO} \quad AF := AO + OF \quad BF := AB - AF \quad EF := \sqrt{BF \cdot AF} \quad DR := \frac{AF \cdot CD}{EF}$$

$$DO := AO - AD \quad OR := DR + DO \quad KQ := \frac{CD \cdot AO}{OR} \quad OK := \frac{AO \cdot KQ}{CD}$$

$$CK := AO - OK \quad QP := \sqrt{CK^2 - KQ^2} \quad OQ := \frac{DO \cdot KQ}{CD} \quad EH := AO - HO$$

$$AP := AO - (OQ + QP) \quad AP - CK = 0$$




$$CJ := \frac{BC \cdot CK}{AO} \quad AQ := AO - OQ \quad SO := CK - OQ \quad BS := AO - SO$$

$$BJ := BC - CJ \quad JS := \sqrt{BJ^2 - BS^2} \quad JS - KQ = 0 \quad SN := SO + OQ - QP \quad JN := \sqrt{JS^2 + SN^2}$$

$$UV := \frac{EH \cdot CJ}{BC} \quad JV := \frac{CJ}{2} \quad JU := \sqrt{JV^2 + UV^2} \quad CU := JU$$

$$AT := \frac{AD \cdot CK}{AC} \quad PT := AP - AT \quad MT := \frac{CD \cdot AP}{AC} \quad MP := \sqrt{PT^2 + MT^2}$$

$$MP - JN = 0 \quad MP - JU = 0 \quad MP - CU = 0$$

**Algebraic Names or Definitions:**

$$AD - \frac{AB}{N} = 0 \quad BD - \frac{AB \cdot (N-1)}{N} = 0 \quad CD - \frac{AB \cdot \sqrt{N-1}}{N} = 0 \quad BC - \frac{AB \cdot \sqrt{N-1}}{\sqrt{N}} = 0 \quad AC - \frac{AB}{\sqrt{N}} = 0 \quad BH - \frac{AB \cdot \sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AO - \frac{AB}{2} = 0 \quad HO - \frac{AB}{2 \cdot \sqrt{N}} = 0 \quad HL - \frac{AB \cdot \sqrt{N-1}}{2 \cdot N} = 0 \quad LO - \frac{AB}{2 \cdot N} = 0 \quad OF - \frac{AB}{2 \cdot \sqrt{N}} = 0 \quad AF - \frac{AB \cdot (\sqrt{N}+1)}{2 \cdot \sqrt{N}} \quad BF - \frac{AB \cdot (\sqrt{N}-1)}{2 \cdot \sqrt{N}} = 0$$

$$EF - \frac{AB \cdot \sqrt{(N-1)}}{2 \cdot \sqrt{N}} = 0 \quad DR - \frac{AB \cdot \sqrt{N-1} \cdot (\sqrt{N}+1)}{N \cdot \sqrt{(\sqrt{N}-1) \cdot (\sqrt{N}+1)}} = 0 \quad DO - \frac{AB \cdot \sqrt{(N-2)^2}}{2 \cdot N} = 0 \quad OR - \frac{AB \cdot (\sqrt{N}+2)}{2 \cdot \sqrt{N}} = 0 \quad KQ - \frac{AB \cdot \sqrt{N-1}}{\sqrt{N} \cdot (\sqrt{N}+2)} = 0$$

$$OK - \frac{AB \cdot \sqrt{N}}{2 \cdot (\sqrt{N}+2)} = 0 \quad QP - \frac{AB}{\sqrt{N} \cdot (\sqrt{N}+2)} = 0 \quad OQ - \frac{AB \cdot \left[\sqrt{(N-2)^2}\right]}{2 \cdot \sqrt{N} \cdot (\sqrt{N}+2)} = 0 \quad EH - \frac{AB \cdot (\sqrt{N}-1)}{2 \cdot \sqrt{N}} = 0$$

$$AP - \frac{AB}{\sqrt{N}+2} = 0 \qquad CK - \frac{AB}{\sqrt{N}+2} = 0$$

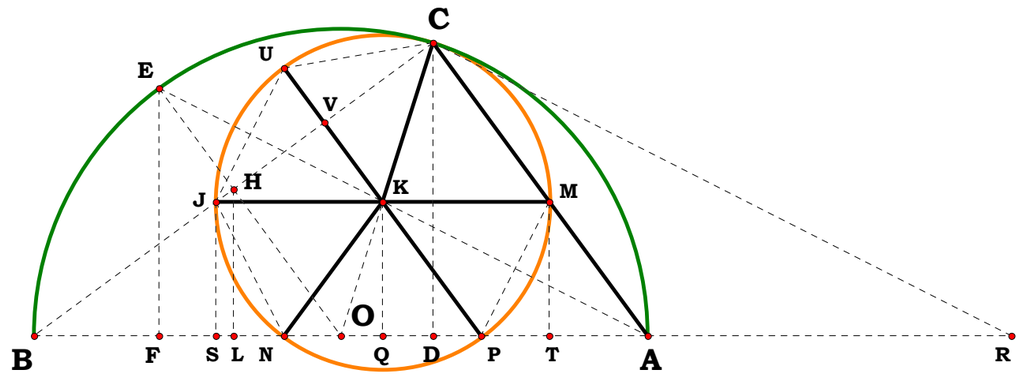
**In logic, things which have the same name are equal. CK equals AP.**

$$CJ - \frac{2\cdot AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad AQ - \frac{AB\cdot\left(\sqrt{N}+1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad SO - \frac{AB\cdot\left(2\cdot\sqrt{N}-N+2\right)}{2\cdot N+4\cdot\sqrt{N}} = 0 \quad BS - \frac{AB\cdot(N-1)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad BJ - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}+2} = 0$$

$$JS - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad JS - KQ = 0 \quad SN - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad JN - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \quad UV - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad JV - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JU - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \quad CU - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \quad AT - \frac{AB}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad PT - \frac{AB\cdot\left(\sqrt{N}-1\right)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \quad MT - \frac{AB\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

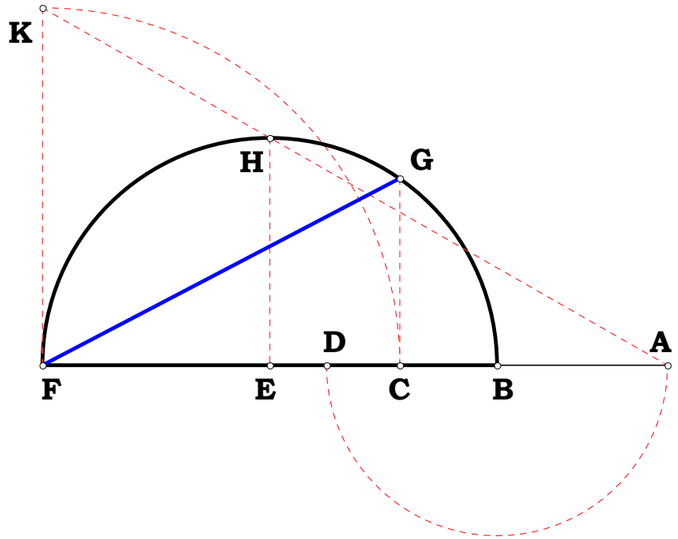
$$MP - \frac{\sqrt{2}\cdot AB\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \quad MP - JN = 0 \quad MP - JU = 0 \quad MP - CU = 0$$



**111293 To Square A Circle Off The Base Of A Right Triangle.**

**Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?**

**Using the approximation, $\pi = 22/7$, square the circle off the base of a right triangle.**




$BF := 1 \quad BE := \frac{BF}{2} \quad EH := BE \quad BD := \frac{3}{4} \cdot BE \quad AB := BD$

$AE := AB + BE \quad AF := AB + BF \quad FK := \frac{EH \cdot AF}{AE} \quad CF := FK \quad BC := BF - CF$

$CG := \sqrt{BC \cdot CF} \quad FG := \sqrt{CF^2 + CG^2} \quad \pi\_A := \frac{FG^2}{BE^2}$

$\pi = 3.14159265359$

$\pi\_A = 3.142857142857$

$\frac{\pi}{\pi\_A} = 0.999597662505843 \qquad \pi\_A - \frac{22}{7} = 0$

DE = 0.39839 in.
BF = 3.18715 in.
AB = 1.19518 in.
AB·BF = 3.80922 in$^2$
FG = 2.82511 in.
BE = 1.59357 in.
BF = 3.18715 in.

$\frac{FG^2}{BF} = 0.78571$

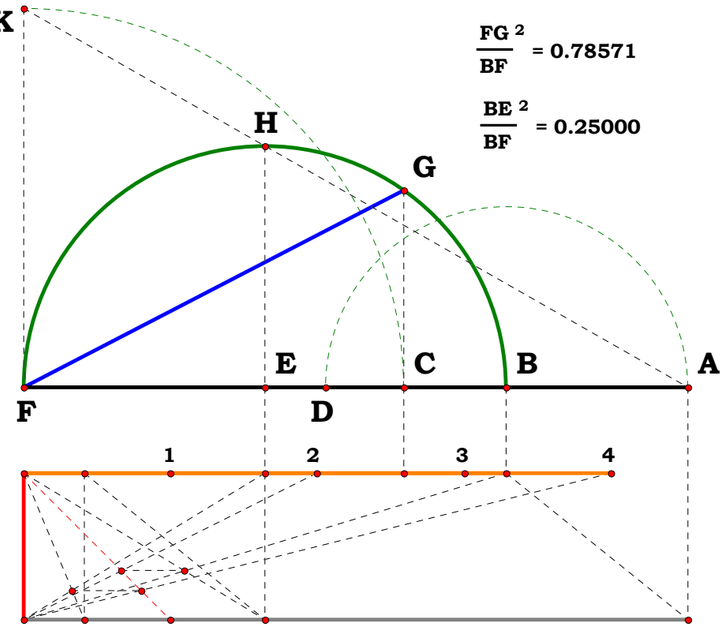
$\frac{BE^2}{BF} = 0.25000$

$\frac{22}{\frac{FG^2}{BF}} = 28.00000$

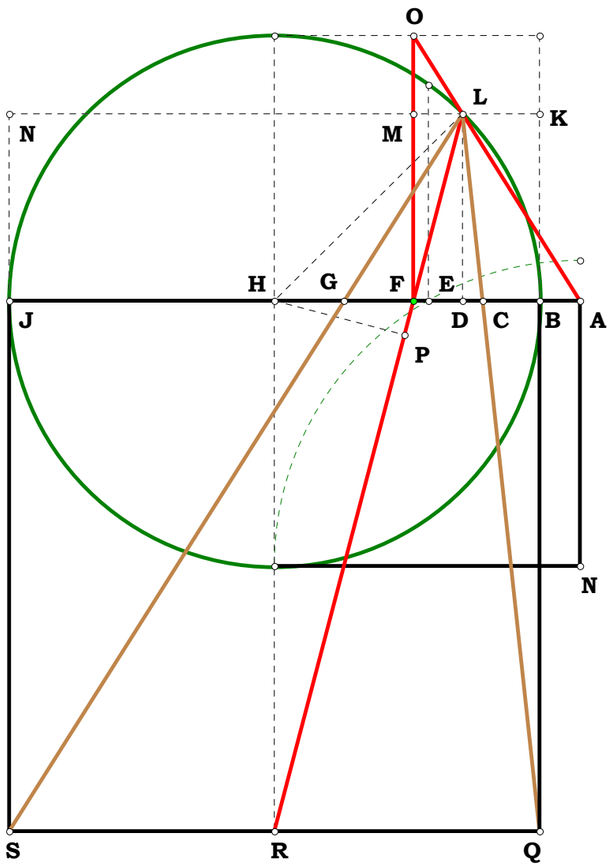
$\frac{7}{\frac{BE^2}{BF}} = 28.00000$

FK = 2.50419 in.

$\left(\frac{FK}{BF}\right) \cdot 28 = 22.00000$



111893A




**Given:**

$N_1 := 2 \quad N_2 := 1.5$

**describe AB.**

$$BJ := N_1 \quad BH := \frac{BJ}{2} \quad BD := \frac{BH}{N_2} \quad HJ := BH$$

$$DH := BH - BD \quad HR := BJ \quad DJ := DH + HJ$$

$$DL := \sqrt{BD \cdot DJ} \quad DF := \frac{DH \cdot DL}{DL + HR} \quad FO := BH \quad BF := BD + DF$$

$$MO := FO - DL \quad LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF$$

**Define each step in the description of AB.**

$$BJ - N_1 = 0 \quad BH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2 \cdot N_2} = 0 \quad HJ - \frac{N_1}{2} = 0 \quad DH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$$

$$HR - N_1 = 0 \quad DJ - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2} = 0 \quad DL - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2} = 0$$

$$DF - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0 \quad FO - \frac{N_1}{2} = 0 \quad BF - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 2\right)}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0$$

$$MO - \frac{N_1 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right)}{2 \cdot N_2} = 0 \quad LM - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)} = 0$$

$$AF - \frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right) \cdot \left(2 \cdot N_2 - 2 \cdot \sqrt{2 \cdot N_2 - 1}\right)} = 0 \quad AB - \frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(2 \cdot {N_2}^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1\right)} = 0$$

BH-$\frac{N_1}{2}$ = 0.00000 in.

BD-$\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

HJ-$\frac{N_1}{2}$ = 0.00000 in.

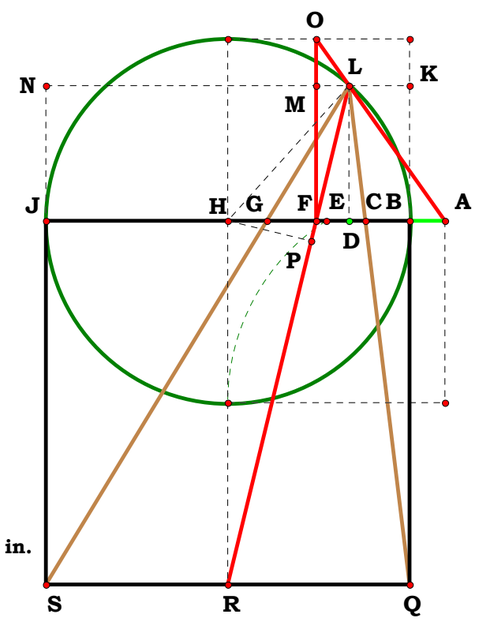
DH-$\frac{N_1 \cdot (N_2-1)}{2 \cdot N_2}$ = 0.00000 in.

HR-$N_1$ = 0.00000 in.

DJ-$\frac{N_1 \cdot (2 \cdot N_2-1)}{2 \cdot N_2}$ = 0.00000 in.

DL-$\frac{N_1 \cdot \sqrt{2 \cdot N_2-1}}{2 \cdot N_2}$ = 0.00000 in.

DF-$\frac{N_1 \cdot (N_2-1) \cdot \sqrt{2 \cdot N_2-1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2+\sqrt{2 \cdot N_2-1}\right)}$ = 0.00000 in.




$N_1$ = 2.40000 in.

$N_2$ = 3.01550

FO-$\frac{N_1}{2}$ = 0.00000 in.

BF-$\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2-1}+2\right)}{2 \cdot \left(2 \cdot N_2+\sqrt{2 \cdot N_2-1}\right)}$ = 0.00000 in.

MO-$\frac{N_1 \cdot \left(N_2-\sqrt{2 \cdot N_2-1}\right)}{2 \cdot N_2}$ = 0.00000 in.

LM-$\frac{N_1 \cdot (N_2-1) \cdot \sqrt{2 \cdot N_2-1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2+\sqrt{2 \cdot N_2-1}\right)}$ = 0.00000 in.

AF-$\frac{N_1 \cdot (N_2-1) \cdot \sqrt{2 \cdot N_2-1}}{\left(2 \cdot N_2+\sqrt{2 \cdot N_2-1}\right) \cdot \left(2 \cdot N_2-2 \cdot \sqrt{2 \cdot N_2-1}\right)}$ = 0.00000 in.

AB-$\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2-1}-1\right)}{2 \cdot \left(\left(2 \cdot N_2{}^2-2 \cdot N_2-N_2 \cdot \sqrt{2 \cdot N_2-1}\right)+1\right)}$ = 0.00000 in.

BJ = 2.40000 in.

BH = 1.20000 in.

BD = 0.39794 in.

HJ = 1.20000 in.

$\frac{BH}{BD}$ = 3.01550

DH = 0.80206 in.

HR = 2.40000 in.

DJ = 2.00206 in.

DL = 0.89258 in.

DF = 0.21743 in.

FO = 1.20000 in.

BF = 0.61537 in.

MO = 0.30742 in.

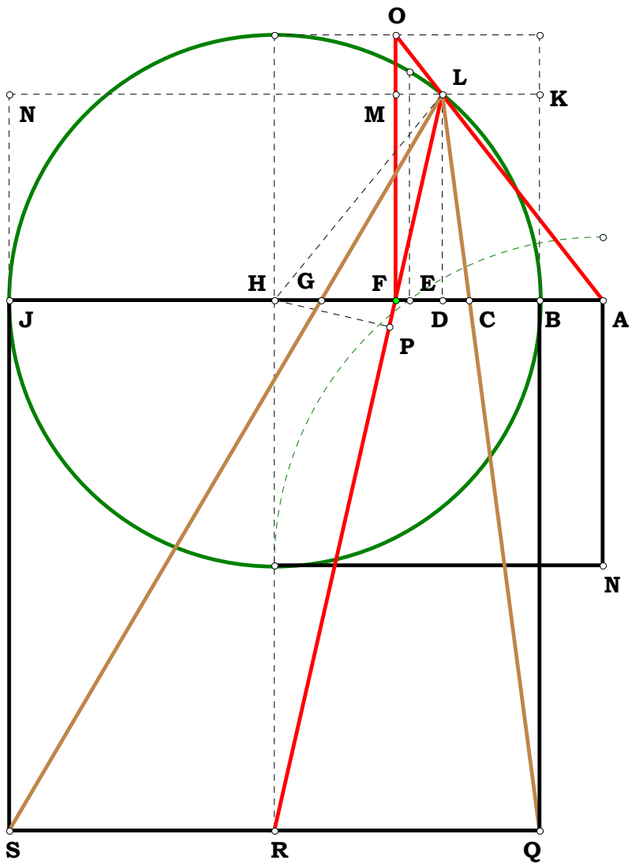
LM = 0.21743 in.

AF = 0.84873 in.

AB = 0.23336 in.

## 111893 Exploring Cube Roots Plate B

**Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root. Notice in this write-up I chose the wrong point to proportion. I get the right answers, but the equations are more complicated. Compare features to plate A.**




$N_1 := 2.4 \quad N_2 := 3.01550$

$BJ := N_1 \quad BH := \frac{BJ}{2} \quad HL := BH \quad BF := \frac{BH}{N_2} \quad FH := BH - BF \quad HR := BJ \quad FR := \sqrt{FH^2 + HR^2}$

$FP := \frac{FH^2}{FR} \quad PH := \frac{HR \cdot FP}{FH} \quad LP := \sqrt{HL^2 - PH^2} \quad FL := LP - FP \quad DF := \frac{FH \cdot FL}{FR} \quad DL := \frac{HR \cdot FL}{FR}$

$FO := BH \quad FM := DL \quad MO := FO - FM \quad LM := DF \quad AF := \frac{LM \cdot FO}{MO} \quad AB := AF - BF \quad BQ := BJ$

$BK := DL \quad BD := BF - DF \quad KQ := BQ + BK \quad KL := BD \quad BC := \frac{KL \cdot BQ}{KQ} \quad DJ := BJ - BD \quad LN := DJ \quad JS := BJ$

$JN := DL \quad NS := JS + JN \quad GJ := \frac{LN \cdot JS}{NS} \quad BG := BJ - GJ \quad AC := AB + BC \quad AG := AB + BG \quad AJ := AB + BJ$

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

$BJ - N_1 = 0 \quad BH - \frac{N_1}{2} = 0 \quad HL - \frac{N_1}{2} = 0 \quad BF - \frac{N_1}{2 \cdot N_2} = 0 \quad FH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0 \quad HR - N_1 = 0$

$FR - \frac{N_1 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}}{2 \cdot N_2} = 0 \quad FP - \frac{N_1 \cdot (N_2 - 1)^2}{2 \cdot N_2 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0 \quad PH - \frac{N_1 \cdot (N_2 - 1)}{\sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0$

$$LP - \frac{N_1 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3}}{2 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0 \qquad FL - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{2 \cdot N_2 \cdot \sqrt{5 \cdot N_2^2 - 2 \cdot N_2 + 1}} = 0$$

$$DF - \frac{N_1 \cdot (N_2 - 1) \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{2 \cdot N_2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0 \qquad DL - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{5 \cdot N_2^2 - 2 \cdot N_2 + 1} = 0$$

$$FO - \frac{N_1}{2} = 0 \qquad FM - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{5 \cdot N_2^2 - 2 \cdot N_2 + 1} = 0 \qquad MO - \frac{N_1 \cdot \left(7 \cdot N_2^2 - 6 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} + 3\right)}{2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0$$

$$LM - \frac{N_1 \cdot (N_2 - 1) \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{2 \cdot N_2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0 \qquad AF - \frac{N_1 \cdot (N_2 - 1) \cdot \left(N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot N_2 \cdot \left(6 \cdot N_2 - 7 \cdot N_2^2 + 2 \cdot N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 3\right)} = 0$$

$$AB - \frac{N_1 \cdot \left(3 \cdot N_2 + N_2^2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 4 \cdot N_2^2 - N_2^3 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 2\right)}{2 \cdot \left(3 \cdot N_2 - 2 \cdot N_2^2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 6 \cdot N_2^2 + 7 \cdot N_2^3\right)} = 0 \qquad BQ - N_1 = 0$$

$$BK - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - 1\right)}{5 \cdot N_2^2 - 2 \cdot N_2 + 1} = 0 \qquad BD - \frac{N_1 \cdot \left[2 \cdot N_2 + N_2^2 + (1 - N_2) \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} + 1\right]}{2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0$$

$$KQ - \frac{N_1 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^2 + 6 \cdot N_2 - 3}\right)}{5 \cdot N_2^2 - 2 \cdot N_2 + 1} = 0 \qquad KL - \frac{N_1 \cdot \left[2 \cdot N_2 + N_2^2 + (1 - N_2) \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} + 1\right]}{2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0$$

$$BC - \frac{N_1 \cdot \left[2 \cdot N_2 + N_2^2 - \sqrt{N_2^2 + 6 \cdot N_2 - 3} \cdot (N_2 - 1) + 1\right]}{2 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^2 + 6 \cdot N_2 - 3}\right)} = 0 \qquad DJ - \frac{N_1 \cdot \left(9 \cdot N_2^2 - 6 \cdot N_2 + N_2 \cdot \sqrt{N_2^2 + 6 \cdot N_2 - 3} - \sqrt{N_2^2 + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot \left(5 \cdot N_2^2 - 2 \cdot N_2 + 1\right)} = 0$$

$$LN - \frac{N_1 \cdot \left(9 \cdot N_2^{\,2} - 6 \cdot N_2 + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot \left(5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1\right)} = 0 \qquad JS - N_1 = 0$$

$$JN - \frac{N_1 \cdot \left(2 \cdot N_2 - N_2^{\,2} + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - 1\right)}{5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1} = 0 \qquad NS - \frac{N_1 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}\right)}{5 \cdot N_2^{\,2} - 2 \cdot N_2 + 1} = 0$$

$$GJ - \frac{N_1 \cdot \left(9 \cdot N_2^{\,2} - 6 \cdot N_2 + N_2 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} + 1\right)}{2 \cdot N_2 \cdot \left(4 \cdot N_2 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}\right)} = 0 \qquad BG - \frac{3 \cdot N_1 - N_1 \cdot N_2 + N_1 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3}}{6 \cdot N_2} = 0$$

$$AC - \frac{3 \cdot N_1 - 5 \cdot N_1 \cdot N_2^{\,2} + 9 \cdot N_1 \cdot N_2^{\,3} + 6 \cdot N_1 \cdot N_2^{\,4} + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(9 \cdot N_1 \cdot N_2^{\,2} - 6 \cdot N_1 \cdot N_2^{\,3} - 8 \cdot N_1 \cdot N_2 + N_1\right) - 5 \cdot N_1 \cdot N_2}{36 \cdot N_2^{\,2} - 72 \cdot N_2^{\,3} + 52 \cdot N_2^{\,4} - \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(2 \cdot N_2^{\,3} + 12 \cdot N_2^{\,2} - 6 \cdot N_2\right)} = 0$$

$$AG - \frac{N_1 + N_1 \cdot N_2^{\,2} - 4 \cdot N_1 \cdot N_2^{\,3} + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(4 \cdot N_1 \cdot N_2^{\,2} - 3 \cdot N_1 \cdot N_2 + N_1\right) - 2 \cdot N_1 \cdot N_2}{6 \cdot N_2 - 12 \cdot N_2^{\,2} + 14 \cdot N_2^{\,3} - 4 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot N_2^{\,2}} = 0$$

$$AJ - \frac{13 \cdot N_1 \cdot N_2^{\,3} - 16 \cdot N_1 \cdot N_2^{\,2} - 2 \cdot N_1 + \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot \left(N_1 \cdot N_2 - 3 \cdot N_1 \cdot N_2^{\,2}\right) + 9 \cdot N_1 \cdot N_2}{6 \cdot N_2 - 12 \cdot N_2^{\,2} + 14 \cdot N_2^{\,3} - 4 \cdot \sqrt{N_2^{\,2} + 6 \cdot N_2 - 3} \cdot N_2^{\,2}} = 0$$

## Etc.

BF- $\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

AF- $\frac{N_1 \cdot (N_2-1) \cdot \left( \left( N_2^2-2 \cdot N_2-N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)+1 \right)}{2 \cdot N_2 \cdot \left( \left( \left( 6 \cdot N_2-7 \cdot N_2^2 \right)+2 \cdot N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-3 \right)}$ = 0.00000 in.

AB- $\frac{N_1 \cdot \left( \left( \left( \left( 3 \cdot N_2+N_2^2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-4 \cdot N_2^2-N_2^3 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-2 \right)}{2 \cdot \left( \left( 3 \cdot N_2-2 \cdot N_2^2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3}-6 \cdot N_2^2 \right)+7 \cdot N_2^3 \right)}$ = 0.00000 in

FH- $\frac{N_1 \cdot (N_2-1)}{2 \cdot N_2}$ = 0.00000 in.

FR- $\frac{N_1 \cdot \sqrt{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}}{2 \cdot N_2}$ = 0.00000 in.

FP- $\frac{N_1 \cdot (N_2-1)^2}{2 \cdot N_2 \cdot \sqrt{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}}$ = 0.00000 in.

PH- $\frac{N_1 \cdot (N_2-1)}{\sqrt{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}}$ = 0.00000 in.

LP- $\frac{N_1 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3}}{2 \cdot \sqrt{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}}$ = 0.00000 in.

FL- $\frac{N_1 \cdot \left( \left( \left( 2 \cdot N_2-N_2^2 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-1 \right)}{2 \cdot N_2 \cdot \sqrt{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}}$ = 0.00000 in.

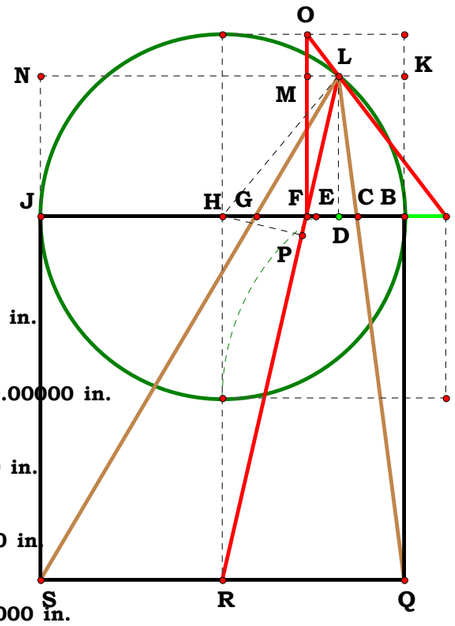
DF- $\frac{N_1 \cdot (N_2-1) \cdot \left( \left( \left( 2 \cdot N_2-N_2^2 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-1 \right)}{2 \cdot N_2 \cdot \left( \left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1 \right)}$ = 0.00000 in.

DL- $\frac{N_1 \cdot \left( \left( \left( 2 \cdot N_2-N_2^2 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-1 \right)}{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}$ = 0.00000 in.

FM- $\frac{N_1 \cdot \left( \left( \left( 2 \cdot N_2-N_2^2 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-1 \right)}{\left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1}$ = 0.00000 in.

MO- $\frac{N_1 \cdot \left( \left( 7 \cdot N_2^2-6 \cdot N_2-2 \cdot N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)+3 \right)}{2 \cdot \left( \left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1 \right)}$ = 0.00000 in.

LM- $\frac{N_1 \cdot (N_2-1) \cdot \left( \left( \left( 2 \cdot N_2-N_2^2 \right)+N_2 \cdot \sqrt{\left( N_2^2+6 \cdot N_2 \right)-3} \right)-1 \right)}{2 \cdot N_2 \cdot \left( \left( 5 \cdot N_2^2-2 \cdot N_2 \right)+1 \right)}$ = 0.00000 in.




$N_1$ = 2.40000 in.

$N_2$ = 1.86107

Animate Point

BJ = 2.40000 in.

BH = 1.20000 in.

BF = 0.64479 in.

$\frac{BH}{BF}$ = 1.86107

FH = 0.55521 in.

FR = 2.46338 in.

FP = 0.12514 in.

PH = 0.54092 in.

LP = 1.07117 in.

FL = 0.94603 in.

DF = 0.21322 in.

DL = 0.92169 in.

FM = 0.92169 in.

MO = 0.27831 in.

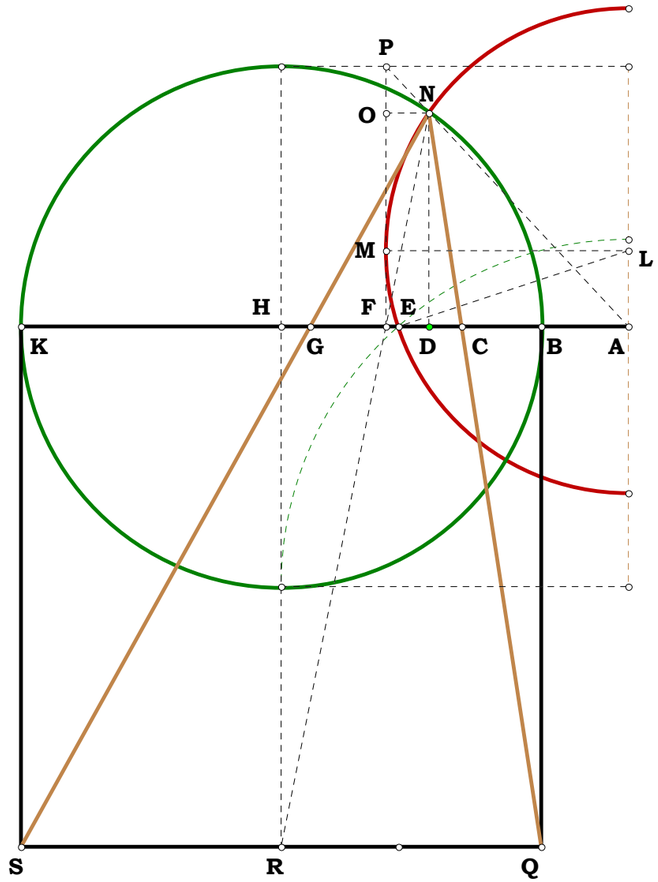
LM = 0.21322 in.

AF = 0.91936 in.

AB = 0.27457 in.

## 111893 Exploring Cube Roots Plate C

**If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.**




$N_1 := 4 \quad N_2 := 1.2 \quad BK := N_1$

$BH := \frac{BK}{2} \quad BD := \frac{BH}{N_2} \quad DK := BK - BD$

$DN := \sqrt{BD \cdot DK} \quad BQ := BK \quad KS := BK \quad HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN} \quad GK := \frac{DK \cdot KS}{KS + DN} \quad BG := BK - GK$

$DH := BH - BD \quad FH := \frac{DH \cdot HR}{HR + DN} \quad BF := BH - FH$

$CF := BF - BC \quad AL := CF \quad DF := BF - BD$

$NO := DF \quad FP := BH \quad PO := FP - DN$

$AD := \frac{NO \cdot DN}{PO} \quad AB := AD - BD$

$AF := AB + BF \quad LM := AF \quad EL := AF \quad AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2} \quad AE_2 := \sqrt{AB \cdot AK} \quad AE_1 - AE_2 = 0$

$N_1 := 4 \qquad N_2 := 1.2 \qquad BK := N_1$

$BH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2 \cdot N_2} = 0 \quad DK - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2} = 0 \quad DN - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2} = 0 \quad BQ - N_1 = 0 \quad KS - N_1 = 0$

$HR - N_1 = 0 \quad BC - \frac{N_1}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \quad GK - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \quad BG - \frac{N_1 \cdot (\sqrt{2 \cdot N_2 - 1} + 1)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \quad DH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0$

$FH - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}} = 0 \quad BF - \frac{N_1 \cdot (\sqrt{2 \cdot N_2 - 1} + 2)}{2 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0 \quad CF - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0 \quad AL - \frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0$

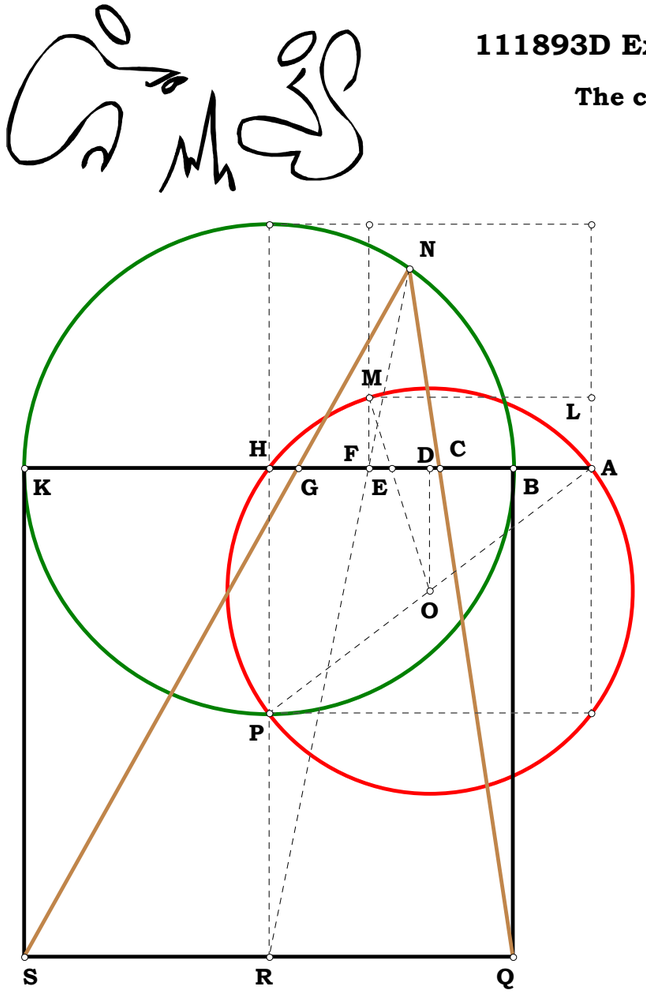
$DF - \frac{\sqrt{2 \cdot N_2 - 1} \cdot N_1 \cdot (N_2 - 1)}{2 \cdot N_2 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0 \quad NO - \frac{\sqrt{2 \cdot N_2 - 1} \cdot N_1 \cdot (N_2 - 1)}{2 \cdot N_2 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0 \quad FP - \frac{N_1}{2} = 0 \quad PO - \frac{N_1 \cdot (N_2 - \sqrt{2 \cdot N_2 - 1})}{2 \cdot N_2} = 0$

$AD - \frac{N_1 \cdot (N_2 - 1) \cdot (\sqrt{2 \cdot N_2 - 1})^2}{2 \cdot N_2 \cdot (N_2 - \sqrt{2 \cdot N_2 - 1}) \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})} = 0 \quad AB - \frac{N_1 \cdot (\sqrt{2 \cdot N_2 - 1} - 1)}{2 \cdot (2 \cdot {N_2}^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1)} = 0$

$AF - \frac{N_1 \cdot (N_2 - 1) \cdot (2 \cdot N_2 + 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1)}{2 \cdot (3 \cdot N_2 + \sqrt{2 \cdot N_2 - 1} - 6 \cdot {N_2}^2 + 4 \cdot {N_2}^3 - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1})} = 0 \quad EL - AF = 0 \quad LM - AF = 0 \quad AK - \frac{N_1 \cdot \left[4 \cdot {N_2}^2 - (2 \cdot N_2 - 1)^{\frac{3}{2}} - 4 \cdot N_2 + 1\right]}{2 \cdot (2 \cdot {N_2}^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1} + 1)} = 0$

$AE_1 - \sqrt{\left[\frac{{N_1}^2 \cdot (N_2 - 1)^2 \cdot (2 \cdot N_2 + 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1)^2}{4 \cdot (3 \cdot N_2 + \sqrt{2 \cdot N_2 - 1} - 6 \cdot {N_2}^2 + 4 \cdot {N_2}^3 - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_2 - 1})^2} - \frac{{N_1}^2 \cdot (2 \cdot N_2 - 1)}{4 \cdot (2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1})^2}\right]} = 0$

$AE_2 - \sqrt{\left[\frac{{N_1}^2 \cdot (\sqrt{2 \cdot N_2 - 1} - 1) \cdot \left[4 \cdot {N_2}^2 - (2 \cdot N_2 - 1)^{\frac{3}{2}} - 4 \cdot N_2 + 1\right]}{4 \cdot (2 \cdot N_2 - 2 \cdot {N_2}^2 + N_2 \cdot \sqrt{2 \cdot N_2 - 1} - 1)^2}\right]} = 0 \qquad AE_1 - AE_2 = 0$




## 111893D Exploring Cube Roots

**The circle AO passes through point M. FM equals half of CG.**

$N_1 := 3 \quad N_2 := 2$

$AB := N_1 \quad BK := N_2 \quad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

$$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0$$

$$AK - \left(N_1 + N_2\right) = 0$$

$$AC - \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}} = 0 \quad AG - \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} = 0 \quad CG - \left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}\right] = 0$$

$$CF - \frac{\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}}{2} = 0 \quad BH - \frac{N_2}{2} = 0 \quad AH - \frac{2 \cdot N_1 + N_2}{2} = 0 \quad HP - \frac{N_2}{2} = 0 \quad AP - \frac{\sqrt{2 \cdot {N_1}^2 + 2 \cdot N_1 \cdot N_2 + {N_2}^2}}{\sqrt{2}} = 0$$

$$AO - \frac{\sqrt{2}}{4} \cdot \sqrt{2 \cdot {N_1}^2 + 2 \cdot N_1 \cdot N_2 + {N_2}^2} = 0 \quad DO - \frac{N_2}{4} = 0 \quad AF - \frac{\left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} + \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}\right]}{2} = 0 \quad AD - \frac{2 \cdot N_1 + N_2}{4} = 0$$

$$DF - \frac{\left[2 \cdot \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - N_2 - 2 \cdot N_1 + 2 \cdot \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}\right]}{4} = 0 \quad FM - \frac{\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}}{2} = 0$$

$$MO - \frac{\sqrt{2}}{4} \cdot \sqrt{2 \cdot {N_1}^2 + 2 \cdot N_1 \cdot N_2 + {N_2}^2} = 0$$

$$MO^2 - \frac{4 \cdot \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{2}{3}} + 2 \cdot {N_1}^2 + {N_2}^2 - 4 \cdot N_1 \cdot \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}} - 4 \cdot N_2 \cdot \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}} \ldots}{+ 2 \cdot N_1 \cdot N_2 + 4 \cdot \left({N_1}^3 + N_2 \cdot {N_1}^2\right)^{\frac{2}{3}} - 4 \cdot N_1 \cdot \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}} = 0$$

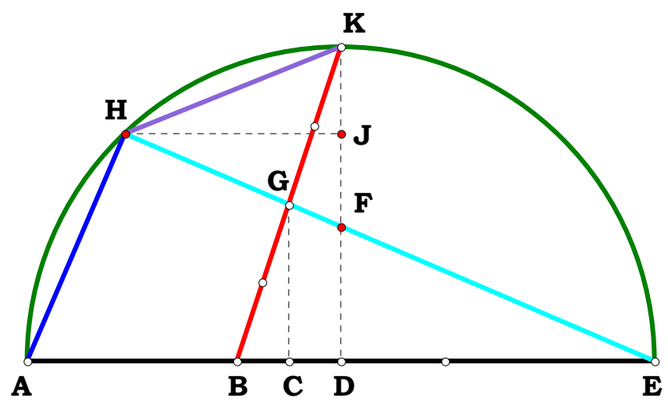
$$\frac{\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{2}{3}}}{2} + \frac{\left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{2}{3}}}{2} - \frac{N_1 \cdot \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}}{2} - \frac{N_1 \cdot \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}}{2} - \frac{N_2 \cdot \left[{N_1}^2 \cdot \left(N_1 + N_2\right)\right]^{\frac{1}{3}}}{2} = 0$$

**Generalize the work of 07/25/93 for dividing the base AE with K constant.**

$N_1 := 3 \quad N_2 := 5 \quad AE := 1$

$\alpha := 1 .. N_1 - 1 \quad \beta := 1 .. N_2 - 1$

**112493 POR Series IV**




$AB := \frac{AE}{N_1} \quad AD := \frac{AE}{2} \quad DK := AD \quad DE := AD$

$BD := AD - AB \quad BK := \sqrt{BD^2 + DK^2} \quad BG := \frac{BK}{N_2} \quad BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK} \quad AC := AB + BC \quad CE := AE - AC \quad DF := \frac{CG \cdot DE}{CE} \quad EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2} \quad AH := \frac{DF \cdot AE}{EF} \quad EH := \frac{DE \cdot AE}{EF} \quad GH := EH - EG \quad FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF} \quad HJ := \frac{DE \cdot FH}{EF} \quad DJ := DF + FJ \quad JK := DK - DJ \quad HK := \sqrt{HJ^2 + JK^2}$

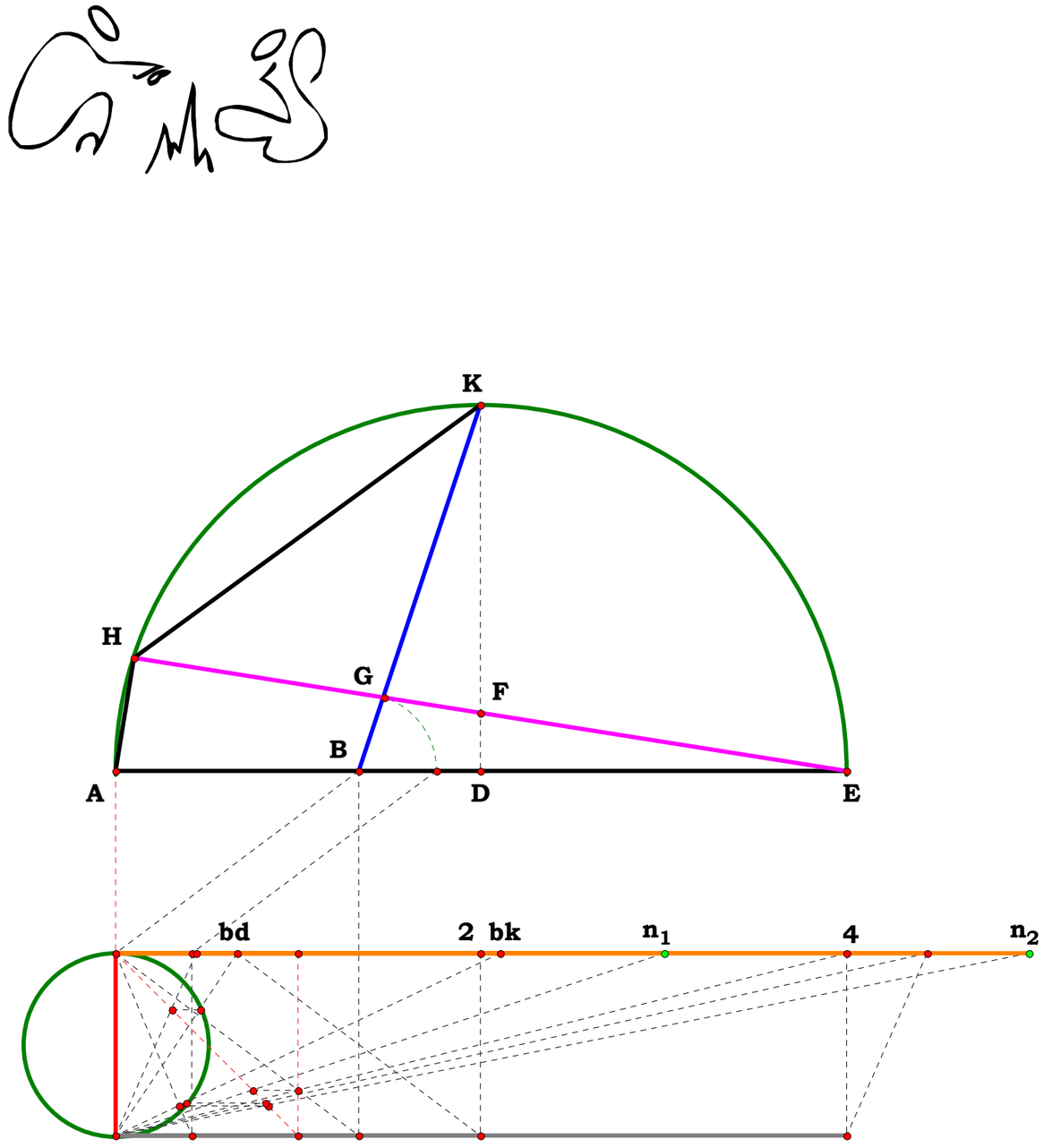
$\frac{AH}{HK} = 0.265 \quad \frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265 \quad SeriesAH_{\alpha,\beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

$SeriesAH = \begin{pmatrix} 0.265 & 0.707 & 1.591 & 4.243 \\ 0.53 & 1.414 & 3.182 & 8.485 \end{pmatrix}$

$\frac{EH}{GH} = 2.85 \quad N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesEH_{\alpha,\beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$SeriesEH = \begin{pmatrix} 2.85 & 3 & 3.643 & 6 \\ 1.65 & 2 & 2.786 & 5.25 \end{pmatrix}$




$HK = 2.35269$

$\frac{AH}{HK} = 0.26517$

$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} = 0.26517$

$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} - \frac{AH}{HK} = 0.00000$

$EH = 3.95105$

$GH = 1.38633$

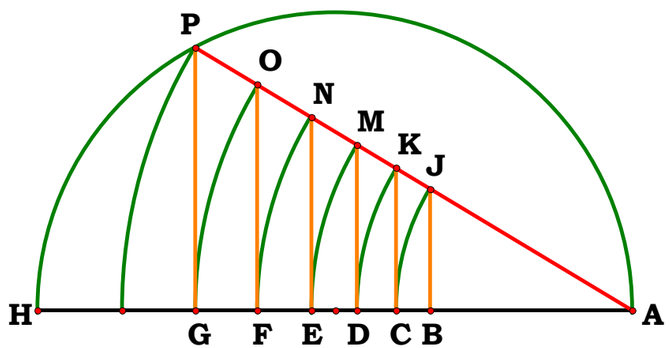
$\frac{EH}{GH} = 2.85000$

$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} = 2.85000$

$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} - \frac{EH}{GH} = 0.00000$

**120293.MCD POR Roots and Powers**
**(Pyramid of Ratio Series V)**

**Is the progression noticed in 11_29_93 a continuous phenomenon?**




$$AH := 5 \qquad \delta := 1 .. 7 \qquad AP_\delta := \frac{AH}{\delta}$$

$$AG_\delta := \frac{(AP_\delta)^2}{AH} \qquad AO_\delta := AG_\delta \qquad AF_\delta := \frac{(AG_\delta)^2}{AP_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AO_\delta} \qquad AN_\delta := AF_\delta \qquad AD_\delta := \frac{(AE_\delta)^2}{AN_\delta} \qquad AM_\delta := AE_\delta$$

$$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta} \qquad AK_\delta := AD_\delta \qquad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta} \qquad \Delta := 1 .. 6$$

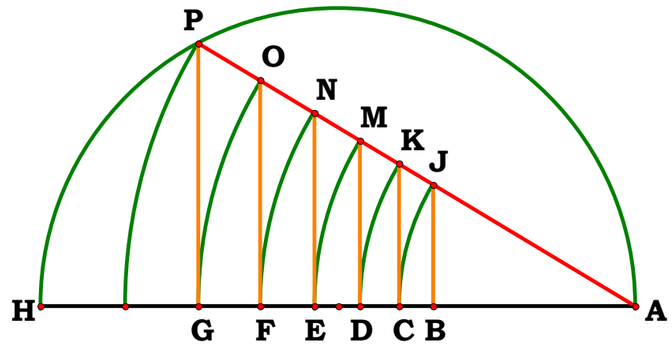
$$\frac{AH}{AP_\Delta} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} =$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

$$\left(\frac{AH}{1},\frac{AH}{1^2},\frac{AH}{1^3},\frac{AH}{1^4},\text{etc}\right)$$
$$\left(\frac{AH}{2},\frac{AH}{2^2},\frac{AH}{2^3},\frac{AH}{2^4},\text{etc}\right)$$
$$\left(\frac{AH}{3},\frac{AH}{3^2},\frac{AH}{3^3},\frac{AH}{3^4},\text{etc}\right)$$
$$\left(\frac{AH}{4},\frac{AH}{4^2},\frac{AH}{4^3},\frac{AH}{4^4},\text{etc}\right)$$
$$\left(\frac{AH}{5},\frac{AH}{5^2},\frac{AH}{5^3},\frac{AH}{5^4},\text{etc}\right)$$

$$\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}} = \frac{AC_\Delta}{AB_\Delta} =$$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 |




| $AP_\Delta =$ | $AG_\Delta =$ | $AF_\Delta =$ | $AE_\Delta =$ | $AD_\Delta =$ | $AC_\Delta =$ | $AB_\Delta =$ |
|---|---|---|---|---|---|---|
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2.5 | 1.25 | 0.625 | 0.313 | 0.156 | 0.078 | 0.039 |
| 1.667 | 0.556 | 0.185 | 0.062 | 0.021 | $6.859\cdot10^{-3}$ | $2.286\cdot10^{-3}$ |
| 1.25 | 0.313 | 0.078 | 0.02 | $4.883\cdot10^{-3}$ | $1.221\cdot10^{-3}$ | $3.052\cdot10^{-4}$ |
| 1 | 0.2 | 0.04 | $8\cdot10^{-3}$ | $1.6\cdot10^{-3}$ | $3.2\cdot10^{-4}$ | $6.4\cdot10^{-5}$ |
| 0.833 | 0.139 | 0.023 | $3.858\cdot10^{-3}$ | $6.43\cdot10^{-4}$ | $1.072\cdot10^{-4}$ | $1.786\cdot10^{-5}$ |

$$AP_\Delta = \quad AG_\Delta\cdot\frac{AH}{AP_\Delta} = AF_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^2 : AE_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^3 = AD_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^4 = AC_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^5 = AB_\Delta\cdot\left(\frac{AH}{AP_\Delta}\right)^6 =$$

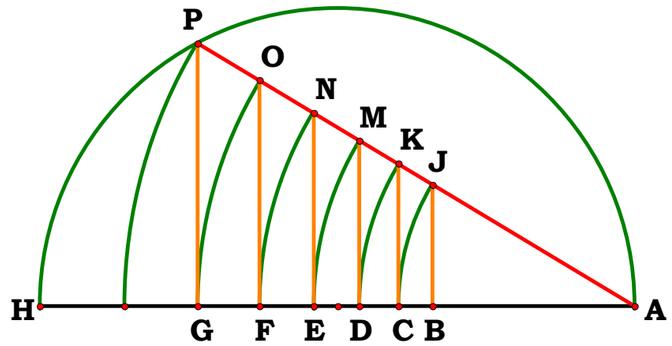
| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| 1.667 | 1.667 | 1.667 | 1.667 | 1.667 | 1.667 | 1.667 |
| 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.833 | 0.833 | 0.833 | 0.833 | 0.833 | 0.833 | 0.833 |

$$\frac{AH^2}{AP_\Delta} = \left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH =$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 15 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |

$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{3} =$$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 27 | 27 | 27 | 27 | 27 | 27 | 27 |
| 64 | 64 | 64 | 64 | 64 | 64 | 64 |
| 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| 216 | 216 | 216 | 216 | 216 | 216 | 216 |




$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}} =$$

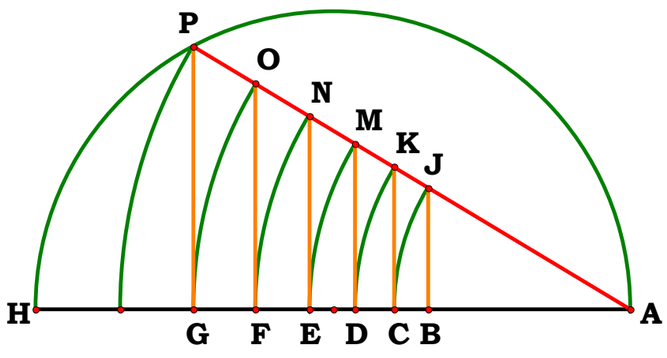
| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 128 | 128 | 128 | 128 | 128 | 128 | 128 |
| $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ | $2.187 \cdot 10^3$ |
| $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ | $1.638 \cdot 10^4$ |
| $7.812 \cdot 10^4$ | $7.812 \cdot 10^4$ | $7.812 \cdot 10^4$ | $7.812 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ | $7.813 \cdot 10^4$ |
| $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ | $2.799 \cdot 10^5$ |

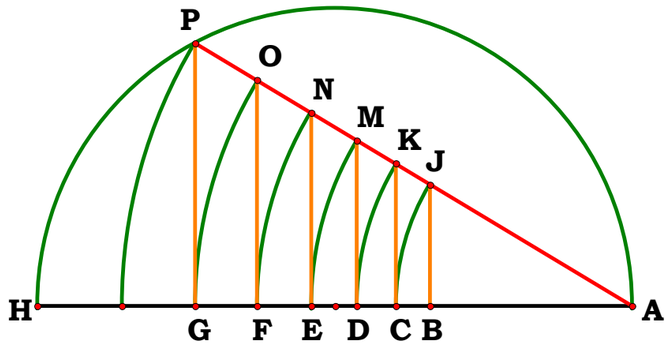
$$GH_\delta := AH - AG_\delta \qquad GP_\delta := \sqrt{AG_\delta \cdot GH_\delta} \qquad FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta} \qquad EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta} \qquad DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta}$$

$$CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta} \qquad BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$$

| $GP_\Delta =$ | $FO_\Delta =$ | $EN_\Delta =$ | $DM_\Delta =$ | $CK_\Delta =$ | $BJ_\Delta =$ | $\frac{GP_\Delta}{\Delta^5} =$ | $\frac{FO_\Delta}{\Delta^4} =$ | $\frac{EN_\Delta}{\Delta^3} =$ | $\frac{DM_\Delta}{\Delta^2} =$ | $\frac{CK_\Delta}{\Delta} =$ | $BJ_\Delta =$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2.165 | 1.083 | 0.541 | 0.271 | 0.135 | 0.068 | 0.068 | 0.068 | 0.068 | 0.068 | 0.068 | 0.068 |
| 1.571 | 0.524 | 0.175 | 0.058 | 0.019 | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ |
| 1.21 | 0.303 | 0.076 | 0.019 | $4.728 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ |
| 0.98 | 0.196 | 0.039 | $7.838 \cdot 10^{-3}$ | $1.568 \cdot 10^{-3}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ |
| 0.822 | 0.137 | 0.023 | $3.804 \cdot 10^{-3}$ | $6.34 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ |

| $\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5} =$ | $\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4} =$ | $\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3} =$ | $\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2} =$ | $\frac{CK_\Delta}{\frac{AH}{AP_\Delta}} =$ | $BJ_\Delta =$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.068 | 0.068 | 0.068 | 0.068 | 0.068 | 0.068 |
| $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ | $6.466 \cdot 10^{-3}$ |
| $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ |
| $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ | $3.135 \cdot 10^{-4}$ |
| $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ |

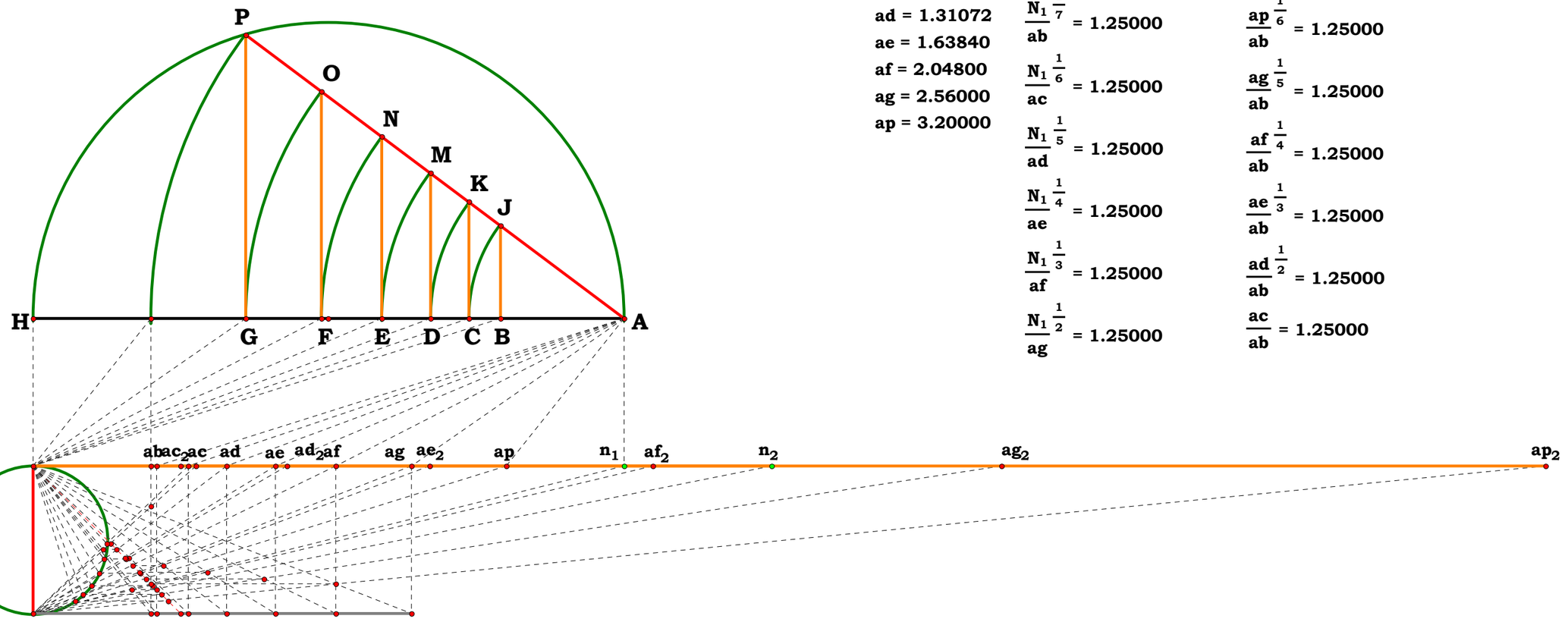





| $GP_{\Delta}$ = | $FO_{\Delta}$ = | $EN_{\Delta}$ = | $DM_{\Delta}$ = | $CK_{\Delta}$ = | $BJ_{\Delta}$ = |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 2.165 | 1.083 | 0.541 | 0.271 | 0.135 | 0.068 |
| 1.571 | 0.524 | 0.175 | 0.058 | 0.019 | $6.466 \cdot 10^{-3}$ |
| 1.21 | 0.303 | 0.076 | 0.019 | $4.728 \cdot 10^{-3}$ | $1.182 \cdot 10^{-3}$ |
| 0.98 | 0.196 | 0.039 | $7.838 \cdot 10^{-3}$ | $1.568 \cdot 10^{-3}$ | $3.135 \cdot 10^{-4}$ |
| 0.822 | 0.137 | 0.023 | $3.804 \cdot 10^{-3}$ | $6.34 \cdot 10^{-4}$ | $1.057 \cdot 10^{-4}$ |

$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = 1.083 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = 0.541 \qquad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = 0.271$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = 0.524 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = 0.175 \qquad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = 0.058$$




$ab = 0.83886$
$ac = 1.04858$
$ad = 1.31072$
$ae = 1.63840$
$af = 2.04800$
$ag = 2.56000$
$ap = 3.20000$

$\frac{N_1}{ap} = 1.25000$

$\frac{N_1}{ab}\frac{1}{7} = 1.25000$

$\frac{N_1}{ac}\frac{1}{6} = 1.25000$

$\frac{N_1}{ad}\frac{1}{5} = 1.25000$

$\frac{N_1}{ae}\frac{1}{4} = 1.25000$

$\frac{N_1}{af}\frac{1}{3} = 1.25000$

$\frac{N_1}{ag}\frac{1}{2} = 1.25000$

$\frac{ap}{ab}\frac{1}{6} = 1.25000$

$\frac{ag}{ab}\frac{1}{5} = 1.25000$

$\frac{af}{ab}\frac{1}{4} = 1.25000$

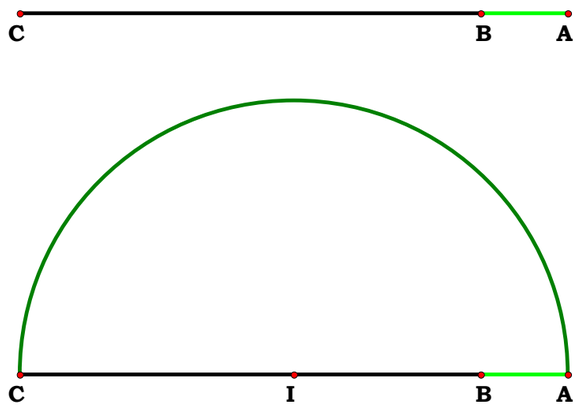
$\frac{ae}{ab}\frac{1}{3} = 1.25000$

$\frac{ad}{ab}\frac{1}{2} = 1.25000$

$\frac{ac}{ab} = 1.25000$

## 120493 Exponential Series M^(1/2^N)

**A number is no more than a digital name used with the stipulation that the indexing system is further qualified by using as standard difference. In other words that the concept of ratio will employ a name called a number. Ratio, however, is independent of the naming convention. Given any ration, say, M, describe a two prime exponential series, M^1/2^N, where N is any whole ratio.**

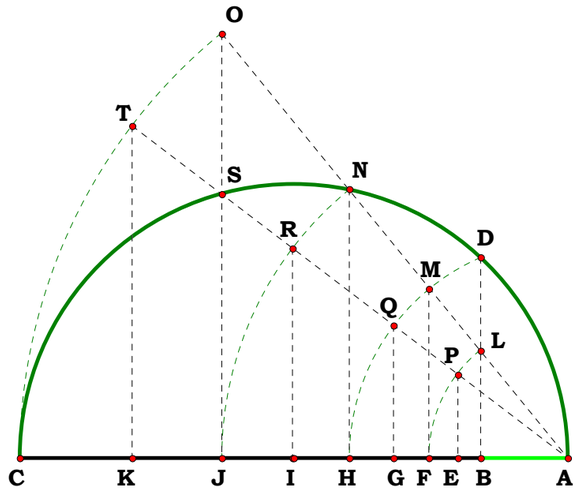



**To use the digital indexing system to apply names, let AC be the thing with which we seek to name an exponential series on. AB is our unit. As a number is a ratio, numbers are two dimensional.**

$N_1 := 1$ $N_2 := 5$ $AB := N_1$ $AC := N_2$

**The circle is a two dimensional object which is capable of producing every ratio between two differences.**

$BC := AC - AB$ $BD := \sqrt{BC \cdot AB}$ $AH := \sqrt{AB^2 + BD^2}$ $CH := AC - AH$ $HN := \sqrt{CH \cdot AH}$




$AJ := \sqrt{AH^2 + HN^2}$ $CJ := AC - AJ$ $JS := \sqrt{CJ \cdot AJ}$ $AK := \sqrt{JS^2 + AJ^2}$

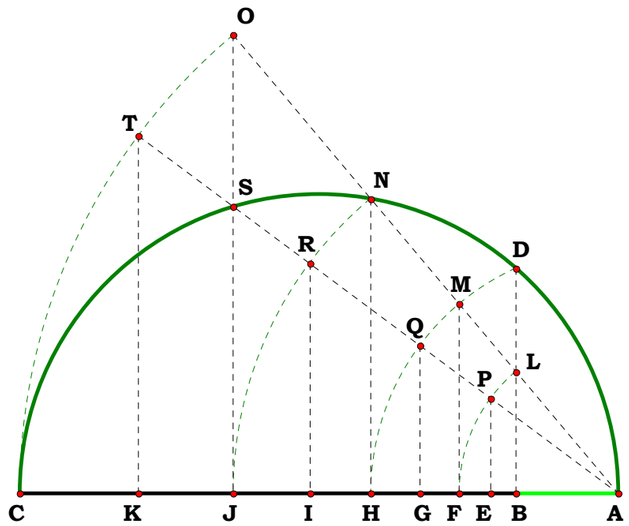
$AI := \frac{AJ^2}{AK}$ $AG := \frac{AH^2}{AI}$ $AF := \frac{AH^2}{AJ}$ $AE := \frac{AF^2}{AG}$ $\left(\frac{AC}{AB}\right)^0 - \frac{AB}{AB} = 0$

$AC^{\frac{1}{8}} - AE = 0$ $AC^{\frac{2}{8}} - AF = 0$ $AC^{\frac{3}{8}} - AG = 0$ $AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0$ $AC^{\frac{6}{8}} - AJ = 0$ $AC^{\frac{7}{8}} - AK = 0$ $AC^{\frac{8}{8}} - AC = 0$

$$\left(\frac{AC}{AB}\right)^{\frac{1}{8}}-\frac{AE}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{2}{8}}-\frac{AF}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{3}{8}}-\frac{AG}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{4}{8}}-\frac{AH}{AB}=0$$

$$\left(\frac{AC}{AB}\right)^{\frac{5}{8}}-\frac{AI}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{6}{8}}-\frac{AJ}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{7}{8}}-\frac{AK}{AB}=0 \quad \left(\frac{AC}{AB}\right)^{\frac{8}{8}}-\frac{AC}{AB}=0$$




| | | | |
|---|---|---|---|
| $AC^0 = 1.00000$ | $N_1 = 1.00000$ | $\frac{AB}{AB} = 1.00000$ | AC = 3.95000 in. |
| $AC^{\frac{1}{8}} = 1.24752$ | AE = 1.24752 | $\frac{AE}{AB} = 1.24752$ | AB = 0.67333 in. |
| $AC^{\frac{2}{8}} = 1.55630$ | AF = 1.55630 | $\frac{AF}{AB} = 1.55630$ | AE = 0.83998 in. |
| $AC^{\frac{3}{8}} = 1.94151$ | AG = 1.94151 | $\frac{AG}{AB} = 1.94151$ | AF = 1.04790 in. |
| $AC^{\frac{4}{8}} = 2.42207$ | AH = 2.42207 | $\frac{AH}{AB} = 2.42207$ | AG = 1.30727 in. |
| $AC^{\frac{5}{8}} = 3.02157$ | AI = 3.02157 | $\frac{AI}{AB} = 3.02157$ | AH = 1.63084 in. |
| $AC^{\frac{6}{8}} = 3.76946$ | AJ = 3.76946 | $\frac{AJ}{AB} = 3.76946$ | AI = 2.03450 in. |
| $AC^{\frac{7}{8}} = 4.70247$ | AK = 4.70247 | $\frac{AK}{AB} = 4.70247$ | AJ = 2.53807 in. |
| $AC^{\frac{8}{8}} = 5.86641$ | AC = 5.86641 | $\frac{AC}{AB} = 5.86641$ | AK = 3.16629 in. |

$AC^0 - N_1 = 0.00000$

$AC^{\frac{1}{8}} - AE = 0.00000$

$AC^{\frac{2}{8}} - AF = 0.00000$

$AC^{\frac{3}{8}} - AG = 0.00000$

$AC^{\frac{4}{8}} - AH = 0.00000$

$AC^{\frac{5}{8}} - AI = 0.00000$

$AC^{\frac{6}{8}} - AJ = 0.00000$

$AC^{\frac{7}{8}} - AK = 0.00000$

$AC^{\frac{8}{8}} - AC = 0.00000$

**Alternate method of creating an exponential series.**

$N_1 = 2.97175$

$N_1^{\frac{8}{8}} = 2.97175$

$N_1^{\frac{7}{8}} = 2.59350$

$N_1^{\frac{6}{8}} = 2.26339$

$N_1^{\frac{5}{8}} = 1.97530$

$N_1^{\frac{4}{8}} = 1.72388$

$N_1^{\frac{3}{8}} = 1.50446$

$N_1^{\frac{2}{8}} = 1.31296$

$N_1^{\frac{1}{8}} = 1.14585$

$N_1^{\frac{9}{8}} = 3.40517$

$N_1^{\frac{10}{8}} = 3.90181$

$N_1^{\frac{11}{8}} = 4.47087$

$N_1^{\frac{-1}{8}} = 0.87272$

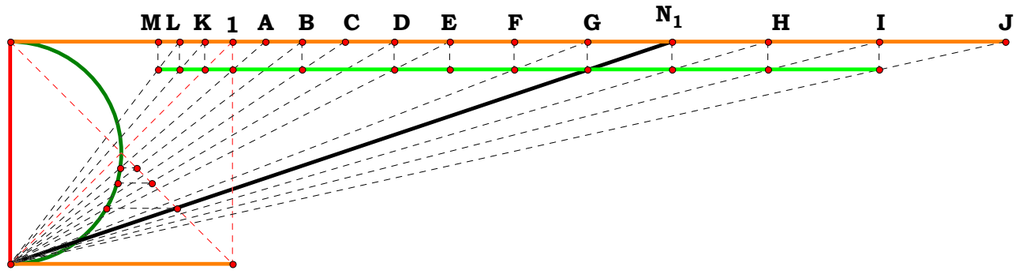
$N_1^{\frac{-2}{8}} = 0.76163$

$N_1^{\frac{-3}{8}} = 0.66469$

A = 1.14585
B = 1.31296
C = 1.50446
D = 1.72388
E = 1.97530
F = 2.26339
G = 2.59350
H = 3.40517
I = 3.90181
J = 4.47087
K = 0.87272
L = 0.76163
M = 0.66469

$N_1^0 = 1.00000$

$N_1^{\frac{8}{8}} - N_1 = 0.00000$




$N_1^{\frac{-1}{8}} - K = 0.00000$

$N_1^{\frac{-2}{8}} - L = 0.00000$

$N_1^{\frac{-3}{8}} - M = 0.00000$

$N_1^{\frac{1}{8}} - A = 0.00000$

$N_1^{\frac{2}{8}} - B = 0.00000$

$N_1^{\frac{3}{8}} - C = 0.00000$

$N_1^{\frac{4}{8}} - D = 0.00000$

$N_1^{\frac{5}{8}} - E = 0.00000$

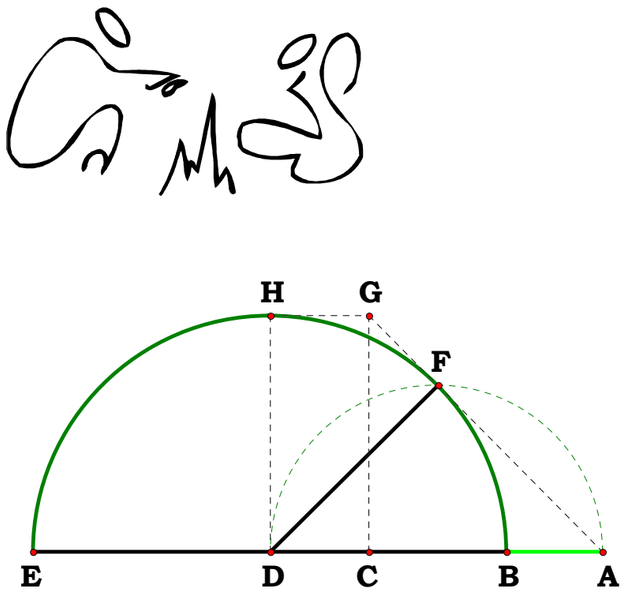
$N_1^{\frac{6}{8}} - F = 0.00000$

$N_1^{\frac{7}{8}} - G = 0.00000$

$N_1^{\frac{9}{8}} - H = 0.00000$

$N_1^{\frac{10}{8}} - I = 0.00000$

$N_1^{\frac{11}{8}} - J = 0.00000$




**120693 Alternate Method: Square Root**
**Common Segment Common Endpoint**

$N_1 := 6 \quad N_2 := 4$

$AB := N_1 \quad BE := N_2$

$AE := AB + BE \quad BD := \frac{BE}{2}$

$DF := BD \quad AD := BD + AB$

$AF := \sqrt{AD^2 - DF^2} \quad AC := AF$

$\sqrt{AB \cdot AE} - AC = 0$

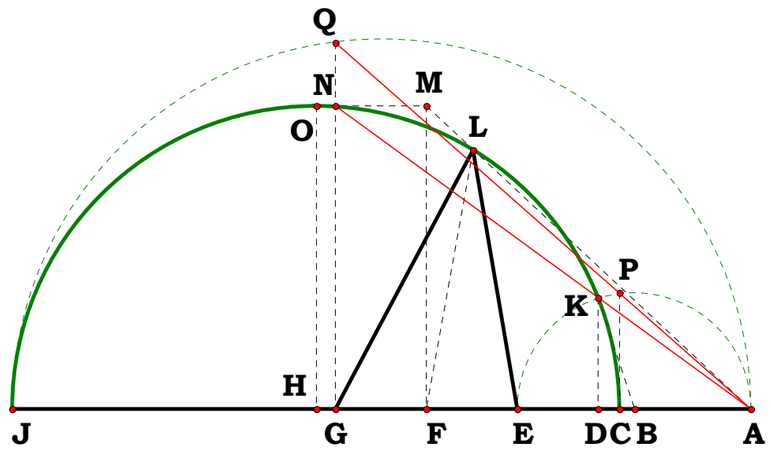
$AE - (N_1 + N_2) = 0 \quad BD - \frac{N_2}{2} = 0$

$DF - \frac{N_2}{2} = 0 \quad AD - \frac{2 \cdot N_1 + N_2}{2} = 0$

$AF - \sqrt{[N_1 \cdot (N_1 + N_2)]} = 0 \quad AC - \sqrt{[N_1 \cdot (N_1 + N_2)]} = 0$

**120693B Gruntwork IV on the Delian Solution.**

**Are A, P and Q collinear? Are A, K and N collinear?**




$N_1 := 5 \qquad N_2 := 6$

$AC := N_1 \qquad CJ := N_2 \qquad AJ := AC + CJ$

$AE := \left(AC^2 \cdot AJ\right)^{\frac{1}{3}} \qquad AG := \left(AC \cdot AJ^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad GJ := CJ - CG \quad GN := \sqrt{CG \cdot GJ}$

$AB := \frac{AE}{2} \quad CE := AE - AC \quad CH := \frac{CJ}{2}$

$BK := AB \quad HK := CH \quad HJ := CH \quad AH := AJ - HJ \quad BH := AH - AB \quad BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$

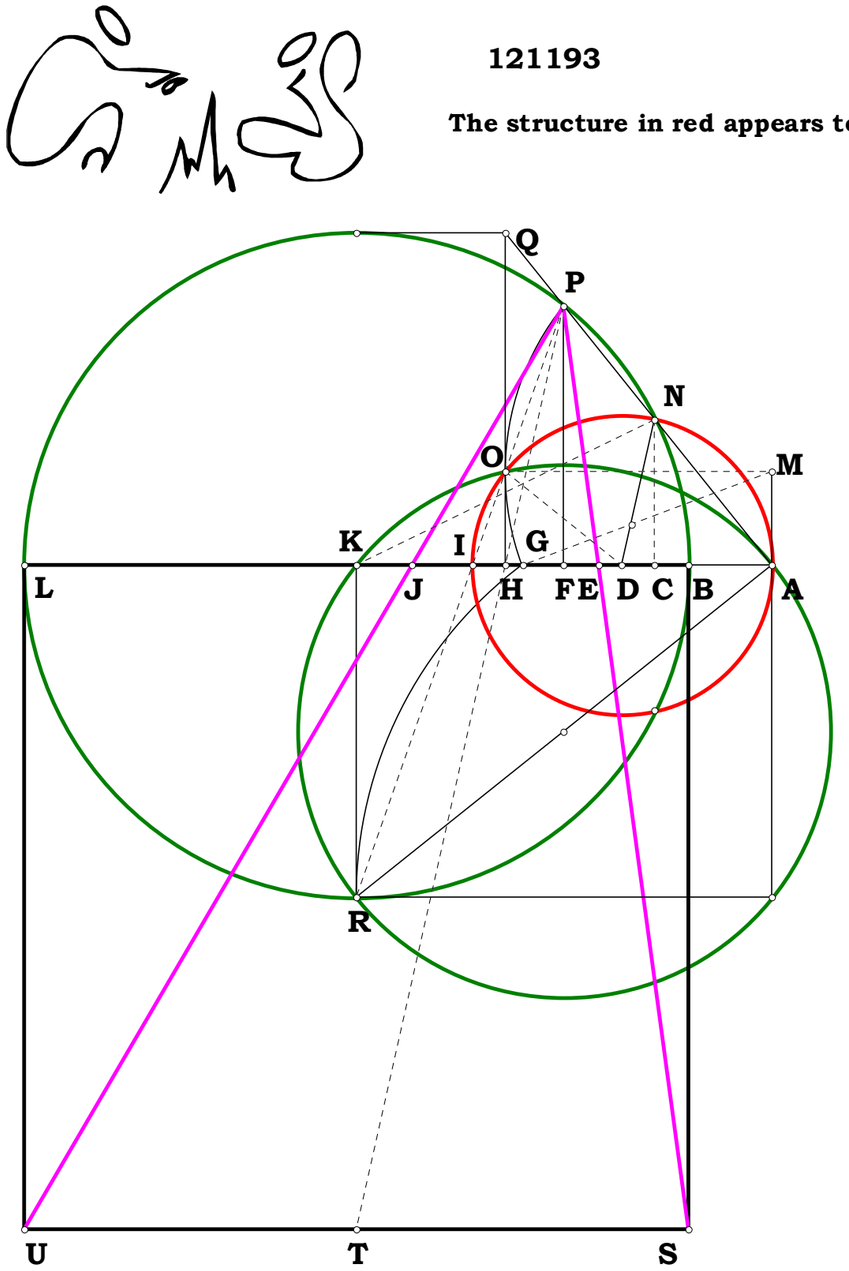
$AD := AB + BD \quad DE := AE - AD \quad DK := \sqrt{AD \cdot DE} \quad GQ := \sqrt{AG \cdot GJ} \quad CP := \sqrt{AC \cdot CE}$

$\frac{AG}{GN} - \frac{AD}{DK} = 0 \qquad \frac{AG}{GQ} - \frac{AC}{CP} = 0$

$$AJ - \left(N_1 + N_2\right) = 0 \qquad AE - \left(N_1^{\,3} + N_1^{\,2} \cdot N_2\right)^{\frac{1}{3}} = 0 \qquad AG - \left(N_1^{\,3} + 2 \cdot N_1^{\,2} \cdot N_2 + N_1 \cdot N_2^{\,2}\right)^{\frac{1}{3}} = 0$$

$$CG - \left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - N_1\right] = 0 \qquad GJ - \left[N_1 + N_2 - \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}\right] = 0$$

$$GN - \sqrt{\left[\left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}} - N_1\right] \cdot \left[N_1 + N_2 - \left[N_1 \cdot \left(N_1 + N_2\right)^2\right]^{\frac{1}{3}}\right]} = 0$$




**121193**

**The structure in red appears to be a constant.**

$N := 6 \quad AB := 1 \quad AL := AB \cdot N$

$BL := AL - AB \quad BK := \frac{BL}{2} \quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB \quad BJ := AJ - AB \quad JL := BL - BJ \quad EJ := AJ - AE \quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ \quad BF := BL - FL \quad FP := \sqrt{BF \cdot FL} \quad KR := BK \quad KL := BK$

$FK := FL - KL \quad IK := \frac{FK \cdot KR}{KR + FP} \quad AK := BK + AB \quad AI := AK - IK \quad AD := \frac{AI}{2}$

$KT := BL \quad FH := \frac{FK \cdot FP}{KT + FP} \quad AF := BF + AB \quad AH := AF + FH \quad HI := AI - AH$

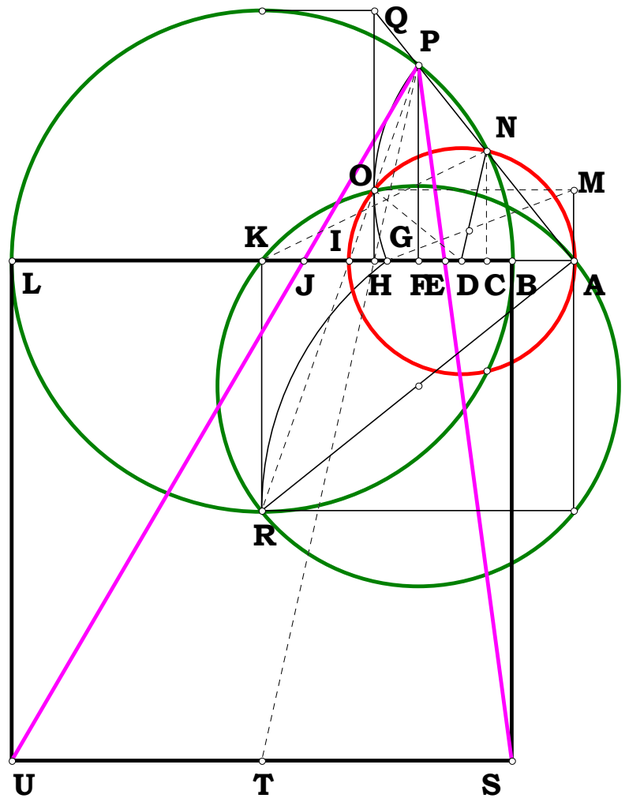
$HO := \sqrt{AH \cdot HI} \quad DN := AD \quad KN := BK \quad DK := AK - AD \quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK \quad CI := AI - AC \quad CN := \sqrt{AC \cdot CI} \quad \frac{KR}{IK} - \frac{HO}{HI} = 0 \quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

$$AB := 1 \quad AL - N = 0 \quad BL - (N-1) = 0 \quad BK - \frac{N-1}{2} = 0 \quad AE - N^{\frac{1}{3}} = 0 \quad AJ - N^{\frac{2}{3}} = 0$$

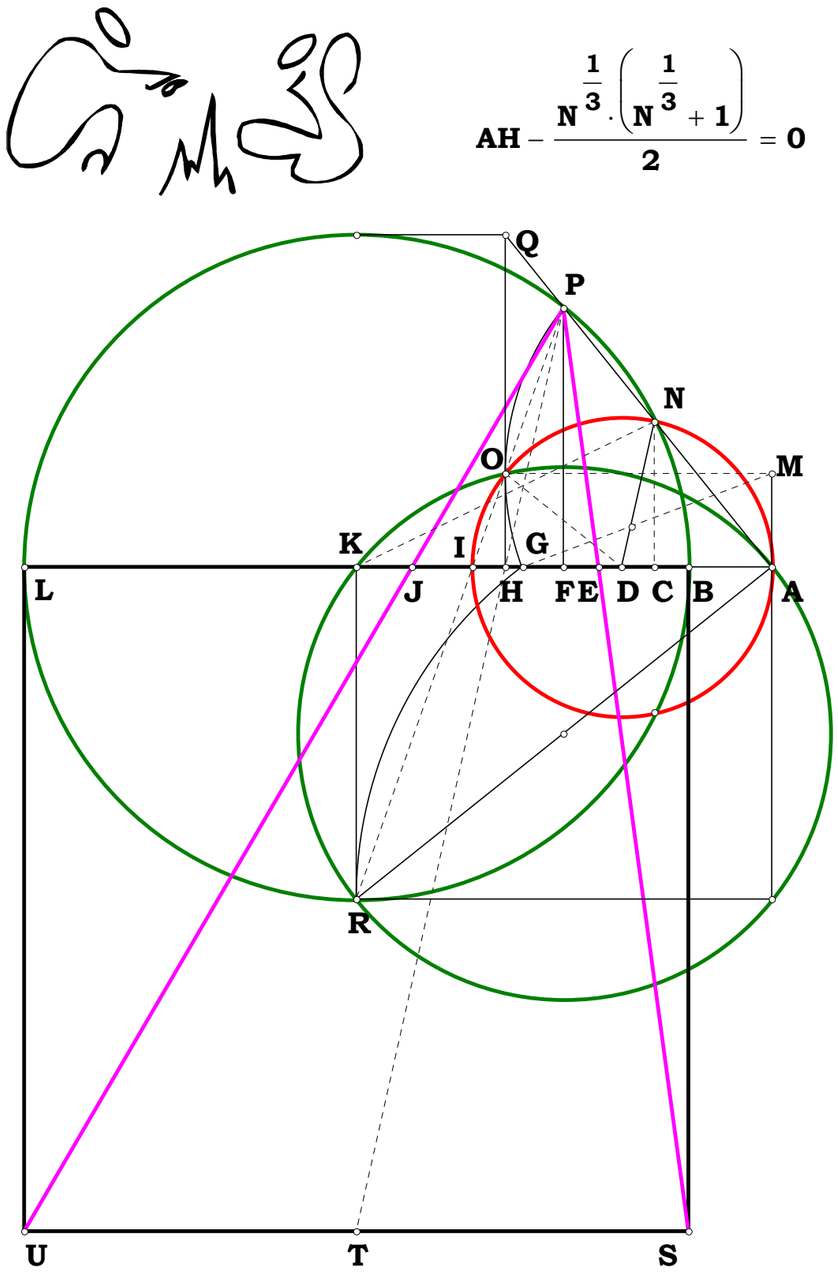
$$BE - \left(N^{\frac{1}{3}} - 1\right) = 0 \quad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \quad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \quad EJ - N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) = 0$$

$$FJ - \frac{N \cdot \left(N^{\frac{1}{3}} - 1\right)}{N^{\frac{2}{3}} + 1} = 0 \quad FL - \frac{N^{\frac{2}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0 \quad BF - \frac{N-1}{N^{\frac{2}{3}} + 1} = 0 \quad FP - \frac{N^{\frac{1}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0$$




$$FK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \quad IK - \frac{\left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$AK - \frac{N+1}{2} = 0 \quad AI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{2}{3}} + 1\right)}{N^{\frac{1}{3}} + 1} = 0 \quad AD - \frac{N + N^{\frac{1}{3}}}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \quad FH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \quad AF - \frac{\left(N^{\frac{1}{3}}\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{2}{3}} + 1} = 0$$




$$AH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{2} = 0 \qquad HI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad HO - \frac{\sqrt{N^{\frac{2}{3}} - 2 \cdot N + N^{\frac{4}{3}}}}{2} = 0$$

$$DK - \frac{N^{\frac{4}{3}} + 1}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad CK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

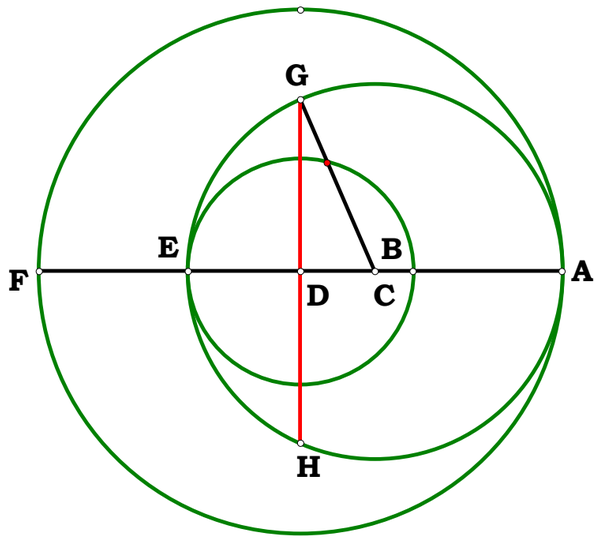
$$AC - \frac{\left(N^{\frac{1}{3}}\right)^3 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{4}{3}} + 1} = 0 \qquad CI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)^2}{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$CN - \frac{N^{\frac{2}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)}{\left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$\frac{KR}{IK} - \frac{N^{\frac{1}{3}} + 1}{N^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{N - 1} = 0$$

## 121293 The Square Root

**Square root by common segment common midpoint. Given AFand BE is GH their root?**




$N_1 := 5 \quad N_2 := 3$

$AF := N_1 \quad BE := N_2$

$AD := \frac{AF}{2} \quad BD := \frac{BE}{2} \quad AB := AD - BD$

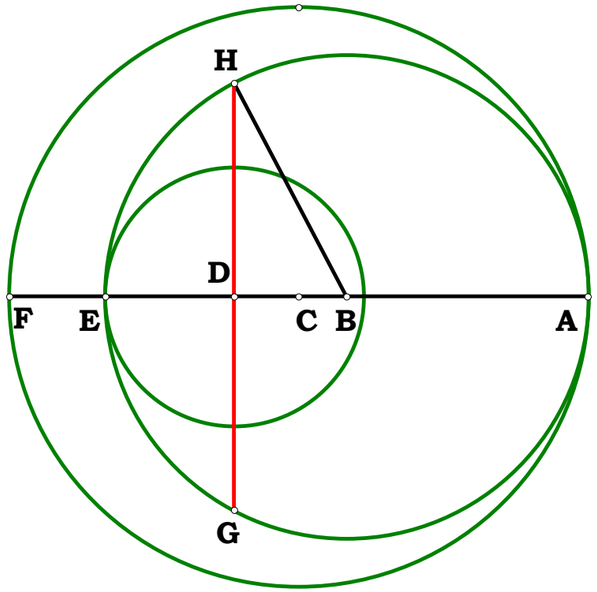
$AE := BE + AB \quad AC := \frac{AE}{2} \quad CG := AC$

$CD := AD - AC \quad GH := 2 \cdot \sqrt{CG^2 - CD^2}$

$GH - \sqrt{AF \cdot BE} = 0$

$GH - \sqrt{N_1 \cdot N_2} = 0$

**121293B Generalize The Previous Square Root Figure**




$$N_1 := 1 \quad N_2 := 3 \quad N_3 := 2$$

$$AF := N_1 \quad DF := \frac{AF}{N_2} \quad AD := AF - DF$$

$$DE := \frac{DF}{N_3} \quad AE := AD + DE \quad AB := \frac{AE}{2}$$

$$BD := AD - AB \quad BH := AB$$

$$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$$

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

12_16_93, using 12_04_93.MCD

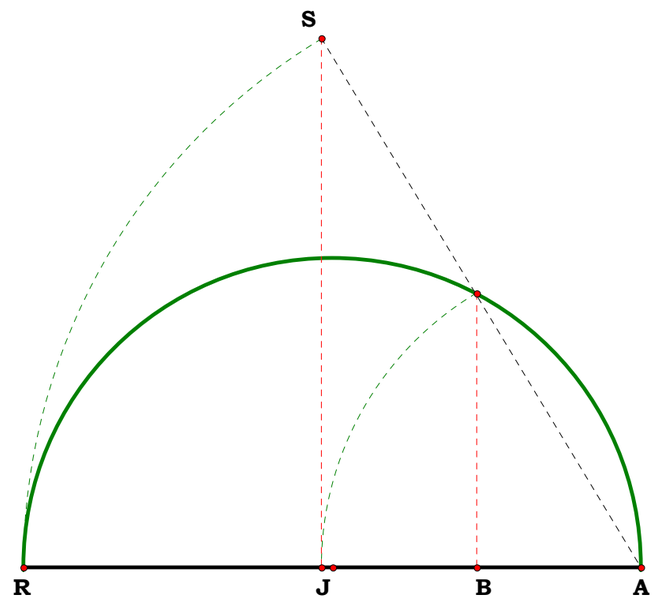
$AR := 10 \quad \Delta := 5 \quad \delta := 2 .. \Delta + 1 \quad AB_\delta := \frac{AR}{\delta}$

$AJ_\delta := \sqrt{AB_\delta \cdot AR} \quad JR_\delta := AR - AJ_\delta$

$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \quad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$

| $AB_\delta$ = | $AJ_\delta$ = |
|---|---|
| 5 | 7.071 |
| 3.333 | 5.774 |
| 2.5 | 5 |
| 2 | 4.472 |
| 1.667 | 4.082 |

**The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.**



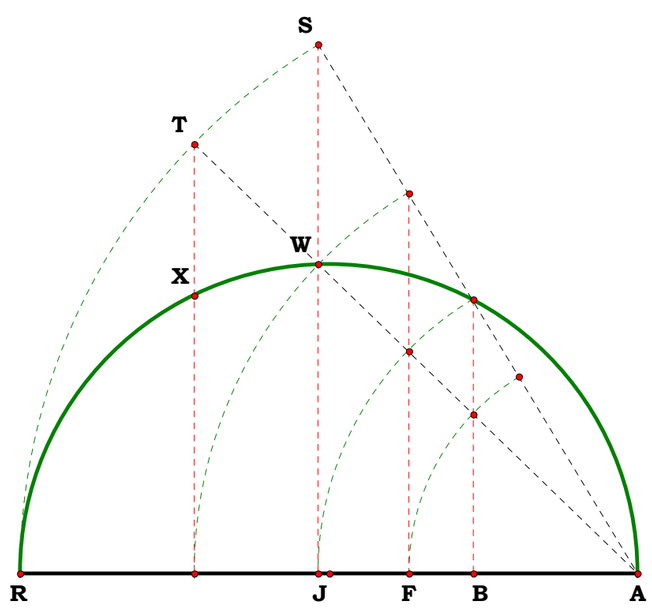

# Euclidean Exponential Series

$AT := AR \quad AN_\delta := AW_\delta \quad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$

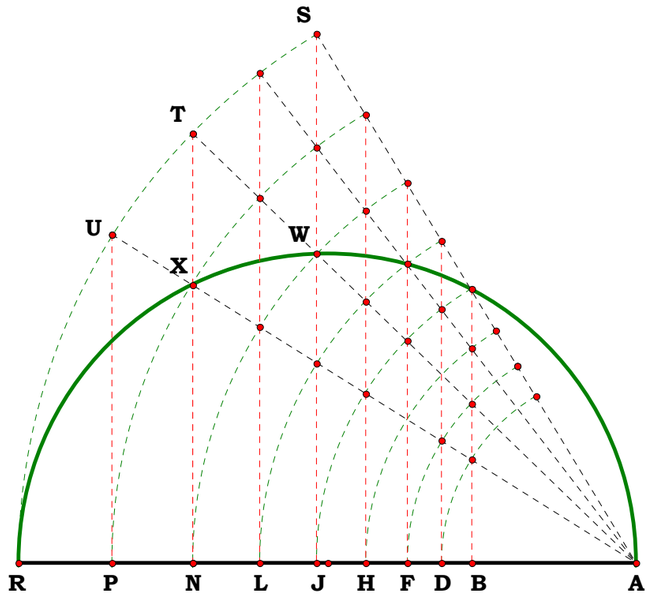
$NR_\delta := AR - AN_\delta \quad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$

$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$

| $AB_\delta$ = | $AF_\delta$ = | $AJ_\delta$ = | $AN_\delta$ = |
|---|---|---|---|
| 5 | 5.946 | 7.071 | 8.409 |
| 3.333 | 4.387 | 5.774 | 7.598 |
| 2.5 | 3.536 | 5 | 7.071 |
| 2 | 2.991 | 4.472 | 6.687 |
| 1.667 | 2.608 | 4.082 | 6.389 |




**What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.**




$$AU := AR \quad AP_\delta := AX_\delta \quad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta} \quad AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \quad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$$

$$PR_\delta := AR - AP_\delta \quad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta} \quad AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$$

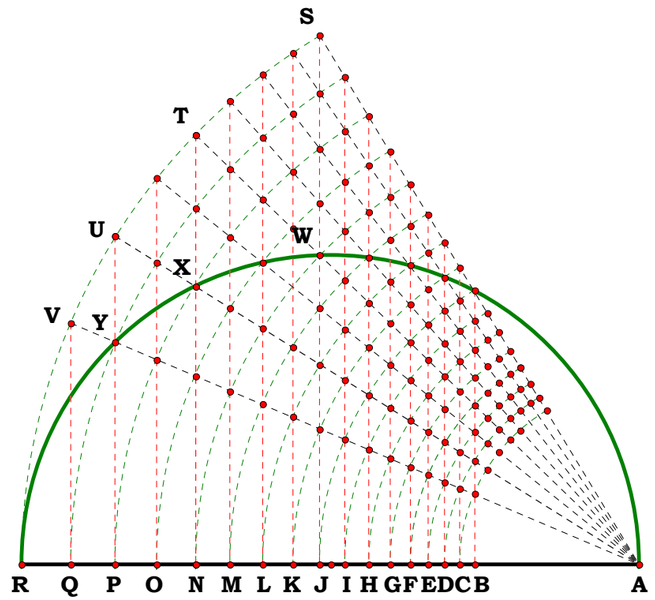
| $AB_\delta$ = | $AD_\delta$ = | $AF_\delta$ = | $AH_\delta$ = | $AJ_\delta$ = | $AL_\delta$ = | $AN_\delta$ = | $AP_\delta$ = |
|---|---|---|---|---|---|---|---|
| 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.333 | 3.824 | 4.387 | 5.033 | 5.774 | 6.623 | 7.598 | 8.717 |
| 2.5 | 2.973 | 3.536 | 4.204 | 5 | 5.946 | 7.071 | 8.409 |
| 2 | 2.446 | 2.991 | 3.657 | 4.472 | 5.469 | 6.687 | 8.178 |
| 1.667 | 2.085 | 2.608 | 3.263 | 4.082 | 5.107 | 6.389 | 7.993 |

$$AV := AR \quad AQ_\delta := AY_\delta \quad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta} \quad AM_\delta := \frac{(AN_\delta)^2}{AO_\delta}$$

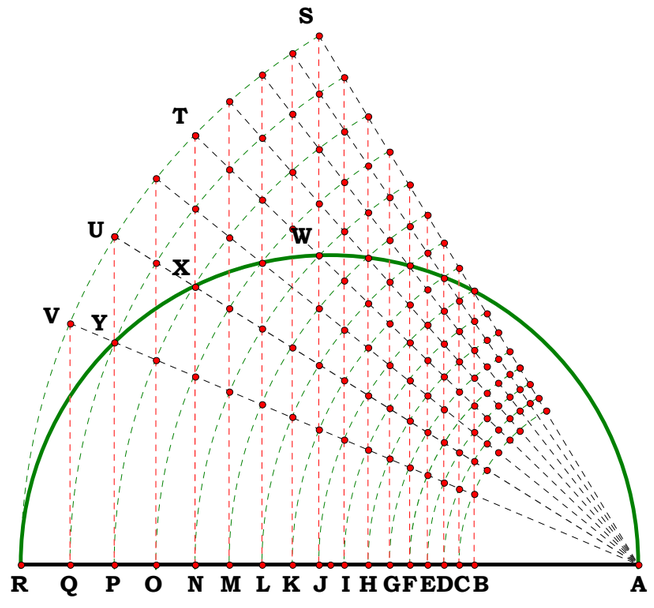
$$AK_\delta := \frac{(AL_\delta)^2}{AM_\delta} \quad AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \quad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \quad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$$

| $AB_\delta$ = | $AC_\delta$ = | $AD_\delta$ = | $AE_\delta$ = | $AF_\delta$ = |
|---|---|---|---|---|
| 5 | 5.221 | 5.453 | 5.694 | 5.946 |
| 3.333 | 3.57 | 3.824 | 4.096 | 4.387 |
| 2.5 | 2.726 | 2.973 | 3.242 | 3.536 |
| 2 | 2.212 | 2.446 | 2.704 | 2.991 |
| 1.667 | 1.864 | 2.085 | 2.332 | 2.608 |



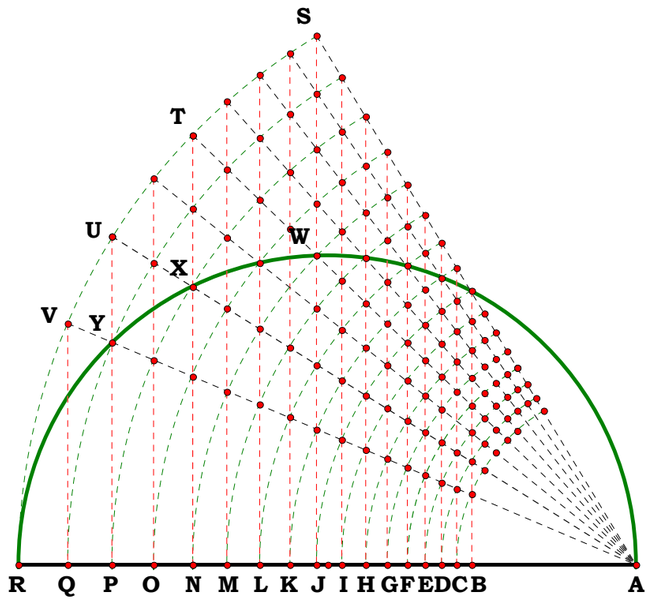
| $AG_\delta$ = | $AH_\delta$ = | $AI_\delta$ = | $AJ_\delta$ = | $AK_\delta$ = | $AL_\delta$ = | $AM_\delta$ = | $AN_\delta$ = | $AO_\delta$ = | $AP_\delta$ = | $AQ_\delta$ = |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.209 | 6.484 | 6.771 | 7.071 | 7.384 | 7.711 | 8.052 | 8.409 | 8.781 | 9.17 | 9.576 |
| 4.699 | 5.033 | 5.39 | 5.774 | 6.184 | 6.623 | 7.094 | 7.598 | 8.138 | 8.717 | 9.336 |
| 3.856 | 4.204 | 4.585 | 5 | 5.453 | 5.946 | 6.484 | 7.071 | 7.711 | 8.409 | 9.17 |
| 3.307 | 3.657 | 4.044 | 4.472 | 4.945 | 5.469 | 6.047 | 6.687 | 7.395 | 8.178 | 9.043 |
| 2.918 | 3.263 | 3.65 | 4.082 | 4.566 | 5.107 | 5.713 | 6.389 | 7.147 | 7.993 | 8.941 |




**Values found by the investigator of**
**12_14_93**

| $AQ_\delta$ = | $\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$ | $AP_\delta$ = | $\left(AB_\delta \cdot AR^{7}\right)^{\frac{1}{8}}$ = | $AN_\delta$ = | $\left(AB_\delta \cdot AR^{3}\right)^{\frac{1}{4}}$ | $AL_\delta$ = | $\left[\left(AB_\delta\right)^{3} \cdot AR^{5}\right]^{\frac{1}{8}}$ | $AI_\delta$ = | $\left[\left(AB_\delta\right)^{9} \cdot AR^{7}\right]^{\frac{1}{16}}$ = |
|---|---|---|---|---|---|---|---|---|---|
| 9.576 | 9.576 | 9.17 | 9.17 | 8.409 | 8.409 | 7.711 | 7.711 | 6.771 | 6.771 |
| 9.336 | 9.336 | 8.717 | 8.717 | 7.598 | 7.598 | 6.623 | 6.623 | 5.39 | 5.39 |
| 9.17 | 9.17 | 8.409 | 8.409 | 7.071 | 7.071 | 5.946 | 5.946 | 4.585 | 4.585 |
| 9.043 | 9.043 | 8.178 | 8.178 | 6.687 | 6.687 | 5.469 | 5.469 | 4.044 | 4.044 |
| 8.941 | 8.941 | 7.993 | 7.993 | 6.389 | 6.389 | 5.107 | 5.107 | 3.65 | 3.65 |

| $AH_\delta$ = | $\left[\left(AB_\delta\right)^{5} \cdot AR^{3}\right]^{\frac{1}{8}}$ | $AG_\delta$ = | $\left[\left(AB_\delta\right)^{11} \cdot AR^{5}\right]^{\frac{1}{16}}$ = | $AF_\delta$ = | $\left[\left(AB_\delta\right)^{3} \cdot AR\right]^{\frac{1}{4}}$ = |
|---|---|---|---|---|---|
| 6.484 | 6.484 | 6.209 | 6.209 | 5.946 | 5.946 |
| 5.033 | 5.033 | 4.699 | 4.699 | 4.387 | 4.387 |
| 4.204 | 4.204 | 3.856 | 3.856 | 3.536 | 3.536 |
| 3.657 | 3.657 | 3.307 | 3.307 | 2.991 | 2.991 |
| 3.263 | 3.263 | 2.918 | 2.918 | 2.608 | 2.608 |




$AE_\delta =$

| |
|---|
| 5.694 |
| 4.096 |
| 3.242 |
| 2.704 |
| 2.332 |

$\left[(AB_\delta)^{13} \cdot AR^3\right]^{\frac{1}{16}}$

| |
|---|
| 5.694 |
| 4.096 |
| 3.242 |
| 2.704 |
| 2.332 |

$AD_\delta =$

| |
|---|
| 5.453 |
| 3.824 |
| 2.973 |
| 2.446 |
| 2.085 |

$\left[(AB_\delta)^7 \cdot AR\right]^{\frac{1}{8}}$

| |
|---|
| 5.453 |
| 3.824 |
| 2.973 |
| 2.446 |
| 2.085 |

$= AC_\delta =$

| |
|---|
| 5.221 |
| 3.57 |
| 2.726 |
| 2.212 |
| 1.864 |

$\left[(AB_\delta)^{15} \cdot AR\right]^{\frac{1}{16}} =$

| |
|---|
| 5.221 |
| 3.57 |
| 2.726 |
| 2.212 |
| 1.864 |

$AQ_\delta =$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$\left(AB_\delta \cdot AR^{15}\right)^{\frac{1}{16}}$

| |
|---|
| 9.576 |
| 9.336 |
| 9.17 |
| 9.043 |
| 8.941 |

$= AP_\delta =$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$\left[(AB_\delta)^2 \cdot AR^{14}\right]^{\frac{1}{16}}$

| |
|---|
| 9.17 |
| 8.717 |
| 8.409 |
| 8.178 |
| 7.993 |

$AO_\delta =$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$\left[(AB_\delta)^3 \cdot AR^{13}\right]^{\frac{1}{16}} =$

| |
|---|
| 8.781 |
| 8.138 |
| 7.711 |
| 7.395 |
| 7.147 |

$AN_\delta =$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$\left[(AB_\delta)^4 \cdot AR^{12}\right]^{\frac{1}{16}}$

| |
|---|
| 8.409 |
| 7.598 |
| 7.071 |
| 6.687 |
| 6.389 |

$= AM_\delta =$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$\left[(AB_\delta)^5 \cdot AR^{11}\right]^{\frac{1}{16}}$

| |
|---|
| 8.052 |
| 7.094 |
| 6.484 |
| 6.047 |
| 5.713 |

$= AL_\delta =$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$\left[(AB_\delta)^6 \cdot AR^{10}\right]^{\frac{1}{16}} =$

| |
|---|
| 7.711 |
| 6.623 |
| 5.946 |
| 5.469 |
| 5.107 |

$AK_\delta =$

| |
|---|
| 7.384 |
| 6.184 |
| 5.453 |
| 4.945 |
| 4.566 |

$\left[(AB_\delta)^7 \cdot AR^9\right]^{\frac{1}{16}}$

| |
|---|
| 7.384 |
| 6.184 |
| 5.453 |
| 4.945 |
| 4.566 |

$= AJ_\delta =$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$\left[(AB_\delta)^8 \cdot AR^8\right]^{\frac{1}{16}}$

| |
|---|
| 7.071 |
| 5.774 |
| 5 |
| 4.472 |
| 4.082 |

$AI_\delta =$

| |
|---|
| 6.771 |
| 5.39 |
| 4.585 |
| 4.044 |
| 3.65 |

$\left[(AB_\delta)^9 \cdot AR^7\right]^{\frac{1}{16}}$

| |
|---|
| 6.771 |
| 5.39 |
| 4.585 |
| 4.044 |
| 3.65 |

$= AH_\delta =$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$\left[(AB_\delta)^{10} \cdot AR^6\right]^{\frac{1}{16}} =$

| |
|---|
| 6.484 |
| 5.033 |
| 4.204 |
| 3.657 |
| 3.263 |

$AG_\delta = \left[(AB_\delta)^{11} \cdot AR^5\right]^{\frac{1}{16}}$ $AF_\delta = \left[(AB_\delta)^{12} \cdot AR^4\right]^{\frac{1}{16}} =$

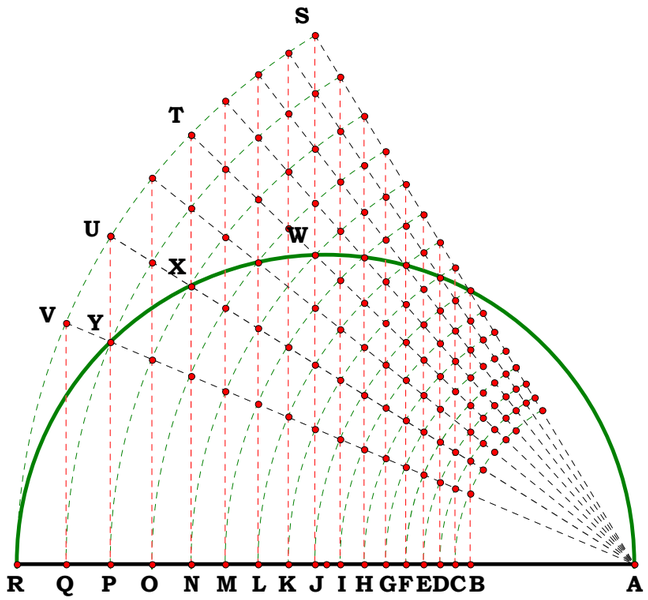
| $AG_\delta$ | $\left[(AB_\delta)^{11} \cdot AR^5\right]^{\frac{1}{16}}$ | $AF_\delta$ | $\left[(AB_\delta)^{12} \cdot AR^4\right]^{\frac{1}{16}}$ |
|---|---|---|---|
| 6.209 | 6.209 | 5.946 | 5.946 |
| 4.699 | 4.699 | 4.387 | 4.387 |
| 3.856 | 3.856 | 3.536 | 3.536 |
| 3.307 | 3.307 | 2.991 | 2.991 |
| 2.918 | 2.918 | 2.608 | 2.608 |

$AE_\delta = \left[(AB_\delta)^{13} \cdot AR^3\right]^{\frac{1}{16}}$ $AD_\delta = \left[(AB_\delta)^{14} \cdot AR^2\right]^{\frac{1}{16}} =$

| $AE_\delta$ | $\left[(AB_\delta)^{13} \cdot AR^3\right]^{\frac{1}{16}}$ | $AD_\delta$ | $\left[(AB_\delta)^{14} \cdot AR^2\right]^{\frac{1}{16}}$ |
|---|---|---|---|
| 5.694 | 5.694 | 5.453 | 5.453 |
| 4.096 | 4.096 | 3.824 | 3.824 |
| 3.242 | 3.242 | 2.973 | 2.973 |
| 2.704 | 2.704 | 2.446 | 2.446 |
| 2.332 | 2.332 | 2.085 | 2.085 |

$AC_\delta = \left[(AB_\delta)^{15} \cdot AR\right]^{\frac{1}{16}}$ $AB_\delta = \left[(AB_\delta)^{16} \cdot AR^0\right]^{\frac{1}{16}} =$

| $AC_\delta$ | $\left[(AB_\delta)^{15} \cdot AR\right]^{\frac{1}{16}}$ | $AB_\delta$ | $\left[(AB_\delta)^{16} \cdot AR^0\right]^{\frac{1}{16}}$ |
|---|---|---|---|
| 5.221 | 5.221 | 5 | 5 |
| 3.57 | 3.57 | 3.333 | 3.333 |
| 2.726 | 2.726 | 2.5 | 2.5 |
| 2.212 | 2.212 | 2 | 2 |
| 1.864 | 1.864 | 1.667 | 1.667 |




**Resultant Equation**

$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

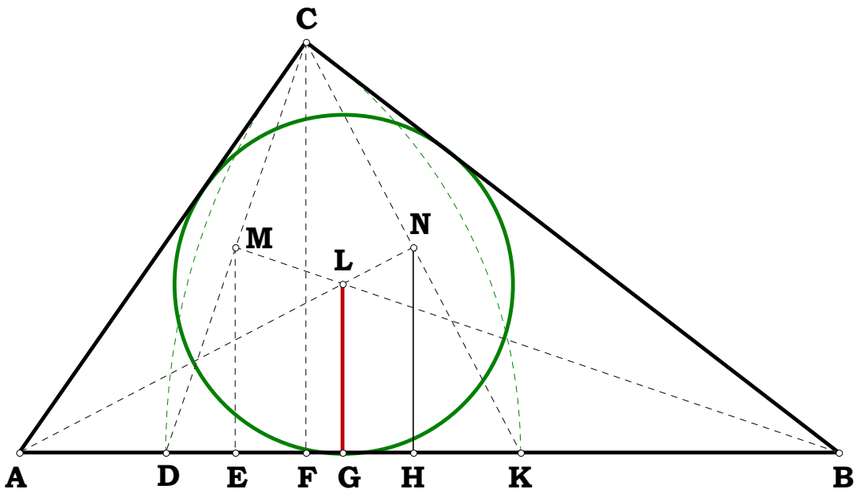
**or**

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

**depending on direction of transcription.**

## 040694 Inscribing A Circle In A Given Triangle

**Given three sides of a triangle, what is the length of the inscribed radius?**




$$AB := 3 \qquad BC := 4 \qquad AC := 5$$

$$AK := AC \qquad BD := BC \qquad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB}$$

$$FK := AK - AF \qquad CF := \sqrt{AC^2 - AF^2}$$

$$CK := \sqrt{FK^2 + CF^2} \qquad AN := \frac{CF \cdot AK}{CK}$$

$$AH := \frac{CF \cdot AN}{CK} \qquad HN := \frac{FK \cdot AN}{CK} \qquad BF := AB - AF$$

$$DF := BD - BF \qquad CD := \sqrt{CF^2 + DF^2} \qquad BM := \frac{CF \cdot BD}{CD} \qquad BE := \frac{CF \cdot BM}{CD} \qquad GL := \frac{HN \cdot AB}{AH + BE}$$

$$S_1 := \begin{pmatrix} AB \\ BC \\ AC \end{pmatrix} \quad S_2 := \begin{pmatrix} BC \\ AC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} AC \\ AB \\ BC \end{pmatrix} \quad \delta := 0 \,..\, 2$$

$$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

DE = 0.58830

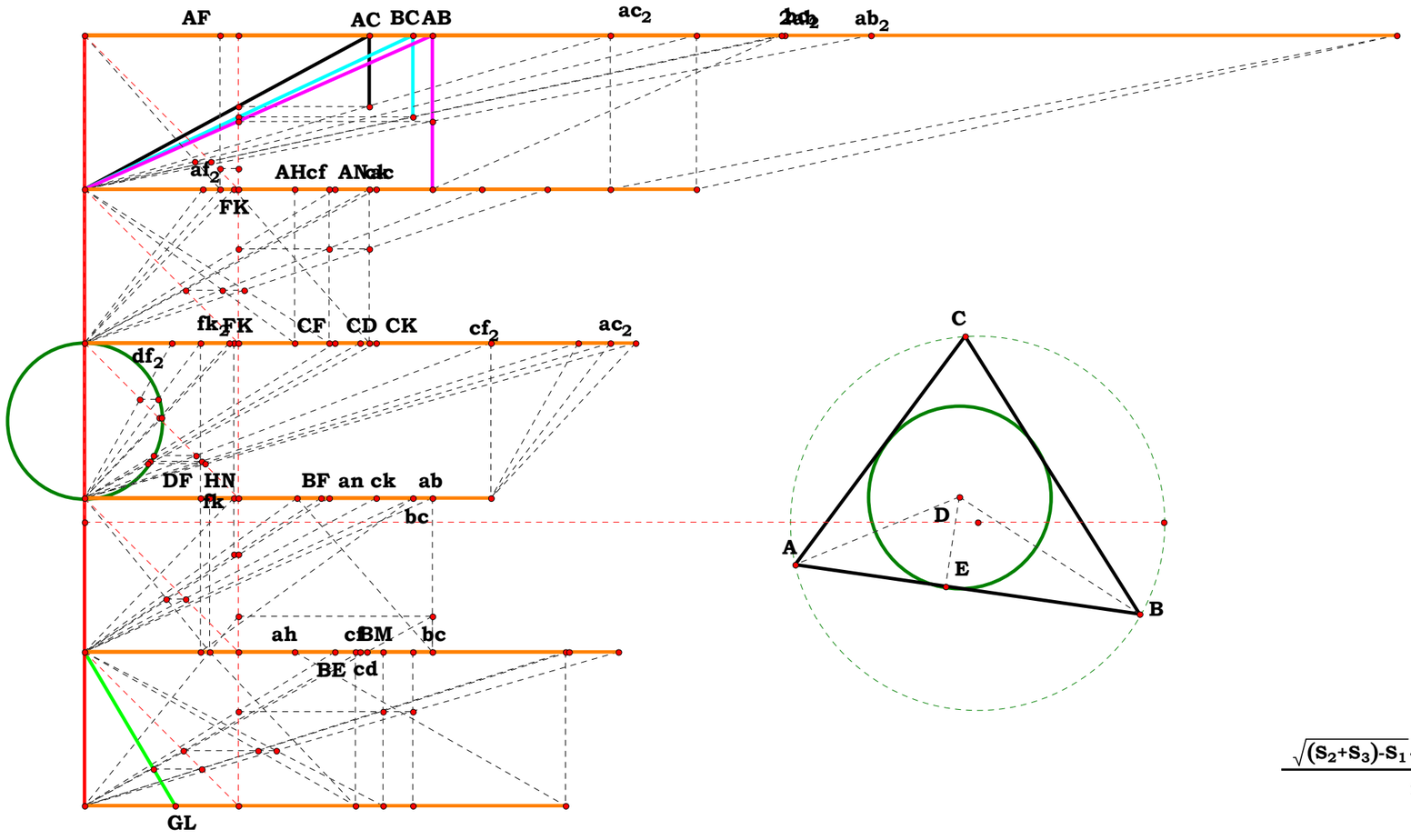
GL = 0.58830

DE-GL = 0.00000

AB = 2.25959

BC = 2.13235

AC = 1.84623





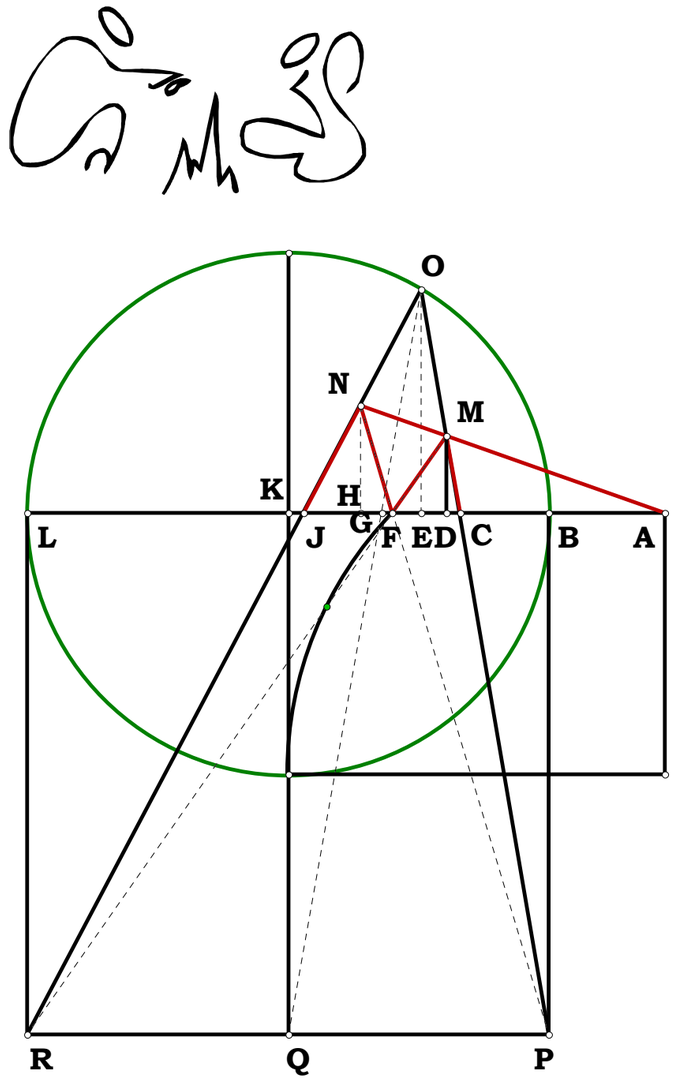


$S_1$ = 2.25959

$S_2$ = 2.13235

$S_3$ = 1.84623

$$\frac{\sqrt{(S_2+S_3)-S_1}\cdot\sqrt{(S_1+S_3)-S_2}\cdot\sqrt{(S_1+S_2)-S_3}}{2\cdot\sqrt{S_1+S_2+S_3}}-GL = 0.00000$$




**042194 The Cradle**

**Are A, M, N colinear?**

$N := 5 \quad AB := 1 \quad AL := AB \cdot N$

$AF := \sqrt{AB \cdot AL} \quad AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \quad BP := BL \quad LR := BL \quad FL := AL - AF$

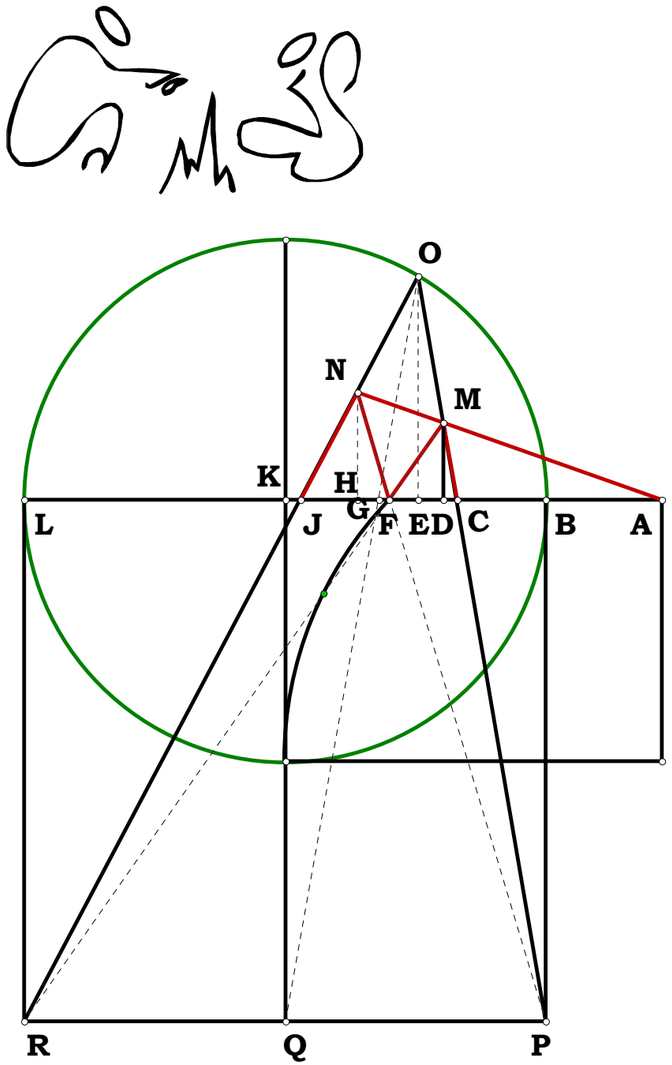
$BC := AC - AB \quad BJ := AJ - AB \quad JL := BL - BJ$

$BF := AF - AB \quad FJ := AJ - AF \quad CF := AF - AC$

$FG := \frac{BF \cdot FJ}{BF + JL} \quad GN := \frac{BP \cdot FG}{BF} \quad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \quad AD := AC + CD \quad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$




$$AL - N = 0 \qquad AF - \sqrt{N} = 0 \qquad AC - N^{\frac{1}{3}} = 0 \qquad AJ - N^{\frac{2}{3}} = 0 \qquad BL - (N-1) = 0 \qquad FL - \left(N - \sqrt{N}\right) = 0$$

$$BC - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left[N - 1 - \left(N^{\frac{2}{3}} - 1\right)\right] = 0 \qquad BF - \left(\sqrt{N} - 1\right) = 0$$

$$FJ - \left(N^{\frac{2}{3}} - \sqrt{N}\right) = 0 \qquad CF - \left[\sqrt{N} - (N)^{\frac{1}{3}}\right] = 0 \qquad FG - \frac{\sqrt{N}\cdot\left(\sqrt{N} - 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$

$$GN - \frac{\sqrt{N}\cdot(N-1)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad CD - \frac{N^{\frac{1}{3}}\cdot\left(N^{\frac{1}{3}} - 1\right)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$

$$DM - \frac{N^{\frac{1}{3}}\cdot(N-1)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad AD - \frac{\left(N^{\frac{1}{6}}\right)^3\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$$

$$AG - \frac{N^{\frac{2}{3}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0 \qquad \frac{AG}{GN} - \frac{N^{\frac{1}{6}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N - 1} = 0 \qquad \frac{AD}{DM} - \frac{N^{\frac{1}{6}}\cdot\left(\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + 1\right)}{N - 1} = 0$$

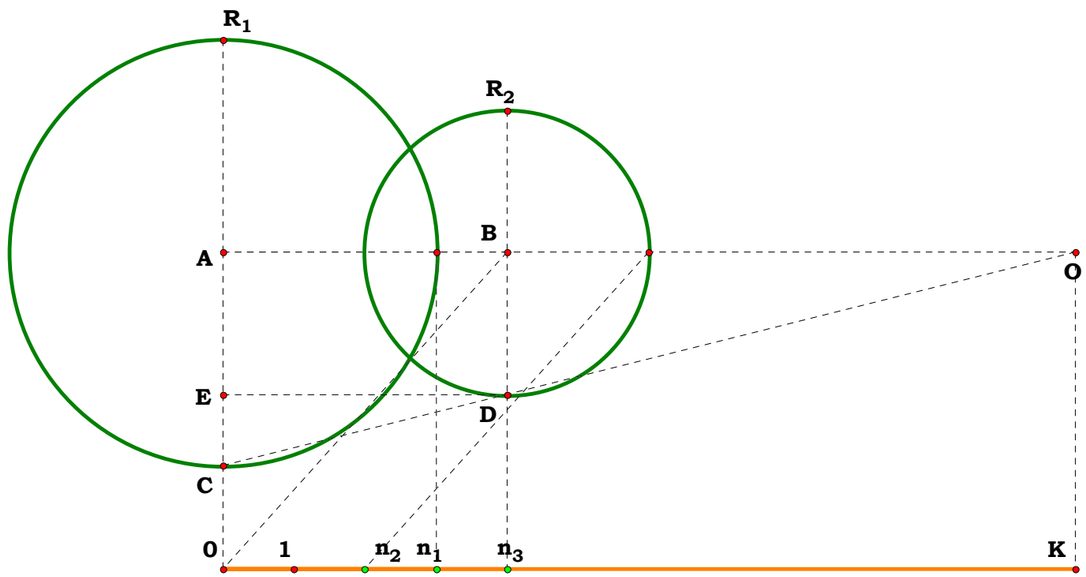
$N_1 := -3 \quad N_2 := 2 \quad N_3 := -7$

$R_1 := \sqrt{N_1^{\,2}} \quad R_2 := \sqrt{N_2^{\,2}}$

**The above is one way using a numbered line for values that do not have negative values.**

$AC := R_1 \quad BD := R_2 \quad AB := N_3$

**042694 Tangents and Similarity Points.**



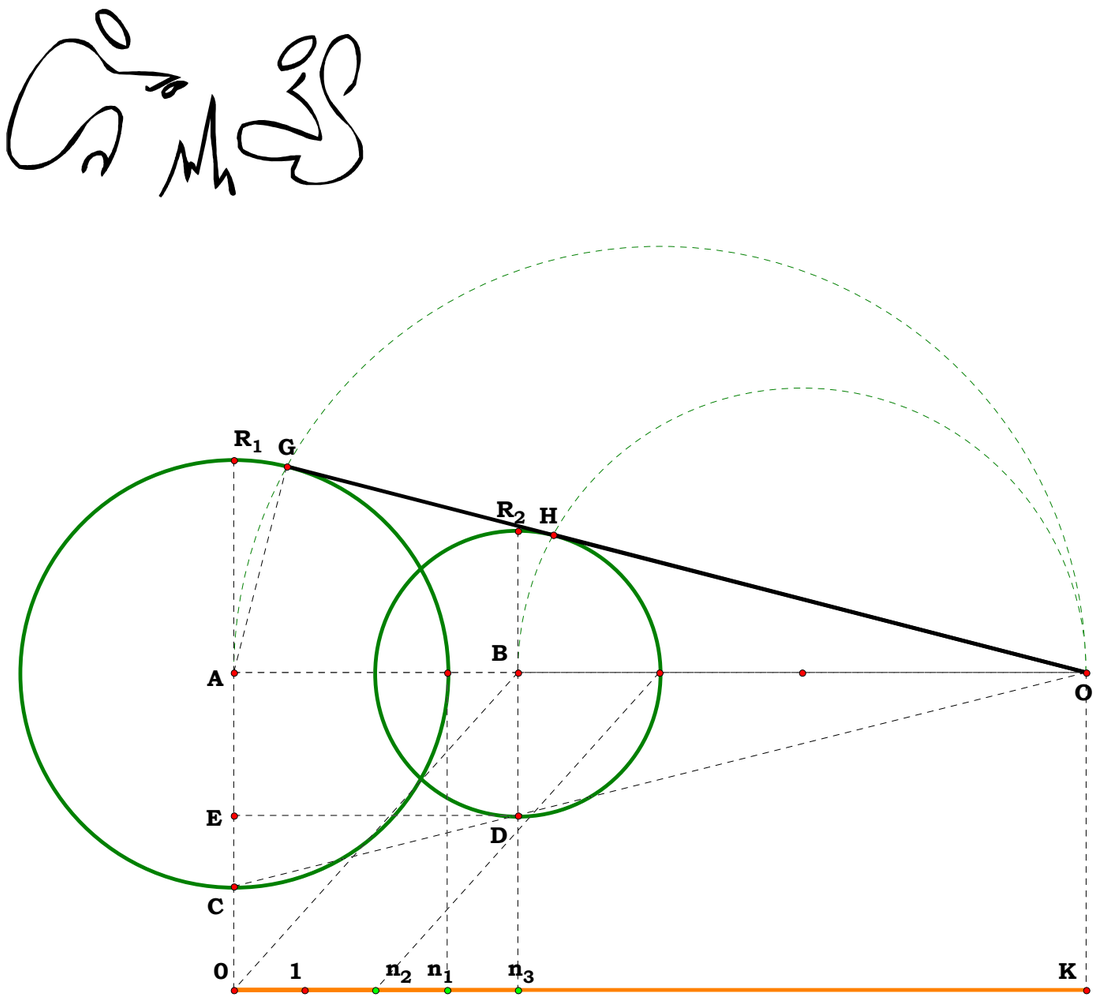

**What is the length of the AO, O being the similarity point?**

$DE := AB \quad AE := BD \quad CE := AC - AE$

$AO := \dfrac{DE \cdot AC}{CE} \qquad AO = -21$

$\dfrac{N_3 \cdot R_1}{R_1 - R_2} = -21$

$AO - \dfrac{N_3 \cdot R_1}{R_1 - R_2} = 0$




**What is the length of the tangent GO?**

$$\mathbf{AG} := \mathbf{R_1} \qquad \mathbf{GO} := \sqrt{\mathbf{AO}^2 - \mathbf{AG}^2}$$

$$\mathbf{GO} - \frac{\mathbf{R_1} \cdot \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 + 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2}}{\sqrt{(\mathbf{R_1} - \mathbf{R_2})^2}} = 0$$

**What is the length of the tangent HO?**

$$\mathbf{BH} := \mathbf{R_2} \qquad \mathbf{HO} := \sqrt{\left(\frac{\mathbf{N_3} \cdot \mathbf{R_1}}{\mathbf{R_1} - \mathbf{R_2}} - \mathbf{N_3}\right)^2 - \mathbf{R_2}^2}$$

$$\mathbf{HO} - \frac{\mathbf{R_2} \cdot \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 + 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2}}{\sqrt{(\mathbf{R_1} - \mathbf{R_2})^2}} = 0$$

**What is the length of line tangent to tangent of these circles?**

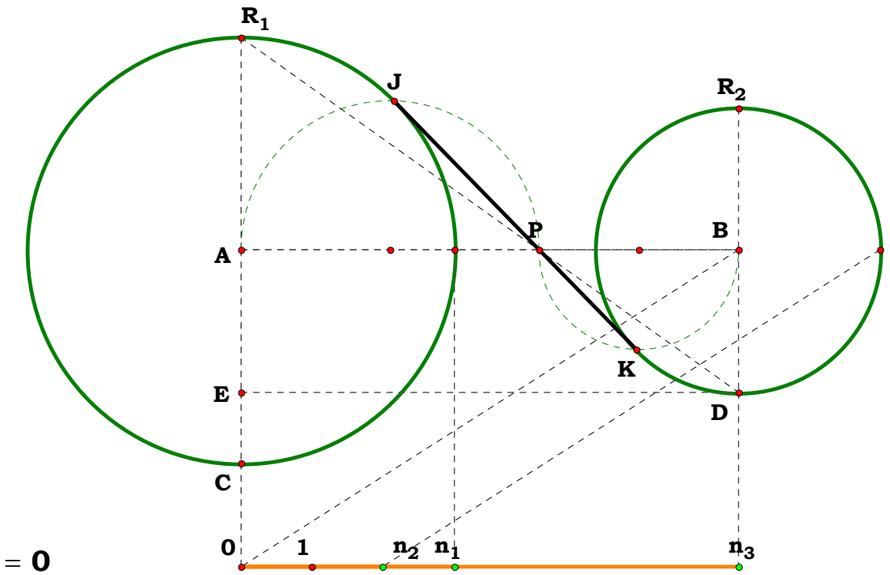
$$\mathbf{GH} := \frac{\mathbf{GO} \cdot \mathbf{AB}}{\mathbf{AO}} \qquad \mathbf{GH} - \sqrt{\mathbf{N_3}^2 - \mathbf{R_1}^2 + 2 \cdot \mathbf{R_1} \cdot \mathbf{R_2} - \mathbf{R_2}^2} = 0$$

**What are the names of the tangents AP and BP to the similarity point P?**

$$AP := \frac{N_3 \cdot R_1}{R_1 + R_2} \qquad BP := N_3 - \frac{N_3 \cdot R_1}{R_1 + R_2} \qquad BP - \frac{N_3 \cdot R_2}{R_1 + R_2} = 0$$

**What is JP?**

$$JP := \sqrt{AP^2 - R_1^2} \qquad JP - \frac{R_1 \cdot \sqrt{N_3^2 - R_1^2 - 2 \cdot R_1 \cdot R_2 - R_2^2}}{R_1 + R_2} = 0$$

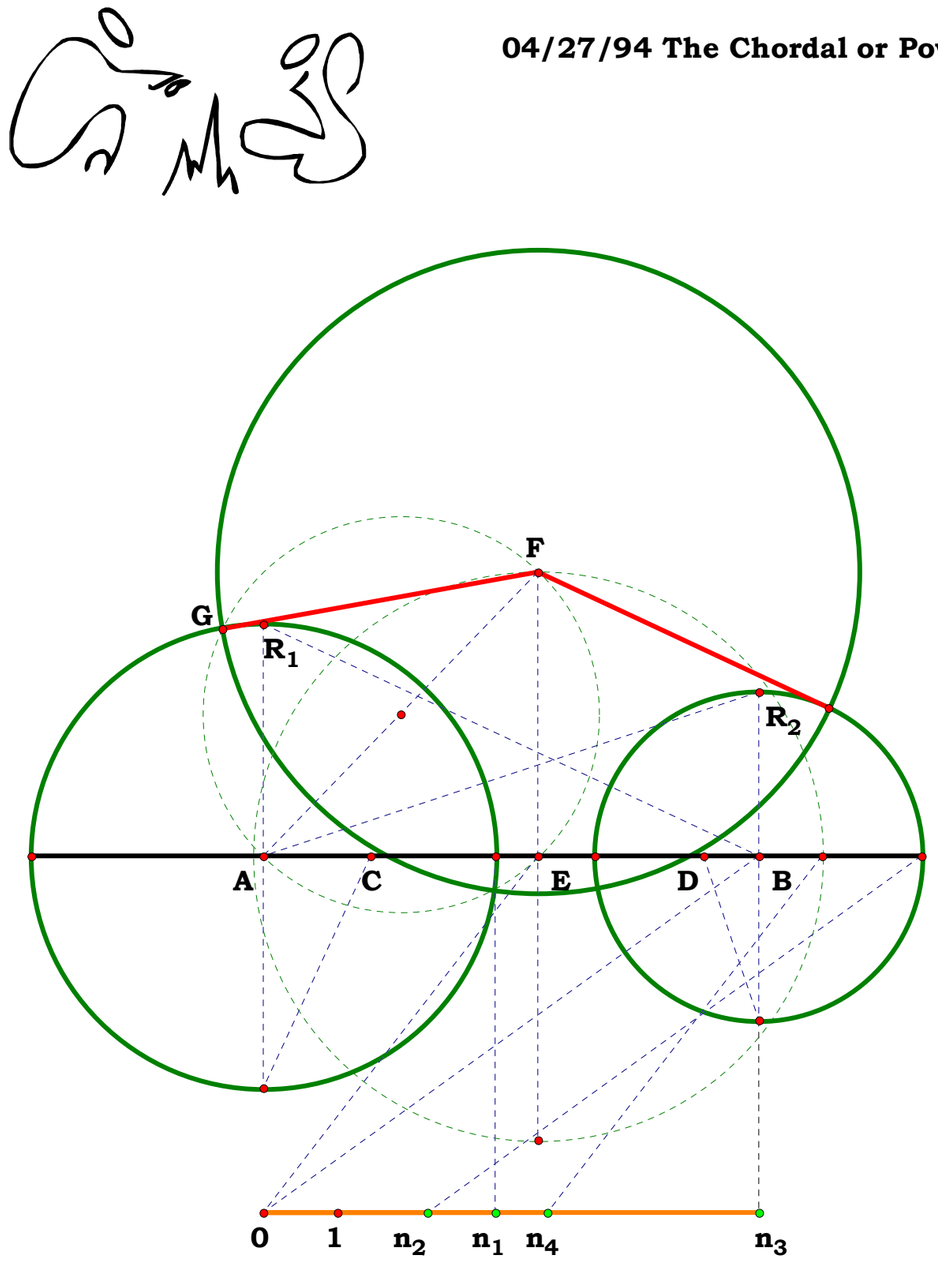



**What is KP?**

$$KP := \frac{JP \cdot BP}{AP} \qquad KP - \frac{R_2 \cdot \sqrt{N_3^2 - R_1^2 - 2 \cdot R_1 \cdot R_2 - R_2^2}}{R_1 + R_2} = 0$$

**What is JK?**

$$JK := \frac{JP \cdot AB}{AP} \qquad JK - \sqrt{N_3^2 - R_1^2 - 2 \cdot R_1 \cdot R_2 - R_2^2} = 0$$




## 04/27/94 The Chordal or Power Line of two Circles

**The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to an ordered process, so I took a couple of minuets (Bach) and developed my own method.**
**One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.**
**Given two circles find their chordal or power line given just their radius and difference between their centers. Then, pick a spot on the power line and write the equation for the tangent circle.**

$N_1 := 3 \qquad N_2 := 2 \qquad N_3 := 6 \qquad R_1 := \sqrt{N_1^2} \qquad R_2 := \sqrt{N_2^2} \qquad AB := N_3$

$$AC := \frac{R_1^2}{AB} \qquad BD := \frac{R_2^2}{AB} \qquad CD := AB - (AC + BD) \qquad CE := \frac{CD}{2}$$

$$AE := AC + CE \qquad BE := AB - AE$$

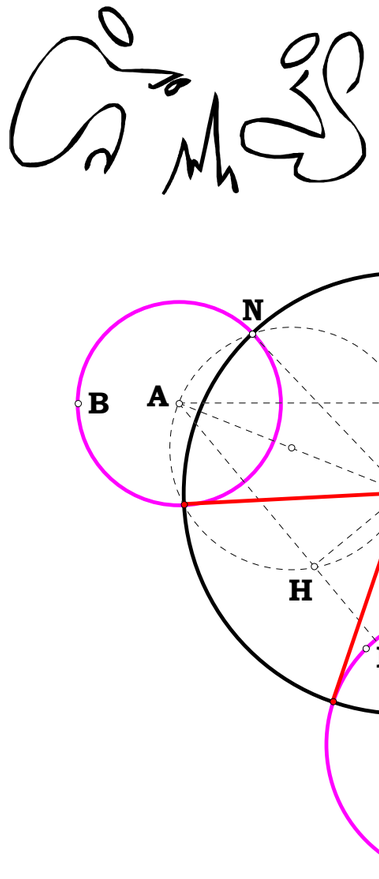
$$AE - \frac{N_3^2 + R_1^2 - R_2^2}{2 \cdot N_3} = 0$$

$$BE - \frac{N_3^2 - R_1^2 + R_2^2}{2 \cdot N_3} = 0$$

**If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.**

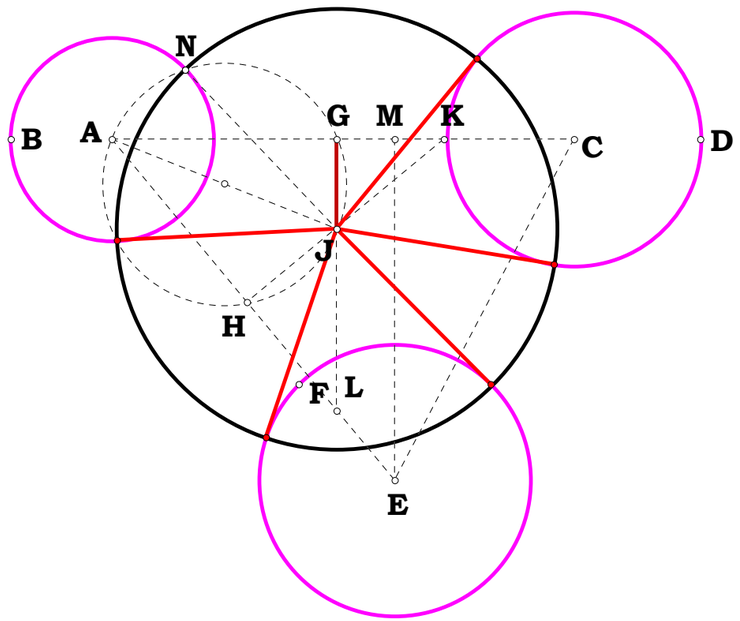
$N_4 := .698 \qquad AF := \sqrt{AE^2 + N_4^2} \qquad AG := R_1 \qquad FG := \sqrt{AF^2 - R_1^2}$

$$FG - \frac{\sqrt{N_3^2 \cdot \left(N_3^2 + 4 \cdot N_4^2 - 2 \cdot R_1^2 - 2 \cdot R_2^2\right) + \left(R_1 - R_2\right)^2 \cdot \left(R_1 + R_2\right)^2}}{2 \cdot \sqrt{N_3^2}} = 0$$



## 042894 Power Point

**Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.**




$N_1 := 3 \quad N_2 := 1 \quad N_3 := 2 \quad AC := 7.81447 \quad AE := 6.96686 \quad CE := 5.33279$

$R_1 := \sqrt{N_1^2} \quad R_2 := \sqrt{N_2^2} \quad R_3 := \sqrt{N_3^2} \quad D_1 := AC \quad D_2 := AE \quad D_3 := CE$

$$AG := \frac{\sqrt{\left(R_1^2 + D_1^2 - R_2^2\right)^2}}{2 \cdot D_1} \qquad AH := \frac{\sqrt{\left(R_1^2 + D_2^2 - R_3^2\right)^2}}{2 \cdot D_2}$$

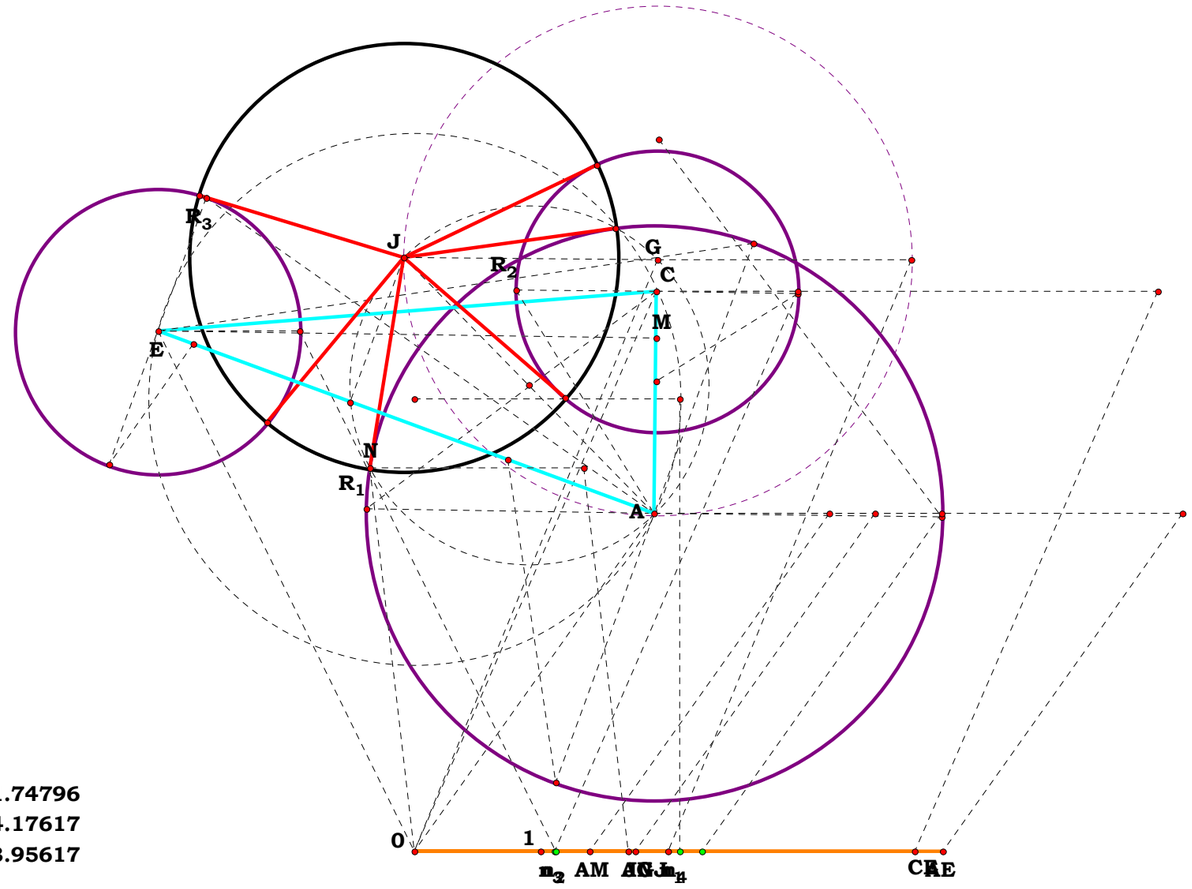
$$AM := \frac{\sqrt{\left(D_2^2 + D_1^2 - D_3^2\right)^2}}{2 \cdot D_1}$$

$$EM := \sqrt{AE^2 - AM^2} \qquad AK := \frac{AE \cdot AH}{AM} \qquad GK := AK - AG \qquad GJ := \frac{AM \cdot GK}{EM}$$

$$GJ - \frac{2 \cdot D_1^2 \cdot \sqrt{\left(D_2^2 + R_1^2 - R_3^2\right)^2} - \sqrt{\left(D_1^2 + D_2^2 - D_3^2\right)^2} \cdot \sqrt{\left(D_1^2 + R_1^2 - R_2^2\right)^2}}{2 \cdot \sqrt{D_1^2} \cdot \sqrt{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_2 - D_1 + D_3\right) \cdot \left(D_1 + D_2 + D_3\right)}} = 0$$

$AJ := \sqrt{AG^2 + GJ^2} \quad AN := R_1 \quad JN := \sqrt{AJ^2 - AN^2}$

$$JN - \sqrt{\frac{\begin{array}{l} D_1^2 \cdot R_3^2 \cdot \left(R_1^2 + R_2^2 - R_3^2 - D_1^2\right) + D_1^2 \cdot D_2^2 \cdot \left(R_2^2 + R_3^2 - D_3^2\right) \ \ldots \\ + \ D_2^2 \cdot R_1^2 \cdot \left(D_3^2 + R_2^2 - R_3^2\right) + D_1^2 \cdot D_3^2 \cdot \left(R_1^2 + R_3^2\right) + D_3^2 \cdot R_1^2 \cdot \left(R_2^2 + R_3^2 - D_3^2 - R_1^2\right) \ \ldots \\ + \ D_2^2 \cdot R_2^2 \cdot \left(R_3^2 - D_2^2 - R_2^2\right) + D_3^2 \cdot R_2^2 \cdot \left(D_2 - R_3\right) \cdot \left(D_2 + R_3\right) - D_1^2 \cdot R_1^2 \cdot R_2^2 \end{array}}{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 + D_2 + D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_1 - D_2 - D_3\right)}} = 0$$




$N_1 = 2.27017$
$N_2 = 1.11001$
$N_3 = 1.12235$
$N_4 = 2.09689$

$R_1 = 2.27017$
$R_2 = 1.11001$
$R_3 = 1.12235$

$AC = 1.74796$
$AE = 4.17617$
$CE = 3.95617$
$AM = 1.38576$
$GJ = 2.00571$
$JN = 1.68883$

$D_1 = 1.74796$
$D_2 = 4.17617$
$D_3 = 3.95617$

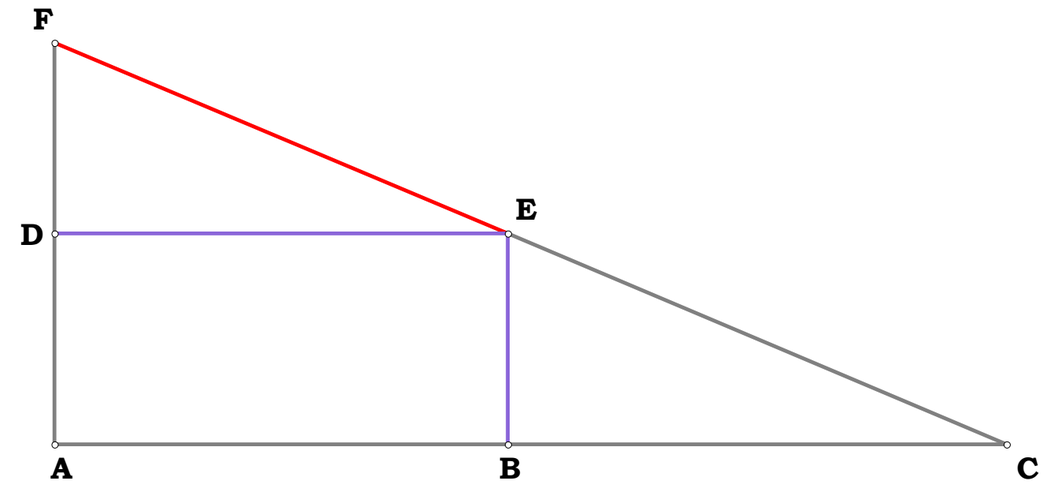
Animate Points

$$\frac{\sqrt{((D_1^2+D_2^2)-D_3^2)^2}}{2\cdot D_1}\text{-AM} = 0.00000$$

$$\frac{2\cdot D_1^2\cdot\sqrt{((D_2^2+R_1^2)-R_3^2)^2}-\sqrt{((D_1^2+D_2^2)-D_3^2)^2}\cdot\sqrt{((D_1^2+R_1^2)-R_2^2)^2}}{2\cdot\sqrt{D_1^2}\cdot\sqrt{((D_1+D_2)-D_3)\cdot((D_1-D_2)+D_3)\cdot((D_2-D_1)+D_3)\cdot(D_1+D_2+D_3)}}\text{-GJ} = 0.00000$$

$$\sqrt{\frac{(D_1^2\cdot R_3^2\cdot((R_1^2+R_2^2)-R_3^2-D_1^2)+D_1^2\cdot D_2^2\cdot((R_2^2+R_3^2)-D_3^2))+(D_2^2\cdot R_1^2\cdot((D_3^2+R_2^2)-R_3^2)+D_1^2\cdot D_3^2\cdot(R_1^2+R_3^2)+D_3^2\cdot R_1^2\cdot((R_2^2+R_3^2)-D_3^2-R_1^2))+((D_2^2\cdot R_2^2\cdot(R_3^2-D_2^2-R_2^2)+D_3^2\cdot R_2^2\cdot(D_2-R_3)\cdot(D_2+R_3))-D_1^2\cdot R_1^2\cdot R_2^2)}{(((D_1+D_2)-D_3)\cdot(D_1+D_2+D_3)\cdot((D_1-D_2)+D_3)\cdot(D_1-D_2-D_3))}}\text{-JN} = 0.00000$$

042994




**Given AB, AF, BE, what is EF?**

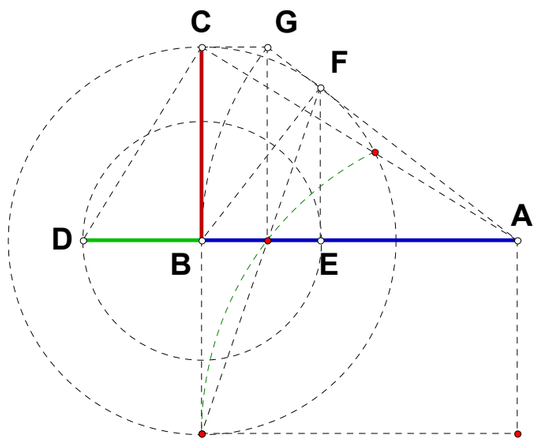
$N_1 := 2 \quad N_2 := 3 \quad N_3 := 5 \quad AF := N_1 \quad BE := N_2 \quad AB := N_3$

$AD := BE \quad DE := AB \quad DF := AF - AD \quad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$

$EF - \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + N_3^2} = 0$

**043094 Division $N^2$**




$N_1 := 5 \qquad N_2 := 2$

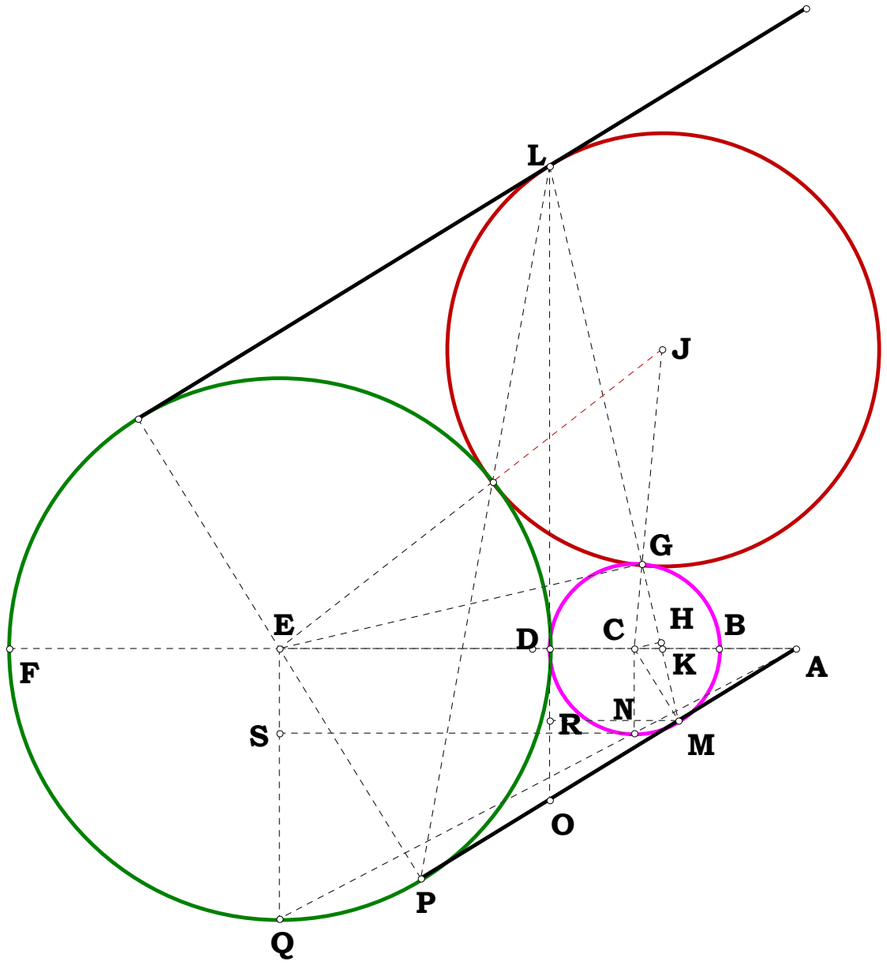
$AB := N_1 \qquad BC := N_2 \qquad AC := \sqrt{AB^2 + BC^2}$

$CD := \frac{BC \cdot AC}{AB} \qquad BD := \sqrt{CD^2 - BC^2}$

$\frac{N_2^2}{N_1} - BD = 0$

## 050194 Two Circles And A Parallel

**Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.**




$R_1 := 3 \quad R_2 := 2$

$DE := R_1 \quad BC := R_2 \quad CN := BC \quad EQ := DE \quad CD := BC \quad CE := CD + DE$

$ES := CN \quad NS := CE \quad SQ := EQ - ES \quad AE := \frac{NS \cdot EQ}{SQ} \quad AD := AE - DE \quad EP := DE$

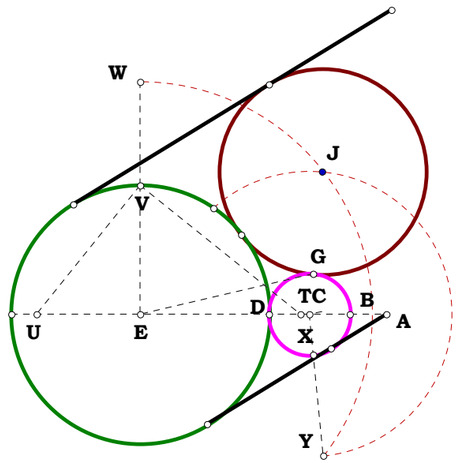
$AP := \sqrt{AE^2 - EP^2} \quad DO := \frac{EP \cdot AD}{AP} \quad DL := \frac{DO \cdot DE}{CD} \quad AC := AD - CD$

$AM := \frac{AP \cdot AC}{AE} \quad AO := \frac{AE \cdot AD}{AP} \quad MO := AO - AM \quad MR := \frac{AD \cdot MO}{AO} \quad RO := \frac{DO \cdot MR}{AD}$

$DR := DO - RO \quad LR := DR + DL \quad ML := \sqrt{MR^2 + LR^2} \quad DK := \frac{MR \cdot DL}{LR}$

$CK := DK - CD \quad CH := \frac{LR \cdot CK}{ML} \quad CM := BC \quad MH := \sqrt{CM^2 - CH^2}$

$MG := 2 \cdot MH \quad GL := ML - MG \quad GJ := \frac{CM \cdot GL}{MG}$




$R_3 := \frac{R_1^2}{4 \cdot R_2} \quad GJ - R_3 = 0$

$ET := 4 \cdot BC \quad EV := DE \quad EU := \frac{DE^2}{ET}$

$VW := EU \quad XY := EU \quad EV := DE \quad CX := BC$

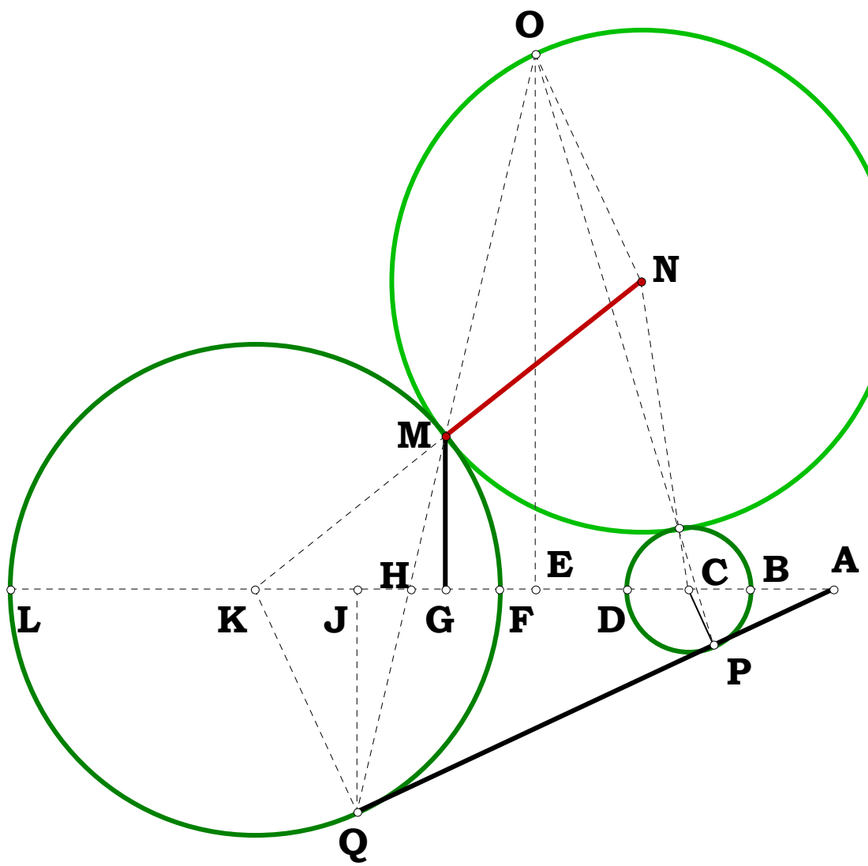
$EW := EV + VW \quad CY := CX + XY$

$GJ_2 := EU \quad GJ - GJ_2 = 0$

**The Algebraic name for GJ suggests a simpler method of construction.**

## 050494 Two Circles And A Tangent

**Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.**




$$R_1 := 3 \quad R_2 := 2 \quad D := 2 \quad dx := 2$$

$$FK := R_1 \quad BC := R_2 \quad CH := D \quad FL := 2 \cdot FK \quad AK := \frac{D \cdot R_1}{R_1 - R_2}$$

$$EK := \frac{R_1^{\,2} + D^2 - R_2^{\,2}}{2 \cdot D} \quad AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2} \quad FG := \frac{FL}{dx}$$

$$GL := FL - FG \quad GM := \sqrt{FG \cdot GL} \quad AJ := \frac{AQ \cdot AQ}{AK} \quad AF := AK - FK$$

$$FJ := AJ - AF \quad JL := FL - FJ \quad JQ := \sqrt{FJ \cdot JL} \quad GJ := FJ - FG$$

$$QM := \sqrt{(JQ + GM)^2 + GJ^2} \quad GH := \frac{GJ \cdot GM}{JQ + GM} \quad HM := \frac{QM \cdot GM}{JQ + GM}$$

$$EF := EK - FK \quad EH := EF + FG + GH \quad HO := \frac{HM \cdot EH}{GH} \quad MO := HO - HM$$

$$KM := FK \quad MN := \frac{KM \cdot MO}{QM}$$

$$MN = -4.5$$

$$MN - \frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

$$\sqrt{\left[\frac{(4 \cdot R_1 \cdot D) - dx \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2 \cdot dx \cdot (R_2 + D - R_1) - 4 \cdot D}\right]^2} = 4.5$$

$AB := 2.38542$

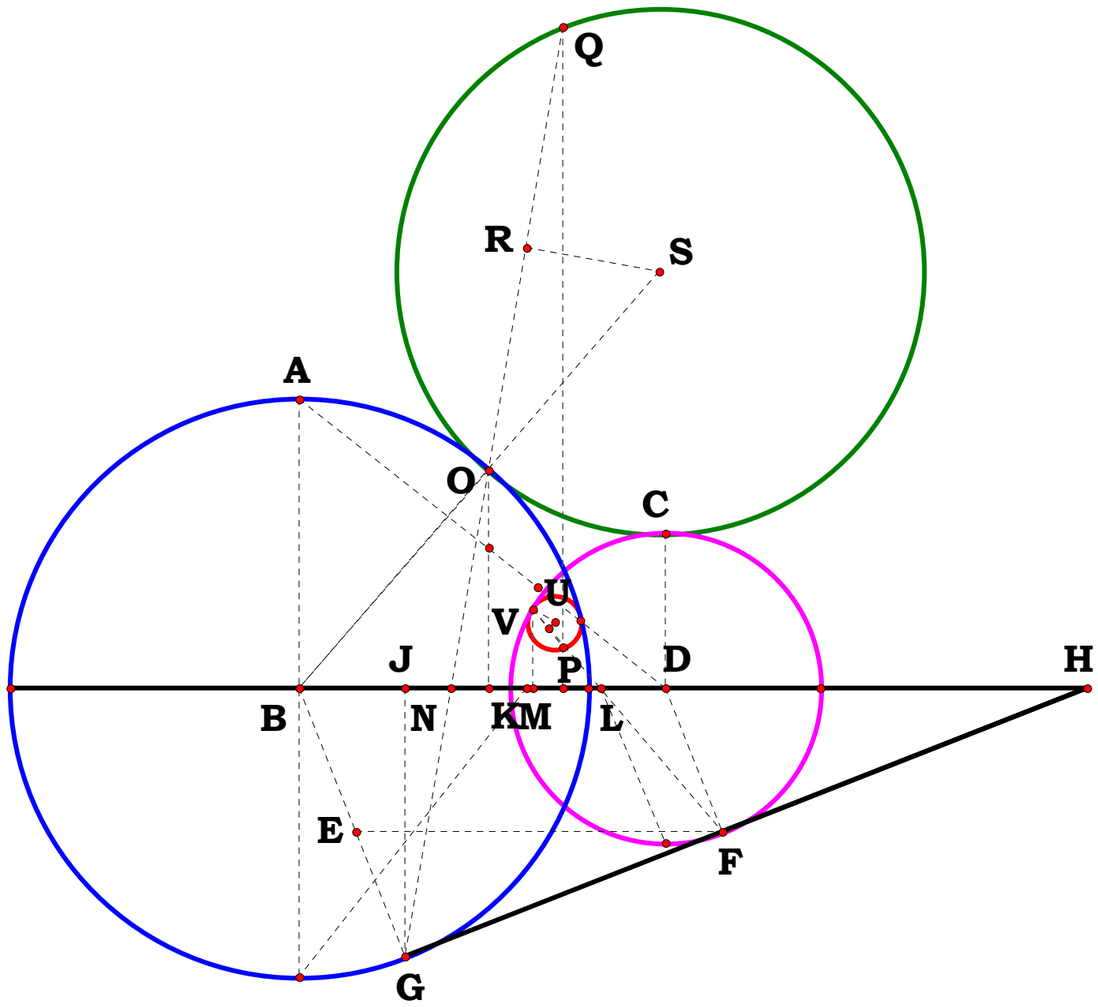
$CD := 1.72917$

$BD := 3.17708$

$dx := 1.22729$

$$\frac{N_3^2 + R_1^2 - R_2^2}{2 \cdot N_3}$$

From 042794 power line.




$$EF := BD \quad EG := AB - CD \quad BH := \frac{EF \cdot AB}{EG} \quad BJ := \frac{AB^2}{BH} \quad KL := \frac{2 \cdot AB}{dx}$$

$$JL := AB - BJ \quad JK := JL - KL \quad GJ := \sqrt{(2 \cdot AB - JL) \cdot JL}$$

$$KO := \sqrt{(2 \cdot AB - KL) \cdot KL} \quad KN := \frac{JK \cdot KO}{GJ + KO} \quad BP := \frac{BD^2 + AB^2 - CD^2}{2 \cdot BD}$$

$$JN := \frac{JK \cdot GJ}{GJ + KO} \quad NP := BP - (BJ + JN) \quad NO := \sqrt{KO^2 + KN^2}$$

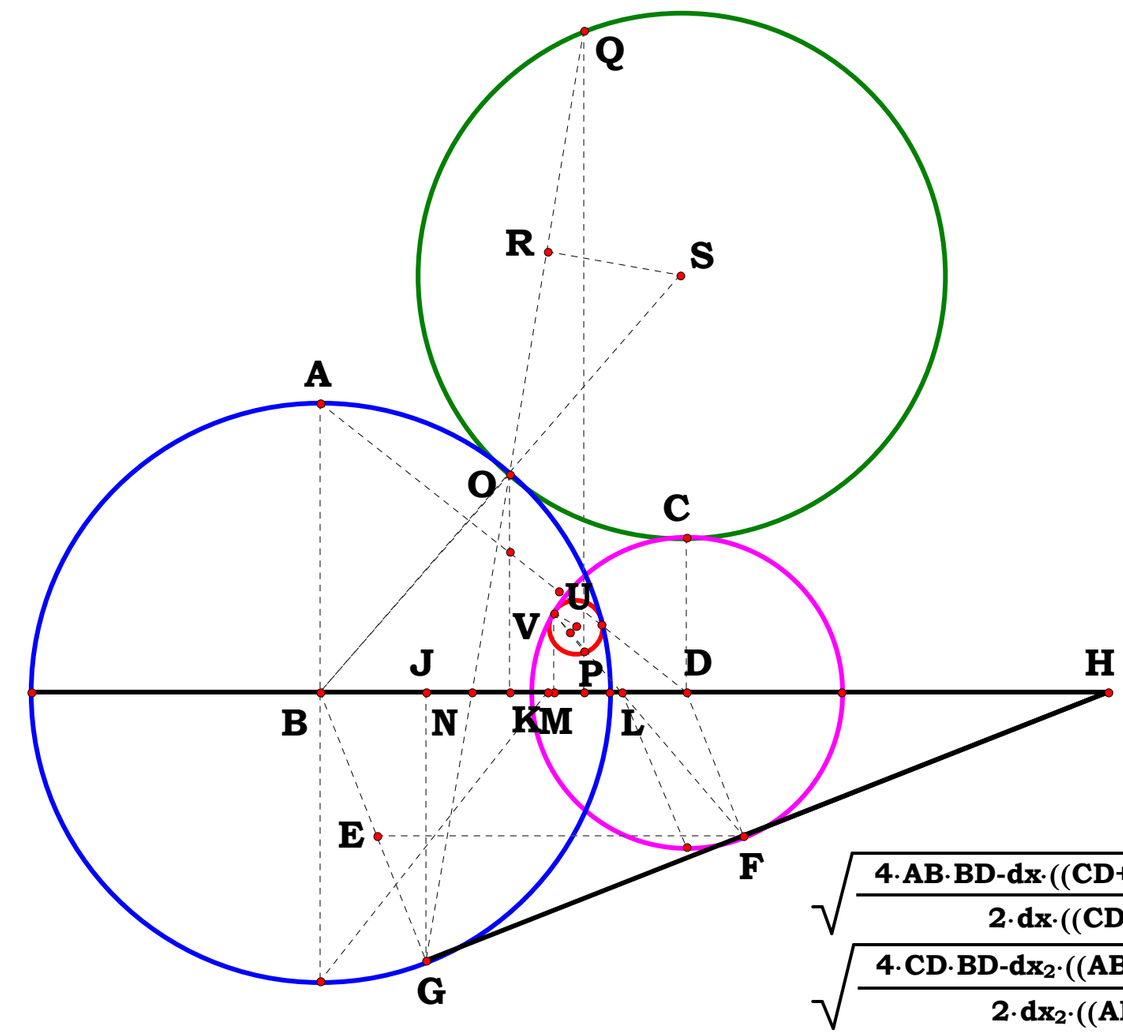
$$QN := \frac{NO \cdot NP}{KN} \quad OQ := QN - NO \quad OR := \frac{OQ}{2} \quad GO := \sqrt{(GJ + KO)^2 + JK^2}$$

$$SO := \frac{AB \cdot OR \cdot 2}{GO} \qquad SO = -4.204157$$

$R_1 := AB \quad R_2 := CD \quad D := BD$

$$SO - \frac{\left(4 \cdot R_1 \cdot D\right) - dx \cdot \left(R_2 + D - R_1\right) \cdot \left(R_2 + R_1 - D\right)}{2 \cdot dx \cdot \left(R_2 + D - R_1\right) - 4 \cdot D} = 0$$

$$\sqrt{\left[\frac{\left(4 \cdot R_1 \cdot D\right) - dx \cdot \left(R_2 + D - R_1\right) \cdot \left(R_2 + R_1 - D\right)}{2 \cdot dx \cdot \left(R_2 + D - R_1\right) - 4 \cdot D}\right]^2} = 4.204157$$




**AB = 1.90625 in.**
**CD = 1.02083 in.**
**BD = 2.41667 in.**
**dx = 5.77984**
**SO = 1.73133 in.**

**KL = 0.65962 in.**

**UV = 0.16880 in.**
**TM = 0.14146 in.**
**$dx_2$ = 14.43321**

$$\sqrt{\frac{4\cdot AB\cdot BD-dx\cdot((CD+BD)-AB)\cdot((CD+AB)-BD)^{2}}{2\cdot dx\cdot((CD+BD)-AB)-4\cdot BD}}-SO = 0.00000\ in.$$

$$\sqrt{\frac{4\cdot CD\cdot BD-dx_2\cdot((AB+BD)-CD)\cdot((AB+CD)-BD)^{2}}{2\cdot dx_2\cdot((AB+BD)-CD)-4\cdot BD}}-UV = 0.00000\ in.$$

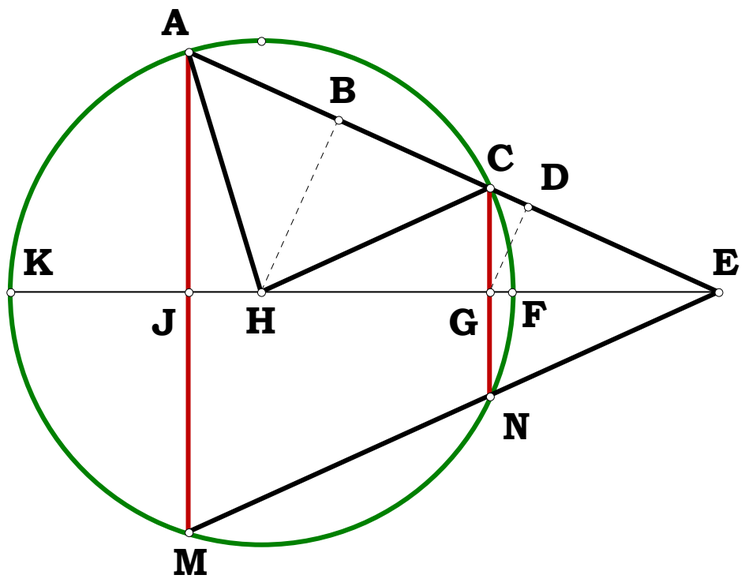
## 050694 A Ratio In Trisection

**What is AJ to CG?**

$N := 6$

$FH := 1$

$CE := FH$




$CG := \frac{FH}{N}$ $EG := \sqrt{CE^2 - CG^2}$ $CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2}$ $EH := 2 \cdot EG$ $BH := \frac{DG \cdot EH}{EG}$ $CH := FH$

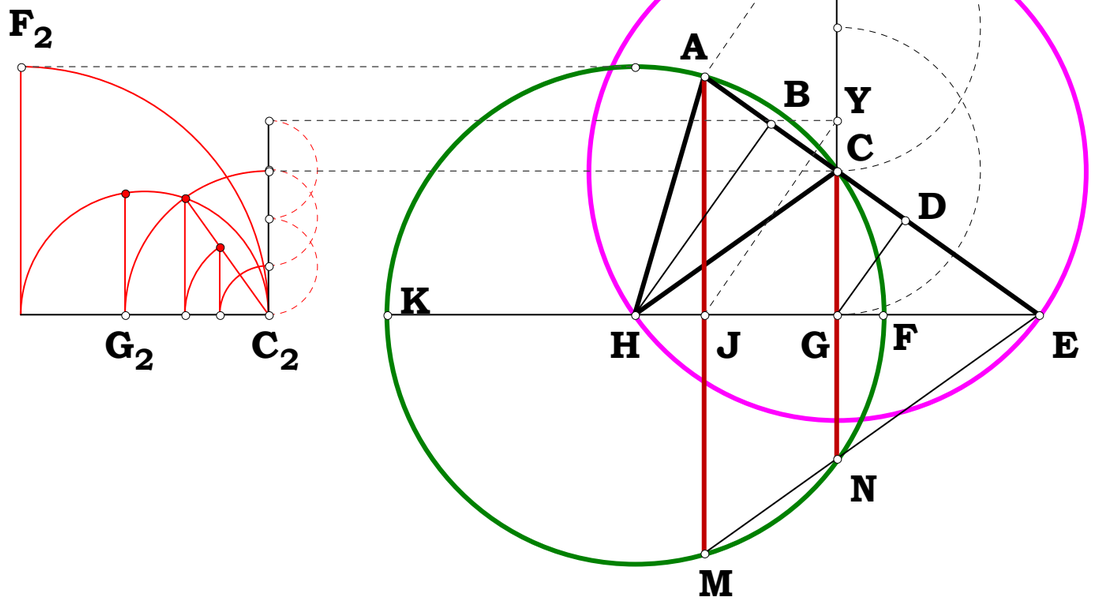
$BC := \sqrt{CH^2 - BH^2}$ $AC := 2 \cdot BAE := AC + CE$ $AJ := \frac{CG \cdot AE}{CE}$ $3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0$

$3 \cdot CG - 4 \cdot CG^3 - AJ = 0$ $AJ - \frac{(3 \cdot N^2 - 4)}{N^3} = 0$ $AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

**The resultant equation suggests this construction.**

$$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG^3}{FH}\right) - \frac{AJ}{FH} = 0.00000$$

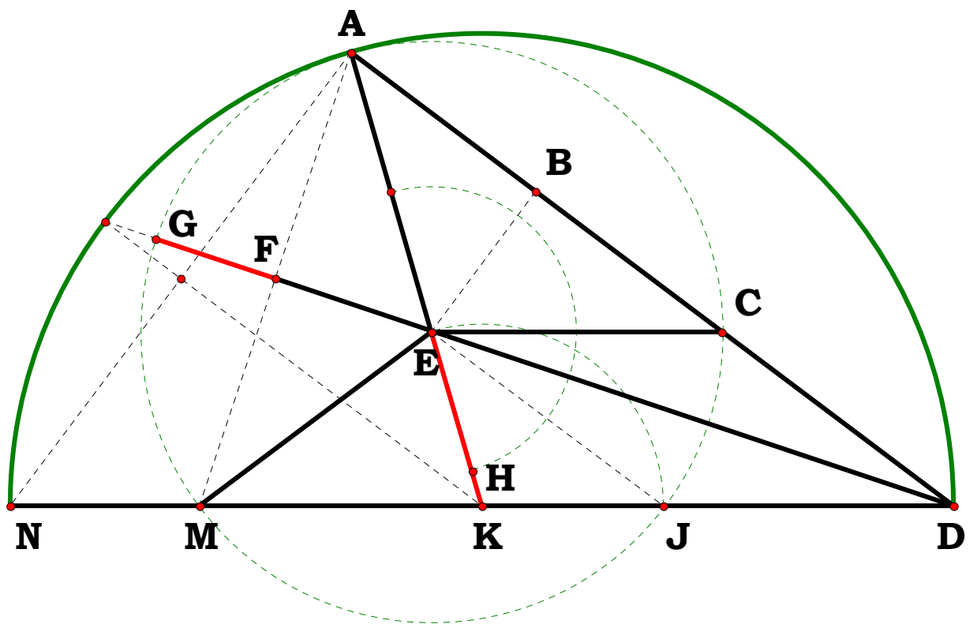
**AJ-XY = 0.000 in.**




$\frac{3 \cdot N^2 - 4}{N^3} - AJ = 0$

## 050794 A Trisection Ratio

**In trisection, what is the ratio of FG/EK?**




**Given any angle to trisect, one the simply solves for N.**

$$N := 6$$

$$AE := 1 \quad EH := \frac{AE}{2} \quad HK := \frac{AE}{N} \quad AK := AE + EH + HK$$

$$EJ := AE \quad EK := EH + HK \quad AD := \frac{EJ \cdot AK}{EK} \quad CD := AE$$

$$AC := AD - CD \quad BC := \frac{AC}{2} \quad CE := AE \quad BE := \sqrt{CE^2 - BC^2}$$

$$BD := CD + BC \quad DE := \sqrt{BD^2 + BE^2} \quad DF := \frac{BD \cdot AD}{DE}$$

$$EG := AE \quad DG := DE + EG \quad FG := DG - DF$$

$$EF := AE - FG \qquad DK := AE + EK$$

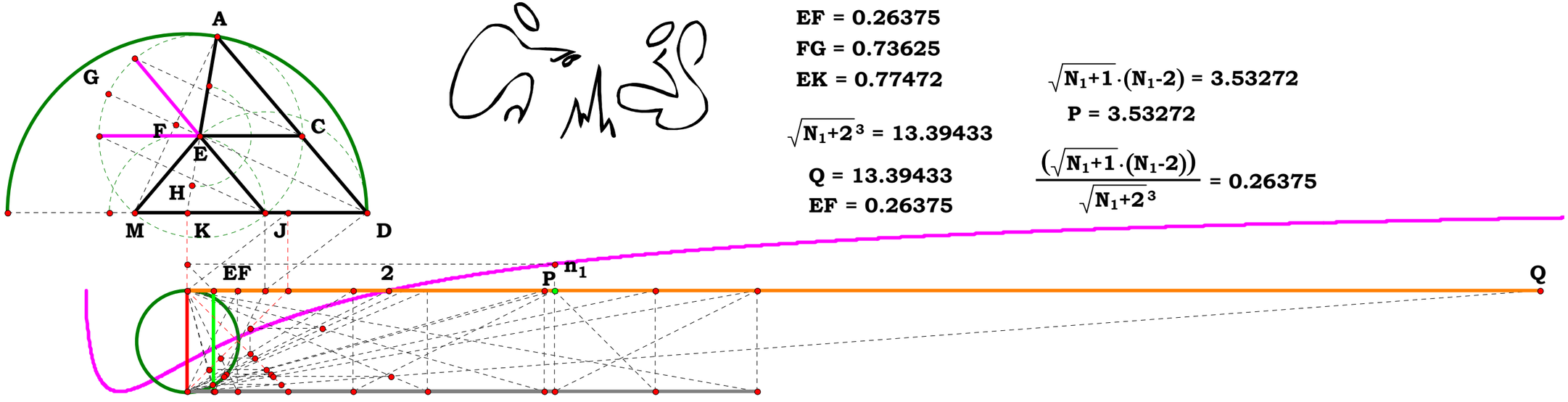
**Algebraic Names.**

$$\frac{FG}{EK} - \frac{2 \cdot N \cdot \left[\sqrt{N+1} \cdot (2-N) + (N+2)^{\frac{3}{2}}\right]}{(N+2)^{\frac{5}{2}}} = 0 \qquad \frac{EK}{EF} - \frac{(N+2)^{\frac{5}{2}}}{2 \cdot N \cdot \sqrt{N+1} \cdot (N-2)} = 0$$

**The gist of the story is this. Once one knows the ratio between difference parts of a figure where one has been set up as a unit, for example, AE as unit and EF as a ratio of the figure, then one knows the multiple of the unit by which to project any of the remaining unknowns; i.e. EK/HK. -1 to 1.**

$$EF - \frac{AE \cdot \sqrt{N+1} \cdot (N-2)}{(N+2)^{\frac{3}{2}}} = 0 \qquad EK - \frac{AE \cdot (N+2)}{2 \cdot N} = 0 \qquad \frac{EF}{EK} - \frac{2 \cdot N \cdot \sqrt{N+1} \cdot (N-2)}{(N+2)^{\frac{5}{2}}} = 0$$

$$FG - \frac{AE \cdot \left[\sqrt{N+1} \cdot (N+2)^2 + \sqrt{N+2} \cdot (N+1) \cdot (2-N)\right]}{\sqrt{N+1} \cdot (N+2)^2} = 0 \qquad \frac{AE}{EF} - \frac{\left(\sqrt{N+2}\right)^3}{\sqrt{N+1} \cdot (N-2)} = 0 \qquad \frac{EF}{AE} \cdot \frac{\left(\sqrt{N+2}\right)^3}{\sqrt{N+1} \cdot (N-2)} = 1 \qquad \frac{EK}{HK} - \frac{N+2}{2} = 0$$




$AE = 1.00000$

$EF = 0.26375$

$FG = 0.73625$

$EK = 0.77472$

$\sqrt{N_1+1}\cdot(N_1-2) = 3.53272$

$P = 3.53272$

$\sqrt{N_1+2}^3 = 13.39433$

$Q = 13.39433$

$EF = 0.26375$

$\frac{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}{\sqrt{N_1+2}^3} = 0.26375$

$N_1 = 3.64002$

$\frac{\left(2\cdot N_1\cdot\left(\sqrt{N_1+1}\cdot(2-N_1)+(N_1+2)^{\frac{3}{2}}\right)\right)}{(N_1+2)^{\frac{5}{2}}}-\frac{FG}{EK} = 0.00000$

$\frac{(N_1+2)^{\frac{5}{2}}}{\left(2\cdot N_1\cdot\sqrt{N_1+1}\cdot(N_1-2)\right)}-\frac{EK}{EF} = 0.00000$

$\frac{AE\cdot\sqrt{N_1+1}\cdot(N_1-2)}{\sqrt{N_1+2}^3}-EF = 0.00000$

$\frac{AE\cdot(N_1+2)}{2\cdot N_1}-EK = 0.00000$

$\frac{\left(2\cdot N_1\cdot\sqrt{N_1+1}\cdot(N_1-2)\right)}{(N_1+2)^{\frac{5}{2}}}-\frac{EF}{EK} = 0.00000$

$\frac{AE\cdot\left(\sqrt{N_1+1}\cdot(N_1+2)^2+\sqrt{N_1+2}\cdot(N_1+1)\cdot(2-N_1)\right)}{\sqrt{N_1+1}\cdot(N_1+2)^2}-FG = 0.00000$

$\frac{\sqrt{N_1+2}^3}{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}-\frac{AE}{EF} = 0.00000$

$\frac{\sqrt{N_1+2}^3}{\left(\sqrt{N_1+1}\cdot(N_1-2)\right)}\cdot\left(\frac{EF}{AE}\right) = 1.00000$

## 051694A Tangent Diameter and Circles

**Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.**

**The first variable is just the chosen place for the tangent circles. The second is the number of circles to construct.**

$$CF := 2 \quad CE := 1 \quad N_1 := 4 \quad N_2 := 2 \quad CD := \frac{CE}{N_1} \quad DE := CE - CD \quad EJ := CE \cdot N_2$$

$$DJ := \sqrt{DE^2 + EJ^2} \quad JG := \frac{EJ^2}{DJ} \quad BE := CE \quad EG := \sqrt{EJ^2 - JG^2} \quad BG := \sqrt{BE^2 - EG^2}$$

$$BJ := BG + JG \quad JK := CE \quad BD := BJ - DJ \quad DH := \frac{JK \cdot BD}{BJ}$$

**Algebraic Names:**

$$CD - \frac{1}{N_1} = 0 \quad DE - \frac{(N_1 - 1)}{N_1} = 0 \quad EJ - N_2 = 0 \quad DJ - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{N_1} = 0$$

$$JG - \frac{N_1 \cdot N_2^2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0 \quad EG := \frac{(N_1 - 1) \cdot N_2}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}$$

$$BG - \sqrt{\frac{\left(N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2\right)}{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}} = 0$$

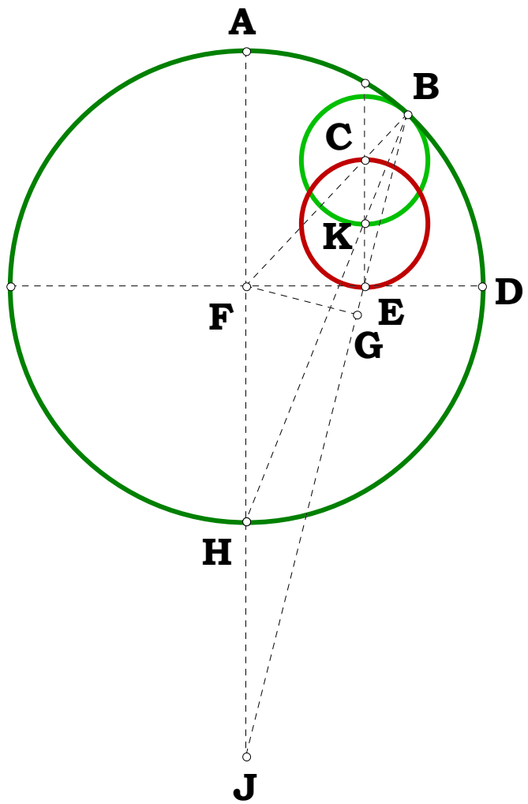
$$BJ - \frac{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)}{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0$$

$$BD - \frac{\left(2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 2 \cdot N_1 - N_2^2 + 1} - 1\right)}{\left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1\right)} = 0$$

$$DH - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1}{N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)} = 0$$

## 05/16/94B Tangent Diameter and Circles

**Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.**




$$N_1 := 2 \quad N_2 := 2 \qquad AF := 1$$

$$DF := AF \quad DE := \frac{DF}{N_1} \quad AJ := AF \cdot N_2$$

$$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$$

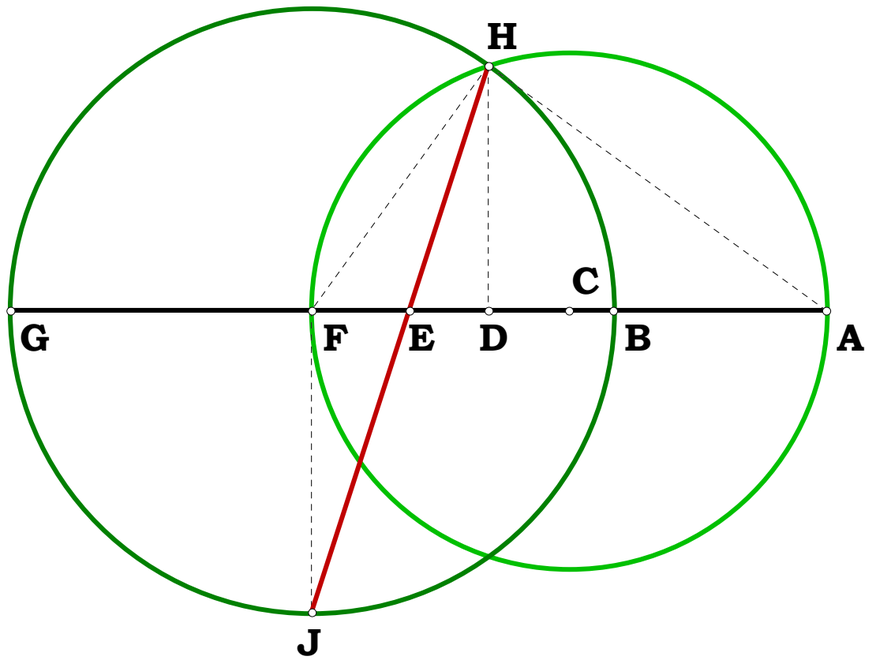
$$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$$

$$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$$

$$BJ := BE + EJ \quad KE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot KE\right]^2} \qquad BC - KE = 0$$

$$BC - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 4 \cdot N_1 \cdot N_2 - N_2^2 + 2 \cdot N_2} - 1}{N_1 \cdot \left(N_1 + N_1 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 + \sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 4 \cdot N_1 \cdot N_2 - N_2^2 + 2 \cdot N_2}\right)} = 0$$

# 102794 Trivial Method Square Root




**AE is the square root of AB x AG.**

$N := 5 \quad AB := 1$

$AG := AB \cdot N \quad BG := AG - AB \quad BF := \frac{BG}{2}$

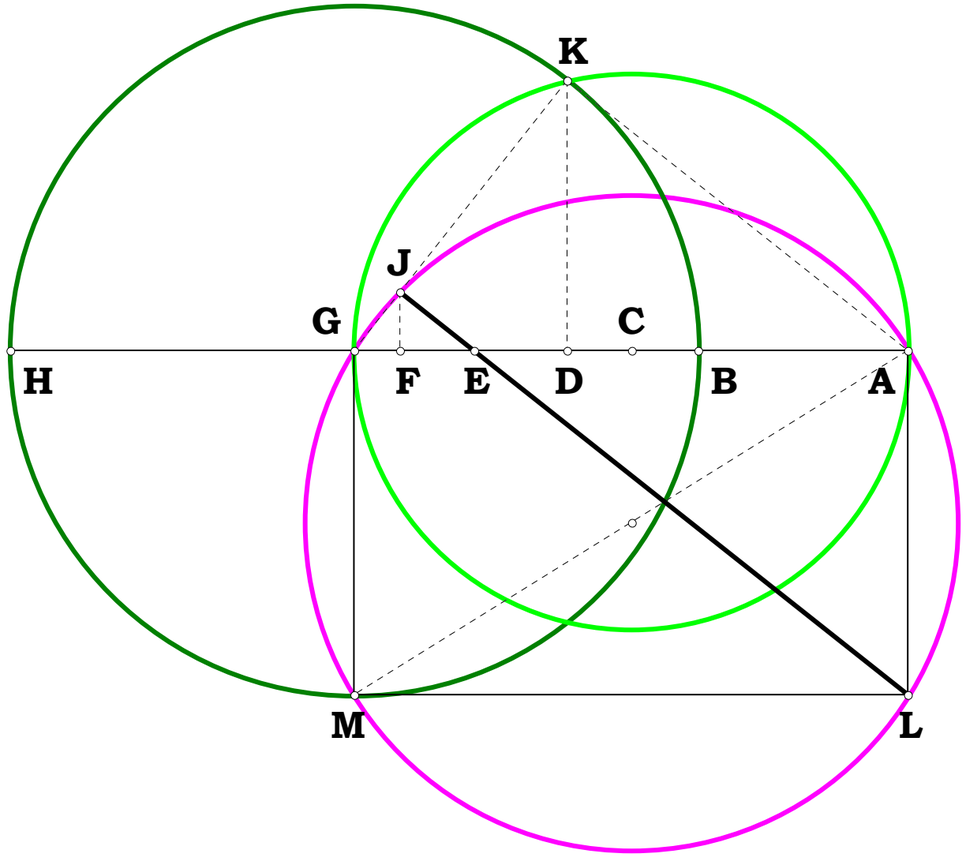
$AF := AB + BF \quad FH := BF \quad DF := \frac{FH^2}{AF} \quad AD := AF - DF$

$BD := AD - AB \quad DG := BG - BD \quad FJ := BF$

$DH := \sqrt{BD \cdot DG} \quad DE := \frac{DF \cdot DH}{DH + FJ} \quad AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0$

# 102894 Trivial Method Square Root




**AE is the square root of AB x AH.**

$N := 5 \quad AB := 1 \quad AH := AB \cdot N \quad BH := AH - AB$

$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

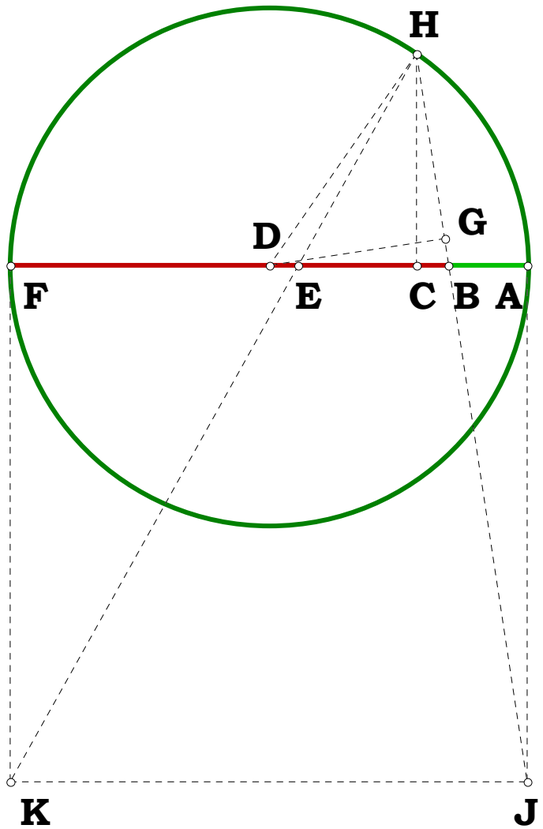
$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad JL := \sqrt{GL^2 - GJ^2}$

$FG := \frac{DG \cdot GJ}{GK} \quad AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL} \quad AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0$

## 103194 Square Root of a Segment

**Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square. Language begins with a naming convention.**




$N := 22 \quad AB := 1$

$AF := AB \cdot N \quad BF := AF - AB \quad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \frac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \frac{CF \cdot CH}{(CH + FK)}$

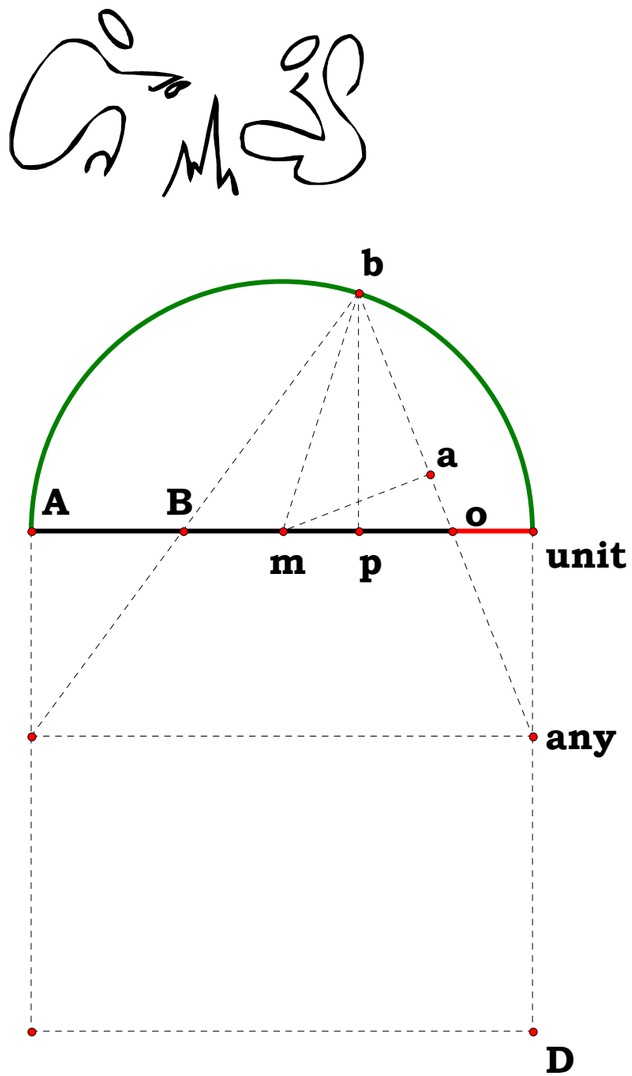
$EF := CF - CE \quad BE := BC + CE \quad DF := BF - \vdots CD := AD - AC \quad BE^2 - EF = 0$

$$BE - \frac{N + N \cdot \sqrt{4 \cdot N - 3} - 2}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} = 0 \qquad EF - \left( \frac{2 \cdot N^2 - \sqrt{4 \cdot N - 3} - 2 \cdot N + 1}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} \right) = 0$$

$AE := AB + BE \qquad unit := AB$

$$\frac{AE}{unit} - \frac{BE}{unit} = 1 \qquad \sqrt{\frac{EF}{unit}} - \frac{BE}{unit} = 0 \qquad \left( \frac{BE}{unit} \right)^2 - \frac{EF}{unit} = 0$$

**There is a difference between a unit, and the number 1. A unit is any difference, when it is called a standard, then it is given the name of 1.**




**Adding a multiplyer.**

**Let OA be any difference what so ever and let 1 be any difference what so ever and called a standard, a unit, or again a definition. It then follows that:**

$N_1 := 2.77083 \quad N_2 := .53125 \quad N_3 := N_1 + N_2 \quad N_4 := 1.35844$

$Ao := N_1 \quad unit := N_2 \quad m := \frac{N_3}{2} \quad any := N_4 \quad mo := m - unit \quad anyo := \sqrt{any^2 + unit^2}$

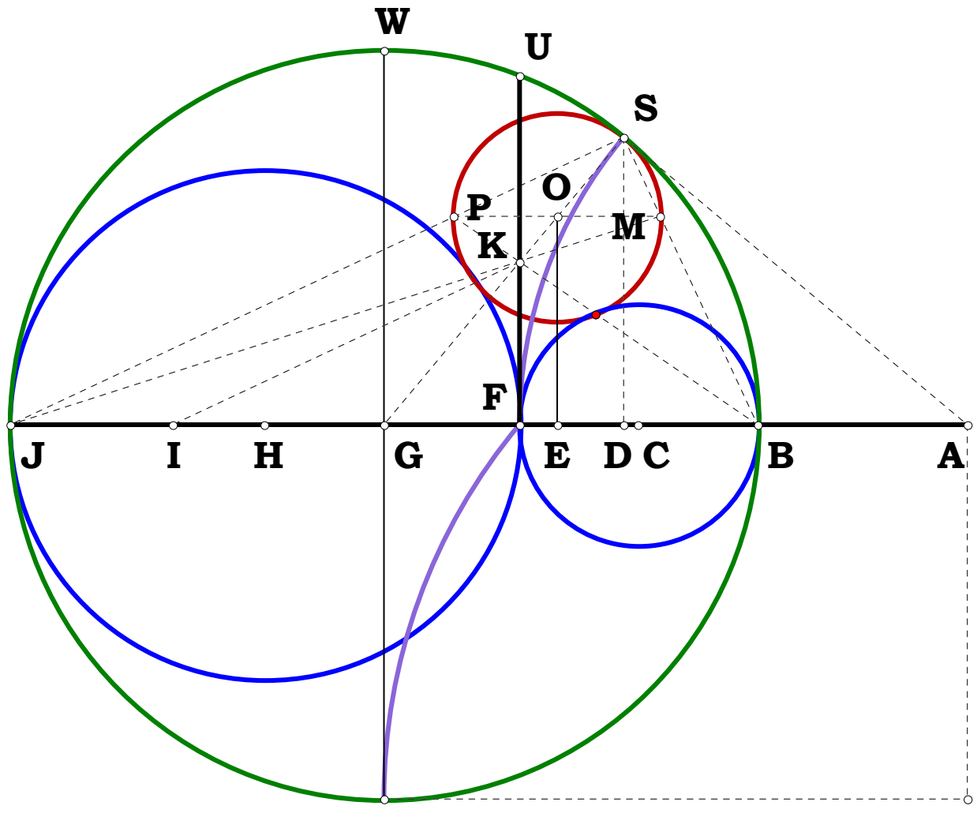
$ao := \frac{unit \cdot mo}{anyo} \quad ma := \frac{any \cdot mo}{anyo} \quad ab := \sqrt{m^2 - ma^2}$

$po := \frac{unit \cdot (ab + ao)}{anyo} \quad Ap := N_3 - (po + unit) \quad bp := \sqrt{(po + unit) \cdot Ap}$

$AB := \frac{Ap \cdot any}{any + bp} \quad Bo := N_3 - (AB + unit)$

$$\sqrt{\frac{AB}{unit}} \cdot \frac{N_3}{any} - \frac{Bo}{unit} = 0$$

## 122494 Power Line At Square Root




**In this square root figure, what is the Algebraic name of the tangent circle OS?**

$$N := 5 \quad AB := 1 \quad AJ := AB \cdot N$$

$$AF := \sqrt{AB \cdot AJ} \quad BJ := AJ - AB \quad BG := \frac{BJ}{2}$$

$$AG := AB + BG \quad GS := BG \quad DG := \frac{GS^2}{AG}$$

$$FG := AG - AF \quad BD := BG - DG \quad DJ := BJ - BD$$

$$DS := \sqrt{BD \cdot DJ} \quad FK := \frac{DS \cdot FG}{DG} \quad BF := AF - AB$$

$$BK := \sqrt{BF^2 + FK^2} \quad FI := \frac{DJ \cdot FK}{DS} \quad BI := FI + BF$$

$$BP := \frac{BK \cdot BJ}{BI} \quad KP := BP - BK \quad MP := \frac{BJ \cdot KP}{BK} \quad OS := \frac{MP}{2}$$

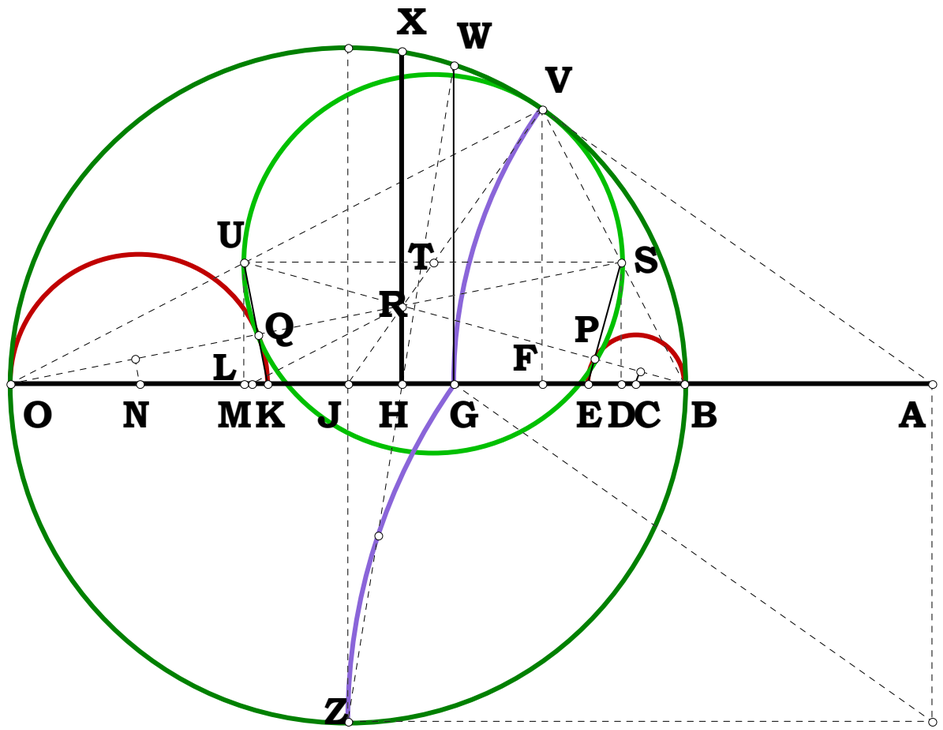
**Algebraic Names**

$$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0 \qquad OS - \frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(N + \sqrt{N} + 1\right)} = 0$$

**122595 Two Prime Exponential Series**
**Developed Through The Powerline Progression**

$\Delta := 8$ $\delta := 1 .. \Delta$

$N := 5$ $AB := 1$




$$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB \quad BJ := \frac{BO}{2} \quad JZ := BJ$$

$$JV := BJ \quad JO := BJ \quad BG_1 := AG - AB \quad GO_1 := BO - BG_1$$

$$GW_1 := \sqrt{BG_1 \cdot GO_1} \quad GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$$

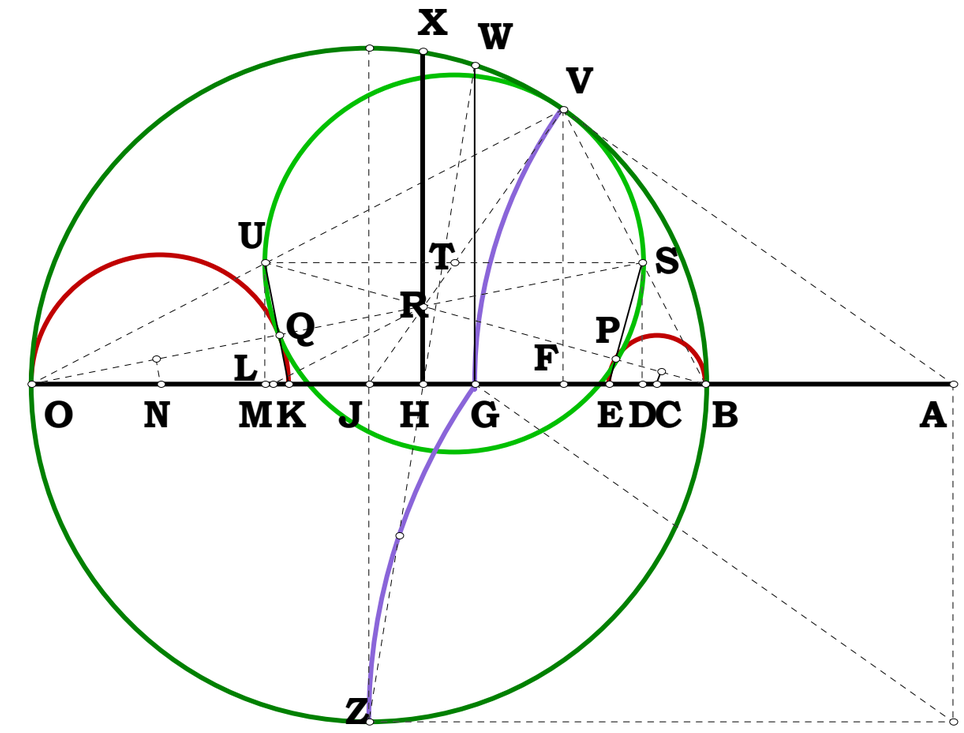
$$\begin{pmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{pmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - \left(BG_\delta + GH_\delta\right) \\ \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]} \\ BJ - \left(BG_\delta + GH_\delta\right) \\ \dfrac{\left[BJ - \left(BG_\delta + GH_\delta\right)\right] \cdot \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}}{JZ + \sqrt{\left(BG_\delta + GH_\delta\right) \cdot \left[BO - \left(BG_\delta + GH_\delta\right)\right]}} \end{bmatrix}$$

$$HJ := BJ - BG_\Delta \quad FJ := \frac{(N-1)^2}{2 \cdot (N+1)} \quad BF := BJ - FJ \quad FO := FJ + JO$$

$$FV := \sqrt{BF \cdot FO} \quad HR := \frac{FV \cdot HJ}{FJ} \quad BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2}$$

$$HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR \quad SU := \frac{BO \cdot RU}{BR} \quad TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR} \quad BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$$

$$N^{\frac{1}{2^\Delta}} - AE = 0$$




$$LU := \frac{HR \cdot BU}{BR} \quad BL := \frac{BH \cdot BU}{BR} \quad HO := JO + HJ$$

$$OR := \sqrt{HR^2 + HO^2} \quad DS := LU \quad OS := \frac{OR \cdot DS}{HR}$$

$$DO := \frac{HO \cdot DS}{HR} \quad QS := \frac{DO \cdot SU}{OS} \quad OQ := OS - QS$$

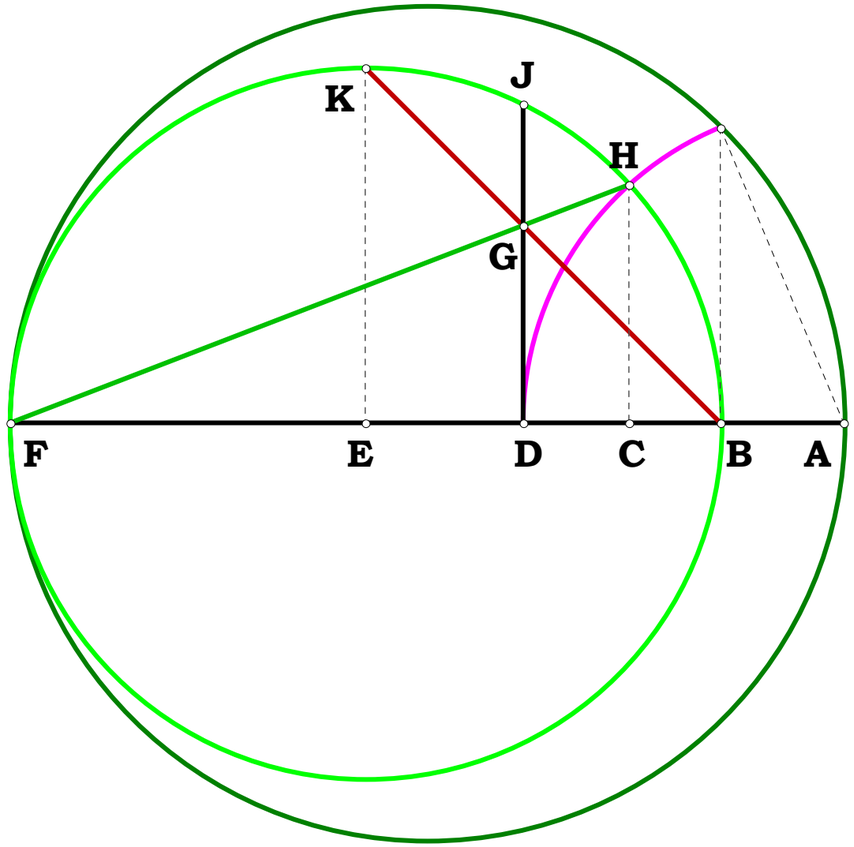
$$KO := \frac{OS \cdot OQ}{DO} \quad AK := AO - KO$$

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$

## 122694 Is Point G on DJ?

**Is G, the intersection of FH and BK, on DJ?**

$\frac{(N-1)^2}{2\cdot(N+1)}$ From 122494




$N := 5 \quad AB := 1 \quad AF := AB \cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EF := BE \quad EK := BE$

$AD := \sqrt{AB \cdot AF} \quad CE := \frac{(N-1)^2}{2\cdot(N+1)} \quad CF := CE + EF$

$DF := AF - AD \quad BC := BF - CF \quad CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF}$

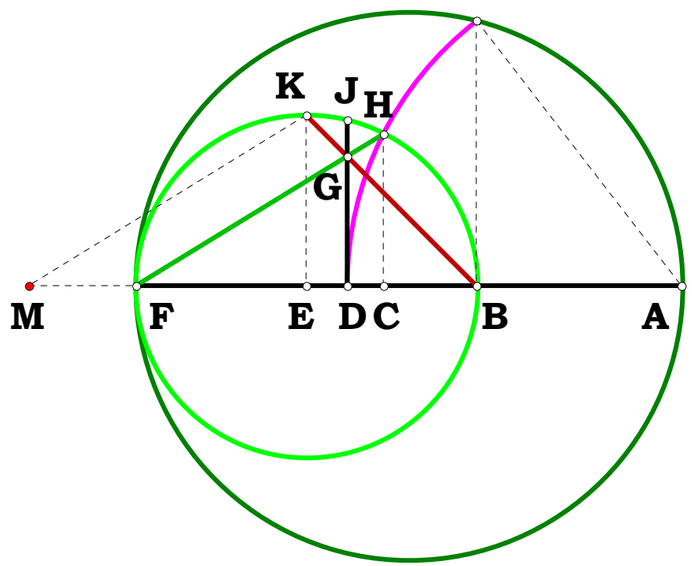
$BD := AD - AB \quad DG_2 := \frac{EK \cdot BD}{BE}$

$DG_1 - DG_2 = 0$

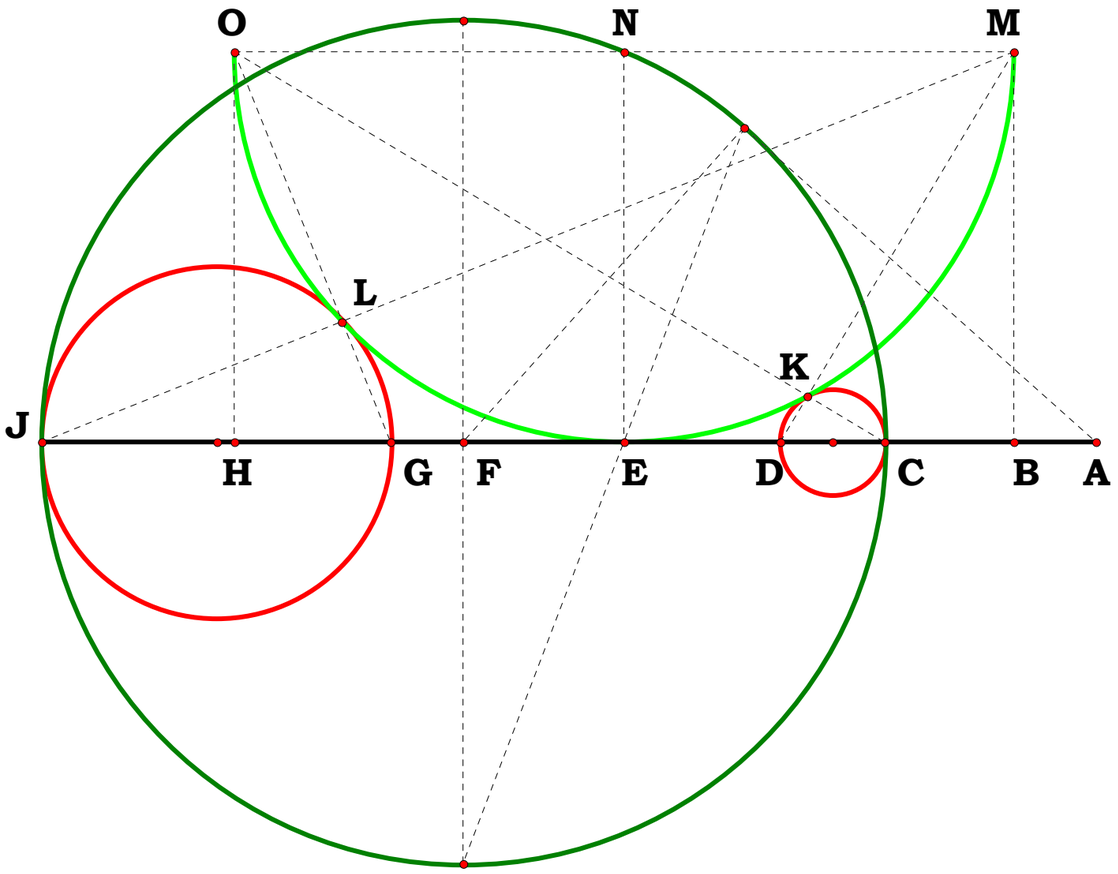
**Alternately;**

$EM := \frac{CF \cdot BE}{CH} \quad DG_3 := \frac{BE \cdot BF}{EM + BE}$

$DG_3 - DG_1 = 0$



**010695 Alternate Method Quad Roots**




$N := 5 \quad AC := 1 \quad AJ := AC \cdot N$

$CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

$MN := EN \quad NO := EN \quad BE := EN \quad EH := EN$

$BJ := BE + EJ \quad MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO$

$ML := \frac{BJ \cdot MO}{MJ} \quad JL := MJ - ML \quad GJ := \frac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ$

$\left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

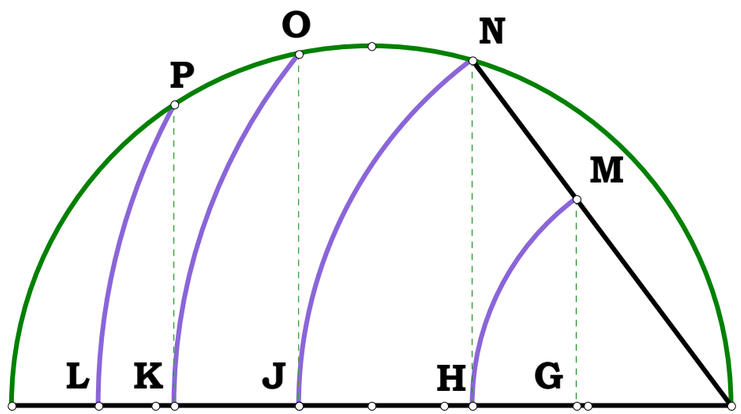
$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \frac{CH \cdot MO}{CO} \quad CK := CO - KO$

$CD := \frac{CO \cdot CK}{CH} \quad AD := AC + CD$

$\left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0 \qquad N^{\frac{3}{4}} - AG = 0 \qquad N^{\frac{1}{4}} - AD = 0$

## 040195 Exponential Series-Roots and Powers

**I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.**




$$N_1 := 5 \quad AB := 1 \quad AF := AB \cdot N_1$$

$$\delta := 0 \,..\, 3$$

$$N_2 := 3 \quad AJ_0 := AB \cdot N_2 \quad AN := AJ_0 \quad AJ_1 := \frac{AN^2}{AF} \quad AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF}$$

$$AD_0 := AJ_0 \quad DF_0 := AF - AD_0 \quad DO_0 := \sqrt{AD_0 \cdot DF_0} \quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$

$$\sum_\delta \left[ \frac{AF}{AJ_\delta} - \left( \frac{N_1}{N_2} \right)^{\delta+1} \right] = 0$$

$$\sum_\delta \left[ \frac{AF}{AD_\delta} - \left( \frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$

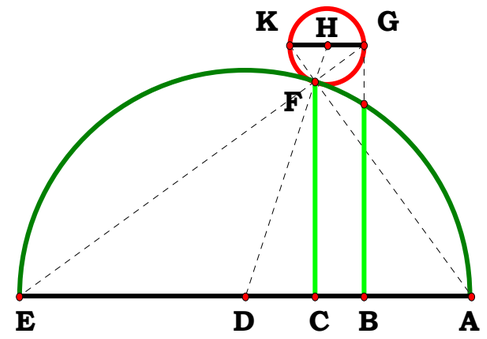
## 091395 A Study In Placement

**Given AE, AB, AC what is GH?**

$N_1 := 8.028 \quad N_2 := 3.044 \quad AE := 1 \quad AB := \frac{AE}{N_1} \quad AC := \frac{AE}{N_2}$

$CE := AE - AC \quad BC := AC - AB \quad GK := \frac{AE \cdot BC}{CE} \quad GH_1 := \frac{GK}{2}$

$$GH_1 - \frac{(N_1 - N_2)}{2 \cdot N_1 \cdot (N_2 - 1)} = 0$$

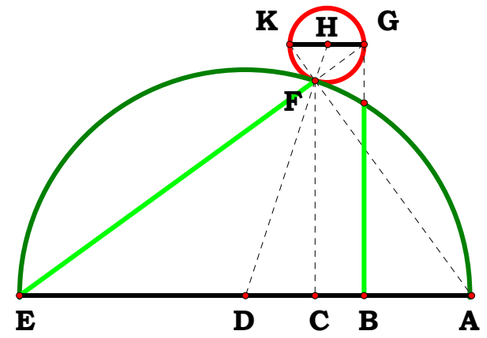



**Given AE, AB, EF what is GH?**

$N_1 := 4.22537 \quad N_2 := 1.23589 \quad AD := \frac{AE}{2} \quad AB := \frac{AE}{N_1} \quad DE := AD \quad EF := \frac{AE}{N_2} \quad CE := \frac{EF^2}{AE}$

$BD := AD - AB \quad BE := BD + DE \quad EG := \frac{EF \cdot BE}{CE} \quad FG := EG - EF \quad GH_2 := \frac{AD \cdot FG}{EF}$

$$\frac{N_1 \cdot N_2^2 - N_1 - N_2^2}{2 \cdot N_1} - GH_2 = 0$$

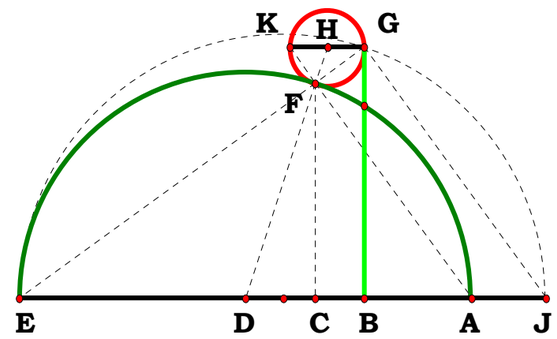



**Given AE, AB, BG what is GH?**

$N_1 := 4.22537 \quad N_2 := 1.80385 \quad AB := \frac{AE}{N_1} \quad BG := \frac{AE}{N_2} \quad BE := AE - AB$

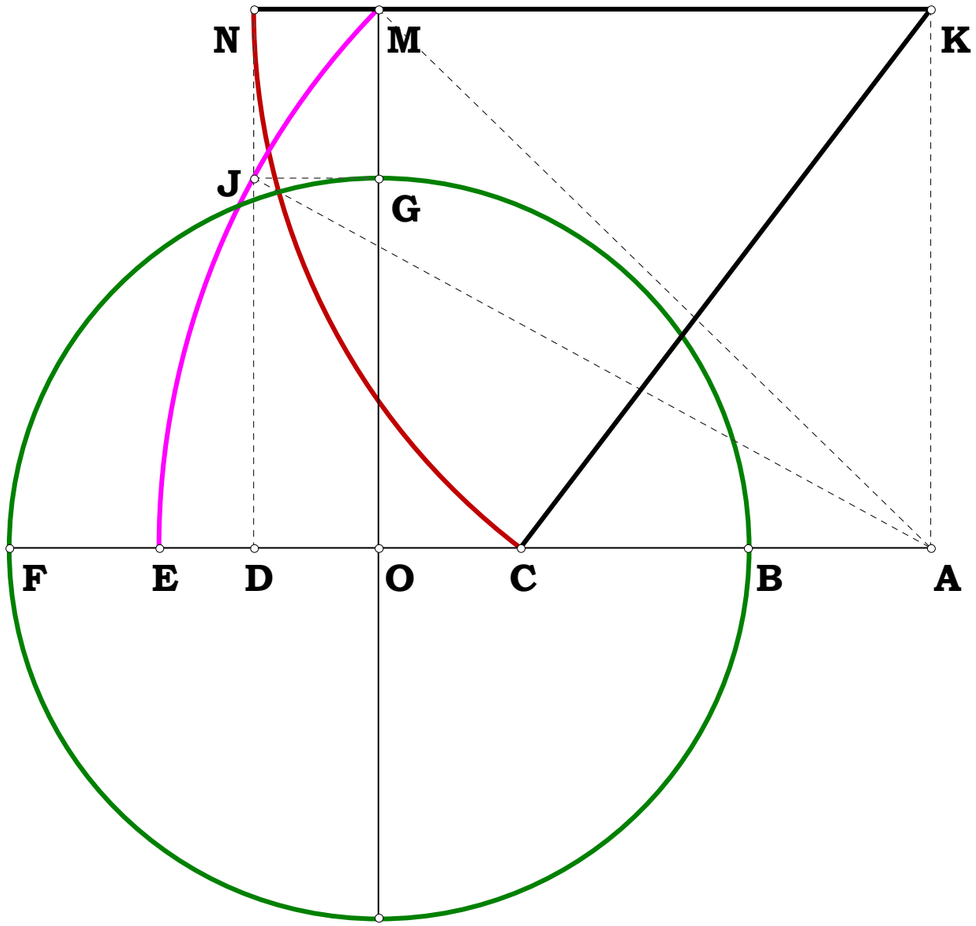
$BJ := \frac{BG^2}{BE} \quad CE := \frac{BE \cdot AE}{BE + BJ} \quad BC := BE - CE \quad GK := \frac{AE \cdot BC}{CE} \quad GH_3 := \frac{GK}{2}$

$$GH_3 - \frac{(N_1^2 - N_1 \cdot N_2^2 + N_2^2)}{2 \cdot N_1 \cdot N_2^2 \cdot (N_1 - 1)} = 0$$



**101495 Alternate Method Square Root**

**For any AK is AC the root of AB x AF?**




$N_1 := 5 \quad N_2 := 4$

$AB := 1 \quad AF := AB \cdot N_1 \quad AK := AB \cdot N_2 \quad BF := AF - AB$

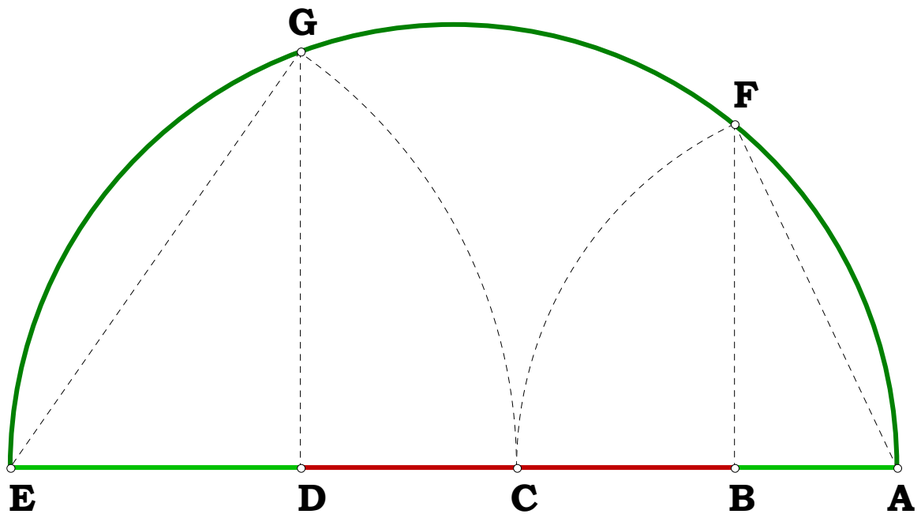
$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$GO := BO \quad DJ := B(AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$KN := AD \quad CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

$\sqrt{AB \cdot AF} - AC = 0$

**102095 Four Times The Square**




$$N_1 := 2 \quad AE := 1 \quad AB := \frac{AE}{N_1}$$

$$BE := AE - AB \quad BF := \sqrt{AB \cdot BE} \quad AF := \sqrt{AB^2 + BF^2}$$

$$AC := AF \quad CE := AE - AC \quad EG := CE \quad DE := \frac{EG^2}{AE}$$

$$AD := AE - DE \quad DG := \sqrt{AD \cdot DE} \qquad BD := AE - (AB + DE) \quad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1$$

**Algebraic Names:**

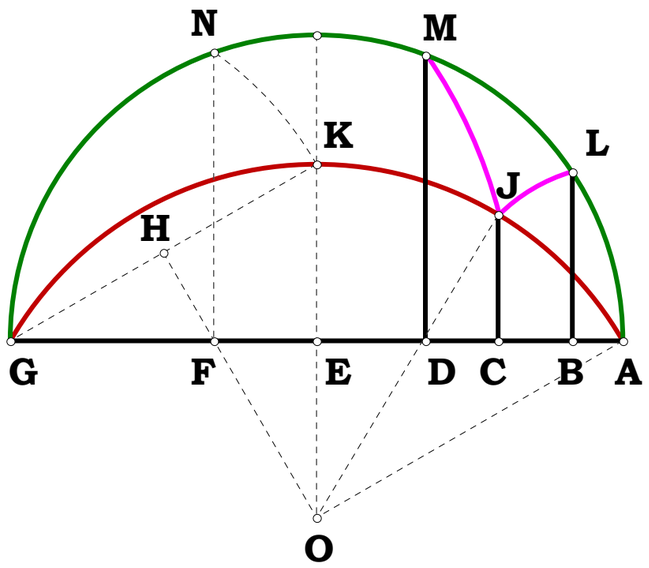
$$\frac{1}{N_1} - AB = 0 \quad 1 - \frac{1}{N_1} - BE = 0 \quad \sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0 \quad \sqrt{\frac{N_1}{N_1^2}} - AF = 0$$

$$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0 \quad 1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0 \quad 2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$$

$$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0 \qquad \frac{\sqrt{\left[N_1^2 \cdot \left(\sqrt{\frac{1}{N_1}} - 1\right)^2 \cdot \left(2 - \sqrt{\frac{1}{N_1}}\right) \cdot \sqrt{\frac{1}{N_1}}\right]}}{N_1} - DG = 0$$

## 110195 A Modification Of The Square Root Figure, Gemini Roots

**On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.**




$N_1 := 5 \quad N_2 := 2 \qquad AG := 1$

$AE := \frac{AG}{2} \quad EG := AE \quad EF := \frac{AG}{2 \cdot N_1} \quad AF := AE + EF$

$FG := EG - EF \quad FN := \sqrt{AF \cdot FG} \quad GN := \sqrt{FN^2 + FG^2} \quad GK := GN$

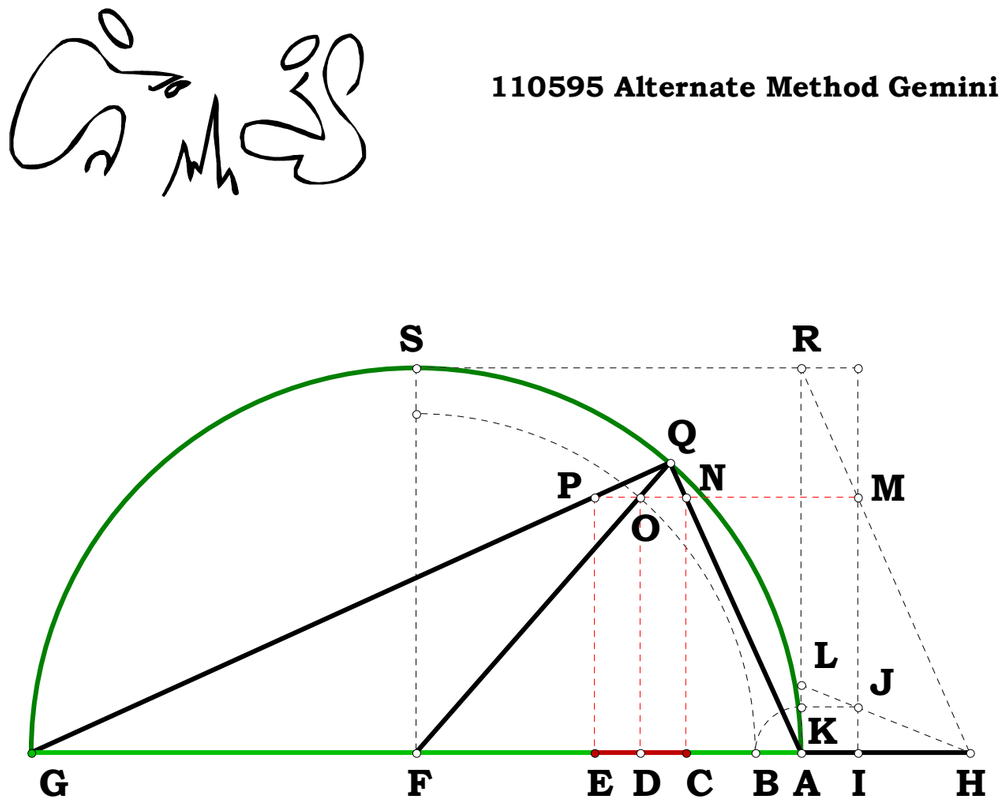
$EK := \sqrt{GK^2 - EG^2} \quad EO := \frac{EG \cdot EF}{EK} \quad OK := EO + EK \quad DE := \frac{AE}{N_2} \quad DO := \sqrt{DE^2 + EO^2}$

$DJ := OK - DO \quad CD := \frac{DE \cdot DJ}{DO} \quad CE := CD + DE \quad CJ := \frac{EO \cdot DJ}{DO} \quad AC := AE - CE \quad AJ := \sqrt{AC^2 + CJ^2}$

$AL := AJ \quad AB := \frac{AL^2}{AG} \quad CG := AG - AC \quad GJ := \sqrt{CG^2 + CJ^2} \quad GM := GJ \quad DG := \frac{GM^2}{AG}$

$BD := AG - (AB + DG)$

$$\frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$$



**110595 Alternate Method Gemini Roots**




$N_1 := 6 \quad N_2 := 7$

$AG := 1 \quad AF := \frac{AG}{2} \quad AR := AF \quad FQ := AF \quad FG := AF$

$AL := \frac{AR}{N_1} \quad IM := \frac{AR}{N_2} \quad AK := \frac{AL \cdot IM}{AR}$

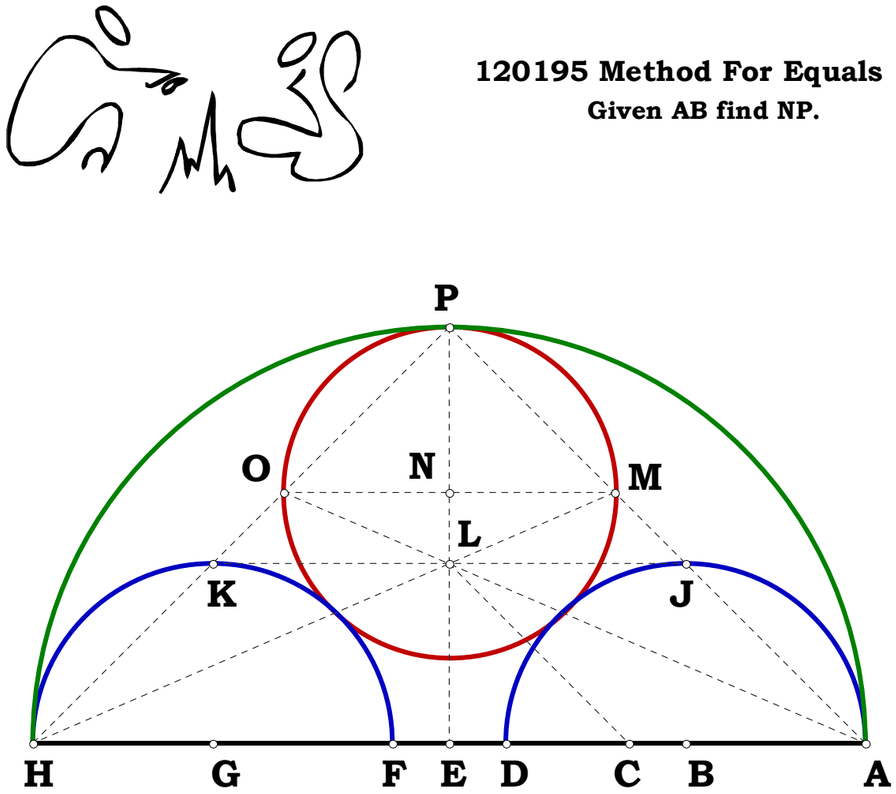
$DO := IM \quad AB := AK \quad BF := AF - AB \quad FO := BF$

$OQ := FQ - FO \quad NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO} \quad NP - 2 \cdot AK = CD := AK \quad DE := AK$

$DF := \sqrt{FO^2 - DO^2} \quad AD := AF - DF \quad AC := AD - CD \quad EG := FG + DF - DE$

$CE := NP \quad \frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$

$\frac{\sqrt{AC \cdot EG}}{CE} = 3$



**120195 Method For Equals**

**Given AB find NP.**



$$N_1 := 5$$

$$AH := 1 \quad AE := \frac{AH}{2} \quad EH := AE \quad EP := AE \quad AP := \sqrt{2 \cdot AE^2}$$

$$AB := \frac{AE}{N_1} \quad CE := AB \quad CH := EH + CE \quad CL := \sqrt{2 \cdot CE^2}$$

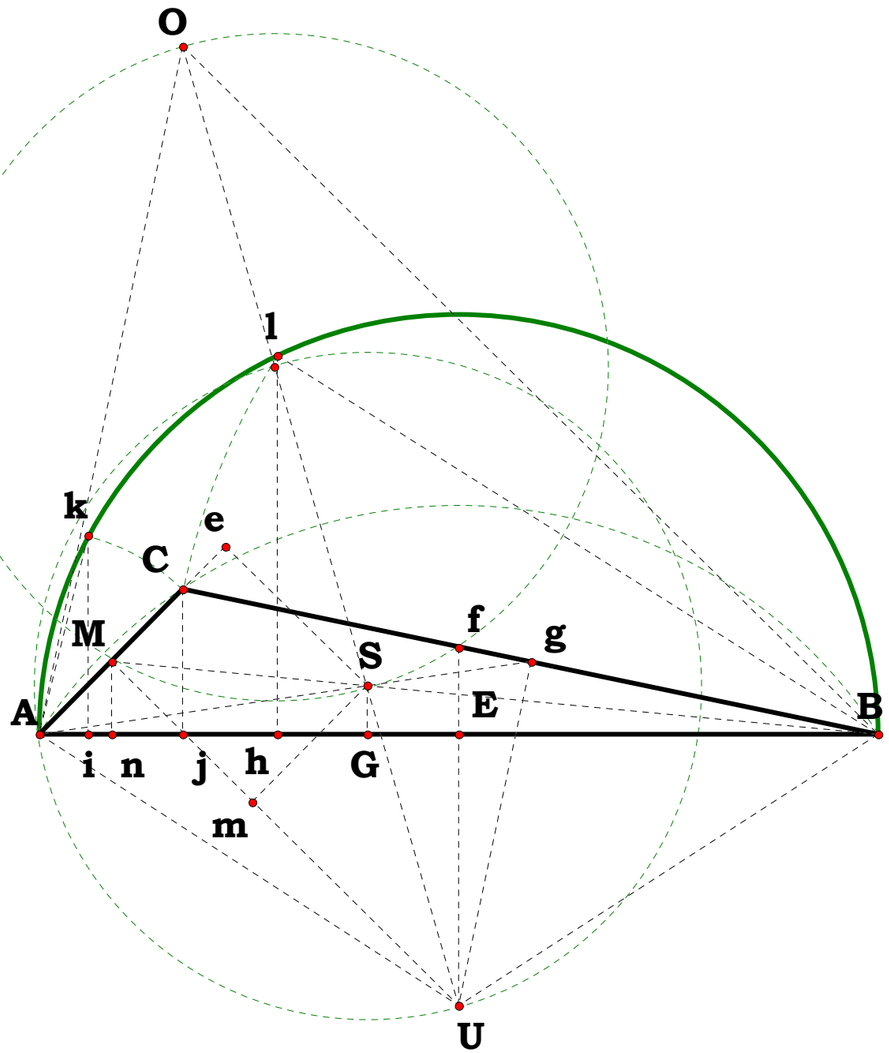
$$AM := \frac{CL \cdot AH}{CH} \quad MP := AP - AM \quad NP := \frac{EP \cdot MP}{AP}$$

$$\frac{1}{2} \cdot \frac{(N_1 - 1)}{(N_1 + 1)} - NP = 0 \qquad \frac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$$

**12_07_95.MCD**

**Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.**

**Scholia:** ***100 Great Problems of Elementary Mathematics*** **H. Dorrie Problem 27 Euler's Line**




$$\delta := 0 .. 2 \quad AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$$

$$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta \quad Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$$

$$Ah_\delta := AB_\delta - Bh_\delta \quad hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2}$$

$$Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2} \quad BE_\delta := AE_\delta \quad Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2}$$

$$Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta} \quad fg_\delta := Bf_\delta - Bg_\delta \quad Ug_\delta := \text{if}\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right)$$

$$BU_\delta := \text{if}\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right] \quad AM_\delta := \frac{AC_\delta}{2} \quad An_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta}$$

$$Bn_\delta := AB_\delta - An_\delta \quad nM_\delta := \sqrt{(AM_\delta)^2 - (An_\delta)^2} \quad BM_\delta := \sqrt{(nM_\delta)^2 + (Bn_\delta)^2}$$

$$BS_\delta := \frac{2 \cdot BM_\delta}{3} \quad BG_\delta := \frac{Bn_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{nM_\delta \cdot BS_\delta}{BM_\delta}$$

$$\mathbf{TRIANGLE} := (\mathbf{Side\_1} + \mathbf{Side\_2} > \mathbf{Side\_3}) \cdot (\mathbf{Side\_1} + \mathbf{Side\_3} > \mathbf{Side\_2}) \cdot (\mathbf{Side\_2} + \mathbf{Side\_3} > \mathbf{Side\_1})$$

$$\mathbf{AG}_\delta := \mathbf{AB}_\delta - \mathbf{BG}_\delta \qquad \mathbf{AS}_\delta := \sqrt{(\mathbf{AG}_\delta)^2 + (\mathbf{GS}_\delta)^2} \qquad \mathbf{MS}_\delta := \mathbf{BM}_\delta - \mathbf{BS}_\delta$$

$$\mathbf{AU}_\delta := \mathbf{BU}_\delta \qquad \mathbf{MU}_\delta := \sqrt{(\mathbf{AU}_\delta)^2 - (\mathbf{AM}_\delta)^2} \qquad \mathbf{Ae}_\delta := \frac{1}{2} \cdot \frac{(\mathbf{AS}_\delta)^2}{\mathbf{AM}_\delta} + \frac{1}{2} \cdot \mathbf{AM}_\delta - \frac{1}{2} \cdot \frac{(\mathbf{MS}_\delta)^2}{\mathbf{AM}_\delta}$$

**The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.**

$$\mathbf{eM}_\delta := \mathbf{Ae}_\delta - \mathbf{AM}_\delta \qquad \mathbf{Sm}_\delta := \mathbf{eM}_\delta \qquad \mathbf{Se}_\delta := \sqrt{(\mathbf{AS}_\delta)^2 - (\mathbf{Ae}_\delta)^2} \qquad \mathbf{Mm}_\delta := \mathbf{Se}_\delta$$

$$\mathbf{Um}_\delta := \mathbf{if}\left[\mathbf{AC}_\delta < \sqrt{(\mathbf{BC}_\delta)^2 + (\mathbf{AB}_\delta)^2}\,,\ \mathbf{MU}_\delta - \mathbf{Mm}_\delta\,,\ \mathbf{MU}_\delta + \mathbf{Mm}_\delta\right]$$

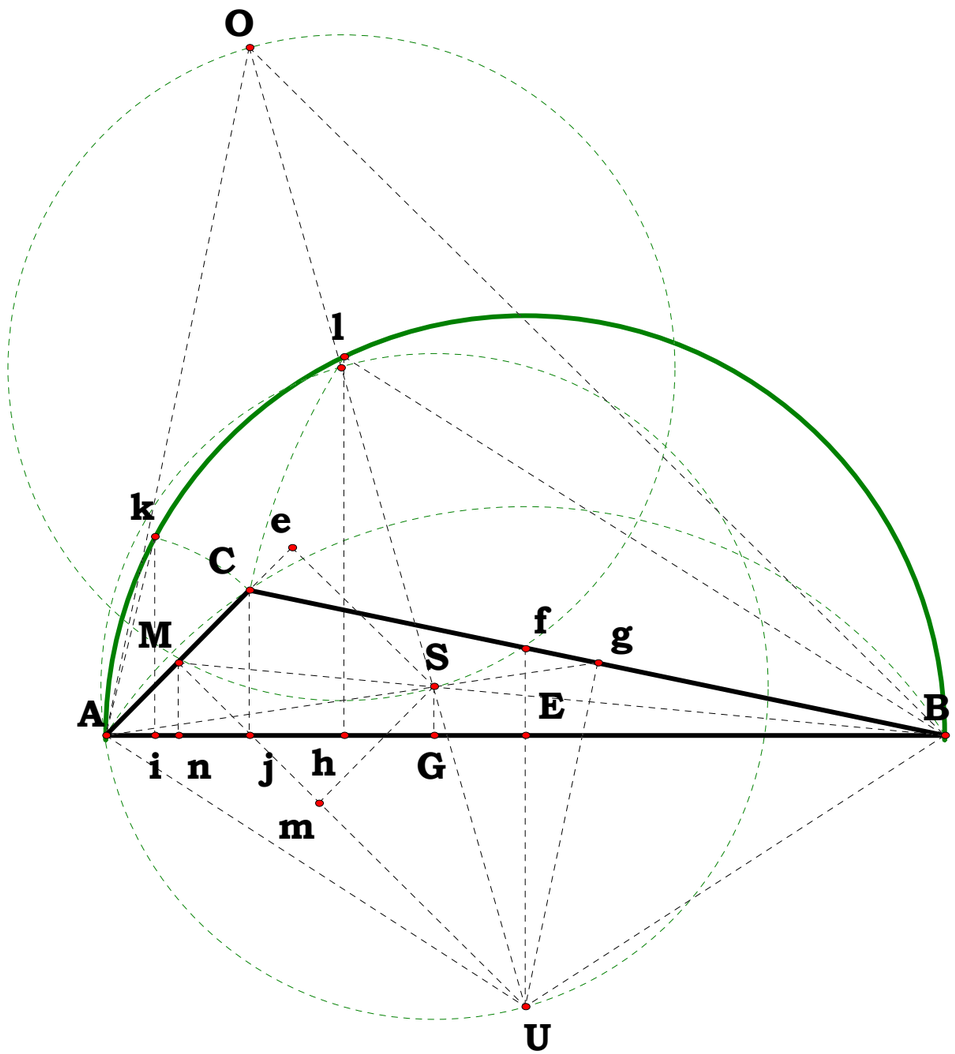
$$\mathbf{SU}_\delta := \sqrt{(\mathbf{Um}_\delta)^2 + (\mathbf{Sm}_\delta)^2} \qquad \mathbf{UO}_\delta := 3 \cdot \mathbf{SU}_\delta$$

**Due to the way in which certain lines lay, the above switch was needed.**

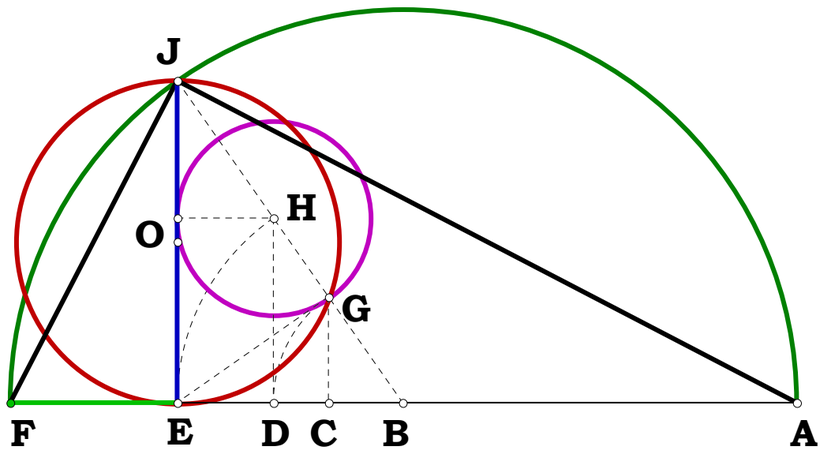
**Is this a TRIANGLE = 1 ?**

**Side_1 ≡ 14.07583 Side_2 ≡ 3.42377 Side_3 ≡ 11.91932**

| $\mathbf{SU}_\delta$ = | $\mathbf{UO}_\delta$ = | $\mathbf{AU}_\delta$ = |
|---|---|---|
| 5.58256 | 16.74768 | 8.382562 |
| 5.58256 | 16.74768 | 8.382562 |
| 5.58256 | 16.74768 | 8.382562 |



**122095 Just For Fun**




$$N := 3$$

$$EF := 1 \quad EJ := EF \cdot N \quad AE := \frac{EJ^2}{EF} \quad AF := AE + EF$$

$$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF \quad BH := BE \quad BJ := \sqrt{EJ^2 + BE^2}$$

$$BD := \frac{BE \cdot BH}{BJ} \quad BG := BD \quad BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD$$

$$HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG \quad EG_2 := \sqrt{EJ^2 - GJ^2}$$

$$\frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1 \quad \left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$$

$$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2+1\right)^2} = 0$$

## 122195 Pascal's Triangle With Exponential Division

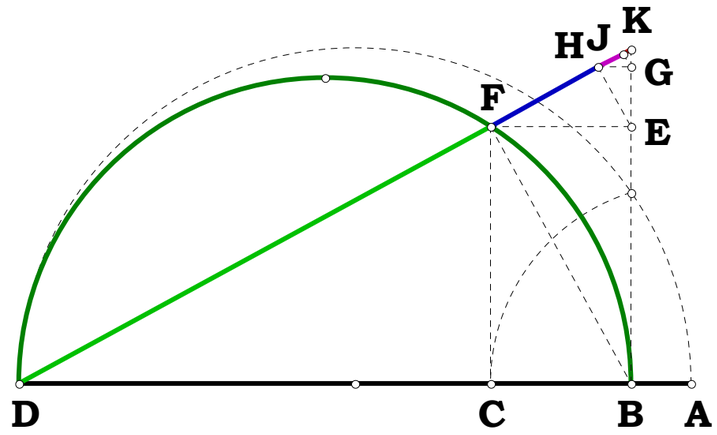
$N := 7$

$AB := 1 \quad AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD} \quad BD := AD - AB \quad CD := AD - AC$

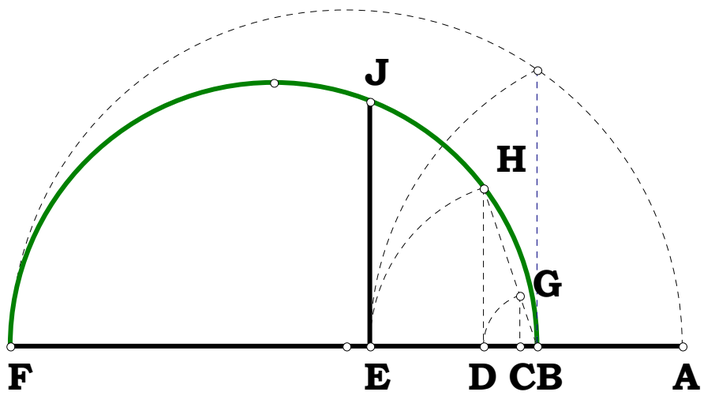
$BC := BD - CD \quad CF := \sqrt{BC \cdot CD} \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$

$\frac{DK}{FK} - \left(\sqrt{N} + 1\right) = 0 \quad \frac{DK}{HK} - \left(\sqrt{N} + 1\right)^2 = 0 \quad \frac{DK}{JK} - \left(\sqrt{N} + 1\right)^3 = 0$




**A more civil figure.**




$AB := 1 \quad AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF} \quad BE := AE - AB \quad BH := BE$

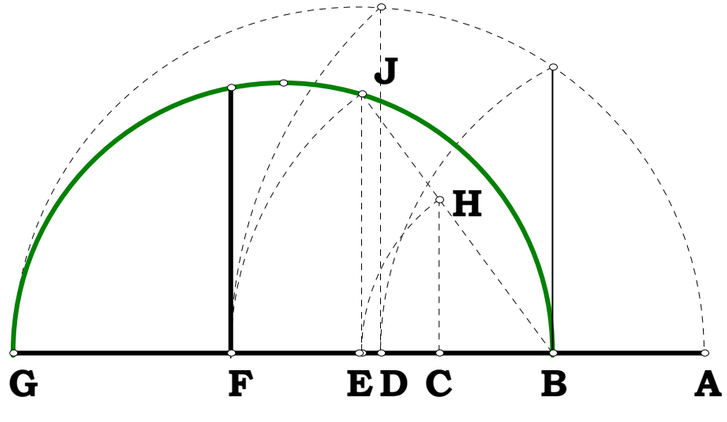
$BD := \frac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \frac{BD \cdot BG}{BE}$

$\frac{BF}{BE} - \left(\sqrt{N} + 1\right) = 0 \quad \frac{BF}{BD} - \left(\sqrt{N} + 1\right)^2 = 0 \quad \frac{BF}{BC} - \left(\sqrt{N} + 1\right)^3 = 0$

$AB := 1 \quad AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BF := AF - AB$

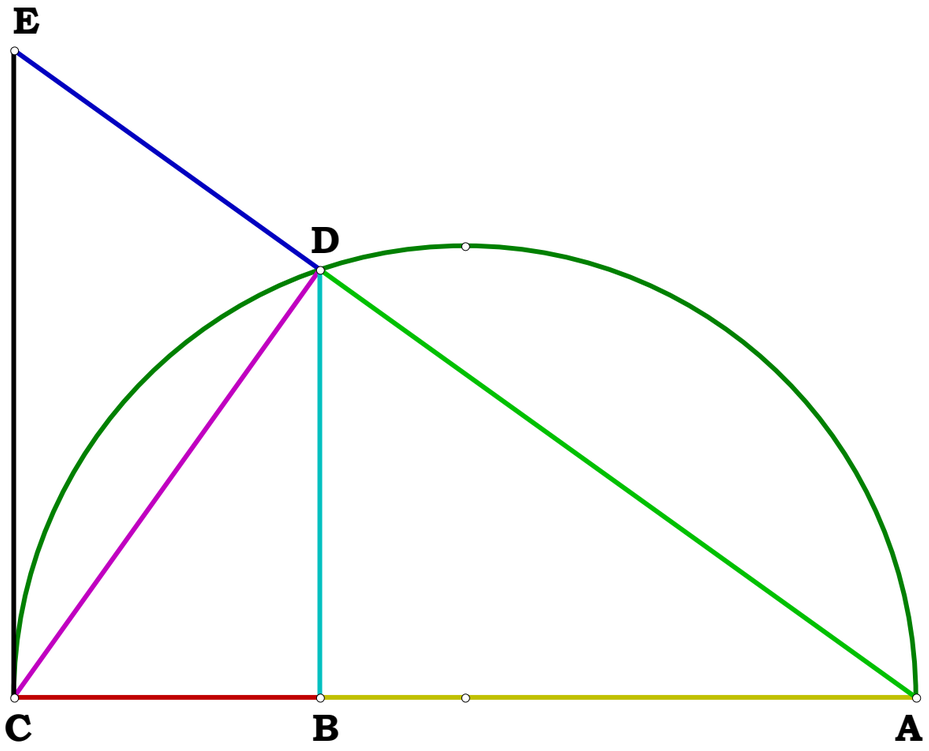
$BJ := BF \quad BE := \frac{BJ \cdot BF}{BG} \quad BH := BE \quad BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N - 1}{N^{\frac{3}{4}} - 1} = 0 \quad \frac{BG}{BE} - \frac{(N - 1)^2}{\left(N^{\frac{3}{4}} - 1\right)^2} = 0 \quad \frac{BG}{BC} - \frac{(N - 1)^3}{\left(N^{\frac{3}{4}} - 1\right)^3} = 0$



**122995**

**Given AC and CE find BC.**




$N_1 := 4 \quad N_2 := 9$

$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$

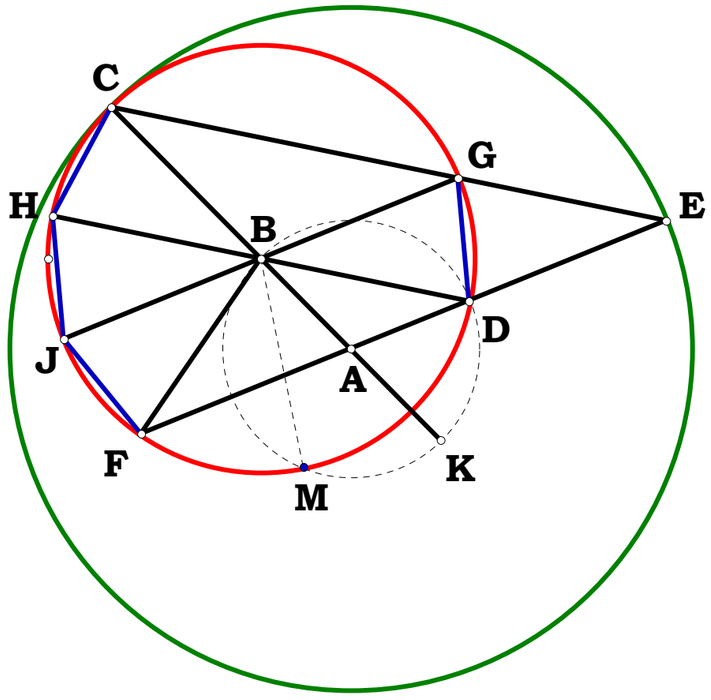
$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$

$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \quad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \quad AB - \left( N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} \right) = 0$

$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \quad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

## 010496 The Archamedian Paper Trisector- Without the Numbers.




**Given any circle AB.**

**Given any circle BC such that BC ≤ 2AB.**

**Construct AE such that AE = AC.**

**As AC = AB + BC and AD = AB so too DE = BC.**

**Construct DH parallel to BD. Construct CE.**

**As AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD.**

**From here one can take two paths.**

**Construct GJ parallel to EF.**

**As CE is parallel to DH, DG = CH.**

**As GJ is parallel to EF, DG = FJ.**

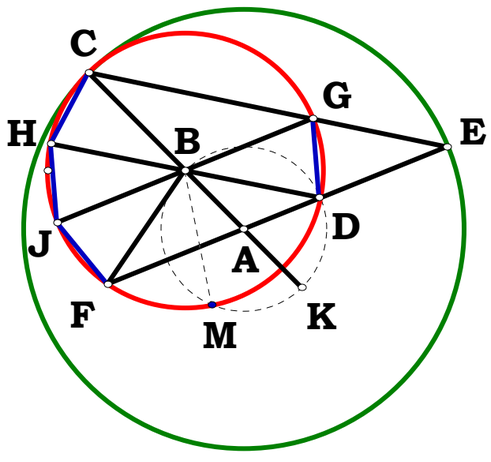
**As ∠ HBJ is opposite and equal to ∠ GBD, DG = HJ, therefore ∠ DG is $\frac{1}{3}$ CF.**

**As CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.**

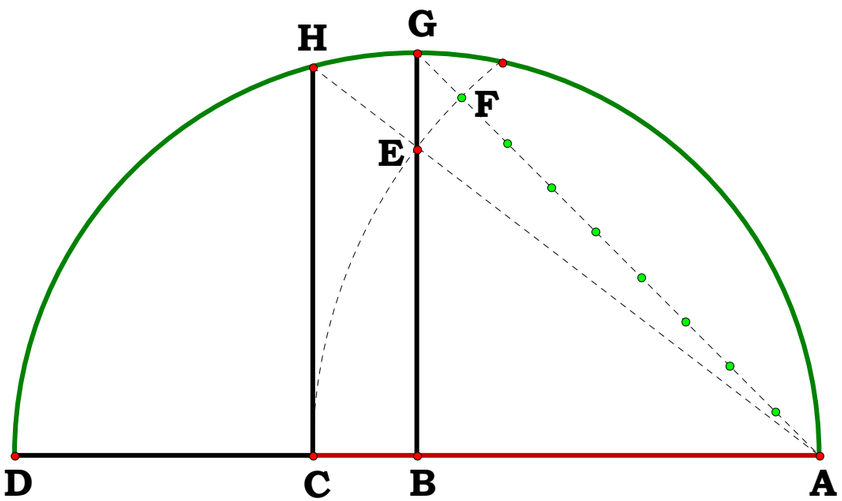
**By construction DK = KM.**

**As DH is parallel to CE, CH = DG.**

**As DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.**



# 010796 Rusty Cubes




**A rusty Compass construction for the duplication of the cube.**

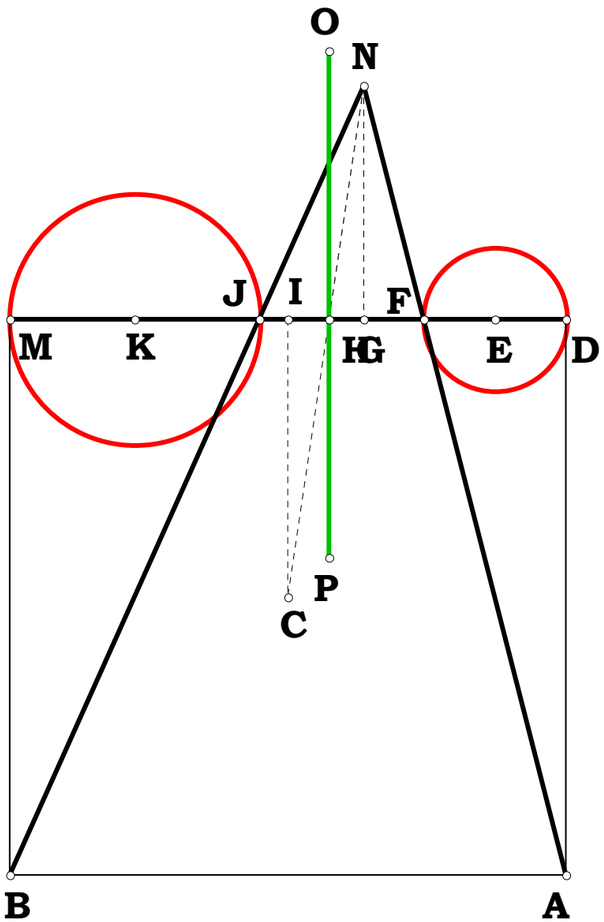
$$AD := 2 \quad AB := \frac{AD}{2} \quad AG := \sqrt{2 \cdot AB^2} \quad AF := \frac{AG}{9} \cdot 8$$

$$AC := AF \quad AC = 1.257079$$

$$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.259921 \quad \frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002261$$

**I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.**

**010896a Alternate Method Power Line**




$R_1 := 2 \quad R_2 := 5 \quad D := 4$

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF \quad AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \quad GJ := FJ - FG \quad DI := \frac{DM}{2}$

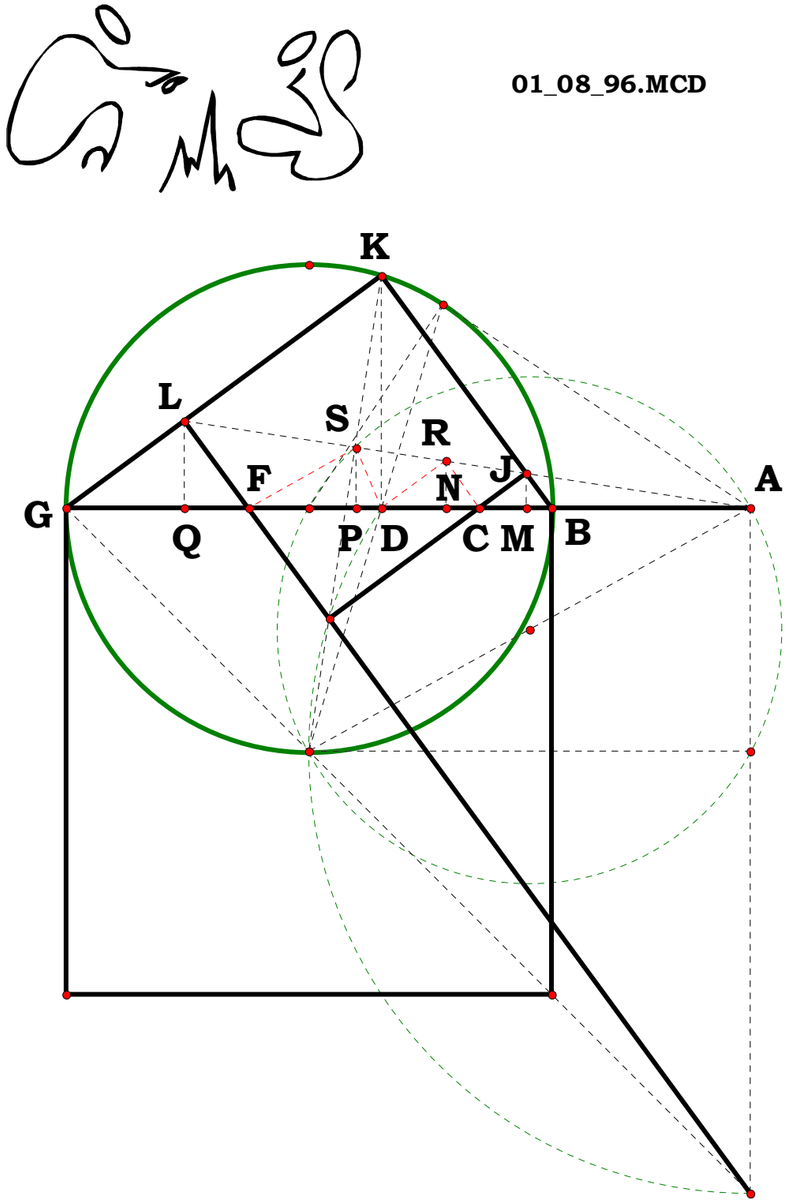
$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad GN := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

$HM := DM - DH$

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$




01_08_96.MCD

**The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.**

$$N = 5 \quad AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB$$

$$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$$

$$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$$

$$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$$

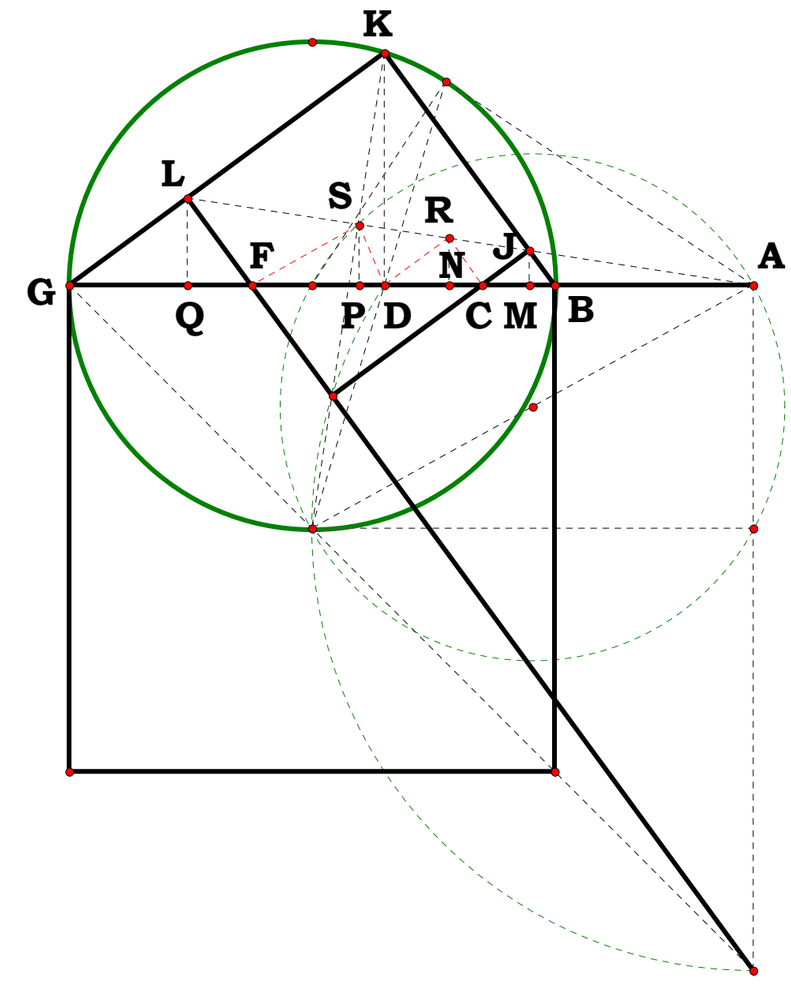
$$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$$

$$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$$

$$\frac{GL}{BJ} = 5 \qquad \frac{AG}{AB} = 5$$

**Plug in AG here. AB will become "1".**

$$N \equiv 5$$




$$\frac{GK}{GL}-\frac{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}}{\left(\frac{AG}{AB}\right)^{\frac{3}{4}}}=0 \qquad \frac{GK}{GL}-\left(\frac{N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}}{N^{\frac{3}{4}}}\right)=0$$

$$\frac{BK}{BJ}-\left[\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}\right]=0 \qquad \frac{BK}{BJ}-\left(N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}\right)=0$$

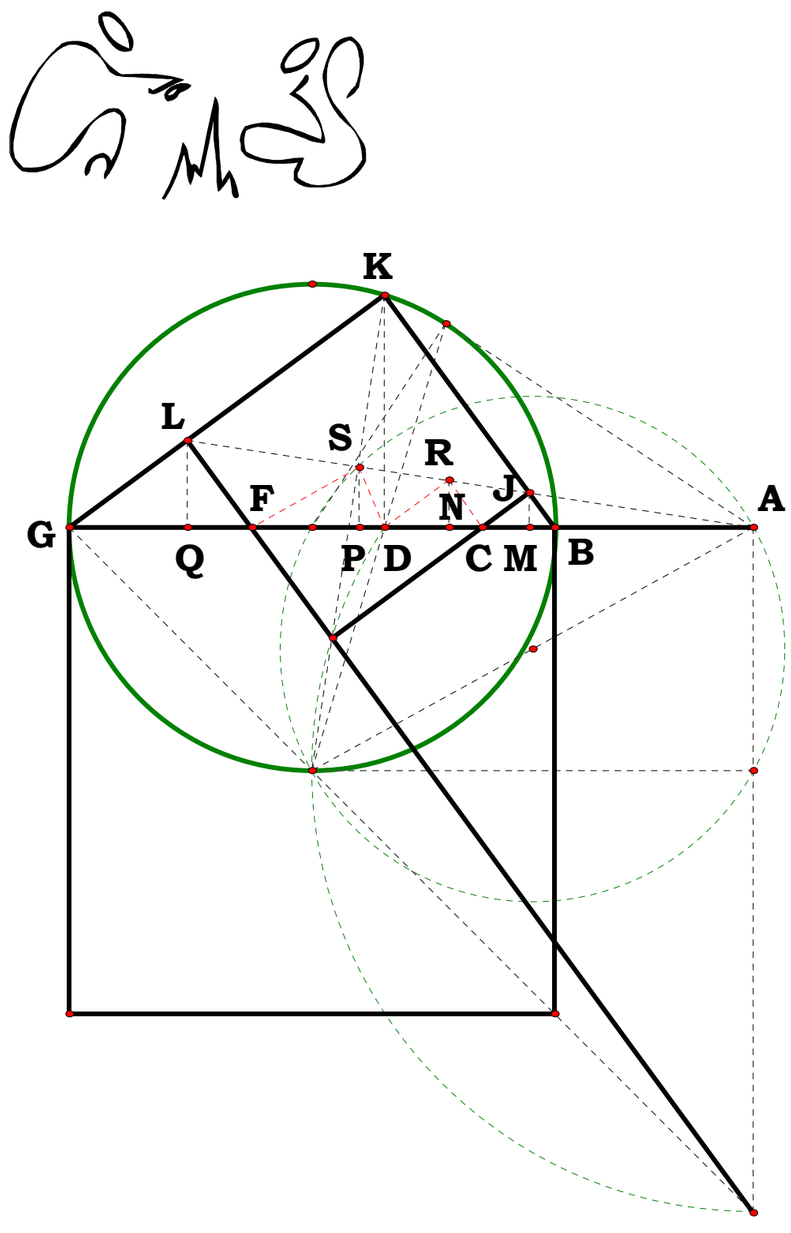
$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG} \quad CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

$$\frac{BG}{BM}-\left[\left(\frac{AG}{AB}\right)^{\frac{5}{4}}+\frac{AG}{AB}+\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}\right]=0$$

$$\frac{BG}{BM}-\left(N^{\frac{5}{4}}+N+N^{\frac{3}{4}}+N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}\right)=0$$

$$\frac{BG}{CN}-\left[\frac{AG}{AB}+\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}\right]=0$$

$$\frac{BG}{CN}-\left(N+N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}\right)=0$$




$$\frac{BG}{DP}-\left[\left(\frac{AG}{AB}\right)^{\frac{3}{4}}+\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{2}{4}}\right]=0$$

$$\frac{BG}{DP}-\left(N^{\frac{3}{4}}+N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}\right)=0$$

$$\frac{BG}{FQ}-\left[\left(\frac{AG}{AB}\right)^{\frac{2}{4}}+\left(\frac{AG}{AB}\right)^{\frac{1}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AG}{AB}\right)^{\frac{0}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{1}{4}}+\left(\frac{AB}{AG}\right)^{\frac{2}{4}}+\left(\frac{AB}{AG}\right)^{\frac{3}{4}}\right]=0$$

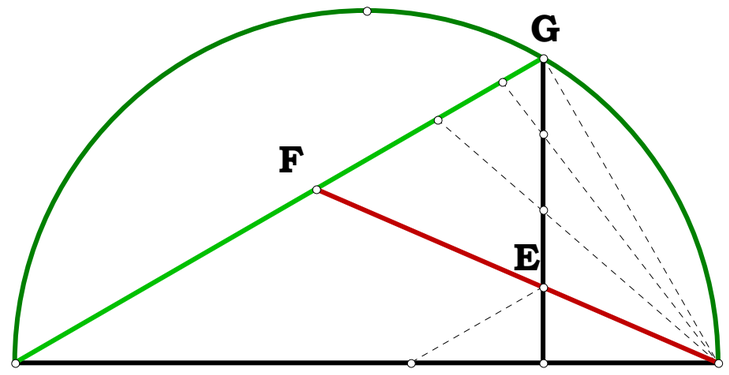
$$\frac{BG}{FQ}-\left(N^{\frac{2}{4}}+N^{\frac{1}{4}}+N^{\frac{0}{4}}+N^{\frac{0}{4}}+N^{\frac{-1}{4}}+N^{\frac{-1}{4}}+N^{\frac{-2}{4}}+N^{\frac{-3}{4}}\right)=0$$

$$\frac{AG}{BM}-\left(\frac{AG^{\frac{6}{4}}+AG^{\frac{4}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{5}{4}}-AB^{\frac{6}{4}}}\right)=0 \qquad \frac{AG}{BM}-\left(\frac{N^{\frac{3}{2}}+N}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{CN}-\left(\frac{AG^{\frac{5}{4}}+AG^{\frac{3}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{4}{4}}-AB^{\frac{5}{4}}}\right)=0$$

$$\frac{AG}{CN}-\left(\frac{N^{\frac{5}{4}}+N^{\frac{3}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0 \qquad \frac{AG}{DP}-\left(\frac{AG^{\frac{4}{4}}+AG^{\frac{2}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{3}{4}}-AB^{\frac{4}{4}}}\right)=0 \qquad \frac{AG}{DP}-\left(\frac{N+N^{\frac{2}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

$$\frac{AG}{FQ}-\left(\frac{AG^{\frac{3}{4}}+AG^{\frac{1}{4}}\cdot AB^{\frac{2}{4}}}{AG^{\frac{1}{4}}\cdot AB^{\frac{2}{4}}-AB^{\frac{3}{4}}}\right)=0 \qquad \frac{AG}{FQ}-\left(\frac{N^{\frac{3}{4}}+N^{\frac{1}{4}}}{N^{\frac{1}{4}}-N^{\frac{0}{4}}}\right)=0$$

**011396 Pyramid of Ratios VI, Moving the Point**




$N_1 := 5 \quad N_2 := 7 \qquad N_1 := N_1 \quad N_2 := N_2$

$AD := 1 \quad AB := \frac{AD}{N_1} \quad BD := AD - AB \qquad BG := \sqrt{AB \cdot BD} \quad BE := \frac{BG}{N_2}$

$BC := \frac{BD \cdot BE}{BG} \quad AE := \sqrt{AB^2 + BE^2} \quad AC := AB + BC \quad AF := \frac{AE \cdot AD}{AC}$

$EF := AF - AE \quad \frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0$

$\Delta := 2 .. N_1 \quad \delta := 2 .. N_2 \quad SeriesAF_{\Delta,\delta} := \frac{\Delta \cdot \delta}{(\Delta - 1) \cdot (\delta - 1)}$

$$SeriesAF = \begin{pmatrix} 4 & 3 & 2.666667 & 2.5 & 2.4 & 2.333333 \\ 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 2.666667 & 2 & 1.777778 & 1.666667 & 1.6 & 1.555556 \\ 2.5 & 1.875 & 1.666667 & 1.5625 & 1.5 & 1.458333 \end{pmatrix}$$

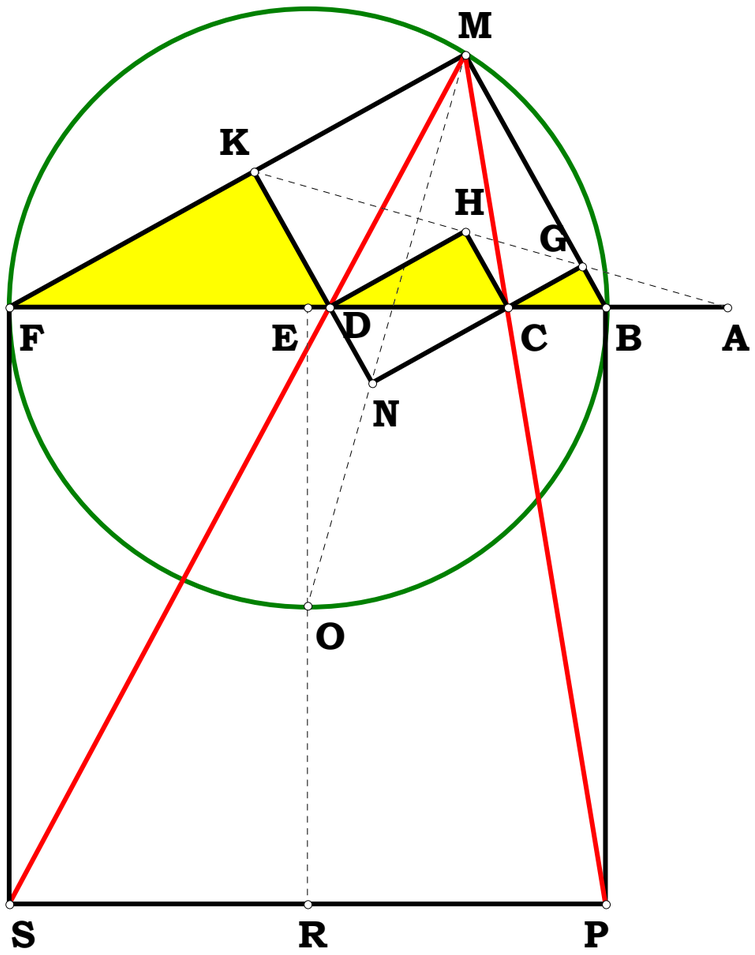
$DG := \sqrt{BD^2 + BG^2} \quad CE := \sqrt{BC^2 + BE^2} \quad DF := \frac{CE \cdot AD}{AC} \quad GF := DG - DF \quad \frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$

$SeriesDG_{\Delta,\delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)}$

$$SeriesDG = \begin{pmatrix} 3 & 2 & 1.666667 & 1.5 & 1.4 & 1.333333 \\ 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 5 & 3 & 2.333333 & 2 & 1.8 & 1.666667 \\ 6 & 3.5 & 2.666667 & 2.25 & 2 & 1.833333 \end{pmatrix}$$

011696

**The figure cuts the base in Cube Roots and provides some interesting ratios.**




$$N := 10$$

$$AG := N \quad AB := \frac{AG}{N} \quad BG := AG - AB \quad BO := \frac{BG}{2}$$

$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad BC := AC - AB$$

$$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BF := AF - AB \quad FG := BG - BF$$

$$HJ := \frac{BC \cdot BG}{BC + FG} \quad BD := HJ \quad DG := BG - BD$$

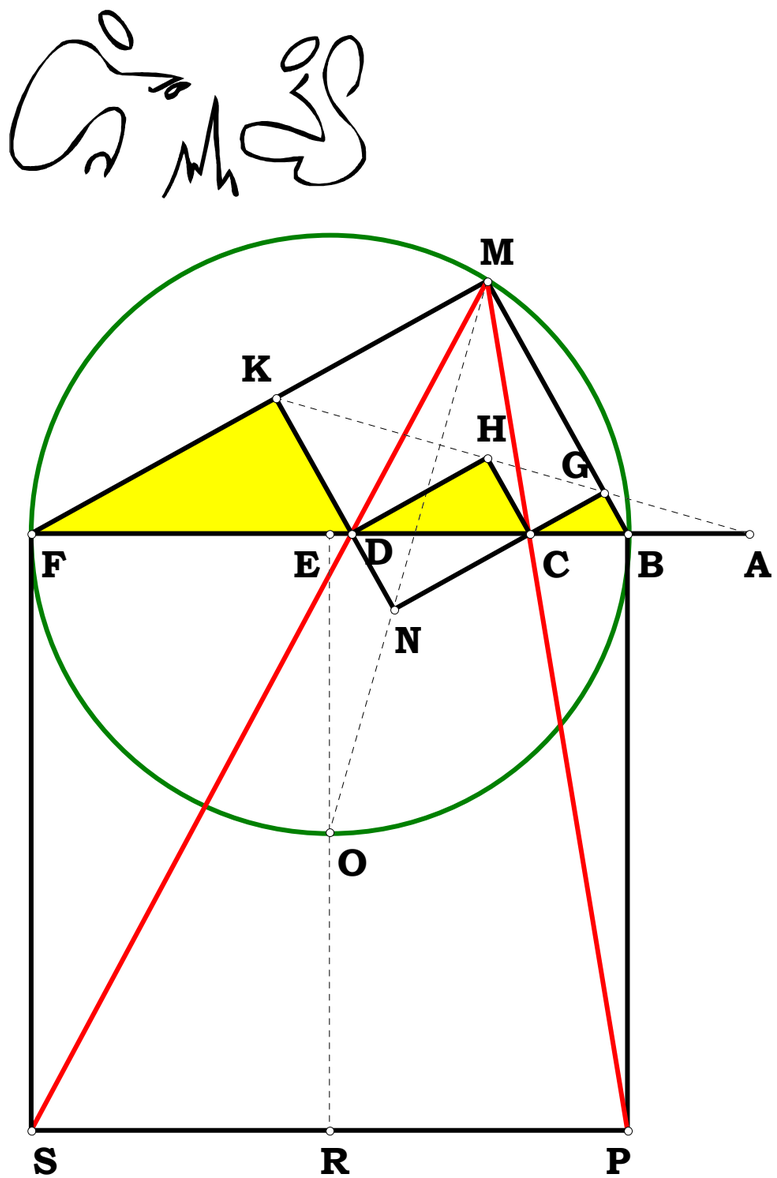
$$DJ := \sqrt{BD \cdot DG} \quad GJ := \sqrt{DJ^2 + DG^2} \quad BJ := \sqrt{DJ^2 + BD^2}$$

$$GN := \frac{GJ \cdot FG}{BG} \quad BM := \frac{BJ \cdot BC}{BG}$$

$$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$$

$$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.679602 \qquad \frac{GJ}{GN} = 1.679602 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.679602$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796024 \qquad \frac{BJ}{BM} = 7.796024 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796024$$




$$CF := AF - AC \quad BP := \frac{BD \cdot BC}{BG} \quad CD := \frac{BD \cdot CF}{BG} \quad FR := \frac{BD \cdot FG}{BG}$$

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.981959$$

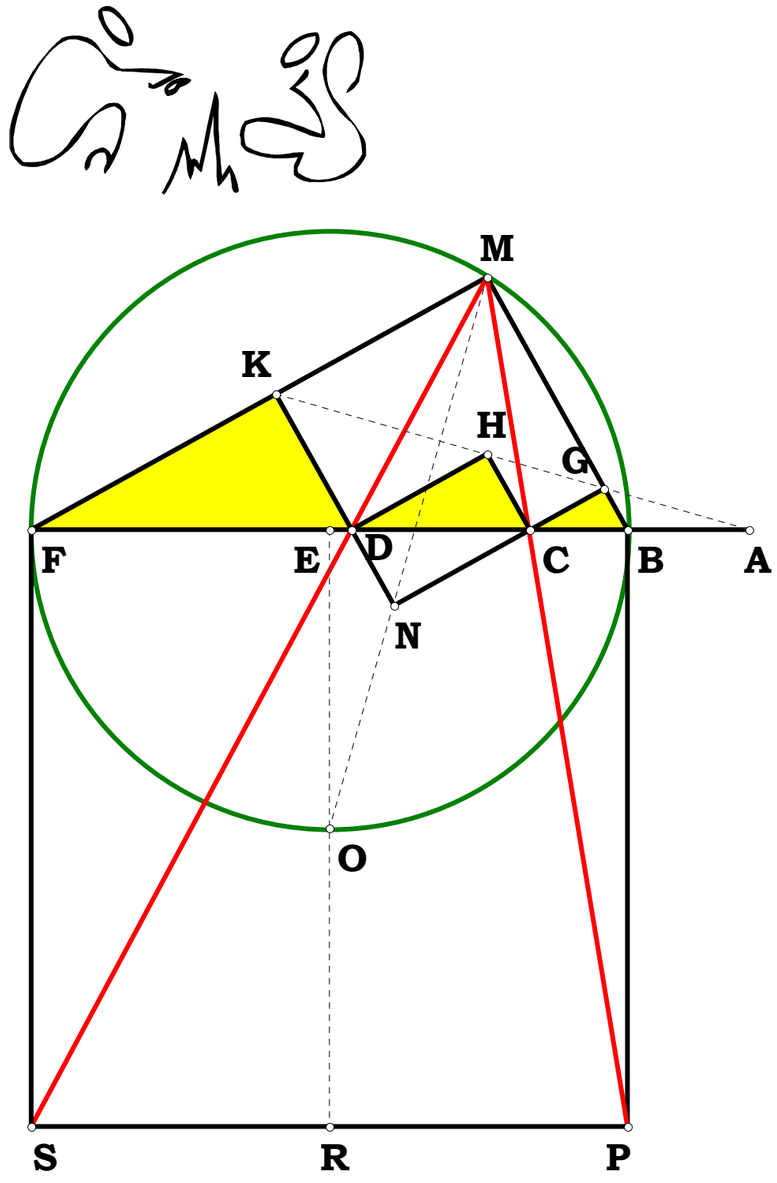
$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.981959 \qquad \frac{BG}{BP} = 43.981959$$

$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.414617$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.414617 \qquad \frac{BG}{CD} = 20.414617$$

$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.475626$$

$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.475626 \qquad \frac{BG}{FR} = 9.475626$$



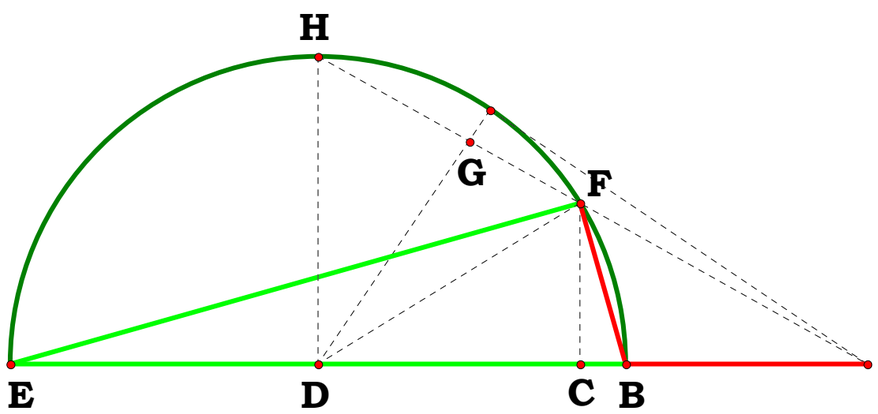

$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.868844 \qquad \frac{AG}{BP} = 48.868844 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.868844$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.682908 \qquad \frac{AG}{CD} = 22.682908 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.682908$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528473 \qquad \frac{AG}{FR} = 10.528473 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528473$$

## 011796A Right Triangle In A Given Ratio

**Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.**




$N := 3$

$AB := 1 \quad AE := AB \cdot N \quad BE := AE - AB$

$BD := \frac{BE}{2} \quad DF := BD \quad DE := BD \quad AD := AB + BD$

$DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

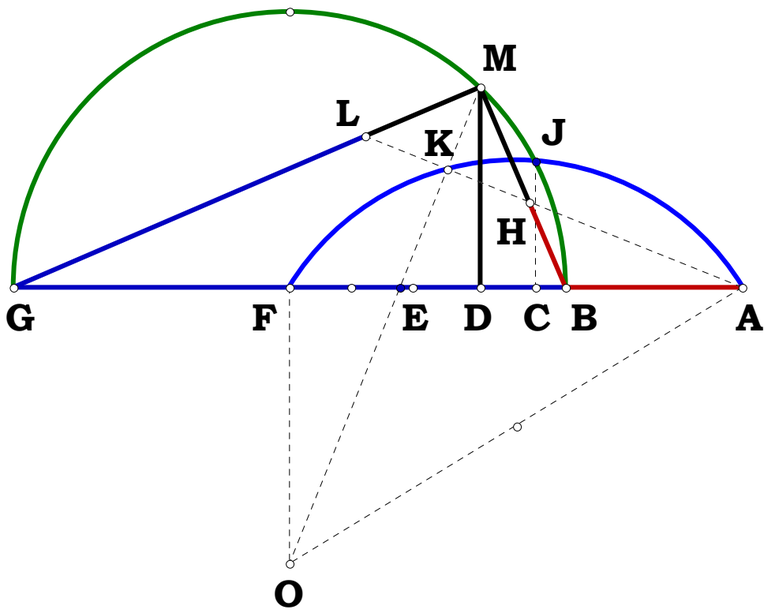
$S_1 := AD \quad S_2 := AF \quad S_3 := DF \quad CD := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad BC := BD - CD$

$CE := CD + DE \quad CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2} \quad AC := AD - CD$

$\frac{AE}{AB} - \frac{EF}{BF} = 0 \quad AC - \frac{N^2 + N}{N^2 + 1} = 0 \quad BF - \frac{N - 1}{\sqrt{N^2 + 1}} = 0 \quad EF - \frac{N^2 - N}{\sqrt{N^2 + 1}} = 0$

## 011796B Divide The Sides Of A Triangle In A Given Ratio

**Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.**




$N_1 := 5 \quad N_2 := 9$

$AB := 1 \quad AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \frac{BG}{2}$

$AC := \frac{N_1^2 + N_1}{N_1^2 + 1} \quad AF := AB + BF \quad CF := AF - AC$

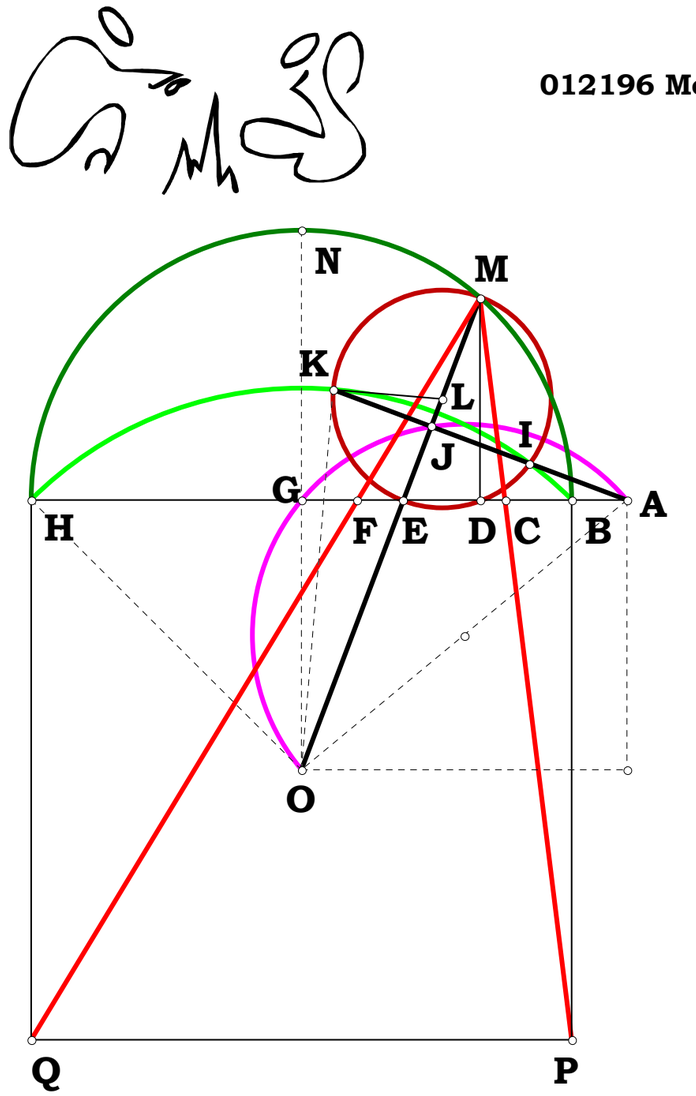
$BC := AC - AB \quad BD := \frac{CF}{N_2} + BC \quad DF := BF - BD$

$DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF \quad EF := \frac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \frac{EO \cdot (DM + FO)}{FO} \quad EK := \frac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO) \quad HK := KM \quad HM := \sqrt{2 \cdot KM^2}$

$BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM \quad GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM$

$\frac{AG}{AB} - \frac{GL}{BH} = 0$




# 012196 More On Cube Roots

$N := 5 \qquad BH := 1$

$BP := BH \quad HQ := BH$

$BG := \frac{BH}{2} \quad GO := BG \quad GN := BG \quad NO := BH \quad GH := BG$

$BE := \frac{BG}{N} \quad EG := BG - BE \quad EO := \sqrt{EG^2 + GO^2}$

$MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO \quad EL := \frac{EM}{2} \quad LK := EL$

$LO := EO + EL \quad LJ := \frac{LK^2}{LO} \quad EJ := EL - LJ$

$AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH \quad AB := AH - BH$

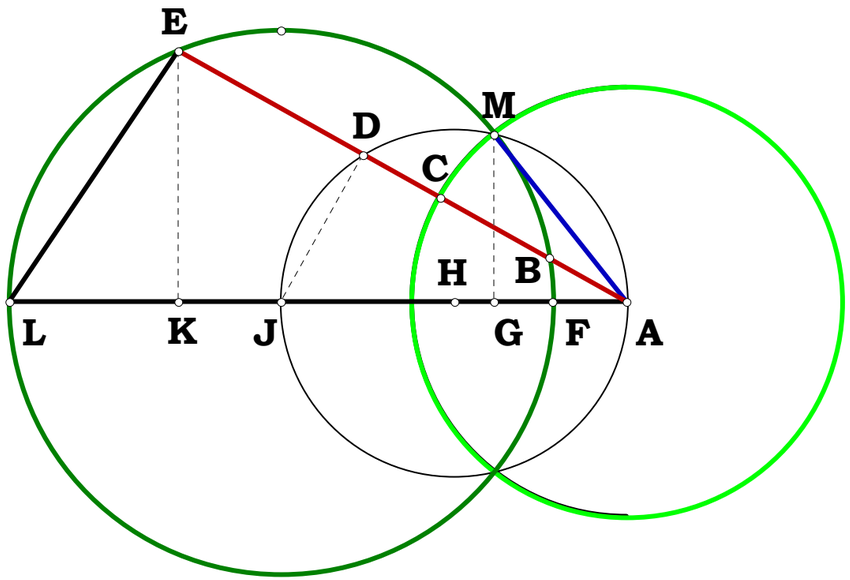
$DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO} \quad BD := BG - (EG + DE)$

$BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD \quad DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0 \qquad (AB \cdot AH^2)^{\frac{1}{3}} - AF = 0$

# 012296 Trivial Method Square Root

**For any E between M and L, AM is the square root of AB x AE.**




$N_1 := 6 \quad N_2 := 3$

$AF := 1 \quad AL := AF \cdot N_1 \quad FL := AL - AF \quad FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL} \quad AJ := AF + FJ \quad AG := \frac{AM^2}{AJ}$

$GL := AL - AG \quad GK := \frac{GL}{N_2} \quad FG := AG - AF$

$FK := GK + FG \quad KL := FL - FK \quad EK := \sqrt{FK \cdot KL}$

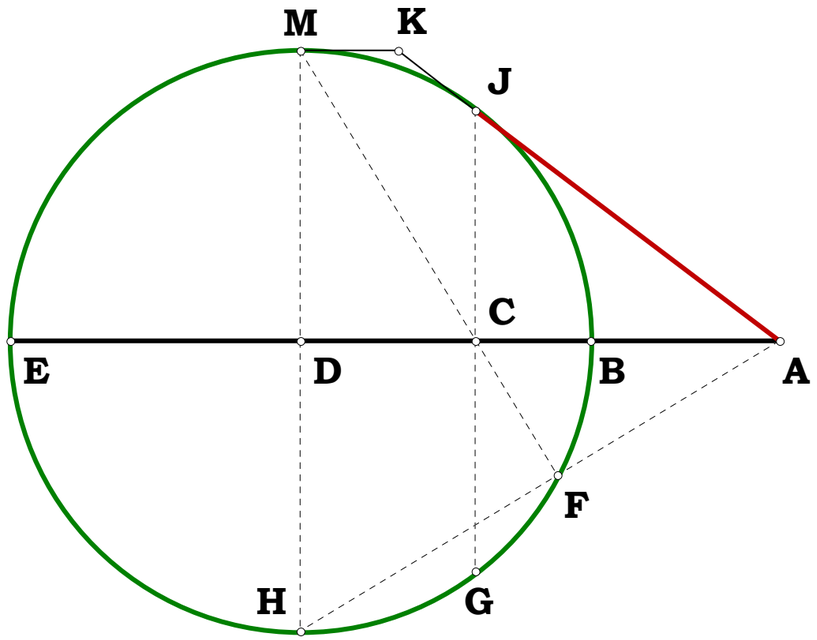
$AK := FK + AF \quad AE := \sqrt{AK^2 + EK^2} \quad AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD \quad BD := DE \quad AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

## 012496 Tangent

**The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.**




$$N := 5 \quad AB := 1 \quad AE := AB \cdot N$$

$$BE := AE - AB \quad BD := \frac{BE}{2} \quad AD := AB + BD$$

$$DM := BD \quad DH := BD \quad CD := \frac{DH \cdot DM}{AD}$$

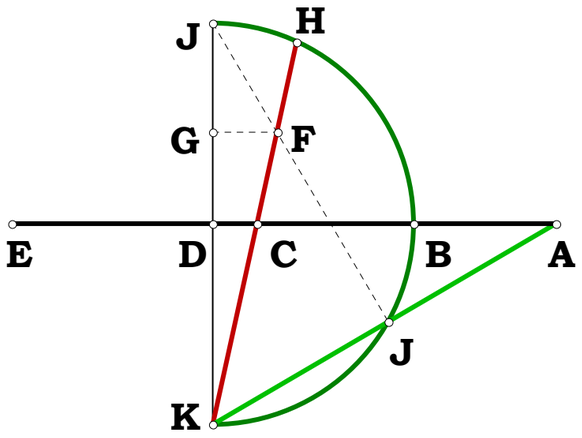
$$BC := BD - CD \quad CE := BE - BC$$

$$CJ := \sqrt{BC \cdot CE} \quad AJ := \sqrt{(AB + BC)^2 + CJ^2}$$

$$AJ - \sqrt{N} = 0 \qquad AJ - \sqrt{AB \cdot AE} = 0$$

$$AC := AB + BC \qquad 2 \cdot AB \cdot \frac{N}{(1 + N)} - AC = 0$$

# 012596 On Cubes




**Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.**

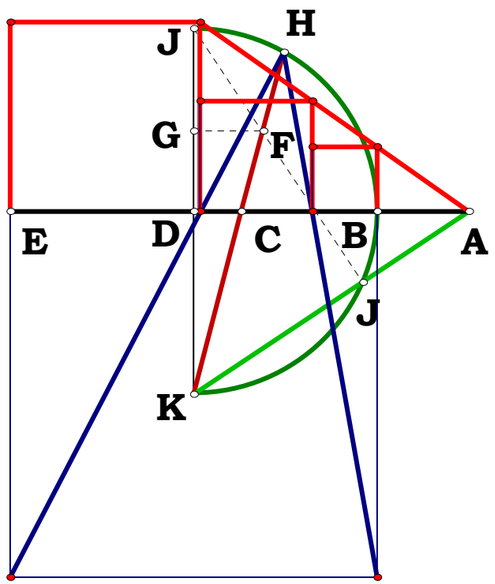
$N := 9 \qquad BE := 1$

$BD := \frac{BE}{2} \quad DK := BD \quad DJ := BD \quad JK := BE \quad DE := BD$

$BC := \frac{BD}{N} \quad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$

$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$

$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$




$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$

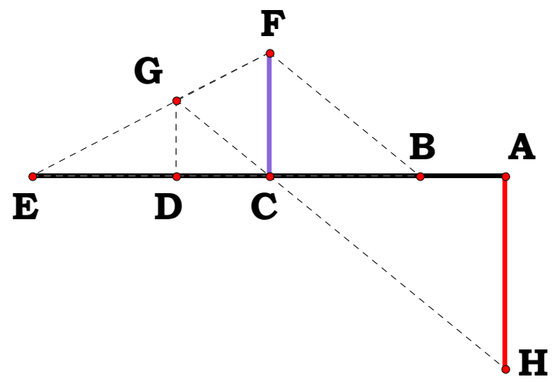
$AB := AE - BE$

$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$\frac{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 1}{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 2} \qquad \frac{1}{8 \cdot N^3 - 12 \cdot N^2 + 6 \cdot N - 2}$$

**012996 Linear division** $\dfrac{N_1\cdot\left(N_2+2\cdot N_3\right)}{2\cdot\left(N_2+N_3\right)}$



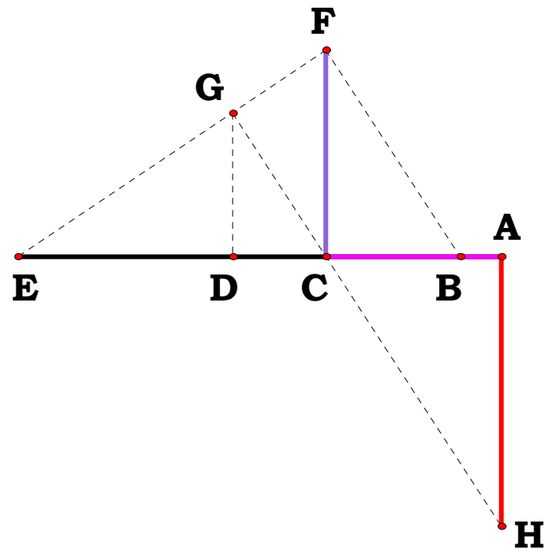

$N_1 := 2 \quad N_2 := 3 \quad N_3 := 9$

$AE := N_1 \quad AH := N_2 \quad AC := \dfrac{AE}{2} \quad CF := N_3 \quad BC := \dfrac{AC\cdot CF}{AH} \quad CE := AC$

$BE := CE + BC \quad CD := \dfrac{BC\cdot CE}{BE} \quad DE := CE - CD \quad AD := AC + CD \quad DG := \dfrac{AH\cdot CD}{AC} \quad BC - \dfrac{N_1\cdot N_3}{2\cdot N_2} = 0$

$DE - \dfrac{N_1\cdot N_2}{2\cdot\left(N_2+N_3\right)} = 0 \quad AD - \dfrac{N_1\cdot\left(N_2+2\cdot N_3\right)}{2\cdot\left(N_2+N_3\right)} = 0 \quad CD - \dfrac{N_1\cdot N_3}{2\cdot\left(N_2+N_3\right)} = 0 \quad DG - \dfrac{N_2\cdot N_3}{N_2+N_3} = 0$

**Linear division** $\dfrac{N_2\cdot\left(N_1-N_3\right)^2}{N_1\cdot N_2-N_2\cdot N_3+N_3\cdot N_4}$



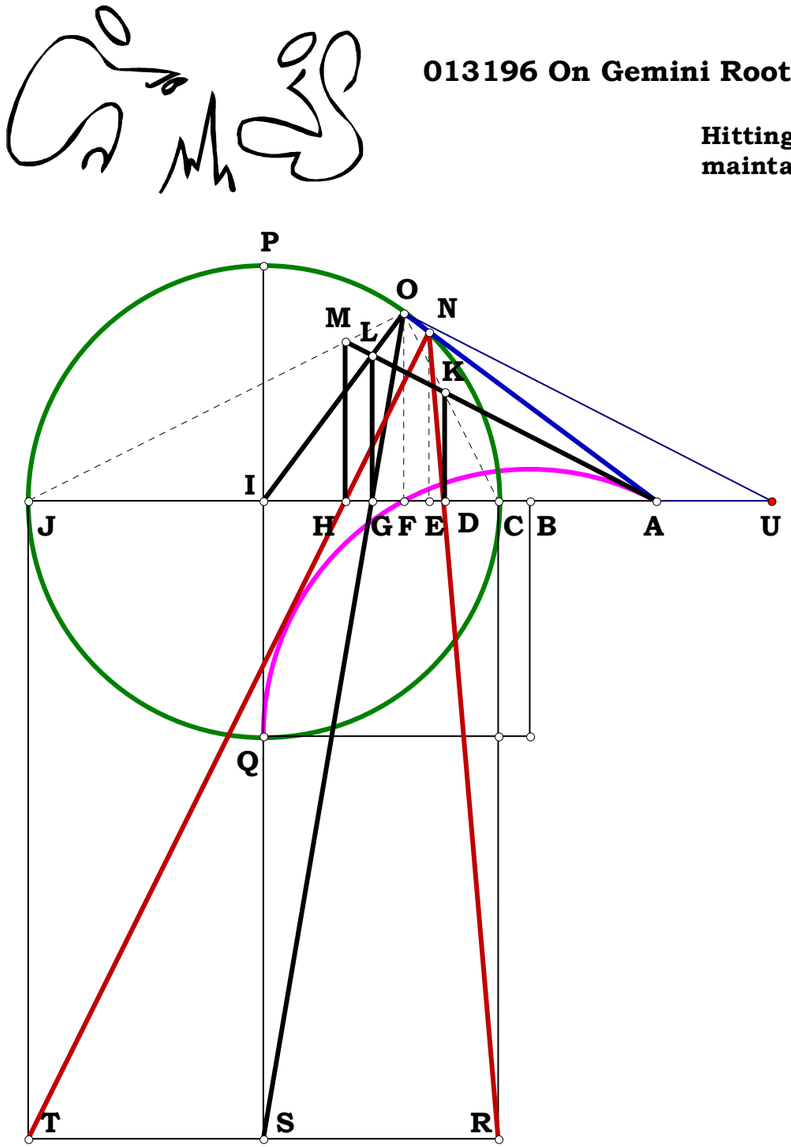

$N_1 := 4 \quad N_2 := 4 \quad N_3 := 2 \quad N_4 := 3$

$AE := N_1 \quad AH := N_2 \quad AC := N_3 \quad CF := N_4 \quad BC := \dfrac{AC\cdot CF}{AH} \quad CE := AE - AC$

$BE := CE + BC \quad CD := \dfrac{BC\cdot CE}{BE} \quad DE := CE - CD \quad AD := AC + CD \quad DG := \dfrac{CF\cdot CE}{BE}$

$BC - \dfrac{N_3\cdot N_4}{N_2} = 0 \quad DE - \dfrac{N_2\cdot\left(N_1-N_3\right)^2}{N_1\cdot N_2-N_2\cdot N_3+N_3\cdot N_4} = 0 \quad AD - \dfrac{N_3\cdot\left(N_1\cdot N_2+N_1\cdot N_4-N_2\cdot N_3\right)}{N_1\cdot N_2-N_2\cdot N_3+N_3\cdot N_4} = 0$

$CD - \dfrac{N_3\cdot N_4\cdot\left(N_1-N_3\right)}{N_1\cdot N_2-N_2\cdot N_3+N_3\cdot N_4} = 0 \quad DG - \dfrac{N_2\cdot N_4\cdot\left(N_1-N_3\right)}{N_1\cdot N_2-N_2\cdot N_3+N_3\cdot N_4} = 0$




**013196 On Gemini Roots**

**Hitting AO from any RT while maintaining Gemini Roots.**

$N_1 := 5 \quad N_2 := 3 \quad BC := 1$

$BJ := BC \cdot N_1 \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IJ := CI \quad BF := \sqrt{BC \cdot BJ} \quad AB := BF$

$AF := AB + BF \quad CF := BF - BC \quad FJ := CJ - CF$

$FO := \sqrt{CF \cdot FJ} \quad CR := CJ \cdot N_2 \quad HS := CR$

$FI := FJ - IJ \quad FG := \frac{FI \cdot FO}{FO + HS} \quad AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2} \quad GO := \frac{OS \cdot FO}{HS + FO} \quad AJ := AF + FJ \quad GL := \frac{HS \cdot GO}{OS} \quad FU := \frac{AG \cdot FO}{GL}$

$AH := \frac{FU \cdot AJ}{FU + FJ} \quad DK := \frac{FO \cdot (AF - CF)}{FU - CF} \quad AD := \frac{AG \cdot DK}{GL} \quad AC := AF - CF \quad CD := AD - AC$

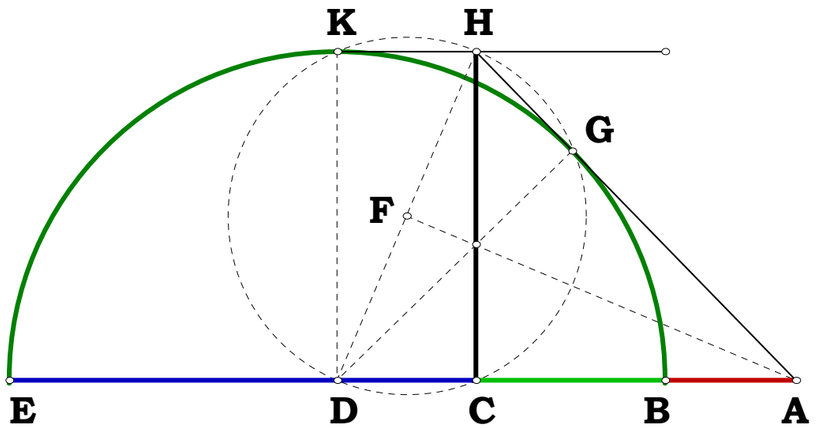
$CH := AH - AC \quad DH := CH - CD \quad HJ := CJ - CH \quad EN := \frac{CR \cdot DH}{CD + HJ} \quad CE := \frac{CD \cdot (CR + EN)}{CR}$

$AE := AC + CE \quad \frac{AF}{FO} - \frac{AE}{EN} = 0 \quad BD := BC + CD \quad BH := BC + CH$

$$\sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$$

## 020296 Find A Segment

**Given BE and BC such that $\sqrt{(AB + BE) \cdot AB} = AB + BC$, find AB.**




$N_1 := 2 \qquad N_2 := 5$

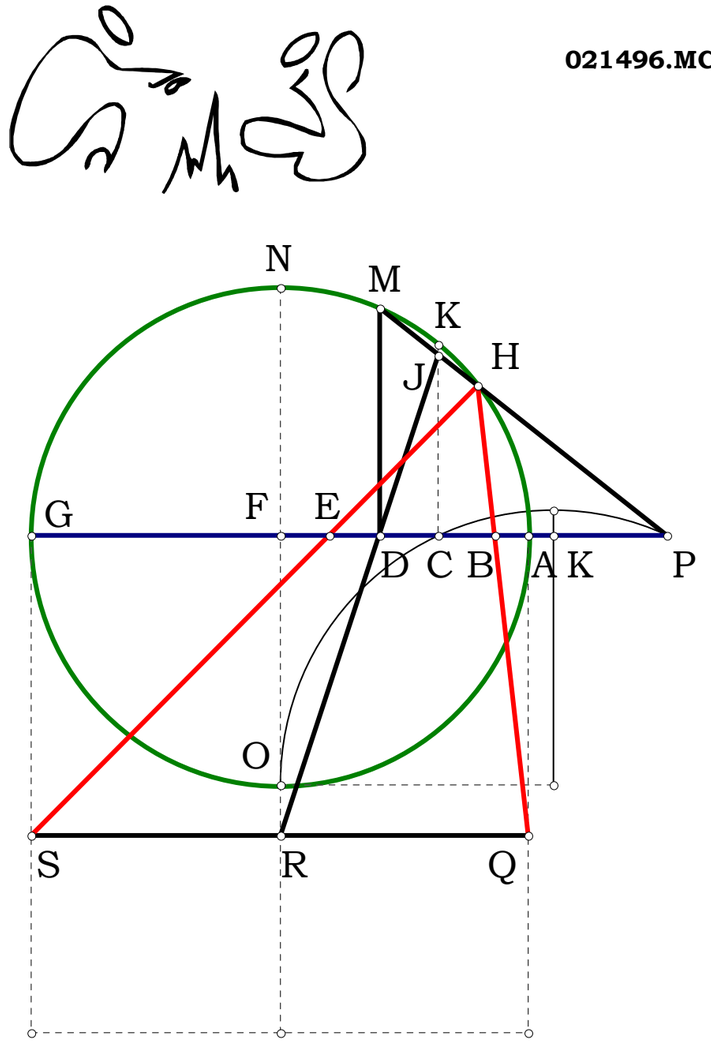
$BC := N_1 \qquad BE := N_2 \qquad BD := \frac{BE}{2} \qquad CD := BD - BC$

$CH := BD \qquad DH := \sqrt{CD^2 + CH^2} \qquad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \qquad AB := AD - BD$

$$AB - \frac{N_1^2}{N_2 - 2 \cdot N_1} = 0$$

$$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$$




**021496.MCD Or, the 17 decimal place rustic solution.**

**Use iteration to find any root pair for BE. Remember that when N is set to 2, we have cube roots.**

$$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$$

$$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$$

$$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$$

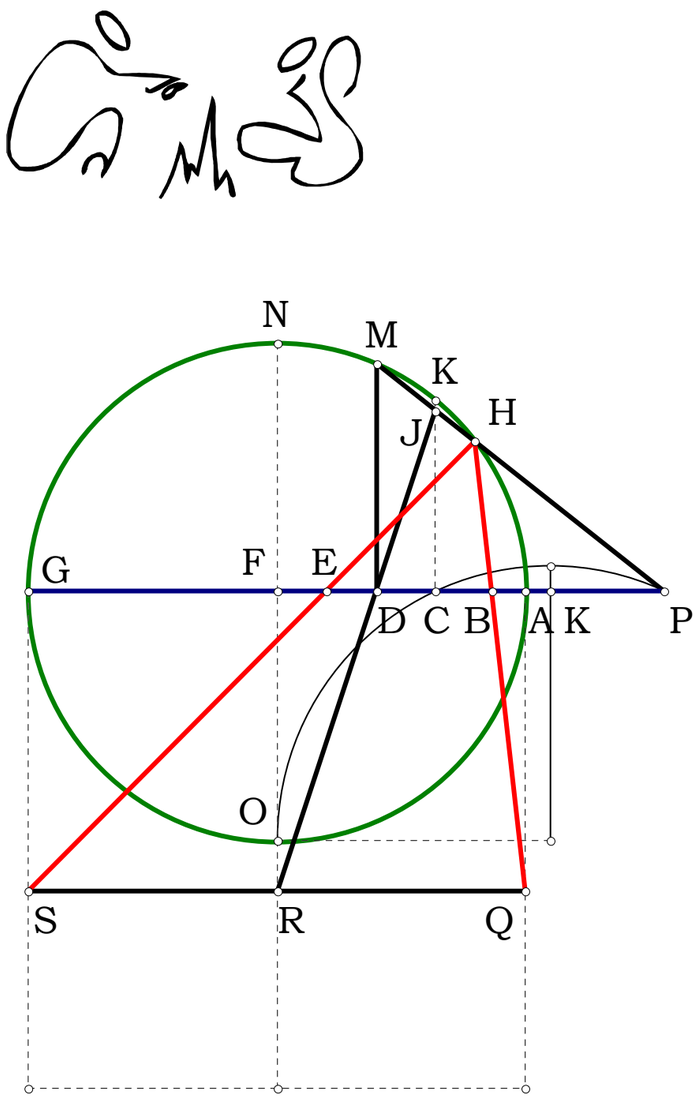
$$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$$

$$N := 2 \quad GN := CG \cdot N \quad IO := GN \quad CM := GN$$

$$\Delta := 40 \quad \delta := 0 .. \Delta$$

$$\begin{pmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{pmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC\right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK}\right)\right]} \end{bmatrix}$$

$$\begin{pmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{pmatrix} := \begin{pmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{pmatrix}$$




$$AK := \sqrt{AE^2 + EK^2} \quad AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2} \quad AJ := \frac{AK^2}{AL} \quad AQ := \frac{AF_\Delta \cdot AJ}{AL} \quad CQ := AQ - AC$$

$$IQ := CI - CQ \quad JQ := \sqrt{CQ \cdot IQ} \quad CD := \frac{CQ \cdot CM}{CM + JQ} \quad HI := \frac{IQ \cdot IO}{IO + JQ} \quad DH := CI - (CD + HI)$$

$$BD := BC + CD \quad BH := BC + CD + DH$$

$$\frac{DH}{\sqrt{CD \cdot HI}} = 1$$

$$BE - \sqrt{BD \cdot BH} = 0$$

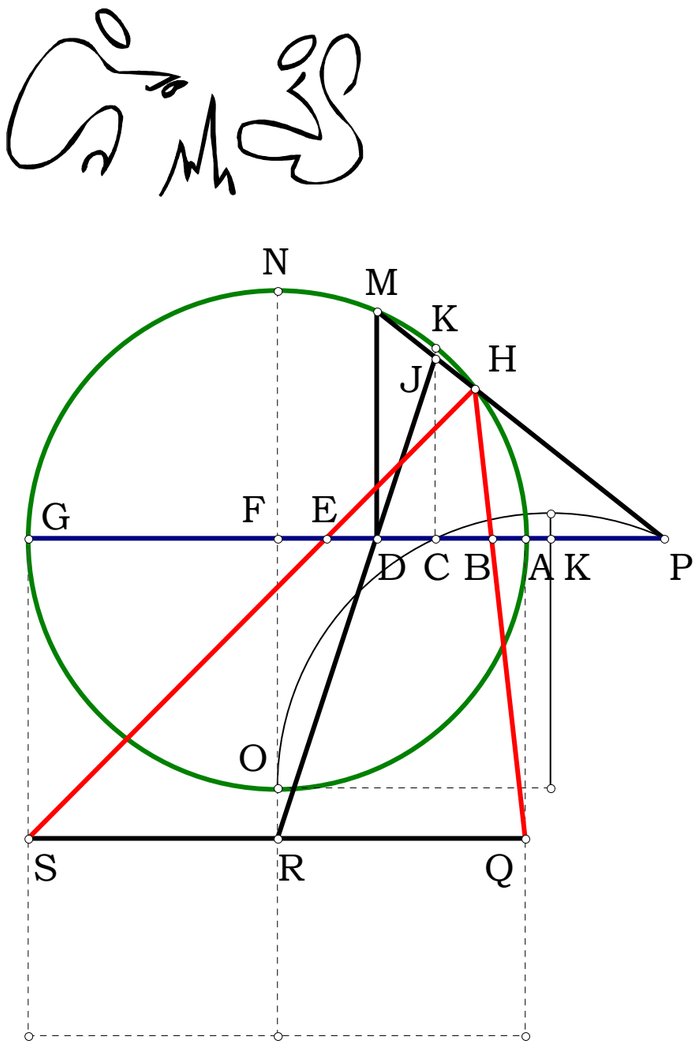
**The next two equations are for the Delian Problem only.**
**Resolution set to max of the program.**

$$(BC^2 \cdot BI)^{\frac{1}{3}} - BD = 0 \qquad (BC \cdot BI^2)^{\frac{1}{3}} - BH = 0$$

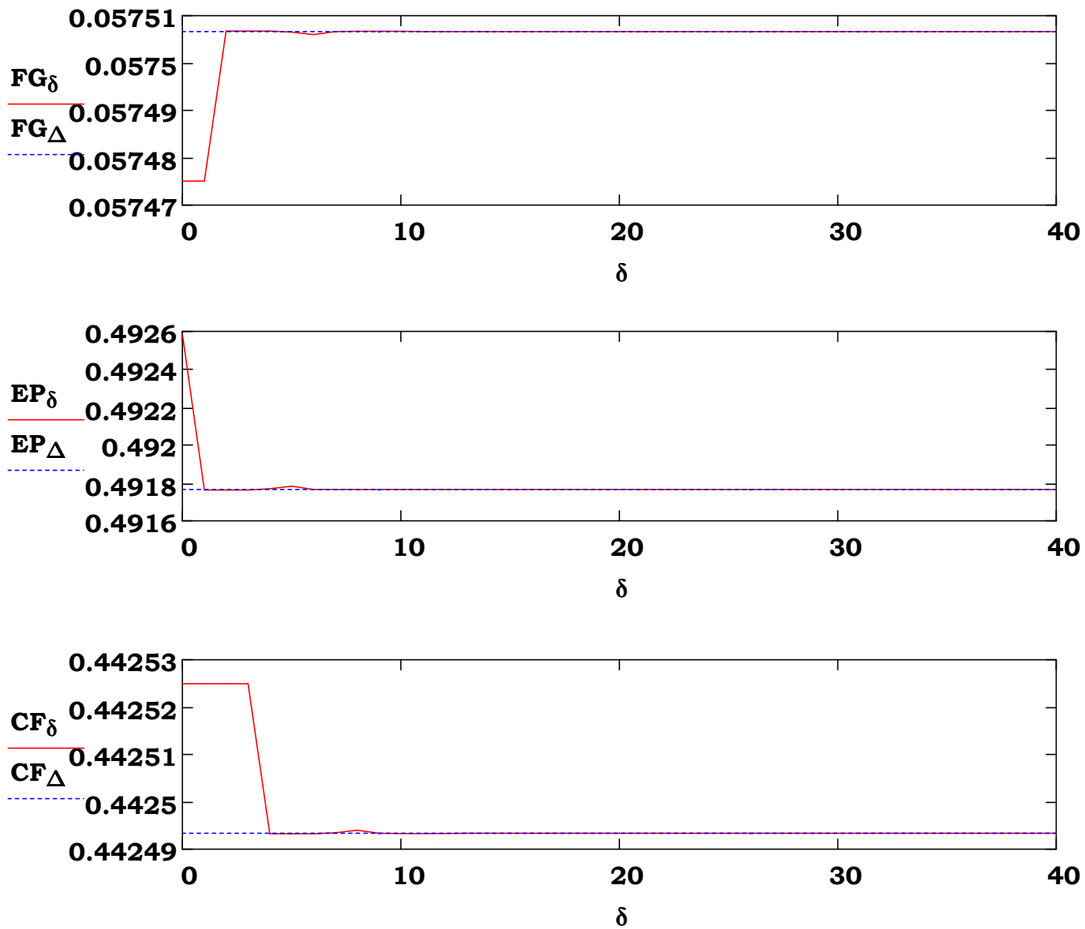
$$BD = 1.2599210498948732 \qquad 2^{\frac{1}{3}} = 1.2599210498948732$$

**17 decimal places. Good to the limits of the program.**

$$BH = 1.5874010519681994 \qquad 4^{\frac{1}{3}} = 1.5874010519681994$$



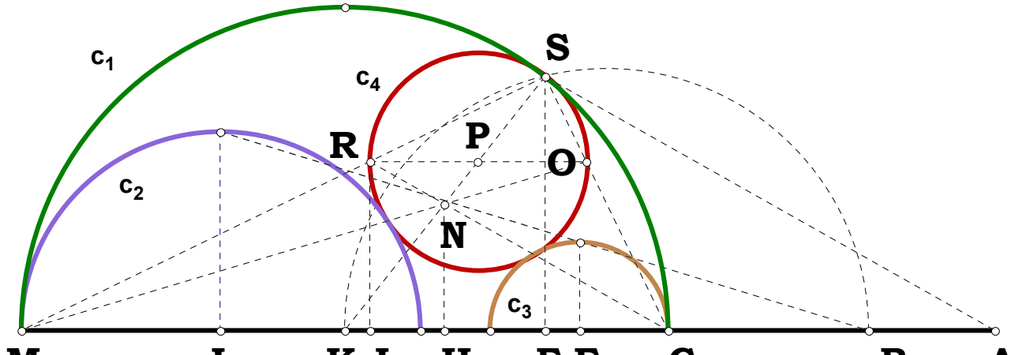

**A graphics peek.**




**The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist. The solution is only good to material differences, on the atomic level, so to speak.**

**041496 Method for Unequals**

**Given $c_1$ , $c_2$ , $c_3$, find $c_4$. I had this sketched out in 95, but if I put it there I would have had a lot of document links to redo in "The Quest."**




$$N_1 := 7 \qquad N_2 := 3$$

$$CM := 1 \quad CK := \frac{CM}{2} \quad CE := \frac{CM}{N_1}$$

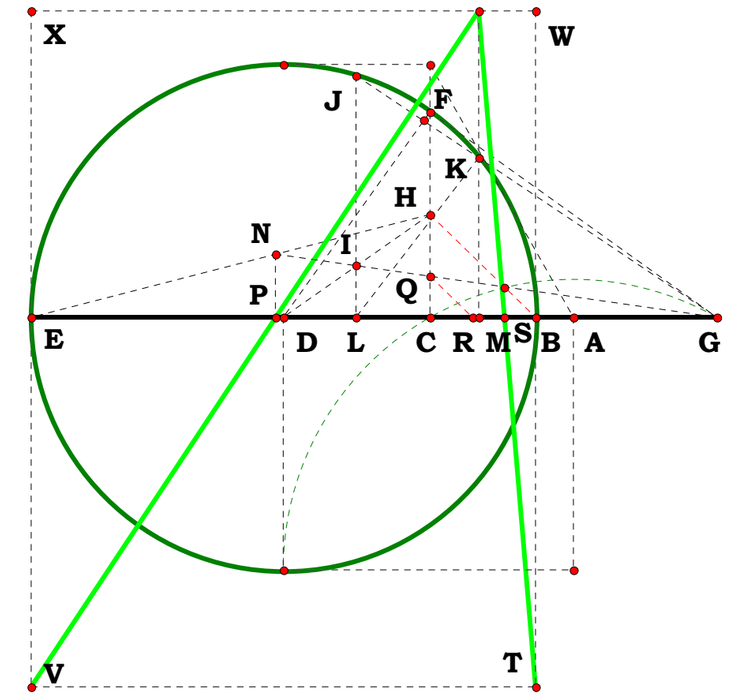
$$LM := \frac{CM}{N_2} \quad EL := CM - (CE + LM)$$

$$BL := \frac{EL \cdot LM}{LM - CE} \quad BM := BL + LM \quad BC := BM - CM \quad BK := \frac{CM}{2} + BC \quad R_1 := LM \quad R_2 := CE \quad D := EL \quad KS := CK \quad EH := \frac{\left(R_2^{\,2} + D^2 - R_1^{\,2}\right)}{2 \cdot D}$$

$$FK := \frac{KS^2}{BK} \quad CF := CK - FK \quad FM := CM - CF \quad FS := \sqrt{CF \cdot FM} \quad HK := CK - (CE + EH) \quad CH := CK - HK \quad HN := \frac{FS \cdot HK}{FK} \quad AF := \frac{CH \cdot FS}{HN}$$

$$JR := \frac{FS \cdot CM}{AF + FM} \quad RO := \frac{CM \cdot (FS - JR)}{FS} \quad PS := \frac{RO}{2} \quad PS - \frac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$$

041596.MCD




## On Gemini Roots

$N_1 := 3 \qquad N_2 := 4$

$AB := N_1 \quad BE := N_2 \qquad BD := \frac{BE}{2}$

$AE := AB + BE \qquad AC := \sqrt{AB \cdot AE} \qquad BC := AC - AB$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE} \qquad CD := BD - BC$

$CG := \frac{CF^2}{CD} \qquad BG := CG - BC \qquad EG := BG + BE$

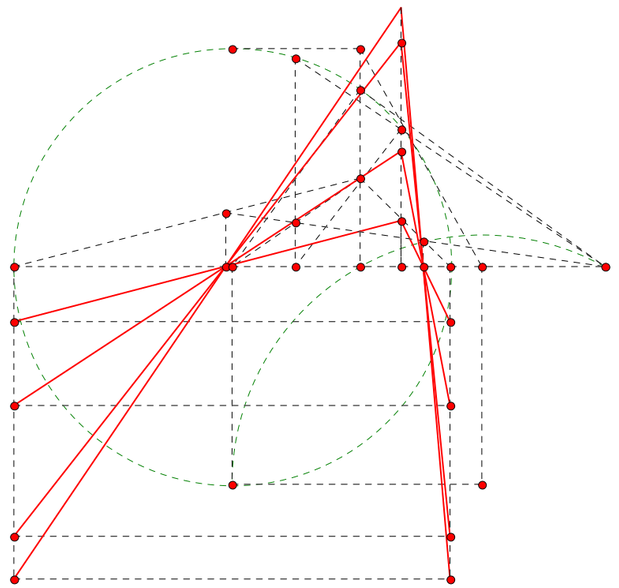
$CH := \frac{1}{2} \cdot CF \qquad DH := \sqrt{CH^2 + CD^2} \qquad DI := \frac{1}{2} \cdot DH$

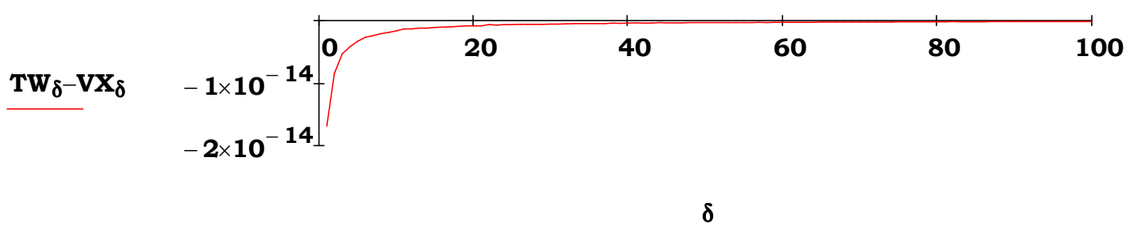
$DL := \frac{CD \cdot DI}{DH} \qquad BL := BD - DL \qquad EL := BE - BL \qquad JL := \sqrt{BL \cdot EL} \qquad GL := BL + BG \qquad GJ := \sqrt{JL^2 + GL^2}$

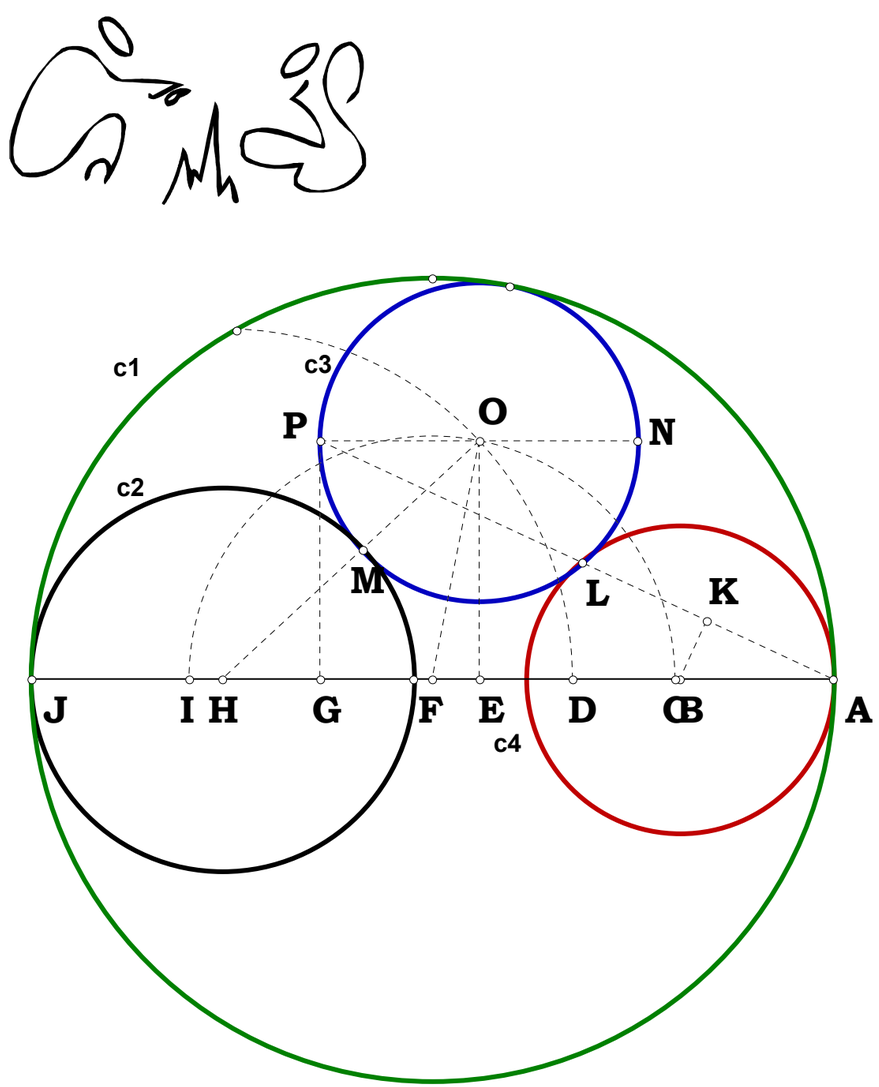
$GK := \frac{BG \cdot EG}{GJ} \qquad GM := \frac{GL \cdot GK}{GJ} \qquad BM := GM - BG \qquad EM := BE - BM \qquad IL := \sqrt{DI^2 - DL^2} \qquad CO := \frac{GL \cdot CH}{IL}$

$NP := \frac{CH \cdot EG}{(CO + CE)} \qquad EP := \frac{CE \cdot NP}{CH} \qquad CQ := \frac{IL \cdot CG}{GL} \qquad CR := \frac{BC \cdot CQ}{CH} \qquad GR := CG - CR \qquad BS := \frac{CR \cdot BG}{GR}$

$\delta := 1 .. 100 \qquad E_\delta := \frac{BE}{\delta} \qquad BT_\delta := E_\delta \qquad EV_\delta := E_\delta \qquad TW_\delta := \frac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$








## 041696 Given Three Radii

**Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.**

$N_1 := 4 \quad N_2 := 16$

$AJ := 1 \quad AF := \frac{AJ}{2} \quad HJ := \frac{AJ}{N_1} \quad NO := \frac{AJ}{N_2}$

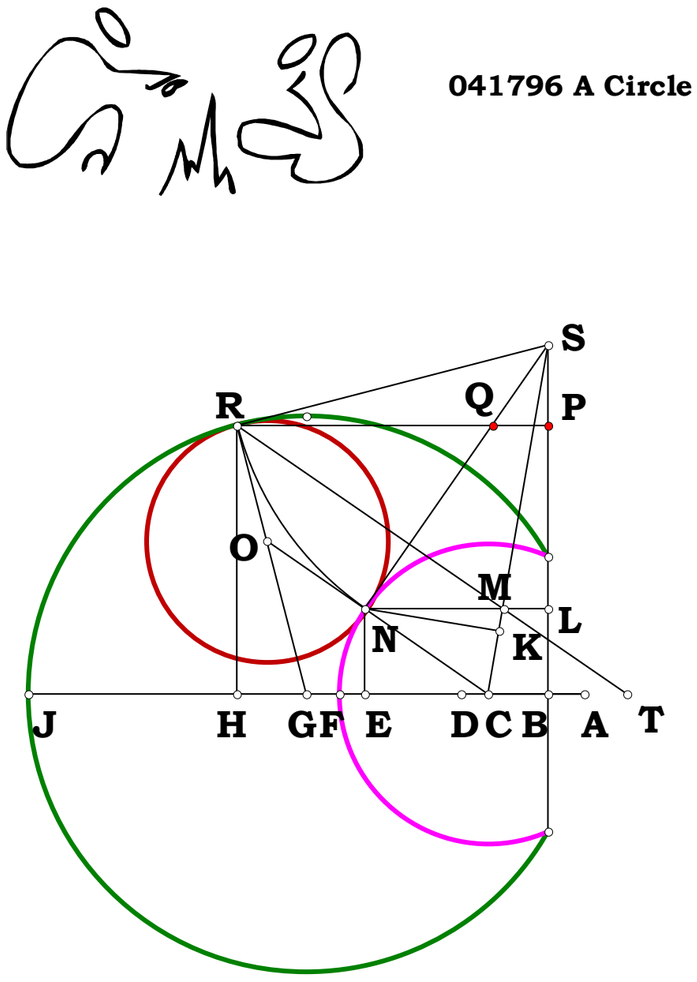
$HM := HJ \quad MO := NO \quad HO := HM + MO$

$FO := AF - NO \quad AH := AJ - HJ \quad FH := AH - AF$

$S_1 := FH \quad S_2 := FO \quad S_3 := HO \quad EH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad EO := \sqrt{HO^2 - EH^2} \quad OP := NO$

$EG := OP \quad AE := AH - EH \quad AG := AE + EG \quad GP := EO \quad AP := \sqrt{AG^2 + GP^2}$

$PL := \frac{AG \cdot (NO + OP)}{AP} \quad AL := AP - PL \quad AB := \frac{AP \cdot AL}{2 \cdot AG} \quad AB - \frac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot (N_2 \cdot N_1 - 2 \cdot N_2 - 4)} = 0$




**041796 A Circle In A Crescent**

$R_1 := 4.65667$ $R_2 := 2.50804$ $D_1 := 3.05114$ $D_2 := 5.20739$

$AG := R_1$ $AJ := 2 \cdot R_1$ $CF := R_2$ $CG := D_1$ $BG := \frac{\left(R_1^2 + D_1^2 - R_2^2\right)}{2 \cdot D_1}$

$BC := CG - BG$ $FG := CG - CF$ $AF := AG - FG$ $AD := \frac{AF}{2}$ $AB := AG - BG$

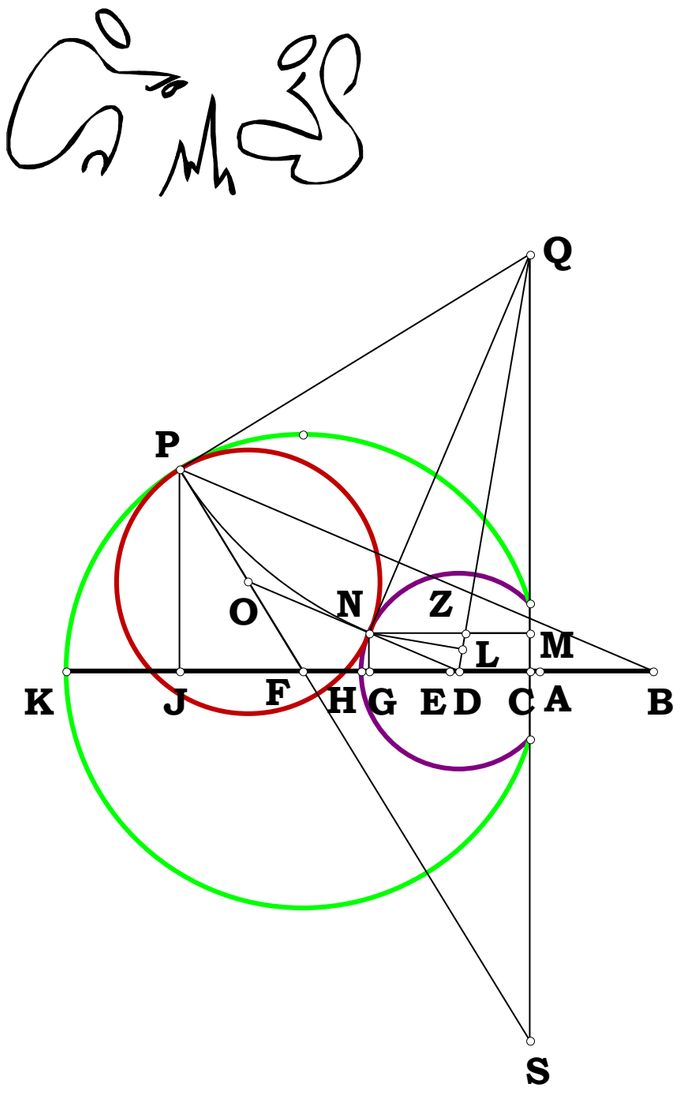
$BJ := AJ - AB$ $BH := D_2$ $AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{AG^2 - GH^2}$

$AP := HR$ $PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := AP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$ $KN := \sqrt{CN^2 - CK^2}$ $KM := \frac{BC \cdot KN}{BS}$

$SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$ $BL := BS - SL$ $EN := BL$ $CE := \sqrt{CN^2 - EN^2}$

$HT := \frac{CE \cdot HR}{EN}$ $GT := HT - GH$ $GO := \frac{AG \cdot CG}{GT}$ $OR := AG - GO$ $OR = 2.005063$




**041796b A Circle In A Crescent**

$R_1 := 5.79438 \quad R_2 := 2.40272 \quad D_1 := 3.82231 \quad D_2 := 6.96436$

$AF := R_1 \quad AK := 2 \cdot R_1 \quad FP := AF \quad FK := AF$

$DH := R_2 \quad DF := D_1 \quad CD := \frac{DH^2 + DF^2 - AF^2}{2 \cdot DF}$

$FH := DF - DH \quad AH := AF - FH \quad AE := \frac{AH}{2} \quad EH := AE \quad AD := DF - AF$

$DE := AE - AD \quad DN := DH \quad AC := CD - AD \quad CK := AK - AC \quad CJ := D_2 \quad CF := CK - FK$

$CF := DF - CD \quad FJ := CJ - CF \quad JP := \sqrt{FP^2 - FJ^2} \quad FS := \frac{FP \cdot CF}{FJ} \quad PS := FS + FP$

$QS := \frac{FP \cdot PS}{JP} \quad PQ := \frac{FJ \cdot QS}{FP} \quad CS := \frac{JP \cdot CF}{FJ} \quad CQ := QS - CS \quad DQ := \sqrt{CD^2 + CQ^2}$

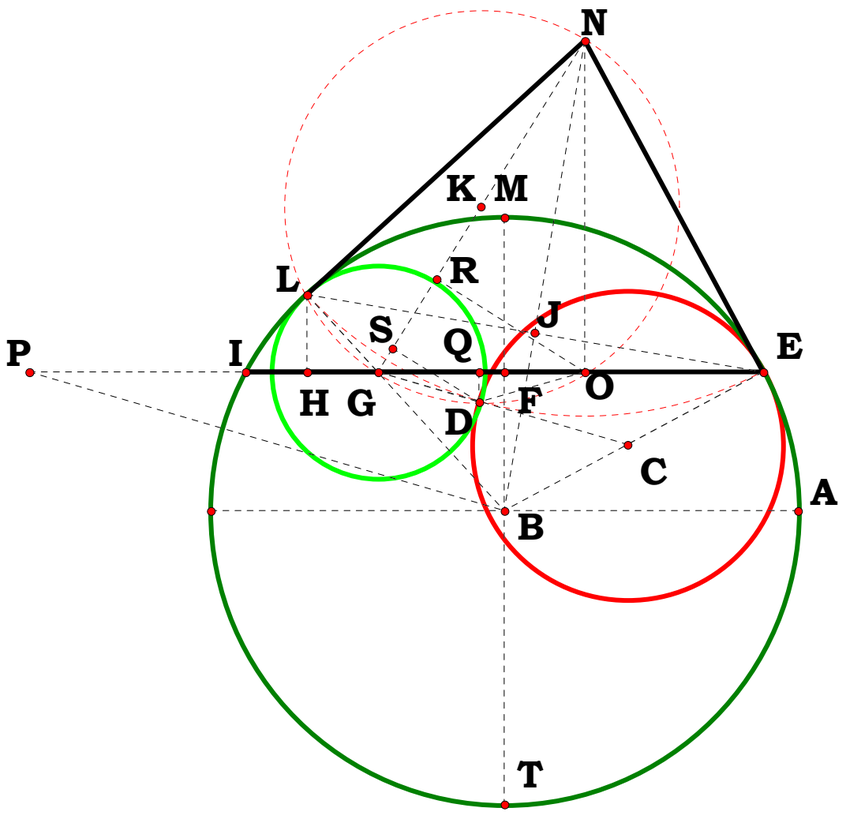
$DL := \frac{DH^2}{DQ} \quad LN := \sqrt{DN^2 - DL^2} \quad LZ := \frac{CD \cdot LN}{CQ} \quad QZ := DQ - DL + LZ \quad MQ := \frac{CQ \cdot QZ}{DQ}$

$CM := CQ - MQ \quad GN := CM \quad DG := \sqrt{DN^2 - GN^2} \quad BJ := \frac{DG \cdot JP}{GN} \quad BF := BJ - FJ$

$FO := \frac{FP \cdot DF}{BF} \quad OP := FP - FO \quad OP = 2.915191$

042296a

**Given BF as a ratio to BM and EG as a ratio to EI, what is CE?**




$N_1 := 9.8425 \quad D_1 := .26381 \quad D_2 := .74461$

$MT := N_1 \quad AB := \frac{MT}{2} \quad BM := AB \quad BE := AB \quad BL := AB$

$MF := D_1 \cdot N_1 \quad BF := N_1 - MF - AB \quad EF := \sqrt{\sqrt{\left(BE^2 - BF^2\right)^2}}$

$EI := 2 \cdot EF \quad EG := EI \cdot D_2 \quad FG := EG - EF \quad BG := \sqrt{BF^2 + FG^2} \cdot \frac{FG}{\sqrt{FG^2}} \quad GL := BL - BG$

$DG := GL \quad GH := \frac{FG \cdot GL}{BG} \quad HL := \sqrt{GL^2 - GH^2} \quad EH := EG + GH \quad EL := \sqrt{EH^2 + HL^2}$

$JL := \frac{EL}{2} \quad BJ := \sqrt{BL^2 - JL^2} \quad LN := \frac{BL \cdot JL}{BJ} \quad GN := \sqrt{LN^2 + GL^2} \quad JN := \sqrt{LN^2 - JL^2}$

$EJ := JL \quad EN := \sqrt{JN^2 + EJ^2} \quad S_1 := EG \quad S_2 := E \, S_3 := GN \quad GO := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$NO := \sqrt{GN^2 - GO^2} \quad NR := \frac{NO^2}{GN} \quad GS := \frac{DG^2}{GN} \quad RS := GN - (NR + GS) \quad DT := RS$

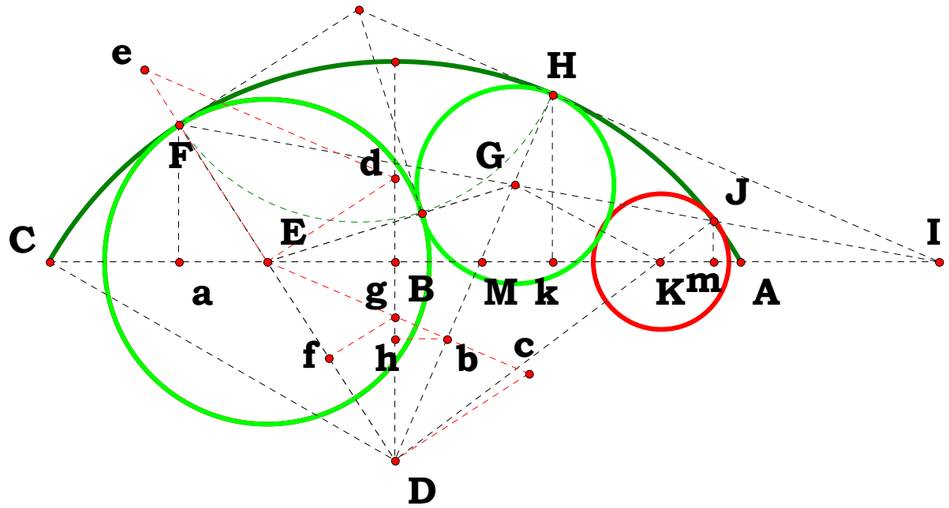
$DS := \sqrt{DG^2 - GS^2} \quad RT := DS \quad OR := \sqrt{NO^2 - NR^2} \quad OT := OR - RT \quad DO := \sqrt{DT^2 + OT^2}$

$S_1 := GO \quad S_2 := DG \quad S_3 := DO \quad OQ := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} \quad GQ := GO - OQ$

$DQ := \sqrt{DO^2 - OQ^2} \quad FP := \frac{GQ \cdot BF}{DQ} \quad CE := \sqrt{\left(\frac{BE \cdot EG}{FP + EF}\right)^2} \quad CE = 2.583347$

042296.MCD

**Place EF and GH and find JK.**




$N_1 := 11.59521 \quad N_2 := 3.34617 \quad D_1 := .812152 \quad D_2 := 3.19428$

$AC := N_1 \quad AB := \frac{AC}{2} \quad BC := AB \quad BD := N_2 \quad Aa := AC \cdot D_1 \quad Ba := Aa - AB$

$CD := \sqrt{BD^2 + BC^2} \quad EF := \frac{CD \cdot \left(\sqrt{CD^2 - Ba^2} - BD\right)}{\sqrt{CD^2 - Ba^2}} \quad DE := CD - EF$

$BE := \sqrt{DE^2 - BD^2} \quad AE := AB + BE \quad GH := \frac{AE - EF}{D_2} \quad EG := EF + GH$

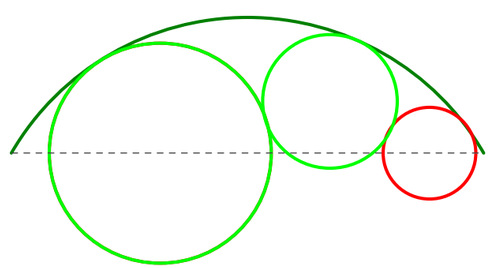
$DG := CD - GH \quad Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG} \quad Eb := \sqrt{DE^2 - Db^2} \quad Ec := \frac{DE^2}{Eb}$

$Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD} \quad Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$

$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db} \quad DM := \sqrt{BD^2 + BM^2}$

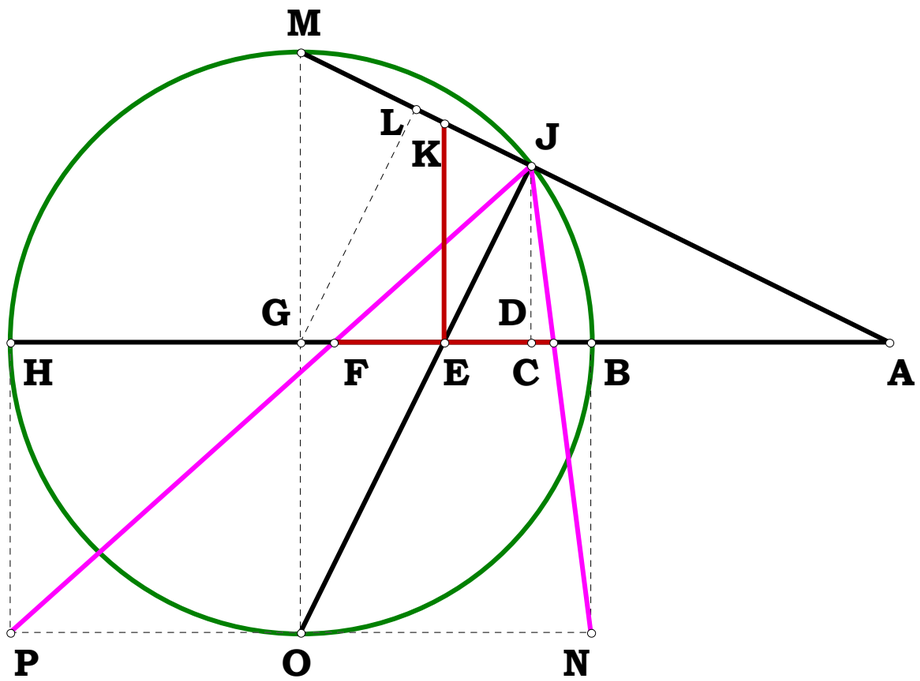
$Bk := \frac{BM \cdot CD}{DM} \quad HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM} \quad Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk} \quad HI := \sqrt{Hk^2 + Ik^2} \quad Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$

$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI} \quad Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm} \quad JK = 1.126755$



042396a

**Is CF always equal to EK?**




$N_1 := 3$ $AB := 1$

$AH := AB \cdot N_1$ $BH := AH - AB$ $BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

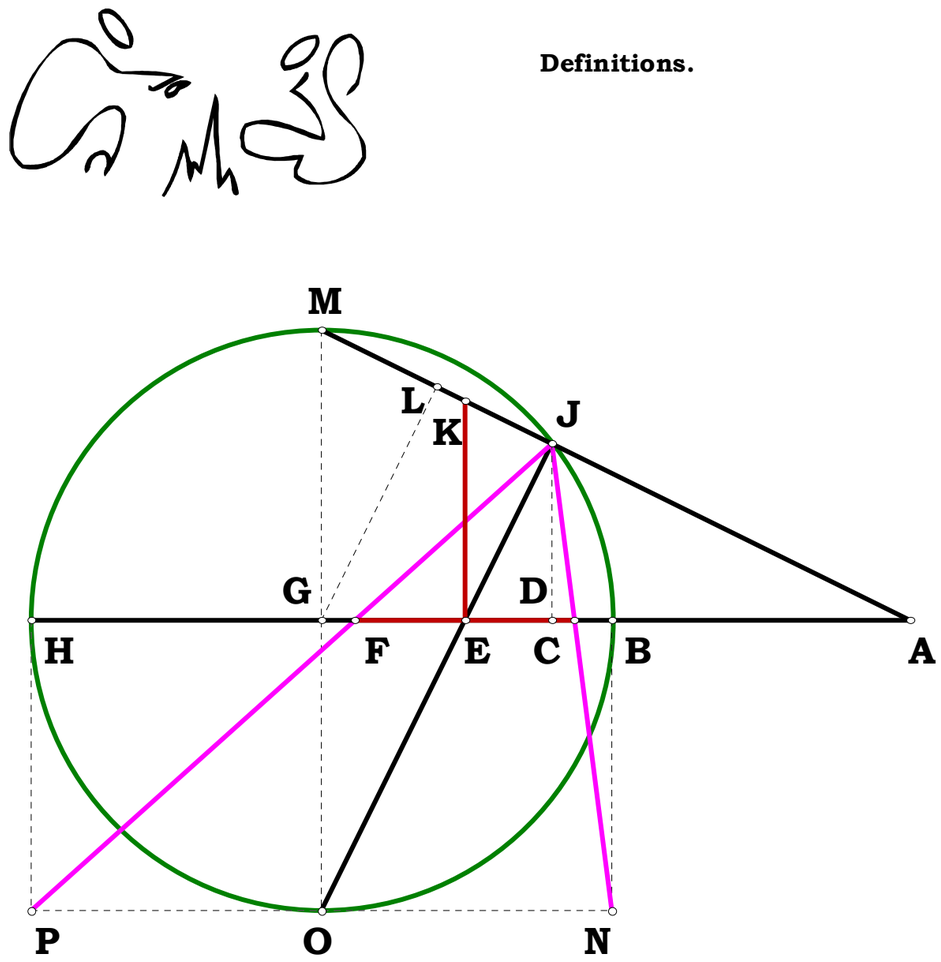
$GM := BG$ $GH := BG$ $AG := AH - GH$ $AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$

$BD := AD - AB$ $DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$

$DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$ $CE := \frac{CF}{2}$ $BE := BC + CE$

$AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$




**Definitions.**

$$BH - (N_1 - 1) = 0 \qquad BG - \frac{N_1 - 1}{2} = 0 \qquad AG - \left(\frac{1}{2}\cdot N_1 + \frac{1}{2}\right) = 0$$

$$AM - \frac{1}{2}\cdot\sqrt{2\cdot N_1^{\,2} + 2} = 0 \qquad AL - \frac{1}{2}\cdot\frac{(N_1 + 1)^2}{\sqrt{2\cdot N_1^{\,2} + 2}} = 0 \qquad LM - \frac{1}{2}\cdot\frac{(N_1 - 1)^2}{\sqrt{2\cdot N_1^{\,2} + 2}} = 0$$

$$AJ - \frac{2}{\sqrt{2\cdot N_1^{\,2} + 2}}\cdot N_1 = 0 \qquad AD - (N_1 + 1)\cdot\frac{N_1}{(N_1^{\,2} + 1)} = 0 \qquad BD - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0$$

$$DH - N_1^{\,2}\cdot\frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0 \qquad DJ - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)}\cdot N_1 = 0 \qquad BC - \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0$$

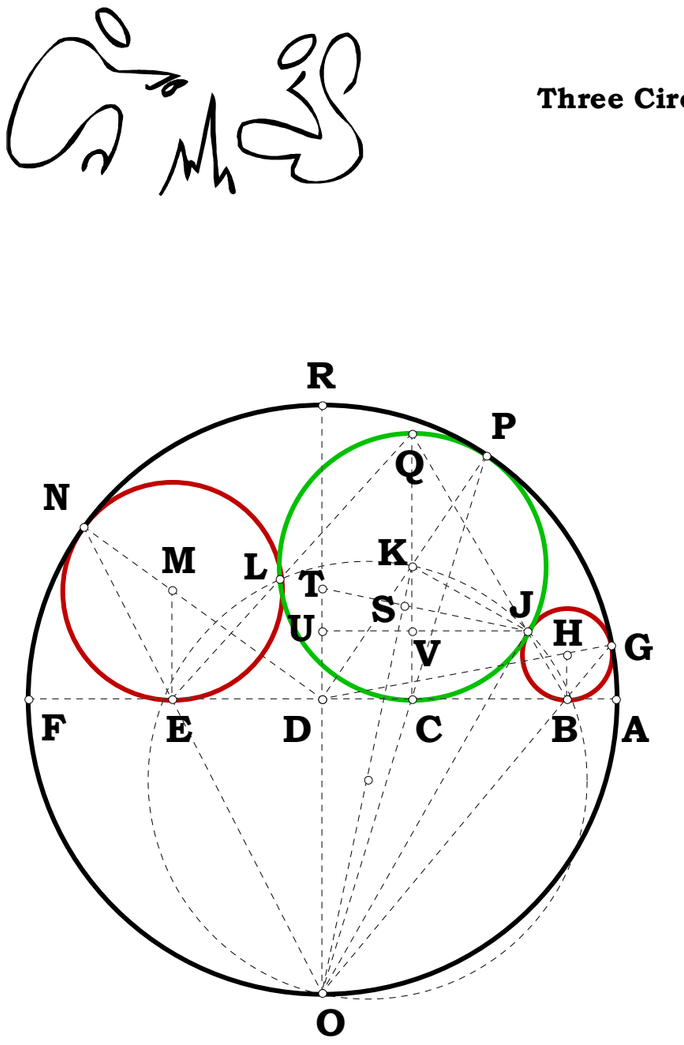
$$DF - 2\cdot N_1^{\,3}\cdot\frac{(N_1 - 1)}{\left[(N_1 + 1)^2\cdot(N_1^{\,2} + 1)\right]} = 0 \qquad CD - 2\cdot(N_1 - 1)\cdot\frac{N_1}{\left[(N_1^{\,2} + 1)\cdot(N_1 + 1)^2\right]} = 0$$

$$CF - 2\cdot N_1\cdot\frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad CE - N_1\cdot\frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad BE - \frac{(N_1 - 1)}{(N_1 + 1)} = 0$$

$$AE - 2\cdot\frac{N_1}{(N_1 + 1)} = 0 \qquad EK - 2\cdot(N_1 - 1)\cdot\frac{N_1}{(N_1 + 1)^2}$$

$$EK - CF = 0 \qquad EK = 0.75$$

**Meditation : Do these equations satisfy the requirement that a definition must contain both form and matter?**




**Three Circles 042496**

**Given AC, find CK and BH.**

$N_1 := 9.8425 \qquad D_1 := .36802$

$AF := N_1 \quad AD := \frac{AF}{2} \quad AC := AF \cdot D_1 \quad DO := AD \quad OR := AF$

$CD := AD - AC \quad CO := \sqrt{CD^2 + DO^2} \quad PO := \frac{DO \cdot OR}{CO} \quad CP := PO - CO \quad CK := \frac{DO \cdot CP}{PO}$

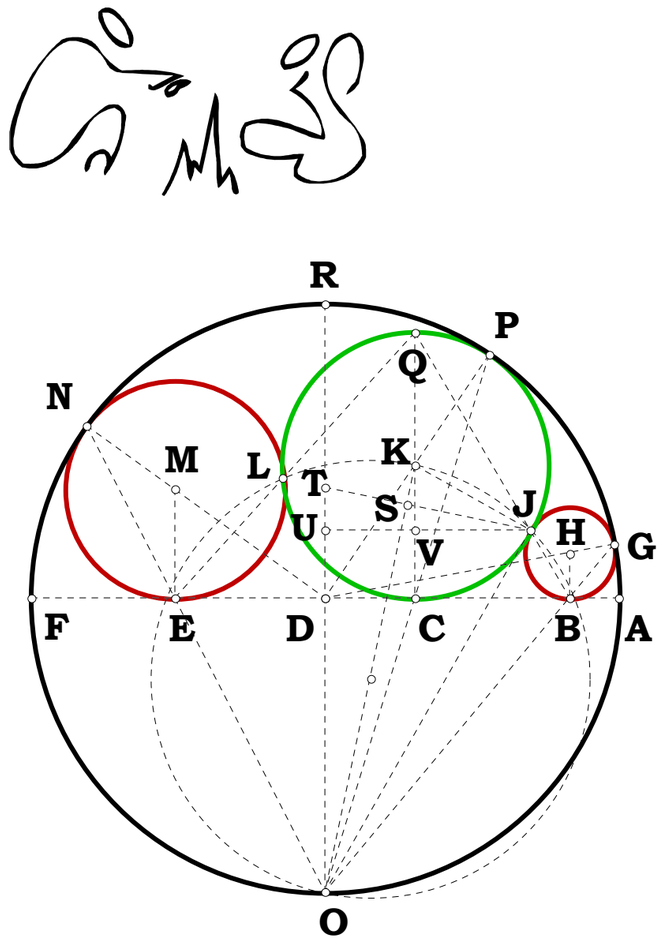
$JK := CK \quad KO := \sqrt{CD^2 + (DO + CK)^2} \quad JO := \sqrt{KO^2 - JK^2} \quad KS := \frac{JK^2}{KO} \quad SO := KO - KS$

$JS := \frac{JK \cdot SO}{JO} \quad ST := \frac{CD \cdot SO}{DO + CK} \quad JT := JS + ST \quad TO := \frac{KO \cdot ST}{CD} \quad TU := \frac{CD \cdot JT}{KO} \quad DU := TO - (DO + TU)$

$CV := DU \quad CQ := 2 \cdot CK \quad QV := CQ - CV \quad BH := \frac{CK \cdot CV}{QV} \quad BH = 0.812843$

**Some Algebraic Names, or Definitions.**

$$BH - N_1 \cdot \frac{D_1 \cdot (D_1 - 1) \cdot \left(2 \cdot D_1 - 2 \cdot D_1^2 - 2 \cdot \sqrt{2} \cdot D_1 + \sqrt{2} - 2\right)}{2 \cdot D_1 + 6 \cdot D_1^2 - 16 \cdot D_1^3 + 8 \cdot D_1^4 - 2 \cdot \sqrt{2} \cdot D_1 + \sqrt{2} + 2} = 0$$




$$AD - \frac{N_1}{2} = 0 \quad AC - N_1 \cdot D_1 = 0 \quad CD - \frac{N_1 \cdot (1 - 2 \cdot D_1)}{2} = 0 \quad CO - \frac{N_1 \cdot \sqrt{(2 \cdot D_1^{\,2} - 2 \cdot D_1 + 1)}}{\sqrt{2}} = 0$$

$$PO - \frac{N_1 \cdot \sqrt{2}}{2 \cdot \sqrt{2 \cdot D_1^{\,2} - 2 \cdot D_1 + 1}} = 0 \quad CP - N_1 \cdot \frac{\sqrt{2} \cdot D_1 \cdot (1 - D_1)}{\sqrt{2 \cdot D_1^{\,2} - 2 \cdot D_1 + 1}} = 0 \quad CK - N_1 \cdot (D_1 - D_1^{\,2}) = 0$$

$$KO - N_1 \cdot \frac{\sqrt{(2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1)}}{\sqrt{2}} = 0 \quad KS - N_1 \cdot \frac{\sqrt{2} \cdot D_1^{\,2} \cdot (D_1 - 1)^2}{\sqrt{2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1}} = 0$$

$$SO - N_1 \cdot \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1}} = 0 \quad JS - N_1 \cdot \frac{D_1 \cdot (1 - D_1)}{\sqrt{2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1}} = 0$$

$$ST - N_1 \cdot \frac{\sqrt{2} \cdot (1 - 2 \cdot D_1)}{2 \cdot (2 \cdot D_1 - 2 \cdot D_1^{\,2} + 1) \cdot \sqrt{2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1}} = 0 \quad JO - N_1 \cdot \frac{1}{\sqrt{2}} = 0$$

$$JT - N_1 \cdot \frac{D_1 + D_1^{\,2} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,4} - \sqrt{2} \cdot D_1 + \frac{\sqrt{2}}{2}}{(2 \cdot D_1 - 2 \cdot D_1^{\,2} + 1) \cdot \sqrt{2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1}} = 0 \quad TO - N_1 \cdot \frac{1}{2 \cdot D_1 - 2 \cdot D_1^{\,2} + 1} = 0$$

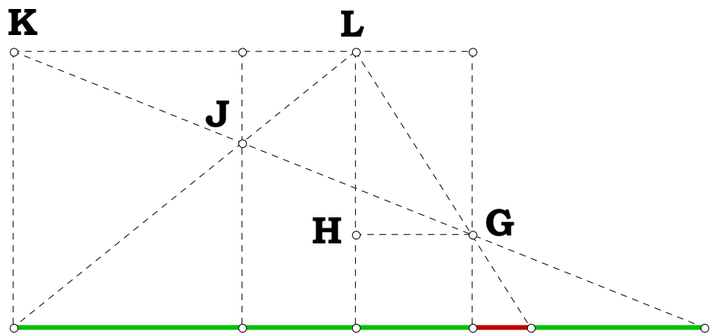
$$TU - N_1 \cdot \left[ \frac{1}{2 \cdot D_1 - 2 \cdot D_1^{\,2} + 1} - \frac{(3 \cdot \sqrt{2} - 2) \cdot D_1^{\,2} - 2 \cdot \sqrt{2} \cdot D_1^{\,3} + (2 - \sqrt{2}) \cdot D_1 + 1}{2 \cdot (2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1)} \right] = 0 \quad DU - N_1 \cdot \left[ \frac{(3 \cdot \sqrt{2} - 2) \cdot D_1^{\,2} - 2 \cdot \sqrt{2} \cdot D_1^{\,3} + (2 - \sqrt{2}) \cdot D_1 + 1}{2 \cdot (2 \cdot D_1^{\,4} - 4 \cdot D_1^{\,3} + 2 \cdot D_1^{\,2} + 1)} - \frac{1}{2} \right] = 0$$

$$CQ - N_1 \cdot 2 \cdot (D_1 - D_1^{\,2}) = 0 \quad QV - N_1 \cdot \frac{D_1 \cdot (D_1 - 1) \cdot (2 \cdot D_1 + 6 \cdot D_1^{\,2} - 16 \cdot D_1^{\,3} + 8 \cdot D_1^{\,4} - 2 \cdot \sqrt{2} \cdot D_1 + \sqrt{2} + 2)}{2 \cdot (4 \cdot D_1^{\,3} - 2 \cdot D_1^{\,4} - 2 \cdot D_1^{\,2} - 1)} = 0$$

**One Over N + One 042596**

**Construct 1/(N+1)**

**In this construction, $N_2$ has to be something, but it can be anything and will not change the equation.**




$$N_1 := 2.817 \quad N_2 := 3 \quad AC := 1$$

$$AF := AC \cdot N_1 \quad CF := AF - AC \quad CE := \frac{CF}{2} \quad AE := AC + CE \quad FK := N_2 \quad EJ := \frac{FK \cdot AE}{AF}$$

$$DL := FEF := CE \quad DF := \frac{EF \cdot DL}{EJ} \quad CG := \frac{FK \cdot AC}{AF} \quad CD := CF - DF \quad DH := CG$$

$$HL := DL - DH \quad BC := \frac{CD \cdot CG}{HL} \quad CF := N_1 - 1 \quad CE := \frac{1}{2} \cdot (N_1 - 1) \quad AE := \frac{1}{2} \cdot (1 + N_1)$$

$$FK := N_2 \quad EJ := \frac{1}{2} \cdot N_2 \cdot \frac{(1 + N_1)}{N_1} \quad DF := \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 \quad CG := \frac{N_2}{N_1} \quad CD := \frac{(N_1 - 1)}{(1 + N_1)}$$

$$HL := N_2 \cdot \frac{(N_1 - 1)}{N_1} \quad \frac{1}{N_1 + 1} - BC = 0$$

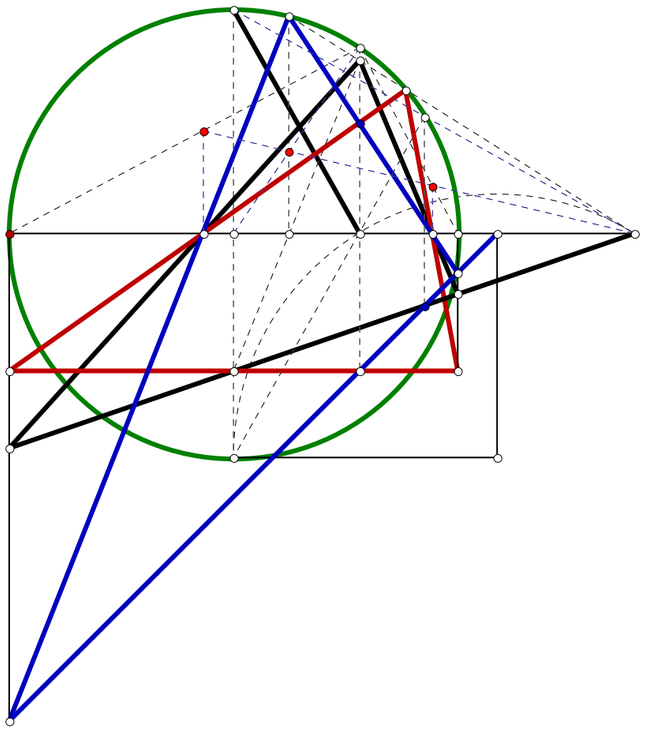
04_26_96.MCD

# Three Base Theorem.

$N_1 := .64966$ $N_2 := 7.51417$ $N_3 := 1.92131$

$BC := N_1$ $CI := N_2$ $CG := \frac{CI}{2}$ $BI := BC + CI$

**Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.**



$BE := \sqrt{BC \cdot BI}$ $BM := N_3$

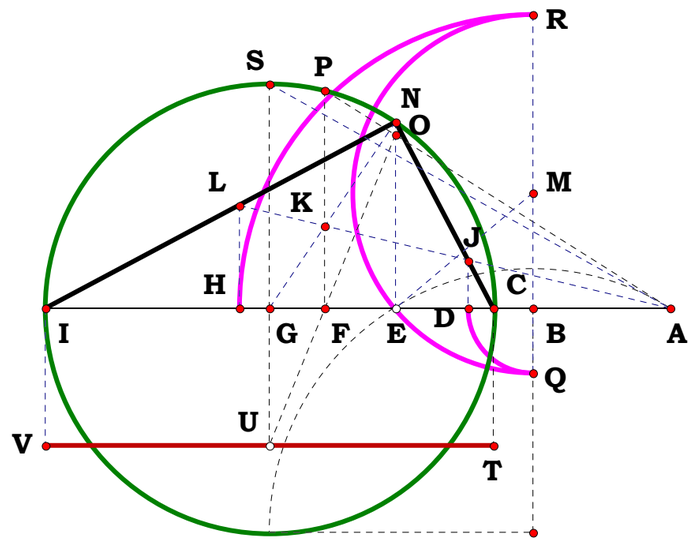
$EM := \sqrt{BM^2 + BE^2}$ $BD := EM - BM$

$BH := BM + EM$ $GN := CG$ $CE := BE - BC$

$EI := CI - CE$ $EN := \sqrt{CE \cdot EI}$ $EH := BH - BE$

$EG := EI - CG$ $AE := \frac{EN^2}{EG}$ $HI := EI - EH$

$HL := \frac{EN \cdot HI}{EI}$ $AG := AE + EG$



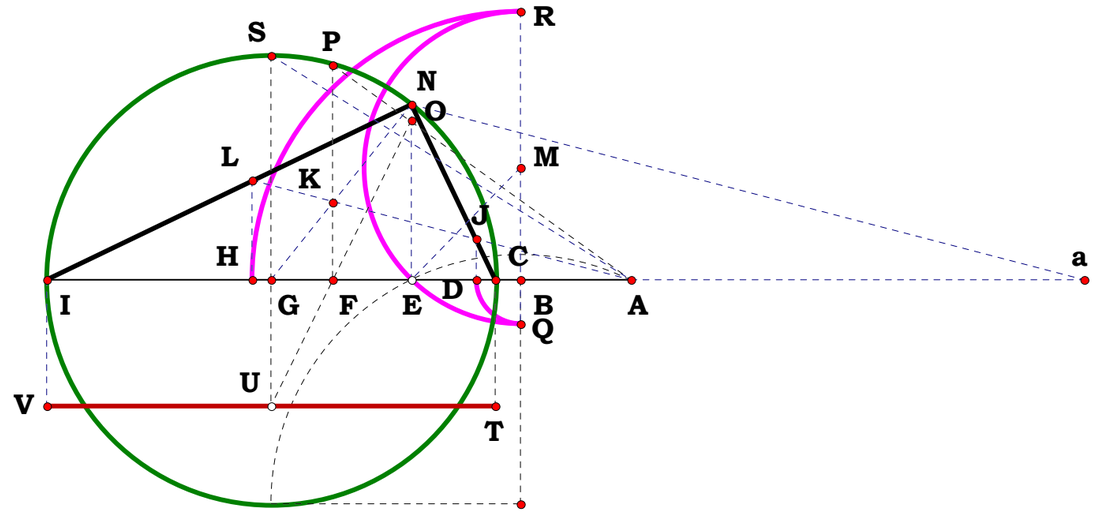
$$AH := AE + EH \qquad Ea := \frac{AH \cdot EN}{HL}$$

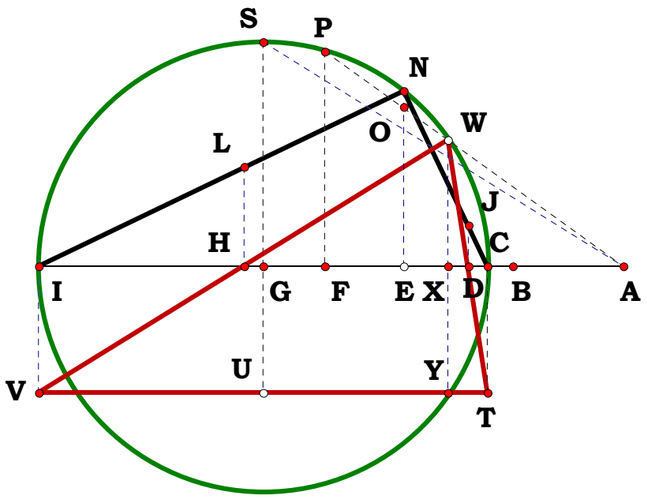
$$FG := \frac{EG \cdot AG}{(Ea + EG)} \qquad CF := CG - FG$$

$$FI := CG + FG \qquad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \qquad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \qquad GU := \frac{EO \cdot FG}{EF}$$




$$AC := AE - CE \qquad AI := AC + CI \qquad AP := \sqrt{AF^2 + FP^2}$$

$$AW := \frac{AC \cdot AI}{AP} \qquad AX := \frac{AF \cdot AW}{AP} \qquad CX := AX - AC \qquad XI := CI - CX$$

$$WX := \sqrt{CX \cdot XI} \qquad XG := CG - CX \qquad YU := XG \qquad UV := CG$$

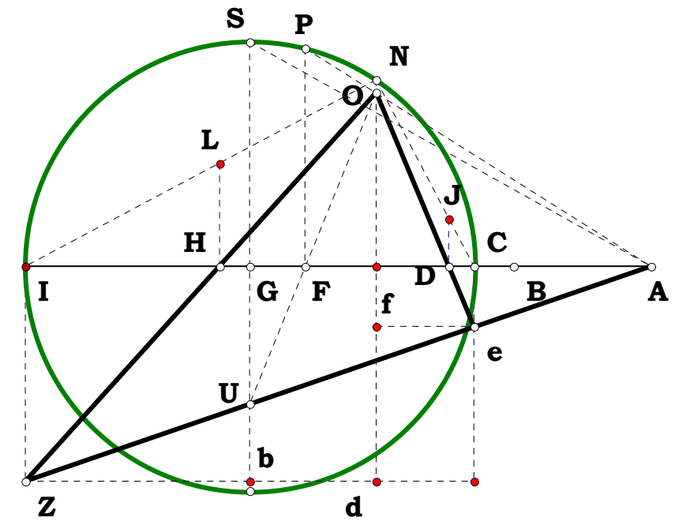
$$YV := YU + UV \qquad XH := \frac{YV \cdot WX}{WX + GU} \qquad CH := AH - AC \qquad \frac{CH}{XH + CX} = 1$$

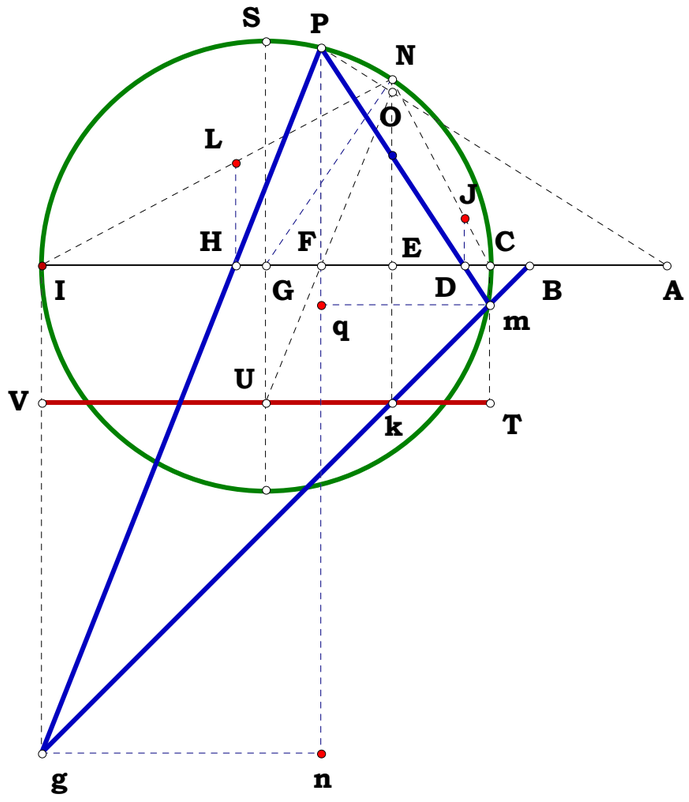
$$CD := BD - BC \qquad DX := \frac{CX \cdot WX}{WX + GU}$$

$$\frac{CD}{CX - DX} = 1$$

$$IZ := \frac{GU \cdot AI}{AG} \qquad Ed := IZ \qquad \frac{\frac{EI \cdot EO}{EO+Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \qquad Ef := Ce \qquad \frac{CD}{\frac{CE \cdot Ce}{EO+Ef}} = 1$$




$$Ek := GU \qquad Ig := \frac{Ek \cdot BI}{BE} \qquad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := I_{|}gn := FI \qquad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \qquad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$

## A Root Figure 042796

**CD + BC is the square root of $\sqrt{BC \cdot BF}$. What is BC?**

$N_1 := 7.72583 \quad N_2 := 1.65429 \quad N_3 := 3.88233$

$CF := N_1 \quad CE := \frac{CF}{2} \quad CD := N_2 \quad FK := N_3$

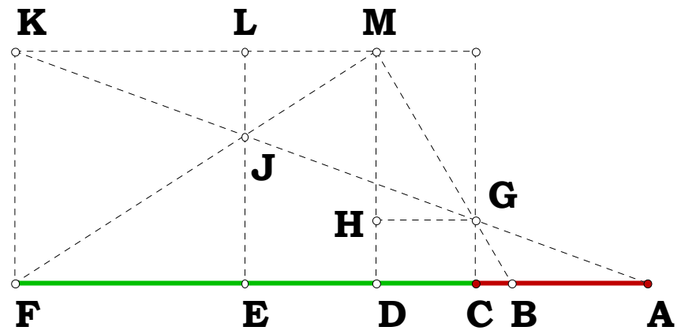
$DM := FK \quad EL := FK \quad DF := CF - CD$

$EF := \frac{CF}{2} \quad EJ := \frac{DM \cdot EF}{DF} \quad JL := EL - EJ$

$KL := EF \quad AF := \frac{KL \cdot FK}{JL} \quad AC := AF - CF \quad CG := \frac{FK \cdot AC}{AF}$

$DH := CG \quad HM := DM - DH \quad BC := \frac{CD \cdot DH}{HM}$

$BF := BC + CF \quad BD := BC + CD \quad \sqrt{BC \cdot BF} - BD = 0$




**Once again, FK makes the equation possible, but disappears in the result.**

$$\left[\frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2}\right]^2 - \frac{N_2^2}{N_1 - 2 \cdot N_2} \cdot \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0$$

**Definitions.**

$$CF - N_1 = 0 \quad DF - (N_1 - N_2) = 0 \quad EF - \frac{N_1}{2} = 0 \quad EJ - \frac{N_1 \cdot N_3}{2 \cdot (N_1 - N_2)} = 0 \quad JL - \frac{N_3 \cdot (N_1 - 2 \cdot N_2)}{2 \cdot (N_1 - N_2)} = 0 \quad AF - \frac{N_1 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0 \quad AC - \frac{N_1 \cdot N_2}{N_1 - 2 \cdot N_2} = 0$$

$$CG - \frac{N_2 \cdot N_3}{N_1 - N_2} = 0 \quad HM - \frac{N_3 \cdot (N_1 - 2 \cdot N_2)}{N_1 - N_2} = 0 \quad BC - \frac{N_2^2}{N_1 - 2 \cdot N_2} = 0 \quad BF - \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0 \quad BD - \frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0$$

042896.MCD

**I had finally decided to write this up in Oct. of 94, and being punctual, here it is.**

**Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.**

**Process Summary will use a $5^{th}$ root series for an example.**

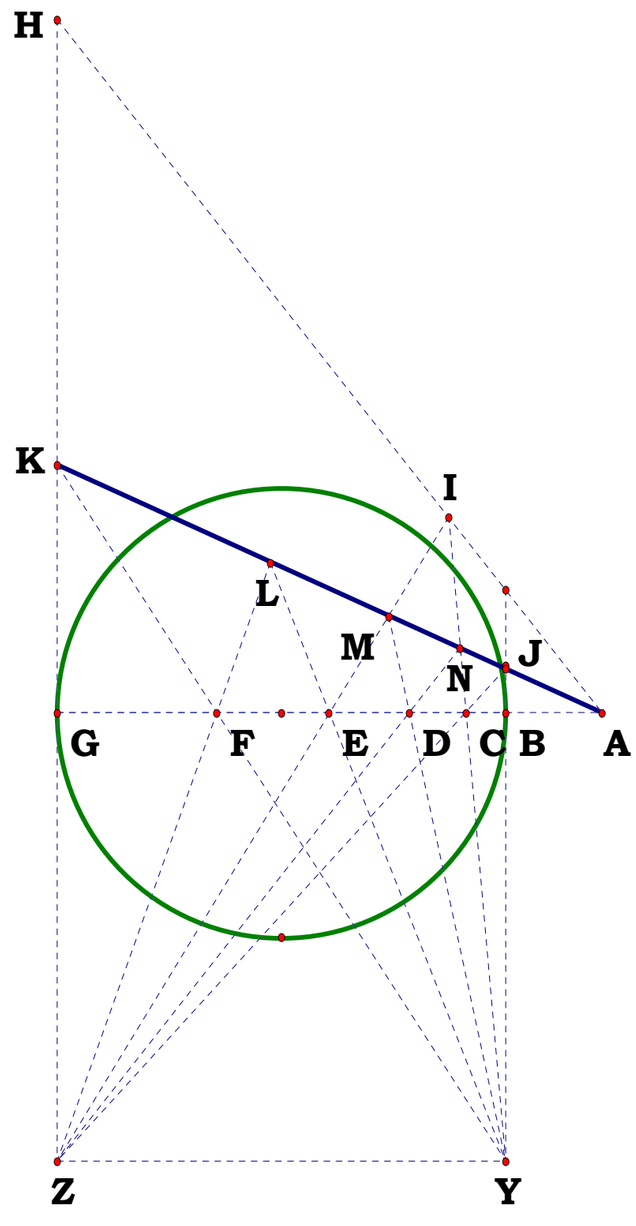
$N_1 := 2$ $N_2 := 6$

$AB := N_1$ $AG := N_2$ $AE := \left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}$

$BG := AG - AB$ $GZ := BG$ $YZ := BG$

$BY := BG$ $BE := AE - AB$ $EG := BG - BE$

$GH := \frac{BY \cdot EG}{BE}$



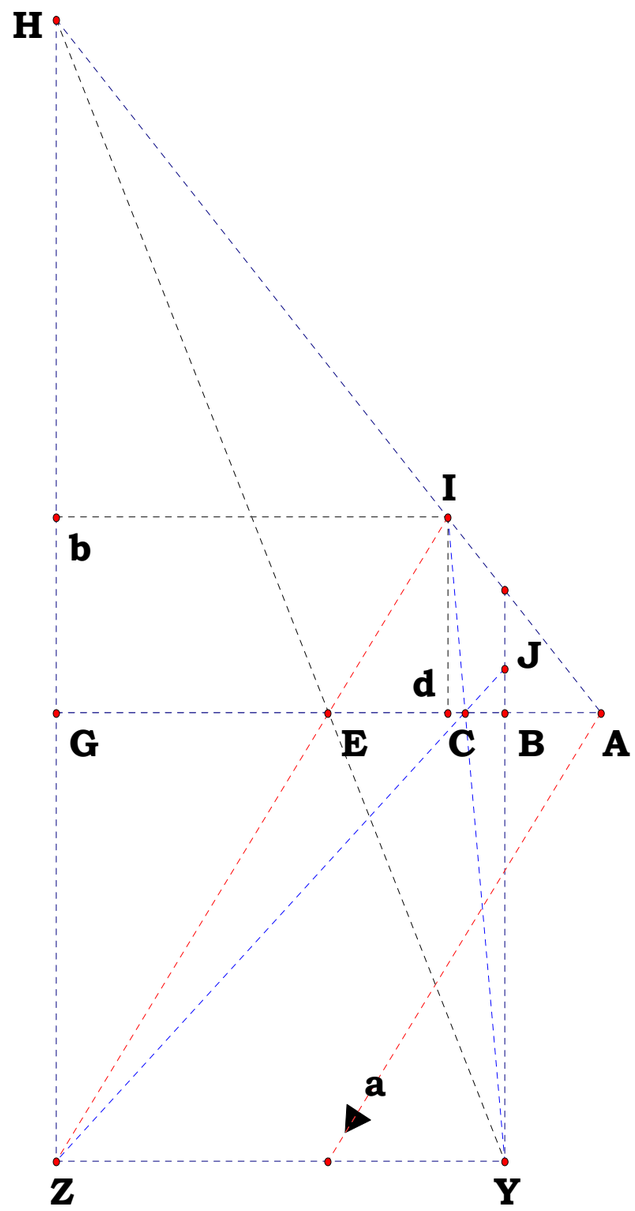
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

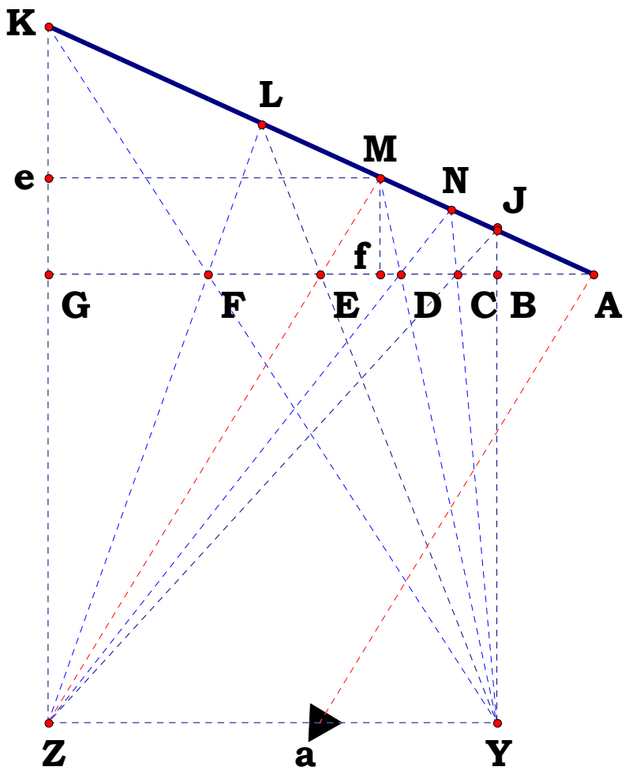
$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$






$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

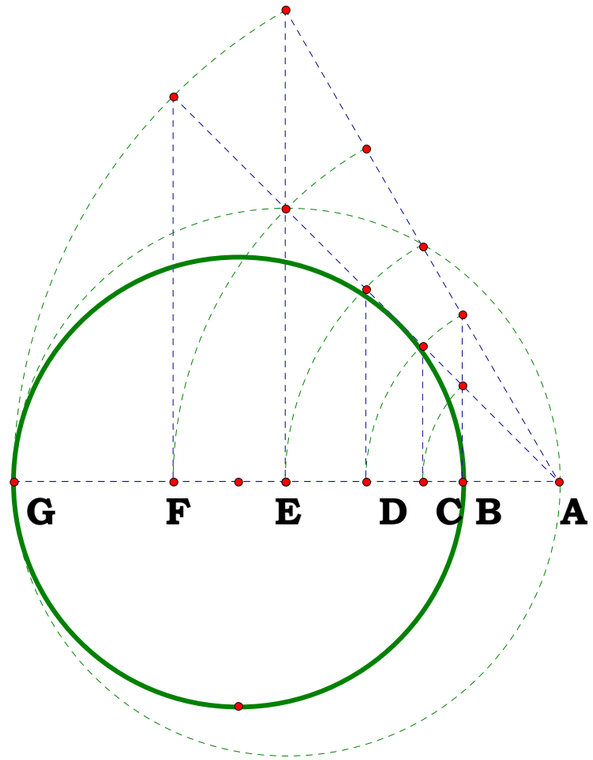
$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

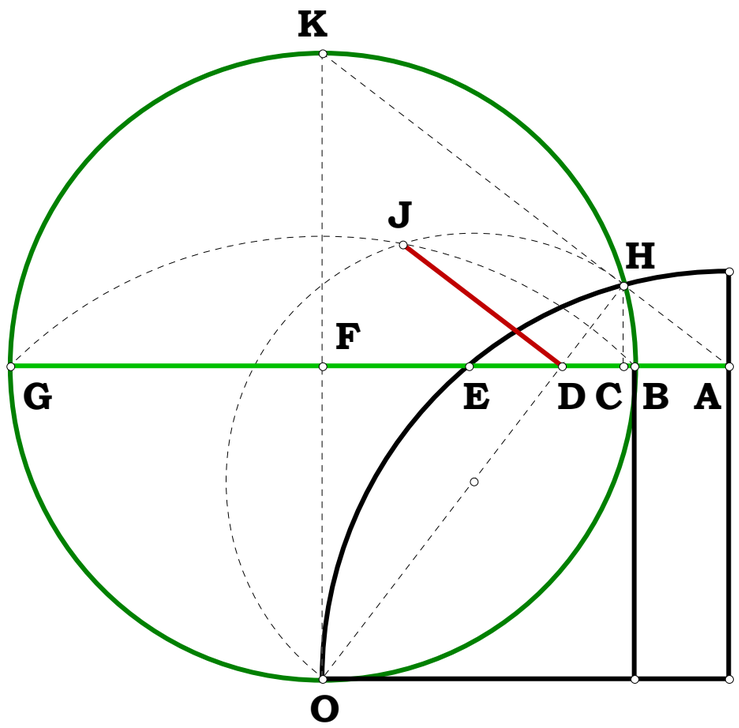
$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$




**If any of a prime root series can be given exactly, every root of the series can be determined exactly.**

**What is DJ? 042996**

**DJ is the Geometric name, what is its Algebraic name?**




$$N := 5 \quad AB := 1 \quad AG := AB \cdot N \quad BG := AG - AB$$

$$BF := \frac{BG}{2} \quad FK := FO := BF \quad AF := BF + AB$$

$$DF := \frac{FK \cdot FO}{AF} \quad AK := \sqrt{AF^2 + FK^2} \quad KO := BG$$

$$HO := \frac{AF \cdot KO}{AK} \quad DO := \frac{AK \cdot FO}{AF} \quad DH := HO - DO$$

$$DJ := \sqrt{DH \cdot DO}$$

$$AG := N \quad BG := N - 1 \quad BF := \frac{N - 1}{2} \quad AF := \frac{1}{2} \cdot N + \frac{1}{2}$$

$$DF := \frac{1}{2} \cdot \frac{(N - 1)^2}{(N + 1)} \quad AK := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2}$$

$$HO := (N + 1) \cdot \frac{(N - 1)}{\sqrt{2 \cdot N^2 + 2}} \quad DO := \frac{1}{2} \cdot \sqrt{2 \cdot N^2 + 2} \cdot \frac{(N - 1)}{(N + 1)}$$

$$DH := 2 \cdot N \cdot \frac{(N - 1)}{\left[\sqrt{2 \cdot N^2 + 2} \cdot (N + 1)\right]}$$

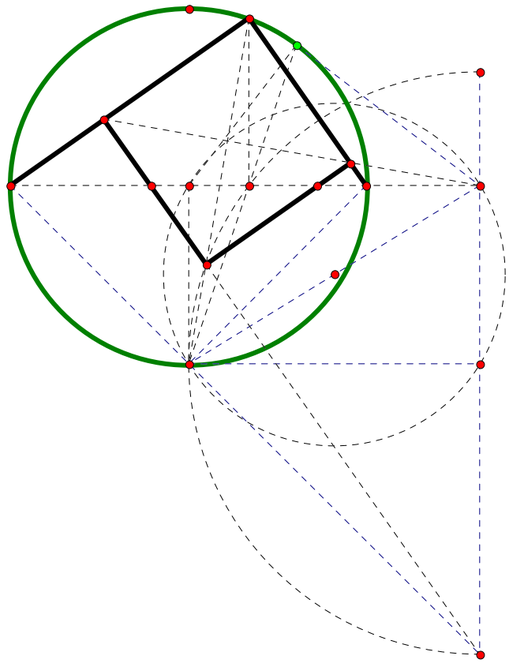
$$DJ := \sqrt{N} \cdot \frac{(N - 1)}{(N + 1)}$$

043096.MCD

**Geometric Exponential Series of the form**

$$\frac{\sum_{\delta} N^{\frac{\text{Root}-\delta}{\text{Root}}}}{N^{\frac{\text{Root}-1}{\text{Root}}}}, \sum_{\delta} N^{\frac{\text{Root}-\delta}{\text{Root}}} \quad \text{and} \quad \frac{N^{\frac{\delta+2}{\text{Root}}} + N^{\frac{\delta}{\text{Root}}}}{N^{\frac{1}{\text{Root}}} - N^{\frac{0}{\text{Root}}}}$$

**Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.**



$N := 4$ $\text{Root} := 4$ $M := 1$ $BG := M$ $AB := M$

$AG := AB + BG$ $BO := \frac{BG}{2}$

$AC := \left(AB^{\text{Root}-1} \cdot AG\right)^{\frac{1}{\text{Root}}}$ $AF := \left(AB \cdot AG^{\text{Root}-1}\right)^{\frac{1}{\text{Root}}}$

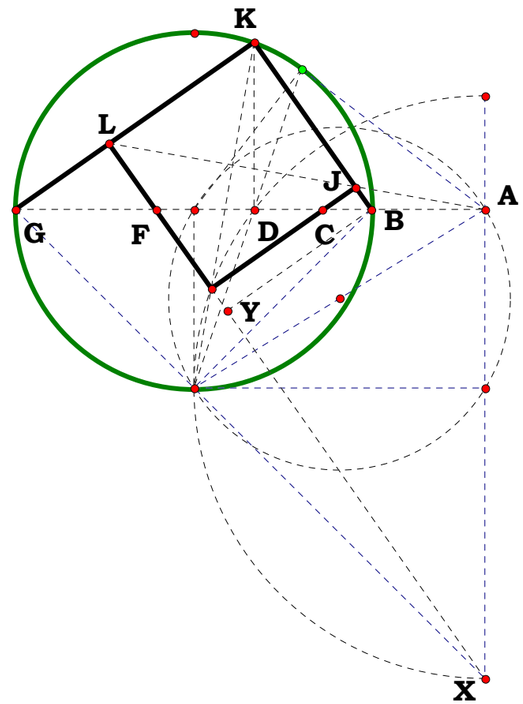
$BC := AC - AB$ $FG := AG - AF$ $FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX}$ $BD := \frac{FY \cdot BG}{FX}$ $AD := BD + AB$

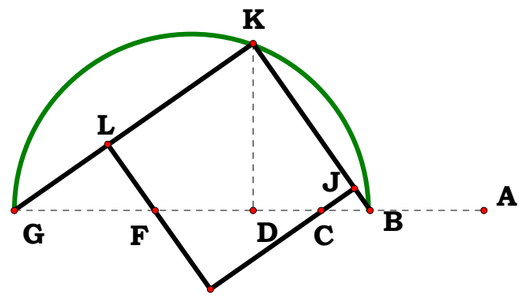
$DG := AG - AD$ $DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2}$ $GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG}$ $GL := \frac{GK \cdot FG}{BG}$



**Plug in BG here as N. AB as M. Plug in root series also.**




$N \equiv 4 \quad Root \equiv 4 \quad \delta := 1 .. Root \quad M \equiv 1$

$GL = 1.376805 \quad BJ = 0.275361 \quad \frac{GL}{BJ} = 5 \quad \frac{AG}{AB} = 5$

$$\frac{\sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}}}{Root-1} = 2.415024 \quad \frac{GK}{GL} = 2.415024 \quad \sum_{\delta} \left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}} = 8.075118 \quad \frac{BK}{BJ} = 8.075118$$

$$\left(\frac{AG}{AB}\right)^{Root}$$

$$BM := \frac{BD \cdot BC}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$$

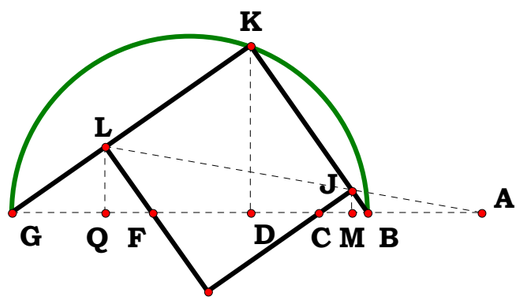
$\frac{AG}{FQ} = 9.768976$

$\frac{AG}{BM} = 32.66454$

**On the left is the first and last of the series, on the right is the entire series.**

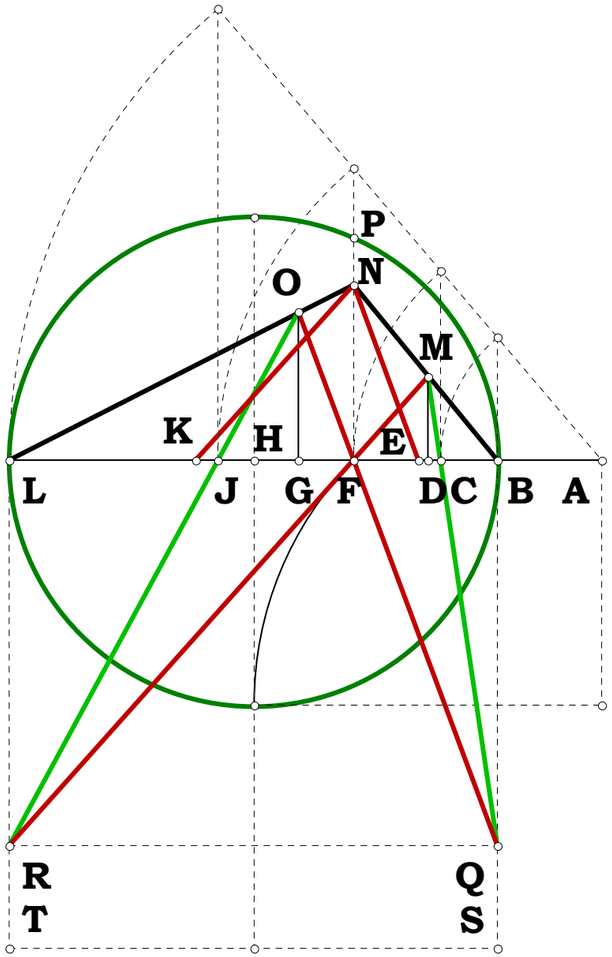
$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}} =$$

| |
|---|
| 9.768976 |
| 14.608026 |
| 21.844094 |
| 32.66454 |



## 122096 Alternate Method Quad Roots

**If FN:FP as BQ:BS then quad roots series can be divided off in the figure.**




$$N_1 := 2 \quad N_2 := .2$$

$$AB := 1 \qquad AL := AB \cdot N_1$$

$$BL := AL - AB \quad BS := BL \quad LT := BL$$

$$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$$

$$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$$

$$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$$

$$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$$

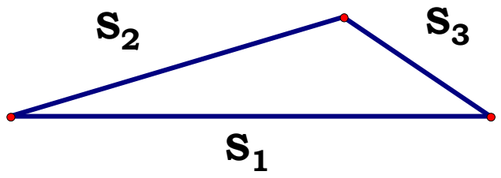
$$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$$

$$AJ := AL - JL$$

$$\left(AB \cdot AL^3\right)^{\frac{1}{4}} - AJ = 0$$

**04_03_97.MCD**

**Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?**

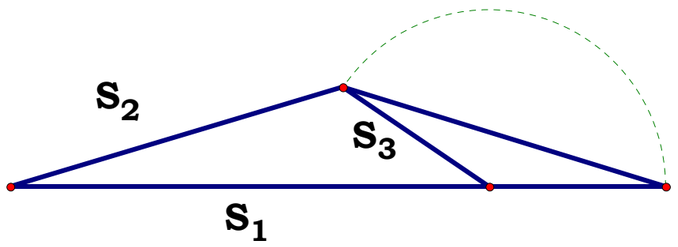



$n := 1 .. 3$

$S_1 := \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $S_2 := \begin{pmatrix} b \\ c \\ a \end{pmatrix}$ $S_3 := \begin{pmatrix} c \\ a \\ b \end{pmatrix}$

$Is\_This\_a\_Triangle := \left(S_{1_1} + S_{2_1} > S_{3_1}\right) \cdot \left(S_{1_1} + S_{3_1} > S_{2_1}\right) \cdot \left(S_{2_1} + S_{3_1} > S_{1_1}\right)$

**As was learned in school, the area of a triagle is given by** $\frac{1}{2} \cdot B \cdot H$.




**From 04_02_97.MCD I show that, for a given side, the height is given by;**

$$H_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

**And since** $B := S_1$

**Area is defined as**

$$A_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{4}$$

$\frac{1 \cdot B_n \cdot H_n}{2} - A_n =$

| |
|---|
| 0 |
| 0 |
| 0 |

$A_n =$

| |
|---|
| 2.904738 |
| 2.904738 |
| 2.904738 |

$H_n =$

| |
|---|
| 2.904738 |
| 1.936492 |
| 1.452369 |

**What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.**

$$\mathbf{Acute}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S2}_n\right)^2} > \mathbf{S3}_n\ ,\ 1\ ,\ 0\right] \qquad \mathbf{Acute2}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S3}_n\right)^2} > \mathbf{S2}_n\ ,\ 1\ ,\ 0\right]$$

$\mathbf{Acute}_n =$

| |
|---|
| 0 |
| 1 |
| 1 |

$\mathbf{Acute2}_n =$

| |
|---|
| 1 |
| 0 |
| 1 |

$\mathbf{H}_n =$

| |
|---|
| 2.904738 |
| 1.936492 |
| 1.452369 |

$\mathbf{S1}_n =$

| |
|---|
| 2 |
| 3 |
| 4 |

$\mathbf{S2}_n =$

| |
|---|
| 3 |
| 4 |
| 2 |

$\mathbf{S3}_n =$

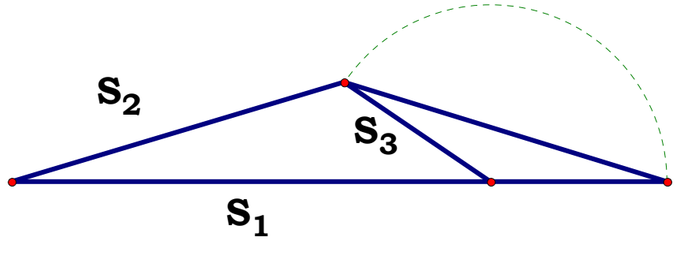
| |
|---|
| 4 |
| 2 |
| 3 |

**if we halve** $\angle\,\mathbf{S1S2}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

| |
|---|
| 4.357106 |
| 3.872983 |
| 1.452369 |

**if we halve** $\angle\mathbf{S1S3}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

| |
|---|
| 5.809475 |
| 1.936492 |
| 2.178553 |




**Is_This_a_Triangle = 1**

**Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.**

$$\left(\mathbf{S2}_n > \mathbf{S3}_n\right) - \left[\frac{\left(\mathbf{B}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n > \frac{\left(\mathbf{B}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n\right] =$$

| |
|---|
| 0 |
| 0 |
| 0 |

**Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.**

$$H_n = \frac{\sqrt{S1_n + S2_n + S3_n} \cdot \sqrt{-S1_n + S2_n + S3_n} \cdot \sqrt{S1_n - S2_n + S3_n} \cdot \sqrt{S1_n + S2_n - S3_n}}{2 \cdot S1_n}$$

**Given $S_1$, $S_2$ and $\sqrt{S_1^2 + S_2^2} > S_3$, find $S_3$.**

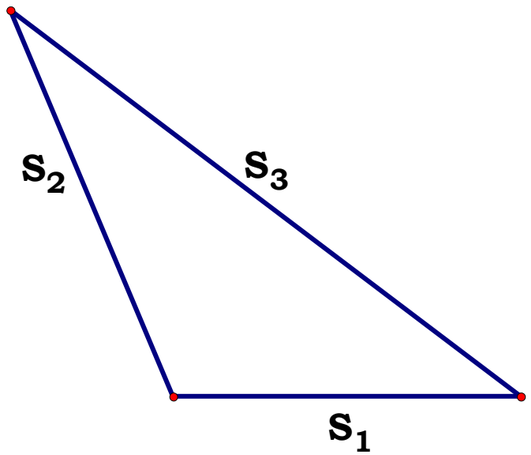
$Acute_n =$

| |
|---|
| 0 |
| 1 |
| 1 |

$Is\_This\_a\_Triangle = 1$

$$S4_n := \sqrt{(S2_n)^2 - (H_n)^2}$$

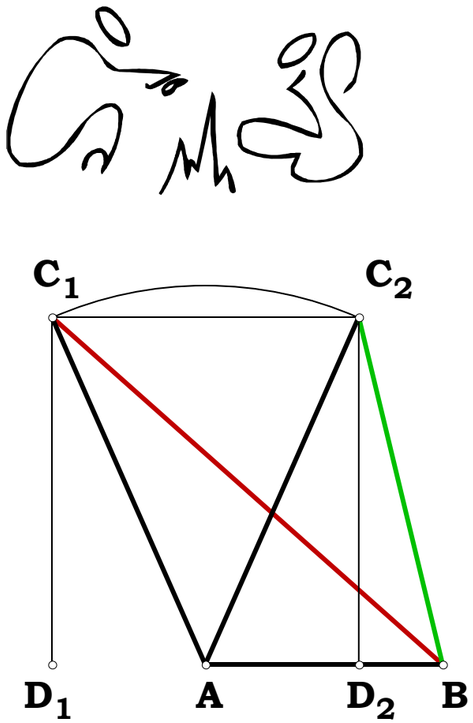
$$Sx_n := if(Acute_n, S1_n - S4_n, S1_n + S4_n)$$




$a \equiv 2$ $b \equiv 3$ $c \equiv 4$ ← Plug your values in here.

$$S3_n := \sqrt{(H_n)^2 + (Sx_n)^2}$$

| $S1_n =$ | $S2_n =$ | $S3_n =$ |
|---|---|---|
| 2 | 3 | 4 |
| 3 | 4 | 2 |
| 4 | 2 | 3 |




## 040497 Triangles

**Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.**

$$AB := 5 \qquad AC := 4 \qquad CD := 3$$

$$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$$

$$BC_1 := \sqrt{CD^2 + BD_1{}^2} \qquad BC_2 := \sqrt{CD^2 + BD_2{}^2}$$

$$BC_1 = 8.213252 \qquad BC_2 = 3.813461$$

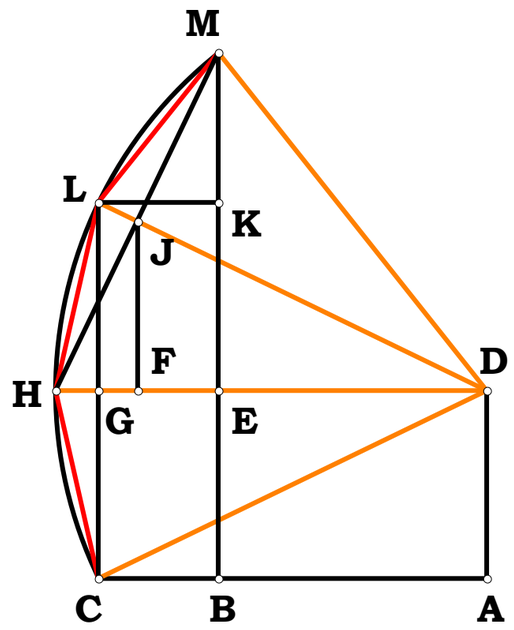
$$S_1 := AB \quad S_2 := AC \quad S_3 := BC_1$$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

$$S_1 := AB \quad S_2 := AC \quad S_3 := BC_2$$

$$\frac{\sqrt{S_1 + S_2 + S_3}\cdot\sqrt{-S_1 + S_2 + S_3}\cdot\sqrt{S_1 - S_2 + S_3}\cdot\sqrt{S_1 + S_2 - S_3}}{2\cdot S_1} - CD = 0$$

042897




$N_1 := 3.14854$ $N_2 := 6.50875$ $AD := N_1$ $AC := N_2$

$CD := \sqrt{AD^2 + AC^2}$ $DH := CD$

$CG := AD$ $DG := AC$ $GH := DH - DG$ $CH := \sqrt{GH^2 + CG^2}$

$HJ := CG$ $DJ := DG$

$FH := \dfrac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH}$ $EF := FH$ $DE := DH - (EF + FH)$

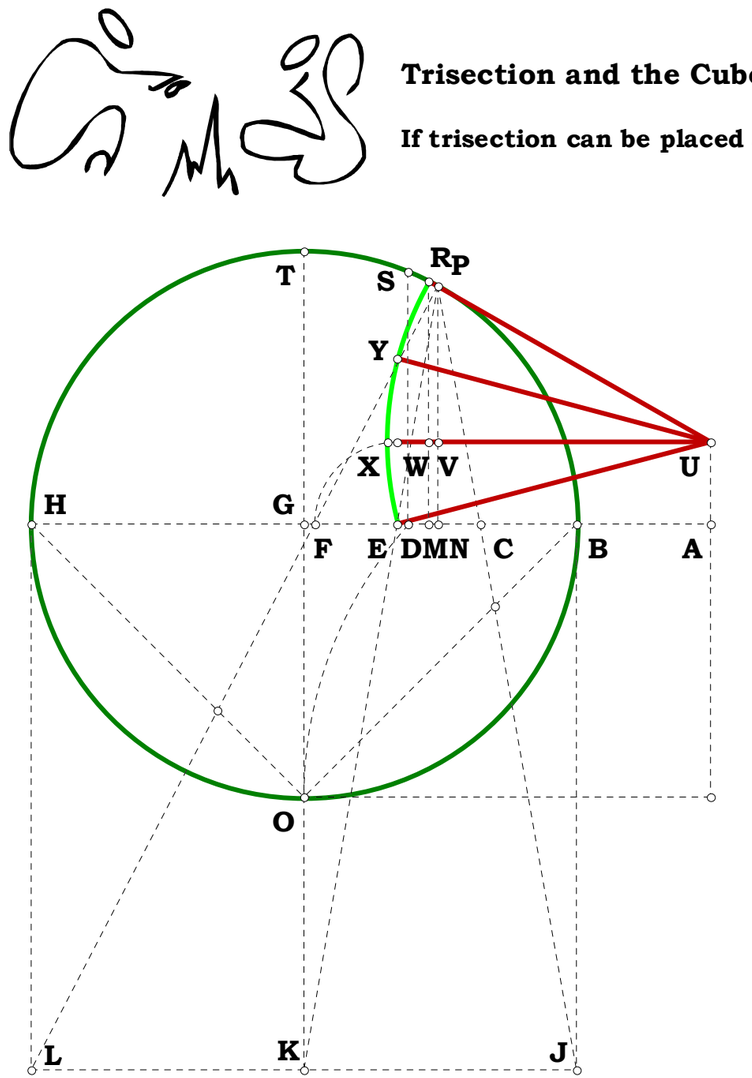
$AB := DE$ $EG := DG - DE$ $LM := CH$ $LK := EG$

$KM := \sqrt{LM^2 - LK^2}$ $BE := AD$ $BK := 2 \cdot BE$ $BM := BK + KM$

**Some definitions:**

$$\sqrt{N_1^2 + N_2^2} - CD = 0 \qquad \sqrt{N_1^2 + N_2^2} - N_2 - GH = 0 \qquad \sqrt{\left[2 \cdot \left(N_1^2 + N_2^2 - N_2 \cdot \sqrt{N_1^2 + N_2^2}\right)\right]} - CH = 0 \qquad \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} - FH = 0 \qquad \frac{\left(N_2^2 - N_1^2\right)}{\sqrt{N_1^2 + N_2^2}} - DE = 0$$

$$2 \cdot N_1 + \frac{\sqrt{2 \cdot N_2^3 \cdot \sqrt{N_1^2 + N_2^2} + N_1^4 + 5 \cdot N_1^2 \cdot N_2^2 - 2 \cdot N_2 \cdot \left(N_1^2 + N_2^2\right)^{\frac{3}{2}} - 2 \cdot N_1^2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}}}{\sqrt{N_1^2 + N_2^2}} - BM = 0$$




**Trisection and the Cube Roots 042997**

**If trisection can be placed at RUE, then PV is proportional to RW.**

$$N := 5 \quad AB := 1 \quad AH := AB \cdot N$$

$$BH := AH - AB \quad BJ := BH$$

$$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \quad AF := \left(AB \cdot AC^2\right)^{\frac{1}{3}} \quad CF := AF - AC \quad CE := \frac{CF}{2}$$

$$AE := AC + CE$$

$$AU := CE \quad NV := AU \quad MW := AU$$

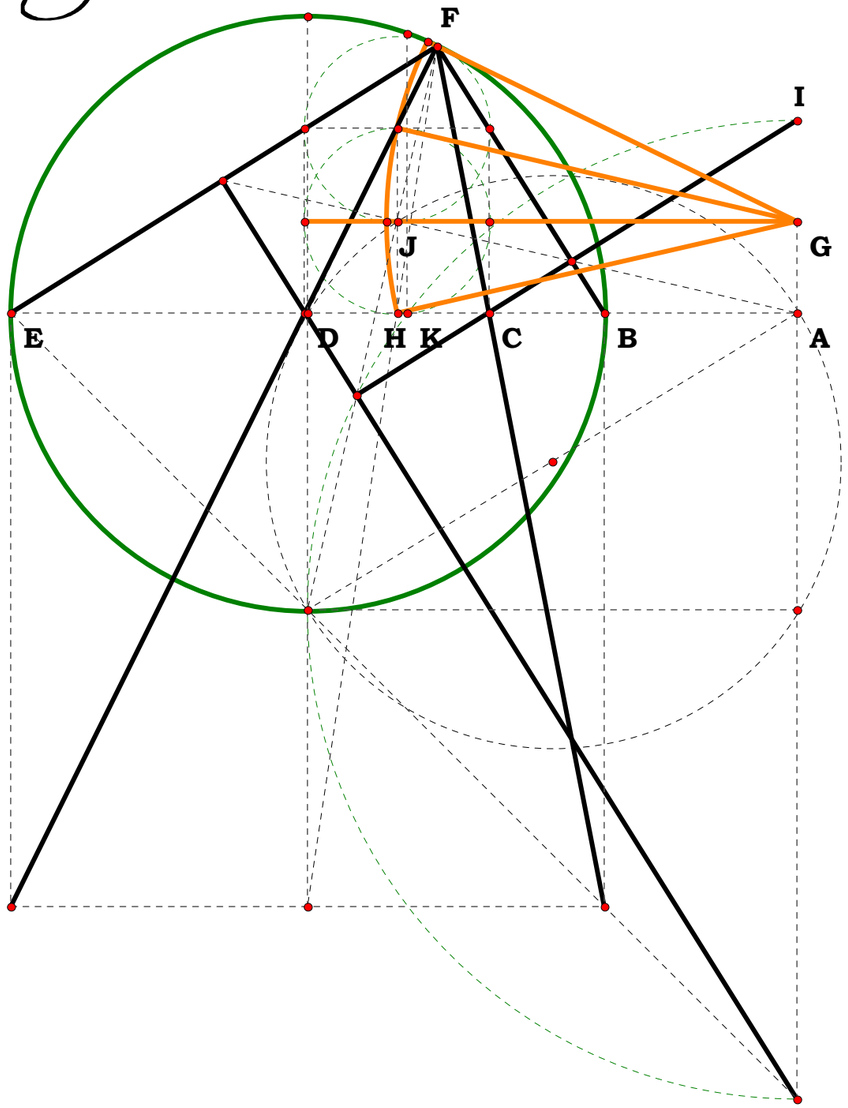
**(For the next two equations see 042897.)**

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} \qquad MR := 2 \cdot AU + AU \cdot \sqrt{\frac{5 \cdot \left(\frac{AE}{AU}\right)^2}{\left(\frac{AE}{AU}\right)^2 + 1} - \frac{4 \cdot \frac{AE}{AU}}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}} + \frac{1}{\left(\frac{AE}{AU}\right)^2 + 1}}$$

$$RW := MR - MW \quad BC := AC - AB \quad FH := AH - AF$$

$$CN := \frac{BC \cdot CF}{BC + FH} \quad NP := \frac{BJ \cdot CN}{BC} \quad PV := NP - NV \quad AN := AC + CN \quad UV := AN \quad UW := AM \quad \frac{RW \cdot UV}{UW} - PV = 0$$

**From a Single Point.**




**AB = 3.24039 cm**
**AC = 5.17367 cm**
**AD = 8.26039 cm**
**AE = 13.18872 cm**
**AK = 6.53732 cm**

$\frac{AK}{AB} = 2.01745$

$\frac{AE}{AB} = 4.07011$

$\frac{AC}{AB} = 1.59662$

$\frac{AD}{AB} = 2.54920$

$\frac{AE}{AB}\frac{1}{3} - \frac{AC}{AB} = 0.00000$

$\frac{AE}{AB}\frac{2}{3} - \frac{AD}{AB} = 0.00000$

$\frac{AE}{AB}\frac{1}{2} - \frac{AK}{AB} = 0.00000$

**m∠FGH = 38.82047°**
**m∠JGH = 12.94016°**

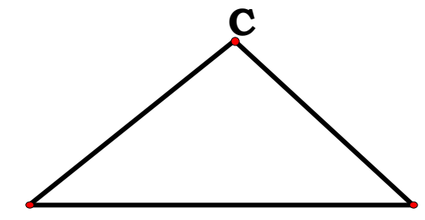
$\frac{m\angle FGH}{m\angle JGH} = 3.00000$

Start with H. Construct the two tangent circles. With those project to G, from the tangent at F and the enter line of the first circle for the tangent string; which will also construct the unit line.
Project from I perpendicular to BF to etc. etc.

## An Indeterminate Problem Reduced To An Equation 072903

**Page 5 of *A Treatise on Algebraic Geometry* by Rev. Dionysius Lardner, 1831**

**Given the base AB, and the sum of the sides (AC and BC) of a triangle, to find the vertex (C).**




**Let AB = *a*, AC = *y*, and CB = *x*, and the the excess of the sum of the sides above the base be *d*.**

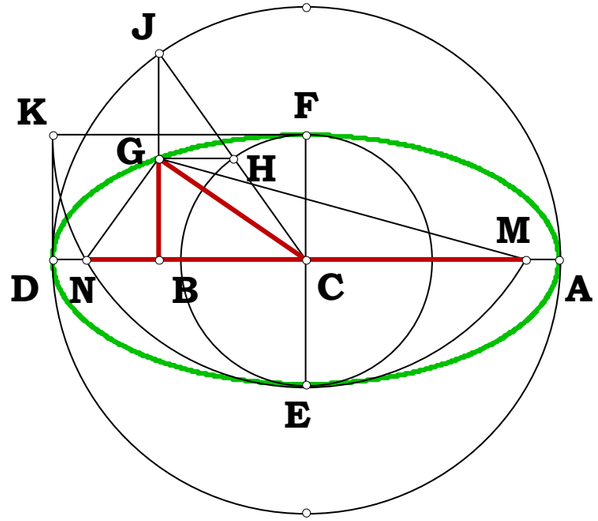
$\therefore\ y + x = a + b.$

**Any values of *y* and *x*, which fulfill the conditions of this equation, represent the sides of the triangle, whose vertex solves the problem.**

**The ability to render a solution to a problem is first tendered by determining if the problem is indeed a problem first in grammar. The problem as stated makes an elementary grammar error, that of that of number itself. The sum of two sides yields a plurality, while the "the vertex" is decidely singular. Even so, anyone with any wit at all does not tender an answer to a problem by giving synonyms. y and x are synonyms for a and b. Any triangle can be seen as three potential ellipses, one for each side, while the remaining two as the sum of foci. Therefor, there was no indeterminate problem save 3, two grammatical, and the third a pretense of an answer. A vertex is not a difference, therefore it cannot possibly solve any problem.**

### 09/11/97 The Ellipse

**Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.**

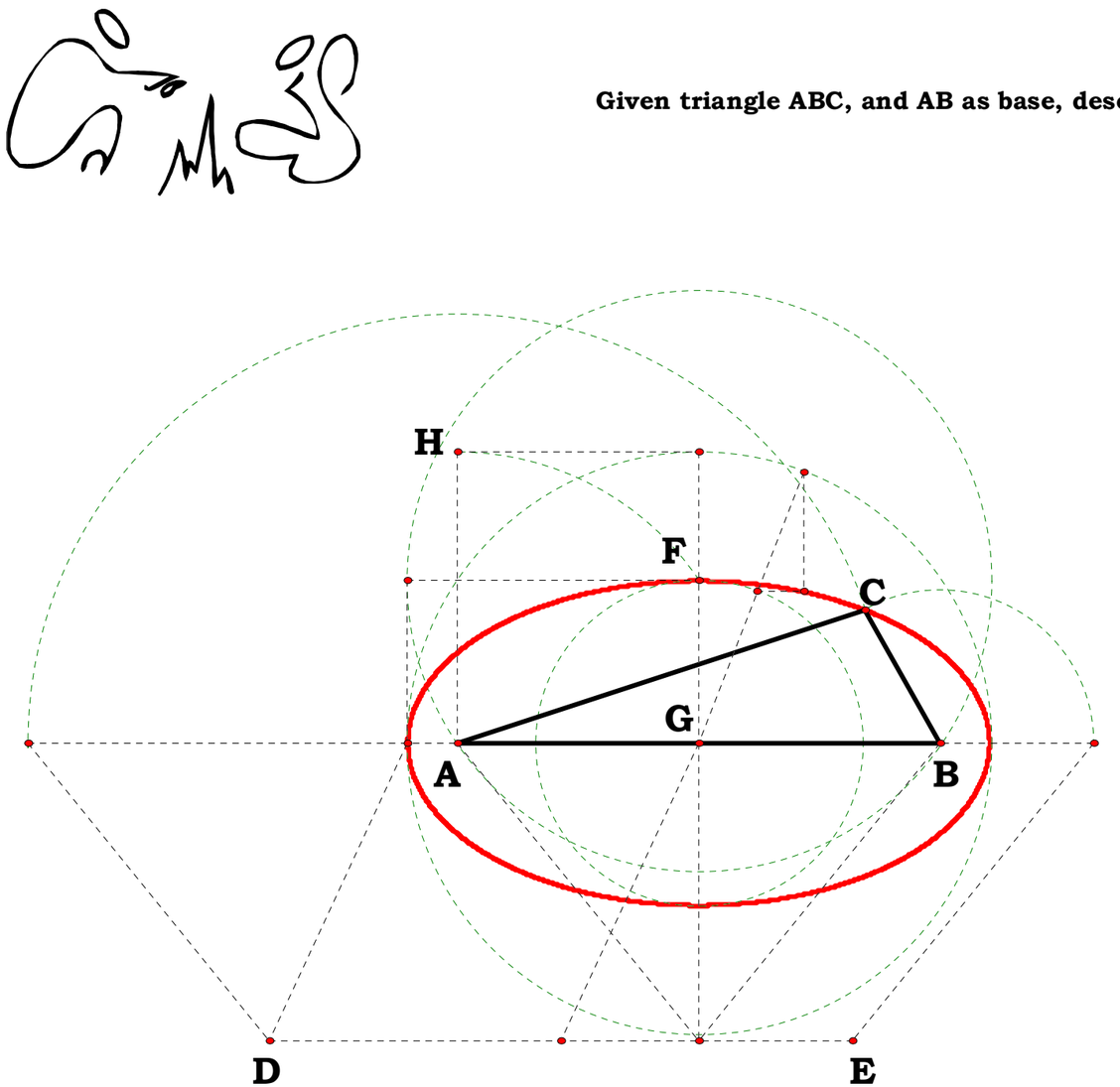



$N_1 := 2.028 \quad N_2 := 1.035$

$AD := 3.333 \quad EF := \frac{AD}{N_1} \quad AB := \frac{AD}{N_2} \quad BD := AD - AB \quad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \quad BC := AC - AB \quad CH := \frac{EF}{2} \quad CJ := AC \quad BG := \frac{BJ \cdot CH}{CJ} \quad CG := \sqrt{BG^2 + BC^2}$

$MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2} \qquad \frac{AD \cdot \sqrt{4 \cdot N_2 - 4 + N_2^2 \cdot N_1^2 - 4 \cdot N_2 \cdot N_1^2 + 4 \cdot N_1^2}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{AD \cdot \sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0 \qquad \frac{AD \cdot \sqrt{\left(N_1^2 - 1\right)}}{N_1} - MN = 0$




**Given triangle ABC, and AB as base, describe the Ellipse**

$S_1 := 8.14917 \qquad S_2 := 7.23745 \qquad S_3 := 2.58277$

$AB := S_1$

$AC := S_2$

$BC := S_3$

$DE := AC + BC$

$AH := \frac{DE}{2} \qquad AG := \frac{AB}{2} \qquad FG := \sqrt{AH^2 - AG^2}$

$$FG - \frac{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2} = 0$$

**The ratio of the ellipse is thus;**

$$\frac{AH}{FG} - \frac{(S_2 + S_3)}{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}} = 0$$

**From any point on DE, one can find everything and not once think about x and y.**

020298

$\Delta := 4 \quad \delta := 0 .. \Delta - 1$

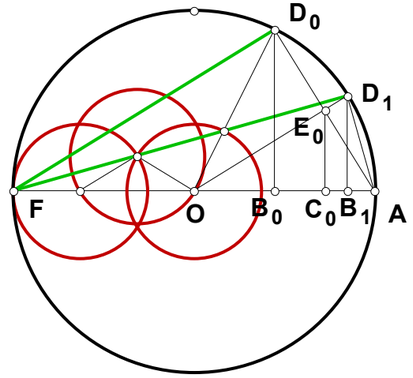
$N := .656 \quad AF := 2.3754$

$AO := \frac{AF}{2} \quad AB_0 := N \quad BD_0 := \sqrt{AB_0 \cdot (AF - AB_0)}$

$CE_0 := \frac{BD_0}{2} \quad AC_0 := \frac{AB_0}{2}$

$OC_0 := AO - AC_0$

$OE_0 := \sqrt{(CE_0)^2 + (OC_0)^2}$




$\sqrt{(AB_\delta)^2 + (BD_\delta)^2}$

$AD_\delta := \sqrt{(AB_\delta)^2 + (BD_\delta)^2} \qquad AD = \begin{pmatrix} 1.248304 \\ 0.648825 \\ 0.327541 \\ 0.164163 \end{pmatrix}$

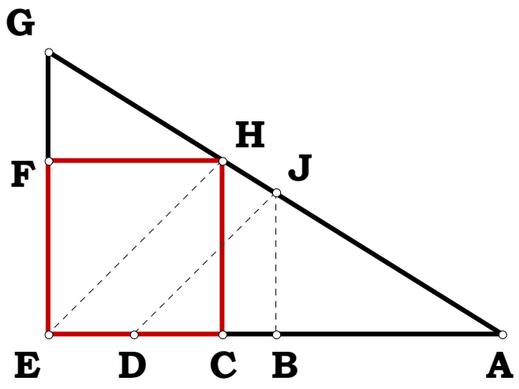
**Length of cord by progressive bisections.**

$$\begin{pmatrix} OB_{\delta+1} \\ AB_{\delta+1} \\ BD_{\delta+1} \\ CE_{\delta+1} \\ AC_{\delta+1} \\ OC_{\delta+1} \\ OE_{\delta+1} \end{pmatrix} := \begin{bmatrix} OC_\delta \cdot \frac{AO}{OE_\delta} \\ AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{OE_\delta} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{(2 \cdot OE_\delta)} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta + OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot \sqrt{AO \cdot \frac{(AF \cdot OE_\delta + 4 \cdot OC_\delta \cdot AO - OC_\delta \cdot AF)}{OE_\delta}} \end{bmatrix}$$

**I have no idea why I did this figure, it was so long ago.**

**A Square In A Triangle 021098**

**What is the Algebraic Name for the square as given in a right triangle?**
**What is the Algebraic name for the ratio AE/AC?**




$$N_1 := 2.98958 \qquad N_2 := 1.86690$$

$$AE := N_1 \quad EG := N_2$$

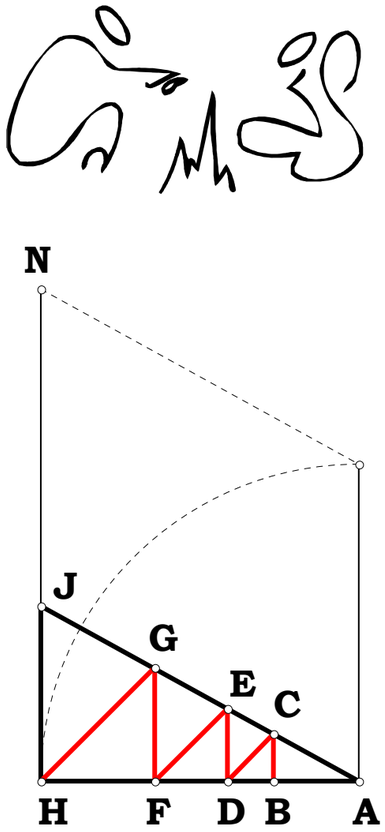
$$AB := \frac{AE}{2} \quad BJ := \frac{EG}{2} \quad BD := BJ$$

$$AD := AB + BD \quad CE := BD \cdot \frac{AE}{AD}$$

$$AC := AE - CE \quad FG := EG - CE$$

$$AB - \frac{N_1}{2} = 0 \qquad BJ - \frac{N_2}{2} = 0 \qquad AD - \left(\frac{N_1}{2} + \frac{N_2}{2}\right) = 0 \qquad CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0 \qquad AC - \frac{{N_1}^2}{N_1 + N_2} = 0$$

$$FG - \frac{{N_2}^2}{N_1 + N_2} = 0 \qquad \frac{AE}{AC} - \left(\frac{N_1 + N_2}{N_1}\right) = 0 \qquad \frac{EG}{FG} - \frac{N_1 + N_2}{N_2} = 0$$




## Alternate Method Root Series 022598

**Given a length and a unit, raise that length to any whole power.**

**Given for the third power.**

$N_1 := 4 \qquad N_2 := 3$

$AH := N_1 \quad HN := AH \cdot N_2$

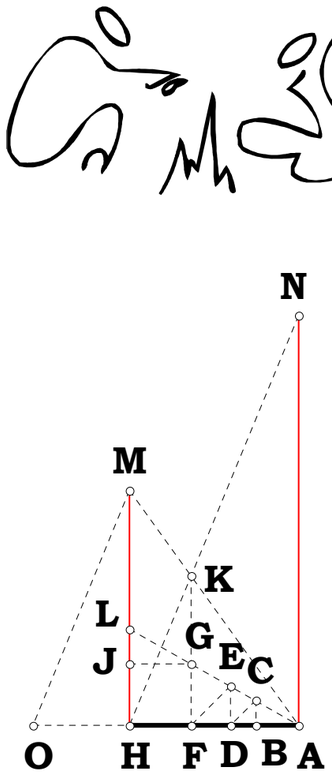
$HJ := HN - AH \quad FH := \frac{AH \cdot HJ}{AH + HJ} \quad AF := AH - FH$

$FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$

$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$

$\frac{AH}{AF} - {N_2}^1 = 0 \qquad \frac{AH}{AD} - {N_2}^2 = 0 \qquad \frac{AH}{AB} - {N_2}^3 = 0$

${N_2}^3 = 27$



**Sum DIvided by One Powered 022598B**




$$N_1 := 5 \quad N_2 := 6 \quad AH := 1 \quad HM := AH \cdot N_1 \quad AN := AH \cdot N_2$$

$$HO := \frac{AH \cdot HM}{AN} \quad AO := AH + HO \quad AF := \frac{AH^2}{AO}$$

$$FH := AH - AF \quad FG := FH \quad DF := \frac{AF \cdot FG}{AF + FG} \quad AD := AF - DF$$

$$DE := DF \quad BD := \frac{AD \cdot DE}{AD + DE} \quad AB := AD - BD$$

$$\frac{AH}{AB} - \left( \frac{N_1 + N_2}{N_2} \right)^3 = 0$$

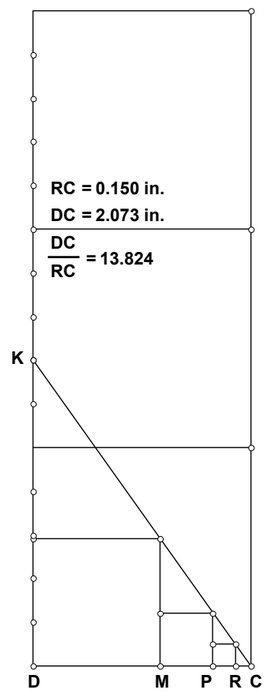
**Doing the Math**

**These plates sat, never actually being wrote up, in their directory, but included in the Delian Quest. Because they are so elementary, I assumed that doing math with a geometric figure was known. I was a bit conflicted about this, however, working 12 hours a day for years on end tends to dull the senses. Then, I got to thinking about them again in 2007. I even did a couple of searches on the internet to see if anyone had actually developed doing the math with a simple geometric figure and could not find anything. Then I found scraps in old books found on the Internet Archive where certain operations were fragmented and really undeveloped. Then I started to realize and understand that BAM was not developed as a language. If it had been, there would be no talk of non-Euclidean Geometry, there would only be embarrassment of its memory.**

**One can see that it is directly derived from plate A on this date. BAM (Basic Analog Mathematics) has its roots in exponential series.**

4
3+4/5
3+3/5
3+2/5
3+1/5
3
2+4/5
2+3/5
2+2/5
2+1/5
2
1+4/5
1+3/5
1+2/5
1+1/5
Show
Hide

M^N




$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

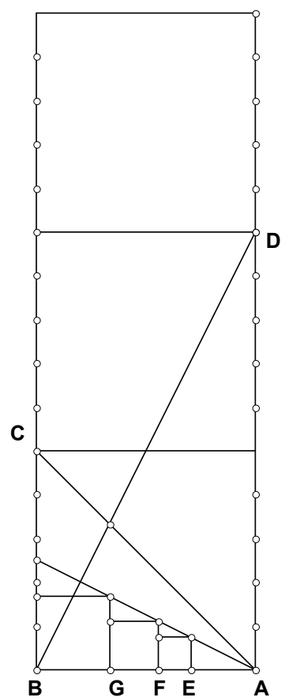
$\left(2+\frac{1}{5}\right)^3 = 10.648$

$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

$\left(1+\frac{1}{5}\right)^3 = 1.728$




Given line AB, divide it by the equation

$\frac{BC+AD^{\,w}}{AD} = 5.06250$

where W is a whole number.

2+4/5
2+3/5
2+2/5
2+1/5
2
1+4/5
1+3/5
1+2/5
1+1/5

BC

4
3+4/5
3+3/5
3+2/5
3+1/5
3

AD

4
3+4/5
3+3/5
3+2/5
3+1/5
3

$\frac{AB}{AG} = 1.50000$

$\frac{AB}{AF} = 2.25000$

$\frac{AB}{AE} = 3.37500$

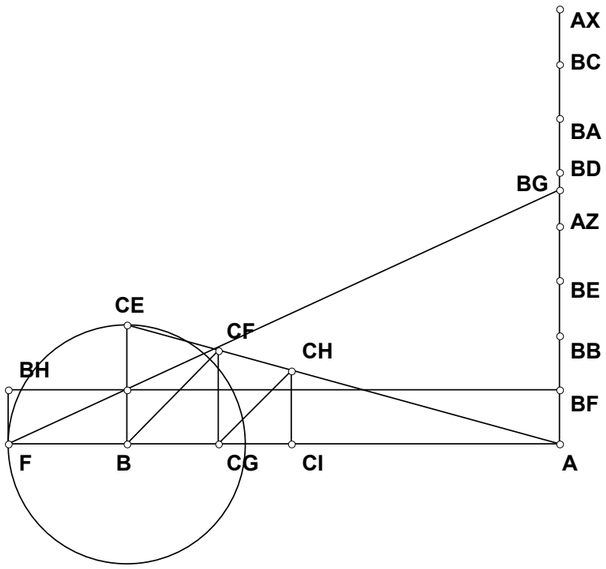
$\frac{BC+AD^{\,1}}{AD} = 1.50000$

$\frac{BC+AD^{\,2}}{AD} = 2.25000$

$\frac{BC+AD^{\,3}}{AD} = 3.37500$

**022698**

**Now ain't this just typically human. Some of the more defined plates that led to Basic Analog Mathematics promptly get wrote up in 0816 2015. I am on the ball! It appears I never even bothered to do a pdf file of these. They are, however, not fundamentally distinct from some previous write ups except, these are series format. So, I think I will forgo the write ups again!**




Move BG->AX
Move BG->BC
Move BG->BA
Move BG->BD
Move BG->AZ
Move BG->BE
Move BG->BB

$\frac{AF}{FB} = 4.672$

$\frac{AF}{ACI} = 2.060$

AB = 3.348 in.

FB = 0.912 in.

$\frac{AF}{ACG} = 1.619$

AF = 4.260 in.

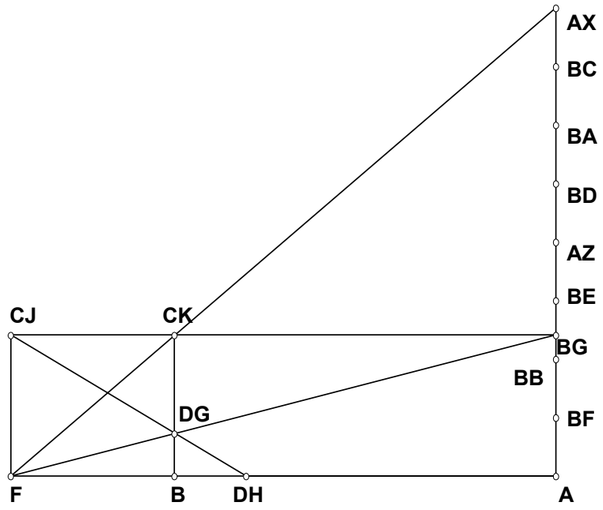
$\frac{AF}{AB} = 1.272$

$\frac{AF^3}{AB} = 2.060$

ACG = 2.632 in.

ACI = 2.068 in.

$\frac{AF^2}{AB} = 1.619$




Move BG->AX
Move BG->BC
Move BG->BA
Move BG->BD
Move BG->AZ
Move BG->BE
Move BG->BB
Move BG->BF

AF = 4.271 in.

DHF = 1.840 in.

BGA = 1.105 in.

AB = 2.985 in.

BF = 1.286 in.

AAX = 3.670 in.

$\frac{AF}{AB} = 1.431$

$\frac{AAX}{BGA} = 3.321$

$\left(\frac{1}{\frac{AAX}{BGA} - 1}\right) \cdot BF = 0.554$ in.

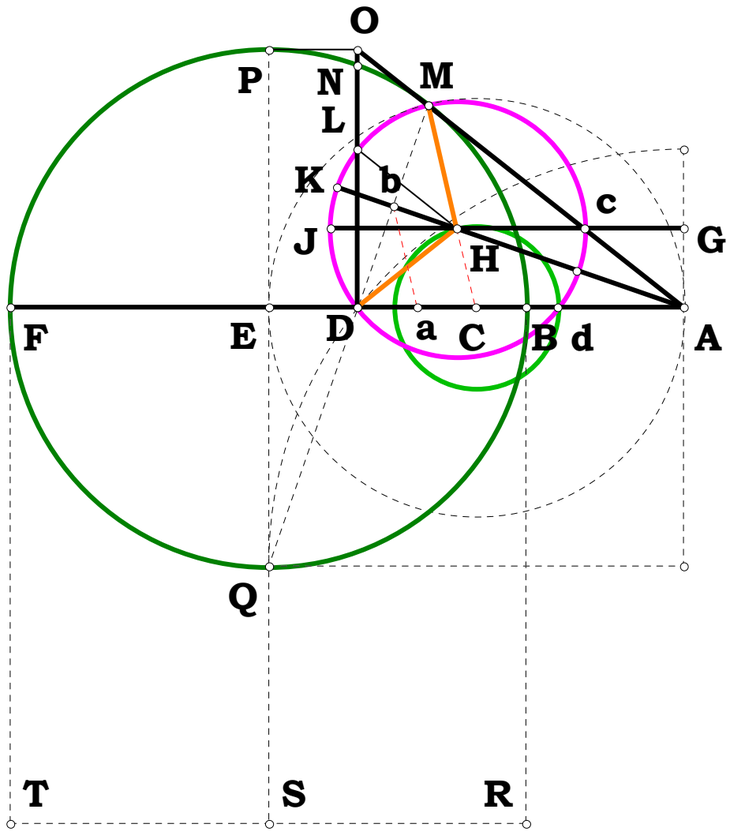
$\frac{AF}{DHF} = 2.321$

$\frac{AAX}{BGA} - 1 = 2.321$

$\frac{DHF}{AF} = 0.431$

## A Square Root Figure And Triseciton 042398

**In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?**




$$N_1 := 2.64583 \qquad N_2 := 11.29771$$

$$AB := N_1 \quad AF := N_2 \quad BF := AF - AB$$

$$AD := (AB \cdot AF)^{\frac{1}{2}} \quad BE := \frac{BF}{2} \quad BD := AD - AB \quad DE := BE - BD$$

$$EQ := BE \quad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \quad PQ := BF \quad QM := \frac{EQ \cdot PQ}{DQ}$$

$$DM := QM - DQ \quad AE := AB + BE \quad AC := \frac{AE}{2} \quad Db := \frac{DM}{2}$$

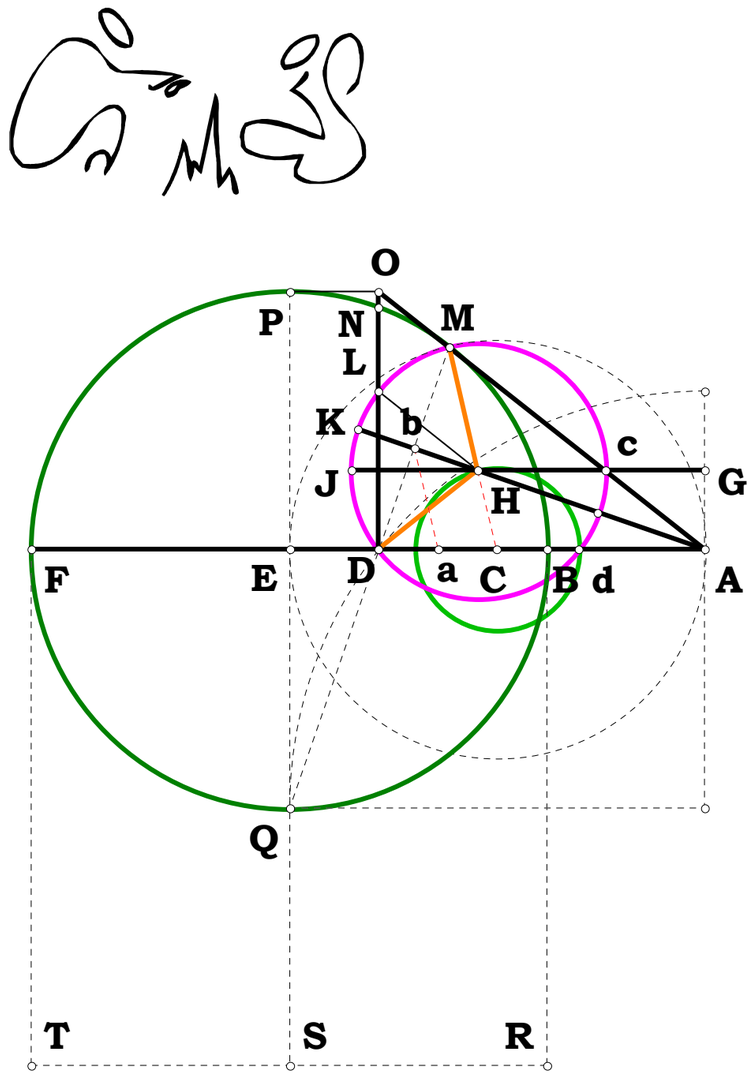
$$CM := AC \quad ab := \frac{CM \cdot Db}{DM} \quad CD := AD - AC$$

$$Ca := \frac{CD}{2} \quad Aa := AC + Ca \quad CH := \frac{ab \cdot AC}{Aa}$$

$$AM := AD \quad Ac := \frac{AM \cdot CH}{CM} \quad HM := CM - CH \qquad HM - Ac = 0$$

$$Ac - \frac{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right)}{N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}} = 0$$

$$HM - \frac{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right)}{N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}} = 0$$




**Definitions:**

$$AB - N_1 = 0 \quad AF - N_2 = 0 \quad BF - (N_2 - N_1) = 0 \quad AD - (N_1 \cdot N_2)^{\frac{1}{2}} = 0$$

$$BE - \frac{N_2 - N_1}{2} = 0 \quad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0 \quad DE - \frac{N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot N_2}}{2} = 0$$

$$DQ - \frac{\sqrt{\left[(N_1 + N_2) \cdot \left(N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot N_2}\right)\right]}}{\sqrt{2}} = 0 \quad QM - \frac{\sqrt{2} \cdot (N_1 - N_2)^2}{2 \cdot \sqrt{(N_1 + N_2) \cdot \left(N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot N_2}\right)}} = 0$$

$$DM - \frac{N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2} + N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2} - 2 \cdot \sqrt{2} \cdot N_1 \cdot N_2}{\sqrt{(N_1 + N_2) \cdot \left(N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot N_2}\right)}} = 0 \quad AE - \frac{N_1 + N_2}{2} = 0 \quad AC - \frac{N_1 + N_2}{4} = 0$$

$$Db - \frac{N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2} + N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2} - 2 \cdot \sqrt{2} \cdot N_1 \cdot N_2}{2 \cdot \sqrt{(N_1 + N_2) \cdot \left(N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot N_2}\right)}} = 0 \quad ab - \frac{N_1 + N_2}{8} = 0$$

$$CD - \frac{4 \cdot \sqrt{N_1 \cdot N_2} - N_2 - N_1}{4} = 0 \quad Ca - \frac{4 \cdot \sqrt{N_1 \cdot N_2} - N_2 - N_1}{8} = 0 \quad Aa - \frac{N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}}{8} = 0$$

$$CH - \frac{(N_1 + N_2)^2}{4 \cdot \left(N_1 + N_2 + 4 \cdot \sqrt{N_1 \cdot N_2}\right)} = 0$$

$$R := \frac{\sqrt{N_2}\cdot\left(1+N_2\right)}{1+N_2+4\cdot\sqrt{N_2}}$$

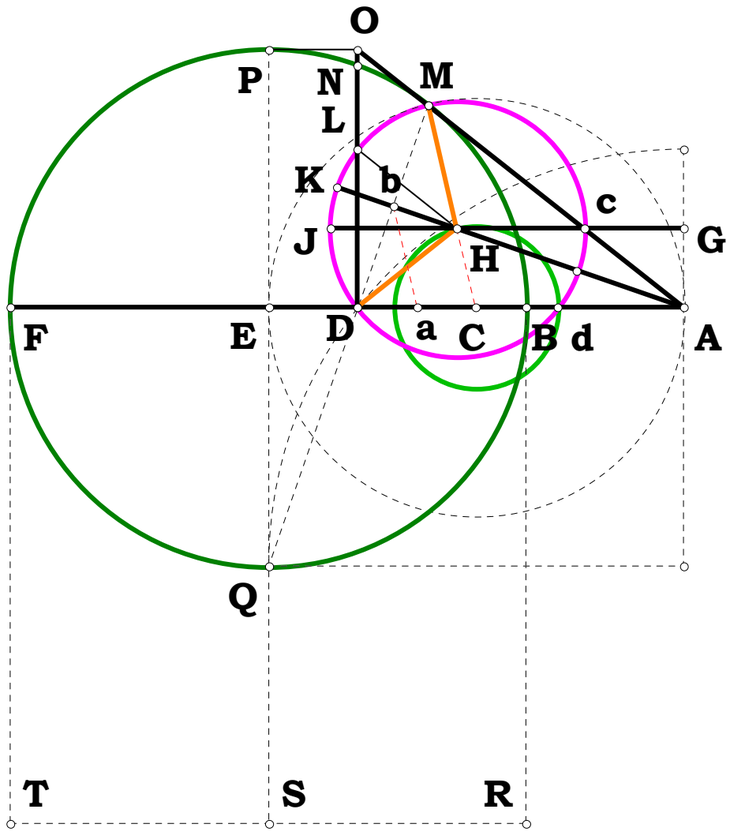
$$\frac{N_2-N_1}{2}-BE=0$$

$$\left[\frac{R}{3}+\left(\frac{R}{3}+\frac{2\cdot R^2}{3}+\frac{R^3}{27}+\sqrt{\frac{50\cdot R^2}{27}-\frac{44\cdot R^3}{27}-\frac{R^4}{9}-\frac{4\cdot R}{9}+\frac{1}{27}}\right)^{\frac{1}{3}}+\frac{\frac{R^2}{9}+\frac{4\cdot R}{3}-\frac{1}{3}}{\left(\frac{R}{3}+\frac{2\cdot R^2}{3}+\frac{R^3}{27}+\sqrt{\frac{50\cdot R^2}{27}-\frac{44\cdot R^3}{27}-\frac{R^4}{9}-\frac{4\cdot R}{9}+\frac{1}{27}}\right)^{\frac{1}{3}}}\right]^2=11.29771$$

$$N_2-\left[\frac{R}{3}+\left(\frac{R}{3}+\frac{2\cdot R^2}{3}+\frac{R^3}{27}+\sqrt{\frac{50\cdot R^2}{27}-\frac{44\cdot R^3}{27}-\frac{R^4}{9}-\frac{4\cdot R}{9}+\frac{1}{27}}\right)^{\frac{1}{3}}+\frac{\frac{R^2}{9}+\frac{4\cdot R}{3}-\frac{1}{3}}{\left(\frac{R}{3}+\frac{2\cdot R^2}{3}+\frac{R^3}{27}+\sqrt{\frac{50\cdot R^2}{27}-\frac{44\cdot R^3}{27}-\frac{R^4}{9}-\frac{4\cdot R}{9}+\frac{1}{27}}\right)^{\frac{1}{3}}}\right]^2=0$$

## A Square Root Figure And Triseciton 042398

**In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?**




$N_1 := 2.64583 \qquad N_2 := 8.65188$

$AB := N_1 \qquad BF := N_2 \qquad AF := N_1 + N_2$

$AD := (AB \cdot AF)^{\frac{1}{2}} \qquad BE := \frac{BF}{2} \qquad BD := AD - AB \qquad DE := BE - BD$

$EQ := BE \qquad DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \qquad PQ := BF \qquad QM := \frac{EQ \cdot PQ}{DQ}$

$DM := QM - DQ \qquad AE := AB + BE \qquad AC := \frac{AE}{2} \qquad Db := \frac{DM}{2}$

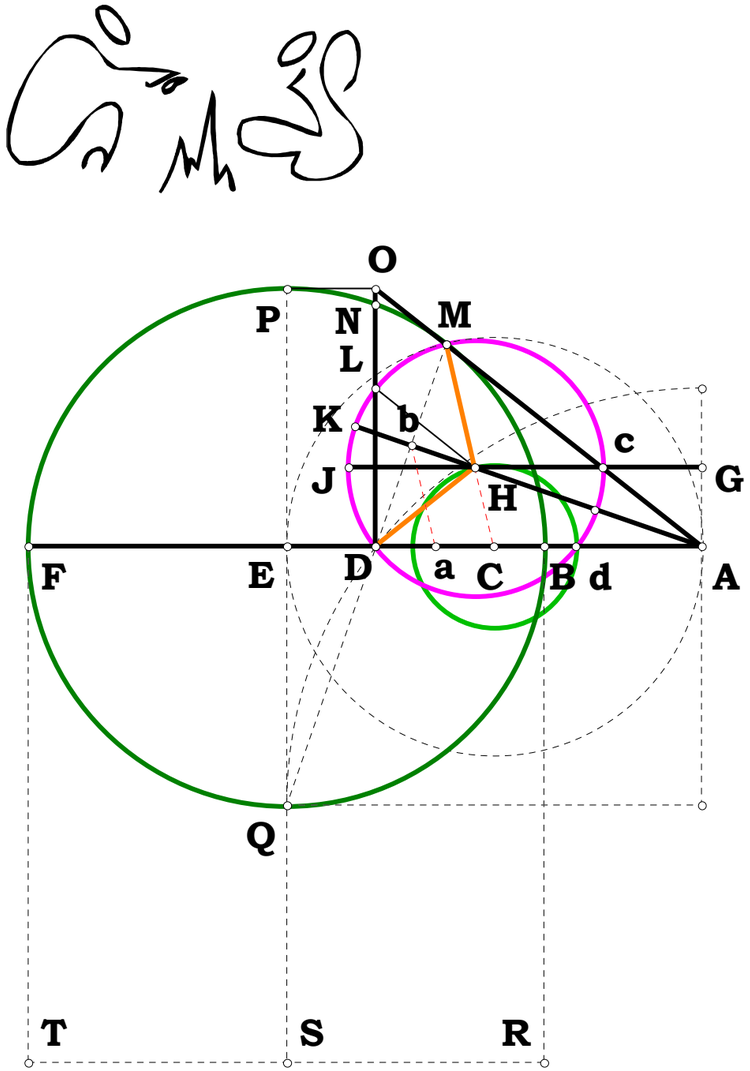
$CM := AC \qquad ab := \frac{CM \cdot Db}{DM} \qquad CD := AD - AC$

$Ca := \frac{CD}{2} \qquad Aa := AC + Ca \qquad CH := \frac{ab \cdot AC}{Aa}$

$AM := AD \qquad Ac := \frac{AM \cdot CH}{CM} \qquad HM := CM - CH \qquad HM - Ac = 0$

$$Ac - \frac{\sqrt{N_1 \cdot (N_1 + N_2)} \cdot (2 \cdot N_1 + N_2)}{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}} = 0$$

$$HM - \frac{\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2)}{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}} = 0$$




**Definitions:**

$$AB - N_1 = 0 \qquad BF - N_2 = 0 \qquad AF - (N_1 + N_2) = 0 \qquad AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0 \qquad BE - \frac{N_2}{2} = 0$$

$$BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right] = 0 \qquad DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}}{2} = 0 \qquad AE - \frac{2 \cdot N_1 + N_2}{2} = 0$$

$$DQ - \frac{\left[(2 \cdot N_1 + N_2) \cdot \left(2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)\right]^{\frac{1}{2}}}{\sqrt{2}} = 0 \qquad AC - \frac{2 \cdot N_1 + N_2}{4} = 0$$

$$QM - \frac{\sqrt{2} \cdot N_2^{\,2}}{2 \cdot \sqrt{(2 \cdot N_1 + N_2) \cdot \left(2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)}} = 0 \qquad ab - \frac{2 \cdot N_1 + N_2}{8} = 0$$

$$DM - \frac{\sqrt{2} \cdot \left[\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^{\,2}\right]}{\sqrt{(2 \cdot N_1 + N_2) \cdot \left[2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot (N_1 + N_2)}\right]}} = 0$$

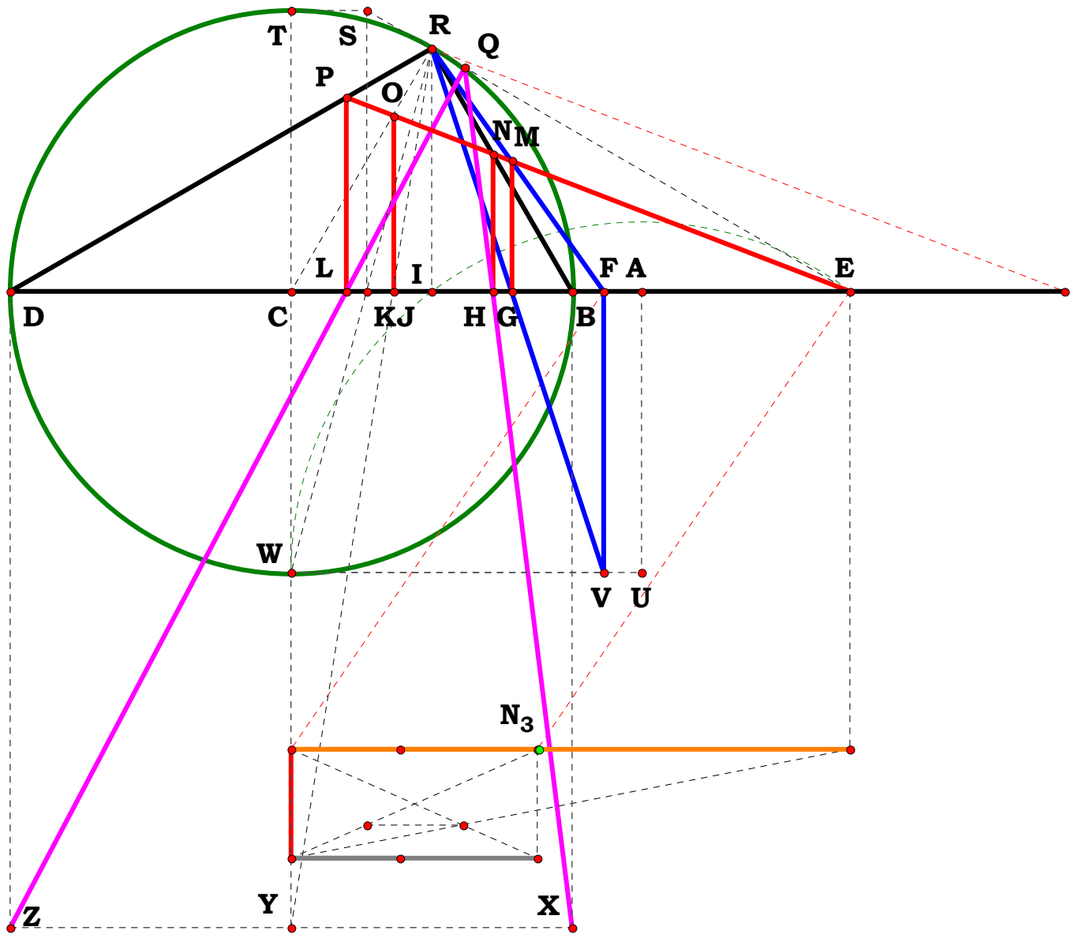
$$Db - \frac{\sqrt{2} \cdot \left[\sqrt{N_1^{\,2} + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^{\,2}\right]}{2 \cdot \sqrt{(2 \cdot N_1 + N_2) \cdot \left[2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1 \cdot (N_1 + N_2)}\right]}} = 0$$

$$CD - \frac{4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1} - N_2 - 2 \cdot N_1}{4} = 0 \qquad Ca - \frac{4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1} - N_2 - 2 \cdot N_1}{8} = 0$$

$$Aa - \frac{2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}}{8} = 0 \qquad CH - \frac{(2 \cdot N_1 + N_2)^2}{4 \cdot \left(2 \cdot N_1 + N_2 + 4 \cdot \sqrt{N_1^{\,2} + N_2 \cdot N_1}\right)} = 0$$

**On Gemini Roots 072499**

**CE is to EF as CY is to CW**




$N_1 := 1.51292 \quad N_2 := 10.77334 \quad N_3 := 3$

$AB := N_1 \quad AD := N_2$

$BD := AD - AB \quad BC := \frac{BD}{2} \quad CW := BC \quad CT := BC$

$FV := BC \quad AI := \sqrt{AB \cdot AD} \quad BI := AI - AB \quad DI := BD - BI$

$IR := \sqrt{BI \cdot DI} \quad AE := AI \quad AC := AB + BC \quad ED := AD + AE$

$CE := AC + AE \quad EF := \frac{CE}{N_3} \quad BE := AE + AB \quad EI := AE + AI$

$FI := EI - EF \quad FG := \frac{FI \cdot FV}{FV + IR} \quad EG := EF + FG \quad GI := FI - FG$

$GM := \frac{FV \cdot GI}{FI} \quad Ia := \frac{EG \cdot IR}{GM} \quad EL := \frac{Ia \cdot ED}{Ia + DI} \quad BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI \quad BH := \frac{BI \cdot BE}{Ba} \quad EH := BE + BH \quad CI := AC - AI$

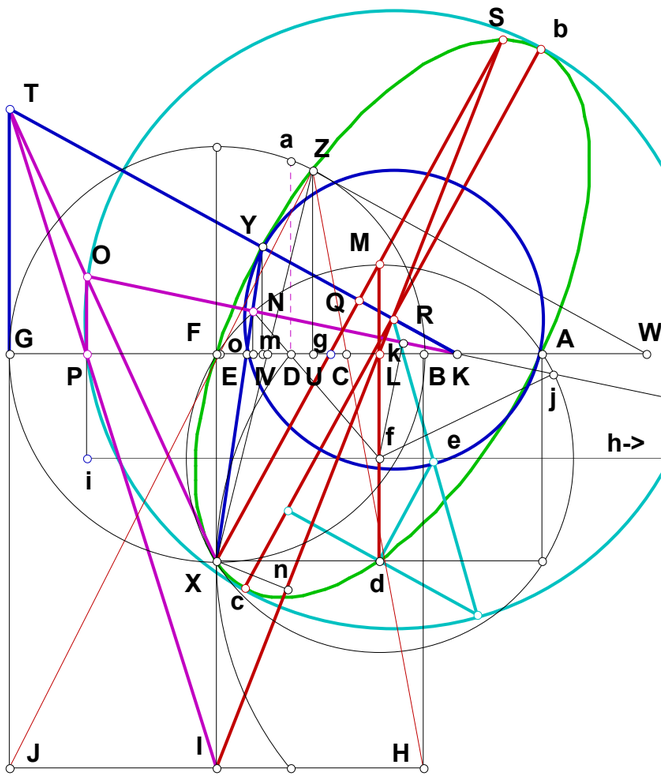
$JO := \frac{IR \cdot CE}{CI + Ia} \quad CJ := \frac{CI \cdot JO}{IR} \quad JI := CI - CJ \quad CY := \frac{IR \cdot CJ}{JI}$

$\frac{CY}{CW} - \frac{CE}{EF} = 0 \qquad \frac{CE}{EF} = 3 \qquad \frac{CY}{CW} = 3$

**Note: Needs lipstick and a dress. Parse it. If I get around to it, I will redo the graphics.**

## A Delian Solution 081199

**What are the minor and major axis for the ellipse that will give point Z for the cube root?**




$$AB := 1 \quad N := 16 \quad AG := AB \cdot N \quad BG := AG - AB$$

$$BF := \frac{BG}{2} \quad FG := BF \quad AF := AB + BF \quad FX := BF \quad Mf := \frac{\sqrt{AF^2 + FX^2}}{2}$$

$$Lf := \frac{FX}{2} \quad ML := Mf - Lf \quad FL := \frac{AF}{2} \quad Xd := FL \quad df := Lf \quad IX := FX$$

$$Md := Mf + df \quad MX := \sqrt{Xd^2 + Md^2} \quad SX := \frac{MX \cdot IX}{IX - ML} \quad Lg := \frac{FL \cdot ML}{ML + FX}$$

$$QX := \frac{SX}{2} \quad Fg := FL - Lg \quad Xg := \frac{MX \cdot Fg}{Xd} \quad Qg := QX - Xg$$

$$Kg := \frac{Xg \cdot Qg}{Fg} \quad GK := FG + Fg + Kg \quad GJ := BG \quad GT := \frac{Fg \cdot GK}{FX}$$

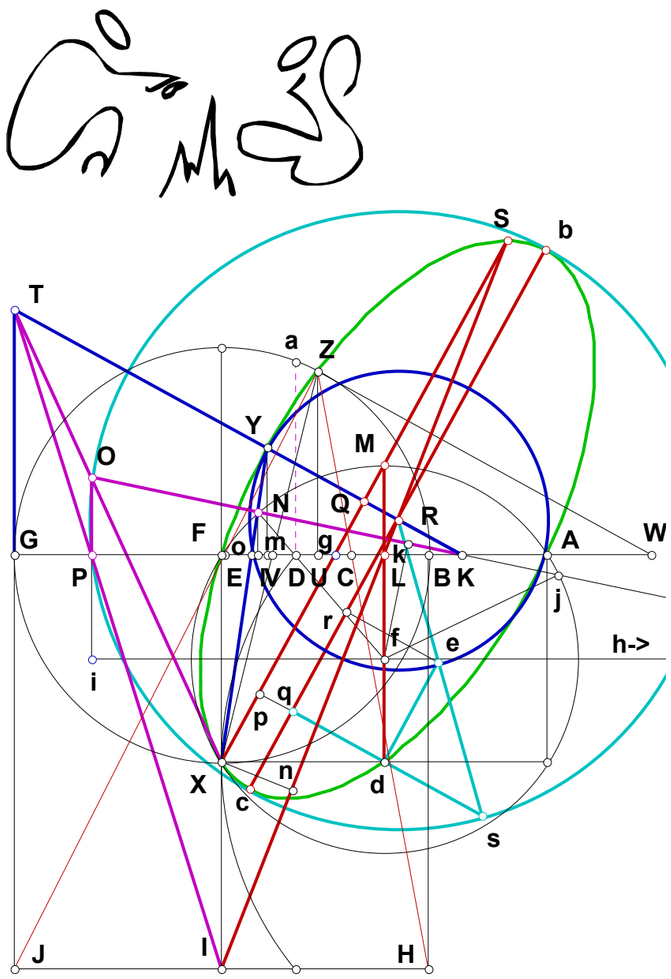
$$JT := GJ + GT \quad IJ := BF \quad FP := \frac{IJ \cdot GJ}{JT} \quad OP := \frac{IX \cdot GT}{JT}$$

$$KP := Fg + Kg + FP \quad Pi := Lf \quad Oi := OP + Pi \quad hi := \frac{KP \cdot Oi}{OP} \quad fi := FP + FL$$

$$fh := hi - fi \quad KO := \sqrt{KP^2 + OP^2} \quad hk := \frac{KP \cdot fh}{KO} \quad fk := \frac{OP \cdot fh}{KO} \quad Nf := Mf$$

$$Nk := \sqrt{Nf^2 - fk^2} \quad Nh := hk + Nk \quad Oh := \frac{KO \cdot Oi}{OP} \quad NO := Oh - Nh$$

$$KN := KO - NO$$




$$N1 := \frac{OP \cdot KN}{KO} \quad K1 := \frac{KP \cdot KN}{KO} \quad FK := KP - FP \quad F1 := FK - K1$$

$$NX := \sqrt{(FX + N1)^2 + F1^2} \quad XY := \frac{NX \cdot IX}{IX - N1} \quad Fm := \frac{F1 \cdot XY}{NX} \quad Km := FK - Fm$$

$$Fo := \frac{F1 \cdot FX}{FX + N1} \quad Xo := \frac{NX \cdot Fo}{F1} \quad mo := Fm - Fo \quad Ym := \frac{FX \cdot mo}{Fo}$$

$$FI := 2 \cdot BF \quad IL := \sqrt{FL^2 + FI^2} \quad IS := \frac{IL \cdot IX}{IX - ML} \quad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

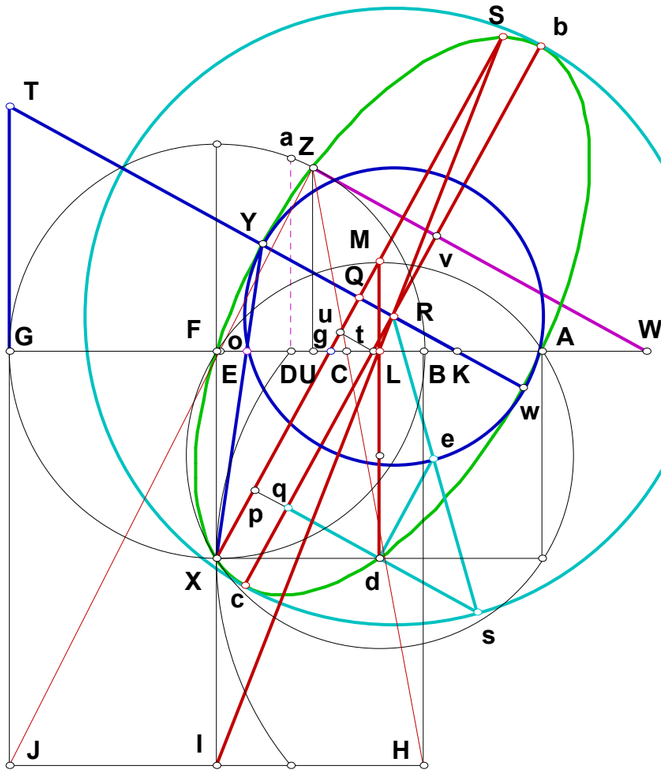
$$Xn := \sqrt{IX^2 - In^2} \quad QR := \frac{Xn \cdot QX}{IS - In} \quad KT := \sqrt{GK^2 + GT^2} \quad KY := \frac{KT \cdot Ym}{GT}$$

$$KQ := \frac{GK \cdot Qg}{GT} \quad RY := KY - KQ + QR \quad Xd := FL \quad dp := \frac{GK \cdot Xd}{KT}$$

$$pq := QR \quad dq := dp - pq \quad Xp := \frac{GT \cdot Xd}{KT} \quad Qp := QX - Xp \quad Rq := Qp$$

$$er := dq \quad Re := RY \quad Rr := \sqrt{Re^2 - er^2} \quad Rs := \frac{Re \cdot Rq}{Rr}$$

**Is the segment Zv equal to the perpendicular for the ellipse?**




$$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \quad AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \quad BC := AC - AB \quad BE := AE - AB$$

$$CE := BE - BC \quad EG := BG - BE \quad CU := \frac{BC \cdot CE}{BC + EG} \quad BU := BC + CU$$

$$GU := BG - BU \quad UZ := \sqrt{BU \cdot GU} \quad UW := \frac{GK \cdot UZ}{GT} \quad Gg := GK - Kg$$

$$tu := QR \quad gt := \frac{KT \cdot tu}{GK} \quad Gt := Gg + gt \quad GW := GU + UW$$

$$Wt := GW - Gt \quad tv := \frac{GT \cdot Wt}{KT} \quad Kt := GK - Gt \quad Rt := \frac{GT \cdot Kt}{KT}$$

$$Rv := tv - Rt \quad bc := 2 \cdot Rs \quad Rc := Rs \quad cv := Rc + Rv \quad Yw := 2 \cdot RY$$

$$WZ := \frac{KT \cdot UZ}{GT} \quad Wv := \frac{GK \cdot tv}{GT} \quad Zv := \left|WZ - Wv\right|$$

$$N_1 := Yw \quad N_2 := bc \quad N_3 := cv \quad N_4 := bc$$

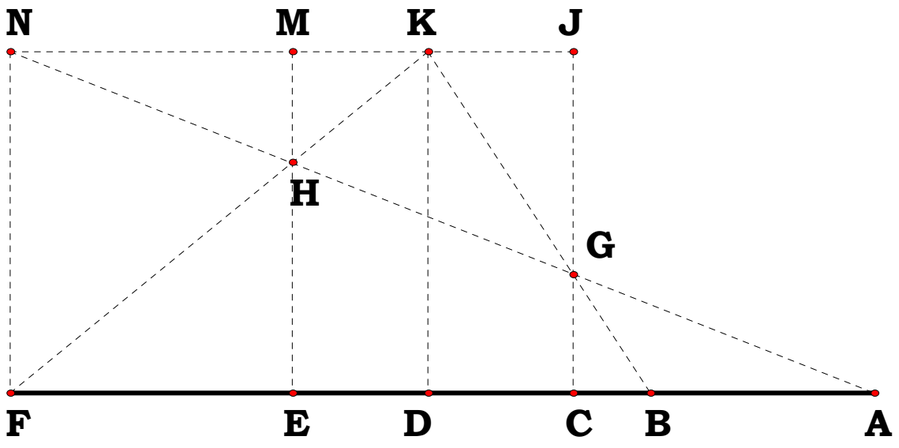
$$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2} - Zv = 0 \qquad \frac{AC}{2 \cdot AB} - 2^{\frac{1}{3}} = 0$$

$\sqrt{N_3 \cdot \left(N_4 - N_3\right)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

081899

**Promptly writing this up in 0816 2015**

**Exponential series by changing the unit, in other words, the same way as done inside a circle only this circle is getting smaller.**




$$N_1 := 5.05354 \quad N_2 := 14.49917 \quad N_3 := 5.71500$$

$$AC := N_1 \quad AF := N_2 \quad FN := N_3 \quad CF := AF - AC$$

$$CG := \frac{FN \cdot AC}{AF} \quad EF := \frac{CF}{2} \quad EH := \frac{FN \cdot (AC + EF)}{AF}$$

$$KN := \frac{EF \cdot FN}{EH} \quad JK := CF - KN \quad BC := \frac{JK \cdot CG}{FN - CG} \quad BF := BC + CF$$

$$AD := AC + JK \quad BD := BC + JK$$

$$\left(\frac{AF}{AC}\right)^2 - \frac{BF}{BC} = 0 \quad \sqrt{\frac{BF}{BC}} - \frac{AF}{AC} = 0$$

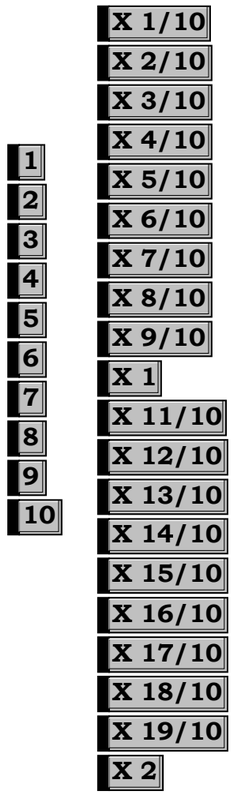
$$CF - (N_2 - N_1) = 0$$

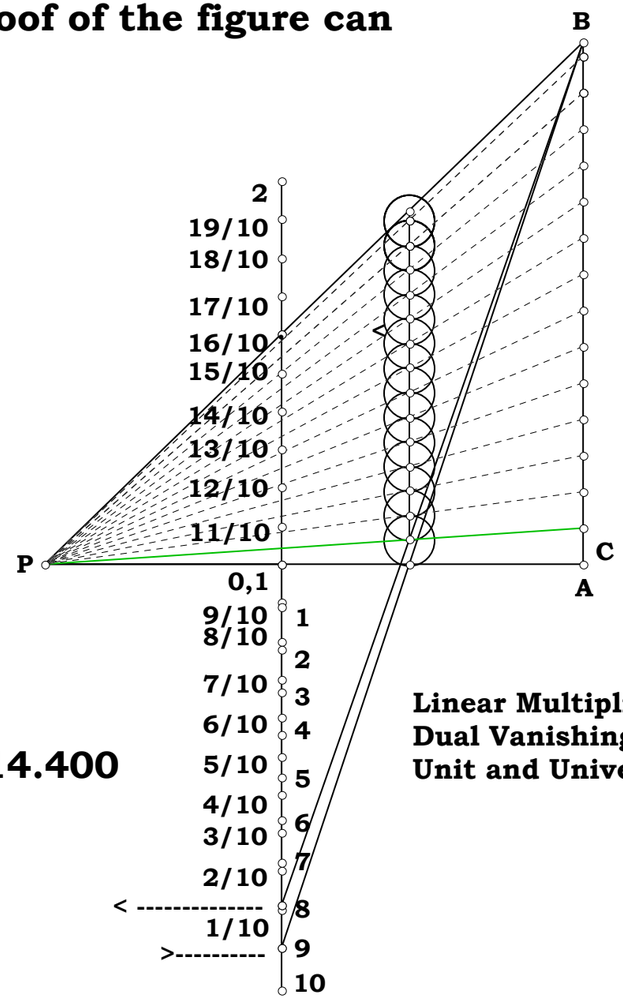
$$CG - \frac{N_3 \cdot N_1}{N_2} = 0 \quad EF - \frac{N_2 - N_1}{2} = 0 \quad EH - \frac{N_3 \cdot (N_1 + N_2)}{2 \cdot N_2} = 0$$

$$KN - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \quad JK - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0 \quad BC - \frac{{N_1}^2}{N_1 + N_2} = 0$$

$$BF := \frac{{N_2}^2}{N_1 + N_2} \quad AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0 \quad BD - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$$

**Since the figure only uses proportion, which has been proven any proof of the figure can be left for an exersize.**




$$\frac{AB}{AC} = 14.400$$

**Linear Multiplication and Division,**
**Dual Vanishing Point Method**
**Unit and Universe of Discourse.**

**In process. POR something or other.**

$N_1 := 4 \quad N_2 := 5 \quad AC := 1 \quad \delta := 1 .. N_2$

$AB := \frac{AC}{N_1} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC} \quad BE_\delta := \frac{BD \cdot \delta}{N_2} \quad CE_\delta := \sqrt{(BE_\delta)^2 + BC^2}$

$CG_\delta := \frac{BC \cdot AC}{CE_\delta} \quad AD := \sqrt{AB^2 + BD^2} \quad AG_\delta := \frac{BE_\delta \cdot AC}{CE_\delta} \quad EG_\delta := CG_\delta - CE_\delta \quad EH_\delta := \frac{BE_\delta \cdot EG_\delta}{CE_\delta}$

$GH_\delta := \frac{BC \cdot EH_\delta}{BE_\delta} \quad BH_\delta := BE_\delta + EH_\delta \quad DH_\delta := BD - BH_\delta \quad DG_\delta := \sqrt{(GH_\delta)^2 + (DH_\delta)^2}$

$BK_\delta := \frac{AB \cdot BE_\delta}{BD} \quad CK_\delta := BC + BK_\delta \quad CJ_\delta := \frac{CE_\delta \cdot AC}{CK_\delta} \quad EJ_\delta := CJ_\delta - CE_\delta$

$AG_\delta = \frac{\delta}{\sqrt{\delta^2 + N_1 \cdot N_2^2 - N_2^2}} =$

| $AG_\delta =$ | |
|---|---|
| 0.114708 | 0.114708 |
| 0.225018 | 0.225018 |
| 0.327327 | 0.327327 |
| 0.419314 | 0.419314 |
| 0.5 | 0.5 |

$DG_\delta = (N_2 - \delta) \cdot \sqrt{\frac{(N_1 - 1)}{\left[N_1 \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)\right]}} =$

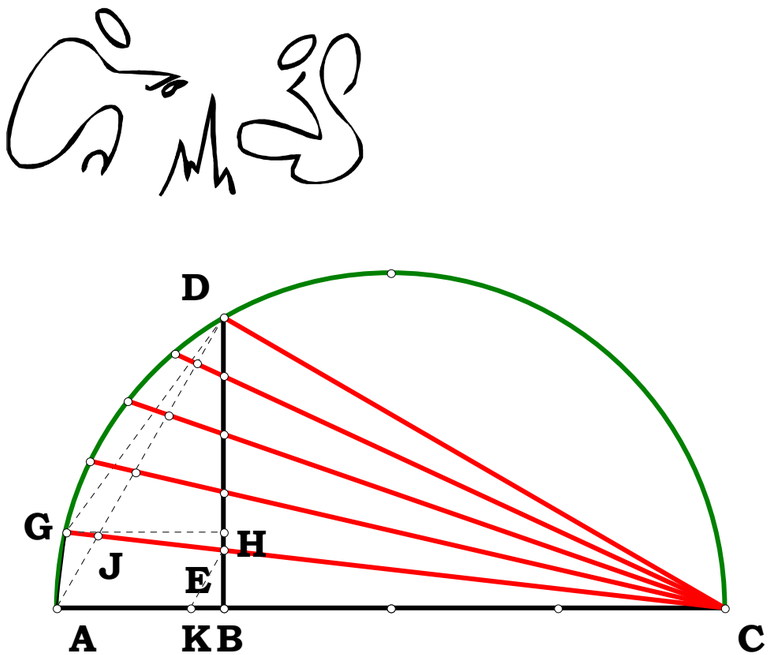
| $DG_\delta =$ | |
|---|---|
| 0.39736 | 0.39736 |
| 0.292306 | 0.292306 |
| 0.188982 | 0.188982 |
| 0.090784 | 0.090784 |
| 0 | 0 |

$EG_\delta = \frac{(N_2^2 - \delta^2) \cdot \sqrt{N_1 - 1}}{N_2 \cdot N_1 \cdot \sqrt{\delta^2 + N_2^2 \cdot N_1 - N_2^2}} =$

| $EG_\delta =$ | |
|---|---|
| 0.238416 | 0.238416 |
| 0.204614 | 0.204614 |
| 0.151186 | 0.151186 |
| 0.081706 | 0.081706 |
| 0 | 0 |

$CG_\delta = N_2 \cdot \frac{(N_1 - 1)}{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}} =$

| $CG_\delta =$ | |
|---|---|
| 0.993399 | 0.993399 |
| 0.974355 | 0.974355 |
| 0.944911 | 0.944911 |
| 0.907841 | 0.907841 |
| 0.866025 | 0.866025 |




$BE_\delta =$

| |
|---|
| 0.086603 |
| 0.173205 |
| 0.259808 |
| 0.34641 |
| 0.433013 |

$\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2} =$

| |
|---|
| 0.086603 |
| 0.173205 |
| 0.259808 |
| 0.34641 |
| 0.433013 |

$EJ_\delta =$

| |
|---|
| 0.188746 |
| 0.135837 |
| 0.088192 |
| 0.043481 |
| 0 |

$\frac{(N_2 - \delta) \cdot \sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{N_2 \cdot N_1 \cdot (N_2 \cdot N_1 - N_2 + \delta)} =$

| |
|---|
| 0.188746 |
| 0.135837 |
| 0.088192 |
| 0.043481 |
| 0 |

$AD = 0.5 \sqrt{\frac{1}{N_1}} = 0.5$

$CJ_\delta =$

| |
|---|
| 0.943729 |
| 0.905577 |
| 0.881917 |
| 0.869616 |
| 0.866025 |

$\frac{\sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{(N_2 \cdot N_1 - N_2 + \delta)} =$

| |
|---|
| 0.943729 |
| 0.905577 |
| 0.881917 |
| 0.869616 |
| 0.866025 |

$\frac{AD}{AG_\delta} =$

| |
|---|
| 4.358899 |
| 2.222049 |
| 1.527525 |
| 1.192424 |
| 1 |

$\frac{\sqrt{\frac{(\delta^2 + N_2^2 \cdot N_1 - N_2^2)}{N_1}}}{\delta} =$

| |
|---|
| 4.358899 |
| 2.222049 |
| 1.527525 |
| 1.192424 |
| 1 |

$\frac{CG_\delta}{EG_\delta} =$

| |
|---|
| 4.166667 |
| 4.761905 |
| 6.25 |
| 11.111111 |
| $7.800463 \cdot 10^{15}$ |

$\text{if}\left(N_2^2 - \delta^2, \frac{N_2^2 \cdot N_1}{N_2^2 - \delta^2}, 0\right) =$

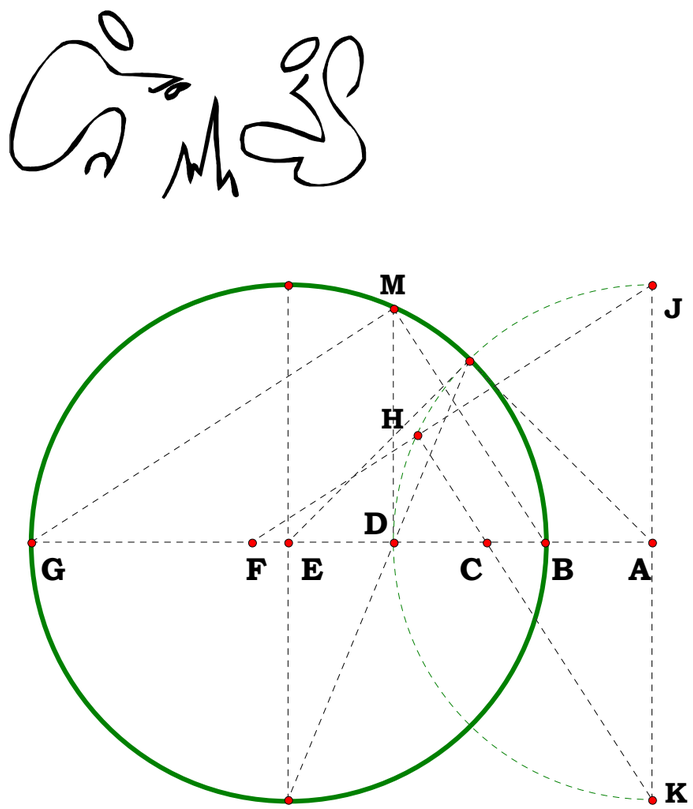
| |
|---|
| 4.166667 |
| 4.761905 |
| 6.25 |
| 11.111111 |
| 0 |

$\text{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right) =$

| |
|---|
| 5 |
| 6.666667 |
| 10 |
| 20 |
| 0 |

$\text{if}\left(EJ_\delta, \frac{N_2 \cdot N_1}{N_2 - \delta}, 0\right) =$

| |
|---|
| 5 |
| 6.666667 |
| 10 |
| 20 |
| 0 |




**07/09/00 Alternate Method Quad Roots**

$N_1 := 1.79201 \quad N_2 := 10.41743$

$AB := N_1 \quad AG := N_2 \quad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \quad BG := AG - AB$

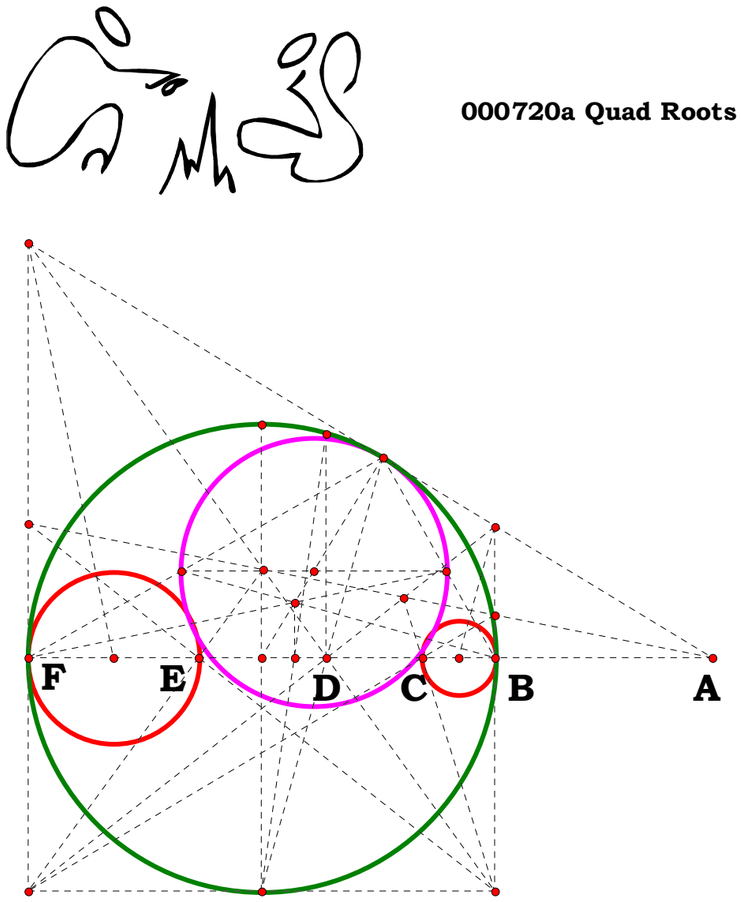
$DG := BG - BD \quad DM := \sqrt{BD \cdot DG}$

$AJ := AD \quad AK := AD \quad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \quad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$

$(AB \cdot AG^3)^{\frac{1}{4}} - AF = 0 \quad (AB^3 \cdot AG)^{\frac{1}{4}} - AC = 0$




**000720a Quad Roots via Tangent Circles.**

$N_1 := 3.73926 \quad N_2 := 11.78259$

$AB := N_1 \quad AF := N_2 \quad AD := \sqrt{AB \cdot AF}$

$BF := AF - AB \quad Bd := \frac{BF}{2}$

$BD := AD - AB \quad Dd := Bd - BD$

$DH := \sqrt{Bd^2 + Dd^2} \quad HL := \frac{Bd \cdot BF}{DH}$

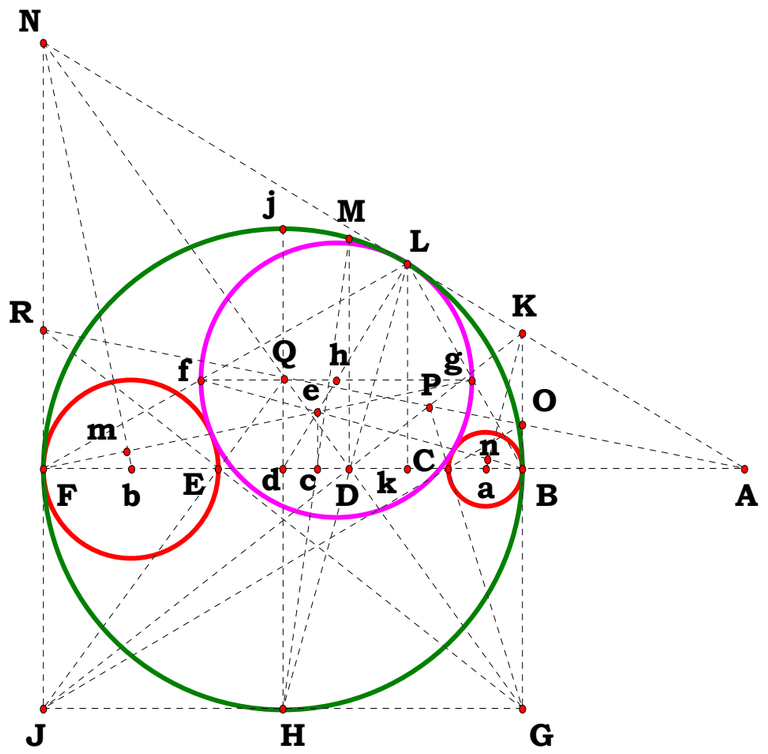
$DL := HL - DH \quad Dk := \frac{Dd \cdot DL}{DH}$

$Bk := Bd - (Dd + Dk) \quad Lk := \frac{Bd \cdot Dk}{Dd}$

$JN := \frac{Bd \cdot BF}{BD} \quad GK := \frac{Bd \cdot BF}{(Bd + Dd)}$

$FN := JN - Bd \quad BK := GK - Bd$

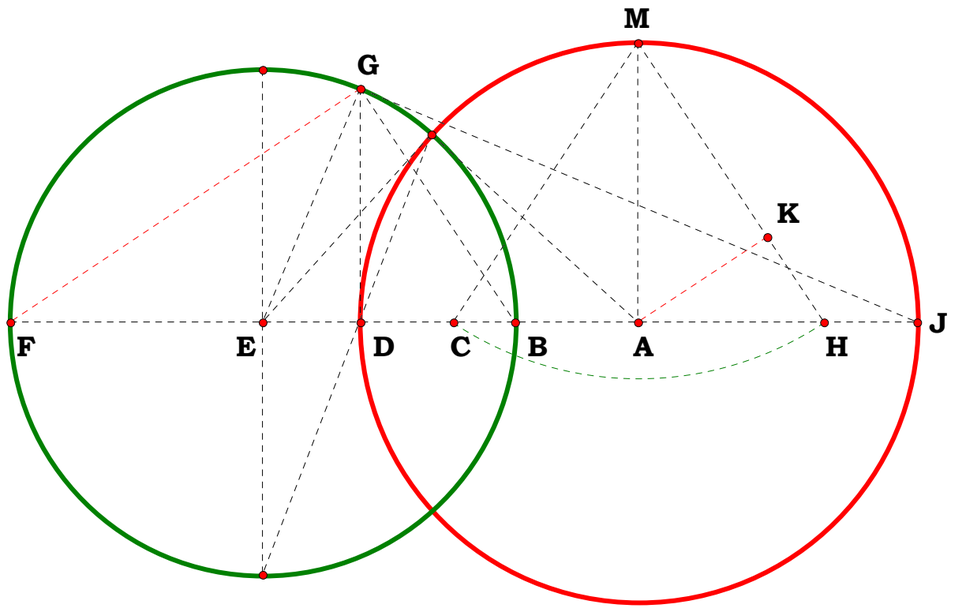
$Ak := Bk + AB \quad AF - \frac{BF \cdot FN}{FN - BK} = 0 \quad \frac{AF}{FN} - \frac{Ak}{Lk} = 0$ i.e., A, K, L and N are colinear.




$DF := Bd + Dd \quad DM := \sqrt{BD \cdot DF} \quad cd := \frac{Dd \cdot Bd}{Bd + DM} \quad dk := Bd - Bk \quad ce := \frac{Lk \cdot cd}{dk} \quad Fb := \frac{ce \cdot FN}{Bd + cd} \quad Ba := \frac{ce \cdot BK}{Bd - cd} \quad AE := AF - 2 \cdot Fb$

$AC := AB + 2 \cdot Ba \quad \left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AC = 0 \quad \left(AB \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$ etc., etc.

**OOO720b Quad Roots by equal angles.**




$N_1 := 2.07320$

$N_2 := 10.53987$

$AB := N_1 \quad AF := N_2$

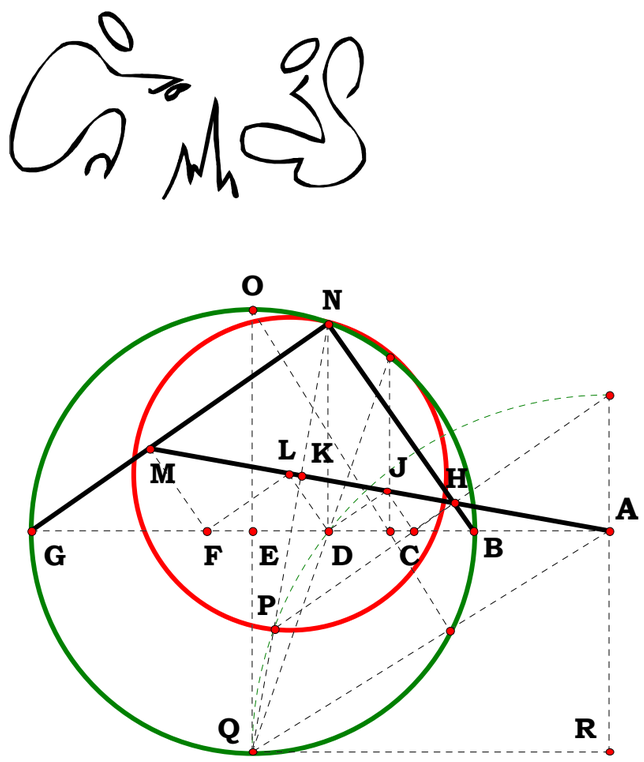
$AD := \sqrt{AB \cdot AF} \quad BF := AF - AB$

$AM := AD \quad DF := AF - AD$

$BD := AD - AB \quad DG := \sqrt{DF \cdot BD}$

$BG := \sqrt{DG^2 + BD^2} \quad AC := \frac{BD \cdot AD}{DG}$

$AC - \left(AB^3 \cdot AF\right)^{\frac{1}{4}} = 0$




**08/01/00 Alternate Method Quad Roots**

$N := 5 \quad AB := 1 \quad AG := AB \cdot N$

$BG := AG - AB \quad BE := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \quad DE := AE - AD \quad NY := DE$

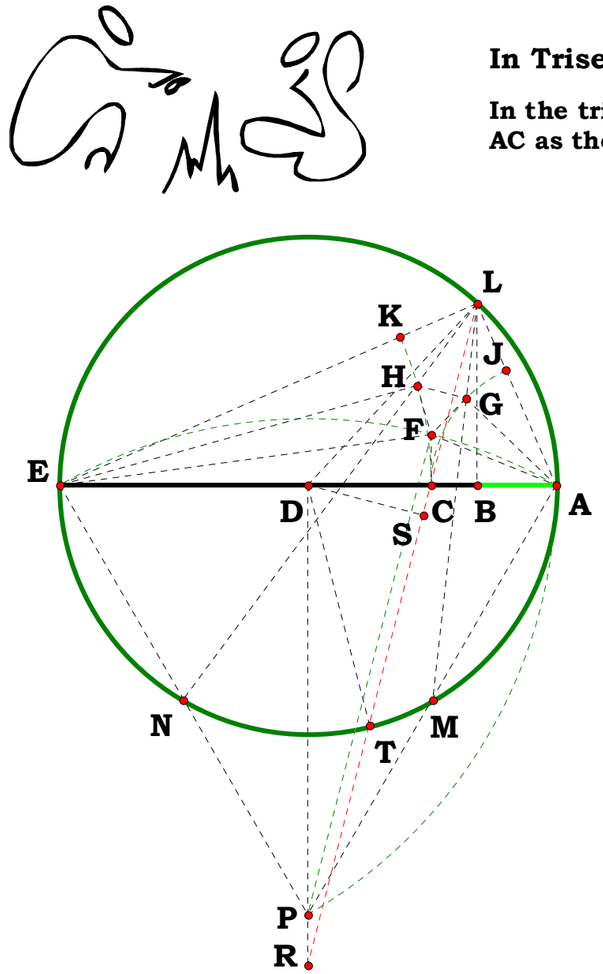
$BD := AD - AB \quad DG := BG - BD \quad EQ := BE$

$DN := \sqrt{BD \cdot DG} \quad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \quad OQ := BG \quad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \quad NP := NQ - PQ \quad MN := \sqrt{\frac{NP^2}{2}} \quad BN := \sqrt{BD^2 + DN^2} \quad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \quad FG := \frac{BG \cdot GM}{GN} \quad AF := AG - FG \quad \left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$




## In Trisection What Is AB? 08/02/00

**In the trisection figure given and given AC as the Unit what is AB?**

$$AC := .884 \quad AE := 3.521 \quad AD := \frac{AE}{2} \quad EP := AE \quad DE := AD \quad DP := \sqrt{EP^2 - DE^2}$$

$$FP := EP \quad CE := AE - AC \quad CD := CE - DE \quad CF := \sqrt{FP^2 - CD^2} - DP \quad PR := CF \quad DR := DP + PR$$

$$CR := \sqrt{CD^2 + DR^2} \quad CS := \frac{CD^2}{CR} \quad DS := \sqrt{CD^2 - CS^2} \quad DL := AD \quad LS := \sqrt{DL^2 - DS^2} \quad RS := CR - CS$$

$$LR := RS + LS \quad BD := \frac{CD \cdot LR}{CR} \quad AB := AD - BD \quad ST := LS \quad RT := RS - ST$$

**In trisection the length RT to the similarity point is equal to the radius of the circle.**

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

$$AD - \frac{AE}{2} = 0 \quad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \quad CF - \left( \frac{\sqrt{4 \cdot AC \cdot AE - 4 \cdot AC^2 + 3 \cdot AE^2}}{2} - \frac{\sqrt{3} \cdot AE}{2} \right) = 0$$

$$CE - (AE - AC) = 0 \quad CD - \left( \frac{1}{2} \cdot AE - AC \right) = 0 \quad DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0$$

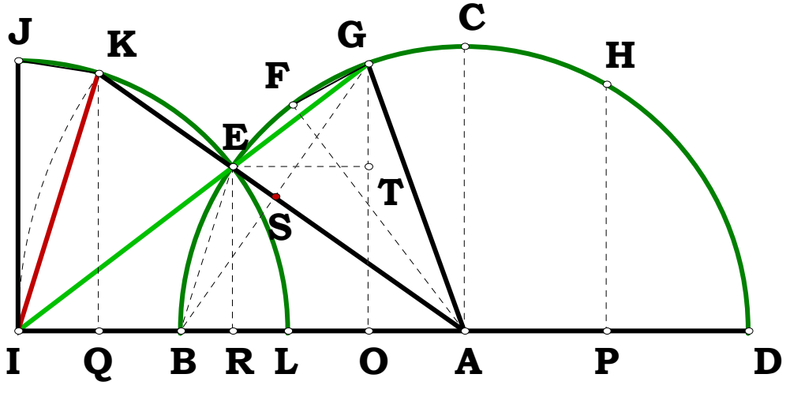
$$CR - AE = 0 \quad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \quad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \quad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0 \quad RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0$$

$$BD - \left( \frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3 \right) = 0 \quad AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \quad AB \cdot AE^2 - AC^2 (3 \cdot AE - 2 \cdot AC) = 0$$

## 080300 Trisection

**If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.**




$N := 3 \qquad BD := 2$

$AB := \frac{BD}{2} \quad AD := AB \quad AP := \frac{AD}{2} \quad BP := AB + AP \quad BO := \frac{BP}{N} \quad AE := AB$

$DO := BD - BO \quad GO := \sqrt{BO \cdot DO} \quad BG := \sqrt{GO^2 + BO^2} \quad BS := \frac{BG}{2} \quad ER := BS \quad TO := ER$

$GT := GO - TO \quad AS := \sqrt{AB^2 - BS^2} \quad ES := AE - AS \quad BR := ES \quad OR := BO - BR$

$ET := OR \quad IO := \frac{ET \cdot GO}{GT} \quad BI := IO - BO \quad AI := BI + AB \quad BE := \sqrt{ER^2 + BR^2}$

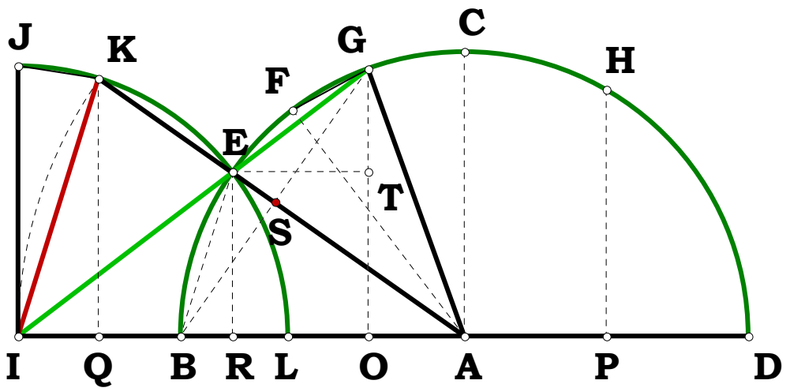
$GE := BE \quad GI := \frac{GE \cdot GO}{GT} \quad EI := GI - GE \quad AK := AI \quad IK := EI \quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

$EK := AK - AE \quad \frac{EK}{IQ} = 2$

**Some Algebraic Names:**

$$\frac{3 \cdot BD}{4 \cdot N} - BO = 0 \qquad \frac{BD}{4} \cdot \frac{(4 \cdot N - 3)}{N} - DO = 0 \qquad \frac{BD}{(4 \cdot N)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N - 3} - GO = 0 \qquad \frac{BD}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BG = 0 \qquad \frac{BD}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N}} - BS = 0$$

$$\frac{BD}{4} \cdot \sqrt{3} \cdot \frac{\left(\sqrt{4 \cdot N - 3} - \sqrt{\frac{1}{N}} \cdot N\right)}{N} - GT = 0 \qquad \frac{BD}{4} \cdot \sqrt{\frac{(4 \cdot N - 3)}{N}} - AS = 0 \qquad \frac{BD}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N - 3)}{N}}\right] - ES = 0 \qquad \frac{BD}{4} \cdot \frac{\left[3 - 2 \cdot N + \sqrt{\frac{(4 \cdot N - 3)}{N}} \cdot N\right]}{N} - OR = 0$$




$$\frac{-BD}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N-3}\cdot\sqrt{N}-2\cdot\sqrt{4\cdot N-3}\cdot N^{\left(\frac{3}{2}\right)}+4\cdot N^{2}-3\cdot N\right]}{\left[N^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)\right]}-IO=0$$

$$\frac{-BD}{2}\cdot\frac{\left(\sqrt{4\cdot N-3}-2\cdot\sqrt{N}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-BI=0$$

$$\frac{-BD}{\left(-2\cdot\sqrt{4\cdot N-3}+2\cdot\sqrt{N}\right)}\cdot\sqrt{N}-AI=0$$

$$\frac{BD}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N}}\cdot\sqrt{4\cdot N-3}}-BE=0$$

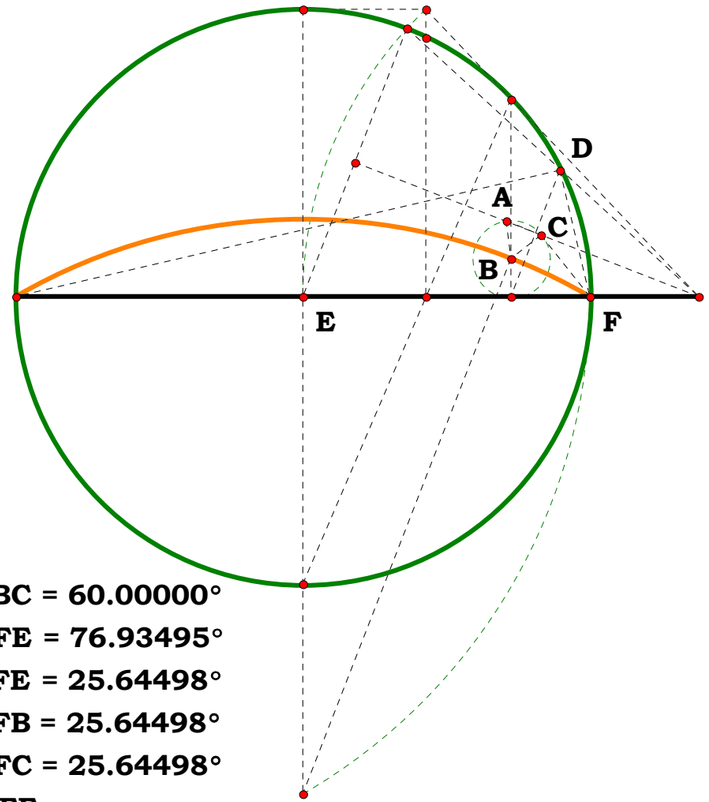
$$\frac{BD}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N}+\sqrt{4\cdot N-3}\right)}{\sqrt{N}}}\cdot\frac{\sqrt{4\cdot N-3}}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-GI=0$$

$$\frac{-BD}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N-3}}{\sqrt{N}}}\cdot\frac{\sqrt{N}}{\left(-\sqrt{4\cdot N-3}+\sqrt{N}\right)}-EI=0$$

$$\frac{BD}{4}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-IQ=0$$

$$\frac{BD}{2}\cdot\frac{\left(2\cdot\sqrt{N}-\sqrt{4\cdot N-3}\right)}{\left(\sqrt{4\cdot N-3}-\sqrt{N}\right)}-EK=0$$

**000804A Trisection and Square Roots**



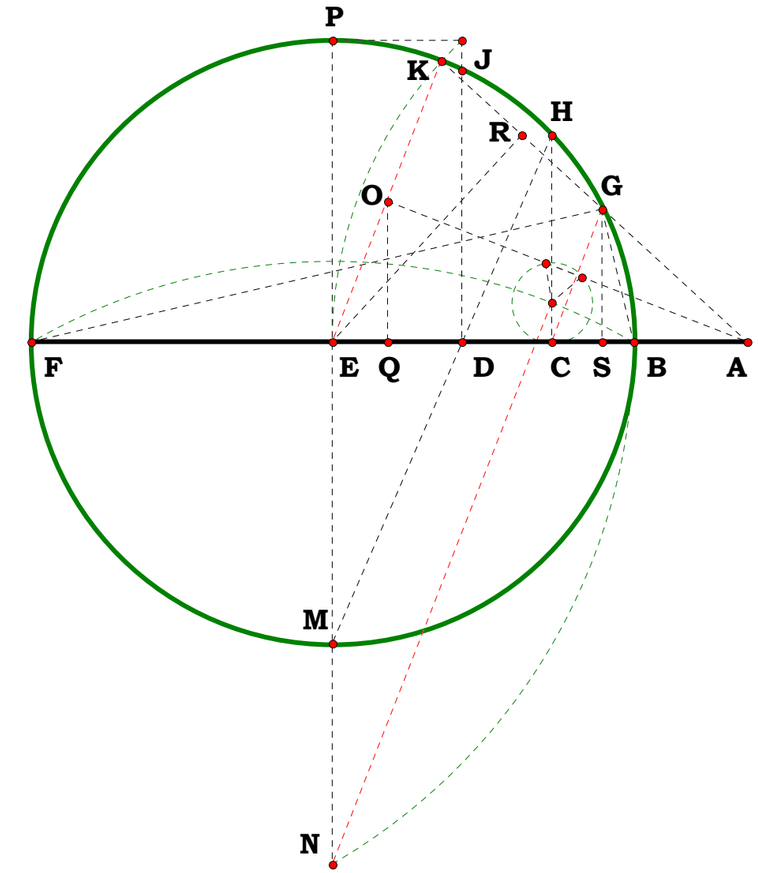

**With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?**

**I am going to figure this out and then I am going to order the equations a little different at the start to see what happens to all the definitions.**

**m∠ABC = 60.00000°**
**m∠DFE = 76.93495°**
**m∠BFE = 25.64498°**
**m∠CFB = 25.64498°**
**m∠DFC = 25.64498°**

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

**m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°**




$N_1 := 1.90557 \quad N_2 := 12.01265$

$AB := N_1 \quad AF := N_2 \quad AD := \sqrt{AB \cdot AF} \quad BD := AD - AB$

$BF := AF - AB \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$

$BE := \frac{BF}{2} \quad EO := \frac{BE}{2} \quad AE := BE + AB \quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \quad CD := \frac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \quad KG := \frac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$

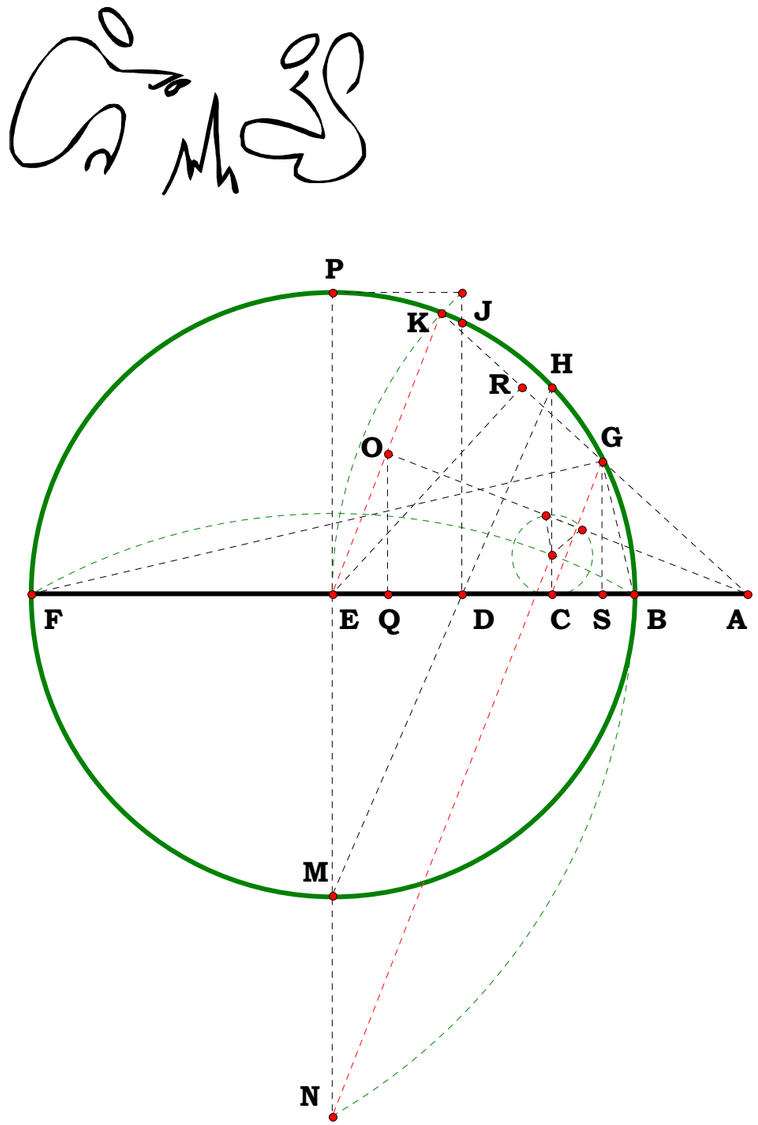
$CS := \frac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - AB$

**Definitions:**

$AB - N_1 = 0 \quad AF - N_2 = 0 \quad AD - \sqrt{N_1 \cdot N_2} = 0 \quad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0$

$BF - (N_2 - N_1) = 0 \quad DF - \left(N_2 - \sqrt{N_1 \cdot N_2}\right) = 0 \quad DJ - \sqrt{\sqrt{N_1 \cdot N_2} \cdot (N_1 + N_2) - 2 \cdot N_1 \cdot N_2} = 0$

$BE - \frac{N_2 - N_1}{2} \quad EO - \frac{N_2 - N_1}{4} = 0 \quad AE - \frac{N_1 + N_2}{2} = 0 \quad EQ - \frac{(N_1 - N_2)^2}{8 \cdot (N_1 + N_2)} = 0$




$$DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}}{2} = 0 \qquad DM - \frac{\sqrt{\left[N_1^{\,2} + N_2^{\,2} + 2\cdot N_1\cdot N_2 - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2)\right]}}{\sqrt{2}} = 0$$

$$HM - \frac{\sqrt{2}\cdot(N_1 - N_2)^2}{2\cdot\sqrt{N_1^{\,2} + N_2^{\,2} - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2) + 2\cdot N_1\cdot N_2}} = 0$$

$$DH - \frac{(N_1 + N_2)\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})}} = 0$$

$$CD - \frac{\sqrt{N_1\cdot N_2}\cdot(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}) - 4\cdot N_1^{\,2}\cdot N_2 - 4\cdot N_1\cdot N_2^{\,2}}{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})} = 0$$

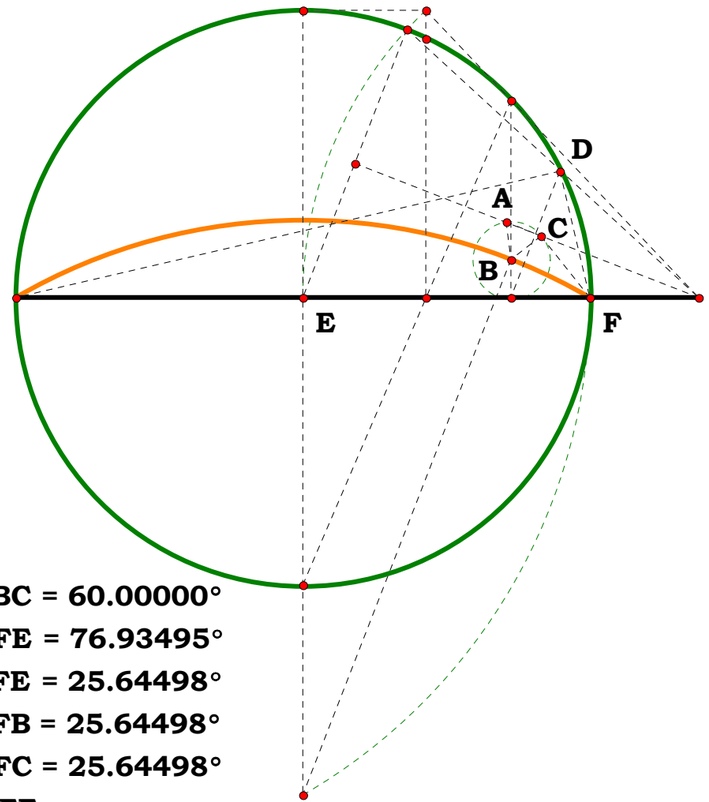
$$CE - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad BC - \frac{N_1\cdot(N_2 - N_1)}{N_1 + N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(N_1 - N_2)^2}}{2} = 0$$

$$KG - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad AG - \frac{2\cdot N_1\cdot N_2}{N_1 + N_2} = 0 \qquad CS - \frac{N_1\cdot N_2\cdot(N_1 - N_2)^2}{(N_1 + N_2)^3} = 0$$

$$AS - \frac{N_1\cdot N_2\cdot(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2})}{(N_1 + N_2)^3} = 0$$

$$BS - \frac{N_1^{\,2}\cdot(N_2 - N_1)\cdot(N_1 + 3\cdot N_2)}{(N_1 + N_2)^3} = 0$$

**000804B Trisection and Square Roots**




**With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?**

**m∠ABC = 60.00000°**

**m∠DFE = 76.93495°**

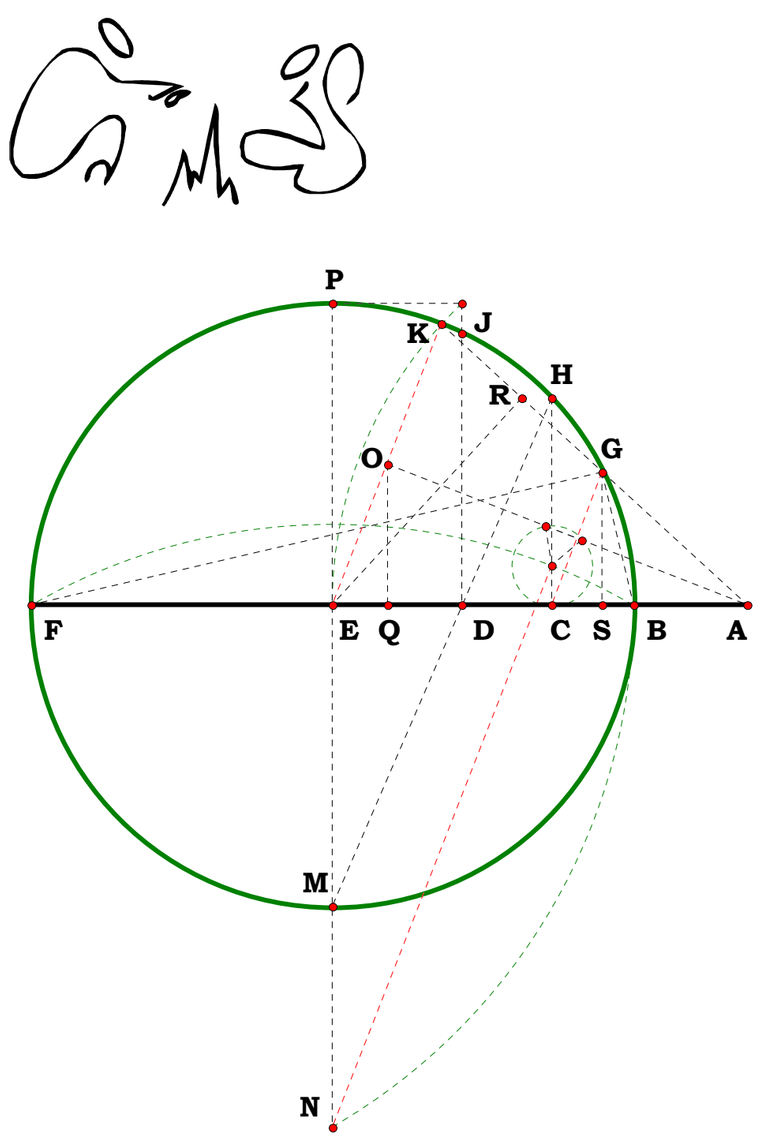
**m∠BFE = 25.64498°**

**m∠CFB = 25.64498°**

**m∠DFC = 25.64498°**

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

**m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°**




$$N_1 := 1.90557 \qquad N_2 := 10.10708$$

$$AB := N_1 \quad BF := N_2 \quad AF := AB + BF \quad AD := \sqrt{AB \cdot AF}$$

$$BD := AD - AB \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$$

$$BE := \frac{BF}{2} \quad EO := \frac{BE}{2} \quad AE := BE + AB \quad EQ := \frac{EO^2}{AE}$$

$$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \frac{BE \cdot BF}{DM}$$

$$DH := HM - DM \quad CD := \frac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$$

$$EN := \sqrt{BF^2 - BE^2} \quad KG := \frac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$$

$$CS := \frac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - AB$$

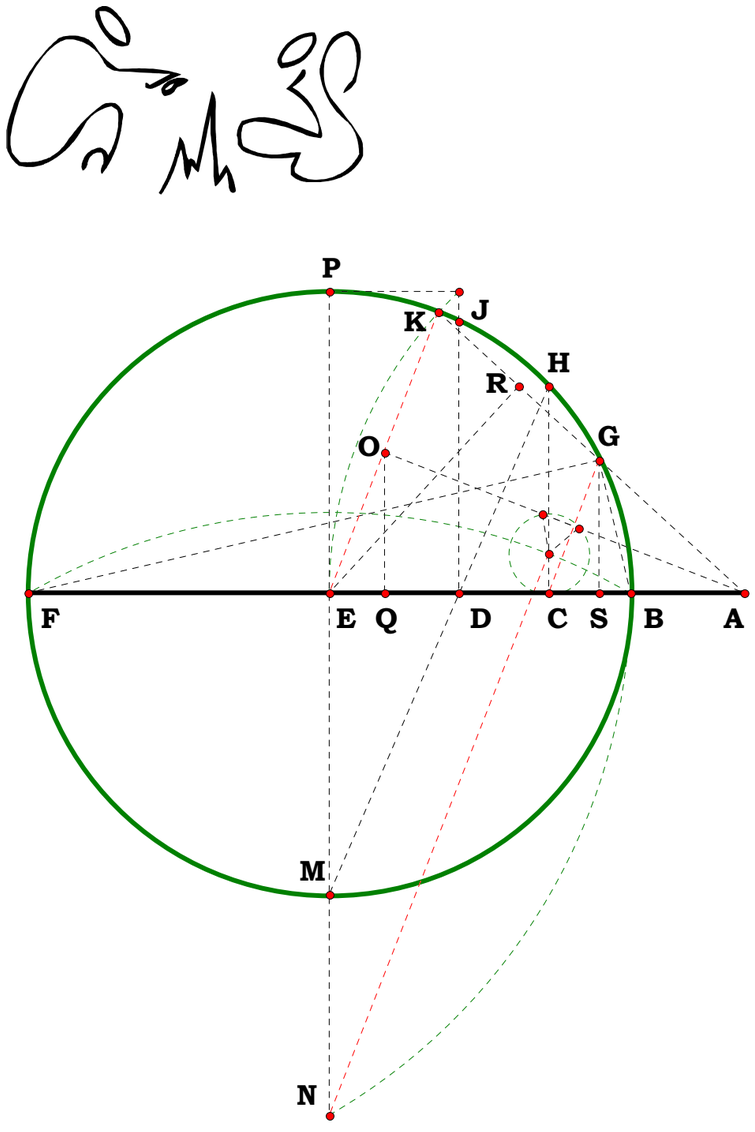
**Definitions:**

$$AB - N_1 = 0 \quad BF - N_2 = 0 \quad AF - (N_1 + N_2) = 0$$

$$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0 \quad BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right] \quad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$$

$$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0 \quad BE - \frac{N_2}{2} = 0 \quad EO - \frac{N_2}{4} = 0$$

$$AE - \frac{2 \cdot N_1 + N_2}{2} = 0 \quad EQ - \frac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0 \quad DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$$




$$DM - \frac{\sqrt{2\cdot(2\cdot N_1+N_2)^2-4\cdot\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}}{2}=0$$

$$HM - \frac{N_2^2}{\sqrt{2\cdot(2\cdot N_1+N_2)^2-4\cdot\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{N_1^2+N_2\cdot N_1}\cdot(2\cdot N_1+N_2)-2\cdot N_1^2-2\cdot N_1\cdot N_2\right]}{\sqrt{(2\cdot N_1+N_2)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot(N_1+N_2)}\right]}}=0$$

$$CD - \frac{\sqrt{N_1^2+N_2\cdot N_1}\cdot\left(8\cdot N_1^2+8\cdot N_1\cdot N_2+N_2^2\right)-4\cdot N_1\cdot(2\cdot N_1+N_2)\cdot(N_1+N_2)}{(2\cdot N_1+N_2)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1^2+N_2\cdot N_1}\right)}=0$$

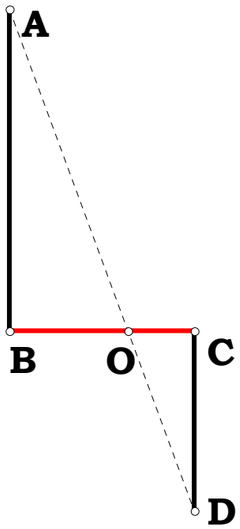
$$CE - \frac{N_2^2}{2\cdot(2\cdot N_1+N_2)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{N_2^2}}{2}=0$$

$$KG - \frac{N_2^2}{2\cdot(2\cdot N_1+N_2)}=0 \qquad AG - \frac{2\cdot N_1\cdot(N_1+N_2)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot N_2^2\cdot(N_1+N_2)}{(2\cdot N_1+N_2)^3}=0$$

$$AS - \frac{N_1\cdot(N_1+N_2)\cdot\left(8\cdot N_1^2+8\cdot N_1\cdot N_2+N_2^2\right)}{(2\cdot N_1+N_2)^3}=0$$

$$BS - \frac{N_1^2\cdot N_2\cdot(4\cdot N_1+3\cdot N_2)}{(2\cdot N_1+N_2)^3}=0$$

## 08/07/00 Proportion Series II




**Divide BC into the same ratio as AB:CD.**

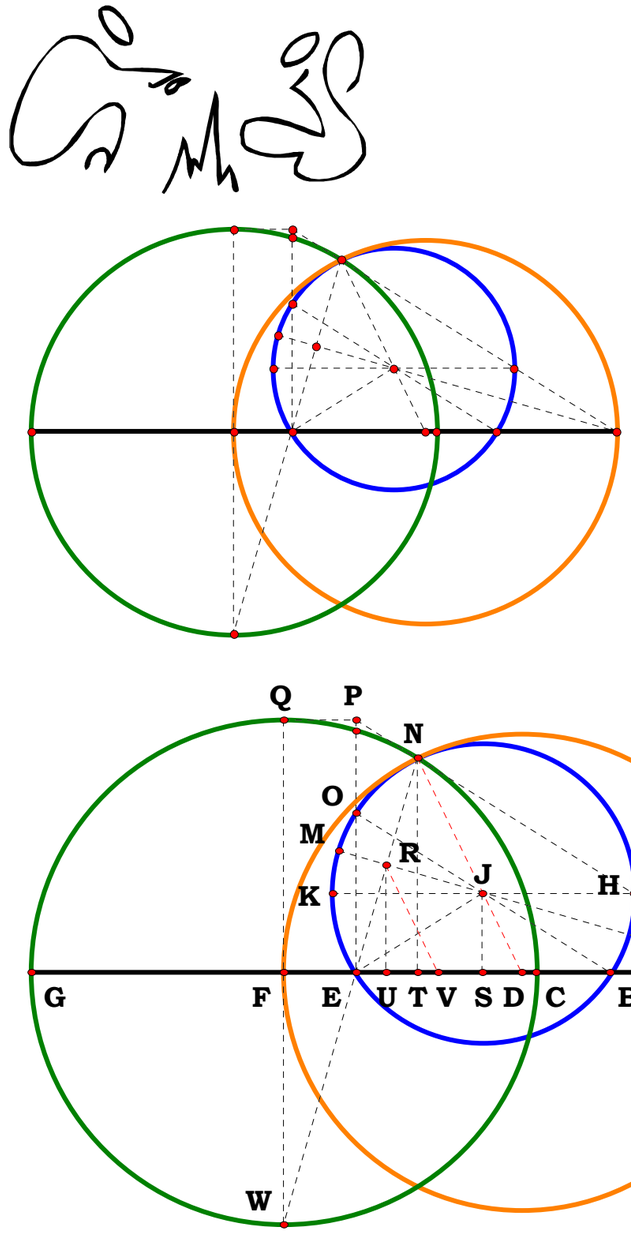
$$N_1 := 9 \qquad N_2 := 2 \qquad N_3 := 5$$

$$AB := N_1 \qquad CD := N_2 \qquad BC := N_3$$

$$BO := \frac{AB \cdot BC}{AB + CD} \qquad CO := \frac{CD \cdot BC}{AB + CD}$$

$$BO + CO - BC = 0$$

$$\frac{AB}{CD} - \frac{BO}{CO} = 0 \qquad BO - \frac{N_1 \cdot N_3}{N_1 + N_2} = 0 \qquad CO - \frac{N_2 \cdot N_3}{N_1 + N_2} = 0$$




**000822 Square Root and the Archimedean Paper Trisecter.**

**This square root figure affords another approach to proofing the Archimedean Paper Trisecter.**

$N_1 := 3.76152 \qquad N_2 := 12.22818$

$AC := N_1 \qquad AG := N_2 \qquad AE := \sqrt{AC \cdot AG} \qquad CG := AG - AC$

$CF := \frac{CG}{2} \qquad AF := AC + CF \qquad AD := \frac{AF}{2} \qquad CE := AE - AC$

$EF := CF - CE \qquad EW := \sqrt{EF^2 + CF^2} \qquad NW := \frac{CF \cdot CG}{EW}$

$EN := NW - EW \qquad EU := \frac{EF \cdot EN}{2 \cdot EW} \qquad AU := AE - EU$

$NT := \frac{CF \cdot EN}{EW} \qquad UV := \sqrt{\left(\frac{AD}{2}\right)^2 - \left(\frac{NT}{2}\right)^2} \qquad AV := AU - UV$

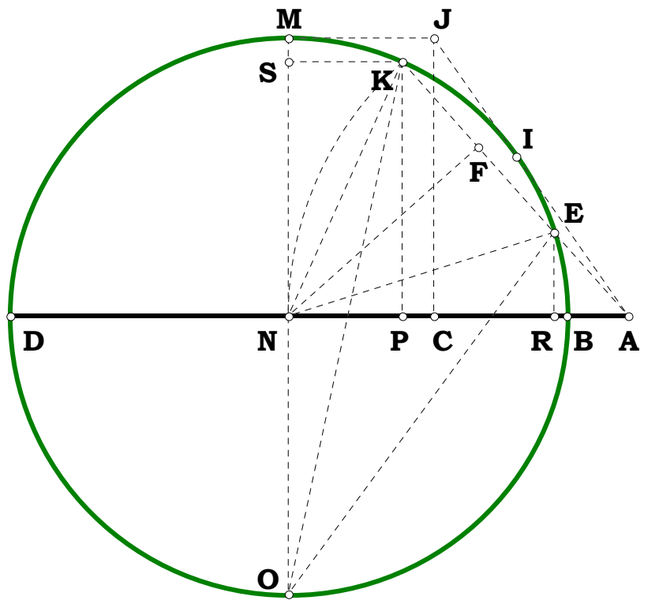
$JS := \frac{NT \cdot AD}{2 \cdot AV} \qquad DJ := \frac{AD \cdot JS}{NT} \qquad BJ := AD - DJ$

$DS := \frac{2 \cdot UV \cdot JS}{NT} \qquad AS := AD + DS \qquad BS := \sqrt{BJ^2 - JS^2} \qquad AB := AS - BS$

$AB - BJ = 0$

## 08/23/00 Trisection In A Square Root Figure

**Given the square root figure drawn for trisection, what is AR given AB and AD? A slightly different apprach than the one on 04.**




$$N := 4 \quad AB := 2 \quad AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD}$$

$$BD := AD - AB \quad BN := \frac{BD}{2} \quad KN := BN$$

$$AN := AB + BN \quad AK := AN \quad AP := \frac{AK^2 + AN^2 - KN^2}{2 \cdot AN}$$

$$AF := \frac{AP \cdot AN}{AK} \quad FK := AK - AF \quad EF := FK \quad AE := AK - 2 \cdot (EF)$$

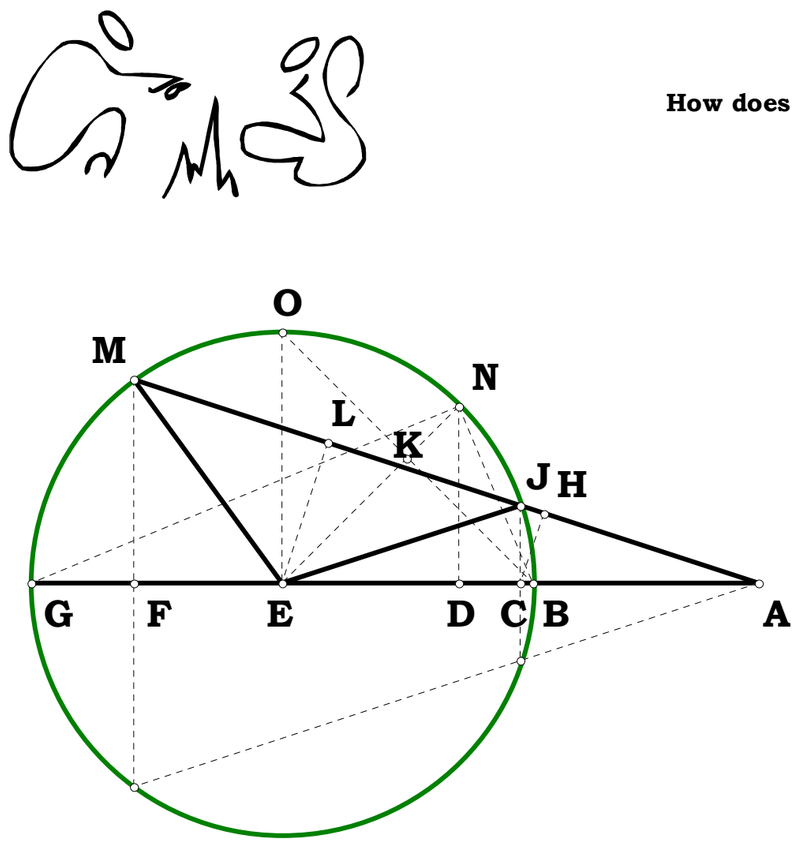
$$AR := \frac{AP \cdot AE}{AK} \quad AB \cdot N \cdot \frac{\left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} - AR = 0 \quad BR := AR - AB$$

$$AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} - BR = 0$$

**Does KS = FK?**

$$BP := AP - AB \quad DP := BD - BP \quad NP := BN - BP \quad KS := NP \quad KS - FK = 0$$

$$\frac{AB \cdot (N-1)^2}{4 \cdot (N+1)} - KS = 0$$




## 09/03/00 Ratios In Trisection

**How does BF vary with BC? How does DF vary with BC?**

$N_1 := 4 \quad N_2 := 8$

$BG := 1 \quad BE := \frac{BG}{2} \quad EM := BE \quad BO := \sqrt{2 \cdot BE^2} \quad EN := BE \quad EK := \frac{BE \cdot BE}{BO}$

$KN := EN - EK \quad BK := \frac{BO}{2} \quad BN := \sqrt{BK^2 + KN^2} \quad BD := \frac{BN^2}{BG} \quad BC := BD \cdot \frac{N_1}{N_2}$

$CG := BG - BC \quad CJ := \sqrt{BC \cdot CG} \quad AJ := BE \quad AC := \sqrt{AJ^2 - CJ^2} \quad AB := AC - BC$

$AE := AB + BE \quad JH := \frac{CJ^2}{AJ} \quad AH := AJ - JH \quad AL := \frac{AH \cdot AE}{AC} \quad JL := AL - AJ$

$LM := JL \quad AM := AL + LM \quad AF := \frac{AH \cdot AM}{AC} \quad BF := AF - AB \quad DF := BF - BD$

$$BF - \frac{1}{8} \cdot \left(7 \cdot \sqrt{2} - 10\right) \cdot \left(N_1 - 4 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{2}\right) \cdot \frac{\left(2 \cdot N_1 - 2 \cdot N_2 - N_2 \cdot \sqrt{2}\right)^2}{N_2^3} = 0$$

$$\frac{\left(3 - 2 \cdot \sqrt{2}\right) \cdot \left(2 \cdot N_2 - 2 \cdot N_1 + \sqrt{2} \cdot N_2\right)^2 \cdot \left(4 \cdot N_2 - N_1 + 2 \cdot \sqrt{2} \cdot N_2\right)}{2 \cdot N_1 \cdot N_2^2} - \frac{BF}{BC} = 0$$

$$\frac{N_1 \cdot \left(2 - \sqrt{2}\right)}{4 \cdot N_2} - BC = 0$$

$$\frac{\left(N_1 - N_2\right) \cdot \left[\left(7 \cdot \sqrt{2} - 10\right) \cdot \left(12 \cdot \sqrt{2} \cdot N_2^2 + 17 \cdot N_2^2 + 2 \cdot N_1^2 - 10 \cdot N_1 \cdot N_2 - 6 \cdot N_1 \cdot \sqrt{2} \cdot N_2\right)\right]}{4 N_2^3} - DF = 0$$

$$\frac{\left(N_1 - N_2\right) \cdot \left[\left(2 \cdot \sqrt{2} - 3\right) \cdot \left(2 \cdot N_1^2 - 10 \cdot N_1 \cdot N_2 - 6 \cdot N_1 \cdot \sqrt{2} \cdot N_2 + 17 \cdot N_2^2 + 12 \cdot \sqrt{2} \cdot N_2^2\right)\right]}{N_1 \cdot N_2^2} - \frac{DF}{BC} = 0$$

**Midpoints and Similarity Points 09/18/00**

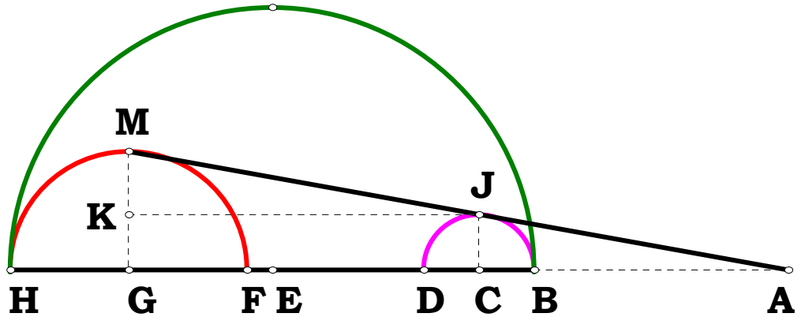
**What is AE given the radius of the two circles and the difference between their centers? (External Unit).**

$N_1 := 2 \quad N_2 := 3 \quad N_3 := 8 \quad BC := N_1 \quad GH := N_2 \quad CG := N_3$

$CJ := BC \quad GM := GH \quad GK := CJ \quad BH := BC + CG + GH \quad BE := \frac{BH}{2}$

$KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH$

$AB := AH - BH \quad AE := AB + BE \quad AE - \frac{N_3 \cdot N_2 - 2 \cdot N_1 \cdot N_2 + N_1^{\,2} + N_3 \cdot N_1 + N_2^{\,2}}{2 \cdot (N_2 - N_1)} = 0$




**What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).**

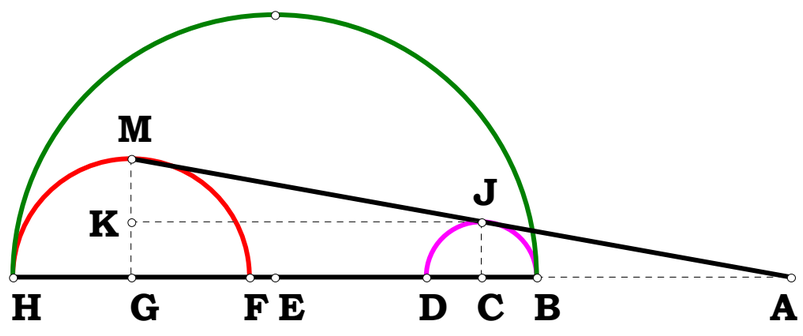
$N_1 := 3 \quad N_2 := 7 \quad N_3 := 4 \quad N_4 := 9$

$BH := 1 \quad BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2} \quad GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$

$CJ := BC \quad GM := GH \quad GK := CJ \quad KM := GM - GK \quad JK := CG \quad AG := \frac{JK \cdot GM}{KM}$

$AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$

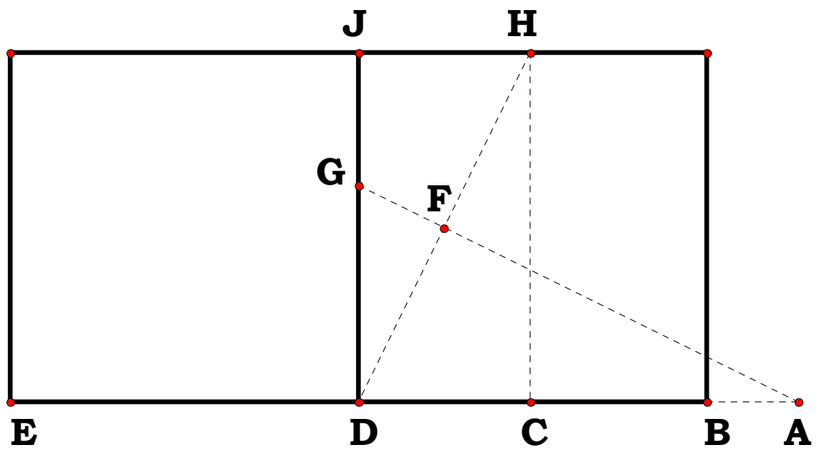
$AE - \frac{(N_3 \cdot N_2 - 4 \cdot N_3 \cdot N_1 + N_1 \cdot N_4)}{2 \cdot (N_3 \cdot N_2 - N_1 \cdot N_4)} = 0$



**000920 Squaring**

**Is AC the square root of AB x AE?**

**Given BC, find AB such that AB x AE is the square root.**




$N_1 := 11.69458 \quad N_2 := 2.96916$

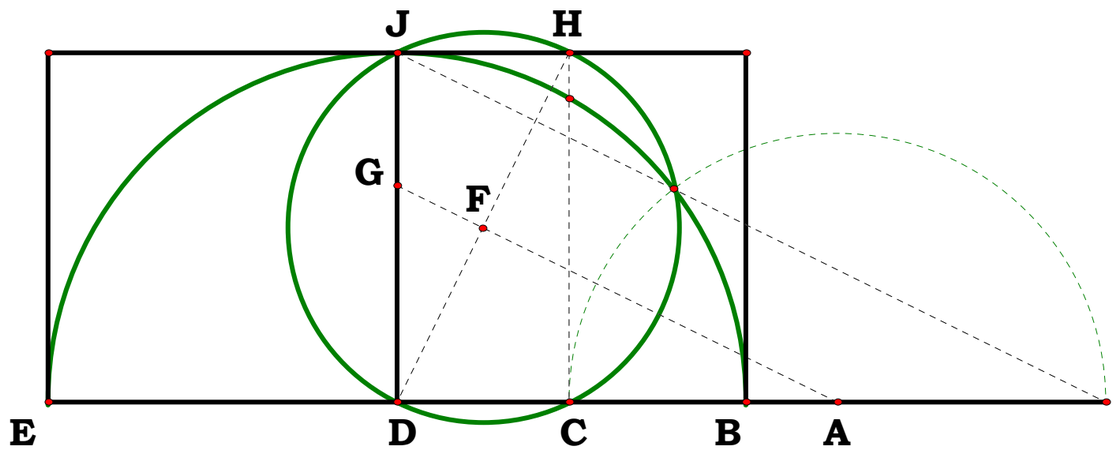
$BE := N_1 \quad BC := N_2 \quad BD := \frac{BE}{2} \quad CD := BD - BC$

$CH := BD \quad DH := \sqrt{BD^2 + CD^2} \quad DG := \frac{DH^2}{2BD}$

$AD := \frac{BD \cdot DG}{CD} \quad AE := AD + BD \quad AB := AE - BE$

$AC := BC + AB$

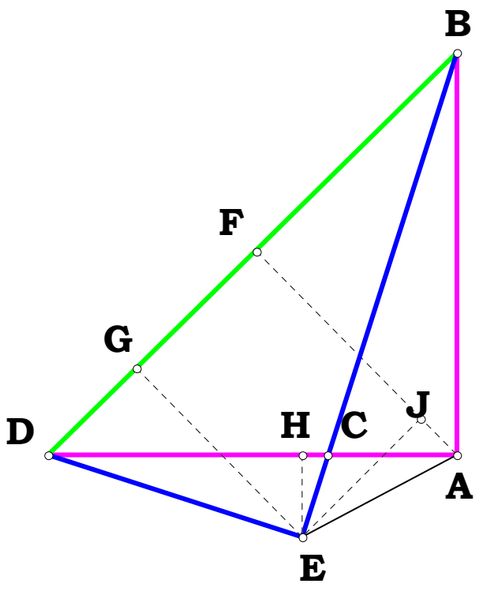
$AC - \sqrt{AB \cdot AE} = 0$



**11/13/00 For Two Right Triangles.**

**Given AB, DE, AD find BE, AC, CD, CE, BC.**

**BAD and BED are right.**




$N_1 := 3 \quad N_2 := 5 \quad N_3 := 1$

$AB := N_1 \quad AD := N_2 \quad DE := N_3$

$BD := \sqrt{AB^2 + AD^2} \quad BF := \frac{AB^2}{BD} \quad DG := \frac{DE^2}{BD} \quad BE := \sqrt{BD^2 - DE^2}$

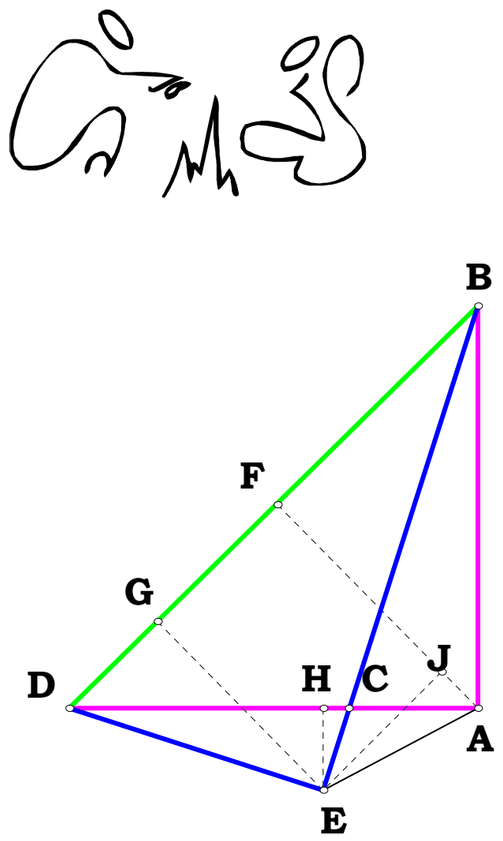
$AF := \sqrt{AB^2 - BF^2} \quad EG := \sqrt{DE^2 - DG^2} \quad FG := BD - (BF + DG)$

$EJ := FG \quad FJ := EG \quad AJ := AF - FJ \quad AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD \quad S_2 := DE \quad S_3 := AE \quad AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$EH := \sqrt{AE^2 - AH^2} \quad CH := \frac{EH \cdot AH}{AB + EH} \quad AC := AH - CH$

$CE := \frac{AC \cdot DE}{AB} \quad CD := AD - AC \quad BC := BE - CE$




**Definitions:**

$$BE - \sqrt{N_1^2 + N_2^2 - N_3^2} = 0$$

$$\frac{N_1 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - AC = 0$$

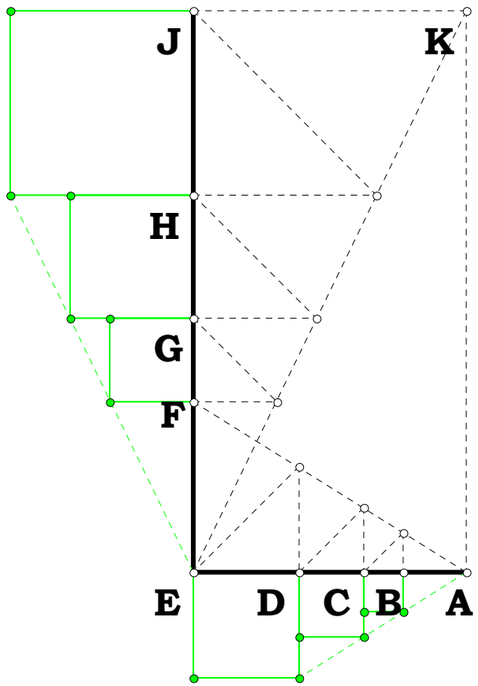
$$N_2 + \frac{N_1 \cdot N_2 \cdot N_3^2 - N_2 \cdot \left(N_1^3 + N_1 \cdot N_2^2\right) + N_1^2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CD = 0$$

$$\frac{N_3 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CE = 0$$

$$BC - \left(\sqrt{N_1^2 + N_2^2 - N_3^2} + \frac{N_2 \cdot N_3^3 - N_2^3 \cdot N_3 - N_1^2 \cdot N_2 \cdot N_3 + N_1 \cdot N_3^2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}}\right) = 0$$

**Means On Means 11/28/00**

**Modify 02/28/98 for Mean proportionals between E and J.**




$$AE := 1 \quad N := 3 \quad EJ := AE \cdot N \quad JK := AE$$

$$HJ := \frac{JK \cdot EJ}{JK + EJ} \quad EH := EJ - HJ \quad GH := \frac{EH \cdot HJ}{EH + HJ}$$

$$EG := EH - GH \quad FG := \frac{EG \cdot GH}{EG + GH} \quad EF := EG - FG$$

$$DE := \frac{EF \cdot AE}{EF + AE} \quad AD := AE - DE \quad CD := \frac{AD \cdot DE}{AD + DE}$$

$$AC := AD - CD \quad BC := \frac{AC \cdot CD}{AC + CD} \quad AB := AC - BC$$

$$M := 0 .. 3 \quad P := 0 .. 3 \quad AEAB_{M,P} := \left[ \frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$$

$$AEAB = \begin{pmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.5625 & 34.328125 \\ 1 & 2.6875 & 7.222656 & 19.410889 \\ 1 & 2.265625 & 5.133057 & 11.629581 \end{pmatrix}$$

$$AEAB_{3,3} - \frac{AE}{AB} = 0 \quad AEAB_{3,2} - \frac{AE}{AC} = 0 \quad AEAB_{3,1} - \frac{AE}{AD} = 0 \quad AEAB_{3,0} - \frac{AE}{AE} = 0$$

## Multiplication and Division-Line By A Line 11/29/00

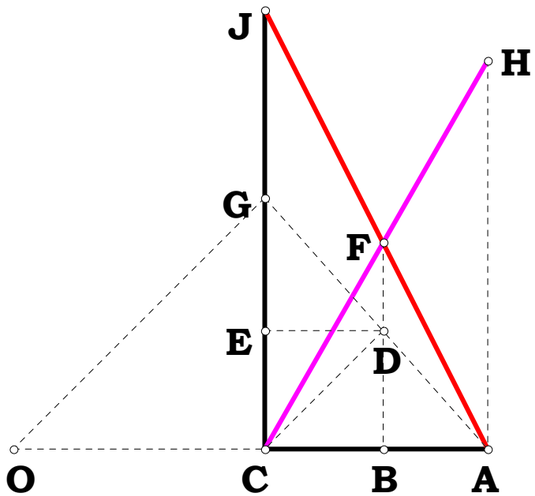
**Given some unit, and two differences, multiply or divide the one difference by the other.**
**For Division:**

$$AC := 1 \quad N_1 := 3 \quad N_2 := 12 \quad AH := N_1$$

$$CJ := N_2 \quad AB := \frac{AH}{(CJ + AH)} \cdot AC$$

$$BC := AC - AB \quad BD := BC \quad CG := \frac{BD \cdot AC}{AB}$$

$$CG - \frac{N_2}{N_1} = 0 \quad CG = 4$$




**For Multiplication:**

$$AC := 1 \quad N_1 := 5 \quad N_2 := 7 \quad AH := N_1$$

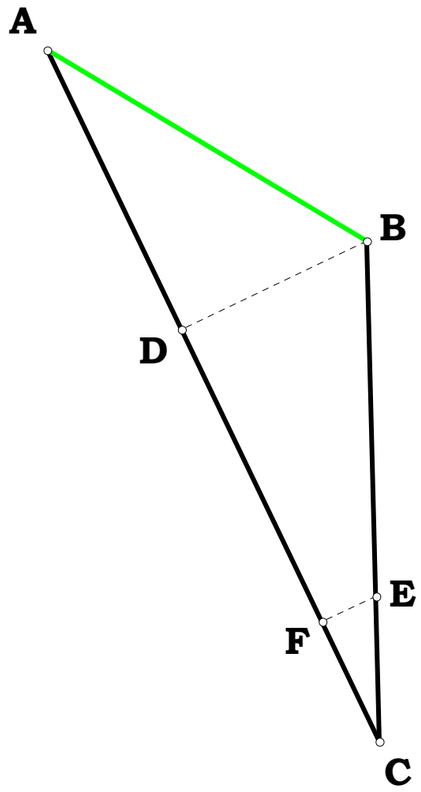
$$CG := N_2 \quad CO := CG \quad BD := \frac{CG \cdot AC}{AC + CO}$$

$$BC := BD \quad AB := AC - BC \quad BF := \frac{AH \cdot BC}{AC}$$

$$CJ := BF \cdot \frac{AC}{AB} \quad CJ - N_1 \cdot N_2 = 0 \quad CJ = 35$$

120500

**From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.**




$$N_1 := 5 \quad N_2 := 25 \quad N_3 := 1 \quad N_4 := .5$$

$$BC := N_1 \quad AC := N_2 \quad CE := N_3 \quad EF := N_4$$

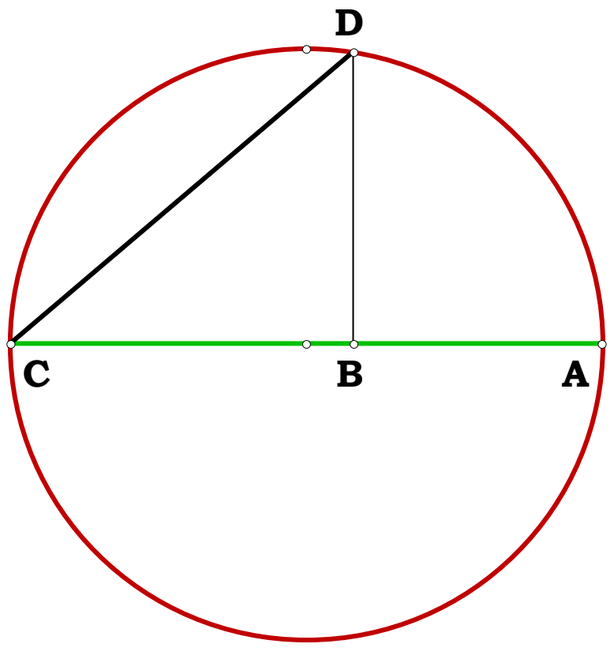
$$BD := \frac{EF \cdot BC}{CE} \quad CF := \sqrt{CE^2 - EF^2} \quad CD := \frac{CF \cdot BC}{CE}$$

$$AD := AC - CD \quad AB := \sqrt{BD^2 + AD^2}$$

$$AB - \frac{\sqrt{N_1^2 \cdot N_3 + N_2^2 \cdot N_3 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2}}}{\sqrt{N_3}} = 0$$

**010101 Square Root, common segment common endpoint.**

**Alternate method for common segment common endpoint square root.** $\sqrt{AC \cdot BC} = CD$



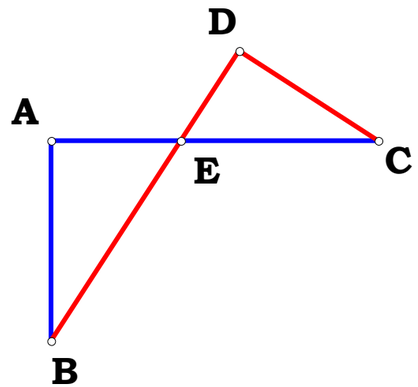

$N_1 := 5 \quad N_2 := 3 \quad AC := N_1 \quad BC := N_2$

$AB := AC - BC \quad BD := \sqrt{AB \cdot BC} \quad CD := \sqrt{BD^2 + BC^2}$

$\sqrt{N_2 \cdot N_1} - CD = 0$

## Three Given Five Taken 042101

**Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?**




$N_1 := 2 \quad N_2 := 3 \quad N_3 := 4$

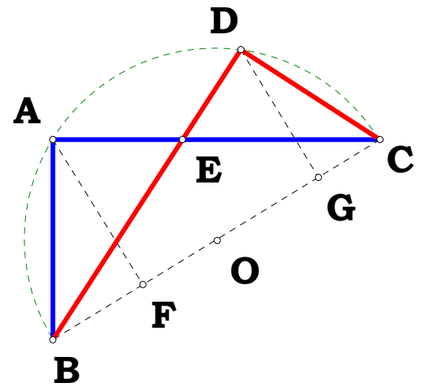
$AB := N_1 \quad CD := N_2 \quad AC := N_3$

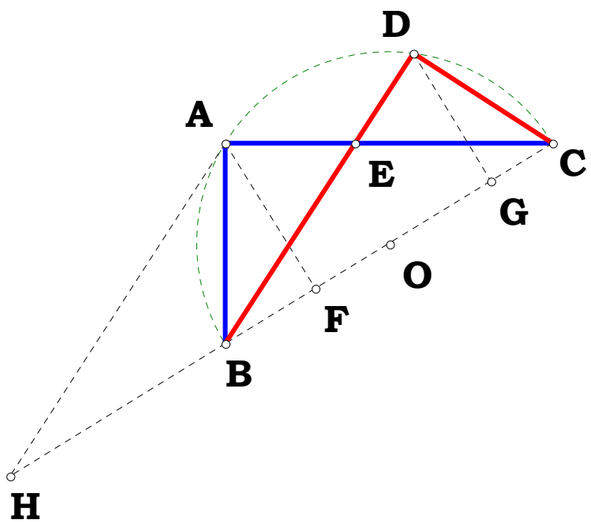
$BC := \sqrt{AB^2 + AC^2} \quad CG := \frac{CD^2}{BC} \quad BF := \frac{AB^2}{BC}$

$BG := BC - CG \quad CF := BC - BF$

$AF := \sqrt{AB^2 - BF^2} \quad DG := \sqrt{CD^2 - CG^2}$

$FH := \frac{BG \cdot AF}{DG} \quad CH := CF + FH$



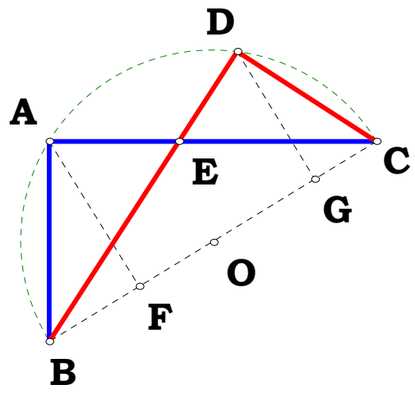




$BD := \sqrt{BG^2 + DG^2}$

$AH := \frac{BD \cdot FH}{BG} \quad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \quad CE := \frac{AC \cdot BC}{CH}$

$AE := AC - CE$




**Some Algebraic Names:**

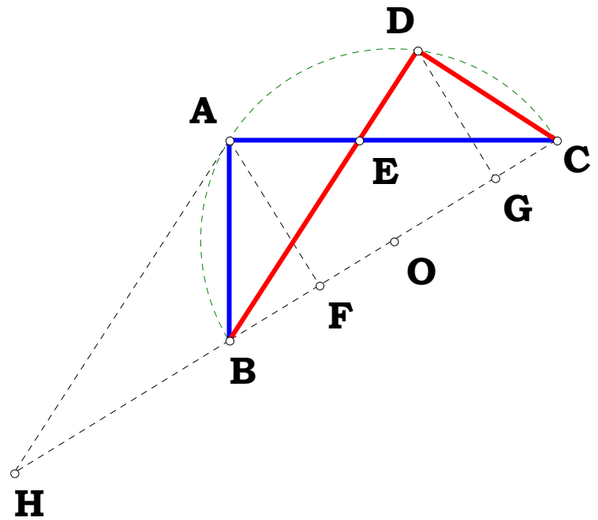
$$\sqrt{N_1^2 + N_3^2} - BC = 0 \qquad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$$

$$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0 \qquad \frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0$$

$$\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0$$

$$\frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$$

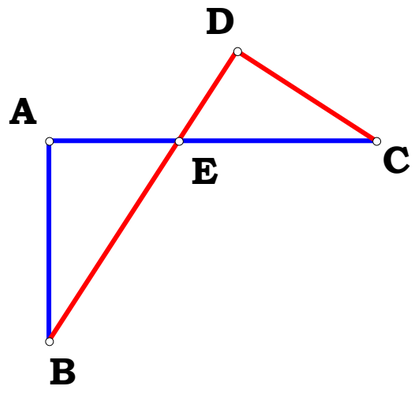
$$N_2 \cdot \sqrt{\frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\left(N_1^2 + N_3^2\right)}} - DG = 0$$




$$\frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0$$

$$\left[\frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} + \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}}\right] - CH = 0 \qquad N_3 \cdot \frac{N_1}{N_2} - AH = 0$$

**The Five Sought:**



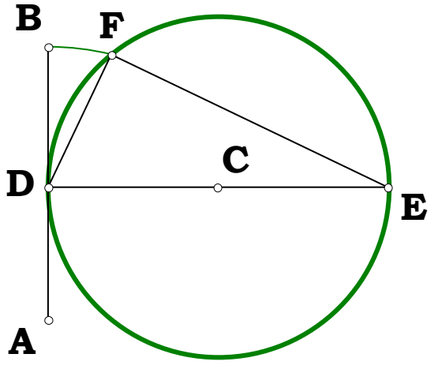

$$\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - DE = 0$$

$$N_1 \cdot \frac{\left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - BE = 0$$

$$\frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1} - CE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - AE = 0$$




042201

**Given AB as unit, AD and DC,**
**what is EF and DF?**

$N_1 := 2.052 \quad N_2 := .62 \quad AB := 1.802$

$AD := \frac{AB}{N_1} \quad CD := AB \cdot N_2 \quad DE := 2CD \quad AF := AB \quad CF := CD$

$AC := \sqrt{AD^2 + CD^2} \quad S_1 := AF \quad S_2 := AC \quad S_3 := CF$

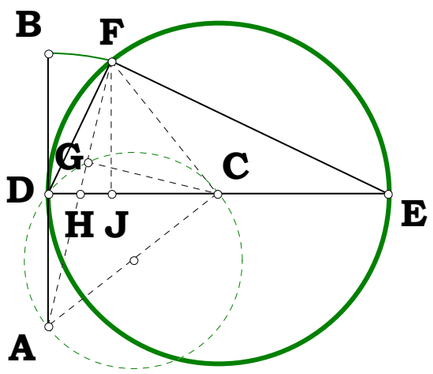
$AG := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CG := \sqrt{AC^2 - AG^2}$

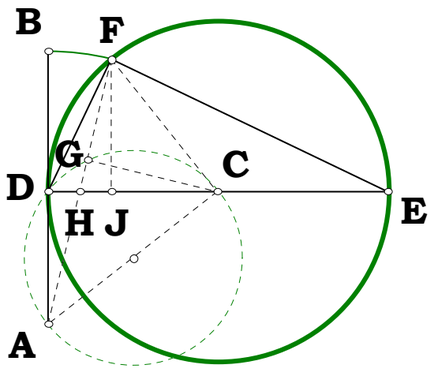
$L_1 := AD \quad L_2 := CG \quad L_3 := CD$

$DH := L_3 - \frac{L_2 \cdot \left(L_1^2 + L_3^2\right)}{L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1}$

$AH := L_1 \cdot \frac{\left(L_1^2 + L_3^2\right)}{\left(L_3 \cdot L_2 + \sqrt{L_1^2 + L_3^2 - L_2^2} \cdot L_1\right)} \quad FH := AF - AH \quad HJ := \frac{DH \cdot FH}{AH}$

$DJ := DH + HJ \quad EJ := DE - DJ \quad FJ := \frac{AD \cdot FH}{AH} \quad EF := \sqrt{FJ^2 + EJ^2} \quad DF := \sqrt{DJ^2 + FJ^2}$






**Some Algebraic Names:**

$$\frac{AB}{N_1} - AD = 0 \qquad AB \cdot N_2 - CD = 0$$

$$(2AB) \cdot N_2 - DE = 0 \qquad AB \cdot \frac{\sqrt{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}}{N_1} - AC = 0$$

$$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^{\,2}\right)}{N_1^{\,2}} - AG = 0$$

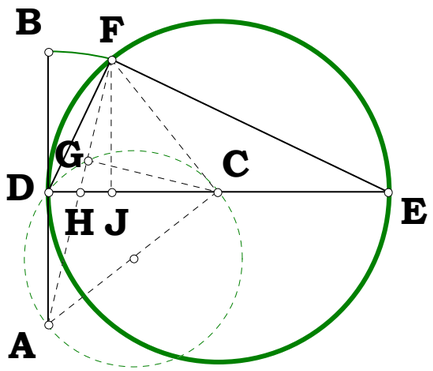
$$AB \cdot \frac{\sqrt{\left(2 \cdot N_1^{\,2} \cdot N_2 + N_1^{\,2} - 1\right) \cdot \left(2 \cdot N_1^{\,2} \cdot N_2 - N_1^{\,2} + 1\right)}}{2 \cdot N_1^{\,2}} - CG = 0$$

$$AB \cdot \frac{N_2 \cdot N_1^{\,3} + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}}}{N_1^{\,3} + N_1^{\,2} \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + N_1} - DH = 0$$

$$2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - AH = 0$$

$$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^{\,2} \cdot N_1^{\,2}\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^{\,2} + 4 \cdot N_1^{\,4} \cdot N_2^{\,2} - 1 - N_1^{\,4}} + 1 + N_1^{\,2}\right)} - FH = 0$$

$$\frac{-1}{2}\cdot AB\cdot\left(\frac{6\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2\cdot N_2{}^2+1}{N_1{}^2\cdot N_2+N_1{}^4\cdot N_2+N_1{}^3\cdot N_2{}^2\cdot\sqrt{4\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2-1}}-\frac{N_1{}^4\cdot N_2{}^2+3\cdot N_1{}^2\cdot N_2{}^2-N_1{}^2+1}{N_1{}^4\cdot N_2{}^3+N_1{}^2\cdot N_2}\right)-HJ=0$$




$$\frac{1}{2}\cdot AB\cdot\frac{N_1{}^3\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2-1}}{N_1\cdot\left(N_1{}^2\cdot N_2{}^2+1\right)}-DJ=0$$

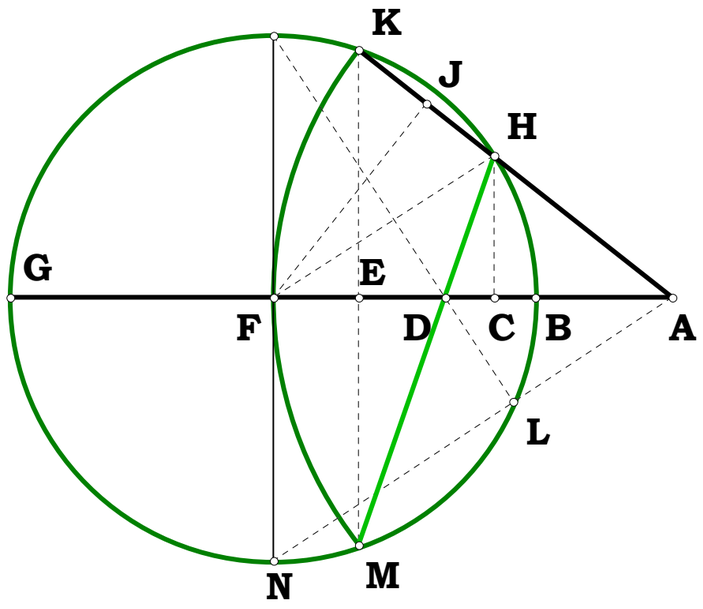
$$\frac{1}{2}\cdot AB\cdot\frac{4\cdot N_1{}^3\cdot N_2{}^3-N_1{}^3\cdot N_2+3\cdot N_1\cdot N_2+\sqrt{4\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2-1}}{N_1\cdot\left(N_1{}^2\cdot N_2{}^2+1\right)}-EJ=0$$

$$\frac{-1}{2}\cdot AB\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1{}^2+4\cdot N_1{}^4\cdot N_2{}^2-1-N_1{}^4}+1-N_1{}^2+2\cdot N_2{}^2\cdot N_1{}^2\right)}{\left[N_1\cdot\left(1+N_2{}^2\cdot N_1{}^2\right)\right]}-FJ=0$$

$$AB\cdot\frac{\sqrt{N_2\cdot\left[N_1\cdot N_2\cdot\left(4\cdot N_1{}^2\cdot N_2{}^2-N_1{}^2+3\right)+\sqrt{4\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2-1}\right]}}{\sqrt{N_1{}^3\cdot N_2{}^2+N_1}}-EF=0$$

$$AB\cdot\frac{\sqrt{\left[N_2\cdot\left(N_1{}^3\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1{}^4\cdot N_2{}^2-N_1{}^4+2\cdot N_1{}^2-1}\right)\right]}}{\sqrt{N_1\cdot\left(N_1{}^2\cdot N_2{}^2+1\right)}}-DF=0$$

042401




**Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?**

$$N := 5 \qquad AB := 1 \qquad AG := AB \cdot N$$

$$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF \qquad AK := AF$$

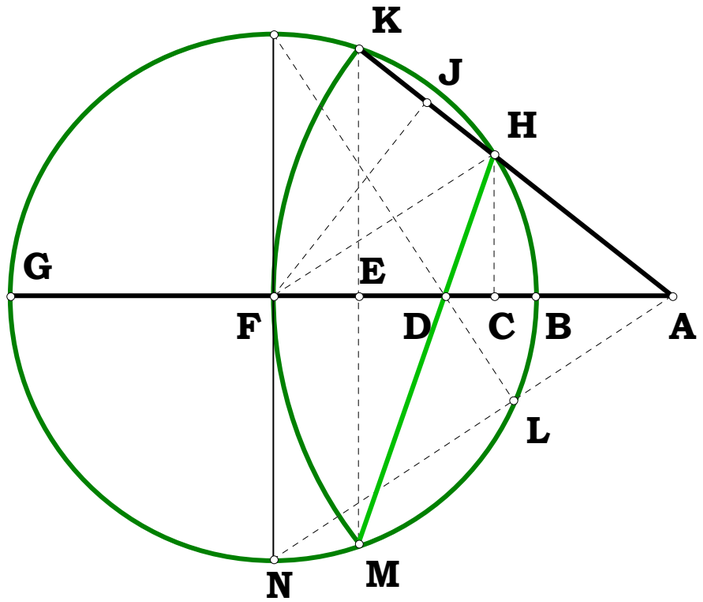
$$FK := BF \qquad AE := \frac{2AK^2 - FK^2}{2AF} \qquad AJ := AE \qquad JK := AK - AJ \qquad HJ := JK$$

$$AH := AK - (JK + HJ) \qquad AC := \frac{AE \cdot AH}{AK} \qquad CE := AE - AC$$

$$BE := AE - AB \qquad EG := BG - BE \qquad EK := \sqrt{BE \cdot EG} \qquad BC := AC - AB$$

$$CG := BG - BC \qquad CH := \sqrt{BC \cdot CG} \qquad DE := \frac{CE \cdot EK}{EK + CH}$$

$$DF := 2 \cdot DE \qquad HM := \sqrt{CE^2 + (EK + CH)^2}$$




**Some Algebraic Names:**

$$AB\cdot N - AB - BG = 0 \qquad \frac{1}{2}\cdot AB\cdot N - \frac{1}{2}\cdot AB - BF = 0$$

$$\frac{1}{2}\cdot AB + \frac{1}{2}\cdot AB\cdot N - AF = 0 \qquad \frac{1}{4}\cdot AB\cdot \frac{\left(N^2 + 6\cdot N + 1\right)}{(1+N)} - AE = 0$$

$$\frac{1}{4}\cdot AB\cdot \frac{\left(1 - 2\cdot N + N^2\right)}{(1+N)} - JK = 0 \qquad 2\cdot AB\cdot \frac{N}{(1+N)} - AH = 0$$

$$AB\cdot \frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^3}\cdot N - AC = 0 \qquad \frac{1}{4}\cdot AB\cdot \left(N^2 + 6\cdot N + 1\right)\cdot \frac{(N-1)^2}{(1+N)^3} - CE = 0$$

$$\frac{1}{4}\cdot AB\cdot (N+3)\cdot \frac{(N-1)}{(1+N)} - BE = 0 \qquad \frac{1}{4}\cdot AB\cdot (3\cdot N+1)\cdot \frac{(N-1)}{(1+N)} - EG = 0$$

$$AB\cdot (3\cdot N+1)\cdot \frac{(N-1)}{(1+N)^3} - BC = 0 \qquad \frac{1}{4}\cdot AB\cdot \sqrt{(N+3)\cdot (3\cdot N+1)}\cdot \frac{(N-1)}{(1+N)} - EK = 0$$

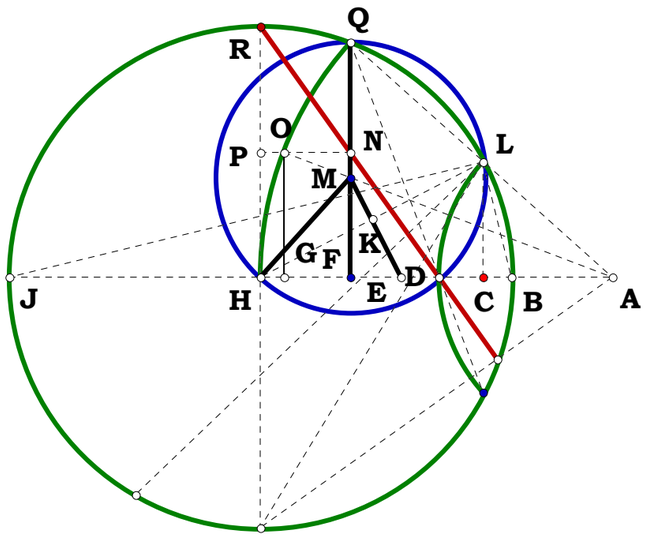
$$AB\cdot N^2\cdot (N+3)\cdot \frac{(N-1)}{(1+N)^3} - CG = 0 \qquad AB\cdot \sqrt{(N+3)\cdot (3\cdot N+1)}\cdot N\cdot \frac{(N-1)}{(1+N)^3} - CH = 0$$

$$\frac{1}{4}\cdot AB\cdot \frac{(N-1)^2}{(1+N)} - DE = 0 \qquad \frac{1}{2}\cdot AB\cdot \frac{(N-1)^2}{(N+1)} - DF = 0$$

$$\frac{1}{2}\cdot AB\cdot (N-1)\cdot \frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^2} - HM = 0$$

042501

**What is the Algebraic name of the circle HM? Does point N divide DR in half?**




$$N := 5.768 \quad AB := .583 \quad AJ := AB \cdot N$$

$$BJ := AJ - AB \quad BH := \frac{BJ}{2} \quad HR := BH$$

$$HP := \frac{HR}{2} \quad GO := HP \quad AH := AB + BH$$

$$AO := AH \quad AG := \sqrt{AO^2 - GO^2}$$

$$HQ := BH \quad AQ := AH \quad FH := \frac{HQ^2}{2 \cdot AH}$$

$$AF := AH - FH \quad FM := \frac{GO \cdot AF}{AG}$$

$$HJ := BH \quad FJ := FH + HJ \quad BF := BJ - FJ$$

$$FQ := \sqrt{BF \cdot FJ} \quad MQ := FQ - FM \quad HM := \sqrt{FH^2 + FM^2} \quad HM - MQ = 0$$

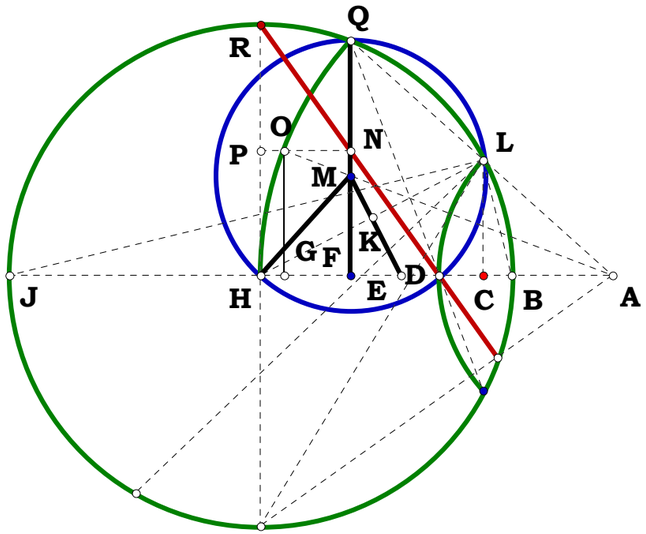
$$DH := \frac{HR^2}{AH} \quad \frac{DH}{2} - FH = 0$$

**Some Algebraic Names:**

$$AB \cdot N - AB - BJ = 0 \quad \frac{1}{2} \cdot AB \cdot (N - 1) - BH = 0 \quad \frac{1}{4} \cdot AB \cdot (N - 1) - HP = 0$$

$$\frac{1}{2} \cdot AB \cdot (1 + N) - AH = 0 \quad \frac{1}{4} \cdot AB \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - AG = 0 \quad \frac{1}{4} \cdot AB \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$$

$$\frac{1}{4} \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - AF = 0 \quad \frac{1}{4} \cdot (N - 1) \cdot AB \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} - FM = 0$$




$$\frac{1}{4}\cdot AB\cdot(N+3)\cdot\frac{(N-1)}{(1+N)}-BF=0 \qquad \frac{1}{4}\cdot AB\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)}-FJ=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)}-FQ=0$$

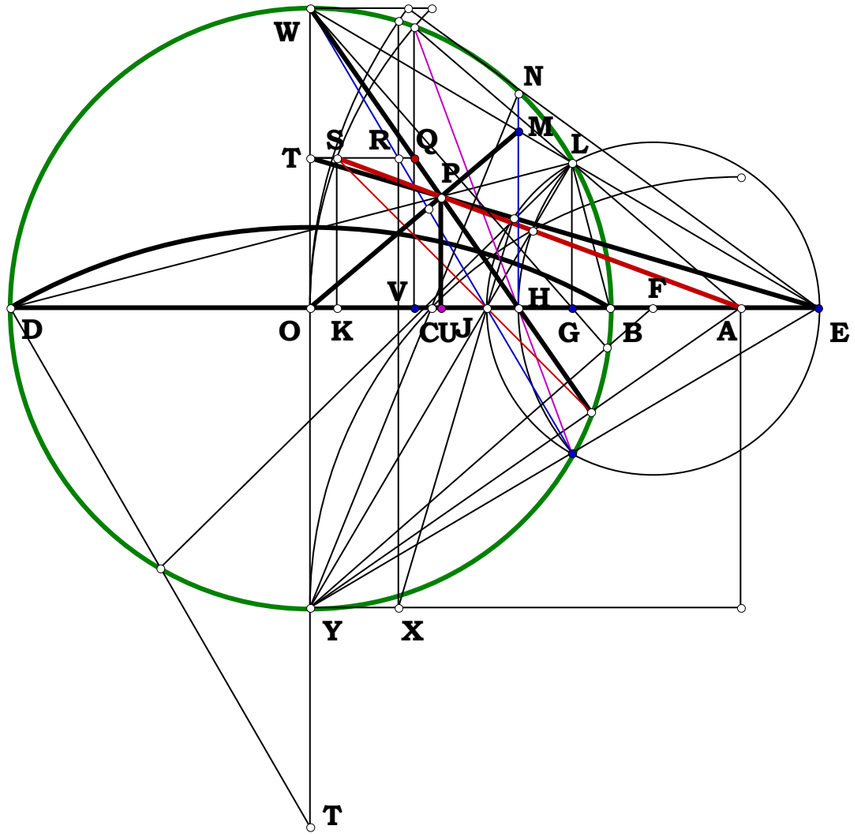
$$\frac{1}{2}\cdot(1+N)\cdot AB\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-MQ=0 \qquad \frac{1}{2}\cdot AB\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-HM=0$$

$$HM-MQ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-DH=0 \qquad \frac{DH}{2}-FH=0$$

## Four Lines To A Point 042901

**Does the difference OU and PU each have but one Algebraic name?**




$N := 5 \qquad AB := 1 \qquad AD := AB \cdot N$

$BD := AD - AB \qquad BO := \frac{BD}{2} \qquad OW := BO$

$OY := BO \qquad DO := BO \qquad AO := AB + BO$

$HO := \frac{OY^2}{AO} \qquad AH := AO - HO \qquad AL := AH$

$OL := BO \qquad GO := \frac{OL^2 + AO^2 - AL^2}{2AO}$

$BG := BO - GO \qquad DG := GO + DO$

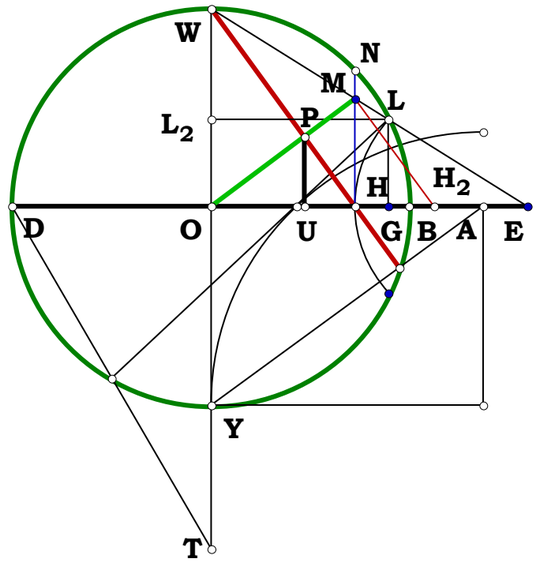
$GL := \sqrt{BG \cdot DG} \qquad EO := \frac{GO \cdot OW}{OW - GL}$

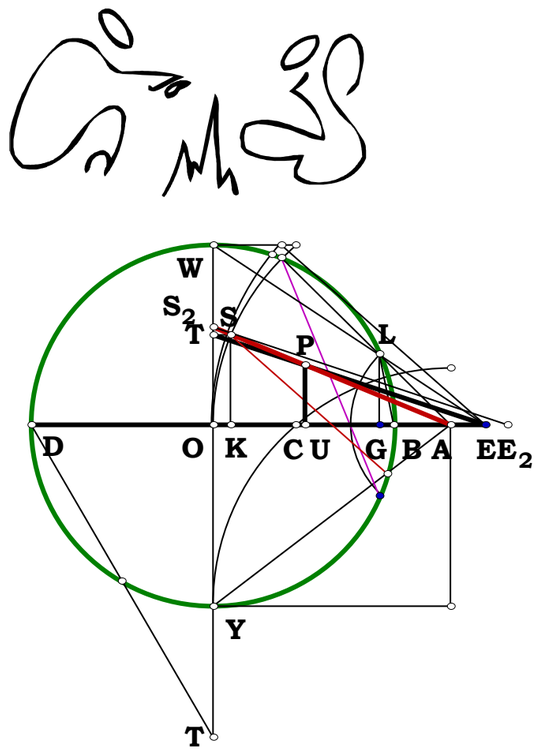
$BE := EO - BO \qquad BH := BO - HO$

$EG := BE + BG \qquad EH := BE + BH$

$HM := \frac{GL \cdot EH}{EG} \qquad HH_2 := \frac{HO \cdot HM}{OW} \qquad OU := \frac{HO^2}{HH_2 + HO}$

$UP := \frac{HM \cdot OU}{HO}$






$$OT := \frac{OW}{2} \quad KS := OT \quad AS := AO$$

$$AK := \sqrt{AS^2 - KS^2} \quad OS_2 := \frac{KS \cdot AO}{AK}$$

$$OE_2 := \frac{EO \cdot OS_2}{OT} \quad AS_2 := \sqrt{AO^2 + OS_2^2}$$

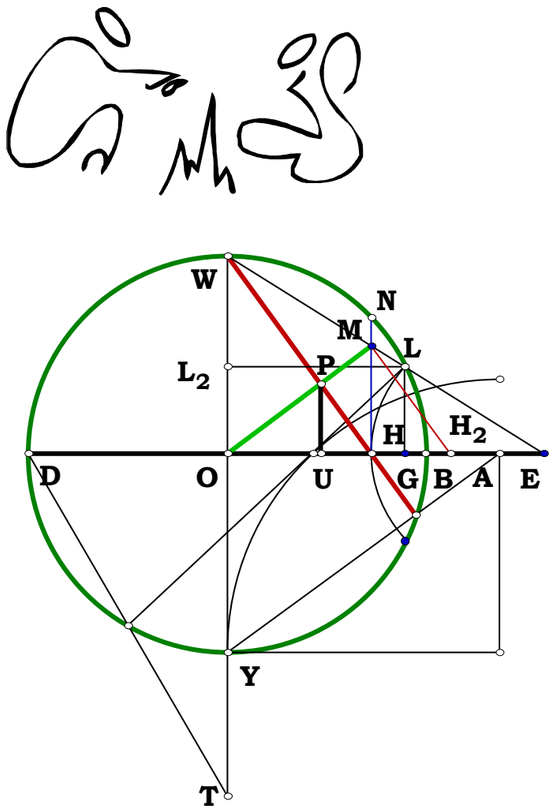
$$AE_2 := OE_2 - AO \quad AE := EO - AO$$

$$AP := \frac{AS_2 \cdot AE}{AE_2} \quad AU := \frac{AO \cdot AE}{AE_2}$$

$$UO := AO - AU \quad PU := \frac{OS_2 \cdot AU}{AO}$$

$$UO - OU = 0$$

$$PU - UP = 0$$




**Some Algebraic Names:**

$$AB\cdot(N-1)-BD=0 \qquad \frac{AB\cdot(N-1)}{2}-BO=0 \qquad \frac{1}{2}\cdot AB\cdot(1+N)-AO=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)^2}{(1+N)}-HO=0 \qquad 2\cdot AB\cdot\frac{N}{(1+N)}-AH=0 \qquad \frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3}-GO=0$$

$$AB\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3}-BG=0 \qquad AB\cdot(N-1)\cdot N^2\cdot\frac{(N+3)}{(1+N)^3}-DG=0$$

$$\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^3}-GL=0$$

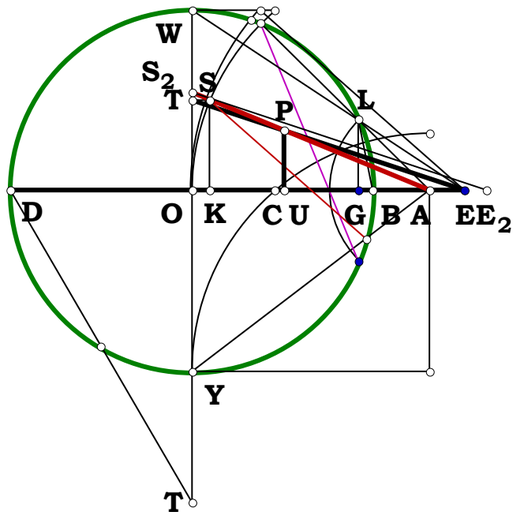
$$\frac{1}{2}\cdot AB\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-EO=0$$

$$AB\cdot(N-1)\cdot\frac{\left[\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N-3\cdot N-1\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-BE=0 \qquad AB\cdot\frac{(N-1)}{(1+N)}-BH=0$$

$$AB\cdot\frac{(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\left(N^2+4\cdot N+1\right)}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)^3\right]}-EG=0$$

$$AB\cdot(N-1)^2\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot(1+N)\right]}-EH=0 \qquad N\cdot(N-1)\cdot\frac{AB}{(1+N)}\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HM=0$$

$$AB\cdot\frac{(N-1)^2}{(1+N)^2}\cdot N\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left(N^2+4\cdot N+1\right)}-HH_2=0 \qquad \frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-OU=0$$




$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UP=0$$

$$\frac{1}{4}\cdot AB\cdot(N-1)-OT=0\qquad\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-AK=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{(N-1)\cdot(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OS_2=0$$

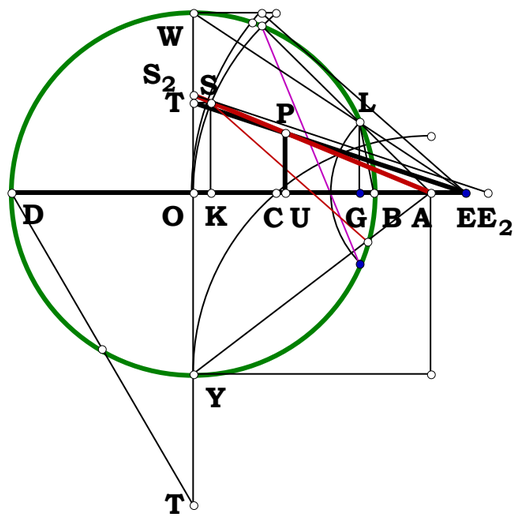
$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-OE_2=0$$

$$(1+N)^2\cdot\frac{AB}{\sqrt{3+10\cdot N+3\cdot N^2}}-AS_2=0$$

$$\frac{\left[AB\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}}-\frac{1}{2}\cdot AB\cdot(1+N)-AE_2=0$$

$$-AB\cdot N\cdot\frac{\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}{\left[3\cdot N+1+3\cdot N^2+N^3-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]}-AE=0$$

$$\frac{-2\cdot AB\cdot N\cdot(1+N)\cdot\left[N^2+6\cdot N-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AP=0$$
$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$




$$-\frac{AB\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[6\cdot N-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{10\cdot N-3\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+2+\left(8\cdot N^2+10\cdot N^3+2\cdot N^4\right)}-AU=0$$

$$-\frac{AB\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[6\cdot N-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{10\cdot N-3\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+8\cdot N^2+10\cdot N^3+2\cdot N^4-\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+2}-AU=0$$

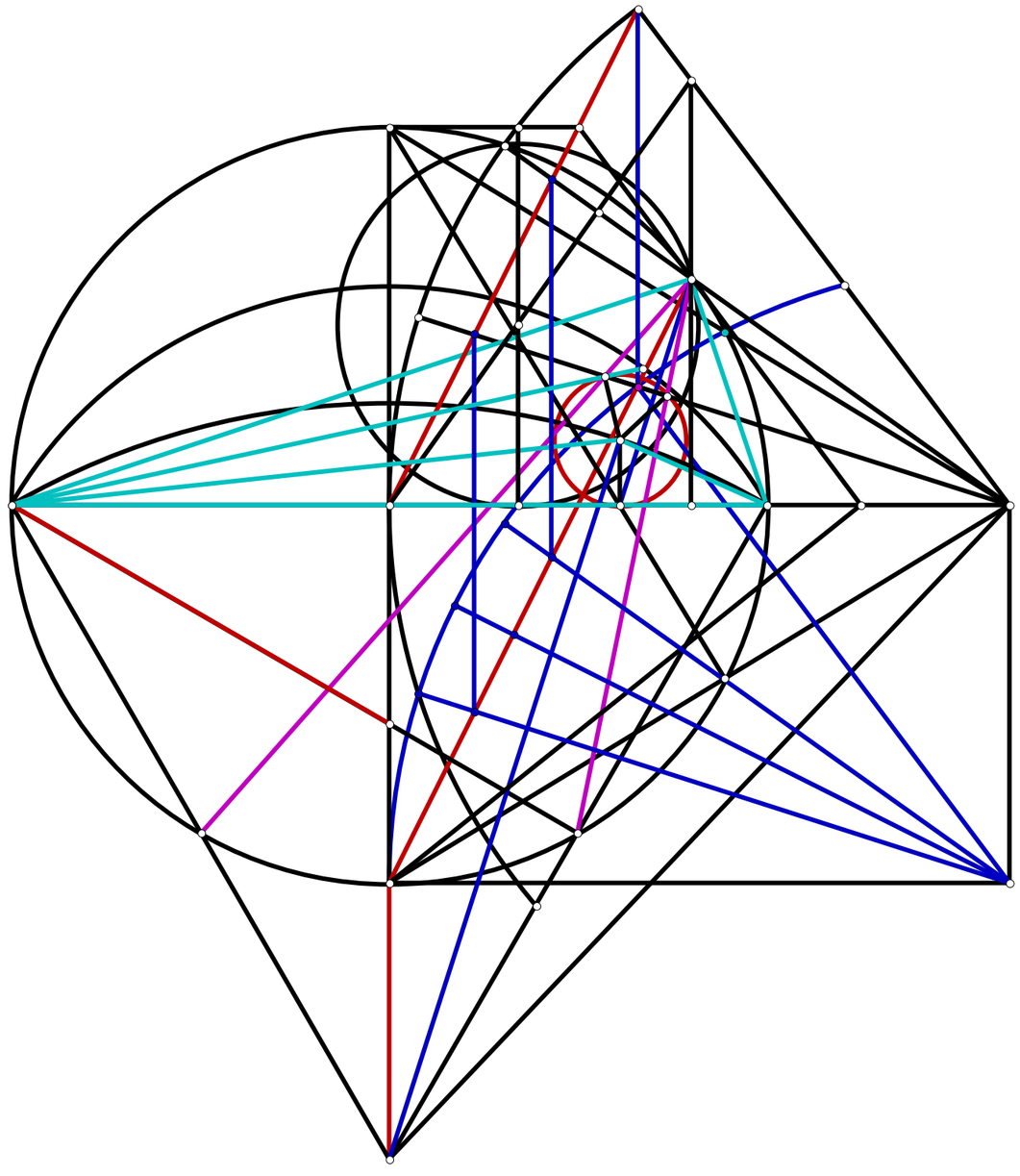
$$\frac{-AB\cdot N\cdot\left[N^2+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\right]\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{\left(2\cdot N^4+10\cdot N^3+8\cdot N^2+10\cdot N+2\right)-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\ \ldots}-AU=0$$

$$+\ -\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^2-\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N^3$$

$$\frac{1}{2}\cdot AB\cdot(N-1)^2\cdot\frac{\left(N^2+4\cdot N+1\right)}{\left[7\cdot N\cdot(N+1)+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-UO=0$$

$$N\cdot(N-1)\cdot AB\cdot\frac{\left[N+\sqrt{(N+3)\cdot(3\cdot N+1)}+1\right]}{\left[7\cdot N+7\cdot N^2+2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+1+N^3\right]}-PU=0$$

**0105 01 Just an Illustration**



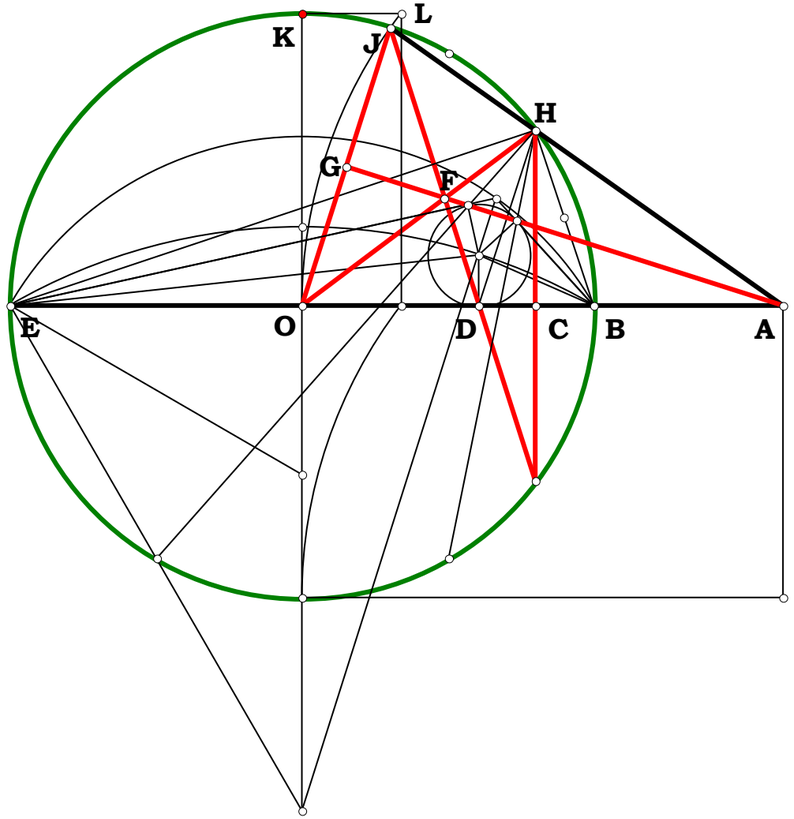
**050601.MCD**

**Just some Algebraic Names**

$AB := .818 \quad N := 4.082$

$AE := AB \cdot N \quad BE := AE - AB \quad BO := \frac{BE}{2} \quad AO := AB + BO$

$AJ := AO \quad JO := BO \quad GO := \frac{JO}{2} \quad AG := \sqrt{AO^2 - GO^2}$




$AP := \frac{AG^2}{AO} \quad OP := AO - AP$

$NO := 2 \cdot OP \quad AN := AO - NO$

$JM := NO \quad HO := BO$

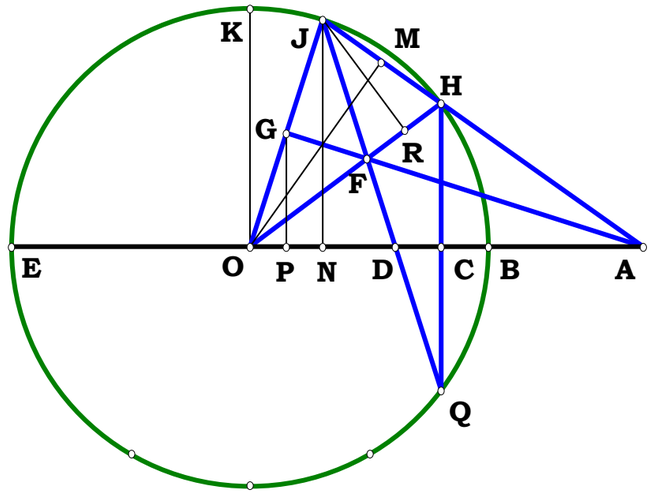
$HJ := 2 \cdot JM \quad AH := AJ - HJ$

$AC := \frac{AN \cdot AH}{AJ} \quad CH := \sqrt{AH^2 - AC^2}$

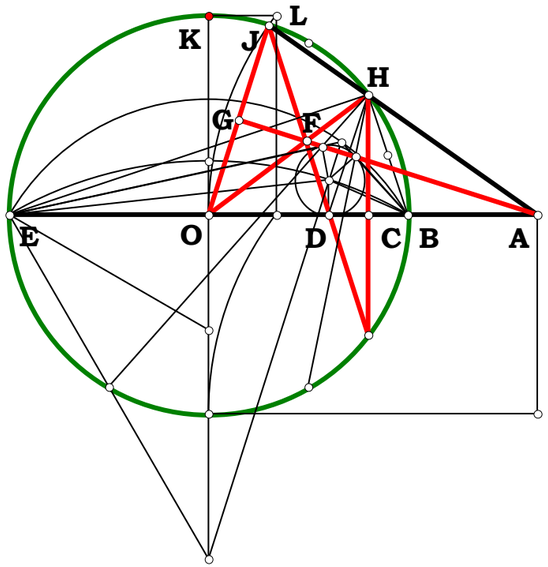
$HQ := 2 \cdot CH \quad CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH} \quad CQ := CH$

$JQ := \sqrt{(CQ + JN)^2 + CN^2} \quad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO} \quad JR := \sqrt{JO^2 - OR^2}$




$FO := \frac{JO \cdot GO}{OR} \quad FJ := FO \quad DQ := \frac{JQ \cdot CQ}{CQ + JN} \quad DF := JQ - (DQ + FJ) \quad FH := HO - FO \quad FG := \frac{JR \cdot GO}{OR} \quad AF := AG - FG$




$$AE - AB\cdot N = 0 \qquad BE - (AB\cdot N - AB) = 0 \qquad BO - \frac{1}{2}\cdot AB\cdot (N-1) = 0$$

$$AO - \frac{1}{2}\cdot AB\cdot (1+N) = 0 \qquad AG - \frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} = 0$$

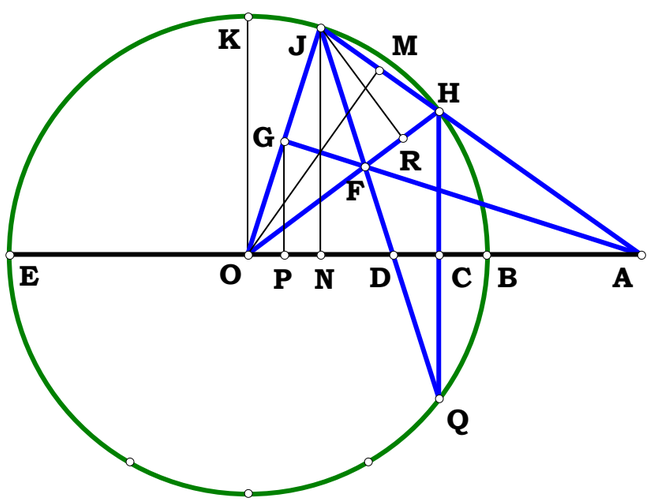
$$AP - \frac{AB}{8}\cdot (N+3)\cdot\frac{(3\cdot N+1)}{(1+N)} = 0 \qquad OP - \frac{AB}{8}\cdot\frac{(N-1)^2}{(1+N)} = 0 \qquad NO - \frac{AB}{4}\cdot\frac{(N-1)^2}{(1+N)} = 0$$

$$AN - \frac{AB}{4}\cdot\frac{\left(1+6\cdot N+N^2\right)}{(1+N)} = 0 \qquad HJ - \frac{AB}{2}\cdot\frac{(N-1)^2}{(1+N)} = 0 \qquad AH - 2\cdot AB\cdot\frac{N}{(1+N)} = 0$$

$$AC - AB\cdot\left(1+6\cdot N+N^2\right)\cdot\frac{N}{(1+N)^3} = 0 \qquad CH - AB\cdot N\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot\frac{(N-1)}{(1+N)^3} = 0$$

$$HQ - 2\cdot AB\cdot N\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot\frac{(N-1)}{(1+N)^3} = 0 \qquad CN - \frac{AB}{4}\cdot\left(N^2+6\cdot N+1\right)\cdot\frac{(N-1)^2}{(1+N)^3} = 0$$

$$JN - \frac{AB}{4}\cdot(N-1)\cdot\sqrt{3\cdot N+1}\cdot\frac{\sqrt{N+3}}{(1+N)} = 0 \qquad JQ - \frac{AB}{2}\cdot(N-1)\cdot\frac{\left(N^2+6\cdot N+1\right)}{(1+N)^2} = 0$$




$$OR - \frac{AB}{4}\cdot\left(N^2+6\cdot N+1\right)\cdot\frac{(N-1)}{(1+N)^2} = 0 \qquad JR - \frac{AB}{4}\cdot(N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{(1+N)^2} = 0$$

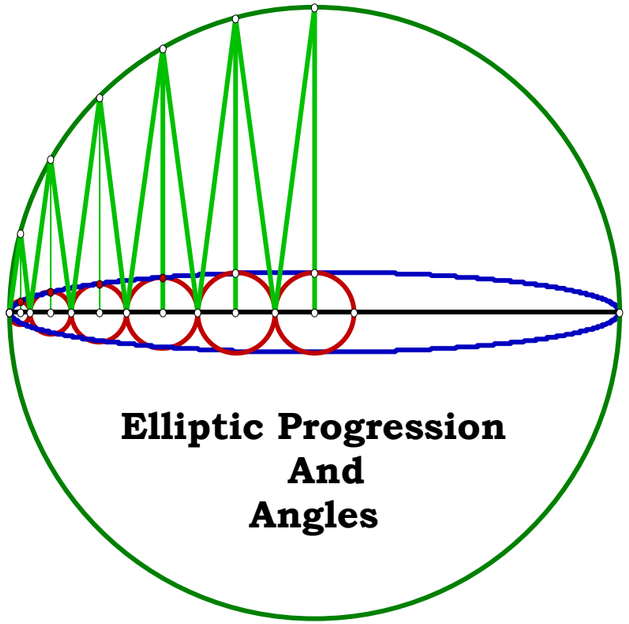
$$FO - \frac{AB}{2}\cdot(1+N)^2\cdot\frac{(N-1)}{\left(N^2+6\cdot N+1\right)} = 0 \qquad DQ - 2\cdot N\cdot(N-1)\cdot\frac{AB}{(1+N)^2} = 0$$

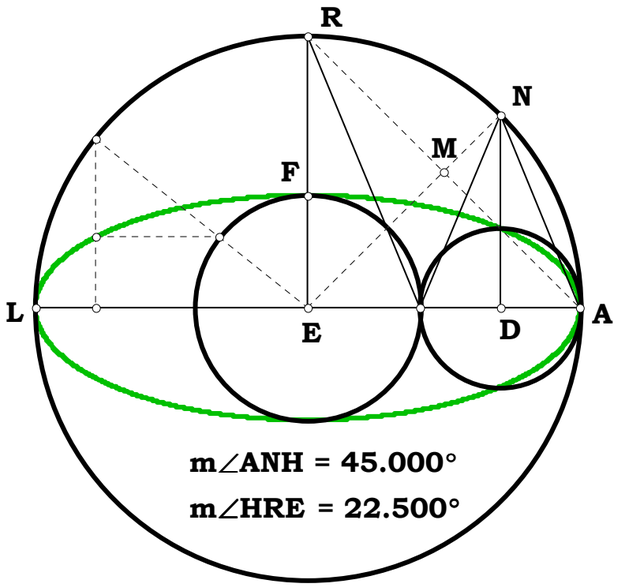
$$DF - 2\cdot N\cdot(N-1)\cdot\frac{AB}{\left(N^2+6\cdot N+1\right)} = 0 \qquad FH - 2\cdot N\cdot(N-1)\cdot\frac{AB}{\left(N^2+6\cdot N+1\right)} = 0$$

$$FG - \frac{AB}{4}\cdot(N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{\left(N^2+6\cdot N+1\right)} = 0 \qquad GO - \frac{AB}{4}\cdot(N-1) = 0$$

$$AF - \left[\frac{AB}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)} - \frac{1}{4}\cdot AB\cdot(N-1)^2\cdot\sqrt{N+3}\cdot\frac{\sqrt{3\cdot N+1}}{\left(N^2+6\cdot N+1\right)}\right] = 0$$

**Angles by Ellipse: 0507013**



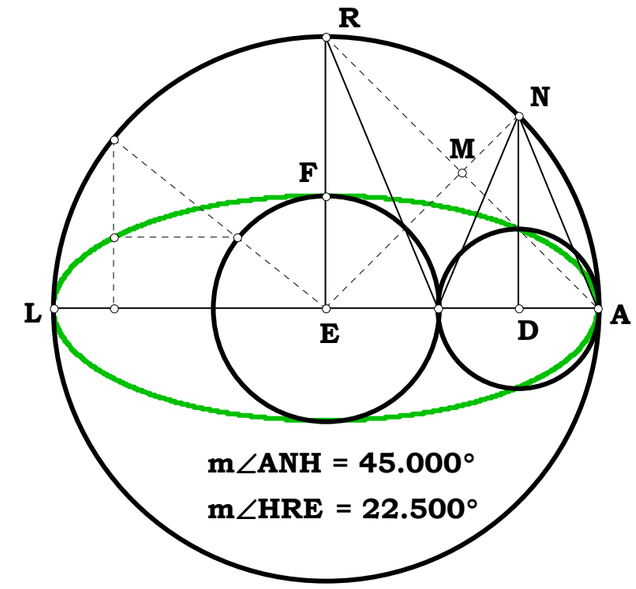

$$N := 1$$

$$AL := N \quad AE := \frac{AL}{2} \quad AR := \sqrt{2 \cdot AE^2}$$

$$AM := \frac{AR}{2} \quad EN := AE \quad EM := AM$$

$$MN := EN - EM \quad AN := \sqrt{AM^2 + MN^2}$$

$$AD := \frac{AN^2}{AL} \quad EF := \frac{AL - (4 \cdot AD)}{2}$$




**Some Algebraic Names.**

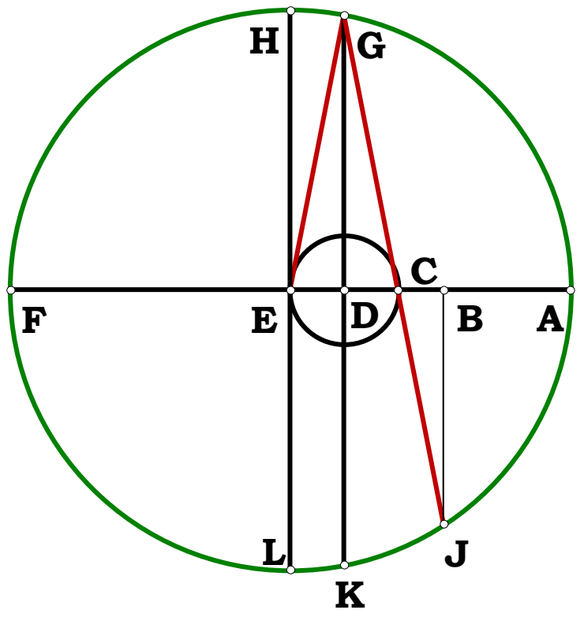
$$\mathbf{AR} - \frac{\mathbf{N}}{2}\cdot\sqrt{2} = 0 \qquad \mathbf{AM} - \frac{\mathbf{N}}{4}\cdot\sqrt{2} = 0 \qquad \mathbf{MN} - \frac{\mathbf{N}}{4}\cdot\left(2-\sqrt{2}\right) = 0 \qquad \mathbf{AN} - \frac{\mathbf{N}}{2}\cdot\sqrt{2-\sqrt{2}} = 0$$

$$\mathbf{AD} - \frac{\mathbf{N}\cdot\left(2-\sqrt{2}\right)}{4} = 0 \qquad \mathbf{EF} - \left(\frac{1}{2}\cdot\mathbf{N}\cdot\sqrt{2} - \frac{1}{2}\cdot\mathbf{N}\right) = 0 \qquad (2\cdot\mathbf{AD} - \mathbf{EF}) - \left(\frac{3}{2}\cdot\mathbf{N} - \mathbf{N}\cdot\sqrt{2}\right) = 0$$

**Some Algebraic Names Where N = 1.**

$$\mathbf{AR} - \frac{\sqrt{2}}{2} = 0 \qquad \mathbf{AM} - \frac{\sqrt{2}}{4} = 0 \qquad \mathbf{MN} - \frac{2-\sqrt{2}}{4} = 0 \qquad \mathbf{AN} - \frac{\sqrt{2-\sqrt{2}}}{2} = 0$$

$$\mathbf{AD} - \frac{2-\sqrt{2}}{4} = 0 \qquad \mathbf{EF} - \left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = 0 \qquad (2\cdot\mathbf{AD} - \mathbf{EF}) - \left(\frac{3}{2} - \sqrt{2}\right) = 0$$




**Elliptic Progression Outtake One 0507011**

**A method of trisection Algebraically.**

$$N := M$$

$$N \geq 4 = 1 \quad AF := 1 \quad AE := \frac{AF}{2} \quad DE := \frac{AF}{N} \quad AD := AE - DE \quad DF := AF - AD$$

$$DG := \sqrt{AD \cdot DF} \quad CD := DE \quad EG := AE \quad CO := \frac{CD^2}{EG} \quad CG := EG \quad CJ := CG - 4 \cdot CO$$

$$BC := \frac{CD \cdot CJ}{CG} \quad AB := AE - (2 \cdot DE + BC) \quad BJ := \frac{DG \cdot BC}{CD} \quad BD := BC + CD$$

$$JK := \sqrt{DG^2 - 2 \cdot DG \cdot BJ + BJ^2 + BD^2} \qquad \frac{JK}{2 \cdot DE} = 1$$ Some Algebraic Names,

**Part of this demonstration may be something of a reductio ad absurdum, if one supposed that CJ were not true. I suppose I need a plate to demonstrate it.**

$$AF \cdot \frac{(N-2)}{2 \cdot N} - AD = 0 \quad AF \cdot \frac{(N+2)}{2 \cdot N} - DF = 0 \quad AF \cdot \frac{\sqrt{(N-2) \cdot (N+2)}}{2 \cdot N} - DG = 0$$

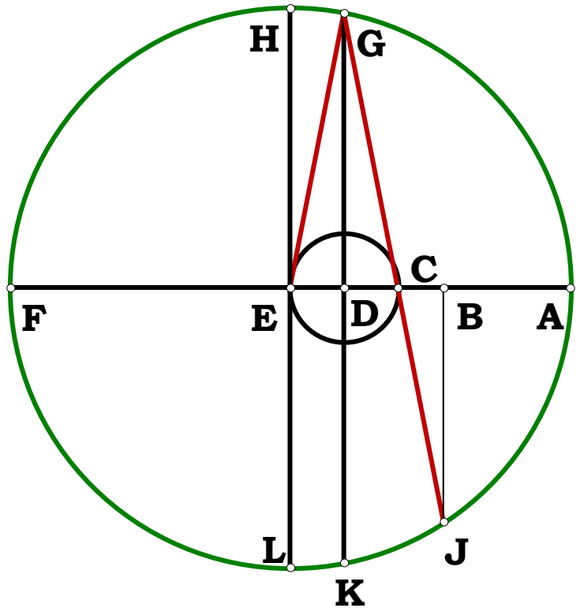
$$\frac{2AF}{N^2} - CO = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4)}{2 \cdot N^2} - CJ = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4)}{N^3} - BC = 0$$

$$AF \cdot \frac{(N+2) \cdot (N-4)^2}{2 \cdot N^3} - AB = 0$$

**One of the meanings of trisection is solving for the following equation when given AF and AB.**

$$\frac{AF}{AB} - \frac{2 \cdot N^3}{(N+2) \cdot (N-4)^2} = 0 \quad AF \cdot \frac{(N-4) \cdot (N+4) \cdot \sqrt{(N-2) \cdot (N+2)}}{2 \cdot N^3} - BJ = 0$$

$$AF \cdot \frac{2 \cdot \left(N^2 - 8\right)}{N^3} - BD = 0 \qquad \frac{2 \cdot AF}{N} - JK = 0$$




$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}-\frac{AF}{AB}=0 \qquad \frac{AF}{AB}=6$$

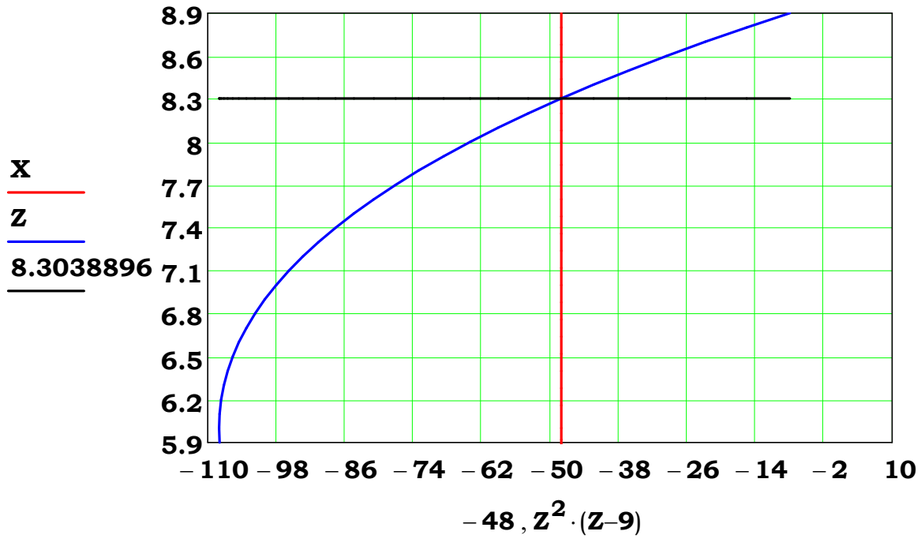
$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}-6=0$$

$$\frac{2\cdot M^3}{(M+2)\cdot(M-4)^2}=6$$

$$9M^2-M^3-48=0$$

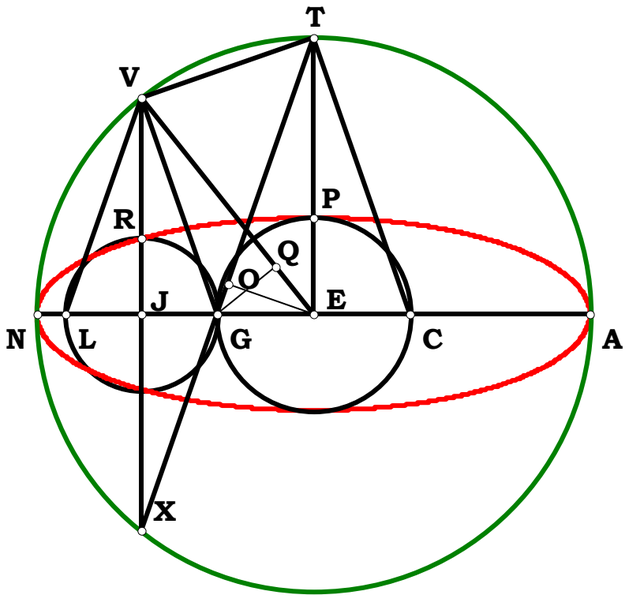
$$DE-\frac{AF}{M}=0$$

$M \equiv 8.303889634816388$ $Z := 5.9, 6 .. 8.9$



**Elliptic Progression Outtake Two 0507012**

**Angles TEV and EVJ equals CTG.**




$$N := 6.381 \quad AN := 2.792$$

$$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN$$

$$EG := \frac{AN}{N} \quad EP := EG \quad GT := \sqrt{EG^2 + ET^2}$$

$$GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO$$

$$JX := \frac{ET \cdot GX}{GT} \quad GJ := \frac{EG \cdot GX}{GT}$$

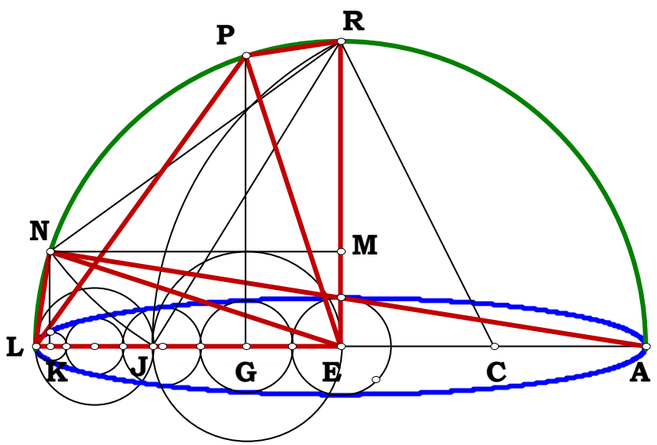
$$\frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV := JX$$

$$EJ := EG + GJ$$

$$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2}$$

$$\frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \quad EQ := \frac{EJ \cdot EG}{EV} \quad GQ := \frac{JV \cdot EQ}{EJ} \quad GQ - GJ = 0 \quad TV - \frac{2 \cdot AN}{\sqrt{N^2 + 4}} = 0$$

**Outtake Three: Alternate Method: Pentasection Or Irrational Rationals 0507013**




$$AL := 1 \qquad AE := \frac{AL}{2} \quad AC := \frac{AE}{2} \quad CE := AC \quad ER := AE \quad CR := \sqrt{CE^2 + ER^2}$$

$$CJ := CR \quad EJ := CJ - CE \quad JR := \sqrt{EJ^2 + ER^2} \quad NR := JR \quad EN := AE \quad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$$

$$KN := EM \quad EK := \sqrt{EN^2 - KN^2} \quad EL := AE \quad KL := EL - EK \quad LN := \sqrt{KL^2 + KN^2}$$

$$EG := \frac{EJ}{2} \quad GL := EL - EG \quad AG := AE + EG \quad GP := \sqrt{AG \cdot GL} \quad PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2}$$

$$PR - LN = 0 \quad AN := \sqrt{AL^2 - LN^2}$$

*Irrational* means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.

**Definitions:**

$$\frac{1}{2} - AE = \frac{1}{4} - AC = 0 \quad \frac{1}{4} \cdot \sqrt{5} - CR = 0 \quad \frac{1}{4} \cdot \sqrt{5} - \frac{1}{4} - EJ = 0 \quad \frac{1}{4} \cdot \sqrt{10 - 2 \cdot \sqrt{5}} - JR = 0$$

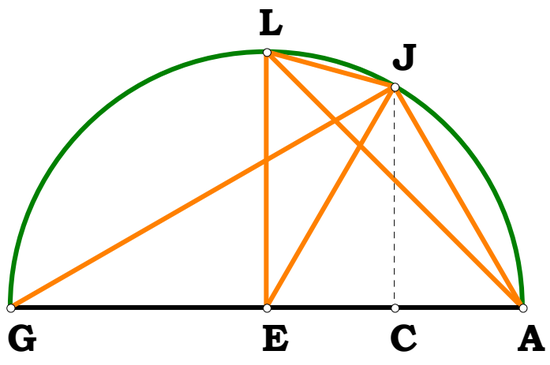
$$\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EM = 0 \quad \frac{1}{2} \cdot \sqrt{\frac{5}{8} + \frac{1}{8} \cdot \sqrt{5}} - EK = 0 \quad \frac{1}{2} - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - KL = 0$$

$$\frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - LN = 0 \quad \frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} - EG = 0 \quad \frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} - GL = 0$$

$$\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} - AG = 0 \quad \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} - GP = 0 \quad \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - PR = 0$$

$$\frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} - AN = 0$$

**Outtake Four: Some Names 0507014**




$$AG := 1 \quad AE := \frac{AG}{2} \quad AC := \frac{AE}{2}$$

$$CG := AG - AC \quad CJ := \sqrt{AC \cdot CG}$$

$$EL := AE \quad CE := AC$$

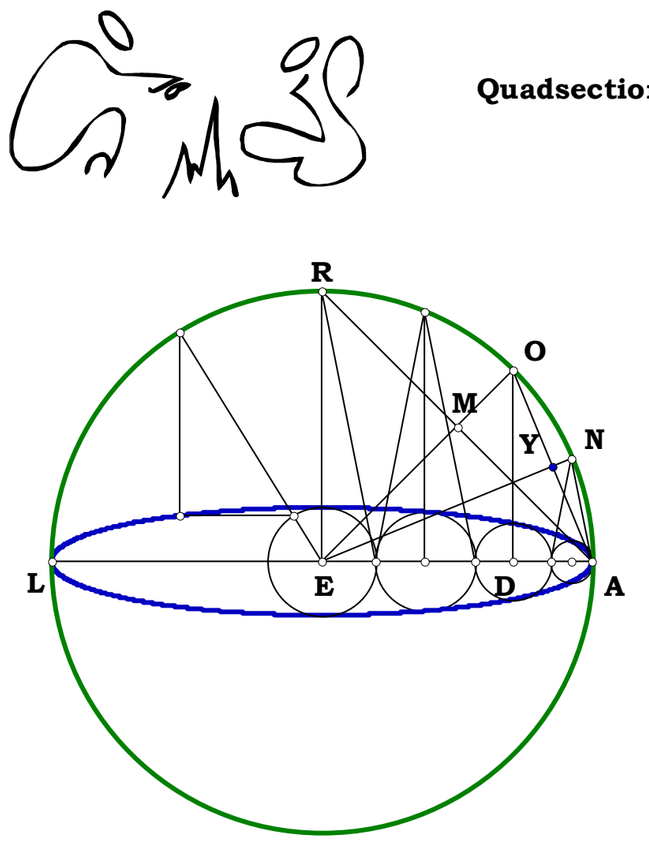
$$JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$$

$$AJ := \sqrt{AC^2 + CJ^2} \quad GJ := \sqrt{CG^2 + CJ^2}$$

$$AL := \sqrt{AE^2 + EL^2}$$

$$AE - \frac{1}{2} = 0 \quad AC - \frac{1}{4} = 0 \quad CG - \left(1 - \frac{1}{4}\right) = 0 \quad CJ - \frac{1}{4} \cdot \sqrt{3} = 0 \quad JL - \left(\frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}\right) = 0$$

$$AJ - \frac{1}{2} = 0 \quad GJ - \frac{1}{2} \cdot \sqrt{3} = 0 \quad AL - \frac{1}{2} \cdot \sqrt{2} = 0$$




**Quadsection: 0507013**

$$AL := 2.5 \quad AE := \frac{AL}{2} \quad AR := \sqrt{2 \cdot AE^2}$$

$$AM := \frac{AR}{2} \quad EO := AE \quad EM := AM$$

$$MO := EO - EM \quad AO := \sqrt{AM^2 + MO^2}$$

$$AY := \frac{AO}{2} \quad EY := \sqrt{AE^2 - AY^2}$$

$$EN := AE \quad NY := EN - EY \quad AN := \sqrt{AY^2 + NY^2}$$

$$AP := \frac{AN^2}{AL} \quad NP := \sqrt{AN^2 - AP^2} \quad ES := \frac{NP \cdot AE}{AL - AP}$$

**Some Algebraic Names.**

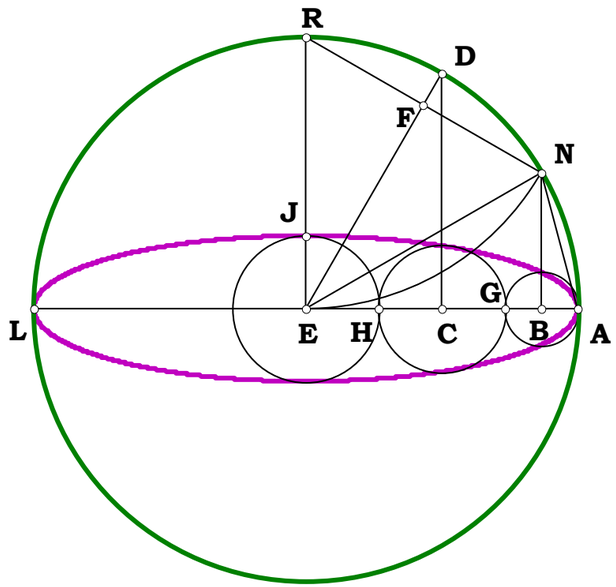
$$AR - \frac{AL}{2} \cdot \sqrt{2} = 0 \quad AM - \frac{AL}{4} \cdot \sqrt{2} = 0 \quad MO - \frac{AL}{4} \cdot \left(2 - \sqrt{2}\right) = 0 \quad AO - \frac{AL}{2} \cdot \sqrt{2 - \sqrt{2}} = 0$$

$$AY - \frac{AL}{4} \cdot \sqrt{2 - \sqrt{2}} = 0 \quad EY - \frac{AL}{4} \cdot \sqrt{2 + \sqrt{2}} = 0 \quad NY - \frac{AL}{4} \cdot \left(2 - \sqrt{2 + \sqrt{2}}\right) = 0$$

$$AN - \frac{AL}{2} \cdot \sqrt{2 - \sqrt{2 + \sqrt{2}}} = 0 \quad \frac{2 - \sqrt{2 + \sqrt{2}}}{4} \cdot AL - AP = 0 \quad \frac{AL}{4} \cdot \sqrt{2 - \sqrt{2}} - NP = 0$$

$$\frac{1}{2} \cdot AL \cdot \frac{\sqrt{2 - \sqrt{2}}}{2 + \sqrt{2 + \sqrt{2}}} - ES = 0$$

**Trisection: 0507013**




$N := 4$

$AL := N \quad AE := \frac{AL}{2} \quad ER := AE$

$NR := ER \quad FN := \frac{NR}{2} \quad EN := AE$

$EF := \sqrt{EN^2 - FN^2} \quad DE := AE$

$DF := DE - EF \quad DN := \sqrt{DF^2 + FN^2}$

$AN := DN \quad AB := \frac{AN^2}{AL} \quad EL := AE$

$BL := AL - AB \quad BN := \sqrt{AN^2 - AB^2}$

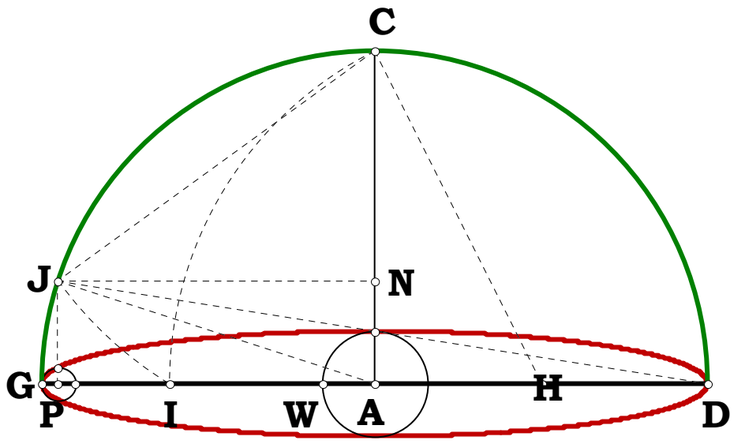
$EJ := \frac{BN \cdot EL}{BL} \quad GH := AE - (EJ + 2 \cdot AB)$

**Some Algebraic Names.**

$FN - \frac{N}{4} = 0 \qquad EF - \frac{N}{4} \cdot \sqrt{3} = 0 \qquad DF - \frac{N \cdot (2 - \sqrt{3})}{4} = 0 \qquad AN - \frac{\sqrt{2} \cdot N \cdot (\sqrt{3} - 1)}{4} = 0 \qquad \frac{N \cdot (2 - \sqrt{3})}{4} - AB = 0$

$\frac{N \cdot (\sqrt{3} + 2)}{4} - BL = 0 \qquad \frac{\sqrt{N^2}}{4} - BN = 0 \qquad \frac{\sqrt{N^2}}{2 \cdot \sqrt{3} + 4} - EJ = 0 \qquad \frac{N + \sqrt{3} \cdot N - \sqrt{N^2}}{2 \cdot (\sqrt{3} + 2)} - GH = 0$

**Pentasection 050701**




$N := 3.333 \quad DG := N$

$AD := \frac{DG}{2} \quad AC := AD$

$AG := AD \quad AH := \frac{AD}{2}$

$CH := \sqrt{AH^2 + AC^2}$

$HI := CH \quad AI := HI - AH \quad CI := \sqrt{AI^2 + AC^2} \quad CJ := CI \quad CN := \frac{CJ^2}{2 \cdot AC}$

$AN := AC - CN \quad JP := AN \quad JN := \sqrt{CJ^2 - CN^2} \quad AP := JN \quad AW := \frac{JP \cdot AD}{AD + AP}$

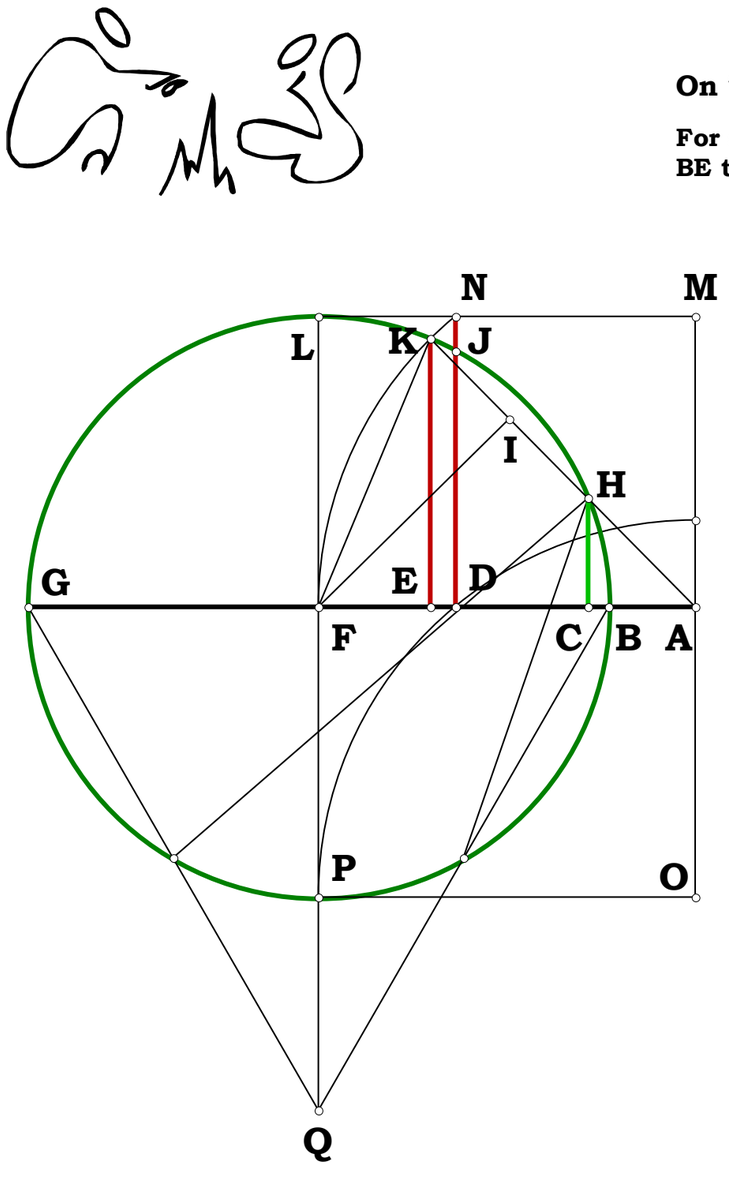
**Definitions:**

$AD := \frac{N}{2} \quad AC := \frac{N}{2} \quad AG := \frac{N}{2} \quad AH := \frac{N}{4} \quad \frac{\sqrt{5} \cdot N}{4} - CH = 0$

$\frac{\sqrt{5} \cdot N}{4} - HI = 0 \quad \frac{N \cdot (\sqrt{5} - 1)}{4} - AI = 0 \quad \frac{\sqrt{2} \cdot N \cdot \sqrt{5 - \sqrt{5}}}{4} - CI = 0$

$\frac{N \cdot (5 - \sqrt{5})}{8} - CN = 0 \quad \frac{N \cdot (\sqrt{5} - 1)}{8} - AN = 0$

$\frac{\sqrt{N^2} \cdot \sqrt{2 \cdot \sqrt{5} + 10}}{8} - JN = 0 \quad \frac{N \cdot (\sqrt{5} - 1)}{2 \cdot \sqrt{2 \cdot \sqrt{5} + 10} + 8} - AW = 0$




## On Trisection 051301

**For any given trisection what is the Algebraic names of BC and BE taking BG as unit?**

$N := 9 \quad BG := 1 \qquad AG := BG \cdot N$

$AB := AG - BG \quad BF := \frac{BG}{2} \quad AF := AB + BF \quad AN := AF \quad AK := AN$

$FK := BF \quad S_1 := AF \quad S_2 := AK \quad S_3 := FK$

$AE := \frac{S_2^{\,2} + S_1^{\,2} - S_3^{\,2}}{2 \cdot S_1} \quad AI := AE \quad IK := AK - AI \quad HI := IK \quad AH := AK - (HI + IK)$

$AC := \frac{AE \cdot AH}{AK} \quad BC := AC - AB \quad BE := AE - AB$

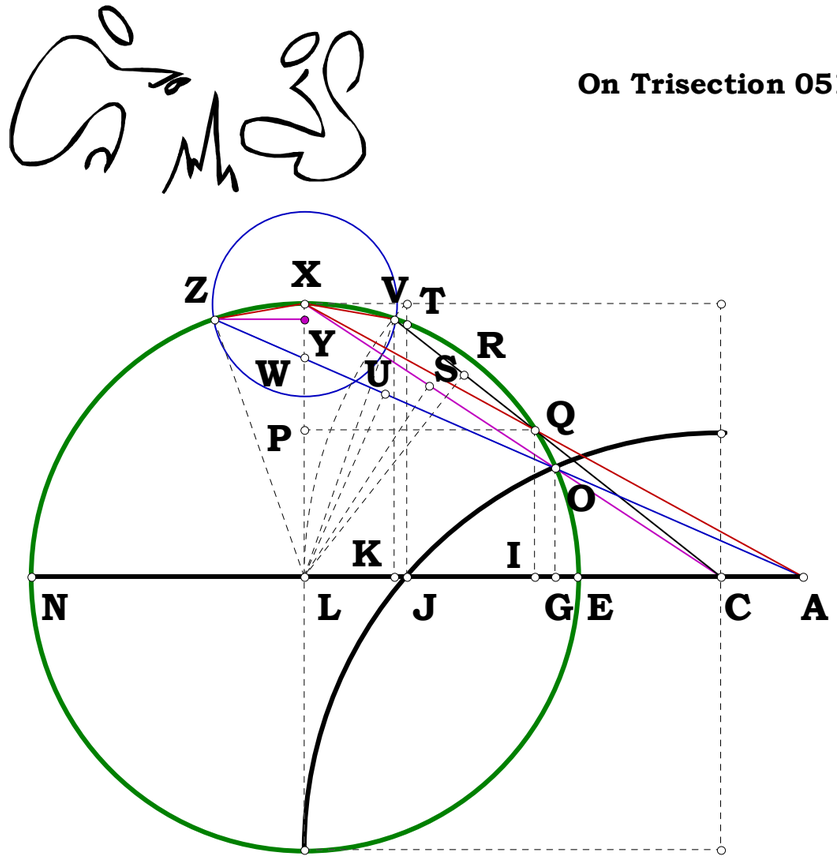
**Some Algebraic Names:**

$N - 1 - AB = 0 \quad \frac{1}{2} - BF = 0 \quad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0 \quad \frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0$

$\frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0 \quad 2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \quad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \quad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

$\frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$




**On Trisection 051401.MCD**

**For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?**

$N := 5 \quad CE := 1 \quad CN := CE \cdot N$

$EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$CL := CE + EL$

$CV := CL \quad S_1 := CL \quad S_2 := CV$

$S_3 := LV \quad CK := \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} \quad CR := CK \quad RV := CV - CR$

$QR := RV \quad CQ := CV - (QR + RV) \quad CI := \frac{CK \cdot CQ}{CV} \quad EI := CI - CE$

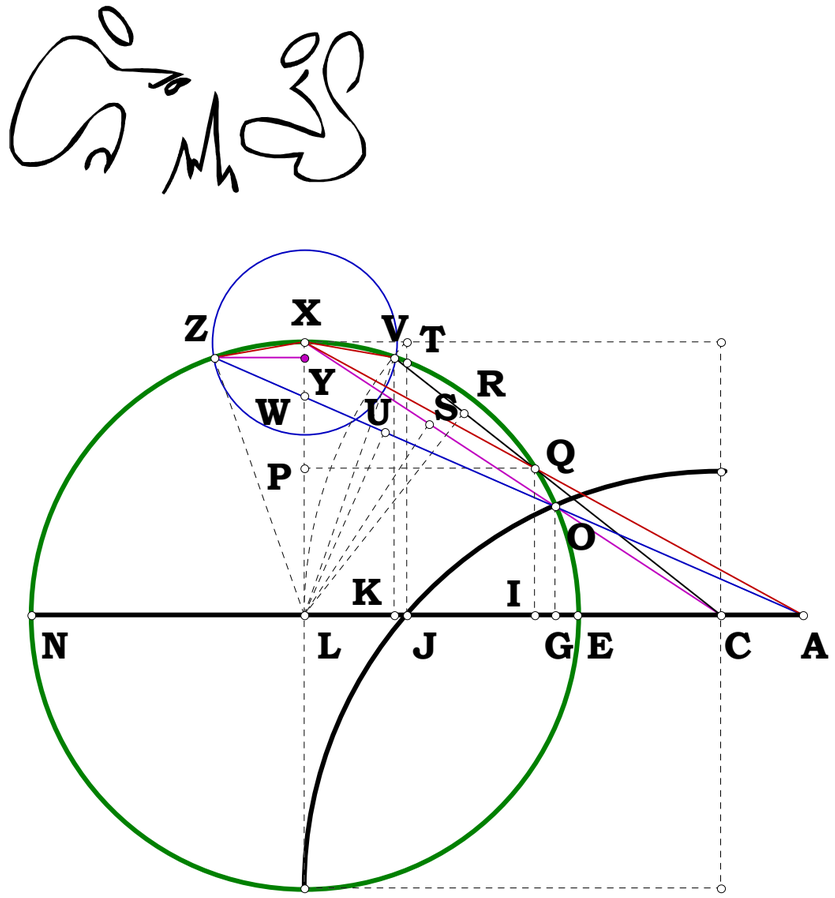
$IN := EN - EI \quad IQ := \sqrt{EI} \quad LP := IQ \quad LX := IL := EL - EI \quad PX := LX - LP$

$AL := \frac{IL \cdot LX}{PX} \quad AE := AL - EL \quad AC := AE - CE \quad AI := AC + CI \quad AX := \sqrt{AL^2 + LX^2}$

$AQ := \frac{AX \cdot AI}{AL} \quad QX := AX - AQ \quad CX := \sqrt{CL^2 + LX^2} \quad CS := \frac{CL^2}{CX} \quad SX := CX - CS$

$OS := SX \quad CO := CX - (SX + OS) \quad GO := \frac{LX \cdot CO}{CX} \quad CG := \frac{CL \cdot CO}{CX} \quad EG := CG - CE$

$AG := AE + EG \quad AO := \sqrt{AG^2 + GO^2} \quad AU := \frac{AG \cdot AL}{AO} \quad OU := AU - AO \quad UZ := OU \quad AZ := AO + (OU + UZ)$




$$LW := \frac{GO \cdot AL}{AG} \quad AW := \frac{AO \cdot AL}{AG} \quad WZ := AZ - AW \quad WY := \frac{GO \cdot WZ}{AO}$$

$$YZ := \frac{AG \cdot WZ}{AO} \quad YX := LX - (LW + WY) \quad XZ := \sqrt{YX^2 + YZ^2}$$

**Some Algebraic Names:**

$$\frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \qquad \frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} - CK = 0$$

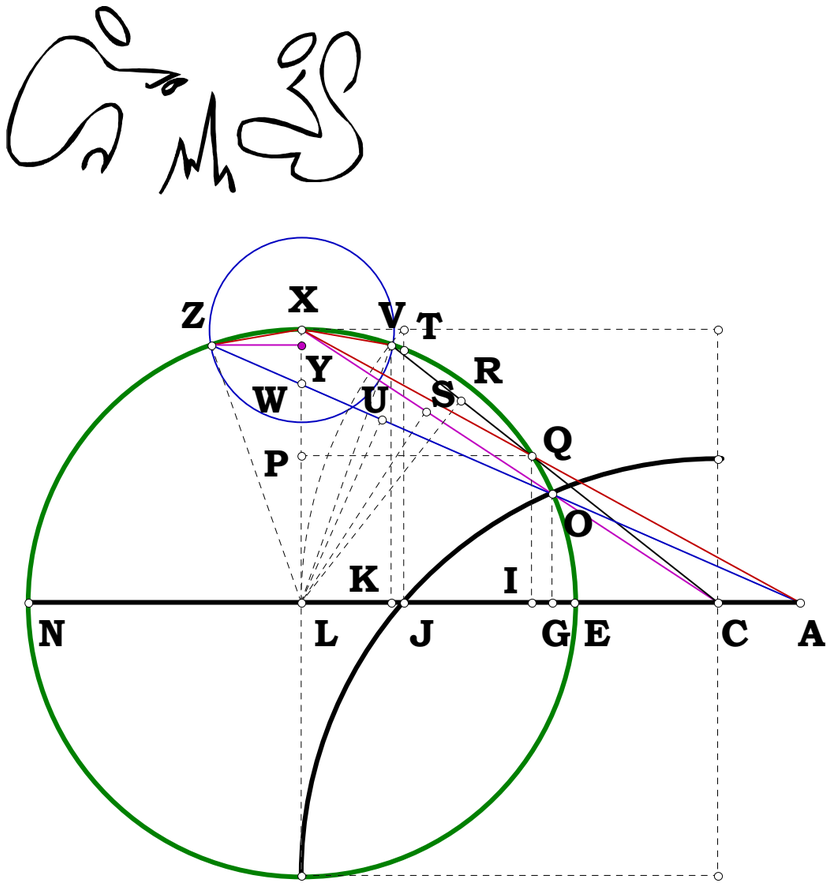
$$\frac{1}{4} \cdot CE \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{(1 + N)} - RV = 0 \qquad 2 \cdot CE \cdot \frac{N}{(1 + N)} - CQ = 0$$

$$CE \cdot \frac{N^3 + 6N^2 + N}{(N + 1)^3} - CE - EI = 0 \qquad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)^3} \cdot N - CI = 0$$

$$CE \cdot N^2 \cdot \frac{\left(-3 + 2 \cdot N + N^2\right)}{(1 + N)^3} - IN = 0 \qquad (N - 1) \cdot N \cdot CE \cdot \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(1 + N)^3} - IQ = 0$$

$$\frac{1}{2} \cdot CE \cdot \frac{\left(2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4 + 1\right)}{(1 + N)^3} - IL = 0$$

$$\left[\frac{1}{2} \cdot CE \cdot (N - 1) \cdot \frac{\left[N^3 + 3 \cdot N^2 - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N + 1\right]}{(1 + N)^3}\right] - PX = 0$$




$$\frac{1}{2}\cdot CE\cdot\left(N^2+4\cdot N+1\right)\cdot\frac{(N-1)^2}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AL=0$$

$$CE\cdot(N-1)\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-3\cdot N-1\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AE=0$$

$$-CE\cdot N\cdot\frac{\left[-N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+6\cdot N+1-\sqrt{(N+3)\cdot(3\cdot N+1)}+N^2\right]}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}-AC=0$$

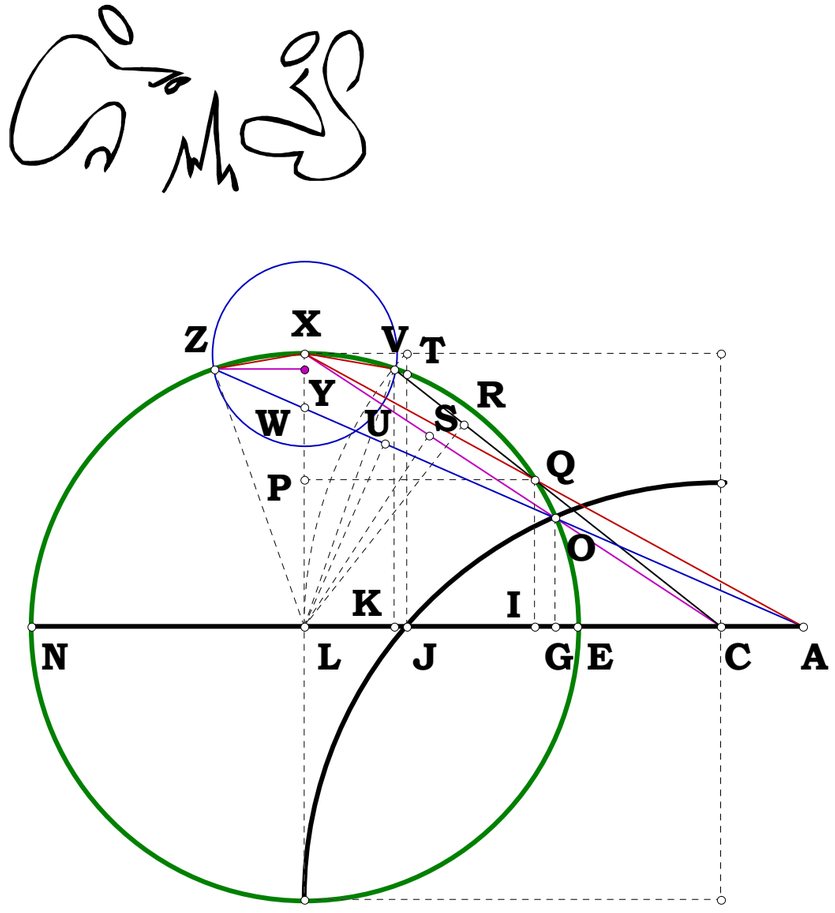
$$\frac{CE\cdot(N-1)^2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+4\cdot N+1\right)}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot(1+N)^3}-AI=0$$

$$\frac{1}{2}\cdot CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}-AX=0$$

$$CE\cdot(N-1)\cdot\sqrt{2}\cdot\sqrt{\frac{(1+N)^3}{\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}\cdot N\cdot\frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{(1+N)^3}-AQ=0$$

$$\sqrt{\frac{\left[2\cdot(1+N)^3\right]}{\left[\begin{matrix}N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\+\ 3\cdot N+1\end{matrix}\right]}}\cdot CE\cdot(N-1)\cdot\frac{\left[\begin{matrix}\left(N^3+3\cdot N^2\right)-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\\+\ 3\cdot N+1\end{matrix}\right]}{2\cdot(1+N)^3}\ \ldots=0$$

$$+\ -QX$$




$$\frac{CE}{2}\cdot\sqrt{2}\cdot\sqrt{\left(1+N^2\right)}-CX=0 \qquad \frac{1}{4}\cdot CE\cdot(1+N)^2\cdot\frac{\sqrt{2}}{\sqrt{1+N^2}}-CS=0$$

$$\frac{1}{4}\cdot\sqrt{2}\cdot CE\cdot\frac{(N-1)^2}{\sqrt{1+N^2}}-SX=0 \qquad \frac{\sqrt{2}}{\sqrt{1+N^2}}\cdot CE\cdot N-CO=0 \qquad CE\cdot N\cdot\frac{(N-1)}{\left(1+N^2\right)}-GO=0$$

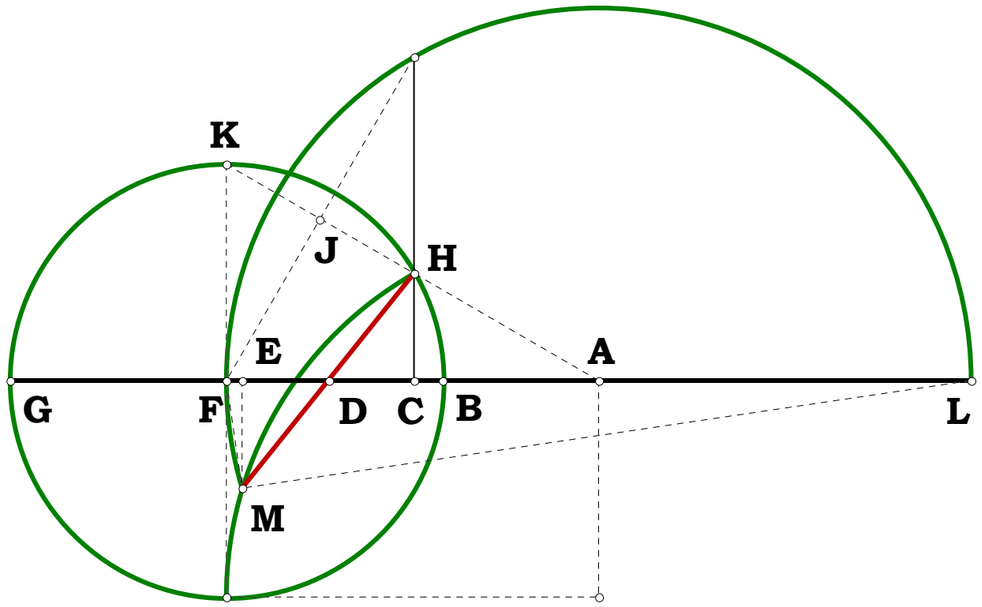
$$CE\cdot\frac{N^2+N}{\left(1+N^2\right)}-CG=0 \qquad CE\cdot\frac{(N-1)}{\left(1+N^2\right)}-EG=0$$

$$CE\cdot(N-1)^2\cdot N\cdot\frac{\left[N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}-2\cdot N+\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}{\left[\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\cdot\left(1+N^2\right)\right]}-AG=0$$

$$\frac{CE\cdot(N-1)}{2}\cdot\sqrt{\frac{\left[\left[2\cdot N^4+14\cdot N^3-N^3\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+32\cdot N^2\right]-7\cdot N^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\ \ldots\right]}{\left[+\ -7\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+14\cdot N+2-\sqrt{(N+3)\cdot(3\cdot N+1)}\right]}}{\left[(1+N)\cdot\left[N^3+3\cdot N^2-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]\right]}-XZ=0$$

**Segment DF And HM 052201**

**Given AB and AG, what is HM and DF?**




$$N := 7.111 \quad AB := .375$$

$$AG := AB \cdot N \quad BG := AG - AB \quad BF := \frac{BG}{2} \quad FK := BF \quad AF := AB + BF$$

$$AK := \sqrt{AF^2 + FK^2} \quad AJ := \frac{AF^2}{AK} \quad JK := AK - AJ \quad HJ := JK$$

$$AH := AK - (JK + HJ) \quad AC := \frac{AF \cdot AH}{AK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot AF$$

$$EF := \frac{FL}{2} - \frac{\sqrt{-4EM^2 + FL^2}}{2} \quad AE := AF - EF \quad CE := AE - AC \quad CH := \frac{FK \cdot AH}{AK}$$

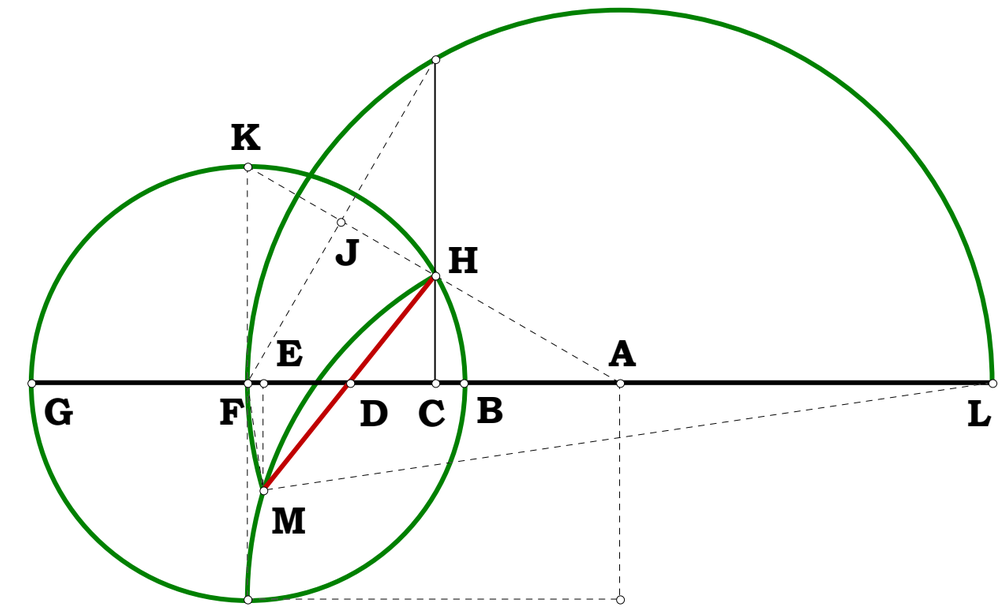
$$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$$

**Some Algebraic Names:**

$$AB \cdot N - AG = 0 \quad AB \cdot N - AB - BG = 0 \quad \frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - BF = 0 \quad \frac{1}{2} \cdot AB \cdot N - \frac{1}{2} \cdot AB - FK = 0$$

$$\frac{1}{2} \cdot AB + \frac{1}{2} \cdot AB \cdot N - AF = 0 \quad \frac{1}{2} \cdot AB \cdot \sqrt{2} \cdot \sqrt{1 + N^2} - AK = 0 \quad \frac{1}{4} \cdot AB \cdot (1 + N)^2 \cdot \frac{\sqrt{2}}{\sqrt{1 + N^2}} - AJ = 0$$

$$\frac{1}{4} \cdot \sqrt{2} \cdot AB \cdot \frac{\left(1 + N^2 - 2 \cdot N\right)}{\sqrt{1 + N^2}} - JK = 0 \quad AB \cdot \sqrt{2} \cdot \frac{N}{\sqrt{1 + N^2}} - AH = 0 \quad AB \cdot (1 + N) \cdot \frac{N}{1 + N^2} - AC = 0$$




$$\frac{1}{4}\cdot AB\cdot N-\frac{1}{4}\cdot AB-EM=0 \qquad AB+AB\cdot N-FL=0$$

$$\left(\frac{1}{2}\cdot AB+\frac{1}{2}\cdot AB\cdot N\right)-\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-EF=0$$

$$\frac{1}{4}\cdot AB\cdot\sqrt{3\cdot N^2+10\cdot N+3}-AE=0$$

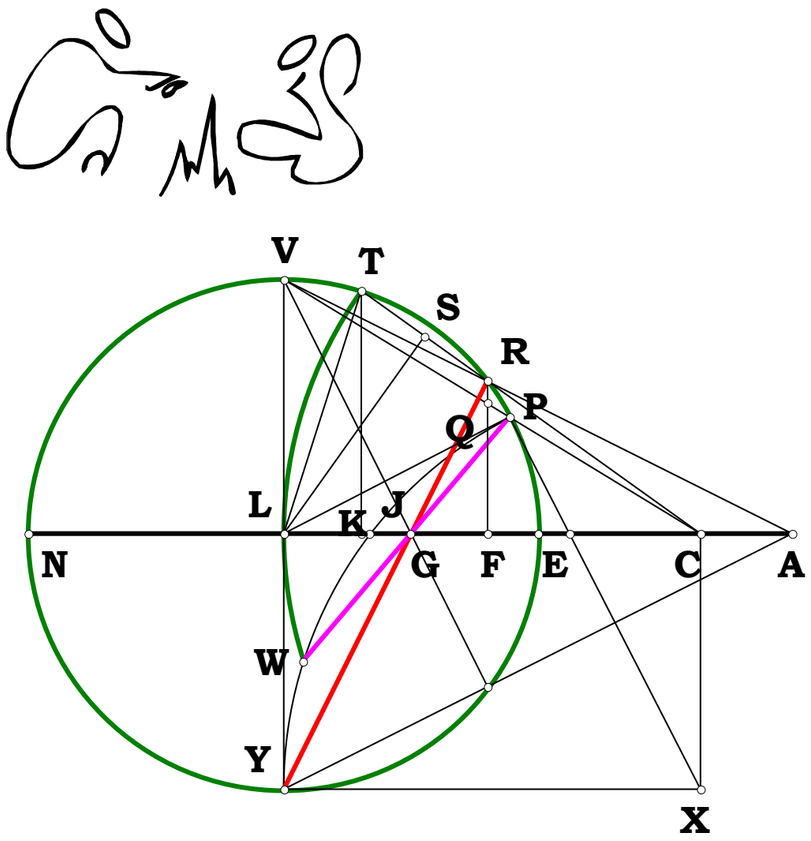
$$\frac{1}{4}\cdot AB\cdot\frac{\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)}{\left(1+N^2\right)}-CE=0$$

$$(N-1)\cdot AB\cdot\frac{N}{\left(1+N^2\right)}-CH=0$$

$$\frac{1}{2}\cdot AB\cdot\sqrt{(1+N)\cdot\frac{\left[N^3+3\cdot N^2-2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N+3\cdot N+1\right]}{\left(1+N^2\right)}}-HM=0$$

$$\frac{1}{4}\cdot\left(\sqrt{3\cdot N^2+10\cdot N+3}+\sqrt{3\cdot N^2+10\cdot N+3}\cdot N^2-4\cdot N-4\cdot N^2\right)\cdot\frac{AB}{\left(N^2+4\cdot N+1\right)}-DE=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{\left(3\cdot N+3\cdot N^2+1+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\right)}{\left(N^2+4\cdot N+1\right)}-DF=0$$




**Point of Intersection 052701**

**Do RY and PW intersect at G?**

$N := 5 \quad CE := 1 \quad CN := CE \cdot N \quad EN := CN - CE \quad EL := \frac{EN}{2} \quad LV := EL$

$LT := EL \quad LY := EL \quad CL := CE + EL \quad CT := CL \quad S_1 := CL \quad S_2 := CT \quad S_3 := LT$

$KL := \frac{S_3^{\,2} + S_1^{\,2} - S_2^{\,2}}{2 \cdot S_1} \quad CK := CL - KL \quad CS := CK \quad ST := CT - CS \quad RS := ST$

$CR := CT - (ST + RS) \quad CF := \frac{CK \cdot CR}{CT} \quad FR := \sqrt{CR^2 - CF^2} \quad FQ := \frac{LV \cdot CF}{CL} \quad FL := CL - CF$

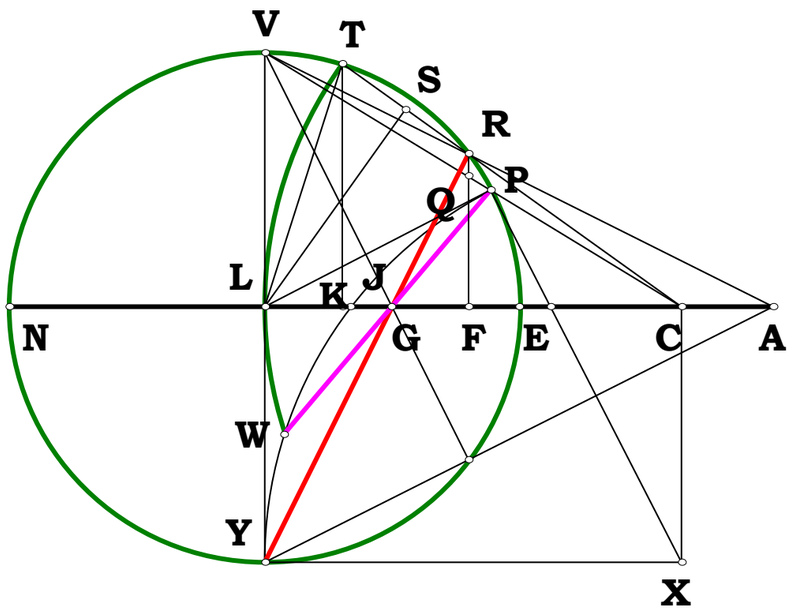
$GL := \frac{FL \cdot LY}{(LY + FR)}$

**Some Algebraic Names:**

$CE \cdot N - CN = 0 \quad CE \cdot N - CE - EN = 0 \quad \frac{1}{2} \cdot CE \cdot N - \frac{1}{2} \cdot CE - EL = 0 \quad \frac{1}{2} \cdot CE + \frac{1}{2} \cdot CE \cdot N - CL = 0 \quad \frac{1}{4} \cdot CE \cdot \frac{(N-1)^2}{(1+N)} - KL = 0$

$\frac{1}{4} \cdot CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)} - CK = 0 \quad \frac{1}{4} \cdot CE \cdot \frac{\left(1 - 2 \cdot N + N^2\right)}{(1+N)} - ST = 0 \quad 2 \cdot CE \cdot \frac{N}{(1+N)} - CR = 0 \quad CE \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1+N)^3} \cdot N - CF = 0$

$CE \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot \frac{(N-1)}{(1+N)^3} - FR = 0 \quad CE \cdot (N-1) \cdot \left(1 + 6 \cdot N + N^2\right) \cdot \frac{N}{(1+N)^4} - FQ = 0 \quad \frac{1}{2} \cdot CE \cdot \frac{\left(1 + 2 \cdot N - 6 \cdot N^2 + 2 \cdot N^3 + N^4\right)}{(1+N)^3} - FL = 0$




$$GL := \frac{1}{2}\cdot\frac{CE\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{\left[N^3+3\cdot N^2+2N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}$$

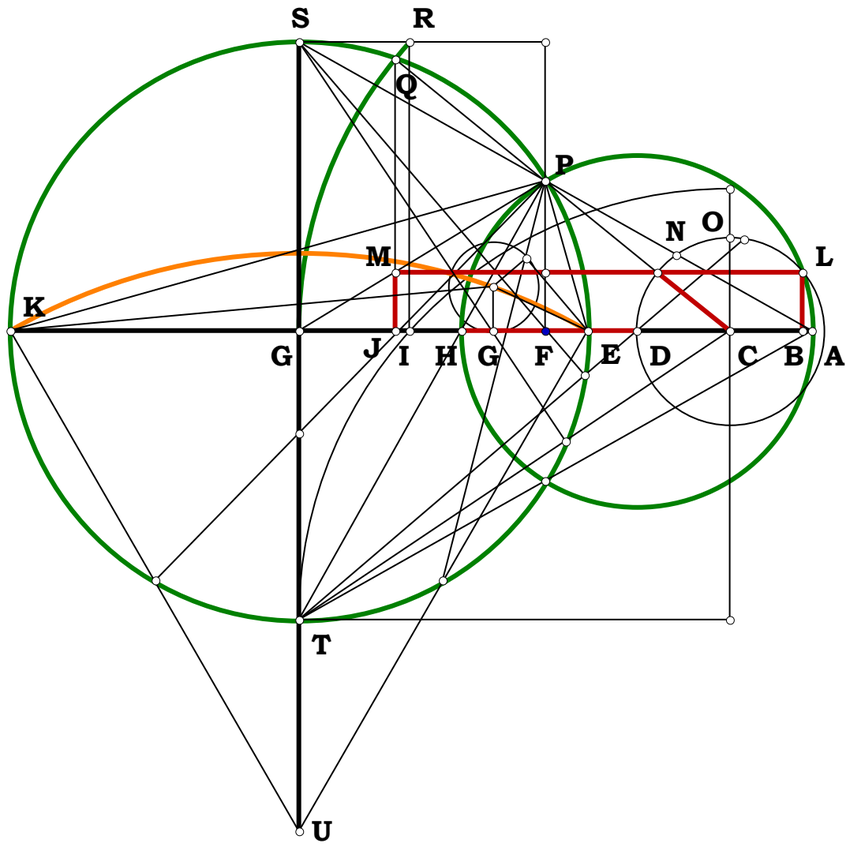
**From Segment DF And HM 052201:**

$$\frac{1}{2}\cdot\frac{CE\cdot\left(N^3+3\cdot N^2+3\cdot N-2N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{\left(N^2+4\cdot N+1\right)}-GL=0$$

**Which does reduce to,**

$$\frac{\frac{1}{2}\cdot\frac{CE\cdot\left(N^3+3\cdot N^2+3\cdot N-2N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{\left(N^2+4\cdot N+1\right)}}{\frac{1}{2}\cdot\frac{CE\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{\left[N^3+3\cdot N^2+2N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N+1\right]}}=1 \qquad 1=1$$

0530011




**What is CD and BL?**

$N_1 := 1 \quad N_2 := 5$

$CE := N_1 \quad EK := N_2 \quad EG := \frac{EK}{2} \quad CG := CE + EG$

$CQ := CG \quad PQ := \frac{EG^2}{CQ} \quad CP := CQ - PQ \quad GT := EG \quad CF := \frac{CP^2 + CG^2 - EG^2}{2 \cdot CG}$

$PF := \sqrt{CP^2 - CF^2} \quad GS := EG \quad FG := CG - CF \quad FS := \sqrt{FG^2 + GS^2}$

$DG := \frac{GS \cdot GT}{FG} \quad CD := CG - DG \quad BL := \frac{PF \cdot CD}{CP}$

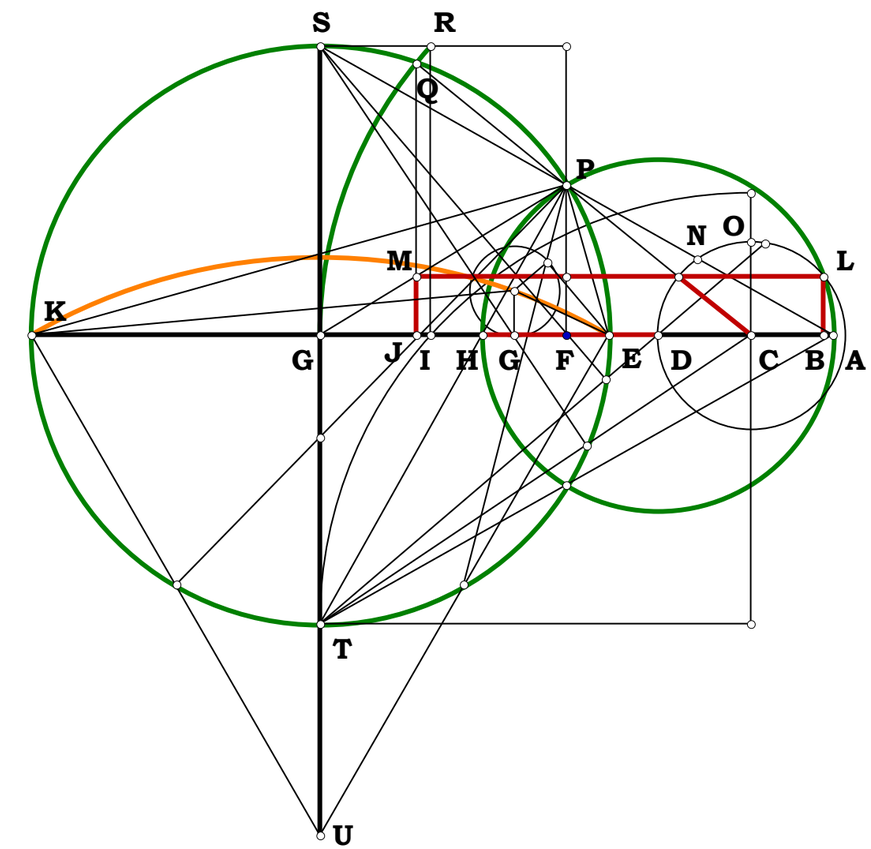
**Some Algebraic Names;**

$CE := N_1 \quad EK := N_2 \quad EG - \frac{N_2}{2} = 0 \quad CG - \left(N_1 + \frac{N_2}{2}\right) = 0$

$PQ - \frac{1}{2} \cdot \frac{N_2^2}{(2 \cdot N_1 + N_2)} = 0 \quad CP - 2 \cdot N_1 \cdot \frac{(N_1 + N_2)}{(2 \cdot N_1 + N_2)} = 0$

$CF := N_1 \cdot (N_1 + N_2) \cdot \frac{\left(8 \cdot N_1^2 + 8 \cdot N_1 \cdot N_2 + N_2^2\right)}{(2 \cdot N_1 + N_2)^3}$

$PF - (N_1 + N_2) \cdot N_1 \cdot \sqrt{(N_2 + 4 \cdot N_1) \cdot (3 \cdot N_2 + 4 \cdot N_1)} \cdot \frac{N_2}{(2 \cdot N_1 + N_2)^3} = 0$




$$FG - \frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$FS - \sqrt{\left[\frac{1}{2}\cdot N_2{}^2\cdot\frac{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}{\left(2\cdot N_1+N_2\right)^3}\right]^2 - \left(\frac{N_2}{2}\right)^2}=0$$

$$FS - \left(\frac{1}{2}\right)\cdot N_2\cdot\sqrt{2}\cdot\frac{\sqrt{\left(138\cdot N_2{}^2\cdot N_1{}^4+116\cdot N_2{}^3\cdot N_1{}^3+54\cdot N_2{}^4\cdot N_1{}^2+12\cdot N_2{}^5\cdot N_1\right)\ \ldots}}{\left(2\cdot N_1+N_2\right)^3}$$
$$\frac{+\ N_2{}^6+32\cdot N_1{}^6+96\cdot N_1{}^5\cdot N_2}{}=0$$

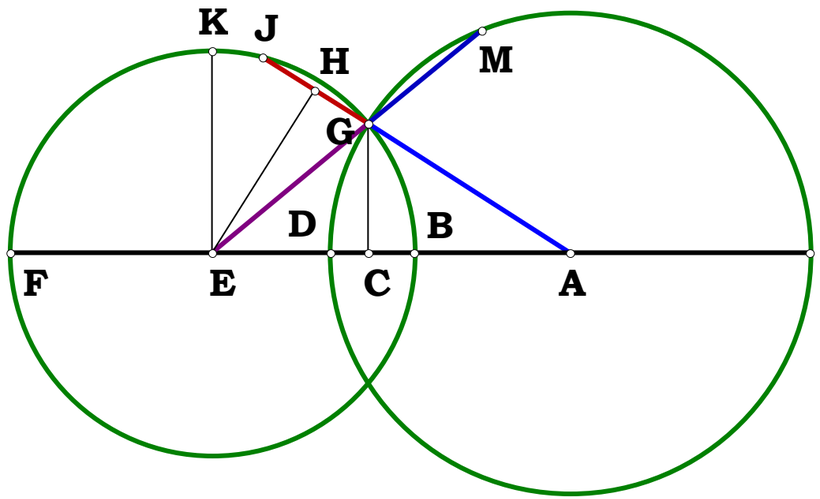
$$DG - \frac{\left(2\cdot N_1+N_2\right)^3}{\left[2\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)\right]}=0$$

$$CD - N_1\cdot\left(N_1+N_2\right)\cdot\frac{\left(2\cdot N_1+N_2\right)}{\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$

$$BL - \frac{\left(N_1+N_2\right)\cdot N_1\cdot\sqrt{\left(N_2+4\cdot N_1\right)\cdot\left(3\cdot N_2+4\cdot N_1\right)}\cdot N_2}{2\cdot\left(2\cdot N_1+N_2\right)\cdot\left(6\cdot N_1{}^2+6\cdot N_1\cdot N_2+N_2{}^2\right)}=0$$

**A Small Extrapolation 060101**

**Given AE, AG, and EG, what is the Algebraic name of the segment GJ?**




$S_1 := 6.00604 \quad S_2 := 4.02167 \quad S_3 := 3.38667$

$AE := S_1 \quad AG := S_2 \quad EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2AE}$$

$$AH := \frac{AC \cdot AE}{AG} \quad GH := AH - AG$$

$HJ := GH \quad GJ := GH + HJ$

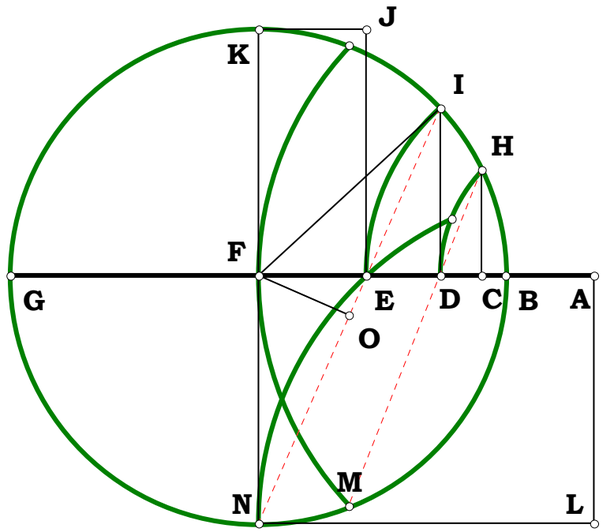
**Some Algebraic Names:**

$$\frac{S_2^2 + S_1^2 - S_3^2}{2S_1} - AC = 0 \qquad \frac{S_1^2 + S_2^2 - S_3^2}{2S_2} - AH = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{2S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0 \qquad \frac{S_1^2 - S_2^2 - S_3^2}{S_3} - GM = \ ?$$

**Units From Both Sides 060201**

**Start with AB as unit and find. . . . then start with . . . . as unit and find AB.**




$N := 5.727 \quad AB := .573 \quad AG := AB \cdot N$

$BG := AG - AB \quad BF := \frac{BG}{2} \quad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \quad BE := AE - AB$

$FN := BF \quad EF := BF - BE \quad EN := \sqrt{FN^2 + EF^2}$

$NO := \frac{FN^2}{EN} \quad NI := 2 \cdot NO \quad EI := NI - EN \quad \frac{1}{2} \cdot AB \cdot (N - 1) - BF = 0$

$DE := \frac{EF \cdot EI}{EN} \quad AB \cdot (N - 1) - BG = 0$

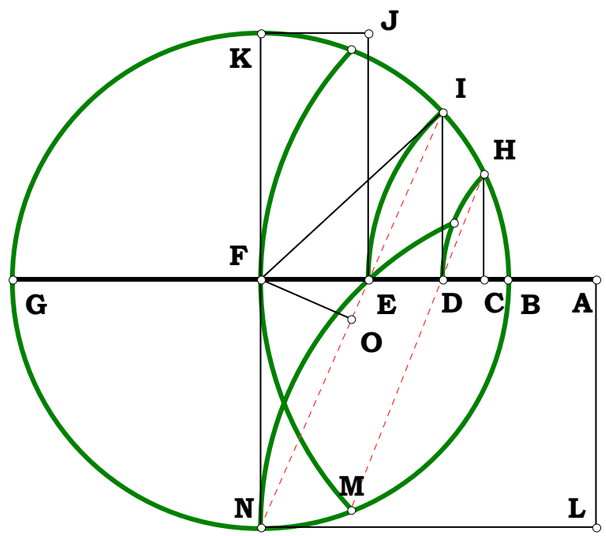
$AB \sqrt{N} - AE = 0 \quad AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$

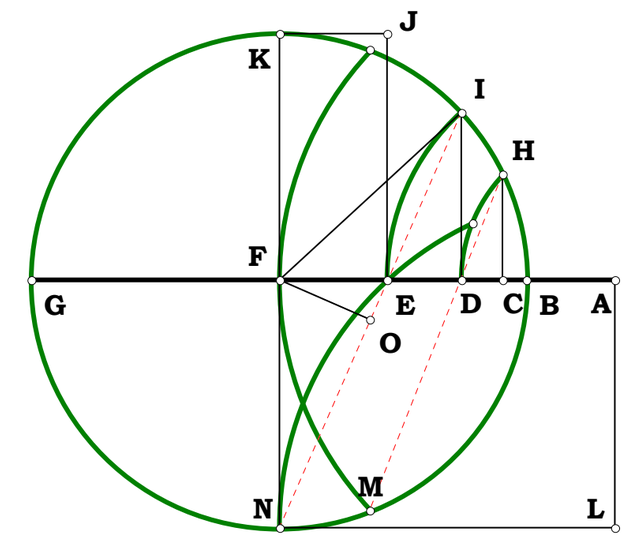
$\frac{1}{2} \cdot AB \cdot (1 + N) - AF = 0$

$N_2 := 2 \quad BG_2 := 1 \quad BF_2 := \frac{BG_2}{2} \quad BE_2 := \frac{BF_2}{N_2} \quad EF_2 := BF_2 - BE_2$

$FN_2 := BF_2 \quad EN_2 := \sqrt{EF_2^2 + FN_2^2} \quad NP_2 := \frac{EN_2}{2} \quad LN_2 := \frac{EN_2 \cdot NP_2}{EF_2}$

$AF_2 := LN_2 \quad AB_2 := AF_2 - BF_2$





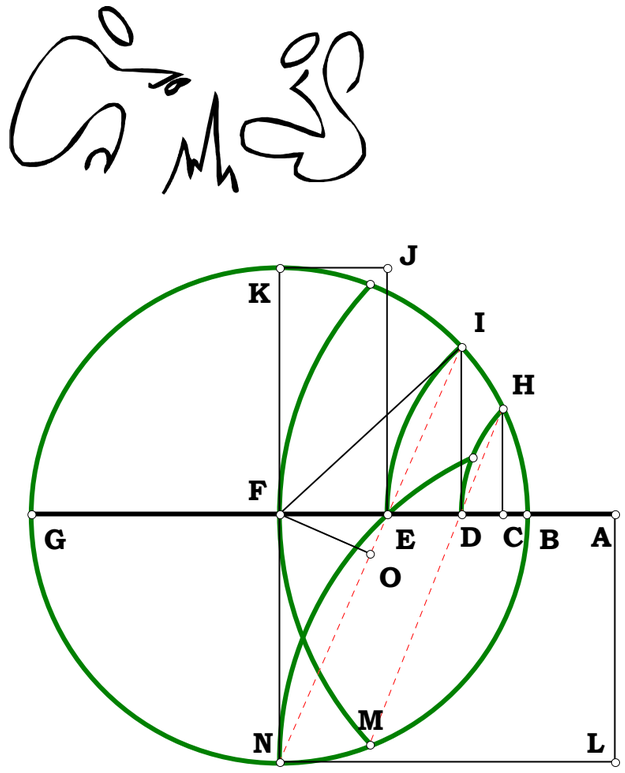

$$\frac{BG_2}{2} - BF_2 = 0 \qquad \frac{BG_2}{(2 \cdot N_2)} - BE_2 = 0 \qquad \frac{BG_2}{2} \cdot \frac{(N_2 - 1)}{N_2} - EF_2 = 0$$

$$\frac{BG_2}{2} \cdot \frac{\sqrt{2 \cdot N_2^{\,2} - 2 \cdot N_2 + 1}}{N_2} - EN_2 = 0 \qquad \frac{BG_2}{4} \cdot \frac{\sqrt{2 \cdot N_2^{\,2} - 2 \cdot N_2 + 1}}{N_2} - NP_2 = 0$$

$$\frac{BG_2}{4} \cdot \frac{\left(2 \cdot N_2^{\,2} - 2 \cdot N_2 + 1\right)}{\left[N_2 \cdot (N_2 - 1)\right]} - LN_2 = 0 \qquad \frac{BG_2}{4 \cdot N_2 \cdot (N_2 - 1)} - AB_2 = 0$$

$$AI := AE \qquad FI := BF \qquad AD := \frac{AI^2 + AF^2 - FI^2}{2AF} \qquad BD := AD - AB$$

$$AD - 2 \cdot AB \cdot \frac{N}{(N + 1)} = 0 \qquad BD - AB \cdot \frac{(N - 1)}{(1 + N)} = 0$$




$$N_3 := 3.364 \quad BG_3 := 2.708 \quad BF_3 := \frac{BG_3}{2} \quad BD_3 := \frac{BF_3}{N_3} \quad DG_3 := BG_3 - BD_3$$

$$DI_3 := \sqrt{BD_3 \cdot DG_3} \quad DF_3 := BF_3 - BD_3 \quad FN_3 := BF_3 \quad EF_3 := \frac{DF_3 \cdot FN_3}{FN_3 + DI_3}$$

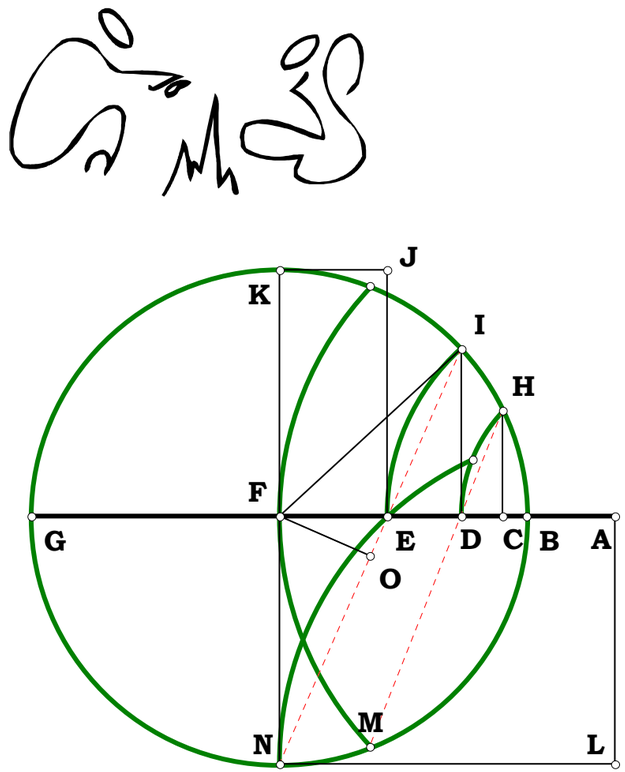
$$EN_3 := \sqrt{EF_3^{\,2} + FN_3^{\,2}} \quad NP_3 := \frac{EN_3}{2} \quad LN_3 := \frac{EN_3 \cdot NP_3}{EF_3} \quad AF_3 := LN_3$$

$$AB_3 := AF_3 - BF_3$$

$$BD_3 - \frac{1}{2} \cdot \frac{BG_3}{N_3} = 0 \quad DG_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(2 \cdot N_3 - 1)}{N_3} = 0 \quad DI_3 - \frac{1}{(2 \cdot N_3)} \cdot BG_3 \cdot \sqrt{2 \cdot N_3 - 1} = 0 \quad DF_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{N_3} = 0$$

$$EF_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{(N_3 - 1)}{(N_3 + \sqrt{2 \cdot N_3 - 1})} = 0 \quad NP_3 - \frac{1}{4} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} = 0 \quad EN_3 - \frac{1}{2} \cdot BG_3 \cdot \sqrt{2} \cdot \sqrt{\frac{N_3}{(N_3 + \sqrt{2 \cdot N_3 - 1})}} = 0$$

$$LN_3 - \frac{1}{2} \cdot BG_3 \cdot \frac{N_3}{(N_3 - 1)} = 0 \quad AB_3 - \frac{1}{2} \cdot \frac{BG_3}{(N_3 - 1)} = 0$$




$$AH := AD$$

$$FH := BF \qquad AC := \frac{AH^2 + AF^2 - FH^2}{2 \cdot AF}$$

$$BC := AC - AB$$

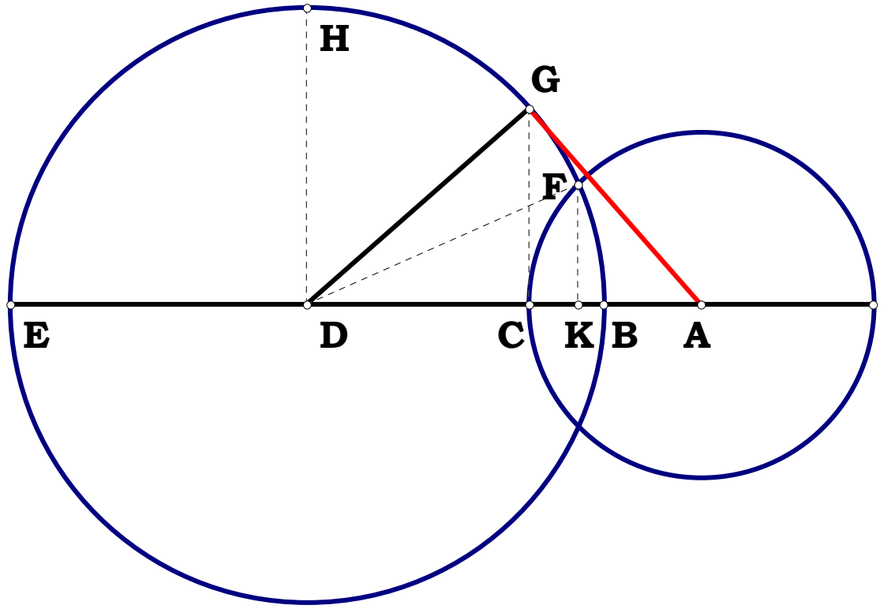
$$AB \cdot \left(\sqrt{N} - 1\right) - BE = 0$$

$$AC - AB \cdot N \cdot \frac{\left(6 \cdot N + 1 + N^2\right)}{(N+1)^3} = 0$$

$$BC - AB \cdot (3 \cdot N + 1) \cdot \frac{(N-1)}{(N+1)^3} = 0$$

**Isolating A Problem 060301**

**If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?**




$N := 4 \quad BE := 1$

$BD := \frac{BE}{2} \quad BC := \frac{BE}{N} \quad CE := BE - BC$

$CG := \sqrt{BC \cdot CE} \quad CD := BD - BC \quad AC := \frac{CG^2}{CD}$

$AF := AC \quad AD := AC + CD \quad DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2AD} \quad BK := BD - DK$

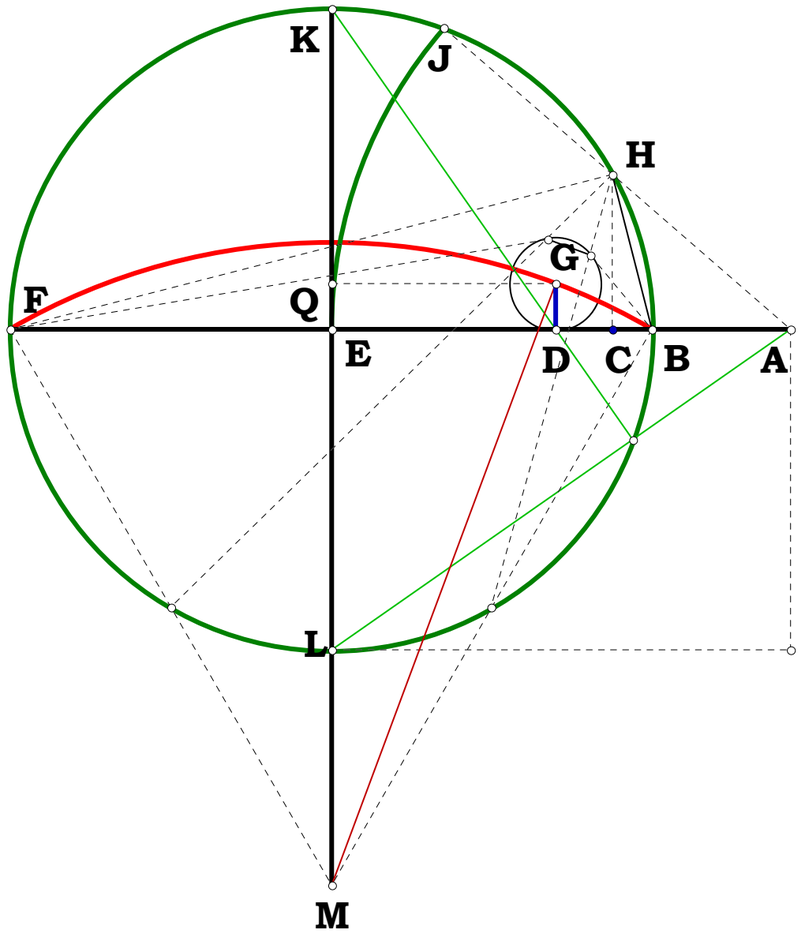
$CK := BC - BK$

$BE \cdot \frac{(N-1)}{N} - CE = 0 \quad BE \cdot \frac{\sqrt{(N-1)}}{N} - CG = 0 \quad BE \cdot \frac{(N-2)}{2 \cdot N} - CD = 0$

$BE \cdot \frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0 \quad BE \cdot \frac{N}{2 \cdot (N-2)} - AD = 0 \quad BE \cdot \frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$BE \cdot \frac{(3 \cdot N - 2)}{N^3} - BK = 0 \quad BE \cdot \frac{(N-1) \cdot (N-2)}{N^3} - CK = 0 \quad \frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

**For Any AB 061001**




**For any AB, AF what is DG?**

$N := 4.39 \quad AB := .615 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad EK := BE$

$AE := AB + BE \quad DE := \frac{EK^2}{AE} \quad EF := BE$

$FM := BF \quad EM := \sqrt{FM^2 - EF^2} \quad GM := FM$

$GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2} \quad EQ := MQ - EM \quad DG := EQ$

**Some Algebraic Names:**

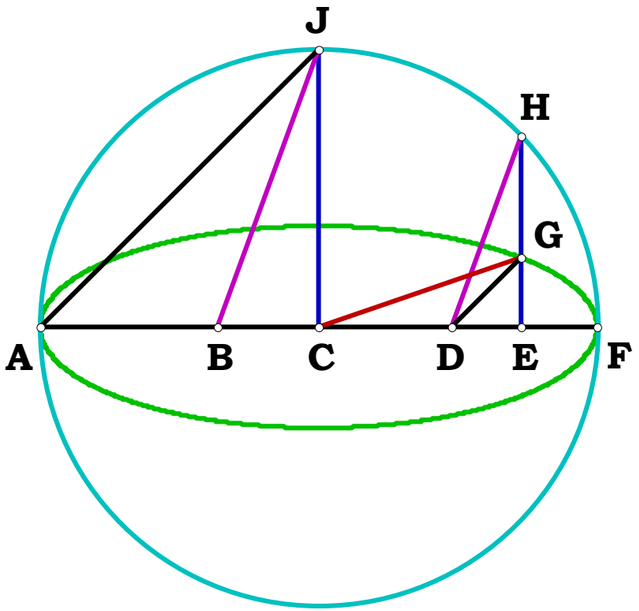
$AB \cdot (N - 1) - BF = 0 \quad \frac{AB \cdot (N - 1)}{2} - BE = 0 \quad \frac{1}{2} \cdot AB \cdot (N + 1) - AE = 0$

$\frac{1}{2} \cdot AB \cdot \frac{(N - 1)^2}{(N + 1)} - DE = 0 \quad \frac{1}{2} \cdot \sqrt{3} \cdot AB \cdot (N - 1) - EM = 0$

$\frac{AB \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)\right]}{2 \cdot (N + 1)} - MQ = 0$

$\frac{AB \cdot (N - 1) \cdot \left[\sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \sqrt{3} - \sqrt{3} \cdot N\right]}{2 \cdot (N + 1)} - DG = 0$

**Elipse By Parallels 082601**




$N_1 := 4 \quad N_2 := 1$

$AF := 1 \quad AC := \frac{AF}{2} \quad CJ := AC$

$BC := \frac{AC}{N_1} \quad AE := \frac{AF}{N_2}$

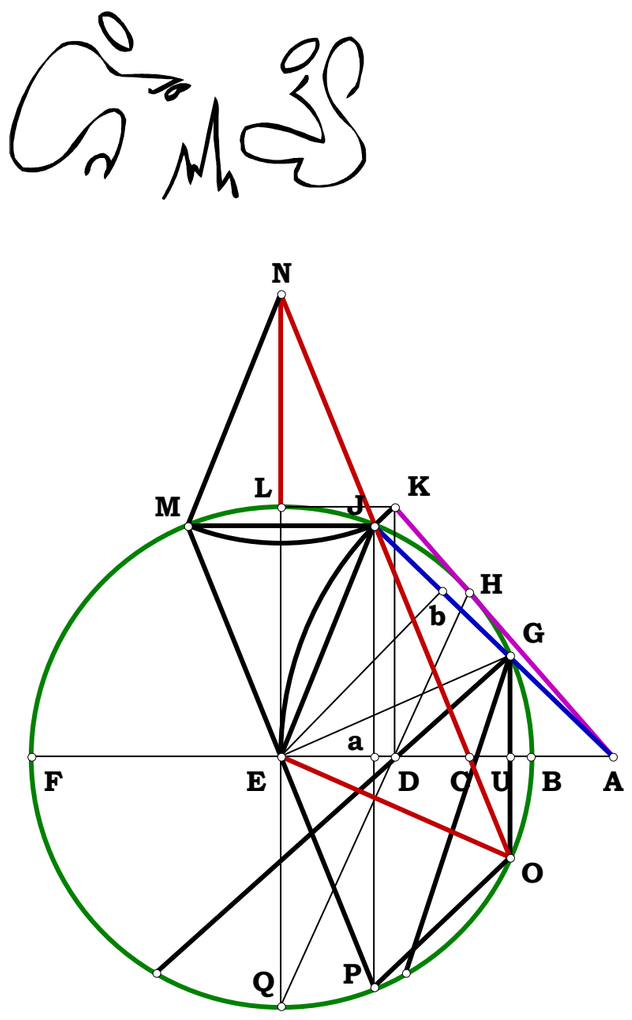
$EF := AF - AE \quad EH := \sqrt{AE \cdot EF} \quad EG := \frac{BC \cdot EH}{CJ}$

$CE := AE - AC \quad CG := \sqrt{CE^2 + EG^2}$

**Some Algebraic Names:**

$BC - \frac{1}{(2 \cdot N_1)} = 0 \quad AE - \frac{1}{N_2} = 0 \quad EF - \left(1 - \frac{1}{N_2}\right) = 0 \quad EH - \frac{\sqrt{N_2 - 1}}{N_2} = 0 \quad EG - \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2} = 0$

$CE - \left(\frac{1}{N_2} - \frac{1}{2}\right) = 0 \quad CG - \frac{1}{2} \cdot \frac{\sqrt{4 \cdot N_1^2 - 4 \cdot N_1^2 \cdot N_2 + N_1^2 \cdot N_2^2 + 4 \cdot N_2 - 4}}{(N_1 \cdot N_2)} = 0$




**Just Another Proof Of Paper 010202**

$N := 7 \quad AB := 1 \quad AF := AB \cdot N$

$BF := AF - AB \quad BE := \frac{BF}{2} \quad AE := BE + AB$

$AJ := AE \quad EJ := BE \quad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \quad GJ := 2 \cdot Gb \quad AG := AJ - GJ$

$Aa := AE - Ea \quad AU := \frac{Aa \cdot AG}{AJ}$

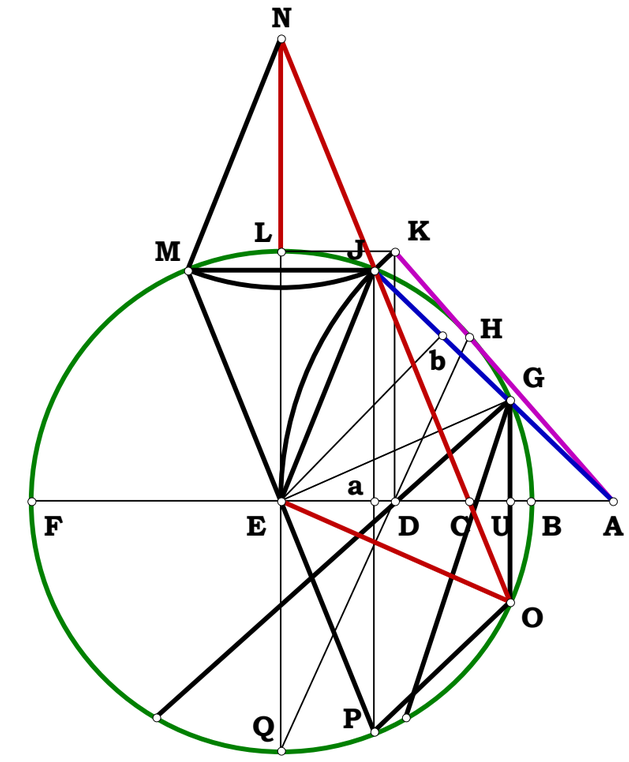
$Ja := \sqrt{AJ^2 - Aa^2} \quad GU := \frac{Ja \cdot AG}{AJ}$

$Ua := Aa - AU \quad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \quad JN - BE = 0$

From 4/29/94 $\quad OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \quad NO := JO + JN \quad EU := Ua + Ea \quad EN := \sqrt{NO^2 - EU^2} \quad EL := BE \quad LN := EN - EL$




**Some Algebraic Names;**

$$N - 1 - BF = 0 \quad \frac{N-1}{2} - BE = 0 \quad \frac{N+1}{2} - AE = 0 \quad \frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0$$

$$\frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \quad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0 \quad \frac{2\cdot N}{N+1} - AG = 0 \quad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0$$

$$\frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0 \quad \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0$$

$$\frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0 \quad \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \quad \frac{N-1}{2} - JN = 0$$

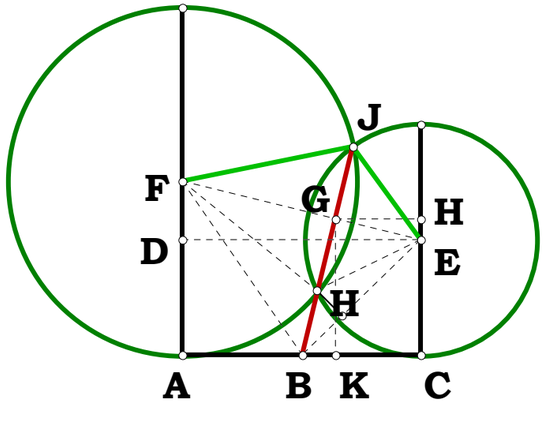
$$\frac{(N-1)^2}{2\cdot(N+1)} - OP = 0 \quad \frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \quad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$

**Easy Power-Line** 06_20_02

**For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.**




$AC := 5 \quad AF := 4$

$CE := 3 \quad FH := AF \quad EH := CE$

$AD := CE \quad DF := AF - AD$

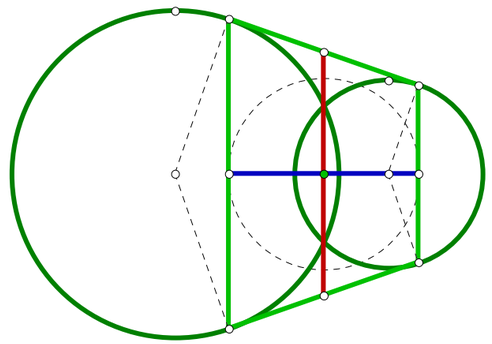
$EJ := CE \quad FJ := AF$

$DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$$EG := \frac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF}$$

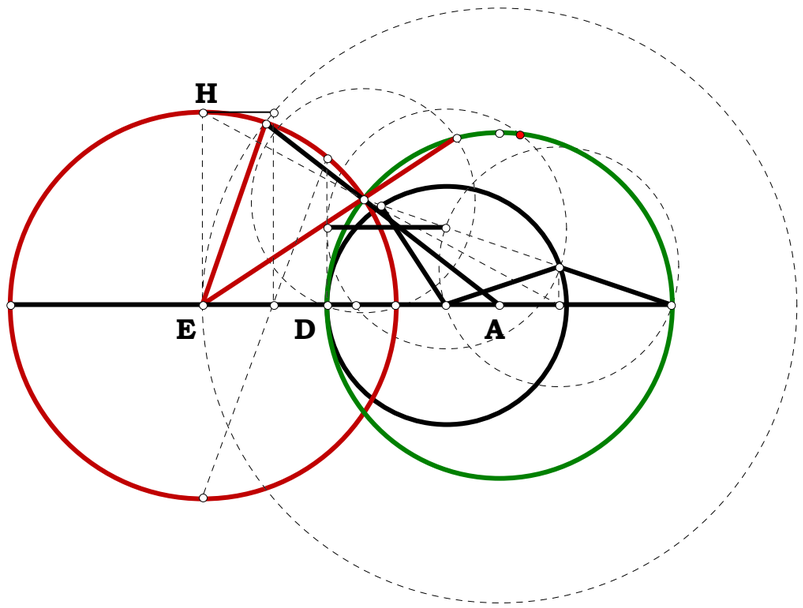
$$EH := \frac{DF \cdot EG}{EF} \quad CH := CE + EH \quad GH := \frac{DE \cdot EG}{EF} \quad CK := GH \quad KG := CH$$

$$BK := \frac{DF \cdot KG}{DE} \quad BC := CK + BK \quad BC - \frac{AC}{2} = 0$$



**021603**

**From AD project a trisection to EH.**



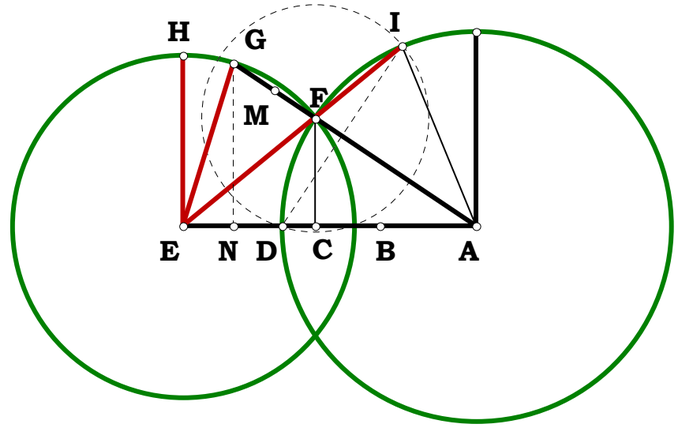

$$AD := .969 \quad AB := \frac{AD}{2} \quad N := 1.51$$

$$BD := AD - AB \quad BC := \frac{BD}{N}$$

$$CD := BD - BC \quad AC := AB + BC$$

$$AF := AD \quad CF := \sqrt{AF^2 - AC^2}$$

$$FI := \sqrt{CD^2 + CF^2}$$

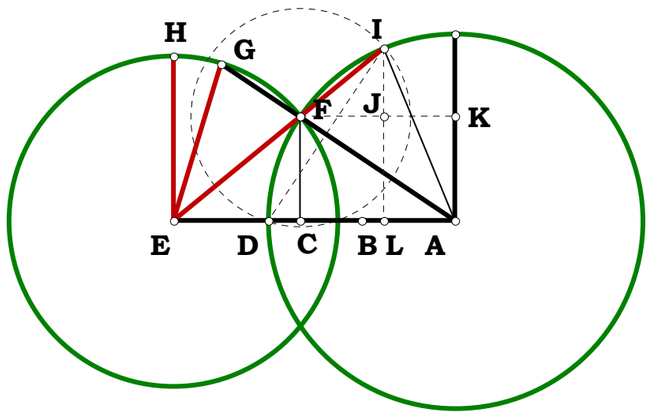


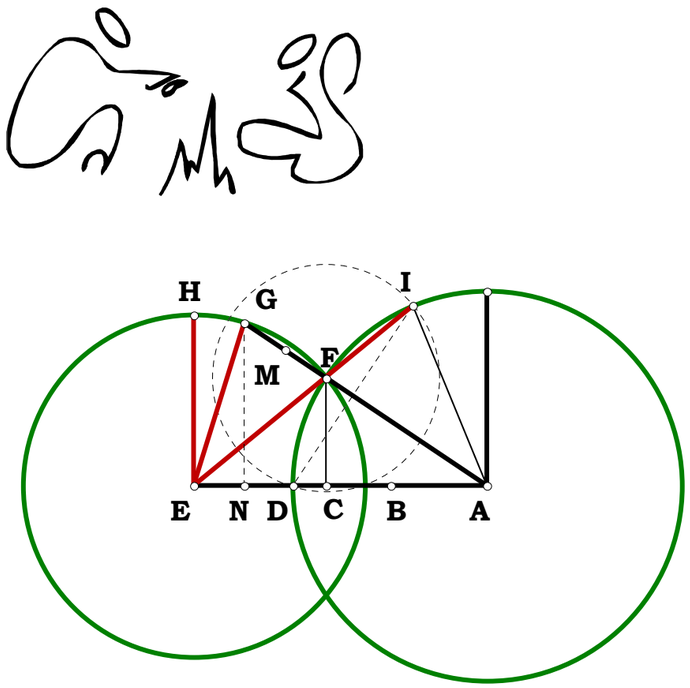

$$DI := 2 \cdot CF \quad AI := AD \quad AL := \frac{\left(2AD^2 - DI^2\right)}{2 \cdot AD}$$

$$FJ := AC - AL \quad IL := \sqrt{AI^2 - AL^2} \quad IJ := IL - CF$$

$$EL := \frac{FJ \cdot IL}{IJ} \quad AE := AL + EL \quad EI := \frac{FI \cdot IL}{IJ}$$

$$EF := EI - FI$$






$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF} \qquad AG := AF + FG \qquad AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \qquad FM := \frac{FG}{2} \qquad FM - EN = 0$$

**Some Algebraic Names;**

$$BC - \frac{AD - AB}{N} = CD - (N-1)\cdot\frac{(AD - AB)}{N} \qquad AC - \frac{(AB \cdot N + AD - AB)}{N} = 0$$

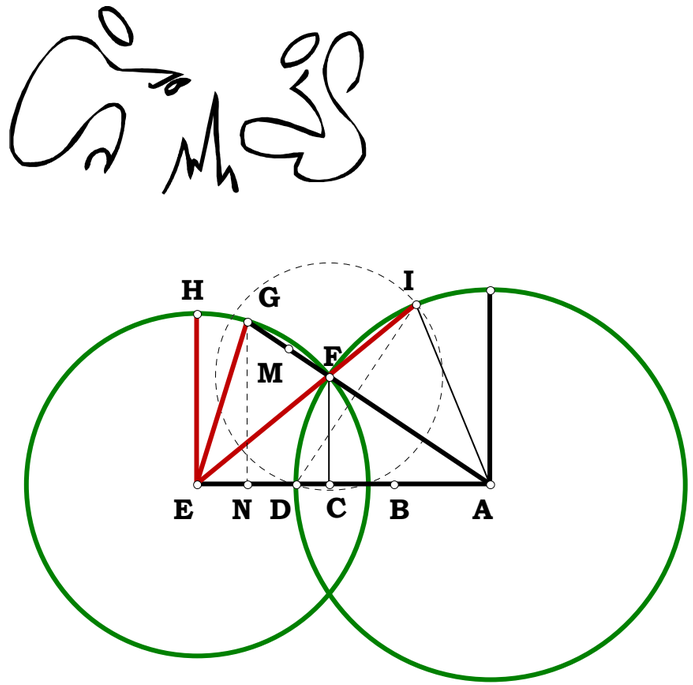
$$CF - \frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0 \qquad FI - \sqrt{2}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot\frac{AD}{N}} = 0$$

$$DI - 2\cdot\frac{\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)}}{N} = 0$$

$$AL - \frac{-\left(AD^2 \cdot N^2 - 4 \cdot AD \cdot N \cdot AB + 4 \cdot AB^2 \cdot N - 2 \cdot N^2 \cdot AB^2 - 2 \cdot AD^2 + 4 \cdot AD \cdot AB - 2 \cdot AB^2\right)}{\left(AD \cdot N^2\right)} = 0$$

$$FJ - (N-1)\cdot(AD - AB)\cdot\frac{(AD \cdot N + 2 \cdot AD - 2 \cdot AB + 2 \cdot AB \cdot N)}{\left(AD \cdot N^2\right)} = 0$$

$$IL - \frac{(AB \cdot N + AD - AB)}{\left(AD \cdot N^2\right)}\cdot\sqrt{4}\cdot\sqrt{(N-1)\cdot(AD - AB)\cdot(AD \cdot N + AD - AB + AB \cdot N)} = 0$$




$$IJ - -\sqrt{(N-1)\cdot(AD-AB)\cdot(AD\cdot N+AD-AB+AB\cdot N)}\cdot\frac{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}{\left(AD\cdot N^2\right)}=0$$

$$EL - \frac{-2\cdot(N-1)\cdot(AD-AB)\cdot(AD\cdot N+2\cdot AD-2\cdot AB+2\cdot AB\cdot N)\cdot(AB\cdot N+AD-AB)}{AD\cdot\left[N^2\cdot(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)\right]}=0$$

$$AE - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}=0$$

$$EI - 2\cdot\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot\frac{(AB\cdot N+AD-AB)}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)}=0$$

$$EF - -\sqrt{2}\cdot\sqrt{(N-1)\cdot(AD-AB)\cdot\frac{AD}{N}}\cdot AD\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}=0$$

$$FG - -2\cdot AD\cdot(AD-AB)\cdot\frac{(N-1)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}=0$$

$$AG - -AD^2\cdot\frac{N}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}=0$$

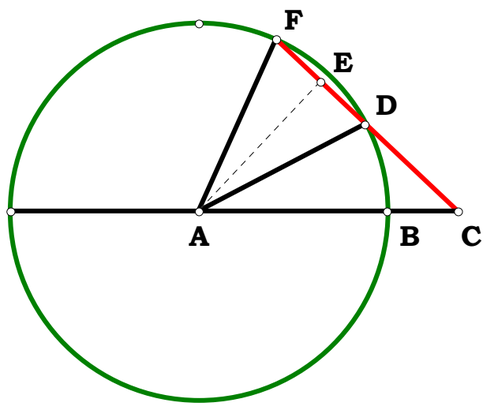
$$AN - (AB\cdot N+AD-AB)\cdot\frac{AD}{(2\cdot AB\cdot N+2\cdot AD-2\cdot AB-AD\cdot N)}=0$$

$$EN - -AD\cdot(N-1)\cdot\frac{(AD-AB)}{(-2\cdot AB\cdot N-2\cdot AD+2\cdot AB+AD\cdot N)}=0$$

$$FM - EN = 0$$

**Given AC, AB and either point of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?**

**032304**




$N_1 := 1.708 \quad N_2 := 1.649$

$N_3 := 1.24$

$AC := N_1 \quad CF := N_2$

$AF := N_3$

$$DF_1 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

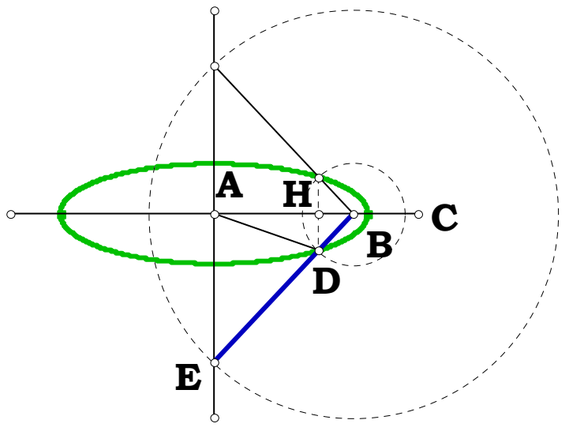
$CD := CF - D N_2 := CD$

$$DF_2 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

$DF_1 = 0.812333 \qquad DF_2 = -0.812333 \qquad DF_1 + DF_2 = 0$

041904

## Straight Line Ellipse




### Cardinal

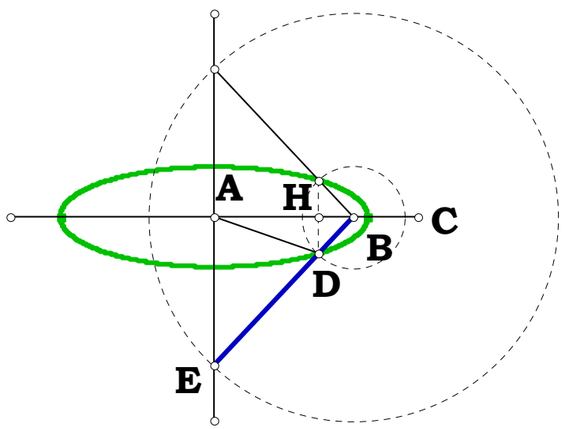
$$U := 1 \quad N_1 := 3 \quad N_2 := 2$$

$$AC := U \quad BE := AC \quad BD := \frac{BE}{N_1}$$

$$AB := \frac{AC}{N_2} \quad AE := \sqrt{BE^2 - AB^2} \quad BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$$

$$AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$$

$$\frac{U}{(N_1 \cdot N_2)} \cdot \sqrt{(N_1)^2 - 2 \cdot N_1 + (N_2)^2} - AD = 0 \quad \frac{\sqrt{(N_1)^2 - 2 \cdot N_1 + (N_2)^2}}{N_1 \cdot N_2} - \frac{AD}{U} = 0$$




### Ordinal

$$N_1 := 1.344 \quad N_2 := .3 \quad N_3 := .5$$

$$AC := N_1 \quad BD := N_2 \quad AB := N_3$$

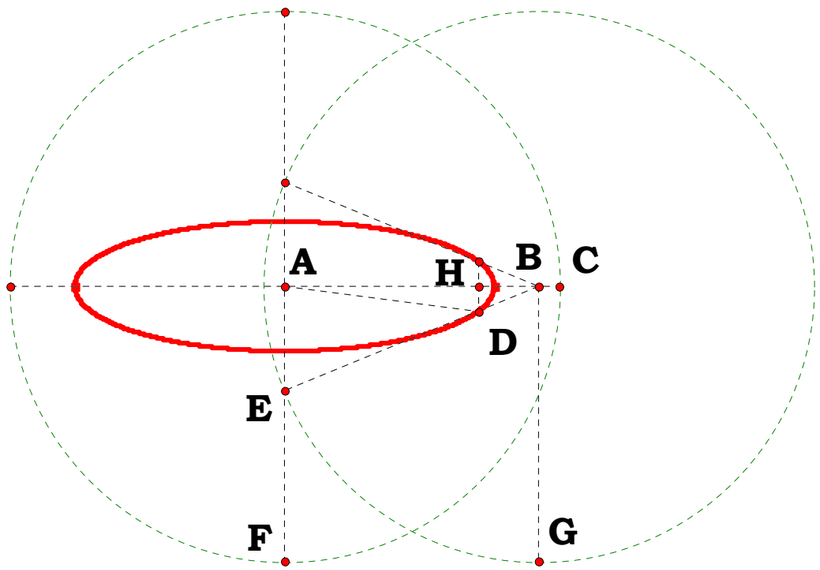
$$AF := AC \quad BE := AC \quad AE := \sqrt{BE^2 - AB^2}$$

$$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE} \quad AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$$

$$\sqrt{\frac{1}{N_1} \cdot \left[ (N_3)^2 \cdot N_1 - 2 \cdot (N_3)^2 \cdot N_2 + (N_2)^2 \cdot N_1 \right]} - AD = 0$$

**Straight Line Ellipse**

**041904B**




$N_1 := 1.344 \quad N_2 := .415 \quad N_3 := 1.102$

$AC := N_1 \quad BD := N_2 \quad AB := N_3 \quad AF := AC$

$BE := AC \quad AE := \sqrt{BE^2 - AB^2}$

$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$

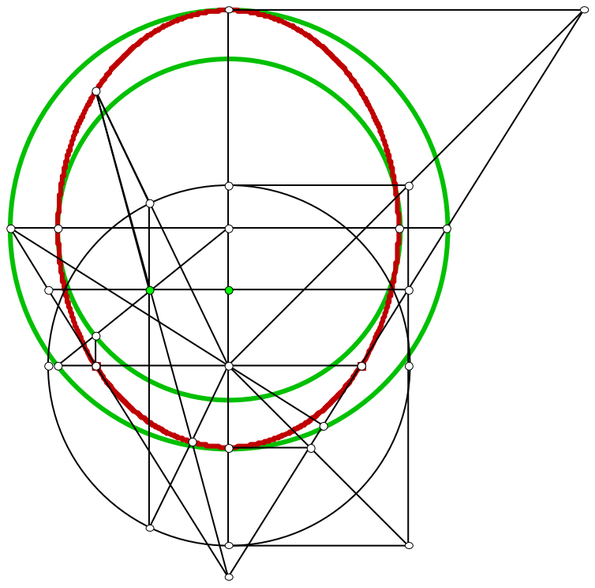
$AH := AB - BH$

$AD := \sqrt{AH^2 + DH^2}$

$$AD := \sqrt{\frac{1}{N_1} \cdot \left[ (N_3)^2 \cdot N_1 - 2 \cdot (N_3)^2 \cdot N_2 + (N_2)^2 \cdot N_1 \right]}$$

**Another Ellipse 031405a**

**The locus formed by N and I as determined by L provides an ellipse.**
**Privide an Algebraic name for the Major and Minor Axis.**



$N_1 := 1.25 \quad N_2 := 3$

$AC := N_1 \quad AD := N_2$

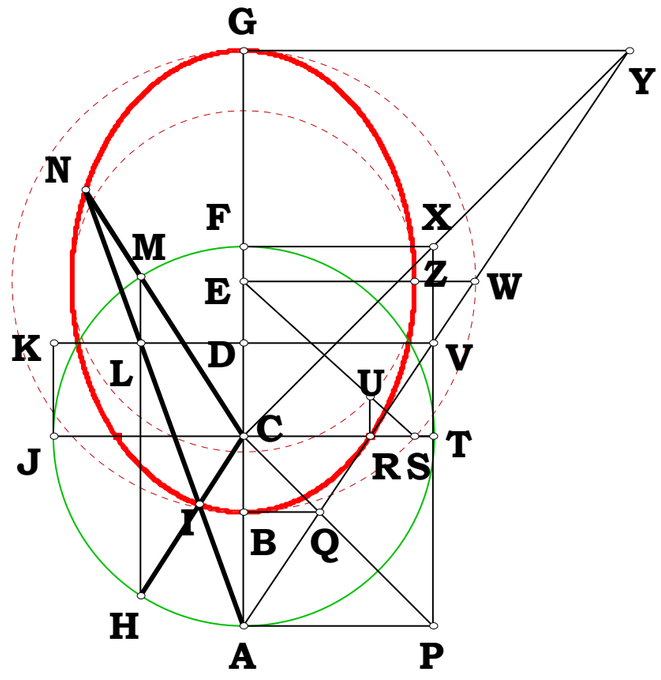
$DV := AC \quad AV := \sqrt{AD^2 + DV^2} \quad AF := 2 \cdot AC$

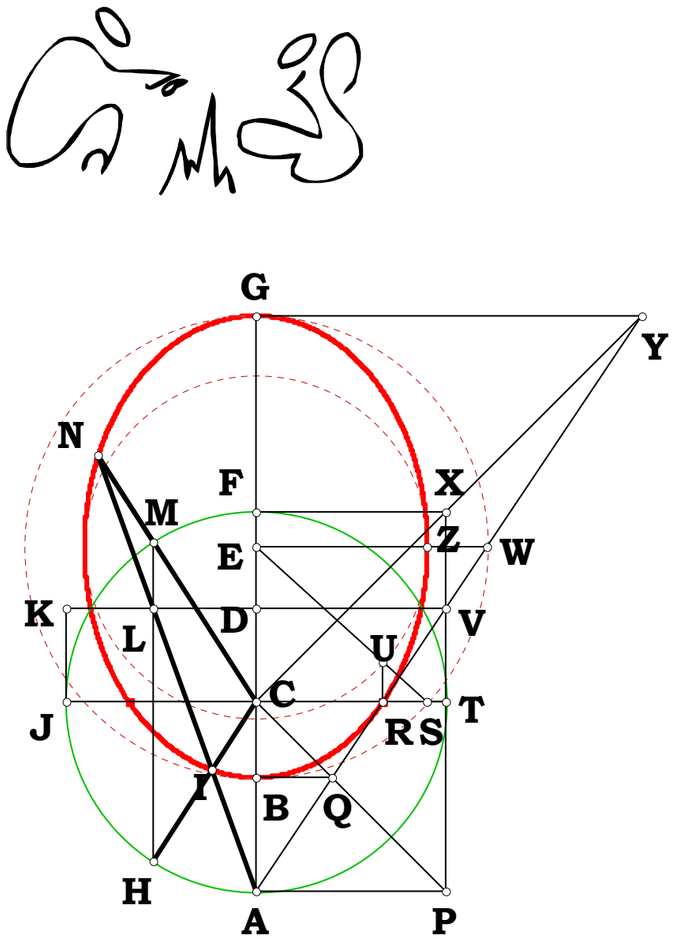
$VX := AF - AD \quad AY := \frac{AV \cdot AC}{AC - VX} \quad AG := \frac{AD \cdot AY}{AV}$

$BC := \frac{AC \cdot AC}{AC + AD} \quad CG := AG - AC \quad BG := BC + CG$

$BE := \frac{BG}{2} \quad CE := BE - BC \quad ES := BE$

$CS := \sqrt{ES^2 - CE^2} \quad CR := \frac{DV \cdot AC}{AD}$






$$EU := \frac{ES \cdot CR}{CS}$$

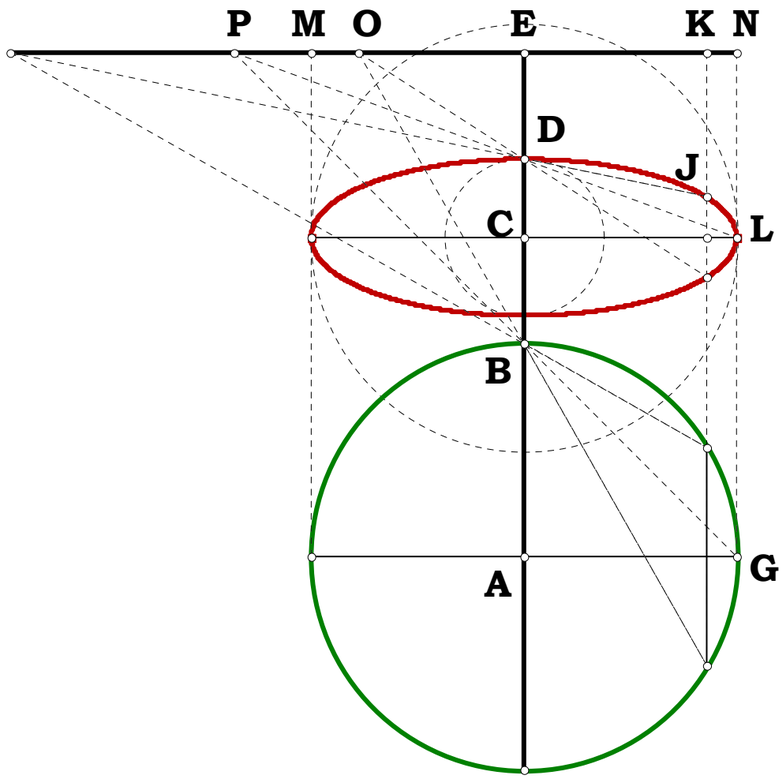
$$EZ := EU$$

$$EZ - \frac{N_1^{\,2} \cdot \sqrt{N_2^{\,2} - N_1^{\,2}}}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$

$$BG - \frac{2 \cdot N_1^{\,2} \cdot N_2}{(N_1 + N_2) \cdot (N_2 - N_1)} = 0$$

**032305a**

**An Ellipse**




$N_1 := 1 \quad N_2 := 2 \quad N_3 := .5$

$AB := N_1 \quad MN := 2 \cdot AB$

$BE := N_2 \quad BD := N_3$

$DE := BE - BD \quad AG := AB$

$$EP := \frac{AG \cdot BE}{AB} \quad EN := AB$$

$$NL := \frac{DE \cdot (EP + EN)}{EP}$$

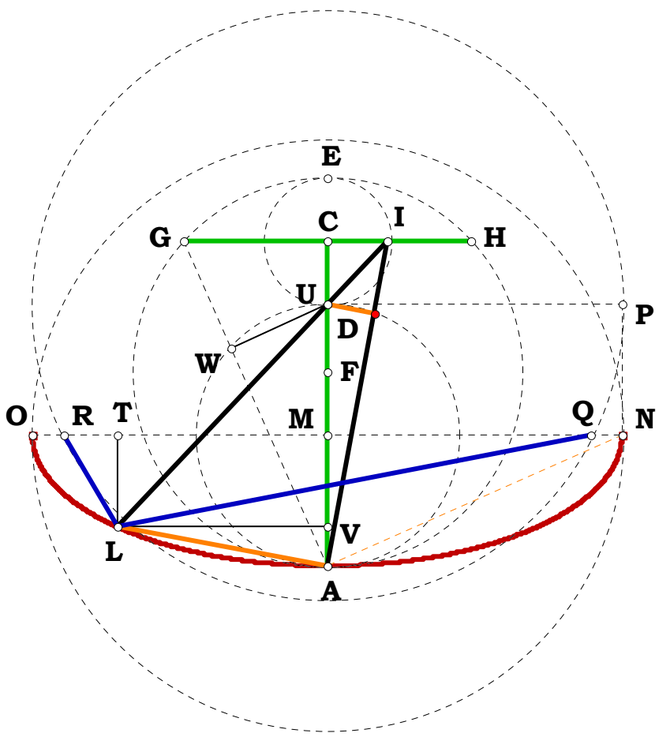
$CE := NL \quad CD := CE - DE$

$$N_1 \cdot \frac{(N_2 - N_3)}{N_2} - CD = 0$$

**032905**

**Elipse Projected From a Perpendicular.**

**Let AC be some perpendicular on some line GH.**




$N_1 := 1.9167 \quad N_2 := .3244 \quad N_3 := .437$

$AC := N_1 \quad CD := N_2 \quad CE := CD \quad CG := \sqrt{CE \cdot AC} \quad GH := 2 \cdot CG$

$GI := N_3 \quad CI := CG - GI \quad AD := AC - CD \quad AI := \sqrt{AC^2 + CI^2} \quad AM := \frac{AD}{2}$

$DM := AM \quad DU := \frac{CI \cdot AD}{AI} \quad AU := \frac{AC \cdot AD}{AI} \quad IU := AI - AU \quad AL := \frac{DU \cdot AI}{IU}$

$IL := \sqrt{AI^2 + AL^2} \quad DI := \sqrt{DU^2 + IU^2} \quad DL := IL - DI \quad DV := \frac{CD \cdot DL}{DI}$

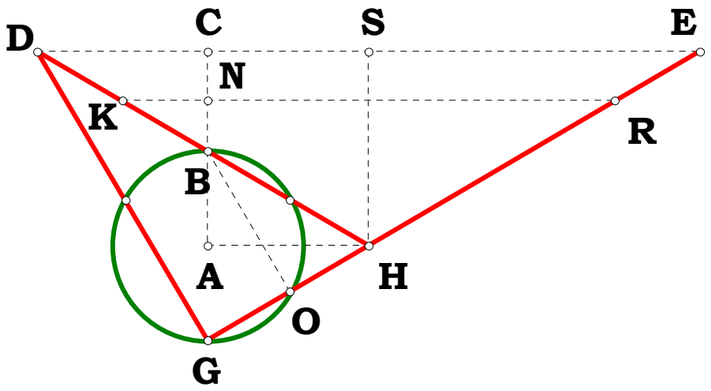
$MV := DV - DM \quad AV := AM - MV \quad LV := \sqrt{AL^2 - AV^2} \quad MT := LV$

$AG := \sqrt{CG^2 + AC^2} \quad DW := \frac{CG \cdot AD}{AG} \quad AW := \frac{AC \cdot AD}{AG} \quad GW := AG - AW$

$AN := \frac{DW \cdot AG}{GW} \quad MN := \sqrt{AN^2 - AM^2} \quad ON := 2 \cdot MN \quad ON - \sqrt{(N_2 - N_1)^2 \cdot \frac{N_1}{N_2}} = 0$

033005c

$N^3$ and More




$N_1 := 1 \quad N_2 := 2 \quad N_3 := 4 \quad A := N_2 \quad N := N_3$

AB is the unit  AH/AB is the constant one.  BC/BN is constant two.

$AB := N_1 \quad AH := N_2 \quad BH := \sqrt{AB^2 + AH^2} \quad GO := \frac{AB \cdot 2 \cdot AB}{BH} \quad HO := BH - GO$

$DH := \frac{BH \cdot BH}{HO} \quad AC := \frac{AB \cdot DH}{BH} \quad BO := \sqrt{BH^2 - HO^2} \quad DG := \frac{BO \cdot BH}{HO}$

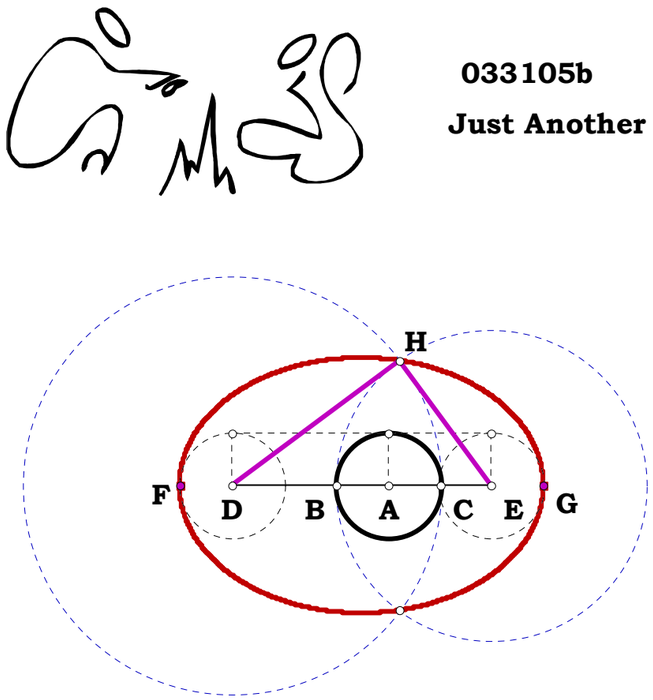
$DE := \frac{BH \cdot DG}{AB} \quad CD := \frac{DG^2}{DE} \quad BC := AC - AB \quad CE := DE - CD$

$\frac{CE}{BC} - \left(\frac{AH}{AB}\right)^3 = 0 \quad \frac{N_2^3}{N_1^3} - \frac{CE}{BC} = 0$

More $\left(A^3 \cdot N\right) - (N - 1) \cdot A$  Let $N_3$ be N

$BN := \frac{BC}{N_3} \quad KN := \frac{CD}{N_3} \quad CG := AC + AB \quad GN := 2 \cdot AB + \frac{BC}{N_3} \quad NR := \frac{CE \cdot GN}{CG}$

$\left[\frac{N_2^3}{N_1^3} \cdot N_3 - \frac{N_2 \cdot (N_3 - 1)}{N_1}\right] - \frac{NR}{BN} = 0 \quad A^3 \cdot N - (N - 1) \cdot A = 26 \quad \frac{NR}{BN} = 26$



033105b

**Just Another Ellipse**




**Given the difference between the foci and difference between the proportional radii, etc., etc.**

$$N_1 := 1.708 \quad N_2 := .693 \quad N_3 := 1.032$$

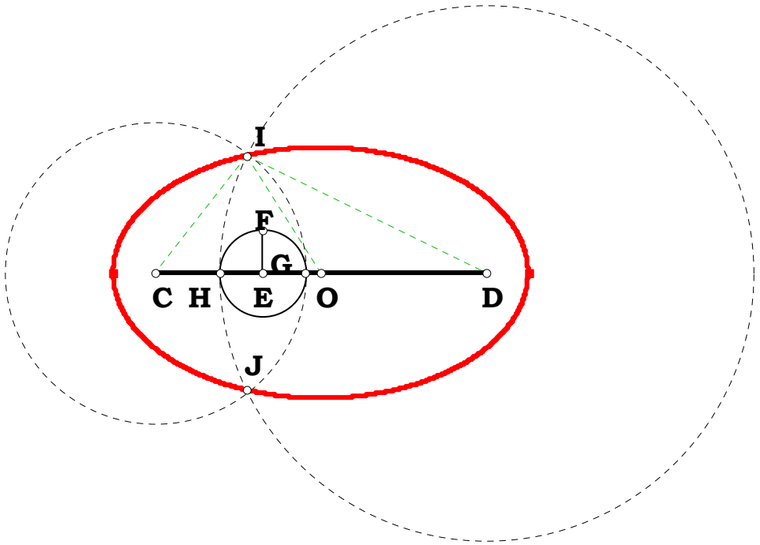
$$DE := N_1 \quad BC := N_2 \quad FG := DE + BC \quad AB := \frac{BC}{2} \quad AD := N_3$$

$$CD := AD + AB \quad BE := DE - AD + AB \quad EH := BE \quad DH := CD$$

$$DJ := \frac{DH^2 + DE^2 - EH^2}{2 \cdot DE} \qquad HJ := \sqrt{DH^2 - DJ^2}$$

$$\frac{\sqrt{-N_3 \cdot (N_3 - N_1) \cdot N_2 \cdot (N_2 + 2 \cdot N_1)}}{N_1} - HJ = 0$$

041205

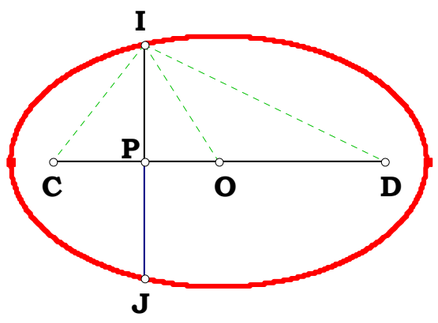



$N_1 := 2.188 \quad N_2 := .278 \quad N_3 := 3.095$

$CD := N_1 \quad CO := \frac{CD}{2} \quad EF := N_2 \quad CE := \frac{CD}{N_3}$

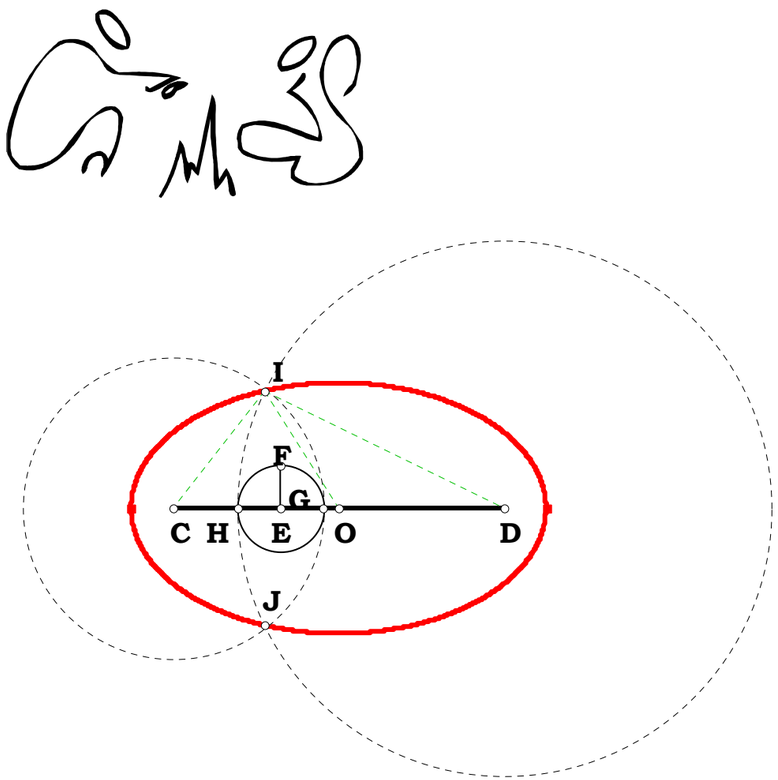
$CG := CE + EF \quad DH := CD - CG + 2 \cdot EF$

$CI := CG \quad DI := DH \qquad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$




$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$

$$2 \cdot \frac{\sqrt{N_2 \cdot (N_1 + N_2) \cdot (N_3 - 1)}}{N_3} - IP = 0$$

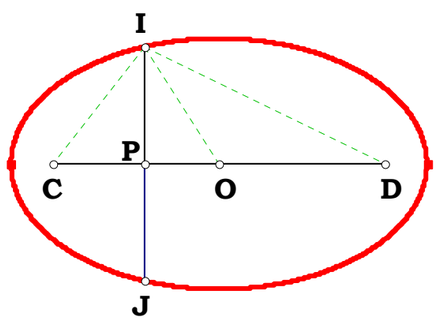



$$N_1 := 2.188 \quad N_2 := .278 \quad N_3 := .707$$

$$CD := N_1 \quad CO := \frac{CD}{2} \quad EF := N_2$$

$$CE := N_3 \quad CG := CE + EF \quad DH := CD - CG + 2 \cdot EF$$

$$CI := CG \quad DI := DH \quad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$



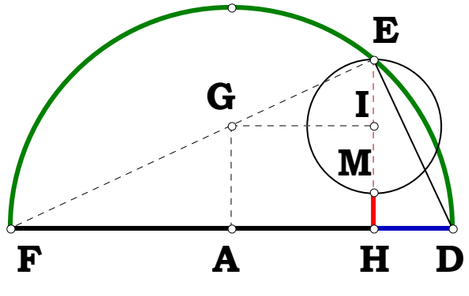

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$

$$2 \cdot \frac{\sqrt{-CE \cdot EF \cdot (CD + EF) \cdot (-CD + CE)}}{CD} - IP = 0$$

$$2 \cdot \frac{\sqrt{-N_3 \cdot N_2 \cdot (N_1 + N_2) \cdot (-N_1 + N_3)}}{N_1} - IP = 0$$

$$IO := \frac{1}{2} \cdot \sqrt{4 \cdot CE^2 + 4 \cdot EF^2 + CD^2 - 4 \cdot CD \cdot CE + 4 \cdot CD \cdot EF}$$

041305b




$N^3$ N Cubed

$N_1 := 1 \quad N_2 := 36$

$DH := N_1 \quad FH := N_2 \quad DF := DH + FH \quad AD := \frac{DF}{2}$

$EH := \sqrt{DH \cdot FH} \quad HI := \frac{EH \cdot AD}{FH} \quad HM := EH - 2 \cdot (EH - HI)$

$\frac{FH}{HM} - \left(\frac{DH}{HM}\right)^3 = 0$

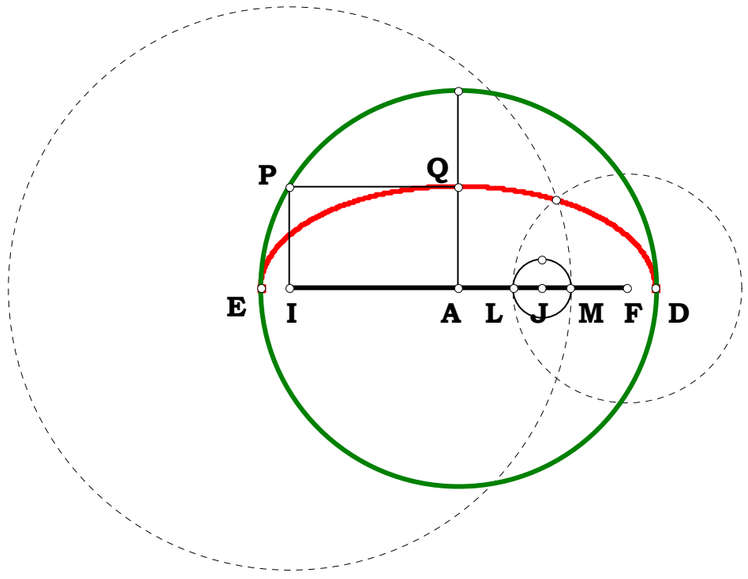
$DF := \sqrt{DH^2 + EH} \quad EF := \sqrt{FH^2 + EH^2} \quad \frac{FH}{HM} - \left(\frac{EF}{DF}\right)^3 = 0$

$\frac{FH}{HM} = 216 \quad \left(\frac{FH}{DH}\right)^{1.5} = 216$

**042205B**

**Given the major axis and the difference between the two foci, whatis the minor axis?**

$N_1 := 2.604 \quad N_2 := 2.234$



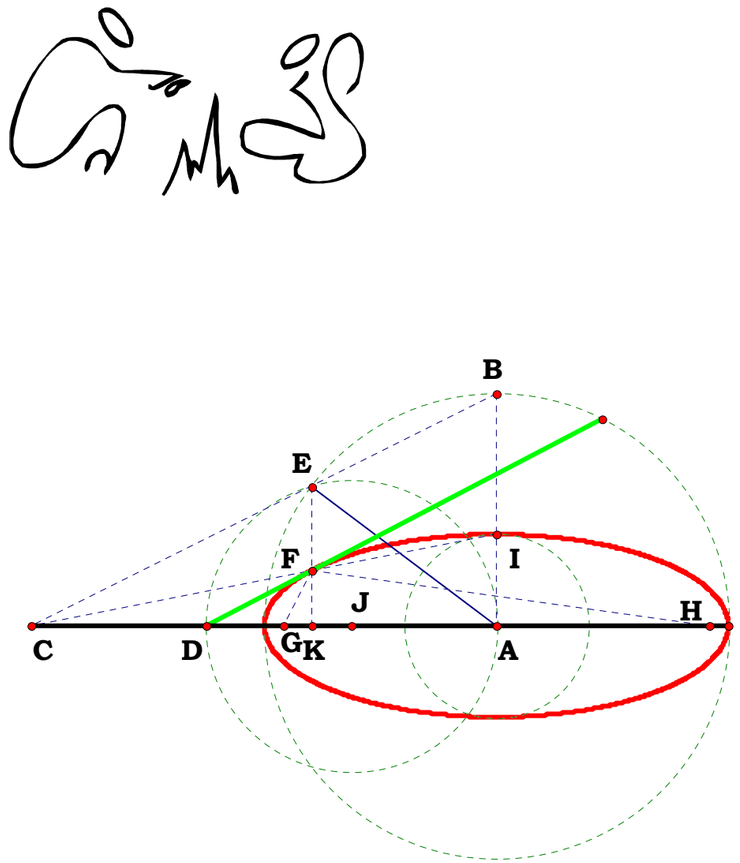

$DE := N_1 \quad FI := N_2 \quad EI := \frac{DE - FI}{2}$

$DI := DE - EI \quad EP := \sqrt{EI \cdot DI}$

$AQ := EP$

$AQ - \frac{1}{2} \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)} = 0 \quad N_2 := AQ$

$FI - \sqrt{N_1^{\,2} - 4 \cdot N_2^{\,2}} = 0$




**062007 C.mcd**

**Tangent from Major Axis**

$N_1 := 3.17500 \quad N_2 := 1.24993 \quad N_3 := 4.23333$

$AB := N_1 \quad AI := N_2 \quad AD := N_3$

$AJ := \frac{AD}{2}$ From 080193 $EK := \frac{AB \cdot \sqrt{(2 \cdot AJ - AB) \cdot (2 \cdot AJ + AB)}}{2 \cdot AJ}$

$AK := \sqrt{AB^2 - EK^2} \quad DK := AD - AK \quad FK := \frac{AI \cdot EK}{AB} \quad DF := \sqrt{DK^2 + FK^2}$

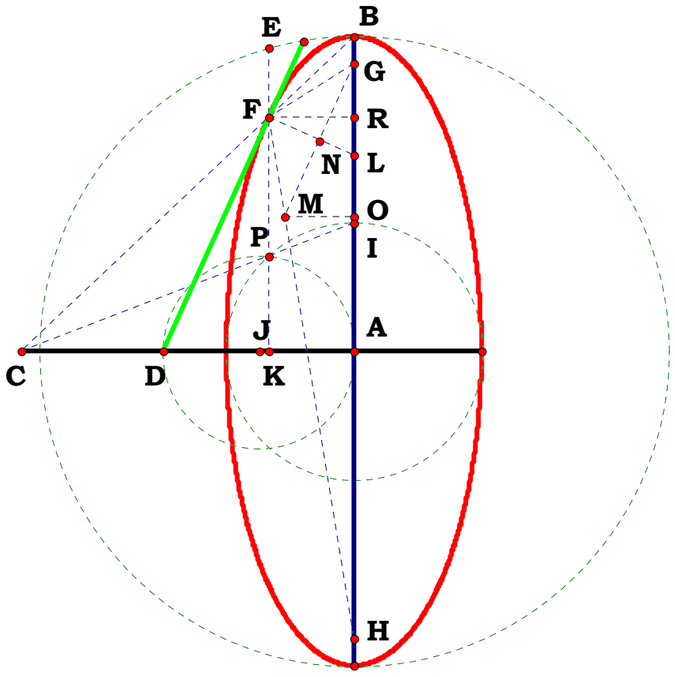
$AG := \sqrt{AB^2 - AI^2} \quad AH := AG \quad HK := AH + AK \quad GK := AG - AK$

$FG := \sqrt{FK^2 + GK^2} \quad FM := FG \quad FH := \sqrt{HK^2 + FK^2} \quad HM := FH - FM$

$HO := \frac{HK \cdot HM}{FH} \quad GH := 2 \cdot AH \quad GO := GH - HO$

$MO := \frac{FK \cdot HM}{FH} \quad \frac{GO}{MO} - \frac{DK}{FK} = 0 \quad N_2 \cdot \frac{\sqrt{(N_3 - N_1) \cdot (N_3 + N_1)}}{N_3} - FK = 0$

062007 D.mcd




**Tangent from Minor Axis**

$N_1 := 3.78354 \quad N_2 := 1.53747 \quad N_3 := 2.43417$

$AB := N_1 \quad AI := N_2 \quad AD := N_3$

$AJ := \frac{AD}{2} \quad PK := \frac{AI \cdot \sqrt{(2 \cdot AJ - AI) \cdot (2 \cdot AJ + AI)}}{2 \cdot AJ} \quad AK := \sqrt{AI^2 - PK^2}$

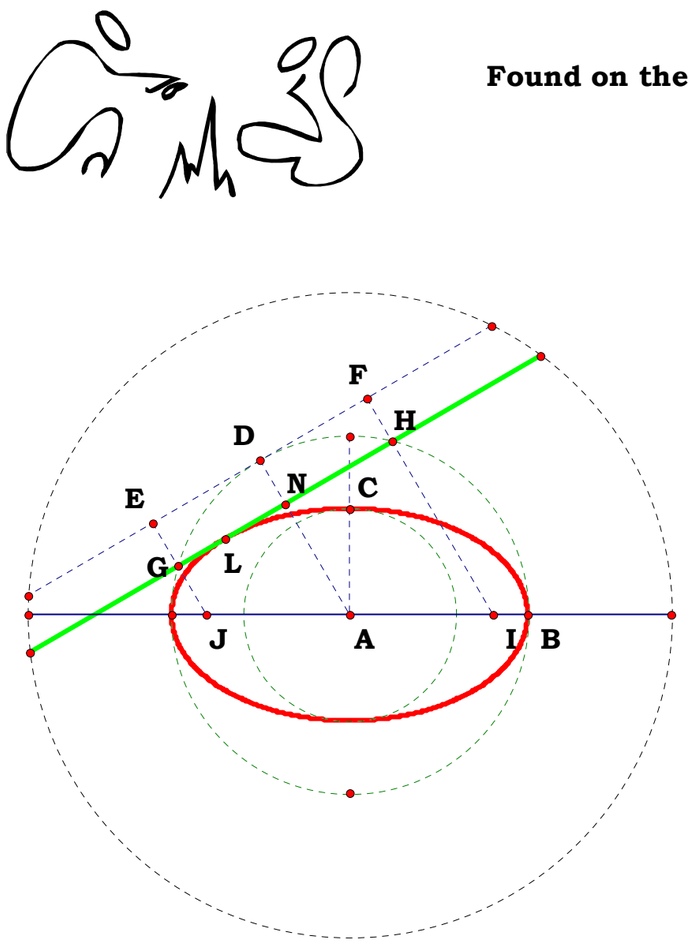
$DK := AD - AK \quad FK := \frac{PK \cdot AB}{AI} \quad DF := \sqrt{DK^2 + FK^2} \quad AG := \sqrt{AB^2 - AI^2}$

$AH := AG \quad HR := AH + FK \quad GR := AG - FK \quad FG := \sqrt{AK^2 + GR^2} \quad FM := FG$

$\frac{\sqrt{(N_3 - N_2) \cdot (N_3 + N_2)}}{N_3} \cdot N_1 - FK = 0 \qquad Major := N_2 \cdot \frac{\sqrt{(N_3 - N_1) \cdot (N_3 + N_1)}}{N_3}$

$FH := \sqrt{HR^2 + AK^2} \quad HM := FH - FM \quad HO := \frac{HR \cdot HM}{FH} \quad GH := 2 \cdot AH$

$GO := GH - HO \quad MO := \frac{AK \cdot HM}{FH} \quad \frac{GO}{MO} - \frac{FK}{DK} = 0$



**Found on the Internet 062407**




**Found the construction, now I explore it with Algebra.**

$N_1 := 2.98979 \quad N_2 := 1.77791 \quad N_3 := 1.83972$

$AB := N_1 \quad AC := N_2 \quad EJ := N_3$

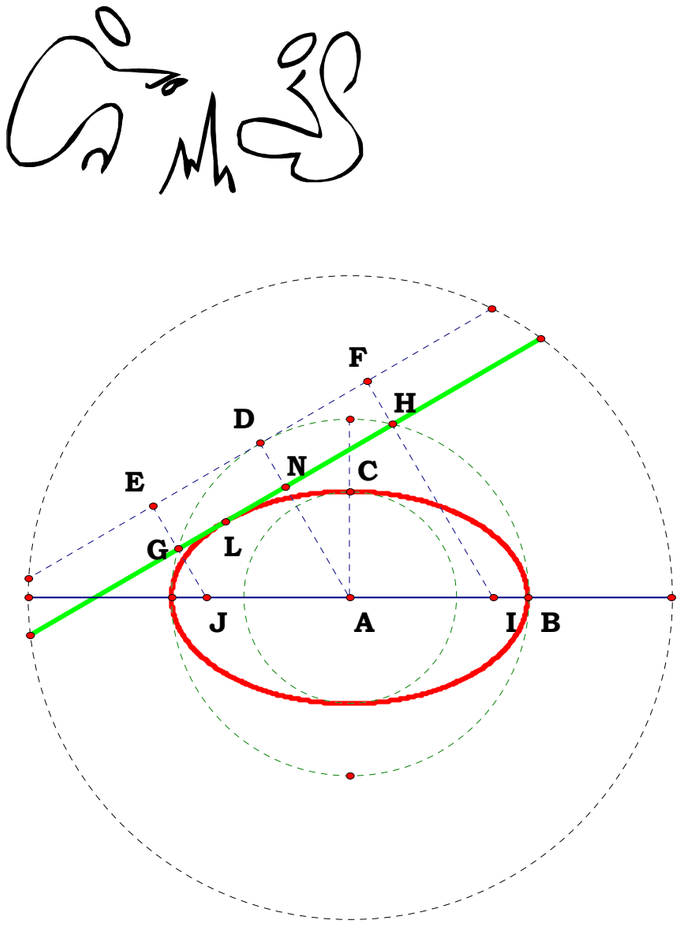
$AJ := \sqrt{AB^2 - AC^2} \quad FI := (AB - EJ) + AB$

$DE := \sqrt{AJ^2 - (AB - EJ)^2} \quad AN := \dfrac{\sqrt{DE^2 \cdot (DE + AB) \cdot (-DE + AB)}}{DE}$

$DN := AB - AN \quad GJ := EJ - DN \quad HI := FI - DN \quad GL := \dfrac{GJ \cdot 2 \cdot DE}{GJ + HI}$

$HL := 2 \cdot DE - GL \quad JL := \sqrt{GL^2 + GJ^2} \quad IL := \sqrt{HI^2 + HL^2}$

$(JL + IL) - 2 \cdot AB = 0$




**Definitions:**

$$AJ - \sqrt{N_1^2 - N_2^2} = 0 \qquad FI - (2 \cdot N_1 - N_3) = 0$$

$$DE - \sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2} = 0 \qquad \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - AN = 0$$

$$N_1 - \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - DN = 0$$

$$N_3 - N_1 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - GJ = 0$$

$$\left(N_1 - N_3 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}\right) - HI = 0$$

$$\left(N_3 - N_1 + \sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}\right) \cdot \frac{\sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2}}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}} - GL = 0$$

$$\sqrt{-N_2^2 + 2 \cdot N_1 \cdot N_3 - N_3^2} \cdot \frac{\left(\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2} - N_3 + N_1\right)}{\sqrt{N_1^2 + N_2^2 - 2 \cdot N_1 \cdot N_3 + N_3^2}} - HL = 0$$

$$N_1 \cdot \frac{\sqrt{2 \cdot N_1^2 + N_2^2 + 2 \cdot N_3^2 + 2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2} \cdot (N_3 - N_1) - 4 \cdot N_1 \cdot N_3}}{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2}} - JL = 0$$

$$N_1 \cdot \frac{\sqrt{2 \cdot N_1^2 + N_2^2 + 2 \cdot N_3^2 + 2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2} \cdot (N_1 - N_3) - 4 \cdot N_1 \cdot N_3}}{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_3 + N_2^2 + N_3^2}} - IL = 0$$

# My Name is John.

Hello. My name is John and I am going to explain how to multiply and divide a line by a line in Geometry. Now, if you are going to ask me if I am a geometer, I have to reply by myth. Explanation by myth is one the ancient Greek's methods of teaching by discourse.

Once upon a time, God created man; They created him male and female, in the image of God. Or one can say, male and female created They him, which is rather awkward, but it does have that ancient New England flair to it. At any rate, once upon a time is not this time. It came to pass as men multiplied on the earth that men started to work for a living and not being god's themselves needed a way to designate each other and so individuals, which are not, by definition man started calling each other by their craft. That is where we got Mr. Smith and Mr. Clark, etc. A vestige of this remains today. Not being man, we tend to think of each other by an assigned craft. I work in a factory, but my name is Clark. The conflict of course is why I spend the entirety of my wages in therapy.

Now this works to my advantage. I have learned that individuals calling themselves geometers (I am personally hoping for the day I become part of man) cannot multiply and divide a line by a line. So, I guess one could say, that a geometer is someone who cannot do the math, which is really a sign for some serious expenditure on therapy—and, if those in mind—field knew what they were doing, the outlay would be advantageous. Too bad they cannot define a man. Now a non-Euclidean Geometer is someone who not only cannot do the math, they demand, as part of their initiation rights, that one will never be able to do the math. So, in due respect to non-Euclidean Geometers, please stop reading and go back to your scribbling—and contradicting yourself. Doing geometry inside of or on the outside of a tennis ball, or a Frisbee, makes me think that one has spent way to many days on the court, spiking one's tea, and certainly missing the ball.

Now, if your like me, a factory worker, and someone were to give you two lines and say,

Hey, you (He is hairy and has a club). Here are two lines, show me how to multiply one by the other, and after that, show me how to divide one by the other.

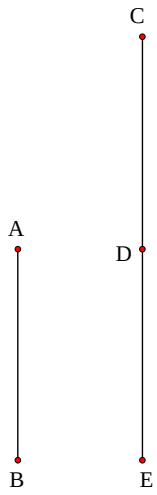
I would look at the man, think for a moment and draw a blank. What the heck does he mean? Then I would say, I am sorry, but I don't understand

what you mean. The man would leave off and I would go get another cup of coffee.

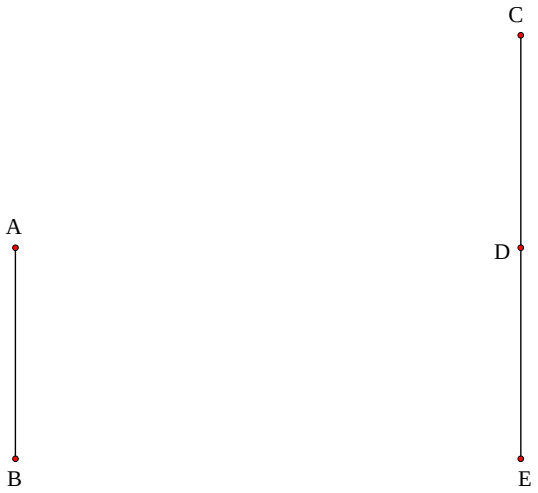
If I were a bit strange, I would consult Euclid's ***Elements*** and find to my dismay, the chap could do the math, but seems to have left this off for some reason, probably because it was too easy (so who don't lie for a friend?). Now, I happen to have in my possession a number of unpublished manuscripts which does have the answer in them and they are full of doing the math. I acquired them from the God's (and for those of you interested, the Delian Problem does have a solution—and it has something to do with Plato under extending himself). If it should be discovered that I am stealing a bit of fire, and giving it to man, please don't tell where you got it from. I have learned from first hand experience, you don't want to mess with Them—they be giants—really, really, big giants.

Now I am not going to explain this exactly as it was explained to me, as I have a poor memory. Please bear with me.

If I were given two lines, and asked to compare them, I would look at them and say;

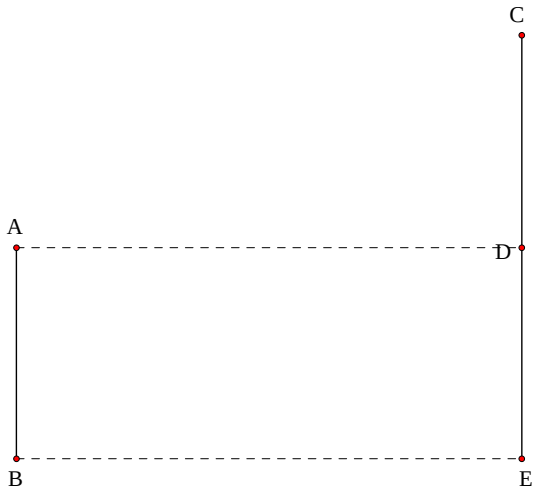



well, AB is shorter than CE. I mean, what can you do with two lines anyway. Reminds me of when I was a kid asking my mother what could I do with seven cents, realizing early on I was three cents short of a dime. If I were Euclid I would subtract one from the other and find that CE — AB = CD, or if you're a top down programmer, CE — AB = DE. If I move CE off a ways,

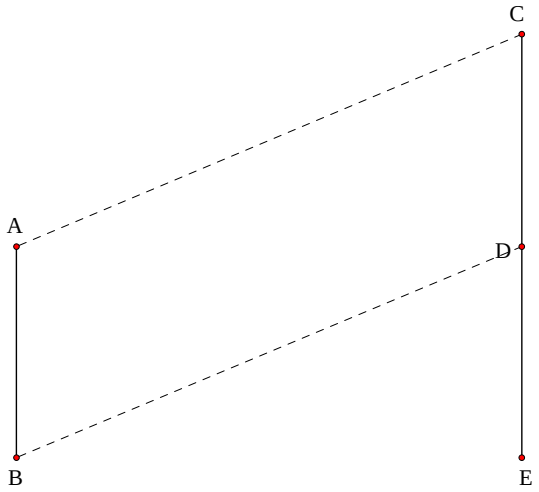



I would say that CE — AB = CD, or DE which ever you choose. Non-Euclidean Geometers, like Einstein, claim that this equality, this simultaneity, is not true and that at some point of moving AB and CE apart, as if it were part of the equation, does mysterious things to these segments. It amounts to a thief's logic—moving CE off sufficiently will make AB infinitely greater than CE 'cause we exact a kind of tribute on it and subtract that tribute as we go. It amounts to constructing a square say, of 25 square inches or so, and claiming if we repeat it enough, well, it just plain disappears—we wore it out. While on the other hand, there are those who claim that if I assert a point an infinite number of times, I can create a line. You know, like waving a knife in the air an infinite number of times an making a salad. This is the kind of mentality that makes credit card lenders rich. As I said, non-Euclidean Geometers are really crooked bankers in disguise—or really lousy cooks. A basic fact of abstraction, when you really know that a boundary is not the difference (a point is that which has not part), a form is in fact absolute, you know you can never attribute difference to that form, the form is applied as a boundary to any given difference—material. The cut is not the cutted! Wow, that was trashy!

Now if I had AB, and wanted to construct CE from it.



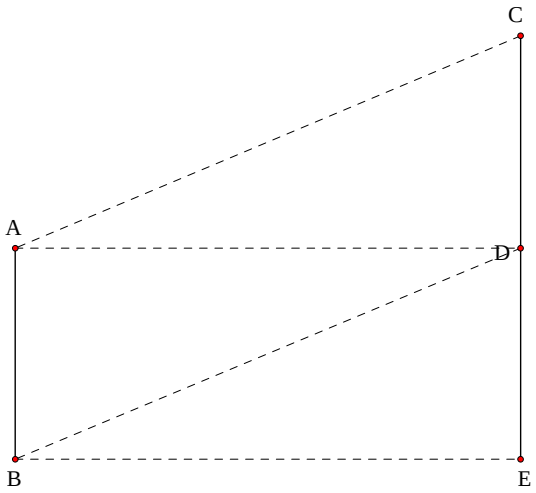

I could transfer one segment at a time



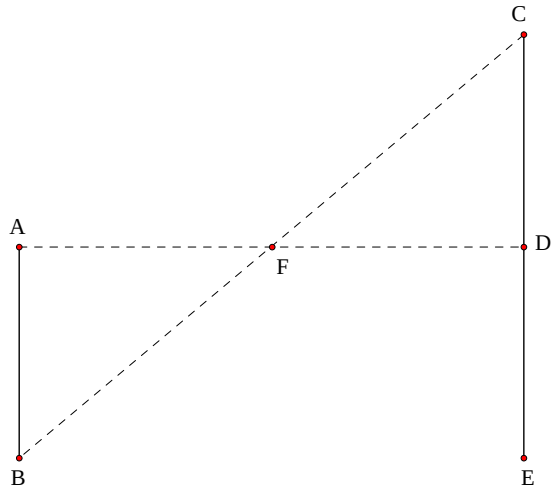

using parallel lines, but this is not multiplication, it is multiple processes, or simply addition. Parallel lines gives us the ability to do multiple additions, which is again not multiplication. One sign of that is that we have to assert each unit point in constructing CE. We have to assert each unit point just to do the parallels. Duh!

One of the things our ancient quibbling buddies, the Greeks, did tell us is that in order to multiply and divide, we have to have a unit. This is just part of plain simple Arithmetic. And they also said that when dealing with numbers in multiplication and division we were dealing with square and oblong (rectangular) numbers. Keep these ideas in mind. A square, an oblong, and a unit. Euclid drew a number of them. We will have need of them. For the moment let us learn what they did say about ratio,

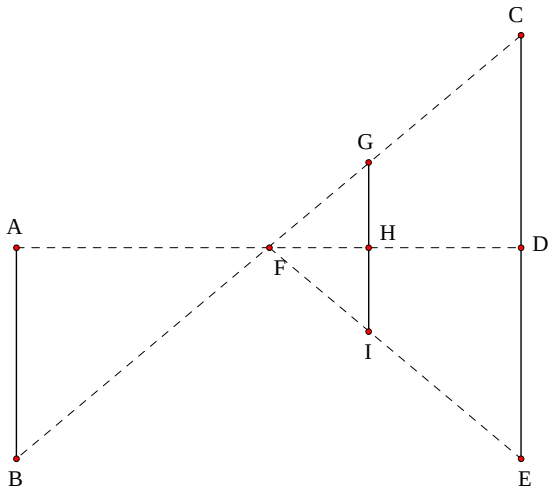
which we will also need. Now, if in constructing CE, we stayed up too late;—




and made a mistake in drawing—or were simply dyslexic;



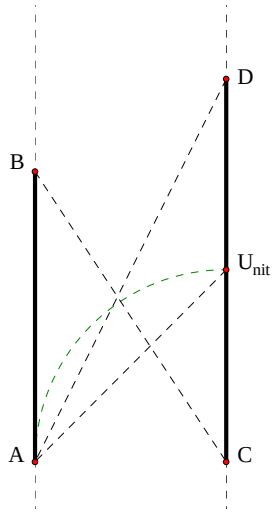

we would discover the ratio. As AB is to CD, so AF is to DF. And by George—(if you remember, he too was a hairy fellow and curious), One learns how to take any multiple and divide another segment of any length by the same multiple. From multiple addition, we have a kind of multiple division, but it is not division, it is still just a plain ratio, of anther segment.



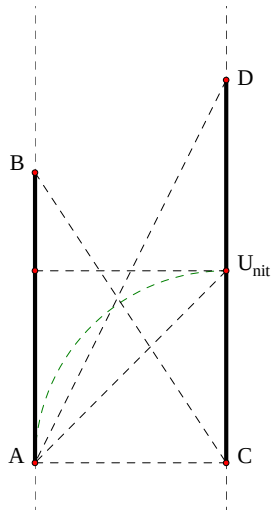

Now, as AB is to GH, so to DE is to HI, etc., etc. This is all fine and good, but, we still have not really learned how to multiply and divide. That is because these ratio's work regardless of the notion of unit, or square. Unless you are a crooked banker or a non-Euclidean Geometer, or a bad cook, this relationship is always true. There is one, and only one, difference between two points.

We are building our ideas up, one standard at a time. Intellectually, we fail, at the point we cannot abstract and use a standard—or what Plato called ***form*** because a boundary is not a difference and by definition (not a difference) always true. The divergence of language itself, starts with the inability to establish a standard even for a name. Many linguists call it the "growth" of language when meaning changes, but then they are non-Euclidean Geometers at heart also. What do they say of a government that has got its constitution saying exactly the opposite of what is written? If you want to reduce them to rubble, ask them outright, *Why can one word be or not be predicated of another*? Or again, if definition is conventional, and meaning can never be conventional, what in the heck does meaning have to do with definition? or even language? They will either get a funny look on their face mumbling to themselves, or start babbling non-sense to you. I have some books by the gods on that topic also. It is really simple, . . . but not here, not now.

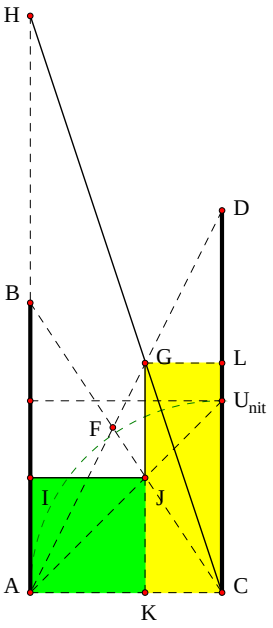
Multiplication and division rely on a standard in unit. So lets add that and see where we go.




At the outset the figure is very shy and unassuming. If you saw it laying in the street, you would hardly be pressed to pick it up. We have placed our segments the difference of our chosen unit apart, and we do have a square. No offence to Descartes who tried to find what I am doing, we don't have a number line, but a lined number. First time I ever seen a studious use of cross hairs actually miss the target.

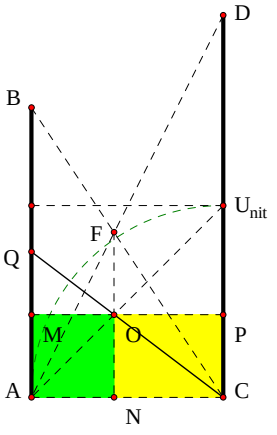



It don't look like much, but it can not only multiply and divide, one can use it to do much in the way of exponential manipulation as well. Let us take a closer look as to what the figure tells us.




$$\frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} - \frac{\text{(Area IJKA)}}{\text{(Area KGLC)}} = 0.00000$$

This is how we perform multiplication. Given AC as our unit, AB × CD = AH. In order to see this using the Arithmetic Grammar system, We divide AC by AC and get 1, our Unit. We then divide AB by AC which gives AB in terms of our unit. We then divide CE by AC and acquire that in units, and again for AH. We will find that by using the notion of Unit, Square and Oblong Numbers, which is incorporated in the idea of ratio, we can Multiply. And we can do what no binary calculator will ever do, we do it exactly. What about division?




$$\frac{\text{(Area MANO)}}{\text{(Area ONCP)}} - \frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} = 0.00000$$

Wouldn't you know it, there is a triplicate ratio in the figure! Right under our pencil. Didn't Euclid write that it was the hardest thing to do in geometry? Well, I have never taken geometry in school and set out to comprehend the triplicate ratio, guess I got somewhere. Going through our steps as before, we find that AB ÷ CD = AQ. Each of these steps is proven individually in Euclid. I suspect he was like Plato and wanted to see if his readers were smart enough to add and subtract ideas. And again, no binary computer will ever be up to Geometry, as Geometry is exact.

One can do a whole lot with this figure, through various projections. One can do a lot in the way of exponential manipulation. Try that with cross hairs! Some of the methods one will find in those unpublished books I was talking about. I don't know how long the gods will let me work on them, in fact, if it were not for Them, I would have been killed over thirty years ago. Imagine that, I am a walking contradiction, a living dead man. At any rate, I hope you have fun playing with the figure.

Now this is not the place to show the solution to the Delian Problem. My god, if one is just learning the simple four, by adding multiplication and division to our list of addition and subtraction, it may be too difficult realize a revolution in Euclidean Geometry based upon a standard long ago recognized but left unemployed—just like these. I will put the idea in the Geometer's Sketchpad file.

I hope I have made it clear that through multiple addition and subtraction, one leads into the understanding of ratio, just like Euclid did, but it is still a step away from multiplication and division. Those depend upon a respect for, and understanding of a standard in definition. We learn to add, and subtract. These teach us ratio—it is part of them. We learn about the units which is taught by them also. This then leads to multiplication and division and our primary four are thus established.

I do have some food for thought though. Using the facts of conventions in language, can you count the ways non-Euclidean geometries commit self-referential errors in simple logic? Apparently not, they are popular. Maybe it has something to do with linguist waving their knife in the air constructing sentences. What is prediction? Maybe I will read it to you sometime. The solution was once written on a Temple "Know Thyself." I will say this, as a sense system, the human mind is suppose to abstract form and create things with it. To deny form as the foundation for thought is simply a sign of dysfunction. I know, look at me.

# Multiplication And Division of Lines

**1. An unit is that by virtue of which each of the things that exist is called one.**
**Euclid's Elements**

312

**The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.**

**John Clark**

© JClark8659@hotmail.com

Printed via import to MS Word.

# Following the Yellow Brick Road

**Multiplication**

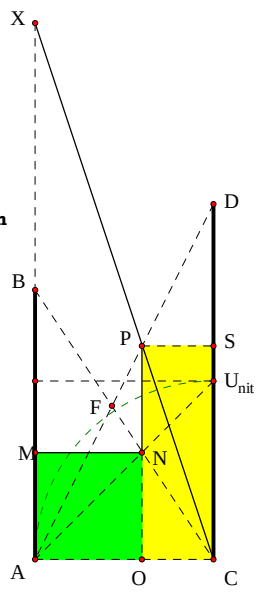
**Unit = 1.00000**

**AB = 1.51095**

**CD = 1.99270**

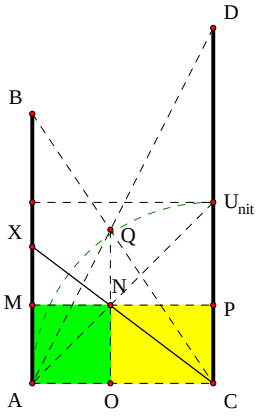
**AX = 3.01087**

**AB·CD-AX = 0.00000**




$$\frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} - \frac{\text{(Area MNOA)}}{\text{(Area OPSC)}} = 0.00000$$

**In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.**




$$\frac{\text{(Area MAON)}}{\text{(Area NOCP)}} - \frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} = 0.00000$$

**Division**

**Unit = 1.00000**

**AB = 1.48921**

**CD = 1.96403**

**AX = 0.75824**

$$\frac{\text{AB}}{\text{CD}} - \text{AX} = 0.00000$$

# Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

# Contents

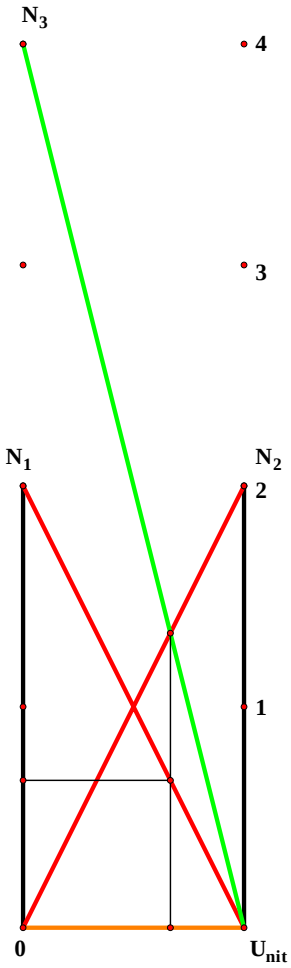
| Function | Page |
|---|---|
| $(N_1 \cdot N_2) - N_3 = 0.00000$ | Link to 1 |
| $\frac{N_1}{N_2} = 1.13725$ $\frac{N_1+N_2}{N_1} = 1.87931$ $\frac{N_1+N_2}{N_2} = 2.13725$ | Link to 2 |
| $\frac{1}{N_1} - N_3 = 0.00000$ | Link to 3 |
| $\frac{N_1}{N_2^2} - N_4 = 0.00000$ $\frac{N_1}{N_2^3} - N_5 = 0.00000$ | Link to 4 |
| $2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$ | Link to 5 |
| $\frac{N_1^2}{(N_2+N_1) \cdot N_2} - N_3 = 0.00000$ | Link to 6 |
| $(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$ | Link to 7 |
| $N_3^2 - \frac{BC}{BD} = 0.00000$ $N_3^3 - \frac{BC}{BE} = 0.00000$ $N_3^4 - \frac{BC}{BF} = 0.00000$ | Link to 8 |
| $\frac{N4_2}{N2_2} = 1.39420$ $\frac{N2_2}{N1_2} = 1.39420$ $\frac{N1_2}{Unit_2} = 1.39420$ $\frac{Unit_2}{N3_2} = 1.39420$ | Link to 9 |
| $\frac{N_1^2}{N_2} - N_5 = 0.00000$ $\frac{N_1^3}{N_2^2} - N_6 = 0.00000$ $\frac{N_2^2}{N_1} - N_7 = 0.00000$ | Link to 10 |

Link to Introdution

| Function | Page |
|---|---|
| $\frac{N_1^2}{N_2^2} - N_5 = 0.00000$ $N_2^2 - N_6 = 0.00000$ | Link to 11 |
| $\sqrt{2} - N_5 = 0.00000$ $\frac{\sqrt{2} \cdot N_1}{N_2} - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{\sqrt{2}} - N_7 = 0.00000$ | Link to 12 |
| $N_5 - 2^{0.75} = 0.00000$ $\left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$ | Link to 13 |
| $N_1^{0.5} - N_2 = 0.00000$ $N_1^{0.25} - N_3 = 0.00000$ $N_1^{0.125} - N_4 = 0.00000$ | Link to 14 |
| $\frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$ $\frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$ $N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$ | Link to 15 |
| $\frac{N_1^2}{(N_1+N_2) \cdot N_2} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+N_2} - M_1 = 0.00000$ | Link to 16 |
| $\frac{N_1^2}{N_2 \cdot (N_1+1)} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+1} - M_1 = 0.00000$ | Link to 17 |
| $\frac{N_1^2}{N_2^4} - N_7 = 0.00000$ $N_2^4 - N_8 = 0.00000$ $N_2^3 - N_{25} = 0.00000$ | Link to 18 |
| $\frac{N_2^4}{N_1} - N_{26} = 0.00000$ $\frac{N_1}{N_2} - N_{27} = 0.00000$ $\frac{N_2^7}{N_1} - N_8 = 0.00000$ | |

# Contents


| Function | Page | Function | Page |
|---|---|---|---|
| $N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$ | Link to 19 | | |
| $N_1 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_5 = 0.00000$ $N_1 \cdot \left(\frac{N_2}{N_1+N_2}\right) - N_6 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_7 = 0.00000$ | Link to 20 | | |
| $N_1^{\frac{1}{2}} - N_2 = 0.00000$ $N_1^{\frac{1}{4}} - N_3 = 0.00000$ $N_1^{\frac{1}{8}} - N_4 = 0.00000$ | Link to 21 | | |
| $N_1^{\frac{8}{8}} - N_1 = 0.00000$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$ | Link to 22 | | |
| $N_1^{\frac{5}{8}} - N_4 = 0.00000$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$ | | | |
| $N_1^{\frac{2}{8}} - N_7 = 0.00000$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$ | | | |




$N_1 = 2.00000$

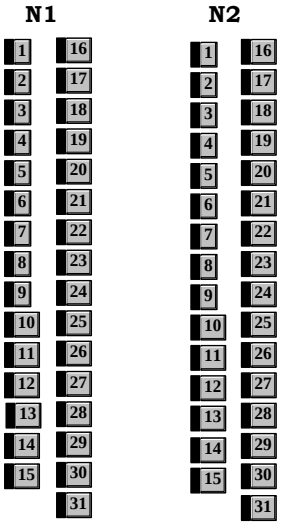
$N_2 = 2.00000$

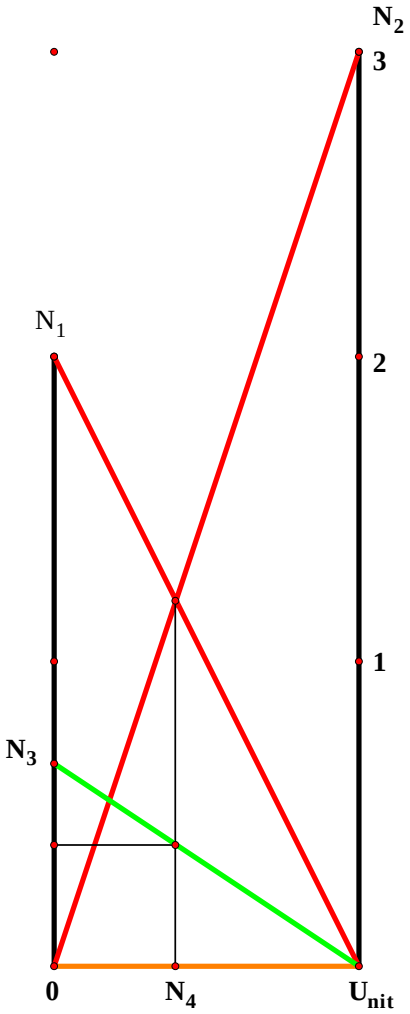
$N_1 \cdot N_2 = 4.00000$

$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

**Multiply N1 by N2**






$N_1 = 2.00000$

$N_2 = 3.00000$

$\frac{N_1}{N_2} = 0.66667$

$N_3 = 0.66667$

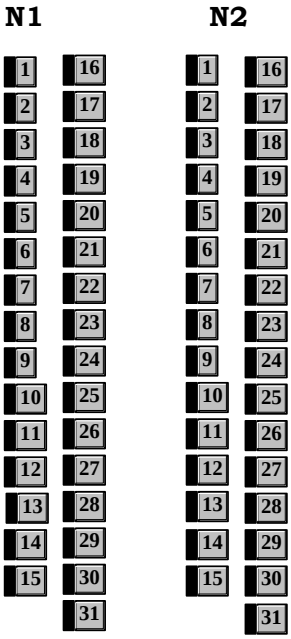
$\frac{U_{nit}}{0N_4} = 2.50000$

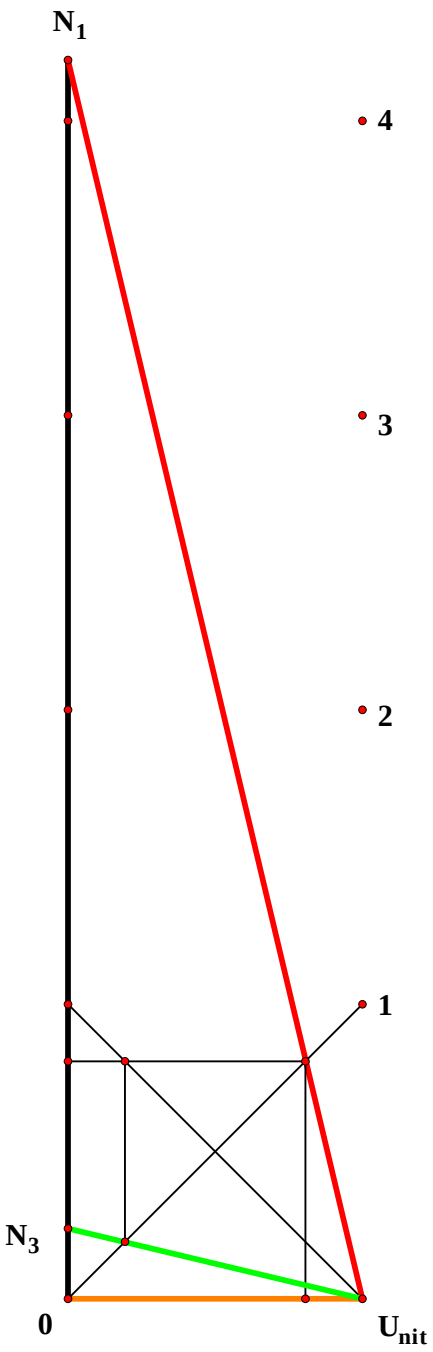
$\frac{N_1+N_2}{N_1} = 2.50000$

$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$

$\frac{N_1+N_2}{N_2} = 1.66667$

**Divide**
**N1 by N2**






**Find the Recprocal N1**

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

**N1**

| | |
|---|---|
| 1 | 16 |
| 2 | 17 |
| 3 | 18 |
| 4 | 19 |
| 5 | 20 |
| 6 | 21 |
| 7 | 22 |
| 8 | 23 |
| 9 | 24 |
| 10 | 25 |
| 11 | 26 |
| 12 | 27 |
| 13 | 28 |
| 14 | 29 |
| 15 | 30 |
| | 31 |

$N_1 = 3.00000$

$N_2 = 2.00000$

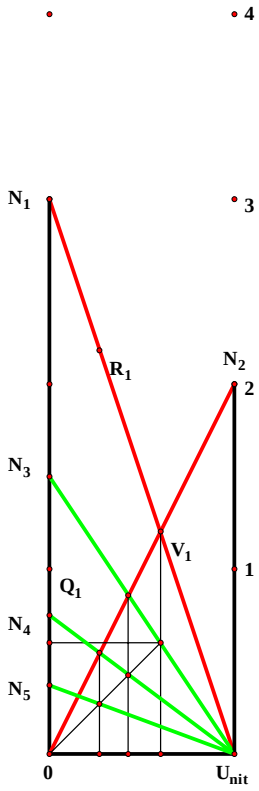
$\frac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

$N_4 = 0.75000$ $\quad \frac{N_1}{N_2^2} - N_4 = 0.00000$

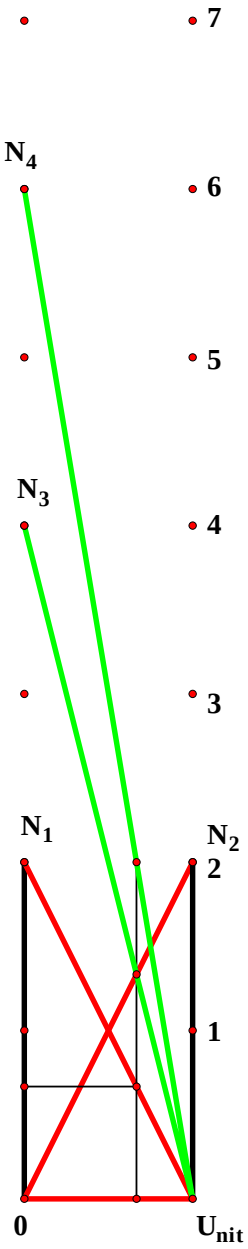
$N_5 = 0.37500$ $\quad \frac{N_1}{N_2^3} - N_5 = 0.00000$

**etc.**




**Divide N1 by a Power of N2**

| | N1 | N2 |
|---|---|---|
| 1 | 16 | 16 |
| 2 | 17 | 17 |
| 3 | 18 | 18 |
| 4 | 19 | 19 |
| 5 | 20 | 20 |
| 6 | 21 | 21 |
| 7 | 22 | 22 |
| 8 | 23 | 23 |
| 9 | 24 | 24 |
| 10 | 25 | 25 |
| 11 | 26 | 26 |
| 12 | 27 | 27 |
| 13 | 28 | 28 |
| 14 | 29 | 29 |
| 15 | 30 | 30 |
| | 31 | 31 |




$N_1 = 2.00000$

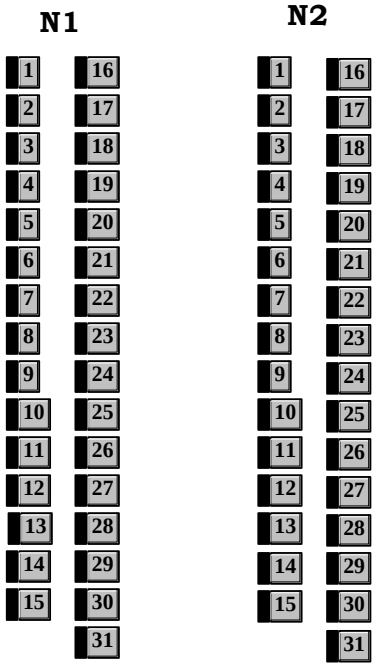
$N_2 = 2.00000$

$N_1 \cdot N_2 = 4.00000$

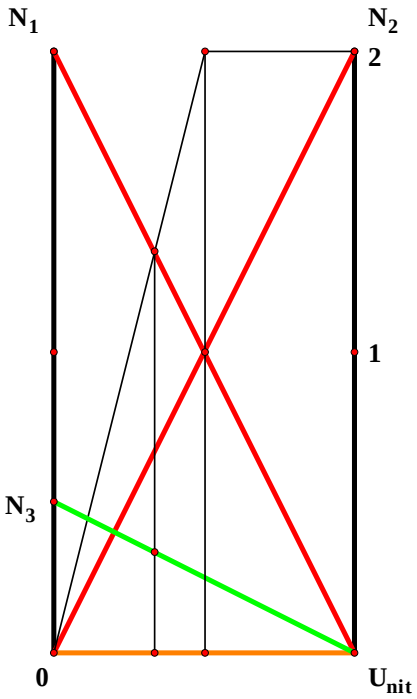
$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



3



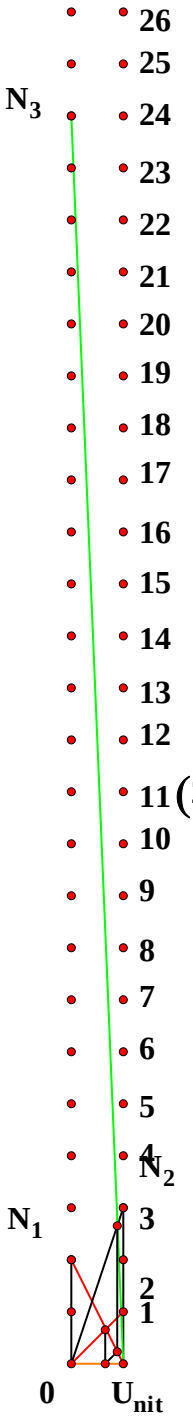

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$

| N1 | | N2 | |
|---|---|---|---|
| 1 | 16 | 1 | 16 |
| 2 | 17 | 2 | 17 |
| 3 | 18 | 3 | 18 |
| 4 | 19 | 4 | 19 |
| 5 | 20 | 5 | 20 |
| 6 | 21 | 6 | 21 |
| 7 | 22 | 7 | 22 |
| 8 | 23 | 8 | 23 |
| 9 | 24 | 9 | 24 |
| 10 | 25 | 10 | 25 |
| 11 | 26 | 11 | 26 |
| 12 | 27 | 12 | 27 |
| 13 | 28 | 13 | 28 |
| 14 | 29 | 14 | 29 |
| 15 | 30 | 15 | 30 |
| | 31 | | 31 |

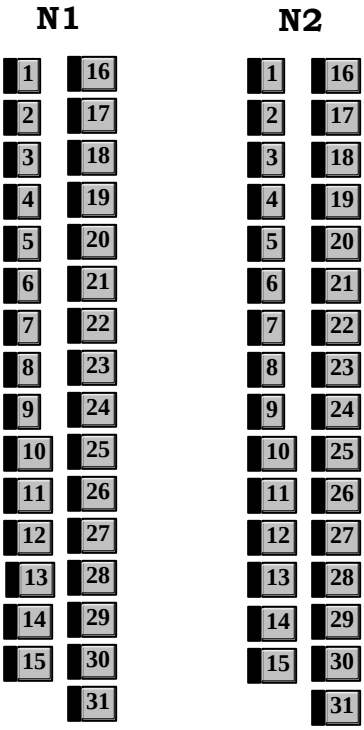



$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$



$\frac{BC}{BV_1} = 0.81650$ $\quad N_3^{-0.5} = 0.81650$

$N_3^{0} = 1.00000$

$\frac{BC}{BU_1} = 1.22474$ $\quad N_3^{0.5} = 1.22474$

$\frac{BC}{BP_1} = 1.50000$

$\frac{BC}{BR_1} = 2.25000$ $\quad N_3^{2} - \frac{BC}{BR_1} = 0.00000$

$\frac{BC}{BS_1} = 3.37500$ $\quad N_3^{3} - \frac{BC}{BS_1} = 0.00000$

$\frac{BC}{BT_1} = 5.06250$ $\quad N_3^{4} - \frac{BC}{BT_1} = 0.00000$

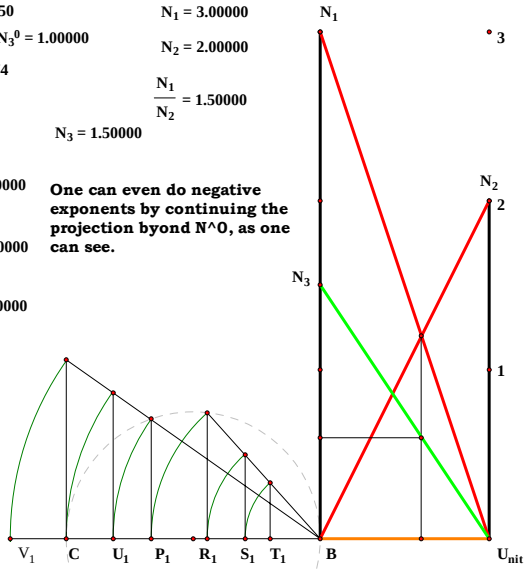
$N_1 = 3.00000$

$N_2 = 2.00000$

$\frac{N_1}{N_2} = 1.50000$

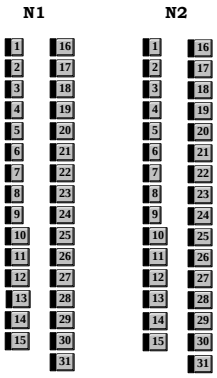
$N_3 = 1.50000$

**One can even do negative exponents by continuing the projection beyond N^0, as one can see.**




**Adding a figure to a figure.**

**I tossed this together, so it is not perfect--it just looks good**



**How to find a Proportional Unit for a given pair of Magnitudes**

$Unit_2 = 1.00000$

$N1_2 = 1.39420$

$N2_2 = 1.94380$

$N3_2 = 0.71726$

$N4_2 = 2.71004$

$\frac{N1_2}{N2_2} - N3_2 = 0.00000$

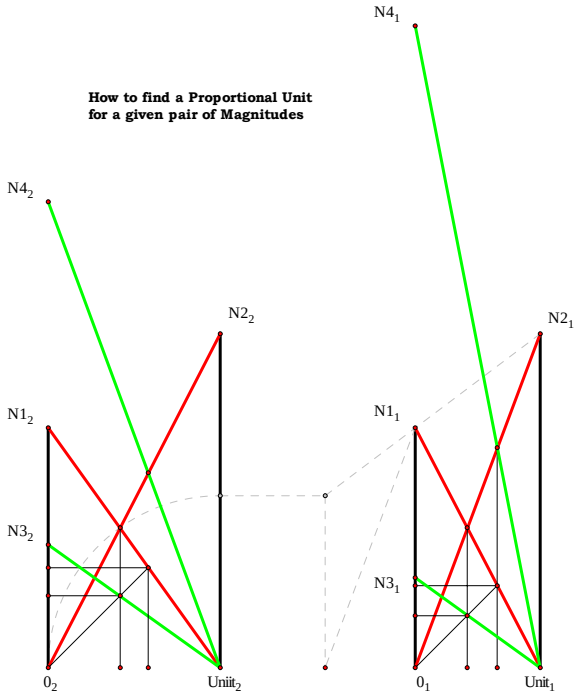
$N1_2 \cdot N2_2 - N4_2 = 0.00000$

$\frac{N4_2}{N2_2} = 1.39420$

$\frac{N2_2}{N1_2} = 1.39420$

$\frac{N1_2}{Unit_2} = 1.39420$

$\frac{Unit_2}{N3_2} = 1.39420$




$Unit_1 = 1.00000$

$N1_1 = 1.91861$

$N2_1 = 2.67492$

$N3_1 = 0.71726$

$N4_1 = 5.13213$

$\frac{N1_1}{N2_1} - N3_1 = 0.00000$

$N1_1 \cdot N2_1 - N4_1 = 0.00000$

$\frac{N4_1}{N2_1} = 1.91861$

$\frac{N2_1}{N1_1} = 1.39420$

$\frac{N1_1}{Unit_1} = 1.91861$

$\frac{Unit_1}{N3_1} = 1.39420$

**Unit = 1.00000**

$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 0.66667$

$N_4 = 6.00000$

$N_5 = 1.33333$

$N_6 = 0.88889$

$N_7 = 4.50000$

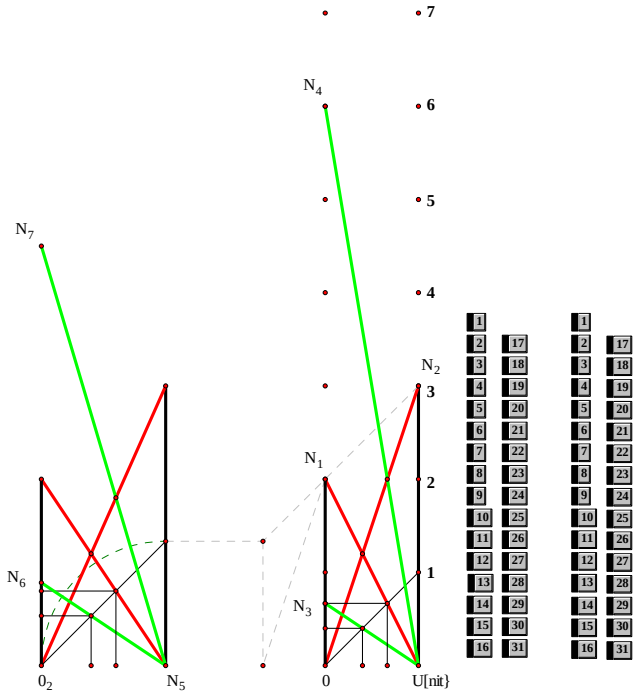
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1^2}{N_2} - N_5 = 0.00000$$

$$\frac{N_1^3}{N_2^2} - N_6 = 0.00000$$

$$\frac{N_2^2}{N_1} - N_7 = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.30952**

**$N_3$ = 0.86598**

**$N_4$ = 4.61905**

**$N_5$ = 0.74992**

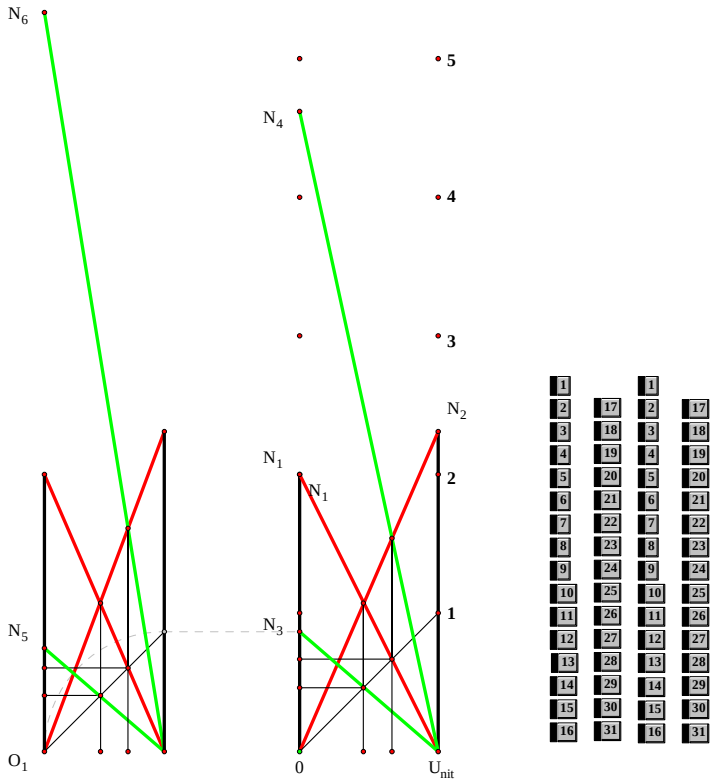
**$N_6$ = 5.33390**

$\frac{N_1}{N_2} - N_3 = 0.00000$

$N_1 \cdot N_2 - N_4 = 0.00000$

$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_2^2 - N_6 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.43939**

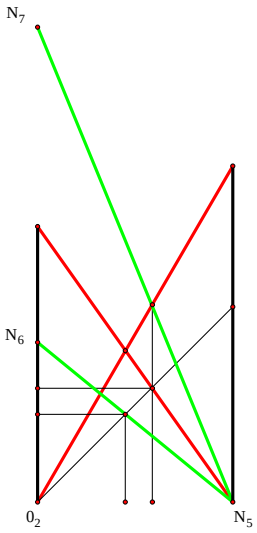
**$N_3$ = 0.81988** $\frac{N_1}{N_2}-N_3 = 0.00000$

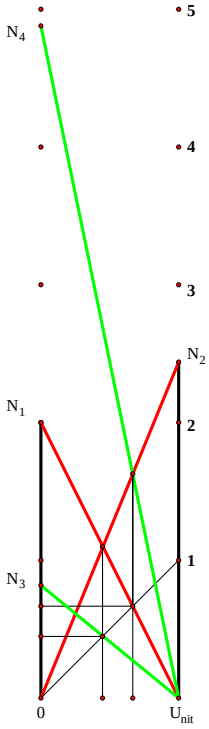
**$N_4$ = 4.87879** $N_1 \cdot N_2 - N_4 = 0.00000$

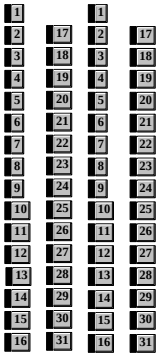
**$N_5$ = 1.41421** $\sqrt{2}-N_5 = 0.00000$

**$N_6$ = 1.15948** $\frac{\sqrt{2}\cdot N_1}{N_2}-N_6 = 0.00000$

**$N_7$ = 3.44982** $\frac{N_1 \cdot N_2}{\sqrt{2}}-N_7 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 2.00000** $\quad \frac{N_1}{N_2} - N_3 = 0.00000$

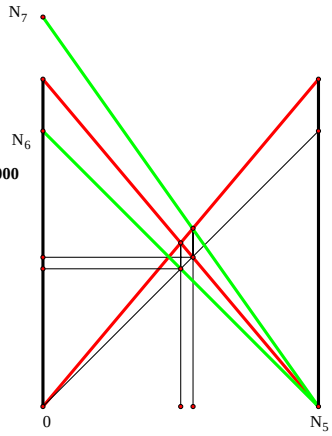
**$N_3$ = 1.00000**

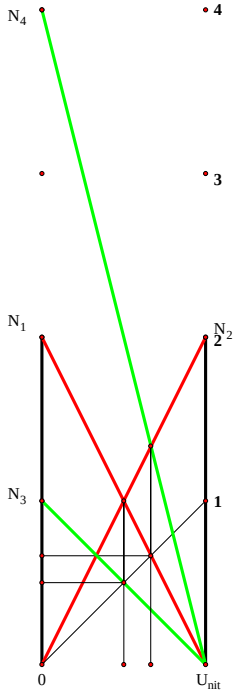
**$N_4$ = 4.00000** $\quad N_1 \cdot N_2 - N_4 = 0.00000$

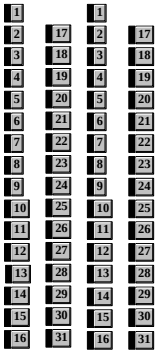
**$N_5$ = 1.68179** $\quad N_5 - 2^{0.75} = 0.00000$

**$N_6$ = 1.68179** $\quad \left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

**$N_7$ = 2.37841** $\quad \frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$



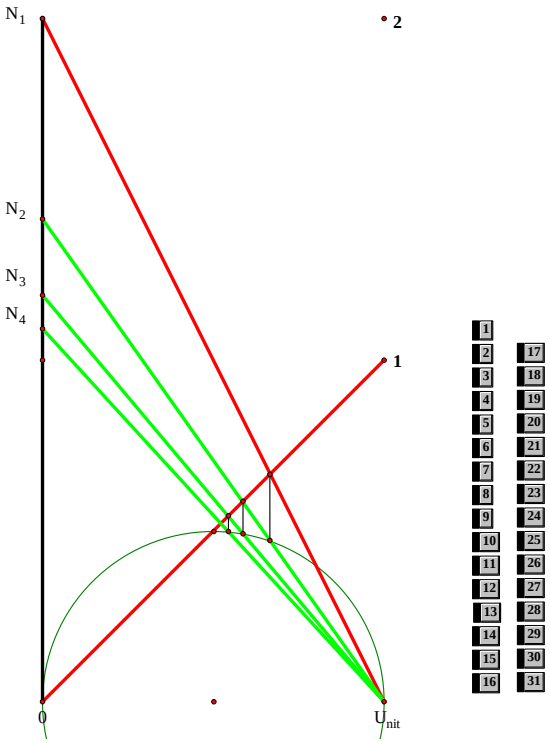
**Unit = 1.00000**

**$N_1$ = 2.00000**

**$N_2$ = 1.41421** **$N_1^{0.5}$-$N_2$ = 0.00000**

**$N_3$ = 1.18921** **$N_1^{0.25}$-$N_3$ = 0.00000**

**$N_4$ = 1.09051** **$N_1^{0.125}$-$N_4$ = 0.00000**



**Unit = 1.00000**

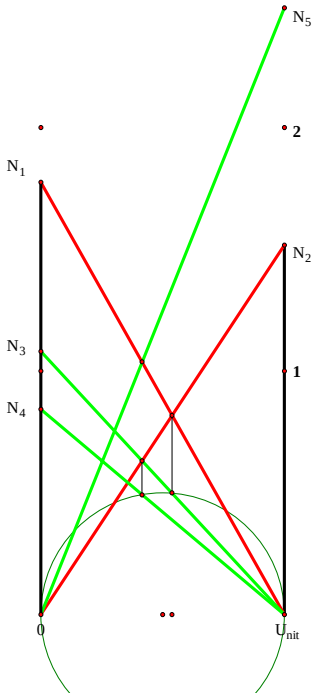
**$N_1$ = 1.77542**

**$N_2$ = 1.51695**

**$N_3$ = 1.08185** $\quad \frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

**$N_4$ = 0.84450** $\quad \frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

**$N_5$ = 2.48947** $\quad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$




| | | | |
|---|---|---|---|
| 1 | 17 | 1 | 17 |
| 2 | 18 | 2 | 18 |
| 3 | 19 | 3 | 19 |
| 4 | 20 | 4 | 20 |
| 5 | 21 | 5 | 21 |
| 6 | 22 | 6 | 22 |
| 7 | 23 | 7 | 23 |
| 8 | 24 | 8 | 24 |
| 9 | 25 | 9 | 25 |
| 10 | 26 | 10 | 26 |
| 11 | 27 | 11 | 27 |
| 12 | 28 | 12 | 28 |
| 13 | 29 | 13 | 29 |
| 14 | 30 | 14 | 30 |
| 15 | 31 | 15 | 31 |
| 16 | | 16 | |

**Unit = 1.00000**

**$N_1$ = 1.00000**

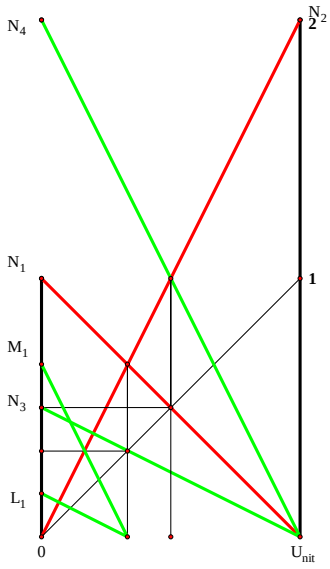
**$N_2$ = 2.00000** $\frac{N_1}{N_2}-N_3 = 0.00000$

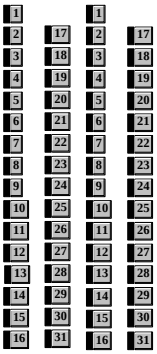
**$N_3$ = 0.50000**

**$N_4$ = 2.00000** $N_1 \cdot N_2 - N_4 = 0.00000$

**$L_1$ = 0.16667** $\frac{N_1^2}{(N_1+N_2)\cdot N_2}-L_1 = 0.00000$

**$M_1$ = 0.66667** $\frac{N_1^2 \cdot N_2}{N_1+N_2}-M_1 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 2.00000**

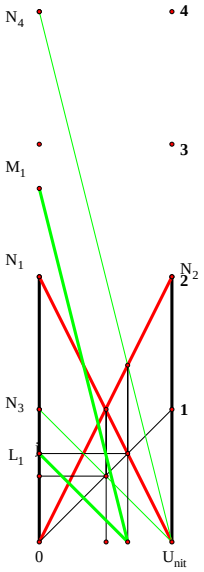
**$N_2$ = 2.00000** $\frac{N_1}{N_2}$**-$N_3$ = 0.00000**

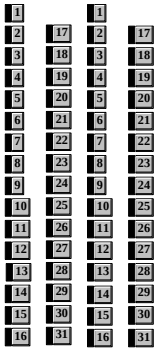
**$N_3$ = 1.00000**

**$N_4$ = 4.00000** **$N_1 \cdot N_2 - N_4 = 0.00000$**

**$L_1$ = 0.66667** $\frac{N_1^2}{N_2 \cdot (N_1+1)}$**-$L_1$ = 0.00000**

**$M_1$ = 2.66667** $\frac{N_1^2 \cdot N_2}{N_1+1}$**-$M_1$ = 0.00000**



$U_1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 1.00000$ $\quad \frac{N_1}{N_2} = 1.00000$

$N_4 = 4.00000$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$U[1] / U_2 = 0.25000$

$N_5 = 2.00000$

$N_6 = 2.00000$

$N_7 = 0.25000$ $\quad \frac{N_1^2}{N_2^4} - N_7 = 0.00000$

$N_8 = 16.00000$ $\quad N_2^4 - N_8 = 0.00000$

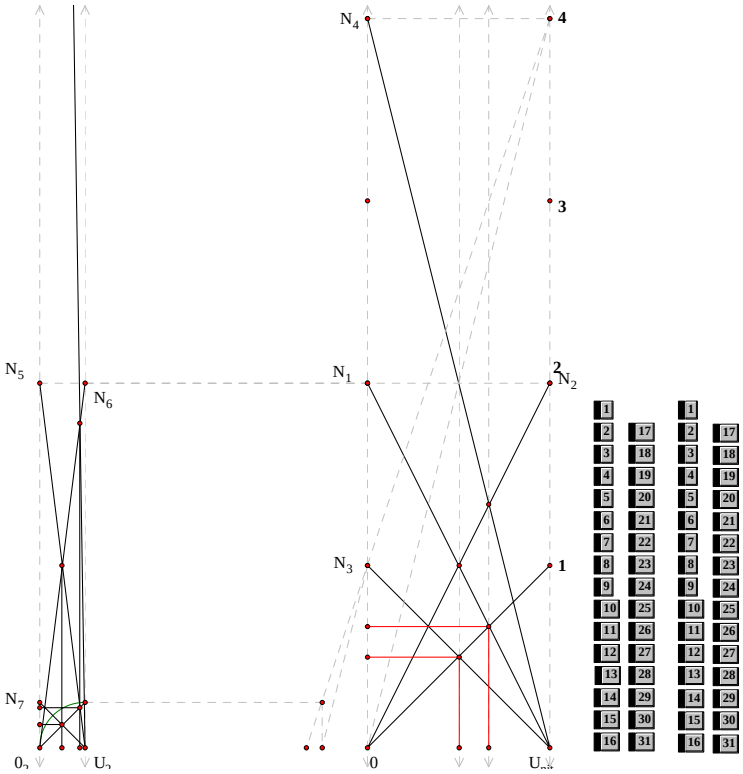
$U_2 = 1.00000$

$N_{25} = 8.00000$ $\quad N_2^3 - N_{25} = 0.00000$

$N_{26} = 8.00000$ $\quad \frac{N_2^4}{N_1} - N_{26} = 0.00000$

$N_{27} = 1.00000$ $\quad \frac{N_1}{N_2} - N_{27} = 0.00000$

$N_8 = 64.00000$ $\quad \frac{N_2^7}{N_1} - N_8 = 0.00000$



**Unit = 1.00000**

**$N_1$ = 1.72159**

**$N_2$ = 2.12500**

**$N_3$ = 0.81016**

**$N_4$ = 3.65838**

**$N_5$ = 0.57386**

**$N_6$ = 1.14773**

**$N_7$ = 0.70833**

**$N_8$ = 1.41667**

**$N_9$ = 1.21946**

**$N_{10}$ = 2.43892**

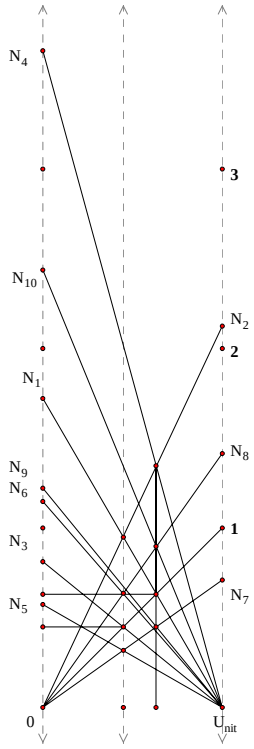
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

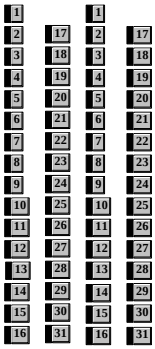
$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1}{N_2} = 0.81016$$

$$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$$N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$



**Unit = 1.00000**

**$N_1$ = 1.37056**

**$N_2$ = 1.73096**

**$N_3$ = 0.79179** $\frac{N_1}{N_2} = 0.79179$

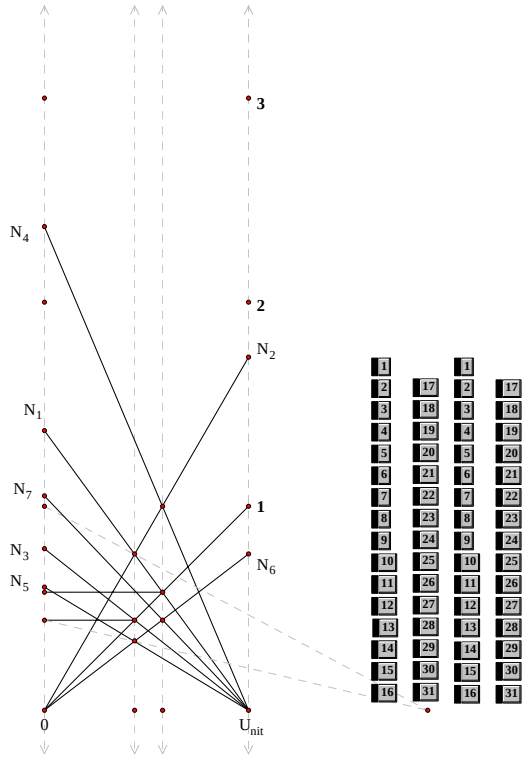
**$N_4$ = 2.37239** $N_1 \cdot N_2 = 2.37239$

$\frac{N_1}{N_2} = 0.79179$

**$N_5$ = 0.60565** $N_1 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000$

**$N_6$ = 0.76491** $N_1 \cdot \left( \frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000$

**$N_7$ = 1.04835** $N_1 \cdot N_2 \cdot \left( \frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000$



**Unit = 1.00000**

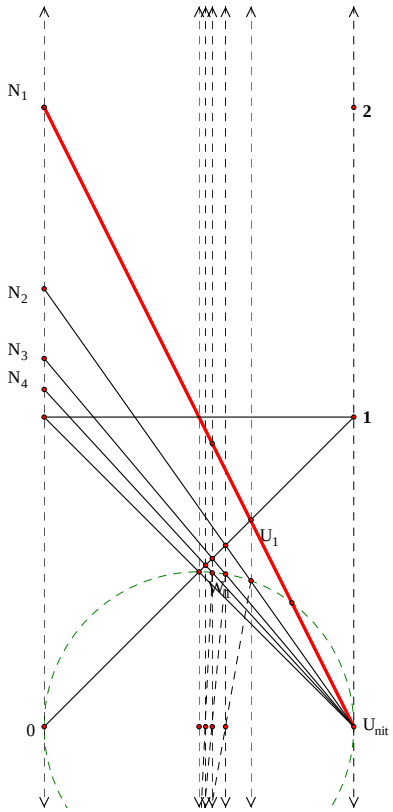
**$N_1$ = 2.00000**

**$N_2$ = 1.41421** $\quad N_1^{\frac{1}{2}} - N_2 = 0.00000$

**$N_3$ = 1.18921** $\quad N_1^{\frac{1}{4}} - N_3 = 0.00000$

**$N_4$ = 1.09051** $\quad N_1^{\frac{1}{8}} - N_4 = 0.00000$

**etc.**




1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

**Unit = 1.00000**

**Eight root series.**

**One can project as long as they like.**

$N_1 = 2.00000$ $N_1^{\frac{8}{8}} - N_1 = 0.00000$

$N_2 = 1.83401$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$

$N_3 = 1.68179$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$

$N_4 = 1.54221$ $N_1^{\frac{5}{8}} - N_4 = 0.00000$

$N_5 = 1.41421$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$

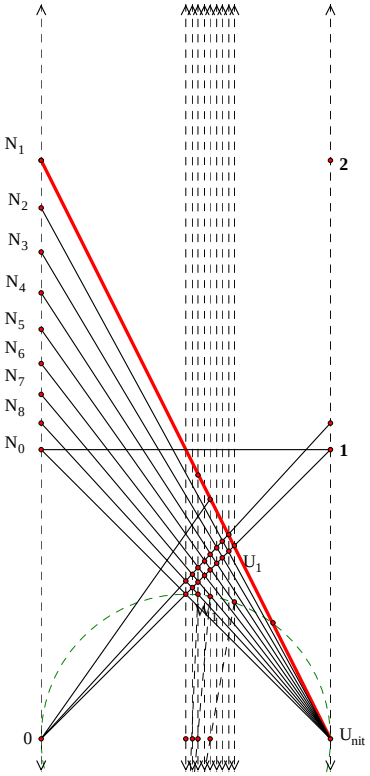
$N_6 = 1.29684$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$

$N_7 = 1.18921$ $N_1^{\frac{2}{8}} - N_7 = 0.00000$

$N_8 = 1.09051$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$

$N_0 = 1.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$

**etc.**





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The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

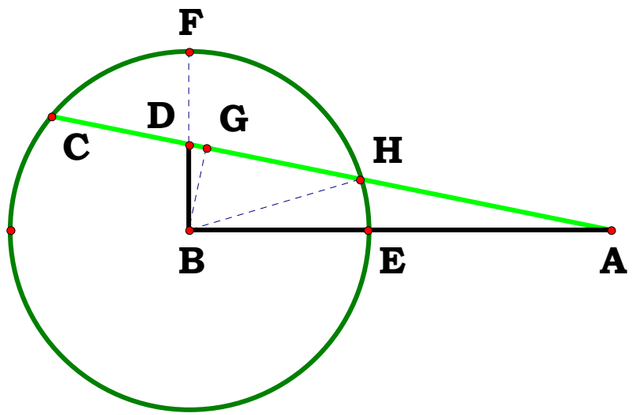
Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.

080521




$N_1 := 3.86292 \qquad N_2 := 1.905 \qquad N_3 := .74482$

$AB := N_1 \qquad BE := N_2 \qquad BD := N_3$

$AD := \sqrt{AB^2 + BD^2} \qquad DG := \frac{BD^2}{AD} \qquad BH := BE$

$BG := \frac{AB \cdot BD}{AD} \qquad GH := \sqrt{BH^2 - BG^2}$

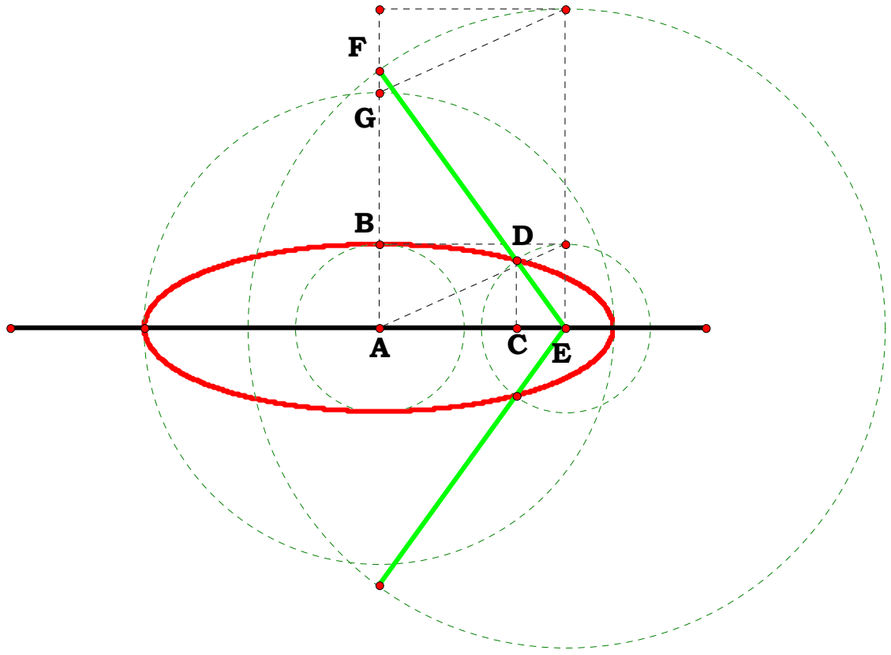
$AC := AD + GH - DG$

$$\frac{\sqrt{AB^2 \cdot BE^2 + BD^2 \cdot BE^2 - AB^2 \cdot BD^2} + AB^2}{\sqrt{AB^2 + BD^2}} - AC = 0$$

$$AC - \frac{N_1^2 + \sqrt{N_1^2 \cdot N_2^2 - N_1^2 \cdot N_3^2 + N_2^2 \cdot N_3^2}}{\sqrt{N_1^2 + N_3^2}} = 0$$

**For a straight line ellipse and three givens.**

**a: AB, AC, CD**




$N_1 := 1.40187 \quad N_2 := 2.31398 \quad N_3 := 1.13348$

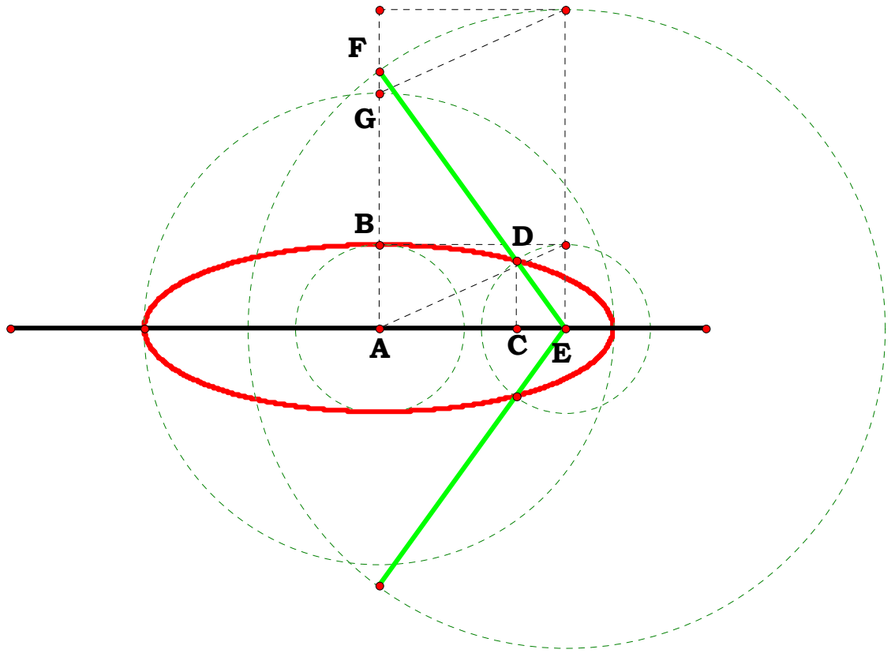
$AB := N_1 \quad AC := N_2 \quad CD := N_3$

$DE := AB \quad CE := \sqrt{DE^2 - CD^2} \quad DF := \frac{DE \cdot AC}{CE} \quad BG := DF - AB$

$$BG - N_1 \cdot \left( \frac{N_2}{\sqrt{N_1^2 - N_3^2}} - 1 \right) = 0$$

**For a straight line ellipse and three givens.**

**b: CE, AC, CD.**




$N_1 := .8249 \quad N_2 := 2.31398 \quad N_3 := 1.13348$

$CE := N_1 \quad AC := N_2 \quad CD := N_3$

$AB := \sqrt{CD^2 + CE^2}$

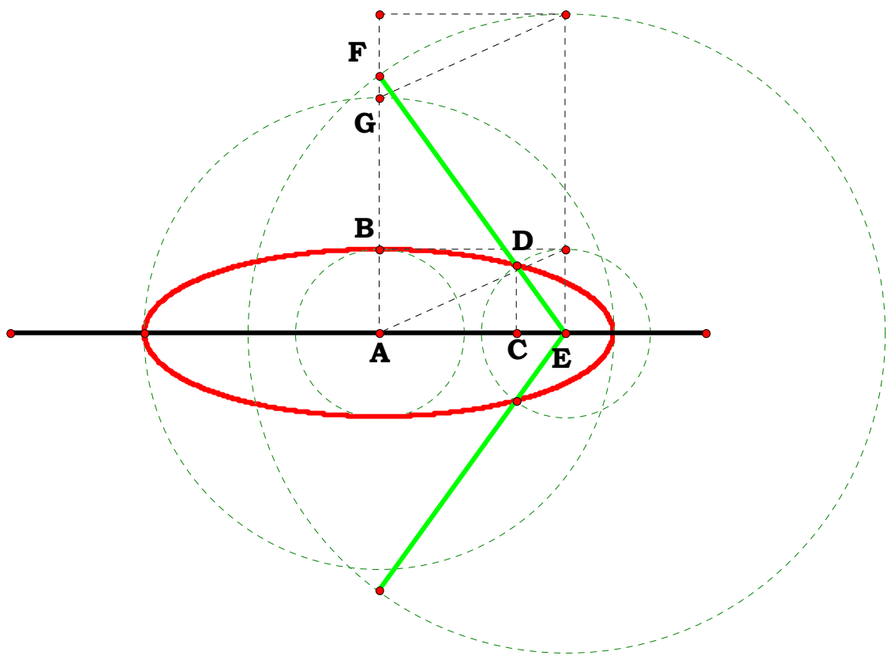
$DE := AB \quad DF := \frac{DE \cdot AC}{CE} \quad BG := DF - AB$

$$BG - \frac{\sqrt{N_1^2 + N_3^2} \cdot (N_2 - N_1)}{N_1} = 0$$

**060208 c.mcd**

**For a straight line ellipse and three givens.**

**c: AB, AC, CE**

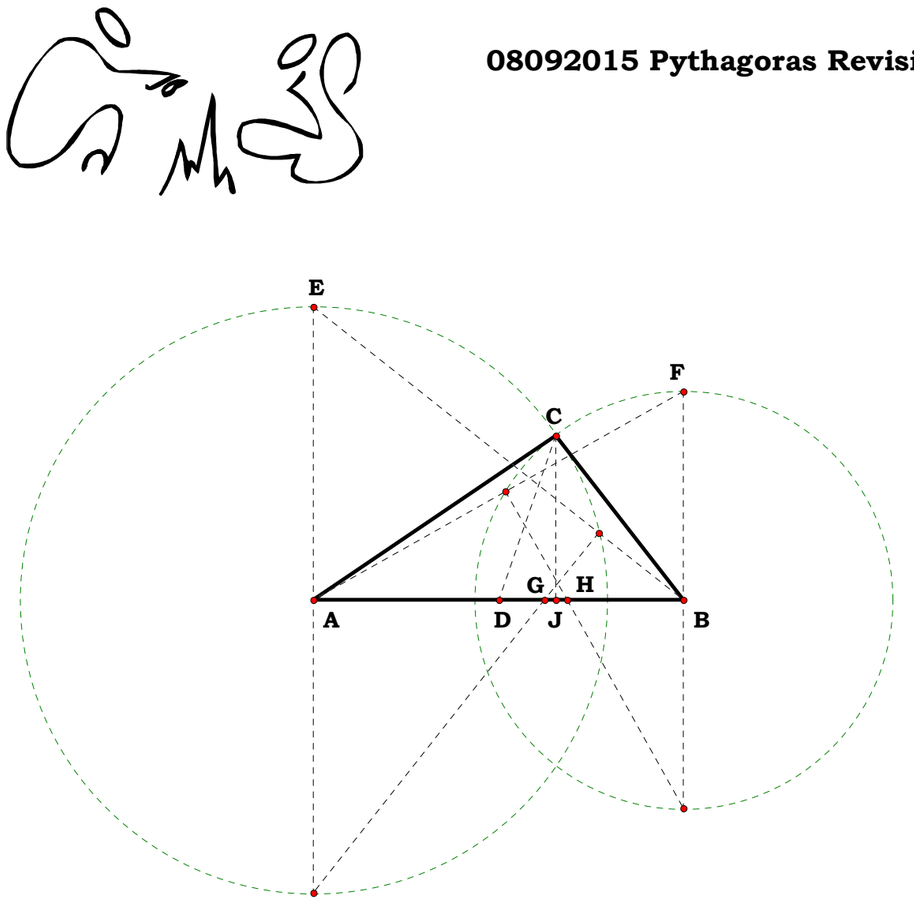



$N_1 := 1.40187 \quad N_2 := 2.31398 \quad N_3 := .8249$

$AB := N_1 \quad AC := N_2 \quad CE := N_3$

$DE := AB \quad DF := \frac{DE \cdot AC}{CE} \quad BG := DF - AB$

$$BG - \frac{N_1 \cdot (N_2 - N_3)}{N_3} = 0$$



**08092015 Pythagoras Revisited Again!**

**One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.**




$AB := 6.20183 \quad AC := 4.89358 \quad BC := 9.20468$

$$GA := \frac{AC^2}{AB} \qquad HB := \frac{BC^2}{AB} \qquad GH := AB - (GA + HB) \qquad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \qquad CJ := \sqrt{AC^2 - JA^2} \qquad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$

$$JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

**Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.**

$$AJ := \sqrt{AC^2 - CJ^2} \qquad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \qquad BJ := \sqrt{BC^2 - CJ^2} \qquad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

$AB := 2.38542$

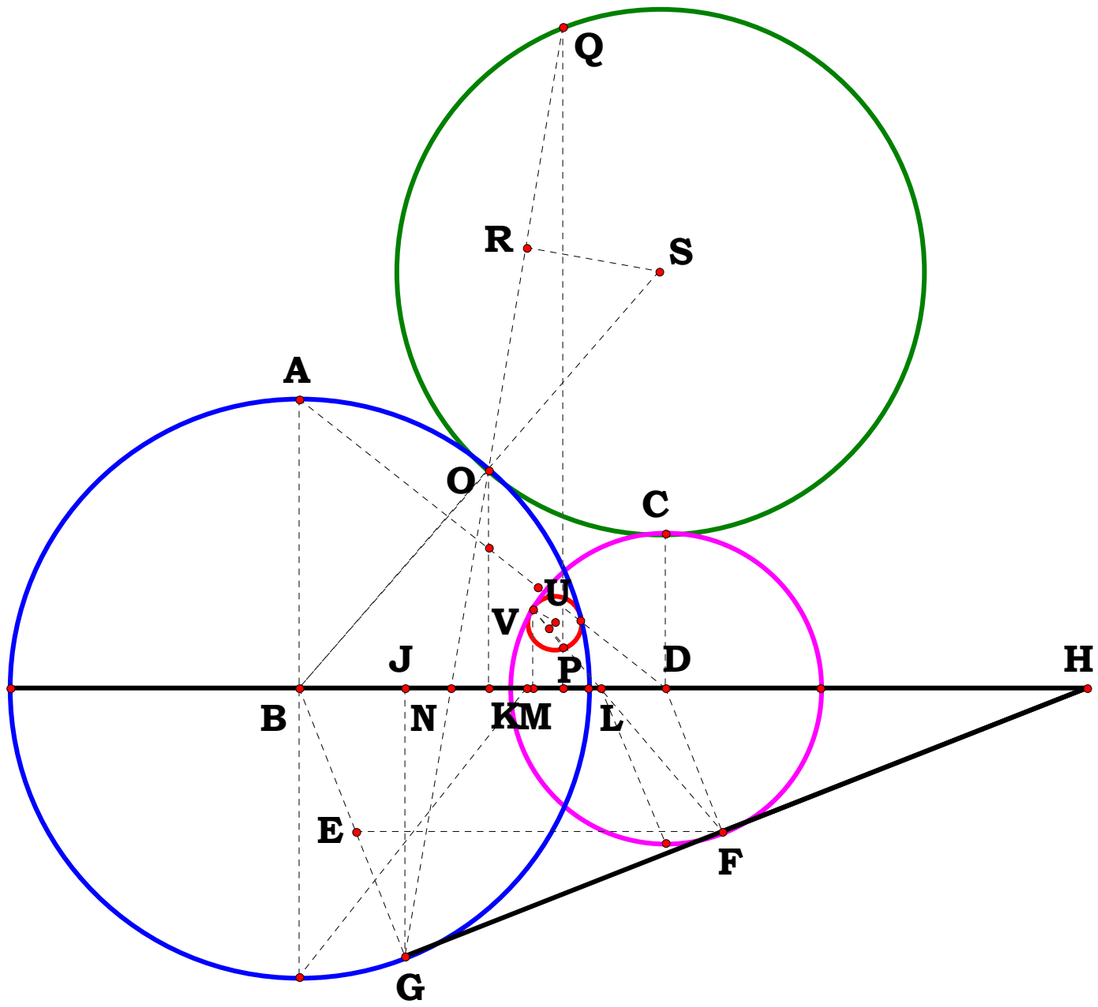
$CD := 1.72917$

$BD := 3.17708$

$dx := 1.22729$

$$\frac{N_3^2 + R_1^2 - R_2^2}{2 \cdot N_3}$$

From 042794 power line.




$$EF := BD \quad EG := AB - CD \quad BH := \frac{EF \cdot AB}{EG} \quad BJ := \frac{AB^2}{BH} \quad KL := \frac{2 \cdot AB}{dx}$$

$$JL := AB - BJ \quad JK := JL - KL \quad GJ := \sqrt{(2 \cdot AB - JL) \cdot JL}$$

$$KO := \sqrt{(2 \cdot AB - KL) \cdot KL} \quad KN := \frac{JK \cdot KO}{GJ + KO} \quad BP := \frac{BD^2 + AB^2 - CD^2}{2 \cdot BD}$$

$$JN := \frac{JK \cdot GJ}{GJ + KO} \quad NP := BP - (BJ + JN) \quad NO := \sqrt{KO^2 + KN^2}$$

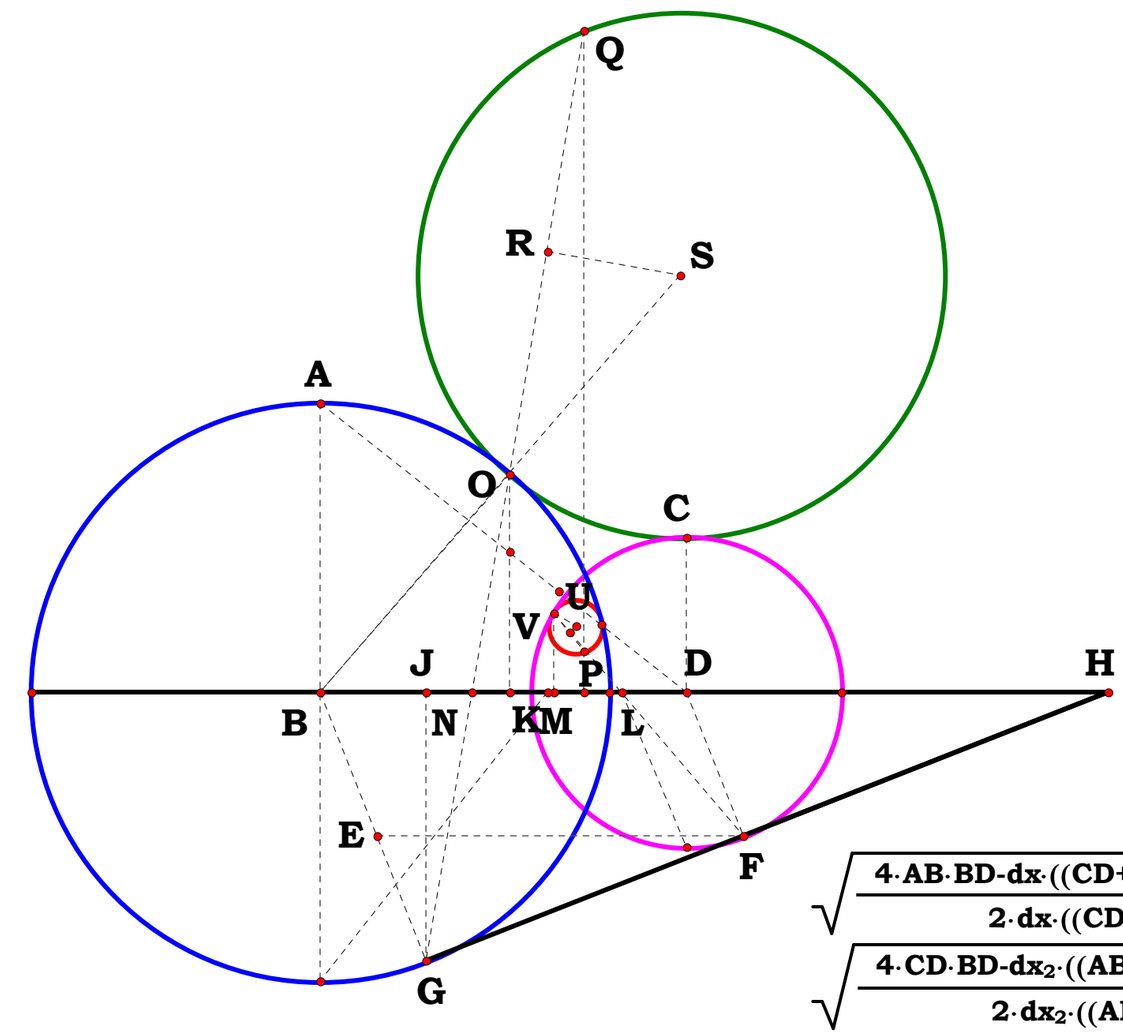
$$QN := \frac{NO \cdot NP}{KN} \quad OQ := QN - NO \quad OR := \frac{OQ}{2} \quad GO := \sqrt{(GJ + KO)^2 + JK^2}$$

$$SO := \frac{AB \cdot OR \cdot 2}{GO} \qquad SO = -4.204157$$

$R_1 := AB \quad R_2 := CD \quad D := BD$

$$SO - \frac{\left(4 \cdot R_1 \cdot D\right) - dx \cdot \left(R_2 + D - R_1\right) \cdot \left(R_2 + R_1 - D\right)}{2 \cdot dx \cdot \left(R_2 + D - R_1\right) - 4 \cdot D} = 0$$

$$\sqrt{\left[\frac{\left(4 \cdot R_1 \cdot D\right) - dx \cdot \left(R_2 + D - R_1\right) \cdot \left(R_2 + R_1 - D\right)}{2 \cdot dx \cdot \left(R_2 + D - R_1\right) - 4 \cdot D}\right]^2} = 4.204157$$




**AB = 1.90625 in.**
**CD = 1.02083 in.**
**BD = 2.41667 in.**
**dx = 5.77984**
**SO = 1.73133 in.**

**KL = 0.65962 in.**

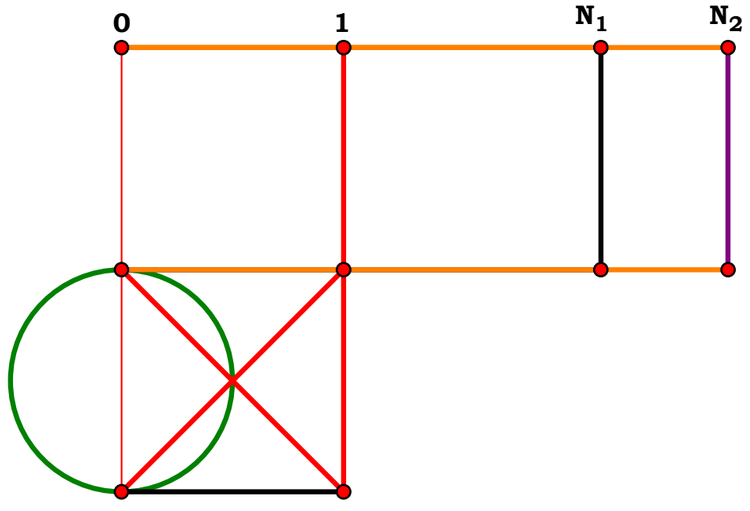
**UV = 0.16880 in.**
**TM = 0.14146 in.**
**$dx_2$ = 14.43321**

$$\sqrt{\frac{4\cdot AB\cdot BD-dx\cdot((CD+BD)-AB)\cdot((CD+AB)-BD)^{2}}{2\cdot dx\cdot((CD+BD)-AB)-4\cdot BD}}-SO = 0.00000\ in.$$

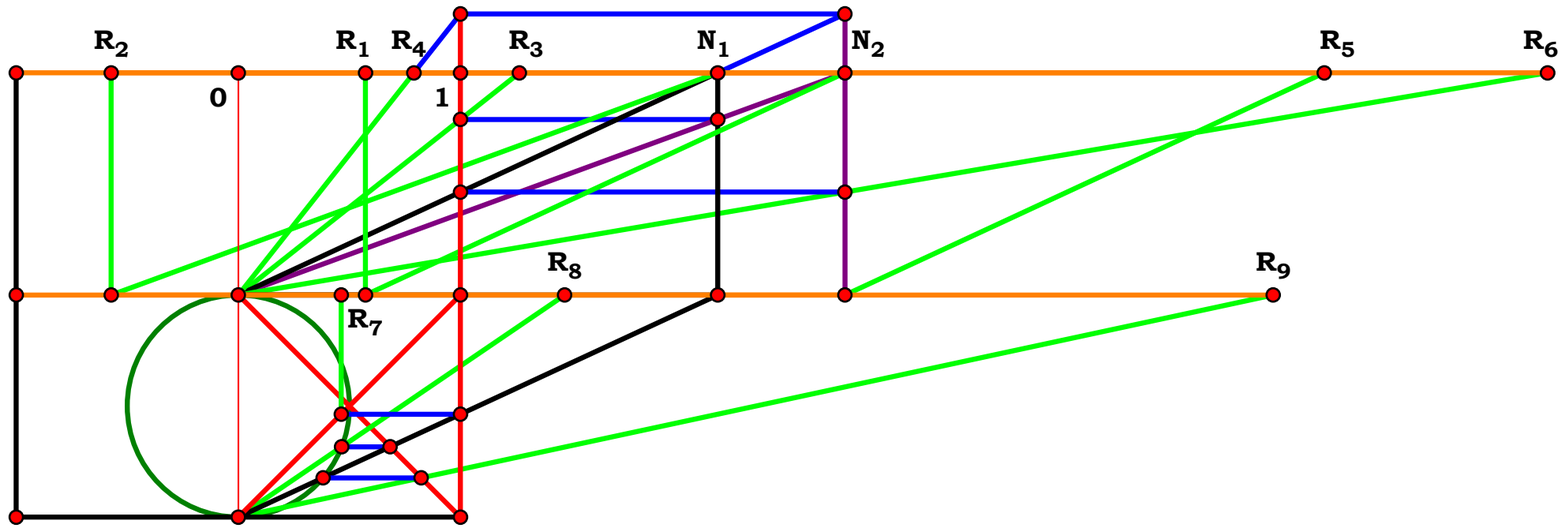
$$\sqrt{\frac{4\cdot CD\cdot BD-dx_2\cdot((AB+BD)-CD)\cdot((AB+CD)-BD)^{2}}{2\cdot dx_2\cdot((AB+BD)-CD)-4\cdot BD}}-UV = 0.00000\ in.$$

# Basic Analog Mathematics.

**Let 0 to 1 be the given Unit and N1 and N2 be any two given differences:**




**With the given analog (figure), it is required to render the products of these two differences using the paradigms sum, differences, and ratio:--**




$N_2 - N_1 - R_1 = 0.00000$

$N_1 - N_2 - R_2 = 0.00000$

$N_1 + N_2 - R_5 = 0.00000$

$\frac{N_2}{N_1} - R_3 = 0.00000$

$\frac{N_1}{N_2} - R_4 = 0.00000$

$\frac{1}{N_1} - R_7 = 0.00000$

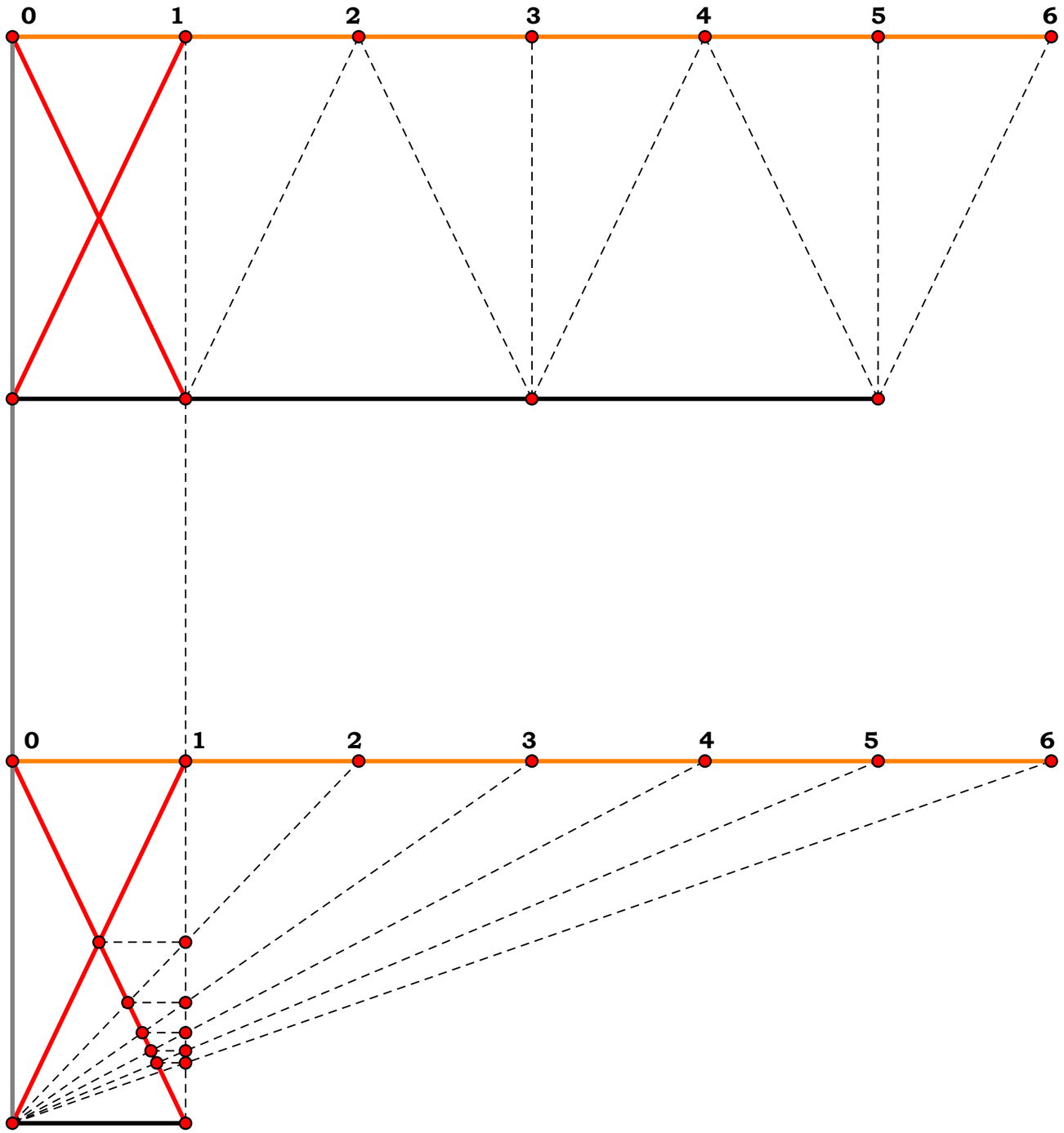
$N_1 \cdot N_2 - R_6 = 0.00000$

$\sqrt{N_1} - R_8 = 0.00000$

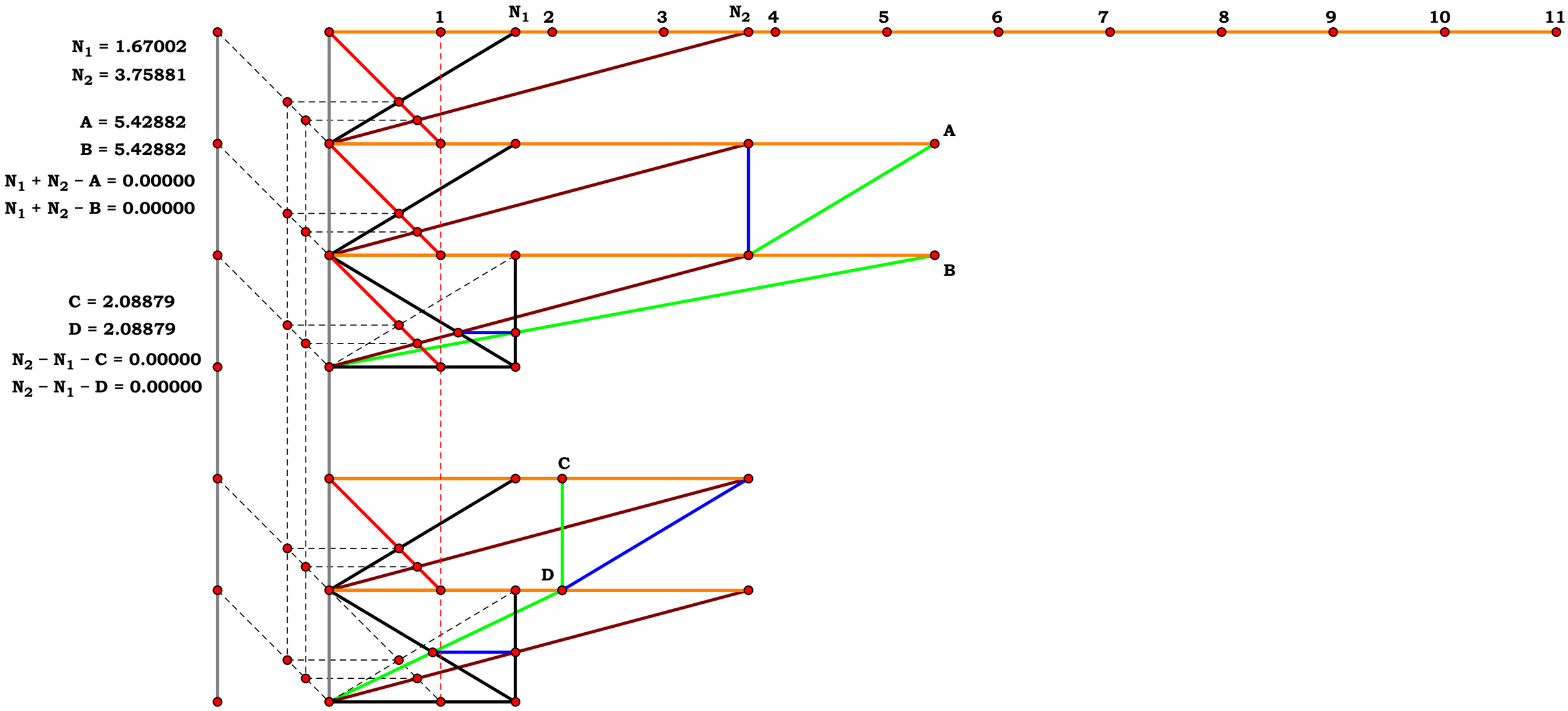
$N_1^2 - R_9 = 0.00000$

# Complete Induction.

**Complete induction, if the term is to make any sense at all, simply means recursion. These display just two methods of costructing an arithmetic ratio and series in analogically.**



## Addition and Subtraction.



# Multiplication and Division

$N_1 = 2.30957$

$N_2 = 4.43080$

$A = 10.23326$

$B = 10.23326$

$C = 10.23326$

$D = 10.23326$

$N_1 \cdot N_2 - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$N_1 \cdot N_2 - C = 0.00000$

$N_1 \cdot N_2 - D = 0.00000$

$E = 0.52125$

$F = 0.52125$

$G = 0.52125$

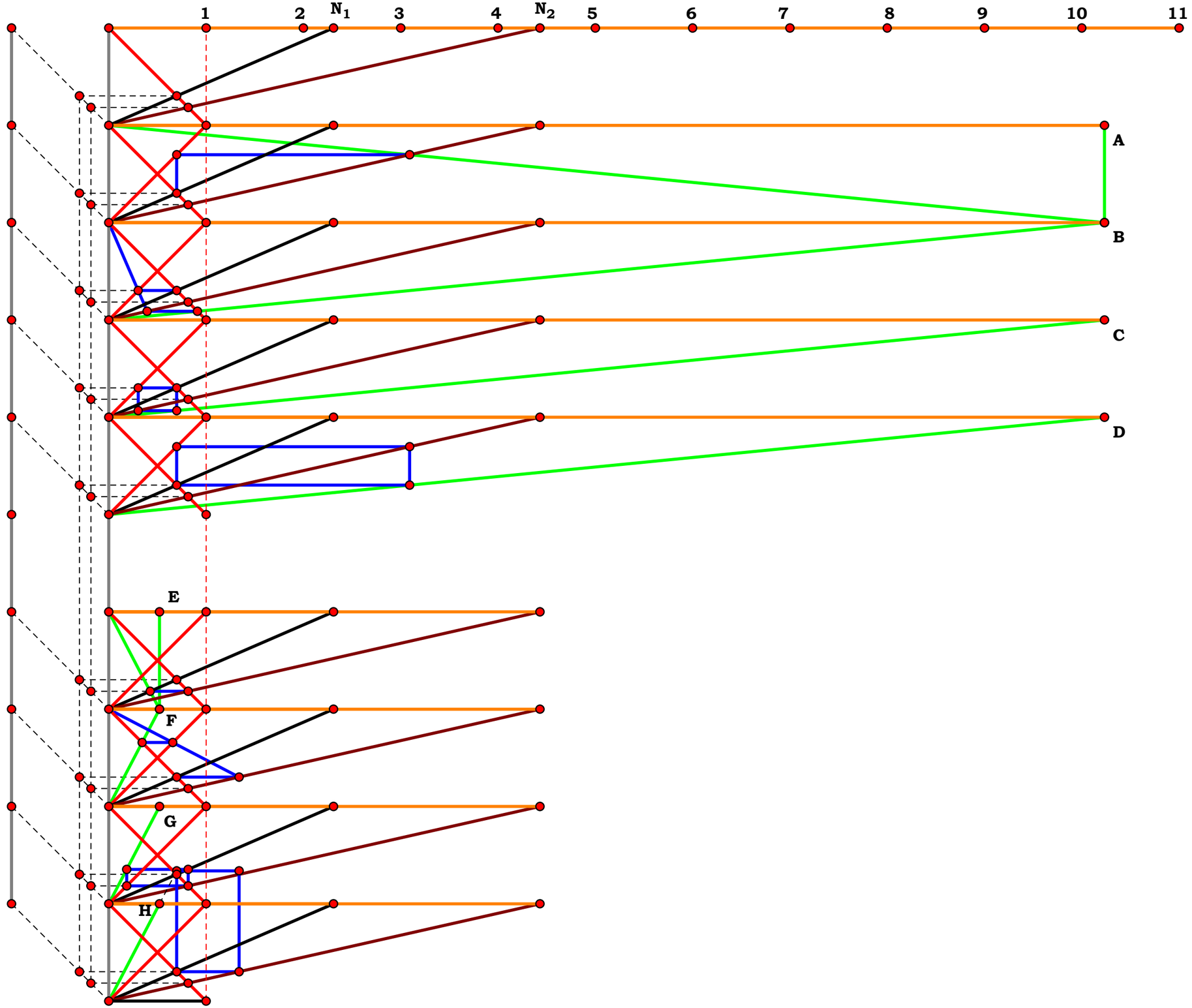
$H = 0.52125$

$\frac{N_1}{N_2} - E = 0.00000$

$\frac{N_1}{N_2} - F = 0.00000$

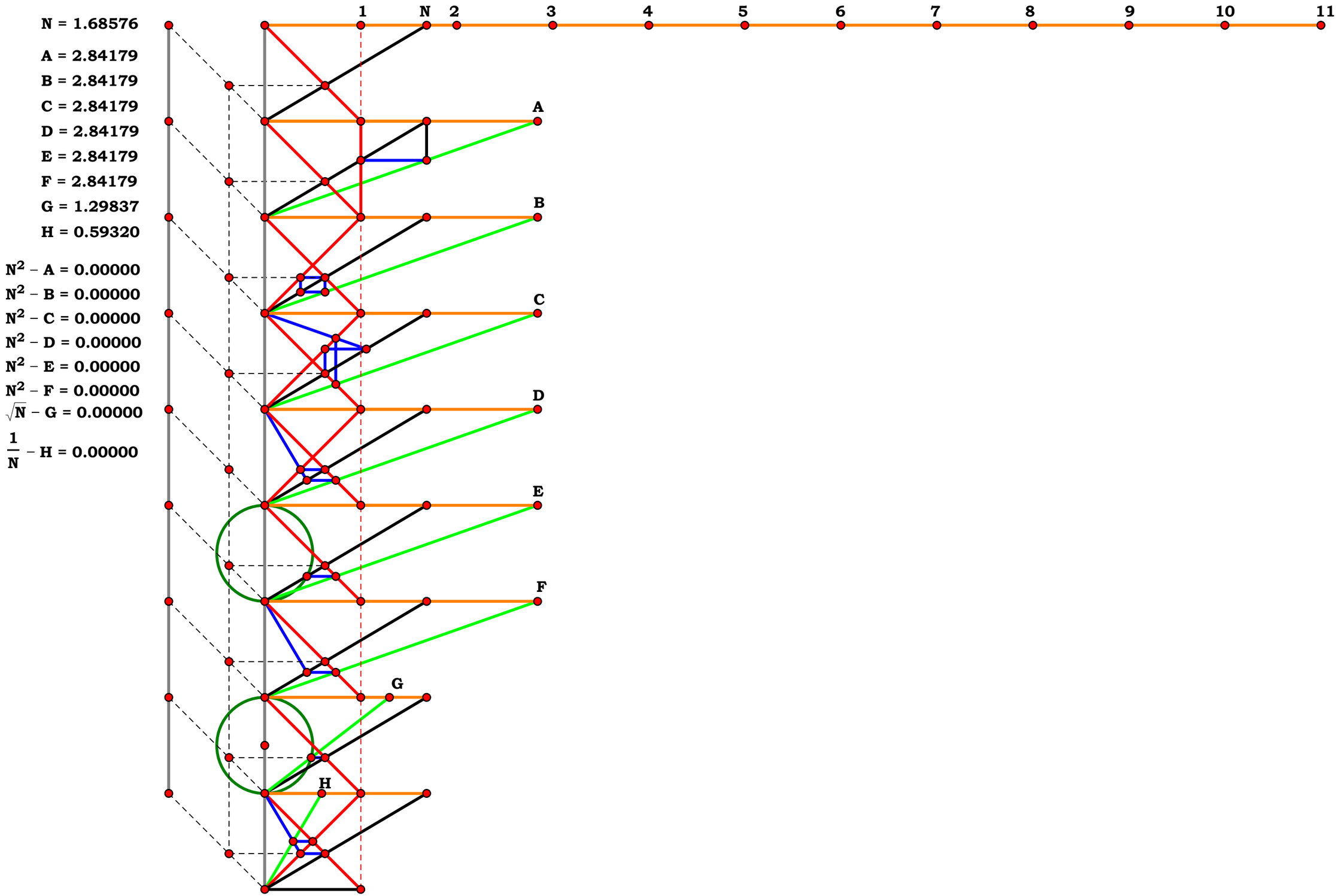
$\frac{N_1}{N_2} - G = 0.00000$

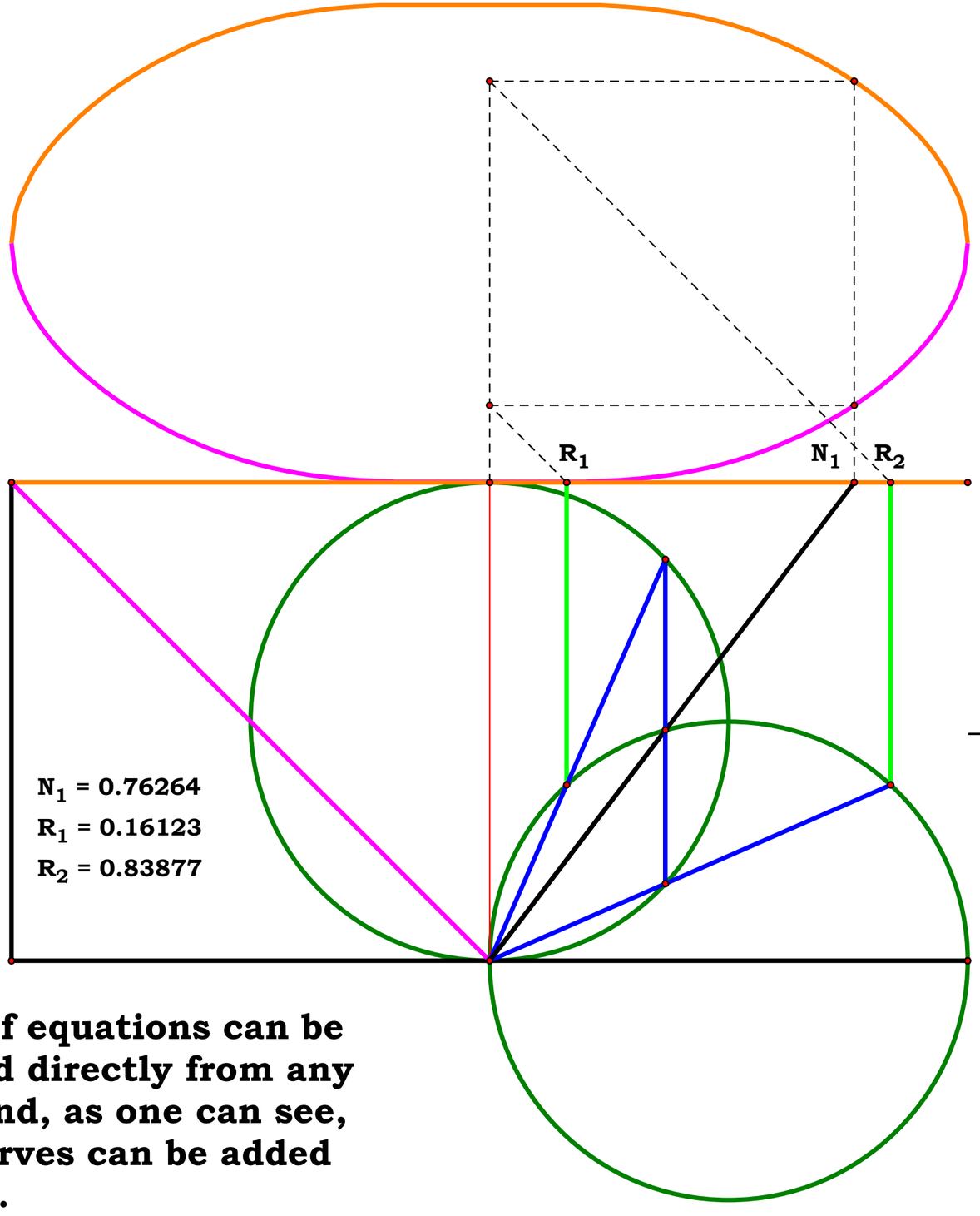
$\frac{N_1}{N_2} - H = 0.00000$




**One can pipe results from glyph to glyph as shown, and also, the glyph itself does not have to be the same size as the others, the pipe can proportion individual glyphs for the whole system. Piping allows for the construction of complex computational structures.**

# Square, Root and Reciprocal.






**3OBT10AR0**

**3OBT10AR3**

$$\frac{N_1^2 + 1 - \sqrt{2 \cdot N_1^2 + 1 - 3 \cdot N_1^4}}{2 \cdot N_1^2 + 2} = 0.16123$$

$$\frac{N_1^2 + 1 + \sqrt{2 \cdot N_1^2 - 3 \cdot N_1^4 + 1}}{2 \cdot N_1^2 + 2} = 0.83877$$

$$\frac{N_1^2 + 1 + \sqrt{2 \cdot N_1^2 - 3 \cdot N_1^4 + 1}}{2 \cdot N_1^2 + 2} - \frac{N_1^2 + 1 - \sqrt{2 \cdot N_1^2 + 1 - 3 \cdot N_1^4}}{2 \cdot N_1^2 + 2} = 0.67755$$

$$\frac{\sqrt{2 \cdot N_1^2 - 3 \cdot N_1^4 + 1}}{N_1^2 + 1} = 0.67755$$

**Curves of equations can be projected directly from any glyph. And, as one can see, these curves can be added together.**

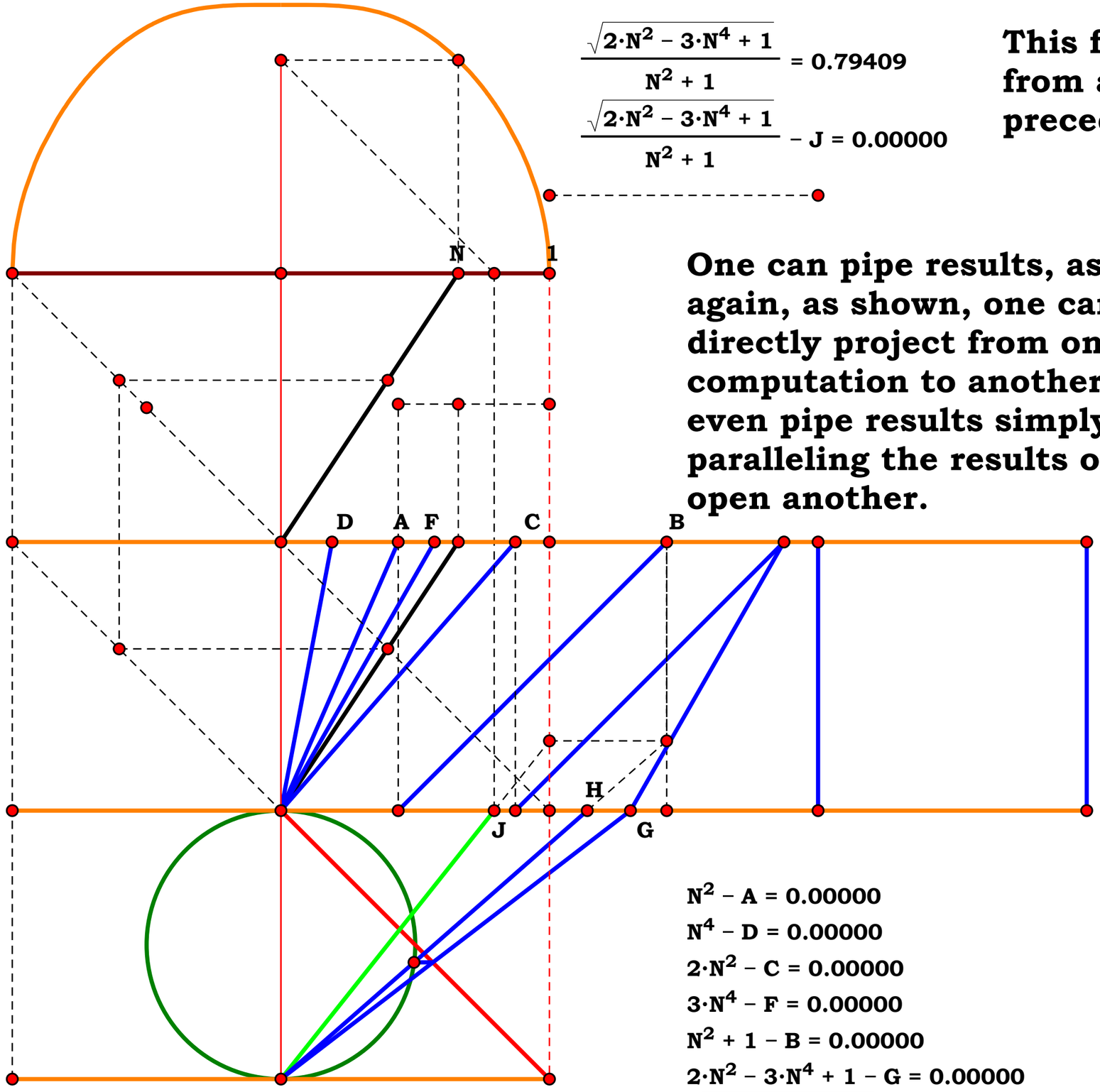
**This figure is one of the hundreds, or thousands of glyphs I demonstrate in BAM. Many glyphs even function by moving parts relative to other values. A single glyph can be a complex equation in of itself.**

$$\frac{\sqrt{2\cdot N^2 - 3\cdot N^4 + 1}}{N^2 + 1} = 0.79409$$

$$\frac{\sqrt{2\cdot N^2 - 3\cdot N^4 + 1}}{N^2 + 1} - J = 0.00000$$

**This figure is constructed from an equation on the preceding page.**

**One can pipe results, as shown, or again, as shown, one can sometimes directly project from one line of computation to another. One can even pipe results simply by paralleling the results on one line to open another.**




N = 0.66074
A = 0.43658
B = 1.43658
C = 0.87315
D = 0.19060
F = 0.57180
G = 1.30136
H = 1.14077
J = 0.79409

$N^2 - A = 0.00000$

$N^4 - D = 0.00000$

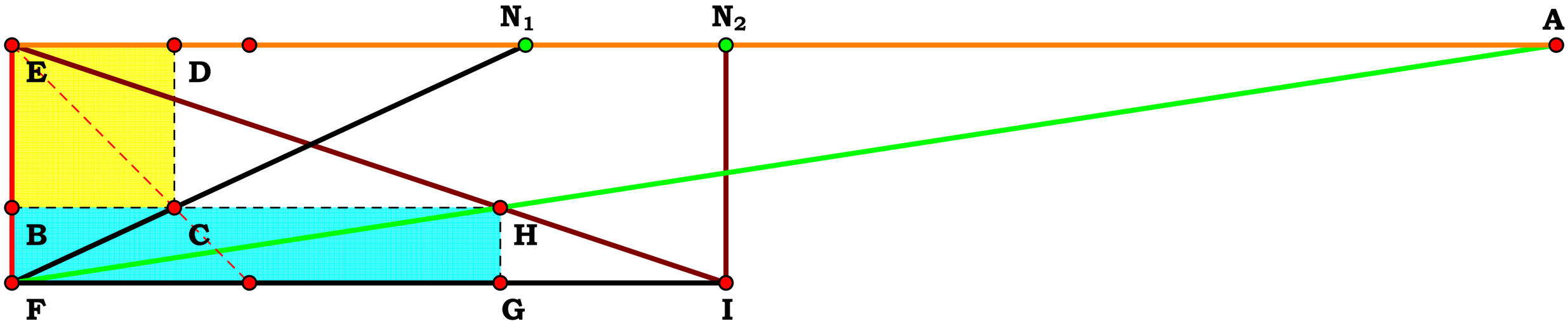
$2\cdot N^2 - C = 0.00000$

$3\cdot N^4 - F = 0.00000$

$N^2 + 1 - B = 0.00000$

$2\cdot N^2 - 3\cdot N^4 + 1 - G = 0.00000$

$\sqrt{2\cdot N^2 - 3\cdot N^4 + 1} - H = 0.00000$




$N_1 = 2.16287$

$N_2 = 3.00688$

$A = 6.50351$

$N_1 \cdot N_2 - A = 0.00000$

$$\frac{\text{Area } \triangle FN_1E}{\text{Area BCDE}} - \frac{\text{Area } \triangle EIF}{\text{Area BFGH}} = 0.00000$$

$$\frac{\text{Area } \triangle EIF}{\text{Area } \triangle FN_1E} - \frac{\text{Area BFGH}}{\text{Area BCDE}} = 0.00000$$

$$\frac{N_2}{N_1} - \frac{\text{Area BFGH}}{\text{Area BCDE}} = 0.00000$$

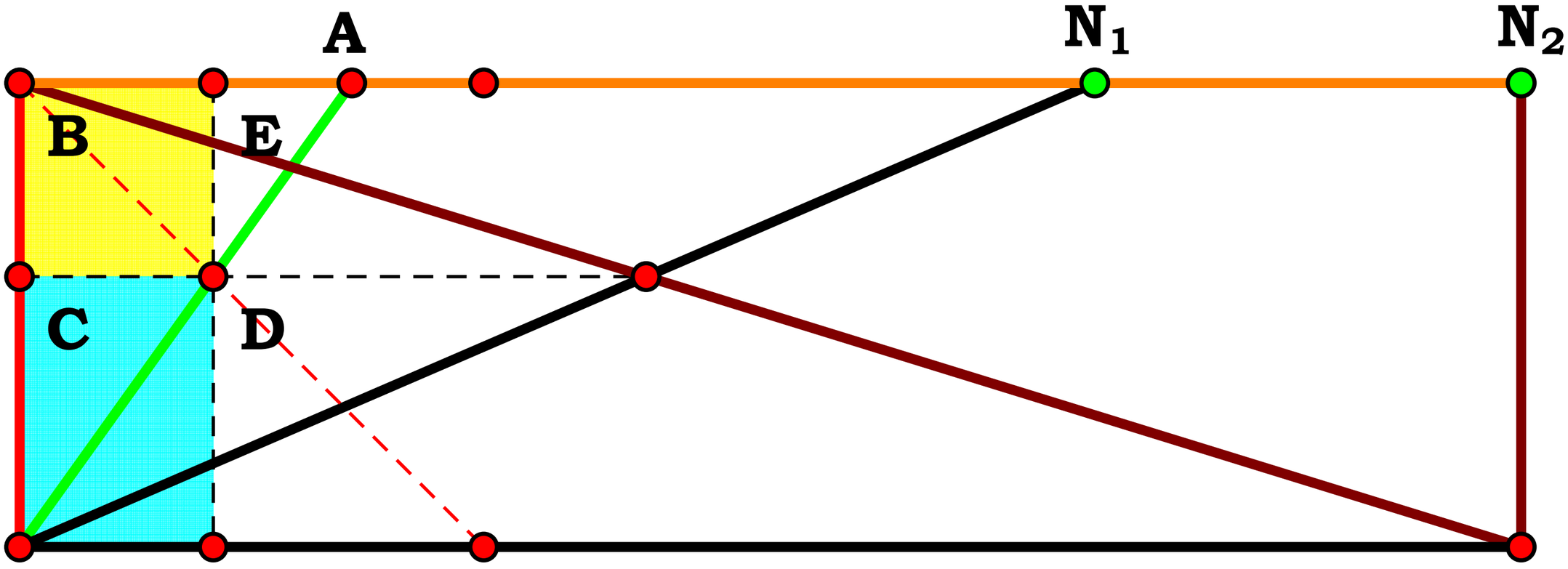
$$\frac{N_2}{N_1} - \frac{\text{Area } \triangle EIF}{\text{Area } \triangle FN_1E} = 0.00000$$

Area $\triangle FN_1E = 14.89003$ cm$^2$

Area BCDE = 6.43862 cm$^2$

Area BFGH = 8.95113 cm$^2$

Area $\triangle EIF = 20.70049$ cm$^2$




$N_1 = 2.31671$

$N_2 = 3.23544$

$A = 0.71604$

$\dfrac{N_1}{N_2} - A = 0.00000$

Area BCDE = 2.36319 $cm^2$

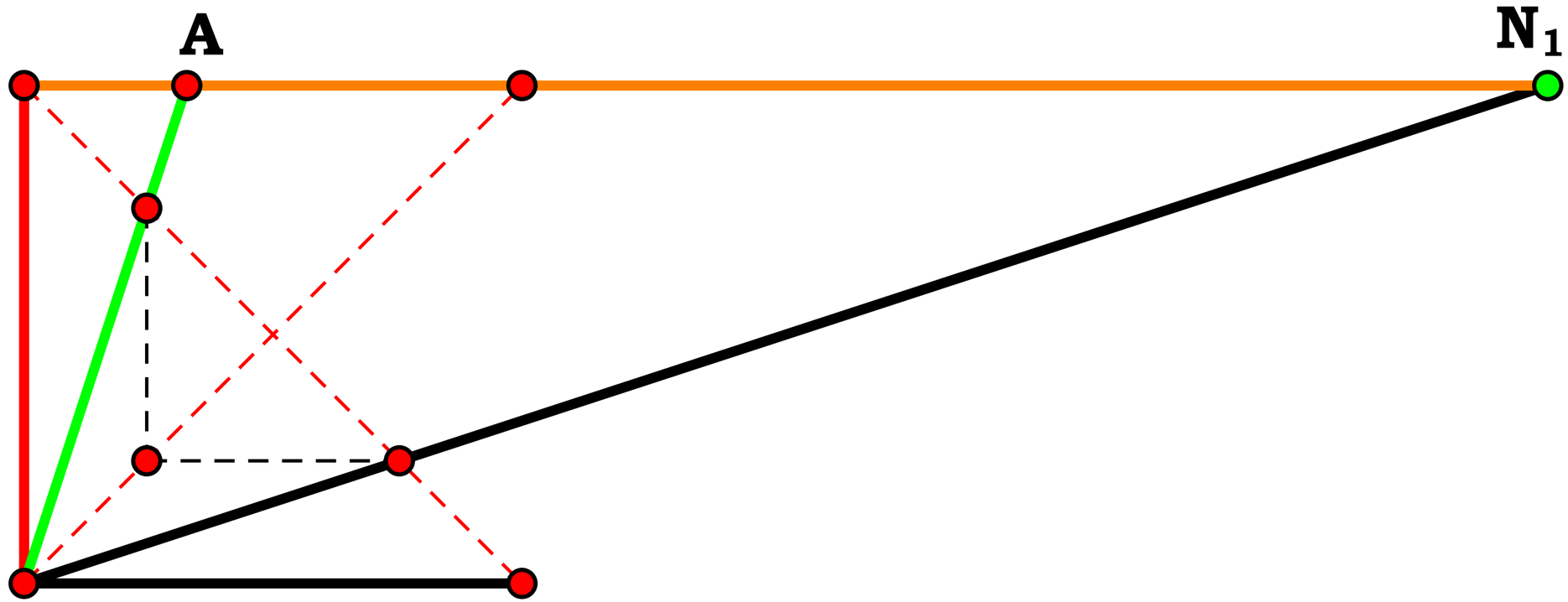
Area DCFG = 3.30036 $cm^2$

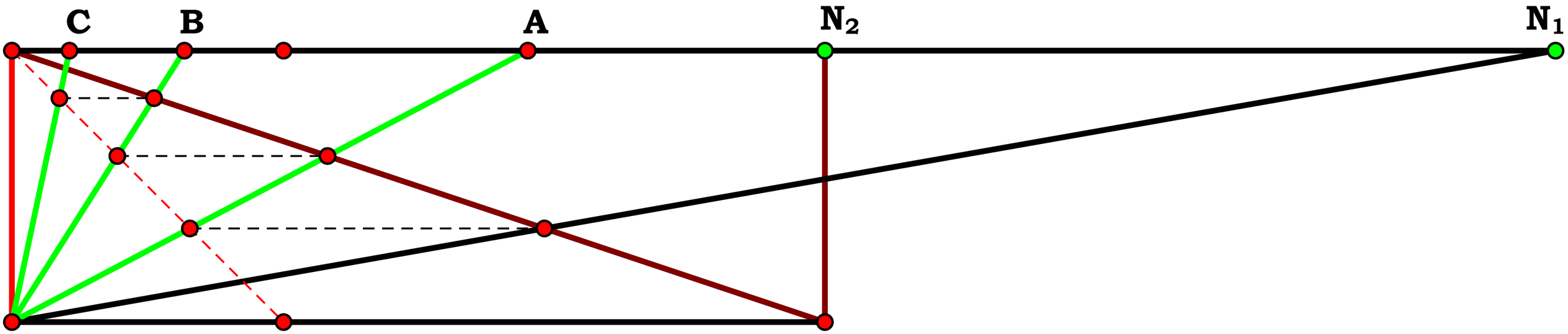
$\dfrac{\text{Area BCDE}}{\text{Area DCFG}} - \dfrac{N_1}{N_2} = 0.00000$

$N_1 = 3.06033$

$A = 0.32676$

$\frac{1}{N_1} - A = 0.00000$





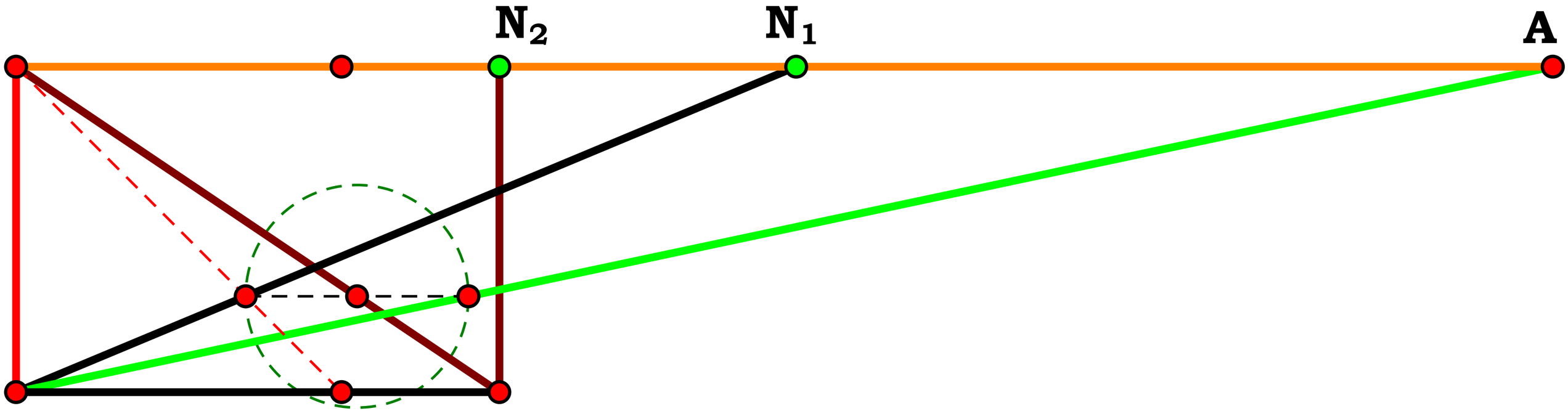

$N_1 = 5.68273$

$N_2 = 2.99267$

$$\frac{N_1}{N_2} - A = 0.00000 \qquad A = 1.89888$$

$$\frac{N_1}{N_2^2} - B = 0.00000 \qquad B = 0.63451$$

$$\frac{N_1}{N_2^3} - C = 0.00000 \qquad C = 0.21202$$

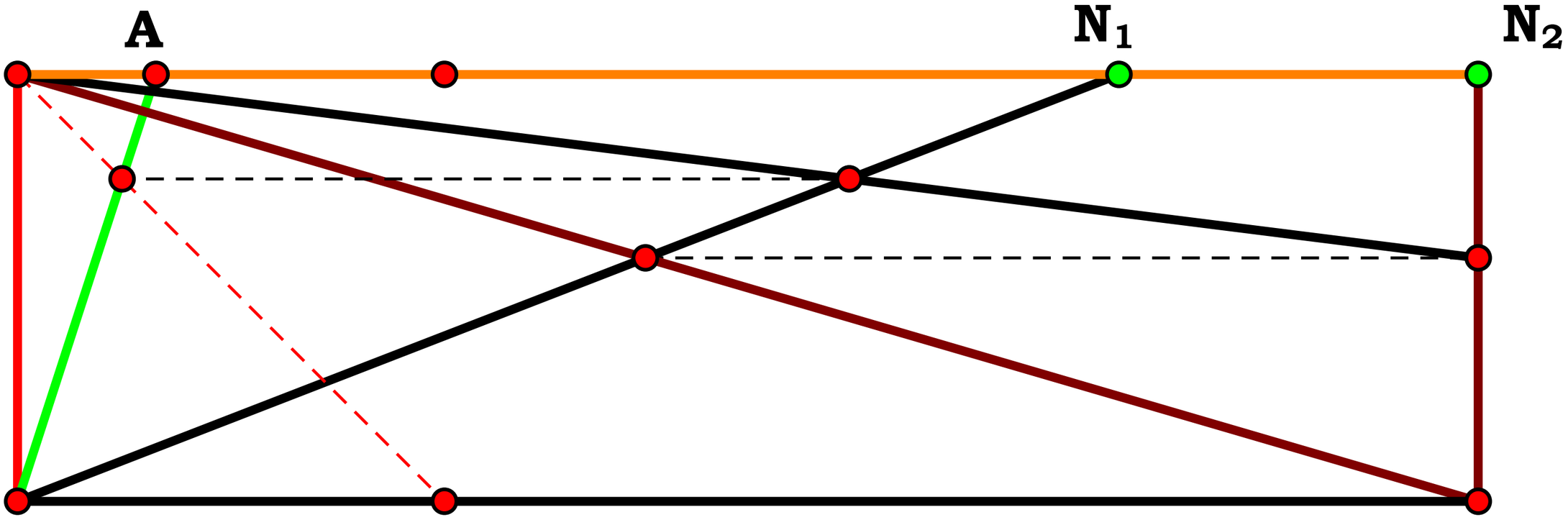



$N_1 = 2.39690$

$N_2 = 1.48490$

$A = 4.72142$

$2\cdot(N_1\cdot N_2) - N_1 - A = 0.00000$

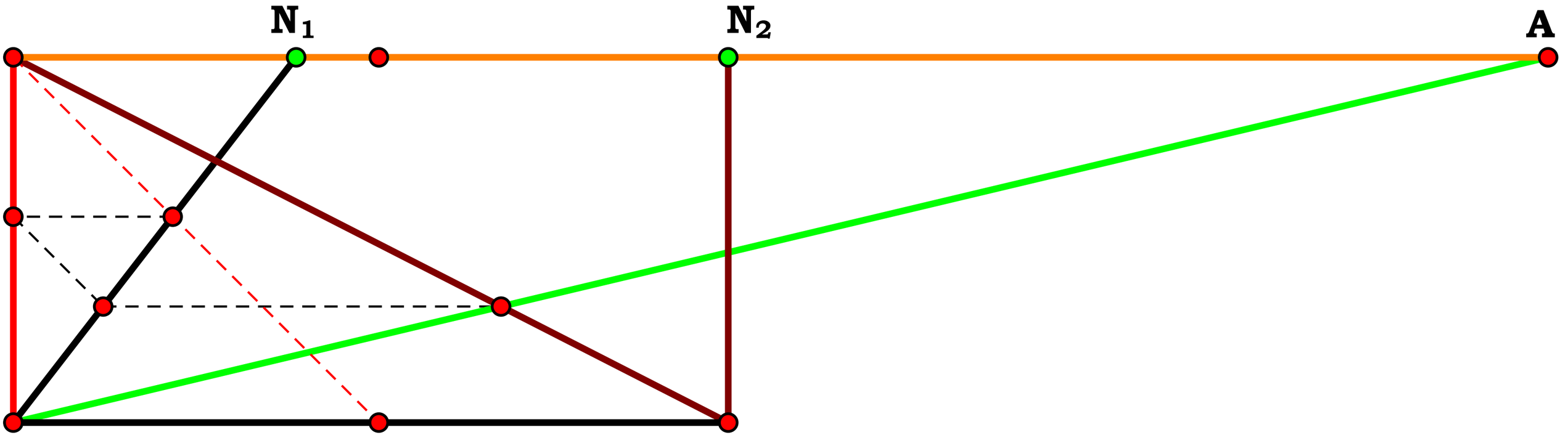



$$N_1 = 2.57987$$

$$N_2 = 3.42162$$

$$A = 0.32412$$

$$\frac{N_1^{\,2}}{(N_2 + N_1)\cdot N_2} - A = 0.00000$$

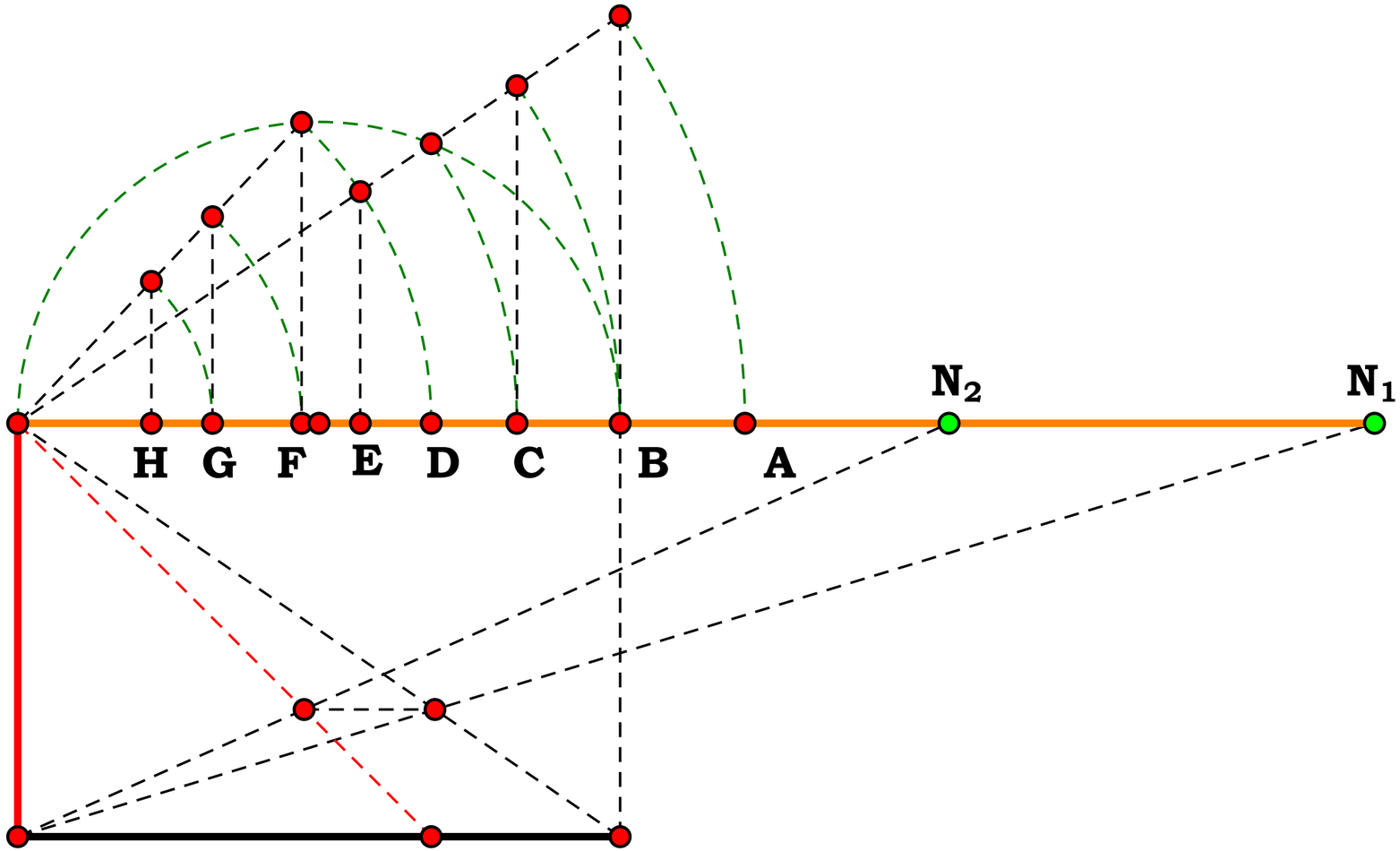



$N_1 = 0.77392$

$N_2 = 1.95611$

$A = 4.19932$

$2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2 - A = 0.00000$




$N_1 = 3.28137$

$N_2 = 2.25211$

$\frac{N_1}{N_2} = 1.45702$

$\left(\frac{N_1}{N_2}\right)^{1.5} - A = 0.00000$

$\frac{N_1}{N_2} - B = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{0.5} - C = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{0} - D = 0.00000$

$A = 1.75873$

$B = 1.45702$

$C = 1.20707$

$D = 1.00000$

$\left(\frac{N_1}{N_2}\right)^{-0.5} - E = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-1} - F = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-2} - G = 0.00000$

$\left(\frac{N_1}{N_2}\right)^{-3} - H = 0.00000$

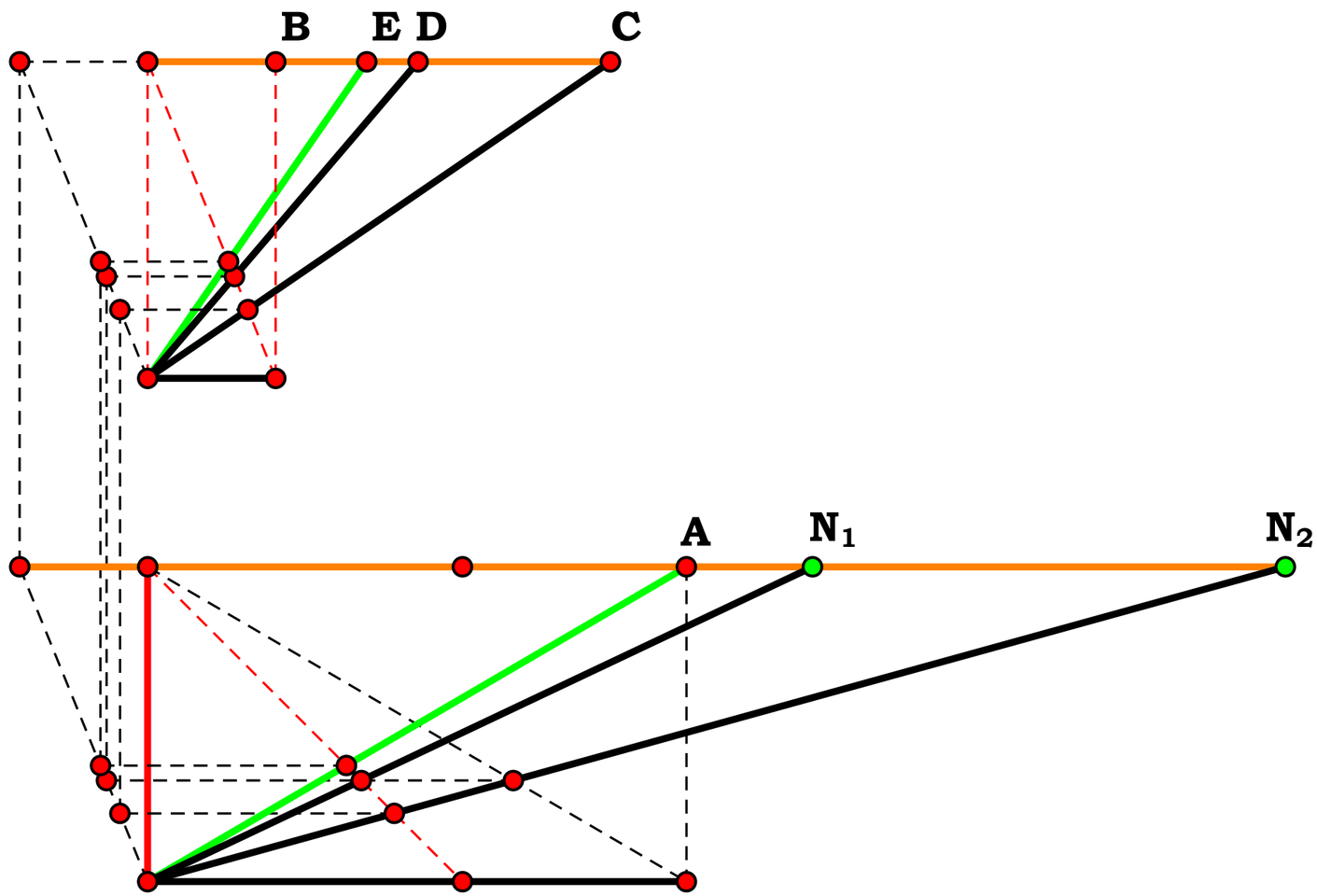
$E = 0.82845$

$F = 0.68633$

$G = 0.47105$

$H = 0.32330$

**Free Proportional Units through piping.**




$A = 1.71133$

$B = 0.40667$

$C = 1.46977$

$D = 0.85884$

$E = 0.69595$

$N_1 = 2.11188$

$N_2 = 3.61413$

$\frac{N_2}{N_1} = 1.71133$

$\frac{N_2}{N_1} - A = 0.00000$

$\frac{D}{B} = 2.11188$

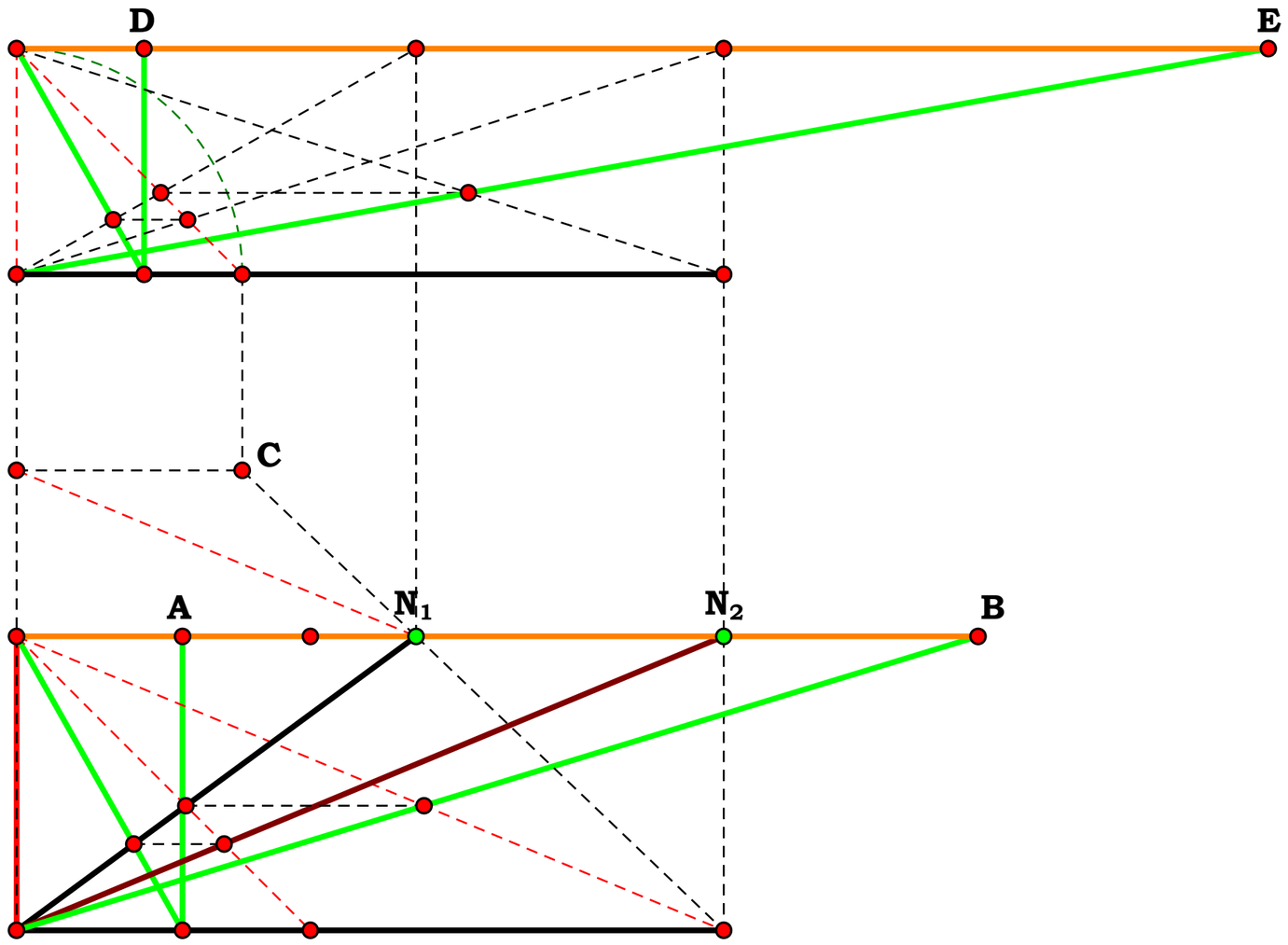
$\frac{C}{B} = 3.61413$

$\frac{E}{B} = 1.71133$

$\frac{D}{B} - N_1 = 0.00000$

$\frac{C}{B} - N_2 = 0.00000$

$\frac{E}{B} - \frac{N_2}{N_1} = 0.00000$




$N_1 = 1.35967$

$N_2 = 2.40682$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$\frac{N_1^2}{N_2} - C = 0.00000$

$\frac{N_1^3}{N_2^2} - D = 0.00000$

$\frac{N_2^2}{N_1} - E = 0.00000$

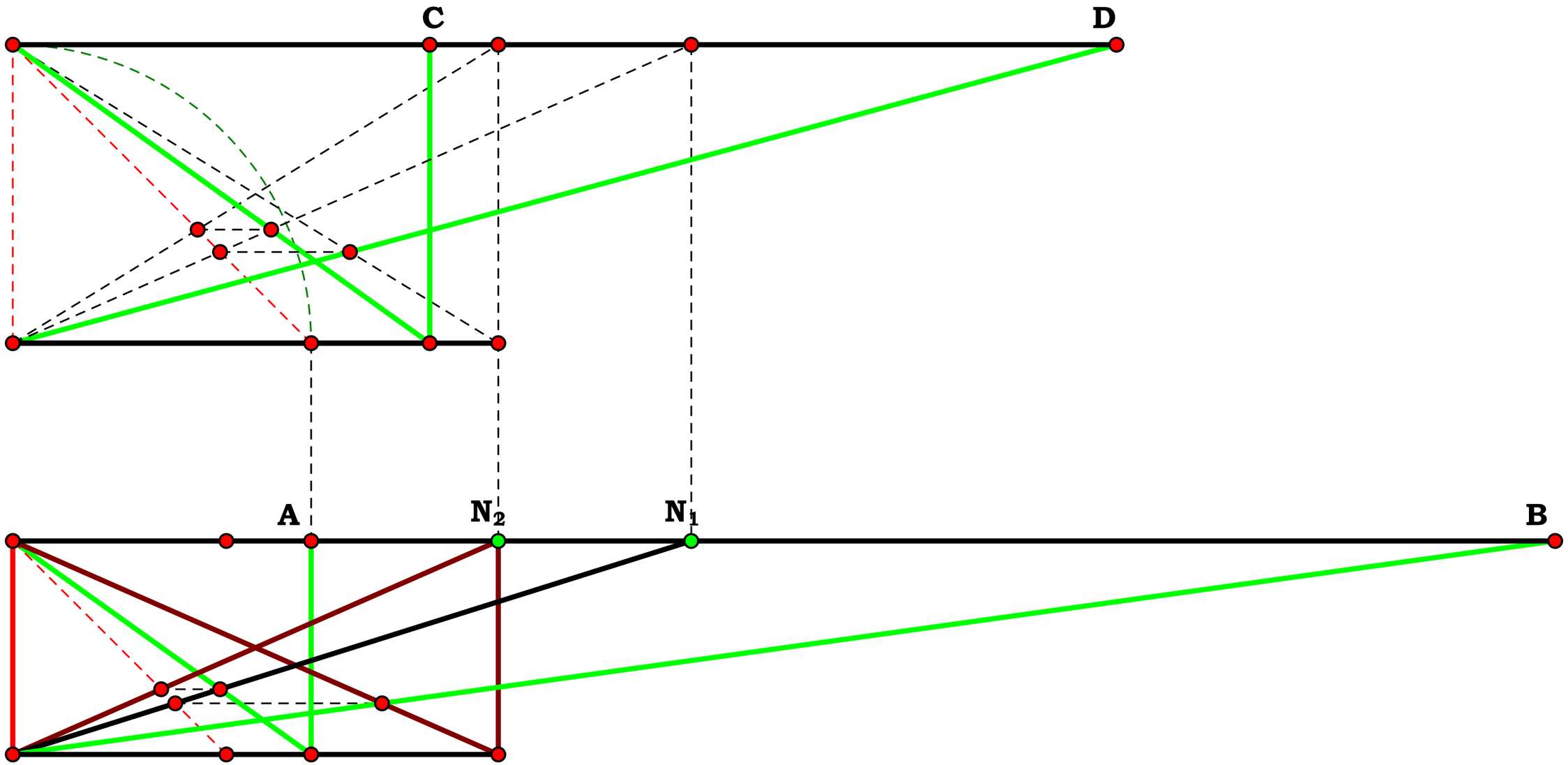
$A = 0.56492$

$B = 3.27248$

$C = 0.76811$

$D = 0.43392$

$E = 4.26045$




$N_1 = 3.17699$

$N_2 = 2.27348$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$\left(\frac{N_1}{N_2}\right)^2 - C = 0.00000$

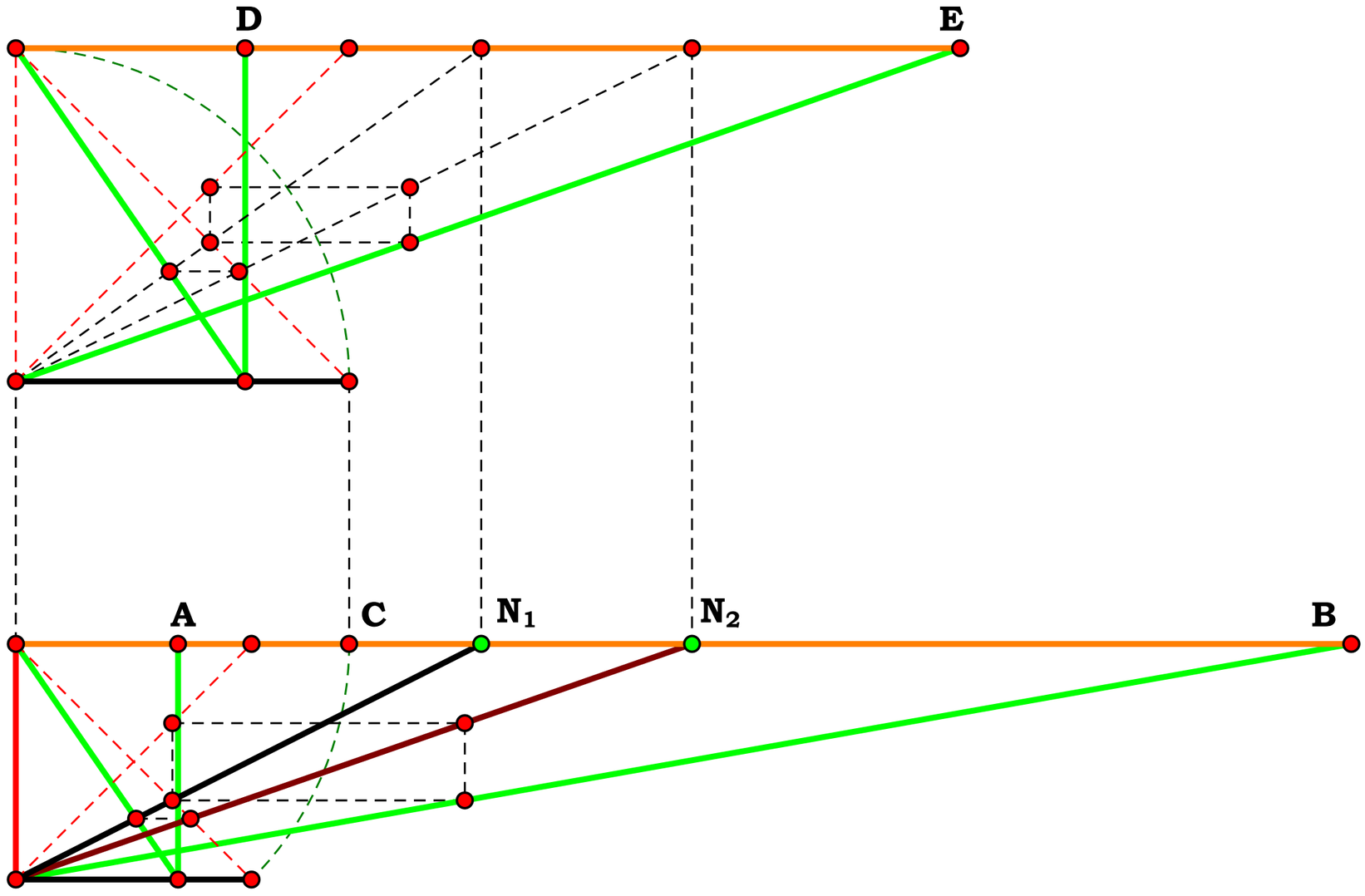
${N_2}^2 - D = 0.00000$

$A = 1.39741$

$B = 7.22280$

$C = 1.95276$

$D = 5.16869$




$N_1 = 1.97504$

$N_2 = 2.86973$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$\sqrt{2} - C = 0.00000$

$\frac{\sqrt{2} \cdot N_1}{N_2} - D = 0.00000$

$\frac{N_1 \cdot N_2}{\sqrt{2}} - E = 0.00000$

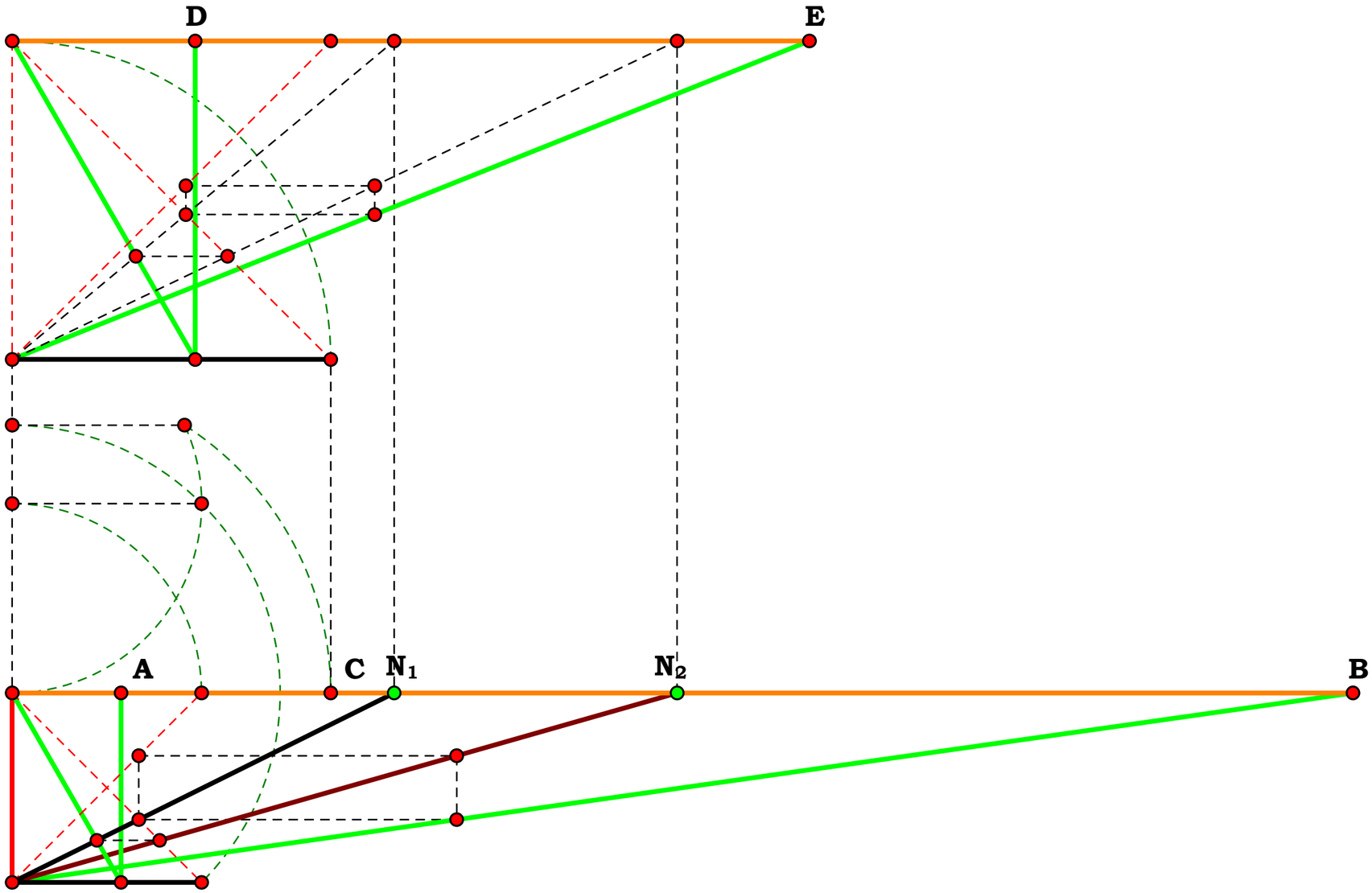
$A = 0.68823$

$B = 5.66782$

$C = 1.41421$

$D = 0.97331$

$E = 4.00776$




$N_1 = 2.01635$

$N_2 = 3.51044$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$2^{0.75} - C = 0.00000$

$\frac{N_1 \cdot 2^{0.75}}{N_2} - D = 0.00000$

$\frac{N_1 \cdot N_2}{2^{0.75}} - E = 0.00000$

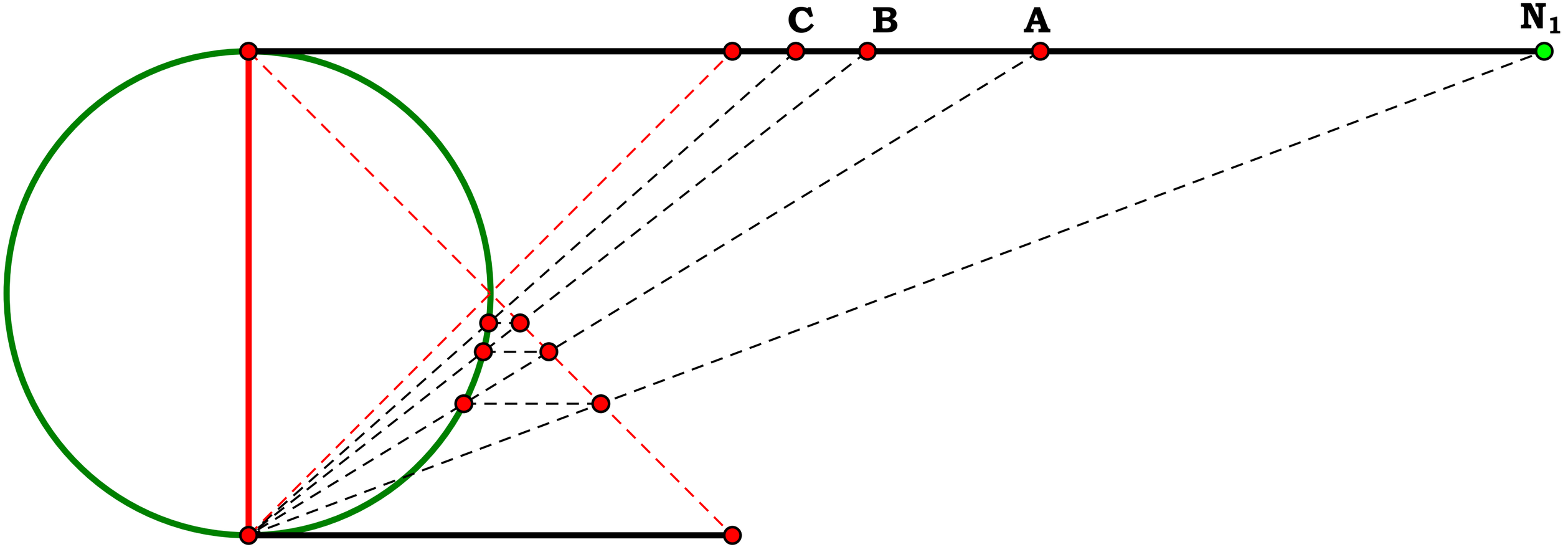
$A = 0.57439$

$B = 7.07830$

$C = 1.68179$

$D = 0.96600$

$E = 4.20878$




$N_1 = 2.67783$

$N_1 = 2.67783$

$N_1^{0.5} - A = 0.00000$

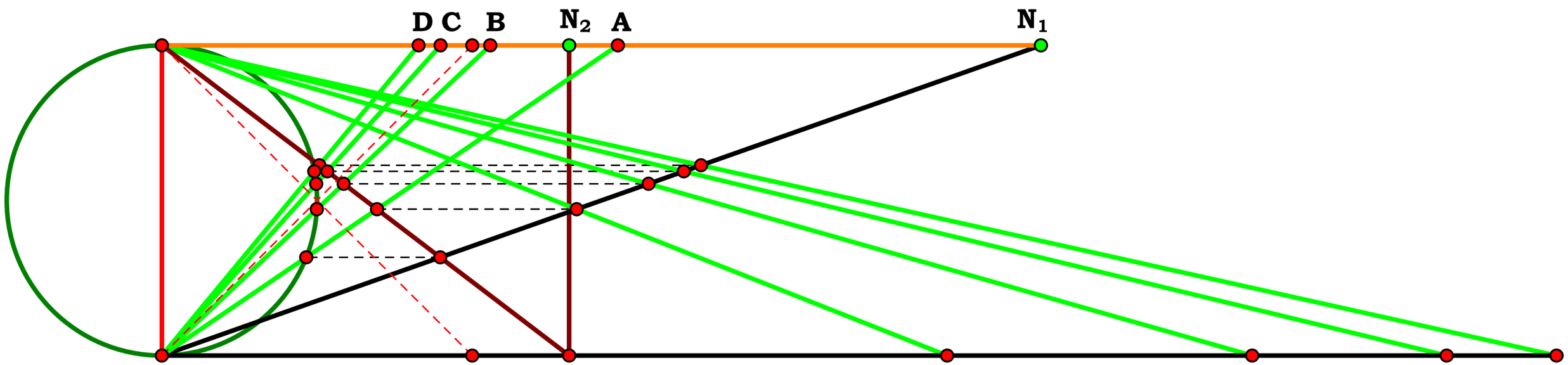
$N_1^{0.25} - B = 0.00000$

$N_1^{0.125} - C = 0.00000$

$A = 1.63641$

$B = 1.27922$

$C = 1.13103$




$N_1 = 2.83229$

$N_2 = 1.31233$

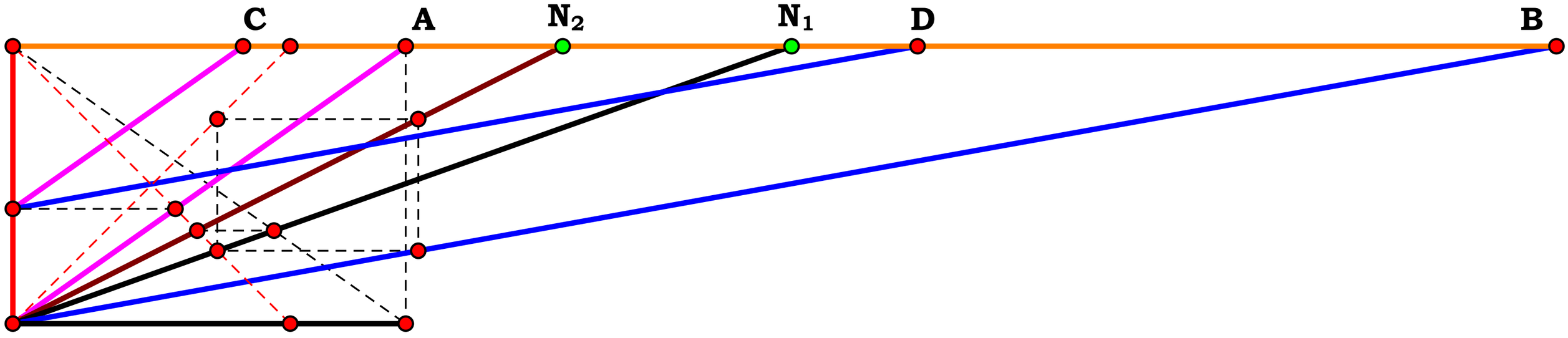
$$\left(\frac{N_1}{N_2}\right)^{0.5} - A = 0.00000 \qquad A = 1.46909$$

$$\left(\frac{N_1^{0.25}}{N_2^{0.75}}\right) - B = 0.00000 \qquad B = 1.05804$$

$$\frac{N_1^{0.125}}{N_2^{0.875}} - C = 0.00000 \qquad C = 0.89791$$

$$\frac{N_1^{0.0625}}{N_2^{0.9375}} - D = 0.00000 \qquad D = 0.82717$$

| E | F | G | H |
|---|---|---|---|
| $N_1^{0.5} \cdot N_2^{1.5} - E = 0.00000$ | $E = 2.53006$ | | |
| $N_1^{0.75} \cdot N_2^{1.75} - F = 0.00000$ | $F = 3.51299$ | | |
| $N_1^{0.875} \cdot N_2^{1.875} - G = 0.00000$ | $G = 4.13951$ | | |
| $N_1^{0.9375} \cdot N_2^{1.9375} - H = 0.00000$ | $H = 4.49351$ | | |




$N_1 = 2.80709$

$N_2 = 1.98213$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\frac{N_1^2}{(N_1 + N_2) \cdot N_2} - C = 0.00000$$

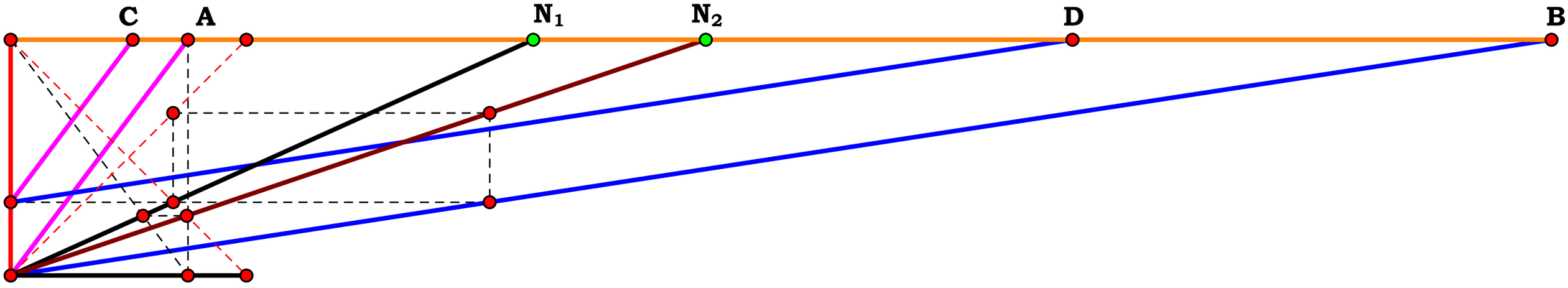
$$\frac{N_1^2 \cdot N_2}{N_1 + N_2} - D = 0.00000$$

$A = 1.41619$

$B = 5.56402$

$C = 0.83007$

$D = 3.26121$




$N_1 = 2.21664$

$N_2 = 2.94845$

$$\frac{N_1}{N_2} - A = 0.00000$$

$$N_1 \cdot N_2 - B = 0.00000$$

$$\frac{N_1^2}{N_2 \cdot (N_1 + 1)} - C = 0.00000$$

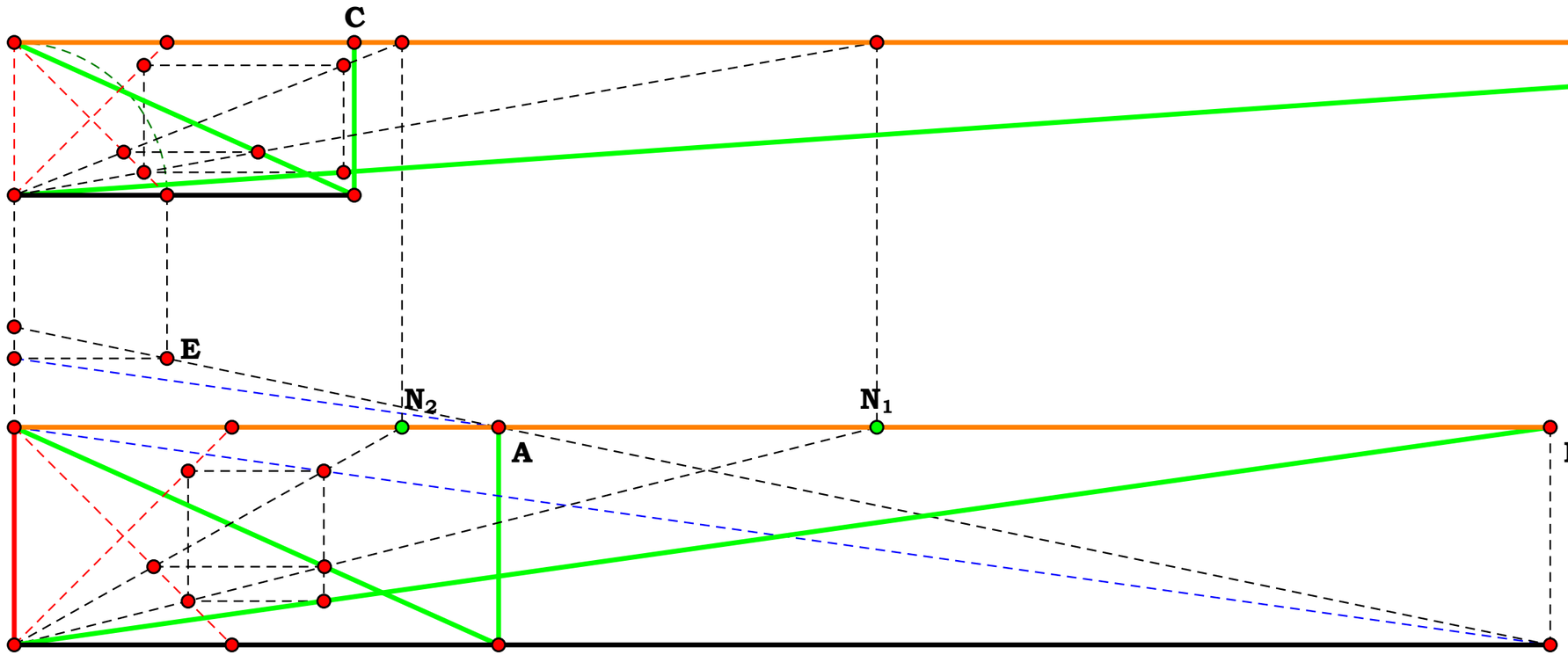
$$\frac{N_1^2 \cdot N_2}{N_1 + 1} - D = 0.00000$$

$A = 0.75180$

$B = 6.53565$

$C = 0.51808$

$D = 4.50383$




$N_1 = 3.96039$

$N_2 = 1.78041$

$\frac{N_1}{N_2} - A = 0.00000$

$N_1 \cdot N_2 - B = 0.00000$

$\frac{N_1^2}{N_2^4} - C = 0.00000$

$N_2^4 - D = 0.00000$

$\frac{N_1}{N_2^3} - E = 0.00000$

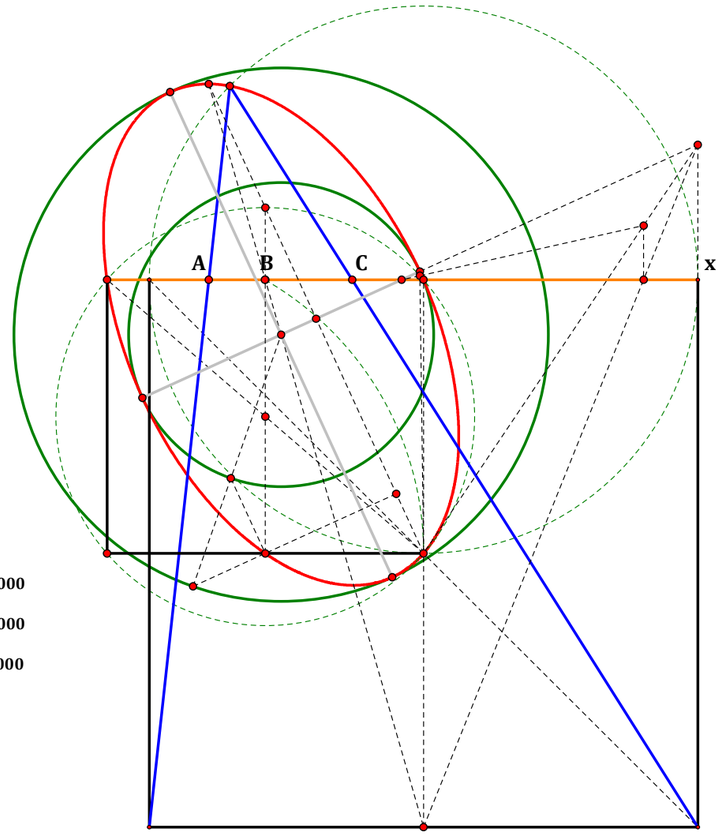
$A = 2.22443$

$B = 7.05112$

$C = 1.56097$

$D = 10.04803$

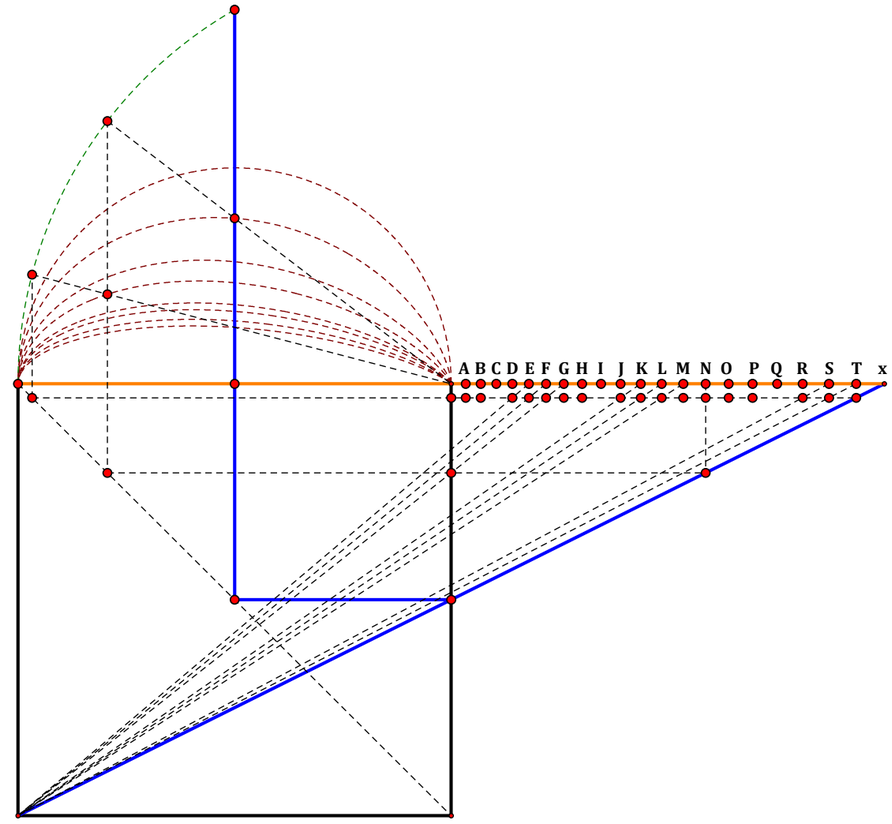
$E = 0.70174$




x = 14.00000
A = 2.41014
B = 3.74166
C = 5.80879
$x^{\frac{1}{3}} - A = 0.00000$
$x^{\frac{1}{2}} - B = 0.00000$
$x^{\frac{2}{3}} - C = 0.00000$

x = 2.00000

A = 1.03356
B = 1.06824
C = 1.10409
D = 1.14114
E = 1.17943
F = 1.21901
G = 1.25992
H = 1.30220
I = 1.34590
J = 1.39107
K = 1.43775
L = 1.48599
M = 1.53586
N = 1.58740
O = 1.64067
P = 1.69573
Q = 1.75263
R = 1.81145
S = 1.87224
T = 1.93506

3·7 = 21.00000
y = 21.00000
$x^{\frac{1}{y}} - A = 0.00000$
$x^{\frac{2}{y}} - B = 0.00000$
$x^{\frac{3}{y}} - C = 0.00000$
$x^{\frac{4}{y}} - D = 0.00000$
$x^{\frac{5}{y}} - E = 0.00000$
$x^{\frac{6}{y}} - F = 0.00000$
$x^{\frac{7}{y}} - G = 0.00000$
$x^{\frac{8}{y}} - H = 0.00000$
$x^{\frac{9}{y}} - I = 0.00000$

$x^{\frac{10}{y}} - J = 0.00000$
$x^{\frac{11}{y}} - K = 0.00000$
$x^{\frac{12}{y}} - L = 0.00000$
$x^{\frac{13}{y}} - M = 0.00000$
$x^{\frac{14}{y}} - N = 0.00000$
$x^{\frac{15}{y}} - O = 0.00000$
$x^{\frac{16}{y}} - P = 0.00000$
$x^{\frac{17}{y}} - Q = 0.00000$
$x^{\frac{18}{y}} - R = 0.00000$
$x^{\frac{19}{y}} - S = 0.00000$
$x^{\frac{20}{y}} - T = 0.00000$